Commentary on Building Code Requirements for Reinforced Concrete (ACI 318-71)

Reported by ACI Committee 318

EDWARD COHEN
Chairman

GEORGE F. LEYH
Secretary

W. C. E. BECKER
W. BURR BENNETT, JR.
DELMAR L. BLOEM*
FRANK B. BROWN
T. Z. CHASTAIN
WILLIAM D. CROMARTIE
OWEN L. DELEVANTE
JAMES N. DE SERIO
FRANK G. ERSKINE
NOEL J. EVERARD
PHIL M. FERGUSON
ASHBY T. GIBBONS, JR.
WILLIAM A. HEITMANN
EIVIND HOGNESTAD
EUGENE P. HOLLAND
FRITZ KRAMRISCH
T. Y. LIN
MICHAEL A. LOMBARD
ROBERT F. MAST
WILLIAM V. MERKEL
ROBERT B. B. MOORMAN
KEITH O. O'DONNELL
DOUGLAS E. PARSONS
EDWARD O. PFRANG
W. G. PLEWES
RAYMOND C. REESE
THEODORE O. REYHNER
PAUL F. RICE
FRANCISCO ROBLES
PAUL ROGERS
JOHN A. SBAROUNIS
MORRIS SCHUPACK
CHESTER P. SIESS
I. J. SPEYER
JOHN P. THOMPSON
M. P. VAN BUREN
A. CARL WEBER
GEORGE WINTER
ALFRED ZWEIG

Because the 1971 ACI Building Code is written as a legal document so that it may be incorporated verbatim or adopted by reference in a general building code, it cannot present background details or suggestions for carrying out its requirements or intent. It is the function of this Commentary to fill this need.

The Commentary discusses some of the considerations of the committee in developing the Code with emphasis given to the explanation of new or revised provisions that may be unfamiliar to Code users.

References to much of the research data referred to in preparing the Code are cited for the user desiring to study individual questions in greater detail. Other documents that provide suggestions for carrying out the requirements of the Code are also cited.

The chapter and section numbering of the Code are followed throughout.

Keywords: admixtures; aggregates; anchorage (structural); beam-column frame; beams (supports); building codes; cements; cold weather construction; columns (supports); combined stress; composite construction (concrete to concrete); composite construction (concrete and steel); compressive strength; concrete construction; concretes; concrete slabs; construction joints; continuity (structural); cover; curing; deep beams; deflections; drawings; earthquake resistant structures; embedded service ducts; flexural strength; floors; folded plates; footings; formwork (construction); frames; hot weather construction; inspection; joists; lightweight concretes; loads (forces); load tests (structural); materials; mixing; mix proportioning; modulus of elasticity; moments; pipe columns; pipes (tubes); placing; precast concrete; prestressed concrete; prestressing steels; quality control; reinforced concrete; reinforcing steels; roofs; serviceability; shear strength; shear walls; shells (structural forms); spans; specifications; splicing; strength; strength analysis; structural analysis; structural design; T-beams; torsion; walls; water; welded wire fabric.

*Deceased

The content of this Commentary is the responsibility of the committee which prepared it. Institute authority attaches only to standards adopted as provided in the Bylaws.

PREFACE

Committee 318 has felt that to be of greatest benefit a Commentary on the 1971 Code should be available at the same time as final copies of the Code become available. To achieve this, it was realized that the work had to be done concurrently with that on the Code itself and had to be kept up to date as the Code was amended. This could not be achieved entirely with volunteer effort. Noel J. Everard, a member of Committee 318, was commissioned on a consulting basis to prepare a first draft of the complete commentary.

After study and comment by committee members, the subcommittee chairmen were each asked to prepare a second draft for their particular chapters taking into account the comments received.

An editorial task group of George F. Leyh, Ashby T. Gibbons, and Samuel J. Henry prepared three subsequent drafts, each time taking into account the comments received on previous drafts and the amendments to the Code which were made as a result of the formal discussion period, further study by committee members, and the discussion at the 1970 ACI Fall Convention where the Code was approved for submission to letter ballot of the ACI membership. The editorial task group was assisted at some of its meetings by Gordon Plewes and Noel Everard. Richard D. Gaynor acted as Chairman of Subcommittee 4 through the last two drafts after the death of Delmar L. Bloem.

This task could not have been completed on time without the dedicated efforts of the members of Committee 318, particularly the subcommittee chairmen as well as the individuals named above.

CONTENTS

INTRODUCTION

This Commentary discusses some of the considerations of Committee 318 in developing the provisions contained in "Building Code Requirements for Reinforced Concrete (ACI 318-71)," hereinafter called the Code or the 1971 Code. Emphasis is given to the explanation of new or revised provisions that may be unfamiliar to Code users. In addition, comments are included for some items contained in previous editions of the Code to make the present Commentary independent of the Commentary for ACI 318-63. Comments on specific provisions are made under the corresponding chapter and section numbers of the Code.

The Commentary is not intended to provide a complete historical background concerning the development of the ACI Code,* nor is it intended to provide a detailed résumé of the studies and research data reviewed by the committee in formulating the provisions of the Code. However, references to some of the research data are provided for those who wish to study the background material in depth.

As the name implies, "Building Code Requirements for Reinforced Concrete (ACI 318-71)" is meant to be used as part of a legally adopted building code and as such must differ in form and substance from documents that provide detailed specifications, recommended practice, complete design procedures, or design aids.

The Code is intended to cover all buildings of the usual types, both large and small. Requirements more stringent than the Code provisions may be desirable for unusual construction. The Code and this Commentary cannot replace sound engineering knowledge, experience and judgment.

A building code states only the minimum requirements necessary to provide for public health and safety. The ACI Code is based on this principle. For any structure, the owner or the structural designer may require the quality of materials and construction to be higher than the minimum requirements necessary to protect the public and stated in the Code. However, lower standards are not permitted.

This Commentary directs attention to other documents that provide suggestions for carrying out the requirements and intent of the Code. However, neither those documents nor this Commentary are intended as a part of the Code.

The Code has no legal status unless it is adopted by government bodies having the police power to regulate building design and construction. Where the Code has not been adopted, it may serve as a reference to good practice even though it has no legal status.

The Code provides a means of establishing minimum standards for acceptance of designs and construction by a legally appointed Building Official or his designated representatives. The Code and Commentary are not intended for use in settling disputes between the Owner, Engineer, Architect, Contractor, or their agents, Subcontractors, Material Suppliers, or Testing Agencies. Other ACI publications, such as "Specifications for Structural Concrete for Buildings" (ACI 301) are written specifically for use as contract documents for construction.

Committee 318 recognizes the desirability of standards of performance for individual parties involved in the contract documents. Available for this purpose are the plant certification programs of the Prestressed Concrete Institute and the National Ready Mixed Concrete Association, and the qualification standards of the American Society of Concrete Constructors. In addition, "Recommended Practice for Inspection and Testing Agencies for Concrete and Steel As Used in Construction (ASTM E 329-70)" recommends performance requirements for inspection and testing agencies.

The National Board of Accreditation in Concrete Construction has been formed to initiate a program of accreditation for testing laboratories, contractors, and concrete suppliers. The accreditation plans have not been formalized as of June 1971 but it appears that, for testing laboratories, the accreditation will be based on ASTM E 329. For contractors or material suppliers, it likely will be based on a record of satisfactory experience or on the existing qualification standards and plant certification programs.

Illustrations of the application of the Code requirements in structural design may be found in the documents listed in the Bibliography that follows.

References

1. ACI Committee 340, *Ultimate Strength Design Handbook*, SP-17, American Concrete Institute, Detroit, 1967, V. 1, 176 pp.

2. ACI Committee 340, *Ultimate Strength Design Handbook*, V. 2, *Columns*, SP-17A, American Concrete Institute, Detroit, 1970, 226 pp.

*For a history of the ACI Building Code see Kerekes, Frank, and Reid, Harold B., Jr., "Fifty Years of Development in Building Code Requirements for Reinforced Concrete," ACI JOURNAL, *Proceedings* V. 50, No. 6, Feb. 1954, p. 441. For a discussion of code philosophy see Siess, Chester P., "Research, Building Codes, and Engineering Practice," ACI JOURNAL, *Proceedings* V. 56, No. 5, May 1960, p. 1105.

3. ACI Committee 317, *Reinforced Concrete Design Handbook—Working Stress Method*, SP-3, American Concrete Institute, Detroit, 3rd Edition, 1965, 271 pp. (Note: Only those procedures related to the design of beams for flexure without axial load apply to the 1971 Code. Specifically, the column design tables and charts *do not* apply.)

4. Reese, R. C., *Columns by Ultimate Strength Design*, Concrete Reinforcing Steel Institute, Chicago, 1967, 213 pp. [Designs are based on ACI 318-63 and may require some modification to meet the 1971 ACI Code. For instance, the Code changes designs for values of P_u equal to or less than $0.10f_c'A_g$ (small axial load with flexure).]

5. Reese, R. C., *Floor Systems by Ultimate Strength Design*, Concrete Reinforcing Steel Institute, Chicago, 289 pp. (Designs are based on ACI 318-63 and may require some modification to meet the 1971 ACI Code. Generally, values included will be found to be conservative with respect to the 1971 Code.)

6. Reese, R. C., *CRSI Design Handbook* (*Working Stress Design*), Concrete Reinforcing Steel Institute, Chicago, Ill., 1965, 389 pp. (Designs are based on ACI 318-63 and may not conform to the 1971 ACI Code. In particular, procedures for column design provided in this manual do not conform to the 1971 Code.)

7. "Ultimate Strength Design of Reinforced Concrete Columns," *Engineering Bulletin* EB0009.01, Portland Cement Association, Skokie, 1969, 40 pp. (Note that the PCA tables do not contain an understrength factor ϕ, hence M_u/ϕ and P_u/ϕ must be used when designing with these aids.)

CHAPTER 1—GENERAL REQUIREMENTS

1.1—Scope

The American Concrete Institute "Building Code Requirements for Reinforced Concrete (ACI 318-71)," hereinafter referred to as the Code, provides minimum requirements for any reinforced concrete design or construction that is regulated by a general code of which it forms a part. The Code should supersede conflicting requirements on concrete design and construction in the general code.

Prestressed concrete is included under the definition of reinforced concrete. Provisions of the Code apply to prestressed concrete except for those which are stated to apply specifically to nonprestressed concrete.

Appendix A of the Code contains provisions for design and detailing of special earthquake resistant structures.

Some special structures involve unique problems which are not covered by the Code. However, many Code provisions, such as the concrete quality and design principles, are applicable for these structures.

1.2—Permits and drawings

The provisions regarding preparation of plans, specifications, and issuance of permits are, in general, consistent with those of most general codes and are intended as supplements thereto.

The Code lists some of the most important items of information that should be included on the plans. The Code does not imply an all inclusive list, and additional items may be required by the Building Official.

"Building Official" is the term used by many general codes to identify the person charged with administration and enforcement of the provisions of the building code. However, such terms as "Building Commissioner" or "Building Inspector" are variations of the title, and the term "Building Official" as used in the ACI Code is intended to include those variations as well as others which are used in the same sense.

The ACI Code accepts well documented computer programs as means of obtaining a structural analysis or design, in lieu of detailed calculations. The extent of input and output information required will vary, according to the specific requirements of individual Building Officials. However, when a well documented computer program has been used by the designer, only skeleton data should normally be required. This should consist of sufficient input and output data and other information to allow the Building Official to perform a detailed review and make comparisons using another program or longhand calculations. Input data should be identified as to member designation, applied loads, and span lengths. The related output data should include member designation and the shears, moments, and reactions at key points in the span. For column design, it is desirable to include moment magnification factors in the output where applicable.

The Code permits model analysis to be used to supplement structural analysis and design calculations. Documentation of the model analysis should be provided with the related calculations. Model analysis is most effective as a tool for predicting the behavior of actual structures when performed by an engineer or architect having experience in this technique.

1.3—Inspection

1.3.1 — Inspection is important since the proper performance of the structure depends on construction which accurately represents the design and meets Code requirements, within the tolerances allowed. In the public interest, local building ordinances should require the owner to provide adequate inspection for all types of construction.

While the Code requires inspection to be done by a competent engineer or architect, or their representatives, it does not intend to set detailed responsibility in this respect. The clause in Section 103 of the 1963 Code "preferably the one responsible for its design" has been omitted from the 1971 Code because of undesirable legal implications. It obviously would be desirable if inspection of construction were done by or under the supervision of the engineer or architect who participated in the design.

When conditions will not permit such an arrangement, the owner may provide proper inspection of construction through his architects or engineers or through separate inspection organizations with demonstrated capability for performing the inspection operation. The degree of inspection required should be set forth in the contracts between the owner, architect, engineer, and contractor. Adequate fees should be provided consistent with the work and equipment necessary to properly perform the inspection.

While it is recognized that sometimes the inspection is done independently of the designer, it is recommended that the designer be employed to at least oversee inspection and observe the work to see that his design requirements are properly executed.

By "inspection," the Code does not mean that the inspector should supervise the construction. Rather it means that the one employed for inspection should visit the project with the frequency necessary to observe the various stages of work and ascertain that it is being done in compliance with contract documents and Code requirements. The frequency should be at least enough to provide general knowledge of each operation, whether this be several times a day or once in several days.

Inspection in no way relieves the contractor from his obligation to follow the plans and specifications implicitly and to provide the designated quality and quantity of materials and workmanship for all job stages. The inspector should be present as frequently as he deems necessary to explain and interpret design requirements; to judge whether the quality and quantity of the work complies with the contract documents; to counsel on possible ways of obtaining the desired results; to see that the general system proposed for formwork appears proper (though it remains the contractors responsibility to design and build adequate forms and to leave them in place until it is safe to remove them); to see that reinforcing steel is properly installed; to see that concrete is of the correct quality, properly placed, and cured; and to see that tests for quality control are being made as specified.

The Code prescribes minimum requirements for inspection of all structures within its scope. It is not a construction specification and any user of the Code can require higher standards of inspection than cited in the legal code if he feels additional requirements are necessary.

Recommended procedures for concrete inspection are given in detail in "Recommended Practice for Concrete Inspection (ACI 311-64)" and *ACI Manual of Concrete Inspection,* (SP-2).

1.3.2 — The term "ambient temperature" means the temperature of the environment to which the concrete is directly exposed. Concrete temperatures as used in this section may be taken as the air temperature near the surface of the concrete; however, during mixing and placing it is practical to measure the temperature of the mixture.

1.3.3 — A permanent record of inspection in the form of a job diary is required by this section, in case questions subsequently arise concerning the structural elements. Photographs documenting job progress may also be desirable.

1.4—Approval of special systems of design and construction

New methods of design, new materials, and new uses of materials must undergo a period of development before being specifically covered in a Code. Hence, good systems or components might be excluded from use by implication if means were not available to obtain acceptance. This section permits proponents to submit data substantiating the adequacy of their system or component to a "board of examiners." Such a board should be created and named in accordance with local laws, and should be headed by a competent structural engineer. It is recommended that all board members be directly associated with, and competent in, the fields of structural design or construction.

For special systems considered under this section, specific tests, load factors, deflection limits, and other pertinent requirements should be set by the board of examiners, and should be consistent with the intent of the Code.

The provisions of this section do not apply to model tests used to supplement calculations under Section 1.2.2 or to strength evaluation of existing structures under Chapter 20.

For consistent application of a code, it is necessary that terms be defined where they have particular meanings in the Code. The definitions given are for use in application of the Code only and do not always correspond to ordinary usage. For example, deformed reinforcement is defined as that meeting Sections 3.5.1, 3.5.6, 3.5.7, or 3.5.8. No other bar or fabric qualifies. This definition permits accurate statement of anchorage lengths. Bars or wire not meeting the deformation requirements or fabric not meeting the spacing requirements are "plain reinforcement," for Code purposes, and may be used only for spirals.

The use of sand replacement for fine aggregate in lightweight concrete has brought about the need for a definition for this type of concrete. The term "sand-lightweight concrete" has generally been used in this case. Partial sand replacement is also used in the sense that all of the fine aggregate is not replaced by sand.

Reinforced concrete has been defined to include prestressed concrete. Heretofore, reinforced concrete and prestressed concrete were often treated as different materials. Integration of provions common to both is an effort to avoid overlapping and conflicting provisions. Although the behavior of a prestressed member with unbonded tendons may vary from that of members with continuously bonded tendons, bonded and unbonded prestressed concrete along with conventionally reinforced concrete are combined under the generic term "reinforced concrete."

Provisions for some uses of plain concrete, such as plain concrete footings, are included in the Code.

The differentiation between columns and walls is based on the principal use rather than on arbitrary relationships of height, thickness, and width. The Code, however, permits walls to be designed using the principles stated for column design, as well as by the empirical method in Chapter 14.

While a wall always separates areas or materials, it may also be used to resist horizontal or vertical forces or bending. For example, a retaining wall or a basement wall serves to separate air, water, soil, or other materials, while it may also support various combinations of loads.

A column is normally used as a main vertical member carrying axial loads combined with bending and shear. It may, however, form a small part of an enclosure or separation.

The term "compression member" is used in the Code to designate any member in which the primary stress is longitudinal compression. Such a member need not be vertical but may have any directional orientation in space. Bearing walls and columns qualify as compression members under this definition.

A number of definitions for loads are given in this chapter as the Code contains requirements that must be met at various load levels. The terms "dead load" and "live load" refer to the unfactored loads specified or defined by the local building code. The loads used to proportion a member for adequate strength are defined as "design loads" and are always factored loads, using the load factors specified in either Section 9.3 or Section 8.10. When the Code refers to design moments, design shears, etc., their values must be determined using design loads (with load factors). Service loads (loads without load factors) are to be used where stipulated in the Code to proportion or investigate members for adequate serviceability, such as in Section 9.5, Control of Deflections.

CHAPTER 3—MATERIALS

3.2—Cements

3.2.1—In previous ACI Codes, there was an implied warning that special attention should be given to moist curing when portland blast-furnace slag cement or portland-pozzolan cement is used. Since specified strength requirements for these types of cements are now the same as imposed on their counterpart portland cements in ASTM C 150, this admonition does not appear in the 1971 ACI Code.

3.2.2 — Depending on the circumstances, this provision may simply mean the same type of cement or it may mean cement from the identical source. The latter would be the case if the standard deviation* of strength tests used in establishing the required overdesign was based on one particular type of cement from one particular source. In the case of a plant that has determined the standard deviation from tests involving cement obtained from several sources, the former interpretation would apply.

*See ACI Committee 214, "Recommended Practice for Evaluation of Compression Test Results of Field Concrete (ACI 214-65)," American Concrete Institute, Detroit, 1965, 28 pp. (This standard also appear in the *ACI Manual of Concrete Practice*.)

3.3—Aggregates

3.3.1—It is recognized that aggregates conforming to nationally recognized specifications are not always economically available and that, in some instances, noncomplying materials have a long history of satisfactory performance. Such nonconforming materials are permitted with special approval when acceptable evidence of satisfactory performance is provided. It should be noted, however, that satisfactory performance in the past does not guarantee good performance under other conditions and in other localities. Whenever possible, aggregates conforming to the designated nationally recognized specifications should be used.

3.3.2—The size limitations on aggregates are provided to insure proper encasement of bars and to minimize honeycomb. A new provision limits the maximum size of aggregate to one-third of the depth of the slab, as recommended by ACI Committee 301. Note that the limitations on maximum size of the aggregate may be waived if, in the judgement of the engineer, the workability and methods of consolidation of the concrete are such that the concrete can be placed without honeycomb or void. In this instance, the engineer in charge of inspection must decide whether or not the limitations on maximum size of aggregate may be waived.

3.4—Water

3.4.1 — A new provision has been added concerning chloride ion content of water (including that portion of the mixing water contributed as free moisture on the aggregates) to be used in prestressed concrete or in concrete with aluminum embedment. No numerical quantities are stipulated. It is suggested that a chloride ion content greater than 400 or 500 ppm might be considered dangerous and ACI Committee 222, Corrosion of Metals in Concrete, recommends that levels well below these values be maintained, if practicable.

Chloride ions contained in the aggregate and in admixtures should be considered in evaluating the acceptability of total chloride ion content of the mixing water.

3.4.2—The method for determining the acceptability of nonpotable mixing water is prescribed including reference to ASTM C 109, which prescribes procedures for preparing and testing mortar cubes. Normally such tests will be needed only when satisfactory experience with the suspect water is nonexistent or inadequate.

3.5—Metal reinforcement

Extensive consolidation of ASTM standards has permitted simplification and shortening of this section from that contained in the 1963 Code.

3.5.1—This section contains two exceptions to the 1968 ASTM specification for reinforcing bars. The first exception requires that for bars with a specified yield strength, f_y, exceeding 60,000 psi, the stress f_y must be measured at a strain of 0.35 percent. This is a change from the 1963 ACI Code, Section 1505(a), which, for ultimate strength designs only, required either a proof stress at a strain not to exceed 0.30 percent for steels with f_y in excess of 60,000 psi, or a reduction in usable yield strength.

The 1971 Code continues to exempt reinforcing steels of 60,000 psi or less from the additional proof test on the basis of the results of an extensive series of stress strain tensile tests on Grade 60 reinforcement in the complete range of bar sizes, sampled from all types of producing mills in all areas of the country.

The tests were under the sponsorship of the American Iron and Steel Institute and were concluded in 1969. Strengths were measured at strains of 0.003, 0.0035, and 0.005. Although average strengths were well above minimum specified yield strength f_y at these strains, normal scatter permitted study of some results in which the bars barely met f_y at the ASTM prescribed strain of 0.005. Stress at 0.003 or 0.0035 was generally closer to f_y than the underweight-understrength tolerances permitted at 0.005 strain under ASTM specifications used in the 1963 ACI Code. ASTM specifications cited in the 1971 Code changed the basis of computing yield strength from actual area (permitted to be underweight 3½ percent for lots, or 6 percent for individual bars) to nominal area, effectively upgrading required strengths 3½ to 6 percent. It was concluded that no exception to the ASTM specifications is required for bars with yield strengths f_y of 60,000 psi or less.

The exception was retained for bars with specified f_y greater than 60,000 psi because the tests did not include Grade 75 bars, but was liberalized to allow the proof stress to be that at a strain of 0.0035 rather than 0.0030, in recognition of the shape of reinforcing bar stress-strain curves observed. The requirement that f_y be the stress corresponding to a strain of 0.35 percent also applies to plain or deformed wire if the yield strength specified exceeds 60,000.

The second exception to the ASTM Specifications in Section 3.5.1 requires that yield strength correspond to that obtained using tests of full-size bars. Tests indicate that standard mill specimens show higher strength than tests on actual reinforcement specimens.

This requirement may be met by various procedures including, but not limited to: (a) tests on full-size bars or (b) tests on standardized small size turned down specimens, the results of which can be correlated on an adequate and conservative statistical basis with results of tests on full-size bars. All measurement of yield strength whether by full section testing or by the correlation method of (b), shall be based on the nominal area of the bar.

3.5.2—Plain bars are permitted only for spiral reinforcement, either as lateral reinforcement in columns, as torsional reinforcement, or for confining reinforcement for splices.

3.5.3—Welding of reinforcing bars should not be indiscriminately executed without regard to steel weldability and proper welding procedures. When welding is called for, the job specifications should cover these items. The important consideration is that the specified procedure and steel weldability are compatible. AWS D12.1 gives authoritative recommended practices on this, including preheat and interpass temperatures and types of electrodes for various ranges of carbon and manganese content. If it is desired to restrict the steel chemistry to a given range to suit a specified procedure, ASTM reinforcing bar specifications for the steel must be supplemented to cover this.

3.5.10—High strength bars for prestressing are defined by minimum physical requirements accepted by the Prestressed Concrete Institute.

3.6—Admixtures

3.6.1—Attention is called here to the possible adverse effects of excessive chloride ions in the presence of aluminum, and in prestressed concrete. Admixtures containing any chloride, other than that which may be contributed as impurities from admixture ingredients, should not be used in prestressed concrete or in concrete which will have aluminum embedments. Research indicates that any amount of chloride ion in such concrete may be harmful.

3.8—Specifications cited in the Code

The specifications listed were the latest editions at the time the Code was prepared. Since these specifications are revised frequently, generally in minor details only, the user of the Code should check directly with the sponsoring society if it is desired to refer to the latest edition.

Standard specifications or other material to be legally adopted by reference into a building code must refer to a specific document. This can be done by simply using the complete serial designation since the first part indicates the subject and the second part the year of adoption. All of the documents referred to in other parts of the Code are listed, with the title and complete serial designation in Section 3.8. In the other sections of the Code, the designations do not include the date so that all may be kept up-to-date by simply revising this one section.

ACI publications outline excellent procedures for design and construction but are not in the legal form for direct adoption in a code. For this reason they are listed in the Commentary and not the Code. Detailed recommendations for acceptable practices are available in the following standards, committee reports, and special publications of the American Concrete Institute:

Standards and recommendations*

ACI 211.1-70 Recommended Practice for Selecting Proportions for Normal Weight Concrete

ACI 211.2-69 Recommended Practice for Selecting Proportions for Structural Lightweight Concrete

ACI 214-65 Recommended Practice for Evaluation of Compression Test Results of Field Concrete

ACI 301-66 Specifications for Structural Concrete for Buildings

ACI 302-69 Recommended Practice for Concrete Floor and Slab Construction

ACI 306-66 Recommended Practice for Cold Weather Concreting

ACI 311-64 Recommended Practice for Concrete Inspection

ACI 315-65 Manual of Standard Practice for Detailing Reinforced Concrete Structures

ACI 347-68 Recommended Practice for Concrete Formwork

ACI 307-69 Specification for the Design and Construction of Reinforced Concrete Chimneys

ACI 506-66 Recommended Practice for Shotcreting

ACI 517-70 Recommended Practice for Atmospheric Pressure Steam Curing

ACI 525-63 Minimum Requirements for Thin-Section Precast Concrete Construction

ACI 605-59 Recommended Practice for Hot Weather Concreting

ACI 614-59 Recommended Practice for Measuring, Mixing, and Placing Concrete

Committee reports*

Guide for Structural Lightweight Concrete (ACI Committee 213, Aug. 1967)

Structural Plain Concrete (ACI Committee 322, Apr. 1967)

Tentative Recommendations for Prestressed Concrete (ACI Committee 323, Jan. 1958)

*All ACI current standards, except ACI 315, and most current ACI committee reports appear in the *ACI Manual of Concrete Practice.*

Tentative Recommendations for Concrete Members Prestressed with Unbonded Tendons (ACI Committee 423, Feb. 1969)

Tentative Recommendations for Design of Composite Beams and Girders for Buildings (ACI Committee 333, Dec. 1960)

Design and Construction of Circular Prestressed Concrete Structures (ACI Committee 344, Sept. 1970)

Deflections of Reinforced Concrete Flexural Members (ACI Committee 435, June 1966)

Deflections of Prestressed Concrete Members, (ACI Committee 435, Subcommittee 5, Dec. 1963)

Allowable Deflections (ACI Committee 435, Subcommittee 1, June 1968)

Suggested Design Procedures for Combined Footings and Mats (ACI Committee 436, Oct. 1966)

Tentative Recommendations for the Design of Reinforced Concrete Members to Resist Torsion (ACI Committee 438, Jan. 1969)

Consolidation of Concrete (ACI Committee 609, Apr. 1960)

Guide to Joint Sealants for Concrete Structures (ACI Committee 504, July 1970)

Special publications

ACI Manual of Concrete Inspection, SP-2 (Reported by Committee 311, 5th Edition, revised 1967)

Reinforced Concrete Design Handbook, SP-3 (Reported by Committee 317, 3rd Edition, 1965) *

Formwork for Concrete, SP-4 (by M. K. Hurd under direction of Committee 347, 1969)

Ultimate Strength Design Handbook, V. 1, SP-17 (Reported by Committee 340, 2nd Printing, 1968)

Ultimate Strength Design Handbook, V. 2, Columns, SP-17A (Reported by Committee 340, 1970)

Torsion of Structural Concrete, SP-18 (Reported by Committee 438, 1968)

*Only those procedures in SP-3 related to the design of beams for flexure without axial load apply to this code. Specifically, the column design tables and charts do not apply.

CHAPTER 4—CONCRETE QUALITY

The requirements for proportioning of concrete mixes and the criteria for acceptance of concrete are based on the philosophy that the Code is intended primarily to protect the safety of the public. Chapter 4 describes procedures by which concrete of adequate quality can be obtained and provides procedures for checking the quality of the concrete during and after its placement in the work.

4.1—General

The basic premises governing the designation and evaluation of concrete strength are presented. It is emphasized that the average strength of concrete produced must always exceed the specified value of f_c' that was used in the structural design phase. This is based on probabilistic concepts, and is intended to insure that adequate strength will be developed in the structure.

4.2—Selection of concrete proportions

Detailed recommendations for proportioning concrete are given in the publications, "Recommended Practice for Selecting Proportions for Normal Weight Concrete" (ACI 211.1) and "Recommended Practice for Selecting Proportions for Structural Lightweight Concrete" (ACI 211.2).

4.2.1—The selected water-cement ratio must be low enough to satisfy both the strength criteria (Sections 4.2.2, 4.2.3, or 4.2.4) and the durability requirements (Sections 4.2.5, 4.2.6, and 4.2.7). The Code does not include provisions for especially severe exposures, such as to acids or high temperatures, nor is it concerned with aesthetical considerations such as surface finishes. Items like these, which are beyond the scope of the Code, must be covered in the contract documents. Concrete ingredients and proportions must be selected to meet the minimum requirements stated in the Code and the additional requirements of the contract documents.

4.2.2—A significant modification has been made in the procedure for establishing concrete proportions. Emphasis has been placed on the use of trial batches or experience as the basis for selecting the required water-cement ratio.

The Code emphasizes a statistical basis for establishing the average strength required to assure attainment of the strength level f_c' that was used in the structural design stage. If an applicable standard deviation* for strength tests of the concrete is known, this establishes the average strength level for which the concrete must be proportioned. Otherwise, the proportions must be selected to produce an excess of average

*See ACI Committee 214, "Recommended Practice for Evaluation of Compression Test Results of Field Concrete (ACI 214-65)," American Concrete Institute, Detroit, 1965, 28 pp. (This standard also appears in the *ACI Manual of Concrete Practice.*)

strength, sufficient to allow for a high degree of variability in the strength tests.

Section 4.2.2.1 refers to the fact that the standard deviation used in the calculation of required average strength must have been developed under "similar conditions to those expected." This requirement is extremely important to assure acceptable concrete. Concrete for background tests to determine standard deviation is considered to have been "similar" to that required if it was made with the same general types of ingredients under no more restrictive conditions of control over material quality and production methods than will exist on the proposed work, and if its specified strength did not deviate more than 1000 psi from the f_c' required. A change in the type of concrete or a major increase in the strength level may increase the standard deviation. Such a situation might occur with a change in type of aggregate (i.e., from natural aggregate to lightweight aggregate or vice versa) or a change from non-air-entrained concrete to air-entrained concrete. Also, there may be an increase in standard deviation when the average strength level is raised by a significant amount, although the increment of increase in standard deviation should be somewhat less than directly proportional to the strength increase. When there is reasonable doubt, any estimated standard deviation used to calculate the required average strength should always be on the conservative (high) side.

Standard deviation may be computed either from a single group of 30 or more successive tests of a given class of concrete meeting the above criteria or from two groups of such tests which, taken together, comprise a total of 30 or more. In the latter case, a "statistical" average value of standard deviation is to be used, calculated by usual statistical methods.

The amounts by which average strength f_{cr} should exceed the specified strength f_c' have been calculated by procedures outlined in the report of ACI Committee 214, "Recommended Practice for Evaluation of Compressive Strength Tests of Field Concrete." The listed values represent the highest average values required to meet all three of the following criteria, using the maximum standard deviation from the range shown in each case:

1. a probability of less than 1 in 10 that a random individual strength test will be below the specified strength f_c'
2. a probability of 1 in 100 that an average of 3 consecutive strength tests will be below the specified strength f_c'
3. a probability of 1 in 100 that an individual strength test will be more than 500 psi below the specified strength f_c'

Using values of "t" from Table 4 of the ACI Committee 214 Standard,* formulas for calculating the required average strengths reduce to the following for the respective criteria above:

1. $f_{cr} = f_c' + 1.282\,\sigma$
2. $f_{cr} = f_c' + \dfrac{2.326\,\sigma}{\sqrt{3}} = f_c' + 1.343\,\sigma$
3. $f_{cr} = f_c' - 500 + 2.326\,\sigma$

where

f_{cr} = average strength to be used as the basis for selecting concrete proportions, psi

f_c' = strength level used in the design of the structure, psi, as defined in Section 2.1 of the Code (specified f_c')

σ = standard deviation of individual strength tests, psi

It can be seen that Criterion 2 always produces a required average strength higher than Criterion 1. Criterion 2 will produce a higher required average strength than will Criterion 3 for low to moderate standard deviations, up to about 500 psi. For higher standard deviations, however, Criterion 3 governs, i.e., limiting the expected frequency of tests more than 500 psi below the specified f_c' to 1 in 100.

The indicated average strength levels are intended to reduce the probability of concrete strength being questioned on any of the following usual bases: (1) too many tests below specified f_c'; (2) strength averaging below specified f_c' for an appreciable period (three consecutive tests); or (3) an individual test being disturbingly low (more than 500 psi below specified f_c').

4.2.4—Estimation of the water-cement ratio from the generalized Table 4.2.4 requires special permission. This is due to the fact that different combinations of ingredients produce concretes which vary considerably in strength level attained at a given water-cement ratio. Therefore, a single table relating concrete strength to water-cement ratio must, of necessity, be very conservative. In the interest of economy, the approximate method should be applied only for relatively small and unimportant structures.

4.2.5—A table of required air contents for air-entrained concrete has been included in the Code, based on "Recommended Practice for Selecting Proportions for Normal Weight Concrete" (ACI 211). The values correspond to an air content in the mortar phase of the concrete of about 9 to 10

*See "Recommended Practice for Evaluation of Compression Test Results of Field Concrete (ACI 214-65)."

percent, which has been shown to provide optimum protection against damage from freezing and thawing. The entrained air will not protect coarse aggregates that undergo disruptive volume changes when frozen in a saturated condition.

Note that for lightweight aggregate concrete, the specified concrete strength f_c' must be at least 3000 psi, except as provided for in Section 4.2.6.

4.2.7 — The sulfate resisting cement required should preferably be Type V or, if Type V is unavailable, it should be Type II. If neither of these types is available, the cement selected should have a tricalcium aluminate content of less than 8 percent for moderate sulfate resistance and less than 5 percent for high sulfate resistance. Note that sulfate resisting cement will not increase resistance to some chemically aggressive solutions, for example, ammonium nitrate. The project specifications should cover all special cases. Although not specifically mentioned in the Code, attention is directed here to numerous researches indicating that the judicious employment of a good quality fly ash (ASTM C 618, Class F) improves the sulfate resistance of concrete.

4.2.9 — The procedures of the 1963 Code for establishing permissible shear stresses and required bar development lengths for lightweight concrete have been modified. Previously, except when low values were used, splitting tensile strength tests were required for use in calculating a ratio F_{sp} for establishing the reduction of shear stresses in relation to those allowed for normal weight concrete. The equivalent of that procedure is still permitted, but a more direct approach is also given in the 1971 Code. Tensile splitting tests are not required if shear and torsion stresses, cracking moment, modulus of rupture, and bar development lengths are based on the reasonable assumption that, for a given compressive strength, the tensile strength of lightweight aggregate concrete (with or without sand replacement) is a fixed proportion of that for normal weight concrete.* The percentage of normal weight concrete shear stress permitted is 75 if all lightweight aggregate is used, or 85 if natural sand is combined with lightweight coarse aggregate to produce sand-lightweight concrete. Linear interpolation is used for partial sand replacement of fine aggregate. (See Sections 9.5.2.2, 11.3, and 12.5(c).) Alternatively, the shear and torsion stress, cracking moment, modulus of rupture and bar development lengths for lightweight aggregate concrete may be upgraded if tests made in accordance with Section 4.2.9 demonstrate that the tensile strength is higher than the assumed conservative percentages stated above. In any case, the test for splitting tensile strength is used only for laboratory determination of its relationship to the compressive strength. It is not intended for control of, or acceptance of, strength properties of the concrete in the field. If use of the splitting tensile strength of lightweight aggregate concrete yields calculated permissible shear values greater, or bar development lengths less, than allowed for normal weight concrete, the values for normal weight concrete must be used.

4.3—Evaluation and acceptance of concrete

Effort has been made in the Code to provide a clear-cut basis for judging the acceptability of the concrete as well as to indicate a course of action to be followed when the results of strength tests are questionable.

4.3.1 — Samples for strength tests must be taken on a strictly random basis if they are to measure properly the acceptability of the concrete. The choice of times of sampling or the batches of concrete to be sampled must be made on the basis of chance alone within the period of placement in order to be representative. If batches to be sampled are selected on the basis of appearance, convenience, or other possibly biased criteria, the statistical concepts lose their validity. Obviously, not more than one test (average of two cylinders made from a sample) should be taken from a single batch, and water may not be added after the sample is taken.

4.3.2 — For small quantities of a given class of concrete, the Building Official may waive strength test requirements if adequate evidence of satisfactory strength is provided such as strength test results from the same type of concrete supplied on the same day by the same supplier and under comparable conditions in other work.

4.3.3 — A single set of criteria is given for acceptability of strength and is applicable to all concrete used in structures designed in accordance with the 1971 Code, regardless of design method used. The concrete strength is considered to be satisfactory as long as averages of any three consecutive tests remain above the specified f_c' and no individual test falls below the specified f_c' by more than 500 psi. Strength tests failing to meet these criteria will occur occasionally (probably about once in 100 tests) even though strength level and uniformity are satisfactory. Allowance should be made for such statistically normal deviations in deciding whether or not the strength level being produced is adequate. Although comparable in terms of the probability of failure, the criterion of minimum individual cylinder strength of 500 psi less than f_c' adapts itself more readily to small

*See Hanson, J. A., "Tensile Strength and Diagonal Tension Resistance of Structural Lightweight Concrete," ACI JOURNAL, *Proceedings* V. 58, No. 1, July 1961, pp. 1-40 (See also Reference 11 6) numbers of tests than did the ACI 318-63 requirement that "not more than 10 percent of the strength tests shall have values less than the specified strength." For example, if only five tests are made on a small job, it is apparent that, if any of them is more than 500 psi below f_c' the criterion is not met. However, in view of the small test population, it is impossible to know whether or not a 10 percent limit on tests below f_c' could be achieved.

4.3.4 — Positive guidance has been provided in the Code concerning the interpretation of tests of field-cured cylinders. Researchers have shown that cylinders protected and cured to simulate good field practice should test not less than about 85 percent of the standard laboratory moist-cured cylinders. This percentage has been set merely as a rational basis for judging the adequacy of field curing. The comparison is made between the actual measured strengths of companion job-cured and laboratory-cured cylinders, not between job-cured cylinders and the specified value of f_c'. However, results for the job-cured cylinders are considered satisfactory if they exceed the specified f_c' by more than 500 psi even though they fail to reach 85 percent of the strength of companion laboratory-cured specimens.

4.3.5 — Instructions have been provided concerning the procedure to be followed when strength tests have failed to meet the specified acceptance criteria. For obvious reasons, these instructions cannot be dogmatic. The Building Official must apply judgment as to the true significance of low test results and whether or not they indicate need for concern. If further investigation is deemed necessary, such investigation may include nondestructive tests, or in extreme cases, strength tests of cores taken from the structure. Nondestructive tests, such as by impact hammer, of the concrete in place may be useful in determining whether or not a portion of the structure actually contains low strength concrete. Such tests are of value primarily for comparisons within the same job rather than as quantitative measures of strength. For cores, if required, conservatively safe acceptance criteria have been provided which, if met, should assure structural adequacy for virtually any type of construction.[4.2,4.3] Lower strength may, of course, be tolerated under many circumstances, but this again becomes a matter of judgment on the part of the Building Official. When the core tests fail to provide assurance of structural adequacy, it may be practicable, particularly in the case of floor or roof systems, for the Building Official to resort to a load test (Chapter 20) as final arbiter. Short of load tests, if time and conditions permit, an effort may be made to improve the strength of the concrete in place by supplemental wet curing. Effectiveness of such a treatment must, of course, be verified by further strength evaluation using procedures previously discussed.

It should be noted that core tests having an average of 85 percent of the specified strength are entirely realistic. To expect core tests to be equal to f_c' is not realistic, since differences in the size of specimens, conditions of obtaining samples, and procedures for curing do not permit equal values to be obtained.

The Code, as stated, concerns itself with assuring structural safety, and the instructions in Section 4.3 are aimed at that objective. It is not the function of the Code to assign responsibility for strength deficiencies, whether or not they are such as to require corrective measures.

References

4.1. Dikeou, J. T., "Fly Ash Increases Resistance of Concrete to Sulfate Attack," *Research Report* No. C-1224, Concrete and Structures Branch, Division of Research, U. S. Bureau of Reclamation, Jan. 1967, 25 pp.

4.2. Bloem, Delmar L., "Concrete Strength Measurement—Cores Vs. Cylinders," ASTM Proceedings, 1965, pp. 668-696.

4.3. Bloem, Delmar L., "Concrete Strength in Structures," ACI Journal, *Proceedings* V. 65, No. 3, Mar. 1968, pp. 176-187.

CHAPTER 5—MIXING AND PLACING CONCRETE

5.1—Preparation of equipment and placing of concrete

This section calls attention to the necessity of using clean equipment and for thoroughly cleaning forms and reinforcement before proceeding to deposit concrete. In particular, sawdust and wood blocks that collect inside of forms should be flushed out, and reinforcement must be thoroughly cleaned of mud. Excess water should be removed from the forms.

5.2—Mixing of concrete

Concrete of uniform and satisfactory quality requires the materials to be thoroughly mixed. The necessary time for mixing will depend on many factors including batch size, stiffness of the batch, size and grading of the aggregate, and the efficiency of the mixer.

Excessively long mixing times may grind the aggregates, and this should be avoided.

5.3—Conveying

The Code requires that conveying equipment be capable of supplying concrete continuously and reliably under all conditions and for all procedures of placement. Those provisions apply to all placement methods, including pumps, belt conveyors, pneumatic systems, wheelbarrows, buggies, crane buckets, and tremies.

Recent reports have indicated that serious loss in strength of concrete can result when it is pumped through pipe made of aluminum or aluminum alloys. Hydrogen gas generated by the reaction between the cement alkalies and aluminum eroded from the interior of the pipe surface has been shown to cause strength reduction as much as 50 percent. Hence, equipment made of aluminum or aluminum alloys should not be used for pump lines, tremies, or chutes other than short chutes, such as those used to remove concrete from a truck mixer.

5.4—Depositing

Rehandling concrete can cause segregation of the materials. Hence the Code cautions against this practice. Retempering of partially set concrete with the addition of water should not be permitted. This does not preclude the practice, recognized in ASTM C 94, of adding water to mixed concrete to bring it up to the specified slump range so long as prescribed limits on the maximum mixing time and water-cement ratio are not violated.

When placing conditions are difficult, such as in deep or heavily reinforced members, the use of mortar batches will aid in preventing honeycomb and poor bonding of the concrete with the reinforcement. When used, the mortar should contain the same ratio of fine aggregate to cement and the same water-cement ratio as the concrete to be placed. The mortar should be placed immediately before depositing the concrete and must be plastic and neither stiff nor fluid when the concrete is placed.

5.5—Curing

In addition to requiring a minimum curing temperature and time interval as contained in the 1963 Code, the Code provides a specific criterion in Section 4.3.4 for judging the adequacy of field curing. At the test age for which the strength is specified (usually 28 days), field-cured cylinders should produce strength not less than 85 percent of that of the standard, laboratory-cured cylinders. For a reasonably correct comparison to be made, field-cured cylinders and companion laboratory-cured cylinders must come from the same sample. Field-cured cylinders must be cured under conditions identical to those of the structure. If the structure is protected from the elements, the cylinder should be protected similarly. That is, cylinders related to members not directly exposed to the weather should be cured adjacent to those members and provided with the same degree of protection and type of curing. Obviously, the field cylinders should not be treated more favorably than the elements they represent. (See Code and Commentary, Section 4.3.4 for additional information.)

If the field-cured cylinders do not provide satisfactory strength by this comparison, measures should be taken to improve the curing of the structure. If the tests indicate a possible serious deficiency in strength of concrete in the structure, core tests may be required, with or without supplemental wet curing, to check the structural adequacy, as provided in Section 4.3.5.

5.5.2 — This section applies whenever an accelerated curing method is used, whether for precast or cast-in-place elements. The ultimate compressive strength f_c' of steam cured concrete is not as high as that of similar concrete continuously cured under moist conditions at moderate temperatures. Also, the elastic modulus E_c of steam-cured specimens may vary from that of specimens moist-cured at normal temperatures. When steam curing is to be used, it is advisable to base the mix proportions on steam-cured test cylinders.

Accelerated curing procedures require careful attention to obtain uniform and dependable results. It is essential that moisture loss during the curing process be prevented.

5.6—Cold weather requirements

Details of approved procedures are available in "Recommended Practice for Cold Weather Concreting," (ACI 306).

5.7—Hot weather requirements

Details of approved procedures are available in "Recommended Practice for Hot Weather Concreting," (ACI 605).

References

5.1 Newlon Jr., Howard, and Ozol, A., "Delayed Expansion of Concrete Delivered by Pumping Through Aluminum Pipe Line," *Concrete Case Study* No. 20; Virginia Highway Research Council, Oct. 1969.

CHAPTER 6—FORMWORK, EMBEDDED PIPES, AND CONSTRUCTION JOINTS

Because proper design, construction, and removal of forms is an involved subject, only the basic requirements are discussed in this Commentary. For detailed information, the reader should refer to the work of ACI Committee 347 in "Recommended Practice for Concrete Formwork (ACI 347-68)" and *Formwork for Concrete*, ACI Special Publication No. 4.

In determining the time for removal of forms, consideration should be given to the construction loads and to the possibilities of deflections. The construction loads are frequently at least as great as the design live loads. At early ages, a structure may be strong enough to support the applied load but may deflect sufficiently to cause permanent damage.

Conduits and pipes not harmful to the concrete can be embedded therein, but the work must be done in such a manner that the structure will not be endangered. Empirical rules are given for safe installations for common conditions, but special designs must be made for other than common conditions. The contractor should not be permitted to install conduits, pipes, ducts, or openings that are not shown on the plans or not approved by the architect or engineer.

The Code prohibits the use of aluminum in structural concrete unless it is effectively coated or covered. Aluminum reacts with concrete and, in the presence of chloride ions, may also react electrolytically with steel causing cracking and/or spalling of the concrete. Aluminum electrical conduits present a special problem since stray electric current speeds up the adverse reaction.

For the integrity of the structure, it is important that all joints be carefully constructed as and where shown on the plans or called for in the specifications. Any variation therefrom should be approved by the architect or engineer.

The delay in placing concrete above columns and walls is provided to permit the concrete to settle and prevent cracking at the underside of the floor system. The restriction on the location of joints is intended to place the joints where they will cause the least weakness in the structure.

CHAPTER 7—DETAILS OF REINFORCEMENT

General

Good structural details have always been vital to satisfactory reinforced concrete structures. Over the years, a standard practice for reinforcement details was developed gradually. In the 1956 Code, the details of connections for structural elements, bar cutoffs, splices, and bar bending were based on a stress of 20,000 psi for steel and equal bond in tension and compression varying directly with concrete strength only. For columns and short span one-way slabs, higher yield strength steels were permitted with higher working stresses under the same assumptions for bond.

Since the 1956 Code, ACI Committee 318 has collected reports of previous research and practice with high yield strength steels, suggested new research needed, received reports on new research, and translated the results into new Code provisions which create new standards for details of reinforcement.

The research findings that bond generally varies with bar diameter and stress, tensile or compressive, as well as concrete strength, and that anchorage bond is not directly proportional to anchorage length, make necessary a whole family of new reinforcement detailing standards. The use of a single bond stress value for all size bars was attractively simple, but has been shown to be incorrect.

In Chapter 7, the Code provides separately for tensile and compressive splices, size and yield stress level of bars, smooth and deformed welded wire fabric, welded or mechanically connected tension splices, end-bearing compression splices, concrete area between splices, and splices laterally confined by auxiliary reinforcement, reflecting in each case specific findings from research.

Research projects have yielded results responsible for a number of specific Code provisions including bending radii for #14 and #18 bars, spiral spacers, end-bearing compression splices, bundled bars, and column ties.

7.1—Hooks and bends

This section is a consolidation of provisions affecting hooks and bends from several sections of the 1956 Code. Bending provisions for the #14 and #18 bars and the 90 deg stirrup or tie hook with a six bar diameter extension were added in 1963.

A number of new provisions are given in the 1971 Code for hooks and bends. Standard hooks are described in terms of the inside diameter of bend since this is easier to measure than the radius of bend.

A broad survey of bending practices, a study of ASTM bend test requirements, and a pilot study of bending Grade 60 and Grade 75 #14 and #18 bars were considered in establishing the minimum diameter of bend for each grade. The primary consideration was feasibility of bending without undue breakage. The provision against hot bending contained in the 1956 Code was relaxed on advice of metallurgists that proper use of heat would not be unsafe.

The Code user is cautioned against combining minimum diameter of bend with extreme cominations of maximum size bar, minimum concrete strengths, no lateral or confining auxiliary reinforcement, and maximum tensile stress in the bars. This is particularly important with #14 and #18 bars.

Since some ASTM specifications do not provide for bend tests of the bars to recommended bend diameters, the designer should make sure that the bends he calls for can safely be made with the grades of steel specified.

7.1.2—The note on special fabrication appearing in the 1963 Code was deleted since bar sizes #14 and #18 are now commonly bent cold.

7.1.3.1 For each size of bar commonly used as stirrups or ties, minimum bend diameters are prescribed based on accepted industry practice in the United States. Use of the recommended sizes for 90 deg and 135 deg stirrup and tie hooks conforming to Section 7.1.1.3 will permit multiple bending on standard stirrup bending equipment. The 1963 Code permitted bend diameters as small as two bar diameters. This minimum value was increased because it is a far more severe bend than required by ASTM bend tests, and could seriously damage the bars. The 1963 ACI Code anchorage requirement that stirrups be hooked *tightly* around longitudinal reinforcement has been revised. (See Section 12.13 1.3.) In actual application, the 1963 Code section could have required different diameters and hooks on each end of the same stirrup, and so was seldom specified.

7.1.3.3 — Welded wire fabric, or plain or deformed wire, can be used for ties and stirrups. The wire at welded intersections does not have the same uniform ductility and bendability as in areas which were not heated. These effects of the welding temperature are usually dissipated in a distance of approximately four wire diameters. Minimum bend diameters permitted are in most cases the same as those required in the ASTM bend tests for wire material.

7.1.4 — This section requires that all bends be made cold unless otherwise permitted by the Engineer. In this sense the Engineer is the engineer or architect employed by the owner to perform inspection. For unusual bends exceeding ASTM bend test requirements special fabrication may be required. It may be necessary to bend bars that have been embedded in concrete, and it usually is not possible to provide a pin of the minimum diameter specified in the Code at the point of bend. Such bending can not be done without authorization of the inspecting engineer. If he so authorizes he will determine whether or not the bars can be bent cold without damage or if heating is necessary. If heating is permitted it must be controlled to avoid splitting of the concrete or damage to the bars. When bars are not embedded in thin sections, temperatures ranging from 600 to 800 F are usually satisfactory to permit bending without damage to the bars or the concrete.

7.2—Surface conditions of reinforcement

Specific limits on rust are based on latest tests, plus a review of earlier tests and recommendations published by Federal agencies using concrete. Reference 7.1 provides guidance with regard to the effects of rust and mill scale on bond characteristics of deformed reinforcing bars. Research has shown that a normal amount of rust increases bond. Normal rough handling generally removes rust which is loose enough to injure bond.

7.3—Placing reinforcement

7.3.1 — Specifications of approved materials and description of devices for supporting reinforcement (appearing in the 1963 Code) were deleted since specification of required performance was considered sufficient. "Tack" welding (welding crossing bars) can seriously weaken a bar at the point welded by creating a metallurgical notch effect. This operation can be performed safely only when the material welded and welding operations are under continuous competent control, as in the manufacture of welded wire fabric.

7.3.2 — An intermediate tolerance was added to provide more uniform effect and to encourage more realistic applications and enforcement. The use of higher stresses requires closer limits on effective depth, particularly in shallow members, and tolerances ideally should be proportional to the depth. However, three fixed limits were considered more practicable for enforcement and for simplicity of instructions to field placing crews.

Since the effective depth and clear concrete cover are components of total depth, the tolerances on these dimensions are directly related. Generally accepted practice, as reflected in other ACI standards, have established tolerances on total depth (formwork or finish) and fabrication of truss bent reinforcing bars and closed ties, stirrups, and spirals. Bar supports and spacers, factory made to tolerances of a lower order of magnitude, are standard in ¼ in. increments. When an accumulation of tolerances may develop, resulting in excessive reduction of effective depth or cover, the Engineer should indicate which dimension is critical. The additional requirement that the cover shall not be reduced by more than one-third of the specified cover is necessary, particularly for the lesser cover permitted in precast construction and shells.

7.3.3 — This provision concerning draped fabric has remained essentially unchanged through a long series of ACI Codes. It permits use of the lighter styles of welded wire fabric which are flexible enough to drape between supports to be exempt from requirements of Sections 12.2 and 12.3. Omission of these anchorage requirements in lightly reinforced, short span slabs (where temperature and shrinkage requirements often control the minimum amount of reinforcement required) has resulted in economy without any known adverse effects on structural performance.

7.4—Spacing of reinforcement

7.4.1 — The spacing limits in this section have been developed from successful practice over many years, remaining essentially unchanged through many codes. The minimum limits were established to permit concrete to flow readily into spaces between bars and between bars and forms without honeycomb, and to ensure against concentration of bars on a line that might result in shear or shrinkage cracking.

7.4.2 — The successful practice of "bundling" standard size bars for large girders, and laboratory tests on bundled column bars led to the provisions for bundling bars in the 1963 Code. The provisions of this section are fundamentally the same as those contained in the 1963 Code, with some exceptions. Bond research[7.4] showed that bar cutoffs for girders and splices for columns should be staggered. Bundled bars should be tied, wired, or otherwise fastened together to ensure remaining in position whether vertical or horizontal.

A limitation that bars larger than #11 not be bundled in beams or girders has been added, since the ACI Code applies primarily to buildings. The United States Bureau of Public Roads design criteria for reinforced concrete bridges by ultimate design[7.2] permits two-bar bundles of bars up to #18 in bridge girders or columns, usually more massive than those in buildings. Conformance to crack control requirements in the 1971 Code will effectively preclude bundling of bars larger than #11 as tensile reinforcement. The Code phrasing "bundled in contact, assumed to act as a unit," is intended to preclude bundling more than two bars in the same plane. Typical bundle shapes are triangular, square, or L-shaped patterns for three- or four-bar bundles. As a practical caution, bundles more than one bar deep in the plane of bending may not be hooked or bent as a unit. Where end hooks are required, it is preferable to stagger them. Bending and hooking of bundles must be established in this manner, even at supports.

7.4.3 — These maximum spacing limits have remained essentially unchanged for many years, even though extended to massive sections with #18 bars.

7.4.4 — These requirements for minimum bar spacing like those in Section 7.4.1 were developed originally to provide access for concrete placing in columns. Use of the bar diameter as a factor in establishing the minimum spacing permitted extension of the original provision to larger bars.

7.4.5 — The Commentary on Sections 7.4.1 and 7.4.4 is applicable here. See also Section 7.5.4.

7.4.6 — The provisions for limiting the spacing of pretensioning steel were in Chapter 26, Prestressed Concrete, of the 1963 Code. They have been incorporated in this section of the 1971 Code to consolidate all provisions for reinforcement spacing in one chapter. The requirements are essentially the same as in the 1963 Code.

7.4.7 — When ducts for post-tensioning steel in a beam are arranged closely together vertically, provision must be made to prevent the steel, when tensioned, from breaking through the duct. Horizontal disposition of ducts must allow proper placement of concrete. Generally a clear spacing of one and one-third times the size of the coarse aggregate, but not less than 1 in., proves satisfactory. Where concentration of steel or ducts tends to create a weakened plane in the concrete cover, reinforcement should be provided to control cracking.

7.5—Splices in reinforcement—General

General provisions are included in this section for splices of reinforcement as related to bond, to overall detailing of structural members and to general specifications. (For discussion on bond research, see the Commentary for Chapter 12.)

Because each end of a splice introduces stress concentrations which tend to induce early splitting, splice lengths for Classes B, C, and D splices have been made larger than ordinary anchorage lengths. Restrictions on the width of member or the lateral spacing of splices have been added because some minimum area of concrete is needed between adjacent lap splices for full anchorage capacity to be developed. (The two bars that form a lap splice can be in contact or spaced, as desired, but must conform to Section 7.5.4.) When the lateral spacing between lap splices is more than 6 in. and the splice is located further than 3 in. from an edge, the development length, l_d, may be multiplied by 0.8 [see Section 12.5(d)], and splices are likewise shortened. Shortening is allowed in this case because of less tendency to split in the plane of the bars.

For ductility, lap splices should be adequate to develop more than the yield strength of the steel; otherwise, a member is subject to sudden splice failure when the yield strength of the steel is reached and no "toughness" is obtainable in the member. The lap splice lengths specified in the Code satisfy this ductility requirement.

Splices should, if possible, be located away from points of maximum tensile stress. The Code encourages this practice by increasingly severe requirements for splices at higher stress and for splices bunched unfavorably.

7.5.1 — The Code requires all welding of reinforcement to conform to AWS D12.1 (see Code Section 3.8.2). Primarily, these requirements demand that the chemical analysis of the reinforcement be secured and that the entire welding operation, including method, material, amount of preheat, if any, etc., be compatible with the chemical analysis.

7.5.2 — Research on lap splices of #14 and #18 bars is limited. There are insufficient data to establish lap lengths for either tensile or compressive lap splices for #14 or #18 bars. Compression splices of #14 or #18 bars to smaller bars used as dowels into footings are allowed. (See Code Section 15.6.8.)

7.5.3 — The increased length of lap required for bars in bundles is based on the reduction in the exposed perimeter of the bars.

7.5.4 — If individual bars in noncontact lap splices are too widely spaced, an unreinforced section is created. Forcing the potential crack to follow a zigzag line (5 to 1 slope) is considered a minimum precaution. The 6 in. maximum spacing is added because most research available on the lap splicing of modern deformed bars was conducted with reinforcement which was within this spacing.

7.5.5.1 A full welded splice, which is defined in this section, is primarily intended for large bars (#6 and larger) in main members. The minimum tensile capacity required will ensure sound welding, adequate also for compression. The requirement of 125 percent of specified yield strength was contained in the 1963 Code. It is desirable that splices be capable of developing the ultimate strength of the bars spliced, but practical limitations make this ideal condition difficult to attain. The maximum reinforcement stress used in design under the Code is the yield strength. To ensure sufficient capacity in splices so that yielding can be achieved in the member and thus brittle failure avoided, the 25 percent increase beyond the specified yield strength was selected as both an adequate minimum for safety and a practicable maximum for economy.

7.5.5.2 Full positive connections are also required to develop 125 percent of the yield strength, in tension or compression as required, for the same reasons as given for full welded splices in the Commentary for Section 7.5.5.1.

7.5.5.3 The use of welded splices or positive connections of less capacity than 125 percent of yield strength is permitted if the minimum design criteria of Section 7.6.3.2 are met. Therefore, lap welds of reinforcing bars, either with or without backup material, welds to plate connections, and end-bearing splices are allowed under certain conditions.

7.6—Splices in tension

This section consolidates material from the 1963 Code and provides new considerations related to the development length concept. (See Chapter 12 of the Code and Commentary.)

Development lengths, l_d, for tension bars in normal weight concrete are given in Chapter 12. For lightweight concrete and sand-lightweight concrete, the multipliers in Chapter 12 must be used. Splices are classed as Types A, B, C, and D, and multipliers of l_d for various classes are provided in Section 7.6.1. Note that Class D splices must be enclosed within a spiral conforming to Section 12.5(d). The various tension lap splice conditions are summarized in Table 7-1.

The splice requirements for situations described in Sections 7.6.2 and 7.6.3 were established to permit maximum economy consistent with a uniform structural capacity for the overall member. Spirals around splices greatly increase resistance against splitting and improve splice strength. Stirrups, or ties, unless closely spaced, are only partially effective for this purpose.

The requirements in this section apply to tensile splices where tension is the critical stress condition. Thus, the term "regions of maximum moment or high computed stress" is intended to

TABLE 7-1—TENSION LAP SPLICES

Member	Maximum stress	Percent spliced	Lap	Section	Notes
Flexure, with or without axial compression	$>0.5f_y$ tension	>50	$1.7l_d$ (Class C)	7.6.3.1.1	Avoid if possible
		≤ 50	$1.3l_d$ (Class B)	7.6.3.1.1	
	$\leq 0.5f_y$ tension	>75	$1.3l_d$ (Class B)	7.6.3.2.1	Preferred
		≤ 75	$1.0l_d$ (Class A)	7.6.3.2.1	
Tension tie (welded or positive connection preferred)		Stagger if possible	$2.0l_d$ (Class D)	7.6.1 7.6.2	Avoid if possible. Spiral Required

describe a cross section in a member other than a tension tie at which all of the tensile reinforcement is designed for the yield strength f_y. This section covers overall requirements of detailing tensile splices under various conditions.

Section 7.6 encourages the staggering of all splices in all types of members. This section identifies explicitly three less-than-ideal arrangements of splices for the total reinforcement in practical elements and, by implication, the ideal arrangement. The most severe condition, requiring special precautions, is the tension tie, and the next is splicing all reinforcement in a member at the cross section of maximum tensile stress. Conditions of intermediate severity involve splicing all reinforcement at any cross section or any reinforcement at the cross section of maximum tensile stress. Implicitly, the preferred splice layout uses staggered splices all located away from the section of maximum tensile stress. The following sections provide explicit requirements for the conditions of splice layout in practical members.

7.6.1 *Classification of tension lap splices*—This section presents the basis for calculation of lap lengths and the minimum lap length in terms of tensile development length l_d for the full f_y as given in Section 12.5. Four classes of splices are defined. Note that Class D is specified only for tension ties and must include a spiral covering, although no additional strength is allowed to be credited for the spiral. However, if a Class A, B, or C splice is enclosed within a spiral, Section 12.5(d) allows a 0.75 factor in computing l_d.

7.6.2 *Splices in tension tie members* — Note that staggered splices are recommended.

The usual limited concrete cover on all sides of tension tie members leads to a member having a minimum of concrete available to resist splitting. In the absence of specific test data for this condition, provisions are made more rigorous than for ordinary splices in beams where cover in at least one direction is not limited. The specified length lowers the splitting stress and the specified spiral increases the splitting resistance.

7.6.3 *Tension splices in other members*

7.6.3.1 This section encourages location of splices away from regions of maximum moment or high computed tensile stress. A lap splice of any portion of the total steel at points of maximum stress must be at least $1.3l_d$ (Class B) in length or if enclosed in the prescribed spiral, $1.0l_d$, where l_d is based on f_y.

If more than one-half of the bars are spliced at these points, lap splices must be at least $1.7l_d$ (Class C) or, if enclosed in spirals, $1.3l_d$. A welded splice or positive connection must develop at least 125 percent of the specified yield strength.

Note that temperature steel, except at points away from the center of slabs on ground with unrestrained ends, generally is considered at maximum stress f_y.

7.6.3.2 Required tensile splice lap lengths may be reduced if the splices are located in regions of low computed stress (where bar stress, computed using the design method of Section 8.1.1., is always less than $0.5f_y$). The type of splice required depends on the percentage of bars spliced.

When the alternate design method of Section 8.10 is used, Section 8.10.4 stipulates that the bar stress can be taken as less than $0.5f_y$ only when the reinforcing provided is more than twice that required. If it is not, tensile splices must be in accordance with Section 7.6.3.1.

7.6.3.2.1 Economical Class A splices are permitted when the bar stress is less than $0.5f_y$ and no more than 75 percent of the steel area is spliced within one lap length (splices are staggered in a member).

7.6.3.2.2 Class B splices are required in regions where f_s is less than $0.5f_y$ and where it is

TABLE 7-2—COMPRESSION LAP SPLICE LENGTHS PER ACI 318-63

f_y	Minimum lap length*		Calculated lap length required to satisfy development bond for full f_y	
	$f_c' \geqq 3000$	$f_c' < 3000$	$f_c' = 2300$	$f_c' = 1300$
40	$20d_b$	$26.7d_b$	$16d_b$	$21.4d_b$
50	$20d_b$	$26.7d_b$	$20d_b$	$26.7d_b$
60	$24d_b$	$32\ d_b$	$24d_b$	$32.0d_b$
75	$30d_b$	$40\ d_b$	$30d_b$	$40.0d_b$

d_b = bar diameters

*Note that the minimum lap lengths will control length of splice in all practical conditions The minimum lap lengths for $f_c' \geqq 3000$ psi are based on the calculated lap lengths for 2300 psi concrete, while those for $f_c' < 3000$ psi are based on 1300 psi concrete Only if concrete with $f_c' < 1300$ psi were used, or if splices were to be stressed before concrete strength reached 1300 psi, would the calculated development length control.

necessary to splice more than 75 percent of the steel at one cross section. This provision enables the designer to accommodate a construction sequence requiring all bars spliced at one location.

7.6.3.2.3 See Commentary on Section 7.5.5.3. This section describes the situation where welded splices or positive connections of less capacity than 125 percent of the specified yield strength of the bar may be used. It provides a relaxation in the splice requirements where the splices or connections are staggered and excess reinforcement area is available. The criterion of twice the computed tensile stress is used to cover sections containing partial tensile splices with various percentages of total steel continuous.

7.7—Splices in compression

Recent bond research has been primarily related to bars in tension. Bond behavior of compression bars is not complicated by the problem of transverse tension cracking and thus compression splices do not require provisions as strict as those specified for tension splices. The minimum lengths specified for column splices in the 1956 Code have been carried forward and extended to compression bars in beams and to higher strength steels as in the 1963 Code.

7.7.1 *Lap splices in compression*

7.7.1.1 Essentially, lap requirements are repeated from the 1963 Code. The basic lap splice requirements in the 1963 Code consist of minimum lap lengths and an ultimate development bond stress. See Table 7-2.

The 1963 Code values have been modified to recognize various degrees of confinement and to permit design with steels having up to 80 ksi yield strength. Tests[7.3,7.4] have shown that splice strengths in compression depend considerably on end bearing and hence do not increase proportionally in strength when the splice length is doubled. Accordingly, for yield strengths above 60 ksi, lap lengths have been significantly increased, except

TABLE 7-3—COMPARISON OF COMPRESSION LAP SPLICE REQUIREMENTS—1963 VERSUS 1971 CODE IN BAR DIAMETERS

	Minimum lap splice lengths $f_c' \geqq 3000$				Calculated lap required by bond for full f_y with $f_c' = 2300$	
	1963 Code	1971 Code				
f_y	All bars	Spiral column	Tied Column	Loose	1963 Code	1971 Code*
40	20	15.0	16.6	20	16	16.7
50	20	18.75	20.75	25	20	20.85
60	24	22.5	24.9	30	24	25.0
75	30	32.6	36.2	43.5	30	31.2
80	—	36.0	39.9	48.0	—	33.3

*For $f_c' = 2300$ psi for splices of loose bars or bars in tied columns.

where there are spiral enclosures (as in spiral columns) where the increase is only about 10 percent at 75 ksi.

For steel yield strengths up to 60 ksi, lap lengths for bars enclosed by spirals have been reduced, but those within adequate ties have been slightly modified from the 1963 Code requirements. For splices without surrounding ties or spirals, laps have been increased for steels with yield strengths above 40 ksi. See Table 7-3. The values obtained from the tables are based on normal weight concrete, as most test data available on lap splices in compression are related to this concrete.

7.7.1.2 Reduced lap lengths are allowed when the splice is enclosed throughout its length by minimum ties as defined here.

Compression splice lengths may be multiplied by 0.83 for tied compression members when the tie area throughout the lap length is at least $0.0015hs$, but the splice length may not be less than 12 in.

The tie legs perpendicular to each direction are computed separately and the requirement must be satisfied in each direction. This is illustrated in Fig. 7-1, where four legs are effective in the narrow direction or two legs in the wide direction. This calculation is critical in one direction which normally can be determined by inspection.

7.7.1.3 Compression lap lengths may be reduced when the lap splice is enclosed throughout its length by spirals because there is increased splitting resistance. Spirals should meet requirements of Sections 7.12.2 and 10.9.2.

7.7.2 — General requirements for use of end-bearing splices are repeated from ACI 318-63. A tolerance of 3 deg has been included, representing practice based on tests of full size members containing #18 bars.

A further limitation to members with enclosing reinforcement has been added to ensure a

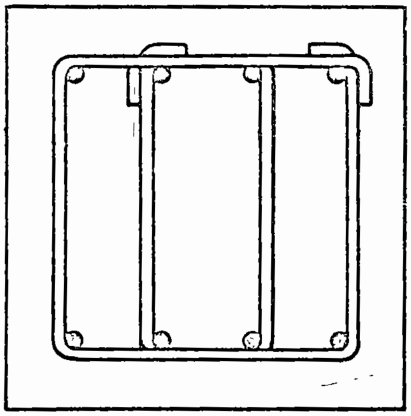


Fig. 7-1—Tie legs which cross the axis of bending are used to compute effective area. In the case shown, four legs are effective

minimum shear resistance in sections containing end-bearing splices. End-bearing splices for compression bars have most commonly been used in columns.

Data concerning end-bearing splices appear in a report by Wiss, Janney, Elstner and Associates, 330 Pfingsten Rd., Northbrook, Ill. 60062.

7.8—Splices of welded plain wire fabric

The strength of lap splices of welded plain wire fabric is dependent primarily on the anchorage obtained from the cross wires rather than on the length of wire in the splice. For this reason, in the 1963 Code, the lap was specified in terms of overlap of cross wires rather than in wire diameters or inches. The 2 in. additional lap required is to assure overlapping of the cross wires and to provide space for satisfactory consolidation of the concrete between them. Splice requirements for welded plain wire fabric are repeated from the 1963 Code.

7.9—Splices of deformed wire and welded deformed wire fabric

Splice requirements for deformed wire and welded wire fabric are provided. The splice formulas stated are based on available tests.[7.22] The anchorage value of the wire deformations can be computed singly or, where cross wires are present in the splice length, as additive to the cross wire anchorage.

7.10—Special details for columns

7.10.1 — This material is repeated from Section 805 of the 1963 Code. Offset bending of bundled bars is prohibited for practical reasons.

7.10.2 — This requirement for lap spliced dowels with column faces offset 3 in. or more, together with Section 7.5.2, precludes offsetting 3 in. or more in columns reinforced with #14 and #18 bars since lap splices are prohibited for such bars except as provided for footing dowels in Chapter 15.

7.10.3 — A minimum tensile capacity is required even where analysis indicates compression only. If end-bearing splices are employed without added splice bars, the maximum amount of reinforcement spliced at one point in any face of the columns is three-fourths. Couplers designed for compression may be used at one cross section for all reinforcement, provided they possess a tensile capacity of one-fourth of the specified yield stress for the bars coupled. Any combination of splices, splice bars, or staggered splices with continuing unspliced bars may be employed, provided the required minimum tensile capacity is maintained.

For the conditions where stress may vary from f_y compressive to one-half f_y or less in tension, minimum tensile capacity of twice the calculated tension (computed using the design method of Section 8.1.1) must be maintained in each face of the column at any section containing splices. See also Commentary for Section 7.10.5.

7.10.4 — Where calculated tension can exceed one-half f_y, splice requirements are the same as for a tensile splice in Section 7.6.

7.10.5 — This section establishes a minimum tensile capacity where splices are located in a reinforced concrete column regardless of other design requirements.

7.10.6 — This section provides an effective maximum of 50 percent transmission of load by end bearing on ends of metal cores. The section encourages, thereby, provision of some tensile capacity at such splices (up to 50 percent), since the remainder of the compression stress must be transmitted by welds, dowels, splice plates, etc. This change should ensure that splices in composite columns meet requirements for tensile capacity similar to those for reinforced concrete columns.

7.11—Connections

Confinement is needed at connections to assure that the flexural capacity of the members can be developed without deterioration of the joint under repeated loadings.[7.21]

7.12—Lateral reinforcement

7.12.2 — In the 1971 Code, some of the previous restrictions on clear spacing and maximum spacing for spiral reinforcement have been removed.

A provision has been added to the 1971 Code requiring ties above the termination of the spirals in a column if enclosure by beams or brackets is not available on all sides of the column. These ties are to enclose the longitudinal column reinforcement and the portion of bars from beams bent into the column for anchorage. The Code allows spirals to be terminated at the level of lowest horizontal reinforcement framing into the column. However, if one or more sides of the column are not enclosed by beams or brackets, ties are required from the termination of the spiral to the bottom of the slab or drop panel. If beams or brackets enclose all sides of the column but are of different depths, the ties should extend from the spiral to the level of the horizontal reinforcement of the shallowest beam or bracket framing into the column.

For cast-in-place construction the minimum diameter of spiral reinforcement was increased to ⅜ in. in the 1971 Code, as this is the smallest size that can be used in a column with 1½ in. or more cover and having concrete strengths of 3000 psi or more if the minimum clear spacing (pitch) for placing concrete is to be maintained.

Standard spiral sizes are ⅜ in., ½ in., and ⅝ in. diameter for hot rolled or cold drawn material, plain or deformed.

The lap length required for splices in spirals has been changed from the 1963 Code requirements and is now 48 spiral reinforcement diameters rather than 1½ turns. Thus, splices for spirals are similar to those required for tension splices of other reinforcement.

7.12.3 — Pilot tests on full size, axially loaded tied columns containing full length bars (no splices) showed no appreciable difference between ultimate strengths of columns with the full tie requirements and no ties at all. The tests did not include a comprehensive range of column sizes, bar sizes, bundled bars, spliced bars, moment-axial load ratios, etc., but they do indicate that Code requirements previous to the 1963 Code were unnecessarily strict. The 1956 Code required, for every vertical bar, "lateral support *equivalent* to that provided by a 90-deg corner of a tie."

The 1963 Code liberalized the tie requirements by increasing the permissible included angle from 90 to 135 deg and exempting bars which are within 6 in. on each side of adequately tied bars. All bars must be enclosed within ties. Circular ties are specifically permitted in place of all other ties where all bars may be enclosed thereby.

The new provisions permit the cores of columns to be freed considerably of the previously required maze of ties. Since spliced bars and bundled bars were not included in the tests, it would be prudent to provide at least a set of ties at each end of lap spliced bars, above and below end-bearing splices, and at minimum spacings immediately below sloping regions of offset bent bars.

Standard tie hooks are intended for use with deformed bars only, and should be staggered where possible. The minimum size of ties has been increased and related to the size of longitudinal bars. Provision for enclosure above the usual termination of ties has been included. Where longitudinal bars are arranged in a circular pattern, only one circular tie per specified spacing is required. This requirement can be satisfied by a continuous circular tie (helix) at larger pitch than permitted for spirals under Section 10.9.2, the maximum pitch being equal to the required tie spacing.

7.12.4 — Precast columns with cover less than 1½ in., prestressed columns without longitudinal bars, columns smaller than minimum dimensions prescribed in former Codes, columns of concrete with small size coarse aggregate, wall-like columns, and other special cases may require special designs for lateral reinforcement. Smooth or deformed wire, W-4, D-4, or larger, or welded wire fabric consisting of such wire may be utilized for ties or spirals. If such special columns are considered as spiral columns for load capacity in design, the ratio of spiral reinforcement ρ_s must conform to Section 10.9.2.

7.12.5 — Compression reinforcement in beams or girders must be enclosed to prevent buckling; similar requirements for such enclosure have remained essentially unchanged through several codes except for minor clarification. Provision for use of welded wire fabric for such enclosing reinforcement has been added in this section.

7.12.6 — This is a new section requiring that any lateral reinforcement in members subject to stress reversal or torsion at supports be in closed form. Further, it requires that such lateral reinforcement must enclose the main reinforcement to increase resistance against buckling and splitting.

7.13—Shrinkage and temperature reinforcement

So-called shrinkage and temperature reinforcement is required at right angles to the principal reinforcement to prevent excessive cracking and to tie the structure together to assure its acting as assumed in the design. The amounts specified are empirical but have been used satisfactorily for many years.

Deformed bars of 60,000 psi steel are recognized on the same basis as welded wire fabric.

The provisions of this section apply to "structural floor and roof slabs" only and not to slabs on ground.

Shrinkage and temperature reinforcement specification references have been changed to conform to the 1968 ASTM specifications. Provisions for use of steel with yield strength up to 80,000 psi have been added. Splices and end anchorages must be designed for the full specified yield strength.

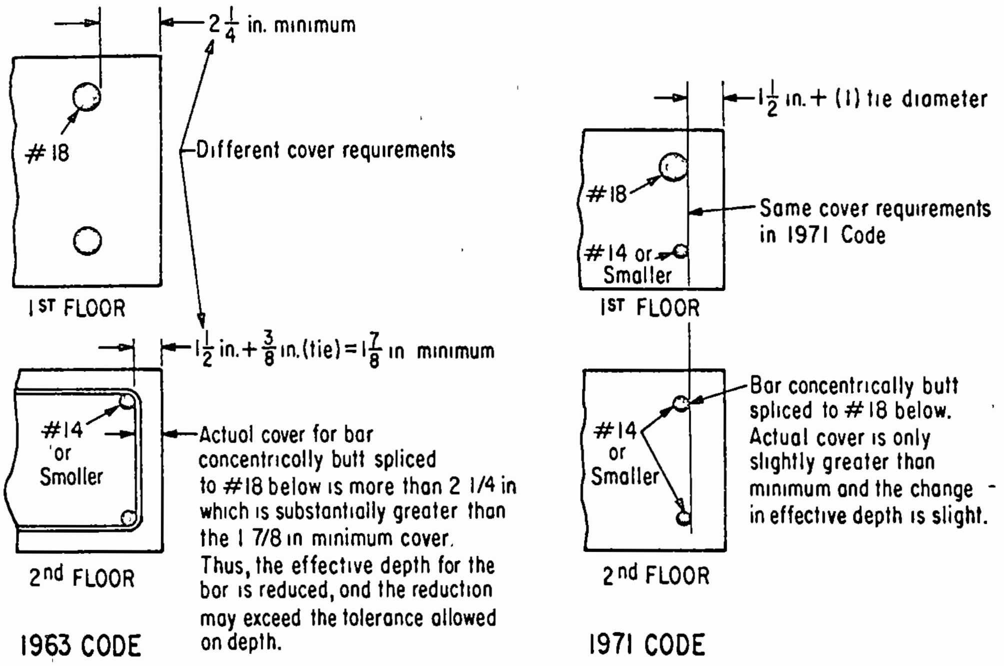


Fig. 7-2—Cover requirements of 1963 and 1971 Codes compared. Note simpler bar arrangement permitted by 1971 Code

7.14—Concrete protection for reinforcement

Concrete cover as protection of reinforcement against weather and other effects is measured from the concrete surface to the outer-most surface of the steel to which the cover requirement applies. Where minimum cover is prescribed for a class of structural member, it is measured to the outer edge of stirrups, ties, or spirals if transverse reinforcement encloses main bars; to the outermost layer of bars if more than one layer is used without stirrups or ties; to the metal end fitting or duct on post-tensioned prestressing steel.

7.14.1 — Cover requirements for cast-in-place, precast, and prestressed concrete, which were in separate chapters of the 1963 Code, are now combined in this section. The lesser thicknesses for precast construction reflect the greater convenience of control for porportioning, placing, and curing inherent in precasting.

For #18 bars in columns, the previous requirement of one-bar-diameter cover was deleted and these bars now have the same cover requirements as #14 or smaller size bars. As column bars now have identical cover requirements, smaller size column vertical bars can be centered on a #18 bar for a full bearing butt splice without an excessive reduction in the effective depth of the section (see Fig. 7-2). With the 1963 Code cover requirements, the reduction in effective depth often exceeded the tolerance permitted.

The phrase "concrete surfaces exposed to the weather" refers to direct exposure to temperature and moisture changes. Slab or thin shell soffits are not usually considered directly "exposed" unless subject to alternate wetting and drying, including that due to condensation conditions or direct leakage from exposed top surface, run off, or similar effects.

7.1.4, 7.14.3, 7.14.4 — These sections generally repeat requirements that were contained in the 1963 Code.

References

7.1. Kemp, E. L.; Brezny, F. S.; and Unterspan, J. A., "Effect of Rust and Scale on the Bond Characteristics of Deformed Reinforcing Bars," ACI Journal, *Proceedings* V. 65, No. 9, Sept. 1968, pp. 743-756.

7.2. Erickson, E. L., and Paulet, E. G., "Strength and Serviceability Criteria—Reinforced Concrete Bridges—Ultimate Design," U. S. Bureau of Public Roads, Aug. 1966, 81 pp. (Superintendent of Documents, U. S. Government Printing Office, Washington, D. C. 20402)

7.3 Pfister, James F., and Mattock, Alan H., "High Strength Bars as Concrete Reinforcement. Part 5: Lapped Splices in Concentrically Loaded Columns," *Journal*, PCA Research and Development Laboratories, V. 5, No. 2, May 1963, pp. 27-40.

7.4 ACI Committee 408, "Bond Stress—The State of the Art," ACI Journal, *Proceedings* V. 63, No. 11, Nov. 1966, pp. 1161-1188. (Also in *ACI Manual of Concrete Practice.*)

7.5. Atlas, A.; Bianchini, A. C.; Yasin, K.; and Kesler, C. E., "Studies of Welded Wire Fabric for Reinforced Concrete," *T&AM Report* No. 624, University of Illinois, 1962, 76 pp.

7.6. Hribar, John A., and Vasko, Raymond C., "End Anchorage of High Strength Steel Reinforcing Bars," ACI Journal, *Proceedings* V. 66, No. 11, Nov. 1969, pp. 875-883.

7.7. Committee on Continuously Reinforced Concrete Pavement, "Test Investigation of Splices in Continuously

Reinforced Concrete Pavement," *Bulletin* No. 3, Concrete Reinforcing Steel Institute, Chicago, May 1963, 26 pp.

7.8 Sanders, Ned M., "Splice Length in Reinforced Concrete," Thesis submitted to the Graduate School, University of Colorado, 1963.

7.9. Skillen, Marvin L., "Strength of Lapped Splices in Wide Beams," Thesis submitted to the Graduate School, University of Colorado, 1963.

7.10. Ferguson, Phil M., and Matloob, Farid N., "Effect of Bar Cut-off on Bond and Shear Strength of Reinforced Concrete Beams," ACI JOURNAL, *Proceedings*. V 56, No. 1, July 1959, pp. 5-24.

7.11. Ferguson, Phil M., and Thompson, J. Neils, "Development Length of High Strength Reinforcing Bars in Bond," ACI JOURNAL, *Proceedings* V. 59, No. 7, July 1962, pp. 887-992.

7.12. Hognestad, Eivind, and Rostasy, Ferdinand S., "Pilot Bond Tests of Large Reinforcing Bars," ACI JOURNAL, *Proceedings* V. 57, No. 5, Nov. 1960, pp. 576-579.

7.13 Watstein, David, and Mathey, Robert G., "Investigation of Bond in Beam and Pull-Out Specimens with High Yield Strength Deformed Bars," ACI JOURNAL, *Proceedings* V. 57, No. 9, Mar. 1961, pp. 1071-1090.

7.14. Bresler, B., and Gilbert, P. H., "Tie Requirements for Reinforced Concrete Columns," ACI JOURNAL, *Proceedings* V. 58, No. 5, Nov. 1961, pp. 555-570. Also, Discussion, *Proceedings* V. 58, No. 6, June 1962, Part 2, pp. 897-907.

7.15 Diaz de Cossio, Roger, and Rosenblueth, Emil, "Reinforced Concrete Failures During Earthquakes," ACI JOURNAL, *Proceedings* V. 58, No. 5, Nov. 1961, pp. 571-590.

7.16. *Concrete Manual*, U. S. Bureau of Reclamation, 7th Edition, 1963, 642 pp.

7.17. Hanson, N. W., and Reiffenstuhl, Hans, "Concrete Beams and Columns with Bundled Reinforcement," *Transactions*, ASCE, V. 125, 1960, pp. 889-904. Also, *Development Department Bulletin* D25, Portland Cement Association.

7.18. Hadley, Homer M., Discussion of "Concrete Beams and Columns with Bundled Reinforcement" by N. W. Hanson and Hans Reiffenstuhl, *Transactions*, ASCE, V. 125, 1960, pp. 905-909.

7.19. *Reinforced Concrete—A Manual of Standard Practice*, Concrete Reinforcing Steel Institute, Chicago, 15th Edition, 1963, 80 pp.

7.20. "Proceedings of Thirty-Seventh Annual Meeting," Concrete Reinforcing Steel Institute, Chicago, 1961, 77 pp.

7.21. Hanson, Norman W., and Conner, Harold W., "Seismic Resistance of Reinforced Concrete Beam-Column Joints," *Proceedings*, ASCE, V. 93, ST5, Oct. 1967, pp. 533-560.

7.22. Lloyd, John P., and Kesler, C. E., "Behavior of One-Way Slabs Reinforced with Deformed Wire and Deformed Wire Fabric," *T&AM Report* No. 323, University of Illinois, 1969, 129 pp.

CHAPTER 8—ANALYSIS AND DESIGN—GENERAL CONSIDERATIONS

8.1—Design methods

8.1.1 — The general design provisions of the Code are referred to as strength design and are similar to the ultimate strength design method contained in the 1963 Code. Some modifications have been made and a number of additional topics are covered to reflect up-to-date knowledge gained from research and experience. The strength design method requires service loads to be increased by specified load factors and computed theoretical strengths to be reduced by specified ϕ factors, given in Chapter 9.

8.1.2 — An alternate method of design employing load factors and ϕ factors equal to unity is, however, permitted for nonprestressed members. The method is outlined in Section 8.10 and is similar to the working stress design method of previous Codes. For members subjected to flexure without axial load the method is identical to that given in the 1963 Code. Differences in procedure occur in all other cases, including design of columns, design for shear, anchorage length, and splices.

Although prestressed members may not be designed for strength under the provisions of the alternate design method of Section 8.10, Chapter 18 permits linear stress-strain assumptions for computing service load stresses and transfer stresses for use in serviceability control.

8.1.3 — The general serviceability requirements of the Code, such as the requirements for deflection control and crack control, must be met regardless of whether the general design method of the Code or the alternate design method of Section 8.10 is used in proportioning for strength.

8.2—Required loading

8.2.1 — The provisions in the Code are suitable for live, wind, and earthquake loads such as those recommended in "Building Code Requirements for Minimum Design Loads in Building and Other Structures," ANSI A-58.1, of the American National Standards Institute (ANSI). If the loads specified by the general building code (of which ACI 318-71 forms a part) differ from those of ANSI, the general building code governs. However, if the nature of the loads contained in the local code differ extremely from ANSI loads, some provisions of this Code may need modification to reflect the difference.

8.2.2 — This section points out that the integral structural parts shall be designed to resist the total assumed lateral load. For example, a reinforced concrete shear wall is an integral structural part. Some partitions, because of their method of construction or because of the likelihood of their being moved, are not integral structural parts in the sense of this section.

8.2.3 — This section reads as in previous Codes, but information is accumulating on the magnitudes of these various effects and on procedures for dealing with them. Current[8.6] papers in the ACI JOURNAL relate to computation of creep and shrinkage effects.

8.3—Modulus of elasticity

8.3.1 — Studies by Pauw[8.8] indicate that the modulus of elasticity of concrete weighing between 90 and 155 lb per cu ft can be represented with acceptable accuracy by the general formula stated. It is assumed that the aggregates used produce structural concrete.

Tests of members made with normal weight aggregates (145 pcf) and lightweight aggregates (90 to 115 pcf), having the same cylinder strengths show that the differences in moduli of elasticity of the concrete do not affect the ultimate strength of a member.

8.3.2 — The value $E_s = 29 \times 10^6$ psi for nonprestressing steel represents a realistic average value obtained from many tests. For prestressing steel, the manufacturer's literature should be consulted to obtain E_s.

8.4—Frame analysis and design—General

8.4.1 — Design load is factored load which means that the factored live load such as $1.7L$ is used in Section 8.5.1.2 for adjacent span loading and for alternate span loading. When the alternate design method of Section 8.10 is used design loads contain load factors of unity for both dead load and live load. In all cases elastic analysis is used to obtain moments, shears, reactions, etc.

8.4.2 — The suggested moment coefficients generally give reasonably conservative values for the stated conditions whether the flexural members are simply supported or are part of a frame. Because the load patterns that produce critical values for moments in columns of frames differ from those for maximum negative moments in beams, column moments must be evaluated separately. (See Section 8.5.4.)

8.5—Frame analysis and design—Details

8.5.1 — The Code permits the far ends of columns to be assumed as fixed for the purpose of analysis under dead and live loads. The assumption does not apply to wind load analysis. However, in analysis for wind load or similar lateral loads, simplified methods (such as the portal method) may be used to obtain the moments, shears, and reactions for structures that are symmetrical. For unsymmetrical or very tall structures, more rigorous methods should be used.

For gravity load analysis it can be assumed that all columns are fixed at the far ends, and the designer must investigate conditions of pattern loading to obtain the most severe cases. The Appendix of SP-3 (working stress design handbook) contains a simple method that applies regardless of span lengths. The method is equivalent to a frame analysis by moment distribution or slope-deflection method with the far ends of all columns fixed.

The two-cycle moment distribution method described in Reference 8.3 also provides a convenient means of determining moments and shears under these provisions.

The foregoing methods of analysis constitute a "first order" method since the effects of deflections and axial deformations are not included. Therefore, beam and column moments must be amplified for column slenderness in accord with Section 10.11.

8.5.2 — The beam moments obtained at the center lines of columns may be reduced to those at the face of supports for design purposes. Reference 8.3 provides an acceptable method of accomplishing this end.

8.5.3.1 Although any reasonable assumptions may be made with regard to stiffness, more accurate results are obtained in a second order analysis (see Commentary for Chapter 10) when the transformed cracked sections of beams are used rather than the gross sections in computing moments of inertia.

The provision in the 1963 Code which allows the torsional stiffness to be neglected in frame analysis under certain conditions has been removed because of misinterpretations, and the Code is now silent on this matter. It has been customary to neglect torsional stiffness in frame analysis when torsional stiffness is small compared to flexural stiffness, and, except for the provisions of Chapter 13, this practice is allowed by the Code. Because of the approximations used in design, the neglect of this small factor will have no important effect on the flexural moments. In members subjected to significant torsional stresses, torsional stiffness should be considered in determining torsional moments.

8.5.3.2 Stiffness coefficients for haunched members can be obtained from References 8.4 and 8.5.

8.5.4 — Columns subjected to significant biaxial bending should be designed for simultaneous moments about both rectangular axes. Mimimum eccentricities need be used only about one axis at a time.

8.6—Redistribution of negative moments in continuous nonprestressed flexural members

Section 1502(d) of the 1963 Code permitted a 10 percent adjustment of negative moments at supports of flexural members. Use of moment redistribution provides for savings in reinforcement quantities. Experience with the use of this provision has been satisfactory. The Code now increases the possible percentage of moment redistribution with appropriate limitations on reinforcement ratio to ensure ductility and limitations on crack widths. This liberalization was justified by additional knowledge of ultimate and service load behavior obtained from tests and analytical studies.

Moment redistribution is dependent on adequate ductility in plastic hinge regions. These plastic hinge regions develop at points of maximum moment and cause a shift in the elastic moment diagram. The result is a reduction in the values of negative moments in the plastic hinge region and an increase in the values of positive moments from those computed by elastic analysis. Since negative moments are determined for one loading arrangement and positive moments for another, each section has a reserve capacity that is not fully utilized for any one loading condition. The plastic hinges permit the utilization of the full capacity of more cross sections of a flexural member at ultimate loads.

Using conservative values of ultimate concrete strains and lengths of plastic hinges derived from extensive tests, flexural members with small rotation capacity were analyzed for moment redistribution varying from 10 to 20 percent, depending on the reinforcement ratio. The results were found to be conservative (see Fig. 8-1).

Studies by Cohn[8.1] and Mattock[8.2] support this conclusion and indicate that cracking and deflection of beams designed for moment redistribution are not more severe than they are for beams designed by the elastic theory distribution of moments. Also, these studies indicated that adequate rotation capacity for the moment redistribution allowed by the Code is available if the members satisfy the Code requirements.

Moment redistribution does not apply to members designed under Section 8.10.

8.7—Requirements for T-beams

This section contains provisions identical with those of previous Codes as concerns dimensions related to stiffness and flexural calculations. Special provisions related to T-beams and other flanged members are stated in Section 11.7.2 with regard to torsion.

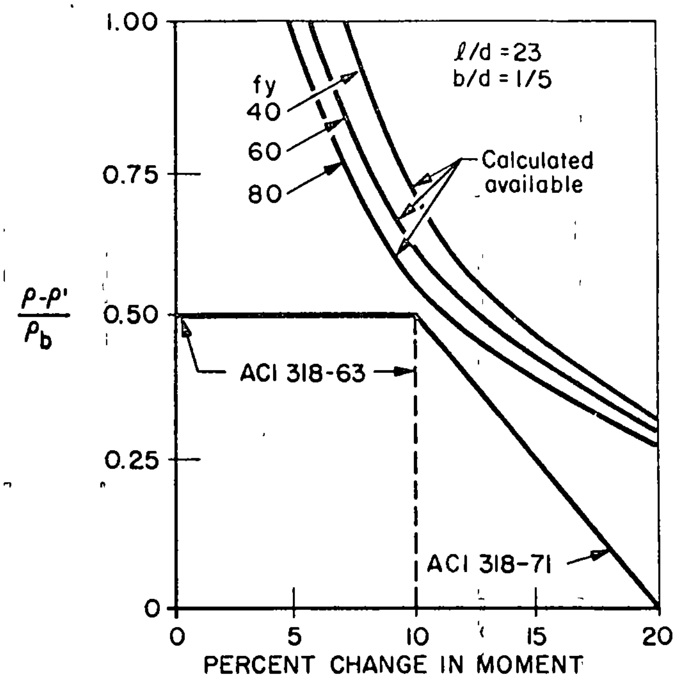


Fig. 8-1—Allowable moment redistribution for minimum rotation capacity

8.8—Concrete joist floor construction

8.8.2 — A minor change has been made in comparison with the depth provisions of the 1963 Code. The maximum depth of joists is now based on the minimum width of the joist. A limit on the maximum spacing of joists is justified by the special provisions permitting higher shear stresses and less concrete protection for the reinforcement for these relatively small, secondary members. The limits are empirical and are based on successful performance in the past. The 30 in. maximum spacing of joists follows the standards of the joist industry as given in "Types and Sizes of Forms for One-Way Concrete Joist Construction," *NBS Voluntary Product Standard* No. PS 16-69, and "Forms for Two-Way Concrete Joist Floor and Roof Construction," *Simplified Practice Recommendation* No. R265-63, U.S. Department of Commerce.

8.8.8 — As in the 1963 Code, a 10 percent greater concrete shear stress is permitted for joists in comparison with other members. The increase is justified on the basis of satisfactory performance of joist construction with higher shear stresses as were designed under previous ACI Codes, which allowed shear stresses comparable to these increased values.

8.9—Separate floor finish

This section is similar to Section 907(b) of the 1963 Code except that the 1971 Code does not specify an additional thickness for wearing surfaces subjected to unusual conditions of wear. Whether or not the separate finish is structural, the need for added thickness for unusual wear is left to the discretion of the designer.

As in previous editions of the Code a floor finish may only be considered for strength purposes if it is cast monolithically with the slab, but permission is now given to include a separate finish in the structural thickness if composite action is insured in accordance with Chapter 17.

All floor finishes may be considered for nonstructural purposes such as cover, fireproofing, etc. Provisions should be made, however, to insure that the finish will not spall off, thus causing decreased cover.

8.10—Alternate design method

The design method permitted by Section 8.10 allows the use of load factors and ϕ factors equal to one and is similar to the working stress design method contained in the 1963 Code. The procedure for flexure is much the same as 1963 as explained in Section 8.10.1, while for other structural effects the capacity is restricted to a percentage of the design capacity given in various Code chapters. In the 1963 Code, the ultimate strength design equations were in many cases divided by safety factors and the resulting equations restated in the working stress design sections of that Code. The 1971 Code does not contain separate sets of equations for working stress design but in the Alternate Design Method simply gives percentages for modifying the strength design capacities given in other parts of the Code.

In view of the simplifications permitted, the Alternate Design Method of Section 8.10 is intended to give results slightly more conservative than the designs obtained using the basic design method of the Code. It allows unity load and capacity reduction factors for both design and analysis, which may result in relative magnitudes for moments, shears, and axial loads which differ from those of the general method. Also, the working stress design provisions of the 1963 Code allowed by this method have not been updated as thoroughly as the basic strength design method provisions of the Code.

The Alternate Design Method does not apply to prestressed concrete design, except for investigation under service load in accordance with Section 8.10.1.

In general, the resulting flexural designs will be similar to those obtained by working stress design under the 1963 Code. Designs for bearing, anchorage length (bond), and axial load differ from results obtained by the 1963 Code and the magnitude of difference will depend on the provisions of the Code chapters covering these structural effects.

8.10.1 — This section applies only to members subjected to flexure without axial load. The procedure used is the well-known straight-line theory with flexural compressive stresses in the concrete limited to $0.45f_c'$. Tensile stresses in the steel are limited to 20,000 psi for Grades 40 and 50 steel and to 24,000 psi for Grade 60 and higher strength steel. One exception of long standing exists in the case of one-way slabs having clear span lengths 12 ft or less that are reinforced with #3 deformed bars or welded wire fabric having a diameter not exceeding ⅜ in. In this case, the allowable tensile stress is the lesser of $0.5f_y$ or 30,000 psi.

In transforming compression steel to equivalent concrete for flexural design, $2E_s/E_c$ must be used in locating the neutral axis and calculating moments of inertia. The lesser of twice the calculated stress in the compression steel or the allowable stress is then used to calculate the contribution of the compression steel in computing the resisting moment at service loads.

This procedure may be used for all sectional shapes with or without compression reinforcement when axial load is not present. Since small axial compression loads tend to increase the moment capacity of a section, such axial loads may be disregarded in most cases concerning beams. When doubt exists as to whether or not the axial compression may be disregarded, the member should be investigated using Section 8.10.2.

Deep beams must be designed in accordance with Section 10.7.

8.10.2 — This new procedure for designing members subjected to axial load with or without flexure provides more consistent safety factors than the working stress design method contained in the 1963 Code. Design aids, based on the 1963 Code, for columns by the working stress design method do not satisfy the 1971 Code.

The slenderness effects provided in Section 10.10 apply here, with $2.5P$ substituted for P_u when dead load plus live load governs. In Eq. (10-5) for long column considerations, ϕ is taken as 1.0 when this section is used for design.

8.10.4 — In computing development lengths the provisions of Chapter 12 govern. Similarly, splice lengths under Chapter 7 are multiples of development length and apply here also. Where M_t and V_u are referenced in Chapter 12, equivalent values are obtained by multiplying the service load resisting moment capacity and the service load shear force by 2.0 when gravity loads govern. The quantity $(d - a/2)$ may be taken as $0.85d$.

8.10.5 — When lateral loads such as wind or earthquake combined with live and dead load govern the design, members may be proportioned for 75 percent of capacities required in Section

8.10. This is similar to the provisions of previous Codes which in working stress design allowed a one-third increase in stresses with these combinations of loads.

When appropriate, due regard must be given to earth, liquid, or other loads in earthquake or wind analyses.

8.10.6 — This section calls attention to the fact that all other provisions of the Code, except those allowing moment redistribution, apply to the alternate method of design. This includes control of deflections and control of cracking, as well as all of the provisions related to slender columns in Chapter 10. Note that the Commentary for Chapter 10 presents an alternative "Modified R-Factor Method" for slender columns. That procedure may also be used with the alternate method of design in lieu of the moment magnification method of Section 10.10, if desired.

References

8.1. Cohn, M. Z., "Rotational Compatibility in the Limit Design of Reinforced Concrete Continuous Beams," *Flexural Mechanics of Reinforced Concrete*, SP-12, American Concrete Institute/American Society of Civil Engineers, Detroit, 1965, pp. 143-180. Also, *Development Department Bulletin* D101, Portland Cement Association.

8.2. Mattock, A. H., "Redistribution of Design Bending Moments in Reinforced Concrete Continuous Beams," *Proceedings*, Institution of Civil Engineers (London), V. 13, 1959, pp. 35-46.

8.3. "Continuity in Concrete Building Frames," Portland Cement Association, Skokie, 4th Edition, 1959, 56 pp.

8.4. "Handbook of Frame Constants," Portland Cement Association, Skokie, 32 pp.

8.5 "Continuous Concrete Bridges," Portland Cement Association, Skokie, 106 pp.

8.6. Fintel, M., "Effects of Column Creep and Shrinkage in Tall Structures—Prediction of Inelastic Column Shortening," ACI JOURNAL, *Proceedings* V. 66, No. 12, Dec. 1969, pp. 957-967.

CHAPTER 9—STRENGTH AND SERVICEABILITY REQUIREMENTS

9.1—General

9.1.1 — The provisions of this chapter correspond closely with the fundamentals related to ultimate strength design of the 1963 Code. Refinements have developed in certain areas because of subsequent experience and research.

The Code requires that the structural strength be adequate to support the anticipated factored loads and that serviceability of the structure at service load level be assured.

As in the 1963 Code, the margin of structural safely is provided in two ways. First, the applied loads are multiplied by load factors to provide for excess load effects from such possible sources as overloads and simplified assumptions in structural analysis.

A greater factor is applied to live load than to dead load, since dead load can be determined with reasonable accuracy whereas live load is often more uncertain and subject to change during the life of the structure. The overall (average) load factor must be large enough to make failures very unlikely, but overall factors must not be unreasonably high. The prescribed factors were devised with this criterion in mind.

The load factors given are the minimum recommended by the Committee. Some increase may be appropriate if the anticipated loads or the simplifying assumptions are materially different from those encountered in the usual design situations. See footnote to Section 8.2 of the Code.

Second, the theoretical capacity of the structural element is reduced by a capacity reduction factor ϕ. The coefficient provides for the possibility that small adverse variations in material strengths, workmanship, and dimensions, while individually within acceptable tolerances and limits of good practice, may combine to result in undercapacity.

In assigning capacity reduction factors for various members, the degree of ductility and the importance of the member are considered as well as the degree of accuracy with which the strength of the member can be evaluated. Columns have lower ϕ factors than beams, since the failure of a column can be sudden and catastrophic while a beam failure normally is preceded by increased deflections and cracking.

Further, a beam may not support as large a loaded area as a column. Because spirally reinforced columns are usually more ductile than tied columns, the ϕ factor for the former is greater than for the latter.

If special circumstances require greater reliance on the strength of particular members than encountered in usual practice, some reduction in the prescribed capacity reduction factors or increase in the prescribed load factors may be appropriate for those members.

With the development of high strength materials and more sophisticated methods of design that provide depths of sections somewhat less than those used in the past, it becomes increasingly important to give attention to control of deflections at service loads.

9.2—Strength

9.2.1 — The reasons for different ϕ factors for various types of members were discussed in connection with Section 9.1. The degree of ductility is an important consideration in selecting capacity reduction factors, as is the importance of the member.

9.2.1.2 For axial loads smaller than that which produces a balanced condition, failure is initiated by yield of the tensile steel and takes place in an increasingly more ductile manner as the ratio of axial-load to moment decreases. Finally, when the axial load vanishes, the member becomes the same as one governed by Section 9.2.1.1 ($\phi = 0.90$). Hence, for small axial loads it is reasonable to permit an increase in the ϕ factor from that required by Sections 9.2.1.2 (a) or 9.2.1.2 (b).

For rectangular sections with steel along the end faces, balanced conditions are simple to compute (see Section 10.3.3). For other shapes or reinforcement configurations, ACI Committee 340 and the Portland Cement Association, as a convenience, have taken the balanced axial load as the smallest axial load for which the outermost tension bar yields.

After studying design charts and tables, ACI Committee 318 selected a value for the axial load P_u below which the ϕ factor could safely be increased for most compression members. The value selected was $0.10f_c'A_g$, which may be used for sections with symmetrical reinforcement provided f_y does not exceed 60,000 psi or the distance γh, between A_s and A_s', is not less than $0.7h$. (In practice and in design aids, the definition $(h - d' - d_s)/h$ has been used as "γ".) Where these conditions are not satisfied, P_b must be calculated and Section 9.2.1.2 (d) used.

Fig. 9-1 illustrates the variation in ϕ for symmetrically reinforced compression members with

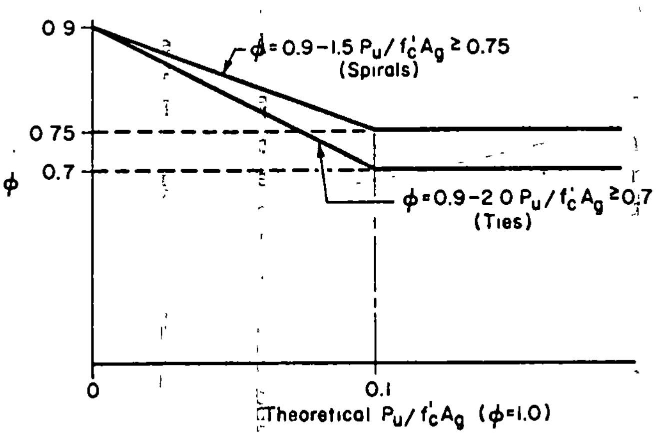


Fig. 9-1—Variation of ϕ for symmetrically reinforced compression members, $g \geq 0.7$, $f_y \leq 60{,}000$ psi

lateral reinforcement meeting Code requirements and with γ and f_y limited as previously mentioned. The related equations are shown thereon.

Fig. 9-1 may also be used for any member when the theoretical value of balanced axial load P_b is greater than $0.10f_c'A_g$.

When there is a compressive load on the member but P_b falls in the tension zone, as may be the case with high steel ratios, low values of γ and high steel yield strengths, no increase in ϕ is permitted. Hence, for this case, $\phi = 0.75$ for spirally reinforced, or $\phi = 0.70$ for tied compression members, must be used for the entire interaction relationship between M_u and P_u in the range where P_u is a compressive force.

In all cases in which axial tension load occurs, with or without bending, $\phi = 0.9$ is allowed to be used.

9.2.1.4 The ϕ factor for bearing on concrete in this section does not apply to post-tensioning anchorage bearing plates (see Commentary on Section 18.11.3).

9.2.2—Development lengths for reinforcement (which correspond in principle to the bond provisions of ACI 318-63) are provided for in Chapter 12, and do not require a ϕ factor. The capacity reduction factor of 0.85 was a consideration already introduced in deriving the equations for anchorage length. Likewise, ϕ factors are not required for splices (Chapter 7) since splice lengths are expressed in multiples of the development lengths given in Chapter 12.

9.3—Required strength

9.3.1 — The provisions of Section 9.3 provide for such sources of possible excess load effects as variations in load, assumptions in structural analysis and simplifications in calculations. Consideration is given as well to combinations of load effects.

Because of the more comprehensive 1971 Code provisions, additional research and experience, and improved concrete and steel quality control, load factors have been decreased from 1.5 to 1.4 for dead load and from 1.8 to 1.7 for live load, providing an average reduction of approximately 6 percent from the 1963 Code values.

The basic load factors are given in Eq. (9-1), (9-2), and (9-3) in the Code.

According to Section 9.3.3, when the effects of earthquake must be considered, $1.1E$ is to be substituted for W in which case Eq. (9-2) and (9-3) become:

$$U = 0.75\ (1.4D + 1.7L + 1.87E)$$

and

$$U = 0.9D + 1.43E$$

When lateral loads (H) due to earth (or other granular) pressure are present, the governing equations become:

$$U = 1.4D + 1.7L + 1.7H$$

$$U = 0.9D + 1.7H$$

and

$$U = 1.4D + 1.7L$$

When lateral loads F due to liquids are present, the governing equations are:

$$U = 1.4D + 1.7L + 1.4F$$

$$U = 0.9D + 1.4F$$

and

$$U = 1.4D + 1.7L$$

It should be noted that in applying Eq. (9-1), (9-2), (9-3) and other load factor provisions of Section 9.3, due regard should be paid to sign, since the standard effects of D, L, W, E, H, and F may, on occasion, be of opposing sense, thus producing tensile axial loads, negative reactions, or reverse bending.

The designer should consider the effect of differential settlement, creep,* shrinkage, and temperature where necessary. Denoting the worst effects of axial force, shear, moment, and torsion due to the combination of these conditions as X, it is then necessary to investigate the design using the equation:

$$U = 0.75\,(1.4D + 1.7L + 1.4X)$$

When impact is present, as may be the case for parking buildings, loading docks, warehouse floors, elevator shafts, etc., impact effects should be considered and impact loads, if any, included with live loads in the various equations for required strength.

9.4—Design strengths for reinforcement

An upper limit of 80,000 psi is placed on reinforcing yield strength (other than prestressing tendons) in Section 9.4.2. Committee 318 did not choose to recommend any strength above 80,000 psi without adding other restrictions since this steel strength is about equal to the ultimate strain in concrete multiplied by the modulus of elasticity of steel. At present the highest yield strength covered by ASTM standards is 75,000 psi, and this grade is not widely used.

9.5—Control of deflections

9.5.1 *General* — This section is concerned only with the deflections or deformations which may occur at service load levels. Where long-time deflections are computed, only the dead load and that portion of the live load which is sustained need be considered.

Two methods are given for controlling deflections. For nonprestressed beams and one-way slabs, and for composite members, provision of a minimum overall thickness as required by Table 9.5(a) will satisfy the requirements of the Code for members *not* supporting or attached to partitions or other construction likely to be damaged by large deflections. For nonprestressed two-way construction, minimum thicknesses as required by Sections 9.5.3.1, 9.5.3.2, and 9.5.3.3 will satisfy the requirements of the Code.

For nonprestressed members which do not meet these minimum thickness requirements or which support or are attached to partitions or other construction likely to be damaged by large deflections, and for all prestressed concrete flexural members, deflections must be calculated by the procedures described or referred to in the appropriate sections of the Code and are limited to the values in Table 9.5 (b).

9.5.2 *Nonprestressed one-way construction*

9.5.2.2 The calculation of immediate deflections for uncracked prismatic beams is relatively simple; the usual methods or formulas for elastic deflections may be used with a constant value of E_cI_g along the length of the beam. However, if the beam is cracked at one or more sections or if its depth varies along the span, exact calculation becomes much more complicated. The I_e procedure described in this Code section and developed in Reference 9.8 was selected as being relatively simple and sufficiently accurate for use with the limiting values in Table 9.5 (b) to control deflections.[9.1, 9.6, 9.9]

It is noted that for additional load increments, such as live load, I_c must be computed for the total moment, and the deflection increment computed from the total deflection, as indicated by Eq. (29) to (32) in References 9.9 and 9.10. For simplicity in the case of continuous beams, the Code procedure suggests a simple averaging of positive and negative moment values for I_e. In certain cases, a weighted average relative to the moments may be preferable, such as the methods suggested in Reference 9.10.

For normal weight concrete, the value of f_r required for the calculation of the cracking moment is given as $7.5\sqrt{f_c'}$. Modifying factors based on the splitting tensile strength f_{ct} are given for "all-lightweight" and "sand-lightweight" concretes. For a lightweight aggregate from a given source, it is intended that appropriate values of f_{ct} should be obtained in advance of design, but tests for f_{ct} are not required for subsequent acceptance of concrete during construction. Indirect control will

*For Creep considerations, see *Symposium on Creep of Concrete*, SP-9, American Concrete Institute, Detroit, 1964, 160 pp. See also Reference 9.9.

be maintained through the normal compressive strength test requirements.

9.5.2.3 Shrinkage and creep due to sustained loads cause additional deflections over and above those which occur when loads are first placed on the structure. The additional deflections are called "long-time deflections." Such deflections are influenced by temperature, humidity, curing conditions, age at time of loading, quantity of compression reinforcement, magnitude of the sustained load, and other factors. Although no simple procedure can take into account all of these factors, the expression given in this section is considered satisfactory for use with the procedures of Section 9.5.2.2, for the calculation of immediate deflections, and with the limits given in Table 9.5(b).[9.2] More comprehensive analysis based on authoritative information such as the reports of ACI Committee 435,[9.1,9.4] and ACI Committee 209[9.9] may, however, be used.

It should be noted that the deflection computed in accordance with this section is the *additional* long-time deflection due to the dead load and that portion of the live load which will be sustained for a sufficient period to cause significant time-dependent deflections. Where this time is less than about 2 years, the curves[9.2] of Fig. 9-2 may be of use in estimating the *additional* long-time deflection.

9.5.3 *Nonprestressed two-way construction*

9.5.3.1, 9.5.3.2, and 9.5.3.3 Deflections of two-way systems of construction of the types considered in Chapter 13 need not be computed if the minimum overall thickness requirements of these sections are satisfied. Eq. (9-6), (9-7), and (9-8) yield thicknesses consistent with those found from experience to provide satisfactory control of deflections for flat slabs, flat plates, and conventional two-way slabs supported on stiff beams. The equations provide for a transition from slabs on stiff beams to slabs

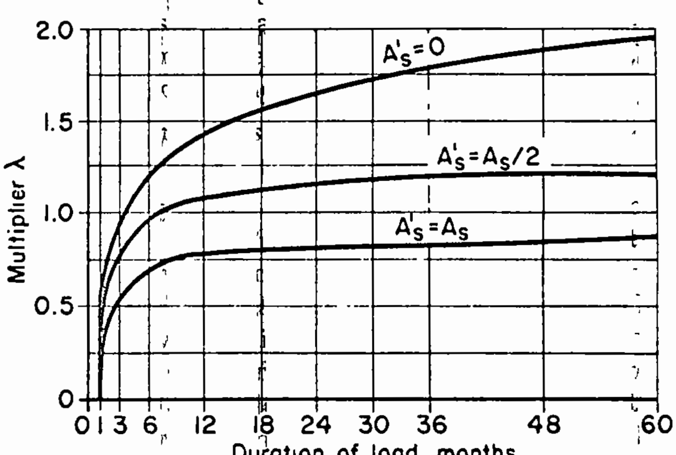


Fig. 9-2—Multipliers for long-term deflections

without beams and involve also a term to adjust the thickness as a function of the design yield strength of the reinforcement. The effect of yield strength in these equations is different than in Footnote (b) to Table 9.5(a) because the degree of cracking has been observed to be less in two-way slabs than in beams and one-way slabs, with a consequent smaller effect of steel stress or strain on the stiffness of the element. This conclusion was reached and the form of the expression involving yield strength in Eq. (9-6), (9-7), and (9-8) was chosen after study of the results of the extensive tests on floor slabs described in the references for Chapter 13 of this Commentary.

9.5.3.4 The calculation of deflections for slabs is complicated even if linear elastic behavior can be assumed. For immediate deflections, the values of E_c and I_e specified in Section 9.5.2.2 may be used.[9.9] However, other procedures and other values of the stiffness, *EI*, may be used if they result in predictions of deflection in reasonable agreement with the results of comprehensive tests. Such a procedure, for example, is described in Reference 9.3.

Since the available data on long-time deflections of slabs are too limited to justify more elaborate procedures, this section requires the additional long-time deflection to be computed using the multiplier given in Section 9.5.2.3.

9.5.4 *Prestressed concrete*— The deflection of any prestressed concrete flexural member must be computed and compared with the allowable values in Table 9.5(b).

9.5.4.1 Immediate deflections of prestressed concrete members may be calculated by the usual methods or formulas for elastic deflections using the moment of inertia of the gross (uncracked) concrete section and the modulus of elasticity for concrete specified in Section 8.3.1. Since this method assumes that the concrete is uncracked, it may be unconservative for members having a relatively large tension stress in the concrete as permitted by Section 18.4.2.3. Hence, Section 18.4.2.3 requires calculation of deflection based on the cracked section.

It has also been shown in Reference 9.1 that the I_e method can be used to compute deflections of partially prestressed members loaded above the cracking load. In this case, the cracking moment must, of course, take into account the effect of prestress. A method for predicting the effect of nonprestressed tension steel in reducing creep camber is also given in Reference 9.1, with approximate forms referred to in References 9.9 and 9.10.

9.5.4.2 The calculation of long-time deflection of prestressed concrete flexural members is complicated. Any suitable method, such as that described in References 9.4, 9.9, and 9.12 may be used.

9.5.5 *Composite members* — Since few tests have been made to study the immediate and long-time deflections of composite members, the rules given in Sections 9.5.5.1 and 9.5.5.2 are based on the judgment of the committee and on experience.

If any portion of a composite member is prestressed or if the member is prestressed after the components have been cast, the provisions of Section 9.5.4 apply and deflections shall always be calculated. For nonprestressed members, deflections need be calculated and compared with the limiting values in Table 9.5(b) only when the thickness of the member or the precast part of the member is less than the minimum thickness given in Table 9.5(a). In unshored construction the thickness of concern depends on whether the deflection before or after the attainment of effective composite action is being considered.*

Table 9.5(a) This table, based on the use of reinforcement having yield strengths of 60,000 psi, is referred to only in Sections 9.5.2 and 9.5.5. It may be used in lieu of calculation of deflections only for the types of members covered by those sections and only if these members do *not* support and are *not* attached to partitions or other construction likely to be damaged by large deflections.

The notes beneath the table are essential to its use for reinforced concrete members constructed with structural lightweight concrete and/or with reinforcement having a yield strength not equal to 60,000 psi. If both of these conditions exist, the corrections in Footnotes (a) and (b) shall both be made.

The modification for lightweight concrete in Footnote (a) is based on studies of the results and discussions in Reference 9.5. No correction is given for concretes weighing between 120 and 145 lb per cu ft since the correction term would be close to unity in this range.

The modification for yield strength in Footnote (b) is based on judgment, experience, and studies of the results of tests and of unpublished analyses. The simple expression given is approximate but should yield conservative results for the types of members considered in the table, for typical reinforcement ratios, and for values of f_y between 40 and 80 ksi.

If the minimum thickness obtained using this table is considered excessive, the designer has the option of computing deflections in accordance with Sections 9.5.1 and 9.5.5.

Table 9.5 (b) This table is the result of an effort by Committee 318 to simplify the very extensive set of limitations which would be required to cover all possible types of construction and conditions of loading. (See, for example, Reference 9.6.) It should be noted that for members supporting or attached to other elements, the limitations given in this table relate only to supported or attached *nonstructural* elements. For those structures in which structural members are likely to be affected by deflection or deformation of members to which they are attached in such a manner as to affect adversely the strength of the structure, these deflections and the resulting forces should be considered explicitly in the analysis and design of the structures as required by Section 9.5.1.

Where long-time deflections are computed, the portion of the deflection before attachment of the nonstructural elements may be deducted. In making this correction use may be made of the curves in Fig. 9-2 for members of usual sizes and shapes.

References

9.1. ACI Committee 435, "Deflections of Reinforced Concrete Flexural Members," ACI JOURNAL, *Proceedings* V. 63, No. 6, June 1966, pp. 637-674. Also *ACI Manual of Concrete Practice,* Part 2, 1968.

9.2. Yu, W. W., and Winter, G., "Instantaneous and Long-Time Deflections of Reinforced Concrete Beams under Working Loads," ACI JOURNAL, *Proceedings* V. 57, No. 1, July 1960, pp. 29-50.

9.3. Vanderbilt, M. D.; Sozen, M. A.; and Siess, C. P., "Deflections of Multiple-Panel Reinforced Concrete Floor Slabs," *Proceedings,* ASCE, V. 91, ST4, Aug. 1965, pp. 77-101.

9.4. Subcommittee 5, ACI Committee 435, "Deflections of Prestressed Concrete Members," ACI JOURNAL, *Proceedings* V. 60, No. 12, Dec. 1963, pp. 1697-1728. Also *ACI Manual of Concrete Practice,* Part 2, 1968.

9.5. ACI Committee 213, "Guide for Structural Lightweight Aggregate Concrete," ACI JOURNAL, *Proceedings* V. 64, No. 8, Aug. 1967, pp. 433-464. Also, Discussion by Ralph N. McManus and Committee Closure, ACI JOURNAL, *Proceedings* V. 65, No. 2, Feb. 1968, pp. 151-155. Also *ACI Manual of Concrete Practice,* Part 1. 1970.

9.6. Subcommittee 1, ACI Committee 435, "Allowable Deflections," ACI JOURNAL, *Proceedings* V. 65, No. 6, June 1968, pp. 433-444.

9.7. Lin, T. Y., "Load Factors in Ultimate Design of Reinforced Concrete," ACI JOURNAL, *Proceedings* V. 48, No. 10, June 1952, pp. 881-900.

9.8. Branson, Dan E., "Instantaneous and Time-Dependent Deflections on Simple and Continuous Reinforced Concrete Beams," *HPR Report* No. 7, Part 1, Alabama Highway Department, Bureau of Public Roads, Aug. 1963 (1965), pp. 1-78.

* In Chapter 17, it is stated that distinction need not be made between shored and unshored members. This refers to strength calculations, not to deflections.

9.9. Subcommittee 2, ACI Committee 209, "Prediction of Creep, Shrinkage, and Temperature Effects in Concrete Structures," *Designing for the Effects of Creep, Shrinkage, and Temperature in Concrete Structures*, SP-27, American Concrete Institute, Detroit, 1971.

9.10. Branson, D. E., Discussion of "Proposed Revision of ACI 318-63: Building Code Requirements for Reinforced Concrete" by ACI Committee 318, ACI JOURNAL, *Proceedings* V. 67, No. 9, Sept. 1970, pp. 692-695.

9.11. Shaikh, A. F., and Branson, D. E., "Non-Tensioned Steel in Prestressed Concrete Beams," *Journal*, Prestressed Concrete Institute, V. 15, No. 1, Feb. 1970, pp. 14-36.

9.12. Branson, D. E.; Meyers, B. L.; and Kripanarayanan, K. M., "Time-Dependent Deformation of Noncomposite and Composite Prestressed Concrete Structures," Symposium on Concrete Deformation, *Highway Research Record* 324, Highway Research Board, 1970, pp. 15-43.

CHAPTER 10—FLEXURE AND AXIAL LOADS

10.1—Scope and 10.2—Assumptions

While Chapter 10 follows the broad lines of ACI 318-63, numerous minor changes have been made.

Detailed formulas for determination of ultimate loads and moments have been eliminated in view of their general availability in reference texts and design aids.

10.2.3 — The maximum concrete compressive strain at ultimate strength has been measured in many tests of both plain and reinforced concrete members. Though the maximum strain decreases somewhat with increasing compressive strength of the concrete, a value of 0.003 is reasonably conservative.

10.2.4 —This section assumes that below the design yield strength of the reinforcement, steel stress is proportional to strain, and that the effect of strain hardening of the steel beyond the yield point is neglected.

10.2.6 — This clause recognizes the inelastic behavior of concrete at high stress, that is, that the stress-strain relationship for concrete is not a straight line but some form of curve. The actual distribution of concrete compressive stress in any particular case is complex and usually not known. However, research has shown that the important properties of the concrete stress distribution can be approximated closely using any one of several different assumptions as to the form of stress distribution. This clause permits any particular stress distribution to be assumed in design if shown to result in predictions of ultimate strength in reasonable agreement with the results of comprehensive tests.

10.2.7 — The equivalent rectangular concrete stress distribution specified in this clause is an expedient which enables the designer to obtain in a simple manner the total concrete compressive force and its centroid. It does not purport to represent the actual distribution of stress in the concrete compression zone, but provides essentially the same results as those obtained in comprehensive tests.

10.3—General principles and requirements

Using the equivalent rectangular concrete stress distribution together with the other assumptions specified in this Code, equations for the design of members subject to flexure or combined flexure and axial load are derived in the paper, "Rectangular Concrete Stress Distribution in Ultimate Strength Design."[10.18] This paper also gives the derivations of strength equations for cross sections other than rectangular.

It is required that the ratio of the tension reinforcement in a flexural member not exceed 75 percent of the ratio of tension reinforcement which corresponds to balanced conditions, that is, to the simultaneous occurrence of yield of the tension reinforcement and crushing of the concrete at ultimate strength. In other words, $A_s f_y$ of the tension steel is limited to three-quarters of the total compressive force at balanced conditions, whether that compressive force is from a rectangular stress block of a rectangular member, or that plus overhanging flanges, or that plus compressive steel. This requirement applies to members of any cross-sectional shape with or without compression reinforcement.

The wording of Section 10.3.2 is intended to assure an interpretation of the requirements for flanged members and members with compressive reinforcement, which is consistent with that used for rectangular members with tension reinforcement only. In the past, a less conservative interpretation has been used for flanged members and members with compressive reinforcement.

The limitation on tensile reinforcement insures that flexural members designed by the strength procedures will have a ductile character. Among other reasons, this is desirable because long-term experiences with reinforced concrete in the field pertain to relatively ductile structures. Furthermore, there has been implicit reliance on the ductile behavior of structures to accommodate those factors which are not normally considered in design calculations, such as the differential settlement of foundations.

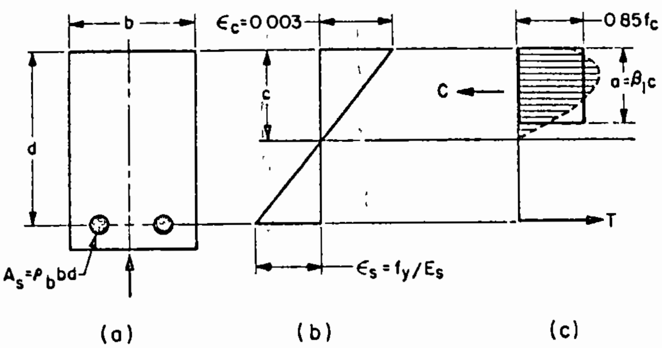
Ductile behavior must continue to be insured and, therefore, a limit on the net amount of tension reinforcement provided in flexural members has been set at $0.75\rho_b$. This limit is further reduced to $0.5\rho_b$ where negative moments are redistributed in design (see Section 8.6). These numerical limits remain the same as in the 1963 Code.

10.3.2, 10.3.3, and 10.3.4 — The reinforcement ratio ρ_b which produces balanced conditions under flexure, depends on the shape of the cross section and the location of the reinforcement.

Derivations for ρ_b in a rectangular member with tension reinforcement only, in a rectangular member with tension and compression reinforcement and in a flanged beam are given in Fig. 10-1a, 10-1b, and 10-1c. Balanced reinforcement ratios for these members are summarized in Table 10-1. Similar approaches can be made for other shapes symmetrical about the vertical axis. With sections unsymmetrical about a vertical axis, the neutral axis will be inclined and twist will develop unless the member is laterally braced against rotations.

10.3.6 — Minimum eccentricities are provided to account for accidental eccentricities due to imperfect positioning of members and reinforcement, nonuniformity of materials, and minor discrepancies between assumptions made in the analysis and actual behavior.

The minimum eccentricities of $0.05h$ for spirally reinforced members and $0.10h$ for tied members



For a rectangular beam

$$c = d\left(\frac{0.003}{0.003 + \frac{f_y}{E_s}}\right) = d\left(\frac{0.003}{0.003 + \frac{f_y}{29 \times 10^6}}\right) = d\left(\frac{87{,}000}{87{,}000 + f_y}\right)$$

$$T = \rho_b b d f_y = C = b\beta_1 c\, 0.85 f_c' = 0.85 f_c' b \beta_1 d \frac{87{,}000}{87{,}000 + f_y}$$

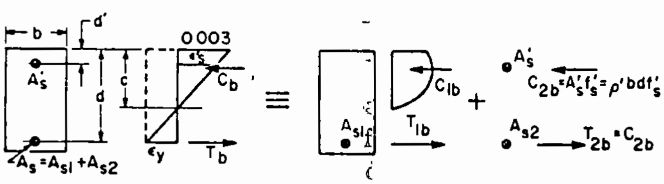
Where C = compressive force

T = tensile force

Solving

$$\rho_b = \frac{\beta_1 0.85 f_c'}{f_y} \cdot \frac{87{,}000}{87{,}000 + f_y}$$

Fig. 10-1a—Expression for ρ_b, rectangular beam without compression reinforcement



$$(T_b = \rho_b b d f_y) = (T_{1b} = \bar{\rho}_b b d f_y) + (T_{2b} = \rho' b d f_s')$$

$$\rho_b = \bar{\rho}_b + \rho' f_s' / f_y$$

For rectangular beam (Fig 10-1a)

$$c = d\left(\frac{87{,}000}{87{,}000 + f_y}\right) \text{ or } 1/c = \frac{1}{d}\left(\frac{87{,}000 + f_y}{87{,}000}\right)$$

From strain triangle

$$f_s' = E_s \epsilon_s' = E_s\left[\left(\frac{c - d'}{c}\right)0.003\right] = 87{,}000(1 - d'/c)$$

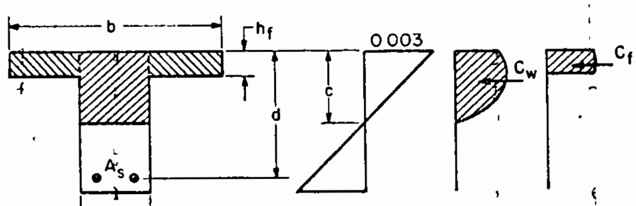
$$= 87{,}000\left(1 - \frac{d'}{d}\,\frac{87{,}000 + f_y}{87{,}000}\right) \leq f_y$$

$$= 87{,}000\left[1 - \frac{d'}{d}(1 + f_y/87{,}000)\right]$$

Where

$\bar{\rho}_b$ = balanced reinforcement ratio for a rectangular beam of width without compression reinforcement

Fig. 10-1b—Expression for ρ_b, rectangular beam with compression reinforcement



$C_w = b_w \beta_1 c\, 0.85 f_c'$ $\qquad$ $C_f = 0.85 f_c'(b - b_w)h_f$

$C_w = T_w = A_{sw} f_y$ $\qquad$ $C_f = T_f = A_{sf} f_y$

$T_w = \frac{\bar{\rho}_b}{b_w d} f_y$ $\qquad$ $T_f = \rho_f b_w d f_y$

$T_w = \rho_b b_w d f_y$ $\qquad$ $T = T_w + T_f = \rho_b b d f_y$

$$\rho_b b d f_y = \rho_b b_w d f_y + \rho_f b_w d f_y$$

$$\rho_b = \frac{b_w}{b}(\bar{\rho}_b + \rho_f)$$

Where

subscript w refers to web with width b_w, subscript f refers to flange with width $b - b_w$

$\bar{\rho}_b$ = balanced reinforcement ratio for a rectangular beam of width b_w without compression reinforcement

$\rho_f = \frac{A_{sf}}{b_w d}$ reinforcement ratio for tension steel, A_{sf}, to develop the compressive strength of the flanges $(b - b_w)h_f$

$A_{sf} = \frac{C_f}{f_y} = \frac{0.85 f_c'(b - b_w)h_f}{f_y}$

Fig. 10-1c—Expression for ρ_b, T-beam

TABLE 10-1—BALANCED REINFORCEMENT RATIO ρ_b

For rectangular beams with tension steel only

$$\rho_b = \bar{\rho}_b = \frac{0.85\,\beta_1 f_c'}{f_y}\;\frac{87{,}000}{87{,}000 + f_y}$$

For rectangular beams with compression steel

$$\rho_b = \bar{\rho}_b + \rho' \frac{f_s'}{f_y}$$

where

f_s' = stress in compression steel

$$f_s' = 87{,}000\left(1 - \frac{d'}{d}\;\frac{87{,}000 + f_y}{87{,}000}\right) \leq f_y$$

For T-Beams without compression steel

$$\rho_b = \frac{b_w}{b}(\bar{\rho}_b + \rho_f)$$

Where ρ_f is the reinforcement ratio for tension steel area necessary to develop the compressive strength of overhanging flanges (see Fig. 10-1c).

are the same as in the 1963 Code, the 1956 Code, and the January 1956 report of the ACI-ASCE Committee 427(327), Ultimate Load Design. The $0.05h$ has also been specified for steel encased compression members because of the similarity to spirally reinforced members.

Elimination of the minimum column size provisions of the 1963 Code, Section 912(a), as well as consideration of dimensional tolerances made it advisable to establish a 1 in. minimum eccentricity for compression members in addition to the previously mentioned $0.05h$ or $0.10h$ minimums for spirally reinforced and tied columns, respectively. It was intended that such eccentricity should be applied about each of the principal axes of the column cross section, but not about both simultaneously.

Corner and other columns exposed to known moments about each axis simultaneously should be designed for biaxial bending, taking into account the minimum eccentricity where applicable.[10.1, 10.2, 10.13, 10.17]

10.4—Distance between lateral supports of flexural members

Tests have shown that laterally unbraced reinforced concrete beams of any reasonable dimensions, even when they are very deep and narrow, will not fail prematurely by lateral buckling.[10.10] This holds true provided the load is well centered in regard to the vertical midplane of the beam. For this reason it was found possible to increase the maximum distance between lateral supports from the value $32b$, prescribed in previous editions of the Code, to $50b$ in the 1963 Code. This value has been retained in the 1971 Code.

Laterally unbraced beams are frequently loaded slightly off center (the "lateral eccentricity" mentioned in the Code) or with slight inclination. Stresses and deformations set up by such loading imperfections become detrimental for narrow, deep beams, the more so as the unsupported length increases. Lateral supports closer together than $50b$ may be required by actual loading conditions.

10.5—Minimum reinforcement of flexural members

Section 10.5.1 is concerned with beams, which for architectural or other reasons, are much larger in cross section than required by strength considerations. With very small reinforcement ratios, the computed bending moment as a reinforced concrete section becomes less than that of the corresponding plain concrete section computed from its modulus of rupture. Failure in such a case is quite sudden.

To prevent such failure, there is required a minimum steel ratio, $\rho = 200/f_y$, which, for 40,000 and 60,000 psi yield point steels, becomes 0.5 percent and 0.33 percent, respectively.

The $200/f_y$ value was derived by equating the strength computed by the modulus of rupture of the plain concrete section to the strength computed as a reinforced section and solving for ρ. This minimum reinforcement must be provided wherever positive reinforcement is needed, except where the reinforcement is one-third greater than required by analysis. This exception provides sufficient extra reinforcement for safety in large members where $200bd/f_y$ would be excessive.

In Section 10.5.2, the minimum reinforcement required for slabs is a little less than that required for beams, since an overload would be distributed laterally and a sudden failure would be less likely. The structural reinforcement should, however, be at least equal to the shrinkage and temperature reinforcement.

10.6—Distribution of flexural reinforcement in beams and one-way slabs

This is a new provision requiring distribution of reinforcement in zones of concrete tension. Several bars at moderate spacing are much more effective in controlling cracking than one or two larger bars of equivalent area.

In Section 10.6.2, a formula is given which will provide a distribution that will reasonably control flexural cracking.

Many structures designed by working stress methods and with low steel stress served their intended functions with very limited flexural cracking. When high strength reinforcing steels are used at high service load stresses, however, visible cracks must be expected, and steps must be taken in detailing of the reinforcement to control cracking. To assure protection of reinforcement against corrosion, and for aesthetic reasons, many fine hair cracks are preferable to a few wide cracks.

Only deformed reinforcement is permitted as main reinforcement in the 1971 Code. The best crack control is obtained when the steel reinforcement is well distributed over the zone of maximum concrete tension. It is prudent to use relatively small bar sizes. In major T-beams, part of the negative reinforcement should be placed in the flanges near the web; otherwise, only a few wide cracks may extend into the slab even when numerous fine cracks exist directly over the web.

Control of cracking is particularly important when reinforcement with a yield strength in excess of 40,000 psi is used. Current good detailing practices will usually lead to adequate crack control even when reinforcement of 60,000 psi yield is used. With careful attention to steel details, entirely satisfactory structures have been built, particularly in Europe, with design yield strengths exceeding 80,000 psi, the limiting yield strength considered by the 1971 Code.

Early investigations of crack width in beams and members subjected to axial tension indicated that crack width was proportional to steel stress and reinforcing bar diameter, but was inversely proportional to reinforcement percentage.

Recent extensive laboratory work[10.3-10.5] involving modern deformed bars has confirmed that crack width at service loads is proportional to steel stress. However, the significant variables reflecting steel detailing were found to be thickness of concrete cover and the area of concrete in the zone of maximum tension surrounding each individual reinforcing bar.

Crack width is inherently subject to wide scatter even in careful laboratory work and is influenced by shrinkage and other time-dependent effects. Great accuracy in crack control computations is not warranted. Simple approximate crack control equations will suffice. The objective of crack control computations is to arrive at reasonable reinforcing details as indicated by laboratory tests and practical experience. Eq. (10-2) is written in a form emphasizing reinforcing details rather than crack width w, per se. It is based on the Gergely-Lutz expression:

$$w = 0.076\beta f_s \sqrt[3]{d_c A_1}$$

in which f_s is in ksi, and w is in units of 0.001 in. To simplify practical design, an approximate value of 1.2 was used for β, the ratio of distances to the neutral axis from the extreme tension fiber and from the centroid of the main reinforcement.

The effective tension area of concrete surrounding the main reinforcement is defined as having the same centroid as that reinforcement. Moreover, this area is to be bounded by the surfaces of the cross section and a straight line parallel to the neutral axis. Computation of the effective area per bar, A_1, is illustrated by the example shown in Fig. 10-2, in which the centroid of the main reinforcement is located 3.64 in. from the bottom of the beam. The effective tension area is then taken as twice 3.64 in. times the beam width b. Divided by the number of bars, this gives 17.5 sq in. per bar.

10.6.4—The numerical limitations of $z = 175$ and 145 kips per in. for interior and exterior exposure, respectively, correspond to limiting crack widths of 0.016 and 0.013 in. Although a number of studies have been conducted, clear experimental evidence is not available regarding the crack width beyond which a corrosion danger exists. Exposure tests indicate that concrete quality, adequate compaction, and ample concrete cover may be of greater importance for corrosion protection than crack width at the concrete surface. The limiting values for z were, therefore, chosen primarily to give

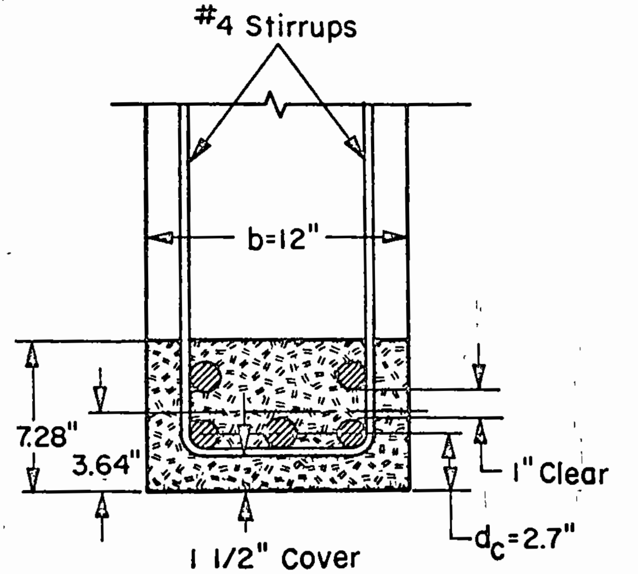


Fig. 10-2—Beam with five #11 bars

reasonable reinforcing details in terms of practical experiences with existing structures.

10.6.5 — For relatively deep girders, a light longitudinal reinforcement should be added near the vertical faces in the tension zone to control cracking in the web. Without such auxiliary steel, only a few wide cracks may extend into the web even when the zone of maximum tension contains numerous fine and well distributed cracks.

One-way slabs — Recent laboratory tests[10.6] have shown that the Gergely-Lutz expression applies reasonably to one-way slabs. The average ratio β is about 1.35 for floor slabs, rather than the value 1.2 used for beams. Accordingly it would be consistent to reduce the maximum values for z by the factor 1.2/1.35.

Two-way slabs — For two-way acting structural slabs, flat slabs, and plates, Code Section 13.5.1 restricts spacing of reinforcement to $2h$.

ACI Committee 224, Cracking, has made the following recommendation.

Results of extensive tests on slabs and plates indicate that the flexural cracking behavior in concrete structural floors under two-way action is significantly different from that in one-way members.[10.22-10.24] In two-way acting slabs, flat slabs, and plates, the possible crack width, which may develop, can be predicted by:

$$w = K\beta f_s \sqrt{M_I}$$

where:

M_I = the grid index and is given by $(d_{b1}s_2/\rho_{t1})$

w = crack width caused by flexural load, in.

f_s = 40 percent of the design yield strength f_y', ksi

d_{b1} = diameter of bar or wire in direction "1" closest to the concrete outer fibers, in.

s_2 = spacing of bar or wire in perpendicular direction "2", in.

"1" = direction for which crack control check is to be made

ρ_{t1} = active steel ratio = A_{s1}/A_{t1}

A_{s1} = area of tensile reinforcement in direction "1" per foot width of slab, sq in.

A_{t1} = concrete stretched area in direction "1" = $12(2C_1 + d_{b1})$, sq in.

C_1 = clear concrete cover to steel bar or wire in direction "1". in.

The coefficient K has been determined experimentally to have a value of 2.8×10^{-5} for uniformly loaded two-way acting slabs and plates. To simplify the calculations, a β of 1.30 can be assumed, although the value of β can vary between 1.20 and 1.35 in most cases.

The value of K given was determined for slabs fully restrained at the boundaries. For simply supported slabs, the value of K should be multiplied by 1.6. However, since most slabs, even those assumed to be simply supported, have some degree of rotational restraint at the edges, $K = 2.8 \times 10^{-5}$ can be used for most cases.

The allowable crack width is to correspond to the value given above consistent with the exposure conditions.

10.7—Deep flexural members

The Code does not contain detailed requirements for designing deep beams for flexure except that nonlinearity of strain distribution and lateral buckling must be considered.

Suggestions for the design of deep beams are given in the following papers: Chow, Li; Conway, Harry; and Winter, George, "Stresses in Deep Beams," *Transactions*, ASCE, V. 118, 1953, pp. 686-708; and "Design of Deep Beams," *Concrete Information* ST-66, Portland Cement Association.

Recently, the subject has gained attention with regard to computer methods of analysis using the finite element approach.

10.8—Limiting dimensions for compression members

The detailed requirements of Sections 912(a) and 913(a) in ACI 318-63 requiring certain minimum sizes for compression members have been eliminated to allow wider utilization of reinforced concrete compression members in smaller size and lightly loaded structures, such as low rise residential and light office buildings. The engineer should recognize the need for careful workmanship, as well as the increased significance of shrinkage stresses with small sections.

10.8.2, 10.8.3, and 10.8.4 — The quantity of reinforcement, both vertical and spiral, is based on the gross column area and core area, and the allowable column load is based on the gross area. In some cases, however, the gross area is larger than necessary to carry the design load. The basic idea of Sections 10.8.2, 10.8.3, and 10.8.4 is that it is satisfactory to design a column of sufficient size to carry the load and then simply add concrete around it without increasing the reinforcement to meet the minimum percentages required by Section 10.9.1. This additional concrete must not be considered as carrying load, but simply as an architectural treatment.

10.9—Limits for reinforcement of compression members

10.9.1 — This section prescribes the amount of longitudinal reinforcement for noncomposite compression members.

Minimum reinforcement ratio — Since the design methods for columns incorporate separate terms for the load carried by the concrete and by the reinforcement, it is necessary to specify some minimum amount of reinforcement to insure that only *reinforced concrete* columns are designed by these procedures. Reinforcement is necessary to provide resistance to bending, which may exist whether or not computations show that bending exists, and to reduce the effects of creep and shrinkage of the concrete under sustained compressive stresses. Tests have shown that creep and shrinkage tend to transfer load from the concrete to the reinforcement, with a consequent increase in stress in the reinforcement, and that this increase is greater as the ratio of reinforcement decreases. Unless a lower limit is placed on this ratio, the stress in the reinforcement may increase to the yield level under sustained service loads. This phenomenon was emphasized in the report of ACI Committee 105[10.21] and minimum reinforcement ratios of 0.01 and 0.005 were recommended for spiral and tied columns, respectively. However, in all editions of the Code since 1936, the minimum ratio has been 0.01 for both types of columns.

Maximum reinforcement ratio — The extensive tests of the ACI Column Investigation[10.21] included reinforcement ratios no greater than 0.06. Although other tests with as much as 17 percent reinforcement in the form of bars produced results similar to those made previously, it is necessary to note that the loads in these tests were applied through bearing plates on the ends of the columns and the problem of transferring a proportional amount of the load to the bars was thus minimized or avoided. Maximum ratios of 0.08 and 0.03 were recommended by ACI Committee 105[10.21] for spiral and tied columns, respectively. In the 1936 Code, these limits were made 0.08 and 0.04, respectively. In the 1956 Code [Section 1104(b)], the limit for tied columns with bending was raised to 0.08. Since the 1963 Code, it has been required that bending be considered in the design of all columns, and the maximum ratio of 0.08 has been applied to both types of columns. This limit can be considered a practical maximum for bar reinforcement in terms of economy and requirements for placing.

Minimum number of bars — This section requires a minimum of six bars for circular compression members and four for rectangular compression members. For other shapes one bar should be provided at each apex or corner, and proper lateral reinforcement provided. For example, tied triangular columns should contain at least three bars.

10.9.2 — The effect of spiral reinforcement in increasing the load-carrying capacity of the concrete within the core does not come into play until the column has been subjected to a load and deformation sufficient to cause the concrete shell outside the core to spall off. The amount of spiral required by Eq. (10-3) was intended to provide additional load-carrying capacity for concentrically loaded columns equal to or slightly greater than the capacity that was lost when the shell spalled off. This principle was recommended by ACI Committee 105[10 21] and has been a part of the Code since 1963. The derivation of Eq. (10-3) is given in the report. Tests and experience show that columns containing the amount of spiral reinforcement required by this section exhibit considerable toughness and ductility.

10.10—Slenderness effects in compression members

Sections 10.10 and 10.11 dealing with slenderness provisions have been entirely rewritten, based on recommendations of ACI-ASCE Committee 441, Reinforced Concrete Columns.[10.7] This recommendation calls for the use of improved structural analysis procedures wherever possible or practical. In place of such improved analysis it provides for an approximate design method based on a moment magnifier principle and similar to the procedure used as part of the American Institute of Steel Construction specifications. After study of the normal range of variables in column design, limits of applicability were set which eliminate from consideration as slender columns a large percentage of columns in braced frames and substantial numbers of columns in unbraced frames. The accuracy of the approximate design procedure was established through a series of comparisons with analytical and test results. Over the total range of slender compression members, the proposed procedure is more rational, more accurate, and more consistent than the reduction factor method used in the 1963 Code. Because the moment magnification method calls the attention of the designer to the basic phenomenon in slender compression members and allows him to evaluate the additional moment requirements in restraining members, a superior and safer design results.

Because results of an extensive series of studies of slender compression members in frames[10 8] indicated that a somewhat modified and carefully limited reduction factor method could give reasonable accuracy in treatment of slenderness effects, such a procedure is included in this Commentary after treatment of the detailed provisions of Section 10.10 and 10.11.

10.10.1 — ACI Committee 441 endorsed the position that the slenderness effect provisions should encourage improvement in the structural analysis since the basic need for any slenderness effect provision stems from weaknesses in conventionally used methods of frame analysis. The Column Committee's studies indicated that many of the analysis shortcomings affect the short columns as much or more than slender compression members.

The following elements are regarded as minimum requirements for an adequate rational frame analysis for design of compression members under Section 10.10.1:

(a) The structure may be idealized as a plane frame of linear elements. In structures containing structural walls, a better estimate of moments and deflections will be obtained if the stiffness of the wall is considered in the analysis.

(b) Realistic moment-curvature relationships must be used to provide accurate values of deflections and secondary moments. A linear approximation of the moment-curvature relationship defined by Eq. (10-7) will be acceptable, although use of a more accurate relationship is encouraged. The effect of duration of loads on deformations must be considered.

(c) The analysis must consider the influence of the axial load on the rotational stiffness of the member.

(d) The maximum moments in the compression member must be determined considering the effects of member and frame deflections and rotations. The possibility of having a maximum moment occur at sections other than the ends of the member must be considered.

(e) Because of the complexity of the problem, any proposed analysis used under the provisions of Section 10.10.1 should be checked against the limited test results available and should show accuracy at least comparable with the more approximate provisions of Section 10.11.

10.11—Approximate evaluation of slenderness effects

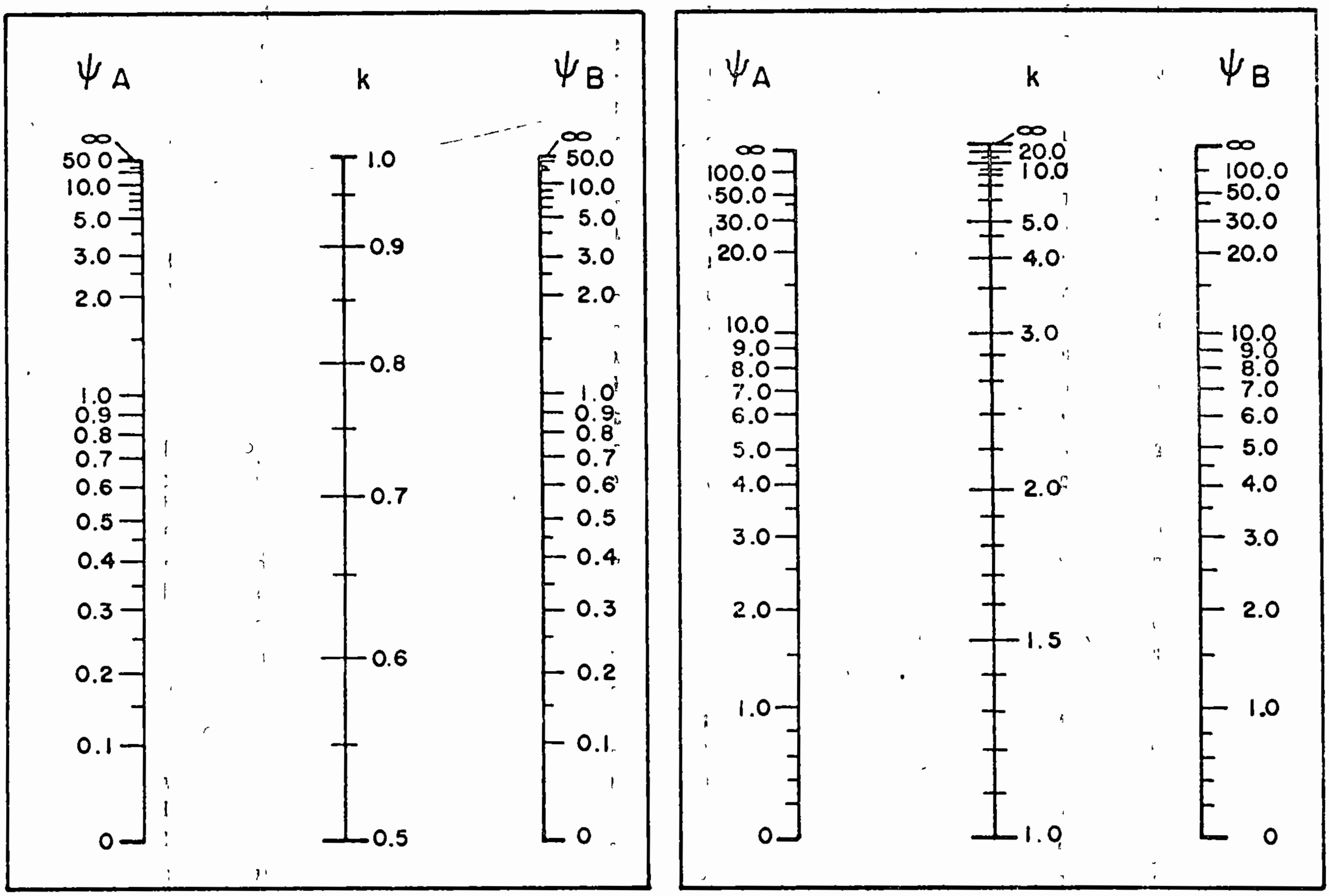
This section describes an approximate slenderness-effect design procedure based on the moment magnifier concept. The moments computed in an ordinary frame analysis are multiplied by a "moment magnifier" which is a function of the axial load P_u and the critical buckling load for the column P_c. The procedure embodies some of the main elements of the working stress design procedure for steel beam columns as included in the AISC specifications for structural steel for buildings.[10.9]

10.11.1 and 10.11.2 — These provisions are essentially unchanged from the 1963 Code, although simplified and condensed.

10.11.3 — This section requires the use of effective length factors in computing slenderness effects. The fundamental equations for the design of slender compression members were derived for hinged ends and must be modified to account for the effect of end restraints. This is done by using an "effective length," kl_u, in the computation of slenderness effects, as has been used for beam-column design in the AISC specifications[10.9] since 1963. Comparisons with more precise computer solutions indicate this procedure is especially accurate in the unbraced frame.

Committee 441 proposed that the effective length be computed in a more or less standard way by use of the Jackson and Moreland Alignment Charts (Fig. 10-3), which allow graphical determination of k for a column of constant cross section in a multibay frame.[10.10,10.13] The effective length is a function of the relative stiffness at each end of the compression member and studies have indicated that the effects of widely varying beam and column reinforcement percentages and of beam cracking should be considered in determining these relative stiffnesses.

Because the behavior of braced and unbraced frames is so different, it is necessary to have one



(a) Braced Frames

(b) Unbraced Frames

ψ = Ratio of $\sum k$ of compression members to $\sum k$ of flexural members in a plane at one end of a compression member

k = Effective length factor

Fig. 10-3—Effective length factors

set of effective length factors for completely braced frames and another set for completely unbraced frames. In actual fact, there is rarely such a thing as a completely braced or a completely unbraced frame. For the purposes of applying Section 10.11.3, a compression member braced against sidesway is a member in a story in which the bracing elements such as shearwalls, shear trusses, or other types of lateral bracing, have a total stiffness, resisting lateral movement of a story, at least six times the sum of the stiffnesses of all the columns resisting lateral movement in the story under consideration, so that lateral deflections of the story are not large enough to significantly affect the column strength. What constitutes adequate bracing in a given case must be left to the judgment of the engineer, depending on the arrangement of the structure in question. A value of k less than 1.2 for columns not braced against sidesway, normally would not be realistic.

10.11.4 — This section provides lower and upper slenderness ratio limits for use with the moment magnification method. The lower limits indicate that many stocky and sufficiently restrained compression members can essentially develop the full cross-sectional strength. The lower limits were determined from a study of a wide range of columns and correspond to lengths for which a slender member strength of at least 95 percent of the cross-sectional strength can be developed.

While elimination of slenderness considerations for these members may result in strength inaccuracies of up to 5 percent, the designer's job is considerably simplified, since studies[10.7] of a series of actual structures indicate that slenderness effects could be neglected for about 90 percent of the columns in the braced frames and 40 percent of the columns in the sway frames studied. An upper limit is imposed on the slenderness ratio of columns designed by the moment magnification method of Section 10.11. No similar limit is imposed if design is carried out according to Section 10.10.1. The limit of $kl_u/r = 100$ represents the upper range of actual tests of slender compression members in frames.

10.11.5—This section presents the slender column approximate design equations. These equations are based on the concept of a moment magnifier δ which amplifies the column moments to account for the effect of axial loads on these moments. The column cross section is then designed for the axial load and the amplified moment. In application, δ is a function of the ratio of the axial load in the column to the assumed critical load of the column, the ratio of column end moments, and the deflected shape of the columns.

In computing δ, the factor C_m is an equivalent moment correction factor. The derivation of the moment magnifier assumes that the maximum moment is at or near midheight of the column. If the maximum applied moment occurs at one end of the column, design must be based on an "equivalent uniform moment," C_mM_2, which would lead to the same maximum moment when magnified.[10.7]

In defining the critical load, the main problem is the choice of a stiffness parameter EI, which reasonably approximates the stiffness variations due to cracking, creep, and the nonlinearity of the concrete stress-strain curve. The Design Subcommittee of Committee 441[10.7] recommended that where more precise values are not available, EI be defined by Eq. (10-7) and (10-8). These expressions approximate the lower limits of EI for practical cross sections and hence are conservative for secondary moment calculations. They were derived for small e/h values and high P_u/P_o values, where the effect of axial load is most pronounced. P_o is the theoretical axial load capacity of a short compression member. Since experimental work involved the theoretical capacity, P_u/ϕ is used in Eq. (10-5).

The approximate nature of these expressions is shown in Fig. 10-4, where they are compared with values derived from load-moment-curvature diagrams for the case of no sustained load ($\delta_d = 0$). Eq. (10-7) represents the lower limit of the practical range of stiffness values. This is especially true for the heavily reinforced columns. Eq. (10-8) is

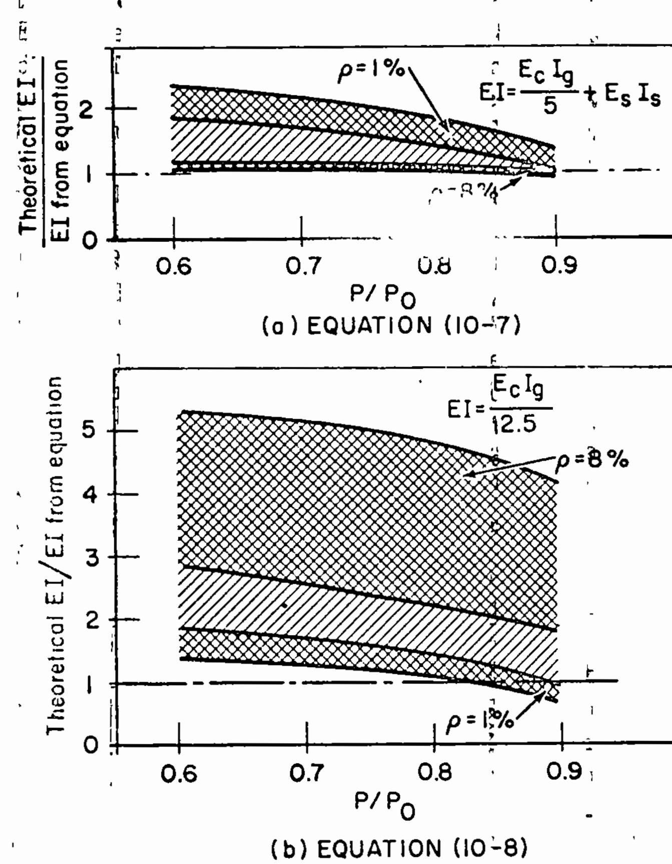


Fig. 10-4—Comparison of equations for EI with EI values from moment-curvature diagrams

simpler to use but greatly underestimates the effect of reinforcement in heavily reinforced columns. However, in many cases, when reinforcement percentages are low, or slenderness effects not very substantial, its relative simplicity may be desirable.

Creep due to sustained loads tends to reduce the effective value of EI. This is taken into account by dividing the EI term by $(1 + \beta_d)$ where β_d is the ratio of dead load moment to total load moment. This factor gives a correct trend when compared to both analyses and tests of columns under sustained loads.

Note that the Code states that EI in Eq. (10-6) may be taken as either value obtained from Eq. (10-7) or (10-8) in lieu of a more precise calculation. In this respect, the Code refers to a more accurate value of EI as obtained from moment-curvature relationships, based on the integration of acceptable nonlinear stress-strain diagrams for concrete in flexure. Any stress-strain function which provides agreement with test data may be used (see Code Section 10.2.6). The more accurate values of EI may be used for designing columns or walls under the provisions stated in Chapter 10.

When the alternate design method of Section 8.10 is used, P_u/ϕ in Eq. (10-5) is taken as $2.5P$ when gravity loads govern and as $1.875P$ when lateral loads with gravity loads govern the design, where P is the unfactored design load in the compression member.

10.11.5.1 When a story of a structure fails in a lateral instability mode, one floor translates relative to another as a unit. Thus, the deflections and hence the amount of moment magnification must be related for all compression members in the story. This section provides a procedure for computing an effective moment magnifier for the entire story. However, since any individual compression member in the story could also be overloaded while being braced against lateral instability by the other members, it is also necessary to check individual heavily loaded members using the effective length factors for braced frames.

10.11.5.2 When biaxial bending occurs in a compression member, the component moments about each of the principal axes must be magnified. The magnification factors (δ) are computed considering the buckling load P_c about each axis separately, based on the appropriate effective lengths (kl_u) and the related stiffness (EI). The clear column height may differ in each direction, and the stiffness ratios $\Sigma_{cols}/\Sigma_{beams}$ may also differ. Thus, the different buckling capacities about the two axes are reflected in different magnification factors.

The moments about each of the two axes are magnified separately, and the cross section is then proportioned. References 10.1, 10.2, and 10.17 provide guidance in this respect. Note that the design moment, $M_c = \delta M_2$, refers to the "larger end moment" with respect to bending about one axis. It will usually be necessary, therefore, to magnify the moments at both ends of a column subjected to biaxial bending, and to investigate both conditions at both ends.

In the case of compression members which are subject to transverse loading between supports, it is possible that the maximum moment will occur at a section away from the end of the member. If this occurs, the value of the largest calculated moment occurring anywhere along the member should be used for the value of M_2 in Eq. (10-4). In accordance with the last sentence of Section 10.11.5, C_m must be taken as 1.0 for this case.

10.11.6 — This provision (similar to one in ACI 318-63) allows computed moments to be used in determining conditions of curvature and restraint when design must be based on minimum eccentricity This eliminates what would otherwise be a discontinuity between columns with computed eccentricities less than minimum eccentricity and columns with computed eccentricities equal to or greater than minimum eccentricity.

10.11.7 — The strength of a laterally unbraced frame is governed by the stability of the columns and by the degree of end restraint provided by the beams in the frame. If plastic hinges form in the restraining beam, the structure approaches a mechanism and its axial load capacity is drastically reduced. This section provides that the designer make certain that the restraining flexural members have the capacity to resist the amplified column moments. The ability of the moment magnification method to provide a good approximation of the actual magnified moments at the member ends in a sway frame is a significant improvement over the 1963 Code reduction factor method.

Modified R Method*

The 1963 Code used a column reduction factor R (Section 916) and an effective length h' for unbraced columns [Section 915(d)]. The modified R values listed below, within the limits noted, lead to an accuracy equal to that of the "moment magnifier" method of Section 10.11.5. Hence they may be used as an alternate method within the stated limits. (Note that for design both the axial load and the moment must be divided by the appropriate factor R.)

If relative lateral displacement of the ends of the member is prevented and the ends of the member are fixed or definitely restrained such that a point of contraflexure occurs between the ends, no

*This section is based on the methods of ACI 318-63, so the notation remains as used in that Code while other notation in both Code and Commentary for ACI 318-71 agrees with "Preparation of Notation for Concrete (ACI 104-71)."

correction for length need be made unless h/r exceeds 54, where h is the actual unsupported length of column and r is the radius of gyration of gross concrete area of a column. For h/r between 54 and 100, the following factor from ACI 318-63 may be used:

$$R = 1.32 - 0.006h/r \leq 1.0$$

If relative lateral displacement of the ends of the members is prevented and the member is bent in single curvature, the following factor, more liberal than ACI 318-63, may be used where the nominal eccentricity does not exceed $0.10t$ where t is the overall thickness of the column

$$R = 1.23 - 0.008h/r \leq 1.0$$

If the nominal eccentricity exceeds $0.10t$, the factor as in ACI 318-63 should be

$$R = 1.07 - 0.008h/r \leq 1.0$$

In both the above paragraphs, no increase in R is justified where tension governs the design, unless axial load is less than $0.10f_c'bt$. Then, the transition to $R = 1$ for flexure without axial load may be patterned after Section 9.2.1.2(c).

For members where (1) relative lateral displacement of the ends is not prevented, (2) with h'/r not exceeding 40, and (3) with bracing beams having a negative moment steel ratio of at least $p = 0.01$, the reduction factor, where design is governed by lateral loads of short duration, should be

$$R = 1.07 - 0.008h'/r \leq 1.0$$

For other loads of longer duration, the factor should be

$$R = 0.97 - 0.008h'/r \leq 1.0$$

These R values generally are more restrictive than those in ACI 318-63 and are primarily for use with columns restrained at each end where $h' = h(0.78 + 0.22r') \leq h$ and r' is the average of ΣK of columns to ΣK of floor members taken at the two ends of the column.

For the restraining beams in both these cases, the design should be based on taking from the column additional lateral load total moment of

$$M = \text{Col. } M_L$$
$$= \text{Nom. } P_L e_N (1 - P_L/P_o)/(R - P_L/P_o)$$

where

M_L = long column end moment

P_L = long column ultimate load

e_N = nominal eccentricity, as for a short column

P_o = theoretical axial load capacity of short column

This equation is based on similar triangles from Fig. 10-5.

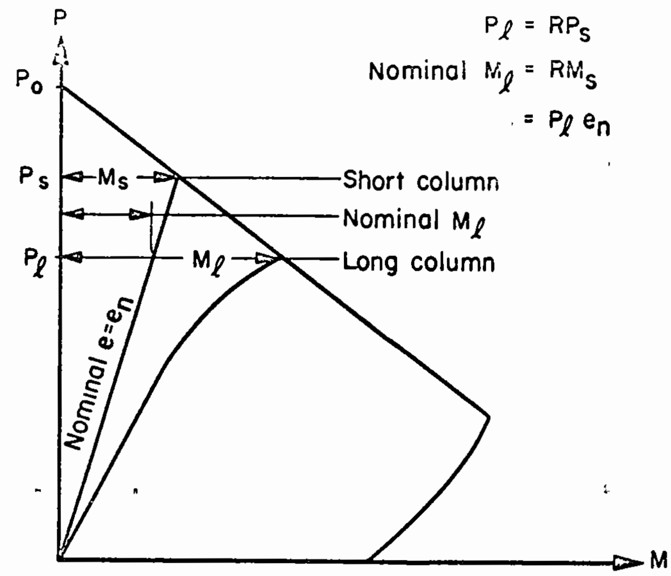


Fig. 10-5—Approximation for M_l for use in determining beam design moment

10.13—Transmission of column loads through floor system

The requirements of this section are based on a paper on the effect of floor concrete strength on column strength.[10 20] The provisions mean that where the column concrete strength does not exceed the floor concrete strength by more than 40 percent, no special precautions need be taken. For higher column concrete strengths, methods in Paragraphs (a) or (b) must be used for corner or edge columns and methods in Paragraphs (a), (b), or (c) for interior columns with adequate restraint on all four sides.

10.14—Bearing

This section deals with bearing stresses on concrete supports which are not laterally reinforced to resist splitting stresses. The provisions are similar to but more liberal than the bearing provisions of ACI 318-63. Work by Hawkins[10,12] indicates the liberalization to be justified. (See also Section 15.6.)

10.14.2 — When the supporting area is wider than the loaded area on all sides, the surrounding concrete confines the bearing area, resulting in an increase in the permissible bearing stress.

This section gives no minimum depth for the supporting pier. The supporting pier should satisfy the shear provisions of Section 11.10, which will control the minimum depth of the support.

10.14.3 — When the top of the support is sloped or stepped, advantage may still be taken of the fact that the supporting pier is larger than the loaded area, provided that the pier does not slope away at too great an angle. Fig. 10-6 illustrates the application of the frustum to find A_2. The frustum

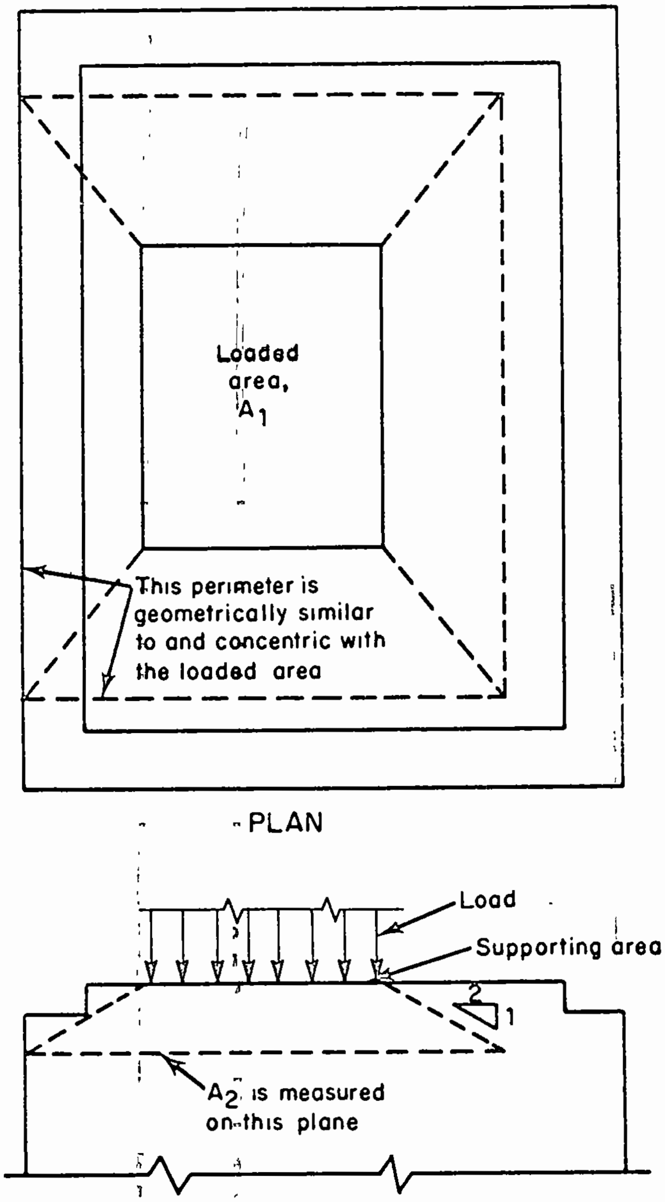


Fig. 10-6—Application of frustum to find A_2 in stepped or sloped supports

should not be confused with the path by which a load spreads out as it travels downward through the support. Such a load path would have steeper sides. However, the frustum described has somewhat flat side slopes, to insure that there is concrete immediately surrounding the zone of high stress at the bearing.

10.14.4 — Post-tensioning anchorages are normally laterally reinforced, in accordance with Section 18.11.3.

10.15—Composite compression members

10.15.1 — Composite columns are defined without reference to obsolete classifications of combination, composite, or concrete-filled pipe column. Reference to other metals used for reinforcement has been omitted because they are seldom used now with concrete in construction.

10.15.2-10.15.3 — These sections give rules for determining the strength of composite cross sections. The same rules that are used for computing ultimate load interaction functions for reinforced concrete sections can be applied to composite sections. Interaction charts for concrete-filled tubing would have a form identical to those of ACI SP-7[10.11] and the *Ultimate Strength Design Handbook, V. 2, Columns,*[10.13] but with γ (formerly g) slightly greater than 1.0.

The requirement that loads assigned to concrete must be developed by direct bearing against the concrete effectively eliminates the old combination column as a composite column under the new definition. Direct bearing can be developed through lugs, plates, or reinforcing bars welded to the structural shape or tubing before the concrete is cast. Flexural compression stress need not be considered a part of direct compression load to be developed by bearing. Simply wrapping concrete around a structural steel shape would stiffen the shape, but it would not necessarily increase its strength.

The rules of Section 10.11.2 for estimating the radius of gyration are overly conservative for concrete-filled tubing, and an alternate procedure is provided in this section. The *EI* formula suggested is consistent with Section 10.11.5, and provides a conservative estimate of the concrete stiffness. It leads to excess moment magnification and conservative estimates of strength.

10.15.4 — Steel encased, concrete sections should have a metal wall thickness large enough to maintain longitudinal yield stress before buckling outward.

10.15.5 — Concrete encasement that is laterally contained by a spiral is obviously useful for carrying load, and the size of spiral required can be regulated on the basis of the strength of the concrete *outside* the spiral by means of the same reasoning that applies for columns reinforced only with longitudinal bars. The radial pressure guaranteed by the spiral insures interaction among concrete, reinforcing bars, and a steel core such that longitudinal bars will both stiffen and strengthen the cross section.

10.15.6 — Concrete encasement that is laterally contained by tie bars is likely to be rather thin along at least one face of a steel core section, and complete interaction among the core, the concrete, and any longitudinal reinforcement should not be assumed. Concrete will probably separate from smooth faces of the steel core. To maintain the concrete encasement, it is reasonable to require more lateral tie steel than that needed for ordinary reinforced concrete columns. Due to the probable separation at high strains between the steel core and the concrete encasement, longitudinal bars will be ineffective in stiffening cross sections even though they would be useful in sustaining compression forces. Finally, the yield strength of the steel core should be limited to that which exists at strains below those that can be sustained without spalling of the concrete encasement. It has been assumed that axially-compressed concrete will not spall at strains less than 0.0018. The yield strength of $0.0018 \times 29{,}000{,}000$, or 52,000 psi, represents an upper limit of the useful maximum steel stress.

10.16—Special provisions for walls

This section recognizes that bearing walls can be designed as compression members. The unique reinforcement requirements for walls are included. Ties may be eliminated for steel ratios less than $0.01A_g$ or when the vertical reinforcement is not required to resist compression stress. All other provisions for compression members must be satisfied.

References

10.1. Weber, D. C., "Ultimate Strength Design Charts for Columns with Biaxial Bending," ACI JOURNAL, *Proceedings* V. 63, No. 11, Nov. 1966, pp. 1205-1230.

10.2. Parme, A. L.; Nieves, J. M.; and Gouwens, A., "Capacity of Reinforced Rectangular Columns Subjected to Biaxial Bending," ACI JOURNAL, *Proceedings* V. 63, No. 9, Sept. 1966, pp. 911-923.

10.3. Gergely, P., and Lutz, L. A., "Maximum Crack Width in Reinforced Concrete Flexural Members," *Causes, Mechanism, and Control of Cracking in Concrete*, SP-20, American Concrete Institute, Detroit, 1968, pp. 1-17.

10.4. Kaar, P. H., "High Strength Bars as Concrete Reinforcement, Part 8: Similitude in Flexural Cracking of T-Beam Flanges," *Journal*, PCA Research and Development Laboratories, V. 8, No. 2, May 1966, pp. 2-12. Also, *Development Department Bulletin* D106, Portland Cement Association.

10.5. Base, G. D.; Reed, J. B.; Beeby, A. W.; and Taylor, H. P. J., "An Investigation of the Crack Control Characteristics of Various Types of Bar in Reinforced Concrete Beams," *Research Report* No. 18, Cement and Concrete Association, London, Dec. 1966, 44 pp.

10.6 Lloyd, John P.; Rejali, Hassen M.; and Kesler, C. E., "Crack Control in One-Way Slabs Reinforced with Deformed Wire Fabric," ACI JOURNAL, *Proceedings* V. 66, No. 5, May 1969, pp. 366-376.

10.7. MacGregor, James C.; Breen, John E.; and Pfrang, Edward O., "Design of Slender Concrete Columns," ACI JOURNAL, *Proceedings* V. 67, No. 1, Jan. 1970, pp. 6-28.

10.8. Ferguson, Phil M., "Long Columns in Frames—Computer Analyses, Part 4: Code Recommendations," University of Texas at Austin, Oct. 1968, 16 pp.

10.9. "Commentary and Specifications for the Design, Fabrication, and Erection of Structural Steel for Buildings," American Institute for Steel Construction, New York, 1969.

10.10. "Guide to Design Criteria for Metal Compression Members," Column Research Council, Fritz Engineering Laboratory, Bethlehem, Pa., 2nd Edition, 1966.

10.11. Everard, Noel J., and Cohen, Edward, *Ultimate Strength Design of Reinforced Concrete Columns*, SP-7, American Concrete Institute, Detroit, 1964, 182 pp.

10.12. Hawkins, N. M., "The Bearing Strength of Concrete Loaded Through Rigid Plates," *Magazine of Concrete Research* (London), V. 20, No. 62, Mar. 1968, pp. 31-40.

10.13. ACI Committee 340, *Ultimate Strength Design Handbook, V. 2, Columns*, SP-17A, American Concrete Institute, Detroit, 1970, 226 pp.

10.14. Whitney, Charles S., "Plastic Theory of Reinforced Concrete Design," *Transactions*, ASCE, V. 107, 1942, pp. 251-326.

10.15. ACI-ASCE Committee 327, "Ultimate Strength Design," ACI JOURNAL, *Proceedings* V. 52, No. 5, Jan. 1956, pp. 504-524.

10.16. Whitney, C. S., and Cohen, Edward, "Guide for Ultimate Strength Design of Reinforced Concrete," ACI JOURNAL, *Proceedings* V. 53, No. 5, Nov. 1956, pp. 455-490.

10.17. Bresler, Boris, "Design Criteria for Reinforced Concrete Columns Under Axial Load and Biaxial Bending," ACI JOURNAL, *Proceedings* V. 57, No. 5, Nov. 1960, pp. 481-490.

10.18. Mattock, A. H.; Kriz, L. B.; and Hognestad, E., "Rectangular Concrete Stress Distribution in Ultimate Strength Design," ACI JOURNAL, *Proceedings* V. 57, No. 8, Feb. 1961, pp. 875-928. Also *Development Department Bulletin* D49, Portland Cement Association.

10.19. Hansell, William, and Winter, George, "Lateral Stability of Reinforced Concrete Beams," ACI JOURNAL, *Proceedings* V. 56, No. 3, Sept. 1959, pp. 193-214. (Discussion Mar. 1960, pp. 957-967). Also, Sant, Jagadish K., and Bletzacker, Richard W., "Experimental Study of Lateral Stability of Reinforced Concrete Beams," ACI JOURNAL, *Proceedings* V. 58, No. 6, Dec. 1961, pp. 713-736.

10.20. Bianchini, Albert C.; Woods, Robert E.; and Kesler, Clyde E., "Effect of Floor Concrete Strength on Column Strength," ACI JOURNAL, *Proceedings* V. 56, No. 11, May 1960, pp. 1149-1169.

10.21. "Reinforced Concrete Column Investigation—Tentative Final Report of Committee 105," ACI JOURNAL, *Proceedings* V. 29, No. 5, Feb. 1933, pp. 275-282.

10.22. Nawy, Edward G., "Crack Width Control in Welded Fabric Reinforced Centrally Loaded Two-Way Concrete Slabs," *Causes, Mechanism, and Control of Cracking in Concrete*, SP-20, American Concrete Institute, Detroit, 1968, pp. 211-235.

10.23. Nawy, Edward G., and Orenstein, G. S., "Crack Width Control in Reinforced Concrete Two-Way Slabs," *Proceedings*, ASCE, V. 96, ST3, Mar. 1970, pp. 701-721.

10.24. Nawy, Edward G., and Blair, Kenneth W., "Further Studies on Flexural Crack Control in Structural Slab Systems," *Cracking, Deflection, and Ultimate Load of Concrete Slab Systems*, SP-30, American Concrete Institute, Detroit, 1971.

10.25. Furlong, R. W., "Design of Steel Encased Concrete Beam-Columns," *Proceedings*, ASCE, V. 94, ST1, Jan. 1968, pp. 267-281.

10.26. Faber, O., "Savings to be Effected by the More Rational Design of Encased Stanchions as a Result of Recent Full Size Tests," *The Structural Engineer* (London), V. 34, No. 3, Mar. 1956, pp. 88-109.

10.27. Malhotra, H. L., and Stevens, R. S., "Fire Resistance of Encased Stanchions," *Proceedings*, Institution of Civil Engineers (London), V. 27, Jan. 1964, pp. 77-98.

CHAPTER 11—SHEAR AND TORSION

This chapter includes shear provisions for both nonprestressed and prestressed concrete members. The torsion provisions apply only to nonprestressed concrete. Special requirements are also included for brackets and corbels, deep beams, and shear walls. A section of the chapter is devoted to the shear-friction concept, which is particularly applicable to design of reinforcing details in precast structures. Shear provisions for slabs and footings include a new procedure for shearhead reinforcement. New provisions are provided for columns receiving moment from beams or slabs.

11.1—General reinforcement requirements

11.1.1 and 11.1.2 — Stirrup reinforcement restrains the growth of inclined cracking, and hence increases ductility and provides a warning in situations where in an unreinforced web the sudden formation of inclined cracking might lead directly to distress. Such reinforcement is of great value if a member is subjected to an unexpected tensile force or catastrophic loading. Accordingly, a minimum area of shear reinforcement not less than that given by Eq. (11-1) or (11-2) is required wherever the nominal ultimate shear stress v_u is greater than 1/2 of v_c. Three types of members are excluded from this requirement: slabs; floor joists; and wide, shallow beams.

Other members may be excluded if it is shown by appropriate tests that the required capacity can be developed when shear reinforcement is omitted.

This provision for minimum area of shear reinforcement is new for nonprestressed concrete members. Eq. (11-1) may also be applied to prestressed concrete members, but it will generally require greater minimum web reinforcement in typical building members than Eq. (11-2), which was retained from the 1963 Code as an alternate.

If a nonprestressed member is subjected to a torsional stress greater than $1.5\sqrt{f_c'}$, the minimum amount of transverse web reinforcement for the combined shear and torsion is $50b_ws/f_y$. The differences in the definition of A_v and the symbol A_t used in Section 11.8 should be carefully noted; A_v is the area of two legs of a closed stirrup, while A_t is the area of only one leg of a similar closed stirrup.

11.1.3 — Limiting the design yield strength of shear and torsion reinforcement to 60,000 psi provides a control on diagonal crack width. Moreover, higher strength reinforcement may be brittle near sharp bends.

11.1.4 and 11.1.5 — These provisions for types and maximum spacings of shear reinforcement are essentially unchanged from the 1963 Code, except that welded wire fabric and spirals are expressly permitted as web reinforcement in the 1971 Code.

11.1.6 — Both longitudinal and closed transverse reinforcement is needed to resist the diagonal tension stresses, and the ultimate torsional strength will not be increased if either is absent. The stirrups must be closed, since inclined cracking due to torsion may appear on all faces of a member.

11.1.7 — It is essential that shear and torsion reinforcement be adequately anchored at both ends, to be fully effective on either side of any potential inclined crack. This generally requires a hook or bend at the end of the reinforcement.

11.2—Shear strength

11.2.1 — The 1971 Code continues the practice established in 1963 of assessing shear strength of reinforced concrete members based on an average or nominal ultimate shear stress $v_u = V_u/b_wd$. This practice applies also to prestressed concrete members. However, because the position of the centroid of the prestressing tendons may vary in prestressed concrete beams, the value of d used in computing v_u need not be taken as less than $0.80h$.

Tests have indicated that Eq. (11-3) may also be applied to members with circular sections. The definition of d as the distance from the extreme compression fiber to the centroid of the longitudinal reinforcement in the opposite half of the member is intended to cover the case of a circular section subjected only to transverse loads.

As a change in format from the 1963 Code, it should be noted that the capacity reduction factor ϕ is included in Eq. (11-3) rather than in the strength computations. This change does not affect the results.

11.2.2 — Shear capacity near a concentrated load on reaction is increased if compression is introduced into the member. The Code, therefore, calls for computation of a maximum shear stress at a distance d from the support in reinforced concrete members, and at a distance $h/2$ in prestressed concrete members.

It should be noted, however, that under some reaction conditions such as that shown in Fig. 11-1, diagonal cracking can take place at the support face and even into the support. The provision regarding shear stress computation at a distance d does not apply in such cases.

11.2.3 and 11.2.4 — In a member without shear reinforcement, shear is assumed carried by the concrete web. In a member with shear reinforcement, shear is assumed carried by the concrete compression zone and the shear reinforcement.

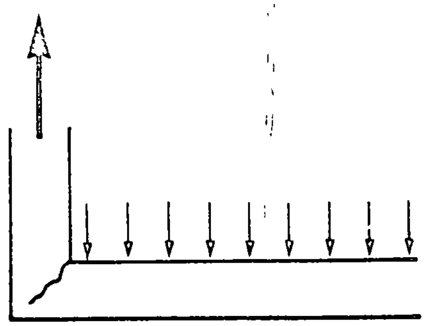


Fig. 11-1—Diagonal cracking at support face

The shear carried by the concrete, v_c, is assumed equal in both cases and is taken equal to the shear causing significant inclined cracking. These assumptions are discussed in the ACI-ASCE Committee 326 (now 426) report,[11.1] and in recent papers.[11.2,11.3]

There are two types of inclined cracking that occur in concrete beams. In recent years they have come to be called web-shear and flexure-shear cracks. These two types of inclined cracks are illustrated in Fig. 11-2.

Web-shear cracking begins from an interior point in a member, due to principal tensile stresses exceeding the tensile strength of the concrete. Flexure-shear cracking is initiated by flexural cracking. When flexural cracking occurs, the shear stresses in the concrete above the crack are increased. The flexure-shear crack develops when the combined shear and tensile flexural stresses cause a principal tensile stress exceeding the tensile strength of the concrete.

Flexure-shear cracking is the type that generally occurs when a nonprestressed concrete beam is loaded to its ultimate capacity. Web-shear cracking occurs (although infrequently) near the supports of deep beams with thin webs, or near the inflection point or bar cutoff points of continuous beams, particularly if the beam is subjected to axial tension.

Both types of inclined cracking may be observed when prestressed concrete beams are subjected to loads greater than the maximum service load. Flexure-shear cracking is the more typical type in prestressed members, particularly those subjected to uniform loads. Web-shear cracking may occur in highly prestressed beams with thin webs, particularly when the beam is subjected to large concentrated loads near a simple support.

Because of the different behavior of nonprestressed and prestressed members, and because researchers have approached the inclined cracking problem in different ways, it is necessary to calculate v_c according to Section 11.4 for nonprestress members and Section 11.5 for prestressed concrete members.

In a member of variable depth, the internal shear at any section is increased or decreased by the vertical component of the inclined flexural stresses. Computation methods are outlined in various textbooks and in the 1940 Joint Committee Report.[11.4]

11.3—Lightweight concrete shear and torsion stresses

11.3.1 and 11.3.2 — Two procedures are given by which the provisions for shear may be modified when lightweight aggregate concrete is used.

For normal weight concrete, the splitting tensile strength f_{ct} is approximately equal to $6.7\sqrt{f_c'}$. Therefore, when f_{ct} is specified and determined for a particular lightweight aggregate concrete, the value of $f_{ct}/6.7$ may be substituted for all values of $\sqrt{f_c'}$ affecting v_c, v_{tc}, and M_{cr} in this chapter. Tests[11.5,11.6] have shown this is a valid approach. However, it is felt that the shear stress values for lightweight concrete should not exceed those for normal weight concrete, and therefore the value of $f_{ct}/6.7$ used in computations should not exceed $\sqrt{f_c'}$.

In the 1963 Code, a designer was required to assume that $f_{ct}/\sqrt{f_c'}$ was equal to 4 if the value of f_{ct} was not determined by test. Use of the factors of 0.75 for "all-lightweight" concrete and 0.85 for "sand-lightweight" concrete imply a ratio of 5 and 5.7 for $f_{ct}/\sqrt{f_c'}$, respectively.

These higher values are based on data obtained from tests on many types of structural lightweight aggregate concrete. It should be noted that the

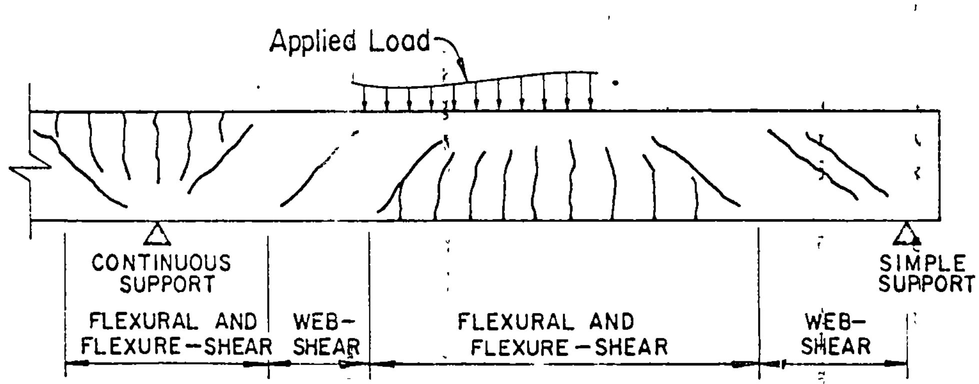


Fig. 11-2—Types of cracking in concrete beams

factors 0.75 and 0.85 apply only to the terms containing $\sqrt{f_c'}$ in the equations of this chapter.

11.4—Nominal permissible shear stress for nonprestressed concrete members

11.4.1 and 11.4.2 — Eq. (11-4) is the basic equation for shear strength of members without shear reinforcement. It was first adopted in the 1963 Code after being endorsed by ACI-ASCE Committee 326. This equation assumes that useful strength is exhausted when inclined cracking first develops in a member.

Designers should recognize that the three variables in this equation, $\sqrt{f_c'}$ (as a measure of concrete tensile strength), ρ_w, and $V_u d/M_u$, are known to affect shear strength, although there are recent data[11.7,11.8] suggesting that Eq. (11-4) overestimates the influence of f_c' and underestimates the influence of ρ_w and $V_u d/M_u$. Further recent information[11.9,11.10] has indicated that shear strength decreases as the overall depth of the member increases.

For most design purposes, it is convenient to assume that the second term of Eq. (11-4) equals $0.1\sqrt{f_c'}$. Thus the Code permits the use of v_c equal to $2\sqrt{f_c'}$, unless a designer chooses to make a detailed analysis.

The minimum value of M_u equal to $V_u d$ in Eq. (11-4) serves the purpose of limiting v_c at and near points of inflection.

11.4.3 and 11.4.4 — Eq. (11-5) and (11-7), for members subjected to axial compression in addition to shear and flexure, are derived in the ACI-ASCE Committee 326 report.[11.1] Because Eq. (11-5) is difficult to apply, a new alternative design provision, Eq. (11-6), is included in the Code.

Another new design provision, Eq. (11-8), is included for the case of axial tension existing with shear and flexure. Values of v_c obtained from these equations are illustrated in Fig. 11-3. These

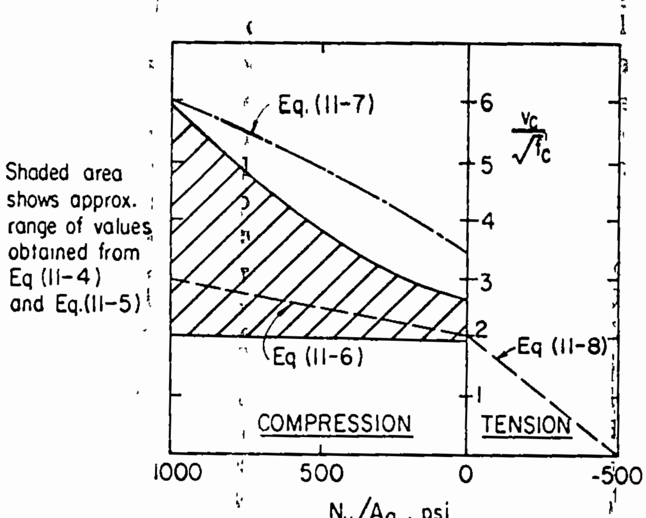


Fig. 11-3—Comparison of design equations for shear and axial load

equations were discussed and comparisons were made with test data in a recent paper.[11.3]

11.4.5 — This section allows torsion to be neglected in design if the nominal ultimate stress due to torsion is less than $1.5\sqrt{f_c'}$. This stress corresponds to about 25 percent of the pure torsional strength of a member without torsion reinforcement. ACI Committee 438[11.11] has pointed out that such simplification is possible because torsion of such magnitude will not cause a significant reduction in ultimate strength in either flexure or shear.

11.5—Nominal permissible shear stress for prestressed concrete members

11.5.1 — Eq. (11-10) is an addition to the Code. It offers a simplified means of computing v_c for prestressed concrete beams having an effective prestress force at least equal to 40 percent of the tensile strength of the flexural reinforcement. Thus, Eq. (11-10) may be applied to some members reinforced with a combination of prestressed tendons and nonprestressed deformed bars. This equation has been discussed in detail in a recent paper.[11.3] It is most applicable to building members subjected to uniform loadings. This equation may give very conservative results when applied to members such as composite I-section bridge girders.

In applying Eq. (11-10) to simply supported members subjected to uniform loads, it is apparent that $V_u d/M_u$ becomes a simple function of d/l, where l is span length, and x, the distance from the section being investigated to the support, expressed as a function of l, given by:

$$\frac{V_u d}{M_u} = \frac{d}{x}\frac{(l-2x)}{(l-x)}$$

Thus for concrete with a compressive strength of 5000 psi, v_c given by Eq. (11-10) can be represented as shown in Fig. 11-4. Similar figures can easily be developed for members of other concrete strength. However, Eq. (11-10) is quite insensitive to concrete strength, and Fig. 11-4 could be used for members with concrete strengths ranging from 4000 to 6000 psi with an error of less than 10 percent.

11.5.2 and 11.5.3 — These sections give the basic design provisions for determining v_c for prestressed concrete beams. Except for a minor change in Eq. (11-11), and conversion to a nominal stress basis, these provisions are the same as in the 1963 Code. Eq. (11-11) and Eq. (11-12) predict the shear stress causing inclined flexure-shear and web-shear cracking, respectively. The lesser of v_{ci} and v_{cw} is the shear stress causing inclined cracking at the section under consideration, and

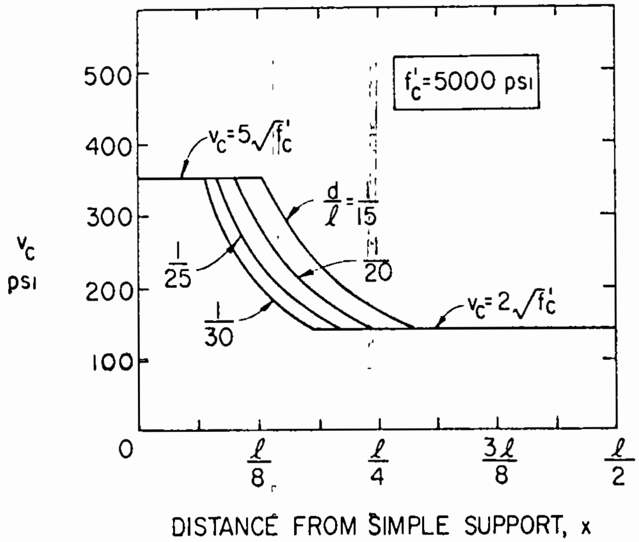


Fig. 11-4—Application of Eq. (11-10) to uniformly loaded prestressed concrete members

this value is assumed equal to the shear stress carried by the concrete, v_c.

Eq. (11-11) predicts flexure-shear cracking as the shear stress due to dead load, live load causing flexural cracking at the section being investigated, and load required to transform the flexural crack into an inclined crack. In the 1963 Code, the corresponding equation used the live load causing flexural cracking at a point $d/2$ toward the reaction from the section being investigated. This modification makes Eq. (11-11) somewhat more conservative.

In computing M_{cr} for substitution into Eq. (11-11), I and y_t are properties of the section resisting the externally applied load. For a composite member, where part of the dead load is resisted noncompositely, appropriate section properties should be used to compute f_d. V_d is then the total shear due to the dead load acting on the noncomposite member plus the superimposed dead load acting on the composite member. For noncomposite uniformly loaded beams, Eq. (11-11) reduces to

$$v_{ci} = 0.6\sqrt{f_c'} + \frac{V_u M_{cr}}{M_u b_w d}$$

Eq. (11-12) predicts web-shear cracking as the shear stress causing a principal tensile stress of approximately $4\sqrt{f_c'}$ at the centroidal axis of the cross section.

11.6—Design of shear reinforcement

11.6.1 and 11.6.2 — These sections continue the previous practice of designing shear reinforcement according to a modified form of the truss analogy. The truss analogy assumes that all of the shear is carried by web reinforcement. However, considerable research on both nonprestressed and prestressed members has indicated that web reinforcement need be designed to carry only the shear exceeding that which causes inclined cracking. The geometry involved in Eq. (11-13) and (11-14) is given in many textbooks.

11.6.3 and 11.6.4 — Similar provisions were used in previous editions of the ACI Code for nonprestressed concrete members. They are now applied equally to prestressed concrete.

11.7—Combined torsion and shear for nonprestressed members

Design criteria for torsion and shear have been presented by ACI Committee 438,[11.11] and have been discussed, along with a presentation of example problems, in recent papers.[11.12,11.13,11.26] Combined shear and torsion for prestressed members is not covered by the Code. Extensive research has been in progress since the late 1960's, but design criteria have not been fully developed.

11.7.1 — Comments for Section 11.4.5 apply here as well. In the development of the torsion design criteria, the effect of restrained warping was ignored. In designing thin-walled open sections, consideration of the torsion carried by restrained warping may be necessary.

11.7.2 — The torsional moment strength of a plain concrete member of rectangular cross section can be expressed by:

$$T_u = \alpha x^2 y f_t$$

where T_u is equal to the design torsional moments, α is a coefficent depending on the ratio y/x, x and y are the smaller and larger dimensions, respectively, of a rectangular cross section, and f_t is the tensile strength of concrete. The coefficient α varies from 0.208 to 1/3 in the elastic theory and from 1/3 to 1/2 in the plastic theory. However, a recent theory based on the bending mechanism of torsional failure[11.12] shows that α can be taken as 1/3. This constant coincides with the maximum value of α in the elastic theory and the minimum value in the plastic theory. For simplicity, therefore, ACI Committee 438[11.11] adopted a value of 1/3.

It is assumed that the torsional strength of a flanged member is equal to the sum of the torsional strengths of the web and flanges. Tests on isolated members have shown that this summation assumption is conservative, provided that the overhanging flange width does not exceed three times the thickness of the flanges. ACI Committee 438 recommends these design rules for beams integral with slabs, as shown in Fig. 11-5a. Since the nominal shear stress due to torsion, v_{tu}, is a measure of the diagonal tension, f_t in the above equation can be replaced by v_{tu}. Rearranging terms results in Eq. (11-16).

The calculation of the quantity $\Sigma x^2 y$ for flanged sections depends on the selection of component rectangles. These rectangles should not

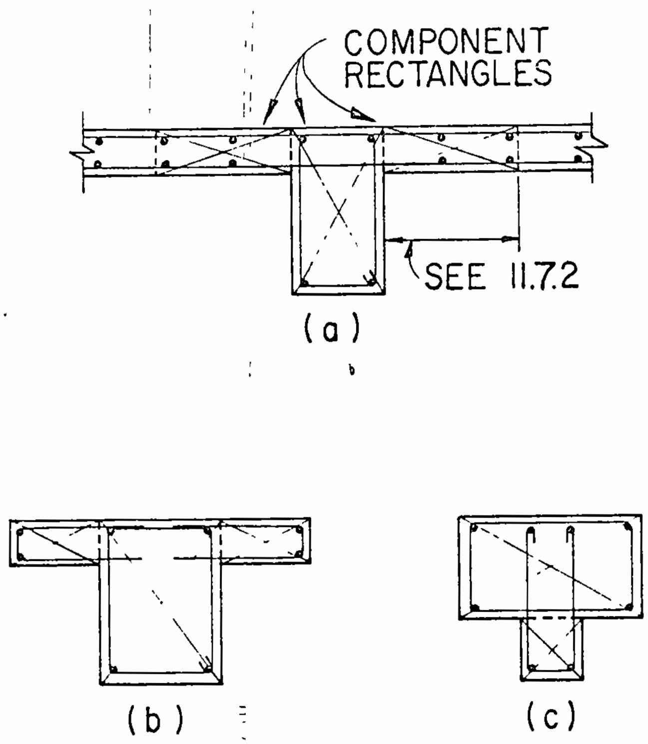


Fig. 11-5—Component rectangles for the calculation of Σx^2y

overlap. In the normal case where the closed stirrups are installed in the stem, as shown in Fig. 11-5b, the quantity Σx^2y should be taken as the x^2y values of the web extending through the overall depth of the section plus the x^2y values of the outstanding flanges. However, in the special case of cross sections such as shown in Fig. 11-5c, it would be more advantageous to install the closed stirrups in the upper wider rectangular portion. In the latter case the Σx^2y value should be taken as the x^2y value of the upper wide component rectangle plus that for the narrow vertical outstanding stem. In the case of members without web reinforcement, the component rectangles should not overlap and may be taken so as to result in the highest possible Σx^2y.

11.7.3 — Eq. (11-16) may be applied to hollow box sections with a wall thickness equal to or greater than $x/4$. If the wall thickness h is less than $x/4$, the torsional strength of a hollow box section will be less than that of a comparable solid beam. This strength reduction is reflected by the factor $4h/x$, which is conservative when compared to test results. This conservatism is desirable because hollow beams with thin walls fail in a brittle manner when subjected to torsion, as compared to a ductile mode of failure for solid beams. Also, the ratio of cracking torque to ultimate torque decreases with decreasing wall thickness.

The minimum wall thickness of $x/10$ prevents excessive flexibility and possible buckling of the wall. If h is less than $x/10$, the design of the cross section should consider the stiffness of the wall.

Box sections, in which the longitudinal torsional reinforcement consists of less than eight bars distributed around the section perimeter, should have, at each interior corner, a fillet with minimum leg size of $x/6$. When the longitudinal torsional reinforcement consists of eight or more bars distributed around the section perimeter, fillets should have a minimum leg size of $x/12$, but not necessarily more than 4 in.

11.7.4 — This provision is analogous to Section 11.2.2.

11.7.5 — In the case of pure torsion, a torsional shear stress of $2.4\sqrt{f_c'}$ is assumed to be contributed by the concrete to the ultimate torsional strength of a beam with web reinforcement. This stress corresponds to a torque equal to about 40 percent of the cracking torque of a beam without web reinforcement. Consequently, it conservatively predicts torsional cracking and failure of an unreinforced web. Such conservatism, however, is justified for two reasons. First, the torsional strength of a beam without web reinforcement may be reduced by up to one-half due to the simultaneous application of a bending moment and a torsional moment. Therefore, by specifying a torsional shear stress which corresponds to 40 percent of the cracking torque, the effect of bending moment on the torsional strength of beams without web reinforcement may be neglected. Second, any member subjected to a large torsional moment should be designed with torsion reinforcement.

In the case of combined torsion, shear, and flexure, the interaction of torsion and shear is taken into account by means of a circular interaction curve.[11.13] The square root factors in Eq. (11-17) and (11-9) were derived on this basis.[11.11,11.26]

The effect of bending is not shown explicitly in Eq. (11-17) and (11-9). However, the adoption of a torsional shear stress, v_{tc}, which corresponds to 40 percent of the cracking torque, also considers the effect of bending. Hence these equations are conservative for any combination of torsion, shear, and bending in beams without stirrups.

11.7.6 — The effect of axial tension on torque at diagonal tension cracking has not been studied experimentally. Since the theoretical effect of axial tension on the cracking torque is similar to its effect on the shear at diagonal tension cracking, the same reduction coefficient used in Eq. (11-8) was applied to torsion.

11.7.7 — Torsional reinforcement should be designed to reach the yield stress before the concrete crushes. Recent test data[11.27] indicates that for pure torsion the maximum torsional stress should be limited to $12\sqrt{f_c'}$. In the case of beams subjected to combined torsion, shear, and bending, it was considered reasonable to assume a circular interaction relationship between the maximum shear stress and the maximum torsion stress, which leads to Eq. (11-18).

11.8—Design of torsion reinforcement

11.8.1 — A simple and conservative method is given in this section. The reinforcement required to resist torsion is simply added to that required to resist shear, bending, and axial forces.

11.8.2 — Torsional reinforcement must consist of longitudinal and closed web reinforcement. The required amount of web reinforcement is given by Eq. (11-19). Present knowledge indicates that closed stirrups may be pairs of U-stirrups meeting the requirements of Section 12.13.4 and that the exterior leg of a stirrup in a spandrel beam may be extended into the slab for development, instead of hooking.

In flanged sections closed stirrups may be placed either in the largest or in all component rectangles. In the first case, the factor $x_1 y_1$ in Eq. (11-19) refers to the dimensions of the closed stirrups placed in the largest rectangle. If closed stirrups are placed in all component rectangles, a limited series of pure torsion tests indicates that Eq. (11-19) may be applied separately to each component rectangle using x, y, x_1, and y_1 for the rectangle under consideration. However, for any component rectangle,

$$v_{tc} + \frac{3 A_o \alpha x_1 y_1 f_y}{s x^2 y} \leq 12\sqrt{f_c'}$$

The overhanging flange width used in design should not exceed three times the flange thickness, and the corresponding stirrup dimension should be taken as flange width minus the concrete cover to the center of the stirrup. Flange stirrup reinforcement should be securely anchored in the web.

11.8.3 — Spacing of the stirrups must be limited to the indicated values to ensure the development of the ultimate torsional strength of the beam, prevent excessive loss of torsional stiffness after cracking, and control crack widths.

11.8.4 — Eq. (11-20) requires that the volume of longitudinal reinforcement be equal to the volume of the web reinforcement required by Eq. (11-19), unless a greater amount of longitudinal reinforcement is required to satisfy the minimum requirement given by Eq. (11-21).

11.8.5 — Longitudinal bars are required in each corner of the stirrups to provide anchorage for the legs of the stirrups, and for convenience in fabricating the reinforcement cage. Corner bars have also been found to be very effective in developing torsional strength and in controlling cracks.

11.8.6 — The required distance $b + d$ beyond the point theoretically required for torsional reinforcement is larger than those commonly used for shear and flexural reinforcement. This is desirable because torsional diagonal tension cracks develop in a spiral form.

11.9—Special provision for deep beams

11.9.1 — Entirely new provisions for the design of deep beams are included in the Code. They are based on the results of more than 250 tests,[11.16-11.17] and are intended to apply only to members loaded at the top or compression face and with a span-depth ratio less than 5. If the loads are applied through the sides or bottom of a member, design for shear should be the same as for ordinary members. The longitudinal tension reinforcement in deep beams should be extended to the support, and adequately anchored by embedment, hooks, or welding to special devices. Truss bars are not recommended.

11.9.2 — As the span-depth ratio of a member without web reinforcement decreases, its shear strength increases above the shear causing diagonal tension cracking. Thus, in Eq. (11-22) it is assumed that diagonal cracking occurs at the same nominal shear stress as for ordinary beams, but that the shear stress carried by the concrete will be greater than the shear stress causing diagonal cracking. The ratio by which it is increased is given by the first term of Eq. (11-22), which shall not exceed a conservatively established limit of 2.5.

Designers should note that shear stresses in excess of the shear stress causing diagonal cracking may result in cracking of unsightly width unless shear reinforcement is provided.

11.9.3 — Based on the analysis carried out at the critical sections specified in this paragraph, it may be determined that the member either does not need shear reinforcement, or that shear reinforcement is required, in which case it shall be used throughout the span.

11.9.4 to 11.9.6 — The inclination of diagonal cracking may be greater than 45 deg in deep beams. Therefore, both horizontal and vertical shear reinforcement is required in a deep beam. The relative amounts of horizontal and vertical shear reinforcement that are selected from Eq. (11-24) may vary, as long as limits on the minimum amount and spacing are observed.

Special attention is directed to the importance of adequate anchorage for the shear reinforcement. Horizontal web reinforcement should be extended to the support and anchored in the same manner as the tension reinforcement. Bearing on the top of deep beams should satisfy requirements similar to those for brackets and corbels.

11.10—Special provisions for slabs and footings

11.10.1 — Differentiation must be made between a long and narrow slab or footing acting as a beam, and a slab or footing subject to two-way action where failure may occur by "punching" along a truncated cone or pyramid around a concentrated load or reaction.

11.10.2 and 11.10.3—Research studied by ACI-ASCE Committee 326[11.1] indicated that the critical section for shear follows the periphery at the edge of the loaded area. The ultimate shear stress acting on this section is a function of $\sqrt{f_c'}$ and the ratio of the side dimension of a square column to the effective slab depth, h/d. The committee pointed out, however, that the variable h/d can be taken into account by assuming a pseudocritical section located at a distance $d/2$ from the periphery of the concentrated load. The ultimate shear stress is then independent of h/d and equal to $4\sqrt{f_c'}$. This method was adopted for the 1963 Code and retained in the 1971 Code because of its simplicity, especially for irregularly shaped column sections and when slab openings are present near the column.

The ultimate shear stress v_u allowed on the unreinforced section may be increased by not more than 50 percent if bars or wires are provided as shear reinforcement, or by not more than 75 percent if shearhead reinforcement is provided.

11.11—Shear reinforcement in slabs and footings

11.11.1 — Recent research has shown that shear reinforcement consisting of bars or wires can work well in thin slabs provided that such reinforcement is anchored as described in Section 12.13. Therefore, the 1971 Code has deleted the previous requirement that shear reinforcement in slabs be considered only 50 percent effective and that it not be considered effective in slabs with a thickness of less than 10 in.

The importance of anchorage details for slab shear reinforcement cannot be overemphasized. Some forms of slab "shearheads" formerly used, such as those consisting of concentric circles of V-shaped wires may not meet anchorage requirements. Extreme care should be taken to assure that shear reinforcement is accurately placed, especially in thin slabs.

It should be noted that shear reinforcement consisting of bars or wires, when used, must be designed to take all shear in excess of $2\sqrt{f_c'}$ which is 1/2 of the permissible v_c for two-way action.

11.11.2 — From recently reported test data, design procedures were formulated for shearhead reinforcement consisting of structural steel shapes in slabs at interior columns.[11.18] Tests in progress indicate that, due to torsional effects, and other peculiarities, the behavior of shearheads at a slab edge differs substantially from that at other locations.

There are three basic criteria which must be considered. First, a minimum flexural capacity must be provided to assure that the required shear capacity of the slab is reached before the flexural capacity of the shearhead is exceeded. Second, the nominal shear stress in the slab at the end of the shearhead reinforcement must be limited. Third, after these two requirements are satisfied, the designer can somewhat reduce the negative slab reinforcement in proportion to the moment contribution of the shearhead at the design section.

The assumed idealized shear distribution along an arm of a shearhead at an interior column is shown in Fig. 11-6. The shear along each of the four arms is taken as $\alpha_v V_c/4$, where α_v is the ratio of the EI value of the shearhead to the EI value of a composite section made up of a portion of the cracked slab, with a width equal to that of the column plus the effective depth of the slab in which the shearhead is embedded, and V_c is the diagonal cracking shear force. However, the peak shear at the face of the column is taken as the total shear applied per arm, $V_u/4$, minus the shear considered carried to the column by the concrete compression zone of the slab. The latter term is expressed as $(V_c/4)(1 - \alpha_v)$, so that it approaches zero for a heavy shearhead and approaches $V_u/4$ when a very light shearhead is used. Eq. (11-26) then follows from the assumption that the inclined cracking shear force V_c is about one-half the ultimate shear force V_u. In this equation, ϕ is the capacity reduction factor for flexure (0.9) and M_p is the required plastic moment capacity of each shearhead arm necessary to assure that ultimate shear is attained as the moment capacity of the shearhead is reached. The quantity l_v is the length from the center of the column to the point at which the shearhead is no longer required, and the distance $c_1/2$, is one-half the dimension of the column in the direction considered.

The test results indicated that slabs containing "under-reinforcing" shearheads failed at a nominal shear stress on a critical section at the end of the

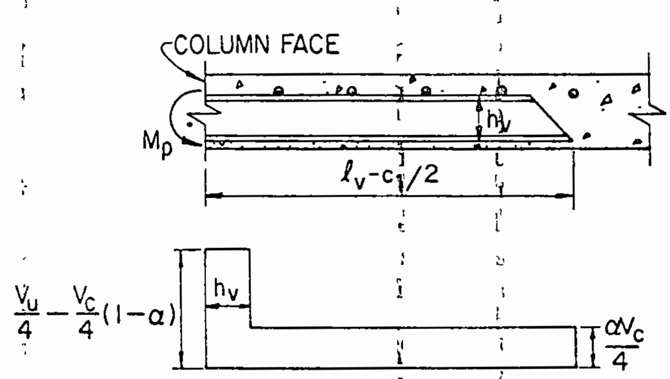


Fig. 11-6—Idealized ultimate shear on shearhead

shearhead reinforcement which was less than the 1963 Code limitation of $4\sqrt{f_c'}$. Although the use of "over-reinforcing" shearheads brought the nominal strength back to about the equivalent of $4\sqrt{f_c'}$, the limited test data suggest that a conservative design is desirable. Therefore, the ultimate shear stress is calculated as $4\sqrt{f_c'}$ on an assumed critical section located inside the end of the shearhead reinforcement.

The design critical section is shown in Fig. 11-7. It is taken through the shearhead arms three-fourths of the distance $[l_v - (c_1/2)]$ from the face of the column to the end of the shearhead. However, this assumed critical section need not be taken any closer than $d/2$ to the column.

For a practical case where the shearhead reinforcement extends beyond the column face a distance equal to the column width, the nominal stress on the section at the end of the shearhead becomes $3.3\sqrt{f_c'}$. For a very long shearhead, the minimum nominal shear stress at its end approaches the value of $3\sqrt{f_c'}$.

If the peak shear at the face of the column is neglected, and the cracking load V_c is again assumed to be about one-half of V_u, the moment contribution of the shearhead, M_v, can be conservatively computed from Eq. (11-27), in which ϕ is the factor for flexure (0.9).

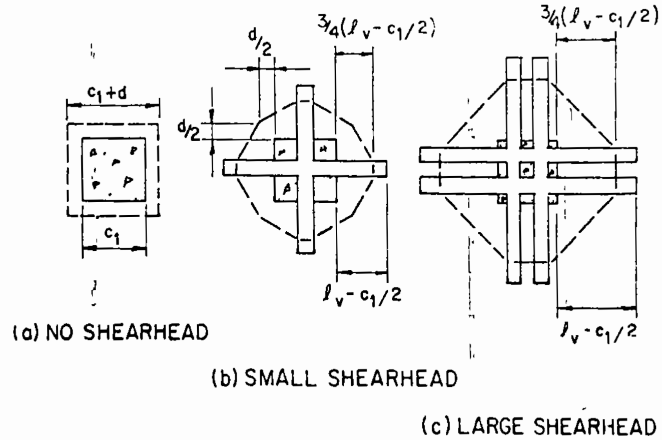


Fig. 11-7—Location of critical section

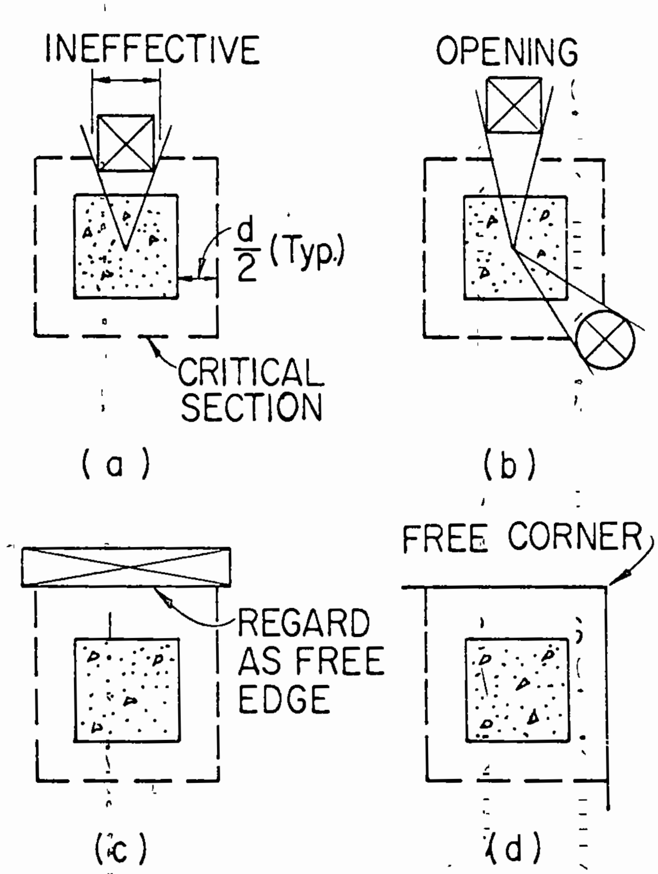


Fig. 11-8—Effect of openings and free edges

11.12—Openings in slabs

Provisions for design of openings in slabs and footings were developed in the ACI-ASCE Committee 326 report.[11.1] Some illustrations of locations of the critical section near openings and free edges are shown in Fig. 11-8. Recent research reported to ACI-ASCE Committee 426 has confirmed that these provisions are conservative.

11.13—Transfer of moments to columns

11.13.1 — Tests[11.19] have shown that the joint region of a beam to column connection in the interior of a building does not need shear reinforcement if the joint is confined on four sides by beams of approximately equal depth. However, joints without lateral confinement, such as at the exterior of a building, need shear reinforcement to prevent deterioration due to shear cracking.

For regions where strong earthquakes may occur, joints may be required to withstand several reversals of loading that develop the flexural capacity of the adjoining beams.

Tests indicated that isolated joints designed for the shear which includes the effect of the tensile and compressive forces of the adjoining beams are able to resist this severe loading.

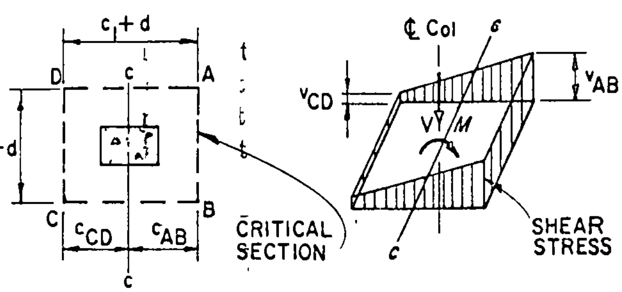
For isolated joints not subjected to load reversals, the tests indicated shear reinforcement is still needed. However, joints designed for 80 percent of the ultimate shear developed the flexural capacity of the adjoining beams.

11.13.2—In recent tests[11.20] it was found that where moment is transferred between a column and a slab, 60 percent of the moment should be considered transferred by flexure across the periphery of the critical section defined in Section 11.10.2, and 40 percent by eccentricity of the shear about the centroid of the critical section. Most of the data in the paper were obtained from tests of square columns, and little other information is available. Fig. 13-2 shows square supports having the same area as some nonrectangular members. For rectangular columns, it is logical to assume that the portion of the moment transferred by flexure increases as the width of the face of the critical section resisting the moment increases. Accordingly, this fraction was taken equal to

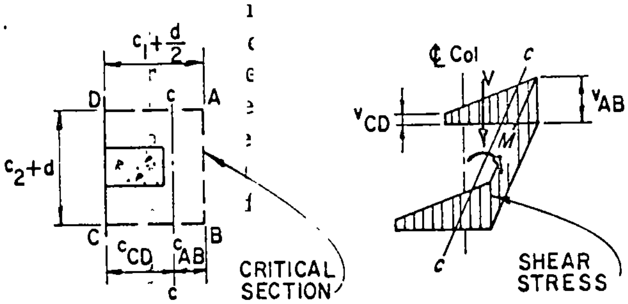
$$\frac{1}{1 + \frac{2}{3}\sqrt{\frac{c_1 + d}{c_2 + d}}}$$

where $(c_2 + d)$ is the width of the face of the critical section resisting the moment, and $(c_1 + d)$ is the width of the face at right angles to $(c_2 + d)$. The remainder is taken by shear.

Since the shear stresses shall be taken as varying linearly about the centroid of the critical section, the stress distribution is assumed as illustrated in Fig. 11-9 for an interior or exterior



(a) INTERIOR COLUMN



(b) EDGE COLUMN

Fig. 11-9—Assumed distribution of shear stress

column. The periphery of the critical section, ABCD, is determined in accord with Section 11.10.2. The resultant shear V and moment M are determined at the centroidal axis, c-c, of the critical section. The maximum shear stress may be calculated from:

$$v_{AB} = \frac{V}{A_c} + \frac{\alpha M c_{AB}}{J_c}$$

or

$$v_{CD} = \frac{V}{A_c} - \frac{\alpha M c_{AB}}{J_c}$$

where

α = fraction of moment between slab and column which is considered transferred by eccentricity of the shear about the centroid of the assumed critical section

$$= \left(1 - \frac{1}{1 + \frac{2}{3}\sqrt{\frac{c_1 + d}{c_2 + d}}}\right)$$

and where, for an interior column, A_c and J_c may be calculated from

A_c = area of concrete in assumed critical section for interior columns

$$= 2d(c_1 + c_2 + 2d)$$

J_c = property of the assumed critical section analogous to polar moment of inertia for interior columns

$$= \frac{d(c_1 + d)^3}{6} + \frac{(c_1 + d)d^3}{6} + \frac{d(c_2 + d)\ (c_1 + d)^2}{2}$$

Similar equations may be developed for A_c and J_c for the cases where columns are located at the edge or corner of a slab.

For investigating the flexural stresses resulting from transfer of bending moment between the slab and column, a conservative method assigns the unbalanced moment not transferred by shear in Section 11.13.2 to the critical slab section described in Section 13.2.4. Often designers concentrate column strip reinforcement near the column to accommodate this unbalanced moment. However, available test data seems to indicate that this practice does not increase shear strength but may be desirable to increase the stiffness of the slab-column junction.

11.14—Special provisions for brackets and corbels

11.14.1—Provisions for the design of brackets and corbels are entirely new in the 1971 Code. They are based on the results of more than 200 tests, and are intended to apply only to members where the distance between the concentrated load and the face of the support is less than d.

11.14.2-11.14.5—Recent papers[11.19,11.21] have described the development of these provisions and have given examples of their use. Eq. (11-28) and (11-29) approximate and simplify the exponential expressions published in Reference 11.21. Designs made in accordance with these provisions will be safe in flexure as well as shear.

Because brackets and corbels are relatively small members, details of bond, anchorage, and bearing are very important. The following rules resulting from experience gained during the test program are recognized as good practice in detailing when using the Code provisions:

1. The tension reinforcement should be anchored as close to the outer face as cover requirements permit. Welding the main bars to special devices such as a cross bar equal in size to the main bar is one method of accomplishing this end.

2. The depth of a corbel measured at the outer edge of the bearing area should be not less than one-half of the required total depth of the corbel.

3. The outer edge of the bearing area should not be closer than 2 in. to the outer edge of the corbel.

4. When corbels are designed to resist horizontal forces, the bearing plate should be welded to the tension reinforcement.

11.15—Shear-friction

This section is new in its entirety. Virtually all previous provisions regarding shear are intended to prevent diagonal tension failures rather than direct shear failures. The purpose of this section is to provide a design method[11.22,11.23] for the instances in which direct shear must be considered, such as in design of reinforcing details for precast concrete structures. An experimental study of shear-friction is reported in a recent paper.[11.24]

11.15.1 and 11.15.2—Uncracked concrete is very strong in direct shear; however, there is always the possibility that a crack will form in an unwanted or unexpected location. The approach is to assume that a crack will form in an unfavorable location, and then to provide reinforcement that will prevent this crack from causing undesirable consequences.

Shear stresses along a crack may be resisted by friction. Because the crack is rough and irregular, the apparent coefficient of friction may be quite high. To develop friction, however, a normal force must be present. This normal force may be obtained by placing reinforcing steel perpendicular to the assumed crack. As shear slip occurs along the crack, the irregularities of the crack will cause the opposing faces to tend to separate, stressing the reinforcing steel in tension. A balancing compressive stress will then exist in the concrete, and friction will be developed along the confined crack.

Successful application of Section 11.15 depends on proper selection of the location of an assumed crack. Some examples are illustrated in Fig. 11-10.

Fig. 11-10a is an end-bearing detail for a precast beam. Stirrups or ties may be needed to enclose the shear-friction steel and prevent a secondary failure plane from forming around the shear-friction steel.

Fig. 11-10b shows a short corbel. Depending on geometry, the shear failure mode may be either diagonal tension or shear-friction. It may be assumed that Eq. (11-30) is applicable when a/d is equal to or less than ½.

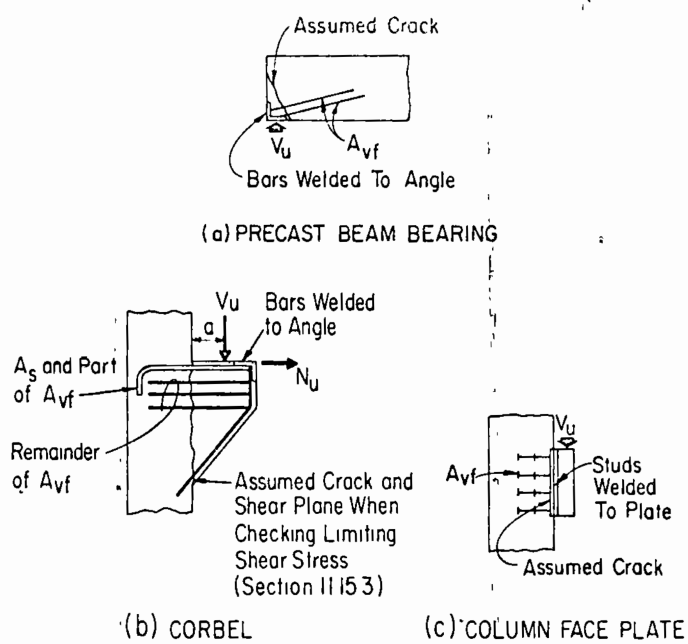


Fig. 11-10—Application of shear-friction

The limiting shear stress specified in Section 11.15.3 should be checked at the interface between the corbel and the column. Tension reinforcement, A_s, should be provided to resist the moment produced by V_u at the face of support and to resist the tensile force N_u.

Fig. 11-10c illustrates a column face plate. The headed studs function as shear-friction steel, and should be firmly anchored into the confined core of the column.

11.15.3 and 11.15.4—The required area of shear-friction reinforcement is determined from Eq. (11-30). An upper limit of $0.2f_c'$, or 800 psi, must be observed for v_u.

11.15.5-11.15.7—If tensile stresses are present across the assumed crack, reinforcement for the tension must be provided in addition to that provided for shear-friction. Unforeseen tension has caused failures, particularly in beam bearings. Causes of tension may be temperature, shrinkage, creep, growth in camber due to prestress and creep, etc.

Since the reinforcing steel acts in tension, it must have full tensile anchorage on both sides of the potential crack. Further, the shear-friction steel anchorage must engage primary steel; otherwise, a potential crack may pass between the shear-friction steel and the body of the concrete. This comment applies particularly to welded headed studs used with steel inserts for connections in precast concrete. Anchorage may be developed by bond, by a welded mechanical anchorage, or by threaded dowels and screw inserts. Space limitations often necessitate a welded mechanical anchorage.

11.16—Special provisions for shear walls

11.16.1—Horizontal shear in the plane of a wall, as a design consideration, is primarily of importance for low shear walls. The design of higher walls, particularly those with uniformly distributed reinforcement, will probably be controlled by flexural considerations. It is, therefore, essential that the flexural capacity of shear walls be computed, along with their shear capacity.

11.16.2 — The values of v_c computed from Eq. (11-32) and (11-33) at a section located a distance $l_w/2$ or $h_w/2$ (whichever is less) above the base apply to that and all sections between this section and the base. The maximum v_u at any section, including the base of the wall, shall not exceed the value given in Section 11.16.5.

11.16.3 — Eq. (11-32) and (11-33) predict the inclined cracking strength at any section through a shear wall. Eq. (11-32) corresponds to the occurrence of a principal tensile stress of approximately $4\sqrt{f_c'}$ at the centroid of the shear wall cross section. Eq. (11-33) corresponds approximately to the occurrence of a flexural tensile stress of $6\sqrt{f_c'}$ at a section $l_w/2$ above the section being investigated.

11.16.4 — Sufficient horizontal shear reinforcement is required to carry the shear stress exceeding v_c. Additional vertical reinforcement is required, because test data indicate that uniformly distributed longitudinal reinforcement as well as transverse shear reinforcement is needed in low shear walls.

11.16.5 — Although the width-to-depth ratio of shear walls is less than that for ordinary beams, recent tests on shear walls with a thickness equal to $l_w/25$ have indicated that ultimate shear stresses up to $12\sqrt{f_c'}$ can be obtained. However, the design shear stress v_u is limited by the Code to a value of $10\sqrt{f_c'}$.

References

11.1. ACI-ASCE Committee 326, "Shear and Diagonal Tension," ACI JOURNAL, *Proceedings* V. 59, No. 1, Jan. 1962, pp. 1-30; No. 2, Feb. 1962, pp. 277-334; and No. 3, Mar. 1962, pp. 352-396. Also, *ACI Manual of Concrete Practice*, Part 2, 1968.

11.2. Bresler, B., and MacGregor, J. G., "Review of Concrete Beams Failing in Shear," *Proceedings*, ASCE, V. 93, ST1, Feb. 1967, pp. 343-372.

11.3. MacGregor, James G., and Hanson, John M., "Proposed Changes in Shear Provisions for Reinforced and Prestressed Concrete Beams," ACI JOURNAL, *Proceedings* V. 66, No. 4, Apr. 1969, pp. 276-288.

11.4. Joint Committee, "Recommended Practice and Standard Specification for Concrete and Reinforced Concrete," *Proceedings*, ASCE, V. 66, No. 6, Part 2, June 1940.

11.5. Hognestad, E.; Elstner, R. C.; and Hanson, J. A., "Shear Strength of Reinforced Structural Lightweight Aggregate Concrete Slabs," ACI JOURNAL, *Proceedings* V. 61, No. 6, June 1964, pp. 643-656.

11.6. Ivey, D. L., and Buth, E., "Shear Capacity of Lightweight Concrete Beams," ACI JOURNAL, *Proceedings* V. 64, No. 10, Oct. 1967, pp. 634-643.

11.7. Kani, G. N. J., "Basic Facts Concerning Shear Failure," ACI JOURNAL, *Proceedings* V. 63, No. 6, June 1966, pp. 675-692.

11.8. Diaz de Cossio, R., and Loera, S., Discussion of "Basic Facts Concerning Shear Failure" by G. N. J. Kani, ACI JOURNAL, *Proceedings* V. 63, No. 12, Dec. 1966, pp. 1511-1514.

11.9. Kani, G. N. J., "How Safe Are Our Large Reinforced Concrete Beams?," ACI JOURNAL, *Proceedings* V. 64, No. 3, Mar. 1967, pp. 128-141.

11.10. MacGregor, James G., Discussion of "How Safe Are Our Large Reinforced Concrete Beams?" by G. N. J. Kani, ACI JOURNAL, *Proceedings* V. 64, No. 9, Sept. 1967, pp. 603-604.

11.11. ACI Committee 438, "Tentative Recommendations for the Design of Reinforced Concrete Members to Resist Torsion," ACI JOURNAL, *Proceedings* V. 66, No. 1, Jan. 1969, pp. 1-8.

11.12. Hsu, T. T. C., and Kemp, E. L., "Tentative Design Criteria for Torsion," ACI JOURNAL, *Proceedings* V. 66, No. 1, Jan. 1969, pp. 12-23.

11.13. Mattock, A. H., "How to Design for Torsion," *Torsion of Structural Concrete*, SP-18, American Concrete Institute, Detroit, 1968, pp. 469-495.

11.14. Hsu, T. T. C., "Torsion of Structural Concrete —Behavior of Reinforced Concrete Rectangular Members," *Torsion of Structural Concrete*, SP-18, American Concrete Institute, Detroit, 1968, pp. 261-306. Also, *Development Department Bulletin* D135, Portland Cement Association.

11.15. Cohen, Edward; Crist, Robert A.; dePaiva, H. A. R.; Mathey, Robert G.; O'Donnell, Keith O.; and Sbarounis, J. A., "Shear Design for Brackets and Deep Beams," presented at the 20th Fall Meeting, American Concrete Institute, Des Moines, Iowa, Nov. 1967.

11.16. dePaiva, H. A. R., and Siess, C. P., "Strength and Behavior of Deep Beams in Shear," *Proceedings*, ASCE, V. 91, ST5, Part 1, Oct. 1965, pp. 19-41.

11.17. Crist, R. A., "Shear Behavior of Deep Reinforced Concrete Beams," *Proceedings*, Symposium on the Effects of Repeated Loading of Materials and Structural Elements (Mexico City, 1966), V. 4, RILEM, Paris, 31 pp. (Published by Instituto Mexicano del Cemento y del Concreto, Mexico D. F. Mexico.)

11.18. Corley, W. G., and Hawkins, N. M., "Shearhead Reinforcement for Slabs," ACI JOURNAL, *Proceedings* V. 65, No. 10, Oct. 1968, pp. 811-824.

11.19. Hanson, N. W., and Conner, H. W., "Seismic Resistance of Reinforced Concrete Beam-Column Joints," *Proceedings*, ASCE, V. 93, ST5, Oct. 1967, pp. 533-560. Also, *Development Department Bulletin* D121, Portland Cement Association.

11.20. Hanson, N. W., and Hanson, J. M., "Shear and Moment Transfer Between Concrete Slabs and Columns," *Journal*, PCA Research and Development Laboratories, V. 10, No. 1, Jan. 1968, pp. 2-16. Also, *Development Department Bulletin* D129, Portland Cement Association.

11.21. Kriz, L. B., and Raths, C. H., "Connections in Precast Concrete Structures—Strength of Corbels," *Journal*, Prestressed Concrete Institute, V. 10, No. 1, Feb. 1965, pp. 16-47.

11.22. Birkeland, P. W., and Birkeland, H. W., "Connections in Precast Concrete Construction," ACI JOURNAL, *Proceedings* V. 63, No. 3, Mar. 1966, pp. 345-368.

11.23. Mast, R. F., "Auxiliary Reinforcement in Precast Concrete Connections," *Proceedings*, ASCE, V. 94, ST6, June 1968, pp. 1485-1504.

11.24. Hofbeck, J. A.; Ibrahim, I. A.; and Mattock, A. H., "Shear Transfer in Reinforced Concrete," ACI JOURNAL, *Proceedings* V. 66, No. 2, Feb. 1969, pp. 119-128.

11.25. Faradji, C., M. J., and Diaz de Cossio, R., "Diagonal Tension in Concrete Members of Circular Sections," *Ingenieria* (Buenos Aires), Apr. 1965, pp. 257-280. (Translated from Spanish as *Foreign Literature Study* No. 466, Portland Cement Association.)

11.26. Discussion of "Tentative Recommendations for the Design of Reinforced Concrete Members to Resist Torsion" by ACI Committee 438, ACI JOURNAL, *Proceedings* V. 66, No. 7, July 1969, pp. 576-588.

11.27. Leonhardt, F.; Walther, R.; and Schelling, A., "Torsion Tests on Reinforced Concrete Beams," *Proceedings*, International Conference on Shear, Torsion, and Bond in Reinforced and Prestressed Concrete, Coimbatore, India, Jan. 1969.

CHAPTER 12—DEVELOPMENT OF REINFORCEMENT

General

The development length concept for reinforcement replaces the dual system contained in the 1963 ACI Code. It is no longer necessary to use the flexural bond concept which placed emphasis on the computation of nominal peak bond stresses. The average bond resistance over the full development length of the bar is more meaningful, partially because all bond tests involve averaging of bond resistance, and partially because uncalculated extreme variations in local bond stresses exist near each flexural crack.

The minimum development length is based on the attainable *average* bond stress over this length. The various l_d lengths in the 1971 Code are based directly upon the 1963 Code permissible bond stresses, with the length increased approximately 20 percent for close spacing as was required for closely spaced splices in the 1963 Code, i.e., $l_d = 1.2 (f_y d_b / 4u_u)$ where u_u is bond stress permissible.

Length l_d reverts essentially to the 1963 code length on applying the 0.8 factor of Section 12.5 (d) for bars spaced at least 6 in. on centers.

It should be recognized that the development lengths specified are required in a large measure by the tendency of highly stressed bars to split thin sections of restraining concrete. A single bar embedded in a mass of concrete does not need as great a value of l_d, although a row of bars, even in mass concrete, can create a weak plane. The Code does not indicate what reductions in l_d might be appropriate in mass concrete, away from corners and edges because insufficient data are available.

In application, the development length concept requires the specified minimum lengths or extensions of reinforcement beyond all points of peak stress in the bars. Such peak stresses generally occur at the points specified in Section 12.1.3.

An understrength factor (ϕ) is not used in this chapter. The specified development lengths already include an allowance for understrength. The required lengths are the same for either the general design method or the alternate design method, since l_d is based on f_y in either case.

12.1—Development requirements—General

12.1.1 — From a point of a peak bar stress some length of bar, or anchorage, is necessary through which to develop the stress. This development length or anchorage is necessary on both sides of such peak stress points, on one side to transfer stress into and on the other to transfer stress out of the reinforcement. Often the bar continues so far on one side of the critical stress point that calculations need involve only the other side, e.g., the negative moment bar continuing on through a support to the middle of the next span.

12.1.3 — Critical sections for a typical continuous beam are indicated with a "c" or an "x" in Fig. 12-1. For uniform loading, the positive reinforcement extending into the support is more apt to be governed by the requirements of Section 12.2.3

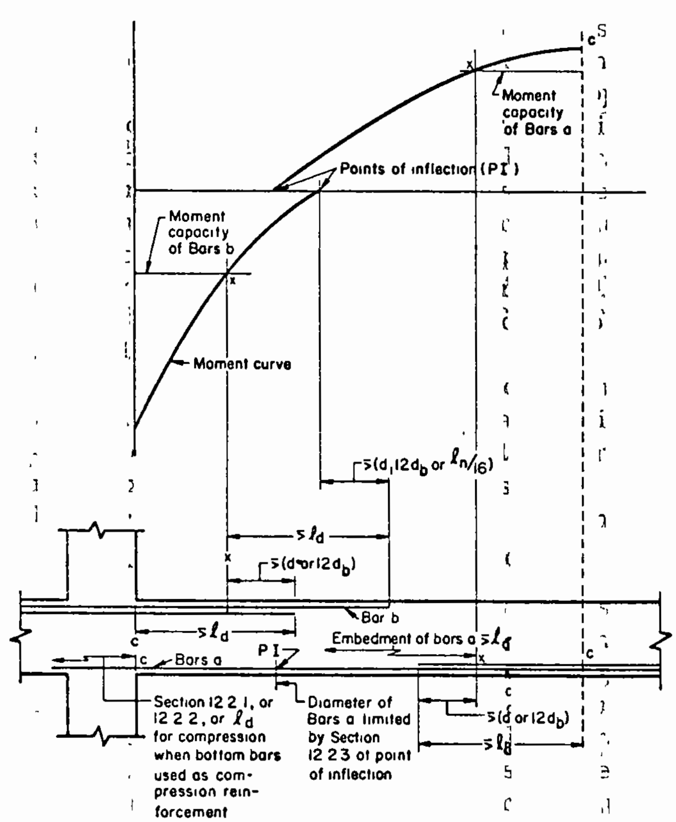


Fig. 12-1—Critical sections for development of reinforcement in a typical continuous beam

than by development length measured from a point of maximum moment or bar cutoff.

12.1.4 — The moment diagrams customarily used in practice are approximate in nature. Some shifting of the location of maximum moments may occur due to changes in loading, settlement of supports, lateral loads or other causes.

It has also been shown that a diagonal tension crack in a flexural member without stirrups may shift the location of the calculated tensile stress approximately a distance d towards a point of zero moment. When stirrups are provided this effect is less severe, although still present to some extent.

To provide for shifts in the location of maximum moments, the Code requires the extension of reinforcement a distance d or $12d_b$ beyond the point at which it is no longer required to resist flexure, except as noted.

Cutoff points of bars to meet this requirement are illustrated in Fig. 12-1.

When bars of different sizes are used in the member, the extension should be in accordance with the diameter of bar being terminated. A bar bent to the far face of the beam and continued there may logically be considered effective, in satisfying this section, to the point where the bar crosses the middepth of the member.

12.1.5 — Peak stresses exist in the remaining bars wherever adjacent bars are cut off, or bent, in tension regions. In Fig. 12-1 an "x" mark is used to indicate the peak stress points remaining in continuing bars after part of the bars have been cut off. If bars are cut off as short as the moment diagrams allow, these peak stresses become the full f_y, which requires a full l_d extension as indicated. This extension may exceed the length required for flexure.

12.1.6 — Evidence of reduced shear capacity and loss of ductility when bars are cut off in a tension zone, as in Fig. 12-1, has accumulated since 1963.

Flexural bars may not be terminated in a tension zone unless special conditions are satisfied. Flexure cracks tend to open early wherever any reinforcing bars are terminated in a tension zone. If the steel stress in the remaining bars and the shear stress are each near their allowable values, diagonal tension cracking tends to develop prematurely from these flexure cracks. Diagonal cracks are less likely to form where shear stress is low (Section 12.1.6.1). Diagonal cracks can be restrained by closely spaced stirrups (Section 12.1.6.2). A lower steel stress reduces the probability of such diagonal cracking (Section 12.1.6.3). The first and second of these specified corrective measures have been relaxed slightly and the third made more restrictive than the counterparts in the 1963 Code, after consideration of recent research. These requirements are not intended to apply to tension splices which are completely covered by Sections 7.6, 12.13.4, and the related Section 12.5.

12.2—Positive moment reinforcement

12.2.1 — Specified amounts of positive moment reinforcement must be carried into the support to care for shifting of moments as loads change.

12.2.2 — When the flexural member is part of the primary lateral load resisting system, loads greater than those anticipated in the design may cause reversal of moment at the support; some positive reinforcement should be well anchored into the support. This anchorage is required to assure ductility of response in the event of serious overstress, such as from blast or earthquake. It is not sufficient to use more bars at lower stresses. This anchorage requirement does not apply to any excess bars provided at the section.

12.2.3—At simple supports and points of inflection, such as those marked "PI" in Fig. 12-1, the diameter of the positive reinforcement must be small enough so that computed development length of the bar l_d does not exceed $M_t/V_u + l_a$, or $1.3\ M_t/V_u + l_a$ under favorable support conditions. Fig. 12-2 illustrates the use of the provision. Note that M_t is theoretical strength of the cross section without the ϕ factor and is not the external moment. The length M_t/V_u corresponds to the development length for the maximum size bar obtained from the previously used flexural bond equation $\Sigma o = V/ujd$, where u is bond stress and jd is moment arm. This anchorage requirement has been relaxed from previous Codes by crediting the available end anchorage length l_a and by including a 30 percent increase for M_t/V_u when the ends of the reinforcement are confined by a compressive reaction.

As an example, consider #11 bars used in Fig. 12-2, under conditions where l_d, as computed by

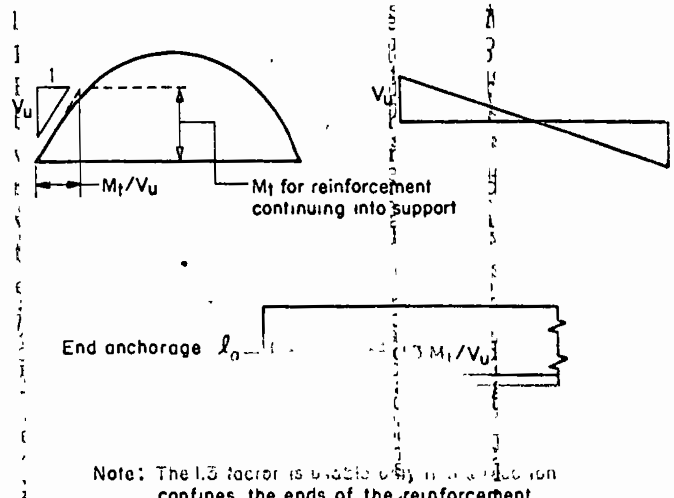


Fig. 12-2—Concept for determining the maximum size of bar at a simple support

Section 12.5, is $0.04A_b f_y/\sqrt{f_c'}$. This bar is satisfactory only if $(0.04A_b f_y/\sqrt{f_c'})$ does not exceed $1.3\, M_t/V_u + l_a$.

The l_a to be used at points of inflection is limited to the effective beam depth d or 12 bar diameters ($12d_b$), whichever is greater (Fig. 12-3). This limitation is added since test data are not available to show that long anchorage length will be fully effective in developing a bar in a short length between the point of inflection and a point of maximum stress.

12.3—Negative moment reinforcement

Fig. 12-4 and 12-5 illustrate two methods of satisfying requirements for anchorage of tension bars beyond the face of support. The anchorage value of a hook, plus an extension beyond the hook, should not be counted as greater than that of a hook unless a larger than minimum radius of bend is used, or the hook is in confined concrete.

Section 12.3.3 provides for possible shifting of the moment diagram at a nominal point of inflection, as discussed under Section 12.1.4 in this Commentary. This requirement may exceed that of Section 12.1.4, and the more restrictive of the two provisions will govern.

12.4—Special members

Special members include brackets, members of variable depth, and any others where steel stress f_s does not decrease linearly in proportion to a decreasing moment. For example, for the bracket shown in Fig. 12-6, an l_d from the support is probably less critical than the required development length for a slightly smaller f_s existing near the load point. In such a case, safety depends largely on the end anchorage provided at the loaded end. Reference 12.1 suggests a welded cross bar of equal diameter as a means of providing good end anchorage. An end hook in the vertical plane has such a large bending radius as to leave essentially a plain concrete corner which is weak. Where brackets are wide, perpendicular to the plane of the figure, and loads are not applied close to the corners, U-shaped bars in a horizontal plane provide effective end hooks.

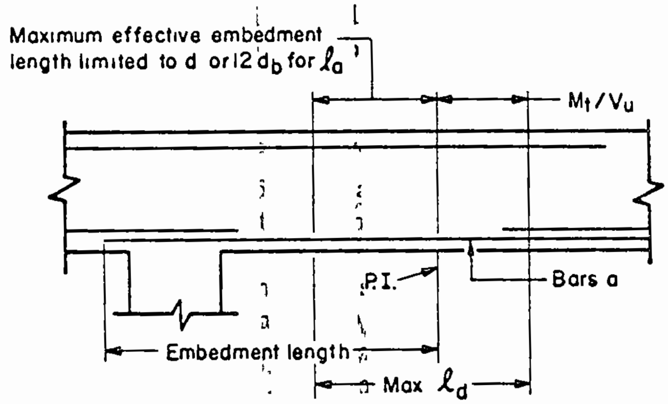


Fig. 12-3—Concept for determining the maximum size of Bar a at point of inflection

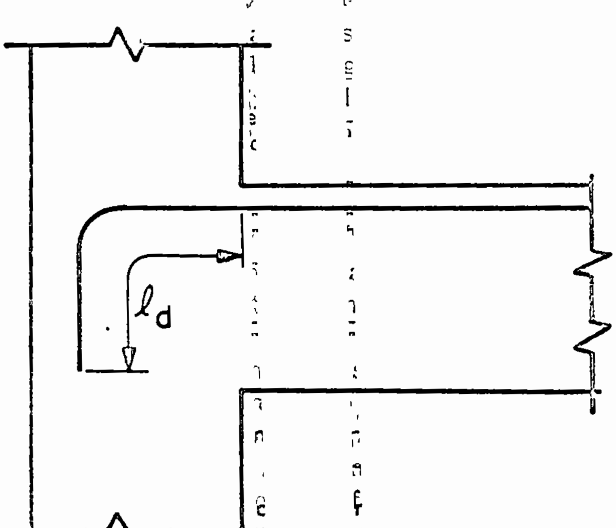


Fig. 12-4—Anchorage into exterior column

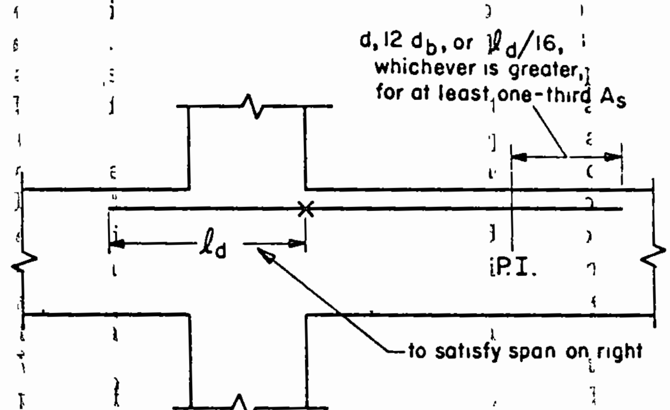


Fig. 12-5—Anchorage into adjacent beam. Note: Usually such anchorage becomes part of adjacent beam reinforcement

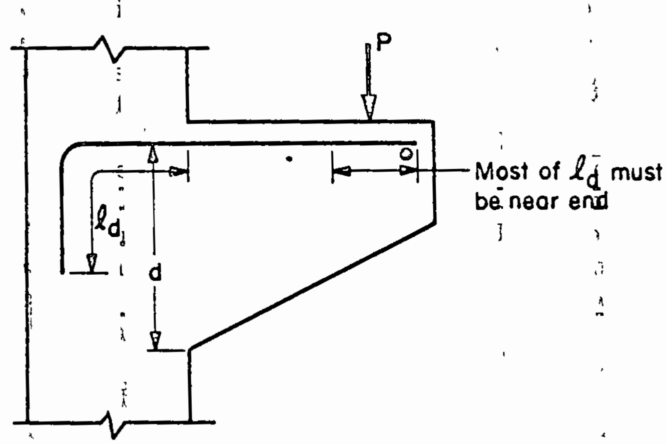


Fig. 12-6—Example of a special member largely dependent on end anchorage

12.5—Development length of deformed bars and deformed wire in tension

This section provides specific l_d requirements to be used with various bar sizes and deformed wire, including modification factors for top bars, yield stress greater than 60 ksi, lightweight concrete, wide lateral spacing between bars, and other conditions.

The factors of Section 12.5(b), 12.5(c), and 12.5(d) are multiplied together when more than one is applicable and provide a composite constant which is multiplied by the basic development length as obtained in Section 12.5(a), for either bars or deformed wire. Thus for many cases the basic l_d for deformed bars is given by $0.04A_bf_y/\sqrt{f_c'}$ but top bars with f_y greater than 60 ksi would require two special factors, as follows:

$$l_d = (0.04A_bf_y/\sqrt{f_c'}) \times 1.4 \times (2 - 60{,}000/f_y)$$

Similarly, the same bars spaced at least 6 in. on centers would permit the use of a third factor:

$$l_d = (0.04A_bf_y/\sqrt{f_c'}) \times 1.4 \times (2 - 60{,}000/f_y) \times 0.8$$

It should be noted that the basic l_d is required to be at least $0.0004d_bf_y$ for #11 or smaller bars, which controls only when bars are very small, usually less than #5 or #6. The lengths obtained from Section 12.5(a) are approximately 20 percent longer than those of the 1963 Code to provide for close spacings of bars, and when multiplied by the 0.8 factor provided in Section 12.5(d) for a wider spacing, become essentially the same as those of the 1963 Code.

Fig. 12-7 shows a typical case where alternate long and short bars are used in one layer. The spacing used in applying Section 12.5(d) for Bars y may be taken the same as for Bars x, since Bars y are developed in Length BC while Bars x are already developed in Length AB.

12.6—Development length of deformed bars in compression

The weakening effect of flexural tension cracks is not present for compression bars and usually end bearing of the bars on the concrete is beneficial. Therefore, shorter development lengths have been specified for compression than for tension.

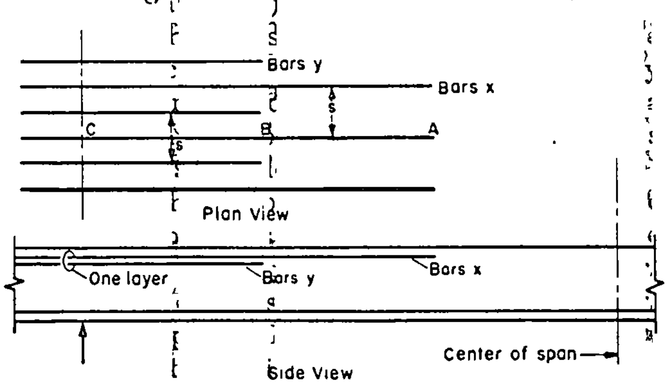


Fig. 12-7—Sketch to clarify meaning of spacing in Section 12.5(d)

12.7—Development length of bundled bars

An increased development length for individual bars is required when three or four bars are bundled together. The extra extension is needed because the grouping makes it more difficult to mobilize resistance from the "core" between the bars.

The designer should also note Section 7.4.2 relating to the cutoff points of individual bars of the bundle and Section 7.5.3 relating to splices of bundled bars.

12.8—Standard hooks

Hook anchorage tests performed at Carnegie Mellon University[12.2] have reinforced the belief, reflected in the 1956 and 1963 ACI Codes, that the anchorage capacity of a hook in mass concrete is typically about the same as that of a straight bar with the same embedment length. Hooks in structural members are often placed relatively close to a free surface where splitting forces roughly proportional to the total bar pull may determine the hook capacity. Additional restraints, therefore, were imposed in updating the anchorage values of hooks from those in the 1963 Code. When $f_c' = 3000$ psi is used, (1) the tensile stress was limited to $0.5f_y$, which is similar to that allowed for Grade 40 steel in the 1963 Code, (2) the tensile force was limited in any bar to 50,000 lb, and (3) the average bond stress over the equivalent embedment length l_e was limited to the allowable bond stress in the 1963 Code. Table 12-1 shows the maximum tensile force which is considered developed by a standard hook with $f_c' = 3000$ psi using the constants for standard hooks, ξ, of this Code. The footnotes indicate the limits which in-

TABLE 12-1—MAXIMUM TENSILE FORCE, POUNDS, DEVELOPED IN STANDARD HOOKS FOR $f_c' = 3000$ PSI

Bar size	$f_y = 60$ ksi Top bars	$f_y = 60$ ksi Other bars	$f_y = 40$ ksi Top bars	$f_y = 40$ ksi Other bars
#3	3,250*	3,250*	2,170*	2,170*
#4	5,920*	5,920*	3,940*	3,940*
#5	9,170*†	9,170*	6,110*	6,110*
#6	10,850	13,020*	8,680*	8,680*
#7	11,830	17,750*	11,830*	11,830*
#8	15,580	23,370*	15,580*	15,580*
#9	19,720	29,580*	19,720*	19,720*
#10	25,040	33,390†	25,040*	25,040*
#11	30,760†	35,880	30,760†	30,760*
#14	40,660†	40,660	40,660†	40,660
#18	48,200‡	48,200‡	48,200‡	48,200‡

*Tensile stress slightly less than $0.5f_y$ for $f_c' = 3000$ psi.
†Bond stress based on equivalent development length, l_e, about equal to or less than specified in Section 1801 of ACI 318-63 for $f_c' = 3000$ psi.
‡Total force limited to less than 50 kips for $f_c' = 3000$ psi.

fluenced the selection of ξ values. The Code no longer includes the clause authorizing hooks to be evaluated as bar extensions.

In compression, hooks are ineffective and cannot be used as anchorage.

12.9—Combination development length

The total development length of a bar simply consists of the sum of all its parts.

12.10—Development of welded wire fabric

12.10.1 — Fig. 12-8 shows the requirements for smooth wire fabric with development wholly dependent on the location of cross wires.

12.10.2 — In the referenced deformed wire fabric specifications, welds are not required to be as strong as those required for smooth wire fabric. Hence, some of the development is assigned to welds and some assigned to the length of deformed wire.

12.11—Development length of prestressing strand

The requirements are intended to provide bond integrity for the load capacity of the member. The provisions are based on tests performed on normal weight concrete members with a minimum cover of 2 in. These tests may not represent the behavior of strand in low water-cement ratio, no-slump concrete. Fabrication methods should insure consolidation of concrete around the strand with complete contact between the steel and concrete. Extra precautions should be exercised when low water-cement ratio, no-slump concrete is used. In general, this section will control only for the design of cantilever and short-span members. The requirement of doubled development length for strand not bonded to the end of the member is also based on recent research.[12.3]

The expression for development length l_d may be rewritten as:

$$l_d = \frac{f_{se}}{3} d_b + (f_{su} - f_{se}) d_b$$

where l_d and d_b are in inches, and f_{su} and f_{se} are in kips per sq in.

The first term represents the transfer length of the strand, i.e., the distance over which the strand must be bonded to the concrete to develop the prestress f_{se} in the strand. The second term represents the additional length over which the strand must be bonded so that a stress f_{su} may develop in the strand at ultimate strength of the member.

The variation of strand stress along the development length of the strand is shown in Fig. 12-9.

The expressions for transfer length, and for the additional bonded length necessary to develop an increase in stress of $(f_{su} - f_{se})$ are based on tests of members prestressed with clean, ¼, ⅜, and ½ in. diameter strands for which the maximum value of f_{su} was 275 kips sq in.[12.3, 12.4]

The transfer length of strand is a function of the perimeter configuration area and surface condition of the steel, the stress in the steel, and the method used to transfer the steel force to the concrete. Strand with a slightly rusted surface can have an appreciably shorter transfer length than clean strand. Gentle release of the strand will permit a shorter transfer length than abruptly cutting the strands.

The section does not apply to plain wires nor to end anchored tendons. The length for smooth wire could be expected to be considerably greater due to the absence of mechanical interlock. Flexural bond failure would occur with plain wire when first slip occurred.

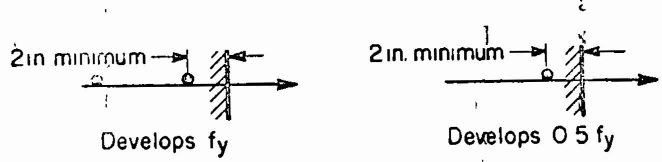


Fig. 12-8—Development of welded wire fabric

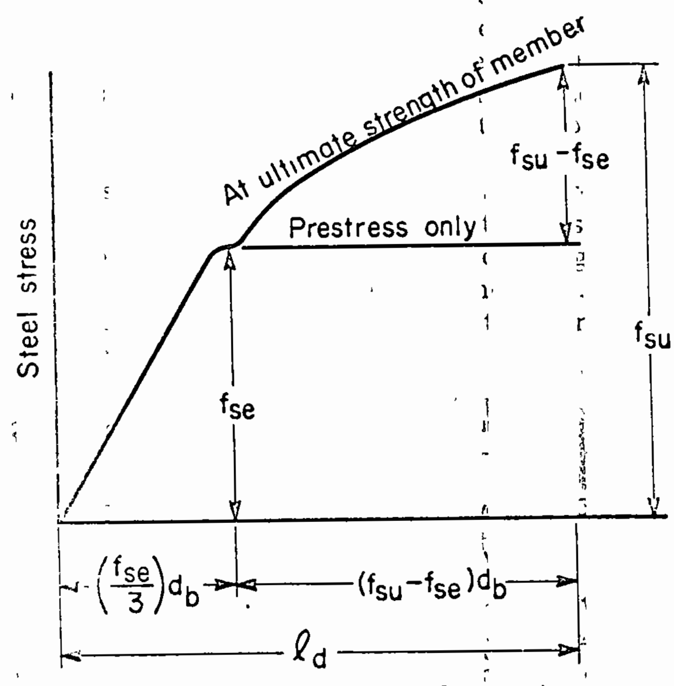


Fig. 12-9—Variation of steel stress with distance from free end of strand

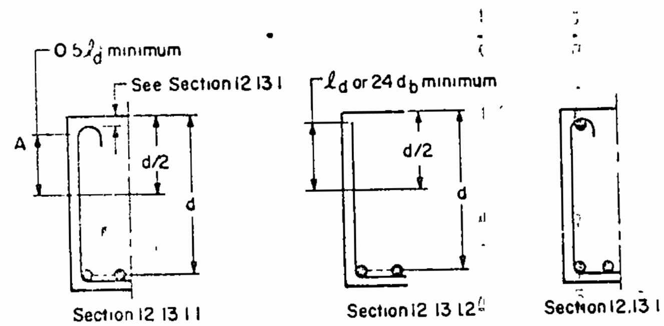


Fig. 12-10—Anchorage of U-stirrups

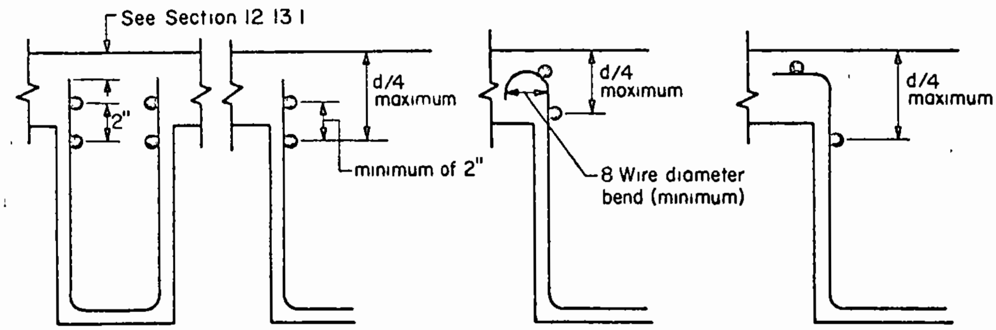


Fig. 12-11—Anchorage of welded plain wire fabric U-stirrups

12.12—Mechanical anchorage

Mechanical end anchorage can be made adequate for strength both for prestressing tendons and for bar reinforcing.

12.13—Anchorage of web reinforcement

12.13.1 — It is especially important for stirrups to be carried as close to the compression face of the member as possible because near ultimate load the flexural tension cracks penetrate deeply. The requirements for anchoring U-stirrups are illustrated in Fig. 12-10.

12.13.1.1 When a standard hook is used, $0.5l_d$ must be provided between $d/2$ and the point of tangency of the hook. Line A in Fig. 12-10 indicates the point of tangency of the hook.

This provision may require a reduction in size and spacing of web reinforcement, or an increase in the effective depth of the beam. It is important to observe this requirement so that the web reinforcement will be effective.

12.13.1.4 The requirements of the Code with regard to anchorage of welded plain wire fabric stirrups are illustrated in Fig. 12-11.

12.13.4 — A new requirement has been added to cover lapping of double U-stirrups to form closed stirrups. Note that the requirement always controls over the provisions of Chapter 7.

References

12.1. ACI Committee 408, "Bond Stress—The State of the Art." ACI Journal, *Proceedings* V. 63, No. 11, Nov. 1966, pp. 1161-1188. (This reference includes a 17-item list of more detailed references). See also Part 2 of *ACI Manual of Concrete Practice.*

12.2. Hribar, J. A., and Vasko, R. C., "End Anchorage of High Strength Steel Reinforcing Bars," ACI Journal, *Proceedings* V. 66, No. 11, Nov. 1969, pp. 875-883.

12.3. Kaar, P., and Magura, D., "Effect of Strand Blanketing on Performance of Pretensioned Girders," *Journal*, Prestressed Concrete Institute, V. 10, No. 6, Dec. 1965, pp. 20-34. Also, *Development Department Bulletin* D97, Portland Cement Association.

12.4. Hanson, N. W., and Kaar, P. H., "Flexural Bond Tests of Pretensioned Beams," ACI Journal, *Proceedings* V. 55, No. 7, Jan. 1959, pp. 783-802. Also, *Development Department Bulletin* D28, Portland Cement Association.

12.5. Kaar, P. H.; LaFraugh, R. W.; and Mass, M. A., "Influence of Concrete Strength on Strand Transfer Length," *Journal*, Prestressed Concrete Institute, V. 8, No. 5, Oct. 1963, pp. 47-67. Also, *Development Department Bulletin* D71, Portland Cement Association.

CHAPTER 13—SLAB SYSTEMS WITH MULTIPLE SQUARE OR RECTANGULAR PANELS

In ACI 318-63 and earlier ACI Codes, the design of reinforced concrete slab systems has been handled in different categories because of the history of development of various types of slabs.[13.1] Chapter 13 of ACI 318-71 represents a major change from this practice in that all slab systems reinforced for structural stresses in more than one direction, with or without beams between supports, are designed on the basis of the same fundamental principles. The design methods given in this chapter are based on analysis of the results of an extensive series of tests[13.2-13.9] and the well established performance record of various slab systems.

Much of Chapter 13 is concerned with the selection and distribution of flexural reinforcement. It is, therefore, advisable before discussing the various rules for design, to caution the designer that the vital problem related to safety of the slab system is the transmission of load from the slab to the columns by flexure, torsion, and shear. Design criteria for shear and torsion in slabs are given in Sections 11.10 through 11.13.

13.1—Scope and definitions

13.1.1 — The fundamental design principles contained in Chapter 13 are applicable to all planar structural systems subjected to transverse loads. However, some of the specific design rules, as well as historical precedents, limit the types of structures to which Chapter 13 is applicable. General characteristics of slab systems which may be designed according to Chapter 13 are described in this section. These include "flat slabs," "flat plates," "two-way slabs," and "waffle slabs." True "one-way slabs," slabs reinforced to resist flexural stresses in only one direction, are excluded.

13.1.4 — A panel, by definition, includes all flexural elements between column center lines. Thus, the column strip includes the beam, if any.

13.1.5 — For monolithic or fully composite construction, the beams include portions of the slab as flanges. Two examples of the rule in this section are provided in Fig. 13-1.

13.2—Design procedures

13.2.1 — This section permits a designer to base a design directly on fundamental principles of structural mechanics, provided he can demonstrate explicitly that all safety and serviceability criteria are satisfied. The design of the slab may be achieved through the combined use of classic solutions based on a linearly elastic continuum, numerical solutions based on discrete elements, or yield-line analyses, including, in all cases, evaluation of the stress conditions around the supports in relation to shear and torsion as well as flexure. The designer must consider that the design of a slab system involves more than its analysis, and justify any deviations in physical dimensions of the slab from common practice on the basis of his knowledge of the expected loads and the reliability of the calculated stresses and deformations of the structure.

13.2.2 — Two design methods are specified in detail in Chapter 13. The Direct Design Method is similar to the empirical method of the previous Codes, but now applies to slabs with beams as well as to flat slabs and flat plates. Some modifications have been made to the limitations for use of the direct design method (see Section 13.3.1). The Equivalent Frame Method is comparable to the elastic analysis of the previous Codes and also is applicable to slabs with and without beams.

13.2.4 — This section is concerned primarily with slab systems without beams. Tests and experience have shown that, unless special measures are taken to resist the torsional and shear stresses, all reinforcement resisting that part of the moments to be transferred to the column by flexure should be placed between lines that are half the slab or drop panel thickness, $h/2$, on each side of the column. The calculated stresses in the slab around the column must conform to the requirements of Sections 11.10, 11.11, 11.12, and 11.13.

13.3—Direct design method

The direct design method consists of a set of rules for the proportioning of slab and beam sec-

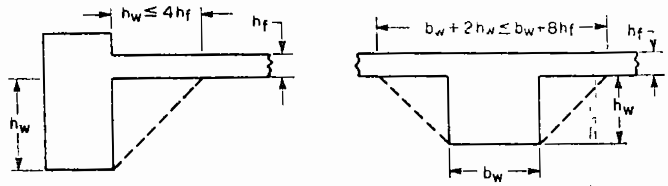


Fig. 13-1—Examples of the portion of slab to be included with the beam under Section 13.1.5

tions to resist flexural stresses. The rules have been developed to satisfy the safety requirements and most of the serviceability requirements simultaneously. Methodologically, the direct design method compares with the "empirical method" for the flat slabs included in preceding editions of the ACI Code. However, the range of applicability of the method has been much extended in relation to the "empirical method."

The direct design method involves three fundamental steps, as follows:

1. Determination of the total design moment (Section 13.3.2)
2. Distribution of the total design moment to design sections for negative and positive moment (Section 13.3.3)
3. Distribution of the negative and positive design moments to the column and middle strips and to the beams, if any (Section 13.3.4)

13.3.1 *Limitations* — The direct design method was developed from considerations of theoretical procedures for the determination of moments in slabs with and without beams, requirements for simple design and construction procedures, and precedents supplied by performance of slab systems. Consequently, the slab system to be designed using the direct design method must conform to the limitations in this section.

13.3.1.1 The primary reason for the limitation in this section is the magnitude of the negative moments at the interior support in a structure with only two continuous spans. The rules given for the direct design method assume tacitly that the slab system at the first interior negative moment section is neither fixed against rotation nor discontinuous.

13.3.1.2 If the ratio of the two spans (long span/short span) of a panel exceeds two, the slab resists the moment in the shorter span essentially as a one-way slab. The limiting ratio has been increased from the 1.33 limit stipulated for the "empirical method" of the 1963 Code.

13.3.1.3 The limitation in this section is related to the possibility of developing negative moments at or near midspan. The limiting variation in span lengths has been increased from that for the 1963 "empirical method."

13.3.1.4 In keeping with previous practice, the designer is permitted to offset the columns within specified limits from a regular rectangular array.

13.3.1.5 This section relates to the effect of pattern loads. No limitation of ratio of live load to dead load was placed on the use of the "empirical method" in 1963, but as stated above, the limitations on span ratios and length to width ratios have been relaxed.

13.3.1.6 The elastic distribution of moments will deviate significantly from those assumed in the direct design method unless the given requirements for stiffness are satisfied.

13.3.1.7 The designer is permitted to use the direct design method even if the structure does not fit the limitations in this section, provided that he can show by analysis that the particular limitation does not apply to that structure. For example, in the case of a slab system carrying a nonmovable load (such as a water reservoir in which the load on all panels is expected to be the same), the designer does not have to satisfy Section 13.3.1.5.

13.3.2 *Total static design moment for a span*

13.3.2.1 Eq. (13-2) follows directly from Nichol's derivation[13.10] with the simplifying assumption that the reactions are concentrated along the faces of the support perpendicular to the span considered. In general, the designer will find it expedient to calculate static moments for two adjacent half panels, which include a column strip with a half middle strip along each side as shown in Fig. 13-2.

13.3.2.2 If the calculated value of l_n is less than $0.65l_1$, the span should be considered as being $0.65l_1$. If a supporting member does not have a rectangular cross section, it is to be treated as a square support having the same area, as illustrated in Fig. 13-3.

13.3.3 *Negative and positive design moments* — The rules for assigning the total design moment to the negative and positive design moments are summarized in Fig. 13-4. The proportions are based on three-dimensional analytical studies of elastic distribution of moments in various slab configurations tempered by the distributions of moments that have been in use.

13.3.3.3 The term α_{ec} in the equations of this section is the flexural stiffness of an equivalent exterior column K_{ec} relative to the flexural stiffness of the slab and the beams, if any. The calculation of K_{ec} is described in Section 13.4.1.5, and rules are given for the calculation of K_c and K_s in Sections 13.4.1.3 and 13.4.1.4, all in connection with the equivalent frame method. Since the use of the direct design method is limited by the requirements of Section 13.3.1, it is permissible to make certain simplifications in the calculation of α_{ec} for use in Section 13.3.3.3, as follows:

1. The stiffness of the slab-beam K_s may be calculated using a uniform cross section between column center lines, disregarding the requirement of Section 13.4.1.4, and the increase in column stiffness K_c provided by the capital may be neglected, disregarding the requirement of Section 13.4.1.3. Since both of these simplifications lead to less stiff elements, one should not be made without the other in order to minimize the change in relative stiffness.

2. The requirement in the last sentence of Section 13.4.1.5 may be waived in calculations of α_{ec} for use with the direct design method.

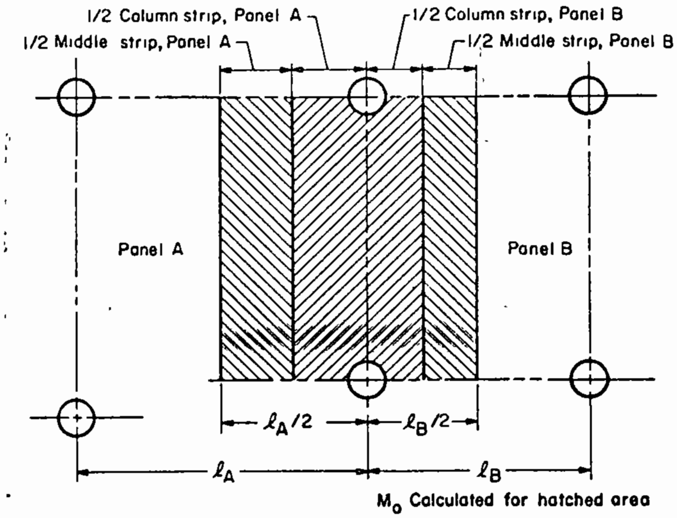


Fig. 13-2—Suggested area to be considered in calculating static moments under Section 13.3.2.1

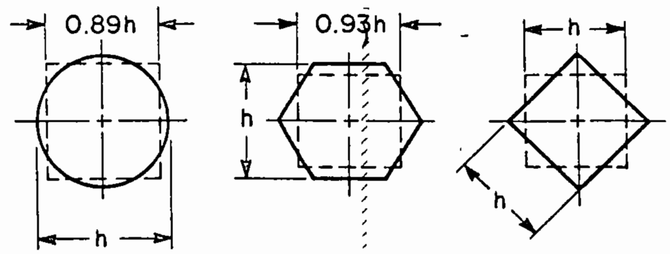


Fig. 13-3—Examples of equivalent square section for nonrectangular supporting members

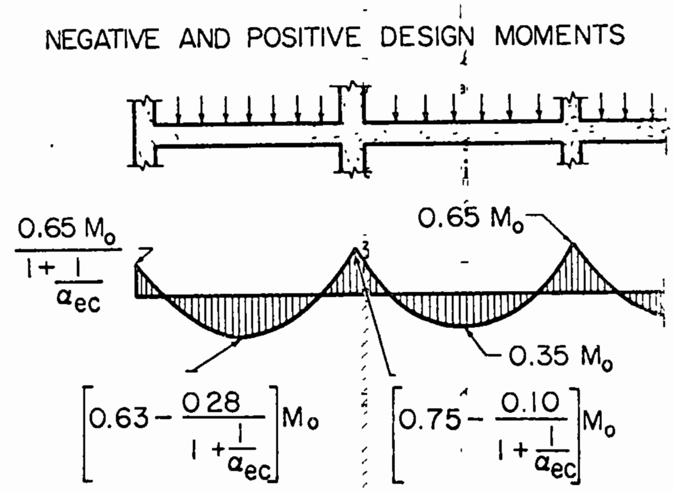


Fig. 13-4—Summary of rules for dividing the total static moment or the total design moment into negative and positive design moments

3. If a corner column is the same size as an adjacent exterior column, it will be acceptable to use for the corner column the value of α_{cc} computed for the adjacent exterior column. This approximation will usually lead to somewhat smaller exterior negative design moments and somewhat larger positive and interior negative design moments than the more rigorous analysis required for the equivalent frame method.

The detailing of the reinforcement transferring the moment from the slab to the exterior column is critical to both the performance and the safety of flat slabs without edge beams or equivalent cantilever slabs. This reinforcement must be placed in accordance with Section 13.2.4.

13.3.3.4 The differences in slab moment on either side of a column or other type of support must be accounted for in the design of the support. If an analysis is made to distribute unbalanced moments, flexural stiffness may be obtained on the basis of the gross plain concrete section of the members involved.

13.3.4 *Design moments and shears on column and middle strips and beams* — The rules given in this section for assigning moments to the middle strip, column strip, and beams, if any, are based on studies of moments in linearly elastic slabs with different beam stiffnesses[13.11] tempered by the moment coefficients that have been used successfully in the past.

Where walls are used as supports along column lines, they can be regarded as very stiff beams with an $\alpha_1 l_2/l_1$ value greater than one. Where the exterior support consists of a wall perpendicular to the direction in which moments are being determined, β_t may be taken as zero if the wall is of masonry without torsional resistance, and β_t may be taken as 2.5 for a concrete wall with great torsional resistance which is monolithic with the slab.

13.3.4.2 The effect of the torsional stiffness parameter β_t is to assign all of the exterior negative design moment to the column strip, and none to the middle strip, unless the beam torsional stiffness is high relative to the flexural stiffness of the supported slab. In the definition of β_t, the shear modulus has been taken as $E_{cb}/2$.

13.3.4.7 and 13.3.4.8 The tributary area for the shear on an interior beam is shown shaded in Fig. 13-5. If the stiffness for the beam $\alpha_1 l_2/l_1$ is less than one, the shear on the beam may be obtained by linear interpolation. For such cases, the beams framing into the column will not account for all the shear force applied on the column. The remaining shear force will produce shear stresses in the slab around the column which must be checked in the same manner as for flat slabs, as required by Section 13.3.4.8. Section 13.3.4.7 does not apply to the calculation of torsional moments on the beams. These moments must be based on the calculated flexural moments acting on the sides of the beam.

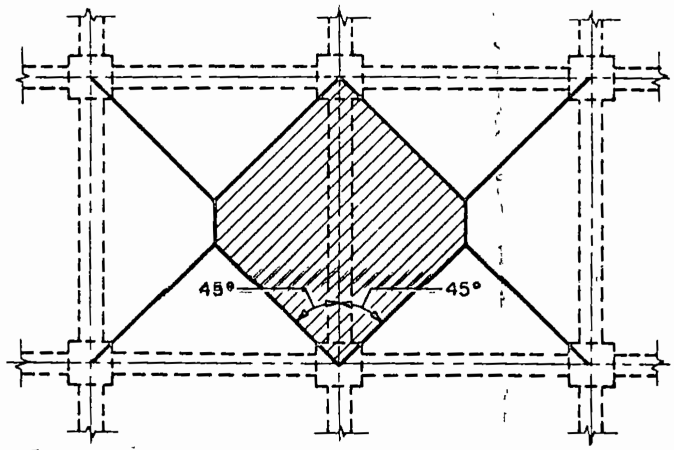


Fig. 13-5—Tributary area for shear on an interior beam

13.3.4.10 Design moments perpendicular to, and at the edge of, the slab structure must be transmitted to the supporting columns or walls. Torsional stresses caused by the moment assigned to the slab should be investigated.

13.3.5 *Moments in columns and walls*

13.3.5.2 Eq. (13-3) refers to two adjoining spans, with one span longer than the other, the full load applied on the longer span and only the dead load applied on the shorter span. The term α_{cc} refers to the flexural stiffness of the columns between the two spans.

In the calculation of α_{cc} it is permissible to make the simplifications given as Items (1) and (2) in the commentary on Section 13.3.3.3. In addition, when applying Eq. (13-3) to determine the moment in an exterior column in a direction parallel to the edge of the panel, it is conservative, and therefore permissible, to use in Eq. (13-3) the value of α_{cc} computed for the adjacent interior column, provided the columns are of the same size.

13.3.6 *Provisions for effects of pattern loading* — The requirements in this section limit the possible increases in moment as a result of pattern loadings at the service load level and are based on analytical and experimental studies summarized in Reference 13.12. Values of the column flexural stiffness α_{min}, required to limit the increase in bending moment caused by pattern loads to less than 33 percent, are listed in Table 13.3.6.1 of the Code as a ratio of the flexural stiffness of the slab system. If the columns of a particular structure do not satisfy the required value of α_{min}, the positive moment in the slab must be increased in accordance with Eq. (13-4).

When applying Eq. (13-4) to moments in the half column strip parallel to an exterior panel edge, it is conservative, and therefore permissible, to use in Eq. (13-4) the value of α_c computed for the adjacent interior column, provided the columns are of the same size.

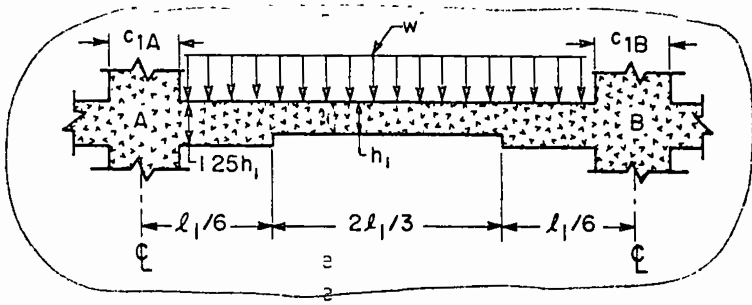
13.4—Equivalent frame method

The equivalent frame method involves the representation of the three-dimensional slab system by a series of two-dimensional frames which are then analyzed for loads, either vertical or horizontal, acting in the plane of the frames. The moments so determined at the critical design sections of the frame are distributed to the slab sections in accordance with Section 13.3.3.

The equivalent frame method is comparable to the "elastic analysis" for flat slabs of previous ACI Codes. On the basis of studies reported in References 13.13, 13.14, and 13.15, the equivalent frame method has been devised to provide a better representation in two dimensions of a three-dimensional system through the stratagem of defining flexural stiffnesses which reflect the torsional rotations possible in the three-dimensional system.

13.4.1.1 The application of the requirements of this section to a regular structure is illustrated

TABLE 13-1—MOMENT DISTRIBUTION CONSTANTS*



Column dimension		Uniform load FEM = Coef. $(wl_2l_1^2)$		Stiffness factor†		Carryover factor	
$\frac{c_{1A}}{l_1}$	$\frac{c_{1B}}{l_1}$	M_{AB}	M_{BA}	k_{AB}	k_{BA}	COF_{AB}	COF_{BA}
	0.00	0.083	0.083	4.00	4.00	0.500	0.500
	0.05	0.083	0.084	4.01	4.04	0.504	0.500
	0.10	0.082	0.086	4.03	4.15	0.513	0.499
0.00	0.15	0.081	0.089	4.07	4.32	0.528	0.498
	0.20	0.079	0.093	4.12	4.56	0.548	0.495
	0.25	0.077	0.097	4.18	4.88	0.573	0.491
	0.30	0.075	0.102	4.25	5.28	0.603	0.485
	0.35	0.073	0.107	4.33	5.78	0.638	0.478
	0.05	0.084	0.084	4.05	4.05	0.503	0.503
	0.10	0.083	0.086	4.07	4.15	0.513	0.503
	0.15	0.081	0.089	4.11	4.33	0.528	0.501
0.05	0.20	0.080	0.092	4.16	4.58	0.548	0.499
	0.25	0.078	0.096	4.22	4.89	0.573	0.494
	0.30	0.076	0.101	4.29	5.30	0.603	0.489
	0.35	0.074	0.107	4.37	5.80	0.638	0.481
	0.10	0.085	0.085	4.18	4.18	0.513	0.513
	0.15	0.083	0.088	4.22	4.36	0.528	0.511
0.10	0.20	0.082	0.091	4.27	4.61	0.548	0.508
	0.25	0.080	0.095	4.34	4.93	0.573	0.504
	0.30	0.078	0.100	4.41	5.34	0.602	0.498
	0.35	0.075	0.105	4.50	5.85	0.637	0.491
	0.15	0.086	0.086	4.40	4.40	0.526	0.526
	0.20	0.084	0.090	4.46	4.65	0.546	0.523
0.15	0.25	0.083	0.094	4.53	4.98	0.571	0.519
	0.30	0.080	0.099	4.61	5.40	0.601	0.513
	0.35	0.078	0.104	4.70	5.92	0.635	0.505
	0.20	0.088	0.088	4.72	4.72	0.543	0.543
0.20	0.25	0.086	0.092	4.79	5.05	0.568	0.539
	0.30	0.083	0.097	4.88	5.48	0.597	0.532
	0.35	0.081	0.102	4.99	6.01	0.632	0.524
	0.25	0.090	0.090	5.14	5.14	0.563	0.563
0.25	0.30	0.088	0.095	5.24	5.58	0.592	0.556
	0.35	0.085	0.100	5.36	6.12	0.626	0.548
0.30	0.30	0.092	0.092	5.69	5.69	0.585	0.585
	0.35	0.090	0.097	5.83	6.26	0.619	0.576
0.35	0.35	0.095	0.095	6.42	6.42	0.609	0.609

*Applicable when $c_1/l_1 = c_2/l_2$. For other relationships between these ratios, the constants will be slightly in error.

†Stiffness is, $K_{AB} = k_{AB}E\frac{l_2h_1^3}{12l_2}$, and $K_{BA} = k_{BA}E\frac{l_2h_1^3}{12l_1}$.

in Fig. 13-6. The shaded areas represent the extent of one interior and one exterior frame in a given direction. See also Reference 13.15.

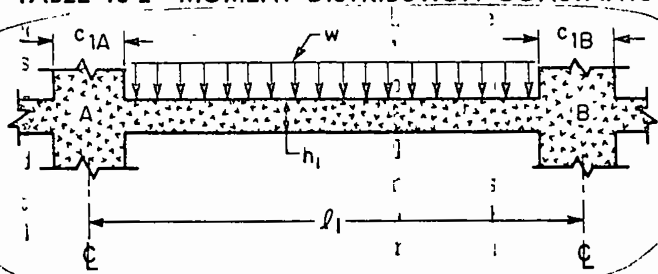
13.4.1.2-13.4.1.7 These sections permit simplifications which may be used in analyzing the frame. The use of a high-speed computer may make it easy to use a comprehensive analysis without resorting to these simplifications.

13.4.1.4 This section stipulates a finite moment of inertia for the slab-beam from the face of the column or capital to the center line of the support to account for the flexibility of the slab on the "sides" of the column in that portion of the span. Fixed-end moments for uniform load, stiffness factors, and carryover factors for slabs without beams and having a varying moment of inertia in accordance with Section 13.4.1.4 are listed in Tables 13-1 and 13-2 (Reference 13.16).

13.4.1.5 This section modifies the column flexural stiffness to account for the torsional flexibility of the slab. The intent of this section is illustrated by the simplified physical model in Fig. 13-7 which represents a column AB, extending above and below the slab, with a portion of the slab CD attached thereto. A moment M applied along CD will cause a torsional rotation of the "cross beam" CD as well as a flexural rotation of the column. Thus, the rotational restraint on the slab-beam which spans in a direction perpendicular to AB and CD, depends on both the torsional rotation of CD and the flexural rotation of AB.

The overall flexibility, $1/K_{ec}$, of the composite element shown in Fig. 13-7 is assumed to be the sum of the flexibility of the columns, $1/\Sigma K_c$, and the torsional flexibility of the "beam", $1/K_t$, as given in Eq. (13-5) in the Code.

TABLE 13-2—MOMENT DISTRIBUTION CONSTANTS*



Column dimension		Uniform load FEM = Coef. ($wl_2l_1^2$)		Stiffness factor†		Carryover factor	
$\frac{c_{1A}}{l_1}$	$\frac{c_{1B}}{l_1}$	M_{AB}	M_{BA}	k_{AB}	k_{BA}	COF_{AB}	COF_{BA}
	0.00	0.088	0.088	4.78	4.78	0.541	0.541
	0.05	0.087	0.089	4.80	4.82	0.545	0.541
	0.10	0.087	0.090	4.83	4.94	0.553	0.541
0.00	0.15	0.085	0.093	4.87	5.12	0.567	0.540
	0.20	0.084	0.096	4.93	5.36	0.585	0.537
	0.25	0.082	0.100	5.00	5.68	0.606	0.534
	0.30	0.080	0.105	5.09	6.07	0.631	0.529
	0.05	0.088	0.088	4.84	4.84	0.545	0.545
	0.10	0.087	0.090	4.87	4.95	0.553	0.544
	0.15	0.085	0.093	4.91	5.13	0.567	0.543
0.05	0.20	0.084	0.096	4.97	5.38	0.584	0.541
	0.25	0.082	0.100	5.05	5.70	0.606	0.537
	0.30	0.080	0.104	5.13	6.09	0.632	0.532
	0.10	0.089	0.089	4.98	4.98	0.553	0.553
	0.15	0.088	0.092	5.03	5.16	0.566	0.551
0.10	0.20	0.086	0.094	5.09	5.42	0.584	0.549
	0.25	0.084	0.099	5.17	5.74	0.606	0.546
	0.30	0.082	0.103	5.26	6.13	0.631	0.541
	0.15	0.090	0.090	5.22	5.22	0.565	0.565
	0.20	0.089	0.094	5.28	5.47	0.583	0.563
0.15	0.25	0.087	0.097	5.37	5.80	0.604	0.559
	0.30	0.085	0.102	5.46	6.21	0.630	0.554
	0.20	0.092	0.092	5.55	5.55	0.580	0.580
0.20	0.25	0.090	0.096	5.64	5.88	0.602	0.577
	0.30	0.088	0.100	5.74	6.30	0.627	0.571
	0.25	0.094	0.094	5.98	5.98	0.598	0.598
0.25	0.30	0.091	0.098	6.10	6.41	0.622	0.593
0.30	0.30	0.095	0.095	6.54	6.54	0.617	0.617

*Applicable when $c_1/l_1 = c_2/l_2$. For other relationships between these ratios, the constants will be slightly in error.

†Stiffness is $K_{AB} = k_{AB}E\,\frac{l_2h_1^3}{12l_1}$, and $K_{BA} = k_{BA}E\,\frac{l_2h_1^3}{12l_1}$.

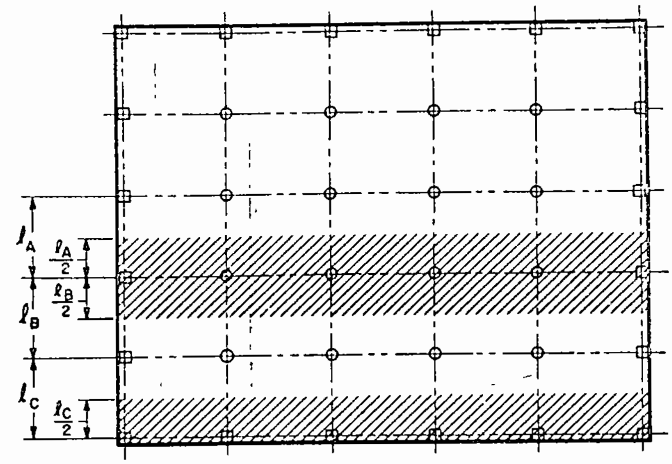


Fig. 13-6—Area to be considered as equivalent frame under Section 13.4

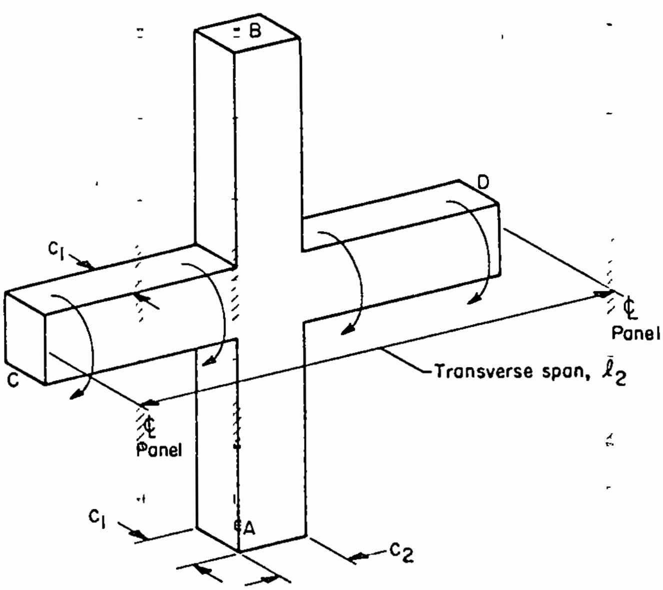


Fig. 13-7—Simplified physical model illustrating the intent of Section 13.4.1.5

The stiffness K_c is based on the length of the column from middepth of slab above to middepth of slab below and its moment of inertia, which is computed on the basis of its cross section, taking into account the increase in stiffness provided by the capital, if any. The column is assumed to be infinitely stiff over the depth of the slab.

The increase in column stiffness provided by the capital may be disregarded when using the direct design method.

Where the exterior edge of a slab system is supported on a concrete wall monolithic with the slab, the flexibility of the wall should replace the column flexibility $1/K_c$ in Eq. (13-5) and the torsional flexibility $1/K_t$ of the wall may be assumed to be zero in the same equation. Where the exterior edge of a slab system is supported on an unreinforced masonry wall, K_{cc} may be taken as zero.

The computation of the torsional stiffness K_t requires several simplifying assumptions. If no

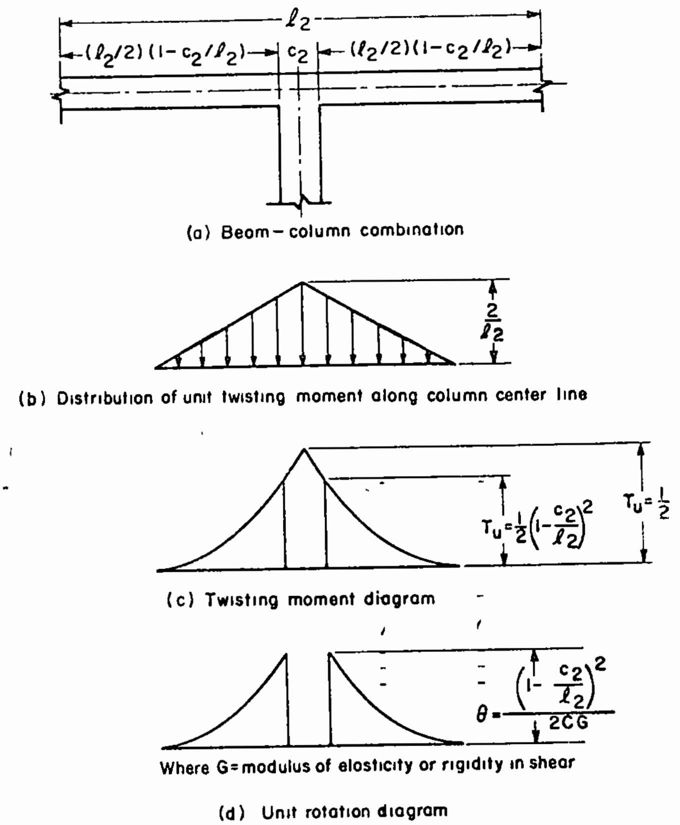


Fig. 13-8—Assumed distribution of unit twisting moment along column center line, twisting moment diagram, and unit rotation diagram

beam frames into the column, a portion of the slab equal to the width of the column or capital is assumed as the effective beam. If a beam frames into the column, T-beam or L-beam action is assumed, with the flanges extending on each side of the beam a distance equal to the projection of the beam above or below the slab but not greater than four times the thickness of the slab. Furthermore, it is assumed that no torsional rotation occurs in the beam over the width of the support.

Studies of three-dimensional analyses of various slab configurations suggest that a reasonable value of the torsional stiffness can be obtained by assuming a moment distribution along the beam CD in Fig. 13-7 which varies linearly from a maximum at the center of the column to zero at the middle of the panel. The assumed distribution of unit twisting moment along the column center line, the twisting moment diagram, and the resulting unit rotation diagram are shown in Fig. 13-8.

Eq. (13-6) is an approximate expression for the stiffness of the torsional member, based on the results of three-dimensional analyses of various slab configurations. The development of this expression and the assumptions on which it is based, as well as the justification for its use, are discussed in References 13.13, 13.14, and 13.15.

The term C is a property of the cross section having the same relationship to the torsional rigidity of a noncircular cross section as does the polar moment of inertia for a circular cross section. The term

$$\left(1 - 0.63\,\frac{x}{y}\right)\frac{x^3 y}{3}$$

in Eq. (13-7) is a conservatively low approximation to the value of C for a rectangular section, assuming elastic behavior (see for example Reference 13.16). Since the value of C obtained by summing the values for each of the component rectangles making up a section will always be less than the theoretically correct value, it is appropriate to subdivide the cross section in such a way as to result in the highest possible value of C.

If a panel contains a beam parallel to the direction in which moments are being determined, the value of K_t obtained from Eq. (13-6) may lead to equivalent column stiffnesses which are too low. In such cases, the value of K_t given by Eq. (13-6) should be increased as follows:

$$K_{ta} = K_t \frac{I_{sb}}{I_s}$$

where K_{ta} is the increased torsional stiffness due to presence of parallel beam, I_s is the moment of inertia of a width of slab equal to the full width between center lines of panels, not considering the parallel beam, and I_{sb} is the moment of inertia of the width of slab used for the calculation of I_s, but including the contribution of that portion of the beam stem extending above or below the slab; the beam as defined in Section 13.1.5 does not apply in this calculation.

13.4.1.8 The use of only three-quarters of the full design live load for maximum-moment loading patterns is based on the fact that maximum negative and maximum positive live load moments cannot occur simultaneously and that redistribution of maximum moments is thus possible before failure occurs. This procedure, in effect, permits some local overstress under the full design live load if it is distributed in the prescribed manner, but still insures that the ultimate capacity of the slab system after redistribution of moment is not less than that required to carry the full design dead and live loads on all panels.

13.4.2 — This section corrects the negative design moments to the face of the supports. The correction is modified at an exterior support in order not to result in undue reductions in the exterior negative moment. Fig. 13-2 illustrates several equivalent rectangular supports for use in establishing faces of supports for design with nonrectangular supports.

13.4.5 — This section is a holdover from many previous Codes and is based on the principle that if two different methods are prescribed to obtain a particular answer, the Code should not require a value greater than the least acceptable value. Due to the long satisfactory experience with designs having total static design moments not exceeding those given by Eq. (13-2), it is considered that these values are satisfactory for design.

13.5—Slab reinforcement

The requirement that the center to center spacing of reinforcement be not more than two times the slab thickness applies only to the reinforcement in solid slabs, and not to that in joists or waffle slabs. This limitation is intended to insure slab action and reduce cracking and to provide for the possibility of loads concentrated on small areas of the slab.

13.5.2 — Bending moments in slabs at spandrel beams can be subject to great variation. If the beam should be built solidly into a masonry wall, the slab could be fixed. If no wall, the slab could be largely simply supported. This regulation provides for unknown conditions that might normally occur in a structure.

13.6—Openings in the slab system

See Commentary for Section 11.12 and Fig. 11-8.

References

13.1. Sozen, M. A., and Siess, C. P., "Investigation of Multiple-Panel Reinforced Concrete Floor Slabs: Design Methods—Their Evolution and Comparison," ACI Journal, *Proceedings* V. 60, No. 8, Aug. 1963, pp. 999-1028.

13.2. Hatcher, D. S.; Sozen, M. A.; and Siess, C. P., "Test of a Reinforced Concrete Flat Plate," *Proceedings*, ASCE, V. 91, ST5, Oct. 1965, pp. 205-231. 14.1

13.3. Guralnick, S. A., and Fraugh, R. W., "Laboratory Study of a Forty-Five-Foot Square Flat Plate Structure," ACI Journal, *Proceedings* V. 60, No. 9, Sept. 1963, pp. 1107-1185. 14.2

13.4. Hatcher, D. S.; Sozen, M. A.; and Siess, C. P., "Test of a Reinforced Concrete Flat Slab," *Proceedings*, ASCE, V. 95, ST6, June 1969, pp. 1051-1072. 14.3

13.5. Jirsa, J. O.; Sozen, M. A.; and Siess, C. P., "Test of a Flat Slab Reinforced with Welded Wire Fabric," *Proceedings*, ASCE, V. 92, ST3, June 1966, pp. 199-224. 14.4

13.6. Gamble, W. L.; Sozen, M. A.; and Siess, C. P., "Test of a Two-Way Reinforced Floor Slab," *Proceedings*, ASCE, V. 95, ST6, June 1969, pp. 1073-1096. 14.5

13.7. Vanderbilt, M. D.; Sozen, M. A.; and Siess, C. P., "Tests of a Modified Reinforced Concrete Two-Way Slab," *Proceedings*, ASCE, V. 95, ST6, June 1969, pp. 1097-1116. 14.6

13.8. Shewmaker, R. E.; Xanthakis, M.; and Sozen, M. A., "Very Small Scale Reinforced Concrete Multi-Panel Flat Slabs," Civil Engineering Studies, Structural Research Series No. 265, University of Illinois, June 1963, 171 pp.

13.9. Xanthakis, M., and Sozen, M. A., "An Experimental Study of Limit Design in Reinforced Concrete Flat Slabs," Civil Engineering Studies, Structural Research Series No. 277, University of Illinois, Dec. 1963.

13.10. Nichols, J. R., "Statical Limitations Upon the Steel Requirement in Reinforced Concrete Flat Slab Floors," *Transactions*, ASCE, V. 77, 1914, pp. 1670-1736.

13.11. Gamble, W. L.; Sozen, M. A.; and Siess, C. P., "Measured and Theoretical Bending Moments in Reinforced Concrete Floor Slabs," Civil Engineering Studies, Structural Research Series No. 246, University of Illinois, June 1962.

13.12. Jirsa, J. O.; Sozen, M. A.; and Siess, C. P., "Pattern Loadings on Reinforced Concrete Floor Slabs," *Proceedings*, ASCE, V. 95, ST6, June 1969, pp. 1117-1137.

13.13. Corley, W. G.; Sozen, M. A.; and Siess, C. P., "The Equivalent-Frame Analysis for Reinforced Concrete Slabs," Civil Engineering Studies, Structural Research Series No. 218, University of Illinois, June 1961

13.14. Jirsa, J. O.; Sozen, M. A.; and Siess, C. P., "The Effects of Pattern Loadings on Reinforced Concrete Floor Slabs," Civil Engineering Studies, Structural Research Series No. 269, University of Illinois, July 1963.

13.15. Corley, W. G., and Jirsa, J. O., "Equivalent Frame Analysis for Slab Design," ACI JOURNAL, *Proceedings* V. 67, No. 11, Nov. 1970, pp. 875-884.

13.16. Seely, F. B., *Advanced Mechanics of Materials*, John Wiley and Sons, Inc., New York, 1932, p. 174.

CHAPTER 14—WALLS

14.1—Structural design of walls

This section requires that walls be designed to resist all loads to which they are subjected, including lateral loads, eccentric axial loads, and wind forces. In general, this chapter applies to walls spanning vertically.

When the resultant load falls within the middle third of the cross section the loads may be considered "reasonably concentric" and the wall may be designed by the empirical method described in this chapter. When the resultant load falls outside of the middle third of the cross section, the wall *must* be designed for combined bending and axial load according to Section 10.16 considering the wall to be a compression member with flexure. In either case, design calculations for walls may be performed using the strength design method of the Code or the alternate design method of Section 8.10.

Eccentric loads and lateral loads are used to determine the total eccentricity of the axial load P_u. If the eccentricity does not exceed $h/6$, Section 14.2 may be applied. The design is then performed considering P_u as an axial load.

14.2—Empirical design of walls

The change in the equation including the change from cubed to squared makes the resulting capacities for Chapters 10 and 14 relatively compatible for members loaded in the middle third of the thickness (see plotted values in Fig. 14-1.) The load capacity or P_u determined from Eq. (14-1) corresponds to the design or factored load.

The detailed provisions of this chapter regarding thickness and minimum reinforcement are similar to those in previous Codes.

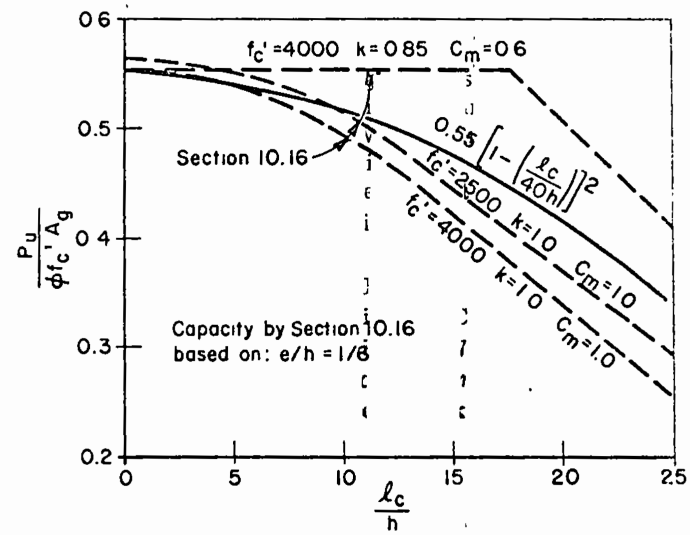


Fig. 14-1 — Empirical design of walls versus Section 10.16

CHAPTER 15—FOOTINGS

15.1—Scope

15.1.1 — While the provisions of this chapter apply to isolated footings which support a single column or wall, most of the provisions are generally applicable to combined footings and mats which support several columns or walls or a combination thereof.

15.1.2 — Additional information concerning combined footings and mats is provided in the Commentary to Section 15.10.

15.2—Loads and reactions

15.2.1 - 15.2.3 — These sections require that "footings shall be proportioned to sustain the applied loads and induced reactions" which include loads, moments, and shears that have to be resisted at the base of the footing or pile cap.

15.2.4 — After the allowable soil pressure or the allowable pile capacity has been determined by principles of soil mechanics and in accord with the local general building code, the size of the base area of a footing on soil or the number and arrangement of the piles shall be established on the basis of service loads (D, L, W, and E) in whatever combination that will govern the design, and without applying any load factors.

In cases in which eccentric loads or moments must be considered, the extreme soil pressure or pile reaction obtained from this loading shall be within the allowable values. Similarly, the resultant reactions due to service loads combined with moments and/or shears caused by wind or earthquake loads may not exceed the increased values that may be permitted by the local building code.

To design a footing or pile cap for strength, the contact pressure or pile reaction due to the factored loading (see Section 8.1) must be determined. This can be done by either of two methods:

1. By factoring the column loads and moments and applying them to the footing or piles to find the soil pressure or pile reaction due to this factored loading condition. In the case of a single concentrically loaded spread footing, the soil reaction q_s due to the factored loading is $q_s = U/A_f$ where U is the factored concentric load to be resisted by the footing, and A_f is the base area of the footing as determined by the principles stated previously using the unfactored loads and the allowable soil pressure.

2. Another approach permits finding the soil pressure or pile reaction due to the factored loading, and then the base area or number of piles, directly by multiplying the allowable soil pressure or pile reaction by the ratio of the factored loads to the unfactored loads. The base area of the footing (or the pile reaction) can then be determined by dividing the factored loads U by the modified allowable soil pressure q_s (or pile reaction). This procedure is particularly useful since the factored loads are used in designing the columns and walls.

In both cases it is important to keep in mind that q_s is only a calculated reaction to the factored loading, used to produce in the footing or pile cap the same required strength conditions regarding flexure, shear, and development length of reinforcement as in any other member.

In the case of eccentric loadings, where conditions produced by various load factors may cause eccentricities and reactions that are different from those obtained for unfactored loads, it is advisable to apply the appropriate load factors directly to the bearing pressures and reactions obtained from the unfactored loading.

Conditions covered by Eq. (9-3) will rarely produce a governing soil pressure but will guard against uplift.

For footings supported on piles, the ratio of the pile reaction under factored loading to the allowable pile capacity is the same as the ratio of soil reaction due to factored loads to the allowable soil pressure.

15.3—Sloped or stepped footings

15.3.1 and 15.3.2 — Attention is drawn to the size of the cap or pedestal (upper portion of a stepped footing) to insure that it is large enough to satisfy design requirements and to design and construct it in such a manner that monolithic action of the upper and lower sections of the footing is obtained.

15.4 — Bending moment

15.4.1 - 15.4.3 — These sections which state the critical locations where maximum bending moments and development length of reinforcement are to be computed for the various conditions of design, are identical with their counterparts in the 1963 Code.

One of the important changes in the 1963 Code was the requirement that reinforcement be provided to resist the total computed moment and bond rather than the 85 percent permitted by the 1956 Code.

Prior to the 1941 Code, it was standard practice to design footings for loading of a trapezoidal area. In 1941 the more correct method described in Section 15.4.1 was adopted. This gave appreciably higher computed moment and bond stresses than the trapezoidal assumption. Although no trouble had been reported with the lesser reinforcement provided under the old method, this reduction was reconsidered and eliminated in the 1963 Code for several reasons, some of which are:

1. There is no theoretical justification for reduced reinforcement.

2. Extensive laboratory tests, simulating footings on soil and piles, give no indication that a reduction in steel is justifiable.

3. The 1963 Code requirements for shear permitted thinner footing slabs than previous Codes. The higher concrete and steel strengths now available permit more flexible footings than previously used.

4. Since most footings are permanently buried and not accessible to inspection, their capacity or performance must be ensured.

5. The safety or load factors of footings should be at least equal to those of other parts of the structure.

15.4.4. — As in previous Codes, the reinforcement in the short direction of rectangular footings must be distributed so that an area of steel [see

Eq. (15-1)] is provided in a band width b, centered about the column center line.

The remaining steel area required in the short direction is to be distributed equally over the two segments outside of band width b, one-half to each segment.

To simplify placement of reinforcement, the intent of the requirement stated above may be satisfied by using an increased area of bars equal to

$$A_{sa} = \frac{2A_{st}\beta}{\beta + 1}$$

where A_{st} is the total area of steel in the short direction required by the Code, and by distributing the bars at equal spacings over the length of the footing.

15.5—Shear and development of reinforcement

15.5.1 — The shear capacity of footings must be determined for the more severe of the two conditions stated in Section 11.10. The critical section is located from the section specified in Section 15.5.1. However, a steel base plate can be so stiffened as to move the reference section to the edge of the plate.

The first condition considers the footing essentially as a wide beam with a potential crack extending in a plane across the entire width. This case is analogous to a conventional beam, and the design proceeds accordingly.

The second condition assumes two-way action, with potential cracking along the surface of a truncated cone or pyramid. The critical section for this case is taken at a distance $d/2$ out from the periphery of the column, pier, pile or other concentrated load as compared with the distance d used prior to the 1963 Code.

When the basic strength design method of the Code is used, shear design is accomplished by using the soil bearing pressure q_s obtained from the factored loads and by using a shear capacity reduction factor $\phi = 0.85$ with the appropriate equations of Chapter 11.

Where necessary, perimeter shear around individual piles shall be investigated in accord with Section 11.10. If shear perimeters overlap, the critical perimeter b_o should be taken as that portion of the smallest envelope of individual shear perimeters which will actually resist the critical shear for the group under consideration. One such situation is illustrated in Fig. 15-1.

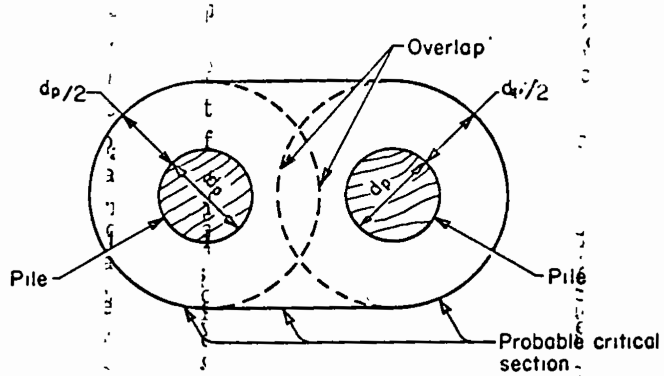


Fig. 15-1—Modified critical section for perimeter shear with overlapping critical perimeters

When the alternate design method of Section 8.10 is applied, the soil bearing pressures or pile reactions are those caused by the service loads. Note that $\phi = 1.0$ is used in the equations of Chapter 11 in this case. Note also that in the 1971 Code, the limiting maximum stresses for shear must be reduced to 50 percent of those computed in accordance with Chapter 11, when footings are designed by Section 8.10. When lateral loads due to wind or earthquake are included in the governing load combination for footings, advantage may be taken of a 25 percent reduction in required capacity, in accordance with Section 8.10.5.

When the intensity of the bearing pressures or pile reactions is not evenly distributed over the base area of the footing, the cross section beyond which the soil pressures or reactions are greatest should be used for proportioning the footing and selecting reinforcement.

15.5.4 — Development lengths are to be calculated according to Chapter 12, regardless of whether the design for strength is made using the method of Chapter 10 or the alternate method of Section 18.10.

15.5.5 — This section is in principle the same as its counterpart in the 1963 ACI Code, except that the "6 in." dimension, previously used to determine whether or not a pile shall be considered as contributing to shear, has been replaced by $d_p/2$. Here, d_p is pile diameter in inches.

15.6—Transfer of stress at base of column or pedestal

15.6.1 — This section states in general that all forces applied at the column base must be transmitted to the footing. All tensile forces, whether created by uplift, moment, or other reason, must be resisted entirely by reinforcement.

Past Codes required the stress in longitudinal bars to be transferred by steel. For compression, this Code is more liberal since steel must be provided only for the excess force which cannot be taken in bearing.

15.6.2 - 15.6.3 — Compressive stresses may, within the permissible limits, be transmitted to the footing by bearing. For strength design, permissible bearing stresses on the actual loaded area will be equal to $0.85\phi f_c'$ (where $\phi = 0.7$), when the loaded area is equal to the [illegible] which it is supported. Therefore, the permissible bearing stresses will be approximately $0.6f_c'$ of the weaker concrete on the common loaded area.

In the common case of a column bearing on a footing larger than the column, bearing stress must be checked on the bottom of the column and the top of the footing. The permissible bearing stress on the column will normally be $0.6f_c'$ of the column concrete. Strength in the lower part of the column must be checked since the column steel cannot be considered effective at the joint because the stress in it is not developed for some distance above the joint unless dowels are provided or the steel is extended into the footing. The permissible bearing stress on the footing may be increased in accordance with Section 10.14.2 and will usually be two times $0.85\phi f_c'$ or approximately $1.2f_c'$ of the footing concrete. The compressive force which exceeds that developed by the permissible concrete bearing stress on the bottom of the column or on the top of the footing must be carried by dowels or extended reinforcement.

Similar procedures apply where a column rests on a pedestal and where a pedestal rests on a footing.

For the alternate design method of Section 8.10, permissible bearing stresses are 50 percent of those for the strength design method.

15.6.5 — The Code does not require that all bars in a column be extended through and be anchored into the footing. However, steel at least equal to $0.005A_g$ or an equal area of properly spliced dowels must extend into the footing with proper anchorage, where A_g is the supported column cross section. At least four bars or dowels must be used, and the diameter of the dowels shall not exceed that of the column bars by more than 0.15 in.

15.6.7 — The shear-friction method given in Chapter 11 may be used to check for transfer of transverse forces from the base of a column to a footing. Shear keys or other devices must be used where additional capacity for transverse force is needed.

15.6.8 — Lap splices of #14 or #18 column bars to dowels from the footing are specifically permitted by this section. The dowel bars must be smaller in size than the #14 or #18 bars. The development length for the dowel into the column must correspond to that required for the #14 or #18 bar.

This provision is an exception to Chapter 7 of the 1971 Code, which prohibits lap splicing of these large bar sizes. This exception results from many years of successful experience with the lap splicing of #14 and #18 column bars with footing dowels and recognition of the practical advantages of this construction method.

15.7—Pedestals and footings of unreinforced concrete

15.7.3—The crack-free entity of a pile cap transmitting concentrated loads of large magnitude is extremely important, because stress redistribution due to the loss of effectiveness of a pile can be critical. For this reason, use of unreinforced concrete for pile caps has been prohibited by the 1971 Code.

15.10—Combined footings and mats

15.10.1 — Any reasonable assumption with respect to the distribution of soil pressure or pile reactions can be used as long as it is consistent with the type of structure and the properties of the soil, and conforms with established principles of soil mechanics (see Section 15.1). Similarly, as prescribed in Section 15.2.4 for isolated footings, the base area or pile arrangement of combined footings and mats must be determined using the unfactored forces and/or moments transmitted by the footing to the soil, considering allowable soil pressures and allowable pile reactions.

Design methods using factored loads and capacity reduction factors can be applied to combined footings or mats, regardless of the soil pressure distribution.

A report of ACI Committee 436, "Suggested Design Procedures for Combined Footings and Mats," was published in the ACI JOURNAL *Proceedings* V. 63, No. 10, Oct. 1966, pp. 1041 to 1056; also *ACI Manual of Concrete Practice*. That report should be used for guidance where appropriate.

References

15.1. Kramrisch, Fritz, and Rogers, Paul, "Simplified Design of Combined Footings," *Proceedings*, ASCE, V. 87, No. SM5, Oct. 1961, p. 19.

15.2. Teng, Wayne C., *Foundation Design*, Prentice-Hall Publishing Co., Englewood Cliffs, N. J., 1962, 466 pp.

CHAPTER 16—PRECAST CONCRETE

General

Precast concrete is simply reinforced concrete cast in units which are assembled and fastened together on the job. The regular provisions for reinforced concrete thus apply except for a few specific variations.

The practice related to design and construction of precast concrete structural elements differs in some respects from that for cast-in-place concrete structural members. Where provisions for cast-in-place concrete apply equally to precast concrete, they have not been repeated in this chapter. Similarly, items related to prestressed concrete in Chapter 18 and composite concrete construction in Chapter 17 that apply for precast concrete (or overrule similar sections given elsewhere in the Code) are not stated in this chapter.

More detailed recommendations concerning precast concrete have been provided in the following ACI standards and reports by ACI committees:

1. "Recommended Practice for Manufactured Reinforced Concrete Floor and Roof Units (ACI 512-67)"
2. "Suggested Design of Joints and Connections in Precast Structural Concrete," by ACI Committee 512, Aug. 1964
3. "Minimum Requirements for Thin-Section Precast Concrete Construction (ACI 525-63)"

In contrast to the 1963 Code, items related to precast concrete concerning aggregates, concrete cover for reinforcement and splicing of reinforcement are now consolidated in other sections of the Code. Larger size aggregate for precast concrete is no longer specifically permitted but waivers of size limits are allowed under Section 3.3.2.

There is no longer a minimum size stated for columns as in previous versions of the ACI Code. However, while fire ratings do not fall within the purview of the ACI Code, the designer is cautioned that the general building code must be consulted in this respect. The actual fire rating is a function of both the cover over the reinforcement and the relationship between the volume of a member and its exposed surface area. In using very small columns, therefore, due consideration must be given to fire ratings. Similarly, when chemical and corrosive considerations are necessary, the designer must use judgment tempered by available test data.

16.1—Scope

The term "under plant controlled conditions" does not specifically imply that precast members must be manufactured in a plant. Structural elements precast at the job site will also qualify under this section if the control of form dimensions, placing of reinforcement, quality control of concrete, and curing procedure are equal to that normally expected in a plant.

The tolerances required by Section 7.3 are considered as a minimum acceptable standard for precast concrete.

16.2—Design

While the design for precast elements is developed as for cast-in-place elements (except as provided in Chapters 17 and 18), special considerations are necessary for precast members. The loads imposed on precast elements during the period from casting to placement may be greater than the actual service loads. Handling procedures may often cause permanent deformations. Hence, special care must be given to the methods of transporting and erecting of precast members.

It is also vitally important to consider the effects of connections and interconnected elements with respect to precast members. The structural behavior of precast elements may differ substantially from that of similar members that are cast-in-place and are monolithic. Design of joints to transmit forces due to shrinkage, creep, temperature, elastic deformation, wind forces, and earthquake forces require particular care in precast elements. The details of such joints are extremely important.

Cracking of concrete must be controlled so that the load carrying capacity will not be reduced.

Precast members may be designed using the general strength design provisions of the Code or the alternate design method provided for in Section 8.10.

16.3—Bearing and nonbearing wall panels

Bearing and nonbearing walls that are precast must be designed according to Chapters 10 or 14. The alternate design method of Section 8.10 may also be used.

16.4—Details

The safety factor of 4.0 provides at least 100 percent impact possibilities during erection. The intent of this factor is to avoid a brittle failure of the insert. It is not intended that any additional ϕ factor or load factor be used.

Reinforcement should be provided adjacent to lifting devices to withstand all temporary forces.

The Code requires adequate performance at service loads and adequate strength under factored loads. However, handling loads should not produce permanent stresses, strains, cracking, nor deflections inconsistent with the provisions of the Code.

The foregoing remarks apply also to prestressed and composite construction.

16.6—Transportation, storage, and erection

It is important that all temporary erection connections, bracing, and shoring be shown on the shop drawings.

CHAPTER 17—COMPOSITE CONCRETE FLEXURAL MEMBERS

17.1—Scope

17.1.1 — The scope of this chapter is intended to include all types of composite concrete flexural members including composite single-T or double-T members, box sections, folded plates, lift slabs, and other structural elements, all of which should conform to the provisions of this chapter. In some cases with fully cast-in-place concrete, it may be necessary to design the interface of consecutive placements of concrete as required for composite members. Composite structural steel-concrete members are not covered in this chapter, since such sections are covered in the "Specification for the Design, Fabrication and Erection of Structural Steel for Buildings," published by the American Institute of Steel Construction (AISC).

17.1.2 — The Code in its entirety applies to composite concrete flexural members except as specifically modified in Chapter 17. For instance, deep composite sections shall be designed in accord with Section 10.7 and 11.9. When composite concrete flexural members are subjected to axial loads, Section 10.8 and 10.9, and 10.10 (or 10.11) apply. The alternate design method of Section 8.10 may also be used.

17.2—General considerations

17.2.1 — This section permits the designer to use any or all of the various components in supporting the load in the most expeditious manner.

17.2.3 — Tests to destruction indicate no difference in strength of shored and unshored members.

17.2.5 — The extent of cracking permitted is dependent on such factors as environment, aesthetics, and occupancy. In addition, composite action must not be impaired.

17.2.6 — The premature loading of precast elements can cause excessive deflections as the result of creep and shrinkage. This is especially so at early ages when the moisture content is high and the strength low.

The transfer of shear by direct bond is essential if excessive deflection from slippage is to be prevented. A shear key is an added mechanical factor of safety but it cannot operate until slippage occurs.

17.3—Shoring

The provisions of Sections 9.5.5.1 and 9.5.5.2 must be considered with regard to deflections of shored and unshored members. Before shoring is removed it should be ascertained that the strength and serviceability characteristics of the structure will not be impaired.

17.5—Horizontal shear

17.5.1 — The full transfer of horizontal shear between segments must be ensured by contact stresses or properly anchored ties, or both.

17.5.2 — Tests[17.1] indicate that horizontal shear does not present a problem in T-beams when the portion below the flange is designed to resist the vertical shear, the interfaces of the components are rough and minimum ties are provided according to Section 17.6.1. The ties must be extended across the joint and fully anchored on both sides of the joint in accord with Section 12.13. These considerations may be used with other segmental shapes.

17.5.3 — The calculated horizontal shear stress represents the force per unit area of interface. When the design is developed using the alternate method of Section 8.10, V_u is the shear due to dead and live load calculated using unity load and ϕ factors. Also, when this method is used and a combination of gravity loads and wind and earthquake loads govern, Section 8.10.5 applies.

17.5.4—The permissible horizontal shear stresses, v_h, apply when the design is based on the load factors and ϕ factors of Chapter 9. When the alternate design method of Section 8.10 is used, the value of v_h should be reduced in accordance with the provisions for shear stresses in Section 8.10.3.

In reviewing composite concrete flexural members for serviceability at service loads and for handling and construction loads, V_u may be replaced by the service load shear or handling load shear in Eq. (17-1). The resulting service load or handling load horizontal shear stress should be compared with the allowable stresses (0.55 v_h) to insure that an adequate factor of safety results.

17.5.5. — Proper anchorage of bars extending across joints is required to insure that contact of the interfaces is maintained.

17.6—Ties for horizontal shear

The minimum areas and maximum spacings are based on test data given in Reference 17.1.

17.7—Measure of roughness

This section conforms with the provisions of Chapter 11, and is based on tests discussed in Reference 17.1.

References

17.1. Saemann, J. C., and Washa, George W., "Horizontal Shear Connections Between Precast Beams and Cast-in-Place Slabs," ACI Journal, *Proceedings* V. 61, No. 11, Nov. 1964, pp. 1383-1409.

17.2. Hanson, N. W., "Precast-Prestressed Concrete Bridges: (2), Horizontal Shear Connections," *Journal* PCA Research and Development Laboratories, V. 2, No. 2, May 1960, pp. 38-58. Also, *Development Department Bulletin* D35, Portland Cement Association.

17.3. Mattock, A. H., and Kaar, P. H., "Precast-Prestressed Concrete Bridges: (4), Shear Tests of Continuous Girders," *Journal*, PCA Research and Development Laboratories, V. 3, No. 1, Jan. 1961, pp. 47-56. Also, *Development Department Bulletin* D45, Portland Cement Association.

17.4. Grossfield, B., and Brinstiel, C., "Tests of T-Beams with Precast Webs and Cast-in-Place Flanges," ACI Journal, *Proceedings* V. 59, No. 6, June, 1962, pp. 843-851.

CHAPTER 18 — PRESTRESSED CONCRETE

18.1—Scope

18.1.1—The provisions in this chapter were developed primarily for structural members such as slabs, beams, and columns which are commonly used in buildings. However, many of the provisions may be applied to other types of construction such as pressure vessels, pavements, pipes, and crossties. The application of the provisions is left to the judgment of the engineer in cases not specifically cited in the Code.

18.1.2 and 18.1.3—The entire Code applies to prestressed concrete except where excluded or in direct conflict with Chapter 18. Some sections of the Code are excluded from use in the design of prestressed concrete for specific reasons. The following discussion provides explanations for such exclusions:

Section 8.6—Section 8.6 of the code is excluded since moment redistribution for prestressed concrete is covered in Section 18.12.

Sections 8.7.2, 8.7.3, and 8.7.4—The empirical provisions of Sections 8.7.2, 8.7.3, and 8.7.4 for T-beams were developed for conventional reinforced concrete and if applied to prestressed concrete would exclude many standard prestressed products in satisfactory use today. Hence, proof by experience permits variations.

By excluding Sections 8.7.2, 8.7.3, and 8.7.4, no special requirements for prestressed concrete T-beams appear in the Code. Instead, the determination of an effective width of flange is left to the experience and judgment of the engineer. Where possible, the flange widths in Sections 8.7.2, 8.7.3, and 8.7.4 should be used unless experience has proven that variations are safe and satisfactory. It is not necessarily conservative in elastic analysis and design considerations to use the maximum flange width as permitted in Section 8.7.2.

Sections 8.7.1 and 8.7.5 provide general requirements for T-beams that are also applicable to prestressed concrete units. The spacing limitations for slab reinforcement are based on flange thickness, which for tapered flanges can be taken as the average thickness.

Section 8.8—The empirical limits for concrete joist floors are justified for conventional reinforced concrete but not for prestressed concrete. Hence, they are excluded in prestressed concrete. Experience and judgment must be used.

Sections 10.3.2, 10.3.3, 10.3.6, 10.5, and 10.9.1 — For prestressed concrete, the limitations on reinforcement given in Sections 10.3.2, 10.3.3, 10.3.6, 10.5, and 10.9.1 are replaced by those in Sections 18.8, 18.9, and 18.14.

Chapter 13—The design of prestressed concrete slabs requires recognition of secondary moments induced by the undulating profile of the prestressing steel. Also volume changes due to the prestressing force can create additional loads on the structure that are not adequately covered in Chapter 13. Because of these unique properties associated with prestressing, many of the design procedures of Chapter 13 are not appropriate for prestressed concrete structures and are replaced by the provisions of Section 18.13.

Chapter 14—The requirements stated for wall design in Chapter 14 are largely empirical, utilizing considerations not intended to apply to prestressed concrete.

18.2—General considerations

18.2.1—As has been past practice in the design of prestressed concrete, the design investigation should include all load stages that may be significant. The three major stages are: (1) initial stage, or prestress transfer stage—when the tensile force in the prestressed steel is transferred to the concrete and stress levels may be high relative to concrete cylinder strength, (2) service load stage—after long-time volume changes have occurred, and (3) the design load stage—when the capacity of the member is checked. There may be other load stages that require investigation. For example, if the cracking load is significant, this load stage may require study, or the handling and transporting stage may be critical.

From the standpoint of satisfactory behavior, the two stages of most importance are those for service load and design load.

Service load stage refers to the loads defined in the general building code, such as live load and dead load, while the design load stage refers to factored loads. When calculating the behavior at the service load stage, the ϕ factors given in Chapter 9 should not be included. It is necessary to investigate service load and design load stages to insure member performance in regard to both serviceability and capacity.[2]

For example, a beam could be prestressed along its longitudinal axis in such a manner that it will support the specified loads without objectionable deflection but the strength could be below adequate safety requirements. Similarly, a design based on strength alone may provide unsatisfactory behavior at service loads, e.g., excessive camber or deflection.

This means that the actual design should be performed for strength, using load factors and understrength factors. Then, an investigation at service load levels is necessary to determine the approximate stresses at service loads. For beams without axial loads, the classical straight-line theory is satisfactory. For sections with axial load, a general analysis considering the stress-strain diagram for concrete is desirable. In this respect, a bilinear approximation of the stress-strain diagram for prestressing steel is advisable. Section 18.3.2 provides assumptions that may be used for investigation at service loads and after transfer of the prestressing force.

18.2.4—This refers to the type of post-tensioning where the tendon makes contact with the prestressed concrete member intermittently. Precautions should be taken to prevent buckling of such members. In particular, if thin webs or flanges are under high precompression, buckling is possible between supports of slender members.

If the tendon is in complete contact with the member being prestressed, or is an unbonded tendon in a duct not excessively larger than the tendon, it is not possible to buckle the member under the prestressing force being introduced.

18.3—Basic assumptions

The provisions which referred to modulus of elasticity of steel and concrete in the 1963 Code have been removed from this chapter. For values of E_s, Section 8.3.2 requires that tests be performed or that data be obtained from the manufacturer. For concrete, E_c may be taken as $E_c = w^{1.5} 33\sqrt{f_c'}$ (see Section 8.3.1).

18.3.2—Assumptions are provided for use in service load investigation and for review of sections at transfer of prestress forces. Note that this section does not apply to the design of compression members in general, but only to members that are prestressed.

18.3.2(c) In considering the area of the open ducts, the critical areas should include those which have coupler sheaths which may be of a larger size than the duct containing the tendon. Also in some instances the trumpet or transition piece from the conduit to the anchorage may be of such a size as to create a critical area.

18.4—Permissible stresses in concrete—Flexural members

Permissible stresses in concrete are given to control serviceability. They do not automatically guarantee adequate structural capacity, which may be checked in conformance with other Code requirements.

18.4.1—These stresses are applicable immediately after transfer of the prestressing force but prior to the occurrence of time-dependent losses such as creep and shrinkage. The concrete stresses at this stage should not exceed the recommended values caused by the force in the steel at transfer reduced by the losses due to elastic shortening of the concrete, some relaxation of the steel, slip at anchorage, plus the stresses due to the weight of the member. Generally, shrinkage is not included at this stage. These stresses apply to both pretensioned and post-tensioned concrete with proper modifications of the losses at transfer that apply.

18.4.1(b) The stress limit $3\sqrt{f'_{ci}}$ refers to locations of tensile stress in a member, other than in the precompressed tension zone, such as at the ends of a simply supported beam near the top. Where the tensile stresses exceed the allowable value the total force in the tensile stress zone should be calculated and reinforcing steel proportioned on the basis of this force at a stress of $0.6f_y$, but not more than 30,000 psi. It should be noted that the effects of creep and shrinkage begin to reduce the tensile stress almost immediately. Some tension remains in these areas after allowance is made for all prestress losses.

18.4.2.2 The precompressed tension zone is that portion of the member cross section in which flexural tension occurs under dead and live loads. Prestressed concrete is usually designed so that the prestress force introduces compression into the zone, thus effectively reducing the magnitude of the tensile stress.

The permissible tensile stress $6\sqrt{f_c'}$ is compatible with the concrete covers required by Section 7.14.1.3. For conditions of corrosive atmosphere, which is defined as an atmosphere in which chemical attack such as seawater, corrosive industrial atmosphere, sewer gas, or other highly corrosive atmospheres are encountered, greater cover than that required by Section 7.14.1.3 should be used, in accordance with Section 7.14.3, and tension stresses reduced to eliminate possible cracking at service loads.

The engineer must use judgment to determine the amount of increased cover and whether reduced tension is required.

The allowable concrete tensile stresses depend on whether or not enough bonded reinforcement is provided to control cracking. Such bonded steel may consist of bonded prestressed or nonprestressed tendons or of bonded reinforcing bars. It should be noted that the control of cracking depends not only on the quantity of reinforcement provided but also on its distribution over the tension zone.

Because of the bonded steel requirements of Section 18.9, it is considered that the behavior of segmental members will be generally comparable to that of similarly constructed monolithic concrete members. Therefore, the permissible tensile stress limits of Sections 18.4.2.2 and 18.4.2.3 apply to both segmental and monolithic members. If deflections are important, the built-in cracks of segmental members should be considered in the computations.

18.4.2.3 The allowable tensile stress $12\sqrt{f_c'}$ represents an increase over the value listed in ACI 318-63 to permit improved service load performance, especially when live loads are of a transient nature. To take advantage of the increased allowable stress, the engineer is required to increase the concrete protection on the reinforcement, as stipulated in Section 7.14.2, and to investigate the deflection characteristics of the member particularly at the load where the member changes from uncracked behavior to cracked behavior.

The load-deflection curves of prestressed concrete members may be idealized into an assumed bilinear curve. The first portion of the curve is a straight line from initial load up to the load that causes cracking of a magnitude sufficient to significantly reduce the member's stiffness. The second portion of the curve proceeds from this point of cracking at a flatter slope as load is increased. The change in slope is a function of the reduction in moment of inertia at cracking. For most usual conditions the change is negligible. In some cases the change is so gradual that the assumption of a bilinear curve is not necessary. However, where the reduction in moment of inertia can be large at cracking, the loss of stiffness, or increase in deflection is large. For this reason, when the higher allowable stress is used, the engineer is directed to compute the deflection, using the cracked cross section and the transformed areas of bonded steel to compute the moment of inertia.

18.4.3—Prestressed concrete is largely a plant-produced manufactured product with rapidly changing technology. This section provides a mechanism whereby development of new products, materials, and techniques need not be inhibited by arbitrary limits on stress which represented the most advanced requirements at the time the Code provisions were adopted. Approvals for the design should be in accordance with Section 1.4 of the Code.

18.5—Permissible stresses in steel

The Code no longer distinguishes between temporary and effective steel stresses, as did the 1963 Code. The reasoning is that the tendon stress immediately after transfer can prevail for a considerable time, even after the structure has been put into service. This stress, therefore, must have an adequate safety factor under service conditions and cannot be considered as a temporary stress. Any subsequent stress drop in the steel due to losses can only improve conditions and, hence, no allowable limit on stress drop has been provided in the Code.

18.6—Loss of prestress

18.6.1—The causes for loss of prestress are listed. For an explanation of how to compute these losses, see the reports of ACI-ASCE Committee 423* and ACI Committee 435.† The lump sum losses of 35,000 psi for pretensioning and 25,000 for post-tensioning that appeared in the report of Committee 423 generally give satisfactory results for many applications.

Actual losses, greater or smaller than the lump sum values, have little effect on the design strength of the member, but affect service load

*ACI-ASCE Committee 423, "Tentative Recommendations for Prestressed Concrete," ACI Journal, *Proceedings* V. 54, No. 7, Jan. 1958, pp. 545-578.

†ACI Committee 435, "Deflections of Prestressed Concrete Members," ACI Journal, *Proceedings* V. 60, No. 12, Dec. 1963, pp. 1697-1728.

behavior, such as deflection and camber, connections, or cracking load. Overestimation of prestress losses can be almost as detrimental as underestimation, since the former can result in excessive camber and horizontal movement.

Data* have been assembled and analyzed to permit computation of the stress loss due to relaxation of tendons composed of stress-relieved wires. Subsequent work on stress-relieved strand conforming to ASTM A 416 indicates relaxation losses of about the same magnitude.

Stabilized strand or wire is material which has smaller relaxation losses than conventional stress-relieved material. While the strand is at the elevated temperature used for the stress-relieving operation, it is subjected to a high tensile force which produces a specific amount of permanent elongation, thus resulting in low relaxation losses after the tendon is put into service. For specific relaxation values of a particular steel the engineer should consult the steel manufacturer.

18.6.2—Friction losses due to wobble and curvature can be computed by Eq. (18-1) and (18-2) of the Code. The coefficients tabulated in Table 18-1 give a range which can be generally expected. Due to the many types of ducts, tendons and wrapping materials available, these values can only serve as a guide. Where rigid conduit is used for instance, the wobble coefficient K can be considered as zero. For large tendons in semirigid type conduit, the wobble factor can also be considered zero. Guidance on the friction that can be expected with particular type tendons and particular type ducts can be obtained from the manufacturers of the tendons. An unrealistically low evaluation of the friction loss can lead to improper camber of the structure and inadequate prestress. Overestimation of the friction may result in extra prestressing force if the estimated friction values are not attained in the field. This could lead to excessive camber and excessive shortening of a member. If the estimated friction factors are determined to be less than those assumed in the design, the stressing force should be adjusted to give only that theoretical prestressing force in the critical portions of the structure required by the design.

TABLE 18-1—FRICTION COEFFICIENTS FOR POST-TENSIONED TENDONS FOR USE IN EQ. (18-1) OR (18-2)

			Wobble coefficient, K	Curvature coefficient μ
Grouted tendons in metal sheathing		Wire tendons	0.0010-0.0015	0.15-0.25
		High strength bars	0.0001-0.0006	0.08-0.30
		7-wire strand	0.0005-0.0020	0.15-0.25
Unbonded tendons	Mastic-coated	Wire tendons	0.001-0.002	0.05-0.15
		7-wire strand	0.001-0.002	0.05-0.15
	Pre-greased	Wire tendons	0.0003-0.002	0.05-0.15
		7-wire strand	0.0003-0.002	0.05-0.15

18.7—Flexural strength

The computation of ultimate flexural strength (now called design strength) may be carried out using the same equations as those provided in the 1963 Code.

1. Rectangular sections, or flanged sections in which the neutral axis lies within the flange (usually where the flange thickness is more than $1.4d\rho_p f_{ps}/f_c'$):

$$M_u = \phi[A_{ps}f_{ps}d(1 - 0.59\omega_p] = \phi\left[A_{ps}f_{ps}\left(d - \frac{a}{2}\right)\right]$$

2. Flanged sections in which the neutral axis falls outside the flange (usually where the flange thickness is less than $1.4d\rho_p f_{ps}/f_c'$):

$$M_u = \phi\left[A_{pf}f_{ps}\left(d - \frac{a}{2}\right) + 0.85f_c'(b - b_w)\,h_f\left(d - \frac{h_f}{2}\right)\right]$$

where

$$A_{pf} = A_{ps} - A_{pw}$$

and

$$A_{pw} = 0.85f_c'(b - b_w)\,h_f/f_{ps}$$

A_{pf} and A_{pw} are those portions of the prestressing steel required to develop the compressive strengths of the overhanging flanges and the web, respectively, where h_f is the flange thickness.

Development and full explanation of these equations are contained in a paper by Warwaruk, Sozen, and Siess.† The introduction of the capacity reduction factor ϕ creates no difficulties in the process of determining the design strength capacity of a member that has been proportioned on the basis of service load requirements. The required design moment can be calculated by multiplying the service load moments by appropriate

*Magura, Donald D.; Sozen, Mete A.; and Siess, Chester P., "A Study of Stress Relaxation in Prestressing Reinforcement," Journal, Prestressed Concrete Institute, V. 9, No. 2, Apr. 1964, pp. 13-57.

†Warwaruk, Joseph; Sozen, Mete A., and Siess, Chester P., "Investigation of Prestressed Reinforced Concrete for Highway Bridges: Part 3 — Strength and Behavior in Flexure of Prestressed Concrete Beams," Bulletin No. 464, Engineering Experiment Station, University of Illinois, Urbana, 1962, 105 pp.

load factors and then dividing by the ϕ factor. Member capacity is then determined by the above formulas without the ϕ factor.

18.7.1—Eq. (18-3) is a shortcut approximation to the more accurate calculation which involves a trial method based on reaching compatability between stresses and strains. The approximate formula may underestimate the capacity of beams with high percentages of steel and, for more accurate evaluations of their capacity, the stress-strain compatability method should be used.

Eq. (18-4) for unbonded members is up-dated on the basis of more recent test data. A recent report* recommended the equation

$$f_{ps} = f_{se} + 10{,}000 + \frac{1.4 f_c'}{100 \rho_p}$$

ACI-ASCE Committee 423, Prestressed Concrete, felt that the slightly more conservative values resulting from dropping the 1.4 multiplier for f_c' would be preferable.

The equation provides a conservative value representing the lower envelope of data on the increase of tendon stress as various members were test-loaded to failure. It should be recognized that very stiff members, such as a post-tensioned deep beam, may not deflect enough to develop the increase in stress reflected by this formula.

Frequently, in practice, prestressed concrete members are proportioned on the basis of stresses at service load and the strength checked for adequate safety using these equations to determine steel stress at design loads (factored loads).

18.8—Steel percentage

18.8.1—The limitations on the reinforcing steel index of 0.3, was originally set by ACI-ASCE Committee 423, Prestressed Concrete, as the dividing line between under-reinforced and over-reinforced members.

When the reinforcing index exceeds 0.3, the strength as predicted by standard equations (without the ϕ) does not correlate well with test results.

18.8.2—The equations to be used for computing the flexural strength of over-reinforced members can be the same as those in the 1963 Code. The following equations satisfy the intent of this section:

For rectangular sections, or flanged sections in which the neutral axis lies within the flange:

$$M_u = \phi\,(0.25 f_c' b d^2)$$

For flanged sections in which the neutral axis falls outside the flange:

$$M_u = \phi[0.25 f_c' b_w d^2 + 0.85 f_c' (b - b_w)\, h_f\, (d - 0.5 h_f)]$$

18.8.3—This provision is a precaution against abrupt flexural failure resulting from rupture of the prestressing steel when failure occurs immediately after cracking. The usual member requires considerable additional load beyond cracking to reach design capacity. Thus, considerable deflection warns that the design capacity is being approached. However, if design capacity occurs shortly after cracking the warning deflection may not occur.

18.9—Minimum bonded reinforcement requirements

The possibility of the formation of excessive cracks in flexural members with unbonded tendons must be taken into consideration. The increase in allowable tensile stress permitted in the Code requires minimum bonded reinforcement for both beams and slabs.

The amount of steel used is proportioned according to Eq. (18-5), based on the amount of tension N_c in the concrete computed on the basis of an uncracked homogeneous concrete section, or Eq. (18-6). These provisions and others on unbonded prestressing have been adapted from a report of ACI-ASCE Committee 423.†

Unbonded reinforcement in accordance with Sections 18.9.1, 18.9.2, and 18.9.3 provides adequate crack control when the allowable tensile stresses of Sections 18.4.2.2 and 18.4.2.3 are used. For two-way slabs the minimum amount is intended in each direction. However, for two-way slabs, Section 18.9.3 permits the amount of bonded steel required, to be decreased when the tension in the precompressed tensile zone at service load does not exceed zero. This is in agreement with the 1963 Code on which current satisfactory design practice has been based. The word "decreased" is used in place of "eliminated" because, although satisfactory behavior is achieved without bonded flexural steel, current practice calls for a nominal amount of bonded reinforcement at joints between slabs and at supporting columns to insure flexural continuity and/or shear resistance.

18.10—Repetitive loads

The effects of repetitive loads cause difficulties primarily in fatigue of the anchorage or tendon material and in loss of bond strength of the concrete. The latter aspect is of importance where

*Yamazaki, Jun; Kattula, Basil T.; and Mattock, Alan H., "A Comparison of the Behavior of Post-Tensioned Prestressed Concrete Beams with and without Bond," Report SM69-3, University of Washington, College of Engineering, Structures and Mechanics, Dec 1969.

†ACI-ASCE Committee 423, "Tentative Recommendations for Concrete Members Prestressed with Unbonded Tendons," ACI Journal, *Proceedings* V. 66, No. 2, Feb. 1969, pp. 81-86.

the available transfer length of pretensioned tendons is short, such as in railroad crossties.

The warning against diagonal tension cracking at repetitive loads lower in magnitude than the static loads used for design indicates that it would be prudent to use at least the minimum shear reinforcement expressed by Eq. (11-1) or (11-2), even though tests or calculations based on static loads show that shear reinforcement is not required.

18.11—End regions

Because the actual stresses are quite complicated around post-tensioning anchorages, the only rational approach is to apply strength design methods.

A refined strength analysis should be used whenever possible, with ϕ being taken as 0.9.

The 1963 Code formula

$$f_{cp} = 0.6f'_{ci}\sqrt[3]{A_2/A_1}$$

but not greater than f'_{ci} may still be used as a guide for determining permissible bearing stress when experimental data or more refined analyses are not available. This formula is used without a ϕ factor.

In the equation for f_{cp}

A_1 = bearing area of anchor plate of post-tensioning steel

A_2 = maximum area of the portion of the anchorage surface that is geometrically similar to, and concentric with, the area of the anchor plate of the post-tensioning steel

f_{cp} = permissible concrete bearing stress under the anchor plate of post-tensioning steel with the end anchorage region adequately reinforced

18.12—Continuity

As member capacity is approached, inelastic behavior at some sections can result in a redistribution of moments in prestressed concrete beams. Recognition of this actual behavior can be advantageous in design under certain circumstances. A rigorous design method for moment redistribution is quite complex. However, recognition of moment redistribution can be accomplished with the simple method of permitting a reasonable adjustment of the elastically calculated design load moments. The amount of adjustment must be kept within predetermined safe limits.

The amount of redistribution allowed depends on the ability of the critical sections to deform inelastically by a sufficient amount. Serviceability under service loads is taken care of by the limiting stresses of Section 18.4. The choice of 0.20 as the largest tension reinforcement index, ω_p, $(\omega + \omega_p - \omega')$, or $(\omega_w + \omega_{pw} - \omega_w')$ for which redistribution of moments is allowed, is in agreement with the requirements for conventionally reinforced concrete of $0.5\rho_b$ stated in Section 8.6.

The secondary bending moments produced by the prestress force in a nonconcordant tendon disappears at the capacity at which, due to plastic hinge formation, the structure becomes statically determinate. Therefore, the design load moments at the critical sections of a continuous prestressed beam are only those due to dead and live loads.

With unbonded tendons, sufficient bonded steel must also be provided to assure the rotational capacity required at the sections where plastic hinges develop. The bonded steel required by Section 18.9 may not be sufficient for this purpose.

18.13—Slab systems

This section does not provide detailed Code provisions for design which will account for such aspects of behavior which are unique to prestressed concrete.

In addition to the more accurate theories for analysis, the classical elastic slab theory or finite elements or finite difference methods are often used for analysis. The frame method also is applicable to square or rectangular panels when columns are relatively stiff and rigidly connected to the slab. Where columns are quite flexible or are not rigidly connected to the slab, the frame method is sometimes simplified into a beam method in which the slab is analyzed as beams in each of the two directions. Simplified methods using average coefficients do not apply for prestressed concrete.

Concerning the strength of prestressed slabs, tests indicate that strength is controlled primarily by the total amount of tendon capacity rather than by tendon distribution. Some tendons should be passed through the columns or around their edges. It is suggested that the maximum spacing of tendons in the column strips should not exceed four times the slab thickness and that the maximum spacing in the middle strips should not exceed six times the slab thickness.

For prestressed flat slabs continuous over two or more spans in each direction, it is suggested that the span-thickness ratio should generally not exceed 42 for floors and 48 for roofs and that these limits may be increased to 48 and 52, respectively, if calculations verify that both short- and long-term deflection, camber, and vibration frequency and amplitude are not objectionable.

Short- and long-term deflection and camber should be computed and checked for the requirements of serviceability of the particular usage of the structure.

The maximum length of a slab between construction joints is generally limited to 100 to 150 ft to minimize the effect of slab shortening, and to avoid excessive loss of prestress due to friction.

18.14—Compression members—Combined axial load and bending

18.14.1—For compression members having less than 225 psi prestress, the minimum vertical reinforcement required in Section 10.9.1 for columns or in Section 10.16 for walls must be provided.

18.14.2—Prestressed concrete compression members will, in most cases, be precast and pretensioned. The high quality control associated with this type of construction may result in smaller column dimensions than those required by the 1963 Code. Since the effects of accidental loads, local buckling, long-column action, shrinkage, creep, and nonuniform temperature distribution must be considered in the design, minimum column dimensions are not required in the Code for either reinforced or prestressed concrete. For somewhat similar reasons, the minimum amounts of reinforcement, specified in Section 10.16 for walls, need not apply to prestressed concrete walls, provided the average prestress is over 225 psi and a complete structural analysis is made to show adequate strength and stability with lower amounts of reinforcement in keeping with production practice. Walls with an average prestress less than 225 psi can be treated as reinforced concrete walls and the provisions of Chapter 14 or Section 10.16 used.

18.15—Corrosion protection for unbonded tendons

Suitable material for corrosion protection of unbonded tendons should have the following properties:

1. Remain free from cracks and not become brittle or fluid over the entire anticipated range of temperatures. In the absence of specific requirements, this is usually taken as 0 to 160 F

2. Chemically stable for the life of the structure

3. Nonreactive with the surrounding materials such as concrete, tendons, wrapping, or ducts

4. Noncorrosive or corrosion inhibiting

5. Impervious to moisture

18.17—Grout for bonded tendons

Grout is the means by which bond is provided between the post-tensioning tendons and the concrete and/or by which corrosion protection of the tendons is assured. Proper grout and grouting procedures, therefore, play an important part in post-tensioned construction. There are various recommended practices for grout materials and grouting procedures, such as those in "Recommended Practice for Grouting Post-Tensioning Tendons," July 1967, Tentative, prepared jointly by the Prestressed Concrete Manufacturers Association of California, Inc., and Western Concrete Reinforcing Steel Institute.

18.17.1—The limitations on water, Section 3.4.1, and on admixtures, Section 3.6.1, are intended to apply to grout. Aluminum powder or other expansive admixtures, when approved, should produce an unconfined expansion not greater than 10 percent.

18.17.5—Quick-set grouts, when approved, may be shown by tests to require shorter periods of protection and the recommendations of the suppliers should be followed.

18.19—Application and measurement of prestressing force

18.19.1—This section contains requirements to insure that the amount of tension assumed for the steel in design is actually placed in the steel. A similar provision appeared in the report of ACI-ASCE Committee 423 and in the 1963 Code.

18.20—Post-tensioning anchorages and couplers

18.20.1—In giving the capacity of anchorages and couplers it is intended that they develop the specified strength of the tendon steel with a minimum amount of permanent deformation and successive set, recognizing that some deformation and set will occur in testing to failure. Bonded tendon anchorages that develop less than 100 percent of the minimum specified strength of the tendon steel should be used only where the bond transfer length equals or exceeds that required to develop the tendon capacity. This bond length, as determined by test, should be provided between the anchorage and the zone where the full prestressing force will be required under service loads and design loads.

18.20.3—For detailed recommendations on tests for static and cyclic loading conditions for tendons and anchorage fittings of unbonded tendons, see Section 4.2.3 of "Tentative Recommendations for Concrete Members Prestressed with Unbonded Tendons" ACI JOURNAL, Feb. 1969.

It is suggested that the elongation requirements for tendon assemblies in Section 4.2.3.2 of this report be reduced from 2½ to 2 percent to conform to more recent recommendations.

19.1—Scope and definitions

Thin shells (except for domes) and folded plates are usually classified as short, intermediate, and long span, depending on the ratio of transverse span to longitudinal span, or the ratio of the radius of curvature in the transverse direction to the longitudinal span.

For short span barrel shells, the load is transferred to the end arches and to edge beams in a complex manner. Arch action is predominate over membrane action. On the other hand, in very long span barrel shells, the shell acts very much as a beam[19.1] having a curved or folded cross section. The ratios of width or radius of curvature to length for which arch action, true shell action, or beam action occur will differ with the shape of the structure.[19.2]

Since this chapter applies to thin shells of all shapes, extensive discussion related thereto is not possible in this Commentary. The reader is therefore referred to the references for further information.

Elastic analysis with respect to shells refers to any method of structural analysis involving assumptions which provide suitable approximations to three-dimensional elastic behavior. Such solutions range from simply satisfying three-dimensional statics[19.1] to a solution involving eighth-order partial differential equations.[19.3]

Elastic analysis reveals the concentration of forces in certain locations. For example, in a short barrel shell, the tension is concentrated near the lower edges. In an elliptical paraboloid the tension builds up to a peak near the corners. In a dome, high tension forces generally occur near the boundaries. If a designer were to design a barrel shell by the ordinary beam or arch theory and not recognize its limitations he could erroneously predict the required amount and distribution of reinforcement. The elastic analysis permits a reasonably realistic evaluation of deflection. This important consideration is frequently overlooked by some designers.

The degree of accuracy required in the analysis of a thin shell structure will depend on certain critical factors. These include: the size of the structure, the geometry and degree of curvature of the surface, the type of boundary conditions, and the nature of the loading.

Modern techniques used in electronic computer solutions include procedures such as the finite element method which result in rigorous solutions[19.4,19.40] predicting principal stresses and their directions at various nodes as well as the maximum shear stresses and their directions at various nodes.

Such techniques provide solutions which are usually superior to conventional solutions inasmuch as the reinforcement may be placed as theoretically required to resist principal tensile stresses and maximum shear stresses. Extensive discussion concerning the directions of such stresses is provided in Reference 19.5 with regard to domes, cylindrical shells, and short and long barrel shells.

Reference 19.6 provides theoretical concepts related to thin shells and References 19.7 through 19.11 provide theoretical as well as practical design concepts related to thin concrete shells. Finite element methods for shells are thoroughly discussed in References 19.12 and 19.13.

The remaining references provide excellent guidance concerning analysis, design, and construction of shells of various types. In particular, Reference 19.15, the report of ACI Committee 334, provides an excellent reference for the design engineer, the constructor, and the field inspector. The *ACI Manual of Concrete Practice*, Part 2, also provides good guidance with regard to shells.

19.2—Assumptions

The designer may assume that concrete is ideally elastic, homogeneous, and isotropic, having identical stress-strain properties in all directions. Further, Poisson's ratio may be assumed equal to zero in the partial differential equations related to the particular type of shell being designed. The word "may" in this section allows the use of simplifications that do not provide errors of large magnitude under usual conditions. However, the knowledgeable analyst-designer may utilize more accurate assumptions.

19.3—General considerations

19.3.1 — Equilibrium investigations are required to insure that statics will be satisfied.

19.3.2 — Solutions which do not satisfy stress and strain compatibility may be used only when extensive experience has proved that safe designs have resulted from their use. Such methods include beam type analysis for barrel shells and folded plates having large ratios of span to width or radius of curvature, simple membrane analysis for domes and shells of revolution, and others in which the partial differential equations related to statics are satisfied while the strain compatibility equations are not precisely satisfied.

However, in complex structures of large radius a more accurate analysis should be used. Greater accuracy of analysis is also warranted in large structures in areas of high wind intensity and in critical earthquake zones. Finite element methods can be used to satisfy statics and strain compatibility as well as to satisfy the boundary conditions. Since numerous simultaneous differential equations result from such an analysis, electronic computers provide the only feasible method of obtaining a solution.

19.3.3 — Model analysis may include strain measurements or photoelastic studies of portions of the shell being analyzed.

Wind tunnel tests of a scaled-down model do not necessarily provide usable results. Many factors enter into model tests besides shape and direct scale.[19.50] Thus, the Building Official should accept results of model tests in lieu of mathematical analysis only when the model tests have been performed under the direction of a proven expert in this area of structural engineering, including expertise in the theory of models and similitude of model and prototype.

19.3.4 — The shell elements must be proportioned to satisfy the strength provisions required by the Code, while the analysis must be made using elastic analysis theory, the elastic theory of models or a combination thereof. The shell thickness, however, is not always dictated by the strength requirements determined from such an analysis, but often by deformation of edge members, structural stability, and required cover over the reinforcement.

The necessary strength must be provided by using either of the two methods prescribed in Section 8.1. The permissible maximum steel ratios, ρ, may not be exceeded.

If the shell is prestressed, then its ultimate capacity must be investigated, as well as its elastic capacity under service loads, the cracking load, and the loads induced at the time of prestressing.

If composite action is involved, the provisions of Chapter 17 must be satisfied. Chapter 16 applies if elements are precast.

19.3.5 — Supporting frames and edge beams must be designed in accord with the general provisions of this Code, using either of the two methods prescibed in Section 8.1. Portions of the shell may be utilized as flanges for transverse arch-frames or longitudinal frames. Such flanges may be curved or sloping. Cantilever action of the flanges must be investigated in determining reinforcement in the flange perpendicular to the longitudinal axis of the supporting member, as required by Chapter 8. In all cases, temperature reinforcement must be used.

19.3.6 — The designer is directed to consider the possible reduction in the buckling capacity due to creep and other factors. Reference 19.17 provides theoretical and practical guidance for the effects of creep.

19.5.4 — The option of increasing the deviation from the line of the principal stress may be applied when excess steel is used, but the total steel area per foot may not exceed $3.6f_c'/f_y$ nor $14{,}500h/f_y$ when the deviation exceeds 10 deg.

Only the membrane stresses need be considered in determining the maximum deviation allowed.

Development of reinforcement must satisfy Chapter 12 and splices must satisfy Chapter 7.

19.5.6 — Typical locations where it may be desirable to concentrate tensile reinforcement rather than distribute it over a zone of varying tensile stress are near the edges of long barrel shells and near the ring beams in domes. Where this is done, minimum distributed reinforcement in the amount of 0.35 percent is specified to control cracking throughout the tensile zone.

19.5.7 — The effects of moments at boundaries and other locations where membrane action must be considered, require that sufficient strength be provided to resist the resulting principal stresses and shear stresses. Axial loads must be included when they are sufficiently large to cause an increase in the reinforcement requirements. If factored axial loads exceed $0.1f_c'bh$ in any section, $\phi=0.7$ must be used, even though there may be only one line of reinforcement in the direction being considered. For lesser axial loads a linear increase in ϕ is used from 0.7 at $P_u/f_c'bh=0.1$ to $\phi=0.9$ at $P_u=0$, in accord with Chapter 9. Principal stresses are converted to forces per foot and then the strength provisions are used. The design stress is then $0.85\phi f_c'$ over the entire cross section of concrete in compression and ϕf_y for steel in tension, except as modified under Section 19.5.

In shells where complex stress patterns may exist a comprehensive mathematical analysis must be utilized. It is dangerous to use simplifications in such cases.

The foregoing discussion regarding principal stress lines and deviations of reinforcement direction therefrom refers to gravity load analysis, that is, dead load, vertical live load, and snow load. The effects of wind load must be considered as well as the effects of earthquake forces when appropriate. Additional reinforcement in different directions (than those required for gravity loads) may be required for such lateral loads. Due consideration must be given to suction effects.

When shell or folded plate elements are precast and connected by cast-in-place segments, composite action must be considered. The dead load forces will cause stresses and deflections due to an entirely different type of behavior than live and wind load, after the segments are connected and respond to loads by shell action.

19.5.8 and 19.5.9 — The designer must consider the possibility or probability of the reinforcement being placed out of line on doubly curved surfaces thus resulting in insufficient splice length if precut reinforcement is used. The designer must provide sufficient reinforcement length to maintain the minimum splice lengths required by Chapter 7.

19.6—Prestressing

Axial forces due to draped prestressed tendons may not lie in one plane, and due consideration must be given to the resulting force components. The provisions of Chapter 18 apply in segments or sections where prestressing is used. The effects of post-tensioning of supporting members on the shell must be taken into account.

19.7—Construction

When early removal of forms is necessary, buckling and deflection must be investigated to establish the modulus of elasticity required before removal of forms. The modulus of elasticity of the concrete must be measured using a stress-strain curve for the field cured specimens. It is not sufficient to determine the modulus from the equation $E_c = 33w^{1.5}\sqrt{f_c'}$ with f_c' determined for the field-cured specimen.

References

19.1. Chinn, James, "Cylindrical Shell Analysis Simplified by Beam Method," ACI JOURNAL, *Proceedings* V. 55, No. 11, May 1959, pp. 1183-1192. Also, Discussion by M. S. Ketchum, A. L. Parme and H. W. Conner, A. Siev, Anton Tedesko, Alfred Zweig, and Author, ACI JOURNAL, *Proceedings* V. 55, Dec. 1959, Part 2, p. 1583.

19.2. Salvadori, Mario G., "Shell Versus Arch Action in Barrel Shells," *Proceedings*, ASCE, V. 81, No. 653, July 1956.

19.3. "Design of Cylindrical Concrete Shell Roofs," *Manual of Engineering Practice* No. 31, American Society of Civil Engineers, New York, 1952, 177 pp. (Prepared by Alfred Parme under the direction of ASCE Structural Division Committee on Masonry and Reinforced Concrete.)

19.4. Scordelis, A. C.; Ramirez, H. D.; and Ngo, D., "Membrane Stresses in Hyperbolic Paraboloid Shells Having a Parallelogram Shape in Plan," ACI JOURNAL, *Proceedings* V. 66, No. 12, Dec. 1969, pp. 994-1000.

19.5. Molke, Eric C., and Kalinka, J. E., "Principles of Concrete Shell Dome Design," ACI JOURNAL, *Proceedings* V. 34, No 5, May-June 1938, pp. 649-707 (with extensive Bibliography).

19.6. Timoshenko, S., and Woinowsky-Krieger, S., *Theory of Plates and Shells*, McGraw-Hill Book Co, New York, 1959, 580 pp.

19.7. Pfluger, Alf, *Elementary Statics of Shells*, Second Edition, McGraw-Hill Book Co., New York, 1961, 122 pp. (Translated from German by Ervin Galantay.)

19 8. Billington, David P., *Thin Shell Concrete Structures*, McGraw-Hill Book Co., New York, 1965, 332 pp.

19.9. Haas, A. M., *Design of Thin Concrete Shells*, John Wiley and Sons, Inc., New York, 1962, 128 pp.

19 10. Gibson, J. E., *Linear Elastic Theory of Thin Shells*, Pergamon Press, New York, 1965, 182 pp.

19.11. Salvadori, Mario, and Levy, Matthys, *Structural Design in Architecture*, Prentice-Hall, Inc., Englewood Cliffs, N.J., 1967, 457 pp. (Example problems and solutions by John J. Farrell.)

19.12. Soare, Mircea, *Application of Finite Difference Equations to Shell Analysis*, Pergamon Press, New York, 1967.

19.13. Zienkiewicz, O. C., and Cheung, Y. K., *The Finite Element in Structural and Continuum Mechanics*, McGraw-Hill Book Co., New York, 1967, 274 pp.

19.14. Mast, Paul E., "Design and Construction of Northlight Barrel Shells," ACI JOURNAL, *Proceedings*, V. 59, No. 4, Apr. 1962, pp. 481-526. (with extensive Bibliography).

19.15. ACI Committee 334, "Concrete Shell Structures — Practice and Commentary," ACI JOURNAL, *Proceedings* V. 61, No. 9, Sept. 1964, pp. 1091-1108. (Corrections and Discussions in ACI JOURNAL, *Proceedings* V. 62, No. 3, Mar. 1965, p. 1755.) Also in *ACI Manual of Concrete Practice*.

19.16. *ACI Manual of Concrete Practice*, Part 2, American Concrete Institute, Detroit, 1968, pp. 334-1 to 334-118.

19.17. ACI Committee 209, *Symposium on Creep of Concrete*, SP-9, American Concrete Institute, Detroit, 1964, 160 pp.

19.18. Candela, Felix, "Simple Concrete Shell Structures," ACI JOURNAL, *Proceedings* V. 48, No. 4, Dec. 1951, pp. 321-332.

19.19. Thurlimann, Bruno, and Johnson, Bruce G., "Analysis and Tests of a Cylindrical Shell Model," *Proceedings*, ASCE, V. 80, No. 434, Apr. 1954.

19.20. Salvadori, Mario G., "Stresses and Displacements in Thin Shells Composed of Cylindrical and Spherical Segments," *Proceedings*, ASCE, V. 79, No. 293, Oct. 1953.

19.21. Ernst, George C., "Stability of Thin-Shelled Structures," ACI JOURNAL, *Proceedings* V. 49, No. 4, Dec. 1952, pp. 277-292.

19.22. Tedesko, Anton, "Construction Aspects of Thin-Shell Structures," ACI JOURNAL *Proceedings* V. 49, No. 6, Feb. 1953, pp. 505-520.

19.23. Whitney, Charles S., "Reinforced Concrete Thin Shell Structures," ACI JOURNAL *Proceedings* V. 49, No. 6, Feb. 1953, pp. 521-536.

19.24. Nervi, Pier Luigi, "Precast Concrete Offers New Possibilities for Design of Shell Structures," ACI JOURNAL, *Proceedings* V. 49, No. 6, Feb. 1953, pp. 537-548.

19.25. Craemer, Herman, "Design of Prismatic Shells," ACI JOURNAL, *Proceedings* V. 49, No. 6, Feb. 1953, pp. 549-563.

19.26. Parme, Alfred, "Solution of Difficult Problems by Finite Differences," ACI JOURNAL, *Proceedings* V. 47, No. 3, Nov. 1950, pp. 237-256.

19.27. Amirikian, Arsham, "Thin-Shell Precast Concrete — An Economical Framing System," ACI JOURNAL, *Proceedings* V. 49, No. 9, May 1953, pp. 775-779.

19.28. Parme, Alfred L., "Hyperbolic Paraboloids and Other Shells of Double Curvature," *Proceedings*, ASCE, V. 82, ST5, Sept. 1956, p. 1057. Also, Discussion by T. Au, W. W. Pagon, S. D. Bannerjee, M. G. Salvadori, and Author, V. 83, ST2, Mar. 1957.

19.29. Oravas, Gunhard, "Analysis of Collar Slabs for Shells of Revolution," *Proceedings*, ASCE, V. 82, ST2, Mar. 1956.

19.30. Salvadori, Mario G., and Sherman, Robert, "Bending Stresses in Edge Stiffened Domes," *Proceedings*, ASCE, V. 82, ST4, July 1956.

19.31. Winter, George, and Pei, Minglung, "Hipped Plate Construction," ACI JOURNAL, *Proceedings* V. 43, No. 5, Jan. 1947, pp. 505-532.

19.32. Ketchum, Milo S., "Design of Shell Structures—Folded Plate Forms," *Consulting Engineer,* V. 16, No. 1, Jan. 1961.

19.33. Ketchum, Milo S., "Design of Shell Structures — Short Shells and Domes of Revolution," *Consulting Engineer,* V. 17, No. 1, July 1962.

19.34. "Design of Barrel Shell Roofs," *Structural Information Sheet* No. 77, Portland Cement Association.

19.35. "Elementary Analysis of Hyperbolic Paraboloid Shells," *Concrete Information Series,* Portland Cement Association.

19.36. "Design of Circular Domes," *Structural Information Sheet* No. 55, Portland Cement Association.

19.37. Fricke, Fred J., "Circular Shell Dome Cast on Earth Mound," *Civil Engineering — ASCE,* V. 25, No. 12, Dec. 1955, p. 831.

19.38. Anderson, Boyd G., "Designing Thin Shell Structures," *Consulting Engineer,* V. 8, No. 6, June 1957.

19.39. Salvadori, Mario G., "Thin Shells," Three articles from *Architectural Record,* V. 126: "Behavior and Forms," July 1954; "Effects of Loads and Forces," Sept. 1954; and "Examples Here and Abroad," Nov. 1954.

19.40. Vlasov, V. Z., "General Theory of Shells and Its Applications in Engineering," *NASA Technical Translation* TT F-99, National Aeronautics and Space Administration, Apr. 1964, 913 pp. (Available from Office of Technical Services, Department of Commerce, Washington, D.C. 20230, $7.00.)

19.41. Scordelis, A. C.; Ramirez, H. D., and Ngo, D., "Membrane Stresses in Hyperbolic Paraboloid Shells Having an Arbitrary Quadrilateral Shape in Plan," ACI JOURNAL, *Proceedings* V. 67, No. 1, Jan. 1970, pp. 36-44.

19.42. Lee, Ti-ta, and Vos, Robert G., "Multiple Folded Plates with Various End Conditions," *Proceedings,* ASCE, V. 94, ST7, July 1968, pp. 1761-1786.

19.43. Lo, Kam S., and Scordelis, A. C., "Finite Segment Analysis of Folded Plates," *Proceedings,* ASCE, V. 95, ST5, May 1969, pp. 831-852.

19.44. Muhlbauer, Karl C., and Beaufait, Fred W., "Behavior of Unsymmetrical Continuous Folded Plates," *Proceedings,* ASCE, V. 95, ST12, Dec. 1969, p. 2793.

19.45. Cheung, Yau Kai, "Folded Plate Structures by Finite Strip Method," *Proceedings,* ASCE, V. 95, ST12, Dec. 1969, p. 2963.

19.46. Johnson, Claude D., and Lee, Ti-ta, "Experimental Study of Non-Prismatic Folded Plates," *Proceedings,* ASCE, V. 94, ST6, June 1968, pp. 1441-1456.

19.47. Scordelis, A. C.; Croy, E. L.; and Stubbs, I. R., "Experimental and Analytical Study of Folded Plate," *Proceedings,* ASCE, V. 87, ST8, Dec. 1961, pp. 139-160.

19.48. Billington, David P., and Mark, Robert, "Small Scale Model Analysis of Thin Shells," ACI JOURNAL, *Proceedings* V. 62, No. 6, June 1965, pp. 673-688.

19.49. Traum, Eliahu, "The Design of Folded Plates," *Proceedings,* ASCE, V. 85, ST8, Oct. 1959, pp. 103-124.

19.50 *Models for Concrete Structures,* SP-24, American Concrete Institute, Detroit, 1970, 495 pp.

CHAPTER 20 — STRENGTH EVALUATION OF EXISTING STRUCTURES

20.1 — Strength evaluation — General

Chapter 20 applies to existing building structures where there is doubt about load-carrying capacity. Typically such doubt may arise if the materials supplied are considered to be deficient in quality, if the construction is suspect, or if the structure does not satisfy the Code in some aspect. In cases of this nature, the Building Official may use Chapter 20 as a guide in an investigation regarding the safety of the structure.

When a strength investigation is made, evaluation by analytical methods is permitted as an alternative to load testing. This was not formally stated in previous Codes. It is included in the 1971 ACI Code, recognizing that in some cases load tests may not be feasible, or may not be the most appropriate method.

20.2 — General requirements for analytical investigation

Section 20.2 points out that in an analytical investigation, the analysis must be based on data gathered concerning the actual dimensions of the structure, the strength of the materials in place and all other pertinent details. The field examination should be thorough. For example, if coring of the concrete is required, sufficient samples should be taken to obtain a reliable average strength indications and to detect possible flaws at critical locations. (Typically, core tests provide about 85 percent of the strength of laboratory-cured cylinders for the same concrete.) Technical problems may be encountered in obtaining a reliable check on the amount, kind, and location of reinforcement.

In some cases the Building Official may deem the analytical procedure to be preferable to load testing. In other cases, analytical evaluation may be the only practicable procedure. Certain members, such as columns and walls, may be difficult to load and the interpretation of the load test results equally as difficult unless severe damage or actual collapse occurs.

The Code states that the investigation shall demonstrate to the Building official's satisfaction that the intent of the Code has been satisfied. The intent of the Code is to ensure public safety. The load factors and capacity reduction factors ϕ provide for possible loads in excess of the specific design loads, complexities involved in the analysis, workmanship variations, materials variations, materials variations, and similar factors which separately may be within tolerances but which might add adversely. In general, it should be shown that the building will have strength close to or in excess of that envisaged in the original design or as required by the Code. This is a matter of judgment considering relevant factors such as the possible consequences of collapse.

20.3 — General requirements for load tests

This section covers general matters concerning load tests of structures. Those provisions are unchanged from previous Codes.

The selection of the portion of the structure to be tested, the test procedure and the interpretation of the results should be done under the direction of a qualified engineer experienced in structural investigations.

20.4 — Load tests on flexural members

Detailed procedures and criteria for load testing of flexural members are given in Section 20.4. The total test load has been changed from $1.3D + 1.7L$ as in ACI 318-63, to $1.2D + 1.44L$, which represents a reduction of about 8 to 15 percent, depending on the ratio of live load to dead load. The new procedure has the advantage, however, that the test load is a constant percentage of the theoretical design strength. This reduction in testing load avoids possible problems in testing of prestressed members where the load values stated in ACI 318-63 might induce inelastic behavior even in a member which proves to have adequate strength capacity.

20.4.5—A general acceptance criterion for the behavior of a structure under the test load is that it shall not show "visible evidence of failure." "Visible evidence of failure" will include cracking, spalling, or deflection of such magnitude and extent that it is obviously excessive and incompatible with the safety requirements for the structure. No simple rules can be developed, for application to all types of structures and conditions. If sufficient damage has occurred that the structure is considered to have failed the test, retesting is not permitted since it is considered that damaged members should not be put into service even at a lower load rating.

If the structure shows no visible evidence of failure, recovery of deflection after removal of the test load is used to determine whether or not the strength of the structure is satisfactory. In the case of a very stiff structure, however, the errors in measurements under field conditions may be of the same order as the actual deflections and recovery. To avoid penalizing a satisfactory structure in such a case, recovery requirements are waived if the maximum deflection is less than $l_t^2/20{,}000h$. The recovery requirements have been elaborated in view of the lower test load and to provide for testing of prestressed concrete constructions.

20.5 — Members other than flexural members

Because the criteria for judging the results of load tests are not well established for Code purposes except for members subjected to flexure only, the analytical method is preferred for the strength evaluation of other types of elements. Load testing of any type of structure is not, however, excluded as an alternative procedure when feasible.

20.6 — Provision for a lower load rating

Except for load tested members that have visibly failed under a test (see Section 20.4.5), the Building Official may permit the use of a structure or member at a lower load rating that is judged to be safe and appropriate on the basis of the test results.

APPENDIX A — SPECIAL PROVISIONS FOR SEISMIC DESIGN

A.1 — Scope

This appendix is new in its entirety since ACI 318-63. All previous editions contained no special provisions for seismic design. The major philosophy here is to minimize seismic forces by producing a ductile energy-absorbing structural system containing elements the strength of which tends to develop through the formation of plastic hinges rather than through less ductile flexural, shear, or compression failures.

The provisions of this appendix are based to some extent on the information and recommendations contained in the 1961 publication: "Design of Multistory Reinforced Concrete Buildings for Earthquake Motions"[A.1] and the 1967 paper "Seismic Resistance of Reinforced Concrete Beam-Column Joints."[A.2] Modifications and extensions have been made based for the most part on the best current engineering practice as represented in the seismic recommendations[A.3,A.4] of the Structural Engineers Association of California, unpublished research data from additional beam-column tests at the Portland Cement Association laboratories, and analysis of studies of damage to buildings resulting from recent catastrophic earthquakes, namely: Skopje (1963),[A.5] Anchorage (1964),[A.6,A.7] and Caracas (1967).[A.8,A.9]

The entire Code applies to this appendix except were specifically excluded or when more stringent requirements are applied therein.

A.1.1—The provisions of this appendix are intended to apply to reinforced concrete structures located in a seismic zone* where major damage to construction has a high probability of occurrence, and designed with a substantial reduction in total lateral seismic forces due to the use of lateral load-resisting systems consisting of ductile moment resisting space frames, with or without special shear walls.

Space frames and shear walls designed according to the body of the Code are satisfactory when the general building code of which the ACI Code is a part requires a ductile moment-resisting space frame in ordinary or tall buildings constructed in seismic probability zones where minor or moderate damage from earthquakes are indicated, providing that ductility reduction factors for lateral seismic forces are not utilized in the design of the space frame or shear walls. In such cases the provisions of Appendix A are not mandatory.

A.1.2—When it is not clear as to which members are the beams and which are the columns at a connection, the most severe conditions govern. If the frame cannot be designed in this manner, it is not covered by Appendix A.

No recommendations are made for the case where the flexural members are slabs (plates) since such members are still under investigation insofar as seismic forces are concerned.

A.1.3—It is important to note that the provisions of this appendix are based on the use of the load factors and capacity reduction factors specified in Chapter 9 and not on the alternate method of Section 8.10.

A.1.4—The state of the art in seismic design is changing rapidly. Studies of seismological data and building response, now made feasible by computers, quite likely will make possible more realistic and more sophisticated methods for determining the requirements for a structure to resist anticipated earthquakes than has been possible heretofore. The provisions of this section are a reaffirmation of the principle of Section 1.4 in the body of the Code with special application to this appendix.

Alternate systems or design methods should be supported by thorough analytical or experimental studies to assure a proper combination of earthquake input and strength and ductility requirements. The acceptance or rejection of such alternate methods lies with the local building authorities.

A.2 — Definitions

The definitions provided in this section apply only to Appendix A.

The term "confined region" is defined primarily for the purposes of anchorage although it describes regions in which some confinement is provided for reasons of ductility. The amount of transverse reinforcement (and therefore confinement) required varies with the type of member and with location of the region in the frame. It is not intended that the term "confinement" in this definition mean sufficient lateral restraint to increase the strength of concrete within the transverse reinforcement.

A.3 — General requirements

A.3.2—Possible reversal of axial forces, reversal of shears, and reversal of bending moments must be considered in the design of frame members, floor and roof systems, and walls.

A.3.3—Specified yield strength of the reinforcement is limited to 60,000 psi because higher strength steels may not have the yield plateau needed for ductility. The steel must not have a higher specified yield strength than that called for in the plans and specifications for each particular placement in the structure. For example, if the plans call for a particular member reinforcement to be Grade 40, then Grade 60 or any other steel must not be used in this situation. The use of higher grade steels may result in inadequate ductility when the member is strained into the plastic range.

A.4 — Assumptions

The general assumptions relating to strength design stated elsewhere in the Code also apply to Appendix A.

A.5 — Flexural members of special ductile frames

A.5.1—The ductility of a flexural member decreases as the steel ratio, ρ, approaches the steel ratio, ρ_b, producing balanced conditions. Necessary ductility is assured by placing an upper limit on ρ. The minimum value of the steel ratio, ρ, is provided to prevent a sudden brittle flexural failure in a lightly reinforced beam where the bending resistance of the concrete acting alone might be greater than that of the reinforced beam after tension cracking occurs.

A.5.2—These minimum provisions allow for shifts in inflection points that are not indicated by combinations of design loads, including seismic forces.

A.5.4—The Code does not require anchorage calculations for top and bottom reinforcement continuous through beam-column connections except for anchorage within each flexural member

*Seismic zone maps are under the jurisdiction of a general building code rather than of ACI 318-71. They do not apply to reinforced concrete frames alone. The maps are used to determine the seismic design loads and special structural requirements for regions of different seismicity. The zones are usually designated as areas of equal probability of risk of damage, such as Zone 0-no damage, Zone 1-minor damage, Zone 2-moderate damage, and Zone 3-major damage.

Reverse loading tests of interior connections conforming to this appendix show that the advantages of continuity offset any theoretical deficiencies in embedment length within the connections. Although strain measurements indicate that the reinforcement was in tension throughout the connections, measured moment capacities on both sides of the connections exceeded the calculated moment strengths based on fully effective compression reinforcement.

A.5.6—Two-thirds of the development length specified in Sections 12.5 (a) and 12.5 (c) will develop the required anchorage based on the yield stress of bars in the confined region because of the additional confinement required by Appendix A.

A.5.7—Web reinforcement is required to prevent a nonductile shear failure before the fully reversible flexural capacity of a member has been developed. Therefore, the stirrups or stirrup-ties perpendicular to the longitudinal reinforcement must be designed to provide for the maximum shears at every section in the member after the formation of plastic hinges due to lateral displacements of the frame in either direction.

The web reinforcement perpendicular to the longitudinal reinforcement is designed according to Chapter 11 for the maximum total design shears V_u throughout the length of the member. Ordinarily, this will be the larger of the shears at each section within the two free bodies of Fig. A-1.

A.5.8—Minimum web reinforcement must be provided throughout the length of the member to protect against shears, moment reversals, and shifts in the points of contraflexure that are not indicated by the loads, including seismic forces assumed in the design.

All of the web reinforcement must be placed perpendicular to the longitudinal steel since inclined bars are effective for shear in one direction only and are, therefore, not suitable in situations where shears are likely to reverse.

A.5.9—Closely spaced web reinforcement is needed at the beam ends to provide the necessary ductility where plastic hinges may form. Eq. (A-1), for the required amount of web reinforcement, is empirical. According to Section 7.12.6, this reinforcement will, almost without exception, consist of closed stirrups or spirals extending completely around all main reinforcement, since this region is almost always within the beam length subject to stress reversals. All of the closed stirrups should preferably be stirrup-ties.

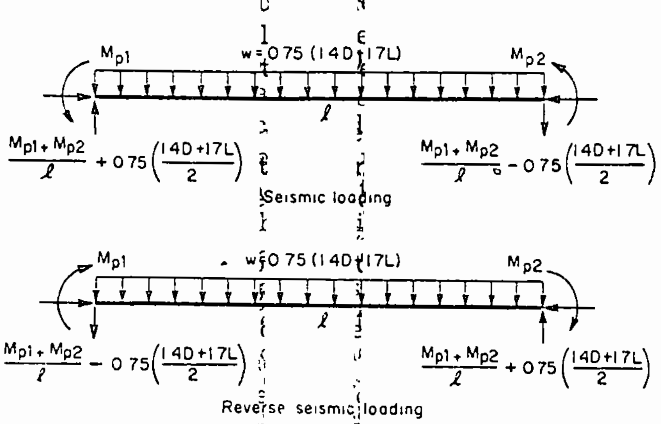


Fig. A-1 — Beam under seismic loading

A.5.10—These stirrup-ties are also effective as web reinforcement.

A.6 — Special ductile frame columns subjected to axial loads and bending

A.6.2—It is desirable to have plastic hinges form in the beams rather than in the columns. The Code, therefore, requires that the moment strengths of the columns exceed those of the beams at a connection except when special provisions are made to allow hinging in one or more columns at a level.

A.6.4.1 This special transverse reinforcement is provided to insure the required ductility should plastic hinges form at column ends, to compensate for strength loss should spalling occur in the concrete cover, and to serve as all or part of the web reinforcement required in this region.

A.6.4.2 Eq. (10-3) is intended to provide confinement and lateral restraint to the core of a column so that the resulting gain in strength replaces the strength loss of an axially loaded column where the concrete cover spalls off. The minimum value specified for the volumetric ratio, ρ_s, provides the necessary ductility of large columns under eccentric loads where the ratio of gross area to core area of the concrete is low.

A.6.4.3 Eq. (A-4) for A_{sh} was developed to determine the cross-sectional area of one leg of a

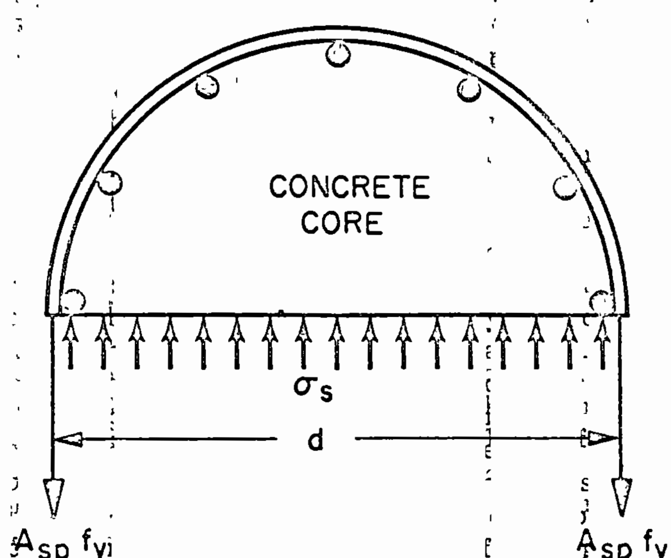


Fig. A-2 — Sectional view of portion of equivalent spiral column

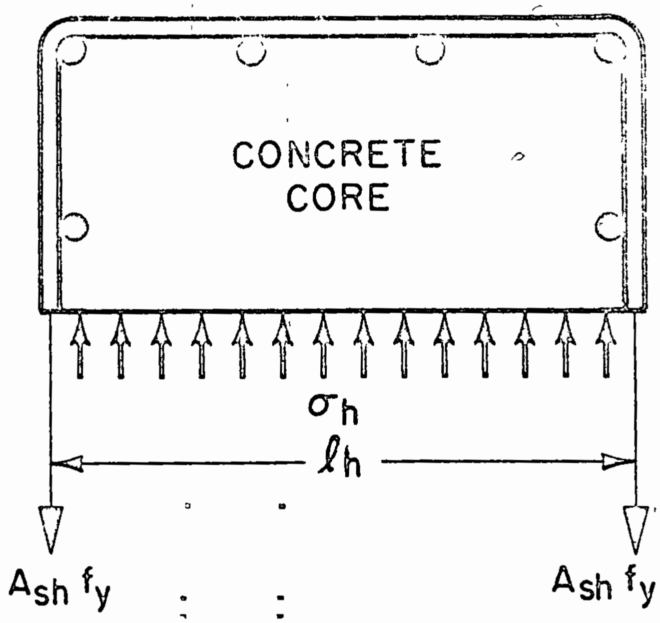


Fig. A-3 — Sectional view of portion of hoop column

single hoop, without supplementary crossties, used to provide confinement to the rectangular core of a compression member. It was devised to provide the same average compressive stress in the rectangular core as would exist in the core of an equivalent circular spiral compression member having equal gross area, core area, center-to-center spacing of lateral reinforcement, and strength of concrete and lateral reinforcement. The derivation of the formula for A_{sh} which follows makes use of Fig. A-2 and A-3.

For free body of Fig. A-2

$$\Sigma V = 0 = \sigma_s s_h d - 2A_{sp}f_y$$

$$\sigma_s = \frac{2A_{sp}f_y}{s_h d}$$

For free body of Fig. A-3:

$$\Sigma V = 0 = \sigma_h s_h l_h - 2A_{sh}f_y$$

$$\sigma_h = \frac{2A_{sh}f_y}{s_h l_h}$$

For equal confinement of concrete:

$$\sigma_s = 0.5\sigma_h$$

since the reduced efficiency of rectangular hoops *may* be as much as 50 percent (see Reference A.1, p. 99):

$$\frac{2A_{sp}f_y}{s_h d} = \frac{A_{sh}f_y}{s_h l_h}$$

$$\frac{2A_{sp}}{d} = \frac{A_{sh}}{l_h} \qquad \text{(A-C1)}$$

For equivalent spiral column:

$$\rho_s \approx \frac{A_{sp}\pi d}{s_h \frac{\pi}{4} d^2} \approx \frac{4A_{sp}}{s_h d}$$

$$d \approx \frac{4A_{sp}}{s_h \rho_s} \qquad \text{(A-C2)}$$

Substituting Eq. (A-C2) into Eq. (A-C1):

$$\frac{s_h A_{sp} \rho_s}{2A_{sp}} = \frac{A_{sh}}{l_h}$$

$$A_{sh} = \frac{l_h \rho_s s_h}{2}$$

which is Code Eq. (A-4). Eq. (A-4) yields twice the volume of hoop steel given by the formula (see Reference A.2, p. 541) used by the Portland Cement Association in the development of their seismic beam-column tests when a single hoop without supplementary crossties is used. The equation provides the same steel volume as the corresponding formulas in the book by Blume, Newmark, and Corning (see Reference A.1, p. 159) and the SEAOC requirements (see Reference A.4, pp. 92-93) under the same condition.

The formula for A_{sh} given above was derived to provide a column that would not lose strength under axial load after the concrete cover spalls off, that has the relatively flat ductile ultimate load curve of a circular spiral column, and that would be otherwise reasonably comparable to a circular spiral column designed under ACI Code requirements. The limited test results now available are inconclusive[A.10] as to the amount of strength closely spaced rectangular hoops add to the core strength of an axially loaded column after the shell is lost. However, strength and ductility under axial load are often not critical under seismic loading, and it has been established (see Reference A.11, p. 126) that circular spiral reinforcement in the quantity required by the Code does not completely replace the strength lost when the cover spalls from a spiral column subject to axial loads and bending. The rectangular hoop tests referred to (Reference A.10, pp. 213-235), nevertheless, show that closely spaced rectangular hoops do very significantly increase the ductility and toughness of column-type members under axial loads or bending moment. The members tested all had reinforcement ratios considerably less than is required by Appendix A. Further, the PCA tests reported by Hanson and Conner (Reference A.2, pp. 533-560) have shown that beam-column connections reinforced with about one-half of the steel required by Eq. (A-4) and conforming with the remaining requirements of Appendix A (particularly Sections A.6.2., A.6.4.4, and A.7.1) perform in a relatively ductile and satisfactory manner under seismic loads. Unfortunately, those tests did not produce data concerning the effectiveness of rectangular hoops under the ultimate axial load conditions assumed in the derivation of Eq. (A-4). The tests referred to herein indicate that further research might lead to a reduction in the volume of steel

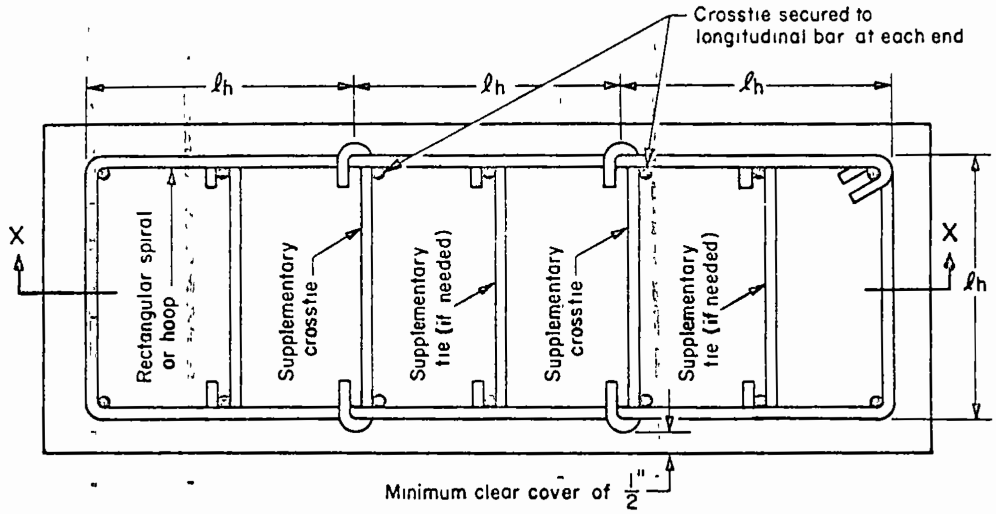


Fig. A-4 — Sectional view of hoop column showing supplementary ties and supplementary crossties

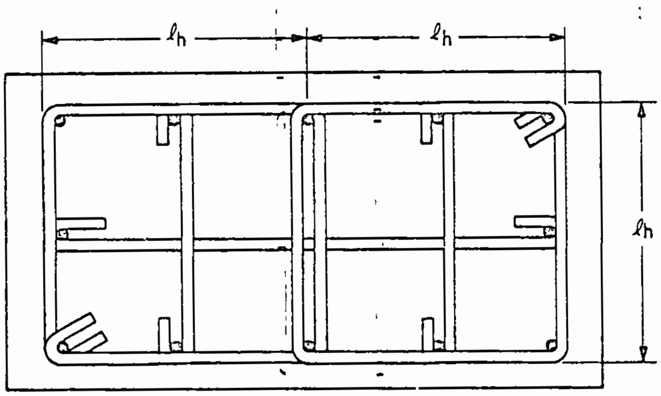


Fig. A-5 — Sectional view of hoop column showing overlapping hoops and supplementary ties

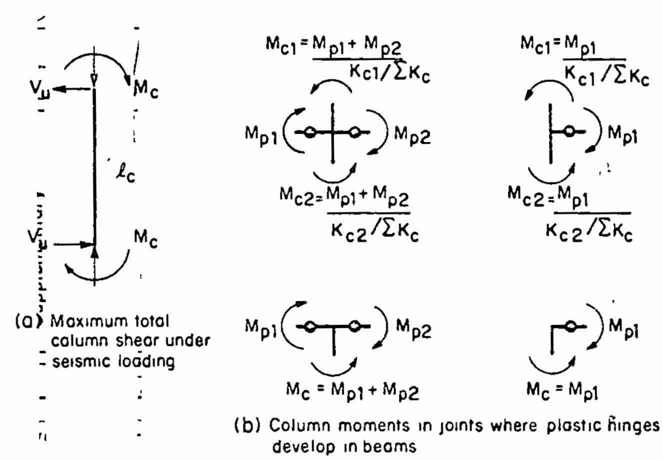


Fig. A-6 — Column under seismic loading

now required by Eq. (A-4) in future revisions of the ACI Code.

Eq. (A-4) has been derived for the case where overlapping hoops or supplementary crossties are not used. Section A.6.4.3 permits overlapping hoops or supplementary crossties to be used to reduce the unsupported length l_h (as in Section X-X of Fig. A-4). A 25 percent reduction in the required steel area through the section is permitted when one supplementary crosstie is used, and a 33 percent reduction when two supplementary crossties are used. This gives some recognition to the apparently superior confinement provided by overlapping hoops and supplementary crossties through bond between the steel and the concrete, and by providing a more favorable restraint on the hoop steel and longitudinal steel.

The limitation on the value of the volumetric ratio, ρ_s, used in Eq. (A-4) is intended to insure the necessary ductility under eccentric loading of large columns where the ratio of gross area to core area is low.

Methods of providing a confined region in a column or connection are shown in Fig. A-4 and A-5. Supplementary ties are provided if required by Sections 7.12.2 and A.6.6. Supplementary crossties used to reduce l_h may be used to satisfy these requirements. A standard 135 deg bend with a 10-bar-diameter extension is required at each end of a hoop.

A.6.5—These columns tend to have high compressive forces due to the cantilever action induced by the rigid elements above and below. Columns that support discontinuous shear walls, braced frames, or similar rigid elements should have special transverse reinforcement extending for the full height of the supporting columns unless a comprehensive analysis shows that the compressive forces in the columns are not structurally significant.

A.6.6—Shear capacity is extremely important in columns under seismic forces.

The transverse reinforcement in a column is designed according to Chapter 11 for the maximum total design shear V_u. This can be computed from the free body of a column shown in Fig. A-6a. The moments M_c both act clockwise or both act counterclockwise under seismic loading.

Their magnitude depends on whether or not plastic hinges develop in the column ends or beam ends. When a plastic hinge develops in a column end, M_c is the plastic moment capacity of the column. When plastic hinges develop in the beams, column moments are shown on the modified free bodies of beam-column joints in Fig. A-6b. Only the moments are shown; shears and axial forces are omitted for clarity. Where two beam moments are shown, they both act clockwise or both act counterclockwise under seismic forces. When two column moments are shown, they both act in opposition to the beam moments.

Under no condition does the transverse reinforcement in a column need to be heavier than that indicated should plastic hinges develop in the top and bottom of the column. Therefore, all cases may be designed for this condition, if desired.

A.6.7—Splices shall preferably be made in the midlength region of columns.

A.7 — Beam-column connections in special ductile frames

The special transverse reinforcement is intended to insure the ductility of the connection, to compensate for strength loss due to spalled concrete, to improve bond of steel to concrete within the connection, and to provide some or all of the shear reinforcement required in this region.

Great care is needed to avoid impractical placement problems in seismic design. This is particularly true for the reinforcing steel in the connections of special ductile frames. If high percentages of reinforcement are used in beams and columns, it may be physically impossible to place the beam and column reinforcement and the required ties in the connections; or if the reinforcement is placeable, it may be impossible to place the concrete in the connection or to get a vibrator into it. It may prove economical to construct a full size model of the reinforcement in a typical connection to investigate its constructibility, unless the designer has had considerable experience in this type of work.

A.7.1—The shear reinforcement within a connection is placed transverse to the main column reinforcement. It is designed according to Chapter 11 for the maximum horizontal total design shear V_u at each horizontal section in the connection. A free body diagram of a typical interior connection is shown in Fig. A-7. Shear reinforcement is proportioned for the total shear force at each horizontal section in the free body. The connection width b used in computing v_u is the width of the beam or column, whichever is larger.

Calculations for the shear stress carried by the concrete v_c within the connection should be

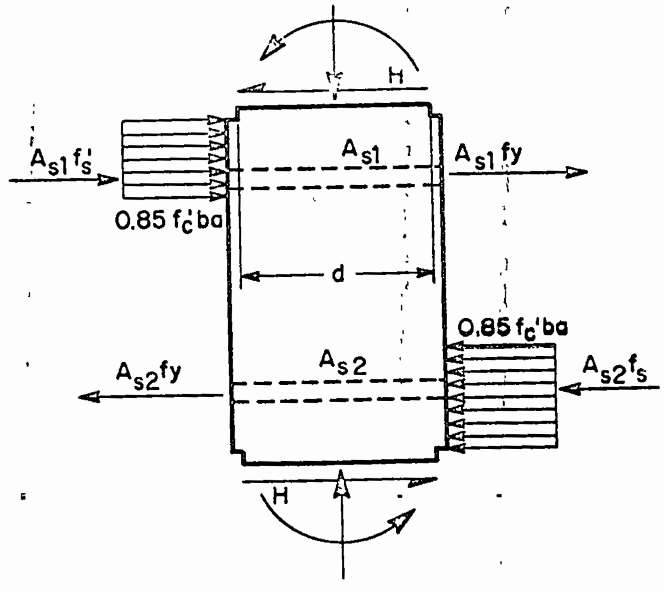


Fig. A-7 — Horizontal shearing forces acting on a connection under seismic loading

based on the specified strength of the concrete actually placed within the connection.

Hoop reinforcement is effective as shear reinforcement. Except as modified by Section A.7.2, the shear reinforcement within a connection must at least equal the hoop requirement of Section A.6.4.

A.7.2—Beams framing into the sides of a connection provide additional restraint and increase the capacity of the connection. A reduction in transverse reinforcement is allowed where beams frame into four sides since they restrain the connection in both directions.

A.8 — Special shear walls

A.8.3—The intent here is to prevent a brittle shear failure in a shear wall undergoing reversing seismic loads.

A.8.4—This section is intended to provide a ductile shear wall when flexure is present without a significantly large axial load.

The wall must be investigated by means of the straight-line theory of stress and strain in flexure using the factored loads of Eq. (9-2) and (9-3), with $1.1E$ substituted for W. If the resulting tension in the concrete exceeds $0.15f_r$, then the minimum area of vertical steel concentrated near the ends of the wall shall be $A_s = (200/f_y)(hd)$. The values for f_r are given is Section 9.5.2.2. This provision is intended to prevent a sudden brittle flexural failure in a lightly loaded shear wall where the bending resistance of the concrete alone might be greater than that of the reinforced wall after tension cracking occurs.

A.8.5—The intent here is to provide a ductile shear wall system when the wall is subjected to flexure and a significantly large axial load.

References

A.1. Blume, J. A.; Newmark, N. M.; and Corning, L. H., *Design of Multistory Reinforced Concrete Buildings for Earthquake Motions,* Portland Cement Association, Skokie, 1961, 318 pp.

A.2. Hanson, N. W., and Conner, H. W., "Seismic Resistance of Reinforced Concrete Beam-Column Joints," *Proceedings,* ASCE, V. 93, ST5, Oct. 1967, pp. 533-560.

A.3. Seismology Committee, "Recommended Lateral Force Requirements and Commentary," Structural Engineers Association of California, 1967, pp. 1-90.

A.4. Seismology Committee, "Recommended Lateral Force Requirements and Commentary, 1968 Revisions and Addendum," Structural Engineers Association of California, 1968, pp. 91-100.

A.5. Sozen, M. A., "Structural Damage Caused by the Skopje Earthquake of 1963," *Bulletin* 279, Civil Engineering Studies, Structural Research Series, University of Illinois, 1964, 39 pp.

A.6. Kunze, W. E.; Sbarounis, J. A.; and Amrhein, J. E., "The March 27 Alaskan Earthquake—Effects on Structures in Anchorage," ACI JOURNAL, *Proceedings* V. 62, No. 6, June 1965, pp. 635-649.

A.7. Clough, R. W., and Benuska, K. L., *FHA Study of Seismic Design Criteria for High-Rise Buildings,* Federal Housing Administration, Aug. 1966, pp. 1-1 through 6-1.

A.8. "Caracas Earthquake Damage Reported by the Portland Cement Association Team," ACI JOURNAL, *Proceedings* V. 65, No. 4, Apr. 1968, pp. 292-294.

A.9. Sozen, M. A., "The Caracas Earthquake of July 29, 1967," ACI JOURNAL, *Proceedings* V. 65, No. 5, May 1968, pp. 394-401.

A.10. Roy, H. E. H., and Sozen, M. A., "Ductility of Concrete," *Flexural Mechanics of Reinforced Concrete,* SP-12, American Concrete Institute/American Society of Civil Engineers, Detroit, 1965 (with Discussion by P. R. Barnard, S. Stockl, and Vitelmo Bertero and C. Felippa), pp. 213-235.

A.11. Hognestad, Eivind, "Inelastic Behavior in Tests of Eccentrically Loaded Short Reinforced Concrete Columns," ACI JOURNAL, *Proceedings* V. 49, No. 2, Oct. 1952, pp. 117-140.

INDEX

Numbers, in most cases, indicate the major subsection in which the subject appears. The number to the left of the decimal, or numbers without decimals, are the chapter number.

EL DISEÑO DE ESTRUCTURAS DE CONCRETO

1. EL DISEÑO ESTRUCTURAL

Una estructura puede concebirse como un sistema; es decir, como un conjunto de partes o componentes que se combinan en una forma ordenada para cumplir una función dada. La función puede ser: salvar un claro, como en los puentes; encerrar un espacio, como sucede en los distintos tipos de edificios; o contener un empuje, como en los muros de retención, tanques o silos. La estructura debe cumplir la función a la que está destinada con un grado de seguridad razonable y de manera que en las condiciones normales de servicio tenga un comportamiento adecuado. Además, deben cumplirse otros requisitos, tales como el de mantener el costo dentro de límites económicos y el de satisfacer determinadas exigencias estéticas.

Un examen de las consideraciones anteriores hace patente la complejidad del diseño de sistemas estructurales. ¿Qué puede considerarse como seguridad razonable, o como resistencia adecuada? ¿Qué requisitos debe satisfacer una estructura para considerar que su comportamiento bajo condiciones de servicio es satisfactorio? ¿Qué es un costo aceptable? ¿Qué vida útil deberá preverse? ¿Es la estructura aceptable estéticamente?.

Estas son algunas de las preguntas que el proyectista tiene en mente al diseñar una estructura. El problema no es sencillo y en su solución el proyectista hace uso de su intuición y su experiencia, apoyando éstas en el análisis y la experimentación.

Si se contemplan en toda su complejidad, puede afirmarse que los problemas de diseño no suelen ser de solución única, sino de solución razonable. En efecto, la labor del ingeniero proyectista tiene mucho de arte. Indudablemente el ingeniero debe aprovechar el cúmulo de información y metodología científica disponible, pero además tiene que tomar en cuenta otros factores que están fuera del campo de las matemáticas y de la física.

El proceso que sigue el proyectista al diseñar una estructura es análogo al utilizado en el diseño de cualquier otro sistema.[1-4,12,17] Son aplicables, por lo tanto, los métodos que aporta la Ingeniería de Sistemas, una de cuyas finalidades es la racionalización del proceso de diseño.

El proceso de diseño de un sistema parte de la formulación de los objetivos que se pretende y de las restricciones que deben tenerse en cuenta. El proceso es cíclico; se parte de consideraciones generales, que se ~~afirma~~ afina en aproximaciones sucesivas a medida que se va acumulando información sobre el problema.

En el diseño de estructuras, una vez planteado el problema, supuestas unas solicitaciones razonables y definidas las dimensiones generales, es necesario ensayar diversas estructuraciones para resolverlo. Es en esta fase del diseño donde la intuición y la experiencia del ingeniero desempeñan un papel primordial. La elección del tipo de estructuración es sin duda uno de los factores que más afecta el costo de un proyecto. Los refinamientos posteriores en el dimensionamiento de secciones son de mucha menor importancia.

La elección de una cierta forma estructural debe ir asociada a la elección del material con que se piensa realizar la estructura. Al hacer esta elección, el proyectista debe tener en cuenta las características de la mano de obra y el equipo disponible, así como también el procedimiento que más se preste al caso. Después de elegir una estructuración tentativa, se idealiza la estructura para estudiar los efectos de las solicitaciones a que puede estar sujeta. Esta idealización es necesaria porque el problema real es siempre más complejo que lo que es práctico analizar.

El análisis estructural implica un conocimiento de las solicitaciones que obran sobre la estructura y de las dimensiones de sus elementos. Estos datos son imprecisos cuando se inicia el diseño, ya que sólo se conocen en forma aproximada las dimensiones que tendrán los elementos. Estos influyen tanto en el valor del peso propio como en el comportamiento estructural del conjunto.

En un proceso cíclico el proyectista va ajustando los datos iniciales a medida que va precisando el análisis. Solamente en la fase final de este proceso hace un cálculo numérico relativamente refinado. El grado de precisión que trate de obtener en este proceso depende de la importancia de la estructura y de la posibilidad de conocer las solicitaciones que obrarán realmente sobre ella. Un vicio frecuente es el de tener un exceso de minuciosidad en casos en que la importancia del problema no lo amerita, en que el conocimiento de las solicitaciones es solamente aproximado, y en que el ahorro que pueda obtenerse gracias al refinamiento en el análisis no lo justifica.

La fase final del diseño consiste en la comunicación de los resultados del proceso descrito a las personas que van a ejecutar la obra. La comunicación de los datos necesarios para la realización del diseño se hace mediante planos y especificaciones. Este aspecto final no debe descuidarse, puesto que el disponer de planos claros y sencillos, y de especificaciones concretas, evita errores y confusiones por parte de los constructores.

Idealmente, el objeto del diseño de un sistema es la optimización del sistema, es decir la obtención de la mejor de todas las soluciones posibles.[1-6,12,13,15,17] El lograr una solución óptima absoluta es prácticamente imposible. Lo que es óptimo bajo un conjunto de circunstancias no lo es bajo otro conjunto; lo que es óptimo para un individuo puede no serlo para otro. Así, como se dijo anteriormente, no existen soluciones únicas, sino solamente reazonables.

Sin embargo puede ser útil optimizar bajo determinado criterio, tales como el de peso o costo mínimos. Si el criterio puede expresarse analíticamente por medio de una función, generalmente llamada "función objetivo" o "función criterio", el problema puede resolverse matemáticamente.

Las técnicas de optimización tienen todavía aplicaciones limitadas, en el diseño estructural, debido a las dificultades matemáticas que suelen involucrar. Sin embargo, es de suponerse que, con el uso cada vez más extendido de la computación electrónica, se vayan perfeccionando estas técnicas de manera que sea posible contar con un grado de refinamiento cada vez mayor. ~~en~~ Los procesos de optimización en el diseño estructural han sido tratados por Spunt, y otros.[5,6,13]

En las consideraciones anteriores, para mayor sencillez, se han tratado los sistemas estructurales como sistemas independientes. De hecho, toda estructura no es sino un subsistema de algún sistema más complejo: un edificio, un complejo industrial, un sistema hidráulico, un sistema de caminos o de comunicación urbana. En un edificio, por ejemplo, pueden distinguirse varios subsistemas además del estructural: las instalaciones eléctricas, de plomería, y ~~este~~ aire acondicionado, los elevadores, los acabados arquitectónicos, la ventanería, etc.

Según el enfoque de sistemas, en el diseño del sistema total debe tenerse en cuenta la interacción entre todos los subsistemas. De esta manera, en el diseño del subsistema estructural deben considerarse no solamente los aspectos de eficiencia estructural, sino también la relación de la estructura con los demás subsistemas. Por ejemplo, puede ser necesario prever pasos para instalaciones que impliquen mayor consumo de materiales que el estrictamente necesario desde el punto de vista estructural. Por otra parte, los enfoques globales o de conjunto implícitos en la concepción de los edificios como sistemas pueden conducir a soluciones de gran eficiencia en las que los componentes estructurales del sistema se diseñan de manera que realicen otras funciones además de las estrictamente estructurales. Así, un muro de carga puede ser también un elemento arquitectónico de fachada y servir de elemento rigidizante.

En el diseño de los subsistemas estructurales para edificios debe tenerse en cuenta su importancia relativa dentro del sistema general. Son ilustrativos los datos de la Tabla 1, basada en información proporcionada en la ref 20.

Se desprende de estos datos la proporción relativamente pequeña del costo total correspondiente a la estructura. Esto indica que en muchas ocaciones no se justifican refinamientos excesivos en el cálaulo estructural, ya que las posibles economías de materiales resultan poco significativas. Lo importante, en efecto, es la optimización del sistema total, como ya se ha indicado, y no la de los subsistemas o componentes considerados individualmente.

TABLA I

DISTRIBUCION APROXIMADA DEL COSTO DE EDIFICIOS ALTOS EN LOS EE. UU. DE AMERICA

CONCEPTO	POR CIENTO
Excavación y cimientos	10
Estructura	25
Instalaciones diversas (electricidad, plomería, aire acondicionado)	30
Elevadores	10
Muros exteriores	12
Acabados diversos	13
	100

Si la optimización de sistemas relativamente sencillos como los sistemas estructurales presenta ciertas dificultades, son aún más graves los problemas que ofrece la optimización rigurosa de sistemas complejos como el de un edificio o una obra urbana en los que intervienen gran número de variables, muchas de ellas de naturaleza psicológica o sociólogica y por lo tanto difícilmente cuantificables. En efecto, la aplicación rigurosa de los métodos del enfoque de sistemas aún no es de uso común.

El interés por el enfoque de sistemas está produciendo un cambio de actitud entre los proyectistas frente al problema de diseño. Por una parte, se tiende a una racionalización creciente del proceso de diseño, lo que implica manipulaciones matemáticas cada vez más refinadas. Por otra, el reconocimiento de la interdependencia entre los diversos subsistemas que integran una obra civil está llevando a un concepto interdisciplinario del diseño. Mientras que antes los diversos subsistemas se diseñaban independientemente, de manera que la coordinación entre ellos solía ser poco satisfactoria, ahora se tiende cada vez más al trabajo de equipo.

El enfoque de sistemas aporta herramientas de gran utilidad en el diseño. Sin embargo, no debe olvidarse que en el proceso de diseño seguirá siendo de gran importancia la intuición y la capacidad creativa e innovadora del proyectista.

2. LAS ESTRUCTURAS DE CONCRETO

Las estructuras de concreto reforzado tienen ciertas características, derivadas de los procedimientos constructivos usados en su fabricación, que las distinguen de las estructuras de otros materiales.

El concreto se fabrica en estado plástico lo que obliga a utilizar moldes para soportarlo mientras adquiere una resistencia suficiente para que la estructura sea autosoportante. Esta característica implica ciertas restricciones, pero al mismo tiempo aporta algunas ventajas. Una de éstas es su "moldeabilidad", propiedad que brinda al proyectista una gran libertad en la elección de formas. Gracias a ello es posible construir estructuras, como los cascarones, que en otro material serían muy difíciles de obtener.

Otra característica importante es la facilidad con que puede obtenerse la continuidad en la estructura, con todas las ventajas que esto supone. Mientras que en estructuras metálicas el logro de continuidad en las conexiones entre los elementos implica serios problemas en el diseño y en la ejecución, en las de concreto reforzado el monolitismo es una consecuencia natural de las característica constructivas.

Existen dos procedimientos principales para construir estructuras de concreto. Cuando los elementos estructurales se forman en su posición definitiva se dice que la estructura ha sido colada "in situ" o colada en el lugar. Cuando los elementos se fabrican en un lugar distinto al de su posición definitiva en la estructura, el procedimiento recibe el nombre de prefabricación.

El primer procedimiento obliga a una secuencia de operaciones determinada, ya que para poder iniciar cada etapa es necesario esperar a que se haya concluido la anterior. Por ejemplo, no puede procederse a la construcción de un nivel en un edificio hasta que el nivel inferior haya adquirido la resistencia adecuada. Además, es necesario, a menudo construir obras falsas muy elaboradas y transportar el concreto fresco del lugar de fabricación a su posición definitiva, operaciones que influyen decisivamente en el costo.

Con el segundo procedimiento se puede economizar tanto en la obra falsa como en el transporte del concreto fresco, y se pueden realizar simultáneamente varias etapas constructiras. Por otra parte, este procedimiento presenta el inconveniente del costo adicional de montaje y transporte de los elementos prefabricados y además el problema de desarrollar conexiones efectivas entre los elementos.

El proyectista debe elegir entre estas dos alternativas guiándose siempre por las ventajas económicas, constructivas y técnicas que pueden obtenerse en cada caso. Cualquiera que sea la alternativa constructiva que escoja, ésta elección influye en forma importante en el tipo de estructuración que se adopte.

Otra característica peculiar de las estructuras de concreto reforzado es el agrietamiento, que debe tenerse en cuenta al estudiar su comportamiento bajo condiciones de servicio.

3. CARACTERISTICAS ACCION-RESPUESTA DE ELEMENTOS

CONCEPTOS GENERALES

Se ha dicho que el objeto del diseño consiste en determinar las dimensiones y características de los elementos de una estructura para que ésta cumpla una cierta función con un grado de seguridad razonable, comportándose además satisfactoriamente bajo condiciones de servicio. Estos requisitos hacen necesario el conocer las relaciones que existen entre las características de los elementos de una estructura (dimensiones, refuerzo, etc.), las solicitaciones que debe soportar y los efectos que dichas solicitaciones producen en la estructura. En otras palabras, es necesario conocer las características acción-respuesta de la estructura estudiada.

Las acciones en una estructura son las solicitaciones a que puede estar sujeta. Entre éstas se encuentran, por ejemplo, el peso propio, las cargas vivas, las presiones por viento, las aceleraciones por sismo y los asentamientos. La respuesta de una estructura, o de un elemento, es su comportamiento bajo una acción determinada. Puede expresarse como deformación, agrietamiento, durabilidad, vibración. Desde luego, la respuesta es función de las características de la estructura, o del elemento estructural considerado.

Si se conocen las relaciones

ACCION ELEMENTOS DE CIERTAS CARACTERISTICAS RESPUESTA

para todas las combinaciones posibles de acciones y características de una estructura se tendrá una base racional para establecer un método de diseño. Este tendrá por objeto el determinar las características que deberá tener una estructura para que, al estar sujeta a acciones determinadas, su comportamiento o respuesta sea aceptable desde los puntos de vista de seguridad a la falla y de utilidad bajo condiciones de servicio.

El problema de la determinación de las relaciones acción-respuesta para estructuras con características cualesquiera, sujetas a toda la gama posible de acciones y combinaciones de estas acciones, es insoluble debido al número infinito de combinaciones que pueden presentarse.

Esta situación ha hecho necesario el desarrollo de métodos mediante los cuales pueda basarse el estudio de una estructura en conjunto en el estudio del comportamiento de sus distintas partes o elementos. Estos métodos, llamados de análisis, permiten determinar las acciones internas en cada uno de los miembros de una estructura resultantes de la aplicación de las solicitaciones exteriores a la estructura total. Esta consideración permite reducir el problema de la determinación de las características acción-respuesta a dimensiones manejables.

Para establecer una base racional de diseño será necesario entonces obtener las características acción-respuesta correspondientes a las solicitaciones que actúan más frecuentemente sobre los distintos elementos estructurales. Con esta información se puede delimitar el rango de las solicitaciones bajo las cuales el elemento se comportará satisfactoriamente en condiciones de servicio. En otras palabras, es necesario establecer las relaciones entre los elementos siguientes.

ACCIONES INTERIORES	CARACTERISTICAS DEL ELEMENTO	RESPUESTAS
Carga axial	tipo de concreto	deformación
Flexión	tipo de refuerzo	agrietamiento
Torsión	Tamaño	durabilidad
cortante	forma	vibración
	restricción	

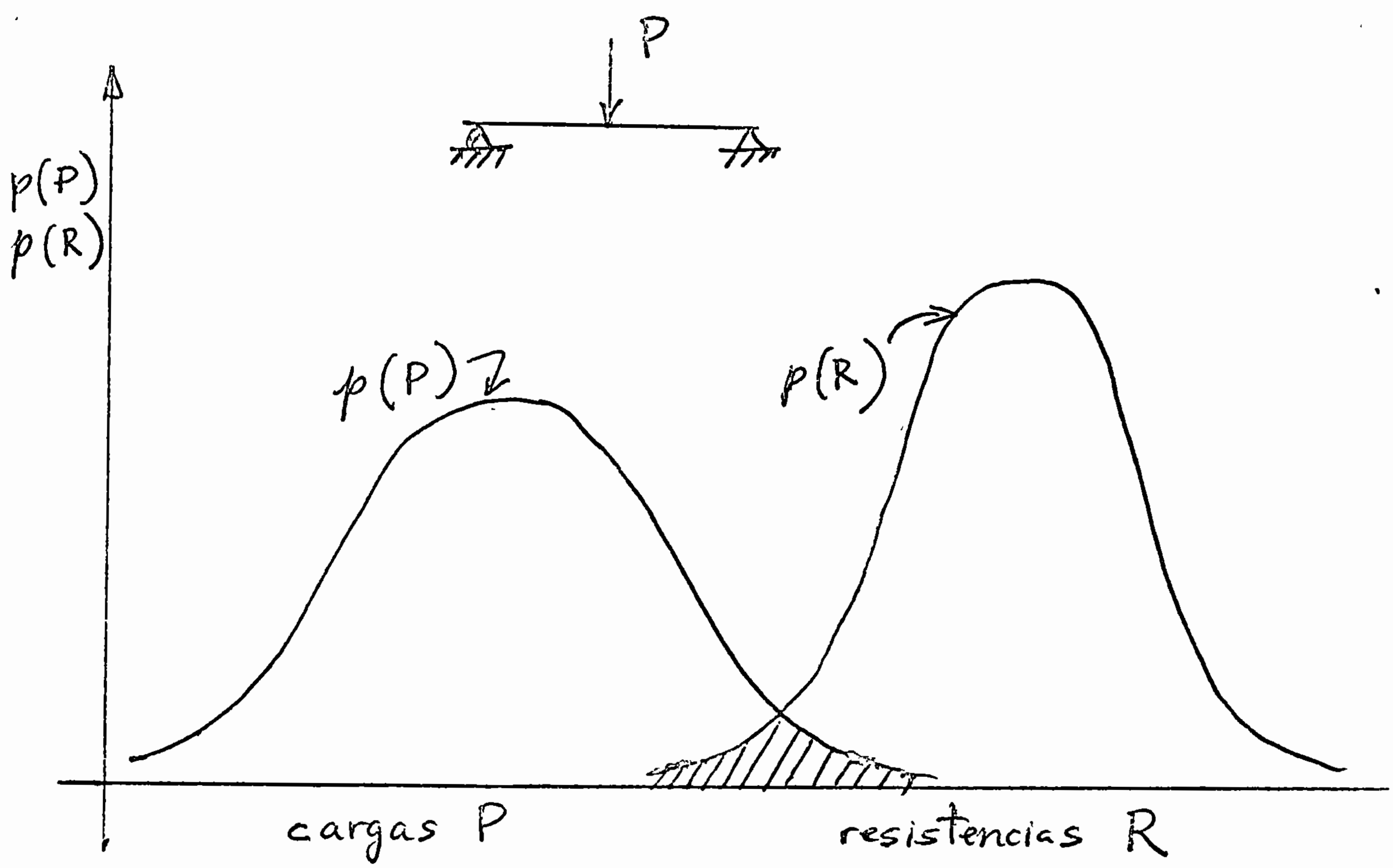
Al valuar la respuesta correspondiente a una acción determinada, es necesario tomar en cuenta el modo de aplicación de la misma, ya que este factor ejerce una influencia muy importante en dicha respuesta. Es decir, la respuesta de una estructura a una acción determinada dependerá de si ésta es instantánea, de corta duración, sostenida, repetida, etc.

En los capítulos siguientes se estudian estas relaciones para las solicitaciones más frecuentes en el caso de estructuras de concreto. La información relativa ha sido obtenida mediante experimento y experiencia, con el transcurso del tiempo.

En los procedimientos de diseño, normalmente el dimensionamiento se lleva a cabo a partir de las acciones interiores, calculadas por medio de un análisis de la estructura. Debe notarse que no siempre es necesario el obtener las acciones interiores inducidas por las solicitaciones exteriores para diseñar satisfactoriamente. Muchos diseños han sido desarrollados directamente a partir del estudio de modelos estructurales. En estos casos se aplican conjuntos de solicitaciones o acciones exteriores, representativas de aquellas a las que el prototipo estará sujeto en la realidad, a un modelo a escala de la estructura por diseñar, y se miden las respuestas del mismo. Para satisfacer la condición de seguridad, el modelo debe resistir solicitaciones, a escala, un tanto mayores que las que se estima deberá soportar la estructura bajo sus condiciones de servicio. Para satisfacer la condición de comportamiento satisfactorio bajo estas condiciones de servicio, las respuestas del modelo a estas solicitaciones deberán estar comprendidas entre los valores considerados como límite de tolerancia. Si una de las dos condiciones no se satisface, se modifican las características del modelo y se repite el proceso.

La primera condición que debe satisfacer un diseño es que la estructura resultante sea lo suficientemente resistente. En términos de las características acción-respuesta, se puede definir la resistencia de una estructura o elemento a una acción determinada como el valor máximo que dicha acción puede alcanzar. Una vez determinada la resistencia a una cierta acción, se compara este valor máximo con el valor correspondiente bajo las condiciones de servicio. De esta comparación se origina el concepto de factor de seguridad o factor de carga. De un modo rudimentario, éste puede definirse como el cociente entre la resistencia y el valor estimado de la acción correspondiente bajo las condiciones de servicio.

El diseño debe garantizar que la estructura tenga un factor de seguridad razonable . Mediante este factor se trata de tomar en cuenta en el diseño la incertidumbre sobre los efectos de ciertas acciones y sobre los valores usados en varias etapas del proceso. Entre las principales incertidumbres se pueden mencionar el desconocimiento de las solicitaciones reales y su distribución, la validez de las hipótesis y simplificaciones utilizadas en el análisis, la diferencia entre el comportamiento real y el supuesto, y la discrepancia entre los valores reales de las dimensiones y de las propiedades de los materiales con las especificadas en el diseño.



Concepto
fig 1.- ~~Concreto~~ de probabilidad de falla

La selección de un factor de seguridad adecuado no es un problema sencillo, debido al gran número de variables y de condiciones que deben tomarse en cuenta. La dificultad principal reside en la naturaleza probabilística tanto de las solicitaciones que obran sobre las estructuras como de las resistencias de éstas. Este carácter aleatorio de solicitaciones y resistencias hace que exista siempre una cierta probabilidad de que se presenten combinaciones de valores tales que la solicitación sea superior a la resistencia. Esto se ilustra en la fig 1, en la que se representan las distribuciones de frecuencias de solicitaciones y resistencias de un elemento estructural, por ejemplo, una viga. El área sombreada representa la probabilidad de falla de la estructura. La probabilidad de falla da una medida significativa del margen de seguridad real de la estructura. Puede expresarse en términos económicos, si se cuenta con los elementos necesarios para estimar el costo de las consecuencias de la falla. La estimación del costo de la falla, junto con el costo de la estructura pueden servir de base para escoger una solución conveniente con un criterio racional que asigne un margen de seguridad de acuerdo con la importancia de la obra. Obviamente el factor de seguridad de una presa debe ser mayor que el de una bodega de chatarra.

Los criterios modernos de diseño moderno están tendiendo a enfoque probabilísticos como el descrito[14], no obstante las dificultades que implican. Por una parte, todavía no se tiene suficiente información sobre la variabilidad tanto de las solicitaciones que deben considerarse como de las resistencias de los materiales y elementos utilizados en las estructuras. Por otra parte, es difícil el problema de asignar precio o valor a las consecuencias de una falla, en términos de posible pérdida de vidas y de costo de reposición. No obstante estas dificultades, el enfoque tiene indudable interés y ya existen proposiciones para formular reglamentos de construcción basados exclusivamente en conceptos probabilísticos. De hecho, ciertos conceptos probabilísticos ya han sido incorporados a algunos reglamentos en relación con la valuación de las características de los materiales y las solicitaciones.[7, 16]

A semejanza con el problema de resistencia, para garantizar que una estructura tenga un comportamiento aceptable bajo condiciones de servicio, se comparan los valores de las respuestas (deformaciones, agrietamiento, durabilidad), correspondientes a las acciones estimadas, con ciertos límites prestablecidos, que la experiencia ha indicado son satisfactorios para el tipo de estructura de que se trata.

El problema es más difícil que el de valuar la resistencia, ya que las deformaciones y el agrietamiento son función de las acciones reales que obran en la estructura, de la historia de carga y de todas aquellas variables que influyen en el comportamiento. El establecer límites razonables para las deformaciones y el agrietamiento para distintos tipos de estructuras es un problema más complejo que el de establecer un factor de seguridad razonable. Los problemas de agrietamiento y deformaciones se tratarán con detalle en un capítulo posterior. Hasta la fecha, la mejor herramienta que posee el diseñador para establecer límites de tolerancia es su experiencia con estructuras semejantes, actuando bajo condiciones similares. Recientemente se han desarrollado algunos procedimientos de cálculo de deformaciones y agrietamiento, pero este campo está todavía en su infancia.

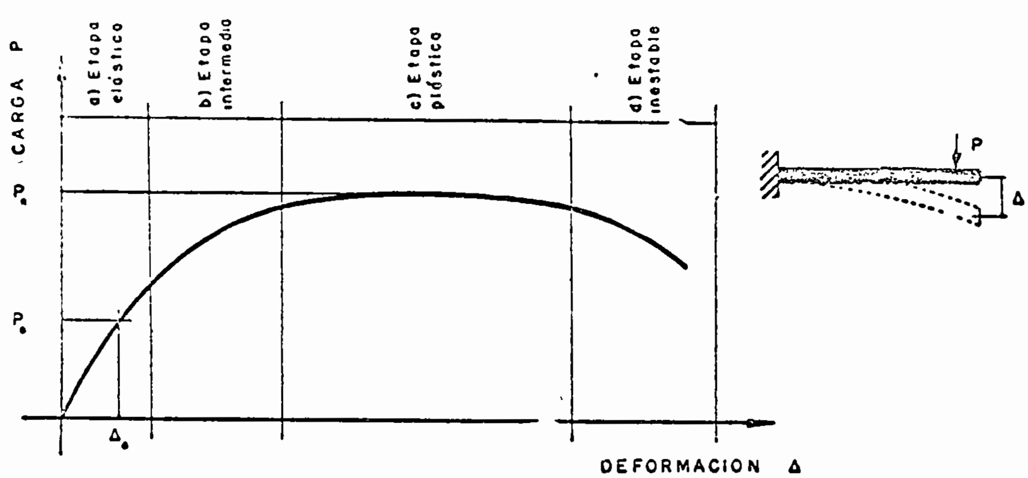


Fig. 1.1. Característica carga-deformación.

fig 2.- Gráfica carga-deformación

ILUSTRACION

Para fijar las ideas anteriores, éstas se aplicarán a un caso específico, Considérese el voladizo mostrado en la fig 2 ~~suejo~~ sujeto a la acción de una carga vertical P que varía desde un valor nulo hasta aquel que produce el colapso. La característica acción-respuesta más inmediata es la curva carga-deflexión presentada también en la figura.

En términos de esta característica es posible definir cuatro etapas en el comportamiento del voladizo:

a) Una etapa inicial elástica, donde las cargas son proporcionales a las deformaciones. Normalmente se pretende que bajo las condiciones permanentes de servicio (excluyendo las cargas de corta duración como viento o sismo), la estructura se encuentra en esta etapa. La carga de servicio se ha marcado en la figura como P_s y la deformación correspondiente como Δ_s.

b) Una etapa intermedia donde la relación carga-deformación ya no es lineal.

c) Una etapa plástica, donde se producen deformaciones relativamente grandes para incrementos pequeños o nulos de las cargas. La resistencia (P_R) se encuentra en esta etapa. Debido a la forma de la curva es difícil establecer cual es la deformación correspondiente a la resistencia.

d) Una etapa inestable, caracterizada por una rama descendente hasta el colapso, donde a mayores deformaciones la carga disminuye.

De la ilustración se puede definir el factor de seguridad como el cociente P_R/P_s. La estructura tendrá una resistencia adecuada si este factor es mayor que un valor predeterminado considerado como aceptable.

Para investigar si el comportamiento bajo condiciones de servicio es satisfactorio se deberá comparar el valor de la deformación correspondiente a P_s con ciertos valores prestablecidos que se estimen tolerables, de acuerdo con experiencias anteriores.

Es interesante hacer notar que en la etapa plástica, a una variación muy pequeña de la carga corresponde una variación importante en la deformación de la estructura. Por lo tanto, si las acciones en esta etapa se determinan a partir de las deformaciones, errores importantes en la estimación de éstas producirán sólo variaciones insignificantes en el valor de la acción. Por el contrario, es difícil predecir en esta etapa el valor de la deformación que corresponderá a una carga determinada.

El ejemplo anterior muestra claramente que es necesario conocer las relaciones acción-respuesta correspondientes a una variación de P desde un valor nulo hasta el que produce el colapso. Esta información permite conocer el grado de seguridad de la estructura y estimar el intervalo de carga bajo el cual el voladizo se comportará satisfactoriamente.

4. LAS SOLICITACIONES

Las principales solicitaciones o acciones exteriores a que puede estar sujeta una estructura son: cargas estáticas debidas a peso propio, a cargas vivas, y a cargas permanentes, así como cargas dinámicas impuestas por un sismo, por la presión de un viento, o por la aplicación repetida de cargas vivas. También se consideran como solicitaciones las deformaciones de la estructura inducidas por asentamiento, por contracción y cambios de temperatura.

Al estimar las solicitaciones es necesario prever las condiciones más desfavorables en que la estructura podrá encontrarse, así como el tiempo en que estará sujeta a estas condiciones desfavorables. Para hacer un análisis riguroso sería necesario conocer las variaciones probables en la intensidad y distribución de las cargas a lo largo de la vida útil de la estructura, cosa difícil de lograr.

En el diseño estructural se ha hecho hincapié en el desarrollo de métodos de análisis de estructuras, pero se han llevado a cabo estudios muy limitados sobre los valores probables de las cargas que actúan. Es aquí donde se pueden cometer los mayores errores, y donde nuestro conocicimiento es más exiguo.

La estimación de las cargas debidas al peso propio puede hacerse con relativa precisión: los errores no serán mayores del 20 por ciento si se han evaluado con cuidado los volúmenes de los materiales y los pesos volumétricos.

En lo que respecta a carga viva, los errores en la estimación pueden ser del 100 por ciento, o aún mayores. La carga viva está especificada comúnmente en los reglamentos de construcción como una carga uniformemente repartida equivalente, con distintas intensidades de acuerdo con el uso considerado, o bien, como una carga móvil idealizada sobre puentes o viaductos. Estos valores equivalentes especificados tienen como base estudios muy limitados. Los efectos de las cargas equivalentes en la estructura pueden ser muy diferentes a los efectos de las cargas reales.

La estimación de cargas laterales debidas a viento o sismo está sujeta aún a mayor incertidumbre. Fácilmente se cometen errores del 300 por ciento o más en la estimación de los efectos de estas solicitaciones.

En el estado actual de nuestro conocimiento puede esperarse solamente que, con base en la experiencia, se especifique un tipo de carga tal que unido a procedimientos adecuados de diseño y construcción proporcione una estructura que se comporte satisfactoriamente.

5. EL ANALISIS DE ESTRUCTURAS DE CONCRETO

Para poder analizar una estructura es necesario idealizarla. Por ejemplo, una idealización frecuente en el análisis de edificios es considerar la estructura como formada por series de marcos planos en dos direcciones. De este modo se reduce el problema real tridimensional a uno de dos dimensiones. Se considera además que las propiedades mecánicas de los elementos en cada marco están concentradas a lo largo de sus ejes. Sobre esta estructura idealizada se aplican las solicitaciones.

Las solicitaciones o acciones exteriores inducen acciones interiores (momentos, fuerzas) de intensidad variable. El propósito fundamental del análisis es valuar las acciones interiores en las distintas partes de la estructura. Para ello es necesario, salvo en estructuras o elementos isostáticos, conocer o suponer la relación entre fuerza y deformación o, en términos más generales, entre acción y respuesta.

La hipótesis más simple que puede hacerse para relacionar carga y deformación, es la de suponer una dependencia lineal. El análisis elástico de estructuras parte de esta hipótesis.

Otra hipótesis relativamente simple que se hace para el análisis de estructuras es la de suponer que las acciones interiores, al llegar a cierto valor crítico de la acción, son independientes de las deformaciones. En esta hipótesis se basa el análisis límite. En él, se tratan de obtener los valores de las acciones para las cuales la estructura se vuelve un mecanismo inestable.

Existen otros tipos de análisis más refinados, con hipótesis menos simples que las anteriores, que se aproximan más a la realidad. Debido a su mayor refinamiento son más laboriosos, aunque con el empleo de computadoras electrónicas se usarán cada vez más.

6. EL DIMENSIONAMIENTO DE ELEMENTOS DE CONCRETO REFORZADO

Se entiende por dimensionamiento la determinación de las propiedades geométricas de los elementos estructurales y de la cantidad y posición del acero de refuerzo.

El procedimiento de dimensionamiento tradicional, basado en esfuerzos de trabajo, consistía en determinar los esfuerzos correspondientes a acciones interiores obtenidas de un análisis elástico de la estructura bajo sus supuestas solicitaciones de servicio. Estos esfuerzos se comparaban con esfuerzos permisibles, especificados como una fracción de las resistencias del concreto y del acero. Se suponía que se lograba así, simultáneamente, un comportamiento satisfactorio en condiciones de servicio y un margen razonable de seguridad.

El factor de seguridad de los elementos de una estructura dimensionados por el método de esfuerzos de trabajo no es uniforme, ya que no puede medirse en todos los casos el factor de seguridad por la relación entre las resistencias de los materiales y los esfuerzos permisibles. En otras palabras, la relación entre la resistencia del material y los esfuerzos de trabajo no es siempre igual a la relación entre la resistencia del elemento y su solicitación de servicio.

El procedimiento más comúnmente utilizado en la actualidad es el a veces denominado método plástico, o de resistencia o de resistencia última. Los elementos o secciones se dimensionan para que tengan una resistencia determinada.

El procedimiento consiste en definir las acciones interiores correspondientes a las condiciones de servicio, mediante un análisis elástico y multiplicarlas por un factor de carga, que puede ser constante o variable para los distintos elementos, para así obtener las resistencias de dimensionamiento. El factor de carga puede introducirse también incrementando las acciones exteriores y realizando después un análisis elástico de la estructura. El dimensionamiento se realiza entonces con las hipótesis de comportamiento inelástico.

El procedimiento de dimensionamiento plástico puede también aplicarse a los resultados de un análisis límite, del cual se obtienen directamente las acciones interiores correspondientes a la carga de la falla que convierte la estructura en un mecanismo. El dimensionamiento a partir de un análisis límite no es todavía de aplicación práctica debido a las incertidumbres que se tienen sobre mecanismos de colapso, la inestabilidad general de la estructura y la capacidad de rotación de los elementos de la misma.

El análisis límite no debe confundirse con el criterio general de dimensionamiento, denominado de estados límites, que es el presentado en las recomendaciones del Comité Europeo de Concreto[16] y en los reglamentos soviéticos[7] e ingleses[19]. El enfoque de estados límites no es sino un formato en el que se consideran todos los aspectos del diseño en una forma ordenada y racional y que permite la fácil incorporación de criterios probabilísticos. De hecho se trata de lograr que las características acción-respuesta de un elemento estructural o de una estructura estén dentro de límites que se consideran aceptables. Según este método, una estructura o un elemento estructural deja de ser útil cuando alcanza un estado, llamado estado límite, en el que deja de realizar la función para la cual fue diseñada. Se distinguen dos grupos de estados límites: a) Los estados últimos, o sea, los correspondientes a la capacidad de carga, y b) los estados límites de servicio, que son los correspondientes a las condiciones normales de servicio. Entre los primeros figuran la falla por ruptura de secciones críticas, inestabilidad, volteo, fatiga, etc. Entre los segundos se cuentan la deflexión y el agrietamiento. El diseño consiste en que la probabilidad de alcanzar dichos se mantenga dentro de un margen razonalbe.

El Comité Europeo de Concreto da recomendaciones específicas al respecto.

REFERENCIAS

1. C. W. Churchman, "The Systems Approach", Dell Publishing Co., Nueva York (1969)

2. A. D. Hall, "Ingeniería de Sistemas", C.E.C.S.A., México (1969)

3. H. Chestnut, "System Engineering Methods", Wiley, Nueva York (1967)

4. R. de Neufville y H. Stafford, "Systems Analysis for Engineers and Managers", McGraw-Hill, Nueva York (1971)

5. Fazlur R. Khan, "Optimization of Building Structures", Proceedings 1966 Illinois Structural Engineering Conference, Structural Engineering in Modern Building Design. University of Illinois Bulletin (1969)

6. N. Khachaturian et al., "Introduction to Structural Optimization" Waterloo University, Waterloo, Ontario, Canada.

7. E. Sigalov y S. Strongin, "Reinforced Concrete", Foreign Language Publishing House, Moscú (1962)

8. "Reglamento de construcciones para el Distrito Federal", Diario Oficial, México, D. F., (1966)

9. F. Robles, "Concreto Reforzado", Sección H del "Manual de diseño de obra civiles", C.F.E., México, D. F. (1970)

10. "ACI Standard 318-71, Building Code Requirements for Reinforced Concrete", ACI, Detroit, Mich. (1971)

11. "Commentary on Building Code Requirements for Reinforced Concrete (ACI 318-71)", ACI, Detroit, Mich. (1971)

12. E. V. Krick, "Introducción a la Ingeniería y al proyecto en la ingeniería", Limusa-Wiley, México (1968)

13. L. Spunt, "Optimum Structural Design", Prentice Hall, Englewood Cliffs, N. Y. (1971)

14. E. B. Haugen, "Probabilistic Approaches to Design", Wiley, Londres (1968)

15. Moreno Bonett, Jauffred, Acosta, "Métodos de Optimización, programación lineal, gráficas," Representaciones y servicios de ingeniería, México.

16. "International recommendations for the design and construction of concrete structures", Comité European du Beton-Federation Internationale de la précontrainte", Cement and Concrete Assn., Londres (1970).

17. E. M. Asimov, "Introduction to Design", Prentice-Hall, Englewood Cliffs, New Jersey (1962)

18. J. R. Wright, "Performance Criteria in Building", Scientitic American, (March 1971)

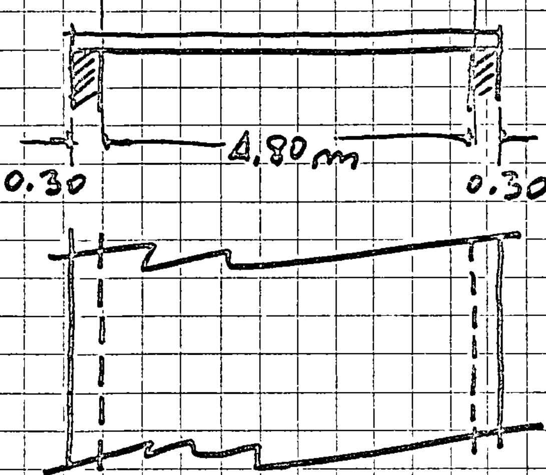
19. A. L. Baker, "Limit Design of Concrete Structures", Cement and Concrete Assn., Londres (1971)

20. L. E. Robertson, "On Tall Buildings", (Proceedings of a Symposium) on Tall Buildings held at the University of Southampton, April, 1966, Pergamon Press, Oxford (1967)

DIMENSIONAMIENTO LOSA SIMPLEMENTE APOYADA EN LADOS OPUESTOS

DATOS

Piso no unido a elementos no estructurales, expuesto a la intemperie.



Cargas

$w_{cv} = 500 \text{ kg/m}^2$

$w_{cm} = 400 \text{ kg/m}^2$

Materiales

Concreto: $f'_c = 200 \text{ kg/cm}^2$

Acero: $f_y = 4000 \text{ kg/cm}^2$

ESPECIFICACIONES Y CONSTANTES

Módulo elasticidad concreto

$E_c = 10\,000\sqrt{f'_c}$ DDF

$E_c = 15\,100\sqrt{f'_c}$ ACI 8.3.1

Usar $E_c = 10\,000\sqrt{f'_c}$

$= 10\,000\sqrt{200} = \underline{\underline{140\,000 \text{ kg/cm}^2}}$

Módulo elasticidad acero

$E_s = \underline{\underline{2.03 \times 10^6 \text{ kg/cm}^2}}$ 8.3.2

Relación modular

$$n = \frac{E_s}{E_c} = \frac{2.03 \times 10^6}{1.4 \times 10^5} = \underline{\underline{15}}$$

ESPECIFICACIONES Y CONSTANTES (cont)

Factores de reducción de capacidad

Flexión : $\phi = \underline{\underline{0.90}}$ 9.2.1.1

Cortante : $\phi = \underline{\underline{0.85}}$ 9.2.1.3

Refuerzo mínimo 10.5.2, 7.13

0.0018 bh (acero principal y transversal)

$s \leq 3h$ (acero principal) 7.4.3

$s \leq 5h$ (acero transversal) 7.13

$s \leq 45\,cm$ (acero principal y transversal)

Refuerzo máximo

$$\rho_{max} = 0.75\,\rho_b$$ 10.3

$$\rho_b = \frac{0.85\,\beta_1 f'_c}{f_y}\;\frac{6117}{6117 + f_y}$$ C 10.3.2, C 10.3.3, C Tabla 10-1

$$\beta_1 = 0.85$$ 10.2.7

$$\rho_b = \frac{0.85(0.85)(200)}{4000}\;\frac{6117}{6117+4000} = \underline{\underline{0.0218}}$$

$$\rho_{max} = 0.75\,(0.0218) = \underline{\underline{0.0163}}$$

Recubrimiento : $r = \underline{\underline{2\,cm}}$ 7.16.1.1

ESPECIFICACIONES Y CONSTANTES (cont)

Esfuerzo cortante que toma el concreto 11.4.1

$v_c = 0.5\sqrt{f'_c} = 0.5\sqrt{200} = \underline{\underline{7.1}}\ kg/cm^2$

ANALISIS (Cálculo de momentos y fuerzas cortantes)

$U = 1.4D + 1.7L$ 9.3.1

$w_u = 1.4\,w_{cm} + 1.7\,w_{cv}$

$= 1.4(0.4) + 1.7(0.5) = \underline{\underline{1.41}}\ T/m$

Claro: $l = 4.80 + h$ 9.5.2.1

Suponiendo $h = 20\ cm$

$l = 4.80 + 0.20 = \underline{\underline{5\ m}}$

$M_u = \frac{1}{8} w_u l^2 = \frac{1.41(5)^2}{8} = \underline{\underline{4.4\ T\text{-}m}}$

$V_u = \frac{w_u l}{2} = \frac{1.41(5)}{2} = \underline{\underline{3.5\ T}}$

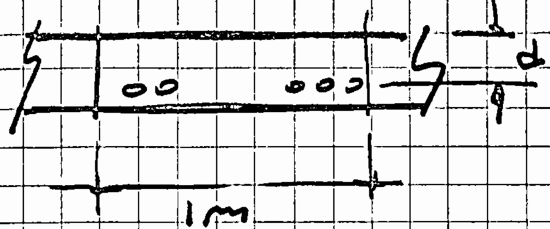
DIMENSIONAMIENTO

Cálculo peralte IMCYC 13.5.1

Suponer $\rho = 0.008$

$(\rho_{min} < 0.008 < \rho_{max})$

Cálculo peralte (cont)



$$M_u = \phi b d^2 f'_c \omega (1 - 0.59\omega)$$

$$\omega = \rho \frac{f_y}{f'_c} = 0.008 \frac{4000}{200} = \underline{\underline{0.16}}$$

$$d = \sqrt{\frac{M_u}{\phi b f'_c \omega (1 - 0.59\omega)}}$$

$$= \sqrt{\frac{4.4(10)^5}{0.90\,(100)(200)(0.16)(1-0.59 \times 0.16)}}$$

$$d = \underline{\underline{13\ cm}}$$

$$h = 13 + \underset{\text{rec}}{2} + \underset{\frac{1}{2}\text{ diám. varilla}}{0.5} = \underline{\underline{15.5\ cm}}$$

Usar : $h = \underline{\underline{15\ cm}}$

Ajuste acero

$$d = 15 - 2.5 = \underline{\underline{12.5\ cm}}$$

Usando gráfica Apéndice A, Texto IMCYC:

$$\frac{M_u}{\phi b d^2 f'_c} = \frac{4.4(10)^5}{0.90(100)(12.5)^2(200)} = \underline{\underline{0.156}}$$

Ajuste acero (cont)

$\omega = \underline{\underline{0.17}}$ (de la gráfica)

$$A_s = \omega \frac{f'_c}{f_y} bd$$

$$= 0.17 \frac{200}{4000} (100)(12.5)$$

$$= \underline{\underline{10.6 \ cm^2/m}}$$

Separación: $s = \dfrac{100 \, A_{sv}}{A_s}$ (A_{sv} = área 1 varilla)

Si $A_{sv} = 1.27 \ cm^2$ (varilla # 4)

$$s_{\#4} = \frac{100\,(1.27)}{10.6} = 12 \ cm$$

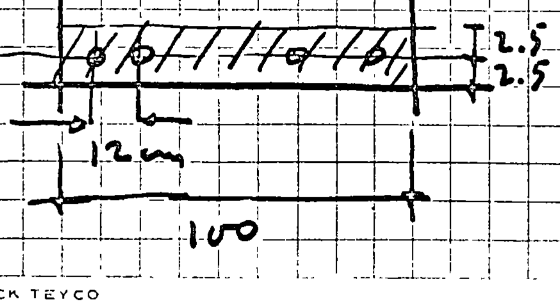
$< 3h = 45$ 7.4.3

$< 45 \ cm$

Control de agrietamiento 10.6.3, 10.6.4

$$z = f_s \sqrt[3]{d_c A}$$

$$f_s = 0.60 \ f_y = 0.60(4000) = \underline{\underline{2400}}$$



$A = \dfrac{5\,(100)}{100/12} = \underline{\underline{60 \ cm^2}}$ C10.6.4

$$z = 2400 \sqrt[3]{2.5\,(60)}$$

$= \underline{\underline{12\,750}} \ kg\text{-}cm < 25\,850$

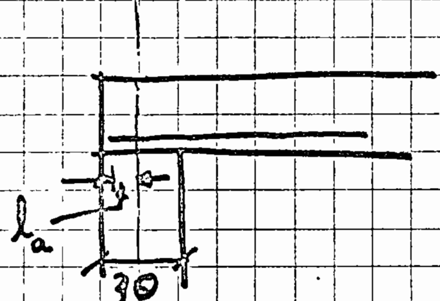
LOSA SIMPLEMENTE APOYADA

Revisión de anclaje

Texto IMCYC 7.6

$$l_d = \frac{0.0594\, A_b f_y}{\sqrt{f'_c}}$$ 12.5(a)

$$= \frac{0.0594\,(1.27)(4000)}{\sqrt{200}} \doteq \underline{\underline{21\ cm}}$$



$$l_d \leq 1.3\,\frac{M_t}{V_u} + l_a$$ 12.2.3

$$l_a = 15 - \underset{rec}{2} = 13\ cm$$

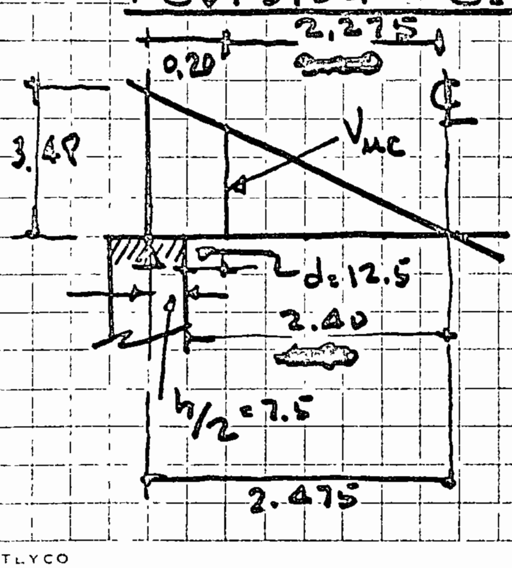
$M_t = M_u = 4.4$ T-m (Si no se cortan varillas.)

$V_u = 3.5$ T

$$1.3\,\frac{M_t}{V_u} + l_a = 1.3\,\frac{4.4}{3.5} + 0.13 = \underline{\underline{1.77}} > l_d$$ ✓

Revisión esfuerzo cortante

11.2



$$l = 4.80 + \overset{h}{0.15} = 4.95\ m$$ 8.5.2.1

$$V_u = \frac{w_u l}{2} = \frac{1.41\,(4.95)}{2} = \underline{\underline{3.48\,T}}$$

$$V_{uc} = 3.48\,\frac{2.275}{2.475} = \underline{\underline{3.2\,T}}$$

$$v_u = \frac{V_u}{\underset{0.85}{\phi} b d} = \frac{3200}{(100)(12.5)} = 2.9 < v_c$$ 11.4.1

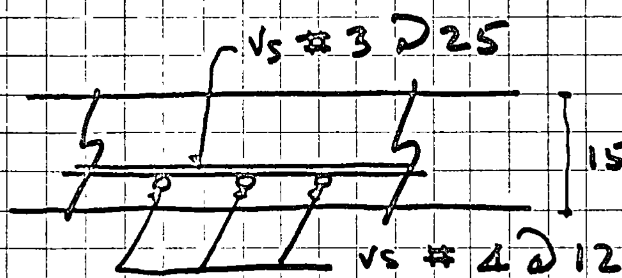
Refuerzo transversal

7.13

$$A_{st} = 0.0018\, bh$$
$$= 0.0018\,(100)(15) = \underline{\underline{2.7\ cm^2/m}}$$

$$S_{\#3} = \frac{100\, A_{sv}}{A_{st}} = \frac{100\,(0.71)}{2.7} = 26.3 \doteq \underline{\underline{25 cm}}$$

$< 5h = 75\ cm$
$< 45\ cm$



Armado

Revisión de deformación

9.5

Peralte máximo admisible sin necesidad de comprobar deformación:

$$h < l/20$$

$$l/20 = \frac{495}{20} = 25\ cm > 15\ cm$$ Tabla 9.5 (a)

$\therefore$ Debe revisarse deflexión.

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] I_{cr} \quad (9\text{-}4)$$ 9.5.2.2

$$M_{cr} = \frac{f_r I_g}{y_t} \quad (9\text{-}5)$$

$$f_r = 2\sqrt{f'_c} = 2\sqrt{200} \doteq \underline{28}\ kg/cm^2$$

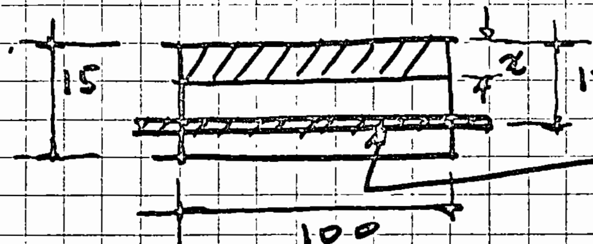
LOSA SIMPLEMENTE APOYADA

$$I_g = \frac{1}{12} b h^3 = \frac{1}{12}(100)(15)^3 = \underline{\underline{28\,000\ cm^4}}$$

$$y_t = h/2 = \underline{\underline{7.5\ cm}}$$

$$M_{cr} = \frac{28\ (28\,000)}{7.5} = \underline{\underline{104\,500}}\ Kg\text{-}cm$$

<u>Cálculo de I_{cr}</u>



$$mA_s = 15\left(\frac{100}{12}\right)1.27$$
$$= 159\ cm^2$$

Profundidad eje neutro (x)

$$\frac{100}{2}(x)^2 = 159\,(12.5 - x) \quad ; \quad x \doteq \underline{\underline{4.9}}\ cm$$ CFE Ad28

Momento inercia sección agrietada

$$\frac{1}{3}(100)(4.9)^3 = 3920$$
$$159\,(12.5 - 4.9)^2 = \underline{9180}$$
$$I_{cr} = \underline{\underline{13\,100\ cm^4}}$$

Momento debido a carga viva:

$$M_a = \frac{1}{8}\,(0.500)(4.95)^2 = \underline{\underline{1.54\ T\text{-}m}}$$

$$\left(\frac{M_{cr}}{M_a}\right) = \frac{104\,500}{154\,000} = 0.68 \ ; \ \left(\frac{M_{cr}}{M_a}\right)^3 = 0.315$$

LOSA SIMPLEMENTE APOYADA

$$I_e = 0.315\,(28\,000) + [1 - 0.315]\,13\,100$$

$$I_e = \underline{\underline{17\,950\ \text{cm}^4}}$$

$$a_{cv} = \frac{5}{384}\,\frac{w\,l^4}{E\,I_e} \qquad w = 5\ \frac{\text{kg}}{\text{cm}}$$

$$= \frac{5\,(5.0)\,(495)^4}{384\,(140\,000)(17\,950)}$$

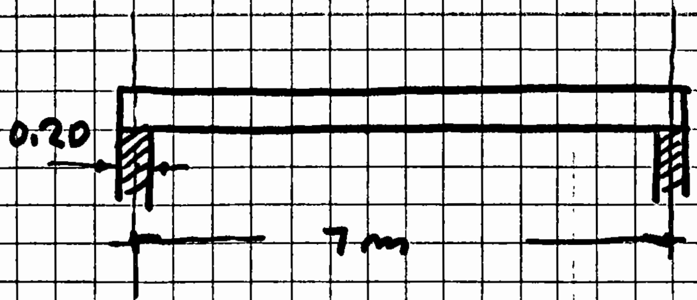
$$= \underline{\underline{1.6}}\ \text{cm}$$

Deformación admisible bajo carga viva en pisos no ligados a elementos no estructurales — Tabla 9.5(b)

$$\frac{l}{360} = \frac{495}{360} = \underline{\underline{1.4}} \leq 1.6 \quad ✓$$

DIMENSIONAMIENTO VIGA RECTANGULAR SIMPLEMENTE APOYADA

DATOS



Sistema de techo ligado a elementos no estructurales; ~~no~~ susceptibles de ser dañados por deflexiones grandes; no expuestos a la intemperie.

Cargas

$$w_{cv} = 2\,T/m$$

$$w_{cm} = 2\,T/m \text{ (incluye peso propio)}$$

Materiales

Concreto : $f'_c = 200\ kg/cm^2$

Acero : $f_y = 4000\ kg/cm^2$ princip.
$f_y = 2300\ kg/cm^2$ estribos

ESPECIFICACIONES Y CONSTANTES

(Ver ejemplo losa libremente apoyada.)

$$E_c = \underline{\underline{140\,000}}\ kg/cm^2$$

$$E_s = \underline{\underline{2.03 \times 10^6}}\ kg/cm^2$$

$$n \doteq 15$$

Factores de reducción de capacidad

Flexión : **0.90**

Cortante : **0.85**

Refuerzo mínimo

$$\rho_{min} = \frac{14}{f_y} = \frac{14}{4000} = \underline{\underline{0.0035}}$$ 10.5.1

Refuerzo máximo

$$\rho_{máx} = 0.75\,\rho_b = \underline{\underline{0.0163}}$$

Recubrimiento

$1\frac{1}{2}" \doteq 3.5$ cm 7.14.1.1

Esfuerzo cortante que toma el concreto

$$v_c = 0.5\sqrt{f'_c} = 7.1 \text{ kg/cm}^2$$ 11.4.1

ó $$v_c = 0.5\sqrt{f'_c} + 176\rho\,\frac{V_u d}{M_u}$$ 11.4.2

$$\frac{V_u d}{M_u} \le 1$$

<u>Esfuerzo cortante a partir del cual debe reducirse a la mitad la separación máxima del refuerzo transversal</u> 11.6.3

$$v_u - v_c = \sqrt{f'_c} = \underline{\underline{14.2}}\ kg/cm^2$$

<u>Esfuerzo cortante máximo admisible con refuerzo</u> 11.6.4

$$v_u - v_c = 2\sqrt{f'_c} = \underline{\underline{28.4}}\ kg/cm^2$$

<u>ANALISIS</u> (Cálculo de momentos y fuerzas cortantes.)

$U = 1.4\,D + 1.7\,L$ 9.3.1

$$w_u = 1.4\,w_{cm} + 1.7\,w_{cv}$$
$$= 1.4\,(2) + 1.7\,(2)$$
$$= 2.8 + 3.4 = \underline{\underline{6.2}}\ Ton/m$$

$l = \underline{\underline{7\,m}}$ (c.a.c. de apoyos) 8.5.2.1

$$M_u = \frac{1}{8}\,w_u\,l^2 = \frac{6.2\,(7)^2}{8} = \underline{\underline{38}}\ Ton\text{-}m$$

$$V_u = \frac{w_u\,l}{2} = \frac{6.2\,(7)}{2} = \underline{\underline{21.7}}\ Ton$$

DIMENSIONAMIENTO

Elección de sección

Suponer $\rho = 0.01$

$(\rho_{min} < 0.01 < \rho_{max})$

$$M_u = \phi\, b d^2 f'_c\, \omega\, (1 - 0.59\,\omega)$$

Usando gráfica Apéndice A, Texto IMCYC:

$$\omega = \rho \frac{f_y}{f'_c} = \frac{0.01\,(4000)}{200} = \underline{\underline{0.2}}$$

$$\frac{M_u}{\phi\, f'_c\, b d^2} = 0.175 \quad \text{(De la gráfica.)}$$

$$d = \sqrt{\frac{M_u}{0.175\,\phi\, f'_c\, b}} = \sqrt{\frac{38\,(10)^5}{0.175\,(0.90)(200)\, b}}$$

$$\underline{\underline{d = \sqrt{\frac{120\,000}{b}}}}$$

$$2V_c = 2v_c\, b d = 2\,(7.1)\, b d = \underline{\underline{14.2\, b d}}$$

VIGA RECTANGULAR SIMPLEMENTE APOYADA

b	d	V_c (kg)
20	78	22 400
25	69	24 500
30	63	26 700
35	59	29 300

Considerar:

$$\boxed{b = 30\ ;\ h = 60}$$

Ajuste acero

supuesto

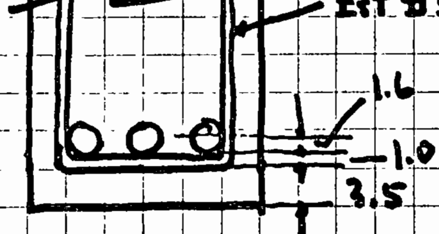
$$d = h - rec = 60 - 6 = 54\ cm$$

$$\frac{M_u}{\phi\, f'_c\, b d^2} = \frac{38(10)^5}{0.90(200)(30)(54)^2} = 0.242$$

$$w = 0.292$$ (gráfica Apendice A, Texto IMCYC)

$$A_s = w \frac{f'_c}{f_y}\, bd$$

$$= \frac{0.292(200)(30)(54)}{4000} = \underline{23.6\ cm^2}$$

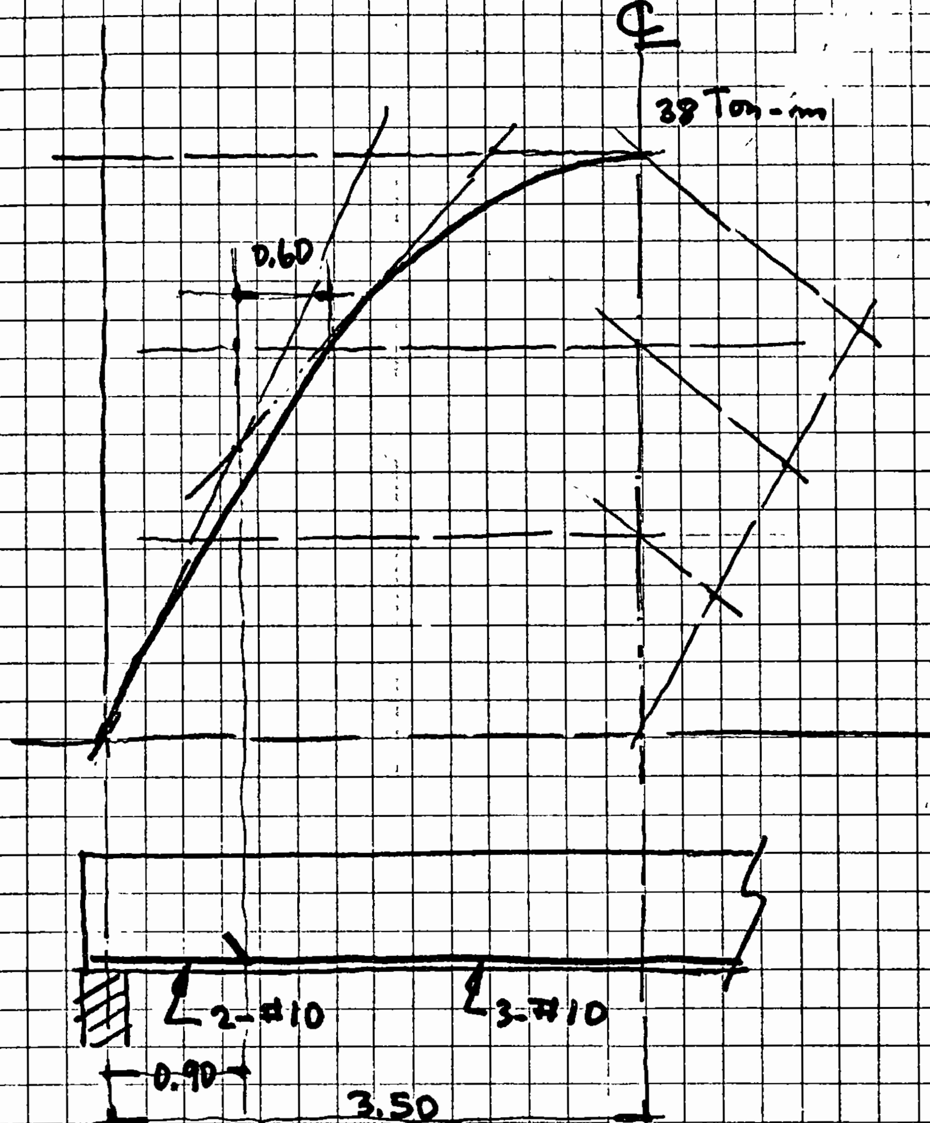


$$\boxed{3\ Vs\ \#10 = 23.8\ cm^2}$$ CFE Ad 3

$$1.6 + 1 + 3.5 = 6.1 \simeq 6\ cm$$

Supuesto

Corte de varillas



CFE Ad 49

Longitud a que deben prolongarse las varillas cortadas a partir de la sección donde teóricamente dejan de ser necesarias. 12.1.4

El mayor de los valores $\left\{ \begin{array}{l} 12\,d_b = 12\,(3.2) = 38\,cm \\ d = 54 \simeq 60\,cm \end{array} \right.$

Revisión de anclaje en el extremo 12.2

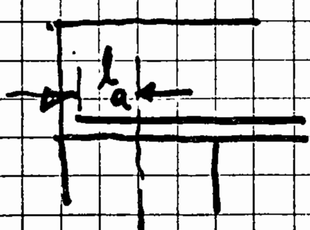
$$l_d = \frac{0.0594 A_b f_y}{\sqrt{f'_c}}$$ ~~12.5 (a)~~ 12.5 (a)

$$= \frac{0.0594(7.94)4000}{\sqrt{200}} = \underline{\underline{134}} \text{ cm}$$

$$M_t = \frac{2}{3} M_u = \frac{2}{3}(38) = 25.3 \text{ Ton-m}$$

$$V_u = 21.7 \text{ Ton}$$

$$1.3 \frac{M_t}{V_u} + l_a$$



$$l_a = 10 - 3.5 = 6.5 \doteq 7 \text{ cm}$$

recubrimiento

$$1.3 \frac{25.3}{21.7} + 0.07 = 1.52 + 0.07 = 1.59$$

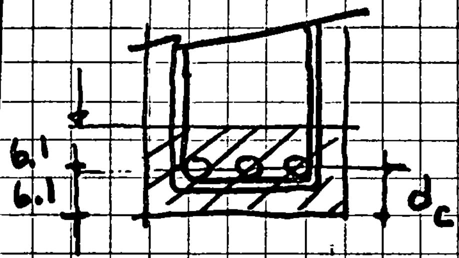
$$1.59 > 1.34 = l_d \quad ✓$$

Control de agrietamiento 10.6.3, 10.6.4

$$z = f_s \sqrt[3]{d_c A}$$

$$f_s = 0.60 f_y = 0.60(4000) = 2400$$

Control de agrietamiento (cont)



$$A = \frac{30\,(12.2)}{3} = 122\ \text{cm}^2$$

$$Z = 2400\ \sqrt[3]{6.1\,(122)}$$

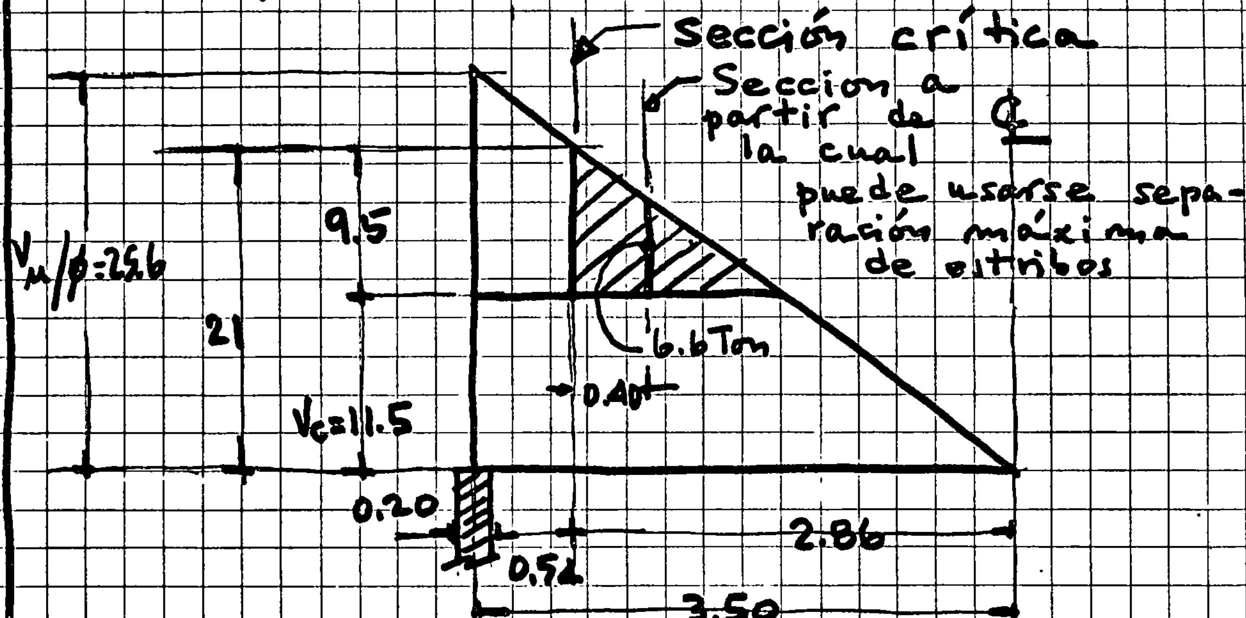
$$= \underline{\underline{20700}} < 31\,250\ \frac{\text{Kg}}{\text{cm}}\ \checkmark$$

Refuerzo transversal

Cap 11

$$V_u/\phi = 21.7/0.85 = \underline{\underline{25.6}}\ \text{Ton}$$

11.2.2



$$V_c = \nu_c b d = 7.1\,(30)\,(54) = 11500\ \text{Kg} = \underline{\underline{11.5}}\ \text{Ton}$$ 11.4.1

$$\frac{V_{uc}}{\phi} = 25.6\,\frac{2.86}{3.50} = \underline{\underline{21}}\ \text{Ton}$$

Separación de estribos: 11.6

$$A_v = \frac{(v_u - v_c)\,bs}{f_y} \qquad (11\text{-}13)$$

$$v_u = \frac{V_u}{\phi\, bd} \qquad 11.2.1$$

$$v_c = \frac{V_c}{bd} \qquad 11.4.1$$

$$A_v = \frac{\left(\frac{V_u}{\phi bd} - \frac{V_c}{bd}\right) bs}{f_y}$$

$$A_v = \left(\frac{V_u}{\phi} - V_c\right)\frac{s}{f_y d}$$

Considerando estribos del #3:

$$A_v = 1.42$$

$$s = \frac{A_v f_y d}{(V_u/\phi - V_c)}$$

$$s_{\#3} = \frac{1.42(2300)(54)}{9500} \doteq \underline{\underline{18}}\ cm < \frac{d}{2} = 27$$

*

Separación transversal mínimo:

$$A_{v\,min} = 3.5\,\frac{bs}{f_y} \qquad (11\text{-}1)$$ 11.1.2

$$s = \frac{A_{v\,min}\,f_y}{3.5\,b}$$

Usando ests. #3 : $A_{v\,min} = 1.42$

$$s = \frac{1.42\,(2300)}{3.5\,(30)} \doteq 31\text{ cm} > \frac{d}{2} = 27$$

∴ Separación máxima del refuerzo transversal:

$$\frac{d}{2} = 27\text{ cm}$$

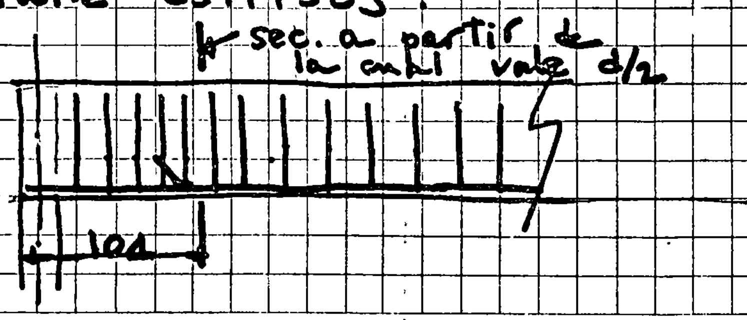
Fuerza cortante $(V_u/\phi - V_c)$ correspondiente a separación máxima:

$$\left(V_u/\phi - V_c\right) = \frac{A_v\,f_y\,d}{s}$$

$$= \frac{1.42\,(2300)\,(54)}{28}$$

$$= 6560\text{ kg} \doteq \underline{\underline{6.6\text{ Ton}}}$$

Detalle estribos:



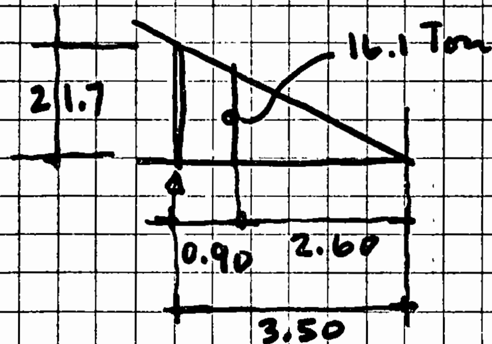
7 ests @ 18

ests. restantes @ 27

(1er estribo a s/2 del paño)

Revisión si se satisfacen condiciones de 12.1.6: 12.1.6

Fuerza cortante en sección de corte de varillas:



$$21.7\,\frac{2.6}{3.5} = \underline{\underline{16.1\,\text{Ton}}}$$

Capacidad de la sección de corte de varillas:

$$V_u/\phi - V_c = \frac{A_v f_y d}{s}$$

$$V_u = \phi\left(\frac{A_v f_y d}{s} + V_c\right)$$

$$V_u = 0.85\left(\frac{1.42(2300)(54)}{18} + 11500\right)$$

$$= 18400\ \text{Kg} = 18.4\ \text{T}$$

$$\frac{16.1}{18.4} = 0.87 > \frac{2}{3}$$

$\therefore$ No se cumple 12.1.6

Revisión considerando

$$v_c = 0.5\sqrt{f'_c} + 176\,\rho\,\frac{V_u d}{M_u}$$ 11.4.2

$$0.5\sqrt{f'_c} = 7.1 \text{ kg/cm}^2$$

$$\rho = \frac{A_s}{bd} = \frac{15.9}{30(54)} = 0.0098$$

$V_u = 16.1$ Ton $\qquad M_u = 16.5$ (gráficamente)

$$\frac{V_u d}{M_u} = \frac{16.1(0.54)}{16.5} = 0.527 < 1$$

$$v_c = 7.1 + 176(0.0098)(0.527)$$
$$= 7.1 + 0.9 = \underline{\underline{8 \text{ kg/cm}^2}}$$

$$V_c = v_c b d = 8(30)(56) = \underline{\underline{13400}}\text{ kg}$$

$$V_u = 0.85\left(\frac{1.42(2300)(58)}{18} + 13400\right)$$
$$= \underline{\underline{19.7}} \text{ Ton}$$

$$\frac{16.1}{19.7} = 0.82 > 2/3$$

Tampoco se cumple 12.1.6

Alternativas:

Anclar barra cortada en el lecho superior

Hacer que se cumpla 12.1.6.2 o 12.1.6.3

Revisión de deflexion 9.5

No es aplicable la Tabla 9.5 (a), porque existe relación con miembros no estructurales susceptibles de ser dañados por deflexiones grandes. Por lo tanto, es necesario revisar la deflexión.

Suponiendo que el 20% de la carga viva actúa en forma constante:

Carga total permanente:

$$w_p = w_{cm} + 0.20(w_{cv})$$

$$= 2 + 0.20\,(2) = \underline{\underline{2.4}} \text{ ton/m}$$

Deflexión immediata debida a w_p:

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] I_{cr} \quad (9\text{-}4)$$

$$M_{cr} = \frac{f_r I_g}{y_t} \quad (9\text{-}5)$$

$$f_r = 2\sqrt{f'_c} = 28.2 \text{ Kg/cm}^2$$

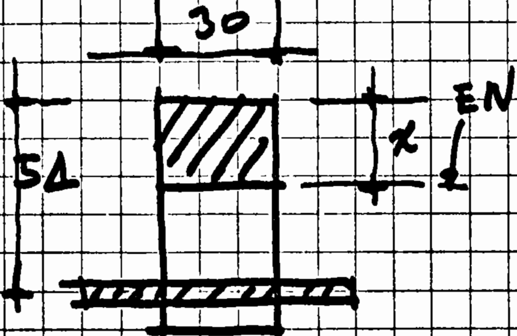
$$I_g = \frac{1}{12} bh^3 = \frac{(30)(60)^3}{12} = 540\,000 \text{ cm}^4$$

$$y_t = 30 \text{ cm}$$

$$M_{cr} = \frac{28.2\,(540\,000)}{30} = \underline{\underline{510\,000}} \text{ Kg-cm}$$

$$M_a = \frac{1}{8} w_p l^2 = \frac{2.4(7)^2}{8} = \underline{\underline{14.7 \times 10^5}} \text{ kg-cm}$$

Cálculo de I_{cr}:



Profundidad EN (x):

$$\frac{30x^2}{2} = 357(54 - x)$$

$$x^2 + 23.8x - 1285 = 0$$

$$x = 26.9 \text{ cm}$$ CFE Ad 28

Momento de inercia (I_{cr}):

$$\frac{1}{3}bx^3 = \frac{30(26.9)^3}{3} = \quad 194\,000$$

$$357(54 - 26.9)^2 = \quad \underline{262\,000}$$

$$I_{cr} = \underline{\underline{456\,000}} \text{ cm}^4$$ CFE Ad 30

$$\left(\frac{M_{cr}}{M_a}\right)^3 = \left(\frac{5.1}{14.7}\right)^3 = \underline{\underline{0.042}}$$

$$I_e = 0.042(540\,000) + (1 - 0.042)(456\,000)$$

$$I_e = \underline{\underline{458\,700}} \text{ cm}^4$$

Deflexión inmediata debida a w_p:

$$a_1 = \frac{5}{384}\frac{w_p l^4}{E I_e}$$

$$= \frac{5(24)(700)^4}{384(140000)(458900)}$$

$$= \underline{\underline{1.2}} \text{ cm}$$

Deflexión diferida adicional debida a w_p:

$$a_2 = 2(a_1) = 2(1.2) = \underline{\underline{2.4}} \text{ cm}$$ 9.5.2.3

Deflexión inmediata debida a carga viva no aplicada en forma continua:

$$w = 0.80\, w_{cv} = 0.80(2) = 1.6 \text{ Ton/m}$$

$$a_3 = 1.2\,\frac{1.6}{2.4} = \underline{\underline{0.8}} \text{ cm}$$

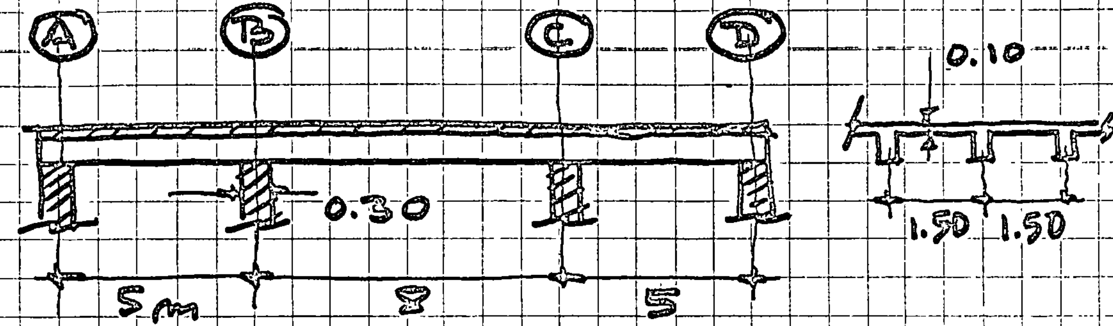
$$a_2 + a_3 = 2.4 + 0.8 = \underline{\underline{3.2}} \text{ cm}$$

$$\frac{l}{480} = \frac{700}{480} = 1.5 \text{ cm} < 3.2$$

$\therefore$ Falta rigidez

DIMENSIONAMIENTO DE UNA VIGA T CONTINUA

DATOS



Sistema de piso; no ligado a elementos no estructurales susceptibles de ser dañados por deflexiones; no expuesto a la intemperie.

Cargas

$w_{cv} = 1.5\ T/m$

$w_{cm} = 0.8\ T/m$

Materiales

Concreto : $f'_c = 200\ kg/cm^2$

Acero :

$f_y = 4000\ kg/cm^2$ (pral.)

$f_y = 2300\ kg/cm^2$ (estribos)

ESPECIFICACIONES Y CONSTANTES

(Ver ejemplo losa libremente apoyada.)

$E_c = 140\,000 \ Kg/cm^2$

$E_s = 2.03 \times 10^6 \ Kg/cm^2$

$n \doteq 15$

Factores de reducción de capacidad

Flexión : 0.90

Fuerza cortante : 0.85

Refuerzo mínimo

$$\rho_{min} = \frac{14}{f_y} = \frac{14}{4000} = \underline{\underline{0.0035}}$$

Refuerzo máximo

$$\rho_{máx} = 0.75\,\rho_b = \underline{\underline{0.0163}}$$

Si se efectuan redistribuciones de momento:

$$\rho_{máx} = 0.50\,\rho_b = \underline{\underline{0.109}}$$

Recubrimiento

$$1\tfrac{1}{2}'' \doteq \underline{\underline{3.5}} \text{ cm}$$ 7.10.1.1

Esfuerzo cortante que toma el concreto

$$v_c = 0.5\sqrt{f'_c} = \underline{\underline{7.1}} \text{ Kg/cm}^2$$ 11.4.1

ó

$$v_c = 0.5\sqrt{f'_c} + 176\rho \frac{V_u d}{M_u}$$ 11.4.2

$$\frac{V_u d}{M_u} \leq 1$$

Esfuerzo cortante a partir del cual debe reducirse a la mitad la separación máxima del refuerzo transversal

$$v_u - v_c = \sqrt{f'_c} = \underline{\underline{14.2}} \text{ Kg/cm}^2$$ 11.6.3

Esfuerzo cortante máximo admisible con refuerzo

$$v_u - v_c = 2\sqrt{f'_c} = \underline{\underline{28.4}} \text{ Kg/cm}^2$$ 11.6.4

——— o ——— o ———

ANALISIS (Cálculo de momentos y fuerzas cortantes.)

Si $w_{cv}/w \geq 0.4$ el Reglamento del D.F. recomienda que se consideren las distribuciones más desfavorables de carga viva. (Art. 183-IV).

$$\frac{w_{cv}}{w} = \frac{1.5}{2.3} = 0.65 > 0.4$$

∴ Debem estudiarse efectos de variaciones de carga viva. 8.5.1

$$w_{cmu} = 1.4(0.8) = \underline{1.1\ T/m}$$

$$w_{cvu} = 1.7(1.5) = \underline{\underline{2.6\ T/m}}$$

(De $U = 1.4D + 1.7L$) 9.3.1

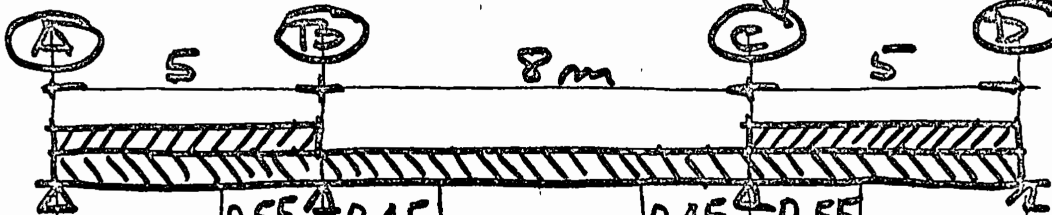
Factores de distribución:

			F.D
$\frac{4}{3}(l_{AB})$	$= \frac{4}{3}(5) = 6.7$		0.45
l_{BC}	$= 8$		0.55
	14.7		1.00

(Rigidez AB: $3\frac{EI}{L}$; Rigidez BC: $4\frac{EI}{L}$)

VIGA T CONTINUA

1ª Condición de carga



	A		B			C			D
F.D.			0.55	0.45		0.45	0.55		
w_{cvu}		2.6			—			2.6	
w_{cmu}		1.1			1.1			1.1	
w_u		3.7			1.1			3.7	
ME	0		+11.6	−5.9					
1ª			−3.1	−2.6					
			0	+1.3					
2ª			−0.7	−0.6					
MF	0		+7.8	−7.8					
RL	9.3		9.3	4.4					
ΔV	−1.9		+1.9	0					
V	7.4		11.2	4.4					
x		2.00			4.00				
+M		+7.4			+8.8 −7.8 +1.0				
i		0, 4.00			2.66				

$$\frac{3.7(5)^2}{8}$$

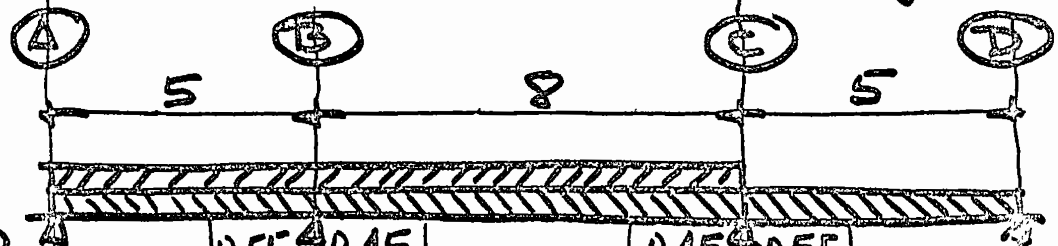
$$\frac{1.1(8)^2}{12}$$

$$x = \frac{V}{w}$$

$$i = \frac{V}{w} \pm \sqrt{\left(\frac{V}{w}\right)^2 - 2\frac{MF}{w}}$$

VIGA T CONTINUA

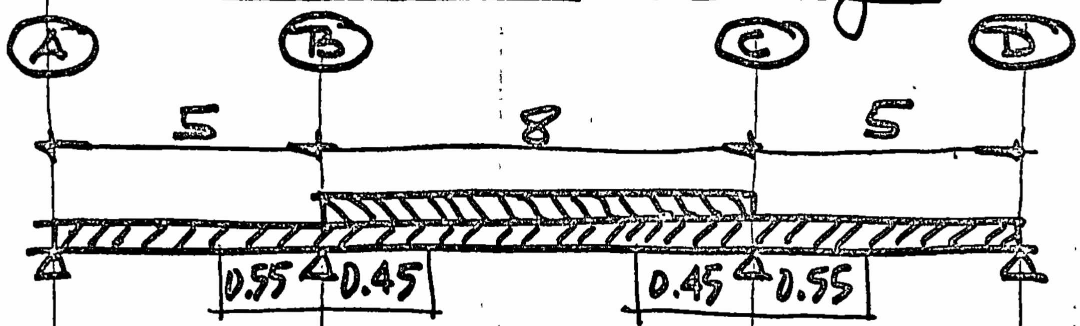
2ª Condición de carga



	A	tramo A–B	B	B	tramo B–C	C	C	tramo C–D	D
F.D.			0.55	0.45		0.45	0.55		
w_{cvu}		2.6			2.6			—	
w_{cmu}		1.1			1.1			1.1	
w_u		3.7			3.7			1.1	
ME	0		+11.6	−19.8		+19.8	−3.4		0
1ª			+4.5	+3.7		−7.4	−9.0		
			0	−3.7		+1.8			
2ª			+2.1	+1.6		−0.8	−1.0		
				−0.4		+0.8			
3ª			+0.2	+0.2		−0.3	−0.5		
MF	0		+18.4	−18.4		+13.9	−13.9		0
RL	9.3		9.3	14.8		14.8	2.7		2.7
ΔV	−3.7		+3.7	+0.6		−0.6	+2.8		−2.8
V	5.6		13.0	15.4		14.2	5.5		−0.1
x		1.52			4.15			2.5	
+M		+4.2			32.0 −18.4 +13.6			13.8 −13.9 −3.4 −3.5	
i		0, 3.04			1.45, 6.85				

VIGA T CONTINUA

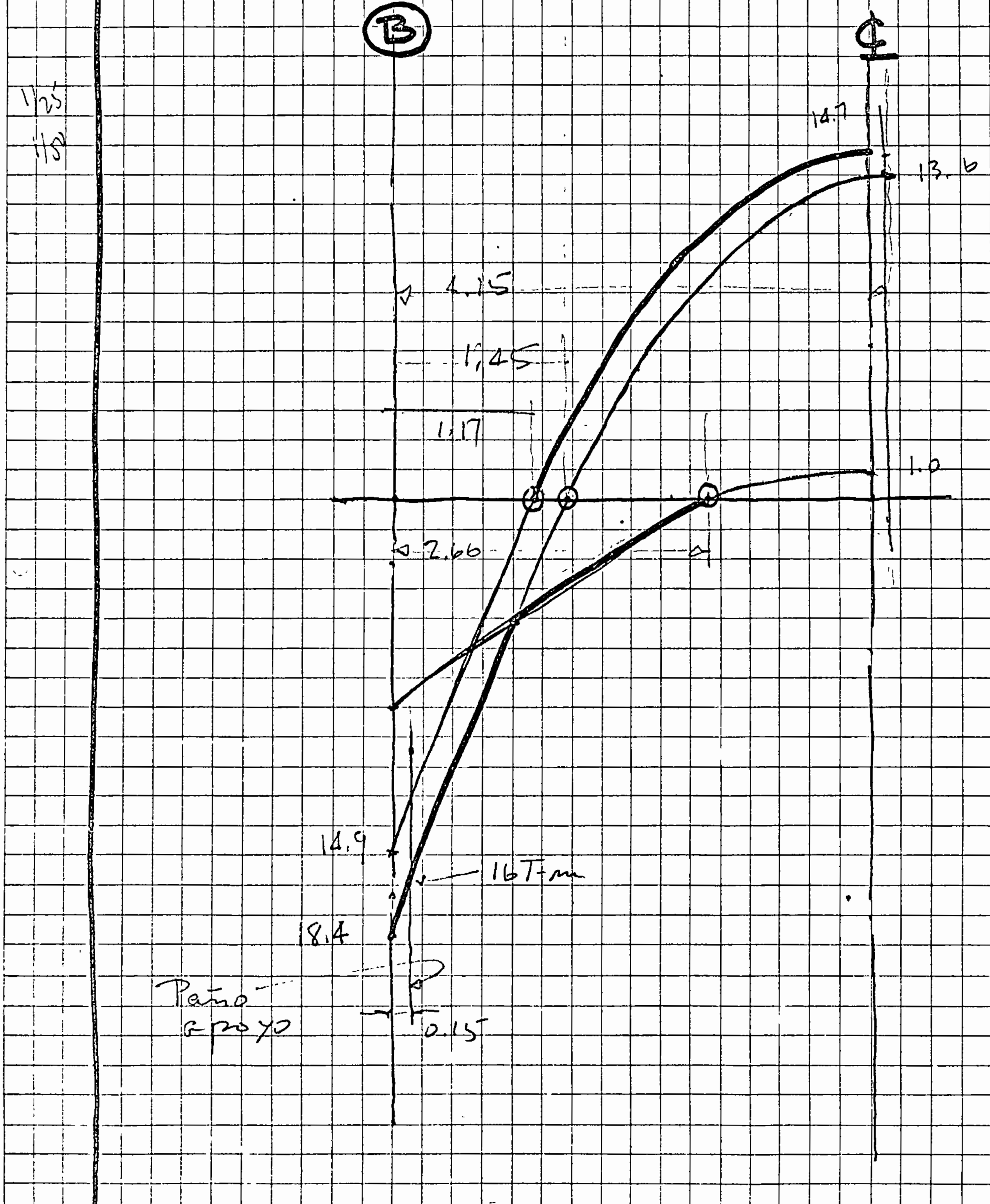
3ª Condición de carga



	A	AB	B	B	BC	C	C	CD	D
w_{CVu}		—			2.6			—	
w_{CMu}		1.1			1.1			1.1	
w_u		1.1			3.7			1.1	
MF	0		+3.4	−19.8		[redacted]	[redacted]		[redacted]
1ª			+9.0	+7.4		[redacted]	[redacted]		
			0	−3.7					
2ª			+2.1	+1.6					
			0	−0.8					
3ª			+0.4	0.4					
MF	0		+14.9	−14.9					
RL	2.7		2.7	14.8		[redacted]			
ΔV	−3.0		+3.0	0					
V	−0.3		+5.7	14.8					
x		2.5			4.0				
+M		−0.8			29.6				
		−3.4			−14.9				
		−4.2			+14.7				
i					1.17				

DIMENSIONAMIENTO

Envolvente de momentos



Elección de sección

Rige el momento en el apoyo B, donde la sección es rectangular.

Considerar momento en el paño del apoyo: 8.5.2.2

$$M_u = \underline{\underline{16}}\ \text{Ton-m} \quad \text{(obtenido gráficamente)}$$

Suponer $\rho = 0.008$ $\quad (\rho_{min} < 0.008 < 0.5\rho_b)$

$$M_u = \phi b d^2 f'_c \omega (1 - 0.59\omega)$$

Usando gráfica Apéndice A, Texto IMCYC:

$$\omega = \rho \frac{f_y}{f'_c} = \frac{0.008(4000)}{200} = \underline{\underline{0.16}}$$

$$\frac{M_u}{\phi f'_c b d^2} = 0.145 \quad \text{(De la gráfica)}$$

$$d = \sqrt{\frac{M_u}{0.145 \phi f'_c b}} = \sqrt{\frac{16(10)^5}{0.145(0.90)(200) b}}$$

$$\underline{\underline{d = \sqrt{\frac{61\,200}{b}}}}$$

$$2V_c = 2v_c bd = 2(7.1)bd = \underline{\underline{14.2\, bd}}$$

DIMENSIONAMIENTO

Envolvente de momentos

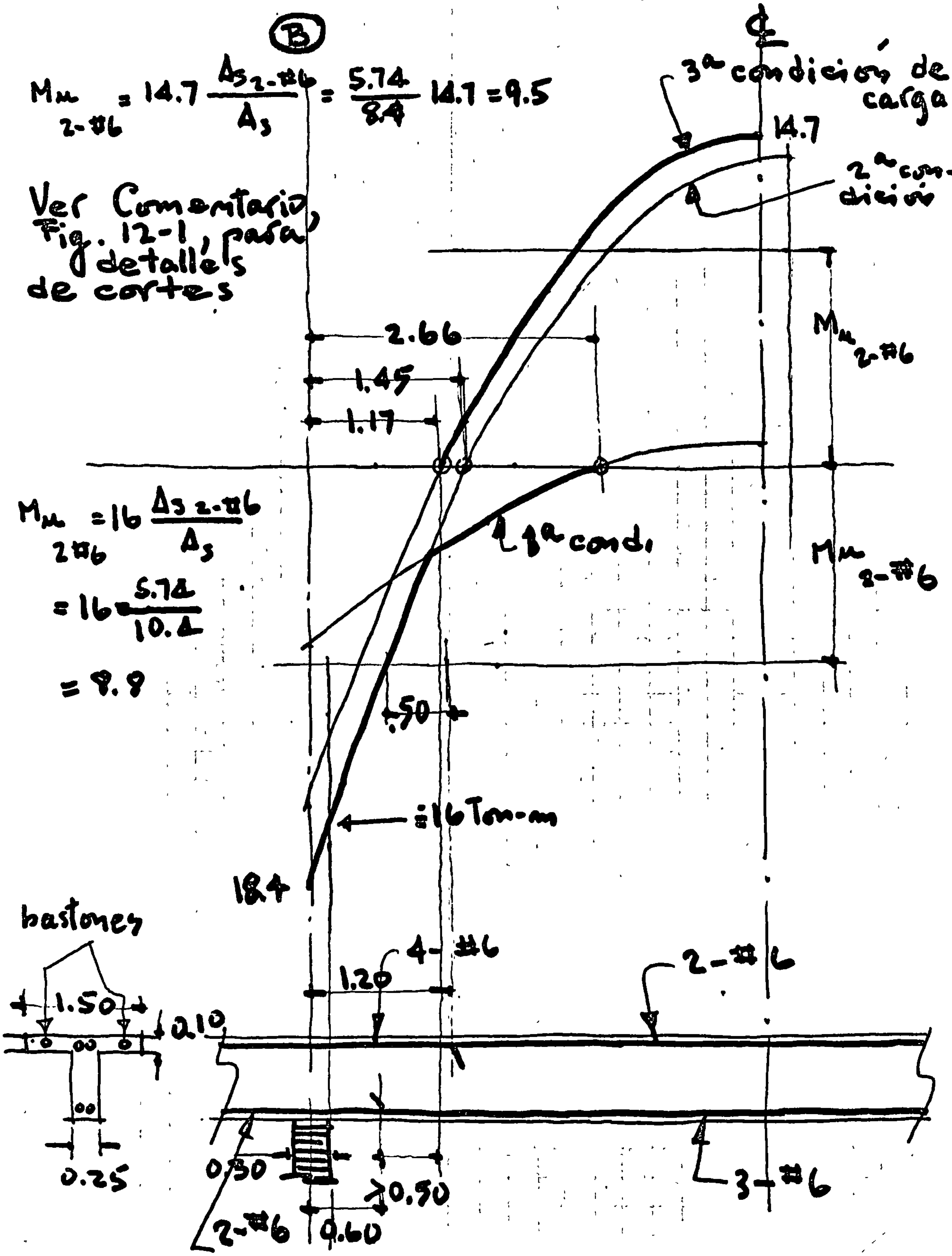
Esc. 1:50

Esc. 1:25

$M_{u\,2\text{-}\#6} = 14.7\,\frac{A_{s\,2\text{-}\#6}}{A_s} = \frac{5.74}{8.4}\,14.1 = 9.5$

Ver Comentario, Fig. 12-1, para detalles de cortes

$M_{u\,2\#6} = 16\,\frac{A_{s\,2\text{-}\#6}}{A_s} = 16\,\frac{5.74}{10.4} = 8.8$



Refuerzo transversal

Como en el ejemplo de la viga rectangular simplemente apoyada.

Revisión de deflexión — 9.5

Deformación admisible — Tabla 9.5(b)

Deformación immediata debida a carga viva = $l/360$

$$= \frac{800}{360} = \underline{\underline{2.2\ \text{cm}}}$$

(Peralte mínimo según Tabla 9.5(a), para no calcular deflexión :

$$l/21 = 800/21 = 38\ \text{cm} < 55$$

∴ podría omitirse el cálculo de deflexión.

Sin embargo, en este ejemplo se efectuará a título ilustrativo.)

VIGA T CONTINUA

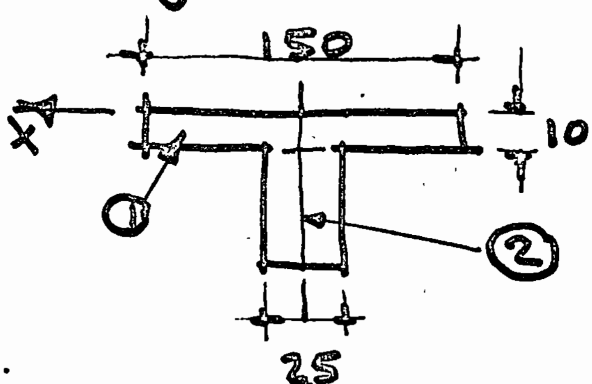
Cálculo de momentos de inercia efectivos

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] I_{cr} \quad (9\text{-}4)$$

$$M_{cr} = \frac{f_r I_g}{y_t} \quad (9\text{-}5)$$

$$f_r = 2\sqrt{f'_c} = 28.2\ \text{Kg/cm}^2$$

Valor de I_g



Parte	Area	y	Ay	$y-\bar{y}$	$(y-\bar{y})^2$
1	1500	5	7500	-11.8	139
2	1125	32.5	36600	+15.7	246
	2625		44100		

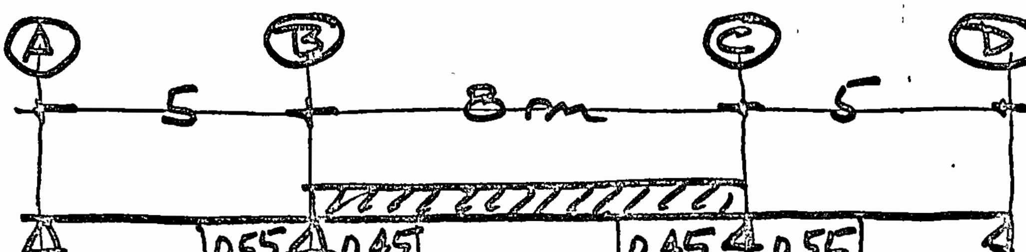
CFE Ad 26

Parte	$A(y-\bar{y})^2$	I_o
1	209 000	12 500
2	277 000	201 500
	486 000	212 000
		486 000

$$\bar{y} = \frac{44100}{2625} = 16.8\ \text{cm}$$

$$I_g = 700\,000\ \text{cm}^4$$

Momentos debidos a carga viva



	A	B		C		D
F.D.		0.55	0.45	0.45	0.55	
w_{cv}		—	1.5 Ton/m		—	
ME	0	0	-8.0	+8.0	0	0
1ª		+4.4	+3.6	-3.6	-4.4	
			-1.8	+1.8		
2ª		+1.0	+0.8	-0.8	-1.0	
			-0.4	+0.4		
3ª		+0.2	+0.2	-0.2	-0.2	
MF	0	+5.6	-5.6	+5.6	-5.6	0
RL = V			6.0	6.0		
x			4.00			

$$+M: \quad 12.0 - 5.6 = 6.4$$

VIGA T CONTINUA

Valor de I_e en los apoyos B y C

$M_a = 5.6$ Ton-m ; $M_{cr} = \dfrac{28.2\,(700\,000)}{16.8}$

$M_{cr} = 11.75$ Ton-m

$M_{cr} > M_a$; $\therefore\ I_e = I_g = \underline{\underline{700\,000\ cm^4}}$

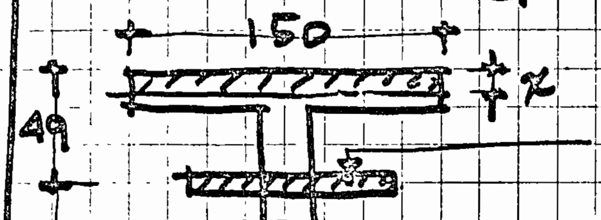
Valor de I_e en el centro del tramo BC

$Y_t = 55 - 16.8 = \underline{\underline{38.2}}$ cm

$M_{cr} = \dfrac{28.2\,(700\,000)}{38.2} = 516\,000$ kg-cm

$M_a = 620\,000$ kg-cm

Valor de I_{cr} :



$15\,(8.6) = 129\ cm^2$

$\dfrac{150\,(x)^2}{2} = 129\,(49 - x)$

$x = \underline{\underline{8.2}}$ cm

VIGA T CONTINUA

$$I_{cr} = \frac{1}{3}(150)(8.2)^3 + 129\,(20.8)^2$$

$$= \underline{242\ 500}\ \text{cm}^4$$

$$\left(\frac{M_{cr}}{M_a}\right)^3 = \left(\frac{5.16}{6.4}\right)^3 = 0.527$$

$$I_e = (0.527)(700\ 000) + (1 - 0.527)\,242\ 500$$

$$= \underline{483\ 000}\ \text{cm}^4$$

$$I_{e_{prom}} = \frac{700\ 000 + 483\ 000}{2}$$

$$= \underline{591\ 500}\ \text{cm}^4$$

$$a = \frac{5\ell^2}{48EI_e}\left[M_m + \frac{1}{10}(M_B + M_c)\right]$$

(Wang y Salmon "Reinforced Concrete Design")

$$a = \frac{5(800)^2}{48(140\ 000)(591\ 500)}\left[6.4(10)^5 - \frac{2(5.6)(10)^5}{10}\right]$$

$$a = \underline{0.4\ \text{cm}} < 2.2\ \text{cm}$$

9

Vigas Continuas y Losas Reforzadas en una Dirección Empleando el DRU

9.1 TIPOS DE CONSTRUCCION

La mayor parte de los miembros de concreto reforzado son estáticamente indeterminados, porque son partes de estructuras monolíticas. Sin embargo, parece que se tiene la impresión equivocada de ignorar muchas formas de construcciones precoladas que están formando una parte importante del mercado actual, que varían desde canales para pisos estáticamente determinados, secciones de doble T, y de otras formas, a sistemas en los que se combinan las unidades precoladas con trabes coladas en el lugar o con losas, para formar construcciones prácticamente monolíticas o compuestas. Con frecuencia, los elementos precolados son también preesforzados.

La forma más usual en la construcción de edificios consiste en losas que se cuelan monolíticamente en los pisos de los entramados que trasmiten las cargas de los pisos a las columnas. La vista en planta de un piso de éstos, en la Fig. 9.1*a*, permite darse cuenta que estas losas están apoyadas en los cuatro lados y que podrían proyectarse convenientemente empleando refuerzo en dos direcciones por los métodos del Cap. 11.

Cuando una losa tiene una longitud mayor del doble de la anchura, generalmente se proyecta como losa continua reforzada en una dirección, pero añadiendo acero especial para los momentos negativos en dirección transversal a las trabes, como se dijo en el Art. 906*e*.* Cuando se proyecta una de estas losas para soportar una

* Debido a que encuentran más sencillo el refuerzo en una dirección, muchos ingenieros proyectan así todos los tableros, excepto los que son casi cuadrados.

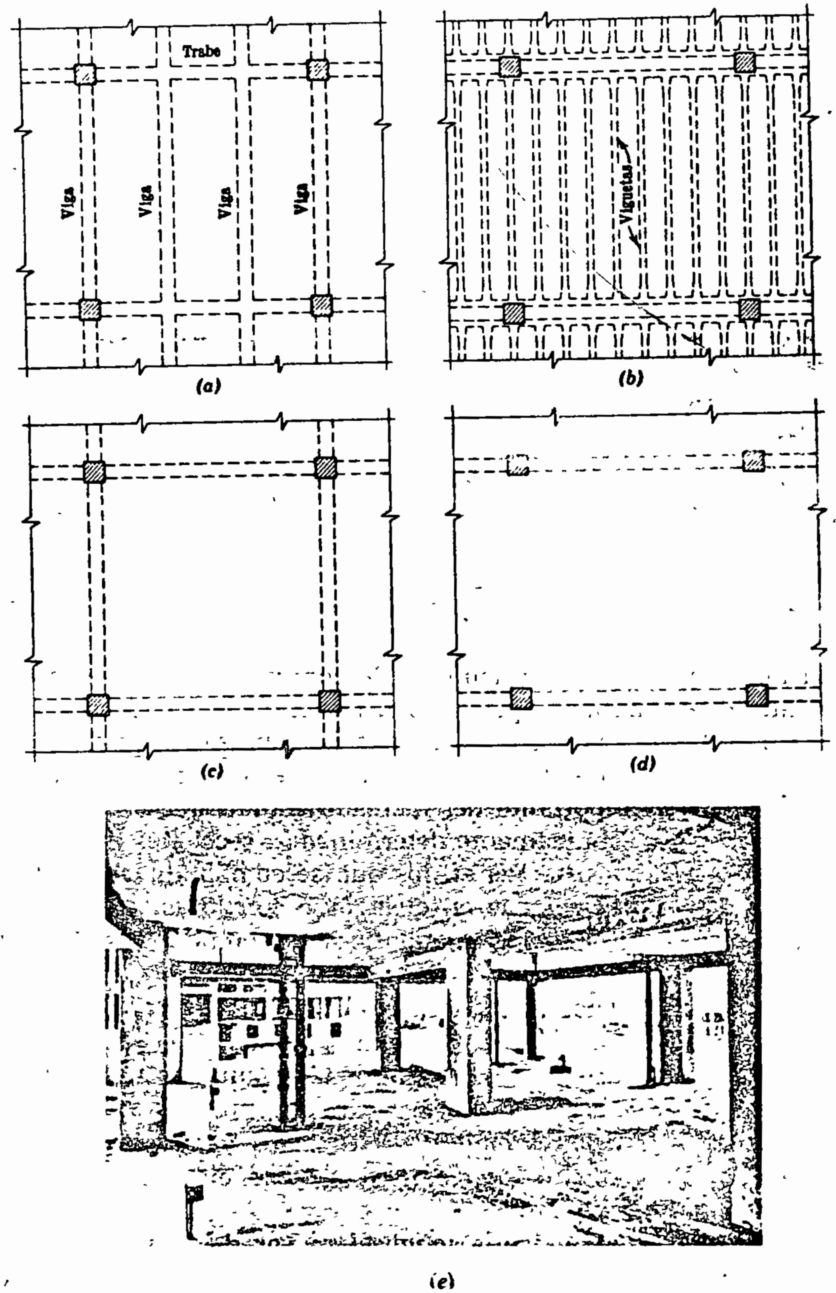


FIG. 9.1. Entramado típico de vigas de los pisos. (*a*) Sistema de vigas y trabes. (*b*) Construcción de viguetas. (*c*) Vigas con losas reforzadas en dos direcciones. (*d*) Vigas con losas reforzadas en una dirección apoyadas en vigas transversal (*e*) Construcción en una dirección. (Cortesía de la Portland Cement Assn.)

carga uniforme, las vigas en que se apoyan también se proyectan para una carga uniforme, ignorando en este proceso la porción de la carga de la losa que se apoya directamente sobre la trabe en la orilla del tablero. Por lo que toca a las vigas, la suposición de la carga uniforme aumenta la seguridad, porque sobreestima la carga sobre la viga. En cuanto a las trabes, la suposición correspondiente es colocar las reacciones de las vigas en la trabe como cargas concentradas y añadir la carga uniforme situada directamente sobre la trabe. Esta suposición no aumenta la seguridad del proyecto de la trabe, especialmente en cuanto al esfuerzo cortante máximo. En la Fig. **11.11*b***, el estudiante debería investigar la diferencia del esfuerzo cortante máximo entre las dos cargas en una trabe simplemente apoyada.

Algunas veces, en claros largos, se usan vigas muy juntas con una losa muy delgada. A este tipo de construcción (Fig. 9.1*b*) se le dan varios nombres, entre ellos: losas aligeradas, losas con forjados, etc., y se facilita con el empleo de cajas o moldes metálicos que se usan entre las viguetas.

Cuando la construcción no lleva otras vigas que las colocadas sobre las columnas, como en la Fig. 9.1*c*, las losas se apoyan únicamente en sus cuatro costados y las vigas y las losas deben proyectarse como se discutió en el Cap. **11**.

En las construcciones ligeras, las vigas a veces corren en una sola dirección, como se muestra en la Fig. 9.1*d* y *e*. En este caso, las vigas trabajan realmente en una sola dirección, excepto en las esquinas donde las vigas soportan las paredes. La construcción con vigas anchas de la Sec. 13.16 es realmente este mismo tipo de construcción, utilizando vigas anchas, de poco peralte.

Dos pisos del tipo de losas con vigas solamente en los muros exteriores y alrededor de las aberturas grandes son económicos; los pisos de placas planas, usados para cargas ligeras y los de losas planas, casi siempre usados para cargas muy pesadas. El sistema de losas levadizas (patentado) es un sistema de piso sin vigas que se cuela en el suelo y que está provisto de collares especiales alrededor de las columnas, para levantarlas y colocarlas en su lugar. Las losas planas y las placas planas se presentan en detalle en el Cap. 13, haciendo mención también de las losas con viguetas en dos direcciones, a las que comúnmente se les llama losas cuadriculadas (Fig. 13.2*a*).

9.2 INTERDEPENDENCIA ENTRE LOS DIFERENTES ELEMENTOS DE LAS ESTRUCTURAS

En todos estos tipos diferentes de construcción el proyectista se enfrenta con un tipo de estructura muy indeterminado, una estructura de tres dimensiones, que no se puede analizar con precisión como una estructura plana.[1-3] Más específicamente, las vigas intermedias de la Fig. 9.1*a* no se pueden analizar exactamente sin considerar la flexión vertical y la rigidez a la torsión de las trabes, así como la rigidez de las columnas. De la misma manera, las vigas que se unen a las columnas tienen momentos en los que influyen las rotaciones de los nudos de las columnas y, por lo tanto, también cualquier torsión que exista en las trabes. Estos aspectos pueden con frecuencia ignorarse como se prescribe en el Art. 905*c*(3), pero el calculista debe darse cuenta que su análisis es solamente aproximado.

Los calculistas comúnmente utilizan métodos aproximados de análisis, pero mejoran como calculistas cuando comprenden la naturaleza de sus aproximaciones y el grado de exactitud o inexactitud que pueden tener. En este capítulo se supone que el lector está familiarizado con los procedimientos ordinarios de distribución de momentos (Apéndice B) y en la construcción de los diagramas de momentos y fuerzas cortantes (Sec. A.5 del Apéndice A).

Las suposiciones ordinarias convencionales son de que el análisis en dos dimensiones es adecuado para la mayor parte de los proyectos. Esta suposición se adapta mejor a las estructuras proyectadas para cargas uniformes en los pisos, que para las que soportan cargas concentradas móviles o cargas de ruedas. Puede suponerse razonablemente que, al soportar una carga uniformemente distribuida, cualquier viga recibe poca ayuda de su vecina, porque la viga vecina probablemente está también completamente cargada.* De la misma manera, en una losa reforzada en una sola dirección puede suponerse que cada faja de la losa soporta la carga que tiene directamente encima de ella cuando toda la losa está cargada.

Cuando las cargas son concentradas y móviles, cada faja de la losa puede soportar un momento diferente y la carga en una sola viga puede depender en mucho de la rigidez de la losa y de las vigas contiguas. El problema de las cargas concentradas en movimiento se discute en el Cap. 14.

Los diagramas de momentos para vigas continuas normalmente indican que existen momentos negativos sobre los apoyos y positivos, cerca de la mitad de los claros, como se indica en la Fig. 9.2*a*. La

* Esta aseveración no es estrictamente cierta, por supuesto En la Fig. 9.1*a*, con una separación igual de las vigas, las vigas que se conectan directamente dentro de la columna son más rígidas, porque las conexiones extremas son más rígidas, mientras que la trabe proporciona solamente un apoyo flexible a la viga vecina. Por lo tanto, las vigas conectadas a las columnas tienden a soportar una carga mayor y, en cierta medida, le quitan carga a las vigas intermedias. Sin embargo, puede notarse que con la carga máxima o carga de ruptura, las vigas del mismo tamaño estén soportando cada una de ellas casi exactamente la misma carga, porque la distribución de la carga cambia después de que comienza a ceder.

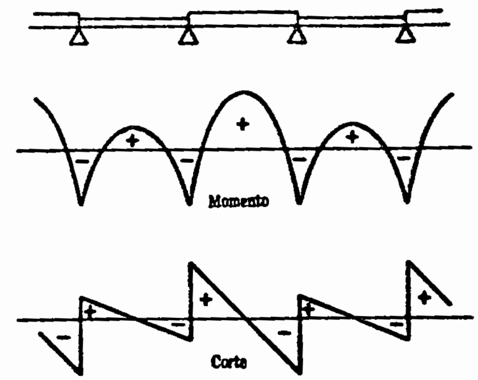


FIG. 9.2. Diagramas típicos del *M* y de la *V* para una viga continua

presencia de columnas cambia el cuadro sólo ligeramente, de manera típica haciendo el momento en las caras opuestas de la columna ligeramente diferente. El diagrama de momentos en cualquier claro puede considerarse como la suma de dos partes; una, el diagrama de momentos correspondiente a una viga simplemente apoyada para ese claro, y la otra, los momentos a lo largo del claro debidos a los momentos negativos de los apoyos, como se bosquejó en la Sec. A.5. Como en estos momentos negativos influyen las cargas de *cualquier* claro, se sigue que el momento en cada punto depende de las cargas en todos los claros. La distribución de cargas para el momento máximo es un asunto muy importante.

El diagrama típico de momentos (Fig. 9.2*b*) difiere sólo ligeramente del que tendría una serie de tramos de vigas simplemente apoyadas. En cualquier tramo, el diagrama de fuerzas cortantes para un sistema de cargas dado, consiste en el correspondiente al de una viga simplemente apoyada, más una corrección constante a la que se le dará el nombre de fuerza cortante de continuidad V_c. Las fuerzas cortantes de continuidad son por lo general, relativamente pequeñas, excepto en los tramos extremos.

9.3 EL PROBLEMA GENERAL DE PROYECTO DE MIEMBROS CONTINUOS

Cada claro de una losa o viga continua requiere un proyecto separado para el momento negativo y para el momento positivo, aunque la costumbre en los Estados Unidos es usar un miembro de peralte constante en un claro determinado y, a menudo, en todos los claros. La Fig. 9.3 muestra las condiciones de proyecto para losas continuas, vigas rectangulares y vigas T, en forma esquemática. Para

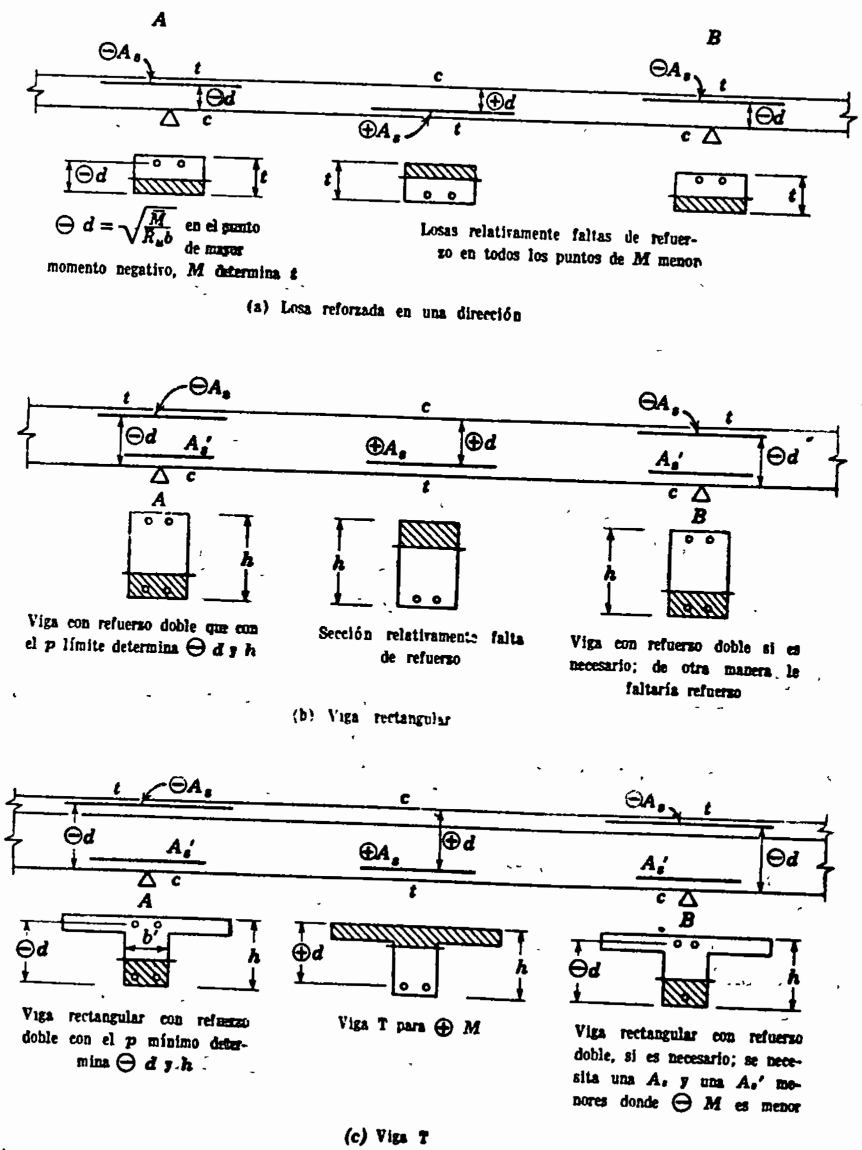


FIG. 9.3. Procedimientos de proyecto para losas continuas y vigas

indicar claramente que en esta etapa no se toma en cuenta la posición del acero, el refuerzo se indica solamente en la vecindad de las zonas de momentos máximos. El momento en A (a la izquierda) se supone que es el momento negativo (máximo) que rige en todos los casos. La letra t designa la cara de tensión.

En las losas más delgadas, el acero de compresión no es tan conveniente, porque quedaría tan cerca del eje neutro que un pequeño

desalojamiento lo haría inefectivo. Por lo tanto, la Fig. 9.3a muestra el momento máximo negativo en A y un porcentaje límite de acero que determina el espesor de la losa, quedando secciones con refuerzo insuficiente en otras partes. Las losas gruesas pueden proyectarse con acero a la compresión, en forma semejante a las vigas rectangulares, si se desea, pero esto no reduce el costo de la losa.

Las vigas rectangulares pueden proyectarse con acero de compresión en los apoyos, como en la Fig. 9.3b o sin él. En el último caso, el proyecto es semejante al expuesto para las losas.

En efecto, las vigas T se convierten en vigas rectangulares invertidas en los apoyos en las que solamente es efectivo en compresión el ancho b' de la nervadura, como se indica en la Fig. 9.3c. Esta zona de compresión restringida se mejora usando una sección con doble refuerzo; y así se recomienda para uso ordinario.

El esfuerzo cortante (tensión diagonal) rara vez rige en el proyecto ordinario de losas reforzadas en una dirección, a pesar del hecho de que la $\bar{v}$ admisible es solamente $2\sqrt{f_c'}$. Los estribos en las losas serían estorbosos, aunque fueran necesarios, y el Reglamento los permite solamente en las losas de 10 plg de espesor o de espesor mayor. Las vigas rectangulares continuas y las vigas T normalmente requieren estribos, pero el tamaño de las nervaduras para las vigas T continuas rara vez estará regido por el esfuerzo cortante, en vez de por el momento, y sólo ocasionalmente, por el tamaño necesario para colocar las varillas.

9.4 DISTRIBUCIONES DE CARGA QUE PRODUCEN MOMENTOS MAXIMOS

(a) Momentos positivos máximos

La teoría elástica es la que se especifica para el cálculo de los momentos de proyecto, excepto por la pequeña, pero importante modificación para el DRU discutida en el Cap. 10.

Pueden usarse líneas de influencia para determinar la distribución de cargas, pero las que son críticas para un análisis elástico son fáciles de deducir de un solo esquema de cargas-flechas, como el de la Fig. 9.4a, en el que las flechas se han exagerado mucho. La idea usual del transporte de momentos conduce directamente al diagrama de momentos de la Fig. 9.4b. Se observa que esta distribución de cargas produce momento positivo cerca de la mitad del claro, en todos los claros de número par. Se deduce, que si estuvieran cargados todos los claros de número par, cada una de estas ca s aumentaría

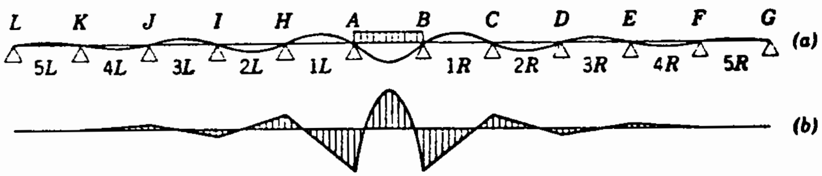


FIG. 9 4 Influencia de la carga en un solo tablero en una viga continua

los momentos positivos en los otros claros cargados. Esta distribución de cargas generalmente se puede describir como sigue:

> **Para obtener el momento máximo positivo cerca de la mitad de un claro, cárguese ese claro y los claros alternados de cada lado, como se muestra en la Fig. 9.5*a*.**

Los factores de carga se aplican a todas las cargas. La carga muerta con su factor de carga se considera que obra siempre* y se incluye en el diagrama de momentos mostrado. Esta distribución de cargas da el momento máximo positivo en todos los claros cargados. El momento positivo máximo en todos los demás claros se obtiene cargando solamente esos claros, como en la Fig. 9.5*b*, quitando las cargas vivas mostradas en la Fig. 9.5*a*. Así, todos los momentos máximos positivos en todos los tramos están determinados por dos distribuciones de carga y por dos distribuciones de momentos. Ampliándolas a los entramados de varios pisos, estas distribuciones se convierten en cargas con formas de tablero de ajedrez (Fig. 9.5*c*), siendo todavía necesario hacer dos distribuciones de carga para obtener el análisis completo.

(b) Momento positivo mínimo (o momento negativo máximo) cerca de la mitad del claro

Como el momento positivo mínimo está simplemente al extremo opuesto de la carga en que el momento positivo es máximo en la viga *AB*, todas las cargas vivas de la Fig. 9.5*a* deben quitarse, y las cargas vivas, con su factor de carga, deben colocarse en los otros claros, como en la Fig. 9.5*b*. Debe advertirse que las dos cargas que ya se discutieron para el momento positivo máximo Fig. 9.5*a* y *b*, dan todos los momentos positivos mínimos necesarios o los momentos negativos máximos cerca de la mitad del claro. El criterio de carga es el siguiente:

> **Omitir la carga viva en el claro que se considera, cargar los claros adyacentes y los alternos que siguen.**

* Excepto cuando contrarresta el efecto del viento o las cargas de los sismos, se reduce el factor de carga.

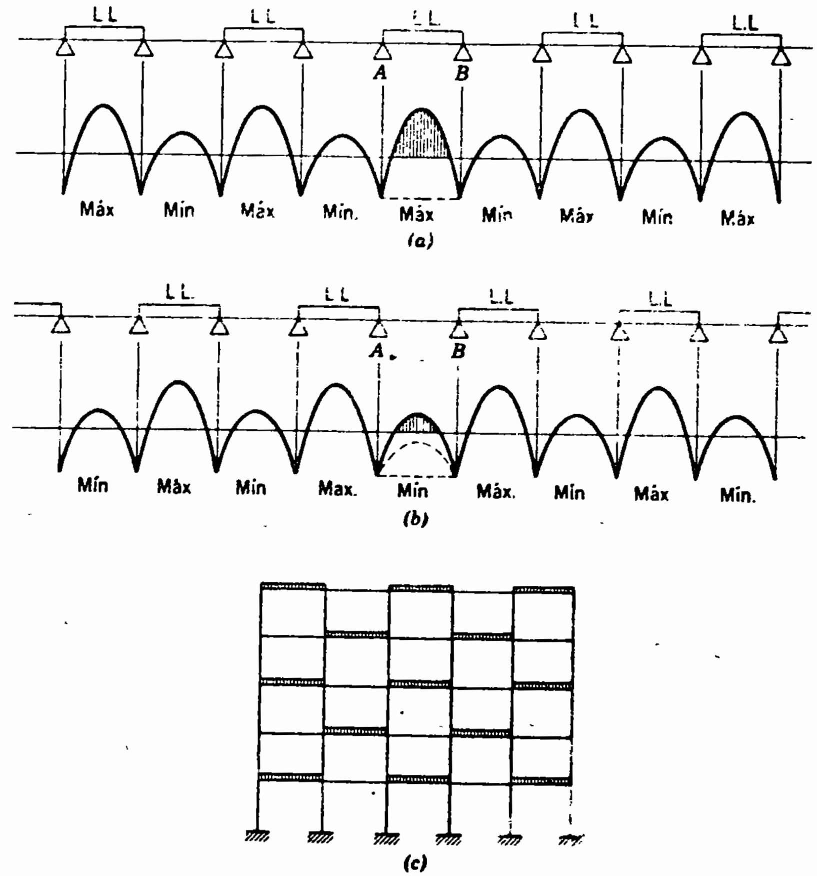


FIG. 9.5. Distribución de carga para (*a*) momento máximo positivo y (*b*) momento mínimo positivo a la mitad del claro. (*c*) Carga en forma de tablero de ajedrez para pisos múltiples

Con un momento por carga muerta menor, el momento a la mitad del claro en *AB* puede ser negativo, como lo indica la curva de rasgos en la Fig. 9.5*b*, especialmente porque el factor de carga. para carga muerta, es menor que para la carga viva.

(c) Momento negativo máximo en el apoyo izquierdo

La carga sobre el claro aislado de la Fig. 9.4 muestra que el *M* negativo se produce en:

B cargando el claro adyacente de la izquierda
D cargando el tercer claro a la izquierda
F cargando el quinto claro de la izquierda

A cargando el claro adyacente de la derecha
I cargando el tercer claro a la derecha
K cargando el quinto claro a la derecha

que se pueden resumir en las siguientes condiciones:

Para obtener el momento máximo negativo en un apoyo dado, cárguense los claros adyacentes a cada lado y los alternos siguientes.

Se deben aplicar a las cargas sus factores correspondientes. Según estas condiciones, aplicándolas al momento máximo en el apoyo izquierdo *A*, da por resultado el diagrama de momentos y cargas mostrado en la Fig. 9.6*a*. Solamente los momentos resultantes adyacentes a *A* tienen importancia porque ninguno de los otros son mínimos ni máximos. Estos momentos son críticos en la cara del apoyo, como se dirá en la Sec. 9.4g.

El ajuste de estos momentos obtenidos por el análisis elástico para adaptarlos al diseño al límite se trata en el Cap. 10.

(d) Momento negativo máximo en el apoyo derecho

Se aplica el mismo criterio a este momento máximo que en el caso anterior. Las cargas se colocan en los claros adyacentes a *B* y en los claros alternos que siguen, como se muestra en la Fig. 9.6*b*. Desafortunadamente, la carga produce máximos solamente cerca del apoyo en *B*.

(e) Claros cargados parcialmente

Debe advertirse que para que se produzcan tanto momentos máximos positivos y negativos es necesario que el claro que se considera, esté completamente cargado.

Los momentos máximos negativos se producen cuando el diagrama de momentos del claro considerado es muy asimétrico. Por otra parte, el momento positivo máximo se obtiene cuando el diagrama de momentos es casi simétrico con relación a la mitad del claro. El momento positivo mínimo a la mitad del claro (o el negativo máximo cuando la carga muerta es pequeña) también produce un diagrama de momentos casi simétrico; en este caso, un diagrama de momentos basado en la carga muerta solamente en el claro en cuestión.

En el proyecto del entramado de un edificio no es costumbre considerar claros cargados parcialmente, porque no aumentan los momentos de proyecto principales. En las estructuras de las carreteras los claros parcialmente cargados tendrían una influencia considera-

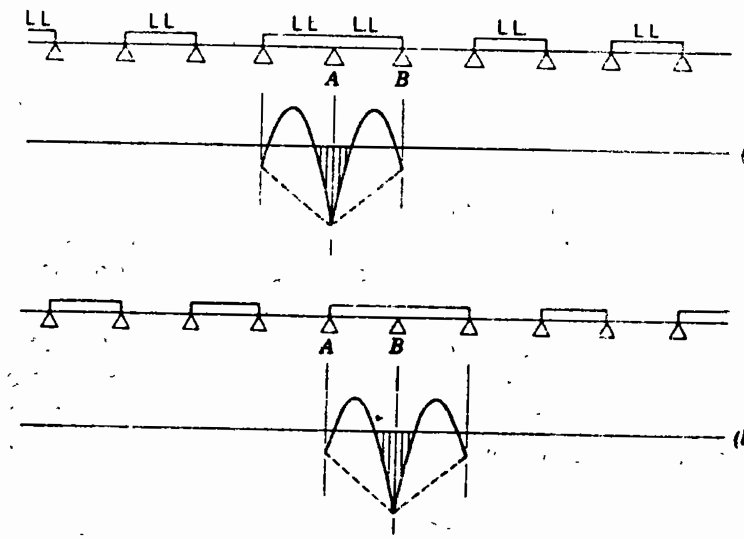


Fig. 9.6. Sistema de cargas para obtener el momento negativo máximo (*a*) en el apoyo de la izquierda *A*; (*b*) en el apoyo de la derecha *B*

ble en el detalle del refuerzo. Los claros cargados parcialmente se mencionan brevemente en la Sec. 9.6 en conexión con los diagramas de momentos máximos.

Algunos reglamentos especifican que se vaya colocando una sola carga móvil en cualquier punto de la estructura como una alternativa de carga para resolver casos especiales.

(f) Simplificaciones permisibles

El reglamento del ACI propugna por el uso de un entramado reducido o simplificado para el análisis de los edificios (Art. 905) y especifica que los momentos se calculen por el análisis elástico para el DRU, así como para el DRT (Art. 1502*c*). Se puede analizar un piso a cada vez considerando fijos los extremos lejanos de las columnas. Al calcular los momentos máximos negativos, la carga viva puede aplicarse solamente sobre los dos claros adyacentes. Lo que permite el uso de procedimientos simplificados para la distribución de momentos, como el procedimiento de dos ciclos dado en la Ref. 1. Estos métodos, en la opinión del autor, son bastante razonables para las estructuras ordinarias, excepto que dan momentos para las vigas y columnas exteriores que tienden a ser demasiado pequeños. No se deberán pasar por alto los factores de carga, tanto para la carga muerta como para la viva, cuando se usa el DRU.

(g) Momento en la cara del apoyo

En la distribución de momentos generalmente se supone que los claros se han considerado de centro a centro de los apoyos. Como en estos cálculos se tratan las reacciones de los apoyos como si estuvieran concentradas en un punto, el diagrama de momentos que resulta dentro del espesor del apoyo es completamente imaginario. Lo que no tiene importancia, porque tanto la teoría como las pruebas demuestran que los momentos críticos están en la *cara* del apoyo [Art. 905*b*(2)].

Muchos ingenieros determinan el momento de proyecto en la cara del apoyo del momento calculado en la línea central del apoyo, deduciendo simplemente $Va/2$, siendo V la fuerza cortante y a la anchura de la columna, como se ilustra en la Fig. 9.7*a*. Lo que equi-

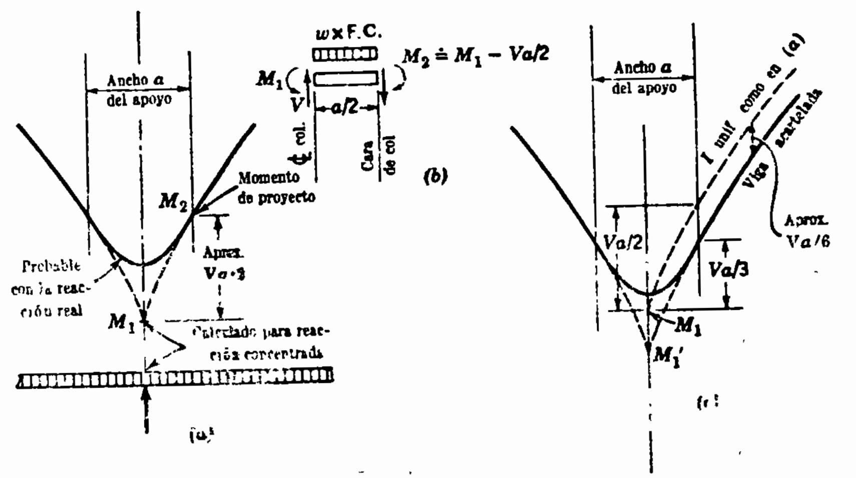


FIG. 9.7. Momento en la cara del apoyo. (*a*) Sin corrección por el aumento de rigidez en el apoyo. (*b*) Diagrama de cuerpo libre estableciendo el procedimiento en (*a*). (*c*) Procedimiento recomendado reconociendo el aumento de la rigidez en el apoyo

vale a tomar a escala el valor del momento del diagrama de momentos, debido a la pequeña longitud de la carga uniforme dentro de la columna (Fig. 9.7*b*) que causaría poco cambio en el momento.

El autor prefiere la corrección más conservadora recomendada por el antiguo Joint Committee Specification en sus especificaciones, es decir, haciendo una reducción de $Va/3$. El razonamiento en que se apoya este procedimiento es el que sigue: el apoyo da rigidez al extremo de la viga como si fuera una cartela. Haciendo un cálculo en el que se considerara el aumento de la rigidez en el extremo, daría por resul-

tado un aumento en el momento negativo en el centro de la columna, como se indica con M_1' en la Fig. 9.7*c*. En vez de calcular M_1' (que podría ser aproximadamente $Va/6$ mayor que M_1), se obtiene un equivalente aproximado aplicando una corrección menor, $Va/3$, al valor original de M_1. El momento de proyecto es entonces $M_1 - Va/3$, como en la Fig. 9.7*c*.

La rigidez adicional en el apoyo también reduce los momentos positivos. Sin embargo, la corrección sería sólo aproximadamente de $Va/6$. Muchos ingenieros consideran esta corrección menos cierta que la de la cara de la columna. Reconocen la menor precisión del cálculo de los momentos positivos, que son siempre sensibles a los valores de la rigidez relativa de la columna y simplemente usan los momentos positivos originales sin ninguna corrección.

(h) Comparación de los momentos de proyecto del DRT con los del DRU

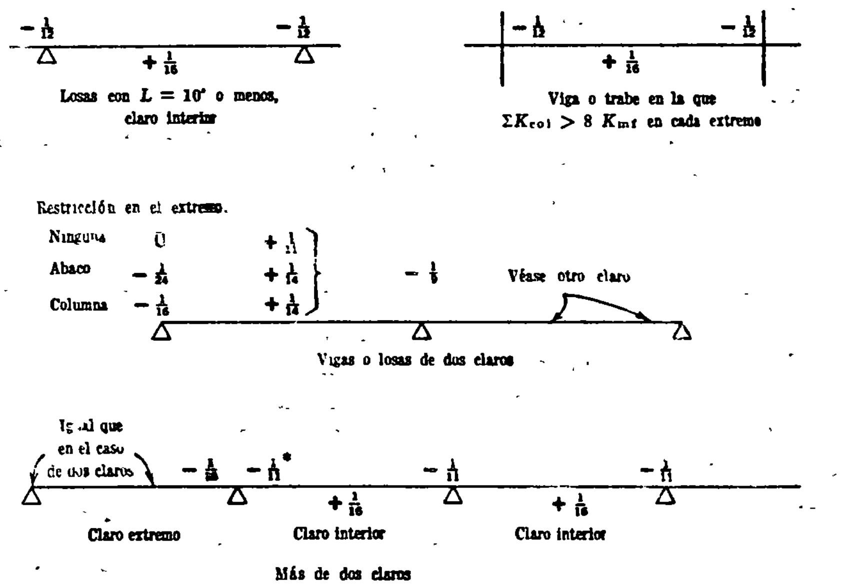
Siempre se ha usado la teoría elástica para el proyecto de las vigas según el DRT, sin usar, por supuesto, factores de carga. En la actualidad no es necesario hacer ningún cambio en el procedimiento del DRT. Cuando se usa el DRU y los factores de carga, la teoría elástica aumenta ligeramente la proporción del momento por carga viva y, por lo tanto, a momentos de proyecto verdaderos (y coeficientes de momentos) que son proporcionalmente más elevados. El principal efecto que esto produce es la necesidad de alargar más las varillas superiores dentro de los claros adyacentes.

Filosóficamente, el uso de la teoría elástica para los momentos y el uso de métodos inelásticos del DRU para el análisis de la sección no son compatibles. Lo que modifica la manera de razonar hacia cierto tipo de análisis de marco inelástico como se discute en el Cap. 10. El Art. 1502*d* del Reglamento permite algunos ajustes en este sentido. Aunque la teoría elástica puede parecer incompatible con el DRU, el error que produce consiste en dar momentos que son demasiado grandes y, por lo tanto es seguro. Este capítulo se basa totalmente en los momentos obtenidos por la teoría elástica.

9.5 COEFICIENTES PARA MOMENTOS

Cualquier momento dado se puede expresar por un momento multiplicado por un coeficiente wL'^2, siendo w la carga total por pie (incluyendo para el DRU la carga muerta multiplicada por su factor de carga y la carga viva por su factor de carga) y L', el claro libre. Los coeficientes para los momentos máximos serán mayores cuando la relación de la carga viva a la carga muerta es grande y cuando la columna o alguna otra restricción en el nudo es relativamente pequeña. Los coeficientes de los momentos negativos pueden también ser grandes cuando los claros adyacentes son más largos o están cargados con cargas más pesadas que el claro en cuestión.

Tomando como base cargas vivas uniformes que no son mayores que tres veces la carga muerta y en claros de longitud "aproximadamente igual (el mayor de los claros adyacentes no debe exceder al menor en más del 20 por ciento)". El Reglamento de Construcción del ACI ha establecido por análisis determinados, coeficientes razonables para momentos, para usarlos en el cálculo de los coeficientes máximos. En el Art. 904*c* se tabulan estos coeficientes y la Fig. 9.8 los presenta en forma esquemática.



* −1/11 en este apoyo solamente si interviene viga o columna en el nudo; de otra manera −1/10.

FIG. 9.8. Coeficientes para momentos del Reglamento del ACI para claros aproximadamente iguales y cuando la carga viva es menor que tres veces la carga muerta

Cuando la relación de la carga muerta a la carga viva es especialmente grande, se puede lograr alguna economía calculando los momentos por procedimientos más precisos. Con el DRU se puede lograr alguna economía por medio de análisis menos precisos y de ajustes que se discuten en el Cap. 10.

El uso de coeficientes para momentos debe limitarse a condiciones relativamente ordinarias, a menos que se usen tablas generales como las del apéndice del *Reinforced Concrete Design Handbook del ACI.*[4]

9.6 DIAGRAMAS DE MOMENTOS MAXIMOS

Para poder determinar la mejor colocación del refuerzo es necesario, para el proyectista, tener en la mente una imagen clara de la extremada variación de momentos que puede existir a lo largo de la viga o losa, la mayor parte de este diagrama se puede formar con los diversos diagramas de momento para los momentos máximos ya ilustrados en las Figs. 9.5 a 9.7. Para un claro interior típico con claros iguales y cargas vivas iguales, estas curvas de momentos se trazan a escala mayor en la Fig. 9.9*a–d* y se agrupan juntos en la Fig. 9.9*e*.

Para las cargas que quedan exactamente opuestas a las que producen el momento máximo negativo, con frecuencia se puede obtener un momento positivo pequeño como lo sugieren las líneas de rayas. Algunos claros parcialmente cargados pueden aumentar los momentos negativos ligeramente, como se indica con las líneas de rayas, pero éstos no son cambios muy importantes.

Una propiedad muy importante de los diagramas de momentos máximos es que, en una porción considerable de la viga, el momento puede cambiar de positivo a negativo, o a la inversa, conforme varían las cargas en los claros adyacentes. Por lo tanto, el proyectista deberá colocar acero de tensión tanto arriba como abajo en esta zona.

Se calculan las posiciones definidas de algunos puntos de inflexión en la Fig. 9.9 en conexión con el doblado de las varillas en la Fig. 9.11 y en la Sec. 9.14.

9.7 FUERZAS CORTANTES MAXIMAS

La carga que produce la fuerza cortante máxima en el apoyo es la misma que produce el momento negativo allí. Por lo tanto, la fuerza cortante en el extremo puede exceder a la de la viga, simplemente apoyada en una cantidad igual a la fuerza cortante de continuidad V_c. En los claros interiores, el valor de V_c será del orden de 3 al 12% de la fuerza cortante en la viga simplemente apoyada, con un promedio bastante aproximado de 8%. En los claros extremos puede llegar a alcanzar un valor hasta del 20% mayor que el de la viga simplemente apoyada. Esta gran fuerza cortante en un claro

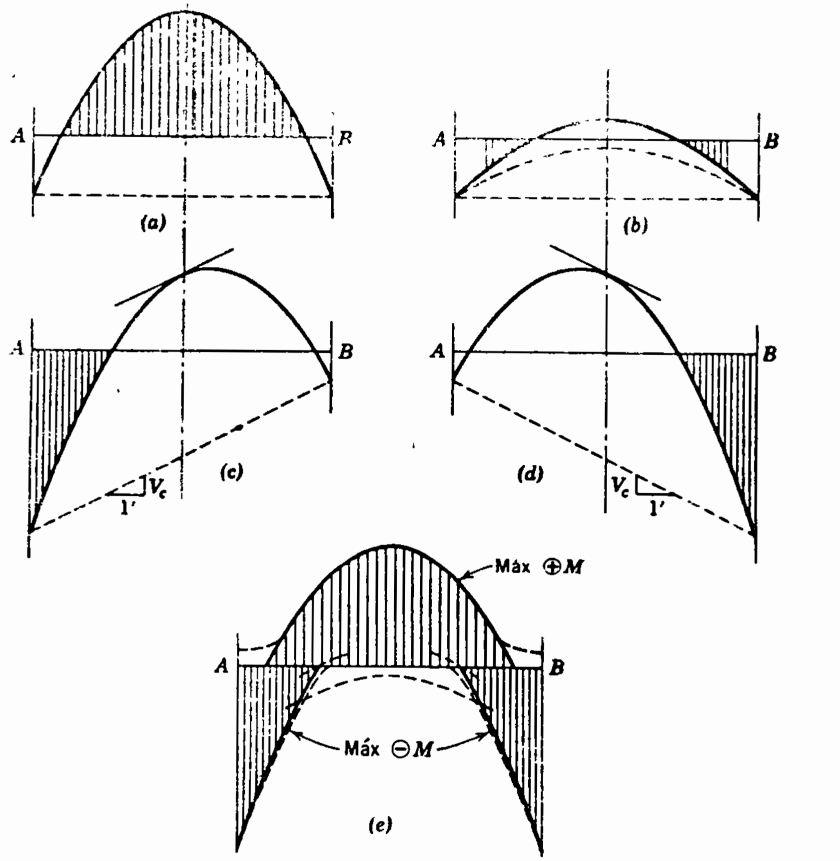


FIG. 9.9. Diagramas de momentos para obtener el momento máximo. (*a*) Momento positivo a la mitad del claro. (*b*) Momento negativo cerca de la mitad del claro. (*c*) Momento negativo en el apoyo izquierdo. (*d*) Momento negativo en el apoyo derecho. (*e*) Diagrama compuesto de los momentos máximos

extremo es aditiva, solamente en la mitad de la viga adyacente al primer soporte interior. Para claros aproximadamente iguales, el Art. 904*c* del Reglamento aconseja 15% de aumento en la fuerza cortante en los miembros extremos, en el primer apoyo interior únicamente. El autor prefiere hacer un aumento nominal en los claros interiores, así como una adición sustancial en el claro extremo.

9.8 PROYECTO DE LOSAS CONTINUAS REFORZADAS EN UNA DIRECCION

Proyéctese una losa continua reforzada en una dirección, apoyada en vigas separadas 12 pies 0 plg, centro a centro, coeficientes para

los momentos del Reglamento del ACI (Art. 904*c*); carga muerta de 25 lb/pie² (más el peso de la losa), carga viva de 200 lb/pie², $f_c' = 3000$ lpc, acero de grado intermedio, con p limitado a $0.18 f_c'/f_y$. Supóngase que la nervadura de la viga tiene una anchura de 12 plg.

SOLUCION

Los coeficientes para momentos que se dan en la Fig. 9 8 indican que el momento negativo en el primer apoyo interior es el máximo, a $0.10wL'^2$. La losa en este punto se proyectará con el porcentaje límite de acero que da una $\bar{R}_u = 0.161 f_c'$ (Fig. 3.8) = 483. El peso de la losa es el que resulta de suponer una t de 6 plg.

$$w_L = 200 \times 1.8\,(\text{F.C.}) = 360 \text{ lb/pie}^2$$
$$w_D = 25 \times 1.5\,(\text{F.C.}) = 38$$
$$\text{Peso de la losa} = 75 \times 1.5 = 112 \quad 94 \text{ (supuesto)}$$
$$w_T = 510 \quad 492 \text{ lb/pie}^2$$
$$L' = 12.0 - 1.0 = 11.0\text{-pies claro libre}$$
$$M_u = 0.10 \times 510 \times 11^2 = 6170 \text{ pies-lb/pie de losa}$$
$$\bar{M} = M_u/\phi = 6170/0.9 = 6880 \text{ pies-lb/pie}$$
$$\bar{R}_u bd^2 = 483 \times 12d^2 = 6880 \times 12$$
$$d = \sqrt{14.2} = 3.77$$

El Art. 808*b* especifica un recubrimiento libre de 0.75 plg.

$$t = d + D/2 + 0.75 = 3.77 + 0.25 + 0.75 = 4.77 \text{ plg para var. No 4.}$$

De donde se obtiene $t = 5$ plg; podría ser solamente de 4.5 plg si se hubiera exagerado mucho el peso. La suposición original de 75 lb/pie² corresponde a una $t = 6$ plg.

Pruébese $t = 5$ plg, peso $= \frac{5}{12}(150) = 63$ lb/pie² × 1.5 F.C. = 94 lb/pie². Lo que indica una disminución de 18 lb/pie², que es 3.5% de la carga total que origina una variación en el momento de 3.5% y aproximadamente de 3 5/2 = 1.7% del cambio necesario en d, ya que d varía como $\sqrt{M}$. (Si se han probado 4.5 plg; el cambio en t no podía haber sido tanto como el 5.6% necesario para cambiar de 4.77 a 4 5 plg).

$$\bar{M} = -0.10 \times 492 \times 11^2/0.9 = 6600 \text{ pies-lb/pie}$$
$$d = \sqrt{6600 \times 12/(483 \times 12)} = \sqrt{13.62} = 3.69$$
$$t = 3.69 + 0.25 + 0.75 = 4.69 \text{ plg, digamos 5 plg como se había estimado.}$$

USESE una $t = 5$ plg, $d = 5 - 0.25 - 0.75 = 4.00$ plg para var. No. 4.

El Reglamento hace una advertencia respecto a las flechas (Tabla 909*b*) si $t < l/35 = 11 \times 12/35 = 3.77$ plg $\ll 5$ plg Corr. Con este diseño no se pueden aprovechar las economías que se pueden obtener con la idea del diseño al límite del Art. 1502*d*, porque se han usado coeficientes aproximados para los momentos.

Todas las demás secciones tienen menos momento y, por lo tanto, todo el acero puede proyectarse suponiendo jd, ligeramente mayor que $0.894d$ de la Fig. 3 8, digamos $jd = 0\ 90d$. (Para momentos muy pequeños puede convenir comprobar a y jd para un valor más económico). Advirtiendo que A_s y $\bar{M}$ se refieren cada uno de ellos a una faja de un pie de anchura.

$$A_s = \frac{\bar{M}}{f_y jd} = \frac{(\bar{M}\text{ en pies-lb})12}{40\,000 \times 0.09 \times 4.00} = \frac{\bar{M}\text{ en pies-lb}}{12\,000}$$

Es conveniente tabular A_s por pulgada de ancho de losa, si la separación de las varillas se va a determinar sin el uso de tablas.

$$A_s/\text{plg} = \frac{\bar{M}\text{ en pies-lb}}{12\,000 \times 12} = \frac{\bar{M}\text{ en pies-lb}}{144\,000}$$

TABLA 9.1. CALCULO DEL ACERO DE LA LOSA

	Claro exterior		Primer apoyo	Interior típico	
	Ext. exterior	Medio claro	Int.	Medio claro	Apoyo
M coef. C	$-\frac{1}{24}$	$+\frac{1}{14}$	$-\frac{1}{10}$	$+\frac{1}{16}$	$-\frac{1}{11}$
$\bar{M} = C \times 66\,000$	−2750 pie-lb/pie	+4720	−6600	+4130	−6000
$A_s/\text{pie} = \bar{M}/12\,000$	0.229 plg²/pie	0.393	0.550	0.345	0.500
$A_s/\text{plg} = \bar{M}/144\,000$	0.0191 plg²/pie	0.0327	0.0458	0 0287	0 0417
Min $+ A_s = 0.005\ bd$	–	0.020 plg²/plg	–	0.020	–
Sep. var No. 4	10.45 plg	6.12	4.36	6.97	4.80
Sep. var No. 5	–	–	6.57	–	7.20
USENSE var. No. 4 a	10	6	4	7	4.5
Alternativa: Use var No. 5 a	–	–	6.5	–	7

siendo $\bar{M} = \text{coef.}\ (492 \times 11^2/0.9) = 66\,000C$. Los cálculos se tabulan en la Tabla 9.1. Adviértase que en el Art. 911*a* se requiere que el porcentaje de acero para el refuerzo positivo sea cuando menos $200/f_y = 200/40\,000 = 0.005$. Las áreas necesarias y la separación de las varillas se dan en la Tabla 9.1, como si el acero superior y el inferior estuvieran totalmente separados, como lo están en la Fig. 9.10*a*.

El proyectista no debe quedar muy contento con este proyecto, en especial con varillas del No. 4, porque con las separaciones prácticas hay un exceso del 6 al 10% de acero para el momento negativo. Sin embargo, con varillas No. 3 las separaciones serían casi la mitad (0.11/0.20), que quedarían más juntas que lo que resulta práctico y las varillas No. 5 dan separaciones satisfactorias solamente en los apoyos interiores. Colocando las varillas como en la Fig. 9.10*a* se puede cambiar el acero superior a varillas del No. 5 en los apoyos inte··s. También sería posible investigar con varillas dobladas de

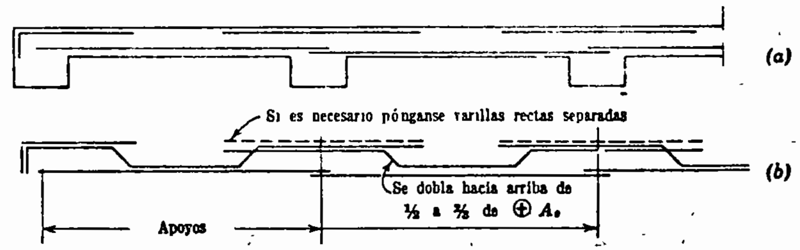


FIG. 9.10. Colocación del acero en la losa. (*a*) Usando solamente varillas rectas. (*b*) Doblando algunas varillas

un tamaño y rectas de otro. Otra posibilidad, ya que se encontró que la relación del claro al espesor era bastante conservadora con respecto a la flecha, podía ser no tomar en cuenta la limitación de $0.18f_c'/f_y$ y usar una losa más delgada en todos los claros. Cuando hay mucha duplicación de miembros iguales, los proyectos alternados, así como las distribuciones alternadas, son con frecuencia la marca de un buen proyectista.

Muchos proyectistas prefieren usar algunas varillas dobladas, porque sienten que de esta manera queda mejor colocado el acero superior. La colocación de la Fig. 9.10*b* es la forma de doblado más común. Las varillas superiores quedan todas realmente en una capa y las inferiores en una capa, pero se dibujan separadas para indicar su forma con más claridad.

Los puntos en que se doblan y en que terminan las varillas en las losas están sujetos a las mismas condiciones que en cualquier otro miembro continuo. Estas consideraciones se discuten con mucho detalle en las Secs. 9.13 a 9.16, en conexión con el proyecto de las vigas T continuas. Aparte de estos análisis, se han elaborado reglas para los casos poco complicados. En el *CRSI Design Handbook*,[5] tiene tabulados muchos proyectos de losas en los que se doblan varillas alternadas inferiores, se doblan hacia arriba, en la forma de la Fig. 9.10*b*, para constituir el refuerzo para el momento negativo. Los puntos de doblado se especifican para los claros simplemente apoyados para los claros extremos y para los claros interiores, respectivamente, en las Figs. D.14, D.15 y D.16 en el Apéndice D. Estos datos son para acero de grado intermedio y f_c' de 3 000 lpc y necesitan algunas modificaciones en las longitudes de anclaje, para ajustarse a los nuevos esfuerzos de adherencia.

Los esfuerzos de adherencia pueden ser críticos en la cara del apoyo, en los puntos de inflexión, donde el acero comienza a desarrollar esfuerzos y en todos los puntos donde el acero de tensión se termina o se dobla. Los detalles de estos cálculos se dan para las vigas T en las Secs. 9.16 y 9.17 Las losas son ligeramente más sencillas, porque sus varillas se doblan generalmente en un solo punto en cada medio claro.

El estudiante debe advertir que se necesita acero para soportar los esfuerzos producidos por las variaciones de temperatura, colocado paralelo a las vigas, cuando menos en la cantidad especificada en el Art. 807*a* del Reglamento. Las varillas para temperatura se usan como separadoras tanto para el acero superior como para el inferior, amarradas al lado inferior de las varillas superiores y al superior de las inferiores. Además, también se deberán usar soportes para las varillas, o silletas, para sostener el acero su nivel correcto.

Se podían haber calculado fácilmente los esfuerzos cortantes en la losa, pero no rigen en las losas reforzadas en una dirección en los claros ordinarios. El proyecto de las vigas rectangulares se discutirá en la Sec. 9.21.

9.9 PROYECTO DE LAS VIGAS T CONTINUAS—GENERALIDADES

Los procedimientos de proyecto para las vigas T continuas se pueden tratar con mayor facilidad aplicándolos a un ejemplo numérico. La mayor parte del resto de este capítulo (hasta la Sec. 9.20) se dedica a los diferentes aspectos de este proyecto, aplicándolos a un claro interior típico, como sigue: Un tablero interior típico de una viga T continua de 20 pies de claro, con columnas cuadradas de 15 plg, se va a proyectar por el DRU para usar la losa proyectada en la Sec. 9.8. Las vigas tienen una separación de 12 pies 0 plg, centro a centro, $t = 5$ plg, $w_L = 200$ lb/pie², $w_D = 88$ lb/pie² (incluyendo el peso de la losa, pero excluyendo el peso de la nervadura), $f_c' = 3000$ lpc, acero A432 ($f_y = 60$ k/plg²) y los coeficientes de los momentos calculados por distribución; cuando se expresan en función de la carga total y del claro libre ($w_T L'^2$), son como sigue: -0.091, $+0.072$, y para los momentos positivos mínimos a la mitad del claro, -0.010.

Existe la posibilidad de hacer diferentes elecciones de b' y d para la nervadura de una viga T, pudiendo constituir cualquiera de ellas un buen proyecto para condiciones específicas. Una de las elecciones de d pudiera ser para aumentar la rigidez y reducir la flecha, según los datos de la Tabla 909b del Reglamento, especialmente para claros largos. Lo que daría un peralte total razonable de $L'/35$. El peralte puede elegirse para que concuerde con el de algún otro miembro, cuya resistencia o rigidez sea más crítica o ajustarse a peraltes necesarios para el paso de ductos. Parecería lógico elegir el peralte más favorable respecto a la economía, recordando que A_s disminuye con el peralte, mientras que el concreto y los moldes son más costosos conforme aumenta el peralte. Sin embargo, la economía de la estructura rara vez puede concordar con la economía general del edificio, porque el aumento de peralte de las vigas significa vigas más pesadas (generalmente) y, por lo tanto, columnas y zapatas más pesadas, muros exteriores más altos con acabados costosos, más peldaños de escalera por piso y, por lo tanto, mayores cubos para las mismas, mayores alturas para los elevadores, etc. En ocasiones se elige la anchura de la nervadura de manera que coincida con la de un muro.

Se puede casi decir que la elección de la b' y la d de las nervaduras es una elección arbitraria. Deben satisfacerse tres condiciones: (1) La capacidad de la sección rectangular invertida que soporta el momento negativo (Fig. 9.3c) debe ser la adecuada para la flexión con una "cantidad razonable" de acero para compresión. (2) Su capacidad para resistir el esfuerzo cortante debe ser la adecuada, pero como su variación es tan grande puede soportarse con estribos, lo que rara vez se restringe en esta disposición. (3) La anchura deberá ser la necesaria para colocar el número requerido de varillas de refuerzo, pero la posibilidad de usar aceros de mayor resistencia o atados de varillas y de dispersar el acero para el momento negativo en el patín, resulta menos restringida que anteriormente. (4) La rigidez debe ser suficiente para mantener las flechas dentro de límites convenientes, pero si se usa acero de compresión, se puede aumentar considerablemente la resistencia de las vigas de poco peralte (Art. 909d del Reglamento).

En este ejemplo, se supone que la flecha no es crítica en un claro de 20 pies muy cargado.* El peralte se elegirá apoyándose en los requisitos para momento negativo. Los coeficientes dados para los momentos indican que el acero, para el momento positivo, será aproximadamente del 75 al 80% del acero de tensión para el momento negativo, ya que jd no será muy diferente para el momento positivo y el negativo. Si se dobla para arriba la mitad del acero para el momento positivo, en la forma indicada en la Fig. 9.12b, la otra mitad continuará en la parte inferior de la viga al interior de la columna, constituyen allí, casi automáticamente, el acero de compresión en la cantidad $A_s' = 0.5 \times 0.8A_s = 0.4A_s$, donde A_s es el acero de tensión para el momento negativo. Traslapando este acero de compresión de dos claros adyacentes, como en la Fig. 9.13d, A_s' se obtiene aproximadamente $0.8A_s$.

Así, el proyectista tiene mucha libertad para elegir la A_s' aproximada que debe usar y las estimaciones como las anteriores son necesariamente aproximadas en esta etapa. (Por ejemplo, el acero para el momento positivo puede convertirse en cinco varillas, lo que significa que hay que doblar hacia arriba 40 o 60%, en vez de la mitad). Para este proyecto se ha elegido el valor de $A_s' = 0.4A_s$, pero el estudiante no debe considerarlo como regla general, ni aún como una buena decisión. No se puede calificar la elección hasta que el proyecto se encuentra más adelantado.

Como se supone que la flecha no es crítica,* el límite de $0.18f_c'/f_y$

* El cálculo de la flecha de la Sec. 7.20 indica la posible necesidad de darle más importancia a la flecha aquí. Si la viga soporta o está unida a tabiques, la flecha resultante aumenta algo así como el 15%.

del Art. 1507, que prescribe $p - p' = 0.0090$, no se tomará en cuenta. Al mismo tiempo, el acero máximo permisible de la Tabla 3.1 ($p_1 = 0.0161$) rara vez es económico. En este proyecto se hará un tanteo con $p - p' = 0.012$.* Para esta condición se puede determinar un valor total de $\bar{R}_u$, suponiendo que A_s' alcanza el esfuerzo f_y y suponiendo el recubrimiento $d' = 0.12d$.

$$A_s' = 0.4A_s = 0.4(0.012bd + A_s'),\quad A_s' = 0.0048bd/0.6 = 0.0080bd$$
$$M_u = A_{s1}f_yj_1d + A_s'f_y(d - d')$$
$$= 0.012bd \times 60\,000 \times 0.9d + 0.008bd \times 60\,000(d - 0.125d)$$
$$= 647bd^2 + 420bd^2 = 1067bd^2 \qquad \bar{R}' = 1067/0.9 = 1186$$

Como este cálculo es francamente aproximado, se redondea $\bar{R}'$, a 1200 en forma relativamente arbitraria. Se necesita más experiencia con el nuevo Reglamento y aceros de alta resistencia, antes de que se pueda recomendar un valor específico sin reservas, pero $\bar{R}'$ se puede determinar muy fácilmente para cualesquiera límites que el proyectista elija.

9.10 PROYECTO DE LAS VIGAS T CONTINUAS— SECCION DE MOMENTO NEGATIVO

Primero se determina el tamaño para el momento como una viga doblemente reforzada.

$$LL = 200 \times 12 \times 1.8 = 4320 \text{ lb/pie lin}$$
$$\text{Losa} + DL = 88 \times 12 \times 1.5 = 1584$$
$$\overline{5900}$$
$$\text{Peso estimado para la nervadura} = 200 \times 1.5 = \cancel{300}\ \ 207$$
$$w_T = \cancel{6200}\ \ 6110 \text{ lb/pie lin}$$
$$M_u = -0.091 \times 6200 \times 20^2 = 225\,000 \text{ pies-lb}$$
$$\bar{M} = 225\,000/0.9 = 250\,000 \text{ pies-lb}$$

Este momento pudiera reducirse por las indicaciones del Art. 1502*d*, pero se usará primero como sea necesario, cuando sólo se disponga de momentos aproximados.

$$b'd^2 \text{ necesario} = \bar{M}/\bar{R} = 250\,000 \times 12/1200 = 2500$$
$$\text{Si } b' = 11.5 \text{ plg},\ d = \sqrt{2500/11.5} = \sqrt{217} = 14.7 \text{ plg}$$
$$b' = 9.5 \text{ plg},\quad d = \sqrt{263} = 16.2 \text{ plg}$$

* Véase l[illegible] a al pie de la Pág. [illegible]

La última relación es la que más se aproxima a la relación usual d/b comprendida entre 1.5 y 2.5, aunque un ancho menor puede dificultar la cuestión del detalle. Añádase un recubrimiento de 1.5 plg + 0.5 plg del estribo +0.5 plg (= $D/2$) = 2.5 plg.

t_b mínimo = 16.2 + 2.5 = 18.7 plg, digamos 19 plg para todo el peralte de la viga.

USESE: $b' = 9.5''$, $d = 19 - 2.5 = 16.5$ plg.

Compruébese el esfuerzo cortante antes de calcular A_s. El esfuerzo cortante es crítico a la distancia d del apoyo y se supondrá un 8% adicional para el esfuerzo cortante correspondiente a la viga simplemente apoyada,* para el esfuerzo cortante producido por la continuidad (aunque no lo exige el Art. 904*c*). (De manera más general, para determinar la escuadría de la nervadura, se usaría el esfuerzo cortante del *extremo* del claro $1.15wL'/2$ para poder usar un tamaño uniforme en los claros exteriores y los interiores).

$$V_u = 6200(10 \times 1.08 - 1.37) = 58\,500 \text{ lb}$$
$$\bar{V}_u = 58\,500/0.85 = 68\,800 \text{ lb}$$
$$\bar{v} = \bar{V}_u/b'd = 68\,800/(9.5 \times 16.5)$$
$$= 439 \text{ lpc} < 10\sqrt{f_c'} = 548 \text{ lpc} \qquad \text{Corr.}$$

Peso de la nervadura (Fig. 9.11*b*), $w_s = 9.5(19 - 5) \times 150/144$
$= 138 \times 1.5FC = 207$ lb/pie lin

Peso revisado $w_T = 6110$ lb/pie lin

La disminución de 1.5% en w_T reduciría el peralte necesario para M aproximadamente el 0.7% y aproximadamente, pero no mucho, permite el uso de $d = 16$ plg y un peralte total de 18.5 plg. Sin embargo, considerando la naturaleza aproximada de la R' supuesta, se puede probar cualquier peralte y no se hará ahora ningún cambio. La anchura de 9.5 plg se adapta muy bien con las medidas de la madera y la relación de la altura a la anchura parece razonable. Algunas veces se usan relaciones tan pequeñas como 1:1 en las vigas pequeñas y tan grandes como 3:1 para vigas grandes.

El acero necesario se calcula como en las Secs. 4.5 y 4.6.

$M_u = -0.091 \times 6110 \times 20^2 = 222\,000$ pie-lb $\quad \bar{M} = 222/0.9 = 247$ pie-k

Para $p - p' = 0.012 \quad a = 0.012bd \times 60\,000/(0.85 \times 3b) = 0.282d$
$jd = 0.859d$

* En la Fig. 9.11*h*,

$$V_c = \frac{(0.091 - 0.048)wL'^2}{L'} = 0.043wL'$$

que es 8.6% del esfuerzo cortante en el extremo de una viga simplemente apoyada y del 10 al 11% del mismo a una distancia d del apoyo.

$$\bar{M}_1 = (0.012 \times 9.5 \times 16.5)60\,000(0.859 \times 16.5)/12\,000 = 133.3 \text{ k-pie}$$

$$\bar{M}_2 = \bar{M} - \bar{M}_1 = 247 - 133.3 = 113.7 \text{ k-pie}$$

$$A_{s1} = \frac{133.3 \times 12}{60 \times 0.859 \times 16.5} = 1.88 \text{plg}^2 (\text{o } p_1bd = 0.012 \times 9.5 \times 16.5 = 1.88)$$

$$A_{s2} = \frac{113.7 \times 12}{60(16.5 - 2.5)} = \underline{1.62 \text{ plg}^2}$$

A_s total negativa = 3.50 plg²

$$c = a/0.85 = 0.282 \times 16.5/0.85 = 5.47 \text{ plg}$$

$$\epsilon_s' = 0.003(5.47 - 2.5)/5.47 = 0.00163$$

$$f_s' = 29 \times 10^3 \times 0.00163 = 47.3 \text{ k/plg}^2)$$

$$A_s' = \frac{113.7 \times 12}{(47.3 - 0.85 \times 3)(16.5 - 2.5)} = 2.24 \text{ plg}^2$$

Debido a $f_s' \ll f_y$, esta A_s' es considerablemente mayor que $0.4A_s$, que originalmente se había elegido como objetivo. Un examen del cálculo de $\bar{R}'$ indica que un f_s' inferior en el segundo término del cálculo de M_u reduciría $\bar{R}'$ entre 100 y 150 unidades. Una revisión de las dimensiones de la viga puede hacerse con esta intensión, pero no sería difícil completar esta área tan grande A_s'. Por lo tanto, no se revisará.

9.11 PROYECTO DE LAS VIGAS T CONTINUAS—PARA EL MOMENTO POSITIVO

El peralte total de 19 plg ya elegido para el momento negativo fija el peralte efectivo a la mitad del claro. Para una capa de acero. (Fig. 9.11*d*),

$$d \text{ positiva} = 19 \text{ plg} - 1.5 \text{ plg recubrimiento} - 0.5 \text{ plg estribo} - 0.5 \text{ plg (para } D/2) = 16.5 \text{ plg}$$

jd de tanteo = $16.5 - t/2 = 14.0$ plg, o $0.9d = 14.85$ plg. Usese un valor mayor.

$$M = +0.072 \times 6110 \times 20^2 = 176\,000 \text{ pies-lb}$$

$$\bar{M} = 176\,000/0.9 = 195\,500 \text{ pies-lb}$$

$$A_s \text{ de tanteo} = \frac{\bar{M}}{f_y jd} = \frac{195\,500 \times 12}{60\,000 \times 14.85} = 2.64 \text{ plg}^2$$

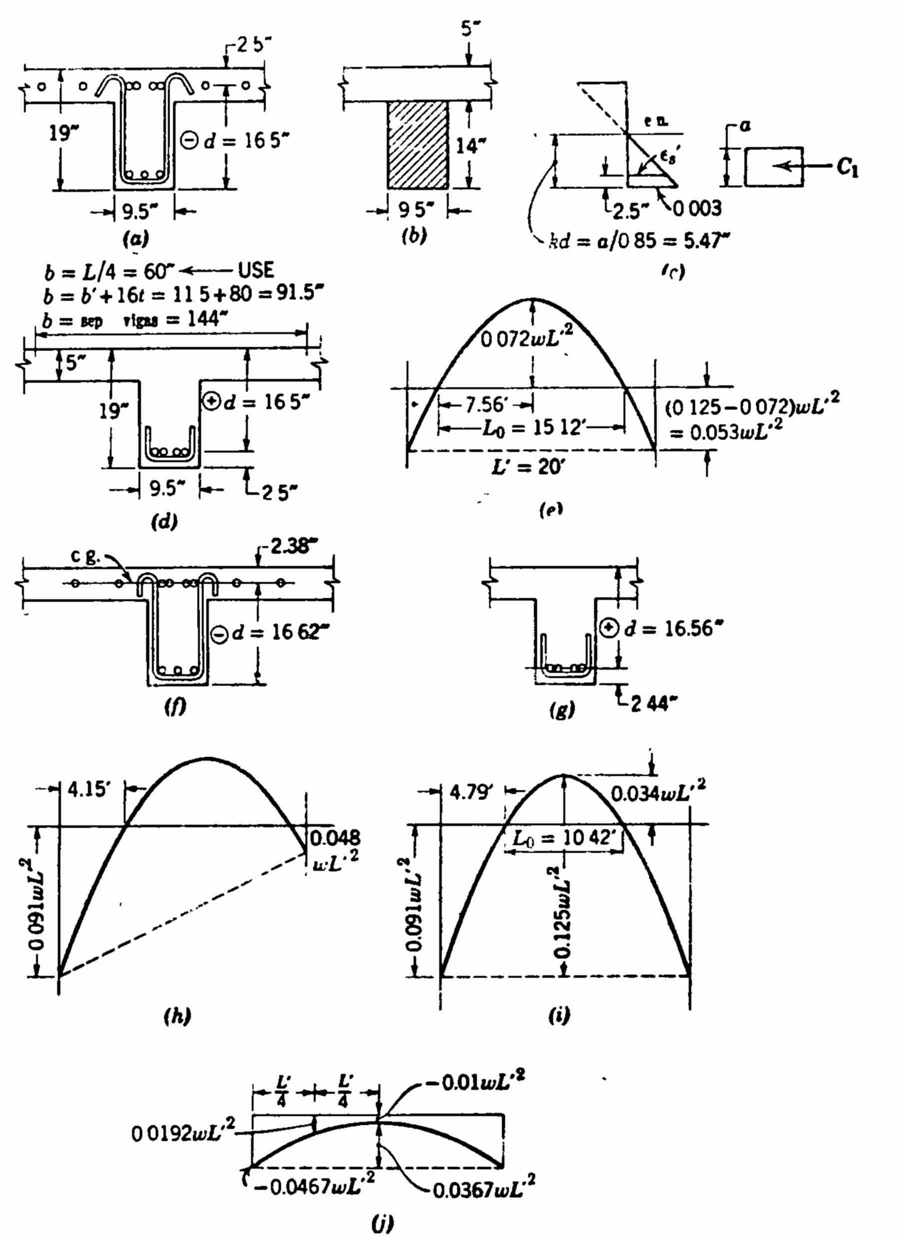


FIG. 9.11. Croquis de proyecto para las vigas T continuas. (*a*) Peralte supuesto para el momento negativo. (*b*) Peso estimado, área sombreada. (*c*) Triángulo de deformaciones para el cálculo de f_s'. (*d*) Peralte supuesto para el momento positivo. (*e*) Posición del P.I. con momento máximo positivo *M*. (*f*) Altura actual al acero superior según se elija. (*g*) Peralte actual al acero inferior como se elija. (*h*) Situación "exacta" del P.I. con momento negativo máximo *M*. (*i*) Situación aproximada del P.I. con momento negativo máximo *M*. (*j*) Momento positivo mínimo cerca de la mitad del claro

Probablemente el área de compresión está limitada a menos del espesor del patín de 60 plg de ancho, funcionando así como una viga rectangular ancha.

$$a = 2.67 \times 60\,000/(0.85 \times 3 \times 60) = 1.05\,\text{plg}^2$$

$$jd = 16.5 - 1.05/2 = 15.98\,\text{plg}$$

$$A_s = 195\,500 \times 12/(60\,000 \times 15.98) = 2.45\,\text{plg}$$

Un ciclo más cambiaría solamente el último dígito. Como a es tan pequeño, la viga evidentemente se clasificaría como falta de refuerzo.

Este acero debe revisarse para ver si satisface el mínimo requerido para el momento positivo por el Art. 911*a*, es decir, comparándolo con $200\,b'd/f_y = 200 \times 9.5 \times 16.5/60{,}000 = 0.53\text{plg}^2$. Las varillas deben disponerse de manera de conservar esta área de acero en toda la longitud de momento positivo, lo que no es problema, ya que es menor que la de 1 var No. 8.

9.12 PROYECTO DE VIGAS T CONTINUAS—ELECCION DE LAS VARILLAS

Los esfuerzos de adherencia pueden ser importantes en la determinación del tamaño adecuado de las varillas. Lo más probable es que la adherencia por flexión sea crítica en el punto de inflexión, aunque ahora el Reglamento permite la alternativa de un cálculo tomando como base la longitud de anclaje. Este punto de inflexión se puede calcular de la Fig. 9.11*e*.

$$0.125wL_0^2 = 0.072wL'^2 = 0.072w \times 20^2$$

$$L_0 = 20\sqrt{8 \times 0.072} = 15.12\,\text{pies}$$

$$V_{\text{P.I.}} = 3670 \times 15.12/2 = 27\,700\,\text{lb}, \quad \bar{V}_{\text{P.I.}} = 27\,700/0.85 = 32\,600\,\text{lb}$$

El esfuerzo de adherencia por flexión permisible para las varillas inferiores es $9.5\sqrt{3000}/D = 520/D$. La fórmula de adherencia por flexión puede resolverse en función del número necesario de varillas de los tamaños comprendidos entre el No. 6 y 11 (y para los números 5 4 y 3 que quedarán del lado de la seguridad).

$$u_u = \bar{V}/(\Sigma o jd) \quad \text{Necesario } \Sigma o = \bar{V}/(u_u jd)$$

$$N\pi D = \bar{V}D/(520 \times 15.98)$$

$$N = \bar{V}/(\pi \times 520 \times 15.98)$$

$$= 32\,600/26\,100 = 1.25 \quad \text{digamos dos varillas.}$$

Necesaria en plg²

Usense 4 No. 6 dobladas = 1.76 plg² 3 No. 8 Rectas = 2.35

4 No. 6 rectas $= \dfrac{1.76}{3.52}$ (2 de un lado, 1 del otro)

2-No. 8 Rectas = 1.58 plg²

2-No. 6 Rectas = 0.88

$\overline{2.46}$

Para el acero del momento positivo, 1 var No. 8 se atará* con 1 var No. 6 en cada esquina de los estribos, ya que no existe espacio para colocar cuatro varillas separadas en el espacio entre los tramos verticales de los estribos que tienen una longitud de $9.5 - 2 \times 1.5$, recubrimiento $- 2 \times 0.5$ estribo $= 5.5$ plg. Con varillas como se dijo antes, el espacio libre entre los atados es de $5.5 - 2 \times 0.75 - 2 \times 1.00 = 2.00$ plg, en realidad más aproximado a 1.5 plg, porque el diámetro sobre las corrugaciones es aproximadamente de ⅛ de plg mayor para cada varilla. La separación para los atados de varillas en el Art. 804*f* es la misma que para varillas aisladas de la misma área combinada. En este caso el área es $0.44 + 0.79 = 1.23$ plg² o aproximadamente el de una varilla No. 10, que tiene un diámetro de 1.13 plg. Por lo tanto, la separación entre varillas es amplia, porque el agregado probablemente tenga un tamaño máximo de 0.75 plg. (Excepto en el caso en que el autor hubiera deseado un ejemplo en el que se usaran dos varillas dobladas, hubiera sido una mejor elección, sujeta a mayores comprobaciones, podría haber sido 2 var. rectas No. 8 más 1 varilla No. 9 doblada).

El acero para momento negativo puede colocarse en una capa atando las 4 varillas No. 6 que se van a doblar hacia arriba formando dos atados adyacentes a los estribos y colocando las otras cuatro varillas rectas en la losa, digamos con una separación de 6 plg. Si importara mucho evitar las grietas en la losa en una dirección perpendicular a la viga, las 2 var. No. 6 de cada lado podrían reemplazarse por 3 var No. 5 a 8 plg, para obtener una distribución más amplia y uniforme del acero.

El acero de compresión (en el apoyo) puede detallarse como 4 varillas No. 8 con las varillas atadas en pares, si se desea simplificar, haciendo de la misma longitud las varillas superiores y las inferiores, como en la Fig. 9.13*d*. En vez de correr una varilla a través de ambas columnas, para que fuera como área efectiva A_s' en claros adyacen-

* Adviértase que los atados de varillas requieren el uso de estribos alrededor de ellos (Art. 804*f*). Si no son necesarios los estribos para el esfuerzo cortante en toda la longitud del tramo, como parece ser el caso de la Sec. 9.18, se hub[illegible] necesitado ponerlos por los atados de todas maneras.

tes una varilla terminará en las columnas. El detalle del acero superior en la Fig. 9.11*f* muestra que el uso de varillas del No. 6 aumenta la *d* negativa en 0.12 plg, a 16.62 plg, pero no se indica ningún cambio en la selección del acero. Para el acero del momento positivo (Fig. 9.11*g*) la *D* promedia es 0.88 plg y *d* se convierte en 16.56 plg, lo que eliminaría la deficiencia (despreciable) que existe allí.

Con no poca frecuencia el proyectista aumenta b' arbitrariamente y aun cambia *d* para mejorar la separación del acero o para cambiar el A_s necesaria o para ajustarse más a los tamaños de las varillas.

El esquema del doblado del acero elegido es el mismo que el mostrado para la losa en la Fig. 9.10, aunque en la actualidad, muchos proyectistas usarían solamente varillas rectas arriba y abajo, sin varillas dobladas.

Hubiera resultado una viga más barata si se hubieran aprovechado las ventajas de las disposiciones con respecto al diseño al límite del Art. 1502*d*. Determinando los momentos tomando en cuenta las modificaciones a la teoría elástica que se recomienda en la Sec. 10.5.

9.13 PROYECTO DE LAS VIGAS T CONTINUAS— REQUISITOS GENERALES PARA EL DOBLADO DE LAS VARILLAS

El constructor debe detallar cada varilla, pero el proyectista queda satisfecho determinando la posición de los puntos de doblado y de los puntos en que terminan con una precisión razonable. En muchas oficinas de proyecto se usan reglas como "dóblese la mitad del acero inferior hacia arriba en la cuarta parte del claro libre," pero presentamos aquí procedimientos más exactos para describir métodos más adaptables a las necesidades que se presentan en muchos casos. Las Figs. D.17, D.18 y D.19 del Apéndice D muestran diagramas de doblado que se usan en los proyectos tabulados en el *CRSI Design Handbook*, según el Reglamento de 1956, con acero de grado intermedio. Es necesario hacer algunas modificaciones de poca importancia por los nuevos esfuerzos de adherencia.

El autor recomienda una actitud muy conservadora respecto al detalle de las varillas. Se han hecho detalles ineficaces en proyectos que de otra manera hubieran sido buenos. El miembro resultante no es mejor que sus detalles.

La posición de los puntos de doblez puede depender de:

1. Los requisitos de los momentos
2. De las longitudes de anclaje
3. De los requisitos "arbitrarios" del Art. 918 del Reglamento

4. De los requisitos de la adherencia.
5. Del uso de varillas dobladas como estribos.

Primero mencionaremos varios de los requisitos de las especificaciones "arbitrarias". El Art. 918*e* requiere que cuando menos la tercera parte del refuerzo total para momento negativo se prolongue más allá del punto de inflexión en más de: (1) $L'/16$ (2) el peralte *d* de la viga. El Art. 918*f* prescribe que cuando menos una cuarta parte del refuerzo positivo en las vigas continuas se prolongue 6 plg dentro del apoyo. Ambas condiciones se deben, según el texto del antiguo Joint Committee Specification "para afrontar contingencias que provengan de la distribución imprevista de las cargas, asentamiento de los apoyos, movimiento de los puntos de inflexión o a la falta de concordancia con las condiciones supuestas que regulan el proyecto de las estructuras elásticas."

También el Art. 918*b* prescribe que todas las varillas que se necesiten para el refuerzo positivo o negativo, deberán prolongarse el peralte de la viga o 12 diámetros más allá del punto en que ya no sean necesarios según los esfuerzos. Parece que el reglamento permite que la longitud quede en la porción doblada de la varilla. El autor prefiere proponer que cuando menos la mitad de esta longitud adicional se considere como otro requisito, como previsión de las modificaciones que pudieran sufrir los diagramas de momentos, como se muestra en líneas de rayas en la Fig. 9.12*a*. Lo que quiere decir que no sólo aplica la longitud requerida como una distancia en línea recta antes de terminar la varilla, sino que también usa cuando menos la mitad de ella como longitud recta necesaria antes del doblado. Esta forma de proceder relativamente severa es la que se usa en la Sec. 9.14.

Disposiciones que satisfacen los requisitos de los momentos, de los anclajes y los del Art. 918 pueden variarse algunas veces ligeramente para aumentar la adherencia o el refuerzo de la nervadura. Por ejemplo, si los esfuerzos de adherencia fueran excesivos en el acero inferior en el punto de inflexión, podría ser posible mover los puntos de doblado hacia arriba, hacia la columna y conservar más acero utilizable para la adherencia; o puede variarse la separación de los puntos de doblado, para que las varillas dobladas sean más útiles como acero de la nervadura.

9.14 PROYECTO DE LAS VIGAS T CONTINUAS— DIAGRAMAS DE MOMENTOS QUE RIGEN EL DOBLADO DE LAS VARILLAS

El diagrama de momentos positivos ya se ha determinado en la Fig. 9.11*e*

Para el momento máximo negativo, el diagrama real es asimétrico y el cálculo de momentos máximos sólo nos proporciona suficientes datos para todo el diagrama. Se puede determinar, en forma razonablemente aproximada, usando un momento negativo en el extremo lejano como ligeramente menor que el momento positivo que lo acompaña en este caso, digamos $-0.048wL'^2$ en vez del valor de $-0.053wL'^2$ mostrado en la Fig. 9.11*e*. El valor 9.11*h* será casi exacto en cuanto se refiere a la posición del P.I. (punto de inflexión). Luego puede usarse el método de la Sec. A.5 para localizar el P.I. a 4.15 pies de la cara de la columna.

Más comúnmente se calcula el P.I. para el diagrama simétrico supuesto de la Fig. 9.11*i*, que es el que se usará aquí.

$$\tfrac{1}{8}wL_0^2 = 0.034w \times 20^2$$

$$L_0 = 20\sqrt{0.272} = 10.42 \text{ pies}$$

Por lo tanto, el P.I. queda a 4.79 pies de la columna. La mayor aproximación que se obtiene al usar un triángulo para la sección de momento negativo de la parábola se justifica cuando el P.I. no está muy cerca de la mitad del claro, digamos, fuera del tercio medio del claro.

El momento positivo mínimo a la mitad del claro es también parte de un diagrama, simétrico, pero en este caso, el diagrama de momentos de la viga simplemente apoyada es solamente para la carga muerta, que puede expresarse como sigue, en función de la carga total w, es decir, 1791 lb/pie lineal de un total de 6110 lb/pie lineal total.

M_s de la viga simplemente apoyada $= 0.125w_DL'^2 =$

$$0.125\,\frac{1791}{6110}\,wL'^2 = 0.0367wL'^2$$

Se hace un esquema del diagrama en la Fig. 9.11*j*. Los diagramas de momentos máximos se unen en la Fig. 9.12*a*.

9.15 PROYECTO DE LAS VIGAS T CONTINUAS— DOBLECES Y PUNTOS EN QUE SE TERMINAN LAS VARILLAS DE TENSION

No es la intención de esta sección considerar simplemente los detalles de la forma en que se colocan las varillas, sino que, la experiencia ha demostrado, que ningún otro tipo de problema pone de manifiest os nociones fundamentales de la manera en que trabaja

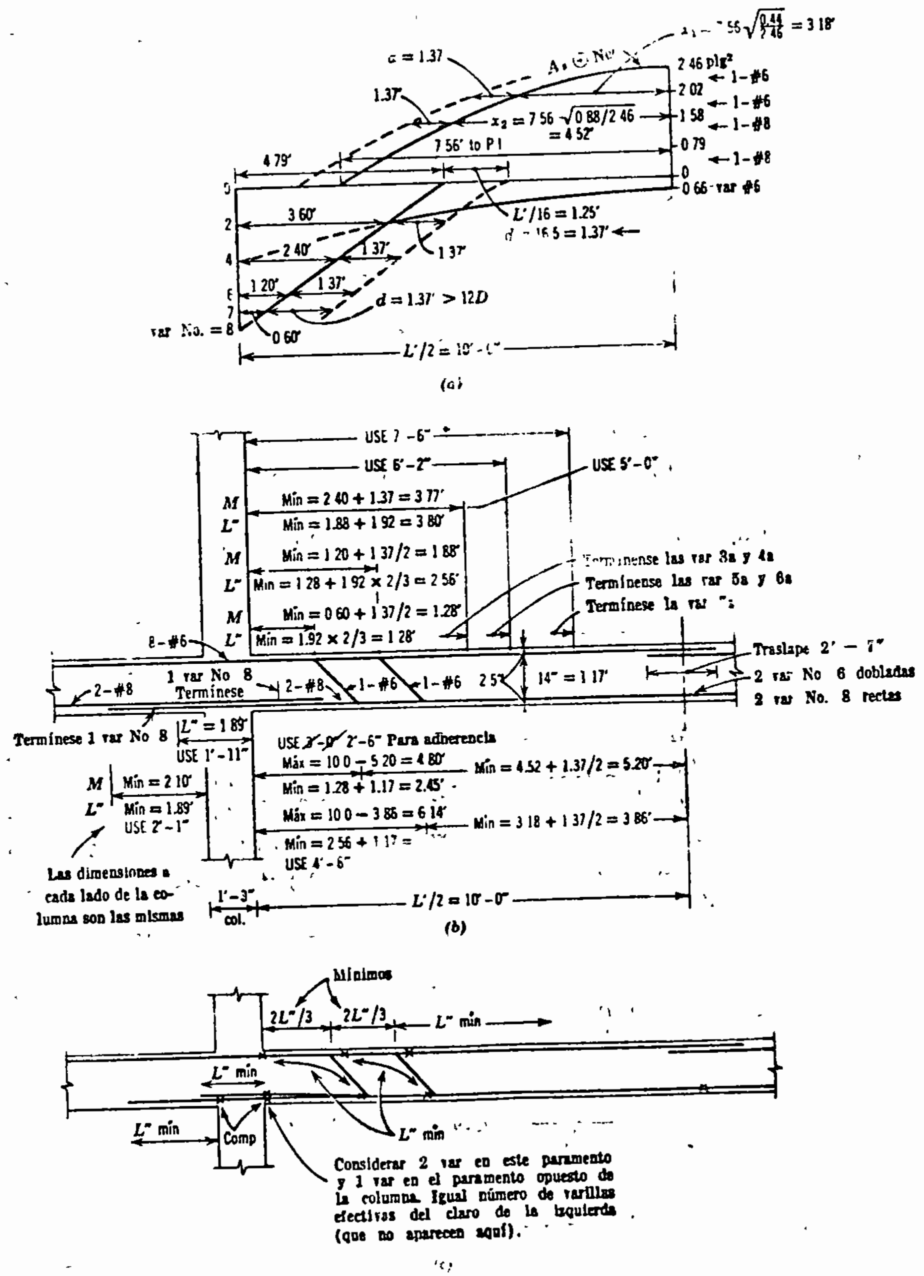


FIG. 9.12. Cálculo de los puntos de terminación y de los dobleces de las varillas. (*a*) Usense los diagramas de momentos máximos para calcular las A_s necesarias. (*b*) Datos relacionados con la colocación de las varillas. (*c*) Requisitos sobre las longitudes de los anclajes separadas por claridad

el concreto reforzado y la manera en que la flexión y el anclaje de las varillas se afectan entre sí. Aun usando varillas rectas sin dobleces se presentan los mismos problemas.

Aunque la mayor parte de los proyectistas probablemente doblarían hacia arriba las dos varillas No. 6 en el mismo punto, en este caso se doblarán una por una, para ilustrar mejor el procedimiento de cálculo. Se recomienda al estudiante hacer un esquema como el de la Fig. 9.12*b* para la tabulación de los valores calculados, aunque la repetición de los títulos de las varillas provocan confusión en los dibujos de proyecto. Como se define como primer doblez de las varillas el de la varilla que queda más cerca del punto de momento máximo, se notará que la primera varilla doblada hacia abajo es la misma que la segunda doblada hacia arriba.

El diagrama del momento positivo determina las distancias mínimas x_1 y x_2 de la línea central del claro para las primeras y segundas varillas dobladas hacia arriba, según las dimensiones puestas en la Fig. 9.12*b*. Estas dimensiones también determinan las distancias máximas de la columna. De la misma manera, el diagrama de los momentos negativos fija la distancia del soporte a que se deben doblar hacia abajo. La distancia entre el acero positivo y el negativo es de 14 plg o de 1.17 pies. A esta distancia se le llama ordenada *y*, con los dobleces usuales de 45°, es igual a la distancia horizontal o abscisa utilizada para hacer la ordenada. La distancia mínima de doblado más esta abscisa, determinan las distancias mínimas al punto de doblado hacia arriba. Por lo tanto, el proyectista tiene algo de libertad de elección; por ejemplo, entre 2.45 y 4.80 pies al segundo punto de doblado hacia arriba.

Hasta ahora, solamente se han considerado las condiciones para los momentos, pero a la longitud de anclaje tiene que dársele la misma atención. Para el acero superior el esfuerzo de adherencia admisible es $6.7\sqrt{f_c'}/D = 6.7\sqrt{3000}/0.75 = 488$ lpc.

$$L'' = \frac{f_y}{4u}D = \frac{60\,000}{4 \times 488} \times 0.75 = 23.0 \text{ plg} = 1.92 \text{ pies}$$

En la Fig. 9.12*c* las cruces indican los puntos de esfuerzos máximos en el acero y los lugares más allá de los cuales se necesitan las longitudes L''. Para el acero inferior, como siempre, las varillas se prolongan lo suficiente para que produzcan un anclaje mucho mayor que el necesario L'' (por inspección, aun sin calcular la distancia para varillas No. 8 con $u = 9.5\sqrt{f_c'}/D$). Para el acero superior que se dobla hacia abajo, las varillas deben correr horizontalmente cuando menos parte de L'', antes de doblarlas hacia abajo. Con acero de

60 k/plg², el autor recomienda que cuando menos dos tercios* de L'' se utilicen como varilla recta, antes de doblarla. Para el primer doblez hacia abajo, la distancia mínima calculada de 1.28 pies, es precisamente la adecuada para este objeto y una longitud de 0.55 pies más larga se eligió al principio en forma relativamente arbitraria. Como este aumento arbitrario en la distancia al primer punto de doblado no forma un anclaje en la segunda longitud más difícil, la siguiente longitud L'' puede medirse con seguridad desde la distancia mínima permisible (para momento) al punto del primer doblez, hacia abajo.† Lo que da el mínimo para anclaje al segundo punto de doblado hacia abajo como 1.28 pies + 1.92 × ⅔ = 2.56 pies.

En este ejemplo los dos puntos de doblado pueden satisfacer fácilmente todos los requisitos. Cuando un proyectista trata de doblar hacia arriba muchas varillas, lo que se acusa por las distancias necesarias que exceden de las distancias máximas disponibles. Esta condición indica que se ha elegido una forma de doblado incorrecta para ese miembro. Cuando, como en este ejemplo, existe una variación entre mínimo y máximo, lo que es una indicación de que el proyectista puede elegir arbitrariamente. También puede significar que se pudieran doblar más varillas si fuera necesario.

Para cualquiera de las 6 varillas No. 6 del acero superior que no se doblan hacia abajo, el anclaje necesario debe ser cuando menos de 1.88 + L'' = 3.80 pies. Sin embargo, si se cortan algunas varillas dentro de la distancia al P.I. para el momento máximo negativo, debe satisfacerse la condición relativamente estricta del Art. 918*c*. Hasta que se conozca más con respecto al efecto de cortar las varillas en las zonas de tensión de las vigas, el autor prefiere prolongar estas varillas cuando menos hasta el P.I. que se produce con la mayor carga. Por lo tanto, no cortará ninguna varilla a una distancia menor que aproximadamente 4.79 pies del apoyo. No todas las varillas pueden cortarse ni aun en este P.I. El momento mínimo a la mitad del claro requiere que algo se quede a todo lo largo del claro y el Art. 918*b*, *e* requiere que se prolongue después del P.I.

Vamos a considerar primero el momento negativo a la mitad del claro. Se puede obtener con suficiente precisión el acero superior que se necesita a la mitad del claro por proporciones de los momentos, ignorando los cambios en *jd*. La A_s negativa a la mitad del claro = (8 varillas) $(0.01wL'^2)/(0.091wL'^2)$ = 0.66 varillas No. 6. Proporcionan un empalme por traslape para 1 No. 6 (la última u octava

* Los ganchos estándar se valúan en 19 k/plg² en el Art. 918*h* y una varilla doblada hacia abajo, probablemente tenga una capacidad un poco mayor.

† Teóricamente, el requisito es una L'' que llegue más allá del punto real de doblado pero calculado para el f_s real reducido

varilla) a la mitad del claro, que requiere $1.92 \times \frac{4}{3} = 2.55$ pies, digamos 2 pies 7 plg. (La L'' de 1.92 pies calculada antes se aumenta por el factor de $\frac{4}{3}$, porque el Art. 805 b permite solamente 0.75 de los esfuerzos usuales de adherencia cuando se usan para el cálculo de empalmes). Parece que esta curva del momento mínimo positivo cruza la curva del momento máximo negativo cerca de la cuarta parte del claro. Si es así, este momento negativo (Fig. 9.11) sería $-0.01wL'^2 - 0.25(0.0367wL'^2) = -0.019wL'^2$ que requiere $0.0192/0.091 \times 8 = 1.48$ varillas. Por lo tanto, parece conveniente correr una segunda varilla No. 6 (la séptima varilla) hasta aproximadamente el punto $\frac{3}{8}$ del claro, digamos a 7 pies 6 plg del apoyo. (Que se puede calcular más exactamente de la parábola, si se desea).

El requisito del Art. 918b es que la tercera parte de la A_s negativa máxima se continúe después de llegar al punto de flexión extremo* $L'/16 = 1.25$ pies o $d = 1.37$ pies, rigiendo la mayor. Así, cuando menos tres varillas deben prolongarse 4.79 al P.I. más $1.37 = 6.16$ pies, digamos 6 pies 2 plg, de las cuales dos ya se han prolongado más adelante por otras razones.

Con 3 varillas No. 6 prolongadas en esta forma y 2 varillas No. 6 dobladas hacia abajo, el punto para cortar las 3 varillas No. 6 intermedias debe fijarse en seguida.

La manera más segura para satisfacer el Art. 918c, es prolongar la tercera y la cuarta varillas hasta donde las cuatro varillas restantes tienen una A_s, igual al doble de la necesaria, es decir, al punto donde dos varillas son suficientes para sólo el momento. Se necesitan 2 varillas (Fig. 9.12a) hasta $3.60 + 1.37 = 4.97$ pies. Por lo tanto, prolónguese 2 varillas No. 6 (la tercera y la cuarta) hasta un poco después del P.I. a 5 pies de distancia y córtese la quinta varilla con la sexta a los 6 pies 2 plg determinadas anteriormente. A 6 pies 2 plg se puede cortar una varilla de tensión debido a que el esfuerzo cortante requerido en el Art. 918c (1) es bajo y es seguro que se satisface, porque estas varillas no están en tensión cuando actúa la carga viva en el claro. Las varillas rectas de la parte inferior se discuten en la varilla) a la mitad del claro. que requiere $1.92 \times \frac{4}{3} = 2.55$ pies, Sec. 9.16.

Consideraremos una alternativa de la colocación de las varillas para insistir en algunos puntos. En la Fig. 9.13a se doblan las dos varillas No. 6 como un solo par. Esta disposición es más sencilla y debe preferirse generalmente por sencillez, pero nótese que la diferencia entre los puntos de doblado mínimos y máximos, se reduce de 2.35 pies y 2.41 pies en la Fig. 9.12b a 0.75 pies aquí. El doblado de varillas separadas es posible hacerlo con frecuencia cuando no es posible doblar pares, pero doblando varillas aisladas se crea mayor número de puntos en los que se producen esfuerzos máximos que hay que vigilar en la longitud de los anclajes.

* El autor interpreta la palabra "extremo" como el extremo para el diagrama de momentos especial que demanda la A_s negativa considerada. Así, si la carga que produce el momento positivo mínimo creado en el P.I. cerca de la mital del claro, el acero que debe prolongarse más allá del P.I. sería la tercera parte del área necesaria por el menor momento negativo (de la carga muerta) en el apoyo.

9.16 PROYECTO DE VIGAS T CONTINUAS — PUNTOS EN QUE SE PUEDEN TERMINAR LAS VARILLAS DE COMPRESION

La longitud de las varillas rectas del No. 8 de la parte inferior está determinada tanto por el momento como por la longitud de anclaje necesaria.

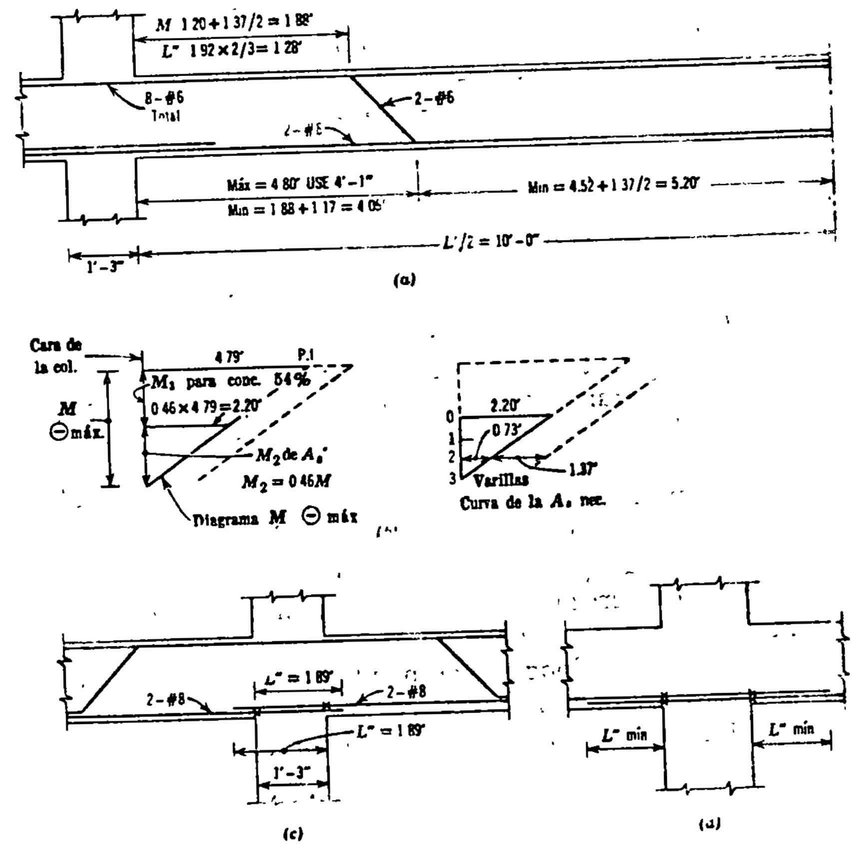


FIG. 9.13. Alternativas de la colocación de las varillas en la viga de la Fig. 9.12. (a) Se doblan dos varillas como un par, (b) Anclaje necesario de A_s' del diagrama para la curva de momentos máximos negativos. (c) Caso A_s' es pequeña. (d) Caso imaginario mostrando los detalles cuando A_s' es grande

Como se necesitan más de dos varillas en cada cara de la columna, son necesarias las varillas de ambas vigas adyacentes. Una No. 8 debe atravesar la columna y prolongarse la longitud de anclaje más allá de la cara más lejana. Esta prolongación debe comprobarse para ver si tiene la longitud necesaria para el momento, para lo cual se requiere un diagrama de momentos para el acero de compresión. En la Sec. 9.10 se obtuvo el valor de $\bar{M}_1$ sin acero de compresión de 133.3 k-pies, que es 133.3/247 = 0.54 del $\bar{M}$ total. Por lo tanto, el diagrama de momentos negativos puede dividirse, como se muestra en la Fig. 9.13*b*, con solamente 2.20 pies (más una distancia *d*) sin necesitar acero de compresión. Las dos varillas No. 8 que corren a lo largo de todo el claro pueden ser suficientes para el mismo menos en 0.73 + 1.37 = 2.10 pies adyacentes al apoyo. Esta longitud en este ejemplo rige sobre la longitud L'' más corta que se determina en el párrafo siguiente y requiere que se prolongue *una* varilla de cada lado a través de la columna y 2.10 pies más allá de la cara lejana, como se indica en la Fig. 9.12*b*. No es necesaria una de las varillas de cada lado más allá de la cara de la columna y puede simplemente prolongarse la longitud L'' dentro de la columna y cortarse.

El proyecto de A_s' en la Sec. 9.10 se basa en $f_s' = 47.3$ k/plg², que, con el esfuerzo permisible de adherencia [Art. 1801*c*(3)] en compresión de $13\sqrt{f_c'}$ se requiere

$$L'' = f_s'D/4u = 47\,300 \times 1.00/(4 \times 13\sqrt{3000}) = 16.6 \text{ plg} = 1.39 \text{ pies}$$

Además, es necesario tomar en cuenta otras circunstancias, porque para la separación transversal, se requiere que las varillas No. 8 se aten en grupos de dos dentro de la columna* y, generalmente, se considera que esto reduce el perímetro. El análisis exacto es difícil, porque la superficie expuesta es invariable. Como un margen razonable para esta complicación, L'' simplemente se alargará 6 plg más a los 1.89 pies. Se espera que la varilla que continúa a través de la columna mantenga su esfuerzo al cruzar la columna y que necesite su L'' más allá de la cara lejana de la columna. La otra, al prolongarse L'' más allá de la cara cercana, se prolonga ligeramente más allá del lado lejano de la columna, pero esta prolongación dentro del claro siguiente, donde no se necesita, puede con toda razón ignorarse en el análisis de ese claro.

Con mucha frecuencia la colocación del acero de compresión puede ser más sencilla. Si se han necesitado 4 varillas No. 8, ambas varillas de un lado podían haber terminado en un punto, como en la Fig. 9.13*d*. Este punto debe también satisfacer los requisitos de momento y de longitud de anclaje. Todavía hubiera resultado más sencilla la colocación

* O colocar el acero en dos capas.

si solamente se hubieran necesitado 2 varillas No. 8 para A_s' como se muestra en la Fig. 9.13*c*. Entonces no hubiera sido necesario el acero adicional de compresión del claro adyacente y sólo es necesario prolongar, *dentro de la columna*, la distancia L''. Entonces el diagrama de momentos no tiene aplicación para determinar la longitud de estas varillas rectas inferiores, porque automáticamente se usa toda la A_s' en toda la longitud del diagrama de momento negativo.

Como anteriormente, para los atados de varillas dentro de la columna, deberá usarse la longitud L'' de 1.89 pies. Otra forma de considerar este armado pudiera ser traslapando varillas como empalmes en compresión, que requieren $24D = 2.00$ pies de traslape. En las columnas gruesas, la diferencia entre los dos procedimientos pudiera ser notable; aquí la diferencia es pequeña. El autor no le considera importancia a la capacidad horizontal de compresión cerca de la mitad de la columna (donde probablemente el peralte efectivo aumenta) y considera L'' de la cara de entrada como el aspecto de importancia crítica.

9.17 PROYECTO DE VIGAS T CONTINUAS—ADHERENCIA POR FLEXION EN EL ACERO DE TENSION

En la Fig. 9.14 se marcan los puntos de esfuerzos máximos de adherencia por flexión. Se llama la atención al Art. 1801*a* que dice que las varillas dobladas no más de *d*/3 del acero longitudinal, pueden contarse en el perímetro. Por lo tanto, los esfuerzos críticos de adherencia por flexión en el acero superior se calcularán a una distancia *d*/3 más allá de los puntos de doblado y, precisamente, más allá de los puntos de corte. Aunque el Reglamento no lo exige en los claros interiores, el autor continuaría usando el esfuerzo cortante correspondiente a una viga simplemente apoyada aumentado en $0.08wL'/2 = 4880$ lb para representar el corte debido a la continuidad, cuando se trate de acero determinado por el momento negativo. Se usará la Ec. 6.3 para comprobación.

En el punto 1 (varillas superiores):

$$V_1 = 1.08wL'/2 = 1.08 \times 6110 \times 20/2 = 65\,700 \text{ lb.}$$

$$\bar{V}_1 = 65\,700/0.85 = 77\,300 \text{ lb}$$

Ec. 6.4. $\quad \text{Mín } N = \bar{V}/(Q\sqrt{f_c'}\pi \times 0.9d)$

$$= 77\,300/(6.7 \times \sqrt{3000} \times 3.14 \times 0.9 \times 16.62)$$

$$= 4.53 \text{ varillas} < 8 \qquad \text{Corr.}$$

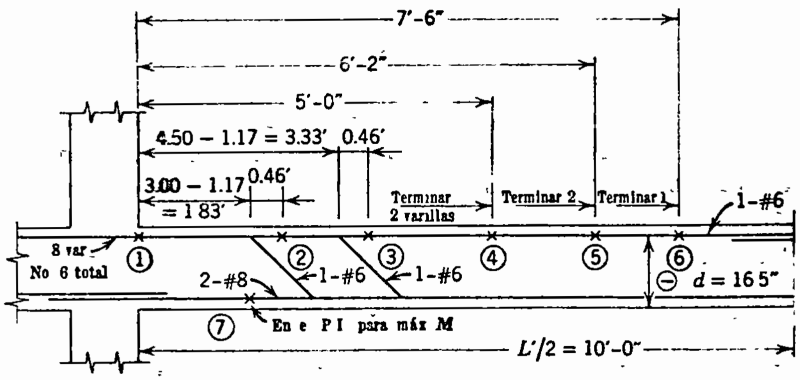


FIG. 9.14. Lugares en los que se debe comprobar la adherencia por flexión

Es evidente que las siete varillas en el punto 2 y las 6 del punto 3 son adecuadas, porque $\bar{V}$ será menor en estos casos. El punto 4 queda en el punto de inflexión y lleva tensión en las varillas solamente cuando el diagrama de momentos varía. En cualquier caso, las cuatro varillas restantes son adecuadas porque $\bar{V}$ no puede ser mayor del 55 o 60% del máximo en el punto 1 y, por lo tanto, la N necesaria no puede ser mayor de 55 o 60% de 4.53. En los puntos 5 y 6 existe tensión solamente cuando la carga viva queda fuera del claro y $\bar{V}$ es evidentemente demasiado pequeña para darle importancia.

Las varillas inferiores están a la tensión máxima cuando la carga es simétrica, lo que anula el esfuerzo cortante producido por continuidad. La fuerza cortante aumenta hacia el apoyo y el número de varillas disminuye; por lo tanto, el lugar peor está en el punto de inflexión para momento máximo positivo (a la izquierda de este punto 7 las varillas están en compresión y las relaciones de adherencia por flexión del Cap. 6 se aplican solamente para las varillas en tensión).

$$V_7 = 6110 \times 7.56 = 46\,200 \text{ lb}, \quad \bar{V} = 46\,200/0.85 = 54\,300 \text{ lb}$$

$$N = \bar{V}/(Q\sqrt{f_c'}\pi \times 0.9d)$$

$$= 54\,300/(9.5\sqrt{3000} \times 3.14 \times 0.9 \times 16.56) = 2.24 \text{ varillas} > 2 \text{ reales}$$

Según la experiencia del autor, en este punto siempre ha sido crítica si la adherencia constituye un problema en algún lugar. Porque resultaría muy molesto a esta etapa poner tres varillas, investigar si la última varilla doblada puede ayudar aquí. Se dobló arriba de 3.0 pies del apoyo, pero se puede contar hasta que esté a $d/3 = 0.46$ del acero superior. Por lo tanto, están disponibles tres varillas a 2.54 pies del

apoyo o 7.46 pies de la mitad del claro. Que está tan cerca del punto de inflexión que se puede despreciar esta pequeña diferencia. Sin embargo, el punto de doblado se puede mover hacia el apoyo (Fig. 9.12b) sin más complicaciones. Por lo tanto, revísese el punto de doblado hacia arriba a 3 pies 0 plg, a 2 pies 6 plg de la columna, lo que todavía satisface el valor mínimo de 2.45 pies.

9.18 PROYECTO DE VIGAS T CONTINUAS—ESTRIBOS

Como en la Sec. 9.12 se eligieron atados de varillas, son necesarios los estribos hasta la mitad de la viga para satisfacer el Art. 804f del Reglamento. Sin embargo, los estribos se consideran como una cuestión del proyecto por esfuerzo cortante.

Bajo el efecto de las cargas que producen los momentos máximos, la viga está sujeta a una fuerza cortante en el extremo igual a la de una viga simplemente apoyada $wL'/2 = 61\,000$ lb, más una fuerza cortante por continuidad, que de nuevo supondremos igual a $0.08wL'/2$ o 4880 lb, actuando ambas en el extremo y a la mitad del claro como se muestra con líneas de rayas en la Fig. 9.15a. La fuerza cortante a la mitad del claro será mayor cuando se quite la carga viva de la mitad izquierda del claro. Esta carga producirá una fuerza cortante por continuidad más pequeña, que se despreciará. Esta fuerza cortante en la viga simplemente apoyada será de 4320 $\times$ 10 $\times$ 5/20 = 10 800 lb. En la Fig. 9.15a la línea llena se usará como diagrama de fuerzas cortantes máximas para proyectar los estribos. Con $b' = 9.5$ plg y $d = 16.62$ plg en el extremo, la fuerza cortante crítica está a la distancia d del apoyo.

$$\bar{v}_0 = \frac{66\,000/0.85}{9.5 \times 16.62} = 495 \text{ lpc}$$

$$\bar{v}_{10 \text{ pies}} = \frac{10\,800/0.85}{9.5 \times 16.56} = 81 \text{ lpc}$$

La pendiente del diagrama de $\bar{v}$ es $(495 - 81)/120 = 345$ lpc/plg. El esfuerzo cortante crítico está a 16.5 plg del apoyo y es de $495 - 3.45 \times 16.6 = 438$ lpc. En la Fig. 9.15b está dibujada la curva del esfuerzo cortante. Los estribos se necesitan en la longitud L_v donde $\bar{v}$ es mayor de $2\sqrt{f_c'}$; además de una longitud adicional igual a $d = 16.6$ plg, según el Art. 1702. Por lo tanto, los estribos son necesarios a lo largo de todo el claro, porque $L_v + d = 112 + 16.6 = 128.6$ plg > 10 pies. Así queda satisfecho automáticamente el

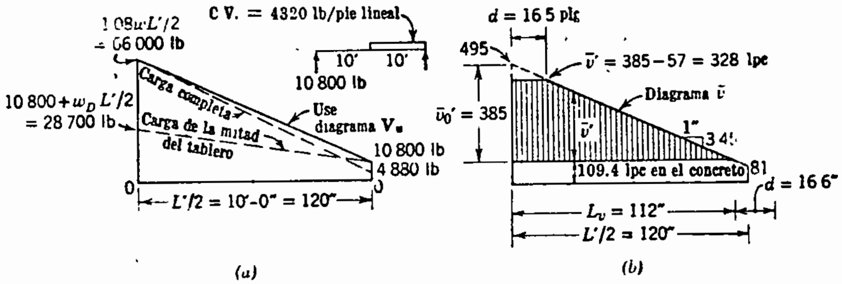


Fig. 9.15. Diagramas de esfuerzos cortantes para el cálculo de los estribos. (*a*) Diagrama V_u. (*b*) Diagrama $\bar{v}$

requisito sobre los atados de varillas; también los requisitos para los estribos sobre las varillas A_s' del Art. 806*c*.

En la Sec. 6.19c se ha hecho con todo detalle el proyecto de estos estribos.

9.19 PROYECTO DE VIGAS T CONTINUAS—COLOCACION DE ESTRIBOS

La correcta colocación de los estribos en las vigas continuas presenta un problema. Para el mejor anclaje de los estribos sería necesario que los ganchos queden donde el concreto está en compresión, que cerca de los apoyos queda abajo. Como la construcción es más sencilla cuando el extremo abierto del estribo queda hacia arriba, la cuestión del anclaje generalmente se ha ignorado. Además de lo fácil de la colocación es el requisito de que el acero de compresión (de abajo) se amarre como se especifica en el Art. 806*c*. Un estribo debe prolongarse completamente alrededor de todas las varillas longitudinales A_s'. Los estribos cerrados, como los de las columnas, serían excelentes y puede ser que se usen más ahora que se necesitan, cuando se usan estribos en las vigas de los tableros (Art. 921). La principal objeción que se les hace, se refiere a la dificultad para meter el refuerzo dentro de los estribos.

9.20 PROYECTO DE VIGAS T CONTINUAS—FLECHAS

Las flechas con las cargas de trabajo constituyen un caso crítico y el tipo de cálculo que se emplea es más del tipo elástico o DRT que del DRU, como se dijo el el Cap. 7. En cualquier viga se complica por la flexión debida a las deformaciones plásticas, bajo la

carga y a la variabilidad de la I producida por los diferentes grados de agrietamiento en las diversas secciones. En las vigas continuas se complica todavía más por la variación de la I efectiva en una viga rectangular invertida en los apoyos a una T en la mitad del claro, como se indica en la Fig. 9.3. Los cálculos para comprobar las flechas son, por lo tanto, más de carácter nominal que preciso y se usará el procedimiento aconsejado en el Art. 909*c*.

Para el acero en el momento positivo $p_w = A_s/b'd = 2.46/(9.5 \times 16.56) = 0.0157$ y $pf_y = 0.0157 \times 60\,000 = 934 > 500$. El valor de pf_y para el acero, para el momento negativo, es todavía mayor. Este criterio del Art. 909*c* indica que ambas secciones deben considerarse agrietadas. Las áreas transformadas mostradas en la Fig. 9.16 se considerarán como efectivas y sus valores de I se promediarán para usarlos.

En la sección de momento negativo, $A_s = 3.52$ plg², $A_s' = 2.35$ plg² y, por tanteos $kd = 6.90$ plg.

Area	y	I
31.7	9.72	2970
18.7	4.40	362
$9.5 \times 6.90 \times 6.90^2/3$		1040
	I_n Total =	4370 plg⁴

En la sección de momento positivo, $A_s = 2.46$ plg², $A_s' = 0.44$ plg² (accidental debido a la colocación del acero para el momento negativo) y, de los momentos de las áreas con relación al eje neutro, $kd = 3.27$ plg.

Area	y	I
22.1	13.29	2940
$60 \times 3.27 \times 3.27^2/3$		698
	I_n Total	3640 plg⁴

$$I \text{ promedio} = (4370 + 3640)/2 = 4000 \text{ plg}^4$$

$$E = w^{1.5}\,33\sqrt{f_c'} = 145^{1.5} \times 33\sqrt{3000} = 3.15 \times 10^6 \text{ lpc}$$

La relación A_s'/A_s es $2.35/3.52 = 0.67$ en el apoyo y $0.44/2.46 = 0.18$ a la mitad del claro para un promedio de 0.42. Con esta base estímese el factor con el que se toman en cuenta los efectos del tiempo en 1.3 para la porción sostenida de la carga. Esta carga sostenida se tomará como la carga muerta (Sec. 9.10) más la mitad de la carga viva, como una carga viva de 200 lb/pie² indica que se trata de una fábrica para industria ligera o para bodega.

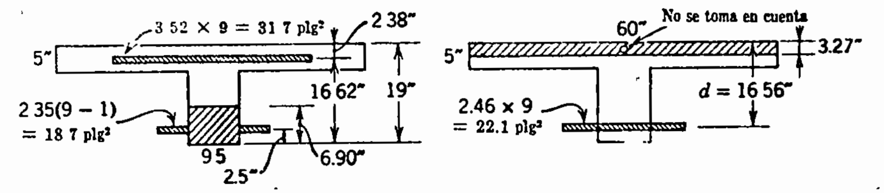


FIG. 9.16. Secciones transversales de la viga consideradas para el cálculo de la flecha

w sostenida $= 88 \times 12 + 138 + 0.5 \times 200 \times 12 = 2400$ lpc

Carga viva temporal $= 0.5 \times 200 \times 12 = 1200$ lb/pie lineal

No se dispone del diagrama de momentos para la carga permanente, aunque para la carga muerta en un claro interior (claros uniformes) sería parecida a la de una viga con los extremos empotrados. El uso del diagrama de momentos de la Fig. 9.11e sería correcto para toda la carga viva y quedaría ligeramente del lado de la seguridad para la carga permanente. Se usará este momento negativo de $0.053wL'^2$.

$$M_n = -0.053 \times 2400 \times 20^2 = -50\,800 \text{ pies-lb}$$

La flecha se calculará como $(5/384)wL'^4/EI$ para una viga simplemente apoyada, restándole el momento negativo $M_nL'^2/8EI$.

$$\begin{aligned} y_s &= (5/384)(2400 \times 20^4 \times 1728)/(3.15 \times 10^6 \times 4000) = && 0.69 \text{ plg} \\ y_n &= -50\,800 \times 20^2 \times 1728/(8 \times 3.15 \times 10^6 \times 4000) = && -0.35 \\ & && \overline{0.34} \\ \text{Adición para el efecto del tiempo } 1.3 \times 0.34 &= && 0.42 \\ \text{Carga total permanente } y &= && \overline{0.76} \text{ plg} \\ \text{C.V. añadida } y = (1200/2400)0.34 &= && 0.17 \\ \text{Flecha total} & && \overline{0.93} \text{ plg} \end{aligned}$$

Para los lugares críticos el Art. 909f sugiere como límite $L'/360 = 0.67$ plg que se coloca sobre la flecha por deformación plástica, más la flecha por carga viva, en el supuesto de que la flecha por carga ocurra antes de que se construyan los tabiques. En este caso, la flecha es un poquito grande, $0.42 + 2 \times 0.17 = 0.76$ plg. Lo que puede reducirse ligeramente usando un mejor diagrama de momentos. La flecha inmediata por carga viva de $2 \times 0.17 = 0.34$ plg queda comprendida bastante dentro del límite aconsejado de $L'/360$ del Art. 909e.

9.21 VIGAS RECTANGULARES CONTINUAS

Las diferencias de proyecto entre las vigas rectangulares continuas y las vigas T continuas son de poca importancia. El proyecto anterior

de la viga T puede también servir de modelo para el proyecto de las vigas continuas rectangulares. Aparte de la diferencia evidente, para proyectar el acero para el momento positivo para una viga rectangular, es necesario llamar la atención a los requisitos especiales para los soportes laterales, como se dan en el Art. 908. A estos miembros les falta la capacidad adicional de la viga T para resistir torsión.

9.22 CLAROS EXTREMOS Y CLAROS IRREGULARES

En los claros extremos, los momentos negativos son menores en el extremo exterior que en el interior, con los puntos de inflexión y el punto de momento positivo máximo movidos hacia el apoyo exterior. En los claros y cargas irregulares los diagramas de momentos máximos son menos simétricos que los que se usan en este capítulo.

Para detallar correctamente los claros extremos y los claros irregulares es necesario tener un mejor conocimiento de los diagramas de momentos asimétricos, pero no es necesaria ninguna teoría adicional sobre concreto reforzado. Cuando se consideran buenas las aproximaciones, el proyectista debe ser más conservador que cuando se conocen exactamente los requisitos de los momentos.

SELECCION DE REFERENCIAS

1. "Continuity in Concrete Building Frames," Portland Cement Association, Chicago, 3rd ed.
2. Phil M. Ferguson, "Analysis of Three-Dimensional Beam-and-Girder Framing," *Jour. ACI*, **22**, Sept. 1950; *Proc.*, **47**, p. 61.
3. R. H. Wood, "Studies in Composite Construction: Part I, The Composite Action of Brick Panel Walls Supported on Reinforced Concrete Beams; Part II, The Interaction of Floors and Beams in Multi-Story Buildings," National Building Studies, *Research Papers No. 13* (1952) and *22* (1955), Her Majesty's Stationery Office, London.
4. ACI Committee 317, *Reinforced Concrete Design Handbook*, ACI, Detroit, 2nd ed., 1955 (now in process of revision).
5. Raymond C. Reese, *CRSI Design Handbook*, Concrete Reinforcing Steel Institute, Chicago, 2nd ed., 1957 (now in process of revision).
6. Raymond C. Reese, "Detailed Design of Reinforced Concrete Members," pp. 39-69 in Sec. 24 of R. W. Abbett (ed.), *American Civil Engineering Practice*, Vol. III; John Wiley and Sons, New York, 1957.
7. Phil M. Ferguson, "Analysis of Beam-and-Girder Framing [illegible] Known Column Settlements," *Jour. ACI*, **24**, Oct. 1952; *Proc.*, **49**, p. 77.

PROBLEMAS

Prob. 9.1. La estructura reducida de la Fig. 9.17*b* debe usarse para el análisis del segundo piso del marco de la Fig. 9.17*a*, porque los coeficientes estándar no son aplicables. Supóngase que todas las vigas tienen $I = 25\,000$ plg^4, las columnas de 16 plg debajo del piso tienen una $I = 5450$ plg^4 y las vigas de 14 plg debajo del piso tienen una $I = 3200$ plg^4. Cada viga soporta una carga muerta de 850 lb/pie lineal y una carga viva de 2150 lb/pie lineal. Por el DRU:

(*a*) Calcúlese el momento máximo negativo en la viga *CD* en *C* y corríjase para obtener el momento de proyecto en la cara de la columna. (Nótese que la simetría con relación a *C* es equivalente a un extremo empotrado para la distribución de momentos).

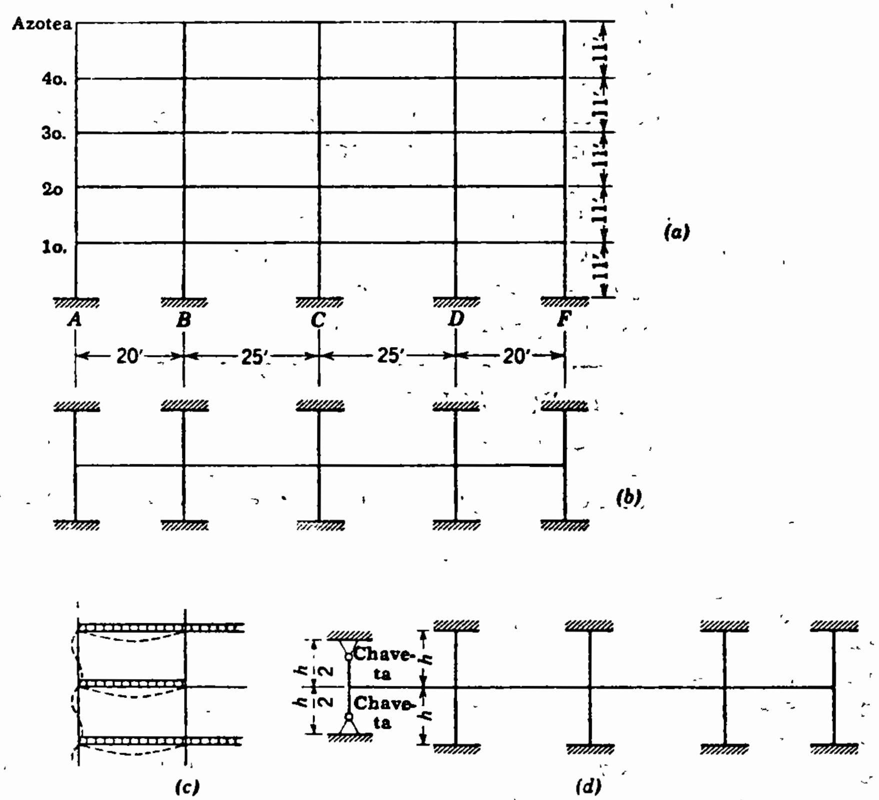


FIG. 9.17. Análisis de la estructura de un edificio. (*a*) La estructura considerada. (*b*) La estructura ordinaria reducida para el cálculo de los momentos en un solo piso. (*c*) Cargas que producen el momento máximo negativo en el extremo exterior de la viga, que también es aproximadamente el máximo en la columna exterior. (*d*) Forma mejorada de estructura reducida para los momentos máximos en el nudo exterior

VIGA T CONTINUA

b	d	$2V_c$
15	64	13600
20	55	15600
25	50	17700
30	45	19200

Sección escogida:

$\boxed{b=25\ ;\ h=55}$

Cálculo del acero en la sección rectangular, paño apoyo B

$d = 55 - 6 = 49$ (rec. supuesto)

$$\frac{M_u}{\phi f'_c b d^2} = \frac{16(10)^5}{0.90(200)(25)(49)^2}$$

$$= 0.148\ ;\quad \omega = 0.17 \quad \text{(de la gráfica)}$$

$$A_s = 0.17\,\frac{f'_c}{f_y}\,bd$$

$$= \frac{0.17(200)(25)(49)}{4000} = 10.4\ cm^2$$

$$\left(p = \frac{10.4}{25(49)} = 0.0085\ ;\ \therefore \rho_{min} < 0.0085 < 0.5\rho_b = 0.0109\right)$$

Cálculo acero sección T, centro tramo BC

8.7, IMCYC 13.5.3

$$\left.\begin{array}{l} l/4 = 800/4 = 200 \\ c.a.c. = 150 \checkmark \\ 16h_l + b_w = 16(10)+25 = 185 \end{array}\right\} \therefore \text{Usar } b = \underline{\underline{150}}\text{ cm}$$

Tanteo inicial

Suponer brazo par interno:

$$z = 0.90d = 0.90(49) = 44 \text{ cm}$$

$$A_s = \frac{M_u}{\phi f_y z} = \frac{14.7(10)^5}{0.90(4000)(44)}$$

$$A_s = \underline{\underline{9.3}}\text{ cm}^2$$

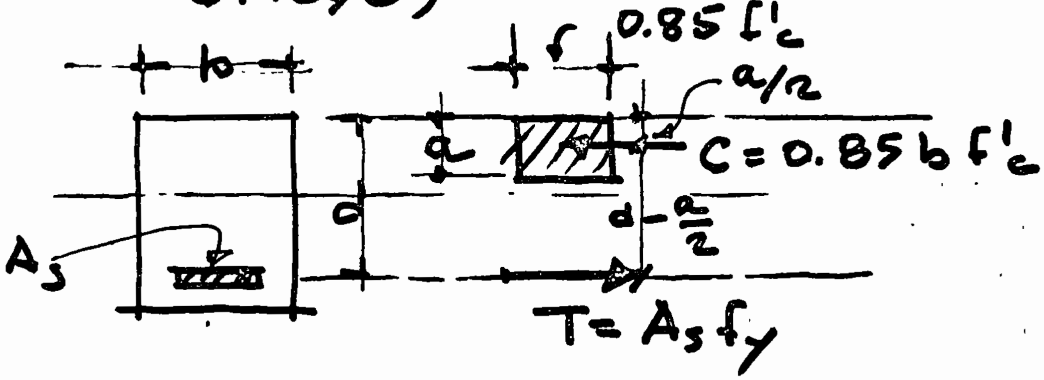
$$T = A_s f_y = 9.3(4000) = \underline{\underline{37\,200}}\text{ kg} = C$$

$$\begin{aligned} C_{patín} &= 0.85 f'_c b h_l \\ &= 0.85(200)(150)(10) \\ &= \underline{\underline{255\,000}}\text{ kg} \end{aligned}$$

$C_{patín} > C$; por lo tanto la sección debe considerarse rectangular.

Determinación área de acero como sección rectangular

(Ej. 13.1 2° procedimiento, Texto IMCyC)



De $C = T$:

$$a = \frac{A_s f_y}{0.85 b f'_c} \quad ; \quad A_s = \frac{M_u}{0.90\, f_y \left(d - \frac{a}{2}\right)}$$

Del tanteo inicial

$$a = \frac{9.3(4000)}{0.85(25)(200)} = 0.88$$

$$A_s = \frac{14.7(10)^5}{0.90(4000)(49 - 0.4)} = \underline{\underline{8.4\ cm^2}}$$

$(\doteq 9.3)$

Refuerzo adoptado

	(B)	℄
A_s	$10.4\ cm^2$	$8.4\ cm^2$
Varillas	$4\#6 = 11.5\ cm^2$	$3\#6 = 8.6\ cm^2$

Observación: En caso de efectuar redistribuciones deba tenerse en cuenta 8.6

Longitud a que deben prolongarse las varillas cortadas a partir de la sección donde teóricamente dejan de ser necesarias: 12.1.4

El mayor de los valores $\begin{cases} 12d_b = 12(1.9) = 23\text{ cm} \\ d = 49 \doteq 50\text{ cm} ✓ \end{cases}$

Revisión de requisitos de anclaje en los puntos de inflexión 12.2.3

$$l_d = \frac{0.0594\,A_b f_y}{\sqrt{f'_c}}$$

$$= \frac{0.0594(2.87)(4000)}{\sqrt{200}}$$

Valor teórico sin ϕ

$$= \underline{\underline{48\text{ cm}}} \doteq \underline{\underline{50\text{ cm}}}$$

$M_t \doteq M_u = 14.7$; $V_u = 5.7$ (gráficamente).

$$\frac{M_t}{V_u} = \frac{14.7}{5.7} > 0.48$$

Longitud de anclaje del refuerzo negativo más allá del punto de inflexión 12.3.3

El mayor de los valores
$$\left\{\begin{array}{l} d = 49 \doteq 50 \\ 12\, d_b = \\ \dfrac{l_n}{16} = \dfrac{800-30}{16} = 48 \end{array}\right.$$

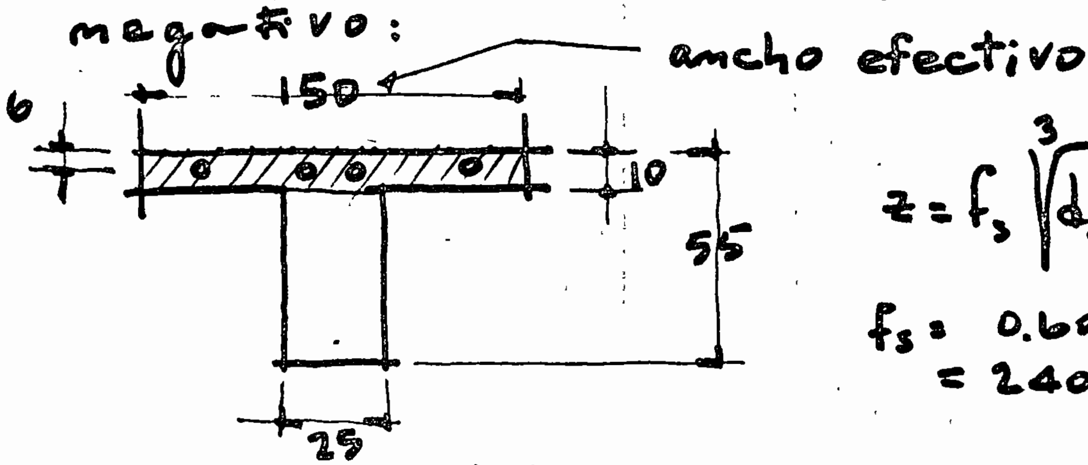
(Ver Comentario, Fig. 12-1, para detalles de cortes de varillas.)

Control de agrietamiento

10.6.3

Como en el ejemplo de la viga rectangular, para la sección de momento positivo. 10.6.4

En la sección de momento negativo:



$$z = f_s \sqrt[3]{d_c A}$$

$$f_s = 0.60 f_y = 2400$$

$$A \doteq \frac{(150)(10)}{4} = 375$$

$$z = \cancel{f_s}\ 2400 \sqrt[3]{6(375)}$$

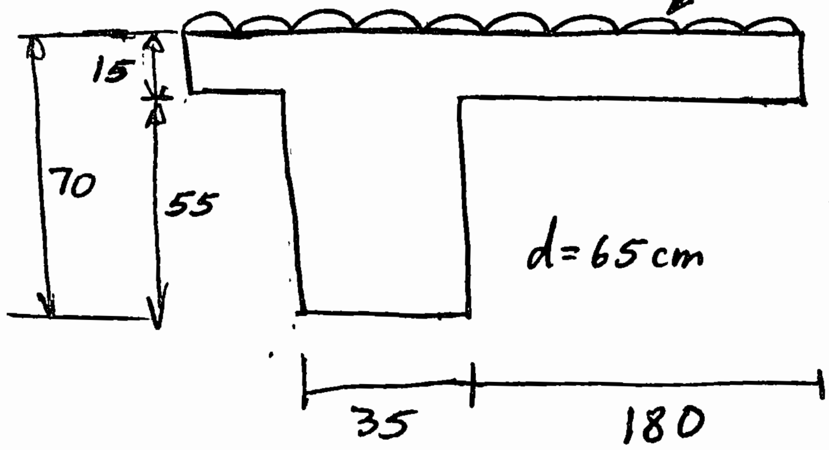
$$= \underline{31400} \text{ Kg/cm} \doteq 31250 \text{ Kg/cm}$$

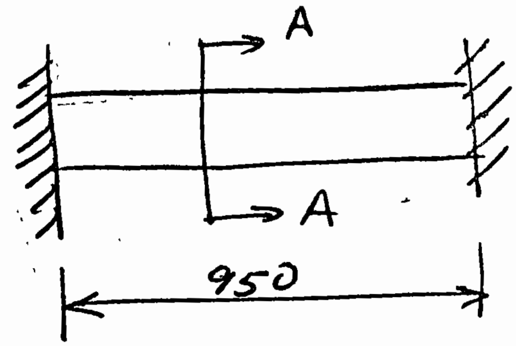
Junio 8, 1972

TORSION

Problema propuesto

$w_u = 1.5 \text{ ton/m}^2$



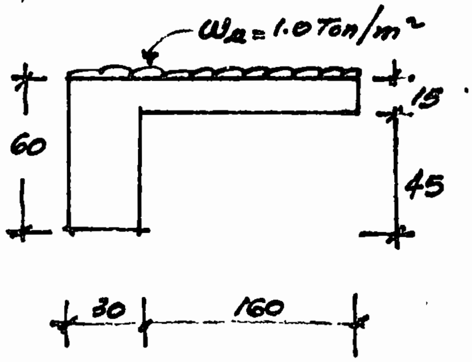


Sección A-A

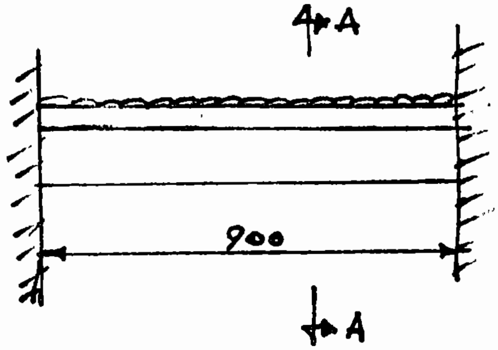
Concreto, $f'_c = 200 \text{ kg/cm}^2$

Acero, $f_y = 4\,200. \text{ kg/cm}^2$

Datos



Sección A-A



Nota: Acotaciones en cm.

CARGAS

$W_u = 1.0$ Ton/m² (No incluye peso propio)

Peso volumétrico del Concreto = 2.4 Ton/m³

Factor de Carga para Peso Propio = 1.2

Peso Propio Losa = $1.2 \times 0.15 \times 2.4 = 0.4$ Ton/m²

Peso Propio Viga = $1.2 \times 0.30 \times 0.45 \times 2.40 = 0.4$ Ton/m

Carga total por m² de losa = $1.0 + 0.4 = 1.4$ Ton/m²

Carga total lineal sobre la viga = $1.4 \times 1.9 + 0.4 = 3.1$ Ton/m

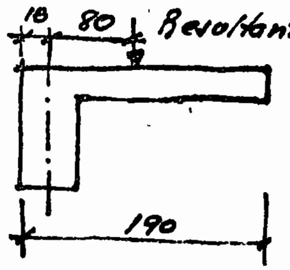
MATERIALES

Concreto, $f'_c = 200$ kg/cm²

Acero, $f_y = 4200$ kg/cm²

Cálculo de Acciones Internas

Momento Torsionante



Momento Torsionante en paño apoyo =
$= 1.4 \times 1.9 \times 0.80 \times 4.50 = 9.6$ ton-m.

Momento Torsionante sección crítica =
$= 1.4 \times 1.9 \times 0.80 \times 3.95 = 8.4$ ton-m.

Momento Flexionante

En los apoyos: $\frac{wl^2}{12} = \frac{3.1 \times 9^2}{12} = 21.0$ Ton-m.

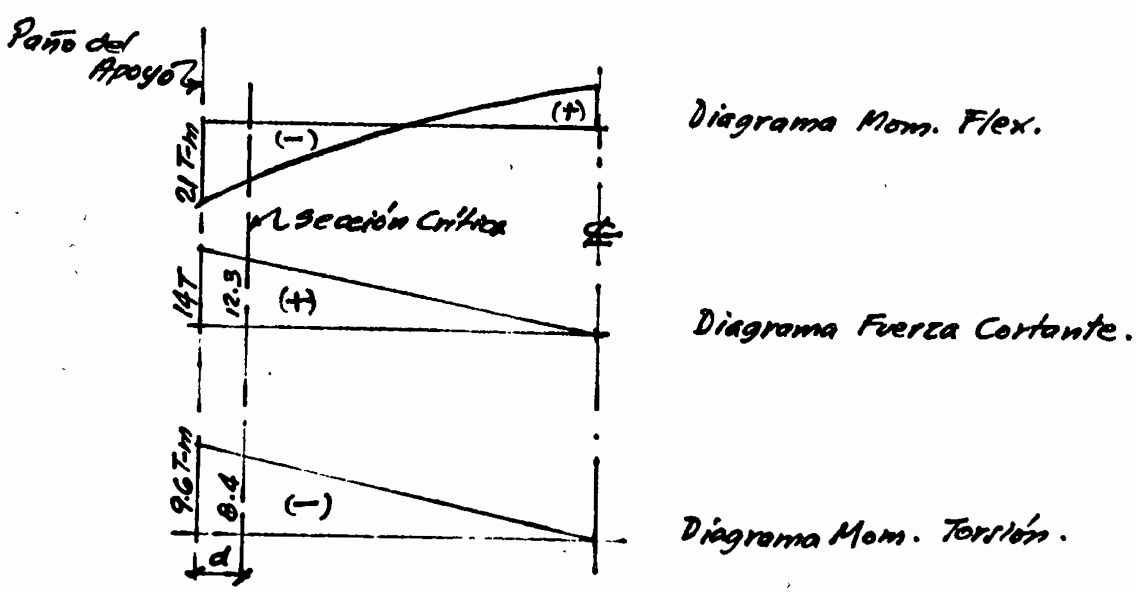
En el centro del claro: $\frac{wl^2}{24} = \frac{3.1 \times 9^2}{24} = 10.5$ Ton-m.

Fuerza Cortante

En los apoyos: $\frac{wl}{2} = \frac{3.1 \times 9}{2} = 14.0$ Ton.

En la sección crítica: $14.0 - wd = 14.0 - 3.1 \times 0.55 = 12.3$ Ton.

Diagramas de Acciones Internas



Calculo del Refuerzo

Refuerzo Transversal

a) Por fuerza cortante

$V_u = 12.3$ Ton $\qquad d = 0.55$ m.

$V_c = v_c\,bd = 0.5\sqrt{f'_c}\,bd = 0.5\sqrt{200} \times 30 \times 55 = 11700$ kg

$V_c = 11.7$ Ton

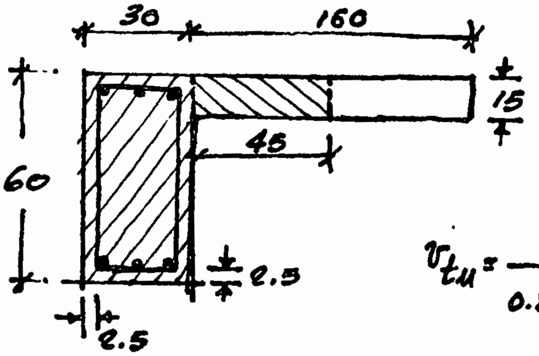
$V'_u = 12.3 - 11.7 = 0.6$ Ton

$$\frac{A_v}{s} = \frac{V'_u}{f_y\,d} = \frac{600}{4200 \times 55} = 0.0026\ \frac{cm^2}{cm}\ (\text{dos ramas})$$

$$v_u = \frac{12\,300}{30 \times 55} = 7.45\ kg/cm^2$$

b) Por Torsión

$T_u = 8.4$ Ton-m.



$$v_{tu} = \frac{3\,T_u}{\phi \sum x^2 y} \qquad \text{Ec. 11.16}$$

$$v_{tu} = \frac{3 \times 840000}{0.85\,(30^2 \times 60 + 15^2 \times 45)} = 46.2\ kg/cm^2$$

$$v_{tc} = \frac{0.64\sqrt{f'_c}}{\sqrt{1 + \left(1.2\,\frac{v_u}{v_{tu}}\right)^2}} = \frac{9.02}{\sqrt{1 + \left(1.2\,\frac{7.45}{46.2}\right)^2}} = 8.80\ kg/cm^2 \qquad \text{Ec. 11.17}$$

$$A_t = \frac{(v_{tu} - v_{tc})\,s \sum x^2 y}{3\,\alpha_t\,x_1 y_1\,(f_y)} \qquad y_1 = 55\ cm, \quad x_1 = 25\ cm \qquad \text{Ec. 11.19}$$

$$\alpha_t = [0.66 + 0.33\,(y_1/x_1)] = .66 + .725 = 1.385 < 1.50$$

$$\frac{A_t}{s} = \frac{(46.2 - 8.80)\,(900 \times 60 + 225 \times 45)}{3 \times 1.385 \times 25 \times 55 \times 4200} = \frac{2.4 \times 10^6}{24 \times 10^6} = 0.1\ \frac{cm^2}{cm}\ (\text{una rama})$$

c) Total

$$\frac{1}{2}\,\frac{A_v}{s} + \frac{A_t}{s} = 0.0013 + 0.1 = 0.1013\ \frac{cm^2}{cm}\ (\text{una rama})$$

Separación de estribos $= \frac{A\,total}{0.1013}$

Con estribos #3 :

$S = \frac{0.71}{0.1013} = 7$ cm.

Con estribos #4 :

$S = \frac{1.27}{0.1013} = 12.5$ cm.

Refuerzo longitudinal

a) Por torsión, ecuación (11-20) :

$$A_L = 2A_t \frac{x_1+y_1}{s} = 0.2\,(25+55) = 16 \text{ cm}^2$$

Por ecuación (11-21),

$$A_L = \left[\frac{28.1\,x\,s}{fy}\left(\frac{v_{tu}}{v_{tu}+v_u}\right) - 2A_t\right]\left(\frac{x_1+y_1}{s}\right)$$

Siendo :

$$2A_t = \frac{3.52\,bw\,s}{fy} = \frac{3.52 \times 30 \times 12.5}{4200} = .32$$

$$A_L = \left[\frac{28.1 \times 30 \times 12.5}{4200}\left(\frac{46.2}{46.2-7.45}\right) - .32\right]\left(\frac{55+25}{12.5}\right) = 17 \text{ cm}^2$$

b) Por flexión

$$As = \frac{Mu}{fy\,z}$$

Suponiendo $z = 0.9d$

Refuerzo Negativo $= \frac{2100000}{4200 \times 0.9 \times 55} = 10.1 \text{ cm}^2$

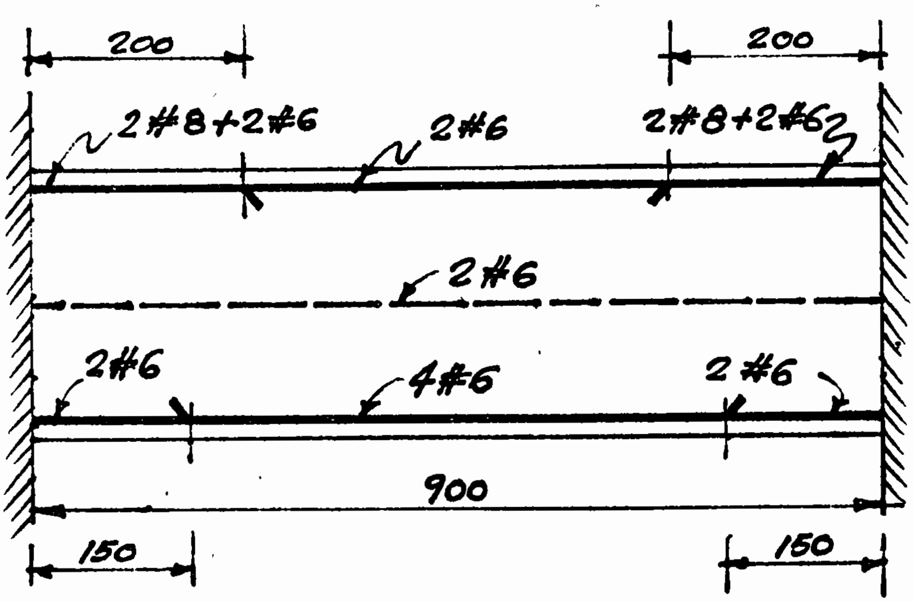
Refuerzo Positivo $= \frac{1050000}{4200 \times 0.9 \times 55} = 5.1 \text{ cm}^2$

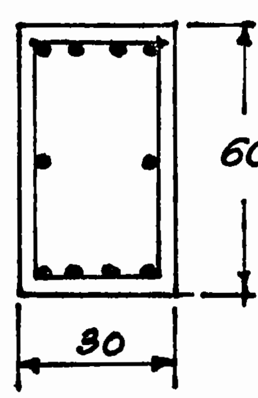
El refuerzo longitudinal de torsión, $A_L = 17\ cm^2$, serán 6 vars. #6, $As = 2.84 \times 6 = 17.04\ cm^2$; colocadas en la siguiente forma:

2 Vars. #6 en el lecho superior
2 Vars. #6 a medio peralte (según 11.8.5, el espaciamiento de varillas longitudinales no deberá exceder de 30 cm.)
2 Vars. #6 en el lecho inferior

DETALLE DEL REFUERZO

	Torsión	Flexión	Total	Vars.	Area Total
Lecho sup.	5.66	10.1	15.76	2#8+2#6	15.8
Medio peralte	5.66		5.66	2#6	5.7
Lecho inf.	5.66	5.1	10.76	4#6	11.4

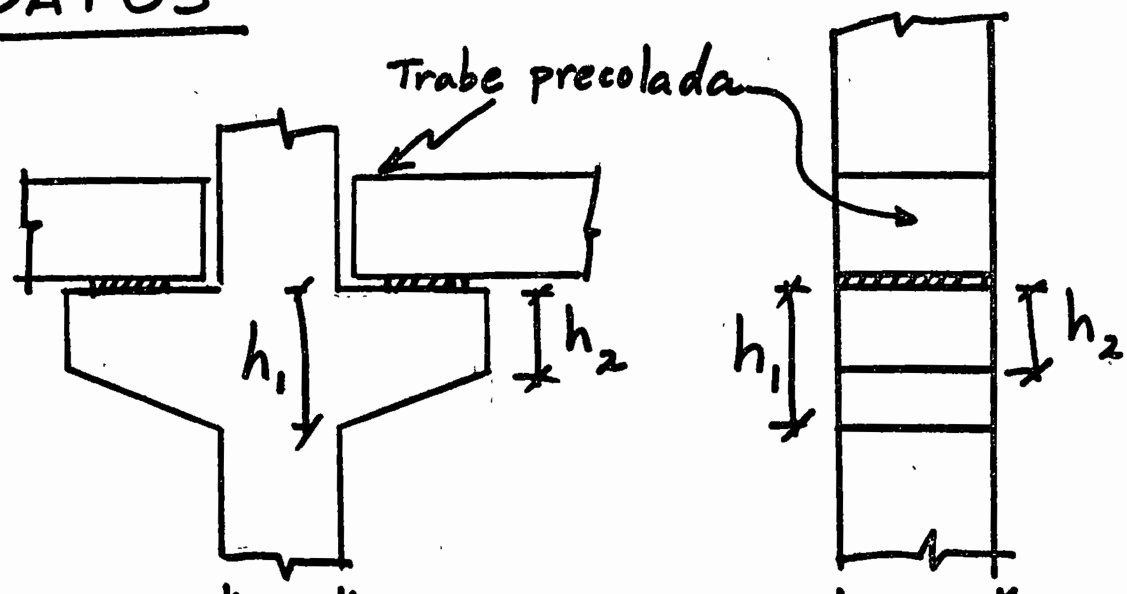




Estribos #4 @ 12.5 cm.

DISEÑO DE UNA MENSULA, ACI-71

DATOS



Longitud trabe = 15 m

CARGAS DE SERVICIO

Carga muerta en trabe = 1400 kg/m
Carga viva en trabe = 2200 kg/m
Carga total = 3 600 kg/m

MATERIALES

Concreto, $f'_c = 350$ kg/cm^2
Acero, $f_y = 2\,800$ kg/cm^2

CALCULO DE ACCIONES INTERNAS

R_1: Reacción por carga permanente

$$R_1 = \frac{w_1 l}{2} = \frac{1.4 \times 15}{2} = 10.5 \text{ ton}$$

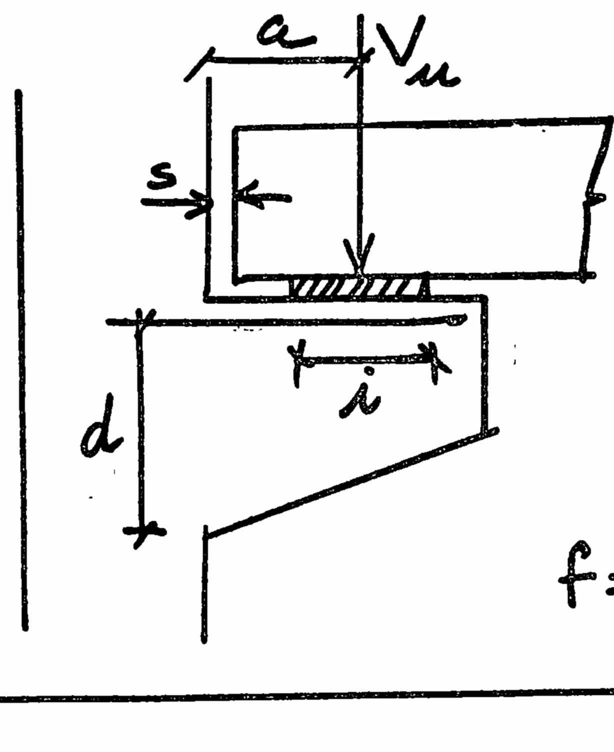
R_2: Reacción por carga viva

$$R_2 = \frac{w_2 l}{2} = \frac{2.2 \times 15}{2} = 16.5 \text{ ton}$$

Carga de diseño,

$$V_u = 1.4 R_1 + 1.7 R_2$$

$$V_u = 1.4 \times 10.5 + 1.7 \times 16.5 = 42.7$$



DETALLES GEOMETRICOS

$$i = \frac{V_u}{b\,f}$$

f = esf. de aplastamiento

$$f = 0.85\,\phi\,f'_c$$

$$f = 0.85 \times 0.70 \times 350 = 208 \text{ kg/cm}^2$$

$$i = \frac{42\,700}{35 \times 208} = 5.9 \approx 6.0$$

$$\underline{\underline{i = 6 \text{ cm}}}$$

Cálculo del claro de cortante "a":

$$a \approx 2s + \frac{1}{2} i = 2 \times 3 + 3 = 9 \text{ cm}$$

$$\underline{\underline{a = 9 \text{ cm}}}$$

Se recomienda $s = 2 \text{ a } 3$ cm

Cálculo del peralte efectivo "d":

Recomendación $\frac{a}{d} = \begin{cases} 0.15 \\ \text{a} \\ 0.40 \end{cases}$

Se selecciona en este caso, $\frac{a}{d} = 0.30$

$$d = \frac{a}{0.30} = \frac{9}{0.3} = \underline{\underline{30 \text{ cm}}}$$

CALCULO DEL REFUERZO

$$v_u = \frac{V_u}{b\,d} = \frac{42700}{35 \times 30} = 40.5\ kg/cm^2$$

$$v_u = 1.7\left[1-0.5\frac{a}{d}\right](1+64\rho_v)\sqrt{f'_c} \quad (11\text{-}29)$$

$$\rho_v = \frac{v_u - 1.7\sqrt{f'_c}\left[1-0.5\frac{a}{d}\right]}{110\sqrt{f'_c}\left[1-0.5\frac{a}{d}\right]}$$

$$\rho_v = \frac{40.5 - 1.7\sqrt{350}\left[1-0.5 \times 0.30\right]}{110\sqrt{350}\left[1-0.5 \times 0.30\right]}$$

$$\underline{\underline{\rho_v = 0.0075}}$$

$$(\rho_v)_{max} = 0.20\frac{f'_c}{f_y} = 0.025$$

$$\rho_v < (\rho_v)_{max} \qquad \underline{OK}$$

$$p_v = \frac{A_s + A_h}{bd}$$

Se considera $A_h = 0.5\,A_s$

$$p_v = \frac{A_s + 0.5 A_s}{bd} \quad , \therefore \quad A_s = \frac{p_v\, bd}{1.5}$$

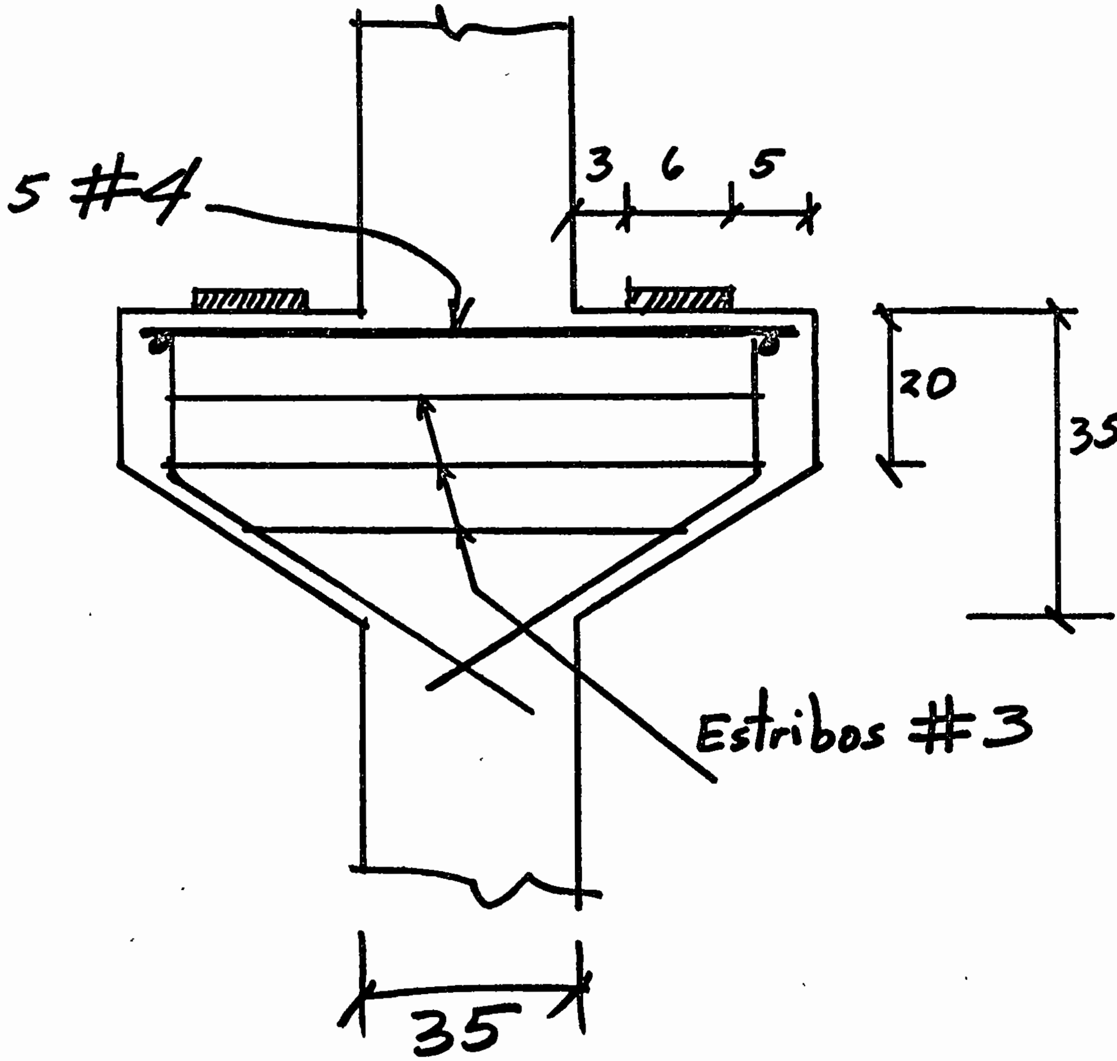
$$A_s = \frac{0.0075 \times 35 \times 30}{1.5} = 5.3\,cm^2$$

$$\underline{\underline{A_s = 5 \text{ Vars. } \#4}}$$

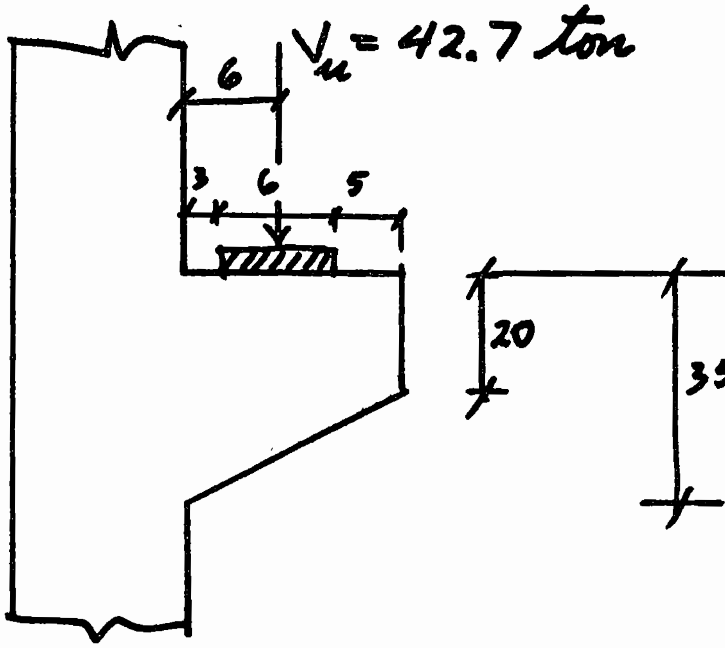
$$A_h = 0.5 \times 5.3 = 2.7\,cm^2$$

$$\underline{\underline{A_h = 2 \text{ Estr. } \#3}}$$

A_h debe repartirse en $\frac{2}{3}$ del peralte efectivo


5 #4
3
6
5
20
35
Estribos #3
35

DISEÑO DE UNA MENSULA USANDO CORTANTE POR FRICCION, ACI-71



$$A_{vf} = \frac{V_u}{\phi f_y \mu} = \frac{42\,700}{0.85 \times 2800 \times 1.4} = 12.8\ cm^2$$

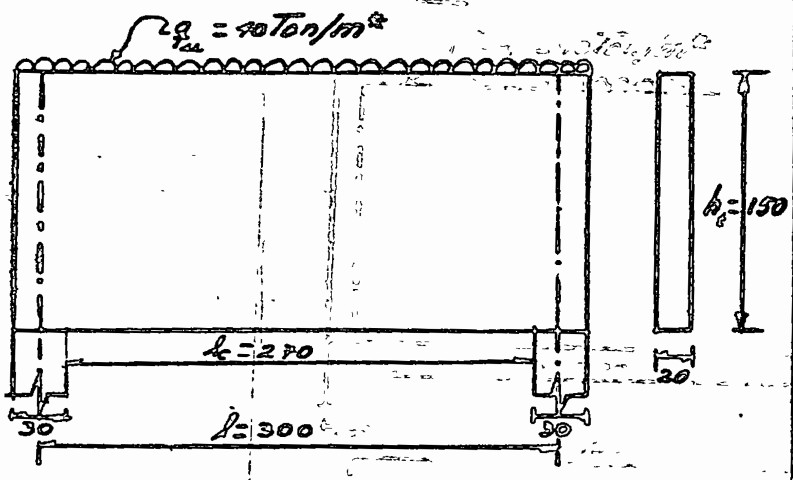
$$\underline{A_{vf} = 12.8\ cm^2}$$

$$A_s = \frac{M_u}{f_y z} = \frac{42\,700 \times 6}{2800 \times 0.9 \times 30} = 3.4\ cm^2$$

$$\underline{A_s = 3.4\ cm^2}$$

EJEMPLO 10.3. DISEÑO DE UNA VIGA DE GRAN PERALTE.	CALCULO: J.S.L.V.	1/4
	REVISO: F.G.V.	
	FECHA: FEB. 1971	

DATOS



*Carga de diseño (q_u) = carga de servicio x factor de carga.

MATERIALES

Concreto. $f'_c = 250\ Kg/cm^2$

Acero. $f_y = 4000\ Kg/cm^2$

DISEÑO POR FLEXION

$$\frac{\ell}{h_t} = \frac{300}{150} = 2 < 3$$

$z = 0.60 h_t$, $\quad z = 0.60 \times 150 = 90\ cm$

Area de acero necesaria

$$A_s = \frac{M_u}{f_y\, z}$$

$$M_u = \frac{q_u\,\ell^2}{8}\ ;\quad M_u = \frac{40 \times 3.0^2}{8} = 45\ Ton\text{-}m$$

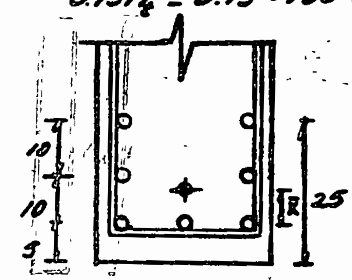
$\therefore\ M_u = 45\ Ton\text{-}m$

EJEMPLO 10.3. DISEÑO DE UNA VIGA DE GRAN PERALTE.	CALCULO: J.S.L.V.	2/4
	REVISO: F.G.V.	
	FECHA: FEB. 1971	

$$A_s = \frac{45 \times 10^5}{4000 \times 90} = 12.5\ cm^2$$

Usar 7#5 ($A_s = 13.93\ cm^2$)

$0.15 h_t = 0.15 \times 150 = 25\ cm$



Centroide del refuerzo

$$\bar{x} = \frac{2 \times 10 + 2 \times 20}{7} = 8.6\ cm$$

$r = 5 + 8.6 = 13.6\ cm$

$d_{real} = 150 - 13.6 = 136.4\ cm$

Revisión de esfuerzos de aplastamiento en el apoyo.

$f_{apl\,perm} = 0.50 f'_c = 0.50 \times 250 = 125\ Kg/cm^2$

$$V_u = \frac{w\ell}{2} = \frac{40 \times 3.0}{2} = 60.0\ Ton$$

$V_u = 60{,}000\ Kg$

$$f_{apl} = \frac{60000}{30 \times 20} = 100\ Kg/cm^2$$

$\therefore\ f_{apl} < f_{apl\,perm}$

Revisión por fuerza cortante.

Sección crítica: $z = 0.15 \ell_c = 0.15 \times 270 = 40\ cm$

Acciones internas en la sección crítica:

$$M_u = 60 \times 0.55 - \frac{40 \times 0.65^2}{2} = 33.0 - 6.1 = 26.9\ Ton\text{-}m$$

$V_u = 60.0 - 40 \times 0.55 = 60.0 - 22 = 38\ Ton$

Porcentaje de refuerzo por flexión:

$$p = \frac{A_s}{bd}\ ;\quad p = \frac{13.93}{30 \times 136} = 0.00342$$

$p = 0.00342$

EJEMPLO 10.3. DISEÑO DE UNA VIGA DE GRAN PERALTE.

CALCULO: J.S.L.V.
REVISO: F.G.V.
FECHA: FEB. 1971

3/4

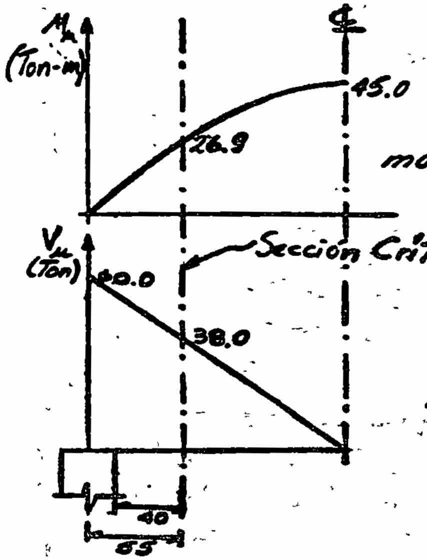


Diagrama de momentos flexionantes.

Diagrama de fuerza cortante.

La resistencia a fuerza cortante del concreto, es:

$$V_c = bd\left(3.5-2.5\frac{M_u}{V_u d}\right)\left(0.5\sqrt{f'_c}+180\,p\,\frac{V_u d}{M_u}\right) \qquad \text{Ec. 10.14}$$

$$1 \le \left(3.5-2.5\frac{M_u}{V_u d}\right) \le 3.5$$

$$3.5-2.5\frac{26.9}{38\times 1.36} = 3.5-1.3 = 2.2$$

$$1 < 2.2 < 3.5$$

$$V_c = 20\times 136\left(3.5-2.5\frac{26.9}{38\times 136}\right)\left(0.5\sqrt{250}+180\times 0.00372\,\frac{38\times 136}{26.9}\right)$$

$$V_c = 20\times 136 \times 2.2 \times 9.1 = 54.4 \text{ Ton}$$

$$(V_c = 54.4 \text{ Ton}) > (V_u = 38.0 \text{ Ton})$$

∴ No requiere refuerzo por Cortante.

REFUERZO MINIMO EN EL ALMA

Refuerzo vertical.

$A_v \ge 0.0015\,bs$; $s \le \frac{d}{5}$ ó 45cm

EJEMPLO 10.3. DISEÑO DE UNA VIGA DE GRAN PERALTE.

CALCULO: J.S.L.V.
REVISO: F.G.V.
FECHA: FEB. 1971

4/4

Usamos varillas #2.5 (2 ramas): $A_v = 0.98\ cm^2$

$s = \frac{A_v}{0.0015b}$; $s = \frac{0.98}{0.0015\times 20} = 32.6cm$

$s_{max} = \frac{d}{5} = \frac{136}{5} = 27cm < 32.6cm < 45cm$ ∴ $s = 27cm$

Refuerzo horizontal.

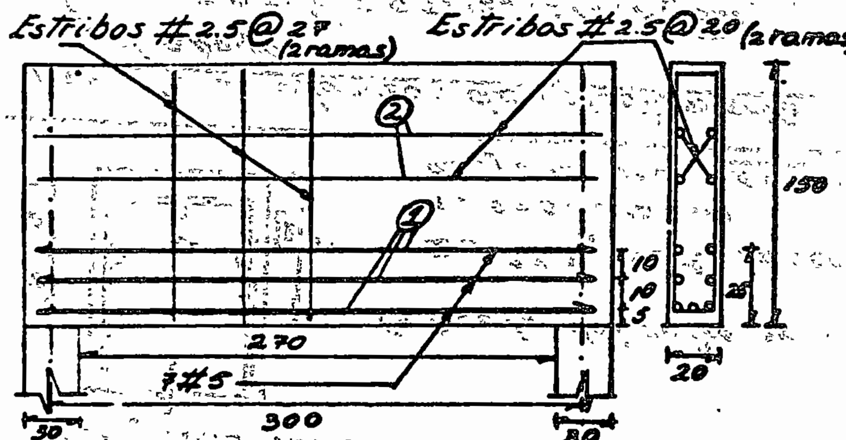
$A_{vh} \ge 0.0025\,bs_h$; $s_h \le \frac{d}{3}$ ó 45cm

Usamos varillas #2.5 (2 ramas): $A_{vh} = 0.98\ cm^2$

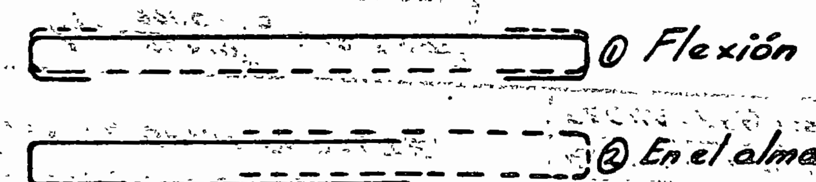
$s_h = \frac{A_{vh}}{0.0025b}$; $s_h = \frac{0.98}{0.0025\times 20} = 19.6cm$

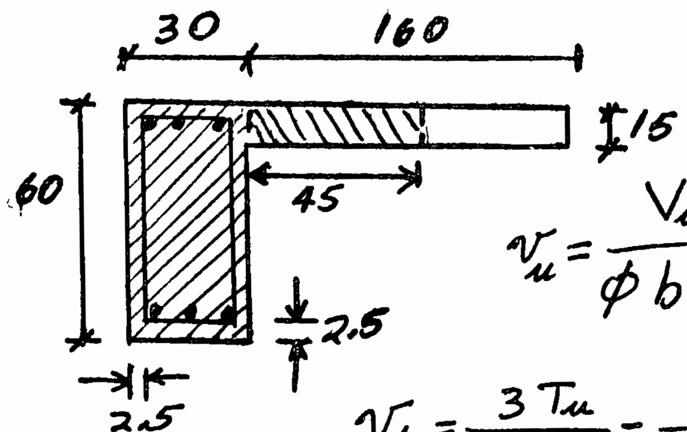
$s_{h\,max} = \frac{d}{3} = \frac{136}{3} = 46cm > 45cm > 19.6cm$ ∴ $s_h = 20.0cm$

CROQUIS DE ARMADO



DETALLE Armado de barras horizontales





$$v_u = \frac{V_u}{\phi b d} = \frac{12\,300}{0.85 \times 30 \times 55} = 8.8 \ kg/cm^2$$ Ec. 11.3

$$v_{tu} = \frac{3 T_u}{\phi \Sigma x^2 y} = \frac{3 \times 840\,000}{0.85(30^2 \times 60 + 15^2 \times 45)} = 46.2 \ kg/cm^2$$ Ec. 11.16

Cálculo del refuerzo

Refuerzo transversal

a) Por fuerza cortante

$V_u = 12.3$ ton

$V_c = \phi v_c b d$

$$v_c = \frac{0.5 \sqrt{f'_c}}{\sqrt{1 + \left(\frac{v_{tu}}{1.2 v_u}\right)^2}} = \frac{0.5\sqrt{200}}{\sqrt{1 + \left(\frac{46.2}{1.2 \times 8.8}\right)^2}} = 1.6 \ kg/cm^2$$ Ec. 11.9

$V_c = 0.85 \times 1.6 \times 30 \times 55 = 2250$ kg $= 2.25$ ton

$V'_u = V_u - V_c = 12.30 - 2.25 = 10.05$ ton

$$\frac{A_v}{s} = \frac{V'_u}{f_y d} = \frac{10\,050}{4200 \times 55} = 0.0044 \ \frac{cm^2}{cm} \text{ (dos ramas)}$$

b) Por torsión

$$v_{tc} = \frac{0.64\sqrt{f'_c}}{\sqrt{1+\left(1.2\frac{v_u}{v_{tu}}\right)^2}} = \frac{9.02}{\sqrt{1+\left(1.2\,\frac{8.8}{46.2}\right)^2}}$$ Ec. 11.17

$$v_{tc} = 8.8 \ kg/cm^2$$

$$A_t = \frac{(v_{tu}-v_{tc})\, s \sum x^2 y}{3\alpha_t\, x_1\, y_1\, (f_y)} \qquad y_1 = 55\,cm \qquad x_1 = 25\,cm$$ Ec. 11.19

$$\alpha_t = \left[0.66 + 0.33\,(y_1/x_1)\right] = 0.66 + 0.725 = 1.385 < 1.50$$

$$\frac{A_t}{s} = \frac{(46.2-8.8)\,(900\times 60 + 225\times 45)}{3\times 1.385\times 25\times 55\times 4200} = 0.1\ \frac{cm^2}{cm} \ (\text{una rama})$$

c) Total

$$\frac{1}{2}\,\frac{A_v}{s} + \frac{A_t}{s} = 0.044 + 0.1 = 0{,}144$$
(una rama)

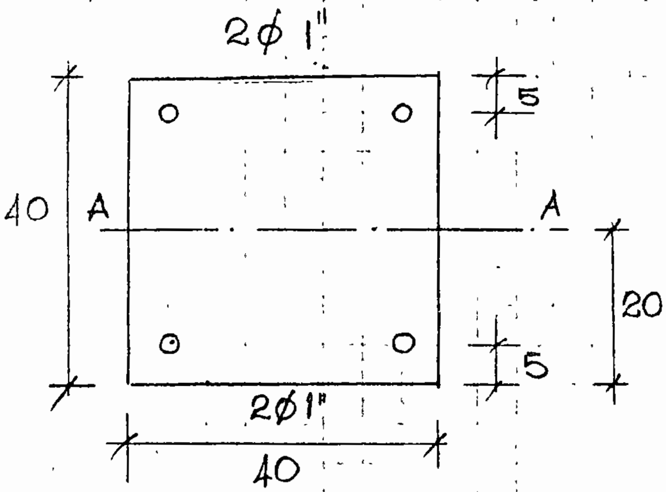
$$\text{Separación de estribos} = \frac{A_{total}}{0.144}$$

Con estribos No. 4

$$s = \frac{1.27}{0.144} \doteq \underline{\underline{9\ cm}}$$

DIAGRAMA DE INTERACCION PARA MIEMBROS SUJETOS A FLEXION Y CARGA AXIAL.

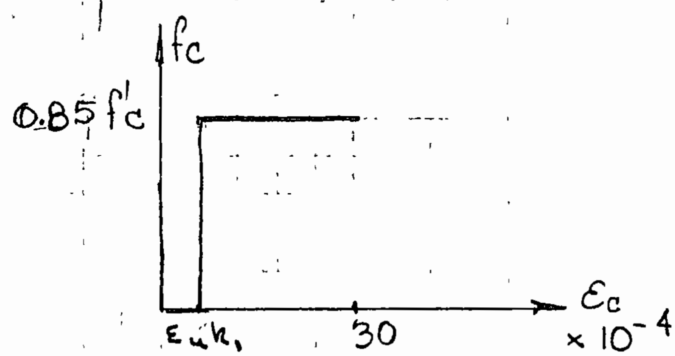
Ejemplo.- Obtener el diagrama de interacción de la sección mostrada, con flexión alrededor del eje A-A.



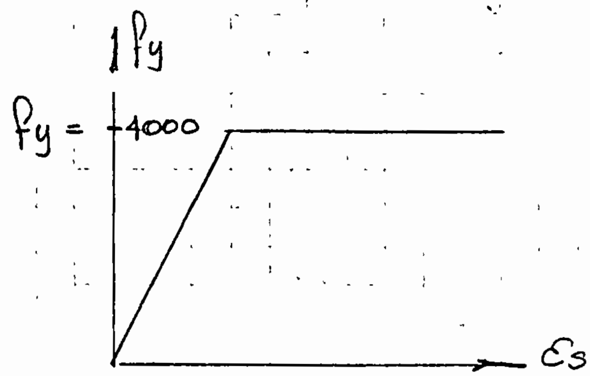
Datos -

$f'_c = 200$ Kg/cm²

$f_y = 4000$ "



Curva esfuerzo deformación del concreto.



Curva esfuerzo-deformación del acero de refuerzo.

1) Cálculo de la carga axial máxima de compresión

$$P_u = (b)(t)(0.85 f'_c) + A_s f_y$$

$$= (40)(40)(0.85\times 200) + (5.07)(4)(4000)$$

$$= 272\,500 + 81{,}120 = 353{,}620 \text{ Kg.}$$

2) Cálculo de la carga axial máxima de tensión.

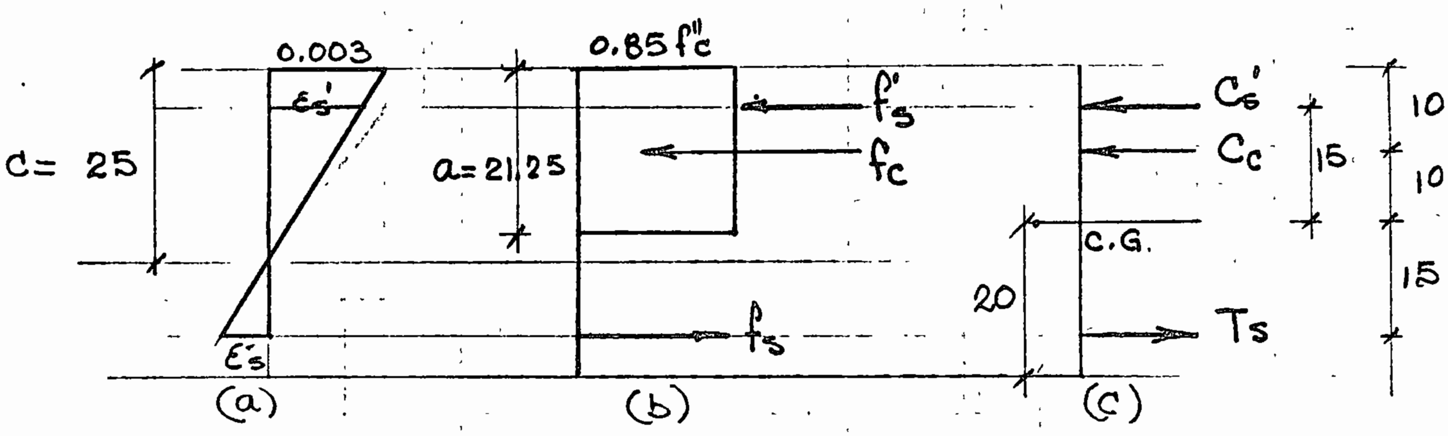
Sólo trabaja el acero de refuerzo.

$$P_u = (A_s)(f_y) = 4(5.07)(4000) = 81{,}120$$

3) Resistencia a carga axial y momento flexionante con una excentricidad cualquiera.

Supongo $c = 25$ cm.

$\therefore$ $a = 0.85\,c = (0.85)(25) = 21.25$ cm.



De la fig. (a)

$$\frac{0.003}{25} = \frac{\varepsilon_s'}{20} \Rightarrow \varepsilon_s' = 0.0024 > 0.002 \quad y \quad f_s' = 4000 \text{ K/c}^2$$

$$\frac{0.003+\varepsilon_s}{35} = \frac{\varepsilon_s}{10} \Rightarrow \varepsilon_s = 0.0012 < 0.002 \qquad f_s = 2{,}400 \text{ Kg/c}^2$$

Las fuerzas actuantes son:

$$T_s = A_s f_s = 2(5.07)(2{,}400) = 24\,400 \text{ Kg.}$$

$$C_s' = -A_s' f_s' = -2(5.07)(4000) = -40\,560 \text{ Kg.}$$

$$C_c = -(0.85)(200)(21.25)(40) = -153\,300 \text{ Kg}$$

$$\Rightarrow P_u = T_s + C_s' + C_c = -169{,}460 \text{ Kg.} \quad (\text{Comp.})$$

Tomando momentos respecto al C.G.

$$(T_s)(15) = (24{,}400)(15) = 365{,}000 \text{ Kg-cm} = M_s$$

$$(C_s')(15) = -(40{,}560)(15) = 610{,}000 \quad '' \quad = M'_{cs'}$$

$$(C_c)(10) = -(153300)(10) = 1{,}533{,}000 \quad '' \quad = M_c$$

$$\Rightarrow \quad M_u = M_s + M_{cs'} + M_c = 2{,}508{,}000 \text{ Kg-cm} = M_u$$

$$e = \frac{M_u}{P_u} = \frac{2{,}508{,}000}{169\,460} = 14.8 \text{ cm.}$$

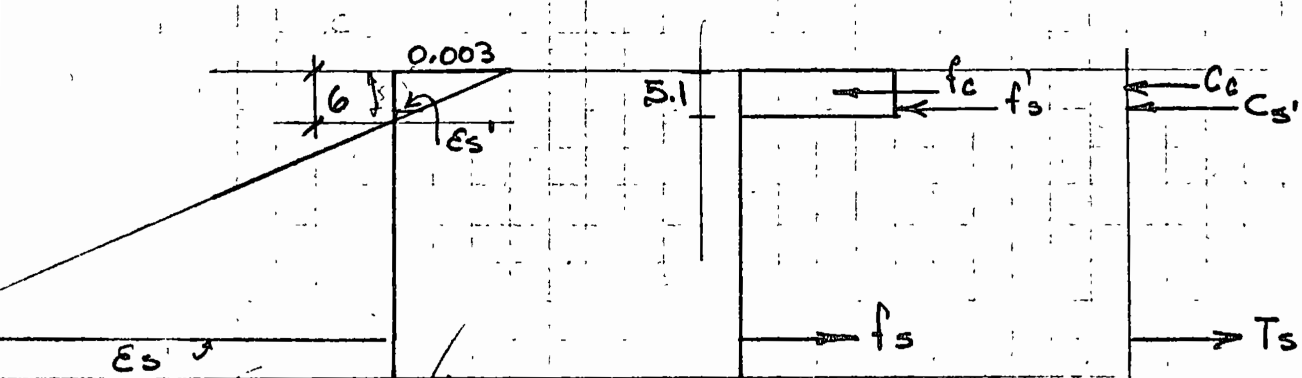
4) Resistencia a carga axial y momento flexionante con una excentricidad dada.

Supongo $e = \infty \quad \therefore \quad P = 0$

Es un caso de flexión pura.

Para $c = 6$ cm

$a = (0.85)(6) = 5.1$ cm.



$$\frac{0.003}{6} = \frac{\varepsilon_s'}{1} \Rightarrow \varepsilon_s' = 0.0005 < 0.002 \Rightarrow f_s' = 1{,}000 \text{ Kg/cm}^2$$

$$\frac{0.003 + \varepsilon_s}{35} = \frac{\varepsilon_s}{29} \qquad \varepsilon_s = 0.0025 > 0.0012 \Rightarrow f_s = 4000 \text{ Kg/cm}^2$$

Las fuerzas actuantes son:

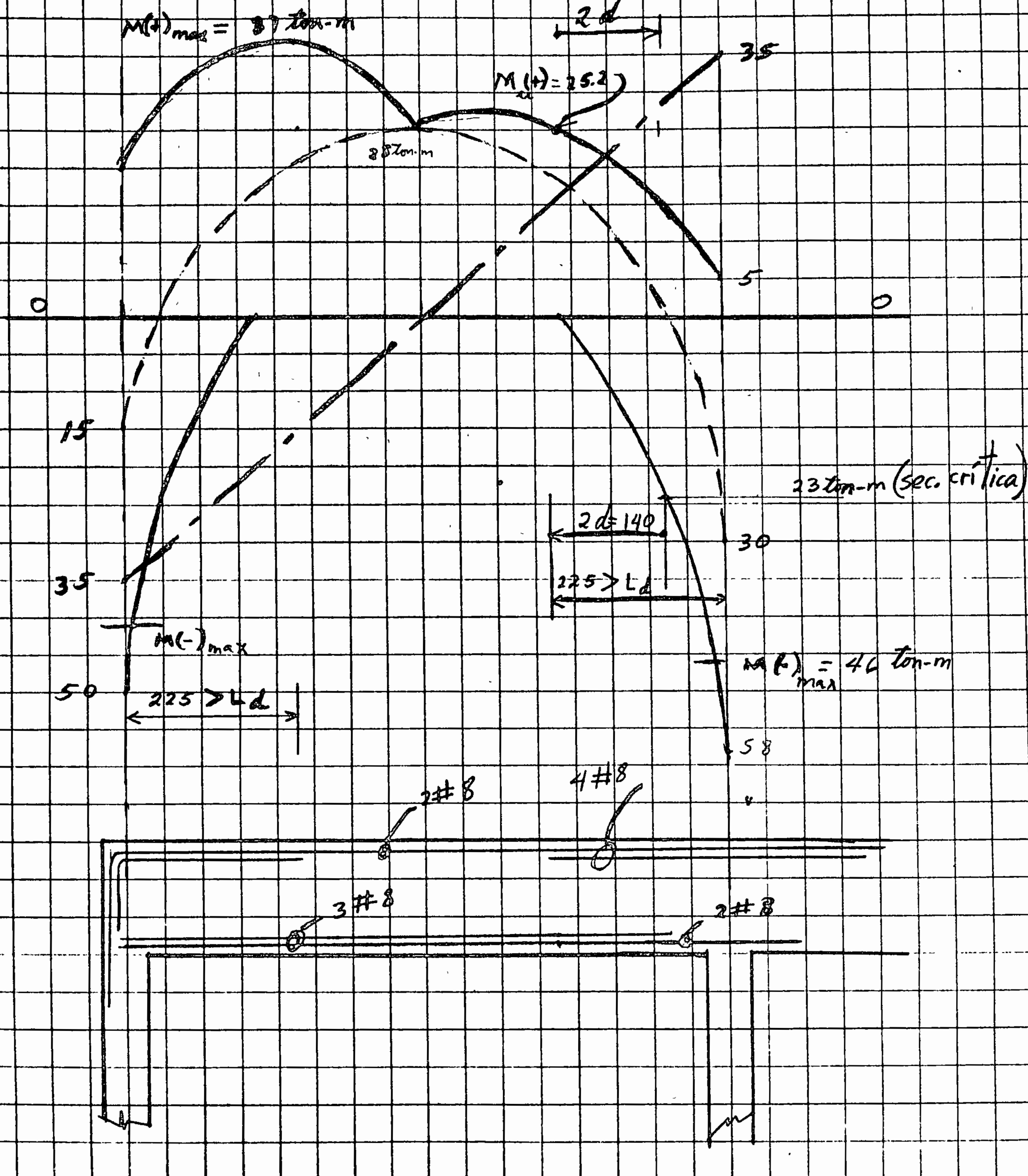
$$T_s = A_s f_s = 2(5.07)(4000) = 40560$$

$$C_s' = -A_s' f_s' = 2(5.07)(1000) = -10{,}140$$

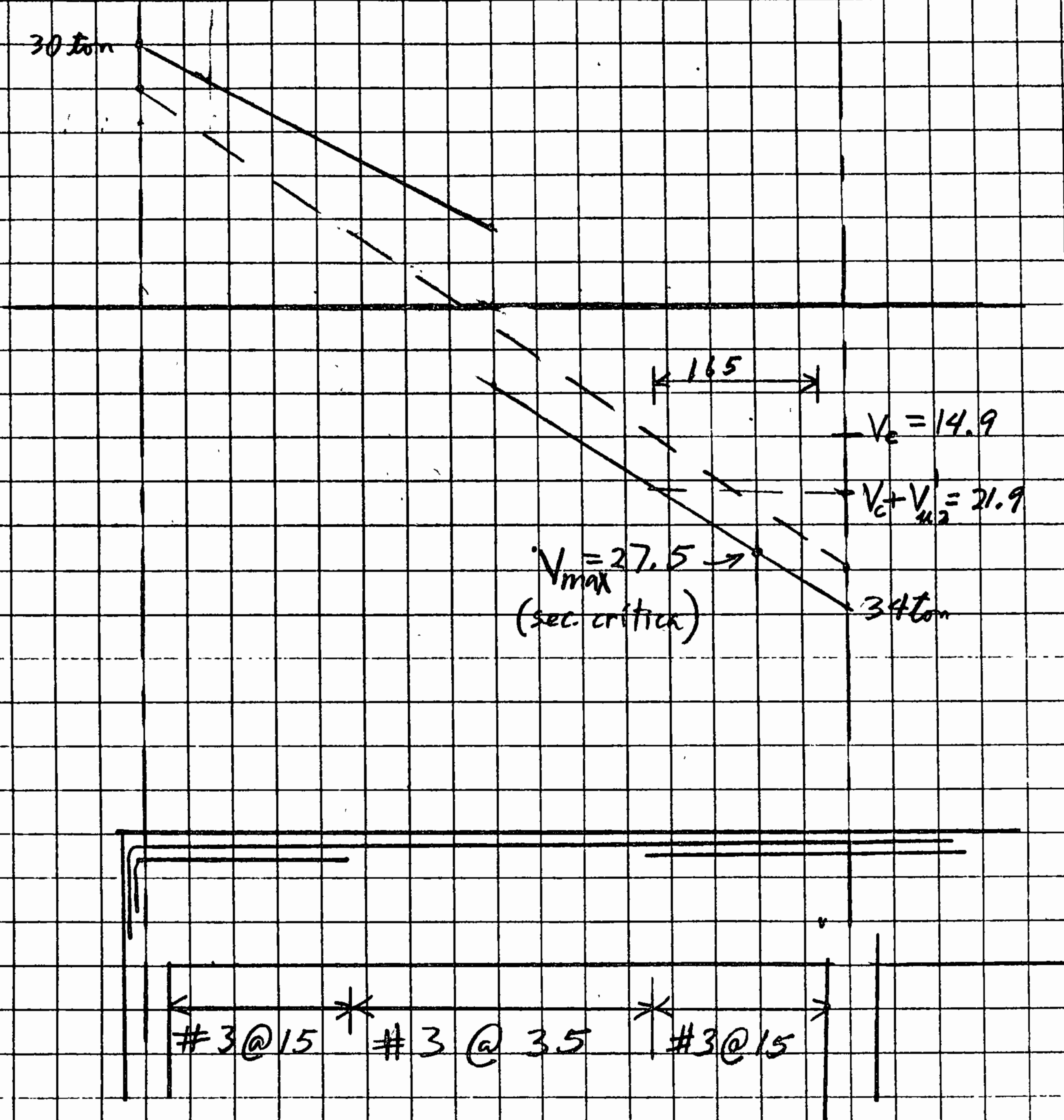
$$C_c = (0.85)(200)(5.1)(40) = -30{,}400$$

$$\Sigma = P_u = +\ 20 \text{ Kg} \doteq 0$$

Ejemplo 13.6 - Flexión



Ejemplo 13.6 - Cortante



14. Sistemas de piso

14.1 Introducción

Las estructuras de edificios de concreto están formadas por diversos elementos que generalmente trabajan como un conjunto ya que dichas estructuras suelen ser continuas o monolíticas. Los elementos clásicos que constituyen una estructura son las losas, las trabes y las columnas, aunque en algunas estructuras sólo existen losas y columnas. Estos elementos han sido estudiados en forma aislada en los capítulos anteriores de este texto. En este capítulo se estudiarán dichos elementos trabajando en conjunto.

Ha sido costumbre analizar las estructuras, por carga vertical, suponiendo que las cargas se transmiten a las losas, las cuales las transmiten a las trabes, y éstas, a su vez, las transmiten a las columnas. O bien, cuando no existen trabes, suponiendo que las losas transmiten directamente las cargas a las columnas. Por lo que se refiere a fuerzas horizontales, tales como fuerzas de viento o sismo, se supone generalmente que son resistidas por la estructura formada por trabes y columnas.

En realidad, tanto las cargas verticales como las horizontales son resistidas por los tres elementos estructurales trabajando en conjunto. Por ejemplo, en el caso de fuerzas horizontales, la contribución de las losas es importante; no es posible aislar las trabes de las losas ya que ambas trabajan simultáneamente bajo la acción de las fuerzas horizontales. En el caso de cargas verticales, se ha demostrado que la distribución de momentos flexionantes en las losas depende de la relación de rigideces entre losas, trabes y columnas; la distribución de momentos en dos sistemas de piso iguales entre sí resulta diferente si las columnas de uno de los sistemas son diferentes de las columnas del otro sistema. Esto se debe al distinto grado de restricción en ambos sistemas de piso.

Por otra parte, ha sido costumbre considerar como sistemas diferentes a los constituidos por losas apoyadas sobre trabes y por losas apoyadas directamente sobre columnas. Como consecuencia de esto, los métodos de análisis y diseño de ambos sistemas difieren en sus principios. La razón es de origen histórico.

Las losas apoyadas sobre columnas se empezaron a construir antes de que se conocieran métodos de análisis, sobre una base completamente empírica. Los métodos que se han usado hasta la fecha son, por lo tanto, de naturaleza empírica. En cambio, las losas apoyadas sobre trabes se empezaron a construir cuando ya se disponía de métodos matemáticos de análisis. Actualmente, se usan modificaciones de dichos métodos que toman en cuenta las características especiales del concreto reforzado. Ahora se reconoce que en realidad los dos sistemas de piso trabajan en la misma forma.

Recientemente se han desarrollado métodos de análisis de estructuras de concreto que consideran, por una parte, el trabajo en conjunto de los elementos estructurales, y por otra, el hecho de que los sistemas de piso actúan en la misma forma cualquiera que sea la rigidez a flexión de las trabes. De esta manera, cuando las losas están apoyadas sobre muros rígidos, se considera que lo están sobre trabes de rigidez a flexión infinita, y cuando están apoyadas directamente sobre columnas, se considera que el sistema de piso tiene trabes de rigidez a flexión nula. Dentro de estos dos casos límite, puede haber trabes de cualquier rigidez.

14.2 Estudios experimentales de sistemas de piso. Principales Variables.

Los elementos de concreto reforzado estudiados en capítulos anteriores han sido elementos isostáticos, en los cuales se han supuesto conocidas las acciones internas. Los sistemas de piso son, por el contrario, elementos altamente hiperestáticos en los que la determinación de tales acciones es un problema complejo. En la sección 12.3 se indicó cómo pueden determinarse los momentos flexionantes en losas aisladas apoyadas sobre elementos infinitamente rígidos. Sin embargo, la distribución de momentos flexionantes en sistemas de piso depende, no solamente de las características propias de la losa, sino también de los otros elementos que constituyen la estructura, como las trabes y las columnas. La distribución de momentos es importante, porque, en general, es necesario conocer dichos momentos para diseñar los elementos estructurales. Por esta razón, se ha estudiado dicha distribución tanto en forma experimental como analítica.

El número de ensayes realizados en estructuras formadas por losas, trabes y columnas es escaso. La serie más extensa es la realizada en la Universidad de Illinois[14.1-14.6] entre los años 1960 y 1963, que incluyó el ensaye de especímenes como el mostrado esquemáticamente en la Fig. 14.1. Los resultados de estos ensayes, en combinación con estudios analíticos, han permitido desarrollar métodos de diseño que toman en cuenta el efecto de las variables más importantes sobre el compotamiento de las estructuras.

Para estudiar la distribución de momentos flexionantes, considérese que en la estructura mostrada en la Fig. 14.2, se aisla la franja de losa comprendida entre los ejes A' y B'. Si se supone que los momentos flexionantes son uniformes a lo ancho de la franja resultante, puede considerarse a esta franja como una viga contínua con una distribución de momentos flexionantes como la mostrada en la Fig. 14.3 en forma cualitativa. Si la losa tiene una carga uniformemente distribuída, w , la viga continua de la Fig. 14.3 tiene una carga por unidad de longitud de un valor $w\ell_2$, donde ℓ_2 es el ancho de la franja entre los ejes A' y B'. En cada uno de los claros de la viga continua, se debe cumplir la siguiente ecuación de equilibrio.

$$M_o = \frac{(w\ell_2)\,\ell_1^2}{8} \qquad (14.1)$$

donde:

M_o = Momento estático total = Momento positivo en el centro del claro, más el promedio de los momentos negativos en los extremos.

$w\ell_2$ = Carga por unidad de longitud

ℓ_1 = Longitud del claro considerado

Por ejemplo, en el claro 2-3:

$$M_o = \frac{M(-)_2 + M(-)_3}{2} + M(+) \qquad (14.2)$$

En realidad, los momentos flexionantes no son uniformes a lo ancho de la franja considerada en la Fig. 14.2. A lo largo del eje de columnas, B, los momentos son mayores que a lo largo de los ejes A' y B'. Esto se debe a que el sistema es más rígido a lo largo del eje B por la presencia de vigas y porque el efecto de restricción de las columnas es máximo en estos ejes y va disminuyendo hacia los extremos de la franja.

La distribución cuantitativa de momentos a lo largo y a lo ancho de las franjas de losa depende de las características de los elementos que forman la estructura (columnas, vigas y losas) y del tipo de carga aplicada. En las siguientes secciones se describe la influencia de estas variables.

14.2.1 Influencia de las columnas

Las columnas influyen sobre la distribución de momentos en la losa por la restricción que ejercen sobre las vigas y la losa, o sea, por el empotramiento parcial que proporcionan a estos elementos estructurales.

Considérese nuevamente la distribución de momentos a lo largo de la franja A' - B' mostrada en la Fig. 14.3. Si la rigidez flexionante de las columnas es grande en comparación con la rigidez flexionante de vigas y losa, la restricción de las columnas en los extremos de la viga continua es grande y los momentos flexionantes en estos extremos son relativamente grandes*. En cambio, si la rigidez flexionante de las columnas es pequeña en comparación con la de vigas y losa, la restricción y los momentos flexionantes en los extremos también son pequeños. En la Fig. 14.4 se comparan cualitativamente estos casos: la Fig. 14.4-a corresponde al primer caso mencionado y la Fig. 14.4-b, al segundo caso. Teóricamente, si la rigidez flexionante de las columnas es nula, los momentos en los extremos de la viga continua son nulos. Este caso, que se muestra en la Fig. 14.4-c, puede presentarse cuando las columnas no son monolíticas ni están unidas rígidamente a las vigas y a la losa.

La rigidez flexionante de las columnas influye también sobre el valor de los momentos flexionantes en otras secciones de la viga contínua de la Fig. 14.3. Los momentos positivos en los claros extremos (1-2 y 3-4) son mayores mientras menores sean los momentos en los extremos de la viga contínua. Por lo tanto, son mayores mientras menor sea la rigidez flexionante de las columnas. Esto se debe a que la Ec. 14.2 debe cumplirse por condiciones de equilibrio, y al disminuir uno de los momentos negativos necesariamente debe aumentar el momento positivo, ya que el momento estático total permanece constante. La influencia de la rigidez flexionante de las columnas sobre los momentos negativos y positivos en claros interiores, como el claro 2-3 de la Fig. 14.3, es menor que sobre los momentos de

---

* Se supone en este capítulo, que el lector está familiarizado con el análisis elemental de estructuras hiperestáticas.

los claros extremos, siempre que la carga de la losa esté uniformemente distribuída en todos los claros. Cuando la carga no está uniformemente distribuída en todos los claros, la influencia de la rigidez flexionante de las columnas es también importante sobre los momentos en claros interiores. Este caso se analizará en la Sec. 14.2.4, que trata del efecto del tipo de carga.

14.2.2 Efecto de la rigidez flexionante de las vigas

La rigidez flexionante de las vigas, comparada con la rigidez flexionante de la losa, influye en la distribución de momentos a lo ancho de la franja. Si las vigas son de peralte grande en comparación con el peralte de la losa, un gran porcentaje del momento total en una sección transversal es resistido por las vigas y un porcentaje pequeño por la losa. En losas planas, en las que no existen vigas, todo el momento es resistido por la losa. Dentro de estos dos casos, el peralte de la viga puede ser de cualquier valor y el momento total se distribuye entre la viga y la losa de acuerdo con su rigidez flexionante.

14.2.3 Efecto de la rigidez torsionante de las vigas

La rigidez torsionante de las vigas proporciona un empotramiento parcial a las losas. Su efecto es especialmente importante en los bordes del sistema de piso, y en tableros interiores cuando un tablero se encuentra cargado y el tablero adyacente descargado. En el primer caso, aumentan los momentos negativos en la losa mientras mayor sea la rigidez torsionante. El segundo caso se analiza al estudiar el efecto del tipo de carga (Secc. 14.2.4).

Para que en un sistema de piso exista el efecto de la rigidez torsionante de las vigas, es necesario que éstas sean monolíticas con la losa y con las columnas. Si no se cumple la primera condición, las vigas no pueden restringir o empotrar a la losa, y no pueden desarrollarse momentos flexionantes en la losa en los bordes del sistema de piso. Si la viga no es monolítica con las columnas, no pueden desarrollarse en ella momentos torsionantes pues giraría libremente en sus extremos.

14.2.4 Influencia del tipo de carga

En un sistema de piso, no necesariamente se encuentran cargados siempre todos los tableros. Es frecuente, por ejemplo, en el caso de bodegas, que algunos tableros soporten carga viva y otros no.

Así como en vigas contínuas existen combinaciones de carga con las cuales los momentos en ciertas secciones son mayores que los correspondientes a carga uniforme en todos los claros de la viga, también en sistemas de piso existen combinaciones desfavorables de carga. Sin embargo, la carga muerta, que siempre es considerable en sistemas de piso, actúa uniformemente en todos los tableros. El incremento de momentos, respecto a los producidos por carga uniforme en todos los tableros, depende de la relación de carga viva a carga muerta. Mientras mayor sea esta relación, mayor es el incremento. En la Fig. 14.5 se presentan combinaciones de carga con las cuales se obtienen momentos positivos y negativos máximos.

El efecto de combinaciones desfavorables de carga está relacionado en forma importante con la rigidez flexionante de las columnas y con la rigidez torsionante de las vigas. Un incremento de estas rigideces aumenta el empotramiento de un tablero de losa dado y por consiguiente este tablero es menos sensible a las condiciones de carga de tableros vecinos.

14.2.5 Comentarios sobre los efectos de las variables

En los párrafos anteriores se han señalado las principales variables que influyen en el comportamiento de sistemas de piso, sin incluir los efectos de las propiedades de los materiales, como resistencia del concreto, porcentaje de refuerzo y límite de fluencia del acero, ni el efecto de la forma de las losas. Se ve que el número de variables es considerable y que el efecto de algunas está relacionada con el efecto de otras; por ejemplo, el efecto del tipo de carga está relacionado con el efecto de las rigideces de columnas y trabes. Esto hace que el tratamiento riguroso de los sistemas de piso sea un problema muy complejo y que sea necesario recurrir a procedimientos simplificados que tomen en cuenta de manera aproximada el efecto de las principales variables.

También se deduce del estudio de las variables, que en general no es suficiente considerar a la losa como un elemento aislado, sino que es necesario tomar en cuenta su interacción con los otros elementos estructurales para estudiar adecuadamente su comportamiento. Los métodos modernos de análisis y diseño están basados en estas consideraciones.

14.3 Análisis de sistemas de piso

En la Secc. 12.3 se señaló que los momentos flexionantes de una losa pueden determinarse resolviendo la siguiente ecuación:

$$\frac{\partial^4 z}{\partial x^4} + \frac{2\,\partial^4 z}{\partial x^2\,\partial y^2} + \frac{\partial^4 z}{\partial y^4} = \frac{w}{N} \qquad (14.3)$$

La resolución de la Ec. 14.3 tiene serias limitaciones cuando se trata de analizar el conjunto de losa, vigas y columnas, ya que no es posible tomar en cuenta variables importantes como la rigidez torsionante de las vigas, y las dimensiones de las vigas y columnas. El método sólo considera losas perfectamente empotradas o completamente libres, y vigas y columnas de ancho nulo. Tampoco es posible tomar en cuenta qué parte de la losa actúa como patín de las vigas ni las características propias del concreto reforzado como agrietamiento y fluencia del refuerzo. Sin embargo, existen algunos métodos que sí permiten tomar en cuenta algunas de estas variables, como la aplicación de diferencias finitas para resolver la Ec. 14.3, ó el método de distribución de momentos, desarrollado por Siess y Newmark[14.7]. Estos métodos requieren el empleo de computadoras de tamaño mediano o grande, pues se llega generalmente a un sistema de ecuaciones simultáneas de número muy elevado. No pueden usarse, por lo tanto, para análisis rutinarios, sino que se emplean para estudiar la influencia de variables en estructuras típicas, con fines de obtener posteriormente métodos simplificados de diseño, y para comparar los resultados experimentales con resultados analíticos y deducir de ahí la influencia del tipo de material.

Los métodos de análisis mencionados anteriormente se han usado para determinar la distribución de momentos flexionantes en estructuras del tipo mostrado en la Fig. 14.1 y estas distribuciones se han comparado con las obtenidas en ensayes[14.8].

La concordancia entre momentos analíticos y experimentales es buena, en general, por lo cual se han podido desarrollar métodos de diseño a partir de análisis teóricos. Ha quedado demostrado que los resultados de análisis de losas ideales de material elástico, homogéneo e isotrópico, son aplicables a losas de concreto reforzado, si se hacen ciertas modificaciones indicadas por los resultados experimentales. Las modificaciones más importantes consisten en tomar en cuenta que el efecto de cargas parciales es menos importante en estructuras de concreto reforzado que en estructuras ideales, y que los momentos negativos en estructuras reales son ligeramente menores que en estructuras ideales y los positivos son ligeramente mayores.

Esto último puede deberse a que la losa se agrieta en las zonas de momento negativo antes que en las zonas de momento positivo, por ser mayores los momentos negativos, y como consecuencia los momentos se redistribuyen de las zonas de momento negativo a las zonas de momento positivo.

14.4 Dimensionamiento de sistemas de piso

El método que se presenta a continuación para el dimensionamien to de sistemas de piso se conoce con el nombre de método de la estructura equivalente, debido a que se basa en el principio de sustituir a la estructura tridimensional, constituida por el sistema de piso, por un marco bidimensional equivalente, constituido por columnas y trabes. El método es similar al presentado en el Reglamento ACI 1941 y ediciones siguientes, pero con modificaciones que se le han hecho para ajustar los momentos calculados a los obtenidos en ensayes.[14.9 y 10]

El método consiste en los pasos que se mencionan a continuación en forma resumida y que se describen con detalle más adelante.

a) Idealización de la estructura tridimensional en marcos bidimensionales constituidos por columnas y trabes.
b) Determinación de las rigideces de los elementos que forman los marcos.
c) Análisis estructural de los marcos.
d) Distribución de los momentos flexionantes y fuerzas cortantes obtenidos en el análisis entre los elementos que forman la estructura tridimensional.
e) Dimensionamiento de los elementos de la estructura.

14.4.1 Idealización de la estructura

En el método de la estructura equivalente se hace una simplificación que consiste en idealizar la estructura por una serie de marcos en dos direcciones, como los que se muestran en las áreas rayadas de la Fig. 14.6 .

Las columnas en de los marcos equivalentes son iguales a las columnas de la estructura modificadas de tal manera que, además de la columna propiamente dicha, incluyen a la trabe perpendicular a la dirección del marco equivalente, como se muestra en la Fig. 14.7. Esta modificación se hace para tomar en cuenta el efecto de restricción por torsión que ejercen las trabes sobre la losa (Sec. 14.2.3). En sistemas de piso sin trabes, se supone que existe una trabe cuyo peralte es igual al de la losa y cuyo ancho es igual al de la columna, capitel o ábaco en la dirección del marco equivalente. En sistemas de piso con trabes, se supone que las trabes transversales, son trabes T o L cuyo ancho del patín es igual a la proyección de la trabe por arriba o por abajo de la losa, pero no mayor que cuatro veces el espesor de la losa.

En el caso de losas apoyadas sobre vigas, las trabes de los marcos equivalentes están formadas por las vigas de la estructura y los tramos de losa comprendidos entre los ejes centrales de los tableros; las vigas y la losa en conjunto constituyen una trábe equivalente cuyas características se definen como se indica en la Sec. 14.4.2. En el caso de losas apoyadas sobre columnas, las trabes de los marcos están formadas por los tramos de losa comprendidos entre los ejes centrales de los tableros. La manera de transformar los tramos de losa en trabes equivalentes se indica también en la Secc. 14.4.2.

14.4.2 <u>Determinación de las rigideces de los elementos</u>

Para calcular las rigideces se consideran únicamente secciones gruesas de concreto sin agrietar y sin tomar en cuenta el acero de refuerzo. A continuación se presentan por separado los métodos de cálculo de rigideces de trabes y columnas en sistemas de piso sin trabes y con trabes. Se presenta únicamente la forma de calcular los valores de 1/EI, ya que a partir de estos valores pueden calcularse las rigideces tomando en cuenta las longitudes de los claros y las condiciones de restricción en los extremos de columnas y trabes. En todos los casos, el valor de E es el del ~~método~~ módulo de elasticidad del concreto, E_c.

a) <u>Trabes del marco equivalente en sistemas de piso sin vigas</u>

El caso ~~es~~ más general de estos sistemas se muestra en la Fig. 14.8-a, la cual representa un sistema que incluye losas, ábaco y capitel.

El momento de inercia de la sección A-A es el de una sección rectangular.

El de la sección B-B, se calcula como el de la sección T mostrada en la Fig. 14.8-c.

La sección CC mostrada en la Fig. 14.8-d es de peralte variable; sin embargo por simplicidad se supone que el momento de inercia, del eje de la columna al extremo del capitel, es constante, e igual al momento de inercia en la sección del ábaco dividido entre el factor $(1-c_2/l_2)^2$, donde $\underline{c_2}$ y $\underline{l_2}$ son las dimensiones del capitel y del claro en dirección transversal a la del marco equivalente considerado. La distribución de valores de $1/EI$ a lo largo del claro se muestra en la Fig. 14.8-a. En las Tablas 14.1 y 14.2 se presentan constantes de distribución de momentos calculados con los criterios (anteriores) para placas planas y losas planas sin capiteles.

b) <u>Trabes del marco equivalente en sistemas de piso con vigas</u>

En estos sistemas el valor de EI es constante en todo el claro. El valor de I se calcula sumando el momento de inercia de una trabe T y el de una sección rectangular, como se muestra en la Fig. 14.9. Los patines de la trabe T tienen un ancho igual a la proyección del alma de trabe por arriba o por abajo de los patines, sin exceder cuatro veces el espesor de la losa, y el ancho de la sección rectangular es igual al ancho promedio de los claros transversales, $\underline{l_2}$, menos el ancho del patín de la trabe T.

c) <u>Columnas del marco equivalente</u>

Se mencionó anteriormente que la columna equivalente está formada por la columna y una trabe que trabaja a torsión restringiendo a la losa, Fig. 14.7. Para calcular la rigidez de este elemento compuesto, se parte de la hipótesis de que su flexibilidad, o sea, el recíproco de su rigidez, es igual a la suma de las flexibilidades a flexión de los tramos de columna y por arriba y por abajo del nivel de piso y de la flexibilidad a torsión de la trabe. Esta consideración, obtenida con base en los ensayes mencionados anteriormente, puede expresarse mediante la siguiente ecuación:

$$\frac{1}{K_{ec}} = \frac{1}{\Sigma K_c} + \frac{1}{K_t} \tag{14.4}$$

donde

K_{ec} = rigidez de la columna equivalente, en momento por unidad de rotación.

ΣK_c = suma de las rigideces a flexión de los tramos de columna comprendidos entre el nivel de piso considerado y los niveles superior e inferior.

K_t = rigidez a torsión de la trabe.

Para calcular la rigidez K_c de cada columna, se supone que el valor de I es constante e igual al de la sección gruesa de cada columna entre la cara superior de la losa y la base del capitel del nivel superior, que I es infinito a lo alto de la losa, y que varía linealmente entre los dos valores anteriores a lo alto del capitel. En la Fig. 14.10 se presenta la variación en los valores de 1/EI de una columna de acuerdo con los criterios anteriores. Para calcular las rigide~~z~~es se considera que la altura de las columnas se mide centro a centro de las losas, como se muestra en la Fig. 14.10.

La rigidez tor~~c~~sionante, K_t, de la trabe unida a la columna, Fig. 14.7, puede calcularse con la siguiente ecuación[14.10]

$$K_t = \sum \frac{9 E_{cs} C}{\ell_2 \left(1 - \frac{c_2}{\ell_2}\right)^3} \qquad (14.5)$$

donde

E_{cs} = módulo de elasticidad del concreto de la losa

$$C = \sum (1 - 0.63 \frac{x}{y}) \frac{x^3 y}{3} \qquad (14.6)$$

x = dimensión total menor de una sección transversal rectangular

y = dimensión total mayor de una sección transversal rectangular

La suma que aparece en la ecuación 14.6 se refiere a los rectángulos en que puede descomponerse la sección T o L de la trabe de la Fig. 14.7

A partir de los valores de K_c y K_t se calcula la rigidez de la columna equivalente, K_{ec} , con la ecuación 14.4.

14.4.3 Análisis estructural de los marcos

Una vez calculadas las rigideces de las trabes y columnas de la estructura equivalente, se efectúa el análisis estructural por los procedimientos usuales para marcos bidimensionales. El análisis por carga vertical puede efectuarse aislando cada uno de los pisos y suponiendo que las columnas de los entrepisos superior e inferior están empotradas en sus extremos opuestos. En el análisis por carga horizontal (viento o sismo) deben considerarse los marcos completos.

Cuando la intensidad de la carga viva no excede de las tres cuartas partes de la intensidad de la carga muerta, o cuando todos los tableros de la losa se encuentran cargados simultáneamente, el análisis estructural se efectúa suponiendo que todos los tableros están cargados. Cuando no se cumplen estas condiciones, el momento positivo máximo en un claro dado se calcula suponiendo que el claro está cargado con las tres cuartas partes de la carga viva y con la carga muerta total, y que los claros adyacentes están cargados únicamente con la carga muerta. El momento negativo máximo en un nudo dado se calcula suponiendo que los dos claros adyacentes al nudo están cargados con las tres cuartas partes de la carga viva total, y los claros siguientes están descargados. En la Fig. 14.11 se ilustra las condiciones desfavorables de carga descritas anteriormente y la simplificación de la estructura que puede hacerse para efectos de cálculo por carga vertical.

En nudos interiores, la sección crítica por momento negativo está localizada en las caras de las columnas, pero a una distancia no mayor de $0.175\ell_1$ del centro de la columna. En apoyos exteriores con capiteles, la sección crítica por momento negativo está localizada a la mitad de la distancia entre la cara de la columna y el extremo del capitel. La sección crítica por momento positivo se considera siempre al centro del claro.

14.4.4 Distribución de momentos flexionantes y fuerzas cortantes

Los momentos flexionantes y fuerzas cortantes obtenidos mediante el análisis descrito en la sección anterior corresponden a las trabes y columnas del marco equivalente. Es necesario distribuir estos momentos y fuerzas cortantes entre los elementos del sistema de piso.

Puesto que las trabes del marco equivalente representan a las franjas del sistema de piso mostradas en la Fig. 14.6, los momentos y cortantes deben distribuirse entre los elementos comprendidos en dichas franjas. Para hacer esta distribución, la franja de piso se divide en una franja de columna y una o dos medias franjas centrales, como se indica en la Fig. 14.12. La franja de columnas incluye las vigas, en caso de que existan.

Una vez hecha la división en franjas señalada anteriormente, se distribuyen los momentos obtenidos en el análisis estructural entre la franja de columnas y las franjas centrales de la manera siguiente. Se calculan los momentos en las franjas de columnas multiplicando los momentos por los porcentajes mostrados en la tabla 14.3. Después se distribuyen los momentos de las franjas de columna entre las vigas, si existen, y los tramos de losa, de acuerdo con lo que se indica en la tabla 14.4; si no existen vigas, todo el momento de las franjas de columnas es resistido por la losa. Por último, se calculan los momentos en las franjas centrales restando los momentos de las franjas de columnas de los momentos totales.

La distribución de momentos en las tablas 14.3 y 14.4 está hecha en base a la relación de claros (l_2/l_1) y a factores que involucran las rigideces a flexión de trabes y losas y la rigidez a torsión de las trabes de borde.

Se ha visto anteriormente que estos parámetros son los que más influyen en el comportamiento de sistemas de piso. Los factores (α_1 y β_t) que aparecen en las tablas se definen de la siguiente manera:

α_1 = relación entre la rigidez a flexión de la sección de la viga y la rigidez a flexión de los tramos de losa de la franja de columnas. La viga debe considerarse de sección T o L, como se muestra en la Fig. 14.9. Puede expresarse como $(E_{cb}I_b/E_{cs}I_s)$, donde E_{cb} es el módulo de elasticidad del concreto de la viga; I_b , el momento de inercia de la viga; E_{cs} , el módulo de elasticidad del concreto de la losa; I_s , el momento de inercia de la losa.

β_t = relación entre la rigidez a torsión de la sección transversal de la viga de borde y la rigidez a flexión de un tramo de losa cuyo ancho es igual al claro centro a centro de apoyos de la viga de borde. Se expresa como $(E_{cb}C/2E_{cs}I_s)$. El término C se define en la ec. 14.6 y los otros términos ya han sido también definidos.

El cálculo de la fuerza cortante en las trabes se hace de la siguiente manera. Cuando el término ($\alpha_1 \ell_2/\ell_1$) es igual o mayor que 1.0, se calcula a partir de áreas tributarias de losa delimitadas por líneas a 45° trazadas a partir de las esquinas de los tableros y por líneas paralelas a los lados largos de los tableros. Cuando α_1 es igual a cero, se supone que las vigas no reciben carga. Para valores intermedios de α_1

se interpreta linealmente.

Los momentos flexionantes de diseño en las columnas son los mismos que los obtenidos en las columnas equivalentes.

(están)

Las trabes de la estructura, especialmente las de borde, bajo la acción de momentos torsionantes debidos a la restricción que proporcionan a las losas. Estos momentos pueden calcularse distribuyendo los momentos de las columnas equivalentes entre las columnas y las trabes, Fig. 14.7, proporcionalmente a sus rigideces K_c y K_t. Obsérvese que el momento en las columnas es el momento total en las columnas equivalentes, ya que los momentos torsionantes en las trabes se transmiten a las columnas como momentos flexionantes.

14.4.5 Dimensionamiento de los elementos de estructura.

Una vez obtenidas las acciones internas (momentos flexionantes, fuerzas cortantes y momentos torsionantes) en los miembros estructurales, se procede al dimensionamiento de los mismos siguiendo los métodos descritos en capítulos anteriores de este texto. En el caso de losas apoyadas directamente sobre columnas y sujetas a cargas horizontales, debe considerarse el problema de la transmisión de momento flexionante de la losa a la columna. Para que el comportamiento de la estructura sea más eficiente a este respecto, se recomienda concentrar el refuerzo de flexión necesario para resistir las fuerzas horizontales en una franja de losa localizada sobre el eje de columnas y cuyo ancho sea igual al ancho de la columna más el grosor de la losa. Deben revisarse los esfuerzos cortantes por penetración tomando en cuenta las cargas verticales y los momentos producidos por las fuerzas horizontales.

REFERENCIAS:

14.1 Hatcher, D. S.; Sozen, M. A.; ~~and~~ y Siess, C. P., "Test of a Reinforced Concrete Flat Plate," Proceedings, ASCE, V. 91, ST5, Oct. 1965, pp. 205-231.

14.2 Guralnick, S. A., ~~and~~ y Fraugh, R. W., "Laboratory Study of a Forty-Five-Foot Square Flat Plate Structure," ACI Journal, Proceedings V. 60, No. 9, Sept. 1963, pp. 1107-1185.

14.3 Hatcher, D. S.; Sozen, M. A.; ~~and~~ y Siess, C. P., "Test of a Reinforced Concrete Flat Slab," Proceedings, ASCE, V. 95, ST6, Jun. 1969, pp. 1051-1072.

14.4 Jirsa, J. O.; Sozen, M. A.; ~~and~~ y Siess, C. P., "Test of a Flat Slab Reinforced with Welded Wire Fabric," Proceedings, ASCE, V. 92, ST3, Jun. 1966, pp. 199-224.

14.5 Gamble, W. L.; Sozen, M. A.; ~~and~~ y Siess, C. P., "Test of a Two-Way Reinforced Floor Slab," Proceedings, ASCE, V. 95, ST6 Jun. 1969, pp. 1073-1096.

14.6 Vanderbilt, M. D.; Sozen, M. A.; ~~and~~ y Siess, C. P., "Tests of a Modified Reinforced Concrete Two-Way Slab," Proceedings, ASCE, V. 95, ST6, Jun. 1969, pp. 1097-1116.

14.7 C. P. Siess y N. M. Newmark, "Rational Analysis and Design of Two-Way Concrete Slabs", ACI Journal, Abril, 1950.

14.8 "Measured ~~y~~ and Theoretical Bending Moments in Reinforced Concrete Floor Slabs," Civil Engineering Studies, Structural Research Series No. 246, University of Illinois, Jun. 1962

14.9 Corley, W. G.; Sozen, M. A.; y Siess, C. P., "The Equivalent-Frame Analysis for Reinforced Concrete Slabs," Civil Engineering Studies, Structural Research Series No. 218, University of Illinois, Jun. 1961.

14.10 Corley, W. G., y Jirsa, J. O., "Equivalent Frame Analysis for Slab Design," ACI Journal, Proceedings V. 67, No. 11, Nov. 1970, pp. 875-884.

Tabla 14.4

Distribución (de los momentos) de las franjas de columnas entre vigas y losas.

Relación de rigideces	Porcentaje que se asigna a la viga	Porcentaje que se asigna a la losa
$(\alpha_1 l_2/l_1) = 0$	0	100
$(\alpha_1 l_2/l_1) \geq 1.0$	85	15

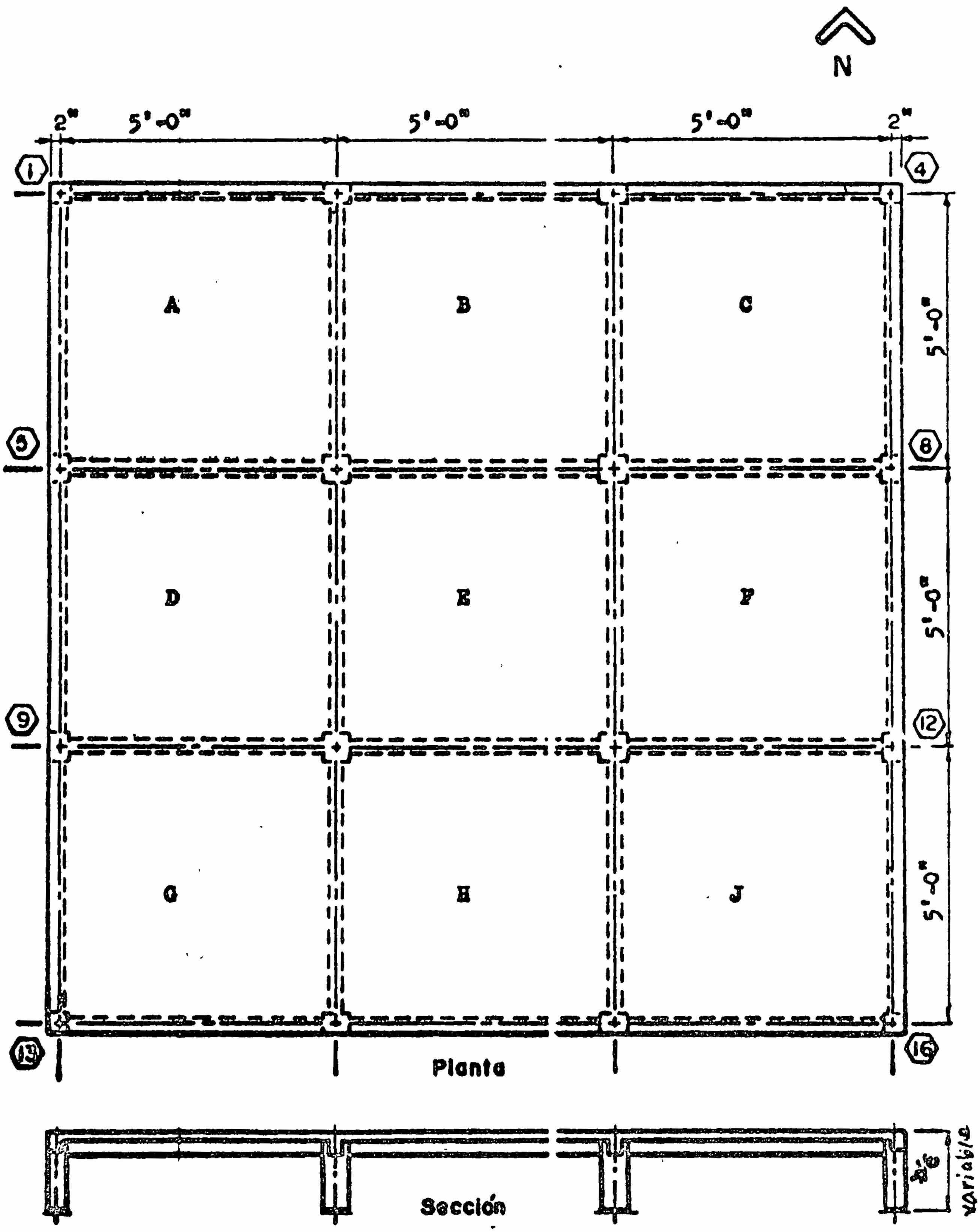
Puede usarse interpolación lineal entre los valores mostrados.

Tabla 14.3

Porcentaje de los momentos totales que se asignan a las franjas de columnas.'

	Relación de rigideces		Valores de l_2/l_1 0.5	1.0	2.0
Momentos negativos en apoyos interiores	$(\alpha_1 l_2/l_1) = 0$		75	75	75
	$(\alpha_1 l_2/l_1) \geq 1.0$		90	75	45
Momentos negativos en apoyos exteriores	$(\alpha_1 l_2/l_1) = 0$	$\beta_t = 0$	100	100	100
		$\beta_t \geq 2.5$	75	75	75
	$(\alpha_1 l_2/l_1) \geq 1.0$	$\beta_t = 0$	100	100	100
		$\beta_t \geq 2.5$	90	75	45
Momentos positivos	$(\alpha_1 l_2/l_1) = 0$		60	60	60
	$(\alpha_1 l_2/l_1) \geq 1.0$		90	75	45

Puede usarse interpolación lineal entre los valores mostrados.



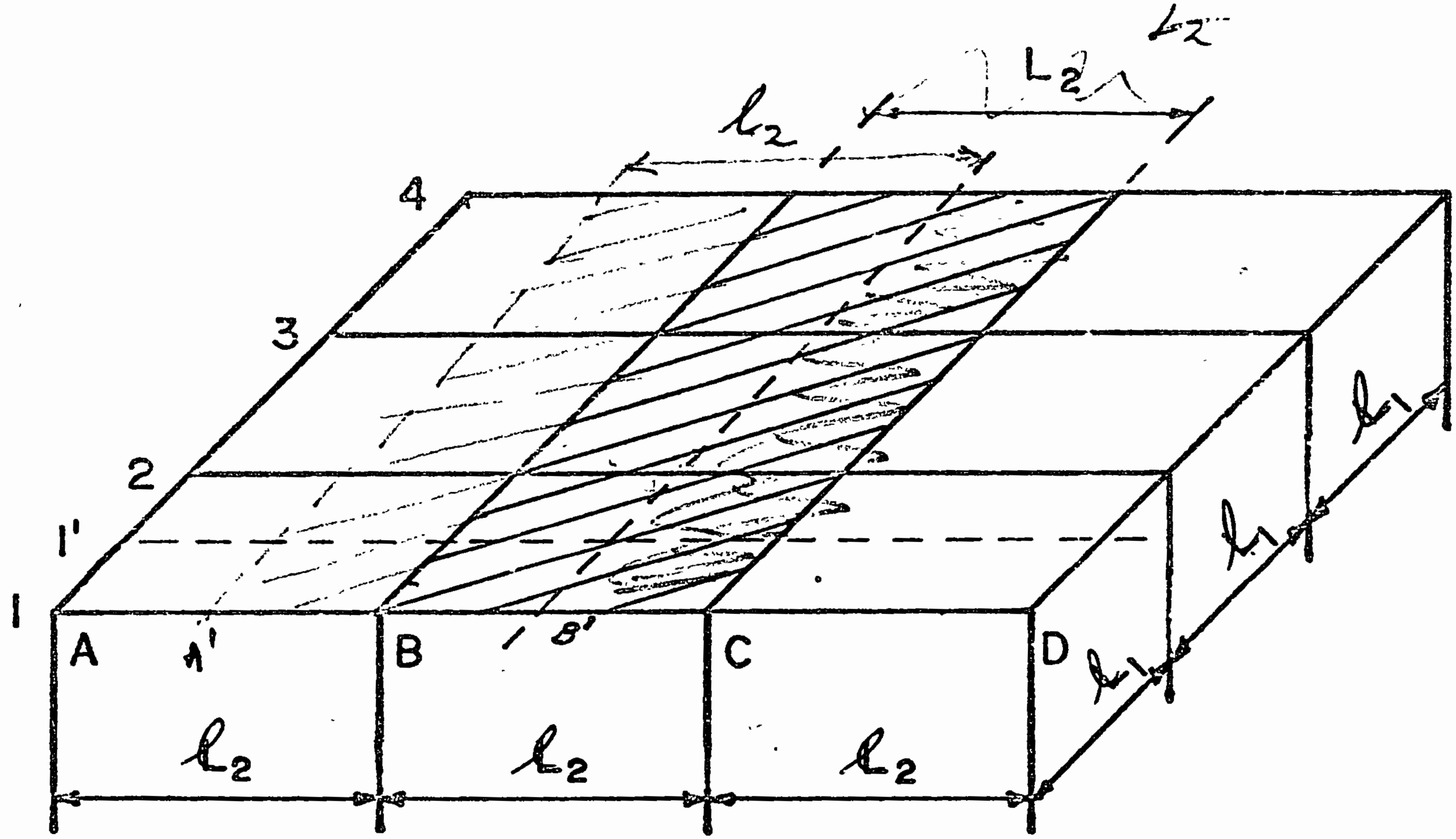
~~Nota: Dimensión h_c = 18-5/8" en losa con vigas rígidas~~
~~h_c = 13-7/8"~~ en losa con vigas flexibles

Fig. 14.1

~~Figura 2.4~~ ~~Fig. 2~~ - Planta típica de las estructuras ensayadas *en la Universidad de Illinois.*

~~2.2~~
F2.9

Traducir a sistema métrico



14.2

Fig. 3 - Estructura de 9 tableros

Franja de una losa en la cual se determinan los momentos flexionantes.

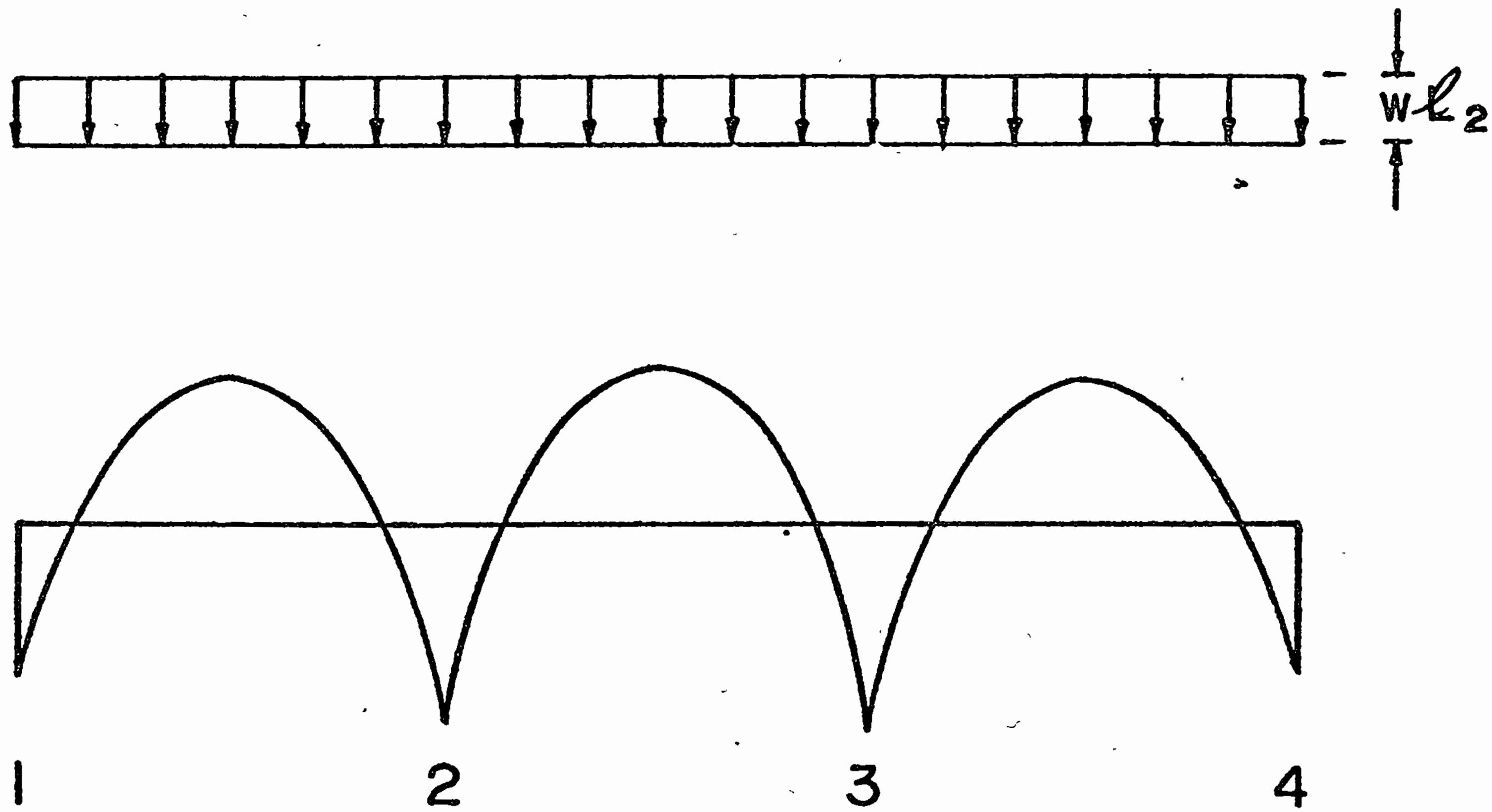


Fig. ~~4~~ 14.3 - Diagrama de momentos en la franja de losa de la Fig. 12.5

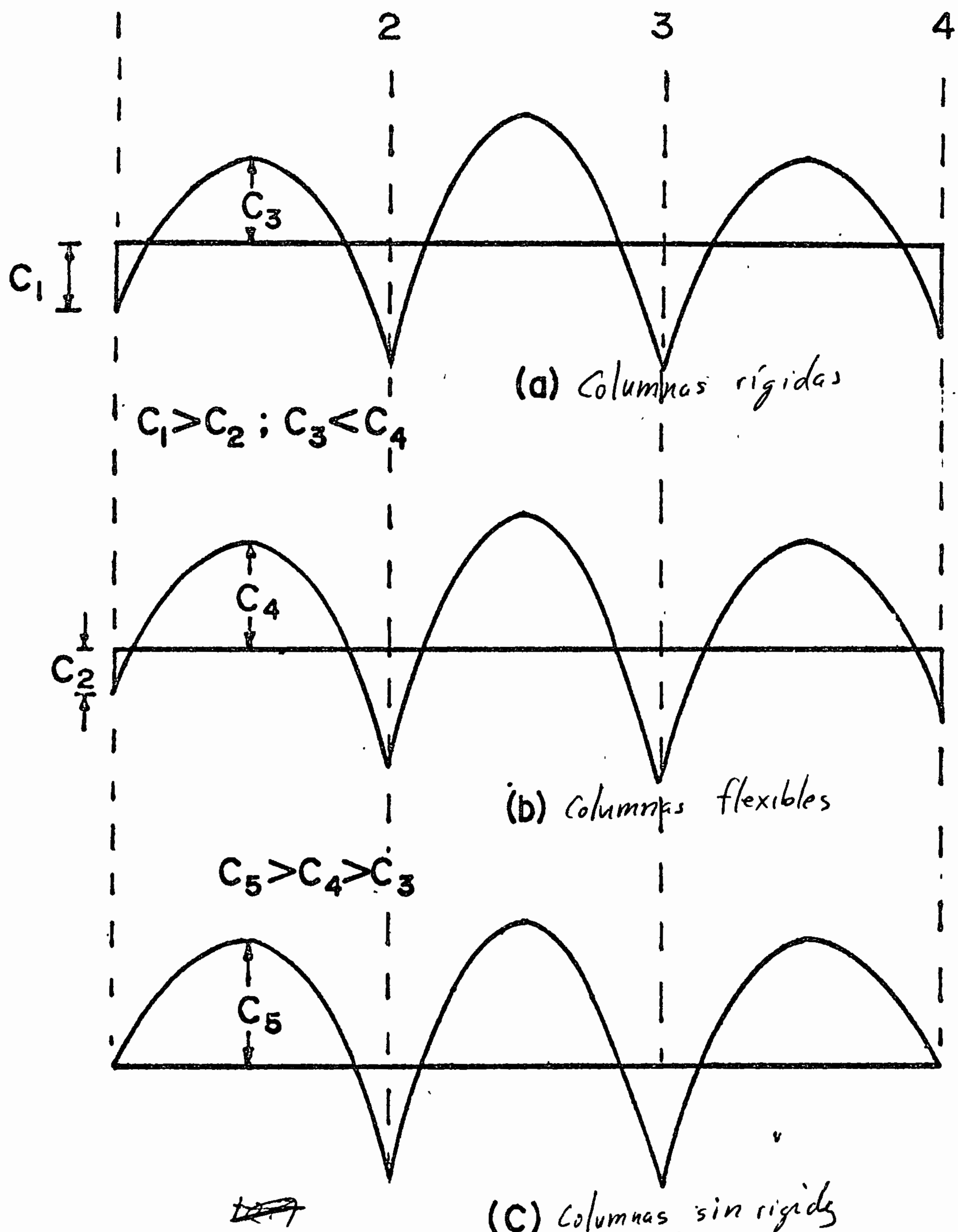
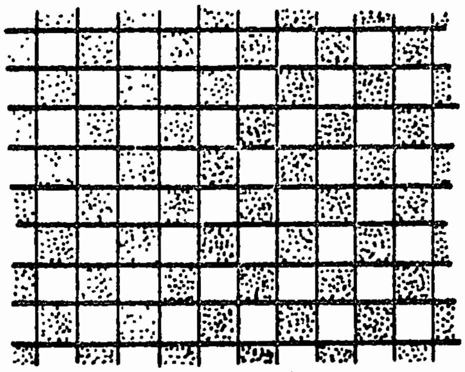
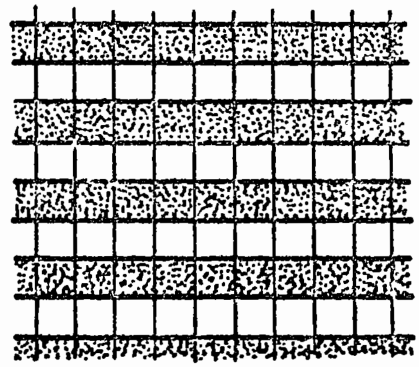


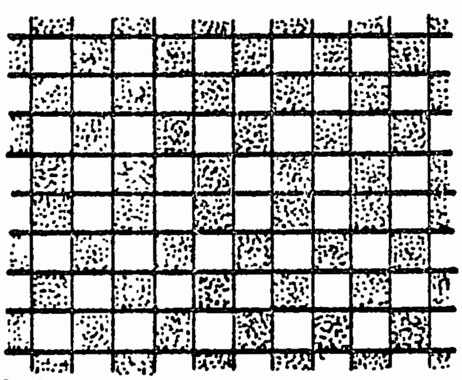
Fig. ~~7~~ 14.4 - Efecto de la rigidez flexionante de las columnas



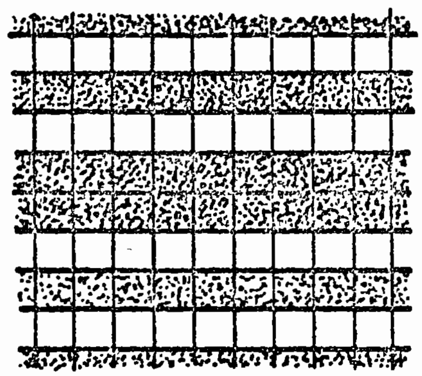
(a) Carga de tablero de ajedrez para momentos positivos máximos



(c) Distribución de carga por franjas para momentos positivos máximos



(b) Carga de tablero de ajedrez para momentos negativos maximos



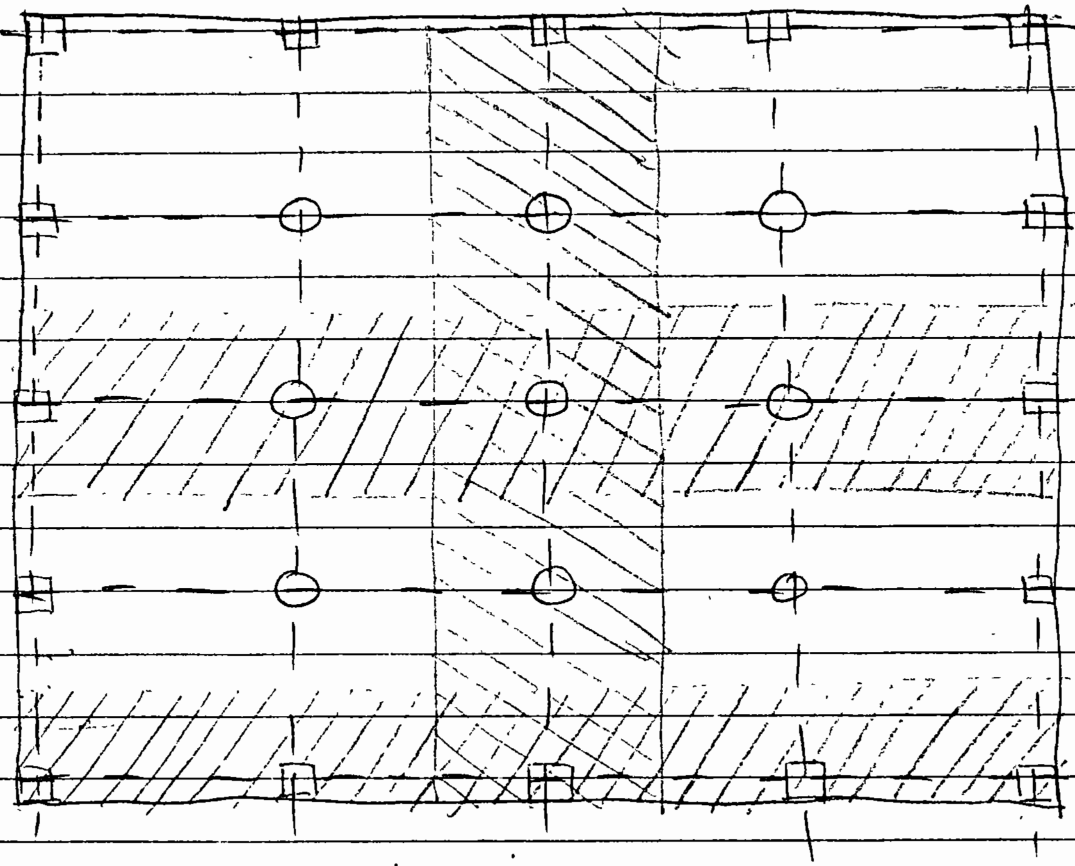
(d) Distribución de carga por franjas para momentos negativos maximos

~~12.7~~

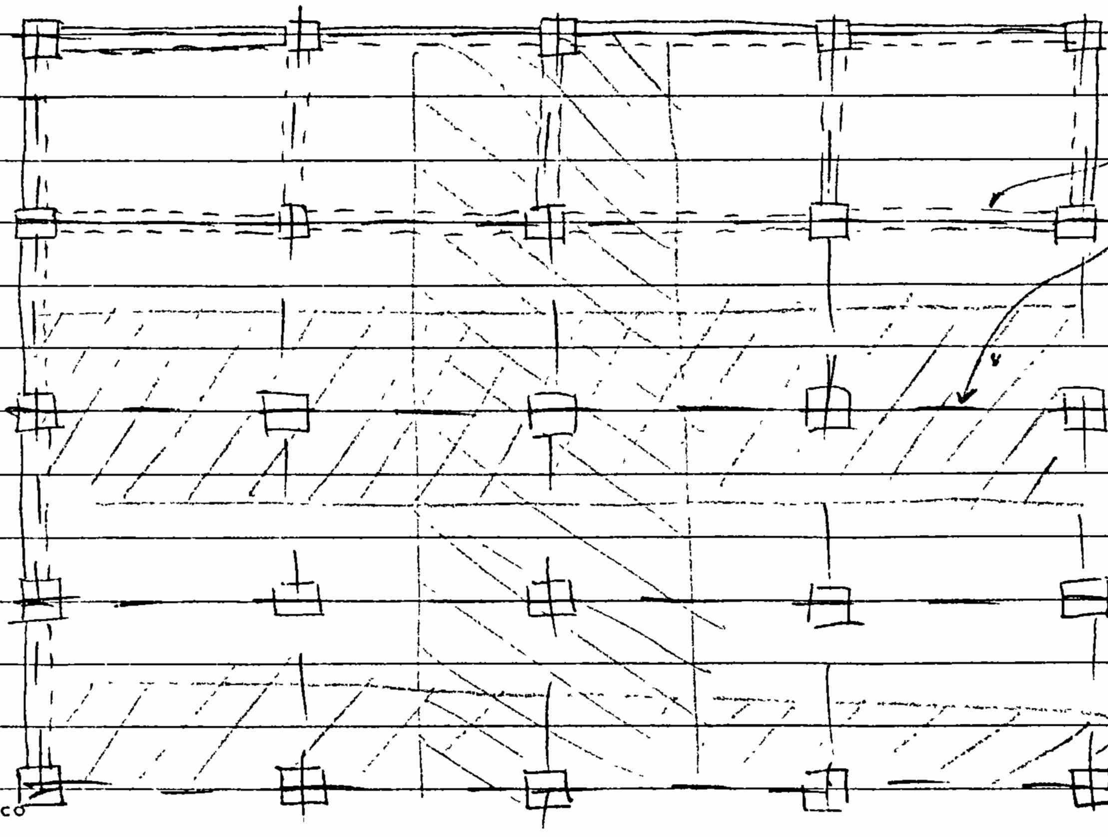
Fig. ~~11~~ – Cargas desfavorables en sistemas de piso

~~12.8~~

14.5



14.6 - b



(para el dibujante) poner vigas en todos los ejes

14.6-a

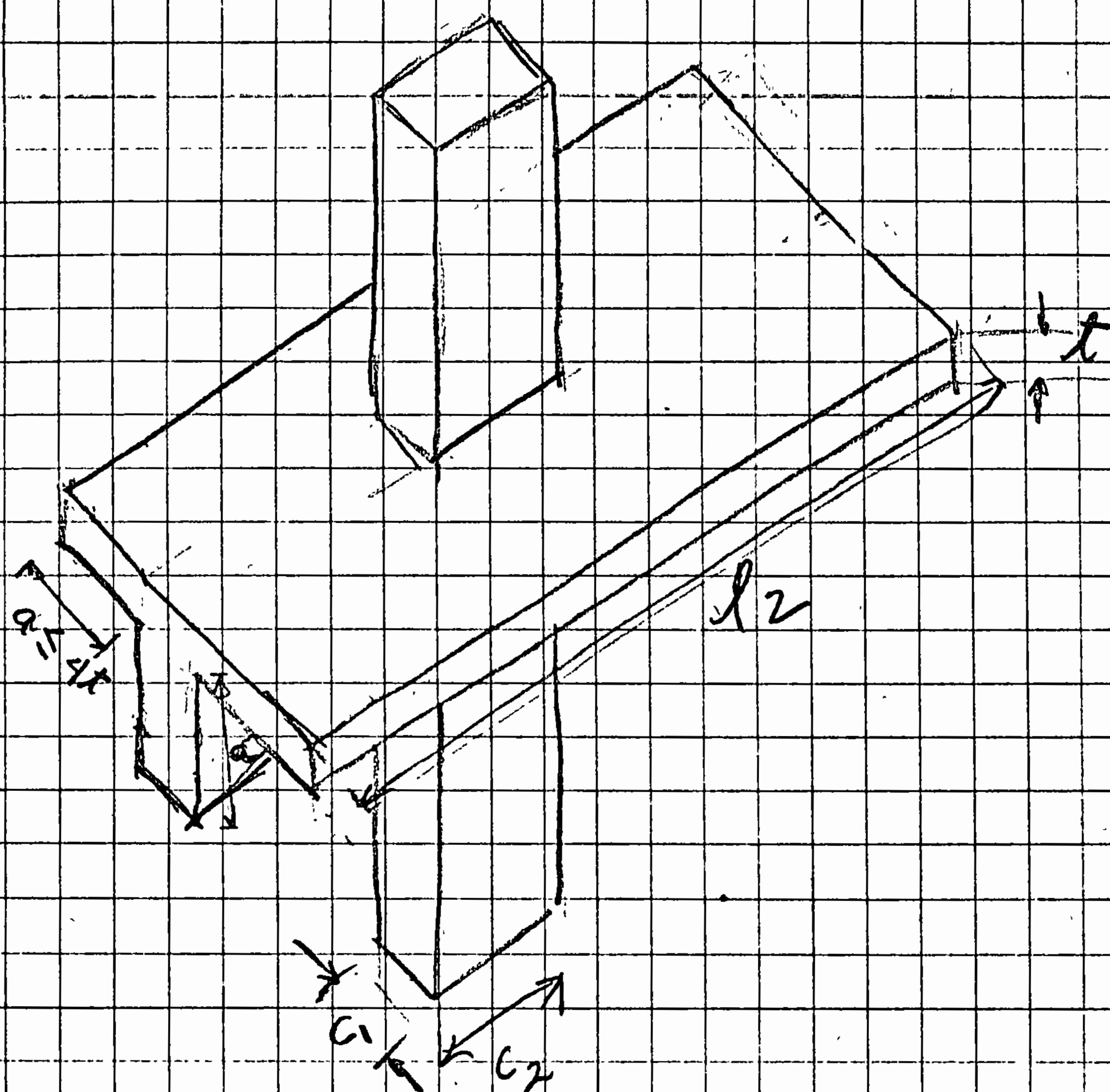
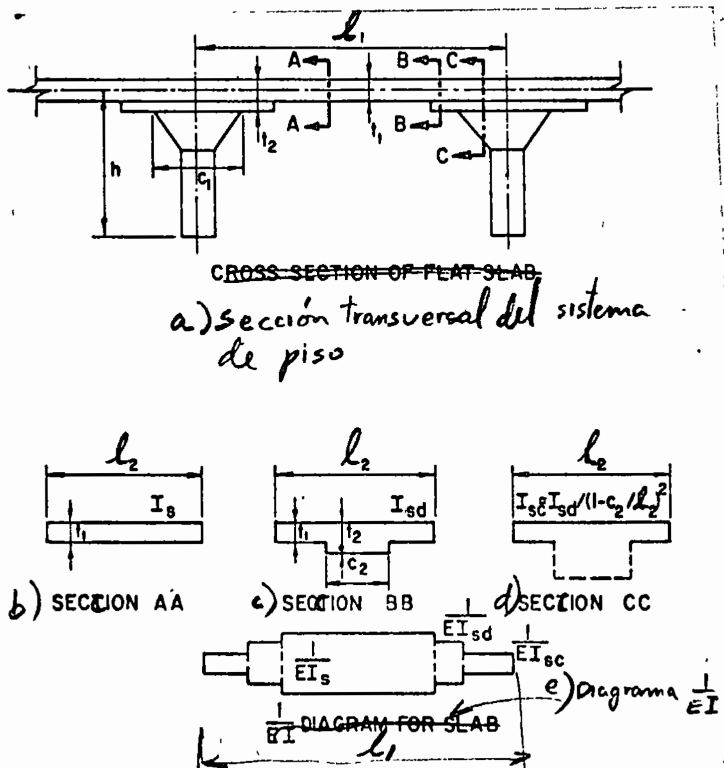


Fig. 14.7 Columna modificada equivalente



~~Fig. 1 — Cross sections for calculating stiffnesses of equivalent frame~~

~~876~~ Fig. 14.8 Cálculo de rigideces de trabes equivalentes en sistemas de piso sin vigas

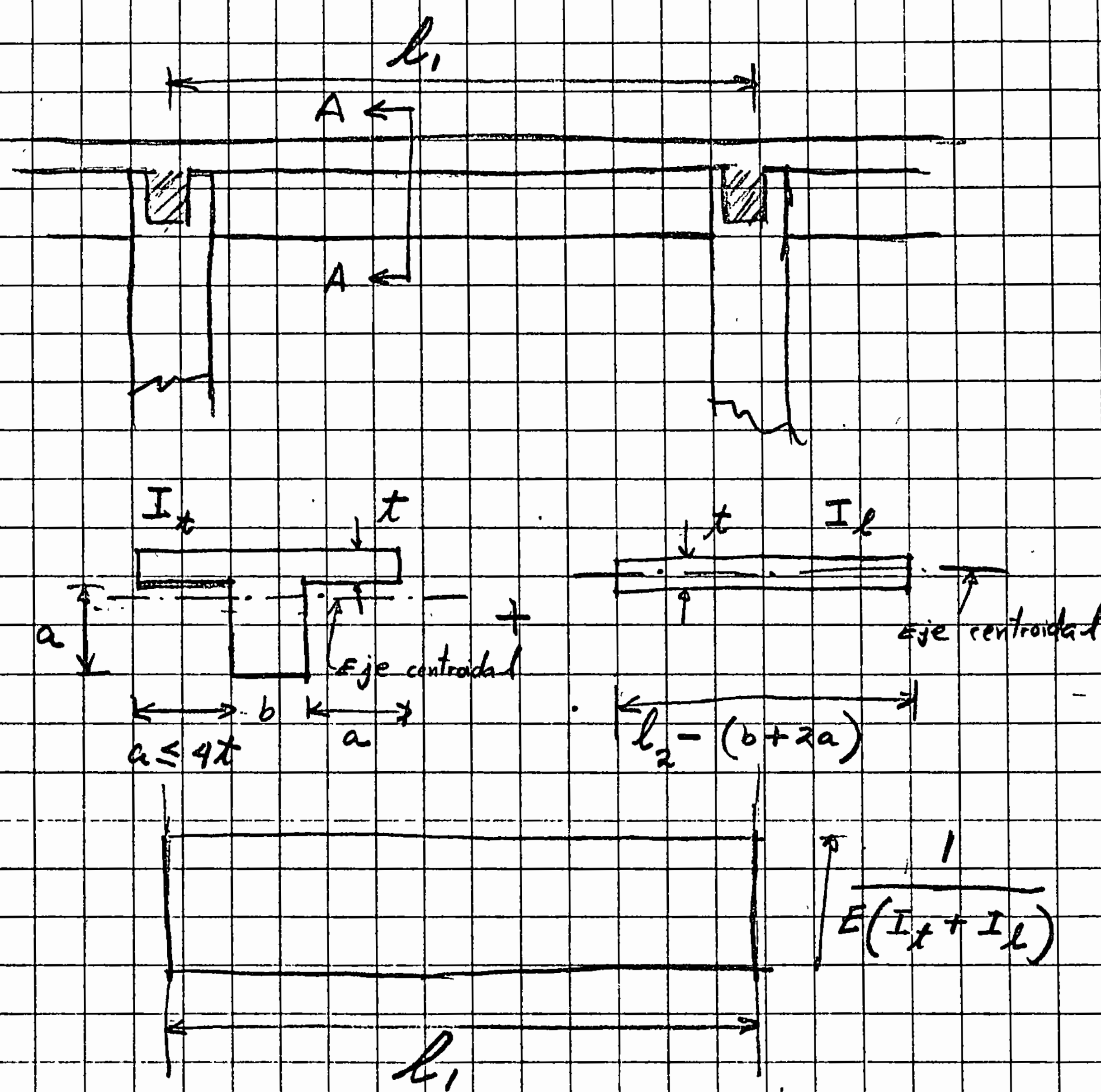


Fig. 14.9 Cálculo de rigidez de la trabe equivalente en sistemas de piso con trabes.

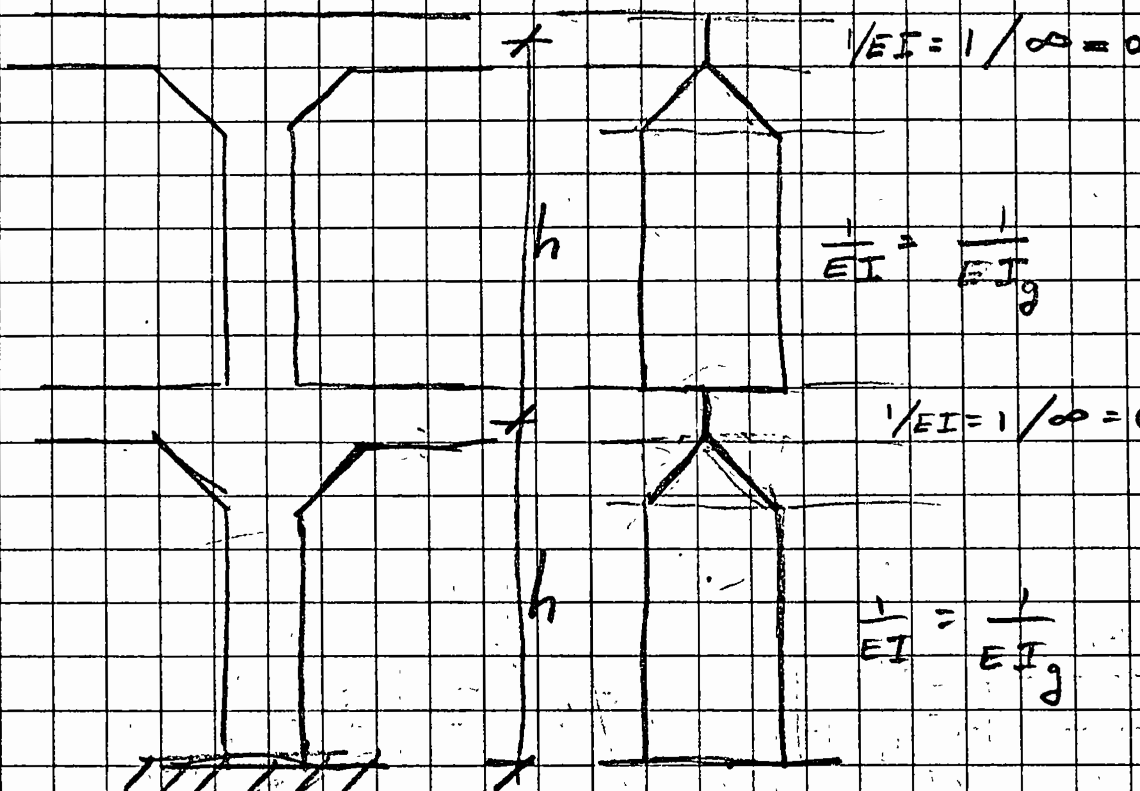
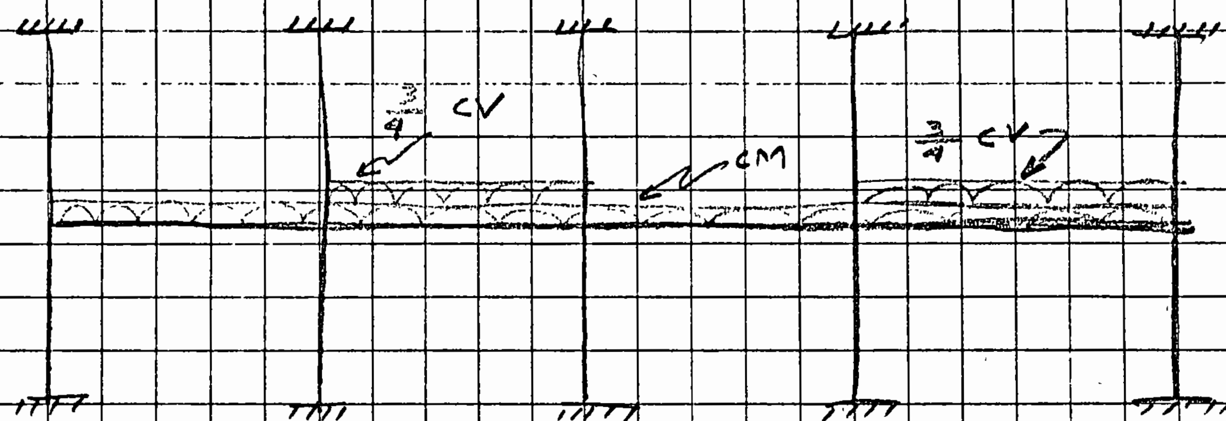
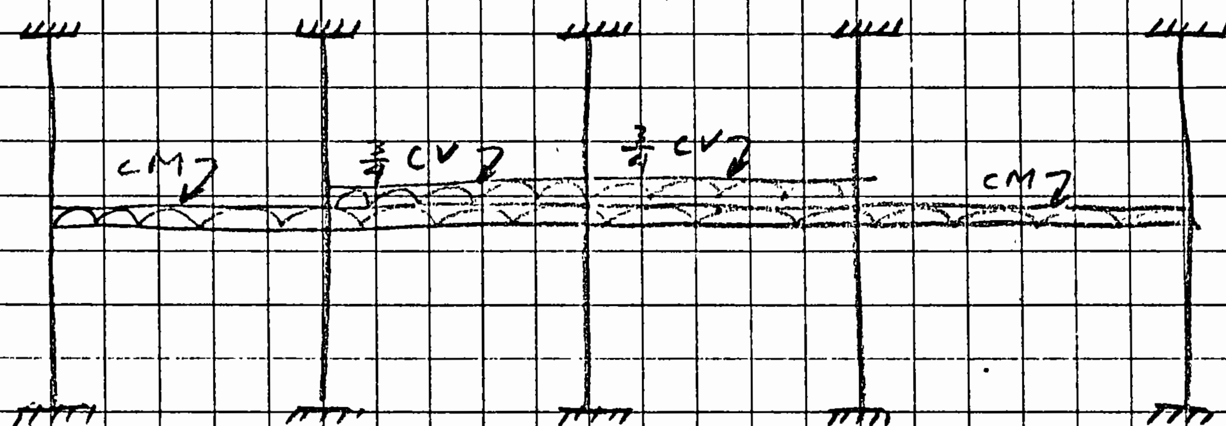


Fig. 14.10. Cálculo de la rigidez a flexión de las columnas equivalentes.

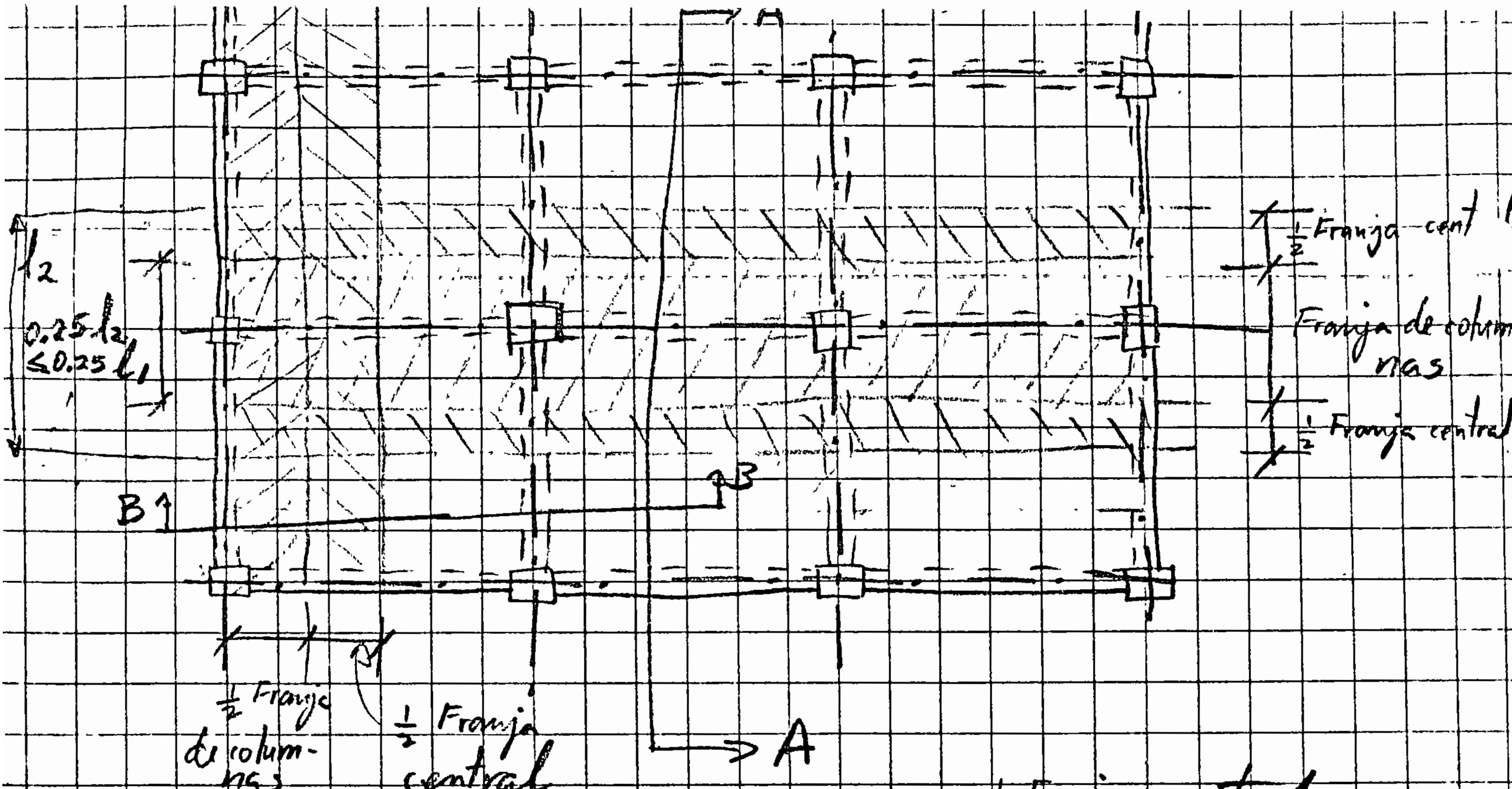


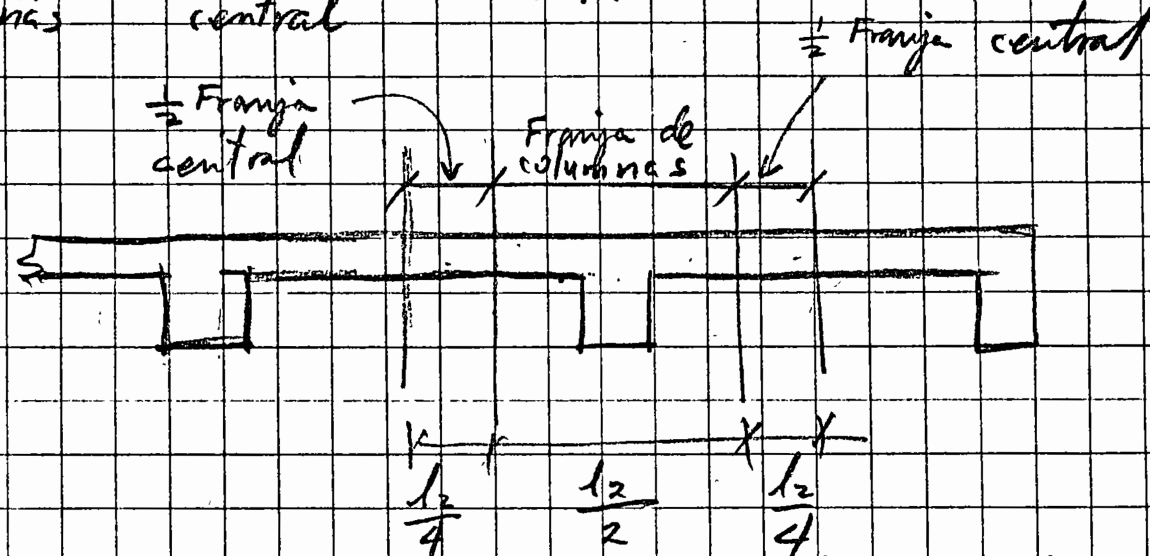
a) Condición de carga desfavorable para momento positivo



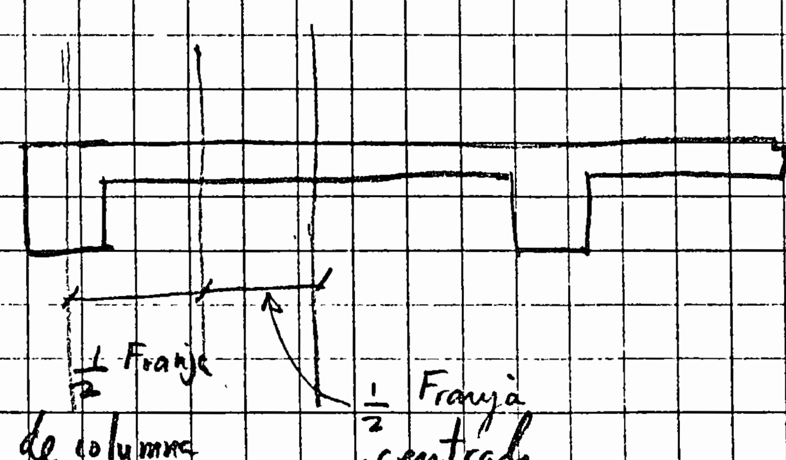
b) Condición de carga desfavorable para momento negativo

Fig. 14.11 - Condiciones desfavorables de carga





Sec. A-A (tablero interior)



Sec. B-B (Tablero de borde)

Fig. 14.12- División del sistema de piso en franjas de columnas y franjas centrales