DIVISION DE EDUCACION CONTINUA
FACULTAD DE INGENIERIA U.N.A.M.

X CURSO INTERNACIONAL DE INGENIERIA SISMICA

ANALISIS DE RIESGO SISMICO

SISMOLOGIA Y TECTONICA DE PLACAS
ONDAS SISMICAS
SISMOMETROS Y SISMOGRAMAS
DETERMINACION DEL EPICENTRO
ESCALAS DE MAGNITUD E INTENSIDAD
CONSTITUCION DE LA TIERRA
SISMICIDAD
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QUE HACER EN CASO DE SISMO

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1. Sismología y Tectónica de Placas
(Teoría del rebote elástico)

SISMOLOGIA Y TECTONICA DE PLACAS

La ciencia que estudia los aspectos relacionados con la ocurrencia de temblores de tierra o sismos es llamada Sismología. Esta es una ciencia joven ya que gran parte de sus métodos e instrumental fueron desarrollados durante este siglo.

A pesar de ésto, la Sismología ha logrado avances notables. Quizá una de sus más valiosas contribuciones al entendimiento de nuestro planeta lo constituya su aportación a la llamada TECTONICA DE PLACAS.

Para esbozar esta teoría consideremos en primer lugar la estructura interna de la tierra. En la figura 1 podemos ver esquemáticamente su constitución. El núcleo terrestre está probablemente compuesto de fierro y niquel. El manto terrestre tiene una composición a base de silicatos ferromagnesianos mientras que la corteza está compuesta por silicatos abundantes en potasio, sodio y calcio. El cascarón más externo de la tierra, el cual comprende la corteza y parte del manto, con un espesor de aproximadamente 100 km parece comportarse como un cuerpo rígido "flotando" en el resto del manto en den-

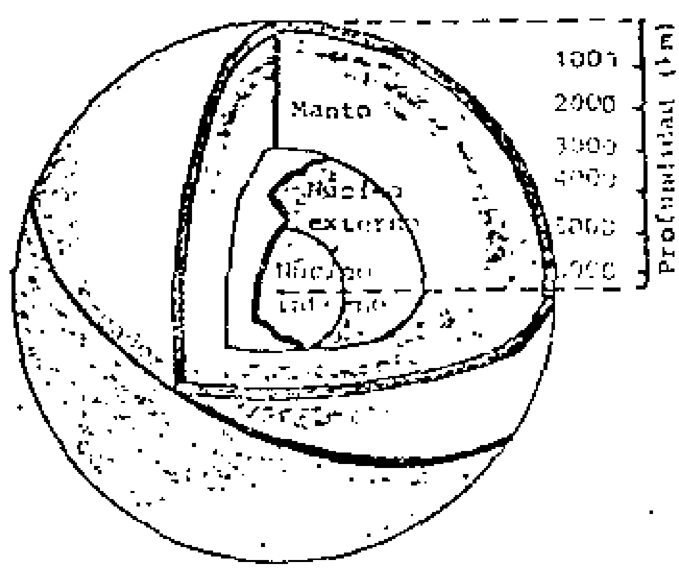


Fig. 1

de pueden presentarse movimientos como si se tratara de un fluido. Esta conducta semejante a la de un fluido tiene sentido solamente en tiempos geológicos, es decir en tiempos del orden de millones de años.

El cascarón exterior llamado litósfera no es continuo sobre la superficie de la tierra sino que está formado por diferentes "placas" en contacto una con otra.

Las placas sufren movimientos relativos debidos a fuerzas de origen aún no completamente conocidos, aplicadas a lo largo de las mismas. Estos mismos esfuerzos producen en algunos de sus márgenes la subducción de una placa bajo la otra y en otras la creación de nueva litósfera (Figura 2). Debido a estos movimientos los continentes han variado su posición relativa a través del tiempo geológico y se cree que en un tiempo estuvieron todos reunidos en un gran continente llamado Pangea. Esto nos explica el ajuste que existe entre, por ejemplo, las costas de Sudamérica y África. ¿Cuál es la distribución geográfica de estas placas? La Figura 3 nos la muestra. Las zonas de creación de nueva litósfera se presentan como cordilleras submarinas y las zonas de subducción forman a menudo trincheras submarinas de

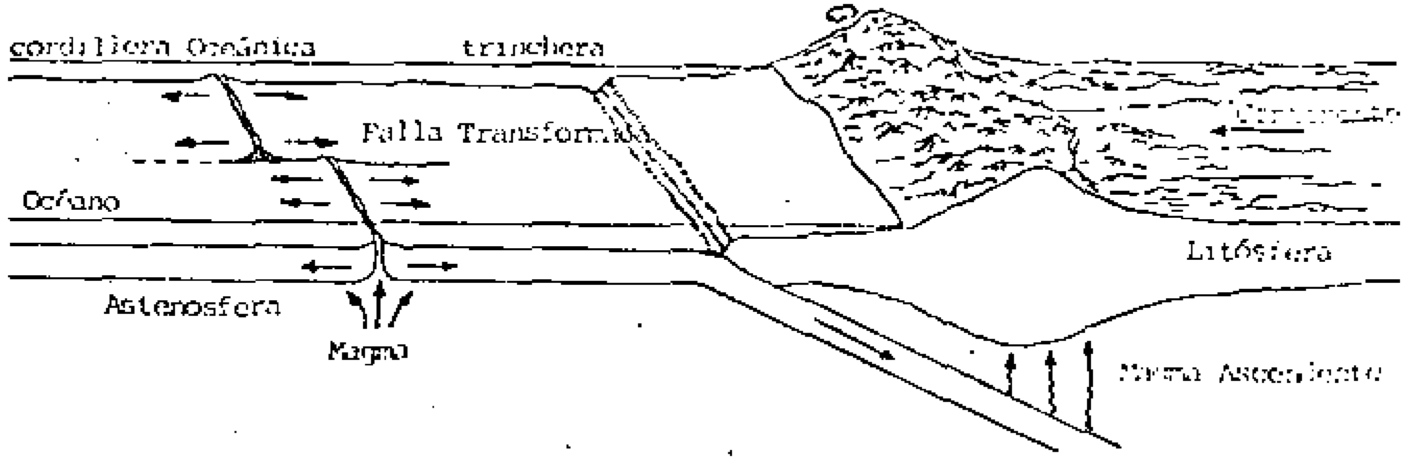


Fig. 2

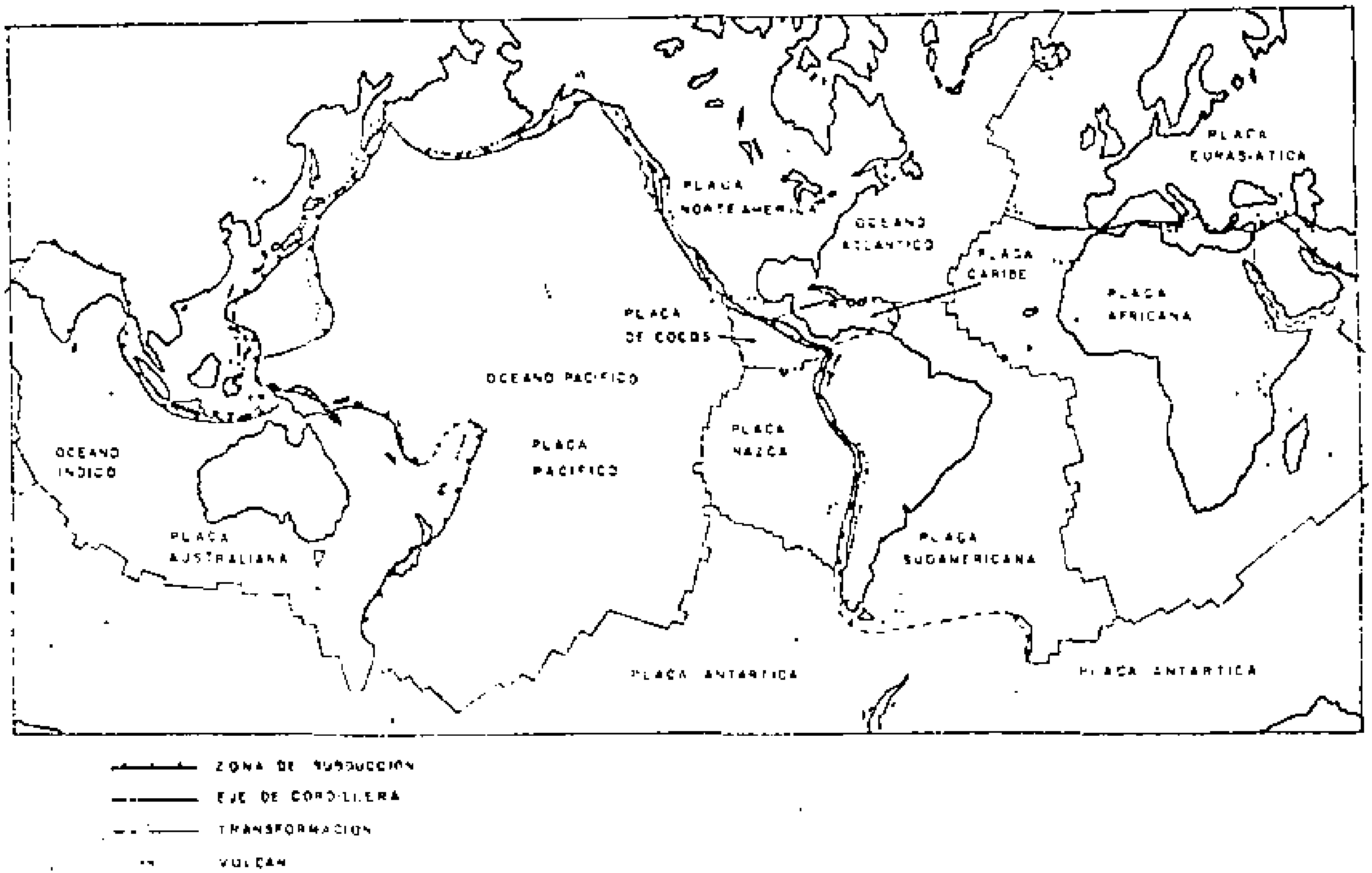

PLACA
EURASIATICA
PLACA
NORTE AMERICA
OCEANO
ATLANTICO
PLACA
CARIBE
PLACA
AFRICANA
PLACA
DE COCOS
OCEANO PACIFICO
PLACA
NAZCA
PLACA
PACIFICO
OCEANO
INDICO
PLACA
AUSTRALIANA
PLACA
SUDAMERICANA
PLACA ANTARTICA
PLACA ANTARTICA
ZONA DE SUBDUCCION
EJE DE CORDILLERA
TRANSFORMACION
VOLCAN

Fig. 3

gran profundidad. Podemos también notar que las diferentes placas no coinciden con los continentes y los océanos, sino que pueden tener corteza continental y oceánica.

No se sabe con certeza que causa los esfuerzos que producen los movimientos de las placas pero se cree que éstos son producidos por transferencia convectiva de calor, de la misma manera como ocurre cuando se hierve agua o cualquier otro líquido. El fluido más cercano a la fuente de calor se expande, se vuelve de esta manera menos denso y tiende por lo tanto a subir a la superficie donde es enfriado y desplazado hacia el fondo por nuevas parcelas ascendentes (Figura 4).

Este tipo de corrientes de convección pueden existir en el manto terrestre aunque no debe por ésto suponerse que el mismo se encuentra en estado de fusión como las lavas. Ya se ha mencionado que ésto sólo tiene sentido en tiempos muy largos. Una manera de visualizar ésto es considerar un cierto

Fig. 4

volumen de roca. Si aplicamos a ésta un esfuerzo tensional por un tiempo corto la roca vuelve a su posición inicial. Si por el contrario aplicamos el esfuerzo por un período prolongado de tiempo la roca quedará deformada permanentemente (Fig. 5). En este último caso la roca "fluye" y se parece, en este sentido, a un fluido, ya que en éstos las deformaciones son permanentes. Esto nos explica también los plegamientos que observamos muchas veces en las cortaduras hechas en las carreteras.

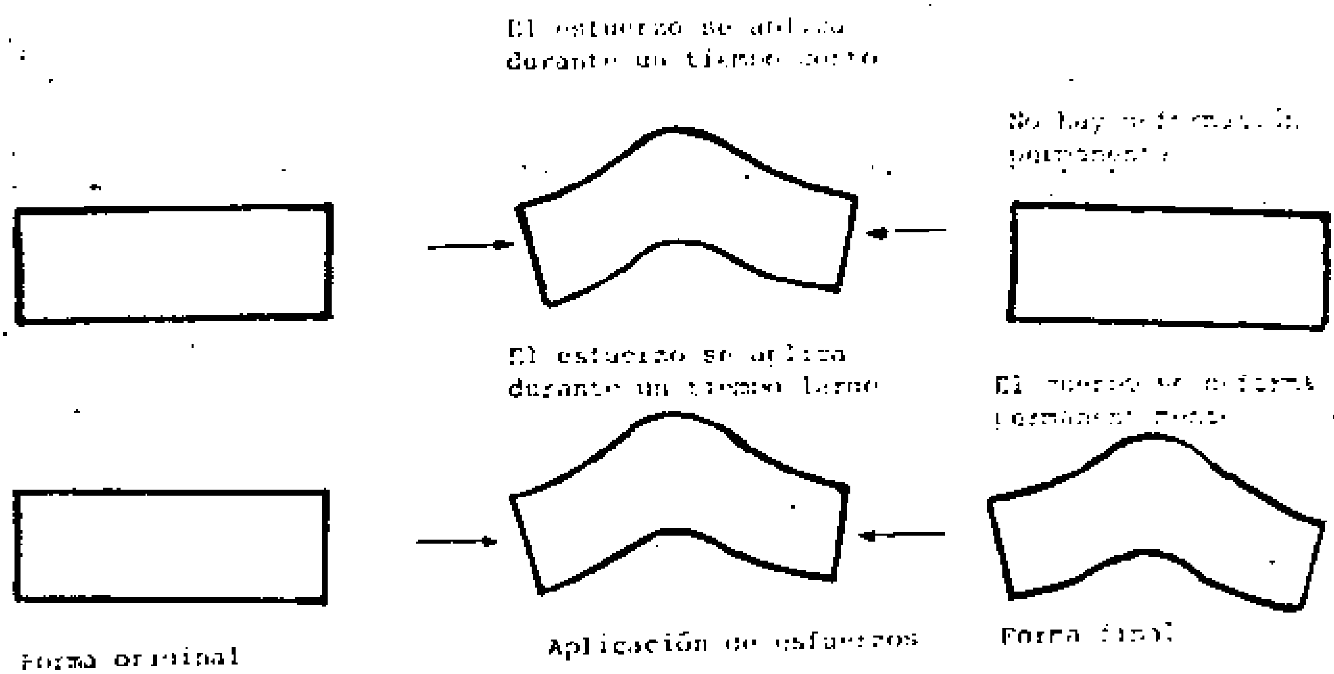


Figura 5

¿Cuál es la relación de ésto con los temblores? En primer lugar notaremos que en una zona de subducción el movimiento de una placa bajo la otra se realiza venciendo las fuerzas de fricción generadas en el contacto entre ambas. A lo largo de este contacto, llamado zona de Wadati-Benioff (WB), el movimiento de una placa contra la otra tiene lugar dicontinuamente, por "brincos". Es ésto precisamente lo que genera los temblores en esas regio nes. Para visualizar estos procesos pensemos en un bloque de cemento sobre una mesa como se muestra en la Figura 6.

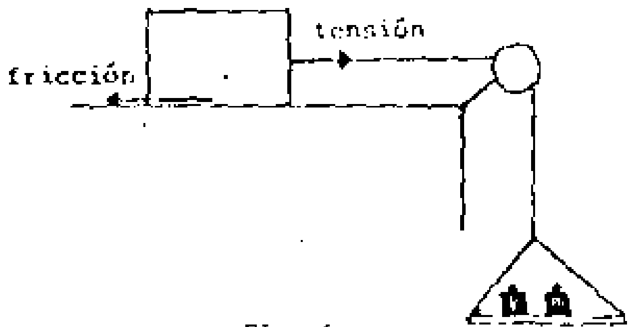


Fig. 6

Si colocamos un peso pequeño en la canastilla el bloque no se moverá debido a la fuerza de fricción entre el bloque y la mesa. Conforme aumentamos el peso la tensión en el cable continúa acumulándose hasta que iguala a la fuerza de fricción, a partir de ese momento el bloque empezará a moverse.

Análogamente en la zona W-B se acumula gradualmente hasta que rebasa un límite, en ese momento comienza a presentarse un fallamiento en algún punto llamado foco desde donde se propaga a toda una superficie (Fig. 7).

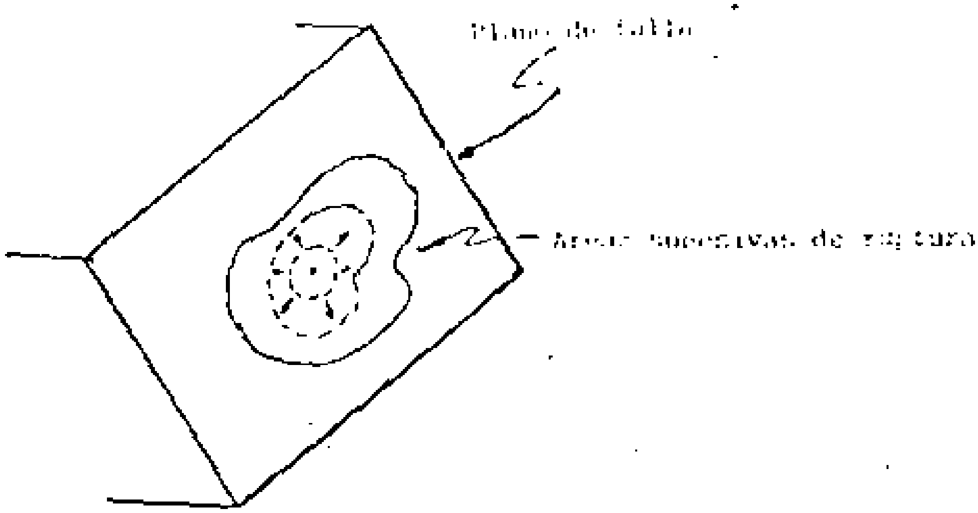


Fig. 7

Este comportamiento puede ser observado cuando el contacto entre placas aflora en la superficie de la tierra como en la famosa Falla de San Andrés en California. De hecho, fue en observaciones hechas en esta falla que pudo deducirse este mecanismo que es conocido como la TEORIA DEL REBOTE ELASTICO. Esto ocurrió durante el sismo de San Francisco en 1906. La Figura 8a muestra las dos placas durante el movimiento lateral que produce la acumulación de esfuerzos. En la Figura 8b los esfuerzos rebasan cierto límite y el fallamiento se produce en un punto y se propaga en ambas direcciones. La Figura 8c muestra la situación después del temblor; existe ahora

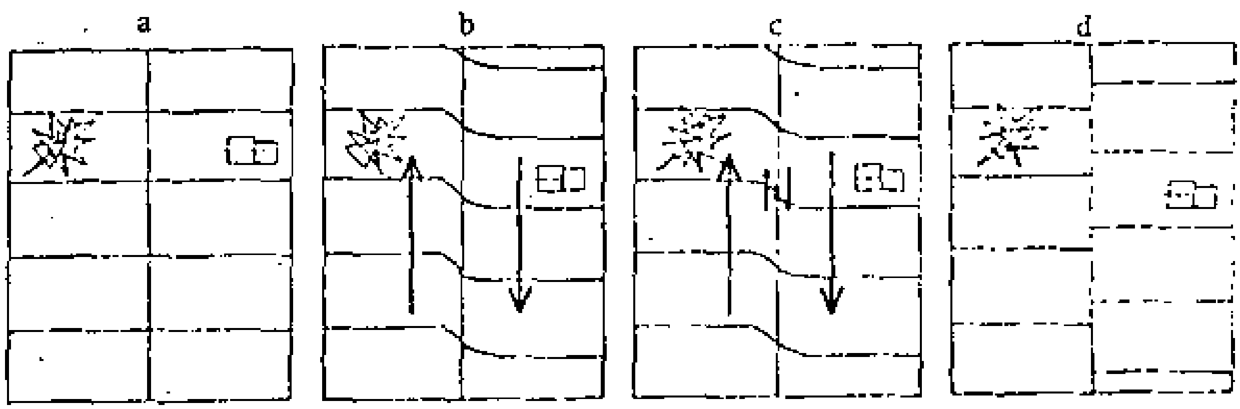


Fig. 6

un desplazamiento permanece entre ambas caras de la falla.

Aunque este proceso puede parecer intuitivamente obvio, en realidad no lo es. Durante mucho tiempo se pensó que el fallamiento de la corteza era un efecto de los temblores y no su origen. Como fuentes de éstos se pensaba en intrusiones de magma o colapso de volúmenes por cambios de densidad de las rocas que componen la corteza. Aunque estos mecanismos pueden ocurrir, se piensa en la actualidad que la mayoría de los temblores en las regiones de subducción se originan por el mecanismo expuesto y son llamados "tectónicos". Otros tipos de sismos están asociados a fenómenos locales como son los volcánicos o algunos otros debido p. ej. al colapso del subsuelo por pérdida de agua, etcétera.

ONDAS SISMICAS

Si desplazamos un diapazón de su posición de equilibrio y lo soltamos repentinamente percibimos su sonido característico (Fig. 9). Lo mismo sucede

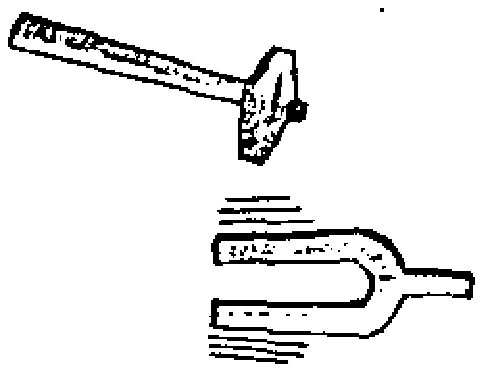


Fig. 9

en la tierra, hemos visto que el fallamiento de la roca consiste precisamente en la liberación repentina de los esfuerzos impuestos al terreno. De esta manera, la tierra es puesta en vibración. Esta vibración es debida a la propagación de ondas como en el caso del diapasón.
Ahora bien, en un sólido pueden transmitirse dos tipos de ondas. El primer tipo de ondas es conocido como compresional porque consiste en la transmisión de compresiones y rarefacciones como en el caso de la transmisión del sonido, en este caso las partículas del medio se mueven en el mismo sentido en que se propaga la onda. El segundo tipo es conocido como ondas transversales o de cizallamiento; las partículas se mueven ahora en dirección perpendicular a la dirección de propagación de la onda.

La figura 10 muestra esquemáticamente la propagación de estas ondas en un bloque sólido.

Las ondas compresionales y transversales han sido llamadas P y S respectivamente por razones que se verán más adelante. Son también conocidas como ondas internas porque pueden viajar en el interior de un sólido elástico.

Además de estas dos clases de ondas pueden existir otros dos tipos más llamadas superficiales. Estas ondas viajan en la superficie de la tierra y su

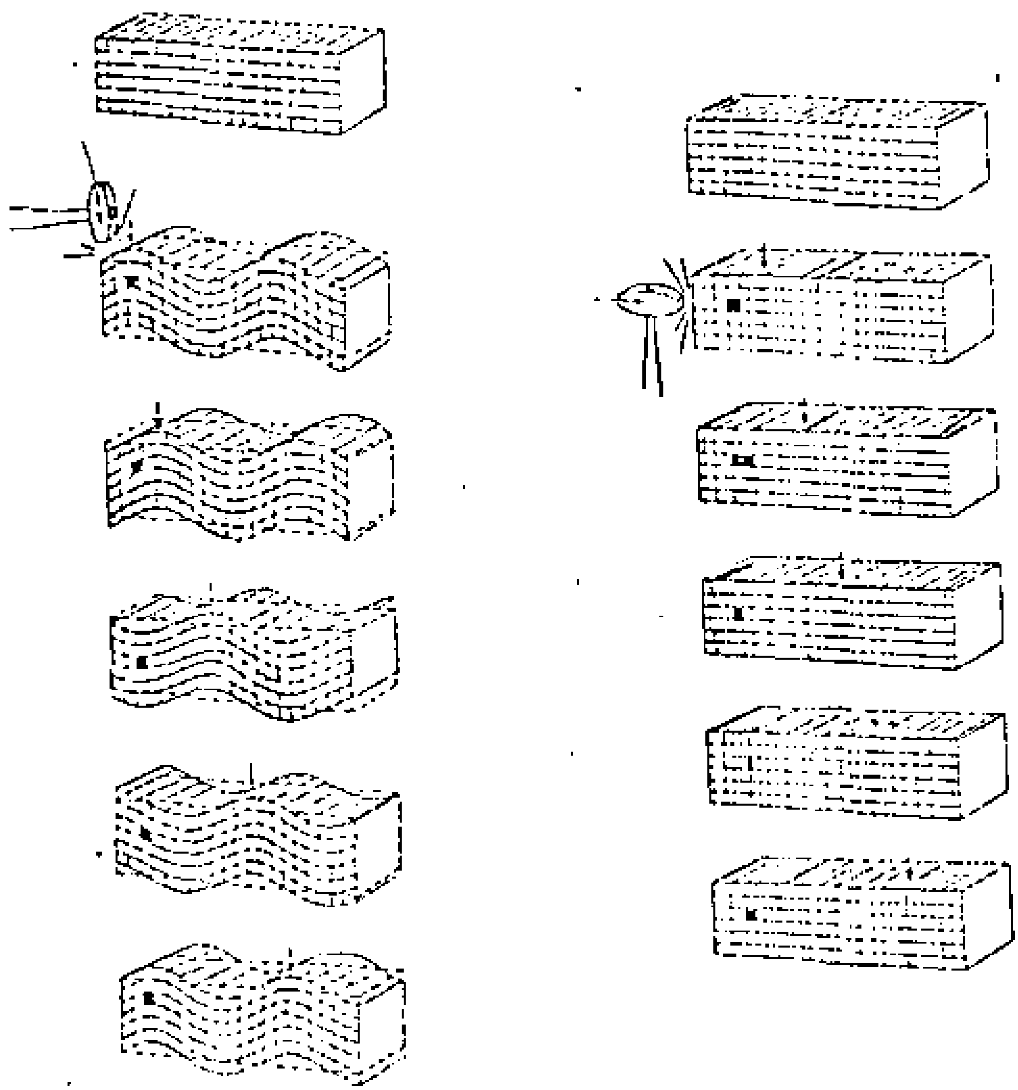


Fig. 10

amplitud decrece con la profundidad. Se les ha denominado con el nombre de los científicos que demostraron teóricamente su existencia: Rayleigh y Love.

Las ondas de Rayleigh se originan en la superficie de un sólido elástico, es decir, estas ondas no podrían generarse en un medio infinito y se caracterizan por la trayectoria elíptica retrógrada que describen las partículas al propagarse la onda. Esta trayectoria ocurre en el plano de propagación de la onda (Fig. 11).

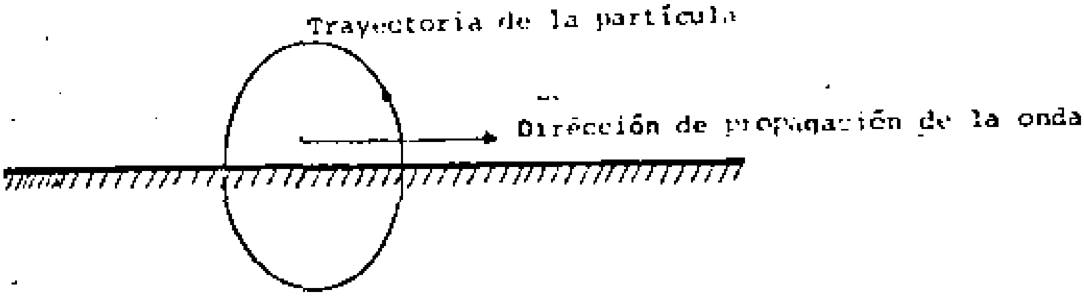


Fig. 11

Por otro lado, las ondas de Love ocurren cuando existe una interfase entre dos medios elásticos de distintas propiedades. Como las ondas S, las ondas de Love ocurren con un movimiento de las partículas perpendicular a la dirección de propagación, sólo que, polarizado en el plano de la superficie terrestre (Fig. 12).

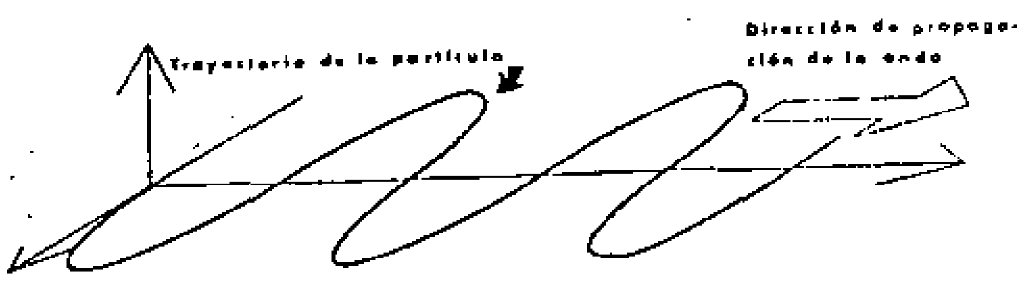


Fig. 12

¿Cuál es la velocidad de estas ondas? Se puede demostrar teóricamente y se observa experimentalmente que la velocidad de las ondas es tal que:

$$V_L < V_s < V_p$$

donde V_p, V_s y V_L son las velocidades de la onda P, S y superficiales respectivamente.

Las velocidades de las diferentes ondas dependen de las características del médio; por ejemplo, en rocas ígneas la velocidad de las ondas P es del orden de 6 km/seg mientras que en rocas poco consolidadas es de aproximadamente 2 km/seg o menor. Así, las ondas P de un terremoto originado en la Costa de Acapulco serían sentidas en la ciudad de México en menos de 2 minutos.

SISMOGRAFOS Y SISMOGRAMAS.

Los mecanismos para detectar los temblores fueron ideados a fines del siglo pasado y perfeccionados a principios de éste. Actualmente estos instrumentos han alcanzado un alto grado de sofisticación, pero al principio básico empleado no ha cambiado. Si tomamos en cuenta que al ocurrir un temblor el suelo se mueve, entonces para poder observar este movimiento tendríamos que estar en un punto fijo fuera de la tierra para no sufrir nosotros mismos ese movimiento y poder detectarlo; ésto obviamente es imposible. Sin embargo, es posible construir un mecanismo que pueda medir este movimiento relativo. El mecanismo consiste de una masa suspendida de un resorte atado a un soporte acoplado al suelo (Fig. 13), cuando el soporte se sacude al paso de las ondas sísmicas, la inercia de la masa hace que ésta permanezca un instante

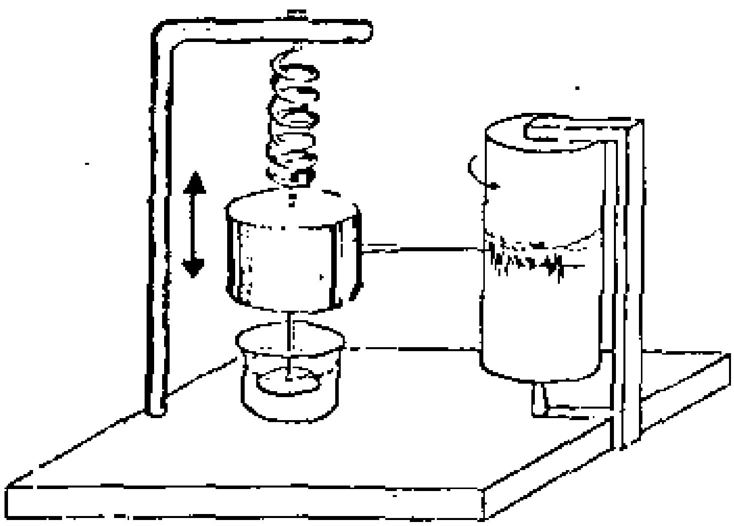
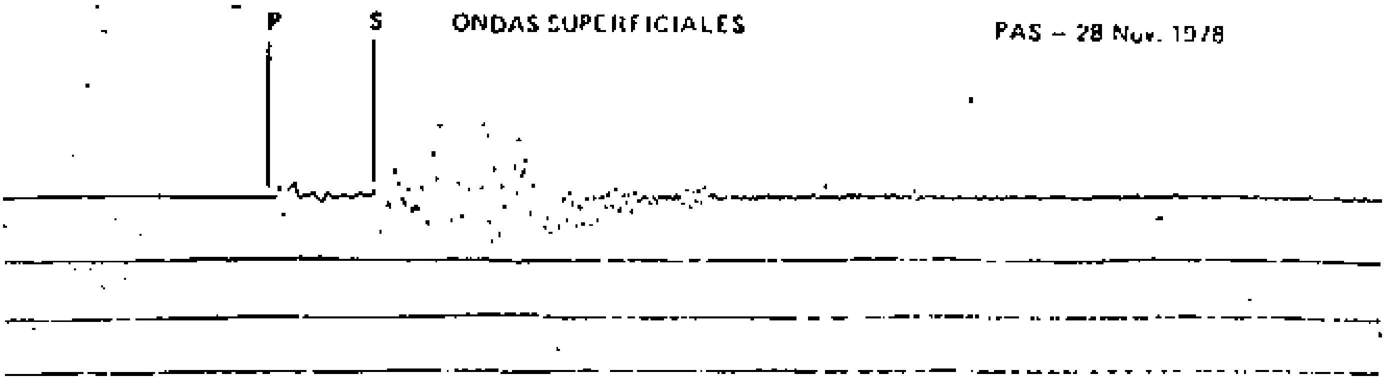


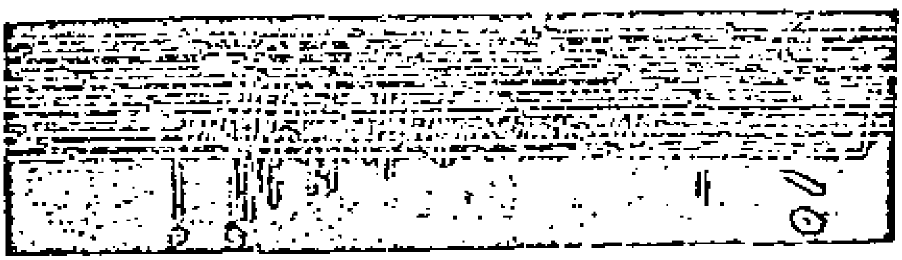
Fig. 13

en el mismo sitio de reposo. Posteriormente cuando la masa sale del reposo, oscila. El movimiento posterior del péndulo no refleja el movimiento del suelo, por lo cual se ha ideado un método para volver a la masa a su sitio original, ésto es lo que se conoce como amortiguamiento del aparato. En la Figura 13 se representa el amortiguamiento como una lámina sumergida en un líquido (comúnmente aceite).

Si se sujeta un lápiz de la masa suspendida para que pueda inscribir sobre un papel pegado sobre un cilindro que gira a velocidad constante, se podrá registrar sucesivamente el movimiento del suelo. El instrumento, hasta aquí descrito, para detectar la componente vertical del movimiento del suelo, se conoce como sismógrafo vertical y el papel donde se inscribe se llama registro o SISMOGRAMA. Sismogramas típicos se muestran en la Figura 14. Los movimientos del suelo también tienen componente horizontal y para me-



Sismograma de la estación sismológica de Pasadena (EE UU.) correspondiente al temblor de Oaxaca del 28 de noviembre de 1978
$\Delta \simeq$ 3 060 km.



Sismograma de la estación sismológica de Tacubaya correspondiente al temblor del día 22 de febrero de 1979, registrado a las 03h 16' 55'' y localizado en el "Eje Volcánico Central". Distancia (Δ) de la estación de Tacubaya 120 km

Fig. 14

dir este movimiento se requiere de péndulos horizontales que oscilan como una puerta que tiene su eje inclinado (Fig. 15a). El sismógrafo horizontal se representa en la (Fig. 15b).

Los sismógrafos que se emplean actualmente, en general tienen masas que pueden ser de unos gramos hasta 100 kg, mientras que los sismógrafos antiguos de amplificación mecánica solían tener grandes masas con el fin de vencer las fuerzas de rozamiento, tal es el caso del sismógrafo horizontal Wiechert de 17000 kg de la estación sismológica de Tacubaya (apéndice A); el amortiguamiento se hace por corrientes parásitas o imánes, etcétera, la amplificación por medio de palancas y galvanómetros y la inscripción en papel ahumado, papel fotográfico o cinta magnética.

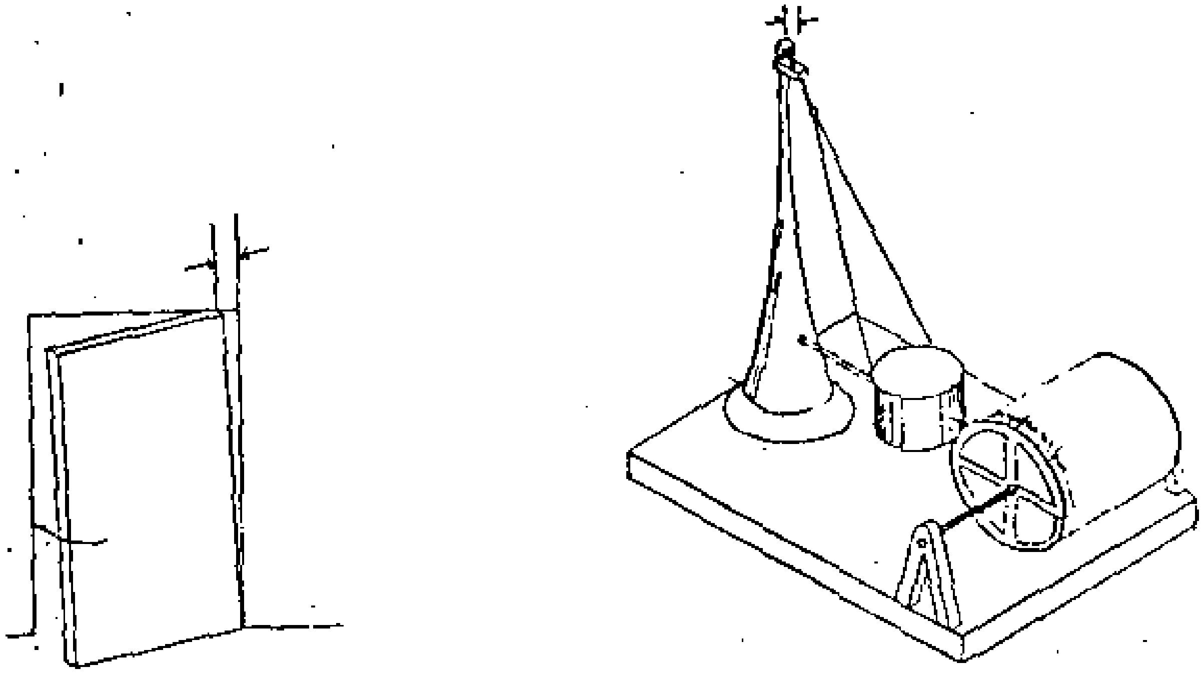


Fig. 15

Los Sismómetros son los sismógrafos cuyas constantes físicas son conocidas de tal manera que se puede conocer el movimiento real del suelo calculado directamente de los sismogramas.

Para determinar con precisión el epicentro de un temblor se requiere del auxilio de varias estaciones sismológicas, por lo cual los observatorios sismológicos requieren por lo menos de tres estaciones sismológicas o formando redes de éstas. Tal como la Red Sismológica Mexicana (ver apéndice B) que controla el Servicio Sismológico Nacional, organismo encargado de la generación de datos e información sismológica. En México existen otras redes de proyectos específicos como RESMAC*, RESNOR**, y SISMEX***. A ni-

* "RED SISMICA MEXICANA DE APERTURA CONTINENTAL" operada por el Instituto de Matemáticas Aplicadas y Sistemas de la UNAM.

** "RED SISMOLOGICA DEL NOROESTE" operada por el Centro de Investigación y Enseñanza Superior de Ensenada, B.C.

*** "SISTEMA DE INFORMACION SISMOTECTONICA DE MEXICO" operada por el Instituto de Ingeniería de la UNAM.

vel mundial existen convenios para el intercambio de datos entre los diferentes observatorios, formando así todas las estaciones la red mundial.

DETERMINACION DEL EPICENTRO

Hemos mencionado que el lugar en que comienza el fallamiento que produce los temblores es llamado foco. A grandes distancias el plano completo de ruptura aparece como un punto y lo llamamos foco; la proyección de éste sobre la superficie terrestre recibe el nombre de epicentro (Figura 16).

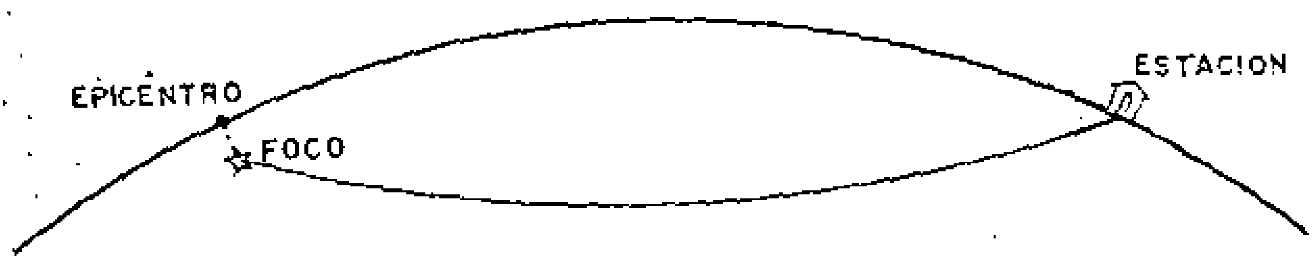


Fig. 16

¿Cómo determinan los sismólogos la ubicación del epicentro?. Ya se dijo que los sismógrafos amplifican e inscriben el movimiento del suelo en una tira de papel (o cualquier otro tipo de material similar) que se llama registro o sismograma. En el sismograma se registran en orden sucesivo de tiempo los diferentes tipos de ondas generadas por un temblor y que arriban a la estación sismológica, como se puede apreciar en la Figura 14. La ubicación del epicentro de un temblor se hace analizando sus registros e identificando los diferentes tipos de ondas como se muestra en la Figura

20. Se ha mencionado ya que la velocidad de las ondas P es mayor que la de las ondas S. Este hecho es utilizado en una de las técnicas más comunes de la Sismología para determinar el epicentro. En efecto, supongamos que la persona A es más veloz que la persona B (Fig. 17). Si ambas empiezan a correr desde el punto 0 en el momento que están juntas a medida que se alejan de 0 la distancia entre ellas será mayor. Puede utilizarse la separación entre ellas en un punto dado para calcular la posición del origen a partir de ese punto.

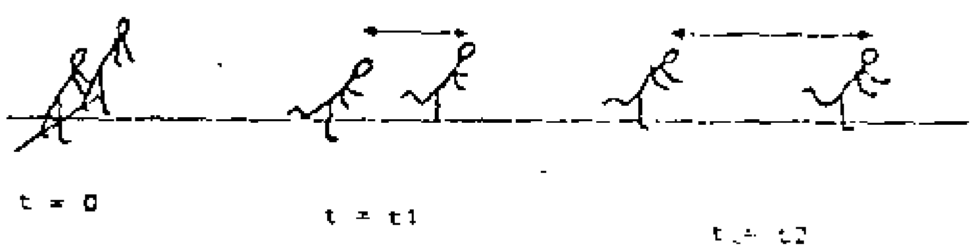


Fig. 17

Sobre la superficie de la tierra, una estación puede proporcionar la distancia al epicentro pero no su dirección de manera que son necesarias al menos tres estaciones para determinarlo sin ambiguedad (Fig. 18).

En la práctica, la intersección de los círculos correspondientes a las tres estaciones no coincide en un solo punto sino comprende una región más o menos grande dependiendo de la calidad de los datos utilizados. La información obtenida de estaciones adicionales es tratada estadísticamente en otras técnicas sismológicas para refinar la posición.

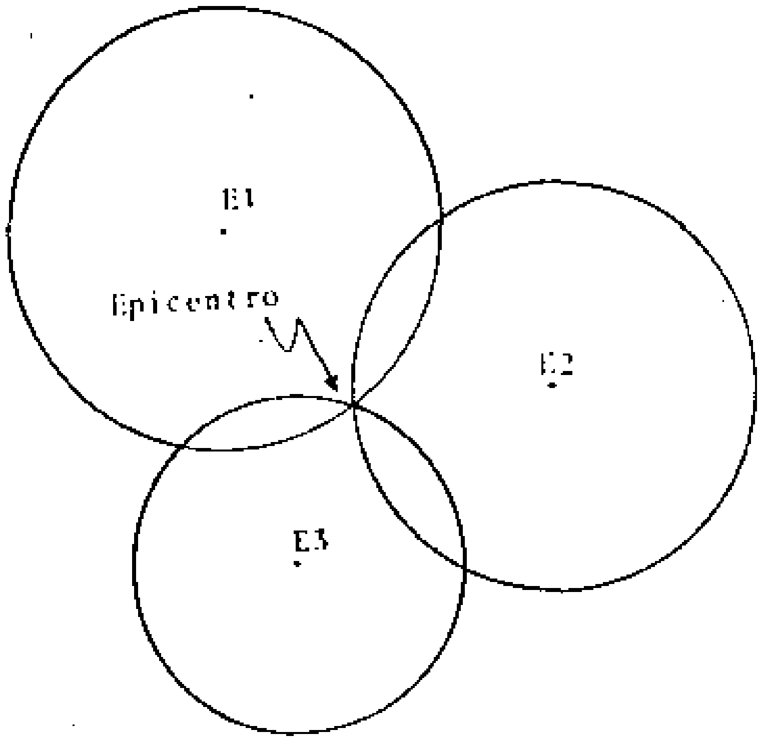


Fig. 18

ESCALA DE MAGNITUD E INTENSIDAD.

Las escalas de magnitud e intensidad son utilizadas para cuantizar o medir los temblores. La escala de magnitud está relacionada con la energía liberada como ondas sísmicas; la de intensidad con los daños producidos por el sismo. Ambas escalas son necesarias puesto que miden aspectos diferentes de la ocurrencia de un temblor. Así la escala de magnitud está relacionada con el proceso físico mismo mientras que la de intensidad lo está con el impacto del evento en la población, las construcciones y la naturaleza.

Como es natural, una clasificación de los temblores por medio de sus efectos, que son observables, fue el primer intento de catalogarlos. Escalas de intensidad fueron propuestas desde los últimos años del siglo pasado. En 1902 Mercalli propuso una tabla, que fue posteriormente modificada en 1931 y desde entonces se le ha llamado escala Modificada de Mercalli (MM). Esta no es la única; pero si la más frecuentemente usada en nuestro continente. Consta de 12 grados como puede apreciarse en la Tabla 1 donde se muestran también las características de cada grado.

Podemos ver que la escala de intensidad es en gran medida subjetiva. No nos da información sobre la energía liberada en el temblor puesto que por ejemplo un sismo pequeño puede causar más daños a una población, si ésta está cercana al epicentro, que uno grande pero a mayor distancia.

Así pues es necesario catalogar temblores de acuerdo con los procesos físicos de la fuente; pero también de manera tal que puedan ser medidos. Desde el punto de vista físico sería conveniente clasificar los temblores de acuerdo con la energía que disipan y aunque podríamos hacerlo, no tenemos instrumentos que puedan medirla directamente.

Resulta entonces necesario encontrar una metodología para poder precisar no sólo el epicentro del sismo sino la magnitud y fecha del mismo.

Poseemos sin embargo sismogramas y éstos pueden ser utilizados para catalogar temblores de una manera racional como se verá a continuación.

De dos temblores ocurridos en el mismo epicentro y registrados en el mismo lugar, el más débil producirá un trazo pequeño en el papel y el más fuerte un trazo grande. Para un mismo sismo y estaciones que se alejan gradualmente del epicentro la traza se hace igualmente menor (Fig. 19).

TABLA 1

ESCALA MODIFICADA DE MERCALLI

I. Microsismo.
Detectado por instrumentos.

II. Sentido por algunas personas
(generalmente en reposo).

III. Sentido por algunas personas dentro de edificios.

IV. Sentido por algunas personas fuera de edificios.

V. Sentido casi por todos.

VI. Sentido por todos.

VII. Las construcciones sufren daño moderado.

VIII. Daños considerables en estructuras.

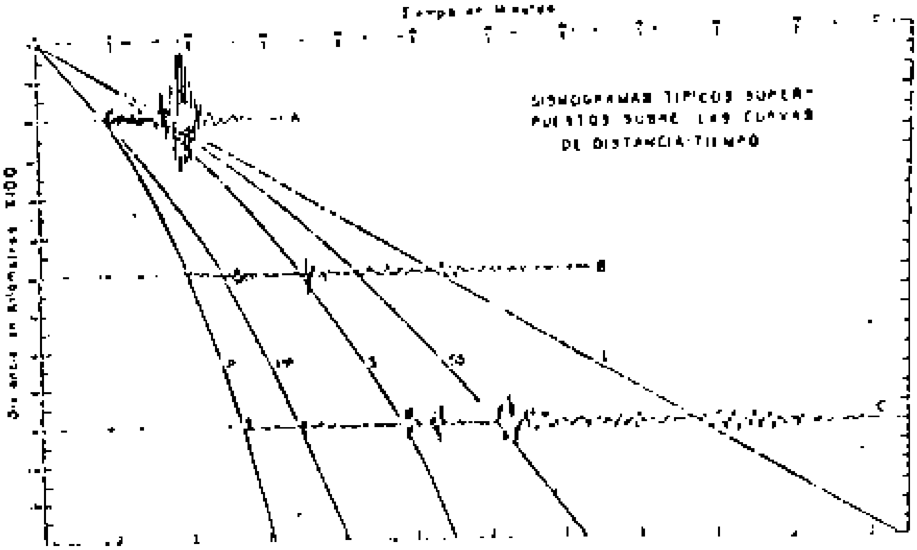
IX. Daños graves y pánico general.

X. Destrucción en edificios bien construidos.

XI. Casi nada queda en pie

XII. Destrucción total.

* Una escala más detallada aparece en el Apéndice D.



Sismogramas típicos superpuestos sobre las curvas de distancia-tiempo

Fig. 19

Si se grafican los valores del logaritmo de la amplitud de la traza contra la distancia, se obtienen gráficas como las mostradas en la Fig. 20. En esa misma figura, la curva más baja representa un temblor más pequeño. Resulta entonces lógico tomar cualquiera de estos sismos como el sismo patrón

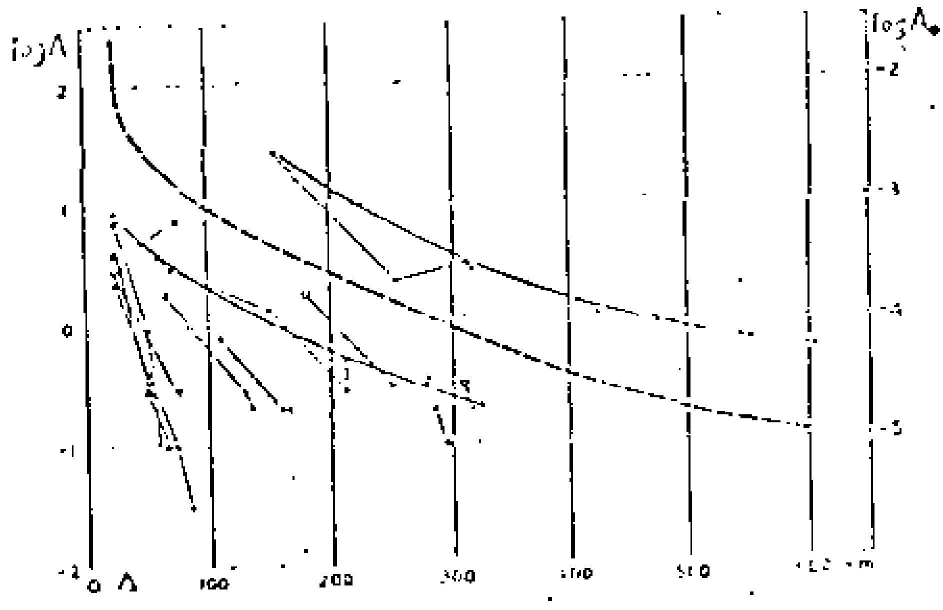


Fig. 20

y asignarle la magnitud cero, los demás pueden ser medidos a partir de éste midiendo la separación entre ellos para cualquier distancia del epicentro. Se tiene entonces que:

$$M = \log a - \log A_o$$

El temblor patrón, de magnitud cero se define como aquel que teniendo su epicentro a 100 km de distancia deja una traza de una micra en un sismógrafo Wood-Anderson elegido también como sismógrafo patrón.

Se tiene ahora una fórmula que nos proporciona un valor relacionado con el "tamaño" del sismo e independiente de los daños que pueda ocasionar. Este mismo valor ha sido relacionado por los sismólogos con la energía liberada por el sismo. Existen diferentes fórmulas que relacionan la energía con la magnitud de un sismo, éstas varían porque la amplitud medida en el sismograma puede ser la de cualquiera de las distancias fases (P, S, superficiales) que son registradas.

Un temblor de magnitud 5.5 libera una energía del orden de magnitud de la de una explosión atómica*, es decir unos 10^{20} ergs. Sin embargo, la energía de un sismo de magnitud 8.5 no es tres veces esa energía sino la equivalente a la de unas 27,000 de estas bombas atómicas, ésto es, la energía aumenta aproximadamente 30 veces por cada grado. Esto puede verse más claramente en las fórmulas que relacionan magnitud y energía; éstas son de la forma:

$$\log E = a + bM$$

donde a y b dependen de la forma en que es calculada M.

Notemos que la escala de magnitud no tiene límites; sin embargo, no se han

* como la de Hiroshima (20 Ktn de TNT).

encontrado temblores mayores de 8.6. Esto está relacionado con el hecho de que la corteza tiene un límite de ruptura más allá del cual ya no pueden acumularse más esfuerzos. Un ejemplo de un temblor de esta magnitud es el de Alaska del 28 de marzo de 1964.

Notemos también que pueden existir temblores de magnitud negativa, puesto que el sismo patrón (de magnitud cero), es elegido, hasta cierto punto arbitrariamente.

La determinación de magnitudes ha sido mejorada en las últimas décadas utilizando la disponibilidad de más información y modelos teóricos. Sin embargo el principio básico es el mismo.

En el Apéndice C se detallan algunos de los sismos mexicanos más destructivos.

LA CONSTITUCION DE LA TIERRA.

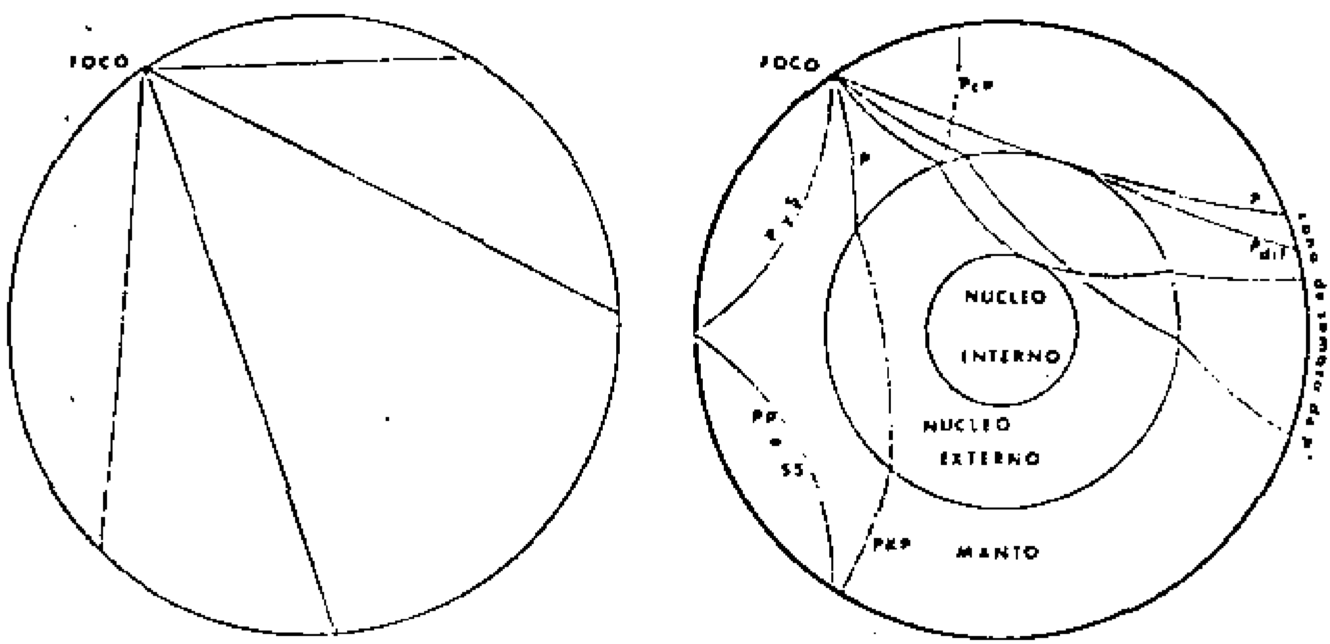
En el primer apartado de este artículo se consideró la estructura de la tierra. ¿Cómo fue posible conocerla si las perforaciones más profundas no alcanzan sino unos pocos kilómetros?. La respuesta está nuevamente en la Sismología.

De igual manera que un médico puede saber si existe fractura en los huesos de un accidentado por medio de rayos X, el sismólogo ha deducido la estructura de la tierra por medio de las ondas sísmicas que viajan a través de ella como los rayos X a través del cuerpo humano.

Supongamos que ocurre un sismo en un punto dado si la tierra fuera completamente homogénea los rayos viajarían en líneas rectas del foco al observador

(Fig. 21a).

Los sismogramas observados serían relativamente simples. Los científicos han hallado que los rayos no viajan en línea recta sino que van curvándose hacia la superficie debido a que la velocidad de las ondas aumenta con la profundidad. (Ver Fig. 21b). Además de ésto se encuentra que éstas sufren refracciones y reflecciones que sólo pueden explicarse si la tierra está compuesta por las diferentes regiones de que se habló en el primer apartado. Los temblores resultan entonces, tener un aspecto positivo y es éste el de darnos a conocer el interior de nuestro planeta.



(a) (b)

Fig. 21

SISMICIDAD

En los últimos 80 años se han podido registrar todos los temblores más importantes (en cuanto a energía) de manera que se han podido hacer estudios cualitativos de la sismicidad de la tierra, así se ha obtenido un esquema global de la sismicidad mundial. El mapa de la Figura 22 muestra la distribución geográfica de sismos. Se puede observar que la mayor parte de energía sísmica se libera en las costas del Océano Pacífico región que se conoce como cinturón de fuego debido a que en esta zona ocurre también gran actividad volcánica. Hay otras regiones, como el Atlántico medio y el cintu-

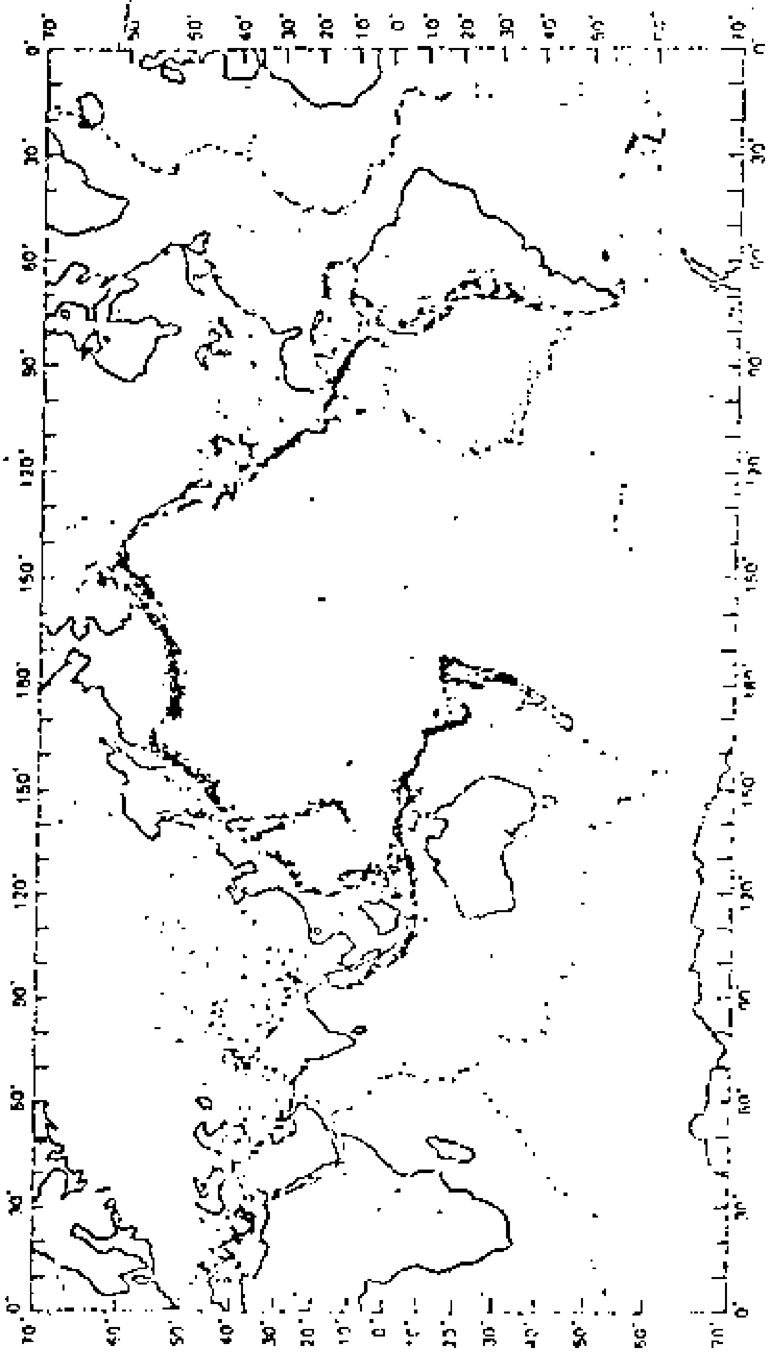


Fig. 22

rón Eurásico pero con una actividad sísmica menor. Nótese que estas franjas coinciden con los límites de placas de la Figura 3. Existen también regiones donde la actividad sísmica es casi nula o desconocida. A estas regiones se les suele llamar escudos. Desde luego que los países que se sitúan en zonas sísmicas serán más afectados por los sismos.

SISMICIDAD GLOBAL.

Observando la actividad sísmica mundial se puede estimar el número de temblores de cierta magnitud que ocurren en un año. Se ha visto que por lo menos ocurren dos grandes terremotos actualmente (ver Tabla II). Por otra parte están ocurriendo varios cientos de miles de temblores de magnitud inferior a 3 que pasan desapercibidos.

TABLA II
PROMEDIO ANUAL DE TEMBLORES

MAGNITUD	NUMERO PROMEDIO
8	2
7	20
6	100
5	3000
4	15000
3	150000

SISMICIDAD DE MEXICO

A fines del siglo pasado se conocía la historia acerca de la actividad sísmica de México desde 1400. Posteriormente con el desarrollo técnico sismológico hacia 1910 se inauguró la red sismológica mexicana (ver apéndice A). De esa fecha a la actualidad se han generado sismogramas que se conservan en el Servicio Sismológico, en la estación Sismológica de Tacubaya.

Durante los últimos 70 años se han registrado y localizado, utilizando los datos de la red sismológica mexicana, sismos ocurridos en la República Mexicana hecho por el cual actualmente se conoce bastante bien la sismicidad de México (Fig. 23).

PREMONITORES Y REPLICAS

Los sismólogos han observado que inmediatamente después de que ocurre un gran temblor, éste es seguido por temblores de menor magnitud llamados réplicas y que ocurren en las vecindades del foco del temblor principal. Se piensa que la ocurrencia de éstos se debe probablemente al reajuste mecánico de la región afectada. Inicialmente la frecuencia de ocurrencia es grande pero decae gradualmente con el tiempo dependiendo de la magnitud del Temblor principal. Por ejemplo para el temblor de Oaxaca del 29 de noviembre de 1978 de magnitud 6.8 inicialmente se observaron hasta 200 réplicas de magnitud mayor que 2.0 diariamente y fue decayendo esta actividad durante 5 meses aproximadamente. El estudio de las réplicas de un gran temblor se ha aprovechado para estimar las dimensiones de la región focal.

Frecuentemente algunos temblores grandes son precedidos por temblores de menor magnitud llamados temblores premonitores que comienzan a fracturar la región focal del gran temblor. No es fácil determinar cuando un temblor queño es un premonitor de un gran temblor ya que se suele confundir con cualquier otro no relacionado. En la generalidad de los casos se sabe que un temblor es premonitor sólo en el contexto de la actividad posterior.

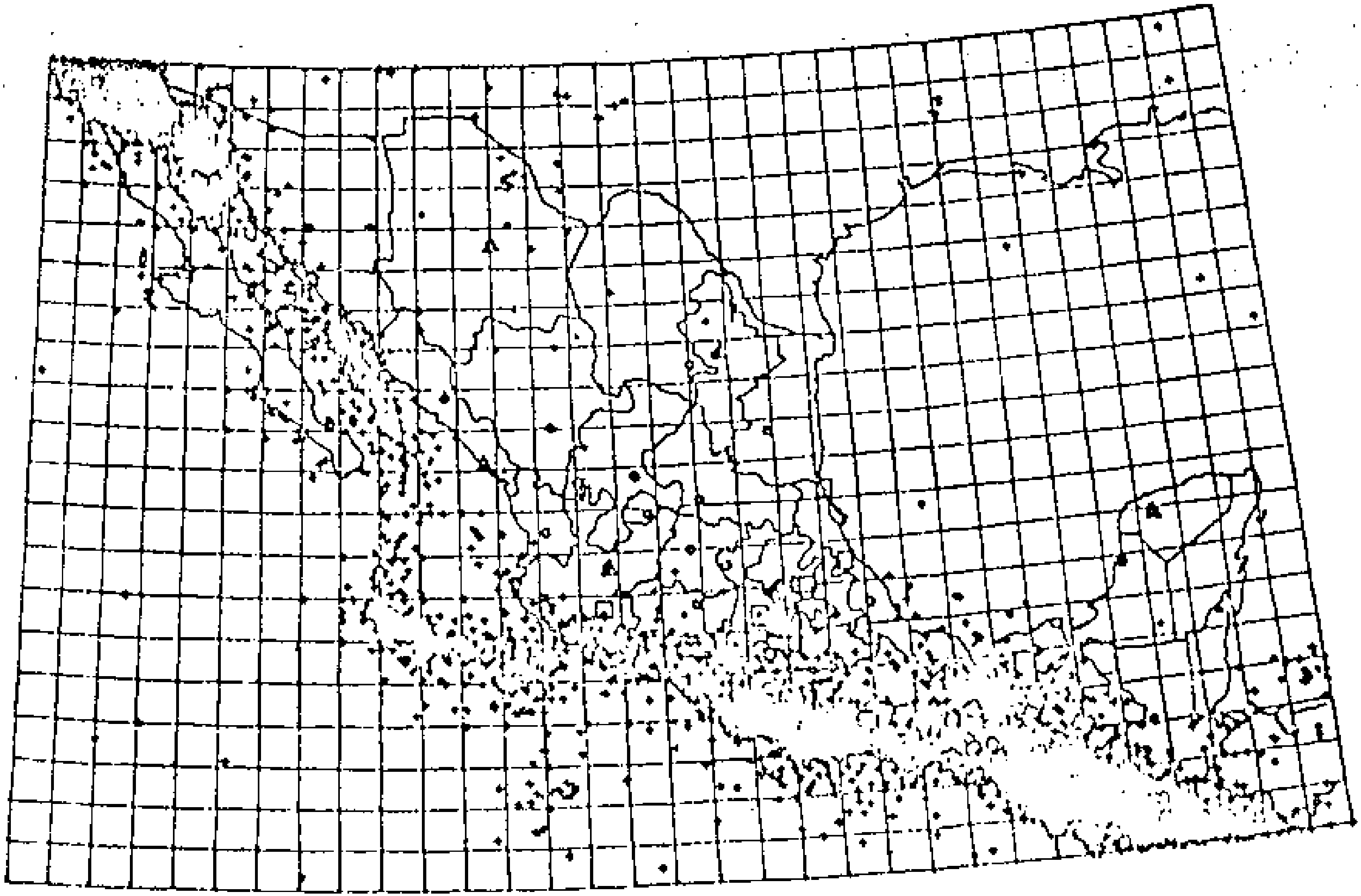


Fig. 23 Sismicidad de México, de 1900 a 1974.

PREDICCION

¿Se pueden predecir los temblores? La respuesta a esta pregunta depende de lo que se entienda por predicción. Año tras año podemos leer en los periódicos las declaraciones hechas por adivinadores, mediums y otras gentes por el estilo, sobre la futura ocurrencia de temblores en algún lugar del planeta. Estas declaraciones distan mucho de ser predicciones. Se ha visto (Tabla II)que en promedio ocurren cerca de 120 temblores de magnitud mayor a 6 anualmente. Se conocen también las zonas sísmicas del planeta, de manera que por ejemplo el afirmar que durante el año de 1981 ocurrirá un temblor en la costa occidental de México no contiene información novedosa ni útil.

En la última década el desarrollo de la sismología ha llevado a los sismólogos a la convicción de que éstos pueden ser predichos. La investigación en este aspecto es relativamente nueva a pesar de la cual se han logrado resultados prometedores.

Existen esencialmente dos maneras de atacar el problema. En una de ellas se estudia la variación de ciertos parámetros físicos debido a la acumulación de los esfuerzos cuya relajación ocasiona el temblor. Así por ejemplo, se ha observado que la región focal sufre una dilatación que altera la velocidad de las ondas que se propagan en ella. Otros de los parámetros que se alteran son por ejemplo resistencia del terreno al paso de corriente eléctrica y el nivel freático. Todos estos factores pueden ser medidos y correlacionados con la ocurrencia final del temblor.

En otra de las formas de enfrentar el problema se ha estudiado la sistematicidad en la ocurrencia de los temblores. Se ha observado que los epicentros a lo largo de una zona de subducción no se distribuyen al azar sino siguiendo un patrón geográfico y temporal. Puede entonces estudiarse la historia sísmica de una región, estimar los períodos de recurrencia de temblores de cierta magnitud y evaluar de esta manera la posibilidad de ocurrencia de un temblor.

Este breve bosquejo trata solamente de poner de manifiesto que los sismólogos actuales se encuentran trabajando sobre bases científicas para la futura predicción de temblores. Cuanto tiempo tomará el desarrollar un sistema eficiente para predecir temblores es difícil de precisar pero seguramente será de algunas décadas. Indudablemente ésto requerirá del desarrollo de nuevas metodologías tanto teóricas como instrumentales.

¿QUE HACER CUANDO OCURRE UN TEMBLOR?

Existen varias medidas que deben tomarse en caso de ocurrir un temblor, pero ante la eminencia de un suceso de esta naturaleza, en regiones sísmicamente activas es mejor prepararse mentalmente para una eventualidad. Por otra parte conviene buscar las condiciones adecuadas de seguridad de los sitios donde se permanece más tiempo como son: la casa, el trabajo, la escuela, etcétera.

La seguridad de las casas en caso de temblores se garantiza construyendo a éstas según los códigos de construcción antisísmica de la región, si en los centros de trabajo se observa poca seguridad en las instalaciones pedir que sean reforzadas. En México las escuelas, y en general, obras civiles son construidas tomando en cuenta el código de construcción pero si se observa alguna anomalía conviene reportarlo a las autoridades competentes. Debe evitarse el colocar objetos pesados o peligrosos como lámparas, etcétera en repisas y lugares elevados a no ser que estén bien sujetos.

Cuando ocurra el temblor, es conveniente tomar en cuenta lo siguiente:

1. Conservar la calma y tratar de serenar a las personas que nos rodean. Evitar dar gritos ya que éstos infunden pánico, y éste es el origen, en muchos casos, de más fatalidades que el temblor mismo.

2. No desplazarse precipitadamente en interiores, es mejor buscar sitios que ofrezcan seguridad (debajo del dintel de puertas, debajo de mesas robustas, lugares con techumbres livianas, etcétera).

3. Tener cuidado de no permanecer debajo de objetos colgantes u objetos mal colocados. Alejarse de las ventanas ya que los vidrios se rompen con las sacudidas, tampoco permanecer cerca de objetos que se puedan desplazar o derribar (como armarios altos, vitrinas, muebles con ruedas, etcétera).

4. En las escuelas, los maestros deben conservar la serenidad y tratar de dar confianza a los alumnos, pedir a éstos que se alejen de las ventanas y, de ser posible, protegerse debajo de las mesas o los dinteles de las puertas. Si están en los patios de recreo pedir que permanezcan lejos de los edificios. Estas explicaciones y un simulacro deberían constituir práctica ordinaria al inicio de clases en todas las escuelas.

5. En otros centros de mucha concentración se aconseja no salir precipitadamente ya que ésta es la respuesta de la mayor parte de personas y se ha visto que causa muchos accidentes personales. Lo mejor es buscar sitios seguros debajo de estructuras reforzadas.

6. Se debe tomar en cuenta que los temblores no duran mucho tiempo, pero a veces ocurren otros. De manera que debe obrarse con cautela al final de uno.

QUE HACER DESPUES DE OCURRIR UN TEMBLOR

Después de ocurrir el temblor se debe revisar si hubo daños y accidentados y proveer ayuda si es necesario. Posteriormente revisar si las diferentes instalaciones eléctricas, gas, agua, etcétera no sufrieron daños. Si es de noche no prender fósforos para alumbrarse hasta no estar seguro que no existen fugas de gas. No use el teléfono si no es para transmitir un mensaje de mucha prioridad.

En las escuelas antes de movilizar a los alumnos conviene inspeccionar el estado de los lugares de acceso: puertas, escaleras, barandales, etcétera. Posteriormente evacuar la escuela para una revisión detallada de sus instalaciones. Esto mismo se debe hacer en caso de edificios altos.

APENDICES

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APENDICE A.

HISTORIA DE LA SISMOLOGIA EN MEXICO.

La República Mexicana está situada en una de las regiones sísmicamente más activas del mundo como se puede apreciar en la Figura 24. El estudio de la actividad sísmica en México es relativamente reciente, sin embargo su observación tiene antecedentes remotos. Sabemos que los primeros pobladores de México se percataron de la actividad sísmica y volcánica en estas regiones y posiblemente hasta existieron personas dedicadas a estudiar estos fenómenos.

En la época de la colonia la descripción de los temblores la hicieron principalmente los monjes en algunos conventos y se encuentran anotadas en algunas obras de Clavijero y Sahagún. Con el uso generalizado de la imprenta se reportaban datos sismológicos en los periódicos de la época con descripciones a veces pintorescas y exageradas. Posteriormente los temblores eran observados por naturalistas, publicistas y por el público en general, pues en todos los folletos antiguos se encuentran notas sobre temblores, cuyas áreas se empezaban a delimitar a medida que las comunicaciones se establecían entre pueblos.

Cuando se instaló la red telegráfica en la República Mexicana los telegrafistas suministraban datos referentes a temblores y se publicaban mensualmente en boletines.

La medición de los temblores por medio de instrumentos se inició a fines del siglo pasado, en la época de Mariano Bárcena, se instaló en el Observatorio Meteorológico Central un sismógrafo del Padre Sechi. Por ese tiempo Juan Orozco y Berra se dedicó a observar estos fenómenos y formar estadísti cas, reuniendo importantes datos de temblores desde tiempos precolombianos, coleccionados con cuidado y publicados en la sociedad científica Antonio Alzate. Sin embargo es hasta el 5 de septiembre de 1910 que por decreto Presidencial se crea e inaugura el Servicio Sismológico Nacional. Este evento se enmarcó dentro de los festejos conmemorativos del primer centenario de la iniciación de la Independencia Nacional. Dicho servicio dependía del Instituto Geológico Nacional.

La red inicial consistió del Observatorio Central de Tacubaya y estaciones ubicadas en Oaxaca, Mérida, Zacatecas, Mazatlán, Guadalajara y Monterrey. Se eligieron como sensores los sismógrafos Wiechert de periodo corto. Básicamente, estos sismógrafos con algunas modificaciones y mejoras continúan operando.

Hacia 1929 el Instituto Geológico Nacional pasó a ser el Instituto de Geología de la UNAM y el Servicio Sismológico formó parte de este nuevo Instituto. En 1949 con la creación del Instituto de Geofísica, el Servicio Sismológico pasó a formar parte de este Instituto.

El Servicio Sismológico vuelve a cobrar vida hacia 1965-1967 cuando se instalaron estaciones de mayor sensibilidad en Tehuantepec PBJ), Vista Hermosa (VHO), Comitán (COM), Toluca (OXM), León (LCG), Presa Infiernillo (PIM), Presa Mal Paso (PMM), Ciudad Universitaria (UNM), Tepostlán (TPM) y Popocatépetl (PPM). También se instaló por 1970 una red de estaciones en el Noroeste, con el fin de observar la actividad sísmica del Golfo de California. Este conjunto de estaciones ahora es controlado por el Centro de Investigaciones y de Educación Superior de Ensenada, Baja California (CICESE).

Actualmente el Servicio Sismológico cuenta con una red de 14 estaciones

(Apéndice B) siendo el Observatorio Central de Tacubaya la estación más completa y donde se recibe la información sismológica de toda la red.

Desde 1910 el Servicio Sismológico ha generado ininterrumpidamente datos que hoy forman el archivo sismológico del país. Ellos constituyen por sí mismos un valioso acervo formando una fuente de información muy importante.

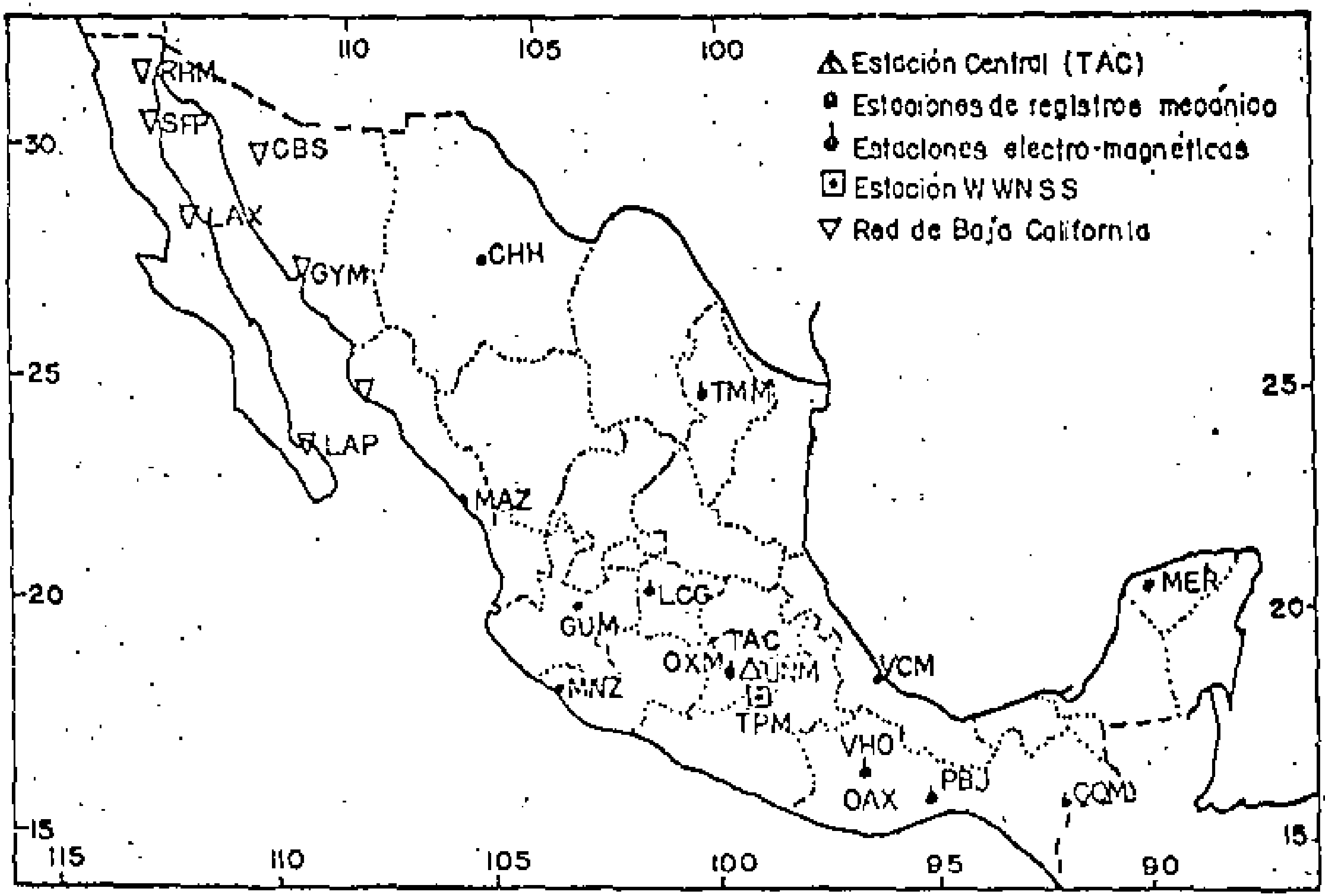
El Servicio Sismológico ha jugado un papel importante en el desarrollo de la Sismología en México, pero además tiene una función social y económica palpable.

DESARROLLO SISMOLOGICO.

En la última década han progresado los estudios de Sismología en México y se han venido formando distintos grupos de trabajo que afrontan los diferentes problemas de Sismología. Existen en la Universidad Nacional Autónoma de México tres de estos grupos de trabajo. El grupo del Instituto de Geofísica, que además de realizar labores de investigación tiene a su cargo el Servicio Sismológico que es el vocero oficial de la UNAM en la divulgación de los parámetros de los temblores. El Instituto de Ingeniería enfoca su trabajo principalmente a problemas de riesgo sísmico y maneja una red de estaciones telemétricas (SISMEX) y el Instituto de Investigaciones Matemáticas Aplicadas y Sistemas, que tiene a su cargo el desarrollo del proyecto de la Red Sísmica de Apertura Continental (RESMAC). Existe otro grupo de trabajo en el Centro de Investigaciones y de Educación Superior de Ensenada, B.C. (CICESE) que enfoca su estudio a la actividad sísmica asociada tanto al Golfo de California como a la falla de San Andrés, igualmente operan la Red Sismológica del Noroeste (RESNOR). Los diferentes grupos tienen comunicación y frecuentemente se realizan simposia donde se dan a conocer los avances en el estudio de la Sismología.

Existe interés en algunas instituciones de enseñanza superior en el interior de la República por el estudio de la Sismología y, recientemente, están enfocando sus esfuerzos por consolidar grupos de trabajo propios para el desarrollo de esta disciplina en sus localidades.

SERVICIO SISMOLOGICO NACIONAL



INSTITUTO DE GEOFISICA DE LA U.N.A.M.

ESCALA DE INTENSIDADES SISMICAS

Escala de Mercalli modificada en 1931 por Harry O. Wood y Frank Neuman.

I. Sacudida sentida por muy pocas personas en condiciones especialmente favorables.

II. Sacudida sentida sólo por pocas personas en reposo, especialmente en los pisos altos de los edificios. Los objetos suspendidos delicadamente pueden oscilar.

III. Sacudida sentida claramente en los interiores, especialmente en los pisos altos de los edificios, muchas personas no la estiman como un temblor. Los carros de motor estacionados pueden moverse ligeramente. Vibración como originada por el paso de un carro pesado. Duración estimable.

IV. Sacudida sentida durante el día por muchas personas en los interiores, por pocas en el exterior. Por la noche algunas despiertan. Estremecimiento de vajillas, vidrieras y puertas; crujimiento de muros. Sensación como de un carro pesado chocando contra un edificio, los carros de motor no en movimiento se balancean claramente.

V. Sacudida sentida casi por todo mundo; muchos despiertan. Algunas piezas de vajillas, vidrieras, etc. se rompen; pocos casos de agrietamiento de aplanados; objetos inestables caen algunas veces. Se observan perturbaciones en los árboles, postes y otros objetos altos. Detención de relojes de péndulo.

VI. Sacudida sentida por todo mundo; muchas personas atemorizadas huyen hacia afuera. Algunos muebles pesados cambian de sitio; pocos ejemplos de caída de aplanados o daño en chimeneas. Daños ligeros.

VII. Todo mundo huye afuera. Daños sin importancia en edificios bien planeados de buena construcción. Daños ligeros en estructuras ordinarias bien construídas; daños considerables en las débiles o mal planeadas; ruptura de algunas chimeneas. Estimado por las personas conduciendo carros de motor.

VIII. Daños ligeros en estructuras de diseño especialmente bueno; considerable en edificios ordinarios con derrumbe parcial; grande en estructuras débilmente construídas. Los muros salen de sus armaduras. Caída de chimeneas, pilas de productos en los almacenes de las fábricas, columnas, monumentos y muros. Los muebles pesados se vuelcan. Arena y lodo proyectados en pequeñas cantidades. Cambio en el nivel del agua de los pozos. Pérdida de control en las personas que guían carros de motor.

APENDICE C

SISMOS IMPORTANTES.

Quizá uno de los fenómenos naturales que más ha impresionado al hombre ha sido los terremotos. Su inquietud ha hecho que desde la antiguedad tratara de explicarse las causas que los originan, pero no fue sino hasta los últimos 100 años que ha ido entendiendo la naturaleza de las fuerzas que ocasionan estos movimientos, como ya se ha explicado.

Los efectos de los temblores a la comunidad y sus edificaciones ha sido la causa del temor a éstos. Esta es la razón por la cual la mayor o menor liberación de energía de los temblores no ha tenido impacto social. Así el temblor de Alaska del 28 de marzo de 1964 de magnitud 8.5, siendo uno de los pocos temblores que han liberado más energía en los últimos 100 años apenas es considerado socialmente, sin embargo el temblor de Tanshan del 27 de julio de 1976, de menor magnitud 6.3 m_b y 7.9 M_s se ha considerado como el terremoto más mortífero de los últimos cuatro siglos y ha tenido un gran impacto social.

Se pueden hacer listas de los temblores más fuertes, pero no nos daría una idea del riesgo a que están sometidos los pueblos asentados en zonas sísmicamente activas. A continuación se mencionan algunos de los temblores que han ocurrido en el mundo (Tabla I) y en la República Mexicana (Tabla II) y que más impacto social han tenido.

IX. Daño considerable en las estructuras de diseño bueno; las armaduras de las estructuras bien planeadas se desploman; grandes daños en los edificios sólidos, con derrumbe parcial. Los edificios salen de sus cimientos. El terreno se agrieta notablemente. Las tuberías subterráneas se rompen.

X. Destrucción de algunas estructuras de madera bien construídas; la mayor parte de las estructuras de mampostería y armaduras se destruyen con todo y cimientos; agrietamiento considerable del terreno. Los rieles se tuercen. Considerables deslizamientos de tierra en las fuertes pendientes. Eyecciones de lodo y arena. Invasión del agua de los ríos sobre las márgenes.

XI. Casi ninguna estructura de mampostería queda en pié. Puentes destruídos. Anchas grietas en el terreno. Las tuberías y subterráneas quedan fuera de servicio. Hundimientos y derrumbes en terreno suave. Gran torsión de vías férreas.

XII. Destrucción total. Ondas visibles sobre el terreno. Perturbaciones de las cotas de nivel. Objetos lanzados en el aire hacia arriba.

TABLA I

Fecha	Magnitud Richter Mb	Ms	Región	No. de Muertos	Datos generales
[illegible], agosto 16	8.6		Chile: Valparaíso Santiago	20 000	
[illegible], diciembre 28	7.5		Italia: Reggio	29 980	
[illegible], diciembre 16	8.6		China: Kansu y Shansi	100 000	
[illegible], septiembre 1	8.3		Japón: Tokio, Yokohama	99 330	Conocido como el terremoto de Kwanto. Tuvo desplazamientos de 4.5 m y grandes incendios le precedieron.
[illegible], mayo 22	8.3		China: Nan Shan	200 000	Grandes fallas, se sintió hasta Pekín
[illegible], diciembre 25	5.6	6.2	Nicaragua: Managua	4 000 - 6 000	Miles de heridos. La ciudad de Managua fue casi totalmente destruida.
[illegible], mayo 30	7.5		Paquistán: Quetta	30 000	Quetta fue totalmente destruida.
[illegible], junio 25	8.3		Chile	28 000	
[illegible], diciembre 26	7.9		Turquía: Erzincan	30 000	Se detectaron movimientos oscilatorios de 3.[illegible] m de desplazamiento con movimientos trepidatorios menores.
[illegible], febrero 29	5.8		Morocco: Agadir	10 000 - 15 000	Es uno de los temblores que más muertos ha ocasionado a pesar de ser poca su magnitud.
[illegible], febrero 4	6.2	7.5	Guatemala: Guatemala	23 000	Se calcula en 76 000 el número de heridos. La mayor intensidad se ubicó en el área de [illegible].
[illegible], agosto 27	6.3	7.9	China: Noreste	655 237	Cerca de [illegible] 000 heridos y daños en el área de Tangshan. Este terremoto es probablemente el más mortífero de los últimos [illegible] siglos y el 2o. más fuerte que registra la historia moderna.

Cont. Tabla 1

Fecha	Magnitud Richter Mb	Ms	Región	No. de muertos	Datos generales
1970, mayo 31	-	7.7	Perú: Huaras Chimbote, Yungay	50 000 - 70 000	Derrumbes, inundaciones. La peor catástrofe registrada en Perú por un terremoto en este siglo.
1978, septiembre 16	-	7.7	Irán	31 000 - 35 000	Muchos heridos y daños considerables en Bozonabad y áreas circunvecinas.

TABLA II

Fecha	Magnitud Richter Ms	Mb	Región	No. de Muertos	Datos generales
[illegible], junio 7	?		Jalisco-Colima	45	Destructor en Cd. Guzmán Jal., ha sido uno de los temblores más fuertes que han ocurrido los últimos 100 años. Se reportaron 45 muertos en el D.F.
[illegible], julio 16	?		Oaxaca-Puebla	?	Grandes daños en Esperanza, Puebla
[illegible], julio 29	7.8		Guerrero: San Marcos	55	Miles de heridos y daños materiales en varios estados. La población más dañada fue San Marcos, Gro.
[illegible], agosto 2	6.3	7.1	Oaxaca: Pinotepa	?	Se estima que hubo varios muertos y miles de heridos. Grandes daños materiales en Pinotepa.
[illegible], enero 30	6.2	7.5	Colima	50	300 heridos y 30 poblaciones afectadas severamente.
[illegible], agosto 28	6.8		Oaxaca-Puebla	600	Miles de heridos y damnificados. Cd. Serdán destruida: daños considerables en las ciudades de Puebla, Orizaba, Oaxaca y México. 77 pueblos dañados seriamente.
[illegible], noviembre 28	6.5		Oaxaca	?	Daños en [illegible], Oaxaca. Es quizá el temblor que más se ha estudiado en México.
[illegible], octubre 24	6.5		Oaxaca: Huajuapan	50	Fuertes daños en la región fronteriza de los estados de Puebla, Oaxaca y Guerrero. Principalmente en Huajuapan de León, Oax.

LOS SIGUIENTES ANEXOS DAN INFORMACION ADICIONAL

SOBRE LOS TEMAS TRATADOS

Deriva Continental. Esparcimiento del Fondo Marino. Tectónica de Placas

A principios de siglo el geofísico alemán Alfredo Wegener propuso la teoría conocida como Deriva Continental, en la que se supone que las masas continentales han sufrido largos desplazamientos horizontales que determinaron la posición y distribución actual de tierras y mares.

Wegener se basó fundamentalmente en la concordancia de algunas líneas de costa, como las de América del Sur y Africa (figura 1); en datos paleoclimáticos y paleontológicos y en la distribución de las cadenas montañosas. Propuso la existencia de un gran continente primario al cual llamó Pangea que en el transcurso de las edades geológicas se fracturó y sus partes navegaron sobre la capa basáltica del piso marino, hasta ocupar diferentes posiciones.

Otros investigadores anteriores ya habían avanzado en la idea de movimientos continentales; sin embargo Wegener en 1912 presentó una considerable cantidad de evidencias y una descripción de las posiciones de los continentes en las diferentes épocas.

La teoría pronto encontró un gran número de objeciones, principalmente por el mecanismo propuesto para explicar el desplazamiento de los continentes así como por la similitud de edades de los océanos, y la existencia de los cratones continentales y de regiones de lentos levantamientos y subsidencias entre otras. Pronto la teoría cayó en desuso.

Con el advenimiento de la Segunda Guerra Mundial las técnicas e instrumentos para la exploración marina experimentaron un fuerte avance, principalmente en los sistemas de navegación así como de registro del fondo, lo que fomentó la investigación de los océanos.

En la década de los cincuenta, las investigaciones oceanográficas se incrementaron enormemente. Se hicieron mediciones de gravimetría, de magnetometría, de flujo de calor de sismología de refracción y reflexión y se tomaron muestras de sedimentos del fondo oceánico. Todos estos estudios aportaron información novedosa cuya interpretación hizo necesaria la revisión cuidadosa de la hipótesis de Wegener.

Dietz en 1961 y Hess en 1962 propusieron la teoría llamada de Esparcimiento del Fondo Oceánico, para explicar sus observaciones sobre el lecho marino. En esta teoría se considera a las dorsales oceánicas como centros generadores de material nuevo proveniente del manto y trasladado a la superficie por corrientes de convección.

La idea general asume que el valle medio de una dorsal oceánica representa una fisura, la cual se rellena por material del manto, probablemente peridotita, que al enfriarse bajo los 500 °C sufre un proceso de serpentinización. Esto hace que disminuya la densidad de 3.3 a 2.6 gr/cm^3 y aumente su volumen, lo que provoca la elevación de las montañas que rodean al valle. Este proceso también justifica el espesor casi uniforme de la corteza oceánica reportado por los estudios sísmicos (Figura 2).

La teoría explica además, la falta de sedimentos antiguos anteriores al Cretácico, los espesores inferiores a 1.3 Km, la distribución de edades (mayores al aproximarse a los continentes) la forma y localización de los guyots (mesetas submarinas, algunas de ellas situadas a gran profundidad), la ausencia de la discontinuidad de Mohorovicic bajo las dorsales y el elevado flujo térmico sobre ellas, y explica la presencia de las fosas profundas en algunas márgenes continentales como zonas en donde el piso oceánico retorna al manto.

La interpretación de las anomalías magnéticas de Campo Total tomadas sobre las cordilleras submarinas empleando las inversiones de polaridad del Campo Geomagnético hecha por Vine y Mattews en 1963, proporcionó un fuerte apoyo a la teoría.

Los años siguientes fueron de intensa actividad y descubrimientos. En 1968 Isacks Oliver y Sykes publicaron un artículo ahora clásico, "Seismology and the New Global Tectonics", en el que se sintetizan la mayoría de las ideas que integran lo que es conocido como Tectónica de Placas o Nueva Tectónica Global.

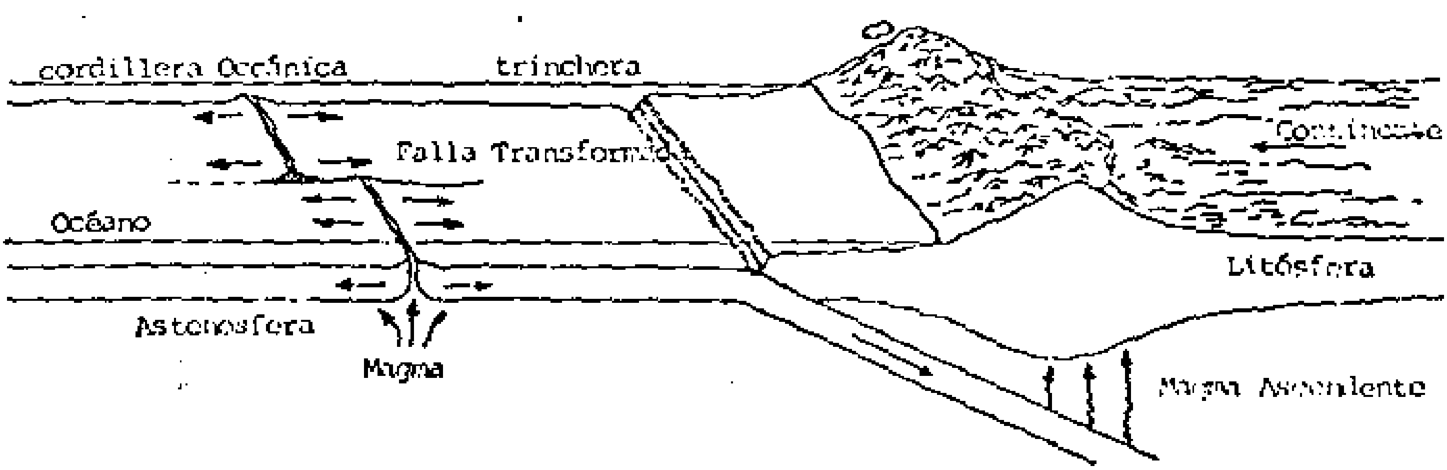
Brevemente, la teoría considera que la tierra posee un casquete externo rígido de unos 100 Km de espesor. Este casquete no es continuo sino que está dividido en varios segmentos (Figura 3) con movimientos relativos entre sí.

Los movimientos a que están somedidas las placas producen en algunos de sus márgenes la subducción de una de ellas bajo la otra, mientras que en otros márgenes se produce la creación de nueva corteza. (Figura 2).

Aunque el origen de los esfuerzos que producen estos movimientos no es bien comprendido es muy posible que sean debido a la transferencia convectiva de calor hacia la superficie. Cualquiera que sea el mecanismo que produce el desplazamiento, el contacto entre las placas es zona de acumulación de esfuerzos y el lugar donde se libera la mayoría de la energía de la superficie terrestre. En la figura 4 puede verse que las zonas sísmica y volcánicamente activas definen los márgenes de las placas. Nótese también que los límites de las placas no necesariamente coinciden con las fronteras océano-continente.

La distribución de hipocentros en un corte transversal a través de una zona de subducción puede verse en la figura 4. La región definida por los focos es llamada zona de Wadati Bennioff en honor a los investigadores que describieron esta relación geométrica en los años 40.

Figura 1.


cordillera Oceánica
trinchera
Falla Transform
Continente
Océano
Litósfera
Astenosfera
Magma
Magma Ascendente

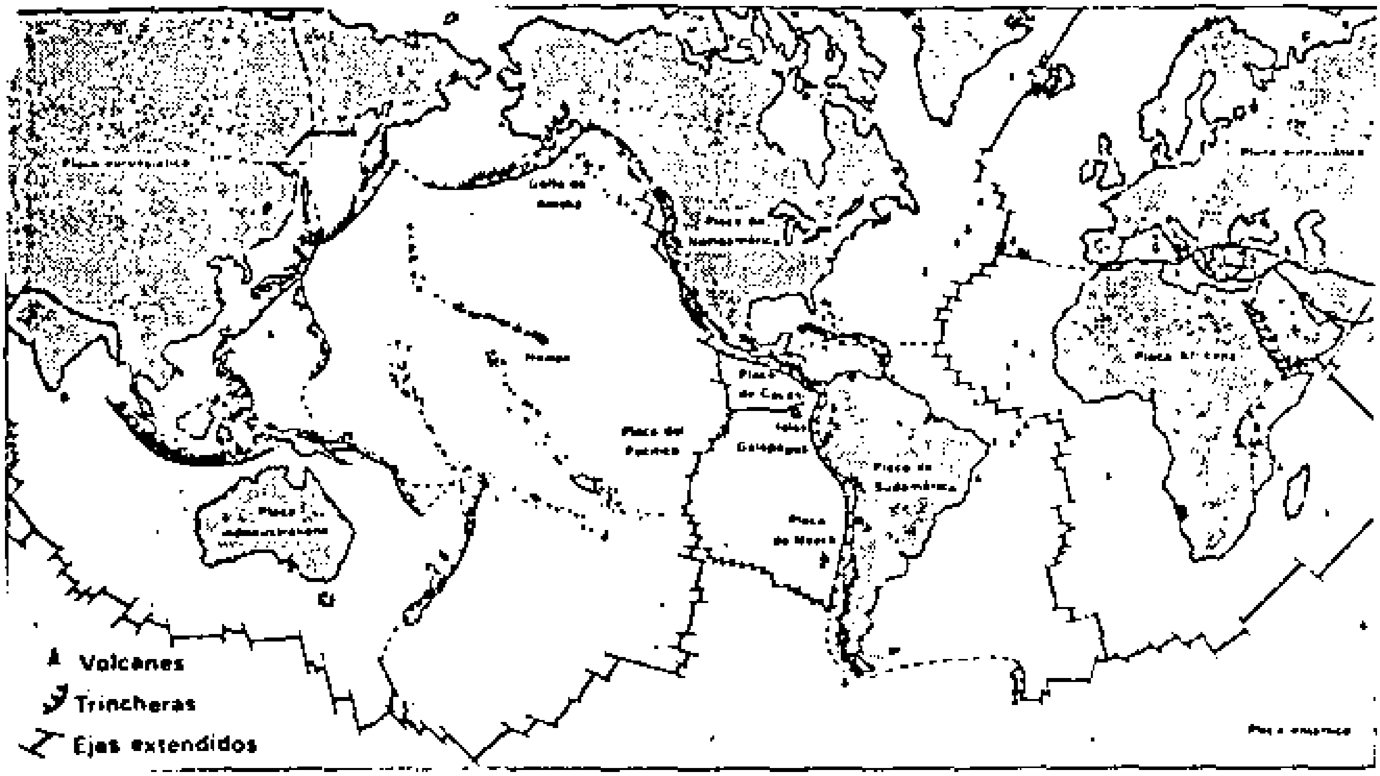
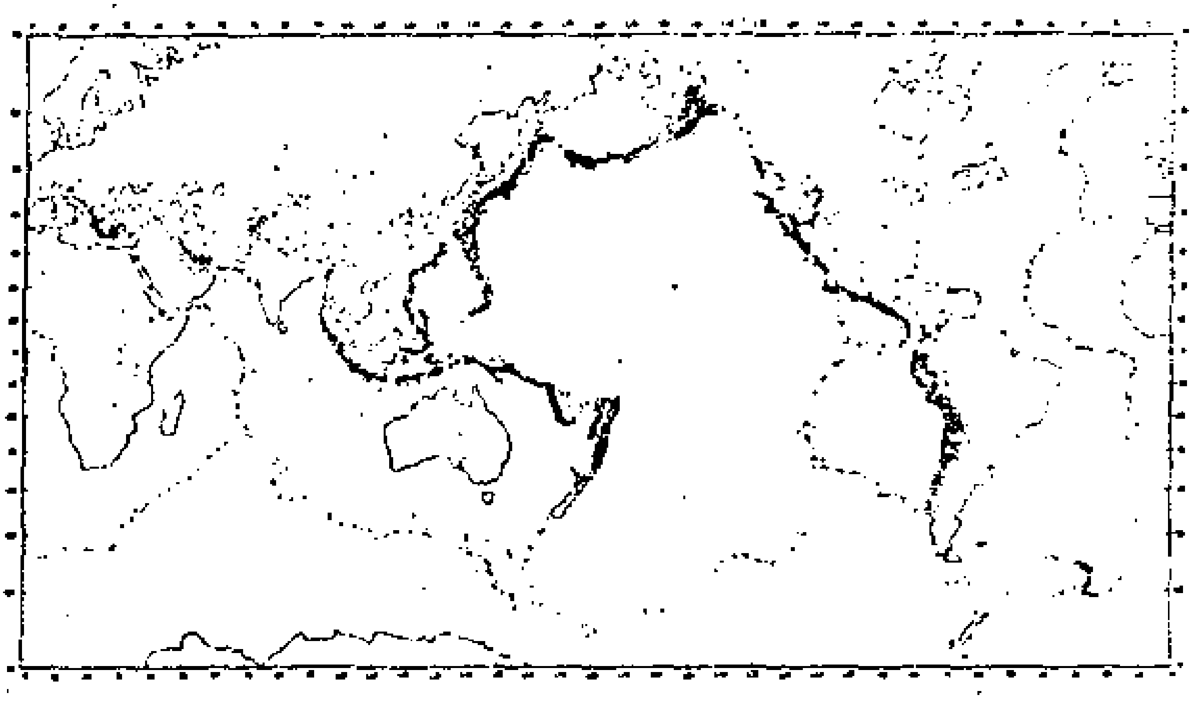


Figura 3.



Algunos de los acontecimientos más importantes en el desarrollo de la Tectónica de Placas.

1620	Se notó la concordancia entre las líneas de costa de África y América del Sur. (Bacon)
1839	Ideas de convección térmica en el manto (Hopkins).
1855	Teoría isostática (Airy).
1858	Mapa mostrando el movimiento de los continentes (Snider).
1859	Teoría isostática (Pratt).
1889	Concepto de la astenósfera (Fisher).
1906	Primeras evidencias de inversiones del Campo Magnético terrestre (Brunhes).
1910	Teoría de movimientos continentales (Taylor).
1912	Teoría de Deriva Continental (Wegener).
1914	Concepto de la astenósfera (Barrel).
1923 26-28	Nuevo método de análisis de temblores, llamado de mecanismos focales o plano de falla (Nakano y Byerly).
1926	Concepto de la astenósfera (Gutenberg).
1929	Establecimiento de una época de polaridad inversa magnética en el Pleistoceno (Matuyama).
1939	Celdas de Convección en el manto (Griggs).
1940	Determinaciones de edades radiométricas K-Ar (Evans).
1944	Mecanismos de deriva continental (Holmes).
1946	Descubrimiento de los guyots en el Océano Pacífico. (Hess).
1949	Descubrimiento de las zonas de Benioff (Benioff).
1950	Estudios de las cordilleras submarinas (Instituto Scripps).
	Comienzo de las mediciones de flujo térmico en el mar (Bullard).
	Empleo de los magnetómetros de Campo total en estudios marinos (Instituto Scripps).
1952	Descubrimiento de las zonas de fractura en el Océano Pacífico (Menard y Dietz).
1955	Investigaciones teóricas del origen del Campo Magnético (Bullard y Elsasser).
	Ideas de fuentes térmicas "Hot spots" (Wilson).
1956	Descubrimiento del valle medio de las dorsales y reconocimiento de los temblores que en ellos ocurren (Ewing y Hazen).
	Demostración de la Deriva Continental con datos Paleomagnéticos (Irving y Runcorn).
1957	Levantamiento marino con gravímetro de superficie.
1957	Año Geofísico Internacional
1958	Reconocimiento de lineamientos magnéticos sobre las dorsales (Mason). Confirmado por Vacquier, 1961 y Mason y Raff 1961.

1960	Primeras perforaciones del proyecto Mohole.
1961-1962	Hipótesis de esparcimiento de los fondos marinos (Hess).
1963	Interpretación de los lineamientos magnéticos empleando las ideas de inversiones del Campo Geomagnético y de esparcimiento de los fondos marinos (Vine y Matthews; Morley).
	Primera escala de tiempo cuantitativa de inversiones del Campo Geomagnético usando edades de K-Ar (Cox).
1960-70	Proyecto Internacional del Manto Superior bajo la dirección del profesor V.V. Beloussov.
1965	Concepto de fallas transformadas (Wilson).
1967-69	Modelo de una Litósfera capaz de soportar tensiones como mecanismo de hundimiento en las trincheras (Elsasser).
1967	Conceptos e implicaciones geométricas de las placas (Mc Kenzie y Parker).
1968	Estudios sobre las placas y sus límites (Morgan).
	Nueva Tectónica global (Isacks).
1969	Concepto de juntas triples (Mc Kenzie y Morgan).
	Relaciones entre la edad, la elevación topográfica y el flujo térmico en los fondos oceánicos (Sclater y Francheteau).
1964	Programas del JOIDES (Muestreo de la tierra profunda)
	Información del proyecto de perforación en mar profundo (Deep sea drilling Project) dentro de los programas del JOIDES (Joint Oceanographic Institutions Deep Earth Sampling).
1970	Proyecto Internacional Geodinámica. Con la participación del Grupo de Trabajo No. 2, Grupo de Estudio No. 2, Placa de Cocos, Comité Mexicano de Geodinámica.
1970	Proyecto CICAR (Cooperative Investigations of the Caribbean and Adjacent Regions). Participación de México.
1971	Proyecto IDOE (International Decade Oceanographic Exploration). Participación de México.

Predicción de Temblores

Como hemos visto, la mayoría de los temblores ocurren en los márgenes de las placas como respuesta a la acumulación de esfuerzos en esas áreas. Los esfuerzos son debidos al movimiento relativo entre placas. Este movimiento, que es del orden de algunos centímetros por año no cambia en plazos cortos de tiempo (geológicamente hablando) por consiguiente es posible estudiar los márgenes sísmicamente activos en términos estadísticos.

Los períodos de recurrencia de los temblores grandes ($M \geq 7.0$) son altamente variables (30 - 100 años) por lo tanto es necesario contar con un record histórico tan completo como sea posible.

Si el período de recurrencia es conocido, la probabilidad de ocurrencia de un temblor es proporcional al tiempo transcurrido desde el último. Este último aspecto nos lleva a lo que se ha definido como tramo de quietud sísmica a falta de una mejor traducción del inglés "seismic gap".

El concepto anterior fue originalmente desarrollado por Fedotov y Mogi y posteriormente por Sykes y Kelleher y colaboradores*. Estos últimos autores* publicaron un mapa de "gaps sísmicos" para el cinturón del Pacífico.

* Kelleher et al. (1973). Journal of Geophysical research.

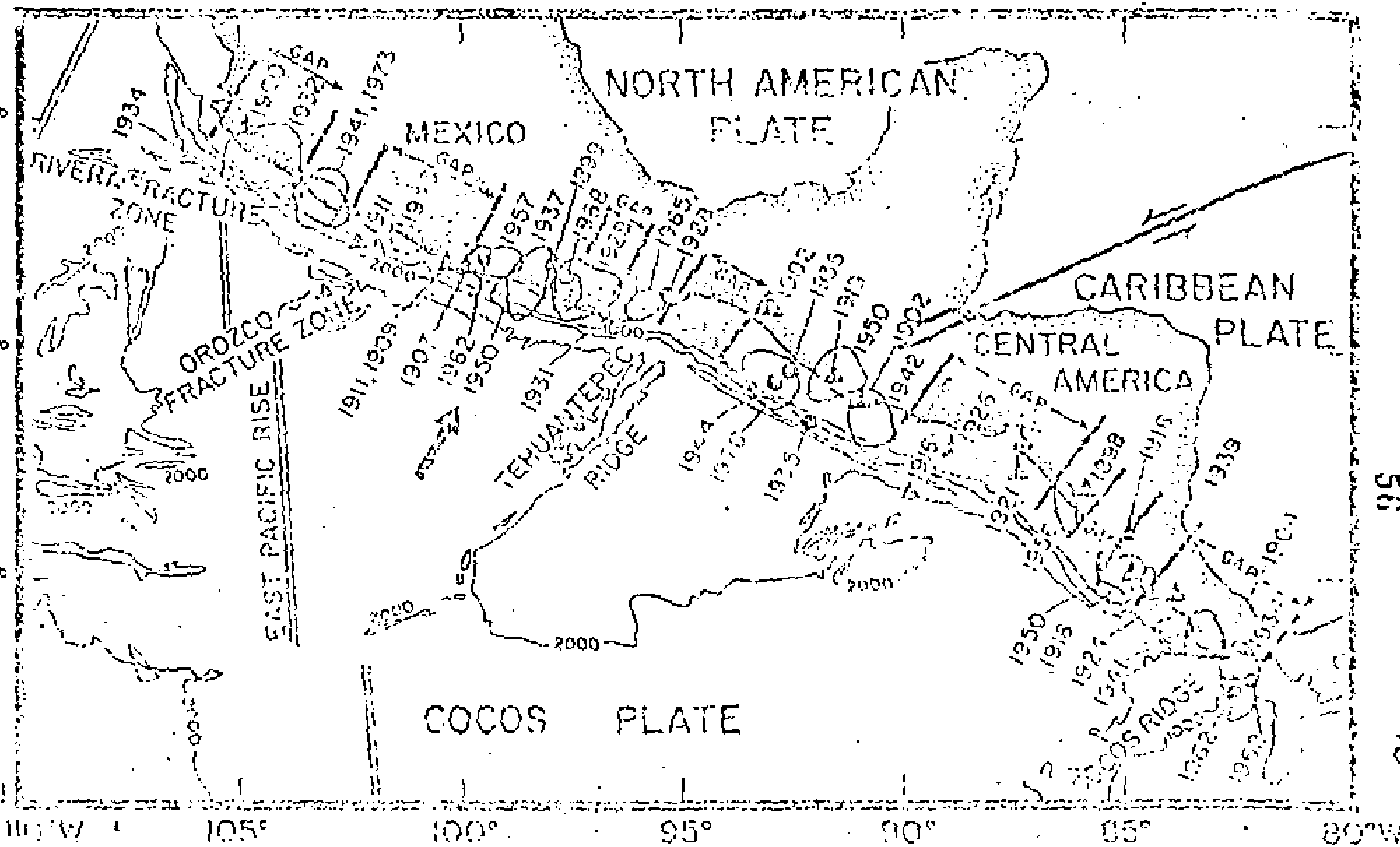
El mapa ha sido revisado recientemente por Mc Cann y colaboradores*. Las siguientes figuras son mapas de "gaps sísmicos" para Mesoamérica, Sudamérica y el Caribe.

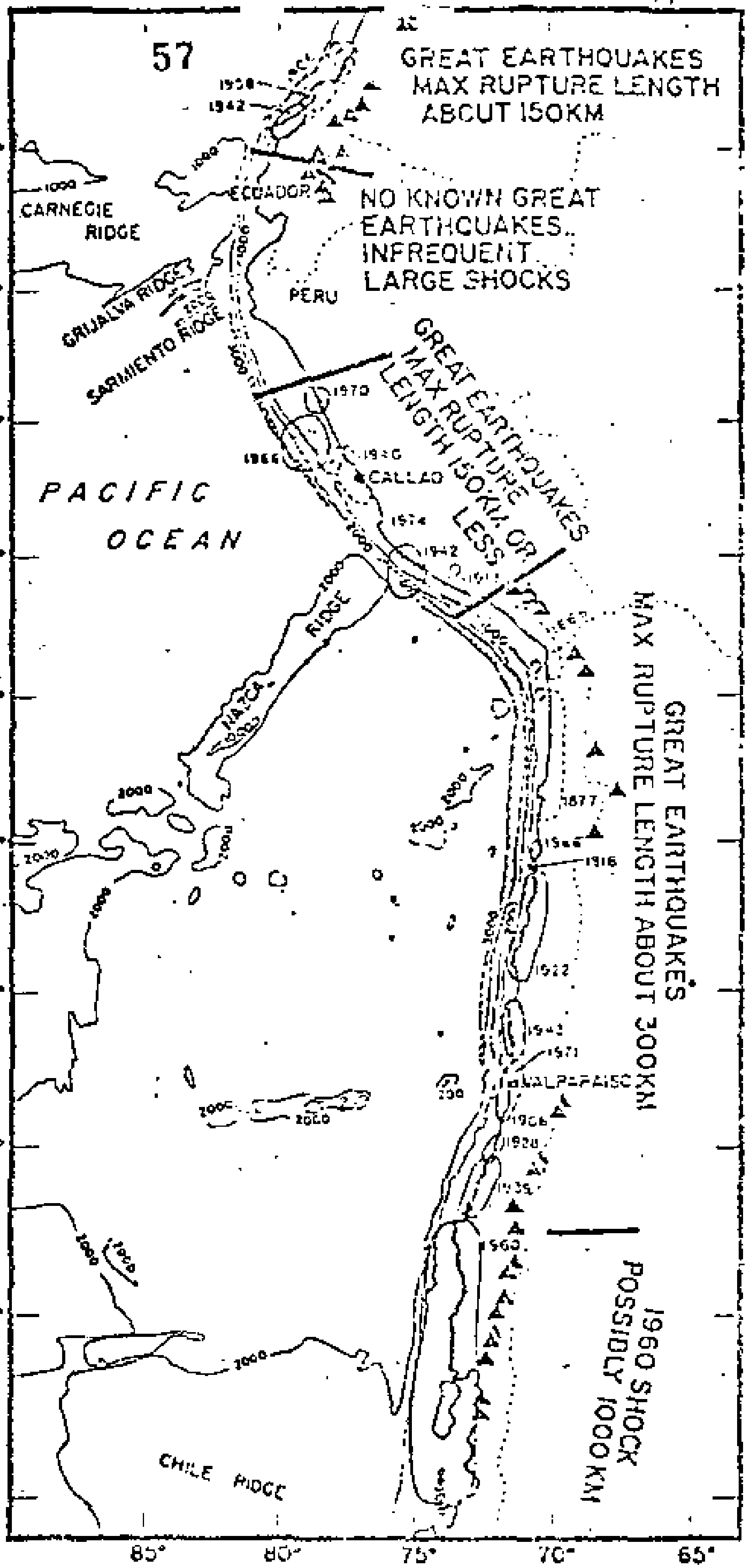
Este tipo de mapas es muy util no sólo para evaluar el riesgo sísmico sino también como parte de un programa de predicción a largo plazo. Una vez que una región es reconocida como "gap", pueden utilizarse técnicas para determinar cambios físicos asociados con la acumulación de esfuerzos.

Los cambios físicos observados se relacionan al aumento de volumen, previo al temblor, llamado dilatancia. Asociado a la dilatancia se encuentran el cambio en la velocidad de las ondas P y S, el aumento de la resistividad eléctrica, el cambio en el campo magnético y el aumento de gas radón en pozos cercanos así como cambios en temperatura y nivel de los fluidos en los mismos.

Para explicar la dilatancia se desarrollaron dos modelos que han sido llamados el modelo americano y el modelo ruso. Ambos coinciden en las fases iniciales pero divergen en la descripción del episodio final. Quizá la situación real sea una combinación de ambos.

* Mc Cann et al. (1979). JGR


NORTH AMERICAN PLATE
MEXICO
CARIBBEAN PLATE
CENTRAL AMERICA
COCOS PLATE
RIVERA FRACTURE ZONE
OROZCO FRACTURE ZONE
EAST PACIFIC RISE
TEHUANTEPEC RIDGE
COCOS RIDGE
GAP
20°
15°
105°
100°
95°
90°
80°W


57
GREAT EARTHQUAKES
MAX RUPTURE LENGTH
ABOUT 150KM
NO KNOWN GREAT
EARTHQUAKES.
INFREQUENT
LARGE SHOCKS
GREAT EARTHQUAKES
MAX RUPTURE
LENGTH 150KM OR
LESS
GREAT EARTHQUAKES
MAX RUPTURE LENGTH ABOUT 300KM
1960 SHOCK
POSSIBLY 1000KM
CARNEGIE
RIDGE
ECUADOR
PERU
GRIJALVA RIDGE
SARMIENTO RIDGE
PACIFIC
OCEAN
CALLAO
NAZCA
RIDGE
VALPARAISO
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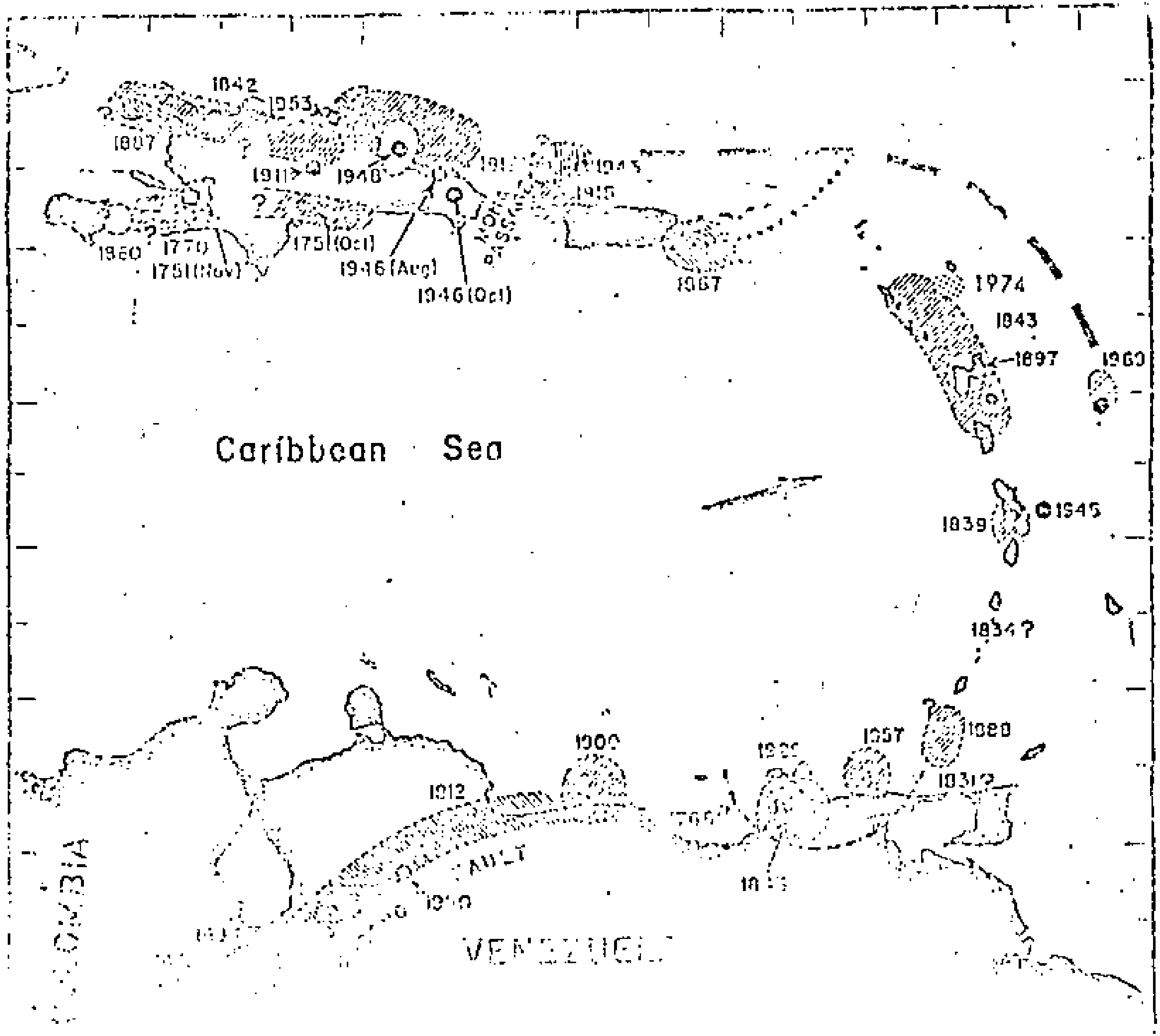

20
18
16
14
12
10
1842
1953
1807
1911
1948
1943
1915
1860
1770
1751 (Nov)
1751 (Oct)
1946 (Aug)
1946 (Oct)
1967
1974
1843
1897
1839
1945
1957
1900
1912
Caribbean Sea
60

Ondas Sísmicas

Dentro de un cuerpo con constantes elásticas λ y μ y densidad ρ pueden viajar dos tipos de ondas llamadas "de cuerpo" o internas:

Longitudinales $V_p = \sqrt{\frac{\lambda + 2\mu}{\rho}}$ onda (de primus)

Transversales $V_s = \sqrt{\mu/\rho}$ onda s (de secundus pues arriba después de p)

En muchos casos $\lambda = \mu$ y $V_p = \sqrt{3}\ V_s$ (Sólido de Poisson)

En un sólido elástico estratificado con una superficie libre se generan además dos tipos de ondas superficiales:

Ondas de Rayleigh con velocidad C_R y ondas de Love con velocidad C_L; puede demostrarse que

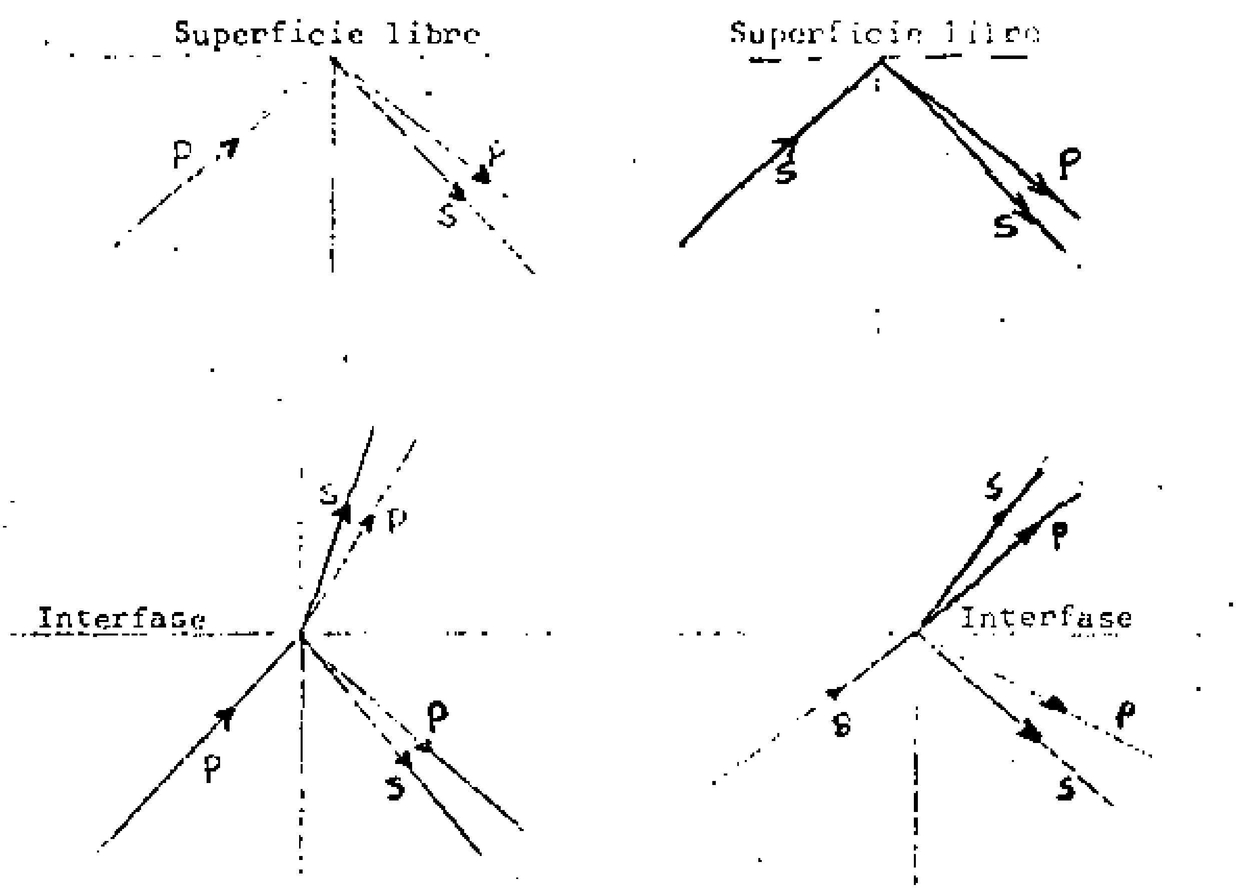
$$C_R < 0.92\ V_s$$

$$V_{S_1} < C_L < V_{S_2}$$

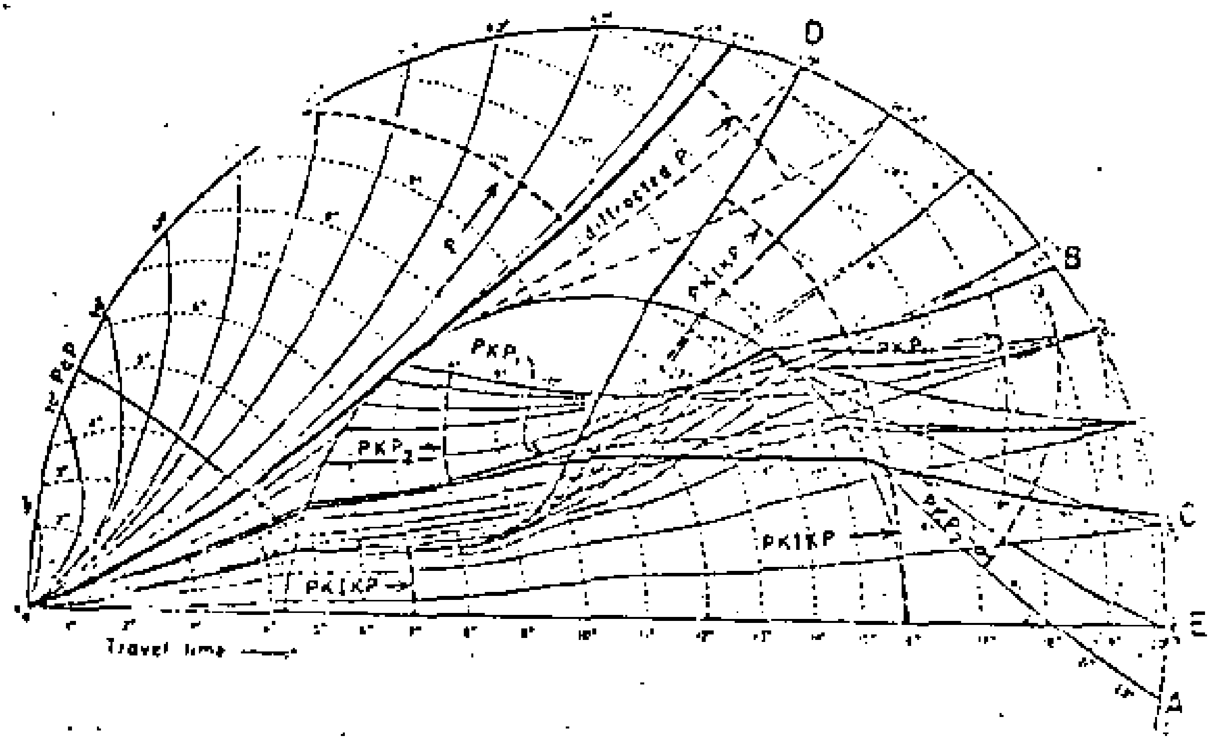
donde V_{S_1} y V_{S_2} son las velocidades transversales de los dos estratos.

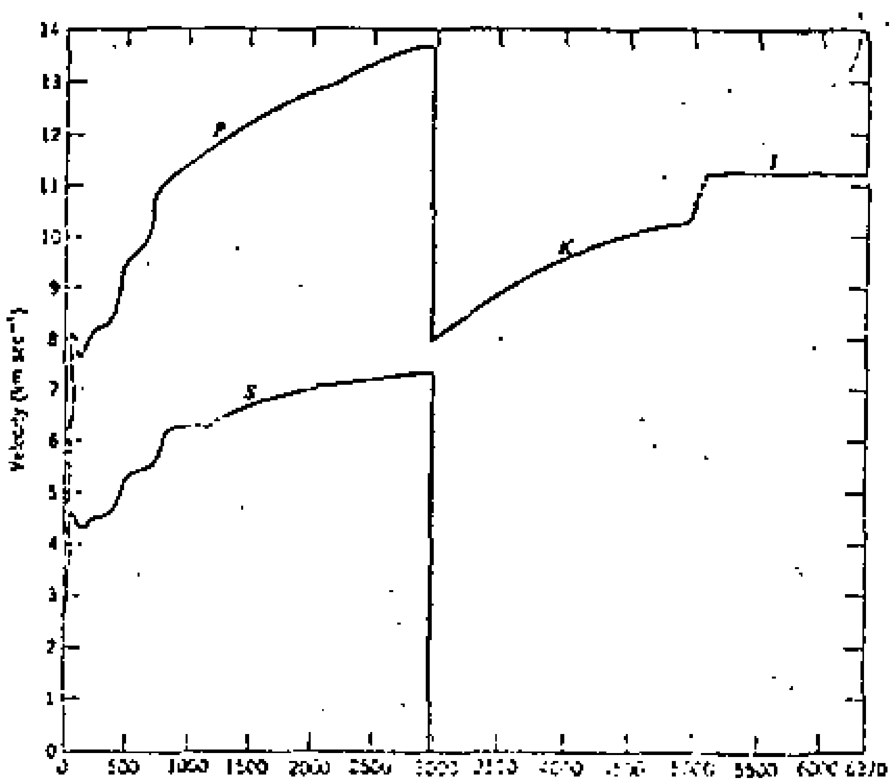
La velocidad de las ondas superficiales depende de la frecuencia y sufren por lo tanto dispersión. La dispersión i.e. la velocidad de grupo depende de la estructura interna del medio y por lo tanto su estudio puede utilizarse para inferir esta estructura.

Las fases arriba descritas quedan registradas en los sismogramas (dependiendo de la posición del instrumento con respecto a la fuente) sin embargo su estructura se complica pues también aparecen fases de ondas reflejadas y refractadas:

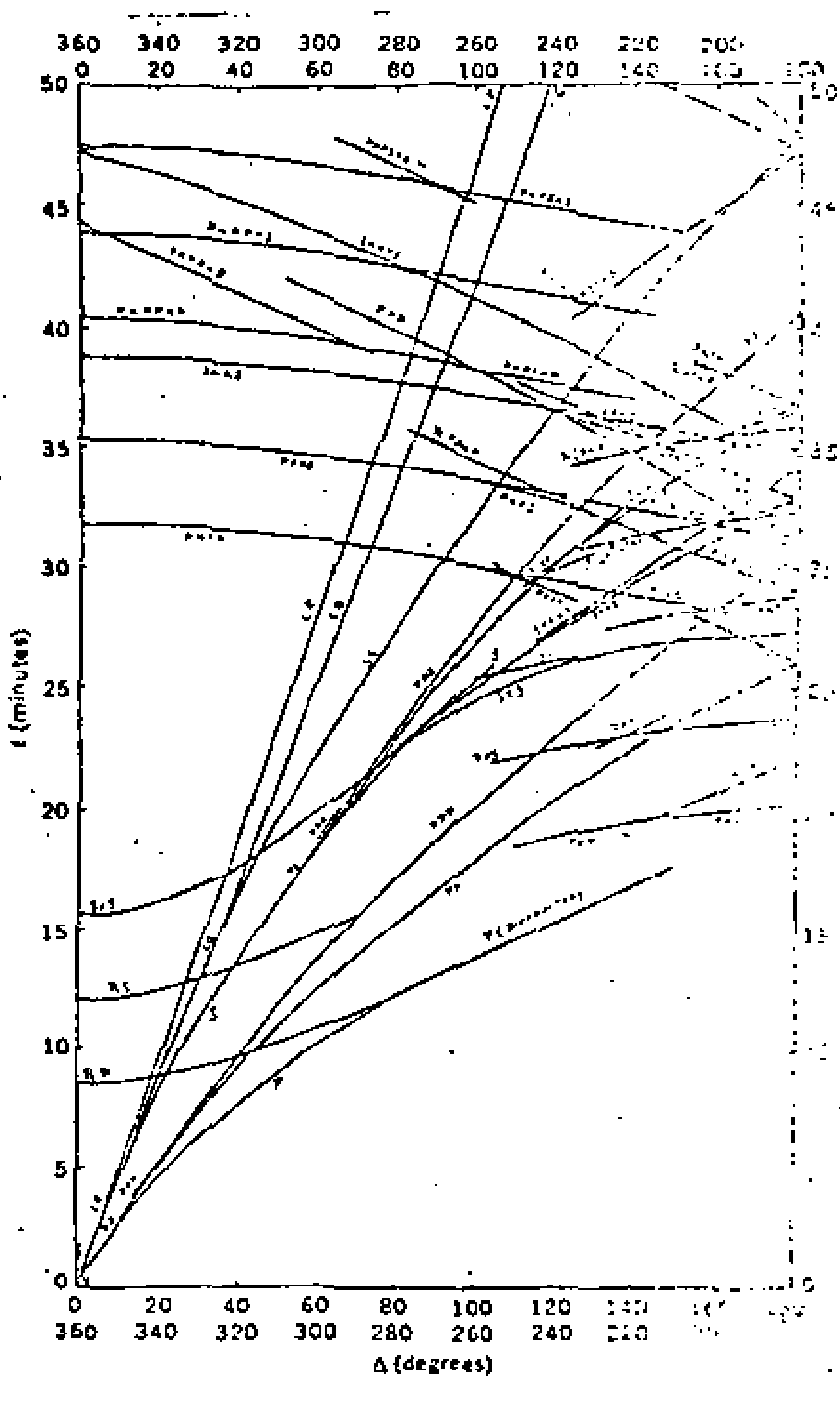


A escala global muchas de estas fases correspondientes a reflexiones y refracciones en el interior del planeta pueden ser identificadas; a continuación pueden verse las trayectorias de algunas de estas ondas y su nomenclatura, así como su distribución de velocidades con la profundidad.





Las diferentes fases pueden identificarse en el registro con ayuda de las curvas de viaje de Jeffreys y Bullen que nos dan el tiempo de arribo en términos de la distancia al foco. Estas curvas fueron dadas por Jeffreys y Bullen en los años 40 y a pesar de la introducción de los computadores modernos han adquirido poca modificación.



DIVISION DE EDUCACION CONTINUA
FACULTAD DE INGENIERIA U.N.A.M.

X CURSO INTERNACIONAL DE INGENIERIA SISMICA

ANALISIS DE RIESGO SISMICO

CONCEPTOS FUNDAMENTALES DE LA TEORIA DE PROBABILIDADES PARA ANALISIS DE RIESGO SISMICO

M. EN I. SONIA E. RUIZ

AGOSTO, 1984.

1

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TEMA

CONCEPTOS FUNDAMENTALES DE LA TEORIA DE PROBABILIDADES PARA ANALISIS DE RIESGO SISMICO

M. en I. Sonia E. Ruiz

CONCEPTOS FUNDAMENTALES DE LA TEORIA DE PROBABILIDADES PARA ANALISIS DE RIESGO SISMICO

INTRODUCCION

La teoría de Probabilidades es una herramienta muy útil para evaluar el riesgo sísmico.

Dado que la naturaleza de los temblores no es de tipo determinístico, el problema debe tratarse mediante modelos probabilístico. Por ejemplo, existen grandes incertidumbres respecto a las coordenadas del foco, magnitd del evento, tiempo de ocurrencia, relaciones magnitud-intensidad, etc.

No es posible decir con certeza cuando ocurrirá un temblor, pero sí podemos decir qué tan probable es que ocurra. El tiempo de ocurrencia de los sismos de diferentes características originados en una determinada fuente puede expresarse mediante un *proceso estocástico*, que es una descripción matemática de la forma en que varía con el tiempo la ocurrencia de ciertos eventos.

Para formular un modelo de sismicidad y estimar sus parámetros sería deseable contar con un número suficiente de registros de movimientos sísmicos fuertes, de las características de su fuente y de su ubicación, sin embargo esta información es muy escasa por lo que se tiene que hacer uso de técnicas estadísticas más refinadas (por ejemplo el teorema de Bayes).

Enseguida se presenta un repaso de los conceptos fundamentales de la teoría de Probabilidades para la mejor comprensión del análisis de riesgo sísmico.

NOCIONES DE TEORIA DE PROBABILIDADES

AXIOMAS FUNDAMENTALES

AXIOMA 1.- La probabilidad de un evento A se encuentra 0 y 1

$$0 \leq P(A) \leq 1$$

AXIOMA 2.- La probabilidad de la unión de dos eventos mutuamente exclusivos es igual a la suma de sus probabilidades

$$P(A \cup B) = P(A) + P(B)$$

Si estos no son mutuamente exclusivos, entonces

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

PROBABILIDAD CONDICIONAL

La probabilidad de que ocurra un evento A, dado que conocemos el resultado de un evento B es igual a

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \qquad (1)$$

De donde

$$P(A \cap B) = P(A|B)\,P(B)$$

Generalizando,

$$P(A \cap B \cap C \ldots \cap N) = P(A|B \cap C \ldots)\,P(B|C \cap \ldots) \ldots \quad P(N)$$

Si los eventos son independientes entre sí, entonces

$$P(A \cap B \cap C \ldots \cap N) = P(A)\,P(B)\,P(C) \ldots P(N)$$

TEOREMA DE PROBABILIDADES TOTALES

Dado un conjunto de eventos mutuamente exclusivos y colectivamente exhaustivos, $B_1, B_2 \ldots B_n$, es posible siempre expresar la probabilidad P(A) de otro evento A como sigue

$$P(A) = P(A \cap B_1) + P(A \cap B_2) + \cdots = \sum_{i=1}^{n} P(A \cap B_i)$$

Entonces

$$P(A) = \sum_{i=1}^{n} P(A|B_i)\,P(B_i) \qquad (2)$$

TEOREMA DE BAYES

La probabilidad condicional de A_j dado que ha ocurrido el evento B es

$$P(A_j|B) = \frac{P(A_j \cap B)}{P(B)} = \frac{P(B \cap A_j)}{P(B)} \qquad (3)$$

Por lo visto en las definiciones anteriores es posible llegar a lo siguiente

$$P(A_j|B) = \frac{P(B|A_j)\ P(A_j)}{\sum_{i=1}^{n} P(B|A_i)\,P(A_i)} \quad , \quad j = 1, 2, \ldots n \qquad (4)$$

Generalmente a la probabilidad resultanto se le llama "*a posteriori*" y a la probabilidad $P(A_j)$ se le llama "*a priori*"

DISTRIBUCION DE PROBABILIDAD DE LAS VARIABLES ALEATORIAS

El comportamiento de una variable aleatoria se describe a través de leyes probabilísticas representadas mediante funciones de distribución de probabilidad.

En el caso de variables aleatorias discretas estas leyes se repre sentan mediante FUNCIONES DE MASA DE PROBABILIDAD. en el caso de variables continuas se utilizan las FUNCIONES DE DENSIDAD DE PROBABILIDAD. Cuando se tratan varias variables a la vez el comportamiento lo determinan leyes de probabilidad CONJUNTAS.

DISTRIBUCION DE PROBABILIDAD MARGINAL

El comportamiento de una (o varias) variable (s) aleatoria(s) se puede obtener a partir de una distribución conjunta, integrando sobre todos los valores de las variables cuyo comportamiento no interesa. La función que representa a este comportamiento es la distribución de probabilidad MARGINAL. Por ejemplo sean X y Y variables aleatorias continuas, con densidad de probabilidades $f_{x,y}$ (x, y); entonces la función de probabilidad marginal de x es igual a

$$f_X(x) = \int_{-\infty}^{\infty} f_{x,y}(x,y)\,dy \qquad (5)$$

Generalizando

$$f_{x_1 x_2}(x_1,x_2) = \iiint_D \cdots \int f_{x_1,x_2,x_3\ldots x_n}(x_1,x_2,x_3\ldots x_n)\,dx\ dx\ dx_3\ldots dx_n$$

DISTRIBUCION DE PROBABILIDAD CONDICIONAL

Si en una función de distribución de probabilidad conjunta algunas variables adquieren valores fijos la función de distribución de probabilidad normalizada resultante representa a la distribución CONDICIONAL. Sean X y Y variables aleatorias continuas y $f_{x,y}$ (x, y) su función de distribución; si Y adquiere el valor y_o, entonces la función de probabilidad condicional de X es igual a

$$f_{X|Y}(x,y_o) = \frac{f_{x,y}(x,y_o)}{f_y(y_o)} \qquad (6)$$

En donde

$$f_y(y_o) = \int_{-\infty}^{\infty} f_{x,y}(x,y_o)\,dx$$

FUNCION DE DISTRIBUCION ACUMULADA

Una forma alternativa de representar el comportamiento de una variable aleatoria es mediante la función de distribución acumulada. El valor de dicha función, $F_X(x)$, es igual a la probabilidad de que la variable aleatoria sea menor o igual que el argumento, es decir

$$F_X(x) = P(X \leq x) = \int_{-\infty}^{x} f_X(u)\,du$$

La función de distribución $f_X(x)$ se puede encontrar a partir de la función acumulada, a través de

$$\frac{dF_X(x)}{dx} = \frac{d}{dx}\left[\int_{-\infty}^{x} f_X(u)\,du\right] = f_X(x)$$

Sus propiedades son las siguientes

$$0 \leq F_X(x) \leq 1$$

$$F_X(-\infty) = 0$$

$$F_X(\infty) = 1.$$

$$F_X(x_2) - F_X(x_1) = P[x_1 < X \leq x_2]$$

MOMENTOS DE UNA VARIABLE ALEATORIA

Media ó valor esperado de una variable continua X

$$m_x = E(x) = \int_{-\infty}^{\infty} x\, f_X(x)\,dx$$

Variancia de una variable continua X

$$\sigma_X^2 = var(x) = \int_{-\infty}^{\infty} (x - m_x)^2 f_X(x)\,dx = E[X^2] - E^2[X]$$

Desviación estandard de una variable continua X;

$$\sigma_X = \sqrt{\sigma_X^2}$$

FUNCIONES DE MASA DE PROBABILIDAD: BINOMINAL Y DE POISSON

Enseguida se tratan dos distribuciones de variables aleatorias discretas ; discontinua de orden finito la BINOMIAL y de orden infinito la de POISSON

DISTRIBUCION BINOMINAL

Se aplica a experimentos de Bernoulli (acepta únicamente dos posibles resultados: éxito y fracaso).

Sea

p = probabilidad de obtener éxito

q = 1 -p = probabilidad de obtener fracaso

La probabilidad de obtener x éxtito al realizar n veces el experimento de Bernoulli es

$$\underbrace{p \cdot p \cdot p \cdot p}_{x} \cdot \underbrace{q \cdot q \cdot q \cdots q}_{n-x} = p^x q^{n-x}$$

Aquí se supuso que los x éxitos ocurren al principio. El número total de formas en que pueden presentarse los x éxitos es igual a las permutaciones de n objetos formados por dos grupos: uno de x objetos iguales y otro de n-x objetos iguales o sea igual a

$$\frac{n!}{x!(n-x)!}$$

o sea que la probabilidad buscada es la siguiente

$$P(x) = \frac{n!}{x!(n-x)!} p^x q^{n-x} \qquad (10)$$

La variable aleatoria x es un número entero entre cero y n

Su media es igual a

$$m_x = \sum_{x=0}^{n} x\, P(x) = np$$

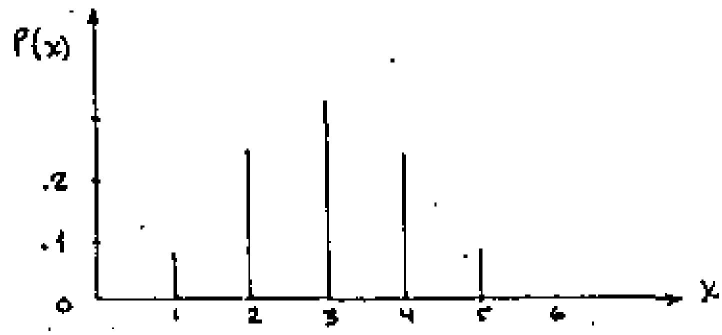
Su variancia es

$$\sigma_x^2 = \sum_{x=0}^{n} x^2 P(x) = npq$$

Su desviación estándard

$$\sigma_x = \sqrt{npq}$$

La representación gráfica de P(x) para n = 6 y p = 0.5 es como sigue



DISTRIBUCION DE POISSON

Si se considera que en la distribución binominal n tiende a infinito mientras que la probabilidad p de éxito tiende a cero, entonces la ec.(10) se convierte en

$$P(x) = \frac{\nu^x}{x!} e^{-\nu} , \quad x = 0,1,2,3 \ldots, \infty \tag{11}$$

Esta distribución de probabilidad se llama de POISSON de parámetro ν

Su media está dada por

$$m_x = \sum_{x=0}^{\infty} x \, P(x) = \nu$$

Su variancia es igual a

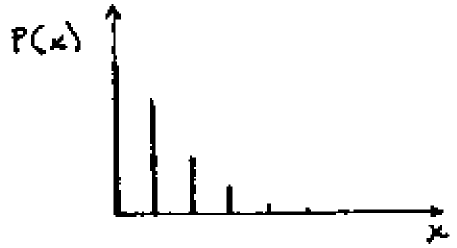
$$\sigma_x^2 = \sum_{x=0}^{\infty} x^2 P(x) = \nu$$

Su desviación estándard

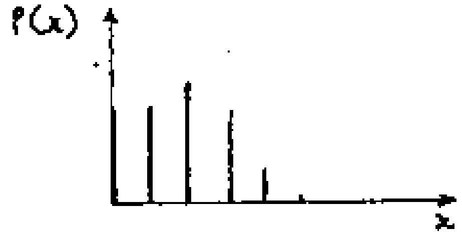
$$\sigma_x = \sqrt{\nu}$$

(La distribución de Poisson de parámetros $\nu = np$ se aproxima a la binomial siempre que $n > 50$ y $n\,p < 5$)

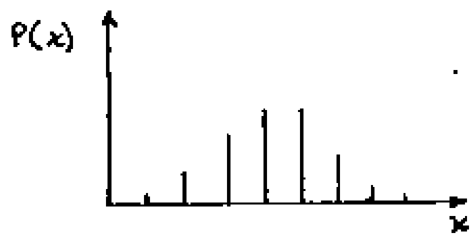
En las siguientes figuras se ilustra la variación de la forma de las distribuciones con el parámetro ν:



$\nu = 0.7$



$\nu = 2.5$



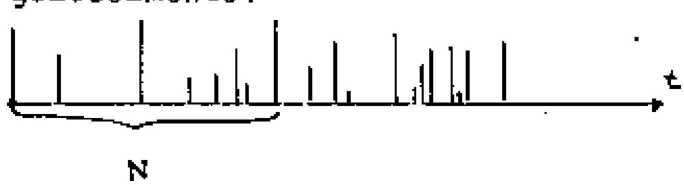
$\nu = 5.0$

PROCESO DE POISSON

Este proceso representa el número de eventos que ocurre en un tiempo t cuando dichos eventos tienen distribución de Poisson; es decir,

$$P(n) = \frac{(\lambda t)^n e^{-\lambda t}}{n!}$$

Ilustrando esto gráficamente:



En un proceso de Poisson, la media de su distribución (de Poisson) es $m_n = \lambda t$. Al parámetro λ se le llama tasa media de ocurrencia del proceso.

Un proceso de Poisson debe satisfacer las siguientes hipótesis:

1. ESTACIONARIEDAD

La probabilidad de un evento en un intervalo *corto* de tiempo $(t, t + \Delta t)$ es aproximadamente $\lambda(\Delta t)$ para cualquier t. Es decir que no importa qué tiempo t se elige para hacer la estimación.

2. NO MULTIPLICIDAD

La probabilidad de 2 o más eventos en un intervalo corto de tiempo es despreciable comparado con $\lambda(\Delta t)$

3. INDEPENCENCIA

El número de eventos en cualquier intervalo de tiempo es independiente de el número en cualquier otro intervalo de tiempo. O sea que la ocurrencia de unos es independiente de los que ocurren en otro intervalo de tiempo.

Varios investigadores han propuesto modelos probabilísticos para describir la ocurrencia de temblores. Algunos de ellos se basan en la construcción de histogramas de tiempos de espera entre eventos sísmicos. Por simplicidad matemática frecuentemente se adopta la hipótesis que la distribución probabilística de tales tiempos de espera es de Poisson. Es decir, para una determinada región, la probabilidad de que ocurran n temblores con magnitud mayor que M en el intervalo de tiempo (0, t) es

$$p(n) = (\lambda_M t)^n e^{-\lambda_M t} / n!$$

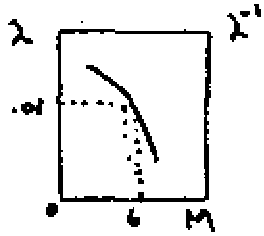
en que λ_M es el número medio de temblores con magnitud mayor que M que ocurran en dicha región por unidad de tiempo. Si se considera n nulo se obtiene $p(0) = e^{-\lambda_M t}$

que representa la probabilidad de que no ocurran temblores con magnitud mayor que M en el intervalo de tiempo t (aún cuando no empiece en cero) es decir la probabilidad de la intensidad máxima en el intervalo t

EJEMPLO

Mediante un estudio estadístico sobre la ocurrencia de sismos en cierta región se estimó que un temblor con una magnitud igual a 6 o mayor tiene un periodo de recurrencia de 100 años. Calcular las probabilidades de que en los próximos 10, 50 y 100 años no ocurra ningún sismo en dicha región cuya magnitud exceda a 6, suponiendo que la ocurrencia de los sismos se pue de modelar mediante un proceso estocástico de Poisson.

$$\lambda_M = \frac{1}{100} = .01$$



$$P(n) = \frac{(.01t)^n \, e^{-.01t}}{n!}$$

Para t = 10 años

$$P(0) = \frac{(.01 \times 10)^0 \, e^{-.01 \times 10}}{0!} = .905$$

Para t = 50 años

$$P(0) = \frac{(.01 \times 50)^0 \, e^{-.01 \times 50}}{0!} = .607$$

Para t = 100 años

$$P(0) = \frac{(.01 \times 100)^0 \, e^{-.01 \times 100}}{0!} = .368$$

Las probabilidades de que ocurra por lo menos un sismo con magnitud mayor que 6 son

$$P_{\text{[illegible]}}(n \geq 1) = 1 - .905 = .095$$

$$P_{\text{[illegible]}}(n \geq 1) = 1 - .607 = .393$$

$$P_{\text{[illegible]}}(n \geq 1) = 1 - .368 = .632$$

FUNCIONES DE DENSIDAD DE PROBABILIDAD:

EXPONENCIAL, NORMAL Y LOGNORMAL

DISTRIBUCION EXPONENCIAL

En la sección anterior se trató el proceso de Poisson. En lo que sigue se describirá la distribución de tiempo de espera entre eventos si ellos se describen mediante un proceso de Poisson.

Si denotamos la variable aleatoria T como el tiempo de la primer ocurrencia de un evento, entonces la probabilidad de que T exceda algún valor de t es igual a la que no ocurran eventos en el intervalo de tiempo t, o sea igual a $1-F_T(t) = p\ (0)$, o sea

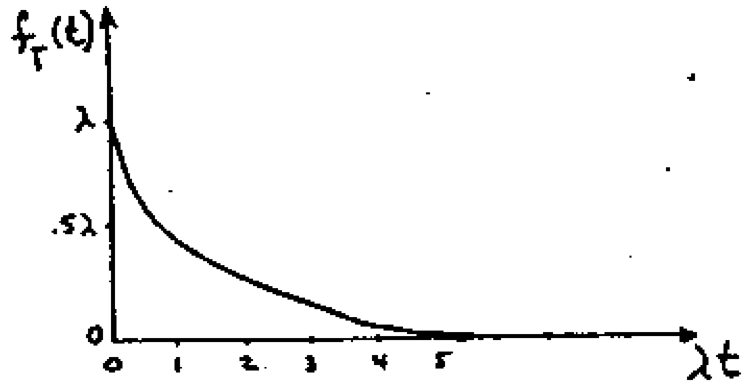
$$1 - F_T(t) = e^{-\lambda_M t}, \quad t \geq 0$$

Entonces

$$F_T(t) = 1 - e^{-\lambda_M t}$$

$$f_T(t) = \frac{dF_T(t)}{dt} = \lambda_M e^{-\lambda_M t}, \quad t \geq 0$$

Que es la distribución exponencial. Esta describe el tiempo de la primer ocurrencia de un evento de Poisson. Pero recordando las propiedades de independencia y estacionariedad del proceso de Poisson $e^{-\lambda_M t}$ es la probabilidad de que no se presenten eventos en ningún intervalo de tiempo t, aunque este no empiece en t = 0. Si utilizamos el tiempo de arrivo del n-ésimo evento como el principio del intervalo de tiempo, entonces $e^{-\lambda_M t}$ es la probabilidad de que el tiempo de ocurrencia del (n+1)-ésimo evento sea mayor t. Es decir, los tiempos de arrivo intermedios de un Proceso de Poisson son independientes y están distribuidos exponencialmente. Su valor esperado, conocido como *período de recurrencia*, es igual a $(\lambda_M)^{-1}$, su varianza es igual a $(\lambda_M^2)^{-1}$



Distribución exponencial

DISTRIBUCION NORMAL

Una de las más importantes funciones dentro de la teoría de Probabilidades es la NORMAL O GAUSSIANA. Esta es aplicable a variables aleatorias continuas dentro del dominio de los números reales.

La función de densidad de probabilidad está dada por

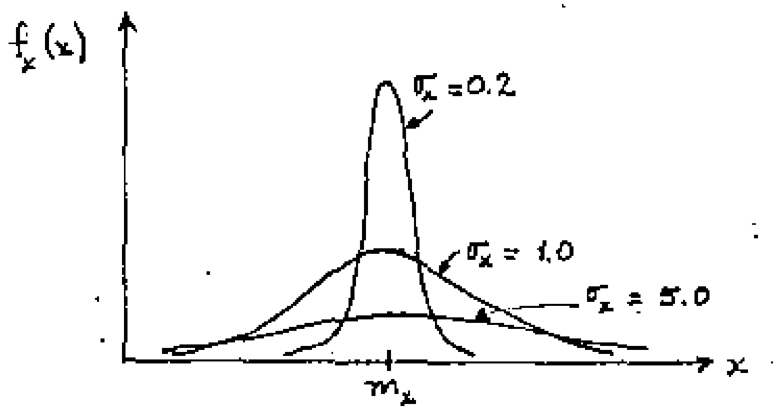
$$f_x(x) = \frac{1}{\sqrt{2\pi}\,\sigma_x} e^{-\frac{(x-m_x)^2}{2\sigma_x^2}} \quad , -\infty < x < \infty \qquad (13)$$

en donde

m_x = media

σ_x^2 = variancia

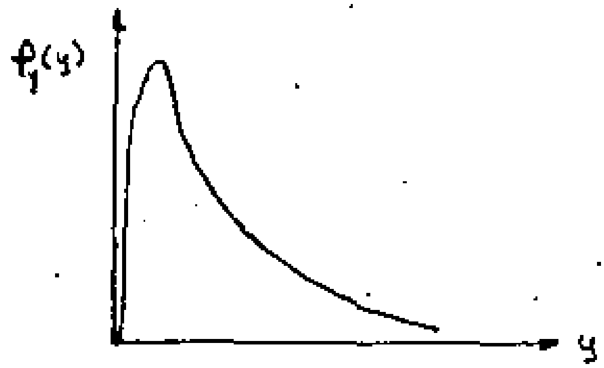
Al examinar esta expresión se deduce que es una función simétrica con respecto a un eje vertical que pasa por m_x, que es asintótica al eje de las abscisas para valores que tiendan a $\pm\infty$, y, que su valor máximo corresponde a m_x. En la siguiente figura se presenta su representación cuando su media permanece constante igual a m_x y su desviación estándar (σ_x) varía



La distribución LOGARITMICO-NORMAL O LOGNORMAL se presenta en el caso de que el logaritmo natural de una variable aleatoria tenga distribución normal. Es decir, si la variable X tiene una función de densidad dada por la ec 13, y si $X = \ln Y$, entonces la función de densidad de Y resulta lognormal y está dada por

$$f_Y(y) = \frac{1}{y\,\sigma_x\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\ln y - m_x}{\sigma_x}\right)^2\right] , \quad y \geq 0 \qquad (14)$$

La siguiente figura muestra la gráfica de una distribución logarítmico-normal con $m_x = 0$ y $\sigma_x = 1$. Esta es de forma asimétrica positiva



Su media está dada por

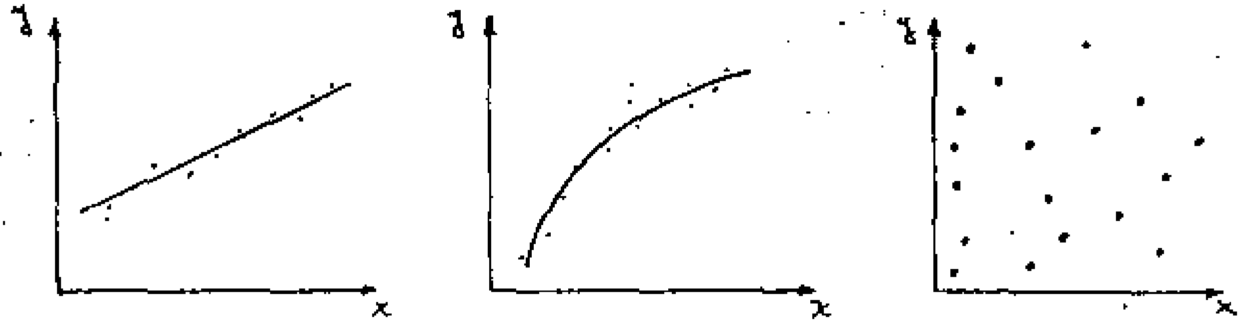
$$m_Y = \int_0^\infty y\, f(y)\, dy = \exp\left(m_x + \sigma_x^2/2\right)$$

Su varianza es igual a

$$\sigma_Y^2 = \exp\left(2m_x + \sigma_x^2\right) \cdot \left(\exp \sigma_x^2 - 1\right)$$

ANALISIS DE REGRESION

Una incógnita importante que debe despejarse en el análisis de regresión es la FORMA GENERAL DE LA EXPRESION MATEMATICA que se piensa puede explicar el comportamiento de cierto fenómeno. Un procedimiento gráfico puede resolver este problema. Dibujando los valores observados de la variable independiente X con los correspondientes valores observados de la variable dependiente y en un sistema de coordenadas rectangulares, se obtiene un conjunto de puntos conocidos como DIAGRAMA DE DISPERSION



a) Relación lineal b) Relación no lineal c) No existe Relación

La CURVA DE REGRESION es aquella a la cual tienden a aproximarse los puntos del diagrama de dispersión. La ecuación de la curva de regresión es la ECUACION DE REGRESION.

En el caso de regresión lineal se tiene una ecuación de regresión de la forma

$$y = a_0 + a_1 x$$

con dos parámetros por determinar a_0 y a_1

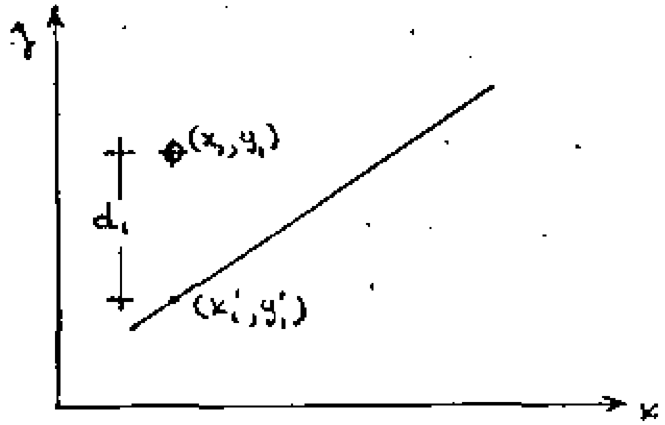
Existen diferentes métodos para determinar estos parámetros aquí se estudiará el método de MINIMOS CUADRADOS

METODO DE MINIMOS CUADRADOS

Se llama DESVIACION, ERROR O RESIDUO a la diferencia de ordenadas de un punto muestral y de la curva de regresión correspondiente a una misma abscisa

$$d_i = y_i - y_i'$$

$$y_i' = a_0 + a_1 x_i'$$



El método de los mínimos cuadrados establece que de todas las curvas de regresión que se pueden ajustar al conjunto de puntos muestrales dados la MEJOR es aquella que tenga la propiedad de que la suma de los cuadrados de sus residuos sea mínima

$$\min \sum_{i=1}^{m} d_i^2$$

Aplicando este criterio para el caso de una recta

$$\min \sum d_i^2 = \min \sum \left[y_i - (a_0 + a_1 x_i) \right]^2$$

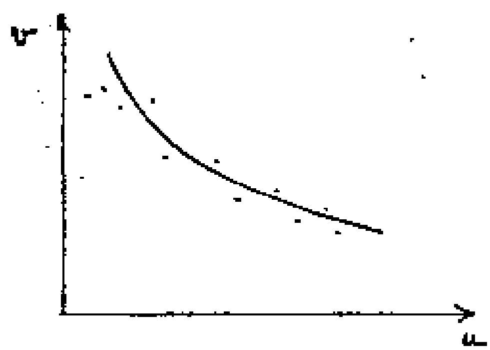
Para encontrar el mínimo se aplica la condición necesaria conocida para que una función tenga un punto extremo, es decir, se iguala a cero las primeras derivadas parciales de la función con respecto a cada una de sus variables.

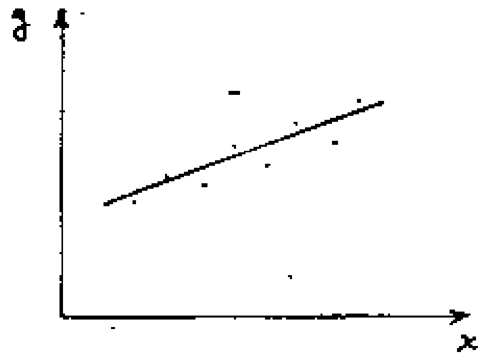
El resultado de esto conduce a un sistema de ecuaciones simultáneas cuya solución es el valor de los parámetros a_0 y a_1.

REGRESION NO LINEAL

Para resolver el caso de regresión no lineal, generalmente conviene MAPEAR los puntos muestrales a un sistema de referencia en donde sí se aproximen a una recta, mediante ecuaciones de TRANSFORMACION

$$x = x(u,v)$$
$$y = y(u,v)$$





Los sistemas de transformación que se usan con mayor frecuencia con los SEMILOGARITMICOS Y LOS LOGARITMICOS

Por ejemplo una transformación SEMILOGARITMICA es como sigue:

$$x = u$$
$$y = \log v$$

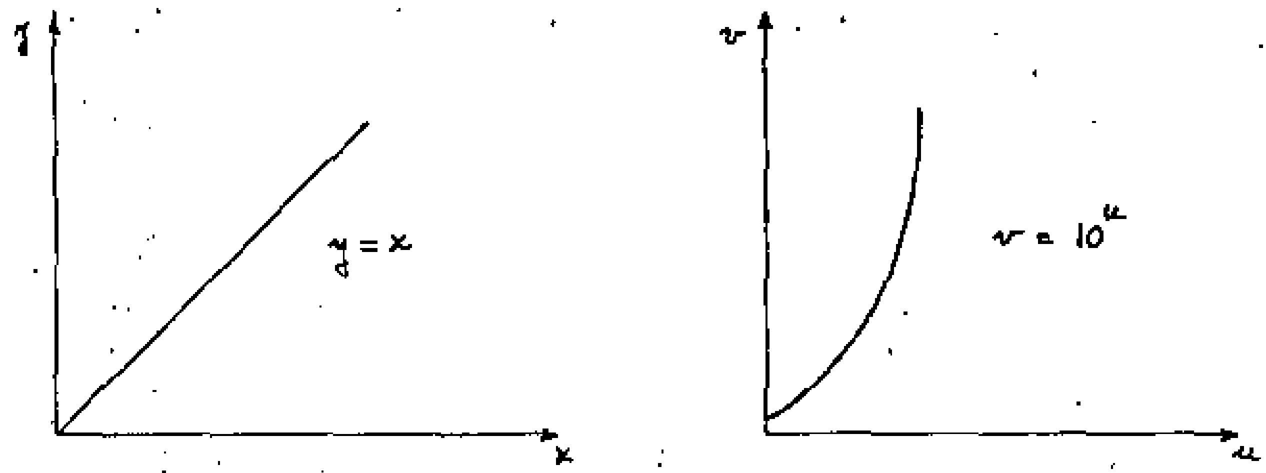
En el sistema x-y el diagrama de dispersión de los puntos mapeados (x_i, y_i) corresponde a una recta cuya ecuación es $y = a_0 + a_1 x$, en el sistema uv la ecuación correspondiente es

$$\log v = a_0 + a_1 u$$
$$v = 10^{a_0 + a_1 u}$$
$$= (10^{a_0})(10^{a_1})^u$$
$$= a\, b^u$$

En donde

$$a = 10^{a_0}$$
$$b = 10^{a_1}$$

Ejemplo. Sea $a_0 = 0$, $a_1 = 1$



DIVISION DE EDUCACION CONTINUA
FACULTAD DE INGENIERIA U.N.A.M.

X CURSO INTERNACIONAL DE INGENIERIA SISMICA

ANALISIS DE RIESGO SISMICO

MODELOS DE RIESGO SISMICO

PARTE I. ESTIMACION BAYESIANA Y EJEMPLO DE APLICACION

DR. MARIO CHAVEZ

AGOSTO, 1984.

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X CURSO INTERNACIONAL DE INGENIERIA SISMICA

MODELOS DE RIESGO SISMICO

PARTE I.- ESTIMACION BAYESIANA Y EJEMPLO DE APLICACION

Mario Chávez*

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1. INTRODUCCION

Los criterios de diseño modernos tienen como objetivo optimizar las estructuras, es decir, conducir a sistemas estructurales en los cuales la utilidad obtenida de su ejecución sea máxima. La determinación de la estructura óptima se puede lograr a través de estudios costo-beneficio que incluyan explícitamente los siguientes parámetros: los beneficios esperados de su construcción, los costos esperados de las posibles fallas, los costos iniciales de diseño y construcción, así como las probabilidades de ocurrencia y las intensidades máximas de las solicitaciones de diseño. En este trabajo se tratará lo relativo a la determinación de las solicitaciones sísmicas en un sitio dado con base en modelos de riesgo sísmico. Los otros parámetros serán discutidos en otras contribuciones de esta reunión.

Las cargas sísmicas producidas en construcciones durante la ocurrencia de un temblor son generadas por el movimiento errático del suelo que rodea la cimentación de una construcción. Dicho movimiento es trasmitido a la estructura como fuerzas de inercia que actúan en los diferentes elementos de la misma. La amplitud de ésas fuerzas depende principalmente de las propiedades dinámicas del sistema estructural y de las características del temblor.

Sin embargo, es ampliamente reconocido que se tienen grandes incertidumbres respecto a la intensidad, localización y el número de temblores futuros, por lo cual el diseño de estructuras en una región de actividad sísmica potencial debe incorporar dichas incertidumbres. Una forma racional de hacer esto último es evaluando los niveles de las intensidades máximas esperadas del terreno en un sitio dado para diferentes lapsos de tiempo. Lo anterior puede lograrse haciendo uso de los llamados modelos de riesgo sísmico, los cuales son modelos matemáticos que en una situación ideal deben sintetizar toda la información sísmica existente sobre la región de interés, así como el estado del arte sobre el proceso de generación de temblores. Los resultados de dichos modelos son generalmente presentados por medio de gráficas o tablas que relacionan los valores de los parámetros del movimiento del suelo (aceleración, velocidad y desplazamiento máximos) en un sitio de interés con lapsos de tiempo llamados periodos de recurrencia.

Una vez que el riesgo sísmico para un sitio ha sido determinado, el calculista estará en posibilidad de analizar el comportamiento de las implicaciones económicas de diferentes[2] diseños bajo varios niveles de la intensidad del movimiento del terreno.

La información sísmica que se mencionó en el párrafo anterior puede dividirse en los siguientes tipos: tectónica, estadística (sismicidad), geofísica y geológica. Como se verá más adelante, con los modelos de riesgo sísmico actuales es muy frecuente utilizar las dos primeras fuentes de información (en particular la relacionada con la sismicidad), mientras que las dos últimas son utilizadas en una forma complementaria e indirecta. Debido a lo anterior en este trabajo se le dará énfasis a la información tectónica y sobre la sismicidad de la región de interés, que es el occidente de México. En el capítulo 2 se presentan brevemente aspectos relacionados con los temblores y la respuesta de las construcciones a los mismos. Una síntesis de los aspectos más importantes sobre la tectónica la sismicidad de esta región se presenta en el capítulo 3.

En el Cap 4 se presenta un modelo de riesgo sísmico cuyos parámetros son estimados aplicando estadística bayesiana. Dicho modelo se ha utilizado recientemente en el Instituto de Ingeniería, UNAM, para determinar las intensidades máximas del terreno (para diferentes lapsos de tiempo) en algunos sitios de México. En el mismo capítulo 5 se presentan los resultados obtenidos de aplicar el modelo mencionado para determinar el riesgo sísmico en un sitio en el estado de Jalisco. Finalmente en el Cap 6 se presentan algunas conclusiones y recomendaciones sobre el riesgo sísmico del occidente de México.

2. TEMBLORES Y LA RESPUESTA DE CONSTRUCCIONES A LOS MISMOS

2.1 Origen de los temblores

Los temblores de interés en ingeniería se consideran de origen tectónico. Esta clase de temblores se generan cuando la corteza terrestre libera energía elástica al desplazarse las superficies laterales de fallas geológicas. Dicha energía se propaga en la forma de ondas sísmicas a través de la corteza terrestre, las cuales se manifiestan por el movimiento caótico de la superficie de la tierra. Este movimiento errático es el que daña las estructuras localizadas en sitios alcanzados por temblores, cuando ésas estructuras no han sido diseñadas para soportarlos.

Se llama foco de un temblor al punto bajo la superficie de la tierra donde se supone que se inicia la ruptura de una falla geológica. La profundidad de ese punto se conoce como profundidad focal y la distancia del foco a un sitio de interés se denomina distancia focal o hipocentro. La proyección vertical del foco sobre la superficie de la tierra se llama epicentro y la distancia de éste a un sitio se le conoce como distancia epicentral. La longitud de ruptura de una falla se denomina longitud de la falla.

2.2 Intensidad de Mercalli y magnitud de un temblor

La intensidad de Mercalli modificada, I(MM), es una medida subjetiva del nivel de daño que sufren las estructuras hechas por el hombre, así como de modificaciones a la geología superficial producidas por un temblor en un sitio o una región dada (ref 1). Dependiendo del nivel de daño causado por un temblor en un sitio, se asigna un número romano que varía del I al XII (de la escala de Mercalli modificada) a un temblor en ése sitio. Se llaman isosistas de un temblor las líneas que unen puntos (sitios) en los cuales ese temblor se manifiesta con una misma intensidad, en la escala de Mercalli modificada. Las isosistas forman parte de los llamados mapas de isosistas de una región para un temblor determinado.

La magnitud, M, de un temblor es una medida objetiva de la energía liberada durante un temblor; en su forma original se definió como

$$M = \log_{10} \frac{A}{A_0} \tag{1.1}$$

donde M es la magnitud del temblor, A es la amplitud máxima (en milímetros) registrados por un sismógrafo Wood-Anderson a una distancia de 100 Km del epicentro y A_0 es una amplitud igual a 10^{-3} milímetros (ref 1). En la actualidad la magnitud de un temblor se obtiene como el promedio de las magnitudes obtenidas por diferentes estaciones sismológicas. Existen algunas variantes de la escala original de magnitudes de los temblores.

La energía sísmica (en ergs) liberada durante un temblor puede calcularse en forma aproximada mediante la expresión semiempírica (ref 1):

$$\log_{10}E = 11.8 + 1.5M \tag{1.2}$$

Dado que la escala de Mercalli modificada no requiere de instrumentos para asignar la intensidad de un temblor en un sitio, los catálogos de I(MM) constituyen la única fuente de información sísmica disponible para algunos sitios o regiones, especialmente para temblores ocurridos antes de principios de siglo. Los catálogos de magnitudes de temblores se inciaron en la tercera década de este siglo, principalmente en el área de California, Estados Unidos. Para otras regiones del mundo dichos catálogos se iniciaron hace unos pocos años. Como se verá en el capítulo 5, la información contenida en esos catálogos constituye una parte esencial de los procedimientos utilizados para predecir las intensidades futuras en algún sitio de interés; por lo cual se han propuesto expresiones semiempíricas que relacionan la magnitud M de un temblor y su correspondiente I(MM), así como algunas características geométricas del temblor (como su profundidad focal etc.). Dichas expresiones se utilizan para generar o complementar catálogos de M en algunos sitios o regiones de interés.

2.3 Acelerogramas e intensidades máximas del terreno

Desde el punto de vista de la ingeniería, la información fundamental para estimar las fuerzas sísmicas que actúan sobre una estructura la proporcionan los acelerogramas, los cuales son gráficas que representan la variación en el tiempo de las aceleraciones del terreno. Un acelerograma, el cual se puede idealizar como una secuencia aleatoria de pulsos de aceleraciones generados por el movimiento del terreno durante un temblor, se obtiene por medio de un acelerógrafo. Los acelerogramas también se pueden simular por medio de computadoras analógicas o digitales. Las variaciones en el tiempo de las velocidades y los desplazamientos del terreno obtienen al integrar el acelerograma correspondiente uno y dos veces respectivamente (ref. 2).

De observaciones de campo y de laboratorio, se ha concluido que las respuestas máximas de las construcciones a temblores (aceleraciones, velocidades, desplazamientos) dependen de la intensidad*, el contenido de frecuencias, la distribución en el tiempo de dicha intensidad y la duración del temblor (ref 1). La intensidad y el contenido de frecuencias de un

* En lo que sigue se entiende por intensidad de un temblor a cualquier parámetro del movimiento del terreno relacionado con la respuesta de una estructura a dicho temblor.

temblor pueden estimarse (en forma aproximada) a partir de su aceleración, velocidad y desplazamiento máximos.

2.4 Espectros de respuesta

Las respuestas máximas de un sistema de un grado de libertad* sujeto a un temblor se pueden obtener resolviendo la ecuación de equilibrio dinámico (expresión matemática que relaciona las fuerzas de inercia, amortiguamiento y elásticas del sistema con la fuerza sísmica correspondiente, ref 1). Otra forma de obtener dichas respuestas es a partir del registro de las aceleraciones de la masa del sistema, en caso que el sistema estructural esté instrumentado.

Se conoce como espectro de respuesta a la curva que resulta de unir los puntos correspondientes a las respuestas máximas a un acelerograma dado de sistemas de un grado de libertad con el mismo amortiguamiento y diferentes periodos naturales de vibración.

De estudios estadísticos efectuados en diferentes lugares (ref 1 y 2) se ha llegado a la conclusión de que las intensidades máximas del terreno están correlacionadas con los espectros de respuesta. En particular, la aceleración máxima del terreno se relaciona satisfactoriamente con las respuestas máximas de sistemas de periodo corto (estructuras rígidas), mientras que la velocidad máxima y el desplazamiento máximos correlacionan adecuadamente con sistemas de periodo intermedio y largo (estructuras flexibles) respectivamente. De aquí se sigue que los espectros de respuesta pueden calcularse a partir de las intensidades máximas del terreno y factores espectrales de amplificación como los propuestos en la ref 1.

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* Un sistema en el cual una sola coordenada define la localización geométrica de su masa, como por ejemplo un pórtico de una crujía, cuya trabe tiene una rigidez infinita.

Con el procedimiento mencionado en el párrafo anterior es posible obtener los espectros de diseño. Los cuales son gráficas que relacionan las envolventes de las respuestas máximas esperadas de sistemas de un grado de libertad sujetos a diferentes temblores con los periodos naturales (o frecuencias naturales) de vibración de dichos sistemas. A partir de los espectros de diseño se pueden calcular las cargas sísmicas que en un instante dado actuarían sobre un sistema estructural.

3. TECTONICA Y SISMICIDAD EN EL OCCIDENTE DE MEXICO

3.1 Tectónica

La estructura tectónica de la región comprendida por los estados de Michoacán, Colima, Jalisco, Nayarit y Sinaloa esta relacionada con el movimiento de subducción de las placas de Cocos y de Rivera bajo la placa Americana, así como con los movimientos de las fallas transformadas y centros de dispersión del Golfo de California refs (3, 4, 5, 6).

Los estudios efectuados en dichas refs permiten concluir que la estructura tectónica de la región es muy compleja, en particular en lo que se refiere al comportamiento tectónico de la triple junta de las mencionadas placas. Lo anterior se debe a que tanto las velocidades de subducción, como los rumbos de las placas de Cocos y de Rivera son muy diferentes, ya que la primera se desplaza con una velocidad de 5 cm/año en la dirección noreste ref (4) mientras la segunda lo hace con una velocidad de 2.1 cm/año en la dirección noroeste ref (3). De lo anterior se sigue que los mecanismos de los temblores en la zona sean muy diferentes, dependiendo de cual sea la placa a que se asocien ref (3).

Así, de los temblores localizados fuera de la costa, algunos se asocian a la zona de la Fractura de Tamayo frente a las costas de Sinaloa y Nayarit, otros a la zona de la fractura de Rivera y la mayoría a la trinchera mesoamericana refs (3, 4, 5, 6). En cuanto a los temblores localizados tierra adentro en la ref (7) se sugiere que se les puede asociar a los movimientos en las fallas Zacamboxo, del Pacífico y Clarión Fig 1; debe mencionarse que no se tiene evidencia concluyente sobre la existencia y por tanto de las características específicas de estas fallas.

3.2 Sismicidad

La sismicidad de la región ha sido motivo de estudios recientes como los de las refs (3, 6, 7, 8). De dichos estudios se puede concluir que se tiene evidencia estadística de la ocurrencia de temblores destructivos en la región de interés, mas no se cuenta con los correspondientes registros del movimiento del terreno.

Entre los temblores con magnitudes mayores o iguales a 7 ocurridos en la región se puede mencionar tres en el siglo pasado (1845, 1858 y 1875) y cuatro en el presente (1911, 1932, 1941 y 1973) refs (7 y 8) Figs 2 a 7.

En la ref (3) se estudia el problema de la localización del temblor de junio de 1932 con magnitud igual a 8.2 y se concluye que dependiendo de si este temblor fue causado por el movimiento de la placa de Cocos o por la placa de Rivera se espera que un temblor con características similares ocurriría (o sea tendría un periodo de recurrencia) en 56 o 140 años a partir de 1932. En la ref (8) se menciona que la región de Jalisco tiene un potencial sísmico muy alto y que el lapso para que ocurran temblores fuertes (magnitud mayor o igual a 7) varía entre 32 y 56 años.

3.3 Conclusiones sobre la tectónica y sismicidad del occidente de México

De lo discutido en los incisos 3.1 y 3.2 se puede concluir lo siguiente:

a) La información actual sobre la geotectónica y la sismicidad del occidente de México solo permite afirmar que la región es sísmicamente activa.

b) Que en la región pueden ocurrir temblores destructivos cuyo periodo de recurrencia varía en un rango de decenas a cientos de años.

c) Se tiene información de tipo estadístico sobre las magnitudes y la localización de temblores ocurridos en la región en el presente siglo y algunos del pasado.

d) No se cuenta con registros del movimiento del terreno en la región; que sean de interés ingenieril.

4. MODELOS DE RIESGO SISMICO

4.1 Modelos de riesgo sísmico

Los modelos de riesgo sísmico son modelos matemáticos que en una situación ideal sintetizan el conocimiento actual sobre el proceso de generación de temblores y que se combinan con la información sismotectónica disponible en una región, para estimar los valores esperados de las intensidades máximas del movimiento del terreno en un sitio dado durante ciertos lapsos (períodos de recurrencia).

Los modelos de riesgo sísmico que se han utilizado en los últimos años en el Instituto de Ingeniería, UNAM, tienen como hipótesis fundamental que las intensidades máximas del terreno en un sitio determinado resultan de las contribuciones de la actividad sísmica de volumenes de la corteza terrestre vecinos al sitio. Sin embargo, los modelos mencionados difieren en la forma en que consideran las incertidumbres inherentes a los diferentes tipos de información sísmica disponible y sobre los modelos conceptuales del proceso de generación de temblores.

Se tiene un tipo de modelos que aquí se denomina bayesianos en los cuales las incertidumbres sobre la información y los modelos mencionados son explícitamente considerados por medio del teorema de Bayes (Apéndice A). Los otros modelos, que llamaremos no bayesianos, manejan las incertidumbres men cionadas apoyandose exclusivamente en la información estadística con que se cuente (generalmente escasa) o en la opinión (generalmente conservadora) de los especialistas para modificar alguna hipótesis sobre el proceso de generación de temblores.

En la Parte II se presenta un modelo de riesgo sísmico no-bayesiano utilizado recientemente. En 4.2 se propone un modelo de riesgo sísmico bayesiano y en el cap 5 un ejemplo de su aplicación a la determinación de las intensidades máximas esperadas del terreno para un sitio en el estado de Jalisco.

4.2 Modelo de riesgo sísmico con parámetros bayesianos

4.2.1 Sismicidad regional y local

La sismicidad regional, v, en un sitio determinado, o sea el efecto que producen los temblores que ocurren en la región vecina al sitio, depende principalmente de la contribución que a la misma hacen los volumenes de la corteza terrestre alrededor de dicho sitio. Dada la atenuación de la intensidad con la distancia, solamente los temblores localizados a algunos cientos de kilómetros del sitio contribuyen en forma importante a esta sismicidad.

Los volúmenes citados se designan como fuentes sísmicas y se caracterizan por las incertidumbres asociadas a la frecuencia de ocurrencia y las magnitudes de los temblores que generan. Se denomina sismicidad local, λ, al proceso de ocurrencia de temblores de diversas magnitudes en una fuente sísmica. De lo anterior se puede concluir que la sismicidad regional para un sitio puede estimarse a partir de las sismicidades locales de las fuentes sísmicas correspondientes.

Por ejemplo si $\lambda(M)$ representa el número de temblores de magnitud igual o mayor que M generados en promedio por unidad de volumen y por año en fuentes sísmicas vecinas a un sitio específico, el efecto de estos temblores en dicho sitio se puede evaluar con la expresión (ref 10)

$$\nu_c(y) = \int_V \lambda\{M(y, R)\}\, dv \qquad (4.1)$$

donde $\nu_c(y)$ es el número medio anual de temblores con intensidad mayores que "y" registrados en el sitio de interés y generados por las fuentes sísmicas, $M\{(y, R)\}$ es la magnitud M que produce una intensidad "y" a la distancia R y dv es un volumen elemental de la corteza terrestre distante R del sitio de interés. El subíndice c de ν significa que las intensidades "y" son calculadas a partir de las magnitudes, profundidades y distancias epicentrales provenientes de catálogos de temblores y de leyes de atenuación que relacionan magnitud-distancia-intensidad como se comenta más adelante.

4.2.2 Modelo de sismicidad local

La sismicidad local, $\lambda(M)$, asociada a una fuente sísmica determinada puede estimarse utilizando un modelo del proceso estocástico (aleatorio) de la ocurrencia de temblores de diversas magnitudes en dicha fuente. Si se adopta una forma para el proceso, la sismicidad local podría calcularse una vez que se estimen los parámetros que definen a dicho proceso.

En este trabajo se supone que la ocurrencia de eventos se puede idealizar mediente un proceso homogeneo de Poisson. Lo anterior implica que la distribución de los tiempos de ocurrencia de temblores es independiente de la historia previa. Esta independencia se supone también por lo que respecta a la distribución espacial de los focos de los eventos sísmicos. De acuerdo con la referencia (11), $\lambda(M)$ se expre sa como

$$\lambda(M) = \alpha\ e^{-\beta M}\ (1 - e^{-\gamma(M_u - M)}) \quad M \leq M_u$$
$$= 0 \qquad\qquad M > M_u \tag{4.2}$$

donde α, β, γ y M_u son parámetros del proceso, los cuales deben estimarse a partir de los datos estadísticos, y de las características sismotectónicas de las fuentes vecinas al sitio o de zonas similares de la tierra. M_u es la magnitud máxima que dichas fuentes pueden generar.

4.2.3 Ecuaciones de atenuación

Las ecuaciones de atenuación son expresiones semiempíricas que relacionan magnitud-distancia-intensidad (aceleraciones, velocidades y desplazamientos máximos) de temblores en terreno firme. Dichas expresiones pueden obtenerse a partir de los datos que existen sobre los parámetros mencionados. En general, los procedimientos utilizados para obtener las leyes de atenuación consisten en ajustar curvas a los datos de eventos ocurridos en diferentes regiones, por lo cual las expresiones así obtenidas reflejan las características geotectónicas de la región para la cual fueran obtenidas (ref 12).

En este estudio se utilizarán leyes de atenuación de la forma siguiente

$$y = b_1\ e^{b_2 M}\ (R + R_0)^{-b_3} \tag{4.3}$$

donde b_1, b_2, b_3 y R_0 son los parámetros estimados del ajuste y R fue definido previamente.

En este trabajo se usarán las leyes de atenuación propuestas en la ref 12 las cuales fueron obtenidas a partir de la información correspondiente a temblores ocurridos en terreno firme de la costa oeste de los Estados Unidos y de la República Mexicana. Las expresiones propuestas en dicha ref para estimar la aceleración, a, y la velocidad, v, máxima del terreno asociadas a un temblor son:

$$a = 5600\, e^{0.8M}\, (R + 40)^{-2} \text{ en } (\text{cm/seg}^2) \quad (4.4a)$$

$$v = 32\, e^{M}\, (R + 25)^{-1.7} \text{ en } (\text{cm/seg}) \quad (4.4b)$$

donde M y R, la magnitud y la distancia del sitio de interés al foco del temblor se obtienen de la información del catálogo de eventos para la región de interés.

En la ref 12 se efectuó un análisis estadístico de los errores de predicción asociados a las expresiones 4.4a y 4.4b. De este estudio se determinó que los logaritmos naturales de los cocientes de las intensidades (a o v) predichas a las registradas instrumentalmente tienen distribución lognormal. Los parámetros de las mencionadas distribuciones fueron: para la aceleración máxima del terreno, media igual a 0.04 y desviación estándar igual a 0.64; para la velocidad máxima del terreno, media igual a 0.124 y desviación estándar igual a 0.74.

4.2.4 Modelo de sismicidad regional

La contribución de una fuente sísmica de volumen elemental Δv a la sismicidad regional ν_c en un sitio, distante R de la fuente, se puede obtener combinando las ecs (4.2) y (4.3), de lo cual resulta la siguiente expresión, (ref 11).

$$\Delta \nu_c(y) = \alpha\, b_1^{r}\, (R+R_0)^{-q}\, y^{-r}\, (1-(y/y_1)^{\epsilon})\, \Delta v, \quad y \leq y_1 \qquad (4.5)$$

$$= 0, \quad y > y_1$$

En (4.5), $r = \beta/b_2$, $\epsilon = \gamma/b_2$, $q = b_3\ r$, y_1 es la intensidad obtenida con la ec 3 para $M = M_u$; α, β y γ son parámetros de la ec (4.2).

Para obtener la contribución de varias fuentes sísmicas a ν_c en un sitio basta con integrar la ec 4.5 con respecto al diferencial de volumen. El resultado de dicha integración no variará en "y" como la ec 4.5; sin embargo, para fines prácticos dicho resultado se puede representar por una expresión similar a su segundo miembro, es decir (ref 11):

$$\nu_c(y) = k\ y^{-r}\ (1-(y/y_1)^{\epsilon}), \quad y \le y_1$$
$$= 0 \quad , \quad y > y_1 \qquad (4.6)$$

donde k, r, ϵ y y_1 son parámetros que se pueden estimar a partir de información estadística de los temblores y de las características sismotectónicas de las fuentes vecinas al sitio de interés o de zonas similares de la tierra. El parámetro y_1 es el valor máximo esperado, que resulta de aplicar la ec 4.3, a la combinación mas desfavorable de M_u y R, para las diferentes fuentes sísmicas vecinas al sitio.

Los valores de ν obtenidos en terminos de los datos estadísticos sobre intensidades calculadas (a partir de magnitudes y distancias) con las ecuaciones de atenuación de 4.2.3 deben corregirse para tomar en cuenta las incertidumbres derivadas de la utilización de dichas leyes. La corrección mencionada se trata en 4.2.6.

Generalmente los catálogos de temblores que contienen la información mencionada en el párrafo anterior están incompletos; es decir, no incluyen todos los eventos que pudieran generar intensidades máximas del terreno capaces de producir daños en las estructuras que se construyan en el sitio de interés. Esto proviene de que sólo contienen información completa para magnitudes por encima de un cierto límite inferior y por lo tanto emplear estos catálogos sin corrección equivale a ignorar la contribución al riesgo de los temblores muy pequeños, pero frecuentes, que pueden ocurrir a distancias muy cortas. Debido a lo anterior resulta necesario introducir una función correctiva f(y), por la cual deberá multiplicarse la sismicidad ν_c obtenida a partir del catálogo; al valor corregido de ν se le designará por ν_{co}; f(y) depende de los parámetros de las leyes de atenuación y de las intensidades máximas calculadas con esas leyes. En forma aproximada f(y) puede representarse así (ref 11)

$$f(y) = 1/(1-b(1-y/y_1^*)^s);\ y \leq y_1^* \qquad (4.7)$$

donde b, s, y y_1^* son definidos como sigue $s > 0$, $0 \leq b \leq 1$ y y_1^* es el valor máximo de y_1 que puede esperarse de las fuentes sísmicas vecinas al sitio de interés. Los valores específicos que adoptan s, b y y_1^* dependen de los parámetros de $\lambda(M)$, de las leyes de atenuación utilizadas y de la distribución espacial de la sismicidad en la zona vecina al sitio.

De la ec 4.7 se puede ver que f(y) es igual a 1/(1-b), para y = 0 e igual a 1 para $y = y_1^*$. Los valores límites manifiestan el hecho de que en general los catálogos de temblores están completos para magnitudes grandes y siendo lo contrario para magnitudes pequeñas.

De lo anterior se sigue que la $\nu(y)$ corregida por incompletez del catálogo se expresa como

$$\nu_{co}(y) = f(y)\ \nu_c(y) \qquad (4.8)$$

donde f(y) y $\nu_c(y)$ estan dados por las ecs (4.7) y (4.6) respectivamente.

4.2.5 Estimación bayesiana de la sismicidad regional $\nu(y)$

En este trabajo la estimación de $\nu(y)$ consistirá en obtener la esperanza de esta varible a partir de la distribución conjunta de probabilidades de los parámetros k, r, ϵ y y_1, que son los que definen a $\nu_c(y)$ de acuerdo a la ec 4.6. Para determinar esta distribución se efectuará el análisis bayesiano de dichos parámetros, que escencialmente consiste en aplicar el teorema de Bayes (Apéndice A) a toda la información geotectónica y sismológica significativa de que se disponga en la región donde se localiza el sitio de interés. Si se representa por Z el vector formado por los parámetros k, r, ϵ, y_1; por Y_N el vector constituido por los N valores de las intensidades y_i observadas durante un lapso t, por f' y f'' a las distribuciones bayesianas inicial y posterior de Z respectivamente; por L la función de verosimilitud del vector Y_N para un Z dado. Se tiene la siguiente expresión (Apéndice A):

$$f''_Z\ (z|Y_N;\ t)\ \alpha\ f'_Z\ (z)\ L\ (Y_N;\ t|z) \qquad (4.9)$$

La distribución $f'_Z(z)$ depende de las distribuciones marginales iniciales de los parámetros de $\lambda(M)$, es decir de α, β, γ y M_u (ver ecs 4.2 y 4.5). En relación a α se puede partir de estimaciones de los valores esperados de $\lambda(M)$ para magnitudes pequeñas en zonas sísmicas que involucren volúmenes considerables de la corteza terrestre. También se puede utilizar la información geofísica sobre la liberación de energía sísmica en regiones de dimensiones intermedias (ref 11). En cuanto a β se puede utilizar valores que provienen de regiones sísmicas semejantes a la de interés, además de que su rango de valores posibles tiene una cota superior (ref 11).

Como no se tiene información sobre los posibles valores de γ, para este parámetro se requiere utilizar distribuciones difusas. Finalmente la distribución inicial de M_u se puede formu lar con base en la información geotectónica de la región vecina al sitio, o si dicha información es escasa o inexistente para las máximas magnitudes observadas en la mencionada región.

Cuando se carece de información significativa en la región sísmica de interés, la distribución $f'_Z(z)$ se puede determinar asignando a los parámetros k, r, ε, y_1 distribuciones marginales uniformes en intervalos que se consideren apropiados; además se supone que dichos parámetros son estadísticamente independientes. De lo anterior se puede obtener que

$$f'(z) = f(k)\ f(r)\ f(\varepsilon)\ f(y_1) \qquad (4.10)$$

donde f(i), i = k, r, ε, y_1 representan las distribuciones marginales de los parámetros k, r, ε, y_1.

Para obtener la función L(.) se supone lo siguiente: que las intensidades observadas (es decir las intensidades calculadas para valores observados de magnitud y distancia) Y_N son variables aleatorias independientes con identica distribución, que dichas intensidades tienen un límite inferior y_o, así como la hipótesis mencionada anteriormente de que la ocurrencia de los temblores constituye un proceso de Poisson(sección 4.2.2). De acuerdo con las hipótesis anteriores se pueden escribir que la función L(.) satisface la siguiente relación de proporcionalidad

$$L(Y_N;\ t|Z) = \nu_{co}(y_o) + \sum_{i=1}^{N} \text{Ln}\nu'_{co}(y_i) \qquad (4.11)$$

En esta relación, $\dot{\nu}_{co}$ (y_o) es la tasa de excedencia correspondiente a y_o calculada con la ec 4.8, t es el lapso de observación de Y_N y $\nu'_{co}(y_i)$ es la derivada de $\nu_{co}(y)$ con respecto a y valuada para $y = y_i$. En el apéndice B se muestra la obtención de la ec (4.11) así como la forma explícita de sus términos.

Sustituyendo las ecs 4.10 y 4.11 en la ec (4.9) es posible obtener la distribución de probabilidades a posteriori de los parámetros k, r, ε, y_1, de lo cual se sigue que la esperanza a posteriori, E"(.), y la variancia a posteriori, $\sigma^{2"}(.)$ de ν_{co} para $y = y_i$ se pueden estimar con las siguientes expresiones:

$$E"(\nu_{co}(y_i)) = \int_\xi \nu_{co}(y_i)\ f_Z^"\ (z|Y_N\ ;\ t)\ dz \tag{4.12}$$

$$\sigma^{2"}(\nu_{co}(y_i)) = \int_\xi (\nu_{co}(y_i) - E"(\nu_{co}(y_i))^2 f_Z^"(z|Y_N\ ;\ t)dz \tag{4.13}$$

en estas ecs dz = dk dr dε dy_1 y ξ es el dominio de Z. La integración de las ecs (4.12, 4.13) se efectuará discretizando ξ por medio de la concentración de las probabilidades bayesianas de Z en los puntos del dominio definidos por valores preseleccionados de los parámetros k, r, ε, y_1; es decir:

$$E"(\nu_{co}(y_i)) = \sum_{m=1}^{n_1} \sum_{n=1}^{n_2} \sum_{s=1}^{n_3} \sum_{\mu=1}^{n_4} \nu_{co}(y_i) f_Z^"\ (z_{m\ n\ s\ \mu}|Y_N;t) \tag{4.14}$$

$$\sigma^{2"}(\nu_{co}(y_i)) = \sum_{m=1}^{n_1} \sum_{n=1}^{n_2} \sum_{s=1}^{n_3} \sum_{\mu=1}^{n_4} (\nu_{co}(y_i) - E"(\nu_{co}(y_i))^2 f_Z^" (z_{m\ n\ s\ \mu}|Y_N,t) \tag{4.15}$$

En las ecs (4.14, 4.15) n_1, n_2, n_3 y n_4 representan el número de valores seleccionados de k, r, ε y y_1 respectivamente, f"(.) es la distribución de probabilidades a posteriori de Z valuada en k_m, r_n, ε_s, y_{1_μ}. En las ecs 4.14 y 4.15 esta implícito que $\nu_{co}(y_1)$ también debe ser valuada en los mismos puntos que $f''(.)$. El coeficiente de variación a posteriori de $\nu_{co}(y_i)$, $V''(\nu_{co}(y_i))$ se puede obtener directamente con la ec 4.16

$$V''(\nu_{co}(y_i)) = \sigma''(\nu_{co}(y_i))/E''(\nu_{co}(y_i)) \qquad (4.16)$$

donde $\sigma''(.)$ es la desviación estandar a posteriori.

4.2.6 Corrección por incertidumbre en las leyes de atenuación

En 4.2.3 se mencionó que los cocientes de las intensidades reales entre las predichas con las leyes de atenuación que se utilizan en este trabajo (ecs 4.4a y 4.4b) tienen una distribución de probabilidades lognormal con medias y desviaciones estándar indicadas en esa sección. Para tomar en cuenta el efecto de las incertidumbres implícitas en las leyes de atenuación en la sismicidad calculada a partir de dichas leyes, en la ref 13 se propuso la siguiente expresión

$$\nu(y) = \int_{\alpha_u}^{\infty} \nu_{co}(y/u)\ f_e(u)\ du \qquad (4.17)$$

donde $\nu(y)$ es la sismicidad en el sitio asociada a la intensidad y corregida por incertidumbre en las leyes de atenuación, $\nu_{co}(y/u)$ es la sismicidad en el sitio sin incluir dicha corrección, $\alpha_u = y/y_1$, y f_e es la función de densidad de probabilidades del cociente de las intensidades reales entre las predichas con las leyes de atenuación.

En el caso que nos ocupa $\nu_{co}(y/u)$ esta dada por la ec 4.8, $f_e(u)$ es la función de densidad de probabilidades de la distribución lognormal y y_1 se definió anteriormente. Combinando las ecs 4.8 y 4.17 se obtiene (ref 13):

$$\nu(y) = C_1 K\, y^{-r} \left(1 - \frac{C_2}{C_1} \left(\frac{y}{y_1}\right)^{\varepsilon}\right) \tag{4.18}$$

$$C_i = e^{Q_i} \left(1 - \Phi\left(\frac{\mathrm{Ln}\,(y/y_1) - m - \sigma^2 r_i}{\sigma}\right)\right)$$

$$i = 1,\ 2$$

$$Q_i = \tfrac{1}{2}\, \sigma^2\, r_i^2 + m\, r_i$$

$$r_1 = r$$

$$r_2 = r - \varepsilon$$

K, r, ε y y_1 son los parámetros de $E''\ (\nu_{co}(y_i))$, m y σ son respectivamente la media y la desviación estándar del error de predicción de las leyes de atenuación (ecs 4.4a y 4.4b) y Φ es la función normal de distribución de probabilidades con media nula y variancia unitaria.

5. RIESGO SISMICO EN UN SITIO EN EL ESTADO DE JALISCO

En este cap se presenta una aplicación del modelo descrito en el cap 4 para la determinación de las intensidades maximas del terreno para diferentes periodos de recurrencia, de un sitio que tiene como coordenadas 21.3°N, 104.4°W, localizado en el estado de Jalisco.

5.1 Información para el sitio

Con base en la información tectónica y sobre la sismicidad de la región (la cual fue discutida en el cap 3) se decidió adoptar como fuente sísmica un prisma de 14 x 14 grados de latitud y longitud con centro en el sitio y 300 kilómetros de profundidad. La información de tipo estadístico sobre los temblores ocurridos en dicha fuente provino de las refs (8, 14).

De las refs (8, 14) se generó un catálogo de temblores con magnitudes mayores o iguales a 4.5 para el lapso 1900 a 1981. Como las magnitudes reportadas en dichas refs (10, 18) incluyen magnitudes asociadas a la trasmisión de ondas de cuerpo (m_b) y de superficie (m_s) para los eventos ocurridos en diversas épocas, fue necesario uniformizar el catálogo a una sóla magnitud.

Para lograr lo anterior se utilizaron eventos registrados de 1900 a 1981 y de los cuales se tenían ambas magnitudes; se efectuó un ajuste de mínimos cuadrados a dichos datos, lo cual condujo a las siguientes ecuaciones:

$$m_s = 1.12\, m_b - 0.705 \qquad m_b < 5.6 \tag{5.1}$$

$$m_s = 1.312 m_b - 1.565 \qquad (5.2)$$

$$m_s \geq 5.6$$

Estas expresiones fueron utilizadas para uniformizar los temblores del catálogo inicial a la magnitud m_s únicamente.

Por otro lado la profundidad de un número importante de eventos del catálogo no fue reportado y dado que se requiere dicha información para poder convertir el catálogo de magnitudes a uno de intensidades (a través de las ecuaciones de atenuación) se utilizaron las siguientes ecuaciones ref (15)

$$(\bar{H} + 40)^{-2} = \frac{1}{n} \sum_{i=1}^{n} (H_i + 40)^{2} \qquad (5.3)$$

$$(\bar{H} + 25)^{-2} = \frac{1}{n} \sum_{i=1}^{n} (H_i + 25)^{-2} \qquad (5.4)$$

para calcular una profundidad promedio $\bar{H}$ de los temblores ocurridos en la fuente sísmica seleccionada.

En las ecs 5.3 y 5.4 H_i es la profundidad reportada para el i-ésimo temblor y n es el número de eventos cuya profundidad sí fue reportada. Las ecs 5.3 y 5.4 son utilizadas para calcular la profundidad de un temblor al cual se le determinará su aceleración o velocidad del terreno en el sitio de interés respectivamente.

De la aplicación de las ecs 5.1 - 5.4 se obtuvieron dos catálogos con magnitudes m_s y con sus profundidades correspondientes para determinar la aceleración y la velocidad del terreno en diferentes sitios de interés.

5.2 Riesgo sísmico en el sitio de interés

El modelo de riesgo sísmico descrito en 4.2 fue aplicado utilizando la información mencionada en 5.1.

A partir de las magnitudes, las coordenadas epicentrales y las profundidades focales de los temblores de los catálogos mencionados al final de 5.1 y haciendo uso de las leyes de atenuación para aceleraciones y velocidades máximas ecs 4.4a, 4.4b, se calcularon las intensidades del terreno correspondientes a los 81 años de observaciones.

A continuación se procedió a calcular la tasa media de excedencia de las intensidades observadas, $\nu_c(y)$ (donde y representa las aceleraciones o las velocidades máximas del terreno). Para ello se ordenaron en forma ascendente las y's y para calcular las $\nu_c(y)$ correspondientes se divide el número de intensidades mayores o iguales a la intensidad de interés entre el lapso de observación, que en este caso es de 81 años.

En la fig 8 se presentan los valores obtenidos de ν_c (o su recíproco el periodo de recurrencia T_r) para la aceleración máxima del terreno para el sitio de interés. En la fig 9 se muestran los correspondientes valores para las velocidades máximas del terreno.

De la localización de los epicentros de los temblores del catálogo con relación a los sitios de interés se decidió proponer que f(y) en la ec 4.7 tendría un valor igual a la unidad. Con esto se supone que el catálogo esta completo en todo el rango de magnitudes utilizadas.

De acuerdo a lo mencionado en el cap 4, para el cálculo de la tasa media de excedencia E" (ν_{co}) se requieren los valores iniciales (a priori) de los parámetros k, r, ε y y_1. Los valores utilizados son los mostrados en la tabla 1. De la inspección de los datos de la muestra para el sitio se decidió que k, r y ε estarían comprendidos en los rangos indicados en dicha tabla. Los límites de y_1 se obtuvieron a través de las ecs 4.4 y se consideran con base en lo discutido en el cap 2 una magnitud máxima esperada de 8.2 (refs 3), y dos valores de R, 24.5 y 40 km, los cuales conducen a un rango de aceleraciones y de velocidades máximas del terreno de 617 a 950 cm/s^2 y de 96 a 150 cm/s respectivamente. De estudios sobre el efecto de y_1 en la estimación de $E''(\nu_{co})$ para los datos del sitio se observó que no era necesario que y_1 fluctuará en los rangos indicados, sino que bastaba con tomar sus límites superiores, por lo cual se decidió utilizar los y_1 indicados en la tabla 1.

En este trabajo se supone que a priori los parámetros k, r, ε y y_1 son estadísticamente independientes por lo cual la probabilidad a priori de cada una de las combinaciones de los diferentes parámetros es igual a 1/1024, donde el denominador se obtiene de multiplicar el número de valores utilizados de cada parámetro es decir, 8x8x8x2, correspondientes a k, r, ε y y_1 respectivamente.

Finalmente los valores de m y σ en la ec son los mencionados en 4.2.3. es decir, m = 0.04 y σ = 0.64 para la aceleración máxima del terreno y m = 0.124 y σ = 0.74 para la velocidad máxima del terreno.

Los resultados obtenidos se presentan en las figs 8 y 9, la primera corresponde a las aceleraciones y la segunda a las velocidades máximas del terreno en el sitio de interés, respectivamente. En esas figs se tienen las curvas de la esperanza de posteriori y el coeficiente de variación de ν_{co},

$E''(\nu_{co})$ y $V''(\nu_{co})$ correspondientes. Las curvas de $E''(\nu_{co})$ tienen la forma de la ec 4.6 con los parámetros k, r, ε y y_1 de la tabla 2. También se muestran en dichas figs las curvas $E''(\nu)$ que representan las esperanzas a posteriori de ν obtenidas después de aplicar las correcciones por incertidumbre en las leyes de atenuación de acuerdo a lo mencionado en 4.2.6. Estas curvas proporcionan los valores finales de las aceleraciones y velocidades máximas del terreno a_r y v_r para el sitio de interés para diferentes ν o T_r. En la tabla 3 se presentan los valores de a_r y v_r para varios periodos de recurrencia obtenidos de las mencionadas curvas.

Comparando los valores de las aceleraciones y velocidades máximas del terreno para periodos de recurrencia de 50, 100 y 500 años obtenidos en este trabajo con los proporcionados en la ref 16 se puede concluir lo siguiente:

a) Las aceleraciones de la ref 16 son 2.42, 2.7 y 3.66 veces menores que las obtenidas en este estudio para los T_r mencionados;

b) Las velocidades de la ref 16 son 1.87, 1.9 y 2.64 veces menores que las obtenidas en el presente trabajo.

Lo anterior quiza se deba a que la información utilizada en este trabajo es mas completa, así como a que las ecuaciones de atenuación utilizadas en la ref 16 eran menos precisas que las usadas en este trabajo.

6. CONCLUSIONES

Con base en lo descrito en los capitulos anteriores, se concluye lo siguiente:

1. Existe evidencia del alto potencial sísmico del occidente de México en particular en los estados de Colima, Jalisco y Michoacán.

2. Debido a lo anterior y a las posibles consecuencias de los efectos de los temblores en las construcciones, seria muy conveniente que a la brevedad posible se realicen estudios de riesgo sísmico detallados para las ciudades más importantes de la región.

3. Sería muy conveniente que a la brevedad posible se propongan reglamentos de diseño sísmico para las ciudades más importantes de la región.

4. Sería muy recomendable que a la brevedad posible se instalen acelerógrafos para registrar los movimientos sísmicos que ocurran en las ciudades importantes y otros sitios del occidente de México.

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PARAMETRO	k_{ini}	k_{fin}	Δk	r_{ini}	r_{fin}	Δr	ε_{ini}	ε_{fin}	$\Delta\varepsilon$	y_{1ini}	y_{1fin}	Δy_1
a	20	27	1	1.50	2.06	0,08	0.10	8.15	1.15	949	951	2
v	0.59	0.73	0.02	1.77	1.84	0.01	0.10	8.15	1.15	149	151	2

TABLA 1 VALORES A PRIORI DE LOS PARAMETROS k, r, ε, y_1 PARA ACELERACIONES (a) Y VELOCIDADES (v) PARA UN SITIO EN EL ESTADO DE JALISCO

PARAMETRO	k''	r''	ε''	y_1''
a	22.2053	1.7922	4.6756	950.00
v	0.6449	1.8033	4.6891	149.99

TABLA 2 VALORES ESPERADOS A POSTERIOR DE LOS PARAMETROS k, r, ε, y_1 PARA ACELERACIONES (a) Y VELOCIDADES (v) PARA UN SITIO EN EL ESTADO DE JALISCO

PARAMETRO	PERIODO DE RECURRENCIA T_r (años)						
	50	100	200	300	400	500	1000
a (cm/s^2)	92	135	202	250	292	330	495
v (cm/s)	10.3	15.2	22.2	28	33	37	54

TABLA 3 ACELERACIONES (a) Y VELOCIDADES (v) MAXIMAS DEL TERRENO PARA VARIOS PERIODOS DE RECURRENCIA (T_r) PARA UN SITIO EN EL ESTADO DE JALISCO

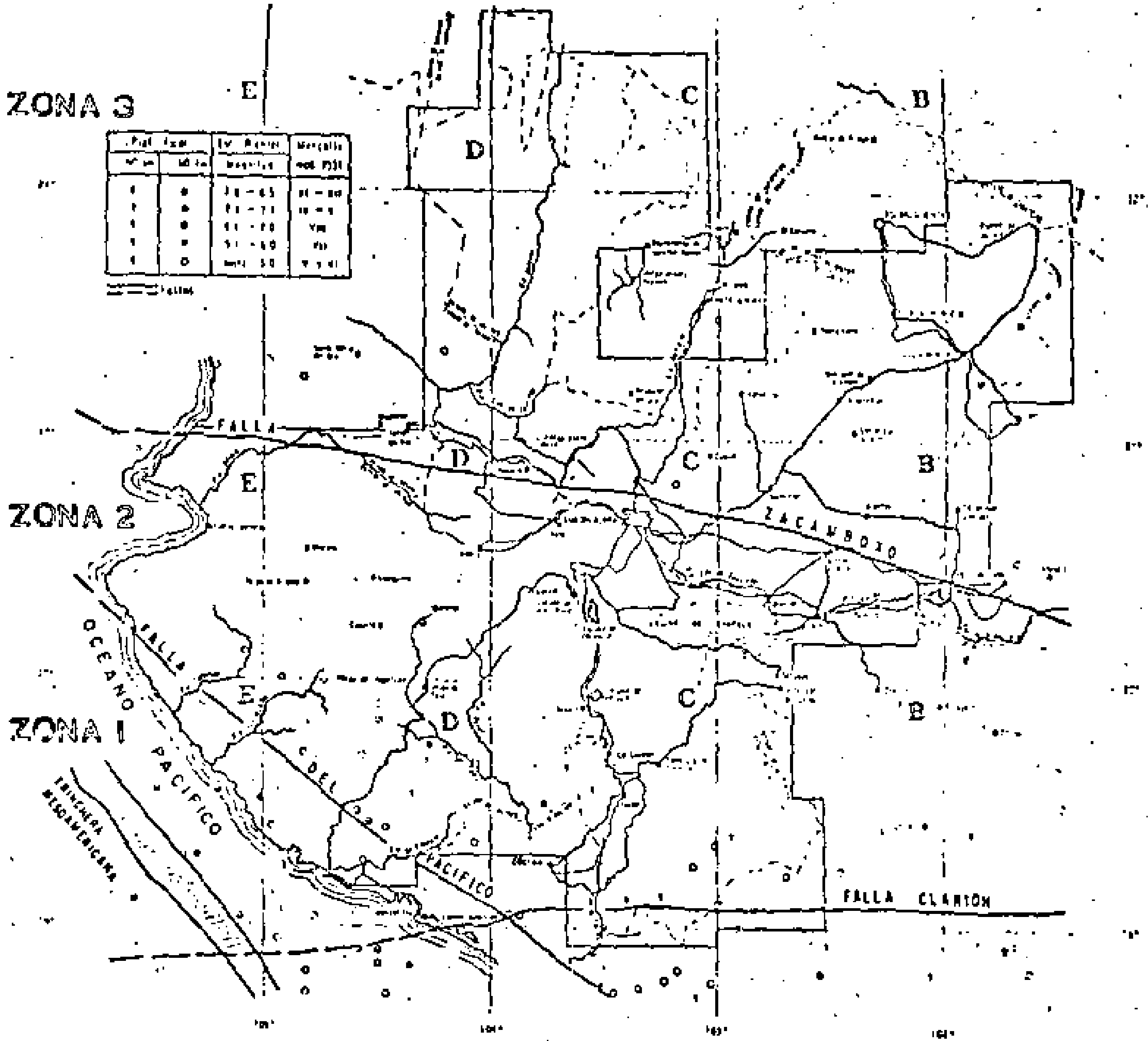

FIG 1 FALLAS DE ZACAMBOXO, CLARION Y DEL PACIFICO (REF 7)

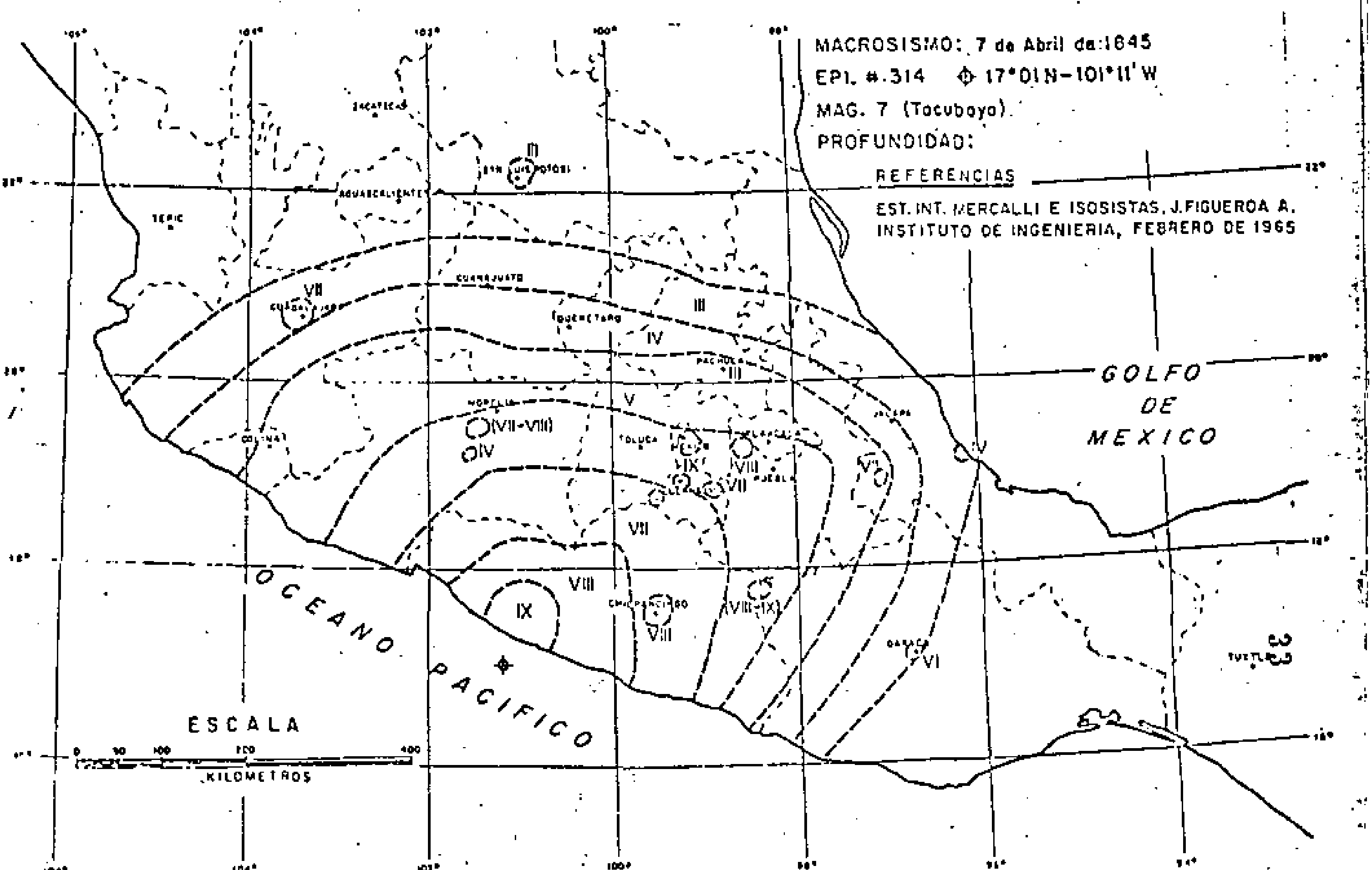

FIG 2 ISOSISTAS DEL TEMBLOR DEL 7 DE ABRIL DE 1845 (REF 7)

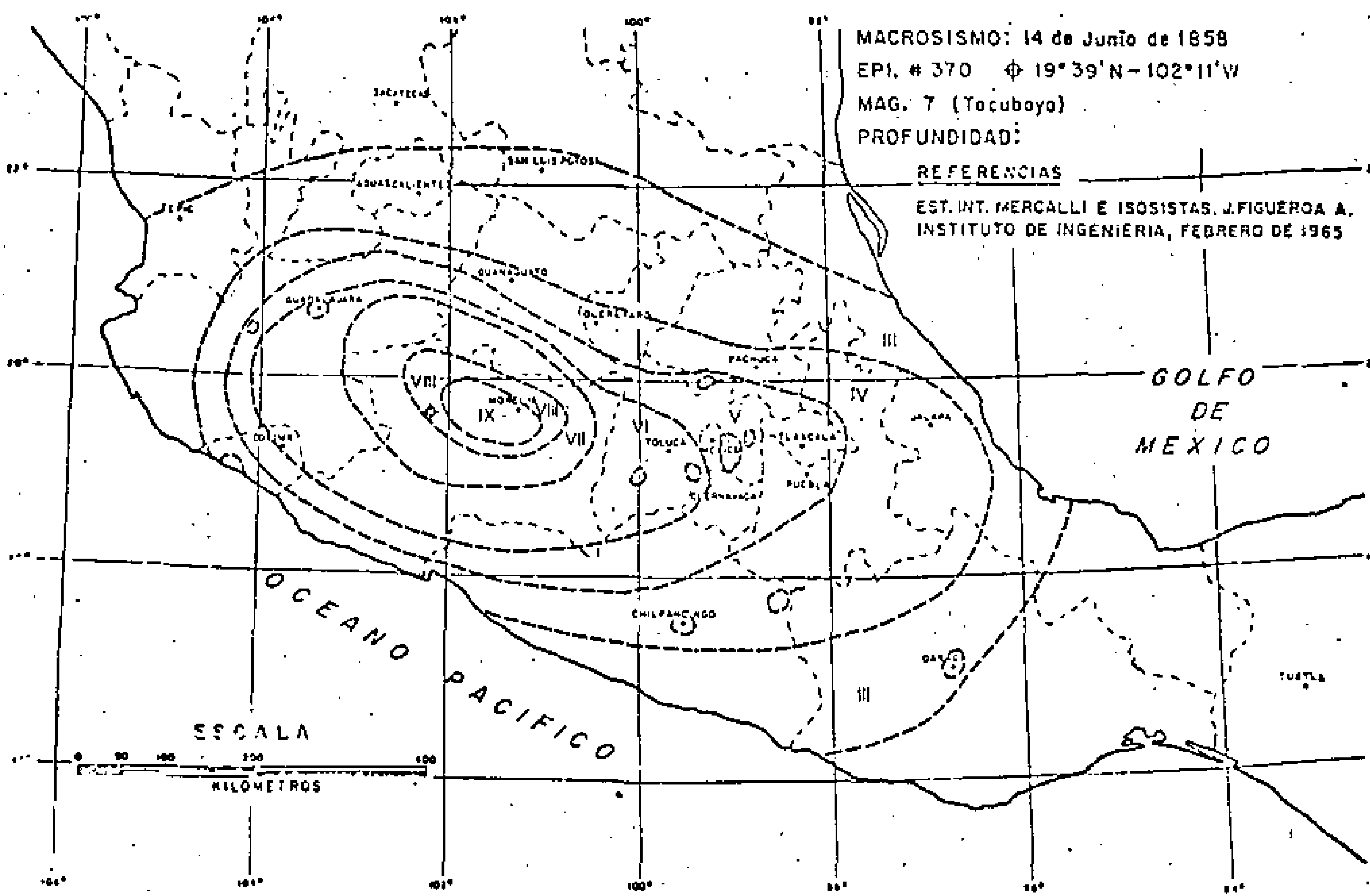

FIG 3 ISOSISTAS DEL TEMBLOR DEL 14 DE JUNIO DE 1858 (REF 7)

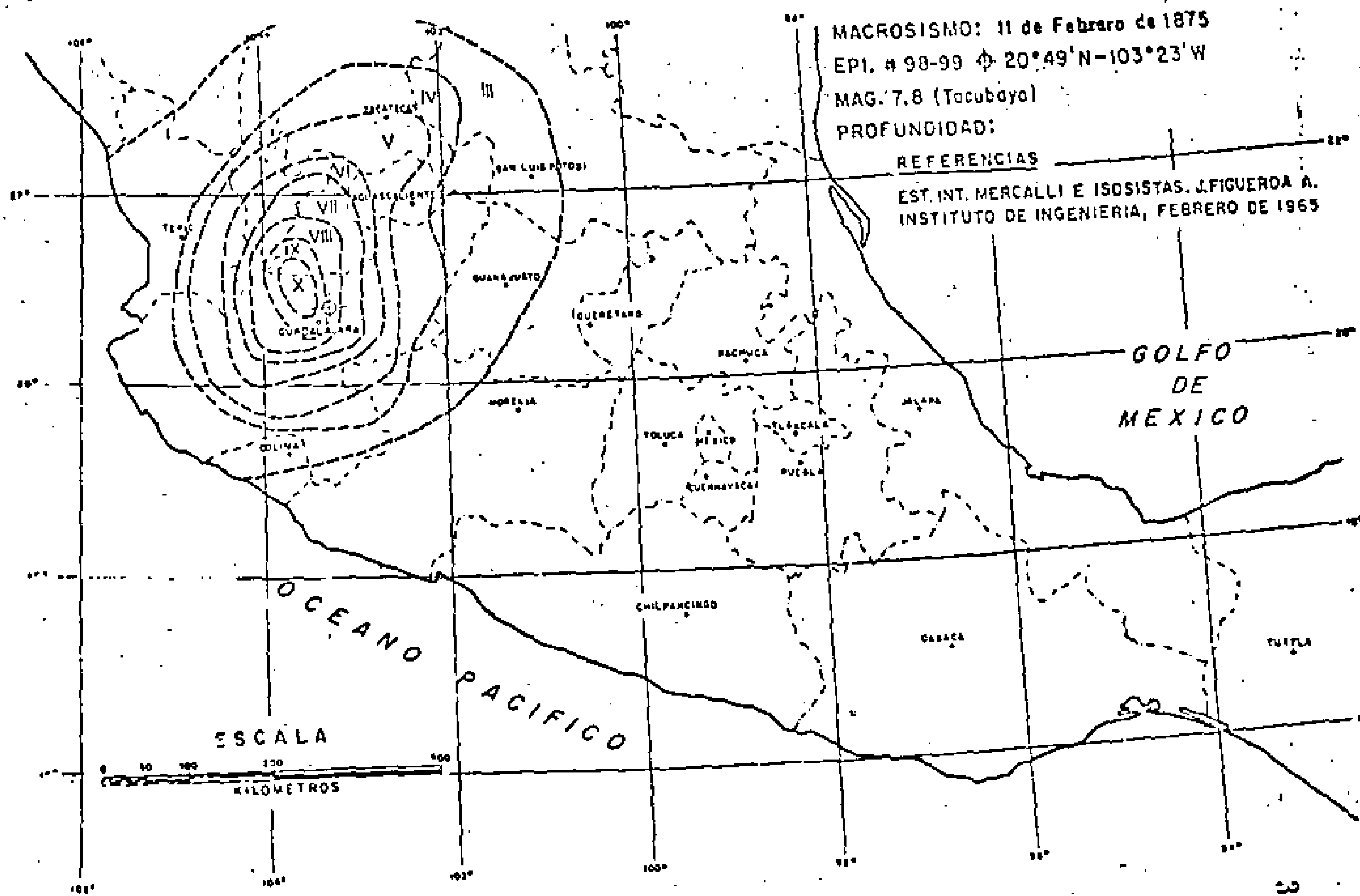

FIG 4 ISOSISTAS DEL TEMBLOR DEL 11 DE FEBRERO DE 1875 (REF 7)

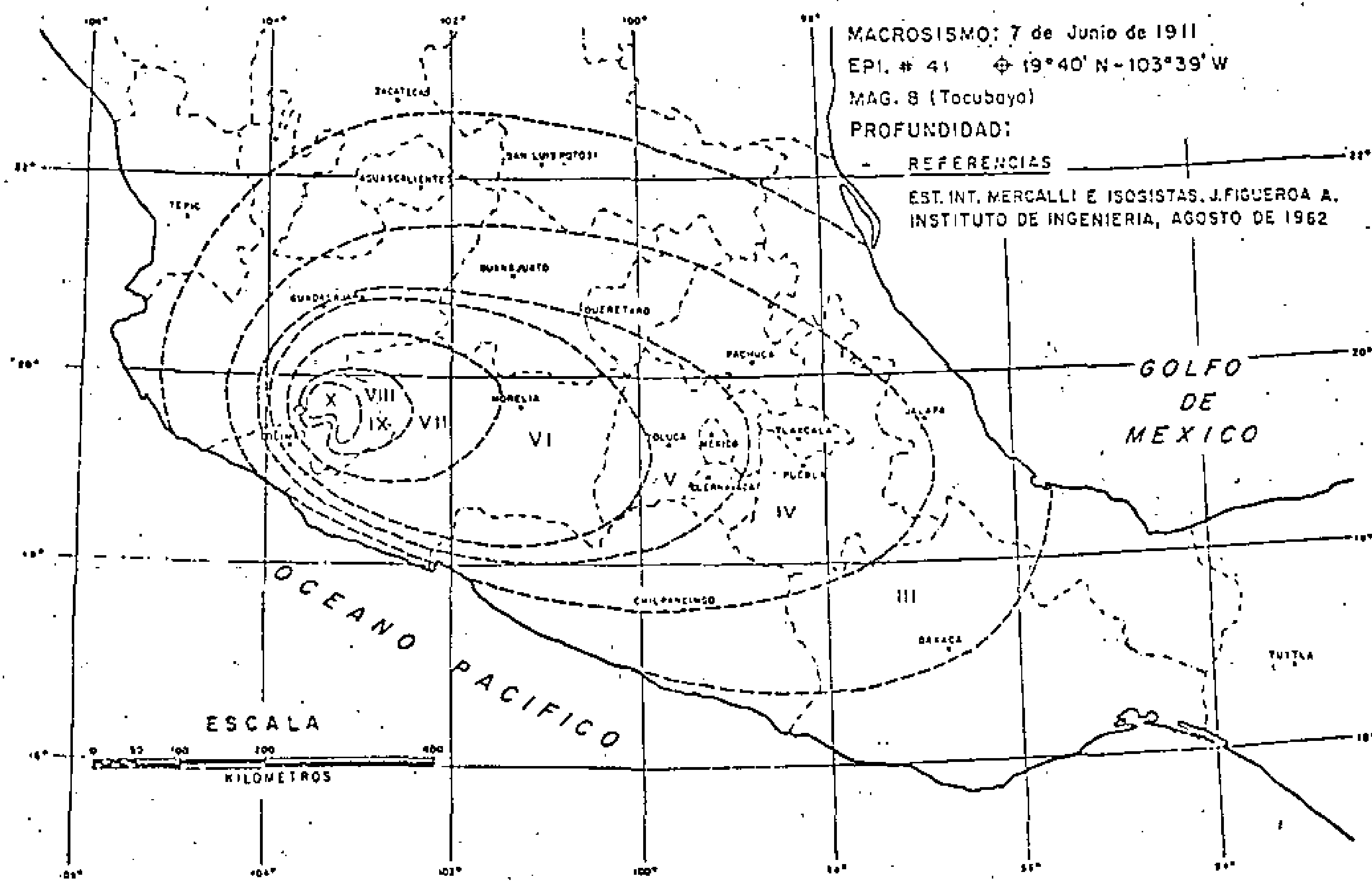

FIG 5 ISOSISTAS DEL TEMBLOR DEL 7 DE JUNIO DE 1911 (REF 7)

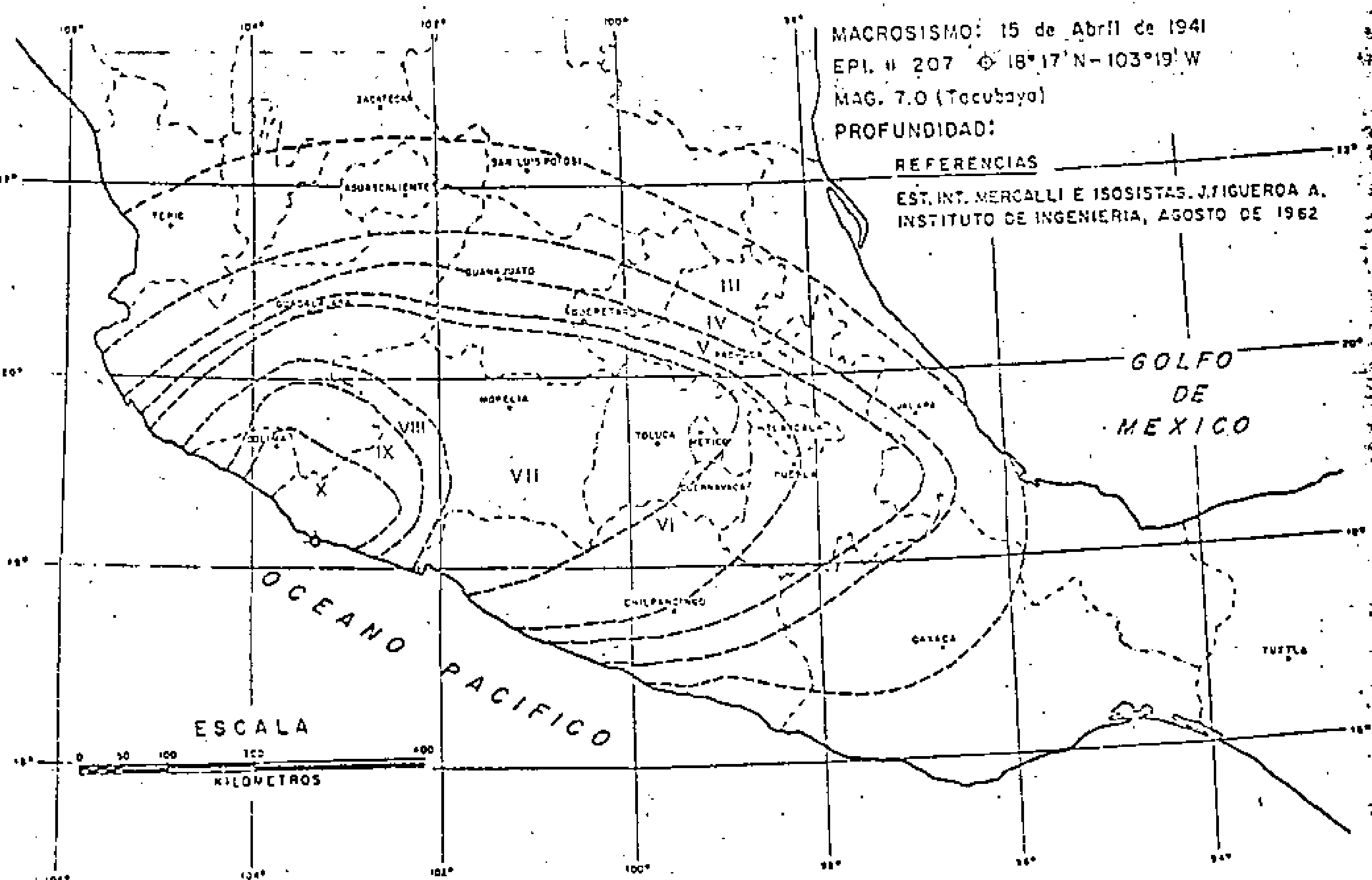

FIG 6 ISOSISTAS DEL TEMBLOR DEL 15 DE ABRIL DE 1941 (REF 7)

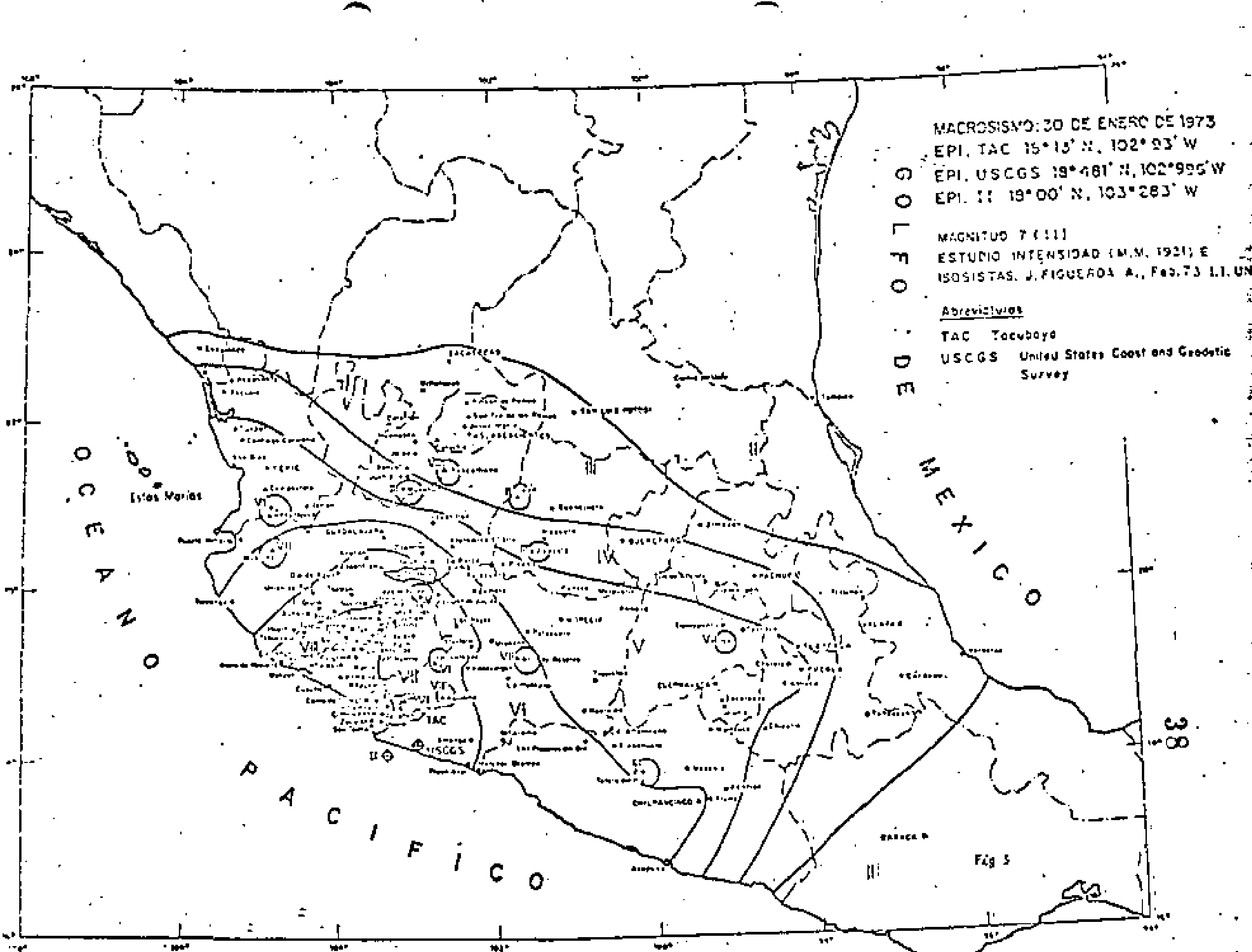

FIG 7. ISOSISTAS DEL ... OR DEL 30 DE ENERO DE 1973 (REF 7)

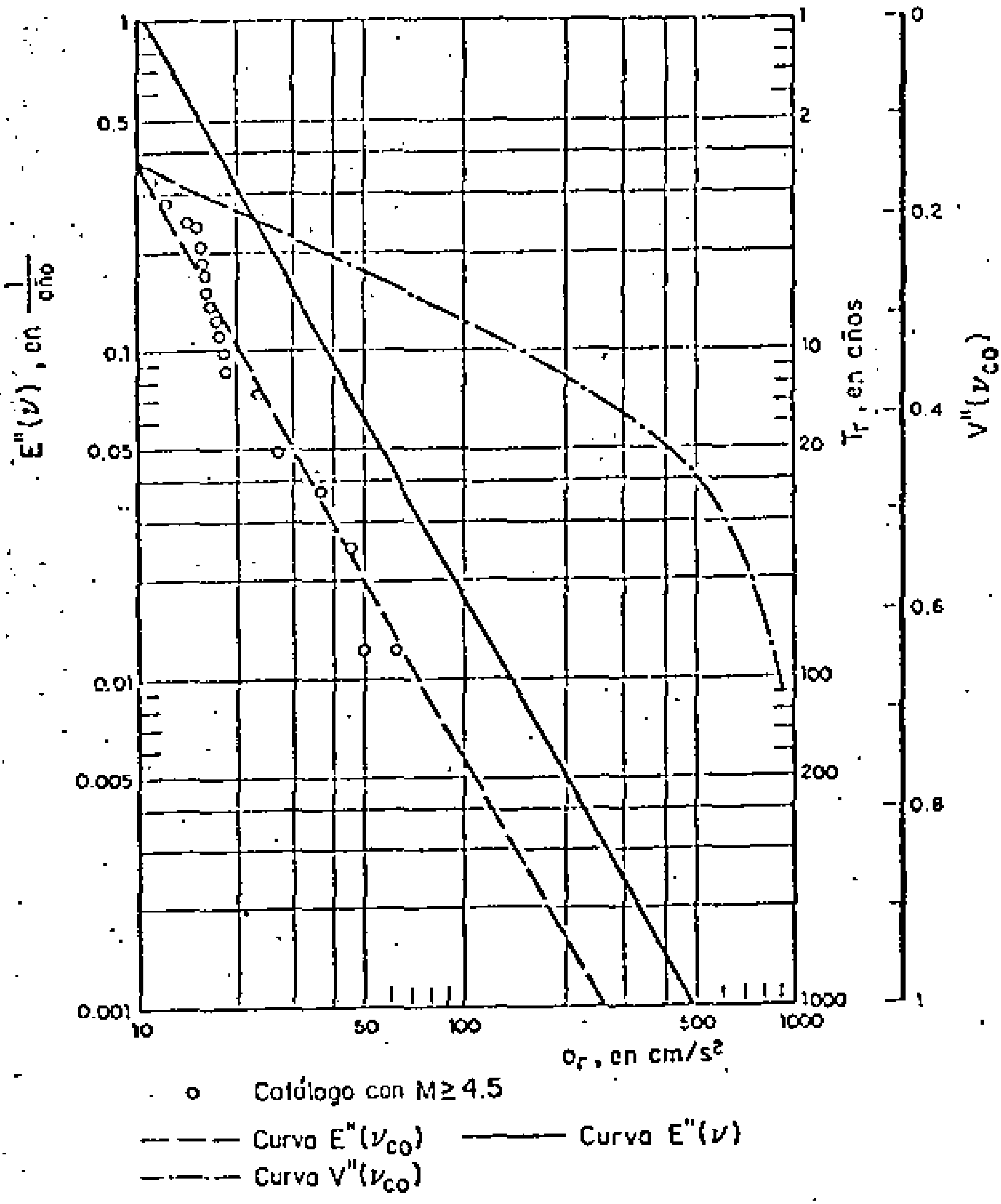

FIG 8 VALORES ESPERADOS A POSTERIOR DE LA TASA MEDIA DE EXCEDENCIA E"(ν) (Y PERIODO DE RECURRENCIA, T_r) VS ACELERACION MAXIMA DEL TERRENO a_r PARA UN SITIO EN EL ESTADO DE JALISCO

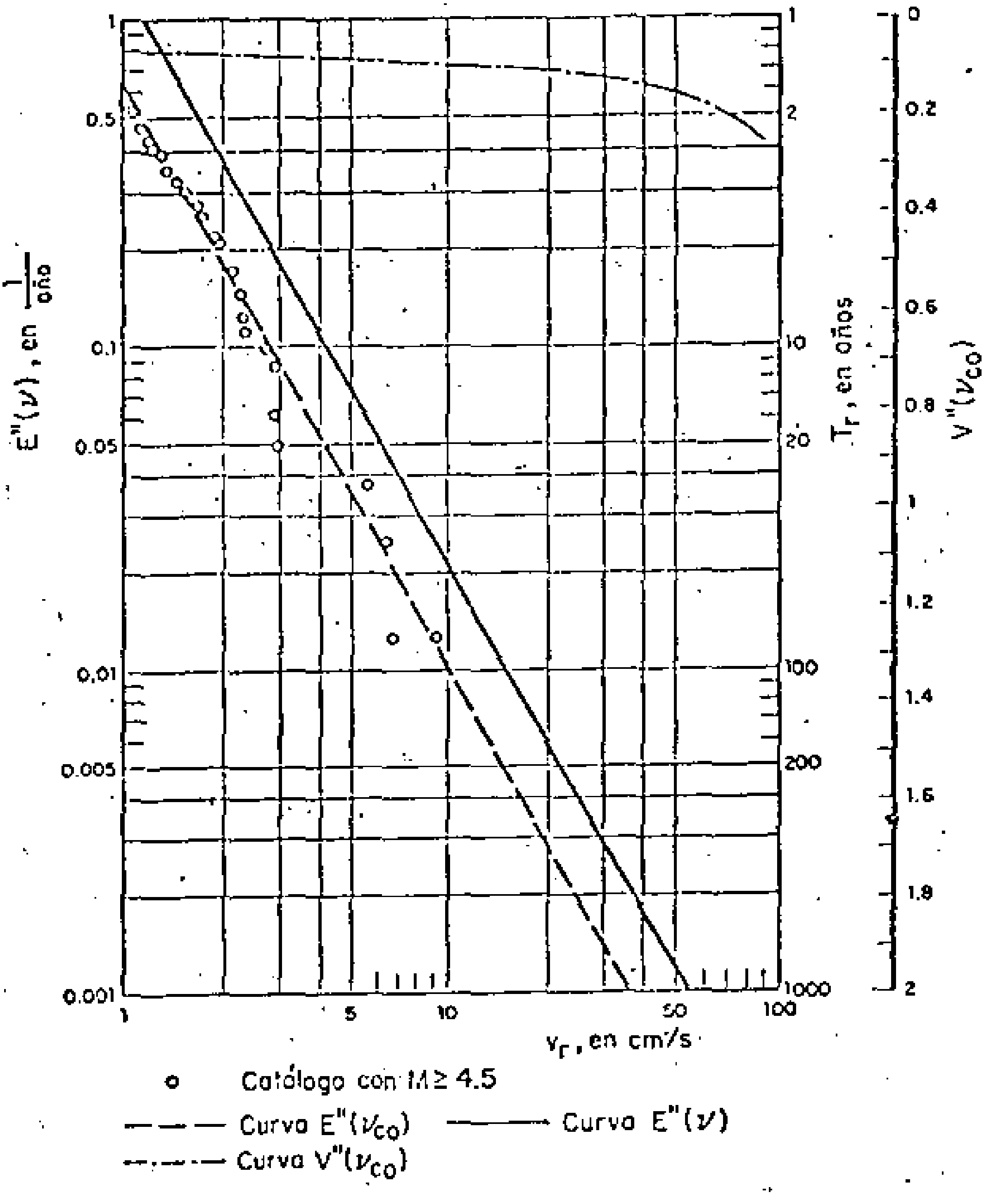

FIG 9 VALORES ESPERADOS A POSTERIOR DE LA TASA MEDIA DE EXCEDENCIA E"(ν) (Y PERIODO DE RECURRENCIA, T_r) VS VELOCIDAD MAXIMA DEL TERRENO v_r PARA UN SITIO EN EL ESTADO DE JALISCO

APENDICE A Teorema de Bayes

Si H_i ($i = 1,n$) representa un conjunto de hipótesis mutuamente excluyentes y colectivamente exhaustivas sobre un fenómeno parcialmente conocido y A es una observación del fenómeno, el teorema de Bayes se puede expresar como (ref A1)

$$P(H_i/A) = P(H_i)\,P\,(A/H_i)/\sum_{j=1}^{n} P(H_j)\,P\,(A/H_j) \qquad \text{(A1)}$$

donde $P(a/b)$ significa la probabilidad de a dado b; $P(H_i/A)$ es la probabilidad a posteriori de que H_i es verdadero dado que se observa A; $P(H_i)$ es la probabilidad inicial de H_i asignada antes de observar A y $P(A/H_i)$ es la probabilidad de A si H_i es verdadera. El término en el denominador del segundo miembro de la ec (A1) proporciona la constante normalizadora para que el primer miembro sea una probabilidad.

Para el caso que se tiene en el cap 3, se desea aplicar el teorema de Bayes para otener la densidad de probabilidades a posteriori de Z (donde Z es el vector formado por los parámetros de la sismicidad regional $\nu_{cc}(y)$) dado que se han observado N temblores con intensidades Y_N durante el lapso t,

A1. Raiffa, H. and Schlaifer R., "Applied Statistical Decision Theory" MIT Press (1968)

A2. Benjamin, J R and Cornell, C A, "Probability, Statistics and Decisions for Civil Engineers", McGraw.Hill (1970)

es decir el primer miembro de la ec A1 se asocia a $f''_Z(Z/Y_N;t)$ de la ec 9.

El término $P(H_i)$ de la ec (A1) corresponde a la densidad de probabilidades inicial (supuesta) de Z_i representado por $f'_Z(Z)$ en la ec (9).

El término $P(A/H_i)$ de la ec (A1) es la probabilidad de que ocurran las intensidades Y_N, en caso de que los parámetros que definen la sismicidad regional, $(\nu_{eo}(y))$, sean precisamente los que se asignaron inicialmente a Z. Se puede demostrar (ref A2) que esta probabilidad es proporcional a la función $L(Y_N;t/Z)$ de la ec (9).

El término en el denominador del segundo miembro de la ec (A1) se omitió en la expresión 9, por lo cual en lugar del símbolo de igualdad de la ec A1 se tiene el de proporcionalidad.

APENDICE B Deducción de la ec 4.11

La función de verosimilitud que se requiere es la probabilidad de que en una región específica durante un lapso t ocurran N temblores con intensidades y_i mayores o iguales a una intensidad y_o, tales que n_1 temblores corresponden al intervalo Δy_1 de intensidades, n_2 al intervalo Δy_2, ...n_k al intervalo Δy_k. Lo anterior puede escribirse como $P(Y_N;t|N)$.

$$P(Y_N;t|N) = P(n_1,n_2,\ldots n_k|N)\ P(N,t) \qquad (B1)$$

De acuerdo a la hipótesis de que la ocurrencia de temblores en la región de interés constituye un proceso de Poisson, se sigue que la intensidad y_i asociada a cada temblor es independiente de la historia previa, por lo cual la distribución conjunta de $n_1,n_2,\ldots n_k$ dado n es multinomial (ref B1)

$$P(n_1,n_2,\ldots n_k|N)=N!\prod_{i=1}^{k} (P_i^{n_i}) / n_i! \qquad (B2)$$

en la ec (B2) P_i es la probabilidad de que cada vez que ocurre un temblor en la región de interés su intensidad y_i se encuentre en el intervalo Δy_i. Si los Δy_i son muy pequeños, los n_i adoptarán valores cero o uno, por lo cual $n_i! = 1$. Sustituyendo este último valor en la ec (B2) conduce a

B1 Igual que ref A2

$$P(n_1,n_2,\ldots n_N|N) = N! \prod_{i=1}^{N} P_i \tag{B3}$$

en la cual

$$P_i = f_Y(y_i)\, dy_i \tag{B4}$$

el término $f_y(y_i)$ de la ec (B4) es la ordenada de la función de densidad de probabilidad de las intensidades para un temblor cualquiera valuada en y_i y dy_i es el intervalo infinintesimal en que se encuentra y_i. La función de densidad mencionada puede obtenerse a partir de la tasa de excedencia de las intensidades observadas $\nu_{co}(y_i)$, de acuerdo a la expresión siguiente.

$$f_Y(y_i) = \frac{1}{\nu_{co}(y_o)} \left.\frac{\partial \nu_{co}(y)}{\partial y}\right|_{y=y_i} \tag{B5}$$

$$= \nu'_{co}(y_i) / \nu_{co}(y_o)$$

sustituyendo la ec (B5) en la ec (B4) y esta última en la ec (B3)

$$P(n_1,n_2,\ldots n_N|N) = N! \prod_{i=1}^{N} (\nu'_{co}(y_i)/(\nu_{co}(y_o))) \tag{B6}$$

donde y_o es la intensidad mínima del catálogo de intensidades observadas, $\nu_{co}(y_o)$ es la ν_{co} (ec 8) correspondiente a y_o, el término $\nu'_{co}(y_i)$ es la derivada de ν_{co} con respecto a y valuada para la intensidad y_i.

Utilizando de nueva cuenta la hipótesis de Poisson sobre la ocurrencia de temblores en la región de interés, la probabilidad, $P(N,t)$, de que durante un lapso t ocurran N temblores con tasa media de excedencia $\nu_{co}(y_o)$ se puede valuar con la expresión siguiente (ref B1)

$$P(N,t) = e^{-\nu_{co}(y_o)t}(\nu_{co}(y_o)t)^N/N! \tag{B7}$$

Sustituyendo las ecs B6 y B7 en la ec B1 se obtiene la ec B8

$$P(Y_N;t \cdot N) = t^N e^{-\nu_{co}(y_\circ)t} \prod_{i=1}^{N} \nu'_{co}(y_i) \qquad (B8)$$

Se puede demostrar (ref 6) que

$$L(Y_N;t|\,Z) \propto P(Y_N;t|N) \qquad (B9)$$

y si se obtiene el Ln de ambos miembros de (B9)

$$L(Y_N;t|Z) \propto -\nu_{co}(y_\circ)\ t + \sum_{i=1}^{N} \mathrm{Ln}\ \nu'_{co}(y_i)$$

DIVISION DE EDUCACION CONTINUA
FACULTAD DE INGENIERIA U.N.A.M.

X CURSO INTERNACIONAL DE INGENIERIA SISMICA

ANALISIS DE RIESGO SISMICO

MODELOS DE RIESGO SISMICO

PARTE 2. ESTIMACION NO BAYESIANA Y EJEMPLO DE APLICACION

ESTUDIOS COSTO - BENEFICIO

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AGOSTO, 1984.

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X CURSO INTERNACIONAL DE INGENIERIA SISMICA

MODELOS DE RIESGO SISMICO

PARTE II.- ESTIMACION NO BAYESIANA Y EJEMPLO DE APLICACION - ESTUDIOS COSTO - BENEFICIO

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1. INTRODUCCION

El objeto de este trabajo es la determinación de *espectros de diseño sísmico* para diversos tipos de terreno en la Planta Geotérmica de Cerro Prieto, B.C. Este sitio se localiza en la vecindad de la ciudad de Mexicali, B.C., y sus coordenadas geográficas son 32°24' de latitud norte y 115°14' de longitud oeste, fig 1.

Los espectros de diseño sísmico son gráficas que relacionan las envolventes de las respuestas máximas esperadas de sistemas estructurales sencillos (idealizados como de un grado de libertad) sujetos a diferentes temblores con los periodos naturales de vibración de dichos sistemas. A partir de los mencionados espectros es posible estimar las solicitaciones sísmicas para las cuales se debe diseñar una estructura localizada en una zona sísmica, tal como el sitio donde se localiza la Planta Geotérmica de Cerro Prieto (PGCP).

Los criterios de diseño que se propongan deben tener por objetivo optimizar las estructuras, es decir, conducir a sistemas estructurales en los cuales la utilidad obtenida de su ejecución sea máxima. La determinación de la estructura óptima se puede lograr a través de estudios costo-beneficio que incluyan explícitamente los siguientes parámetros: los beneficios esperados de su construcción, los costos esperados de las posibles fallas, los costos iniciales de diseñar y construir, así como las probabilidades de ocurrencia (e intensidades máximas) de las solicitaciones.

Aunque se cuenta con las herramientas necesarias para efectuar los estudios costo-beneficio citados arriba (ref 1) su aplicación práctica se ve a veces limitada por las dificultades asociadas con la estimación de las consecuencias de falla o utilidades de la operación de las estructuras proyectadas. Por ello la determinación de la estructura óptima ante solicitación sísmica se llevará a cabo siguiendo el criterio propuesto en la ref 1, el cual se basa en la comparación de las intensidades esperadas de diseño (aceleraciones, velocidades, etc)., en dos sitios diferentes, para un mismo periodo de recurrencia. Este parámetro se define como el recíproco de la tasa media de excedencia de una intensidad dada durante un lapso previamente definido.

Como no se cuenta con un número suficiente de registros del movimiento del terreno en el sitio de interés, ni de información suficiente sobre las características sismotectónicas de las fallas cercanas al sitio, la determinación de los espectros de diseño se efectuará a partir de la información estadística (catálogo) sobre las magnitudes y las coordenadas focales de temblores generados cerca del sitio de interés durante este siglo.

Con la información anterior y utilizando leyes de atenuación (relaciones magnitud-distancia-intensidad) previamente propuestas en la ref 2 se obtienen expresiones que relacionan aceleraciones y velocidades máximas del terreno con tasas de excedencia (periodos de recurrencia).

Las expresiones mencionadas se corrigen para tomar en cuenta las incertidumbres asociadas con las intensidades reales en comparación con las calculadas a través de las leyes de atenuación, así como las incertidumbres relacionadas con el uso de un catálogo que no incluye todos los eventos que pudieran producir intensidades importantes en el sitio de interés. Lo descrito en los párrafos anteriores se tratará con detalle en el cap 3.

Los espectros de diseño para diversos periodos de recurrencia se obtienen a partir de las intensidades máximas del terreno. El espectro de diseño propuesto se selecciona siguiendo las recomendaciones de la ref 1, tal como se indica en el cap 5.

2. GEOLOGIA, TECTONICA Y SISMICIDAD DE LA REGION

2.1 *Geología*

La PGCP se localiza en el Valle de Mexicali (fig 1) el cual forma parte de la provincia geológica llamada del Golfo de California o depresión de Salton (ref 3). Estructuralmente está formado por graberns (depresiones) llenados por aluviones y horsts (levantamientos) de rocas paleozoicas formadas antes de la actividad tectónica (ref 3).

La litología de la región de interés corresponde a la del delta del Río Colorado (refs 3 y 4) y está formda por arenisca, arcillas y lutitas depositadas cíclicamente. Dichos sedimentos se encuentran cubiertos por aluvión de la Sierra Cucapa y áreas adyacentes (ref 3). La distribución horizontal de los sedimentos deltaicos es desconocida debido a la irregularidad horizontal en que se han depositado los sedimentos mencionados.

2.2 *Tectónica*

La estructura tectónica de la región ha sido descrita por un arreglo simple de fallas transformadas y centros de dispersión que se caracterizan por generar enjambres de temblores, actividad volcánica reciente, áreas geotérmicas y depresiones topográficas submarinas (ref 5, 6).

Entre las fallas principales que se localizan en el Valle de Mexicali se pueden mencionar a las siguientes: Imperial, Cerro Prieto, Elsinore (también llamada Laguna Salada). Otras fallas en la zona son las de Agua Blanca, San Miguel, de Sierra Juárez (refs 5, 6). Recientemente se ha localizado un sistema de fallas secundario denominado Volcano en la zona donde se encuentra la PGCP (ref 4). Estas últimas fallas se asocian al movimiento lateral derecho de las fallas Imperial y Cerro Prieto (ref 7).

Una característica de la mayoría de las fallas mencionadas en los párrafos anteriores es que solo se conocen parcialmente sus dimensiones (ref 8, 9).

2.3 *Sismicidad*

En la ref 5 se reportó que la actividad sísmica de la región de interés se concentró en las fallas Imperial, Cerro Prieto, Sierra Juárez y San Miguel.

En particular se observaron temblores cuyos epicentros se alinearon con las fallas Imperial y Cerro Prieto. Estudios posteriores realizados entre 1974 y 1975 mencionados en la ref 10 mostraron actividad sísmica asociada a la falla Cerro Prieto únicamente.

Otros estudios de la sismicidad en la región donde se localiza la PGCP (ref 10) mostraron que la mayoría de los eventos identificados se concentraron en el extremo noroeste de la falla Cerro Prieto y el sureste de la falla Imperial.

Desde el punto de vista ingenieri l recientemente ocurrieron temblores que causaron daños a estructuras; estos temblores fueron los de octubre 15 de 1979 y junio 9 de 1980. El primero tuvo una magnitud de 5.6 (magnitud de ondas de cuerpo) y aceleraciones máximas de hasta 500 cm/seg^2 (ref 11). Su epicentro se localizó cerca de la ciudad de Mexicali (ref 12).

El segundo temblor ocurrió el 8 de junio de 1980 con una magnitud local de 6.7 y aceleraciones máximas del terreno de hasta 663 (cm/seg^2). Su epicentro se localizó cerca de la falla de Cerro Prieto; las coordenadas fueron 32.213° N y 115.028°W (refs 13 y 14). En la ref 13 se describen los daños observados en diversas estructuras en la región afectada. También se describen en la ref (14) los efectos de los mencionados temblores en el terreno, entre los que destaca la formación de pequeños "volcanes" de arena muy fina y agua con elevaciones de hasta 40 cm sobre el nivel original del terreno, lo cual indica que las capas de suelo superficiales en la región de interés son susceptibles de licuarse. Este efecto de licuación puede ser peligroso para estructuras que se desplanten en terrenos susceptibles a tal fenómeno, dado que al fallar el suelo existe el peligro de colapso total o parcial de las estructuras. Por lo anterior se recomienda identificar las formaciones para las que haya sospechas sobre potencial de licuación y efectuar estudios detallados específicos para decidir sobre uso del suelo, es decir, sobre sitios en donde pueden desplantarse estructuras y en donde debe impedirse la construcción.

3. MODELO DE RIESGO SISMICO

3.1 *Sismicidad regional y local*

La sismicidad regional, ν, en un sitio determinado, o sea el efecto que producen los temblores que ocurren en la región vecina al sitio, depende principalmente de la contribución que a la misma hacen los volumenes de la corteza terrestre alrededor de dicho sitio. Dada la atenuación de la intensidad con la distancia, solamente los volumenes localizados a algunos cientos de kilómetros del sitio contribuyen en forma importante a esa sismicidad.

Los volumenes citados se designan como fuentes sísmicas y se caracterizan por las incertidumbres asociadas a la frecuencia de ocurrencia y las magnitudes de los temblores que generan. Se denomina sismicidad local al proceso de ocurrencia de temblores de diversas magnitudes en una fuente sísmica.

De los párrafos anteriores se puede concluir que la sismicidad regional para un sitio puede estimarse a partir de las sismicidades locales de las fuentes sísmicas correspondientes.

La sismicidad local asociada a una fuente sísmica determinada puede estimarse utilizando un modelo del proceso estocástico (aleatorio) de la ocurrencia de temblores de diversas magnitudes en dicha fuente. Si se adopta una forma para el proceso, la sismicidad local podría calcularse una vez que se estimen los parámetros que definen a dicho proceso.

Como se mencionó en el cap 2, la información sobre las características sismotectónicas de la zona vecina al sitio no es suficiente como para definir varias fuentes sísmicas, por lo cual se calculará la sismicidad en el sitio de interés a partir de las intensidades calculadas de los temblores con focos contenidos en un prisma de 12 x 12 grados de longitud y latitud, con centro en el sitio y profundidad de varias decenas de kilómetros. Para estos fines, la sismicidad se definirá en terminos de la tasa media de excedencia de cada intensidad por unidad de tiempo. En forma aproximada la sismicidad de un sitio puede expresarse mediante una expresión de la forma

$$\nu_j(y) = \alpha_j(y^{-r_j} - y_{j\,\text{máx}}^{-r_j}), \quad y \leq y_{j\,\text{máx}} \tag{1}$$

$$\nu_j = 0 \qquad\qquad y > y_{j\,\text{máx}}$$

$$j = a,\ v$$

En la ec 1 $\nu_j(y)$ es el número medio anual de temblores con intensidades mayores que y registrados en el sitio de interés y generados por las fuentes sísmicas vecinas al sitio, α_j, r_j y $y_{j\,\text{máx}}$ son los parametros del proceso, los cuales deben estimarse a partir de los datos estadísticos, y de las características sismotectónicas de las fuentes vecinas; y son las intensidades máximas del terreno (aceleración, a, y velocidad, v) que pueden calcularse a partir de las leyes de atenuación que se presentan en 3.2

Los valores de ν obtenidos en términos de los datos estadísticos sobre intensidades calculadas (a partir de magnitudes y distancias) con las leyes de atenuación de 3.2 deben corregirse para tomar en cuenta las incertidumbres derivadas de la utilización de dichas leyes, así como las incertidumbres relacionadas con el catálogo de temblores empleado. Este no incluye todos los eventos que pudieran generar intensidades del terreno importantes (desde el punto de vista de diseño) en el sitio. Las correcciones mencionadas se tratan en 3.4 y 3.3 respectivamente.

3.2 *Leyes de atenuación*

Las leyes de atenuación son expresiones semiempíricas que relacionan magnitud-distancia-intensidad (aceleraciones, velocidades y desplazamientos máximos) de temblores en terreno firme. Dichas expresiones pueden obtenerse a partir de los datos que existen sobre los parámetros mencionados. En general, los procedimientos utilizados para obtener las leyes de atenuación consisten en ajustar curvas a los datos de eventos ocurridos en diferentes regiones, por lo cual las expresiones así obtenidas reflejan las características geotectónicas de la región para la cual fueran obtenidas (ref 15).

En este trabajo se usarán las leyes de atenuación propuestas en la ref 2, las cuales fueron obtenidas a partir de la información correspondiente a temblores ocurridos en terreno firme de la costa oeste de los Estados Unidos y de la República Mexicana. La elección de dichas leyes de atenuación se justifica dado que la región de interés (ver cap 2) se localiza cerca de la zona de los Estados Unidos mencionada arriba. Las expresiones propuestas en la ref 2 para estimar la aceleración a, y la velocidad, v, máximas del terreno asociadas a un temblor son:

$$a = 5600\, e^{0.8\, M} (R + 40)^{-2} \quad \text{en } (\text{cm/seg}^2) \qquad (2)$$

$$v = 32\, e^{M} (R + 25)^{-1.7} \quad \text{en } (\text{cm/seg}) \qquad (3)$$

donde M y R, la magnitud y la distancia del sitio de interés al foco del temblor se obtienen del catálogo de eventos para la región de interés.

En la ref 2 se efectuó un análisis estadístico de los errores de predicción asociados a las expresiones 2 y 3. De ese estudio se determinó que los logaritmos naturales de los cocientes de las intensidades (a o v) predichas a las registradas instrumentalmente tienen distribución normal.

Los parametros de las mencionadas distribuciones fueron: para la aceleración máxima del terreno, media igual a 0.01 y desviación estándar igual a 0.64; para la velocidad máxima del terreno, media igual a 0.124 y desviación estándar igual a 0.74.

3.3 . *Corrección por incompletez del catalogo*

Como se mencionó en 3.2 la sismicidad en un sitio puede estimarse a partir de la información estadística sobre las magnitudes, coordenadas y profundidades focales de los temblores ocurridos en las fuentes sismicas vecinas. Esta información se utiliza en combinación con las leyes de atenuación (ecs 2 y 3) para evaluar las intensidades máximas del terreno en el sitio de interés. Sustituyendo estas intensidades en la ec 1 es posible evaluar la sismicidad en el sitio, ν_e, asociada a la información estadística utilizada.

Generalmente los catálogos de temblores que contienen la información mencionada en el parrafo anterior están incompletos; es decir, no incluyen todos los eventos que pudieran generar intensidades máximas del terreno capaces de producir daños en las estructuras que se construyan en el sitio de interes. Esto proviene de que sólo contienen información completa para magnitudes por encima de un cierto límite inferior y por lo tanto emplear estos catálogos sin corrección equivale a ignorar la contribución al riesgo de los temblores muy pequeños pero frecuentes, que pueden ocurrir a distancias muy cortas. Debido a lo anterior resulta necesario introducir un factor de corrección, f_c, por el cual deberá multiplicarse la sismicidad ν_e obtenida a partir del catálogo; al valor corregido de ν se le designará por ν_i.

Como se verá más adelante, f_c depende de los parámetros de la sismicidad local, de los parámetros de las leyes de atenuación y de las intensidades máximas calculadas con esas leyes.

Para fines de obtener el factor correctivo f_c, la sismicidad local λ asociada a una fuente sísmica se estimará con la expresión siguiente (ref 15):

$$\lambda(M) = \alpha e^{-\beta M} \tag{4}$$

donde $\lambda(M)$ es la tasa de excedencia de la magnitud M por unidad de volumen y por unidad de tiempo, y α y β son parámetros que dependen de las características sismotectónicas de la fuente sísmica considerada.

Dado que interesa evaluar la sismicidad en terminos de las intensidades máximas en el sitio, se deberá expresar la ec.4 en términos de esas intensidades, para lo cual se utilizarán las ecs. 2 y 3. Estas últimas pueden representarse por

$$y = b_1 e^{b_2 M} (R + R_0)^{-b_3} \tag{5}$$

donde y es la intensidad calculada (aceleración o velocidad) asociada a M, R_0 y b_i ($i = 1,2,3$) son los parámetros de las ecs mencionadas y R fue previamente definido. Despejando M en la ec (5) y sustituyendo en la ec 4 se obtiene

$$\lambda(M(y,R)) = \alpha \gamma y^{-r} (R + R_0)^{-s} \tag{6}$$

donde $\gamma = b_1^r$, $r = \beta/b_2$, $s = \beta b_3/b_2$.

La sismicidad regional ν puede evaluarse con la expresión siguiente (ref 15)

$$\nu(y) = \int_V \lambda(M(y,R))dv \tag{7}$$

donde la integral con respecto al volumen incluye todas las fuentes sísmicas que contribuyen a la sismicidad en el sitio de interés.

Para calcular el factor f_c partiremos de lo siguiente: supongamos que para una región se tiene un catálogo (para un lapso dado) que se considera confiable para temblores con magnitudes mayores a una magnitud seleccionada, digamos M_0. La sismicidad total en el sitio de interés, $\nu_t(y)$, o sea la correspondiente a todos los eventos ocurridos en la región, puede obtenerse como la suma de las sismicidades producidas por los temblores con magnitudes menores o iguales que M_0 y los mayores que dicho valor; a estas dos componentes las llamaremos $\nu_1(y)$ y $\nu_2(y)$ respectivamente; es decir,

$$\nu_t(y) = \nu_1(y) + \nu_2(y) \tag{8}$$

El factor f_c es función de la intensidad y, y se obtiene como el cociente de la sismicidad total que resulta de integrar la ec 7 para un volumen ilimitado, con λ dado por la ec 6, entre la que se obtiene cuando al integrar la ec 7 para un volumen ilimitado se toma λ dado por la ec 6 para $M > M_0$ e igual a $\lambda(M_0)$ para $M \leq M_0$; es decir,

$$f_c(y) = \frac{\nu_{\ }(y)}{\nu_2(y)} \tag{9}$$

en donde $\nu(y)$ está dada por la ec 7 y $\nu_2(y)$ se obtiene como sigue:

$$\nu_2(y) = \int_{R \leq R^*} \lambda(M_0)dV + \int_{R > R^*} \alpha e^{-\beta M(y,R)}\, dV \tag{10}$$

En esta ecuación, R^* es la distancia a la que un temblor con magnitud M_0 produce una intensidad calculada igual a y, de acuerdo con la ec 5.

Para calcular $\nu_t(y)$ se supondrá que la sismicidad es uniforme en un semiespacio limitado por la superficie del terreno*, por lo cual al combinar las expresiones (6) y (7) se obtiene

$$\nu_t(y) = \frac{-4\ \alpha\gamma y^{-r}\ R_0^{\ (3-s)}}{(1-s)\ (2-s)\ (3-s)} \tag{11}$$

$$s > 3$$

* Aunque esta hipótesis no sería la adecuada para evaluar ν_t, se considera aceptable para la estimación de f_c.

La expresión para R^* se puede deducir de la ec 5,

$$R^* = (b_1 \; e^{b_2 M_o / y})^{1/b_3} - R_o \qquad (12)$$

donde todos los parámetros han sido previamente definidos.

Una vez que se tiene el valor de R^* se obtienen las integrales que aparecen en la ec 10:

$$\nu_2(y) = \nu_{21}\,(y) + \nu_{22}\,(y) \qquad (13)$$

en donde

$$\nu_{21}(\;) = \frac{2\,\pi\,\alpha\,R^{*3}\,e^{-\beta\,M_o}}{3} \qquad (14)$$

$$\nu_{22}(y) = -2\pi\,\alpha\,\gamma\,y^{-r}\,\left(\frac{R^{*2}\,R_1^{(1-s)}}{(1-s)} - \frac{2R^*\,R_1^{(2-s)}}{(1-s)\,(2-s)} + \frac{2\,R_1^{(3-s)}}{(1-s)(2-s)(3-s)}\right) \qquad (15)$$

$s > 3$

En la última ecuación,

$$R_1 = (R^* + R_o) \qquad (16)$$

Al sustituir las ecs 11, 13, 14 y 15 en la ec 9 se obtiene:

$$f_c(y) = -\frac{2\,\gamma\,y^{-r}\,R_o^{(3-s)}}{(1-s)\,(2-s)\,(3-s)}$$

$$\left(\frac{R^{*3}\,e^{-\beta M_o}}{3} - \gamma\,y^{-r}\,\left(\frac{R^{*2}\,R_1^{(1-s)}}{(1-s)} - \frac{2R^*\,R_1^{(2-s)}}{(1-s)\,(2-s)} + \frac{2R_1^{(3-s)}}{(1-s)(2-s)(3-s)}\right)\right)^{-1} \qquad (17)$$

$s > 3$

Finalmente, la sismicidad corregida por incompletez $\nu_i(y)$, asociada a la intensidad y, se obtiene como

$$\nu_i(y) = F_c(y)\ \nu_e(y) \tag{18}$$

3.4 Corrección por incertidumbre en las leyes de atenuación

En 3.2 se mencionó que los cocientes de las intensidades reales entre las predichas con las leyes de atenuación que se utilizan en este trabajo (ecs 2 y 3) tienen una distribución de probabilidades lognormal con medias y desviaciones estándar indicadas en esa sección. Para tomar en cuenta el efecto de las incertidumbres implícitas en las leyes de atenuación en la sismicidad calculada a partir de dichas leyes, en la ref 15 se propuso la siguiente expresión

$$\nu_c(y) = \int_{\alpha_u}^{\infty} \nu_p(y/u)\ f_\varepsilon(u)\ du \tag{19}$$

donde $\nu_c(y)$ es la sismicidad en el sitio asociada a la intensidad y corregida por incertidumbre en las leyes de atenuación, $\nu_p(y/u)$ es la sismicidad en el sitio sin incluir dicha corrección, $\alpha_u = y/y_{máx}$, y f_ε es la función de densidad de probabilidades del cociente de las intensidades reales entre las predichas con las leyes de atenuación.

En el caso que nos ocupa $\nu_p(y/u)$ esta dada por la ec 18, $f_\varepsilon(u)$ es la función de densidad de probabilidades de la distribución lognormal y $y_{máx}$ se definió en la sección 3.1. Combinando las ecs 18 y 19 se obtiene (ref 15):

$$\nu_c(y) = c_0\ K_0 + c_1\ K_1\ y^{-r} \tag{20}$$

donde

$$c_0 = 1 - \phi\left(\frac{\operatorname{Ln}(y/y_{máx}) - m}{\sigma}\right)$$

$$K_0 = -\ \alpha\ y_{máx}^{-r}$$

$$c_1 = \exp(Q) \{ \frac{\ln(y/y_{máx}) - m - \sigma^2 r}{\sigma} \}$$

$$K_1 = \alpha$$

$$Q = \frac{1}{2} \sigma^2 r^2 + m\, r$$

α, r, $y_{máx}$ son los parámetros de la ec 18

m, σ son la media y la desviación estandard de las leyes de atenuación (ecs 2 y 3)

Φ es la distribución de probabilidades normal estandarizada acumulada

4. RIESGO SISMICO EN LA PLANTA GEOTERMICA DE CERRO PRIETO

4.1 *Información para el sitio*

La información de tipo estadístico sobre los temblores ocurridos en la fuente sísmica adoptada, que es un prisma de 12 x 12 grados de latitud y longitud con centro en el sitio de interés y algunas decenas de kilómetros de profundidad, provino de las refs 16, 17, 18, 19. De las refs 16 y 17 se generó un catálogo de temblores con magnitudes mayores o iguales a 4.5 para el periodo 1932 a 1980. De las refs 18 y 19 se obtuvieron dos catálogos, ambos para el lapso 1932 a 1974 y para magnitudes mayores o iguales a 5 y 6 respectivamente. En la fig 1 se muestran los epicentros y profundidades de los eventos con magnitudes mayores o iguales a 4.5.

Las intensidades $y_{j\ \text{máx}}$ (j = aceleración, velocidad) es decir, las cotas superiores a las intensidades máximas esperadas del terreno en la región de interés, se estimaron a partir de las ecs 2 y 3 adoptando una magnitud de ondas de cuerpo máxima esperada M = 7.4 y una distancia R = 15 km. El valor de M fue seleccionado con base en las refs 20 y 21 y el valor de R se tomó como un valor característico de la región en cuestión (fig 1). Con los valores de M y R mencionados se obtuvo:

$$a_{máx} = 690 \text{ (cm/seg}^2\text{)}$$

$$v_{máx} = 100 \text{ (cm/seg)}$$

El valor del parámetro β en la ec. 4 fue calculado a partir de la siguiente expresión (ref 22)

$$\beta = 2.3 \ b \qquad (21)$$

donde b es un parámetro que depende de las características sismotectónicas de la región de interés (ref. 23). En este trabajo se tomó un valor de b = 0.87 con base en las refs. 18 y 24, con lo cual se obtuvo un β = 2.

Finalmente los valores de m y σ utilizados en la ec. 20 son los mencionados en 3.2, es decir, m = 0.04 y σ = 0.64 para la aceleración máxima del terreno y m = 0.124 y σ = 0.74 para la velocidad máxima del terreno.

4.2 *Riesgo sísmico en el sitio de interés*

El modelo de riesgo sísmico descrito en el cap. 3 fue aplicado utilizando la información sobre el sitio mencionado en 4.1. Los resultados obtenidos fueron los siguientes:

. En las figs 2 y 3 se presentan los valores calculados de la tasa media de excedencia ν (o su recíproco el periodo de recurrencia, T_r) para las aceleraciones y velocidades máximas del terreno respectivamente. En ambas figs. se tienen los resultados para cada uno de los catálogos, así como las curvas de ajuste correspondientes. Dichas curvas tienen la forma de la ec. 1 con parametros α = 3600, r = 2.23 para aceleraciones máximas (fig 2) y α = 150, r = 2.82 para velocidades máximas (fig 3). Los valores de α y r se calcularon aplicando un criterio de mínimos cuadrados dandole mayor peso a las ν asociadas a las intensidades medias, En las figs 4 y 5 se presentan las curvas ν versus aceleración y velocidad máxima del terreno obtenidas después de aplicar las correcciones por incompletez del catálogo y por incertidumbre en las leyes de atenuación respectivamente. Las curvas resultantes de la corrección por incompletez (llamadas la corrección en las figuras) tienen como parámetros $\alpha = 75\,000$, $r = 2.74$ para aceleraciones (fig 4) y $\alpha = 200$, $r = 2.9$ para velocidades (fig 5). Las curvas obtenidas de la corrección por incertidumbre en las leyes de atenuación (denominadas curvas finales en ambas figuras), proporcionan los valores esperados de las aceleraciones y velocidades máximas del terreno a_r y v_r para el sitio de interés para diferentes ν y T_r. Por ejemplo, se tiene

T_r (años)	a_r (cm/seg^2)	v_r (cm/seg)
30	270	29
50	331	38
100	410	53

5. ESPECTROS DE DISEÑO

En el capítulo 1 se mencionó que los espectros de diseño para un sitio, es decir, las envolventes de las respuestas máximas esperadas de sistemas estructurales sencillos sujetos a diferentes temblores pueden obtenerse a partir de las intensidades máximas del terreno en el sitio. Dado que estas últimas dependen, entre otros factores, de las propiedades mecánicas de los materiales que forman los estratos superficiales (ref 25), en este trabajo se consideran tres tipos de terreno atendiendo a su rigidez. La clasificación propuesta es la siguiente: terreno firme, tal como tepetate, arenisca medianamente cementada, arcilla compacta; terreno intermedio, tal como arenas no cementadas, limos de mediana o alta compacidad, arcillas de mediana compacidad; terrenos compresibles, como arcillas blandas muy compresibles.

Para definir que tipo de terreno se tiene en un sitio especifico se puede aplicar el siguiente criterio (ref 26):

a) Se localizará el nivel del terreno firme, bajo el cual todos los suelos tengan módulos de rigidez mayores que 5×10^4 ton/m^2, o requieren mas de 50 golpes por cada 30 cm, en la prueba de penetración estándar.

b) Para estratos comprendidos entre el nivel del terreno firme y el nivel en que las aceleraciones horizontales del terreno se trasmitan a la construcción se calculará la suma $\Sigma H_i\sqrt{\gamma_i/G_i}$ donde

H_i = espesor del i-ésimo estrato, en m

γ_i = su peso volumétrico en ton/m^3

G_i = módulo de rigidez en ton/m^2.

Si la suma $\Sigma H_i\sqrt{\gamma_i/G_i}$ es menor que 0.20, el terreno se considerará firme.

Si la suma $\Sigma H_i\sqrt{\gamma_i/G_i}$ es mayor que 0.20, y menor que 0.45 el terreno se considerará intermedio.

A falta de información precisa, para la aplicación del criterio anterior puede tomarse para γ_i el valor de 1.5 ton/m^3 y los valores de G_i pueden estimarse como G_i = 0.35 E_i, en que E_i es la pendiente inicial de la curva esfuerzo-deformación de una prueba de compresión simple.

Los terrenos cuyas propiedades se desconozcan se supondrán como terrenos compresibles.

Para la clasificación del tipo de terreno anterior se tomarán en cuenta todos los suelos que se encuentren debajo del nivel en que las aceleraciones horizontales se trasmiten a la construcción; por ejemplo, en el caso de un cajón de cimentación este nivel correspondería al desplante de la losa inferior.

5.1 *Espectros de diseño para terreno firme*

El criterio que se seguirá en este trabajo para calcular espectros de diseño para terreno firme se apoya en las ref 1 y 27. Dicho criterio consiste en multiplicar la aceleración y la velocidad máxima del terreno en un sitio para un periodo de recurrencia dado, a_r y v_r, por los factores de amplificación f_a y f_v (que dependen del amortiguamiento) respectivamente (ref 1). Los valores resultantes, que se denominarán a* y v*, son respectivamente las envolvente de los espectros de diseño de aceleraciones y velocidades para sistemas de un grado de libertad sujetos a diferentes temblores. Estos últimos son congruentes con la sismicidad del sitio de interés.

Dado que las leyes de atenuación utilizadas en este trabajo (cap 3) corresponden a terreno firme, los espectros de diseño obtenidos serán aplicables para estructuras que se apoyen en suelos de esas características.

Las ordenadas A_f, del espectro de diseño de aceleraciones para un periodo de recurrencia T_r se obtienen a partir de a_r, a^* y v^* que ya se definieron, y T, que son los periodos naturales de los sistemas de un grado de libertad. De acuerdo con estos parámetros, las ordenadas $A_f(T)$ se calculan así:

. Para $T = 0$, $A_f = a_r$

. Para $0 < T < T^*$ (en donde $T^* = 2\pi\, v^*/a^*$), se tienen dos casos:

a) Si $a^*/Q > a_r$, A_f varía linealmente con T desde $T = 0$ hasta $T = 0.12$ en que adquiere el valor máximo a^*/Q, que se mantiene constante para $0.12 < T \leq T^*$.

b) Si $a^*/Q < a_r$, A_f varía linealmente con T desde $T = 0$ hasta $T = T^*$, en que adquiere el valor a^*/Q.

. Para $T > T^*$, $A_f = (a^*/Q)(T^*/T)^{1/2}$.

En las expresiones anteriores, Q es el factor de ductilidad, definido para una sección, un miembro o una porción de una estructura con comportamiento elastoplástico, como el cociente de su deformación máxima entre su deformación de fluencia. Q caracteriza la capacidad de absorción de energía por comportamiento inelástico de una estructura. En diseño sísmico los valores de Q usados comúnmente son los siguientes: 1 para sistemas frágiles, 1.5 para mamposterías huecas reforzadas, 2 para mamposterías macizas confinadas y 4 para estructuras de marcos de concreto y acero. En la ref (28) se proponen valores de Q para diversas condiciones de la estructuración y de los materiales utilizados.

Las figs 6 a 9 contienen espectros elásticos para amortiguamientos viscosos con valores 0.02 y 0.05 del crítico, respectivamente, para distintos tipos de terreno así como los correspondientes espectros reducidos por ductilidad de acuerdo con los criterios propuestos arriba para obtener A (T). En las estructuras reales, la parte del amortiguamiento que se presenta para deformaciones pequeñas es muy inferior a los números que suelen citarse como representativos del amortiguamiento "viscoso" para distinto tipo de estructuras; es decir, la mayor parte del amortiguamiento que convencionalmente se asigna a cada tipo de estructura proviene en realidad de disipación de energía mediante comportamiento no lineal histerético. En las páginas que siguen se adopta el criterio de determinar los espectros de diseño sísmico en el sitio de interés mediante calibración con los que la experiencia ha considerado adecuados en otros sitios. En dichos espectros se adopta la convención de separar el amortiguamiento en una parte "viscosa" que no depende del tipo de estructura y en otra que se representa mediante la reducción por ductilidad; la parte viscosa se hace corresponder a 0.05 del crítico. Esta misma convención se adopta aquí; por lo tanto, en lo que sigue los espectros que se proponen para diseño deberán corresponder al amortiguamiento de 0.05 del crítico y al factor Q reductivo por ductilidad que se recomienda para cada tipo de estructura.

En las figs 6 a 8 se presentan los espectros de diseño aceleraciones para terreno firme en la PGCP. Dichos espectros resultaron de aplicar el criterio descrito en los párrafos anteriores a los datos del sitio de interés. Los valores de a_r y v_r utilizados corresponden a períodos de recurrencia T_r de 30, 50 y 100 años y aparecen tabulados al final del cap 4. Los valores de f_a y f_v empleados tomados de la ref 1 son, respectivamente 4.3 y 2.8 para amortiguamiento de 0.02, y 2.6 y 1.9 para amortiguamiento igual a 0.05 (ver tabla 1).

Como se mencionó en el cap 1, los criterios de diseño deben conducir a sistemas estructurales en los cuales la utilidad obtenida de su ejecución sea máxima. También se comentó que la determinación de la estructura óptima requiere de estudios costo-beneficio. Dichos estudios permiten seleccionar de entre un conjunto de posibles soluciones, aquel proyecto que conduce al equilibrio óptimo entre la seguridad y el costo de la estructura. Dado que los parámetros necesarios para la aplicación directa de los estudios costo-beneficio y optimización son muy difíciles de evaluar, en la práctica suele acudirse a criterios de calibración de diseño y seguridad. En este trabajo se utilizará el criterio de calibración propuesto en la ref 1, que consiste en determinar la intensidad de diseño en un sitio a partir de expresiones que relacionan los valores óptimos de diseño para estructuras de características similares en dos sitios con diferentes niveles de riesgo sísmico. Es decir, el criterio supone que se toman como base de comparación las intensidades de diseño en un sitio en donde dichas intensidades hayan sido establecidas a partir de estudios teóricos, experiencia y juicio ingenieril.

El criterio mencionado en el párrafo anterior se traduce en lo siguiente: para definir la intensidad de diseño Y_{D1} (o el periodo de recurrencia respectivo, T_{D1}) de una estructura que se construirá en el sitio 1 se utiliza la expresión

$$Y_{D1}/Y_{D2} = T_{D1}/T_{D2} = (a_{r1}/a_{r2})^{r/(r+1)} \tag{22}$$

la cual relaciona Y_{D1} con la intensidad de diseño Y_{D2} (o el periodo de recurrencia respectivo T_{D2}) que se juzgue satisfactoria para construir una estructura análoga en el sitio 2.

En la ec 22 a_{r1} y a_{r2} son las aceleraciones máximas del terreno en los sitios 1 y 2, asociadas a la misma tasa de excedencia (o periodo de recurrencia) y r es uno de los parámetros de la ec 1.

El criterio de optimización descrito en el párrafo anterior se aplicó con objeto de definir la intensidad de diseño utilizando los datos del sitio de interés, denominado sitio 1. Como sitio 2 se eligió el Distrito Federal para el cual se tiene toda la información que requiere la ec 22.

La tasa de excedencia seleccionada fue de 0.01 (o sea un periodo de recurrencia de 100 años); el valor de r para ambos sitios fue de 2.7, el cual es congruente en los resultados de esta investigación y la ref 1. Los valores a_{r1} = 310 (cm/seg²) y a_{r2} = 100 (cm/seg²) se obtuvieron de la tabla incluida al final del cap 4, y de la ref 1 respectivamente.

Sustituyendo los valores de a_{r1}, a_{r2} y r en la ec 22 se obtiene $Y_{D1} = 2.79\ Y_{D2}$. Si las intensidades de diseño Y_{D1} y Y_{D2} son las ordenadas máximas del espectro de aceleraciones para Q = 1 y ξ = 0.05, el valor de Y_{D2} obtenido de la ref 28 es 210 cm/seg² y por tanto Y_{D1} = 587 cm/seg². Esta última intensidad corresponde aproximadamente a la ordenada máxima del espectro de diseño para un periodo de recurrencia de 17 años.

Dicho periodo de recurrencia es excesivamente corto de acuerdo con la práctica ordinaria teniendo en cuenta los factores de seguridad y métodos de análisis y diseño usuales (ref 28), por lo que se propone adoptar los espectros de diseño de la fig 6 que corresponden a un periodo de recurrencia de 30 años. Estos espectros deberán aplicarse en combinación con los factores de seguridad y criterios de análisis y diseño de la ref 28, que se acaba de mencionar.

5.2 *Espectros de diseño para terrenos intermedio y blando*

La determinación de los espectros de diseño para terrenos de baja rigidez se puede efectuar a partir de los espectros para terreno firme. El criterio que se seguirá para este propósito se basa en las ref 26 y 28. Dicho criterio consiste en multiplicar las ordenadas del espectro de diseño para terreno firme por factores de amplificación. Los valores de estos factores dependen de los periodos, T, de los sistemas estructurales considerados, como se verá más adelante.

El criterio mencionado se apoya en estudios analíticos sobre amplificación dinámica del movimiento del terreno debida a la presencia de mantos blandos, así como en los espectros de respuesta obtenidos a partir de acelerogramas registrados en México, D.F., durante sismos ocurridos en los últimos años. También incluye dicho criterio las incertidumbres asociadas con la determinación de los periodos naturales de vibración de las estructuras, los cuales provienen de la estimación de rigideces tangentes iniciales, así como de la influencia del com portamiento no lineal y de la interacción suelo-estructura (ref 28).

El criterio de construcción de espectros de diseño para terrenos de baja rigidez en un sitio determinado a partir de los espectros para terreno firme se traduce en lo siguiente:

En terrenos de tipo intermedio las ordenadas del espectro correspondiente, A_i, se obtienen partiendo de las ordenadas A_f del espectro para terreno firme como sigue:

. Para $T = 0$, $A_i(T) = 1.6\ A_f(T)$

. Para $0 < T \leq 1.6$, se tienen dos casos:

a) si a^*_i/Q es mayor que $A_i(0)$, en donde $a^*_i = 1.25\ A_f(0.12)$, A_i varía linealmente con T desde $T = 0$ hasta $T = 0.20$, en que adquiere el valor máximo a^*_i/Q, que se mantiene constante para $0.20 < T \leq 1.6$.

b) si $a^*_i/Q < A_i(0)$, A_i varía linealmente con T desde $T = 0$ hasta $T = 1.6$, en que adquiere el valor a^*_i/Q.

. Para $T > 1.6$, $A_i = (a^*_i/Q)\ (1.6/T)^{2/3}$

En terrenos compresibles, las ordenadas de su espectro, A_c, se evalúan como sigue:

. Para $T = 0$, $A_c(T) = 2\ A_f(T)$

. Para $0 < T \leq 2.9$, se tienen dos casos:

a) si a^*_c/Q es mayor que $A_c(0)$, en donde $a^*_c = 1.5\ A_f\ (0.12)$, A_c varía linealmente con T desde $T = 0$ hasta $T = 0.30$, en que adquiere el valor máximo a^*_c/Q, que se mantiene constante para $0.30 < T \leq 2.9$

b) si a^*_c/Q es menor o igual que $A_c(0)$, A_c varía linealmente con T desde $T = 0$

hasta $T = 2.9$, en que adquiere el valor a_c'/Q.

Para $T > 2.9$, $A_c = (a_c'/Q)\,(2.9/T)$

En la fig 9 se presentan los espectros de diseño de aceleraciones para terrenos intermedios y compresible en la PGCP. Dichos espectros resultaron de aplicar el criterio descrito en los párrafos anteriores al espectro de diseño propuesto para terreno firme que es para un periodo de recurrencia de 30 años, (que también se incluye en la fig 9) con un factor de ductilidad $Q = 1$ y un porcentaje de amortiguamiento crítico $\xi = 0.05$. Los espectros de diseño para otros valores de Q y ξ se pueden calcular siguiendo los pasos señalados en los párrafos anteriores.

Los espectros de la fig 9 deberán aplicarse en combinación con los factores de seguridad y criterios de análisis y diseño de la ref 28.

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PERIODO DE RECURRENCIA	MOVIMIENTO DEL TERRENO		AMORTIGUAMIENTO	FACTORES DE AMPLIFICACION		ENVOLVENTES DE LOS ESPECTROS	
T_r (años)	a_r (cm/seg²)	v_r (cm/seg)	ξ	f_a	f_v	a^* (cm/seg²)	v^* (cm/seg)
30	270	29	0.02	4.3	2.8	1161	81
50	331	38	0.02	4.3	2.8	1423	106
100	410	53	0.02	4.3	2.8	1763	146
30	270	29	0.05	2.6	1.9	702	55
50	331	38	0.05	2.6	1.9	860	72
100	410	53	0.05	2.6	1.9	1066	101

TABLA 1 VALORES MAXIMOS DE ACELERACIONES Y VELOCIDADES DEL TERRENO Y SU RELACION CON LAS ENVOLVENTES DE LOS ESPECTROS DE DISEÑO

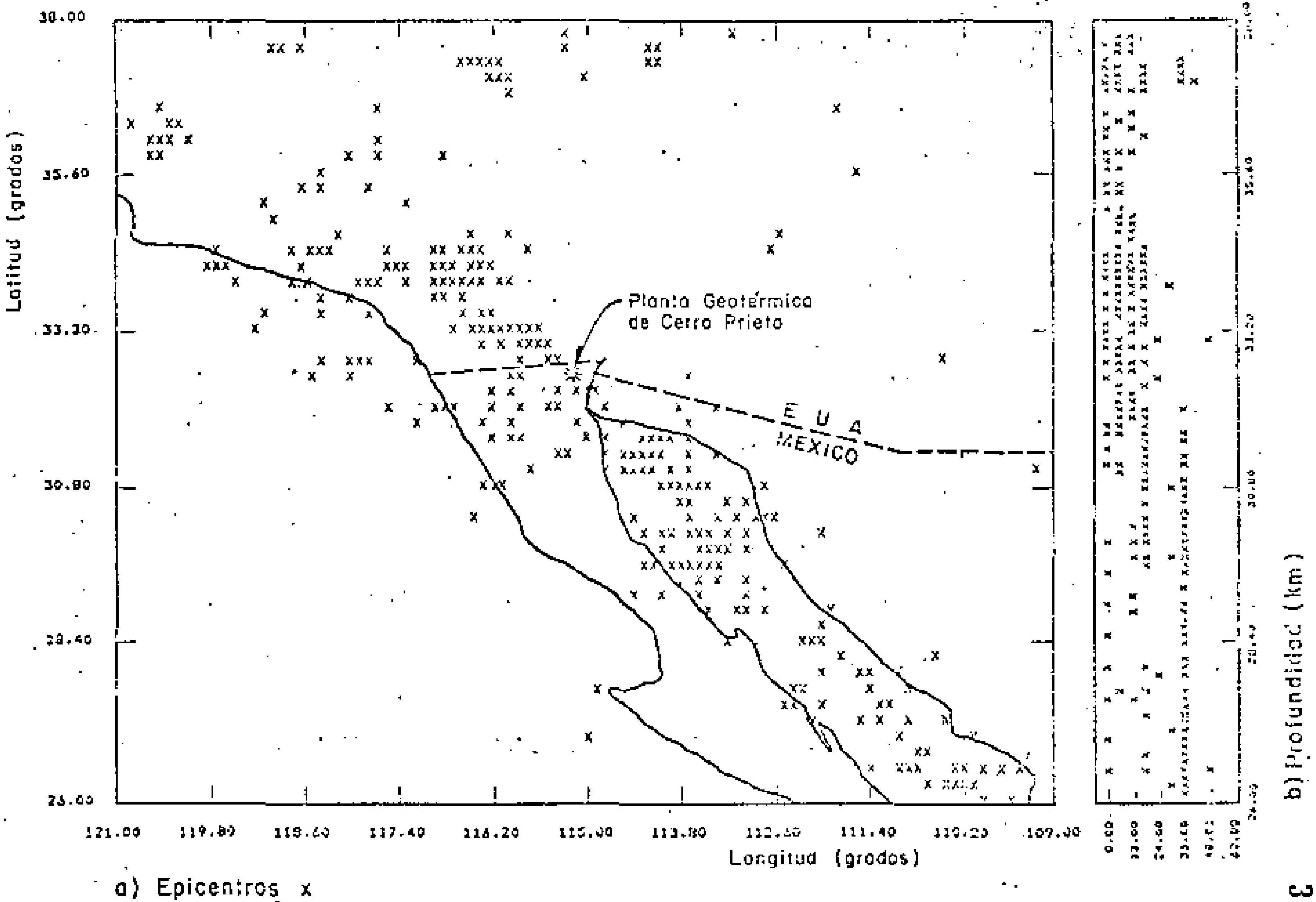

Fig 1. Características de los temblores con magnitud M ≥ 4.5 ocurridos de 1932 a 1980 seleccionados en el presente estudio

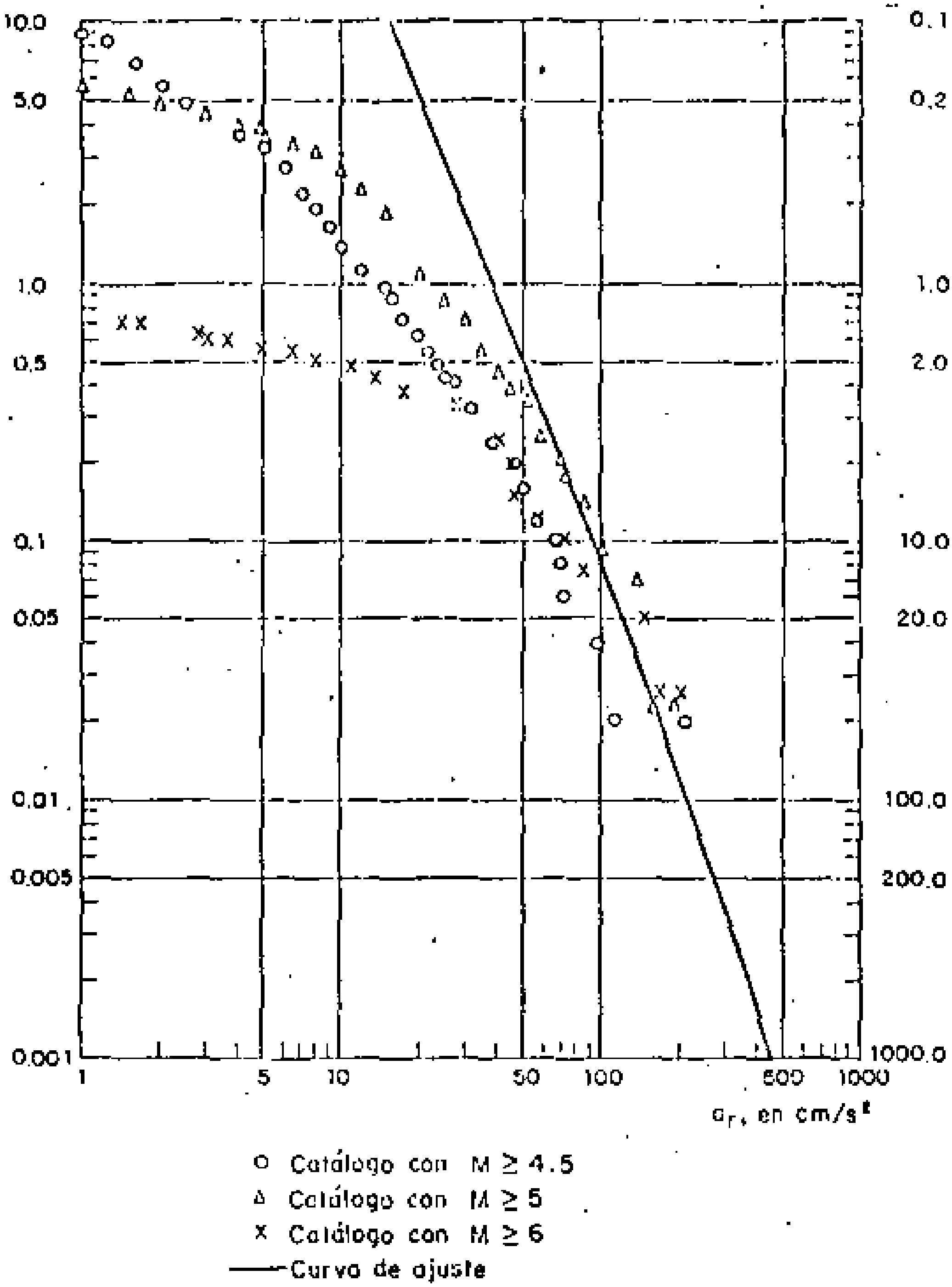

Fig 2. Curva de ajuste de las parejas de valores de la tasa media de excedencia ν y aceleraciones máximas del terreno a_r obtenidas con datos de diversos catálogos, para Cerro Prieto B.C.

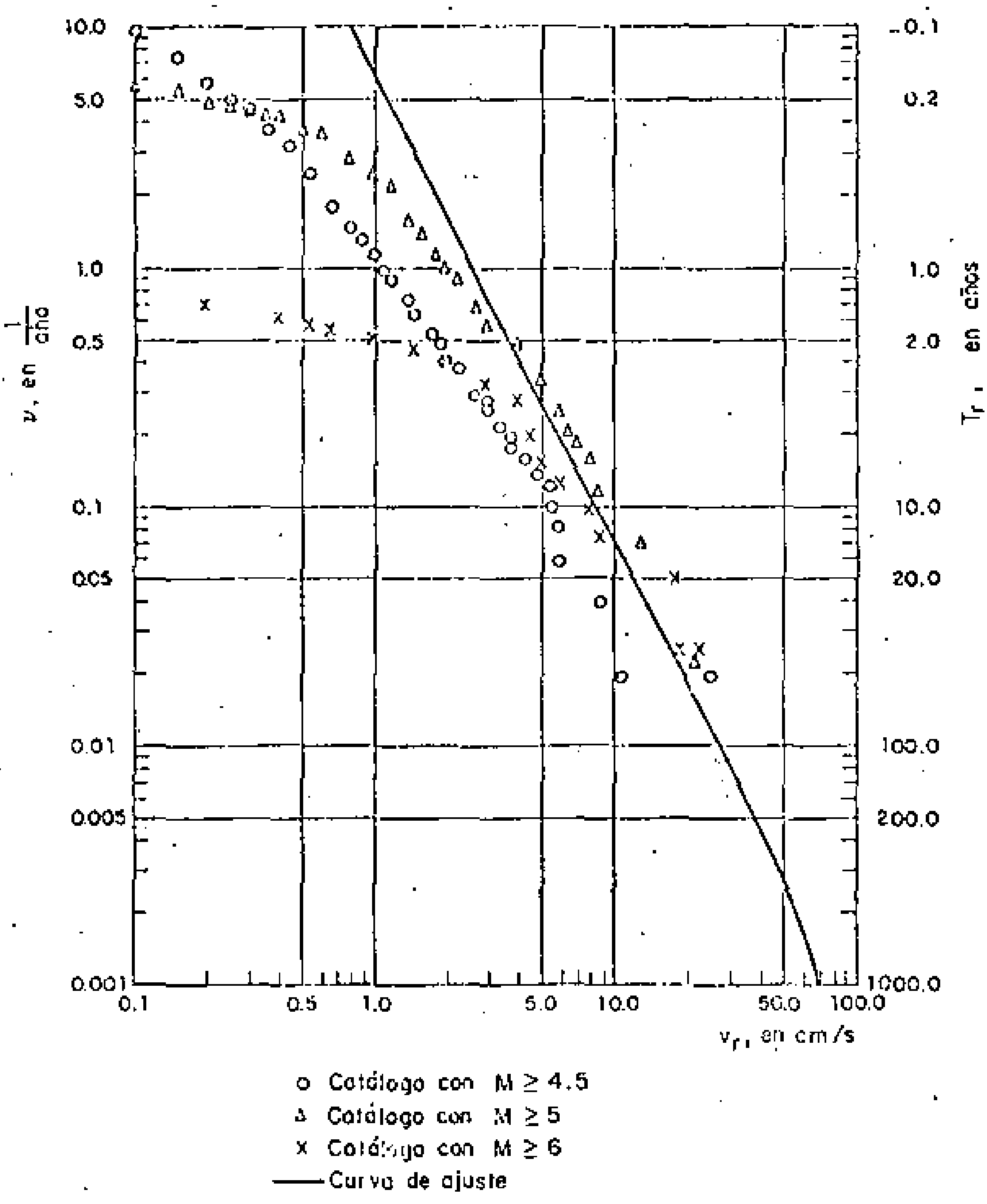

Fig 3. Curva de ajuste de las parejas de valores de la tasa media de excedencia ν y velocidades máximas del terreno v_r obtenidas con datos de diversos catálogos, para Cerro Prieto B. C.

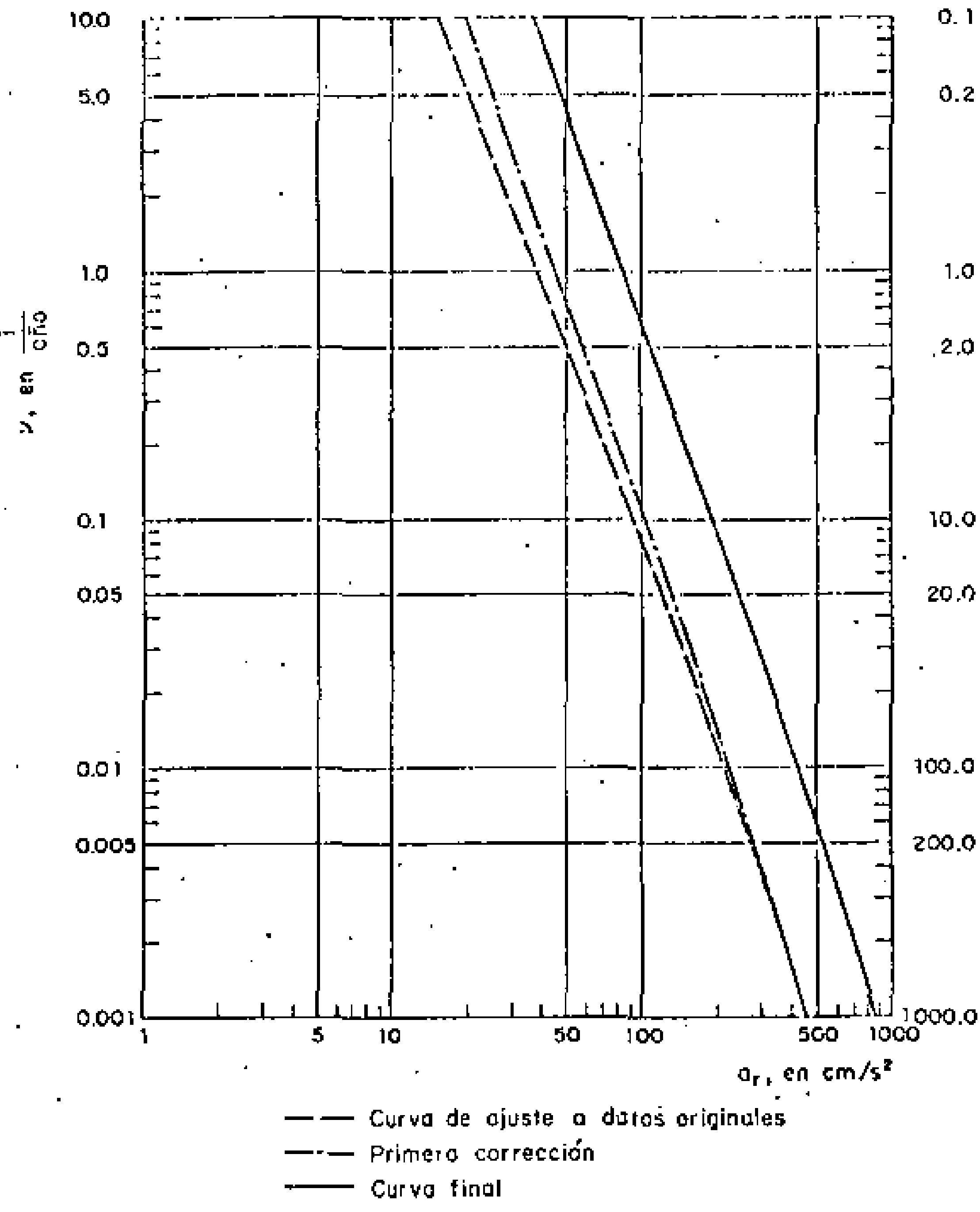

Fig 4. Tasa media de excedencia ν (y periodo de recurrencia T_r) vs aceleración máxima del terreno a_r para Cerro Prieto B. C.

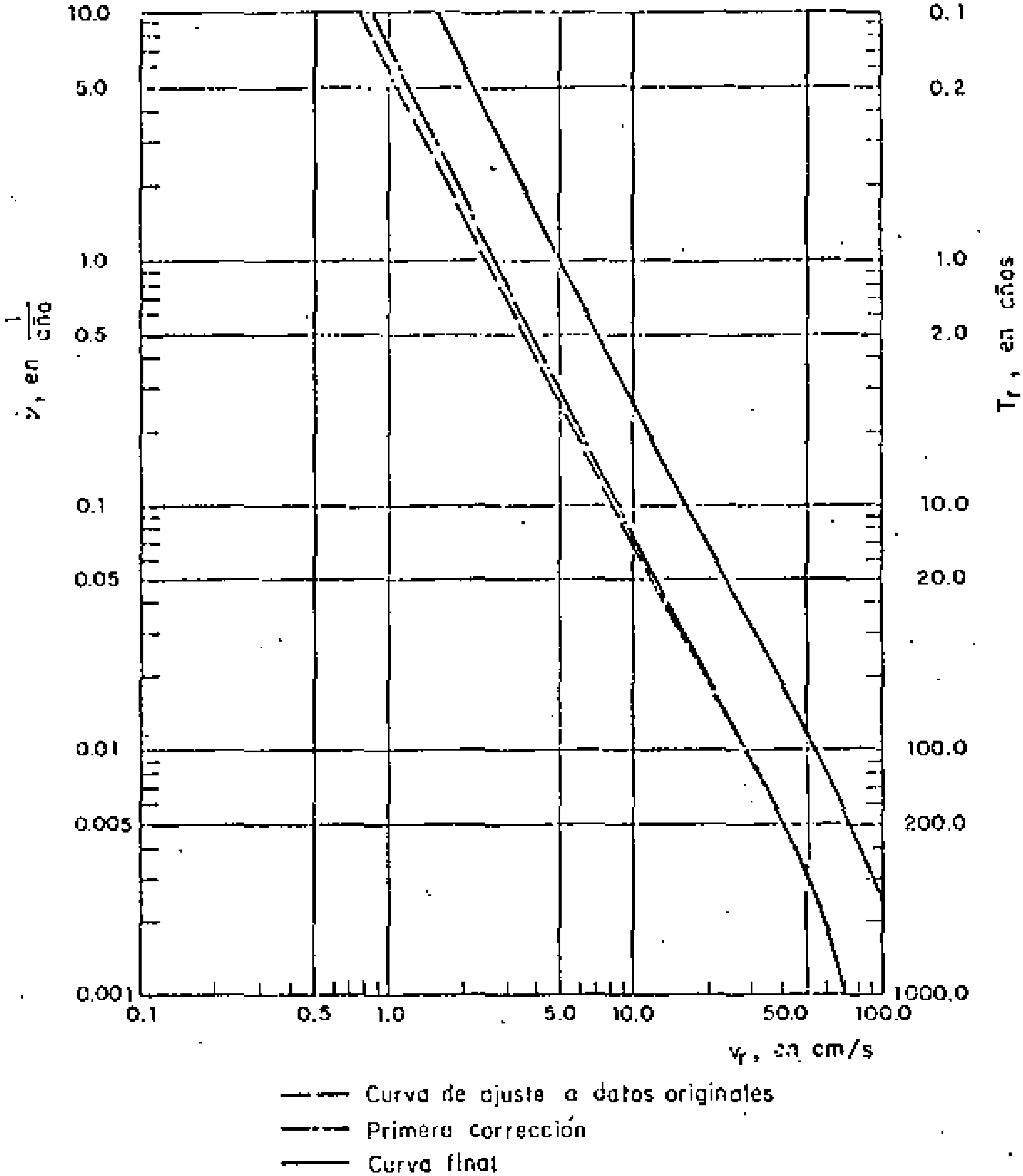

Fig 5. Tasa media de excedencia ν (y periodo de recurrencia T_r) vs velocidad máxima del terreno v_r para Cerro Prieto B. C.

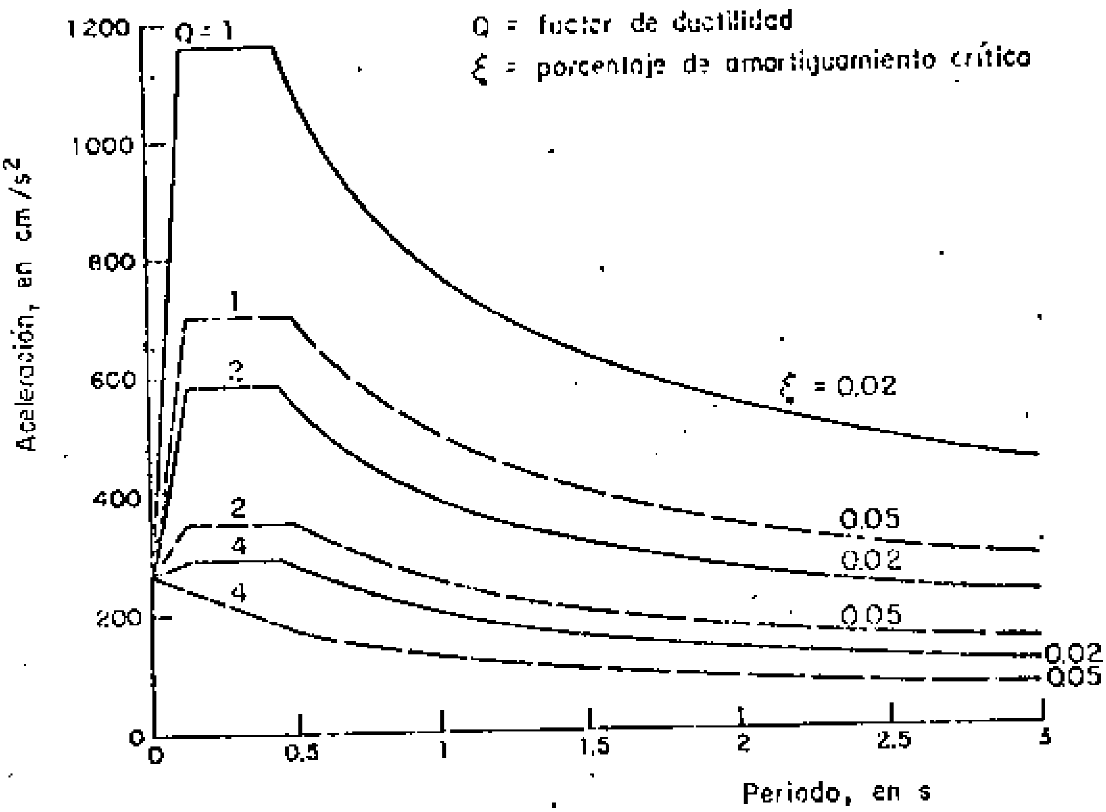

Fig 6. Espectro de diseño para terreno firme, periodo de recurrencia de 30 años, en Planta Geotérmica de Cerro Prieto, B. C.

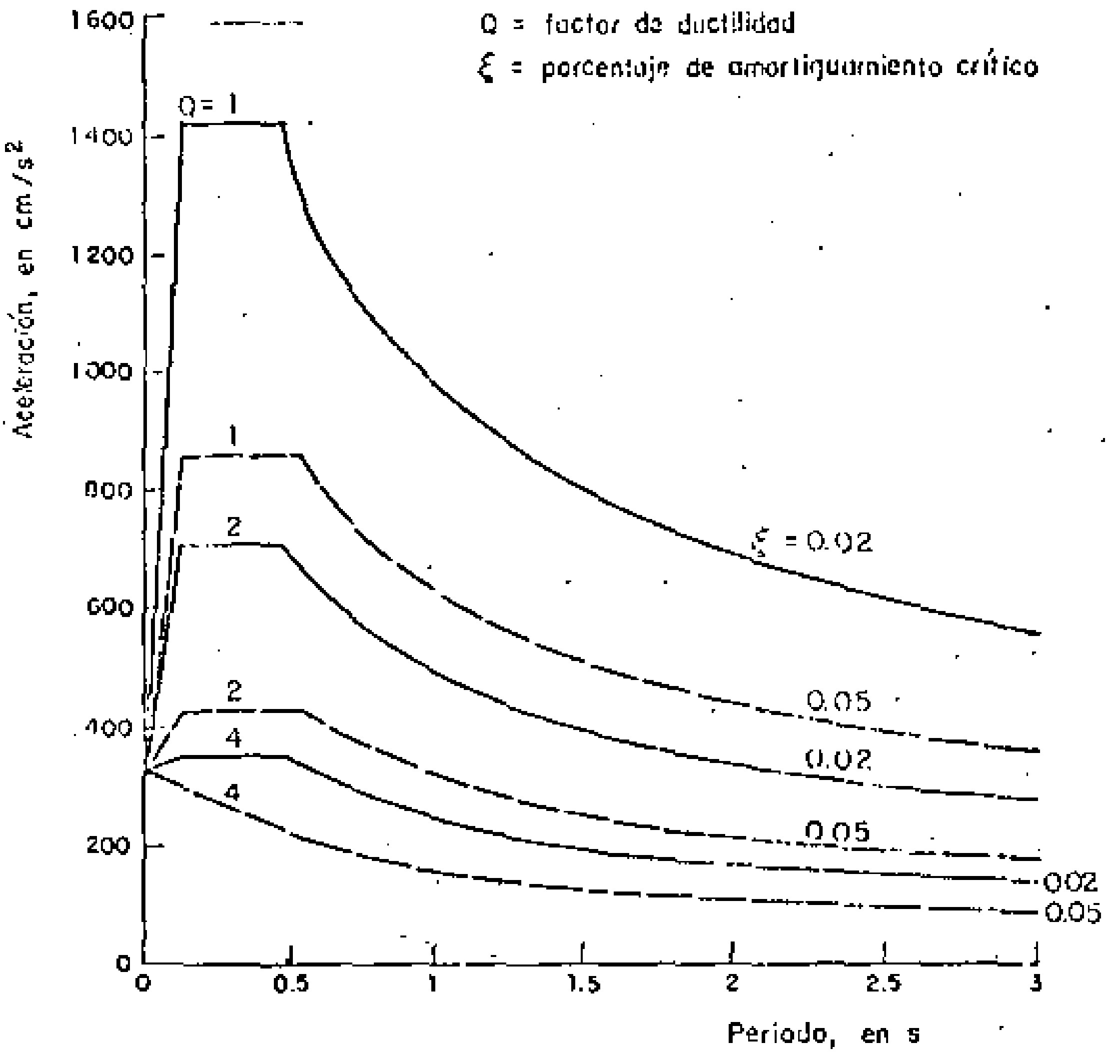

Fig 7. Espectro de diseño para terreno firme, periodo de recurrencia de 50 años, en Planta Geotérmica de Cerro Prieto, B. C.

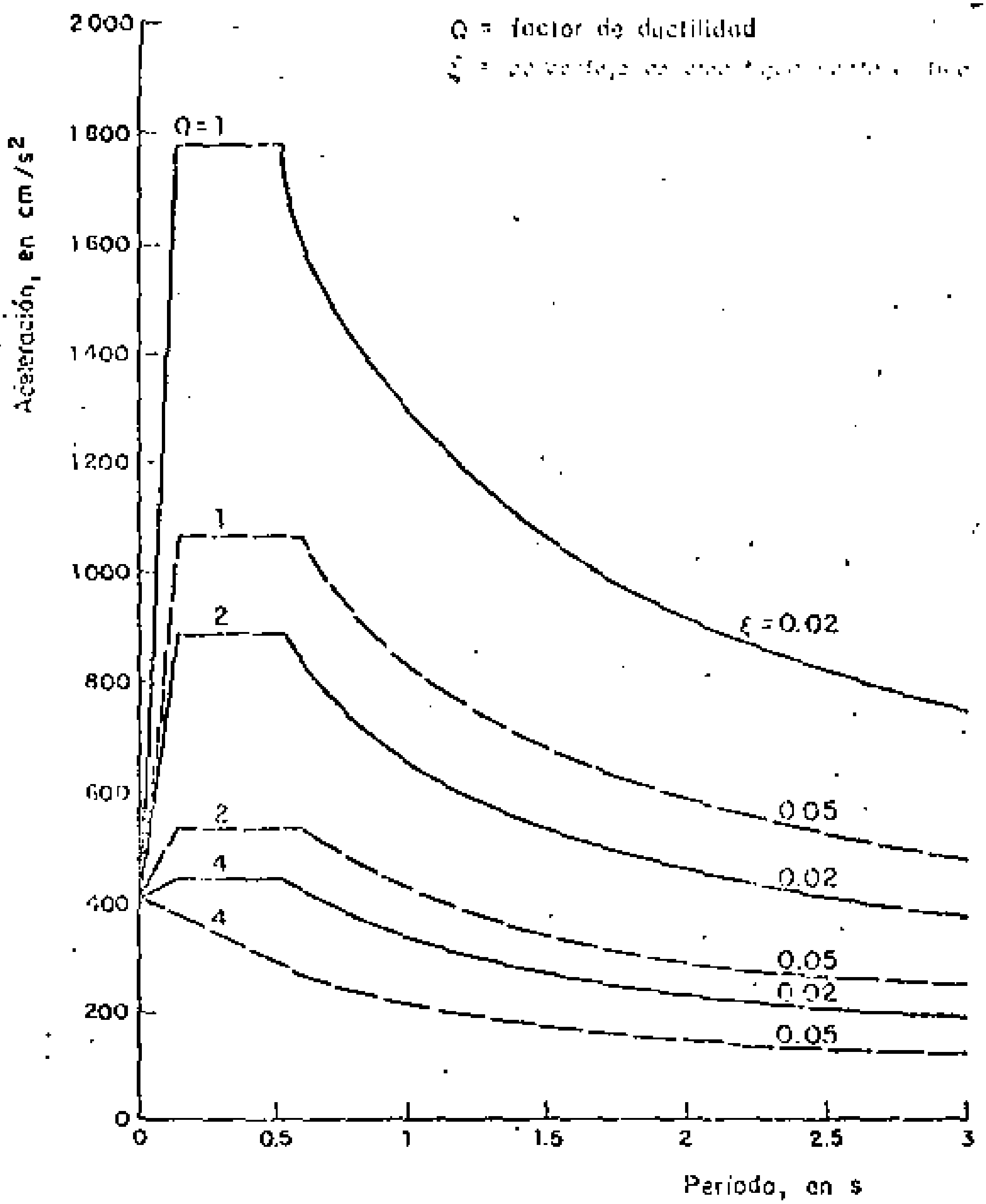

Fig 8. Espectro de diseño para terreno firme, período de recurrencia de 100 años, en Planta Geotérmica de Cerro Prieto, B. C.

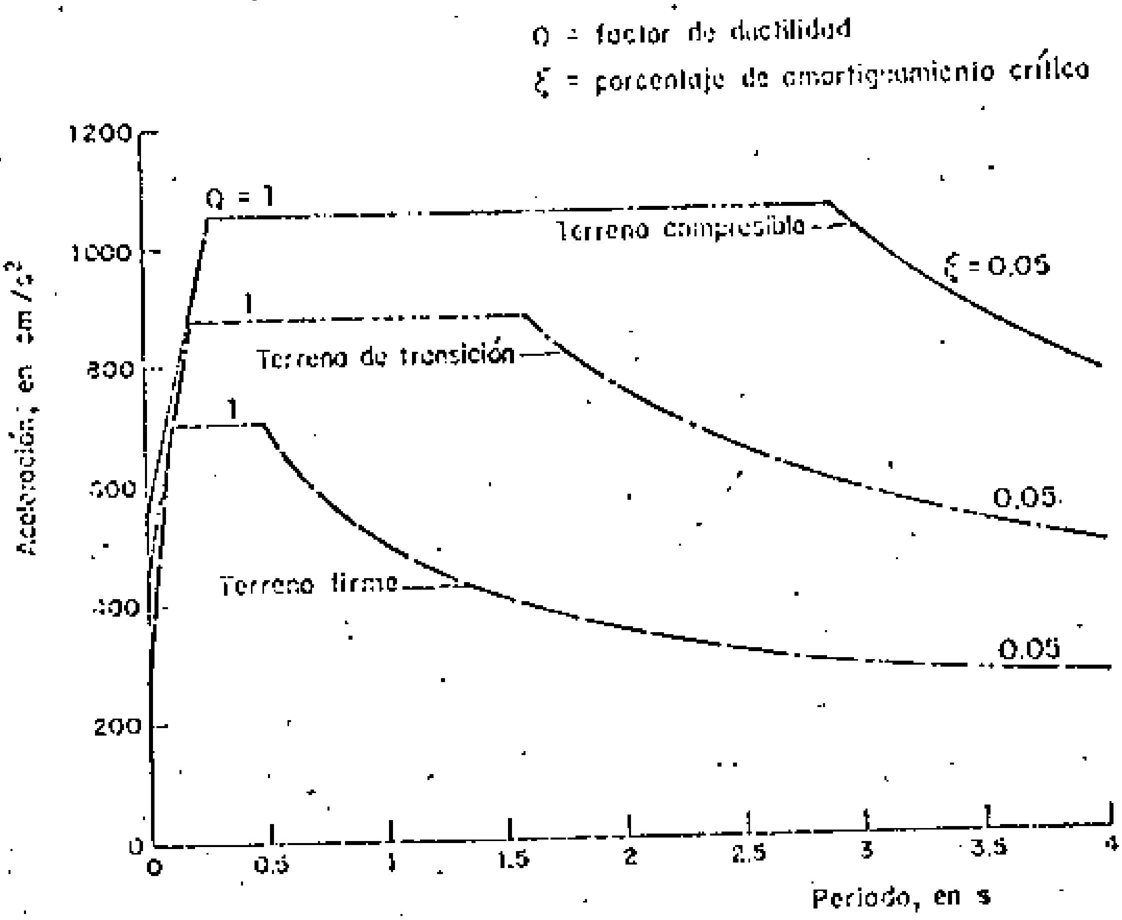

Fig 9. Espectro de diseño para terreno firme, de transición y compresible, periodo de recurrencia de 30 años, en Planta Geotérmica de Cerro Prieto, Baja California

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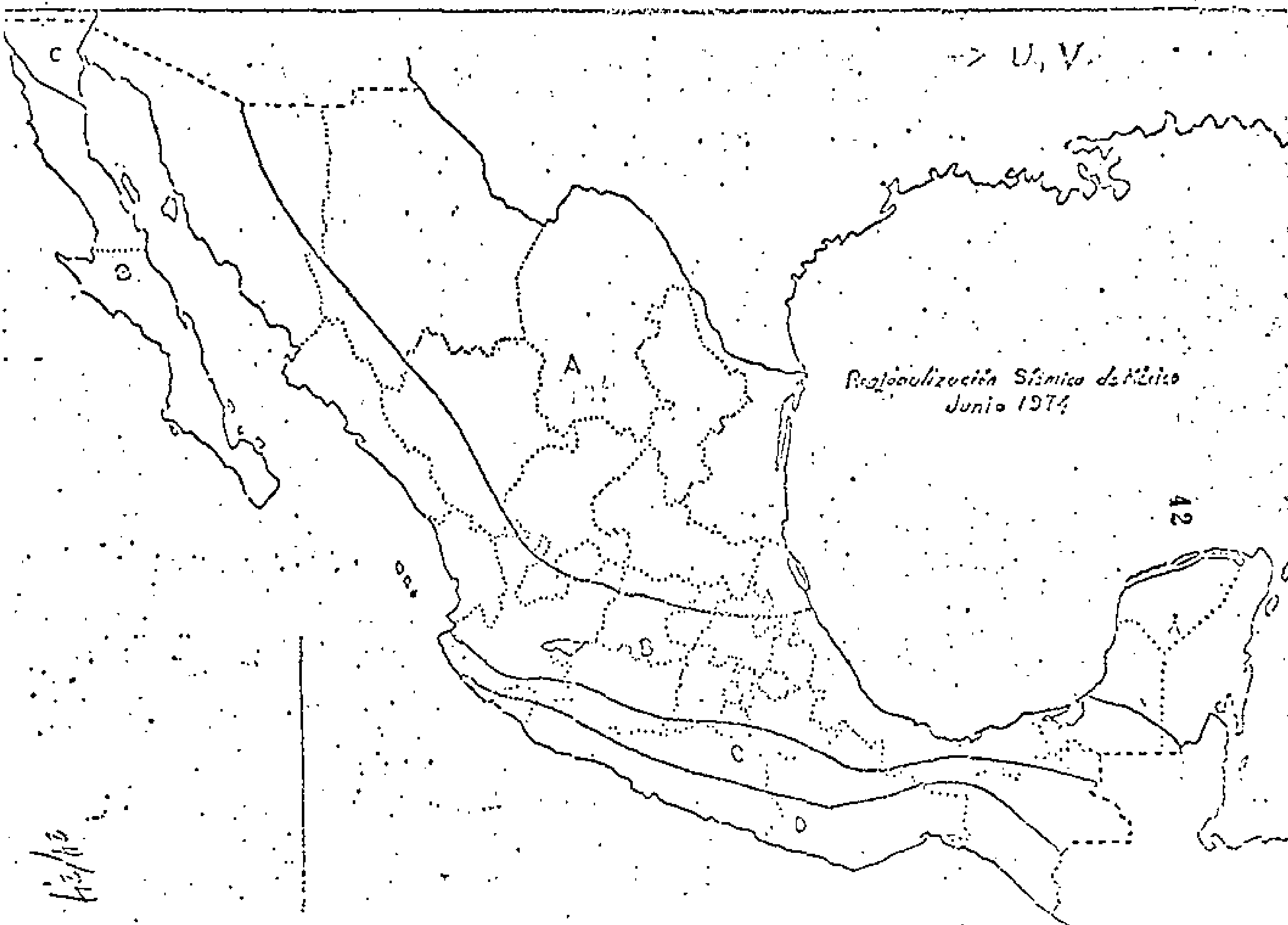
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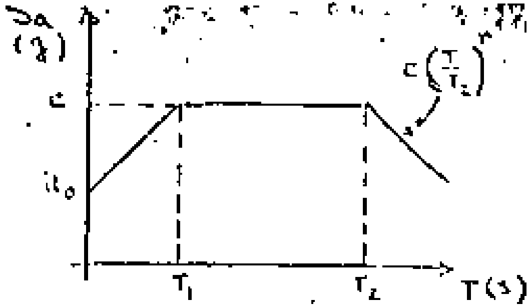

TABLA 10.3

VALORES DE a_o, T_1, T_2 y r.

ZONA DE LA REPUBLICA	TIPO DE SUELO	a_o	c	T_1	T_2	r
A	I	0.03	0.08	0.30	0.8	1/2
	II	0.045	0.12	0.55	2.0	2/3
	III	0.06	0.16	0.75	3.3	1
B	I	0.03	0.16	0.30	0.8	1/2
	II	0.045	0.20	0.50	2.0	2/3
	III	0.06	0.24	0.80	3.3	1
C	I	0.05	0.24	0.25	0.67	1/2
	II	0.08	0.30	0.45	1.6	2/3
	III	0.10	0.36	0.60	2.9	1
D	I	0.09	0.48	0.15	0.55	1/2
	II	0.14	0.56	0.30	1.4	2/3
	III	0.18	0.64	0.45	2.7	1

T_1 y T_2 están en segundos.

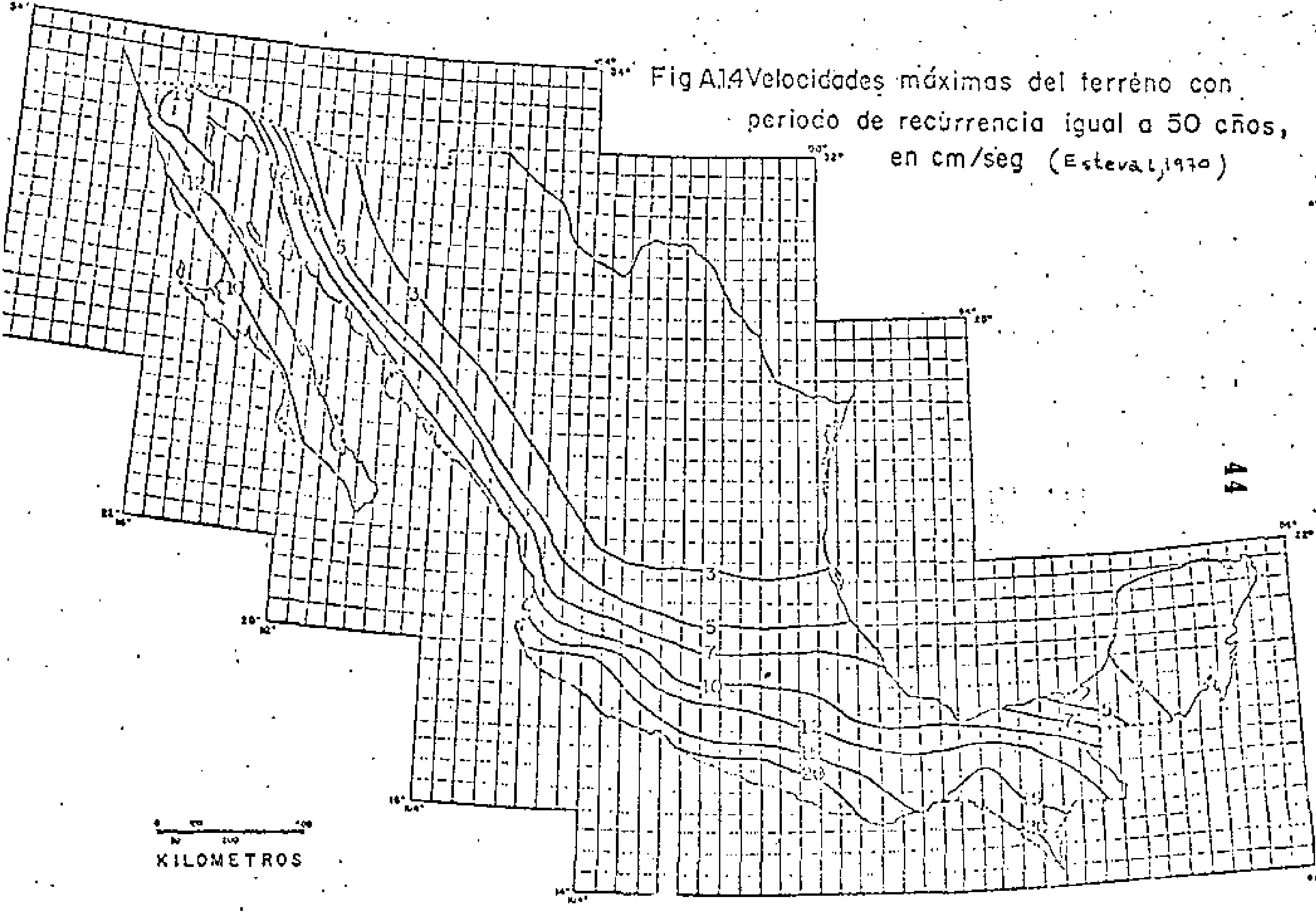

Fig A.1.4 Velocidades máximas del terreno con periodo de recurrencia igual a 50 años, en cm/seg (Esteva, 1970)

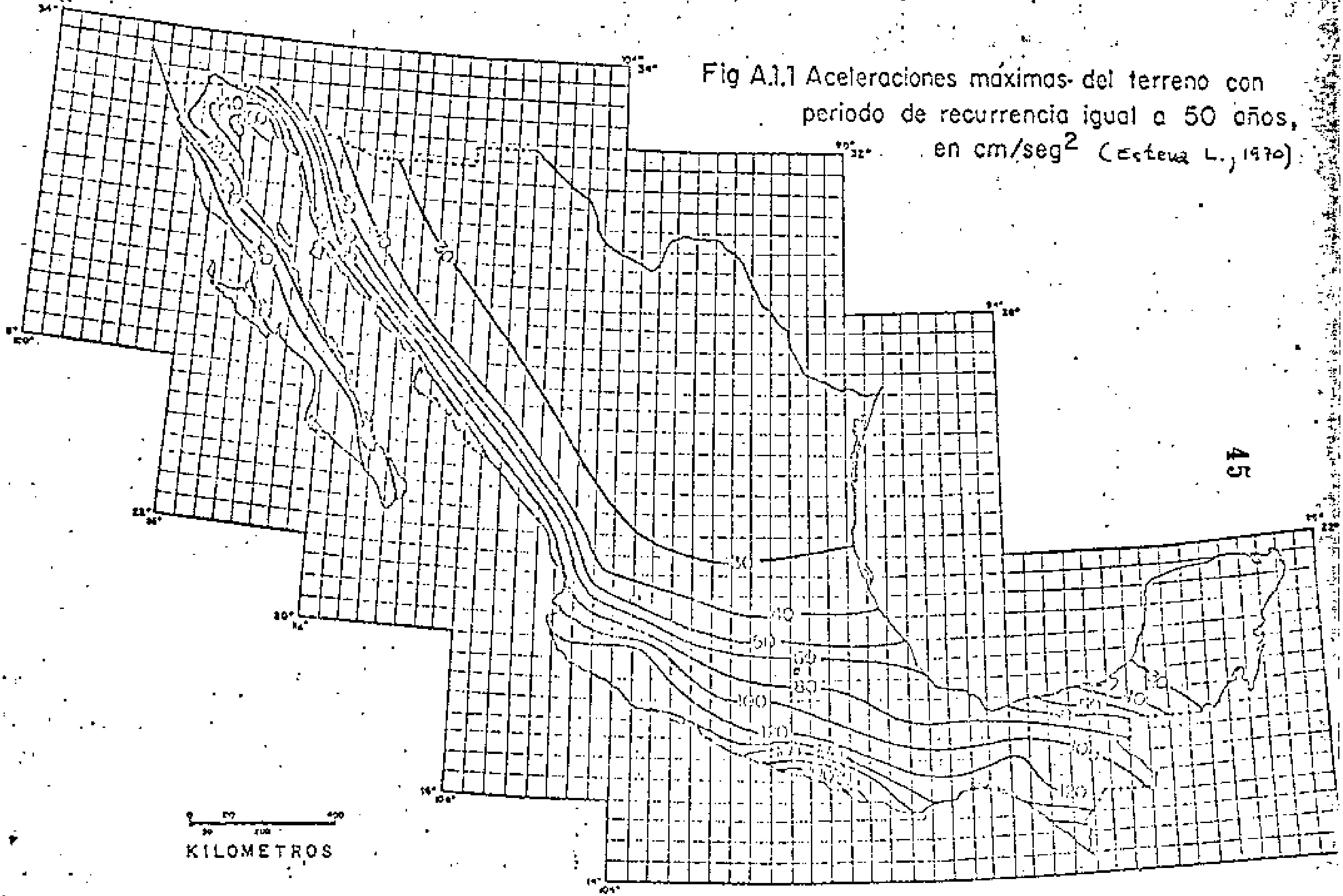

Fig A.1.1 Aceleraciones máximas del terreno con periodo de recurrencia igual a 50 años, en cm/seg^2 (Esteva L., 1970)

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X CURSO INTERNACIONAL DE INGENIERIA SISMICA

ANALISIS DE RIESGO SISMICO

PROPAGACION DE ONDAS ELASTICAS EN UN MEDIO SEMINFINITO

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AGOSTO, 1984.

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PROPAGACION DE ONDAS ELASTICAS EN UN MEDIO SEMINFINITO

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1. INTRODUCCION

Las ondas sísmicas se propagan desde la fuente de acuerdo con las propiedades mecánicas del medio en que viajan y, por supuesto, dependen también de las características de la fuente. La descripción del fenómeno ha podido hacerse de forma satisfactoria al recurrir a simplificaciones e hipótesis que llevan a la formulación de modelos que representan los aspectos más importantes de la propagación de ondas en la tierra. Es usual aceptar que la tierra es un medio elástico lineal, homogéneo e isotrópico. En un medio de esta naturaleza con extensión ilimitada se pueden propagar dos tipos de ondas elásticas: las ondas P o de compresión y las ondas S o de cortante. Las primeras se propagan con mayor velocidad y por eso se les suele llamar primarias mientras que las segundas reciben el nombre de secundarias. Existen diversas soluciones para las ecuaciones que gobiernan el fenómeno de propagación. Así, para una fuente puntual se podría hablar de ondas esféricas, que a grandes distancias de la fuente se pueden represen tar como ondas planas. En algunos casos se modela el pro blema de propagación como bidimensional y las soluciones para una fuente se dan en términos de ondas cilíndricas, que también a grandes distancias son aproximadamente planas. Un buen número de soluciones de las ecuaciones fundamentales puede encontrarse en el excelente texto de Ewing, Jardetzky y Press (1957).

La existencia de una superficie libre introduce reflexiones de las ondas al llegar a esta. Para estudiar la naturaleza de las reflexiones dicha superficie debe considerarse libre de esfuerzos. Dado que a grandes distancias de la fuente las ondas pueden suponerse planas y que para las longitudes de onda de interés la curvatura de la tierra es, comparativamente, pequeña se estudiará el problema de reflexión de ondas planas por la superficie de un medio elástico semininfinito. Dicha superficie se supondrá plana.

A continuación se presentan algunos aspectos de la propagación de ondas en un medio elástico de extensión ilimi tada y se expresan las ecuaciones que gobiernan el fenóme no en términos de potenciales de desplazamiento. Posteriormente se discute la reflexión de ondas planas por la frontera libre de un semiespacio elástico, homogeneo e isotrópico. La incidencia de ondas P y SV armónicas se estudia con detalle. Finalmente se presentan las ondas superficiales de Rayleigh y de Love, las primeras como caso límite en que la velocidad aparente es menor que las velocidades de propagación de las ondas de cuerpo y las segundas como ejemplo de propagación en el caso más simple de un medio estratificado.

2. PROPAGACION DE ONDAS EN UN MEDIO ELASTICO

Puede demostrarse que en un sólido elástico, homogéneo e isotrópico las ecuaciones de movimiento están dadas por

$$(\lambda+2\mu)\frac{\partial^2 u}{\partial x^2} + \mu\left(\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)+(\lambda+\mu)\left(\frac{\partial^2 v}{\partial x\partial y}+\frac{\partial^2 w}{\partial x\partial z}\right) = \rho\,\frac{\partial^2 u}{\partial t^2}$$

$$(\lambda+2\mu)\frac{\partial^2 v}{\partial y^2} + \mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial z^2}\right)+(\lambda+\mu)\left(\frac{\partial^2 u}{\partial x\partial y}+\frac{\partial^2 w}{\partial y\partial z}\right) = \rho\,\frac{\partial^2 v}{\partial t^2} \qquad (1)$$

$$(\lambda+2\mu)\frac{\partial^2 w}{\partial z^2} + \mu\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)+(\lambda+\mu)\left(\frac{\partial^2 u}{\partial x\partial z}+\frac{\partial^2 v}{\partial y\partial z}\right) = \rho\,\frac{\partial^2 w}{\partial t^2}$$

donde u, v, w = desplazamientos en las direcciones x, y, z, respectivamente; λ, μ = constantes de Lamé, ρ = densidad del medio y t = tiempo. Estas ecuaciones pueden escribirse de una manera compacta en notación vectorial, esto es

$$\mu\nabla^2\bar{u} + (\lambda+\mu)\,\nabla\nabla\cdot\bar{u} = \rho\,\ddot{\bar{u}} \qquad (2)$$

donde $\bar{u} = (u, v, w)$ = vector desplazamiento, ∇^2 = operador Laplaciano y ∇ = operador gradiente.

Antes de considerar soluciones generales de las ecuaciones de movimiento dos ejemplos simples permitirán ilustrar las principales características de las ondas planas en un sólido elástico de extensión ilimitada.

Supongase que $u \neq 0$, $v = w = 0$ y que u es solo función de x y del tiempo. Las ecs 1 se reducen a la expresión

$$(\lambda+2\mu) \frac{\partial^2 u}{\partial x^2} = \rho \frac{\partial^2 u}{\partial t^2} \tag{3}$$

una solución de esta ecuación es

$$u = f(t - x/\alpha) + g(t + x/\alpha) \tag{4}$$

donde $\alpha^2 = (\lambda+2\mu)/\rho$ y f,g son funciones de *una sola variable* que pueden describir una forma de onda arbitraria. Un simple análisis de los argumentos de f y g permite establecer que $f(t-x/\alpha)$ representa una onda que viaja en la dirección positiva de x con velocidad α y $g(t+x/\alpha)$ describe una onda que viaja en la dirección negativa. Debe notarse que $f(t-x/\alpha)$ puede representar una onda armónica estacionaria, $\exp[i\omega(t-x/\alpha)]$ donde $i = \sqrt{-1}$ y ω = frecuencia circular del movimiento. Puede demostrarse que la ec 4 representa ondas de compresión o P.

Un segundo ejemplo simple se obtiene si se supone que $u = w = 0$ y que $v = v(x,t)$. De las ecs 1 se obtiene que

$$\mu \frac{\partial^2 v}{\partial x^2} = \rho \frac{\partial^2 v}{\partial t^2} \tag{5}$$

y la solución tiene la misma forma que la ec 4 pero representa ondas que viajan con una velocidad β, donde $\beta^2 = \mu/\rho$. Debe notarse que el movimiento es perpendicular a la dirección de avance. Puede demostrarse que las soluciones de la ec 5 representan ondas de cortante, sin cambio de volumen.

Las ecuaciones de movimiento pueden resolverse de una manera más general por medio de *potenciales de desplazamiento*.

Si el vector desplazamiento se expresa como

$$\bar{u} = \nabla\phi + \nabla\times\bar{\psi}, \text{ con } \nabla\cdot\bar{\psi} = 0 \tag{6}$$

donde ϕ es un potencial escalar y $\bar{\psi}$ es un potencial vectorial, puede demostrarse que la ec 6 representa una solución de la ec 2 (o de la ec 1 en coordenadas rectangulares) si ϕ y $\bar{\psi}$ satisfacen, respectivamente, las ecuaciones de onda:

$$\nabla^2\phi = \frac{1}{\alpha^2} \frac{\partial^2 \phi}{\partial t^2} \tag{7}$$

$$\nabla^2 \bar{\psi} = \frac{1}{\beta^2} \frac{\partial^2 \bar{\psi}}{\partial t^2} \tag{8}$$

Así, por ejemplo, una solución de la ec 7 que representa una onda plana de compresión que viaja en una dirección arbitraria está dada por

$$\phi = f\left(t - \frac{x\ell + ym + zn}{\alpha}\right) \tag{9}$$

donde ℓ, m, n = cosenos de los ángulos formados por la dirección de viaje y los tres ejes coordenados, respectivamente. Si $\bar{r}$ = (x, y, z) y $\bar{n}$ = (ℓ, m, n) donde $\bar{r}$ = vector de posición y $\bar{n}$ = vector unitario que da la dirección de propagación, la ec 9 puede escribirse como

$$\phi = f(t - \bar{r}\cdot\bar{n}/\alpha) \tag{10}$$

Es evidente que soluciones similares pueden encontrarse para los tres componentes del potencial vectorial y representarían ondas de cortante viajando con una velocidad β.

En coordenadas rectangulares la ec 6 se desarrolla como

$$\begin{aligned} u &= \frac{\partial \phi}{\partial x} + \frac{\partial \psi_z}{\partial y} - \frac{\partial \psi_y}{\partial z} \\ v &= \frac{\partial \phi}{\partial y} - \frac{\partial \psi_z}{\partial x} + \frac{\partial \psi_x}{\partial z} \\ w &= \frac{\partial \phi}{\partial z} + \frac{\partial \psi_y}{\partial x} - \frac{\partial \psi_x}{\partial y} \end{aligned} \tag{11}$$

donde $\bar{\Psi} = (\psi_x, \psi_y, \psi_z)$.

Los potenciales de desplazamiento ϕ y $\overline{\psi}$ permiten especificar ondas planas de compresión y cortante, respectivamente, que viajen en cualquier dirección y con cualquier forma. Además, dado el caracter lineal de las ecuaciones involucradas, cualquier combinación de soluciones sigue satisfaciendo las ecuaciones de movimiento de un sólido elástico, homogéneo e isotrópico de extensión ilimitada. La utilidad de este hecho se hace evidente cuando se hace necesario seleccionar una combinación particular de ondas planas que satisfaga una cierta condición de frontera o que describa una fuente. Tal es el caso en el problema que se aborda a continuación.

3. REFLEXION DE ONDAS PLANAS POR LA FRONTERA LIBRE DE UN SEMIESPACIO ELASTICO

Considérese que la frontera libre es el plano yz como se muestra en la fig 1. Además, sin perder generalidad, supóngase que las direcciones de avance de las ondas están alojadas en el plano xz.

Para describir el movimiento debido a ondas de cortante se introduce el concepto de planos de polarización. Así, se descompone el movimiento en la dirección de la coordenada y (ondas polarizadas horizontalmente o SH) y en la dirección perpendicular a la dirección de avance en el plano vertical xz (ondas polarizadas verticalmente o SV). En la propagación de ondas P el movimiento es en la dirección de avance de la onda. Esto se ilustra en la fig 2.

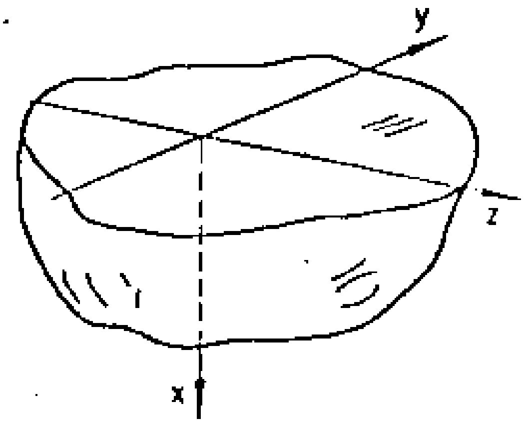

Fig 1. Sistema de coordenadas en el semiespacio elástico

La propagación de ondas SH está gobernada por la ecuación

$$\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial z^2} = \frac{1}{\beta^2} \frac{\partial^2 v}{\partial t^2} \tag{12}$$

Que es precisamente la ecuación de onda en dos dimensiones, en este caso no es necesario recurrir a la formulación del problema en términos de los potenciales de desplazamiento. Puede demostrarse que, en la reflexión de una onda SH plana por una frontera libre, el ángulo de incidencia es igual al ángulo de reflexión y la onda reflejada mantiene la forma de la onda incidente. Si la onda incidente está dada por

$$v^{(i)} = f\left(t + \frac{x \cos\gamma - z \operatorname{sen}\gamma}{\beta}\right) \tag{13}$$

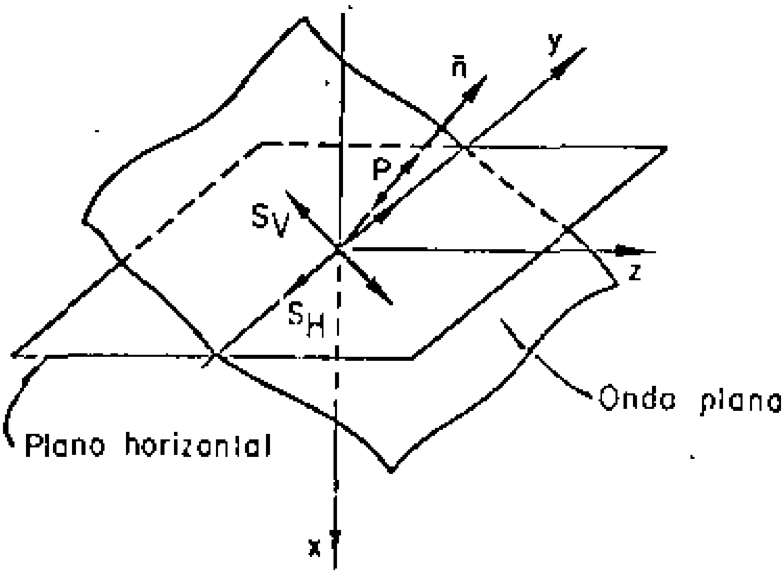

Fig 2. Nomenclatura para ondas planas

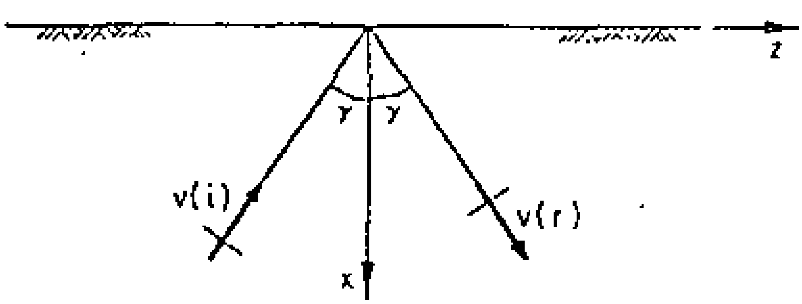

Fig 3. Ondas SH incidente y reflejada

la onda reflejada está dada simplemente por

$$v^{(r)} = f(t - \frac{x \cos \gamma + z \operatorname{sen} \gamma}{\beta}) \tag{14}$$

aquí γ = ángulo de incidencia. Puede verificarse que $v = v^{(i)} + v^{(r)}$ satisface la ec 12 y la condición de que el plano $x = 0$ esté libre de esfuerzo pues los únicos esfuerzos relevantes están dados por

$$\tau_{xy} = \mu \frac{\partial v}{\partial x}, \quad \tau_{yz} = \mu \frac{\partial v}{\partial z} \tag{15}$$

y combinando las ecs 13, 14 y 15 resulta que $\tau_{xy} = 0$ en $x = 0$. Debe observarse que en estas condiciones el movimiento en $x = 0$, la superficie libre, se puede escribir como

$$v_{x=0} = 2 f(t - \frac{z \operatorname{sen} \gamma}{\beta}), \tag{16}$$

por lo que el factor de amplificación es dos.

En la propagación de ondas P y SV el movimiento está en el plano xz, es decir $u = u(x,z,t)$, $w = w(x,z,t)$ y $v = 0$. En este caso las ecuaciones de onda que deben satisfacer los potenciales, si $\psi = \psi_y$, son

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial z^2} = \frac{1}{\alpha^2} \frac{\partial^2 \phi}{\partial t^2} \tag{17}$$

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial z^2} = \frac{1}{\beta^2} \frac{\partial^2\psi}{\partial t^2} \tag{18}$$

Los desplazamientos quedan como

$$u = \frac{\partial\phi}{\partial x} - \frac{\partial\psi}{\partial z} \tag{19}$$

$$w = \frac{\partial\phi}{\partial z} + \frac{\partial\psi}{\partial x} \tag{20}$$

Los esfuerzos que al valuarse en la superficie deben anular se son

$$\sigma_x = \lambda\left(\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\psi}{\partial z^2}\right) + 2\mu\left(\frac{\partial^2\phi}{\partial x^2} - \frac{\partial^2\psi}{\partial x\partial z}\right) \tag{21}$$

$$\tau_{xz} = \mu\left(2\frac{\partial^2\phi}{\partial x\partial z} + \frac{\partial^2\psi}{\partial x^2} - \frac{\partial^2\psi}{\partial z^2}\right) \tag{22}$$

ya que $\tau_{xy} = 0$.

Considérense los casos mostrados en las figs 4 y 5, la incidencia de ondas P y de ondas SV, respectivamente.

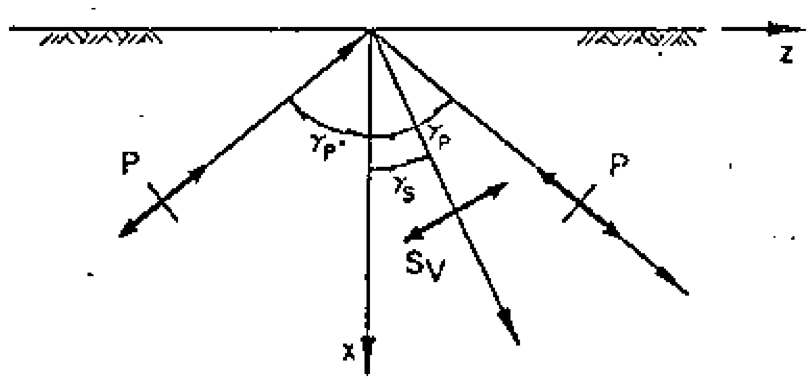

Fig 4. Incidencia de ondas P

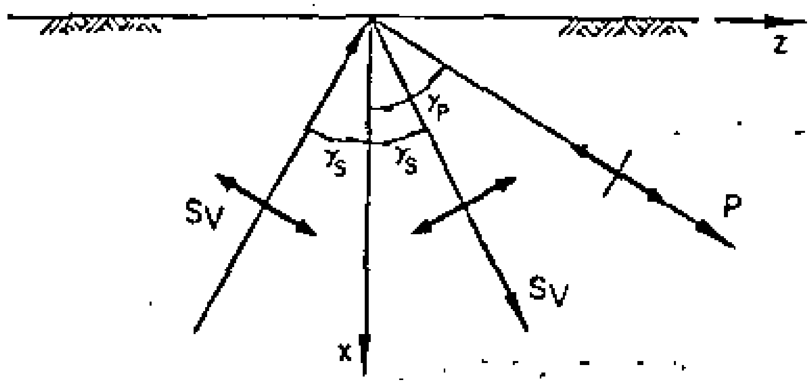

Fig 5. Incidencia de ondas SV

Mediante la técnica de separación de variables se puede demostrar que las soluciones de las ecs 17 y 18 son de la forma

$$\phi = A_o\ e^{Mx}\ e^{Lz}\ e^{\Omega t} \tag{23}$$

$$\psi = B_o\ e^{Kx}\ e^{Lz}\ e^{\Omega t} \tag{24}$$

donde

$$M^2 + L^2 = \frac{\Omega^2}{\alpha^2} \quad y \quad K^2 + L^2 = \frac{\Omega^2}{\beta^2} \tag{25}$$

Hasta ahora K, L, M y Ω con valores complejos son posibles soluciones, pero para los actuales propósitos basta hacer

$$\Omega = i\ \omega \quad y \quad L = -\ i\ell \tag{26}$$

pues al tener a Ω como número imaginario puro se garantiza que los potenciales sean finitos. Al definir L como imaginario negativo con $\ell = \omega/c$ se observa que el producto

$$e^{-i\ell z}\, e^{i\omega t} = e^{i\omega(t-z/c)} \qquad (27)$$

representa una onda armónica que viaja en la dirección positiva de z con una velocidad de fase c, si c es negativa la dirección de viaje es en la dirección negativa de z.
En términos de los ángulos de las figs 4 y 5 se tiene que

$$c = \frac{\alpha}{\operatorname{sen}\gamma_P} = \frac{\beta}{\operatorname{sen}\gamma_S} \qquad (28)$$

Con estas definiciones M y K deben ser o reales o imaginarios pues, de las ecs 25 y 26, se tiene que

$$M^2 = \ell^2 - \omega^2/\alpha^2 = \omega^2(1/c^2 - 1/\alpha^2) \qquad (29)$$

y

$$K^2 = \ell^2 - \omega^2/\beta^2 = \omega^2(1/c^2 - 1/\beta^2) \qquad (30)$$

Así, para $\beta < \alpha < |c|$, M y K son imaginarios; para $\beta < |c| < \alpha$, M es real y K imaginario; para $|c| < \beta < \alpha$, M y K son reales.

Para el primer caso, $\beta < \alpha < |c|$, se tienen los potenciales

$$\phi = (A_1 e^{imx} + A_2 e^{-imx}) e^{-i\ell z}\, e^{i\omega t} \qquad (31)$$

$$\psi = (B_1 e^{ikx} + B_2 e^{-ikx}) e^{-i\ell z} e^{i\omega t} \qquad (32)$$

donde $\ell = \omega/c$, $m = \omega(1/\alpha^2 - 1/c^2)^{1/2}$ y $k = \omega(1/\beta^2 - 1/c^2)^{1/2}$. Si $B_1 = 0$ se tiene el caso mostrado en la fig 4 de incidencia de ondas P. En cambio si $A_1 = 0$ se tendrá incidencia de ondas SV. Sustituyendo las ecs 31 y 32 en las ecs 21 y 22, haciendo que $\sigma_x = \tau_{xz} = 0$ en $x=0$ y resolviendo el sistema de ecuaciones resultante se obtiene que

a) Para $B_1 = 0$

$$\frac{A_2}{A_1} = \frac{4 \cot \gamma_P \cot \gamma_S - (\cot^2 \gamma_S - 1)^2}{4 \cot \gamma_P \cot \gamma_S + (\cot^2 \gamma_S - 1)^2} \qquad (33)$$

$$\frac{B_2}{A_1} = \frac{4 \cot \gamma_P (\cot^2 \gamma_S - 1)}{4 \cot \gamma_P \cot \gamma_S + (\cot^2 \gamma_S - 1)^2} \qquad (34)$$

donde γ_P = ángulo de incidencia y de reflexión de la onda P y γ_S = ángulo de reflexión de la onda SV. Debe recordarse que la velocidad aparente está dada por

$$c = \frac{\alpha}{\operatorname{sen} \gamma_P} = \frac{\beta}{\operatorname{sen} \gamma_S} \qquad (35)$$

b) Para $A_1 = 0$

$$\frac{A_2}{B_1} = - \frac{4 \cot \gamma_S (\cot^2 \gamma_S - 1)}{4 \cot \gamma_P \cot \gamma_S + (\cot^2 \gamma_S - 1)^2} \qquad (36)$$

$$\frac{B_2}{B_1} = \frac{4 \cot \gamma_P \cot \gamma_S - (\cot^2 \gamma_S - 1)^2}{4 \cot \gamma_P \cot \gamma_S + (\cot^2 \gamma_S - 1)^2} \qquad (37)$$

La incidencia de una onda P puede variar de vertical (c infinita) a horizontal ($c = \alpha$) y las ecs 33 y 34 permiten calcular las amplitudes de los potenciales de las ondas reflejadas. Para la incidencia de una onda SV se tiene que $0 \leq \gamma_S \leq \mathrm{sen}^{-1}(\beta/\alpha)$.

Si se toma la energía cinética por unidad de volumen como $\frac{1}{2}\rho(\dot{u}^2 + \dot{w}^2)$, puede calcularse el flujo de energía mediante el producto de la energía cinética por la velocidad de propagación y el área del frente de onda considerado. Para incidencia de una onda P puede demostrarse que las fracciones de energía reflejada como ondas P y SV están dadas, respectivamente, por

$$\frac{A_2^2}{A_1^2} \quad \text{y} \quad \frac{B_2^2 \tan \gamma_P}{A_1^2 \tan \gamma_S}$$

y, similarmente, para incidencia de una onda SV se tiene que

$$\frac{A_2^2 \tan \gamma_S}{B_1^2 \tan \gamma_P} \quad \text{y} \quad \frac{B_2^2}{B_1^2}$$

En las figs 6 y 7 se presentan valores de $\sqrt{E/E_{inc}}$ para incidencia de ondas P y SV, respectivamente, en función del ángulo de incidencia y con diferentes relaciones α/β.

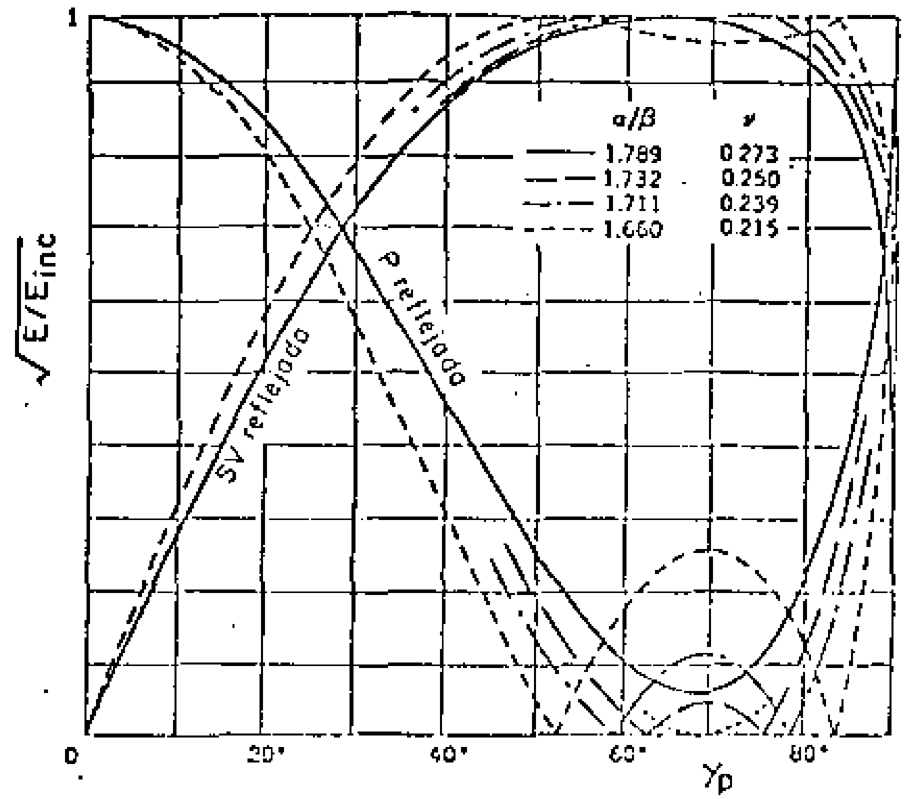

Fig 6. Raíz cuadrada de la relación de energía reflejada a energía incidente para una onda P incidente en una superficie libre

Para el segundo caso, $\beta < |c| < \alpha$, se tienen los potenciales

$$\phi = (A_1 e^{mx} + A_2 e^{-mx})\ e^{-i\ell z}\ e^{i\omega t} \tag{38}$$

$$\psi = (B_1 e^{ikx} + B_2 e^{-ikx}) e^{i\ell z}\ e^{i\omega t} \tag{39}$$

donde $\ell = \omega/c$, $m = |\omega|(1/c^2 - 1/\alpha^2)^{1/2}$ y $k = \omega(1/\beta^2 - 1/c^2)^{1/2}$. Para evitar que ϕ crezca indefinidamente al aumentar x se hace que $A_1 = 0$ por lo que *no hay* onda P incidente en este caso. Mediante un proceso análogo al del caso anterior se obtiene que

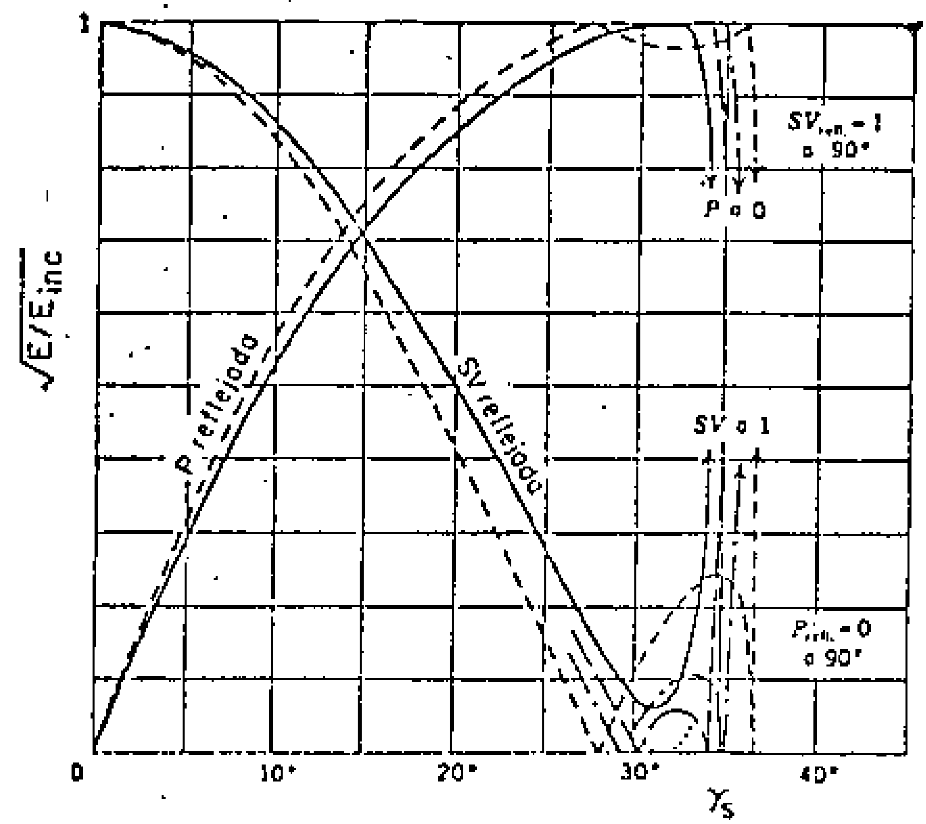

Fig 7. Raiz cuadrada de la relación de energía reflejada a energía incidente para una onda SV incidente en una superficie libre

$$\frac{A_2}{B_1} = -\frac{4\cot\gamma_S(\cot^2\gamma_S-1)}{(\cot^2\gamma_S-1)^2 - 4i(1-c^2/\alpha^2)^{1/2}\cot\gamma_S\ \mathrm{sgn}\,\omega} \tag{40}$$

$$\frac{B_2}{B_1} = -\frac{(\cot^2\gamma_S-1)^2+4i(1-c^2/\alpha^2)^{1/2}\cot\gamma_S\ \mathrm{sgn}\,\omega}{(\cot^2\gamma_S-1)^2-4i(1-c^2/\alpha^2)^{1/2}\cot\gamma_S\ \mathrm{sgn}\,\omega} \tag{41}$$

donde sgn ω =(-1 si $\omega<0$ o 1 si $\omega>1$). En este caso, la incidencia de ondas SV con ángulos de incidencia γ_S mayores que $\mathrm{sen}^{-1}(\beta/\alpha)$ genera ondas P no homogéneas que se atenuan con la profundidad.

4. ONDAS DE RAYLEIGH

Para el tercer caso, $|c| < \beta < \alpha$, se tiene que

$$\phi = A_2 \, e^{-mx} \, e^{-i\ell z} \, e^{i\omega t} \tag{42}$$

$$\psi = B_2 \, e^{-kx} \, e^{-i\ell z} \, e^{i\omega t} \tag{43}$$

donde $m = |\omega|(1/c^2 - 1/\alpha^2)^{1/2}$ y $k = |\omega|(1/c^2 - 1/\beta^2)^{1/2}$. Se han eliminado A_1 y B_1 pues no representan ondas incidentes con potenciales finitos. Las ecuaciones de esfuerzos nulos en $x = 0$ conducen a

$$\frac{A_2}{B_2} = \frac{2i(1-c^2/\beta^2)^{1/2} \operatorname{sgn}\omega}{2 - c^2/\beta^2} \tag{44}$$

$$\frac{A_2}{B_2} = \frac{2 - c^2/\beta^2}{2i(1-c^2/\alpha^2)^{1/2} \operatorname{sgn}\omega} \tag{45}$$

como las ecs 44 y 45 deben ser iguales se obtiene que la velocidad de fase, c, debe satisfacer la siguiente ecuación:

$$\left(2 - \frac{c^2}{\beta^2}\right)^2 - 4\left(1 - \frac{c^2}{\alpha^2}\right)^{1/2} \left(1 - \frac{c^2}{\beta^2}\right)^{1/2} = 0 \tag{46}$$

La raiz real de esta ecuación, c_R, encontrada por vez primera por Rayleigh, da la velocidad de las llamadas ondas de Rayleigh. En la fig 6 se presentan valores de c_R para distintos valores del módulo de Poisson ν. Las ondas de Rayleigh son ondas *superficiales* y debido a ello sufren

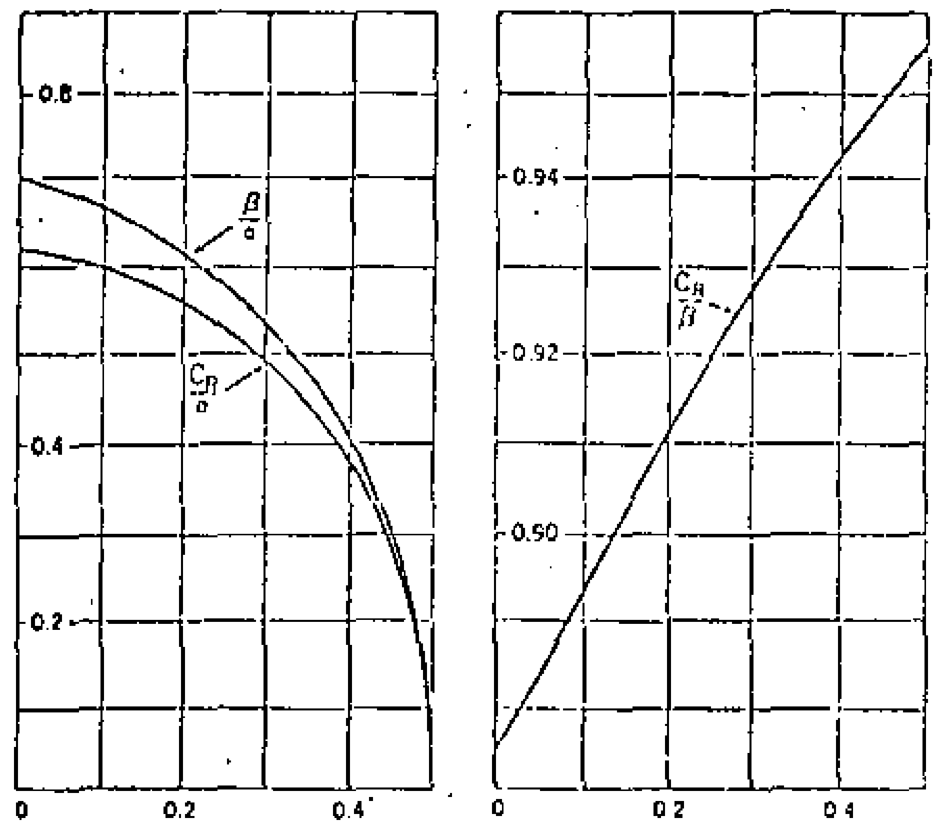

Fig 8. Relaciones β/α, c_R/α y c_R/β como funciones del módulo de Poisson ν

menor atenuación geométrica. Puede demostrarse que el movimiento generado por ondas de Rayleigh hace que las partículas describan trayectorias elípticas con ciclos retrógrados, a diferencia de los ciclos progresivos que se presentan en las ondas superficiales en líquidos. La fig 9 muestra un dibujo esquemático de las ondas superficiales de Rayleigh.

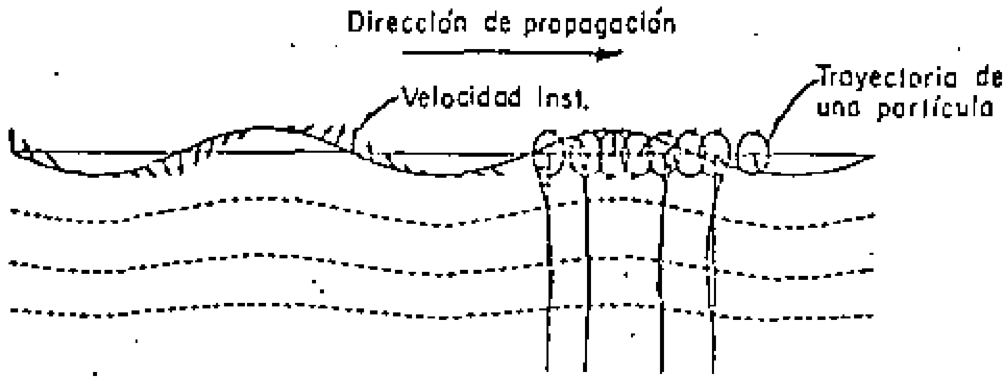

Fig 9. Ondas de Rayleigh

5. ONDAS DE LOVE

Puede demostrarse que la propagación de ondas superficiales (que se atenúen con la profundidad) del tipo SH es imposible en un semiespacio homogéneo. No obstante, las ondas SH superficiales se observan en la superficie de la tierra. Love demostró que una teoría suficiente para explicar las ondas SH superficiales puede desarrollarse si se tiene un estrato homogéneo de espesor uniforme H con propiedades μ_1 y β_1 sobre un semiespacio de propiedades μ_2 y β_2 como se muestra en la fig 10. Supóngase que los desplazamientos son independientes de la coordenada y, y además que la variación con el tiempo está dada por $e^{i\omega t}$. El plano $x = -H$ representa la superficie libre. Las ecuaciones de movimiento (ecs 1) se reducen a

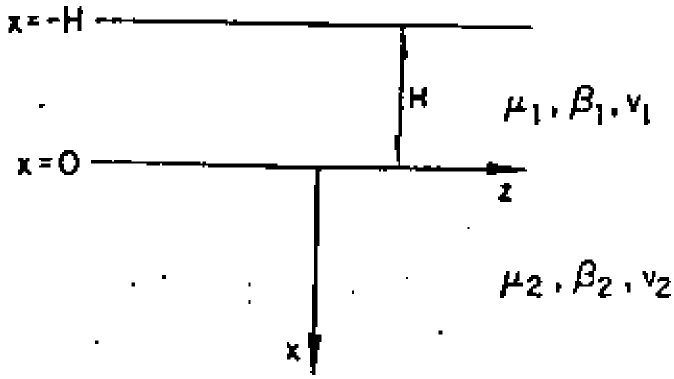

Fig 10. Notación para un estrato sobre un semiespacio elástico

para el estrato y

$$\frac{\partial^2 v_2}{\partial x^2} + \frac{\partial^2 v_2}{\partial z^2} + k^2_{\beta_2} v_2 = 0 \tag{48}$$

donde $k_{\beta_i} = \omega/\beta_i$, $i = 1,2$ para el semiespacio.

Haciendo uso de soluciones del tipo de las ecs 23 y 24 se puede escribir que

$$v_1 = (A\, e^{-k\gamma_1 x} + B\, e^{k\gamma_1 x})\, e^{ik(z-ct)} \tag{49}$$

$$v_2 = C\, e^{-k\gamma_2 x}\, e^{ik(z-ct)} \tag{50}$$

donde $\gamma_1 = (1-c^2/\beta_1^2)^{1/2}$ y $\gamma_2 = (1-c^2/\beta_2^2)^{1/2}$. Se observa que si $c < \beta_2$, $v_2 \to 0$ cuando $x \to \infty$.

Las condiciones de frontera son que $v_1 = v_2$ y $(\tau_{xy})_1 = (\tau_{xy})_2$ en x=0 y que $\tau_{xy} = 0$ en $x = -H$. Estas condiciones conducen a un sistema de ecuaciones homogéneo en A, B y C. Para que se tenga solución diferente de cero el determinante del sistema debe anularse. Así, se tiene que

$$\tan k\gamma_1 H = \frac{\mu_2 \gamma_2}{\mu_1 \gamma_1} = \frac{\mu_2 (1 - c^2/\beta_2^2)^{1/2}}{\mu_1 (c^2/\beta_1^2 - 1)^{1/2}} \tag{51}$$

es la ecuación para obtener la velocidad de las ondas de Love.

Si $\beta_1 < \beta_2$ la ec 51 da valores reales de c, en el intervalo $\beta_1 < c < \beta_2$, que dependen de k y H. Pueden obtenerse ondas de Love de forma general superponiendo ondas de Love del tipo de la ec 49 con diferentes k.

La dependencia de la velocidad de propagación de la frecuencia ocasiona el fenómeno de *dispersión* y, en general, este es el caso en medios estratificados.

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Para ilustrar el problema de propagación estudiemos primeramente el caso unidimensional. La ecuación que gobierna este movimiento es la llamada 'ecuación de onda unidimensional' y se escribe como

$$\frac{\partial^2 \phi}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 \phi}{\partial t^2} \qquad (1)$$

donde ϕ es por ejemplo la elevación de una onda en el agua y c es un coeficiente conocido como celeridad o velocidad de propagación.

Para obtener la solución general de la ec 1 es conveniente hacer el siguiente cambio de variables,

$$u = x - ct \quad \text{y} \quad v = x + ct. \qquad (2)$$

Este método conocido como de D'Alembert conduce a las siguientes transformaciones

$$\frac{\partial \phi}{\partial x} = \frac{\partial \phi}{\partial u} + \frac{\partial \phi}{\partial v} \quad , \quad \frac{\partial^2 \phi}{\partial x^2} = \frac{\partial^2 \phi}{\partial u^2} + 2\frac{\partial^2 \phi}{\partial u \partial v} + \frac{\partial^2 \phi}{\partial v^2}$$

$$\frac{\partial \phi}{\partial t} = -c\frac{\partial \phi}{\partial u} + c\frac{\partial \phi}{\partial v} \quad \text{y} \quad \frac{\partial^2 \phi}{\partial t^2} = c^2\left(\frac{\partial^2 \phi}{\partial u^2} - 2\frac{\partial^2 \phi}{\partial u \partial v} + \frac{\partial^2 \phi}{\partial v^2}\right) \qquad (3)$$

mismas que al substituirse en la ec (1) dan la ecuación

$$\frac{\partial^2 \phi}{\partial u \, \partial v} = 0 \qquad (4)$$

cuya solución general se escribe como

$$\phi = f(u) + g(v) \qquad (5)$$

o bien de las ecs 2 se tiene

$$\phi = f(x-ct) + g(x+ct) \qquad (6)$$

donde f y g son funciones arbitrarias de sus argumentos.

De la interpretación física de la ec 6 se tiene que para un tiempo dado la función $f(x-ct)$ representa una perturbación moviendose en la dirección positiva del x con una velocidad de propagación c. Similarmente la función $g(x+ct)$ representa una perturbación que se mueve con una celeridad c en dirección negativa del eje x.

La anterior interpretación física se ilustra en la siguiente figura

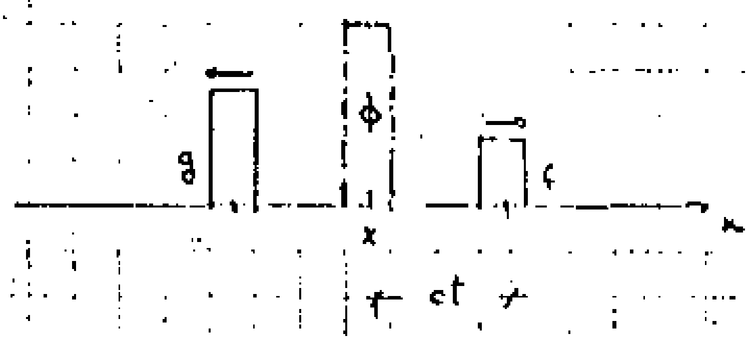


Fig 1

Para definir algunos parámetros de interés en el problema de propagación consideremos, sin pérdida de generalidad una onda armónica que se mueve con una velocidad de propagación constante y sin cambio de forma, en la dirección positiva del eje x. Así

$$\phi = a \operatorname{sen} k(x-ct) \qquad (7)$$

Definiciones:

- A el valor máximo de la perturbación, en este caso 'a', se le denomina amplitud.
- A la forma de la perturbación cuando $t=0$ se le denomina 'perfil de onda'.
- A la distancia para la cual se repite el perfil de onda, 'λ', se le denomina longitud de onda. En este caso

$$\lambda = \frac{2\pi}{k} \qquad (8)$$

o sea la ec. 7 se puede escribir como:

$$\phi = a \operatorname{sen} \frac{2\pi}{\lambda}(x-ct) \qquad (9)$$

- El tiempo que toma una onda en pasar por un punto se llama periodo y se escribe como

$$T = \frac{\lambda}{c} \qquad (10)$$

- La frecuencia n, es el número de ondas que pasan por un punto fijo en una unidad de tiempo, esto es

$$n = \frac{1}{T} \qquad (11)$$

El número de ondas en una distancia 2π, k, se le llama número de onda y resulta ser

$$k = \frac{2\pi}{\lambda} \qquad (12)$$

Las definiciones anteriores son válidas para cualquier perturbación cuyo perfil de onda se repite regularmente.

Ondas Planas
La generalización de la solución dada por la ec 6 a un problema tridimensional permite la definición de ondas planas, esto es, ondas para las cuales la perturbación es constante en todos los puntos contenidos en cada plano perpendicular a la dirección de propagación - frente de onda. Así si la dirección de propagación se define por sus cosenos directores l, m, n, la ec 6 en un espacio tridimensional se escribe como

$$\phi = f(lx + my + nz - ct) + g(lx + my + nz + ct) \qquad (13)$$

Ondas esféricas.
Si estamos interesados en la solución de la ecuación de onda para problemas que poseen simetría esférica. Se tiene que la

ecuación que gobierna al problema es

$$\frac{\partial^2\phi}{\partial r^2} + \frac{2}{r}\frac{\partial\phi}{\partial r} = \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} \qquad (14)$$

o bien

$$\frac{\partial^2(r\phi)}{\partial r^2} = \frac{1}{c^2}\frac{\partial^2(r\phi)}{\partial t^2} \qquad (15)$$

De los conceptos establecidos anteriormente se demuestra que la ec 15 tiene la solución general

$$\phi = \frac{1}{r}f(r-ct) + \frac{1}{r}g(r+ct) \qquad (16)$$

donde f y g son nuevamente funciones arbitrarias de sus argumentos. Nótese que la amplitud de las ondas esféricas decrece con $\frac{1}{r}$.

Se puede demostrar que, en el caso de ondas cilíndricas con simetría axial, la atenuación de las ondas es con $\frac{1}{\sqrt{r}}$

<u>Propagación de Ondas en un Medio Infinito</u>

Las ecuaciones que gobiernan el movimiento de un medio elástico lineal, homogéneo e isótropo, conocidas como ecuaciones de Navier, se pueden escribir como

$$\begin{aligned}
\rho\frac{\partial^2 u}{\partial t^2} &= (\lambda+G)\frac{\partial e}{\partial x} + G\nabla^2 u + \rho X\\
\rho\frac{\partial^2 v}{\partial t^2} &= (\lambda+G)\frac{\partial e}{\partial y} + G\nabla^2 v + \rho Y\\
\rho\frac{\partial^2 w}{\partial t^2} &= (\lambda+G)\frac{\partial e}{\partial z} + G\nabla^2 w + \rho Z
\end{aligned} \qquad (17)$$

donde u, v y w son las componentes del vector desplazamiento, λ y G los coeficientes elásticos del material, ρ la densidad de masa, X Y y Z las componentes del vector de fuerzas de cuerpo por unidad de masa y e la dilatación definida como

$$e = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \qquad (18)$$

El sistema de ecuaciones de movimiento se puede desacoplar si el vector desplazamiento se define como

$$\underset{\sim}{u} = \text{grad}\,\phi + \text{rot}\,\underset{\sim}{\psi} \qquad (19)$$

donde $\underset{\sim}{u}$ es el vector desplazamiento con componentes u, v y w, ϕ y $\underset{\sim}{\psi}$ son potenciales, uno escalar y otro vectorial ambos conocidos como potenciales de Helmholtz.

De las ecs 17 y 19 se encuentra que las ecs 17 se satisfacen simultáneamente si los potenciales ϕ y $\underset{\sim}{\psi}$ son soluciones de las ecuaciones de onda

$$\nabla^2\phi = \frac{1}{C_p^2}\frac{\partial^2\phi}{\partial t^2} \qquad (20)$$

y

$$\nabla^2\underset{\sim}{\psi} = \frac{1}{C_s^2}\frac{\partial^2\underset{\sim}{\psi}}{\partial t^2} \qquad (21)$$

donde

$$C_p = \sqrt{\frac{\lambda + 2G}{\rho}} \quad \text{y} \quad C_s = \sqrt{\frac{G}{\rho}} \qquad (22)$$

De simples transformaciones se puede demostrar, que las ecs. 20 y 21 se pueden escribir respectivamente como

$$\nabla^2 e = \frac{1}{C_p^2}\frac{\partial^2 e}{\partial t^2} \qquad (23)$$

y

$$\nabla^2 \underset{\sim}{w} = \frac{1}{C_s^2}\frac{\partial^2 \underset{\sim}{w}}{\partial t^2} \qquad (24)$$

donde $\underset{\sim}{w}$ es el vector rotación.

Usando los conceptos establecidos en el capítulo anterior se observa que las ecs. 23 y 24 son ecuaciones de onda asociadas a ondas de tipo dilatacional o P y distorcionales o S, respectivamente. De aquí se concluye que en un medio elástico de extensión infinita solo puede existir dos tipos de ondas, las ondas dilatacionales que se propagan con una velocidad C_p y las distorcionales que se propagan con una velocidad C_s.

Con el objeto de discutir más ampliamente el problema de propagación de ondas en un medio infinito sin fuerzas de cuerpo, investiguemos algunas soluciones particulares.

Sea la solución asociada a una onda plana viajando en la dirección positiva del eje x

$$u = a \operatorname{sen} \frac{2\pi}{\lambda}(x - ct)$$

$$v = 0 \qquad (26)$$

$$w = 0$$

Esta solución satisface las ecs 17 si y solo si $c = c_p$, y representa un tren de ondas planas dilatacionales propagándose con una velocidad c_p. Nótese que para este tipo de ondas el movimiento de la partícula coincide con la dirección de propagación y que además este tipo de movimientos solo produce cambios volumétricos.

Consideremos ahora las soluciones

$$u = w = 0$$

$$v = a \operatorname{sen} \frac{2\pi}{\lambda}(x - ct) \qquad (26)$$

o'

$$u = v = 0$$

$$w = a \operatorname{sen} \frac{2\pi}{\lambda}(x - ct) \qquad (27)$$

De la substitución de cualquiera de las soluciones anteriores en las ecs 17 se puede concluir que éstas representan una solución si y solo si $c = c_s$. Además se observa que en el caso de ondas S el movimiento de la partíesta es normal a la dirección de propagación.

Las ondas distorcionales son generalmente polarizadas *, estos, ondas SH en las que el movimiento de la partícula es horizontal y ondas SV en las que el movimiento de la partícula ocurre en el plano vertical. En los casos en que el medio tiene isotropía transversal, como es el caso de muchas formaciones geológicas, en una perturbación propagandose en dirección horizontal la onda SH se propaga mas rápido que la onda SV.

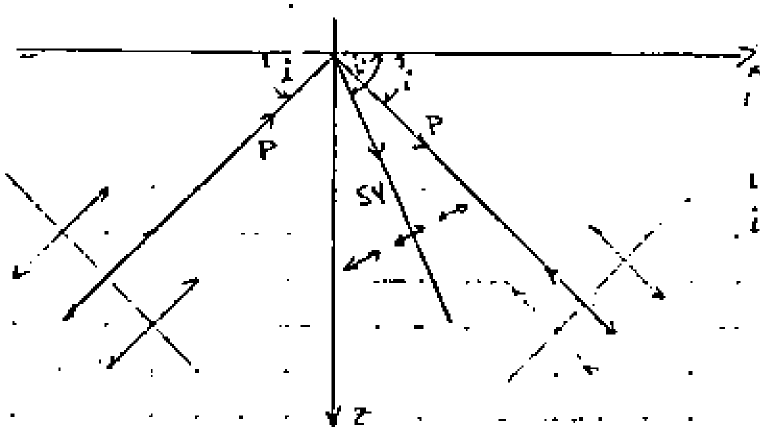
De la existencia de distintas velocidades de propagación asociadas a distintos tipos de onda se puede concluir una propiedad importante de las ondas sísmicas, esto es, que una onda plana arbitraria compuesta de movimientos P y S no se puede propagar. En cada instante las distintas ondas viajan diferentes distancias y es de esta propiedad que la velocidad de propagación de las ondas P y S a cualquier profundidad es facilmente calculable.

Propagación de Ondas en un Semiespacio Elástico

Consideremos ahora el caso de propagación de ondas en un medio que consiste de un semiespacio elástico. Por conveniencia en la formulación escojamos el eje z dirigido verticalmente hacia abajo. Así, la superficie del semiespacio está definida por el plano $z=0$. Esta superficie está libre de esfuerzos por lo que las condiciones de frontera son

$$\sigma_z = \tau_{zx} = \tau_{zy} = 0 \qquad (28)$$

Si consideramos una onda plana dilatacional incidiendo con un ángulo i a la superficie del terreno se puede demostrar que ésta produce 2 ondas reflejadas una P y una SV



i = angulo de incidencia
i' = angulo de reflexión

Así para las ondas dilatacionales se tiene

$$e = \underbrace{f\left(t - \frac{x\cos i - z \operatorname{sen} i}{C_p}\right)}_{\text{incidente}} + \underbrace{A f\left(t - \frac{x\cos i + z \operatorname{sen} i}{C_p}\right)}_{\text{reflejada}} \qquad (29)$$

y para las ondas distorsionales

$$\omega_z = \underbrace{B f\left(t - \frac{x\cos i' + z \operatorname{sen} i'}{C_s}\right)}_{\text{reflejada}} \qquad (30)$$

las condiciones de frontera dadas por las ecs 28 nos

permiten determinar los coeficientes desconocidos A y B'',
definidos como

$$A = \frac{4\sqrt{\xi^2-1}\sqrt{k^2\xi^2-1} - (\xi^2-2)^2}{4\sqrt{\xi^2-1}\sqrt{k^2\xi^2-1} + (\xi^2-2)^2}$$

$$B =$$

(distortional) propagates at the same velocity ($v_s = \sqrt{G/\rho}$) in both the rod and the infinite medium.

3.3 *Waves in an Elastic Half-Space*

In Sec. 3.2 it was found that two types of waves were possible in an infinite elastic medium—waves of dilatation and waves of distortion. In an elastic half-space, however, it is possible to find a third solution for the equations of motion which corresponds to a wave whose motion is confined to a zone near the boundary of the half-space. This wave was first studied by Lord Rayleigh (1885) and later was described in detail by Lamb (1904). The elastic wave described by these investigators is known as the *Rayleigh wave* (*R*-wave) and is confined to the neighborhood of the surface of a half-space. The influence of the Rayleigh wave decreases rapidly with depth.

Rayleigh-Wave Velocity

A wave with the characteristics noted above can be obtained by starting with the equations of motion (Eqs. 3-42, 3-43, and 3-44) and imposing the appropriate boundary conditions for a free surface. We define the surface of the half-space as the *x-y* plane with *z* assumed to be positive toward the interior of the half-space, as shown in Fig. 3-12. For a *plane wave* traveling in the *x*-direction, particle displacements will be independent of the *y*-direction. Displacements in the *x*- and *z*-directions, denoted by *u* and *w* respectively, can be written in terms of two potential functions Φ and Ψ:

$$u = \frac{\partial\Phi}{\partial x} + \frac{\partial\Psi}{\partial z} \qquad \text{and} \qquad w = \frac{\partial\Phi}{\partial z} - \frac{\partial\Psi}{\partial x}$$

The dilatation $\bar{\epsilon}$ of the wave defined by *u* and *w* is

$$\bar{\epsilon} = \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = \frac{\partial}{\partial x}\left(\frac{\partial\Phi}{\partial x} + \frac{\partial\Psi}{\partial z}\right) + \frac{\partial}{\partial z}\left(\frac{\partial\Phi}{\partial z} - \frac{\partial\Psi}{\partial x}\right) = \nabla^2\Phi$$

and the rotation $2\bar{\omega}_y$ in the *x-z* plane is

$$2\bar{\omega}_y = \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} = \frac{\partial}{\partial z}\left(\frac{\partial\Phi}{\partial x} + \frac{\partial\Psi}{\partial z}\right) - \frac{\partial}{\partial x}\left(\frac{\partial\Phi}{\partial z} - \frac{\partial\Psi}{\partial x}\right) = \nabla^2\Psi$$

Now it can be seen that the potential functions Φ and Ψ have been chosen

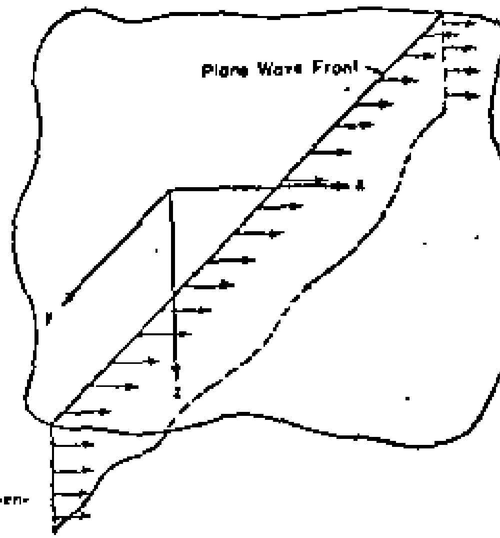


Figure 3-12 Coordinate convention for elastic half-space.

such that Φ is associated with dilatation of the medium and Ψ associated with rotation of the medium.

Substituting *u* and *w* into Eqs. (3-42) and (3-44) yields

$$\rho\frac{\partial}{\partial x}\left(\frac{\partial^2\Phi}{\partial t^2}\right) + \rho\frac{\partial}{\partial z}\left(\frac{\partial^2\Psi}{\partial t^2}\right) = (\lambda + 2G)\frac{\partial}{\partial x}(\nabla^2\Phi) + G\frac{\partial}{\partial z}(\nabla^2\Psi) \tag{3-49}$$

and

$$\rho\frac{\partial}{\partial z}\left(\frac{\partial^2\Phi}{\partial t^2}\right) - \rho\frac{\partial}{\partial x}\left(\frac{\partial^2\Psi}{\partial t^2}\right) = (\lambda + 2G)\frac{\partial}{\partial z}(\nabla^2\Phi) - G\frac{\partial}{\partial x}(\nabla^2\Psi) \tag{3-50}$$

Equations (3-49) and (3-50) are satisfied if

$$\frac{\partial^2\Phi}{\partial t^2} = \frac{\lambda + 2G}{\rho}\nabla^2\Phi = v_p^2\nabla^2\Phi \tag{3-51}$$

and

$$\frac{\partial^2\Psi}{\partial t^2} = \left(\frac{G}{\rho}\right)\nabla^2\Psi = v_s^2\nabla^2\Psi \tag{3-52}$$

Now, by assuming a solution for a sinusoidal wave traveling in the positive *x*-direction, expressions for Φ and Ψ can be written

$$\Phi = F(z)\exp\left[i(\omega t - Nx)\right] \tag{3-53}$$

and

$$\Psi = G(z) \exp [i(\omega t - Nx)] \tag{3-54}$$

The functions $F(z)$ and $G(z)$ describe the variation in amplitude of the wave as a function of depth, and N is the wave number defined by

$$N = \frac{2\pi}{L}$$

where L is the wave length.

Now, substituting the expressions for Φ and Ψ from Eqs. (3-53) and (3-54) into Eqs. (3-51) and (3-52) yields

$$-\frac{\omega^2}{v_P^2} F(z) = -N^2F(z) + F''(z) \tag{3-55}$$

and

$$-\frac{\omega^2}{v_S^2} G(z) = -N^2G(z) + G''(z) \tag{3-56}$$

By rearranging Eqs. (3-55) and (3-56), we get

$$F''(z) - \left(N^2 - \frac{\omega^2}{v_P^2}\right)F(z) = 0 \tag{3-57}$$

and

$$G''(z) - \left(N^2 - \frac{\omega^2}{v_S^2}\right)G(z) = 0 \tag{3-58}$$

where $F''(z)$ and $G''(z)$ are derivatives with respect to z. Now, letting

$$q^2 = \left(N^2 - \frac{\omega^2}{v_P^2}\right) \tag{3-59}$$

and

$$s^2 = \left(N^2 - \frac{\omega^2}{v_S^2}\right) \tag{3-60}$$

Eqs. (3-57) and (3-58) can be rewritten as

$$F''(z) - q^2F(z) = 0 \tag{3-61}$$

and

$$G''(z) - s^2G(z) = 0 \tag{3-62}$$

The solutions of Eqs. (3-61) and (3-62) can be expressed in the form

$$F(z) = A_1 \exp(-qz) + B_1 \exp(qz) \tag{3-63}$$

$$G(z) = A_2 \exp(-sz) + B_2 \exp(sz) \tag{3-64}$$

A solution that allows the amplitude of the wave to become infinite with depth cannot be tolerated; therefore,

$$B_1 = B_2 = 0$$

and Eqs. (3-53) and (3-54) become

$$\Phi = A_1 \exp[-qz + i(\omega t - Nx)] \tag{3-65}$$

and

$$\Psi = A_2 \exp[-sz + i(\omega t - Nx)] \tag{3-66}$$

Now, the boundary conditions specifying no stress at the surface of a half-space imply that $\sigma_z = 0$ and $\tau_{zx} = 0$ at the surface $z = 0$. Therefore, at the surface,

$$\sigma_z = \lambda\bar{\epsilon} + 2G\epsilon_z = \lambda\bar{\epsilon} + 2G\frac{\partial w}{\partial z} = 0$$

and

$$\tau_{xz} = G\gamma_{xz} = G\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right) = 0$$

Using the definitions of u and w and the solutions for Φ and Ψ from Eqs. (3-65) and (3-66), the above equations for boundary conditions can be written

$$\sigma_z|_{z=0} = A_1[(\lambda + 2G)q^2 - \lambda N^2] - 2iA_2GNs = 0 \tag{3-67}$$

and

$$\tau_{xz}|_{z=0} = 2iA_1Nq + A_2(s^2 + N^2) = 0 \tag{3-68}$$

Upon rearranging, Eqs. (3-67) and (3-68) become

$$\frac{A_1}{A_2}\frac{(\lambda + 2G)q^2 - \lambda N^2}{2iGNs} - 1 = 0 \tag{3-69}$$

and

$$\frac{A_1}{A_2}\frac{2qiN}{(s^2 + N^2)} + 1 = 0 \tag{3-70}$$

Now we add these two equations to get

$$\frac{(\lambda + 2G)q^2 - \lambda N^2}{2iGNs} = -\frac{2qiN}{s^2 + N^2} \tag{3-71}$$

and cross-multiply in Eq. (3-71) to obtain

$$4qGsN^2 = (s^2 + N^2)[(\lambda + 2G)q^2 - \lambda N^2] \tag{3-72}$$

Squaring both sides of Eq. (3-72) and introducing q from Eq. (3-59) and s from Eq. (3-60), we get

$$16G^2N^4\left(N^2 - \frac{\omega^2}{v_P^2}\right)\left(N^2 - \frac{\omega^2}{v_S^2}\right) = \left[(\lambda + 2G)\left(N^2 - \frac{\omega^2}{v_P^2}\right) - \lambda N^2\right]^2\left[N^2 + \left(N^2 - \frac{\omega^2}{v_S^2}\right)\right]^2 \tag{3-73}$$

Now, dividing through by G^2N^8, we obtain

$$16\left(1 - \frac{\omega^2}{v_P^2N^2}\right)\left(1 - \frac{\omega^2}{v_S^2N^2}\right) = \left[2 - \left(\frac{\lambda + 2G}{G}\right)\left(\frac{\omega^2}{v_P^2N^2}\right)\right]^2\left(2 - \frac{\omega^2}{v_S^2N^2}\right)^2 \tag{3-74}$$

Then, using the following relationships derived in the footnote* gives

$$\frac{\omega^2}{v_P^2N^2} = \frac{v_R^2}{v_P^2} = \alpha^2K^2 \tag{3-75}$$

$$\frac{\omega^2}{v_S^2N^2} = \frac{v_R^2}{v_S^2} = K^2 \tag{3-76}$$

$$\frac{\lambda + 2G}{G} = \frac{1}{\alpha^2} = \frac{2 - 2\nu}{1 - 2\nu} \tag{3-77}$$

* By definition,

$$N = \frac{2\pi}{L}$$

or

$$L = \frac{2\pi}{N}$$

(Let L_R and v_R be the wave length and velocity, respectively, of the surface wave.)

Eq. (3-74) can be written

$$16(1 - \alpha^2K^2)(1 - K^2) = \left(2 - \frac{1}{\alpha^2}\alpha^2K^2\right)^2(2 - K^2)^2 \tag{3-78}$$

After expansion and rearrangement, Eq. (3-78) becomes

$$K^6 - 8K^4 + (24 - 16\alpha^2)K^2 + 16(\alpha^2 - 1) = 0 \tag{3-79}$$

Equation (3-79) can be considered a cubic equation in K^2 and real valued solutions can be found for given values of ν. The quantity K represents a ratio between the velocity of the surface wave and the velocity of the shear wave.

Also,

$$L_R = \frac{v_R}{f} = \frac{2\pi v_R}{\omega}$$

and, from above,

$$L_R = \frac{2\pi}{N} = \frac{v_R 2\pi}{\omega}$$

therefore,

$$N = \frac{\omega}{v_R}$$

and

$$N^2 = \frac{\omega^2}{v_R^2}$$

Let K and α be defined such that

$$\frac{v_R^2}{v_S^2} = K^2 \quad \text{and} \quad \frac{v_R^2}{v_P^2} = \alpha^2K^2$$

Then

$$\frac{\omega^2}{v_P^2N^2} = \frac{v_R^2}{v_P^2} = \alpha^2K^2$$

and

$$\frac{\omega^2}{v_S^2N^2} = \frac{v_R^2}{v_S^2} = K^2$$

Substitution of v_s and v_p from Eqs. (3-46) and (3-48) gives

$$\frac{1}{\alpha^2} = \frac{v_P^2}{v_S^2} = \frac{\frac{\lambda + 2G}{\rho}}{\frac{G}{\rho}} = \frac{\lambda + 2G}{G}$$

and using

$$\nu = \frac{\lambda}{2(\lambda + G)}$$

we get

$$\frac{\lambda + 2G}{G} = \frac{2 - 2\nu}{1 - 2\nu} = \frac{1}{\alpha^2}$$

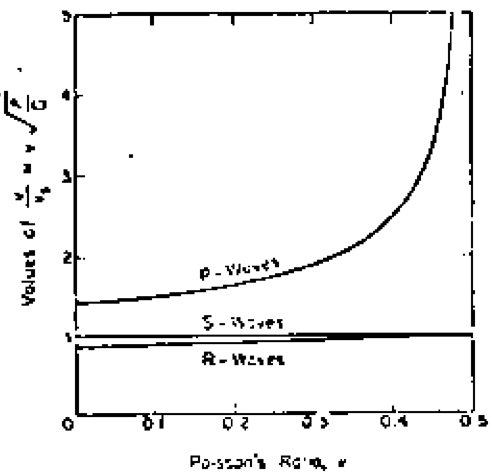


Figure 3-13 Relation between Poisson's ratio, ν, and velocities of propagation of compression (P), shear (S), and Rayleigh (R) waves in a semiinfinite elastic medium (from Richart, 1962).

From this solution it is clear that K^2 is independent of the frequency of the wave; consequently, the velocity of the surface wave is independent of frequency and is nondispersive.

Ratios of v_R/v_S and v_P/v_S can be obtained from Eq. (3-79) for values of Poisson's ratio ν from 0 to 0.5. Curves of these ratios as a function of ν are shown in Fig. 3-13.

Rayleigh-Wave Displacement

So far, a relationship for the ratio of the Rayleigh-wave velocity to the shear-wave velocity has been obtained, but additional information about the Rayleigh wave can be determined by obtaining the expressions for u and w in terms of known quantities. Upon substituting the expressions for Φ and Ψ from Eqs. (3-65) and (3-66) into the expressions for u and w, we get

$$\begin{aligned} u &= \frac{\partial \Phi}{\partial x} + \frac{\partial \Psi}{\partial z} \\ &= -A_1 iN \exp\left[-qz + i(\omega t - Nx)\right] - A_2 s \exp\left[-sz + i(\omega t - Nx)\right] \end{aligned} \tag{3-80}$$

and

$$\begin{aligned} w &= \frac{\partial \Phi}{\partial z} - \frac{\partial \Psi}{\partial x} \\ &= -A_1 iN \exp\left[-qz + i(\omega t - Nx)\right] + A_2 iN \exp\left[-sz + i(\omega t - Nx)\right] \end{aligned} \tag{3-81}$$

From Eq. (3-70) we can get

$$A_2 = -\frac{2qiNA_1}{s^2 + N^2}$$

and substitution of A_2 into Eqs. (3-80) and (3-81) gives

$$u = A_1\left[-iN \exp(-qz) + \frac{2iqsN}{s^2 + N^2}\exp(-sz)\right]\exp i(\omega t - Nx) \tag{3-82}$$

and

$$w = A_1\left[\frac{2qN^2}{s^2 + N^2}\exp(-sz) - q\exp(-qz)\right]\exp i(\omega t - Nx) \tag{3-83}$$

Equations (3-82) and (3-83) can be rewritten

$$u = A_1 N i\left\{-\exp\left[-\frac{q}{N}(zN)\right] + \frac{2\dfrac{q}{N}\dfrac{s}{N}}{\dfrac{s^2}{N^2} + 1}\exp\left[-\frac{s}{N}(zN)\right]\right\} \times \exp i(\omega t - Nx) \tag{3-84}$$

and

$$w = A_1 N\left\{\frac{2\dfrac{q}{N}}{\dfrac{s^2}{N^2} + 1}\exp\left[-\frac{s}{N}(zN)\right] - \frac{q}{N}\exp\left[-\frac{q}{N}(zN)\right]\right\} \times \exp i(\omega t - Nx) \tag{3-85}$$*

Now, from Eqs. (3-84) and (3-85), the variation of u and w with depth can be expressed as

$$U(z) = -\exp\left[-\frac{q}{N}(zN)\right] + \frac{2\dfrac{q}{N}\dfrac{s}{N}}{\dfrac{s^2}{N^2} + 1}\exp\left[-\frac{s}{N}(zN)\right] \tag{3-86}$$

and

$$W(z) = \frac{2\dfrac{q}{N}}{\dfrac{s^2}{N^2} + 1}\exp\left[-\frac{s}{N}(zN)\right] - \frac{q}{N}\exp\left[-\frac{q}{N}(zN)\right] \tag{3-87}$$

* The significance of the presence of i in the expression for u (Eq. 3-84) and its absence in the expression for w (Eq. 3-85) is that the x-component of displacement is 90° out of phase with the z-component of displacement.

The functions $U(z)$ and $W(z)$ represent the spatial variations of the displacements u and w. Equations (3-59) and (3-60) can be rewritten

$$\frac{q^2}{N^2} = 1 - \frac{\omega^2}{N^2 v_P^2} \tag{3-88}$$

and

$$\frac{s^2}{N^2} = 1 - \frac{\omega^2}{N^2 v_S^2} \tag{3-89}$$

and then, using Eqs. (3-75) and (3-76), Eqs. (3-88) and (3-89) can be reduced to

$$\frac{q^2}{N^2} = 1 - \alpha^2 K^2 \tag{3-90}$$

and

$$\frac{s^2}{N^2} = 1 - K^2 \tag{3-91}$$

Now, $U(z)$ and $W(z)$ can be evaluated in terms of the wave number N for any given value of Poisson's ratio. For example, if $\nu = \frac{1}{4}$, $U(z)$ and $W(z)$ are given by

$$U(z) = -\exp\left[-0.8475\,(zN)\right] + 0.5773\exp\left[-0.3933\,(zN)\right] \tag{3-92}$$

and

$$W(z) = 0.8475\exp\left[-0.8475(zN)\right] - 1.4679\exp\left[-0.3933(zN)\right] \tag{3-93}$$

Figure 3-14 shows curves for $U(z)$ and $W(z)$ vs. distance from the surface in wave lengths of the Rayleigh wave (L_R) for Poisson's ratios of 0.25, 0.33, 0.40, and 0.50.

Wave System at Surface of Half-Space

In preceding paragraphs expressions have been determined for the wave velocities of the three principal waves which occur in an elastic halfspace. Knowing these velocities, we can easily predict the order in which waves will arrive at a given point due to a disturbance at another point. In addition to predicting the order of arrival of the waves along the surface, Lamb (1904) described in detail the surface motion that occurs at large distances from a point source at the surface of an ideal medium.

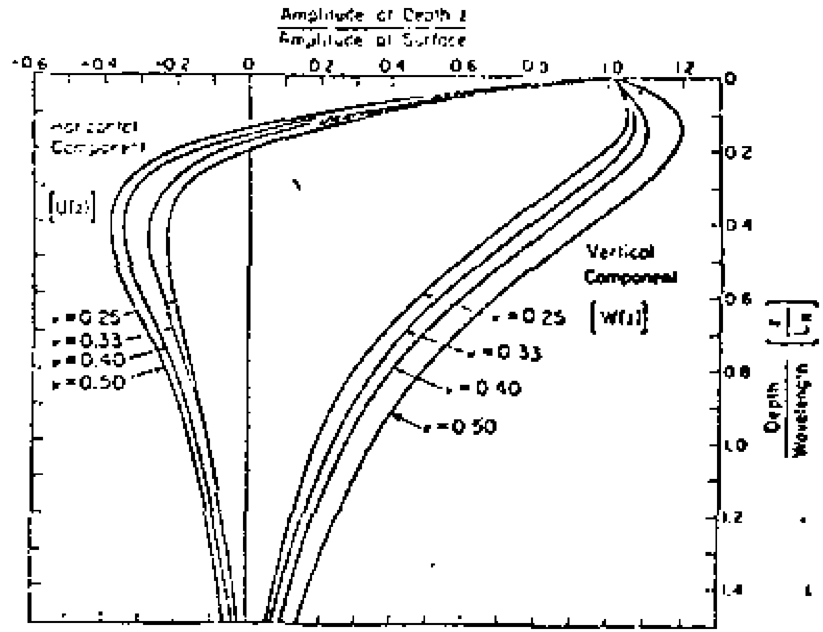


Figure 3-14. Amplitude ratio vs. dimensionless depth for Rayleigh wave.

Under the conditions considered by Lamb, a disturbance spreads out from the point source in the form of a symmetrical annular-wave system. The initial form of this wave system will depend on the input impulse; but if the input is of short duration, the characteristic wave system shown in Fig. 3-15 will develop. This wave system has three salient features corresponding to the arrivals of the *P*-wave, *S*-wave, and *R*-wave. The horizontal and vertical components of particle motion are shown separately in Fig. 3-15. A particle at the surface first experiences a displacement in the form of an oscillation at the arrival of the *P*-wave, followed by a relatively quiet period leading up to another oscillation at the arrival of the *S*-wave. These events are referred to by Lamb as the *minor tremor* and are followed by a much larger magnitude oscillation, the *major tremor*, at the time of arrival of the *R*-wave.

The time interval between wave arrivals becomes greater and the amplitude of the oscillations becomes smaller with increasing distance from the source. In addition, the minor tremor decays more rapidly than the major tremor. It is evident, therefore, that the *R*-wave is the most significant disturbance along the surface of a half-space and, at large distances from the source, may be the only clearly distinguishable wave.

7.10. LOVE WAVES

In the Rayleigh waves examined in the previous section the material particles move in the plane of propagation. Thus, in Rayleigh waves over the half-space $y > 0$ along the surface $y = 0$, propagating in the x-direction, the z-component of displacement w vanishes. It may be shown that surface waves with displacements perpendicular to the direction of propagation (the so-called *SH waves*) is impossible in a homogeneous half-space. However, *SH* surface waves are observed as prominently on the Earth's surface as other surface waves. Love showed that a theory sufficient to include *SH* surface waves can be constructed by having a homogeneous layer of a medium M_1 of uniform thickness H_1, overlying a homogeneous half-space of another medium M.

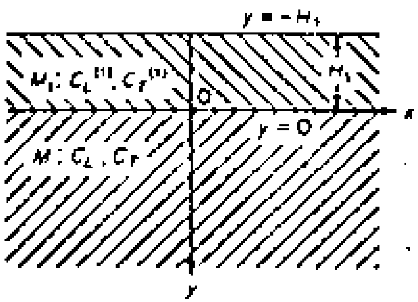


Fig. 7.10:1. A layered half-space.

Using axes as in Fig. 7.10:1, we take $u = v = 0$, and

$$(1) \qquad w = A \exp\left[-k\sqrt{1 - \frac{c^2}{c_T^2}}\, y\right] \exp\{ik(x - ct)\}$$

in M, and

$$(2) \qquad w = \left\{A_1 \exp\left[-k\sqrt{1 - \left(\frac{c}{c_T^{(1)}}\right)^2}\, y\right] + A_1' \exp\left[k\sqrt{1 - \left(\frac{c}{c_T^{(1)}}\right)^2}\, y\right]\right\} \times \exp[ik(x - ct)]$$

in M_1. It is easily verified that these equations satisfy the Navier's equations. If $c < c_T$, then $w \to 0$ as $y \to \infty$, as desired.

The boundary conditions are that w and σ_{zy} must be continuous across the surface $y = 0$, and σ_{zy} zero at $y = -H_1$. On applying these conditions to (1) and (2), we obtain

$$(3) \qquad A = A_1 + A_1'.$$

$$(4) \qquad GA[1 - (c/c_T)^2]^{1/2} = G_1(A_1 - A_1')[1 - (c/c_T^{(1)})^2]^{1/2}.$$

$$(5) \qquad A_1 \exp\{kH_1[1 - (c/c_T^{(1)})^2]^{1/2}\} = A_1' \exp\{-kH_1[1 - (c/c_T^{(1)})^2]^{1/2}\}.$$

Eliminating A from (3) and (4), and then using (5) to eliminate A_1 and A_1', we have

$$\frac{G[1 - (c/c_T)^2]^{1/2}}{G_1[1 - (c/c_T^{(1)})^2]^{1/2}} = \frac{A_1 - A_1'}{A_1 + A_1'} = i \tan\{ikH_1[1 - (c/c_T^{(1)})^2]^{1/2}\}.$$

Hence, we have

$$(6) \qquad G[1 - (c/c_T)^2]^{1/2} - G_1[(c/c_T^{(1)})^2 - 1]^{1/2} \tan\{kH_1[(c/c_T^{(1)})^2 - 1]^{1/2}\} = 0$$

as the equation to give the *SH* surface wave velocity c in the present conditions.

If $c_T^{(1)} < c_T$, Eq. (6) yields a real value of c which lies in the range $c_T^{(1)} < c < c_T$ and depends on k and H_1 (as well as on G, G_1, c_T, and $c_T^{(1)}$) because for c in this range the values of the left-hand-side terms in (6) are real and opposite in sign. Thus, *SH* surface waves can occur under the stated boundary conditions, provided the shear velocity $c_T^{(1)}$ in the upper layer is less than that in the medium M. These waves are called *Love waves*.

Love waves of general shape may be derived by superposing harmonic Love waves of the type (2) with different k. The dependence of the wave speed c on the wave number k introduces a dispersion phenomenon which will be considered later.

PROBLEMS

7.2. Derive Navier's equation in spherical polar coördinates.

7.3. From data given in various handbooks, determine the longitudinal and shear wave speeds in the following materials:

(a) Gases: air at sea level, and at 100,000 ft altitude.

(b) Metals: iron, a carbon steel, a stainless steel, copper, bronze, brass, nickle aluminium, an aluminium alloy, titanium, titanium carbide, berylium, berylium oxide.

(c) Rocks and soils: a granite, a sandy loam.

(d) Wood: spruce, mahogany, balsa.

(e) Plastics: lucite, a foam rubber.

7.4. Sketch the instantaneous wave surface, particle velocities, and particle paths of a Love wave.

7.5. Investigate plane wave propagations in an anisotropic elastic material. Apply the results to a cubic crystal. *Note:*

$$\rho \frac{\partial^2 u_i}{\partial t^2} = C_{ijkl} \frac{\partial^2 u_k}{\partial x_j \partial x_l}, \qquad u_i = A_i e^{-i(\omega t - k_j x_j)}$$

where $\mathbf{k}(k_1, k_2, k_3)$ is the wave vector normal to the wave front.

7.6. Determine the stress field in a rotating, gravitating sphere of uniform density.

EFFECT OF LOCAL SOIL CONDITIONS UPON EARTHQUAKE GROUND MOTIONS

by Robert V. Whitman

1. INTRODUCTION

It has long been recognized that local soil conditions can have a profound effect upon the damage caused by an earthquake. Such an effect was clearly evident in accounts of the great Lisbon earthquake of 1755, and in the accounts of almost every subsequent major earthquake that affected a large city. The effect of soil conditions upon damage during the 1906 San Francisco earthquake was well recognized in studies of that earthquake. The topic received considerable study following the Kanto (Tokyo) earthquake of 1923. The effect of local soil conditions upon earthquake damage is hardly a new problem. The seismic codes of most countries specifically require different earthquake resistance for different soil conditions. Codes now in effect in the United States contain no such requirement, but not because soil conditions are thought to be unimportant. The writers of the U.S. codes recognized the importance of soil conditions, but felt the problem was so complex and poorly understood that adequate code provisions could not be written. Without a doubt, soil conditions will be incorporated into U.S. seismic codes in the very near future.

Much of the earthquake damage to buildings built upon poor soils results from partial or complete failure of the soil. Such failures include slumping of river banks, failure of waterfront retaining structures, large landslides, foundation settlement and foundation failures. Seed (1970) has provided an excellent summary description of such failures. Many such failures are caused by total or partial liquefaction of loose saturated cohesionless soils. The possibility of such failures, especially liquefaction failures, in any given locale or site requires individual study by experts. Appendix B contains a very brief discussion of liquefaction.

This chapter considers the effect of local soil conditions upon earthquake ground motions, and hence upon the shaking of buildings,

when there is no failure of the soil. Field observations and theoretical studies of this effect have been summarized in recent papers by Ohsaki (1969) and Seed (1969). Much is now known about the problem, although by no means is there complete understanding. There are several ways in which this new knowledge can be put to practical use. One way is the development of site-conditioned earthquake motions for input to the analysis of important structures: this approach is now being used in the design of tall buildings in San Francisco and Tokyo. The second way is to guide the development of new building code provisions. This chapter deals primarily with the latter application. That is, the chapter will discuss how the base shear coefficient C should vary with soil conditions. A plot of C vs. T, the fundamental period of the building, will be called a seismic coefficient diagram.

Figure 1 illustrates several different forms of seismic coefficient diagrams incorporating soil conditions. The simplest forms are those in Figures 1a and 1b: here all ordinates are multiplied by a factor that is independent of period. That is:

$$C(T) = S\,C_o(T) \qquad (1)$$

where S is a soil factor and $C_o(T)$ is the seismic coefficient function for a reference soil condition. Ohsaki (1969) has tabulated values of S required by the codes of 13 countries. Table 1 gives examples of such factors, ranging from the very simple table used in Canada to the somewhat complex table in effect in Japan.

Figures 1c through 1f show more complicated proposals for introducing the effect of local soil conditions into seismic coefficient diagrams; now the effect of soil is varied depending upon the period T.

1. Figure 1c comes from the new Chilean code. The curve of C vs. T varies in shape depending upon a parameter T_o. The parameter T_o is related to the characteristic frequency of the site of the building being designed.

2. Figure 1d shows a seismic coefficient diagram proposed by Muto in Japan in 1963. Both the maximum seismic coefficient and the period scale are adjusted in accordance with the type of ground.

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3. According to the proposed curves shown in Figure 1e, low stiff buildings having a small period T would be designed for a larger seismic coefficient if on hard ground than if on soft ground. For tall flexible buildings, the reverse would be true.

4. Figure 1f shows the code provisions developed for Mexico City, so as to account for the effect of the unusually soft and deep clay which underlies much of that city.

Thus, a great variety of methods have been proposed for incorporating the effects of soils conditions into the seismic provisions of building codes. A building official faced with the selection of a suitable provision, or an engineer faced with implementing such provisions, must understand the basic thinking lying behind the various proposals. To develop such basic understanding, it is useful to consider four categories of soil conditions:

I. Shallow soil deposit with a distinct characteristic frequency.
II. Deep deposit of firm soil.
III. Shallow soft soil overlying deep deposit of firm soil.
IV. Deep deposit of soft soil.

While these four cases do not encompass all possible soil conditions, they serve to bring out the fundamental considerations.

2. ROLE AND STATUS OF THEORY

In order to understand adequately the effect of local soil conditions, we must combine interpretations of actual accelerograph records together with theoretical analysis. Within the recent past, it has been necessary to rely very heavily upon theory, since the field data from accelerographs has been very scanty indeed. Because of the many accelerographs which have been installed within the past few years and will be installed within the near future, there soon should be many more records involving a variety of soil conditions. However, theory will continue to be of vital importance in helping to sort out and understand the potentially staggering quantity of rather confusing data.

The theory of ground amplification* as it exists today is by no means perfect. However, in many cases predictions from the theory are in accord with observations (Seed, 1969). There now has been considerable experience in the practical use of the theory, and we understand both its limitations as well as how it can be used. Used with judgement, this theory is a very useful tool for understanding the effects of local soil conditions.

3. CASE I: SHALLOW SOIL DEPOSIT WITH DISTINCT CHARACTERISTIC FREQUENCY

For a uniform soil deposit (Fig. 2a), the fundamental period is given by:

$$T_o = \frac{4H}{C_s} \tag{2}$$

where H = thickness of deposit
C_s = shear wave velocity

Case I is typified by $T_o < 0.5$ sec. The following tabulation indicates typical combinations of C_s and H satisfying this condition.

C_s(m/sec)	H(m)
100 (Very soft clay or silt)	< 12.5
200 (Loose sand, soft clay)	< 25
300 (Dense sand, stiff clay)	< 37.5
400 (Compact sand, hard clay)	< 50

Soil deposits with a depth greater than about 50 meters probably do not belong in Case I. The soil descriptions in the table are intended to give a very general idea of typical shear wave velocities in soils; for further discussion of the evaluation of soil properties for specific cases, see Appendix A and Whitman (1969). Since the soil is nonlinear, the shear wave velocity and hence the fundamental period depend upon the intensity of the earthquake, decreasing as the intensity increases.

* The nature of this theory is outlined in Appendix A.

Theoretical Considerations

The theory of soil amplification may conveniently be used to indicate the expected effects of a shallow soil deposit.

Amplification spectrum: An amplification spectrum is the ratio of the Fourier amplitude spectra for motions atop the soil to the Fourier amplitude spectra for motions of the underlying rock. Thus an amplification spectrum shows how the various frequency components in earthquake motion are amplified by the soil.

Figure 2b shows a typical amplification spectrum for a shallow soil deposit. It is characterized by a predominant peak, which occurs at the period given by Eq. 3. Smaller, unimportant peaks may occur at very small periods. The peak amplification ratio is a function of:

1. The ratio of the seismic impedance of the soil to the seismic impedance of the underlying rock:

$$\frac{(\gamma C_s)_{soil}}{(\gamma C_s)_{rock}} \tag{3}$$

where γ denotes unit weight. As discussed in Appendix A, this factor accounts for the loss of energy back into the underlying rock. The smaller this ratio, the greater the amplification. Thus, for a given rock, the peak amplification ratio increases as the overlying soil becomes softer.

2. The internal damping within the soil: This damping is determined primarily by the magnitude of the dynamic strains which occur within the soil. Thus, the stronger the earthquake, the greater the damping and the smaller the amplification.

One point from the theory is worth emphasizing: the amplification from an outcropping of rock to the surface of soil is less than the amplification from the interface between soil and rock to the surface of the soil. Thus, comparison of motions measured at several depths may overestimate the amplification between the surface outcroppings of different soil or rock.

For cases of interest, the peak amplification ratio between soil and an outcropping of underlying rock is typically between 3 and 6, with the larger values applying to the softer soils during smaller earthquakes.

Peak acceleration: For Case I, T_o lies within the range of the predominant periods in earthquake ground motions. Hence the amplifying effect of a shallow soil layer causes the peak acceleration at the ground surface to exceed that at an outcrop of the underlying rock. Figure 3 compares computed ground motions for the case corresponding to Fig. 2. As the peak of the amplification spectrum increases, the ratio of peak accelerations increases; however, the increase in peak acceleration is less than the peak amplification ratio. For typical cases the computed ratio of peak accelerations is from 1.5 to 4, with the larger values during smaller earthquakes. In Figure 3, note also the obvious change in predominant frequency.

Response spectra: Figure 4 compares response spectra computed from the motions on soil and on rock. At a period corresponding to the fundamental period of the soil, the ordinates of the spectra from soil motions are considerably greater than those for the spectra from rock motions. Thus, a building whose fundamental period is approximately the same as the fundamental period of the soil will respond much more strongly if on the soil than if on the rock.

A diagram formed by taking the ratio of the response spectra at each period is very similar to the amplification spectrum, although the peak of the former is not so high as the peak of the latter.

Field Evidence

While there are many pieces of evidence which support the general conclusions of the theory (see the papers by Ohsaki and Seed), it is not possible at this time to present evidence which totally substantiates the theory. In particular, there are very few instances of records from instruments placed over very different soils in the same immediate vicinity. A sampling of the available evidence is presented in the following subsections.

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Amplification spectra: Figure 5 shows a comparison of actual and predicted amplification spectra (Dobry, 1971). The heavy line is an average of amplification spectra for six earthquakes at a given site, based on measurements made at different depths in Japan. Considering that there are uncertainties in the actual amplification data introduced by the processing of the data, the theoretical curve follows the actual behavior very well.

Peak accelerations: There are a number of examples within the Japanese literature showing that peak acceleration increases as the ground surface is approached, in accordance with the theory. As observed in rather small earthquakes, during which the internal damping is small, this increase is typically in the ratio of 3 or 4. Figure 6 shows peak accelerations observed, mostly in basements of buildings, at various depths beneath the surface of the ground in Tokyo. It should be emphasized that such an increase occurs in rock as well as in soil, because the stiffness of soil decreases near the surface, partly because of weathering and partly because of decrease in overburden stress.

Response spectra: Figure 7 compares response spectra computed from ground motions measured on soft soil and firm soil during the same earthquake. In each diagram, the spectral ordinates have been normalized to the peak acceleration, and hence the effect of soil conditions shows only in the shape and not in the ordinates of the spectra. The shift in the period at which the spectra peak is the result of amplification by the soil.

Damage to buildings: Most of the evidence concerning the effect of local soil conditions is indirect: in the form of differences in damage to buildings founded over different soils (Ohsaki, 1969; Duke and Leeds, 1963). Small buildings, whose fundamental period is in the range from 0.2 to 0.5 second, generally experience greater damage when founded over soft soil than when founded upon firm soil. These observations for the most part are consistent with the theory. However, some of the differences in damage may have resulted from partial failure of softer soils in addition to differences in ground motions.

Perspective from field evidence: Considering the available comparisons between predictions and observations, it may be concluded that the theory may be used to guide the choice of seismic coefficient diagrams for practical work. However, it also is clear that more actual experience is necessary before greater accuracy can be expected from theory.

Microtremor Studies

Kanai and Tanaka (1961) have proposed a method of microzoning based upon measurement of ambient vibrations. The measured vibrations are plotted in the form of an amplitude spectra; in Kanai's original work this spectra was constructed in an approximate way, but more recently Fourier analysis has been used for this purpose. Figure 8 shows some typical results; for identification of the soil types, see Table 1. Both the period and the magnitude of the peak of the spectra are used to determine the seismic zone; the longer the period and the higher the peak, the more severe the expected damage during an earthquake. Kanai has correlated the observed period and amplitude to the four types of ground considered in the Japanese seismic code.

This approach was specifically developed to predict the effect of shallow soil deposits upon damage to buildings having only a few stories. For these conditions, the predictions made by Kanai's approach are entirely consistent with the predictions of amplification theory. Thus, there is a sound reason why Kanai's approach has been in accord with experience during actual earthquakes.

Medvedev's Method

The Russian seismologist Medvedev (1962) has proposed a method for estimating the effect of ground conditions upon earthquake intensity, based upon two factors:

1. The ratio

$$\frac{(\gamma C_0)\ \text{soil}}{(\gamma C_0)\ \text{granite}} \qquad (4)$$

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where C_D is the dilatational, or compressive, wave velocity. The wave velocity for granite serves as a reference against which a soil is rated. The smaller this ratio, the more severe the expected damage during an earthquake.

2. The depth to the water table. The shallower the water table, the greater the expected damage.

These two factors are combined in the equation

$$n = 1.67 \log_{10}\left[\frac{(\gamma C_D)\ \text{rock}}{(\gamma C_D)\ \text{soil}}\right] + e^{-0.04h^2} \qquad (5)$$

where n is the increment in intensity units on a scale equivalent to the modified Mercalli scale, and h is the depth to the water table in meters. Eq.5 typically gives an increase in 1 to 2 intensity units (equivalent to a 2 or 4 fold increase in acceleration) for soft ground as compared to firm ground. Medvedev's method was originally developed for use in connection with shallow soil deposits and buildings having only a few stories.

The relationship between Medvedev's method and amplification theory may be understood by means of the example in Figure 9. When the water table is very low, then the ratio C_D/C_S is the same for both the soil and the rock. Thus ratios 3 and 4 are equivalent, and Medvedev's method and amplification theory will predict the same trends. The soil in Figure 9b has the same C_S as in Figure 9a, and thus amplification theory would predict the same behavior for both cases. Raising the water table means that C_D increases considerably in the soil, and thus the first term in Mevedev's equation decreases. However, this decrease is compensated by an increase in the second term. Thus, Medvedev's two factors taken together give roughly the same result as amplification theory. Moreover, the increases in intensity predicted by Medvedev are consistent with increases in acceleration predicted by amplification theory.

Summary

For the common case of shallow soil deposits, the predictions of amplification theory are generally in accord with actual experience during earthquakes and moreover are in accord with the semi-empirical methods of microzoning proposed by Kanai and Medvedev.

Figure 10 summarizes the effect of local soil and rock conditions upon response spectra (say for 5% damping) at a given distance from the epicenter of an earthquake. With increasing softness of the earth material, the peak of the spectra increases and shifts to a larger period. Thus, the response of low stiff buildings is strongly affected by soil conditions. On the other hand, a shallow soil deposit has little or no effect upon the response of the fundamental period of tall buildings having long natural periods (although the shallow soil will affect the response of the higher modes of such a building).

Based upon current knowledge, a seismic coefficient diagram such as type (b) in Figure 1 should be used to account for differences in near surface earth materials within a small region. That is to say, the soil factor S should be independent of period. There are several reasons for this recommendation.

1. Because of uncertainties in both the fundamental period of the soil and the predominant periods in the input ground motion, it is difficult to predict the predominant period in motion at the top of soil. Use of constant S for T < 0.5 sec. covers these uncertainties.

2. Use of constant S for T > 0.5 sec. recognizes that the contribution of the higher modes will be affected by soil conditions, and provides extra conservatism with regard to the design of tall buildings.

With further research, it may be possible to use a reduced value of S for T > 0.5 sec.

Table 2 gives recommended soil factors. These factors are based upon both theory and experience, and consider possible settlement problems in addition to amplification effects. In the 2nd column of the table, hard crystalline rock found at considerable depth has been taken as the reference; the soil factor for a soft soil is 4. However, it generally is more practical to use surface exposures of rock as a reference (3rd column), and then the soil factor for soft soil is 2.2. In some localities, it may even be desirable to use firm soil as a reference (4th column), in which case the soil factor for soft soil is only 1.6.

4. CASE II DEEP DEPOSIT OF FIRM SOIL

Several areas that have experienced major earthquakes are underlain by more than 100 meters of compact alluvium. Los Angeles, Caracas, Venezuela, and Santiago, Chile are prime examples.

Theoretical Considerations

Amplification spectra: Figure 11 illustrates the general nature of the amplification spectrum for this case. Now several peaks occur within the range of building periods of practical interest.

The fundamental period is greater than in Case I, and tends to coincide with the period of taller structures. Because the shear wave velocity of compact alluvium is rather high (300 to 450 m/sec) the radiation damping also is greater than in Case I, and hence the amplification at the fundamental peak generally is less than in Case I. Nonetheless, this amplification can be quite important.

The higher order peaks typically occur at periods less than 0.5 second; that is, within the same range of periods for which amplification occurred in Case I. Radiation damping is less important for these higher modes, and hence when internal damping is small--as during small earthquakes--the peaks corresponding to these modes may be nearly as high as the fundamental peak.

Peak accelerations: Figure 12 shows computed acceleration at ground surface, for conditions corresponding to Figure 10 (the input is the same as in Fig. 3, but with a peak acceleration of 0.03g). Peak acceleration is increased: typical increases are factors of 1.5 to 3, with the larger values applying to smaller earthquakes. This increase is caused by the higher modes of the soil; these modes have amplification peaks in the range of the predominant periods of the input motion. The fundamental mode does not cause an increase in peak acceleration, but does amplify the longer period components of ground motion.

Response spectra: Figure 13 compares response spectra for motions at the surface of several different depths of compact alluvium. Changing the depth of the alluvium has relatively little effect upon the general position of the spectra for T < 0.5 second. However, increasing the depth of the alluvium has a very significant effect upon the spectra at larger periods corresponding to taller buildings.

Field Evidence

There is, to the author's knowledge, no adequate direct confirmation of these theoretical results, although Gutenberg (1957) has shown that deep deposits amplify the long period components of ground motion. Actual accelerograph records from nearby sites with very different depths of alluvium must be obtained before adequate confirmation is possible.

Observations of damage to buildings during the Caracas earthquake of July 1967 do provide strong indirect confirmation of the theory (Whitman, 1969; Seed et al, 1970). Caracas is underlain by a compact alluvium whose depth generally is less than 100 meters. However, under one portion of the city the depth is as much as 300 meters. Analysis of the patterns of damage shows:

1. For buildings having 8 stories or less, the percentage of buildings damaged is more-or-less constant for all parts of the city.

2. Buildings having more than 8 stories, and particularly those having more than 15 stories, were much more heavily damaged in the part of the city over the very deep alluvium than elsewhere in the city.

These observations show clearly that a great depth of alluvium significantly amplifies the earthquake threat to tall buildings.

<u>Summary</u>

The theory, together with the evidence from the Caracas earthquake shows the need to guard against the strong shaking that can occur when the fundamental period of a tall building coincides with the fundamental period of a deep soil deposit. Thus, the fundamental period of the soil must enter into the code. When differences in depth of soil, rather than differences in the nature of the soil, are of concern, it appears that a seismic coefficient diagram of type (c) in Figure 1 is suitable. An example is the following formula from the Chilean code:

$$C = \begin{cases} C_o & T \le T_o \\ C_o \left[\dfrac{2\,T/T_o}{1+(T/T_o)^2}\right] & T \ge T_o \end{cases} \qquad (6)$$

The soil period T_o must be determined from a combination of experience, careful analysis of earthquake records and theoretical studies. Usually it is not possible to determine T_o by measurement of microtremors, since the high frequencies present in ambient vibrations mask the low frequencies associated with the fundamental period. When using Eq. 6, T_o should always be at least 0.4 even if the fundamental period is smaller than this limit.

5. CASE III SHALLOW SOFT SOIL OVERLYING DEEP DEPOSIT OF FIRM SOIL

<u>Theoretical Considerations</u>

As yet, this case (which is sketched in Figure 14) has not been studied completely from a theoretical standpoint. The effect of the soft shallow deposits enter through the higher modes, and the response of these higher modes is quite sensitive to the details of the analysis--especially the assumptions concerning damping. The theoretical results which have been computed are not entirely satisfactory.

However, in a general way it may be said that Case III is a combination of Case I and Case II. Thus the fundamental mode of the deep compact alluvium will amplify long period motions while the higher modes of the deep alluvium will also amplify shorter period motions. The shallow soil deposits will further amplify the short period motions. With respect to the effect upon buildings, the following can be expected:

1. Buildings with T < 0.5 second. Damage will be greater if these buildings are founded upon the soft soil than if they rest upon firm alluvium. The depth of the firm alluvium beneath a building has little effect upon the damage to that building. Thus conclusions applicable to Case I apply.
2. Buildings with T > 0.5 second. Damage will be greater if a building is founded over a great depth of firm alluvium than if it rests upon a shallow depth of this alluvium. The presence or absence of soft soil near the surface has less effect upon the damage. Thus conclusions applicable to Case II apply.

<u>Field Evidence</u>

Damage in Valdivia and Concepción during the 1960 earthquakes has been studied extensively (Duke and Leeds, 1963, and subsequent studies at the University of Chile). This damage was greatest where there was soft soil at the surface. The great majority of this damage was to 1 and 2 story buildings. Thus the behavior during these earthquakes is consistent with Case I. Hence it is not surprising that predictions based upon Kanai's and Medvedev's methods correlated well with the damage patterns.

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At both of these cities, there exist deep deposits of firm soil. Using amplification theory, attempts have been made to correlate damage to this total depth. However, since there were very few buildings having periods greater than 0.5 second, no such correlation was possible. Moreover, since the theory for a soft shallow layer over a deep stiff layer is still not reliable, the theory often did not show correctly the effect of the shallow layer.

Thus the experience from the 1960 earthquakes showed the effect of shallow soft deposits but gave no indication as to the effect of varying depths of the compact alluvium. However, the effect of the deep alluvium must not be ignored when establishing microregionalization or building code provisions for future construction, because more and more tall buildings certainly will be constructed in these and other cities with similar soil conditions.

Summary

A seismic coefficient diagram for this case must recognize both the effect of shallow soft deposits upon buildings having $T < 0.5$ second and also the effect of deep soil upon buildings having longer periods. These requirements might be met by combining Eqs. 1 and 6:

$$C = \begin{cases} C_0 S & T \leq T_0 \\ C_0 S \left[\dfrac{2T/T_0}{1+(T/T_0)^2} \right] & T \geq T_0 \end{cases} \qquad (7)$$

The soil factor S would be chosen based upon the shear wave velocity of the near-surface soils, while T_0 would bring in the effect of the deep deposit. Such a code provision might apply to many cities, such as Boston for example, where very poor soils at the surface overlie deep deposits of clay. Such a provision probably should be used only for $T < 1.5$ seconds. If the fundamental period of the soil is greater, special provisions such as that described in the next section are warranted. As before, the minimum T_0 is 0.4.

6. CASE IV DEEP DEPOSIT OF SOFT SOIL

Theoretical Considerations

Amplification spectra: Figure 15 shows an amplification spectra for a deep deposit of very soft clay. This spectrum is similar to that in Fig. 10, with one important difference: now the peak amplification at the fundamental mode is distinctly greater than that for the higher modes. This change occurs because, with a deep deposit of soft soil, radiation damping is less important and strains (and hence internal damping) are larger.

Response spectra: Figure 16 compares response spectra from motions measured on top of the soil with that from motions at an outcropping of the underlying hard soil. There is an increase in the ordinates at low periods. However, the remarkable feature is the very great increase in the range from 2 to 2.5 seconds. Now the peak of the spectra has been shifted to a much larger period.

Peak acceleration: In the case corresponding to Figure 16, peak acceleration on top of the soil was twice that on the hard outcropping. In other cases which have been investigated theoretically, peak acceleration is decreased.

Field Evidence

The classic example of this case is the soft deep deposit of clay underlying Mexico City. The examples in Figs. 15 and 16 apply for the soil conditions in Mexico City, and have been confirmed by actual accelerograph records.

It has often been suggested that a deep soft deposit can actually cause a decrease in peak acceleration. While there is little or no field evidence to prove this, such might occur during a strong earthquake when the internal damping within the soil would be increased.

Summary

For this situation, it is appropriate to use a seismic coefficient diagram of Type (f) in Fig 1. Now the seismic coefficient is less for very small periods than for intermediate periods. However, at this time use of such a diagram is justified only for sites where there is considerable actual experience which has been studied in detail.

7. PILE FOUNDATIONS

The evidence currently available suggests that piles usually do not alter the ground motions at the base of a building (Ohsaki, 1969). This is because piles generally are flexible enough to follow the horizontal motions of the soil (for example, see Yamamoto and Seki, 1970) However, piles may improve the ability of the building to resist the effects of the ground motion--by reducing both static settlements (that may use up some of the reserve strength of the building) and dynamic rocking motion. Because of the need for less conservatism, the soil factor S might be reduced somewhat for pile-supported buildings.

Large diameter caissons may be stiff enough to resist following the motions of a soft soil through which they pass (Ohsaki, 1969). Then the amplifying effect of the soil-caisson system will be more like that of a firm soil rather than a soft soil. Thus for caisson-supported buildings, T_o used in Eq. 7 could be somewhat less than the fundamental period of the soil.

Unfortunately, at the present time there are no sound rules for deciding just how much S and T_o might be modified in accordance with these considerations.

8. RESPONSE SPECTRA AND TIME HISTORIES

The emphasis in this chapter has been upon code provisions to

reflect soil conditions However, there is a growing trend toward requiring dynamic analyses for tall or important buildings.

The principles discussed in connection with Cases I, II, and III can be used to suggest the possible form for a general design response spectrum incorporating soil conditions:

$$S_a = \begin{cases} S_{ao}S & T \le T_o \\ S_{ao}\dfrac{T_o}{T}\left[(S-1)\dfrac{T_o}{T} + 1\right] & T \ge T_o \end{cases} \quad (8)$$

where S_a is the spectral acceleration and S_{ao} is the spectral acceleration for the reference soil condition. This equation is plotted in Figure 17. The soil factor S, which brings in the effect of the near-surface soil, might be less than in Table 2, since use of dynamic analysis means less need for conservatism For example, the following values might be used:

Ground condition	S
Exposed rock	1.0
Firm soil	1.3
Soft soil	1.8

The effect of the near-surface soil upon spectral acceleration decreases for $T > T_o$. This is in contrast to Eq. 7 where there was need to account for the contributions from higher modes; when a dynamic analysis is performed, the response of higher modes is introduced directly

For T > 3 seconds and T_o > 1 second, Eq. 8 becomes too conservative Tezcan (1972) has recently presented a more general approach to development of response spectra including soil effects.

Time histories whose spectra lie above the spectra given by Eq. 8

would be suitable as input for dynamic analysis. Great caution should be followed in using individual time histories generated by the theoretical procedures described in Appendix A, since there are uncertainties both in the validity of the procedures and the selection of soil properties. If such procedures are used to generate site-conditioned time histories, it is very essential that a set of time histories be developed by varying the input assumptions.

9. FINAL COMMENTS

The four cases which have been discussed in this chapter certainly do not cover all possible soil conditions, and many problems remain to be solved by further theoretical research plus analysis of accelerograph records. For example, the line of demarcation between Cases II and IV is not at all clear. However, the current understanding of the effect of local soil conditions is almost equal--and perhaps even equal--to the current understanding of the nature and amplitude of earthquake ground motions for average soil conditions. That is to say, the effect of soil conditions can be evaluated with almost as much confidence as can the reference seismic coefficient C_0.

Table 1

EXAMPLES OF SOIL FACTORS

Canada

General	1.0
Soft	1.5

Argentina

Hard	0.75
Medium	1.00
Soft	1.25
Very soft	1.50

Japan

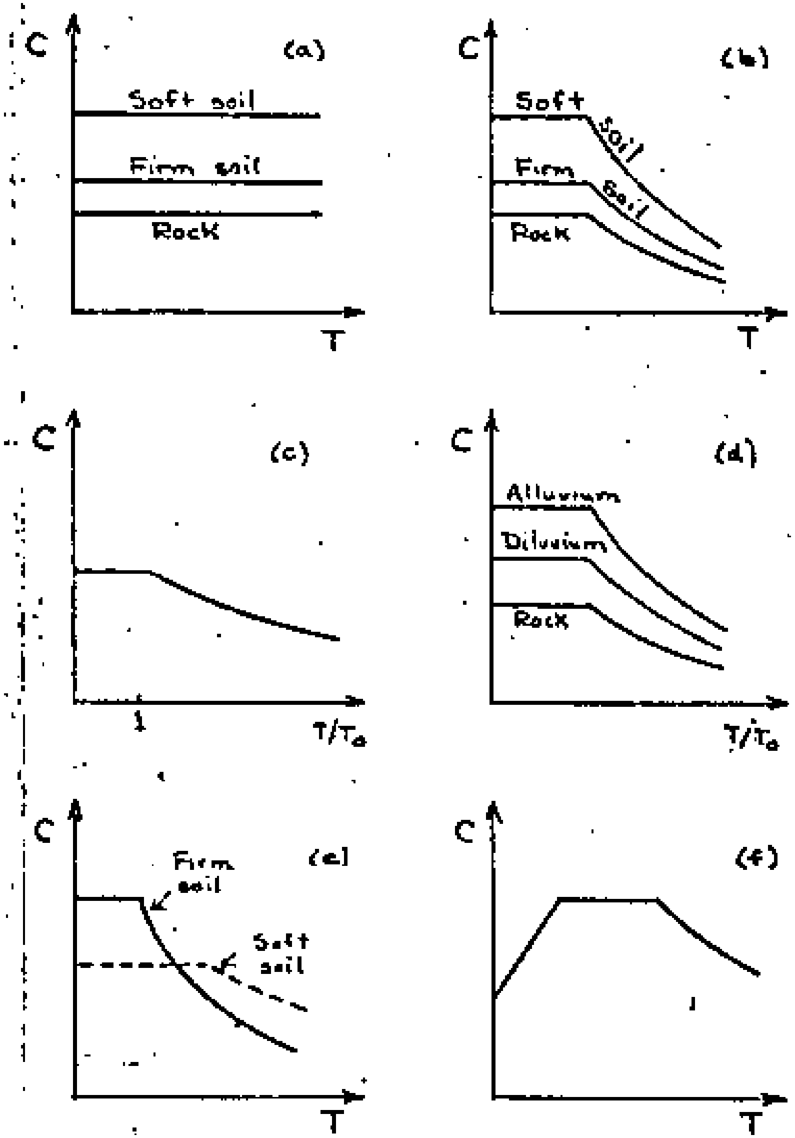
Ground/Structure		Wood	Steel	Reinf. Conc.
I	Rock	0.6	0.6	0.8
II	Diluvium	0.8	0.8	0.9
III	Alluvium	1.0	1.0	1.0
IV	Very soft	1.5	1.0	1.0

Table 2

RECOMMENDED SOIL FACTORS TO ACCOUNT FOR

EFFECT OF NEAR SURFACE SOILS

Ground condition	Reference ground condition: Hard rock	Exposed rock	Firm soil
Hard crystalline rock at depth (C_s > 1200 m/sec)	1.0	0.6	0.4
Exposed rock with minimal weathering (C_s = 700 m/sec)	1.8	1.0	0.7
Firm clay, compact sand/gravel, deeply weathered rock (C_s = 350 m/sec)	2.5	1.4	1.0
Soft clay or silt (C_s = 120 m/sec)	4.0	2.2	1.6



-URE 1 VARIOUS TYPES OF SEISMIC COEFFICIENT DIAGRAMS

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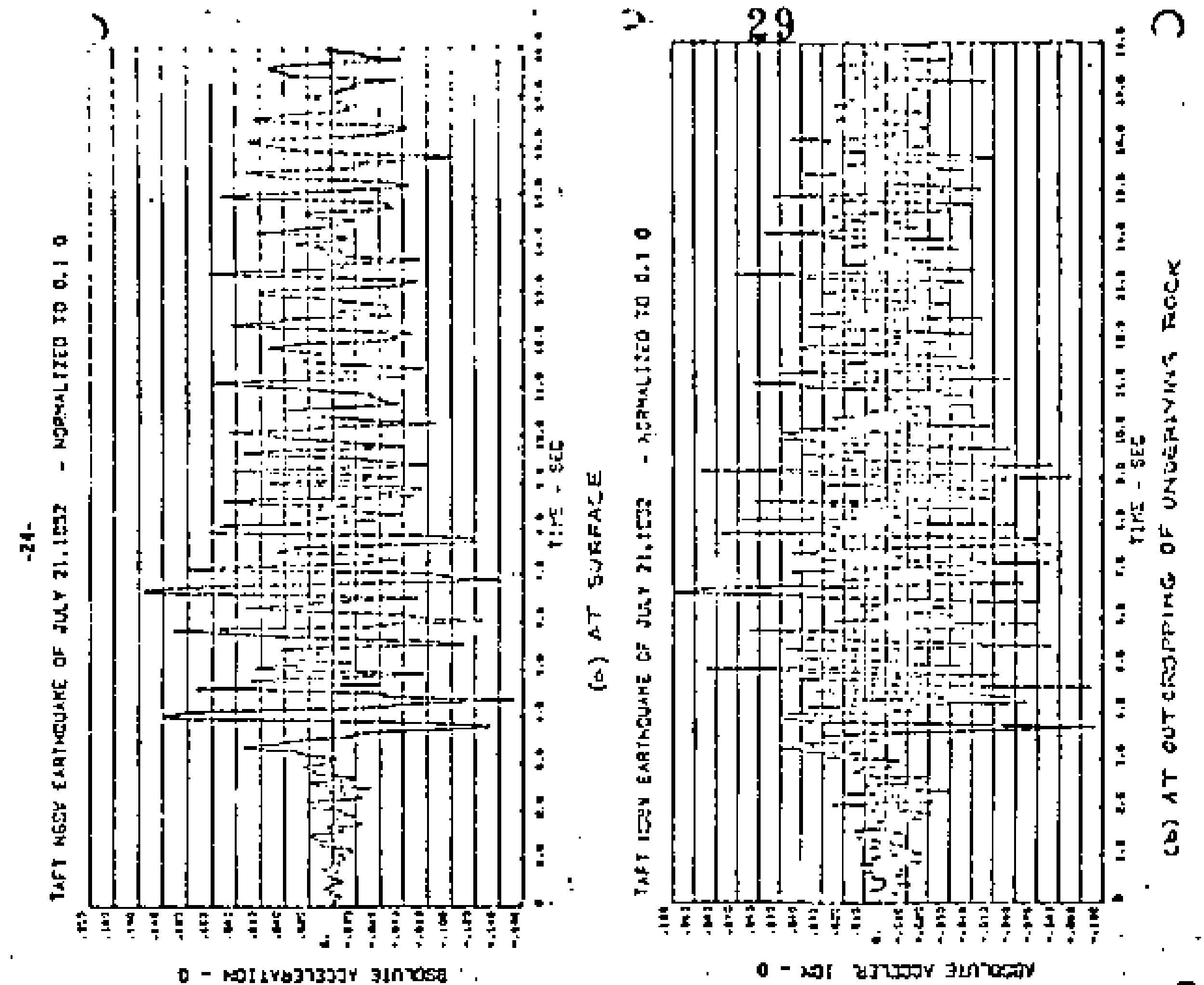

FIGURE 3 INPUT AND COMPUTED SURFACE MOTIONS FOR PROFILE IN FIGURE 2

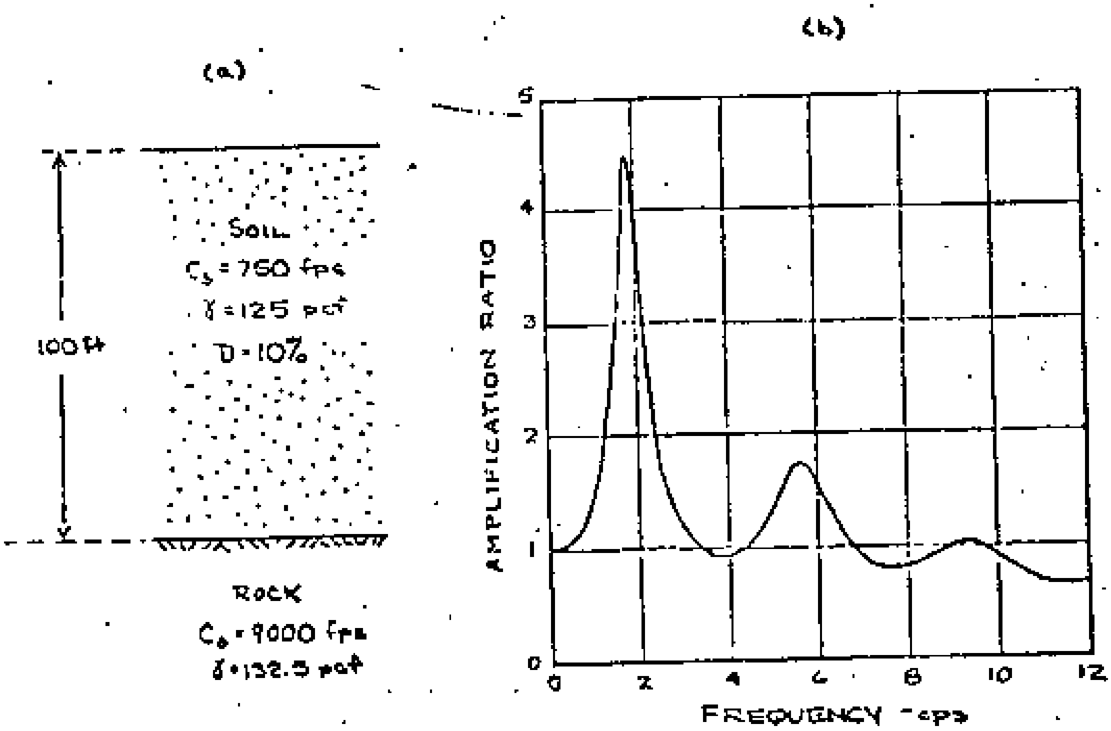

FIGURE 2 AMPLIFICATION RATIO FOR SHALLOW SOIL PROFILE

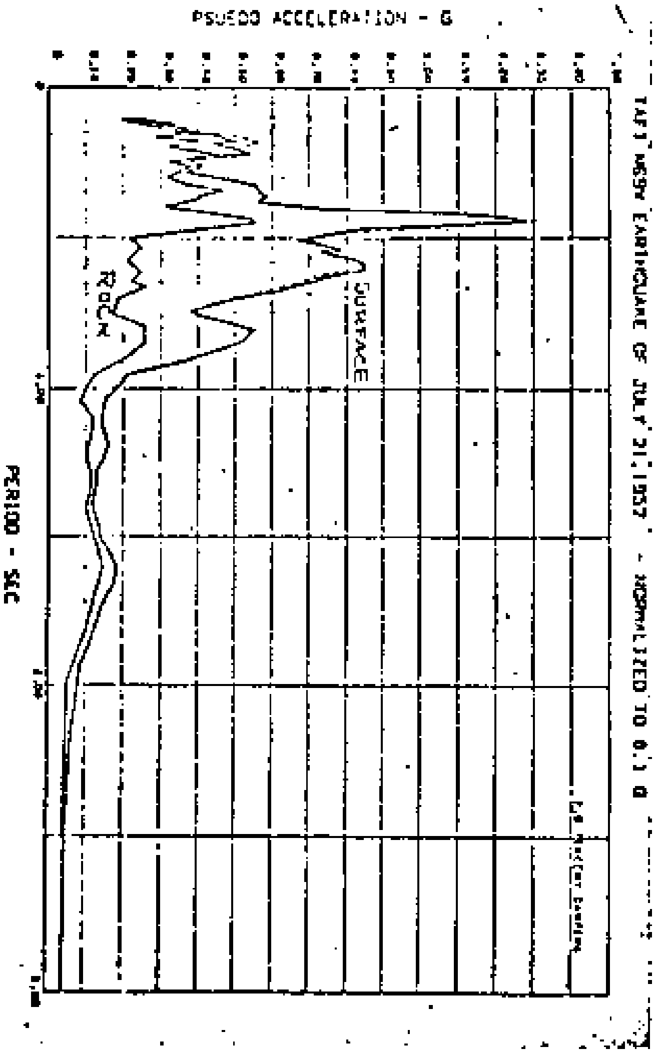

FIGURE 4 COMPARISON OF INPUT AND SURFACE RESPONSE SPECTRA FOR SHALLOW SOIL PROFILE

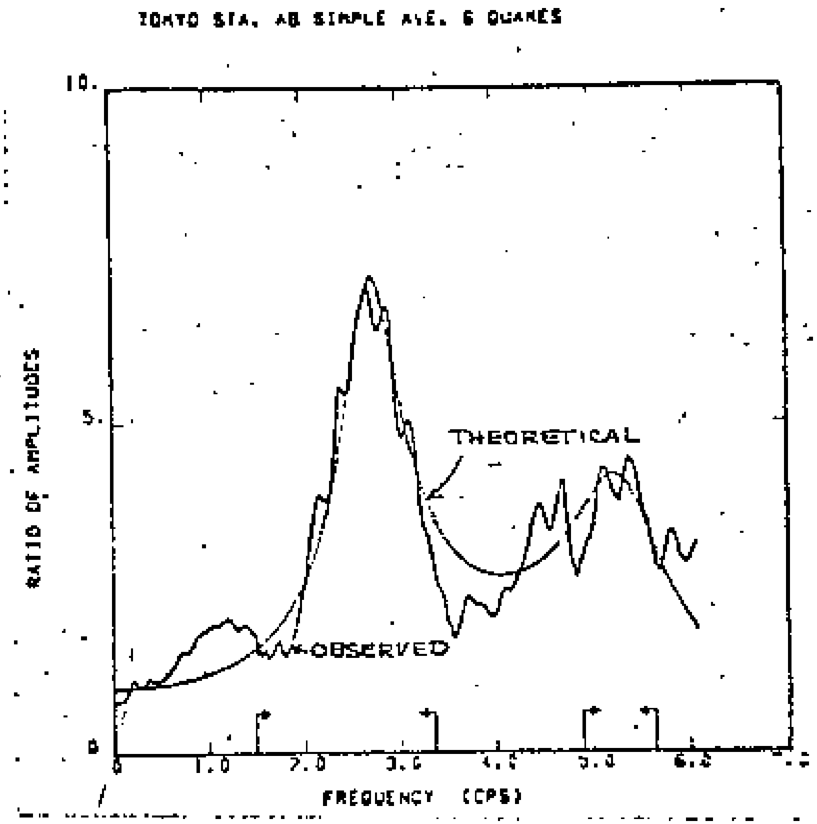

FIGURE 6 THEORETICAL AND OBSERVED AMPLIFICATION CURVES

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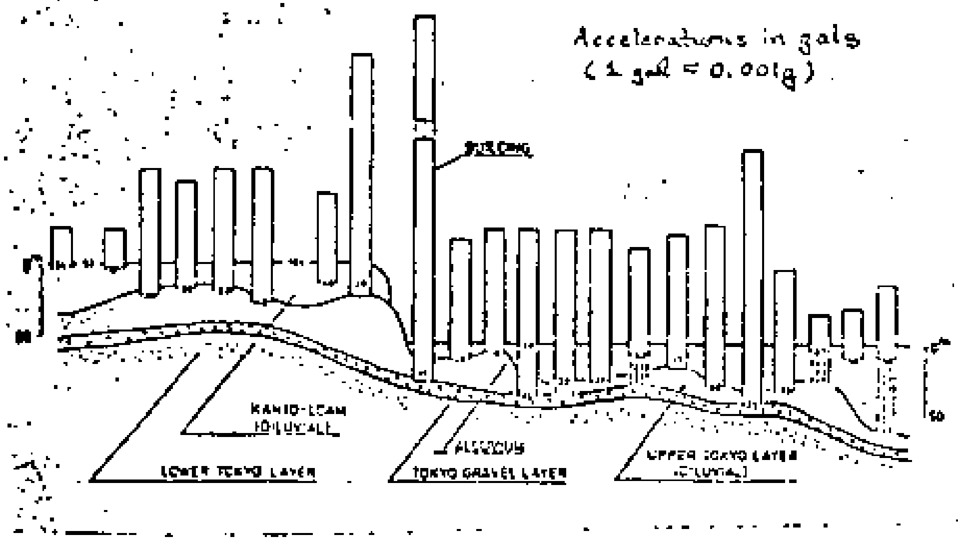
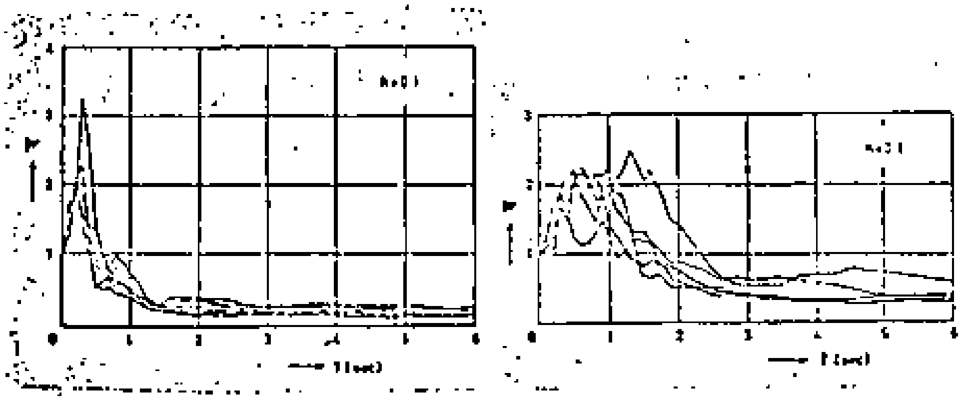
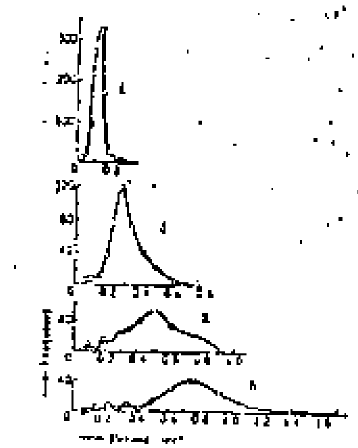


FIGURE 6 PEAK ACCELERATIONS FOR DIFFERENT SOILS AND DEPTHS IN TOKYO DURING EARTHQUAKE IN 1968 (from Ohsaki, 1969)

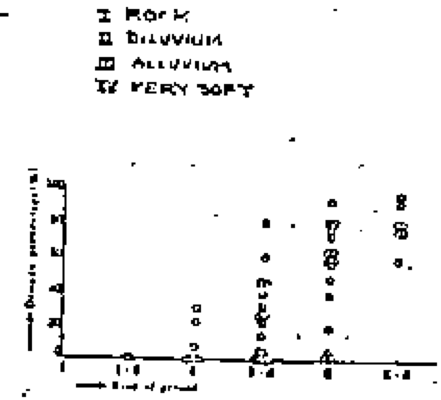


(a) Rock or hard soil (b) Soft multi-layered soil

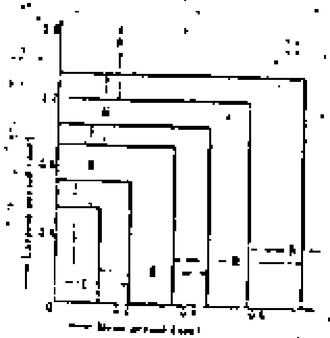
FIGURE 7 RESPONSE SPECTRA (normalized to peak acceleration) FOR DIFFERENT SOIL CONDITIONS IN TOKYO (from Ohsaki, 1969)



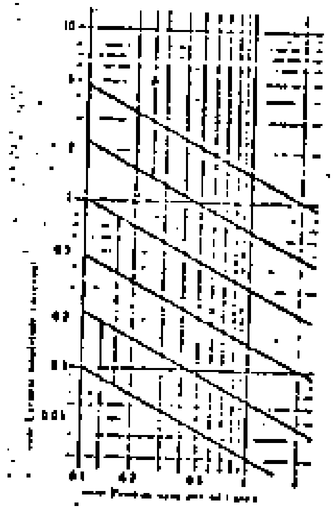
(a) Frequency of occurrence of various periods in microtremor records on different ground



(b). Correlation between type of ground and damage to wooden buildings during 1923 earthquake.



(c) Correlation of mean and largest period with ground type.



(d). Correlation of amplitude and predominant period with ground type

FIGURE 8 USE OF MICROTREMOR MEASUREMENTS TO CLASSIFY GROUND TYPE (after Kanai and Tanaka, 1961)

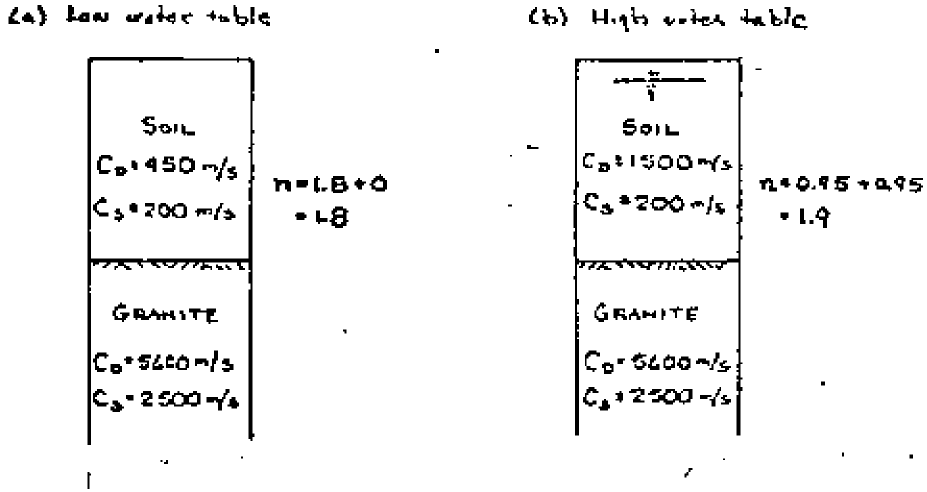


FIGURE 9 EXAMPLE OF MEDVEDEV METHOD

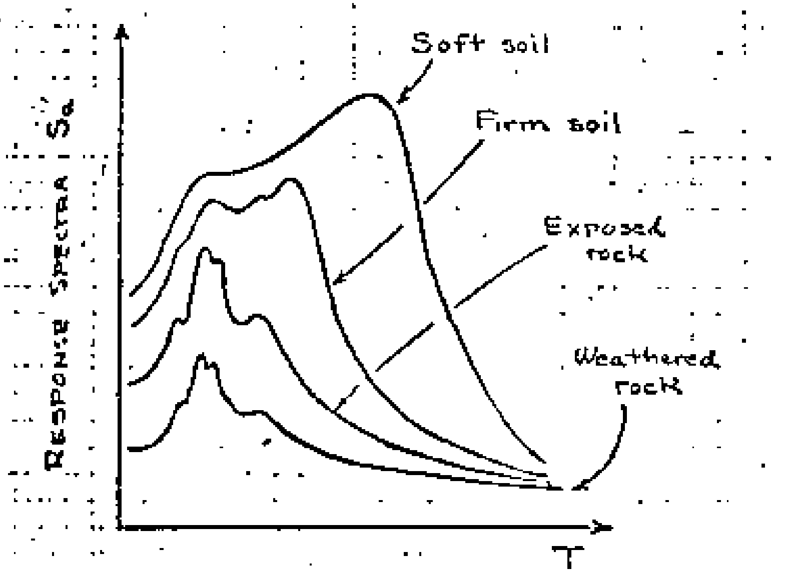


FIGURE 10 SUMMARY OF AMPLIFYING EFFECTS OF SHALLOW SOIL DEPOSIT

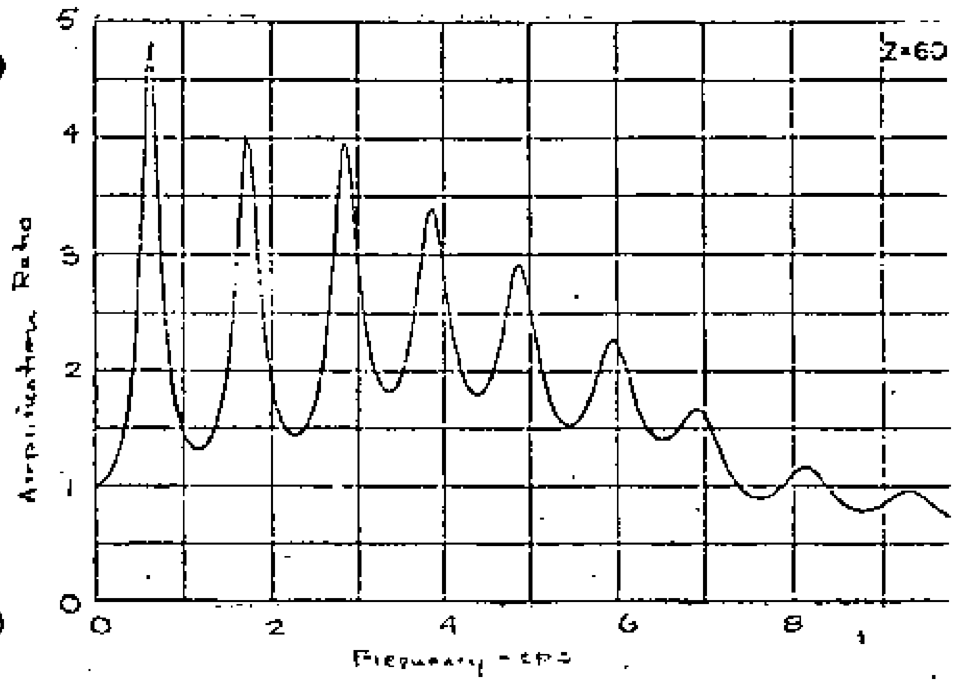


FIGURE 11 AMPLIFICATION SPECTRUM FOR 600 FOOT DEEP SOIL PROFILE

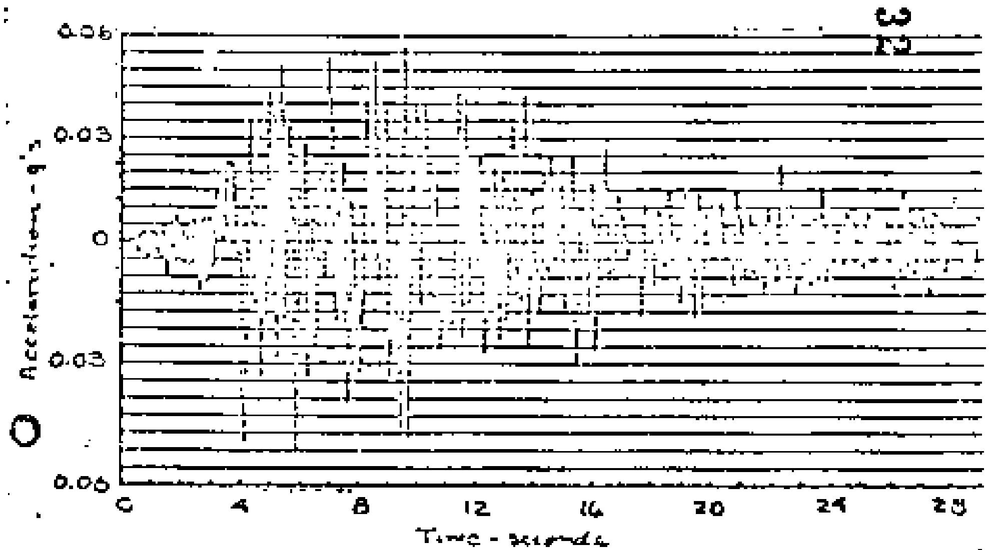


FIGURE 12 ACCELERATION COMPUTED ATOP 450' OF SOIL

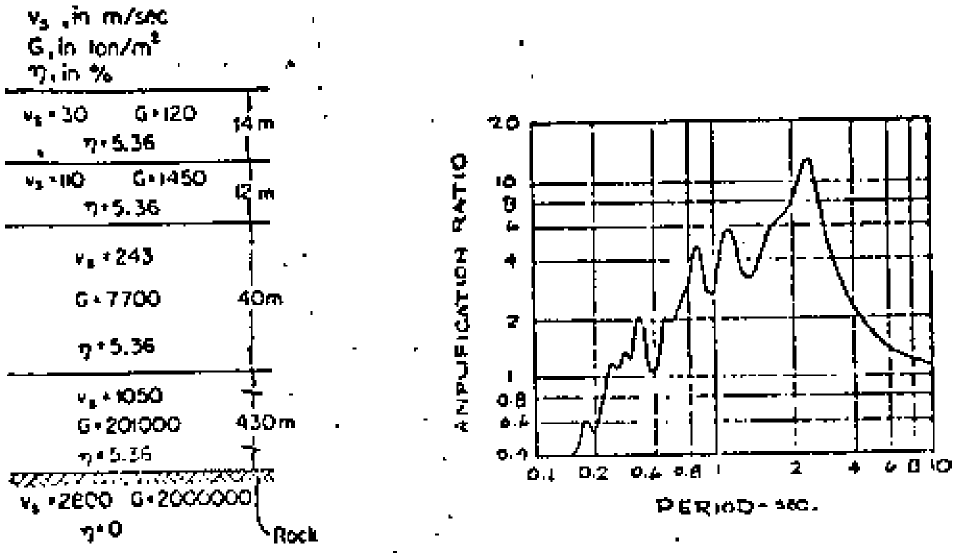


FIGURE 15 AMPLIFICATION SPECTRUM FOR DEEP PROFILE WITH VERY SOFT SOIL

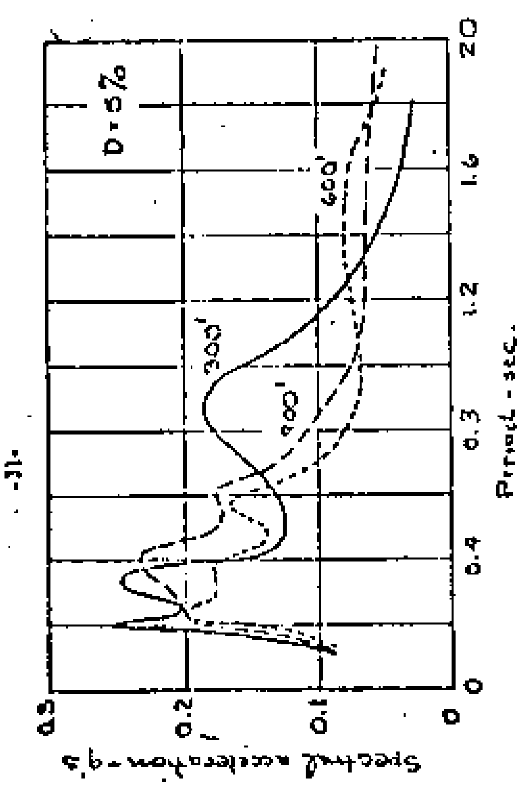


FIGURE 13 RESPONSE SPECTRA (smoothed average for several inputs) FOR DIFFERENT DEEP SOIL PROFILES

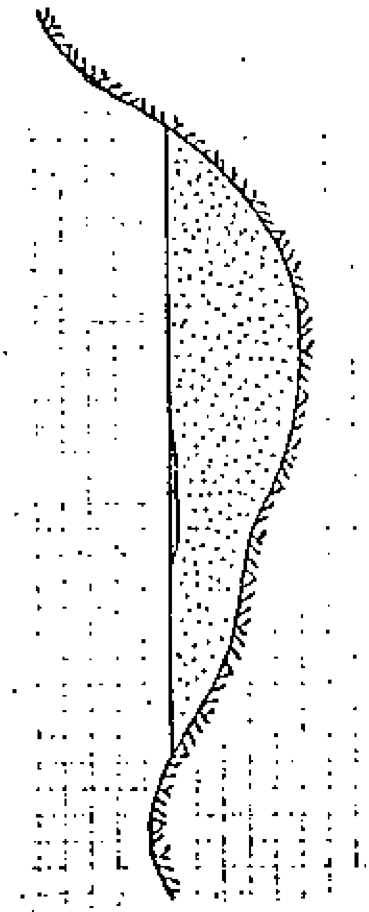


FIGURE 14 SHALLOW STRATA OF SOFT SOIL OVERLYING DEEP DEPOSIT OF FIRM SOIL

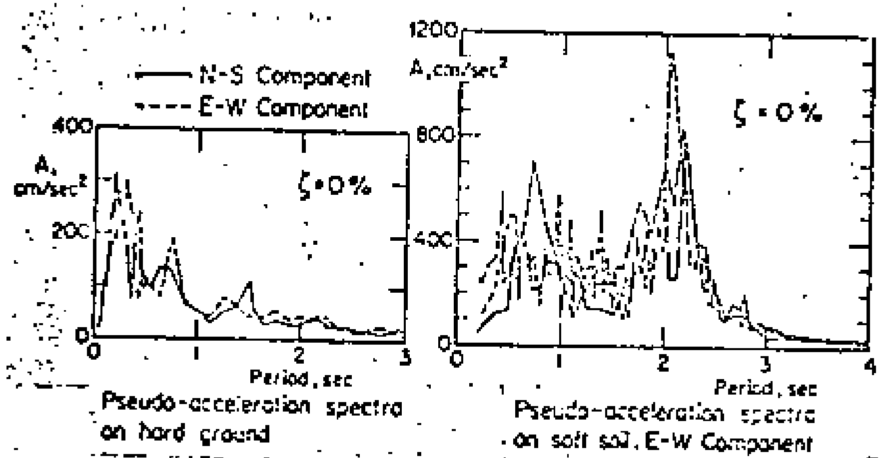


FIGURE 16 RESPONSE SPECTRA FOR HARD AND SOFT GROUND IN MEXICO CITY (from Esteva et al, 1969)

FUNDAMENTALS OF SOIL AMPLIFICATION

by

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March 1969

Preface

The purpose of these notes is to present some of the methods now available to include the effect of local soil conditions in the derivation of design earthquakes or response spectra.

The dynamic characteristics of a soil deposit can be expressed by its transfer function representing the amplification experienced from bottom to top by a sinusoidal steady state motion. The derivation of amplification curves using both a continuous and a discrete solution is presented in II and the relative advantage of each method is discussed.

The general problem of considering an actual earthquake record and filtering it through the soil is discussed in III. Finally, approximate simplified methods are presented to obtain directly response spectra which include the effect of the soil from the knowledge of a response spectrum on firm ground or at bedrock.

Table of Contents

FUNDAMENTALS OF SOIL AMPLIFICATION*

by

J. M. Roesset

I - INTRODUCTION

1.1 Statement of the Problem.

During the initial phases of development, Earthquake Engineering was mainly concerned with developing methods to estimate the response of a structure to given dynamic loads. While there are still many problems to be solved in the area of Structural Dynamics, particularly in the nonlinear range, it is somewhat disturbing to observe the large discrepancy between the accuracy sought by some methods of analysis and the uncertainty in the nature and magnitude of the loads to which the structure will be subjected. This inconsistency has been recognized in the last years and an increasing amount of effort is now being devoted to study the characteristics of earthquake motions as a function of magnitude, distance to the epicenter and local soil conditions. The purpose of this research is to arrive at simple, but realistic methods to represent the characteristics of the ground motion at a particular site. Among these methods one of the most powerful ones is through the use of design response spectra.

Determination of the appropriate earthquake motions at any given site involves two fundamental steps:

1. Evaluation of the seismic risk of the region. For an area with frequent strong earthquakes this step may be relatively easy and engineers may already know that a certain city is periodically subject to earthquakes of some average magnitudes with epicenters at some average distances. For regions with relatively scarce earthquake history, the determination of a design earthquake becomes much more complicated and requires in general geological and seismological studies, which attempt to identify possible sources of earthquakes or active faults.

The results of this step can take different forms, the simplest one being a series of values for probable magnitudes and associated epicentral distances. It is possible from these values, using the formulas suggested by Rosenblueth, to characterize each possible design earthquake by its maximum ground acceleration, velocity and displacement. Newmark has derived a simple approximate method by which the corresponding design spectra can be estimated, knowing these three characteristics. It is possible then to draw response spectra for each design earthquake and to find their average or envelope. Alternatively one can try to generate artificial earthquakes that would have the same average characteristics. It must be realized, however, that if this procedure is used it will not be enough to generate just one sample earthquake for a given set of values of magnitude and epicentral distance. Quite the contrary, a substantial number of samples should be generated and used for each possible earthquake, making the procedure extremely long and costly.

In any case the corresponding design earthquakes or response spectra will apply to an overall region for firm ground conditions.

2. Having obtained one or more earthquake records which could occur at the site on firm ground, or better, a set of design response spectra, the next step is to study how these motions would be modified by the local soil conditions of the particular site where the structure is going to be built. The effect of the soil is going to be one of filtering the motions, increasing their amplitude in some ranges of frequencies and decreasing it in others. This problem is normally referred to as soil amplification and will be the subject of the following discussion.

The particular problem under consideration can then be stated as: Given a soil profile and a design earthquake or response spectrum at bedrock, determine the corresponding earthquake or spectrum at the top of the soil.

It should be noticed that in order to be able to apply these results directly in the dynamic analysis of the structure, it must be assumed that its mass is negligible in comparison to that of the underlying soil. Otherwise a third step is involved corresponding to the problem of soil-structure interaction. In other words it will not be possible to consider the structure and the soil as uncoupled systems.

1.2 General Considerations.

Earthquake motions may be decomposed into a series of waves which propagate from the focus in all directions. Given an infinite medium these waves are basically of two types: dilatational or compressional waves and shear waves. The first are normally called P waves. The second can be decomposed again by projecting the motion in two orthogonal directions. SV waves correspond to motions in a vertical plane, SH waves to horizontal motions. Of course when the direction of propagation is vertical, both SH and SV waves would correspond to horizontal motion.

When the waves propagating through a continuous medium find a free surface, a new type of wave is generated, normally referred to as surface or Rayleigh waves. If, in addition, the medium is not homogeneous, but there is a clear discontinuity at some depth from the free surface, a second type of surface wave, called Love wave, is generated. When there are several surfaces of discontinuities in the properties, other types of waves are created.

The overall problem of following an earthquake as it propagates from its focus is of course a three-dimensional wave propagation problem. By assuming for instance a line source of relatively large length or by considering only the effects at some distance from the epicenter, the problem can be reasonably reduced to a two-dimensional one for SV and P waves and a one-dimensional problem for the propagation of SH waves.

The methods described here relate all of them to the solution of the one-dimensional wave propagation equation. Their basic limitations are thus:

1. Only shear waves are considered, either SH or SV if they are propagating vertically, and only SH if they propagate at an angle. P waves propagating vertically could be considered by replacing the appropriate constants (modulus, wave propagation velocity). Surface waves are, however, neglected.

2. The different layers of soil are assumed to be parallel and extending in the horizontal direction for a distance several times larger than the total depth to bedrock.

In spite of these limitations, the solutions obtained by these methods seem to provide a useful and reasonable estimate of the filtering effect of the soil. Two-dimensional wave propagation problems can now be solved by the use of the finite element method. These techniques offer a promising future. Their application is, however, still limited and there are several questions which still have to be solved before they can be used with confidence.

The filtering effect of the soil can be measured in two different ways:

1. By considering a steady state harmonic oscillation of the soil and the underlying rock and determining the ratio of the amplitude at the free surface of the soil to the amplitude at bedrock or at the outcropping of rock (without any soil on top). This ratio will be a function of the frequency of the motion, and if there is damping, a complex function. It is normally referred to as the Transfer function of the soil. Its modulus is the amplification function, amplification ratio or amplification spectrum.

2. By considering a given earthquake record (time history of acceleration) at bedrock or at the outcropping of rock, and determining the corresponding accelerogram at the free surface of the soil. The result in this case is not only a complete time history of acceleration at the free surface of the soil but also, if so desired, time histories of shear stresses and strains at any point within the soil. It provides therefore a much more complete solution, but it requires considerably

more computer time. Furthermore, because of the reasons previously mentioned, the complete analysis would have to be repeated for each earthquake sample, and it would make little sense to do it for just one record.

Both types of results can be obtained using two different mathematical models:

1. A continuous solution of the differential equation corresponding to the one-dimensional wave propagation problem.
2. A discrete solution replacing each layer of soil by a system of lumped masses and springs and applying standard procedures of Structural Dynamics.

The continuous model offers in general more flexibility and has an economic advantage if the results are desired only at a few points. At present the discrete model requires less computational time when complete histories of accelerations, velocities, strains and stresses are necessary at many points. Both models yield exactly the same results (except for small discrepancies due to different round-off and truncation errors) when:

a) Damping is assumed constant in all modes and viscosity for each layer directly proportional to its modulus and inversely proportional to frequency.

b) The underlying rock is assumed to be rigid or in other words the input motion is considered at bedrock with the soil on top, rather than at the outcropping of rock. For elastic rock, results can still be made to agree if an additional damping is inserted in the discrete model to simulate the loss of energy through radiation in the rock.

In the following pages the basis of both formulations will be presented, considering first the case of steady state periodic motions (determination of the amplification curve), then the case of transient motions. The application of these methods to obtain filtered earth-

quake records at the free surface of the soil is immediate. On the other hand, their application to design response spectra, modifying them to include the effect of the soil, is not so straightforward. This point and approximate solutions are discussed at the end.

II - STEADY STATE PERIODIC MOTIONS

The Amplification Function

II.1 Continuous Solution.

A. Uniform layer. Rigid rock.

Let us consider first a uniform layer of soil resting on rock. The equation of motion corresponding to the one-dimensional wave propagation problem is

$$\rho \frac{\partial^2 u}{\partial t^2} = G \frac{\partial^2 u}{\partial x^2} + \eta \frac{\partial^3 u}{\partial t \partial x^2}$$

where ρ = density or mass per unit volume = $\frac{\gamma}{g}$

γ = unit weight

g = acceleration of gravity

G = shear modulus

η = viscosity constant

$u(x,t)$ = displacement of a point in the soil layer

If the rock is rigid but a displacement $u_G(t)$ is imposed at the base of the soil, the boundary conditions are:

$$\frac{\partial u}{\partial x} = 0 \text{ at } x = 0$$

$$u = u_G(t) \text{ at } x = H$$

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and the initial conditions:

$$u = 0 \text{ at } t = 0$$

$$\frac{\partial u}{\partial t} = 0 \text{ at } t = 0$$

By calling $y = u - u_G$ the relative displacement, the equation can be rewritten as:

$$\rho \frac{\partial^2 y}{\partial t^2} = G \frac{\partial^2 y}{\partial x^2} + \eta \frac{\partial^3 y}{\partial x^2 \partial t} - \rho \frac{\partial^2 u_G}{\partial t^2}$$

with Initial Conditions

$$y = 0, \ \frac{\partial y}{\partial t} = 0 \qquad \text{at } t = 0$$

and Boundary Conditions

$$y = 0 \qquad \text{at } x = H$$

$$\frac{\partial y}{\partial x} = 0 \qquad \text{at } x = 0$$

If $u_G(t) = 0$, the free vibrations can be investigated. Writing then

$$\rho \frac{\partial^2 y}{\partial t^2} = G \frac{\partial^2 y}{\partial x^2} + \eta \frac{\partial^3 y}{\partial x^2 \partial t}$$

and trying a solution of the form

$$y = U(x) \cdot Y(t)$$

where U is a function only of x

Y is a periodic function of t alone

$$\frac{U''}{U} = \rho \frac{\ddot{Y}}{GY + \eta \dot{Y}} = -p^2$$

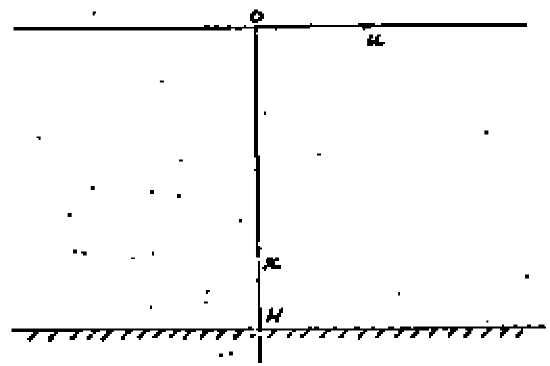

FIG. 1 - UNIFORM LAYER

The natural frequencies of the layer of soil are then given by

$$f_n = \frac{2n-1}{4H}\sqrt{\frac{G}{\rho}} = \frac{(2n-1)C_s}{4H}$$

$$\omega_n = \frac{(2n-1)\pi}{2H}\sqrt{\frac{G}{\rho}} = \frac{(2n-1)\pi C_s}{2H}$$

and the natural periods

$$T_n = \frac{4H}{2n-1}\sqrt{\frac{\rho}{G}} = \frac{4H}{(2n-1)C_s}$$

where $C_s = \sqrt{\frac{G}{\rho}}$ is the shear wave velocity of the soil.

The corresponding modal shapes are

$$U = \sin \frac{(2n-1)\pi}{2H} x$$

If the soil has viscosity $\eta \neq 0$ in order to have harmonic motion we must have

$$\eta < \frac{4H}{(2n-1)\pi}\sqrt{G\rho}$$

A critical value of viscosity can be established for each mode. In particular in order to have at least 1 mode

$$\eta < \eta_{1\,crit} = \frac{4H}{\pi}\sqrt{G\rho}$$

It is important to realize that if there is any viscosity the number of modes will be finite. (Higher modes will have damping higher than critical).

Considering now the forced vibration problem, it is convenient for a steady state periodic motion to represent the base displacement by

$$u_G(t) = C\, e^{i\Omega t}$$

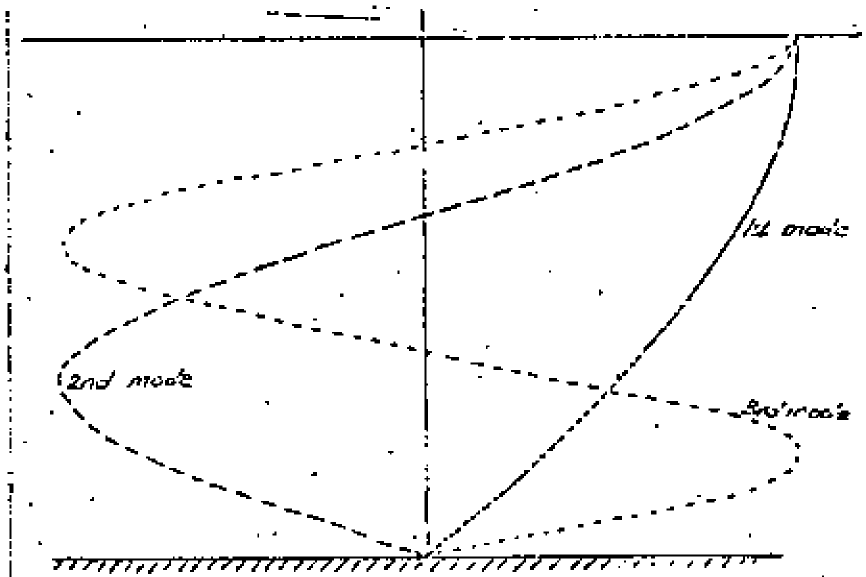

FIG. 2. MODAL SHAPES FOR A UNIFORM LAYER

and the solution by

$$y(x,t) = U(x)e^{i\Omega t}$$

then

$$U = E \cos px + F \sin px + C$$

$$\text{with } p^2 = \frac{\rho\Omega^2}{G + i\eta\Omega}$$

Imposing the boundary conditions

$$F = 0$$

$$E \cos pH + C$$

$$U = C\left(\frac{\cos px}{\cos pH} - 1\right)$$

and $\ddot{y} = -\Omega^2 C\left(\frac{\cos px}{\cos pH} - 1\right)e^{i\Omega t}$

$$\ddot{u} = \ddot{y} + \ddot{u}_G = -\Omega^2 C\,\frac{\cos px}{\cos pH}\,e^{i\Omega t}$$

and at the free surface of the soil the absolute acceleration becomes

$$\ddot{u} = -\Omega^2 C\,\frac{1}{\cos pH}\,e^{i\Omega t}$$

Since the base motion was $\ddot{u}_G = -\Omega^2 C\,e^{i\Omega t}$

$$\ddot{u} = \frac{1}{\cos pH}\,\ddot{u}_G$$

The transfer function of the soil for absolute acceleration at the free surface is then defined as:

$$TF(\ddot{u}) = \frac{1}{\cos pH}$$

It should be noticed that if there is viscosity, p is a complex variable and therefore cos pH has to be interpreted as

$$1/2\,(e^{ipH} + e^{-ipH})$$

or

$$TF(\ddot{u}) = \frac{2}{e^{ipH} + e^{-ipH}}$$

The fact that the transfer function is complex indicates that there is both a change in amplitude and in phase. If only the change in amplitude is considered the Amplification Function is defined as the modulus of the transfer function.

$$A(\Omega) = \frac{2}{|e^{ipH} + e^{-ipH}|}$$

Calling

$$\alpha = \frac{1}{\sqrt{2}}\,H\Omega\,\sqrt{\frac{\rho}{G}}\,\sqrt{\frac{\sqrt{1 + (\eta\Omega/G)^2} - 1}{1 + (\eta\Omega/G)^2}}$$

$$\beta = \frac{1}{\sqrt{2}}\,H\Omega\,\sqrt{\frac{\rho}{G}}\,\sqrt{\frac{\sqrt{1 + (\eta\Omega/G)^2} + 1}{1 + (\eta\Omega/G)^2}}$$

$$A(\Omega) = \frac{1}{\sqrt{\cos h^2\alpha\,\cos^2\beta + \sin h^2\alpha\,\sin^2\beta}}$$

For small values of $\eta\frac{\Omega}{G}$

$$\alpha = \frac{1}{2}\,H\Omega\sqrt{\frac{\rho}{G}}\;\eta\frac{\Omega}{G} \simeq \frac{1}{2}\,\frac{\mu\,\Omega^2}{V_s G}$$

$$\beta = H\Omega\sqrt{\frac{\rho}{G}}$$

and $$A(\Omega) = \frac{1}{\sqrt{\cos^2\beta + \alpha^2 \sin^2\beta}}$$

In particular if there is no viscosity $\eta = 0$

$$A(\Omega) = \frac{1}{\cos\beta}$$

and the amplification will become infinity at $\beta = \frac{(2n-1)\pi}{2}$ which corresponds to $\Omega = \frac{(2n-1)\pi}{2H}\sqrt{\frac{G}{\rho}} = \omega_n$, nth natural frequency of the layer.

On the other hand, if $\eta \neq 0$, the amplification will not become infinity. Two cases can then be considered:

1. If the value of η is constant as Ω increases, the first expression will have to be used since $\eta\frac{\Omega}{G}$ will not be small any longer. As Ω increases $A(\Omega)$ tends to zero, which means that for very large input frequencies the top of the layer remains at rest. The amplification function will have only a finite number of peaks corresponding to those natural frequencies of the layer which have damping less than critical.

For values of Ω such that $\eta\frac{\Omega}{G}$ is still small, the amplification at $\Omega = \omega_n$, nth natural frequency of the layer, becomes

$$A(\omega_n) = \frac{4}{(2n-1)\pi}\ \frac{1}{\eta\omega_n/G} = \frac{2\,[\text{illegible}]}{\pi\eta}\ \frac{1}{(2n-1)^2}$$

This shows that the amplitude of the peak at the second natural frequency of the layer will be $\frac{1}{9}$ of that at the first, the amplitude of the third will be $\frac{1}{25}$ etc.

2. If it is assumed that the viscosity is inversely proportional to the frequency so that $\eta\frac{\Omega}{G} = \tan\delta$ is a constant, for small values of $\eta\frac{\Omega}{G}$

$$A(\omega_n) = \frac{4}{(2n-1)\pi\tan\delta} = \frac{4}{(2n-1)\pi}\cdot\frac{1}{2\beta}$$

with $$\beta = \frac{1}{2}\tan\delta = \text{fraction of critical damping}$$

In this case the amplitude of the second peak will be 1/3 that of the 1st, the amplitude of the third peak 1/5 and so on.

Comparing these results with those for a lumped mass discrete system as normally encountered in Structural Dynamics, we can say that a constant value of viscosity η corresponds to an increasing percentage of damping in each mode, whereas a constant value of $\eta\frac{\Omega}{G}$ corresponds to constant damping in all modes.

Figures 3 and 4 show the amplification curve for a uniform soil layer with the following characteristics:

Depth	$h = 100'$
Shear wave velocity	$c_s = 750$ ft/sec.
Unit weight	$\gamma = 125$ lbs/cubic ft.

8. Uniform layer. Elastic rock

Taking now two sets of axes, one with origin at the free surface of the soil, the second with origin at the top of the rock, the motions in the soil and the rock can be expressed as

$$u_s = E_s\, e^{i(p_s x_s + \Omega t)} + F_s\, e^{-i(p_s x_s - \Omega t)}$$

$$u_r = E_r\, e^{i(p_r x_r + \Omega t)} + F_r\, e^{-i(p_r x_r - \Omega t)}$$

where $$p_s^2 = \frac{\rho_s\Omega^2}{G_s + i\eta_s\Omega}$$

$$p_r^2 = \frac{\rho_r\Omega^2}{G_r + i\eta_r\Omega}$$

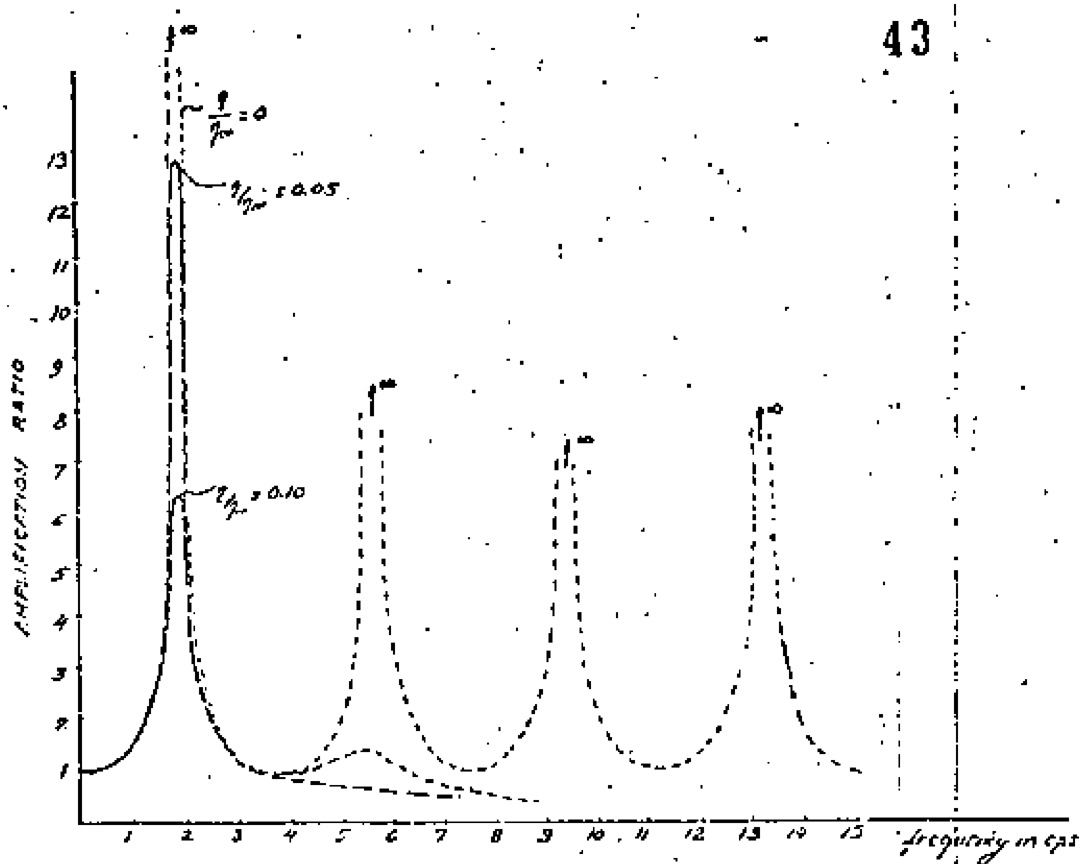

FIG. 3. AMPLIFICATION CURVE FOR UNIFORM LAYER
RIGID ROCK - CONSTANT VISCOSITY

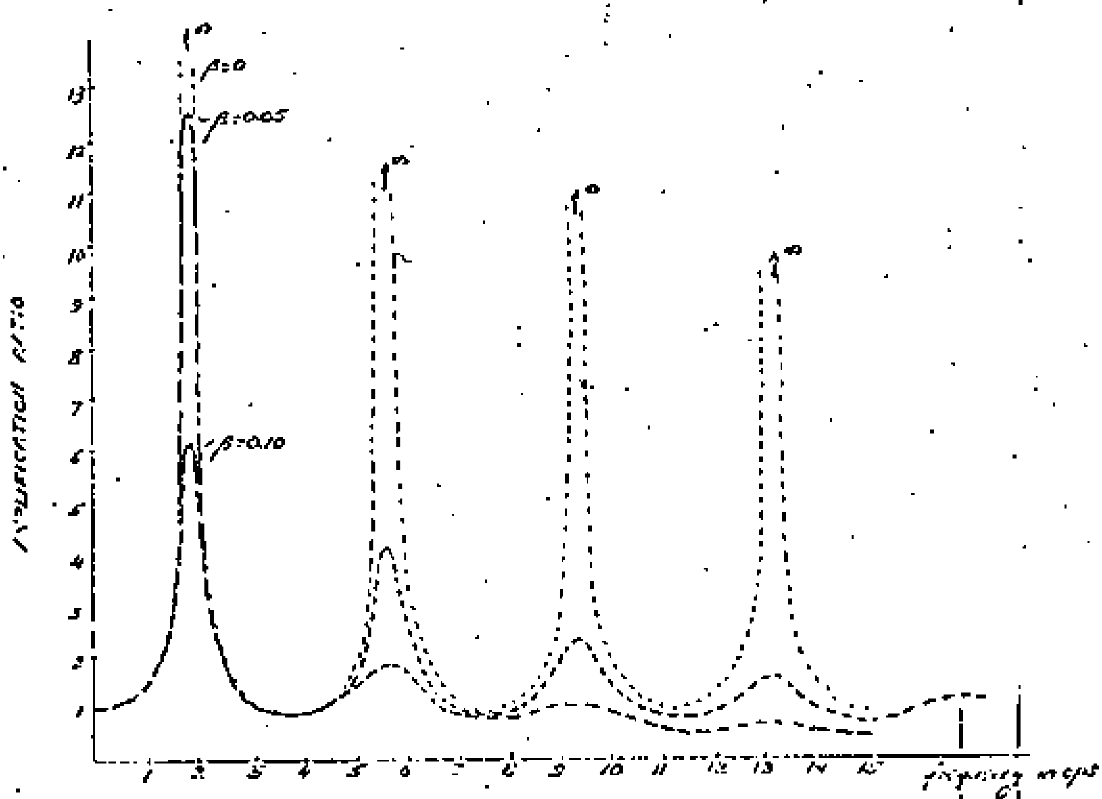

FIG. 4. AMPLIFICATION CURVE FOR UNIFORM LAYER
RIGID ROCK - CONSTANT MODAL DAMPING

The boundary conditions are now

$$\frac{\partial u_s}{\partial x_s} = 0 \quad \text{at } x_s = 0$$

$$u_s(x_s = H) = u_r(x_r = 0)$$

$$(G_s + i\eta_s\Omega)\,\frac{\partial u_s}{\partial x_s}\,(x_s = H) = (G_r + i\eta_r\Omega)\,\frac{\partial u_r}{\partial x_r}\,(x_r = 0)$$

The result is

$$E_s = F_s$$

$$u_s = E_s\, e^{i\Omega t}\left(e^{ip_s x_s} + e^{-ip_s x_s}\right)$$

$$E_s = \frac{2E_r}{e^{ip_sH}(1+\mu) + e^{-ip_sH}(1-\mu)}$$

$$F_r = E_r\,\frac{e^{ip_sH}(1-\mu) + e^{-ip_sH}(1+\mu)}{e^{ip_sH}(1+\mu) + e^{-ip_sH}(1-\mu)}$$

with

$$\mu = \frac{p_s(G_s + i\eta_s\Omega)}{p_r(G_r + i\eta_r\Omega)}$$

$$u_s(x_s = 0) = 2E_s\, e^{i\Omega t}$$

$$u_s(x_s = H) = u_r(x_r = 0) = (E_r + F_r)e^{i\Omega t} = E_s\left(e^{ip_sH} + e^{-ip_sH}\right)e^{i\Omega t}$$

The ratio between the displacement (or acceleration) u at the free surface of the soil, and the displacement (or acceleration) at the interface between rock and soil can then be expressed as

$$TF = \frac{2}{e^{ip_sH} + e^{-ip_sH}}$$

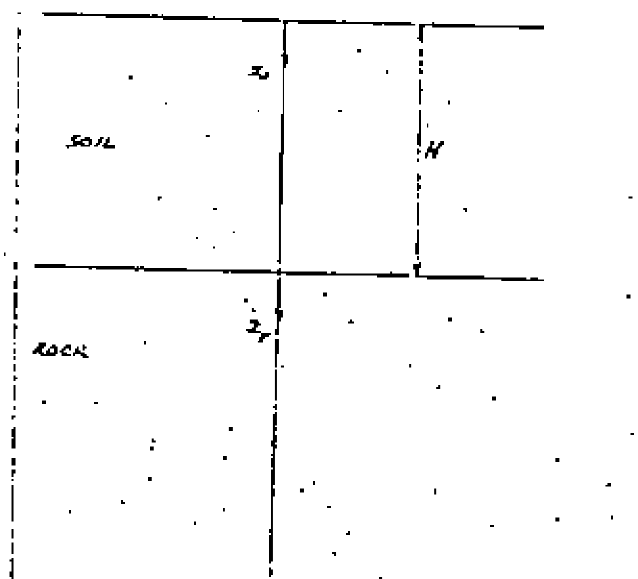

FIGURE 5. UNIFORM LAYER ON ELASTIC ROCK

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which is the same expression previously obtained for rigid rock.

If on the other hand one considers the situation of the rock without soil on top, the motion at the outcropping of rock would be

$$u = 2E_r\, e^{i\Omega t}$$

and the ratio between the displacement (or acceleration) at the free surface of the soil, and the displacement (or acceleration) at the outcropping of rock would be

$$TF = \frac{2}{e^{ip_sH}(1+\alpha)+e^{-ip_sH}(1-\alpha)}$$

A second amplification function can thus be defined by considering the elastic properties of the rock. This function gives now the ratio between amplitudes of motion on top of the soil to the amplitude of the motion that would be felt on top of the rock if the soil were not there.

$$A(\Omega) = \frac{2}{|e^{ip_sH}(1+\alpha)+e^{-ip_sH}(1-\alpha)|}$$

In this case if there is no viscosity $\eta_s = \eta_r = 0$

$$p_s = \frac{\Omega}{C_s} \qquad p_r = \frac{\Omega}{C_r}$$

$$\alpha = \frac{p_sG_s}{p_rG_r} = \frac{G_sC_r}{G_rC_s} = \sqrt{\frac{G_s\rho_s}{G_r\rho_r}} = \sqrt{\frac{G_s\gamma_s}{G_r\gamma_r}} = \frac{\gamma_sC_s}{\gamma_rC_r}$$

and

$$A(\Omega) = \frac{1}{|\cos p_sH + i\alpha \sin p_sH|}$$

At the natural frequencies of the layer $\Omega = \omega_n$ $\cos p_sH = 0$ and $\sin p_sH = (-1)^n$

$$A(\omega_n) = \frac{\gamma_rC_r}{\gamma_sC_s}$$

It can be seen that this amplification ratio does not become infinity even if the soil has no viscosity.

If the rock has no viscosity $\eta_r = 0$, and the soil has a viscosity η_s so that $\eta_s\Omega/G_s$ is small, one can again derive an approximate formula for the amplification at the natural frequencies $\Omega = \omega_n$

$$A(\omega_n) = \frac{\gamma_rC_r}{\gamma_sC_s} \cdot \frac{1}{1+\frac{(2n-1)\pi}{4}\frac{\gamma_rC_r}{\gamma_sC_s}\cdot\frac{\eta_s\omega_n}{G_s}}$$

For the case of constant viscosity η_s

$$A(\omega_n) = \frac{\gamma_rC_r}{\gamma_sC_s} \cdot \frac{1}{1+\frac{\gamma_rC_r}{\gamma_sC_s}\cdot\frac{\eta_s}{\eta_{s\,crit}}(2n-1)^2\frac{\pi}{2}}$$

and for the case of constant $\frac{\eta_s\Omega}{G} = 2\beta$

$$A(\omega_n) = \frac{\gamma_rC_r}{\gamma_sC_s} \cdot \frac{1}{1+\frac{\gamma_rC_r}{\gamma_sC_s}\cdot 2\beta\cdot\frac{(2n-1)\pi}{4}}$$

Figures 6 and 7 show amplification functions with elastic rock for the same uniform layer of soil previously considered ($\gamma_r = 140$, $C_r = 4500$).

The main difference between both cases is the fact that the rock considered as an elastic half space is dissipating energy by radiation. Both solutions are then approximately related by the

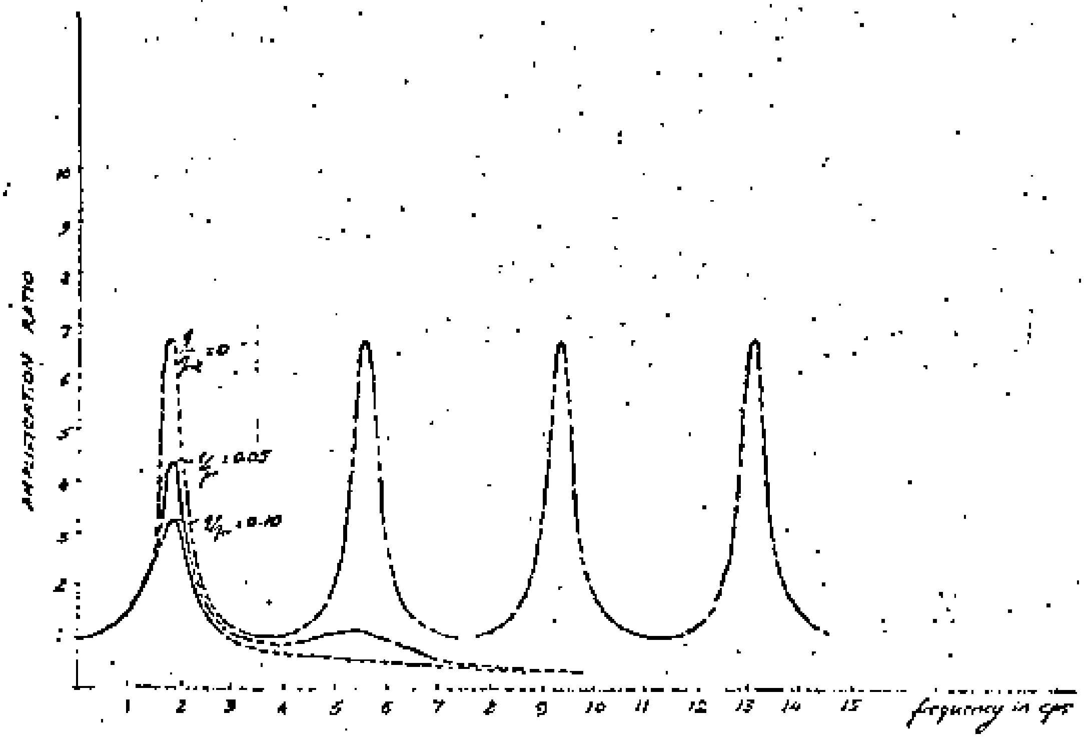

FIG 6. AMPLIFICATION CURVE FOR UNIFORM LAYER.
ELASTIC ROCK - CONSTANT VISCOSITY

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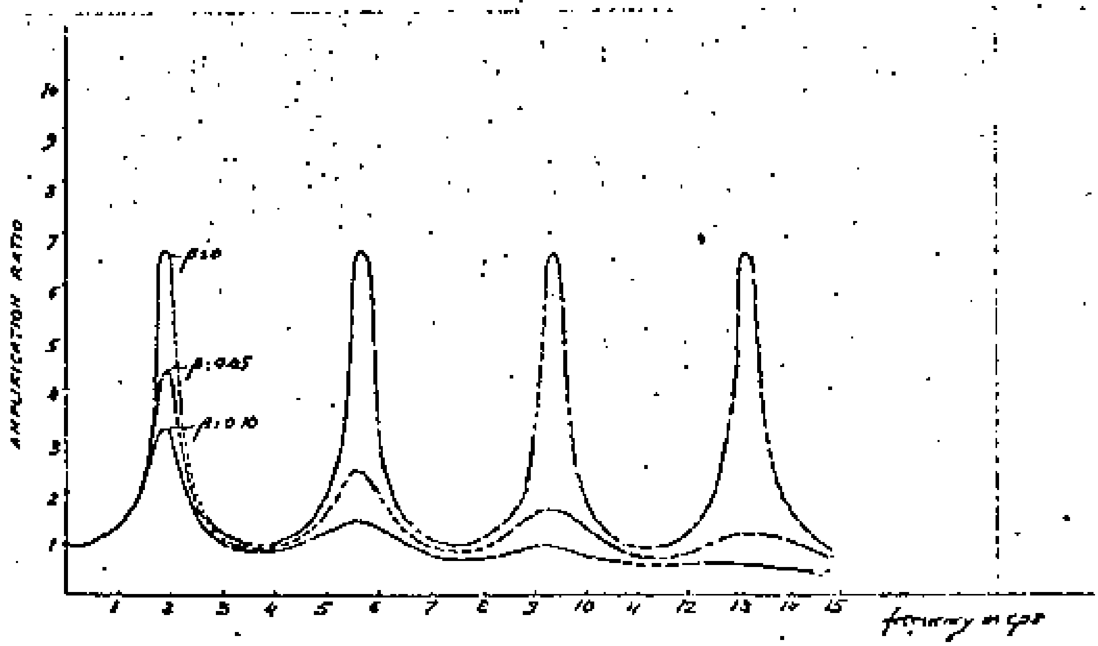

FIG 7. AMPLIFICATION CURVE FOR UNIFORM LAYER
ELASTIC ROCK - CONSTANT MODAL DAMPING.

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expression

$$\frac{1}{A_2(\omega)} = \frac{\gamma_s C_s}{\gamma_r C_r} + \frac{1}{A_1(\omega)}$$

where $A_1(\omega)$ is the amplification function with rigid rock

$A_2(\omega)$ is the amplification function with elastic rock.

This formula can be reproduced by adding to the first case an equivalent radiation damping, function of frequency

$$\beta_{eq} = \frac{2}{\pi} \cdot \frac{\gamma_s C_s}{\gamma_r C_r} \cdot \frac{1}{2n-1}$$

or $$\beta_{eq} = \frac{2}{\pi} \; \frac{\gamma_s C_s}{\gamma_r C_r} \cdot \frac{\omega_1}{\omega}$$

since $\eta \frac{\omega}{G} = 2\beta$ this represents

$$\eta_{eq} = \frac{G}{\omega} 2\beta_{eq} = \frac{4}{\pi} G_s \frac{\gamma_s C_s}{\gamma_r C_r} \cdot \frac{\omega_1}{\omega^2}$$

C. Multilayered System.

When the soil deposit is made of several layers with different properties (Fig. 8) one can define for each layer j

h_j = thickness
G_j = shear modulus
γ_j = unit weight
C_j = shear wave velocity
η_j = viscosity coefficient

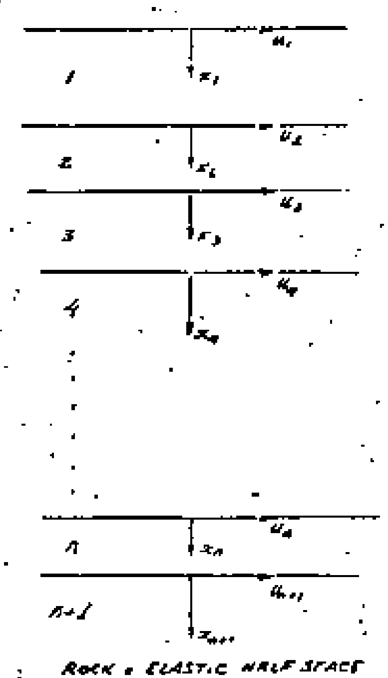

FIG 8: MULTIPLE STRATA OVER ELASTIC ROCK

$$p_j^2 = \frac{\rho_j \Omega^2}{G_j + i\eta_j \Omega}$$

and

$$\nu_j = \frac{p_j(G_j + i\eta_j\Omega)}{p_{j+1}(G_{j+1} + i\eta_{j+1}\Omega)}$$

Displacement in each layer with respect to a local set of coordinate axes with origin at the top of the layer can then be expressed as

$$u_j = E_j\, e^{i(p_j x_j + \Omega t)} + F_j\, e^{-i(p_j x_j + \Omega t)}$$

By establishing compatibility between each layer and the next, one can write

$F_1 = E_1$ (because of the free surface condition)

$$2E_2 = E_1[e^{ip_1h_1}(1+\nu_1) + e^{-ip_1h_1}(1-\nu_1)]$$

$$2F_2 = E_1[e^{ip_1h_1}(1-\nu_1) + e^{-ip_1h_1}(1+\nu_1)]$$

$$2E_3 = E_2(1+\nu_2)e^{ip_2h_2} + F_2(1-\nu_2)e^{-ip_2h_2}$$

$$2F_3 = E_2(1-\nu_2)e^{ip_2h_2} + F_2(1-\nu_2)e^{-ip_2h_2}$$

and

$$2E_{n+1} = E_n(1+\nu_n)e^{ip_nh_n} + F_n(1-\nu_n)e^{-ip_nh_n}$$

$$2F_{n+1} = E_n(1-\nu_n)e^{ip_nh_n} + F_n(1+\nu_n)e^{-ip_nh_n}$$

By replacing into the expressions for E_3, F_3 the values of $E_2 F_2$ in terms of E_1, then these ones into the expressions for E_4, F_4 and so on one can finally obtain

$$E_{n+1} = aE_1$$

$$F_{n+1} = bE_1$$

The amplification function with rigid rock (ratio of displacement or acceleration at top of soil to displacement or acceleration at bedrock) is then

$$A_1(\Omega) = \left| \frac{2E_1}{E_{n+1} + F_{n+1}} \right| = \frac{2}{|a+b|}$$

The amplification function with elastic rock (ratio of displacement or acceleration at top of soil to displacement or acceleration at the outcropping of rock) is

$$A_2(\Omega) = \frac{|2E_1|}{|2E_{n+1}|} = \frac{1}{|a|}$$

The explicit expression for the amplification function in terms of the soil properties becomes too long even for two layers. However, the numerical computation proceeding from layer to layer is simple and adapts itself very well to be programmed in a digital computer.

It is possible to have any kind of viscosity (constant or an arbitrary function of frequency) in any layer. On the other hand for the purpose of comparing the results with those obtained by other methods (modal analysis of the discrete model) a case which becomes easy to interpret is that in which η/G is equal for all layers.

Then if $\frac{\eta_j}{G}$ is constant, independent of frequency, the resulting amplification function will have in each natural frequency a percentage of critical damping increasing linearly with the frequency ω. The magnitude of the amplification in the nth natural frequency will thus be proportional to $1/\omega_n^2$.

If $\frac{\eta_j}{G_j} \cdot \Omega = 2\beta$ is constant, the resulting amplification function will have a constant percentage of critical damping in all the modes. The magnitude of the amplification will thus be proportional to $1/\omega_n$.

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While it is not possible to find an exact simple formula to reproduce the effect of the elastic rock, it has been found that good results can be obtained by taking some average properties for the soil

$$\gamma_{s\ average} = \frac{\sum \gamma_j h_j}{\sum h_j}$$

$$c_{s\ average} = \frac{\sum c_{sj} h_j}{\sum h_j}$$

and writing $\quad \dfrac{1}{A_2(\omega)} = \dfrac{\gamma_{save}\ c_{save}}{\gamma_r\ c_r} + \dfrac{1}{A_1(\Omega)}$

or adding an equivalent radiation damping

$$\beta_{eq} = \frac{2}{\pi} \cdot \frac{\gamma_{save}\ c_{save}}{\gamma_r c_r}\ \frac{\omega_1}{\Omega}$$

(The correct formula would be

$$A_2(\Omega) = \frac{A_1(\Omega)}{1 + \dfrac{\tau_b}{\rho_r c_r \dot{u}_b \Omega}} = \frac{A_1(\Omega)}{1 - \dfrac{\tau_b \Omega}{\rho_r c_r \ddot{u}_b}}$$

where τ_b is the shear stress at the base of the soil
$\dot{u}_b$ the base displacement
$\ddot{u}_b$ the base acceleration.

Figures 9 and 10 show the amplification functions for a multi-layered soil profile with the following characteristics:

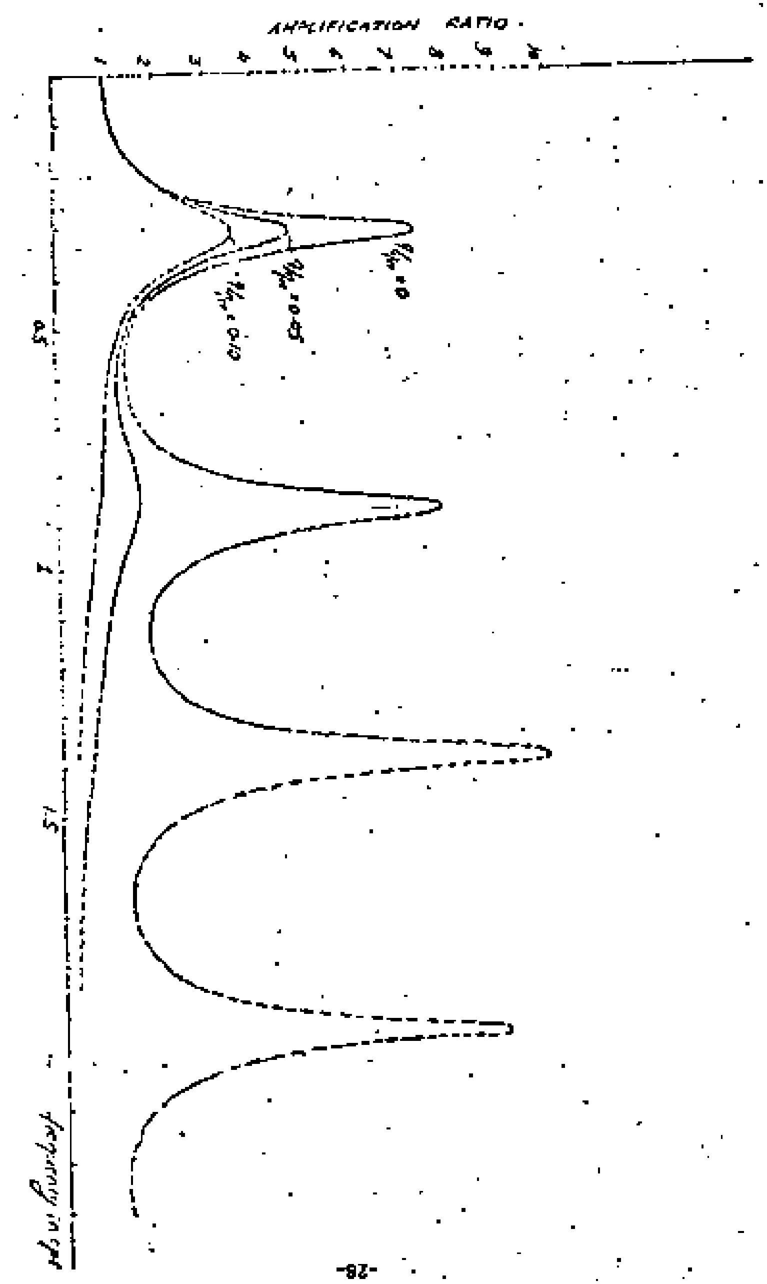


FIG. 9. AMPLIFICATION CURVE FOR LAYERED SOIL ELASTIC ROCK - CONSTANT VISCOSITY RATIO

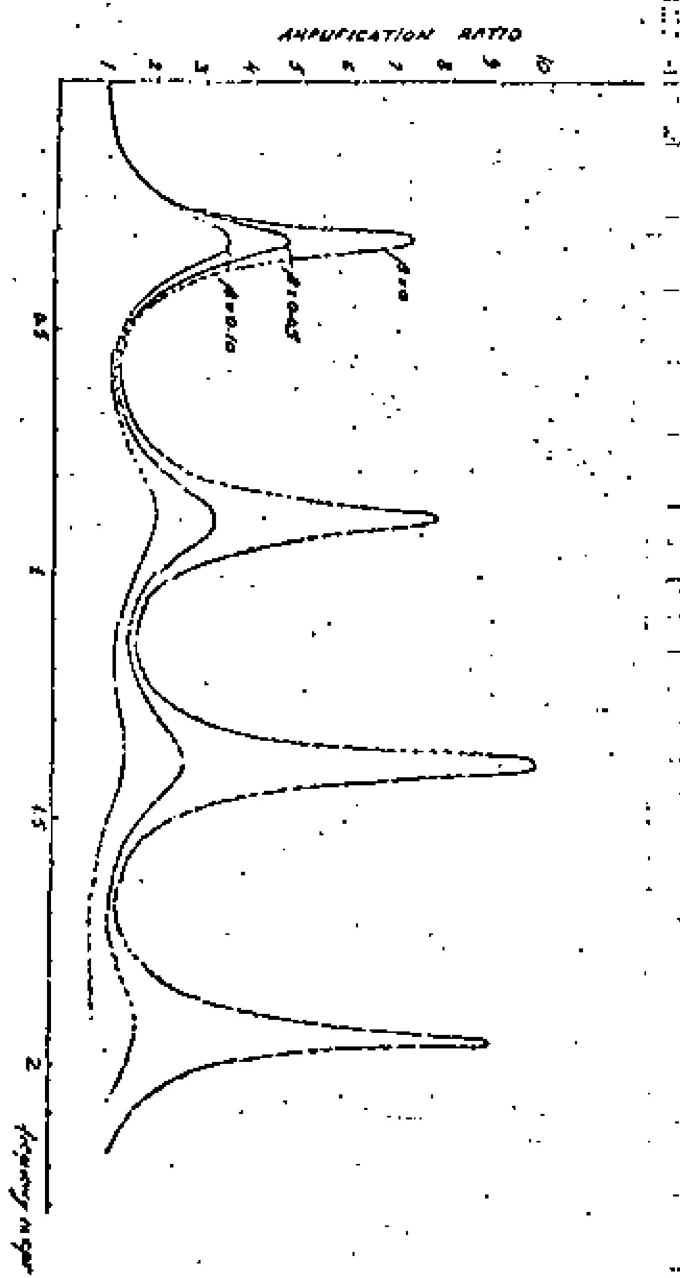
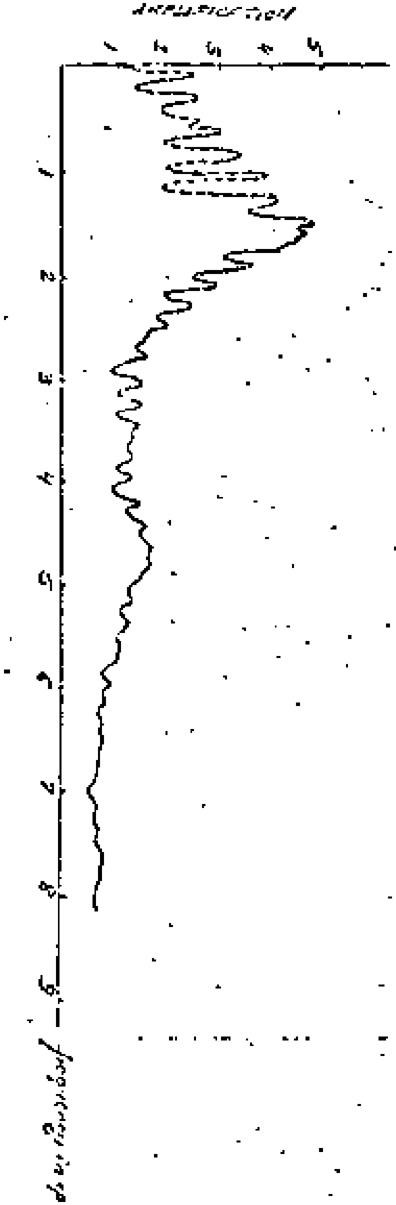


FIG 10. AMPLIFICATION CURVE FOR LAYERED SOIL
ELASTIC ROCK - CONSTANT MODAL DAMPING

Layer No.	Thickness ft.	Shear Wave Velocity ft./sec.	Unit Weight lb./ft.^{3}
1	10	714	100
2	150	697	120
3	70	1200	125
4	500	1300	125
5	400	1500	135
Rock		8000	150

In both cases it has been assumed that the ratio n/G is the same for all layers. It should be noticed that for $n = 0$ the maximum amplification does not occur at the first natural frequency but at the third one. This effect is more evident when different values of damping are considered for each layer, particularly in the case of soft layers of soil over relatively hard strata. Figure 11 shows the amplification curve for a soil profile which would correspond to the conditions at El Centro, California. It can be seen in this case that the amplification curve is basically the product of two functions: one corresponding to the relatively hard bottom of 11,000 feet with a fundamental frequency of about 0.1 cps, the other corresponding to the top 100 feet of soft soil with a fundamental frequency of about 1.5 cps. The amplification due to these top 100 feet is larger than that due to the remainder of the soil and the maximum peak in the combined amplification curve occurs in the range of 1.5 cps. It is important to notice that this simplified approach, lumping several layers of soil with similar properties into one layer with average properties and reducing the total system to just two layers which can be considered independently, can often be successfully applied for preliminary estimates. Of course to be able to treat the two resulting layers independently, multiplying the corresponding amplification functions at each point, it is necessary to be able to treat them as uncoupled, or what is the same, the mass of the top layer should be considerably smaller than that of the bottom layer.



Layer	h	G	γ	β
1	100	650	128	.11
2	1000	2770	139	0.03
3	3000	3520	139	0.005
4	3500	4020	147	0.002
5	3500	7200	107	0.
Rock		10000	172	

FIG 11. AMPLIFICATION CURVE FOR LAYERED SOIL (EL CENTRO)
ELASTIC ROCK - VARIABLE DAMPING IN LAYERS.

II.2 Discrete Model.

The basis for the discrete model is to replace each layer of soil by a series of lumped masses connected by a spring and dashpot. The resulting system (Fig. 12) is of course a familiar one for engineers working in Structural Dynamics.

For any given layer of thickness h_j, shear modulus G_j, unit weight γ_j and viscosity coefficient η_j, replaced by n_j discrete masses, one would have

$$M_1 = \frac{1}{2}\frac{\gamma_j}{g}\cdot\frac{h_j}{n_j}$$

$$M_2 = M_3 = M_4 = \ldots = M_{n_j} = \frac{\gamma_j}{g}\,\frac{h_j}{n_j}$$

Of course at the interface between two layers the total lumped mass would be

$$\frac{1}{2g}\left(\frac{\gamma_j h_j}{n_j} + \frac{\gamma_{j+1}h_{j+1}}{n_{j+1}}\right)$$

$$k_1 = k_2 = k_3 = \ldots = k_{n_j} = \frac{G_j n_j}{h_j}$$

and $$C_1 = C_2 = C_3 \quad \ldots = \frac{n_j \eta_j}{h_j}$$

where C_i are the dashpot constants.

It is thus possible to write the equation of motion for each mass forming a stiffness matrix, a damping matrix and a mass matrix (diagonal). The solution of the problem falls then within the classical methods of Structural Dynamics and requires no further explanation here. It is important to notice, however, that if the dashpot constants are arbitrary (η/G variable from one layer to another) it is necessary to solve the problem by physical integration of the equations of motion.

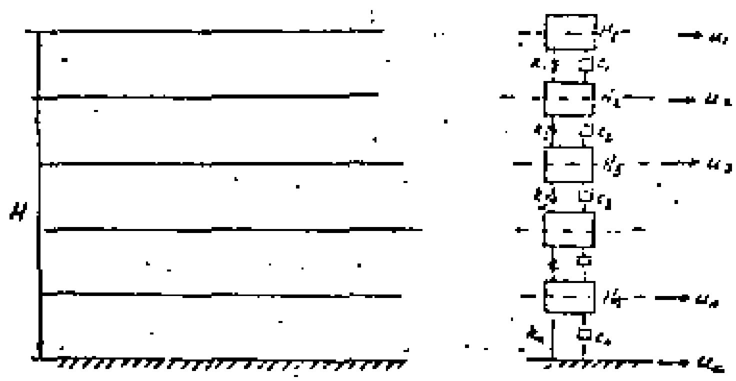


FIG 12. EQUIVALENT LUMPED MASS SYSTEM

In other words it is not possible in general to find an equivalent system with modal damping. If on the other hand the damping matrix can be expressed as a polynomial expression of $K^{-1}M$ or $M^{-1}K$ of the form

$$C = \sum_{r=m}^{r=m+n-1} d_r\, K(M^{-1}K)^r$$

or

$$C = \sum_{r=m}^{r=m+n-1} e_r\, M(K^{-1}M)^r$$

corresponding values of modal damping can be found and a normal modal analysis can be performed with damping in the ith mode,

$$2\beta_i\omega_i = \sum_{r=m}^{r=m+n-1} d_r\, \omega_i^{2r+2}$$

or

$$\beta_i = \frac{1}{2} \sum_{r=m}^{m+n-1} d_r\, \omega_i^{2r+1}$$

It can be shown that for the free vibration problem as the number of masses increases the natural periods and modal shapes tend to those given by the continuous solution. It has been found that a period T is reproduced with sufficient accuracy if the number of masses is

$$n \geq 5 \left(\frac{H}{TC_S} + \sqrt{\frac{H}{TC_S}} \right)$$

For a multilayered soil this condition should be verified for each layer independently and for the total deposit with an average shear wave velocity.

Knowing the natural periods or frequencies and the corresponding modal shapes, the participation factor of each mode for a base motion is

$$\Gamma_n = (-1)^{n-1} \frac{4}{(2n-1)\pi} \quad \text{for a uniform layer.}$$

In general the participation factor can be found by the normal procedures of Structural Dynamics. If M is the mass matrix and ϕ_n the nth eigenvector

$$\Gamma_n = \frac{\phi_n^T M I}{\phi_n^T M \phi_n}$$

where I is a vector with all components unity.

If ϕ_{n1} is the component of the eigenvector at the free surface of the soil calling $g_n = \Gamma_n \phi_{n1}$ the amplification function can be written as

$$A(\Omega) = \sqrt{\left[\sum_{i=1}^{n} \frac{g_i\left[\omega_i^2(\omega_i^2 - \Omega^2) + 4\beta_i^2\omega_i^2\Omega^2\right]}{(\omega_i^2-\Omega^2)^2 + 4\beta_i^2\omega_i^2\Omega^2}\right]^2 + \left[\sum_{i=1}^{n} \frac{g_i 2\beta_i\omega_i\Omega^3}{(\omega_i^2-\Omega^2)^2 + 4\beta_i^2\omega_i^2\Omega^2}\right]^2}$$

or alternatively

$$A(\Omega) = \sqrt{\left(\sum_{i=1}^{n} \frac{g_i\Omega^2(\omega_i^2 - \Omega^2)}{(\omega_i^2-\Omega^2)^2 + 4\beta_i^2\omega_i^2\Omega^2} + 1\right)^2 + \left[\sum_{i=1}^{n} \frac{g_i 2\beta_i\omega_i\Omega^3}{(\omega_i^2-\Omega^2)^2 + 4\beta_i^2\omega_i^2\Omega^2}\right]^2}$$

If all the modes are considered both formulas give the same results. If only the first few modes are included the results will differ slightly in the high frequency range.

Results obtained with these formulas (taking sufficient number of masses and modes) agree with the amplification curves obtained with the continuous model for the case of rigid rock. Adding to the values of modal damping β_i the equivalent radiation damping previously suggested results for elastic rock are again reproduced.

For all practical purposes it can be considered that the continuous and the discrete model will both be applicable to determine amplification functions and will yield the same results provided modal damping can be specified and an additional radiation damping is added to the lumped system. The continuous model is, however, more flexible since it allows for arbitrary variations of damping from one layer to another. Furthermore, it requires in general less computer time. The discrete model has the advantage that it is easier to visualize since it reduces the problem to a classical case of Structural Dynamics.

III - TRANSIENT MOTIONS

The amplification function has several important properties:

a) It gives a clear graphical picture and qualitative information on the effect of the soil. A simple look at the curve is sufficient to determine in what ranges of frequencies the soil can have a serious damaging effect, and in what ranges this effect would not be important or might even be beneficial.

b) It is independent of any given earthquake and it represents therefore a property or characteristic of the soil itself.

c) For some of the methods that will be described here, the determination of the amplification curve is a necessary first step to determine accelerograms or response spectra at the top of the soil. For other methods, however, this step may be bypassed.

In spite of these features the amplification curve is by no means the ultimate goal of this type of studies. From the point of view of the structural designer the main objective is to have a set of response spectra which apply to the surface of the soil or less frequently a set of earthquake records which could characterize the motions to be expected. From the point of view of the soils engineer it is important to be able to estimate the magnitude of shear stresses in the soil during an earthquake to determine the factor of safety against liquefaction and to guide in the selection of suitable values for modulus and damping ratios.

In order to obtain these results several methods are still available using either the continuous or the discrete model.

II.1 - Continuous Solution

Given a certain time history of acceleration representing an earthquake record at the outcropping of rock, or at the interface between soil and rock, the corresponding accelerogram at the free surface of the soil can be obtained by:

a) Obtaining the Fourier transform of the input earthquake.
b) Multiplying it by the Transfer function of the soil.
c) Obtaining the inverse Fourier transform of the resulting function.

The Fourier transform of a function of time f(t) can be visualized as a limiting case of a Fourier series expansion. It is given by the formula

$$F(\omega)=\int_0^{\infty} f(t)e^{-i\omega t}dt=\int_{-\infty}^{+\infty} f(t)e^{-i\omega t}dt \quad (\text{if } f(t)=0 \text{ for } t\leq 0)$$

f(t) is then said to be the inverse Fourier transform of $F(\omega)$

$$f(t)=\frac{1}{2\pi}\int_{-\infty}^{+\infty} F(\omega)e^{i\omega t}\,d\omega$$

It should be noticed that $F(\omega)$ is a complex function, writing it as

$$F(\omega)=C(\omega)-iS(\omega)$$

$$C(\omega)=\int_0^{\infty} f(t)\cos\omega t\,dt \quad (\text{is the cosine transform})$$

$$S(\omega)=\int_0^{\infty} f(t)\sin\omega t\,dt \quad (\text{is the sine transform})$$

If on the other hand it is written as

$$F(\omega)=E(\omega)\,e^{-i\phi(\omega)t}$$

$$E(\omega)=\sqrt{C(\omega)^2+S(\omega)^2}$$

$$\phi(\omega)=\tan^{-1}\frac{S(\omega)}{C(\omega)}.$$

$E(\omega)$ represents the amplitude Fourier spectrum, and $\phi(\omega)$ the phase spectrum. The amplitude spectrum has an important physical meaning. Given two values of frequency, ω_1, ω_2, the area under the curve $E(\omega)$ from ω_1 to ω_2 gives the amplitude of the motion in this range of frequencies. A simple look at the Fourier amplitude spectrum (often referred to for short as Fourier spectrum) gives immediately an idea of the range of frequencies where most of the amplitude of the motion is contained. In fact Hudson has shown that this spectrum is a lower bound to the undamped velocity response spectrum and in general a very good approximation to it (they would coincide if the maximum response occurred after the end of the excitation). Arias has also shown that if $E(\omega)$ is computed for different durations of the earthquake, the envelope of these spectra is an upper bound to the undamped velocity response spectrum.

The amplitude Fourier spectrum is of course closely related to the spectral density function $S(\omega)$

$$S(\omega)=\frac{1}{2\pi}\,\frac{E(\omega)^2}{T}$$

and

$$S^*(\omega)=\frac{1}{\pi}\,\frac{E(\omega)^2}{T}$$

where T is the duration of the excitation f(t)

The area under the spectral density function between two frequencies ω_1 and ω_2 gives a measure of the energy of the excitation in that range.

The Fourier transform has not been used normally in Structural Dynamics. Its determination is, however, extremely fast and simple with a digital computer. Even if this method of analysis is not going to be used, the Fourier transform will provide significant information on the nature of the excitation and will help to answer such questions as how many modes should be included in a modal analysis.

The transfer function of the soil as determined in II is again a complex function. The product of these two complex functions can be accomplished by

Multiplying the Fourier amplitude spectrum of the input by the amplification function of the soil. The result is the Fourier amplitude spectrum of the earthquake at the free surface of the soil.

Adding to the Fourier phase spectrum the change in phase curve of the soil. The result is the Fourier phase spectrum of the output.

The inverse Fourier transform of this product will be again a real function representing the time history of acceleration on top of the soil. Notice that if the input represents the accelerogram at the interface between rock and soil the transfer function corresponding to rigid rock should be used. If on the other hand the input represents the accelerogram which would be recorded on the rock without any soil on top, the transfer function for elastic rock has to be used. The second approach seems more logical. However, at the present time, there is no clear way to determine what the earthquake records should be at bedrock with soil on top or at the outcropping of rock, since most accelerograms of real earthquakes have been obtained on ground (even if firm ground).

Figures 13 through 17 summarize the procedure as outlined. (The phase spectra are not plotted). Figure 13 shows a record of the Taft earthquake and Figure 14 its amplitude Fourier spectrum. Figure 15

FIG 13. ACCELEROGRAM OF TAFT EARTHQUAKE

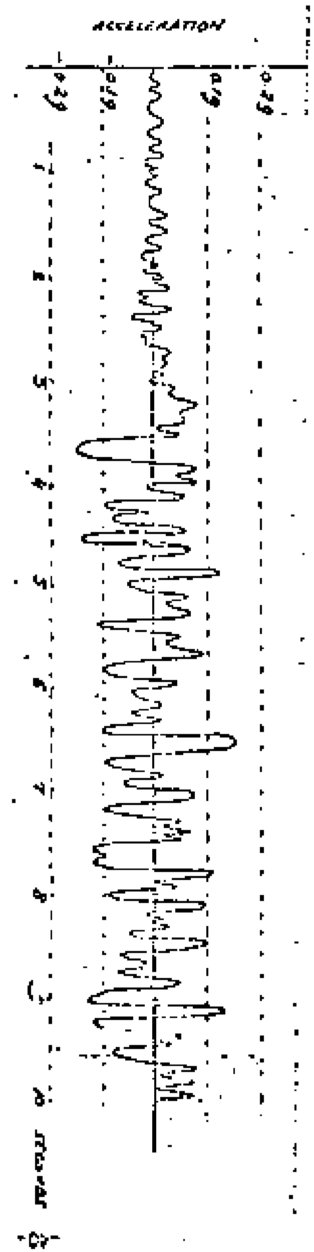
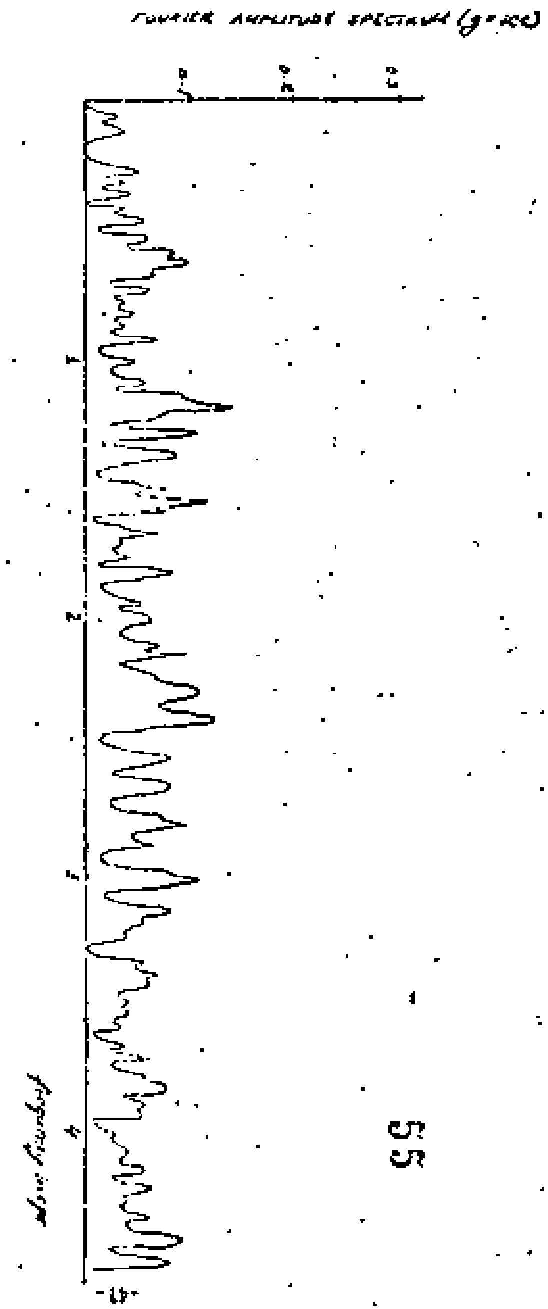


FIG 14 FOURIER AMPLITUDE SPECTRUM FOR TAFT EARTHQUAKE



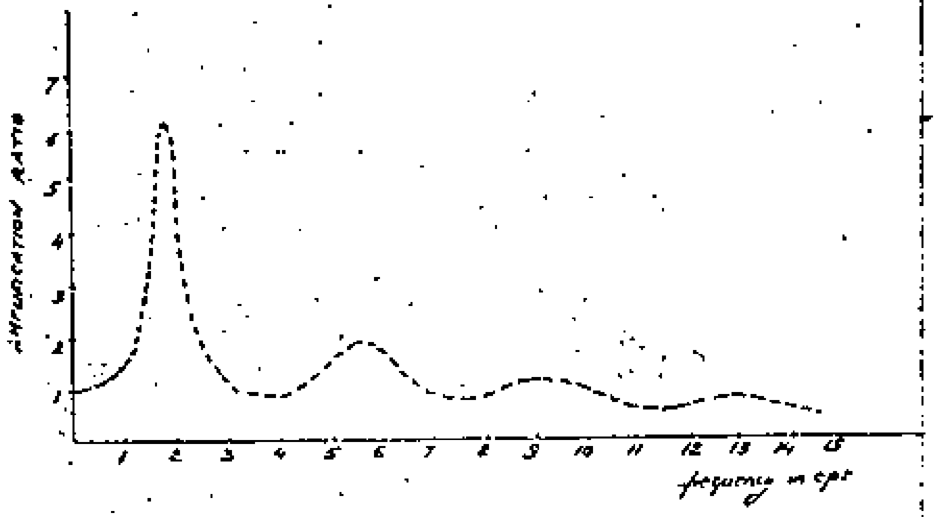


FIGURE 15. AMPLIFICATION FUNCTION OF SOIL

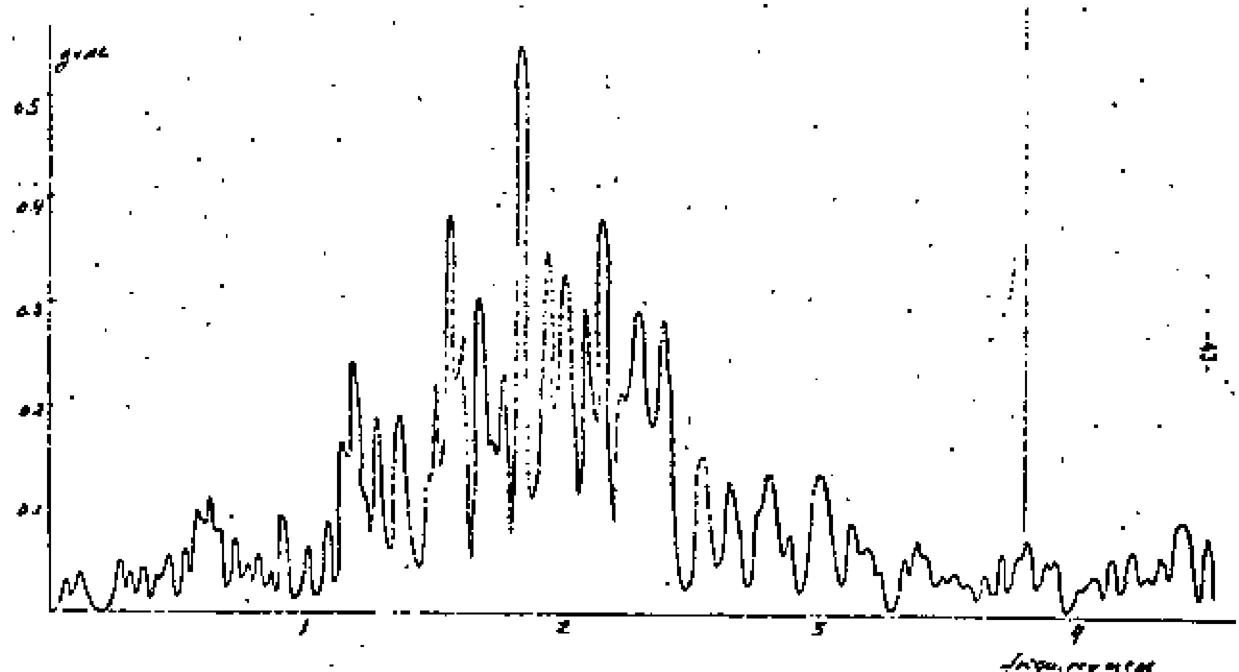


FIG 16. FOURIER AMPLITUDE SPECTRUM OF ACCELEROGRAM AT TOP OF SOIL

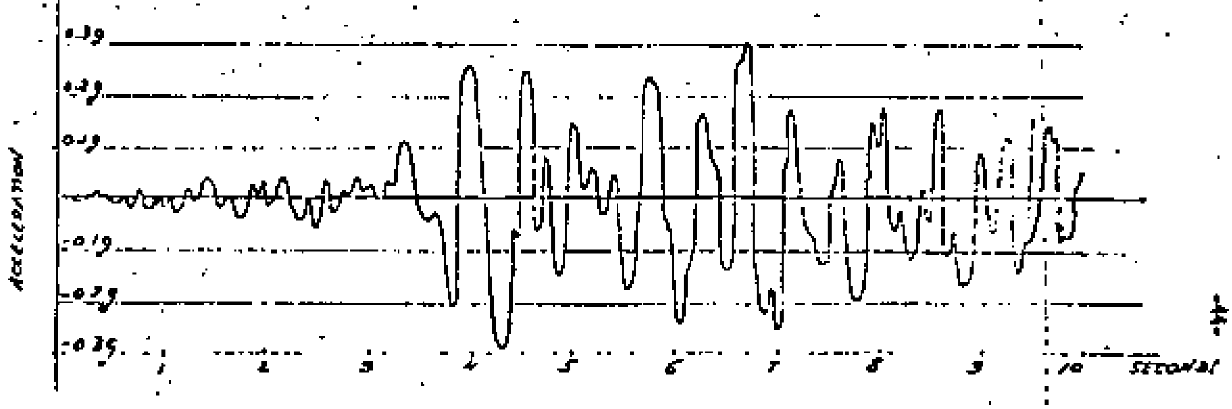


FIGURE 17. ACCELEROGRAM AT TOP OF SOIL

shows the amplification curve for a given soil profile (the uniform layer previously considered with rigid rock). The product of the Fourier spectrum of the input by the amplification function is shown in Figure 17.

Once the earthquake record at the surface of the soil has been obtained, design response spectra can be obtained in two different ways:

a) From the Fourier transform of the output (before inverting it), by multiplying it by the Transfer function of a one-degree-of-freedom linear oscillator, then inverting the result and finding the maximum. The Transfer function of the one-degree-of-freedom system with frequency ω_n and damping β_n is

$$H(\omega) = \frac{\omega_n^2 + 2i\beta_n\omega_n\omega}{\omega_n^2 - \omega^2 + 2i\beta_n\omega_n\omega}$$

b) By integrating through a step-by-step procedure, the equation of motion of a one-degree-of-freedom system

$$\ddot{y} + 2\beta_n\omega_n\dot{y} + \omega_n^2 y = -\ddot{u}_s$$

where $\ddot{u}_s$ is the acceleration time history on top of soil.

The first procedure is normally referred to as integration in the frequency domain whereas in the second case the solution is said to be carried out in the time domain. While the first method would represent a consistent continuation of the procedure followed up to that moment at the present time the second seems more economical as far as computer time is concerned.

Figure 18 shows the pseudo-acceleration response spectrum for the Taft earthquake filtered through the uniform soil deposit. The response spectrum of the input is also shown in the same figure. Figure 19 shows the ratio of both response spectra for 2 and 5%

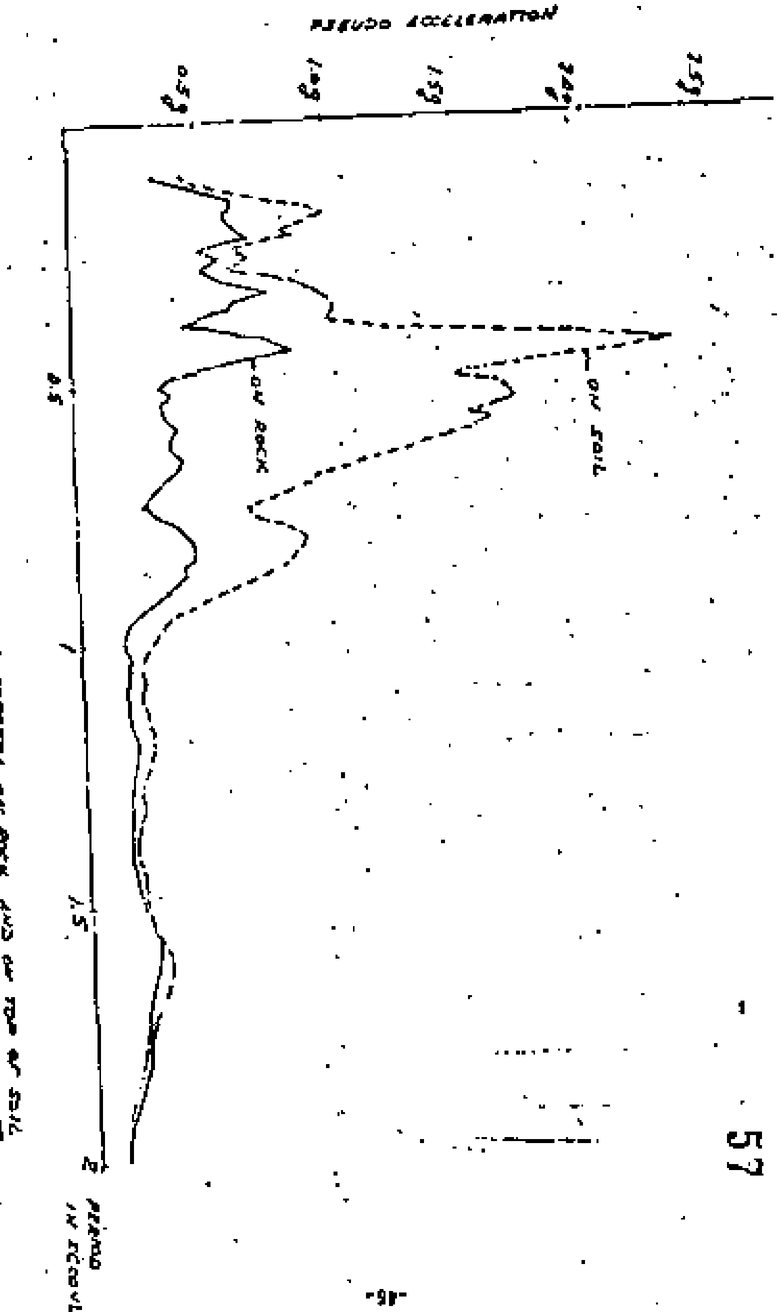


FIG 18. PSEUDO-ACCELERATION RESPONSE SPECTRA ON ROCK AND ON TOP OF SOIL (2% STRUCTURAL DAMPING)

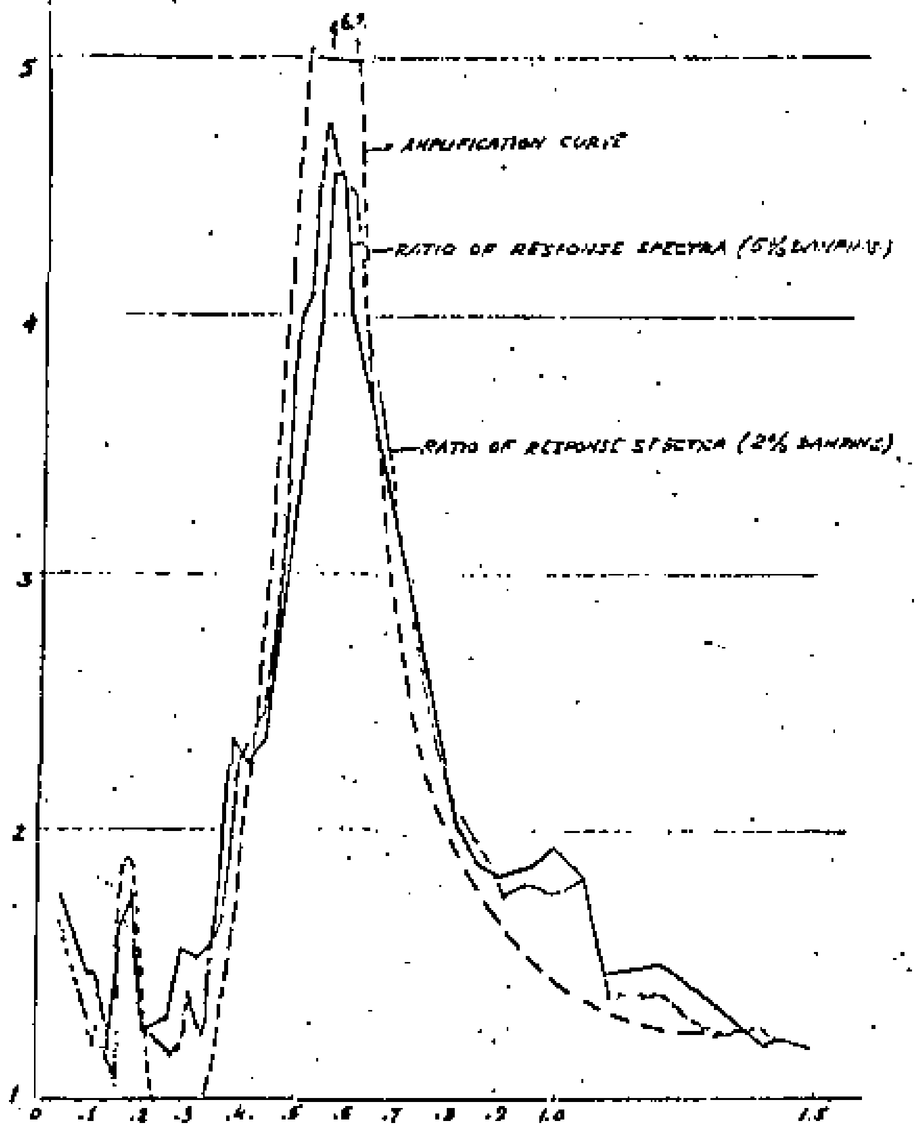


FIGURE 19

damping together with the amplification curve. The similarity between these curves is of course striking, and their relationship will be further discussed later.

III.2 - Discrete Model

In the discrete model the soil is represented by a close-coupled set of masses, springs and dashpots. The equations of motion can be written in terms of relative displacements

$$M_1\ddot{y}_1 + C_1(\dot{y}_1-\dot{y}_2) + k_1(y_1-y_2) = -M_1\ddot{U}_G$$

$$M_2\ddot{y}_2 + C_1(\dot{y}_2-\dot{y}_1) + C_2(\dot{y}_2-\dot{y}_3) + k_1(y_2-y_1) + k_2(y_3-y_3) = -M_2\ddot{U}_G$$

$$\vdots$$

$$M_n\ddot{y}_n + C_{n-1}(\dot{y}_n-\dot{Y}_{n-1}) + C_n\dot{y}_n + k_{n-1}(y_n-y_{n-1}) + k_n y_n = -M_n\ddot{U}_G$$

or in matrix form

$$M\ddot{Y} + C\dot{Y} + KY = -MI\ddot{U}_G$$

where

$$M = \begin{vmatrix} M_1 & & & \\ & M_2 & & \\ & & M_3 & \\ & & & \ddots \\ & & & & M_n \end{vmatrix} \qquad C = \begin{vmatrix} C_1 & -C_1 & & \\ -C_1 & C_1+C_2 & -C_2 & \\ & -C_2 & C_2+C_3 & -C_3 \\ & & -C_{n-1} & C_{n-1}+C_n \end{vmatrix}$$

$$K = \begin{vmatrix} k_1 & -k_1 & & & \\ -k_1 & k_1+k_2 & -k_2 & & \\ & -k_2 & k_2+k_3 & -k_3 & \\ & & & -k_n & k_{n-1}+k_n \end{vmatrix} \quad \text{and } I = \begin{vmatrix} 1 \\ 1 \\ 1 \\ \\ 1 \end{vmatrix}$$

Replacing $\ddot{u}_G$ by the accelerogram of the earthquake at bedrock and integrating numerically this set of differential equations, one can obtain the time history of displacements, velocity or acceleration at any of the masses, or what is equivalent at any point in the soil. Once $\ddot{u}$, absolute acceleration = $\ddot{y} + \ddot{U}_G$ is obtained at the surface of the soil as a function of time, the procedure to determine response spectra is the same as that described for the continuous model, second approach.

The integration of the set of differential equations as outlined above is normally referred to as physical integration of the equations of motion. This procedure is the only one which can be used if the properties of the soil are considered non-linear. On the other hand, for linear systems if modal damping can be specified it is normally preferred to carry out the solution by modal superposition.

If ω_i are the natural circular frequencies of the soil deposit, ϕ_i, its modal shapes (eigenvectors), normalized so that $\phi_i^T M \phi_i = 1$ and Γ_i the participation factor of the ith mode

$$\Gamma_i = \phi_i^T M I = (M_1\phi_{i1} + M_2\phi_{i2} + \dots + M_n\phi_{in})$$

the solution can be expressed as

$$y = \Sigma_i \Gamma_i \phi_i a_i(t)$$

where $a_i(t)$ is the solution of the one-degree-of-freedom equation

$$\ddot{a}_i + 2\beta_i\omega_i\dot{a}_i + \omega_i^2 a_i = -\ddot{u}_G(t)$$

In particular at the surface of the soil

$$\ddot{u}_s = \Sigma_i \Gamma_i \phi_{i1} \ddot{a}_i + \ddot{u}_G = \Sigma g_i \ddot{a}_i + \ddot{U}_G$$

The advantage of this type of solution is that it requires only the solution of a one-degree-of-freedom equation for each mode, once the modal shapes, participation factors and natural frequencies are known. Moreover in general only the first few modes contribute significantly to the solution. For the case of the uniform layer, previously considered with the continuous solution the coefficients g_i are

first mode $g_1 = 1.27$
second mode $g_2 = -0.416$
third mode $g_3 = 0.24$

Only three modes are enough in this case to obtain a good solution. The maximum acceleration in the first mode is of the order of 0.27g, in the second of 0.15g, and in the third of 0.05g.

Response spectra obtained by this method show good agreement with those obtained by the continuous solution, although not as perfect as in the case of the amplification function. The discrepancies are, however, very small and are easy to understand if the large number of computations involved is considered. Each method has its own round-off and truncation errors and they will affect each procedure differently. For all practical purposes the results can, however, be considered equivalent.

Again if the effect of the elastic rock has to be included, it can be done by adding the equivalent radiation damping in each mode.

If the only result desired is the time history of acceleration at the free surface of the soil (or at a small number of points) the continuous solution has an advantage both from the point of view of flexibility (being able to consider different values of damping in each layer) and from the point of view of time of computation. On the other hand, if the time history of acceleration and stresses is desired at many points the modal solution becomes more economical. Damping in the soil does not really come from viscosity but from non-linear hysteretic dissipation of energy. Correspondingly both shear modulus and damping are functions of strain. If a non-linear analysis is to be performed only the discrete model with physical integration of the equations of motion would be applicable. Often, however, the system is considered linear, assuming values of shear modulus and damping, determining histories of strains, computing new values of modulus and damping and cycling until convergence of the process is obtained. For these preliminary runs the discrete model with modal analysis is convenient and it provides a faster, more economical solution. Once appropriate values of modulus and damping have been obtained, the continuous model can be used for a final series of analyses with different values of damping in each layer. (In the modal solutions the values of damping are averaged and expressed as modal damping, constant in all modes).

IV. DERIVATION OF RESPONSE SPECTRA

The methods previously outlined are mainly intended to consider an earthquake at the base of the soil, filter it and obtain the resulting time history of acceleration at the free surface. While it is possible to obtain then design response spectra on top of the soil, the procedure has for this purpose several difficulties:

a) It requires as an input an actual accelerogram, be it that of a real earthquake, scaled or not, or an artificial earthquake obtained by a simulation process. While the area of Earthquake Simulation has seen a considerable progress in the last years, it is still harder to derive the time history of an earthquake corresponding to a certain magnitude and epicentral distance than it is to derive a response spectrum.

b) In order to obtain reliable results the analysis cannot be done for just one input earthquake, but should be repeated for several inputs representing samples of earthquakes with the same average characteristics. The resulting response spectra should finally be smoothed by drawing an average or envelope. The process becomes then too long and expensive.

It would be therefore desirable to have simple and approximate ways by which smooth response spectra on top of the soil would be derived from response spectra at bedrock or on firm ground. Figure 19 showed the amplification curve for a given soil profile and the ratio of response spectra for 2 and 5% of structural damping. The similarity of these curves is apparent. In fact, if the Fourier spectrum were exactly the undamped velocity response spectrum, the amplification curve should coincide with the ratio of response spectra for no damping.

There are, however, several important differences between these two curves:

1. The amplification curve tends to zero as the frequency increases or as the period becomes very small. The ratio of response spectra on the other hand tends to a finite value which is the ratio of the maximum acceleration on top of the soil to the maximum acceleration of the input. (This ratio can be estimated from the design response spectra at bedrock if the modes of the soil are known). The ratio of response spectra is therefore highly dependent on the input earthquake in the high frequency range (or for very short periods, say T smaller than 0.1 seconds).

2. The amplification curve is a function of the soil properties only. The ratio of response spectra on the other hand will depend on the soil properties (periods and damping), the amount of structural damping and the selected earthquake input.

3. The ratio of response spectra is in general smoother than the amplification curve with lower peaks and higher valleys and it becomes smoother as the structural damping increases. For damping values of 20 or 25% the ratio of response spectra is practically constant over a long range of periods. On the other hand for very small values of structural damping or for undamped spectra, the ratio of the response spectra should be close to the amplification curve except in the range of very small periods.

4. It should also be expected that the agreement between the amplification curve and the ratio of response spectra would be better for

high values of damping in the soil since this would tend to eliminate the transients and furnish a motion closer to a periodic one.

In order to determine the applicability of the amplification curve to reproduce the ratio of response spectra, the ratio between both curves b has been obtained at several points for uniform soils with varying fundamental periods and damping, subject to different earthquake inputs. The earthquake records considered are El Centro, Taft and five artificial earthquakes with a Tajimi type spectral density function.

Figure 20 shows for one of the cases studied the amplification curve and the ratio of response spectra for El Centro and Taft earthquakes. Figure 21 shows the average ratio of response spectra for the five artificial earthquakes, together with the 95% confidence levels (mean + 2σ). Most of the points of the curves for El Centro and Taft fall within this band. It must be therefore realized that even within samples of earthquakes with the same properties (magnitude and epicentral distance) a substantial variation is to be expected in the ratio of response spectra.

Figure 22 shows the effect of the natural period of the soil and the amount of damping in the soil in the value of b at different points. For $T = 2T_1$ (1st natural period) the ratio is practically constant, independent of T_1. For $T = T_1$ it has again little variation for periods larger than 0.3 seconds. For $T = 0.5\ T_1$ the variation is large for periods smaller than 0.5 seconds and for $T = \frac{1}{3}\ T_1$ for periods smaller than 0.8 or 0.9 seconds. It should be noticed that in all cases the variation is small for values of T larger than 0.2 or 0.3 seconds. If it is accepted that in this range (say 0 to 0.2 or 0.3 seconds) the maximum acceleration at top of the soil controls the response spectrum, the values of b can be considered only slightly depending on the natural period of the soil over the range of application.

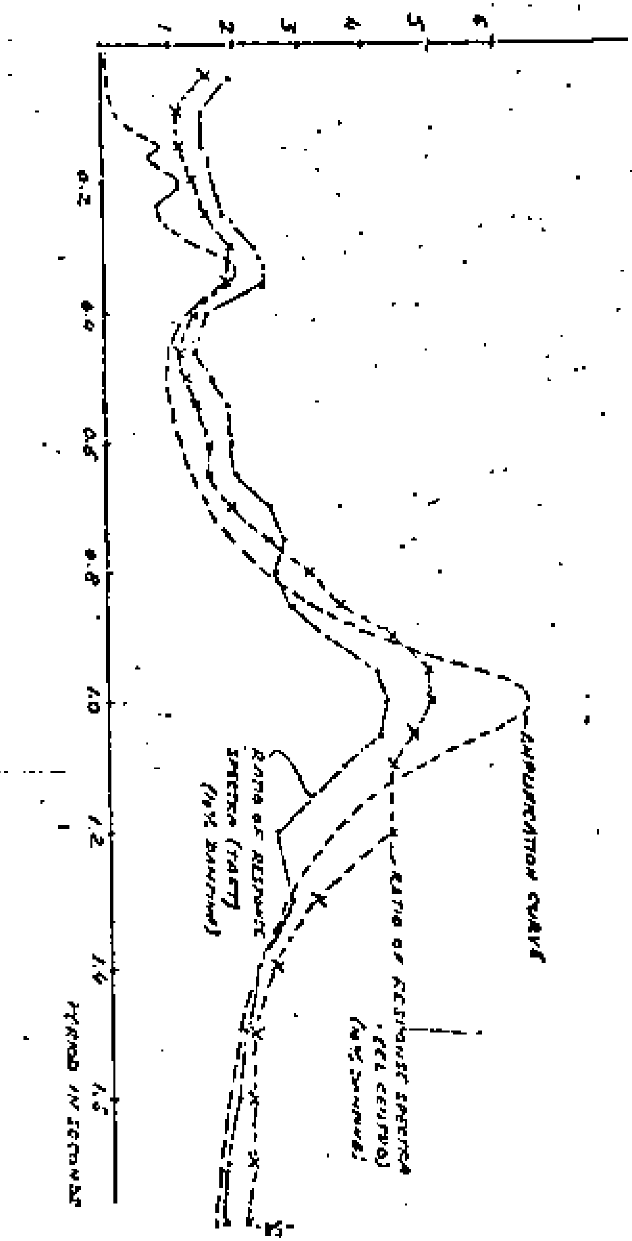


FIG 20. COMPARISON OF AMPLIFICATION CURVE AND RATIO OF RESPONSE SPECTRA
EL CENTRO AND TAFT EARTHQUAKES

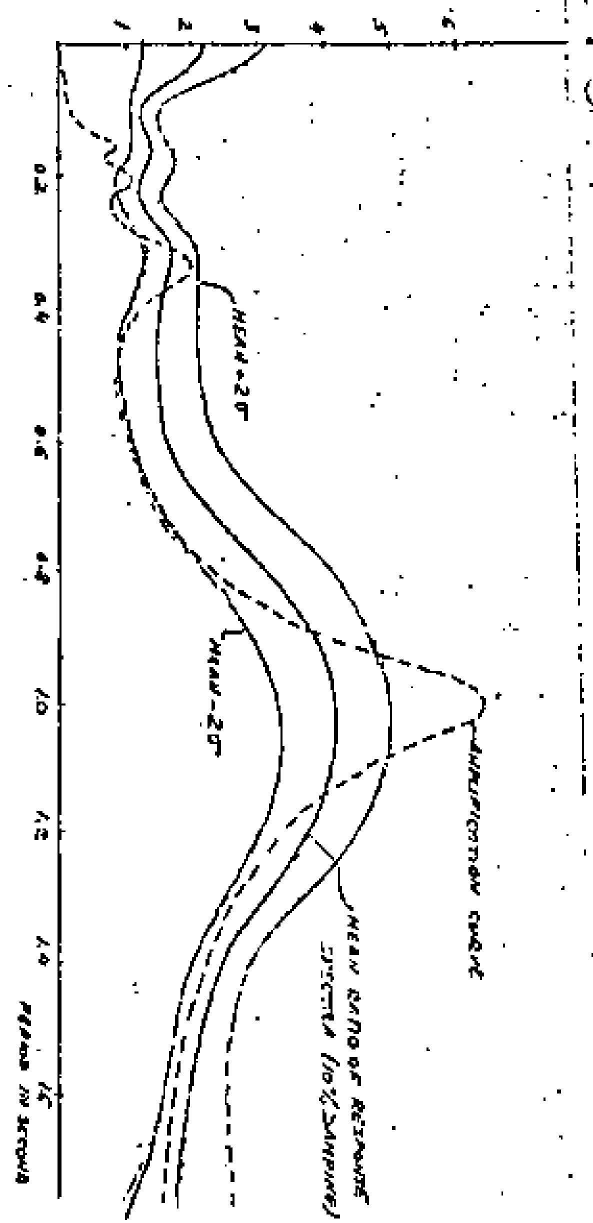
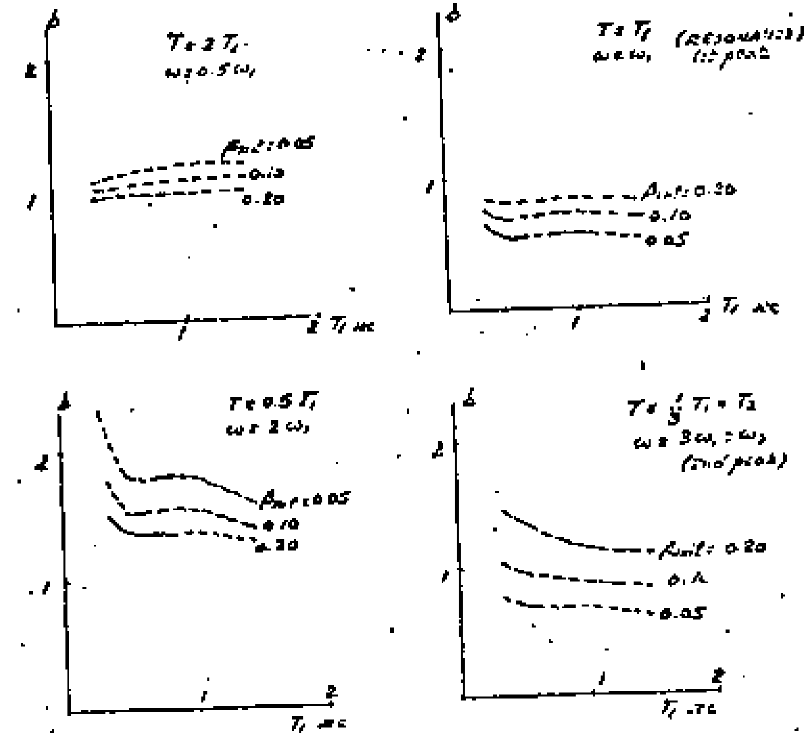


FIG. 21. COMPARISON OF AMPLIFICATION CURVE AND RATIO OF RESPONSE SPECTRA

5 ARTIFICIAL EARTHQUAKES.



β = ratio of response spectra (5% damping) / amplification ratio.

FIG. 22. RATIO BETWEEN RATIO OF RESPONSE SPECTRA AND AMPLIFICATION CURVE AT DIFFERENT POINTS

EFFECT OF NATURAL PERIOD OF SOIL

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Figure 23 shows the effect of structural damping in the values of b. It can be noticed that as it should be expected the value of b is closer to unity as the damping in the soil increases and the structural damping decreases. While the effect of the structural damping is large in the range 0 to 20% in the normal range of structures (say 0 to 5%) this effect may be considered slight, particularly compared to the variation from one input record to another within a family of earthquakes.

Curves like those shown in figures 22 and 23 have been obtained for different values of T/T_1. Using these curves the following procedure is suggested to derive the ratio of response spectra from the amplification curve.

1. At each one of the peaks ($T = T_1$, $T = \frac{1}{3} T_1$, $T = \frac{1}{5} T_1$ etc.) find the value of b from the curves and obtain the corresponding point (multiplying the amplification by the factor b). A horizontal segment is then drawn passing by each one of these points and cutting the peak of the amplification curve if b is smaller than 1.

2. At each one of the valleys ($T = \frac{1}{2} T_1$, $T = \frac{1}{4} T_1$ etc.) the ratio b is obtained from the curves and a point is drawn. These points are then joined by smooth curves to those resulting from step 1.

3. At $T = 2T_1$, the value of b is again found or can be taken approximately equal to 1. For $T > 2 T_1$ the amplification curve can be used. This point is then joined to the point immediately next to T_1 by a smooth curve if the curve is plotted versus period or a straight line if plotted versus frequency.

4. In the range of small periods or large frequencies the response spectrum has to be controlled by the acceleration on top of the soil. The ratio of this acceleration to the maximum input acceleration can be obtained. At a period of 0.1 seconds (or a frequency of 10 cycles per second) this value may be taken as the ratio of response spectra. Between the value at a period of 0.2 seconds and this value a straight line may be drawn if the curve is plotted versus period or a smooth transition curve if plotted versus frequency.

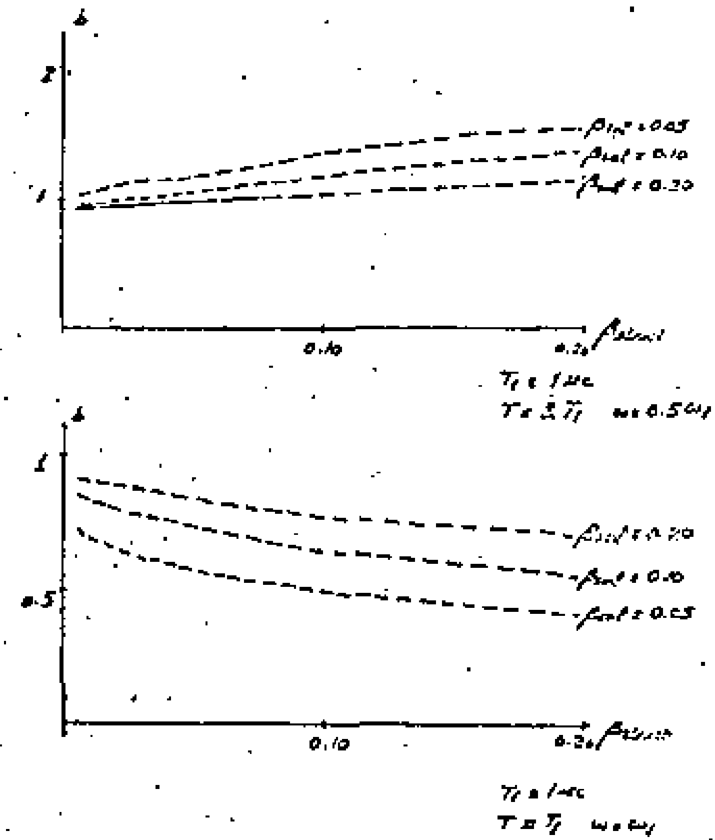

FIG 23. RATIO BETWEEN RATIO OF RESPONSE SPECTRA AND AMPLIFICATION CURVE AT DIFFERENT POINTS

EFFECT OF STRUCTURAL DAMPING

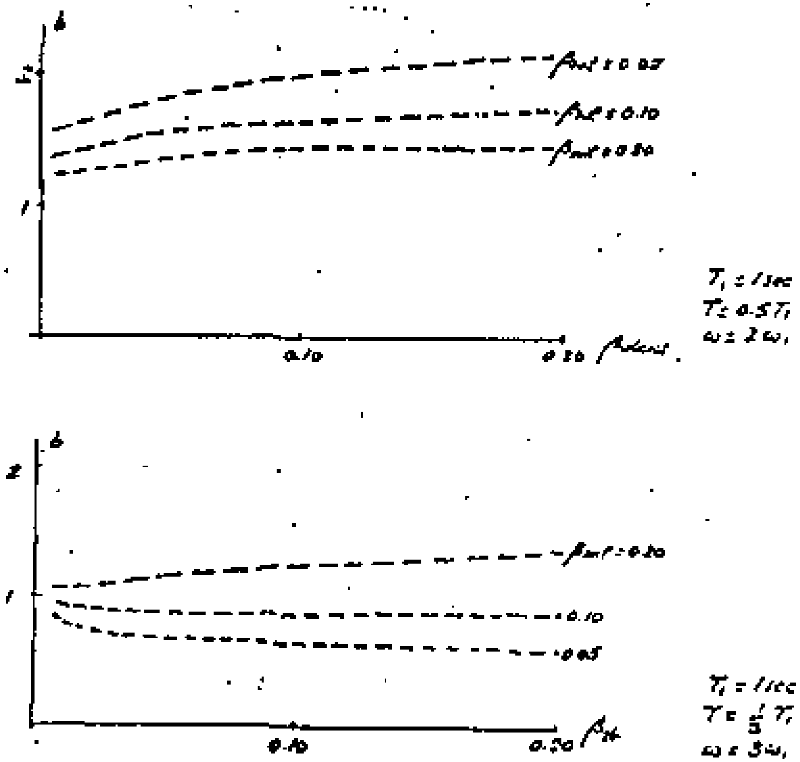

FIG 23 (CONTINUED)

While all the cases considered to derive the curves for the values b have corresponded to a uniform soil profile and further testing is necessary for multilayer systems, it is believed that such a procedure could also be applied for the latter taking the T_1 as the period at which the maximum amplification occurs rather than the first fundamental period.

Figure 24 shows an example of application of the method. Curves of the standard deviation σ have also been obtained. From these curves it is then possible to draw not only the average ratio of response spectra but also confidence levels.

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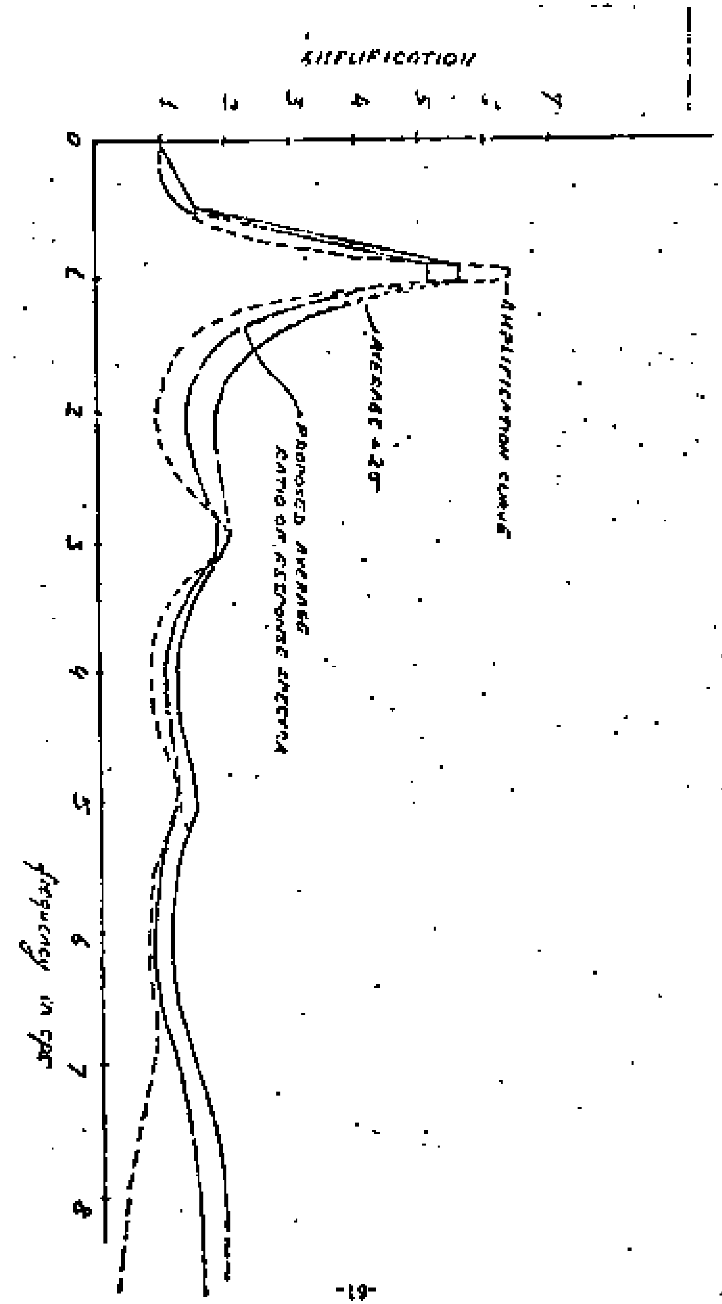

FIGURE 3?, SUGGESTED RATIO OF RESPONSE SPECTRA (2% DAMPING)

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X CURSO INTERNACIONAL DE INGENIERIA SISMICA

ANALISIS DE RIESGO SISMICO

INFLUENCIA DE LAS CONDICIONES LOCALES EN LAS CARACTERISTICAS DE LOS SISMOS

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AGOSTO,1984.

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INFLUENCIA DE LAS CONDICIONES LOCALES EN LAS CARACTERISTICAS DE LOS SISMOS

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ABSTRACT

A critical review is presented of existing literature about influence of some factors in earthquake characteristics. An analysis is made of the conditions adequate for idealizing actual situations by means of given analytical models, and the results of those models are compared with observations obtained during seismics motions. Some studies are proposed oriented to the prediction of the influence of local conditions on the characteristics of earthquake motions.

RESUMEN

Se presenta una revisión crítica de la literatura que trata la influencia de diversos factores en las características de los sismos. Se analizan las condiciones propicias para las que puede idealizarse una situación real, mediante modelos analíticos determinados, y se comparan los resultados obtenidos de dichos modelos con observaciones hechas durante movimientos sísmicos. Se proponen estudios enfocados a predecir la influencia de las condiciones locales en las características de los movimientos sísmicos.

1. INTRODUCCION

El estudio de las características de los temblores que pueden esperarse en un sitio dado es de gran interés para el diseño sísmico de obras civiles. Tales características dependen de factores cuya influencia es variable para cada evento; en algunos las condiciones locales del sitio influyen de manera determinante, mientras que en otros resultan de menor importancia. Esto tiene implicaciones importantes en la solución del modelo analítico que deba aplicarse a cada sitio particular con el fin de estimar la influencia de dichas condiciones locales.

Los movimientos en la superficie se han tratado de predecir mediante diversos modelos analíticos; sin embargo, la mayor parte de ellos se basa en hipótesis excesivamente simplistas, y con frecuencia se omiten características físicas significativas, tales como posición relativa entre el foco y la estación registradora, irregularidades topográficas, etc, lo que impone limitaciones indeseables en la utilización de dichos modelos en algunos casos. En este trabajo se mencionan las condiciones propicias para las que puede idealizarse, con un determinado modelo, una situación real y se comparan los resultados de utilizar dichos modelos con evidencias reales; por ejemplo, el modelo analítico basado en la propagación vertical de un frente de ondas plano de cortante puede reproducir con bastante aproximación movimientos correspondientes a sitios con determinadas características geológicas y topográficas, pero dista mucho de ser válido para otras condiciones. También se presenta una evaluación cualitativa de la influencia que pueden tener las condiciones locales del suelo en las características del movimiento sísmico, y se propone el desarrollo de algunos estudios enfocados a predecir su contribución en dicho movimiento.

2. FACTORES PRINCIPALES QUE INFLUYEN EN LAS CARACTERISTICAS DE LOS SISMOS

Los movimientos sísmicos del suelo dependen de diversos factores, variables en tiempo y espacio, que pueden asociarse a:

Mecanismo generador del temblor

Trayectoria de las ondas sísmicas, desde su origen hasta el sitio en estudio (fig 1)

En la tabla 1 se presentan los principales factores asociados a estos grupos. Cada uno influye en distinta proporción, que varía de un temblor a otro, en el movimiento del terreno.

2.1 *Factores asociados al mecanismo generador del temblor*

Los mecanismos que causan los sismos de origen tectónico no han sido, a la fecha, completamente explicados. La reciente teoría de las placas *tectónicas* (refs 1 y 2), actualmente la más aceptada, explica la ocurrencia de eventos sísmicos en áreas próximas a la periferia de las placas litosféricas (ref 3), pero no en regiones lejanas de estas, donde obviamente las causas de temblores son de naturaleza diferente.

Existen parámetros asociados con la fuente de liberación de energía al presentarse sismos de origen tectónico; desde el punto de vista de ingeniería sísmica, los de mayor interés por su influencia en las características del movimiento del sitio en estudio, son: cantidad de energía liberada, geometría y orientación de la superficie de falla, dirección y sentido de propagación de la ruptura, etc. Algunos ejemplos acerca de la influencia de esos parámetros, la de la trayectoria y de las condiciones locales, se presentan en el cap 3.

2.2 *Factores asociados con la trayectoria de las ondas sísmicas*

La sismología generalmente trata los eventos sísmicos como un problema de radiación de ondas elásticas que se propagan en el interior de la tierra y en su superficie; fundamentalmente se distinguen dos grupos de ondas: de cuerpo y de superficie. Ambas, durante su recorrido, tienen un decremento en su amplitud debido principalmente a:

a) Refracciones y reflexiones

b) Ampliación del frente de onda

c) Atenuación

La ampliación del frente de onda contribuye en diferente proporción a dicho decremento en las ondas de cuerpo que en las de superficie; en el primer caso, la reducción de la amplitud es proporcional al recíproco de la distancia, mientras que en el segundo al recíproco de la raíz cuadrada de la distancia.

La atenuación produce, además de decremento en la amplitud de las ondas, variación en el contenido de frecuencias de estas. Los componentes de alta frecuencia se atenúan más rápidamente que los restantes; es decir, reducen su amplitud a mayor rapidez con la distancia, que otras de menor frecuencia.

Las características de las ondas, durante su trayectoria, se ven influidas por las condiciones del medio a través del cual se propagan, en este caso el suelo. En algunos casos, las condiciones son tales que las características de las ondas en la última etapa de su trayectoria se determinan por las condiciones locales del sitio, y tienen poca influencia los factores relacionados con el mecanismo de falla y con la trayectoria precedente a esta etapa. En el cap 3 se presentan algunos ejemplos acerca de la importancia de las condiciones locales en los movimientos sísmicos, las cuales dan lugar a fenómenos locales de gran magnitud que han ocasionado pérdidas materiales y humanas.

3. INFLUENCIA DE LAS CONDICIONES LOCALES EN LAS CARACTERISTICAS DE LOS MOVIMIENTOS SISMICOS

Las condiciones locales constituyen, en ciertos casos, un factor que influye en las características sísmicas de un sitio dado. Uno de los fenómenos locales más importantes dentro de la ingeniería sísmica, es el de *amplificación* o *atenuación* del movimiento en un punto dado, en relación con el que se tiene en un punto diferente, ubicado en la misma región, pero asociado a características geométricas y geológicas distintas del primero. Dicha modificación del movimiento implica:

1. Cambio en las amplitudes máximas de aceleración, velocidad y desplazamiento

2. Cambio en el contenido de frecuencias del movimiento

En este trabajo se tratan únicamente las condiciones del suelo en situación estable, es decir, no se hace referencia a problemas asociados con la compactación de suelos por vibración ni a los de licuación o pérdidas de la capacidad en cortante por acumulación de presiones de poro excesivas.

Los criterios empleados hasta la fecha para predecir los fenómenos de amplificación dinámica de los movimientos sísmicos en términos de las condiciones locales, pueden agruparse en:

1. Registro de microsismos

2. Extrapolación de registros existentes de temblores fuertes

3. Métodos analíticos

Sin embargo, ninguno de esos criterios ha conducido a resultados satisfactorios en los diversos intervalos de las variables significativas; de ahí el interés por estudiar sus posibilidades y limitaciones mediante la comparación de sus predicciones con las observaciones durante temblores reales y por establecer sus intervalos de validez, así como los criterios de aplicabilidad más general para estimar la influencia de las condiciones locales en las características de los temblores. Esto constituye la materia de los capítulos posteriores.

3.1 *Amplificación de la respuesta sísmica de suelos idealizados mediante modelos unidimensionales*

La aplicabilidad de modelos unidimensionales en el análisis de la amplificación local que sufre la respuesta del suelo ante excitaciones sísmicas, puede ser adecuada siempre y cuando las condiciones idealizadas en dichos modelos sean suficientemente aproximadas a las reales.

El empleo de modelos unidimensionales que se basan en la propagación vertical de ondas de cortante queda limitado a casos en que se cumplan simultáneamente condiciones que queden agrupadas en: *a)* geométricas, *b)* geológicas y *c)* relacionadas con la excitación.

3.1.1 Condiciones geométricas

La estratificación del subsuelo debe ser aproximadamente horizontal (dimensiones horizontales grandes comparadas con la profundidad). Generalmente las ciudades de gran tamaño se encuentran en valles cuyo subsuelo lo constituyen estratos casi horizontales. Las formaciones que presentan esta configuración son generalmente de origen lacustre.

3.1.2 Condiciones geológicas

Debe existir homogeneidad en cada estrato. La constitución del suelo debe ser tal que no aparezcan condiciones inestables (licuación, compactación, deslizamiento de masas de suelo hacia superficies sin soporte, etc) ante la acción de cargas dinámicas.

3.1.3 Condiciones relacionadas con la excitación

La influencia de los diferentes tipos de ondas sísmicas en la región en estudio debe ser tal que dé lugar a un predominio de ondas de cortante con propagación vertical. Las condiciones para que esto suceda son variadas y dependen de factores tales como posición relativa del foco respecto al sitio, magnitud sísmica del evento, etc.

El hecho de que las ondas de cuerpo manifiesten múltiples refracciones al viajar a través del medio (supuesto con estratificación horizontal), y de que la velocidad de dichas ondas aumente con la profundidad, tiene como consecuencia para eventos de foco profundo, que incidan casi verticalmente en la superficie, lo que permite suponer una propagación vertical en la última etapa de su trayectoria cuando es a través de estratos blandos horizontales (línea OAS, fig 2); sin embargo, la proporción en que las ondas de cortante contribuyen al movimiento se relaciona directamente con la posición relativa del foco respecto al sitio, por lo que no es posible en todos los casos despreciar la contribución de otros componentes en las características del movimiento.

La incidencia de las ondas superficiales en la región en estudio es horizontal (línea CS, fig 2). En la fig 3 se presenta el detalle idealizado del caso particular de ondas superficiales predominantes con longitud de onda (λ) comparable al doble de la dimensión horizontal (L) que limita las fronteras verticales de la región en estudio: se observa que la frontera inferior de dicha región, supuestamente horizontal, experimentará en este caso movimientos paralelos a ella, actuando prácticamente en fase, que generarán ondas de corte con propagación vertical a través de los estratos horizontales superiores. Para el caso de longitudes de ondas superficiales, menores que la dimensión horizontal L antes indicada, el movimiento en la frontera inferior no es uniforme, por lo que la suposición de propagación vertical de ondas de cortante para este caso no es válida. Asimismo, debe notarse (fig 3) que la relación de impedancia supuesta entre los dos medios es muy alta. Cuando esto no sucede, las ondas generadas en el manto superficial, debidas al paso de las ondas superficiales, pueden ser de diversos tipos, sin que necesariamente predominen ondas de cortante.

Cuando se satisfacen las condiciones anteriores, los métodos de análisis de modelos unidimensionales usualmente empleados, son: a) los que se basan en el empleo de la ecuación de onda, y b) los que utilizan la técnica de *elementos finitos* en una dimensión. La más reciente revisión de la literatura respecto al tema fue hecha por Faccioli y Reséndiz (ref 4); aquí se omite.

Cuando el contraste entre la velocidad de propagación de ondas del manto superficial y del medio seminfinito sobre el cual se apoya no es muy fuerte, se presenta el problema de fijar a cierta profundidad el espesor de dicho manto y de considerar una frontera inferior que tome en cuenta la retroalimentación de energía; la manera más sencilla de hacer esto último es adicionar en la frontera un amortiguador con constante $(G\rho)^{1/2}$ donde G es la velocidad de propagación de ondas de cortante en el semiespacio y ρ la densidad de masa del mismo (refs 5 y 6).

La profundidad del estrato superficial representa en cada caso particular un problema diferente. Lo más usual hasta la fecha es analizar varios perfiles de suelo, que difieren entre sí en la profundidad y características de la roca basal y de acuerdo con los resultados seleccionar el modelo representativo del lugar. El procedimiento resulta demasiado burdo y los resultados dependen del criterio que se use en la elección de dicho perfil, lo que indica la necesidad de realizar estudios sobre el problema tendientes a definir las características del conjunto modelo representativo-excitación, de manera que las diversas hipótesis sobre dicho conjunto conduzcan a respuestas comparables.

La aplicabilidad de modelos unidimensionales a suelos con una pequeña diferencia de impedancias entre el manto superficial y el que los soporta, es más limitada que para los casos en en que dicha diferencia es elevada, debido a que en estos casos existe mayor incertidumbre respecto a los requisitos expuestos en 3.1.3.

Entre los sitios donde existen suelos caracterizados por un periodo dominante de vibración, normalmente alto, que pueden representarse mediante un modelo unidimensional, se encuentran algunas áreas de la ciudad de México, diversas zonas próximas a las bahías de San Francisco y Tokio, etc. La validez de esta idealización se comprueba mediante estudios relacionados con dichos lugares (refs 7, 8 y 9); sin embargo, la evidencia de los resultados en esos estudios no significa que esté, a la fecha, totalmente entendido el fenómeno de amplificación en tales sitios. Por ejemplo, existen serias dudas sobre las condiciones para las cuales es despreciable la influencia de las ondas superficiales en el movimiento del suelo (que cumple con las condiciones especificadas anteriormente), y resulta adecuada la suposición de propagación vertical de ondas de cortante. Aún más, es claro que para algunos casos en que la teoría de propagación vertical de ondas unidimensionales de cortante produce buenos resultados, como en el Valle de México, la concordancia es producto más bien de la gran diferencia de impedancias que de la validez del modelo que supone la llegada de la energía principalmente en forma de ondas *S* viajando verticalmente; en el caso citado es reconocido el hecho de que, poseyendo los temblores importantes distancias epicentrales grandes, las ondas que llegan son principalmente de tipo superficial, pues las de cuerpo deben haber sufrido atenuación significativa para tales distancias. Asada, Kawakami y Kamiyama (ref 10) deducen de un estudio que se basa en 20 sismos registrados en un mismo sitio, a partir de 1967, cuyo subsuelo está constituido por estratos horizontales, blandos, que las ondas superficiales forman parte importante del movimiento en el lugar; sin embargo, los autores no especifican la posible razón de esta contribución. Rosenblueth (ref 11), en una discusión a dicho estudio, subraya el poco contenido de ondas superficiales en la respuesta de la formación arcillosa del Valle de México, y atribuye tal discrepancia a la diferencia entre las características del suelo en dicho valle y en el sitio analizado en el estudio mencionado.

La necesidad de estudios más profundos que los existentes sobre el tema es obvia, y deberán basarse en observaciones de registros hechos *in situ* o en sitios con características semejantes, por lo que es aconsejable la instalación de instrumentos diseñados para tal idea, y dado que los resultados de estos no serían inmediatos, es aconsejable el estudio teórico del problema bajo suposiciones mejores que las actuales.

3.2 *Influencia de efectos bidimensionales*

Las irregularidades geométricas y la heterogeneidad de las formaciones que se encuentran en algunas regiones, conducen al estudio del problema en espacios de más de una dimensión.

Un factor importante para la evaluación, aunque sea cualitativa, del tipo de respuesta es la posición relativa del foco con respecto al lugar de estudio, ya que de ella depende el tipo y la amplitud de las ondas sísmicas que causan la perturbación.

La solución general del problema, tratado en un espacio bidimensional con procedimientos analíticos, implica serias complicaciones matemáticas, por lo que la mayoría de los estudios existentes al respecto incluyen simplificaciones tales como linealidad, homogeneidad e isotropía de los materiales, tipo de excitación estacionaria, frente de ondas plano, aproximación de sistemas infinitos a finitos mediante fronteras equivalentes, continuidad del medio, etc. La utilidad de los modelos simplificados es grande, ya que permite la evaluación cualitativa de fenómenos reales ante diversas condiciones.

A continuación se describen algunos estudios analíticos sobre formaciones con fronteras irregulares y topografía superficial plana. Por los motivos antes expuestos, las irregularidades de dichas fronteras se han idealizado mediante configuraciones simples.

Trifunac (ref 12) trata el caso de estratos superficiales soportados por suelos cuya impedancia es muy alta comparada con la parte inferior sujetos a excitación de ondas horizontales ocasionadas por una falla vertical superficial, relativamente cercana al sitio en cuestión donde se tiene un perfil como el de la fig 4. El autor lo resuelve como un problema de propagación horizontal de energía. Una de las principales fuentes de error del modelo se indica en el texto, es la suposición de la excitación como un proceso continuo estacionario y no de carácter transitorio. Para este caso, Trifunac predice que las amplitudes de onda decrecen al viajar a traves de un espesor de suelo blando mayor (fig 4, a la derecha del eje x), cambiando su contenido de frecuencias contrariamente a lo que predice respecto a estratos de mayor espesor la teoría de propagación vertical unidimensional de ondas de cortante. Dada la canalización de energía en el estrato superior, debido a la alta impedancia supuesta, la predicción del autor resulta explicable. Esto implica que los máximos de los espectros pueden ocurrir en diferentes periodos para distintas distancias horizontales.

Lo anterior lo comprueban las observaciones de Housner y Trifunac (ref 13) en un análisis de acelerogramas registrados durante el sismo de Parkfield, de magnitud 5.6, ocurrido el 27 de junio de 1966; el epicentro se localizó 20 millas al noroeste de Cholame, Calif, y ocurrió un rompimiento en la superficie a lo largo de la falla de San Andrés hacia el sur (fig 5). Las curvas espectrales calculadas para los sitios 2, 5, 8, 12 y *TEMBLOR* indican diferentes periodos dominantes en sitios con igual geología local. En las figs 6 y 7, correspondientes a las estaciones 5 y 12, se muestran los espectros de respuesta de velocidad para componentes aproximadamente paralelos a la línea de falla. En la estación 5 aparecen máximos en la respuesta espectral para periodos de aproximadamente 0.5 seg, mientras que en la estación 12 ocurren a diferentes periodos, presentándose el máximo en lapsos de 2.2 seg, aproximadamente. La magnitud de respuesta para cada uno de estos dos casos es muy diferente: mucho mayor en el sitio más cercano a la falla de San Andrés (estación 5) que en los más alejados (estaciones 8 y 12).

La ocurrencia de esos efectos puede explicarse mediante la teoría simplificada mencionada, o bien al considerar la influencia de factores geológicos que, vistos dentro de un modelo tridimensional, podrían afectar el movimiento (por ejemplo, fallas geológicas menores, discontinuidades, etc).

Jackson (ref 14) estudia las posibles trayectorias de ondas sísmicas para varias configuraciones geológicas típicas. En cada caso calcula los factores de amplificación correspondientes en la superficie. Para el análisis utiliza técnica de ondas progresivas como una generalización del trazo de rayos y construcción de frente de ondas usadas en óptica geométrica (ref 15).

La concentración de ondas sísmicas puede deberse a la irregularidad geométrica de los estratos del suelo o a heterogeneidad de un volumen relativamente pequeño de suelo ubicado en un espacio que pueda considerarse prácticamente homogéneo. El caso de un frente de ondas de cortante plano que viaja en un material homogéneo e incide en otro, limitado por una frontera cilíndrica, y cuya velocidad de propagación de ondas es menor que la del medio circundante, se muestra en la fig 8a; la trayectoria aparece distorsionada respecto al frente plano con que viajaba inicialmente, dando lugar a concentración de ondas aproximadamente en el centro del semicírculo; la trayectoria calculada de los rayos reflejados es como se indica en la fig 8b, y presenta también una fuerte concentración de ondas. Lo anterior ocasiona amplificación y atenuación de la amplitud de las ondas en la superficie (línea punteada, parte superior de la figura). La amplitud de las ondas incidentes en la frontera circular es unitaria.

Los cambios en la trayectoria de las ondas, ocasionados por no uniformidad del espesor del estrato superior, dependen de la geometría de este, del tipo de ondas que se tratan y de su ángulo de incidencia. Así, la posible trayectoria de ondas de cortante (*SV*) que inciden en una anomalía geológica como la de la fig 9, es diferente a la de las que inciden en una como la de la fig 10; en ambos casos, la refracción de las ondas de compresión es tal que resultan menos concentradas que las de cortante. Para el caso de una depresión en la roca basal (fig 9), la concentración de ondas de cortante en la superficie provoca incrementos en la intensidad del movimiento en ciertas zonas y decrementos en otras. Las amplitudes en la superficie son en general menores que las ondas que llegan a la interfase, debido a la bifurcación de energía que se produce en ella. Se aprecia (fig 9), cualitativamente, que la concentración de ondas puede dar lugar a alteraciones en las amplitudes de ondas que inciden en la superficie. Debe observarse que para los tres casos, el frente de onda incidente se supone plano y se desprecia la contribución de ondas superficiales al movimiento.

El último caso que se menciona ha sido comprobado mediante observaciones en el campo. Un hecho semejante, que podría idealizarse a través de este modelo, ocurrió en Skopje, Yugoslavia, en el temblor de julio 26 de 1963. Poceski (ref 16) informa que la distribución del daño en la ciudad fue bastante irregular. Según este autor se presentaron en la ciudad diferentes intensidades sísmicas que pueden agruparse por zonas; en la fig 12 se muestra la distribución del daño: la zona *I* corresponde al área de mayor destrucción, la *II* indica un daño fuerte, la *III* uno menor y en la *IV* no se presentaron averías. La razón de esta no uniformidad la atribuye Poceski a las características geométricas del suelo; hace ver que los edificios de mayor daño se localizan en zonas de cambio brusco de profundidad del estrato aluvial sobre el que fueron construidos. Por ejemplo, en la parte oeste de la ciudad, la geología muestra depresiones en la roca basal de 15 m en distancias muy pequeñas (fig 11), donde los daños registrados fueron mayores que en las zonas vecinas. La amplitud del movimiento en el estrato superficial de menor espesor fue más pequeña que en el de mayor, por lo que los edificios sobre esta franja crítica, que limita los estratos de espesor diferente, probablemente sufrieron vibraciones torsionantes, imprevistas en su diseño. Esto, aunado al efecto de la concentración de las ondas debido a la irregularidad geométrica del manto superficial, fue probablemente lo que ocasionó la destrucción parcial o total de algunos edificios.

La concentración de efectos destructivos mayores sobre una zona relativamente pequeña dentro del área total afectada, como lo acontecido en Skopje, en 1963, se observó también en Caracas, en 1967. Sozen et *al* (ref 17) presentan un informe detallado del acontecimiento.

El fenómeno de amplificación local ocurrido en sitios cuyo subsuelo está constituido por formaciones con estratigrafía irregular, comúnmente es reproducido mediante modelos analizados con el método de elementos finitos (*refs* 18 y 19). La facilidad de su manejo y sencillez de los conceptos en que se sustenta dicho método, han generalizado su uso en los últimos años y se ha convertido en la herramienta de análisis más socorrida y desarrollada de las que se basan en la discretización del medio continuo (diferencias finitas, etc). Aunque el método de análisis está bastante avanzado, su utilización en problemas de amplificación local puede conducir a resultados lejanos de la realidad, ya que estos dependen del tipo de solicitaciones dinámicas supuestas en el sistema representativo del lugar. Estas solicitaciones no pueden predecirse en general de manera precisa, principalmente por la falta de registros sísmicos correspondientes a arreglos de instrumentos que permiten estudiar la distribución de la energía sísmica que llega a cada sitio en distintos tipos de ondas y la forma en que estas se propagan en la vecindad.

3.3 *Influencia de efectos tridimensionales*

La evidencia de algunos fenómenos ocurridos en diferentes lugares implica la necesidad de métodos analíticos de modelos tridimensionales; la complejidad de estos obviamente es mayor que la de modelos más simples.

La reproducción de las condiciones locales del sitio mediante modelos tridimensionales, se justifica cuando intervienen parámetros de importancia tal que pueden alterar la respuesta de forma significativa respecto a la que se tendría si el problema se tratara en un espacio bidimensional.

El método más usual para tratar problemas tridimensionales es el de elementos finitos; sus limitaciones ya se mencionaron en la sección 3.2.

Igual que en los problemas tratados en dos dimensiones, el modelo analítico adecuado a cada caso particular depende en gran parte del tipo de excitación a que estará sometido el sitio y de sus condiciones locales.

La influencia de diversos factores, vistos en un espacio tridimensional, es clara en el ejemplo siguiente:

Stephenson (ref 20) analizó dos sitios en Hutt Valley, cerca de Wellington, Nueva Zelanda, cuyo subsuelo está constituido por terreno aluvial reciente, saturado, separados 900 m uno de otro, con velocidad de propagación de ondas de cortante promedio de 100 m/seg. Obtuvo las densidades espectrales de aceleración en los dos sitios (figs 13 y 14), las cuales presentan sendas direcciones predominantes de respuesta; observó que existía una elevada correlación estadística entre los movimientos en las direcciones dominantes en los sitios estudiados, según se deduce del espectro de correlación cruzada de la fig 15, lo que sugiere el movimiento de una gran masa de terreno aluvial en oscilación torsionante con respecto a un eje vertical.

Aun cuando el autor no opina sobre las posibles razones de dicho movimiento, es claro que este ejemplo no debería pasar inadvertido para quienes están interesados en problemas similares.

Otro ejemplo acerca de la importancia de estudios en espacios tridimensionales en el caso de Skopje mencionado en la sección anterior, en el que aparentemente los edificios ubicados sobre la zona de transición, que limita los mantos superficiales de diferentes espesores, estuvieron sujetos a esfuerzos torsionantes imprevistos provocados por el movimiento del terreno sobre el cual fueron construidos.

3.4 *Amplificación local debida a irregularidades topográficas*

Las irregularidades de la superficie del suelo pueden alterar el movimiento que esta tendría si fuera horizontal, dando lugar a amplificación o atenuación del movimiento en el sitio. La magnitud de los efectos depende principalmente de la relación entre longitud de ondas sísmicas y dimensiones de las características topográficas. Así, en los casos en que la longitud de las ondas es mucho mayor que la profundidad o altura de las anomalías, la influencia de estas resulta despreciable. En ingeniería sísmica es de gran interés el estudio de ondas de altas frecuencias; esto significa que en la mayoría de los casos la topografía será un factor determinante que influirá en el movimiento del terreno; también juegan un papel muy importante la orientación y el tipo de ondas que llegan al sitio.

Necesariamente el problema debe atacarse en un espacio de más de una dimensión. Existen diversos procedimientos analíticos que resultan útiles desde el punto de vista de la evaluación cualitativa del fenómeno; algunos de ellos se mencionan enseguida.

Aki y Larner (ref 21) plantean el problema general de dispersión* de ondas de cuerpo en un medio estratificado con interfase irregular mediante una ecuación integral de cuya solución se obtiene el campo de desplazamiento. La fig 16 muestra las amplitudes del movimiento en la superficie debido a ondas SH, que inciden a 0° y 35° para diferentes valores de la relación altura/ancho de un promontorio ($2l$). En todos los casos, la relación de la longitud de onda (λ) respecto a la altura (h) es igual a cinco. La forma de la amplificación presenta variaciones importantes para diferentes ángulos de incidencia, de tal manera que para un mismo punto la amplificación en uno y otro casos puede ser muy diferente. En el caso de la incidencia oblicua, la concentración de las ondas provocada por la irregularidad hace que el máximo ocurra en el centro (parte más alta) cuando la pendiente h/l es mayor. El efecto para ondas incidentes P y SV es similar. Para la forma inversa, o sea una depresión en el terreno (fig 17), la amplificación para valores pequeños de h presenta fuertes

* Sinónimo del vocablo inglés SCATTERING

variaciones locales. Es interesante observar la amplificación que para dichos valores pequeños ocurre en el centro, a diferencia de la atenuación que se presenta para valores grandes de h, donde la forma de la amplificación es más uniforme y aumenta a medida que la profundidad de la depresión es mayor. El efecto del ángulo de incidencia en la superficie ha sido estudiado por Trifunac (ref 22) mediante el modelo de un valle aluvial semicilíndrico, cuyo comportamiento supone perfectamente elástico donde llegan ondas *SH*. A partir de estas hipótesis calcula espectros de desplazamiento en función del ángulo de incidencia de las ondas, cuyo efecto es claramente visible en la fig 18. El conocimiento preciso de los ángulos de incidencia en la práctica es un problema complejo. El frente de ondas generalmente se supone plano en los estudios analíticos, sin embargo, la hipótesis no es aplicable en todos los casos.

Boore (ref 23) simula la propagación de ondas *SH* incidentes en una superficie irregular, mediante un método de diferencias finitas. Supone que el material a través del cual se propagan estas es elástico, isótropo y homogéneo. Sus resultados proporcionan idea del tamaño de estructura y longitud de onda para el cual la topografía puede ser importante. En la fig 19 se presenta uno de los tres modelos que él estudia: sus lados tienen inclinación de 35° respecto a la horizontal y altura aproximada de 20 m. La amplitud normalizada, respecto a la que se tendría si la superficie fuera plana, contra la frecuencia de excitación, se presenta en forma gráfica (fig 20) para los puntos 1 y 10 del modelo descrito; el espectro de amplitud de la excitación se muestra en la parte inferior de la figura citada. Las respuestas para las tres diferentes excitaciones son muy similares, excepto para la frecuencia menor (*1 Hz*). A medida que el periodo de las ondas crece, la amplitud normalizada tiende a ser unitaria; es decir, para estos casos la influencia de la topografía es mínima; no sucede igual con las ondas de alta frecuencia, que en la parte más alta del modelo (sitio 1) presentan amplificaciones del orden hasta de un 100 por ciento. Para alturas intermedias la ley atenuación-amplificación es variable, según se observa en la fig 21, donde se presentan atenuaciones hasta de 50 por ciento para diferentes sitios y frecuencias intermedias. Las abscisas representan la frecuencia adimensional fl/β donde

f frecuencia

l dimensión característica de la estructura (no bien definida por el autor)

β velocidad de ondas de cortante, que para este caso se supuso 500 m/seg.

Otro método, comúnmente usado para tratar el problema en cuestión, es el de elementos finitos. En este, el medio continuo se discretiza mediante una red de elementos de forma y tamaño adecuados, cuyas propiedades pueden ser no lineales. Es factible efectuar el análisis de la respuesta dinámica mediante un proceso, paso a paso, de integración de las ecuaciones de movimiento, expresadas en términos de los desplazamientos de ciertos puntos de unión entre los diversos elementos. El método ha sido utilizado para valuar los efectos dinámicos de las condiciones locales del sitio en espacios de dos y tres dimensiones.

Reimer, Clough y Raphael (ref 24) investigan la posible amplificación de las altas aceleraciones del terreno, registradas en Pacoima Dam, California, por influencia de la topografía, mediante un modelo tridimensional de elementos finitos sin considerar la retroalimentación de energía.

El acelerograma registrado en Pacoima Dam durante el temblor de San Fernando, el 9 de febrero de 1971, ha sido motivo de extensos estudios, por ser la aceleración máxima registrada hasta la fecha (1.25 *g*); representa un ejemplo típico de alteración del movimiento causada por irregularidades topográficas. Existe la necesidad de un análisis tridimensional de este problema, ya que de otra manera podría llegarse a conclusiones erróneas. Suponiendo que un análisis bidimensional fuera suficiente, se plantea el problema de considerar un perfil representativo, pero dada la complejidad topográfica del lugar (fig 22) es posible obtener perfiles en los que la estación aparece en una cresta y otros donde aparece en un valle, dando lugar a movimientos muy diferentes en cada caso (figs 16 y 17).

La limitación principal de los análisis tridimensionales numéricos es de tipo económico, ya que los tiempos de procesamiento en computadora requeridos para el análisis son mucho mayores que si estos fueran en un espacio bidimensional.

Numerosas observaciones resultantes de explosiones subterráneas y registros de aceleraciones comprueban la dependencia de la amplificación del movimiento de la frecuencia de excitación y del tamaño de las anomalías del terreno. Davis y West (ref 25) presentan datos de registros reales tomados en la base y parte superior de Kagel Mountain y Josephine Peak durante movimientos pequeños *(after shock)* después del evento principal del sismo de San Fernando en 1971. En la fig 23 se muestra la localización de los sitios mencionados y de los epicentros de los eventos registrados. La geología y topografía de las montañas y la ubicación de los sitios registradores en cada una de ellas se presenta en las figs 24 y 25. A partir de los registros de velocidad se obtuvieron historias de desplazamiento y aceleración, y espectros de seudovelocidad (*PSRV*) con 5 por ciento de amortiguamiento (aproximación cercana a espectros de amplitudes de Fourier). Las relaciones espectrales del movimiento entre la cresta y la base de las montañas, para diferentes valores de frecuencias, se muestran en las figs 26 y 27. Estas indican que la amplificación depende de la frecuencia y su forma es diferente en cada montaña: para la primera (Kagel Mountain), el intervalo de frecuencias de amplificación es menor que para la segunda, y las amplificaciones alcanzan tamaños más elevados principalmente para el componente *NS*. Las relaciones espectrales son diferentes para cada evento debido a las características de estos; sin embargo, muestran similitud entre ellas. Se presenta mayor amplificación en los componentes horizontales que en el vertical.

Debe notarse que los altos valores de amplificación espectral obtenidos de la punta con respecto a la base de las montañas, no están referidos al movimiento que ocurriría en caso de no existir irregularidad topográfica, sino que incluyen el efecto de reducción en la base mostrado en la fig 16.

Es probable que los valores de amplificación espectral fueran menores si la ubicación de la estación de la base fuera diferente, por ejemplo, ubicada en la base, pero a una distancia epicentral mayor. Las relaciones de amplitudes máximas se presentan en la tabla 2.

Los movimientos en la cresta son siempre mayores que en la base; esto puede apreciarse tanto en el dominio de las frecuencias como en el del tiempo. En la fig 28 se comparan los registros de velocidad obtenidos en la cresta y en la base de Kagel Mountain durante el evento *A*. Los sismómetros estaban orientados en la dirección *NS*, o sea transversal a la línea que une al sitio con el epicentro.

Existe una notable diferencia en duración y amplitud de las ondas de cortante para cada estación. Suponiendo una velocidad de ondas de cortante de 10 000 pies/seg y un periodo de 0.5 seg, que es aproximadamente donde ocurre la amplificación máxima, se obtiene una longitud de onda de 5 000 pies, que coincide con el ancho medio de Kagel Mountain, por lo que es posible que las ondas de cortante causen resonancia, lo que justifica la gran amplitud de estas y la aparición de más ciclos de movimiento en el registro correspondiente a la parte elevada de la montaña; sin embargo, no es claro que esta sea la única razón debido a que la amplificación ocurre también en periodos diferentes de 0.5 seg.

Las amplitudes máximas de velocidad de la cresta son cuatro veces mayores que las de la base; la parte posterior de las ondas de cortante que llegan es mucho mayor en la cresta que en la base.

Los sismógrafos de velocidad registrados en la cresta y en la base de Josephine Peak durante el evento *D*, se muestran en la fig 29; en ellos se aprecia que el movimiento fue mayor en la cresta que en la base durante casi todo el registro; sin embargo, la amplificación de las ondas de cortante es menor que en el caso de Kagel Mountain y la duración de estas ondas no es mucho mayor en los dos sismogramas, a diferencia de lo ocurrido en Kagel Mountain. Esto puede atribuirse a varios factores: diferencias de tamaño y forma de las montañas, mayor distancia epicentral de Josephine Peak e influencia del mecanismo de falla, ya que los sismogramas corresponden a eventos diferentes.

El fenómeno de amplificación calculado y observado en montañas, constituidas generalmente por roca, advierte que no solo deben considerarse las condiciones del subsuelo locales como principal contribuyente del fenómeno de amplificación del movimiento, sino además tomar en cuenta las condiciones locales de frontera como posible factor importante para la ocurrencia de efectos dinámicos desconocidos.

Los casos presentados en este capítulo indican la importancia de una reproducción adecuada de situaciones reales a través de modelos analíticos equivalentes, constituidos por dos partes fundamentales:

1. Representación de las condiciones geológicas y geométricas del sitio
2. Características de la excitación probable a que estará sometido dicho sitio, lo cual implica conocer las características de los diferentes tipos de ondas sísmicas contribuyentes al movimiento esperado.

4. INFLUENCIA DEL MECANISMO DE FALLA EN LOS MOVIMIENTOS DEL SUELO

Las características de algunos eventos sísmicos se determinan básicamente por factores asociados al mecanismo de falla. En estos casos, la influencia de las condiciones locales es poco apreciable y difícil de predecir; sin embargo, en las características de los temblores el mecanismo se manifiesta en los contenidos de frecuencia y en los detalles de la historia del movimiento del terreno, puesto que no es posible predecir en detalle los mecanismos de falla, tampoco puede predecirse su influencia en los temblores que pueden originarse en una fuente dada. La incertidumbre sobre esta influencia se engloba en la incertidumbre asociada a las expresiones semiempíricas que se han desarrollado para relacionar parámetros en la fuente con intensidades y distancia (refs 26, 27 y 28).

La evidencia de la influencia de tales factores, puede confirmarse en la práctica a partir de registros con características diferentes hechos en estaciones muy próximas, situadas sobre suelos con condiciones locales similares, o en una misma estación, de eventos originados en una misma fuente o fuentes cercanas entre sí; sin embargo, esto no siempre es posible debido a la escasez de instrumentación sísmica existente, salvo en algunas regiones, como la parte suroeste de Estados Unidos, donde son posibles estudios de esta índole. Algunos de ellos se presentan a continuación:

Udwadia y Trifunac (ref 29) analizaron una serie de 15 eventos registrados en la estación de El Centro, California, la mayoría de ellos cercanos al epifoco, con magnitudes (*ML*) que varían entre 3 y 6.8, y los clasifican en cuatro grupos de acuerdo con el azimut de cada epicentro con respecto a la estación (fig 30); la tabla 3 contiene algunas de sus características.

Los espectros de amplitudes de Fourier, calculados a partir de los registros corregidos de estos sismos, se presentan en las figs 31 a 34; de ellas se deduce que:

a) El grupo I contiene cuatro eventos, tres de los cuáles tienen un mismo epifoco y diferentes magnitudes. Las formas espectrales de los componentes correspondientes de los diversos eventos difieren considerablemente entre sí (fig 31). Esto debe atribuirse a las diferentes características del mecanismo de falla para uno y otro evento, puesto que la trayectoria de las ondas es aparentemente igual en los tres casos. El evento restante (No 38.1) es, de los que se presentan en este estudio, uno de los de menor distancia epicentral; sus amplitudes, sin embargo, no son grandes debido a que su magnitud es 3.0, la menor dentro del intervalo estudiado. La energía total incidente en el sitio de los eventos 38.1 y 55.3 es semejante, como puede observarse a partir del área que encierran las respectivas curvas espectrales.

b) El grupo II contiene cuatro eventos; se asocian en un subgrupo las magnitudes 6.4 y 6.5 (eventos 42.1 y 68.5) y en otro las 5.5 y 5.6 (eventos 51.2 y 53.1); las distancias epicentrales son muy diferentes de un grupo a otro y entre los eventos del mismo grupo. La energía total que llega a la estación es mayor para los eventos más distantes que para los cercanos al foco (fig 32), como consecuencia de las diferencias de magnitud. El evento 68.5 presenta frecuencias predominantes entre 0 y 2 cps para los componentes horizontales, lo que puede atribuirse al predominio de ondas superficiales observadas en la mayor parte del acelerograma correspondiente. La irregularidad de las formas espectrales conduce a pensar en la poca influencia de las condiciones locales sobre estos movimientos.

c) El grupo III incluye el sismo del Valle Imperial, California, ocurrido en 1940, cuyo espectro de amplitudes de aceleración se presenta en la fig 33, junto con otros tres eventos que pertenecen al mismo grupo. En estos se aprecia cierta regularidad en los componentes horizontales, no así en el componente vertical, pues al evento 40.1 se asocian ordenadas muy altas para frecuencias elevadas. Este evento consiste, en realidad, en la superposición de varios, lo que implica la llegada de ondas S y P a intervalos pequeños de tiempo con sus correspondientes altas frecuencias; es posible que el contenido de altas frecuencias obedezca a la corta distancia epifocal del sismo, que implica una baja atenuación en estas ondas. La influencia del mecanismo focal es en este caso muy significativa.

d) El siguiente grupo de eventos se caracteriza por distancias epicentrales (Δ) muy grandes, por lo que es de esperarse la influencia de las ondas superficiales con su contenido de bajas frecuencias (fig 34). A este grupo corresponde el evento 54.5 (ML = 6.3; Δ = 148 km), que posee características comparables a las del evento 66.9 (ML = 6.3; Δ = 150 km), del grupo III. A pesar de las similitudes entre estos dos eventos, sus formas espectrales son muy diferentes, lo que hace ver la influencia del mecanismo de falla y de la trayectoria de las ondas.

Del estudio comparativo, realizado por Trifunac y Udwadia (ref 30), sobre los registros en seis estaciones ubicadas dentro del área metropolitana de los Angeles, Calif, durante los sismos de Borrego Mountain (1968), Lytle Creek (1970) y San Fernando (1971), se deduce que en esta área los registros dependieron fundamentalmente del mecanismo de falla y de la distancia focal, en tanto que las condiciones locales del suelo jugaron un papel menos importante.

La localización de las estaciones y los epicentros de los tres sismos empleados en el estudio se incluyen en la fig 35. Cuatro estaciones (N. Robertson, Hollywood Storage, S. Olive y Water and Power) están dentro del área de Los Angeles y Hollywood, mientras que las otras dos (Castaic y Cedar Springs) se localizan fuera del área metropolitana. (tabla 4).

Los espectros de Fourier de los tres componentes de los movimientos registrados en las seis estaciones se presentan en las figs 36 y 37, las cuales muestran que en ninguna de dichas estaciones se observan periodos dominantes de vibración. En las figuras se identifican los espectros de la siguiente manera: *BM68* sismo de Borrego Mountain (1968); *LC70* sismo de Lytle Creek (1970) y *SF71* sismo de San Fernando (1971). Los espectros correspondientes al componente *NS* de los eventos *BM68* y *SF71* registrados en la estación de Cedar Springs, hacen pensar en periodos dominantes en ese sitio (debido a la similitud que hay entre ellos en el intervalo 0-2.5 Hz); sin embargo, la forma diferente del espectro del sismo *LC70* elimina la duda sobre el posible efecto local. Este ejemplo ilustra la importancia de la seguridad que debe haber sobre el comportamiento real de un sitio antes de emplear cualquier modelo analítico.

Dado que las estaciones 135, 142, 166 y 137 se encuentran cercanas entre sí y que las características locales del suelo son similares, Trifunac y Udwadia (ref 30) comparan los registros obtenidos durante los eventos mencionados a fin de evaluar cualitativamente la influencia de la distancia focal en las características de estos. Las figs 38 y 39 presentan curvas de aceleración, velocidad y desplazamiento en las cuatro estaciones para los sismos *BM68* y *SF71*, correspondientes a sus componentes radial, transversal y vertical.

Para el evento *BM68* son notorias las amplitudes y periodos grandes del componente de desplazamiento transversal; dada la ubicación de las estaciones respecto al epifoco, estas pueden obedecer a la presencia de ondas Love. La forma de los desplazamientos transversales es similar en los cuatro registros, pero su amplitud no, siendo menor en las estaciones Hollywood Storage y N. Robertson, a diferencia de las curvas correspondientes al componente radial, que presentan amplitudes mayores que las de los otros registros. Una explicación puede encontrarse en los cambios de profundidad del terreno aluvial que ocurren entre los dos grupos de estaciones, cuya separación máxima es 12 km. Las estaciones Water and Power y S. Olive están separadas menos de 1 km, por lo que los registros muestran desplazamientos casi iguales. Las ondas en cuestión tienen periodos de 3 a 5 seg que corresponden a longitudes de onda del orden de 10 a 20 km. Esto explica la similitud de registros obtenidos en sitios que distan pocos kilómetros.

Las historias de aceleración registradas, y las de velocidad y desplazamiento calculadas, correspondientes al temblor de San Fernando (1971) en las cuatro estaciones mencionadas, se presentan en la fig 39; las curvas de cada componente para aceleración, velocidad y desplazamiento muestran gran similitud en los cuatro sitios. De acuerdo con la interpretación de Hanks (citado en ref 30), las grandes amplitudes de ondas de desplazamiento radial se deben a ondas de Rayleigh, mientras que las de desplazamiento transversal a ondas de Love. Los contenidos de frecuencias en los acelerogramas de los dos sismos anteriores, obtenidos en una misma estación, son muy diferentes; los espectros de amplitudes de Fourier de los registros correspondientes a los eventos *BM68* y *SF71* se presentan en las figs 40 y 41, respectivamente. La fig 40*a* muestra gran similitud entre los espectros de los sitios *HOLS* y *NROB* entre 0 y 1.2 Hz, y entre los de los sitios *SOLV* y *WOPO* hasta 1.5 Hz (como se mencionó en el análisis en el dominio del tiempo, fig 39). La fig 40*b* muestra espectros similares en los sitios *HOLS*, *SOLV* y *WOPO* para frecuencias menores de 1 Hz, y en la fig 40*c* los espectros en los cuatro sitios son muy parecidos. Las características comunes de los espectros obedecen al predominio de ondas superficiales.

Es erróneo interpretar como periodos dominantes asociados al terreno local aquellos a los que corresponden las máximas ordenadas espectrales en estas figuras. Su justificación, como ya se indicó, se encuentra probablemente en la distancia epicentral grande (≈ 200 km).

Los espectros de los sitios *HOLS* y *NROB* (fig 41a) son similares en frecuencias menores de 2 Hz, siendo aproximadamente iguales a los restantes de 0 a 1.2 Hz. A diferencia de los espectros de Borrego Mountain, aquí los de componentes transversales solo coinciden hasta frecuencias de aproximadamente 1 Hz; más allá de estas, la contribución de las altas frecuencias es significativa, aunque irregular, lo cual puede atribuirse a la proximidad del foco (≈ 40 km).

Un estudio realizado por Hudson (ref 31) revela que para el área de Pasadena, Calif., no existe gran correlación entre las características de los movimientos y la geología local.

El estudio se basa en el análisis de los registros de 18 sismoscopios y de cuatro acelerogramas obtenidos dentro de un área de 40 millas cuadradas durante el sismo de San Fernando del 9 de febrero de 1971. En la fig 42 se muestra la localización de los sitios registradores y la posición del epicentro respecto al área. En esta figura, los segmentos de recta, orientados según la respuesta horizontal máxima, representan los desplazamientos máximos registrados, con una longitud proporcional a estos; como los sitios *DG*, *ER* y *EW* contaban cada uno con dos sismoscopios cercanos, aparecen dos líneas en el mismo punto. Las características generales del terreno en estos sitios se presentan en la tabla 5.

A pesar de las similitudes existentes entre algunos sitios (*FP* y *DG*; *MS*, *RG* y *EW*; *WJS* y *SL*), la diversidad de movimientos es notoria entre ellos (fig 43), lo que hace pensar en la importancia que pueden tener en el movimiento del terreno otros factores tales como la topografía local o la configuración irregular de los mantos del subsuelo (refs 23 y 32), factores tratados en el capítulo anterior.

A fin de comparar diferentes movimientos, se presentan en la fig 44 combinaciones de los espectros de respuesta amortiguados, calculados a partir de los acelerogramas corregidos registrados en los sitios *ATH*, *JPL*, *ML*, *SL*. De esta figura se observa que:

a) En la fig 44*a* aparecen los espectros correspondientes a los sitios *ML* y *ATH*, distantes 0.5 km uno de otro. El sitio *ML* se localiza en la base de un edificio de nueve niveles y el sitio *ATH* en la de uno de dos y medio niveles; sus espectros de respuesta no revelan periodos dominantes que puedan relacionarse con la interacción suelo-estructura. Sus formas son parecidas, no así sus magnitudes, que resultan mayores para el sitio *ML*.

b) La fig 44*b* muestra espectros de los componentes de movimiento de los sitios *SL* y *ATH*, siendo sus formas aproximadamente iguales. Este ejemplo ilustra el caso en que las condiciones locales tienen poca influencia en el movimiento; no sucede lo mismo con el componente *NS*, ni con los movimientos de los sitios *ML* y *JPL* (figs 44*c* y *d*), donde la respuesta resulta menor para el sitio sobre roca cristalina.

Hudson (ref 31) subraya el hecho de que los espectros de respuesta en los mismos sitios serían diferentes ante perturbaciones originadas por un foco con características distintas a las del sismo de San Fernando. Esto denota la influencia que tienen el mecanismo de falla y la trayectoria de las ondas en los movimientos en esa zona. La elaboración de mapas sobre riesgo sísmico en el área de Pasadena, por lo anterior, opina el autor, deberá basarse en la extrapolación de datos acumulados correspondientes a eventos fuertes, lo cual no representa problema, ya que la zona está ampliamente instrumentada; sin embargo, pasará mucho tiempo para que puedan realizarse estudios similares en áreas con escasa instrumentación.

Trifunac (refs 12 y 22) realizó estudios referentes al temblor del Valle Imperial de 1940, basados en registros de las estaciones de Tinemaha y El Centro, Calif. Ese sismo se caracteriza por la multiplicidad de eventos ocurridos en 15 seg (agrupados con el nombre de *EVENTO PRINCIPAL*) seguidos de otros, más espaciados en tiempo, en los siguientes 5 min (*RÉPLICAS*); cuatro días después, se registraron 29 eventos con magnitudes de 2.0 a 5.5, y posteriormente eventos más espaciados y menos fuertes, que no son motivo de estudio en los trabajos citados.

La localización de la zona donde ocurrieron los eventos individuales y la ubicación de sus epicentros se presentan en la fig 45. La estación de Tinemaha se encuentra a 546 km del epicentro del evento principal, por lo que no aparece en la figura.

El evento principal consta de cuatro movimientos fuertes espaciados entre 5 y 7 seg uno del otro (fig 46). Para los componentes horizontales, la presencia de altas frecuencias en los registros indica las llegadas de ondas de cortante (S) (fig 46, parte inferior), el arribo de ondas de compresión (P) es difícil de identificar debido a los efectos de los eventos precedentes y a la cercanía del epifoco. Los tiempos probables de arribo se indican por las flechas superiores en la misma figura. Un análisis más detallado de estos acelerogramas que utiliza el concepto de espectro de respuesta envolvente (*Response Envelope Spectrum: RES*) indica el predominio de ondas superficiales (fig 47). Como se aprecia, este tipo de representación resulta conveniente para el análisis de acelerogramas.

La fig 47 muestra el *RES* de aceleración correspondiente al componente *EW* de los eventos *1A*, *1B* y *1C*. Las curvas de llegada de los primeros modos de las ondas superficiales de Love (línea llena) y de Rayleigh (línea punteada) se dibujaron a partir de sus curvas de dispersión y de las distancias epicentrales, que para este caso son de: 10 km (*1A*), 12.5 km (*1B*) y 15.0 km (*1C*).

Como se aprecia en las últimas figuras mencionadas, la secuencia de eventos da lugar a movimientos que serían diferentes si estos ocurrieran individualmente; las características del movimiento a distancias epicentrales grandes también son alteradas, al respecto, la fig 48 muestra las historias de aceleración, velocidad y desplazamiento durante un tiempo de cinco minutos, correspondientes a la estación de Tinemaha. Trifunac realiza un estudio detallado de estos registros mediante análisis de Fourier con ventana móvil, que es alternativo al análisis que conduce al *RES* antes mencionado. El espectro de Fourier del componente *EW* del sismograma de Tinemaha se presenta en las figs 49*a* y *b*, donde se pueden apreciar periodos dominantes largos (≈ 10 seg) que indican la presencia de ondas superficiales, justificadas por la lejanía de la estación respecto al foco.

5. CONCLUSIONES Y RECOMENDACIONES

El problema de definir la importancia relativa de diversos factores en el movimiento de un sitio, del cual no se cuenta con registros sísmicos, dista mucho de estar resuelto.

En algunos casos, la influencia de las condiciones locales es preponderante sobre otros factores tales como el mecanismo de falla, la trayectoria de las ondas durante su recorrido del foco a la región de interés, etc, en tanto que para otros, sucede lo contrario.

Del análisis de registros obtenidos en sitios bajo diversas condiciones, algunos de ellos revisados en este trabajo, se pueden deducir algunas conclusiones de tipo cualitativo que proporcionan idea sobre la menor o mayor importancia de ciertos factores en el movimiento:

1. Las características del movimiento en la superficie dependen principalmente de los del mecanismo de ruptura, la posición relativa del foco y el sitio, y de las condiciones del suelo a través del cual se propagan las ondas.

2. Es más común que la influencia de las condiciones locales pueda aislarse de las demás en sitios lejanos al foco que en los próximos a este.

3. En lugares a distancias focales muy cortas, la influencia del mecanismo de falla generalmente disminuye la de otros factores.

4. En suelos cuya superficie es aproximadamente plana, la ocurrencia de fenómenos de amplificación local es más significativa si los suelos son blandos que si son firmes.

5. La respuesta de un depósito de suelo blando, que descansa sobre un medio cuya impedancia es mucho mayor que la de él y se parece mucho a la de un sistema de cortante, que para excitaciones sísmicas moderadas responde como si poseyera periodos naturales de vibración, sufre generalmente amplificación cuando se somete a movimientos cuyos periodos dominantes son cercanos al suyo. Cuando el espesor del manto superficial no está perfectamente definido, la amplificación dista de seguir las leyes que corresponderían a sistemas con modos clásicos de vibración por cortante, y no se han definido criterios generales para valuarlo.

6. Las regiones que presentan irregularidades topográficas cuyas dimensiones son comparables a la longitud dominante de las ondas incidentes en tales anomalías, pueden sufrir fenómenos de amplificación y/o atenuación significativa, en sus diferentes partes.

7. En la superficie de depósitos de suelo blando, apoyados sobre roca, delimitados por una frontera inferior con características geométricas irregulares cuyas dimensiones son comparables al espesor del manto superficial, puede haber una concentración de ondas.

A continuación se propone el desarrollo de estudios encaminados a predecir la influencia de las condiciones locales en el movimiento sísmico para algunos modelos sobresimplificados, relacionados con la naturaleza de la excitación sísmica. Debido a lo drástico de las hipótesis que se admiten en algunos de ellos, las conclusiones serán de tipo cualitativo, pero de gran utilidad para entender mejor los diversos fenómenos observados durante los eventos sísmicos.

Tales estudios están relacionados con la excitación en la fuente, con la influencia del medio a través del cual se propaga esta, y con la excitación en la vecindad del sitio o con la forma como las condiciones locales pueden modificarla. En cada uno se da una breve descripción sobre su importancia y posible solución de acuerdo con el estado actual del conocimiento.

5.1 *Condiciones bajo las cuales son importantes las características locales del suelo en los movimientos sísmicos*

La importancia de las condiciones locales del suelo varía (según los estudios indicados en este trabajo) de una situación a otra, sin existir actualmente un criterio fijo que permita definir dicha influencia; sin embargo, es factible realizar estudios de tipo semicualitativo para entender y evaluar las características fundamentales del problema.

Aun cuando algunas conclusiones de tipo cualitativo (también mencionadas en este trabajo) proporcionan idea de las condiciones bajo las cuales las características locales cobran importancia, es deseable contar con estudios más detallados que permitan predecir la influencia de las condiciones locales para diversas magnitudes y posiciones relativas foco-sitio. Estos estudios deberán basarse en registros de eventos diversos para los cuales las características locales del suelo influyeron en diversa medida, y analizar las condiciones bajo las cuales son determinantes las propiedades del suelo local.

5.2 *Atenuación de diferentes tipos de ondas al viajar dentro o por la superficie de la tierra*

El comportamiento de las ondas sísmicas durante su recorrido puede estudiarse separando unas de otras, según sean de cuerpo o de superficie. Dicha separación, a la fecha no ha sido lograda por completo; sin embargo, un criterio simple para distinguir unas de otras es por su tiempo de arribo a las estaciones registradoras y por su contenido de frecuencias, con lo cual es posible deducir, mediante un estudio estadístico, leyes de atenuación correspondientes a ondas con características determinadas, que podrían asociarse a ondas *P*, *S*, *Rayleigh*, *Love* o a combinaciones de estas.

Los estudios más comunes, basados en la observación de registros sísmicos, acerca de la atenuación de ondas durante su recorrido, pertenecen a los intervalos de frecuencias mediano y alto (0.3 a 5.0 Hz), en tanto que para las ondas de bajas frecuencias (< 0.3 Hz), intervalo al cual pertenecen las frecuencias naturales de algunas estructuras civiles importantes, como por ejemplo: puentes de grandes claros, edificios altos, etc, de hecho hay ausencia de conocimientos.

El comportamiento de ondas, propagándose en una superficie plana a través de suelos con diferentes propiedades geológicas ha sido estudiado experimentalmente; sin embargo, es deseable que los resultados sean comparados con otros basados en observaciones de eventos sísmicos de poca intensidad y/o explosiones subterráneas.

5.3 *Contribución de diferentes tipos de ondas en el movimiento*

La modificación del movimiento en la superficie respecto al que ocurre en terreno firme puede en algunos casos predecirse mediante la *teoría de propagación unidimensional de ondas de cortante*. Salvo en situaciones extremas, es poco común que las hipótesis en que esta se sustenta se cumplan rigurosamente, lo que da lugar a discrepancias entre movimientos calculados y observados.

Una de las dudas más frecuentes que se presenta en los casos reales acerca de la aplicabilidad de la teoría, en cuanto a la excitación en la vecindad del sitio, es el predominio de ondas de cortante como hipótesis fundamental.

Algunos estudios experimentales (cap 3) han comprobado una contribución considerable de ondas superficiales en el movimiento de sitios, en los que aparentemente se cumplen los requisitos exigidos para la utilización de este método de análisis. La influencia de ondas que no son de cortante, en la función de amplificación, puede ser en ocasiones lo suficientemente importante como para modificar su forma.

Se propone el estudio de la contribución de diferentes tipos de ondas en el movimiento del terreno a partir del análisis de registros de eventos en sitios que cumplan (aparentemente) las hipótesis de la teoría. Los registros deberán corresponder a movimientos ocurridos a diferentes profundidades en varias verticales dentro de la vecindad en estudio, y a partir de estos análisis deducirse condiciones sobre el predominio de ondas de cortante, así como su relación con el contraste de impedancias entre el manto superficial y el medio que lo soporta.

5.4 *Influencia de la topografía en la propagación y generación de ondas superficiales*

La presencia de fenómenos de amplificación y/o atenuación observados durante eventos sísmicos en lugares con irregularidades topográficas, señala la necesidad de estudios encaminados a explicar dichos fenómenos.

Algunos estudios (cap 3) que utilizan diferentes métodos de análisis, han contribuido a tal explicación. La mayoría de ellos consideran los efectos de la incidencia de ondas de cuerpo en las anomalías topográficas; sin embargo, es necesario investigar los efectos que sobre dichas anomalías tendrían las ondas superficiales (considerando diferentes relaciones de longitudes de onda-dimensiones de irregularidades topográficas), para lo cual se podrían emplear algunos métodos de análisis mencionados en 3.4.

Dentro de este tema deberán estudiarse también las leyes de variación de amplitudes de ondas superficiales con la profundidad, a fin de poder considerar los alcances de su influencia; esto es importante en los casos en que se consideren depresiones en el terreno cuyas dimensiones sean comparables a las longitudes de onda de las incidentes. La necesidad de registros simultáneos a diferentes profundidades, es obvia para este tipo de estudios.

5.5 *Amplificación del movimiento debido a diferentes tipos de ondas*

La amplificación del movimiento de un punto en la superficie de un terreno blando, respecto al de un punto en terreno firme debido a la excitación de ondas de cuerpo, ha sido ampliamente estudiada. Tratamientos similares deberán efectuarse para casos de depósitos aluviales con incidencia horizontal de ondas superficiales, encaminados a predecir la amplificación del movimiento respecto al de un punto ubicado en el terreno firme circundante.

La contribución de las ondas superficiales en el movimiento provocado por sismos de gran intensidad y de foco poco profundo es bastante considerable, por lo que ameritan atención especial los estudios de esta índole.

5.6 *Correlación entre parámetros sísmicos, incluyendo características del mecanismo de falla y condiciones locales del suelo*

Se propone la predicción estadística de espectros de respuesta en sitios con características locales dadas, ubicados en posiciones conocidas respecto al foco y originados por mecanismos de naturaleza supuestamente conocida. Los estudios deberán basarse en un número suficiente de registros clasificados en:

a) Grupo de registros correspondientes a sitios en los cuales las condiciones locales no son importantes y los mecanismos de falla tienen características esenciales similares (magnitud, orientación de falla, momento sísmico, etc). En este caso, se estudiará la influencia de la trayectoria de las ondas en la respuesta de la superficie.

b) Grupo de registros correspondientes a eventos de naturaleza similar, hechos en sitios ubicados en posiciones relativas semejantes respecto al foco, pero con condiciones locales diferentes. En este caso, se estudiará la influencia de dichas condiciones en la respuesta (dadas ciertas características del mecanismo de falla y posición foco-sitio).

Los estudios resultantes del grupo *b* deberán complementar los del inciso *a*, ya que en un caso no se considera la influencia de las condiciones locales y en otro sí. De esta manera será posible plantear expresiones semiempíricas que incluyan la influencia del terreno local y el mecanismo de falla, y estimar su grado de incertidumbre para diferentes intervalos de magnitud, distancia epicentral, profundidad focal, condiciones locales geológicas, etc.

Actualmente, se han realizado estudios, menos completos, que no consideran la influencia del mecanismo de falla; en cualquier caso, la forma de las expresiones será análoga.

6. RECONOCIMIENTO

Se agradece a Luis Esteva sus valiosos comentarios y sugerencias durante el desarrollo de este trabajo.

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TABLA 1. FACTORES PRINCIPALES QUE INFLUYEN EN LAS CARACTERISTICAS DE LOS MOVIMIENTOS SISMICOS

Fuente sísmica

- Características tectónicas de la fuente
- Cantidad de energía liberada
- Orientación de la radiación de ondas
- Esfuerzos estáticos y dinámicos desarrollados en el foco
- Características geométricas de mecanismo

Trayectoria de las ondas sísmicas

- Posición relativa del foco y sitio
- Geología de los estratos existentes entre el foco y la región en estudio
- Geometría de los estratos
- Continuidad del medio (presencia de fallas)
- Comportamiento del suelo ante cargas dinámicas
- Condiciones locales:
  - Topografía de la superficie
  - Relación de impedancia entre el manto superficial y el estrato que lo soporta
  - Geometría del manto superficial
  - Geología
  - Continuidad
  - Comportamiento ante cargas dinámicas

TABLA 2

Evento	Componente*	Aceleración	Velocidad	Desplazamiento
Kagel Mountain (Estaciones 82 y 83)				
A	NS	1.14	2.20	3.15
	Z	1.02	1.26	2.87
	V	1.03	1.57	2.96
B	NS	1.41	3.64	7.95
	Z	1.09	1.60	3.52
	V	1.07	2.45	6.13
C	NS	1.75	3.96	5.70
	Z	2.31	2.65	1.91
	V	1.46	2.84	3.80
Josephine Peak (Estaciones 80 y 81)				
D	NS	0.83	1.80	2.03
	EW	1.50	1.81	2.46
	Z	1.31	1.58	1.82
	V	1.56	2.26	2.22
E	NS	1.40	2.25	1.72
	EW	1.56	2.28	2.24
	Z	1.28	1.80	1.83
	V	1.63	2.54	1.94

* NS, norte-sur; EW, este-oeste, Z, vertical, V, vector resultante

TABLA 3

Evento	Clave	Grupo	Fecha	ML	Distancia epicentral, en km
1	34.2	III	Dic 30, 1934	6.5	60
2	38.1	I	Abr 12, 1938	3.0	16
3	38.4	III	Jun 6, 1938	4.0	71
4	40.1	III	May 18, 1940	6.5	15
5	42.1	II	Oct 21, 1942	6.5	44
6	51.2	II	Ene 23, 1951	5.6	25
7	53.1	II	Ene 14, 1953	5.5	19
8	54.5	IV	Nov 12, 1954	6.3	148
9	55.3	I	Dic 16, 1955	4.3	27
10	55.4	I	Dic 16, 1955	3.9	27
11	55.5	I	Dic 16, 1955	5.5	27
12	56.1	IV	Feb 9, 1956	6.8	119
13	56.2	IV	Feb 9, 1956	6.1	119
14	66.9	III	Ago 7, 1966	6.3	150
15	68.5	II	Abr 8, 1968	6.4	72

TABLA 4

Estación	No	Elevación de la estación, en pies	Elevación de la roca basal, en pies	Espesor del terreno aluvial, en pies	Distancia epicentral, en km		
					Borrego Mountain	Lytle Creek	San Fernando
Castaic	110	2500	2500	0	279	108	31
Hollywood Storage (HOLS)	135	290	-6000	6000(?)	226	74	35
Ro N. Robertson (NROB)	142	170	-10000	10200	230	79	37
646.Olive (SOLV)	166	265	-12000	12300	218	71	42
Water and Power Building (WOPO)	137	390	-12000	12400	218	71	42
Cedar Springs (Allen Ranch)	111	3500	3500	0	166	19	96

TABLA 5

No sitio	Estación	Características locales
1	ATH	900 pies terreno aluvial. Cimentación, edificio de concreto de 2.5 niveles
2	CR	600 pies terreno aluvial
3	DG	400 pies terreno aluvial
4	DG	Presa de gravedad de concreto
5	ER	Roca cristalina sobre terciario
6	ER	Presa de tierra
7	EW	800 pies terreno aluvial
8	EW	Presa de tierra
9	FP	400 pies terreno aluvial
10	GES	300 pies terreno aluvial
11	HES	1000 pies terreno aluvial
12	JPL	400 pies terreno aluvial. Cimentación, edificio de acero de 9 niveles
13	ML	900 pies terreno aluvial. Cimentación, edificio de concreto de 9 niveles
14	MS	800 pies terreno aluvial
15	NM	100 pies terreno aluvial
16	RG	800 pies terreno aluvial
17	SL	Roca cristalina de granito
18	SMC	1200 pies terreno aluvial
19	VT	1000 pies de sedimento antiguo sobre terciario
20	WJS	Brote de roca cristalina

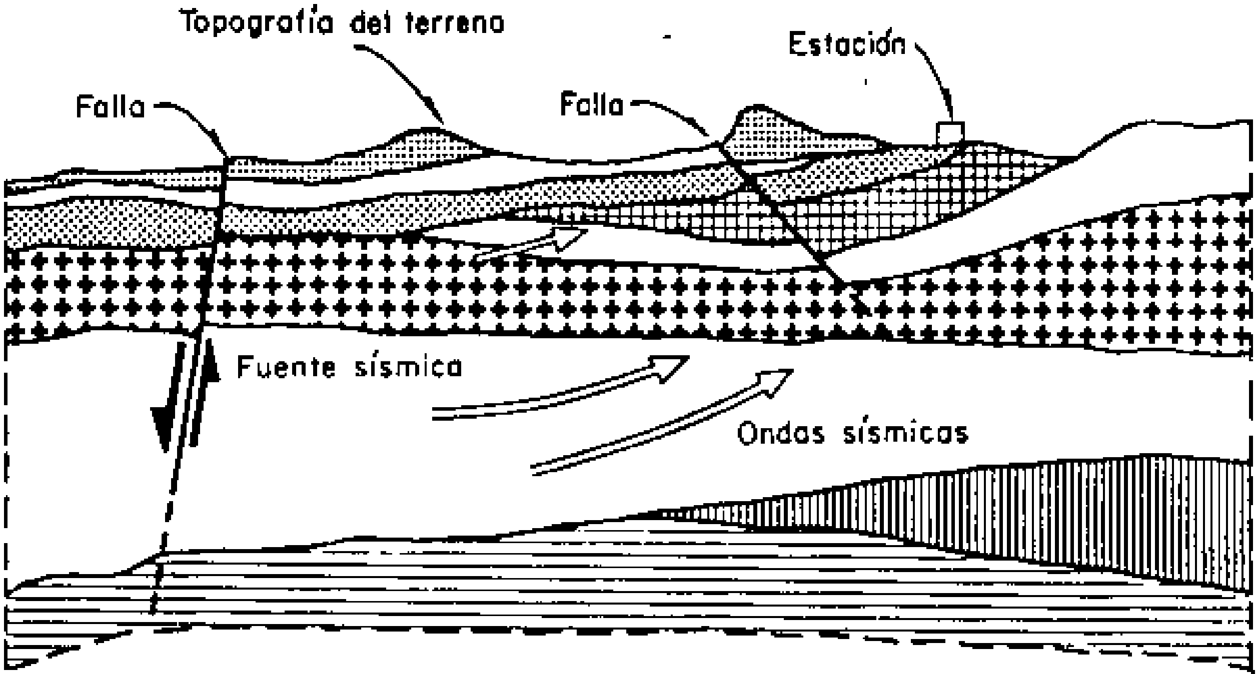


Fig 1. Corte geológico típico

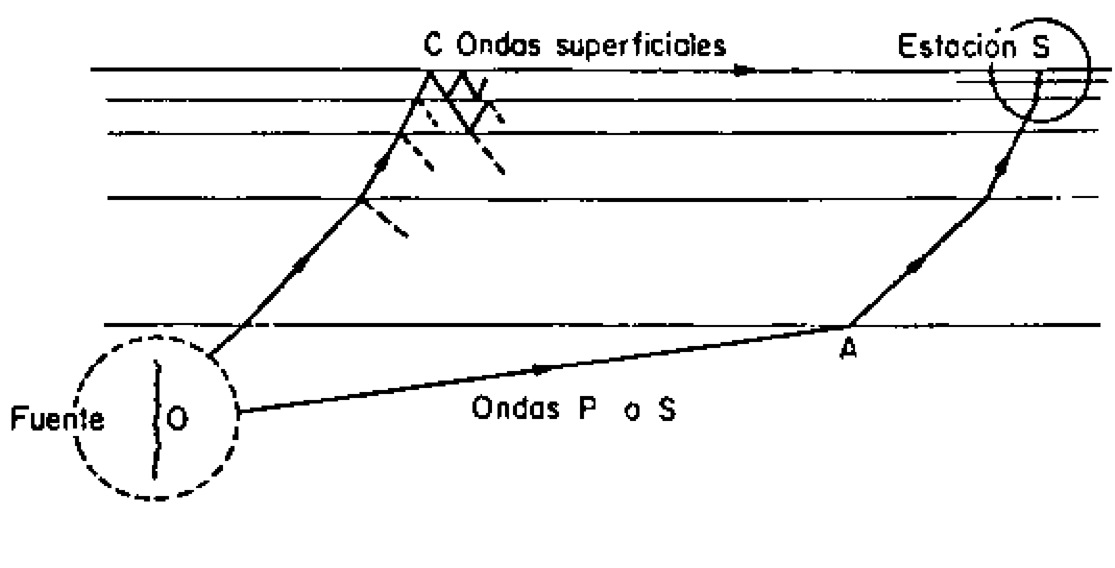


Fig 2. Idealización de la trayectoria de ondas entre el mecanismo de falla y la estación de registro (ref 5)

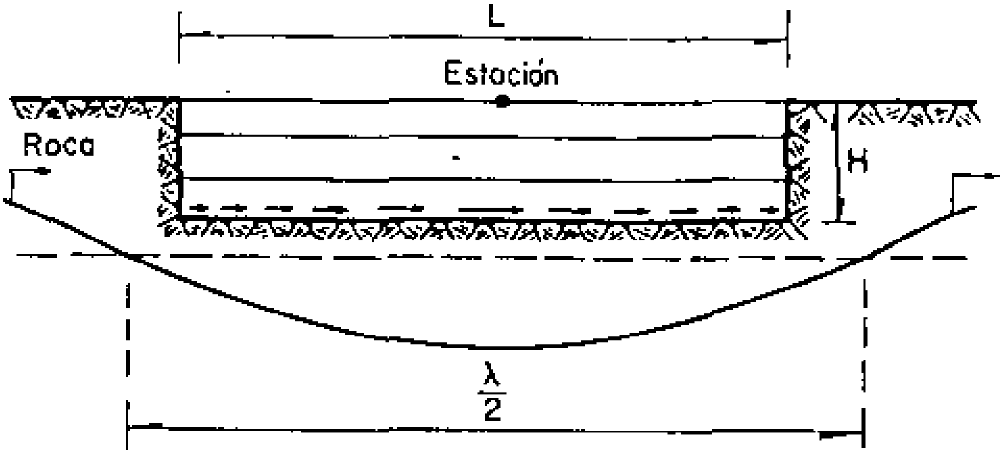


Fig 3. Respuesta de estratos de suelo locales ante componentes de onda incidiendo horizontalmente (ref 5)

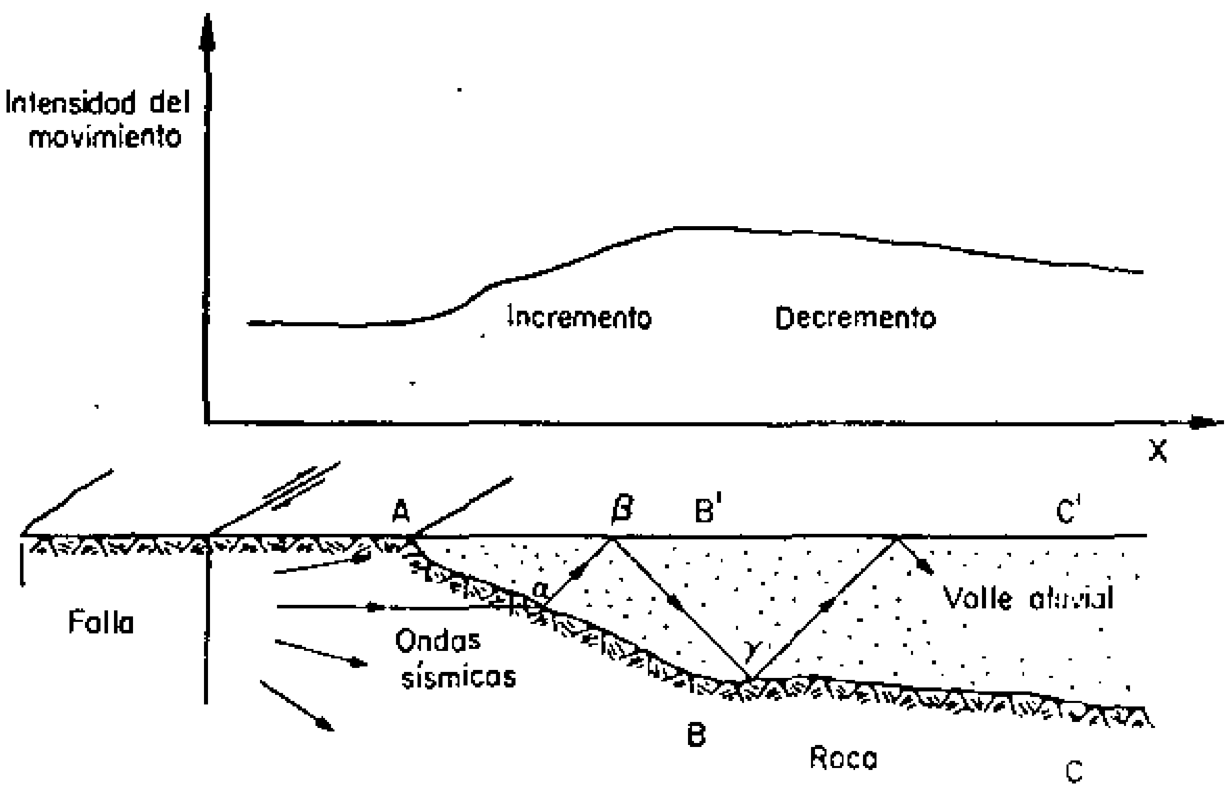


Fig 4. Influencia del subsuelo en la intensidad del movimiento (ref 12)

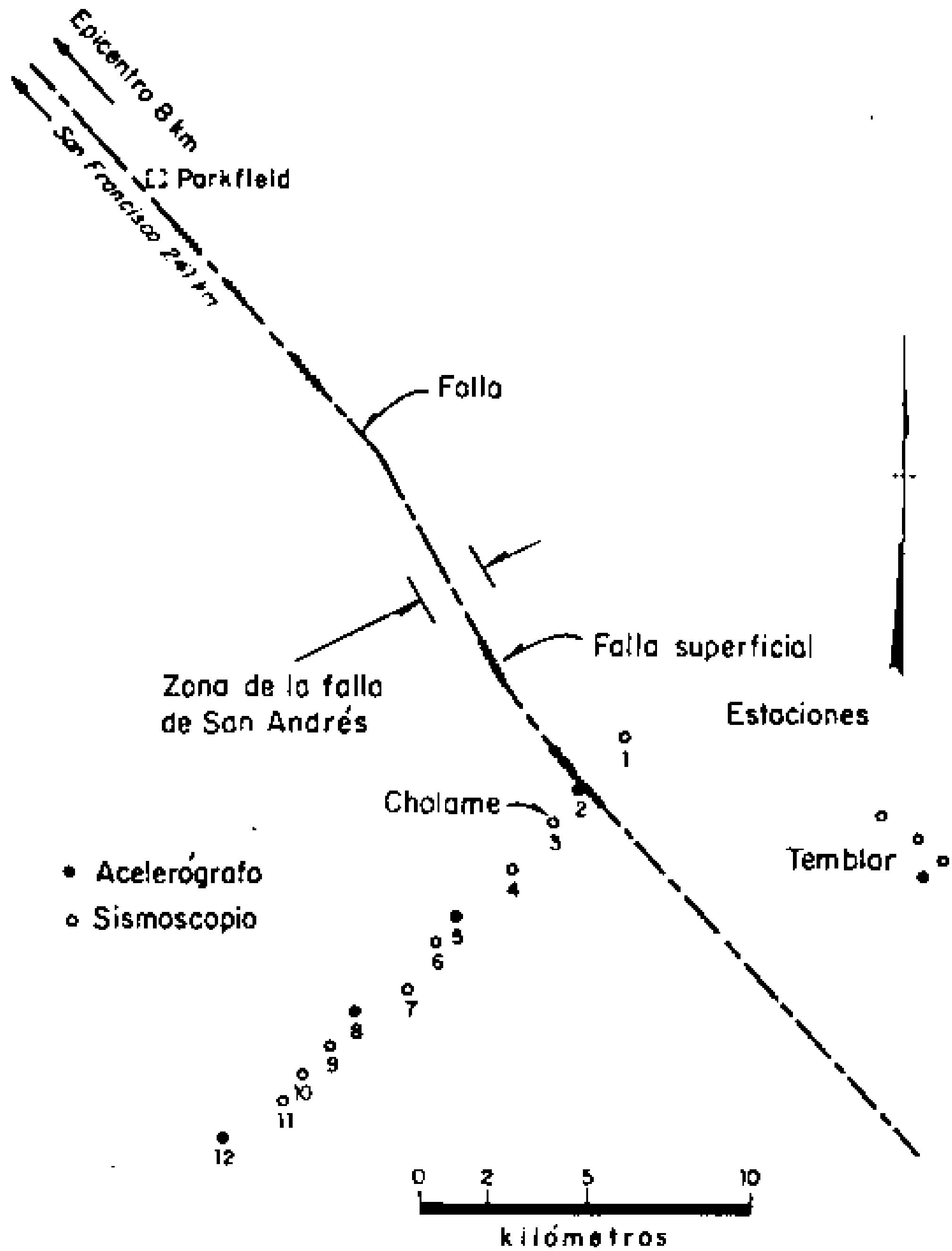


Fig 5. Localización de acelerógrafos y sismoscopios en Cholame (ref 13)

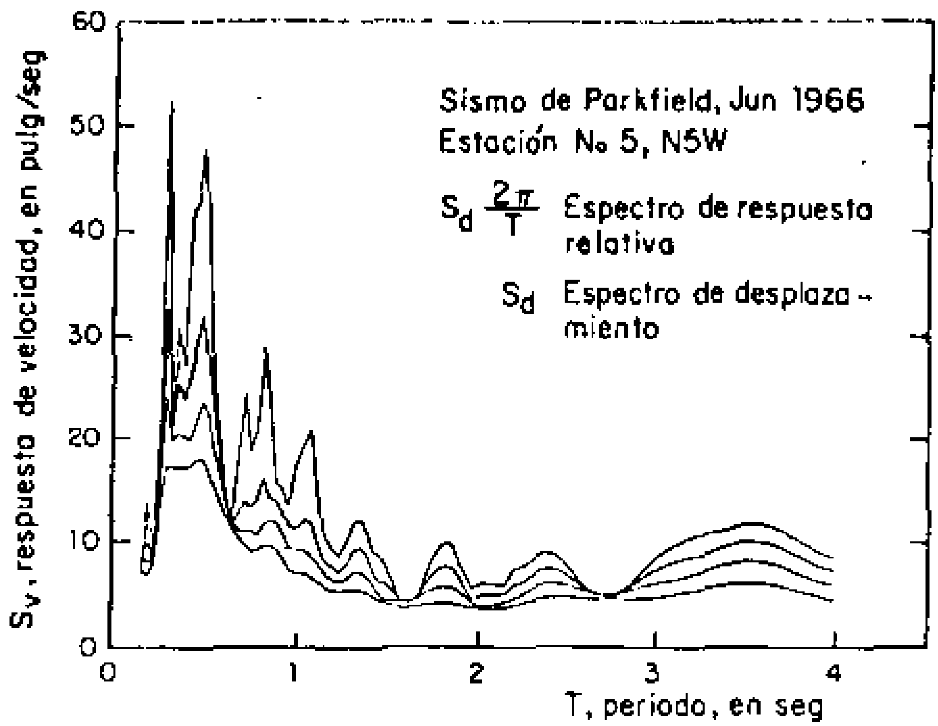


Fig 6. Espectro de respuesta, Est 5, N5W. Las curvas son para 0, 2, 5 y 10 por ciento de amortiguamiento (ref 13)

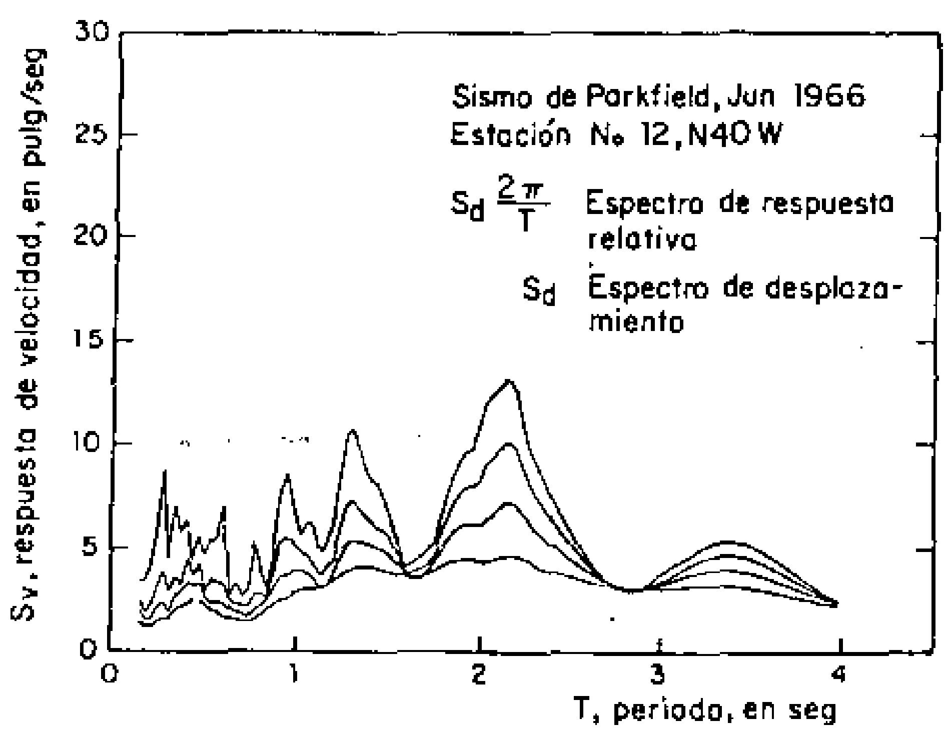


Fig 7. Espectro de respuesta, Est 12, N40W. Las curvas son para 0, 2, 5 y 10 por ciento de amortiguamiento (ref 13)

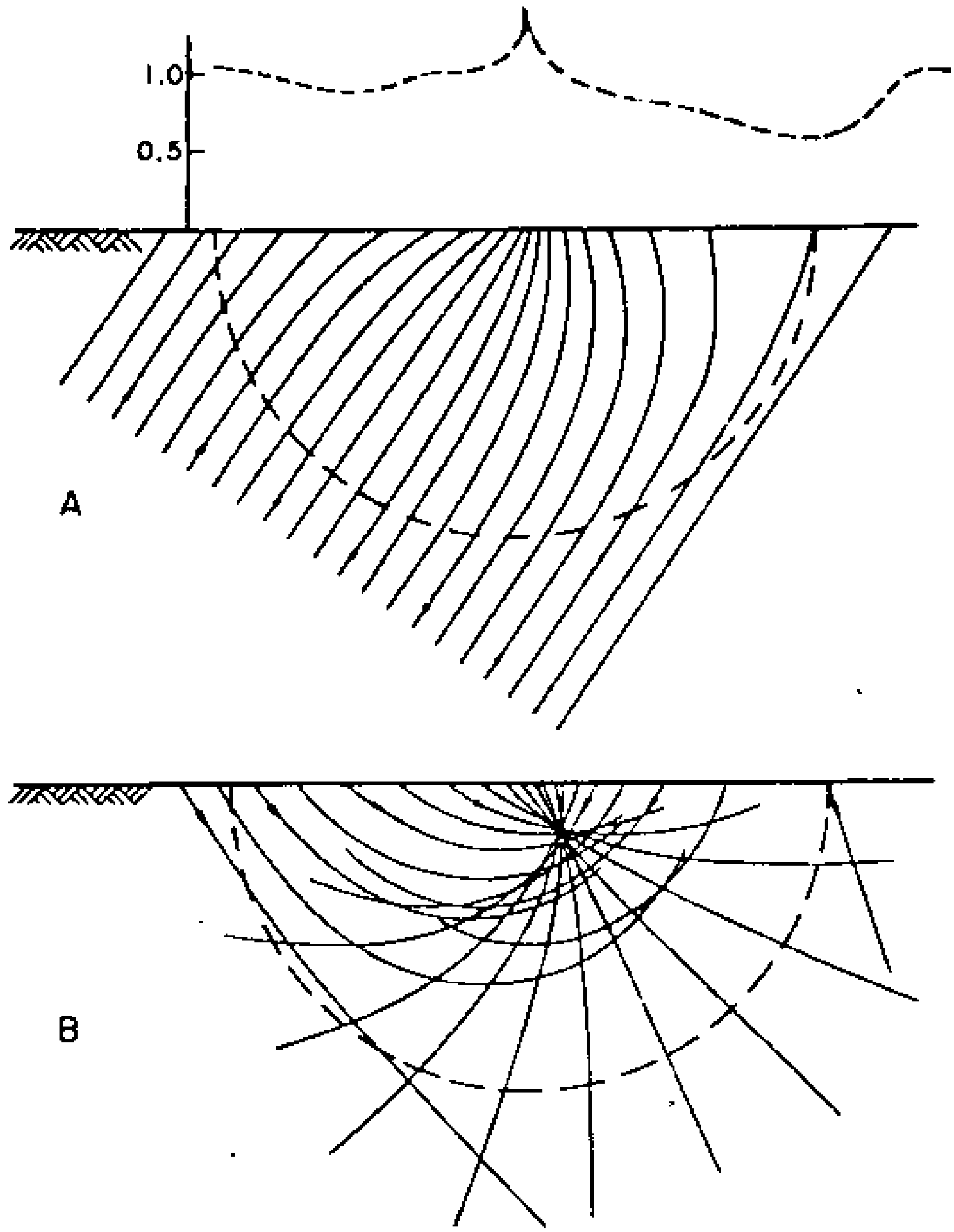


Fig 8. Concentración de ondas sísmicas. A) ondas SV incidentes, B) ondas SV reflejadas (ref 14)

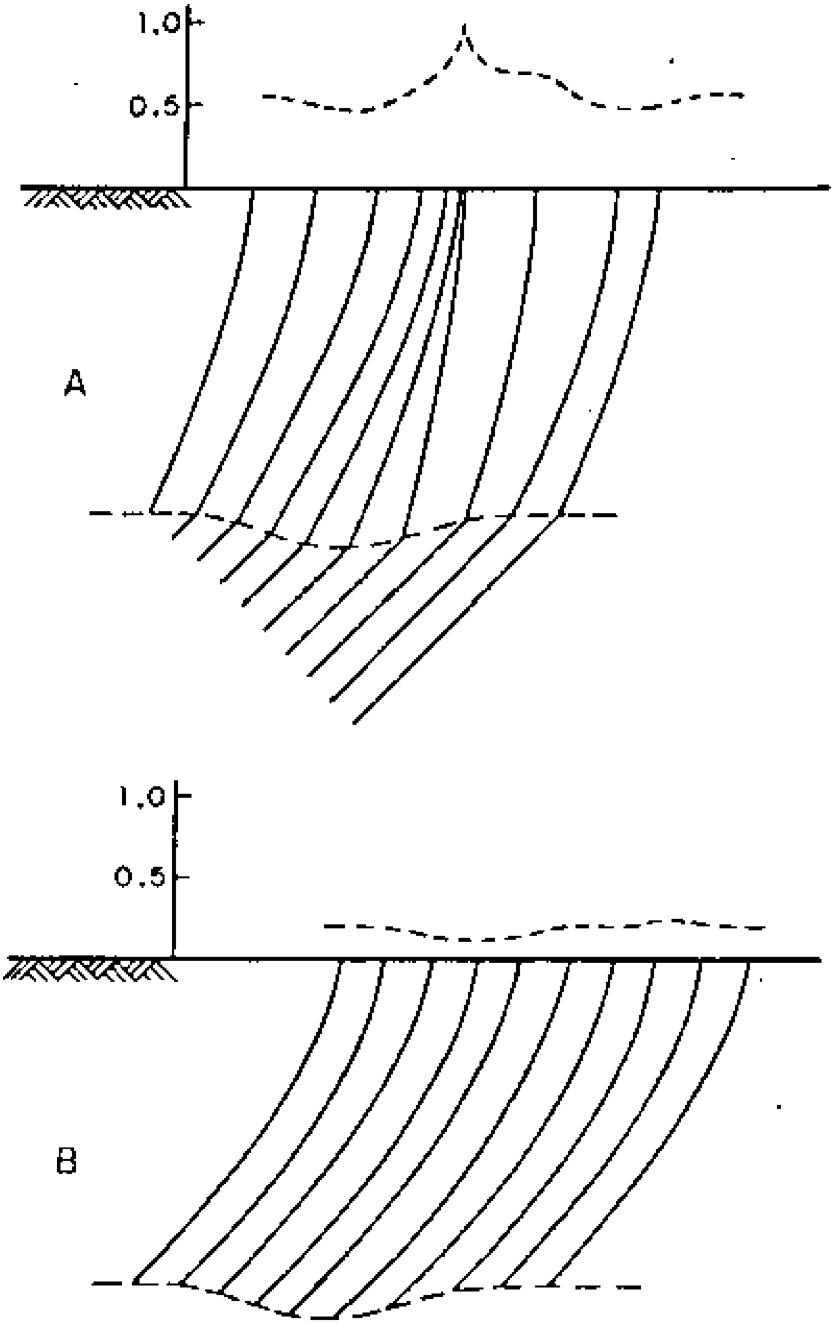


Fig 9. Concentración de ondas, debida a una depresión en la roca basal. A) ondas SV, B) ondas P (ref 14)

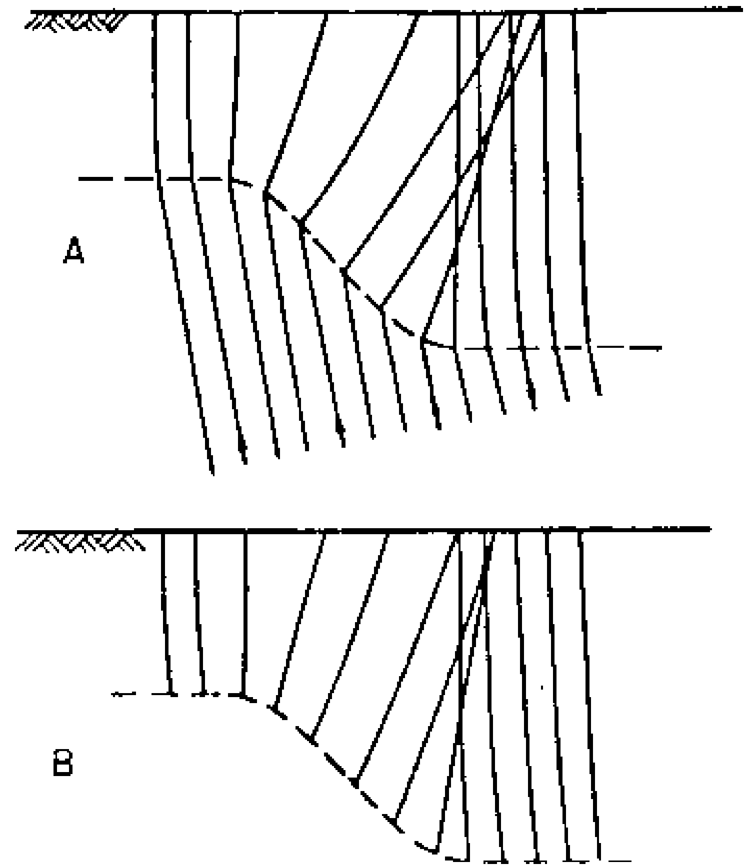


Fig 10. Concentración de ondas sísmicas, semejante a lo ocurrido en Skopje, Yugoslavia. A) ondas SV, B) ondas P (ref 14)

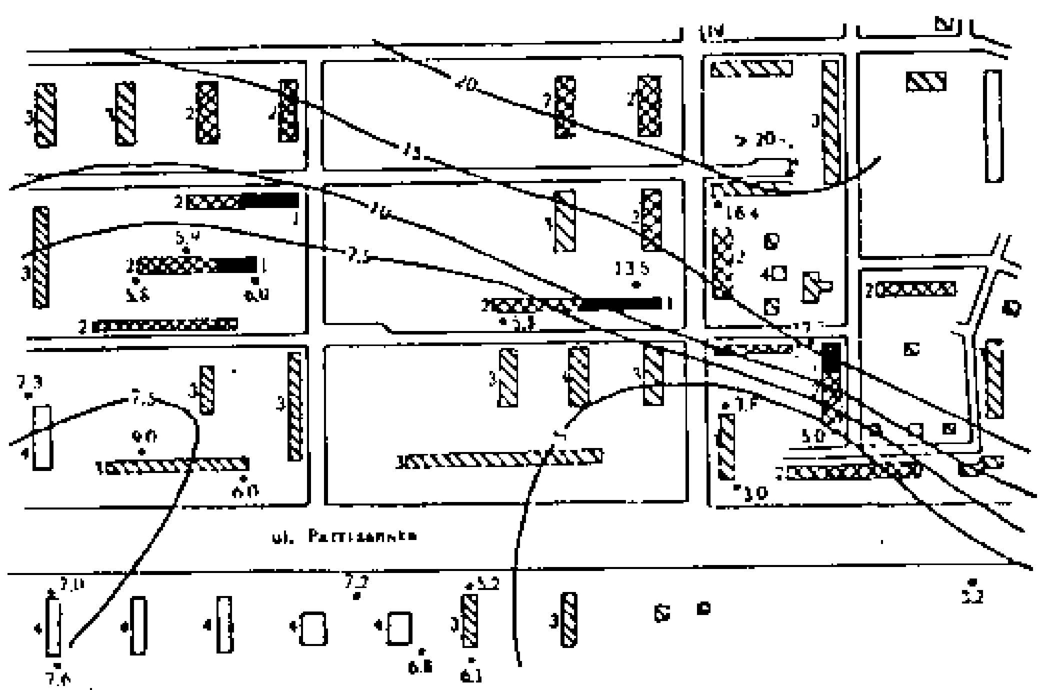


Fig 11. Isolíneas de espesor de terreno aluvial (en metros) y grado de daño ocurrido en Karposh (oeste de la ciudad de Skopje, Yugoslavia). 1. Destrucción; 2. Daño fuerte; 3. Daño menor; 4. Sin daño (ref 16)

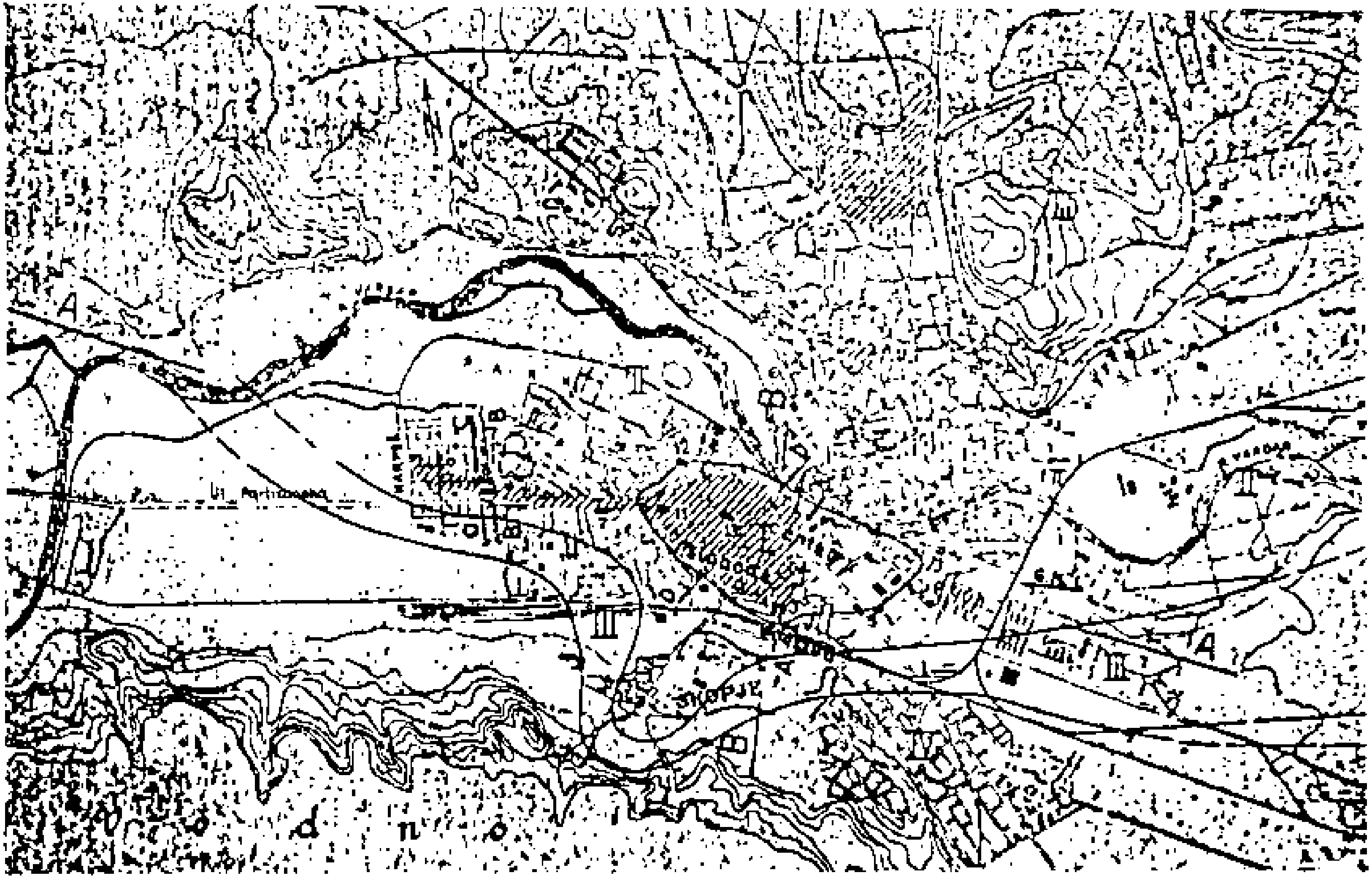


Fig 12. Distribución de daño en Skopje, 1963. Evaluado por Poceski (ref 16)

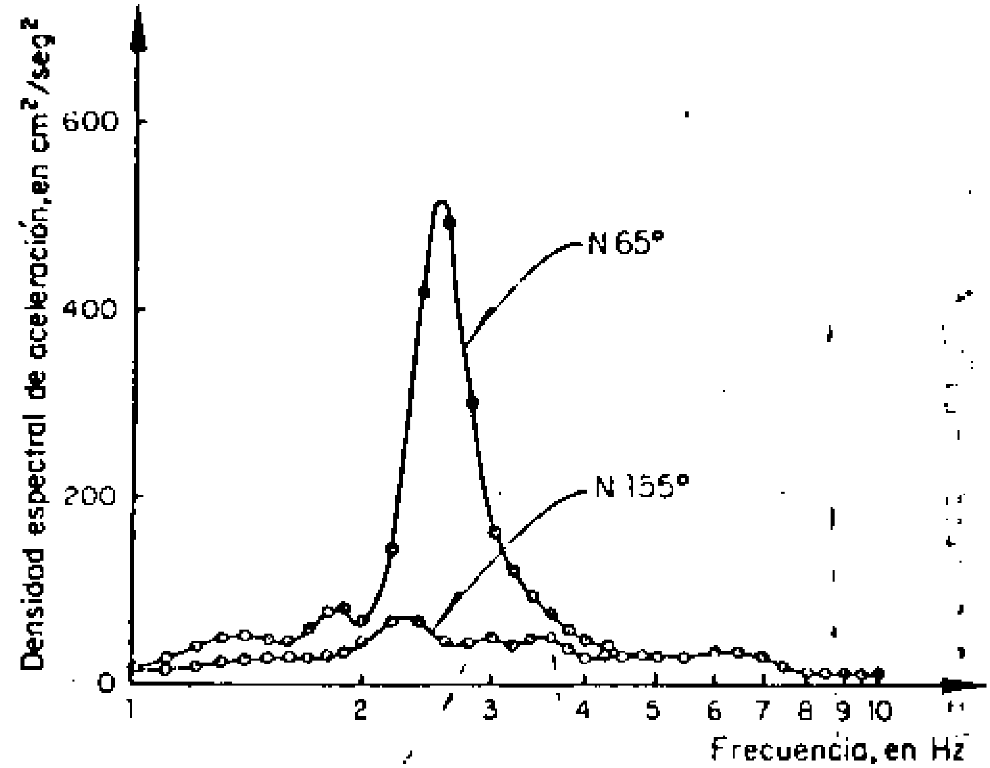


Fig 13. Espectro de aceleración en el sitio 1 (ref 20)

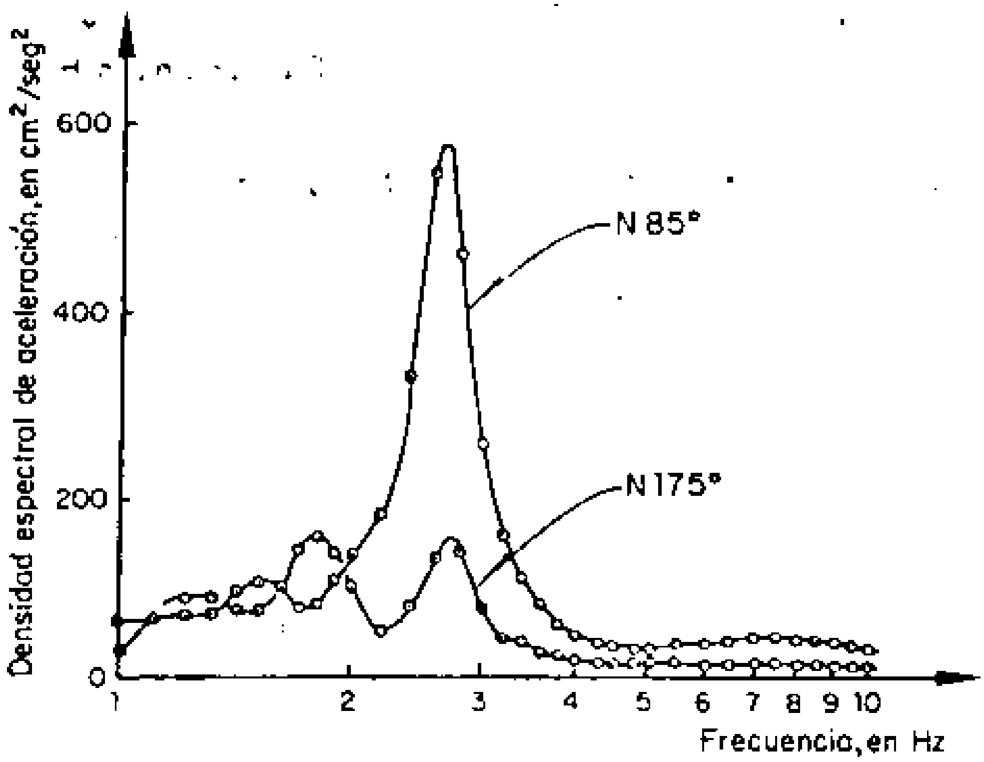


Fig 14. Espectro de aceleracion en el sitio 2 (ref 20)

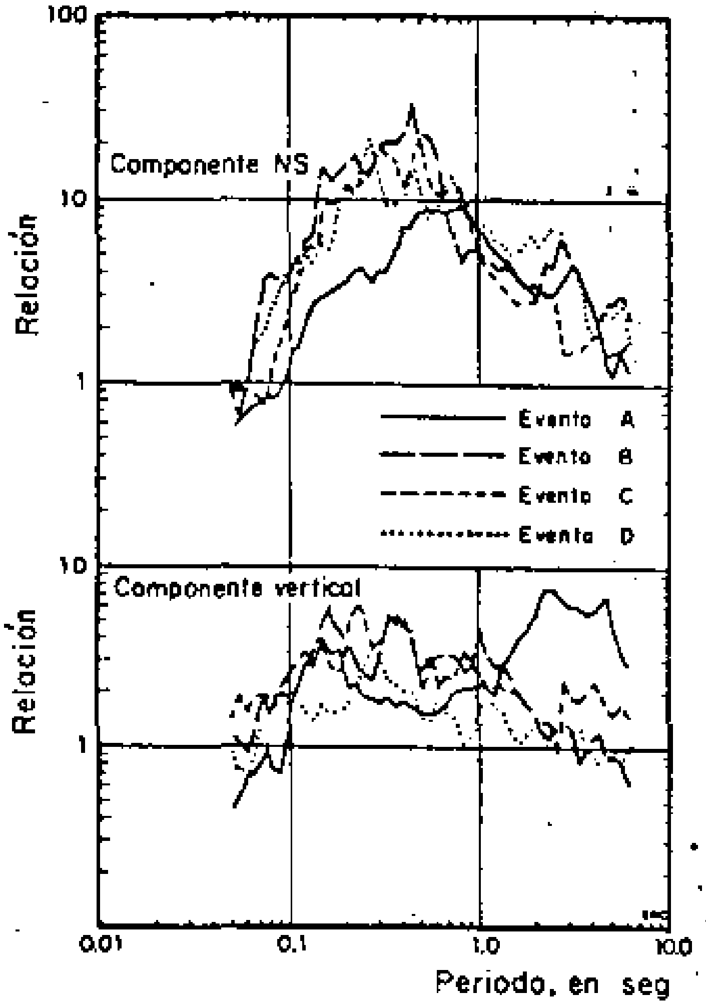


Fig 26. Kagel Mountain

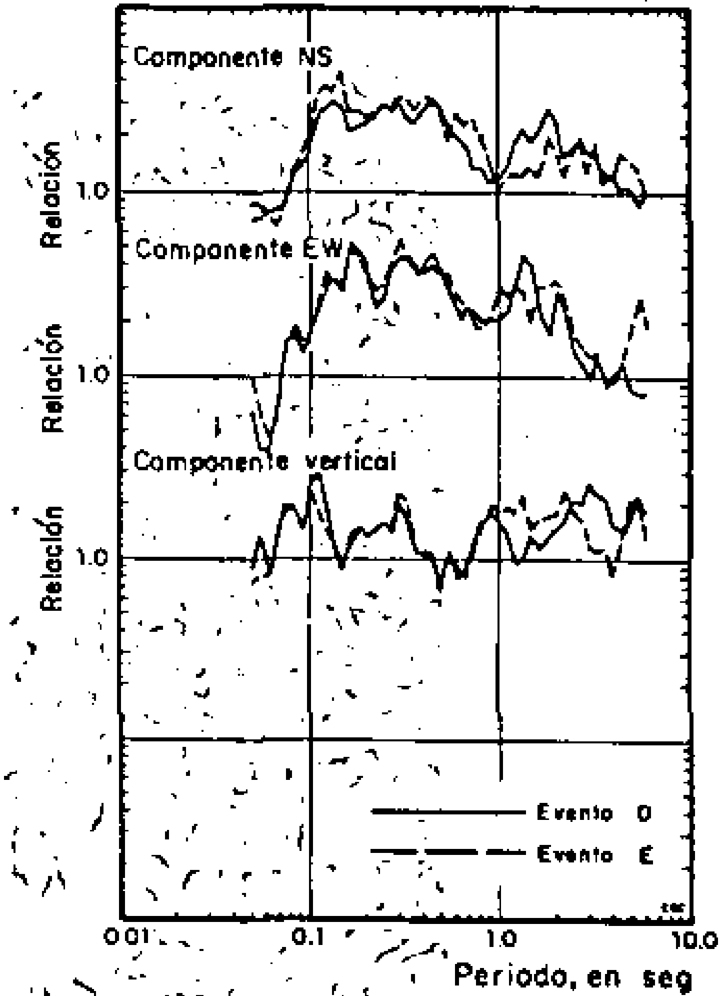


Fig 27. Josephine Peak

Relaciones espectrales entre el movimiento de la cresta y la base de Kagel Mountain y Josephine Peak durante los eventos menores de San Fernando, 1971 (ref 25)

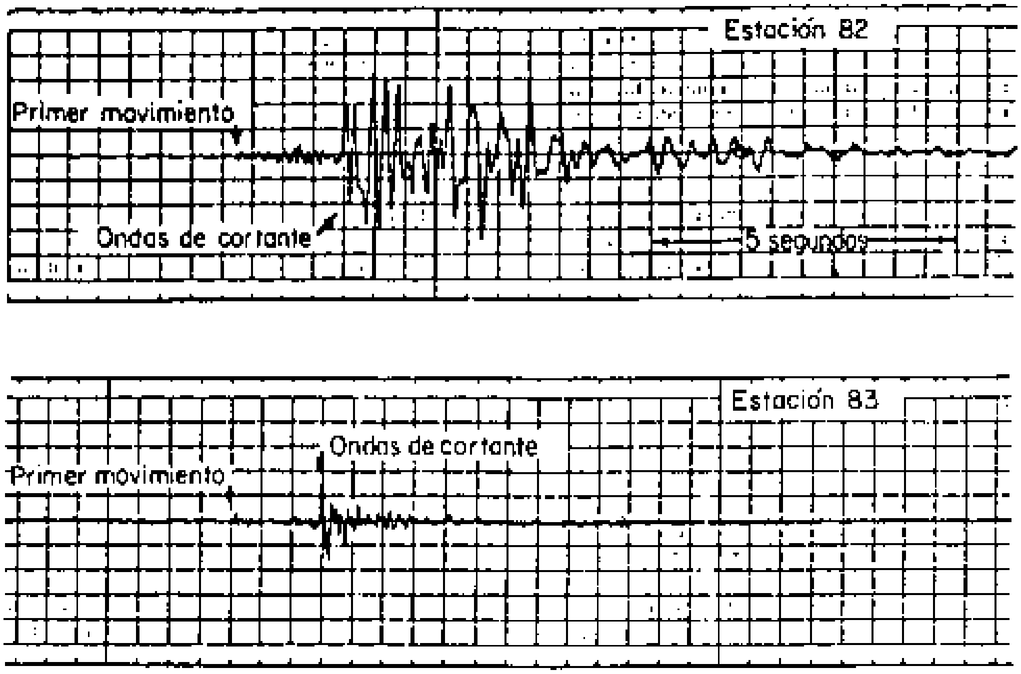


Fig 28. Sismogramas de velocidad, componente NS registrados en la cresta y base de Kagel Mountain, evento A (ref 25)

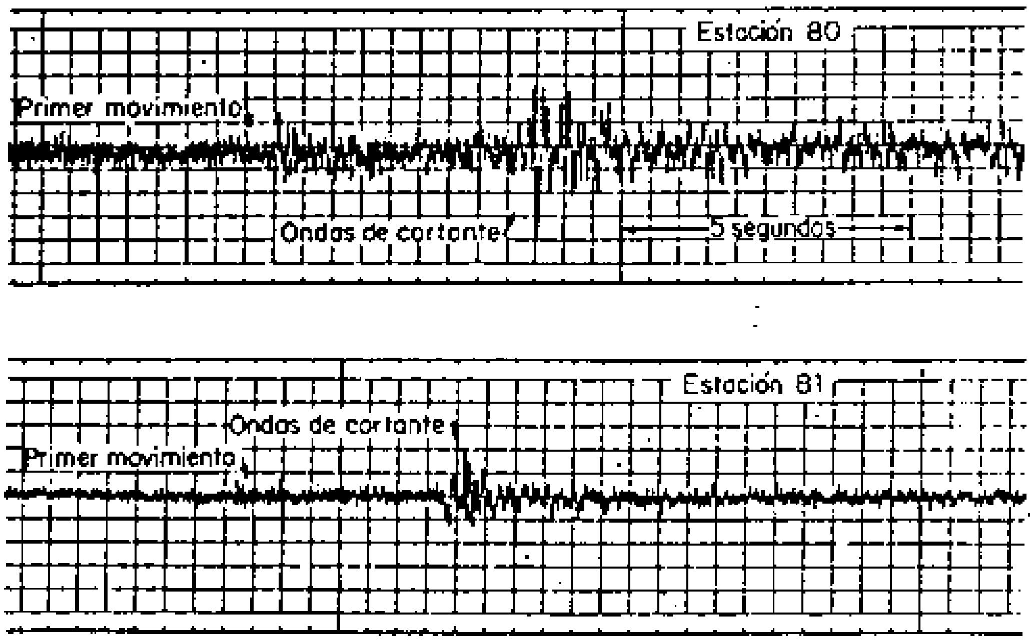


Fig 29. Sismogramas de velocidad, componentes NS registrados en la cresta y base de Josephine Peak, evento D (ref 25)

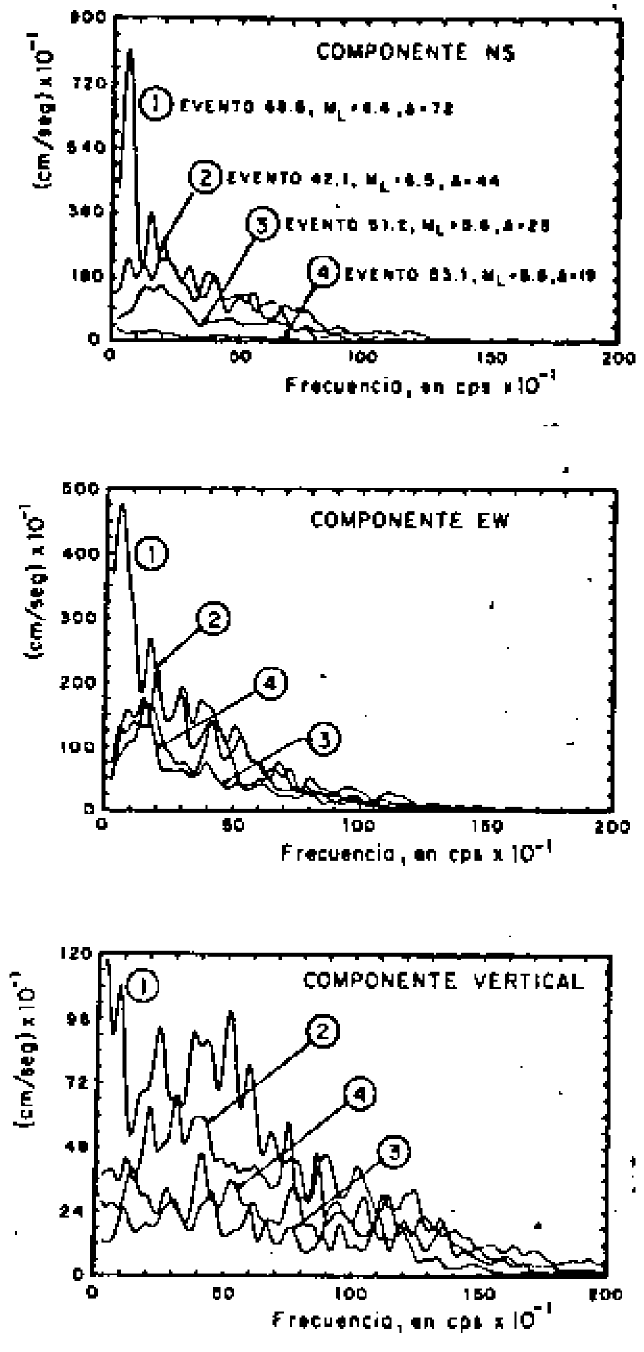


Fig 32. Espectros de amplitudes de Fourier de aceleración del grupo II (ref 29)

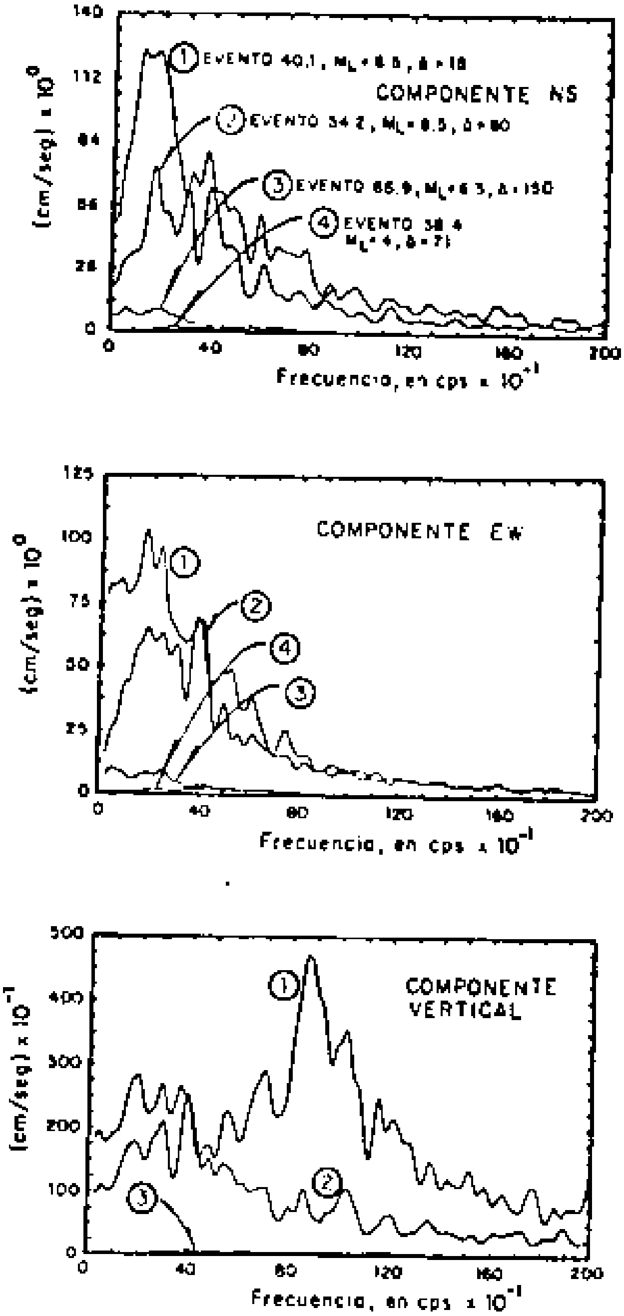


Fig 33. Espectros de amplitudes de Fourier de aceleración del Grupo III (ref 29)

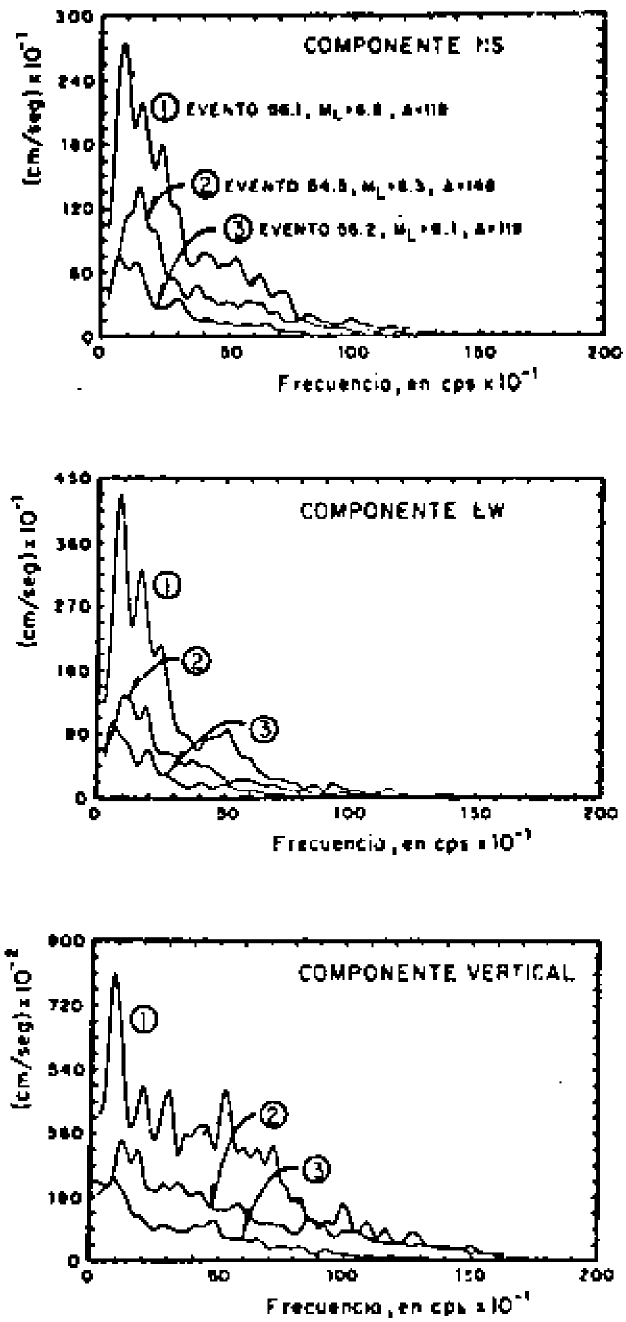


Fig 34. Espectros de amplitudes de Fourier de aceleración del grupo IV (ref 29)

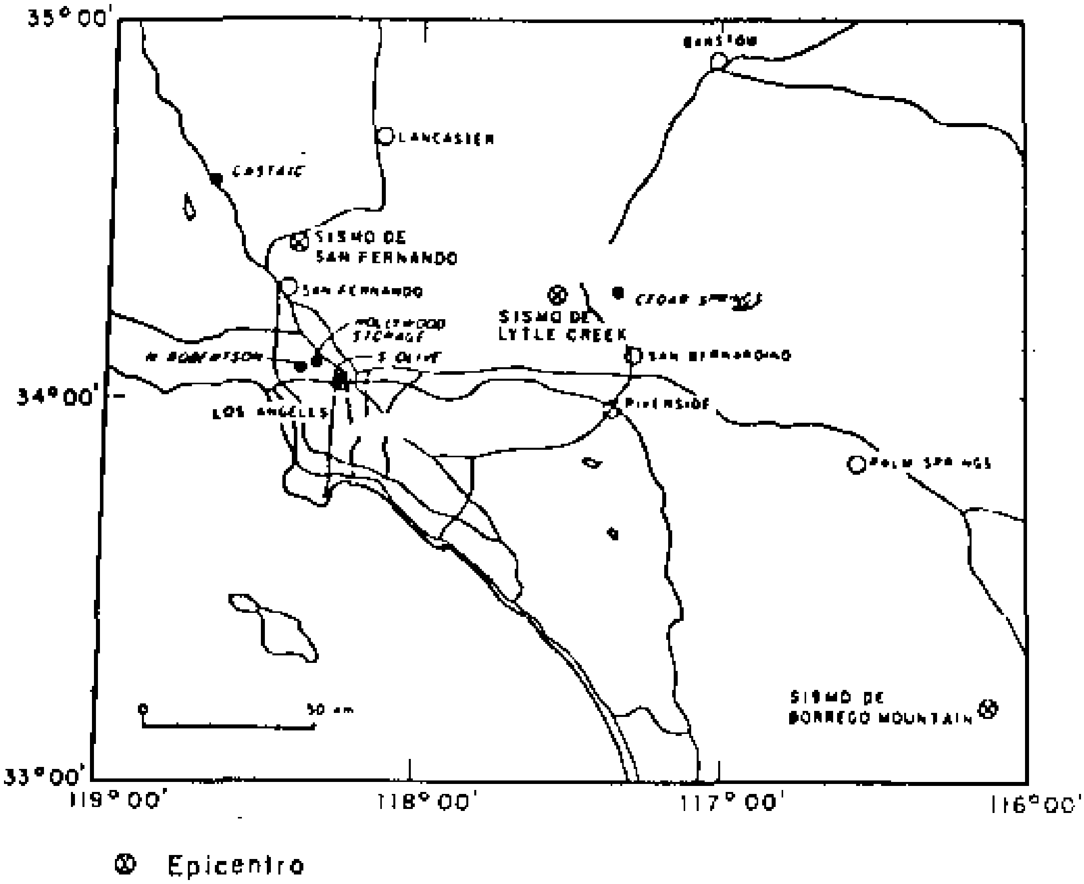


Fig 35. Localización de las seis estaciones y los tres epicentros de los sismos de San Fernando, 1971, Lytle Creek, 1970 y Borrego Mountain, 1968 (ref 29)

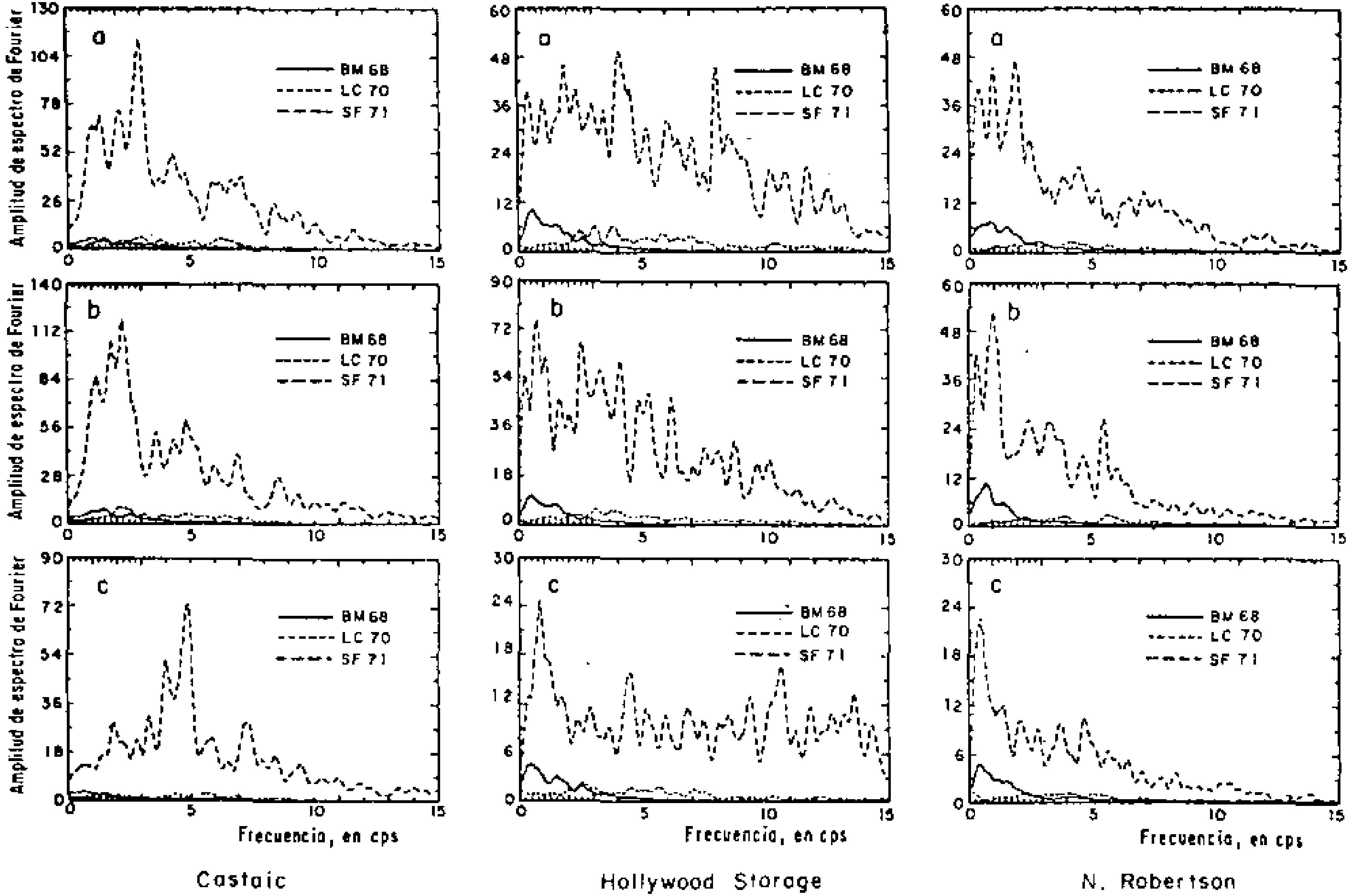


Fig 36. Amplitudes de espectros de Fourier. a) componente Norte, b) componente Este, c) componente vertical (ref 30)

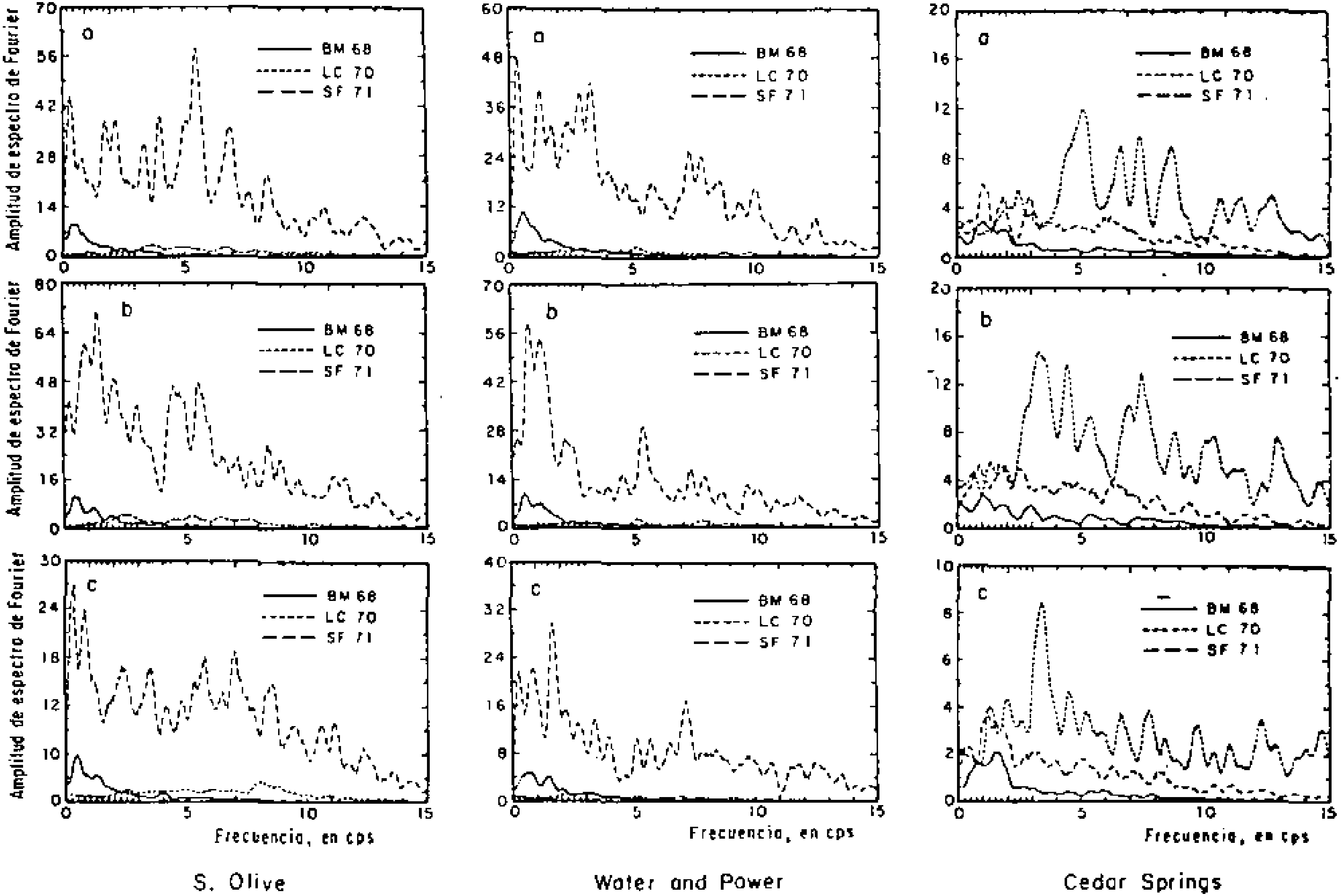


Fig 37. Amplitudes de espectros de Fourier. a) componente Norte, b) componente Este, c) componente vertical (ref 30)

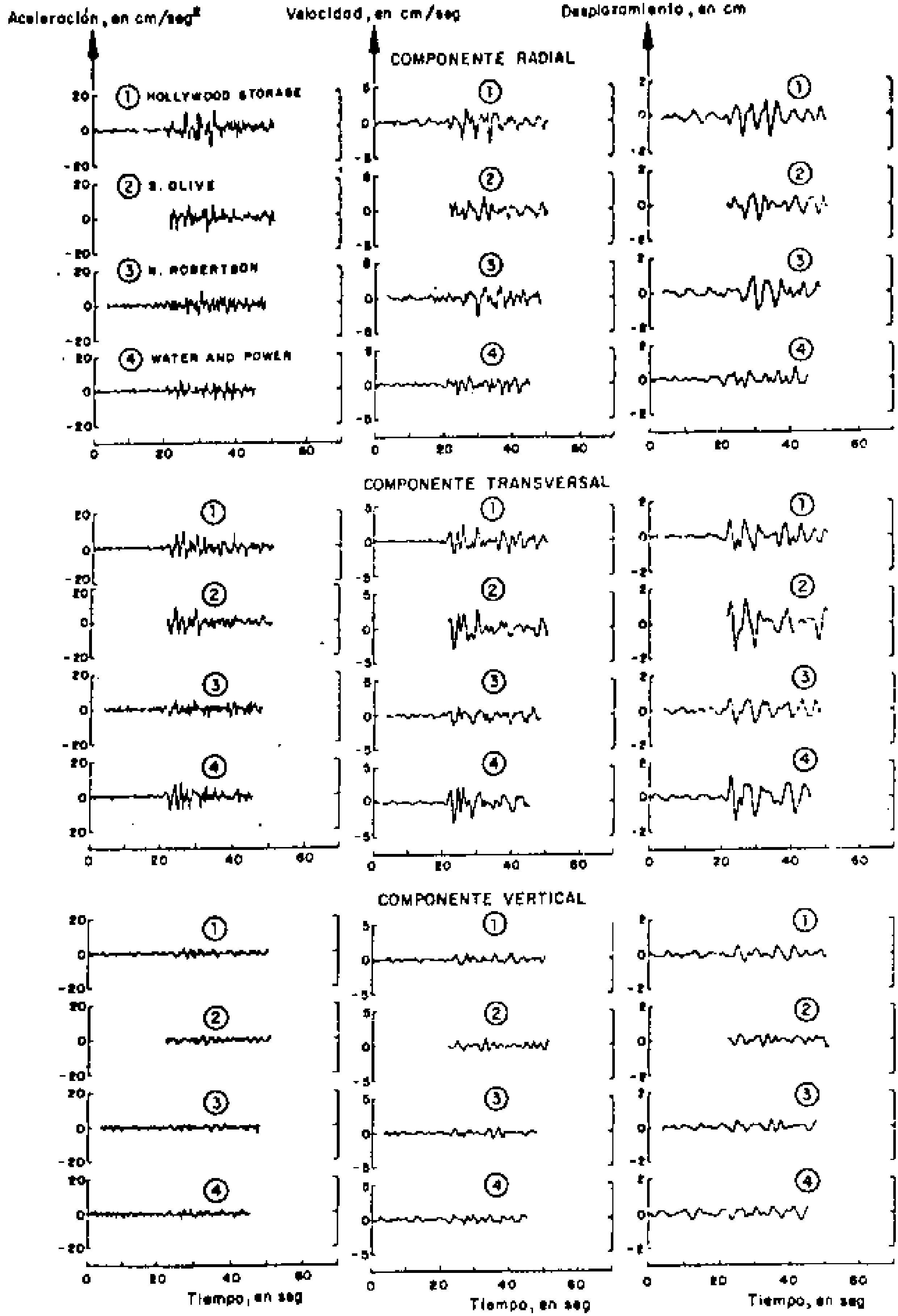


Fig 38. Aceleraciones, velocidades y desplazamientos registrados en cuatro estaciones durante el sismo de Borrego Mountain, 1968 (ref 30)

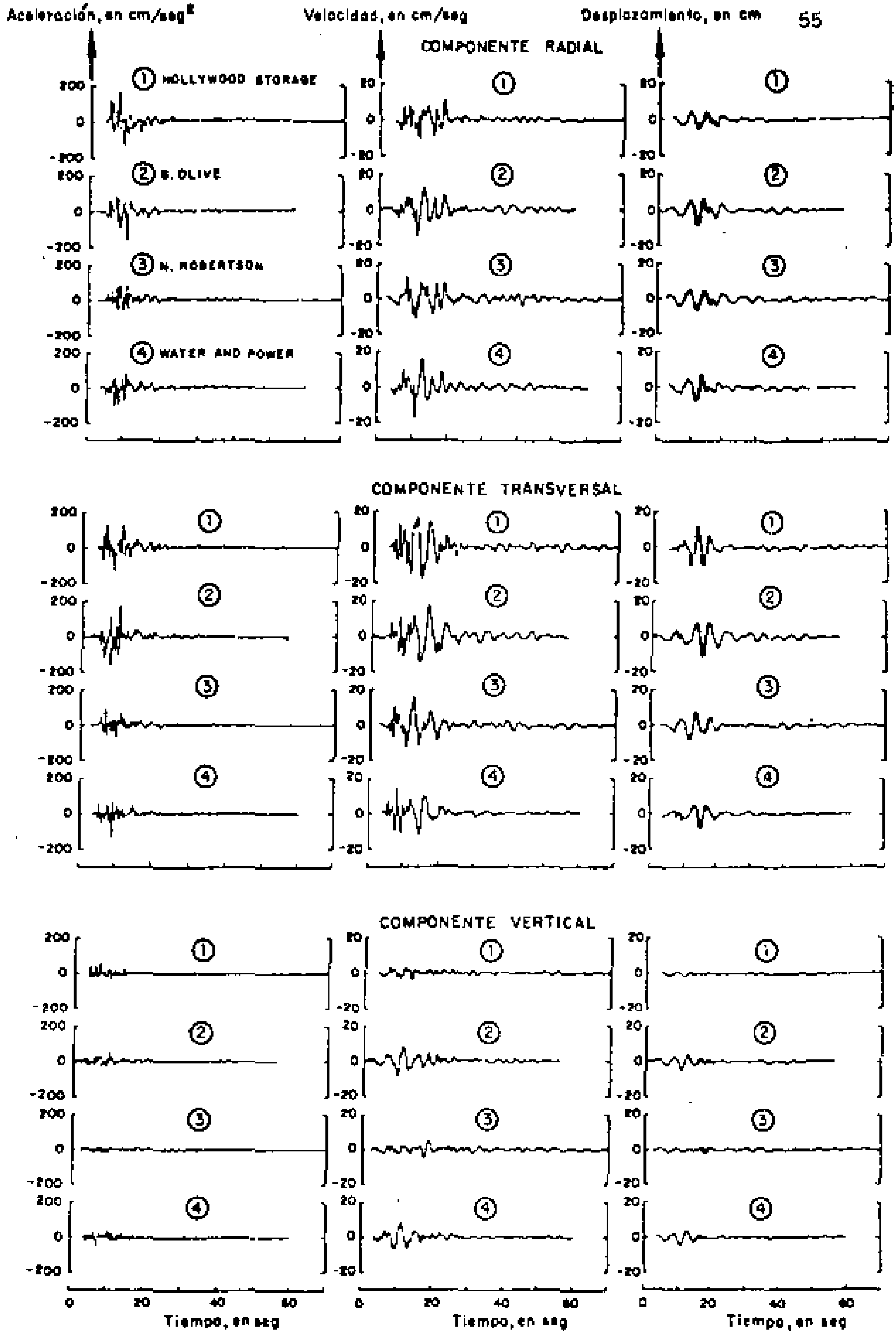


Fig 39. Aceleraciones, velocidades y desplazamientos registrados en cuatro estaciones durante el sismo de San Fernando, 1971 (ref 30)

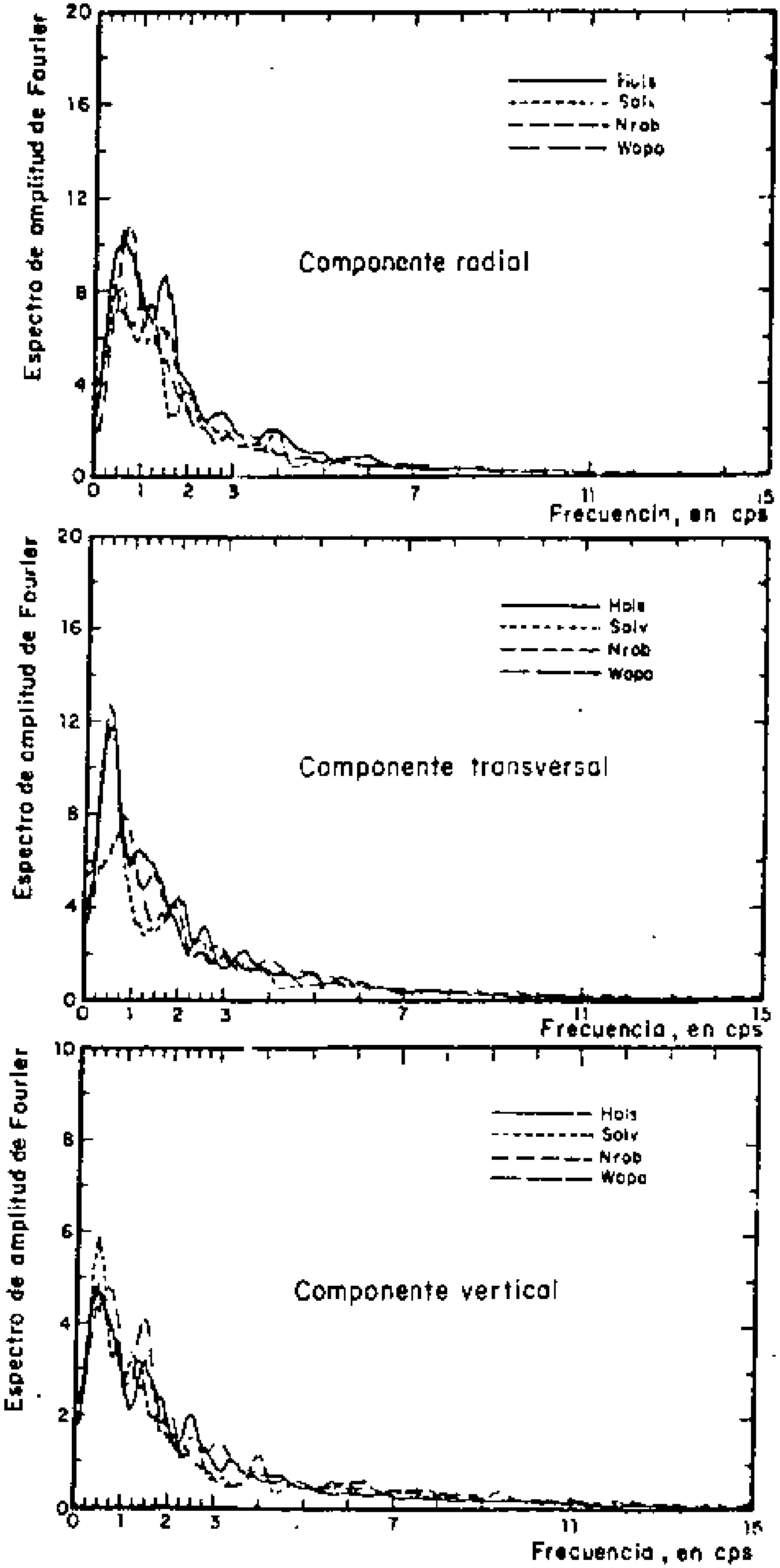


Fig 40. Espectro de amplitudes de Fourier. Sismo de Borrego Mountain (ref 30)

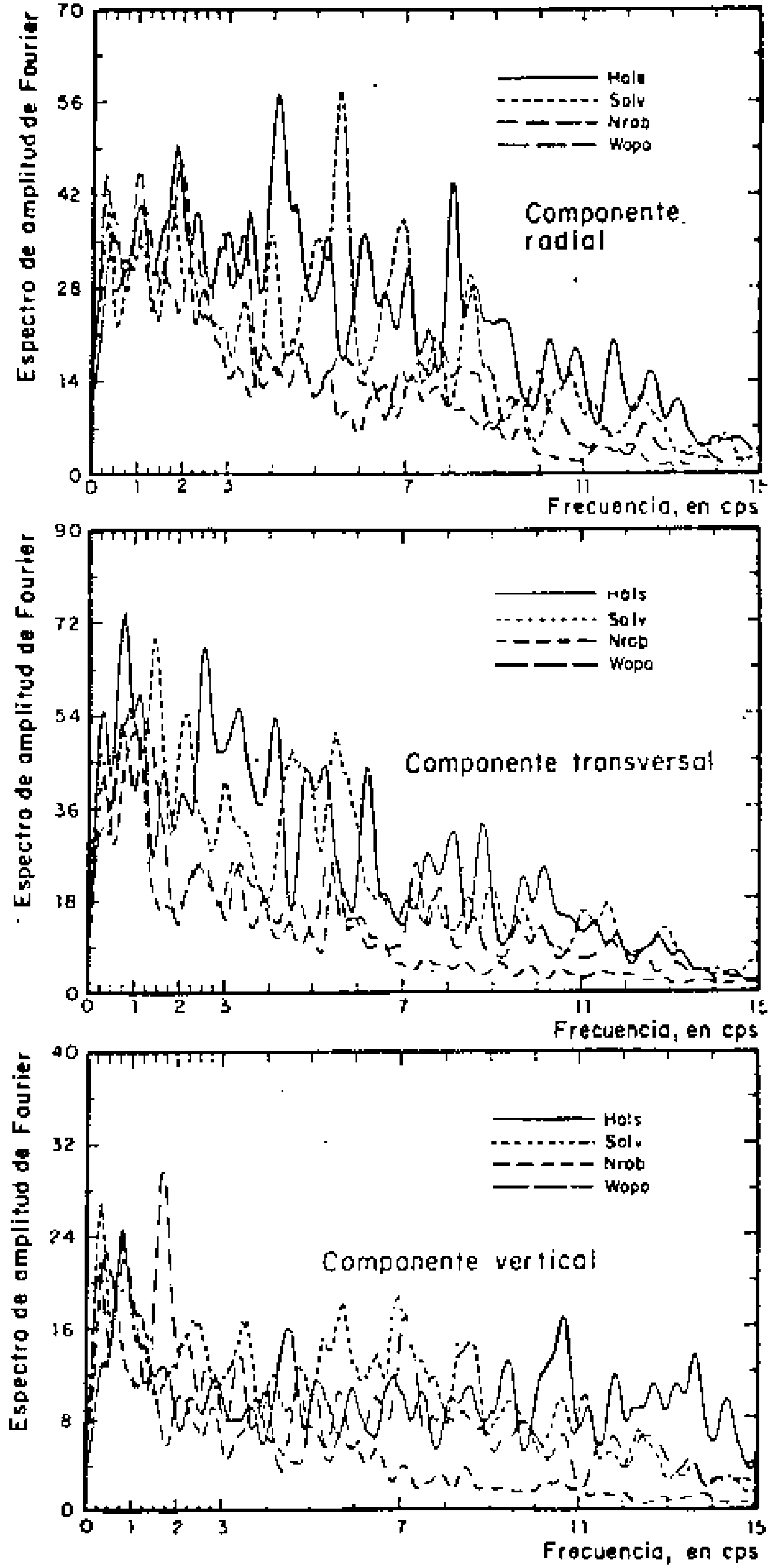


Fig 41. Espectro de amplitudes de Fourier. Sismo de San Fernando (ref 30)

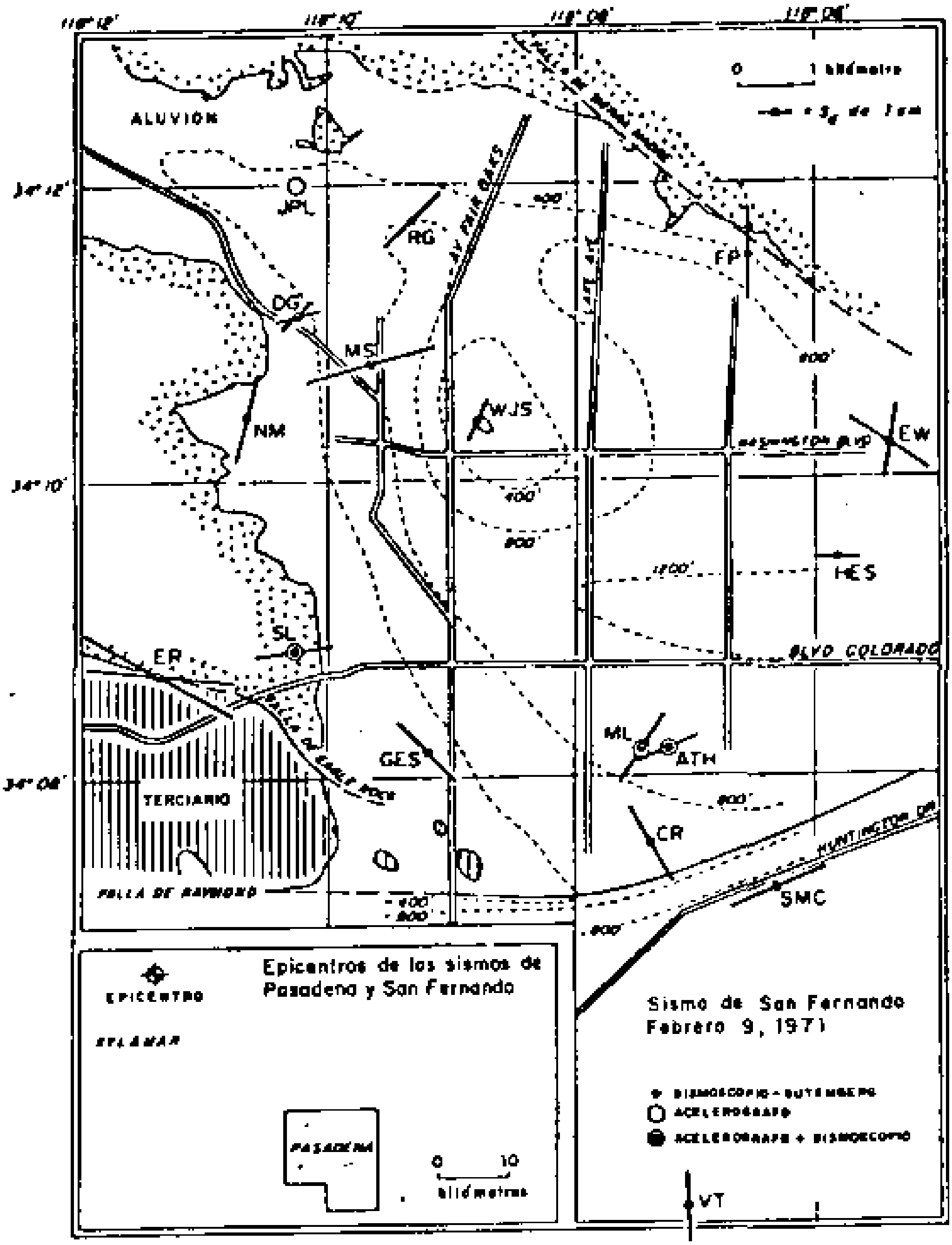
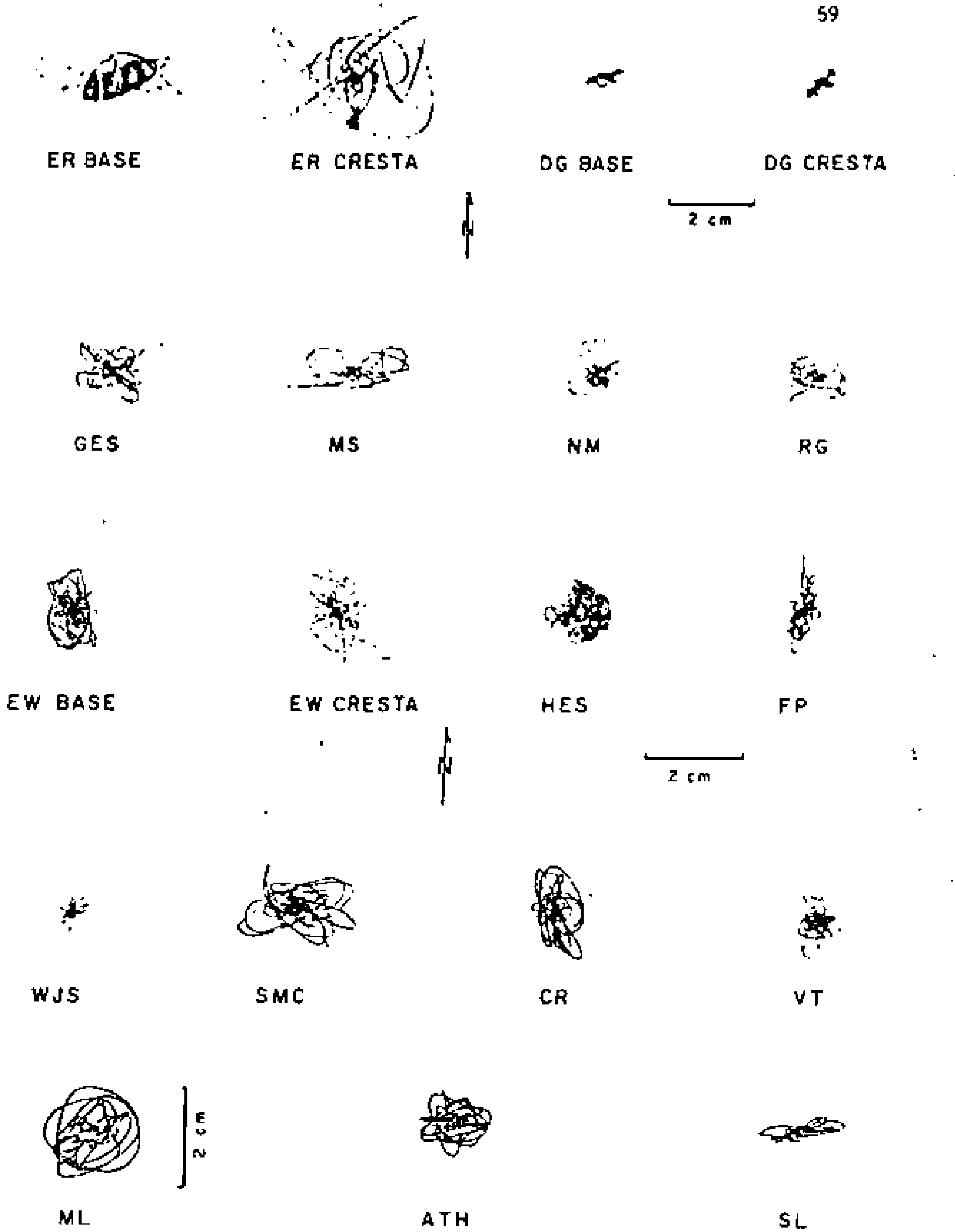


Fig 42. Mapa de localización (ref 32)



Ref 43. Registros de sismoscopios del sismo de San Fernando, febrero 9 de 1971, en el área de Pasadena, Calif (ref 32)

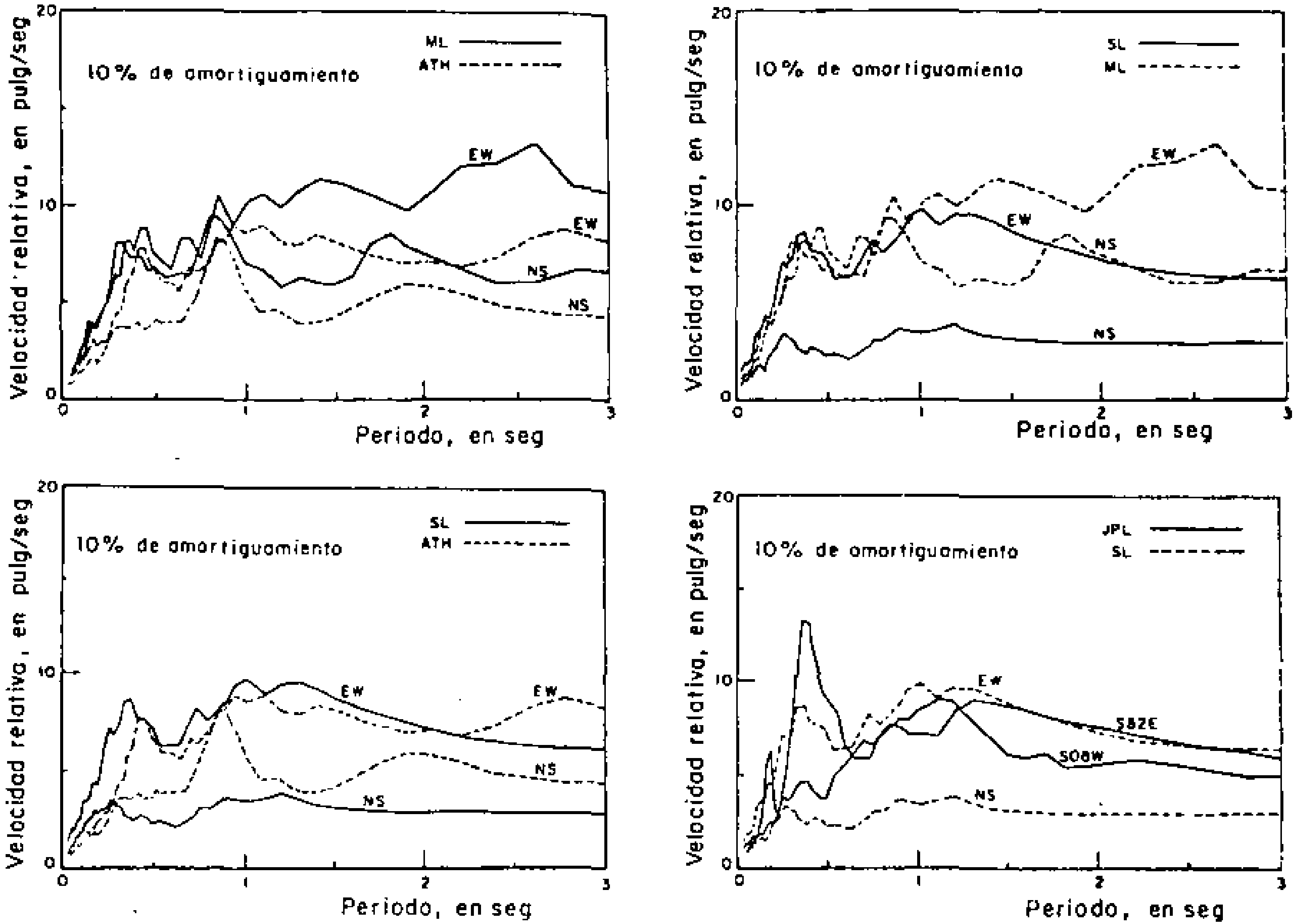


Fig 44. Comparación de espectros de velocidades (ref 32)

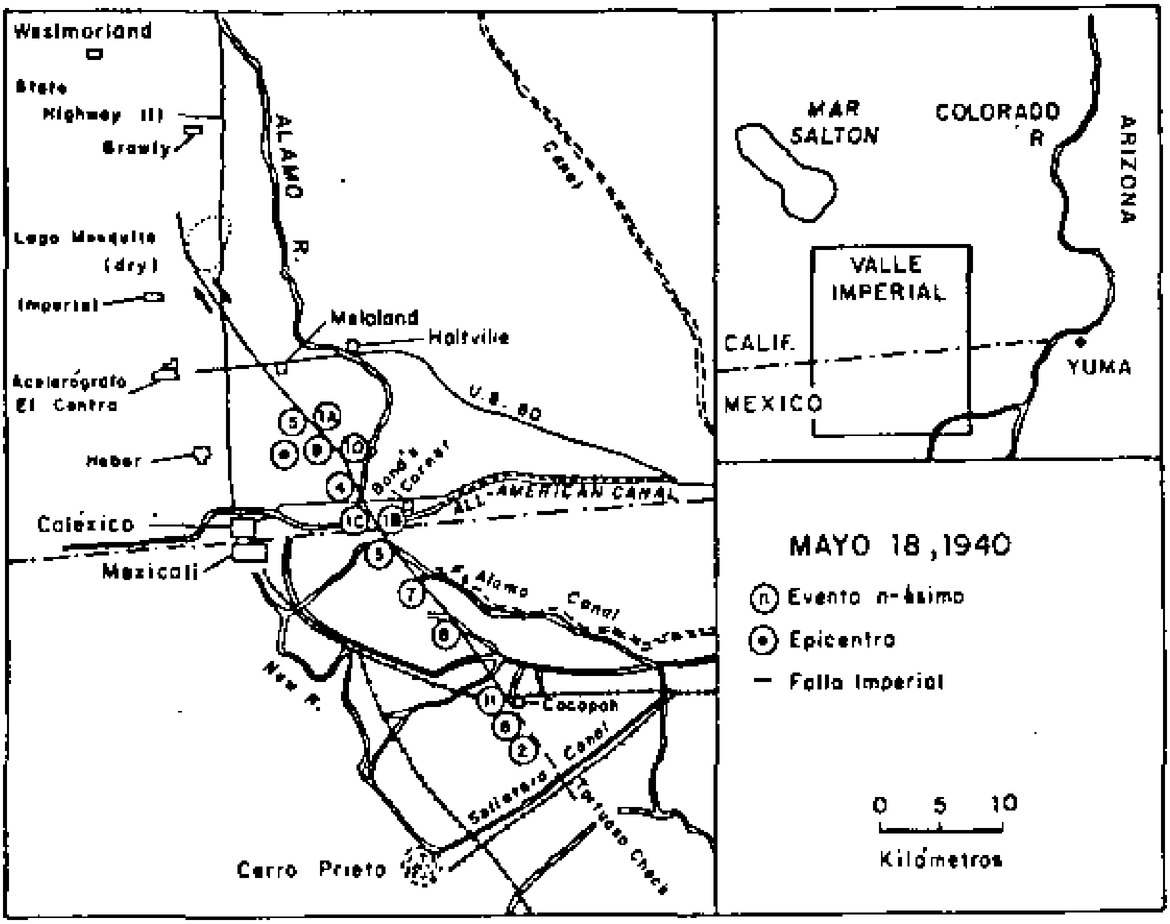


Fig 45. Localización de la falla Imperial, epicentro principal y eventos individuales (ref 35)

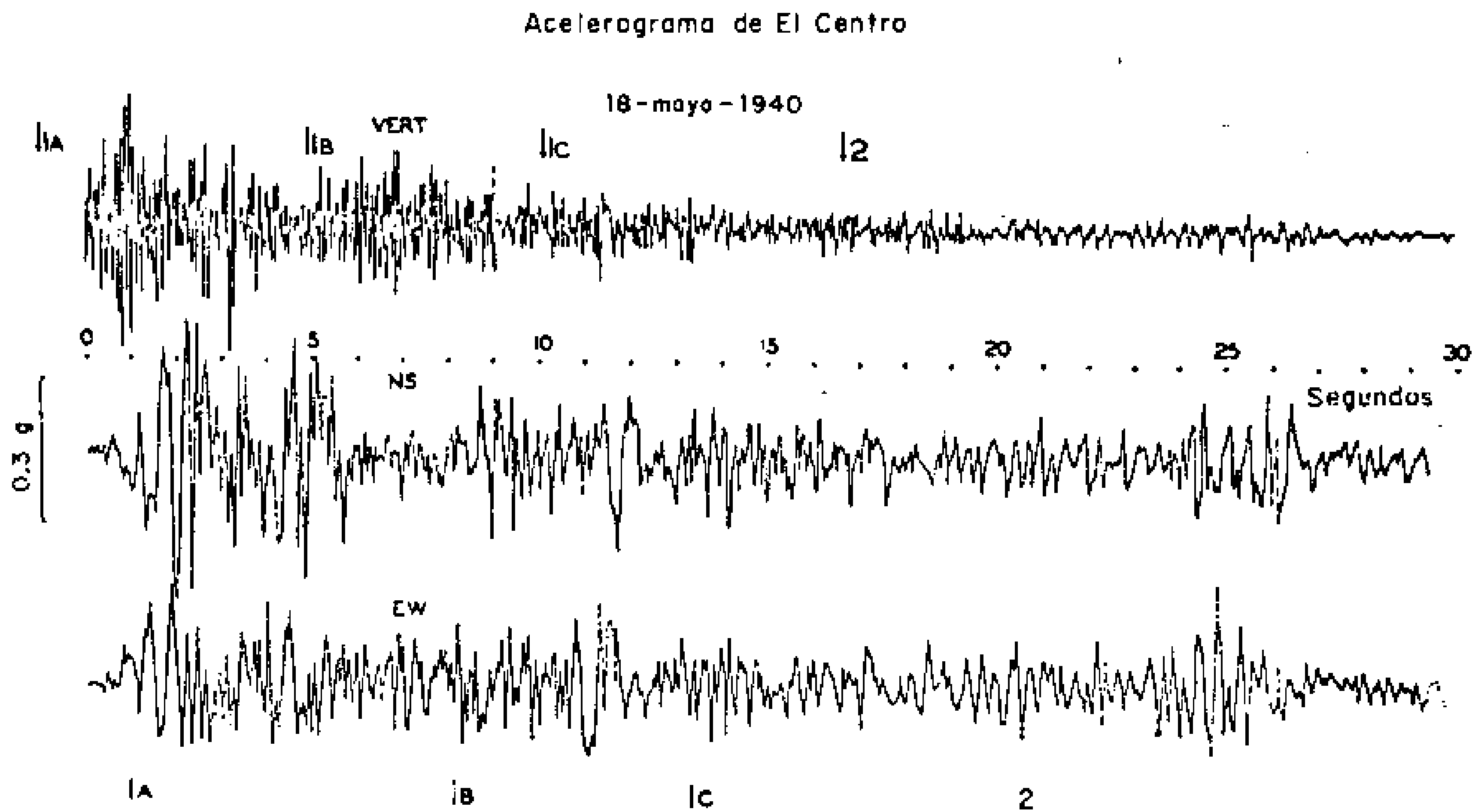


Fig 46. Acelerograma registrado en El Centro, Calif, correspondiente a los eventos 1A, 1B y 1C (ref 12)

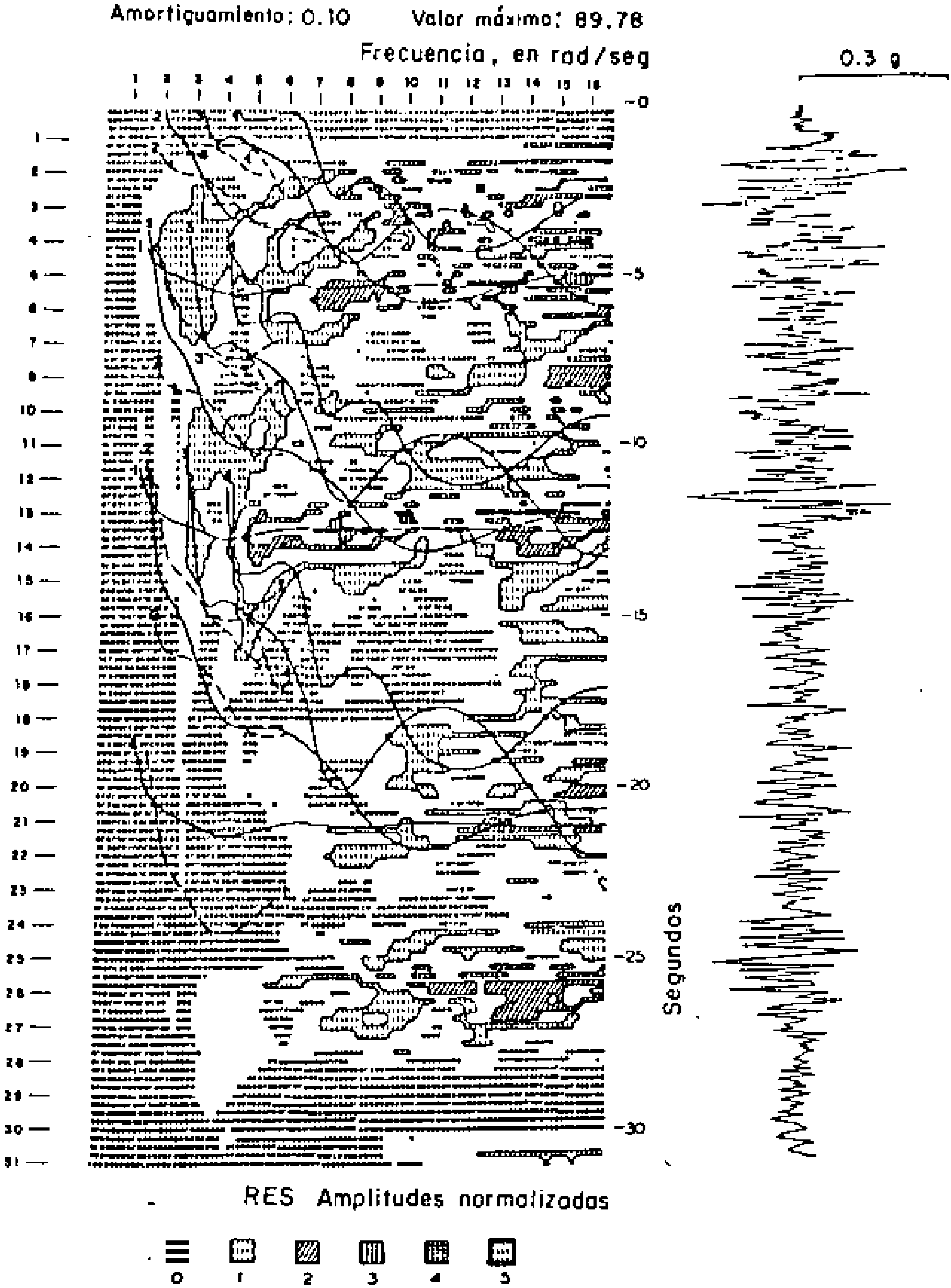


Fig 47. Espectro envolvente de respuesta para los eventos 1A, 1B y 1C, componente EW, Sismo del Valle Imperial, mayo 18 de 1940 (ref 22)

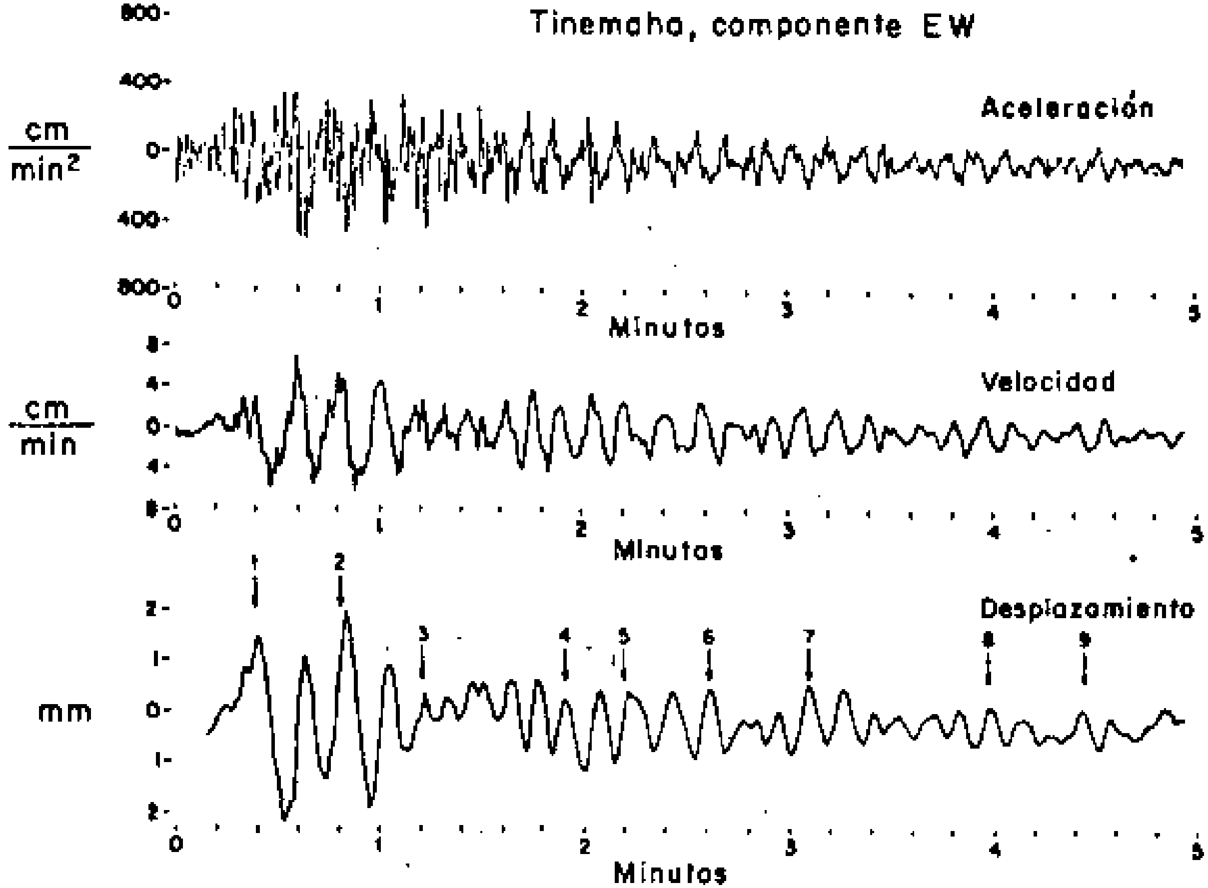
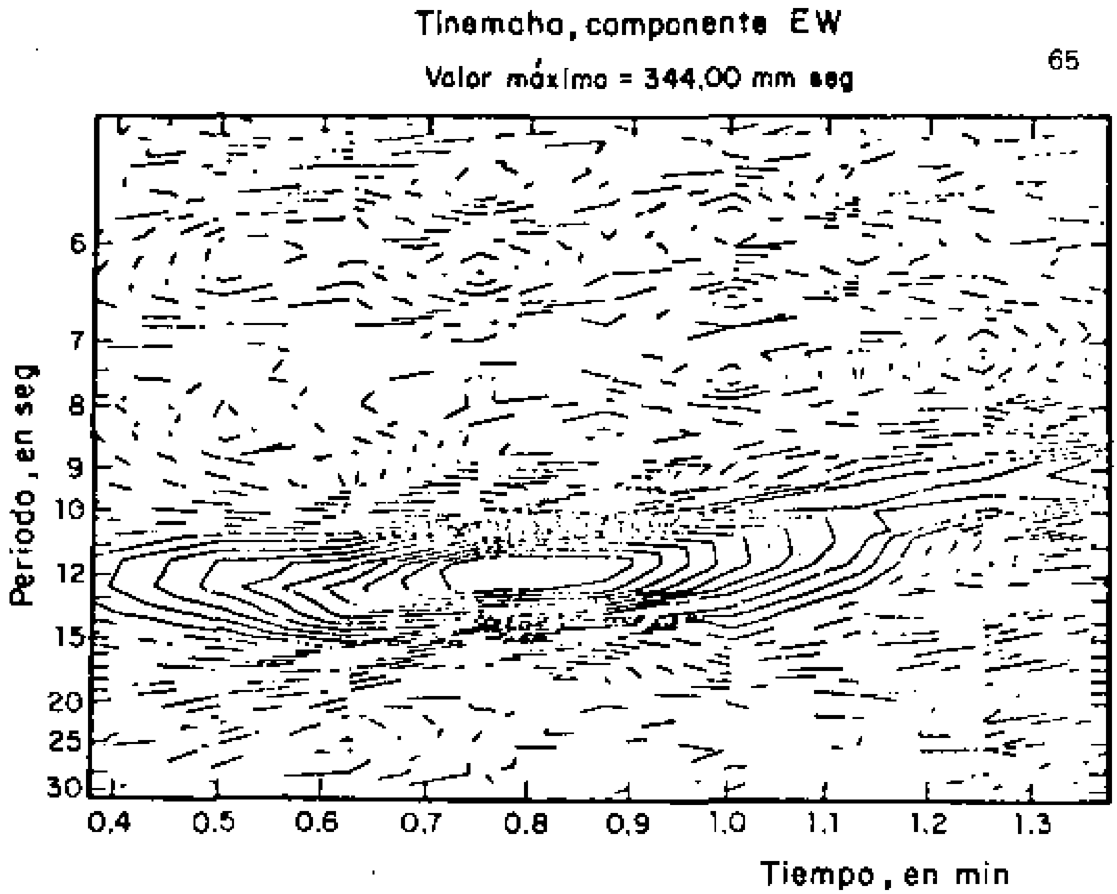


Fig 48. Historia de aceleraciones, velocidades y desplazamientos en Tinemaha, componente EW, durante el sismo del Valle Imperial, mayo 18 de 1940 (ref 12)



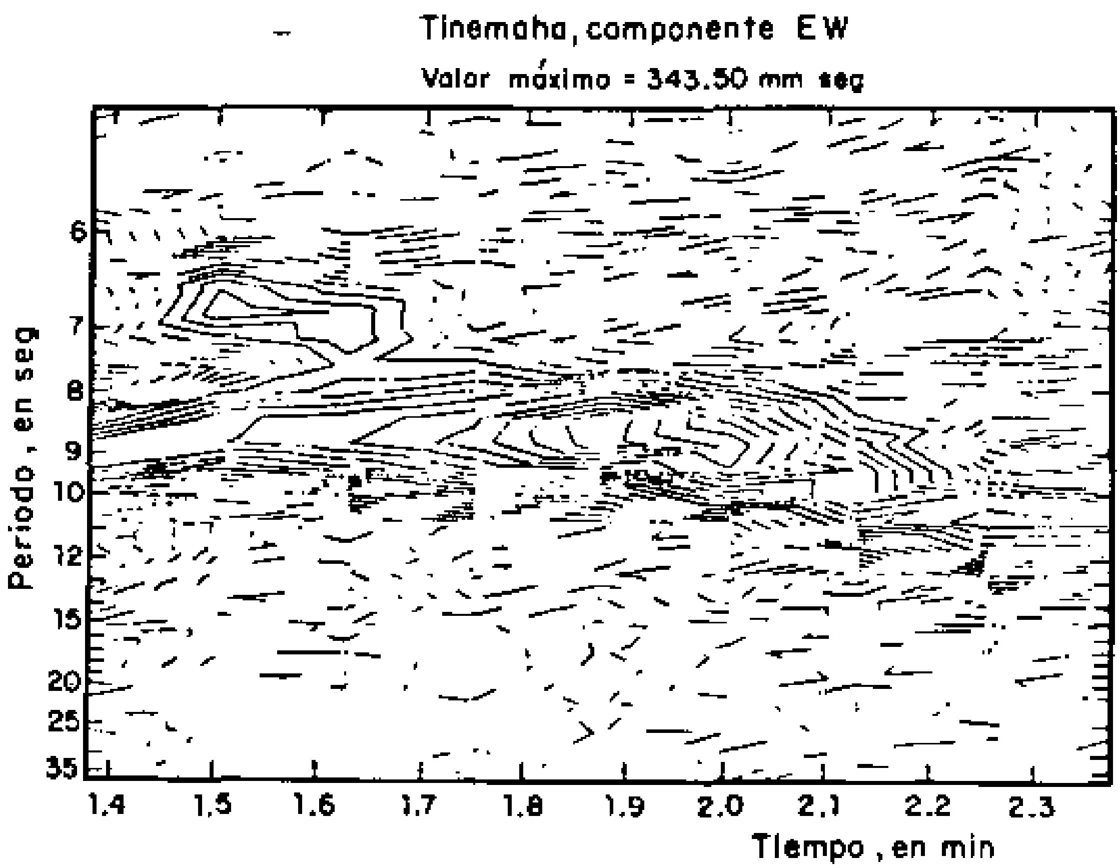


Fig 49. Espectro de Fourier de ventana móvil (w, t, Δt) para el sismograma EW de Tinemaha durante el sismo del Valle Imperial, mayo 18 de 1940 (ref 12)*

DIVISION DE EDUCACION CONTINUA
FACULTAD DE INGENIERIA U.N.A.M.

X CURSO INTERNACIONAL DE INGENIERIA SISMICA

ANALISIS DE RIESGO SISMICO

INFLUENCIA DE LAS CONDICIONES LOCALES EN LAS CARACTERISTICAS DEL MOVIMIENTO SISMICO

DR. FRANCISCO SANCHEZ SESMA

AGOSTO, 1984.

INFLUENCIA DE LAS CONDICIONES LOCALES EN LAS CARACTERISTICAS DEL MOVIMIENTO SISMICO

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INTRODUCCION

Las amplitudes y formas de las ondas sísmicas generadas en un temblor dependen del mecanismo focal y de la cantidad de energía liberada en la zona de ruptura. El mecanismo focal controla la manera en que las ondas son irradiadas en el espacio y en el tiempo. No obstante, las ondas sísmicas una vez emitidas por la fuente sufren modificaciones en su trayecto que dependen de las propiedades mecánicas de los medios en que se propagan y de las dimensiones de las inhomogeneidades o irregularidades con que se encuentren. Si los cambios de las propiedades en una interfase son grandes o si el tamaño de las irregularidades es comparable o mayor que la longitud de onda predominante de las ondas incidentes, se generarán cambios significativos en el movimiento debidos a reflexión, refracción y difracción de las ondas.

Interesa entender la naturaleza de esos cambios porque pueden ocasionar grandes amplificaciones locales y variaciones significativas del movimiento del terreno en distancias relativamente pequeñas. Este efecto es de particular importancia en la respuesta sísmica de estructuras grandes como presas, puentes o lineas de trasmisión. Se trata de estructuras en las que los movimientos diferentes en los apoyos pueden ser muy peligrosos. Existe evidencia del papel que juegan los efectos de las condiciones locales en estudios de la distribución espacial del daño en temblores. Si bien el daño depende de la calidad de las construcciones, en muchos casos los daños severos están asociados a fenómenos de amplificación. En muchos casos se han utilizado en éxito modelos de propagación unidimensional de ondas de cortante cuando la configuración del sitio en estudio esta formada por estratos aproximadamente horizontales. Debe notarse, sin embargo, que el uso indiscriminado de modelos unidimensionales puede dar lugar a errores de importancia cuando las irregularidades locales son significativas pues no se toma en cuenta la naturaleza física del problema.

Los temblores fuertes, de interés en ingeniería sísmica, tienen componentes importantes en la banda de frecuencias de 0.1 Hz a 15 o 20 Hz. Por otra parte, las velocidades de propagación cerca de la superficie de la tierra varian de unos 200 m/s a casi 2 km/s; de manera que las correspondientes longitudes de onda caen en el rango de las decenas de metros a las decenas de kilómetros.

Las irregularidades geológicas y topográficas con dimensiones comparables con las longitudes de onda predominantes tendrán, entonces, considerable influencia en el movimiento. La extensión y detalle con que deben estudiarse las condiciones locales podrá estimarse en términos de las longitudes de onda asociadas con los periodos de oscilación que son más significativos para un análisis particular. Para un edificio alto, una presa o un puente, por ejemplo, las dimensiones locales pueden ser de varios kilómetros. Para una estructura pequeña y rígida, en cambio esas dimensiones pueden ir de las decenas a los cientos de metros.

El problema de calcular el movimiento en la vecindad de una irregularidad topográfica o estratigráfica ante incidencia de ondas sísmicas ha sido tratado como un problema de difracción de ondas elásticas de un cierto tipo. No se han desarrollado aun criterios prácticos para determinar la contribución de los distintos tipos de ondas en un cierto temblor (es usual suponer que se trata de ondas SV propagándose verticalmente). En parte, esto es debido a que las técnicas para tratar la difracción de diversos tipos de ondas elásticas son relativamente recientes. El hecho de que sean pocos los estudios que consideren materiales no lineales es también ilustrativo de las dificultades que en este caso se presentan. La mayoría de los estudios de difracción de ondas elásticas consideran configuraciones bidimensionales y sólo algunos casos de incidencia de ondas SH admiten soluciones analíticas (en el dominio de la frecuencia). Si bien las soluciones bidimensionales son una aproximación, proporcionan información útil sobre la respuesta sísmica de irregularidades; de hecho, algunos resultados preliminares de difracción tridimensional son similares a los obtenidos para dos dimensiones.

Los métodos que se han empleado para estudiar el problema son de varios tipos (de acuerdo con cada caso particular) y en algunos casos son de reciente desarrollo. Se ha empleado el método de los elementos finitos, que permite una gran flexibilidad en el modelado de dominios irregulares y aun de materiales no lineales. Suele ser, sin embargo, costoso y requiere precauciones especiales para tratar las fronteras del dominio y definir apropiadamente la excitación. Los elementos finitos pueden combinarse con esquemas de diferencias finitas en el tiempo o con solución en el dominio de la frecuencia.

Se han aplicado con éxito esquemas de diferencias finitas en el espacio y en el tiempo, sin embargo, algunas de las restricciones mencionadas para los elementos finitos limitan el uso generalizado de esta técnica.

Los métodos de frontera, basados en representaciones integrales y/o en expansiones en términos de familias completas de soluciones, están en desarrollo y es de esperarse que no sufran las desventajas de otras técnicas; en particular, al tratar sólo las fronteras se reduce la dimensionalidad del problema.

Por algún tiempo seguirán faltando criterios para formular excitaciones en términos de tipos de ondas incidentes, ángulos de incidencia, contenidos de frecuencias, etc. No obstante, en tanto estos criterios se formulan, se continúa el desarrollo de las técnicas de análisis de la difracción.

Podría parecer que es aun lejana la aplicación de criterios prácticos en la evaluación de los efectos de una cierta irregularidad en la intensidad sísmica. Es probable que el empleo sistemático de medidas integrales de la intensidad sísmica permita construir tales criterios. Una medida integral con interesantes propiedades es la intensidad de Arias. Los valores relativos de la intensidad de Arias respecto a una intensidad de referencia permitirían definir un índice de efectos topográficos que serviría para estimar la variación espacial de las aceleraciones máximas. Sin embargo, al presente, se carece de suficientes registros para estudiar con finura la variación espacial de la intensidad.

El propósito de estas notas es dar una introducción muy general del problema de estimar los efectos de las condiciones locales en las características de los temblores. Preceden también a algunos textos más especializados que el autor ha reunido aquí.

THE EFFECTS OF LOCAL IRREGULARITIES ON SEISMIC GROUND MOTION

By

Francisco J. Sánchez-Sesma*

ABSTRACT

The effects of topographical and geological irregularities on seismic ground motion are discussed. A short description is given of some of the available techniques to calculate such effects. Some comments are made on the importance of local conditions in the assessment of seismic risk.

INTRODUCTION

It has long been recognized that local topography and geology can significantly affect the ground motion at a site (Figure 1). The contribution of topography could serve to explain the high acceleration recorded at the Pacoima Dam (1.25 g) during the San Fernando, California earthquake of February 9, 1971 (Trifunac and Hudson, 1971; Boore, 1972c). For the aftershocks of the same earthquake Davis and West (1973) in a series of observations have found significative local amplifications due to topographical relief. In a field

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study in the Appalachian Mountains using distant mine blasts as sources, average amplitude ratios between mountaintop and valley sites were determined (Griffiths and Bollinger, 1979). These average ratios showed that the seismic wave amplitudes at the crests were amplified by factors from 1.7 to 3.4. In Figure 2 three seismograms for the same event in the Powell Mountain area are shown. The positions of recording sites are also displayed in the figure.

The effect of soil conditions in ground motion has been observed in well-documented earthquakes (Sozen *et al.*, 1968; Jennings, 1971) and in regression analyses of strong motion data. There is significant evidence that subsurface topography, *i.e.* lateral heterogeneities, are related to localized damage distribution in the Skopje, Yugoslavia earthquake of July 26, 1963 (Poceski, 1969). It has been suggested that focusing of the wave energy, by irregular interfaces, generated large motion amplification in limited zones of the city (Jackson, 1971). Damage statistics of buried utility pipes in the Miyagiken-Oki, Japan earthquake of June 12, 1978 have shown spectacular increase in the number of occurred failures near the cut and fill boundary of a newly developed area (Kubo and Isoyama, 1980).

Local conditions can generate large amplifications and important spatial variations of seismic ground motion. These effects are of particular significance in the assessment of seismic risk, in studies of microzonation, in planning and in the seismic design of important facilities (Esteva, 1977; Ruiz, 1977). In particular, local irregularities can be relevant in calculating the seismic response of long structures (see Figure 3) like dams, bridges or life-line systems (*e.g.* Esquivel and Sánchez-Sesma, 1980; Ruiz and Esteva, 1981).

As pointed out by Trifunac (1980), the strong earthquake shaking of interest in earthquake engineering falls in the frequency range from about 0.1 Hz to about 20 Hz and since the seismic wave velocities near the earth's surface lie in the range from about 0.1 km/s to about 3 km/s, it can be seen that the corresponding wave lengths are from tens of meters to tens of kilometers. Thus, the topographical and geological irregularities of dimensions near to this range will have considerable influence on the corresponding waves. It follows that the extent and detail of local conditions required to study their effects should be considered in terms of the wave lengths associated with the periods of motion which are more important for a particular analysis. For a tall building, a dam or a bridge, for example, these local site dimensions might be of several kilometers. On the other hand, for stiff structures or small buildings, these dimensions can be from tens to hundreds of meters.

Although recent work has emphasized the physical understanding of local effects so that quantitative predictions can be made, as stated in a recent review by Boore (1983), there is still lack of criteria for dealing with the problem taking into account source, path and local conditions. Active research is needed to predict more accurately the local effects, given the source parameters. Indeed, it is encouraging the recent progress on strong motion prediction using mathematical modeling techniques (Aki, 1982). Much of the research is concentrated on the understanding of fault mechanics and wave propagation in the Earth. It is generally accepted that high frequency radiation, which controls accelerations, comes from very localized parts of the fault. A powerful asymptotic theory of high frequency radiation has been recently developed (Madariaga, 1983). Applications of the theory are coming. However, it should be noted that the foci of future earthquakes are not known;

their location, mechanism and amount of released energy can only be speculated in terms of regional seismicity models (Esteva, 1976). On the other hand, the knowledge of geological details is generally small to justify the use of very refined models of wave propagation, particularly for the high frequencies. It is then clear, in view of the mentioned uncertainties, that the problem of seismic risk assessment must be dealt with a probabilistic framework. A promising approach seems to be the use of integral measures of intensity such as the Arias' (1970) using stochastic descriptions of the input and simplified models of the local irregularities (Sánchez-Sesma *et al.*, 1983).

The aim of this work is to review the problem of calculating the effects of topographical and geological irregularities on ground motion given certain input, *i.e.* some kind of seismic waves. For this purpose the current formulation of the problem, the known analytical solutions and the available numerical methods are discussed in brief. This review is by no means complete and reflects the particular trends of the author. Nevertheless, it is hoped that this work could serve to stimulate discussion and interest on the problem.

FORMULATION OF THE PROBLEM

There is no doubt that the source mechanism governs the way in which the released seismic energy is radiated in space and time. However, seismic waves, once emitted by the source, are dependent on the mechanical properties of earth materials and the heterogeneities encountered in their path. This is also true dealing with irregular local conditions. Moderate changes in mechanical impedances or irregularities with size comparable to incident wave lengths can generate significant amplifications and spatial variations of ground motion (Boore, 1972b).

Plane waves are reflected back and refracted forward as they arrive at a plane interface. The amounts of reflected and transmitted energy depend on the mechanical properties of the media involved. Reflection and refraction in elastic wave propagation can well be described by geometrical means. Let us call diffraction to every change in the waves' path that can not be described as reflection or refraction. For studying diffraction of elastic waves it is necessary to solve a boundary value problem for the governing equations of linear elasticity (*e.g.* Achenbach, 1973; Aki and Richards, 1980).

To fix ideas, consider an elastic, homogeneous and isotropic half-space with an irregular surface. Under incidence of elastic waves the irregularity will diffract the incident waves (diffraction is frequently called scattering). Diffracted waves must satisfy, toghether with incident waves, the governing equations (Navier equations) and the boundary conditions (traction-free surface). Moreover, the diffracted fields must satisfy the Sommerfeld (1949) radiation condition at infinity, which means that the diffracted fields must scatter to infinity; *i.e.*, no energy may be radiated from infinity into the irregular region. The Sommerfeld radiation condition has been extended to elastic wave fields by Kupradze (1965).

ANALYTICAL SOLUTIONS

The simplest problems in elastic wave diffraction are the two-dimensional SH-wave problems because they can be analyzed separately from other body waves. The governing equation for this case is the scalar wave equation. Then, analytical solutions can be obtained for geometries of the scatterer which allow separation of variables (Mow and Pao, 1971). Using this method, exact solutions have been obtained for the diffraction of SH-waves by canyons and alluvial valleys with semi-circular (Trifunac, 1971, 1973) or semi-elliptical shapes (Wong and Trifunac, 1974a, b). Even with these simple models of local irregularities, complicated interference patterns were found and the obtained surface displacement fields vary strongly in space (see Figure 4). They are very sensitive to the incidence angle and the frequency. Results for alluvial valleys show the importance of the two-dimensional behaviour which gives much larger amplifications than those obtained from unidimensional calculations. These analytical solutions have shown the importance of the problem and they provide a check for numerical procedures.

For the more difficult cases of P- or SV-incident waves the orthogonal wave functions developed in classical physics are not separable for the half-space surface due to the coupling of boundary conditions. Lee (1978, 1982) overcome this difficulty for a semi-spherical canyon by expanding the spherical wave functions further into a power series which matched all the boundary conditions successfully. However, this approach is limited to small frequencies because the resulting matrix equations, which are infinite, can only be solved approximately for this case.

Under additional simplifying assumptions, other analytical solutions have been obtained using orthogonal wave functions. For an acoustic medium the exact expressions for the scattered fields generated by incidence of P-waves on canyons of semi-circular and semi-spherical shapes (Singh and Sabina, 1977) have been obtained. However, the results are of small utility because the acoustic assumption can hardly be met in real cases. The problem of vertically incident P-waves upon a semi-ellipsoidal three-dimensional scatterer has been exactly solved for an elastic medium in which horizontal displacements are restricted (Sánchez-Sesma, 1983b). Results for vertical displacement are in reasonable agreement with those from more reliable computations for a truly elastic medium (Sánchez-Sesma, 1983a).

Under the assumption of small-slope irregularities a perturbation solution has been obtained for the elastic scattered field by two-dimensional geometries (Gilbert and Knopoff, 1960). The approximation is based on replacing the irregularity by an equivalent stress distribution. An application of this method by Hudson (1967) deals with small-slope three-dimensional scatterers. With this approach reasonable estimates have been obtained of the scattered Rayleigh waves as compared with observations even in cases in which slope angles are as large as 25° or, say, 30° (Hudson and Boore, 1980).

The method of matched asymptotic expansions has no restrictions on the slope of the irregularity. It is based in matching the first terms of an outer expansion of the near field with those of an inner expansion of the far field

(Sabina and Willis, 1975, 1977). Although the method is limited to very small frequencies, results are in qualitative agreement with observations.

NUMERICAL METHODS

A powerful technique has been developed by Aki and Larner (1970) to treat scattering of SH-waves by irregular interfaces. In the Aki-Larner method, incidence is assumed of a plane single-frequency wave which causes a displacement field. This field is represented by superposition of plane waves of unknown complex amplitudes propagating in many directions. Inhomogeneous plane waves are allowed. The total motion is obtained from integration over horizontal wave number. Under the assumption of horizontal periodicity of the irregularity, the integral is replaced by an infinite sum. Truncation of this sum and application of the interface conditions of continuity of stress and displacement leads to a system of linear equations for the complex scattering coefficients. This method, which is restricted to small-slope irregularities, has been applied by Bouchon (1973) to study the effects of two-dimensional irregular topographies on ground motion for incidence of SH, SV and P waves. An extension of the method has been advanced by Bouchon and Aki (1977a, b) to represent with this discrete-wave-number technique near seismic source fields in a layered medium with irregular interfaces. Another extension of the method, now to time domain calculations, has been developed to study the seismic response of alluvial valleys (Bard and Bouchon, 1980a, b) under incidence of SH, P and SV waves. The Aki-Larner technique has been recently used by Bard (1982) to analyze the effects of two-dimensional elevated topography on ground motion. The comprehensive studies by Bard and Bouchon throw light on the subject and on the physics of the problem. The method has been used to model the fields generated by real faults (Bouchon, 1979; Campillo, 1983). It can be used to model ground motion considering together the effects of source, path and local conditions. However, in this case the numerical computations may become very expensive -if they can be performed- for many real cases.

The finite difference method is also a powerful tool in elastic wave propagation studies (Alterman and Karal, 1968; Boore, 1972a). It has been applied to modelate two-dimensional irregular interfaces (Boore *et al.*, 1971) and ridges (Boore, 1972b) in the SH case and also for incident of P and SV-waves upon a sedimentary basin (Harmsen and Harding, 1981) and a step-like topography (Boore *et al.*, 1981). Interesting results have been found concerning the significant generation of Rayleigh surface waves by lateral irregularities. For incidence of P waves upon a surface slot the computations by Ilan and Bond (1981) give good agreement with experiments. It was found that the amplitude of the scattered Rayleigh wave is, as expected, dependent on incidence angle. A finite difference analysis of axisymmetric topographical irregularities has been presented to study the effects of vertically incident shear waves (Zhenpeng *et al.*, 1980). Spectral ratios were obtained and comparison with observations gives reasonable agreement. The finite difference method is theoretically unlimited to model details and nonlinear behaviour of materials, but the size of the problem can easily exceed the capacity of major computing facilities.

The finite element method also allows a detailed description of site topography and layering. It is possible with this method to calculate the response of two-dimensional soil configurations with truly nonlinear stress-strain relations

(Streeter *et al.*, 1974; Joyner and Chen, 1975; Joyner, 1975). The major disadvantage of the method is its low-frequency limit and high cost. Usually, real time analysis must be shortened to avoid the reflections from the artificial boundaries. The use of different transmitting techniques can reduce the spurious waves to some extent (*e.g.* Smith, 1974, 1975; Ayala and Aranda, 1977; Clayton and Engquist, 1977; Castellani *et al.*, 1981; Liao and Wong, 1981). A successful criterion has been developed for damping out the unwanted reflections by means of non-uniform element size (Day, 1977). Finite elements have been used to treat problems of irregular layering (Lysmer and Drake, 1972; Drake, 1972; Aranda and Ayala, 1978) and two-dimensional topographical irregularities (Castellani *et al.*, 1982) under idealized conditions. Nevertheless, a realistic wave analysis is quite costly.

In recent years boundary methods have gained increasing popularity. This fact is mainly due to the availability of high speed computers. Boundary methods are well suited to deal with wave propagation problems because they avoid the introduction of fictitious boundaries and reduce by one the dimensionality of the problem. These facts yield numerical advantages. Moreover, boundary methods can be used together with the finite element method (Zienkiewics *et al.*, 1977). Then, the region modelated with finite elements can be smaller (*e.g.* Ayala and Gómez, 1979; Shah *et al.*, 1982).

There are two main approaches for the formulation of boundary methods; one is based on the use of boundary integral equations (Cruse and Rizzo, 1968a, b; Brebbia, 1978; Cole *et al.*, 1978; Alarcón *et al.*, 1979), and the other, on the use of complete systems of solutions (Herrera and Sabina, 1978; Herrera, 1980). The scattering of incident SH-waves from two-dimensional irregular topographies has been formulated with integral equations by Wong and Jennings (1975) for arbitrarily shaped canyon-like profiles and by Sills (1978) for ridges and mixed shapes. This method has been applied with success to calculate the effects of a dipping layer of alluvium of an SH-wave source on the surface (Wong *et al.*, 1977). Results compare favorably with observations during a full-scale low-amplitude propagation test. A powerful approach which combines the boundary integral equation method with finite differences in time has been presented (Cole *et al.*, 1978) for solving elastodynamic problems. The performance of the method was found to be good in a simple numerical problem. A boundary method has been recently developed and applied to solve two-dimensional scattering of harmonic elastic waves by canyons (Sánchez-Sesma, 1978, 1981; Sabina *et al.*, 1979; Sánchez-Sesma and Rosenblueth, 1979; Wong, 1979, 1982; England *et al.*, 1980; Sánchez-Sesma *et al.*, 1982a), alluvial deposits (Sánchez-Sesma and Esquivel, 1979; Dravinski, 1982a, b, 1983) and ridges (Sánchez-Sesma and Esquivel, 1980; Sánchez-Sesma *et al.*, 1982b) for different types of waves and shapes of the scatterers. The method consists of constructing the scattered fields with linear combinations of members of a c-complete family of wave functions (Herrera and Sabina, 1978). These families of functions, which are solutions of the governing equations of the problem, can be constructed in a very general way, with single or multipolar sources having their singularities outside the region of interest. Coefficients of the linear forms thus constructed are obtained from a least-squares matching of boundary conditions. As pointed out by Wong (1982), the method can be considered as a generalized inverse one. In doing this, Wong suggested a procedure which improves the solution numerically. A general framework for the method is given by a recent algebraic theory of boundary value problems (Herrera, 1979, 1980 a, b).

This approach has recently been extended to three-dimensional problems (Sánchez-Sesma, 1983a). The case of incident elastic waves upon axisymmetric irregularities on the surface of an elastic half-space was formulated using an azimuthal decomposition. The diffracted fields were constructed with multipolar solutions of the reduced Navier equations in spherical coordinates (Takeuchi and Saito, 1972; Aki and Richards, 1980). For a semi-spherical alluvial deposit and vertical incidence of P-waves a very large amplification was found, as compared with the flat layer problem.

Using also multipolar expansions the scattering of harmonic SH-waves by arbitrarily shaped alluvial basins has been solved (Ize *et al.*, 1981). Within the theoretical framework, a least-squares numerical scheme (related to Courant's) is used and it is found to reduce the order of the systems of equations to be solved for the same given accuracy.

CONCLUDING REMARKS

The influence of topographical and geological irregularities on seismic ground motion has been briefly discussed and a short description of some of the available methods to deal with such effects was made.

There is no doubt that local conditions play an important role in the spatial variation of ground shaking and should be explicitly considered in the design of some important facilities, as well as for microzoning. But local conditions are not alone; the source mechanism and the paths of seismic waves give also their part in the assessment of seismic risk. Thus, a more complete description of the problem is needed.

Most of the models of local conditions require the definition of the types, incidence angles and time variation or frequency content of the incoming waves. However, there are at present no general criteria for selecting on a physical basis the characteristics of the input for many practical situations. Such criteria, hopefully, will come from the research on the source high frequency radiation.

ACKNOWLEDGEMENTS

Thanks are given to A. Arias and L. Esteva for the critical reading of the manuscript. The assistance of J. Avilés, S. Chávez-Pérez and G.M. Cortés-Rubio is greatly appreciated.

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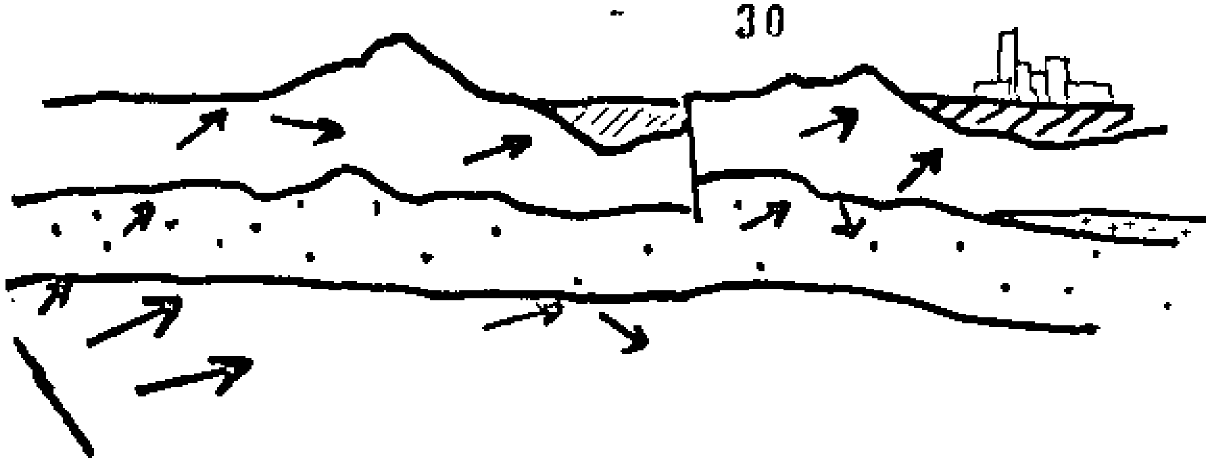

Fig. 1 Local topography and geology

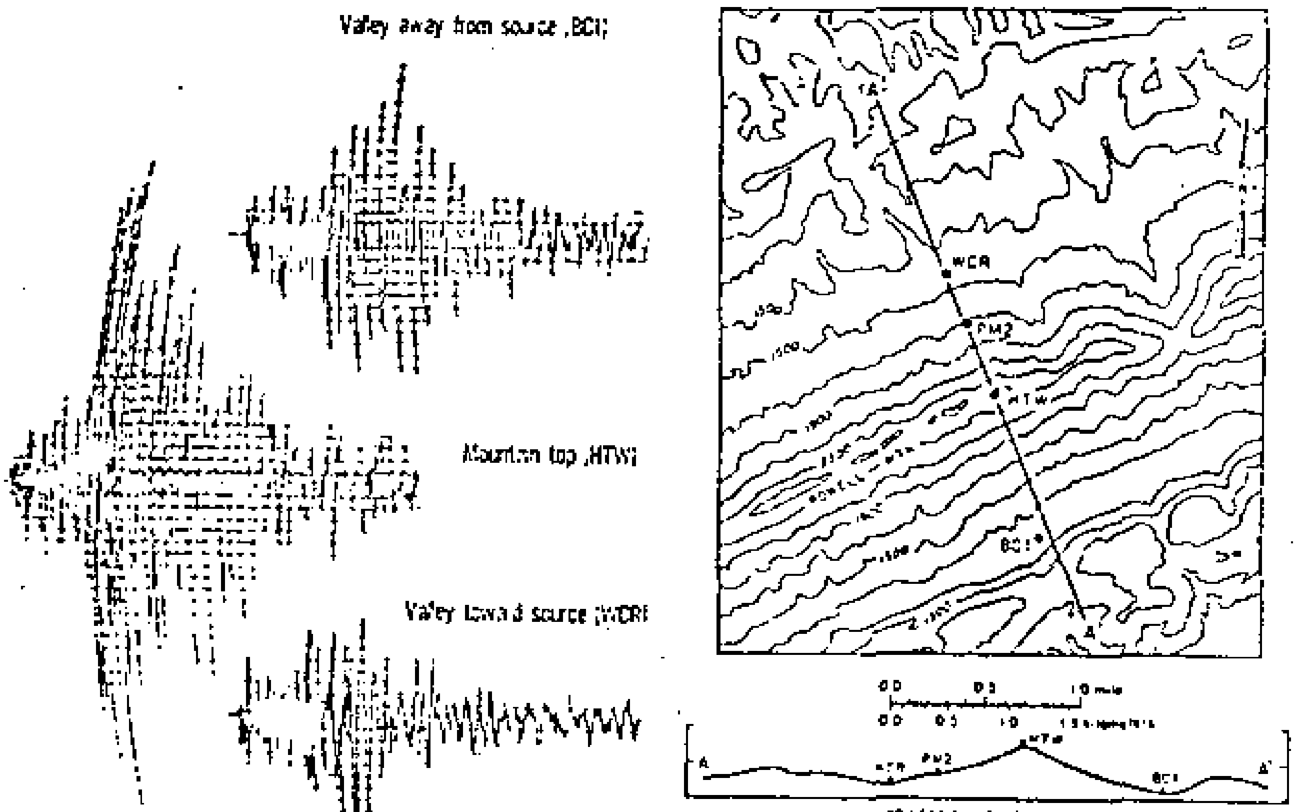

Fig. 2 Seismograms at three stations at the Powell Mountain area and location of the recording sites. Epicentral distance was about 30 km. Topographic contour interval is 200 ft (61 m) (After Griffiths and Bollinger, 1979)

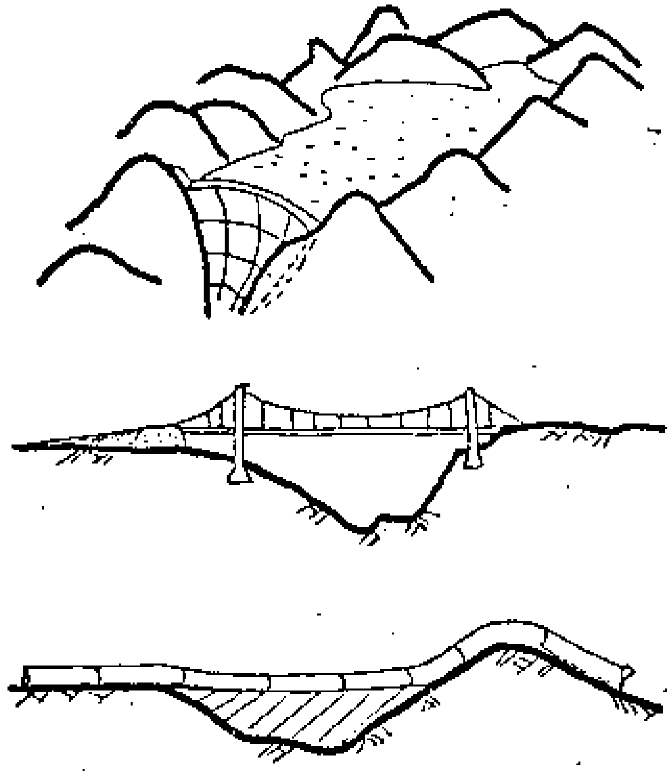

Fig. 3 Long structures at irregular sites: a) dam; b) bridge; and c) a life-line system

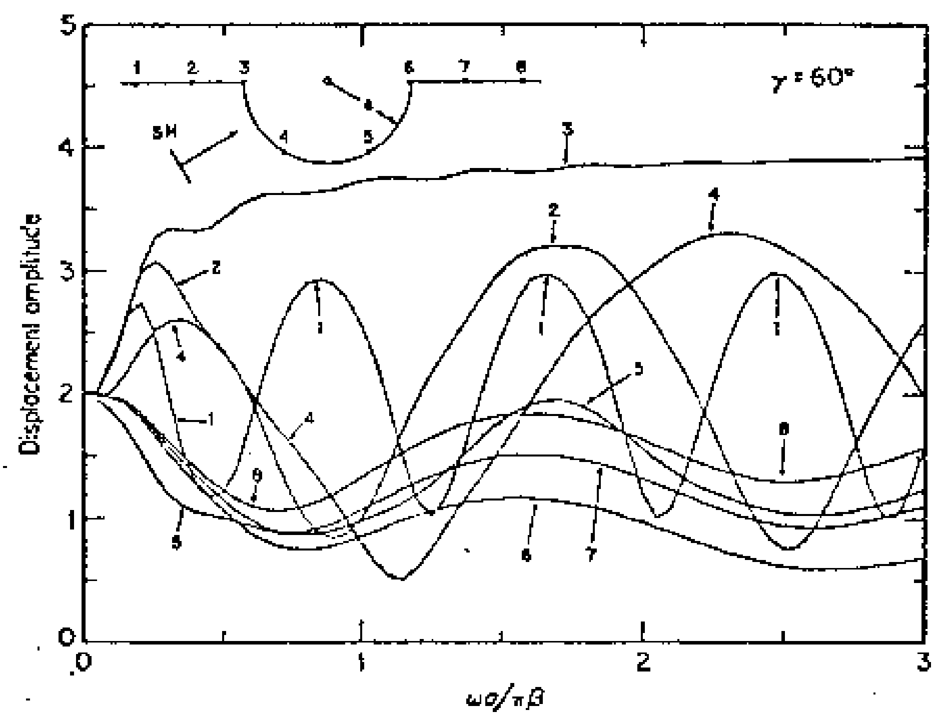

Fig. 4 Displacement amplitudes at points in the surface of a semi-circular canyon. Incidence of harmonic plane SH waves (After Trifunac, 1971)

DIVISION DE EDUCACION CONTINUA
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X CURSO INTERNACIONAL DE INGENIERIA SISMICA

ANALISIS DE RIESGO SISMICO

DIFFRACTED WAVES AND DISPLACEMENT FIELD OVER

TWO-DIMENSIONAL ELEVATED TOPOGRAPHICS

AGOSTO, 1984.

Francisco J Sánchez-Sesma

Geophys. J. R. astr. Soc. (1982) 71, 731–760

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Diffracted waves and displacement field over two-dimensional elevated topographies

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Received 1982 April 14; in original form 1981 April 16

Summary. The Aki-Larner technique is used to perform, in both the time and frequency domains, an analysis of the effects of two-dimensional elevated topography on ground motion. Incident plane *SH*-, *SV*- and *P*-waves are considered and the respective influences of surface geometry, elastic parameters and the incident wave characteristics, as long as they remain within the limits of applicability of the A-L technique, are investigated in some detail.

Besides the well-known amplification/deamplification effect related to the surface curvature, wave scattering phenomena on the convex parts of the surface are shown to contribute significantly to the disturbances in the displacement field around the topographic structure. These scattered waves are *SH* in the case of incident *SH*-waves, and mainly Rayleigh waves in the *P* case, while both Rayleigh and horizontal *P*-waves, sometimes of large amplitude, develop in the *SV* case. The frequency dependence of this scattering, though complex, seems to be mainly controlled by the horizontal scale of the topographic structure. The parameter study points out the regular and intuitive behaviour of this wave scattering in both *SH* and *P* cases, while it exhibits a puzzling complexity for incident *SV*-waves, which is interpreted as resulting from the importance of the *S–P* reflections on mountain slopes in that case.

As to the ground motion, some general features may be pointed out. The amplification on mountain tops, which is systematically greater for incident *S*-waves than for *P*-waves, generally decreases as the average slope decreases or as the angle of incidence increases. Mountain slopes undergo either amplification or deamplification depending on site location, frequency and incidence angle, but they always undergo strong differential motion due to the lateral propagation of the scattered waves and their interference with the primary wave. Finally, all these effects may be greatly enhanced in the case of complex topographies, which moreover give rise to a significant prolongation of ground motion because of the large number of scattered waves.

2

1 Introduction

For many years, a great number of authors have pointed out the importance of site effects in many seismological problems, ranging from source parameter computation to seismic risk assessment. These site effects were at first related to the local sediment cover (resonance of surface layers, sand liquefaction, attenuation of high frequencies, etc.). More recently, however, observations have been made (e.g. Key 1967; Nason 1971; Trifunac & Hudson 1971) showing the influence of topography on surface ground motion.

Besides field studies (Davis & West 1973; Griffiths & Bollinger 1979), or model studies (Rogers, Katz & Bennett 1974; Ilan, Bond & Spivack 1979), the theoretical investigations have focused more on numerical solution rather than on a detailed study of the effects of topography on surface motion. A lot of numerical methods have been developed to investigate the scattered far-field (small perturbations: Gilbert & Knopoff 1960; Hudson & Knopoff 1967; Hudson 1967; MacIvor 1969; small perturbations and finite difference: Aboudi 1971; Alterman & Aboudi 1971), or the displacement field over the topography itself (analytical models: Trifunac 1973; Wong & Trifunac 1974; Singh & Sabina 1977; finite differences: Boore 1972; Alterman & Nathaniel 1975; Ilan 1977; Ilan *et al.* 1979; finite elements: Smith 1975; discrete wavenumber representation: Lamer 1970; Bouchon 1973; matched asymptotic expansion: Sabina & Willis 1975, 1977; integral equations: Wong & Jennings 1975; Sills 1978; Sanchez-Sesma & Rosenblueth 1979; and boundary methods: England, Sabina & Herrera 1980). All these theoretical models, involving a two-dimensional homogeneous elastic half-space with a simple topographic section (except Singh & Sabina 1977, who consider a three-dimensional hemispheric valley), provide results which quantitatively agree with one another, and qualitatively confirm the experimental observations: amplification usually occurs on convex parts of the ground surface (mountain tops or valley edges), and deamplification on concave parts and shadow zones. Our purpose here is therefore not to present yet another numerical method, but to try to gain a better understanding of these topographic effects, through their dependence on parameters such as incident wave type, frequency, incidence angle and anomaly height.

In other respects, all available theoretical results seem to underestimate the observed effects of topography on surface motion, as reported by Davis & West (1973), and Griffiths & Bollinger (1979). Since the model study values corroborate those obtained by numerical methods (Rogers *et al.* 1974; Ilan *et al.* 1979), it is believed that quantitative disagreement between theory and observation is due, at least partly, to the complexity of both incident signal and topography. It is therefore another aim of the present paper to show how a somewhat complicated topography can produce greater effects than isolated ridges or valleys, and that the surface motion at a particular site depends a lot on the topographic features of a wide area around.

We believe that the Aki-Larner method is the best suited to this kind of study, since it is a very tractable and computationally cheap method, and because the discrete wavenumber representation provides useful information on the nature of the scattered elastic field. This

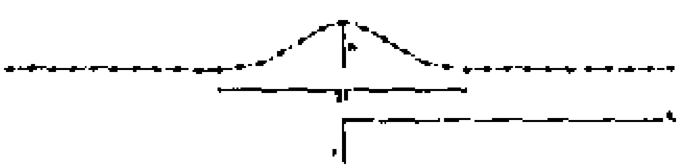

Figure 1. Geometrical shape of the mountain model introduced by Sills (1978), and corresponding to the cases investigated in Figs 2–17. The open circles indicate the sites where the synthetic seismograms of Figs 2, 3 and 14 are computed.

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technique has already been presented with many details by Larner (1970), Aki & Larner (1970) and Bouchon (1973), and its reliability carefully discussed. The limitation of our investigations to topographic features having slopes lower than 40°, and to incident body waves only, with rather small incidence angles so as to avoid shadow zones, together with the experimental observations of Rogers *et al.* (1974) and Ilan *et al.* (1979) showing that the radiation lobes of the diffracted waves are directed essentially downwards, allow us to think that the Rayleigh ansatz error does not alter the accuracy of our results too much.

2 Effects of simple topography on incident plane waves — a parameter study

2.1 GENERALITIES

Our purpose is to evaluate the surface displacement and the scattered elastic field produced by a plane body wave impinging upon a two-dimensional topographic feature lying over an otherwise isotropic homogeneous elastic half-space, for various topography, half-space and incident wave characteristics.

Our computations rely on the same theoretical formulation as those presented by Bouchon (1973), except that we do not correct the displacement field for the residual stresses, since we remain well within the validity limits of the A-L technique. Furthermore, in order to investigate the ground motion in the time domain, we follow the approach described by Bard & Bouchon (1980a), making the computation at equally spaced frequencies having an imaginary part equal to half the frequency step. The disturbances induced by the structure periodicity L (required by the A-L method) are avoided by a suitable choice of both the frequency range and the time window.

Topographic effects then depend on three kinds of parameters:

2.1.1 *Topographic shape*

This is the most difficult to quantify. For a better understanding of the diffraction and amplification/deamplification mechanisms, only simple topographic profiles are considered at first. The 'pseudo-realistic' mountain model of Sills (1978) is chosen, defined through the equation:

$$\zeta(x) = h(1-a)\cdot\exp(-3a) \qquad \text{with} \qquad a = (x/l)^2$$

this topography is completely defined by its half-width l and its height h. Its shape is illustrated in Fig. 1.

2.1.2 *Half-space elastic parameters*

These reduce to the shear and compressional velocities, respectively denoted as β and α, since the material density has no influence.

2.1.3 *Incident wavefield characteristics*

These are wave type, incidence angle, azimuth and frequency spectrum. Only incident plane waves are considered here, for simplicity and because nearfield studies introduce a lot of new parameters related to the source mechanism. Moreover, only incident body waves are taken into account. The effect of incident wave azimuth is not considered: only in plane and out of plane motions are investigated.

The remaining independent dimensionless parameters considered in this paper are therefore: the topography shape ratio h/l, the Poisson ratio ν, the dimensionless frequency

4

$\eta = 2l/\lambda$ (which is scaled to the S wavelength $\lambda = \beta/f$), and the incidence angle θ, measured from the vertical axis.

For time domain studies, we generally choose an input signal in form of a Ricker wavelet, having a time dependence:

$$f(t) = (a - 0.5)\exp(-a) \quad \text{with} \quad a = |\pi(t - t_s)/t_p|^2$$

where t_s is the time of maximum amplitude, and t_p the characteristic period. We also sometimes use a combination of several Ricker wavelets.

2.2 GENERAL CHARACTERISTICS OF TOPOGRAPHIC EFFECTS IN THE VERTICAL INCIDENCE CASE

In this section the case of P, SV and SH plane waves vertically incident on two topographic features is investigated, a mountain having a shape ratio h/l equal to 0.375, and a symmetric depression. Materials with Poisson's ratio $\nu = 0.25$ are considered until Section 2.5.

2.2.1 *Time domain study*

Fig. 2 illustrates the time history of ground motion at several sites located over the topographic elevation and some distance apart, for an incident Ricker signal having a characteristic dimensionless frequency of $\eta_p = 1.83$, in each of the three SH, SV and P incident cases. The well-known amplification pattern on mountain tops appears on each of the three components, but it is more important in the case of incident S-waves (43 per cent for SH and 30 per cent for SV) than in the case of incident P-waves (only 10 per cent). Moreover the computed motions do not exhibit much change in signal waveform or duration. On the contrary, on mountain slopes, significant alterations appear; while the ground displacement amplitude is reduced, its duration is increased. The amplitude reduction is more important for horizontal components (the maximum attenuation is about 25 per cent for SH, 40 per cent for SV and only 10 per cent for incident P-waves). In the SH case, seismograms away from the topography show an outward propagating signal having roughly the same shape as the incident signal and an amplitude of about 20 per cent at a distance $2l$ from the mountain top. The phase velocity of this diffracted wave, together with the space dependence of its amplitude, which fits an $x^{-1/2}$ decay, are consistent with a horizontal SH-wave generated near the mountain top. In the P and SV cases, seismograms exhibit a somewhat greater complication, because of the outward propagation of two such diffracted waves. Their phase velocities, vertical/horizontal component ratio, and spatial decay ($x^{-1/2}$ for the fastest wave, and nearly constant for the slowest), allow their identification as a horizontal P-wave and Rayleigh wave, respectively. The relative importance of these two waves undergoes great changes with the incident wave type. In the SV case, the horizontal P-wave is the main disturbance and is associated with a horizontal displacement around 20 per cent of the direct one (measured at the same reference site $x = 2l$), whereas in the P case,

Figure 2. Response of the mountain illustrated in Fig. 1 ($h/l = 0.375$) to vertically incident, respectively, SH, SV and P Ricker wavelets of characteristic dimensionless frequency $\eta_p = 1.83$. The traces represent the time history of the displacement at surface receivers, spaced from 0 to $2.5l$ from the mountain top (numbers on the left represent the site location in dimensionless values x/l). In the SH case, only the transverse horizontal component, v, is represented, while in the P and SV cases, both horizontal motion along the x-axis (u, lower trace) and vertical motion (w, upper trace), are depicted. The bottom traces represent the ground motion for a plane free surface. The dimensionless length of the time window is 3.33. Here $\nu = 0.25$.

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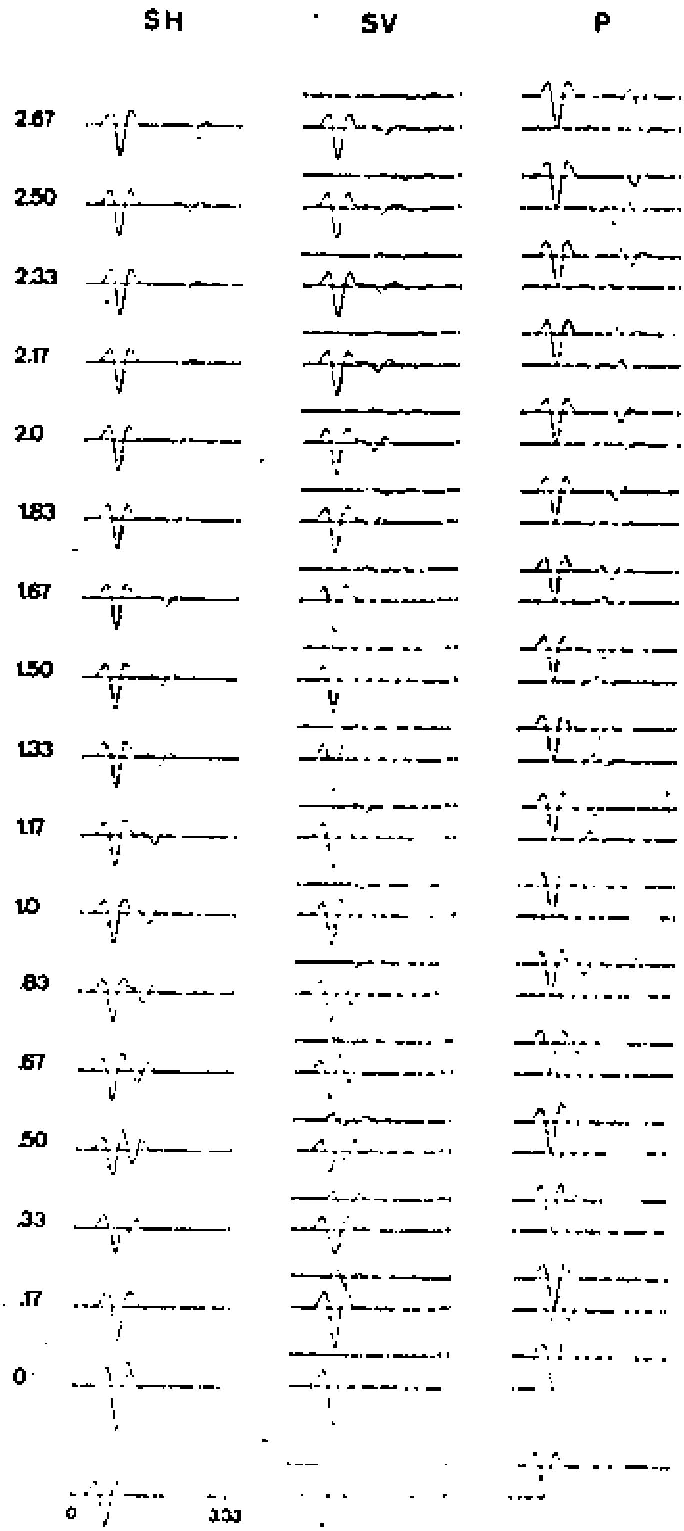

Incident signal : $\eta_0 = 1.83$

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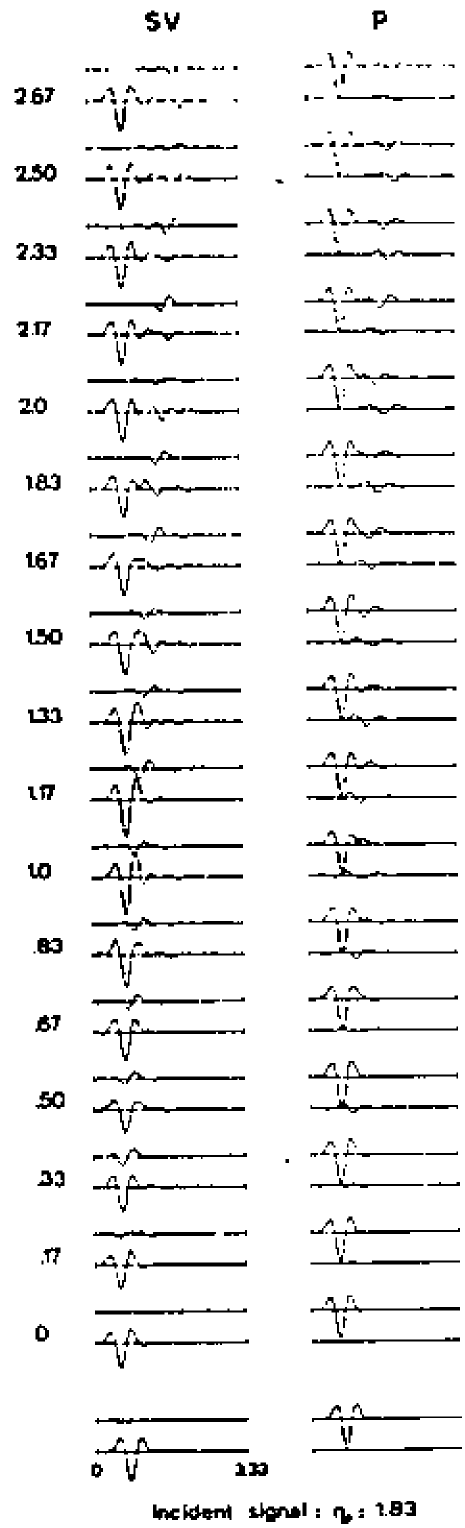

Figure 3. The same as Fig. 2, but for a depression of same shape ratio. Only the *SV* and *P* cases are illustrated.

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the most important disturbance is the Rayleigh wave, which produces a vertical displacement around 25 per cent of the direct one. The deamplification pattern at the mountain base, reported by some authors, does not show up here, probably because the incident signal has a relatively short duration, so that direct signal and diffracted waves are well separated and do not interfere. On the contrary, the crest/flank amplification ratio reaches rather large values for incident *S*-waves (2 in the *SH* case, 2.2 in the *SV* one).

The case of *P*- and *SV*-waves vertically incident on a depression is illustrated in Fig. 3. The bottom of the canyon undergoes deamplification in both cases, the amount of which is greater on the horizontal (20 per cent) than on the vertical motion (10 per cent). The synthetic seismograms on canyon flanks exhibit only a slight deamplification in the *SV* case, and no changes as to signal duration and waveform. On the contrary, the ground motion at sites distant from the valley borders reveals an outward propagation of diffracted waves generated at the valley edges, which themselves undergo a clear amplification (around 20 per cent for incident *SV*-waves).

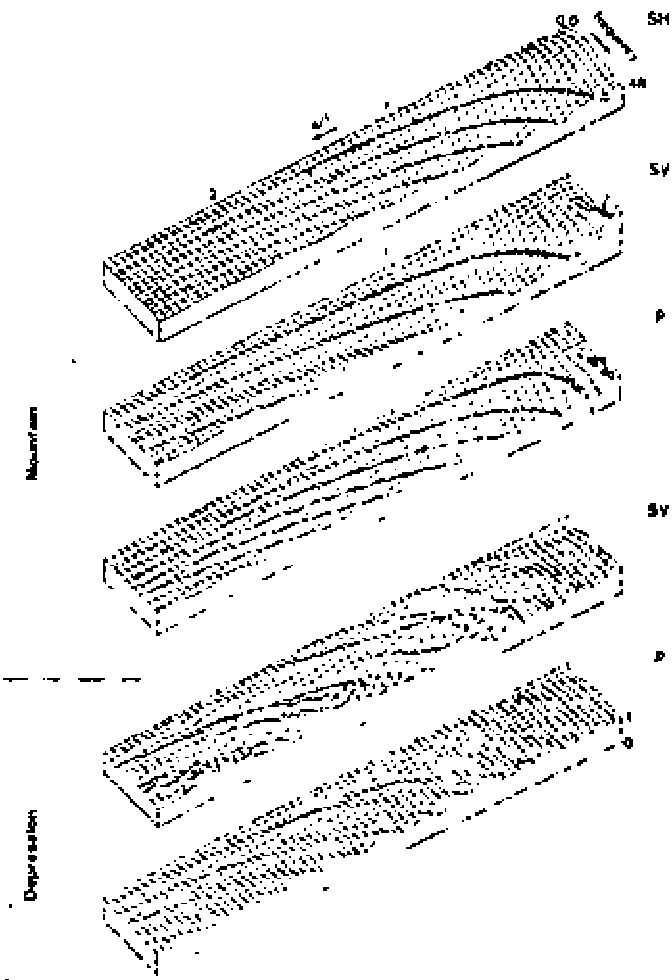

Figure 4. Amplitude of the ground motion as a function of site location and frequency. The three upper diagrams depict the mountain case (Fig. 2) and the two lower the depression case (Fig. 3). The top diagram represents the horizontal transverse component *v* for the *SH* case, the second and fourth one the horizontal component *u* in the *SV* case, and the third and fifth the vertical component *w* in the *P* case. The null amplitude reference and the scale are indicated for each diagram. The topographic outline is not shown so as to see the displacement amplitude variations better.

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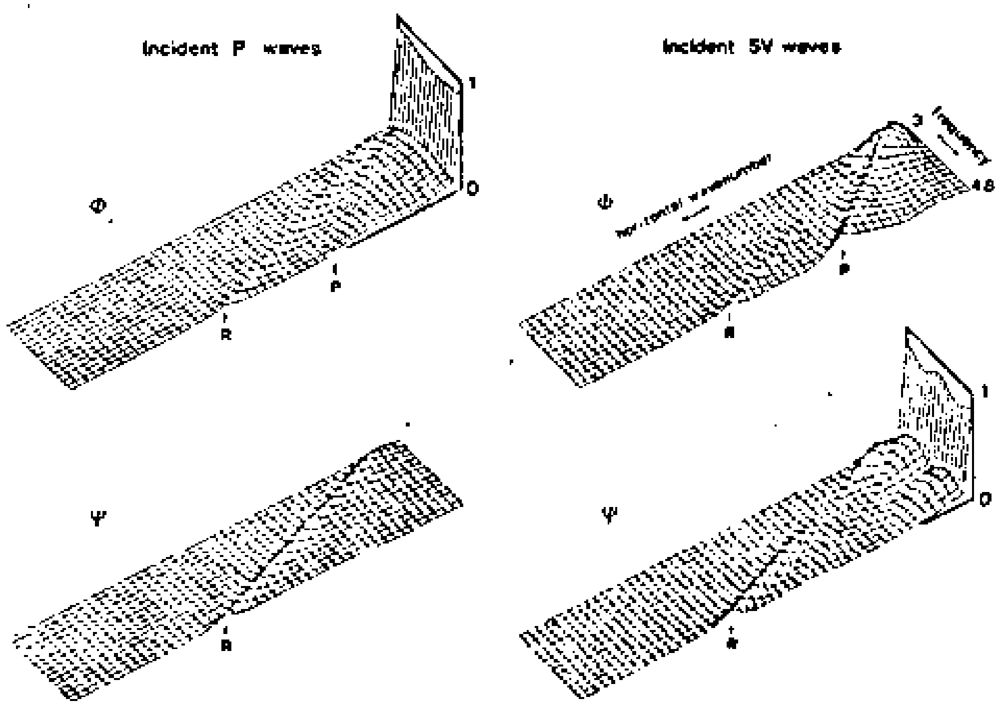
2.2.2 *Frequency-domain study*

It is important to get an idea of the frequency at which topographic effects (amplification or deamplification, diffraction of scattered waves) become significant. Theoretical calculations of Gilbert & Knopoff (1960), Hudson (1967) and Hudson & Boore (1980) relate the amplitude of the diffracted wave with the spatial Fourier transform of the topographic cross-section, so that this 'starting frequency' depends, for a given topographic shape, essentially on its width (in comparison to the wave velocity), while the shape ratio acts as an amplitude factor. Furthermore, experimental observations (Rogers *et al.* 1974) and theoretical computations (Boore 1972; Bouchon 1973) show surprisingly low values for these starting frequencies. On mountain tops, there appear spectral amplifications greater than 10 per cent for wavelengths about four times the total width of the topographic feature. Though our purpose here was not to investigate in detail the very low frequency domain, we may note that, for incident *P*- and *SV*-waves as well as for *SH*-waves, our computations happen to confirm these very low values of the 'starting frequency', and its main control by the topographic width (see Fig. 8b).

However, our interest here is in the 'medium frequency' domain, i.e. wavelengths between 2 or 3 times the mountain width, and its height. Fig. 4 represents the 'raw' displacement spectra in the *SH*, *SV* and *P* cases corresponding to Figs 2 and 3 (these are not the true displacement spectra because they must be corrected for the complex frequency, which attenuates the amplitude on elevated sites and lessens the importance of late arrivals). The low-frequency behaviour is consistent with the time domain observations reported



$h/l = .375$: $\nu = .25$

Figure 5. Elastic potential characteristics for the topographic structure illustrated in Fig. 1, impinged by vertical *P*-waves (left) and *SV*-waves (right). The compressional (ϕ) and rotational (ψ) potentials of the scattered waves, computed at a $z = 0$ altitude, are shown as a function of frequency (dimensionless step: 0.3). *L* is the structure periodicity involved by the use of the A-L technique. The scale on the right of the upper left and bottom right diagrams represents the amplitude of the incident *P*- or *SV*-waves, respectively.

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above, but higher frequencies give rise to significant changes, essentially on mountain slopes and valley edges; these changes follow an oscillatory pattern, due to the interaction between the direct and diffracted waves, as noted by Rogers *et al.* (1974). At a given site, the frequency spacing of lobes of constructive and destructive interference is inversely proportional to the time delay between the direct wave and the main diffracted wave. For instance, the frequency spacing at the same site is larger on the horizontal displacement for incident *SV*-waves than on the vertical one for incident *P*-waves, since the former is controlled mainly by the horizontally diffracted *P*-wave, and the latter by the Rayleigh wave. It is thus very difficult to predict whether amplification or deamplification will occur systematically at a given site; nevertheless, a constant feature is that mountain slopes always undergo strong differential strains, the amount of which increases with the frequency (at least within the frequency range investigated here).

On another hand, these oscillations converge towards the onset site of the corresponding diffracted waves. Fig. 4 thus indicates that the scattered *SH*-, *P*- and Rayleigh-waves originate at mountain tops and valley edges, that is, on convex parts of the topography. Also noticeable on this figure are the great high-frequency disturbances appearing in the horizontal

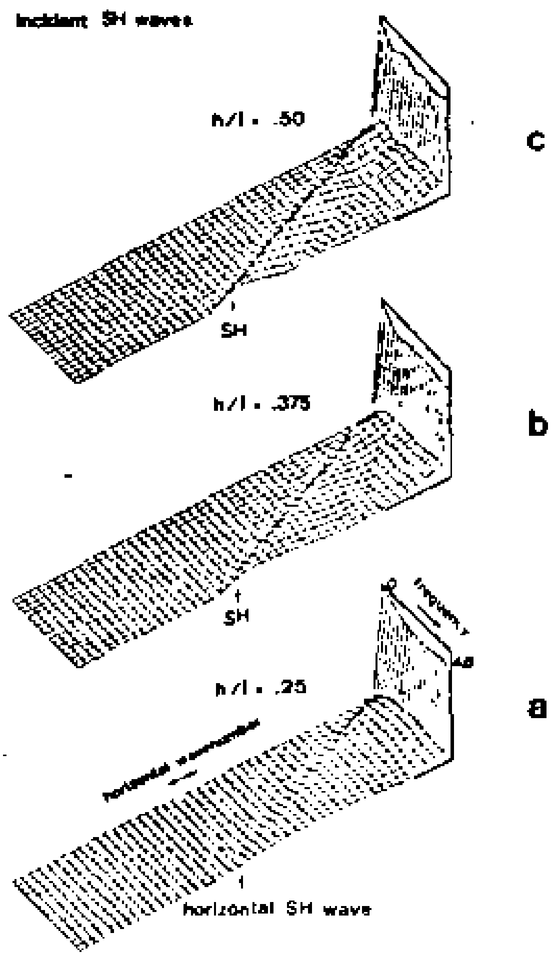

Figure 6. Variations of the scattered wavefield characteristics with the shape ratio in the *SH* case. The diagrams represent the amplitude of the scattered waves, computed at a $z = 0$ altitude, as a function of the horizontal wavenumber and of the frequency. These amplitudes are normalized to that of the incident wave.

displacement field for the canyon case. They are due to the Rayleigh ansatz error, which should be greatly reduced in more realistic models, where the presence of other interfaces would allow the presence of an upgoing diffracted field. However, in this paper, we have focused our interest on elevated topographic features, for which the Rayleigh ansatz error is rather small and localized.

The diagrams in Figs 5 and 6(b) illustrate the dependence of the scattered elastic field (represented by the elastic potentials ϕ and ψ in the *P* and *SV* cases, and the displacement in the *SH* case, each being computed at a $z = 0$ altitude) on both horizontal wavenumber and frequency. This scattered field is clearly separated into a main vertically reflected wave, homogeneous waves corresponding to the rays obliquely reflected on mountain slopes, and the horizontally diffracted *SH*-, *P*- and Rayleigh-waves. Although it is very difficult to distinguish between a horizontal *SV*-wave and a Rayeligh wave because of their small velocity difference, our opinion is that only a very small amount of energy is diffracted as a pure horizontal *SV*-wave, since the ψ/ϕ amplitude ratio, which is about 1.5 for the slowest branch, fits very well the theoretical Rayleigh curve (1.47 for $\nu = 0.25$). The amplitudes of both the vertically reflected wave and the oblique ones exhibit a quite strong dependence on frequency, but, in each case, minima of the vertical waves correspond to maxima of the homogeneous waves. Such a pattern suggests that the amount of energy propagating downwards remains roughly constant whatever the frequency. As to the waves diffracted along the surface, their amplitude does not depend strongly on the frequency, except for the Rayleigh wave in the *SV* incident case, which vanishes for dimensionless frequencies between 1.5 and 2.4. The reasons for this disappearance are not yet clear, but it explains the weakness of the Rayleigh wave on Fig. 2 in the *SV* incident case. Nevertheless, for other frequencies, the diffracted Rayleigh wave is larger for incident *SV*-waves than for incident *P*-waves, especially in the low-frequency domain. On another hand, the horizontal *P*-wave is almost non-existent in the *P* incident case.

As a partial conclusion, the topographic effects may be summarized as follows: the energy of the incident plane waves is first focused on convex parts of the surface topography where the displacement amplitude undergoes significant amplification over a rather broad frequency range. Although a large amount of this energy is reflected downwards as vertical and oblique homogeneous waves, the differential motion and strain induced by this local amplification make these convex parts radiate energy outwards in the form of surface waves (Rayleigh waves or surface *SH*- and *P*-waves). The fact that the mountain or valley slopes of the topographic feature investigated in this section, remain lower than 30°, and thus cannot reflect *SH* and *P* incident rays into horizontal waves, which we nevertheless observe, supports this idea. Furthermore, the large amplitude of the horizontal *P*-wave in the *SV* incident case may result from the superposition of a classical geometric reflection of incident *SV*-waves on mountain slopes, and of this radiation from mountain tops. Therefore, except on mountain tops and valley bottoms where no diffracted waves arrive in the case of a single topographic feature, and where the displacement amplitude is thus mainly controlled by focusing or defocusing, the amplitude spectra are strongly governed by the interference between the direct and diffracted waves.

2.3 EFFECT OF MOUNTAIN HEIGHT

We consider in this section the case of plane *SH*-waves vertically incident upon two mountains having respective shape ratios 0.25 and 0.5, and the case of *P*- and *SV*-waves incident upon this last mountain, and we compare them with the results obtained in the previous section ($h/l = 0.375$). The corresponding horizontal wavenumber frequency diagrams are shown in Fig. 6 for the *SH* case, and Fig. 7 for the *P* and *SV* cases (see Fig. 5 for comparison).

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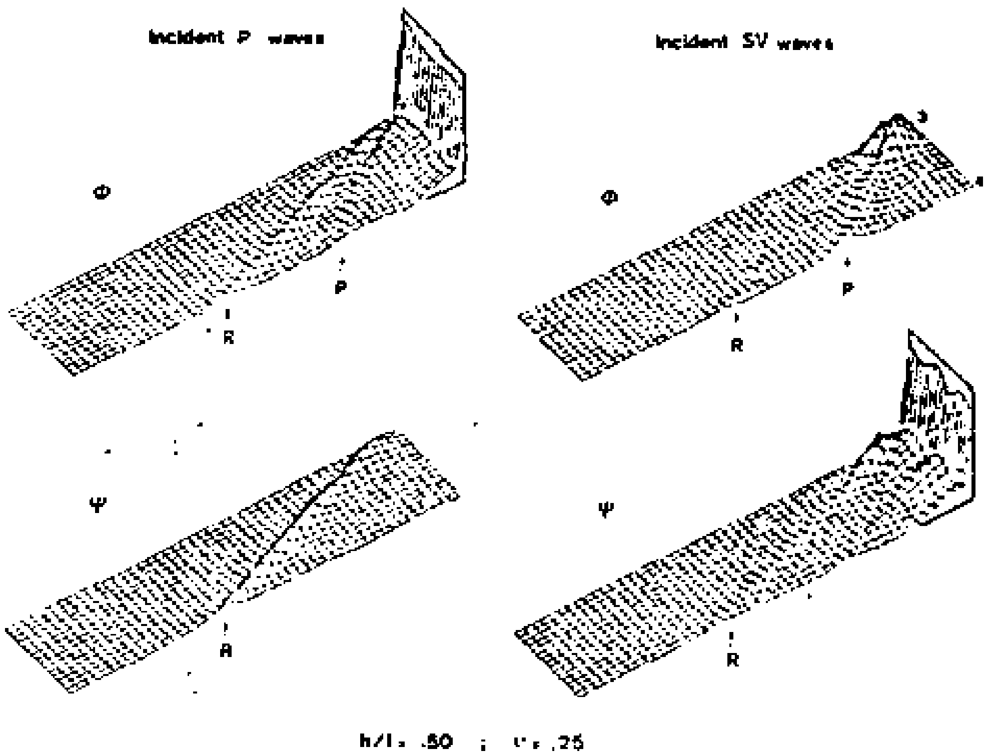

Figure 7. The same as Fig. 5, but for a higher elevation (h/l = 0.50).

2.3.1 *Diffracted wavefield*

The coupled frequency dependence of vertically and obliquely reflected waves mentioned above appears to be controlled by the mountain height: the frequency spacing of adjacent lobes gets narrower as the height increases. This phenomenon is interpreted as an interference scheme between vertical or subvertical waves reflected on the mountain top, and those reflected on each flat side of the topography. Such a hypothesis is supported by the measurement of the frequency spacing, which corresponds to a travel-time difference around $2\,h/c$, where c is the adequate wave velocity (β for incident S-waves, α for incident P-waves). This interference pattern has only little influence on the surface displacement, and has most effect on the deep underground motion below the topographic feature.

The horizontal diffracted SH-wave clearly strengthens as mountain height increases. Its amplitude is about 20–30 per cent larger for the highest elevation than for the medium-size one. This is only slightly lower than would result from a linear increase. Moreover, this ratio shows a trend to increase with frequency.

The horizontal P-wave scattered by incident P-waves also builds up as the shape ratio increases, but the Rayleigh wave still remains the most important feature. On the contrary, this regular strengthening is not observed in the SV incident case. The highest mountain scatters a greater amount of horizontal P-waves (up to 30 per cent more) only in the low-frequency domain, and the phenomenon is reversed for higher frequencies ($\eta > 3$).

The diffracted Rayleigh wave exhibits a behaviour very similar to that of the horizontal P-wave. In the incident P case, the Rayleigh wave shows a significant magnification (between 15 and 45 per cent) for the highest mountain and this magnification slightly increases with the frequency. On the other hand, for incident SV waves, the magnification (around 30 per

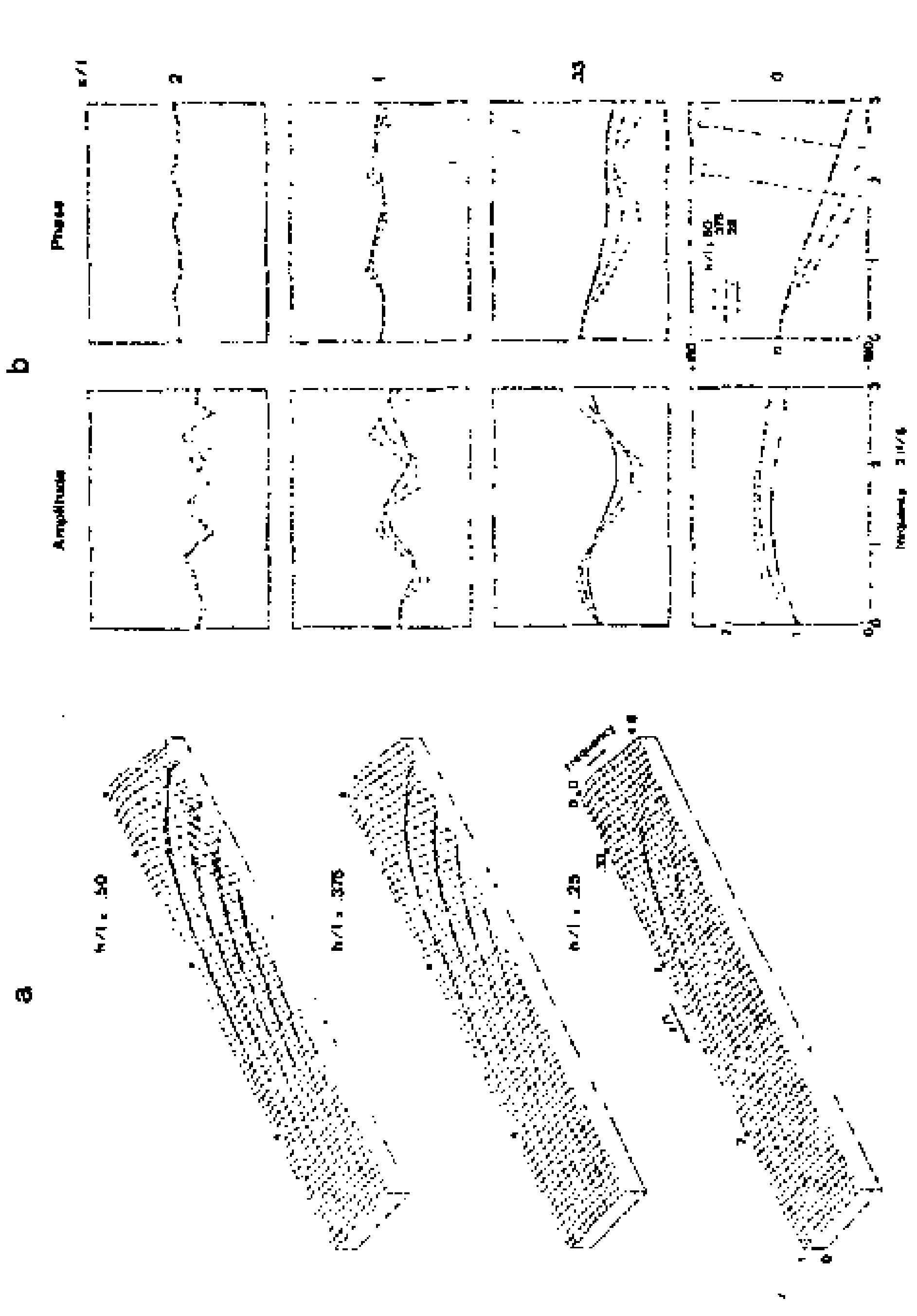

Figure 8. Variations of the displacement field with the shape ratio in the *SH* case. (a) Ground motion amplitude as a function of site location and frequency. (b) Detailed amplitude and phase spectra at four representative sites, the location of which is indicated by small dots on left diagrams.

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2.4 SOME ASPECT...

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cent) appears only at low frequencies ($\eta > 1.5$), and the trend is reversed for $\eta > 2.5$: higher elevations scatter less Rayleigh waves. The vanishing of the Rayleigh wave for dimensionless frequencies around 2 may be observed for both shape ratio values. This phenomenon therefore does not appear to be connected with the mountain height.

The regular strengthening, especially at high frequencies, of the wave scattering in both *SH* and *P* cases thus contrasts with its high-frequency weakening for incident *SV*-waves. Since this weakening does not occur at low frequencies, the reversal may be due, in our opinion, to the building up of horizontal interference phenomena within the mountain as both frequency and mountain height increase. It thereby must appear earlier in the *SV* case because of the *S–P* reflections on mountain slopes.

2.3.2 *Displacement field*

The frequency dependence of the displacement field in the *SH* incident case is shown in Fig. 8(a) for the three mountain heights. These diagrams bear obvious ressemblance to one another, except for the oscillatory pattern, the amplitude of which is related to the amplitude of the diffracted *SH*-wave. The amplitude spectra at four particular sites located on mountain crest, slope, base and some distance apart, are detailed on Fig. 8(b). As noticed above, the general shape of these spectra depends only slightly on mountain height, the main effect of which is to enhance the amplification or deamplification patterns. For instance, the maximum amplification on the mountain top is 36 per cent for the lowest, 53 per cent for the medium height and 65 per cent for the highest, in each case for a dimensionless frequency around 1.6; on another hand, the maximum deamplification on the mountain base is, respectively, 20, 30 and 42 per cent for dimensionless frequencies about 0.6.

It is interesting as well to look at the displacement phase versus frequency curves (Fig. 8b). For instance, the phase difference between mountain top ($x/l = 0$) and slope ($x/l = 0.33$) increases significantly for dimensionless frequencies around 1.8, and this increase enlarges with increasing mountain height. Such a phenomenon appears in a particularly strong way in the time domain, as shown in Fig. 9: the largest differential motions are concentrated around the site located at $x/l = 0.33$, and, for the highest mountain, the displacements on each side of this point are almost completely out of phase. Assuming for instance a 1 km wide, 250 m high hill, and a shear velocity of 3 km s^{-1}, these differential motions will act over a length of about 120 m for frequencies of about 6 Hz, that is within the frequency band of interest in earthquake engineering. Such differential motions may therefore be very important from an engineering point of view, as they may induce large torsional strain on big man-made structures.

2.4 SOME ASPECTS OF THE DEPENDENCE ON THE INCIDENCE ANGLE

Many authors have already investigated in some detail the displacement field over topographic features impinged upon by oblique and grazing *SH* plane waves (Trifunac 1973; Wong & Trifunac 1974; Wong & Jennings 1975; Sills 1978; Sanchez-Sesma & Rosenblueth 1979; England *et al.* 1980). Furthermore, a similar thorough investigation in the *P* and *SV* cases would need to be very fine, because of the great sensitivity of *P–SV* coupling on the incidence angle, especially in the *SV* case. Our aim here is therefore not to perform an exhaustive study of such a dependence, but only to give some indications about the qualitative and quantitative changes which may occur in the diffracted wavefield and ground displacement with varying incidence angle. We have thus investigated the elastic field scattered by a medium-size mountain ($h/l = 0.375$) illuminated by plane *SH*-waves with 30° and 60° incidence angles, and by *P*- and *SV*-waves at the critical angle $\theta = 35.66°$. This

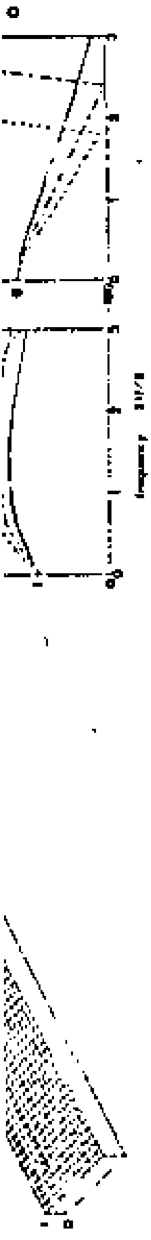

Figure 8. Variations of the displacement field with the shape ratio in the *SH* case. (a) Ground motion amplitude as a function of site location and frequency. (b) Detailed amplitude and phase spectra at four representative sites, the location of which is indicated by small dots on left diagrams.

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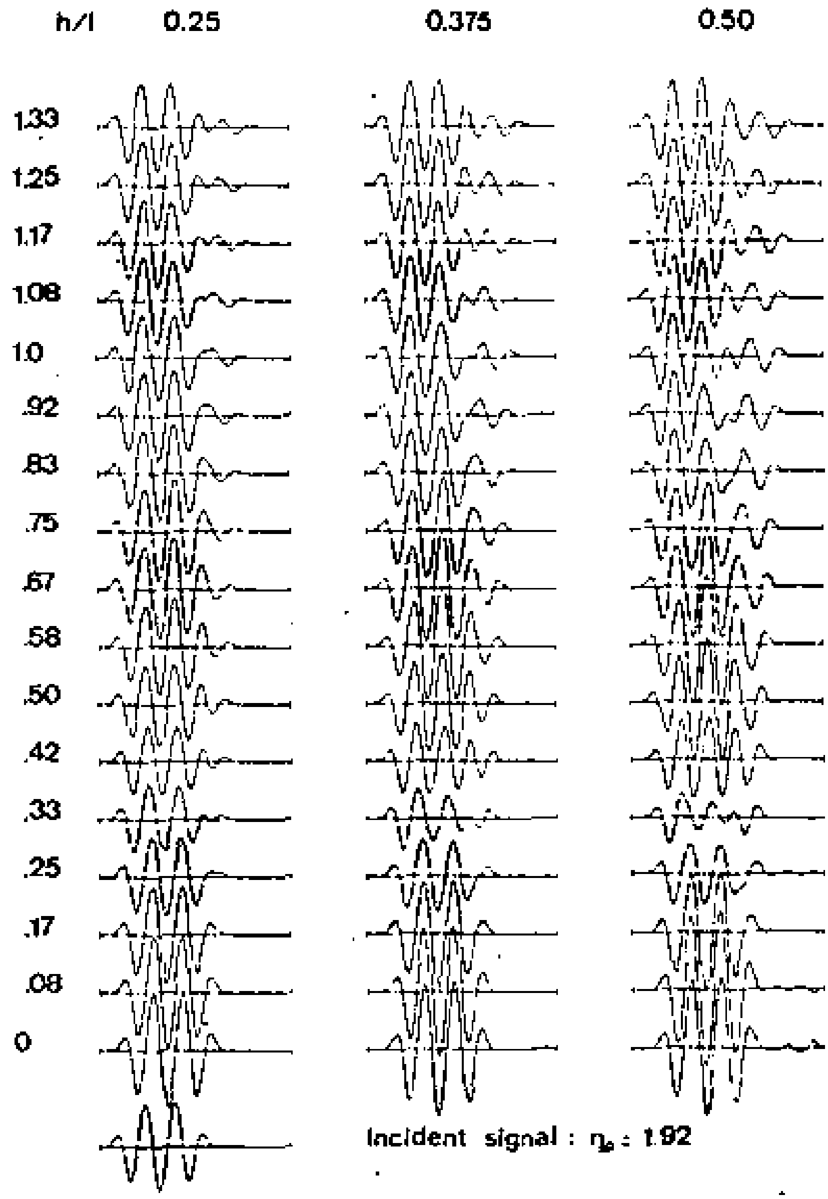
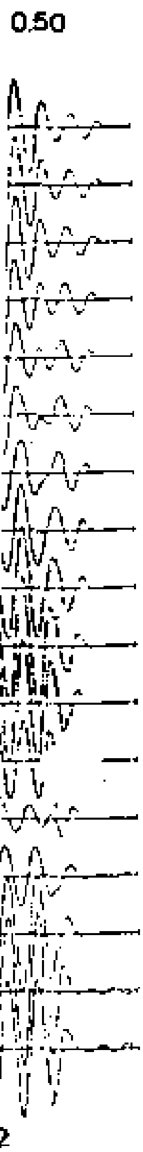


Figure 9. Dependence of ground motion on the shape ratio in the time domain. The incident signal is a combination of Ricker wavelets and is depicted by the bottom left trace. Numbers on the left are the dimensionless values of the horizontal distance between the recording site and mountain top.

particular value is chosen on the basis of our preliminary results (not reported here) which give evidence of a great enhancement of the forward scattered Rayleigh wave around this angle in the *SV* case, thus corroborating theoretical results obtained by Hudson & Knopoff (1967).

The most striking feature of the diffracted field is the directivity effect. In the *SH* case (Fig. 10), the forward scattered horizontal *SH*-wave exhibits an increase with respect to the vertical incidence case, which is about 100 per cent for $\theta = 30^\circ$ and reaches 350 per cent for $\theta = 60^\circ$. The *P* and *SV* cases (Figs 11 and 12) show the same qualitative behaviour for the



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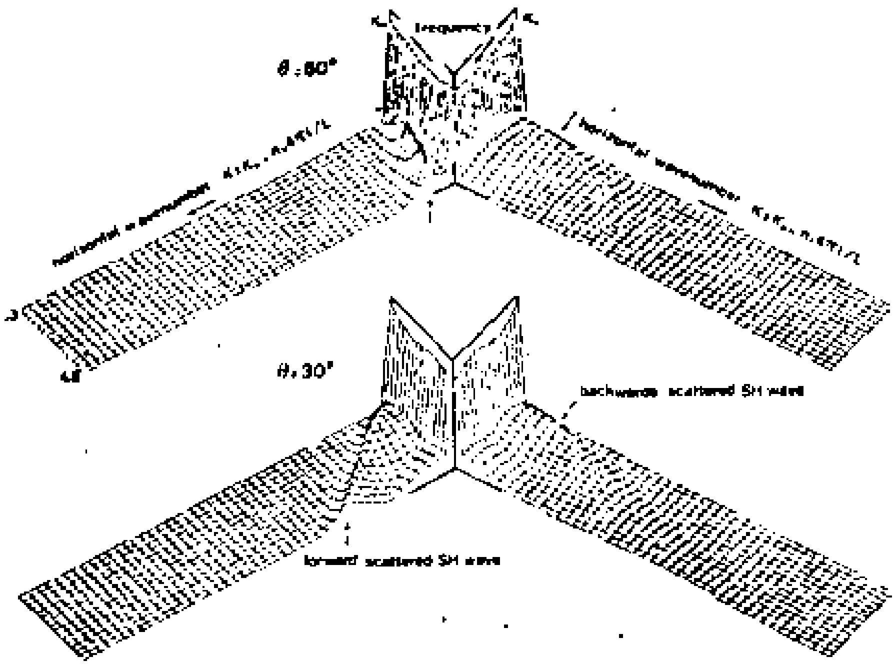


Figure 10. Variations of the scattered wavefield characteristics with the incidence angle in the *SH* case. These diagrams represent the amplitude of the scattered waves, computed at a $z = 0$ altitude, as a function of the horizontal wavenumber and of the frequency. The left part of the diagrams correspond to forward scattered waves, and the right one to the back scattered waves. K_0 is the horizontal wavenumber of the incident plane wave.

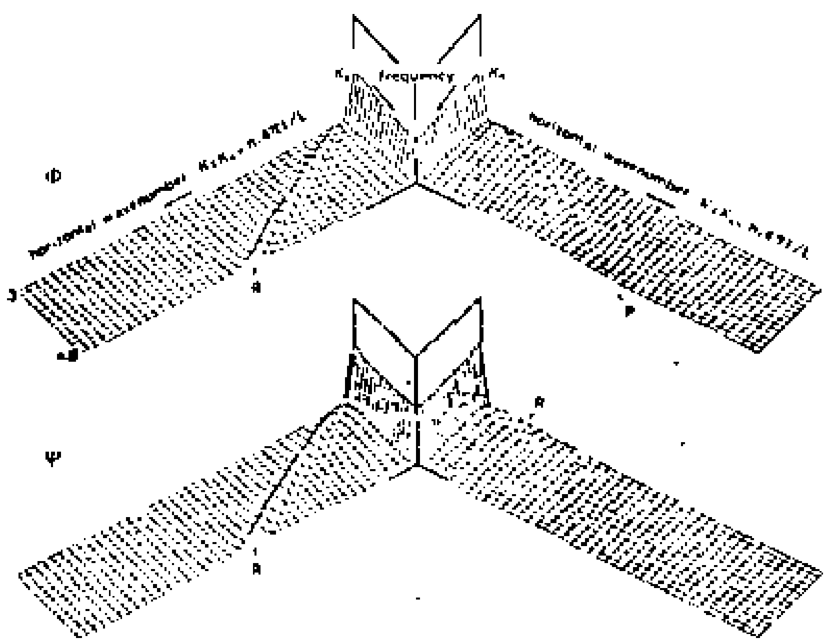
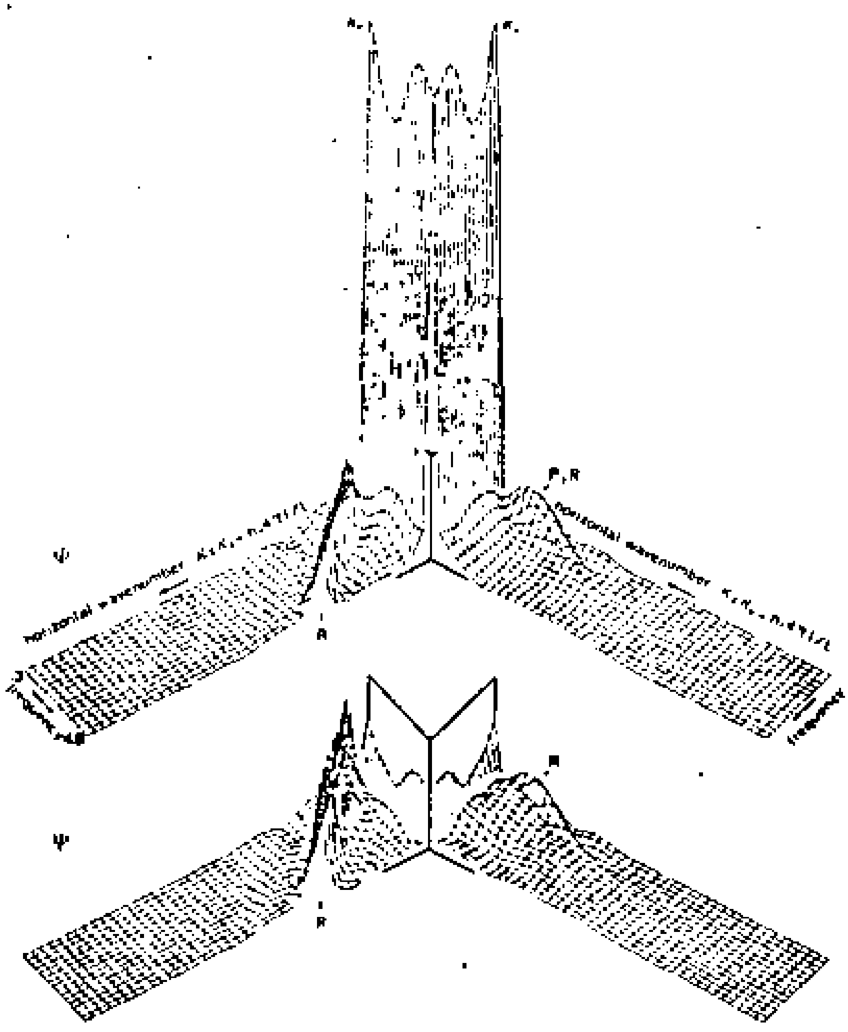


Figure 11. Elastic potential characteristics for obliquely incident *P*-waves. See Figs 5 and 10 for further explanations.

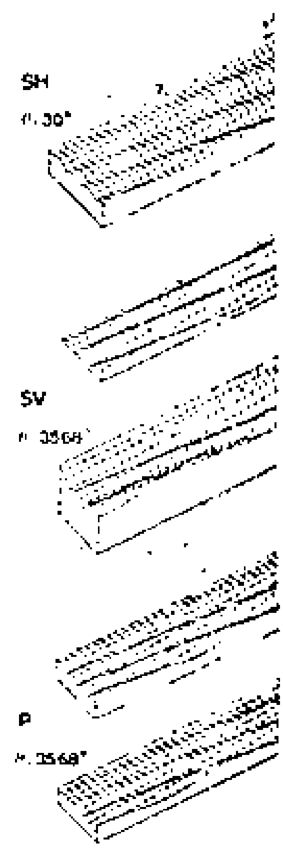


Incident SV waves : h/l = .375 ; ν = .25 ; θ = 35.68°

Figure 12. The same as Fig. 11, but for *SV*-waves incident at the critical angle. The scale is the same as in Fig. 11.

forward scattered Rayleigh wave; at that particular incidence angle, this enhancement is greatly emphasized in the *SV* case (Fig. 12), since it reaches one order of magnitude, while it ranges between 50 and 150 per cent in the *P* incident case. However, the ratio of the disturbance induced by the forward scattered Rayleigh wave to the surface displacement in the absence of the topographic feature is roughly the same in the two cases. Moreover, in each of the *SH*, *SV* and *P* cases, high frequencies seem to favour this enhancement of the forward scattered waves, and we may perhaps infer from the two *SH* diagrams (Fig. 10) that the frequency at which this reinforcement starts is greater for large incidence angles. Finally, for the particular *SV* case, the forward scattered Rayleigh wave no longer exhibits any vanishing or significant decrease for any frequency window. We do not observe any forward scattered *P*-wave in the *P* incident case.

For the back scattered field, we still observe a similar behaviour in each of the three cases, but quantitative values strongly differ from one another. The back scattered horizontal



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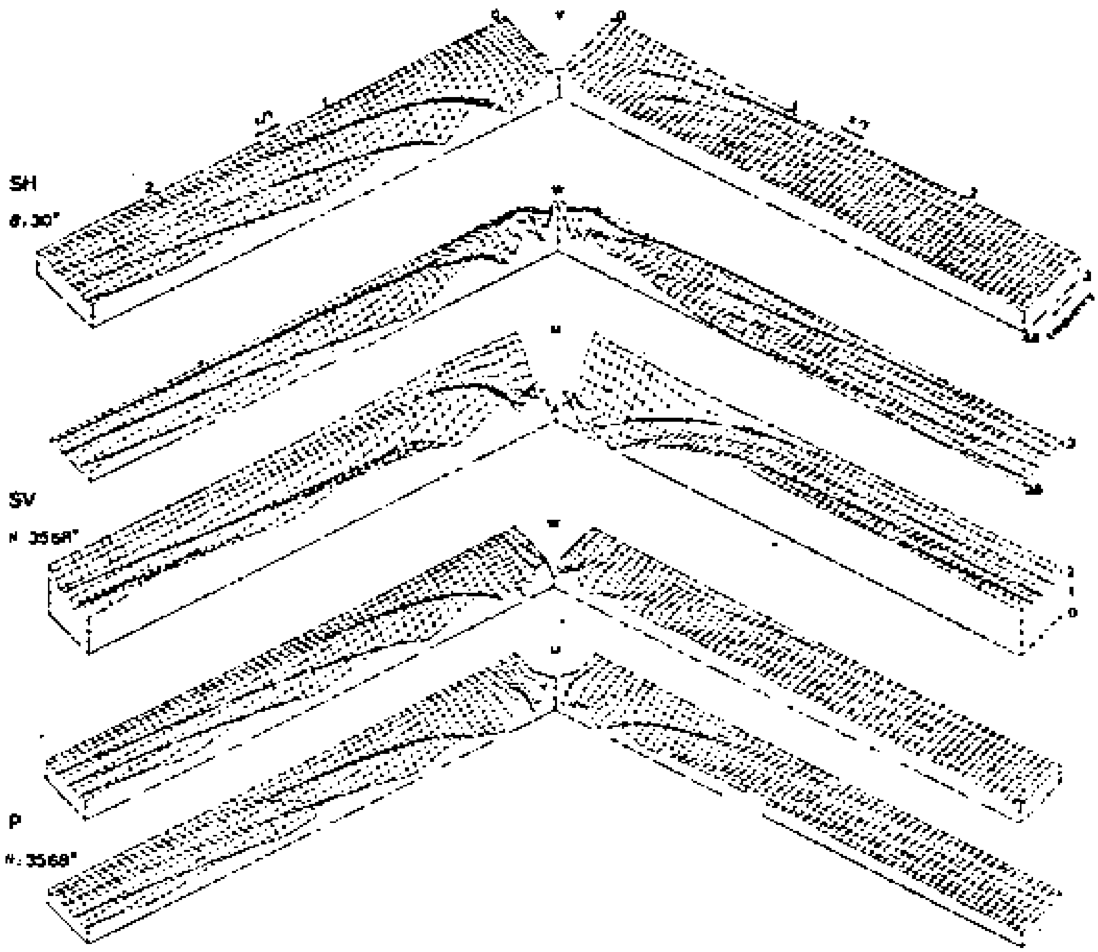

Figure 13. Ground motion amplitude as a function of frequency and site location, in, respectively, oblique *SH* (top diagram, transverse component *v*), *SV* (second, vertical component *w* and third diagram, horizontal component *u*) and P cases (fourth, *w*, and fifth, *u*). The incident waves propagate in the positive *x*-direction, that is from right to left.

SH-, horizontal *P*- and Rayleigh-waves are essentially confined to the low-frequency domain, and this cut-off frequency seems to lessen as the incidence angle increases ($\eta = 3.0$ for $\theta = 30°$, and $\eta = 1.2$ for $\theta = 60°$). But, whereas in the *SH* and *P* cases the amplitude of the corresponding back scattered waves is reduced a lot, except in the very low-frequency domain ($\eta < 0.6$) where it is only slightly lower than in the vertical incidence case, the amplitude of the back scattered waves in the *SV* case, for $\eta < 0.6$, is 300 per cent greater than in the vertical incidence case; the decrease with frequency is then very sharp, since the waves almost disappear for $\eta > 1.5$.

We may notice that these results are in good agreement with those of the small perturbation method: we have used the analytical formulation of Hudson (1967) to compute the amplitude of the Rayleigh wave scattered by the present mountain impinged by *P*- or *SV*-waves, and these calculations do predict both the strengthening of the forward scattered Rayleigh waves together with the high-frequency broadening of this scattering mechanism, and, concerning the back scattered Rayleigh wave, its confinement to lower frequencies together with its reduction in the *P* case and its amplification for *SV*-waves at critical angle.

The strengthening of the forward scattered Rayleigh wave in the P case is also consistent with experimental results of Rogers *et al.* (1974).

As for the surface displacement field (Fig. 13), we still observe that the most complicated case is the SV one. In both P and SH cases, the point of maximum amplification is slightly shifted away from the mountain top towards the 'far' side. The near side exhibits a main deamplification pattern, and the oscillation pattern on the far side is enhanced owing to the strengthening of the forward scattered waves. These features are conspicuously

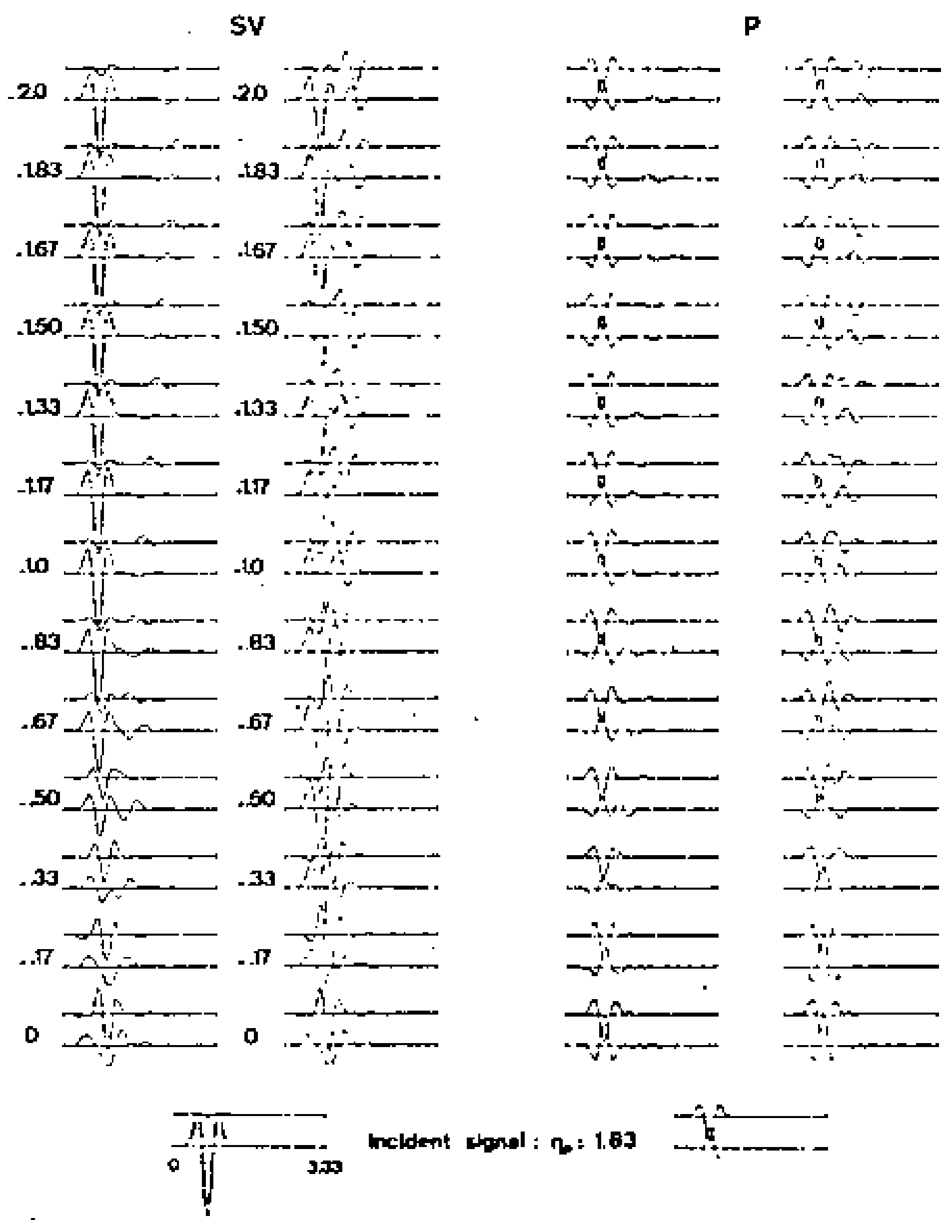

Figure 14. Time response of a medium-size mountain (h/l = 0.375) to obliquely incident (θ = 35.68°) SV and P Ricker wavelets of characteristic frequency 1.83. The bottom traces depict the ground motion for a plane free surface. Numbers on the left are the dimensionless values of the horizontal distance between the recording site and mountain top.

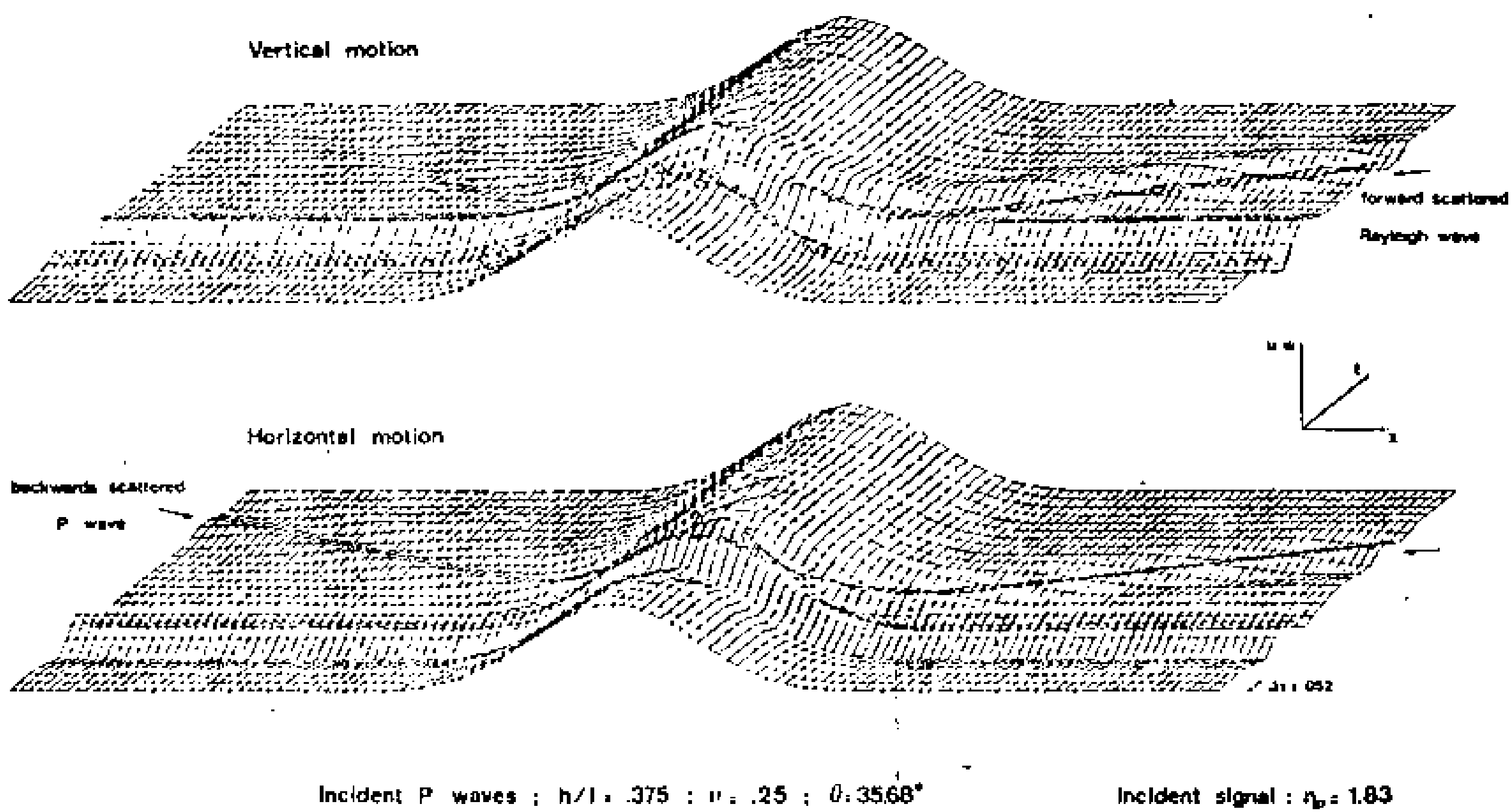

Figure 15. Spatial (x) and temporal (t) evolution of surface displacement components around a topographic structure impinged by oblique P-waves. The dots indicate the location of the sites where seismograms of Fig. 14 are computed.

emphasized in the SV case. The deamplification pattern on the near side is combined with the great sensitivity of the surface horizontal displacement on the incidence angle (that is with the local surface slope), and, at low frequencies, with the interference between the back scattered Rayleigh and P-waves and the primary wave. The mountain top here exhibits a low frequency strong deamplification pattern, and the point of maximum amplification is located very near the middle of the slope. The vertical displacement is maximum near mountain top, and slightly greater on the far side than on the near one.

Synthetic seismograms in Fig. 14 provide a good illustration of all these features. The directivity effect appears very clearly for both P and SV cases. For instance, the forward scattered Rayleigh wave is five times larger than the back scattered one in the SV case. For incident P-waves, this ratio is greater since the back scattered Rayleigh is very weak. On the contrary, Fig. 15 shows a very clear back scattered P-wave, in this case, while there is no forward scattered such wave. The maximum displacement values undergo important changes over the topography, for both vertical and horizontal components in the SV case, and mainly for the horizontal component in the P case. As an example, the deamplification on the near side reaches 80 per cent in the SV case for the horizontal motion. An interesting feature is the motion at the mountain top in the SV case where horizontal or vertical components appear to be completely decoupled: the horizontal motion is of rather low frequency while the vertical motion attains much higher frequencies, and the apparent arrival time of the SV-wave is different on the two components (about 0.4 dimensionless time later on the vertical one).

2.5 ASPECTS OF THE INFLUENCE OF THE POISSON RATIO

The vast majority of Earth components have a Poisson ratio very near 0.25. Nevertheless, this value may be significantly greater for some surface materials, such as poorly consoli-

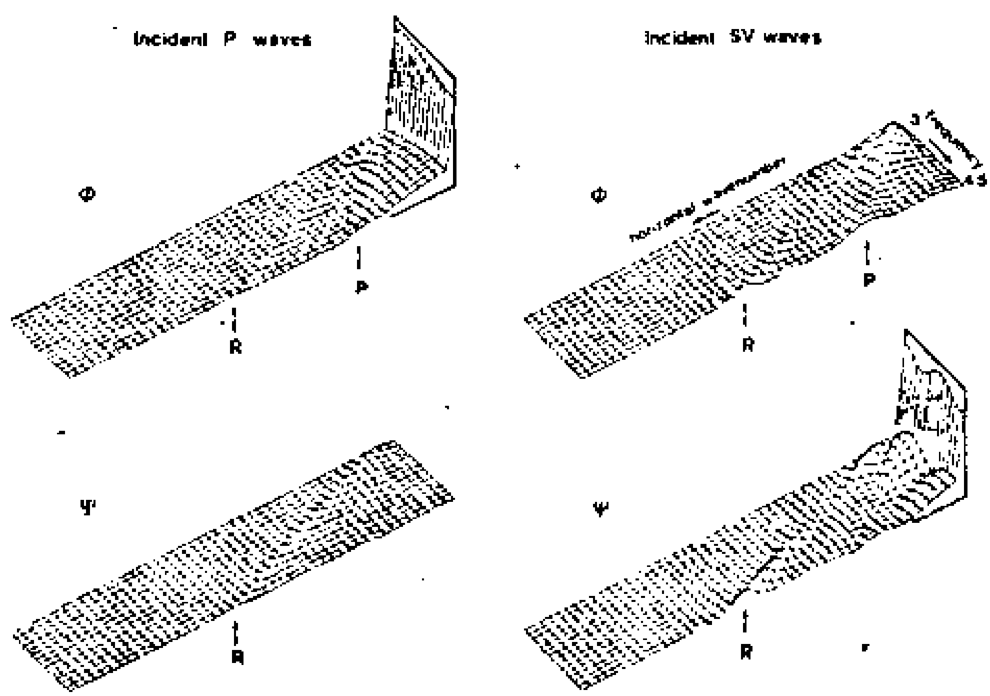

Figure 16. The same as Fig. 5, for a mountain of same shape but having a larger Poisson ratio, i.e. $\nu = 0.40$.

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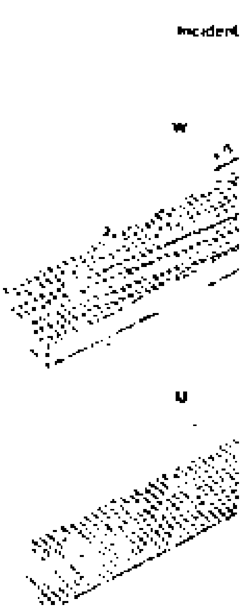

Figure 17. Displaceme
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dated sediments or artificial fills. In order to get an idea of the upper limit of the changes in the diffraction and amplification/deamplification phenomena due to the Poisson ratio, we have investgated the case of a medium size mountain (h/l = 0.375) having a Poisson ratio of 0.40 (that is a shear to compressional velocity ratio of 0.41) impinged by vertical *P*- and *SV*-waves, for a dimensionless frequency up to 4.5.

As shown in Figs 16 and 17, these variations of the Poisson ratio induce significant variations on the scattered Rayleigh wave. For incident *P*-waves, its amplitude is much smaller for $\nu = 0.40$, especially at high frequencies: around one-half for $\eta < 2.5$, and one-third for $\eta > 3$. In the *SV* incident case, we observe a slight reduction too in the low frequency domain ($\eta < 1.5$), then the Rayleigh wave disappears, as in the $\nu = 0.25$ case, for dimensionless frequencies between 1.5 and 2.4, and, for higher frequencies, its amplitude is similar in both cases, and even a little greater for $\nu = 0.40$.

The horizontally diffracted *P*-wave is still very weak for incident *P*-waves, though slightly greater in the high Poisson ratio case. For incident *SV*-waves, its amplitude is comparable in both cases at dimensionless frequencies below 2.5, and it is then significantly lower (around one-half) in the high Poisson ratio case.

Also noticeable are a slight decrease in the amplitude of the homogeneous waves for low-frequency incident *P*-waves, and their opposite small increase for low-frequency incident *SV*-waves.

These variations in the diffracted field are obviously reverberating on the surface motion. At some distance from the topographic feature, the weakening of the main scattered waves (the *P*-wave for incident *SV*-waves, and the Rayleigh one in the *P* incident case), together with the slight increase in the amplitude of the other diffracted wave (respectively Rayleigh and *P*), somewhat lessen and disturb the oscillating pattern of the horizontal and vertical displacement spectra (Fig. 17).

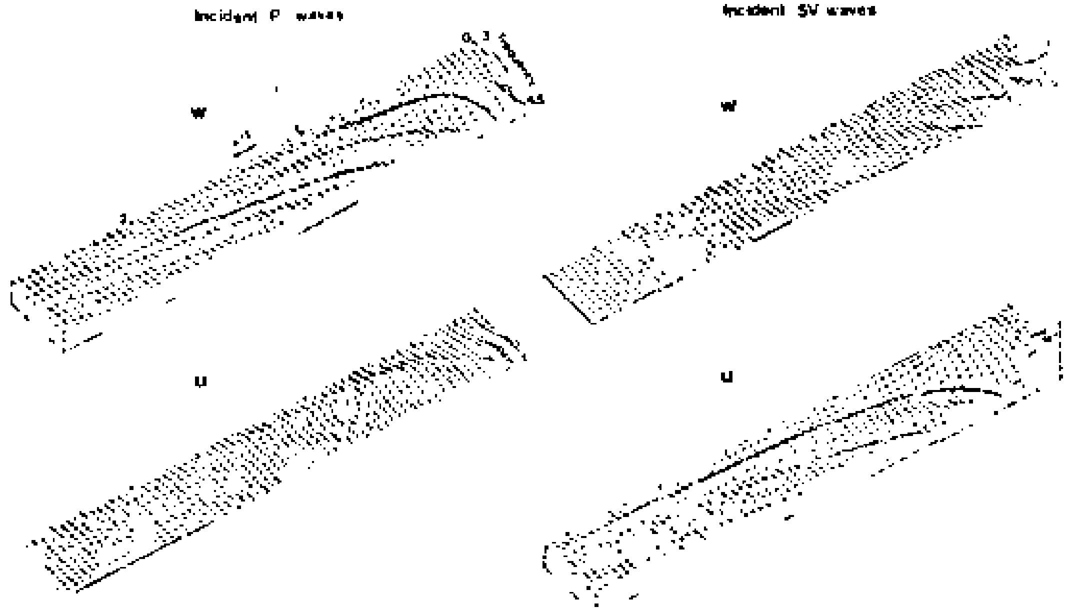

Figure 17. Displacement amplitude as a function of site location and frequency for the large Poisson ratio case ($\nu = 0.40$), and for incident *P*-waves (left) and *SV*-waves (right). The top diagrams represent the vertical component, and the bottom diagrams the horizontal component.

The displacement field over the anomaly itself also exhibits significant variations. On the mountain top, the overall shape of the spectrum remains roughly the same in both cases for both P and SV incident waves, but the high-frequency deamplification in the P incident case is much more important for $\nu = 0.40$ (up to 70 per cent for $\eta = 4.0$), and for incident SV-waves, the amplification is always greater when $\nu = 0.40$. These features may be explained by the changes in energy focusing, which is smaller in the former case and greater in the latter, because of the variations in the propagation direction of the reflected SV- and P-waves, respectively.

On mountain flanks, the spectrum shape of the main component of displacement (i.e. the vertical for incident P-waves, and vice versa), remains roughly similar, but differential motions are greater, for incident P-waves, for $\nu = 0.25$ and, in the SV incident case, for $\nu = 0.40$ (that is, the amplification is greater on the mountain top, and deamplification greater near the base). Nevertheless, these differential strains are in some sense balanced by the other component of motion, which is higher for $\nu = 0.40$ in the P incident cases and vice versa for incident SV-waves.

3 Topographies having a more complex geometric shape

The above reported results only deal with isolated topographic features, having a rather simple and smooth geometric shape. Unfortunately, such cases are met in nature only very occasionally, and topographic features most often either present a pronounced three-dimensional character, the investigation of which is beyond our present purpose, or, when roughly two-dimensional, consist of a complex alternation of mountains and valleys of quite different heights, slopes and widths. Of course, a theoretical study such as the present one cannot include every particular case that may occur in reality; notwithstanding, it is

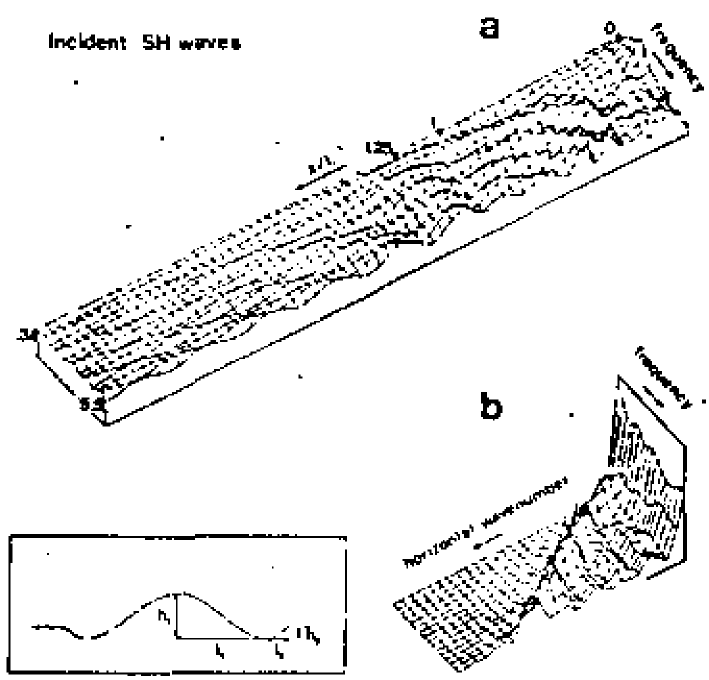

Figure 18. Displacement field and wavefield characteristics for the topographic structure illustrated in the bottom left box, and specified in the text. (a) Displacement amplitude as a function of site location and frequency. (b) Scattered waves amplitude as a function of horizontal wavenumber and frequency.

interesting to have some rough ideas about the modifications undergone by the scattered wavefield and the surface motion when the topographic structure has a somewhat higher degree of complexity than the above investigated mountain model.

Fig. 18(b) illustrates the *SH* scattered field of a cosine-shaped mountain of shape ratio $h_1/l_1 = 0.5$, flanked on each side by a small cosine-shaped depression of same shape ratio h_2/l_2. The mountain width is four times the depression width: $l_1/l_2 = 4$. Although this diagram is much more complicated than those shown on Fig. 6, its main characteristic remains the existence of a horizontally diffracted *SH*-wave. The great difference is, however, that its amplitude shows a near dependence on frequency, combined with a strong intensification with respect to the isolated mountain case (up to 3.5 times larger). Moreover, the frequencies at which these high amplitudes occur correspond to integer values of the mountain width over wavelength ratio $2l_1/\lambda$, thus indicating some kind of lateral resonance effect. This great amplitude horizontal *SH*-wave has obviously a strong influence on the displacement field (Fig. 18a), and induces strong differential strains on mountain slopes at much lower frequencies than in the isolated mountain case (Fig. 8a). Furthermore, the low-frequency amplification on the mountain top is greatly enhanced (up to 80 per cent spectral

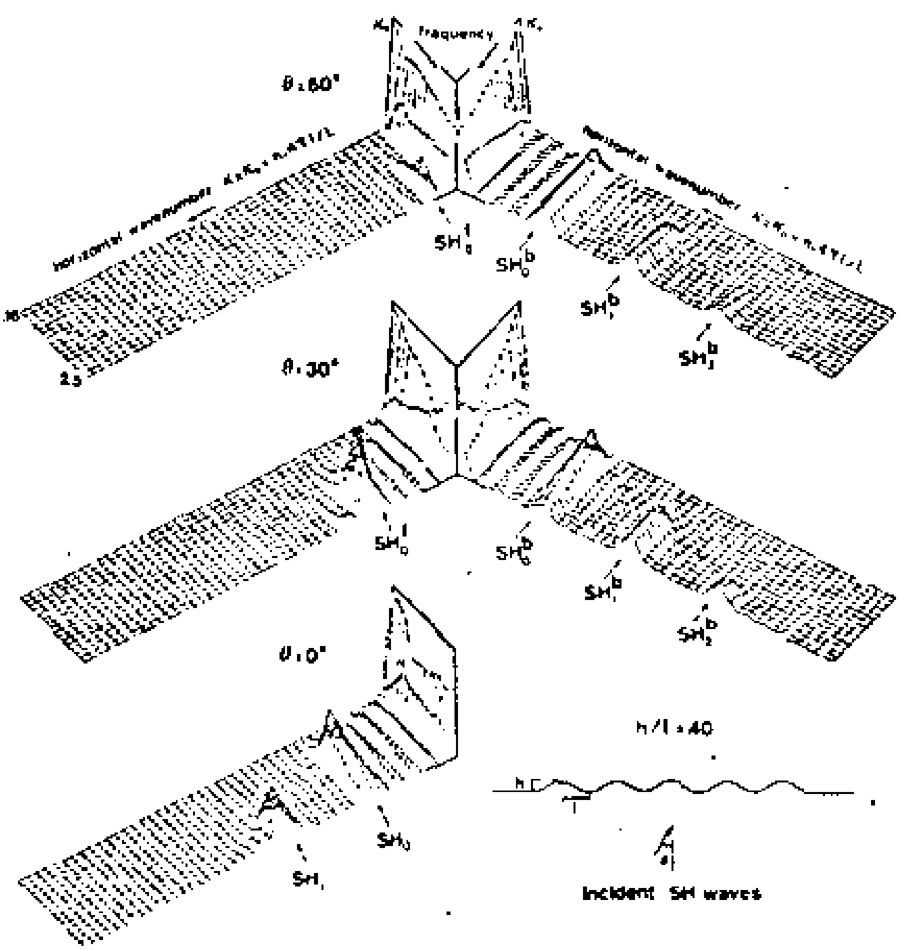

Figure 19. Scattered waves amplitude as a function of horizontal wavenumber and frequency, for the topographic structure illustrated on bottom right corner when impinged by *SH*-waves under various incidence angles. See Fig. 10 and text for further explanations and comments.

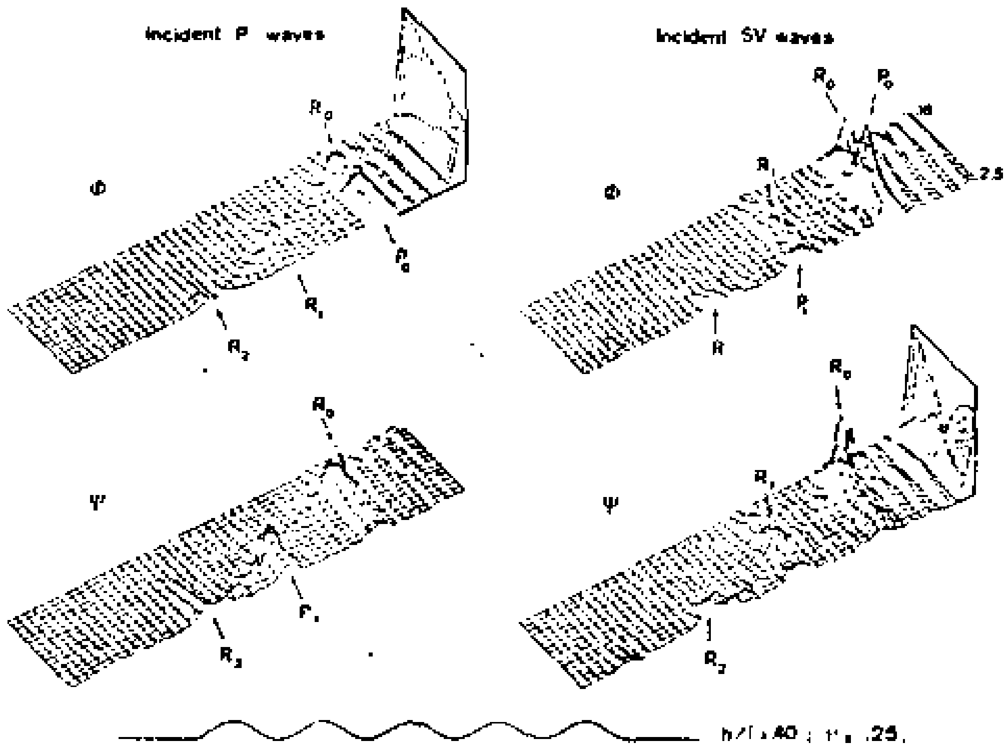

Figure 20. The same as Fig. 5, but for the topographic structure illustrated in Fig. 19.

amplification for $\eta = 2l_1/\lambda = 2$), probably because of a greater energy trapping in the mountain, due to the lateral small depressions.

The frequency-dependent strengthening of the diffracted wave has led us to investigate another topographic feature, involving some kind of periodicity in its geometric shape. Such a case is not unrealistic, since some mountain ranges, such as for instance the Appalachian mountains, exhibit a very clear sequence of two-dimensional ridges and valleys having similar widths and heights. We have chosen a series of five cosine-shaped mountains of the same shape ratio, $h/l = 0.4$.

Figs 19 and 20 present the elastic field scattered by such a topography for vertical and oblique *SH*-waves, and for vertically incident *P*- and *SV*-waves, respectively. The resonance effect mentioned above is now very clear in each case, and affects the three diffracted waves (*SH*, *P* and Rayleigh). The corresponding frequencies are $f_n = (n+1)\cdot c/2l$ (where c is respectively β, α and $c_R = 0.92\beta$) in the vertical incidence case. They are shifted for the *SH* oblique incidence cases, according to the relation:

$$f_n = (n+1)\,\frac{\beta}{2l}\,\frac{1}{1+\epsilon\sin\theta} \qquad \text{with} \qquad n = 0, 1, 2, \ldots$$

$\epsilon = -1$ for the forward scattered wave $SH_n^f(f_n^f)$,

$\epsilon = +1$ for the backward scattered wave $SH_n^b(f_n^b)$.

This equation may be derived in a straightforward way from a constructive interference pattern between waves diffracted at the different ridges. In the oblique incidence case, the backward scattered wave appears therefore at much lower frequencies than the forward scattered wave:

$$\frac{f_n^t}{f_n^b} = \frac{1+\sin\theta}{1-\sin\theta}.$$

Moreover, the diffracted wave strengthening at resonance frequencies is particularly strong, since their amplitude is up to 5 times greater than in the case illustrated in Figs 3 and 4.

The detailed examination of these diagrams displays some complementary features. First, in each case, the amplitude of the scattered field shows a peak around

$$k = 2\pi \cdot \frac{n+1}{2l} \cdot \frac{1}{1+\epsilon\sin\theta},$$

for frequencies higher than the corresponding resonance frequency f_n: the resonance thus also affects the obliquely scattered waves. However, the weakening of this peak with increasing frequency indicates that this effect is maximum on the horizontally diffracted waves.

Furthermore, the 'fundamental' resonance gives rise in each case to larger amplitude for the corresponding diffracted wave than the 'higher' modes. This feature is particularly enhanced for the Rayleigh wave in the SV case, where the second resonance is very weak.

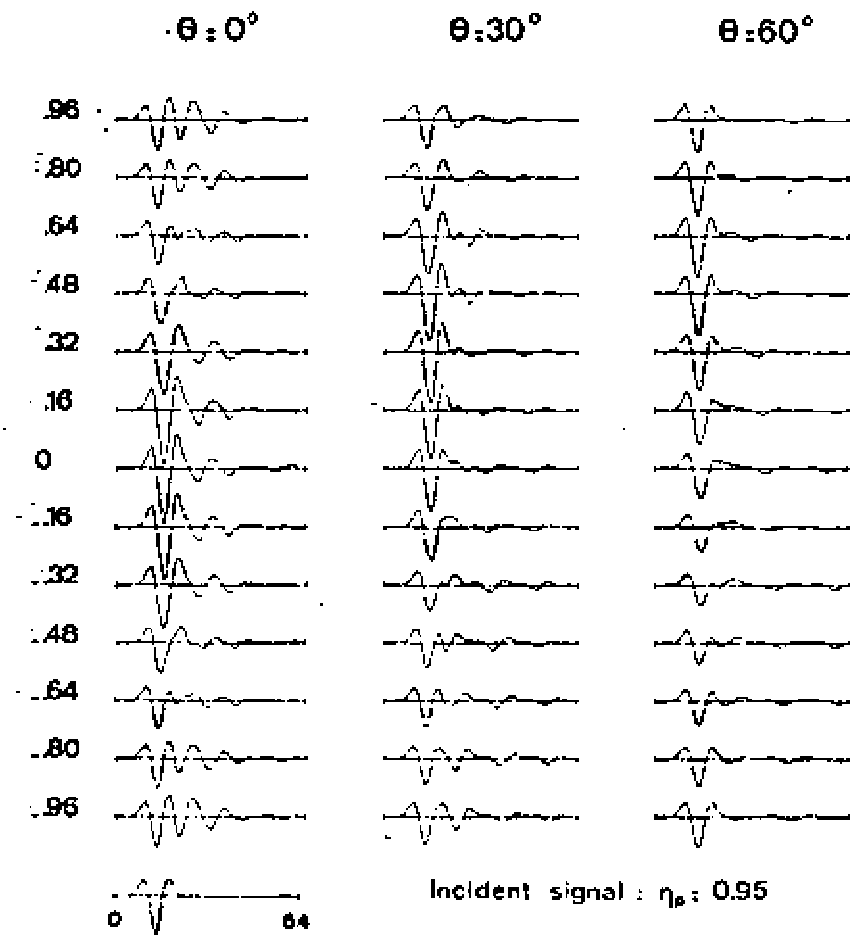

Figure 21. Response of the topographic structure illustrated in Fig. 19 to an SH Ricker wavelet of characteristic frequency 0.95, incident under various angles. The ground motion is shown only for the central hill because of the structure periodicity.

This may perhaps be related with the above mentioned vanishing of the diffracted Rayleigh wave for dimensionless frequencies between 1.5 and 2.5.

Finally, as to the dependence of the diffracted waves both on wave type and incidence angle, these diagrams confirm what might be inferred from previous sections. The *SV*-waves still possess the larger scattering power (at least for the fundamental mode), especially for the diffracted *P*-wave: its amplitude in the *SV* case is more than twice that in the *P* case,

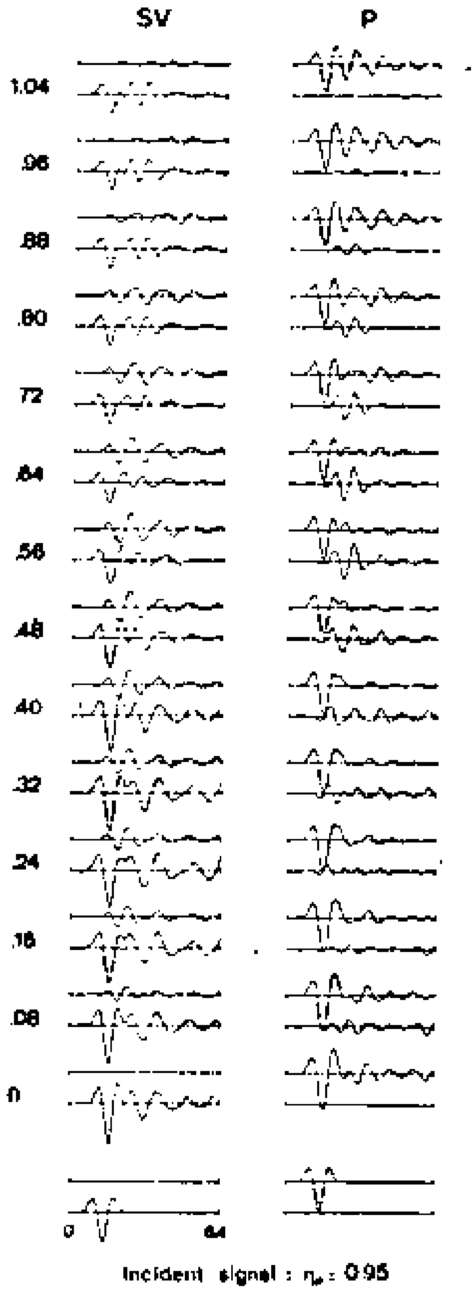

Figure 22. Response of the topographic structure illustrated in Fig. 19 to vertically incident *SV* and *P* Ricker wavelets of characteristic frequency 0.95. As in Fig. 21, the ground motion is computed only over the central hill.

while the fundamental Rayleigh wave is 50 per cent larger for incident *SV*-waves too. From another point of view, the amplitude of the backward scattered *SH*-wave diminishes as the incidence angle increases, and the contrary for the forward scattered wave. One may reasonably think that the diffracted waves follow a similar behaviour for obliquely incident *P*-waves.

The time response of such a topography is illustrated in Fig. 21 for the *SH* cases, and in Fig. 22 for the *P* and *SV* cases. The synthetic seismograms are shown only over the central hill because the topography periodicity induces a strong periodicity in the displacement field (the only difference is that motion lasts somewhat longer on the lateral hills than on the central one).

A common striking feature is the strong prolongation of the displacement duration. This property results from the arrival of the waves scattered at each hill. As seen in Fig. 21, this prolongation lessens as the incidence angle increases. Nevertheless, as the resonance frequencies are considerably shifted for obliquely incident waves, we may think that the duration may be longer for oblique waves in the case of high-frequency incident signals. However, for the particular incident signal investigated here, the main scattered waves in the oblique *SH* cases appear very clearly to be the back scattered ones.

The displacement amplitude variations over the topography are still much greater in the *S* cases than in the *P* case. In the vertical incidence case, the amplification always occurs on the mountain top (+43 per cent in the *SH* case, +33 per cent in the *SV* case, and +20 per cent in the *P* case), but the location of maximum deamplification differs according to the incident wave type: in the *SV* and *P* cases, these are valleys (−37 per cent and −8 per cent, respectively), whereas they are hill slopes in the *SH* cases (−25 per cent at mid-slope). In each case, the differential motion is the most important on mountain slopes, because of the rapid frequency and amplitude changes in the displacement field. This appears very clearly in the *SH* case.

In the case of oblique incident waves, the results follow what has been found for simple topographies. The near side undergoes increasing deamplification when the incidence angle increases, and so does the maximum amplification, shifted on the far side, with increasing incidence angle.

In conclusion, we think that the complexity of the topography has little effect on the primary wave amplitude, but it significantly prolongates and complicates the end of the signal because of the number and amplitude of scattered waves. This may be important for complex and long incident signals, for which scattered and direct waves may strongly interfere.

Such interferences may explain the considerable scatter in experimental data, such as those of Griffiths & Bollinger (1979), for which the surrounding topography (Appalachian Mountains) actually exhibits some kind of two-dimensional periodicity, as assumed here. Anyhow, our results fail to explain the large amplification values they generally observed for the crest/base or crest/slope amplitude ratios in the time domain.

4 Conclusion

The Aki-Larner technique has been used to perform, in both the time and frequency domains, a circumstantial analysis of the effects of two-dimensional topographies on ground motion. Incident plane *SH*-, *SV*- and *P*-waves, having wavelengths comparable with the horizontal and vertical sizes of the topographic feature and longer have been considered. The respective influences of the surface geometry, of the elastic parameters of the half-space and of the incident wave characteristics, have been investigated in some detail, to get a clear view of the physics of the topographic effects.

Essentially two such effects, both appearing at very long wavelengths (about four times the topographic width), show up for the topographic structures investigated in this paper: the first concerns the now classical amplification/deamplification pattern, related to the local surface curvature, and the second consists of a wave diffraction scheme on the convex parts of the topography.

The well-known amplification on mountain top is observed in each of the three *SH, SV* and *P* cases; its amount is more important for incident *S*-waves than for *P*-waves, which is consistent with the experimental observations of Davis & West (1973), and Griffiths & Bollinger (1979). Yet this amplification does not occur systematically, and it is shown to depend significantly on the characteristics of both the incident waves and the topographic structure. Although this dependence may vary from one case to another (and especially for incident *SV*-waves), the following features are generally observed: the amplitude spectrum exhibits a rather flat maximum for wavelengths comparable with, or slightly shorter than the mountain width; this amplification generally decreases with increasing incidence angle (and *SV*-waves incident at the critical angle give rise to a significant deamplification on mountain tops), it increases with mountain height, at least for the rather smooth topographies investigated here, and it also shows a significant but complex dependence on the Poisson ratio. This amplification scheme is obviously replaced by a deamplification one for concave topographies, but we did not investigate this latter case in detail.

The other effect of topographic irregularities is the diffraction of body and surface waves, which are generated on the convex parts of the topography and then propagate outwards along the ground surface. The type and the amplitude of these waves depend on the incident wave type. In the *SH* case, we observe a horizontally diffracted *SH*-wave; in the *P* case, we have mainly a Rayleigh wave, with a weak horizontal *P*-wave; in the *SV* case, both *P*- and Rayleigh-waves are present and their amplitude may be very large for some values of the frequency and/or the incidence angle. Incident *SV*-waves clearly possess the greatest scattering power, and seem to be associated with the most complicated diffraction scheme. Whereas intuitive reasoning concerning the behaviour of the diffracted waves (strengthening as mountain height increases, rather smooth frequency dependence, strengthening of the forward scattered waves and weakening of the back scattered as incidence angles increases) agree quite well with the computed results in both *SH* and *P* cases, they completely fail for incident *SV*-waves. This particular behaviour, together with the large amplitude of the scattered *P*-wave in the *SV* case, should, in our opinion, be related with the importance and the sensitivity of the *S–P* reflections, as previously underlined by Bouchon (1978) and Bard & Bouchon (1980b). These scattered waves control the displacement field over a wide area around the topographic irregularity, and may be of great importance in earthquake engineering studies. For instance, their lateral propagation along mountain slopes, as well as their interference with the primary wave, may result in strong differential strain, over distances and for frequencies in the range of man-made structures. For complex topographies (which is the general case), the large number of scattered waves may result in a significant prolongation of the ground motion.

In conclusion, it must be underlined that, in spite of the extreme simplicity of the geometrical models investigated here, the topographic effects exhibit a great complexity due to their dependence on the incident wave characteristics. From the results presented in this paper, it is thus very difficult to infer general rules concerning the displacement field over topographic irregularities. Moreover, real topographies may be affected by other phenomena than those treated in the present study: for instance, steeper topographies may give rise to a lateral resonance pattern making the topographic structure oscillate as a whole. In other respects, the two-dimensional assumption, which is only seldom met with in reality, may lead to an underestimation of the actual topographic effects, especially those affecting the primary wave amplitude. Finally, another important factor may be the presence of layering under the topographic structure, which, in our opinion, may enhance the amplitude of the scattered body *SH*- or *P*-waves by converting them into Love or Rayleigh waves. Such effects could perhaps explain the quantitative discrepancy which still exists between the numerous theoretical results and the experimental observations.

Acknowledgments

I am particularly grateful to Michel Bouchon for discussions and encouragement throughout the whole course of this work; he also provided the basic computer programs and critically read the manuscript.

This work was supported by the French Ministry of Industry and by the Institut National d'Astronomie et de Géophysque.

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DIVISION DE EDUCACION CONTINUA
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X CURSO INTERNACIONAL DE INGENIERIA SISMICA

ANALISIS DE RIESGO SISMICO

THE EFFECTS OF LOCAL IRREGULARITIES ON SEISMIC GROUND MOTION

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AGOSTO, 1984.

THE EFFECTS OF LOCAL IRREGULARITIES ON SEISMIC GROUND MOTION

By

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ABSTRACT

The effects of topographical and geological irregularities on seismic ground motion are discussed. A short description is given of some of the available techniques to calculate such effects. Some comments are made on the importance of local conditions in the assessment of seismic risk.

INTRODUCTION

It has long been recognized that local topography and geology can significantly affect the ground motion at a site (Figure 1). The contribution of topography could serve to explain the high acceleration recorded at the Pacoima Dam (1.25 g) during the San Fernando, California earthquake of February 9, 1971 (Trifunac and Hudson, 1971; Boore, 1972c). For the aftershocks of the same earthquake Davis and West (1973) in a series of observations have found significative local amplifications due to topographical relief. In a field

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study in the Appalachian Mountains using distant mine blasts as sources, average amplitude ratios between mountaintop and valley sites were determined (Griffiths and Bollinger, 1979). These average ratios showed that the seismic wave amplitudes at the crests were amplified by factors from 1.7 to 3.4. In Figure 2 three seismograms for the same event in the Powell Mountain area are shown. The positions of recording sites are also displayed in the figure.

The effect of soil conditions in ground motion has been observed in well-documented earthquakes (Sozen *et al.*, 1968; Jennings, 1971) and in regression analyses of strong motion data. There is significant evidence that subsurface topography, *i.e.* lateral heterogeneities, are related to localized damage distribution in the Skopje, Yugoslavia earthquake of July 26, 1963 (Poceski, 1969). It has been suggested that focusing of the wave energy, by irregular interfaces, generated large motion amplification in limited zones of the city (Jackson, 1971). Damage statistics of buried utility pipes in the Miyagiken-Oki, Japan earthquake of June 12, 1978 have shown spectacular increase in the number of occurred failures near the cut and fill boundary of a newly developed area (Kubo and Isoyama, 1980).

Local conditions can generate large amplifications and important spatial variations of seismic ground motion. These effects are of particular significance in the assessment of seismic risk, in studies of microzonation, in planning and in the seismic design of important facilities (Esteva, 1977; Ruiz, 1977). In particular, local irregularities can be relevant in calculating the seismic response of long structures (see Figure 3) like dams, bridges or life-line systems (*e.g.* Esquivel and Sánchez-Sesma, 1980; Ruiz and Esteva, 1981).

As pointed out by Trifunac (1980), the strong earthquake shaking of interest in earthquake engineering falls in the frequency range from about 0.1 Hz to about 20 Hz and since the seismic wave velocities near the earth's surface lie in the range from about 0.1 km/s to about 3 km/s, it can be seen that the corresponding wave lengths are from tens of meters to tens of kilometers. Thus, the topographical and geological irregularities of dimensions near to this range will have considerable influence on the corresponding waves. It follows that the extent and detail of local conditions required to study their effects should be considered in terms of the wave lengths associated with the periods of motion which are more important for a particular analysis. For a tall building, a dam or a bridge, for example, these local site dimensions might be of several kilometers. On the other hand, for stiff structures or small buildings, these dimensions can be from tens to hundreds of meters.

Although recent work has emphasized the physical understanding of local effects so that quantitative predictions can be made, as stated in a recent review by Boore (1983), there is still lack of criteria for dealing with the problem taking into account source, path and local conditions. Active research is needed to predict more accurately the local effects, given the source parameters. Indeed, it is encouraging the recent progress on strong motion prediction using mathematical moleling techniques (Aki, 1982). Much of the research is concentrated on the understanding of fault mechanics and wave propagation in the Earth. It is generally accepted that high frequency radiation, which controls accelerations, comes from very localized parts of the fault. A powerful asymptotic theory of high frequency radiation has been recently developed (Madariaga, 1983). Applications of the theory are coming. However, it should be noted that the foci of future earthquakes are not known;

their location, mechanism and amount of released energy can only be speculated in terms of regional seismicity models (Esteva, 1976). On the other hand, the knowledge of geological details is generally small to justify the use of very refined models of wave propagation, particularly for the high frequencies. It is then clear, in view of the mentioned uncertainties, that the problem of seismic risk assessment must be dealt with a probabilistic framework. A promising approach seems to be the use of integral measures of intensity such as the Arias' (1970) using stochastic descriptions of the input and simplified models of the local irregularities (Sánchez-Sesma *et al.*, 1983).

The aim of this work is to review the problem of calculating the effects of topographical and geological irregularities on ground motion given certain input, *i.e.* some kind of seismic waves. For this purpose the current formulation of the problem, the known analytical solutions and the available numerical methods are discussed in brief. This review is by no means complete and reflects the particular trends of the author. Nevertheless, it is hoped that this work could serve to stimulate discussion and interest on the problem.

FORMULATION OF THE PROBLEM

There is no doubt that the source mechanism governs the way in which the released seismic energy is radiated in space and time. However, seismic waves, once emitted by the source, are dependent on the mechanical properties of earth materials and the heterogeneities encountered in their path. This is also true dealing with irregular local conditions. Moderate changes in mechanical impedances or irregularities with size comparable to incident wave lengths can generate significant amplifications and spatial variations of ground motion (Boore, 1972b).

Plane waves are reflected back and refracted forward as they arrive at a plane interface. The amounts of reflected and transmitted energy depend on the mechanical properties of the media involved. Reflection and refraction in elastic wave propagation can well be described by geometrical means. Let us call diffraction to every change in the waves' path that can not be described as reflection or refraction. For studying diffraction of elastic waves it is necessary to solve a boundary value problem for the governing equations of linear elasticity (e.g. Achenbach, 1973; Aki and Richards, 1980).

To fix ideas, consider an elastic, homogeneous and isotropic half-space with an irregular surface. Under incidence of elastic waves the irregularity will diffract the incident waves (diffraction is frequently called scattering). Diffracted waves must satisfy, toghether with incident waves, the governing equations (Navier equations) and the boundary conditions (traction-free surface). Moreover, the diffracted fields must satisfy the Sommerfeld (1949) radiation condition at infinity, which means that the diffracted fields must scatter to infinity; *i.e.*, no energy may be radiated from infinity into the irregular region. The Sommerfeld radiation condition has been extended to elastic wave fields by Kupradze (1965).

ANALYTICAL SOLUTIONS

The simplest problems in elastic wave diffraction are the two-dimensional SH-wave problems because they can be analyzed separately from other body waves. The governing equation for this case is the scalar wave equation. Then, analytical solutions can be obtained for geometries of the scatterer which allow separation of variables (Mow and Pao, 1971). Using this method, exact solutions have been obtained for the diffraction of SH-waves by canyons and alluvial valleys with semi-circular (Trifunac, 1971, 1973) or semi-elliptical shapes (Wong and Trifunac, 1974a, b). Even with these simple models of local irregularities, complicated interference patterns were found and the obtained surface displacement fields vary strongly in space (see Figure 4). They are very sensitive to the incidence angle and the frequency. Results for alluvial valleys show the importance of the two-dimensional behaviour which gives much larger amplifications than those obtained from unidimensional calculations. These analytical solutions have shown the importance of the problem and they provide a check for numerical procedures.

For the more difficult cases of P- or SV-incident waves the orthogonal wave functions developed in classical physics are not separable for the half-space surface due to the coupling of boundary conditions. Lee (1978, 1982) overcome this difficulty for a semi-spherical canyon by expanding the spherical wave functions further into a power series which matched all the boundary conditions successfully. However, this approach is limited to small frequencies because the resulting matrix equations, which are infinite, can only be solved approximately for this case.

Under additional simplifying assumptions, other analytical solutions have been obtained using orthogonal wave functions. For an acoustic medium the exact expressions for the scattered fields generated by incidence of P-waves on canyons of semi-circular and semi-spherical shapes (Singh and Sabina, 1977) have been obtained. However, the results are of small utility because the acoustic assumption can hardly be met in real cases. The problem of vertically incident P-waves upon a semi-ellipsoidal three-dimensional scatterer has been exactly solved for an elastic medium in which horizontal displacements are restricted (Sánchez-Sesma, 1983b). Results for vertical displacement are in reasonable agreement with those from more reliable computations for a truly elastic medium (Sánchez-Sesma, 1983a).

Under the assumption of small-slope irregularities a perturbation solution has been obtained for the elastic scattered field by two-dimensional geometries (Gilbert and Knopoff, 1960). The approximation is based on replacing the irregularity by an equivalent stress distribution. An application of this method by Hudson (1967) deals with small-slope three-dimensional scatterers. With this approach reasonable estimates have been obtained of the scattered Rayleigh waves as compared with observations even in cases in which slope angles are as large as 25° or, say, 30° (Hudson and Boore, 1980).

The method of matched asymptotic expansions has no restrictions on the slope of the irregularity. It is based in matching the first terms of an outer expansion of the near field with those of an inner expansion of the far field

(Sabina and Willis, 1975, 1977). Although the method is limited to very small frequencies, results are in qualitative agreement with observations.

NUMERICAL METHODS

A powerful technique has been developed by Aki and Larner (1970) to treat scattering of SH-waves by irregular interfaces. In the Aki-Larner method, incidence is assumed of a plane single-frequency wave which causes a displacement field. This field is represented by superposition of plane waves of unknown complex amplitudes propagating in many directions. Inhomogeneous plane waves are allowed. The total motion is obtained from integration over horizontal wave number. Under the assumption of horizontal periodicity of the irregularity, the integral is replaced by an infinite sum. Truncation of this sum and application of the interface conditions of continuity of stress and displacement leads to a system of linear equations for the complex scattering coefficients. This method, which is restricted to small-slope irregularities, has been applied by Bouchon (1973) to study the effects of two-dimensional irregular topographies on ground motion for incidence of SH, SV and P waves. An extension of the method has been advanced by Bouchon and Aki (1977a, b) to represent with this discrete-wave-number technique near seismic source fields in a layered medium with irregular interfaces. Another extension of the method, now to time domain calculations, has been developed to study the seismic response of alluvial valleys (Bard and Bouchon, 1980a, b) under incidence of SH, P and SV waves. The Aki-Larner technique has been recently used by Bard (1982) to analyze the effects of two-dimensional elevated topography on ground motion. The comprehensive studies by Bard and Bouchon throw light on the subject and on the physics of the problem. The method has been used to model the fields generated by real faults (Bouchon, 1979; Campillo, 1983). It can be used to model ground motion considering together the effects of source, path and local conditions. However, in this case the numerical computations may become very expensive -if they can be performed- for many real cases.

The finite difference method is also a powerful tool in elastic wave propagation studies (Alterman and Karal, 1968; Boore, 1972a). It has been applied to modelate two-dimensional irregular interfaces (Boore *et al.*, 1971) and ridges (Boore, 1972b) in the SH case and also for incident of P and SV-waves upon a sedimentary basin (Harmsen and Harding, 1981) and a step-like topography (Boore *et al.*, 1981). Interesting results have been found concerning the significant generation of Rayleigh surface waves by lateral irregularities. For incidence of P waves upon a surface slot the computations by Ilan and Bond (1981) give good agreement with experiments. It was found that the amplitude of the scattered Rayleigh wave is, as expected, dependent on incidence angle. A finite difference analysis of axisymmetric topographical irregularities has been presented to study the effects of vertically incident shear waves (Zhenpeng *et al.*, 1980). Spectral ratios were obtained and comparison with observations gives reasonable agreement. The finite difference method is theoretically unlimited to model details and nonlinear behaviour of materials, but the size of the problem can easily exceed the capacity of major computing facilities.

The finite element method also allows a detailed description of site topography and layering. It is possible with this method to calculate the response of two-dimensional soil configurations with truly nonlinear stress-strain relations

(Streeter *et al.*, 1974; Joyner and Chen, 1975; Joyner, 1975). The major disadvantage of the method is its low-frequency limit and high cost. Usually, real time analysis must be shortened to avoid the reflections from the artificial boundaries. The use of different transmitting techniques can reduce the spurious waves to some extent (*e.g.* Smith, 1974, 1975; Ayala and Aranda, 1977; Clayton and Engquist, 1977; Castellani *et al.*, 1981; Liao and Wong, 1981). A successful criterion has been developed for damping out the unwanted reflections by means of non-uniform element size (Day, 1977). Finite elements have been used to treat problems of irregular layering (Lysmer and Drake, 1972; Drake, 1972; Aranda and Ayala, 1978) and two-dimensional topographical irregularities (Castellani *et al.*, 1982) under idealized conditions. Nevertheless, a realistic wave analysis is quite costly.

In recent years boundary methods have gained increasing popularity. This fact is mainly due to the availability of high speed computers. Boundary methods are well suited to deal with wave propagation problems because they avoid the introduction of fictitious boundaries and reduce by one the dimensionality of the problem. These facts yield numerical advantages. Moreover, boundary methods can be used together with the finite element method (Zienkiewics *et al.*, 1977). Then, the region modelated with finite elements can be smaller (*e.g.* Ayala and Gómez, 1979; Shah *et al.*, 1982).

There are two main approaches for the formulation of boundary methods; one is based on the use of boundary integral equations (Cruse and Rizzo, 1968a, b; Brebbia, 1978; Cole *et al.*, 1978; Alarcón *et al.*, 1979), and the other, on the use of complete systems of solutions (Herrera and Sabina, 1978; Herrera, 1980). The scattering of incident SH-waves from two-dimensional irregular topographies has been formulated with integral equations by Wong and Jennings (1975) for arbitrarily shaped canyon-like profiles and by Sills (1978) for ridges and mixed shapes. This method has been applied with success to calculate the effects of a dipping layer of alluvium of an SH-wave source on the surface (Wong *et al.*, 1977). Results compare favorably with observations during a full-scale low-amplitude propagation test. A powerful approach which combines the boundary integral equation method with finite differences in time has been presented (Cole *et al.*, 1978) for solving elastodynamic problems. The performance of the method was found to be good in a simple numerical problem. A boundary method has been recently developed and applied to solve two-dimensional scattering of harmonic elastic waves by canyons (Sánchez-Sesma, 1978, 1981; Sabina *et al.*, 1979; Sánchez-Sesma and Rosenblueth, 1979; Wong, 1979, 1982; England *et al.*, 1980; Sánchez-Sesma *et al.*, 1982a), alluvial deposits (Sánchez-Sesma and Esquivel, 1979; Dravinski, 1982a, b, 1983) and ridges (Sánchez-Sesma and Esquivel, 1980; Sánchez-Sesma *et al.*, 1982b) for different types of waves and shapes of the scatterers. The method consists of constructing the scattered fields with linear combinations of members of a c-complete family of wave functions (Herrera and Sabina, 1978). These families of functions, which are solutions of the governing equations of the problem, can be constructed in a very general way, with single or multipolar sources having their singularities outside the region of interest. Coefficients of the linear forms thus constructed are obtained from a least-squares matching of boundary conditions. As pointed out by Wong (1982), the method can be considered as a generalized inverse one. In doing this, Wong suggested a procedure which improves the solution numerically. A general framework for the method is given by a recent algebraic theory of boundary value problems (Herrera, 1979, 1980 a, b).

This approach has recently been extended to three-dimensional problems (Sánchez-Sesma, 1983a). The case of incident elastic waves upon axisymmetric irregularities on the surface of an elastic half-space was formulated using an azimuthal decomposition. The diffracted fields were constructed with multipolar solutions of the reduced Navier equations in spherical coordinates (Takeuchi and Saito, 1972; Aki and Richards, 1980). For a semi-spherical alluvial deposit and vertical incidence of P-waves a very large amplification was found, as compared with the flat layer problem.

Using also multipolar expansions the scattering of harmonic SH-waves by arbitrarily shaped alluvial basins has been solved (Ize *et al.*, 1981). Within the theoretical framework, a least-squares numerical scheme (related to Courant's) is used and it is found to reduce the order of the systems of equations to be solved for the same given accuracy.

CONCLUDING REMARKS

The influence of topographical and geological irregularities on seismic ground motion has been briefly discussed and a short description of some of the available methods to deal with such effects was made.

There is no doubt that local conditions play an important role in the spatial variation of ground shaking and should be explicitly considered in the design of some important facilities, as well as for microzoning. But local conditions are not alone; the source mechanism and the paths of seismic waves give also their part in the assessment of seismic risk. Thus, a more complete description of the problem is needed.

Most of the models of local conditions require the definition of the types, incidence angles and time variation or frequency content of the incoming waves. However, there are at present no general criteria for selecting on a physical basis the characteristics of the input for many practical situations. Such criteria, hopefully, will come from the research on the source high frequency radiation.

ACKNOWLEDGEMENTS

Thanks are given to A. Arias and L. Esteva for the critical reading of the manuscript. The assistance of J. Avilés, S. Chávez-Pérez and G.M. Cortés-Rubio is greatly appreciated.

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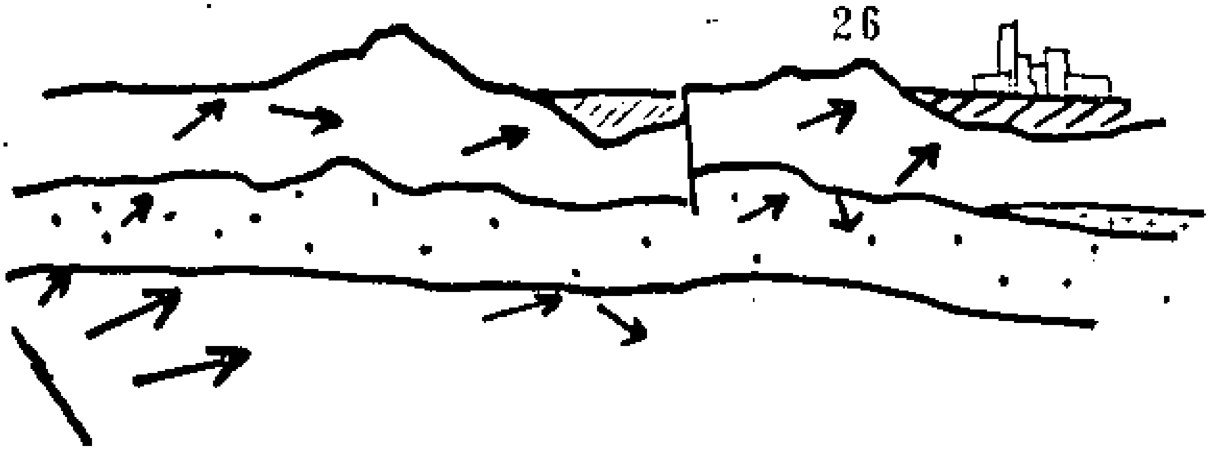

Fig. 1 Local topography and geology

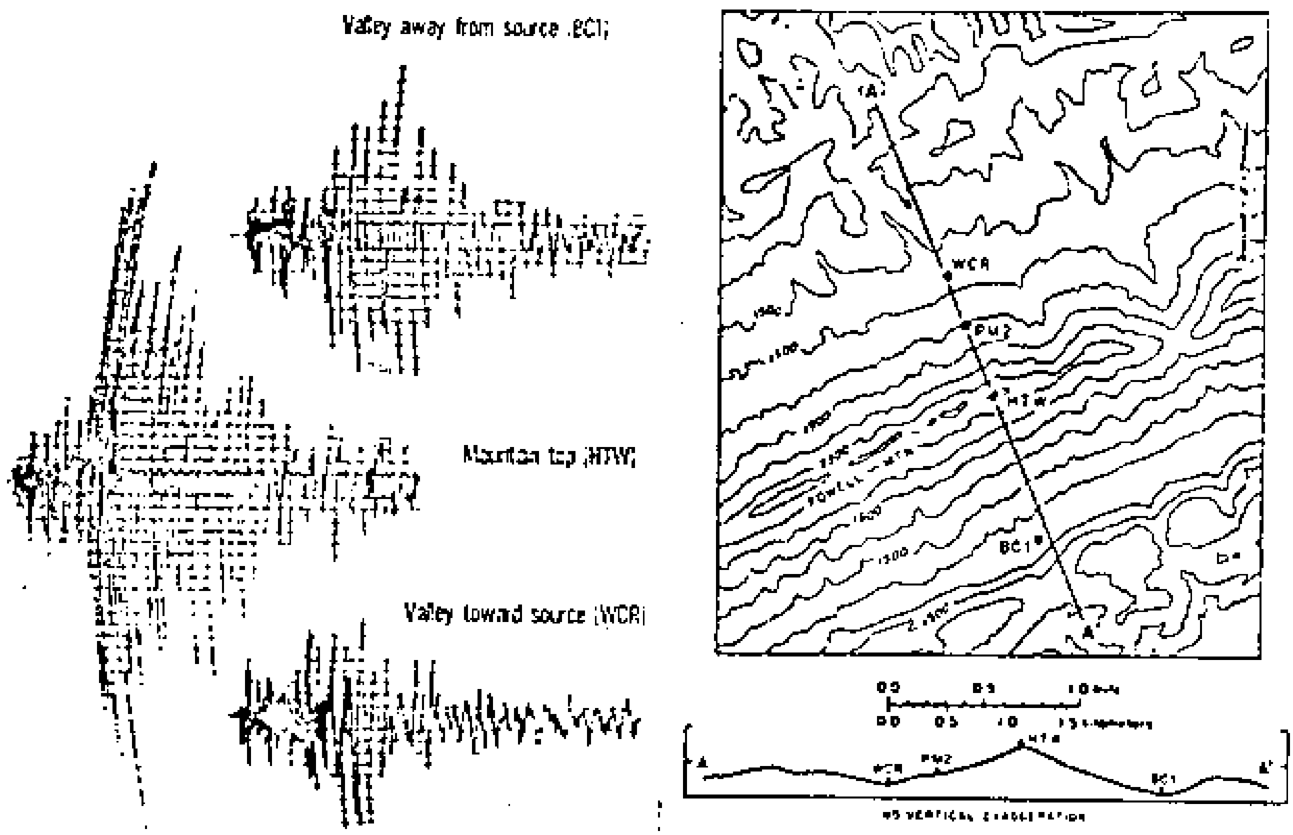

Fig. 2 Seismograms at three stations at the Powell Mountain area and location of the recording sites. Epicentral distance was about 30 km. Topographic contour interval is 200 ft (61 m) (After Griffiths and Bollinger, 1979)

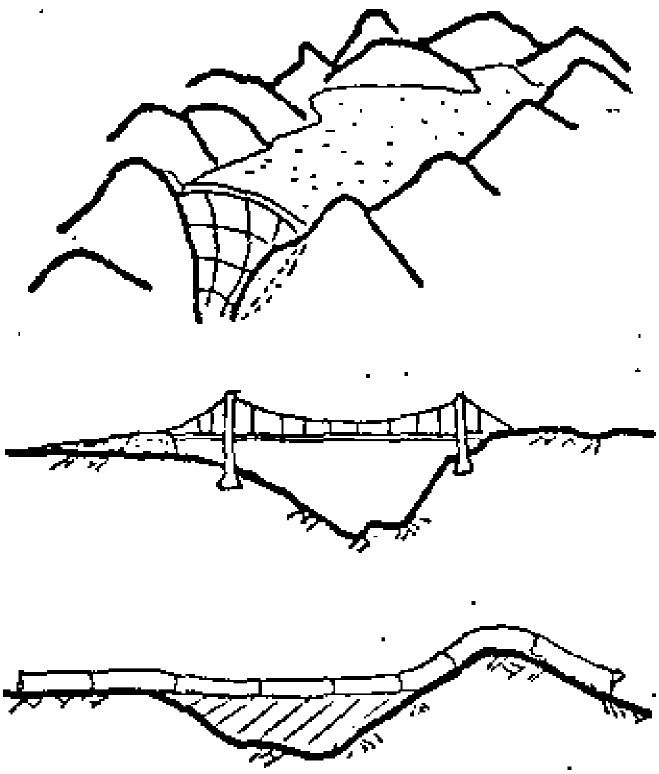

Fig. 3 Long structures at irregular sites: a) dam; b) bridge; and c) a life-line system

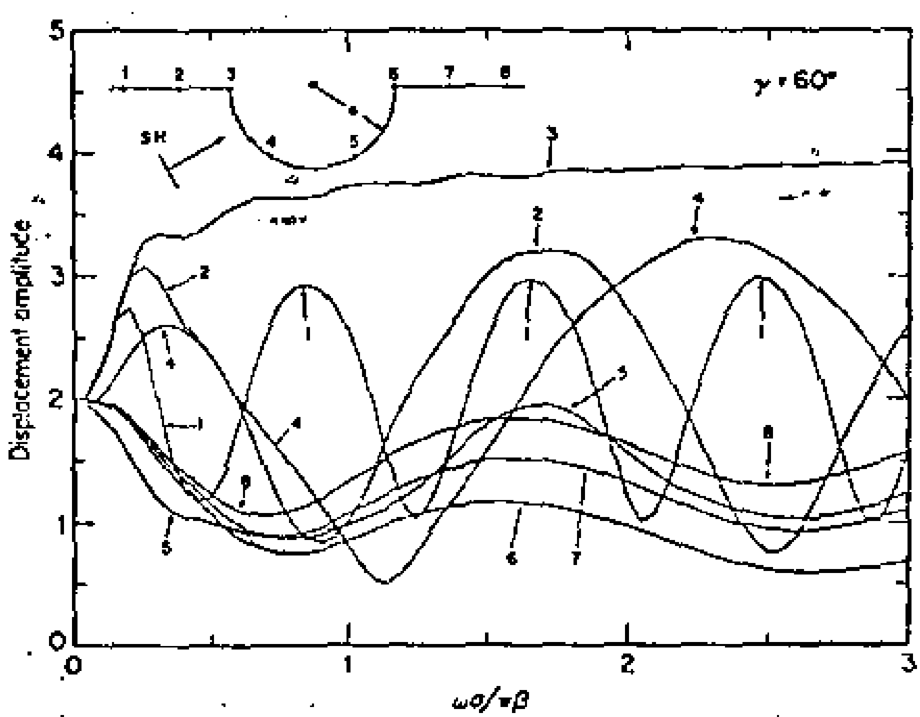

Fig. 4 Displacement amplitudes at points in the surface of a semi-circular canyon. Incidence of harmonic plane SH waves (After Trifunac, 1971)

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X CURSO INTERNACIONAL DE INGENIERIA SISMICA

ANALISIS DE RIESGO SISMICO

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DR. FRANCISCO SANCHES SESMA

AGOSTO, 1984.

EARTHQUAKE ENGINEERING AND STRUCTURAL DYNAMICS, VOL. 7, 441–450 (1979)

GROUND MOTION AT CANYONS OF ARBITRARY SHAPE UNDER INCIDENT SH WAVES

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SUMMARY

A method for calculating the two-dimensional scattering of incident SH waves by canyons of arbitrary shape is presented. The problem is formulated in terms of a Fredholm integral equation of the first kind with the integration path outside the boundary. Point-source discretization and a least-squares scheme are used. Numerical results are compared with the known analytic solution for a semi-cylindrical canyon. Spatial variations of surface amplitudes are computed for triangular and half-cycle sinusoidal canyons as well.

INTRODUCTION

Influence of local conditions on ground motion due to earthquakes has been recognized as a significant feature in seismic design.[1,2] Topographic effects can introduce appreciable differences on seismic movements between neighbouring sites, even when the distance from the active fault and predominant incident angles do not change.

The subject has been treated in the literature as a problem of elastic-wave diffraction. Analytic solutions for semi-cylindrical and semi-elliptical canyons under incident SH waves have been obtained.[3,4] An acoustic approximation has been used for a similar problem where only vertically incident P waves are considered in detail.[5] By means of matched asymptotic expansions, expressions for the solution of the SH-wave diffraction problem have also been obtained.[11] This solution is restricted to low frequencies. Another approach assumes periodicity of surface shape and uses discretized integral equations to describe the movement in the neighbourhood of topographic irregularities with small slopes under low-frequency incident waves.[6] The SH-wave diffraction problem for canyons of arbitrary shape has been formulated in terms of a Fredholm integral equation of the second kind and applied to the study of topographic effects at Pacoima Canyon[7]. Large amplifications and decrements were found in harmonic analysis but computed response spectra showed small differences, save for the range of high frequencies. For the same problem a multipole expansion in terms of Hankel functions about the origin and the least-squares method have been used.[13] Comparison of results with some known exact solutions yields very good agreement.

In the present work an alternative method is developed for solving SH-wave diffraction problems in arbitrarily shaped canyons. The method makes use of an integral representation of scattered waves as a simple layer potential[8] applied at interior points thus avoiding singularities in the kernel of the integral equation. The idea is similar to Copley's[9] for Weber's equation and has been applied by De Mey[10] to the solution of Laplace's interior problem.

The resulting Fredholm integral equation of the first kind is *discretized* with line sources concentrated at an interior curve. Emphasis is on field representation rather than on solution of the integral equation itself. Thus, the Fredholm integral equation serves as an intermediate step in the problem formulation and provides a useful tool to investigate some of the singularities of the operator.

Once the boundary conditions are established, a system of equations is obtained which is solved in the least-squares sense.

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0098-8847/79/0507-0441$01.00

Received 1 August 1978
Revised 19 March 1979

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The method has been applied to solve scattering and diffraction of P and SV waves by canyons[17] as well as ground motion of alluvial valleys under incident SH waves.[18]

Questions arise on completeness of the set of sources and convergence of the method. We will not deal with them in this work. It can be shown that the set of line sources located on an interior curve is complete so that no additional sources are required to approximate the field outside the canyon and its boundary. The proof is similar to that used by Millar[16] to establish the completeness of the multipole expansion and its normal derivatives. Given that the set is complete, it can be shown that the least-squares solution, when applied on the boundary as done here, leads to an approximate solution that converges uniformly in the mean to the exact solution as the number of sources tends to infinity.

For the semi-cylindrical canyon numerical results obtained by this method are compared with Trifunac's exact solution.[3] Convergence is illustrated showing results for increasing number of sources. Results are provided also for half-cycle sinusoidal and triangular canyons.

PROBLEM FORMULATION

For the propagation of polarized SH waves, displacements in the z direction satisfy the scalar wave equation

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=\frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} \tag{1}$$

where $c=(\mu/\rho)^{\frac{1}{2}}$ is the shear wave velocity, μ the shear modulus, and ρ the mass density of the medium. For harmonic waves of the form $u\exp(\mathrm{i}\omega t)$, equation (1) can be transformed into the Helmholtz equation

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+k^2 u=0 \tag{2}$$

where $k=\omega/c$ is the wave number and ω the circular frequency.

The traction-free boundary condition implies that at the free surface

$$\frac{\partial u}{\partial \mathbf{n}}=0 \tag{3}$$

where $\mathbf{n}$ is the normal vector to the half-space surface or to the canyon surface (Figure 1).

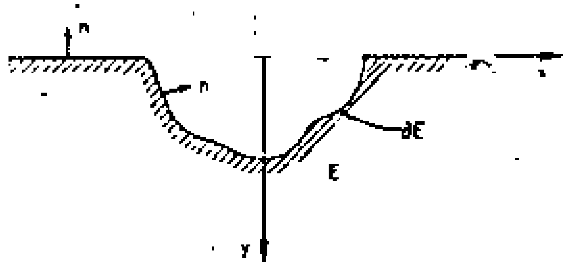

Figure 1. Canyon geometry

Assume that the solution has the form

$$u=u^{(0)}+u^{(d)} \tag{4}$$

where $u^{(0)}$ is the free-field displacement (without surface irregularities) and $u^{(d)}$ the contribution of the diffracted waves.

For illustration, consider a plane wave of unit amplitude that propagates towards the half-space surface

$$u^{(i)}=\exp\left[\mathrm{i}\omega\left(t-\frac{x}{c_x}+\frac{y}{c_y}\right)\right] \tag{5}$$

where $c_x=c/\sin\gamma$, $c_y=c/\cos\gamma$ and γ is the incidence angle (Figure 2). To satisfy the free boundary conditions at $y=0$, a reflected wave must be given by

$$u^{(r)}=\exp\left[\mathrm{i}\omega\left(t-\frac{x}{c_x}-\frac{y}{c_y}\right)\right] \tag{6}$$

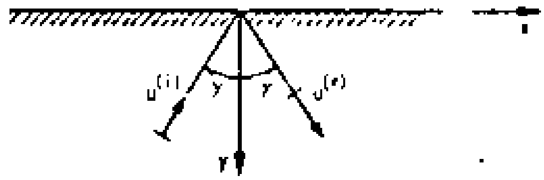

Figure 2. Incident and reflected plane SH waves, free-field

Thus the free-field solution is obtained as $u^{(0)} = u^{(i)} + u^{(r)}$, which may be written as

$$u^{(0)} = 2\cos\left(\frac{\omega y}{c_y}\right)\exp i\omega\left[\left(t - \frac{x}{c_x}\right)\right] \tag{7}$$

Assume that displacement $u^{(d)}$ can be expressed as a simple layer potential at interior curve C

$$u^{(d)}(P) = \int_C \sigma(Q)\, G(P, Q)\, \mathrm{d}S_Q \tag{8}$$

where $Q \in C$, $P \in E \cup \partial E$ (Figure 3), $\sigma(Q)$ is the simple layer density—an unknown function to be determined from the boundary conditions—and $G(P, Q)$ is Green's function for a point Q in the half-space, that is, $G(P, Q)$ satisfies

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k^2\right) G(P, Q) = -\delta(|\bar{r} - \bar{r}_0|) \tag{9}$$

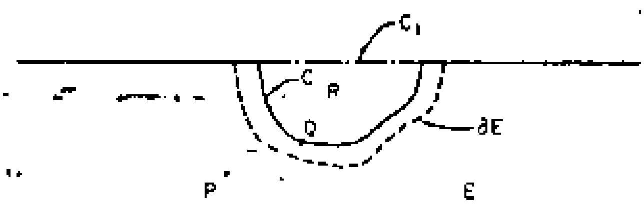

Figure 3. Definition of regions R and E and curves C, C_1 and ∂E

with the free-surface condition

$$\frac{\partial G}{\partial y} = 0 \quad \text{at } y = 0 \tag{10}$$

where $\delta(\cdot)$ is Dirac's delta function, $\bar{r}$ the position vector of point P and $\bar{r}_0$ the position vector of point Q.

Green's function is given by

$$G(P, Q) = \frac{i}{4}[H_0^{(2)}(kr_1) + H_0^{(2)}(kr_2)]\exp(i\omega t) \tag{11}$$

where $H_0^{(2)}(\cdot)$ is Hankel's function of the second kind and order zero, $r_1 = [(x - x_0)^2 + (y - y_0)^2]^{\frac{1}{2}}$, the distance from point $Q(x_0, y_0)$ to point $P(x, y)$, and $r_2 = [(x - x_0)^2 + (y + y_0)^2]^{\frac{1}{2}}$ is the distance from P to the image point of Q with co-ordinates $(x_0, -y_0)$. In this equation Hankel's function represents cylindrical SH waves that propagate towards infinity with speed c and satisfy Sommerfeld's radiation condition.[12]

From equations (4) and (6) one can write

$$u(P) = u^{(0)}(P) + \int_C \sigma(Q)\, G(P, Q)\, \mathrm{d}S_Q \tag{12}$$

where $P \in E \cup \partial E$.

Replacing equation (12) in equation (3) for $P \in \partial E$, the resulting Fredholm integral equation of the first kind is

$$\int_C \sigma(Q)\frac{\partial G(P, Q)}{\partial n_P}\,\mathrm{d}S_Q = -\frac{\partial u^{(0)}(P)}{\partial n_P}, \quad P \in \partial E \tag{13}$$

where $\mathbf{n}_P$ is the normal vector to boundary ∂E at point P.

To investigate conditions for which the solution of (13) is non-unique, we will look for non-trivial solutions σ of the homogeneous problem

$$\int_C \sigma(Q)\frac{\partial G(P,Q)}{\partial \mathbf{n}_P}\mathrm{d}S_Q = 0, \quad P \in \partial E \tag{14}$$

Let ϕ be the solution of

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}+k^2\phi = 0 \tag{15}$$

in a region R, limited by C and C_1 (Figure 3). For a point P outside this region, applying Green's theorem[13] we get

$$\int_{C\cup C_1}\left[\frac{\partial\phi(Q)}{\partial \mathbf{n}_Q}G(P,Q)-\phi(Q)\frac{\partial G(P,Q)}{\partial \mathbf{n}_Q}\right]\mathrm{d}S_Q = 0 \tag{16}$$

Let $\phi = 0$ at C and $\partial\phi/\partial\mathbf{n} = 0$ at C_1; that is, ϕ is an eigenfunction of the problem

$$\left.\begin{aligned}\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}+k^2\phi = 0 \quad &\text{in } R\\ \phi = 0 \quad &\text{at } C\\ \frac{\partial\phi}{\partial\mathbf{n}} = 0 \quad &\text{at } C_1\end{aligned}\right\} \tag{17}$$

Furthermore, by construction

$$\frac{\partial G(P,Q)}{\partial \mathbf{n}_Q} = 0 \quad \text{at } C_1 \tag{18}$$

Then, from equation (16), for P outside region R we get (Q)

$$\int_C \frac{\partial\phi(G)}{\partial \mathbf{n}_Q}G(P,Q)\mathrm{d}S_Q = 0 \tag{19}$$

For $P\in\partial E$, from equation (19) we may write

$$\int_C \frac{\partial\phi(Q)}{\partial \mathbf{n}_Q}\frac{\partial G(P,Q)}{\partial \mathbf{n}_P}\mathrm{d}S_Q = 0 \tag{20}$$

That is, there are non-trivial solutions $\sigma(Q) = \partial\phi(Q)/\partial\mathbf{n}_Q$ when k coincides with the eigenvalues of the problem defined by equations (17).

In particular, when R is a semi-circle with radius a_0 the eigenvalues are given by

$$k_{m,n} = \frac{j_{m,n}}{a_0} \tag{21}$$

where $j_{m,n}$ is the nth root of the equation $J_m(x) = 0$, $J_m(\cdot)$ is the Bessel function of the first kind and order m, $m = 0, 1, 2, \ldots$ and $n = 1, 2, \ldots$.

Given an arbitrary region R, there exists an infinite sequence of eigenvalues k for which the problem of equations (17) has a continuous solution.[18]

Nevertheless, there may be other non-trivial solutions of equation (14). The eigenfunctions of the problem defined by equations (17) provide only some of them. This suggests use of a suitable numerical approach to avoid the lack of uniqueness.

We seek a solution in the least-squares sense, that is, one for which the mean square error

$$\int_{\partial E}\left|\frac{\partial u^{(0)}(P)}{\partial \mathbf{n}_P}+\frac{\partial u^{(d)}(P)}{\partial \mathbf{n}_P}\right|^2\mathrm{d}S_P \tag{22}$$

is minimum. This criterion and the source-discretization scheme shown below lead us to a method which, apparently, does not suffer lack of uniqueness. Thus, emphasis is laid on field representation rather than on the solution of the integral equation itself. On the other hand, it can be shown that the set of line sources is complete if we take a *dense* subset on C; it suffices to select equally spaced points. Uniform convergence to the exact solution in the mean is thus ensured. These ideas parallel those in the work of Millar.[14]

SOURCE-DISCRETIZATION AND NUMERICAL SOLUTION

Let $\sigma(Q)$ be of the form

$$\sigma(Q) = \sum_{n=1}^{N} b_n\, \delta(|Q - Q_n|) \tag{23}$$

where N is the number of sources of amplitude b_n at points $Q_n \in C$. Equations (12) and (13) can then be written as

$$u(P) = u^{(0)}(P) + \sum_{n=1}^{N} b_n\, G(P, Q_n) \tag{24}$$

and

$$\sum_{n=1}^{N} b_n \frac{\partial G(P, Q_n)}{\partial \mathbf{n}_P} = -\frac{\partial u^{(0)}(P)}{\partial \mathbf{n}_P} \tag{25}$$

In order to find the N unknowns b_n, $n = 1, 2, \ldots, N$, impose the condition from equation (13) at M points P_m on the boundary of the canyon

$$\sum_{n=1}^{N} b_n \frac{\partial G(P_m, Q_n)}{\partial \mathbf{n}_{P_m}} = -\frac{\partial u^{(0)}(P_m)}{\partial \mathbf{n}_{P_m}}, \quad m = 1, 2, \ldots, M \tag{26}$$

Equations (26) represent the standard problem of linear algebra of M equations with N unknowns

$$[\mathbf{A}_{mn}]\{\mathbf{b}_n\} = \{\mathbf{f}_m\} \tag{27}$$

Good results can be obtained when $M = N$, but this case requires care in the choice of curve C. For instance, when k coincides with or is very close to the eigenvalues of the interior problem (17), numerical difficulties arise (e.g. unrealistically large displacements or ill-conditioned operator). Modifying the choice of curve C changes the interior problem and removes the singularity.

It is convenient, though, to take $M > N$ and solve in the least-squares sense[14] by means of

$$[\mathbf{A}^*_{mn}]^{\mathrm{T}}[\mathbf{A}_{mn}]\{\mathbf{b}_n\} = [\mathbf{A}^*_{mn}]^{\mathrm{T}}\{\mathbf{f}_m\} \tag{28}$$

where $[\mathbf{A}^*_{mn}]^{\mathrm{T}}$ is the transposed conjugate of the coefficients matrix. The resulting system of equations is of order $N \times N$.

Once the values of b_n, $n = 1, 2, \ldots, N$, are obtained, equation (24) allows us to calculate the displacement at any point of the region E and its boundary. Solution of equation (28) is equivalent to obtaining a minimum for the mean square error in expression (22).

RESULTS

In order to gauge the method's accuracy, displacements at point of a semi-cylindrical canyon have been calculated for several incidence angles and normalized frequencies

$$\eta = \frac{ka}{\pi} = \frac{2a}{\lambda} \tag{29}$$

where λ is the incident wave length, a the radius of the canyon and thus η is the canyon width-to-wave-length ratio.

Values of real and imaginary parts of u at some points are presented in Table I for $\eta = 2{\cdot}0$ and $\gamma = 30, 60, 90$ degrees using several values of N for the calculation. Comparison is provided with the exact values given by Trifunac's solution.[3] For the interior curve C, a semi-circumference with radius $0{\cdot}8a$ and 99 collocation points at the boundary were used.

Table 1. Comparison of results to exact solution, semi-cylindrical canyon, $\eta = 2·00$

x/a	$M = 10$		$M = 15$		$M = 20$		Exact	
				[$\gamma = 30°$]				
−1·50	0·74091	−1·15704	0·70448	−1·13299	0·70317	−1·12942	0·70319	−1·12941
−1·00	−3·40383	−0·18822	−3·46605	−0·37223	−3·47828	−0·42692	−3·47834	−0·42714
−0·50	1·28720	0·48138	1·25202	0·49594	1·25098	0·49759	1·25097	0·49748
0·00	1·62731	−1·78459	1·59742	−1·76556	1·59831	−1·76382	1·59843	−1·76378
0·50	−1·79672	0·20781	−1·76972	0·20801	−1·76331	0·20677	−1·76347	0·20669
1·00	1·43898	0·38959	−1·42650	0·63539	−1·42391	0·72468	−1·42407	0·72470
1·50	−0·12401	2·04079	−0·05495	2·06556	−0·05054	2·06386	−0·05053	2·06384
				[$\gamma = 60°$]				
x/a	$M = 10$		$M = 15$		$M = 20$		Exact	
−1·50	−1·83283	2·46628	−1·76901	2·55157	−1·76450	2·55561	−1·76450	2·55559
−1·00	2·92025	−3·02770	2·85667	−2·74221	2·80912	−2·63566	2·80893	−2·63559
−0·50	2·64131	−1·31221	2·72037	−1·24558	2·73097	−1·24294	2·73078	−1·24296
0·00	−2·40835	0·07178	−2·44644	0·14278	−2·45132	0·14889	−2·45120	0·14878
0·50	1·51314	−0·37793	1·51624	−0·37835	1·52035	−0·37156	1·52027	−0·37135
1·00	0·70125	−0·67186	0·47134	−0·86228	0·38808	−0·89220	0·38792	−0·89198
1·50	−0·94335	−0·85681	−0·99280	−0·74464	−0·99267	0·73442	−0·99266	−0·73446
				[$\gamma = 90°$]				
x/a	$M = 10$		$M = 15$		$M = 20$		Exact	
−1·50	−3·41897	−0·42424	−3·40347	−0·18771	−3·39576	−0·17178	−3·39574	−0·17174
−1·00	4·70441	0·08372	4·13223	0·19777	3·92585	0·27071	3·92598	0·27041
−0·50	−3·53399	−0·48741	−3·48763	−0·44017	−3·47836	−0·42708	−3·47834	−0·42725
0·00	2·60129	0·04269	2·66496	0·13720	2·65254	1·49088	2·65257	0·14950
0·50	−1·53581	0·61482	−1·43244	0·71927	−1·42397	0·72545	−1·42407	0·72483
1·00	−0·06127	0·69389	−0·01917	0·76622	−0·04852	0·84105	−0·04860	0·84038
1·50	0·95417	0·13408	0·99010	0·23138	0·99196	0·24013	0·99201	0·24020

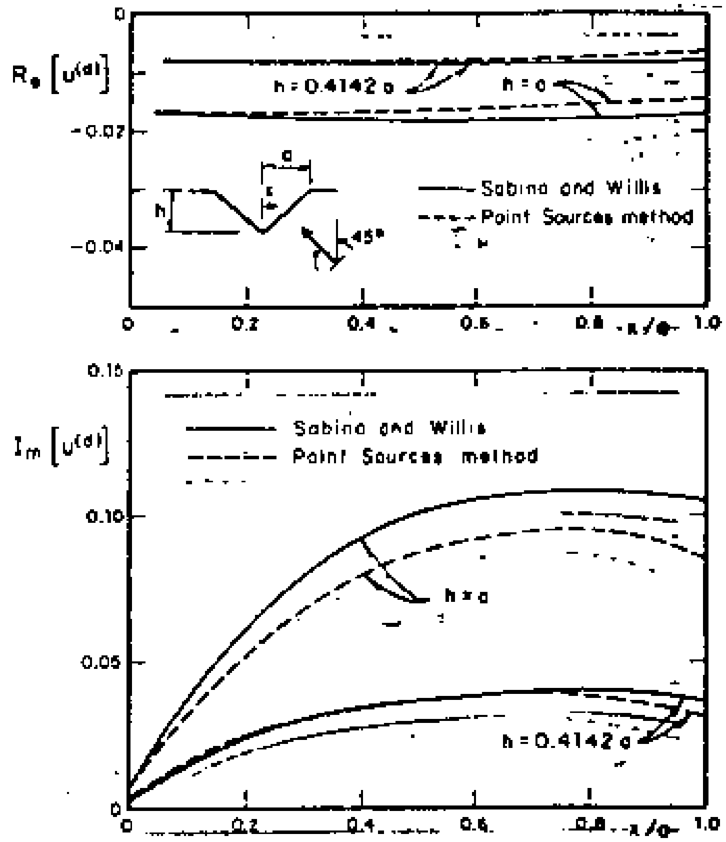

Figure 4. Comparison for real and imaginary parts of $u^{(d)}$ between numerical solution and solution obtained using matched asymptotic expansions[11] at part of the surface of the triangular canyon for $\eta = 0·1/\pi$

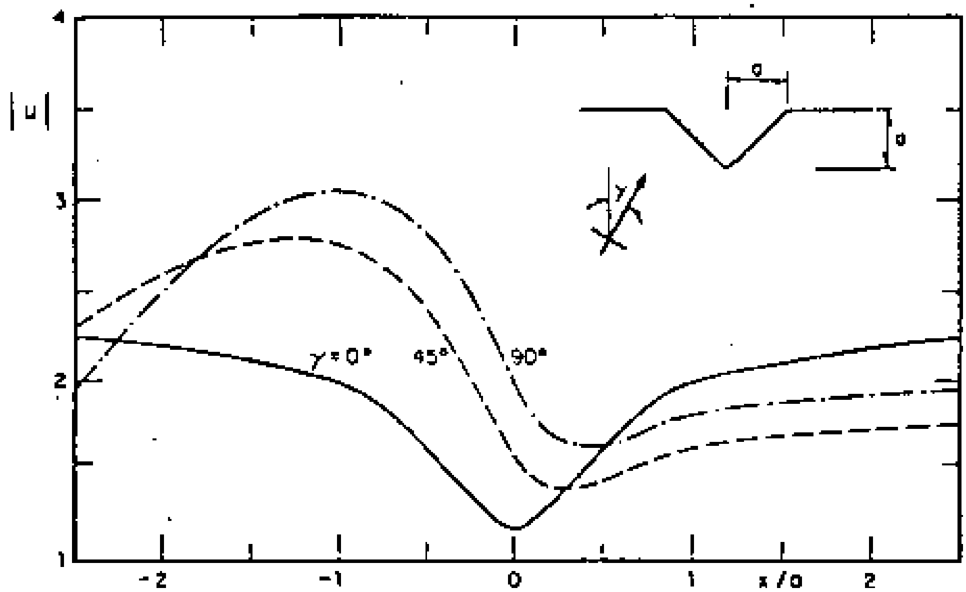

Figure 5. Displacement amplitudes at the surface of a triangular canyon with 45° slopes for different incidence angles γ and normalized frequency $\eta = 0{\cdot}25$

The method also has been applied to a semi-elliptical canyon;[10] agreement with the published exact solution[4] is excellent.

Real and imaginary parts of the diffracted field at a portion of the surface of a triangular canyon for two different depths, incidence angle $\gamma = -45$ degrees and normalized frequency $\eta = 0{\cdot}1/\pi$ are shown in Figure 4. The solution is compared with the one obtained by means of matched asymptotic expansions.[11]

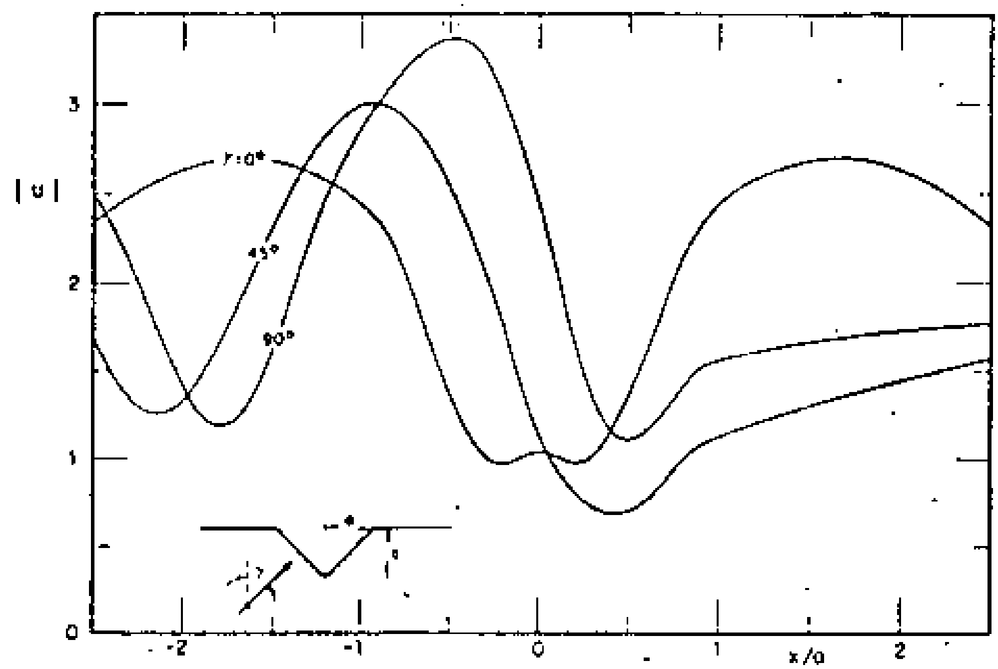

Figure 6. Displacement amplitudes at the surface of a triangular canyon with 45° slopes for different incidence angles γ and normalized frequency $\eta = 0{\cdot}5$

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Both curves show a similar trend. The fit with the shallow canyon is quite good whereas that for the deeper one is less good although one cannot ensure that the matched asymptotic solution is exact for $\eta = 0{\cdot}1/\pi$. At any rate we are comparing solutions at the very low frequency range and quantities that are very small.

Figures 5–7 show displacement amplitudes at the surface of a triangular canyon with 45 degree slopes and depth a for three normalized frequencies ($\eta = 0{\cdot}25$, $0{\cdot}5$, $1{\cdot}0$) and three incidence angles ($\gamma = 0$, 45,

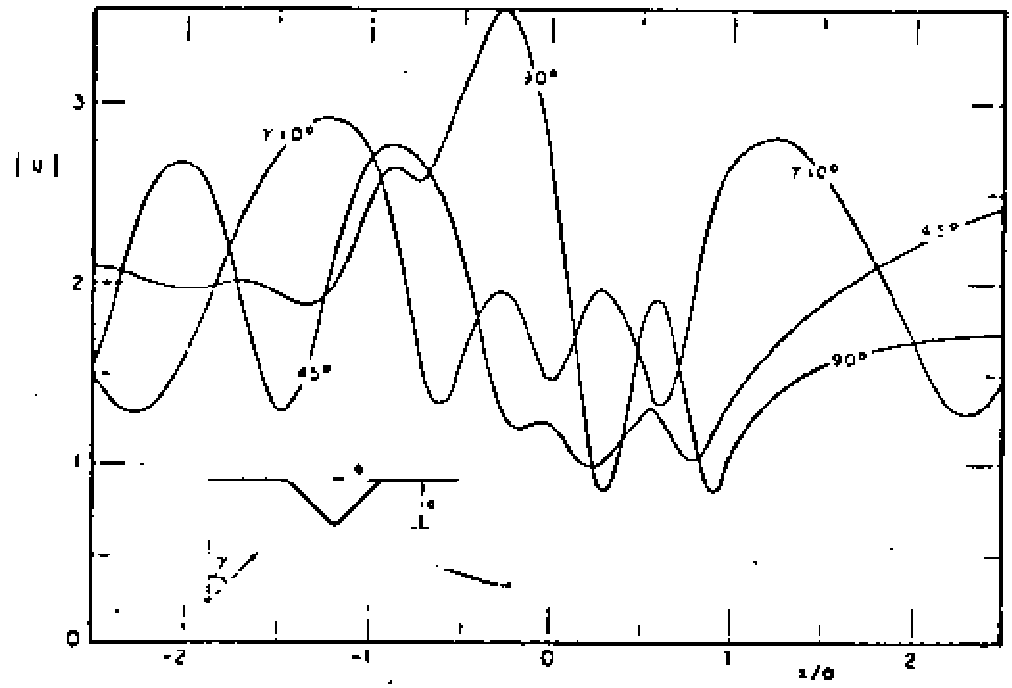

Figure 7. Displacement amplitudes at the surface of a triangular canyon with 45° slopes for different incidence angles γ and normalized frequency $\eta = 1{\cdot}0$

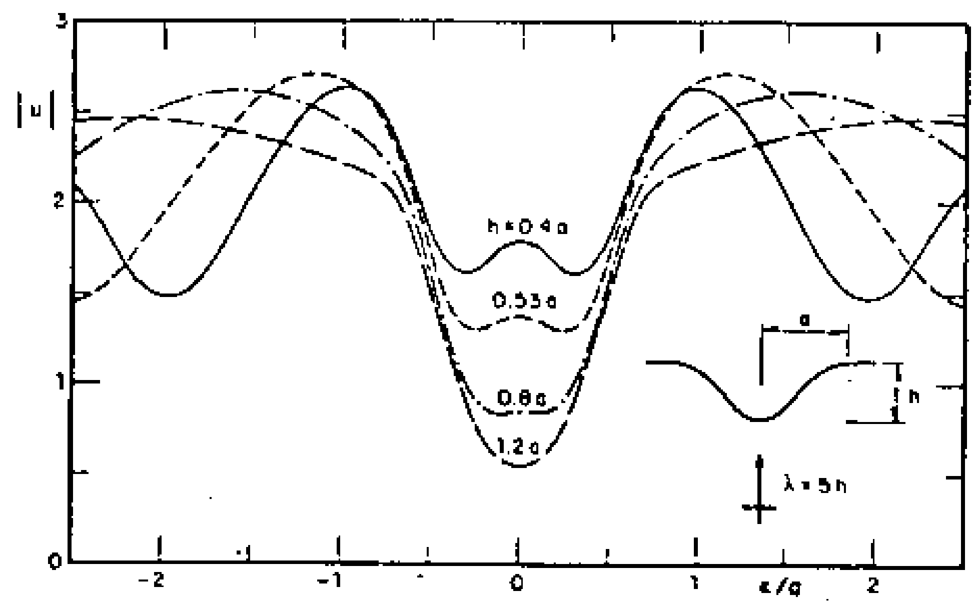

Figure 8. Displacement amplitudes at the surface of sine-shaped canyons with different slopes and vertical incidence

90 degrees). Sources were set along lines parallel to the slopes and separated from them at a distance of $0{\cdot}07a$. At the boundary, 99 collocation points were taken. For the calculation, the vertex of the canyon has been smoothed with a segment of circumference tangent to the slopes.

To show the influence of the slope on the amplitude spectra, surface amplitudes of sinusoidal and triangular canyons of different depths were calculated, keeping the wave length $\lambda = 5h$ constant for vertical incidence. Figures 8 and 9 display the amplitude distribution for different slopes. In these cases 19 sources and 99 collocation points were employed. As previously, the triangular canyon vertex has been smoothed.

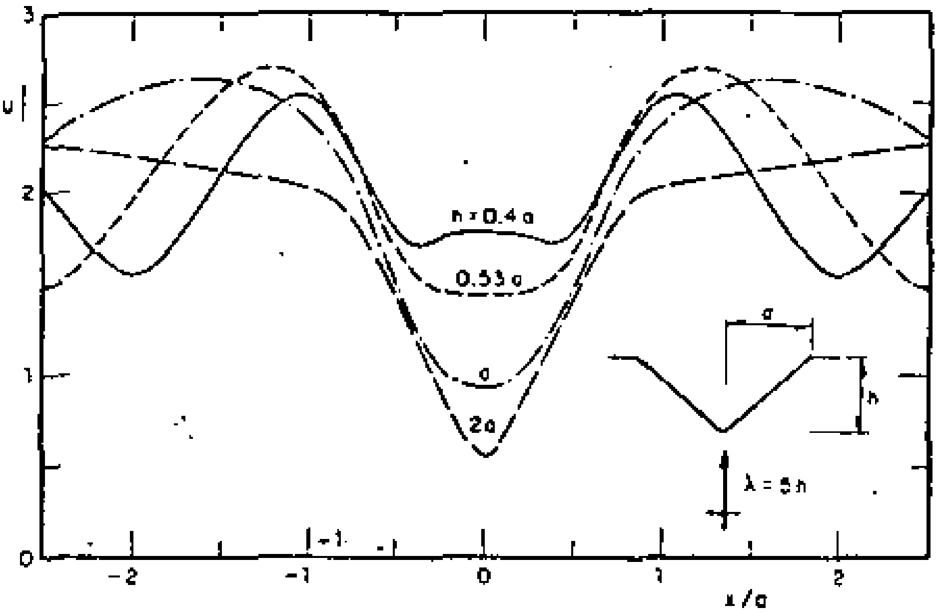

Figure 9. Displacement amplitudes at the surface of triangular canyons with different slopes and vertical incidence

CONCLUSIONS

A method to solve SH wave diffraction problems at arbitrary shaped canyons has been presented. The method is based on setting a Fredholm integral equation of the first kind defined on a curve different from the boundary, thus getting a regular kernel. Emphasis is laid on the field representation rather than on the solution of the integral equation itself.

Source-discretization and the least-squares method lead to an efficient numerical scheme which provides stable, accurate results. The number of sources to be used depends on the accuracy desired and is an increasing function of frequency. Choice of curve C following the canyon boundary seems to be generally the best decision.

Amplification phenomena noted for various topographic features[2,4,5,7] also show up for triangular canyons. Within the range covered here for vertical incidence, reductions of about 50–70 per cent and amplifications of about 10–30 per cent at the edges have been obtained.

With incidence angles $\gamma = 45$ and 90 degrees the canyon acts as a barrier of seismic waves. This feature must be considered in bridge or reservoir design, as large differences in the support motions can take place.

The influence of canyon slope is significant. This effect can be appreciated in Figures 8 and 9 where reductions grow with slope. The barrier effect for oblique incident seismic waves also increases with slope; canyon borders near the half-space surface tend to behave like a quarter of space, especially for high frequencies which can be associated with 100 per cent amplification.

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ACKNOWLEDGEMENTS

The treatment of the eigenvalue problem presented was suggested by A. A. Minzoni. Thanks are given to G. Ayala, I. Herrera, F. J. Sabina and S. K. Singh for their comments and suggestions, and to J. A. Esquivel for his assistance in various aspects of this work.

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ANALISIS DE RIESGO SISMICO

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DR. FRANCISCO SANCHES SESMA

AGOSTO, 1984.

Bulletin of the Seismological Society of America, Vol. 73, No. 6, pp. 1621–1636, December 1983

DIFFRACTION OF ELASTIC WAVES BY THREE-DIMENSIONAL SURFACE IRREGULARITIES

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Abstract

A boundary method is applied to study the scattering and diffraction of elastic waves by three-dimensional surface irregularities on the surface of a half-space. The method makes use of the c-completeness of a family of wave functions in order to construct with linear combinations the diffracted fields. Boundary conditions are satisfied in a least-squares sense. For axisymmetric scatterers, an azimuthal decompositon is presented. Some numerical results for vertical incidence of *P* waves are reported.

Introduction

Topographical and geological irregularities can induce large amplifications and variations in ground motion during earthquakes. The focusing and scattering of the energy carried by seismic waves seem to be the causes of such effects. Large differences of motion between nearby places may be significant in the response of important facilities, like bridges, dams, and life-line systems. Amplification caused by soil deposits has been recognized as a subject of paramount importance in microzonation studies (Esteva, 1977).

Many authors have studied the problem of two-dimensional irregularities for various incident wave fields (e.g., Gilbert and Knopoff, 1960; McIvor, 1969; Boore, 1972; Bouchon, 1973; Wong and Jennings, 1975; Sabina and Willis, 1977; Sills, 1978; Sánchez-Sesma, 1978; Sánchez-Sesma and Rosenblueth, 1979; Boore *et al.*, 1981; Wong, 1982; Dravinski, 1982b; Bard, 1982; Sánchez-Sesma *et al.*, 1982b). Of particular significance are the analytical solutions for incident *SH*-wave fields on semi-circular (Trifunac, 1971, 1973) and semi-elliptical (Wong and Trifunac, 1974a, b) irregularities at the surface of a half-space.

Three-dimensional problems have received less attention. This is due to the increased difficulties which arise in solving this class of problems. A theoretical development has been presented (Hudson, 1967) for three-dimensional small slope cavities or inclusions at the surface of an elastic half-space, in which equivalent surface loads are used for representing the scattered fields. With this first-order perturbation approach, reasonable estimates have been obtained of the scattered Rayleigh waves as compared with observations (Hudson and Boore, 1980). A finite difference analysis of axisymmetric irregularities has been presented (Zhenpeng *et al.*, 1980) for vertically incident shear waves. Spectral ratios were obtained and comparison with some observations gives reasonable agreement. With an acoustic approximation, the exact solution has been obtained for a semi-spherical cavity under the incidence of *P* waves (Singh and Sabina, 1977). However, such an approximation can be applied in very few cases. For the same geometry and considering an elastic medium, a solution has been presented for incident *P* and *S* waves (Lee, 1978, 1982). The method consists of a very ingenious and laborious matching of coefficients of the series expansions of the involved wave fields. It seems, however, that the latter approach is limited to small frequencies.

In this paper, the scattering and diffraction of elastic waves by a three-dimensional irregularity on the surface are considered. A boundary method recently developed for two-dimensional problems (Sánchez-Sesma, 1978, 1981; Sánchez-Sesma and Rosenblueth, 1979; Sánchez-Sesma and Esquivel, 1979, 1980; Wong, 1979, 1982; England *et al.*, 1980; Dravinski, 1982a, b; Sánchez-Sesma *et al.*, 1982a, b) is extended here to three-dimensional cases. The method consists of constructing the scattered fields with linear combinations of members of a c-complete family of wave functions (Herrera and Sabina, 1978) which are solutions of Navier's equations. Coefficients of the linear forms thus constructed are obtained from a collocation, least-squares matching of boundary conditions. As pointed out by Wong (1982), the method can be considered as a generalized inverse one. In doing this, Wong suggested a procedure which improves the solution numerically.

The families of solutions can be formed in a very general way with sources having their singularities outside the region of interest and fulfilling radiation conditions if the region is infinite. A general framework for the method is given by a recent algebraic theory of boundary value problems (Herrera, 1979, 1980a, b).

It is convenient that the mentioned solutions satisfy boundary conditions on the free surface of the half-space leaving only the region of the irregularity for numerical treatment. However, in some cases this requirement introduces severe restrictions in numerical calculations because of the lengthy computations needed to obtain such solutions. It has been found in a recent, two-dimensional analysis of the scattering of P, SV and Rayleigh waves (Sánchez-Sesma *et al.*, 1982a) that the use of solutions that actually do not satisfy free-boundary conditions gives results which agree very well with those of Wong (1979, 1982) for a semi-circular canyon. These solutions for the displacement fields were constructed using only Hankel and trigonometric functions. Of course, the numerical treatment included the free boundary. However, the computational effort, even with the addition of part of the half-space surface, was greatly reduced. A relatively small part of the free surface (three times the radius of the canyon at both sides of it) needed to be considered to obtain convergent results for the range of frequencies considered; i.e., $\omega \lesssim \pi\beta/2a$, where a = radius and β = velocity of shear waves (Sánchez-Sesma *et al.*, 1982a).

In this work, the three-dimensional scattered displacement fields are constructed with linear combinations of solutions of Navier's equations, which are given in terms of spherical Hankel and Bessel functions, associated with Legendre and trigonometric functions (Takeuchi and Saito, 1972; Aki and Richards, 1980). Since each of these solutions does not satisfy in itself the free-boundary conditions, the numerical treatment is extended to part of the half-space surface, as has already been mentioned. These solutions have been widely used in seismology to deal, for example, with the free oscillations of the earth. Here, the origin of the spherical coordinate system is on the surface of the half-space.

In the present approach, axial symmetry of the scatterer is assumed in order to allow azimuthal decomposition. That is to say, the problem is split into "two-dimensional" problems. For normal incidence of P or SV waves, only one azimuthal number is required. For nearly vertical incidences or long wavelengths of the apparent incident surface field, only a few terms of the azimuthal decomposition are needed to give good results.

Some numerical examples are given for vertically incident P waves on different surface irregularities.

THE PROBLEM

Consider the elastic half-space and a three-dimensional surface irregularity represented in Figure 1 by regions E and R, respectively. Let $\partial_1 E$ and $\partial_1 R$ be the free boundaries of the regions, and $\partial_2 E = \partial_2 R$ be the common boundary between them. Under incidence of elastic waves, the irregularity generates diffracted fields that should be superimposed on the free-field solution, that is to say, on the fields in *absence* of irregularity.

Under the assumption of time dependence given by $\exp(i\omega t)$, where $i = \sqrt{-1}$, $\omega =$ circular frequency, and $t =$ time, the displacement vector, $\bar{u}$, must satisfy the reduced Navier equation, which in vector form is given by

$$\mu\nabla^2\bar{u} + (\lambda + \mu)\nabla\nabla\cdot\bar{u} + \rho\omega^2\bar{u} = 0. \tag{1}$$

Here λ, μ = Lamé constants, and ρ = mass density. These constants should be particularized for each medium.

We can write the total fields as

$$\bar{u}^E = \bar{u}^{(0)} + \bar{u}^{(s)} \tag{2}$$

for the region E, and

$$\bar{u}^R = \bar{u}^{(r)} \tag{3}$$

for region R. In equation (2) $\bar{u}^{(0)}$ is the displacement vector of the free-field solution. In equations (2) and (3), $\bar{u}^{(s)}$ and $\bar{u}^{(r)}$ represent the scattered and refracted fields in the half-space and the irregularity, respectively.

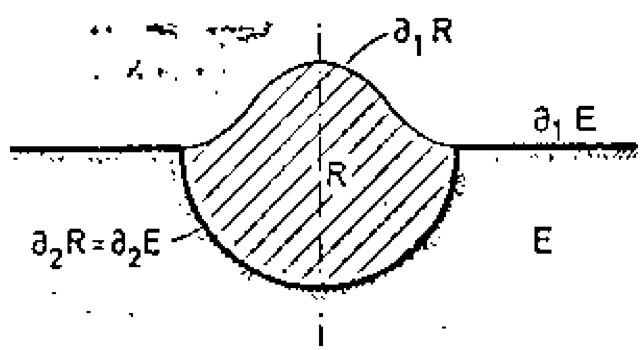

FIG. 1. Definition of regions E and R and their boundaries.

The boundary conditions that must be satisfied by the total fields are given by

$$\bar{\sigma}(\bar{u}^E) = 0 \quad \text{in} \quad \partial_1 E, \tag{4}$$

$$\bar{\sigma}(\bar{u}^R) = 0 \quad \text{in} \quad \partial_1 R, \tag{5}$$

$$\bar{\sigma}(\bar{u}^E) = \bar{\sigma}(\bar{u}^R) \quad \text{in} \quad \partial_2 E, \quad \text{and} \tag{6}$$

$$\bar{u}^E = \bar{u}^R \quad \text{in} \quad \partial_2 E. \tag{7}$$

In these equations, $\bar{\sigma}$ stands for the traction vector associated with a particular displacement field and with the normal vector to the boundary. Assume, for definiteness, that the normal vector on the common boundary is directed toward R. The first two of equations (4) to (7) are the boundary conditions of free surface, while the last two are conditions of continuity of tractions and displacements, respectively, along $\partial_1 E = \partial_1 R$.

In addition, $\bar{u}^{(s)}$ must satisfy the Sommerfeld-Kupradze elastic radiation condition at infinity (Sommerfeld, 1949; Kupradze, 1965) which means that no energy may be radiated from infinity of the prescribed singularities of the field.

Method of Solution

Let $B^E = \{\bar{w}_1{}^E, \bar{w}_2{}^E, \ldots\}$, and $B^R = \{\bar{w}_1{}^R, \bar{w}_2{}^R, \ldots\}$ be c-complete (Herrera and Sabina, 1978) families of solutions of equation (1) for the external (E) and internal (R) regions, respectively. the members of B^E satisfy, in addition, the elastic radiation condition.

The introduction of the concept of c-completeness allows constructing systems of solutions which are complete with respect to boundary values independently of the specific region considered (Herrera, 1980b; Herrera and Sabina, 1978).

We can approximate the scattered and refracted displacements $\bar{u}^{(s)}$ and $\bar{u}^{(r)}$ by linear combinations of members of the families B^E and B^R, respectively. Then, equations (2) and (3) can be written in the forms

$$\bar{u}^E = \bar{u}^{(0)} + \sum_{j=1}^{N} A_j{}^N \bar{w}_j{}^E, \quad \text{and} \tag{8}$$

$$\bar{u}^R = \sum_{j=1}^{M} B_j{}^M \bar{w}_j{}^R. \tag{9}$$

Here, $A_j{}^N$ and $B_j{}^M$ are unknown coefficients, and M, N are the orders of the approximations.

Substituting equations (8) and (9) in equations (4) to (7), a set of equations is obtained for each point on the boundaries. By imposing that these equations be satisfied at a finite number of points on the boundaries, we obtain a system of linear equations for the unknown coefficients, in which the independent parts are given in terms of the free-field solution. It is convenient to form an overdetermined system by choosing the "collocation" points in such a way that we obtain more equations than unknowns and solve the system in the least-squares sense (Sánchez-Sesma and Rosenblueth, 1979). With appropriate weighting factors and careful location of collocation points, this discrete procedure is equivalent to minimizing the quadratic error on the boundary conditions, integrated along the boundaries, in which the resulting integrals are implicitly calculated numerically. High order integration schemes can be used (e.g., England *et al.*, 1980) to improve the solution. Moreover, further numerical improvement can be achieved if the procedure suggested by Wong (1982) is applied. It consists of selecting a square matrix of order $N \times N$, where N is the number of unknowns, with the maximum determinant, and then express the remaining rows of the whole system in terms of the previously selected matrix. Further details can be found in the work by Wong (1982).

The Free-Field

Assume incidence of plane waves in which the normal to the plane wave front is parallel to the xz plane. The elastic half-space occupies the region $z > 0$, and the free surface lies in the plane $z = 0$. Consider also a spherical reference system with coordinates r, θ, ϕ as shown in Figure 2. The unit vectors $\hat{e}_r$, $\hat{e}_\theta$, and $\hat{e}_\phi$ of the spherical system are also drawn in this figure.

It can be shown that the free-field solution, for harmonic incident P, SV, SH, and Rayleigh waves, is given in the Cartesian reference frame by expressions which have the form

$$f(z)\exp[i\omega(t - x/c)] \tag{10}$$

for displacements and stresses. Here $f(z)$ can have different forms depending on the type of wave and component of the field considered. In expression (10), c = apparent phase velocity in the x direction (for Rayleigh waves, $c = c_R$ = velocity of these waves). With coordinate transformation, the components of the free-field can be expressed in the spherical reference system. It is found that for *in-plane* motion (P, SV, and Rayleigh waves), displacements u_r and u_θ are even whereas u_ϕ is odd with respect to the azimuthal angle ϕ. The stresses σ_{rr}, $\sigma_{\theta\theta}$, $\sigma_{\phi\phi}$, and $\sigma_{r\theta}$ are also even; the remaining stresses $\sigma_{r\phi}$ and $\sigma_{\theta\phi}$ are odd, with respect to ϕ. The situation is

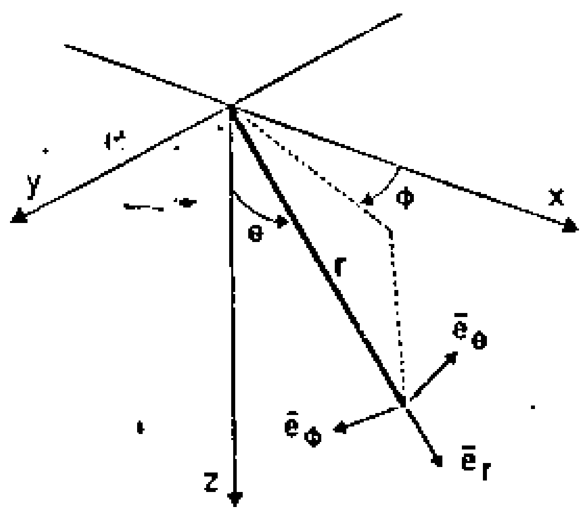

FIG. 2. Cartesian and spherical coordinate systems, and unitary vectors in the spherical system.

completely reversed for *out-of-plane* incident SH waves. These properties of the plane incident fields will allow an azimuthal decomposition.

Now, consider the identity

$$\exp(-ilx) = \sum_{k=0}^{\infty} \epsilon_k(-i)^k J_k(lr\sin\theta)\cos k\phi \tag{11}$$

where ϵ_k = Neumann factor (=1 if $k = 0$; =2 *if* $k > 0$), $J_k(\cdot)$ = Bessel function of the first kind and order k, and $l = \omega/c$ = horizontal wavenumber. This makes it clear that the "propagation factor" of the free-field is even with respect to ϕ.

From equation (11) and the aforementioned coordinate transformation, the even and odd parts in ϕ of any component of the field can be written, respectively, in the general forms

$$\sum_{k=0}^{\infty}\sum_{j=0}^{k} f_k^{(j)}[\cos[(k+j)\phi] + \cos[(k-j)\phi]], \tag{12}$$

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and

$$\sum_{k=0}^{\infty} \sum_{j=1}^{2} g_k^{(j)}\{\sin[(k+j)\phi] - \sin[(k-j)\phi]\}. \tag{13}$$

Here, $f_k^{(j)}$ and $g_k^{(j)}$ are known functions of r and θ. To illustrate this result, let us consider the compoment σ_{rr} of the stress tensor of the free-field under incidence of *in-plane* waves. In this case, the factors of expression (12) are given by

$$f_k^{(j)} = \tfrac{1}{2}\epsilon_k(-i)^k J_k(lr \sin\theta)a_j, \; j = 0, 1, 2 \tag{14}$$

where

$$
\begin{aligned}
a_0 &= \tfrac{1}{2}(\sigma_{xx} + \sigma_{yy})\sin^2\theta + \sigma_{zz}\cos^2\theta \\
a_1 &= \sigma_{xz}\sin 2\theta \\
a_2 &= \tfrac{1}{2}(\sigma_{xx} - \sigma_{yy})\sin^2\theta.
\end{aligned} \tag{15}
$$

Here, σ_{xx}, σ_{yy}, σ_{zz}, and σ_{xz} are the nonzero components of the stress tensor of the free-field *without* the propagation factor e^{-ilx}. Clearly, these stresses are all functions of z and are all known. But as $z = r\cos\theta$, the statement that $f_k^{(j)}$ are functions of r and θ follows from the analysis of equations (14) and (15).

The Scattered Fields

The well-known solutions of Navier's equation in spherical coordinates which traditionally have been used to study the free oscillations and surface waves of the earth (Takeuchi and Saito, 1972; Aki and Richards, 1980) are used here to construct the scattered fields of our problem.

Independent solutions of the reduced Navier equation are given by

$$\mathbf{w}^T = y_1^T(r)\mathbf{T}_n^m(\theta, \phi), \tag{16}$$

$$\mathbf{w}^P = y_1^P(r)\mathbf{R}_n^m(\theta, \phi) + y_3^P(r)\mathbf{S}_n^m(\theta, \phi), \tag{17}$$

and

$$\mathbf{w}^S = y_1^S(r)\mathbf{R}_n^m(\theta, \phi) + y_3^S(r)\mathbf{S}_n^m(\theta, \phi) \tag{18}$$

where,

$$\mathbf{R}_n^m(\theta, \phi) = Y_n^m(\theta, \phi)\hat{e}_r, \tag{19}$$

$$\mathbf{S}_n^m(\theta, \phi) = \frac{\partial Y_n^m}{\partial\theta}\hat{e}_\theta + \frac{1}{\sin\theta}\frac{\partial Y_n^m}{\partial\phi}\hat{e}_\phi, \quad \text{and} \tag{20}$$

$$\mathbf{T}_n^m(\theta, \phi) = \frac{1}{\sin\theta}\frac{\partial Y_n^m}{\partial\phi}\hat{e}_\theta - \frac{\partial Y_n^m}{\partial\theta}\hat{e}_\phi \tag{21}$$

are the surface vector harmonics in spherical coordinates. In these equations

$$Y_n^m(\theta, \phi) = P_n^m(\cos\theta)e^{im\phi} \tag{22}$$

where $P_n^m(\cdot)$ = Legendre function, and $m = 0, \pm1, \pm2, \ldots, \pm n$. The radial functions are given by

$$y_1^T(r) = \frac{1}{r}[kr\zeta_n(kr)] \tag{23}$$

$$y_1^P(r) = \frac{1}{r}[n\zeta_n(qr) - qr\zeta_{n+1}(qr)] \tag{24}$$

$$y_3^P(r) = \frac{1}{r}[\zeta_n(qr)] \tag{25}$$

$$y_1^S(r) = \frac{1}{r}[-n(n+1)\zeta_n(kr)] \tag{26}$$

$$y_3^S(r) = \frac{1}{r}[-(n+1)\zeta_n(kr) + kr\zeta_{n+1}(kr)] \tag{27}$$

where $\zeta(\cdot)$ may be either $j_n(\cdot)$ = spherical Bessel function of order n (for bounded regions) or $h_n^{(2)}(\cdot)$ = spherical Hankel function of the second kind and order n (for unbounded regions), $k = \omega/\beta$ = S wavenumber, $q = \omega/\alpha$ = P wave (number), $\beta = \sqrt{\mu/\rho}$ is the S-wave velocity, and $\alpha = \sqrt{(\lambda + 2\mu)/\rho}$ is the P-wave velocity. The notation used by Takeuchi and Saito (1972) is adopted here; the superscripts T, P, and S have been added to mean toroidal S waves, and spheroidal P and S waves, respectively.

Expressions of the associated stress components can be found in the literature (e.g., Takeuchi and Saito, 1972; Mow and Pao, 1971; Lee, 1982).

Assume that the displayed solutions form c-complete families of solutions in the sense introduced by Herrera (1979, 1980a, b) for the internal and external regions.

The linear forms of equations (8) and (9) will then be double sums over m and n, the azimuthal and radial numbers, respectively. But, as has already been mentioned, for axisymmetric scatterers, the azimuthal decomposition applies and the whole problem may be "split" into two-dimensional problems with only r and θ dependence for each azimuthal number.

The Azimuthal Decomposition

If the shape of the irregularity is independent of ϕ, that is to say, axisymmetric with respect to the z axis, the orthogonality of azimuthal functions allows a complete decomposition of the problem in terms of the azimuthal number. The *evenness* and *oddness* properties of the free-field solution also hold for the scattered fields introduced in the last section by putting $\cos m\phi$ or $\sin m\phi$, respectively in the place of $e^{im\phi}$ in equation (22). Moreover, in the case of axisymmetric irregularities, boundary conditions will also have these properties.

Assume we are dealing with an *even* boundary condition which has an associated equation of the form

$$\sum_m \{\sum_n A_{mn} g_{mn}(r, \theta)\} \cos m\phi = F(z) \exp[-ikx]. \tag{28}$$

Here, A_{mn} represents the unknowns, g_{mn} the contributions of the scattered fields, and $F(z)$ is some function, also even, of the free-field solution. Taking into account equations (11) and (12) and the azimuthal orthogonality, an equation is obtained of the form

$$\sum_n A_{mn} g_{mn} = \sum_{j=0}^{2} [f^{(j)}_{m+j} + f^{(j)}_{m-j}] \tag{29}$$

for each m, where $m = 0, 1, 2, \ldots$ The terms on the right-hand side of the equation are all known. All boundary conditions are decomposed in the same way leading to a "two-dimensional" problem for each azimuthal number.

For vertically incident plane waves, i.e., with no x dependence, only one azimuthal number is required. In this case, for P waves, only $m = 0$ is needed; for SV waves, it suffices to take $m = 1$. If the incident fields are nearly vertical, only a few azimuthal terms are needed to give good results. For almost grazing incidences or for Rayleigh waves in which the horizontal wavenumber is large, this approach would require many azimuthal numbers. However, even in this critical case, four or five azimuthal terms can give a good approximation if the horizontal wavelengths of the incident field are of the order of the maximum horizontal dimension of the irregularity. The degree of approximation of the azimuthal decomposition can be estimated with equation (11).

Examples

In this section, some results are presented for vertical incidence of P waves. Figures 4 to 7 correspond to the normalized amplitudes of vertical and horizontal displacements on the surface of a semi-spherical cavity (Figure 3). Note that in the absence of irregularity, displacements would be vertical and with relative amplitude of two. Normalized frequencies are given by

$$\eta_q = \frac{qa}{\pi} = \frac{2a}{\Lambda_P} \tag{30}$$

where a = radius of the cavity, q = wavenumber, and Λ_P = wavelength of the incident P wave. Thus, η_q is the diameter-to-wavelength ratio. A Poisson coefficient $\nu = 0.25$ was taken for the calculations. Results for $\eta_q = 0.25, 0.50, 0.75$, and 1.00 are given in Figures 4, 5, 6, and 7, respectively. Fifty to 70 collocation points were used, and they were uniformly placed along the boundary of the cavity and in a part of the flat surface with a length of three radii. The orders of the expansions used were 10 for the first case and 15 for the next three. The behavior of the solution, for vertical displacements, is similar to that obtained for SH waves in the two-dimensional case (e.g., Trifunac, 1973, Figures 6 to 8). Note that horizontal displacements are generated up to 80 per cent of the amplitude of the incident field and, according to the symmetry of the problem, they are zero at the center of the cavity. Horizontal displacements also evidence that important mode conversion takes place

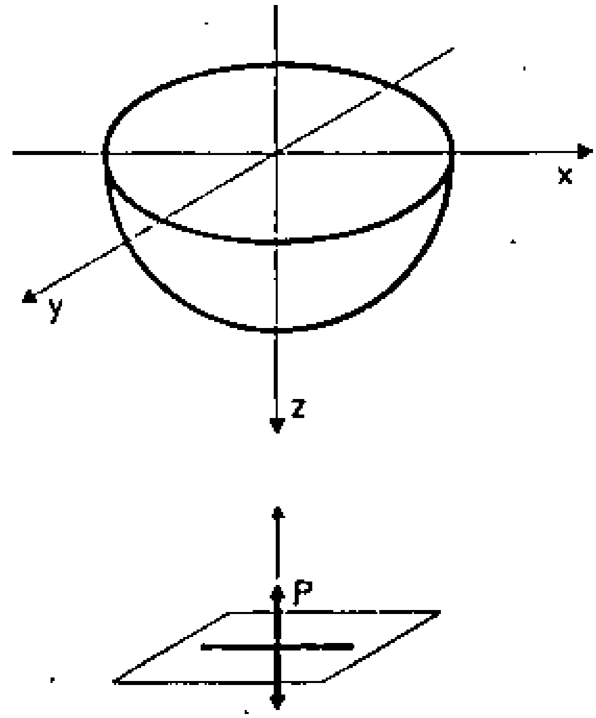

FIG. 3. Semi-spherical cavity on the half-space surface and vertically incident P wave.

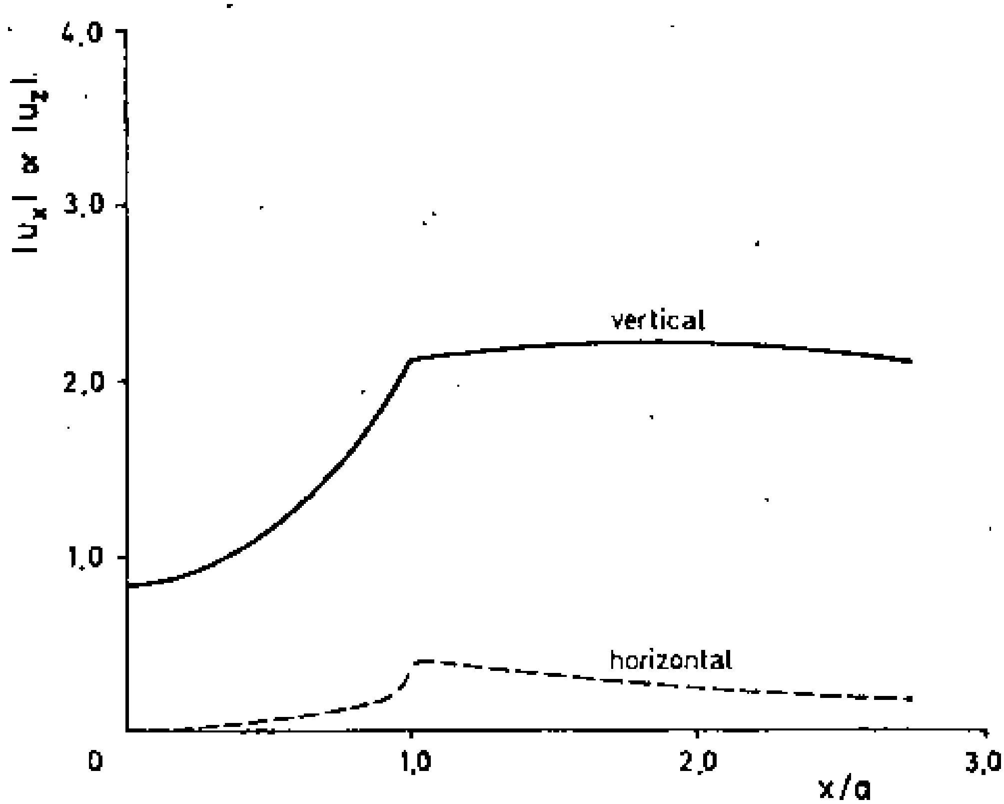

FIG. 4. Surface amplitudes of vertical and horizontal displacements. Semi-spherical cavity. Normalized frequency $\eta_0 = 0.25$. Poisson coefficient $\nu = 0.25$.

at the irregularity. Errors were calculated in terms of surface tractions; the maximum was inferior to 5 per cent of the value of the largest principal stress in the free-field solution.

Figure 8 shows the amplitudes of vertical and horizontal displacements on the flat surface of an alluvial deposit of semi-spherical shape for vertical incidence of P waves with $\eta_v = 0.50$. Material properties are given by $\mu_R/\mu_E = 0.3$, $\rho_R/\rho_E = 0.6$ for shear modulus and mass density, respectively. The subscripts R and E denote the deposit and the half-space, respectively. Poisson coefficients were assumed to be $\nu_R = 0.3$ and $\nu_E = 0.25$. In this case, the collocation points were placed as follows: 20 along the common boundary of the deposit and the half-space; 40 on the surface of

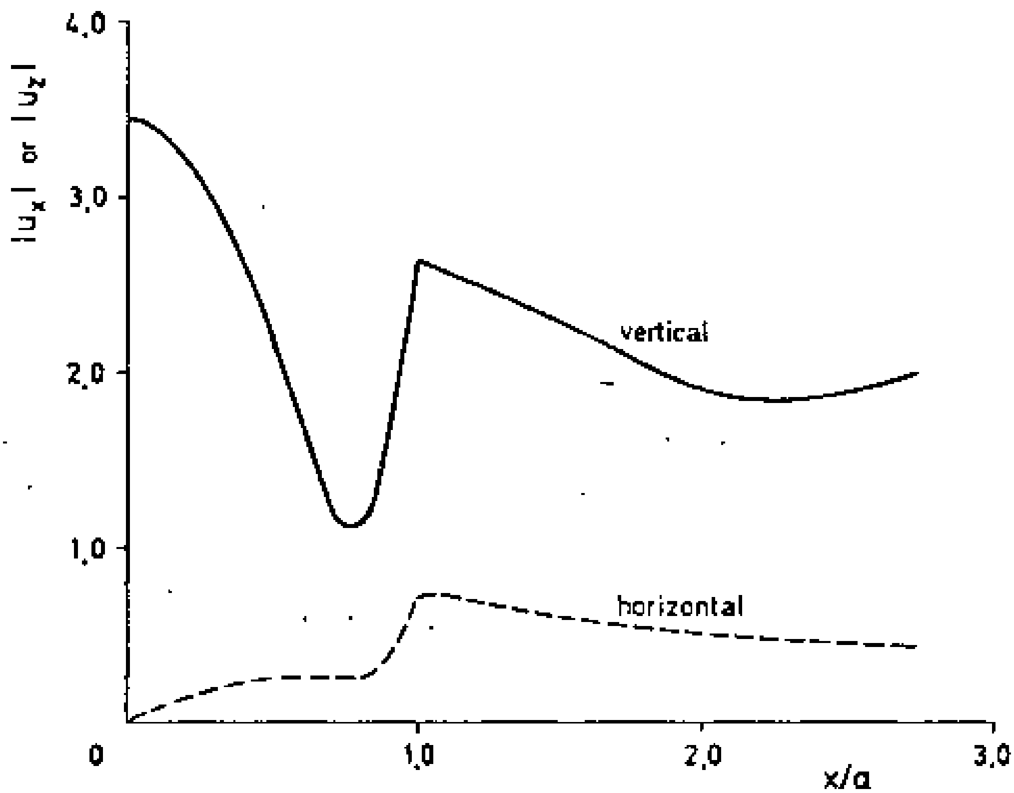

FIG. 5. Surface amplitudes of vertical and horizontal displacements. Semi-spherical cavity. Normalized frequency $\eta_v = 0.50$. Poisson coefficient $\nu = 0.25$.

the half-space over a length of three radii; and 13 on the free surface of the deposit. The orders of the expansions were 15 for the two regions. As can be seen in Figure 8, the amplification at the center of the deposit is about 170 per cent (as compared with the free-field which gives two). Horizontal motion is excited as in the cavity case and here, at $x/a = 0.5$, is about the 120 per cent of the amplitude of the incident field. For a flat layer with the same properties and thickness a, the motion would be amplified 60 per cent (actually, that value is 3.23) which means that the focusing generated by diffraction gives, in this case, amplification in the center of about three times the one calculated for a single flat layer.

Similar results were obtained for an alluvial deposit with the material properties of the previous example, but a different shape. The interface shape is given by the equation

$$z = h[1 - 3\xi^2 + 2\xi^3] \quad 0 \leq \xi \leq 1 \tag{31}$$

where h = maximum depth of the deposit, $\xi = 1/a(x^2 + y^2)^{1/2}$, and $2a$ = maximum horizontal dimension of the deposit. In Figure 9, the displacement amplitudes are displayed for a shape factor of $h/a = 0.5$ and normalized frequency of $\eta_u = 1.0$. The number of collocation points and the orders of the expansions used are equal to those of the previous case. Although the normalized frequency is here two times the

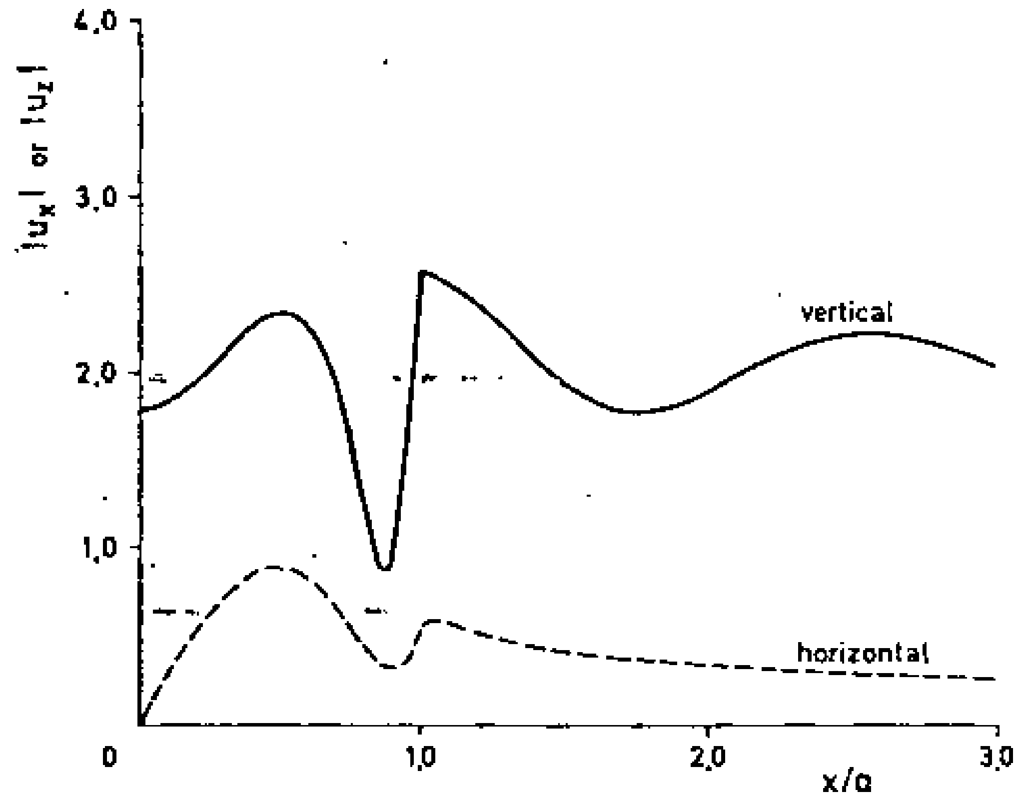

FIG. 6. Surface amplitudes of vertical and horizontal displacements. Semi-spherical cavity. Normalized frequency $\eta_u = 0.75$. Poisson coefficient $\nu = 0.25$.

value used for the last example, the depth-to-wavelength ratio is the same in both cases. This fact could serve to explain the similarities of Figures 8 and 9. However, the shapes of the interfaces, the frequency and the properties of materials chosen for the examples are arbitrary. Any generalization would require a parametric study to investigate, for example, the effects of interface depth and curvature, frequency and type of the excitation, and various material properties on ground motion. For the examples of alluvial deposits, calculated surface tractions are generally less than 5 per cent of the value of the greatest principal stress in the free-field solution. Differences of interface tractions for the two media are less than 2 per cent of the mentioned reference value.

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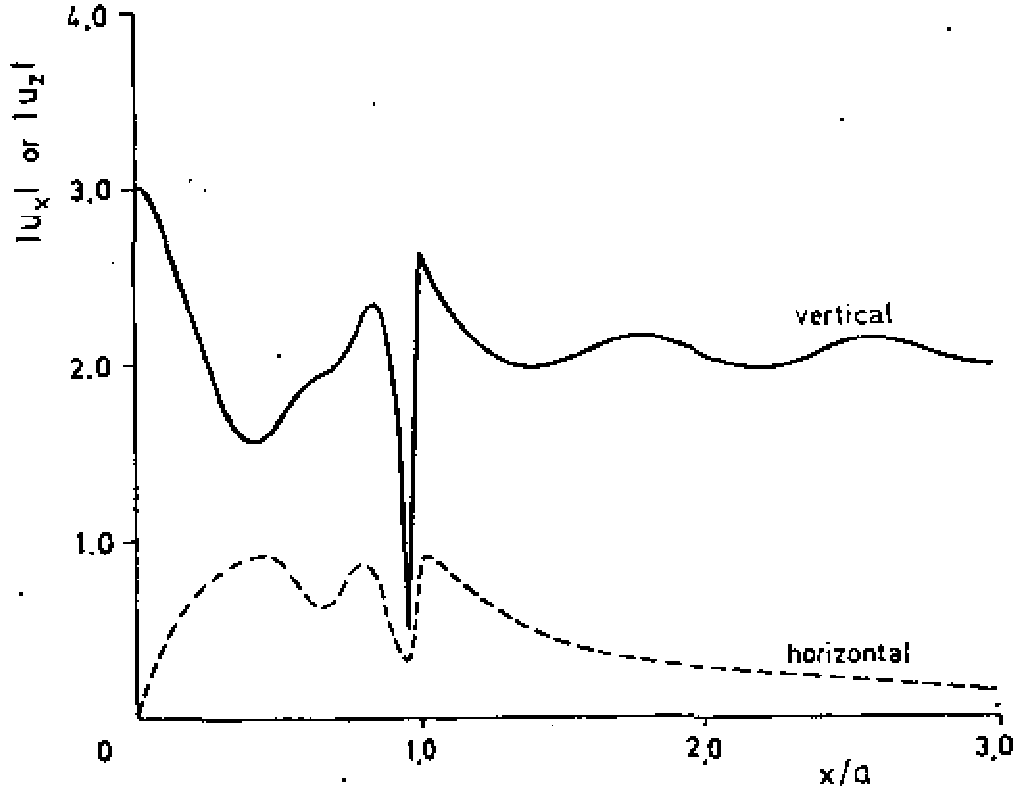
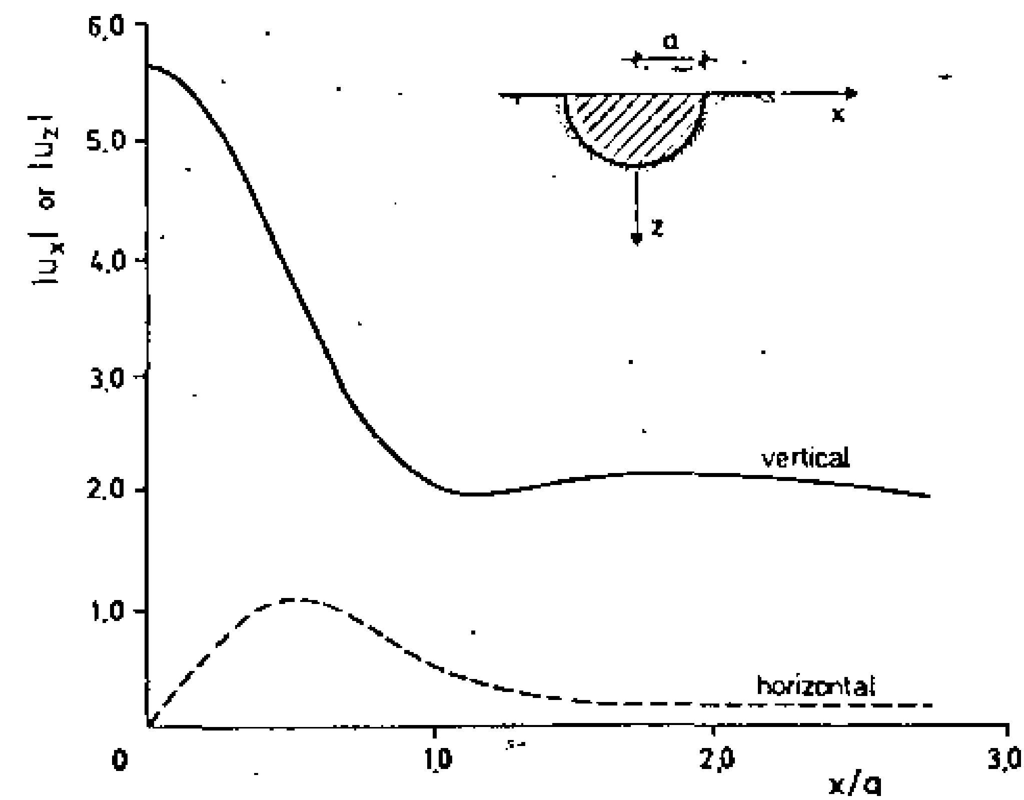


FIG. 7. Surface amplitudes of vertical and horizontal displacements. Semi-spherical cavity. Normalized frequency η_p = 1.50. Poisson coefficent ν = 0.25.



A ridge was also studied with a shape given by the negative of equation (31). Figure 10 shows the displacement amplitudes for vertical incidence of P waves with a normalized frequency $\eta_p = 0.5$. Poisson's coefficient is $\nu = 0.3$. Here, 90 collocation points were used distributed as follows: 30 in the fictitious interface; 20 in the surface of the ridge itself; and 40 in part of the free surface in a length of $3a$, where $2a$ = maximum horizontal dimension of the ridge. The orders of expansions were 15 for the internal and external regions. Residual tractions do not exceed 4 per cent of the reference value. An amplification of vertical motion of 60 per cent at the top of the ridge and a reduction of about 25 per cent near the base can be observed. In this example, horizontal motion has small relative amplitudes. The results for the

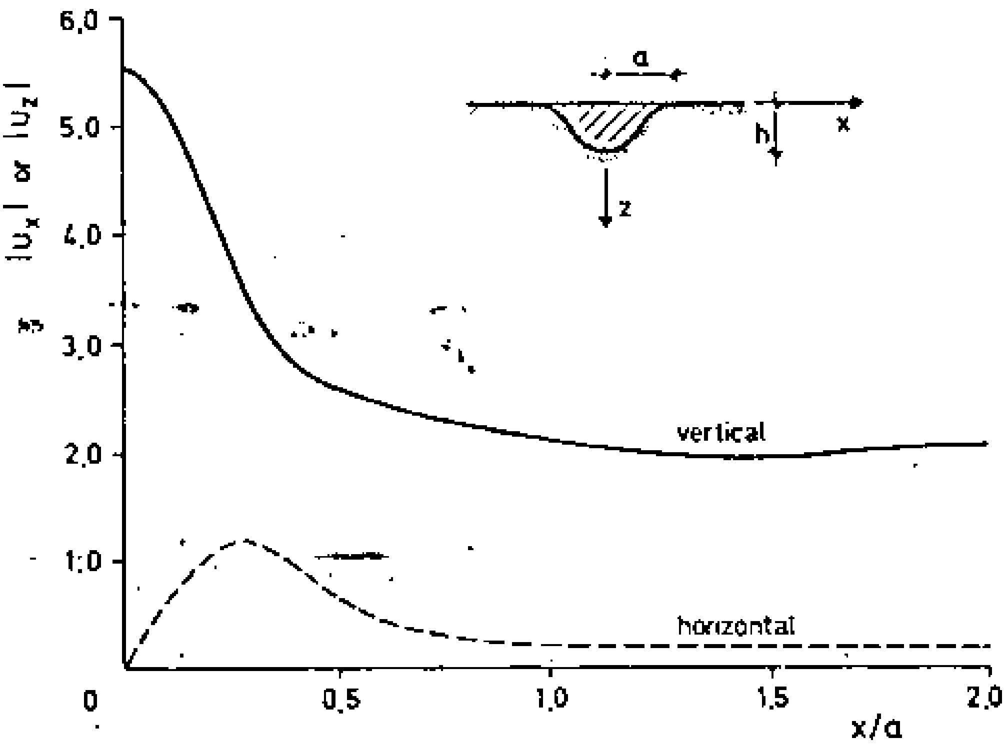


FIG. 9. Surface amplitudes of vertical and horizontal displacements. Axisymmetric alluvial deposit with $h/a = 0.5$. Normalized frequency $\eta_p = 1.0$. Poisson coefficients $\nu_R = 0.30$ and $\nu_E = 0.25$. Material properties $\mu_R/\mu_E = 0.3$ and $\rho_R/\rho_E = 0.6$.

ridge studied are similar to those obtained for incident SH waves on a two-dimensional ridge (Sills, 1978; Sánchez-Sesma *et al.*, 1982b).

The results presented in this section were obtained from a "trial and error" procedure to determine the order of expansions and the number and location of collocation points. This procedure was based upon the error analysis on boundary conditions and the stability of the calculated surface field. Typically, when the residual tractions are less than, say, 5 per cent of the reference value, the calculated displacement fields of various analyses do not present significative changes. Additional research is needed to generate rules for practical assessment of the calculation parameters.

FIG. 8. Surface amplitudes of vertical and horizontal displacements. Semi-spherical alluvial deposit. Normalized frequency $\eta_p = 0.5$. Poisson coefficients $\nu_R = 0.30$ and $\nu_E = 0.25$. Material properties $\mu_R/\mu_E = 0.3$ and $\rho_R/\rho_E = 0.6$.

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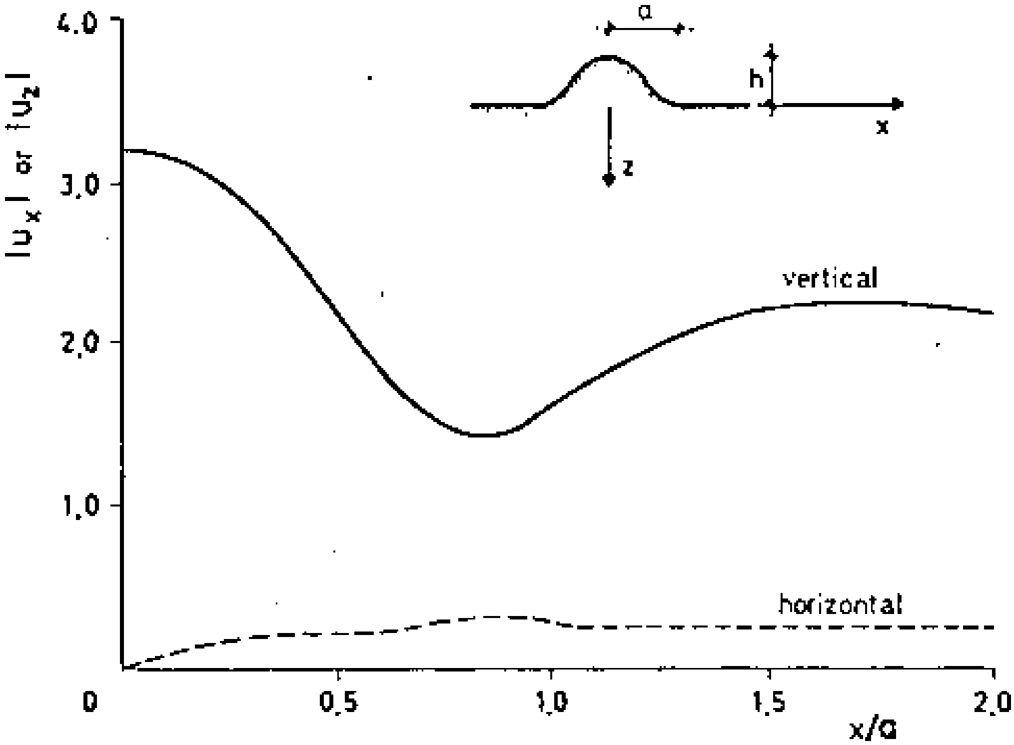

FIG. 10. Surface amplitudes of vertical and horizontal displacements. Axisymmetric ridge with $h/a = 0.5$. Normalized frequency $\eta_p = 0.5$. Poisson coefficient $\nu = 0.30$.

CONCLUSIONS

A boundary method has been applied to solve the scattering and diffraction of elastic waves by axisymmetric three-dimensional surface irregularities. The method consists of approximating the scattered fields with linear combinations of members of a c-complete family of wave functions which are solutions of Navier's equations.

An azimuthal decomposition has been presented which allows to "split" the problem into "two-dimensional" ones.

Some examples were solved for vertical incidence of P waves which have shown a satisfactory performance of the method, with small errors in boundary conditions, for the set of parameters chosen. Although a more complete parametric study is needed, to deal, for example, with the spatial phase variation of the surface displacement fields and the behavior of the solution for long distances from the scatterer, the obtained results confirm the significance of the problem.

ACKNOWLEDGMENTS

I am grateful to R. Madariaga and A. Deschamps for helpful discussions, and to D. M. Boore, E. Faccioli, J. W. Leonard, and F. Ovando-Shelley for the critical reading of the manuscript. The help of G. Bianchi and L. V. Hellina is appreciated.

This work was done while the author was on leave from the National University of Mexico at the Institute de Physique du Globe L.E.G.S.P., Université P. et M. Curie of Paris, France, and at the Dipartimento di Ingegneria Strutturale of the Technical University of Milan (Politecnico di Milano), Italy. The author acknowledges financial support by these institutions.

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Manuscript received 4 April 1983

X CURSO INTERNACIONAL DE INGENIERIA SISMICA

ANALISIS DE RIESGOS SISMICO

EFECTOS DE IRREGULARIDADES TOPOGRAFICAS TRIDIMENSIONALES EN EL MOVIMIENTO SISMICO

DR. FRANCISCO SANCHES SESMA

AGOSTO, 1984.

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EFECTOS DE IRREGULARIDADES TOPOGRAFICAS TRIDIMENSIONALES EN EL MOVIMIENTO SISMICO

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RESUMEN

Se aplica un método de frontera para resolver el problema de difracción de ondas elásticas por irregularidades topográficas tridimensionales en un semiespacio elástico, isótropo y homogéneo. En dicho método, los campos difractados se construyen mediante combinaciones lineales de funciones, que forman una familia completa de soluciones de la ecuación reducida de Navier, las cuales están dadas en términos de funciones esféricas de Bessel y de Hankel conjuntamente con funciones trigonométricas y de Legendre. Los coeficientes indeterminados de las formas lineales así construidas, se obtienen mediante el método de colocación con mínimos cuadrados aplicado a las condiciones de frontera de la irregularidad y de una zona finita de la superficie libre del semiespacio.*

Se supone simetría axial de las irregularidades, con el fin de trasformar el problema tridimensional en bidimensional al permitirse la descomposición azimutal en las soluciones generadas.

Se estudia el caso de incidencia normal de ondas P y SV a dos tipos de irregularidades: valles aluviales y promontorios. Los resultados numéricos que se presentan corresponden a las amplitudes de los desplazamientos en la superficie libre incluyendo la irregularidad.

INTRODUCCION

No hay dudas acerca del importante papel que juegan las condiciones topográficas y geológicas locales en las características de los temblores. La topografía y la estratigrafía en un sitio dado pueden hacer que la variación espacial del movimiento sísmico sea significativa y que lleguen a presentarse grandes amplificaciones en ciertos lugares debidas a la concentración de la energía sísmica.

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Se ha sugerido que la gran aceleración registrada en la presa Pacoima (1.25g) durante el temblor de San Fernando, California, en 1971 se haya debido a efectos topográficos (3, 28). Para las réplicas posteriores a este temblor, Davis y West (5) encontraron amplificaciones locales significativas generadas por la topografía. En estudios de campo con explosiones se han encontrado amplificaciones en la cima de promontorios de 1.7 a 3.4 veces las amplitudes registradas en sitios cercanos (9).

Hay evidencia de que las heterogeneidades laterales están relacionadas con la distribución espacial del daño ocurrido durante el temblor de Skopje, Yugoslavia, en 1963 (15). Parece ser que las interfases irregulares de la estratigrafía local generaron focalización de las ondas y, por ende, grandes amplificaciones en zonas limitadas de la ciudad (12).

Los efectos de las condiciones locales son de singular importancia en la estimación del riesgo sísmico, en estudios de planeación y microrregionalización y en el diseño sísmico de obras importantes (8, 16). En particular, las irregularidades locales deben considerarse al calcular la respuesta sísmica de estructuras largas como presas, puentes y líneas vitales (7, 17).

Muchos autores han estudiado el problema de irregularidades bidimensionales para varios tipos de ondas incidentes (p. ej. 2, 4, 23, 27, 29, 30) y ello ha permitido entender más del fenómeno de amplificación local. En contraste, pocos han sido los estudios que tratan casos tridimensionales. Para irregularidades con pendientes pequeñas se ha aplicado con éxito un método de perturbaciones de primer orden (11). El caso de una cavidad semiesférica en la superficie de un semiespacio se ha resuelto analíticamente bajo la hipótesis de que el medio es acústico (24). Sin embargo, esta condición es difícil de encontrar en la práctica por lo que los resultados son de utilidad limitada. Para la misma geometría pero con un medio elástico Lee (14) ha obtenido resultados para incidencia normal y oblicua de ondas P y S. Este método involucra un tratamiento muy ingenioso, y laborioso, de los coeficientes de los desarrollos en serie de los campos de ondas para satisfacer las condiciones de frontera. Parece ser, sin embargo, que este método está limitado a bajas frecuencias.

En este trabajo se estudia la difracción de ondas elásticas por irregularidades tridimensionales en la superficie de un semiespacio. Para ello se extiende un método de frontera desarrollado recientemente para problemas bidimensionales (p. ej. 6, 18, 20, 21, 23, 29). El método consiste en la superposición de la solución del problema en ausencia de irregularidad, llamada de campo libre, con la generada por las ondas difractadas que se representa mediante combinaciones lineales de soluciones que for man una familia completa de la ecuación reducida de Navier (10, 26). Los coeficientes de las formas lineales así construidas se obtienen de manera que el error cuadrático en las condiciones de frontera sea mínimo. Las soluciones mencionadas están dadas en términos de funciones esféricas de Bessel o de Hankel, asociadas con funciones de Legendre y trigonométricas (1, 26). Como cada una de esas soluciones no satisface por sí sola las condiciones de frontera libre en la superficie plana del semiespacio, el tratamiento numérico se extiende a parte de la superficie libre. Esta forma de proceder ha dado resultados satisfactorios en un problema bidimensional de incidencia de ondas P, SV y de Rayleigh (22).

Se ha supuesto simetría axial de la irregularidad, de manera que ello permite la descomposición azimutal. Es decir, el problema se puede resolver como una secuencia de problemas bidimensionales. Para incidencia normal de ondas P o SV sólo se requiere un número azimutal. Si la longitud horizontal aparente del campo incidente es grande, por ejemplo 2 veces la dimensión horizontal de la irregularidad, entonces unos 4 ó 5 términos de la descomposición azimutal darán buenos resultados.

En este escrito se presenta brevemente el método de solución (19) y se dan algunos resultados numéricos para el caso de incidencia vertical de ondas P o SV a diferentes irregularidades superficiales.

FORMULACION DEL PROBLEMA

Considérese un semiespacio elástico, lineal, homogéneo e isótropo con una irregularidad superficial axisimétrica como se muestra en la Fig 1. Las regiones exterior e interior se denotan por E y R, respectivamente. Sean $\partial_1 E$ y $\partial_1 R$, las fronteras libres de ambas regiones y $\partial_2 E = \partial_2 R$ la frontera común entre ellas. Ante incidencia de ondas elásticas se generan campos difractados y refractados en E y R, respectivamente. Los primeros deberán superponerse a la solución de campo libre para dar el campo de desplazamientos en el exterior.

Bajo la hipótesis de dependencia armónica del tiempo, el vector de desplazamientos, $\bar{u}$, debe satisfacer la ecuación reducida de Navier:

$$\mu \nabla^2 \bar{u} + (\lambda + \mu) \nabla \nabla \cdot \bar{u} + \rho \omega^2 \bar{u} = 0 \qquad (1)$$

donde ∇ = operador gradiente, λ, μ = constantes de Lamé, ρ = densidad de masa y ω = frecuencia circular. Las condiciones de frontera para $\bar{u}$ son tracciones nulas en $\partial_1 E$ y $\partial_1 R$ y continuidad de tracciones y desplazamientos en $\partial_2 E = \partial_2 R$. Adicionalmente, el campo difractado debe satisfacer la condición de irradiación de Sommerfeld-Kupradze (13, 25).

Sean las soluciones de la ec 1 de las formas

$$\bar{u}^{E} = \bar{u}^{(0)} + \sum_{j=1}^{3} \sum_{n=0}^{N} \sum_{m=0}^{n} A_{jnm}^{N} \bar{w}_{jnm}^{E} \qquad (2)$$

$$\bar{u}^{R} = \sum_{j=1}^{3} \sum_{n=0}^{N} \sum_{m=0}^{n} B_{jnm}^{N} \bar{w}_{jnm}^{R} \qquad (3)$$

para las regiones E y R, respectivamente. En la ec 2, $\bar{u}^{(0)}$ = vector de desplazamientos de campo libre y $\bar{w}_{jnm}^{E}$ = elemento del conjunto completo de soluciones de la ec 1 en E que satisface la condición de irradiación al infinito. En la ec 3, $\bar{w}_{jnm}^{R}$ = elemento del conjunto completo de soluciones de la ec 1 en R. A_{jnm}^{N} y B_{jnm}^{N} son coeficientes indeterminados y N es el orden de las aproximaciones.

A partir de las ecs 2 y 3 y las condiciones de frontera se obtienen ecuaciones para cada punto de la frontera (3 en $\partial_1 E$ o $\partial_1 R$ o 6 en $\partial_2 E = \partial_2 R$). Imponiendo dichas condiciones en un número suficiente de puntos de *colocación* y buscando que el *error cuadrático* a lo largo de la frontera sea mínimo se obtienen los coeficientes de las ecs 2 y 3. Este criterio de colocación y mínimos cuadrados ha sido probado en problemas bidimensionales (20, 22, 23).

CAMPO LIBRE Y CAMPOS DIFRACTADOS

Puede demostrarse (19) que los campos libres y difractados admiten representación en series de Fourier con respecto a la coordenada azimutal ϕ, que aparece en la Fig 2 donde se muestra el sistema de coordenadas esféricas. En el caso del campo libre la representación se obtiene a partir del desarrollo del factor de propagación exponencial en una serie de funciones de Bessel y cosenos de múltiplos de ϕ. Por otra parte, los campos difractados son de tres tipos: esferoidales de ondas P y S y toroidales de ondas S y son, en general, vectores de la forma

$$f_n(r)\ \bar{F}_n^m\ (\theta,\ \phi) \qquad (4)$$

donde $f_n(r)$ = función escalar que se expresa en sus diferentes formas mediante funciones esféricas de Bessel o de Hankel y $\bar{F}_n^m$ = función vectorial que se expresa con funciones trigonométricas de θ y la función

$$Y_n^m\ (\theta,\ \phi) = P_n^m\ (\cos\ \theta)\ e^{im\phi} \qquad (5)$$

y sus primeras derivadas. En la ec 5 P_n^m (.) = función de Legendre con m = 0, ± 1, ± 2, ..., ± n. Puede verse que también para los campos difractados se tienen expresiones en donde aparecen separadamente senos y cosenos de $m\phi$, donde m = número azimu tal. Si el difractor es axisimétrico se encuentra que la ortogonalidad de las funciones trigonométricas del ángulo azimutal ϕ permite resolver el problema como *bidimensional*, para cada número azimutal. Para incidencia vertical de ondas P o S basta tomar $m = 0$ ó 1, respectivamente.

RESULTADOS NUMERICOS

Se presentan resultados numéricos para el caso de incidencia normal de ondas SV y P a dos tipos de irregularidades topográficas: valles aluviales y promontorios. Las ondas incidentes, así como las difractadas, son función de las frecuencias normalizadas

$$\eta_k = \frac{ka}{\pi} = \frac{2a}{\Lambda_k} \quad \text{o} \quad \eta_q = \frac{qa}{\pi} = \frac{2a}{\Lambda_q} \qquad (6)$$

donde Λ_k, Λ_q = longitud de las ondas incidentes SV y P, respectivamente, a = semiancho de la irregularidad, k, q = número de onda de cortante y compresión, respectivamente y η_k, η_q = frecuencia normalizada de cortante y compresión, respectivamente. Todos los resultados que se presentan corresponden a ondas incidentes con frecuencia normalizada η_k o η_q igual a 1.0.

Para determinar el orden de los desarrollos así como el número y la posición de los puntos de colocación se emplea un procedimiento de *prueba y error*. Dicho procedimiento se basa en el análisis del error en las condiciones de frontera y de la estabilidad del campo de superficie libre. Para los cálculos, el orden de los desarrollos fue de 10 y el número de puntos de colocación de 30 distribuidos uniformemente en $\partial_2 R$, $\partial_1 R$ y una porción de $\partial_1 E$ con longitud de $2a$, con lo cual se obtuvieron tracciones residuales que no excedieron el 6% del máximo esfuerzo en la solución de campo libre.

En la Fig 3 se presentan resultados para el caso de incidencia de ondas P a un promontorio definido por

$$z = h\left[1 - 3\xi^2 + 2\xi^3\right],\ 0 \leq \xi \leq 1 \qquad (7)$$

donde h = máxima altura o profundidad de la irregularidad y $\xi = (x^2 + y^2)^{1/2}/a$. Las amplitudes de los desplazamientos corresponden a una relación de aspecto, h/a, igual a 1.0 y un módulo de Poisson, ν_E, igual a 0.25. Para el mismo ejemplo, pero para incidencia de ondas SV se obtuvieron los resultados que se muestran en la Fig 4. En ambos casos se presentan amplificaciones importantes con órdenes máximos de 2 para ondas P y 2.5 para SV.

En las Fig 5 y Fig 6 se presentan resultados para un valle aluvial, definido por la ec 7, cuando inciden ondas P y SV, respectivamente. En ambos casos los parámetros son $h/a = 0.5$, $\mu_R/\mu_E = 0.25$, $\rho_R/\rho_E = 0.75$, $\nu_R = 0.30$ y $\nu_E = 0.25$. En este ti po de irregularidad también se presentan amplificaciones importantes, con órdenes máximos de 2.0 para ondas P y 3.0 para SV.

Para la frecuencia adimensional considerada, en el promontorio se encontraron importantes amplificaciones y reducciones de las amplitudes de los desplazamientos, mientras que en el valle solo se encontraron amplificaciones.

CONCLUSIONES

Se ha aplicado un método de frontera para resolver el problema de difracción de ondas elásticas por irregularidades topográficas tridimensionales. Los resultados obtenidos muestran la generación de amplificaciones importantes en la superficie libre de las irregularidades, por lo que la influencia de estas en el movimiento sísmico puede ser significativa en diseño.

En este trabajo se estudiaron irregularidades axisimétricas e incidencia vertical. Es importante estudiar los casos de incidencia oblicua de ondas de cuerpo así como de ondas superficiales. Es de esperarse que los efectos de cambios del ángulo de incidencia sean significativos.

En análisis posteriores será importante estudiar la variación de los resultados para un rango de frecuencias suficientemente amplio.

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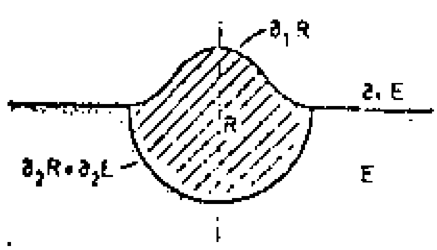

Fig 1. Definición de las regiones E y R y sus fronteras

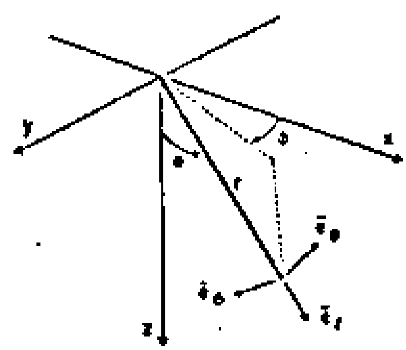

Fig 2. Sistema de coordenadas cartesianas y esféricas. Vectores unitarios en el sistema esférico

DIVISION DE EDUCACION CONTINUA
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X CURSO INTERNACIONAL DE INGENIERIA SISMICA

ANALISIS DE RIESGO SISMICO

BAYESIAN ANALYSIS OF SEISMIC HAZARD: AN UPDATING OF CONCEPTS, CRITERIA AND MODELS

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AGOSTO, 1984.

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BAYESIAN ANALYSIS OF SEISMIC HAZARD: AN UPDATING OF CONCEPTS, CRITERIA AND MODELS

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INTRODUCTION

Probabilistic models of hazard and risk constitute the basis for rational engineering decisions in the face of uncertainty. They provide a framework for the definition of quantitative criteria and rules for measuring safety, for balancing it with costs (both, initial and maintenance) and benefits, for making decisions relative to maximum tolerable risks and for stating policies with regards to desirable marginal expenditures aimed at protecting human lives.

Probabilities are usually deemed as properties of nature. According to this conception, evaluating them is a matter of observing a phenomenon a large number of times, quantifying the corresponding outcomes and plotting their histograms; if the number of observations is small, the probabilistic model itself is said to be uncertain or else the validity of a probabilistic formulation of the decision making process is denied. Consistently with the bayesian approach, the uncertainty about a probabilistic model of nature can be assimilated to the uncertainties intrinsic to that model** and used together with the

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** What we understand by *intrinsic* here depends on the variables that we use to describe the model: a large portion of the uncertainty *intrinsic* to the generation of earthquakes along a fault described by its size and rate of slip would be dispelled if we had a time history of the state of stresses throughout a region including the fault. That part of the uncertainty would now be ascribed to the model adopted, while the remaining portion would be called *intrinsic*.

latter with the aim of making decisions based on what we know and how well we know it. Thus, probability is not a property of nature (or not only, at least), but of our uncertainty about its past, present and future. The probability distributions that we would determine on the exclusive basis of direct statistical observations are conventionally branded as *objective* probabilities, while the uncertainty that arises from the practical difficulties to collect enough *objective* information constitutes the realm of *subjective* probabilities.

The arguments favoring and detracting the validity of subjective probabilities for making of engineering decisions have been discussed in extenso elsewhere (1, 2). Those arguments support the adoption of a bayesian framework for assimilating information and making decisions for hazardous environments. Rather than repeating those arguments, this paper concentrates on recent developments and criteria which have resulted from the experience gained in practical applications. The following concepts are given special attention:

a) Recent discoveries which contradict widely accepted assumptions about the shape of magnitude-recurrence curves.

b) Improved criteria for processing statistical information from seismic sources similar to those of interest when trying to propose prior probability distributions of the forms and parameters of seismicity models.

c) Study of simplified seismicity models represented by stochastic processes other than Poisson.

d) Discussion of decision criteria related to safety levels and analysis of the implications of bayesian uncertainty.

MAGNITUDE-RECURRENCE CURVES

Statistical studies about the occurrence of earthquakes in large regions of the earth have led to frequency recurrence curves similar to those depicted in fig. 1, which will be represented in this paper by an expression of the following form:

$$\lambda(M) = \alpha e^{-\beta M}(1 - e^{\gamma(M - M_u)}), \quad M < M_u \qquad (1)$$
$$= 0 \qquad\qquad\qquad\qquad , \quad M \geq M_u$$

Here, $\lambda(M)$ is the mean number of earthquakes with magnitude larger than M occuring per unit time in a given (unit) volume of the earth's crust within a given seismic source, M_u is the upper bound to the magnitudes that can be generated in the seismic source of interest, and α, β, γ are parameters obtained by statistical curve fitting. α and β determine λ for small values of M, and if the parenthesis in the second member of eq. 1 is taken as unity (*i.e.* if γ is taken as infinity), we obtain the well known Gutenberg-Richter exponential expression, which provides the possibility of estimating an upper bound to the expected rates of occurrence of large magnitude earthquakes on the basis of the statistical information about small magnitudes.

Careful studies recently developed show that at least in some regions eq. 1 does not hold. For instance, ref. 3 reports the results of analyzing the catalogues of NOAA Hypocentral Data File and of Preliminary Determination of Epicenters from 1963 to 1981 related to the Mexican subduction zone ($H \leq 65$ km; fig. 2). Figure 3 shows some results of the analysis for the Oaxaca portion of the subduction zone: solid squares represent numbers of earthquakes having body-wave magnitudes (m_b) larger than given values, while the solid circles correspond to surface-wave magnitudes (M_s); hollow squares and circles correspond to numbers of earthquakes in magnitude increments of 0.1. It is seen that, whereas magnitude-recurrence curves similar to those of fig. 1 are valid for body-wave magnitudes, the data of surface-wave magnitudes lead to a bulge in the range of large values of the latter; this bulge precludes the possibility of extrapolating magnitude-recurrence statistics from small to large magnitude intervals. Similar situations have been found elsewhere (5-7). When m_b values are used this effect is concealed because of saturation.

An explanation for the bulge in the $\lambda(M_s)$ curve for the Oaxaca seismic province can be provided by a model similar to that proposed in ref. 4. The plate interface in the region can be subdivided into a few large simple fault zones (40 to 100 km in length), the lateral boundaries of which terminate in barriers imposed by geometry or by abrupt strength variations. Each fault ruptures cyclically over its entire dimension without giving rise to smaller events in the manner required by eq. 1. The aftershocks, the background seismicity, and the foreshocks, taken together, follow eq. 1, but the maximum magnitudes of these sequences are well below the mainshock magnitude.

For the purpose of seismic hazard analysis, the following equation is used to represent magnitude-recurrence curves as shown in fig. 4:

$$\lambda(M) = \alpha(e^{-\beta M} - e^{-\beta M_1}) + \delta(1 - e^{\gamma(M - M_u)}) \qquad (2)$$

The first and second terms in this equation represent the background and mainshock activities, respectively; each of them must be taken equal to zero if $M \geq M_1$ or $M \geq M_u$, respectively. According to the observations, M can be substantially lower than M_u, but when trying to fit the parameters of eq. 2 to a set of data, practically equal results are obtained if M_1 is made equal to M_u, thus reducing by one the number of parameters to be estimated.

For the purpose of bayesian estimation of the parameters of $\lambda(M)$ in eqs. 1 and 2, it is necessary to propose prior joint probability distributions of those parameters. This is better achieved if a change of variables is performed, such that the new parameters can be taken as stochastically independent in their joint bayesian distribution, and if their marginal probability density functions can be easily related with indirect observations (by comparison with other seismic regions) or with quantitative models of the process of energy accumulation and release at seismic sources similar to that of interest.

When eq. 1 is applicable, the requirements in the foregoing paragraph are nearly satisfied if $\lambda(M)$ is expressed in terms of E_0, β, γ and M_u, where E_0 is the energy dissipated per unit time by generation of earthquakes with magnitude larger than a threshold magnitude M_0 , and the other parameters were defined above. The prior marginal probability density functions of E_0, β and M_u can be postulated independently; those of E_0 and M_u may be based on both physical considerations and observations in similar regions, whereas that of β will in general be based exclusively on the latter type of concepts, and very little can be said a priori about γ. If the natural logarithm of the energy dissipated by a shock of magnitude M is expressed as $A + BM$, then E_0 is given as follows:

$$E_0 = D_1(e^{k_1 M_u} - e^{k_1 M_0}) + D_2 \; (e^{k_2 M_u} - e^{k_2 M_0}) \qquad (3)$$

where $D_1 = -\alpha\beta e^{A} k_1^{-1}$, $D_2 = -\alpha(\gamma - \beta)e^{A - \gamma M_u} k_2^{-1}$

$k_1 = B - \beta$, $k_2 = B - \beta + \gamma$

From eq. 3, α can be obtained as a function of E_0, β, γ, M_u, for any specified M_0.

Similarly, if eq. 2 is applicable, and it is decided to take $M_1 = M_u$, it may be convenient to take as independent parameters E_0, μ, β, γ and M_u, where μ is the ratio of the energies dissipated by the first and second processes considered by eq. 1. These energies are, respectively,

$$E_{01} = D_3 \; (e^{k_3 M_u} - e^{k_3 M_0}) \qquad (4a)$$

$$E_{02} = D_4 \; (e^{k_4 M_u} - e^{k_4 M_0}) \qquad (4b)$$

where $D_3 = -\alpha\beta e^{A} k_3^{-1}$, $D_4 = -\gamma\delta e^{A - \gamma M_u} k_4^{-1}$

$k_3 = B - \beta$, $k_4 = B + \gamma$

Given E_0, μ, β, γ and M_u, it is easy to obtain α and δ.

MULTI-PARAMETER BAYESIAN SEISMIC HAZARD ANALYSIS

The assessment of seismic hazard in practice is based as a rule on information about concepts of different nature, including, among others, statistical data about the activity of seismic sources and qualitative or quantitative descriptions of the geotectonic environment. The latter information is usually taken by beologists and geophysicists as the basis for making estimates of maximum magnitudes that can be generated at given sources. These estimates are obtained by extrapolation of magnitudes observed in other regions with comparable geotectonic conditions; they include significant subjective components, such as selecting the sources from which extrapolations are made, or deciding about the maximum length of a particular fault that can rupture during the largest possible earthquake. The estimates mentioned tend systematically to err on the safe side, which is reasonable under minimax decision rules: risk to facilities should be kept to a minimum, regardless of cost.

If decisions are to account for a proper balance between costs, benefits and risk, the latter must be expressed in quantitative terms. This entails assigning quatitative measures to uncertainties tied to hazard estimates (maximum possible magnitude, rate of activity) based on geology, and defining probabilistic models capable of assimilating the mentioned uncertainties with those attached to predicting the seismic history on the basis of maximum possible magnitude and rate of activity. All this can be achieved through bayesian analysis, which is not a substitute for geotectonic and statistical information, but an efficient tool for processing available knowledge consistently. A well founded criticism which can be made to the subjective assignment of probabilities by individual experts to alternate hypotheses concerning the nature and parameters of seismotectonic processes is the possibility of arriving at prior bayesian distributions which do not reflect the levels of uncertainty implied by the available information. In fact, we even lack a criterion for judging about the consistency between that information and the assigned probabilities. Under some consitions, the analysis of statistical data for a number of seismotectonic regions similar to that of interest may serve to gage the consistency in question, as shown below. In other, we shall have to pursue consistency by a mechanism permiting the analysis by a group of experts of the subjective probabilities assigned (including, of course, the basis for those assignments) by a number of independent experts or groups of experts.

In this section and in the following, we consider that the seismicity of a given source is defined by its activity rate $\lambda(M)$, and therefore we deal with the problem of making bayesian estimates of the parameters of $\lambda(M)$ in eqs. 1 and 2. This section is devoted to the practical application of Bayes theorem, while next section concentrates on the formulation of the prior distribution of the mentioned parameters.

Let ε be the vector of parameters that determine $\lambda(M)$ (for instance, if we work with eq. 1, $\varepsilon^T = [\alpha\ \beta\ \gamma\ M_u]$). If H represents the observed seismic history, Bayes theorem states that

$$f''_{\varepsilon}(e|H) = K f'_{\varepsilon}(e) p(H|e) \tag{5}$$

where f'_{ε} and f''_{ε} are respectively prior and posterior joint bayesian probability density functions of ε, $p(H|e)$ is the likelihood of H given e and K is a normalizing constant such that the multiple integral of $f''_{\varepsilon}(e|H)$ over the region of definition of ε equals unity.

In general, H will consist of the magnitudes and times (M_i, t_i; $i = 1, \ldots, N$) of earthquakes occurred during time interval t. If H is an observed realization of the process of occurrence of earthquakes above the threshold value M_0, and it is assumed to be a Poisson process with all magnitudes identically distributed and mutually independent, then

$$p(H|e) \propto e^{-\lambda(M_0)t} \prod_{i=1}^{N} (-\lambda'(M_i)) \tag{6}$$

where $\lambda' = d\lambda/dM$. An equation similar to eq. 6 is given in ref. 1 for the case in which seismicity is expressed by the rate of exceedance of different intensity values at a given site.

The determination of K in eq. 5 implies carrying out a multifold integration, unless f_ϵ' can be factorized. Otherwise, in practical applications we may have to recourse to numerical integration, Monte Carlo simulation or discrete distribution techniques (8). In some cases, and provided the conditional probability density function of magnitudes given the occurrence of a random event is independent of time and previous history, it may prove advantageous to split the parameter estimation process in two steps: first, estimating the parameters of the conditional distribution of M, and second, estimating a parameter proportional to the energy dissipated per unit time. If eq. 1 is adopted, this is tantamount to obtaining initially the joint bayesian p.d.f. of β, γ and M_u, and then the conditional p.d.f. of α for given values of these parameters. For the first part, and starting for instance from eq. 1, one obtains that the conditional p.d.f. of M given that an earthquake occurs is as follows:

$$f_M(m) = -Ae^{-\beta m}\left[\beta + (\gamma - \beta)e^{-\gamma(M_u - m)}\right] \qquad (7)$$

where $A = e^{\beta M_0}\left[1 - e^{-\gamma(M_u - M_0)}\right]^{-1}$. The likelihood function to be used in eq. 5 is given by eq. 8.

$$p(H|c) \propto \prod_{i=1}^{N} f_M(m_i;\ \beta,\ \gamma,\ M_u) \qquad (8)$$

For the purpose of updating the conditional distribution of α for given values of β, γ and M_u, we can use as statistical information the number of earthquakes with magnitudes greater than the threshold value M occurred during time interval t. If we substitute M_u for M in eq. 1 and then express the latter in the form $\lambda(M_0) = \alpha G(\beta, \gamma, M_u)$, the conditional distribution of α can be readily obtained from that of $\lambda(M_0)$: updating of the latter distribution on the basis of the statistical information is straightforward (9).

The criterion described by eqs. 5-8 is applied in the following to a fictitious example represented by a simulated record of earthquake magnitudes. A hypothetical source with $f_M(m)$ given by eq. 7 with $\beta = 2$, $\gamma = 3$, $M_0 = 5.5$, $M_u = 7.5$ was assumed and a set of 150 stochastically independent magnitude values was simulated from that p.d.f. In order to study the variability of the seismicity estimates with the sample size, Bayes theorem was applied to determine the joint distribution of β, γ and M_u, considering successively the complete set of simulated magnitudes and two subsets of sizes 50 and 100. These simulated magnitude values are shown in a cumulative magnitude-recurrence plot in fig. 7 together with the theoretical $\lambda(M)$ curve from which the simulated record was obtained. The bayesian probability distributions of β, γ and M_u were supposed to be discretized at all combinations of the following feasible values of each parameter: $\beta = 1.8, 2.6$; $\gamma = 0.5, 2, 4$; $M_u = 7.3, 8$. The feasible values assumed for γ were obtained by inspection from the simulated records of fig. 7; those for γ were arbitrarily adopted, and the value $M_u = 7.3$ was proposed from the condition that it should have to be greater than the maximum simulated value, *i.e.*, 7.18. Thus, the domain of the multiparameter bayesian distribution considered is that of the twelve cases included in Table 1. The prior bayesian probabilities are 1/12 for each case. The posterior distributions derived from application of eqs. 5-8 to the three sample sizes ($N = 50, 100, 150$) are shown in the same table. Table 2 summarizes the means and variation coefficients of β, γ, M_u as well as of the complementary cumulative magnitude distributions, $\bar{F}_M(m)$. The corresponding deterministic values are shown for comparison in the last column. Although the means of all parameters and functions seem to approach systematically their deterministic values, significant random deviations disturb those trends. The deviations corresponding to β, γ and M_u are more pronounced than those affecting $\bar{F}_M(m)$. Similar statements can be made about the systematic decrease in the variation coefficients with increasing sample size. Further analysis of fig. 7 and of the results of similar cases shows that adoption of discretized distributions of the seismicity parameters must be done with caution, considering, among others, the following problems.

a) In some cases the statistical information may warrant posterior probability distributions having a pronounced peak in the vicinity of one or more parameters (for instance, those cases in which β is very well defined). Such a peak cannot be reproduced unless the discrete mesh adopted a priori includes one or more points sufficiently close to its maximum; otherwise, the posterior distribution will overestimate uncertainty with respect to the mentioned parameters.

b) In regions where the slopes of the likelihood function of the seismicity parameters given the sample are large the posteior bayesian probability masses may concentrate excessively on one or few points, failing to represent the uncertainty determined by the continuous posterior distribution corresponding to the rigorous solution.

Bayesian analysis of seismicity under assumptions more general than Poisson process is a topic practically unexplored, in spite of the significantly more sophisticated models which have been developed (10-12), capable of representing effects as complex and as relevant as systematic variation of hazard with time, aftershock sequences and more general types of clustering. This omission results from the complexities arising from having to cope simultaneously with too many parameters as well as from the difficulties involved in obtaining likelihood functions.

If fore- and aftershocks are ignored, one can think of adopting renewal process models other than Poisson.

Reference 9 deals with the bayesian analysis of a renewal process with inter-arrival times (T) distributed in accordance with a gamma function:

$$f_T\ (t) = \frac{\nu}{(k-1)!}\ (\nu t)^{k-1}\ e^{-\nu t} \qquad (9)$$

where ν and k are the parameters to be estimated. Only very simple cases are covered, assuming k known. It is obtained, for instance, that if the prior p.d.f. of ν is gamma with parameters ρ and μ, and if H is expressed as the time T_n elapsed between $n+1$ consecutive events, then the posterior p.d.f. of ν is also gamma, now with parameters $\rho + nk$ and $\mu + t_n$. In the latter case, only a portion of the relevant statistical information is used. In most cases, specially if seismic activity has been low during the observation interval, significant information is provided by the durations of the intervals elapsed from the initiation of observations to the first of the $n+1$ events considered and from the last of those events to the end of the observation interval. Here, the posterior p.d.f. of ν has to be obtained by application of eq. 5 with the following likelihood function:

$$p(H|c) = f_{T_1}(t_1) \cdot f_{T_{n-1}}(t_n - t_1) \cdot [1 - F_T(t_f - t_n)] \qquad (10)$$

The first factor in the second member of this equation is the probability density function of the waiting time to the first (delayed) event. T_1 can be taken as the excess life in a renewal process at an arbitrary value of time that tends to infinity and its p.d.f. can be obtained as $\frac{1}{\bar{T}}|1 - F_T(t)|$, where T is the random time between consecutive events, F_T is its cumulative probability distribution function and $\bar{T}$ is its expected value (13). The second factor is the probability density function of the time between events 1 and n, evaluated at $t_n - t_1$, and t_f in the last factor is the end of the observation interval. For the particular case where the statistical record reports no events during the time interval $(0, t_f)$, the second member of eq. 10 must be replaced with $p\{T_1 \geq t_f\}$. This particular case is typical of seismic gaps, and is studied in ref. 9. The results show that the posterior expected value of T_1, the waiting time to the first event, conditional to no occurrence of events up to instant t may start decreasing as t grows, in accordance with the behaviour of ordinary renewal processes; however, as time goes on and no events occur, the statistical evidence makes the estimated risk to decrease, and therefore the conditional expected value of T_1 to increase with t. The smaller the value of k in eq. 9, the faster the decreasing of risk estimates.

A summary is presented in ref. 2 of the results of an analysis of the influence of k on the ratio of the present value of expected failure costs for gamma and Poisson processes. The structures considered have deterministic properties. Because the coefficient of variation of the waiting time between events is inversely proportional to the square root of k, the greater k the lesser the uncertainty about the waiting time to next event. The ratio of expected failure costs for gamma and Poisson processes is smaller than unity for small values of the time elapsed since last event and grows with that time. The greater k the more pronounced are these variations.

A likelihood function as given by eq. 10 is meaningful if we define as *events* all earthquakes with magnitudes larger than a given value; if we assume that the distribution of each magnitude is independent of time and history (renewal process with independent random selection), bayesian estimates of risk can be made on the basis of eq. 8 together with application of eq. 10 for all earthquakes with magnitudes larger than M_0; but the assumption of independent selection is inconsistent with the concept that the times required for the accumulation of the energy amount necessary to produce shocks of given magnitudes should grow with those magnitudes, and that the time elapsed since the occurrence of a small magnitude earthquake should not affect significantly the energy available for large shocks. If we could filter out fore- and aftershocks (which is much more easily said than done), it would be reasonable to conceive the ocurrence of earthquakes above a given magnitude M_0 as a renewal process with parameter k dependent on M_0. A low value of M_0 considers the occurrence of many small and a few large earthquakes; a random event would very unlikely give place to a significant change in the energy available for future shocks, and the assumption of Poisson process (k = 1) would be warranted. For large M_0 values this assumption is untenable; therefore, we arrive at the problem of assimilating a set of statistical data relative to different magnitude ranges, and using it as an ensemble for estimating the parameters of a complex process, which cannot be represented as a renewal process with independent random selection. The problem is solved if we relate the hazard at any instant with the previous history of energy dissipation. For this purpose we adopt an extended version of a seismicity model proposed in ref. 10.

Suppose first that the seismic record does not contain any fore- or aftershock. Represent that record by a Markov process with system states specified by the values of the hazard function $\lambda(t)$, where $\lambda(t)dt$ is the conditional probability that there will be one earthquake (of any magnitude) during time interval t, $t + dt$ given that none has occurred. It is assumed that the distribution of the energy (or magnitude) of an earthquake is independent of history; it is also assumed that $\lambda(t)$ decreases sharply every time there is an earthquake and that the size of the discontinuity can be deterministically related to the energy of the earthquake producing it, in terms of parameters to be estimated.

A typical sample of the process (where, of course, the values of λ are non-observable variables) is shown in fig. 5. If λ is deterministically related to the strain energy stored in the system, and if this energy increases at constant rate during time intervals between shocks, it follows that all segments of the $\lambda(t)$ curve can be obtained by translation of the initial hazard function. Also, the size of each discontinuity can be determined from the corresponding earthquake energy by considering that the time origin of the $\lambda(t)$ curve is translated an amount equal to the length of time required to store the mentioned energy:

$$\tau_{k+1} - \tau_k = E_{k+1}/\omega$$

Here, E_i is the energy liberated by the i-th shock and ω is the rate of energy accumulation. The latter is one of the parameters to be estimated.

Before talking about the problem of bayesian estimation of parameters, let us discuss a possible family of hazard functions and their application when trying to relate E with jumps in λ. A particular case of the Weibull distribution corresponds to the following p.d.f. (13):

$$f(t) = \frac{k}{\upsilon}\left(\frac{t}{\upsilon}\right)^{k-1} \exp\left(-\left(\frac{t}{\upsilon}\right)^{k}\right) \qquad \begin{array}{l} \upsilon \geq 0 \\ k \geq 1 \\ t \geq 0 \end{array} \tag{11}$$

If the waiting time between earthquakes is assumed to possess this distribution, the hazard function is:

$$\lambda(t) = \frac{k}{\upsilon}\left(\frac{t}{\upsilon}\right)^{k-1} \tag{12}$$

If $k = 1$, one has Poisson process with $\lambda(t) = 1/\upsilon$. For other values of k one gets the types of hazard function shown in fig. 6.

Let us take for simplicity $A = k/\upsilon^{k}$, $\pi = k - 1$. Then $\lambda = At^{\pi}$. Immediately prior to the $(k+1)$th shock, the hazard function is $\lambda'_k = At^{\pi}$, and just after a shock with energy E_k it is $\lambda_{k+1} = A(t - E/\omega)^{\pi}$. Thus, λ_{k+1} can be expressed in terms of λ'_k:

$$\lambda_{k+1} = A\left[\left(\frac{\lambda'_k}{A}\right)^{1/\pi} - E_{k+1}/\omega\right]^{\pi}$$

Suppose λ_0, A and π were known. Then time history (t_1, E_1; t_2, E_2; ... t_n, E_n) would determine $\lambda(t)$ for any t. The only function we need in order to complete the model is the conditional p.d.f. of energy for any shock. This may have for instance a form consistent with eq. 1. It will be represented as $f_E(e; \varepsilon)$, where ε is a vector of parameters to be estimated.

If eq. 1 applies, $\varepsilon^T = \{\alpha\ \beta\ \gamma\ M_u\}$. Given $B^T = \{\lambda_0\ A\ \pi\ \varepsilon^T\ \omega\}$ (the vector of parameters to be estimated) one can obtain their likelihood. Once this is done, the problem of bayesian updating of the prosed seismicity model is solved (at least in theory). The mentioned likelihood is obtained as follows:

$$L(t_1, t_2, \ldots, t_n; e_1, e_2, \ldots, e_n | \tau_0, A, \pi, \varepsilon, \omega) =$$
$$\prod_{j=1}^{n} f_T(t_j - \tau_{j-1}) f_E(e_j)(1 - F_T(t_f - \tau_n))$$

where t_j is the time of occurrence of the j-th earthquake, t_f is the end of the observation interval, f_T is the probability density function of the waiting time from the fictitious time origin of the hazard function (see fig. 5), f_E the p.d.f. of the energy liberated by a randomly selected earthquake, e_j the energy liberated by the j-th earthquake; because τ_j is a deterministic function of τ_0, A, π, and all observed values t_k, e_k, k = 1, ..., n, the second member in the last equation is a function of the variables in vector B defined above.

ON THE CONSISTENCY OF PRIOR BAYESIAN DISTRIBUTIONS OF SEISMICITY PARAMETERS

The prior distribution of the seismicity parameters of a potential seismic source is an efficient description of the estimates that a team of experts can make concerning those parameters before looking at the local seismic record. That distribution should account for all available knowledge about the local geologic structures and tectonic processes, as well as for the seismicity patterns observed at other regions with similar geologic and tectonic characteristics. Also, that distribution summarizes the extrapolations that a capable team of experts in the geophysical sciences can make on the basis of observations throughout the globe; therefore, it must reflect the degree of uncertainty tied to those extrapolations. A simple way of expressing that uncertainty is by formulating a comprehensive and mutually exclusive set of hypotheses about the possible models of the seismic process at a source and assigning to each hypothesis a weight, taken by definition as proportional to the bayesian probability of its being the correct representation of the natural process (in reality, due to the fact that the set of hypotheses does not cover the universe of all possible models, we understand by "the correct representation of the natural process" that model within the set which best represents nature). But no uniquelly determined criteria have been proposed capable of transforming information other than direct statistical data into prior bayesian probability distributions: no doubt, subjective probabilities can be used as descriptors of degrees of belief, and then used for decision making under uncertain risk conditions, but the decision maker faced with stating subjective probability distributions reflecting his beliefs does not know what a subjective probability should measure, what consistency rules should it satisfy and how well founded are his degrees of belief. This kind of problems requires immediate attention by those interested in decision making on the basis of quantitative risk information. A partial solution can be provided by having groups of experts assigning, evaluating and correcting probability values and calibrating the final decisions with those that would be obtained by a rational analysis of the mentioned probabilities. In some cases the adequacy of given probability distributions of seismicity parameters assigned on the basis of the information available for a number of *comparable* regions can be judged by contrasting them with the statistics observed on those regions. The concepts and criteria which may be used for such evaluations are exemplified in the sequel.

Take a set of potential seismic sources corresponding to regions with similar geologic and tectonic characteristics. They may differ in some quantitative aspects, such as size of faults, intensity of faulting (cross area of faults per square kilometer of ground surface) or rate of crust deformation or fault slip; they may, instead, be undistinguishable as far as the mentioned features are concerned. In the first case the statistical seismicity information can be used to estimate the parameters of a predictive model of expected seismic activity in sources of the type considered in terms of quantitative measures of the geologic and tectonic structures and processes. In the second case all the regions may be considered to be samples obtained from a population characterized by the probability distribution of the seismicity parameters of a randomly selected region. This distribution, if known, might be taken as the prior bayesian probability distribution of the mentioned parameter for a new region of interest, assumed to belong to the same population. Thus, the regions included in the original set shall be called *auxiliary regions* and the collective statistical information available for them shall be used as the basis for stating a prior probability distribution for the seismicity parameters of the new region. The statistical information for the latter should then be combined with the mentioned prior probability distribution in order to obtain the corresponding posterior distribution through use of eqs. 5-8.

Returning to the set of auxiliary regions, let ε_i, i = 1, ..., N, be the vector of parameters of an expression similar to the second member of eqs. 1 or 2, that is, of a given magnitude-recurrence expresion, $\lambda_M(m; \varepsilon_i)$, valid for the *i-th* auxiliary region. Let $f_{\varepsilon|\mu}(e|m)$ be the bayesian p.d.f. of vector ε for a randomly selected region, and μ the parameters determining that p.d.f. Because μ is uncertain we have to solve the problem of obtaining its bayesian probability distribution $f_\mu(m)$; for this purpose we shall recourse to the observed seismic history h at all the regions. If this information were sufficient to permit obtaining accurate estimates of ε_i for $i = 1, \ldots, N$, the bayesian estimation of $f_\mu(m)$ would be formulated as follows:

$$f''_{\mu|\varepsilon}(m|e) = K_1 f'_\mu(m) f_{\varepsilon|\mu}(e|m) \qquad (15)$$

Here, $f_{\varepsilon|\mu}(e|m)$ is the likelihood function of $\mu = m$ conditioned to $\varepsilon = e$, K_1 is a normalizing constant and f'_μ, $f''_{\mu|\varepsilon}$ are respectively prior and posterior bayesian density functions of μ. As a rule, h does not suffice for making sufficiently accurate estimates of ε. In other words, ε is not observable and therefore eq. 15 cannot be applied directly. We must instead try to obtain a posterior distribution of μ conditioned to the observed seismic history: $H = h$. This is expressed as follows:

$$f''_{\mu|H}(m|h) = K_2 f'_\mu(m) p_{H|\mu}(h|m) \qquad (16)$$

where K_2 is a normalizing constant, f'_μ and $f''_{\mu|H}$ are prior and posterior distributions and, in accordance with the rules of conditional probabilities,

$$p_{H|\mu}(h|m) = \int p_{H|\varepsilon}(h|e) f_{\varepsilon|\mu}(e|m)\,de \qquad (17)$$

The marginal posterior p.d.f. of ε for a randomly selected region is, therefore,

$$f''_{\varepsilon|H}(e|h) = \int f_{\varepsilon|\mu}(e|m) f''_{\mu|H}(m|h)\,dm \qquad (18)$$

The latter p.d.f. would be taken as the prior p.d.f. of ε for a new seismic region assumed to belong to the same population as the original set.

The team of experts analyzing the characteristics of the different regions may assume a priori a certain degree of correlation among vectors ε_i and ε_j. This assumption will be reflected in $f_{\varepsilon|\mu}(e|m)$. Thus, if ε_i and ε_j are independent,

$$f_{\varepsilon|\mu}(e|m) = \prod_{i=1}^{N} f_{\varepsilon_i|\mu}(e_i|m) \tag{19}$$

while in a more general case the correlation matrix may form part of μ. Either for practical reasons or on the grounds of geophysical similarity, it may be justified to assume at least some parameters of the correlation matrix. In the extreme case when all regions are so similar that they are assumed to possess the same ε, we obtain the condition of perfect correlation, which is tantamount to taking together all the regions and all the statistical information and using it as a single block in the estimation of the vector ε valid throughout all regions.

The use of eqs. 15-18 is illustrated in the following for a hypothetical set of two regions. For simplicity, it is assumed that we are interested only in the rate of activity above a given threshold value, and therefore vectors ε_i will have only one component each, the corresponding rate λ_i, $i = 1, 2$. The complete set of seismicity parameters is therefore $\varepsilon^T = [\lambda_1 \lambda_2]$. The observed history is described by N_i and t_i, $i = 1, 2$, the number of events observed and the duration of the observation interval for each region. The following values are assumed here:

$N_1 = 3$, $t_1 = 50$ years, $N_2 = 3$, $t_2 = 80$ years; the corresponding estimates of λ_i are therefore 0.06 and 0.0375, respectively.

λ_1 and λ_2 are assumed independent, with their bayesian distributions discretized at three points. The vector μ of parameters of the bayesian distributions of λ_1 and λ_2 is $\mu^T = [E(\lambda_1), E(\lambda_2), V(\lambda_1), V(\lambda_2)]$, where $E(\cdot)$ denotes expectation and $V(\cdot)$ variation coefficient.

The distribution of μ is discretized in the following values: $E(\lambda_i) = 0.02, 0.08$; $V(\lambda_i) = 0.2, 0.6$; $i = 1, 2$. The prior distribution of μ assigns equal values to each of the possible combinations of these parameter values. Table 3 summarizes the probability mass function of ε_j for each $E(\lambda_i)$ and $V(\lambda_i)$.

The likelihood functions for each $\mu = m$ are obtained as follows:

$$p_{H|\mu}(h|m) = \sum_{ij}\sum p(h|\ell_i, \ell_j)\, p(\lambda_1 = \ell_i, \lambda_2 = \ell_j|m)$$

and because the prior distribution of μ is uniform, the posterior distribution is proportional to the likelihood function:

m: $\bar{\lambda}_i$, V_{λ_i}	$p(h\|m)$	$p''(m\|h)$
0.02, 0.2	8.209×10^{-3}	0.217
0.02, 0.6	6.435×10^{-3}	0.170
0.08, 0.2	1.269×10^{-2}	0.363
0.08, 0.6	9.416×10^{-3}	0.249

The marginal probability mass function of ϵ is, therefore,

$$p''_{\lambda_i|H}(\ell_i|h) = \sum_j p_{\lambda_i|\mu}(\ell_i|m_j)p''_{\mu|H}(m_j|h)$$

which leads to:

ℓ_i	$p''(\ell_i\|h)$
0.01	0.0399
0.02	0.4200
0.08	0.4799
0.16	0.0595

$E''(\lambda_i) = 0.0567$

$V''(\lambda_i) = 0.633$

The foregoing analysis assumed λ_1 independent from λ_2. An alternate assumption might have considered $\lambda_i = r_i\lambda$, with r_i and λ independent and the distribution of r_i prescribed. Considering this case, taking the distribution of r_i as discretized at points 0.7 and 1.3 with equal probabilities, and the prior distribution of λ as discretized at points 0.02 and 0.08, also with equal probabilities, we obtain the following posterior distribution of λ: $p_\lambda(0.02) = 0.332$, $p_\lambda(0.08) = 0.668$. From this distribution and that of r_i we obtain for λ the following posterior distribution:

ℓ_i	$p_\lambda(\ell_i)$
0.014	0.166
0.026	0.166
0.056	0.332
0.104	0.332

$E''(\lambda_i) = 0.06$

$V''(\lambda_i) = 0.576$

If r_j is taken deterministically equal to unity, $E''(\lambda_i) = 0.057$ and $V''(\lambda_i) = 0.503$.

DECISION MAKING UNDER UNCERTAIN RISK

Several criteria have been proposed for decision making under risk conditions (2, 14-18). The simplest are based on a conventional cost-benefit approach aimed at optimizing an objective function made by the algebraic sum of initial construction costs, expected benefits and expected costs of damage (14); these criteria fail to account for failure consequences which cannot be easily translated into a common scale of values (for instance, a monetary scale). Foremost among alternate decision criteria are those which optimize a utility function under acceptable risk restrictions and those which optimize a utility function that accounts for those losses which can easily be translated into utilities and then make decisions about additional investments for decreasing risk related to consequences not covered by the objective function, such as human lives, government prestige or panic (15-18). These criteria represent better than cost-benefit models the real human attitudes in the face of risk. Their application in practice is conditioned to the possibility of making numerical estimates of risk as well as of defining acceptable risk levels or adequate marginal investments oriented to risk reduction. These are very difficult problems even when risk is well determined (in the sense that a stochastic model of the failure process is available with precisely known form and parameters); they lead to much wider difficulties when uncertainty about hazard estimates is recognized and evaluated, and risk is expressed in terms of a bayesian distribution of the form and parameters of the random process of failure events.

A very simple case is adopted in the following for the purpose of discussing the implications of bayesian uncertainty on the selection of the intensity of the design earthquake for an engineering system. Let y_D be that intensity, and assume that the resulting system is deterministic and has only one failure mode; that is, it remains undamaged under the action of any intensity smaller than y_D, but fails otherwise. Suppose also that seismic activity is a Poisson process and that the mean rate of occurrence of earthquakes with intensities equal to or greater than y is given by the equation $\nu(y) = Ky^{-r}$, where K and r are determined from a seismic hazard analysis, and that the present value of the expected cost of failures is obtained under the assumed policy that the system will be rebuilt immediately after each failure, under specifications identical to the original ones.

The present value of the expected failure cost is then $D = D_c\nu(y_D)/\theta$, where D_c is the nominal cost of failure in case it takes place and θ is the present value discount coefficient (effective interest rate). If the initial construction cost C varies with y_D as $A_1 + A_2\, y_D$, the value of y_D that optimizes the conventional cost-benefit objective function is

$$y_D = \left(\frac{r\, D_c\, K}{A_2\, \theta}\right)^{\frac{1}{r+1}} \qquad (20$$

The failure risk corresponding to this design intensity is $\nu_D = Ky_D^{-r}$. If the adopted decision criterion imposes an upper bound ν_a on the allowable risk, then the design value must be the largest of $(K/\nu_a)^{1/r}$ and the value given by eq. 20.

Making decisions when ν_D is uncertain, that is, when either K or r or both are imperfectly known, is far more complicated, in particular when risk aversion considerations (of which the specification of an upper bound to allowable risk is a particular case) are relevant.

If all risk aversion considerations can be neglected, conventional cost-benefit analysis can be applied, but taking the present value of expected failure costs equal to $D_c E(\nu(y_D))/\theta$, where $E(\cdot)$ is the expectation with respect to the bayesian distribution of K and r. For the particular case when r is known and K has a gamma distribution with variation coefficient $V_k = \rho^{-1/2}$, the marginal distribution of the number of events during a given time interval is negative binomial (19), such that the probability of occurrence of at least one earthquake with intensity greater than y_D during time interval t is equal to

$$p_F = 1 - \left(\frac{\rho}{E(\nu(y_D))t + \rho}\right)^{\rho} \tag{21}$$

The values obtained from this expression are compared in Table 4 with those of $\tilde{p}_F$, which result from the assumption of a Poisson process with a mean rate equal to $E(\nu(y_D))$. In all cases the failure probabilities which account for uncertainty in $\nu(y_D)$ are smaller than those which neglect it. The effect is more pronounced for the longer time intervals.

The writer is not aware of any attempt to obtain a formal solution for the case when risk aversion is important and risk is uncertain. In fact, there may be no solution when an upper bound ν_a to allowable risk is specified, because in general we will find a non-zero probability that the actual risk $\nu(y_D)$ be greater than ν_a. Therefore, we must accept to formulate decisions on the basis of a specified probability that $\nu(y_D)$ exceeds ν_a; but this requires that decision makers develop the feeling required to specify that probability. Besides, proper consideration should be given to the differences between nominal, real and perceived risk; we are far from possessing the necessary criteria and tools.

If adeuate risk aversion functions expressed in terms of nominal risk were available, we could approach the problem within the framework of a conventional cost-benefit analysis where the present value of expected failure costs would incorporate a risk dependent factor.

Assume, for instance, that the present value of the expected failure cost is obtained multiplying D by a risk aversion function

$$\phi(\nu(y_D)) = 1 + cw^n \tag{22}$$

where $w = \nu(y_D)/E(\nu(y_0))$, c and n are adequate shape parameters, y_0 is a reference intensity value and $E(\cdot)$ is the expected value with respect to the bayesian distribution of ν. Several sample ϕ curves are shown in fig. 8. If $\nu(y) = Ky^{-r}$, with r known and K uncertain, and the initial construction cost is $C = A_1 + A_2 y_D$, the objective function to be minimized is

$$U = \frac{1}{\theta} E\left[\nu(y_D)\phi(\nu(y_D))\right] + A_1 + A_2 y_D \tag{23}$$

In non-dimensional form, eq. 23 can be expressed as

$$\frac{U}{D_c E(\nu(y_0))} = \frac{1}{\theta} E(k\eta^{-r}(1 + c\eta^{-n})) + a_1 + a_2\eta \tag{24}$$

where $k = K/\bar{K}$, $\eta = y_D/y_0$, $a_1 = A_1/(D_c E(\nu(y_0)))$ and $a_2 = A_2 y_0/(D_c E(\nu(y_0)))$. If K has a lognormal distribution with variation coefficient V_k, the value of η which optimizes eq. 24 satisfies the equation

$$\eta = \left[\frac{r}{a_2\theta}\ (1 + A\eta^{-rn})\right]^{\frac{1}{r+1}} \tag{25}$$

with $A = c(n+1)(1+V_k^2)^{\frac{n(n+1)}{2}}$. If $A = 0$, eq. 25 is equivalent to eq. 20, the solution for the case when risk aversion is neglected. If $\eta_1 = y_{D1}/y_0$ is the solution of eq. 25 for this case, we can obtain the ratio η/η_1 of the design intensities for both cases considered:

$$\frac{\eta}{\eta_1} = (1 + A\eta^{-rn})^{\frac{1}{r+1}} \tag{26}$$

This ratio can be used as the basis for the calibration procedure which is necessary in order to obtain the risk aversion functions implicit in conventional intuitive decisions in the face of uncertain risk conditions. Probably, a more appealing parameter to be used in this calibration procedure is the ratio of the mean values of the failure rates which correspond to η and η_1. From eq. 25, this ratio is a function of η_1; it is tabulated in Table 5 as a function of A, η and η_1 for $r = 2$, and Table 6 as a function of V_k for $c = 0.01$, $n = 2$, $r = 2$ and $\eta_1 = 0.01$. The sensitivity to the various parameters is evident. In particular, it is seen that variation coefficients of 0.5 in the bayesian distribution of $\nu(y_D)$ (which are far from exceptional in practical cases) may lead to values of about 2.5 in the ratio of the design intensities obtained considering and neglecting risk aversion, or of 1.25 in the ratio of the corresponding expected values of the failure rate.

These conclusions give only an indication of the corrective factors to be expected from the use of more refined representations of the $\nu(y)$ function.

CONCLUDING REMARKS

So called "deterministic" criteria for assessment of seismic hazard are not in fact deterministic. They emphasize the assessment of maximum possible events on the basis of geophysical and seismological information as well as on extrapolations of knowledge derived from other seismic sources. These criteria are called deterministic, not because they are devoid of uncertainty, but because they circumvent uncertainty analysis by stressing the determination of conservative earthquake intensities for design. They try to minimize risk, but they are not concerned about providing quantitative measures of it. The ralations between cost and risk are only vaguely and intuitively accounted for.

Bayesian criteria emphasize quantitative analysis of hazard and risk. Their sources of information do not differ from those utilized by deterministic criteria, but bayesian analysis permits dealing explicitly with uncertainties attached to analysis of statistical data, to interpretation of geophysical information and to extrapolation of knowledge derived from other seismic sources. These criteria recognize that the available information, or that which can be collected at a reasonable expense (geophysical and seismological), is not sufficient, as a rule, to determine unique model of nature to be used without question for making risk related decisions. They formulate decision making criteria in terms of a comprehensive (within reasonable simplifications) set of feasible models of the seismic activity process, and they permit the study of the concepts and algorithms necessary for testing the consistency of relative weights, or probabilities, assigned to alternate assumptions about adequate models of nature. One important asset of bayesian criteria is their ability to produce seismicity estimates with different degrees of uncertainty, on the basis of different amounts of information, and to assess the value of additional studies by considering the potential reduction of uncertainties and the expected impact of that reduction on the expected costs, benefits and risks tied to the ultimate decision.

Bayesian criteria also provide a framework for rational decision making. Introducing probabilistic models to represent the state of knowledge of a decision maker serves to foster the search for rational decision rules, as well as to develop the models and plan the studies required for obtaining the information having the highest value for the decision at hand.

Several alternate criteria for formal decision making under seismic risk have been developed in the last few years. They make use of concepts related to cost-benefit studies, maximum tolerable risks and marginal expenditures per human life saved. They all require probabilistic models of risk that account for objective and subjective uncertainty, and they all assume the availability of scales of values adequate for expressing the consequences of structural damage or failure. Because the latter are still far from well developed, decisions for seismic risk problems in the next few years will probably not be made exclusively on the basis of the formal criteria advocated in this paper. But the limitations of informal decisions, devoid of quantitative probabilistic descriptions of hazard and risk, support the recommendation that informal methods should not be applied without the guidance of quantitative measures of likelihood and uncertainty. It is very likely that an interaction between formal and informal criteria will improve our knowledge and understanding of the scales of values and decision rules that best serve the interests and goals of the human groups to be affected by the decisions under discussion.

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TABLE 1. *Posterior bayesian probabilities for three sample sizes*

case	Assumption			Probabilities		
	β	γ	Mu	N=50	100	150
1	1.8	0.5	7.3	0.0407	0.0363	0.2550
2		✓	8.0	0.0083	0.0021	0.0154
3		2	7.3	0.0039	0.0008	0.0266
4		✓	8.0	0.0010	0	0.0004
5		4	7.3	0.0013	0.0001	0.0052
6		✓	8.0	0.0007	0	0.0001
7	2.6	0.5	7.3	0.2037	0.1331	0.0015
8		✓	8.0	0.1761	0.1539	0.0141
9		2	7.3	0.1820	0.2439	0.1205
10		✓	8.0	0.1207	0.1179	0.0680
11		4	7.3	0.1529	0.2138	0.4297
12		✓	8.0	0.1081	0.0979	0.0635

TABLE 2. *Means and coefficients of variation of seismicity parameters and of complementary cumulative magnitude distributions*

Parameter X	E(X) N = 50	100	150	V(X) N = 50	100	150	Deterministic values
β	2.56	2.57	2.36	0.072	0.062	0.156	2
γ	1.88	2.14	2.57	0.752	0.656	0.592	3
M_u	7.59	7.56	7.41	0.045	0.047	0.035	7.5
$\overline{F}_M(6.0)$	0.257	0.259	0.288	0.110	0.088	0.113	0.36
$\overline{F}_M(6.5)$	0.062	0.063	0.075	0.253	0.194	0.211	0.13
$\overline{F}_M(6.75)$	0.029	0.030	0.038	0.346	0.261	0.257	0.072
$\overline{F}_M(7.0)$	0.012	0.012	0.016	0.477	0.377	0.335	0.038

TABLE 3. *Discrete bayesian distributions of λ_i for each $E(\lambda_i)$ and $V(\lambda_i)$*

$E(\lambda_i)$	$V(\lambda_i)$	λ_i = 0.01	0.02	0.08	0.16
0.02	0.2	0.0229	0.973	0.0038	0
	0.6	0.2057	0.760	0.0343	0
0.08	0.2	0	0.0305	0.947	0.0229
	0.6	0	0.2743	0.520	0.2057

TABLE 4. *Values of* $p_F/\tilde{p}_F$

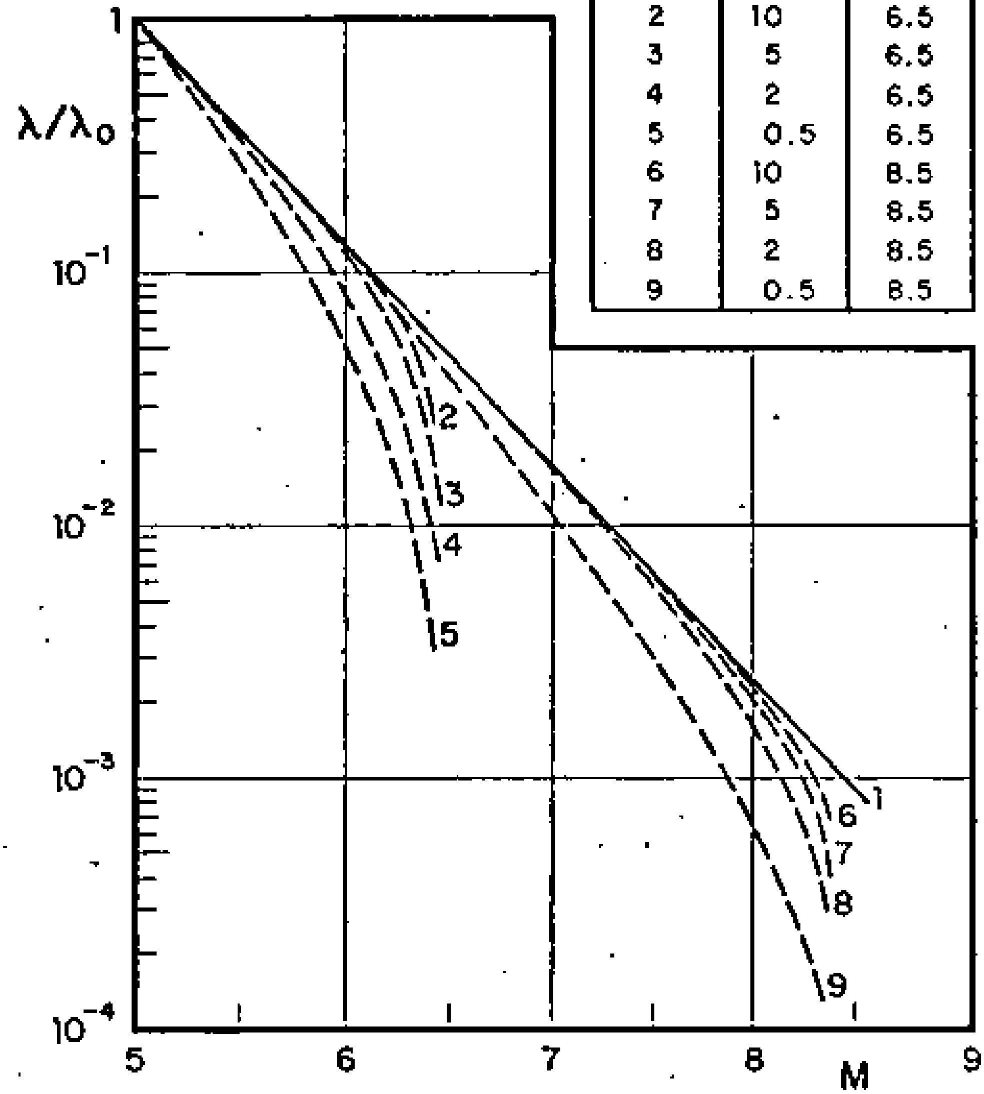
$E(v)t$	p				
	0.1	0.3	1	3	10
0.001	0.9950	0.9983	0.9995	0.9998	1.0
0.01	0.9533	0.9838	0.9951	0.9984	0.9995
0.1	0.7037	0.8689	0.9553	0.9845	0.9953
1.0	0.3373	0.5630	0.7910	0.9146	0.9721

TABLE 5. *Values of* $E(\nu(y_D)) \,/\, E(\nu(y_{DI}))$ *for* $r = 2$

A	n	η_1						
		1	0.5	0.2	0.1	0.05	0.02	0.01
10^{-7}	2	1.0	1.0	1.0	0.999	0.990	0.799	0.465
	4	1.0	2.0	0.977	0.589	0.235	0.0631	0.0234
10^{-2}	2	0.994	0.919	0.518	0.257	0.120	0.0426	0.0193
	4	0.994	0.713	0.218	0.0808	0.0296	0.0078	0.0029

TABLE 6. *Values of* $E(\nu(y_D)) \,/\, E(\nu(y_{DI}))$ *and* η/η_1 *for*
$c = 0.01,\quad n = 2,\quad r = 2,\quad \eta_1 = 0.01$

v_k	0	0.10	0.20	0.30	0.50	0.70	1.0
$E(\nu(y_D))/E(\nu(y_{DI}))$	0.191	0.189	0.185	0.178	0.159	0.137	0.107
η/η_1	2.288	2.300	2.325	2.370	2.508	2.701	3.057



Curve	γ	M_u
1	∞	
2	10	6.5
3	5	6.5
4	2	6.5
5	0.5	6.5
6	10	8.5
7	5	8.5
8	2	8.5
9	0.5	8.5

Fig 1. Influence of γ and M on $\lambda(M)$ for $\beta = 2$

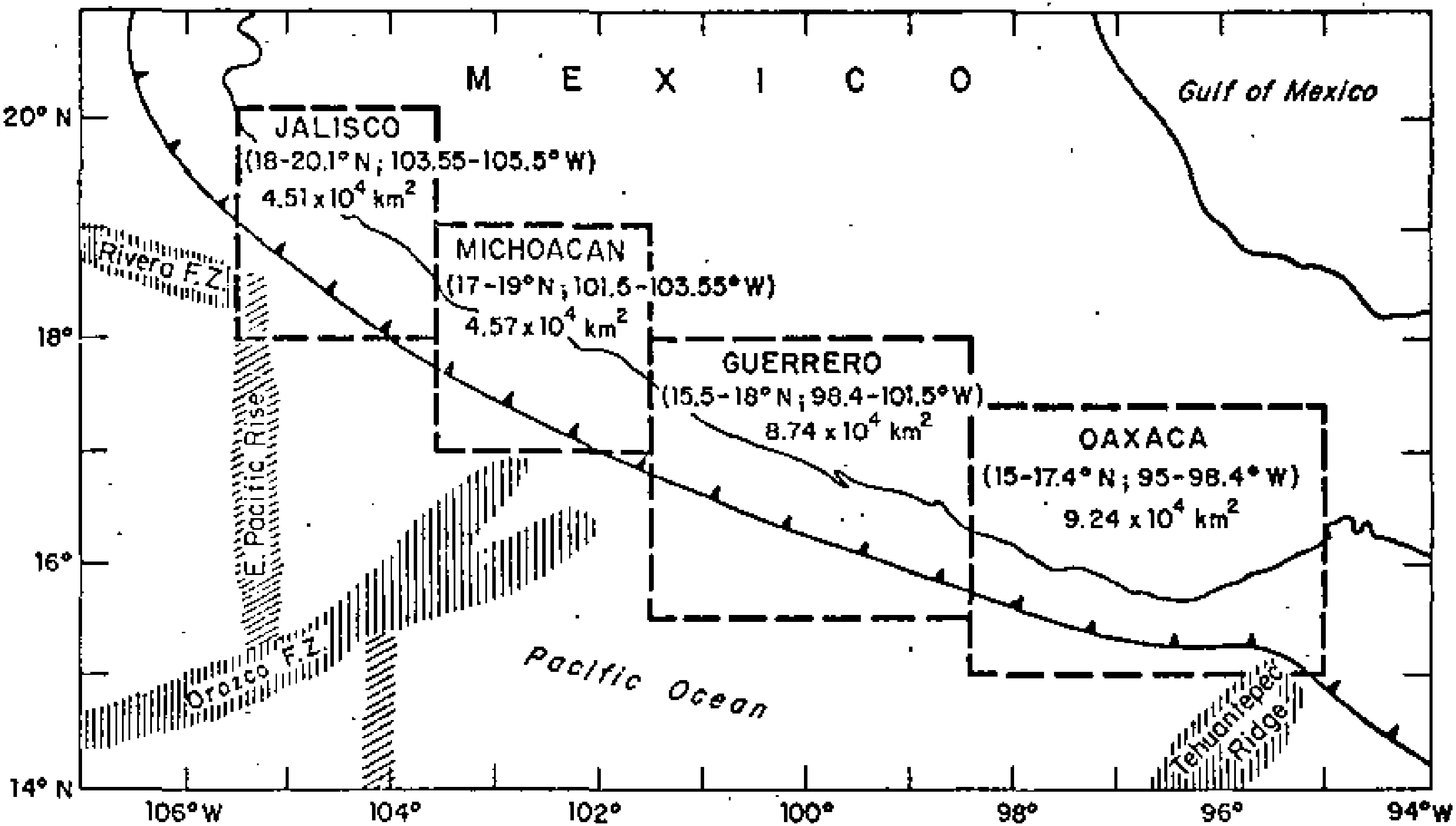


Fig 2. Mexican subduction zone

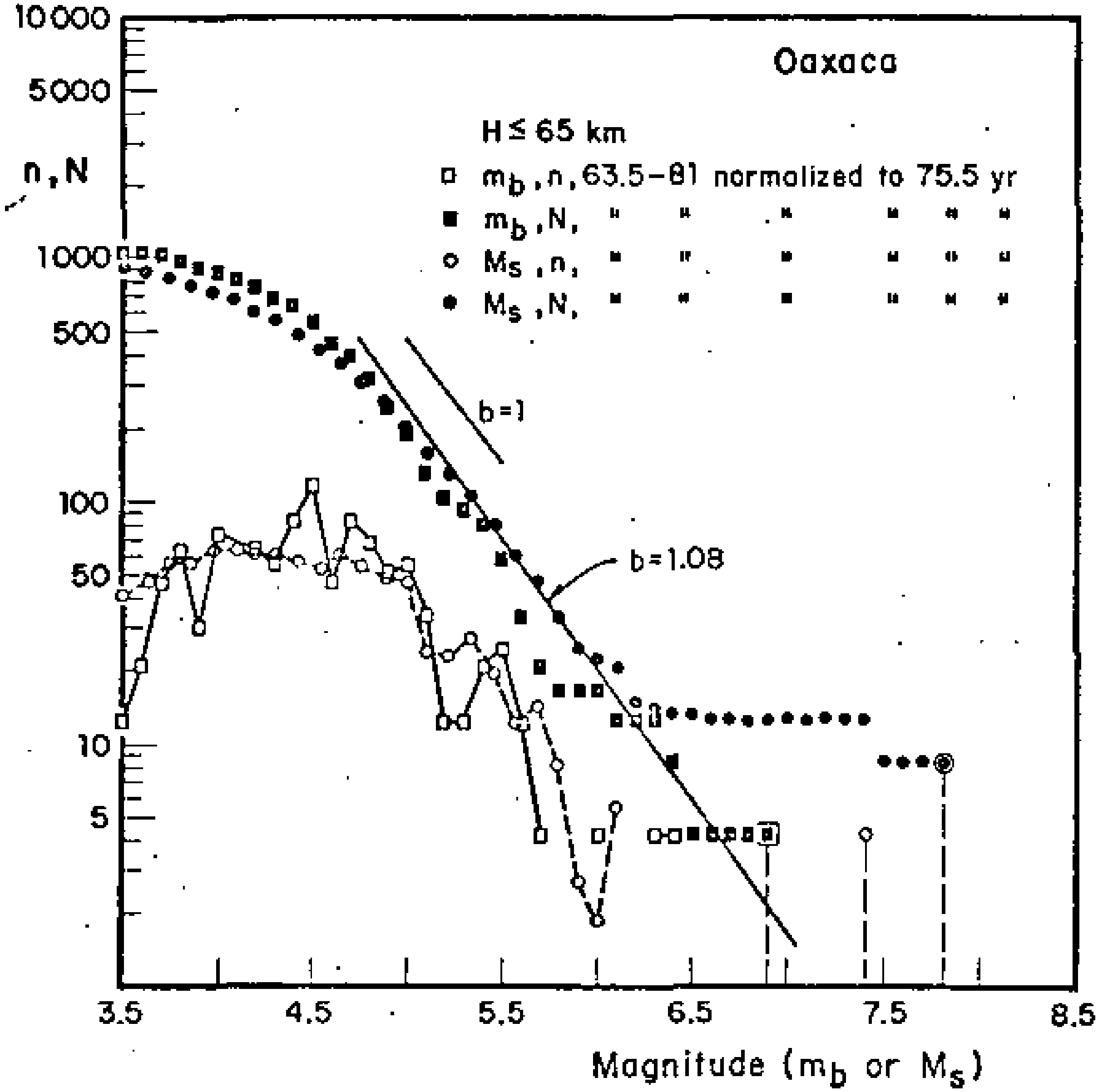
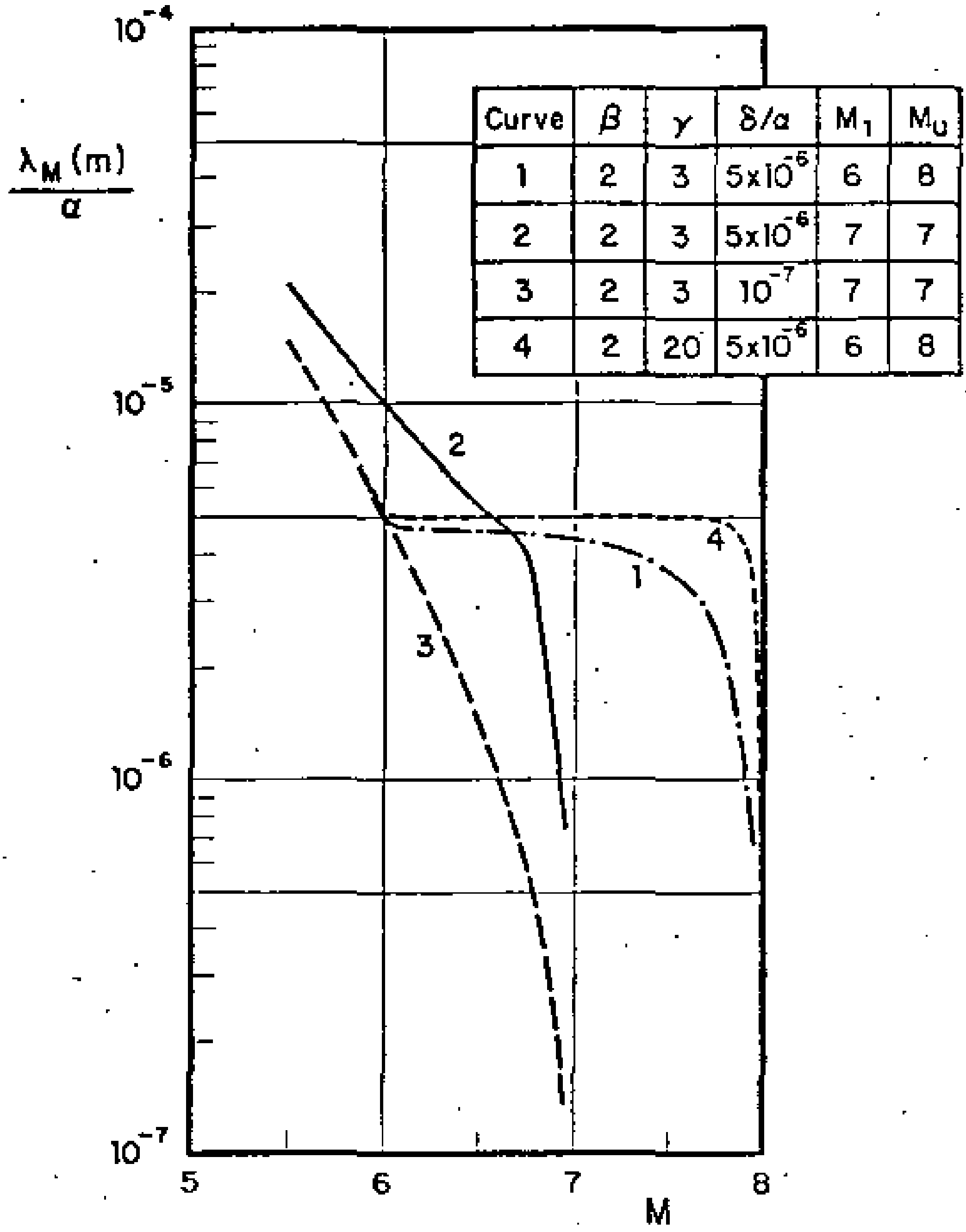


Fig 3. Magnitude-recurrence data for the Oaxaca subduction zone (taken from ref. 3)



Curve	β	γ	δ/α	M_1	M_U
1	2	3	5×10^{-6}	6	8
2	2	3	5×10^{-6}	7	7
3	2	3	10^{-7}	7	7
4	2	20	5×10^{-6}	6	8

Fig 4. General shapes of magnitude recurrence curves given by eq 2

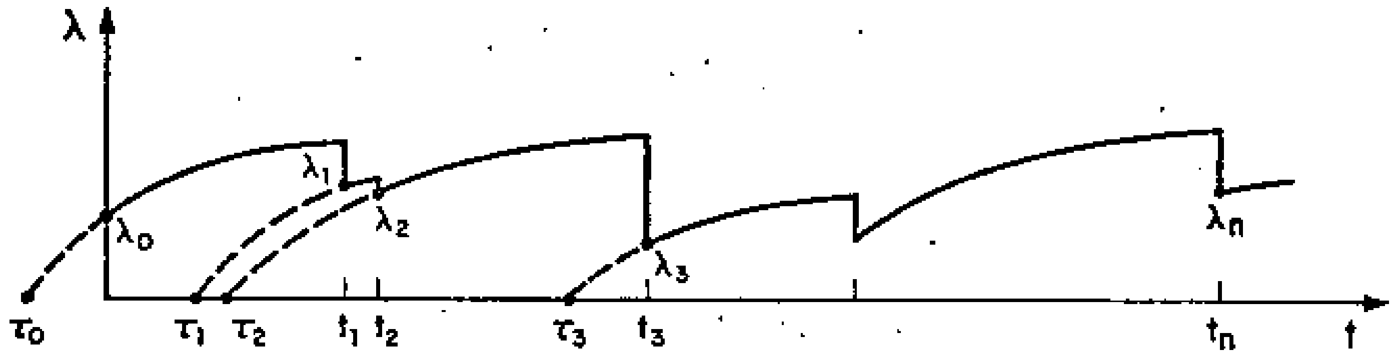


Fig 5. Hazard function for a sample of a Markov process model of seismicity neglecting clustering

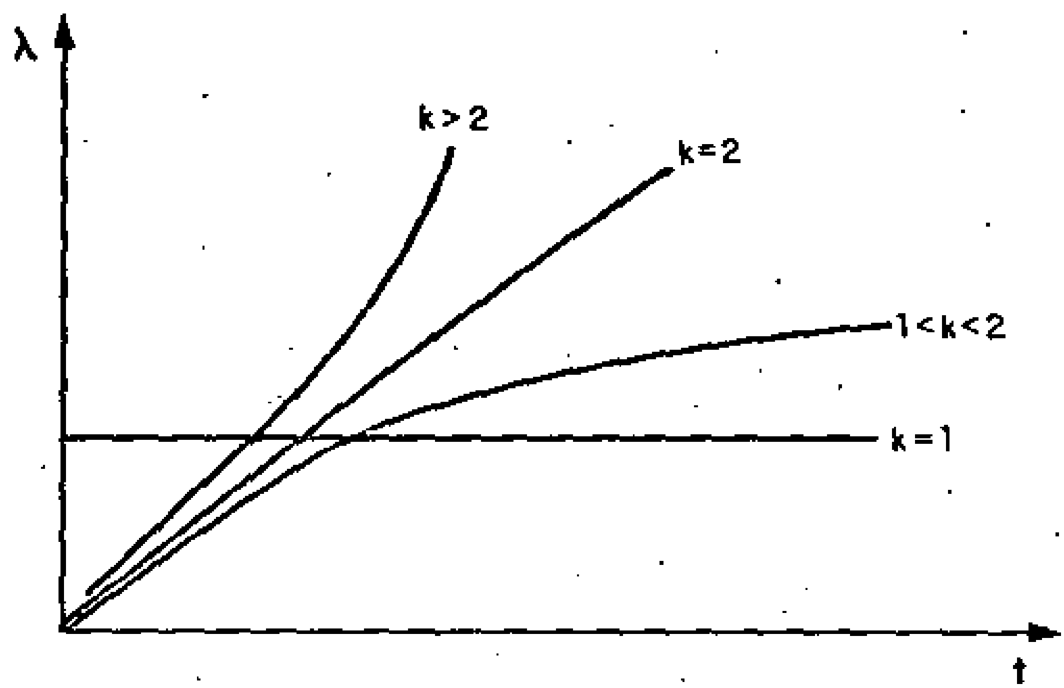


Fig 6. Shapes of hazard functions for Weibull distribution of inter-arrival times

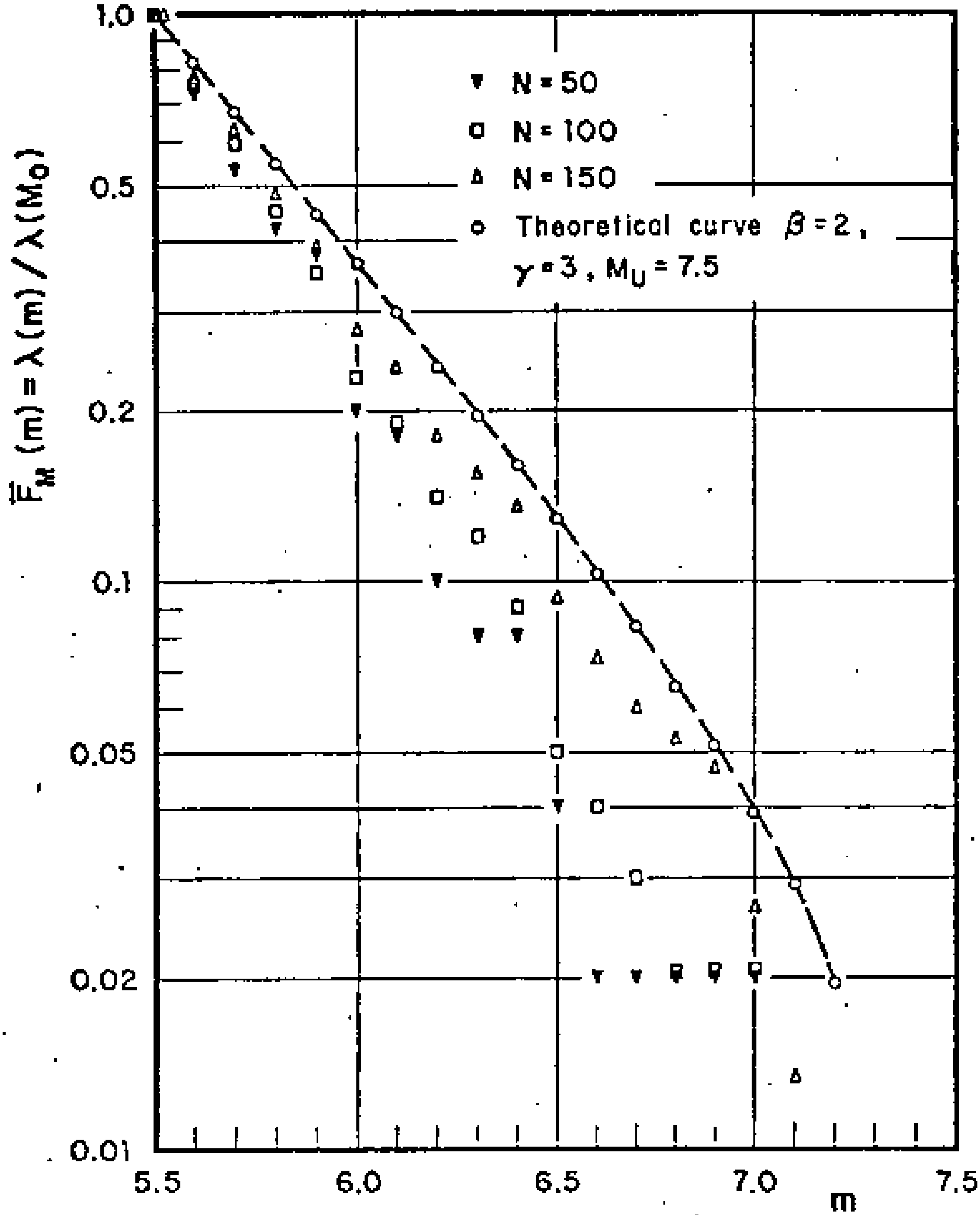


Fig 7. Summary of simulated seismic records

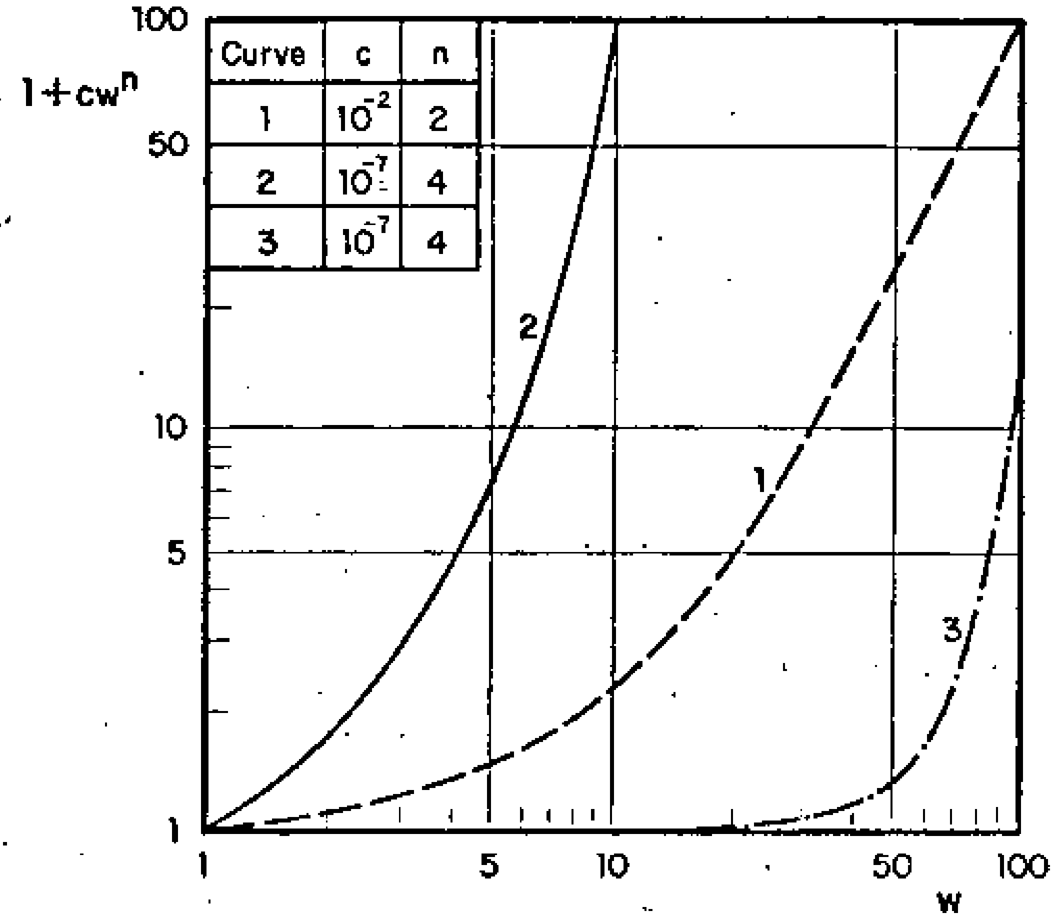
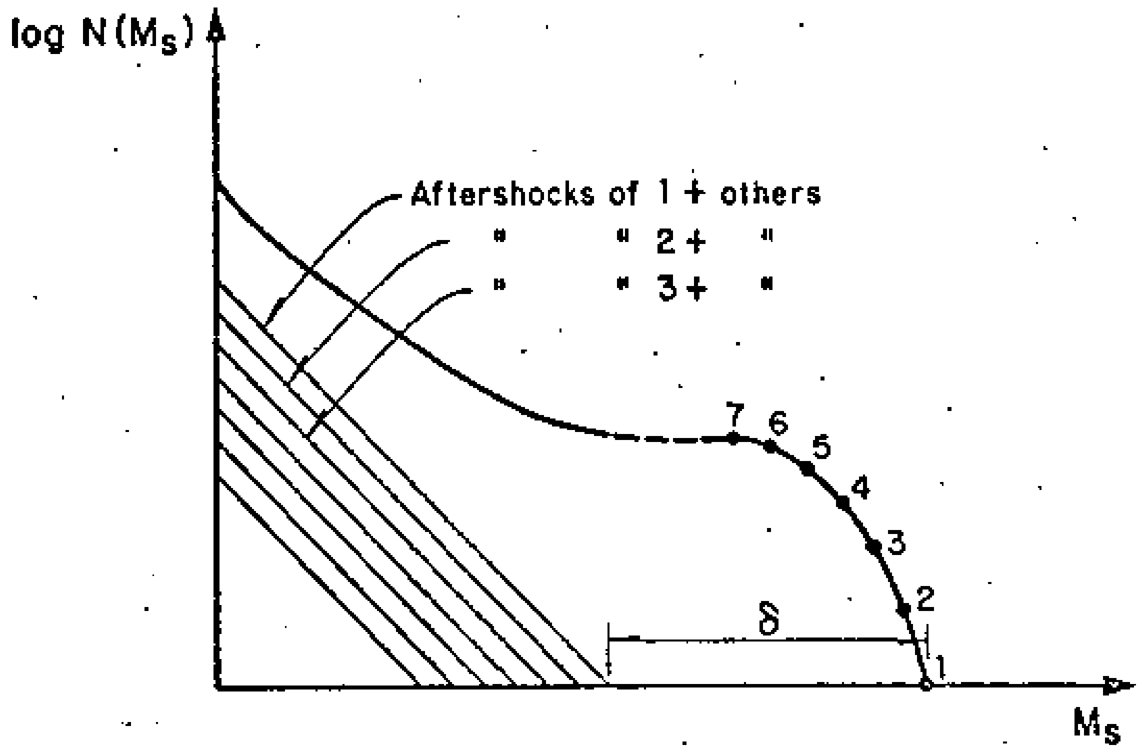


Fig 8. Utility functions for risk aversion


log N(Ms)
Aftershocks of 1 + others
" " 2 + "
" " 3 + "
7
6
5
4
3
2
1
δ
Ms

DIVISION DE EDUCACION CONTINUA
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X CURSO INTERNACIONAL DE INGENIERIA SISMICA

ANALISIS DE RIESGO SISMICO

SEISMICITY

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AGOSTO, 1984.

Chapter 6

SEISMICITY

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6.1 ON SEISMICITY MODELS

Rational formulation of engineering decisions in seismic areas requires quantitative descriptions of seismicity. These descriptions should conform with their intended applications: in some instances, simultaneous intensities during each earthquake have to be predicted at several locations, while in others it suffices to make independent evaluations of the probable effects of earthquakes at each of those locations.

The second model is adequate for the selection of design parameters of individual components of a regional system (the structures in a region or country) when no significant interaction exists between response or damage of several such individual components, or between any of them and the system as a whole. In other words, it applies when the damage — or negative utility — inflicted upon the system by an earthquake can be taken simply as the addition of the losses in the individual components.

The linearity between monetary values and utilities implied in the second model is not always applicable. Such is the case, for instance, when a significant portion of the national wealth or of the production system is concentrated in a relatively narrow area, or when failure of life line components may disrupt emergency and relief actions just after an earthquake. Evaluation of risk for the whole regional system has then to be based on seismicity models of the first type, that is, models that predict simultaneous intensities at several locations during each event; for the purpose of decision making, nonlinearity between monetary values and utilities can be accounted for by means of adequate scale transformations. These models are also of interest to insurance companies, when the probability distribution of the maximum loss in a given region during a given time interval is to be estimated.

Whatever the category to which a seismic risk problem belongs, it requires the prediction of probability distributions of certain ground motion characteristics (such as peak ground acceleration or velocity, spectral density, response or Fourier spectra, duration) at a given site during a single shock or of maximum values of some of those characteristics in earthquakes occurring during given time intervals. When the reference interval tends to infinity, the probability distribution of the maximum value of a given characteristic approaches that of its maximum *possible* value. Because different systems or subsystems are sensitive to different ground motion characteristics, the term *intensity characteristic* will be used throughout this chapter to mean a particular parameter or set of parameters of an earthquake motion, in terms of which the response is to be predicted. Thus, when dealing with the failure probability of a structure, intensity can be alternatively measured — with different degrees of correlation with structural response — by the ordinate of the response spectrum for the corresponding period and damping, the peak ground acceleration, or the peak ground velocity.

In general, local instrumental information does not suffice for estimating the probability distributions of maximum intensity characteristics, and use has to be made of data on subjective measures of intensities of past earthquakes, of models of *local seismicity*, and of expressions relating characteristics with magnitude and site-to-source distance. Models of local seismicity consist, at least, of expressions relating magnitudes of earthquakes generated in given volumes of the earth's crust with their return periods. More often than not, a more detailed description of local seismicity is required, including estimates of the maximum magnitude that can be generated in these volumes, as well as probabilistic (stochastic process) models of the possible histories of seismic events (defined by magnitudes and coordinates).

This chapter deals with the various steps to be followed in the evaluation of seismic risk at sites where information other than direct instrumental records of intensities has to be used: identifying potential sources of activity near the site, formulating mathematical models of local seismicity for each source, obtaining the contribution of each source to seismic risk at the site and adding up contributions of the various sources and combining information obtained from local seismicity of sources near the site with data on instrumental or subjective intensities observed at the site.

The foregoing steps consider use of information stemming from sources of different nature. Quantitative values derived therefrom are ordinarily tied to wide uncertainty margins. Hence they demand probabilistic evaluation, even though they cannot always be interpreted in terms of relative frequencies of outcomes of given experiments. Thus, geologists talk of the maximum magnitude that can be generated in a given area, assessed by looking at the dimensions of the geological accidents and by extrapolating the observations of other regions which available evidence allows to brand as similar to the one of interest; the estimates produced are obviously uncertain, and the degree of uncertainty should be expressed together with the most probable value. Following nearly parallel lines, some geophysicists estimate the energy that can be liberated by a single shock in a given area by making quantitative assumptions about source dimensions, dislocation amplitude and stress drop, consistent with tectonic models of the region and, again, with comparisons with areas of similar tectonic characteristics.

Uncertainties attached to estimates of the type just described are in general extremely large: some studies relating fault rupture area, stress drop, and magnitude (Brune, 1968) show that, considering not unusually high stress drops, it does not take very large source dimensions to get magnitudes 8.0 and greater, and those studies are practically restricted to the simplest types of fault displacement. It is not clear, therefore, that realistic bounds can always be assigned to potential magnitudes in given areas or that, when this is feasible, those bounds are sufficiently low, so that designing structures to withstand the corresponding intensities is economically sound, particularly when occurrence of those intensities is not very likely in the near future. Because uncertainties in maximum feasible magnitudes and in other parameters defining magnitude-recurrence laws can be as significant as their mean values when trying to make rational seismic design decisions, those uncertainties have to be explicitly recognized and accounted for by means of adequate probabilistic criteria. A corollary is that geophysically based estimates of seismicity parameters should be accompanied with corresponding uncertainty measures.

Seismic risk estimates are often based only on statistical information (observed magnitudes and hypocentral coordinates). When this is done, a wealth of relevant geophysical information is neglected, while the probabilistic prediction of the future is made to rely on a sample that is often small and of little value, particularly if the sampling period is short as compared with the desirable return period of the events capable of severely damaging a given system.

The criterion advocated here intends to unify the foregoing approaches and rationally to assimilate the corresponding pieces of information. Its philosophy consists in using the geological, geophysical, and all other available non-statistical evidence for producing a set of alternate assumptions concerning a mathematical (stochastic process) model of seismicity in a given source area. An initial probability distribution is assigned to the set of hypotheses, and the statistical information is then used to improve that probability assignment. The criterion is based on application of *Bayes theorem*, also called the *theorem of the probabilities of hypotheses*. Since estimates of risk depend largely on conceptual models of the geophysical processes involved, and these are known with different degrees of uncertainty in different zones of the earth's crust, those estimates will be derived from stochastic process models with uncertain forms or parameters. The degree to which these uncertainties can be reduced depends on the limitations of the state of the art of geophysical sciences and on the effort that can be put into compilation and interpretation of geophysical and statistical information. This is an economical problem that should be handled, formally or informally, by the criteria of decision making under uncertainty.

Available criteria for the evaluation of the contribution of potential seismic sources to the risk at a site make use of *intensity attenuation* expressions that relate intensity characteristics with magnitude and distance from site to source. Depending on the application envisaged, the intensity characteristic to be predicted can be expressed in a number of manners, ranging from a subjective index, such as the *Modified Mercalli intensity*, to a combination of one or more quantitative measures of ground shaking (see Chapter 1).

A number of expressions for attenuation of various intensity characteristics with distance have been developed, but there is little agreement among most of them (Ambraseys, 1973). This is due in part to discrepancies in the definitions of some parameters, in the ranges of values analyzed, in the actual wave propagation properties of the geological formations lying between source and site, in the dominating shock mechanisms, and in the forms of the analytical expressions adopted a priori.

Most intensity-attenuation studies concern the prediction of earthquake characteristics on rock or firm ground, and assume that these characteristics, properly modified in terms of frequency-dependent soil amplification factors, should constitute the basis for estimating their counterparts on soft ground. Observations about the influence of soil properties on earthquake damage support the assumption of a strong correlation between type of local ground and intensity in a given shock. Attempts to analytically predict the characteristics of motions on soil given those on firm ground or on bedrock have not been too successful, however (Crouse, 1973; Hudson and Udwadia, 1973; Salt, 1974), with the exception of some peculiar cases, like Mexico City (Herrera et al., 1965), where local conditions favor the fulfillment of the assumptions implied by usual analytical models. The following paragraphs concentrate on prediction on intensities on *firm ground*; the influence of local soil is discussed in Chapter 4.

6.2.1 Intensity attenuation on firm ground

When isoseismals (lines joining sites showing equal intensity) of a given shock are based only on intensities observed on homogeneous ground conditions, such as *firm ground* (compact soils) or bedrock, they are roughly elliptical and the orientations of the corresponding axes are often correlated with local or regional geological trends (Figs. 6.1–6.3). In some regions — for instance near major faults in the western United States — those trends are well defined and the correlations are clear enough as to permit prediction of intensity in the near and far fields in terms of magnitude and distance to the generating fault or to the centroid of the energy liberating volume. In other regions, such as the eastern United States and most of Mexico, isoseismals seem to elongate systematically in a direction that is a function of the epi-

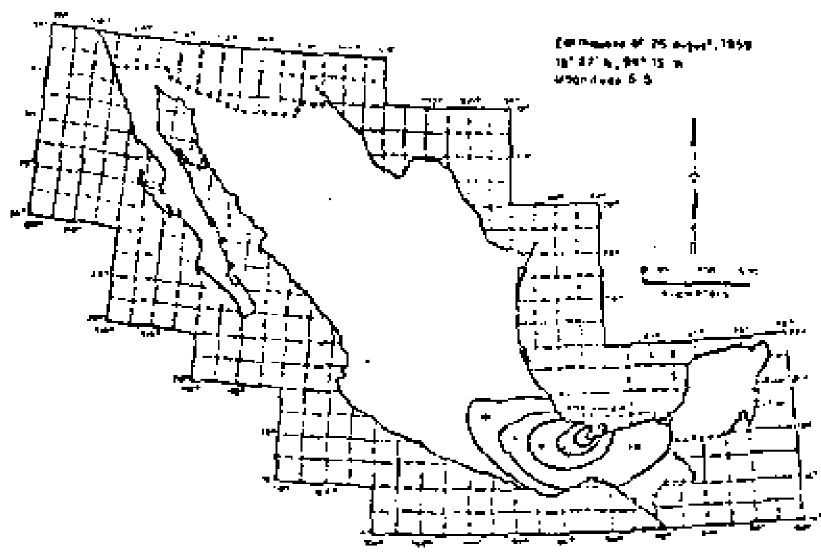


Fig. 6.1. Isoseismals of an earthquake in Mexico. (After Figueroa, 1963.)

central coordinates (Bollinger, 1973; Figueroa, 1963). In that case, intensity should be expressed as a function of magnitude and coordinates of source and site. For most areas in the world, intensity has to be predicted in terms of simple — and cruder — expressions that depend only on magnitude and distance from site to instrumental hypocenter. This stems from inadequate knowledge of geotectonic conditions and from limited information concerning the volume where energy is liberated in each shock.

A comparison of the rates of attenuation of intensities on firm ground for shocks on western and eastern North America has disclosed systematic differences between those rates (Milne and Davenport, 1969). This is the source of a basic, but often unavoidable, weakness of most intensity-attenuation expressions, because they are based on heterogeneous data, recorded in different zones, and the very nature of their applications implies that the less is known about possible systematic deviations in a given zone, as a consequence of the meagerness of local information, the greater weight is given to predictions with respect to observations.

6.2.1.1 Modified Mercalli intensities

An analysis of the Modified Mercalli intensities on firm ground reported for earthquakes occurring in Mexico in the last few decades leads to the fol-

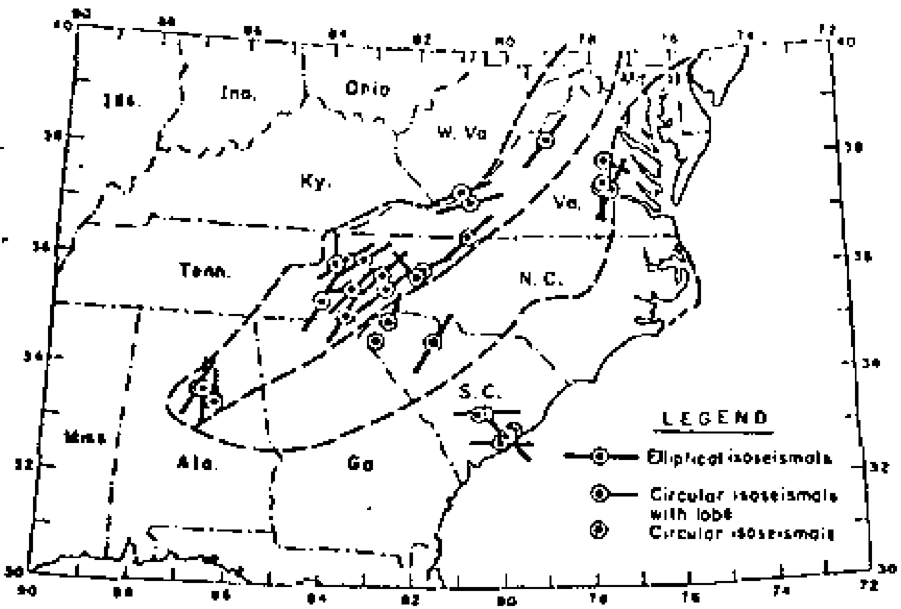


Fig. 6.2. Elongation of isoseismals in the southeastern United States. (After Bollinger, 1973.)

lowing expression relating magnitude M, hypocentral distance R (in kilometers) and intensity I (Esteva, 1968):

$$I = 1.45\,M - 5.7 \log_{10} R + 7.9 \qquad (6.1)$$

The prediction error, defined as the difference between observed and computed intensity, is roughly normally distributed, with a standard deviation of 2.04, which means that there is a probability of 60% that an observed intensity is more than one degree greater or smaller than its predicted value.

6.2.1.2 Peak ground accelerations and velocities

A few of the available expressions will be described. Their comparison will show how cautiously a designer intending to use them should proceed.

Housner studied the attenuation of peak ground accelerations in several regions of the United States and presented his results graphically (1969) in terms of fault length (in turn a function of magnitude), shapes of isoseismals and areas experiencing intensities greater than given values (Fig. 6.4 and 6.5).

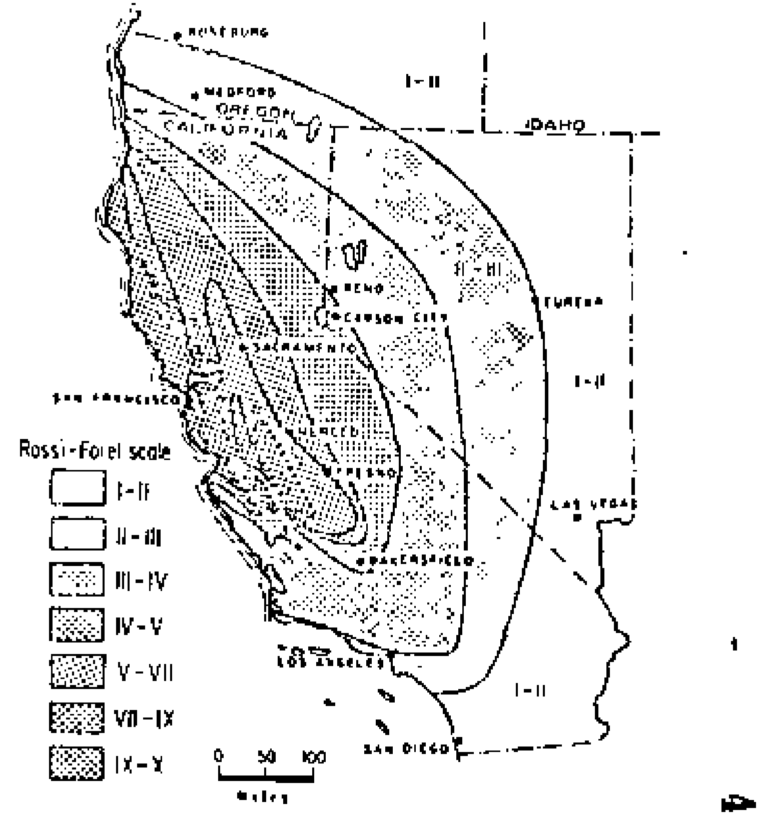


Fig. 6.3. Isoseismals in California. (After Bolt, 1970.)

He showed that intensities attenuate faster with distance on the west coast than in the rest of the country. This comparison is in agreement with Milne and Davenport (1969), who performed a similar analysis for Canada. From observations of strong earthquakes in California and in British Columbia, they developed the following expression for a, the peak ground acceleration, as a fraction of gravity:

$$a/g = 0.0069\,e^{1.6M}/(1.1\,e^{1.1M} + R^2) \qquad (6.2)$$

Here, R is epicentral distance in kilometers. The acceleration varies roughly as $e^{1.6M} R^{-2}$ for large R, and as $e^{0.5M}$ where R approaches zero. This reflects to some extent the fact that energy is released not at a single point but from a finite volume. A later study by Davenport (1972) led him

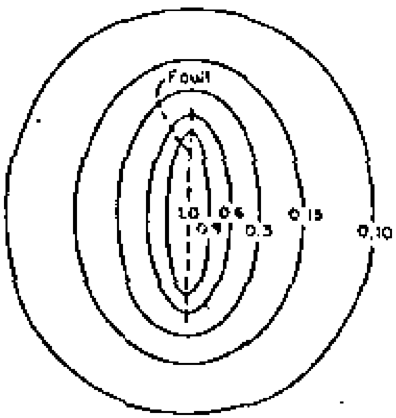


Fig. 6.4. Idealized contour lines of intensity of ground shaking. (After Housner, 1969.)

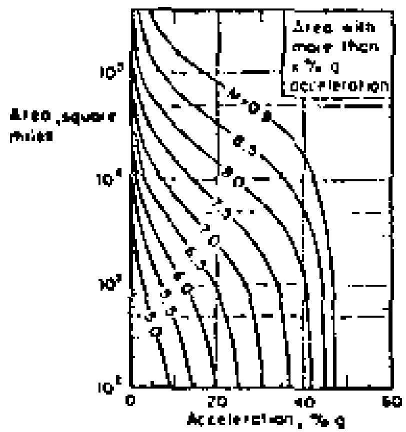


Fig. 6.5. Area in square miles experiencing shaking of x %g or greater for shocks of different magnitudes. (After Housner, 1969.)

to propose the expression:

$$a/g = 0.279\, e^{0.80M}/R^{1.64} \tag{6.3}$$

The statistical error of this equation was studied by fitting a lognormal probability distribution to the ratios of observed to computed accelerations. A standard deviation of 0.74 was found in the natural logarithms of those ratios.

Esteva and Villaverde (1973), on the basis of accelerations reported by Hudson (1971, 1972a,b), derived expressions for peak ground accelerations and velocities, as follows:

$$a/g = 5.7\, e^{0.8M}/(R + 40)^2 \tag{6.4}$$

$$v = 32\, e^{M}/(R + 25)^{1.7} \tag{6.5}$$

Here v is peak ground velocity in cm/sec and the other symbols mean the same as above. The standard deviation of the natural logarithm of the ratio of observed to predicted intensity is 0.64 for accelerations and 0.74 for velocities. If judged by this parameter, eqs. 6.3 and 6.4 seem equally reliable. However, as shown by Fig. 6.6, their mean values differ significantly in some ranges.

With the exception of eq. 6.2, all the foregoing attenuation expressions are products of a function of R and a function of M. This form, which is acceptable when the dimensions of the energy-liberating source are small com-

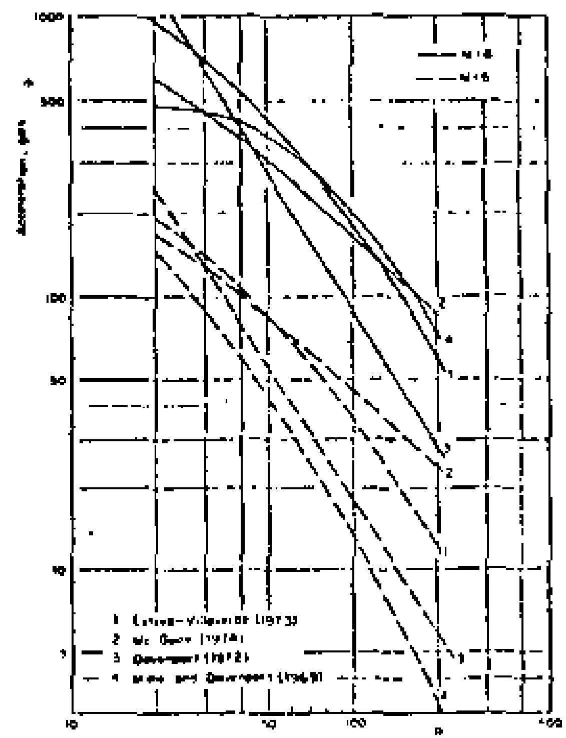


Fig. 6.6. Comparison of several attenuation expressions.

pared with R, is inadequate when dealing with earthquake sources whose dimensions are of the order of moderate hypocentral distances, and often greater than them. Although equation errors (probability distributions of the ratio of observed to predicted intensities) have been evaluated by Davenport (1972) and Esteva and Villaverde (1973), their dependence on M and R has not been analyzed. Because seismic risk estimates are very sensitive to the attenuation expressions in the range of large magnitudes and short distances, more detailed studies should be undertaken, aiming at improving those expressions in the mentioned range, and at evaluating the influence of M and R on equation error. Information on strong-motion records will probably be scanty for those studies, and hence they will have to be largely based on analytical or physical models of the generation and propagation of seismic waves. Although significant progress has been lately attained in this direction (Trifunac, 1973) the results from such models have hardly influenced the practice of seismic risk estimation because they have remained either unknown to or imperfectly appreciated by engineers in charge of the corresponding decisions.

6.2.1.3 Response spectra

Peak ground acceleration and displacement are fairly good indicators of the response of structures possessing respectively very high and very small natural frequencies. Peak velocity is correlated with the response of intermediate-period systems, but the correlation is less precise than that tying the former parameters; hence, it is natural to formulate seismic risk evaluation and engineering design criteria in terms of spectral ordinates.

Response spectrum prediction for given magnitude and hypocentral or site-to-fault distance usually entails a two-step process, according to which peak ground acceleration, velocity and displacement are initially estimated and then used as reference values for prediction of the ordinates of the response spectrum. Let the second step in the process be represented by the operation $y_s = \alpha y_g$, where y_s is an ordinate of the response spectrum for a given natural period and damping ratio, and y_g is a parameter (such as peak ground acceleration or velocity) that can be directly obtained from the time-history record of a given shock regardless of the dynamic properties of the systems whose response is to be predicted. For given M and R, y_g is random and so is $y_s/y_g = \alpha$; the mean and standard deviation of y_s depend on those of y_g and α and on the coefficient of correlation of the latter variables. As shown above, y_g can only be predicted within wide uncertainty limits, often wider than those tied to y_s (Esteva and Villaverde, 1973). The coefficient of variation of y_s given M and R can be smaller than that of y_g only if α and y_g are negatively correlated, which is often the case: the greater the deviation of an observed value of y_g with respect to its expectation for given M and R, the lower is likely to be α. In other words, it seems that in the intermediate range of natural periods the expected values of spectral ordinates for given damping ratios can be predicted directly in terms of magnitude and focal distance with narrower (or at most equal) margins of uncertainty than those tied to predicted peak ground velocities. For the ranges of very short or very long natural periods, peak amplitudes of ground motion and spectral ordinates approach each other and their standard errors are therefore nearly equal.

McGuire (1974) has derived attenuation expressions for the conditional values (given M and R) of the mean and of various percentiles of the probability distributions of the ordinates of the response spectra for given natural periods and damping ratios. Those expressions have the same form as eqs. 6.4 and 6.5, but their parameters show that the rates of attenuation of spectral ordinates differ significantly from those of peak ground accelerations or velocities. For instance, McGuire finds that peak ground velocity attenuates in proportion to $(R + 25)^{-1.20}$, while the mean of the pseudovelocity for a natural period of 1 sec and a damping ratio of 2% attenuates in proportion to $(R + 25)^{-0.58}$. These results stem from the way that frequency content changes with R and lead to the conclusion that the ratio of spectral velocity should be taken as a function of M and R.

TABLE 6.1

McGuire's attenuation expressions $y = b_1\, 10^{b_2 M}(R + 25)^{-b_3}$

y	b_1	b_2	b_3	$V(y)$ = coeff. of var. of y
a gals	472.3	0.278	1.301	0.548
v cm/sec	5.64	0.401	1.202	0.696
d cm	0.393	0.434	0.885	0.883
Undamped spectral pseudovelocities				
T = 0.1 sec	11.0	0.278	1.346	0.941
0.5	3.05	0.291	1.001	0.636
1.0	0.631	0.378	0.519	0.768
2.0	0.0768	0.469	0.419	0.989
5.0	0.0834	0.564	0.897	1.344
5% damped spectral pseudovelocities				
T = 0.1 sec	10.09	0.233	1.341	0.651
0.5	5.74	0.356	1.197	0.591
1.0	0.432	0.393	0.701	0.703
2.0	0.122	0.466	0.675	0.941
5.0	0.0706	0.557	0.938	1.193

Table 6.1 summarizes McGuire's attenuation expressions and their coefficients of variation for ordinates of the pseudovelocity spectra and for peak ground acceleration, velocity and displacement. Similar expressions were derived by Esteva and Villaverde (1973), but they are intended to predict only the maxima of the expected acceleration and velocity spectra, regardless of the periods associated with those maxima. No analysis has been performed of the relative validity of McGuire's and Esteva and Villaverde's expressions for various ranges of M and R.

6.3 LOCAL SEISMICITY

The term *local seismicity* will be used here to designate the degree of seismic activity in a given volume of the earth's crust; it can be quantitatively described according to various criteria, each providing a different amount of information. Most usual criteria are based on upper bounds to the magnitudes of earthquakes that can originate in a given seismic source, on the amount of energy liberated by shocks per unit volume and per unit time or on more detailed statistical descriptions of the process.

6.3.1 Magnitude-recurrence expressions

Gutenberg and Richter (1954) obtained expressions relating earthquake magnitudes with their rates of occurrence for several zones of the earth. Their results can be put in the form:

$$\lambda = \alpha e^{-\beta M} \tag{6.6}$$

where λ is the mean number of earthquakes per unit volume and per unit time having magnitude greater than M and α and β are zone-dependent constants; α varies widely from point to point, as evidenced by the map of epicenters shown in Fig. 6.7, while β remains within a relatively narrow range, as shown in Fig. 6.8. Equation 6.6 implies a distribution of the energy liberated per shock which is very similar to that observed in the process of microfracturing of laboratory specimens of several types of rock subjected to gradually increasing compressive or bending strain (Mogi, 1962; Scholz, 1968). The values of β determined in the laboratory are of the same order as those obtained from seismic events, and have been shown to depend on the heterogeneity of the specimens and on their ability to yield locally. Thus, in heterogeneous specimens made of brittle materials many small shocks precede a major fracture, while in homogeneous or plastic materials the number of small shocks is relatively small. These cases correspond to large and small β-values, respectively. No general relationship is known to the writer between β and geotectonic features of seismic provinces; complexity of crustal structure and of stress gradients precludes extrapolation of laboratory results; and statistical records for relatively small zones of the earth are not, as a rule, adequate for establishing local values of β. Figure 6.8 shows that for very high magnitudes the observed frequency of events is lower than predicted by eq. 6.6. In addition, Rosenblueth (1969) has shown that β cannot be smaller than 3.46, since that would imply an infinite amount of energy liberated per unit time. However, Fig. 6.8 shows that the values of β which result from fitting expressions of the form 6.6 to observed data are smaller than 3.46; hence, for very high values of M (above 7, approximately) the curve should bend down, in accordance with statistical evidence.

Expressions alternative to eq. 6.6 have been proposed, attempting to represent more adequately the observed magnitude-recurrence data (Rosenblueth, 1964; Merz and Cornell, 1973). Most of these expressions also fail to recognize the existence of an upper bound to the magnitude that can be generated in a given source. Although no precise estimates of this upper bound can yet be obtained, recognition of its existence and of its dependence on the geotectonic characteristics of the source is inescapable. Indeed, the prac-

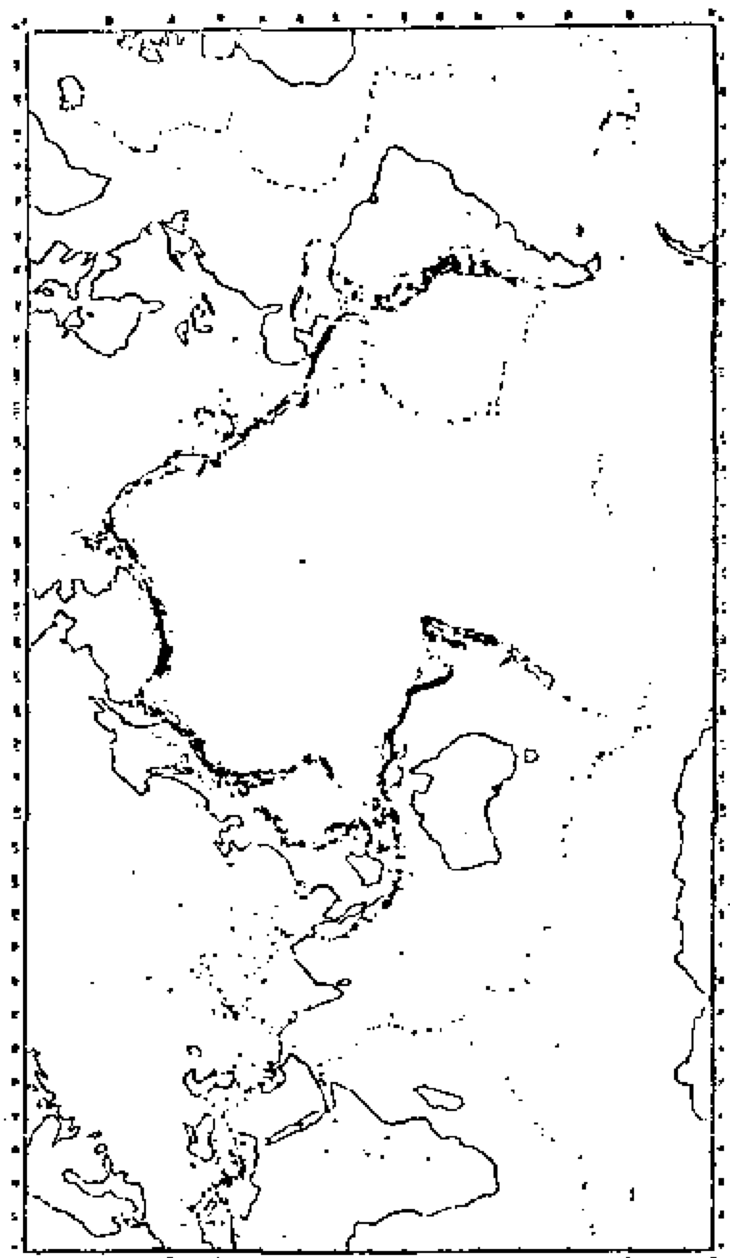


Fig. 6.7. Map showing epicenters for the interval 1961–1967. (After Newmark and Rosenblueth, 1971.)

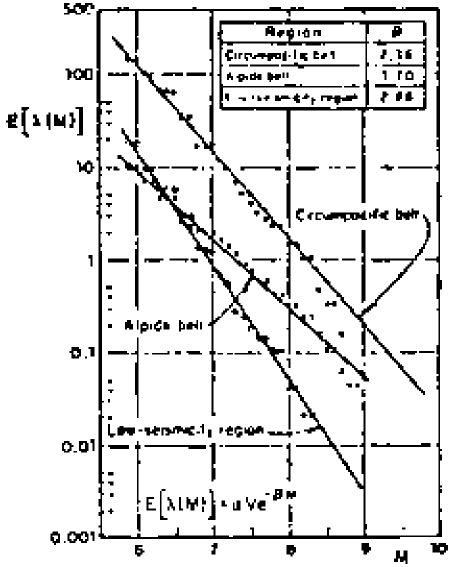


Fig. 6.8. Seismicity of macrozones. (After Esteva, 1968.)

tice of seismic zoning in the Soviet Union has been based on this concept (Gzovsky, 1962; Ananiin et al., 1968) and in many countries design spectra for very important structures, such as nuclear reactors or large dams, are usually derived from the assumption of a maximum credible intensity at a site; that intensity is ordinarily obtained by taking the maximum of the intensities that result at the site when at each of the potential sources an earthquake with magnitude equal to the maximum feasible value for that source is generated at the most unfavourable location within the same source. When this criterion is applied no attention is usually paid to the uncertainty in the maximum feasible magnitude nor to the probability that an earthquake with that magnitude will occur during a given time period. The need to formulate seismic-risk-related decisions that account both for upper bounds to magnitudes and for their probabilities of occurrence suggests adoption of magnitude recurrence expressions of the form:

$$\begin{aligned} \lambda &= \lambda_L G^*(M) && \text{for } M_L \leqslant M \leqslant M_U \\ &= \lambda_L && \text{for } M < M_L \\ &= 0 && \text{for } M > M_U \end{aligned} \qquad (6.7)$$

where M_L = lowest magnitude whose contribution to risk is significant, M_U = maximum feasible magnitude, and $G^*(M)$ = complementary cumulative probability distribution of magnitudes every time that an event ($M \geqslant M_L$) occurs. A particular form of $G^*(M)$ that lends itself to analytical derivations is:

$$G^*(M) = A_0 + A_1 \exp(-\beta M) - A_2 \exp[-(\beta - \beta_1)M] \qquad (6.8)$$

where:

$$A_0 = A\beta_1 \exp[-\beta(M_U - M_L)]$$

$$A_1 = A(\beta - \beta_1) \exp(\beta M_L)$$

$$A_2 = A_\beta \exp(-\beta_1 M_U + \beta M_L)$$

$$A = \{\beta[1 - \exp[-\beta_1(M_U - M_L)]] - \beta_1 \{1 - \exp[-\beta(M_U - M_L)]\}\}^{-1}$$

As M tends to M_L from above, eq. 6.7 approaches eq. 6.6. Adoption of adequate values of M_U and β_1 permits satisfying two additional conditions: the maximum feasible magnitude and the rate of variation of λ in its vicinity. When $\beta_1 \to \infty$, eq. 6.8 tends to an expression proposed by Cornell and Vanmarcke (1969).

Yegulalp and Kuo (1974) have applied the theory of extreme values to estimating the probabilities that given magnitudes are exceeded in given time intervals. They assume those probabilities to fit an extreme type-III distribution given by:

$$\begin{aligned} F_{M_{max}}(M|t) &= \exp[-C(M_U - M)^K t] && \text{for } M \leqslant M_U \\ &= 0 && \text{for } M > M_U \end{aligned} \qquad (6.9)$$

Here $F_{M_{max}}(M|t)$ indicates the probability that the maximum magnitude observed in t years is smaller than M, M_U has the same meaning as above, and C and K are zone-dependent parameters. This distribution is consistent with the assumption that earthquakes with magnitudes greater than M take place in accordance with a Poisson process with mean rate λ equal to $C(M_U - M)^K$. Equation 6.9 produces magnitude recurrence curves that fit closely the statistical data on which they are based for magnitudes above 5.2 and return periods from 1 to 50 years, even though the values of M_U that result from pure statistical analysis are not reliable measures of the upper bound to magnitudes, since in many cases they turn out inadmissibly high.

For low magnitudes, only a fraction of the number of shocks that take place is detected. As a consequence, λ-values based on statistical information lie below those computed according to eqs. 6.6 and 6.8 for M smaller than about 5.5. In addition, Fig. 6.9, taken from Yegulalp and Kuo (1974), shows that the numbers of detected shocks fit the extreme type III in eq. 6.9 better than the extreme type-I distribution implied by eq. 6.6., coupled with the assumption of Poisson distribution of the number of events. It is not

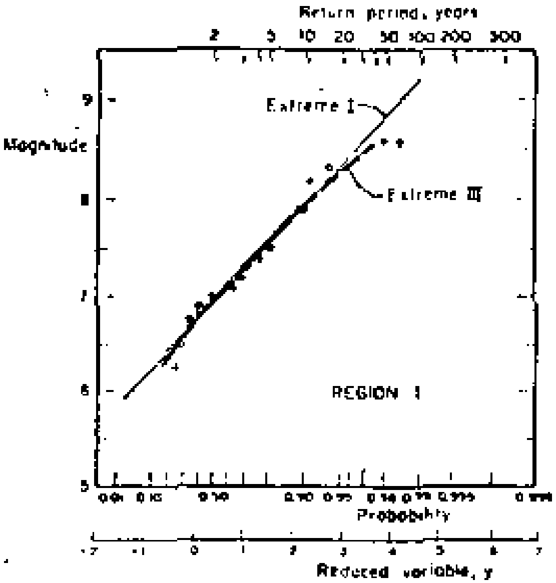


Fig. 6.9. Magnitude statistics in the Aleutian Islands region. (After Yegulalp and Kuo, 1974.)

clear what portion of the deviation from the extreme type-I distribution is due to the low values of the detectability levels and what portion comes from differences between the actual form of variation of λ with M and that given by eq. 6.6. The problem deserves attention because estimates of expected losses due to nonstructural damage may be sensitive to the values of λ for small magnitudes (say below 5.5) and because the evaluation of the level of seismic activity in a region is often made to depend on the recorded numbers of small magnitude shocks and on assumed detectability levels, i.e. of ratios of numbers of detected and occurred earthquakes (Kaila and Narain 1971; Kaila et al., 1972, 1974).

None of the expressions for λ presented in this chapter possess the desirable property that its applicability over a number of non-overlapping regions of the earth's crust implies the validity of an expression of the same form over the addition of those regions, unless some restrictions are imposed on the parameters of each λ. For instance, the addition of expressions like 6.6 gives place to an expression of the same form only if β is the same for all terms in the sum. Similar objections can be made to eq. 6.8. In what follows these forms will be preserved, however, as their accuracy is consistent with

the amount of available information and their adoption offers significant advantages in the evaluation of regional seismicity, as shown later.

6.3.2 Variation with depth

Depth of prevailing seismic activity in a region depends on its tectonic structure. For instance, most of the activity in the western coast of the United States and Canada consists of shocks with hypocentral depths in the range of 20–30 km. In other areas, such as the southern coast of Mexico, seismic events can be grouped into two ensembles: one of small shallow shocks and one of earthquakes with magnitudes comprised in a wide range, and with depths whose mean value increases with distance from the shoreline (Fig. 6.10). Figure 6.11 shows the depth distribution of earthquakes with magnitude above 5.9 for the whole circum-Pacific belt.

6.3.3 Stochastic models of earthquake occurrence

Mean exceedance rates of given magnitudes are expected averages during long time intervals. For decision-making purposes the times of earthquake occurrence are also significant. At present those times can only be predicted within a probabilistic context.

Let t_i (i = 1, ..., n) be the unknown times of occurrence of earthquakes generated in a given volume of the earth's crust during a given time interval, and let M_i be the corresponding magnitudes. For the moment it will be assumed that the risk is uniformly distributed throughout the given volume, and hence no attention will be paid to the focal coordinates of each shock.

Classical methods of time-series analysis have been applied by different researchers attempting to devise analytical models for random earthquake sequences. The following approaches are often found in the literature:

(a) Plotting of histograms of waiting times between shocks (Knopoff, 1964; Aki, 1963).

(b) Evaluation of Poisson's index of dispersion, that is of the ratio of the sample variance of the number of shocks to its expected value (Vere-Jones, 1970; Shlien and Toksöz, 1970). This index equals unity for Poisson processes, is smaller for nearly periodic sequences, and is greater than one when events tend to cluster.

(c) Determination of autocovariance functions, that is, of functions representing the covariance of the numbers of events observed in given time intervals, expressed in terms of the time elapsed between those intervals (Vere-Jones, 1970; Shlien and Toksöz, 1970). The autocovariance function of a Poisson process is a Dirac delta function. This feature is characteristic for the Poisson model since it does not hold for any other stochastic process.

(d) The hazard function $h(t)$, defined so that $h(t)\,dt$ is the conditional probability that an event will take place in the interval (t, t + dt) given that

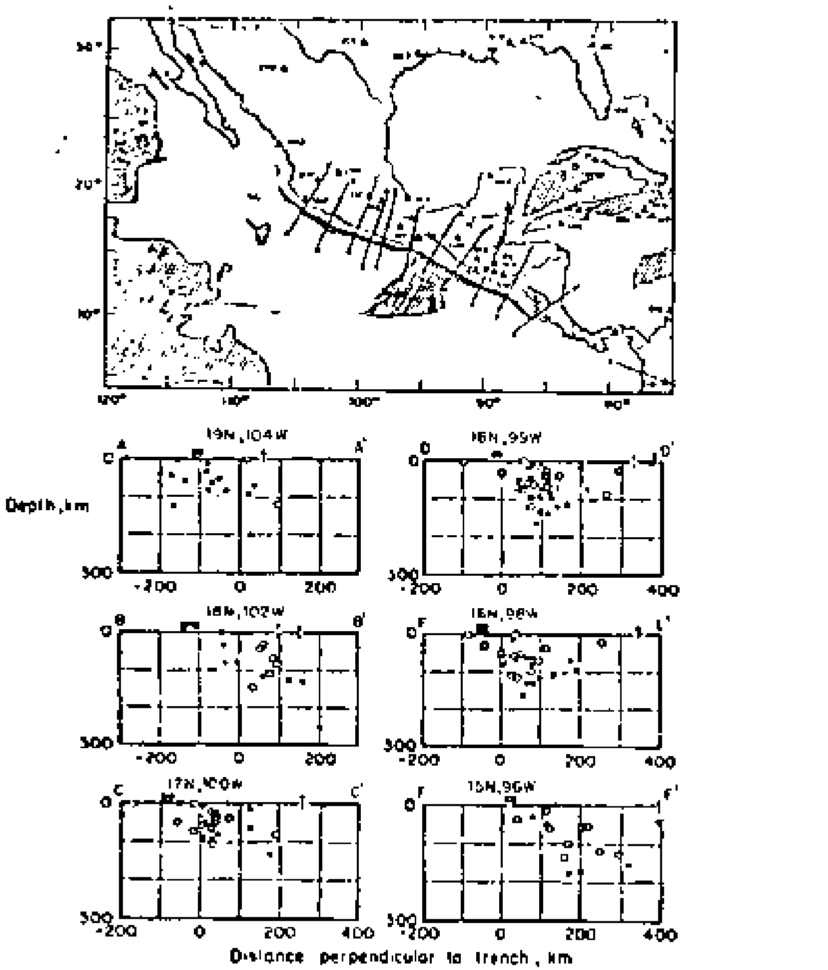


Fig. 6.10. Earthquake hypocenters projected onto a series of vertical sections through Mexico (After Molnar and Sykes, 1969.)

no events have occurred before t. If $F(t)$ is the cumulative probability distribution of the time between events:

$$h(t) = f(t)/[1 - F(t)] \tag{6.10}$$

where $f(t) = \partial F(t)/\partial t$.

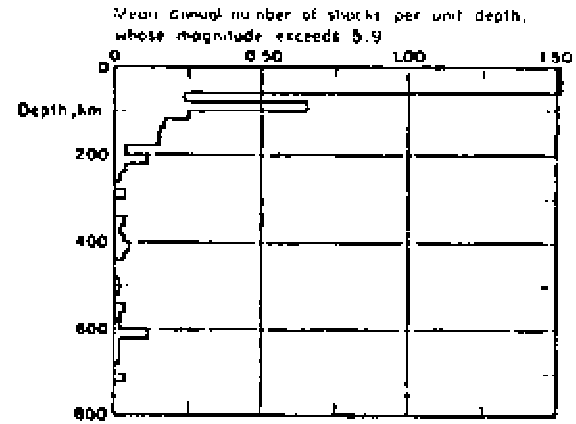


Fig. 6.11. Variation of seismicity with depth, Circum-Pacific Belt. (After Newmark and Rosenblueth, 1971.)

For the Poisson model, $h(t)$ is a constant equal to the mean rate of the process.

6.3.3.1 Poisson model

Most commonly applied stochastic models of seismicity assume that the events of earthquake occurrence constitute a Poisson process and that the M_i's are independent and identically distributed. This assumption implies that the probability of having N earthquakes with magnitude exceeding M during time interval $(0, t)$ equals:

$$p_N = [\exp(-\nu_M t)(\nu_M t)^N]/N! \tag{6.11}$$

where ν_M is the mean rate of exceedance of magnitude M in the given volume. If N is taken equal to zero in eq. 6.11, one obtains that the probability distribution of the maximum magnitude during time interval t is equal to $\exp(-\nu_M t)$. If ν_M is given by eq. 6.6, the extreme type-I distribution is obtained.

Some weaknesses of this model become evident in the light of statistical information and of an analysis of the physical processes involved: the Poisson assumption implies that the distribution of the waiting time to the next event is not modified by the knowledge of the time elapsed since the last one, while physical models of gradually accumulated and suddenly released energy call for a more general renewal process such that, unlike what happens in the Poisson process, the expected time to the next event decreases as time goes on (Esteva, 1974). Statistical data show that the Poisson assumption may be acceptable when dealing with large shocks throughout the world (Ben-Menahem, 1960), implying lack of correlation between seismicities of different regions; however, when considering small volumes of the earth, of the order of those that can significantly contribute to seismic risk at a site, data often contradict Poisson's model, usually because of clustering of earthquakes in time: the observed numbers of short intervals between events are significantly higher than predicted by the exponential distribution, and values of Poisson's index of dispersion are well above unity (Figs. 6.12 and 6.13). In some instances, however, deviations in the opposite direction have been observed: waiting times tend to be more nearly periodic, Poisson's index of dispersion is smaller than one, and the process can be represented by a renewal model. This condition has been reported, for instance, in the southern coast of Mexico (Esteva, 1974), and in the Kamchatka and Pamir—Hindu Kush regions (Gaisky, 1966 and 1967). The models under discussion also fail to account for clustering in space (Tsuboi, 1958; Gajardo and Lomnitz, 1960), for the evolution of seismicity with time, and for the systematic shifting of active sources along geologic accidents (Allen, Chapter 3 of this book). On account of its simplicity, however, the Poisson process model provides a valuable tool for the formulation of some seismic-risk-related decisions, particularly of those that are sensitive only to magnitudes of events having very long return periods.

6.3.3.2 Trigger models

Statistical analysis of waiting times between earthquakes does not favor the adoption of the Poisson model or of other forms of renewal processes, such as those that assume that waiting times are mutually independent with lognormal or gamma distributions (Shlien and Toksöz, 1970). Alternative models have been developed, most of them of the 'trigger type' (Vere-Jones, 1970), i.e. the overall process of earthquake generation is considered as the superposition of a number of time series, each having a different origin, where the origin times are the events of a Poisson process. In general, let N be the number of events that take place during time interval $(0, t)$, τ_m = origin time of the mth series, $W_m(t, \tau_m)$ the corresponding number of events up to instant t, and n_t the random number of time series initiated in the interval $(0, t)$. The total number of events that occur before instant t is then:

$$N = \sum_{m}^{n_t} W_m(t, \tau_m) \tag{6.12}$$

If origin times are distributed according to a homogeneous Poisson process with mean rate ν, and all W_m's are identically distributed stochastic processes with respect to $(t - \tau_m)$, it can be shown (Parzen, 1962) that the mean and variance of N can be obtained from:

$$E(N) = \nu \int_0^t E[W(t, \tau)]\mathrm{d}\tau \tag{6.13}$$

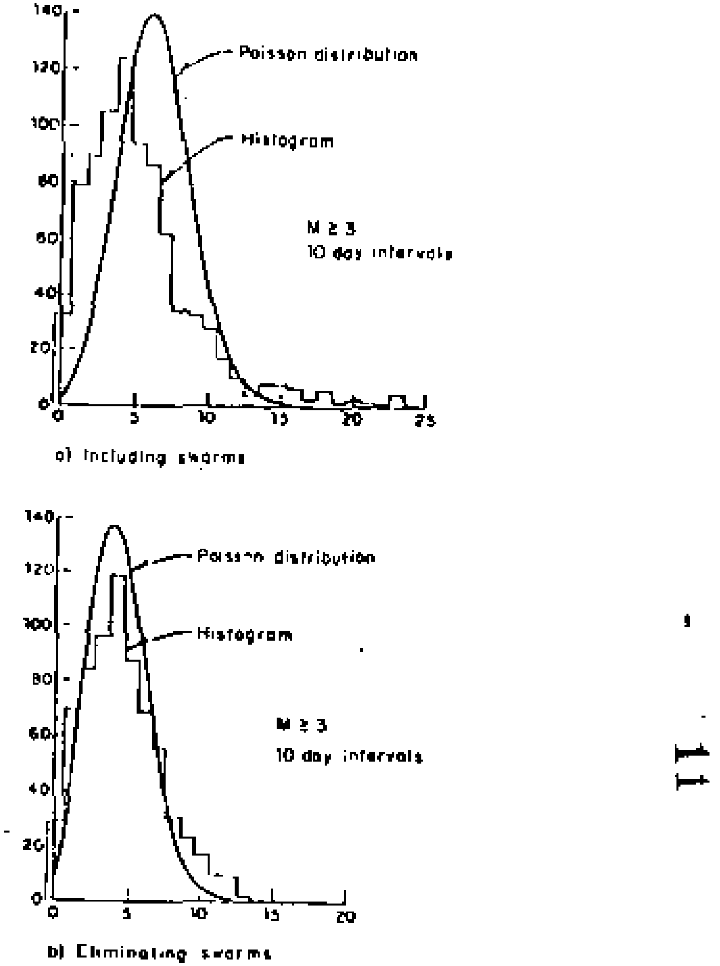


Fig. 6.12. Evaluation of *Poisson* process assumption. (After *Knopoff*, 1964.)

$$\mathrm{var}(N) = \nu \int_0^t E[W^2(t, \tau)]\mathrm{d}\tau \tag{6.14}$$

Parzen (1962) gives also an expression for the probability generating function $\psi_N(Z; t)$ of the distribution of N in terms of $\psi_W(Z; t, \tau)$, the generat-

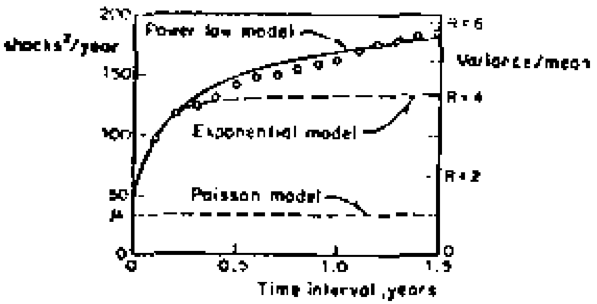


Fig. 6.13. Variance-time curve for New Zealand shallow shocks. (After Vere-Jones, 1966.)

ing function of each of the component processes:

$$\psi_N(Z;t) = \exp\{-\nu t + \nu \int_0^t \psi_W(Z;t,\tau)d\tau\} \tag{6.15}$$

where:

$$\psi_W(Z;t,\tau) = \sum_{n=0}^{\infty} Z^n\, P\{W(t,\tau) = n\} \tag{6.16}$$

and the probability mass function of N can be obtained from $\psi_N(Z;t)$ by recalling that:

$$\psi_N(Z;t) = \sum_{n=0}^{\infty} Z^n P\{N = n\}$$

expanding ψ_N in power series of Z, and taking $P\{N = n\}$ equal to the coefficient of Z^n in that expansion. For instance, if it is of interest to compute $P\{N = 0\}$, expansion of $\psi_N(Z;t)$ in a Taylor's series with respect to $Z = 0$ leads to:

$$\psi_N(Z;t) = \psi_N(0;t) + Z\psi_N'(0;t) + \frac{Z^2}{2!}\psi_N''(0;t) + \ldots \tag{6.17}$$

where the prime signifies derivative with respect to Z. From the definition of ψ_N, $P\{N = 0\} = \psi_N(0;t)$.

Because the component processes of 'trigger'-type time series appear overlapped in sample histories, their analytical representation usually entails study of a number of alternative models, estimation of their parameters, and comparison of model and sample properties — often second-order properties (Cox and Lewis, 1966).

Vere-Jones models. Applicability of some general 'trigger' models to represent local seismicity processes was discussed in a comprehensive paper by Vere-Jones (1970), who calibrated them mainly against records of seismic activity in New Zealand. In addition to simple and compound Poisson processes (Parzen, 1962), he considered Neyman-Scott and Bartlett-Lewis models, both of which assume that earthquakes occur in clusters and that the number of events in each cluster is stocastically independent of its origin time. In the Neyman-Scott model, the process of clusters is assumed stationary and Poisson, and each cluster is defined by p_N, the probability mass function of its number of events, and $\Lambda(t)$, the cumulative distribution function of the time of an event corresponding to a given cluster, measured from the cluster origin. The Bartlett-Lewis model is a special case of the former, where each cluster is a renewal process that ends after a finite number of renewals. In these models the conditional probability of an event taking place during the interval $(t, t + dt)$, given that the cluster consists of N shocks, is equal to $N\lambda(t)dt$, where $\lambda(t) = \partial\Lambda(t)/\partial t$.

Because clusters overlap in time they cannot easily be identified and separated. Estimation of process parameters is accomplished by assuming different sets of those parameters and evaluating the corresponding goodness of fit with observed data.

Various alternative forms of Neyman-Scott's model were compared by Vere-Jones with observed data on the basis of first- and second-order statistics: hazard functions, interval distributions (in the form of power spectra) and variance time curves. The statistical record comprises about one thousand New Zealand earthquakes with magnitudes greater than 4.5, recorded from 1942 to 1961. Figures 6.13—6.15 show results of the analysis for shallow New Zealand shocks as well as the comparison of observed data with sev-

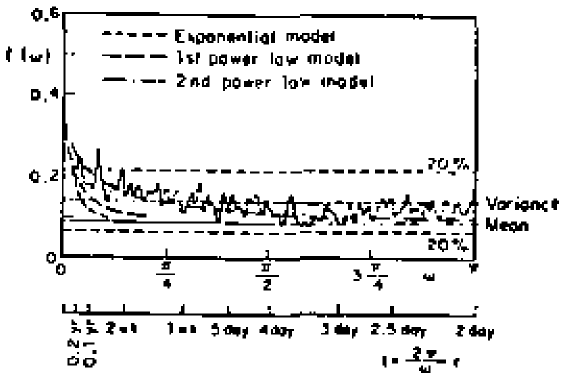


Fig. 6.14. Smoothed periodogram for New Zealand shallow shocks. (After Vere-Jones, 1966.)

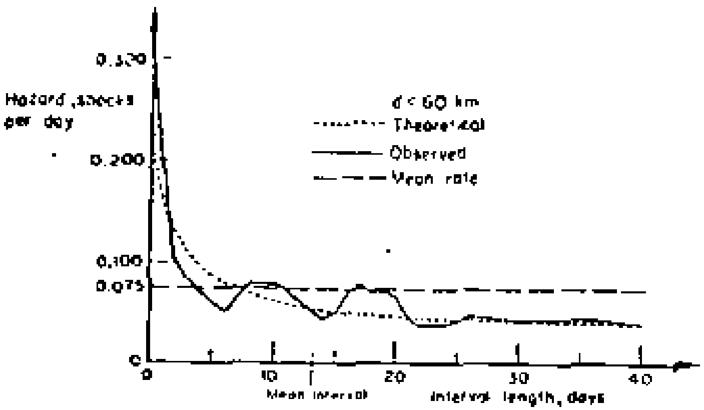


Fig. 6.15. Hazard function for New Zealand shallow shocks. (After Vere-Jones, 1970.)

eral alternative models. The process of cluster origins is Poisson in all cases, but the distributions of cluster sizes (N) and of times of events within clusters differ among the various instances; in the Poisson model no clustering takes place (the distribution of N is a Dirac delta function centered at $N = 1$) while in the exponential and in the power-law models the distribution of N is extremely skewed towards $N = 1$, and $\Lambda(t)$ is taken respectively as $1 - e^{-\lambda t}$

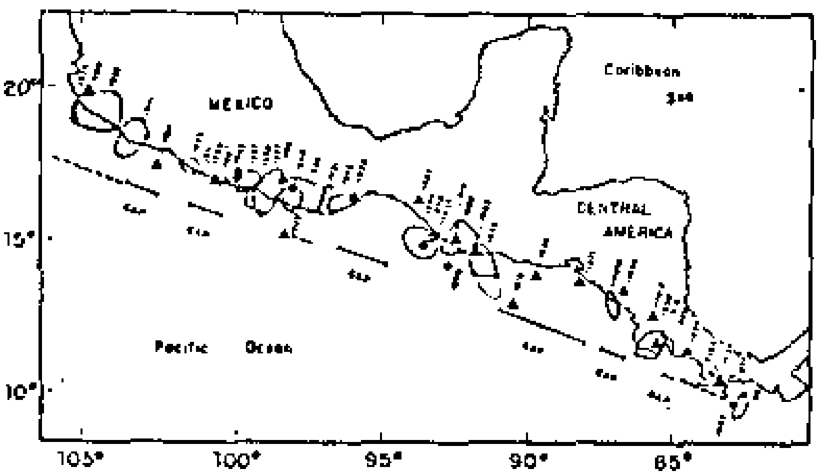


Fig. 6.16. Rupture zones and epicenters of large shallow Middle American earthquakes of this century. (After Kelleher et al., 1973.)

and $1 - [c/(c + t)]^{\delta}$ for $t \geq 0$, and as zero for $t < 0$, where λ, c, and δ are positive parameters. In Figs. 6.13–6.15, δ = 0.25, c = 2.3 days, and λ = 0.061 shocks/day. The significance of clustering is evidenced by the high value of Poisson's dispersion index in Fig. 6.13, while no significant periodicity can be inferred from Fig. 6.14. Both figures show that the power-law model provides the best fit to the statistics of the samples. A similar analysis for New Zealand's deep shocks shows much less clustering: Poisson's dispersion index equals 2, and the hazard function is nearly constant with time.

Still, data reported by Gaisky (1967) have hazard functions that suggest models where the cluster origins as well as the clusters themselves may be represented by renewal processes. Mean return periods are of the order of several months, and hence these processes do not correspond, at least in the time scale, to the process of alternate periods of activity and quiescense of some geological structures cited by Kelleher et al. (1973), which have led to the concept of 'temporal seismic gaps', discussed below.

Simplified trigger models. Shlien and Toksöz (1970) proposed a simple particular case of the Neyman-Scott process: they lumped together all earthquakes taking place during non-overlapping time intervals of a given length and defined them as clusters for which $\lambda(t)$ was a Dirac delta function. Working with one-day intervals, they assumed the number of events per cluster to be distributed in accordance with the discrete Pareto law and applied a maximum-likelihood criterion to the information consisting of 35 000 earthquakes reported by the USCGS from January 1971 to August 1968. The model proposed represents reasonably well both the distribution of the number of earthquakes in one-day intervals and the dispersion index. However, owing to the assumption that no cluster lasts more than one day, the model fails to represent the autocorrelation function of the daily numbers of shocks for small time lags. The degree of clustering is shown to be a regional function, and to diminish with the magnitude threshold value and with the focal depth.

Aftershock sequences. The trigger processes described have been branded as reasonable representations of regional seismic activity, even when aftershock sequences and earthquake swarms are suppressed from statistical records, however arbitrary that suppression may be. The most significant instances of clustering are related, however, to aftershock sequences which often follow shallow shocks and only rarely intermediate and deep events. Persistence of large numbers of aftershocks for a few days or weeks has propitiated the detailed statistical analysis of those sequences since last century. Omori (1894) pointed out the decay in the mean rate of aftershock occurrence with t, the time elapsed since the main shock; he expressed that rate as inversely proportional to $t + q$, where q is an empirical constant. Utsu (1961) proposed a more general expression, proportional to $(t + c)^{-\zeta}$ where ζ is a constant; Utsu's proposal is consistent with the power-law expression for $\Lambda(t)$ presented above.

Lomnitz and Hax (1966) proposed a clustering model to represent aftershock sequences; it is a modified version of Neyman and Scott's model, where the process of cluster origins is non-homogeneous Poisson with mean rate decaying in accordance with Omori's law, the number of events in each cluster has a Poisson distribution, and $A(t)$ is exponential. All the results and methods of analysis described by Vere-Jones (1970) for the stationary process of cluster origins can be applied to the nonstationary case through a transformation of the time scale. Fitting of parameters to four aftershock sequences was accomplished through use of the second-order information of the sample defined on a transformed time scale. By applying this criterion to earthquake sets having magnitudes above different threshold values it was noticed that the degree of clustering decreases as the threshold value increases.

The magnitude of the main shock influences the number of aftershocks and the distribution of their magnitudes and, although the rate of activity decreases with time, the distribution of magnitudes remains stable throughout each sequence (Lomnitz, 1966; Utsu, 1962; Drakopoulos, 1971). Equation 6.6 represents fairly well the distribution of magnitudes observed in most aftershock sequences. Values of β range from 0.9 to 3.9 and decrease as the depth increases. Since values of β for regular (main) earthquakes are usually estimated from relatively small numbers of shocks generated throughout crust volumes much wider than those active during aftershock sequences, no relation has been established among β-values for series of both types of events. The parameters of Utsu's expression for the decay of aftershock activity with time have been estimated for several sequences, for instance those following the Aleutian earthquake of March 9, 1957, the Central Alaska earthquake of April 7, 1958, and the Southeastern Alaska earthquake of July 10, 1958 (Utsu, 1962), with magnitudes equal to 8.3, 7.3, and 7.9, respectively; c (in days) was 0.37, 0.40, and 0.01, while ξ was 1.05, 1.05 and 1.13, respectively. The relationship of the total number of aftershocks whose magnitude exceeds a given value with the magnitude of the main shock was studied by Drakopoulos (1971) for 140 aftershock sequences in Greece from 1912 to 1968. His results can be expressed by $N(M) = A \exp(-\beta M)$, where $N(M)$ is the total number of aftershocks with magnitude greater than M, and A is a function of M_0, the magnitude of the main shock:

$$A = \exp(3.62\,\beta + 1.1M_0 - 3.46) \tag{6.16}$$

Formulation of stochastic process models for given earthquake sequences is feasible once this relationship and the activity decay law are available for the source of interest. For seismic-risk estimation at a given site the spatial distribution of aftershocks may be as significant as the distribution of magnitudes and the time variation of activity, particularly for sources of relatively large dimensions.

6.3.3.3 Renewal process models

The trigger models described are based on information about earthquakes with magnitudes above relatively low thresholds recorded during time intervals of at most ten years. The degrees of clustering observed and the distributions of times between clusters cannot be extrapolated to higher magnitude thresholds and longer time intervals without further study.

Available information shows beyond doubt that significant clustering is the rule, at least when dealing with shallow shocks. However, there is considerable ground for discussion on the nature of the process of cluster origins during intervals of the order of one century or longer. While lack of statistical data hinders the formulation of seismicity models valid over long time intervals, qualitative consideration of the physical processes of earthquake generation may point to models which at least are consistent with the state of knowledge of geophysical sciences. Thus, if strain energy stored in a region grows in a more or less systematic manner, the hazard function should grow with the time elapsed since the last event, and not remain constant as the Poisson assumption implies. The concept of a growing hazard function is consistent with the conclusions of Kelleher et al. (1973) concerning the theory of periodic activation of *seismic gaps*. This theory is partially supported by results of nearly qualitative analysis of the migration of seismic activity along a number of geological structures. An instance is provided by the southern coast of Mexico, one of the most active regions in the world. Large shallow shocks are generated probably by the interaction of the continental mass and the subductive oceanic Cocos plate that underthrusts it, and by compressive or flexural failure of the latter (Chapter 2). Seismological data show significant gaps of activity along the coast during the present century and not much is known about previous history (Fig. 6.16). Along these gaps, seismic-risk estimates based solely on observed intensities are quite low, although no significant difference is evident in the geological structure of those regions with respect to the rest of the coast, save some transverse faults which divide the continental formation into several blocks. Without looking at the statistical records a geophysicist would assign equal risk throughout the area. On the basis of seismicity data, Kelleher et al. have concluded that activity migrates along the region, in such a manner that large earthquakes tend to occur at seismic gaps, thus implying that the hazard function grows with time since the last earthquake. Similar phenomena have been observed in other regions; of particular interest is the North Anatolian fault where activity has shifted systematically along it from east to west during the last forty years (Allen, 1969).

Conclusions relative to activation of seismic gaps are controversial because the observation periods have not exceeded one cycle of each process. Nevertheless, those conclusions point to the formulation of stochastic models of seismicity that reflect plausible features of the geophysical processes.

These considerations suggest the use of renewal-process models to represent sequences of individual shocks or of clusters. Such models are characterized because times between events are independent and identically distributed. The Poisson process is a particular renewal model for which the distribution of the waiting time is exponential. Wider generality is achieved, without much loss of mathematical tractability, if inter-event times are supposed to be distributed in accordance with a gamma function:

$$f_T(t) = \frac{\nu}{(k-1)!}(\nu t)^{k-1}\, e^{-\nu t} \qquad (6.19)$$

which becomes the exponential distribution when $k = 1$. If $k < 1$, short intervals are more frequent and the coefficient of variation is greater than in the Poisson model; if $k > 1$, the reverse is true. Shlien and Toksöz (1970) found that gamma models were unable to represent the sequences of individual shocks they analyzed; but these authors handled time intervals at least an order of magnitude shorter than those referred to in this section.

On the basis of hazard function estimated from sequences of small shocks in the Hindu-Kush, Vere-Jones (1970) deduces the validity of 'branching renewal process' models, in which the intervals between cluster centers, as well as those between cluster members, constitute renewal processes.

Owing to the scarcity of statistical information, reliable comparisons between alternate models will have to rest partially on simulation of the process of storage and liberation of strain energy (Burridge and Knopoff, 1967; Veneziano and Cornell, 1973).

6.3.4 Influence of the seismicity model on seismic risk

Nominal values of investments made at a given instant increase with time when placing them at compound interest rates, i.e. when capitalizing them. Their real value — and not only the nominal one — will also grow, provided the interest rate overshadows inflation. Conversely, for the purpose of making design decisions, nominal values of expected utilities and costs inflicted upon in the future have to be converted into present or actualized values, which can be directly compared with initial expenditures. Descriptions of seismic risk at a site are insufficient for that purpose unless the probability distributions of the times of occurrence of different intensities — or magnitudes at neighbouring sources — are stipulated; this entails more than simple magnitude-recurrence graphs or even than maximum feasible magnitude estimates.

Immediately after the occurrence of a large earthquake, seismic risk is abnormally high due to aftershock activity and to the probability that damage inflicted by the main shock may have weakened natural or man-made structures if emergency measures are not taken in time. When aftershock activity has ceased and damaged systems have been repaired, a normal risk level is attained, which depends on the probability-density functions of the waiting times to the ensuing damaging earthquakes.

For the purpose of illustration, let it be assumed that a fixed and deterministically known damage D_0 occurs whenever a magnitude above a given value is generated at a given source. If $f(t)$ is the probability-density function of the waiting time to the occurrence of the damaging event, and if the risk level is sufficiently low that only the first failure is of concern, the expected value of the actualized cost of damage is (see Chapter 9):

$$\bar{D} = D_0 \int_0^{\infty} e^{-\gamma t} f(t)\,dt \qquad (6.20)$$

where γ is the discount (or compound interest) coefficient and the overbar denotes expectation. If the process is Poisson with mean rate ν, then $f(t)$ is exponential and $\bar{D} = D_0\ \nu/\gamma$; however, if damaging events take place in clusters and most of the damage produced by each cluster corresponds to its first event, the computation of $\bar{D}$ should make use of the mean rate ν corresponding to the clusters, instead of that applicable to individual events. Table 6.11 shows a comparison of seismic risk determined under the alternative assumptions of a Poisson and a gamma model ($k = 2$), both with the same mean return period, k/ν (Esteva, 1974). Three descriptions of risk are presented as functions of the time t_0 elapsed since the last damaging event: $\bar{T}_1$, the expected time to the next event, measured from instant t_0; the expected value of the present cost of failure computed from eq. 6.20, and the hazard function (or mean failure rate). Since clustering is neglected, risk of aftershock occurrence must be either included in D_0 or superimposed on that displayed in the table.

This table shows very significant differences among risk levels for both processes. At small values of t_0, risk is lower for the gamma process, but it

TABLE 6.11

Comparison of Poisson and gamma processes

$t_0\,\nu/k$	$\bar{T}_1\,\nu/k$	Poisson process, $k = 1$ D/D_0 $\gamma k/\nu = 10$	Poisson process, $k = 1$ D/D_0 $\gamma k/\nu = 100$	hk/ν	$\bar{T}_1\,\nu/k$	Gamma process, $k = 2$ D/D_0 $\gamma k/\nu = 10$	Gamma process, $k = 2$ D/D_0 $\gamma k/\nu = 100$	hk/ν
0					1.0	0.0276	0.0004	0
0.1					0.92	0.0511	0.0036	0.367
0.2					0.86	0.0675	0.0059	0.667
0.5					0.75	0.0973	0.0100	1.333
1	1.0	0.0909	0.0099	1.0	0.67	0.120	0.0132	2.000
2					0.60	0.139	0.0156	2.667
5					0.54	0.154	0.0179	3.333
10					0.52	0.160	0.0187	3.633
					0.50	0.167	0.0196	4.000

grows with time, until it outrides that for the Poisson process, which remains constant. The differences shown clearly affect engineering decisions.

6.4 ASSESSMENT OF LOCAL SEISMICITY

Only exceptionally can magnitude-recurrence relations for small volumes of the earth's crust and statistical correlation functions of the process of earthquake generation be derived exclusively from statistical analysis of recorded shocks. In most cases this information is too limited for that purpose and it does not always reflect geological evidence. Since the latter, as well as its connection with seismicity, is beset with wide uncertainty margins, information of different nature has to be evaluated, its uncertainty analyzed, and conclusions reached consistent with all pieces of information. A probabilistic criterion that accomplishes this is presented here: on the basis of geotectonic data and of conceptual models of the physical processes involved, a set of alternate assumptions can be made concerning the functions in question (magnitude recurrence, time, and space correlation) and an initial probability distribution assigned thereto; statistical information is used to judge the likelihood of each assumption, and a posterior probability distribution is obtained. How statistical information contibutes to the posterior probabilities of the alternate assumptions depends on the extent of that information and on the degree of uncertainty implied by the initial probabilities. Thus, if geological evidence supports confidence in a particular assumption or range of assumptions, statistical information should not greatly modify the initial probabilities. If, on the other hand, a long and reliable statistical record is available, it practically determines the form and parameters of the mathematical model selected to represent local seismicity.

6.4.1 Bayesian estimation of seismicity

Bayesian statistics provide a framework for probabilistic inference that accounts for prior probabilities assigned to a set of alternate hypothetical models of a given phenomenon as well as for statistical samples of events related to that phenomenon. Unlike conventional methods of statistical inference, Bayesian methods give weight to probability measures obtained from samples or from other sources; numbers, coordinates and magnitudes of earthquakes observed in given time intervals serve to ascertain the probable validity of each of the alternative models of local seismicity that can be postulated on the grounds of geological evidence. Any criterion intended to weigh information of different nature and different degrees of uncertainty should lead to probabilistic conclusions consistent with the degree of confidence attached to each source of information. This is accomplished by Bayesian methods.

Let H_i ($i = 1, ..., n$) be a comprehensive set of mutually exclusive assumptions concerning a given, imperfectly known phenomenon and let A be the observed outcome of such a phenomenon. Before observing outcome A we assign an initial probability $P(H_i)$ to each hypothesis. If $P(A|H_i)$ is the probability of A in case hypothesis H_i is true, then Bayes' theorem (Raiffa and Schlaifer, 1968) states that:

$$P(H_i|A) = P(H_i)\frac{P(A|H_i)}{\sum_i P(H_i)P(A|H_i)} \tag{6.21}$$

The first member in this equation is the (posterior) probability that assumption H_i is true, given the observed outcome A.

In the evaluation of seismic risk, Bayes' theorem can be used to improve initial estimates of $\lambda(M)$ and its variation with depth in a given area as well as those of the parameters that define the shape of $\lambda(M)$ or, equivalently, the conditional distribution of magnitudes given the occurrence of an earthquake. For that purpose, take $\lambda(M)$ as the product of a rate function $\lambda_L = \lambda(M_L)$ by a shape function $G^*(M,B)$, equal to the conditional complementary distribution of magnitudes given the occurrence of an earthquake with $M \geq M_L$, where M_L is the magnitude threshold of the set of statistical data used in the estimation, and B is the vector of (uncertain) parameters $B_1, ..., B_r$ that define the shape of $\lambda(M)$. For instance, if $\lambda(M)$ is taken as given by eq. 6.8, B is a vector of three elements equal respectively to β, β_1, and M_u; if eq. 6.9 is adopted, B is defined by k and M_u.

The initial distribution of seismicity is in this case expressed by the initial joint probability density function of λ_L and B: $f'(\lambda_L, B)$. The observed outcome A can be expressed by the magnitudes of all earthquakes generated in a given source during a given time interval. For instance, suppose that N earthquakes were observed during time interval t and that their magnitudes were $m_1, m_2, ..., m_N$. Bayes' expression takes the form:

$$f''(\lambda_L, B|m_1, ..., m_N; t) = f'(\lambda_L, B)\frac{P[m_1, m_2, ..., m_N; t|\lambda_L, B]}{\iint P[m_1, m_2, ..., m_N; t|l, b] f'(l, b) dl db} \tag{6.22}$$

where $f''(.)$ is the posterior probability density function, and l and b are dummy variables that stand for all values that may be taken by λ_L and B, respectively. Estimation of λ_L can usually be formulated independently of that of the other parameters. The observed fact is then expressed by N_L, the number of earthquakes with magnitude above M_L during time t, and the following expression is obtained, as a first step in the estimation of $\lambda(M)$:

$$f''(\lambda_L|N_L; t) = f'(\lambda_L)\frac{P(N_L; t|\lambda_L)}{\int P(N_L; t|l) f'(l) dl} \tag{6.23}$$

6.4.1.1 Initial probabilities of hypothetical models

Where statistical information is scarce, seismicity estimates will be very sensitive to initial probabilities assigned to alternative hypothetical models: the opinions of geologists and geophysicists about probable models, about the parameters of these models, and the corresponding margins of uncertainty should be adequately interpreted and expressed in terms of a function f', as required by equations similar to 6.22 and 6.23. Ideally, these opinions should be based on the formulation of potential earthquake sources and on their comparison with possibly similar geotectonic structures. This is usually done by geologists, more qualitatively than quantitatively, when they estimate M_U. Initial estimates of λ_L are seldom made, despite the significance of this parameter for the design of moderately important structures (see Chapter 9).

Analysis of geological information must consider local details as well as general structure and evolution. In some areas it is clear that all potential earthquake sources can be identified by surface faults, and their displacements in recent geological times measured. When mean displacements per unit time can be estimated, the order of magnitude of creep and of energy liberated by shocks and hence of the recurrence intervals of given magnitudes can be established (Wallace, 1970; Davies and Brune, 1971), the corresponding uncertainty evaluated, and an initial probability distribution assigned. The fact that magnitude-recurrence relations are only weakly correlated with the size of recent displacements is reflected in large uncertainties (Petrushevsky, 1966).

Application of the criterion described in the foregoing paragraph can be unfeasible or inadequate in many problems, as in areas where the abundance of faults of different sizes, ages, and activity, and the insufficient accuracy with which focal coordinates are determined preclude a differentiation of all sources. Regional seismicity may then be evaluated under the assumption that at least part of the seismic activity is distributed in a given volume rather than concentrated in faults of different importance. The same situation would be faced when dealing with active zones where there is no surface evidence of motions. Hence, consideration of the overall behavior of complex geological structures is often more significant than the study of local details.

Not much work has been done in the analysis of the overall behavior of large geological structures with respect to the energy that can be expected to be liberated per unit volume and per unit time in given portions of those structures. Important research and applications should be expected, however, since, as a result of the contribution of plate-tectonics theory to the understanding of large-scale tectonic processes, the numerical values of some of the variables correlated with energy liberation are being determined, and can be used at least to obtain orders of magnitude of expected activity along plate boundaries. Far less well understood are the occurrence of shocks in apparently inactive regions of continental shields and the behavior of complex continental blocks or regions of intense folding, but even there some

progress is expected in the study of accumulation of stresses in the crust.

Knowledge of the geological structure can serve to formulate initial probability distributions of seismicity even when quantitative use of geophysical information seems beyond reach. Initial probability distributions of local seismicity parameters λ_L, B in the small volumes of the earth's crust that contribute significantly to seismic risk at a site, can be assigned by comparison with the average seismicity observed in wider areas of similar tectonic characteristics, or where the extent and completeness of statistical information warrant reliable estimates of magnitude-recurrence curves (Esteva, 1969). In this manner we can, for instance, use the information about the average distribution of the depths of earthquakes of different magnitudes throughout a seismic province to estimate the corresponding distribution in an area of that province, where activity has been low during the observation interval, even though there might be no apparent geophysical reason to account for the difference. Similarly, the expected value and coefficient of variation of λ_L in a given area of moderate or low seismicity (as a continental shield) can be obtained from the statistics of the motions originated at all the supposedly stable or aseismic regions in the world.

The significance of initial probabilities in seismic risk estimates, against the weight given to purely statistical information, becomes evident in the example of Fig. 6.16: if Kelleher's theory about activation of seismic gaps is true, risk is greater at the gaps than anywhere else along the coast; if Poisson models are deemed representative of the process of energy liberation, the extent of statistical information is enough to substantiate the hypothesis of reduced risk at gaps. Because both models are still controversial, and represent at most two extreme positions concerning the properties of the actual process, risk estimates will necessarily reflect subjective opinions.

6.4.1.2 Significance of statistical information

Estimation of λ_L. Application of eq. 6.23 to estimate λ_L independently of other parameters will be first discussed, because it is a relatively simple problem and because λ_L is usually more uncertain than M_U and much more so than β.

A model as defined by eq. 6.19 will be assumed to apply. If the possible assumptions concerning the values of λ_L constitute a continuous interval, the initial probabilities of the alternative hypotheses can be expressed in terms of a probability-density function of λ_L. If, in addition, a certain assumption is made concerning the form of this probability-density function, only the initial values of $E(\lambda_L)$ and $V(\lambda_L)$ have to be assumed. It is advantageous to assign to $\nu = k/E(T)$ a gamma distribution. Then, if ρ and μ are the parameters of this initial distribution of ν, if k is assumed to be known, and if the observed outcome is expressed as the time t_n elapsed during $n + 1$ consecutive events (earthquakes with magnitude $\geq M_L$), application of eq. 6.23 leads to the conclusion that the posterior probability function of ν is also gamma, now with parameters $\rho + nk$ and $\mu + t_n$. The initial and the posterior expected values of ν are respectively equal to ρ/μ, and to $(\rho + nk)/(\mu + t_n)$. When initial uncertainty about ν is small, ρ and μ will be large and the initial and the posterior expected values of ν will not differ greatly. On the other hand, if only statistical information were deemed significant, ρ and μ should be given very small values in the initial distribution, and $E(\nu)$, and hence λ_L, will be practically defined by n, k, and t_n. This means that the initial estimates of geologists should not only include expected or most probable values of the different parameters, but also statements about ranges of possible values and degrees of confidence attached to each.

In the case studied above only a portion of the statistical information was used. In most cases, especially if seismic activity has been low during the observation interval, significant information is provided by the durations of the intervals elapsed from the initiation of observations to the first of the $n + 1$ events considered, and from the last of these events until the end of the observation period. Here, application of eq. 6.23 leads to expressions slightly more complicated than those obtained when only information about t_n is used.

The particular case when the statistical record reports no events during at least an interval $(0, t_0)$ comes up frequently in practical problems. The probability-density function of the time T_1 from t_0 to the occurrence of the first event must account for the corresponding shifting of the time axis. Furthermore, if the time of occurrence of the last event before the origin is unknown, the distribution of the waiting time from $t = 0$ to the first event coincides with that of the *excess life* in a renewal process at an arbitrary value of t that approaches infinity (Parzen, 1962). For the particular case when the waiting times constitute a gamma process, T_1 is measured from $t = 0$, T is the waiting time between consecutive events, and it is known that $T_1 \geq t_0$, the conditional density function of $\tau_1 = (T_1 - t_0)/E(T)$ is given by eq. 6.24 (Esteva, 1974), where $u_0 = t_0/E(T)$:

$$f_{\tau_1}(u|T_1 \geq t_0) = \frac{\sum_{m=1}^{k} \frac{k}{(m-1)!} [k(u + u_0)]^{m-1}}{\sum_{m=1}^{k} \sum_{n=1}^{m} \frac{1}{(n-1)!} (ku_0)^{n-1}} e^{-ku} \tag{6.24}$$

Consider now the implications of Bayesian analysis when applied to one of the seismic gaps in Fig. 6.16, under the conditions implicit in eq. 6.24. An initial set of assumptions and corresponding probabilities was adopted as described in the following. From previous studies referring to all the southern coast of Mexico, local seismicity in the gap area (measured in terms of λ for $M \geq 6.5$) was represented by a gamma process with $k = 2$. An initial

probability density function for ν was adopted such that the expected value of $\lambda(6.5)$ for the region coincided with its average throughout the complete seismic province. Two values of ρ were considered: 2 and 10, which correspond to coefficients of variation of 0.71 and 0.32, respectively. Values in Table 6.III were obtained for the ratio of the final to the initial expected values of ν, in terms of u_0.

The last two columns in the table contain the ratios of the computed values of $E''(T_1)$ and $E'(T)$ when ν is taken as equal respectively to its initial or to its posterior expected value. This table shows that, for $\rho = 10$, that is, when uncertainty attached to the geologically based assumptions is low, the expected value of the time to the next event keeps decreasing, in accordance with the conclusions of Kelleher et al. (1973). However, as time goes on and no events occur, the statistical evidence leads to a reduction in the estimated risk, which shows in the increased conditional expected values of T_1. For $\rho = 2$, the geological evidence is less significant and risk estimates decrease at a faster rate.

6.4.1.3 Bayesian estimation of jointly distributed parameters

In the general case, estimation of B will consist in the determination of the posterior Bayesian joint probability function of its components, taking as statistical evidence the relative frequencies of observed magnitudes. Thus, if event A is described as the occurrence of N shocks, with magnitudes $m_1, ..., m_N$, and b_i $(i = 1, ..., r)$ are values that may be adopted by the components of vector B being estimated, eq. 6.21 becomes:

$$f_B''(b_1, ..., b_r|A) = \frac{f_B(b_1, ..., b_r)P(A|b_1, ..., b_r)}{\int ... \int f_B(u_1, ..., u_r)P(A|u_1, ..., u_r)du_1, ..., du_r} \tag{6.25}$$

where $P(A|u_1, ..., u_r)$ is proportional to:

$$\prod_{i=1}^{N} g(m_i|u_1, ..., u_r)$$

and $g(m) = -\partial G^*(m)/\partial m$.

Closed-form solutions for f'' as given by eq. 6.25 are not feasible in general. For the purpose of evaluating risk, however, estimates of the posterior first and second moments of f'' can be obtained from eq. 6.25, making use of available first-order approximations (Benjamin and Cornell, 1970; Rosenblueth, 1975). Thus, the posterior expected value of B_i is given by $\int f_{B_i}''(u)\, u\, du$, where $f_{B_i}''(u_i) = \int ... \int f_B''(u_1, ..., u_r)\, du_1, ..., du_r$ and the multiple integral is of order $r - 1$, because it is not extended to the dominion of B_i. Hence:

$$E''(B_i) = \frac{E_B'[B_i P(A|B_1, ..., B_r)]}{E_B'[P(A|B_1, ..., B_r)]} \tag{6.26}$$

TABLE 6.III

Bayesian estimates of seismicity in one seismic gap

$u_0 = t_0/E(T)$	$E''(\nu)/E'(\nu)$		$E''(T_1 \mid T_1 > t_0)/E'(T)$	
	$\rho = 2$	$\rho = 10$	$\rho = 2$	$\rho = 10$
0	1.0	1.0	0.75	0.75
0.1	0.95	0.99	0.78	0.74
0.5	0.75	0.94	0.91	0.71
1	0.58	0.87	1.14	0.73
5	0.20	0.51	3.11	1.05
10	0.11	0.36	5.47	1.55
20	0.06	0.22	10.50	2.48

where E' and E'' stand for initial and posterior expectation, and subscript B means that expectation is taken with respect to all the components of B. Likewise, the following *posterior moments* can be obtained:

Covariance of B_i and B_j

$$\mathrm{Cov}''(B_i, B_j) = \frac{E'_B[B_i B_j P(A \mid B_1, \ldots, B_r)]}{E'_B[P(A \mid B_1, \ldots, B_r)]} - E''(B_i)E''(B_j) \tag{6.27}$$

Expected value of $\lambda(M)$

$$\begin{aligned} E''[\lambda(M)] &= E''(\lambda_1)E''[G^*(M; B)] \\ &= E''(\lambda_1)\frac{E'_B[G^*(M; B)P(A \mid B_1, \ldots, B_r)]}{E'_B[P(A \mid B_1, \ldots, B_r)]} \end{aligned} \tag{6.28}$$

Marginal distributions. The posterior expectation of $\lambda(M)$ is in some cases all that is required to describe seismicity for decision-making purposes. Often, however, uncertainty in $\lambda(M)$ must also be acounted for. For instance, the probability of exceedance of a given magnitude during a given time interval has to be obtained as the expectation of the corresponding probabilities over all alternative hypotheses concerning $\lambda(M)$. In this manner it can be shown that, if the occurrence of earthquakes is a Poisson process and the Bayesian distribution of λ_L is gamma with mean $\bar{\lambda}_L$ and coefficient of variation V_L, the marginal distribution of the number of earthquakes is negative binomial with mean $\bar{\lambda}_L$. In particular, the marginal probability of zero events during time interval t — equivalently, the complementary distribution function of the waiting time between events — is equal to $(1 + t/t^*)^{-r^*}$, where $r^* = V_L^{-2}$ and $t^* = r^*/\bar{\lambda}_L$. The marginal probability-density function of the waiting time, that should be substituted in eq. 6.20, is $\bar{\lambda}_L(1 + t/t^*)^{-r^*-1}$, which tends to the exponential probability function as r^* and t^* tend to infinity (and $V_L \to 0$) while their ratio remains equal to $\bar{\lambda}_L$.

Bayesian uncertainty tied to the joint distribution of all seismicity parameters $(\lambda_L, B_1, \ldots, B_r)$ can be included in the computation of the probability of occurrence of a given event Z by taking the expectation of that probability with respect to all parameters:

$$P(Z) = E_{\lambda_L, B}[P(Z); \lambda_L, B_1, \ldots, B_r)] \tag{6.29}$$

When the joint distribution of λ_L, B stems from Bayesian analysis of an initial distribution and an observed event, A, this equation adopts the form:

$$P''(Z) = \frac{E'_{\lambda_L, B}[P(Z \mid \lambda_L, B)P(A \mid \lambda_L, B)]}{E'_{\lambda_L, B}[P(A \mid \lambda_L, B)]} \tag{6.30}$$

where ' and " stand for initial and posterior, respectively.

Spatial variability. Figure 6.17 shows a map of geotectonic provinces of Mexico, according to F. Mooser. Each province is characterized by the large-scale features of its tectonic structure, but significant local perturbations to the overall patterns can be identified. Take for instance zone 1, whose seismotectonic features were described above, and are schematically shown in Fig. 6.18 (Singh, 1975): the Pacific plate underthrusts the continental block and is thought to break into several blocks, separated by faults transverse to the coast, that dip at different angles. The continental mass is also

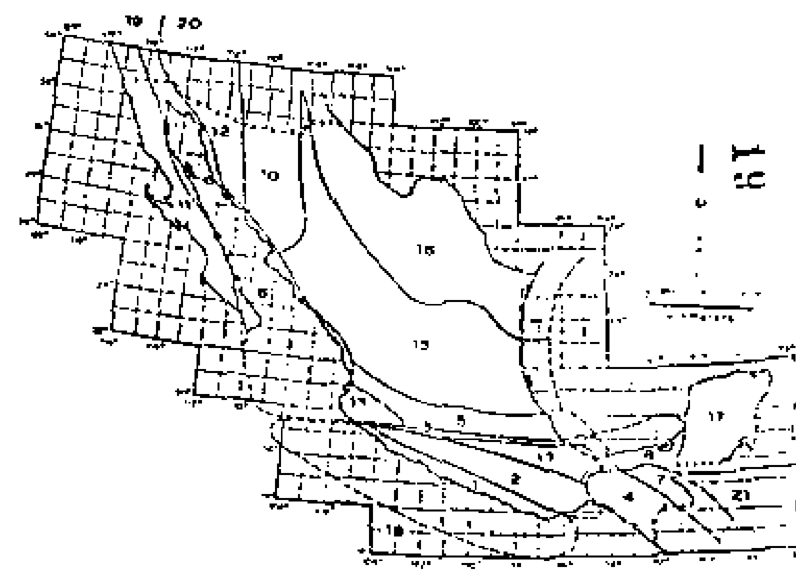
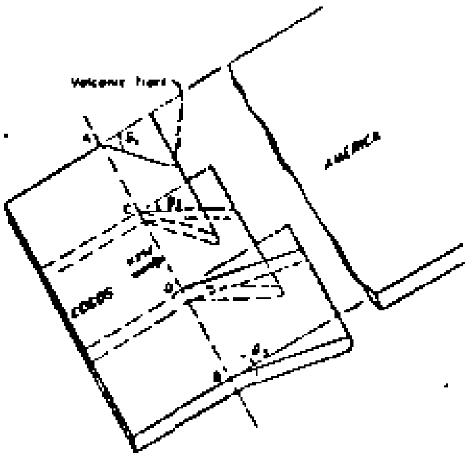


Fig. 6.17. Seismotectonic provinces of Mexico. (After F. Mooser.)



$\theta_1 > \theta_2 > \theta_3$

Fig. 6.18. Schematic drawing of the segmenting of Cocos plate as it subducts below American plate. (After Singh, 1974.)

made up of several large blocks. Seismic activity at the underthrusting plate or at its interface with the continental mass is characterized by magnitudes that may reach very high values and by the increase of mean hypocentral depth with distance from the coast; small and moderate shallow shocks are generated at the blocks themselves. Variability of statistical data along the whole tectonic system was discussed above and is apparent in Fig. 6.10. Bayesian estimation of local seismicity averaged throughout the system is a matter of applying eq. 6.21 or any of its special forms (eqs. 6.22 and 6.23), taking as statistical evidence the information corresponding to the whole system. However, seismic risk estimates are sensitive to values of local seismicity averaged over much smaller volumes of the earth's crust; hence the need to develop criteria for probabilistic inference of possible patterns of space variability of seismicity along tectonically homogeneous zones.

On the basis of seismotectonic information, the system under consideration can first be subdivided into the underthrusting plate and the subsystem of shallow sources; each subsystem can then be separately analyzed. Take for instance the underthrusting plate and subdivide it into s sufficiently small equal-volume subzones. Let ν_L be the rate of exceedance of magnitude M_L throughout the main system, ν_{L_i} the corresponding rate at each subzone, and define p_i as ν_{L_i}/ν_L, with p_i independent of ν_L (p_i is equal to the probability that an earthquake known to have been generated in the overall system originated at subzone i). Initial information about possible space variability of

ν_{L_i} can be expressed in terms of an initial probability distribution of p_i and of the correlation among p_i and p_j for any i and j. Because $\Sigma \nu_{L_i} = \nu_L$, one obtains $\Sigma p_i = 1$. This imposes two restrictions on the initial joint probability distribution of the p_i's: $E'(p_i) = 1$, var$'$ $\Sigma p_i = 0$. If all p_i's are assigned equal expectations and all pairs p_i, p_j, $i \neq j$ are assumed to possess the same correlation coefficient $\rho_{ij} = \rho'$, the restrictions mentioned lead to $E'(p_i) = 1/s$ and $\rho' = -1/(s-1)$. Posterior values of $E(p_i)$ and ρ_{ij} are obtained according to the same principles that led to eqs. 6.25–6.28. Statistical evidence is in this case described by N, the total number of earthquakes generated in the system, and n_i ($i = 1, ..., s$) the corresponding numbers for the subzones. Given the p_i's, the probability of this event is the multinomial distribution:

$$P[A|p_1, ..., p_s] = \frac{N!}{n_1!, ..., n_s!}\, p_1^{n_1} \cdots p_s^{n_s} \tag{6.31}$$

If the correlation coefficients among seismicities of the various subzones can be neglected, each p_i can be separately estimated. Because p_i has to be comprised between 0 and 1, it is natural to assign it a beta initial probability distribution, defined by its parameters n_i' and N_i', such that $E'(p_i) = n_i'/N_i'$ and var$'(p_i) = n_i'(N_i' - n_i')/[N_i'^2(N_i' + 1)]$ (Raiffa and Schlaifer, 1968). The parameters of the posterior distribution will be:

$$n_i'' = n_i' + n_i,\ N_i'' = N_i' + N$$

Take for instance a zone whose prior distribution of λ_L is assumed gamma with expected value λ_L' and coefficient of variation V_L'. Suppose that, on the basis of geological evidence and of the dimensions involved, it is decided to subdivide the zone into four subzones of equal dimensions; a-priori considerations lead to the assignment of expected values and coefficients of variation of p_i for those subzones, say $E'(p_i) = 0.25$, $V'(p_i) = 0.25$ ($i = 1, ..., 4$). From previous considerations for $s = 4$ take $\rho_{ij}' = -1/3$ for $i \neq j$. Suppose now that, during a given time interval t, ten earthquakes were observed in the zone, of which 0, 1, 3, and 6 occurred respectively in each subzone. If the Poisson-process model is adopted, λ_L' and V_L' can be expressed in terms of a fictitious number of events $n' = V_L'^{-2}$ occurred during a fictitious time interval $t' = n'/\lambda_L'$; after observing n earthquakes during an interval t, the Bayesian mean and coefficient of variation of λ_L will be $\lambda_L'' = (n' + n)/(t' + t)$, $V_L'' = (n' + n)^{-1/2}$ (Esteva, 1968). Hence:

$$\lambda_L'' = (V_L'^{-2} + 10)/(V_L'^{-2}\lambda_L'^{-1} + t), \qquad V_L'' = (V_L'^{-2} + 10)^{-1/2}$$

Local deviations of seismicity in each subzone with respect to the average λ_L can be analyzed in terms of p_i ($i = 1, ..., 4$); Bayesian analysis of the proportion in which the ten earthquakes were distributed among the subzones proceeds according to:

$$E''(p_i|A) = \frac{E'[p_i P(A|p_1, ..., p_4)]}{E'[P(A|p_1, ..., p_4)]} \tag{6.32}$$

The expectations that appear in this equation have to be computed with respect to the initial joint distribution of the p_i's. In practice, adequate approximations are required. For instance, Benjamin and Cornells' (1970) first-order approximation leads to $E''(p_1) = 0.226$, $E''(p_4) = 0.204$.

If correlation among subzone seismicities is neglected, and statistical information of each subzone is independently analyzed, when the p_i's are assigned beta probability-density functions with means and coefficients of variation as defined above, one obtains $E''(p_1) = 0.206$, $E''(p_4) = 0.311$, which are not very different from those formerly obtained; however, when $E'(p_i) = 0.25$ and $V'(p_i) = 0.5$, the first criterion leads to $E''(p_i) = 0.206$, $E''(p_4) = 0.314$, while the second produces 0.131 and 0.116, respectively. Part of the difference may be due to neglect of p_0', but probably a significant part stems from inaccuracies of the first-order approximation to the expectations that appear in eq. 6.32; alternate approximations are therefore desirable.

Incomplete data. Statistical information is known to be fairly reliable only for magnitudes above threshold values that depend on the region considered, its level of activity, and the quality of local and nearby seismic instrumentation. Even incomplete statistical records may be significant when evaluating some seismicity parameters; their use has to be accompanied by estimates of detectability values, that is, of ratios of the numbers of events recorded to total numbers of events in given ranges (Esteva, 1970; Kaila and Narain, 1971).

6.5 REGIONAL SEISMICITY

The final goal of local seismicity assessment is the estimation of regional seismicity, that is, of probability distributions of intensities at given sites, and of probabilistic correlations among them. These functions are obtained by integrating the contributions of local seismicities of nearby sources, and hence their estimates reflect Bayesian uncertainties tied to those seismicities. In the following, regional seismicity will be expressed in terms of mean rates of exceedance of given intensities; more detailed probabilistic descriptions would entail adoption of specific hypotheses concerning space and time correlations of earthquake generation.

6.5.1 Intensity-recurrence curves

The case when uncertainty in seismicity parameters is neglected will be discussed first. Consider an elementary seismic source with volume dV and local seismicity $\lambda(M)$ per unit volume, distant R from a site S, where intensity-recurrence functions are to be estimated. Every time that a magnitude M shock is generated at that source, the intensity at S equals:

$$Y = \epsilon Y_p = \epsilon b_1 \exp(b_2 M) g(R) \tag{6.33}$$

(see eqs. 6.4 and 6.5), where ϵ is a random factor and Y and Y_p stand for actual and predicted intensities, b_1 and b_2 are given constants, and $g(R)$ is a function of hypocentral distance. The probability that an earthquake originating at the source will have an intensity greater than y is equal to the probability that $\epsilon Y_p > y$. If Y_p is expressed in terms of M and randomness in ϵ is accounted for, one obtains:

$$\nu(y) = \int_{\alpha_U}^{\alpha_L} \nu_p(y/u) f_\epsilon(u) du \tag{6.34}$$

where ν and ν_p are respectively mean rates at which actual and predicted intensities exceed given values, $\alpha_U = y/y_U$, $\alpha_L = y/y_L$, y_U, and y_L are the predicted intensities that correspond to M_U and M_L, and f_ϵ the probability-density function of ϵ. If eq. 6.33 is assumed to hold:

$$\nu_p(y) = K_0 + K_1 y^{-r_1} - K_2 y^{-r_2} \tag{6.35}$$

where:

$$K_i = [b_1 g(R)]^{r_i} A_i \lambda_L dV \quad (i = 0, 1, 2) \tag{6.36}$$

$$r_0 = 0, \quad r_1 = \beta/b_2, \quad r_2 = (\beta - \beta_1)/b_2 \tag{6.37}$$

Substitution of eq. 6.35 into 6.34, coupled with the assumption that $\ln \epsilon$ is normally distributed with mean m and standard deviation σ leads to:

$$\nu(y) = c_0 K_0 + c_1 K_1 y^{-r_1} - c_2 K_2 y^{-r_2} \tag{6.38}$$

where:

$$c_i = \exp(Q_i) \left[\phi\left(\frac{\ln \alpha_L - u_i}{\sigma}\right) - \phi\left(\frac{\ln \alpha_U - u_i}{\sigma}\right)\right] \tag{6.39}$$

ϕ is the standard normal cumulative distribution function, $Q_i = 1/2\, \sigma^2 r_i^2 + m r_i$, and $u_i = m + \sigma^2 r_i$. Similar expressions have been presented by Merz and Cornell (1973) for the special case of eq. 6.8 when $\beta_1 \to \infty$ and for a quadratic form of the relation between magnitude and logarithm of exceedance rate. Closed-form solutions in terms of incomplete gamma functions are obtained when magnitudes are assumed to possess extreme type-III distributions (eq. 6.9).

Intensity-recurrence curves at given sites are obtained by integration of the contributions of all significant sources. Uncertainties in local seismicities can be handled by describing regional seismicity in terms of means and variances of $\nu(y)$ and estimating these moments from eq. 6.34 and suitable first- and second-moment approximations. Influence of these uncertainties in design decisions has been discussed by Rosenblueth (in preparation).

6.5.2 *Seismic probability maps*

When intensity-recurrence functions are determined for a number of sites with uniform local ground conditions the results are conveniently represented by sets of seismic probability maps, each map showing contours of intensities that correspond to a given return period. For instance, Figs. 6.19 and 6.20 show peak ground velocities and accelerations that correspond to 100 years return period on *firm* ground in Mexico. These maps form part of a set that was obtained through application of the criteria described in this chapter. Because the ratio of peak ground accelerations and velocities does not remain constant throughout a region, the corresponding design spectra will not only vary in scale but also in shape (frequency content); in other words, seismic risk will usually have to be expressed in terms of at least the values of two parameters (for instance, as in this case, peak ground accelerations and velocities that correspond to various risk levels (return periods)).

6.5.3 *Microzoning*

Implicit in the above criteria for evaluation of regional seismicity is the adoption of intensity attenuation expressions valid on firm ground. Scatter of actual intensities with respect to predicted values was ascribed to differences in source mechanisms, propagation paths, and local site conditions; at least the latter group of variables can introduce systematic deviations in the

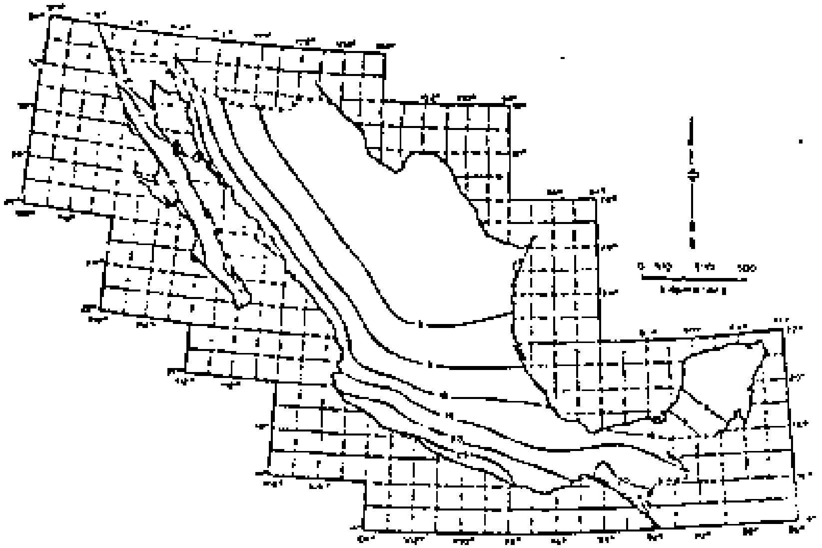


Fig. 6.19. Peak ground velocities with return period of 100 years (cm/sec).

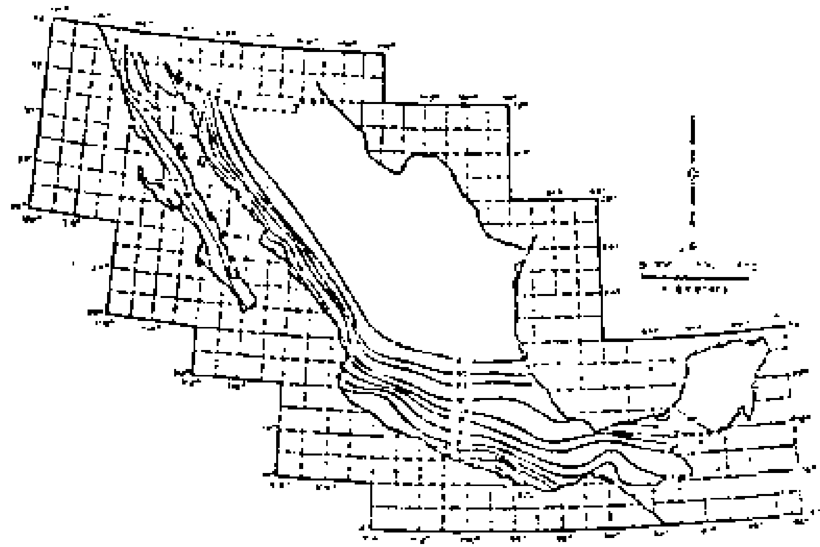


Fig. 6.20. Peak ground accelerations with return period of 100 years (cm/sec^2)

ratio of actual to predicted intensities; and geological details may significantly alter local seismicity in a small region, as well as energy radiation patterns, and hence regional seismicity in the neighbourhood. These systematic deviations are the matter of microzoning, that is, of local modification of risk maps similar to Figs. 6.19 and 6.20.

Most of the effort invested in microzoning has been devoted to study of the influence of local soil stratigraphy on the intensity and frequency content of earthquakes (see Chapter 4). Analytical models have been practically limited to response analysis of stratified formations of linear or nonlinear soils to vertically traveling shear waves. The results of comparing observed and predicted behavior have ranged from satisfactory (Herrera et al., 1965) to poor (Hudson and Udwadia, 1972). Topographic irregularities, as hills or slopes of firm ground formations underlying sediments, may introduce significant systematic perturbations in the surface motion, as a consequence of wave focusing or dynamic amplification. The latter effect was probably responsible for the exceptionally high accelerations recorded at the abutment of Pacoima dam during the 1971 San Fernando earthquake.

Present practice of microzoning determines seismic intensities or design parameters in two steps. First the values of those parameters on firm ground are estimated by means of suitable attenuation expressions and then they are amplified according to the properties of local soil; but this implies an arbitrary decision to which seismic risk is very sensitive: selecting the boundary between soil and firm ground. A specially difficult problem stems when trying to fix that boundary for the purpose of predicting the motion at the top of a hill or the slope stability of a high cliff (Rukos, 1974).

It can be concluded that rational formulation of microzoning for seismic risk is still in its infancy and that new criteria will appear that will probably require intensity attenuation models which include the influence of local systematic perturbations. Whether these models are available or the two-step process described above is acceptable, intensity-recurrence expressions can be obtained as for the unperturbated case, after multiplying the second member of eq. 6.34 by an adequate intensity-dependent corrective factor.

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Chapter 7

TSUNAMIS

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7.1 INTRODUCTION

7.1.1 Some data

Tsunamis are the long water waves (with wave 'periods' in approximately the 5−60 minute range) generated impulsively by mechanisms such as underwater tectonic displacements associated with earthquakes, high-speed subaqueous slides, rock-slides into reservoirs, bays or the ocean, and exploding islands. They may be caused by the tectonic displacement of an entire body of water such as a lake (Wiegel and Camotim, 1962).

The horizontal component of velocity V_h at which the water is displaced from the source by one of the mechanisms mentioned above is important, with the speed being measured relative to $\sqrt{gd}$ (where g is the acceleration of gravity and d is the water depth). However, as long as the Froude number $(V_h/\sqrt{gd})$ is high, theory and hydraulic experiments show it is not as important as the amount of water that is displaced (see, for example, Wiegel et al., 1970; Hatori, 1970).

It is likely that the major cause of large-scale catastrophic tsunamis is a rapidly occurring tectonic displacement of the ocean bottom, with the displacement having a substantial vertical component (dip-slip), as shown in Fig. 7.1 (Iida, 1970; see also, Balakina, 1970 and Watanabe, 1970). One would expect that strike-slips would have to occur through a seamount or submarine cliff to generate a tsunami, and then, owing to the rapid decrease of the ground displacement with distance from the fault (Bonilla, 1970), it is unlikely that major tsunamis would be generated by this mechanism as the waves would be rather short (Garcia, 1972). However, earthquakes associated with strike-slip faults (as well as with other types of faulting) may trigger a submarine earthquake, which in turn may generate a tsunami.

Tsunamis are important because of the loss of life and great property damage that result from large ones. More than 27,000 people were killed and 10,000 houses destroyed in Japan by the tsunami of June 15, 1896 (Leet, 1948). A great tsunami which struck Chile, Hawaii, California, Japan and all other coastal areas bordering the Pacific Ocean, occurred in conjunction with the Chilean earthquake of May 23, 1960 (Committee for Field Investigation

ISSN 0185 - 2345

INSTITUTO DE INGENIERIA UNAM

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SERIES DEL INSTITUTO DE INGENIERIA

No. 471

SEPTIEMBRE 1983

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ABSTRACT

In a previous work the authors developed a method for elastic seismic analysis of structures; it was named quasidynamic method. In this paper such method is generalized to include elastoplastic behavior in terms of ductility factors, which are used to reduced the design spectra.

It is found that this new method leads to better results than those obtained with the seismic static criterium established in the Mexico City Seismic Code, when they are compared with the corresponding ones computed with the dynamic modal spectral procedure given in the same code.

The proposed method is simple and can be applied to both regular and irregular buildings and, therefore, is useful for practising engineers related with seismic design.

RESUMEN

En una investigación previa efectuada por los autores se estudió, en forma paramétrica, la respuesta sísmica de edificios irregulares en elevación con comportamiento elástico. A fin de generalizar el método de análisis ahí propuesto, en este trabajo se estudia la influencia del comportamiento elastoplástico, definido en términos de los espectros de diseño reducidos por ductilidad.

Se muestra que este procedimiento conduce a mejores resultados que los del método estático establecido en el Reglamento de Construcciones para el Distrito Federal, tomando como base de comparación los resultados del método dinámico del mismo Reglamento.

El método ha sido denominado *cuasidinámico* y se puede emplear para analizar edificios regulares e irregulares en elevación; es de aplicación sencilla y útil para ingenieros que se dedican al diseño sísmico de edificios.

NOTACION

$A(T_1,Q)$ aceleración espectral de diseño para el modo fundamental

A_i coeficiente para distribución de aceleraciones

B dimensión en planta del edificio, en la dirección analizada

B' ancho de la base del muro

C_1 factor de participación aproximado del modo fundamental

F_i fuerza sísmica que actúa en la masa i

F_i^* fuerza sísmica cuasidinámica aplicada en la masa i

H altura total de la estructura

M_{d_c} momento sísmico de volteo dinámico, en la bse de la estructura, calculado considerando todos los modos de vibrar

M_{d_r} momento sísmico de volteo dinámico, en el nivel r, calculado considerando todos los modos de vibrar

M_{e_r} momento sísmico estático de volteo en el nivel r

M_r^* momento de volteo cuasidinámico en el nivel r

P_i fuerza sísmica horizontal que actúa estáticamente en la masa i

Q factor de ductilidad

T_1 periodo fundamental

V_{d_o} cortante sísmica dinámica basal, calculada considerando todos los modos de vibrar

V_{d_r} cortante sísmica dinámica en el entrepiso r, calculada considerando todos los modos de vibrar

V_{dl_o} igual que V_{d_o}, pero empleando solo el modo fundamental

V_{e_o} cortante sísmica estática basal

V_{e_r} cortante sísmica estática en el entrepiso r

V_o cortante sísmica basal no corregida

V_o^* cortante sísmica basal cuasidinámica

V_r cortante sísmica de entrepiso no corregida

V_r^* cortante sísmica cuasidinámica en el entrepiso r

W_i peso de la masa i

W_T carga gravitacional total de la estructura.

c coeficiente sísmico

g aceleración de la gravedad

h_i altura de la masa i respecto a la base del edificio

m_i masa concentrada en el nivel i

n número total de niveles donde están concentradas las masas

r índice para identificar entrepisos

x_i desplazamiento de la masa i causado por la fuerza P_i

$\ddot{x}_i$ aceleración de la masa i (excitación dinámica)

α factor correctivo para cortante basal

β_i parámetros para definir al factor correctivo α

λ exponente para ajuste de aceleraciones

1. INTRODUCCION

El método de análisis sísmico estático del Reglamento de Construcciones para el Distrito Federal (ref 1), se puede aplicar indistintamente para edificios regulares o irregulares en elevación. Sin embargo, si la distribución de fuerzas sísmicas laterales se hiciera tomando en cuenta la variación de masas y rigideces con la altura, como lo hace el método dinámico, el análisis sería más adecuado.

En la ref 2 se tiene un criterio normativo que considera la distribución irregular de masas y rigideces mediante un método iterativo, en el cual las fuerzas estáticas laterales dependen de la configuración deformada del sistema. El criterio establece que si al aplicar dicho método la respuesta difiere en más de treinta por ciento en dos ciclos consecutivos, debe hacerse un análisis sísmico dinámico modal.

En investigaciones anteriores (refs 3 a 5) se ha estudiado el comportamiento de edificios irregulares en elevación, cuando están sometidos a solicitaciones sísmicas. Ha sido común encontrar que la respuesta dinámica modal espectral es menor que la obtenida con los métodos sísmicos estáticos, excepto en algunos casos de estructuras con reducciones bruscas de rigidez y masa.

El método que se presenta en esta investigación se formuló inicialmente para estudiar la respuesta elástica de modelos estructurales de cortante y flexión (ref 4); su bondad se verificó al aplicarlo a diferentes estructuraciones con o sin irregularidades en elevación. En este trabajo se generaliza el método a fin de incluir el efecto inelástico, para lo cual se emplean los espectros de diseño del Reglamento (ref 1), reduciéndolos con factores de ductilidad iguales a 2, 4 y 6. Para estudiar la aproximación del método se toma como base de comparación la respuesta dinámica espectral con la participación de todos los modos.

En el cap 2 de este informe se presenta y verifica el método y se discuten los ajustes requeridos para generalizar su aplicación al diseño sísmico. Por sus características, se ha denominado método cuasidinámico de análisis sísmico de edificios.

En el cap 3 se exponen las conclusiones relevantes obtenidas y se recomienda incluir a futuro el método cuasidinámico en el Reglamento de Construcciones para el Distrito Federal, como una opción más de análisis sísmico.

2. METODO CUASIDINAMICO PARA ANALISIS SISMICO DE EDIFICIOS

El método que aquí se propone para calcular la respuesta sísmica de estructuras parte del análisis sísmico estático convencional, el cual se modifica para lograr una distribución de fuerzas cortantes más acorde con las que se obtendrían mediante un análisis dinámico modal espectral. Esa modificación se basa parcialmente en los resultados de estudios paramétricos de marcos rígidos (sistemas de cortante) y muros de cortante acoplados (sistemas de flexión).

2.1 *Descripción del método*

El método cuasidinámico consiste en lo siguiente:

a) Considerar que los edificios están empotrados en su base y calcular las fuerzas sísmicas horizontales mediante el método estático convencional (ref 1)

$$P_i = \frac{W_i h_i}{\sum_{i=1}^{n} W_i h_i} \frac{c}{Q} W_T$$

para estudiar la respuesta elástica de modelos estructurales de cortante y flexión (ref 4); su bondad se verificó al aplicarlo a diferentes estructuraciones con o sin irregularidades en elevación. En este trabajo se generaliza el método a fin de incluir el efecto inelástico, para lo cual se emplean los espectros de diseño del Reglamento (ref 1), reduciéndolos con factores de ductilidad iguales a 2, 4 y 6. Para estudiar la aproximación del método se toma como base de comparación la respuesta dinámica espectral con la participación de todos los modos.

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$$P_i = \frac{W_i h_i}{\sum_{i=1}^{n} W_i h_i} \frac{c}{Q} W_T$$

donde

P_i fuerza sísmica horizontal que actúa estáticamente en la masa i

Q factor de ductilidad

W_i peso de la masa i

W_T carga gravitacional total de la estructura

c coeficiente sísmico

h_i altura de la masa i respecto a la base del edificio

n número total de niveles donde están concentradas las masas

b) Obtener los desplazamientos x_i que producen las fuerzas P_i en cada nivel, y con ellos determinar la aceleración de cada masa i mediante la ecuación

$$\ddot{x}_i = A(T_1, Q)\, C_1 x_i$$

con

$$C_1 = \frac{\sum_{i=1}^{n} m_i x_i}{\sum_{i=1}^{n} m_i x_i^2}$$

donde

$A(T_1,Q)$ aceleración espectral de diseño para el modo fundamental

C_1 factor de participación aproximado del modo fundamental

T_1 periodo fundamental

m_i masa concentrada en el nivel i

Calcular la aceleración de la manera antes indicada implica que la configuración dada por los desplazamientos x_i se tome como una aproximación de la forma característica del primer modo.

El periodo fundamental se puede determinar empleando cualquier procedimiento de cálculo; proporciona resultados satisfactorios el que se basa en el cociente de Schwartz (ref 4)

$$T_1 = 2\pi\left(\frac{1}{g} \sum_{i=1}^{n} W_i x_i^2 \Big/ \sum_{i=1}^{n} P_i x_i\right)^{1/2}$$

donde g es la aceleración de la gravedad.

c) Calcular la fuerza sísmica lateral que actúa en cada masa, provocada por la aceleración $\ddot{x}_i$

$$F_i = m_i \ddot{x}_i$$

A partir de esta, se obtienen las fuerzas cortantes para cada entrepiso r

$$V_r = \sum_{i=r}^{n} F_i$$

donde el subíndice r indica que el nivel r está inmediatamente arriba del entrepiso r, siendo la base el nivel 0.

Las fuerzas cortantes V_r se aproximan a las cortantes calculadas mediante el modo fundamental en un análisis dinámico (V_{d1_r}). Para lograr una cortante basal que se asemeje a la que resultaría de la contribución de todos los modos (V_{d_o}), debe hacerse la corrección que sigue.

d) Multiplicar la cortante basal, V_o, por un factor correctivo, α, para obtener una cortante basal corregida, V_o^*, que se aproxime a la cortante V_{d_o}

$$V_o^* = \alpha V_o$$

Se ha encontrado (ref 4) que el factor correctivo α depende de la relación de cortantes basales V_o/V_{e_o} (V_{e_o} es la cortante basal obtenida con las fuerzas P_i), y del espectro de diseño asociado con el sitio y tipo de suelo donde se desplante la estructura. En esta investigación se observó que α también depende del factor de ductilidad que se asigne a la estructura.

En el subcap 2.2 se determinan las ecuaciones para calcular α, correspondientes a los espectros de diseño del Reglamento de Construcciones para el Distrito Federal.

e) Calcular las nuevas fuerzas laterales corregidas, F_i^*, que actúan en cada masa. Esto se hará de acuerdo con lo que se establece en el subcap 2.3.

f) Obtener las fuerzas cortantes de entrepiso, V_r^*, utilizando las fuerzas F_i^* y los momentos de volteo correspondientes. Estas fuerzas cortantes se denominan cortantes sísmicas cuasidinámicas de entrepiso.

2.2 *Determinación de los factores correctivos, α, para los espectros de diseño del Distrito Federal*

El factor correctivo α se define como el cociente de la fuerza cortante basal que se obtiene en un análisis dinámico modal espectral, incorporando el efecto de todos los modos, V_{d_o}, entre la fuerza cortante basal correspondiente al primer modo, V_{d1_o}, es decir

$$\alpha = V_{d_o}/V_{d1_o}$$

En la ref 4 se determinó este factor utilizando los espectros elásticos de diseño (Q = 1) del Reglamento de Construcciones para el Distrito Federal (ref 1). En este trabajo se verificó también que, al aplicar el método a las estructuras reales de la ref 3, los resultados fueron satisfactorios.

En la presente investigación, el factor α se calcula para las estructuras obtenidas con las combinaciones posibles de masa y rigidez que se indican en figs 1 y 2 (la masa M_c-1 solo se combina con la rigidez K_c-3), considerando que puedan estar desplantadas en suelo firme, de transición o compresible del Distrito Federal. A cada modelo estructural se le asignan factores de ductilidad de 1,2,4 y 6.

Después de analizar varias funciones envolventes de los puntos mostrados en las figs 3 a 14, se decidió proponer la siguiente ecuación general para definir

$$\alpha = 1 + \beta_1 \exp(\beta_2 (V_o/V_{e_o} - \beta_3)^{\beta_4}), \text{ para } \beta_3 \leq V_o/V_{e_o} \leq 1 \qquad (1)$$

donde β_1, β_2, β_3 y β_4 son constantes que dependen del tipo de suelo y del factor de ductilidad. En la tabla 1 se dan los valores obtenidos de estas constantes para los distintos factores de ductilidad y zonas geotécnicas del Distrito Federal, al ajustar la ec 1 a los resultados de cada una de las figs 3 a 14.

Al notar la similitud entre algunas curvas, se decidió reducirlas en número a fin de simplificar la aplicación del método. De esta manera, se optó por emplear las siguientes

a) Terreno firme

Para todos los modelos de las figs 1 y 2 con Q = 1 y Q = 2, 4 y 6, excepto el K_P-3

$$\alpha = 1 + 1.5 \exp\{-4.9(V_o/V_{e_o} - 0.108)^{0.75}\} \qquad (2)$$

Con factores de ductilidad 2, 4 y 6, para el modelo K_P-3

$$\alpha = 1 + 1.7 \exp\{-4.95(V_o/V_{e_o} - 0.108)\} \qquad (3)$$

b) Terrenos de transición y compresible

Para cualquier factor de ductilidad y todos los modelos de las figs 1 y 2, se empleará la ec 3.

En la fig 15 se muestran las curvas de α para fines de diseño dadas por las ecs 2 y 3; se observará que son envolventes de las presentadas en las figs 3 a 14.

2.3 *Obtención de cortantes de entrepiso cuasidinámicas*

En investigaciones recientes de los autores (refs 4 y 5), las fuerzas sísmicas cuasidinámicas en edificios irregulares en elevación se obtuvieron distribuyendo el cortante basal, V_o^*, en función de factores de ponderación que dependían del peso y el desplazamiento, x_i, de cada masa

$$F_i^* = \frac{W_i x_i}{\sum\limits_{i=1}^{n} W_i x_i} V_o^* \qquad (4)$$

A fin de comprobar si esta distribución de fuerzas era adecuada al emplear espectros reducidos por ductilidad, se analizaron paramétricamente los modelos estructurales de las figs 1 y 2.

Para los muros de cortante acoplados se emplearon relaciones de esbeltez de 1.2 y 3.6; la primera con 30 m de altura y 10 niveles, y la segunda con 90 m de altura y 30 niveles, por lo que ya no sería aplicable el método estático del Reglamento. Los marcos rígidos se analizaron con relaciones de esbeltez de 2 y 4; la primera con 30 m de altura y 10 niveles, y la segunda con 60 m de altura y 20 niveles. En todos los casos se emplearon valores del coeficiente Q iguales a 1, 2, 4 y 6; resulta pertinente aclarar que el valor Q = 1 no es estrictamente un factor de ductilidad ya que implica comportamiento estructural elástico; además, no es factible emplear Q = 6 cuando se usan muros pues sería prácticamente imposible desarrollar, con este tipo de estructuración, las deformaciones requeridas para alcanzar esa ductilidad.

La respuesta estructural se calculó con el método cuasidinámico y los métodos estático y dinámico espectral del Reglamento (ref 1). Al comparar los resultados, tomando como base de referencia la respuesta dinámica, se encontró que el cuasidinámico da mejores soluciones que el estático, aunque para determinadas estructuras subestima con más frecuencia la respuesta en los entrepisos superiores; en dichas estructuras, el estático sobrestimó la respuesta en los niveles restantes con mayor grado que el cuasidinámico.

Así, por ejemplo, en la fig 16 se presenta la comparación de respuestas para el modelo (K_F-2, M_F-2) con H/B = 1.2 y Q = 1, cuando se analizó con el espectro de diseño para terreno compresible; se aprecia que la sobrestimación de cortantes del estático es hasta 3.4 veces la del dinámico, en tanto que la del cuasidinámico es 1.6.

En la fig 17 se muestra la comparación de respuestas para el modelo (K_c-2, M_c-3) con H/B = 4 y Q = 6, cuando se analizó para la zona de terreno compresible. La sobrestimación de cortantes con el estático es hasta cinco veces mayor que la del dinámico, mientras que la del cuasidinámico llega solo a dos.

En ambas figuras se nota también que el cuasidinámico sobrestima de manera más uniforme con la altura que el estático, lo cual conduce a diseños con factor de seguridad sensiblemente igual para cada entrepiso.

En las tablas 2 a 7 se presenta el número de casos con error (entendidos como aquellos donde el cortante cuasidinámico es menor que el dinámico), obtenido al calcular las fuerzas cortantes cuasidinámicas con la ec 4; las comparaciones se hicieron para cada entrepiso.

En las tablas 2 a 4 puede notarse que para muros acoplados, la frecuencia de subestimación a que conduce la ec 4 aumenta cuando se incrementa la relación de aspecto, y se tienen mayores porcentajes de error al crecer el factor de ductilidad hasta Q = 4. Se advierte también que existe mayor número de errores en la zona de terreno firme y son más grandes que en los de transición y compresible.

En marcos rígidos, la frecuencia de error varía poco en terrenos firme y compresible (tablas 5 y 7) cuando cambia la relación de aspecto, y varía mucho en terreno de transición (tabla 6). Para factores de ductilidad de 1, 2 y 4, independientemente de la relación de aspecto, el número de casos con error se mantiene casi constante, y disminuye para Q = 6. En terreno firme, el error no excede del 20 por ciento; mientras que en los de transición y compresible, no rebasa un porcentaje de 10.

Al comparar los errores consignados en las tablas 2 a 4 con los de las tablas 5 a 7, se aprecia que las estructuras con marcos rígidos presentan menos errores y estos son menores que los obtenidos para estructuras con muros de cortante acoplados.

De lo anterior se deduce que el tipo de suelo, estructuración, relación de aspecto y factor de ductilidad influyen directamente en la aproximación de la respuesta cuasidinámica.

En la tabla 8 se presenta el caso donde se tuvo mayor subestimación de la respuesta con el método cuasidinámico. Las fuerzas cortantes y momentos de volteo calculados con los tres métodos corresponden al modelo con rigidez K_F-2, masa M_F-2, relación H/B = 3.6 y Q = 4, desplantado en terreno firme. La subestimación del cuasidinámico alcanzó hasta un 52 por ciento en el entrepiso superior, mientras que la del estático fue de 16 por ciento.

Después de examinar detalladamente la respuesta de todos los modelos, se decidió proponer otra distribución de fuerzas más efectiva.

Se buscó, como punto de partida, una distribución de aceleraciones cuasidinámicas que fuera función de los desplazamientos x_i e incrementara la magnitud de las fuerzas sísmicas en la parte superior del edificio. Se encontró que eso se logra al modificar cada x_i, multiplicándola por el factor A_i definido como

$$A_i = \left(1 - \frac{x_{i-1}}{x_i}\frac{h_{i-1}}{H}\right)^{\lambda} - \left(1 - \frac{x_{i-1}}{x_i}\frac{h_i}{H}\right)^{\lambda} \qquad (5)$$

donde

H altura total del edificio

λ exponente para ajuste de aceleraciones

De esta manera, las fuerzas sísmicas se obtienen mediante la ecuación

$$P_i^* = \frac{W_i A_i x_i}{\sum_{i=1}^{n} W_i A_i x_i} V_o^* \qquad (6)$$

A fin de asignar valores al parámetro λ, se estudiaron minuciosamente las estructuras cuyas subestimaciones fueron mayores al emplear la ec 4. Para ello se aplicó la ec 6 con valores de λ comprendidos entre 0.25 y 1.0.

En la tabla 9 se exponen los valores de las cortantes de entrepiso obtenidos mediante la ec 6 para la estructura K_F-2, M_F-2 y distintos valores de λ. Se puede apreciar que con λ = 0.66, la respuesta cuasidinámica solo presenta un caso de subestimación, del orden de 0.6 por ciento, en el piso superior. Se observa, además, que el valor de λ = 0.75 proporciona resultados bastante aceptables, con subestimación de 1.06 contra 1.25 ton únicamente en la parte superior.

En la tabla 10 se muestra la comparación entre cortantes de entrepiso y momentos de volteo, calculados mediante los tres métodos mencionados, en la cual se utiliza la ec 4 para obtener fuerzas cuasidinámicas. Los resultados corresponden al modelo (K_F-4, M_F-2) con H/B = 3.6, Q = 4 y terreno firme. Se observa que la subestimación alcanzó un valor hasta del 40 por ciento en el entrepiso superior. En la tabla 11 se presentan los cortantes de entrepiso obtenidos con diferentes valores de λ; se puede ver que para λ = 0.75, los resultados son satisfactorios.

A fin de conocer la sobrestimación que se introduce al emplear la ec 6, se estudiaron aquellos casos en los cuales la ec 4 conducía a mayor exceso en la respuesta. El peor caso correspondió al modelo (K_F-3, M_F-3) con H/B = 3.6; Q = 6 y espectro de terreno firme; la sobrestimación alcanzó 162.0 por ciento y ocurrió en el entrepiso 22 (tabla 12). Se observa que en dicho modelo, el valor adecuado para λ debería ser mayor de 1 (tabla 13) pues con λ = 1 se tienen cortantes cuasidinámicas aún mayores que las dinámicas.

Con fines ilustrativos, se presentan también los resultados del modelo (K_c-2, M_c-3), con H/B = 4.0, Q = 1 y terreno firme. En este caso, el método cuasidinámico con la ec 4 da respuestas con error en los dos entrepisos superiores (tabla 14). En la tabla 15 se muestran las cortantes correspondientes a distintos valores de λ; se nota que con λ = 0.9 se evitan las subestimaciones (lo mismo se concluyó para Q = 2, 4 y 6). Si con este modelo se usara λ = 1.0, se tendrían errores de 10 a 13 por ciento en el entrepiso superior solamente.

Al hacer este tipo de análisis para los modelos con marcos rígidos, se encontró que la mayoría de los casos quedaba cubierta con λ = 1.0. Las excepciones (aparte de las del párrafo anterior) requirieron que λ fuera igual a 0.9; aunque con $\lambda = 1.0$, los errores en el entrepiso superior eran tan solo de 3 por ciento para el caso más desfavorable: modelos (K_c-2, M_c-2) y (K_c-3, M_c-3).

Finalmente, un análisis exhaustivo de todos los casos condujo a proponer los siguientes valores de λ , con los cuales se eliminaron prácticamente todas las subestaciones y las pocas que quedaron fueron insignificantes

a) Para marcos rígidos, desplantados en cualquier tipo de terreno: $\lambda = 1.0$

b) Para muros de cortante acoplados, excepto los modelos que tienen K_F-3 , se empleará un valor de λ de acuerdo con el tipo de terreno: $\lambda = 0.75$, en suelo firme; $\lambda = 0.9$, en suelos de transición y compresible

c) Para muros de cortante acoplados, modelados con K_F-3, y en todo tipo de terreno: $\lambda = 0.9$

Vale la pena señalar que la ec 5 está inspirada en una del código chileno que tiene la forma (ref 6)

$$A_i = (1 - \frac{h_i - 1}{H})^{\lambda} - (1 - \frac{h_i}{H})^{\lambda} \qquad (7)$$

con $\lambda = 0.5$. Al principio de los análisis realizados dentro de este trabajo, se intentó usar la ec 7 ajustando el valor de λ, ya que con $\lambda = 0.5$ se sobrestima excesivamente la respuesta; sin embargo, al interpretar los resultados se intuyó la modificación que condujo a la ec 5, la cual funcionó mejor que la ec 7.

2.4 *Resultados*

A fin de mostrar la eficacia del método cuasidinámico con los valores de λ propuestos, se presentan únicamente los resultados de los modelos (K_F-2, M_F-2), (K_F-3, M_F-3) y (K_c-3, M_c-1), en suelos firme y compresible, con factores de ductilidad 1 y 6, relaciones de aspecto 1.2 y 3.6 en muros de cortante acoplados y 2 y 4 en marcos rígidos (figs 18 a

29). Las respuestas (fuerzas cortantes y momentos de volteo) del método dinámico se obtuvieron con el criterio de la raíz cuadrada de la suma de cuadrados de la respuesta de cada modo de vibrar.

En el estudio se incorporó también la modificación de las fuerzas cortantes del método estático, permitida por el Reglamento del Distrito Federal cuando se toma en cuenta el periodo fundamental de la estructura. En las figs 18 a 29 se muestran los casos en que dicha modificación mejora sustancialmente los resultados del método estático.

La notación utilizada en dichas figuras es

- V fuerzas cortantes de entrepiso
- M momentos de volteo
- r número de entrepiso
- e estático
- d dinámico
- * cuasidinámico

La tilde sobre V o M significa que son valores reducidos

En esas figuras se aprecia que los resultados del método cuasidinámico en general se apegan más a los del dinámico que los del estático, modificado o no; esta concordancia fue mayor en casi todos los demás casos estudiados. Conviene destacar que en algunos casos, la modificación de fuerzas cortantes del estático conduce a una respuesta mejor que la cuasidinámica para los entrepisos superiores, pero de todas maneras sobrestima en exceso la de los demás entrepisos. Asimismo, es importante señalar que en ciertos casos la modificación del método estático puede ocasionar subestimación (figs 19, 20 y 23).

En las figs 18 a 29 se muestran también los momentos de volteo obtenidos con los métodos estático y cuasidinámico reducidos de acuerdo con el Reglamento; esta reducción se basa en el hecho de que los momentos calculados con integración del diagrama de fuerzas cortantes resultan mayores que los dinámicos. En el caso del método estático dicha reducción se aplicó a las fuerzas cortantes reducidas.

En las figuras mencionadas se aprecia que los valores así estimados resultan mejores que los originales, y la reducción es mayor en los niveles inferiores. Solo en dos casos del cuasidinámico se obtuvieron valores para la base ligeramente menores que los dinámicos (menos del uno por ciento).

Se nota también que en los modelos de marcos rígidos, la reducción condujo a una distribución más uniforme de la relación de momentos de volteo, lo cual permite mantener un factor de seguridad casi constante.

Con base en las observaciones hechas en este estudio, puede afirmarse que el método cuasidinámico es apropiado para estudiar la respuesta sísmica de los edificios desplantados en cualquiera de las zonas sísmicas del Distrito Federal con distintas relaciones de esbeltez y factor de ductilidad menor de 6.

3. CONCLUSIONES Y RECOMENDACION

3.1 *Conclusiones*

Se ha presentado la extensión del método cuasidinámico para calcular la respuesta sísmica de edificios irregulares en elevación, empleando los espectros reducidos por ductilidad del Reglamento de Construcciones para el Distrito Federal.

El ajuste del factor correctivo α se hizo para cada zona geotécnica del Distrito Federal y cuatro factores de ductilidad (Q = 1, 2, 4 y 6). Sin embargo, por la similitud encontrada entre algunas de las curvas elaboradas para tal fin, se decidió emplear curvas comunes que cubrieran los puntos correspondientes a varias de ellas, lo que permitió tener dos en total (fig 15). Así, para edificios desplantados en terreno firme, se calculará α con las ecs 2 o 3, según sea la estructuración, y para los de transición y compresible se utilizará la ec 3.

Con las ecuaciones propuestas en este trabajo para calcular α, se ha incrementado el intervalo de aplicación del método cuasidinámico, ya que en la investigación precedente (ref 4) se requería que V_o/V_{e_o} fuera mayor o igual a 0.2 y actualmente esa cota inferior se ha disminuido. Tal es el caso, por ejemplo, de la zona de terreno firme donde el intervalo de aplicación es

$$\beta_3 = 0.108 \leq V_o/V_{e_o} \leq 1.0$$

Para los terrenos de transición y compresible, los valores de β_3 pueden obtenerse de la tabla 1.

Otras conclusiones de interés acerca del método cuasidinámico son

a) Casi siempre da resultados bastante mejores que el estático

b) Es aplicable a estructuras regulares o irregulares en elevación hasta de 30 pisos con $H/B \leq 3.6$, para edificios de muros de cortante acoplados, y hasta de 20 pisos con $H/B \leq 4$, para edificios con marcos rígidos

c) Resulta práctico y sencillo de utilizar en los despachos de cálculo; lo más complicado es obtener la configuración de desplazamientos estáticos x_i, lo cual no debe ser difícil en la actualidad para los ingenieros calculistas

d) En algunos casos donde fue aplicable, la reducción que permite el Reglamento de las fuerzas cortantes calculadas con el método estático condujo a subestimaciones en los entrepisos superiores, tal como sucedió en el modelo (K_P-2, M_P-2).

e) El criterio de reducción de momentos de volteo es aplicable al método cuasidinámico

f) Con el fin de aplicar el método cuasidinámico en otras ciudades, es necesario obtener las ecuaciones para calcular α, correspondientes a los espectros de diseño que se tengan en los reglamentos respectivos, mediante un trabajo de investigación semejante al que se presentó en este informe.

3.2 *Recomendación*

Como resultado de todo lo discutido en este trabajo, se recomienda estudiar la conveniencia de incluir el método cuasidinámico en el Reglamento de Construcciones para el Distrito Federal como procedimiento opcional para el análisis sísmico de edificios. En tal caso, será necesario simplificar aún más el método y hacer ajustes de curvas para α que den valores medios, en vez de envolventes a fin de no cometer sistemáticamente error del lado de la seguridad.

4. RECONOCIMIENTO

Los autores expresan su agradecimiento a Joel García y Raúl Paredes por la ayuda prestada, organizando la información que se empleó en esta investigación.

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TABLA 1. CONSTANTES β_i PARA CALCULAR EL FACTOR CORRECTIVO α EN DISTINTAS ZONAS DEL D F

ZONA D F	β_i	Q=1	Q=2	Q=4	Q=6
FIRME I	β_1	1.500	1.700	1.800	1.700
	β_2	-4.900	-6.000	-5.430	-5.800
	β_3	0.108	0.108	0.108	0.108
	β_4	0.750	1.000	1.000	1.000
TRANSICION II	β_1	1.100	1.150	1.500	1.700
	β_2	-5.300	-5.360	-5.920	-7.400
	β_3	0.160	0.160	0.160	0.116
	β_4	0.750	0.750	1.000	1.000
COMPRESIBLE III	β_1	1.050	1.100	1.700	1.670
	β_2	-5.320	-5.400	-6.260	-7.400
	β_3	0.175	0.175	0.160	0.116
	β_4	0.750	0.750	1.000	1.000

TABLA 2. NUMERO DE CASOS CON ERROR AL CALCULAR LAS FUERZAS CORTANTES CUASIDINAMICAS (EC 4) Y COMPARARLAS, EN CADA NIVEL, CON LAS DINAMICAS. MUROS DE CORTANTE ACOPLADOS

Error, en porcentaje	Terreno firme							
	H/B = 1.2				H/B = 3.6			
	Q=1	Q=2	Q=4	Q=6	Q=1	Q=2	Q=4	Q=6
0 - 2		5	3	4	6	5	5	
2 - 4	1	5	3	4	1	3	3	5
4 - 6			6	2	3		1	3
6 - 8	1	1	1	1	3	5	3	
8 - 10			2	2	7	3	4	3
10 - 15			4	4	6	8	4	6
15 - 20		1	1	1	9	6	7	6
20 - 25				1	2	3	6	3
25 - 30			1	1	3	5	5	4
30 - 40					5	4	3	4
> 40					1	1	3	1
Total	2	12	21	20	46	43	44	35
Total de comparaciones	220	220	220	220	360	360	360	360

TABLA 3. NUMERO DE CASOS CON ERROR AL CALCULAR LAS FUERZAS CORTANTES CUASIDINAMICAS (EC 4) Y COMPARARLAS, EN CADA [illegible], CON LAS DINAMICAS. MUROS DE CORTANTE ACOPLADOS

Error, en porcentaje	Terreno de transición							
	H/B = 1.2				H/B = 3.6			
	Q=1	Q=2	Q=4	Q-6	Q=1	Q=2	Q=4	Q=6
0 - 2			3	1	5	7	2	3
2 - 4			3		4	4	5	1
4 - 6			2	1	2	4	2	
6 - 8		1	1	1	1	2	3	2
8 - 10				2	3	1	4	2
10 - 15			3	2	5	6	4	1
15 - 20			3	1	1	3	3	1
20 - 25			1			1	3	
25 - 30					1		1	1
30 - 40						1	1	
Total		1	16	8	22	29	38	11
Total de comparaciones	220	220	220	220	360	360	360	360

TABLA 4. NUMERO DE CASOS CON ERROR AL CALCULAR LAS FUERZAS CORTANTES CUASIDINAMICAS (EC 4) Y COMPARARLAS, EN CADA NIVEL, CON LAS DINAMICAS. MUROS DE CORTANTE ACOPLADOS

Error, en porcentaje	Terreno compresible							
	H/B = 1.2				H/B = 3.6			
	Q=1	Q=2	Q=4	Q=6	Q=1	Q=2	Q=4	Q=6
0 - 2			3	1	6	6	5	1
2 - 4		1	2		4	5	4	
4 - 6			3	1	1	6	5	2
6 - 8			3	1	3	2	3	1
8 - 10				2	1	2	4	
10 - 15			2	2	2	5	5	2
15 - 20			4	1	1	2	3	
20 - 25			1				3	1
25 - 30						1	1	
30 - 40							1	
Total		1	18	8	18	29	34	7
Total de comparaciones	220	220	220	220	360	360	360	360

TABLA 5. NUMERO DE CASOS CON ERROR AL CALCULAR LAS FUERZAS CORTANTES CUASIDINAMICAS (EC 4) Y COMPARARLAS, EN CADA NIVEL, CON LAS DINAMICAS. MARCOS RIGIDOS

Error, en porcentaje	Terreno firme							
	H/B = 2				H/B = 4			
	Q=1	Q=2	Q=4	Q=6	Q=1	Q=2	Q=4	Q=6
0 - 2	4	4	5		1	2	2	3
2 - 4	1	1	1		1	1	1	2
4 - 6	1	1	1	2	1	1	1	1
6 - 8	1	1		4	4	4	4	
8 - 10			1		1	1	1	1
10 - 15	6	5	5	2	2	2	2	1
15 - 20		1	1		1	1	1	
Total	13	13	14	8	11	12	12	8
Total de comparaciones	130	130	130	130	260	260	260	260

TABLA 6. NUMERO DE CASOS CON ERROR AL CALCULAR LAS FUERZAS CORTANTES CUASIDINAMICAS (EC 4) Y COMPARARLAS, EN CADA NIVEL, CON LAS DINAMICAS. MARCOS RIGIDOS

Error, en porcentaje	Terreno de transición							
	H/B = 2				H/B = 4			
	Q=1	Q=2	Q=4	Q=6	Q=1	Q=2	Q=4	Q=6
0 - 2	7	8	10		1	1	3	
2 - 4	1	2	2		2	2	2	
4 - 6	2	1	1		1	1	1	
6 - 8	1	2	1		1			
8 - 10			1			1	1	
Total	11	13	15		5	5	7	
Total de comparaciones	130	130	130	130	260	260	260	260

TABLA 7. NUMERO DE CASOS CON ERROR AL CALCULAR LAS FUERZAS CORTANTES CUASIDINAMICAS (EC 4) Y COMPARARLAS, EN CADA NIVEL, CON LAS DINAMICAS. MARCOS RIGIDOS

Error, en porcentaje	Terreno compresible							
		H/B = 2				H/B = 4		
	Q=1	Q=2	Q=4	Q=6	Q=1	Q=2	Q=4	Q=6
0 - 2	4	7	10		9	7	8	
2 - 4	3	3	2		4	4	5	
4 - 6		2	4		2	4	3	
6 - 8	1	1	1		1		1	
8 - 10			1			1	1	
Total	8	13	18		16	16	18	
Total de comparaciones	130	130	130	130	260	260	260	260

TABLA 8. COMPARACION DE LA RESPUESTA PARA EL MODELO (K_F-2, M_F-2), CON H/B=3.6, Q=4, EN TERRENO FIRME

ENTREPISO	CORTANTE, ton			MOMENTO, ton-m			ERROR (%) (METODO CUASIDINAMICO)*				ERROR (%) (METODO ESTATICO)	
							POR NIVEL		RESPECTO AL BASAL		POR NIVEL	
	V_r^*	V_{d_r}	V_{e_r}	M_r^*	M_{d_r}	M_{e_r}	V	M	V	M	V	M
30	0.597	1.253	1.044	1.791	3.759	3.133	-52.364	-52.364	-2.775	-0.144	-16.674	-16.674
29	1.354	2.281	2.367	5.854	10.445	10.232	-40.638	-43.956	-3.921	-0.337	3.729	-2.032
28	2.255	3.214	3.946	12.618	19.730	22.071	-29.826	-36.044	-4.055	-0.522	22.798	11.843
27	3.278	4.072	5.761	22.452	31.348	39.375	-19.496	-28.379	-3.357	-0.653	41.498	25.241
26	4.402	4.868	7.790	35.657	45.075	62.724	-9.578	20.894	-1.272	-0.691	60.015	39.154
25	5.606	5.614	10.011	52.474	60.720	92.755	-0.153	-13.580	-0.036	-0.405	78.301	52.758
24	6.868	6.321	12.401	73.079	78.126	129.959	8.664	-6.461	2.316	-0.370	96.208	66.345
23	8.148	6.976	14.941	97.584	97.167	174.732	16.754	0.430	4.958	0.031	113.556	79.873
22	9.494	7.651	17.609	126.047	117.745	227.603	23.739	7.031	7.766	0.609	130.136	93.304
21	10.807	8.295	20.381	158.469	139.795	286.752	30.256	13.359	10.625	1.370	145.705	106.353
20	12.110	8.936	23.233	194.000	163.780	359.466	35.523	19.304	12.426	2.312	160.047	119.540
19	13.383	9.583	26.157	234.940	188.190	434.938	39.652	24.846	16.071	3.430	172.962	132.173
18	14.610	10.242	29.117	278.776	214.542	514.290	42.646	29.940	16.473	4.712	184.297	144.377
17	15.780	10.918	32.096	326.116	242.375	620.579	44.524	34.550	20.561	6.143	193.964	156.041
16	16.883	11.616	35.073	376.765	271.750	725.799	45.343	38.644	22.277	7.704	201.938	167.084
15	17.911	12.336	38.026	430.499	302.739	839.879	45.193	42.202	23.590	9.373	208.250	177.427
14	18.877	13.078	40.934	487.071	335.425	962.679	44.190	45.210	24.443	11.175	212.992	187.003
13	19.716	13.839	43.774	546.221	369.894	1094.000	42.467	47.669	24.857	12.936	216.300	195.760
12	20.485	14.615	46.525	607.676	406.229	1233.575	40.167	49.589	24.625	14.779	218.341	203.665
11	21.162	15.398	49.166	671.161	444.504	1381.072	37.435	50.991	24.380	16.628	219.306	210.700
10	21.747	16.179	51.674	736.402	484.773	1536.095	34.417	51.906	23.550	18.460	219.399	216.869
9	22.241	16.946	54.030	803.176	527.072	1698.184	31.247	52.375	22.396	20.252	218.830	222.192
8	22.649	17.667	56.209	871.072	571.403	1866.812	28.054	52.445	20.986	21.965	217.799	226.707
7	22.974	18.386	58.192	939.995	617.735	2041.338	24.954	52.169	19.405	23.642	216.501	230.463
6	23.223	19.027	59.957	1009.665	665.999	2221.259	22.055	51.602	17.749	25.213	215.114	233.523
5	23.404	19.593	61.481	1079.878	716.083	2405.702	19.452	50.803	16.119	26.689	213.792	235.953
4	23.526	20.067	62.744	1150.454	767.836	2593.934	17.232	49.831	14.626	28.070	212.665	237.624
3	23.598	20.435	63.723	1221.248	821.071	2785.105	15.476	48.738	13.376	29.359	211.829	239.204
2	23.633	20.634	64.398	1292.147	875.573	2978.298	14.256	47.577	12.471	30.561	211.338	240.154
1	23.643	20.806	64.746	1363.077	931.114	3172.536	13.638	46.392	12.001	31.690	211.190	240.725

* El error negativo indica que existe subestimación con respecto a la respuesta dinámica

TABLA 9. COMPARACION DE CORTANTES DE ENTREPISO, EN TONELADAS, PARA DIFERENTES VALORES DEL PARAMETRO λ. MODELO (K_F-2, M_F-2) CON H/B=3.6, EN TERRENO FIRME Y Q=4

ENTRE-PISO	V_{d_r}	V_r^*	λ							
			0.25	0.33	0.50	0.66	0.75**	0.80	0.90	10
30	1.25312	0.59694	2.58875	2.25487	1.69092	1.24530	1.06361	0.96605	0.79427	0.65029
29	2.28149	1.35434	4.61928	4.12053	3.24277	2.51825	2.20926	2.03971	1.73385	1.46834
28	3.21271	2.25489	6.43019	5.83723	4.76465	3.84506	3.44107	3.21579	2.80205	2.43463
27	4.07157	3.27778	8.10603	7.46130	6.26925	5.21530	4.74192	4.47477	3.97558	3.52395
26	4.86500	4.40176	9.67626	9.00714	7.75[illegible]0	6.61406	6.09160	5.79334	5.22091	4.71313
25	5.61438	5.64579	11.15231	10.43441	9.20651	8.02263	7.47016	7.15147	6.54541	5.97915
24	6.32026	6.86616	12.53722	11.88498	10.61913	9.42375	8.85754	8.52869	7.89640	7.29874
23	6.99629	8.16545	13.83198	13.20751	11.99052	10.80164	10.23579	9.90479	9.26317	8.64270
22	7.65138	9.48762	15.03700	14.44[illegible]82	13.28199	12.14[illegible]63	11.58915	11.26329	10.62699	10.01285
21	[illegible]	10.80719	16.15192	15.60738	14.51536	13.43396	12.90267	12.58544	11.97005	11.36791
20	[illegible]	12.11043	17.17706	16.70073	15.67458	14.66515	14.14452	13.86641	13.27681	12.69736
19	9.55278	13.38250	18.11352	17.66729	16.75439	15.82773	15.36376	15.03613	14.53387	13.98743
18	10.24155	14.60957	18.96248	18.56713	17.75160	16.91437	16.49157	16.23737	15.72509	15.22757
17	10.91646	15.77980	19.72594	19.38097	18.64377	17.91968	17.54053	17.31214	16.85231	16.39146
16	11.61613	16.88325	20.40662	20.11040	19.49002	18.83997	18.50654	18.30437	17.89626	17.46435
15	12.33619	17.91129	21.01745	20.75746	20.23056	19.67267	19.38462	19.20931	18.85385	18.49300
14	13.07618	18.87739	21.53269	21.32511	20.88573	20.41690	20.17309	20.02415	19.72095	19.41145
13	[illegible]	19.71641	21.98444	21.81469	21.45039	21.07285	20.87103	20.74740	20.49464	20.23529
12	14.61479	20.46506	22.36859	22.23623	21.95170	21.64706	21.47529	21.37920	21.17389	20.96218
11	15.37769	21.16186	22.69023	22.56629	22.36802	22.12730	21.99934	21.92122	21.75938	21.59171
10	16.17861	21.74673	22.95344	22.87774	22.71329	22.53229	22.43592	22.37631	22.25318	22.12506
9	16.94418	22.24122	23.16382	23.10992	22.97224	22.86182	22.79203	22.74873	22.65903	22.56529
8	17.69707	22.64692	23.32693	23.27055	23.21073	23.17170	23.07381	23.04403	22.98214	22.91721
7	18.38611	22.97420	23.44847	23.42557	23.37513	23.31851	23.28791	23.25803	23.22907	23.18720
6	19.02700	23.22339	23.53425	23.52129	23.49236	23.45971	23.44200	23.43092	23.40778	23.38332
5	19.57598	23.40416	23.59052	23.58415	23.56933	23.55356	23.54470	23.53915	23.52752	23.51519
4	20.06748	23.52557	23.62335	23.62086	23.61531	23.60597	23.60550	23.60333	23.59876	23.59390
3	20.43540	23.59794	23.63856	23.63028	23.63700	23.63552	23.63471	23.63421	23.63314	23.63200
2	20.68429	23.63294	23.64342	23.64342	23.64342	23.64342	23.64342	23.64342	23.64342	23.64342
1	20.80596	23.64342	23.64342	23.64342	23.64342	23.64342	23.64342	23.64342	23.64342	23.64342

** Valor seleccionado

TABLA 10. COMPARACION DE LA RESPUESTA PARA EL MODELO (K_F-4, M_F-2), CON H/B=3.6, Q=4, EN TERRENO FIRME

ENTREPISO	CORTANTE, ton			MOMENTO, ton-m			ERROR (%) (METODO CUASIDINAMICO)*				ERROR (%) (METODO ESTATICO)	
							POR NIVEL		RESPECTO AL BASAL		POR NIVEL	
	V_r^*	V_{d_r}	V_{e_r}	M_r^*	M_{d_r}	M_{e_r}	V	M	V	M	V	M
30	0.615	1.039	1.044	1.846	3.117	3.133	-40.785	-40.785	-1.542	-0.094	0.476	0.496
29	1.390	2.138	2.367	6.015	9.518	10.232	-34.971	-36.802	-2.722	-0.231	10.700	7.545
28	2.309	3.231	3.946	12.941	19.143	23.071	-29.535	-32.390	-3.354	-0.409	22.157	15.295
27	3.356	4.241	5.761	23.008	31.835	37.355	-21.794	-27.729	-3.407	-0.501	34.274	23.620
26	4.513	5.309	7.790	36.548	47.400	52.724	-14.977	-22.504	-2.893	-0.714	46.736	32.527
25	5.766	6.235	10.011	53.846	65.639	95.755	-9.265	-17.967	-1.870	-0.776	59.271	41.310
24	7.094	7.226	12.401	75.127	86.371	129.959	-1.635	-13.018	-0.453	-0.740	71.615	50.467
23	8.479	8.141	14.941	100.563	109.440	174.782	4.144	-8.112	1.228	-0.584	83.521	59.706
22	9.902	9.040	17.609	130.269	134.731	227.603	9.540	-3.212	3.138	-0.294	94.790	68.935
21	11.345	9.927	20.391	164.303	162.160	288.752	14.277	1.322	5.158	0.141	105.303	78.066
20	12.769	10.804	23.238	202.667	191.666	356.465	16.359	5.739	7.218	0.724	115.077	87.024
19	14.215	11.666	26.157	245.312	223.210	436.959	21.844	9.902	9.274	1.455	124.210	95.752
18	15.609	12.509	29.117	292.141	256.744	524.290	24.787	13.787	11.253	2.330	132.773	104.207
17	16.959	13.337	32.095	343.017	292.237	620.579	27.156	17.376	13.179	3.342	140.659	112.354
16	18.253	14.173	35.073	397.777	329.698	725.799	28.795	20.649	14.850	4.481	147.475	120.141
15	19.489	15.039	38.026	456.245	367.218	839.878	29.595	23.571	16.195	5.729	152.[illegible]	127.475
14	20.662	15.934	40.934	518.231	410.928	962.679	29.672	26.117	17.204	7.063	156.694	134.269
13	21.761	16.629	43.774	583.512	454.902	1094.000	29.312	26.272	17.949	8.466	160.176	140.491
12	22.775	17.712	46.525	651.838	501.163	1233.575	28.584	30.065	18.423	9.918	162.667	146.142
11	23.698	18.587	49.166	722.933	549.702	1381.072	27.495	31.514	18.597	11.403	164.512	151.240
10	24.522	19.453	51.674	796.498	600.492	1536.095	26.058	32.641	18.446	12.902	165.641	155.806
9	25.241	20.305	54.030	872.222	653.499	1698.184	24.309	33.470	17.961	14.398	166.084	159.864
8	25.854	21.138	56.209	949.785	709.674	1866.812	22.313	34.029	17.163	15.871	165.919	163.423
7	26.360	21.937	58.192	1028.864	765.962	2041.328	20.162	34.323	16.094	17.306	165.271	166.513
6	26.760	22.685	59.957	1109.144	825.202	2221.257	17.764	34.396	14.829	18.626	164.302	169.151
5	27.060	23.360	61.431	1190.374	884.524	2405.702	15.938	34.269	13.463	19.999	163.166	171.364
4	27.266	23.939	62.744	1272.130	949.540	2593.934	13.908	33.973	12.116	21.235	162.100	173.176
3	27.397	24.396	63.723	1354.320	1014.143	2785.105	12.301	33.543	10.920	22.390	161.206	174.626
2	27.461	24.707	64.398	1436.703	1090.109	2979.298	11.137	33.015	10.014	23.473	160.625	175.741
1	27.481	24.863	64.746	1519.145	1147.190	3172.536	10.531	32.423	9.527	24.485	160.415	176.548

* El error negativo indica que existe subestimación con respecto a la respuesta dinámica

TABLA 11. COMPARACION DE CORTANTES DE ENTREPISO, EN TONELADAS, PARA DIFERENTES VALORES DEL PARAMETRO λ. MODELO (K_F-4, M_F-2) CON H/B=3.6, EN TERRENO FIRME Y Q=4

ENTRE-PISO	V_{d_r}	$V_r^λ$	λ = 0.25	λ = 0.33	λ = 0.50	λ = 0.66	λ = 0.75**	λ = 0.80	λ = 0.90	λ = 1.0
30	1.03902	0.61526	2.45538	2.15216	1.65432	1.22830	1.05307	0.96112	0.79801	0.65291
29	2.13733	1.30977	4.37944	4.09372	3.23714	2.52684	2.22260	2.05520	1.75232	1.48862
28	3.23056	2.30877	6.54097	5.93705	4.64535	3.90892	3.49712	3.26734	2.84502	2.46964
27	4.29063	3.355[illegible]	8.50989	7.71013	6.45793	5.35542	4.85718	4.57701	4.05533	3.53335
26	5.30654	4.51[illegible]	10.14171	9.42028	8.06608	6.84272	6.28305	5.96335	5.36282	4.81074
25	6.29520	5.75573	11.90665	11.04753	9.65899	8.35984	7.75541	7.40812	6.74740	6.13236
24	7.22650	7.09369	13.33434	12.64725	11.22343	9.88594	9.23490	8.88920	8.18779	7.52716
23	8.14130	8.47867	14.87387	14.15441	12.74770	11.40422	10.76221	10.38758	9.66341	8.97412
22	9.03978	9.90214	16.27739	15.58423	14.22037	12.89912	12.26016	11.88497	11.15459	10.45271
21	9.92741	11.34475	17.58[illegible]	16.93158	15.63239	14.35550	13.73171	13.36333	12.64159	11.94194
20	10.80445	12.75604	18.79765	18.19239	16.97376	15.76046	15.16193	14.80671	14.10654	13.42236
19	11.66149	14.21494	19.92117	19.36640	18.23905	17.10345	16.53872	16.20175	15.53403	14.87677
18	12.50026	15.60948	20.95277	20.47160	19.42948	18.37702	17.85196	17.53738	16.91086	16.28996
17	13.33487	16.95868	21.89960	21.45163	20.52695	19.57644	19.09335	18.80670	18.22813	17.65108
16	14.17256	18.25349	22.76502	22.37173	21.55261	20.70249	20.26745	20.00794	19.48223	18.95472
15	15.02357	19.48919	23.55570	23.21481	22.50139	21.75405	21.37070	21.13927	20.66903	20.19546
14	15.83402	20.66193	24.26595	23.97660	23.36700	22.72294	22.39030	22.18768	21.77756	21.36119
13	16.92795	21.76059	24.89493	24.65649	24.14684	23.60372	23.32145	23.14910	22.79835	22.44052
12	17.71249	22.77533	25.44975	25.25477	24.83900	24.39223	24.15865	24.01579	23.72357	23.42397
11	18.58735	23.69808	25.92629	25.77261	25.44707	25.08636	24.89877	24.78349	24.54718	24.30375
10	19.45272	24.52174	26.32897	26.21193	25.95956	25.68440	25.53890	25.44922	25.26475	25.07387
9	20.30543	25.24128	26.66113	26.57564	26.39037	26.18496	26.07883	26.01199	25.87407	25.73074
8	21.13780	25.85424	26.92694	26.86774	26.72882	26.59633	26.52020	26.47302	26.37536	26.27344
7	21.93697	26.35972	27.13142	27.09321	27.00960	26.91660	26.86668	26.83546	26.77126	26.70379
6	22.66495	26.75997	27.28059	27.25821	27.20903	27.15400	27.12433	27.10585	27.06737	27.02691
5	23.36040	27.06011	27.38148	27.37016	27.34515	27.31702	27.30179	27.29228	27.27243	27.25149
4	23.93893	27.26843	27.44216	27.43767	27.42772	27.41647	27.41035	27.40632	27.39851	27.39003
3	24.39538	27.39681	27.47163	27.47077	27.46840	27.46571	27.46425	27.46333	27.46140	27.45936
2	24.70902	27.46098	27.48085	27.48085	27.48085	27.48085	27.48085	27.48085	27.48085	27.48085
1	24.86267	27.48085	27.48085	27.48085	27.48085	27.48085	27.48085	27.48085	27.48085	27.48085

** Valor seleccionado

TABLA 12. COMPARACION DE LA RESPUESTA PARA EL MODELO (K_F-3, M_F-3), CON H/B=3.6, Q=6, EN TERRENO FIRME

ENTREPISO	CORTANTE, ton			MOMENTO, ton-m			ERROR (%) (METODO CUASIDINAMICO)* POR NIVEL		ERROR (%) (METODO CUASIDINAMICO)* RESPECTO AL BASAL		ERROR (%) (METODO ESTATICO) POR NIVEL	
	V_r^*	V_{d_r}	V_{s_r}	M_r^*	M_{d_r}	M_{s_r}	V	M	V	M	V	M
30	1.937	1.758	3.396	5.811	5.273	10.187	10.193	10.193	0.930	0.039	92.181	97.181
29	3.805	2.832	6.678	17.225	13.474	30.222	24.370	27.639	5.048	0.273	135.855	124.294
28	5.594	3.644	9.848	34.008	23.912	59.764	53.505	42.219	10.113	0.735	170.216	142.930
27	7.295	4.090	12.904	55.892	35.710	98.476	78.378	56.517	16.625	1.469	215.535	175.736
26	8.895	4.326	15.847	82.577	47.870	145.017	105.620	72.503	23.698	2.526	264.324	205.027
25	10.333	4.522	18.677	113.727	59.980	202.046	129.635	89.609	30.402	3.912	313.045	236.856
24	11.749	4.711	21.393	148.975	71.908	265.226	149.380	107.175	36.503	5.609	354.073	270.233
23	12.983	5.025	23.997	187.924	83.809	338.216	153.373	124.229	41.275	7.578	377.561	303.556
22	14.075	5.370	26.487	230.148	96.019	417.677	162.096	139.691	45.149	9.762	393.228	334.994
21	15.018	5.731	28.864	275.203	108.837	504.269	162.043	152.859	48.148	12.108	403.634	363.327
20	15.807	6.081	31.126	322.622	122.496	597.653	159.945	163.373	50.445	14.566	411.915	387.894
19	16.437	6.372	33.279	371.935	137.101	697.488	157.968	171.286	52.206	17.092	422.270	408.741
18	16.912	6.616	35.316	422.672	152.649	803.436	155.622	176.891	53.403	19.653	433.780	426.328
17	17.242	6.809	37.240	474.397	169.062	915.137	153.211	180.606	54.110	22.223	446.905	441.316
16	17.451	6.999	39.051	526.751	186.230	1032.311	149.335	182.851	54.212	24.784	457.946	454.322
15	17.745	7.698	42.447	579.987	204.205	1159.651	130.508	184.071	52.109	27.350	451.333	467.835
14	18.010	8.663	45.616	634.016	223.584	1296.500	107.424	183.569	48.377	29.872	425.370	479.870
13	18.246	9.635	48.359	688.755	244.875	1442.177	88.989	181.268	44.561	32.307	402.965	463.945
12	18.454	10.557	51.276	744.118	268.383	1596.005	74.602	177.260	40.959	34.625	385.694	494.675
11	18.635	11.376	53.766	800.023	294.234	1757.304	63.617	171.900	37.653	36.912	372.646	497.247
10	18.790	12.108	56.030	856.392	322.411	1925.394	55.189	165.627	34.658	38.864	362.765	497.187
9	18.919	12.770	58.067	913.150	352.801	2099.596	48.154	158.929	31.894	40.783	354.716	475.122
8	19.025	13.389	59.879	970.226	385.259	2279.232	42.079	151.837	29.234	42.575	347.231	491.610
7	19.109	13.978	61.463	1027.554	419.645	2463.622	36.706	144.863	26.612	44.245	339.706	487.073
6	19.173	14.529	62.822	1085.073	455.830	2652.087	31.966	138.044	24.088	45.798	332.390	481.815
5	19.219	15.014	63.953	1142.732	493.677	2843.947	28.006	131.474	21.310	47.240	325.946	476.075
4	19.250	15.414	64.859	1200.483	533.023	3038.524	24.092	125.222	19.900	48.579	320.790	470.055
3	19.269	15.723	65.538	1258.289	573.677	3235.139	22.551	119.337	18.390	49.828	316.828	463.930
2	19.278	15.948	65.991	1316.122	615.442	3433.112	20.877	113.850	17.269	50.997	313.783	457.629
1	19.280	16.077	66.217	1373.963	658.120	3631.763	19.923	108.771	16.613	52.101	311.867	451.837

* El error negativo indica que existe subestimación con respecto a la respuesta dinámica

TABLA 13. COMPARACION DE CORTANTES DE ENTREPISO, EN TONELADAS, PARA DIFERENTES VALORES DEL PARAMETRO λ. MODELO (K_F-3, M_F-3), CON H/B=3.6, EN TERRENO FIRME Y Q=6

ENTRE-PISO	V_{d_r}	V_r^*	λ								V_{er} Modificado
			0.25	0.33	0.50	0.66	0.75	0.80	0.90**	1.0	
30	1.75774	1.93700	5.19756	4.77937	3.98971	3.27373	2.94613	2.75297	2.41122	2.09403	[illegible]
29	2.83151	3.80470	8.44981	7.91611	6.87076	5.87278	5.39351	5.12329	4.59550	4.10045	[illegible]
28	3.64434	5.59425	10.79491	10.25026	9.15033	8.05920	7.52538	7.21068	6.59673	6.00709	[illegible]
27	4.08952	7.29481	12.59561	12.08290	11.02314	9.93871	9.39552	9.07122	8.42945	7.80114	[illegible]
26	4.32392	8.89495	14.02004	13.55898	12.58695	11.56564	11.04374	10.72878	10.09785	9.46993	[illegible]
25	4.52168	10.38337	15.16206	14.76032	13.89850	12.97174	12.48978	12.19616	11.60164	11.00140	[illegible]
24	4.71141	11.74931	16.07976	15.73830	14.99426	14.17734	13.74581	13.48068	12.93866	12.38436	[illegible]
23	5.02483	12.98280	16.81221	16.52792	15.89962	15.19670	14.82011	14.58696	14.10620	13.60833	[illegible]
22	5.37014	14.07492	17.38766	17.15484	16.63371	16.04078	15.71909	15.51856	15.10187	14.66629	[illegible]
21	5.73116	15.01809	17.82801	17.63932	17.21227	16.71924	16.44880	16.27924	15.92449	15.55033	[illegible]
20	6.08066	15.80658	18.15175	17.99887	17.64979	17.24204	17.01642	16.87428	16.57530	16.25767	[illegible]
19	6.37190	16.43746	18.37620	18.25044	17.96159	17.62151	17.43221	17.31257	17.05996	16.79025	[illegible]
18	6.61621	16.91248	18.51987	18.41291	18.16654	17.87541	17.71288	17.60999	17.39236	17.15942	[illegible]
17	6.80924	17.24174	18.60550	18.51044	18.29139	18.03233	17.88762	17.79598	17.60205	17.39434	[illegible]
16	6.99911	17.45127	18.66641	18.57971	18.37977	18.14307	18.01073	17.92688	17.74935	17.55905	[illegible]
15	7.69526	17.74510	18.76674	18.71517	18.54913	18.35098	18.23954	18.16870	18.01815	17.85597	[illegible]
14	8.48766	18.00997	18.89943	18.83134	18.69580	18.53262	18.44063	18.38185	18.25649	18.12081	[illegible]
13	9.65459	18.24613	18.97624	18.93000	18.82151	18.69012	18.61540	18.56762	18.46536	18.35419	[illegible]
12	10.55724	18.45429	19.04875	19.01277	18.92793	18.82445	18.76530	18.72736	18.64592	18.55699	[illegible]
11	11.37558	18.63518	19.10847	19.08122	19.01664	18.93737	18.89182	18.86254	18.79947	18.73033	[illegible]
10	12.10767	18.78977	19.15676	19.13679	19.08924	19.03050	18.99659	18.97474	18.92752	18.87355	[illegible]
9	12.77004	18.91930	19.19494	19.18089	19.14729	19.10552	19.08131	19.06566	19.03176	18.99430	[illegible]
8	13.38873	19.02522	19.22427	19.21489	19.19235	19.16418	19.14777	19.13714	19.11406	19.08847	[illegible]
7	13.97830	19.10920	19.24595	19.24010	19.22600	19.20828	19.19792	19.19119	19.17654	19.16024	[illegible]
6	14.52891	19.17321	19.26117	19.25786	19.24985	19.23974	19.23380	19.22993	19.22150	19.21208	[illegible]
5	15.01444	19.21946	19.27108	19.26946	19.26553	19.26054	19.25760	19.25568	19.25148	19.24678	[illegible]
4	15.41361	19.25041	19.27682	19.27620	19.27469	19.27276	19.27162	19.27088	19.26925	19.26741	[illegible]
3	15.72308	19.26831	19.27953	19.27939	19.27904	19.27860	19.27834	19.27816	19.27778	19.27736	[illegible]
2	15.94918	19.27768	19.28032	19.28032	19.28032	19.28032	19.28032	19.28032	19.28032	19.28032	[illegible]
1	16.07732	19.28032	19.28032	19.28032	19.28032	19.28032	19.28032	19.28032	19.28032	19.28032	[illegible]

** Valor seleccionado

TABLA 14. COMPARACION DE LA RESPUESTA PARA EL MODELO (K_c-2, M_c-3), CON H/B=4, Q=1, EN TERRENO FIRME

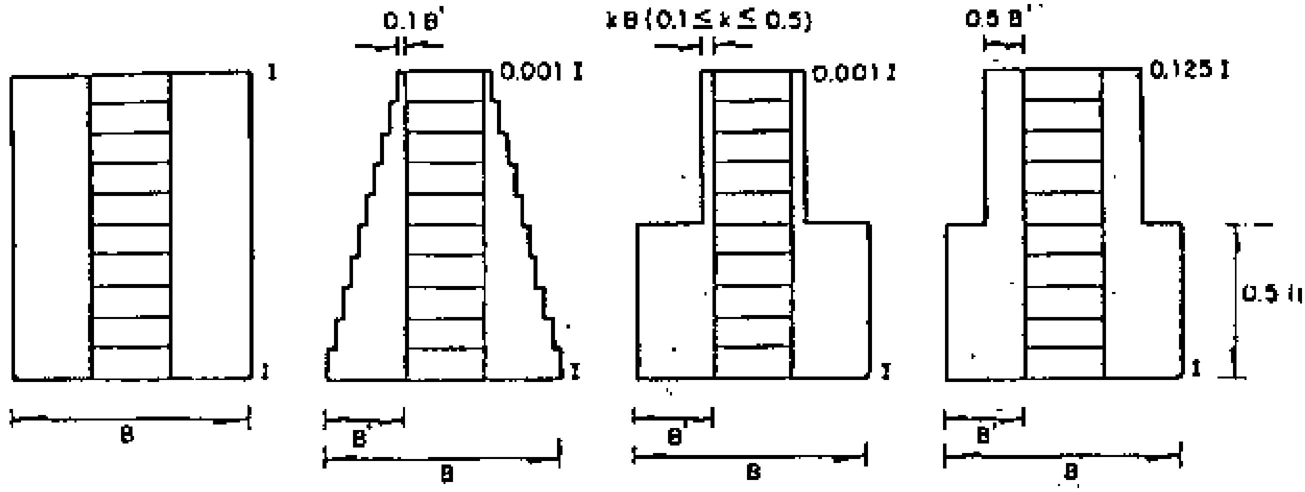
ENTREPISO	CORTANTE, ton			MOMENTO, ton-m			ERROR (%) (METODO CUASIDINAMICO)*				ERROR (%) (METODO ESTATICO)	
							POR NIVEL		RESPECTO AL BASAL		POR NIVEL	
	V_r^*	V_{d_r}	V_{e_r}	M_r^*	M_{d_r}	M_{e_r}	V	M	V	M	V	M
20	12.015	14.534	35.206	36.044	43.602	105.618	-17.333	-17.333	-1.968	-0.143	142.233	142.233
19	23.523	24.768	68.651	106.612	117.393	311.572	-5.028	-9.184	-0.973	-0.205	177.177	165.409
18	34.508	31.901	100.337	210.135	210.357	612.582	8.172	-0.106	2.037	-0.004	214.529	191.211
17	44.951	33.048	130.261	344.989	317.916	1003.365	18.144	8.516	5.374	0.514	242.362	215.607
16	54.836	43.750	138.426	509.497	439.394	1478.643	25.340	15.955	8.662	1.330	262.116	236.519
15	64.145	48.563	184.830	701.931	573.908	2033.133	32.086	22.307	12.175	2.430	280.600	254.261
14	72.861	52.543	209.474	920.514	719.557	2661.555	38.664	27.928	15.874	3.814	298.657	269.885
13	80.970	56.082	232.357	1163.425	874.486	3359.627	44.378	33.041	19.446	5.493	314.316	284.069
12	88.459	59.314	253.481	1428.802	1037.306	4119.070	49.137	37.742	22.772	7.430	327.357	297.093
11	95.313	62.281	272.844	1714.742	1206.947	4937.601	53.039	42.073	25.810	9.637	338.036	309.098
10	101.523	65.176	290.447	2019.309	1392.704	5808.940	55.766	46.041	28.399	12.081	345.633	320.115
9	107.076	68.187	306.289	2340.536	1564.390	6727.806	57.032	49.613	30.385	14.730	349.189	330.059
8	111.963	71.313	320.371	2676.425	1752.242	7688.919	57.002	52.743	31.761	17.539	349.246	338.805
7	116.177	74.453	332.692	3024.956	1946.646	8636.997	56.040	55.373	32.600	20.464	346.846	346.255
6	119.709	77.542	343.254	3384.084	2147.958	9716.759	54.379	57.549	32.947	23.459	342.667	352.372
5	122.555	80.529	352.055	3751.748	2356.436	10772.924	52.187	59.213	32.836	26.480	337.178	357.170
4	124.710	83.308	359.096	4125.877	2572.172	11850.213	49.698	60.404	32.349	29.486	331.048	360.708
3	126.175	85.698	364.376	4504.401	2794.974	12943.342	47.232	61.161	31.626	32.441	325.186	363.093
2	126.969	87.468	367.897	4885.307	3024.230	14047.033	45.160	61.599	30.863	35.319	320.607	364.483
1	127.986	95.343	376.698	5269.265	3261.235	15177.126	34.238	61.573	25.505	38.108	295.099	365.330

* El error negativo indica que existe subestimación con respecto a la respuesta dinámica

TABLA 15. COMPARACION DE CORTANTES DE ENTREPISO, EN TONELADAS, PARA DIFERENTES VALORES DEL PARAMETRO λ. MODELO (K_c-2, M_c-3), CON H/B=4, EN TERRENO FIRME Y Q=1

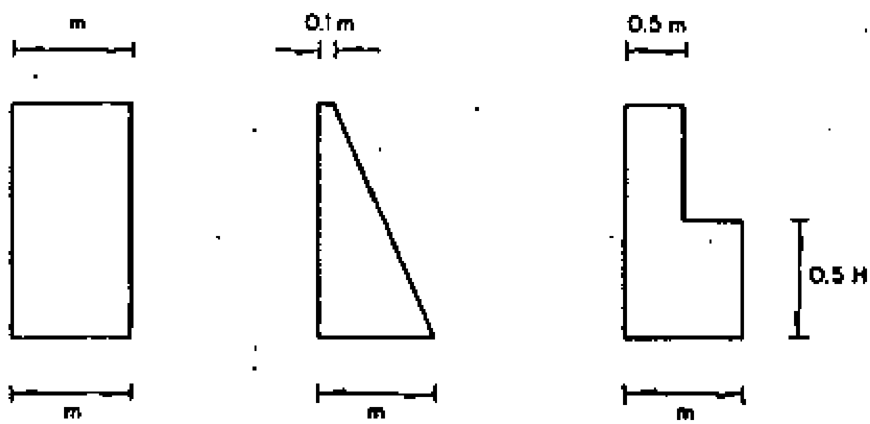
ENTRE-PISO	V_{d_r}	V_r^*	λ							
			0.25	0.33	0.50	0.66	0.75	0.80	0.90	1.0 **
20	14.53398	12.01474	33.25431	30.40989	25.10393	20.37514	18.24287	17.03934	14.80268	12.73965
19	24.76798	23.52261	53.51159	49.90035	42.90295	36.34151	33.26640	31.49465	28.12563	24.99854
18	31.90063	34.50757	66.11879	64.39610	56.98324	49.77298	46.29832	44.76623	40.33671	36.60573
17	38.04776	44.95133	79.42951	75.87460	68.64254	61.40265	57.83632	55.72594	51.53991	47.59155
16	43.74793	54.93601	88.52596	85.26965	78.52666	71.61363	69.14593	66.07359	61.76705	57.93714
15	49.56274	64.14460	96.00489	93.10665	87.01368	80.63850	77.39056	75.43319	71.51726	67.62463
14	52.54500	72.86106	102.23407	99.71496	94.34632	88.63219	85.68011	83.88793	80.27233	76.63739
13	56.08218	80.97044	107.45920	105.31670	100.70031	95.70456	93.09323	91.49746	88.25391	84.95998
12	59.31367	88.45882	111.84948	110.07020	106.19358	101.93784	99.68895	98.30649	95.47745	92.57816
11	62.23079	95.31342	115.53439	114.09313	110.92228	107.39553	105.51324	104.34988	101.95442	99.47899
10	65.17617	101.52253	118.60912	117.47590	114.96028	112.12842	110.60313	109.65570	107.69372	105.65066
9	68.18725	107.07559	121.14892	120.28991	118.34709	116.17823	114.98923	114.24726	112.70262	111.08264
8	71.31300	111.96310	123.21403	122.59293	121.17177	119.58001	118.69752	118.14445	116.98732	115.76567
7	74.45340	116.17673	124.85375	124.43263	123.47561	122.34387	121.75061	121.36470	120.55353	119.69175
6	77.54228	119.70928	126.10917	125.84913	125.25407	124.55631	124.16668	123.92382	123.40684	122.85433
5	80.52705	122.55486	127.01552	126.87722	126.55961	126.18168	125.97087	125.83721	125.55381	125.24921
4	83.30767	124.70969	127.60447	127.54864	127.41918	127.26469	127.17772	127.12239	127.00458	126.87727
3	85.69805	126.17457	127.91047	127.89914	127.87268	127.84082	127.82277	127.81125	127.78661	127.75983
2	87.46806	126.96867	127.98596	127.98596	127.98596	127.98596	127.98596	127.98596	127.98596	127.98596
1	95.34264	127.98596	127.98596	127.98596	127.98596	127.98596	127.98596	127.98596	127.98596	127.98596

** Valor seleccionado



Caso K_F-1 Caso K_F-2 Caso K_F-3 Caso K_F-4

a) Rigideces, con I = 12.5 m^4



Caso M_F-1 Caso M_F-2 Caso M_F-3

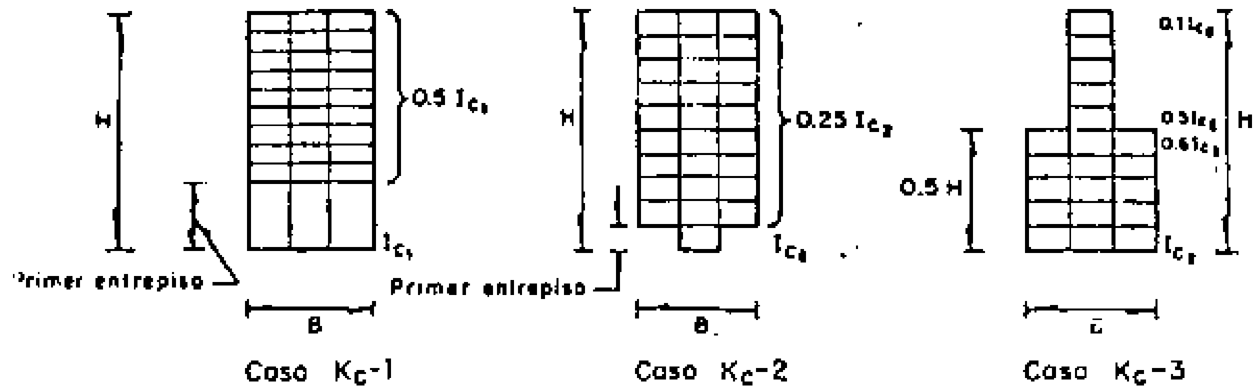
b) Masas, con m = 10 ton-s^2/m

Para H/B = 1.2 , n = 10

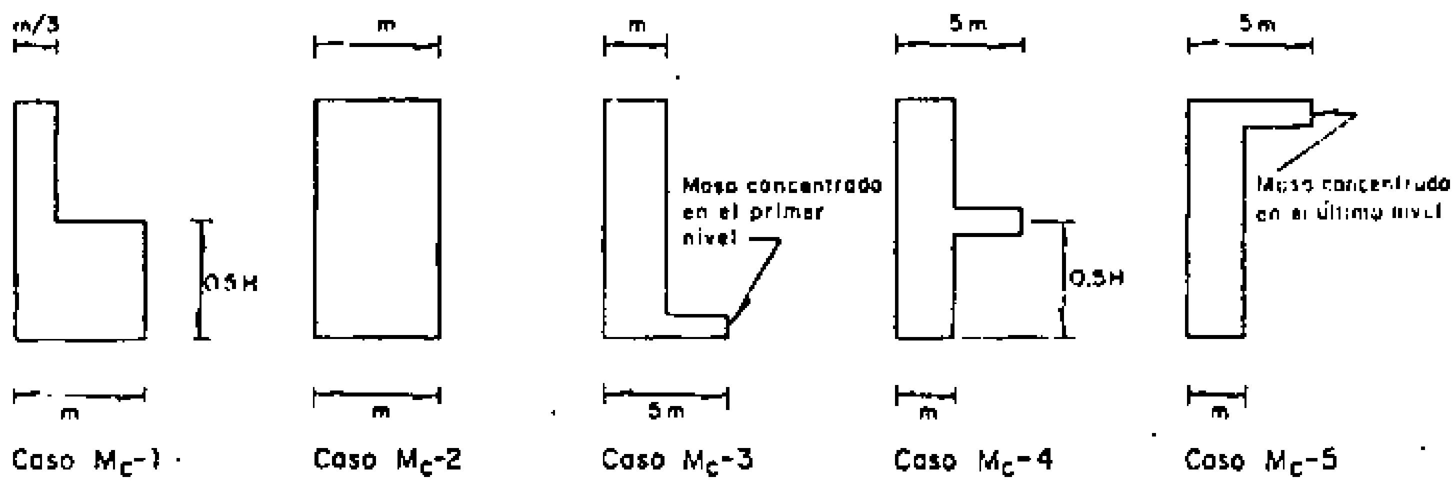
Para H/B = 3.6 , n = 30

n = número de pisos

Fig 1. Variación de parámetros en los modelos de muros de cortante acoplados



a) Rigideces con $I_{c_1} = 0.02\ m^4$; $I_{c_2} = 0.034\ m^4$



b) Masas, con $m = 10.0\ ton\text{-}s^2/m$

Para $H/B = 2$, $n = 10$
Para $H/B = 4$, $n = 20$
n = número de pisos

Fig 2. Variación de parámetros en los modelos de marcos

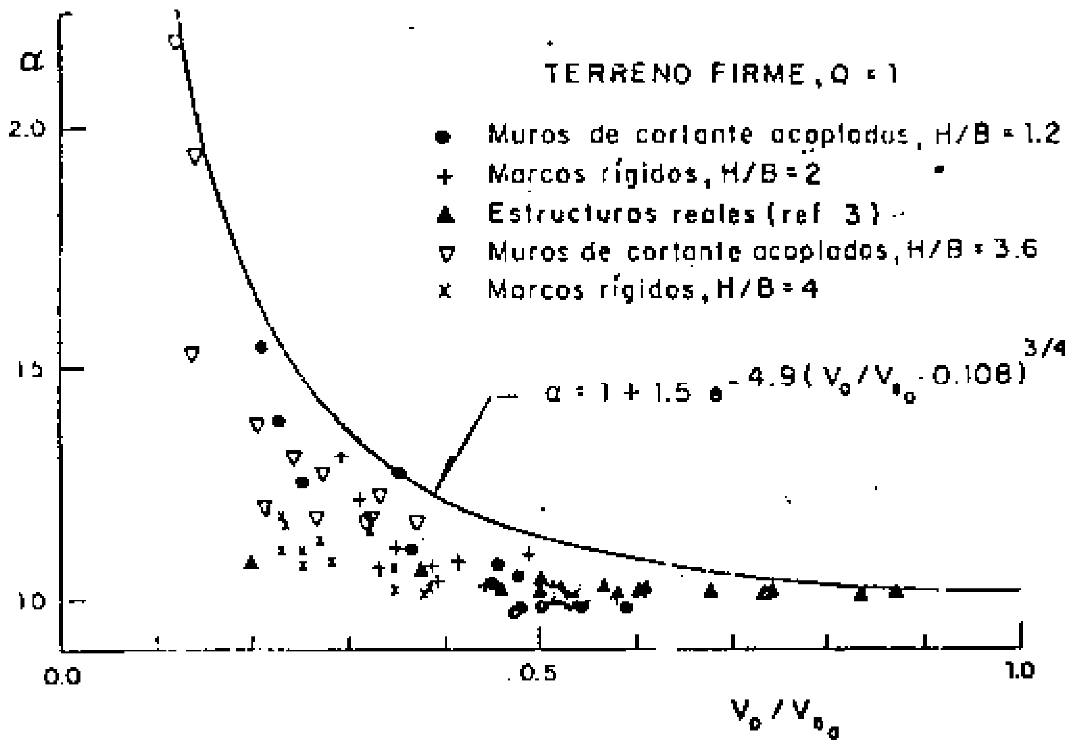


Fig 3. Factor correctivo α para terreno firme en el D F

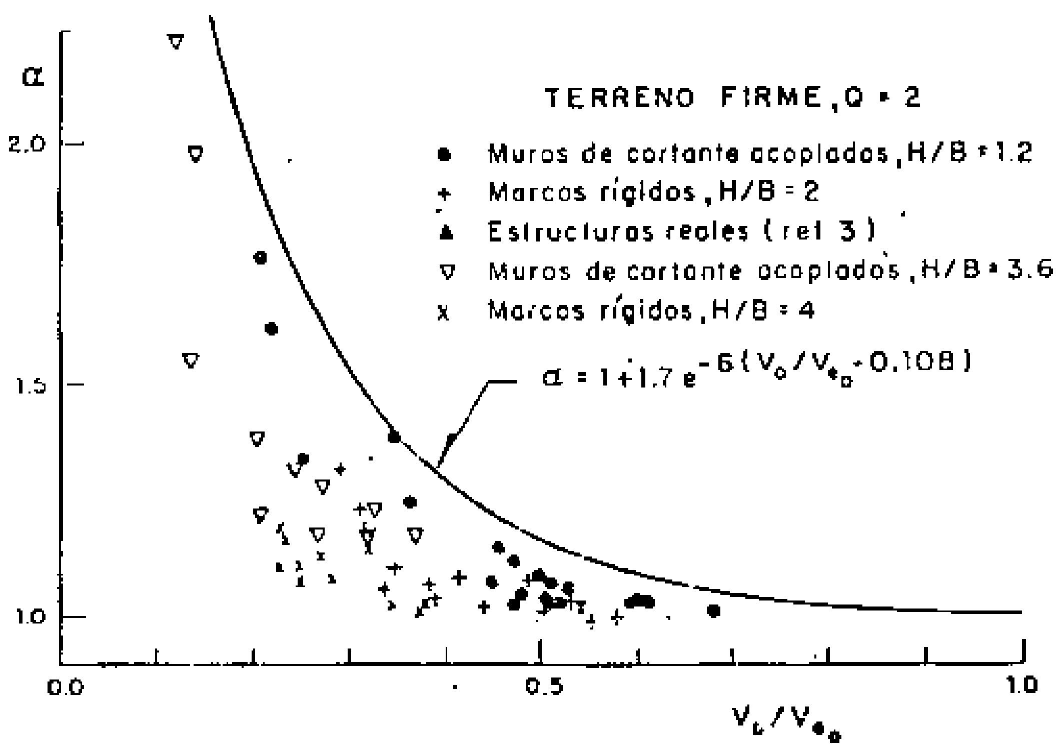


Fig 4. Factor correctivo α para terreno firme en el D F

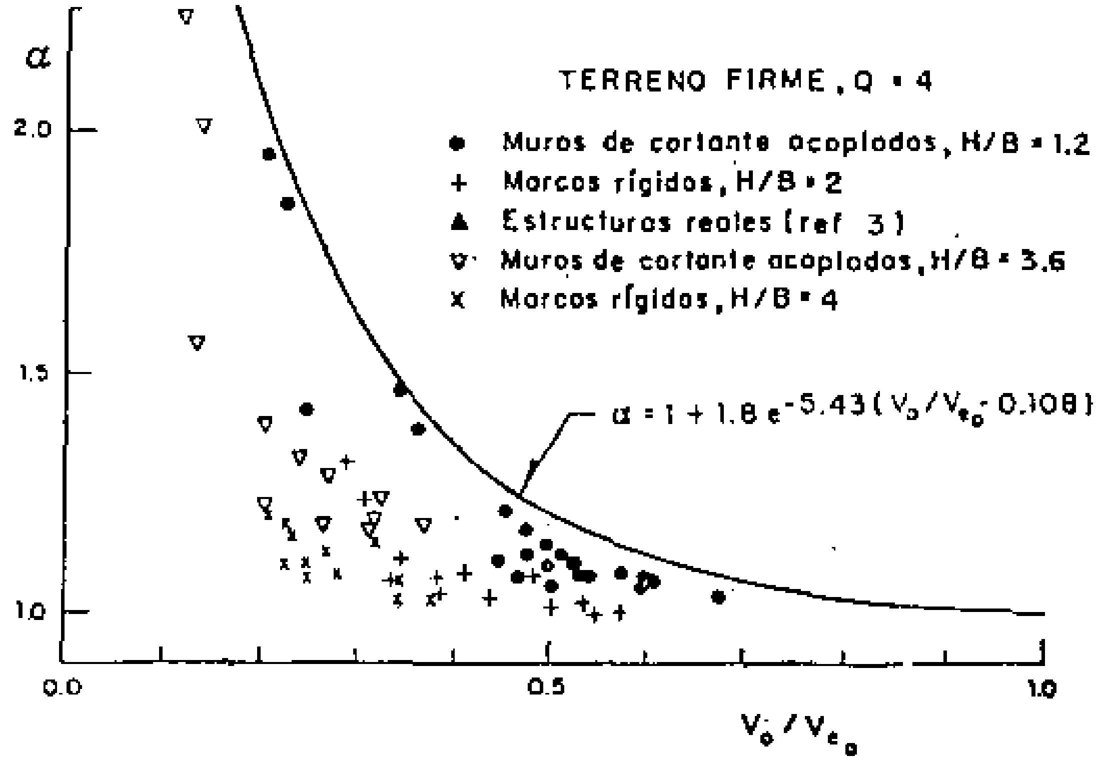


Fig 5. Factor correctivo α para terreno firme en el D F

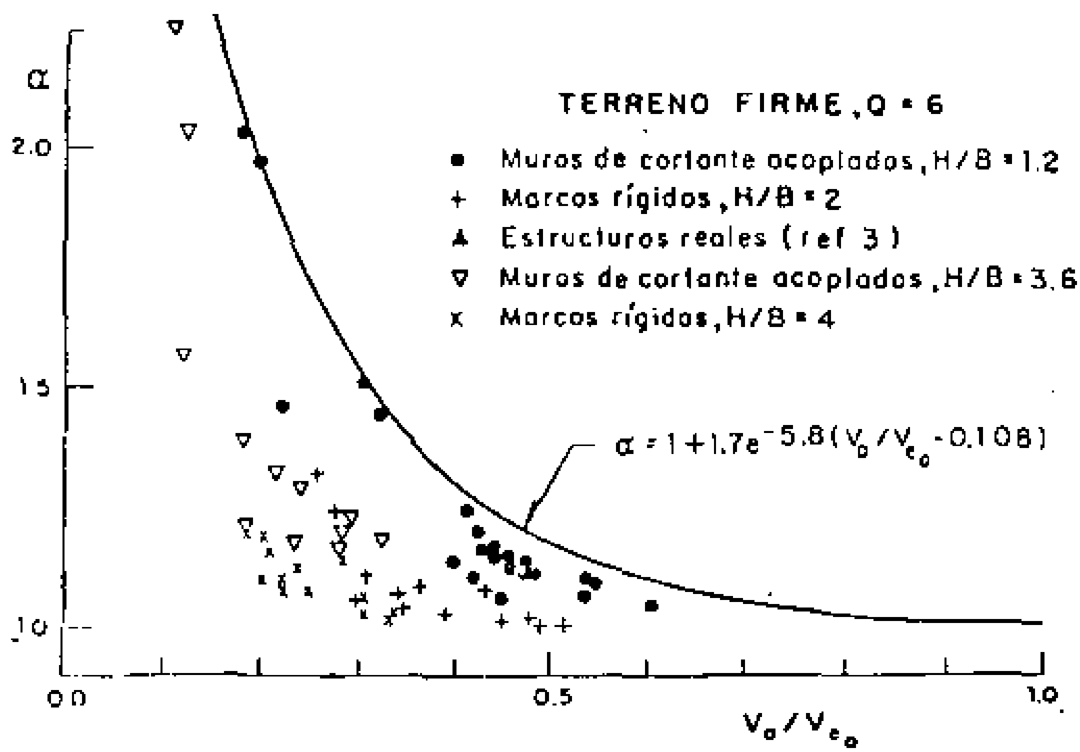


Fig 6. Factor correctivo α para terreno firme en el D F

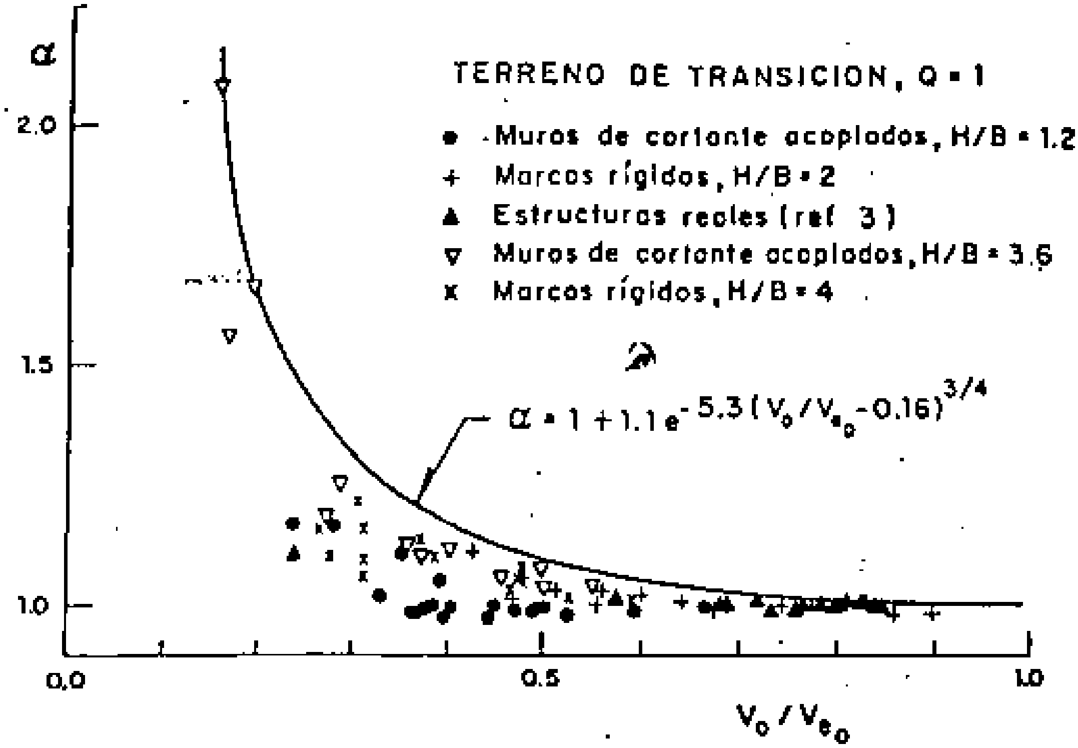


Fig 7. Factor correctivo α para terreno de transición en el D F

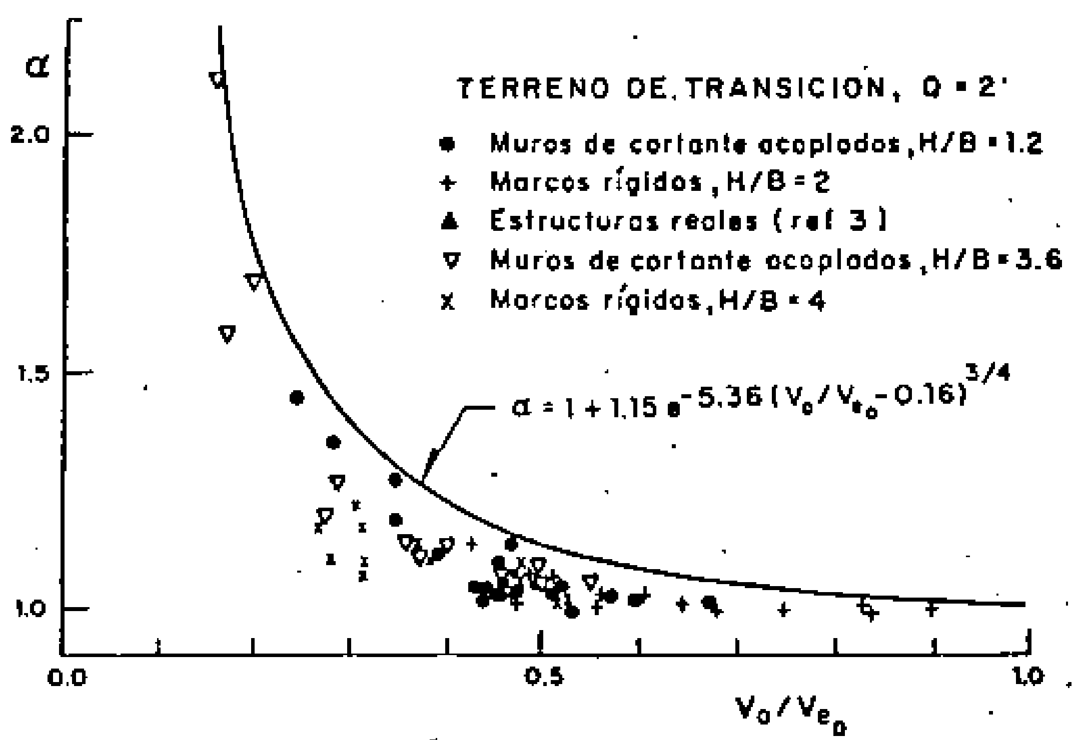


Fig 8. Factor correctivo α para terreno de transición en el D F

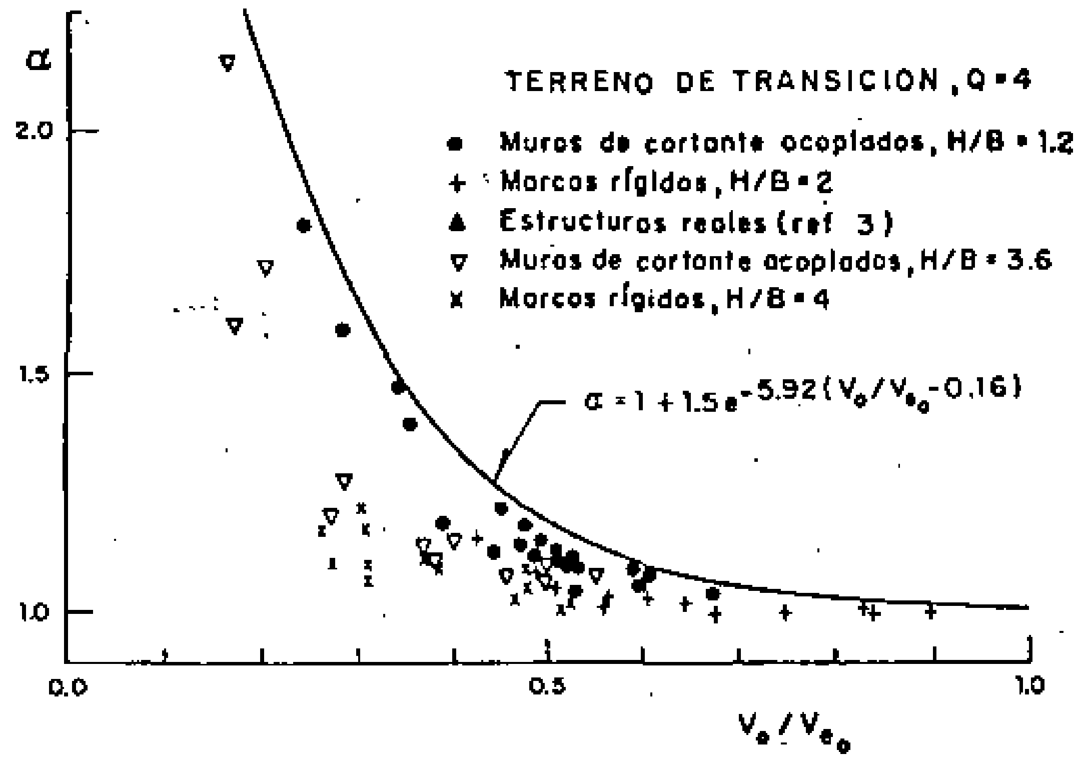


Fig 9. Factor correctivo α para terreno de transición en el D F

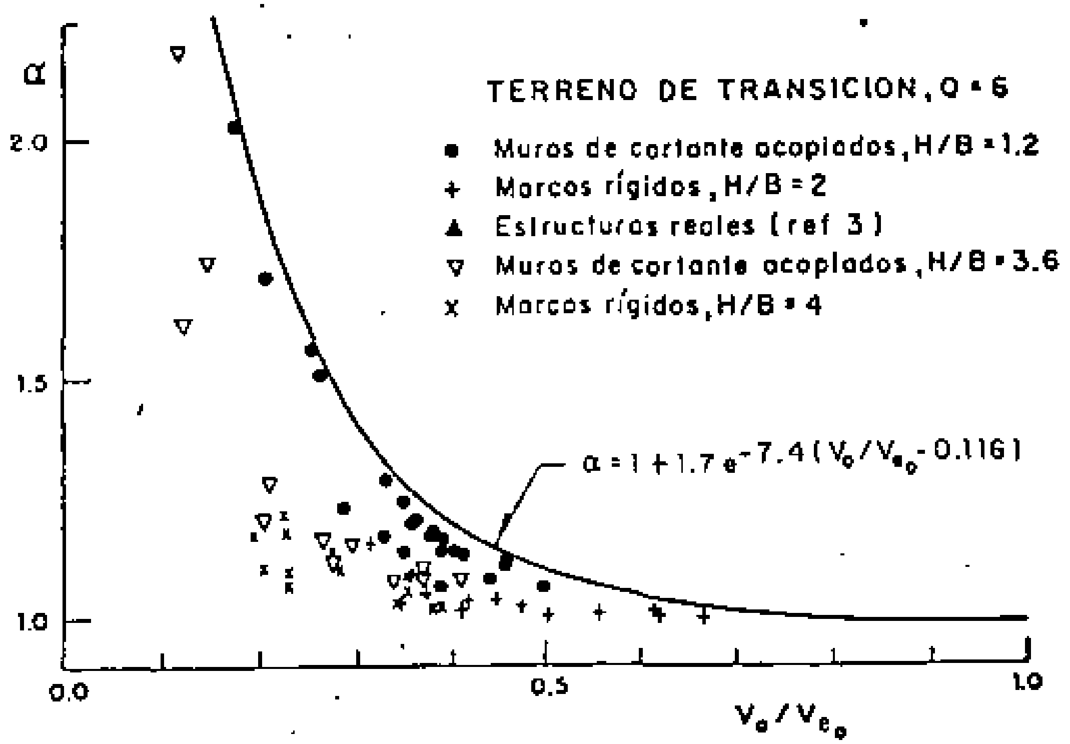


Fig 10. Factor correctivo α para terreno de transición en el D F

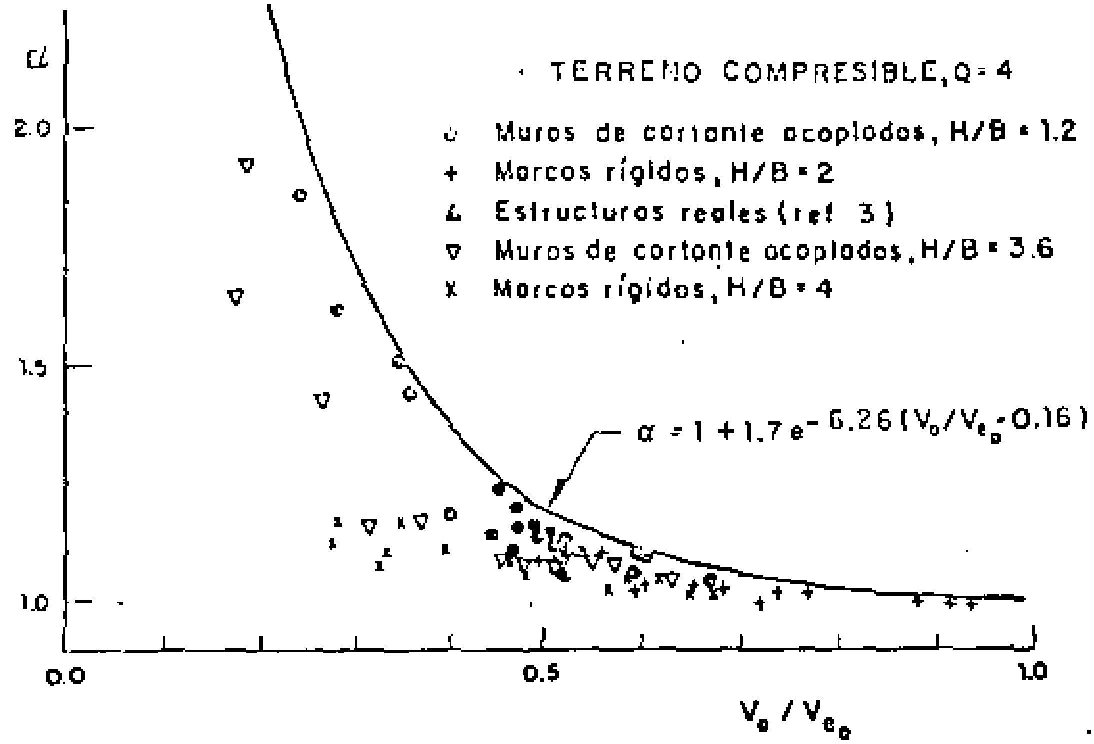


Fig 11. Factor correctivo α para terreno compresible en el D F

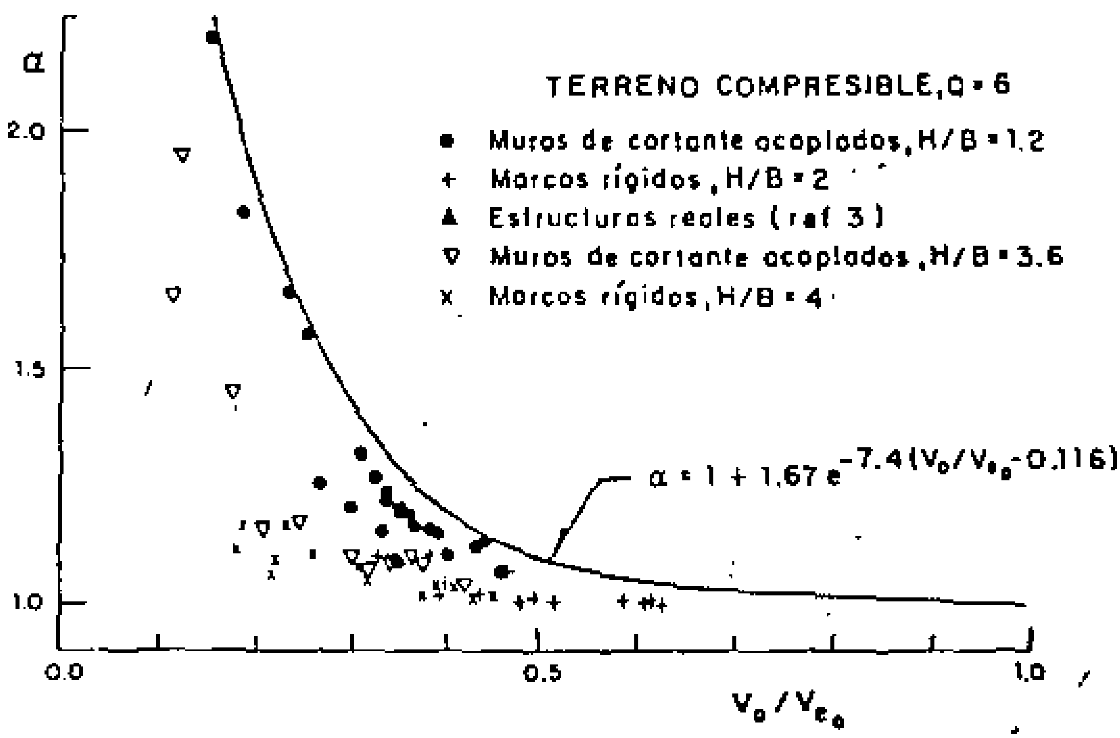


Fig 12. Factor correctivo α para terreno compresible en el D F

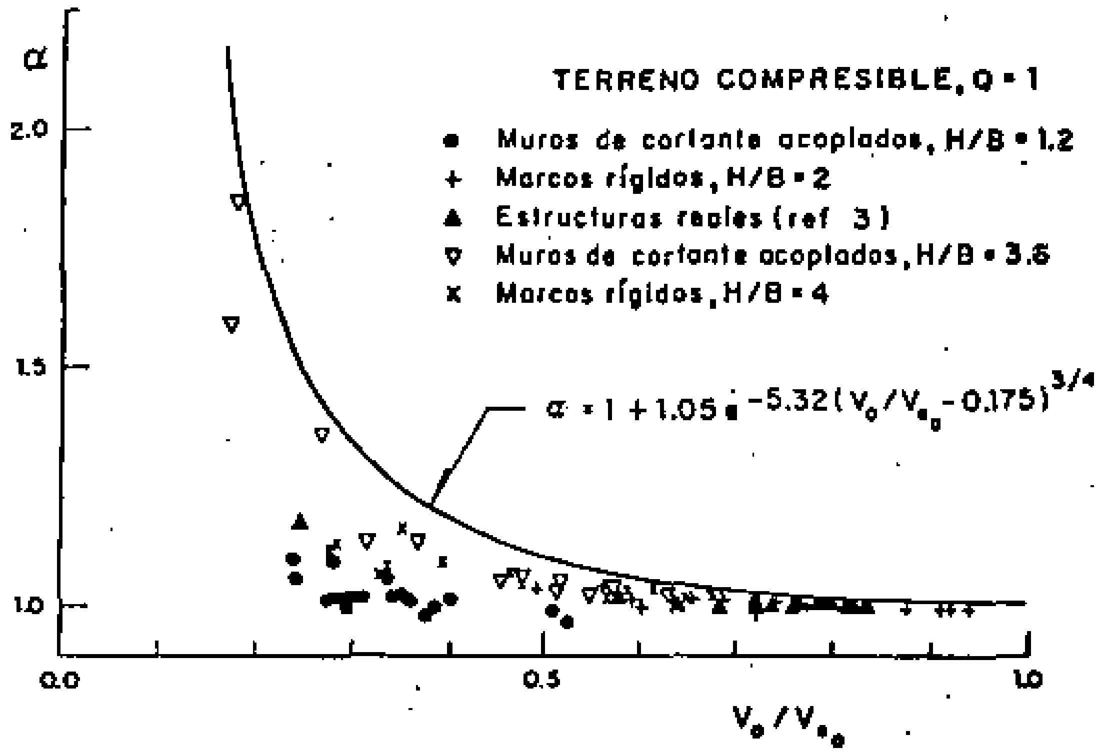


Fig 13. Factor correctivo α para terreno compresible en el D F

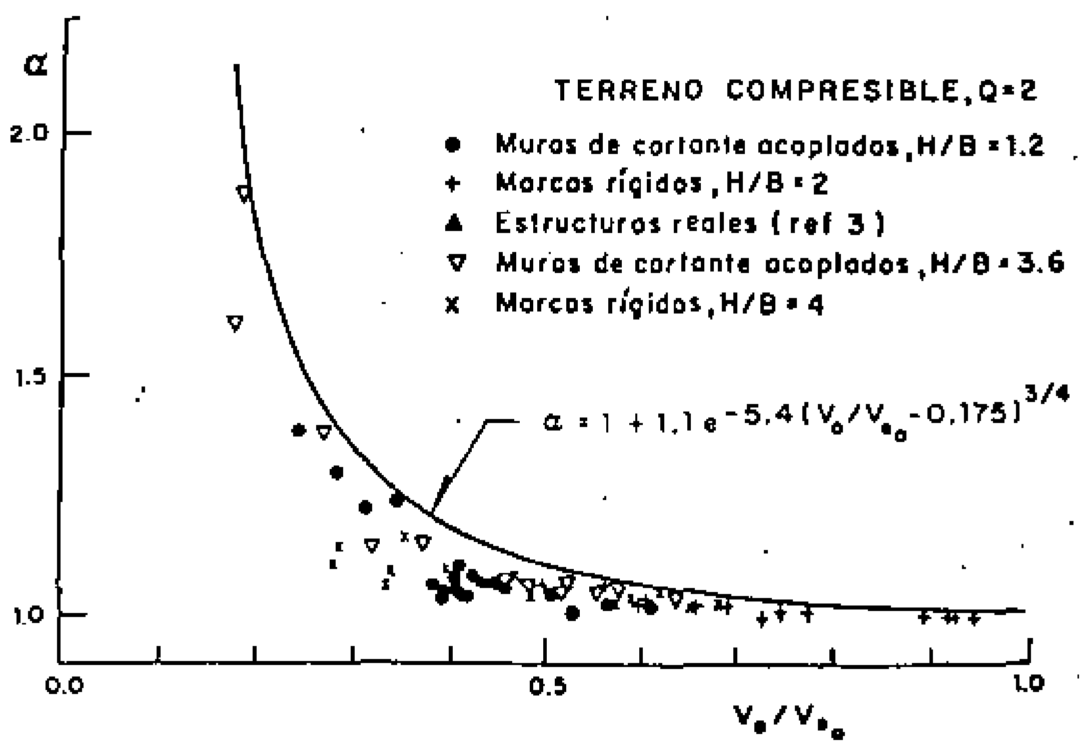


Fig 14. Factor correctivo α para terreno compresible en el D F

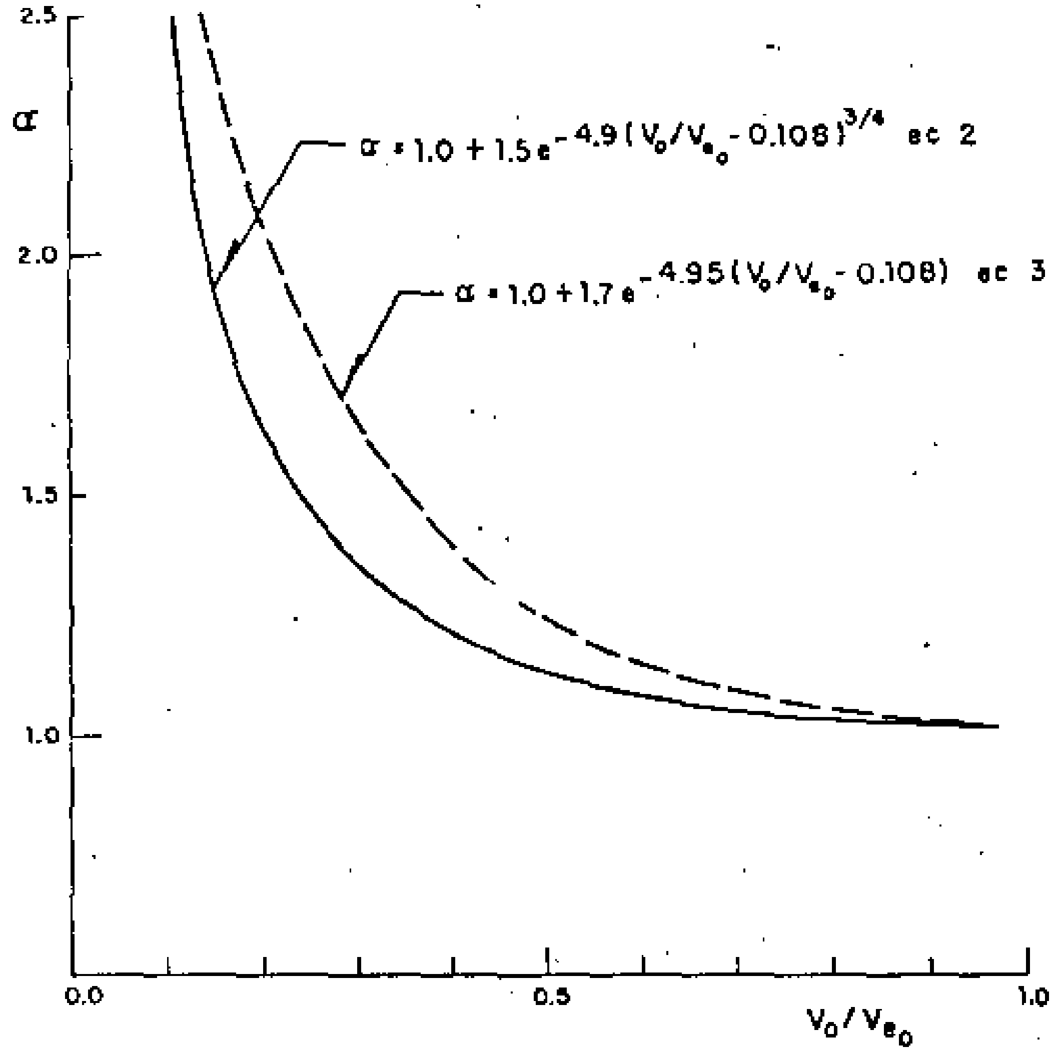


Fig 15. Envolventes de diseño para factor correctivo α

Espectro de diseño para terreno compresible del DF, con Q=1

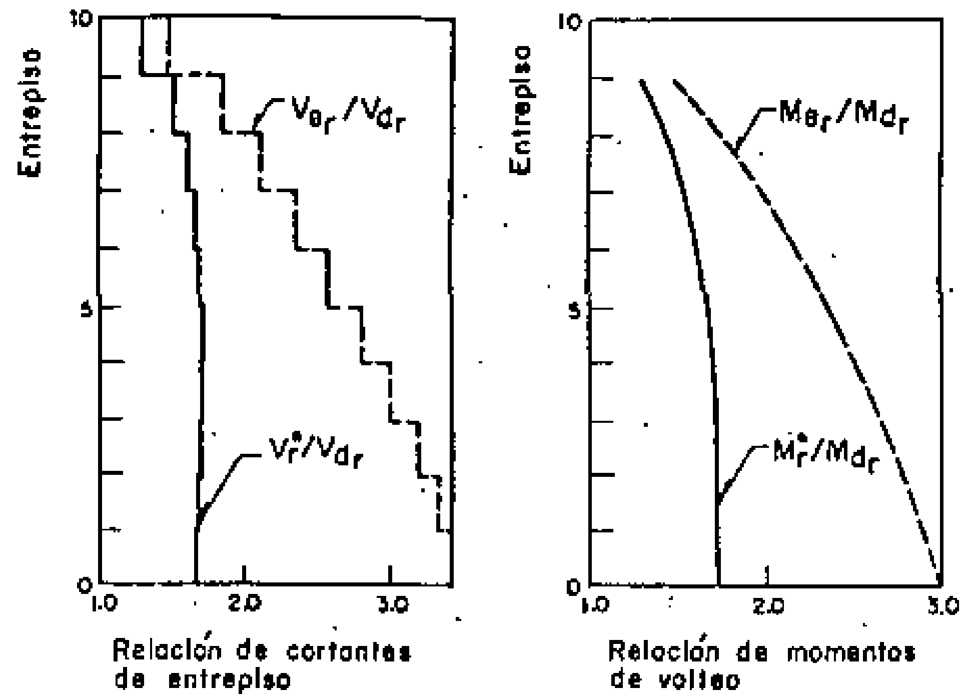


Fig 16. Comparación de resultados para el modelo (K_E-2, M_E-2) con espectro de terreno compresible, factor de ductilidad de Q=1 y relación de aspecto H/B=1.2

Espectro de diseño para terreno compresible del DF, con Q=6

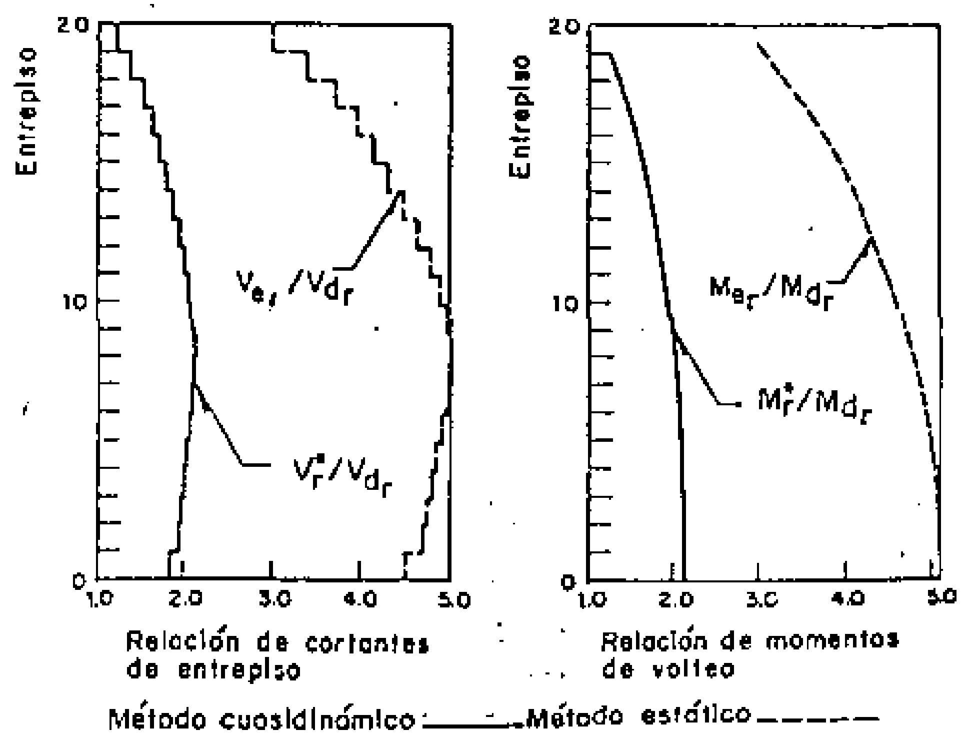


Fig 17. Comparación de resultados para el modelo (K_c-2, M_c-3) con espectro de terreno compresible, factor de ductilidad de Q=6 y relación de aspecto H/B=4

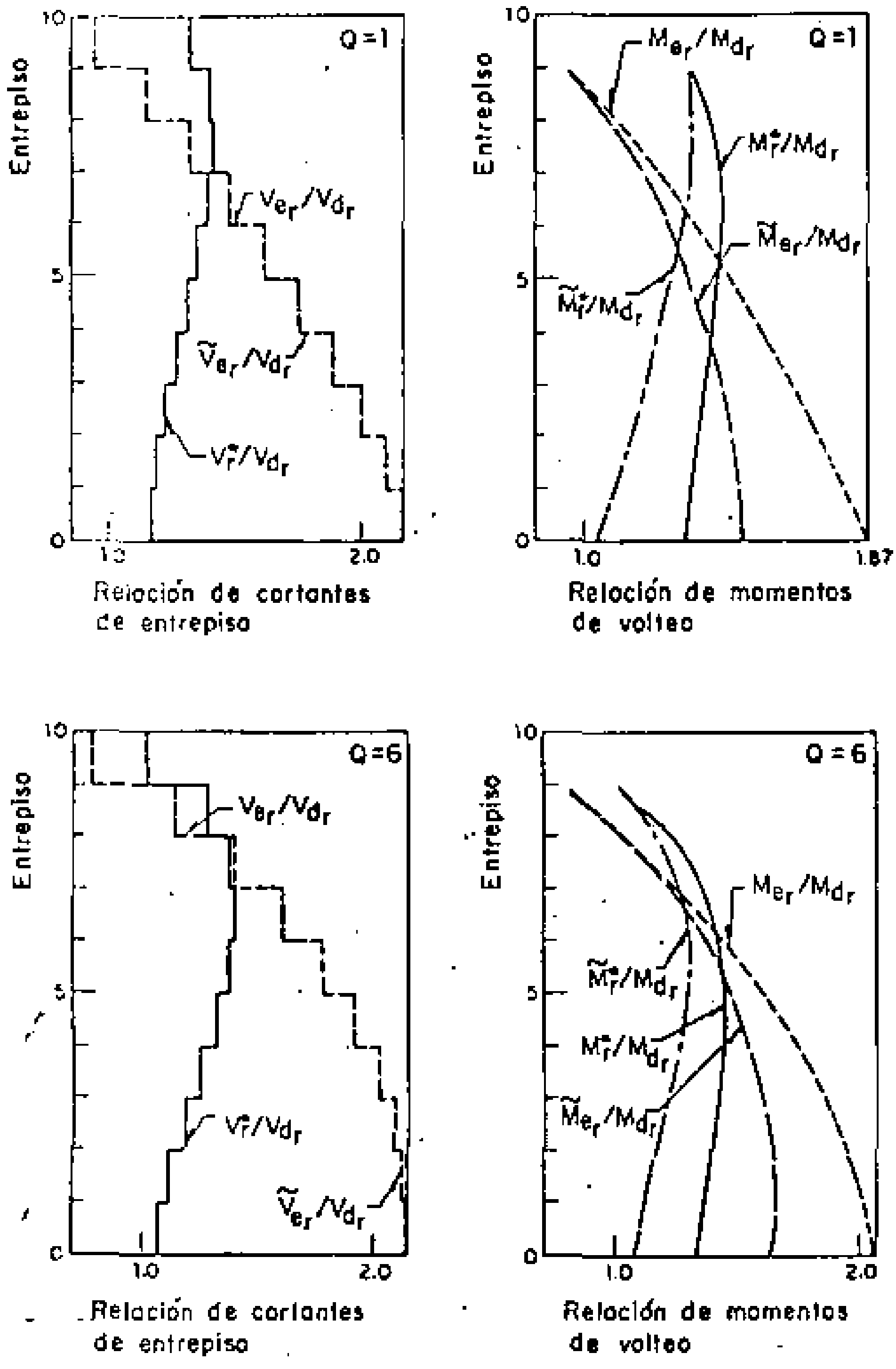


Fig 18. Comparaciones para el modelo (K_F-2, M_F-2) con relación de aspecto H/B=1.2 en terreno firme

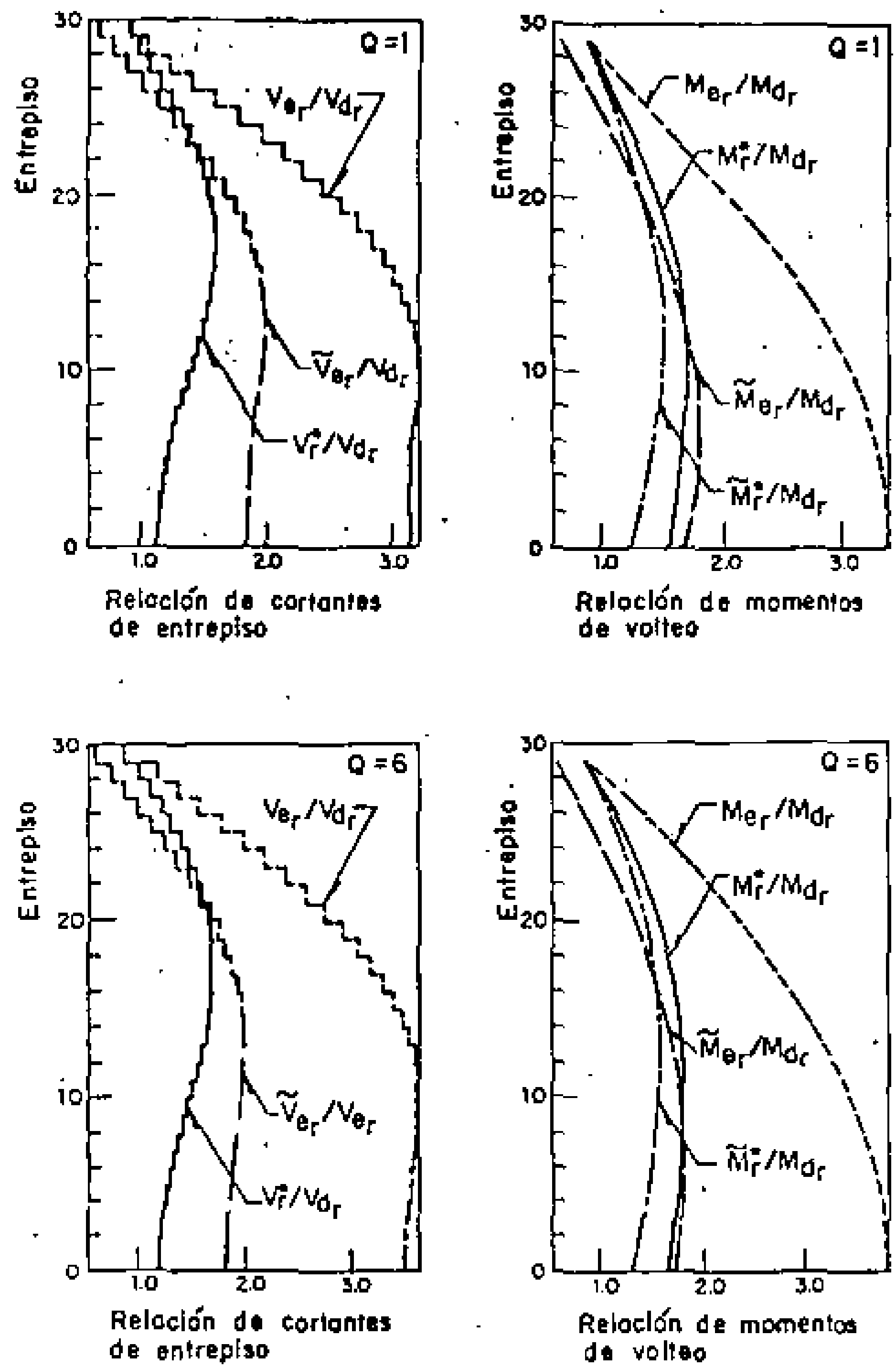


Fig 19. Comparaciones para el modelo (K_F-2, M_F-2) con relación de aspecto H/B=3.6 en terreno firme

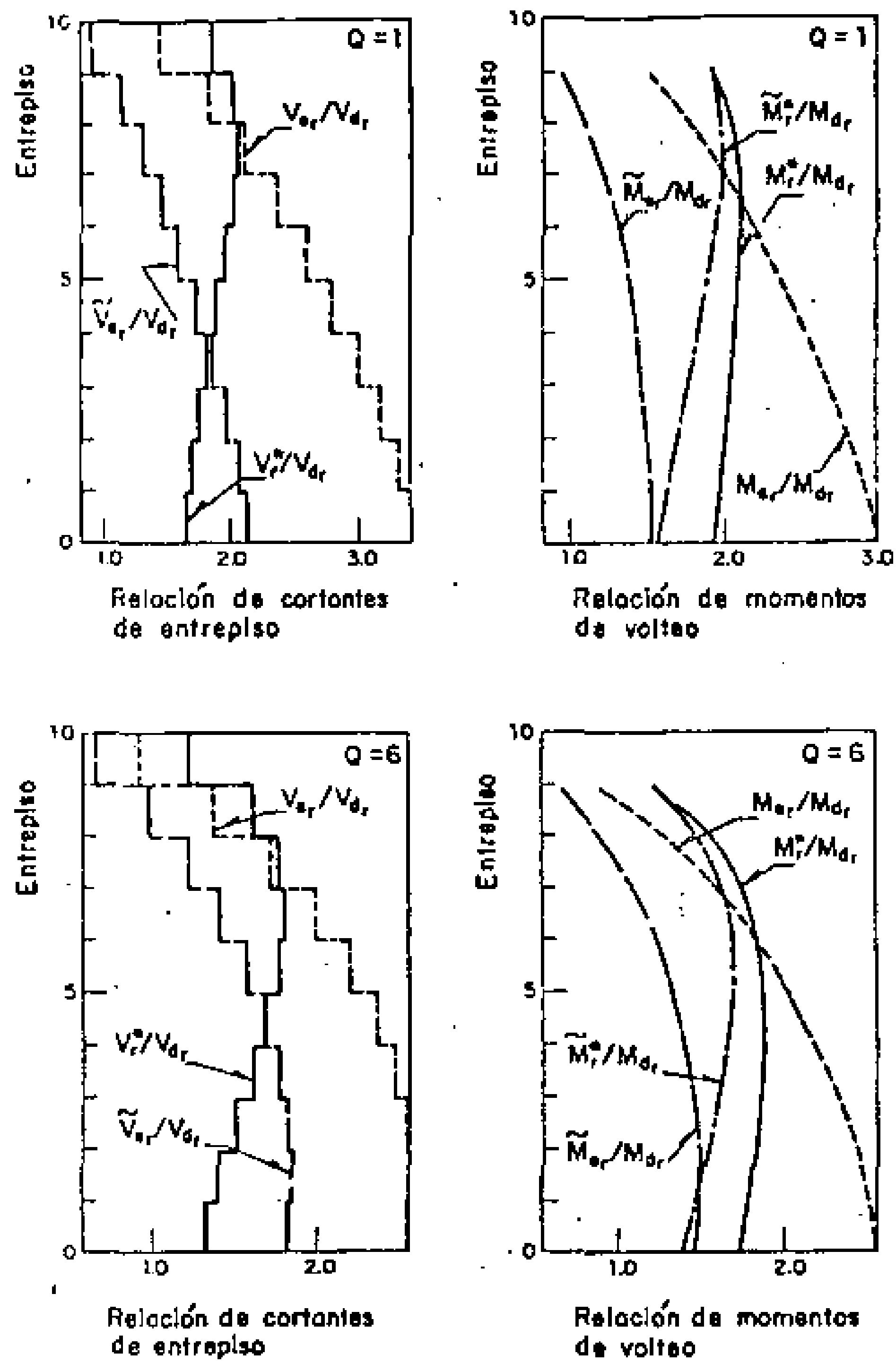


Fig 20. Comparaciones para el modelo (K_F-2, M_F-2) con relación de aspecto H/B=1.2 en terreno compresible

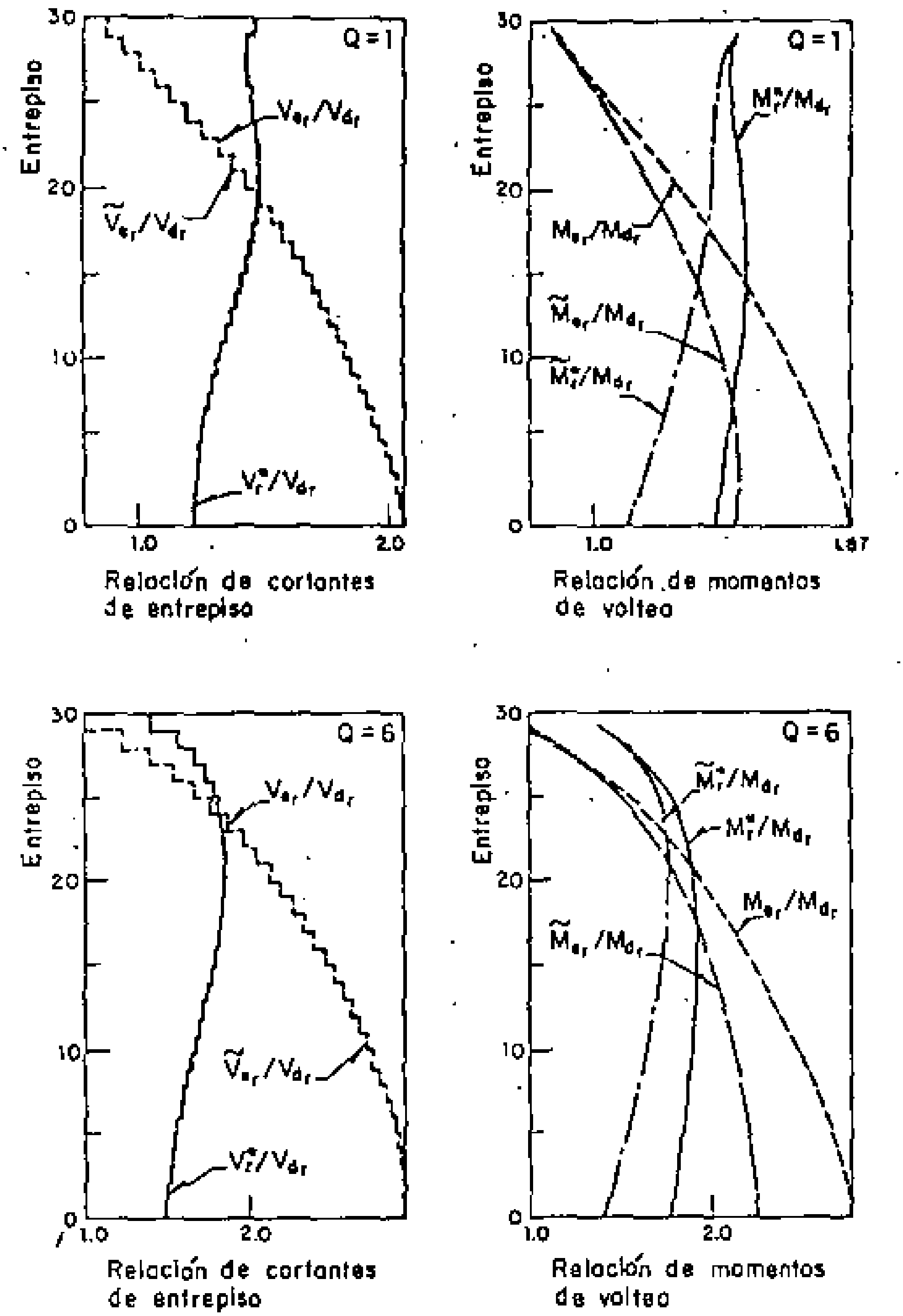


Fig 21. Comparaciones para el modelo (K_F-2, M_F-2) con relación de aspecto H/B=3.6 en terreno compresible

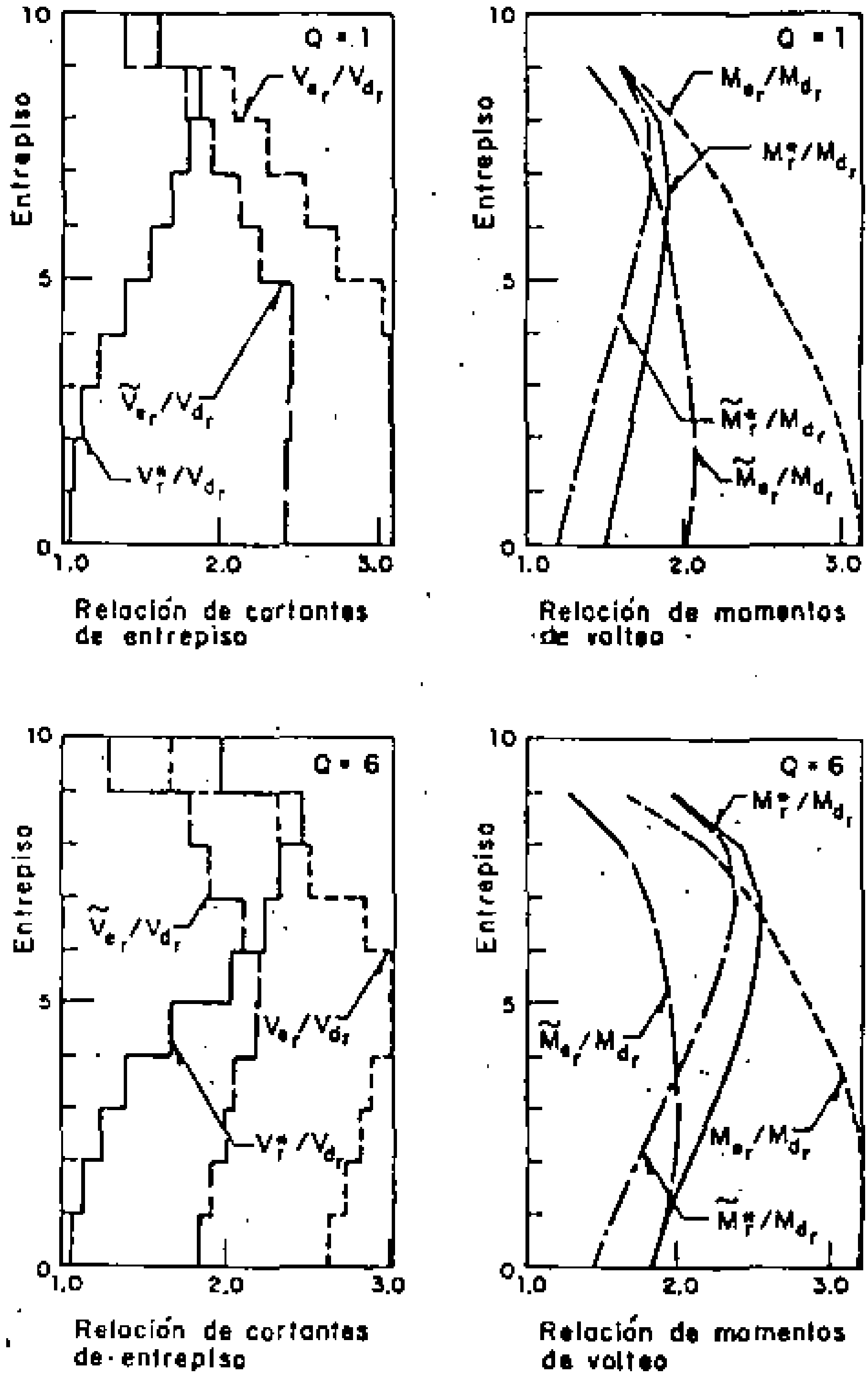


Fig 22. Comparaciones para el modelo (K_F=3, M_F=3) con relación de aspecto H/B=1.2 en terreno firme

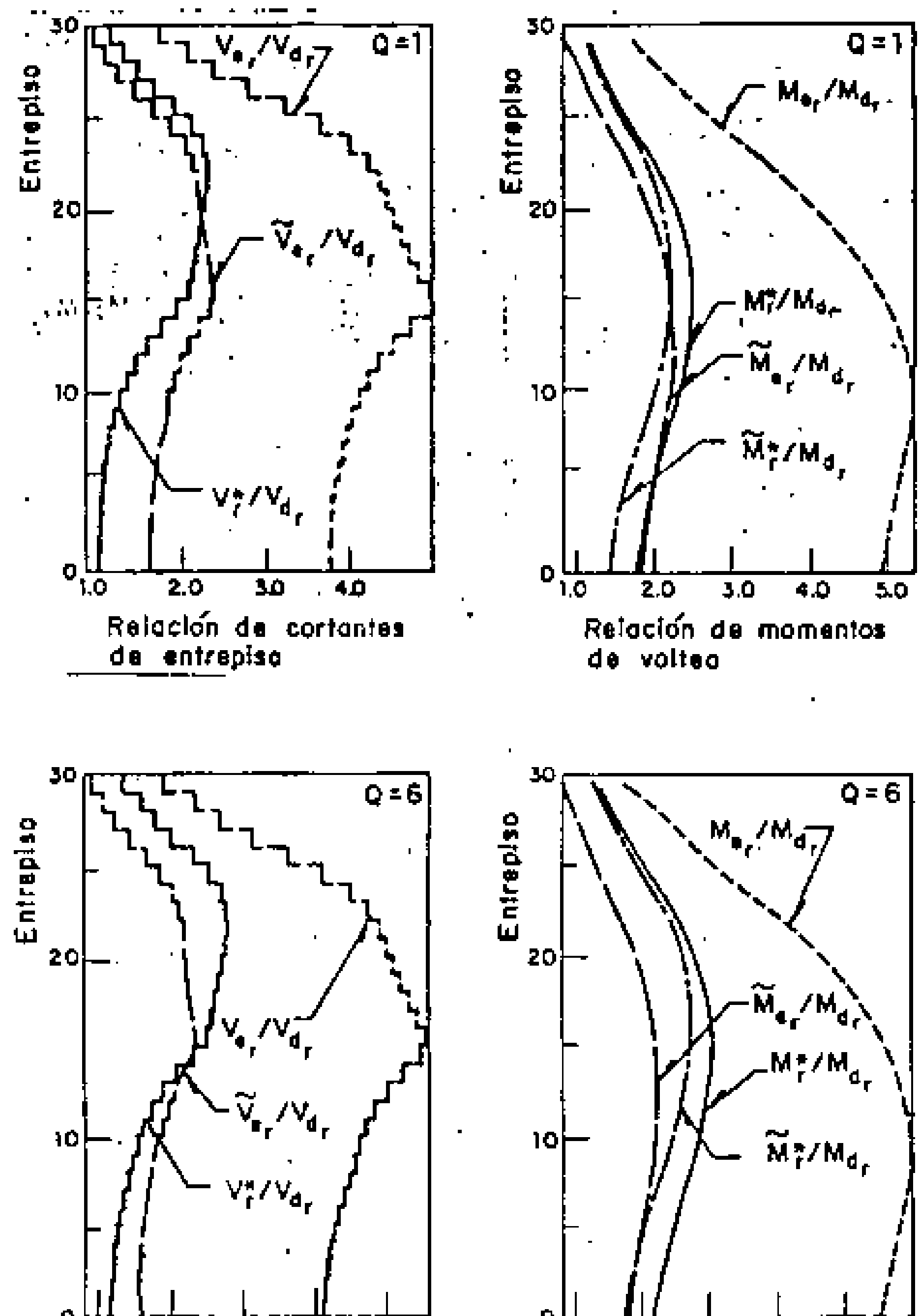


Fig 23. *Comparaciones para el modelo (K_F-3, M_F-3) con relación de aspecto H/B=3.6 en terreno firme*

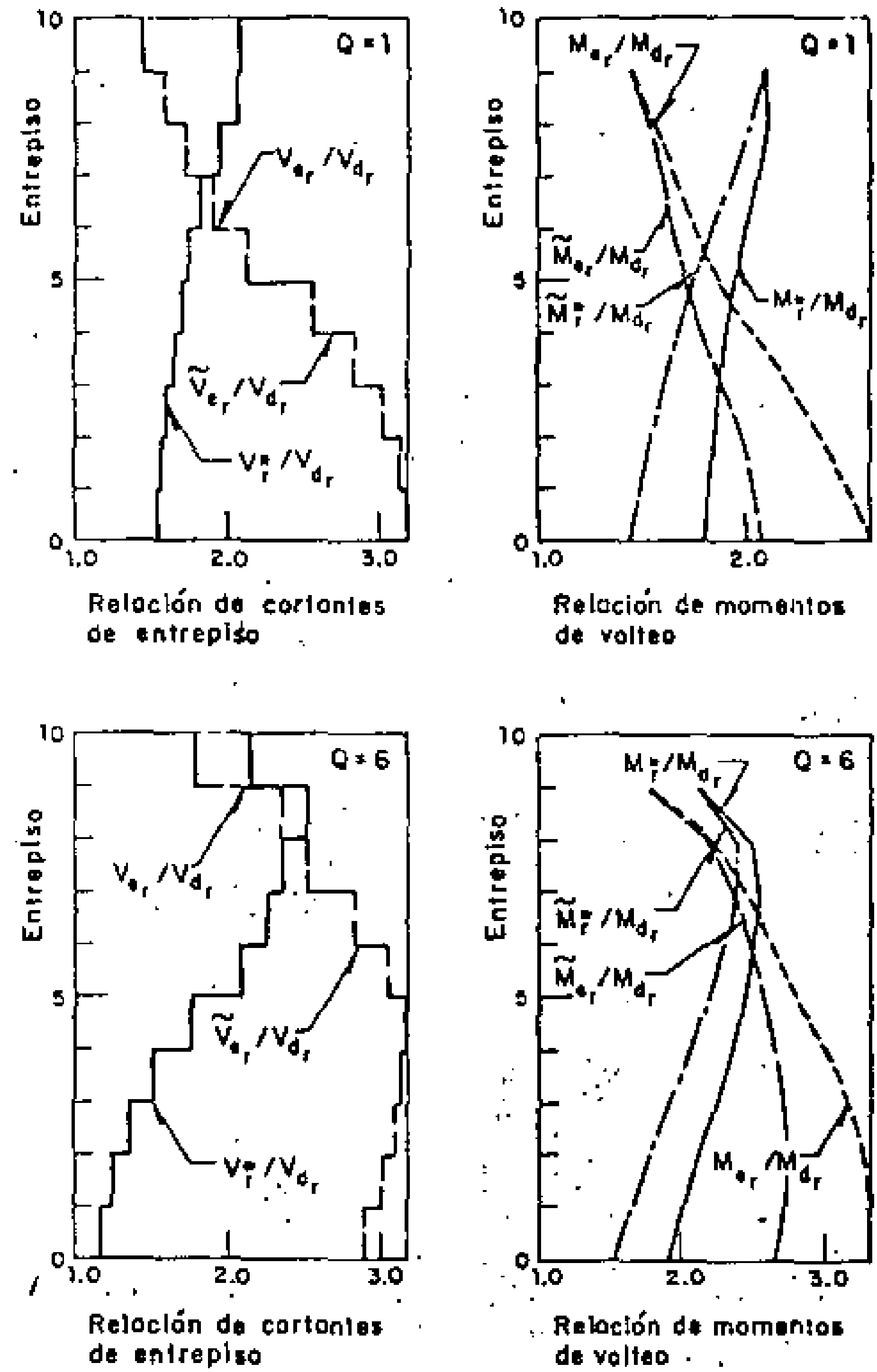


Fig 24. Comparaciones para el modelo (K_F-3, M_F-3) con relación de aspecto H/B=1.2 en terreno compresible

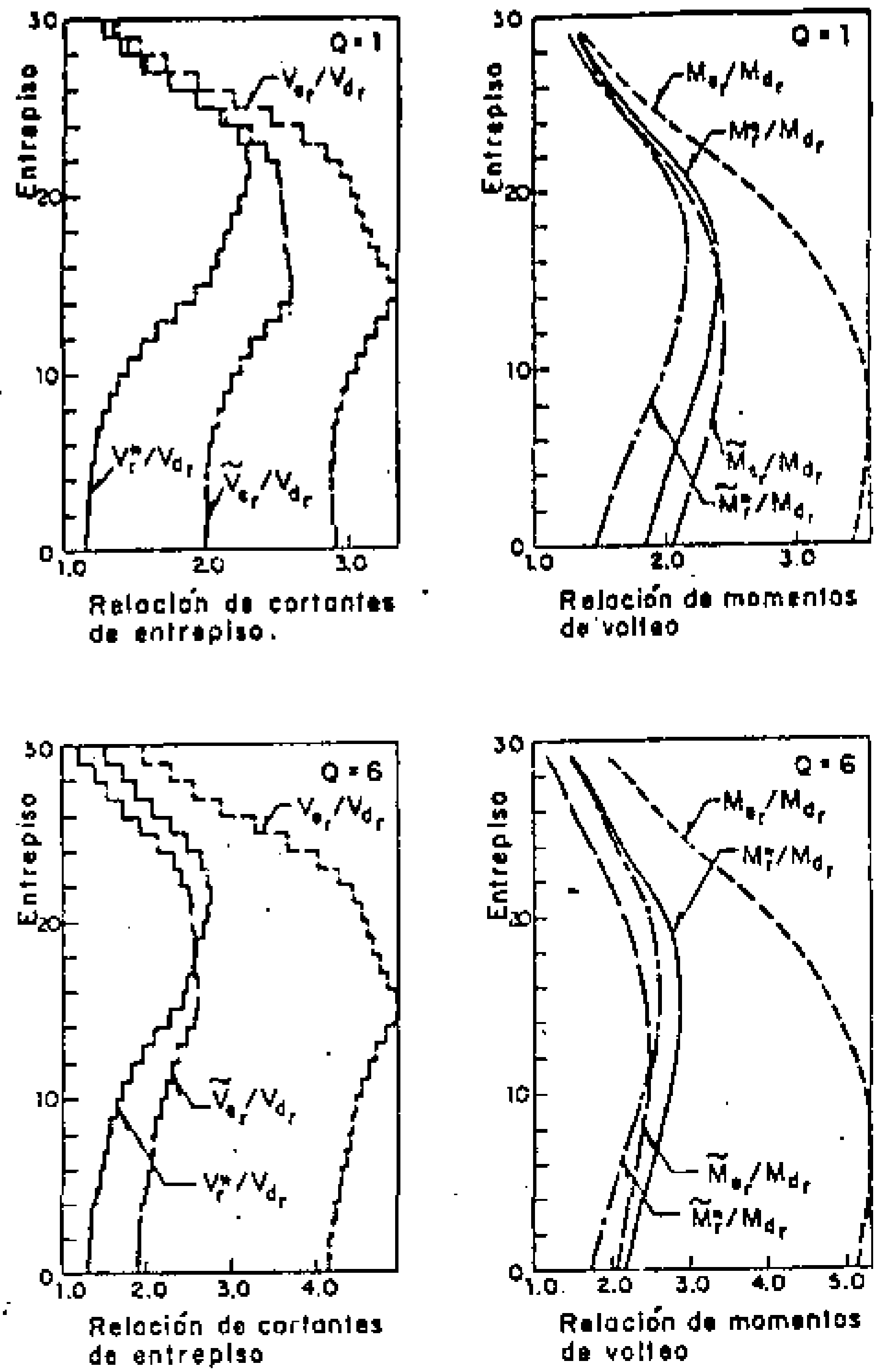


Fig 25. Comparaciones para el modelo (K_F-3, M_F-3) con relación de aspecto H/B=3.6 en terreno compresible

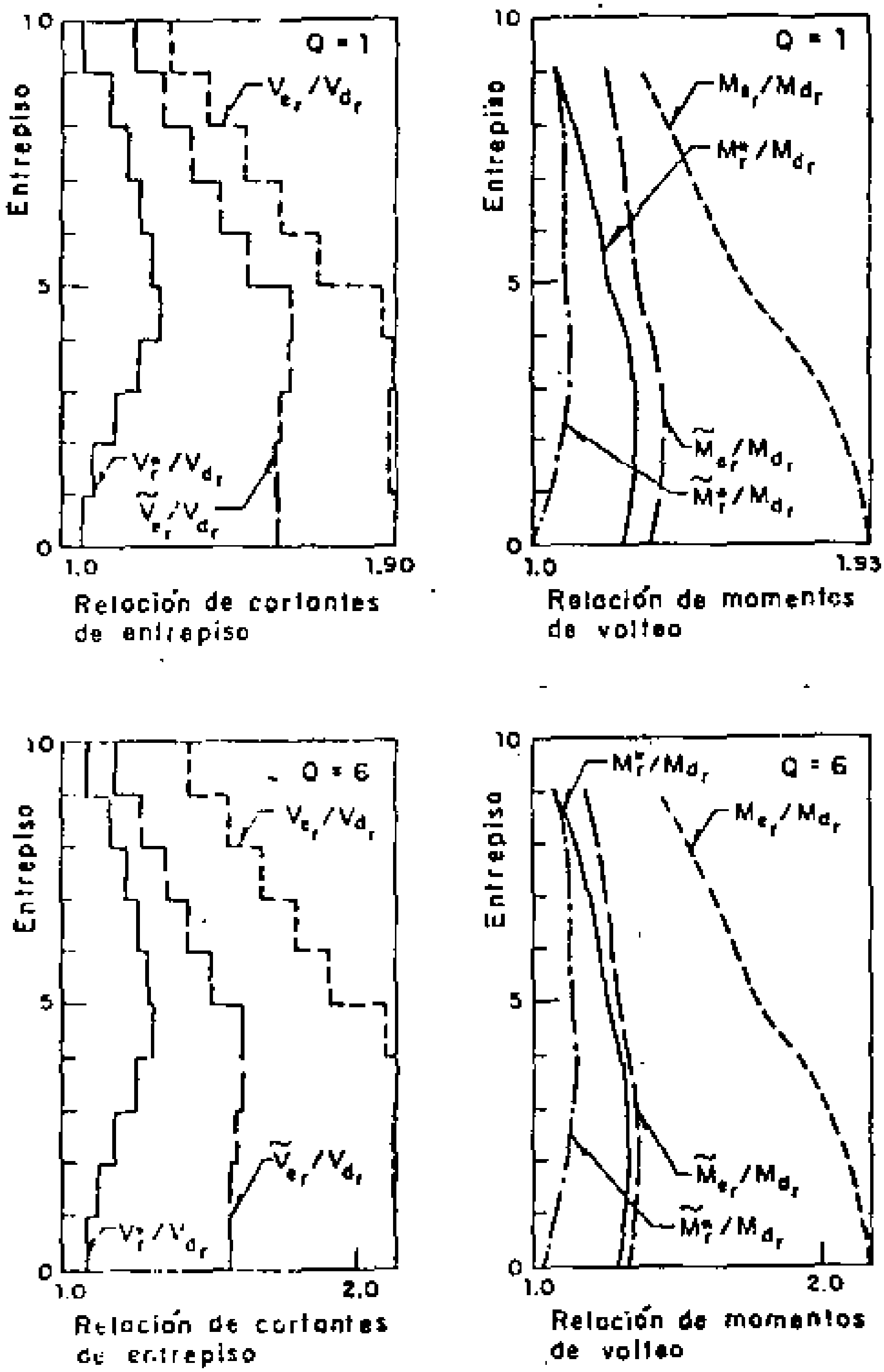


Fig 26. Comparaciones para el modelo (K_c=3, H_c=1) con relación de aspecto H/B=2 en terreno firme

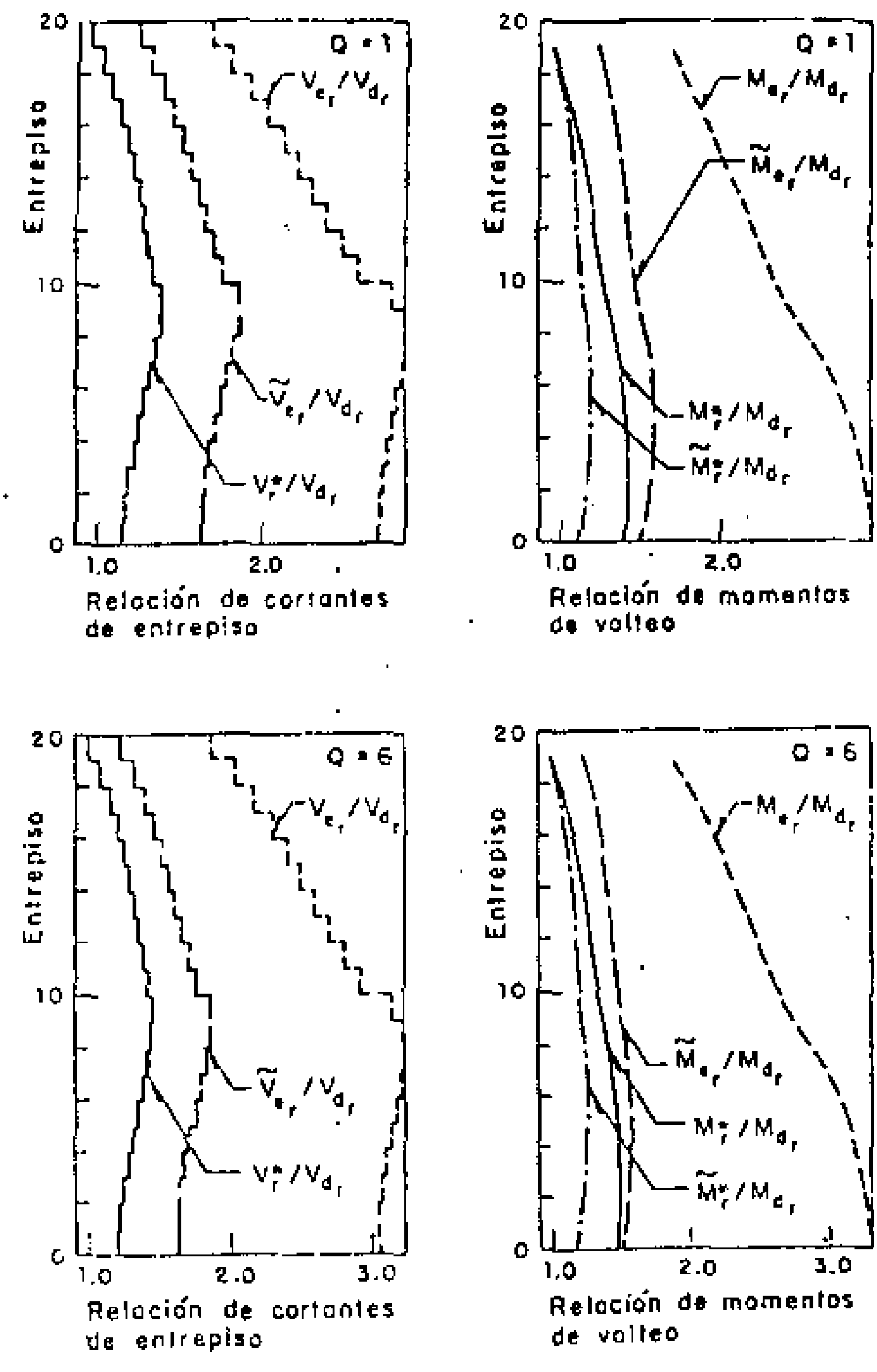


Fig 27. Comparaciones para el modelo (K_c-3, M_c-1) con relación de aspecto H/B=4 en terreno firme

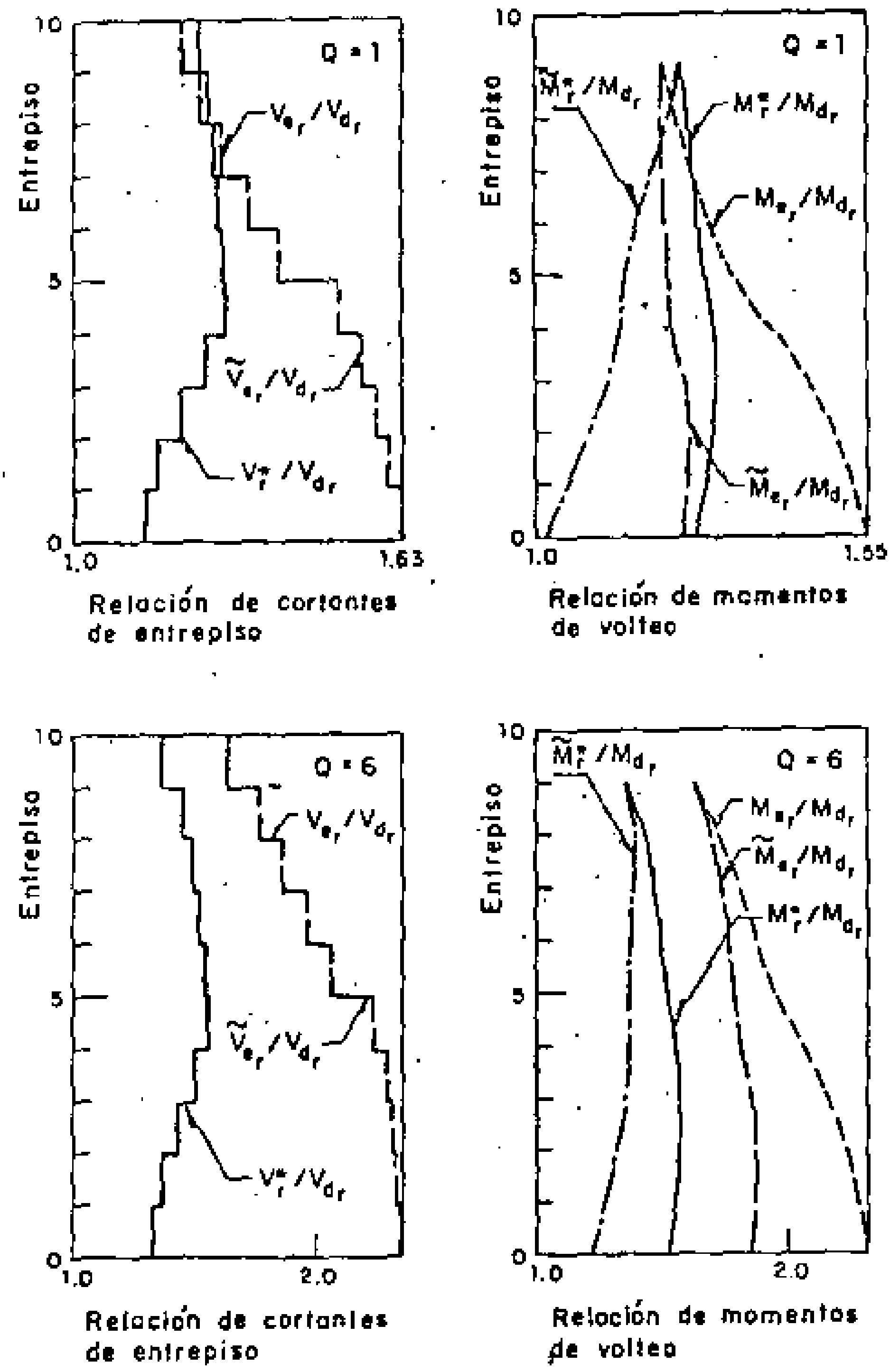


Fig 28. Comparaciones para el modelo ($K_c=3$, $M_c=1$) con relación de aspecto H/B=2 en terreno compresible

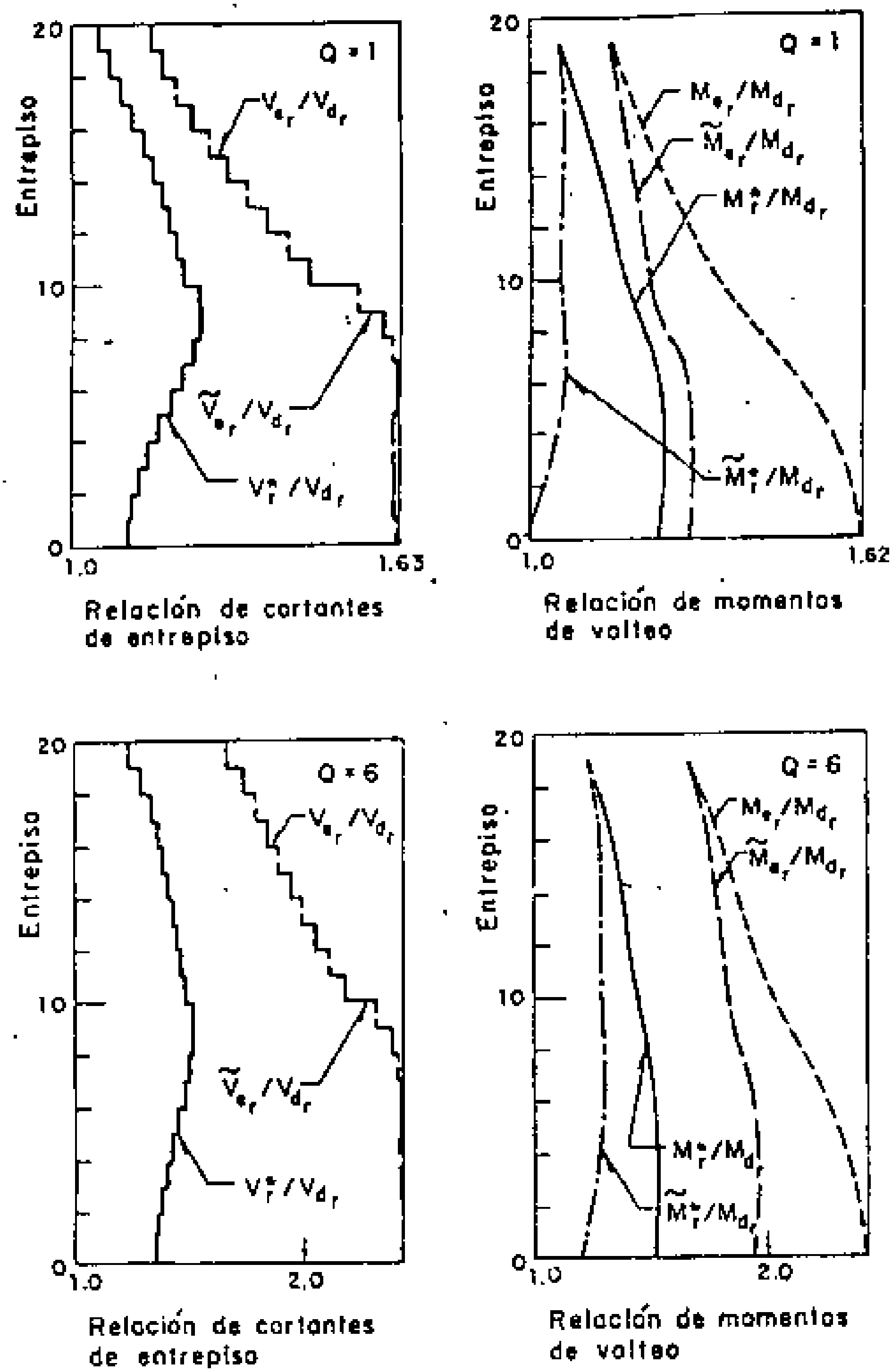


Fig 29. Comparaciones para el modelo $\{K_c=3,\ M_c=1\}$ *con relación de aspecto H/B=4 en terreno compresible*

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