DIRECTORIO DE PROFESORES DEL CURSO GEOESTADISTICA

1980.

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CURSO: "GEOESTADISTICA"
FECHA: Del 10 al 14 de marzo
HORARIO: De 9.00 a 18.00 h.
COORDINADOR: M. en C. Juán José Obregón Andría
Ing. Mario Vargas Flores.

T E M A R I O	PROFESOR	FECHA	HORARIO
INTRODUCCION	Dr. Juan M. Berlanga Gutiérrez	10 de marzo	
-Geoestadística, su filosofía y objetivos fundamentales	"	"	9.00 a 10.00 h.
-Variable aleatoria y funciones de distribución	"	"	10.00 a 12.00 h.
-Diferencias básicas entre la estadística clásica y la geoestadística	"	"	12.00 a 13.00 h.
-La variable aleatoria regionalizada	"	"	15.00 a 16.30 h.
-Ejercicios	M. en C. Juan José Obregón Andría Ing. Mario Vargas Flores	"	16.30 a 18.00 h.
ANALISIS ESTRUCTURAL			
-El variograma, su interpretación, cálculo y modelo	Dr. Juan M. Berlanga Gutiérrez	11 de marzo	9.00 a 13.00 h.
-Ejercicios	M. en C. Juan José Obregón Andría Ing. Mario Vargas Flores	"	15.00 a 18.00 h.
KRIGING			
-Cálculo de la confiabilidad de un estimador lineal	Dr. Juan M. Berlanga Gutiérrez	12 de marzo	9.00 a 11.00 h.
-La variancia de estimación, una nueva herramienta en la exploración	"	"	11.00 a 13.00 h.
-La técnica del kriging	"	"	15.00 a 16.30 h.
-Ejercicios	M. en C. Juan José Obregón Andría Ing. Mario Vargas Flores	"	16.30 a 18.00 h.
EVALUACION DE RESERVAS			
-El kriging universal	Dr. Juan M. Berlanga Gutiérrez	13 de marzo	9.00 a 13.00 h.
-Estimación de reservas in-situ	"	"	15.00 a 16.30 h.
-Estimación de reservas recuperables	"	"	16.30 a 18.00 h.

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"GEOESTADISTICA"

T E M A R I O	PROFESOR	FECHA	HORARIO
TECNICAS RECIENTES DE SIMULACION			
-Simulación numérica de yacimientos mineros	Dr. Juan M. Berlanga Gutiérrez	14 de marzo	9.00 a 13.00 h.
-Uso de programas de cómputo en la aplicación del kriging y en la simulación numérica.	"	"	15.00 a 18.00 h.

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C U R S O D E : G E O E S T A D I S T I C A

Dr. J.M. Berlanga

10 - 14 marzo, 1980

- CAPITULO I -

INTRODUCCION A LA GEOESTADISTICA

1 - Geoestadística, su filosofía y objetivos fundamentales.

El objetivo principal de este curso es dar a conocer a aquellos profesionales de las ciencias de la tierra, los conceptos básicos de una nueva teoría denominada "Geoestadística". Georges Matheron (1962) fue el primero en darle formalidad a esta teoría, la cual definió como: "la aplicación de las funciones aleatorias al reconocimiento y estimación de fenómenos naturales".

La premisa básica en geoestadística es considerar que las variables de fenómenos naturales son de carácter mixto, es decir, están compuestas de dos partes: una estructural y otra aleatoria. La figura 1 ilustra una gráfica de mediciones de porcentaje de mineral efectuados a lo largo de cierta dirección en un yacimiento minero. El eje vertical denota el porcentaje de mineral y el eje horizontal representa una cierta dirección, x. Dos características pueden apreciarse: una local, de comportamiento errático o aleatorio, y otra general, con cualidades estructurales.

Por ejemplo, un proceso de mineralización puede presentar una estructura global y además seguir ciertas leyes que bien pueden caracterizarse como geológicas, metalogenéticas, o bien, como una combinación de ambas; en particular, existen zonas donde las concentraciones de mineral son altas, así como zonas de baja concentración (Fig. 1), siendo ésto posible, únicamente, si las concentraciones poseen un cierto grado de continuidad. Dependiendo del tipo de depósito, el grado de continuidad podrá ser más o menos aparente.

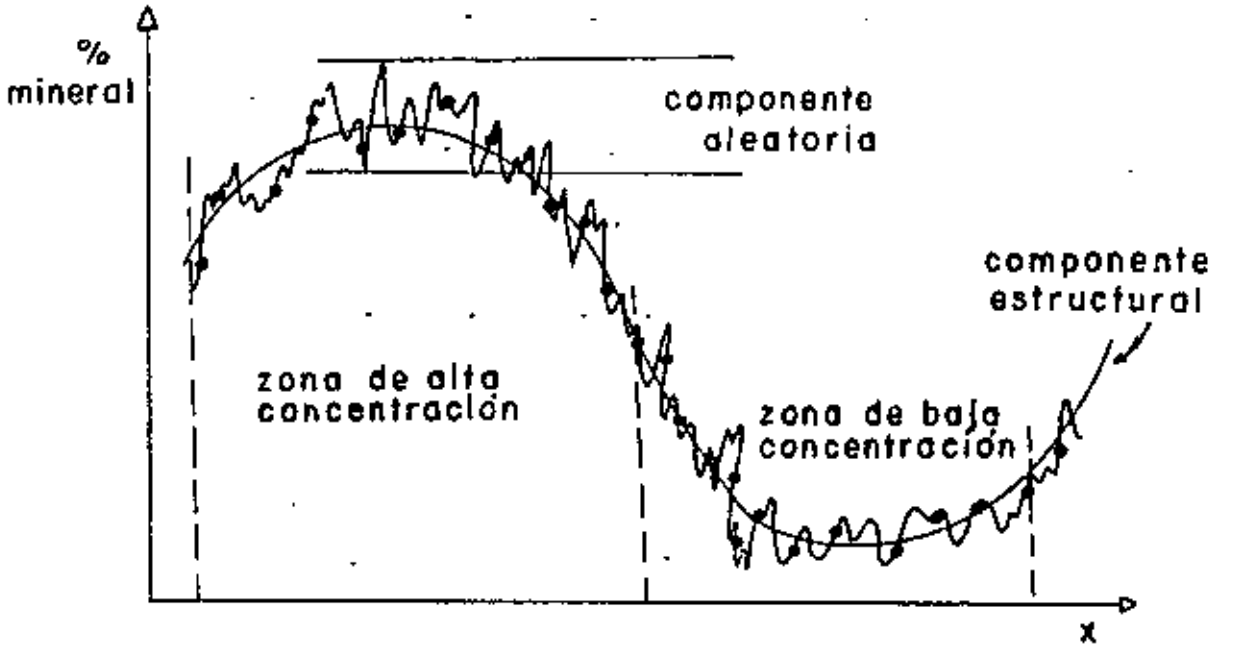


Figura 1

En efecto, si no fuese por la presencia de esta continuidad, cualquier estimación y consecuentemente cualquier selección serían imposibles de realizar. En nuestro proceso, sin embargo, la mineralización no es tan caótica como para anular cualquier método de estimación, ni lo suficientemente regular como para permitir el empleo de métodos determinísticos. Es por ello, que una estimación realista debe necesariamente tomar en cuenta ambos aspectos, el estructural y el aleatorio, el primero siendo observado primordialmente por geólogos y el segundo por estadísticos.

De aquí que el nombre de Geoestadística -propuesto por Matheron- defina al campo que sintetiza estos dos aspectos, y que abre un camino hacia la solución de problemas de evaluación de depósitos mineros. Un estudio geoestadístico se inicia con un "análisis estructural". Este consiste en el ajuste de una función -denominada variograma- a las variabilidades espaciales (in-situ) de los parámetros estu-

diados. Como ejemplos de estos últimos podríamos citar: la concentración media de mineral en un cierto volumen de roca, el espesor de una formación geológica, la permeabilidad de una roca, etc.

Es obvio que un yacimiento mineral de cobre porfírico no puede ser estimado de la misma manera que un yacimiento de fosfatos. El procedimiento de estimación debe de tomar en cuenta la estructura de la variabilidad espacial de cada yacimiento, así como la manera particular con la cual el yacimiento fue muestreado. Tomando en cuenta estas peculiaridades, es posible asignar a cada valor estimado, un intervalo de confianza. El método "Kriging" (en honor a Danie G. Krige) de estimación, toma en cuenta todos estos factores y además permite la estimación de reservas in-situ. Kriging proporciona el "mejor" estimador lineal e insesgado (en inglés: best linear unbiased estimator BLUE). "Mejor" es entendido aquí, en el sentido de que minimiza la variancia de estimación del error.

El siguiente paso en el estudio geoestadístico corresponde a la evaluación de la proporción de reservas in-situ que pueden ser recobradas dentro de un marco económico y tecnológico. La evaluación de reservas recobrables debe tomar en cuenta el método de selección y evaluación a emplear.

Finalmente, es posible en geoestadística, realizar simulaciones de yacimientos, o más concretamente, generar un modelo con las mismas características estructurales de la variable(s) estudiada. Dentro de ciertos límites, es posible entonces, examinar las consecuencias que implicarían el uso de diversas técnicas de extracción por medio de la simulación de éstas dentro del modelo.

La aceptación de la geoestadística, especialmente en ingeniería minera, se debe a la coherencia y efectividad de las soluciones

que ella ofrece a los diversos problemas encontrados en la práctica.

2 - Conceptos importantes en estadística.

A continuación se presentan una serie de definiciones de la estadística, los cuales nos servirán como punto de apoyo para el entendimiento formal de la teoría geoestadística. En esta sección se introducen los conceptos de variable aleatoria, función de distribución acumulativa y función de densidad, principalmente.

Una variable aleatoria (en adelante v.a.) denotada por $X(\cdot)$ o simplemente X, es una función con dominio en el espacio Ω y contradominio en la línea de los reales $\mathbb{R}$ (ver figura 2).

Si pensáramos en términos de experimentos aleatorios, Ω estaría formada por la totalidad de los resultados obtenibles al realizar dichos experimentos. La función o variable aleatoria (v.a.) X asociaría un número real a cada uno de los resultados del experimento.

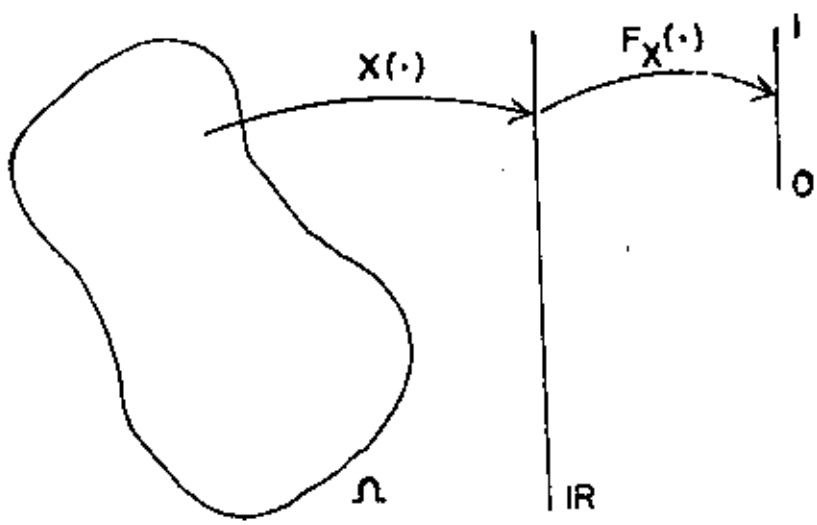


Figura 2.

Consideremos, por ejemplo, el experimento de efectuar un "volado" con una moneda, en donde una variable aleatoria podría definirse de la siguiente manera:

Si $\Omega = \{\omega_1 = \text{águila}, \omega_2 = \text{sol}\}$ conjunto total de resultados obtenibles.

entonces $X(\omega_1) = 1$, $X(\omega_2) = 0$.

La v.a. X asocia un número real (0 ó 1) a cada resultado del experimento.

Nota: Los números reales asociados a cada resultado del experimento se representan, generalmente, con letras minúsculas, ésto es, $X(\omega)=x$.

La función de distribución acumulativa (en adelante f.d.a.) de una v.a. X, denotada $F_X(\cdot)$ es una función con dominio en $\mathbb{R}$ y contradominio en el intervalo $[0,1]$ (ver figura 2), y que puede definirse como:

$$F_X(x) = \Pr\{X \leq x\} = \Pr\{\omega = X(\omega) \leq x\}, \qquad \forall\, x \in \mathbb{R}$$

donde $\Pr\{X \leq x\}$ representa la probabilidad de que la v.a. X adquiera todos los valores posibles menores o iguales a x.

El uso del término: "función de distribución acumulativa" en la definición de $F_X(\cdot)$, está totalmente justificado. $F_X(\cdot)$ es, primero que todo, una función; es una distribución ya que ella nos indica la forma en la cual los valores de la variable aleatoria x se encuentran distribuidos, y es acumulativa ya que ella presenta la distribución de los valores en forma acumulativa.

En el ejemplo de la moneda, si la v.a. X fuera definida como el número de veces en que cae "águila", entonces $F_X(\cdot)$ sería igual a:

$$F_X(\cdot) = \begin{cases} 0 & \text{si } x < 0 \\ \frac{1}{2} & \text{si } 0 \leq x < 1 \\ 1 & \text{si } x \geq 1 \end{cases}$$

Gráficamente, $F_X(\cdot)$ tendría la siguiente forma:

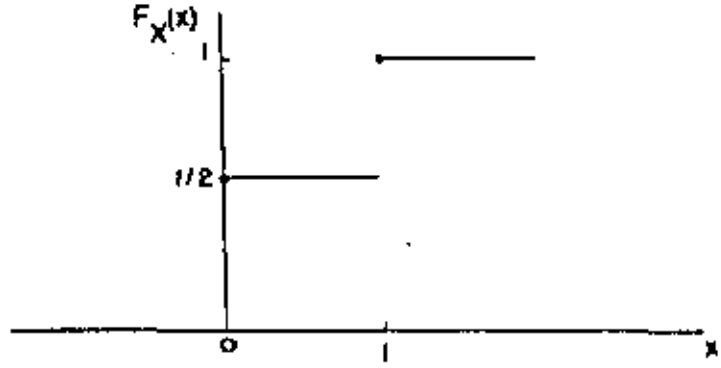


Figura 3

No cualquier función $F_X(\cdot)$ puede ser una función de distribución acumulativa; para serlo, ella debe de satisfacer las siguiente propiedades:

i) $F_X(-\infty) \equiv \lim_{x \to -\infty} F_X(x) = 0$

$F_X(\infty) \equiv \lim_{x \to \infty} F_X(x) = 1$

ii) $F_X(\cdot)$ debe de ser una función no-decreciente, es decir:

$$F_X(a) \leq F_X(b) , \qquad \forall\ a < b$$

iii) $F_X(\cdot)$ debe de ser contínua por la derecha, es decir:

$$\lim_{0 < h \to 0} F_X(x+h) = F_X(x)$$

Nota: A cada variable aleatoria X le corresponde una y sólo una función de distribución acumulativa. Sin embargo, a diferentes variables aleatorias X, Y, Z,... les pueden corresponder la misma función de distribución.

Las funciones de densidad (en adelante f.d.) permiten describir con mayor simplicidad la distribución de valores de las variables aleatorias. El significado de estas funciones depende del tipo de variables aleatorias (discretas o contínuas) al que están asociadas.

CASO A .- Variable aleatoria discreta

Una v.a. X será discreta si el rango de X es contable. Esto implica que su correspondiente f.d.a. $F_X(\cdot)$ esté definida como discreta (figura 3).

Si X es una v.a. discreta con valores $x_1, x_2, \ldots, x_n, \ldots$, entonces la función denotada por $f_X(\cdot)$ y definida como:

$$f_X(x) = \begin{cases} \Pr\{X = x\} & \text{si } x = x_j\ , \quad j = 1,2,\ldots,n,\ldots \\ 0 & \text{si } x \neq x_j\ , \quad j = 1,2,\ldots,n,\ldots \end{cases}$$

se le conoce como "función discreta de densidad" de la v.a. X. $f_X(\cdot)$ es una función con dominio en $\mathbb{R}$ y contradominio en el intervalo $[0,1]$.

En el ejemplo de la moneda, la función de densidad asociada a la función de distribución acumulativa representada en la figura 3, tendría la siguiente forma:

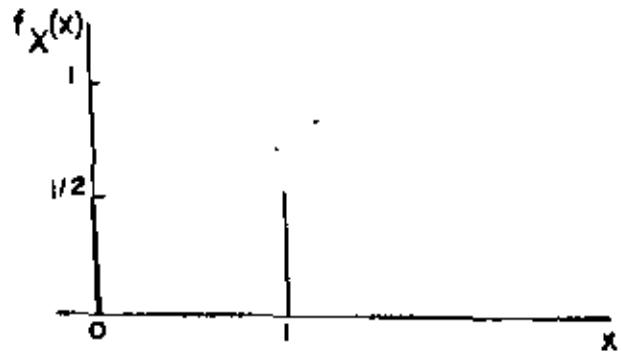


Figura 4

Si X es una v.a. discreta, las siguientes relaciones pueden derivarse fácilmente:

$$F_X(x) = \sum_{(j=x_j<x)} f_X(x_j)$$

$$f_X(x_j) = F_X(x_j) - \lim_{0<h\to 0} F_X(x_j-h)$$

Para que $f_X(\cdot)$ sea una función de densidad de una v.a. discreta X, ésta debe satisfacer las siguientes condiciones:

i) $f_X(x_j) > 0$ $\quad j = 1,2,\ldots,n,\ldots$

ii) $f_X(x) = 0$ $\quad x \neq x_j$, $j = 1,2,\ldots,n,\ldots$

iii) $\sum_j f_X(x_j) = 1$ donde la suma se lleva a cabo sobre todos los puntos $x_1, x_2, \ldots, x_n$

CASO B .- Variable aleatoria contínua.

Se dice que una v.a. X es continua si existe una función $f_X(\cdot)$ tal que $F_X(x) = \int_{-\infty}^{x} f_X(u)du$, $\forall$ x.

La función de distribución acumulativa de una v.a. contínua X, $F_X(x)$, es entonces absolutamente contínua.

La función $f_X(\cdot)$ en la expresión anterior se denomina función probabilística de densidad o simplemente función de densidad. $f_X(\cdot)$ es una función con dominio en $\mathbb{R}$ y contradominio en el intervalo $[0, \infty)$.

Si X es una v.a. contínua, las siguientes relaciones se satisfacen:

$$F_X(x) = \int_{-\infty}^{x} f_X(u)du$$

$$f_X(x) = \frac{d}{dx} F_X(x)$$

Si $f_X(\cdot)$ es una función de densidad, ella debe de cumplir con las siguientes condiciones:

i) $f_X(x) \geq 0 \quad \forall\, x$

ii) $\int_{-\infty}^{\infty} f_X(u)du = 1$

Es importante hacer notar que las interpretaciones de las funciones de densidad en los casos discreto y contínuo, no son las mismas, ésto es, en el caso discreto $f_x(\cdot)$ representa una probabilidad ya que por definición:

$$f_X(x) = Pr\left\{X = x\right\}$$

y en el caso contínuo, habiéndose establecido la relación:

$$f_X(x) = \frac{d}{dx} F_x(x) = \lim_{\Delta x \to 0} \frac{F_X(x + \Delta x) - F_X(x - \Delta x)}{2\Delta x}$$

ó $2\Delta x f_X(x) \approx F_X(x + \Delta x) - F_X(x - \Delta x) = Pr\left\{x - \Delta x < X \leq x + \Delta x\right\}$

la probabilidad de que X esté en un "pequeño" intervalo $x - \Delta x < X \leq x + \Delta x$, conteniendo al valor x, es aproximadamente igual a $f_X(\cdot)$ multiplicado por la longitud del intervalo.

Dos conceptos sumamente importantes en la solución de problemas de variables aleatorias y funciones de densidad y de los cuales haremos constante uso en geostadística son los conceptos de media y variancia.

Media .- La media de una v.a. X, denotada por $E\left[X\right]$, se define como:

$$E\left[X\right] = \sum_j x_j f_X(x_j) \qquad \text{caso discreto}$$

$$E\left[X\right] = \int_{-\infty}^{\infty} x f_X(x)dx \qquad \text{caso contínuo}$$

Variancia .- La variancia de una v.a. X, denotada $Var[X]$, se define como:

$$Var[X] = \sum_j (x_j - E[X])^2 f_X(x_j) \qquad \text{caso discreto}$$

$$Var[X] = \int_{-\infty}^{\infty} (x - E[X])^2 f_X(x)dx \qquad \text{caso contínuo}$$

El concepto de media de una v.a. X puede extenderse al de una función de una variable aleatoria $g(X)$, es decir:

$$E[g(X)] = \int_{-\infty}^{\infty} g(x) f_{g(x)}(x)dx = \int_{-\infty}^{\infty} g(x) f_X(x)dx$$

si $g(X) = X^r$ entonces $E[X^r]$ se denomina r-ésimo momento;
si $g(X) = (X - E[X])^r$ entonces $E[(X - E[X])^r]$ se denomina r-ésimo momento central;
si $g(X) = (X - E[X])^2$ entonces $E[(X - E\,X)^2] = \int_{-\infty}^{\infty} (x - E[X])^2 f_X(x)dx = Var[X]$

La variancia de la v.a. X puede escribirse también en términos del operador $E[\,]$, es decir:

$$Var[X] = E[(X - E[X])^2] = E[X^2] - (E[X])^2$$

De acuerdo a lo anterior, $Var[X]$ es también el segundo momento central de la v.a. X.

Nota: La media se interpreta como una medida de tendencia central y la variancia como una medida de dispersión.

Los siguientes son ejemplos de las distribuciones paramétricas más comúnmente empleadas en estadística.

A .- Variable aleatoria discreta

(1) Se dice que la v.a. X sigue una distribución Bernoulli, $X \sim B(p)$, si la función $f_X(\cdot)$ está dada por:

$$f_X(x) = \begin{cases} p^x(1-p)^{1-x} & \text{para } x = 0 \text{ ó } 1 \\ 0 & \text{para otros valores de } x \end{cases}$$

donde p es un parámetro igual a $E[X]$.

(2) Se dice que la v.a. X sigue una distribución Binomial, $X \sim B_i(n,p)$, si la función $f_X(\cdot)$ está dada por:

$$f_X(x) = \begin{cases} \binom{n}{x} p^x(1-p)^{n-x} & \text{para } x = 0,1,\ldots,n \\ 0 & \text{para otros valores de } x. \end{cases}$$

donde p y n son dos parámetros, p satisface $0 \leq p \leq 1$ y $n = 0,1,2,\ldots$ pertenece al conjunto de los enteros positivos.

$$E[X] = np \ , \quad Var[X] = np(1-p)$$

(3) Se dice que la v.a. X sigue una distribución Poisson, $X \sim P(\lambda)$, si la función $f_X(\cdot)$ está dada por:

$$f_X(x) = \begin{cases} \frac{e^{-\lambda}\lambda^x}{x!} & \text{para } x = 0,1,2,\ldots \\ 0 & \text{para otros valores de } x. \end{cases}$$

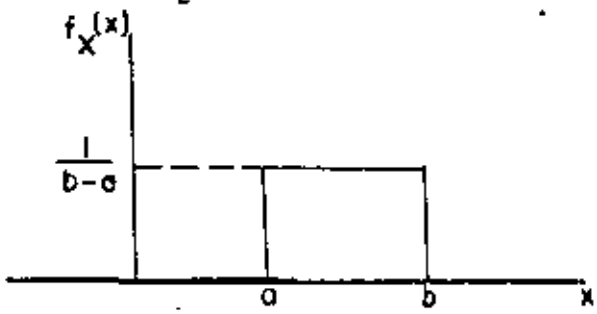
donde el parámetro λ satisface $\lambda > 0$.

$$E[X] = \lambda \ , \quad Var[X] = \lambda$$

B. Variable aleatoria contínua.

(1) Se dice que la v.a. X sigue una distribución Uniforme $X \sim U(a,b)$, si la función $f_X(\cdot)$ está dada por:

$$f_X(x) = \begin{cases} \dfrac{1}{b-a} & \text{para } a \leq x \geq b \\ 0 & \text{para otros valores de } x \end{cases}$$



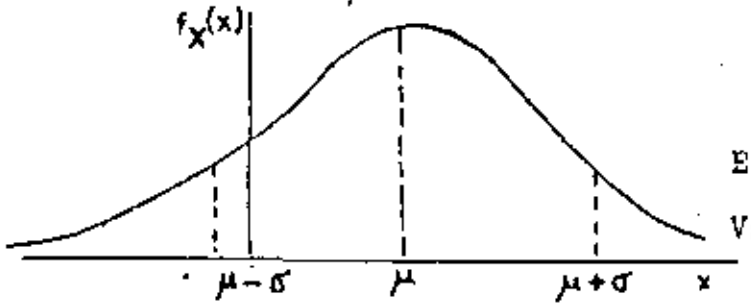
donde:

$$E[X] = \frac{a+b}{2}, \quad Var[X] = \frac{(b-a)^2}{12}$$

(2) Se dice que la v.a. X sigue una distribución Normal $X \sim N(\mu, \sigma^2)$ si la función $f_X(\cdot)$ está dada por:

$$f_X(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \qquad \forall\ x \in (-\infty, \infty)$$

donde los parámetros μ y σ satisfacen $-\infty < \mu < \infty$ y $\sigma > 0$.



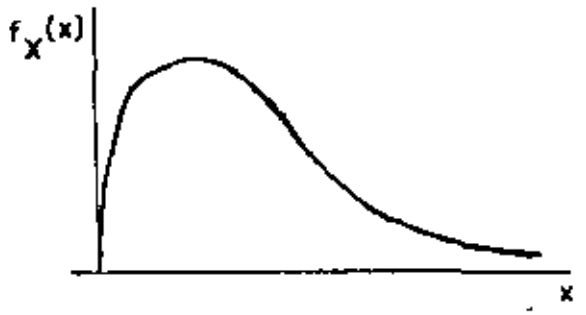
$$E[X] = \mu$$
$$Var[X] = \sigma^2$$

si $\mu = 0$ y $\sigma = 1$ se dice que la v.a. X tiene una distribución Gaussiana o standard, es decir:

$$f_X(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$$

(3) Se dice que la v.a. X sigue una distribución Lognormal, $X \sim \log(m, s^2)$, si la función $f_X(\cdot)$ se define como:

$$f_X(x) = \begin{cases} \dfrac{1}{x\sqrt{2\pi}\sigma} \exp\left(-\dfrac{1}{2\sigma^2}\{\ln x - \mu\}^2\right) & , \forall\, x \in (0,\infty) \\ 0 \quad \text{para otros valores de x.} \end{cases}$$



donde μ y σ son los parámetros de una v.a. Y con distribución normal.

$$E[X] = e^{\mu + \sigma^2/2} \qquad Var[X] = e^{2\mu + 2\sigma^2} - e^{2\mu + \sigma^2}$$

Si $X \sim \log(m, s^2) \Rightarrow Y = \ln X \sim N(\mu, \sigma^2)$

donde:

$$m = e^{\mu + \sigma^2/2} \qquad y \qquad s^2 = m^2(e^{\sigma^2} - 1)$$

(4) Se dice que la v.a. X sigue una distribución Gamma $X \sim \gamma(\lambda, r)$ si la función $f_x(\cdot)$ es igual a:

$$f_X(x) = \begin{cases} \dfrac{\lambda}{(r)} (\lambda x)^{r-1} e^{-\lambda x} & , \forall\, x \,[0,\infty) \\ 0 \quad \text{para otros valores de x.} \end{cases}$$

donde λ y r son dos parámetros que satisfacen $\lambda > 0$, $r > 0$.

$$E[X] = r/\lambda \quad , \qquad Var[X] = r/\lambda^2$$

En las ciencias de la tierra, en general, se han analizado estadísticamente una gran cantidad de variables. Ejemplos de las distribuciones seguidas por las variables aleatorias más comunes, se muestran en la tabla 1.

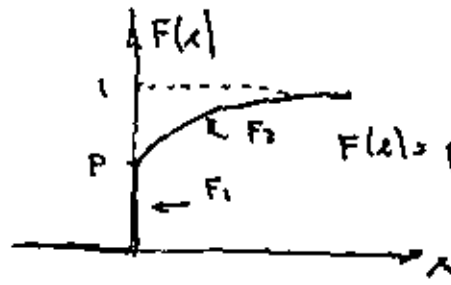
Table 5.4 Examples of geological population distributions

1. Normal distributions
   - Topographic relief
   - Firmness of beach sand
   - Pebble sphericity for fixed particle size
   - Pebble roundness for fixed pebble size
   - Water levels in well through time
   - Drainage density (miles of stream per square mile of drainage-basin area)
   - Specific gravity of rock specimens from a granite pluton
   - Packing density of grains in sandstone
   - Various dimensions (binge length, etc.) of invertebrate fossils
   - Angle of slope on beach foreshores†
   - Angle of slope in valley walls†
   - Angle of dip of sandstone cross-beds†
   - Porosity of sandstones (expressed as percent void space)
   - Percentages of abundant minerals in rocks
   - Percentage of frosted grains in some dune sand
   - Percentage of moisture in sediments
   - Percentage of some chemical elements or oxides in rocks
   - Mean values based on n observations from normal or nonnormal densities
2. Lognormal distributions
   - Particle-size distributions (by weight or number frequency) of some sediments
   - Thickness of sedimentary beds
   - Length of first-order streams in drainage basins of given order
   - Permeability of sedimentary rocks (sandstone, limestone)
   - Concentrations of trace elements in rocks
   - Lengths of beach segments on some cliffed coasts
   - Areas of river placer deposits
3. Gamma distributions
   - Thickness of sedimentary beds
   - Sand-shale ratio and clastic ratio of some stratigraphic units
   - Percentage of rare components in some rocks (organic matter, trace elements, moisture, heavy minerals)
   - Particle sphericity over a large particle-size range
   - Particle roundness over a large particle-size range
4. Circular normal distributions ≈ normal distrib.
   - Orientation of rock joints and fractures
   - Orientation of particle axes in sediments
   - Direction of dip in sandstone cross-beds
5. Binomial distributions
   - Abundant minerals in rocks expressed as number of grains in subsamples of fixed size
   - Abundant fossils in rocks in subsamples of fixed size
   - Occurrence of cross-beds in sandstone (0 = not present, 1 = present)
6. Poisson distributions
   - Rare minerals in rocks expressed as number of grains in subsamples of fixed size
   - Number of alpha particles emitted per unit time from radioactive sediments
   - Sizes of invertebrate fossils in a "death" population

† These items are normal approximations of circular distributions.

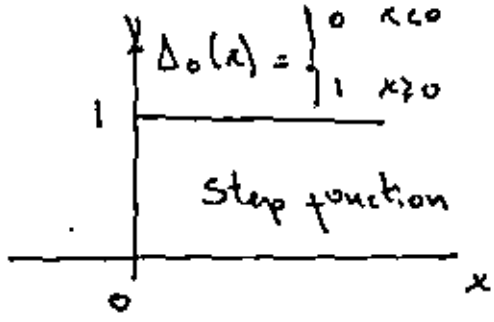
If $F(x) = \sum_{i=1}^{k} \lambda_i F_i(x)$ $\Rightarrow$ is a dist. funct $\Rightarrow$ $\sum \lambda_i = 1$; $\lambda_i > 0 \ \forall i$,

similarly $f(x) = \sum_{i=1}^{k} \lambda_i f_i(x)$ 14. bis.



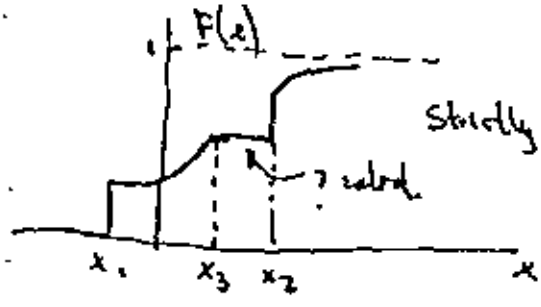
$F(x) = p F_1(x) + (1-p) F_2(x)$.

F_1 is called the Dirac distribution



$F(x) = p \Delta_0(x) + (1-p) F_2(x)$

each Dirac distr. would be introduced in any case where discontinuities were observed



$$\Delta_K(x) = \begin{cases} 0 & x_K < x_K \\ 1 & x \geq x_K \end{cases}$$

$$F(x) = \sum_{i=1}^{l} \lambda_i \Delta_i(x) + \lambda_0 F_0(x).$$

$$\sum_{i=1}^{l} \lambda_i + \lambda_0 = 1$$

} In general

Fitting theoretical models to experimental distributions.

Sample $\rightsquigarrow \hat{F}(x)$ | how do we pass

Population $\rightsquigarrow F(x)$ | from $\hat{F}$ to F?

Los conceptos anteriores pueden extenderse sin dificultad, de una variable aleatoria a varias variables aleatorias. Sean, por ejemplo, X_1, X_2, ..., X_k , k variables aleatorias todas ellas definidas en el mismo espacio Ω . La función acumulativa de distribución conjunta de $X_1,\ldots,X_k$ denotada por: $F_{X_1,\ldots,X_k}(x_1,\ldots,x_k)$ sería igual a: $\Pr\{X_1 \leq x_1,\ X_2 \leq x_2,\ldots,\ X_k \leq x_k\}$ para todos los valores $x_1,\ldots,x_k$. Esta función conjunta tiene su dominio en el espacio euclidiano E^k y su contradominio en el intervalo $[0,1]$.

Similarmente, la función de densidad conjunta de K v.a. estaría definida de la siguiente forma:

$$f_{X_1,\ldots,X_k}(x_1,\ldots,x_k) = \Pr\{X_1 = x_1,\ldots,\ X_k = x_k\}$$

Tanto $f_{X_1,\ldots,X_k}(\cdot)$ como $F_{X_1,\ldots,X_k}(\cdot)$ tendrían propiedades similares a las descritas para los casos de una variable aleatoria.

Una propiedad muy importante en problemas de varias variables aleatorias es el concepto de independencia.

Se dice que las v.a. $X_1,\ldots,X_k$ son independientes sí y sólo si $F_{X_1,\ldots,X_k}(x_1,\ldots,x_k) = F_{X_1}(x_1)\,F_{X_2}(x_2)\ldots F_{X_k}(x_k)$, para todo valor $x_1,\ldots,x_k$.

Y para el caso particular de dos v.a., X e Y, las siguientes definiciones nos serán útiles:

(i) Covariancia (X,Y) ó $\mathrm{Cov}(X,Y) = E\left[(X - E[X])\,(Y - E[Y])\right]$

(ii) Correlación (X,Y) ó $\rho_{X,Y} = \mathrm{Cov}(X,Y)/\sqrt{\mathrm{Var}[X]\ \mathrm{Var}[Y]}$

$\forall\ \mathrm{Var}[X] > 0$ y $\mathrm{Var}[Y] > 0$.

Diremos que dos v.a. X e Y guardan no-correlación entre ellas toda vez que $Cov(X,Y) = 0$ ó bien $\rho_{XY} = 0$.

Por último, mencionaremos como ejemplo de densidades multivariables a la distribución bi-normal. Dos v.a. X e Y siguen una distribución normal bivariable si su función de densidad conjunta es igual a:

$$f_{XY}(x,y) = \frac{1}{2\pi\sigma_x\sigma_y\sqrt{1-\rho^2}} \exp\left\{\frac{-1}{2(1-\rho)^2}\left[\left(\frac{x-\mu_x}{\sigma_x}\right)^2 - 2\rho\frac{x-\mu_x}{\sigma_x}\frac{y-\mu_y}{\sigma_y} + \left(\frac{y-\mu_y}{\sigma_y}\right)^2\right]\right\}$$

donde $\infty < x < \infty$, $-\infty < y < \infty$ y σ_x, σ_y, μ_x, μ_y, y ρ son cinco parámetros tales que: $\sigma_x > 0$, $\sigma_y > 0$, $-\infty < \mu_x < \infty$, y $-\infty < \mu_y < \infty$.

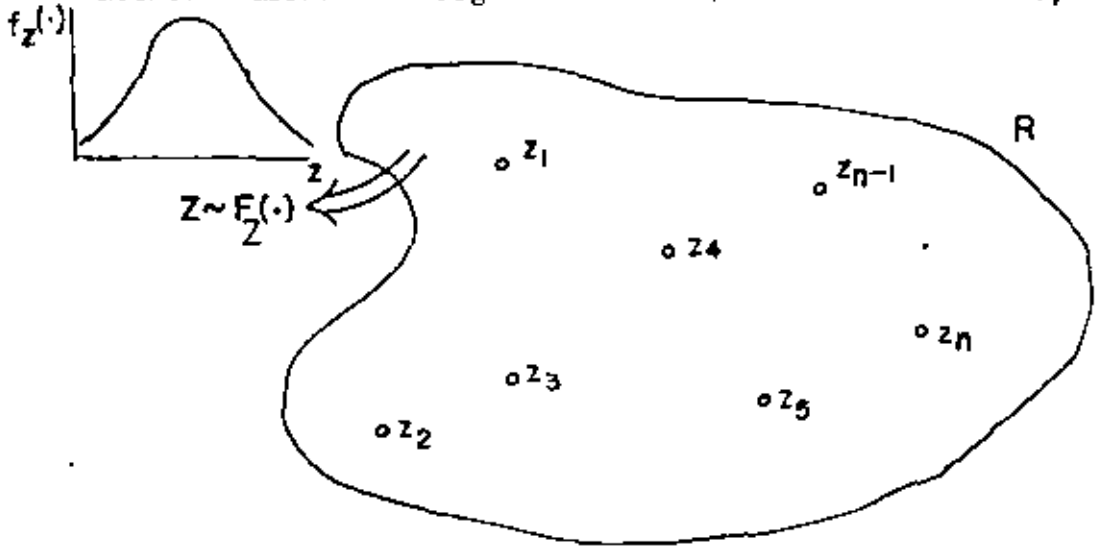
3 - Diferencias básicas entre la estadística y la geoestadística.

Tradicionalmente, en el estudio de los fenómenos naturales se hacía frecuente uso de conceptos básicos de la estadística. Se consideraba, por ejemplo, a cada uno de los valores muestreados, dentro de una cierta región R y para un determinado fenómeno natural, como diferentes realizaciones independientes de una misma variable aleatoria Z. Se asumía que todos los valores muestreados provenían de una misma distribución. La metodología consistía en inferir de los valores muestreados una distribución a partir de la cual se evaluaría reservas, por ejemplo. Estos métodos, por su simplicidad en las suposiciones, producían, sin embargo, resultados frecuentemente incoherentes. Además, no se tomaban en cuenta en ellos conceptos tan importantes como el de correlación entre valores muestreados.

Actualmente, en geoestadística, se considera que cada valor muestreado en un punto (o soporte) $x = (u, v, w)$ de una región R representa únicamente una realización de una variable aleatoria $Z(x)$. Si n valores fueran muestreados, ellos representarían realizaciones de n diferentes variables aleatorias $Z(x_1)$, $Z(x_2)$, ..., $Z(x_n)$, cada una de las cuales tendría asociada su propia función de distribución $F_{Z(x_1)}(\cdot)$, $F_{Z(x_2)}(\cdot)$, ..., $F_{Z(x_n)}(\cdot)$.

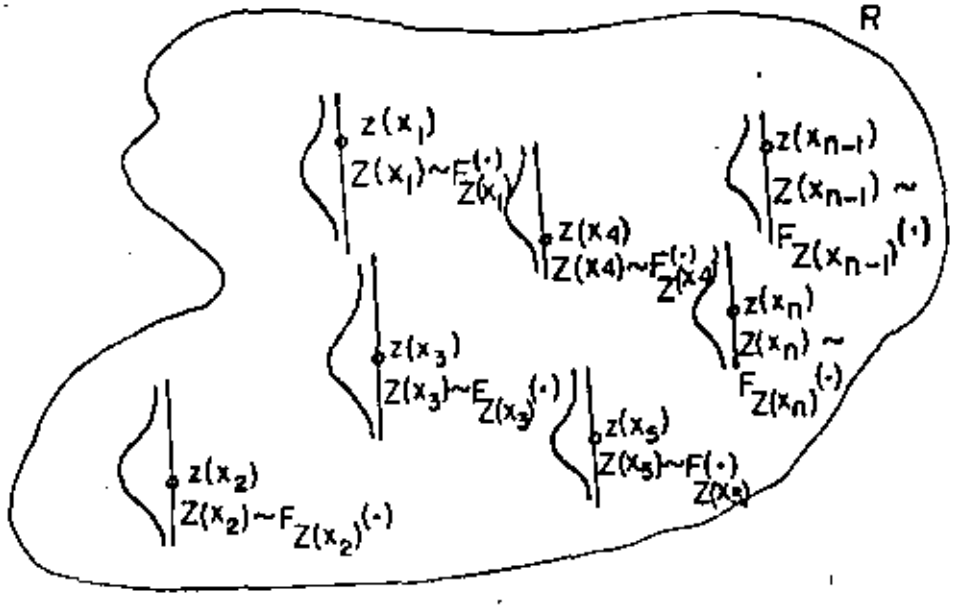
Estas ideas se representan esquemáticamente en las figuras 5-A y 5-B.

Variable aleatoria regionalizada.- Las variables aleatorias $Z(x_1)$, ..., - $Z(x_n)$, por estar distribuídas en el espacio, se les conoce con el nombre de "variables aleatorias regionalizadas" (en adelante v.a.r.),



Concepto tradicional

(A)



(B)

Figura 5. Fenómeno natural caracterizado por la distribución espacial de una Variable Aleatoria Regionalizada.

y al fenómeno representado por estas variables aleatorias regionalizadas - $Z(x_i)$ se le denomina "regionalización", ejemplo de lo cual podríamos - citar:

(i) El precio del barril de petróleo puede ser visto como una varia-- ble distribuída en el tiempo (espacio de una dimensión).

(ii) El espesor de un estrato puede considerarse como una variable - distribuída en dos dimensiones.

(iii) El porcentaje de mineral en un bloque de volumen V de un cierto depósito podría ser caracterizado por una distribución en tres - dimensiones.

Al conjunto de v.a.r. $\{ Z(x_1), \ldots, Z(x_n) \}$, representado - de ahora en adelante como $Z(x)$, se le conoce como "función aleatoria" - (en adelante f.a.), y se distingue (como toda variable aleatoria) por tener asociada una función acumulativa de distribución conjunta.

$$F_{Z(x)}(\cdot) = F_{Z(x_1), \ldots, Z(x_n)}(z(x_1), \ldots, z(x_n)) =$$
$$= P_r \{ Z(x_1) \leq z(x_1), \ldots, Z(x_n) \leq z(x_n) \}$$

Nota: Ambos aspectos, el estructural y el aleatorio se encuentran im-- plícitos en las funciones aleatorias.

De la misma manera en que los operadores $E[\ ]$, $Var[\ ]$, etc., definidos en la sección anterior, se aplicaron a variables aleatorias, ellos también pueden aplicarse a variables aleatorias regionalizadas. - Consideremos la v.a.r. $Z(x_i)$, en el punto x_i de la región $\mathbb{R}$:

(i) $E\left[Z(x_i)\right]$.- Si la función de distribución de $Z(x_i)$ tiene media (o esperanza), entonces esta media es una función de x_i y se escribe: $E\left[Z(x_i)\right] = m(x_i)$.

(ii) $\text{Var}\left[Z(x_i)\right]$.- Si la variancia de la v.a.r. $Z(x_i)$ existe entonces se define como el momento central de segundo orden, y se escribe:

$$\text{Var}\left[Z(x_i)\right] = E\left[\left\{Z(x_i)-m(x_i)\right\}^2\right]$$

Al igual que la media de $Z(x_i)$, la variancia es generalmente una función de x_i.

(iii) $\text{Cov}\left[Z(x_i),\ Z(x_j)\right]$.- Si las variancias de los v.a.r. $Z(x_i)$ y $Z(x_j)$ existen, entonces la covariancia de las dos variables aleatorias también existe y es una función de las dos localizaciones x_i y x_j.

$$\text{Cov}\left[Z(x_i),\ Z(x_j)\right] = E\left[\left\{Z(x_i) - m(x_i)\right\}\left\{Z(x_j) - m(x_j)\right\}\right]$$

(iv) Variograma $2\gamma\left[Z(x_i),\ Z(x_j)\right]$.- La función variograma se define como la variancia del incremento de dos variables aleatorias regionalizadas, es decir:

$$2\gamma\left[Z(x_i),\ Z(x_j)\right] = 2\gamma(x_i, x_j) \triangleq \text{Var}\left[Z(x_i) - Z(x_j)\right]$$

La función $\gamma(x_i, x_j)$ se denomina entonces, semi-variograma. El variograma también puede definirse como aquella función que expresa la estructura de intercorrelación de una variable aleatoria regionalizada.

Es conveniente hacer notar que los conceptos introducidos en esta sección, han sido presentados como una extensión natural de nociones básicas de la estadística. Estos conceptos constituyen el lenguaje de la geoestadística.

Hipótesis de la Geoestadística.- Volvamos a la figura 5-B y pensemos por un momento si sería imposible inferir estadísticamente la distribución $F_{Z(x_i)}(\cdot)$ de la v.a.r. $Z(x_i)$ en el punto x_i, contando únicamente con la realización $z(x_i)$. Obviamente, esto no es posible como tampoco es posible determinar la distribución de la variable aleatoria, "resultado de echar un volado con una moneda", con sólo una realización. Inferir la forma de $F_{Z(x_i)}(\cdot)$ requeriría contar con muchas realizaciones $z_1(x_i), z_2(x_i), \ldots, z_m(x_i)$ de la v.a.r. $Z(x_i)$. Dado que en la práctica estamos limitados a contar con una realización de la v.a.r. $Z(x_i)$, en el punto x_i, parece ser como si nos encontráramos en un camino sin salida, es entonces cuando resulta necesario adoptar ciertas hipótesis.

Cuatro diferentes hipótesis relacionadas con la función aleatoria $Z(x)$ pueden adoptarse:

(i) Estacionariedad estricta.- Una f.a. $Z(x)$ se dice que es estacionaria, en el sentido estricto, si su función acumulativa de distribución conjunta $F_{Z(x)}(\cdot)$ permanece constante bajo efectos de translación. En otras palabras, la función acumulativa de distribución conjunta $F_{Z(x)}(\cdot) = F_{Z(x_1), Z(x_2), \ldots, Z(x_n)}(z(x_1),$ -

$z(x_2), \ldots, z(x_n)$) de las v.a.r. $Z(x_1), \ldots, Z(x_n)$, será idéntica a la f.a.d.c. $F_{Z(x+h)} = F_{Z(x_1+h), \ldots, Z(x_n+h)}(z(x_1+h), \ldots, z(x_n+h))$ de las v.a.r. $Z(x_1+h), \ldots, Z(x_n+h)$, para cualquier vector de translación h. Esta hipótesis equivaldría a considerar como iguales a todos los momentos de los conjuntos de v.a.r. $\{Z(x_1), \ldots, Z(x_n)\}$ y $\{Z(x_1+h), \ldots, Z(x_n+h)\}$, para cualquier valor h.

La hipótesis se ilustra gráficamente en las figuras 6-A y 6-B para el caso de dos variables aleatorias regionalizadas, $\{Z(x_1), Z(x_2)\}$.

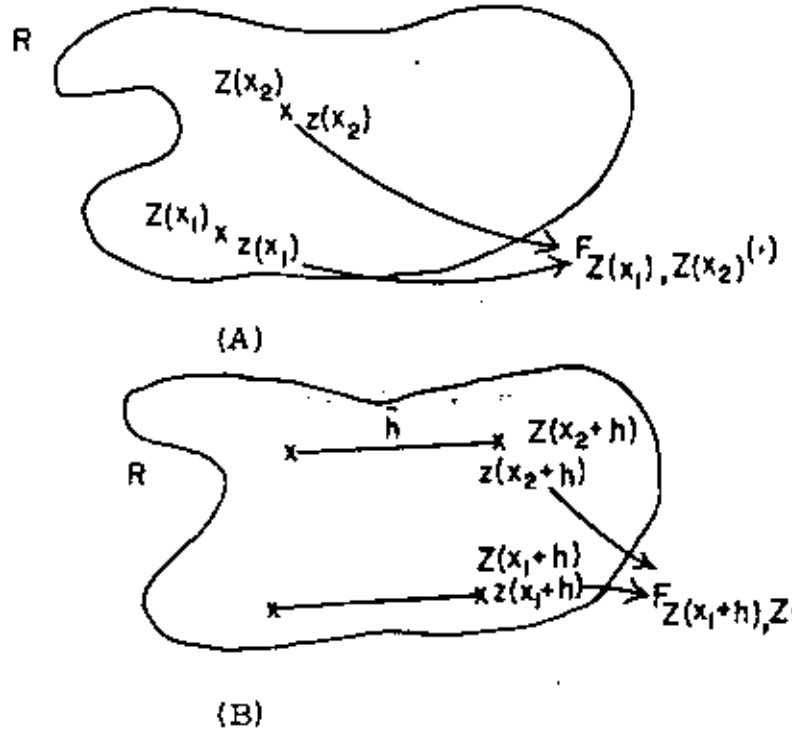


Figura 6. Distribuciones $F_{Z(x_1), Z(x_2)}(\cdot)$ y $F_{Z(x_1+h), Z(x_2+h)}(\cdot)$ son idénticas bajo efectos de translación.

(ii) Estacionaridad de segundo orden.- Una f.a. $Z(x)$ se dice ser estacionaria de segundo orden cuando:

(a) $E\left[Z(x_i)\right]$ existe y no depende del punto o soporte x_i

$E\left[Z(x_i)\right] = m$ (constante), $\forall\, x_i$

(b) Para cada par de v.a.r. $\left\{Z(x_i),\ Z(x_i+h)\right\}$ la covariancia existe y sólo depende del vector de traslación h.

$$C(h) = \text{Cov}\left[Z(x_i),\ Z(x_i+h)\right] = E\left[\left\{Z(x_i) - E\left[Z(x_i)\right]\right\} \cdot \left\{Z(x_i+h) - E\left[Z(x_i+h)\right]\right\}\right] = E\left[\left\{Z(x_i) - m\right\}\left\{Z(x_i+h) - m\right\}\right]$$

$$C(h) = E\left[Z(x_i) \cdot Z(x_i+h)\right] - m^2, \quad \forall\, x_i$$

Estacionaridad de la covariancia implica estacionaridad de la variancia y del variograma.

$$\text{Var}\left[Z(x_i)\right] = E\left[\left(Z(x_i) - m\right)^2\right] = C(o), \quad \forall\, x_i$$

$$\gamma(h) = 1/2\ E\left[\left(Z(x_i) - m - \left\{Z(x_i+h) - m\right\}\right)^2\right]$$

$$= 1/2\ E\left[Z^2(x_i) - 2Z(x_i)\,Z(x_i+h) + Z^2(x_i+h)\right]$$

$$= 1/2\ E\left[Z^2(x_i)\right] - E\left[Z(x_i)\,Z(x_i+h)\right] + 1/2\ E\left[Z^2(x_i+h)\right]$$

$$\gamma(h) = C(o) - C(h), \quad \forall\, x_i.$$

Como se observa en la última expresión, $\gamma(h)$ y $C(h)$, son dos herramientas que permiten expresar la correlación entre las dos v.a.r. $Z(x_i)$ y $Z(x_i+h)$, separadas por el vector h. Estacionaridad equivale a considerar, por ejemplo, una mineralización homogénea, donde la correlación entre los datos $z(x_i)$ y $z(x_i')$ no depende de su posición particular, sino de la distancia que los separa.

Nota: Esta hipótesis asume la existencia de la covariancia C(h) - y por lo tanto de la variancia $\text{Var}\left[Z(x_i)\right] = C(o) < \infty$.

(iii) Hipótesis intrínseca.- Una f.a. Z(x) se dice ser intrínseca si:

(a) Su media existe y no depende del punto x_i.

$$E\left[Z(x_i)\right] = m, \quad \forall\, x_i$$

(b) Para todo vector h, el incremento $Z(x_i) - Z(x_i+h)$ tiene - variancia finita (o existe) y no depende del punto x_i.

$$\text{Var}\left[Z(x_i)-Z(x_i+h)\right] = E\left[(Z(x_i)-Z(x_i+h))^2\right] =$$

$$= 2\,\gamma(h), \quad \forall\, x_i.$$

La existencia de la función variograma representa una hipótesis - más fácil de satisfacer que la existencia de la covariancia. Muchos fenómenos físicos (Caso-Tiempos de Reflexión) presentan una capacidad infinita de dispersión, donde ambas la variancia y la - covariancia no existen, pero para las cuales es posible definir - una función variograma.

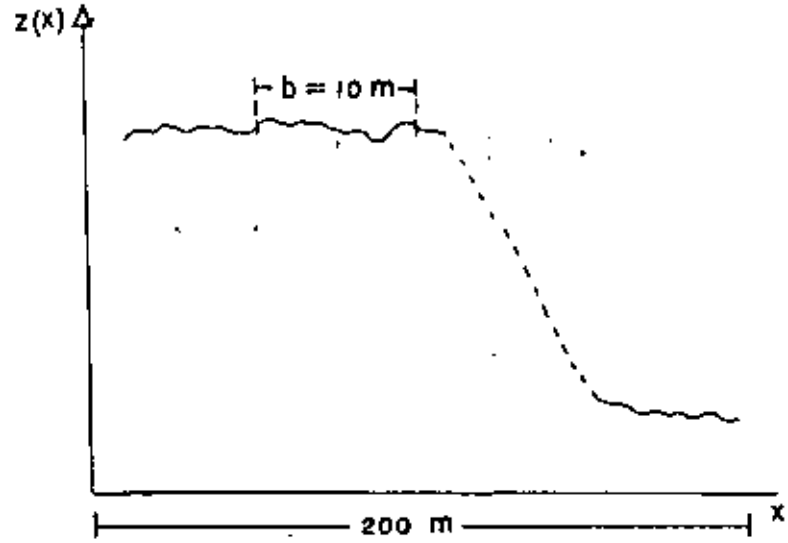
Por lo tanto, estacionaridad de segundo orden implica la hipótesis intrínseca, pero no lo contrario. La función variograma puede - expresarse en términos de la covariancia, y no así esta última en función del variograma.

$$\gamma(h) = C(o) - C(h)$$

$$C(h) \neq f(\gamma(h))$$

(iv) Quasi - estacionaridad.- Esta es la limitación de la hipótesis de estacionaridad de segundo orden (o de la hipótesis intrínseca, si -

sólo la función $\gamma(h)$ es asumida) a distancias $|h| \leq b$. En la práctica, el límite b puede representar la extensión de una zona homogénea, o el diámetro de la zona considerada para propósitos de estimación (Figura 7).



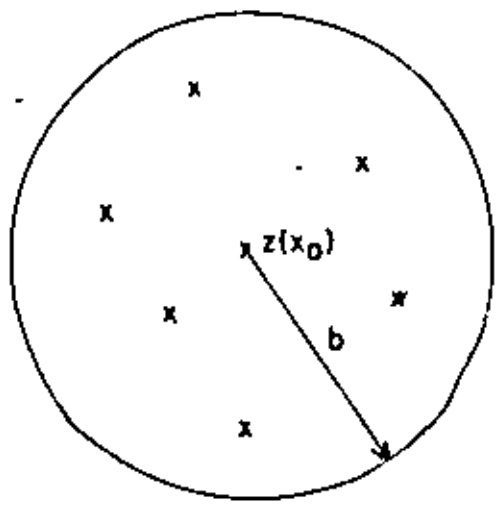


Figura 7. Vecindad de "quasi" estacionaridad.

Nota: En lo que resta de este curso asumiremos la hipótesis de estacionariedad de segundo orden en todas las funciones aleatorias $Z(x)$.

- CAPITULO II -

VARIOGRAMA

Anatomía del variograma.

La definición del variograma como la variancia de la diferencia de dos variables aleatorias regionalizadas sugiere las siguientes propiedades:

$\gamma(o) = 0$

$\gamma(h) = \gamma(-h)$ el variograma es una función par. Con el objeto de entender el comportamiento de la función $\gamma(h)$ es necesario observar primero el comportamiento de la función $C(h)$. Intuitivamente sabemos que el grado de correlación entre las variables $Z(x_i)$ y $Z(x_i+h)$ generalmente decrece a medida que la distancia que las separa se incrementa. De acuerdo con esto, y observando la relación

$$\gamma(h) = C(o) - C(h)$$

podemos deducir que la función $\gamma(h)$ debe de incrementarse con h (ver figura 8).

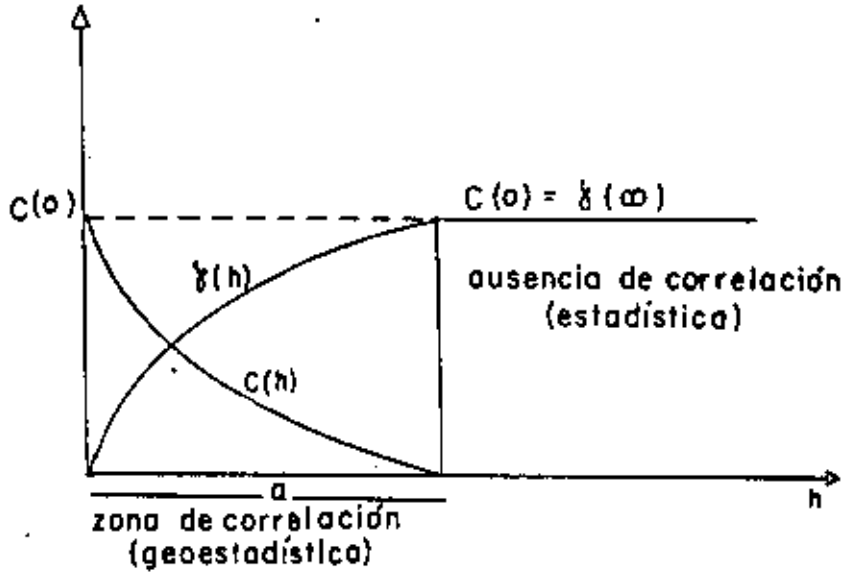


Figura 8

La distancia "a" a partir de la cual C(h) resulta prácticamente igual a cero, se denomina "rango" y representa el punto de transición entre el estado en el cual existe correlación (o zona de influencia) $|h| \leq a$ y el estado en el cual hay ausencia de correlación $|h| \geq a$.

A medida que h crece, la función $\gamma(h)$ resulta, generalmente, más o menos estable alrededor de un límite llamado "meseta" (o sill, en inglés), el cual es simplemente la variancia de la v.a.r. $Z(x_i)$

$$\gamma(\infty) = \text{Var}\left[Z(x_i)\right] = C(o) - C(\infty)$$

Aquellos variogramas caracterizados por un rango "a" y una meseta C(o) se les conoce como "modelos de transición", ya que permiten identificar las zonas de transición definidas anteriormente

Estrictamente hablando, y dado que h representa un vector, la función $\gamma(h)$ denota al conjunto de semi-variogramas $\gamma(|h|, \alpha)$, obtenido al hacer variar el ángulo dirección α. Estudiando $\gamma(h)$ en varias direcciones α, es probable determinar la existencia de posibles anisotropías, tales como el cambio de rango $a(\alpha)$ con la dirección α. Consideremos el ejemplo de las lentes mineralizadas de la figura 9, donde:

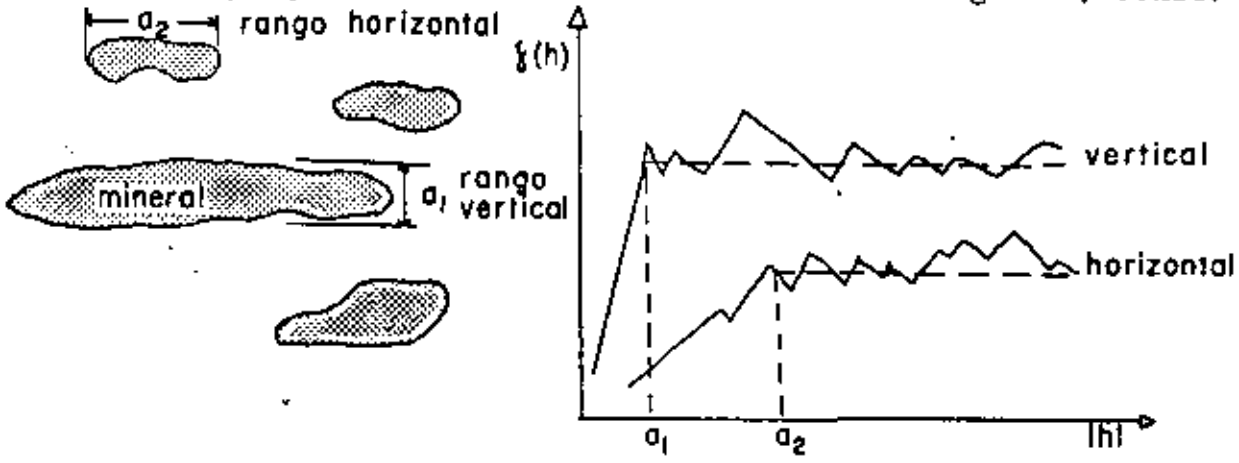


Figura 9 - Variograma como herramienta estructural.

el semi-variograma calculado en la dirección vertical muestra un rango a_1, menor al rango a_2 del semi-variograma calculado en la dirección horizontal. Los rangos a_1 y a_2 pueden interpretarse como el valor medio del ancho y el valor medio del largo de las lentes mineralizadas, respectivamente. El rango del variograma representa, en promedio, características morfológicas de lentes mineralizadas.

Comportamiento del variograma al origen.- El comportamiento del variograma cerca del origen está relacionado con la continuidad y la regularización de la f.a. Z(x). Cuatro diferentes comportamientos pueden observarse:

(i) Parabólico - $\gamma(h) \sim A|h|^2$, comportamiento característico de variabilidades sumamente regulares.

(ii) Lineal - $\gamma(h) \sim A|h|$

(iii) Discontinuo al origen - $\gamma(h)$ no tiende hacia cero cuando $h \longrightarrow 0$, aunque por definición $\gamma(0) \triangleq 0$. La variabilidad entre dos valores $z(x)$ y $z(x+h)$ muy cercanos uno del otro puede ser muy alta e incrementarse con el grado de discontinuidad en el origen de $\gamma(h)$. Esta discontinuidad al origen de $\gamma(h)$ se denomina "efecto pepita" (o "nugget effect" en inglés) y puede deberse a mediciones erróneas y/o a micro-variabilidades.

iv) Efecto "pepita" puro - Este es el caso donde $\gamma(h)$ aparece exclusivamente como una discontinuidad en el origen, $\gamma(0) = 0$ y $\gamma(h) = Co$, para $h > \epsilon$.

Este efecto corresponde exclusivamente al estado total de ausencia de correlación

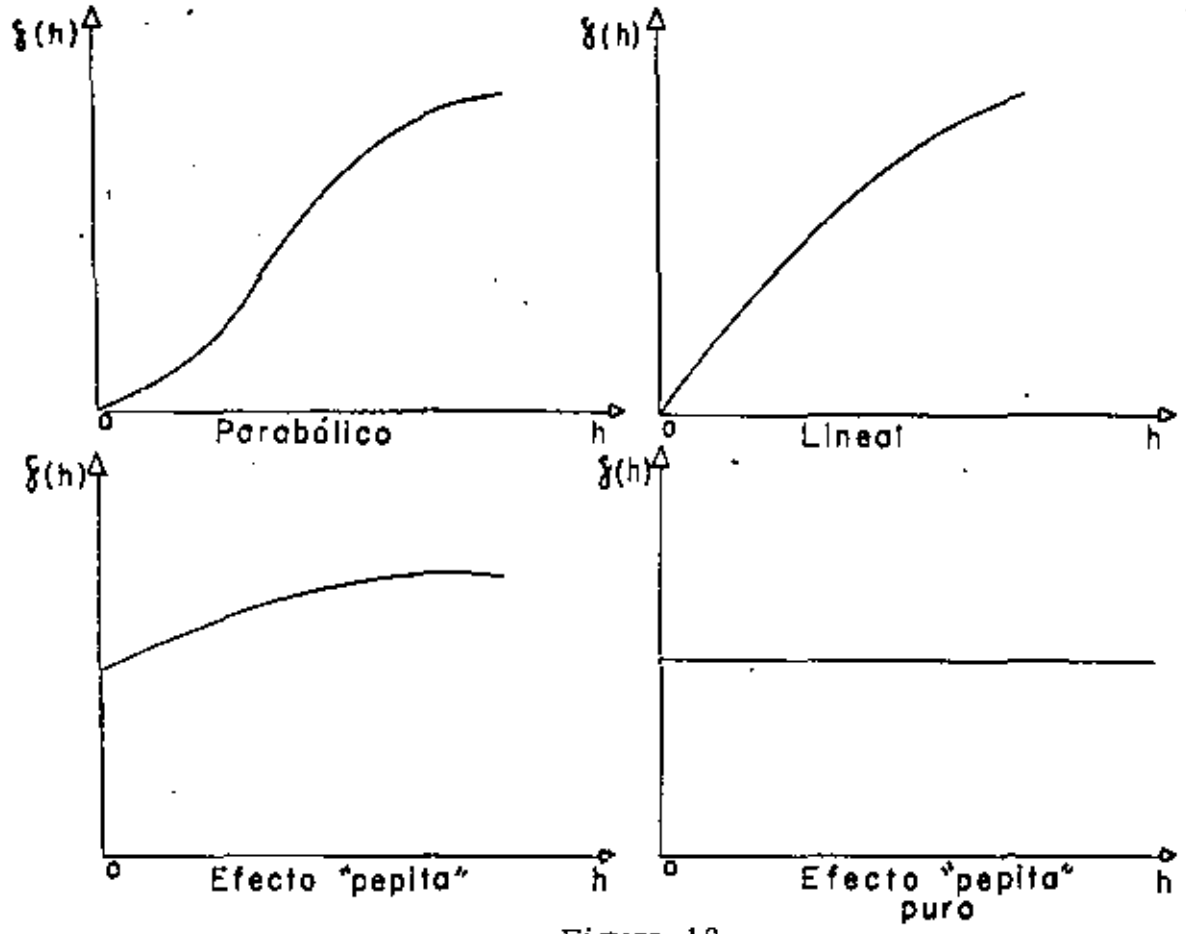


Figura 10

Orígenes de variabilidad.- La variabilidad entre las v.a.r. $Z(x_i)$ y $Z(x_i+h)$, representada por el variograma tiene diversos orígenes los cuales están íntimamente ligados a las diversas escalas de medida, por ejemplo:

(i) A nivel de punto ($h \approx 0$) existe una variabilidad causada por errores en las mediciones mismas.

(ii) A nivel petrográfico ($|h| < 1$ cm) una segunda variabilidad puede existir producida por transición de un elemento a otro.

(iii) A nivel de estratos o lentes mineralizadas ($|h| < 100$ m) una tercera variabilidad aparece debida a alteraciones de los estratos o de los lentes con material de desperdicio.

(iv) A nivel de cuenca geológica o de provincia metalífera ($|h| < 100$ km) una cuarta variabilidad surge como consecuencia de la distribución de los yacimientos a partir de la orogénesis de la provincia.

Todas estas fuentes o estructuras de variabilidad y posiblemente muchas más, actúan simultáneamente y para cualquier distancia h, y por ello se les llama "estructuras anidadas".

Observar simultáneamente todas estas variabilidades requeriría contar con una gran cantidad de información cubriendo todos los rangos de variabilidad, desde uno hasta 100 km., lo cual en la práctica, nunca ocurre.

Bajo la hipótesis de estacionaridad de segundo orden, las estructuras anidadas pueden ser representadas como la suma de un cierto número de semi-variogramas (o de covarianclas), cada uno caracterizando una variabilidad a una cierta escala (ver figura 11).

$$\gamma(h) = \gamma_o(h) + \gamma_1(h) + \gamma_2(h) + \ldots + \gamma_n(h)$$

Modelos de variogramas.- A continuación se presentan las principales funciones empleadas en la representación de variogramas. No cualquier función f(h) puede ser una función-variograma. Unicamente aquellas funciones "definidas positiva y condicionalmente" (Matheron, 1971) pueden emplearse como variogramas.

Como se mencionó anteriormente, las dos características principales del semi-variograma $\gamma(h)$ ...

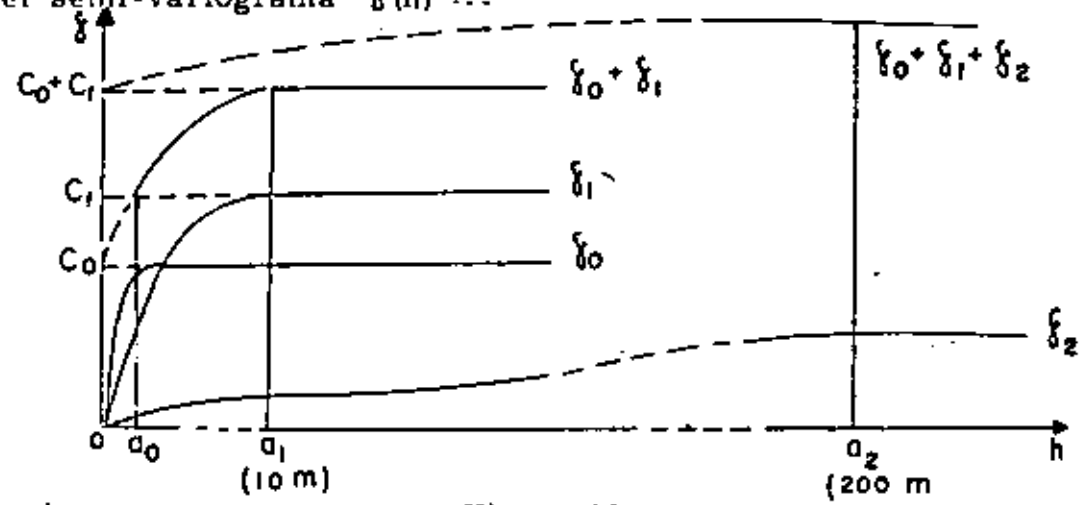


Figura 11
Estructuras Anidadas

son su comportamiento al origen (parabólico, lineal y nugget effect) y la presencia o ausencia de una meseta o sill para valores de $|h| \geq a$. - Atendiendo a estas características, los modelos teóricos más comúnmente empleados pueden clasificarse como:

(i) Modelos con meseta (o modelos de transición) y comportamiento - lineal al origen:

(a) Modelo esférico

(b) Modelo exponencial

y comportamiento parabólico al origen:

(c) Modelo Gaussiano

(ii) Modelo sin meseta (la función aleatoria correspondiente será intrínseca donde ni la variancia, ni la covariancia existen).

(a) Modelos de forma $|h|^{\Theta}$, con $\Theta \in (0, 2)$

(b) Modelo logarítmico

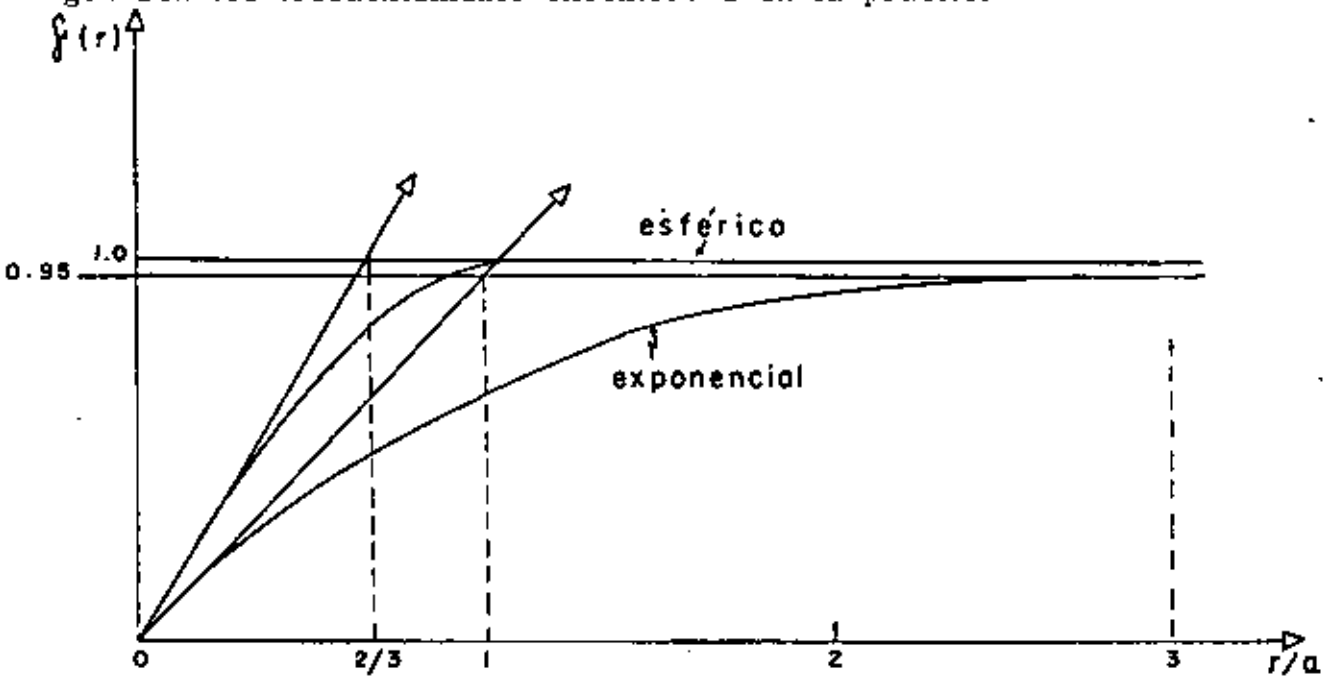
Por el momento consideraremos únicamente modelos isotrópicos, - esto es, aquéllos cuyas funciones aleatorias $Z(x)$ presentan la misma - variabilidad espacial en toda dirección.

Los modelos que a continuación se presentan están normalizados, - es decir, corresponden a f.a. $Z(x)$ con variancia $\mathrm{Var}\left[Z(x)\right] = 1$. Para obtener modelos con "sill" $C(o) = C \neq 1$ bastará multiplicar las expresiones dadas de $\gamma(h)$ por C.

Modelo esférico $$\gamma(r) = \begin{cases} 3/2\ (r/a) - 1/2\ (r/a)^3, & \forall_r \in [0, a] \\ 1 = \text{sill}, & \forall\ r \geq a \end{cases}$$

Modelo exponencial $$\gamma(r) = 1 - e^{-r/a}$$

Estos dos modelos de variogramas con comportamiento lineal al - origen son los más frecuentemente encontrados en la práctica.



La diferencia entre los modelos esférico y exponencial es la distancia (r) a la cual sus tangentes al origen intersectan el sill C(0); $r = \frac{2a}{3}$ dos tercios del rango a, para el modelo esférico; $r = \frac{a}{3}$ - un tercio del rango a, para el modelo experimental. Estos modelos - presentan frecuentemente "nugget-effect".

Modelo Gaussiano $\gamma(r) = 1 - e^{-(r/a)^2}$

Este modelo de comportamiento parabólico cerca del origen es - raramente encontrado en aplicaciones prácticas.

Los siguientes modelos corresponden a f.a. Z(x) con capacidad - ilimitada de dispersión y esto es, Z(x) es intrínseco.

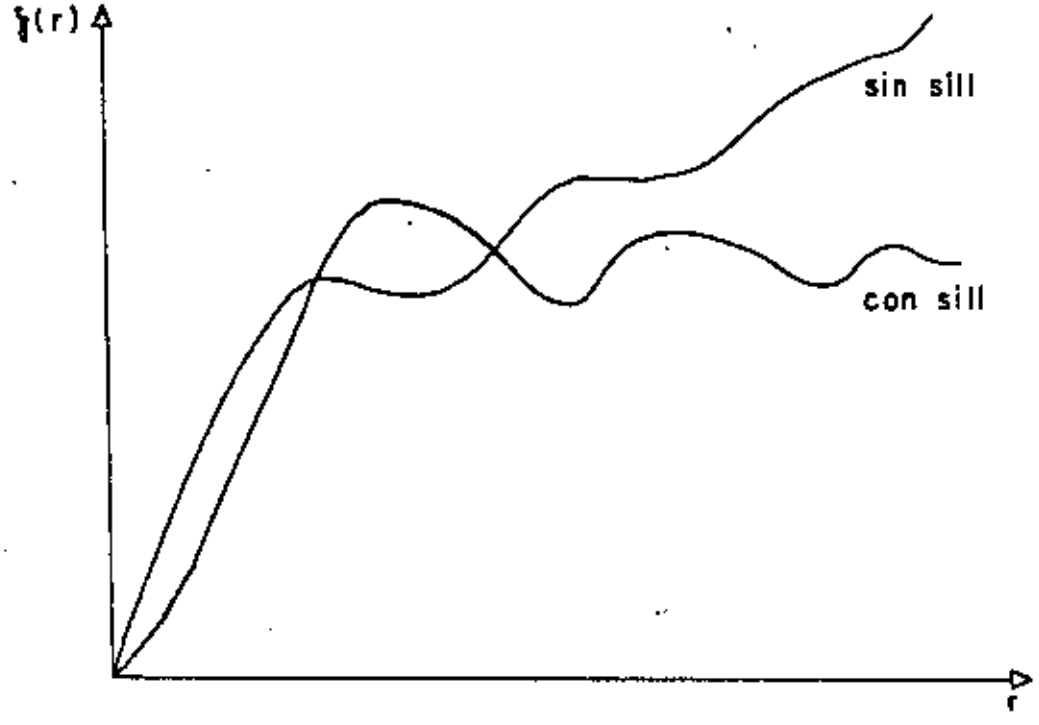
Modelo del tipo r^θ $\quad \gamma(r) = r^\theta$, $\theta \in (0, 2)$

Modelo logarítmico $\quad \gamma(r) = \log r$.

El modelo logarítmico o modelo De Wijs fue aplicado extensamente durante los años 60's. Sus características analíticas así como el hecho de que las primeras aplicaciones de la geoestadística a diferentes minerales (oro y uranio principalmente) produjeron variogramas sin sill, lo - hicieron "popular".

Modelo con efecto de agujero

Un semi-variograma presenta "el efecto de agujero" cuando su crecimiento no es monótono. Este efecto puede aparecer en modelos con o sin sill. Un modelo con efecto de agujero, con sill y con comportamiento parabólico al origen tendría la siguiente forma: $\gamma(r) = 1 - \frac{\text{sen } r}{r}$



Este comportamiento se ha observado frecuentemente en aquellos depósitos mineros donde existe una sucesión de zonas ricas y pobres.

Fenómenos anisotrópicos.- Un fenómeno anisotrópico se caracteriza por presentar diversas variabilidades en cada dirección α. La función estructural $\gamma(h) = \gamma(|h|, \alpha)$ depende entonces de la dirección α y del módulo $|h|$. Cuando la función $\gamma(|h|, \alpha)$ depende únicamente del módulo $|h|$, el fenómeno se nombra isotrópico.

En la práctica, el carácter anisotrópico de la función $\gamma(h)$ se manifiesta por la existencia de direcciones preferenciales al momento de la creación del fenómeno estudiado. Estas direcciones preferenciales son conocidas generalmente, de antemano, como por ejemplo, la dirección vertical en un depósito formado por depositación deltáica.

Los modelos isotrópicos presentados en la versión anterior, dependen exclusivamente del módulo $r = |h|$ del vector h. En esta versión, los modelos anisotrópicos serán presentados empleando el método de "reducción" al caso isotrópico. Para el caso de "anisotropía geométrica", la reducción se hará por medio de una transformación lineal y para el caso de "anisotropía zonal", la reducción se hará por separación de cada variabilidad direccional.

(i) Anisotropía Geométrica.

Un semi-variograma $\gamma(h) = \gamma(h_u, h_v)$ presenta anisotropía geométrica, cuando la anisotropía puede ser reducida a isotropía aplicando una transformación lineal a las coordenadas:

$$\underset{\text{anisotrópico}}{\gamma(h_u, h_v)} = \underset{\text{isotrópico}}{\gamma'(h_u', h_v')} = \gamma'(|h'|)$$

donde $h_u' = a_{11} h_u + a_{12} h_v$

$h_v' = a_{21} h_u + a_{22} h_v$

o en forma matricial $h' = Ah$ donde A representa la matriz de transformación.

Como ejemplo consideremos los semivariogramas γ_{α_1} y γ_{α_2} de la figura 12, calculados en las direcciones α_1 y α_2, en el espacio de dos dimensiones. Los dos semi-variogramas han sido representados por modelos esféricos con sill igual a 1 y rangos iguales a a_{α_1} y a_{α_2}, dado que $\gamma_{\alpha_1} \neq \gamma_{\alpha_2}$, el fenómeno representado es anisotrópico.

Con el objeto de hacer coincidir las dos curvas, basta multiplicar la distancia a lo largo de la dirección α_1, h_1 por el radio de afinidad - $a_{\alpha 2} / a_{\alpha_1}$ o alternativamente, multiplicar la distancia a lo largo de - la dirección α_2, h_2 por $a_{\alpha_1} / a_{\alpha 2}$. En efecto, para los dos - modelos esféricos:

$$\gamma_{\alpha_1}(h_1) = \frac{3}{2a_{\alpha_1}} h_1 - \frac{1}{2a_{\alpha_1}^3} h_1^3, \quad \forall\, h_1 \leq a_{\alpha_1}$$

y

$$\gamma_{\alpha_2}(h_2) = \frac{3}{2a_{\alpha_2}} h_2 - \frac{1}{2a_{\alpha_2}^3} h_2^3, \quad \forall\, h_2 \leq a_{\alpha_2}$$

se obtendría lo siguiente:

$$\gamma_{\alpha_1}(h_1) = \frac{3}{2\ \alpha_2}\left(h_1 \frac{a_{\alpha 2}}{a_{\alpha_1}}\right) - \frac{1}{2a_{\alpha_2}^3}\left(h_1 \frac{a_{\alpha_2}}{a_{\alpha_1}}\right)^3, \forall\, h_1 \frac{a_{\alpha_2}}{a_{\alpha_1}} \leq a_{\alpha_2}$$

ó

$$\gamma_{\alpha_1}(h_1) = \gamma_{\alpha_2}(h_1') \quad \text{donde} \quad h_1' = h_1 \frac{a_{\alpha_2}}{a_{\alpha_1}}$$

El cambio de coordenadas $h_1' = h_1 \frac{a_{\alpha 2}}{a_{\alpha_1}}$ permite caracterizar la variabilidad en las direcciones α_1 y α_2 por medio de un sólo modelo esférico con rango a_{α_2}.

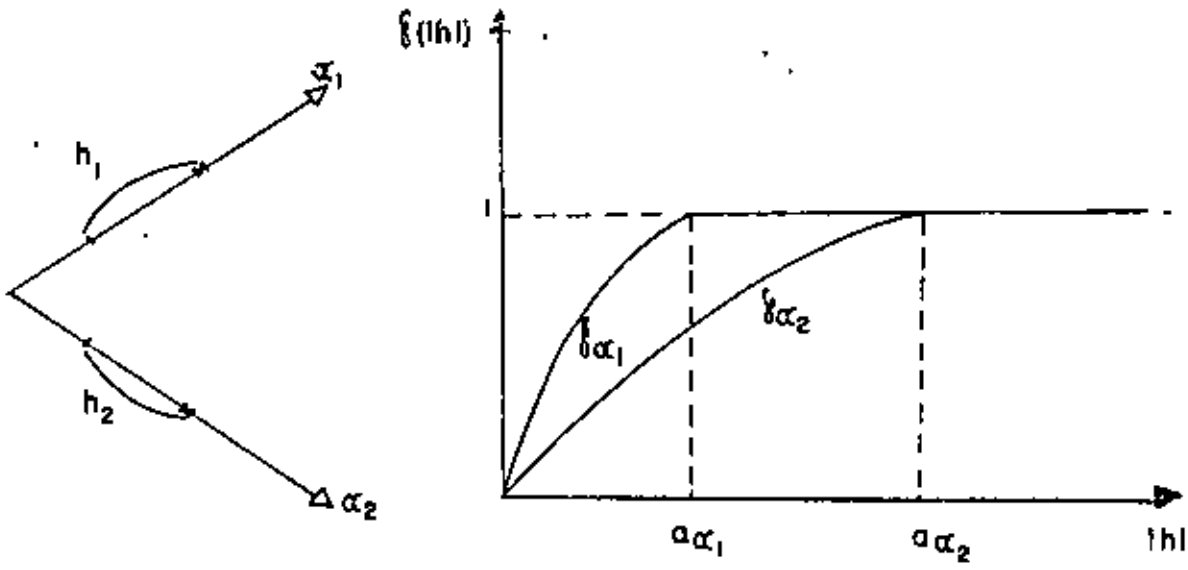


Figura 12 - Anisotropía Geométrica.

Considerando n direcciones $\alpha_1, \ldots, \alpha_n$, bastaría con graficar los rangos de los variogramas en función de sus direcciones para deducir la posible existencia de una transformación lineal. Si la gráfica pudiera aproximarse a una elipse, entonces una transformación lineal existiría y permitiría transformar la elipse en círculo lo cual equivaldría a considerar un fenómeno isotrópico. En caso contrario, el modelo de anisotropía zonal sería adoptado.

(ii) Anisotropía Zonal

El modelo de anisotropía zonal consiste en definir al semi-variograma asociado al fenómeno en estudio como una estructura anidada, es decir, como la suma de otros semi-variogramas donde cada semi-variograma se permite exhibir su propia anisotropía.

$$\gamma(h) = \sum_{i=1}^{n} \gamma_i(h)$$

Por ejemplo, el semi-variograma $\gamma_j(h)$, (h-vector) puede representar -

un fenómeno con anisotropía geométrica, el semi-variograma $\gamma_Q(h)$ - puede ser isotrópico en 2 dimensiones $\gamma(h_u, h_v) = \gamma\left(\sqrt{h_u^2 + h_v^2}\right) = \gamma(|h|)$, o bien el semi-variograma $\gamma_K(h)$ puede depender únicamente de la - distancia h_u, $\gamma_K(h) = \gamma_K(h_u)$, $\forall\, h_v$.

El modelo de anisotropía zonal es el más usado en la práctica, ya que por su flexibilidad puede ajustarse a cualquier tipo de anisotropía.

La práctica del análisis estructural.- Antes de iniciar un estudio - geoestadístico es recomendable familiarizarse con ambos, la naturaleza - del fenómeno que se estudia (geología, petrografía, técnicas de operación, presupuesto) y con los datos disponibles. Esta fase preliminar es esencial tanto en el análisis estructural como en la formulación correcta del problema. Por tal motivo, siempre es deseable poder llevar a cabo un análisis estadístico elemental de la información. Histogramas y diagra-- mas de correlación, entre otras muchas técnicas, ayudan en la detección de datos erróneamente muestreados. Esto es de especial importancia - ya que cualquier error en los datos se reflejaría sistemáticamente en - cada etapa del análisis geoestadístico.

Una parte fundamental de la formulación del problema la constituye la definición de la variable aleatoria regionalizada, la cual comprende los siguientes aspectos:

(i) Significado de los datos o de la variable que se estudia.

(ii) Soporte o volumen en el cual la variable está definida. Para un - ingeniero petrolero, el soporte sería por ejemplo un núcleo de -

roca al cual se le ha determinado sus características petrográfi- cas, porosidad, permeabilidad, etc.

Para un ingeniero minero, el soporte sería el volumen de roca - donde el porcentaje medio de un mineral se desea estimar.

(iii) Extensión o dominio del campo sobre el cual la distribución espacial de la variable se definirá. Esta extensión puede cubrir una cuenca geológica, un yacimiento petrolero o una zona de mineralización primaria.

Cálculo del variograma.- Analizados los datos y definida la variable procedemos al cálculo del variograma. Recordemos que el variograma ha sido definido como la variancia de la diferencia de dos variables - aleatorias regionalizadas, estando distanciadas una de la otra

$$2\gamma(h) = \mathrm{Var}\left[Z(x+h) - Z(x) \right]$$

un vector h. Un semi-variograma calculado a partir de datos experimentales $\gamma^*(h)$ podría estar dado por la siguiente fórmula:

$$\gamma^*(h) = 1/2\,N' \sum_{i=1}^{N'} \left[z(x_i+h) - z(x_i) \right]^2$$ donde N' representa el número de pares de datos separados por el vector h.

En la práctica, sabemos que los datos pueden estar distribuídos - en 1, 2 y 3 dimensiones, y que además pueden presentarse regular o - irregularmente espaciados. También sabemos que la confiabilidad en el - variograma como función estructural será mayor cuanto mayor sea el - número de datos disponibles. No es sorprendente, en algunos casos, - emplear datos del orden de los cientos o de los miles, lo cual de no ser

insergado y que además, paralelamente al caso discreto, la variancia de estimación estará dada por:

$$\boxed{\sigma_E^2 = \overline{C}(V, V) + \overline{C}(v, v) - 2\,\overline{C}(V, v)} \qquad (2)$$

Empleando la relación $C(h) = C(0) - \gamma(h)$, la expresión anterior resulta:

$$\boxed{\sigma_E^2 = 2\,\overline{\gamma}(V, v) - \overline{\gamma}(V, V) - \overline{\gamma}(v, v)} \qquad (3)$$

donde $\overline{\gamma}(V, v)$, por ejemplo, representa el valor medio de $\gamma(h)$ cuando un extremo del vector h describe el dominio V(x) y el otro extremo describe independientemente el dominio v(x').

CASO HIBRIDO

En este caso se trata de estimar el valor medio $z_v(x)$ de un bloque de volumen V por medio de una combinación lineal z_K^* de n datos conocidos $\{z(x_i'),\ i=1, \ldots, n\}$. En términos de variables aleatorias regionalizadas tenemos:

$$Z_V(x) = 1/V \int_{V(x)} Z(y)dy, \quad Z_K^* = \sum_{i=1}^{n} \lambda_i\, Z(x_i')$$

Z_K^* será insesgado si la condición $\sum_{i=1}^{n} \lambda_i = 1$ se cumple ya que $E[Z_V] = m$ y $E[Z_K^*] = E\left[\sum_{i=1}^{n} \lambda_i\, Z(x_i)\right] = m \sum_{i=1}^{n} \lambda_i$

Procediendo análogamente a los dos casos anteriores

$$\sigma_E^2 = E\left[(Z_V - Z_K^*)^2\right]$$

$$\sigma_E^2 = 2\sum_{i=1}^{n} \lambda_i\, \overline{\gamma}(x_i, V) - \overline{\gamma}(V, V) - \sum_{i=1}^{n}\sum_{\ell=1}^{n} \overline{\gamma}(x_i - x_\ell) \qquad (4)$$

Sustituyendo en la expresión de σ_E^2 obtenemos:

$$\sigma_E^2 = 1/K^2 \sum_{j=1}^{K}\sum_{\ell=1}^{K} C(x_j - x_\ell) + 1/n^2 \sum_{i=1}^{n}\sum_{\ell=1}^{n} C(x_i' - x_\ell') - 2/Kn \sum_{j=1}^{K}\sum_{i=1}^{n} C(x_j - x_i')$$

Denotando por $\overline{C}$ (K, n) al valor medio de la covariancia C(h) cuando un extremo del vector h describe al conjunto de puntos $\{z(x_j),\ j=1,\ \ldots,\ K\}$ y el otro extremo describe independientemente al conjunto de puntos $\{z(x_i'),\ i=1,\ \ldots,\ n\}$, es decir,

$$\overline{C}\ (K,\ n) = 1/Kn \sum_{j=1}^{K}\sum_{i=1}^{n} C(x_j - x_i'),$$

entonces la expresión anterior puede escribirse como:

$$\boxed{\sigma_E^2 = \overline{C}\ (K,\ K) + \overline{C}\ (n,\ n) - 2\overline{C}\ (K,\ n)} \qquad (1)$$

CASO CONTINUO

Consideremos ahora a los K puntos x_j localizados dentro del volumen V con centro en el punto x, y a los n puntos x_i' dentro del volumen v con centro en el punto x'. Si K y n tienden hacia infinito, entonces las medias aritméticas z_K y z_K^* tenderán hacia los valores medios en V y v de la variable puntual z(y), esto es:

$$z_k \longrightarrow z_V(x) = 1/V \int_{V(x)} z(y)\, d_y, \quad y \quad z_k^* \longrightarrow z_v(x') = 1/v \int_{v(x')} z(y)\, d_y$$

Los valores medios $z_V(x)$ y $z_v(x')$ son interpretados como realizaciones particulares de las dos v.a.r. $Z_V(x)$ y $Z_v(x')$. Bajo la hipótesis de estacionaridad de segundo orden, es fácil demostrar que $Z_v(x')$ es

El valor z_k^* se interpreta como una realización de la v.a.r. - Z_K^*, y el error desconocido $z_k - z_k^*$ denota una realización particular - de la v.a.r. $Z_K - Z_K^*$. Bajo la hipótesis de estacionaridad de segundo orden, la condición de "estimador insesgado" se cumple ya que:

$$E\left[Z_K\right] = E\left[1/K \sum_{j=1}^{K} Z(x_j)\right] = 1/K \sum_{j=1}^{K} E\left[Z(x_j)\right] = 1/K \sum_{j=1}^{K} m = m$$

$$E\left[Z_K^*\right] = E\left[1/n \sum_{i=1}^{n} Z(x_i')\right] = 1/n \sum_{i=1}^{n} E\left[Z(x_i')\right] = 1/n \sum_{i=1}^{n} m = m$$

lo que implica $E\left[Z_K - Z_K^*\right] = 0$

Para la variancia de estimación se tiene:

$$\sigma_E^2 = E\left[(Z_K - Z_K^*)^2\right] = E\left[Z_K^2\right] + E\left[Z_K^{*2}\right] - 2\,E\left[Z_K Z_K^*\right]$$

donde

$$E\left[Z_K^2\right] = E\left[\left(1/K \sum_{j=1}^{K} Z(x_j)\right)^2\right] = E\left[1/K^2 \sum_{j=1}^{K} \sum_{l=1}^{K} Z(x_j)\, Z(x_l)\right] =$$

$$= 1/K^2 \sum_{j=1}^{K} \sum_{l=1}^{K} E\left[Z(x_j)\, Z(x_l)\right] = 1/K^2 \sum_{j=1}^{K} \sum_{l=1}^{K} \left[C(x_j - x_l) + m^2\right]$$

dado que

$$C(h) = C(x-y) = E\left[Z(x)\, Z(y)\right] - m^2$$

Similarmente

$$E\left[Z_K^{*2}\right] = 1/n^2 \sum_{i=1}^{n} \sum_{l=1}^{n} \left[C(x_i' - x_j') + m^2\right], \quad y$$

$$E\left[Z_K\, Z_K^*\right] = 1/Kn \sum_{j=1}^{K} \sum_{i=1}^{n} \left[C(x_j - x_i') + m^2\right]$$

como se demostrará en seguida.

Sea Z(x) una función aleatoria y estacionaria de segundo orden, con media m, covariancia C(h) y semi-variograma $\gamma(h)$.

CASO DISCRETO.

Se desea estimar la media aritmética z_k de un conjunto de valores desconocidos $\{z(x_j),\ j=1,\ 1,\ \ldots,\ k\}$, es decir:

$$z_k = 1/k \sum_{j=1}^{k} z(x_j)$$

Se empleará como estimador lineal a z_k^*, definido como la media aritmética de un conjunto de datos conocidos $\{z(x_i'),\ i' = 1,\ 2,\ \ldots,\ n\}$ (Figura 17).

$$z_k^* = 1/n \sum_{i=1}^{n} z(x_i')$$

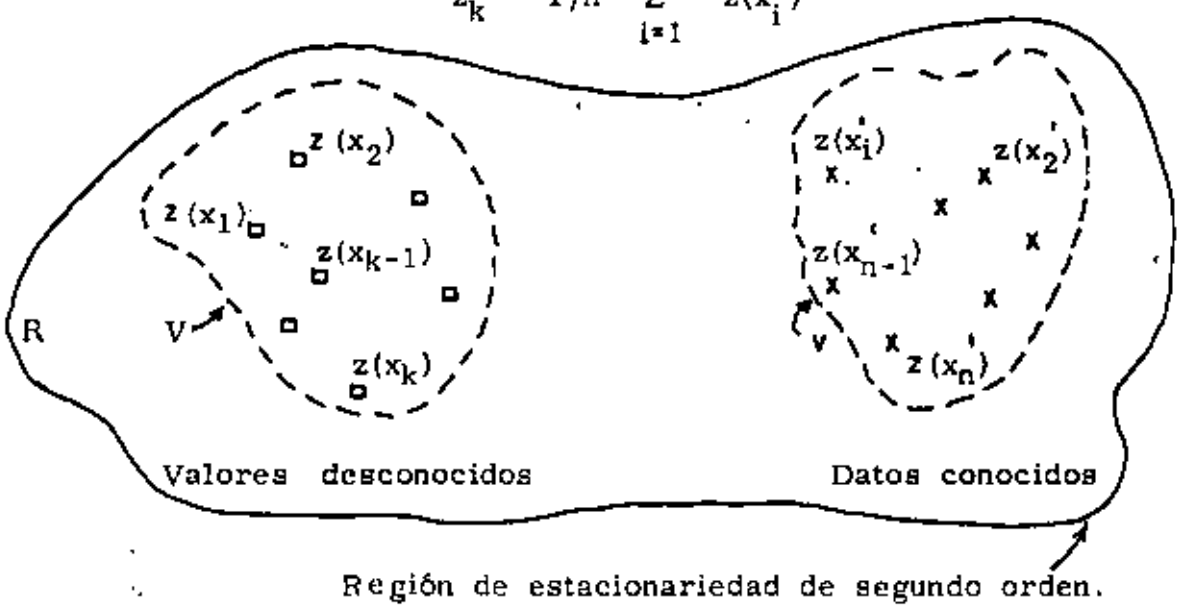


Figura 17

(o la función de distribución, si ésta es conocida) pueden ofrecernos una idea de la calidad de la estimación. La media m_E caracteriza el valor medio de los errores y la variancia σ_E^2 representa una medida de dispersión de los errores alrededor de la media. Por lo tanto, un buen método de estimación será aquel que:

(i) Posea una media de los errores m_E cercana a cero, propiedad que guarda un estimador "insesgado" (o unbiased, en inglés), y

(ii) Muestre una dispersión mínima o muy concentrada alrededor de la media, es decir, con una variancia de estimación cercana a cero - $\sigma_E^2 \approx 0$. .

Consideremos por un momento el problema de estimar el valor Z_V, por ejemplo el porcentaje medio $Z_V(x)$ de cobre en un bloque de tamaño V centrado en el punto x, a partir de un conjunto de n datos $\{Z(x_i),\ i = 1 \text{ a } n\}$. El estimador Z estará definido como una función de los datos: $Z^* = f(Z(x_1),\ Z(x_2),\ \ldots,\ Z(x_n))$.

El cálculo de los momentos de primero y segundo orden del error $Z_V - Z^*$, requerirá del conocimiento de la función de distribución conjunta $F_{Z(x_1),\ \ldots,\ Z(x_n)}(\cdot)$, la cual, al menos en la etapa de estimación, es imposible de derivar. Esto nos restringe a la clase de estimadores lineales.

$$Z^* = \sum_{i=1}^{n} \lambda_i Z(x_i)$$

donde, ahora sí, la media y la variancia del error pueden calcularse empleando la función variograma $\gamma(h)$ (o la función covariancia $C(h)$),

- CAPITULO III -

EL METODO KRIGING DE ESTIMACION

La Variancia de estimación.- Todo método de estimación introduce implícitamente un error de estimación derivado del simple hecho de que la cantidad por estimar z no coincide con el valor estimado z^* r = z - z^*.

Así como z(x) se interpreta como una realización de la función aleatoria Z(x), el error $r(x_i)$ puede interpretarse como una realización de la variable aleatoria regionalizada $R(x_i) = Z(x_i) - Z^*(x_i)$, en el punto x_i. Por otra parte, si la f.a. Z(x) es estacionaria, entonces la función aleatoria error R(x) será estacionaria y los dos errores $r(x_i)$ y $r(x_j)$ serán considerados como dos realizaciones diferentes de la misma f.a. $R(x) = Z(x) - Z^*(x)$.

Bajo la hipótesis de estacionaridad de segundo orden, si los errores $r(x_1), \ldots, r(x_n)$ fueran conocidos en una cierta zona de control, sería posible, a través del histograma de los n valores, inferir la función de distribución de R(x), o al menos sería posible inferir la media $m_E = E\left[R(x)\right]$ y la variancia del error ó <u>variancia de estimación</u> $\text{Var}\left[R(x)\right] = \sigma_E^2$.

El error $r(x_j)$, $(j \neq 1, 2, \ldots, n)$ introducido al tratar de estimar el valor $z(x_j)$ en el punto x_j por medio del valor $z^*(x_j)$ permanecería desconocido, sin embargo, la media y la variancia de los errores -

INPUT DATA EMPLOYED IN THIS RUN

77

COORDINATES (X,Y,Z)	VARIABLE 1 **ZINC**	VARIABLE 2 **PB**
(0.00, 0.00, 2.00)	1.00	10.00
(0.00, 0.00, 6.00)	2.00	20.00
(0.00, 0.00, 10.00)	3.00	30.00
(0.00, 0.00, 16.00)	4.00	40.00
(0.00, 12.00, 2.00)	5.00	50.00
(0.00, 16.00, 6.00)	6.00	60.00
(0.00, 19.00, 9.00)	7.00	70.00

NUMBER OF VARIABLES = 2
TOTAL NUMBER OF DATA POINTS = 7

SEMI-VARIOGRAM **ZINC** ***PAGE 1 1 *** GAM-V3 ***
(IRREGULAR 3 DIMENSIONAL GRID)

78

GLOBAL STATISTICS
MEAN = 4.00000 VARIANCE = 0.40000E 01 NUMBER OF DATA = 7

LAG = 4.00 LAG TOLERANCE = 2.00

	DIRECTION 1	DIRECTION 2	DIRECTION 3	DIRECTION 4
	LONGITUDE = 0.0	LONGITUDE = 45.0	LONGITUDE = 0.0	LONGITUDE = 45.0
	LATITUDE = 0.0	LATITUDE = 45.0	LATITUDE = 0.0	LATITUDE = 90.0
	TOLERANCE = 0.1	TOLERANCE = 0.1	TOLERANCE = 89.0	TOLERANCE = 90.0
	DIRECTION STATISTICS	DIRECTION STATISTICS	DIRECTION STATISTICS	DIRECTION STATISTICS
	MEAN= 2.00000	MEAN= 5.50000	MEAN= 2.50000	MEAN= 1.50000
	VARIANCE= 0.66667E 00	VARIANCE= 0.25000E 00	VARIANCE= 0.12500E 01	VARIANCE= 0.25000E 00
	# OF DATA= 3	# OF DATA= 2	# OF DATA= 4	# OF DATA= 2
LAG	NC DIST. VARIO.	NC DIST. VARIO.	NC DIST. VARIO.	NC DIST. VARIO.
1	0 0.00 0.0000E 00	0 0.00 0.0000E 00	0 0.00 0.0000E 00	0 0.00 0.0000E 00
2	2 4.00 0.5000E 00	2 4.95 0.5000E 00	0 0.00 0.0000E 00	0 0.00 0.0000E 00
3	2 7.00 0.1250E 01	1 9.90 0.2000E 01	0 0.00 0.0000E 00	0 0.00 0.0000E 00
4	1 10.00 0.2000E 01	0 0.00 0.0000E 00	1 12.65 0.4500E 01	1 12.00 0.8000E 01
5	1 14.00 0.4500E 01	0 0.00 0.0000E 00	3 15.80 0.6333E 01	1 16.00 0.8000E 01
6	0 0.00 0.0000E 00	0 0.00 0.0000E 00	6 19.34 0.7583E 01	0 0.00 0.0000E 00

SEMI-VARIOGRAM **PB** ***PAGE 1 *** GAM-V3 ***
(IRREGULAR 3 DIMENSIONAL GRID)

79

GLOBAL STATISTICS
MEAN = 40.00000 VARIANCE = 0.40000E 03 NUMBER OF DATA = 7

LAG = 4.00 LAG TOLERANCE = 2.00

	DIRECTION 1	DIRECTION 2	DIRECTION 3	DIRECTION 4
	LONGITUDE = 0.0	LONGITUDE = 45.0	LONGITUDE = 0.0	LONGITUDE = 45.0
	LATITUDE = 0.0	LATITUDE = 45.0	LATITUDE = 0.0	LATITUDE = 90.0
	TOLERANCE = 0.1	TOLERANCE = 0.1	TOLERANCE = 89.0	TOLERANCE = 90.0
	DIRECTION STATISTICS	DIRECTION STATISTICS	DIRECTION STATISTICS	DIRECTION STATISTICS
	MEAN= 20.00000	MEAN= 55.00000	MEAN= 25.00000	MEAN= 15.00000
	VARIANCE= 0.66667E 02	VARIANCE= 0.25000E 02	VARIANCE= 0.12500E 03	VARIANCE= 0.25000E 02
	# OF DATA= 3	# OF DATA= 2	# OF DATA= 4	# OF DATA= 2
LAG	NC DIST. VARIO.	NC DIST. VARIO.	NC DIST. VARIO.	NC DIST. VARIO.
1	0 0.00 0.0000E 00	0 0.00 0.0000E 00	0 0.00 0.0000E 00	0 0.00 0.0000E 00
2	2 4.00 0.5000E 02	2 4.95 0.5000E 02	0 0.00 0.0000E 00	0 0.00 0.0000E 00
3	2 7.00 0.1250E 03	1 9.90 0.2000E 03	0 0.00 0.0000E 00	0 0.00 0.0000E 00
4	1 10.00 0.2000E 03	0 0.00 0.0000E 00	1 12.65 0.4500E 03	1 12.00 0.8000E 03
5	1 14.00 0.4500E 03	0 0.00 0.0000E 00	3 15.80 0.6333E 03	1 16.00 0.8000E 03
6	0 0.00 0.0000E 00	0 0.00 0.0000E 00	6 19.34 0.7583E 03	0 0.00 0.0000E 00

E USAGE OBJECT CODE= 17376 BYTES, ARRAY AREA= 1856 BYTES, TOTAL AREA AVAILABLE= 126976 BYTES

GNOSTICS NUMBER OF ERRORS= 0, NUMBER OF WARNINGS= 0, NUMBER OF EXTENSIONS= 16

PILE TIME= 0.20 SEC, EXECUTION TIME= 0.06 SEC, 0.40.44 SUNDAY 18 MAR 79 WATFIV - JAN 1976 V1L5

*STOP

```
      UD(IK)=UD(IK)/NN
      IV2=KMAX*(KD-1)
C
C       SEMI-VARIOGRAMS
C
      DO 31 K=1,KMAX
      IJK=K+IV2+IV0
      NCC=MAXO(1,NC(IJK))
      D(IJK)=D(IJK)/NCC
      G(IJK)=G(IJK)/NCC
   31 CONTINUE
   30 CONTINUE
    3 CONTINUE
C
C               IF IS .NE. 1 EDIT VARIABLE/VARIABLE
C               MAX.5 DIRECTIONS/PAGE. IF KMAX .LE.50, EDITION ON
C               A SINGLE PAGE
C
      IF(IS.EQ.1)GO TO 5
      IMP=(NDI-1)/5+1
      IDM=FLOAT(NDI)/FLOAT(IMP)+0.9999
      DO 41 IV=1,NV
      DO 42 IM=1,IMP
      WRITE(IOUT,2001)NAM(IV),IM
      WRITE(IOUT,2002)U(IV),V(IV),N(IV)
      ID1=1+IDM*(IM-1)
      ID2=MIN0(NDI,IDM*IM)
      WRITE(IOUT,2003) PAS,TOL,(IA,ID,ID=ID1,ID2)
      WRITE(IOUT,2005) (IA,ALP(ID),ID=ID1,ID2)
      WRITE(IOUT,2004) (IA,BET(ID),ID=ID1,ID2)
      WRITE(IOUT,2006) (IA,TET(ID),ID=ID1,ID2)
      KD0=NDI*(IV-1)
      KD1=ID1+KD0
      KD2=ID2+KD0
      WRITE(IOUT,2007) (IA,ID=ID1,ID2)
      WRITE(IOUT,2008) (IA,UD(IK),IK=KD1,KD2)
      WRITE(IOUT,2009) (IA,VD(IK),IK=KD1,KD2)
      WRITE(IOUT,2010) (IA,ND(IK),IK=KD1,KD2)
      WRITE(IOUT,2011) (IA,ID=ID1,ID2)
      IK0=NDI*KMAX*(IV-1)+KMAX*(ID1-1)
      IKD=IK0+KMAX*(ID2-ID1)
      DO 43 K=1,KMAX
      IK1=K+IK0
      IK2=K+IKD
   43 WRITE(IOUT,2012)IK,(NC(IK),D(IK),G(IK),IK=IK1,IK2,KMAX)
   42 CONTINUE
   41 CONTINUE
C
 2000 FORMAT(1H1,'DUPLICATED DATA **GAM-V3**  DATA',I4,
     1' X=',F7.2,' Y=',F7.2,' Z=',F7.2
     2,'  DATA',I4,' X=',F7.2,' Y=',F7.2,' Z=',F7.2)
 2001 FORMAT(1H1,55X,'SEMI-VARIOGRAM',5X,A8,10X,'***PAGE # ',I2,
     13X,'*** GAM-V3 ***'/56X,'**********' ',20X,
     2'(IRREGULAR 3 DIMENSIONAL GRID)'//)
 2002 FORMAT(1H ,5X,'GLOBAL STATISTICS'/1H ,6X,'MEAN = ',F10.5,
     15X,'VARIANCE = ',E11.5,5X,'NUMBER OF DATA = ',I4//)
 2003 FORMAT(1H ,16X,'LAG = ',F7.2,5X,'LAG TOLERANCE = ',F7.2///1H
     15X,'|',5(A1,5X,'DIRECTION',I2,5X,'|'))
 2004 FORMAT(1H ,5X,'|',5'   ,1X,'LATITUDE = ',F7.1,2X,'|'))
 2005 FORMAT(1H ,5X,'|',5(A1,'LONGITUDE = ',F7.1,2X,'|'))
 2006 FORMAT(1H ,5X,'|',5(A1,'TOLERANCE = ',F7.1,2X,'|'))
 2007 FORMAT(1H ,5X,'|',5(A1,'DIRECTION STATISTICS ','|'))
 2008 FORMAT(1H ,5X,'|',5(A1,'MEAN= ',F10.5,5X,'|'))
 2009 FORMAT(1H ,5X,'|',5(A1,'VARIANCE= ',E11.5,'|'))
 2010 FORMAT(1H ,5X,'|',5(A1,'# OF DATA= ',I4,6X,'|'))
 2011 FORMAT(1H ,' LAG |',5(A1,' NC   DIST.   VARIO. |'))
 2012 FORMAT(1H ,I4,' |',5(I4,1X,F6.2,1X,E10.4,'|'))
C
C
C
C
    5 RETURN
      END
$DATA
```

```
      IF(TET(KD).LE.0. .OR. TET(KD).GT.90.) TET(KD)=45.0
      A=PI*ALP(KD)/180.0
      B=PI*BET(KD)/180.0
      T=PI*TET(KD)/180.0
      CX(KD)=COS(A)
      CY(KD)=SIN(A)
      CZ(KD)=COS(B)
      CT(KD)=COS(T)
      IF(CT(KD).LE.0.001) CT(KD)=0.
      IJ0=KMAX*(KD-1)
C
C       INITIALIZATIONS
C
      DO 1 IV=1,NV
      I=KD+NDI*(IV-1)
      IJ=IJ0+KMAX*NDI*(IV-1)
      KKD(I)=0
      UD(I)=0.
      VD(I)=0.
      ND(I)=0
      DO 1 K=1,KMAX
      IJK=IJ+K
      NC(IJK)=0
      G(IJK)=0.
      D(IJK)=0.
    1 CONTINUE
      DO 11 IV=1,NV
      IJ=KD+IV
      IF(VR(IJ)-TEST)12,12,13
   12 N(IV)=0
      U(IV)=0.
      V(IV)=0.
      GO TO 11
   13 N(IV)=1
      U(IV)=VR(IJ)
      V(IV)=VR(IJ)*VR(IJ)
   11 CONTINUE
      ND1=ND-1
C
C                COMPUTE VARIOGRAM.  NEW FIRST DATA
C
      DO 2 I=1,ND1
      IND=0
      DO 20 IV=1,NV
      IP=I+ND*(IV-1)
      VR1=VR(IP)
      IF(VR1.LE.TEST)GO TO 20
      IND=1
      N(IV)=N(IV)+1
      U(IV)=U(IV)+VR1
      V(IV)=V(IV)+VR1*VR1
   20 CONTINUE
      IF(IND.EQ.0)GO TO 2
      I1=I+1
      DO 21 J=I1,ND
C
C                                        NEW SECOND DATA
C
      DX=X(J)-X(I)
      DY=Y(J)-Y(I)
      DZ=Z(J)-Z(1)
      H=SQRT(DX*DX+DY*DY+DZ*DZ)
      IF(H.LT.0.001*TOL) GO TO 25
      K=INT(H/PAS+0.5)+1
      IF(K.GT.KMAX.OR.ABS(H-(K-1)*PAS).GT.TOL)GO TO 21
C
C                                LOOK FOR DIRECTION
C
      DO 22 KD=1,NDI
      CD=(DX*CX(KD)+DY*CY(KD)+DZ*CZ(KD))1/H
      IF(ABS(CD).GE.CT(KD))GO TO 23
   22 CONTINUE
      GO TO 21
   23 IKD=K+KMAX*(KD-1)
      DO 24 IV=1,NV
      IP=I+ND*(IV-1)
      JP=J+ND*(IV-1)
      IF(VR(IP).LE.TEST.OR.VR(JP).LE.TEST)GO TO 24
      IJK=IKD+KMAX*NDI*(IV-1)
      NC(IJK)=NC(IJK)+1
      D(IJK)=D(IJK)+H
      VRR=VR(IP)-VR(JP)
      G(IJK)=G(IJK)+0.5*VRR*VRR
      IK=KD+NDI*(IV-1)
      KKD(IK)=1
   24 CONTINUE
      GO TO 21
   25 WRITE(IOUT,2000)I,X(I),Y(I),Z(I),J,X(J),Y(J),Z(J)
   21 CONTINUE
      DO 26 IV=1,NV
      DO 26 KD=1,NDI
      IK=KD+NDI*(IV-1)
      IF(KKD(IK).EQ.0) GO TO 26
      ND(IK)=ND(IK)+1
      IP=I+ND*(IV-1)
      UD(IK)=UD(IK)+VR(IP)
      VD(IK)=VD(IK)+VR(IP)*VR(IP)
      KKD(IK)=0
   26 CONTINUE
    2 CONTINUE
C
C                RESULTS
C
      DO 3 IV=1,NV
C
C          GLOBAL STATISTICS
C
      IF(N(IV).EQ.0)GO TO 3
      V(IV)=(V(IV)-U(IV)*U(IV)/N(IV))/N(IV)
      U(IV)=U(IV)/N(IV)
      IVD=KMAX-NDI*(IV-1)
      IV1=NDI*(IV-1)
      DO 30 KD=1,NDI
      IK=KD+IV1
      NN=ND(IK)
C
C          DIRECTIONAL STATISTICS
C
      IF(NN.EQ.0) GO TO 30
      VD(IK)=(VD(IK)-UD(IK)*UD(IK)/NN)/NN
```

```
GNATFIV                                                                 4
C                                                                       5
C                                                                       6
C                                                                       7
      DIMENSION VR(14),NC(80),G(80),D(80),X(40),Y(40),Z(40),            8
     *          U(2),V(2),N(2),UD(8),VD(8),MD(8),ALP(4),              9
     *          BET(4),TET(4)                                         10
      DOUBLE PRECISION NAM(10)                                        11
      COMMON INP,IOUT,TEST,NV,NAM                                     12
      INP=5                                                           13
      IOUT=6                                                          14
      KMAX=6                                                          15
      IS=0                                                            16
      TEST=0.                                                         17
C                                                                     18
C     READ INPUT DATA                                                 19
C                                                                     20
      READ(INP,1000) (NAM(I),I=1,10)                                  21
      READ(INP,1001) NV,ND,NDI                                        22
      READ(INP,1002) (ALP(I),I=1,NDI),(BET(I),I=1,NDI),(TET(I),I=1,NDI) 23
      READ(INP,1003) (X(I),Y(I),Z(I),VR(I),VR(I+ND),I=1,ND)           24
C                                                                     25
C     PRINT OUT DATA                                                  26
C                                                                     27
      WRITE(IOUT,2000) (NAM(I),I=1,NV)                                28
      WRITE(IOUT,2001) (X(I),Y(I),Z(I),VR(I),VR(I+ND),I=1,ND)         29
      WRITE(IOUT,2002) NV,ND                                          30
C                                                                     31
      CALL GAMV3(VR,X,Y,Z,ND,KMAX,4.0,2.0,NDI,ALP,BET,TET,NC,G,D,     32
     *           U,V,N,UD,VD,MD,IS)                                   33
C                                                                     34
 1000 FORMAT(10A8)                                                    35
 1001 FORMAT(8I5)                                                     36
 1002 FORMAT(8F10.0)                                                  37
 1003 FORMAT(5F10.0)                                                  38
 2000 FORMAT(1H1,44X,'INPUT DATA EMPLOYED IN THIS RUN'///22X,         39
     1'COORDINATES  (X,Y,Z) ',10X,'VARIABLE : ',A8,10X,'VARIABLE : ', 40
     2A8//)                                                           41
 2001 FORMAT(1H ,17X,'(',F8.2,',',F8.2,',',F8.2,')',10X,F10.2,        42
     110X,F10.2)                                                      43
 2002 FORMAT(//17X,'NUMBER OF VARIABLES = ',I4,/1H ,16X,              44
     1'TOTAL NUMBER OF DATA POINTS = ',I4)                            45
C                                                                     46
      STOP                                                            47
      END                                                             48
C                                                                     49
C                                                                     50
C                                                                     51
C                                                                     52
      SUBROUTINE GAMV3(VR,X,Y,Z,ND,KMAX,PAS,DP,NDI,ALP,BET,TET,NC,G,  53
     *D,U,V,N,UD,VD,MD,IS)                                            54
C                                                                     55
C  CALCULATIONS OF SEMI-VARIOGRAMS IN A 3 DIM.SPACE ALONG NDI         56
C  GIVEN DIRECTIONS.  GROUPING INTO CLASSES OF ANGLE AND              57
C  DISTANCE.  NV VARIABLES                                            58
C                                                                     59
C  PARAMETERS                                                         60
C       VR(ND*NV)       DATA STORED COLUMNWISE                        61
C       X,Y,Z(ND)       COORDINATES OF DATA (CENTRAL POINT)           62
C       ND              NUMBER OF DATA                                63.
C       KMAX            MAXIMUM NUMBER OF LAGS                        64.
C       PAS             AMPLITUDE OF LAG                              65.
C       DP              TOLERANCE OF DISTANCE = AMPLITUDE OF A CLASS  66.
C                       IF DP .LE. 0 THEN: DP=PAS/2.                  67.
C       NDI             NUMBER OF DIRECTIONS TO BE CONSIDERED         68.
C       ALP(NDI)        LONGITUDE OF THE DIRECTION (FROM OX AXIS)     69.
C       BET(NDI)        LATITUDE (ANGLE WITH THE UPWARD VERTICAL)     70.
C       TET(NDI)        TOLERANCE OF ANGLE (HALF OF THE CONE'S ANGLE) 71.
C                       IF TET NOT IN (0, 90), THEN : TET = 45.       72.
C                       (ALL ANGLES ARE EXPRESSED IN DECIMAL DEGREES) 73.
C       NC(KMAX*NDI*NV) # OF PAIRS OF DATA/LAG/DIRECTION/VARIABLE     74.
C       G(KMAX*NDI*NV)  SEMI-VARIOGRAM                                75.
C       D(KMAX*NDI*NV)  MEAN INTERDISTANCE OF THE NC PAIRS            76.
C       U,V,N(NV)       GLOBAL MEAN, VARIANCE, AND TOTAL # OF         77.
C                       NON BLANK DATA/VARIABLE                       78.
C       UD,VD,MD(NDI*NV) DIRECTIONAL MEAN, VARIANCE, AND # OF DATA    79.
C                        USED FOR THE COMPUTATION OF CORRESPONDING     80.
C                        SEMI-VARIOGRAM                               81.
C                                                                     82.
C                                                                     83.
C  WORKING VARIABLES                                                  84.
C       CX,CY,CZ,CT(NDI)    SINE AND COSINE OF ANGLES                 85.
C       KKD(NDI*NV)         INDICATOR OF DIRECTIONAL STATISTICS       86.
C                                                                     87.
C                                                                     88.
C  OPTIONS      IS.NE.1          EDIT RESULTS  1 PAGE/VARIABLE        89.
C                                MAXIMUM 5 DIRECTIONS/PAGE/VARIABLE   90.
C                                                                     91.
C                                                                     92.
C  COMMONS      INP                                                   93.
C               IOUT             PRINTER                              94.
C               TEST             INDICATOR OF BLANK DATA              95.
C                                VR.LE.TEST                           96.
C               NV               NUMBER OF VARIABLES                  97.
C               NAM(NV)          NAME OF VARIABLES (A8)               98.
C                                                                     99.
C                                                                     100.
C     WARNING 1                                                       101.
C          GAM-V3 CONSIDERS ND*(ND-1)/2 PAIRS. FOR ND .GT. 500        102.
C          OTHER SUBROUTINES SHOULD BE USED.  GAM-V1 OR               103.
C          CLAS-2 + GAM-V2.                                           104.
C                                                                     105.
C     WARNING 2                                                       106.
C          A PAIR OF DATA IS AFFECTED TO A SINGLE CONE, THE FIRST     107.
C          ONE IN WHICH IT FALLS                                      108.
C                                                                     109.
C                                                                     110.
C                                                                     111.
      DIMENSION VR(1),X(1),Y(1),Z(1),ALP(1),BET(1),TET(1)             112
      DIMENSION G(1),D(1),NC(1),U(1),V(1),N(1),UD(1),VD(1),MD(1)      113
      DIMENSION KKD(8),CX(4),CY(4),CZ(4),CT(4)                        114
      DOUBLE PRECISION NAM(10)                                        115
      COMMON INP,IOUT,TEST,NV,NAM                                     116
      DATA IA/' '/                                                    117
      PI=3.14159265                                                   118
      TOL=DP                                                          119
      IF(TOL.LE.0)TOL=PAS/2.                                          120
C                                                                     121
      DO 1 KD=1,NDI                                                   122
```

LAG= 10.000 TOLERANCE FOR THE LAG= * 50

VARIABLE	**ZINC**	**PB **
MEAN	1.6042	3.4842
VARIANCE	0.39092E 00	0.43294E 00
NUMB OF DATA	19	19

LAG	NC	VARIOGRAM	DISTANCE	NC	VARIOGRAM	DISTANCE
1	10	2.55	0.13050E 00	9	2.51	0.12278E 00
2	25	10.18	0.13182E 00	21	10.30	0.10024E 00
3	19	20.17	0.41526E 00	21	20.34	0.27150E 00
4	14	30.20	0.56214E 00	14	30.71	0.35357E 00
5	14	39.85	0.74393E 00	16	39.69	0.39875E 00
6	7	48.89	0.30571E 00	9	49.50	0.15111E 00
7	11	59.14	0.51182E 00	13	58.53	0.36085E 00
8	9	68.99	0.59722E 00	11	69.42	0.48662E 00
9	10	79.76	0.66750E 00	10	79.62	0.62000E 00
10	11	90.00	0.57045E 00	11	90.00	0.68545E 00

DUPLICATED DATA*** GAM-V2 *** DATA 20 X= 0.0000 Y= 0.0000 DATA 40 X= 0.0000 Y= 0.0000

MEAN = 1.59210 VARIANCE = 0.24179E 00 NUMB OF DATA = 38
TOLERANCE OF DIRECTION = 45.DEGREES ,LAG = 10.00 , LAG TOLERANCE = * 5.00

	DIRECTION 1 0.0DEGREES			DIRECTION 2 45.0DEGREES			DIRECTION 3 90.0DEGREES			DIRECTION 4 135.0DEGREES		
LAG	NC	DISTANCE	VARIOGRAM	NC	DISTANCE	VARIOGRAM	NC	DISTANCE	VARIOGRAM	NC	DISTANCE	VARIOGRAM
1	10	2.12	0.10250E 00	3	4.24	0.13023E 00	11	2.82	0.41364E-01	0	0.00	0.00000E 00
2	5	10.03	0.27000E-01	27	10.37	0.12741E 00	41	10.09	0.10110E 00	1	14.14	0.00000E 00
3	33	19.58	0.28954E 00	4	21.92	0.59250E 00	40	19.16	0.18062E 00	0	0.00	0.00000E 00
4	25	30.17	0.55850E 00	5	27.44	0.26600E 00	32	28.85	0.71875E-01	0	0.00	0.00000E 00
5	9	39.89	0.64278E 00	13	38.95	0.67769E 00	36	39.63	0.12625E 00	0	0.00	0.00000E 00
6	12	49.43	0.47917E 00	6	49.50	0.33583E 00	40	49.86	0.11162E 00	0	0.00	0.00000E 00
7	17	58.57	0.40333E 00	10	59.26	0.54500E 00	43	59.36	0.16081E 00	0	0.00	0.00000E 00
8	20	68.03	0.35300E 00	10	68.59	0.65000E 00	42	69.62	0.32206E 00	0	0.00	0.00000E 00
9	15	79.65	0.15200E 00	11	79.71	0.69591E 00	26	79.27	0.21327E 00	0	0.00	0.00000E 00
10	30	89.63	0.82667E-01	12	90.16	0.59333E 00	15	89.57	0.28233E 00	0	0.00	0.00000E 00

MEAN = 3.12051 VARIANCE = 0.46728E 00 NUMB OF DATA = 39
TOLERANCE OF DIRECTION = 45.DEGREES ,LAG = 10.00 , LAG TOLERANCE = 5.00

	DIRECTION 1 0.0DEGREES			DIRECTION 2 45.0DEGREES			DIRECTION 3 90.0DEGREES			DIRECTION 4 135.0DEGREES		
LAG	NC	DISTANCE	VARIOGRAM	NC	DISTANCE	VARIOGRAM	NC	DISTANCE	VARIOGRAM	NC	DISTANCE	VARIOGRAM
1	9	2.04	0.56111E-01	3	4.24	0.14333E 00	13	2.92	0.54231E-01	0	0.00	0.00000E 00
2	3	11.18	0.19333E 00	22	10.48	0.96591E-01	42	9.78	0.10845E 00	1	14.14	0.24500E 00
3	37	19.66	0.36824E 00	5	22.34	0.38300E 00	43	19.25	0.16480E 00	0	0.00	0.00000E 00
4	28	30.14	0.50339E 00	4	27.93	0.12875E 00	37	28.54	0.33067E 00	0	0.00	0.00000E 00
5	18	40.79	0.26028E 00	15	38.94	0.34933E 00	37	39.56	0.44662E 00	0	0.00	0.00000E 00
6	19	49.66	0.35947E 00	8	50.03	0.16000E 00	40	49.83	0.27037E 00	0	0.00	0.00000E 00
7	18	58.82	0.59472E 00	13	58.31	0.35615E 00	45	59.19	0.32022E 00	0	0.00	0.00000E 00
8	20	68.08	0.11600E 01	12	69.06	0.49667E 00	43	69.42	0.47620E 00	0	0.00	0.00000E 00
9	16	79.37	0.13050E 01	11	79.58	0.62909E 00	25	79.34	0.38640E 00	0	0.00	0.00000E 00
10	31	89.75	0.11116E 01	12	90.16	0.71000E 00	13	89.98	0.32654E 00	0	0.00	0.00000E 00

CORE USAGE OBJECT CODE= 33920 BYTES,ARRAY AREA= 3152 BYTES,TOTAL AREA AVAILABLE= 126976 BYTES

DIAGNOSTICS NUMBER OF ERRORS= 0, NUMBER OF WARNINGS= 0, NUMBER OF EXTENSIONS= 36

COMPILE TIME= 0.48 SEC,EXECUTION TIME= 0.87 SEC, 2.21.09 THURSDAY 15 FEB 79 HATFIV - JAN 1976 V11S

$STOP

63

MEAN = 1.59210 VARIANCE = 0.24179E 00 NUMB OF DATA = 38

LAG	DIRECTION 1 NC	DIRECTION 1 VARIOGRAM	DIRECTION 2 NC	DIRECTION 2 VARIOGRAM	DIRECTION 3 NC	DIRECTION 3 VARIOGRAM	DIRECTION 4 NC	DIRECTION 4 VARIOGRAM
1	27	0.[illegible]E-01	32	0.16312E 00	24	0.22021E 00	24	0.10104E 00
2	18	0.93275E-01	28	0.33236E 00	14	0.35571E 00	14	0.24500E 00
3	9	0.16384E 00	25	0.41360E 00	7	0.45286E 00	6	0.20333E 00
4	0	0.00000E 00	22	0.24636E 00	0	0.00000E 00	0	0.00000E 00
5	0	0.00000E 00	18	0.18770E 00	0	0.00000E 00	0	0.00000E 00
6	0	0.00000E 00	14	0.21321E 00	0	0.00000E 00	0	0.00000E 00
7	0	0.00000E 00	11	0.19955E 00	0	0.00000E 00	0	0.00000E 00
8	0	0.00000E 00	8	0.34187E 00	0	0.00000E 00	0	0.00000E 00
9	0	0.00000E 00	4	0.40375E 00	0	0.00000E 00	0	0.00000E 00
10	0	0.00000E 00	0	0.00000E 00	0	0.00000E 00	0	0.00000E 00

64

MEAN = 3.12051 VARIANCE = 0.46728E 00 NUMB OF DATA = 39

LAG	DIRECTION 1 NC	DIRECTION 1 VARIOGRAM	DIRECTION 2 NC	DIRECTION 2 VARIOGRAM	DIRECTION 3 NC	DIRECTION 3 VARIOGRAM	DIRECTION 4 NC	DIRECTION 4 VARIOGRAM
1	29	0.42241E-01	34	0.31397E 00	26	0.39365E 00	26	0.21750E 00
2	19	0.12316E 00	31	0.67030E 00	15	0.60633E 00	16	0.52594E 00
3	9	0.22333E 00	27	0.57278E 00	6	0.76667E 00	7	0.50143E 00
4	0	0.00000E 00	23	0.53413E 00	0	0.00000E 00	0	0.00000E 00
5	0	0.00000E 00	19	0.53862E 00	0	0.00000E 00	0	0.00000E 00
6	0	0.00000E 00	15	0.55567E 00	0	0.00000E 00	0	0.00000E 00
7	0	0.00000E 00	11	0.53045E 00	0	0.00000E 00	0	0.00000E 00
8	0	0.00000E 00	7	0.50429E 00	0	0.00000E 00	0	0.00000E 00
9	0	0.00000E 00	4	0.32125E 00	0	0.00000E 00	0	0.00000E 00
10	0	0.00000E 00	0	0.00000E 00	0	0.00000E 00	0	0.00000E 00

NU OF LINES = 2 OF COLUMNS = 4 OF LEVELS = 10
DA MEAN=0.23662E 01 VARIANCE=0.93990E 00 NUMBER= 77
MAXIMUM LAG = 10

65

LAG	N.PAIRS	VARIO/LINES	N.PAIRS	VARIO/COLUMNS	N.PAIRS	VARIO/LEVELS
1	37	0.33243E-01	54	0.11537E 00	67	0.63726E 00
2	0	0.00000E 00	37	0.30919E 00	60	0.55091E 00
3	0	0.00000E 00	20	0.57250E 00	52	0.80654E 00
4	0	0.00000E 00	0	0.00000E 00	45	0.12966E 01
5	0	0.00000E 00	0	0.00000E 00	37	0.13206E 01
6	0	0.00000E 00	0	0.00000E 00	29	0.16202E 01
7	0	0.00000E 00	0	0.00000E 00	22	0.18325E 01
8	0	0.00000E 00	0	0.00000E 00	14	0.18725E 01
9	0	0.00000E 00	0	0.00000E 00	7	0.74357E 00
10	0	0.00000E 00	0	0.00000E 00	0	0.00000E 00

66

LAG= 10.000 TOLERANCE FOR THE LAG= 5.000

VARIABLE	**ZINC**	**PB**
MEAN	1.5000	2.7150
VARIANCE	0.73689E-01	0.25489E 00
NUMB OF DATA	19	20

LAG	NC	VARIOGRAM	DISTANCE	NC	VARIOGRAM	DISTANCE
1	11	2.82	0.41364E-01	13	2.92	0.54231E-01
2	31	10.19	0.53548E-01	34	10.03	0.11338E 00
3	24	19.00	0.84533E-01	27	19.04	0.22956E 00
4	23	28.83	0.56655E-01	26	28.77	0.41327E 00
5	20	39.35	0.86730E-01	22	39.45	0.67136E 00
6	17	48.94	0.80294E-01	19	48.84	0.23763E 00
7	18	59.06	0.95003E-01	20	58.80	0.22675E 00
8	14	69.50	0.12444E 00	16	69.25	0.19544E 00
9	8	78.50	0.61875E-01	8	78.50	0.12487E 00
	4	87.50	0.11500E 00	4	87.50	0.18125E 00

	(	0.00,	80.00,	[illegible]	2.40
	(	0.00,	77.00,	[illegible]	2.90
	(	0.00,	73.00,	0.00)	2.80
	(	0.00,	69.00,	0.00)	3.00
	(	0.00,	66.00,	0.00)	3.20
	(	0.00,	57.00,	0.00)	3.70
	(	0.00,	59.00,	0.00)	3.70
	(	0.00,	51.00,	0.00)	3.30
	(	0.00,	37.00,	0.00)	3.20
	(	0.00,	30.00,	0.00)	3.00
	(	0.00,	27.00,	0.00)	2.80
	(	0.00,	23.00,	0.00)	2.50
	(	0.00,	20.00,	0.00)	2.30
	(	0.00,	19.00,	0.00)	2.40
	(	0.00,	11.00,	0.00)	2.20
	(	0.00,	9.00,	0.00)	2.60
	(	0.00,	0.00,	0.00)	2.80

SECOND DRILLHOLE

	(	95.00,	95.00,	95.00)	3.70
	(	91.00,	91.00,	91.00)	3.90
	(	90.00,	90.00,	90.00)	4.00
	(	85.00,	85.00,	85.00)	4.40
	(	78.00,	78.00,	78.00)	4.30
	(	65.00,	65.00,	65.00)	4.20
	(	56.00,	56.00,	56.00)	4.00
	(	46.00,	46.00,	46.00)	3.90
	(	39.00,	39.00,	39.00)	3.20
	(	31.00,	31.00,	31.00)	3.60
	(	26.00,	26.00,	26.00)	3.60
	(	23.00,	23.00,	23.00)	3.80
	(	17.00,	17.00,	17.00)	3.50
	(	15.00,	15.00,	15.00)	3.30
	(	10.00,	10.00,	10.00)	3.00
	(	8.00,	8.00,	8.00)	0.00
	(	3.00,	-3.00,	3.00)	2.90
	(	2.00,	2.00,	2.00)	2.70
	(	1.00,	1.00,	1.00)	2.20
	(	0.00,	0.00,	0.00)	2.00

NUMBER OF VARIABLES = 2 TOTAL NUMBER OF DATA POINTS = 40

SEMI-VARIOGRAM 1 ***PAGE: 1 *** GAM-1 ***
********** (REGULAR 1- DIMENSIONAL GRID)

		ZINC		**PB**	
MEAN		1.5000		2.7750	
VARIANCE		0.736895E-01		0.254896 00	
NUMBER OF DATA		19		20	
LAG	DISTANCE	NC	VARIOGRAM	NC	VARIOGRAM
1	10.000	17	0.30000E-01	19	0.50261E-01
2	20.000	16	0.55687E-01	18	0.10750E 00
3	30.000	15	0.82667E-01	17	0.16265E 00
4	40.000	14	0.80714E-01	16	0.24719E 00
5	50.000	13	0.80846E-01	15	0.30833E 00
6	60.000	12	0.84583E-01	14	0.39357E 00
7	70.000	12	0.58750E-01	13	0.46923E 00
8	80.000	11	0.57727E-01	12	0.54333E 00
9	90.000	10	0.62000E-01	11	0.50500E 00
10	100.000	9	0.83889E-01	10	0.39350E 00

SEMI-VARIOGRAM 2 ***PAGE: 1 *** GAM-1 ***
********** (REGULAR 1- DIMENSIONAL GRID)

		ZINC		**PB**	
MEAN		1.6842		3.4842	
VARIANCE		0.392926 00		0.432946 00	
NUMBER OF DATA		19		19	
LAG	DISTANCE	NC	VARIOGRAM	NC	VARIOGRAM
1	10.000	17	0.27353E-01	17	0.44706E-01
2	20.000	16	0.90625E-01	16	0.46250E-01
3	30.000	15	0.155000 00	15	0.14733E 00
4	40.000	14	0.25107E 00	14	0.15500E 00
5	50.000	13	0.37385E 00	14	0.24250E 00
6	60.000	12	0.47458E 00	13	0.29808E 00
7	70.000	11	0.54636E 00	12	0.39500E 00
8	80.000	10	0.62600E 00	11	0.42773E 00
9	90.000	10	0.66350E 00	10	0.45800E 00
10	100.000	9	0.59722E 00	9	0.52111E 00

```
C                                                                       779.
C               IF IS .NE. 1 EDIT VARIABLE/VARIABLE                     780.
C               MAX.6 DIRECTIONS/PAGE. IF KMAX .LE.50, EDITION ON       781.
C               A SINGLE PAGE                                           782.
C                                                                       783.
      IF(IS.EQ.1)GO TO 5                                                784.
      IMP=(NDI-1)/5+1                                                   785.
      IDM=FLOAT(NDI)/FLOAT(IMP)+0.9999                                  786.
      DO 41 IV=1,NV                                                     787.
      DO 42 IM=1,IMP                                                    788.
      WRITE(IOUT,2001)NUM(IV),IM                                        789.
      WRITE(IOUT,2002)U(IV),V(IV),N(IV)                                 790.
      ID1=1+IDM*(IM-1)                                                  791.
      ID2=MIN0(NDI,IDM*IM)                                              792.
      WRITE(IOUT,2003)ALPHA,PAS,TOL,(IA,ID,ID=ID1,ID2)                  793.
      WRITE(IOUT,2004)(ALP(ID),ID=ID1,ID2)                              794.
      WRITE(IOUT,2005)(IA,ID=ID1,ID2)                                   795.
      IK0=NDI*KMAX*(IV-1)+KMAX*(ID1-1)                                  796.
      IKM=IK0+KMAX*(ID2-ID1)                                            797.
      DO 43 K=1,KMAX                                                    798.
      IK1=K+IK0                                                         799.
      IK2=K+IKM                                                         800.
   43 WRITE(IOUT,2006)K,(NC(IK),D(IK),G(IK),IK=IK1,IK2,KMAX)            801.
   42 CONTINUE                                                          802.
   41 CONTINUE                                                          803.
C                                                                       804.
 2000 FORMAT(1H1,'DUPLICATED DATA*** GAM-V2 ***   DATA',I4,            805.
     1' X= ',F9.4,' Y= ',F9.4                                           806.
     2,'  DATA  ',I4,' X= ',F9.4,' Y= ',F9.4)                           807.
 2001 FORMAT(1H1,45X,' SEMI - VARIOGRAM          ',A8,10X,'***PAGE:',I2, 808.
     110X,'*** GAM-V2 ***'/                                             809.
     247X,'*************** ',10X,'(IRREGULAR 2 DIMENSIONAL GRID   )'/)   810.
 2002 FORMAT(1H ,'    MEAN = ',F10.5,5X,' VARIANCE = ',E11.5,5X,' NUMB ' 811.
     1,'OF  DATA   = ',I4)                                              812.
 2003 FORMAT(1H ,'TOLERANCE OF DIRECTION = ',F4.0,'DEGREES',' ,LAG = ',  813.
     1F7.2,' , LAG  TOLERANCE      = ',F7.2/1H ,5X,'|',5(A1,5X,          814.
     2' DIRECTION',I2,6X,'|'))                                          815.
 2004 FORMAT(1H ,5X,'|',5(3X,F7.1,'DEGREES',7X,'|'))                    816.
 2005 FORMAT(1H ,' LAG |',5(A1, 'NC DISTANCE   VARIOGRAM|'))            817.
 2006 FORMAT(1H ,1X,I3,' |',5(1X,I3,1X,F7.2,1X,E11.5,'|'))              818.
C                                                                       819.
    5 RETURN                                                            820.
      END                                                               821.
C                                                                       822.
C                                                                       823.

$DATA                                                                   824.
```

INPUT DATA EMPLOYED IN THIS RUN

COORDINATES (X,Y,Z)	VARIABLE : **ZINC**	VARIA
FIRST DRILLHOLE		
(0.00, 95.00, 0.00)	1.60	

```
      DO 2 I=1,NDI                                                      719.
      IND=0                                                             720.
      DO 20 IV=1,NV                                                     721.
      IP=I+NDI*(IV-1)                                                   722.
      VR1=VR(IP)                                                        723.
      IF(VR1.LE.TEST)GO TO 20                                           724.
      IND=1                                                             725.
      N(IV)=N(IV)+1                                                     726.
      U(IV)=U(IV)+VR1                                                   727.
      V(IV)=V(IV)+VR1*VR1                                               728.
   20 CONTINUE                                                          729.
      IF(IND.EQ.0)GO TO 2                                               730.
      I1=I+1                                                            731.
      DO 21 J=I1,NDI                                                    732.
C                                                                       733.
C                                        NEW LAG                        734.
C                                                                       735.
      DX=X(J)-X(I)                                                      736.
      DY=Y(J)-Y(I)                                                      737.
      H=SQRT(DX*DX+DY*DY)                                               738.
      IF(H.LT.0.001*TOL) GO TO 25                                       739.
      K=INT(H/PAS+0.5)+1                                                740.
      IF(K.GT.KMAX.OR.ABS(H-(K-1)*PAS).GT.TOL)GO TO 21                  741.
C                                                                       742.
C                                          NEW DIRECTION                743.
C                                                                       744.
      DO 22 KD=1,NDI                                                    745.
      COSD=(DX*CAN(KD)+DY*SAN(KD))/H                                    746.
      IF(ABS(COSD).GE.CDA)GO TO 23                                      747.
   22 CONTINUE                                                          748.
      GO TO 21                                                          749.
   23 IK0=K+KMAX*(KD-1)                                                 750.
      DO 24 IV=1,NV                                                     751.
      IP=I+NDI*(IV-1)                                                   752.
      IP1=J+NDI*(IV-1)                                                  753.
      IF(VR(IP).LE.TEST.OR.VR(IP1).LE.TEST)GO TO 24                     754.
      IK=IK0+NDI*KMAX*(IV-1)                                            755.
      NC(IK)=NC(IK)+1                                                   756.
      D(IK)=D(IK)+H                                                     757.
      VRR=VR(IP1)-VR(IP)                                                758.
      G(IK)=G(IK)+0.5*VRR*VRR                                           759.
   24 CONTINUE                                                          760.
      GO TO 21                                                          761.
   25 WRITE(IOUT,2000)I,X(I),Y(I),J,X(J),Y(J)                           762.
   21 CONTINUE                                                          763.
    2 CONTINUE                                                          764.
C                                                                       765.
C                 RESULTS                                             766.
C                                                                       767.
      DO 3 IV=1,NV                                                      768.
      IF(N(IV).EQ.0)GO TO 3                                             769.
      V(IV)=(V(IV)-U(IV)*U(IV)/N(IV))/N(IV)                             770.
      U(IV)=U(IV)/N(IV)                                                 771.
      IK0=NDI*KMAX*(IV-1)+1                                             772.
      IK1=IK0+KMAX-NDI-1                                                773.
      DO 30 IK=IK0,IK1                                                  774.
      NCC=MAX0(1,NC(IK))                                                775.
      D(IK)=D(IK)/NCC                                                   776.
   30 G(IK)=G(IK)/NCC                                                   777.
    3 CONTINUE                                                          778.
```

```
C                                  OF DISTANCE  IF DP .EQ. 0 THEN DP=PAS/2.
C                 NDI              NUMBER OF DIRECTIONS TO BE CONSIDERED
C                 ALP(NDI)         ANGLES DEFINING THE DIRECTONS
C                 DA               TOLERANCE OF ANGLE. IF DA .EQ.0
C                                  THEN DA = 45 DEGREES.
C                 NC(KMAX*NDI*NV)  NUMBER OF PAIRS/LAG/DIRECTION/VARIABLE
C                 G(KMAX*NDI*NV)   SEMI - VARIOGRAM
C                 D(KMAX*NDI*NV)   MEAN DISTANCE
C                 U(NV)            MEAN     |
C                 V(NV)            VARIANCE|PER VARIABLE (VR(I) > TEST)
C                 N(NV)            # DATA   |
C
C  OPTIONS        IS.NE.1          EDIT RESULTS  1 PAGE/VARIABLE
C                                  MAXIMUM 5 DIRECTIONS/PAGE/VARIABLE
C
C
C  COMMONS        INP
C                 IOUT             PRINTER
C                 TEST             INDICATOR OF BLANK DATA
C                                  VR.LE.TEST
C                 NV               NUMBER OF VARIABLES
C                 NAM(NV)          NAME OF VARIABLES (A8)
C
      DIMENSION VR(1),X(1),Y(1)
      DIMENSION ALP(1),CAN(10),SAN(10)
      DIMENSION G(1),D(1),NC(1),U(1),V(1),N(1)
      DOUBLE PRECISION NAM(10)
      COMMON INP,IOUT,TEST,NV,NAM
      DATA IA/' '/
      PI=3.14159265
      TOL=DP
      IF(TOL.LE.0)TOL=PAS/2.
      DALPHA=DA
      IF(DA.LE.0)DALPHA=45.
      DO 1 KD=1,NDI
      ALPHA=PI*ALP(KD)/180.
      CAN(KD)=COS(ALPHA)
    1 SAN(KD)=SIN(ALPHA)
      THETA=PI*DALPHA/180.
      CDA=COS(THETA)
      KMM=NV*KMAX*NDI
      DO 10 IK=1,KMM
      NC(IK)=0
      D(IK)=0.
   10 G(IK)=0.
      DO 11 IV=1,NV
      IJ=ND*IV
      IF(VR(IJ)-TEST)12,12,13
   12 N(IV)=0
      U(IV)=0.
      V(IV)=0.
      GO TO 11
   13 N(IV)=1
      U(IV)=VR(IJ)
      V(IV)=VR(IJ)*VR(IJ)
   11 CONTINUE
      NDI=ND-1
C
C                 COMPUTE VARIOGRAM.  NEW DATA
C
```

```
      33 G(IK)=G(IK)/NCC                                                600.
       3 CONTINUE                                                       601.
         IF(IS.EQ.1)GO TO 5                                             602.
C                                                                       603.
C              EDIT (5 VARIABLES/PAGE)  IF IS .NE. 1                    604.
C              50 LAGS / PAGE                                           605.
C                                                                       606.
350      INP=(NV-1)/5+1                                                 607.
351      IVI=FLOAT(NV)/FLOAT(INP)+0.9999                                608.
352      DO 41 IN=1,INP                                                 609.
353      IV1=1+IVI*(IN-1)                                               610.
354      IV2=MIN0(NV,IVI*IN)                                            611.
355      WRITE(IOUT,2001)NOL,IN ,PAS,TOL                                612.
356      WRITE(IOUT,2002)(NAM(IV),IV=IV1,IV2)                           613.
357      WRITE(IOUT,2003)(U(IV),IV=IV1,IV2)                             614.
358      WRITE(IOUT,2004)(V(IV),IV=IV1,IV2)                             615.
359      WRITE(IOUT,2005)(N(IV),IV=IV1,IV2)                             616.
360      WRITE(IOUT,2006)(A,IV=IV1,IV2)                                 617.
361      DO 42 K=1,KMAX                                                 618.
362      IK1=K+KMAX*(IV1-1)                                             619.
363      IK2=K+KMAX*(IV2-1)                                             620.
364     42 WRITE(IOUT,2007)K,(NC(IK),D(IK),G(IK),IK=IK1,IK2,KMAX)       621.
365     41 CONTINUE                                                     622.
C                                                                       623.
366    2000 FORMAT(1H1,'*DUPLICATED DATA *** GAM-V1 ***   DATA ',       624.
       1I4,' X= ',F9.4,' Y= ',F9.4                                      625.
       2,'  DATA  ',I4,' X= ',F9.4,' Y= ',F9.4)                        626.
367    2001 FORMAT(1H1,48X,'  SEMI - VARIOGRAM      ',I4,10X,'***PAGE:',I2,627.
       110X,'*** GAM-V1 ***'/1H ,                                       628.
       250X,'**********',I3,'(IRREGULAR 1 DIMENSIONAL GRID  )'/         629.
       3 1H ,'  LAG=',F10.3,' TOLERANCE FOR THE LAG=',F10.3)           630.
368    2002 FORMAT(1H ,'VARIABLE',5(6X,A8,9X))                         631.
369    2003 FORMAT(1H ,'  MEAN  ',5(7X,F9.4,9X))                       632.
370    2004 FORMAT(1H ,'VARIANCE',5(6X,E11.5,6X))                      633.
371    2005 FORMAT(1H ,'NUMB OF DATA',4(5X,I4,15X))                    634.
372    2006 FORMAT(/1H ,2X,'LAG  |',5(A1,'NC VARIOGRAM DISTANCE'       635.
       1,'  |'))                                                       636.
373    2007 FORMAT(1H ,1X,I4,2X,'|',5(1X,I3,1X,F7.2,1X,E11.5,'|'))     637.
C                                                                       638.
374       5 RETURN                                                     639.
375       END                                                          640.
C                                                                       641.
C                                                                       642.
C                                                                       643.
C                                                                       644.
C                                                                       645.

376       SUBROUTINE GAMV2(VR,X,Y,ND,KMAX,PAS,OP,NDI,ALP,DA,NC,G,D,U,V,N,IS) 646.
C                                                                       647.
C                    TWO - DIMENSIONS SEMI-VARIOGRAM                    648.
C  IRREGULARLY SPACED DATA.  GROUPING INTO CLASSES OF ANGLE AND DISTANCE 649.
C                                                                       650.
C  PARAMETERS   VR(ND*NV)        DATA STORED COLUMNWISE                 651.
C                                                                       652.
C               X(ND)            COORDINATES OF DATA (CENTER POINT)    653.
C               Y(ND)                                                  654.
C               ND               NUMBER OF DATA                        655.
C               KMAX             MAXIMUM NUMBER OF LAGS                656.
C               PAS              AMPLITUDE OF LAG                      657.
C               OP               TOLERANCE DISTANCE = AMPLITUDE OF CLASS 658.
```

```
      6,'S
   73 2001 FORMAT(1H ,I4,4X,I5,6X,E11.5,4X,I5,6X,E11.5,6X,I5,6X,E11.5)
      C
   79    5 RETURN                                                    54
   80      END
      C
      C
      C
      C
      C
   81      SUBROUTINE GAMV1(VR,X,Y,ND,KMAX,PAS,DP,NC,G,D,U,V,N,NOL,IS)
      C
      C                  ONE - DIMENSION SEMI-VARIOGRAM
      C  IRREGULARLY SPACED DATA ALONG A LINE.  GROUPING INTO CLASSES OF
      C  DISTANCES.  DATA ARE NOT NECESSARILY ORDERED  ALONG THE LINE.
      C  THIS ROUTINE CAN BE USED TO COMPUTE AN ISOTROPIC SEMI-VARIOGRAM
      C  IN A 2-DIMENSIONAL SPACE
      C
      C  PARAMETERS   VR(ND*NV)    DATA STORED COLUMNWISE
      C
      C              X(ND)         COORDINATES OF DATA (CENTER POINT)
      C              Y(ND)
      C              ND            NUMBER OF DATA ON THE LINE
      C              KMAX          MAXIMUM NUMBER OF LAGS
      C              PAS           AMPLITUDE OF LAG
      C              DP            TOLERANCE DISTANCE = AMPLITUDE OF A CLASS
      C                            OF DISTANCE. IF DP .EQ. 0 THEN DP=PAS/2.
      C              NC(KMAX*NV)   NUMBER OF PAIRS OF DATA/LAG/VARIABLE
      C              G(KMAX*NV)    SEMI - VARIOGRAM
      C              D(KMAX*NV)    MEAN DISTANCE OF PAIRS IN THE CLASS
      C              U(NV)         MEAN     |
      C              V(NV)         VARIANCE|PER VARIABLE (VR(I) > TEST))
      C              N(NV)         # DATA   |
      C              NOL           NUMBER OF THE LINE OR VARIOGRAM
      C
      C  OPTIONS     IS.NE.1       EDIT RESULTS.  IF KMAX .LT. 50 A MAXIMUM
      C                            OF 5 VARIABLES ARE EDITED/PAGE
      C
      C  COMMON      INP
      C              IOUT          PRINTER
      C              TEST          INDICATOR OF BLANK DATA :
      C                            VR .LE. TEST
      C              NV            NUMBER OF VARIABLES
      C              NAM(NV)       NAME OF VARIABLES (A8)
      C
  282       DIMENSION VR(1),X(1),Y(1)
  283       DIMENSION G(1),NC(1),D(1),U(1),V(1),N(1)
  284       DOUBLE PRECISION NAM(10)
  285       COMMON INP,IOUT,TEST,NV,NAM
  286       DATA IA/' '/
      C
      C                  INITIALIZE
      C
  287       TOL=DP
  288       IF(TOL.LE.0)TOL=PAS/2.
  289       KM1=NV*KMAX
  290       DO 1 IK=1,KM1
  291       D(IK)=0.
  292       NC(IK)=0
```

```
  293     1 G(IK)=0.                                                  55
  294       DO 11 IV=1,NV                                                 540.
  295       IJ=ND*IV                                                      541.
  296       IF(VR(IJ)-TEST)12,12,13                                       542.
  297    12 N(IV)=0                                                       543.
  298       U(IV)=0.                                                      544.
  299       V(IV)=0.                                                      545.
  300       GO TO 11                                                      546.
  301    13 U(IV)=VR(IJ)                                                  547.
  302       V(IV)=VR(IJ)*VR(IJ)                                           548.
  303       N(IV)=1                                                       549.
  304    11 CONTINUE                                                      550.
      C                                                                   551.
      C            SEMI - VARIOGRAM AND STATISTICS CALCULATION            552.
      C                                                                   553.
  305       ND1=ND-1                                                      554.
  306       DO 2 I=1,ND1                                                  555.
  307       IND=0                                                         556.
  308       DO 20 IV=1,NV                                                 557.
  309       IJ=I+ND*(IV-1)                                                558.
  310       VR1=VR(IJ)                                                    559.
  311       IF(VR1.LE.TEST)GO TO 20                                       560.
  312       IND=1                                                         561.
  313       N(IV)=N(IV)+1                                                 562.
  314       U(IV)=U(IV)+VR1                                               563.
  315       V(IV)=V(IV)+VR1*VR1                                           564.
  316    20 CONTINUE                                                      565.
  317       IF(IND.EQ.0)GO TO 2                                           566.
  318       I1=I+1                                                        567.
  319       DO 21 J=I1,ND                                                 568.
  320       H=SQRT((X(I)-X(J))*(X(I)-X(J))+(Y(I)-Y(J))*(Y(I)-Y(J)))       569.
  321       IF(H.LT.0.001*TOL) GO TO 23                                   570.
  322       K=INT(H/PAS+0.5)+1                                            571.
  323       IF(K.GT.KMAX.OR.ABS(H-(K-1)*PAS).GT.TOL)GO TO 21              572.
  324       DO 22 IV=1,NV                                                 573.
  325       IJ=I+ND*(IV-1)                                                574.
  326       JI=J+ND*(IV-1)                                                575.
  327       IF(VR(IJ).LE.TEST.OR.VR(JI).LE.TEST)GO TO 22                  576.
  328       IK=K+KMAX*(IV-1)                                              577.
  329       NC(IK)=NC(IK)+1                                               578.
  330       D(IK)=D(IK)+H                                                 579.
  331       VRR=VR(IJ)-VR(JI)                                             580.
  332       G(IK)=G(IK)+0.5*VRR*VRR                                       581.
  333    22 CONTINUE                                                      582.
  334       GO TO 21                                                      583.
  335    23 WRITE(IOUT,2000)I,X(I),Y(I),J,X(J),Y(J)                       584.
  336    21 CONTINUE                                                      585.
  337     2 CONTINUE                                                      586.
      C                                                                   587.
      C              RESULTS                                              588.
      C                                                                   589.
  338       DO 3 IV=1,NV                                                  590.
  339       IF(N(IV).EQ.0)GO TO 3                                         591.
  340       V(IV)=(V(IV)-U(IV)*U(IV)/N(IV))/N(IV)                         592.
  341       U(IV)=U(IV)/N(IV)                                             593.
  342       IK0=KMAX*(IV-1)+1                                             594.
  343       IK1=IK0+KMAX-1                                                595.
  344       DO 30 IK=IK0,IK1                                              596.
  345       NCC=MAX0(1,NC(IK))                                            597.
  346       D(IK)=D(IK)/NCC                                               598.
  347                                                                     599.
```

```
     1BX,'*** GAM-2 ***'/50X,'**************** ',20X,
     2'( REGULAR 2-DIMENSIONAL GRID  )'/)
 2001 FORMAT(1H ,'     MEAN = ',F10.5,5X,' VARIANCE = ',E11.5,5X,' NUMB
     1,'  OF  DATA = ',I4)
 2002 FORMAT(1H ,6X,'|',6(A1,2X,'DIRECTION ',I2,3X,'|'))
 2003 FORMAT(1H ,' LAG  |',6(A1,'NC   VARIOGRAM   |'))
 2004 FORMAT(1H ,1X,I3,2X,'|',6(I4,2X,E11.5,1X,'|'))
C
    5 RETURN
      END
C
C
C
C
C
      SUBROUTINE GAM3(VR,NLI,NCO,NIV,KMAX,NC,G,U,V,N,IS)
C
C                    THREE - DIMENSIONAL SEMI-VARIOGRAM
C   REGULARLY SPACED DATA WITH POSSIBLE BLANKS.  1 VARIABLE
C
C   PARAMETERS    VR(NLI*NCO*NIV)  DATA STORED COLUMNWISE
C                 NLI              NUMBER OF LINES
C                 NCO              NUMBER OF COLUMNS
C                 NIV              NUMBER OF LEVELS
C                 KMAX             MAXIMUM NUMBER OF LAGS
C                 NC(KMAX*3)       NUMBER OF PAIRS/LAG/LINE/COLUMN/LEVEL
C                 G(KMAX*3)        SEMI-VARIOGRAMS
C                 U                MEAN        |
C                 V                VARIANCE    | OF DATA NOT BLANK
C                 N                NUMBER      |
C
C   OPTIONS       IS.NE.1          IF IS .NE. 1 EDIT RESULTS (1 PAGE
C                                  IF KMAX .LE. 50)
C
C
C   COMMON        IOUT             PRINTER
C                 TEST             INDICATOR OF BLANK DATA: VR .LE. TEST
C
C
C
      DIMENSION VR(1),NC(1),G(1)
      DOUBLE PRECISION DUMMY(40)
      COMMON INP,IOUT,TEST,NDUMMY,DUMMY
      U=0.
      V=0.
      N=0
      KM=3*KMAX
      DO 1 K=1,KM
      NC(K)=0.
    1 G(K)=0.
      DO 2 L=1,NIV
      N3=MINO(NIV,L+KMAX)
      DO 2 J=1,NCO
      N2=MINO(NCO,J+KMAX)
      DO 2 I=1,NLI
      ID1=I+NLI*(J-1)+NLI*NCO*(L-1)
      N1=MINO(NLI,I+KMAX)
      VR1=VR(ID1)
      IF(VR1.LE.TEST)GO TO 23
C
C           COMPUTE MEAN, VARIANCE, AND NUMBER OF DATA
```



```
C
      U=U+VR1
      V=V+VR1*VR1
      N=N+1
      I0=I+1
      J0=J+1
      L0=L+1
      IF(I.EQ.N1)GO TO 24
C
C                 VARIOGRAM/LINE
C
      DO 20 I1=I0,N1
      ID2=ID1+I1-I
      IF(VR(ID2).LE.TEST)GO TO 20
      K=I1-I
      NC(K)=NC(K)+1
      G(K)=G(K)+(VR(ID2)-VR1)*(VR(ID2)-VR1)
   20 CONTINUE
   24 IF(J.EQ.NCO)GO TO 25
C
C                 VARIOGRAM/COLUMN
C
      DO 21 J1=J0,N2
      ID2=ID1+NLI*(J1-J)
      IF(VR(ID2).LE.TEST)GO TO 21
      K=KMAX+(J1-J)
      NC(K)=NC(K)+1
      G(K)=G(K)+(VR(ID2)-VR1)*(VR(ID2)-VR1)
   21 CONTINUE
   25 IF(L.EQ.NIV)GO TO 23
C
C                 VARIOGRAM/LEVEL
C
      DO 22 L1=L0,N3
      ID2=ID1+NLI*NCO*(L1-L)
      IF(VR(ID2).LE.TEST)GO TO 22
      K=2*KMAX+(L1-L)
      NC(K)=NC(K)+1
      G(K)=G(K)+(VR(ID2)-VR1)*(VR(ID2)-VR1)
   22 CONTINUE
   23 CONTINUE
    2 CONTINUE
      DO 3 K=1,KM
    3 G(K)=G(K)/2/MAX0(1,NC(K))
      V=(V-U*U/MAX0(1,N))/MAX0(1,N)
      U=U/MAX0(1,N)
      IF(IS.EQ.1)GO TO 5
C
C                 EDIT  IF IS .NE. 1
C
      WRITE(IOUT,2000)NLI,NCO,NIV,U,V,N,KMAX
      DO 4 K=1,KMAX
    4 WRITE(IOUT,2001)K,(NC(K1),G(K1),K1=K,KM,KMAX)
C
 2000 FORMAT(1H1,53X,'SEMI - VARIOGRAM 3-D',35X,'*** GAM-3 ***'/1H ,
     153X,'**************** ',
     2'***'/1H ,'NUMBER OF LINES   :',I4,' OF COLUMNS  :',I4,' OF LEVEL'
     3,'S  :',I4/1H ,'DATA     :     MEAN=',E11.5,' VARIANCE=',E11.5,
     4' NUMBER=',I5/1H ,'MAXIMUM LAG =',I4/1H ,' LAG    N.PAIRS   VARIO'
     5,'/LINES      N.PAIRS   VARIO/COLUMNS      N.PAIRS   VAR  [illegible]'
```

```
                    MAXIMUM OF 6 DIRECTIONS/VARIABLE/PAGE
                                                                    50
COMMONS     INP
            IOUT            PRINTER #
            TEST            INDICATOR OF BLANK DATA :
                            VR.LE.TEST
            NV              NUMBER OF VARIABLES ;
            NAM(NV)         NAME OF VARIABLES (A8)

COMMENTS      INDICATORS ID, JD TELL THE POSITION OF THE POINT
              NEAREST TO POINT (I,J) IN THE DIRECTION CONSIDERED
              FOR EXAMPLE:

                     N-S   (//COLUMNS )    ID=1  JD=0
                     W-E   (//LINES   )    ID=0  JD=1
                    NW-SE  (1- DIAGONAL)   ID=1  JD=1
                    NE-SW  (2- DIAGONAL)   ID=-1 JD=1

      DIMENSION VR(1),NC(1),G(1),U(1),V(1),N(1),ID(1),JD(1)
      DOUBLE PRECISION NAM(10)
      COMMON INP,IOUT,TEST,NV,NAM
      DATA IA/' '/
      KDI=NV*KMAX*NDI
      DO 1 IV=1,NV
      U(IV)=0.
      V(IV)=0.
    1 N(IV)=0
      DO 10 IK=1,KDI
      NC(IK)=0
   10 G(IK)=0.

                COMPUTE VARIOGRAM. NEW DATA

      DO 2 I=1,NLI
      DO 2 J=1,NCO
      IJ=I+NLI*(J-1)

                COMPUTE STATISTICS

      IND=0
      DO 3 IV=1,NV
      IP0=NLI*NCO*(IV-1)
      IP=IJ+IP0
      VR1=VR(IP)
      IF(VR1.LE.TEST)GO TO 3
      IND=1
      N(IV)=N(IV)+1
      V(IV)=V(IV)+VR1*VR1
      U(IV)=U(IV)+VR1
    3 CONTINUE
      IF(IND.LE.0)GO TO 2

                                    NEW DIRECTION

      DO 30 KD=1,NDI
      J3=JD(KD)
      I3=ID(KD)
      I1=I
      J1=J
```

```
                                    NEW LAG                          302.
C                                                                    303.
      DO 31 K=1,KMAX                                                 304.
      I1=I1+I3                                                       305.
      IF(I1.LT.1.OR.I1.GT.NLI)GO TO 30                               306.
      J1=J1+J3                                                       307.
      IF(J1.LT.1.OR.J1.GT.NCO)GO TO 30                               308.
      IJ1=I1+NLI*(J1-1)                                              309.
      IK0=K+KMAX*(KD-1)                                              310.
      DO 32 IV=1,NV                                                  311.
      IP0=NLI*NCO*(IV-1)                                             312.
      IP=IJ+IP0                                                      313.
      IP1=IJ1+IP0                                                    314.
      IF(VR(IP).LE.TEST.OR.VR(IP1).LE.TEST)GO TO 32                  315.
      IK=IK0+KMAX*NDI*(IV-1)                                         316.
      NC(IK)=NC(IK)+1                                                317.
      VRR=VR(IP1)-VR(IP)                                             318.
      G(IK)=G(IK)+0.5*VRR*VRR                                        319.
   32 CONTINUE                                                       320.
   31 CONTINUE                                                       321.
   30 CONTINUE                                                       322.
    2 CONTINUE                                                       323.
C                                                                    324.
C               RESULTS                                              325.
C                                                                    326.
      DO 4 IV=1,NV                                                   327.
      IF(N(IV).EQ.0)GO TO 4                                          328.
      V(IV)=(V(IV)-U(IV)*U(IV)/N(IV))/N(IV)                          329.
      U(IV)=U(IV)/N(IV)                                              330.
      IK0=NDI*KMAX*(IV-1)+1                                          331.
      IK1=IK0+KMAX*NDI -1                                            332.
      DO 40 IK=IK0,IK1                                               333.
   40 G(IK)=G(IK)/MAX0(1,NC(IK))                                     334.
    4 CONTINUE                                                       335.
      IF(IS.EQ.1)GO TO 5                                             336.
C                                                                    337.
C               IF IS .NE. 1 EDIT VARIABLE/VARIABLE  MAXIMUM OF 6    338.
C               DIRECTIONS/PAGE.  IF KMAX .LE. 50, EDITION ON        339.
C               A SINGLE PAGE                                        340.
C                                                                    341.
      IMP=(NDI-1)/6+1                                                342.
      IDM=FLOAT(NDI)/FLOAT(IMP)+0.9999                               343.
   42 DO 43 IV=1,NV                                                  344.
      DO 44 IM=1,IMP                                                 345.
      WRITE(IOUT,2000)NAM(IV),IM                                     346.
      WRITE(IOUT,2001)U(IV),V(IV),N(IV)                              347.
      ID1=1+IDM*(IM-1)                                               348.
      ID2=MIN0(NDI,IDM*IM)                                           349.
      WRITE(IOUT,2002)(IA,ID,ID=ID1,ID2)                             350.
      IK0=NDI*KMAX*(IV-1)+KMAX*(ID1-1)                               351
      IKM=IK0+KMAX*(ID2-ID1)                                         352
      WRITE(IOUT,2003)(IA,ID=ID1,ID2)                                353
      DO 45 K=1,KMAX                                                 354
      IK1=K+IK0                                                      355
      IK2=K+IKM                                                      356
   45 WRITE(IOUT,2004)K,(NC(IK),G(IK),IK=IK1,IK2,KMAX)               357
   44 CONTINUE                                                       358
   43 CONTINUE                                                       359
C                                                                    360
 2000 FORMAT(1H1,45X,'    SEMI - VARIOGRAM      ',A8,10X,'***PAGE',I2, 361
```

```
      C
            KM=NV*KMAX
            DO 1 IK=1,KM
            NC(IK)=0
          1 G(IK)=0.
            DO 11 IV=1,NV
            IJ=ND*IV
            IF(VR(IJ)-TEST)12,12,13
         12 N(IV)=0
            U(IV)=0.
            V(IV)=0.
            GO TO 11
         13 U(IV)=VR(IJ)
            V(IV)=VR(IJ)*VR(IJ)
            N(IV)=1
         11 CONTINUE
      C
      C                  COMPUTE VARIOGRAM AND STATISTICS
      C
            ND1=ND-1
            DO 2 I=1,ND1
            IND=0
            DO 20 IV=1,NV
            IJ=I+ND*(IV-1)
            VR1=VR(IJ)
            IF(VR1.LE.TEST)GO TO 20
            IND=1
            N(IV)=N(IV)+1
            U(IV)=U(IV)+VR1
            V(IV)=V(IV)+VR1*VR1
         20 CONTINUE
            IF(IND.EQ.0)GO TO 2
            I1=I+1
            JM=MIN0(I+KMAX,ND)
            DO 21 J=I1,JM
            K=J-I
            DO 22 IV=1,NV
            IJ=I+ND*(IV-1)
            JI=J+ND*(IV-1)
            IF(VR(IJ).LE.TEST.OR.VR(JI).LE.TEST)GO TO 22
            IK=K+KMAX*(IV-1)
            NC(IK)=NC(IK)+1
            VRR=VR(IJ)-VR(JI)
            G(IK)=G(IK)+0.5*VRR*VRR
         22 CONTINUE
         21 CONTINUE
          2 CONTINUE
      C
      C                  RESULTS
      C
            DO 3 IV=1,NV
            IF(N(IV).EQ.0)GO TO 3
            V(IV)=(V(IV)-U(IV)*U(IV)/N(IV))/N(IV)
            U(IV)=U(IV)/N(IV)
            IK0=KMAX*(IV-1)+1
            IK1=IK0+KMAX-1
            DO 30 IK=IK0,IK1
         30 G(IK)=G(IK)/MAX0(1,NC(IK))
          3 CONTINUE
            IF(IS.EQ.1)GO TO 5
```

```
      C
      C                  EDIT RESULTS (6 VARIABLES PER PAGE)  49
      C                  IF  IS .NE. 1
      C
            INP=(NV-1)/6+1
            INV=FLOAT(NV)/FLOAT(INP)+0.9999
            DO 4 IN=1,INP
            IV1=1+INV*(IN-1)
            IV2=MIN0(NV,INV*IN)
            WRITE(IOUT,2000)NDL,IN
            WRITE(IOUT,2001)(NAM(IV),IV=IV1,IV2)
            WRITE(IOUT,2002)(U(IV),IV=IV1,IV2)
            WRITE(IOUT,2003)(V(IV),IV=IV1,IV2)
            WRITE(IOUT,2004)(N(IV),IV=IV1,IV2)
            WRITE(IOUT,2005)(IA,IV=IV1,IV2)
            DO 41 K=1,KMAX
            IK1=K+KMAX*(IV1-1)
            IK2=K+KMAX*(IV2-1)
            D=K*PAS
         41 WRITE(IOUT,2006) K,D,(NC(IK),G(IK),IK=IK1,IK2,KMAX)
          4 CONTINUE
      C
       2000 FORMAT(1H1,48X,'    SEMI-VARIOGRAM      ',I4,10X,'***PAGE:',I2,
           110X,'*** GAM-1 ***'/1H ,50X,'**************** ',10X,
           2'(REGULAR 1- DIMENSIONAL GRID )'/)
       2001 FORMAT(1H ,16X,6(5X,A3,6X))
       2002 FORMAT(1H ,4X,'  MEAN ',4X,6(4X,F9.4,6X))
       2003 FORMAT(1H ,4X,'VARIANCE',3X,6(3X,E11.5,5X))
       2004 FORMAT(1H ,' NUMBER OF DATA ',6(5X,I4,13X))
       2005 FORMAT(1H ,' LAG  DISTANCE ','|',6(A1,'NC   VARIOGRAM   |'))
       2006 FORMAT(1H ,1X,I2,2X,F8.3,2X,'|',6(I4,2X,E11.5,1X,'|'))
      C
          5 RETURN
            END
      C
      C
      C
      C
      C
            SUBROUTINE GAM2(VR,NLI,NCO,KMAX,NDI,ID,JD,NC,G,U,V,N,IS)
      C
      C                  TWO-DIMENSIONAL SEMI-VARIOGRAM
      C  REGULAR SPACING WITH POSSIBLE BLANKS.  NV VARIABLES
      C
      C  PARAMETERS   VR(NLI*NCO*NV)   ARRAY OF DATA STORED COLUMNWISE
      C                                VARIABLE / VARIABLE
      C               NLI              NUMBER OF LINES OF THE NETWORK
      C               NCO              NUMBER OF COLUMNS OF THE NETWORK
      C               KMAX             MAXIMUM NUMBER OF LAGS
      C               NDI              NUMBER OF DIRECTION TO BE CONSIDERED
      C               ID(NDI)          LINE INDICATOR OF DIRECTION
      C               JD(NDI)          COLUMN INDICATOR OF DIRECTION
      C               NC(KMAX*NDI*NV)  NUMBER OF PAIRS/LAG/DIRECTION/VARIABLE
      C               G(KMAX*NDI*NV)   SEMI - VARIOGRAM
      C               U(NV)            MEAN     ]
      C               V(NV)            VARIANCE|PER VARIABLE (BLANKS
      C               N(NV)            # DATA  |    NOT CONSIDERED)
      C
      C  OPTIONS      IS.NE.1          EDIT RESULTS
```

```
//GAM1 JOB
C
C                                                          96
C
C
      DIMENSION VR(100),NC(120),G(120),D(120),N(2),U(2),V(2),
     *          ID(6),JD(6),X(40),Y(40),Z(40),ALP(6),VR1(40),
     *          VR2(40),XP(20),YP(20),ZP(20)
      DOUBLE PRECISION NAM(10)
      COMMON INP,IOUT,TEST,NV,NAM
      INP=5
      IOUT=6
      TEST=0.
      KMAX=10
      IS=0
C
C  READ INPUT DATA
C
      READ(INP,1000) (NAM(I),I=1,10)
      READ(INP,1001) NV,ND,NDI,NDH
      READ(INP,1001) (ID(I),JD(I),I=1,NDI)
      READ(INP,1002) (ALP(I),I=1,NDI)
      READ(INP,1002) (X(I),Y(I),Z(I),VR(I),VR(I+ND),I=1,ND)
C
C  ARRANGE OF DATA BELONGING TO EACH DRILL HOLE
C
      DO 1 I=1,NDH
      VR1(I)=VR(I)
      VR1(I+NDH)=VR(I+ND)
      VR2(I)=VR(I+NDH)
      VR2(I+NDH)=VR(I+ND+NDH)
      XP(I)=X(I+NDH)
      YP(I)=Y(I+NDH)
    1 ZP(I)=Z(I+NDH)
C
C  PRINT OUT DATA
C
      WRITE(IOUT,2000) (NAM(I),I=1,NV)
      WRITE(IOUT,2003)
      WRITE(IOUT,2001) (X(I),Y(I),Z(I),VR(I),VR(I+ND),I=1,NDH)
      WRITE(IOUT,2004)
      WRITE(IOUT,2001) (XP(I),YP(I),ZP(I),VR2(I),VR2(I+NDH),I=1,NDH)
      WRITE(IOUT,2002) NV,ND
C
C  CALL SUBROUTINES
C
C
C     SEMI-VARIOGRAM SUBROUTINES
C     ONE-DIMENSIONAL, TWO-DIMENSIONAL, AND THREE-DIMENSIONAL
C     CONFIGURATIONS ARE CONSIDERED IN THIS CASE
C     GAM1 IS CALLED THICE (ONE TIME PER DRILL HOLE)
C     GAM2 CONSIDERS TWO-DIMENSIONAL DATA AS BEING PROJECTED
C     ON A HORIZONTAL PLANE
C
      CALL GAM1(VR1,NDH,KMAX,NC,G,U,V,N,10.,1,IS)
      CALL GAM1(VR2,NDH,KMAX,NC,G,U,V,N,10.,2,IS)
      CALL GAM2(VR,4,10,KMAX,NDI,ID,JD,NC,G,U,V,N,IS)
      CALL GAM3(VR,2,4,10,KMAX,NC,G,DUM1,DUM2,N,IS)
C
C     1-D AND 2-D CONFIGURATIONS OF IRREGULAR SPACED DATA
C     ARE HANDLED BY SUBROUTINES GAMV1 AND GAMV2                    47
C
      CALL GAMV1(VR1,X,Y,NDH,KMAX,10.0,0.,NC,G,D,U,V,N,1,IS)
      CALL GAMV1(VR2,XP,YP,NDH,KMAX,10.,0.,NC,G,D,U,V,N,2,IS)
      CALL GAMV2(VR,X,Y,ND,KMAX,10.0,0.,NDI,ALP,0.,NC,G,D,U,V,N,IS)
C
C  NOTE THAT THE SAME SET OF DATA WAS USED, AS SAKE OF
C  ILLUSTRATION, IN CONSTRUCTING BOTH REGULAR SPACED DATA
C  SEMI-VARIOGRAMS AND IRREGULAR SPACED DATA SEMI-VARIOGRAMS
C
 1000 FORMAT(10A8)
 1001 FORMAT(8I5)
 1002 FORMAT(5F10.0)
 2000 FORMAT(///44X,'INPUT DATA EMPLOYED IN THIS RUN'///,
     1 20X,'COORDINATES  (X,Y,Z) ',10X,'VARIABLE : ',A8,
     2 20X,'VARIABLE : ',A8//)
 2001 FORMAT(1H ,17X,'(',F8.2,',',F8.2,',',F8.2,')',10X,F10.2,19X,F10.2)
 2002 FORMAT(///17X,'NUMBER OF VARIABLES = ',I4,10X,
     1'TOTAL NUMBER OF DATA POINTS = ',I4//)
 2003 FORMAT(//12X,'FIRST DRILLHOLE')
 2004 FORMAT(///12X,'SECOND DRILLHOLE')
      STOP
      END
C
C
C
C
      SUBROUTINE GAM1(VR,ND,KMAX,NC,G,U,V,N,PAS,NOL,IS)
C
C                1 - DIMENSIONAL SEMI-VARIOGRAM
C  REGULARLY SPACED DATA WITH POSSIBLE BLANKS.  NV VARIABLES
C
C  PARAMETERS   VR(ID*NV)     ARRAY OF DATA STORED COLUMNWISE
C                             VARIABLE/VARIABLE
C               ND            NUMBER OF DATA ON THE LINE
C               KMAX          MAXIMUM NUMBER OF LAGS
C               NC(KMAX*NV)   NUMBER OF PAIRS/LAG AND/VARIABLE
C               G(KMAX*NV)    SEMI - VARIOGRAM
C               U(NV)         MEAN    |
C               V(NV)         VARIANCE|FOR EACH VARIABLE (BLANKS
C               N(NV)         # DATA  |          NOT CONSIDERED)
C               PAS           LENGTH OF THE UNIT LAG
C               NOL           NUMBER OF THE LINE OR VARIOGRAM
C
C  OPTIONS      IS.NE.1       EDIT RESULTS.  IF KMAX .LT. 50, A MAXIMUM
C                             OF 6 VARIABLES ARE EDITED PER PAGE
C
C  COMMON       INP
C               IOUT          PRINTER
C               TEST          INDICATOR OF BLANK DATA: VR .LE. TEST
C               NV            NUMBER OF VARIABLES
C               NAM(NV)       NAME OF VARIABLES (A8)
C
      DIMENSION VR(1),NC(1),G(1),N(1),U(1),V(1)
      DOUBLE PRECISION NAM(10)
      COMMON INP,IOUT,TEST,NV,NAM
      DATA IA/' '/
C
C             INITIALIZE
```

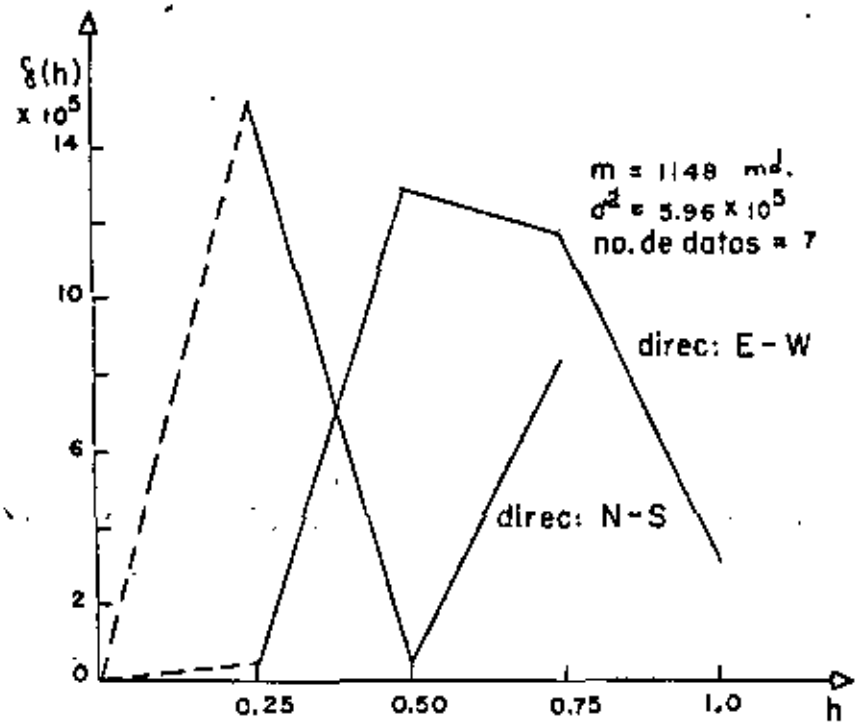


Figura 16 - Semi-variograma de Permeabilidades.

CONCLUSION

El objetivo del análisis estructural es detectar, analizando los diversos variogramas experimentales, las principales características estructurales del fenómeno regionalizado que se estudia. La información obtenida de este análisis debe ser comparada con las características del fenómeno conocidas por otras evidencias (geológicas, mineralógicas, procedimientos de muestreo, etc.). Bajo ninguna circunstancia, un estudio estructural deberá reemplazar una campaña geológica (o geofísica) de exploración, por lo contrario, el estudio debe ser guiado por la geología. El análisis estructural complementa y enriquece el conocimiento geológico del fenómeno cuantificando la información para usos posteriores de estimación.

$$\gamma(h) = \begin{cases} 0.30\left(\frac{3}{2}\left(\frac{h}{18}\right) - \frac{1}{2}\left(\frac{h}{18}\right)^3\right) \forall\ h \leq 18\ \text{Km.} \\ 0.30 \quad 18\ \text{km} \leq h \leq 30\ \text{Km.} \end{cases}$$

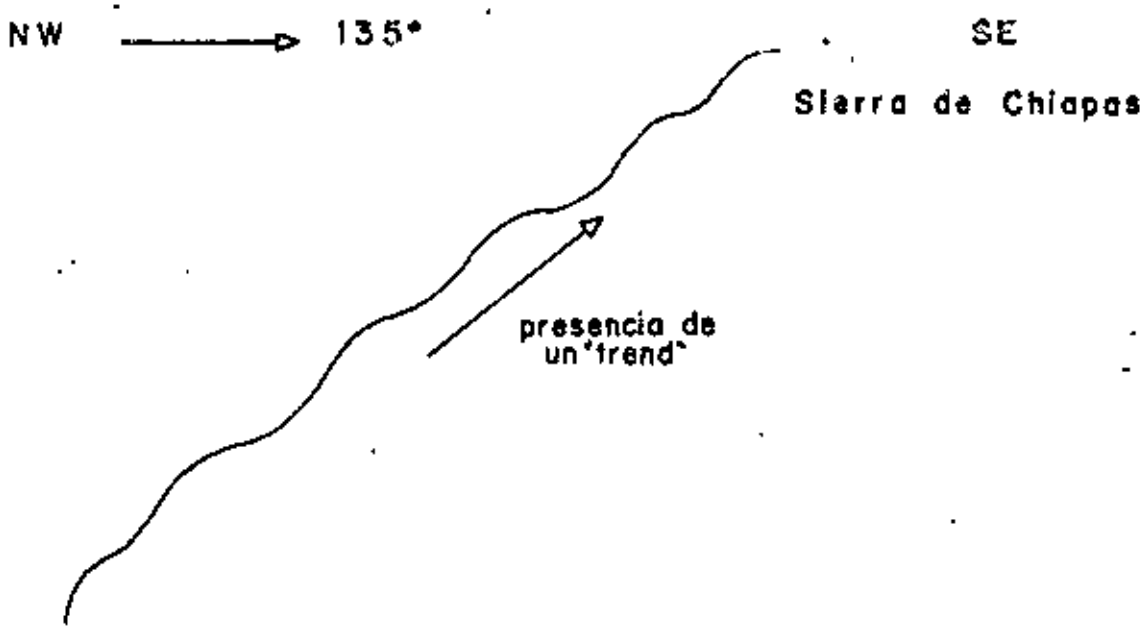
CASO: Permeabilidades "Campo Acuatempa"

El efecto de pepita puro suele "aparecer" en aquellos casos donde el modelo de transición $\gamma_o(h)$, sí existe, tiene un rango "a" mucho menor al de la dimensión del soporte de los datos, $a << v$. La dimensión del soporte v cubre toda la variabilidad del $\gamma_o(h)$, aparentando las características del efecto de pepita puro.

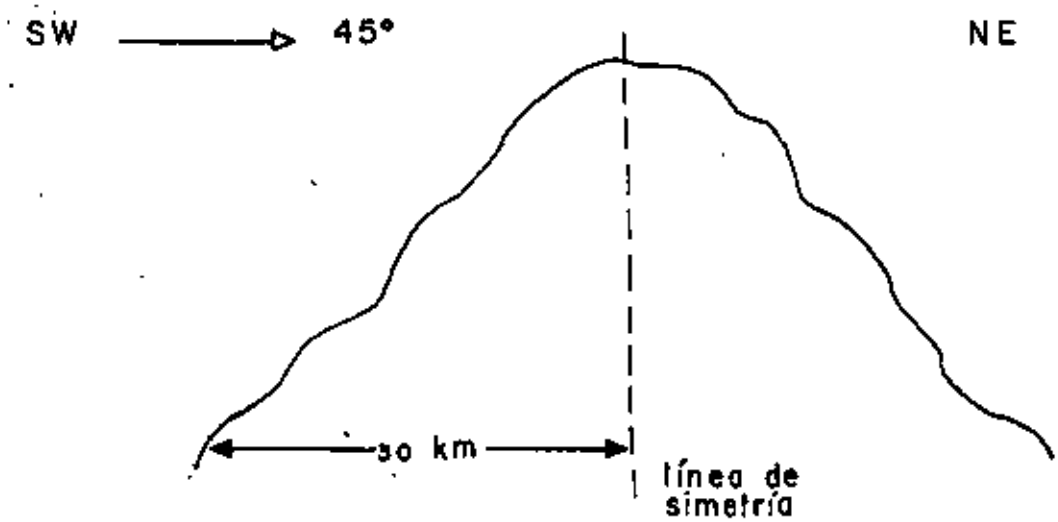
Es frecuente asumir equivocadamente, la hipótesis del efecto de pepita puro, cuando la razón del comportamiento de $\gamma_o(h)$ no es sino la escasez de información o el suavizamiento de los datos.

La Figura 16 muestra un semi-variograma experimental obtenido a partir de datos de permeabilidad de 7 pozos del campo Acuatempa. Este es el caso donde por la escasez de información se generó un efecto de fluctuación alrededor de lo que se suponía el "sill", simulando el efecto de pepita puro.

Cada dato de permeabilidad está asociado a un soporte v aproximadamente igual al volumen de un cilindro de radio, el radio de drene del pozo, y de altura, el espesor de formación. Si el modelo de transición $\gamma_o(h)$ existiese en este caso, éste tendría como rango una dimensión a mucho menor a la dimensión v.



(A)



(B)

Figura 15

Perfil estructural de la Formación Tamabra, sureste de México.

(iii) Para pares de puntos con distanciamientos, entre sí, menores a 30 Km. el fenómeno puede considerarse como estacionario (o quasi estacionario) e isotrópico, y puede caracterizarse por un semi-variograma teórico (esférico) con rango, a = 18 Km y sill C(o) = 0.30.

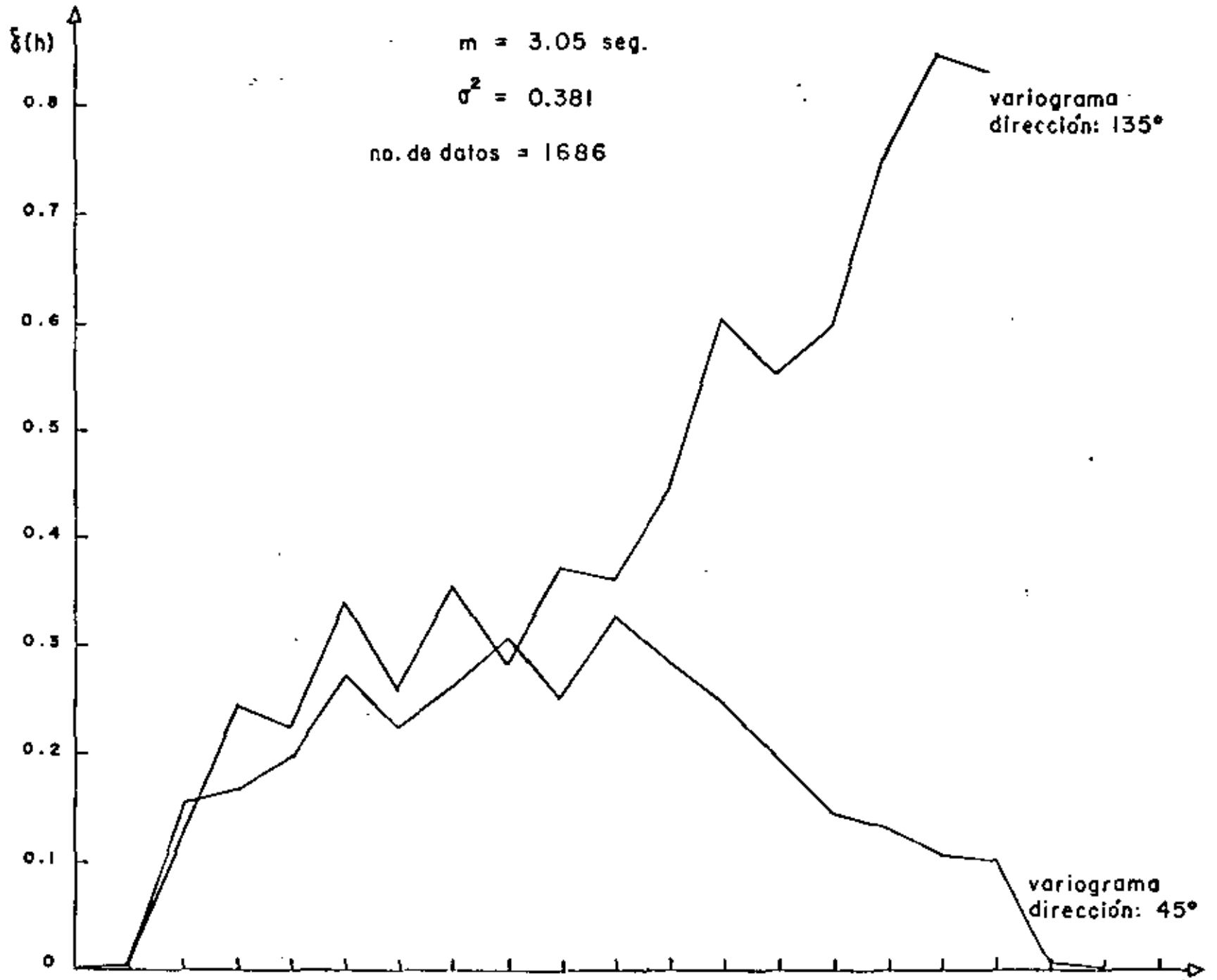

γ̂(h)
m = 3.05 seg.
σ² = 0.381
no. de datos = 1686
variograma
dirección: 135°
variograma
dirección: 45°
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
6
12
18
24
30
36
42
48
54
h

FIGURA 14

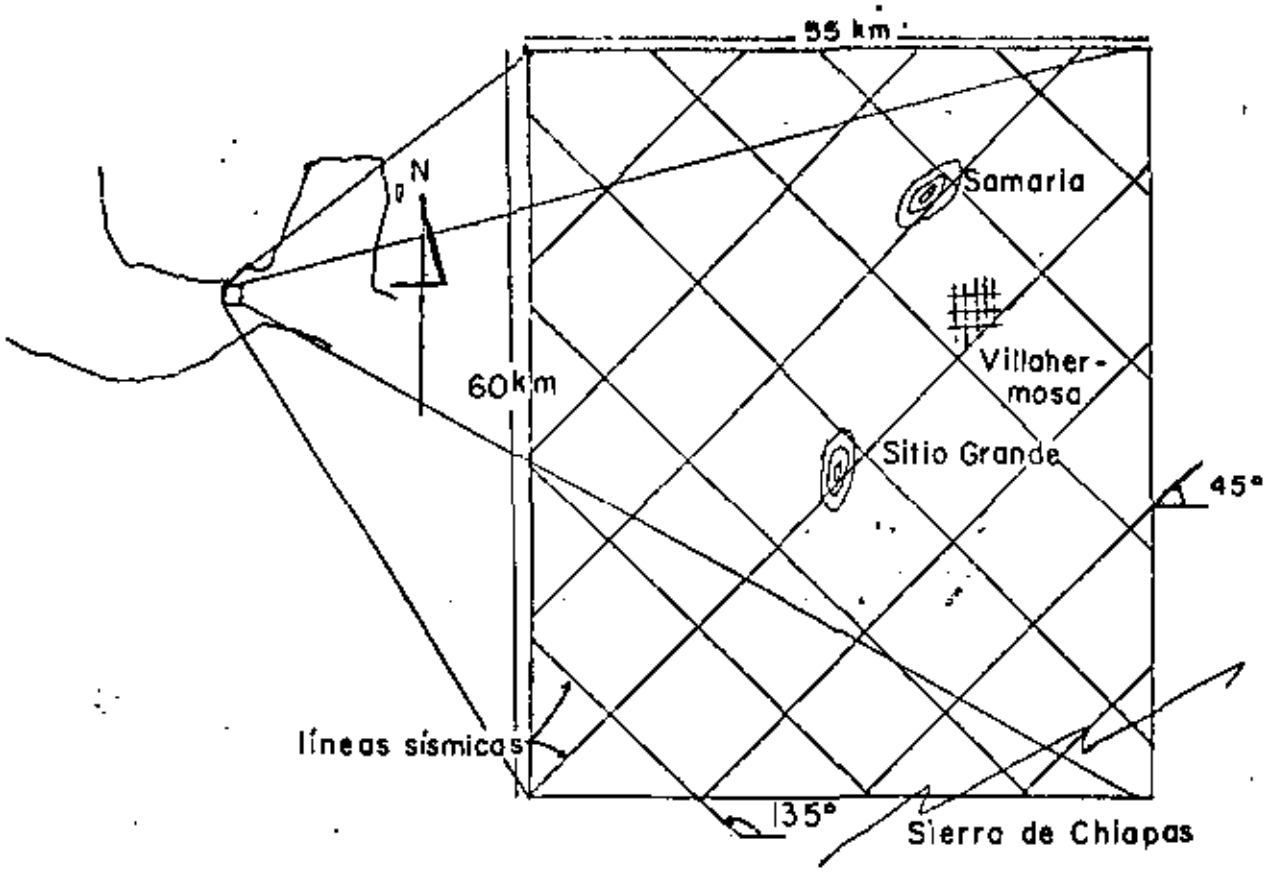


Figura 13

El semi-variograma obtenido en la dirección 45° muestra una caída brusca en sus valores, también a partir de h = 30 Km., lo cual sugiere la presencia, globalmente hablando, de una estructura simétrica (con respecto a la línea NW-SE, ver figuras 15-A y 15-B, la cual está generada, como se sabe, por el empuje de un domo salino subyaciente.

por las computadoras, haría del análisis estructural una técnica totalmente impráctica. Por estos motivos, el cálculo de los semi-variogramas experimentales se efectúa empleando programas de cómputo. Seis subrutinas, una para cada uno de los casos mencionados arriba, se listan a continuación.

CASO: Tiempos de reflexión

A partir de información estructural obtenida en el Distrito Villahermosa de la Zona Sur que comprende los campos Sitio Grande, Samaria y otros (Figura 13), se evaluaron semi-variogramas experimentales de tiempos de reflexión a lo largo de dos direcciones 45 y 135 grados, con respecto a la línea oeste-este. Estos semi-variogramas presentan las siguientes características (Figura 14):

(i) En los dos casos (45° y 135°) no existe presencia aparente de "nugget effect", (al menos a la escala de los datos experimentales).

(ii) Ambos presentan un fenómeno de transición entre el origen y una distancia de cerca de 30 Km. El semi-variograma obtenido en la dirección 135° muestra un incremento brusco en sus valores a partir de h = 30 Km. indicando con ello la presencia de un "trend", el cual conocíamos de antemano por evidencias geológicas. Se sabe, por ejemplo, que la formación reflectora (Formación Tamabra) aflora hacia el sureste en la Sierra de Chiapas.

donde $\bar{\gamma}(x_i, V)$ denota el valor medio de $\gamma(h)$ cuando un extremo del vector h está fijo en el punto x_i y el otro extremo describe independientemente el volumen V.

Comentarios

1.- Estas fórmulas de la variancia de estimación son completamente generales para cualesquiera que sean los dominios v y V.

2.- La función $2\gamma(h)$ puede interpretarse también como la variancia de estimación generada al tratar de estimar la variable $Z(x)$ por medio de la variable $Z(x+h)$:

$$\sigma_E^2 = E\left[(Z(x+h) - Z(x))^2\right] = 2\gamma(x+h,x) - \cancel{\gamma(x,x)}^{=0} - \cancel{\gamma(x+h,x+h)}^{=0}$$

$$= 2\gamma(h)$$

3.- Las fórmulas de σ_E^2 expresan cuatro conceptos esenciales e intuitivos que cualquier buen proceso de estimación debería de expresar:

(i) Análisis del término $\bar{\gamma}(V, V)$ (ecuación 3). Dado que $\bar{\gamma}(h)$ se incrementa con h, entonces $\bar{\gamma}(V, V)$ se incrementará con el tamaño de V. Considerando al dominio v y a la distancia (v, V) fijos, será más fácil estimar el valor medio de un bloque V, que el valor asociado a un punto desconocido (V = x). Si V permanece fijo, el término $\bar{\gamma}(V, V)$ y consecuentemente σ_E^2, dependerán de la geometría de V.

(ii) Análisis del término $\bar{\gamma}(V, v)$ (ecuación 3). Si la distancia (v, V) se incrementa, así lo hace el término $\bar{\gamma}(v, V)$ - y por consiguiente σ_E^2 también se incrementa.

(iii) Análisis del término $\bar{\gamma}(v, v)$ (ecuación 3. Si el dominio v crece, $\bar{\gamma}(v,v)$ también crece y σ_E^2 disminuye. Considerando fijos a los volúmenes V y v, y a la distancia (v, V), la variancia de estimación dependerá de la configuración de la información v. En la Figura 18, por ejemplo, el bloque V estará mejor estimado por las dos muestras v_1 y v_2 cuando éstas están separadas, que por las dos muestras v_1' y v_2' cuando éstas están juntas. En efecto, $\bar{\gamma}(v_1' + v_2', v_1' + v_2')$ será menor que $\bar{\gamma}(v_1 + v_2, v_1 + v_2)$. Esta noción intuitiva de la importancia en la configuración de los datos formalizada en geoestadística a través del término $\bar{\gamma}(v, v)$, es ignorada por los métodos más comunes de estimación.

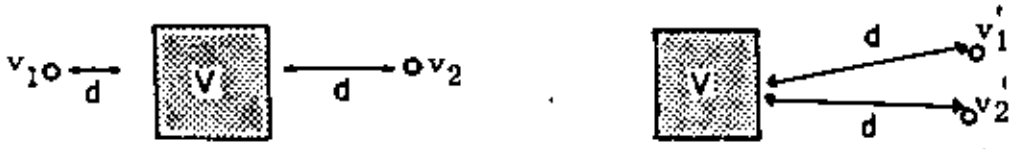


Figura 18

(iv) Análisis de la función $\gamma(h)$. La calidad de la estimación dependerá obviamente de las características estructurales del fenómeno. En yacimientos petrolíferos, la variación en los valores de la permeabilidad será mucho más conti-

nua en una dirección horizontal que en una dirección vertical. Considerando esta anisotropía en el semi-variograma, se asignará, en la estimación de V, un mayor peso a la muestra v_1 que se encuentra en el mismo estrato que V, que a la muestra v_2 de un estrato diferente (Figura 19).

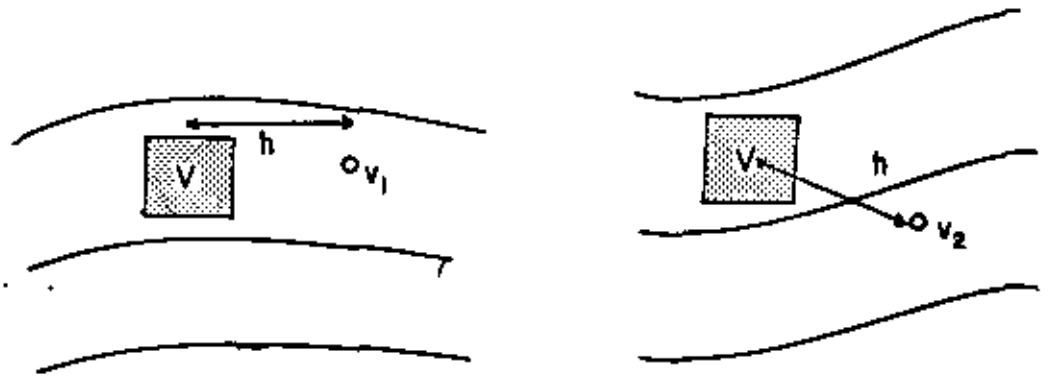


Figura 19

5.- La fórmula (*) expresa σ_E^2 como una función lineal de los n pesos λ_i. El método Kriging de estimación determina el conjunto óptimo de pesos λ_i, tales que σ_E^2 es minimizada y la condición de insesgamiento ($\Sigma\lambda_i = 1$) es satisfecha. Por ello, kriging es reconocido como el mejor estimador lineal insesgado.

6.- La represión de σ_E^2 en la fórmula (4) es general, ya que puede aplicarse sobre cualquier volumen V y para cualquier conjunto $\{\lambda_i,\ i=1,\ \ldots,\ n\}$ sujeto a la condición $\Sigma\lambda_i=1$. De aquí que esta fórmula pueda emplearse en el cálculo de la variancia de estimación de otros métodos lineales de estimación, tales como el método de "ponderación con respecto al inverso del cuadrado de la distancia".

Cálculo de los valores medios $\overline{\gamma}$.

La función $\overline{\gamma}(v, V)$ introducida en la sección anterior se emplea en el cálculo de la variancia de estimación y también, como veremos más adelante, forma parte fundamental del método Kriging de estimación.

$$\overline{\gamma}(v, V) = 1/(vV)\int_{v} d_x \int_{V} \gamma(x-x')\, d_{x'}$$

Existen dos métodos para evaluar la función $\overline{\gamma}$:

(i) Mediante el cálculo numérico, los dominios v y V pueden discretizarse permitiendo emplear sumatorias (Σ) en vez de integrales.- El uso constante de las computadoras ha hecho de este método el más empleado.

(ii) Mediante el cálculo directo de los integrales.- Asumiendo un cierto modelo $\gamma(h)$ -- exponencial, esférico, lineal, etc. - y ciertos dominios v y V , es posible calcular las integrales que aparecen en la definición de $\overline{\gamma}$. Las expresiones que resultan de resolver las integrales se les conoce por "funciones auxiliares" y se representan por medio de gráficas o bien directamente a través de su expresión analítica.

Funciones auxiliares.

Existen cuatro funciones auxiliares básicas representadas por las letras α , χ, F y H, y definidas sobre dominios de forma rectangular en

una y dos dimensiones.

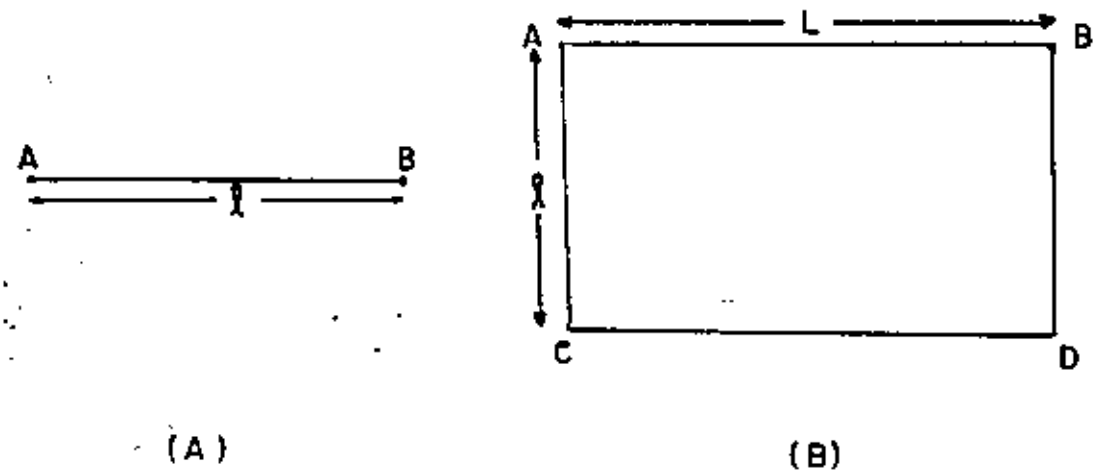
Una dimensión.- Sea AB el segmento de longitud L representado por la Figura 20 A, y $\overline{\gamma}(h)$ un cierto modelo de variabilidad estructural.- La función auxiliar $\chi(L)$ se define como el valor medio de $\gamma(h)$ cuando un extremo del vector h está fijo en el punto A y el otro extremo describe el segmento AB.

$$\chi(L) = \overline{\gamma}(A, AB) = 1/L \int_0^L \gamma(u)\, du$$

La función F(L) se define como el valor medio de $\gamma(h)$ cuando los dos extremos del vector h, describen independientemente el segmento AB.

$$F(L) = \overline{\gamma}(AB, AB) = 1/L^2 \int_0^L du \int_0^L \gamma(u-u')\, du'$$

Dos dimensiones.- Sea ABCD el rectángulo ($L \times \ell$) representado en la Figura 20 B. La función auxiliar $\alpha(L; \ell)$ se define como el valor -



(A) (B)

Figura 20

medio de $\gamma(h)$ cuando un extremo del vector h describe el lado AC y el otro extremo describe independientemente el lado BD.

$$\alpha(L;\ell) = \bar{\gamma}(AC, BD)$$

Procediendo similmente, podemos definir las siguientes funciones auxiliares:

$$\alpha(\ell;L) = \bar{\gamma}(AB, CD)$$

$$\chi(L;\ell) = \bar{\gamma}(AC, ABCD)$$

$$F(L;\ell) = \bar{\gamma}(ABCD, ABCD)$$

$$H(L;\ell) = \bar{\gamma}(A, ABCD)$$

Existe un gran número de relaciones entre las diversas funciones auxiliares, las cuales, por falta de tiempo, no se expondrán aquí.

Para el caso específico de un modelo isotrópico lineal $\gamma(h) = r$, $r = |h|$ se tienen las siguientes funciones auxiliares:

(i) Una dimensión

$$\chi(L) = L/2, \qquad F(L) = L/3$$

(ii) Dos dimensiones

$$\alpha(L;1) = 1/3\ u + 2/3\ L^2/\ell^2\ (L-u) + L^2/\ell \log \frac{\ell+u}{L}$$

$$\chi(L;1) = \frac{1}{6}\frac{L^3}{\ell^2} + u\left(\frac{1}{4} - \frac{L^2}{6\ell^2}\right) + \frac{1}{3}\frac{L^2}{\ell}\log\frac{\ell+u}{L} + \frac{1}{12}\frac{l^2}{L}\log\frac{L+u}{\ell}$$

$$F(L;1) = u\left(\frac{1}{5} - \frac{1}{15}\frac{L^2}{\ell^2} - \frac{1}{15}\frac{\ell^2}{L^2}\right) + \frac{1}{15}\left(\frac{L^3}{\ell^2} + \frac{\ell^3}{L^2}\right) + \frac{1}{6}\frac{L^2}{\ell}\log\frac{\ell+u}{L}$$

$$+ \frac{1}{6}\frac{\ell^2}{L}\log\frac{L+u}{\ell}$$

$$H(L.;\ell) = \frac{1}{3}u + \frac{\ell^2}{6L}\log\frac{L+u}{\ell} + \frac{L^2}{6\ell}\log\frac{\ell+u}{L}$$

donde $u = \sqrt{L^2+\ell^2}$

Para el caso particular donde $L=\ell$ se tiene

$$\alpha(\ell;\ell) = 1.0765\,\ell, \qquad \gamma(\ell;\ell) = 0.7351\,\ell$$

$$F(\ell;\ell) = 0.5213\,\ell, \qquad M(\ell;\ell) = 0.7652\,\ell$$

Nota: Expresiones similares para los casos de modelos exponencial, logarítmico, esférico y r^θ pueden encontrarse en Mining Geostatistics (Journel & Huijbregts-1978).

Ejemplo de aplicación

1.- La variancia de estimación asociada a la evaluación de un segmento AB de longitud ℓ a partir de una muestra central O se obtiene empleando la fórmula 3 como sigue:

$$\sigma_E^2 = 2\,\bar{\gamma}(O,\ AB) - \bar{\gamma}(AB,\ AB) - \bar{\gamma}(O,\ O)$$

donde, debido a la simetría $\bar{\gamma}(O,\ AB) = \bar{\gamma}(O,\ OA) = \bar{\gamma}(O,\ OB) = \gamma(\ell/2)$ y $\bar{\gamma}(AB,\ AB) = F(\ell)$, $\bar{\gamma}(O,\ O) = \gamma(0) = 0$

Finalmente, la variancia de estimación resulta igual a $\sigma_E^2 = 2\gamma(\ell/2) - F(\ell)$

2.- La variancia de estimación de un segmento AB de longitud ℓ estimado por dos muestras localizadas a los extremos del segmento. Sea $E = \{A + B\}$ las dos muestras, entonces:

$$\sigma_E^2 = 2\,\bar{\gamma}(E,\ AB) - \bar{\gamma}(AB,\ AB) - \bar{\gamma}(E,\ E)$$

donde debido a la simetría

$$\bar{\gamma}(E, AE) = \bar{\gamma}(A, AB) = \bar{\gamma}(B, AB) = X(\ell),$$

$$\bar{\gamma}(AB, AB) = F(\ell) \quad y$$

$$\bar{\gamma}(E, E) = \bar{\gamma}(A, E) = 1/2 \left[\gamma(A, A) + \gamma(A, B) \right] =$$

$$= 1/2 \ \gamma(\ell)$$

finalmente

$$\sigma_E^2 = 2X(\ell) - F(\ell) - \gamma(\ell)/2$$

El Método Kriging

El método Kriging es una técnica de estimación local la cual proporciona el mejor estimador lineal insesgado de las características desconocidas del fenómeno en estudio. El objetivo de la estimación local es encontrar el mejor estimador del valor medio de una variable regionalizada asociada a un dominio limitado de dimensiones menores a las dimensiones de la zona de quasi-estacionaridad del fenómeno. Una estimación global, por lo contrario, considera dimensiones mayores a las del límite de quasi-estacionaridad, llegando a abarcar en ocasiones zonas heterogéneas.

La información requerida por el método Kriging consiste de: un conjunto de datos (permeabilidades, porosidades, porcentajes de mineral, tiempos de reflexión, etc.), e información estructural, es decir, el modelo del variograma que caracteriza la variabilidad de la zona estudiada.

El Sistema Kriging. - Sea Z(x) una función aleatoria, de soporte puntual y estacionaria de segundo orden con media $E\left[Z(x)\right] = m$ covariancia $E\left[Z(x+h)\,Z(x)\right] - m^2 = C(h)$ y variograma $E\left[\left\{Z(x+h) - Z(x)\right\}^2\right] = 2\gamma(h)$.

El objetivo es estimar el valor medio de la variable regionalizada $Z_V(x_o)$ asociada al dominio $V(x_o)$ con centro en el punto x_o. Los datos experimentales pueden estar dados por el conjunto de valores $\left\{Z_{v_\alpha}\right.$, $\alpha = 1, 2, \ldots, n$ donde cada valor Z_{v_α} está definido sobre el soporte v_α con centro en el punto x_α.

El valor $Z_V(x_o)$ será estimado linealmente, a partir de los n datos experimentales, por el estimador Z_K^*.

$$Z_K^* = \sum_{\alpha=1}^{n} \lambda_\alpha Z_{v_\alpha}$$

Los n coeficientes λ_α se calcularán asegurando que el estimador Z_K^* sea insesgado y que la variancia de estimación sea mínima.

Para satisfacer la condición de insesgamiento basta imponer la condición $\sum_{\alpha=1}^{n} \lambda_\alpha = 1$, ya que sólo así se garantiza que el valor esperado de Z_V sea igual al valor esperado de Z_K^*, es decir,

$$E\left[Z_K^*\right] = E\left[\sum \lambda_\alpha Z_v\right] = m \sum \lambda_\alpha = m = E\left[Z_V\right]$$

En cuanto a la variancia de estimación σ_E^2, se tiene la siguiente expresión:

$$\sigma_E^2 = E\left[(Z_V - Z_K^*)^2\right] = E\left[Z_V^2\right] - 2E\left[Z_V\, Z_K^*\right] + E\left[Z_K^{*2}\right]$$

donde

$$E\left[Z_V^2 \right] = \overline{C}(V, V) + m^2 \quad \text{(ver desarrollo de la fórmula de } \sigma_E^2\text{)}$$

$$E\left[Z_V Z_K^* \right] = \sum_{\alpha} \lambda_\alpha \overline{C}(v_\alpha, V) + m^2 \qquad \text{y}$$

$$E\left[Z_K^{*2} \right] = \sum_{\alpha} \sum_{\beta} \lambda_\alpha \lambda_\beta \overline{C}(v_\alpha, v_\beta) + m^2$$

Sustituyendo en la expresión de σ_E^2 se obtiene

$$\sigma_E^2 = \overline{C}(V, V) - 2\sum_{\alpha} \lambda_\alpha \overline{C}(v_\alpha, V) + \sum_{\alpha} \sum_{\beta} \lambda_\alpha \lambda_\beta \overline{C}(v_\alpha, v_\beta)$$

Aplicando el método de los multiplicadores de Lagrange es posible encontrar el conjunto óptimo de coeficientes λ_α sujetos a la condición $\Sigma \lambda_\alpha = 1$. Al igualar a cero las n derivadas parciales:

$$\partial / \partial X \left[\sigma_E^2 - 2\mu \Sigma \lambda_\alpha \right] \quad , \quad \forall \ \alpha = 1 \text{ a } n$$

y al considerar la función restricción $\Sigma \lambda_\alpha = 1$, se define un sistema lineal de (n+1) ecuaciones y (n+1) incógnitas (los n coeficientes λ_α más el multiplicador de Lagrange μ), el cual se denomina "sistema Kriging".

$$\sum_{\beta=1}^{n} \lambda_\beta \cdot \overline{C}(v_\alpha, v_\beta) - \mu = \overline{C}(v_\alpha, V) \quad , \quad \forall \ \alpha = 1 \text{ a } n$$

$$\sum_{\beta=1}^{n} \lambda_\beta = 1$$

Una vez resuelto el sistema para los coeficientes λ_α, la obtención de la variancia de estimación mínima σ_K^2 es inmediata:

$$\sigma_K^2 = \overline{C}(V, V) + \mu - \sum_{\alpha=1}^{n} \lambda_\alpha \overline{C}(v_\alpha, V)$$

Haciendo uso de la relación $C(h) = C(0) - \gamma(h)$, el sistema Kriging también puede expresarse en función del variograma $\gamma(h)$:

$$\sum_{\beta=1}^{n} \lambda_\beta \, \gamma(v_\alpha , v_\beta) + \mu = \bar{\gamma}(v_\alpha , V), \qquad \forall \alpha = 1 \text{ a } n$$

$$\sum_{\beta=1}^{n} \lambda_\beta = 1$$

y $$\sigma_K^2 = \sum_{\alpha=1}^{n} \lambda_\alpha \bar{\gamma}(v_\alpha , V) + \mu - \bar{\gamma}(V , V)$$

Comentarios:

1.- El sistema Kriging proporciona una solución única toda vez que la matriz de covariancia $\bar{C}(v_\alpha , v_\beta)$ sea una matriz definida positivamente.

2.- El método Kriging es un interpolador exacto, esto es, si el soporte V a estimar coincide con cualquiera de los soportes v_α, entonces el estimador Kriging Z_K^* será idéntico al dato conocido Z_α, asociado al soporte $v_\alpha \equiv V$. Además, la variancia de estimación σ_K^2 será igual a cero.

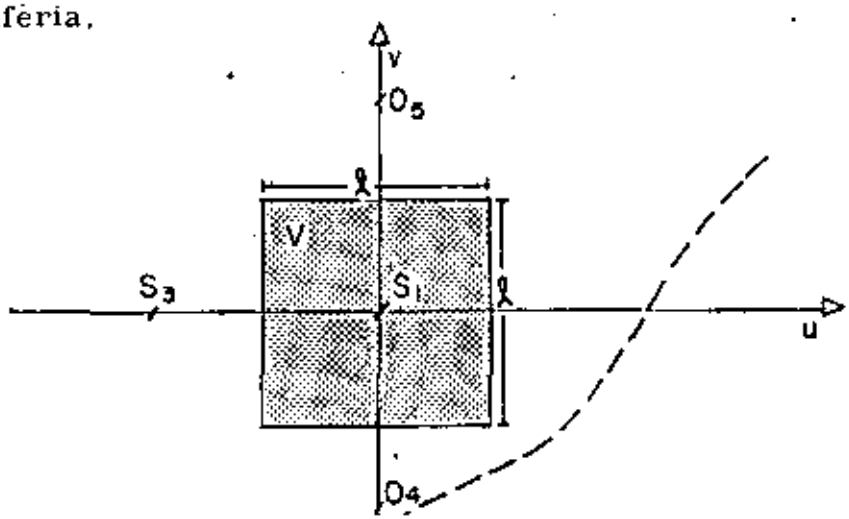
3.- El sistema Kriging es aplicable para cualesquiera que sean los soportes v_α y V, y para cualquiera que sea el modelo estructural $\gamma(h)$ (ó $C(h)$).

4.- El sistema Kriging y la variancia de estimación σ_K^2 dependen -

del modelo estructural $\gamma(h)$ o C(h) y de la posición relativa de los soportes v_α y V, pero no dependen de los valores particulares de los datos Z_{v_α}. Por lo tanto, una vez que la configuración de la información sea conocida, anterior a cualquier perforación (o campaña sismológica), el sistema Kriging puede resolverse y la variancia de estimación pronosticarse. De esta manera, la variancia de estimación puede emplearse como un índice comparativo entre los costos de perforación (o de la campaña sismológica) y las utilidades del pronóstico.

Ejemplo del Método Kriging.- Consideremos en el espacio de dos dimensiones a la función aleatoria Z(u, v) caracterizada por el semi-variograma lineal e isotrópico $\gamma(h) = \gamma(r)$, $r = |h|$. Se desea estimar el valor medio Z_V de un panel cuadrado de lado ℓ_1 a partir de una configuración no-simétrica de cuatro datos de soporte v, tal y como se muestra en la Figura 21.

S_1 es una muestra central y S_3, O_4 y O_5 son muestras localizadas en la periferia.



Por razones de simetría, y debido a que $\gamma(h)$ es isotrópico, los datos O_4 y O_5 recibirán el mismo peso y por lo tanto pueden agruparse y formar el conjunto $S_2 = \{O_4 \cup O_5\}$ de soporte $2v$. El estimador lineal Z_K^* estará definido como:

$$Z_K^* = \sum_{\alpha=1}^{3} \lambda_\alpha Z(S_\alpha) \quad \text{con} \quad Z(S_2) = 1/2 \left[Z(O_4) + Z(O_5) \right]$$

entonces, el sistema Kriging estará formado por las 4 ecuaciones siguientes:

$$\lambda_1 \bar{\gamma}(S_1, S_1) + \lambda_2 \bar{\gamma}(S_1, S_2) + \lambda_3 \bar{\gamma}(S_1, S_3) + \mu = \bar{\gamma}(S_1, V)$$

$$\lambda_1 \bar{\gamma}(S_2, S_1) + \lambda_2 \bar{\gamma}(S_2, S_2) + \lambda_3 \bar{\gamma}(S_2, S_3) + \mu = \bar{\gamma}(S_2, V)$$

$$\lambda_1 \bar{\gamma}(S_3, S_1) + \lambda_2 \bar{\gamma}(S_3, S_2) + \lambda_3 \bar{\gamma}(S_3, S_3) + \mu = \bar{\gamma}(S_3, V)$$

$$\lambda_1 + \lambda_2 + \lambda_3 = 1$$

y la variancia de estimación será igual a:

$$\sigma_K^2 = \lambda_1 \bar{\gamma}(S_1, V) + \lambda_2 \bar{\gamma}(S_2, V) + \lambda_3 \bar{\gamma}(S_3, V) + \mu - \bar{\gamma}(V, V)$$

donde

$$\bar{\gamma}(S_1, S_1) = \bar{\gamma}(S_3, S_3) = \bar{\gamma}(v, v)$$

$$\bar{\gamma}(S_2, S_2) = \bar{\gamma}(O_4, S_2) = 1/2 \left[\bar{\gamma}(v, v) + \gamma(2\ell) \right]$$

(asumiendo que las dimensiones de v son despreciables con respecto a la longitud ℓ).

$$\bar{\gamma}(S_1, S_3) = \bar{\gamma}(S_1, S_2) = \gamma(\ell)$$

$$\bar{\gamma}(S_2, S_3) = \gamma(\ell\sqrt{2})$$

$$\bar{\gamma}(S_1, V) = H(\ell/2;\ \ell/2)$$

$$\bar{\gamma}(S_2, V) = \bar{\gamma}(O_4, V) = 2/\ell^2 \left[\frac{3\ell^2}{4} H\left(\frac{3\ell}{2}, \frac{\ell}{2}\right) - \frac{\ell^2}{4} H\left(\frac{\ell}{2}, \frac{\ell}{2}\right) \right]$$

$$\bar{\gamma}(S_2, V) = \frac{3}{2} H\left(\frac{3}{2}\ell, \frac{\ell}{2}\right) - \frac{1}{2} H\left(\frac{\ell}{2}, \frac{\ell}{2}\right)$$

$$\bar{\gamma}(S_3, V) = \bar{\gamma}(S_2, V)$$

$$\bar{\gamma}(V, V) = F(\ell, \ell)$$

Asumamos ahora, dentro del modelo lineal, las tres alternativas siguientes:

(i) $\gamma(r) = \begin{cases} 0 & \text{si } r = 0 \\ 1 & \text{si } r > 0 \end{cases}$ efecto de pepita puro

(ii) $\gamma(r) = \begin{cases} 0 & \text{si } r = 0 \\ 1/2 + r & \text{si } r > 0 \end{cases}$ efecto de pepita

(iii) $\gamma(r) = \begin{cases} 0 & \text{si} \quad r = 0 \\ 1.92\,r & \text{si} \quad r > 0 \end{cases}$ Modelo lineal con ausencia total del efecto pepita.

Si el soporte v es lo suficientemente pequeño como para considerarlo puntual, y la longitud ℓ es igual a la unidad, la solución del sistema Kriging para cada una de los tres alternativas aportaría los resultados que se presentan en la Tabla 2. Estos resultados fueron calculados al resolver los siguientes sistemas:

$$\left.\begin{aligned} \lambda_2 + \lambda_3 + \mu &= 1 \\ \lambda_1 + \lambda_{2/2} + \lambda_3 + \mu &= 1 \\ \lambda_1 + \lambda_2 + \mu &= 1 \\ \lambda_1 + \lambda_2 + \lambda_3 &= 1 \end{aligned}\right\} \quad \text{efecto pepita puro}$$

$$\left.\begin{aligned} 1.5\,\lambda_2 + 1.5\,\lambda_3 + \mu &= 0.883 \\ 1.5\,\lambda_1 + 1.25\,\lambda_2 + 1.91\,\lambda_3 + \mu &= 1.543 \\ 1.5\,\lambda_1 + 1.91\,\lambda_2 + \mu &= 1.543 \\ \lambda_1 + \lambda_2 + \lambda_3 &= 1 \end{aligned}\right\} \quad \text{efecto pepita}$$

$$\left.\begin{aligned} 1.92\,\lambda_2 + 1.92\,\lambda_3 + \mu &= 0.735 \\ 1.92\,\lambda_1 + 1.92\,\lambda_2 + 1.71\,\lambda_3 + \mu &= 2.0 \\ 1.92\,\lambda_1 + 2.71\,\lambda_2 + \mu &= 2.0 \\ \lambda_1 + \lambda_2 + \lambda_3 &= 1.0 \end{aligned}\right\} \quad \begin{aligned} &\text{ausencia del} \\ &\text{efecto pepita} \end{aligned}$$

Comentarios:

(i) En el caso del efecto de pepita puro, los valores de los coeficientes fueron proporcionales a los soportes $\lambda_1 = \lambda_3 = \lambda_2/2$, o en otras palabras, los coeficientes asociados a cada dato fueron los mismos. Esto no sorprende ya que el efecto de pepita puro caracteriza a un fenómeno donde:

Tabla 2 - Estimación del Valor Medio Z_V

Dirección de incremento del "efecto pantalla"

Efecto Pepita	Kriging	Poli	ID	ID 2
Puro	$\lambda_1 = .25$ $\lambda_2 = .50$ $\lambda_3 = .25$ $\sigma_K^2 = .25$	$\lambda_1 = 1$ $\lambda_2 = 0$ $\lambda_3 = 0$ $\sigma_E^2 = 1$	0.484 0.344 0.172 0.324	0.727 0.182 0.091 0.553
Parcial	$\lambda_1 = .468$ $\lambda_2 = .395$ $\lambda_3 = .136$ $\sigma_K^2 = .442$	idem $\sigma_E^2 = 0.734$	idem 0.780	idem 0.671
Total Ausencia	$\lambda_1 = .624$ $\lambda_2 = .291$ $\lambda_3 = .084$ $\sigma_K^2 = .221$	idem $\sigma_E^2 = 0.468$	idem 1.23	idem 0.754

existe total ausencia de correlación entre los datos.

(ii) A medida que el efecto pepita disminuye, la influencia del dato S_1 se incrementa (λ_1 va de 0.25 a 0.468). Nótese que el

valor del coeficiente de S_2 (de soporte 2v) es siempre mayor que dos veces λ_3, el valor del coeficiente de S_3. Esto es debido al hecho de que S_2 está más cercano a una zona donde existe menor información (a la derecha).

(iii) En la misma Tabla 2 se han incluído los valores de los coeficientes evaluados según otros métodos de estimación (polígonos de influencia, inverso de la distancia e inverso del cuadrado de la distancia).

Estos métodos no toman en cuenta las características estructurales del fenómeno, de aquí que produzcan los mismos resultados en las tres alternativas.

(iv) En todos los casos, Kriging proporciona el mejor estimador. Dependiendo del grado de correlación, alguno de los otros métodos se acerca al método Kriging, pero únicamente un análisis estructural puede decirnos cual de ellos es el más cercano. Una selección afortunada (ID, por ejemplo) en el caso del efecto de pepita puro, hubiera dado resultados tan aceptables como los del Kriging. La misma selección, pero en el caso de total ausencia del efecto pepita, resultaría en errores de estimación de aproximadamente seis veces el error que se produciría con el método Kriging.

Epílogo.- Para aquel lector interesado en la elaboración de un algoritmo del método Kriging, resumiremos a continuación los pasos principales del método:

(i) Selección de los datos Z_{v_α} empleados en la estimación de Z_V.

(ii) Cálculo de las covariancias medias $\overline{C}(v_\alpha, v_\beta)$ ó $\left(\overline{\gamma}(v_\alpha, v_\beta)\right)$.

(iii) Cálculo de las covariancias medias $\overline{C}(v_\alpha, V)$ ó $\left(\overline{\gamma}(v_\alpha, V)\right)$.

(iv) Selección del algoritmo más apropiado para la solución del sistema Kriging.

El diseño de un buen algoritmo debe de minimizar el tiempo de ejecución a la vez que producir resultados aceptables dentro de ciertos límites de aproximación.

Cinco puntos son esenciales en la reducción del tiempo de ejecución:

1.- La reducción de la dimensión del sistema Kriging.

2.- La reducción del número de sistemas a resolver.

3.- La rápida evaluación de los valores medios $\overline{C}$ (ó $\overline{\gamma}$).

4.- La preparación de un archivo de datos convenientemente adaptado al plan del método.

5.- La selección de un buen algoritmo para la solución del sistema.

Por lo que al algoritmo se refiere, el enfoque de estos cinco puntos puede variar drásticamente de un problema a otro, permitiendo introducir, en algunos casos, tremendas simplificaciones. ——

A continuación se lista el programa KRI-3D, ejecución del método Kriging en 3 dimensiones.

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```
      DEFINE FILE 9(100,10,U,K9)
      DEFINE FILE 10(2,4,U,K10)
      DEFINE FILE 11(8,1,U,K11)
      DEFINE FILE 12(8,1,U,K12)
      DEFINE FILE 13(64,4,U,K13)
C
C
C
C
C
C
C
C
C                       **** KRI-3D ****
C                  PROGRAMME  KRI-3D  LATEST VERSION
C  3-D. KRIGING OF UNIT BLOCKS ARRANGED IN A NETWORK OF SUPER-BLOCKS.
C  ALL UNITS WITHIN A SAME S-BLOCK HAVE THE SAME NEIGHBORHOOD OF
C  ESTIMATION.  NV VARIABLES ALL CONSISTENTLY ANALYSED.  DATA ARE
C       PREVIOUSLY ARRANGED IN THE NETWORK BY SSP CLAS4.
C           REF. MINING GEOSTATISTICS PP.361
C
C
C  PARAMETERS   XOB,YOB,ZOB    REAL COORDINATES OF THE ORIGIN, AT CENTER
C               = XYZ(3)       OF S-BLOCK # NY (TOP SW)-SEE FIGURE PP.362
C               NX,NY,NZ       NUMBERS OF S-BLOCKS DEFINING THE THREE
C               = NXYZ(3)      SIDES OF THE NETWORK
C               NZ1,NZ2        HIGHER AND LOWER LEVEL OF S-BLOCKS
C                               TO BE KRIGED.
C               IADO           STARTING ADDRESS ON OUTPUT FILE IO.
C               DBX,DBY,DBZ    DIMENSIONS OF A S-BLOCK ALONG THE THREE
C               =DB(3)         AXES OX,OY,OZ.
C               NSX,NSY,NSZ    # OF UNITS CONSTITUTING EACH SIDE OF A
C                              S-BLOCK  NSB=NSX*NSY*NSZ
C               IEXP(NX*NY)    IF IEXP=1, THE S-BLOCK IS TO BE KRIGED
C                              IF NOT, IT IS NOT KRIGED.  FOR ALL S-
C                              BLOCKS OF A NETWORK LEVEL.
C               NST,C0,AA,C,   |
C               AX,AY,AZ,      | COVARIANCES PARAMETERS, SEE COVA
C               COSAL,SINAL    |
C               NL,NP,IP,IJV    PARAMETERS DEFINING THE KRIGING PLAN,
C                              SEE VOIS.
C               NOB(NA*NP(L))*INTEGER INDEX CHARACTERIZING WHICH
C                              S-BLOCKS OF THE NEIGHBORHOOD ARE TO BE
C                              ASSOCIATED WITH A GIVEN KRIGING WEIGHT.
C               NPB,NUM,NIMB   ARRAYS AND PARAMETER OF THE DATA CLASSI-
C                              FICATION PERFORMED BY CLAS4.
C               XOB,YOB,ZOB    COORDINATES OF THE 64 POINTS DIGITIZING
C                              EACH UNIT. THEY ARE RELATIVE TO THE
C                              CENTER OF S-BLOCK TO BE KRIGED.
C               CBB(NV)        POINT VARIANCE IN A UNIT.
C               XA,YA,ZA,      OUTPUT DATA AND PARAMETERS GIVEN BY
C               NIN,TA,NA      SSP VOIS.
C               A,R,RR,X,IER   MATRICES OF KRIGING, SEE SSP KRIGMS
C               XB,YB,ZB       COORDINATES OF A CENTER OF A S-BLOCK
C                              TO BE KRIGED
C          TK(NX*NY*NSB,NV)    KRIGED VALUES/UNIT/LEVEL/VARIABLE
C          SK(NX*NY*NSB,NV)    CORRESPONDING KRIGING VARIANCE
C          UK,VK,NK(NV)        STATISTICS OF THE KRIGED UNITS
C
C
C  DIRECT ACCESS FILES
C
C            IE                DATA FILE   |
C            IE2               ARRAY NUM   |  PROVIDED BY CLAS4
C            IE3               ARRAY NPB   |
C            IE1               ARRAY IEXP
C            IO                OUTPUT FILE CONTAINS KRIGING RESULTS/LEVEL
C
C  COMMON
C            INP               PUNCHED CARDS READER UNIT
C            IOUT              PRINTER
C            TEST              BLANK (OR NON KRIGED) VALUES
C            NV                # OF VARIABLES
C            NAM(NV)           NAME OF EACH VARIABLE (A8)
C
C       COMMON /STRUC/*        TRANSFER OF PARAMETERS TO SSP COVA
C       COMMON /VOISI/         TRANSFER OF PARAMETERS TO SSP VOIS
C
C
C  OPTIONS
C
C                     IS .EQ. 1   EDIT ONLY STATISTICS OF KRIGED VALUES
C                     IS .NE. 1   ADDITIONAL EDITION OF KRIGED VALUES/
C                                 S-BLOCK.
C
C
C   SUBROUTINES
C            KRIGMS+RELMSM     SET AND SOLVE KRIGING SYSTEM
C            VOIS              SORTING OF DATA IN THE NEIGHBHD.
C            CBARS             CALCULATION OF MEAN COVARIANCES
C            COVA              FUNCTION COVARIANCE
C
C
      DIMENSION NP(3),NOB(18),XOB(512),YOB(512),ZOB(512),IJV(3),IP(81)
      DIMENSION XA( 80),YA( 80),ZA( 80),NIN( 4),TA( 6)
      DIMENSION TK(  32,2),SK(  32,2)
      DIMENSION NUM(   8),NPB(   8),IEXP(    4)
      DIMENSION R(64),RR(64),X(64),A( 20)
      DIMENSION NK(2),VK(2),UK(2),CBB(2),CB1(16),CB2(16),COV(2)
      DIMENSION JL(3),NXYZ(3),XYZ(3),DB(3)
      DOUBLE PRECISION NAM(2)
      COMMON INP,IOUT,TEST,NV
      COMMON/VOISI/NIMB,XB,YB,ZB,IJVL
C
      DIMENSION NST(2),C0(2),AA(1,2),C(1,2),IT(1,2),AX(1,2),AY(1,2)
     1,AZ(1,2)
      COMMON/STRUC/NST,C0,AA,C,IT,AX,AY,AZ,COSAL,SINAL
C
C     READ PARAMETERS AND PRINT TITLE
C
      INP=5
      IOUT=6
      READ(INP,1000)XOB,YOB,ZOB,DBX,DBY,DBZ,NX,NY,NZ,TEST,IS,
     1 NSX,NSY,NSZ
      READ (INP,1010) NZ1,NZ2,IADO
 1000 FORMAT(6F8.3,3I3,F8.3,I1,3I2)
 1010 FORMAT(3I10)
      DSX=DBX/NSX
      DSY=DBY/NSY
      DSZ=DBZ/NSZ
```

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```
      NDB=NBX*NBY*NBZ
      NZ1=MIN0(1,NZ1)
      NZ2=MIN0(NZ,NZ2)
      IAD0=MAX0(1,IAD0)
      WRITE(IOUT,2000)DBX,DBY,DBZ,XDB,YDB,ZDB,NY,NX,NZ,DSX,DSY,DSZ,NSB
C
C     DEFINE NEIGHBORHOOD AND ARRANGE DATA
C
      READ(INP,1001)NL,(IJV(I),I=1,3)
      READ(INP,1002)(NP(L),L=1,NL)
      WRITE(IOUT,2001)NL
      NI=0
      NLI=IJV(1)
      NLC=IJV(1)*IJV(2)
      DO 100 L=1,NL
      N=NP(L)
      READ(INP,1003)(NDB(IJ),IJ=1,N)
      WRITE(IOUT,2002)L,(NDB(IJ),IJ=1,N)
      DO 100 I=1,N
      NI=NI+1
      N2=3*(NI-1)
      LB=(NDB(I)-1)/NLC+1
      JB=(NDB(I)-NLC*(LB-1)-1)/NLI+1
      IB=NDB(I)-NLI*(JB-1)-NLC*(LB-1)
      IP(N2+1)=IB-IJV(1)/2-1
      IP(N2+2)=JB-IJV(2)/2-1
      IP(N2+3)=LB-IJV(3)/2-1
  100 CONTINUE
 1001 FORMAT(4I2)
 1002 FORMAT(40I2)
 1003 FORMAT(40I2)
C
C     READ AND EDIT STRUCTURAL CHARACTERISTICS
C
      READ(INP,1004)NV,NIMB,COSAL,SINAL
      WRITE(IOUT,2003)NV
      READ(INP,1005)(NAM(IV),IV=1,NV)
      DO 103 IV=1,NV
      READ(INP,1006)NST(IV),C0(IV)
      NS=NST(IV)
      READ(INP,1007) (C(I,IV),AA(I,IV),IT(I,IV),AX(I,IV),AY(I,IV),
     1AZ(I,IV),I=1,NS)
 1004 FORMAT(2I2,2F10.0)
 1005 FORMAT(10A8)
 1006 FORMAT(I1,F10.4)
 1007 FORMAT(3(F10.4,F8.3,I1,3F4.2))
      UK(IV)=0.
      VK(IV)=0.
      WK(IV)=0
      CBB(IV)=0.
  103 WRITE(IOUT,2004)NAM(IV),C0(IV),(IT(J,IV),C(J,IV),AA(J,IV),J=1,NS)
C
C     READ # OF DIRECT ACCESS FILES
C
      READ(INP,1003)IE,IE1,IE2,IE3,ID
 1008 FORMAT(5I2)
      NXYZ(1)=NX
      NXYZ(2)=NY
      NXYZ(3)=NZ
      XYZ(1)=XDB
      XYZ(2)=YDB
      XYZ(3)=ZDB
      DD(1)=DBX
      DD(2)=DBY
      DD(3)=DBZ
C
C     DIGITIZE ALL UNITS OF A SUPER-BLOCK INTO POINTS
C
      NDB=64
      I1=0
      DO 14 LS=1,NSZ
      ZS0=DBZ/2.-DSZ/2.-DSZ*(LS-1)
      DO 13 IS=1,NSY
      YS0=DBY/2.-DSY/2.-DSY*(IS-1)
      DO 12 JS=1,NSX
      XS0=-DBX/2.+DSX/2.+DSX*(JS-1)
      X0=XS0-DSX/2.+DSX/16.
      Y00=YS0+DSY/2.-DSY/16.
      Z0=ZS0+DSZ/2.-DSZ/8.
      DO 10 L1=1,2
      X0=X0+(L1-1)*DSX/8.
      Y0=Y00
      DO 10 L2=1,2
      Y0=Y0-(L2-1)*DSY/8.
      DO 11 J=1,4
      DO 11 I=1,4
      I1=I1+1
      XDD(I1)=X0+(J-1)*DSX/4
      YDD(I1)=Y0-(I-1)*DSY/4
   11 ZDD(I1)=Z0
      Z0=Z0-DSZ/4.
   10 CONTINUE
   12 CONTINUE
   13 CONTINUE
   14 CONTINUE
C
C     POINT VARIANCE WITHIN A UNIT
C
      DO 15 I=1,NDB
      DO 15 J=1,NDB
      CALL COVA(XDB(I),YDB(I),ZDB(I),XDB(J),YDB(J),ZDB(J),COF
      DO 15 IV=1,NV
   15 CBB(IV)=CBB(IV)+COV(IV)/4096.0
C
C     -IFFORM KRIGING FOR EACH SUPER-BLOCK
C
      NBL=0
      IAD=IAD0
      N1=NX*NY
      IF(IS.NE.1)WRITE(IOUT,2006)TEST
      DO 1 LB=NZ1,NZ2
      ZB=Z00-DBZ*(LB-1)
      IF(IS.NE.1)WRITE(IOUT,2007)LB,ZB
      NDK=0
      IF((LB-IJV(3)/2) .LT. 1) GO TO 6
      IJVL=IJV(3)/2+1
      L0=LB-IJV(3)/2
      GO TO 7
    6 IJVL=LB
      L0=1
```

```
128       7 L1=MIND(NZ,LB+IJV(3)/2)                                        250.
          C             READ IEXP,NUM,NPB ON FILES                         251.
          C                                                                252.
129         IAD1=LB                                                        253.
130         READ(IE1'IAD1) (IEXP(IJ),IJ=1,N1)                              254.
131         IAD2=NX*NY*(LB-1)+1                                            255.
132         N2=NX*NY*L1-IAD2+1                                             256.
133         DO 5 IJ=1,N2                                                   257.
134         IRC=IAD2+IJ-1                                                  258.
135         READ(IE2'IRC) NUM(IJ)                                          259.
136       5 READ(IE3'IRC) NPB(IJ)                                          260.
          C             NEW SUPER-BLOCK                                    261.
137         DO 2 IB=1,NY                                                   262.
138         DO 3 JB=1,NX                                                   263.
139         IJB=IB+NY*(JB-1)                                               265.
140         IF(IEXP(IJB).EQ.0)GO TO 3                                      266.
141         NBX=NDK+1                                                      267.
142         NDL=NSL+1                                                      268.
143         XB=X00+DBX*(JB-1)                                              269.
144         YB=Y00+DBY*(NY-IB)                                             270.
145         JL(1)=IB                                                       271.
146         JL(2)=JB                                                       272.
147         JL(3)=LB                                                       273.
          C                                                                274.
          C     ARRANGE INFORMATION WITHIN NEIGHBORHOOD                    275.
          C                                                                276.
148         CALL VOIS(NL,NP,IP,JL,NXYZ,NPB,NUM,IE,XA,YA,ZA,NIN,TA,NA)      277.
149         IF(NA.EQ.0)GO TO 37                                            278.
150         IF(NA.GT.1)GO TO 31                                            279.
          C     A SINGLE INFORMATION AVAILABLE - NO KRIGING                280.
151         CALL CBARS(XA,YA,ZA,1,1,NIN,XDB,YDB,ZDB,NDB,NSB,CB1,COV)       281.
152         CALL CBARS(XA,YA,ZA,1,0,NIN,XDB,YDB,ZDB,NDB,NSB,CB2,COV)       282.
153         DO 30 K=1,NSB                                                  283.
154         J=K+NSB*(NDK-1)                                                284.
155         DO 30 IV=1,NV                                                  285.
156         TK(J,IV)=TA(IV)                                                286.
157         IV1=IV+NV*(K-1)                                                287.
158      30 SK(J,IV)=CBB(IV)-2*CB2(IV1)+CB1(IV)                            288.
159         GO TO 34                                                       289.
          C                   SOLVE KRIGING SYSTEM                       290.
160      31 CALL KRIGMS(NA,XA,YA,ZA,NIN,XDB,YDB,ZDB,NDB,NSB,R,RR,A,X,IER,CB1 291.
           1,COV)                                                          292.
161         IF(IER.NE.0)GO TO 36                                           293.
          C                   SOLUTION                                   294.
162         DO 33 K=1,NSB                                                  295.
163         J=K+NSB*(NDK-1)                                                296.
164         DO 33 IV=1,NV                                                  297.
165         TTK=0.                                                         298.
166         IK=(NA+1)*K+(NA+1)*NSB*(IV-1)                                  299.
167         SSK=CBB(IV)-X(IK)                                              300.
168         DO 32 I=1,NA                                                   301.
169         IJ=I+NL*(IV-1)                                                 302.
170         IK=I+(NA+1)*(K-1)+NSB*(NA+1)*(IV-1)                            303.
171         TTK=TTK+X(IK)*TA(IJ)                                           304.
172      32 SSK=SSK-X(IK)*RR(IK)                                           305.
173         TK(J,IV)=TTK                                                   306.
174      33 SK(J,IV)=SSK                                                   307.
          C                                                                308.
          C       STATISTICS OF ALL KRIGED UNITS                           309.
          C                                                                310.
```

```
      34 DO 35 K=1,NSB                                                  311
         J=K+NSB*(NBK-1)                                                312
         DO 35 IV=1,NV                                                  313
         UK(IV)=UK(IV)+TK(J,IV)                                         314
         VK(IV)=VK(IV)+TK(J,IV)*TK(J,IV)                                315
      35 NK(IV)=NK(IV)+1                                                316
         IF(IS.EQ.1)GO TO 3                                             317
C                     EDIT RESULTS/SUPER-BLOCK                         318
         WRITE(IOUT,2009)IB,JB,XB,YB,NA                                 319
         WRITE(IOUT,2016) (NAM(IV),IV=1,NV)                             320
         DO 40 K=1,NSB                                                  321
         J=K+NSB*(NBK-1)                                                322
         WRITE(IOUT,2010) K,(TK(J,IV),IV=1,NV)                          323
      40 WRITE(IOUT,2011) (SK(J,IV),IV=1,NV)                            324
         GO TO 3                                                        325
C                     SINGULAR SYSTEM                                  326
      36 WRITE(IOUT,2008)I3,JB,LB                                       327
         GO TO 38                                                       328
C                     NO INFORMATION                                   329
      37 WRITE(IOUT,2012)IB,JB,LB                                       330
      38 DO 39 K=1,NSB                                                  331
         J=K+NSB*(NBK-1)                                                332
         DO 39 IV=1,NV                                                  333
         TK(J,IV)=TEST                                                  334.
      39 SK(J,IV)=TEST                                                  335.
       3 CONTINUE                                                       336.
       2 CONTINUE                                                       337.
         N3=NBK*NSB                                                     338.
         DO 41 N=1,N3                                                   339.
         WRITE(IO'IAD) (TK(N,IV),SK(N,IV),IV=1,NV)                      340.
      41 IAD=IAD+1                                                      341.
         WRITE(IOUT,2015) LB,IAD,NBK                                    342.
       1 CONTINUE                                                       343.
C                                                                       344.
C     EDIT STATISTICS OF KRIGED VALUES ON THE WHOLE DEPOSIT             345.
C                                                                       346.
         WRITE(IOUT,2013)DSX,DSY,DSZ,NBL                                347.
         DO 4 IV=1,NV                                                   348.
         IF(NK(IV).EQ.0)GO TO 4                                         349.
         VK(IV)=(VK(IV)-UK(IV)*UK(IV)/NK(IV))/NK(IV)                    350.
         UK(IV)=UK(IV)/NK(IV)                                           351.
       4 WRITE(IOUT,2014)NAM(IV),UK(IV),VK(IV),NK(IV)                   352.
         WRITE(IOUT,2017)                                               353.
C                                                                       354.
 2000 FORMAT(1H1,4(/),48X,'****BLOCK KRIGING****'//' NETWORK OF '       355.
     1,'SUPER - BLOCKS  '//' SIZE  =',' DBX=',F8.3,'                    356.
     2' DBY=',F8.3,' DBZ=',F8.3/1H ,'NETWORK ORIGIN ',                  357.
     3' =  X0B=',F9.3,'  Y0B=',F9.3,'  Z0B=',F9.3/1H ,'NETWORK'         358.
     4,' OF ',I3,' LINES  ',I3,' COLUMNS AND ',I3,' LEVELS '/           359.
     5' *KRIGED UNITS*  SIZE  = DSX=',F8.3,' DSY=',F8.3,' DSZ=',        360.
     6F8.3,'* UNITS/S.BLOCK=',I3)                                       361.
 2001 FORMAT(1H ,'KRIGING PLAN   = ',I2,' WEIGHTS     '/1H ,'WEIGHT '   362.
     1,'INDEX OF S.BLOCK IN NEIGHBORHOOD.NOB. ')                        363.
 2002 FORMAT(1H ,5X,I2,5X,40(1X,I2))                                    364.
 2003 FORMAT(1H ,'NUMBER OF VARIABLES TO BE KRIGED = ',I2//48X,         365.
     1'STRUCTURAL CHARACTERISTICS'//11X,'NAME',10X,'NUGGET EFFECT',     366.
     210X,'TYPE',10X,'SILL',10X,'RANGE')                                367.
 2004 FORMAT(1H ,6X,A8,6X,F8.2,15X,(2X,I1,6X,F10.2,5X,F10.2/47X))       368.
 2006 FORMAT(1H1,46X,'**KRIGING RESULTS/SUPER BLOCK**'//1H              369.
     1,'  VALUE   ',E11.5,3X,' CORRESPONDS TO BLANK DATA OR NON KRIGED', 370.
```

```
     2' VALUES'/)                                                          371.
218  2007 FORMAT(//55X,' **LEVEL ',I3,'      Z=',F7.2/)                   372.
219  2008 FORMAT(1H ,'*S.BLOCK IB=',I3,' JB=',I3,' LB=',I3,               373.
     1' VARIABLE ',I2,1X,'SINGULAR SYSTEM*')                               374.
220  2009 FORMAT(/12X,'****S.BLOCK IB=',I2,'  JB=',I2,'  XB=',             375.
     1F7.3,'  YB=',F7.3,6X,'NUMBER OF WEIGHTS INFORMED NA=',               376.
     2I2,'  ****'/)                                                       377.
221  2010 FORMAT(1H0,4X,I4,4X,5(2X,F10.4,4X))                             378.
222  2011 FORMAT(1H ,12X,5(2X,E11.4,3X))                                  379.
223  2012 FORMAT(1H ,'*S.BLOCK IB=',I3,' JB=',I3,' LB=',I3,' NO INFORM')  380.
224  2013 FORMAT(1H1,41/),25X,'****KRIGING OF UNITS  DSX=',F8.3,'  DSY=', 381.
     1F8.3,'  DSZ=',F8.3,' ****'/1H0,2X,'STATISTICS BASED ON',I5,         382.
     2'KRIGED S.BLOCKS'//1H ,'VARIABLE',6X,'MEAN      VARIANCE  ',2X,     383.
     3' NUMBER OF UNITS')                                                  384.
225  2014 FORMAT(1H ,A8,1X,F10.4,4X,E11.4,7X,I4)                          385.
226  2015 FORMAT(1H0,10X,' LB=',I3,5X,'IAD=',I8,5X,' NBK=',I6///)         386.
227  2016 FORMAT(5X,'UNIT #',2X,5(3X,A8,5X))                              387.
228  2017 FORMAT(1H1)                                                     388.
     C                                                                     389.
229       STOP                                                           390.
230       END                                                            391.
     C                                                                     392.
     C                                                                     393.
     C                                                                     394.
     C                                                                     395.

231       SUBROUTINE CBARS(XA,YA,ZA,I,J,NIN,XDB,YDB,ZDB,NDB,NSB,CB,COV)  396.
     C                                                                     397.
     C          CALCULATION OF THE MEAN COVARIANCE BETWEEN THE TWO SETS   398
     C        OF INFORMATION I AND J, OR BETWEEN THE SET I AND THE UNITS  399
     C        IF J=0.        NV VARIABLES ARE CONSIDERED                  400
     C                                                                    401
     C             I         FIRST SET OF INFORMATION                     402
     C             J         SECOND SET OF INFORMATION                    403
     C             J.EQ.0    MEAN COVARIANCE BETWEEN SET I AND THE UNITS  404
     C             NIN(NL+1) GIVES THE ADDRESS OF EACH GROUP OF INFORMATION 405
     C                       WITHIN THE ARRAYS XA,YA,ZA                   406
     C             XDB,YDB,ZDB(NDB*NSB)    COORDINATES OF THE POINTS      407
     C                                     DIGITIZING THE UNITS (J=0)     408
     C             CB(NV*NSB)    MEAN COVARIANCE/VARIABLE/UNIT            409
     C             COV(NV)     COVARIANCE FUNCTION/VARIABLE               410
     C             NDB         # OF DIGITIZING POINTS (=64 IN KRI-3D)     411
     C             NSB         # OF UNITS IN A SUPER-BLOCK                412
     C             NL          # OF GROUPS OF DATA CONSIDERED BY THE KRIGING 413
     C                         PLAN                                       414
     C                                                                    415
     C  SUBROUTINE    :  COVA                                            416
     C                                                                    417
232       DIMENSION XA(1),YA(1),ZA(1),NIN(1),CB(1),COV(1)                418
233       DIMENSION XDB(1),YDB(1),ZDB(1)                                 419
234       COMMON INP,IOUT,TEST,NV                                        420.
235       NC=0                                                           421.
236       NVI=NV*NSB                                                     422
237       DO 100 IV=1,NVI                                                423
238  100  CB(IV)=0.                                                      424
239       IF(I.EQ.0) GO TO 4                                             425.
240       N1=NIN(I)                                                      426
241       N2=NIN(I+1)-1                                                  427.
242       IF(J.EQ.0)GO TO 3                                              428
     C                                                                    429.
```

```
      C     MEAN COVARIANCE BETWEEN  TWO SETS I AND J                        //2   430.
      C                                                                            431.
  243         M1=NINI(I)                                                         432.
  244         M2=NINI(I+1)-1                                                     433.
  245         NCIJ=(M2-M1+1)*(M2-M1+1)                                           434.
  246         DO 1 I2=M1,M2                                                      435.
  247         DO 1 J2=M1,M2                                                      436.
  248         CALL COVA(XA(I2),YA(I2),ZA(I2),XA(J2),YA(J2),ZA(J2),COV)           437.
  249         DO 1 IV=1,NV                                                       438.
  250         CB(IV)=CB(IV)+COV(IV)/NCIJ                                         439.
  251       1 CONTINUE                                                           440.
  252         RETURN                                                             441.
      C                                                                        442.
      C     MEAN COVARIANCE BETWEEN SET I AND THE UNITS                        443.
      C                                                                        444.
  253       3 NCIB=(M2-M1+1)*NDB                                               445.
  254         DO 2 K2=1,NSS                                                    446.
  255         DO 2 I2=M1,M2                                                    447.
  256         DO 2 J1=1,NDB                                                    448.
  257         J2=J1+NDB*(K2-1)                                                 449.
  258         CALL COVA(XA(I2),YA(I2),ZA(I2),XDB(J2),YDB(J2),ZDB(J2),COV)      450.
  259         DO 2 IV=1,NV                                                     451.
  260         IV1=IV+NV*(K2-1)                                                 452.
  261         CB(IV1)=CB(IV1)+COV(IV)/NCIB                                     453.
  262       2 CONTINUE                                                         454.
  263         RETURN                                                           455.
  264       4 WRITE(IOUT,2000)                                                 456.
  265    2000 FORMAT(1H1,'   CBARS NOT EXECUTED BECAUSE I=0 '//)                457.
  266         RETURN                                                           458.
  267         END                                                              459.
      C                                                                        460.
      C                                                                        461.
      C                                                                        462.
      C                                                                        463.

  268         SUBROUTINE COVA(X1,Y1,Z1,X2,Y2,Z2,COV)                           464.
      C                                                                        465.
      C       COVARIANCE FUNCTION BETWEEN TWO POINTS.  NV VARIABLES            466.
      C                                                                        467.
      C  PARAMETERS                                                            468.
      C         X1,Y1,Z1          REAL COORDINATES OF THE FIRST POINT          469.
      C         X2,Y2,Z2          REAL COORDINATES OF THE SECOND POINT         470.
      C         COV(IV)           COVARIANCE VALUE/VARIABLE                    471.
      C                                                                        472.
      C   COMMON /STRUC/                                                       473.
      C         NST(NV)           # OF NESTED STRUCTURES (.LE. 3)/VARIABLE    474.
      C         C0(NV)            NUGGET EFFECT                              475.
      C         A(NST,NV)         RANGES OF THE NESTED STRUCTURES            476.
      C         C(NST,NV)         SILLS                                      477.
      C         IT(NST,NV)=0      NESTED STRUCTURE IS EXPONENTIAL, PARAMETER A 478.
      C                   =1      NESTED STRUCTURE IS SPHERICAL, RANGE A     479.
      C         AX,AY,AZ(NST,NV)  AFFINITY CORRECTION PARAMETERS/STRUCTURE/  480.
      C                           VARIABLE (GEOMETRIC ANISOTROPY)            481.
      C         COSAL,SINAL       ROTATION PARAMETERS OF THE HORIZONTAL      482.
      C                           AXES (X,Y).  IDENTICAL FOR ALL STRUCTURES  483.
      C                           AND ALL VARIABLES                          484.
      C                                                                        485.
      C                                                                        486.
  269         DIMENSION COV(1)                                                 487.
  270         COMMON/ INP,IOUT,TEST,NV                                         488.
```

```
      COMMON/STRUC/NST(2),C0(2),A(1,2),C(1,2),IT(1,2),AX(1,2),
     1AY(1,2),AZ(1,2),COSAL,SINAL
      DO 100 IV=1,NV
  100 COV(IV)=0.
C           ROTATE AXES
      DX=(X2-X1)*COSAL+(Y2-Y1)*SINAL
      DY=(X1-X2)*SINAL+(Y2-Y1)*COSAL
      DZ=Z2-Z1
      H=DX*DX+DY*DY+DZ*DZ
      IF(H.GT.0.001)GO TO 10
      DO 1 IV=1,NV
      NS=NST(IV)
      COV(IV)=C0(IV)
      DO 1 IS=1,NS
    1 COV(IV)=COV(IV)+C(IS,IV)
      RETURN
   10 DO 11 IV=1,NV
      NS=NST(IV)
      DO 12 IS=1,NS
      DX1=DX*AX(IS,IV)
      DY1=DY*AY(IS,IV)
      DZ1=DZ*AZ(IS,IV)
      H=SQRT(DX1*DX1+DY1*DY1+DZ1*DZ1)/A(IS,IV)
      IF(IT(IS,IV).EQ.0)GO TO 13
      IF(H.GE.1.)GO TO 12
      COV(IV)=COV(IV)+C(IS,IV)*(1.-H*(1.5-0.5*H*H))
      GO TO 12
   13 COV(IV)=COV(IV)+C(IS,IV)*EXP(-H)
   12 CONTINUE
   11 CONTINUE
      RETURN
      END
C
C
C
C

      SUBROUTINE VOIS(NL,NP,IP,JL,NXYZ,NPB,NUM,IE,XA,YA,ZA,NIN,TA,NA)
C
C             SORT AND GROUP THE INFORMATION WITHIN THE NEIGHBORHOOD
C             OF A SUPER-BLOCK.  DATA ARE READ ON FILE IE PREPARED BY
C                                SUBROUTINE CLAS4.
C
C  PARAMETERS   NL          # OF GROUPS OF DATA CONSIDERED BY THE
C                           KRIGING PLAN.
C               NP(NL)      # OF S.BLOCKS CONSTITUTING EACH GROUP
C                           NBT=NP(1)+NP(2)+...+NP(NL)
C
C               IP(3*NBT)    INTEGER COORDINATES OF EACH S.BLOCK OF THE
C                            NEIGHBORHOOD/S.BLOCK CENTER OF THE NEIGHBHD.
C
C               JL(3)        INTEGER COOD. OF THE S.BLOCK CENTER OF THE
C                            NEIGHBORHOOD/NETWORK OF S.BLOCK
C               NXYZ(3)      # OF S.BLOCKS ALONG THE 3 SIDES OF NETWORK
C               IE           DATA FILE NUMBER
C               NPB(NY*NX*IJV(3))     # OF DATA RECORDED ON EACH S.BLOCK
C               NUM(NY*NX*IJV(3))    ADDRESS IN FILE IE OF DATA RELATIVE
C                                    TO EACH S.BLOCK
C
C               XA(NIMA)
```

```
C              YA(NIMA)  COORDINATES OF DATA WITHIN THE NEIGHBORHOOD      548.
C              ZA(NIMA)                                                   549.
C              NIN(NL+1) GIVES THE ADDRESS OF EACH GROUP OF               550.
C                        INFORMATION WITHIN THE ARRAYS XA,YA,ZA           551.
C        .     TA(NL*NV) MEAN VALUE OF EACH GROUP/VARIABLE                552.
C              NA        # OF GROUPS EFFECTIVELY INFORMED                 553.
C                                                                         554.
C  COMMON INP,IOUT,TEST                                                   555.
C                                                                         556.
C   COMMON/VOISI/ NIMB        MAX. # OF DATA RECORDED IN A S.BLOCK        557.
C                                                                         558.
C                 XB,YB,ZB  REAL COORD. OF THE S.BLOCK CENTER OF NEIGH.   559.
C                 IJV(3)    # OF BLOCKS CONSTITUTING THE 3 SIDES OF      560.
C                           THE NEIGHBORHOOD.  PREFERABLY ODD INTEGERS    561.
C        NIMA=NBT*NIMB       MAX # OF DATA IN A NEIGHBORHOOD OF NBT       562.
C                            S.BLOCKS.  GIVES THE DIM. OF ARRAYS XA,      563.
C                            YA,ZA(NIMA)                                  564.
C                                                                         565.
C                                                                         566.
302       DIMENSION NP(1),JL(1),IP(1)                                     567.
303       DIMENSION NXYZ(1),NPB(1),NUM(1)                                 568.
304       DIMENSION XA(1),YA(1),ZA(1),NIN(1),TA(1)                        569.
C                                                                         570.
305       DIMENSION T(2)                                                  571.
C                                                                         572.
306       COMMON INP,IOUT,TEST,NV                                         573.
307       COMMON/VOISI/NIMB,XB,YB,ZB,IJVL                                 574.
308       NX=NXYZ(1)                                                      575.
309       NY=NXYZ(2)                                                      576.
310       NZ=NXYZ(3)                                                      577.
311       NA=0                                                            578.
312       IA=0                                                            579.
313       J=0                                                             580.
314       NIN(1)=1                                                        581.
315       N=0                                                             582.
C                                                                         583.
C     NEW GROUP                                                           584.
C                                                                         585.
316       DO 1 IL=1,NL                                                    586.
317       NB=NP(IL)                                                       587.
318       IA=NA+1                                                         588.
319       HA=0.                                                           589.
320       DO 20 IV=1,NV                                                   590.
321       IJ=IA+NL*(IV-1)                                                 591.
322    20 TA(IJ)=0.                                                       592.
C                    NEW S.BLOCK OF THE GROUP                           593.
323       DO 2 N=1,NB                                                     594.
324       N=N+1                                                           595.
325       N1=3*(N-1)                                                      596.
326       I0=JL(1)+IP(N1+1)                                               597.
327       J0=JL(2)+IP(N1+2)                                               598.
328       L0=JL(3)+IP(N1+3)                                               599.
329       LB=IP(N1+3)+IJVL                                                600.
330       IF(I0*(NY+1-I0).LE.0)GO TO 2                                    601.
331       IF(J0*(NX+1-J0).LE.0)GO TO 2                                    602.
332       IF(L0*(NZ+1-L0).LE.0)GO TO 2                                    603.
333       IJL=I0+NY*(J0-1)+NX*NY*(L0-1)                                   604.
334       IAD=NPB(IJL)                                                    605.
335       IF(IAD.EQ.0)GO TO 2                                             606.
336       IDA=(NUM(IJL)-1)*NIMB+1                                         607.
```

```
      DO 3 I=1,IAD                                                      606
C                   READ DATA IN FILE IE                              609
C                                                                     610
      READ(IE'IDA) NUS,XD,YD,ZD,XF,YF,ZF,HL,(T(IV),IV=1,NV)             611
      IF(T(1).LE.TEST)GO TO 31                                          612
      J=J+1                                                             613
      XA(J)=XD+(XF-XD)/2.-XB                                            614
      YA(J)=YD+(YF-YD)/2.-YB                                            615
      ZA(J)=ZD+(ZF-ZD)/2.-ZB                                            616
      NA=NA+HL                                                          617
      DO 30 IV=1,NV                                                     618
      K=IA+HL*(IV-1)                                                    619
   30 TA(K)=TA(K)+T(IV)*HL                                              620
   31 IDA=IDA+1                                                         621.
    3 CONTINUE                                                          622.
    2 CONTINUE                                                          623.
      IF(NA.EQ.0)GO TO 1                                                624.
C                   GROUP IS INFORMED                                   625.
      NIN(IA+1)=J+1                                                     626.
      DO 10 IV=1,NV                                                     627.
      K=IA+HL*(IV-1)                                                    628.
   10 TA(K)=TA(K)/HA                                                    629.
      NA=NA+1                                                           630.
    1 CONTINUE                                                          631.
      RETURN                                                            632.
      END                                                               633.
C                                                                       634.
C                                                                       635.
C                                                                       636.
C                                                                       637.

      SUBROUTINE KRIGMS(NA,XA,YA,ZA,NIN,XDB,YDB,ZDB,NDB,NSB,R,RR,A,X,   638.
     1IER,CB,COV)                                                       639.
C                                                                       640.
C                   COMPUTE KRIGING MATRICES AND SOLVE KRIGING          641.
C                           SYSTEM.     NV VARIABLES                    642.
C                                                                       643.
C                                                                       644.
C  PARAMETERS   NA          # OF INFORMED WEIGHTS. NA+1 IS THE DIMENSION 645.
C                           OF THE STATIONARY SYSTEM                    646.
C               XA,YA,ZA    COORDINATES OF EACH DATA TO BE USED         647.
C                                                                       648.
C               NIN         GIVES ADDRESS OF EACH GROUP OF INFORMATION  649.
C                           WITHIN THE ARRAYS XA,YA,ZA                  650.
C               NSB             # OF UNITS WITHIN A S.BLOCK             651.
C               R((NA+1)*NV*NSB)   RIGHT HAND SIDE KRIGING MATRICES     652.
C               X((NA+1)*NV*NSB)   ARRAY OF SOLUTION PROVIDED BY RELMSM 653.
C               A((NA+1)*(NA+2)/2*NV)  TRIANGULAR LEFT HAND SIDE KRIGING 654.
C                                      MATRICES                         655.
C               KTILT=IER   INDICATOR OF SINGULARITY OF MATRIX A IF.NE.0 656.
C                                                                       657.
C  SUBROUTINES              CBARS                                       658.
C                           RELMSM                                      659.
C                                                                       660.
      DIMENSION XA(1),YA(1),ZA(1),XDB(1),YDB(1),ZDB(1),NIN(1)           661.
      DIMENSION CB(1),COV(1)                                            662.
      DIMENSION R(1),RR(1),A(1),X(1)                                    663.
      COMMON INP,IOUT,TEST,NV                                           664.
      NEQ=NA+1                                                          665.
      NN=(NEQ+1)*NEQ/2                                                  666.
```

115

```
                                                                    667.
C                                        116                        668.
C     SET THE KRIGING MATRICES                                      669.
      IN=0                                                          670.
      DO 1 J=1,NA                                                   671.
      DO 2 I=1,J                                                    672.
      IN=IN+1                                                       673.
      CALL CBARS(XA,YA,ZA,I,J,NIN,XDB,YDB,ZDB,NDB,NSB,CB,COV)       674.
      DO 20 IV=1,NV                                                 675.
      K=IN+NN*(IV-1)                                                676.
   20 A(K)=CB(IV)                                                   677.
    2 CONTINUE                                                      678.
      CALL CBARS(XA,YA,ZA,J,0,NIN,XDB,YDB,ZDB,NDB,NSB,CB,COV)       679.
      DO 10 L=1,NSB                                                 680.
      DO 10 IV=1,NV                                                 681.
      K=J+NEQ*(L-1)+NEQ*NSB*(IV-1)                                  682.
      IV1=IV+NV*(L-1)                                               683.
      R(K)=CB(IV1)                                                  684.
   10 RR(K)=R(K)                                                    685.
    1 CONTINUE                                                      686.
      DO 3 I=1,NA                                                   687.
      IN=IN+1                                                       688.
      DO 3 IV=1,NV                                                  689.
      K=IN+NN*(IV-1)                                                690.
    3 A(K)=1.                                                       691.
      IN=IN+1                                                       692.
      DO 4 IV=1,NV                                                  693.
      K=IN+NN*(IV-1)                                                694.
      A(K)=0.                                                       695.
      DO 4 L=1,NSB                                                  696.
      K=NEQ+NEQ*(L-1)+NEQ*NSB*(IV-1)                                697.
      R(K)=1.                                                       698.
    4 RR(K)=1.                                                      699.
      NSM=NSB                                                       700.
      M=NSB                                                         701.
      ND=(NA+1)*(NA+2)/2*NV                                         702.
      NC=(NA+1)*NSB*NV                                              703.
C                                                                   704.
C     SOLVE THE SYSTEM                                              705.
C                                                                   706.
      CALL RELMSM(X,A,R,M,NSM,NV,IER)                               707.
      RETURN                                                        708.
      END                                                           709.
C                                                                   710.
C                                                                   711.
C                                                                   712.
C                                                                   713.
      SUBROUTINE RELMSM(X,A,B,M,N,NV,KTILT)                         714.
C                                                                   715.
C        SOLUTION OF A SYSTEM OF LINEAR EQUATIONS WITH TRIANGULAR   716.
C        LEFT HAND SIDE MATRIX.  PIVOTS ARE ON DIAGONAL.  SEVERAL RIGHT 717.
C                  HAND SIDE MATRICES.  NV VARIABLES                718.
C  PARAMETERS                                                       719.
C        X(M*N*NV)       - SOLUTION MATRICES                       720.
C        A(N*(N+1)/2*NV)- LEFT HAND SIDE MATRICES                  721.
C                         VERSUS COLUMNWISE                        722.
C                                                                   723.
C        B(M*N*NV)       - INPUT RIGHT HAND SIDE MATRICES          724.
C                                                                   725.
C        NV      - NUMBER OF VARIABLES
```

```
C       M       - NUMBER OF EQUATIONS
C       N       - # OF RIGHT HAND SIDE MATRICES/VARIABLE.
C                 IN KPIV=, NR OF UNITS/S.BLOCK                    117
C       KTILT   - INDICATOR OF SINGULARITY
C                 KTILT=0  EVERYTHING IS OK!!!
C                 KTILT=-1          M .LE. 1
C                 KTILT=K  A NULL PIVOT APPEARED AT THE KTH. ITERATION
C
C
      DIMENSION X(1),A(1),B(1)
      IF(M.LE.1)GO TO 15
C
C     INITIALIZATIONS
C
      TOL=0.0
      KTILT=0
      NM1=(M+1)/2
      NM=N*M
      M1=M-1
      KK=0
C
C     START TRIANGULATION
      DO 7 K=1,M1
      KK=KK+K
      AK=A(KK)
      IF(AK-TOL)2,1,2
    1 KTILT=K
      RETURN
    2 KN1=K-1
      DO 6 IV=1,NV
      KN1=KN1+(IV-1)
      II=KK+KN1+(IV-1)
      PIV=1./A(II)
      LP=0
      DO 5 I=K,M1
      LL=II
      II=II+I
      R=A(II)*PIV
      LP=LP+1
      IJ=II-KN1
      DO 3 J=I,M1
      IJ=IJ+J
      LL=LL+J
    3 A(IJ)=A(IJ)-R*A(LL)
      DO 4 LLB=K,NM,M
      IN=LLB+LP+NM1
      LL1=LLB+NM1
    4 B(IN)=B(IN)-R*B(LL1)
    5 CONTINUE
    6 CONTINUE
    7 CONTINUE
      IJM=IJ-NM*(NV-1)
      IF(A(IJM)-TOL)9,8,9
    8 KTILT=M
      RETURN
C
C     END OF TRIANGULATION
C
C     START SOLVING BACK
C
    9 DO 14 IV=1,NV                                                 118
      KN1=NM*(IV-1)
      IJ=IJM+NM*(IV-1)
      PIV=1./A(IJ)
      DO 10 LLB=M,NM,M
      LL1=LLB+KN1
   10 X(LL1)=B(LL1)*PIV
      I=M
      KK=IJ
      DO 13 II=1,M1
      KK=KK-I
      PIV=1./A(KK)
      I=I-1
      DO 12 LLB=I,NM,M
      LL1=LLB+KN1
C
      IN=LL1
      R=B(IN)
      IJ=KK
      DO 11 J=I,M1
      IJ=IJ+J
      IN=IN+1
   11 R=R-A(IJ)*X(IN)
   12 X(LL1)=R*PIV
   13 CONTINUE
   14 CONTINUE
      RETURN
C
C
   15 KTILT=-1
      RETURN
      END
C
C
C
C
$DATA
```

NETWORK OF SUPER - BLOCKS

SIZE : DBX= 47.500 DBY= 47.500 DBZ= 47.500
NETWORK ORIGIN : XOB= 18.750 YOB= 18.750 ZOB= 71.250
NETWORK OF 2 LINES, 2 COLUMNS AND 2 LEVELS
KRIGED UNITS= SIZE : DSX= 23.750 DSY= 23.750 DSZ= 23.750 UNITS/S.BLOCK= 8
KRIGING PLAN 1 3 WEIGHTS
WEIGHT INDEX OF S.BLOCK IN NEIGHBORHOOD.NOB.

1 14
2 10 11 12 13 15 16 17 18
3 1 2 3 4 5 6 7 8 9 19 20 21 22 23 24 25 26 27

NUMBER OF VARIABLES TO BE KRIGED = 2

STRUCTURAL CHARACTERISTICS

NAME	NUGGET EFFECT	TYPE	SILL	RANGE
ZINC	1.00	0	0.00	1.00
PB	0.00	1	800.00	1200.00

KRIGING RESULTS/SUPER BLOCK

VALUE 0.000000E 00 CORRESPONDS TO BLANK DATA OR NON KRIGED VALUES

**LEVEL 1 Z= 71.25

****S.BLOCK IB= 1 JB= 1 XB= 18.750 YB= 66.250 NUMBER OF WEIGHTS INFORMED NA= 2 ****

UNIT #	**ZINC**	**PB**
1	1.8180	3.6521
	0.4504E-01	0.8365E 02
2	1.8180	3.7109
	0.4504E-01	0.7574E 02
3	1.8180	3.6416
	0.4504E-01	0.9477E 02
4	1.8180	3.6915
	0.4504E-01	0.4981E 02
5	1.8180	3.5220
	0.4504E-01	0.8149E 02
6	1.8180	3.5545
	0.4504E-01	0.7564E 02
7	1.8180	3.4817
	0.4504E-01	0.5030E 02
8	1.8180	3.5392
	0.4504E-01	0.4742E 02

****S.BLOCK IB= 1 JB= 2 XB= 66.250 YB= 66.250 NUMBER OF WEIGHTS INFORMED NA= 3 ****

UNIT #	**ZINC**	**PB**
1	1.7387	4.0463
	0.4504E-01	0.2217E 02
2	1.7387	4.2263
	0.4504E-01	0.9934E 01
3	1.7387	3.8711
	0.4504E-01	0.2252E 02
4	1.7387	4.0453
	0.4504E-01	0.2241E 02
5	1.7387	4.0405
	0.4504E-01	0.2007E 02
6	1.7387	4.1969
	0.4504E-01	0.1472E 02
7	1.7387	3.8838

	0.4504E-01	0.1537E 02
8	1.7387	4.0413
	0.4504E-01	0.1991E 02

****S.BLOCK IB= 2 JB= 1 XB= 18.750 YB= 18.750 NUMBER OF WEIGHTS INFORMED NA= 3 ****

UNIT #	**ZINC**	**PB**
1	1.7387	3.1388
	0.4504E-01	0.3195E 02
2	1.7387	3.3761
	0.4504E-01	0.3248E 02
3	1.7387	2.8935
	0.4504E-01	0.1212E 02
4	1.7387	3.1392
	0.4504E-01	0.3205E 02
5	1.7387	3.1199
	0.4504E-01	0.2585E 02
6	1.7387	3.3679
	0.4504E-01	0.2608E 02
7	1.7387	2.8604
	0.4504E-01	0.6336E 01
8	1.7387	3.1197
	0.4504E-01	0.2586E 02

****S.BLOCK IB= 2 JB= 2 XB= 66.250 YB= 18.750 NUMBER OF WEIGHTS INFORMED NA= 2 ****

UNIT #	**ZINC**	**PB**
1	1.8180	3.6915
	0.4504E-01	0.4988E 02
2	1.8180	3.7109
	0.4504E-01	0.7583E 02
3	1.8180	3.6421
	0.4504E-01	0.5485E 02
4	1.8180	3.6525
	0.4504E-01	0.8391E 02
5	1.8180	3.5399
	0.4504E-01	0.4743E 02
6	1.8180	3.5352
	0.4504E-01	0.7562E 02
7	1.8180	3.4829
	0.4504E-01	0.5032E 02
8	1.8180	3.5229
	0.4504E-01	0.8149E 02

LD= 1 IAD= 33 NBK= 6

**LEVEL 2 Z= 23.75

****S.BLOCK IB= 1 JB= 1 XB= 18.750 YB= 66.250 NUMBER OF WEIGHTS INFORMED NA= 2 ****

UNIT #	**ZINC**	**PB**
1	1.8180 0.4504E-01	3.3745 0.8652E 02
2	1.8180 0.4504E-01	3.4323 0.8464E 02
3	1.8180 0.4504E-01	3.2929 0.5201E 02
4	1.8180 0.4504E-01	3.3495 0.5328E 02
5	1.8180 0.4504E-01	3.2415 0.9772E 02
6	1.8180 0.4504E-01	3.2918 0.9918E 02
7	1.8180 0.4504E-01	3.1352 0.6011E 02
8	1.8180 0.4504E-01	3.1891 0.6499E 02

****S.BLOCK IB= 1 JB= 2 XB= 66.250 YB= 66.250 NUMBER OF WEIGHTS INFORMED NA= 2 ****

UNIT #	**ZINC**	**PB**
1	1.8180 0.4504E-01	3.5028 0.9339E 02
2	1.8180 0.4504E-01	3.5590 0.1130E 03
3	1.8180 0.4504E-01	3.4354 0.6790E 02
4	1.8180 0.4504E-01	3.5034 0.9331E 02
5	1.8180 0.4504E-01	3.3589 0.1117E 03
6	1.8180 0.4504E-01	3.4224 0.1330E 03
7	1.8180 0.4504E-01	3.2763 0.8422E 02
8	1.8180 0.4504E-01	3.3592 0.1116E 03

****S.BLOCK IB= 2 JB= 1 XB= 18.750 YB= 18.750 NUMBER OF WEIGHTS INFORMED NA= 2 ****

UNIT #	**ZINC**	**PB**
1	1.8180 0.4504E-01	3.2020 0.2253E 02
2	1.8180 0.4504E-01	3.2517 0.2925E 02
3	1.8180 0.4504E-01	3.1464 0.8575E 01
4	1.8180 0.4504E-01	3.2040 0.2263E 02
5	1.8180 0.4504E-01	3.0086 0.2418E 02

6	1.8180	3.0725
	0.4504E-01	0.3530E 02
7	1.8180	2.9162
	0.4504E-01	0.3099E 01
8	1.8180	3.0086
	0.4504E-01	0.2404E 02

123 bis

****S.BLOCK IB= 2 JB= 2 XB= 66.250 YB= 18.750 NUMBER OF WEIGHTS INFORMED NA= 2 ***

UNIT #	**ZINC**	**PB**
1	1.8180	3.3504
	0.4504E-01	0.5325E 02
2	1.8180	3.4332
	0.4504E-01	0.8457E 02
3	1.8180	3.2942
	0.4504E-01	0.5201E 02
4	1.8180	3.3754
	0.4504E-01	0.8647E 02
5	1.8180	3.1892
	0.4504E-01	0.6491E 02
6	1.8180	3.2922
	0.4504E-01	0.9911E 02
7	1.8180	3.1354
	0.4504E-01	0.6000E 02
8	1.8180	3.2419
	0.4504E-01	0.9764E 02

IB= 2 IAD= 65 NBK= 4 (124)

****KRIGING OF UNITS DSX= 23.750 DSY= 23.750 DSZ= 23.750 ****

STATISTICS BASED ON 8KRIGED 5.BLOCKS

125

VARIABLE	MEAN	VARIANCE	NUMBER OF UNITS
ZINC	1.7993	0.1208E-02	64
PB	3.4354	0.9627E-01	64

CAPITULO IV

EL KRIGING UNIVERSAL

En lo que hasta ahora hemos visto, se han asumido condiciones de estacionaridad (o de quasi-estacionaridad) en la función aleatoria Z(x). Pero, ¿ qué alternativa puede adoptarse cuando estas condiciones no se satisfacen?. Esto es, cuando el valor esperado de una variable regionalizada depende de la posición de la variable,

$$E\left[Z(x)\right] = m(x)$$

o cuando no existe suficiente información en la zona a estudiar como para asumir condiciones de quasi-estacionaridad. El método "Kriging Universal" nos da una respuesta a esté problema.

Las funciones aleatorias no estacionarias se caracterizan por presentar cierta disposición en sus realizaciones (o valores); estas realizaciones crecen o decrecen más o menos constantemente a lo largo de ciertas direcciones preferenciales. Por ejemplo, el perfil del fondo marino cercano a las márgenes continentales muestra, a medida que se aleja de la costa, una clara tendencia a incrementar su profundidad con el alejamiento. Esta actitud de los valores de las variables aleatorias regionalizadas se denomina "tendencia" (trend o drift, en inglés, y dérive, en francés). (Estrictamente hablando, trend y drift son dos conceptos diferentes -Matheron, 1969- los cuales no discutiremos aquí).

El Kriging Universal es un método que proporciona un estimador lineal insesgado y toma en cuenta, además, la tendencia. Todo ello a partir del conocimiento de ambos, la forma de la tendencia $E\left[Z(x)\right] = m(x)$ y del modelo de la estructura de variabilidad $\gamma(h)$ de la f.a. $Z(x)$.

Por definición, la tendencia $m(x)$ es la media de la f.a. $Z(x)$, $E\left[Z(x)\right] = m(x)$. Evaluar el semi-variograma $\gamma(h)$ de $Z(x)$ implica asumir condiciones de estacionaridad, o bien conocer la fórmula de la función $m(x)$; de otra manera, la estimación de $\gamma(h)$ y $m(x)$ debería de efectuarse simultáneamente y a partir de la realización única $z(x)$, lo cual sería, rigurosamente hablando, imposible.

La forma de la función $m(x)$ puede estar definida como una combinación lineal de funciones pre-establecidas $f_\ell(x)$,

$$m(x) = \sum_{\ell=1}^{k} a_\ell \; f_\ell(x)$$

donde los coeficientes a_ℓ permanecen desconocidos.

La función $m(x)$ podría expresarse de las diversas maneras siguientes: $m(x) = a_1 + a_2 x$ - tendencia lineal (1 dimensión)

$m(x) = a_1 + a_2 x + a_3 x^2$ - tendencia cuadrática (1 dimensión)

$m(x) = a_1 + a_2 u + a_3 v + a_4 u^2 + a_5 uv + a_6 v^2$ - tendencia cuadrática (2 dimensiones)

etc.

Ecuaciones del Kriging Universal.

Consideremos la estimación del valor medio $Z_V(x_0)$, definido en

el bloque $V(x_0)$, a partir de los n datos Z_{v_α} definidos a su vez sobre los soportes v_α ;

La función aleatoria puntual correspondiente $Z(x)$ es no estacionaria y, dentro de una vecindad $R(x_0)$, presenta una tendencia de forma

$$E\left[Z(x)\right] = m(x) = \sum_{\ell=1}^{K} a_\ell \, f_\ell(x)$$

en donde las funciones $f_\ell(x)$ se asumen conocidas.

$R(x_0)$ representa una vecindad centrada en x_0 e incluye el bloque V y los soportes de todos los datos usados en la estimación. Dentro de $R(x_0)$, la función covariancia $C(h)$ o el semi-variograma $\gamma(h)$ son conocidos.

Entonces, un estimador lineal de $Z_V(x_0)$ puede estar dado de la siguiente forma:

$$Z_K^* = \sum_{\alpha=1}^{n} \lambda_\alpha \, Z_{v_\alpha}$$

La condición de insesgamiento se establece a partir de:

$$E\left[Z_V - Z_K^*\right] = E\left[Z_V\right] - E\left[Z_K^*\right] = 0$$

donde

$$E\left[Z_V\right] = 1/V \int_{V(x_0)} E\left[Z(x)\right] dx = \sum_{\ell=1}^{K} a_\ell \; 1/V \int_{V(x_0)} f_\ell(x) dx$$

denotando, en general, al valor medio de la función $f_\ell(x)$ sobre el soporte v por b_v^ℓ, esto es: $b_v^\ell = 1/v \int_v f_\ell(x) dx$

la expresión anterior se reduce a:

$$E\left[Z_V\right] = \sum_{\ell=1}^{K} a_\ell \, b_v^\ell$$

similarmente

$$E\left[Z_K^*\right] = \sum_{\alpha=1}^{n} \lambda_\alpha E\left[Z_{v_\alpha}\right] = \sum_{\alpha=1}^{n} \lambda_\alpha \; 1/v_\alpha \int_{v_\alpha} E\left[Z(x)\right] d_x$$

$$= \sum_{\alpha=1}^{n} \lambda_\alpha \; 1/v \sum_{\ell=1}^{K} a_\ell \int_v f_\ell(x)\, d_x = \sum_{\alpha=1}^{n} \sum_{\ell=1}^{K} \lambda_\alpha \; a_\ell \; b_v^\ell$$

Para obtener la condición de insesgamiento basta con imponer las K condiciones siguientes:

$$\sum_{\alpha=1}^{n} \lambda_\alpha \, b_{v_\alpha}^\ell = b_v^\ell \,, \qquad \forall \; \ell = 1 \text{ a } K$$

Como puede observarse en esta última expresión, en nada intervienen los coeficientes a_ℓ de la tendencia; además, esta condición elimina todos los términos donde la tendencia m(x) aparece permitiendo reducir la expresión de la variancia de estimación:

$$\sigma_K^2 = \mathrm{Var}\left[Z_V - Z_K^*\right] = E\left[\left\{Z_V - Z_K^* - \left(E\left[Z_V\right] - E\left[Z_K^*\right]\right)\right\}^2\right]$$

$$= E\left[\left(Z_V - Z_K^*\right)^2\right]$$

Repitiendo el procedimiento seguido en el método Kriging (bajo condiciones de estacionaridad), la variancia de estimación puede escribirse también en función de la covariancia:

$$E\left[(Z_V - Z_K^*)^2\right] = \overline{C}(V,V) - 2\sum_{\alpha}\lambda_\alpha \overline{C}(V, v_\alpha) + \sum_{\alpha}\sum_{\beta}\lambda_\alpha\lambda_\beta\overline{C}(v_\alpha, v_\beta)$$

La minimización de la variancia de estimación estará sujeta, en este caso, a las K condición de insesgamiento establecidas arriba. Empleando el método de los multiplicadores de Lagrange se obtiene un sistema de n+K ecuaciones lineales con n+K incógnitas (los n coeficientes , y los K multiplicadores de Lagrange μ_k). .Este sistema se denomina - "Sistema del Kriging Universal":

$$\sum_{\beta=1}^{n}\lambda_\beta\,\overline{C}(v_\alpha, v_\beta) - \sum_{\ell=1}^{K}\mu_\ell\, b_V^\ell = \overline{C}(v_\alpha, V), \quad \forall\ \alpha = 1 \text{ a } n$$

$$\sum_{\beta=1}^{n}\lambda_\beta\, b_{v_\beta}^\ell = b_V^\ell, \quad \forall\ \ell = 1 \text{ a } K$$

donde la mínima variancia de estimación correspondiente queda igual a:

$$\sigma_K^2 = \overline{C}(V,V) + \sum_{\ell=1}^{K}\mu_\ell\, b_V^\ell - \sum_{\alpha=1}^{n}\lambda_\alpha\,\overline{C}(v_\alpha, V)$$

o escrito en términos de la función semi-variograma $\gamma(h)$:

$$\sum_{\beta}\lambda_\beta\overline{\gamma}(v_\alpha, v_\beta) + \sum_{\ell}\mu_\ell b_{v_\alpha}^\ell = \overline{\gamma}(v_\alpha, V), \quad \forall\ = 1 \text{ a } n$$

$$\sum_{\beta=1}^{n}\lambda_\beta\, b_{v_\beta}^\ell = b_V^\ell, \quad \forall\ \ell = 1 \text{ a } K$$

y

$$\sigma_K^2 = \sum_{\alpha=1}^{n}\lambda_\alpha\,\overline{\gamma}(v_\alpha, V) + \sum_{\ell}\mu_\ell b_V^\ell - \overline{\gamma}(V,V)$$

El caso del Kriging estacionario puede verse como un caso particular del Universal Kriging cuando K es idéntica a 1, es decir, para -

tendencias de la forma

$$m(x) = a_1 f_1(x) = a_1, \text{ con } f_1(x) = 1, \qquad \forall_x.$$

Estimación de Reservas in-Situ

Entre las herramientas de la geoestadística, aparte del variograma y de la variancia de estimación, existe otra herramienta conocida como "variancia de dispersión" la cual es de gran utilidad en la solución de aquellos problemas relacionados con la estimación de reservas.

En ingeniería minera, por ejemplo, conocer el valor medio del porcentaje de mineral en un cierto bloque V de un yacimiento, es de poca utilidad si no se tiene también una idea de la variabilidad o dispersión de los valores del porcentaje de mineral asociados a las unidades de producción v, localizadas dentro de V. En la explotación del yacimiento, la medida de la variación diaria de la producción es uno de los parámetros más importantes que se consideran en la selección adecuada del equipo (molinos, transportadoras, etc.).

Variancia de Dispersión

Consideremos un bloque V centrado en el punto x y dividido en N unidades iguales $v(x_i)$ centrados en los puntos x_i: $V = \sum_{i=1}^{N} v_i = Nv$.

Sea $z(y)$ una variable regionalizada puntual. El valor medio de la variable en cada unidad $v(x_i)$ será:

$$\overline{C}\,(V(x),\ V(x)) = \overline{C}\,(V,\ V), \qquad \forall_x$$

y $$\overline{C}\,(v(y),\ v(y)) = \overline{C}\,(v,\ v), \qquad \forall_y$$

Sustituyendo en la última expresión de la variancia de dispersión $D^2(v/V)$, obtenemos:

$$D^2(v/V) = 1/V \int_{V(x)} \left\{ \overline{C}(V,\ V) + \overline{C}(v,\ v) - 2\overline{C}(V(x),\ v(y)) \right\} d_y$$

donde

$$1/V \int_{V(x)} \overline{C}\,(V(x),\ v(y))\ d_y = \overline{C}\,(V(x),\ V(x)) = \overline{C}(V,\ V)$$

Finalmente

$$\boxed{D^2(v/V) = \overline{C}(v,\ v) - \overline{C}(V,\ V)}$$

o en términos del semi-variograma $\gamma(h)$

$$\boxed{D^2(v/V) = \overline{\gamma}(V,\ V) - \overline{\gamma}(v,\ v)}$$

La propiedad de aditividad de la variancia de dispersión puede ser establecida como una consecuencia de la linearidad en la expresión de $D^2(v/V)$. Esta propiedad encontrada experimentalmente por D. G. Krige estableco lo siguiente:

$$D^2(v/G) = D^2(v/V) + D^2(V/G), \text{ si } v \subset V \subset G.$$

La dispersión de la unidad v dentro del depósito G es igual a la suma de la dispersión de v dentro del bloque V y la dispersión de los bloques V dentro del depósito G. Esta propiedad se le conoce como "Relación de Krige".

Nota: La variancia de dispersión aumenta cuando el tamaño del soporte - disminuye.

$$D^2(v/G) \geq D^2(V/G) \qquad \text{si} \qquad v < V$$

Si v es puntual y V es infinitamente grande, entonces $D^2(0/\infty) = -$ $\bar{\gamma}(\infty, \infty) - \bar{\gamma}(0, 0) = \gamma(\infty) - \gamma(0) = C(0)$.

Estimación de Reservas In-Situ

Una vez efectuadas las estimaciones locales, el siguiente paso le corresponde a la estimación global, es decir, la estimación correspondiente a toda la zona de estudio D (Figura 22).

La estimación global puede llevarse a cabo simplemente ponderando las diversas estimaciones locales

$$Z_D^* = 1/D \sum_i V_i Z_{V_i}^*$$

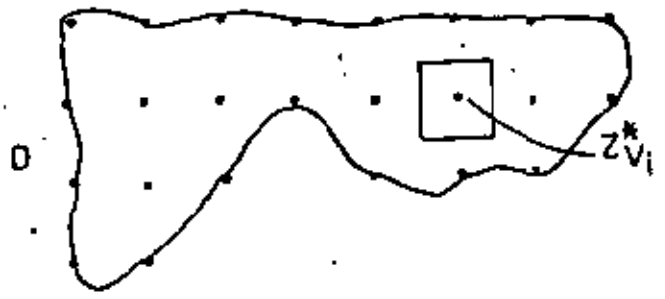

Figura 22

Las variancias de estimación locales, sin embargo, no pueden combinarse tan fácilmente como los estimadores locales.

Si $Z_D^* = 1/D \sum_{i=1}^{N} V_i Z_{V_i}^*$ representa la combinación de los N valores $Z_{V_i}^*$ obtenidos por el método Kriging, y $\sigma_{KV_i}^2 = E\left[(Z_{V_i} - Z_{V_i}^*)^2 \right]$ representa la variancia de estimación de cada unidad V_i, entonces, la variancia de estimación global asociada al estimador Z_D^* es igual a:

$$\sigma_{ED}^2 = E\left[(Z_D - Z_D^*)^2 \right] = E\left[\left\{ 1/D \sum_i V_i (Z_{V_i} - Z_{V_i}^*) \right\}^2 \right]$$

$$= 1/D^2 \left[\sum_i V_i^2 \sigma_{KV_i}^2 + \sum_i \sum_{j \neq i} V_i V_j E\left[(Z_{V_i} - Z_{V_i}^*)(Z_{V_j} - Z_{V_j}^*) \right] \right]$$

Cada vez que los estimadores $Z_{V_i}^*$ y $Z_{V_j}^*$ $(j \neq i)$ han considerado información común, los dos errores de estimación $\left[Z_{V_i} - Z_{V_i}^* \right]$ y $\left[Z_{V_j} - Z_{V_j}^* \right]$ están correlacionados. Por lo tanto, algunas de las covariancias $E\left[(Z_{V_i} - Z_{V_i}^*)(Z_{V_j} - Z_{V_j}^*) \right]$ son diferentes de cero y su suma no puede despreciarse frente al término $1/D^2 \sum_i V_i^2 \sigma_{KV_i}^2$ de la expresión de la variancia de estimación global σ_{ED}^2.

El cálculo de la expresión $E\left[(Z_{V_i} - Z_{V_i}^*)(Z_{V_j} - Z_{V_j}^*) \right]$ es sumamente tedioso. Sin embargo, es posible obtener buenas aproximaciones en los casos particulares siguientes:

(i) Malla regular.- Si la zona D está dividida en una malla regular de N celdas (Figura 23), donde cada celda d_i de la malla tiene una muestra en su centro, y si esta muestra es la única que interviene en la estimación de $Z_{V_i}^*$, entonces los errores $\left[Z_{V_i} - Z_{V_i}^* \right]$ y $\left[Z_{V_j} - Z_{V_j}^* \right]$ serán independientes, ya que ninguno de ellos empleará datos comunes. Por lo tanto, la variancia de estimación global puede expresarse como $\sigma_{ED}^2 = 1/N \ \sigma_{KV}^2$, ya que todas las variancias de estimación locales $\sigma_{KV_i}^2$ serán idénticas.

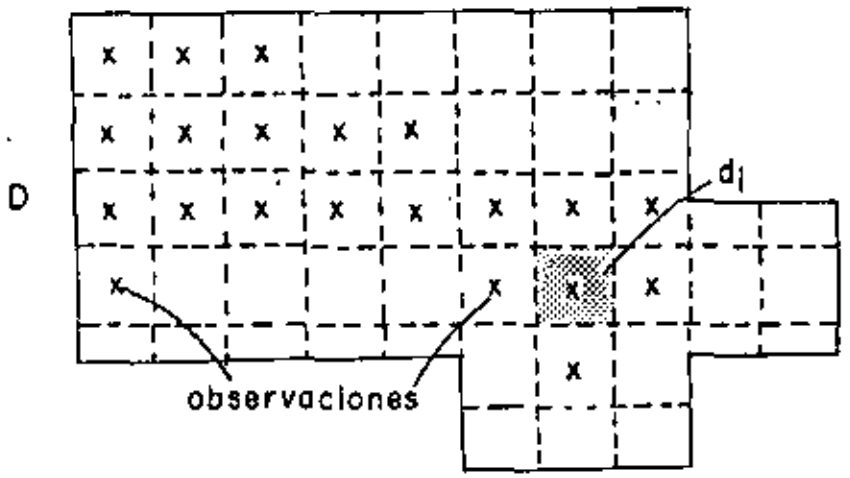

Figura 23

(ii) Malla regular con datos uniformemente distribuídos.- Si la zona D está dividida en una malla regular de N celdas (Figura 24), donde cada celda d_i de la malla contiene una muestra localizada al azar y con la misma probabilidad de situarse en cualquier punto de la celda, y si esta muestra es la única que interviene en la

estimación de $Z_{V_i}^*$, entonces los errores $\left[Z_{V_i} - Z_{V_i}^* \right]$ y $\left[Z_{V_j} - Z_{V_j}^* \right]$ serán independientes ya que ninguno de ellos empleará información común. Por lo tanto, la variancia de estimación global puede expresarse como $\sigma_{ED}^2 = 1/N^2 \sum_i \sigma_{KV_i}^2$, ya que cada variancia de estimación local $\sigma_{KV_i}^2$ dependerá de la posición de la muestra en la celda d_i.

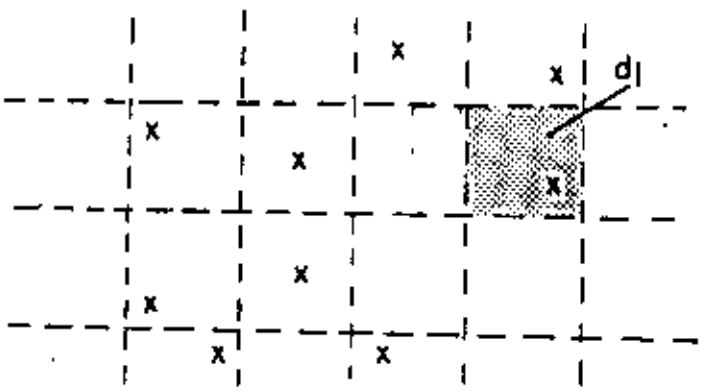

Figura 24

(iii) Malla irregular. - Si la zona D está dividida en una malla irregular (Figura 25), donde cada celda d_i contiene una muestra localizada en su centro, y si esta muestra es la única que interviene en la estimación de $Z_{V_i}^*$, entonces los errores $\left[Z_{V_i} - Z_{V_i}^* \right]$ y $\left[Z_{V_j} - Z_{V_j}^* \right]$ serán como en los ejemplos anteriores, independientes. La variancia de estimación global queda igual a:

$$\sigma_{ED}^2 = 1/D \sum_i V_i^2 \sigma_{KV_i}^2$$

ya que cada variancia de estimación local $\sigma_{KV_i}^2$ dependerá del -

volumen V_i asociado a la celda d_i.

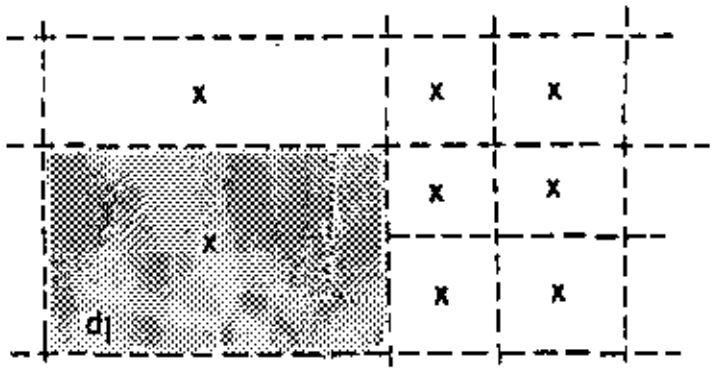

Figura 25

En los ejemplos que hemos visto, se ha asumido implícitamente - que la geometría o el volumen de la zona o depósito es conocida. En - aquellos casos donde exista cierta incertidumbre asociada a la extensión - del depósito a estimar, otros métodos, tales como "el método transitivo" (Matheron, 1971), deben aplicarse.

Evaluación de Reservas Recobrables

Una vez determinadas las reservas in-situ, es necesario determi-- nar qué porcentaje de estas reservas puede ser explotado siguiendo un - determinado contexto técnico y/o económico.

En un proyecto minero, por ejemplo, son diversos los factores que afectan la estimación de las reservas recobrables, entre los que se puede citar:

- el criterio de selección, el cual puede estar orientado hacia la maximización de las ganancias o hacia la satisfacción de las demandas

del mercado.

- los parámetros adoptados para alcanzar tal selección, entre los que se pueden citar la ley de corte, el espesor mínimo minable, etc.
- Las restricciones tecnológicas del proyecto minero; si la explotación es a cielo abierto, un bloque V puede ser minado únicamente si todos los bloques arriba de éste, en un cono con vértice en V, han sido previamente minados.
- el soporte (tamaño y forma) de la unidad selectiva (bloque de varios cientos o miles de toneladas).
- la información disponible al tiempo de la selección.

La influencia en la selección por parte del soporte y del nivel de información está expresada en las variancias de estimación y de dispersión, lo cual representa una de las principales aportaciones de la geoestadística a la ingeniería minera.

Algunos factores semejantes podrían citarse en ingeniería petrolera. Además de los criterios ya mencionados arriba, el criterio de selección podría estar dictado por aspectos políticos y/o sociales. La saturación de aceite (o de gas) sería uno de los parámetros más importantes en la selección y por ende en la estimación de las reservas recobrables. Como ejemplos de restricciones tecnológicas se podrían citar la profundidad del yacimiento a estimar, su localización (marina o terrestre), etc. Por último, el soporte y el nivel de información serían dos factores de vital importancia en la evaluación de la porosidad y de la permeabilidad,

ambos parámetros fundamentales para la determinación de las caracterís ticas de yacimientos petroleros.

Método de Estimación.

La estimación del volumen de mineral recobrable en un depósito - G, basada en los datos del porcentaje de mineral medidos en núcleos de longitud constante $z_c(x)$, podría realizarse empleando un histograma de - los valores $z_c(x)$ (Figura 26). En el eje de las abscisas estarían referi dos todos los valores de $z_c(x)$, y en el de las ordenadas, las frecuencias de estos valores. El histograma de los valores $z_c(x)$ tendría una me-- dia experimental m*, una variancia de dispersión $s^2(c/G)$, y una cierta - forma (asimétrica, por ejemplo). Considerando ahora un cierto valor - z_0 en las abscisas, el área achurada representaría la proporción de - núcleos con porcentaje del mineral mayor a z_0 ($z_c(x) \geq z_0$), lo cual al - multiplicarse por el volumen total de roca proporcionaría un estimador - del valor de la reserva recobrable.

Suponiendo ahora conocidos los valores $z_v(x)$ (porcentaje de mine- ral asociado al soporte de volumen v), un histograma (Figura 26) con - media experimental m*, variancia de dispersión $s^2(v/G)$, y de cierta for ma (simétrica, por ejemplo), podría ser construído. El área punteada - representaría, en este caso, la proporción de bloques de volumen v con - porcentaje de mineral $z_v(x) \geq z_0$.

Lo que se observa en estos dos histogramas no puede clasificarse de ninguna manera como trivial. Sabemos que en un proyecto minero -

la selección se efectúa en bloques de tamaño v y no en muestras del tamaño de un núcleo. Considerando valores de $m^* > z_0$, cualquier evaluación de las reservas recobrables basada en el histograma de los valores $z_c(x)$ sobreestimaría la proporción real de bloques con valores medios mayores a z_0, o lo que es lo mismo, subestimaría la proporción de metal pronosticado como desperdicio. Esto es debido, entre otras razones, a que la variancia de dispersión de los valores con soporte v es menor a la variancia de dispersión de los valores con soporte c, $s^2(v/G) < s^2(c/G)$.

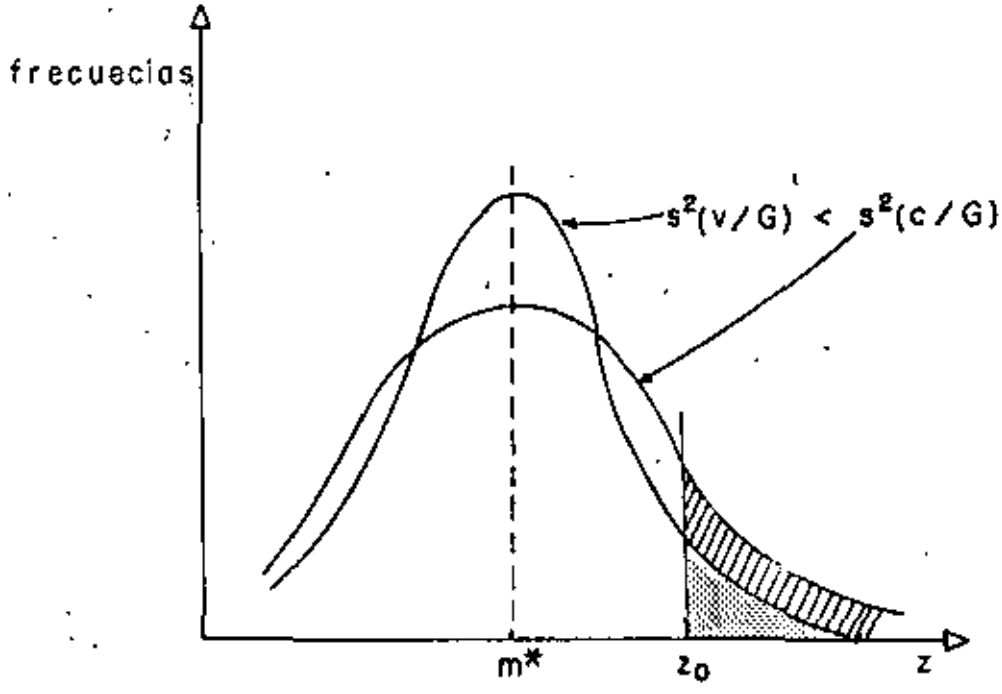

Figura 26 - Histogramas Experimentales

Una de las ventajas de la geoestadística es la de permitirnos evaluar el valor teórico de la variancia de dispersión $D^2(v/G)$ asociada a los valores Z_v, ya sea por medio de la fórmula

$$D^2(v/G) = \bar{\gamma}(G, G) - \bar{\gamma}(v, v)$$

o bien por medio de la relación de Krige

$$D^2(v/G) = D^2(c/v) + D^2(c/G)$$

donde las dos últimas variancias de dispersión pueden obtenerse experimentalmente.

Asimismo, el valor medio de Z_V puede ser estimado por m*, el valor medio de los valores de las muestras disponibles. Sin embargo, no bastan estos dos parámetros (m* y $D^2(v/G)$) para inferir la distribución de Z_V. Es necesario conocer también el tipo de distribución. Si el tipo de distribución no se conoce de antemano, será indispensable adoptar la hipótesis de "permanencia de distribución", la cual consiste en asumir la misma distribución para las dos variables aleatorias, Z_c y Z_V.

Pero, ¿es realmente útil conocer el histograma de los valores reales Z_V?. Sabemos que, excepto en casos muy raros, los valores reales z_v son desconocidos al momento de la selección, y que, en efecto, la selección se basa en los valores estimados z_v^*. Esto significa que la recuperación real consistirá no de aquellas unidades con valores reales $z_v \geq z_0$, sino más bien, de aquellas unidades con valores estimados $z_v^* \geq z_0$, es decir, del área punteada mostrada en la Figura 27.

Esta área punteada, fundada en unidades con valores $z_v^* \geq z_0$, difiere del área achurada correspondiente a una selección perfecta efec-

tuada sobre los valores reales ($z_v \geq z_0$). Esta diferencia resulta más importante a medida que la desviación entre los valores estimados z_v^* y los valores reales z_v se incrementa, esto es, cuando la variancia de estimación $E\left[(Z_v - Z_v^*)^2\right]$ aumenta. Sabemos, por otro lado, que el método Kriging minimiza esta variancia.

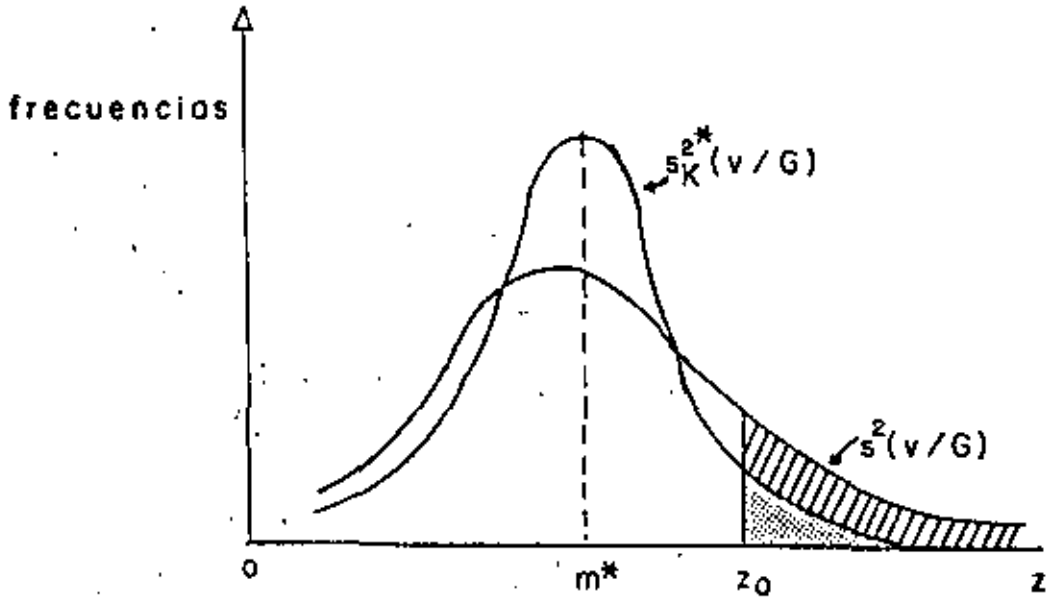

Figura 27 - Distribución de valores reales y estimados.

Si el estimador Z_v^* está definido según el método Kriging, entonces es posible establecer una relación entre las variancias de los valores reales y de los valores estimados $D^2(v/G)$ y $D_K^{2*}(v/G)$ (Journel & Huijbregts, 1978, p. 451), esto es:

$$D^2(v/G) \cong D_K^{2*}(v/G) + \bar{\sigma}_{Kv}^2$$

donde $\bar{\sigma}_{Kv}^2$ es el valor medio de las variancias de estimación, asociadas a las unidades v_i

$$\bar{\sigma}_{K_v}^2 = 1/N \sum_{i=1}^{N} \sigma_{Kv_i}^2$$

A través de esta relación es más fácil inferir la distribución de - Z_v, que a través de la hipótesis de permanencia de la distribución. - Además, se puede deducir que $D_K^{2*}(v/G) \leq D^2(v/G)$, lo que significa - que el método Kriging introduce un efecto de suavizamiento en la estimación y el cual se refleja directamente en el histograma de $z_{v_k}^*$.

El objetivo de la estimación de reservas puede, finalmente, representarse graficando la distribución bi-variable de las funciones aleatorias Z_v y Z_v^*, como se muestra en la Figura 28.

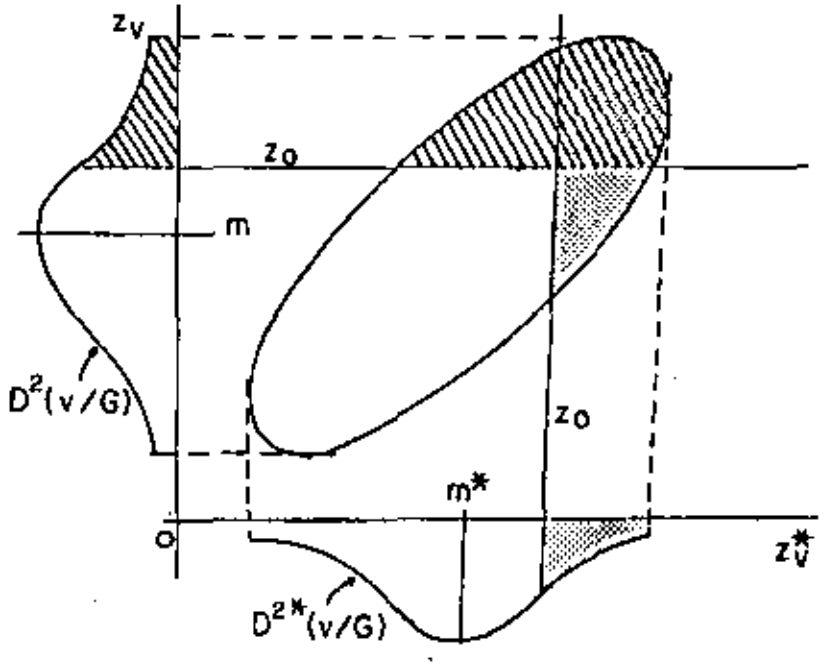

Figura 28 - Distribución bi-variable de valores reales y estimados, - Z_v y Z_v^*.

Los valores reales aparecen a lo largo del eje de las ordenadas y los valores estimados a lo largo del eje de las abscisas. La distribución de los valores reales, con variancia $D^2(v/G)$, puede ser vista -

como la proyección de la distribución bi-variable sobre el eje de las ordenadas. El área achurada representando el valor real del porcentaje de mineral recobrable. Similarmente, la distribución de los valores estimados, con variancia $D^{2*}(v/G)$, puede ser vista como la proyección de la distribución bi-variable sobre el eje de las abscisas. El área punteada denotando el valor estimado del porcentaje de mineral recobrable.

Observando las correspondientes áreas achurada y punteada de la distribución bi-variable, puede verse que se comete un error en la estimación. Primero, un cierto número de unidades estimadas como pobres, resultan arriba del valor z_0; estas unidades se rechazan equivocadamente y corresponden al área achurada, exclusivamente. Segundo, un cierto número de unidades estimadas con valores arriba de z_0 resultan pobres; estas unidades se aceptan erróneamente y corresponden al área punteada, exclusivamente.

El objetivo pues, será la minimización de estas dos áreas, mismo que se obtiene al minimizar la variancia de estimación $E\left[(Z_v - Z_v^*)^2\right]$.

$$z_v(x_i) = 1/v \int_{v(x_i)} z(y)\, d_y$$

y el valor medio de la variable en el bloque V será:

$$z_v(x) = 1/V \int_{V(x)} z(y)\, d_y = 1/N \sum_{i=1}^{N} z_v(x_i)$$

Considerando los N valores $z_v(x_i)$ y la media $z_V(x)$, es posible calcular la variancia experimental, es decir:

$$s^2(x) = 1/N \sum_{i=1}^{N} (z_V(x) - z_v(x_i))^2$$

En la práctica, sin embargo, en la etapa de evaluación, los valores verdaderos $z_v(x_i)$ de las unidades v_i, así como el valor medio $z_V(x)$ son desconocidos. El problema es, otra vez, estimar las dos características principales de la distribución de los valores $z_v(x_i)$, es decir, la media $z_V(x)$ y la variancia $s^2(x)$.

Si $z(y)$ es interpretada como una realización de la función aleatoria $Z(y)$, entonces $z_v(x_i)$ y $z_V(x)$ aparecerán como realizaciones de las funciones aleatorias $Z_v(x_i)$ y $Z_V(x)$, respectivamente.

Consecuentemente, $s^2(x)$ se interpretará como una realización de la f.a. $S^2(x)$ definida en el punto x.

$$S^2(x) = 1/N \sum_{i=1}^{N} (Z_V(x) - Z_v(x_i))^2$$

Bajo la hipótesis de estacionaridad de la función aleatoria puntual Z(y), el valor esperado o media de la f.a. $S^2(x)$ es, por definición, la - variancia de dispersión de las unidades v dentro de V, representada - como:

$$D^2(v/V) = E\left[S^2(x)\right] = E\left[1/N \sum_{i=1}^{N} (Z_V(x) - Z_v(x_i))^2\right]$$

Generalizando para el caso contínuo se tiene:

$$S^2(x) = 1/V \int_{V(x)} (Z_V(x) - Z_v(y))^2 \, d_y$$

y

$$D^2(v/V) = E\left[1/V \int_{V(x)} (Z_V(x) - Z_v(y))^2 \, d_y\right] , \quad v << V$$

o bien

$$D^2(v/V) = 1/V \left[\int_{V(x)} E\left[(Z_V(x) - Z_v(y))^2\right] \, d_y\right]$$

$$= 1/V \int_{V(x)} \sigma_E^2 (V(x), v(y)) \, d_y, \qquad v << V$$

La variancia de dispersión aparece entonces como el valor medio - sobre V(x) de la variancia de estimación introducida al pretender calcular $Z_V(x)$ empleando $Z_v(y)$. Recordando la fórmula de la variancia de estimación, el término $\sigma_E^2 (V(x), v(y))$ puede escribirse como:

$$\sigma_E^2 (V(x), v(y)) = \overline{C}(V(x), V(x)) + \overline{C}(v(y), v(y)) - 2\overline{C}(V(x), v(y))$$

dado que C(h) es estacionaria (no depende de la posición x ó y), podemos escribir:

GEOESTADISTICA

FE DE ERRATAS

FE DE ERRATAS

Pág. 33, Tercero y cuarto renglón:

dice: $r = a'/3$

debe de decir: $r = a = a'/3$ un tercio del rango práctico a', para el modelo exponencial.

Pág. 39, primer renglón:

dice: roca al cual se le ha determinado sus características petrográficas, porosidad, permeabilidad, etc.

debe de decir: roca al cual se le ha determinado, empleando siempre la misma técnica, sus características petrográficas, porosidad, permeabilidad, etc.

Cuarto renglón:

Dice: Donde el porcentaje medio de un mineral se dice estimar.

Debe decir: Donde el porcentaje medio de un mineral se dice estima. (Otra vez, asumiendo que se sigue un mismo método de muestreo).

Séptimo y octavo renglón:

Dice: cuenca geológoca, un yacimiento petrolero o una zona de mineralización primaria.

Debe de decir: cuenca geológica, un yacimiento petrolero o una zona de mineralización.

Pág. 88, onceavo renglón

Dice: 6.- La represión de σ_E^2

Debe de decir: 6.- La expresión de σ_E^2

Pág. 136, noveno renglón:

Dice: ii) Malla regular con datos uniformemente distribuidos.

Debe de decir: ii) Malla regular con datos cuya localización está uniformemente distribuida.

GEOESTADISTICA

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MARZO, 1 9 8 0

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GEOESTADISTICA

EVALUACION DE RESERVAS
(2a. Parte)

DR. JUAN MANUEL BERLANGA GUTIERREZ

MARZO, 1 9 8 0

EJEMPLO DE APLICACION EN UN DEPOSITO MINERO SIMULADO.

Trabajar con depósitos simulados, en contraste con depósitos reales, reporta una serie de ventajas. En un depósito simulado es posible de una manera inmediata y sin tener que "explotar" el depósito, comprobar los resultados obtenidos de las estimaciones con los datos "reales". En un depósito simulado es posible determinar exactamente el efecto de cualquier método de estimación en la evaluación de las reservas recobrables. Un depósito simulado es un depósito perfectamente conocido.

La zona simulada en este estudio consiste de un rectángulo de dimensión 50 u x 10 u, formado por 500 bloques cuadrados v_i (de dimensión u x u), Figura 29. Un total de 60,500 datos (porcentaje de mineral) se simularon en una malla regular de 550x110 nodos, dando un total de 121(=11x11) muestras por bloque v_i. Los 60,500 datos se consideran, para efectos de este ejemplo, como el depósito real G a estudiar.

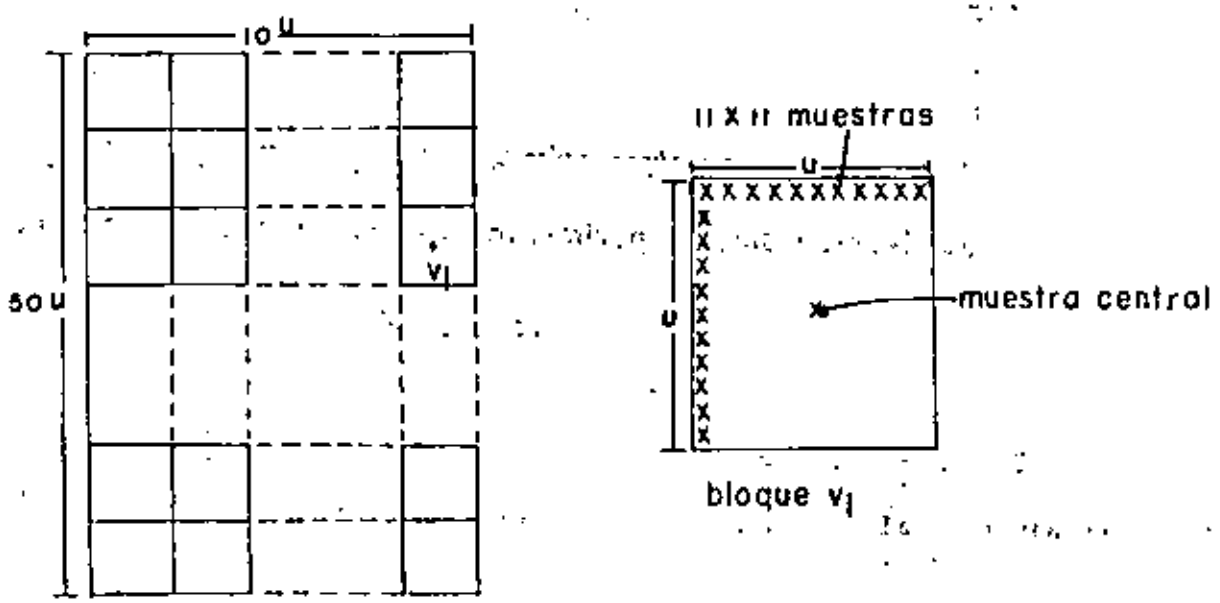
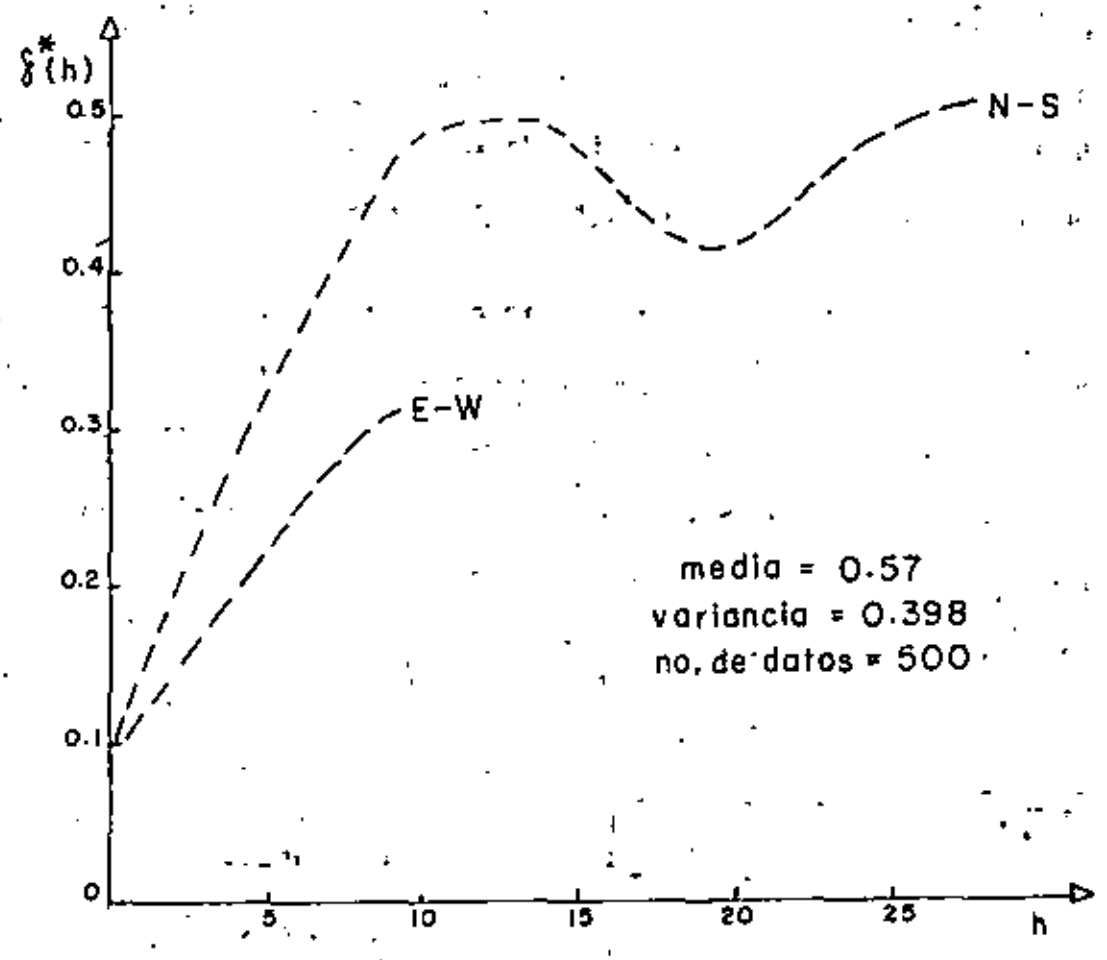


FIGURA 29

Empleando las 500 muestras centrales s_i (una por cada bloque v_i), tal y como si ellas constituyeran los primeros resultados de una campaña de exploración, se obtuvo el semi-variograma experimental $\gamma^*(h)$ de la Figura 30.



Semivariograma experimental de 500 muestras centrales

Figura 30

Un semi-variograma teórico $\gamma(h)$ con efecto de agujero e isotrópico fue ajustado al semi-variograma experimental $\gamma^*(h)$.

$$\gamma(h) = \begin{cases} 0 & \text{si } h=0 \\ 0.1+3.6\left[1-\frac{\text{Sen } 0.041h}{0.041h}\right], & \forall h>0 \end{cases}$$

Empleando el modelo de $\gamma(h)$ se verificó la relación de Krige (ver Tabla 3):

$$D^2(o/G) = D^2(o/v) + D^2(v/G)$$

Las diferentes variancias de dispersión fueron calculadas, para el caso de los 60,500 datos, por medio del valor esperado de las variancias experimentales (esto es, según la definición de $D^2(\cdot/\cdot)$), y para el caso de las 500 muestras, a través de la fórmula

$$D^2(v/G) = \overline{\gamma}(G,G) - \overline{\gamma}(v, v).$$

TABLA 3. Relación de Krige

	Variancia de Dispersión Real		Variancia de Dispersión Estimada	
$D^2(o/G)$	0.3846		0.398	
$D^2(o/v)$	0.1236	+ = 0.3840	0.1270	+ = 0.398
$D^2(v/G)$	0.2604		0.2710	

Cada uno de los porcentajes medios z_{v_i} y errores $\sigma^2_{k_{v_i}}$ de los 500 bloques v_i fueron estimados según el método Kriging. El número de muestras consideradas en la estimación de z_{v_i} fue de 9, según se muestra en la Figura 31.

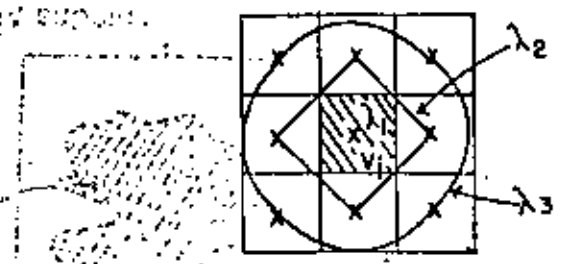


FIGURA 31

El coeficiente λ_1 fue asignado a la muestra central; el coeficiente λ_2 fue asignado al valor promedio de las cuatro muestras de la primer aureola; y λ_3 fue asignado a las cuatro muestras restantes de la aureola externa. Posteriormente, se verificó el efecto de suavizamiento inherente del método Kriging.

$$D^2(v/G) \simeq D_k^2(v/G) + \overline{\sigma^2_{k_{v_i}}}$$

obteniéndose los siguientes resultados:

$$D^2(v/G) = 0.2604,\ D_k^2(v/G) = 0.2349 \text{ y } \sigma^2_{k_{v_i}} = 0.042$$

ó $0.2604 = 0.2349 + 0.042 = 0.2769$

Finalmente, y con el propósito de comparar la eficiencia del método Kriging con otros métodos de estimación, se calcularon curvas de tonelaje, de cantidad de metal recobrado, y de porcentaje de mineral promedio contra varios valores del porcentaje de corte, z_c.

El tonelaje se calculó empleando el estimador Kriging $I^*_{k_{v_i}}$ de la variable aleatoria regionalizada "indicador", I_{v_i}. Esta variable indicador denota la proporción mineralizada del bloque v_i (Figura 32). El tonelaje se define, entonces, como la suma de aquellos valores estimados $I^*_{k_{v_i}}$ asociados a valores $z^*_{k_{v_i}} \geq z_c$, para cada bloque v_i

$$T(z_c) = \sum_{i=1}^{N(z_c)} I^*_{k_{v_i}}$$

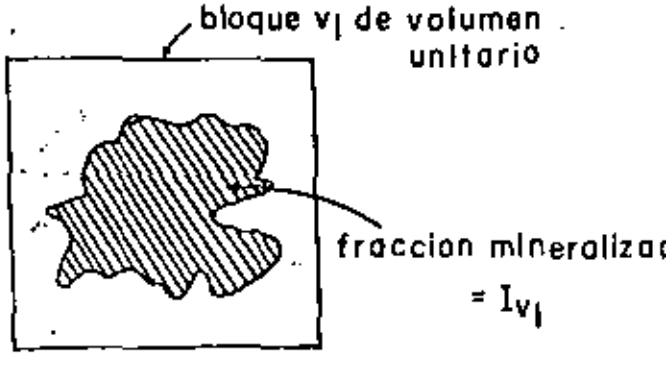


FIGURA 32

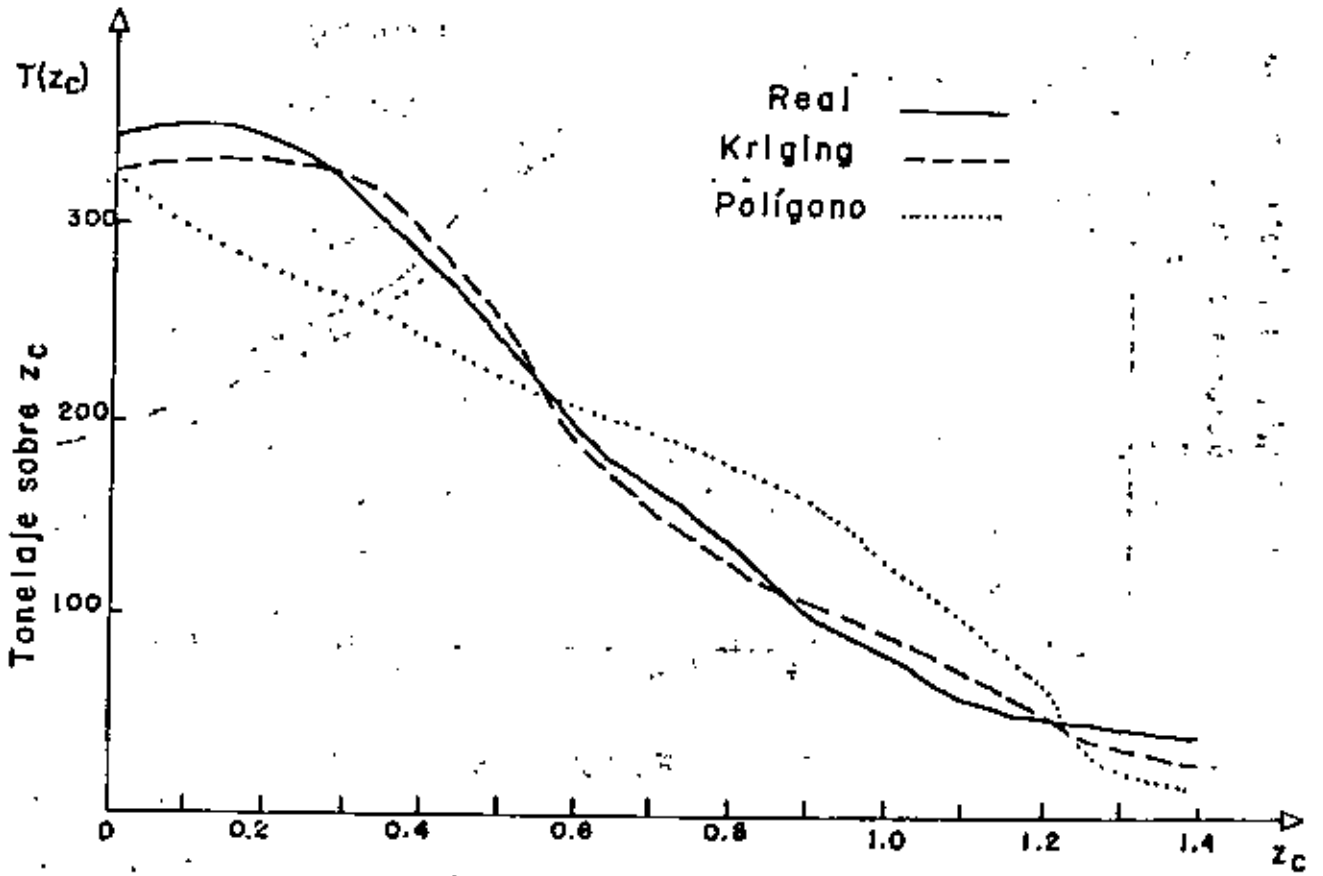


FIGURA 33

La Figura 34 muestra igualmente tres curvas (real, Kriging y polígonos de influencia) todas ellas de cantidad de mineral recobrado vs z_c. Una vez más, el método Kriging proporciona la curva que más se aproxima a la curva real. Por lo que se refiere a la estimación de reservas globales (z_c=0), no existe ninguna diferencia práctica entre los dos métodos de estimación.

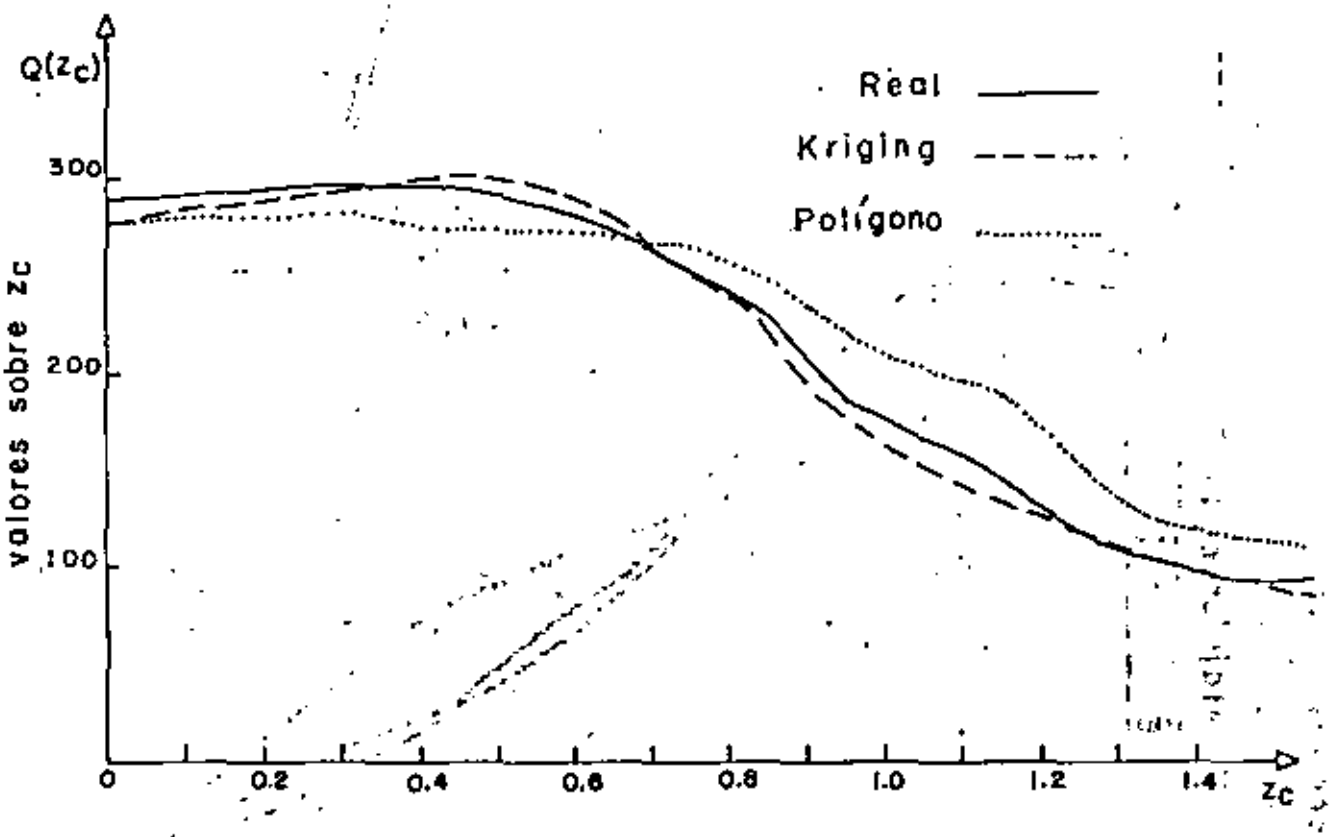


FIGURA 34

En la Figura 35, gráfica de promedio del porcentaje de mineral contra z_c, se observa que la curva obtenida por el método de los polígonos de influencia sobre-estima constantemente a la curva real. En la práctica, y dependiendo del tipo de yacimiento, este error en la estimación del porcentaje promedio podría acarrear pérdidas económicas de considerables consecuencias. El método Kriging, por otro lado, produce una curva más cercana a la curva real.

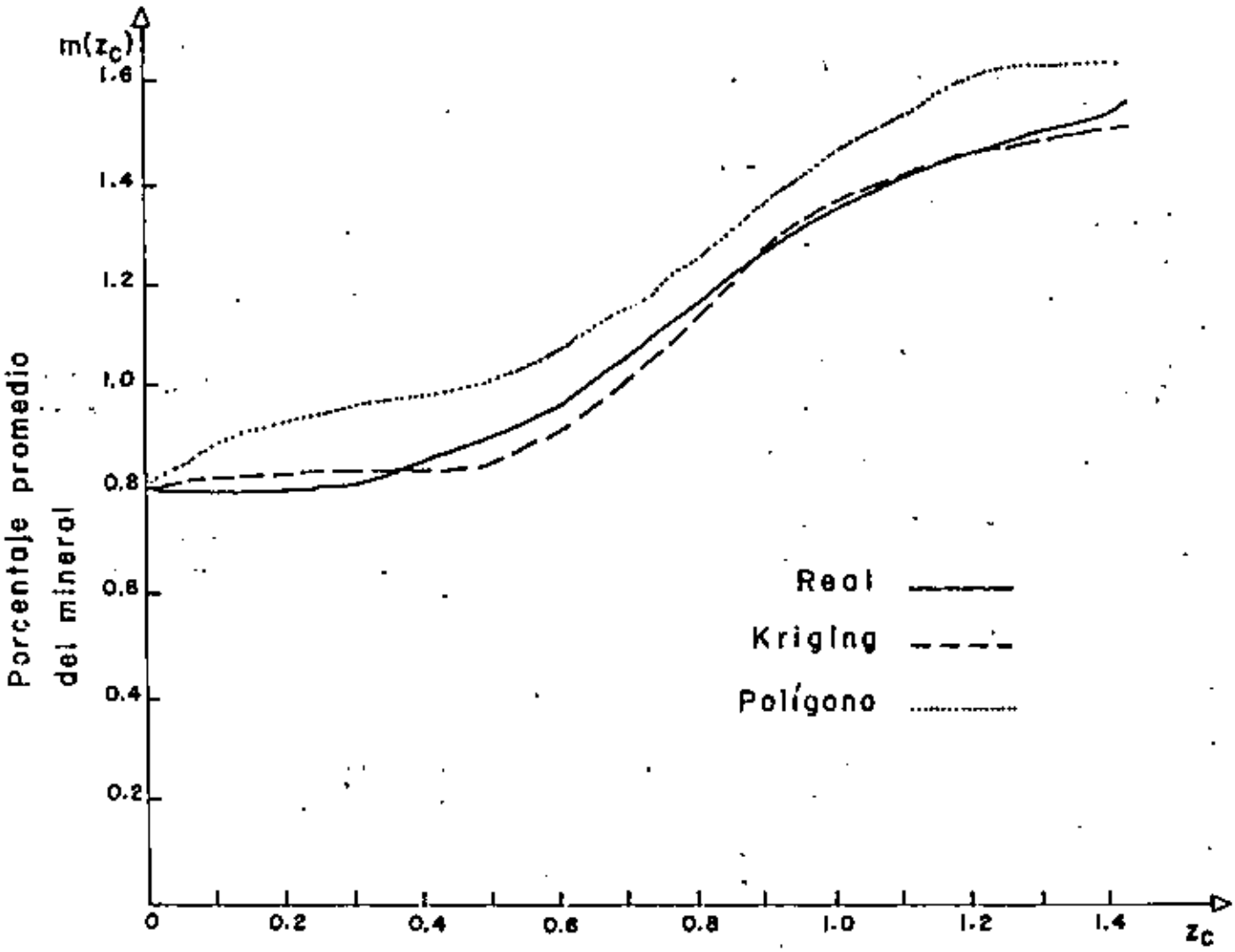

m(zc)
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
Porcentaje promedio del mineral
Real
Kriging
Polígono
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
zc

FIGURA 35

GEOESTADISTICA

CAPITULO V
SIMULACION DE YACIMIENTOS

MARZO 1980.

CAPITULO V

SIMULACION DE YACIMIENTOS

Es bien sabido entre los ingenieros mineros que uno de los parámetros más importantes que intervienen en la selección del equipo minero, por lo que a flexibilidad se refiere, es la dispersión de los porcentajes de mineral extraído en todas las escalas, diaria, mensual, anual, etc. Igualmente, la selección del equipo de excavación depende entre otros factores, de las dispersiones diarias del espesor mineralizado.

Si un yacimiento minero fuese conocido en su totalidad, las dispersiones requeridas, y por ende los métodos más adecuados para su explotación, podrían ser determinados aplicando diversos procesos simulados. Desafortunadamente, en las etapas de exploración y de planeación, se dispone únicamente de información fragmentaria, porcentajes de mineral en un pequeño número de muestras, por ejemplo.

Ahora bien, si no es posible conocer la realidad del yacimiento en suficiente detalle, una alternativa podría ser simularlo siguiendo algún modelo. Pensemos que, en cierta forma, el yacimiento real y las distintas simulaciones del yacimiento no son más que diversas realizaciones de una misma función aleatoria $Z(x)$. El yacimiento simulado $\{z_s(x)\}$ tiene la ventaja sobre el yacimiento real $\{z(x)\}$ de que el primero puede ser conocido en todos los puntos x.

Más aún, es posible restringir la simulación de tal forma que los valores simulados en las localizaciones muestreadas sean idénticos precisamente a los valores muestreados, es decir:

$$z_{sc}(x_\alpha) = z(x_\alpha) \qquad \forall\, x_\alpha$$

Esto se conoce como simulación condicional y puede ser todavía mejorada, agregando toda clase de información cualitativa.

En conclusión, un yacimiento simulado se distingue por satisfacer la condición $z_{sc}(x_\alpha) = z(x_\alpha)$, en toda localización muestreada x_α, y por presentar las mismas características de dispersión (al menos hasta de segundo orden) del yacimiento real. ¿Pero entonces, en que forma se diferencian la simulación y la estimación? Su diferencia se basa en sus objetivos:

(i) La estimación proporciona, en cada punto x, el estimador $z^*(x)$ más cercano al valor real y desconocido $z(x)$. Sin embargo, la estimación no reproduce la variabilidad espacial de los valores reales $\{z(x)\}$.

(ii) No obstante que, en cada punto x, el valor simulado $z_s(x)$ no es el mejor estimador de $z(x)$, el conjunto de valores simulados $\{z_s(x)\}$ o todavía mejor, el conjunto de valores condicionalmente simulados $\{z_{sc}(x)\}$ reproduce los mismos primeros dos momentos (media y covariancia C(h) o semivariograma $\gamma(h)$) que el con-

junto de valores reales $\{z(x)\}$, esto es, reproduce las principales características de dispersión de los valores reales. Por lo que a la variancia de dispersión se refiere, ambos, el yacimiento simulado y el yacimiento real, son idénticos.

En general, los objetivos de la simulación y de la estimación no son compatibles. En la Figura 36 puede observarse que, aunque la curva de estimación $z^*(x)$ se encuentra en promedio más cercana a la curva real $z(x)$, la curva de simulación $z_{sc}(x)$ reproduce mejor las fluctuaciones de la curva real.

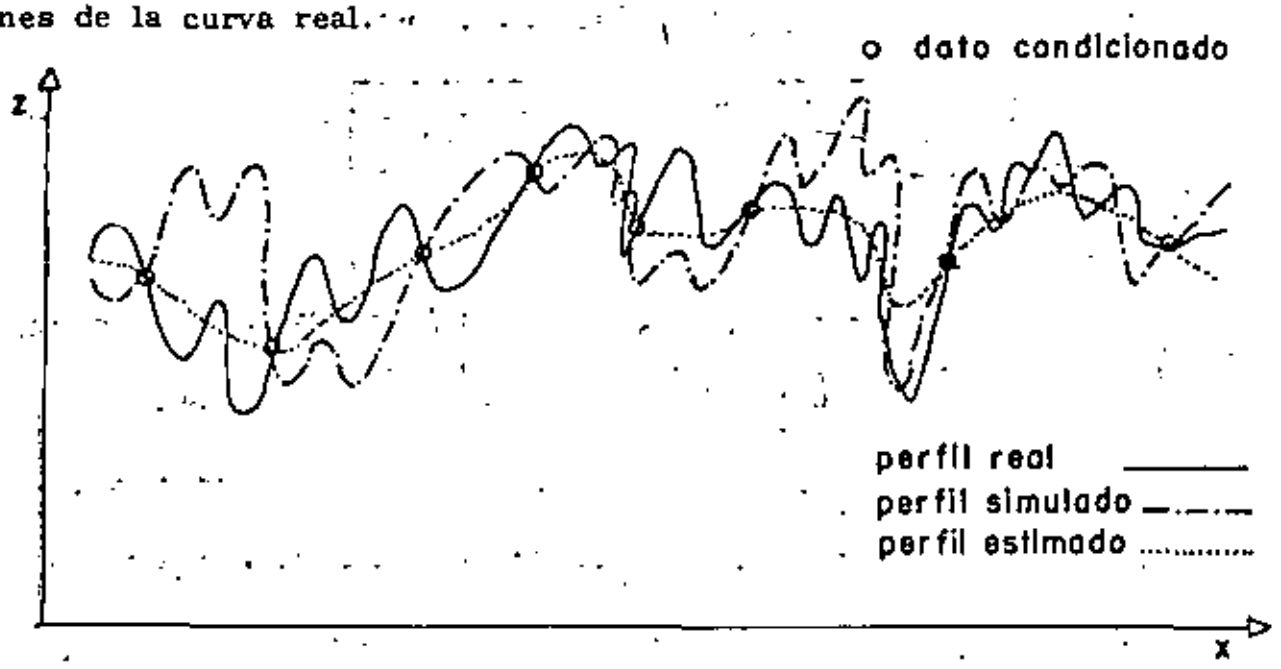


Figura 36 - Perfiles real, simulado y estimado (Kriging).

Teoría de la Simulación Condicional.

Consideremos la función aleatoria estacionaria $Z(x)$, con media m y covariancia $C(h)$ o semi-variograma $\gamma(h)$. El problema con-

siste en formar realizaciones de la función aleatoria $Z_{sc}(x)$ isomórfica a $Z(x)$, es decir, una función aleatoria con media y momentos de segundo orden, $C(h)$ o $\gamma(h)$, iguales a los de la f.a. $Z(x)$. Más aún, las realizaciones $z_{sc}(x)$ deben estar condicionadas, esto es, en cada localización muestreada, el valor simulado debe ser igual al valor muestreado:

$$z_{sc}(x_\alpha) = z(x_\alpha) \qquad \forall x_\alpha \text{ conjunto de datos.}$$

Es posible demostrar, aunque no lo haremos aquí, que cada realización de la f.a. $Z_{sc}(x)$ estará dada por:

$$\boxed{z^*_{sc}(x) = z^*_k(x) + \left[z_s(x) - z^*_{sk}(x) \right]} \qquad (5)$$

donde:

$z^*_k(x)$ es el valor obtenido en el punto x, a partir del conjunto de datos $z(x_\alpha)$, según el método Kriging.

$z_s(x)$ es el valor simulado (no condicional) en el punto x, que representa una realización de la f.a. $Z_s(x)$, la cual es isomórfica a, e independiente de la f.a. $Z(x)$.

$z^*_{sk}(x)$ es el valor obtenido en el punto x, a partir del conjunto de valores simulados $z_s(x_\alpha)$, según el método Kriging.

y $z^*_{sc}(x)$ es el valor condicionalmente simulado en el punto x.

En términos de funciones aleatorias, esta última expresión queda representada como:

$$Z_{sc}(x) = Z^*_k(x) + \left[Z_s(x) - Z^*_{sk}(x) \right]$$

Igualmente y sin pasar por mayores detalles, diremos que, la varianza de estimación de un valor real, $z(x)$ por un valor condicionalmente simulado, $z_{sc}(x)$, es igual a dos veces la varianza de estimación según el método Kriging $\sigma^2 K$, es decir:

$$E\left[(Z(x) - Z_{sc}(x))^2 \right] = 2E\left[(Z(x) - Z_k^*(x)) \right] = 2\sigma_k^2 .$$

De la expresión (5) podemos ver que la obtención de los valores condicionalmente simulados $z_{sc}(x)$, se inicia con la generación de las realizaciones no condicionadas $z_s(x)$, de una función aleatoria $Z_s(x)$, isomórfica a $Z(x)$. Recordemos que las notaciones $z_s(x)$ y $z(x)$ representan realizaciones en el espacio de tres-dimensiones, $x \in \mathbf{R}^3$.

Existen diversas maneras de obtener simulaciones sujetas a satisfacer una función de covariancia dada en una dimensión. Hablando de simulaciones en tres dimensiones, sin embargo, los métodos comunes de simulación resultan, en términos del tiempo de ejecución por computadora, sumamente costosos. Una alternativa la representa el método de "rotación de bandas" (turning bands, en inglés) creado por George Matheron. Este método reduce la simulación en tres dimensiones a varias simulaciones independientes en una dimensión. Las simulaciones en una dimensión se efectúan a lo largo de líneas rectas, mismas que al rotarse en el espacio de tres dimensiones generan otras simulaciones (en una dimensión).

Asumiendo por un momento como realizadas las simulaciones en una dimensión a lo largo de N líneas rectas localizadas en el espacio E^3, la simulación $z_s(x)$ en el punto $x \in E^3$ podrá obtenerse a partir de la proyección del punto x sobre cada una de las N líneas rectas. El valor de $z_s(x)$ será igual a la suma de los N valores observados en los puntos de proyección sobre las N líneas rectas, multiplicada por $1/\sqrt{N}$, esto es:

$$z_s(x) = 1/\sqrt{N} \sum_{i=1}^{N} z_i(x)$$

donde $z_i(x)$ es el valor simulado a lo largo de la i-ésima línea recta y que coincide (dentro de cierta tolerancia) con la proyección del punto x en E^3 sobre la línea recta.

El método de la rotación de bandas se muestra esquemáticamente en la Figura 37.

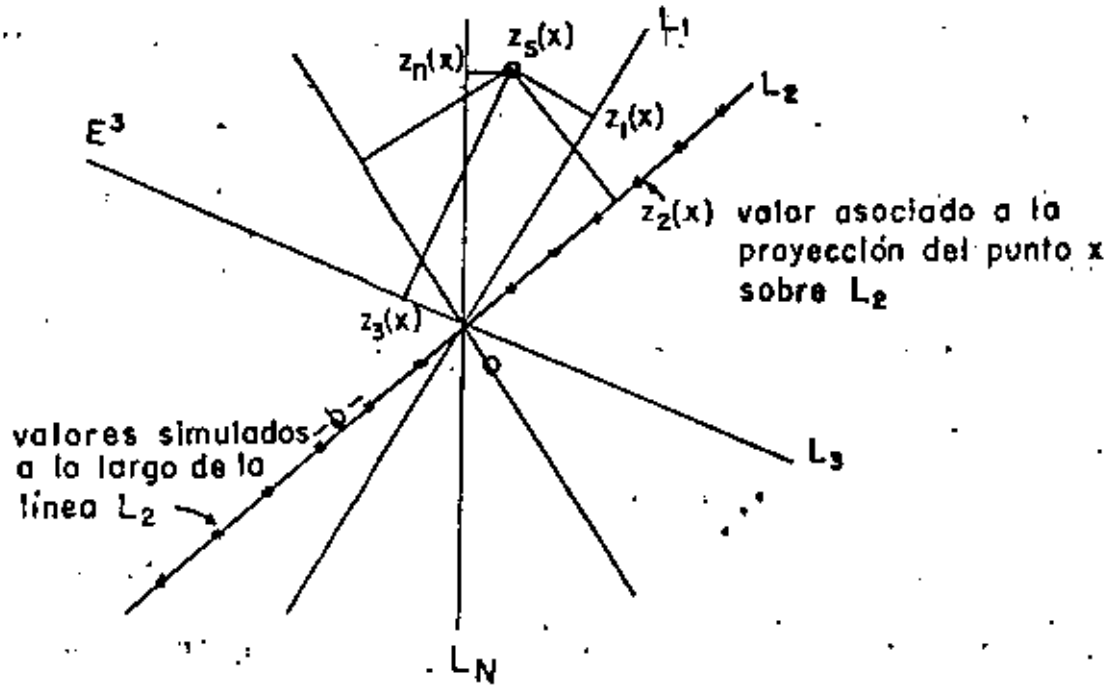


Figura 37 - Bandas Rotantes.

La simulación resultante $z_s(x)$ es una realización de la función aleatoria tridimensional $Z_s(x)$, la cual es estacionaria de segundo orden, tiene como media cero y como covariancia C(h).

Sin entrar en detalles, el procedimiento seguido en la generación de realizaciones $z_s(x)$ de una función aleatoria $Z_s(x)$ con covariancia tri-dimensional C(s), a partir de simulaciones uni-dimensionales es el siguiente:

(i) Dada la función covariancia tri-dimensional e isotrópica C(s), la respectiva función covariancia uni-dimensional $C^{(1)}(s)$ asociada a simulaciones a lo largo de las líneas rectas, se obtiene de la fórmula

$$C^{(1)}(s) = \frac{\partial}{\partial_s} (s\ C(s))$$

(ii) La función covariancia $C^{(1)}(s)$, a su vez, puede representarse como la convolución de una función f(s) y su transpuesta

$$\check{f}(s) = f(-s)$$

$$C^{(1)}(s) = f * \check{f} = \int_{-\infty}^{\infty} f(w)\ f(u+s)\ d_u$$

o lo que es lo mismo, dada la función $C^{(1)}(s)$ es posible encontrar una función f(s) tal que ésta satisfaga la expresión anterior.

(iii) Una función aleatoria uniformemente distribuída Y(u) puede crearse, entonces, aplicando la siguiente fórmula:

$$Y(u) = T * \check{f} = \int_{-\infty}^{\infty} T(r)\ f(r+u)\ d_r$$

EJERCICIO N° 7

Anisotropía Geométrica.

El semi-variograma del ejercicio n° 6 representa la variabilidad estructural de cierta variable aleatoria regionalizada, en la dirección norte-sur. Los valores que a continuación se muestran corresponden al semi-variograma de la misma variable aleatoria regionalizada, pero en la dirección este-oeste.

Grafique los valores de este último semi-variograma sobre la gráfica del semi-variograma N-S y ajuste a dichos valores el modelo teórico más apropiado, definiendo, una vez más, sus respectivos parámetros.

Defina la transformación lineal necesaria para reducir los dos semi-variogramas a un sólo semi-variograma isotrópico.

h_i	$\gamma^*(h_i)$
200′	0.24
282′	0.29
400′	0.38
488′	0.45
564′	0.55
600′	0.64
800′	0.75
↓	↓

$h_i \quad \gamma(h_i)$

continuación

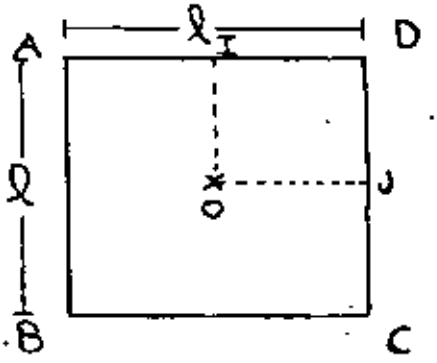
1000'	0.85
1200'	0.87
1400'	0.85
1600'	0.82

EJERCICIO Nº 8

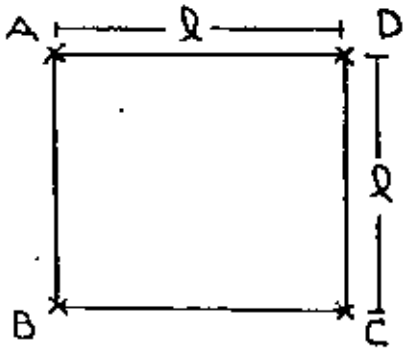
Ejemplo de Aplicación de las Funciones Auxiliares.

Procediendo en forma similar a la descrita en los ejemplos de aplicación (página 92), exprese la variancia de estimación σ_E^2 en términos de funciones auxiliares apropiadas, para cada uno de los casos particulares siguientes:(*)

(a) La variancia de estimación de un cuadrado ABCD de lado l, cuando éste es estimado por una muestra central O.



(b) La variancia de estimación de un cuadrado ABCD de lado "l", cuando éste es estimado por el conjunto de muestras localizadas en cada una de las esquinas del cuadrado.



(*) Asuma condiciones isotrópicas.

(c) Considerando en los dos casos anteriores un semivariograma lineal e isotrópico $\gamma(r)=r$, $r=|h|$, y "1"= 1, calcule los valores correspondientes de σ_E^2 .

EJERCICIO Nº 9

Estimación del Valor Medio de la Porosidad en un Bloque de Roca de Tamaño V.

Consideremos el ejemplo de la página 97. La variable aleatoria regionalizada a tratar representa la porosidad, la cual se ha medido en núcleos (de tamaño v) de pozos perforados en las localizaciones S_1, S_3, O_4 y O_5 (Figura 21).

Empleando esta información, se desea estimar el valor medio de la porosidad en el bloque V, así como la variancia de estimación. Asuma que el tamaño del núcleo v es lo suficientemente pequeño, comparado con el tamaño del bloque V, como para considerarlo puntual.

Los valores medidos son:

localización	porosidad (%)
S_1	22.0
S_3	30.0
O_4	14.0
O_5	30.0

El espaciamiento "1" entre los pozos es igual a 600 m. y el semi-variograma asociado al fenómeno tiene como ecuación:

$$\gamma(r) = \begin{cases} 0 & \text{si } r=0 \\ 1.92\ r & \text{si } r > 0. \end{cases}$$

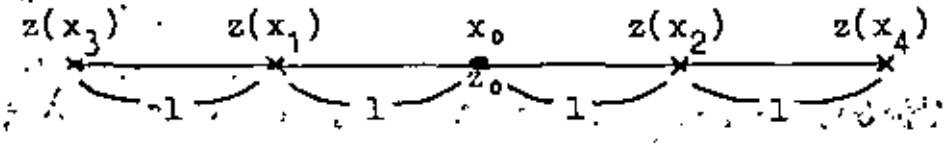
"semivariograma lineal sin efecto de pepita".

EJERCICIO Nº 10

Estimación de Valores Puntuales.

En este ejercicio se pretende mostrar la influencia o efecto de la pendiente de un semi-variograma lineal en los pesos de un estimador.

Se desea estimar el valor puntual $z(x_o)$ empleando cuatro valores puntuales regularmente espaciados y alineados



El estimador $z^*(x_o)$ del valor $z(x_o)$, es una combinación lineal de los cuatro datos:

$$z^*(x_o) = \lambda_1 z(x_1) + \lambda_2 z(x_2) + \lambda_3 z(x_3) + \lambda_4 z(x_4)$$

Debido a condiciones de simetría y de insesgamiento, esta expresión se reduce a:

$$z^*(x_o) = \lambda \frac{z(x_1) + z(x_2)}{2} + (1 - \lambda) \frac{z(x_3) + z(x$$

donde λ es el peso asociado al conjunto de datos $S_1 = \{z(x_1), z(x_2)\}$ y $(1 - \lambda)$ es el peso asociado al conjunto de datos $S_2 = \{z(x_3), z(x_4)\}$.

(a) Exprese la variancia de estimación σ_E^2 en términos de λ, $\bar{\gamma}(S_1, S_1)$, $\bar{\gamma}(S_2, S_2)$, $\bar{\gamma}(S_1, S_2)$, $\bar{\gamma}(S_1, 0)$ y $\bar{\gamma}(S_2, 0)$.

(b) Exprese σ_E^2 en términos de $\gamma(h)$

(c) Si $\gamma(h) = h^\omega$, $\forall \omega \in (0,2)$, exprese σ_E^2 en términos de h, ω y λ.

(d) Exprese σ_E^2 / h^ω en términos de ω y λ.

(e) Si ω se considera constante, ¿qué valor de $\lambda = \lambda_m$ minimiza la expresión σ_E^2 / h^ω ?.

(f) Dé los valores numéricos de λ_m y los correspondientes de σ_E^2 / h^ω para cada uno de los siguientes valores de ω :

ω	λ_m	σ_E^2 / h^ω	$(1 - \lambda_m)$
0^+			
$\frac{1}{2}$			
1			
3/2			
2^-			

Comente los resultados.

NOTA: cuando $\omega \to 0^+$, $\gamma(h) = h^\circ = 1$, representa un modelo

con efecto de pepita puro.

cuando $\omega = 1$, $\gamma(h)=h$, el modelo corresponde a un proceso "Wiener-Levy" (movimiento Browniano); esto es, en un proceso donde el estado presente $z(x)$ es conocido, el estado futuro $z(x+h)$ dependerá exclusivamente de $z(x)$ y no de los estados pasados $z(x-h)$.

donde T(r) representa una sucesión de variables aleatorias uniformemente distribuídas.

Para el caso discreto, las realizaciones y_i a lo largo de una línea recta pueden calcularse según la fórmula

$$y_i = \sum_{k=-\infty}^{\infty} t_{i+k} \cdot f(kb)$$

donde los valores t_{i+k} denotan realizaciones independientes de una variable aleatoria uniformemente distribuída y las cuales pueden ser generadas por la computadora (ver subrutina URAND basada en un algoritmo de Donald Knuth "The Art of Computer Programming", Vol. I). "b" representa el intervalo de separación (ver Figura 37), a lo largo de la línea recta, entre cada valor simulado.

En la práctica, un número impar (2 R+1) de valores elementales t_{i+k} es empleado en el cálculo de cada realización y_i.

$$y_i = \sum_{k=-R}^{R} t_{i+k} \cdot f(kb).$$

Si la función de covariancia tri-dimensional estuviese dada por el modelo esférico,

$$C(s) = \begin{cases} K \left[1 - \frac{3s}{a} + \frac{s^3}{2a^3}\right], & \forall\ s \in [0,\ a] \\ 0, & \forall\ s \geq a \end{cases}$$

entonces, la función de covariancia uni-dimensional $C^{(1)}(s)$ estaría dada

por:

$$C^{(1)}(s) = \begin{cases} K\left[1 - \frac{3s}{a} + \frac{2s^3}{a^3}\right], & \forall\ s \in [0, a] \\ 0 & \forall\ s \geq a \end{cases}$$

y la función f(s) estaría dada, según la condición $C^{(1)}(s) = f * \check{f}$, por

$$f(s) = \begin{cases} \sqrt{12\ K/a^3} \cdot s, & \forall\ s \in [-a/2, +a/2] \\ 0 & \text{para cualquier otro valor de } s. \end{cases}$$

Aplicaciones

La técnica de simulación descrita ha sido de gran utilidad en el análisis y solución de los problemas de ingeniería minera que a continuación se citan:

(i) Determinación del tamaño mínimo de la unidad de selección en la recuperación de recursos in-situ.

(ii) Influencia de la concentración de paneles ricos y paneles pobres en la recuperación de recursos in-situ.

(iii) Influencia de la cantidad de información disponible al momento de la selección de recursos in-situ.

(iv) Determinación de los gastos de producción y de los porcentajes de corte (mismos que varían con el tiempo) y su impacto económico.

Ejemplo de Aplicación de la Simulación en la Selección del Método de Explotación (J. Deraisme, 1977).

En este estudio se analiza un depósito de cobre G, el cual presenta mineralización homogénea. Sus dimensiones en el plano horizontal son 450 m x 450 m; además, consiste de cuatro niveles de bloques paralelepipédicos v de 18 m x 18 m x 5 m (Figura 38). El depósito es conocido a través de muestras de pozos verticales centrados en cada uno de los bloques v. Un modelo de simulación sujeto a las siguientes condiciones fue creado:

(i) Porcentaje medio de los valores simulados m=1% C_u.

(ii) Regionalización isotrópica representada por un modelo esférico con efecto pepita y con rango de aproximadamente 70 m.

La mínima unidad por minar se establece con dimensiones iguales a las del bloque v. Cada bloque v contiene 4,730 toneladas de mineral. Ocho bloques de mineral (35,000 toneladas) se explotarán constantemente cada día. Para tal efecto, se instalará un molino el cual recibirá el mineral extraído.

Tres diferentes alternativas de producción se contemplan:

1. La zona G se minará considerando bancos de 18 m x 18 m x 10 m. Dos palas mecánicas instaladas en el depósito avanzarán paralelamente al frente y no podrán retroceder; las palas deberán extraer

todo el mineral a lo largo del frente (18 x 25 = 450 m) antes de pasar al siguiente frente. Cada pala mecánica podrá extraer - únicamente dos bancos contiguos por día.

II. La explotación se llevará a cabo de la misma forma que en el - método anterior, excepto que:

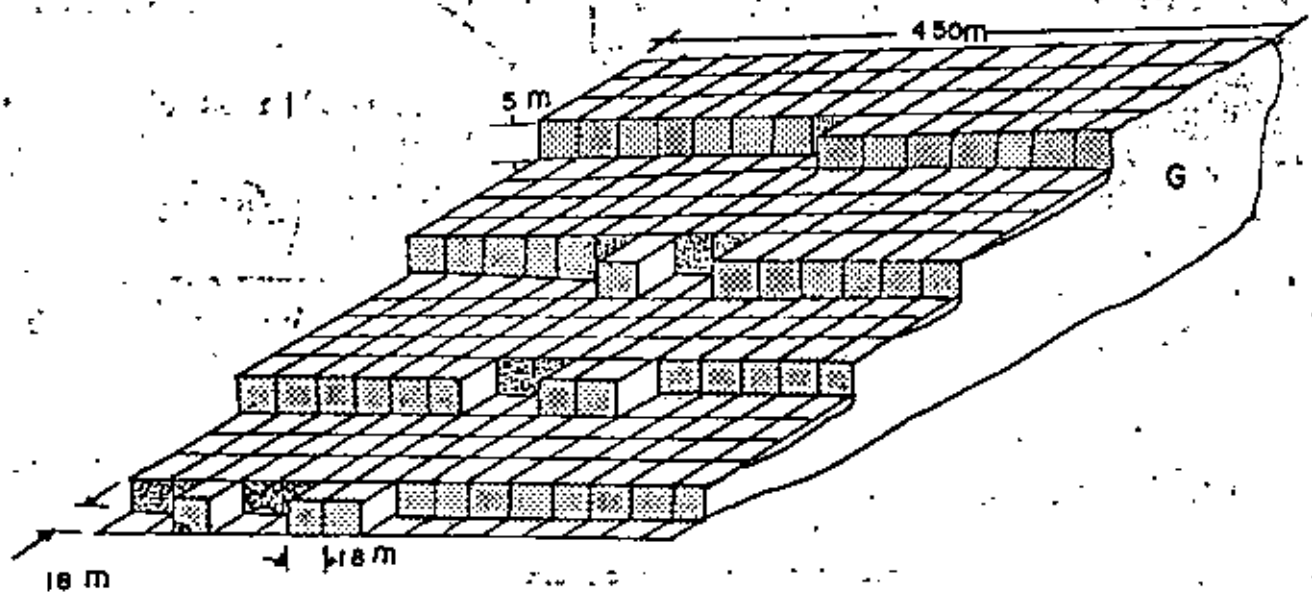


Figura 38
Vista de frentes mineros

en lugar de enviar todos los bloques al molino, algunos de ellos - serán enviados a un depósito de almacenamiento (stockpile).

El depósito de almacenamiento tiene capacidad para almacenar el equivalente a dos días de producción (70,000 Ton.) y está dividido en dos secciones: una para mineral rico y la otra para mineral pobre. Un banco será enviado al depósito de almacenamiento cuando su porcentaje

medio afecte la producción media del día. Cuando un banco sea enviado a la sección de mineral rico, un banco equivalente será tomado de la sección de mineral pobre y enviado al molino, a - fin de satisfacer la demanda diaria (ver Figura 39).

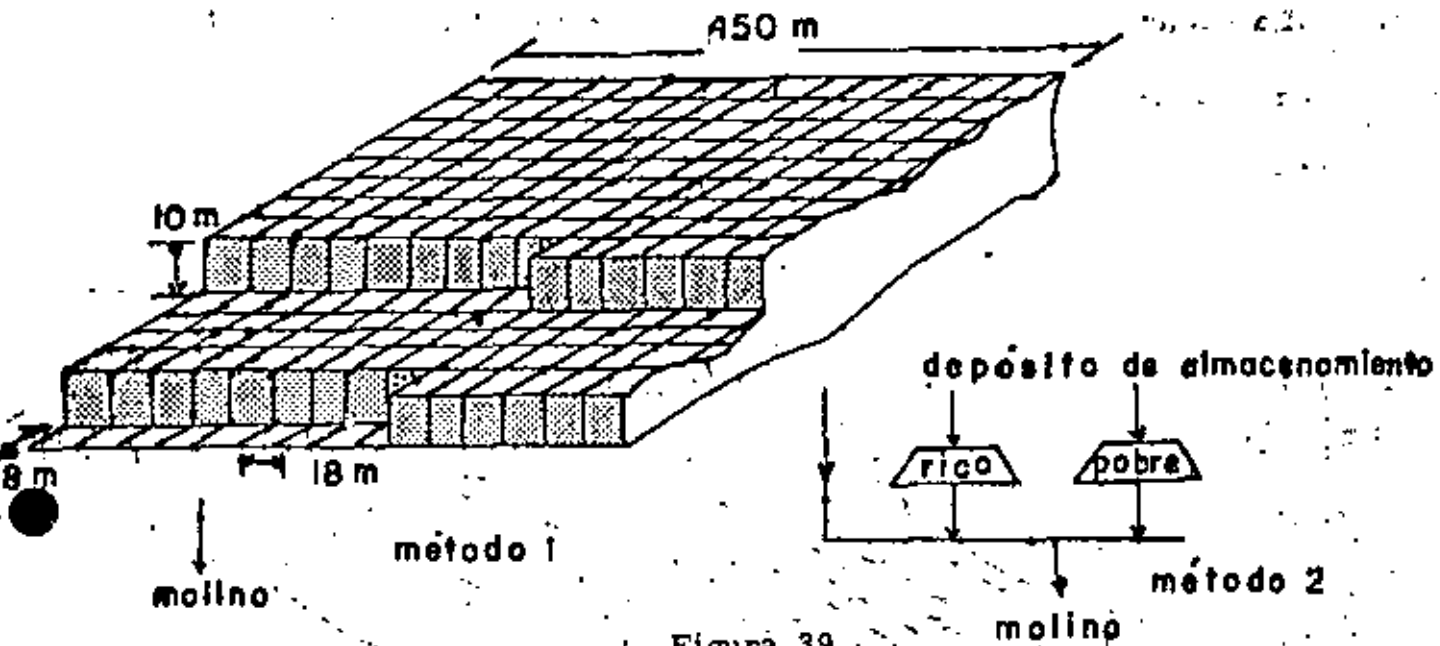


Figura 39

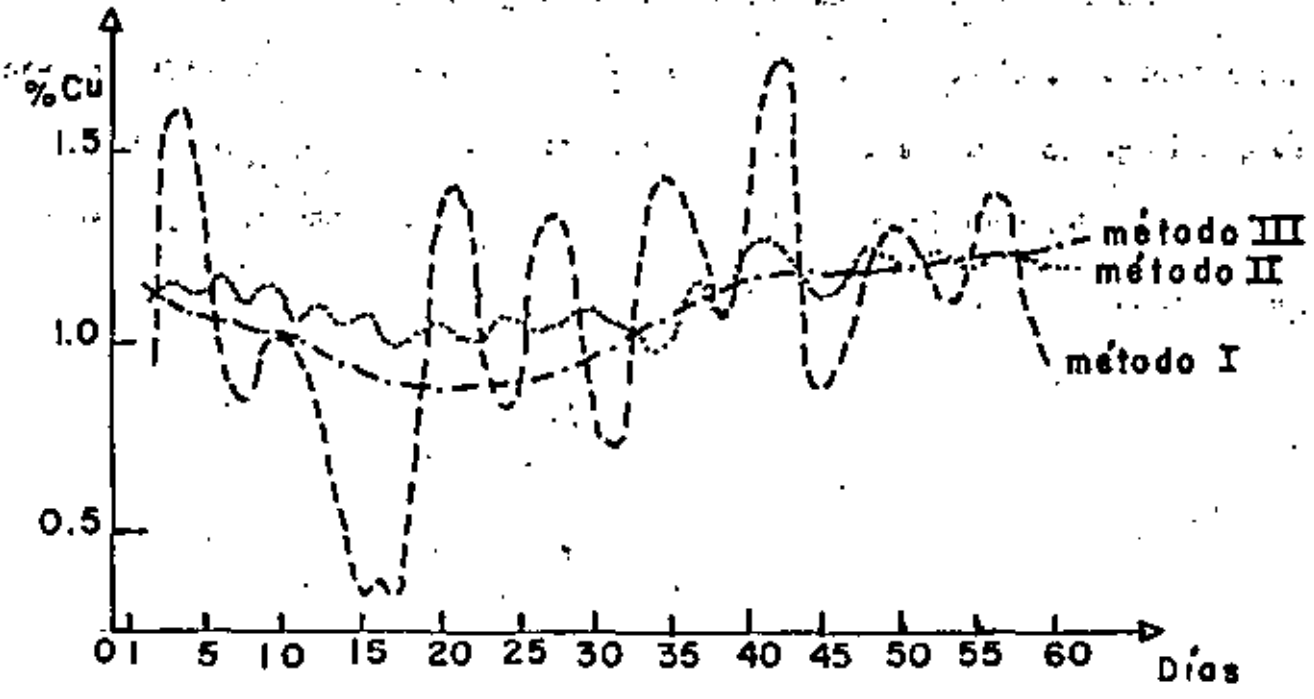
Vista de los frentes minerales

III. La zona G será minada empleando bancos de altura 5 m (Figura 38). En cada uno de los cuatro niveles habrá una pala mecánica con capacidad suficiente para extraer hasta dos bloques de 18 m x 18 m x 5 m cada día. Toda la producción diaria de 35,000 Ton. será enviada directamente al molino. Todo el mineral a lo largo del frente deberá ser extraído antes de pasar al siguiente frente.

Los bloques extraídos deberán ser seleccionados considerando - ambos aspectos, la estabilidad de la producción diaria y el man_

tenimiento al mínimo del desplazamiento total de cada pala mecánica.

Partiendo de los datos originales se efectuó la simulación del depósito. Cada uno de los bloques, según las tres alternativas, fue simulado. Los resultados se muestran en la Figura 40, donde las variaciones diarias (líneas punteadas) del porcentaje de mineral simulado por bloque, se grafican para los primeros 60 días de producción.



Variacion diaria del porcentaje medio de cobre

Figura 40

Observando la gráfica, podemos deducir la inclinación del operador del molino por las alternativas II ó III, y la preferencia del ingeniero encargado de la extracción del mineral por la alternativa I.

La manera más idónea de decidir cual alternativa sería la más

apropiada es por medio de la asignación de costos. Las pérdidas causadas en el molino, si el método I fuese elegido, deberían ser balanceadas con los costos que implicarían la creación de un depósito de almacenamiento (método II) o por los costos que originarían operaciones más flexibles (método III).

La teoría de simulación descrita en este capítulo ha encontrado aplicaciones en el campo de la ingeniería minera, principalmente. Por lo que respecta a otras ciencias de la tierra, aún se esta estudiando la elaboración del tipo de preguntas válidas que puedan plantearse y resolverse dentro del marco de esta teoría.

EJERCICIO Nº 1

Los datos de la Tabla 4 fueron obtenidos empleando registros eléctricos y núcleos de pozos perforados a través de diversas formaciones en el área de "Chicagoland". Los valores dados están expresados en porcentajes.

Sample no.	Log-derived porosity	Core-derived porosity	Sample no.	Log-derived porosity	Core-derived porosity
1	10.0	5.5	26	10.0	9.6
2	9.0	3.6	27	5.0	10.3
3	7.0	3.6	28	7.0	4.5
4	6.0	4.9	29	8.0	6.0
5	9.0	7.1	30	9.0	6.7
6	7.0	2.0	31	5.0	4.1
7	10.0	8.5	32	6.0	4.5
8	5.0	5.7	33	9.0	6.5
9	7.0	2.6	34	7.0	3.5
10	0.0	1.9	35	3.0	2.1
11	5.0	6.1	36	7.0	2.5
12	6.0	0.5	37	7.0	6.6
13	9.0	6.9	38	10.0	3.7
14	8.0	4.3	39	7.0	6.0
15	5.0	3.3	40	5.0	3.4
16	6.0	2.5	41	8.0	2.2
17	6.0	4.8	42	4.0	1.8
18	5.0	2.4	43	5.0	2.9
19	8.0	3.8	44	8.0	2.6
20	15.0	18.4	45	16.0	15.3
21	16.0	14.7	46	4.0	16.9
22	7.0	10.9	47	5.0	15.7
23	12.0	12.5	48	14.0	12.4
24	14.0	16.6	49	21.0	22.9
25	22.0	22.1	50	21.0	21.8

(a) Ordene en forma ascendente los valores de porosidad obtenidos a partir de los registros eléctricos.

(b) Compute el número de muestras que caendentro de cada una de las categorías siguientes:

intervalo de clase	frecuencia	porcentaje
0.0 – 3.0		
3.0 – 6.0		
6.0 – 9.0		
9.0 – 12.0		
12.0 – 15.0		
15.0 – 18.0		
18.0 – 21.0		
21.0 – 24.0		
	50	100 %

(c) Calcule el porcentaje asociado a cada intervalo de clase y grafique la pareja de valores (porcentaje, intervalo de clase), o en otras palabras, obtenga el histograma de frecuencias relativas.

Según la forma del histograma obtenido, ¿qué tipo de distribución, diría ud., sigue la variable aleatoria ϕ ?

Si la variable aleatoria ϕ sigue una distribución log-normal, entonces, el mejor estimador de la media de ϕ estará dado, no por la media aritmética, sino por la media geométrica, la cual se expresa como:

$$\bar{\phi} = \left(\prod_{i=1}^{N} \phi_i \right)^{1/N} = \text{antilog}\left(\frac{1}{N} \sum_{i=1}^{N} \log \phi_i\right)$$

(d) Repita el procedimiento anterior empleando los valores de ϕ obtenidos de los núcleos de pozo.

EJERCICIO Nº 2

Si la variable aleatoria contínua X está normalmente distribuida con media cero y variancia úno, entonces su función de densidad $f_X(x)$ está dada por la expresión:

$$f_X(x) = \left(\frac{1}{2\pi}\right)^{\frac{1}{2}} e^{-x^2/2} \qquad \forall \; x \in (-\infty, \infty)$$

Demuestre que, en efecto, $f_X(x)$ es una función de densidad que satisface las dos condiciones:

(i) $f_X(x) \geq 0 \quad \forall\, x$

(ii) $\int_{-\infty}^{\infty} f_X(u)\, du = 1.$

EJERCICIO Nº 3.

De entre las distribuciones paramétricas que hemos visto, cite aquéllas en las cuales:

(i) La media sea mayor o igual a la variancia

(ii) La media sea igual a la variancia

(iii) La media sea menor o igual a la variancia

(iv) La media pueda ser mayor que, menor que, o igual a la variancia

EJERCICIO Nº 4.

En las primeras etapas del desarrollo de un depósito minero de cobre, 20 valores del porcentaje de mineral se encuentran a nuestra disposición:

z =	0.25	0.32	0.32	0.36	0.39	0.45	0.53	0.70
	0.73	0.78	0.90	0.95	1.13	1.20	1.35	1.55
	2.10	2.94	2.96	5.91	%Cu			

Estos porcentajes, definidos bajo el mismo tamaño de muestra, han sido muestreados empleando en cada uno de ellos la misma técnica. Las localizaciones de los 20 datos no son proporcionadas, por lo tanto asumiremos que cada dato representa una realización independiente de la variable aleatoria Z.

(a) Efectúe un estudio estadístico de los 20 valores z, esto es, calcule la media experimental m y la variancia experimental s^2, empleando las fórmulas:

$$m = 1/20 \sum_{i=1}^{20} z_i$$

$$s^2 = 1/20 \sum_{i=1}^{20} (z_i - m)^2$$

Calcule el número de muestras que caéndentro de cada una de las categorías siguientes:

intervalo de clase	valor medio del intervalo	frecuencia	%	%acc.
(1)	(2)	(3)	(4)	(5)
0.0 - 0.6	0.3			
0.6 - 1.2	0.9			
1.2 - 1.8	1.5			
1.8 - 2.4	2.1			
2.4 - 3.0	2.7			
3.0 -)				
		20		100%

Calcule el porcentaje (columna 4) y el porcentaje acumulado (columna 5) para cada uno de los intervalos de clase.

Grafique las parejas de valores de las columnas 2 y 5 en la hoja adjunta de papel logarítmico-probabilístico.

Una prueba simple para determinar si la variable aleatoria Z sigue una distribución log-normal, consiste en observar si los puntos graficados presentan (mas o menos) cierta alineación.

Cumulative Percent

2
5
10
20
30
40
50
60
70
80
90
95
98
99

10^{-1} 10^{0} 10^{1} 10^{2} 10^{3}

Valor medio del intervalo de clase

Ejercicio nº 4

(b) Asumiendo a m y s^2 como los parámetros de la distribución log-normal, calcule μ y σ^2, los parámetros de la distribución normal asociada a la variable aleatoria $Y = \log Z$.

(c) Como ingeniero experto en ciencias de la tierra responsable de la explotación de este yacimiento, diga ud., ¿cuál sería el siguiente paso hacia la mejor comprensión del yacimiento?.

EJERCICIO N° 5

Construccion de un Semi-variograma:

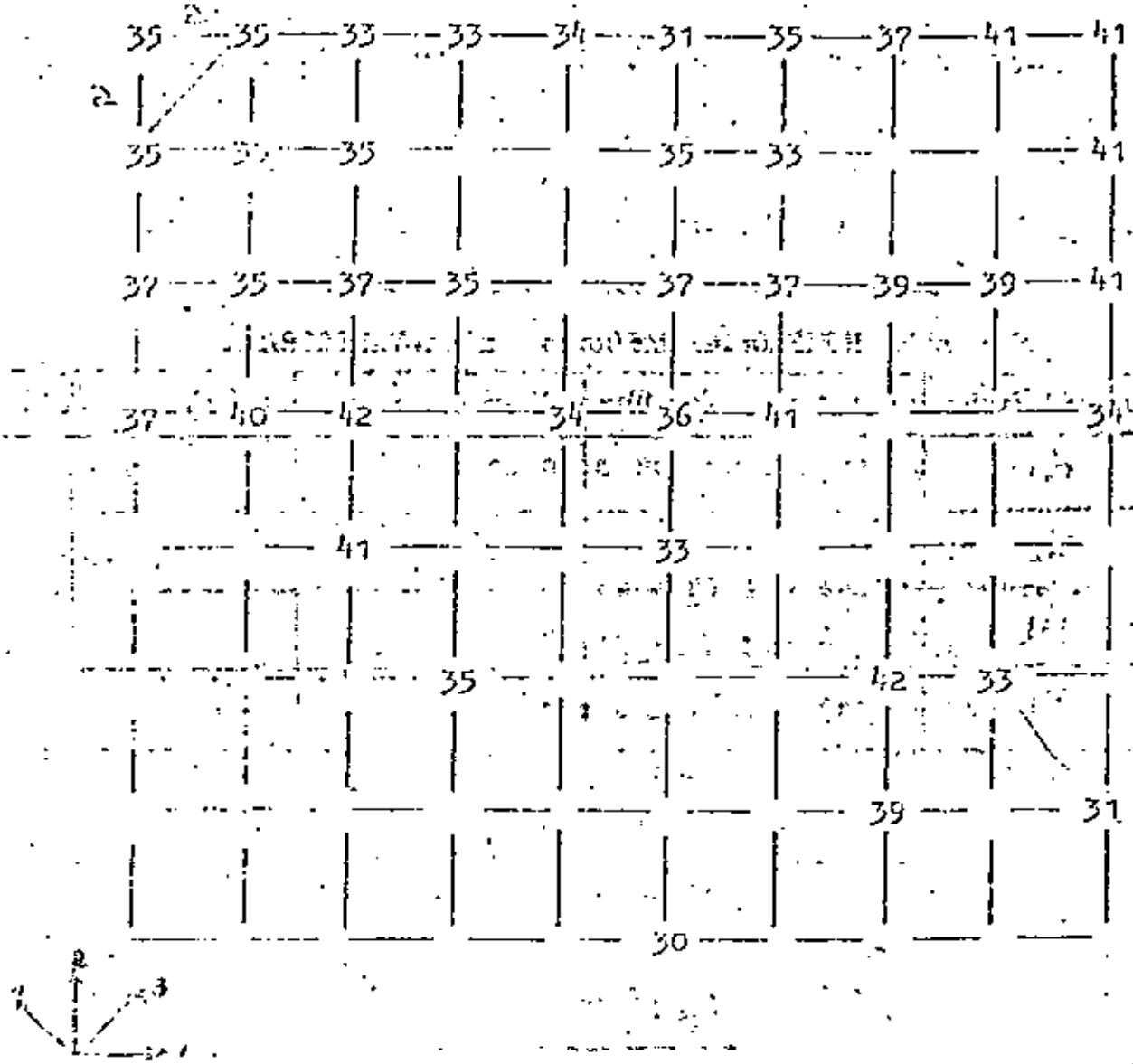
El conjunto de datos empleado en este ejercicio ha sido suficientemente reducido para permitir el cálculo de los diversos semi-variogramas direccionales , ya sea a mano ó empleando la calculadora.

Los datos están localizados en los nodos de la malla cuadrada de lado a. Las direcciones por estudiar son las dos direcciones principales α_1 y α_2, y las dos direcciones diagonales α_3 y α_4. Note que el tamaño del espaciamiento básico a lo largo de las direcciones diagonales es $a\sqrt{2}$, mientras que en las direcciones principales es a.

Complete la tabla de abajo dando los números de pares de datos usados N(i); y los valores correspondientes del semi-variograma experimental $\gamma(i)$, para cada una de las cuatro direcciones y para los tres primeros múltiplos de los espaciamientos básicos. Use la siguiente fórmula para el cálculo de $\gamma(i)$:

$$\gamma(r) = \frac{1}{2N(r)} \sum_{i=1}^{N(r)} \left[z(x_i+r) - z(x_i) \right]^2$$

MALLA DE DATOS



Verifique si la regionalización es isotrópica. Calcule el semi-variograma isotrópico medio combinando los cuatro semi-variogramas experimentales. Use la fórmula

$$\tilde{\gamma}(r) = \sum_{k=1}^{4} N_k(r)\,\gamma_k(r) \Big/ \left(\sum_{k=1}^{4} N_k(r) \right)$$

ejercicio nº 5

Ajuste un modelo lineal al semivariograma medio.

T A B L A

NUMERO DE DATOS - SEMI-VARIOGRAMA

DIRECCION	N(1)	$\gamma(1)$	N(2)	$\gamma(2)$	N(3)	$\gamma(3)$
α_1						
α_2						
α_3						
α_4						

EJERCICIO Nº 6

Los siguientes valores corresponden a un semi-variograma experimental. Grafique los puntos definidos por las parejas $(h_i, \gamma(h_i))$ y ajuste a ellos el modelo teórico más apropiado (esférico, lineal, exponencial, etc.) definiendo los parámetros del modelo.

h_i	$\gamma(h_i)$
200′	0.43
282′	0.57
400′	0.63
488′	0.75
564′	0.85
600′	0.85
800′	0.87
1000′	0.88
1200′	0.87
1400′	0.85
1600′	0.80

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GEOESTADISTICA

EL KRIGAJE UNIVERSAL APLICADO A LA DESCRIPCION DE YACIMIENTOS HETEROGEONEOS

ING. MARIO VARGAS FLORES

MARZO, 1980.

EL KRIGAJE UNIVERSAL APLICADO A LA DESCRIPCION DE YACIMIENTOS HETEROGENEOS

RESUMEN

Actualmente, uno de los problemas más estudiados por los investigadores, es describir un fenómeno distribuido espacialmente, como son: cimas, bases, espesores de una formación geológica; propiedades físicas de las rocas: porosidad y permeabilidad. Parámetros que indican condiciones de saturación de agua, capacidad de flujo, índice de hidrocarburos, etc. Estas distribuciones se representan por alguna función o modelo matemático que result , la mayoría de las veces, tan complicado en su expresión como difícil la solución; o también se utilizan diversos algoritmos de interpolación o el contorneo a mano.

El objetivo de este trabajo es presentar e introducir los conceptos fundamentales en los que se basa la teoría de las VARIABLES REGIONALIZADAS desarrollada por el matemático francés Matheron, así como la técnica de estimación denominada KRIGAJE UNIVERSAL. Esta es una rama de la teoría estadística general, que toma en cuenta las relaciones espaciales del fenómeno natural, y permite la detección de rasgos característicos tales como continuidad, variación en diferentes direcciones y la influencia de la variable alrededor de una vecindad. Considera también, que el valor de una variable es el resultado de dos procesos, es decir, uno determinista y el otro estocástico. Además de ser un interpolador exacto, el krigaje universal proporciona la variancia del valor estimado.

Se presenta, por último, la aplicación al Campo Cactus, consistente en hallar la distribución de la capacidad de flujo, basada en datos de pozo. Esta distribución se encontró en una área determinada así como su variancia. Estos resultados cobran importancia como herramienta accesoria en la toma de decisiones para el desarrollo del campo.

EL KRIGAJE UNIVERSAL APLICADO A LA DESCRIPCION DE YACIMIENTOS HETEROGENEOS

RESUMEN

Se presentan los antecedentes teórico-matemáticos de una reciente teoría geostadística, desarrollada por el matemático francés Georges Matheron y denominada: TEORIA DE LAS VARIABLES REGIONALIZADAS; así también se presenta la técnica del KRIGAJE UNIVERSAL, como un método de evaluación estadística de la variable regionalizada. Por último, se presenta una aplicación práctica a la descripción de un yacimiento petrolífero, el cual es considerado como un sistema heterogéneo.

INTRODUCCION

Uno de los problemas más agudos y estudiados por los investigadores durante mucho tiempo, es describir un fenómeno distribuido espacialmente, como son: cimas, bases y espesores de formaciones geológicas; propiedades físicas de las rocas como porosidad y permeabilidad; parámetros que indican condiciones de saturación de agua, capacidad de flujo, índice de hidrocarburos, etc. Estas distribuciones se intentan, generalmente, expresarlas por alguna función o modelo matemático que resulta, la mayoría de las veces, tan complicado en su expresión como difícil la solución.

En la actualidad, existen diversos algoritmos y técnicas para describir a una variable espacial basados en distintas suposiciones según el problema a resolver. Un método utilizado durante muchos años y que aún se practica, es el contorneo a mano ejecutado por algún técnico, quien realiza la configuración en base a su experiencia, esta técnica adolece de fallas. Otra técnica de uso frecuente, es el ajuste de superficies de tendencias por el método de mínimos cuadrados, pero sufre de deficiencias en cuanto a sus consideraciones primarias, y sólo da buenos resultados en algunos casos (Matheron, 1967).

En la exploración y explotación petroleras, siempre se cuenta con un conjunto finito de valores de la variable en estudio, a partir de estos datos, espacialmente distribuidos, se debe reconstruir el fenómeno con la fidelidad y confiabilidad suficiente para realizar decisiones para un mejor desarrollo del yacimiento. Dentro

de la ingeniería petrolera en donde la perforación es, actualmente, a grandes profundidades y por lo tanto más difícil y costosa, es necesario cuantificar con más precisión y veracidad las variables que se obtienen durante la perforación; por lo tanto, se requieren de nuevas y mejores técnicas para el procesamiento de datos.

El propósito de este trabajo es presentar e introducir los conceptos básicos en los que se apoya la teoría de las variables regionalizadas y la técnica para evaluar estas variables, conocida como KRIGAJE UNIVERSAL. Finalmente, para comprender mejor los conceptos, se aplica la teoría para describir un yacimiento petrolífero, en función de su capacidad de flujo.

PRESENTACION DE LA TEORIA

En condiciones prácticas, para definir y describir un fenómeno natural u otra variable distribuida espacialmente, se cuenta siempre con un número limitado de datos obtenidos por muestreos en la superficie, de perforaciones, de registros eléctricos, etc. Anteriormente a la geostadística de Matheron, el procesamiento de la información se realizaba utilizando la geostadística clásica, sin tomar en cuenta el carácter espacial de la variable. La teoría de las variables regionalizadas (denominada así, precisamente, para acentuar la disposición espacial de la variable estudiada); realiza un estudio estadístico exhaustivo (análisis estructural) considerando la correlación espacial y la dependencia existente entre las variables. Este análisis estructural conduce a resolver problemas de continuidad y zona de influencia de una variable, así como el comportamiento de ésta en distintas direcciones (anisotropía).

Para conocer el valor de la variable en un punto dado, la teoría de las variables regionalizadas hace uso del krigaje universal, en honor al Dr. Krige de la Escuela Sud-africana, que fue uno de los primeros investigadores en considerar la correlación espacial en la evaluación del contenido de oro en las minas sudafricanas. La técnica del krigaje estima el valor de la variable mediante una combinación lineal de la información alrededor de su radio de influencia (datos estadísticamente dependientes). Una gran ventaja de esta técnicaa consiste en la posibilidad de estimar la varianza de cada valor calculado y por lo tanto, conocer el grado de confianza de la distribución.

VARIABLE REGIONALIZADA. La teoría de las variables regionalizadas se basa en una serie de suposiciones que caracterizan a una variable que representa un fenómeno natural espacial. Una variable regionalizada se define como una función numérica con una distribución espacial y que varía continuamente, estos cambios no pueden ser representados por una función matemática sencilla y de fácil solución (Blais and Carlier, 1968; Matheron, 1970). Las principales características de las variables regionalizadas son: localización, anisotropía y continuidad (Matheron, 1963). Dentro de estas características están contenidas la mayoría de las variables que definen un fenómeno geológico, geofísico, petrolero o minero.

Los datos que se poseen de algún fenómeno se consideran que son generados por alguna función de densidad de probabilidad y se reconocen, dentro de esta teoría geostadística, que los datos son realizaciones de una función aleatoria. Debido a que la realización de la variable regionalizada es única y que un proceso natural distribuido espacialmente no se puede repetir, nunca se podrá conocer la función de densidad de probabilidad, pero se tendrá en cuenta que existe una, la cual genera los valores aleatorios.

Otra suposición importante consiste en la correlación espacial que existe entre las variables. Si cerca de un punto de información se recolecta otro dato, éste tendrá una gran probabilidad de repetir el mismo valor o de que tenga uno cercano a la primera información.

Ahora bien, para poder realizar deducciones estadísticas, se supone, primeramente, que la variable es estacionaria de primer orden. Esta clase de estacionaridad no es común en un proceso natural espacial, por lo que la teoría de las variables regionalizadas considera la hipótesis intrínsica; esta hipótesis supone que los incrementos $Y(\vec{x}) - Y(\vec{x}+\vec{h})$ son estacionararios en lugar de $Y(\vec{x})$ (Dagbert and Davis, 1976). Así:

$$E\left[Y(\vec{x}+\vec{h}) - Y(\vec{x})\right] = m(\vec{h}) \qquad - (1)$$

$$\text{var}\left[Y(\vec{x}+\vec{h}) - Y(\vec{x})\right] = 2\gamma(\vec{h}) \qquad - (2)$$

La función $\gamma(\vec{h})$ se denomina el semivariograma o función intrínsica; $m(\vec{h})$ es la deriva lineal.

EL SEMIVARIOGRAMA. El semivariograma define el grado de continuidad

de la variable, da un contenido preciso de la zona de influencia de una muestra y la menor o mayor variación de una variable en distintas direcciones. De la gráfica de un semivariograma se obtiene valiosa información acerca de una variable; según la estructura de la gráfica se pueden obtener el comportamiento y las características esenciales de la variable. Los principales tipos de semivariograma se muestran en la figura 1. En la figura 1.a, la gráfica se comporta de una forma parabólica cerca del origen e indica una variable con alta continuidad. La figura 1.b representa una continuidad "en promedio" de la variable. La figura 1.c muestra una discontinuidad en el origen, llamado "efecto pepita", y es característico de una variable con una continuidad pobre. La figura 1.d es el tipo de semivariograma que corresponde a una variable aleatoria independiente. La anisotropía de una variable se refleja si se grafica en diferentes direcciones (Fig. 1.e). El radio de influencia de una variable se define matemáticamente como:

$$\xi \leq \mathrm{cov}(0) - \gamma(h)$$

donde ξ es un número pequeño y considerado despreciable, en la figura 1.f se ejemplifica este concepto.

El semivariograma no representa la totalidad ni los detalles del fenómeno, pero sí expresa en una forma sintética sus características esenciales (Matheron, 1963).

Físicamente, si se tienen puntos a lo largo de una línea y espaciados regularmente una distancia a, el semivariograma puede ser calculado por retrasos h y múltiplos de a, por medio de la siguiente fórmula:

$$\gamma(h) = \frac{1}{2\,N(h)} \sum_{i=1}^{N(h)} (Z(x_i+h) - Z(x_i))^2$$

donde: $Z(x_i)$ son los datos; x_i son las localizaciones tal que los datos estén disponibles en x_i y x_i+h; $N(h)$ es el número de datos tomados en la suma. En la siguiente figura se ilustra el cálculo del semivariograma (Delfiner, 1979).

DERIVA. Este es un concepto de los más importantes dentro de la teoría de las variables regionalizadas y representa físicamente la tendencia sobre el campo geométrico de la variable, matemáticamente se expresa como el valor medio de la variable Z en la localización x_i (Journel, 1969)..

$$m(x_i) = E\left[Z(x_i)\right] \qquad - (4)$$

Con este concepto de deriva se puede definir el residual, éste es la diferencia de la variable y su valor medio:

$$R(x_i) = Z(x_i) - m(x_i) \qquad - (5)$$

Un estimador de la deriva se obtiene mediante una combinación lineal de funciones conocidas, escogidas arbitrariamente; para una vecindad U, se supone que m(x) varía regularmente, por lo que se puede expresar como una combinación lineal de funciones polinomiales de grado diferentes o funciones senoidales, es decir:

$$m(\vec{x}) = \sum_{i=0}^{k} a_i f_i(\vec{x}) \qquad - (6)$$

Los valores a_i son definidos por estimadores lineales de la forma: $A_i = \lambda_i Z(\vec{x}_i)$, bajo las restricciones que:

$$E\left[A_i\right] = a_i$$
$$E\left[(A_i - a_i)^2\right] \text{ es mínimo de } \lambda_i^j, \text{ para } i=0,1,\ldots,n$$

Es decir, que el estimador tiene que ser insesgado y que el conjunto de pesos λ_i minimizan la variancia de estimación. Entonces, un estimador estadístico de $m(\vec{x})$ será:

$$M(\vec{x}) = \sum_{i=0}^{n} A_i f^i(\vec{x})$$

KRIGAJE UNIVERSAL. Esta técnica permite evaluar con confianza una variable regionalizada. En este caso, la variable regionzalizada es considerada como puntual, por lo que las estimaciones formarán una malla de interpolación que permitirá conocer la distribución de la variable en una área determinada. Se define la estadística $Z^*(x)$ como una combinación lineal de las Z(x) alrededor de una vecindad proporcionada por el semivariograma, o sea:

$$Z^* = \sum_{i=1}^{n} \lambda_i Z(\vec{x}_i) \qquad - (7)$$

Se imponen dos restricciones para lograr la optimización de la estadística Z :

a) Z^* debe ser un estimador insesgado, es decir:

$$E\left[Z^*(\vec{x}_o) - Z(\vec{x})\right] = 0$$

b) La varianza de estimación debe ser mínima con respecto a las λ_i's, o sea:

$$E\left[Z^*(\vec{x}_o) - Z(\vec{x}_o)^2\right] \text{ es un mínimo de } \lambda_i.$$

De la primera restricción se deduce la condición insesgada, expresada por la relación: $\lambda_i = 1$.

Para conseguir la estimación de Z se debe minimizar la variancia de la diferencia $Z^* - Z$, esta variancia está dada por la relación:

$$VAR(Z^*(\vec{x}_o) - Z(\vec{x}_o)) = cov(\vec{x}_o,\vec{x}_o) - 2\sum_{j=1}^{k}\lambda_j cov(\vec{x}_j,\vec{x}_o) + \sum_{j=1}^{k}\sum_{jj=1}^{k}\lambda_j\lambda_{jj}cov(\vec{x}_j,\vec{x}_{jj}) \quad - (8)$$

La minimización de esta relación se logra utilizando el método de los multiplicadores de Lagrange, y se obtiene el siguiente sistema de ecuaciones:

$$-cov(\vec{x}_j,\vec{x}_o) + \sum_{jj=1}^{k}\lambda_{jj}cov(\vec{x}_j,\vec{x}_{jj}) - \sum_{i=0}^{n}\mu_i f^i(\vec{x}_j) = 0 \quad - (9)$$

para $j = 1,2,\ldots,k$.

Donde μ_i son los multiplicadores de Lagrange; $cov(\vec{x}_j,\vec{x}_{jj})$ es la covariancia de los residuales y f^i son funciones escogidas de antemano. Aquí, se supone el conocimiento teórico de la covariancia de los residuales.

Cuando se conoce el semivariograma de los residuales, se tiene una expresión similar a la anterior, pero expresada en términos de la función intrínsica o semiavriograma:

$$\gamma(\vec{x}_j-\vec{x}_o) - \sum_{jj=1}^{k}\lambda_{jj}(\vec{x}_j-\vec{x}_{jj}) - \sum_{i=1}^{n}\mu_i f^i(\vec{x}_j) - \delta = 0 \quad - (10)$$

donde δ es otro multiplicador de Lagrange. Una gran ventaja de este método consiste en la posibilidad de conocer la variancia de la variable en cada punto calculado, mediante la relación:

$$VAR(Z^*(\vec{x}_o) - Z(\vec{x}_o)) = \sum_{j=1}^{k} \lambda_j (\vec{x}_j - \vec{x}_o) + \sum_{i=1}^{n} \mu_i f^i(\vec{x}_o) + \delta$$

Ahora bien, si se supone un semivariograma lineal de la forma:

$$\gamma(\vec{x}_j - \vec{x}_{jj}) = \bar{\omega} |\vec{x}_j - \vec{x}_{jj}| \text{ para } |\vec{x}_j - \vec{x}_{jj}| \leq 2|\vec{r}|$$

donde $|\vec{r}|$ es el radio de influencia de la variable. El sistema de ecuaciones será ahora:

$$|\vec{x}_j - \vec{x}_{jj}| - \sum_{jj=1}^{k} \lambda_{jj} |\vec{x}_j - \vec{x}_{jj}| - \sum_{i=1}^{n} \mu_i f^i(\vec{x}_j)/\bar{\omega} - \delta/\bar{\omega} = 0$$

donde $\bar{\omega}$ es la pendiente del semivariograma de los residuales (Olea, 1972).

El problema del krigaje se reduce, ahora, a encontrar la solución de un sistema de ecuaciones, en donde las incógnitas son las λ_{jj}, μ_i y δ. Para resolver este sistema de ecuaciones, es necesario conocer $\bar{\omega}$ y el radio de influencia de la variable. Esta información se obtiene de las gráficas de semivariogramas de los residuales. Entonces, para poder generar una retícula de valores calculados por krigaje universal, se debe de hacer un análisis estructural de la variable con la ayuda de los semivariogramas de los residuales. En la práctica, el semivariograma experimental de los residuales se ajusta a un modelo teórico, si no se logra un buen ajuste, entonces se intentará otro ajuste experimental con una nueva combinación de grado de deriva, otra función teórica del semivariograma y un nuevo radio de la vecindad o radio de influencia. Otro elemento que se debe conocer son las funciones $f^i(\vec{x})$ que definen a la deriva analíticamente. En la aplicación siguiente se usó una deriva cuadrática de la forma:

$$M(\vec{x}) = a_1\vec{x} + a_2\vec{y} + a_3\vec{x}^2 + a_4\vec{x}\vec{y} + a_5\vec{y}^2$$

Como no se conoce la deriva, sino sólo una estadística, los residuales calculados son residuales estimados y, en base a éstos, se calcula el semivariograma de los residuales.

APLICACION AL CAMPO CACTUS

Para ilustrar la técnica del krigaje universal se utilizaron datos del Campo Cactus, ésta información consistió de valores de capacidad de flujo (kh), obtenidos mediante una nueva técnica de interpretación de las curvas de incremento de presión desarrollada recientemente por Pérez-Rosales (1978), que considera al yacimiento como un medio de naturaleza estocástica y heterogénea. La capacidad de flujo, considerada como variable aleatoria, varía a través del yacimiento de una manera aparentemente continua, y tratar de representar una distribución de esta variable es el objetivo que se logró con la ayuda de las técnicas de la teoría de las variables regionalizadas.

En la figura 2 se muestra la localización de los pozos del campo, se cuenta con 37 datos de capacidad de flujo. La retícula punteada es la superficie que fue estimada. La información se procesó en unidades de darcy-pie.

Para la práctica del krigaje se consideran varias suposiciones para ser aplicadas al yacimiento en estudio. Primeramente, se supone que la variable se comporta de una forma isotrópica; es decir, sus variaciones en todas las direcciones son similares. El modelo del semivariograma teórico se considera lineal de la forma $\bar{\omega}|h|$ por último, para la deriva se adoptó un polinomio de segundo grado. Con estas condiciones prácticas, el semivariograma de los residuales se obtuvo mediante el estimador insesgado:

$$\gamma(h) = \frac{1}{2N(h)} \sum_{i=1}^{N(h)} (R(x_i+h) - R(x_i))^2$$

Donde R^i son los residuales definidos por la expresión (5).

En las figuras 3-6 se muestran los semivariogramas de los residuales con diferentes longitudes de vecindad y dirección. Para la generación de la malla de interpolación por krigaje, fue seleccionado el semivariograma con vecindad de 6 kms y $\bar{\omega} = 15.3$, constante para todas las direcciones.

RESULTADOS

Una vez hallados mediante el semivariograma los siguientes parámetros: radio de influencia y la pendiente del semivariograma $\bar{\omega}$, la generación de la malla por krigaje universal es el siguiente paso.

La obtención de los semivariogramas fue realizada con el programa SEMIVARIOG y la ejecución del krigaje fue hecho con el programa KRIGAJE, ambos programas fueron desarrollados en la Subdirección de Explotación del Instituto Mexicano del Petróleo. En las figuras 7-8 se presentan los resultados obtenidos, la figura 7 es la distribución de la variable regionalizada, en este caso la capacidad de flujo; la figura 8 es el mapa de error esperado, que proporciona el grado de confianza de la distribución de la variable. Como la variable aleatoria $Z^*(x) - Z(x)$ se considera normalmente distribuida, dos veces la desviación estándar dará un intervalo de confianza del 95% con respecto a la superficie. Por ejemplo, si se sobreponen los dos mapas y se localiza la isolínea de 6 en la capacidad de flujo, en el mapa de error esperado se encuentra una isolínea de 2, lo que significa que en ese lugar el valor de la capacidad de flujo tiene un 95% de probabilidad de encontrarse entre 4 y 8 darcy-pie de capacidad de flujo.

La distribución de la capacidad de flujo revela, en general bajos valores, incrementándose hacia la parte inferior del mapa; aunque este incremento debe tomarse con ciertas precauciones debido a las altas variancias que se tienen, pero de algún modo existe el incremento de valores en esa zona. Otra zona importante con altas capacidades de flujo es la zona inferior izquierda.

CONCLUSIONES

Contar con un mapa de una variable distribuida espacialmente en el subsuelo es, desde luego, una arma decisiva para planear y proyectar un yacimiento petrolífero. Ahora bien, las técnicas del krigaje universal, como método para encontrar una distribución espacial de alguna variable aleatoria, proporcionan resultados confiables como consecuencia de la teoría matemática en la que se apoya esta técnica. Una de las características más importantes de la teoría es la posibilidad de conocer la variancia de los valores estimados y que proporcionan un rango de confiabilidad a la distribución. Actualmente, la teoría ofrece perspectivas prometedoras dentro del campo de la ingeniería petrolera. La apliacción al Campo Cactus arroja resultados suficientemente confiables y que adquieren importancia dentro de los programas de recuperación secundaria del yacimiento, como una herramienta accesoria para la toma de decisiones.

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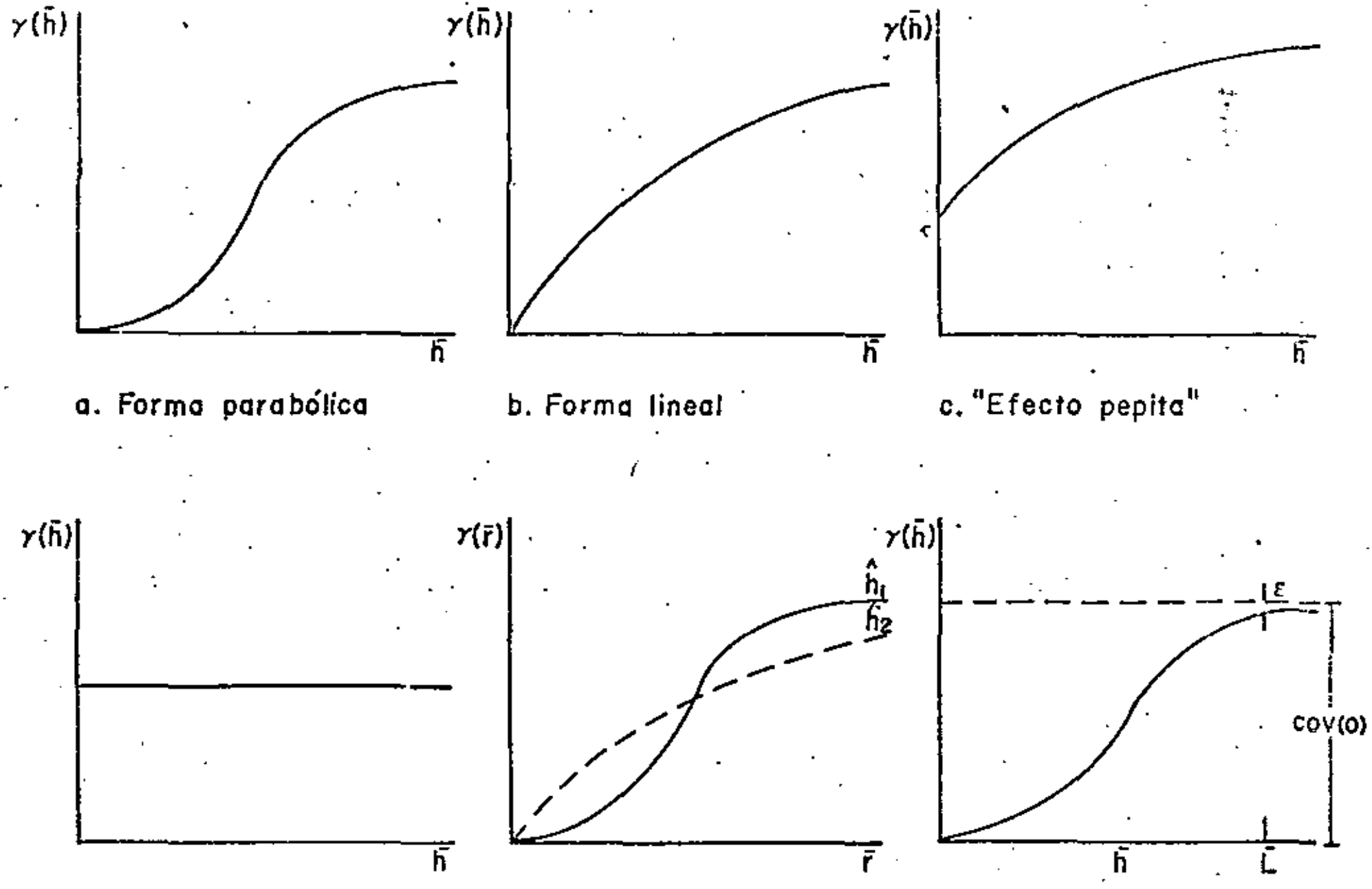


FIG. 1. TIPOS DE ..MIVARIOGRAMAS

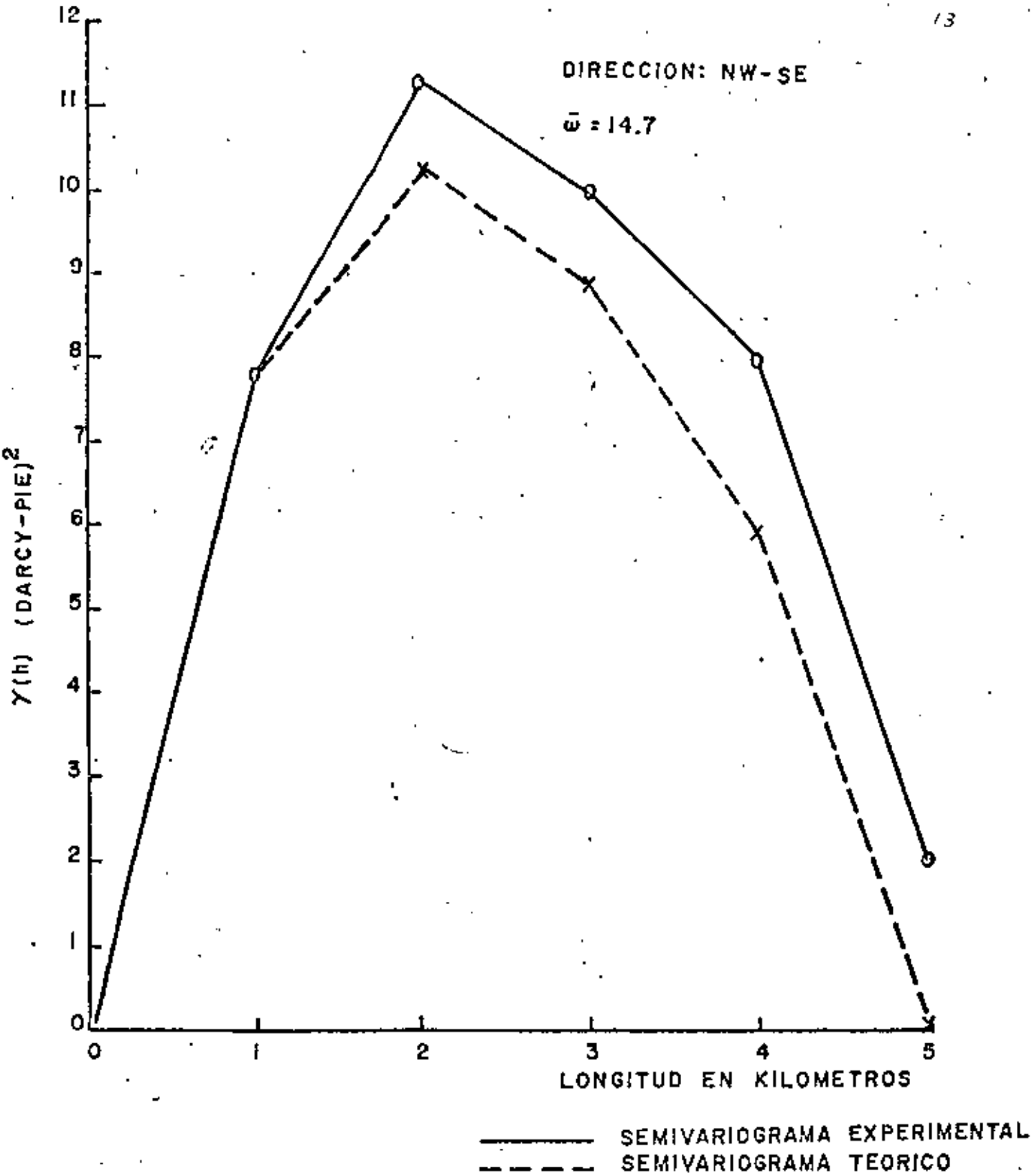


FIG. 3. SEMIVARIOGRAMAS DE LOS RESIDUALES PARA DERIVA CUADRATICA.

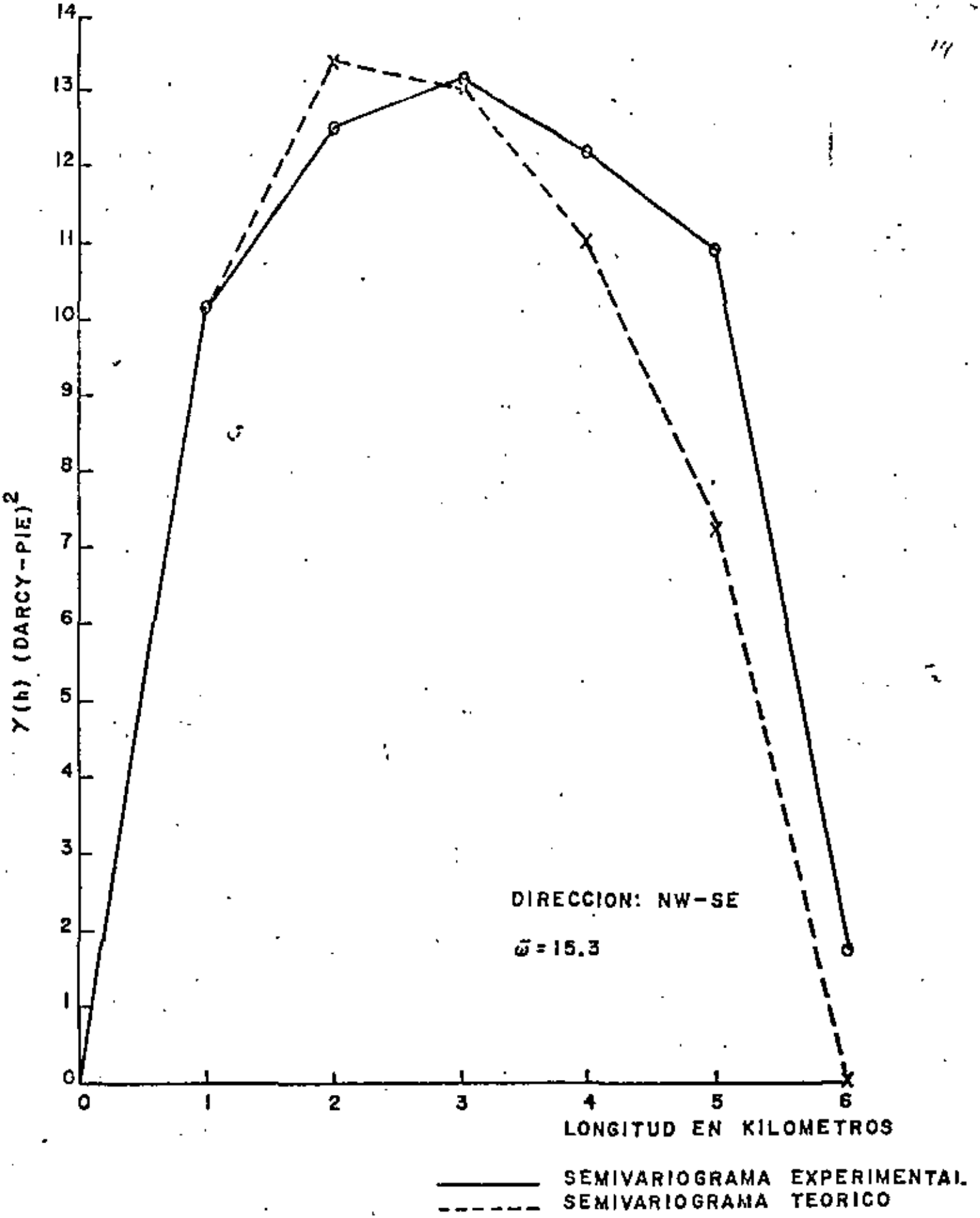


FIG. 4. SEMIVARIOGRAMA DE LOS RESIDUALES PARA DERIVA CUADRATICA

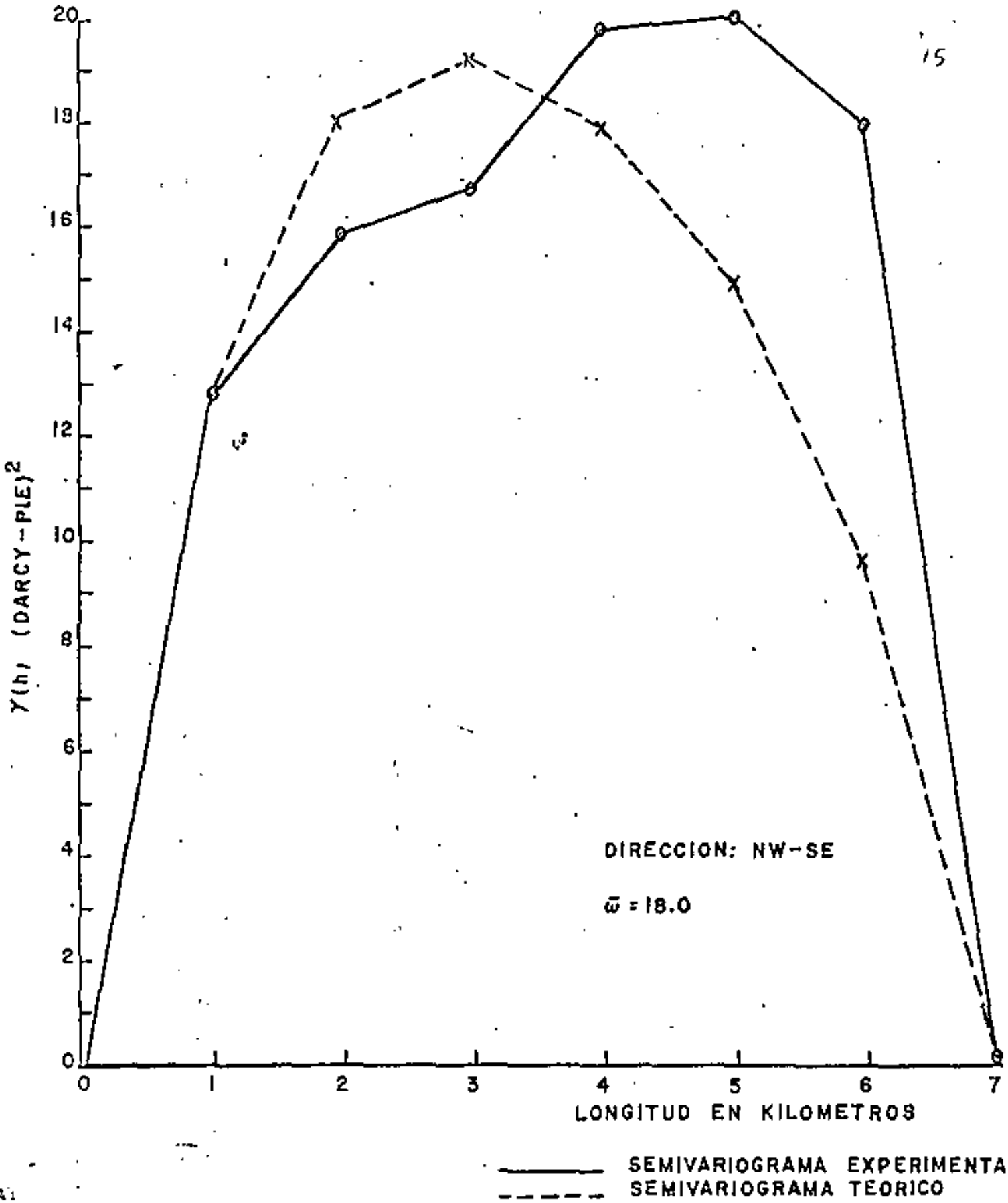


FIG. 5. SEMIVARIOGRAMAS DE LOS RESIDUALES PARA DERIVA CUADRATICA

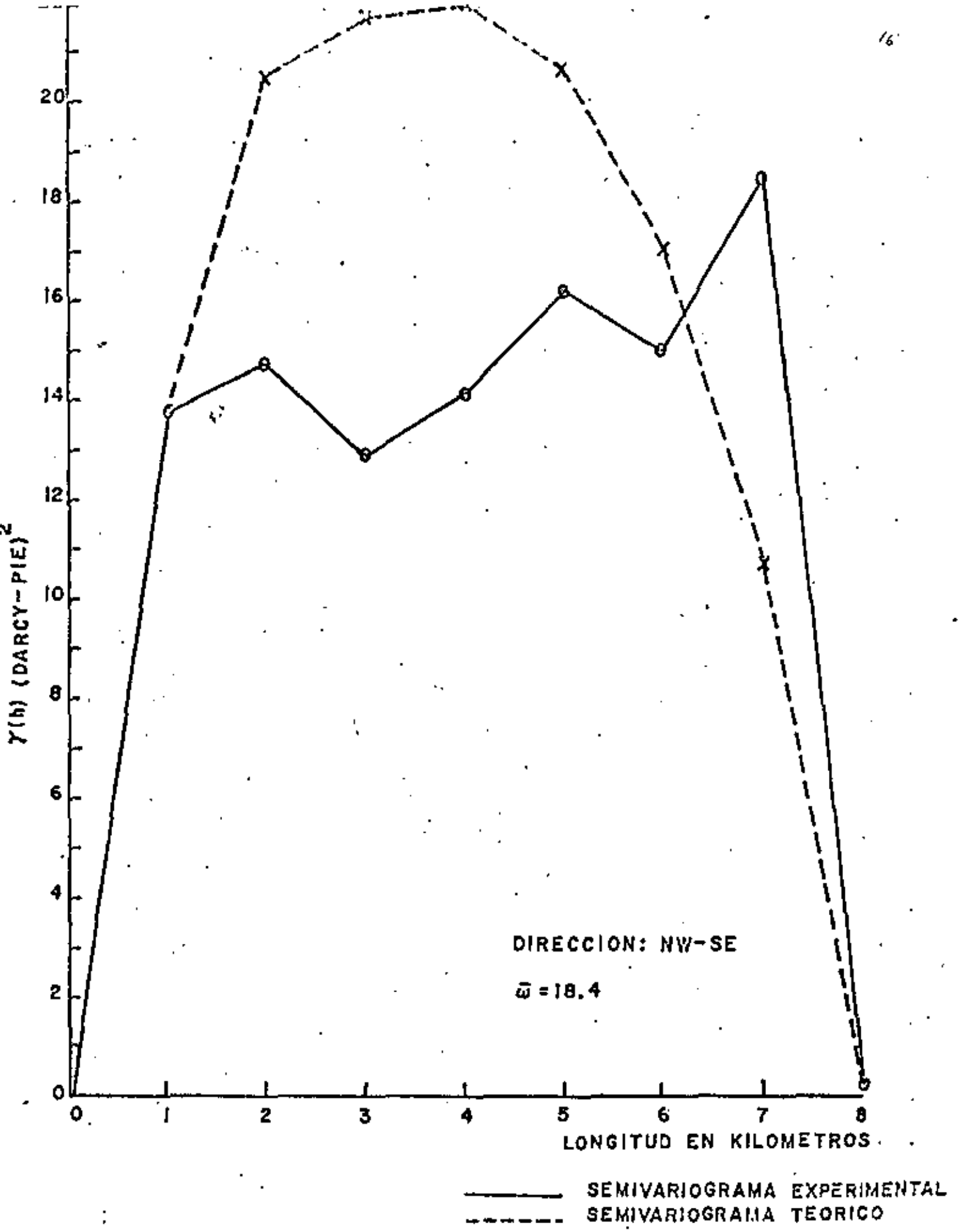


FIG. 6. SEMIVARIOGRAMAS DE LOS RESIDUALES PARA DERIVA CUADRATICA.

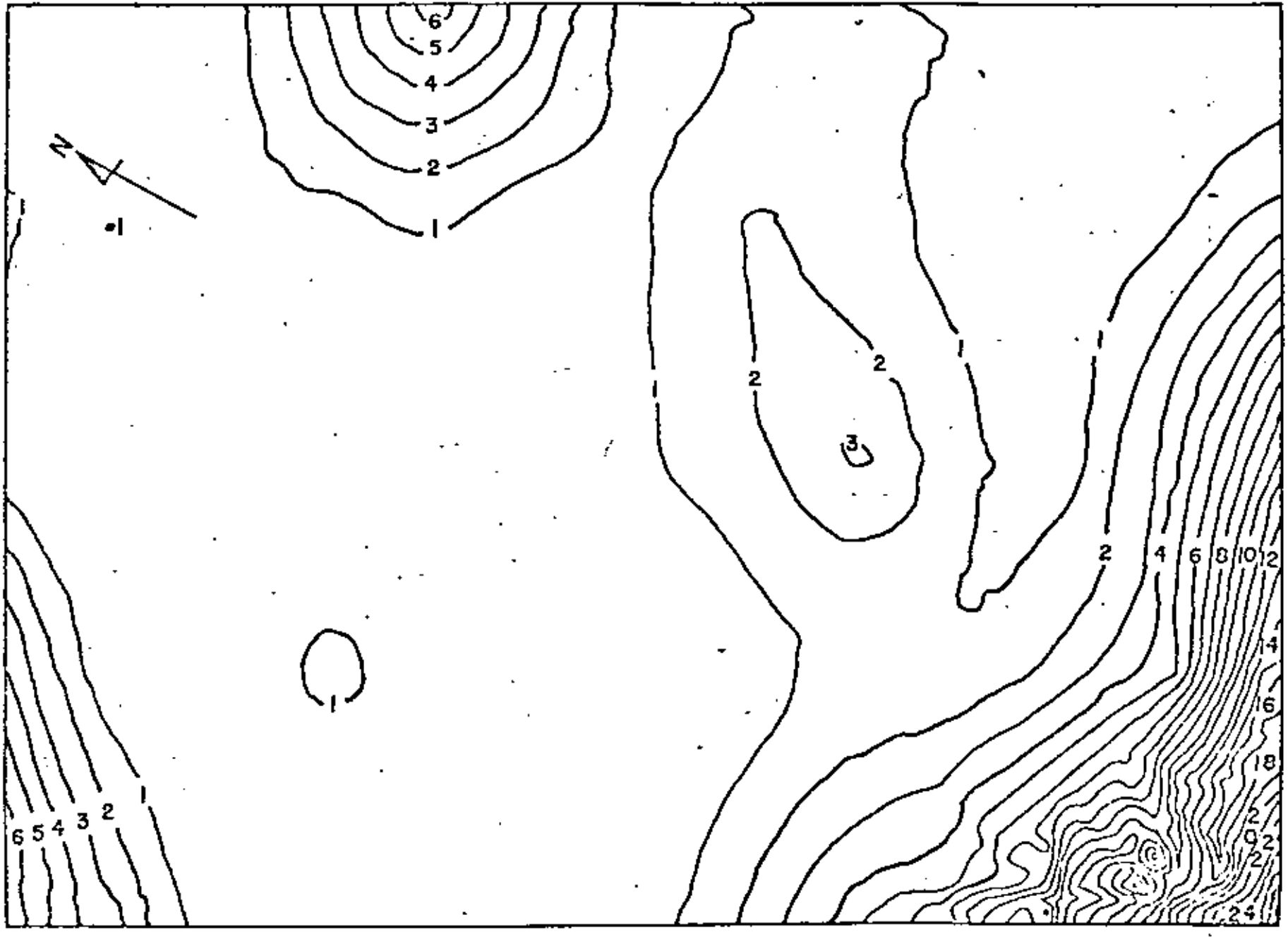


FIG. 7. CAMPO CACTUS. DISTRIBUCION DE KH POR KRIGAJE UNIVERSAL

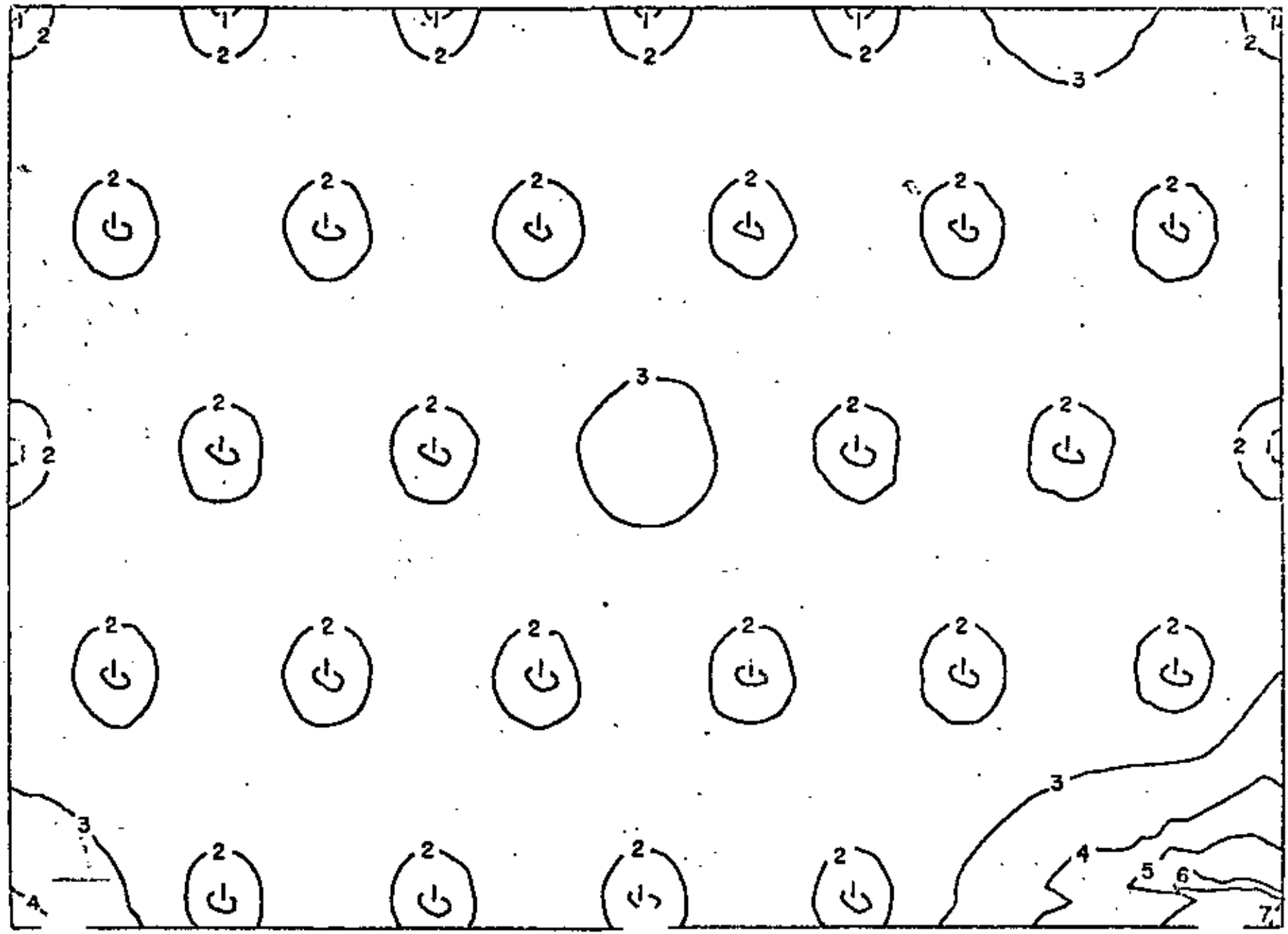
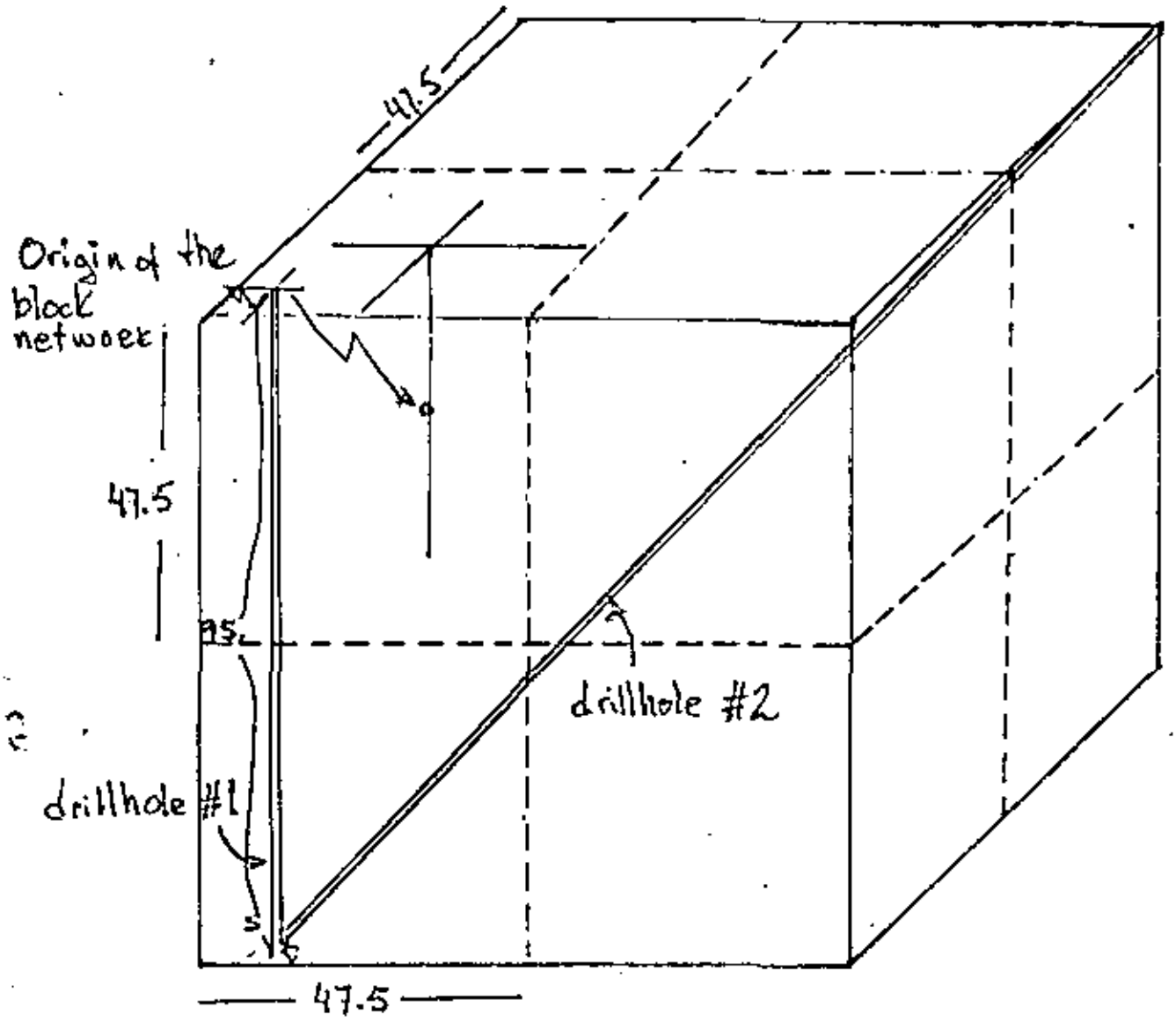


FIG 2 MAPA DE ERROR ESPERADO



XOB = 18.75 DB(1) = 47.5 , MX = 2
YOB = 18.75 DB(2) = " NY = 2
ZOB = 71.25 DB(3) = " NZ = 2

1	3
2	4

5	7
6	8

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English Language Publications on the French School of Geostatistics[1]

Ivan Paunez[2]

INTRODUCTION

Geoscientists have become more aware and concerned about the eventual exhaustion of economic materials as the depletion of known reserves gathers momentum. While the search for alternatives continues, there has been an increasing trend towards the more careful development of planning and budgeting policies in an attempt to optimize and equilibrate the present situation.

Some of the most important developments in this sequence of events have been in the field of ore reserve estimation. The single most outstanding contribution to this field in the last fifteen years must undoubtedly be credited to a group of French geomathematicians who pioneered and synthesized from several disciplines what has become known as "Geostatistics" but what could be called more explicitly the "Theory of Regionalized Variables and its Application to Problems in Estimation."

Since the early sixties a number of contributions have been made to the geostatistical literature. Any prospective researcher wishing to develop expertise in geostatistics is confronted with the difficult task of finding some starting point and then wading through articles that can prove at times very difficult to obtain. It is this starting point that this compilation is intended to supply.

The paradox in the title simply reflects the author's limited grasp of the French language. It should not be construed as a statement concerning the importance or the expertise of authors who publish in languages other than English.

Any omissions are unintentional but only published articles, reasonably accessible in Australia, have been included. Suggestions for additions or

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improvements to the list would be appreciated as contributions to the planned annual updates.

It would be remiss not to acknowledge the vital role of the fifteen International Symposia on the Applications of Computers and Mathematics in the Mineral Industry (APCOM) to date. These must be given credit for continuing to be one of the most effective media for the rapid dissemination of the latest developments in geostatistics.

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BIBLIOGRAFIA

Kriging and Geostatistics: A Review of the Literature Available in English

By

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ABSTRACT

A review of the literature available in English on kriging and geostatistics is given with the references cross-classified by their subject matter for convenience and ease of use.

REVIEW OF LITERATURE

In the last few years there has been a tremendous surge of interest in the minerals industry in kriging and its applications. The technique was originally developed by Dr. D. G. Krige, as a weighted moving average for grade estimation in the South African gold mines. Since then, the mathematical and statistical foundations of geostatistics have been developed by a French research group led by Dr. G. Matheron. The practical applications of kriging are widespread and include grade estimation at a point or of a block, grade control using the data from exploration holes without the need for sampling blast holes, simulation of the deposit to obtain the optimal mine design, contouring, and the optimal spacing and location of drill holes.

The paper provides, as far as is possible, a comprehensive bibliography of the literature available in English. Geostatistical techniques have often been criticized on the grounds that the literature was available only in French and that few case studies had been undertaken. It was decided to restrict the bibliography to the English language to demonstrate the vast amount of material published in English on these techniques. Although much of the early work was published in French, the more important papers have since been republished in English. Recently, many case studies have been published in English. It is hoped that this literature review will dispel any remaining doubts about the practicability or availability of literature on these techniques.

As many papers on grade control and grade estimation use kriging or closely-related techniques, some criteria were needed to decide which papers should be included. With the exception of Krige's early work, using only weighted moving averages, papers were omitted if they did not include a variogram. Even after applying these selection criteria, a great many references remained. To make it possible to locate relevant papers quickly and easily, the references have been cross-classified in Table 1 by their subject matter under the following headings.

1. elementary theory
2. other theoretical papers
3. computer simulations of orebodies
4. sample spacing
5. comparison of kriging with other techniques
6. contouring
7. estimation of the block grade
8. tonnage—cut off grade
9. case studies
10. papers showing the early developments of these techniques

A few papers which give a particularly clear and concise explanation of the principles of kriging have been grouped together under the first heading to help non-mathematicians grasp the fundamentals of geostatistics. The remaining papers on the theoretical aspects of geostatistics have been grouped under the second heading.

Where appropriate, papers have been listed under more than one heading. The main topic of each paper has been marked by an asterisk (*), any additional topics being denoted by a section-mark (§). If a paper covers a topic particularly well, it has been marked with a dagger (†). For example, "Geostatistical ore estimation—a step-by-step case study" by M. David (1971), which considers the effect of varying the size of the sampling grid to be used for the subsequent drilling of a copper/nickel deposit, has been listed under the headings "sample spacing" and "case studies—Cu, Ni", using the abbreviated title "David (1971)". At the end of the paper, an alphabetic listing of all the references is given. Although every effort was made to sight copies of all the papers, some papers were not obtained in time. These papers have been included only in the final alphabetic listing and are marked with a double dagger (‡).

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TABLE I

Classification of main topics

	Clear	Advanced	Simulation	Spacing	Comparisons	Contouring	Block kriging	Grade control	Examples	Early
Agterberg 64					Trend	Trend*				
Agterberg 66						Trend*			Cu	
Agterberg 67								•	Cu	
Agterberg 68a						Trend*				
Agterberg 68b						Trend*				
Agterberg 69					‡	Trend*			Cu, Au	
Agterberg 70	‡				Trend*	Trend				
Agterberg 74	*									
Agterberg 76					•					
Agterberg and Chung 73	§						§		S in coal*	
Akima 75	§									
Alfaro and Huijbregts 74			•			§			Ni, Baux, P	
Alfaro and Miguez 76		•		§		§			Geophysics	
Barnes 77				Poly, IDS				•		
Bilodeau and Mackenzie 77				•						
Blais and Carlier 68	•				§		§	§	U, Fe	
Borgman and Daud 74			Pit?				§			
Borgman and Frahme 76	•								Clay	
Brooker 75a				•			§			
Brooker 75b				•						
Brooker 76a				•			§			
Brooker 76b				•	Poly		§			
Brooker 77a				•	§		§			
Brooker 77b	§						•			
Bubenicek and Haas 69	*						§		Fe, Thick	
Chiles 76			§			§			Sea floor*	
Chiles and Chauvet 74	§					•			Sea floor	
Clark 76a	•								Ni	
Clark 76b	•									
Clark 77a							•			
Clark 77b	•									
Clark and White 77			•						Pb/Zn	
Cortez, Muge and Pereira 74a	•								Fe	
Cortez, Muge and Pereira 74b							§		Au*	
Cortez, Muge and Pereira 77								G/T*	Phosphate	
Dagbert and David 76	•								Pb/Zn	
Dagbert and David 77a								G/T*		
Dagbert and David 77b							•			
Daud 74			§		§		•		Base metal	
David 69	•			§			§		Pb-Zn	
David 71				§			•		Cu, Ni, Thick	
David 72								G/TT		
David 73a			§	§			§	G/T*		
David 73b	•									
David 74a	•									
David 76a	•									
David 76b				•						
David 77	†								Bibliography	
David and Blais 73	*						§		Porph-Cu, Fe	

TABLE 1 (continued)

Classification of main topics

	Clear	Advanced	Simulation	Spacing	Comparisons	Contouring	Block kriging	Grade control	Examples	Early
David and Dagbert 74	§				•	§			Geochem	
David, Dagbert and Belisle 77	•							§		
David, Dowd and Korobow 74			Cond		ID			G/T	Pit*	
Davis 73	•									
Davis 76					Actual		UK			
Delfiner 76		•	§							
Delfiner and Delhomme 75			§		LS, ID	§			Rain*	
Dowd 73a	•			§	§					
Dowd 73b				•			2D			
Dowd 75				§			•			
Dowd and David 76			•							
Dowd and Royle 77	§								Tar sand*	
Dutta and Rao 77	§								Fe complex*	
Fang 77									Coal*	
Fang *et al* 76									Coal*	
Filion 76					Poly*		§		Pb/Zn	
Gignac 75					ID*					
Guarascio 76	§					•			Uranium	
Guarascio and Raspa 74								G/T*	Zn	
Guarascio and Turchi 77								•	Uranium	
Gy 64		§		•					Pb, Pb-Zn	
Haas and Jousselin 76						§			Oil†	
Haas and Viallix 76	•					§				
Hazen 67a										
Huijbregts 73		•								
Huijbregts 75a	•									
Huijbregts 75b					Mined		§	G/T	Moly*	
Huijbregts 76	§						§	G/T*		
Huijbregts and Matheron 71	§					UK†				
Huijbregts and Segovia 73							§	G/T	Cut	
Journel 73	•									
Journel 74a	•						§			
Journel 74b	§		•						Ni, Rain	
Journel 74c			§		Std error		§		Ni, Co, Si, Mg*	
Journel 75	•									
Journel 76		•	§							
Journel 77		•								
Journel and Huijbregts 73	§							§	Ni-Cu*	
Journel and Sans 74		§						•	Phosphate	
Kim and Knudsen 77	•								U	
Kim, Myers, and Knudsen 77	•						§			
Knudsen, Kim and Mueller 75					ID*				Porph-Cu	
Knudsen, Kim and Mueller 78					•					
Koch and Link 71	•									
Krige 51										•
Krige 52										•
Krige 59										•
Krige 60										•

TABLE 1 (continued)

Classification of main topics

	Clear	Advanced	Simulation	Spacing	Comparisons	Contouring	Block kriging	Grade control	Examples	Early
Krige 61	•									•
Krige 62a	•									•
Krige 62b								•		•
Krige 62c									Au	•
Krige 64a									Au	•
Krige 64b						Trend			Au	•
Krige 64c										•
Krige 66a						Trend	•			
Krige 66b	§				•		§		Au	§
Krige 66c	•					Trend			Au, U	
Krige 70b						•				
Krige 73									Cu*	
Krige 76a					•					
Krige 76b	•								Au	
Krige 77	•								Au, Cu	
Krige and Munro 68	•									
Krige and Rendu 75	§						•		Cu/Zn	
Krige and Ueckermann 63					§	§				•
Krige, Watson *et al.* 69					•				Au-U	
Lallement 77				•					Pb, Fe	
Leach 72a	•									
Leach 72b								G/T*		
Leach 75						Trend*				
Link and Koch 74					•				Ni	
Lucerno 71							§		Uranium*	
Marechal 75					§			G/T*	Cu	
Marechal 76a			§				§	*		
Marechal 76b	§							†		
Marechal and Serra 70	•					§	§	G/T	Al	
Marechal and Shrivastava 77	§								Fe*	
Marino and Slama 73					Trend		§		Pit*	
Matern 60		•								
Matheron 61	§			•					Bauxite	
Matheron 63		•								
Matheron 67	•				True	§				
Matheron 70		•								
Matheron 71		•								
Matheron 73		•								
Matheron 76a		•								
Matheron 76b		•								
Miesch 75	•									
Monget and Albuisson 71	•								Gravity	
Montgomery *et al.* 77									Gold*	
Nemec 70									Structure*	
Newton 73				•						
Newton and Royle 73	•				ID				Au	
Olea 74	•					UK				
Olea 75	•									

TABLE 1 (continued)

Classification of main topics

	Clear	Advanced	Simulation	Spacing	Comparisons	Contouring	Block kriging	Grade control	Examples	Early
Olea and Davis 77						•			Oil	
Parker 77a	§				All*			G/T*	Fe, Cu	
Parker 77b				•						
Parker and Sandefur 76 ..	•									
Parker and Switzer 75							§	G/T*	Ni-Cu	
Paoucz 78									Bibliography*	
Pereira, Cortez and Muge 75	§			•					Fe	
Piloski 68						Trend*				
Pirow and Harrison 66			•							
Pirow and Kriga 65					Poly*					
Quinlan and Crosby 77							•			
Rendu 70	•						§			
Rendu 71	§			•						
Rendu 76	§			•						
Royle 71	•									
Royle 77a					Poly*		§	§		
Royle 77b	•			§						
Royle 77c									Sand*	
Royle and Hosgit 74	§						§		Sand*	
Royle and Newton 72			§		Poly*			§		
Royle, Newton and Sarin 72				•			§			
Rutledge 75	•			§		UK				
Rutledge 76					§					
Sabourin 75					Actual		§		Ash, S Coal*	
Sabourin 76	•					UK				
Sandefur and Grant 76				•	Poly				Uranium	
Serra 71	•							G/T	Ni, Co	
Sinclair and Dersaisme 74				§			§		Cu veinf	
Sinclair and Dersaisme 76				§	§		§		Cu*	
Singh 76		•								
Stanley 77									Moly*	
Switzer and Parker 76								G/T*		
Vallee, Belisle and David 77					Mined*			G/T	Cu	
Venter 76										
Watson 71	•					§			Coal*	
Watson 72	•	§				Trend				
Whitten 66					Poly*					
Whitten 77	•									
Le Wijs 53										
Le Wijs 72	•									
Williamson and Mueller 76								G/T*		
Wood 77				•			§			
Zwicky 75									Tar*	

* Main topic of paper.
§ Additional topics of paper.
• The paper covers this topic particularly well.

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Extra

PRINCIPLES OF GEOSTATISTICS

1963

G. MATHERON

ABSTRACT

Knowledge of ore grades and ore reserves as well as error estimation of these values, is fundamental for mining engineers and mining geologists. Until now no appropriate scientific approach to those estimation problems has existed; geostatistics, the principles of which are summarized in this paper, constitutes a new science leading to such an approach. The author criticizes classical statistical methods still in use, and shows some of the main results given by geostatistics. Any ore deposit evaluation as well as proper decision of starting mining operations should be preceded by a geostatistical investigation which may avoid economic failures.

RESUME

Pour tout mineur et géologue minier, la connaissance des teneurs et du tonnage et l'appréciation des erreurs sur ces grandeurs est fondamentale. Or, jusqu'à présent, il n'existait pas d'approche scientifique correcte de ces problèmes.

La *géostatistique*, dont les principes sont résumés dans cet article, est la nouvelle science qui permet cette approche. L'auteur indique les méthodes statistiques antérieures et encore courantes et donne quelquesuns des résultats principaux de la géostatistique.

Toute évaluation de gisement et toute décision de mise en exploitation devrait être précédée d'une étude géostatistique permettant de limiter le risque d'une déconvenue ultérieure.

INTRODUCTION AND SHORT HISTORICAL STATEMENT

GEOSTATISTICS, in their most general acceptation, are concerned with the study of the distribution in space of useful values for mining engineers and geologists, such as grade, thickness, or accumulation, including a most important practical application to the problems arising in ore-deposit evaluation.

Historically geostatistics are as old as mining itself. As soon as mining men concerned themselves with foreseeing results of future works and, in particular as soon as they started to pick and to analyze samples, and compute mean grade values, weighted by corresponding thicknesses and influence-zones, one may consider that geostatistics were born. In so far as they take into account the space characteristics of mineralization, these traditional methods still keep all their merit. Far from disproving them, modern developments of the theory have adopted them as their starting point and have brought them up to a higher level of scientific expression.

However, assuming they could provide a correct evaluation of mean values, the traditional methods failed to express in any way an important

character of mineralizations, which is their variability or their dispersion. Some scores of years ago, classical probability calculus techniques began to be used in order to take into account this characteristic. If an unskillful application of those techniques has sometimes led to absurdities, it remains certain that, on the whole, results have been profitable. In a way this is a paradox, for classical statistical methods, in so far as they are not concerned with the spatial aspect of the studied distributions, actually cannot be applied. As a matter of fact, the South-African school, which has recorded the most remarkable results with Krige, Sichel, used to say, and believed, that they were applying classical statistics. But the methods they were developing differed more and more from classical statistics, and adjusted themselves spontaneously to their object.

The second decisive change appeared when the insufficiency of classical probability calculus was clearly understood as well as the necessity of reintroducing the spatial characters of the distributions. It consisted in realizing on a higher level the synthesis between traditional and statistical methods. Hence, geostatistics started elaborating their own methods and their own mathematical formalism, which is nothing else than an abstract formulation and a systematization of secular mining experience. This formalism has inherited from its statistical origin a language in which one still speaks of variance and covariance, including however in those notions a new content. This similarity in vocabulary must not deceive. At the end of a protracted evolution, the geostatistical theory had to admit that it was facing, instead of random occurrences, natural phenomena distributed in space. And, therefore, its methods are approximately those of mathematical physics and more specially those of harmonic analysis.

INSUFFICIENCY OF CLASSICAL STATISTICAL CONCEPTS

To be brief, we shall limit ourselves, in what follows, to the distribution of ore-grades in a deposit. The results that will be obtained will however have a general range and will be applicable to any character owned by a spatial distribution. In an usual statistical approach, the grades of samples picked in a deposit are classified on a histogram. Such a procedure does not take into account the location of samples in the deposit. But it is not enough to know the frequency of a given ore-grade in a deposit. It is also necessary to know in what way the different grades follow each other on the field, and specially what is the size and the position of economic orebodies! At the starting point of the theory we have to face one fact: the inability of common statistics to take into account the spatial aspect of the phenomenon, which is precisely its most important feature.

More precisely, the aim of the classical probability calculus is the study of aleatory variables. The mere example of the heads or tails game shows clearly what is going on. Let us record $+1$ each time the coin falls on tails and -1 in the opposite case. Before throwing the coin, there is no way of forecasting whether $+1$ or -1 will be recorded; we only know that there is one chance out of two for one or the other of these two opportunities. An aleatory variable has classically two essential properties: 1) The possibility,

theorically at least, of repeating indefinitely the test that assigns to the variable a numerical value; we can for example, throw the coin as often as we want. 2) The independence of each test from the previous and the next ones; if all the 100 first attempts have given tails, there remains however one chance out of two for the 101st attempt to give heads.

It appears clearly that a given ore-grade within a deposit cannot have those two properties. The content of a block of ore is first of all unique. This block is mined only once and there is no possibility of repeating the test indefinitely. When the grade of a sample is concerned, which may be a groove sample of a given size for example, the result is exactly the same, because the grade of a groove located in a point with coordinates (x, y) is unique and well determined. However it is possible to pick a second sample close to the first, then a third one, etc. . . . which shows an apparent possibility of repeating the test. Actually, it is not exactly the same test but a slightly different one. But even assuming this possibility of repetition, the second property will surely not be respected. Two neighboring samples are certainly not independent. They tend, in average, to be both high-grade if they originate from a high-grade block of ore, and vice-versa. This tendency, more or less stressed, expresses the degree of more or less strong continuity in the variation of grades within the mineralized space.

The misunderstanding of this fact and the rough transposition of classical statistics has sometimes led to surprising misjudgments. Around the fifties, in mining exploration, it was advised to draw lots to locate each drilling (i.e., to locate them exactly anywhere). Miners of course went on still using traditional regular grid pattern sampling, and geostatistics could later prove they were right. Or else again, it was urged that the accuracy of ore evaluation of a deposit depended only on the number of samples (and not on their location) and varied as the square root of this number. This unskillful transposition of the theory of errors led to absurdities. For example, if a given deposit is explored by drilling, it would suffice to cut the cores in 5 mm pieces instead of 50 cm pieces to obtain 100 times more samples, and therefore 10 times higher accuracy. This, of course, is wrong. The multiplicity of samples thus obtained is a fallacy, and does nothing more than repeat indefinitely the same information, without yielding anything else. Geostatistics actually show that accuracy is the same with pieces of 5 mm and 50 cms, as every miner understands instinctively.

NOTION OF REGIONALIZED VARIABLE

Thus a grade cannot in any way be assimilated to an aleatory variable. We speak of regionalized variables precisely in order to stress the spatial aspect of the phenomena. A regionalized variable is, *sensu stricto*, an actual function, taking a definite value in each point of space.

In general such a function has properties too complex to be studied easily through common methods of mathematical analysis. From the point of view of physics or geology, a given number of qualitative characteristics are linked to the notion of regionalized variable.

a) In the first place, a regionalized variable is *localized*. Its variations occur in the mineralized space (volume of the deposit or of the strata), which is called *geometrical field* of the regionalization. Moreover such a variable is in general defined on a *geometrical support (bolder)*. In the case of an ore-grade, this support is nothing but the volume of the sample, with its geometrical shape, its size and orientation. If, in the same deposit, the geometrical support is changed, a new regionalized variable is obtained, which shows analogies with the first one, but does not coincide with it.

For instance, samples of 10 Kg corresponding to drill cores are not distributed in the same way as samples of 10 tons corresponding to blasts. Often the case of a punctual support will be considered. A punctual grade, for example, will take value 0 or value +1 according to whether its support will fall into a barren or mineralized grain.

b) Secondly, the variable may show a more or less steady continuity in its spatial variation, which may be expressed through a more or less important deviation between the grades of the two neighboring samples mentioned above. Some variables with a geometrical character (thickness or dip of a geological formation) are endowed with the strict continuity of mathematicians. Fairly often (for grades or accumulations) only a more lax continuity will exist or, in other words, a continuity "in average." In some circumstances, even this "in average" continuity will not be confirmed, and then we shall speak of a *nugget effect*.

c) Lastly the variable may show different kinds of *anisotropies*. There may exist a preferential direction along which grades do not vary significantly, while they vary rapidly along a cross-direction. Those phenomena are well known under the names of runs, or zonalities.

To those general characters, common to any regionalized variable, specific features can be superimposed. For example, in the case of a sedimentary deposit, a *stratification effect*, will be noted. Large-scale stratification provides individualizable and separately minable strata. Inside each strata it may appear by the existence of beds following one another vertically, and separated by discontinuity surfaces. The grade, almost constant or barely varying inside a given bed, will vary abruptly from one bed to another; however common and familiar this phenomenon appears to be, it is still fundamental, and a theoretical formulation of the problem that would not take it into account would miss the point. It will happen as well that to those vertical discontinuities, stressed by jointing, will be added lateral discontinuities, owing to the lenticular endings of beds. This *bed-relaying phenomenon*, when it does exist, shows up at each stratigraphic level a partitioning of the sedimentation area into micro-basins with almost autonomous evolution, and may appear during operation through *grade-limit effect*.

In the same way in stockwerk types of deposits, high-grade veinlets or granules individualized in a more or less impregnated mass will be observed. This *stockwerk effect*, just as the stratification and bed relaying effects, expresses the appearance of a discontinuity net-work within a homogeneous geometrical field. On a very different scale, that of granularity, nugget

effect appears as a phenomenon of the same nature, the net-work of discontinuities being here that one separating barren from mineralized grains.

Those different specific aspects of spatial distribution of regionalized variables—far apart from classical probability calculus—must compulsorily be taken into account by geostatistics. This is made possible owing to a simple mathematical tool: the *variogram*.

THE VARIOGRAM

The variogram is a curve representing the degree of continuity of mineralization. Experimentally, one plots a distance d in abcissa and, in ordinate, the mean value of the square of the difference between the grades of samples picked at a distance d one from the other. Theoretically, let $f(M)$ be the value taken in a point M of the geometrical field V by a regionalized variable defined on a given geometrical support v (in general support v will be small and the limit may be considered as punctual). The *semi-variogram* $\gamma(h)$, or law of dispersion, is defined, for a vectorial argument h, by the expression:

$$\gamma(h) = \frac{1}{2V} \int\int\int_{V} [f(M+h) - f(M)]^2 dV; \qquad (1)$$

In general, the variogram is an increasing function of distance h, since, in average, the farther both samples are one from the other, the more their grades are different. It gives a precise content to the traditional concept of the *influence zone* of a sample. The more or less rapid increase of the variogram represents, indeed, the more or less rapid deterioration of the influence of a given sample over more and more remote zones of the deposit. The qualitative characteristics of regionalization are very well expressed through the variogram:

a) The greater or lesser regularity of mineralization is represented by the more or less regular behavior of $\gamma(h)$, near the origin. It is possible to distinguish roughly four types (Fig. 1). In the first type the variogram has

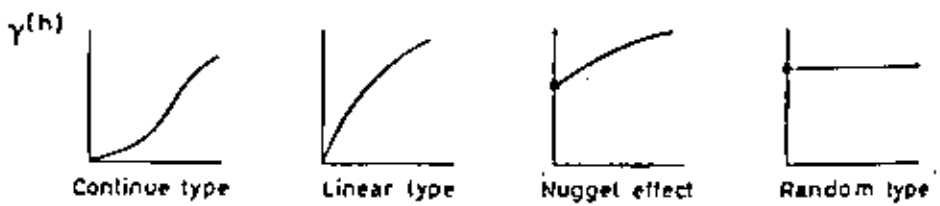

FIG. 1.

a parabolic trend at the origin, and represents a regionalized variable with high continuity, such as a bed-thickness.

The second type, or linear type, is characterized by an oblique tangent at the origin, and represents a variable which has an "in average" continuity. This type is the most common for grades in metalliferous deposits.

The third type reveals a discontinuity at the origin and corresponds to a variable presenting not even an "in average" continuity, but a nugget effect.

The fourth type is a limit case corresponding to the classical notion of random variable. Between type 1 (continuous functional) and type 4 (purely random) appears a range of intermediates, the study of which is the proper object of geostatistics.

b) The variogram is not the same along different directions of the space. Function $\gamma(h)$ defined in (1) does not only depend upon the length, but also upon the direction of vector h. Preferential trends, runs, and shoots are revealed through the study of the distortion of variogram when this direction is altered. Geological interpretation of such anisotropies is often instructive.

c) Structural characters are also reflected in the variogram. For instance, the bed-relaying phenomenon appears in the experimental curve as a level stretch of the variogram beyond a distance, i.e., a range equal to the mean diameter of the autonomous micro-basins of sedimentation. And the

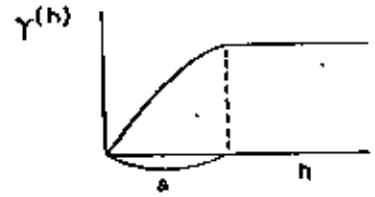

FIG. 2.

fact that these ranges are not the same along different directions makes it possible to determine the directions of elongation, and the average shape of the micro-basins.

This tool, the variogram, does not represent the totality nor the local details of the mineralizing phenomenon, but it expresses in a synthetic form their essential characters. The harmonic analysis of a vibratory phenomenon assigns for each harmonic a phase and an amplitude. The local outline of the phenomenon depends mostly upon phases, but energy depends only upon the square of amplitudes. The spectral curve giving the squares of the amplitudes does not describe the whole phenomenon but gives an account of the essential, i.e., the energetic characteristics. The variogram (or more precisely its Fourier's transformed curve) plays exactly the part of such a spectral curve.

In the following paragraphs, a few of the possible applications of variograms will be run over. It is obviously out of the question to give here a systematic study. I will merely mention some examples and several characteristic formulae. For more details I kindly ask the reader to refer to my "Treatise of Applied Geostatistics."[1]

[1] Editions *Technip*, Paris. Tome I (1962)—Tome II (Le Krigeage) (in press). Tome III (l'effet de pépite et les phénomènes de transition) to be published.

ABSOLUTE DISPERSION (OR INTRINSIC) LAW

The semi-variogram defined in (1) is bound to the geometrical field V of the regionalized variable. If, instead of the total field V, only a portion V' of it would have been considered, a function $\gamma'(h)$ possibly different from $\gamma(h)$ would have been obtained. However, we have the intuitive notion that in a geologically homogeneous geometrical field there might be something intrinsic, independent from location, in the characteristics representing the variabilities of regionalized variable.

Formulated in an accurate way, that intuition leads to the hypothesis of an absolute dispersion or intrinsic law expressed through the equation:

$$\gamma'(h) = \gamma(h)$$

which means that the variogram is independent from the portion V' of the deposit V selected for its calculation. It may be said at once, that this hypothesis is not really essential to the development of the theory and it is possible to eliminate it because of some mathematical complications.[2]

Nevertheless it makes the statement of the theory much easier, and for that reason it will be followed here. A slow deviation of the variogram in space is generally ascertained through experience and if this drift does not take too much importance, the results yielded by the hypothesis of an absolute dispersion law provide an excellent approximation of reality (on the condition that $\gamma(h)$ actually employed has been calculated from the actual portions of the considered deposit).

When this hypothesis is verified, the semivariogram $\gamma(h)$ itself acquires an intrinsic significance. It is often designated under the name of intrinsic (or absolute) *dispersion law* or, more shortly, intrinsic function of regionalized variable.

VARIANCES AND COVARIANCES

Let us consider in the first place, a regionalized variable (which will be called grade in order to simplify) defined in a field V, on a punctual support and submitted to an intrinsic dispersion law $\gamma(h)$. Let $f(M)$ be the value taken by the grade in a point M of the field V. Instead of the punctual grade $f(M)$ we usually are concerned with the grade $y(M)$ of a sample v, of a given size, shape and orientation, picked at point (M).[3] This new variable is deducted from the previous one through an integration performed within the volume v centered in M.

$$y(M) = \frac{1}{v}\int_v f(M+h)dv. \tag{2}$$

To this variable will be bound a parameter measuring its dispersion inside V, called variance, as in classical probability calculus. The mean

[2] Loc. cit., Tome III.

[3] This means that the center of gravity of v is located at point M.

value of the punctual variable inside V being m,

$$m = \frac{1}{V}\int_V f(M)dV$$

the variance of $y(M)$ inside V is defined as the average value within V of the square of the expression $[y(M) - m]$, let:

$$\sigma^2 = \frac{1}{V}\int_V [y(M) - m]^2 dv. \tag{3}$$

It will be noted that this notion has, at the outset, a geometrical and not a probabilistic meaning. It will not deter us from calculating these variances, in the applications, from experimental data with common statistical methods. Should they be taken according to their spatial order, as in integral (3), or previously rearranged in histograms, the same expressions $(y - m)$ are appearing, with the same weights in both the calculation procedures. But, on a conceptual ground, definition (3) has a physical content that the statistical notion has not. From expression (1) of the variogram, of (2), and the definition (3) of the variance, one may deduce, reversing the order of the integrations.

$$\sigma^2 = \frac{1}{V^2}\int_V dV\int_V \gamma(h)dV' - \frac{1}{v^2}\int_v dv\int_v \gamma(h)dv'. \tag{4}$$

Each one of these sextuple integrals has a very clear meaning: it represents the average value of the $\gamma(h)$ inside V (or v) when both the extremities of vector h sweep, each one for its own account, the volume V (or v).

If we write:

$$F(V) = \frac{1}{V^2}\int_V dV\int_V \gamma(h)dV,$$

i.e., $F(V)$ = average value of $\gamma(h)$ inside V.

One gets:

$$\sigma^2 = F(V) - F(v). \tag{5}$$

Thus knowledge of the variogram of punctual grades allows the "a priori" calculation of the variance of any sample v within any portion V of a deposit. It will be noted that this variance does not depend only upon the sizes of volumes v and V, but also upon their shapes and orientation.

Physical meaning of relation (4) is highly instructive. The variance of a macroscopic sample v, considered as the juxtaposition of a great number of microsamples dv, does not depend in any way on the number of those micro-samples nor on their variances, but only on the average value of intrinsic function $\gamma(h)$ inside the geometrical volume v. Classical statistics, considering these micro-samples as independent, should lead to a variance in terms of $1/v$. There does not actually exist any deposit in which 10 ton

is would have a variance a thousand times lower than tha 10 kg

cores. Formula (4) shows why. The grades of micro-samples are not independent at all. They are inserted into a spatial correlation lattice, the nature of which is bound to the more or less steady continuity of mineralization, and which is expressed precisely through the intrinsic dispersion law $\gamma(h)$. The grades of the micro-samples are much less different, on the average, than classical statistics would indicate, and in consequence 10 ton blasts have a much higher variance than the thousandth of the variance of 10 kg cores.

The expression of the variance in form (5) shows a law of additivity. If we consider panel V' and samples v within a field V and if $\sigma^2(V', V)$ $\sigma^2(v, V)$ and $\sigma^2(v, V')$ designate the variances of V' inside V, of v inside V and v inside V', we get:

$$\sigma^2(v, V) = \sigma^2(v, V') + \sigma^2(V', V).$$

This formula is known as *Krige's Formula*. It has been established by D. G. Krige in the case when the grades are distributed according to a (statistical) lognormal law. Its validity is actually not linked to a special statistical distribution law, but only to the existence of an intrinsic dispersion law.

Besides the variance, geostatistics introduce the notion of *covariance*. If $y(M)$ and $z(M + h)$ are the grades of two samples v and v' centered in two points M and $M + h$, covariance (inside V) of y and z is the function of h defined by:

$$\sigma_{vv'} = \frac{1}{V^2}\int_V [y(M) - m][z(M + h) - m]dv.$$

It can be expressed through the variogram with a relation similar to (4):

$$\sigma_{vv'} = F(V) - \frac{1}{vv'}\int_v dv \int_{v'} \gamma(k)dv'. \tag{6}$$

The second integral represents the average value of $\gamma(k)$, when both extremities of vector k sweep, respectively, volume v and volume v', at a distance h one from the other.

Let us consider, as a particular case, the isotropic de Wijs's[1] scheme. It is defined by an intrinsic isotropic function of the form:

$$\gamma(r) = 3a \ln r \tag{7}$$

in which $r = |h|$ represents the modulus of the vectorial argument h, or otherwise the distance between the two points M and $M + h$. When symbol ln represents the natural logarithm, parameter a is called *absolute dispersion*. It characterizes indeed the dispersion of *grades independently* from the shape and the volume of the samples and of the deposit. In the

[1] The starting point of development of the present theory is the original De Wijs's reasoning which is a remarkable example of transition from classical statistics to geostatistics. Reference to "Traite de Geostatistique Appliquee," where bibliographical references will be found.

particular case where the volume of the samples is geometrically similar to the volume V of the deposit, formulae (4) and (7) give:

$$\sigma^2 = a \ln \frac{V}{v}. \tag{8}$$

This formula, which is the *Wijs's formula*, does express a principle of similitude. It ceases to be applicable generally as soon as the deposit is not geometrically similar to the samples. It is however possible to associate to any geometrical volume v its *linear equivalent* d devined by relation:

$$\ln d - \frac{3}{2} = \frac{1}{v^2}\int_v dv \int_v \ln r dv'. \tag{9}$$

Formula (4) entails that sample v has the same variance in any deposit as the linear sample of length d. If D and d are the linear equivalents of the deposit and of the samples respectively, the variance may be set into the form:

$$\sigma^2 = 3a \ln \frac{D}{d}.$$

The linear equivalents have been calculated and tabulated for a certain amount of geometrical figures, and, in addition, we have at our disposal some simple approximation formulae. For example for a rectangle with sides a and b we have:

$$d = a + b.$$

For a parallelogram, with sides a, b, and surface S:

$$d = \sqrt{a^2 + b^2 + 2S}.$$

For a triangle with sides a, b, c, and Surface S:

$$d = \sqrt{\frac{a^2 + b^2 + c^2}{3} + 2S}.$$

For a trapezium with basis $a = \frac{L + l}{2}$,

$$b = \frac{L - l}{2}$$

Median: m
Surface: S

$$d = \sqrt{L^2 + l^2 + m^2 - \frac{l^2 m^2}{3L^2} + 2S}$$

For a rectangular parallelepiped with sides $a > b > c$,

$$d = a + b + \frac{c}{2}.$$

For an oblique parallelepiped with edges r_1, r_2, r_3, faces S_1, S_2, S_3 and volume V, we put up:

$$\begin{cases} R^2 = r_1^2 + r_2^2 + r_3^2 \\ S^2 = S_1^2 + S_2^2 + S_3^2, \end{cases}$$

and we obtain the following approximate equivalent:

$$d = \sqrt{R^2 + 2S + \frac{V^2 R^2}{S^2}}.$$

This notion of linear equivalent allows an easy comparison between samplings of different natures, at least in the case, common in metalliferous deposits, where the law of dispersion has the form (7).

ESTIMATION VARIANCE AND EXTENSION VARIANCE

One of the most practical problems geostatistics are supposed to resolve is the size of the possible error in the evaluation of a deposit. The general characteristics of regionalized variables indicate that this error does not only depend upon the amount of picked samples, but first of all upon their shapes, their sizes and their respective locations, in other words, on the whole, upon *the geometry of achieved mining workings*. These indications get a precise meaning through the geostatistical notion of the estimation variance. Let us suppose that, in order to estimate the real unknown grade z of a deposit or of a panel V, we know the grade x of a given net-work of mining workings *Mw*. The estimation error $(z - x)$ has a simple, well determined value, although unknown, for a given panel V as for the net-work *Mw* located preferentially. In order to make out of this error a regionalized variable, geostatistics consider the panel or the deposit to be estimated as a panel extracted from a very large fictive deposit K. This deposit is supposed to be ruled by the intrinsic dispersion law $\gamma(h)$ defined by the experimental variogram controlled in mining works *Mw*. We shall see that the shape and the sizes assigned to K do not actually intervene. Let us imagine that panel V which is being estimated travels across the large deposit K, drawing with its attached mining works, the error $(z - x)$ then appears as a regionalized variable with an average value equal to zero and a variance:

$$\sigma^2 = \sigma_z^2 + \sigma_x^2 - 2\sigma_{zx}. \tag{10}$$

This variance called *estimation variance* is calculated after variances σ_z^2, σ_x^2 and covariance σ_{zx} of the variables z and x inside the field K, which are themselves given by formulae of type (4) or (6). Field K interferes in the

expression of σ_z^2, σ_x^2 and σ_{zx} by the simple constant $F(K)$ which is eliminated in equation (10), so that the estimation variance σ^2 is independent from the choice of K, and is calculated after the formula:

$$\sigma^2 = \frac{2}{VV'} \int_V dV \int_{V'} \gamma(h) dV' - \frac{1}{V^2} \int_V dV \int_V \gamma(h) dV' - \frac{1}{V'^2} \int_{V'} dv \int_{V'} \gamma(h) dv'. \tag{11}$$

In (11) V' is the volume of the deposit being estimated and V' that of mining works *Mw*. The estimation variance σ^2 is calculated after integration of the intrinsic function $\gamma(h)$ inside the geometrical volumes of the deposit and of the samples. In the same way, as the variogram could give to the concept of the influence zone of a sample a precise content, one may say that the estimation variance (11) can give a precise meaning to the "influence" of mining works over the whole deposit.

In practical calculations, formula (11) should be difficult to use. Mining works usually frame a discontinuous net-work in which the samples themselves may be picked discontinuously (for instance, groove samples cut off according to a regular grid pattern in the drifts on a vein developed at different levels). Volume V' interfering in (11) is the discontinuous volume set up by the lattice of samples actually cut off and analyzed. An influence zone is traditionally assigned to each individual sample, in the center of which it is located and supposed to represent the grade. The error usually performed in extending the grade of such an individual sample to its influence zone can be represented by a type (11) variance, where V is the volume of influence zone and V' that of the sample.

Such a variance is called *elementary extension variance* and can be calculated for a given $\gamma(h)$ in terms of geometrical parameters of the sample and its influence-zone. On condition of certain approximation hypothesis, it is possible to prove that an estimation variance of type (11) can be calculated by composing the elementary extension variances.

In practice, two cases are to be distinguished essentially. The elementary samples network, for an isotropic function $\gamma(h)$ may be isotropic[1] or not. Let us mention, as an easy example of isotropic network, the square grid pattern drilling. The errors made for an isotropic network by extending to each influence zone the grade of its central sample may be considered as independent (in other words having a geostatistical covariance equal to zero). In this case *estimation variance is obtained by dividing the extension variance σ_E^2 of each sample within its influence zone by the number N of these influence zones.*

$$\sigma^2 = \frac{1}{n} \sigma_E^2. \tag{12}$$

[1] More generally, for any given function $\gamma(h)$, the lattice may or not be adjusted to the anisotropy of function $\gamma(h)$. For questions concerning the different types of anisotropy, one should refer to the *Treatise of Applied Geostatistics*.

If, on the contrary the network is not isotropic, we are led to rearrange the samples along lines or planes of maximum density, and to compose extension variances of different natures. For example let us suppose a vein-type deposit developed by drifts and channel sampled. In the first place, we have to consider the extension variance $\sigma_{E_1}^2$ of a channel within the length of a drift from which it has been cut off. If N is the total number of channels, one can see that the estimation variance $(1/N)\sigma_{E_1}^2$ represents the error obtained by extending the grade deduced from channel samples over the mining works themselves. We consider afterwards the extension variance $\sigma_{E_2}^2$ of the grade (supposed to be perfectly well known) of a drift inside its influence zone. The influence zone is here the panel composed by joining both the half-levels located above and below the drift. If n is the number of developed levels, the estimation variance $(1/n)\sigma_{E_2}^2$ represents the error obtained by extending the average grade supposed to be perfectly well known of the mining works to the whole deposit. The resulting estimation variance becomes:

$$\sigma^2 = \frac{1}{N}\sigma_{E_1}^2 + \frac{1}{n}\sigma_{E_2}^2. \tag{13}$$

It is usually necessary to add an additional variance to this expression, representing the sampling and analyses errors. The second term in such an expression is usually broadly predominating. The greater part of the error proceeds from the extension of data from the mining works to the deposit. In particular, it would be no use to increase indefinitely the number N of samples without carrying out supplementary mining works. In fact, the estimation variance coincides very soon with the $(1/n)\sigma_{E_2}^2$ limit below which it cannot decrease.

Tables and graphs giving the numerical values of elementary extension variances have been established [8] for a given number of intrinsic functions (especially for type (7) of de Wijs's function). They allow a fast computation of estimation variances assigned to different drilling and underground exploration schemes.

We offer for example a vein deposit conformable to a type (7) isotropic de Wijs's scheme and developed by drifts. Let us also assume that drifts have been sufficiently well sampled as to reduce the first term of equation (13) to zero.

Let h be the raise between two consecutive levels (measured inside the plane of the vein). The extension variance of a drift of length l within an influence panel lh is proved to be:

$$\sigma_E^2 = a\frac{\pi}{2}\frac{h}{l}.$$

This formula is valid only if h is small compared to l, but it may be used until $h = l$. When $h > l$, it must be replaced by a different formula. Let us assume that lengths $l_1, l_2 \cdots l_n$ are all superior to h. The estimation variance is obtained by weighting the extension variance of each drift within its influence panel by the square of the surface of this panel:

$$\sigma^2 = \frac{l_1^2\sigma_{E_1}^2 + l_2^2\sigma_{E_2}^2 + \cdots}{(l_1 + l_2 + \cdots)^2} = a\frac{\pi}{2}h\frac{l_1 + l_2 + \cdots}{(l_1 + l_2 + \cdots)^2}.$$

The explored mineralized surface being $S = h(l_1 + l_2 \cdots + l_n)$ and the total developed length being $L = l_1 + l_2 + \cdots l_n$, we obtain the following remarkable formula:

$$\sigma^2 = a\frac{\pi}{2}\frac{S}{L^2}.$$

Once the estimation variance has been calculated, one has still to interpret it for practical uses under the form of *conventional error spread*. This aim is reached by allocating to this variance a probabilistic meaning. By implicit reference to a gaussian model, we shall take it that the actual average grade of the deposit is included within a 95% probability in the range $m \pm 2\sigma$, m being the estimated grade. In other cases, particularly if 2σ is not small towards m, we shall take the spread $m \exp(\pm 2\sigma/m)$, by reference to a lognormal model.

These implicit references to probabilistic models are mainly arbitrary. Actually, the notion itself of statistical distribution of an estimation error is doubtlessly meaningless. The only thing which has an objective physical meaning is the variance. This is why we speak about conventional spreads. Their practical interest resides in the fact that they draw a more intuitive picture of the possible errors than variances themselves.

KRIGING

A second application of major importance is provided by a geostatistical procedure called "kriging." It consists in estimating the grade of a panel by computing the weighted average of available samples, some being located inside others outside the panel. The grads of these samples being x_1, x_2, $\cdots x_n$, we attempt to evaluate the unknown grade z of the panel with a linear estimator z^* of the form:

$$z^* = \sum a_i x_i.$$

The suitable weights a_i assigned to each sample are determined by two conditions. The first one expresses that z^* and z must have the same average value within the whole large field V and is written as:

$$\sum a_i = 1.$$

The second condition expresses that the a_i have such values that estimation variance of z by z^*, in other words the kriging variance, should take the smallest possible value.

[8] *Treatise of Applied Geostatistics*, Vol. I, for the de Wijs's functions. Vol. III for the case of a nugget effect.

This is formulated with a linear equation system related to a_i, the coefficients of which are expressed with the help of the variances and covariances of the samples and of the panel. It is thus possible to tabulate, for each intrinsic function, the coefficients and the kriging variance in terms of geometrical parameters, appropriately for different configurations. Numerous drilling and underground work configurations have thus been tabulated in

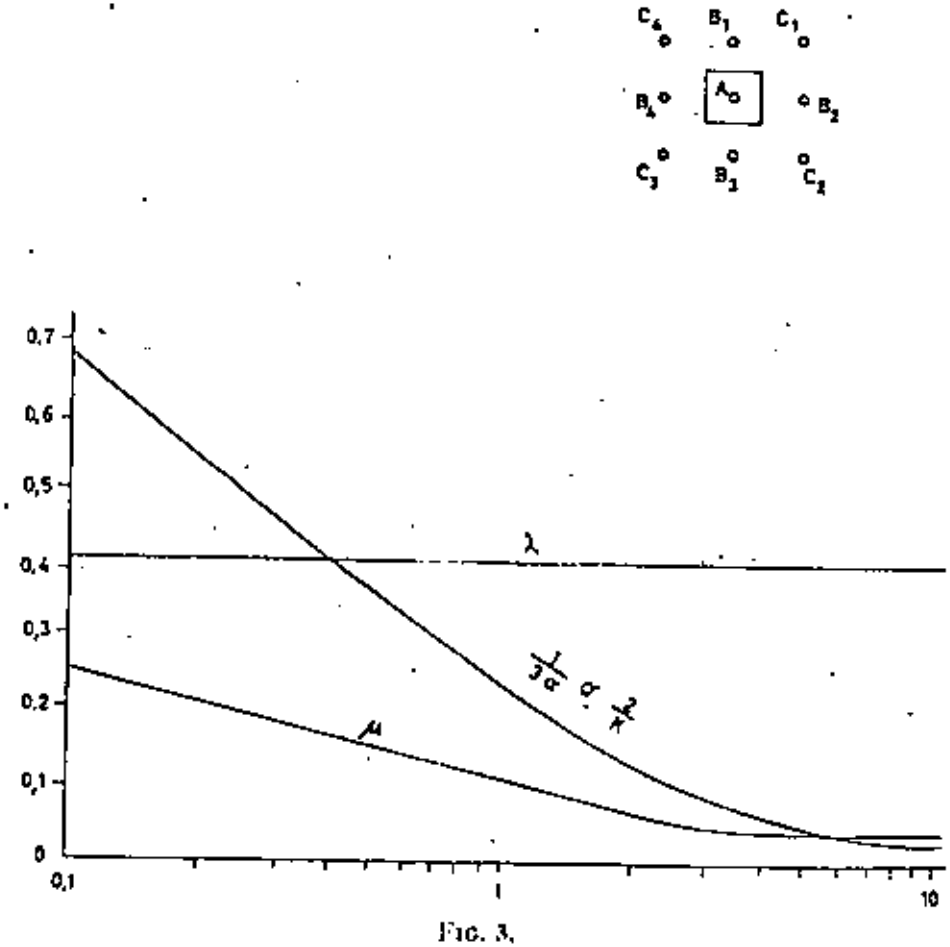

FIG. 3.

the case of an isotropic scheme of de Wijs. For information we show an example in Figure 3. The studied configuration is useful for the appraisal of a deposit explored by drilling, or for open cast selective mining. It consists, in the case of a square grid pattern drilling, in the kriging of the influence blocks of a drilling A with help of the grade of this central drilling A, and those of the 8 nearest drillings rearranged into two "aureoles" $B_1B_2B_3B_4$ and $C_1C_2C_3C_4$. Let a be the grade of A, v and w the average grades of drilli

B and C, the estimator to be used is:

$$a^* = (1 - \lambda - \mu)a + \lambda v + \mu w.$$

In Figure 3, is plotted in abscissa the ratio h/a between the width h of the formation and the size a of the mesh of the drilling grid and the numerical values of λ and μ are read on the curves as well as those of the expression $(1,3\alpha)\sigma_k^2$. The multiplication of this last expression by three times the value of absolute dispersion, 3α, yields the kriging variance.

Theoretically it is advantageous to "krige" each panel by all the samples located in the deposit, inside and outside this panel. In addition to the great complexity of computation which grows very fast, it appears in numerical examples that it is usually unnecessary to take into account remote samples. In general the one or two proximate aureolae of external samples are enough to remove practically the whole effect of remaining external samples. This is, in particular, the case of the configuration studied in Figure 3 where both aureole B and C form an almost perfect screen towards all other external drillings.

One can even notice that for high values of the h/a ratio, the weight μ of the second aureole becomes slight, so that the aureole made out of the four B drillings constitutes a screen by itself alone. This *screen effect* is a general phenomenon and plays an important part in the kriging theory.

From a practical point of view, the advantage of kriging is double. First of all, as a result of the definition itself of this procedure, it leads to achieve the best possible estimation for a given panel, that is to say the estimation with minimal variance. It can pay most appreciable services by improving, for example, the monthly output forecast for different mine-sections, and especially in the case where the mine operator is compelled to supply ores with characteristics as constant as possible.

However appreciable they are, the improvements of accuracy provided by the kriging would not always justify the amount of calculations it requires. In most cases, the major interest of the procedure does not come for the reduction of estimation variances but from its being able to eliminate the cause of systematical error. A deposit seldom happens indeed to be payable in the whole. Only some panels chosen as payable according to the grades of the samples cut off within them, are considered as payable. D. G. Krige [1] has proved that the results based only on inside samples inevitably led to over-estimating rich panels and underestimating poor ones. The geostatistical notion of kriging allows to expound this phenomenon easily and to rectify its effects. The selected panel being a rich one, the aureola of outside samples has, in general, a lower grade than that of inside samples. Yet its influence on the panel to be estimated, is not negligible, since it is allocated a weight different from zero by the kriging. Not to take in account this external aureole inevitably introduce therefore *a cause of systematical error by over-estimation* which can be eliminated by kriging.

[1] D. G. Krige's original reasoning constitutes a second example of an implicit passage from classical statistics to geostatistics. It is essentially based on the fact that the variance of a panel is always lower than that of its inside sampling. For references, see "... (se of Applied Statistics."

THE NUGGET EFFECT

In the presence of a strong nugget effect the general rules outlined in the above paragraphs may suffer some apparent objections. The nugget effect has been defined in Figure 1 by a variogram characterized by a discontinuity at the origin, and corresponding to a regionalized variable that does not have the "in average" continuity. Its nature may be purely granulometrical, as in gold or diamond deposits, or, more generally, it may reveal the existence of discontinuous micro-structures. The presence of veinlets or microfractures with high-grade fillings in a stockwerk may promote such an effect. In gold deposits, the grades of two very close or even adjoining samples may be different if, by chance, one of them contains a large nugget. The smaller the samples, the more important this effect is, and it may reach a considerable magnitude for samples of several liters in volume. A translation of some millimeters only of the geometrical support of a sample is enough for it to contain or not a large nugget able to modify its grade in a proportion of 1 to 10 or 1 to 100. The possibility for a marginal nugget to be embodied, or not, inside a sample appears as an entirely random event. Actually, however, the behavior of the grade can be considered as random locally only. If it were not so, the panels of several thousand tons, on which marginal nuggets have no more detectable effect, would present almost constant grades (their variance being then a million times lower than that of samples of several kilograms). It is well known that actually, even in gold deposits, there are rich panels and poor panels. But this random effect may locally be so strong that it entirely hides the underlaying regionalization. The frequency of some expressions such as "erratic," "monstrous," or "mammoth grades" etc. . . . alluding to an hypothetical anomalous behavior of mineralization in the literature devoted to these deposits is striking. Certainly the classical statisticians were right when they noted that there was no actual anomaly, and that those monster grades, actually existing in the deposit, appeared from time to time in the sampling, with frequency determined by random laws. Historically, a clear distinction between the notions of regionalized and aleatory variables was doubtlessly hampered for a long while by the fascination aroused by this nugget effect. It appears, from the geostatistical point of view, that in fact, the ingenious terminology was not wrong while suggesting the existence of some anomaly; but the aberrant fact is not the presence of some "anomalously" high grades, but rather the locally aleatory behavior of all the grades, high or low, as well as in the deterioration of the spatial correlations grid.* Those mammoth grades of the ingenious terminology are not aberraht by themselves, but the fact that they are not assorted with influence zones is so. And, on the other hand, classical statisticians were right stressing the fact that the apparitions of these aberrant grades are ruled by random laws. But they failed to note that the phenomenon can be considered as aleatory locally only.

Without trying to make a systematical statement,* let us show briefly how geostatistics allow us to represent a nugget effect. Let us examine the

* See *Treatise of Applied Geostatistics*, Vol. III.

$\gamma(r)$ semi-variogram representing the third type of Figure 1. We shall stick here to the case where $\gamma(r)$ is an isotropic function (in other words depending only upon the r modulus of the h vectorial argument). The C discontinuity, or jump, noticed at the origin on the $\gamma(r)$ of a variable with punctual support, is called *nugget constant*. $H(r)$ being Heaviside's function, thus defined:

$$\begin{cases} H(r) = 1 & r > 0 \\ H(r) = 0 & r = 0 \end{cases}$$

the semi-variogram may be divided into 2 components:

$$\gamma(r) = CH(r) + \gamma_1(r). \tag{15}$$

The first component $CH(r)$ represents the pure nugget effect. The second one $\gamma_1(r)$, continuous at the origin, represents the underlaying regionalization. All the variances and the covariances that have to be introduced, may then be calculated as if the variable $x(M)$ with punctual support was the sum:

$$x = x_0 + \epsilon \tag{16}$$

of a theoretical regionalized variable x_0 following the $\gamma(r)$ dispersion law continuous at the origin, and of an aleatory ϵ variable with a zero average and C variance.

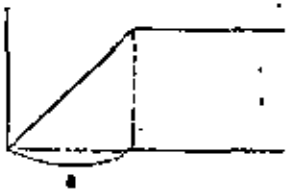

FIG. 4.

The x_0 and the ϵ are independent, and the ϵ assigned to two distinct points even very close, are independent as well. If we limit our study to the variation of the punctual grade x in the proximity of a given point, or, in other words, we consider only the small values of the distance r, $\gamma_1(r)$ will vary so slightly that it might be taken for a constant equal to C. The locally detectable variations are to be assigned almost solely to x_0. That is what we mean when we say that the regionalized variable behaves locally as an aleatory variable. But, on a larger scale, i.e., for higher values of r, the increase of the continuous component $\gamma_1(r)$ can no longer be neglected and the regionalization of x_0 becomes perceptibly apparent.

As a matter of fact, the Heaviside function does not represent with entire satisfaction the random aspect of the behavior of a punctual variable. Unless we suppose the constant C to be infinite, the term $CH(r)$ will lose all influence over the variance of a sample of a size different from zero. It is automatically eliminated in formula (4). It means that the mean value of the ϵ independent aleatory variables, located in infinite number inside an unpunctual support, has compulsorily a zero variance.

The notion of a random variable ε with a punctual support has actually no physical meaning. The actual physical phenomenon will never involve a true discontinuity at the origin but a narrow transition zone in the proximity of $r = 0$. $H(r)$ must be replaced by the transition function $T(r, a)$ defined by:

$$\begin{cases} T(r, a) = \dfrac{r}{a} & \text{if} \quad r \leq a \\ T(r, a) = 1 & \text{if} \quad r > a. \end{cases}$$

The a constant, or *range*, gives the scale of the transition zone, that is to say the size of the nuggets. In the case of homogranular nuggets of same volume v it is shown that:

$$v = \frac{\pi}{3} a^3.$$

The intrinsic function $\gamma(r)$ of a punctual grade is decomposed in the following way:

$$\gamma(r) = CT(r, a) + \gamma_1(r).$$

C is still the nugget constant, and $\gamma_1(r)$ the continuous component.

The punctual grade ε can be given by a sum similar to (16) in which ε is a regionalized variable admitting $CT(r, a)$ as its intrinsic function. Now the ε are only independent for distances superior to the range a. For smaller distances they are bound by a linear variogram. The nugget effect will therefore reflect itself on samples of size v different from zero. If v is large in regard to the grain size a^3 the transition zone will be diluted in the integration volume v, and the nugget effect will yield an additional variance of the type a^3/v. Indeed, let σ_ε^2 (nugget variance) be the share of $CT(r, a)$ for the variance of sample v. According to (4) we have to compute integrals of the type:

$$\frac{C}{v^2} \int\int\int_v dv_1 \int\int\int_v T(r, a) dv_2.$$

If all sizes of v are supposed to be large in regard to a, each point inside v brings to the sextuple integral the following part:

$$C\left(v - \tfrac{1}{2}\pi a^3\right) + \frac{C}{a} \int_0^a 4\pi r^3 dr = C\left(v - \frac{\pi}{3} a^3\right).$$

This is valid only for points located at a distance superior to (a) from the boundary of v; but, when v is large, the boundary points only interfere with superior order terms. With such an approximation, the sextuple integral is equal to $C(1 - (\pi/3)(a^3/v))$.

As the integral inside V is computed in the same way, we finally have

$$\sigma_\varepsilon^2 = C \frac{\pi}{3} \left[\frac{a^3}{v} - \frac{a^3}{V} \right]. \tag{17}$$

Practically a^3/V is negligible and the nugget variance is in terms of a^3/v, i.e., in an inverse ratio to the number of grains contained inside the sample.

Any time a nugget effect does exist, i.e., anytime a regionalized variable shows a locally aleatory behavior, an additional variance is assigned to macroscopic samples, called nugget variance, inversely proportional to their size.

The variance of those samples appears as the sum:

$$\sigma^2 = \sigma_\varepsilon^2 + \gamma,$$

of the nugget variance and of the theoretical variance σ_γ^2 calculated with the continuous component $\gamma_1(r)$ of the intrinsic function.

When v is increasing, the theoretical variance is decreasing much slower than the nugget variance. In the presence of a very strong nugget effect σ_ε^2 may happen to be widely predominating for samples of several kilograms. The underlying regionalization is almost completely hidden at the scale of these samples. If we limit the variation of the volume v in the interval of a few liters up to tens of liters, the experimentally observable variations of the variance will be those of the nugget variance effect only, and we may take the risk to conclude that the variance varies in inverse ratio of the volume.

Whereas if we consider samples of several tens of tons, the term σ_ε^2 decreases and disappears, and the theoretical variance σ_γ^2 becomes prominent. The effect of the underlying regionalization appears again and the variance is steadily decreasing as v is increasing, but much slower than $1/v$.

We have somewhat insisted upon the nugget effect in order to show, through a crucial example, how geostatistical concepts allow us to rediscover the local results that are fluently obtained from common statistical reasoning (nugget variance inversely proportional to volume) but inserting them in the general prospect of an underlying regionalization. As for the practical use of this theory, let us succinctly mention the two following points:

In the presence of a nugget effect, the extension and the estimation variances are both increased by a term $C(\pi/3)(a^3/v)$ inversely proportional to the total volume of available samples and, therefore, in particular to the number n of those samples. In this regard, the additional estimation variance due to the nugget effect behaves itself as the sampling and analysis variances, and may be rearranged with them.

As for the kriging, the nugget effect results in partly removing all the screens. Practically, we are led to use the special forms of kriging called "aleatory kriging" which are not different from those proposed formerly by D. G. Krige himself, in connection with the gold deposit of the Rand, in which the nugget effect is probably very strong.

SEARCH FOR OPTIMUM IN MINING EXPLORATION

Geostatistics are able, through estimation variances, to provide an accurate measurement of the information yielded, by a given amount of underground workings on a deposit. Generally, these workings are expensive, and their cost must be weighed against the economic value of the provided

information. Thus appears the possibility to determine the optimum amount of credits to be allocated for the exploration of a deposit, and particularly the possibility to choose the suitable moment for stopping the exploration, as well as for taking a positive or negative decision towards starting the exploitation of the deposit. These methods, permit one to solve, at least partly, one of the main problems raised by mining exploration, will be published in another connection, and cannot be treated here. Let us only, as a conclusion, stress the fact that they appear as the natural extension of geostatistics. The possibility of their adjustment was bound to the preliminary elucidation and to the thorough scientific study of the different ideas which have been summarized in this paper.

BUREAU DE RECHERCHES GEOLOGIQUE ET MINIERES,
PARIS, FRANCE.
June 10, 1963

un nombre déjà important de gisements de métaux différents (cuivre, fer, phosphate, bauxite, fluorine, nickel, plomb) ont déjà été evalués qu'ils soient stratiformes ou non.

CONCLUSION.

On constate actuellement que le nombre des études géostatistiques est en augmentation ; la raison est probablement que les évaluations géostatistiques sont maintenant reconnues comme les meilleures et que grâce à un bon soutien logistique elles ne coûtent pas plus cher que les évaluations classiques. Il faut bien reconnaître que la géostatistique est maintenant opérationnelle. Dans l'avenir, il semble que la géostatistique doive se développer en intégrant davantage les données géologiques et les données d'exploitation. De nombreux projets sont d'ailleurs à l'étude : détermination de l'influence de la taille des échantillons sur les résultats de l'évaluation, simulation d'une exploitation en utilisant les techniques de simulation conditionnelle.

Mario Jean Flores

GEOSTATISTICS IN PETROLEUM INDUSTRY

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ABSTRACT. Prospection and exploitation of hydrocarbon fields need a large quantity of data and sophisticated processings. Geostatistics and in particular "kriging ", bring an important contribution to working out a satisfactory subsurface model. In practice it is necessary to have a data processing tool for implementing the techniques of kriging and structural analysis, at the various phases of exploration and production (no matter what the variables are). Several complete studies using such a program are presented in this paper : estimation of seismic variables at the major phases of exploration, estimation by layers of the reserves of an oil field, estimation by grid blocks of the reserves of a gas field.

1. INTRODUCTION

Oil prospecting and rational field developing require a good knowledge of the subsurface. The prospected areas are getting more and more hazardous, drilling in such areas is becoming more and more expensive,.. So, it is necessary to get more and more numerical data, to process them with more and more sophisticated techniques in order to work out synthetic information. This evolution has been possible through Data Processing techniques.

In this connection, emphasis was laid on to the new estimation methods gathered in geostatistics, specially the kriging method. Two oil companies, 'Compagnie Française des Petroles', 'Société Nationale des Pétroles d'Aquitaine' have co-operated with 'Ecole des Mines de Paris', to design a package, KRIGEPACK, which can perform various operations such as structural analysis,

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estimation of several parameters at points or in average over polygonal domains, contour-mapping...

This paper aims to display several applications of geostatistics along the various phases of exploration and production ; its purpose is also to point out all the problems including those that are not quite solved by geostatistics.

2. THE VARIOUS PHASES OF PETROLEUM INDUSTRY

Petroleum industry is usually divided into two main phases : exploration and production (or prospection and development).

2.1. Exploration

The Exploration itself is divided into several steps of prospecting :
. the first step aims to locate the areas which seem to be the best within the prospected zone.
. the following steps have to bring more details in the areas of interest.
. the last step must allow the technician to estimate the commercial value of the discovered oil field, before deciding to develop the field.

At each step one seeks to design a morphological model of the subsurface based upon the few available drill holes, and mainly upon data from the geophysical prospection. Seismic techniques are the most common in geophysics. Seismic data can be roughly described as the measurements of the acoustic responses of the subsurface to a vibration induced for example by an explosion. Receivers put along a line measure the times during which the signal is transmitted from the ground to the geological horizons bearing high acoustic impedance contrasts.

Timing the horizons leads to isochron-contour maps which present a distorted image of some geological layers. Distortion is mainly induced by the fact that the paths of the seismic rays are not vertical. These images may nevertheless show the possible reservoir traps : anticlines, wedge, reef...

If velocities are available, the depth contouring map can be drawn. The velocities are calculated from seismic times related to various seismic courses provided that there is at least one well to adjust the depths.

2.2. Production

While developing an oil field one seeks to acquire more and more accurate knowledge of the reservoir parameters, not only the geometrical ones but also the physical ones. The static model of exploration becomes a dynamic model for Production. It is necessary to forecast the production volume related to the time and the life of the oil field.

Most of the new data is derived from the well measurements depth, measurements on cores, loggs (resistivity, sonic velocity, radioactivity ...). A set of variables : height, porosity, permeability, saturation ... is tied up with each well. Each variable can be spread over the whole field to estimate the total reserves or over some parts of the field in order to design a dynamic reservoir model (flow and pressure forecasts for each well).

In some cases it may be necessary to simulate the oil field boundary in order to get a probability distribution of the reserves.

3. ESTIMATION OF SEISMIC VARIABLES

The reported survey about exploration deals with marine seismic prospection. Seismic data are recorded on a boat that carries a transmitter and various receivers spread along a seismic cable some km long.

3.1. Prospection phase

The first seismic survey consisted of lines forming a grid of roughly square elements of 4 km x 4 km (fig. 1). The successive records of a line make up a 'section' on which one can usually follow several geological horizons (fig. 2). Time picking is carried out manually ; the results depend greatly upon the interpreter who must try to reduce the differences between the times at cross lines.

Let's look at the horizon H 1 close to a reservoir (porous layer) that may contain hydrocarbons. The structural analysis clearly exhibit great continuity of the variable (fig. 3). The variograms show a strong curvature in the vicinity of the origin.

To get the estimation from the kriging method we need a theoretical model for the variogram. We choose h^α with $\alpha \leq 1.4$. We do not take into account the anisotropy which, if real, seems to be very slight. The times are estimated at the nodes of a

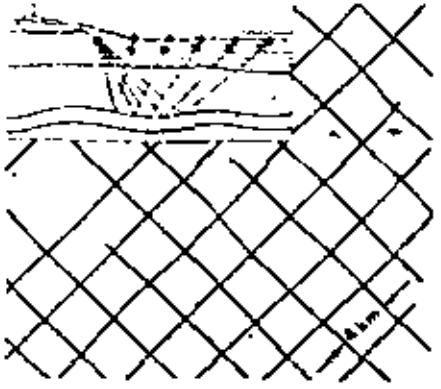

Fig.1. Seismic prospection: lines map

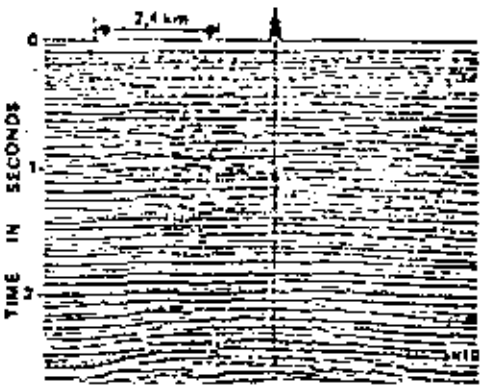

Fig.2. Interpreted seismic record section.

square meshed (1 km x 1 km) grid ; from the grid, a contour map is drawn (fig. 4). It can be noted that other methods using empirical weighing formulas lead in this case to very similar estimations due to the continuity of the variable. The main advantage of the kriging method, in this case, is to provide the variance of the estimate or the error of the estimation, particularly at the center of the grid elements (where the error is maximum). This isochron-contour map (fig. 4) is used to choose the places to drill ; these locations are supposed to be the tops of structure that should be traps.

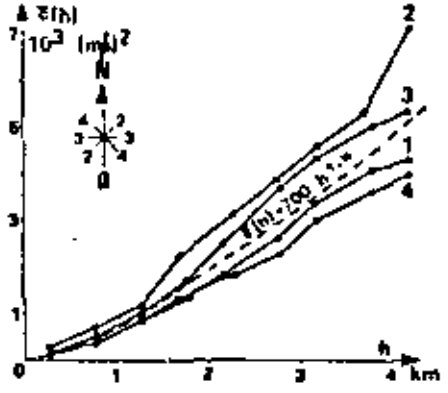

Fig.3. Time variogram of the horizon H 1

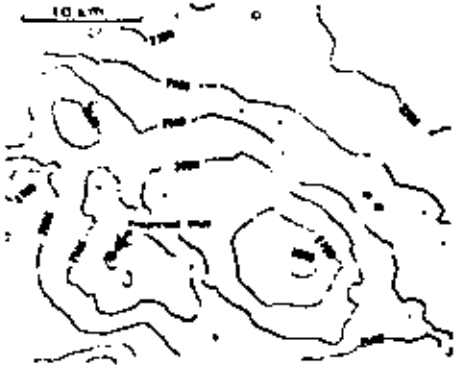

Fig.4. Time contour map of horizon H 1

3.2. Field evaluation phase

If a hole has been drilled through an oil impregnated layer one seeks to evaluate the discovery and to estimate the boundary of this oil field (fig. 5) :

. the lower boundary is a water level, the depth of which can be known from the well (or others in the neighbourhood),

. the upper boundary or the reservoir top is close to a seismic horizon H 2, the depth of which is estimated by means of the data supplied by a second seismic survey limited to the vicinity of the well.

The seismic data are processed in a more sophisticated way than previously :

. time picking for the horizon H 2 is more accurate and the interpretator tries to take the major faults into account,

. velocity analysis provides the apparent velocities (fig. 5).

PICKING : [illegible]

Structural analysis of the velocities presents great interest (fig. 6) :

. the variogram involves a nugget effet estimated at : $C_0 = 2800\ (m/s)^2$ which represents the variance of the measurement errors (i.e. an error range of about 100 m/s which gives an idea of the difference we can expect between two close velocity analyses),

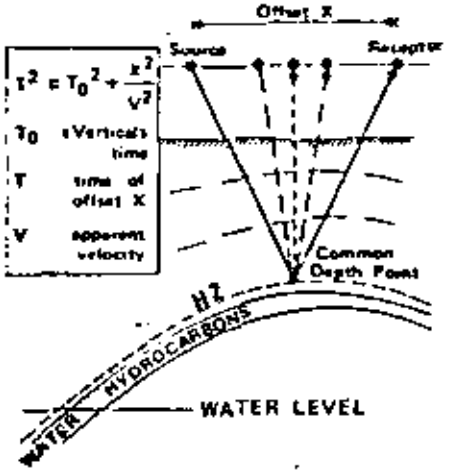

Fig.5. Reservoir cross-section and velocity analysis

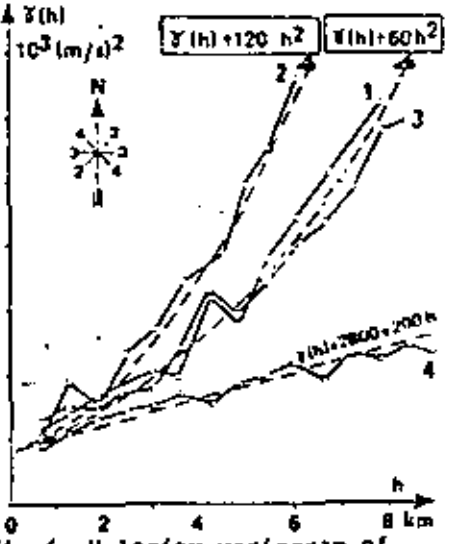

Fig.6. Velocity variogram of horizon H 2

he behaviour of the variogram for greater distance is differe-
t depending on the direction in which it was calculated.
s anisotropy can be interpretated as the effect of a linear
ft minimum in the North West direction ; this variogram,
, corresponds therefore to an estimation of the variogram of
residual variable. In short, the theoretical model that is
t is : $\gamma(h) = C_0 + ph$.

Estimation of the depths requires a multistep process
g. 7) :

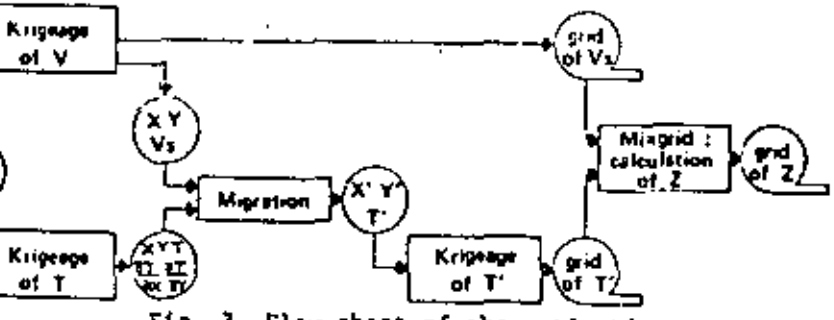

Fig. 7. Flow chart of the estimation

The velocities were estimated first at the data points
selves. The nugget effect results in a rather large smoo-
g out of the data. The display (fig. 8) of the differences
een estimated and measured velocities of the data points
rlines the importance of this smoothing. Let us emphasize
fact that the amount of smoothing is not left to the know-
of the interpreter any longer ; it is a function of the
gram which has been determined experimentally, and to some
t, of the number of data points which are taken in the
ing computation. From such a map, it is possible to detect
mal points, for instance, a lack of homogeneity between
or surveys. In this case, we have found that the first
ity determination on each line was more irregular than the
s, probably due to poor streamer alignment. These points
therefore eliminated. We have also found that a rather
discrepancy exists between the two surveys which supplied
ata, probably due to some uncertainty on the streamer
hs. A computed average of 60 m/s was therefore removed from
elocities of the second survey to make them consistant with
elocities of the first survey. Apart from those corrections,
n see on the map that some lines should have been raised or
ed as they obviously include a D.C. component, for some
wn reason ...

Next, velocities were estimated at regular grid points.
This enabled us to draw a velocity contour map (fig. 9). The
outcome, a fairly regular gradient, looks quite realistic and
has been cross-checked with the velocity surveys in the avail-
ble wells. It is advisable to restrict the estimation process to
the area where the variance of estimate is below a reasonable
limit. In this case the contours have been cut-off at the stan-
dard deviation contour $\sigma_E = 20$ m/s. As expected the standard
deviation grows very quickly as we go away from the data point..

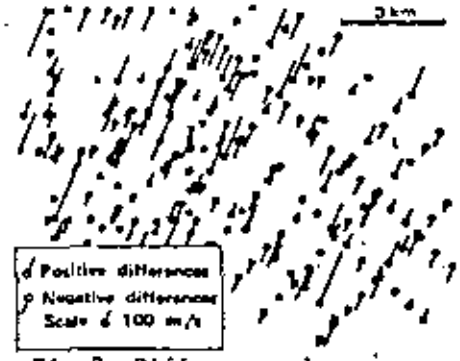

Fig.8. Differences between estimated and measured velocities

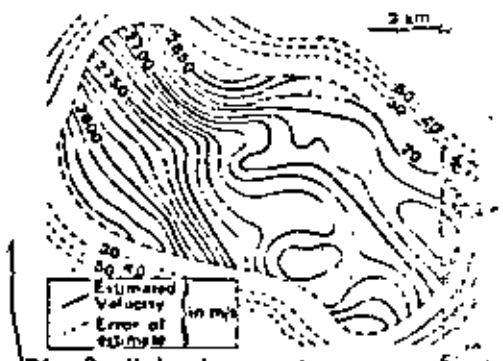

Fig.9. Velocity contour map of horizon H 2

The kriging process was also applied to the times at the
data points, not for smoothing purposes, since the variogram
does not show the nugget effect, but to compute the derivatives
of time with respect to X and Y as a bonus. Adding these time
gradients to the file of smoothed velocities makes it possible
to come up with the migrated data points and the corresponding
vertical times (fig. 10). We can see that the amount of shift
between unmigrated and migrated points should not be overlooked
even in this case of fairly gentle dip (fig. 11).

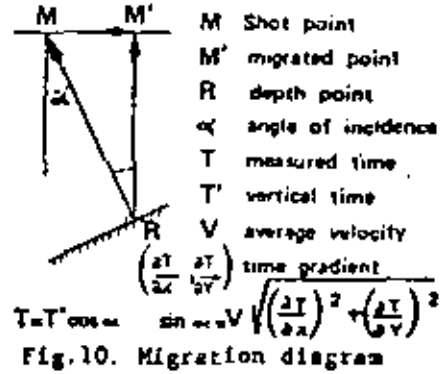

Fig.10. Migration diagram

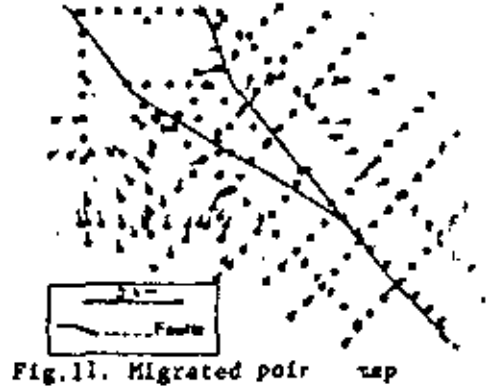

Fig.11. Migrated poir map

The file of migrated points was then used to grid the reflected times with the kriging technique and to draw a time contour map (fig. 12). The program took care of the two main faults which divide the field in 3 parts, and considered the data points located on each side of a fault independently.

Finally we could compute the depths by combining time and velocity grids by means of a specific program called MIXGRID. To complete the depth determinations, the following operations were carried out :
. correction of the times to fit the times recorded in the wells, since the horizon picked was not exactly located at the top of the reservoir,
. in order to reduce the seismic quadratic velocities to geological or true average velocities, the seismic velocities must be lowered by about 10 %,
. product of times and corrected velocities.

With enough control wells we can compute, by kriging, a correction, variable in every grid point, in order to reach the exact depths at all the wells.

The depth grid makes it possible to draw an isobath-contour map which shows as expected the same discontinuities as the isochron map (fig. 13).

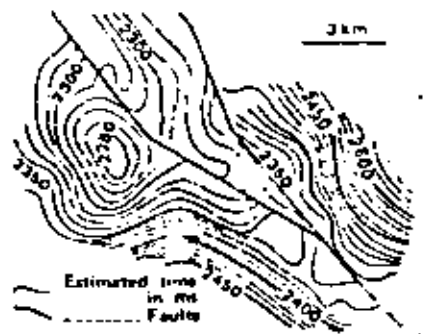

Fig.12. Time contour map of horizon H 2

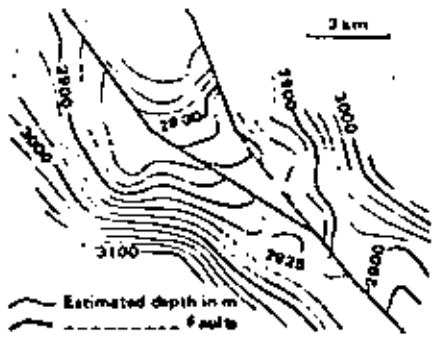

Fig.13. Depth contour map of reservoir top

4. FIELD RESERVES ESTIMATION

The field reserves estimation is undertaken as soon as some wells are available and must be worked out again each time a new well produces different information from that which was expected. The calculation method depends upon the field shape as shown in the two following examples :

4.1. Field estimation by layers

The field we are dealing with is an oil field, the shape of which is a well marked dome (fig. 14). The base of this field is make up of a water level. In this type of field the transition zone (where the water saturation goes from 100 %, at the water level, to a residual saturation of a few %) is particularly important ; therefore the oil porosity is considerably smaller in the lower layers.

In order to estimate the reserves (and to set up a model making it possible to simulate the rise of the water level as the wells are producing) it is necessary to divide the field into thin horizontal layers (20 m). Each of these 7 layers corresponds to :
. various numbers of samples (a flank well does not cut through the upper layers ; conversely a top well does not usually cut through the lower layers ...),
. various boundaries made up of 2 isodepths (difference 20 m) (the depths of the top have been estimated in a previous step by means of all the available data),
. various oil porosity variograms ; one can nevertheless choose the same type of theoretical variogram, the spherical one with a range of 800 m, assuming the true variograms differ only by a multiplicative factor.

The average porosity $\bar{\Phi}$ of every layer is estimated by 'spanned kriging'. The table of the results (fig. 15) emphasizes the decrease of porosity in the lower layers.

Fig.14. Field estimation by layers

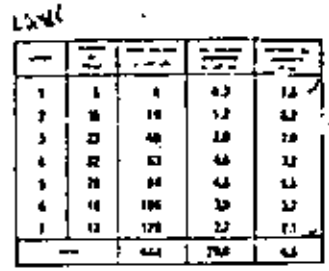

Fig.15. Kriging of the layers' average porosities

It is interesting to connect an estimation variance σ^2_R to the global oil reserves R worked out in this way. The calculation cannot be carried out with the conventional geostatistic methods alone : indeed porosity krigeage takes no account of the uncertainty about the reservoir geometry. In this connection we have used a simulation method to draw lots for :

water level depth between two fixed limits,
. reservoir top depth taking the exact depths known at the wells into account.

In pratice the field boundaries are supposed to be random polygons (fig. 16) making possible the calculation of the distribution of the oil field volume V, then the distribution of the reserves R and the corresponding variance σ_R^2. This survey shows that, in this case, it would not have been necessary to make a kriging estimation of the average porosity for each simulation, because the results are nearly independent of the geometrical boundaries.

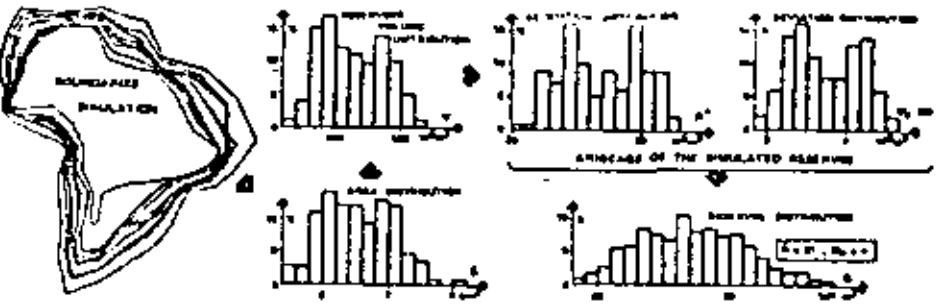

Fig.16. Simulation of the field boundaries : reserves variance calculation

4.2. Field estimation by grid blocks

This example is related to a gas field more complex than the oil field of the previous example : one deals with a dolomitic reservoir situated at the base of an ancient limy "cuesta" (cliff shaped outcrop) covered with discordant posterior deposits (fig. 17). The gas impregnated layer dips steeply into an aquifer (the dip is about 30°). The field therefore makes up a ribbon a few km wide which spreads over about 25 km.

The field has been divided into square blocks (250 m x 250m) making up a grid of 21 rows and 93 columns. The processes that were carried out (fig. 17) involve the following stages :

a. Estimation of the reservoir top depth at the center of the blocks by means of a 'generalized kriging ' method, using :
. the depth Z_i at the various wells (i = 1 to 25),
. the gradient $(\frac{\partial Z_j}{\partial x}, \frac{\partial Z_j}{\partial y})$ of the depth at the wells where a dipmeter logging has been recorded (j = 1 to 14),

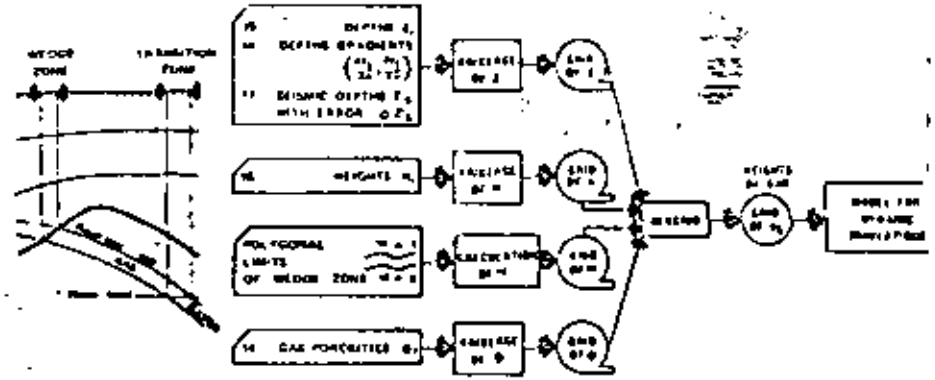

Fig. 17. Flow chart of the estimation by grid blocks of the field reserves.

. the depth Z'_k calculated from the seismic data at "fictitious wells" selected along seismic lines, in areas where information brought by the real wells is not sufficient, mainly outside the field (k = 1 to 17). The estimator which has been chosen is expressed by :

$$Z^* = \sum_{i=1}^{n} p_i Z_i + \sum_{j=1}^{m} \left(q_j \frac{\partial Z_j}{\partial x} + r_j \frac{\partial Z_j}{\partial y} \right) + \sum_{k=1}^{p} s_k Z'_k$$

The weights p_i , q_j , r_j , s_k are solutions of a linear system of equations involving :
. the values of the variogram $\gamma_Z(h)$ for vectors h between wells (real or fictitious),
. the values of the first and second derivatives of the variogram for the same vectors :

$$\frac{\partial \gamma_Z}{\partial x} , \frac{\partial \gamma_Z}{\partial y} , \frac{\partial^2 \gamma_Z}{\partial x^2} , \frac{\partial^2 \gamma_Z}{\partial x \partial y} , \frac{\partial^2 \gamma_Z}{\partial y^2} ,$$

the theoretical model chosen must therefore have a parabolic behaviour in the vicinity of the origin (fig. 18a) ;
. the variance $(\Delta Z'_k)^2$ of the measurement error at the fictitious wells (corresponding to the conventional range $Z'_k \pm 2\Delta Z'_k$) ;
. a quadratic trend.

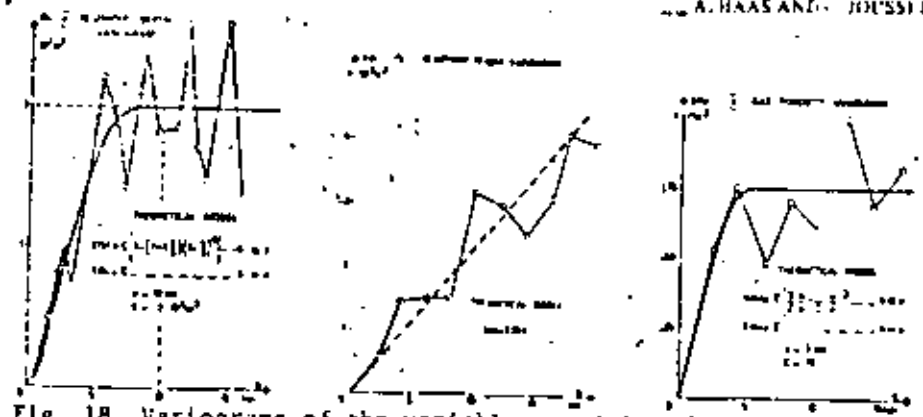

Fig. 18. Variograms of the variables used for the reserves calculation

The grids of estimated values enable the depth contouring (fig. 19). The boundaries of the reservoir may be displayed :
. North, the intersection with the water level (Z_w is about 5000m)
. South, the area where the reservoir vanishes gradually or 'wedge zone'.

Fig. 19. Depth contour map of the reservoir top

b. Estimation of the average height H of the reservoir in each elementary block. The available data are much less numerous than previously : only 15 holes have been drilled through the whole reservoir. (The wells situated in the partially eroded area cannot be used). Under these conditions the choice of a theoretical variogram from the experimental data appears to be a tricky question. The linear model (fig. 18 b) was used for calculating the average heights in the blocks. The corresponding grid is used to get the contouring map (fig. 20).

c. In the partially or completely eroded zone it can be assumed that computed values represent estimations of the average height before erosion. Therefore to take this into account it is necessary to introduce a wedge factor which varies from o to 1 when going from the completely eroded reservoir to the complete reservoir. In practice we use a program that can calculate by simple interpolation an average wedge factor in each block; the

boundaries of the wedge area are introduced in the form of broken lines. It must be noticed that these lines are not accurately known: they are estimated from different sources, especially from qualitative geological information...

Fig. 20. Height contour map of the reservoir

d. Estimation of the average gas porosity in the elementary blocks : as previously, the choice of the variogram (fig. 18 c) and the krigeage must be carried out from a few data : only 14 wells reached the gas reservoir. The contour map of the porosity obtained from the average gridded values displays variations that must be rather far from reality (fig. 21).

e. Average heights : one has to mix the various grids (with the program MIXGRID) in order to calculate the average height h of the reservoir and the corresponding average height of the gaz for every block : $h_G = hO_G$ (fig. 22) ; the following logical operations are carried out :

. if $o = o$ ..... $h = o$ (eroded reservoir)
. if $o \neq o$ and :

$\begin{cases} Z \geq Z_w & \ldots\ldots\ldots\ldots h = o \text{ (water-bearing reservoir)} \\ Z < Z_w \leq Z + Ho & \ldots. h = Z_w - Z \text{ (water-and-gas-bearing reservoir} \\ Z + H\,o < Z_w & \ldots\ldots h = Ho \text{ (gas-bearing reservoir)} \end{cases}$

Fig. 21. Gas porosity contour map

f. Volume : One comes up with the reservoir volume V and the in-situ gas volume in addina together the previous values :
$V = 9$ billionsm^3 , $V_G = 270$ millions m^3
(therefore the average porosity Φ_G is about 3 %).

The standard gas volume is the product of V_G by an expansion factor F which equals about 300 :

$R = F\ V_G \approx 80$ billions standard m^3.

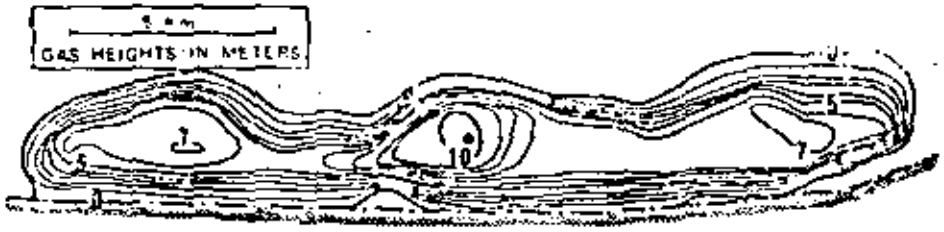

Fig. 22. Gas height contour map

g. The grids previously calculated are the basic data for a model, making it possible to study the behaviour of the field during the production phase. One is particularly interested in the rise of the water level with respect to the time (fig. 23) in order to forecast and possibly prevent the drowning of the wells.

Fig. 23. Forecasts of water level rise during production

5. CONCLUSIONS

The best estimation is not worth more than the data it comes from. And in this connection it must be noticed that the kriging method is a good stimulation to criticize the data.

Besides other advantages already mentioned, the kriging method constitutes a very flexible tool to synthesize many of the data available in the petroleum industry and this feature should be emphasized by the new interactive processing systems.

But, of course, we expect still more from geostatistics, for example, in the field of the co-kriging or more simply about the replacement of the estimation variance with a probability. Some answers to our questions should probably be given very soon.

ACKNOWLEDGMENTS

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Universal Kriging For Ore-Reserve Estimation — Conceptual Background and Application To the Navan Deposit

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Abstract

This paper explains the theory of universal kriging and the concept of regionalized variables and their use in the estimation of ore reserves. To illustrate the application of these methods, the authors have used available drill-hole results from the Navan lead-zinc deposit of Tara Exploration and Development Company Limited in Ireland.

Introduction

KRIGING is a term which by now should be familiar to all those of the mining profession engaged in ore-reserve calculations. Numerous papers have presented the theory, its application and programs to perform it (Matheron, 1967; David, 1970). However, as for every statistical model, this model, developed by Matheron (Matheron, 1963) almost 20 years ago, is based on a series of assumptions. The best known is that of stationarity. To those who discuss theoretical models only, this was a serious drawback, because many natural phenomena on which we would like to perform estimations are known to be non-stationary. This objection was answered by Matheron in 1969 (Matheron, 1969), when he developed his model of universal kriging for precisely the non-stationary case. Since then, several papers (Huijbregts and Matheron, 1970; Olea, 1974) have been published on the subject; we feel, however, that a clear account of the theory and practical applications is still needed. For somebody involved in block calculations, and most of the time worried about a poor model, the different practical implications of hypotheses such as stationarity, intrinsicity or non-stationarity may be in question. What are the steps involved in one method or the other, and when should one use them?

This paper has been written with these questions in mind. It will present applications in one real case to try and show the real impact of the different theories on estimated values. We will present the problems, then consider the theoretical aspects of the question and finally show the practice of kriging.

Presentation of the Case Study — The Earlier Published Results of the Navan Deposit

To illustrate the methods presented in this paper, the authors have used part of the published drill-hole results from the Navan zinc-lead deposit, County Meath, Ireland, available in *World Mining* and *The Northern Miner* to October, 1971.

The company which made the Navan discovery, Tara Exploration and Development Company Limited, and its subsidiary at Navan, Tara Mines Limited, wish it to be made clear that the calculations set out in this paper were not made as part of any calculations of ore reserves by or on behalf of Tara or in connection with the development of the Navan orebody. The development of the method of calculation and the application of those calculations presented here has been quite independent of Tara staff and consultants, who have had no occasion to comment on them.

Tara wishes it to be pointed out that the company's calculations on the tonnage and grade of its ores have had to take into account many additional factors not covered in the authors' study. Such additional factors include subsequent drilling and assaying, separate assay results for each 5 feet of core, separate assays for zinc and lead, variations of specific gravity, and observations on the detailed stratigraphy and geological structure of the enclosing rocks.

The authors view the well-known high-grade zinc-lead orebody at Navan as subhorizontal, noting that the mineralization spreads "with continuity and quite rapidly varying thickness" through dolomitic and

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sandy Carboniferous limestones. From published data (Tara in references), 147 nearly vertical drillholes have been selected by the authors on a regular grid with holes at generally 100-ft intervals (Fig. 1). For each hole, the thickness of the mineralized zone is given and assays provide the average combined Zn-Pb grade along the hole. From these values, a combined Zn-Pb accumulation is computed for each hole. Accumulation is the product of thickness and average grade and is expressed in %* ft. It thus represents a quantity of metal. In the following, interest will be focused on the spatial correlation structure of that variable which is distributed over a two-dimensional field. The target of the present study will be to perform a local estimation, by kriging, of the average accumulation in square blocks. This estimation will be restricted to the south and richer part of the deposit.

Theoretical Considerations

In this section, we will redefine a regionalized variable and the different hypotheses which can be made about them.

Regionalized variable: This is a function which can be defined at any point of a given field. In our case, for a tabular type of deposit, the grade of a mineralized intersection and its thickness, or their product, the accumulation, are two-dimensional regionalized varinbles.

The basic idea of the theory is to consider a function such as $z(x)$, where x is a point or a vector of R^n, as one realization of a random function (RF) $Z(x)$. Thus, we are turning a perfectly well defined unique numerical value into a realization of a random process. We have only one realization of that random function, and the problem we are faced with consists of finding the characteristics of RF $Z(x)$. In order to do this, we have to introduce several hypotheses about the stationarity of $Z(x)$ or its increments.

THREE POSSIBLE HYPOTHESES

Going from the most restrictive to the more general, three sets of possible assumptions are mentioned.

The Weak Stationarity Assumption

This assumption, seldom found in natural phenomena, consists of two conditions:

— the expected value of the regionalized variable is the same all over the field of interest;
— the spatial correlation of the regionalized variable is the same all over the field of interest.

In other words:

$$E[Z(x)] = m(x) = m$$

$$COV[Z(x), Z(x+h)] = K(x, x+h) = K(h)$$

where x and h are vectors or points in R^n; h will simply be the length of vector h.

In this case, the random function has a finite variance:

$$VAR[Z(x)] = K(o)$$

The Intrinsic Assumption

In many deposits, as shown by Krige (1951), such a thing as a finite variance does not exist. The intrinsic assumption considers that the increments $Z(x) - Z(x+h)$ are stationary, rather than the function $Z(x)$ itself. Thus:

$$E[Z(x+h) - Z(x)] = m(h)$$

$$VAR[Z(x+h) - Z(x)] = 2\gamma(h)$$

The expectation $m(h)$ is the linear drift and the function $\gamma(h)$ is called the semi-variogram of the RF.

With such an hypothesis, the increments of the RF have a covariance which is no longer stationary (Matheron, 1971):

$$COV[Z(x_1) - Z(x_2), Z(y_1) - Z(y_2)] =$$
$$\gamma(x_1 - x_2) + \gamma(y_1 - y_2) - \gamma(x_1 - x_2 + y_2 - y_1)$$

If a RF is second-order stationary, it is also intrinsic with a semi-variogram defined by:

$$\gamma(h) = K(o) - K(h)$$

The Hypothesis of Universal Kriging

A third kind of hypothesis, less restrictive than the previous ones, assumes that the second moment of the RF or its increments has some properties of stationarity within a vicinity of restricted size and that the expectation, which is no longer stationary, varies in a regular manner in such a vicinity.

If x, y and $x+h$ are taken in the same vicinity:

$$E[Z(x)] = m(x) \text{ or } E[Z(x) - Z(y)] = m(x) - m(y)$$

with $m(x) = \sum_{l=0}^{k} a_l f_l(x)$, the $f(x)$ being $k+1$ independent functions and the a_l unknown coefficients.

$$COV[Z(x), Z(x+h)] = K(h) \text{ or}$$
$$VAR[Z(x+h) - Z(x)] = 2\gamma(h)$$

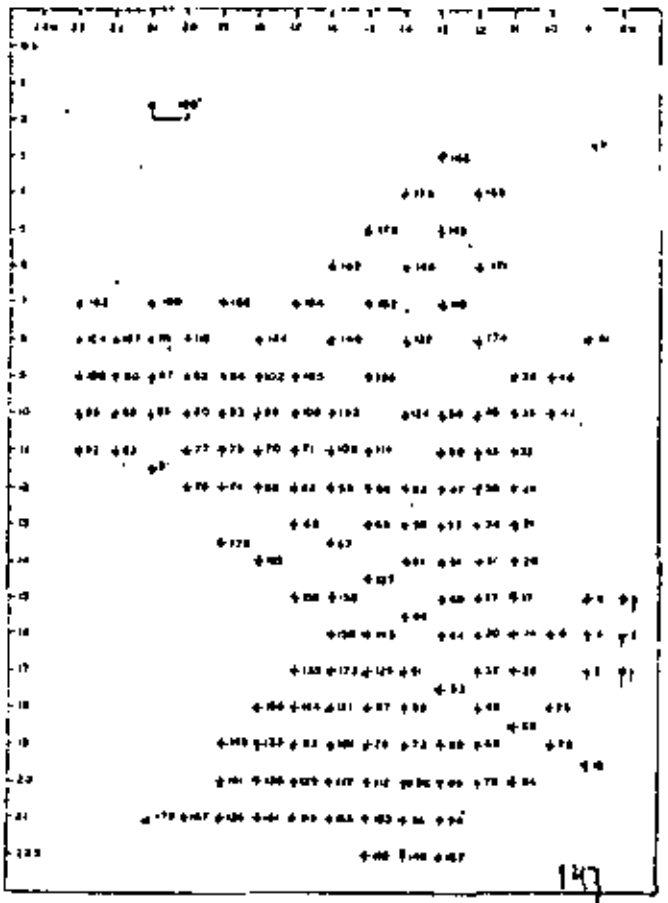

FIGURE 1 — Drill-hole map of the Navan deposit.

In this third kind of hypothesis, not only the covariance or variogram function has to be defined from the experimental values, but also the size of the vicinity where the hypotheses remain valid, the nature of functions $f(x)$, as well as their number, and the values of coefficients a_n, which are functions of the position of the vicinity in the field.

KRIGING FROM A MATHEMATICAL POINT OF VIEW

The purpose of kriging is to obtain the best linear estimator of the average value of $Z(x)$ in a domain v (block, panel, whole deposit or more simply a point) of size (volume, surface or length) v, from components of $Z(x)$ known on a discrete set of points $S = x_1, \ldots, x_n$ within or around v.

The random variable which is estimated is:

$$Z_v = 1/v \int_v Z(x)dx \text{ or } Z_v = Z(x_0)$$

if v is reduced to a point x_0, and the estimator has the form:

$$Z^* = \sum_{i=1}^{n} \lambda_i Z(x_i)$$

Two conditions are imposed on Z^* and thus on λ_i:

— Z^* must be an unbiased estimator — $E[Z_v - Z^*] = 0$;

— the variance of the difference, $Z_v - Z^*$, must exist and be minimal with respect to the λ_i's.

$$\text{VAR}[Z_v - Z^*] = \text{VAR}[Z_v] - 2 \sum_{i=1}^{n} \lambda_i \text{COV}[Z_v, Z(x_i)] + \sum_{i=1}^{n} \sum_{j=1}^{n} \lambda_i \lambda_j \text{COV}[Z(x_i), Z(x_j)]$$

Thus kriging amounts to finding the set of weights, λ_i, which minimizes a quadratic form under the constraint expressed by the unbias condition. According to Lagrange's principle, this is done by introducing unknown coefficients (μ_i) and solving a linear system of equations. The form of that system depends on the hypothesis made on the RF. Before seeing that, we should spend some time on the properties of linear combinations of random variables.

STATISTICAL PROPERTIES OF THE LINEAR COMBINATIONS OF RANDOM VARIABLES

Linear combinations of random variables are random variables (Feller, 1968). The combinations may be defined from components of a RF on a finite set of points S:

$$Z^* = \sum_{i=1}^{n} \lambda_i Z(x_i), \; x_i \in S = [x_1, \ldots, x_n]$$

or an infinite set of points contained in a domain v:

$$Z^* = 1/v \int_v Z(x)\,dx$$

The variance of Z^* can be expressed for each of the previous sets of hypotheses.

If $Z(x)$ is a second-order stationary RF, whatever the λ_i are, Z^* has a variance which is expressed from the covariance of $Z(x)$:

$$\text{VAR}[Z^*] = \sum_{i=1}^{n} \sum_{j=1}^{n} \lambda_i \lambda_j K(x_i - x_j)$$

or

$$\text{VAR}[Z^*] = 1/v^2 \int_v K(x - y)\,dx\,dy$$

If $Z(x)$ is an intrinsic RF, it has been shown (Matheron, 1971) that Z^* has a variance if and only if the condition $\sum_{i=1}^{n} \lambda_i = 0$ is realized. In that case:

$$\text{VAR}[Z^*] = -\sum_{i=1}^{n} \sum_{j=1}^{n} \lambda_i \lambda_j \gamma(x_i - x_j)$$

$Z^* = 1/v \int Z(x)\,dx$ is a linear combination where the weights are all equal to dx/v and, because $1/v \int_v dx = 1$, then Z^* does not have a finite variance.

CONSEQUENCES FOR KRIGING

When $Z(x)$ is a second-order stationary RF, the unbias condition again becomes:

$$m - \sum_{i=1}^{n} \lambda_i m = 0 \text{ or } \sum_{i=1}^{n} \lambda_i = 1$$

In the expression of the estimation variance to minimize:

$$\text{VAR}[Z_v] = 1/v^2 \int_v K(x - y)\,dx\,dy$$

$$\text{COV}[Z_v, Z(x_0)] = 1/v \int_v K(x_i - x)\,dx$$

One Lagrange coefficient, μ, is introduced which corresponds to the unbias condition, and the linear system, which gives the λ_i's, is written:

$$\sum_{i=1}^{n} \lambda_i K(x_i - x_j) = \text{COV}[Z_v, Z(x_i)] + \mu$$

$$\sum_{i=1}^{n} \lambda_i = 1$$

and the kriging variance is:

$$\text{VAR}[Z_v - Z^*] = \text{VAR}[Z_v] + \mu - \sum_{i=1}^{n} \lambda_i \text{COV}[Z_v, Z(x_i)]$$

When $Z(x)$ is an intrinsic RF with constant drift, the unbias condition stands as before:

$$\sum_{i=1}^{n} \lambda_i = 1$$

It should be noted that, when such a condition is realized, the difference $Z_v - Z^*$ has a finite variance, even if Z^* does not, because:

$$1/v \int_v dx - \sum_{i=1}^{n} \lambda_i = 0$$

This variance is written:

$$\text{VAR}[Z_v - Z^*] = -\bar{\gamma}_{vv} + 2 \sum_{i=1}^{n} \lambda_i \bar{\gamma}_{v x_i} - \sum_{i=1}^{n} \sum_{j=1}^{n} \lambda_i \lambda_j \gamma(x_i - x_j)$$

where

$$\bar{\gamma}_{vv} = 1/v^2 \int_v \gamma(x - y)\,dx\,dy$$

and

$$\bar{\gamma}_{v x_i} = 1/v \int_v \gamma(x - x_i)\,dx$$

The linear system which gives λ_i is now written:

$$\sum_{i=1}^{n} \lambda_i \gamma(x_i - x_j) = \bar{\gamma}_{v x_i} - \mu$$

$$\sum_{i=1}^{n} \lambda_i = 1$$

and the kriging variance is:

$$\text{VAR}[Z_v - Z^*] = -\bar{\gamma}_{vv} + \mu + \sum_{i=1}^{n} \lambda_i \bar{\gamma}_{v x_i}$$

At this point, where the drift m(x) has always been assumed to be constant, it should be noted that the weights λ_i only depend on the form of the covariance or variogram function.

When the hypotheses of universal kriging apply, the drift of the RF is represented by the expression:

$$m(\underset{\sim}{x}) = \sum_{l=0}^{k} a_l f_l(\underset{\sim}{x})$$

In a vicinity U, of size U, which includes the domain v and the set S. In that case, the unbias condition on U is written:

$$E[Z_v] = E[Z^*]$$

or:

$$\sum_{l=0}^{k} a_l \, 1/v \int_{\underset{\sim}{v}} f_l(\underset{\sim}{x}) \, dx = \sum_{l=0}^{k} a_l \sum_{i=1}^{n} \lambda_i f_l(\underset{\sim}{x}_i)$$

This relation must stand whatever the unknown values of the a_l coefficients are. This is the universallity condition, which is expressed by k + 1 relations:

$$1/v \int_{\underset{\sim}{v}} f_l(\underset{\sim}{x}) \, dx = \sum_{i=1}^{n} \lambda_i f_l(\underset{\sim}{x}_i)$$

The variance of the difference $Z_v - Z^*$ is expressed as before, depending on the existence of a covariance for Z(x) or a variogram for its increments.

For example, in the intrinsic hypothesis, the kriging system is written:

$$\sum_{i=1}^{n} \lambda_i \gamma(\underset{\sim}{x}_i - \underset{\sim}{x}_j) = \bar{\gamma}_{\underset{\sim}{v}\,\underset{\sim}{x}_j} - \sum_{l=0}^{k} \mu_l f_l(\underset{\sim}{x}_j)$$

$$\sum_{i=1}^{n} \lambda_i f_l(\underset{\sim}{x}_i) = 1/v \int_{\underset{\sim}{v}} f_l(\underset{\sim}{x}) \, dx$$

and the kriging variance is

$$\mathrm{VAR}[Z_v - Z^*] = -\bar{\gamma}_{\underset{\sim}{v}} + \sum_{l=0}^{k} a_l \, 1/v \int_{\underset{\sim}{v}} f_l(\underset{\sim}{x}) \, dx + \sum_{i=1}^{n} \lambda_i \bar{\gamma}_{\underset{\sim}{v}\,\underset{\sim}{x}_i}$$

This completes the formal expressions of the system of equations for any of the three sets of hypotheses.

The Practice of Kriging

Mathematically speaking, kriging only involves solving a system of linear equations, something easily done by a computer program. In fact, the difficulties encountered in kriging do not lie in that mechanical operation, but rather in the adequate choice of the statistical model for the spatial correlation of the variable under study in the deposit or parts of it. Such a choice defines the parameters or functions intervening in the kriging equations; i.e. $\gamma(h)$, $f_l(x)$ and U, which have to be derived from the experimental data available.

These problems will now be reviewed and, to stay in the less restrictive kind of hypothesis, we will assume that the function Z(x) is intrinsic, which means that the increments of Z(x) have a variance, the variogram, which is stationary on vicinities of size U.

On the determination of the variogram when m(x) is constant

When m(x) is assumed to be constant on U, an estimate of the variogram, the experimental variogram, can be computed from the available data, simply by averaging the squared difference between values from samples taken the same distance apart in the same direction. Practical aspects of this computation (precision of the estimate, regularization and grading) will not be examined here and the reader is referred to David (1974) for extensive discussion of the subject.

On the determination of the variogram when m(x) is no longer constant

If m(x) cannot be assumed to be constant on U, we are unable to compute an experimental variogram directly from the values of the realization of Z(x). The calculation has to be performed on the residuals:

$$Y(\underset{\sim}{x}) = Z(\underset{\sim}{x}) - m(\underset{\sim}{x})$$

It implies that we are able to define m(x) from the values of Z(x) on hand. In other words, we have to estimate m(x).

Estimation of the drift: On a vicinity U, we will assume that m(x) varies regularly and has the following expression:

$$m(\underset{\sim}{x}) = \sum_{l=0}^{k} a_l f_l(\underset{\sim}{x})$$

where $f_l(x)$ form a set of independent functions, such as x, x^2, xy or xyz (x, y, z are the coordinates of x in a 3D space). The unknown values, a_l are defined by linear estimators:

$$A_l = \sum_{i=1}^{n} \lambda_{il} Z(\underset{\sim}{x}_i)$$

and m(x) is estimated by:

$$m^*(\underset{\sim}{x}) = \sum_{l=0}^{k} A_l f_l(\underset{\sim}{x}) = \sum_{i=1}^{n} \sum_{l=0}^{k} \lambda_{il} Z(\underset{\sim}{x}_i) f_l(\underset{\sim}{x}_i)$$

As for kriging, this estimator has to be unbiased and it is possible to define a set of weights, λ_{il}, which minimizes the corresponding estimation variance. However, to solve the equations that give this optimal estimator, it is necessary to know the variogram of the RF, which is precisely the ultimate purpose of the study. Nevertheless, a simple unbiased estimator is a valid one and can be derived from least-square methods of trend surface analysis (Agterberg, 1973).

Thus, at every data point of U, an estimated residual is given by:

$$Y^*(\underset{\sim}{x}) = Z(\underset{\sim}{x}) - m^*(\underset{\sim}{x})$$

The variogram of residuals: We are now able to compute an experimental variogram of estimated residuals $\gamma^*(h)$, but Matheron (1969) has shown that such a variogram differs from the underlying variogram of the true residuals $\gamma(h)$ and that the bias is a function of the form of the estimator $m^*(x)$.

An example of such a deviation is given in Figure 2. Here, the RF have intrinsic properties on intervals U of length U = 7 and, on these intervals, the drift m(x) is supposed to be quadratic:

$$f_1(\underset{\sim}{x}) = x \qquad f_2(\underset{\sim}{x}) = x^2$$

The underlying variogram is a spherical one:

$\gamma(h) = C_0 + C(1.5\,h/A - 0.5\,(h/A)^3)$ ..... for $h \leq A$

$\gamma(h) = C_0 + C$ ..... for $h > A$

with $C_0 = 0$, $C = 1$, $A = 2$

The coefficients of the quadratic drift are estimated from eight samples regularly distributed on the interval U. The estimator chosen is the unbiased one, which minimizes the estimation variance when the underlying variogram is a linear one.

Removing the bias: Now, in order to find $\gamma(h)$ from the form of the biased $\gamma^*(h)$, we have to graphically compare its curve with a set of $\gamma_o^*(h)$ defined from various types of variograms $\gamma_o(h)$ and the same type of estimator $m^*(x)$. This graphical comparison is done within a multiplier, as $\gamma^*(h)$ is a linear form of $\gamma(h)$:

if.....$\gamma(h) = \lambda\,\gamma_1(h) + \mu\,\gamma_2(h)$

then.....$\gamma^*(h) = \lambda\,\gamma_1^*(h) + \mu\,\gamma_2^*(h)$

The multiplier is defined by the translation which brings $\gamma^*(h)$ on $\gamma_o^*(h)$ in a semilogarithmic system of coordinates. Details on such procedures are given in Huijbregts (1970)

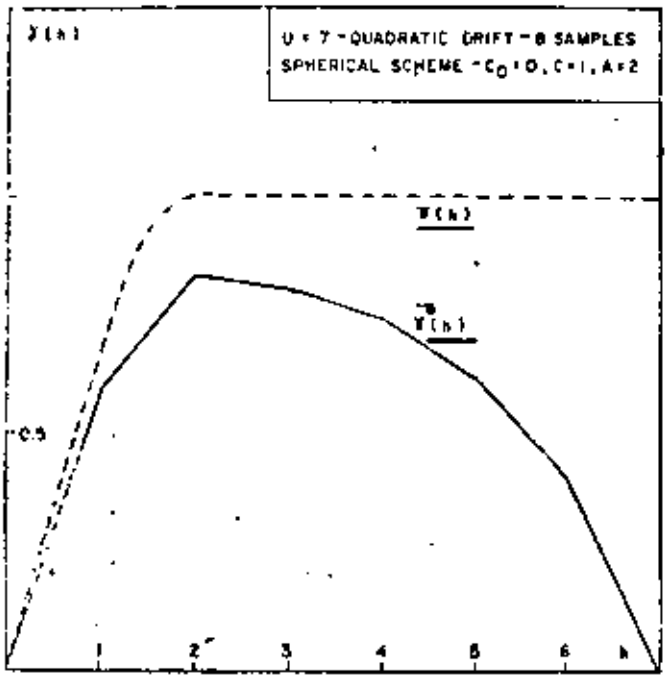

FIGURE 2 — Biased variogram of estimated residuals.

On the form of m(x)

In the two previous sections, we have reviewed the methods which allow us to define the variogram $\gamma(h)$ according to the hypothesis made for the variations of $m(x)$ on U. Here, with two simple examples, we will show that this hypothesis depends in turn on the size of U and that the idea of drift is necessarily related to that of scale.

The first diagram of Figure 3 is a simulated profile of copper grade along a 400-ft DDH sampled at regular 10-ft intervals. If we want to estimate the average grade on large blocks (100 ft to 200 ft) by kriging, the vicinity U will include all the available samples ($U = 400$ ft). On a field of that size, $m(x)$ can be considered as constant and the variogram computed on U will exhibit two transition structures: one of small range (or nugget effect) corresponding to the primary saw-toothed pattern and the other, with a range of about 80 ft, linked to the secondary bumps of greater amplitude.

On the other hand, if the purpose of the study is to krige smaller blocks (20 ft), U will not be greater than 100 ft and, on that vicinity of reduced size, a quadratic drift has to be assumed to indicate that the secondary bumps are no longer structures of random nature.

The second diagram on the same figure shows a reverse situation. In this case, the drift is obvious on the whole profile, but locally the grade can be

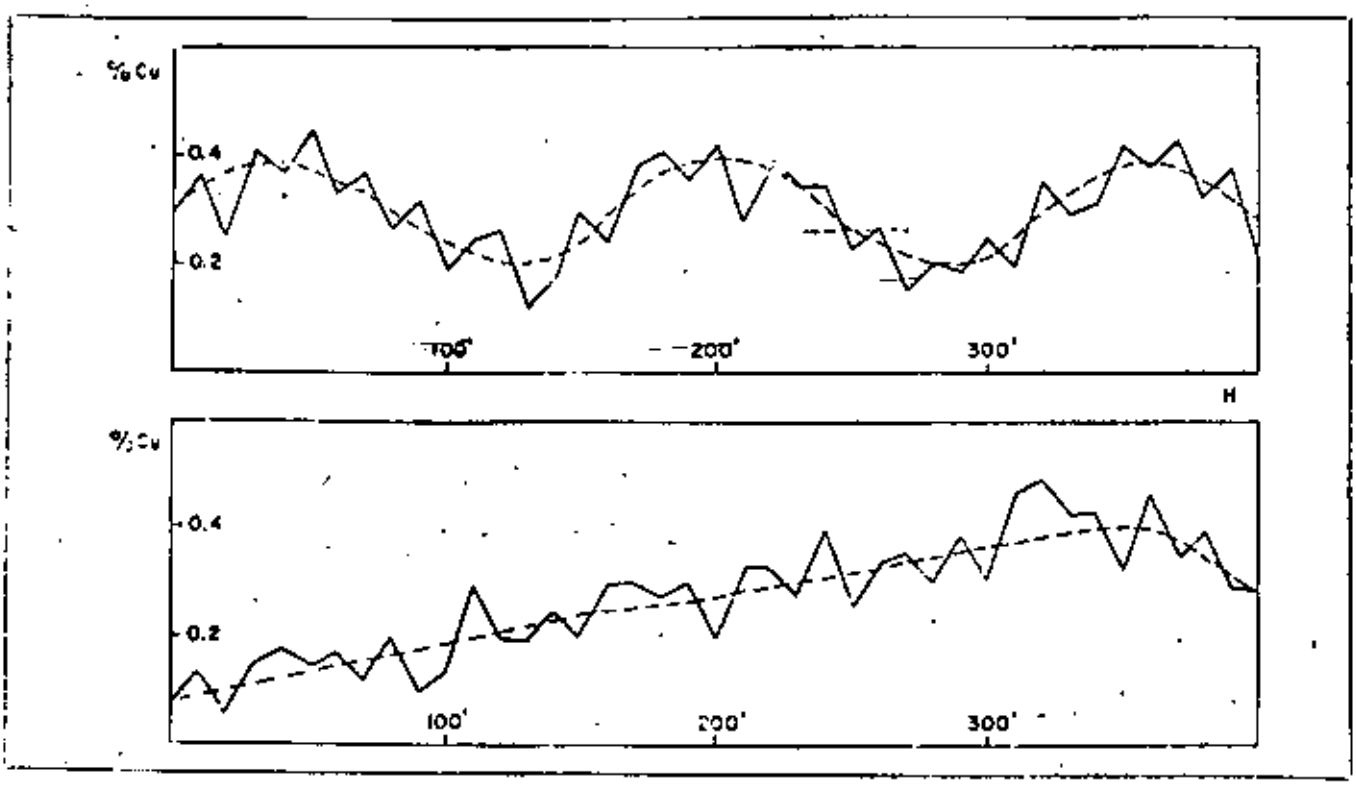

FIGURE 3 — Simulated profiles along drill holes.

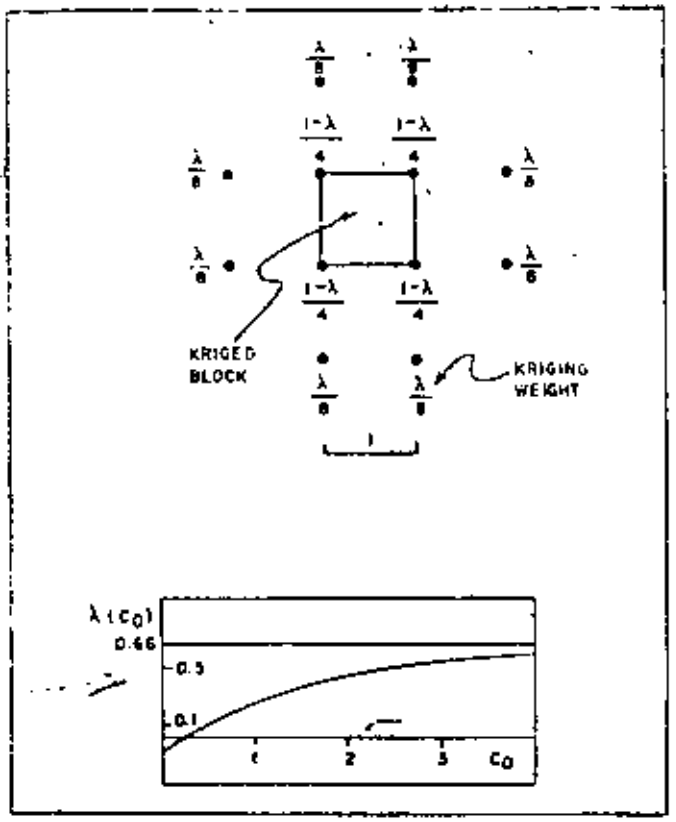

FIGURE 4 — Variation of kriging weights with C_o (Model: $\gamma(h) = C_o + h$).

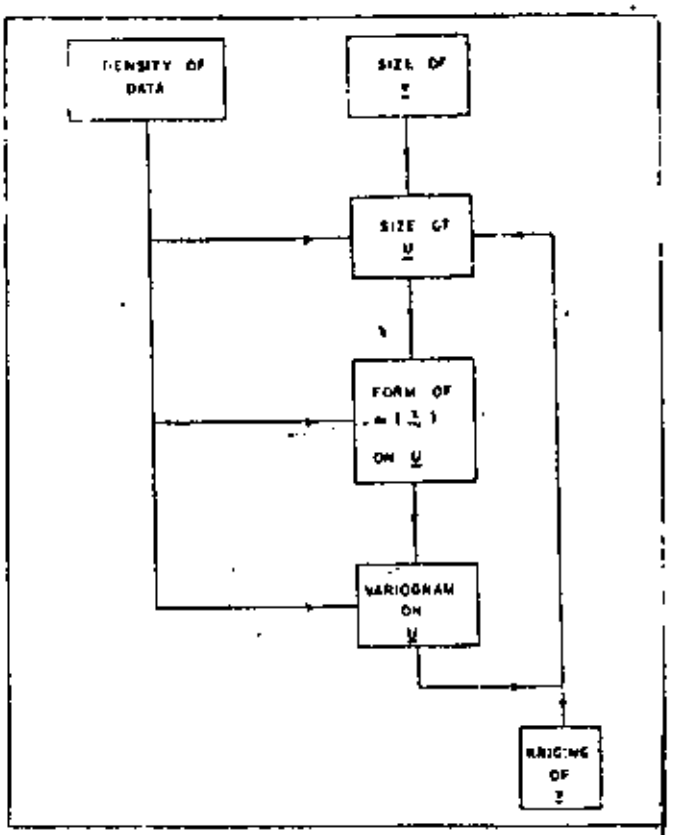

FIGURE 5 — Different steps involved in the kriging of blocks of size v.

assumed to be stationary and, at that scale, simple kriging is relevant.

On the size and definition of U: influence of the variogram on kriging:

The previous section demonstrated that the hypothesis made concerning the behaviour of m(x) has to be related to the size of U. A last question remains: given a block of size v in a deposit, among available samples, which are those actually used in kriging v. In other words, how can we define the extent of U? To answer such a question, one has to investigate the influence of the characteristics of the variogram on the kriging weights λ_i.

To simplify the discussion, we will consider a RF with two transition structures expressed by a spherical variogram of common form:

$\gamma(h) = C_o + C\,(1.5\,h/A - 0.5\,h^3/A^3) \ldots\ldots\ldots h \leq A$

$\gamma(h) = C_o + C \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots h > A$

C_o is the nugget effect (first transition structure of very small range) and C is the sill of the second structure, the range of which is A.

It can be shown that when C_o/C is null or very small and when A is rather large compared to the average distance between samples and to the dimensions of v, a screen effect appears, due to a Markovian property, expressed by the almost linear variation in the first part of the variogram. Roughly speaking, this means that only the samples closest to v actually are used in the weighting procedure, so that U can be reduced to a narrow neighbourhood around v.

When C_o/C increases, the screen effect disappears and, in the extreme case of a pure nugget effect, all samples available contribute with the same weight to the kriging estimate of v, which is merely the arithmetic mean of the value at every sample point.

An example: These results are illustrated by the 2D-kriging pattern of Figure 4. In that simple case, samples are taken at mesh points of a regular square grid and each square is kriged by using the twelve nearest samples. The variogram of the RF is supposed to be of the isotropic and spherical type, with a sufficiently large range with respect to the dimensions of the grid to consider only the linear part of the variogram near the origin. Symmetry of the pattern reduces kriging to the determination of one coefficient λ: the weight of each sample in the outer rim is λ/8 and in the inner one 1-λ/4. The sill, C, of the variogram is kept constant and C_o is progressively increased. The adjacent chart shows the variation of λ(C_o).

When the screen effect is partly removed (C_o/C above 0.2 or 0.3) and when the dimensions of v are close to or greater than the range A, other parameters of γ(h) (mainly the range; the sill C only affects the kriging variance) and the geometry of the kriging pattern have to be considered in order to define U. Extensive discussion on these effects is given in Serra (1967) and Newton (1972).

Conclusion

To recapitulate all the previous statements, when kriging of a block v has to be performed from a set of punctual samples, a strategy can be developed, which is summarized on the flow chart of Figure 5.

As a first assumption, the drift m(x) is supposed to be constant on the whole field or homogeneous fields defined by geological criteria. Then, an ex-

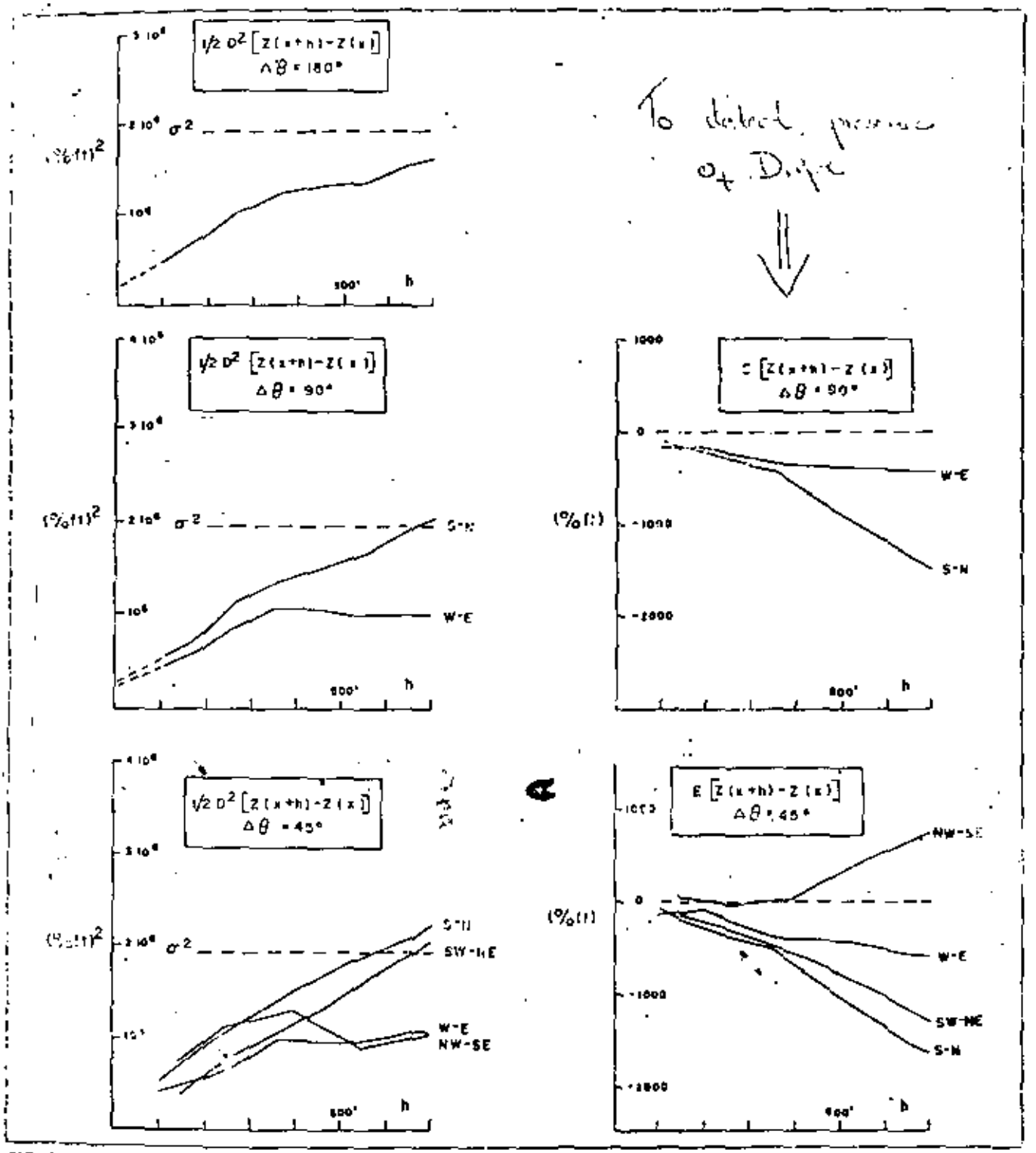

FIGURE 6 — Zn-Pb accumulation in the Navan deposit — experimental variograms.

perimental variogram can be computed on this broad field directly from data available at sample points.

This variogram provides a good appraisal of the magnitude of the nugget effect, if any, and considerations of the previous section can be used to define the size of U, taking into account the dimensions of v and the density of available samples in and around v.

Meaningful assumptions can be made on the variations of m(x) in vicinities of size U, which lead to the calculation of a variogram of estimated residuals from which the true variogram can be derived. As shown in the flow chart, the procedure can be used in a recurrent manner, but generally a single run will provide a sound definition of the parameters needed to perform kriging.

With that theoretical background, we are now in a position to perform the estimations mentioned at the beginning of the paper.

Navan Deposit — The Authors' Experimental Variograms Computations

First Hypothesis

The drift of accumulation is assumed to be constant on the whole deposit. Thus, experimental vario-

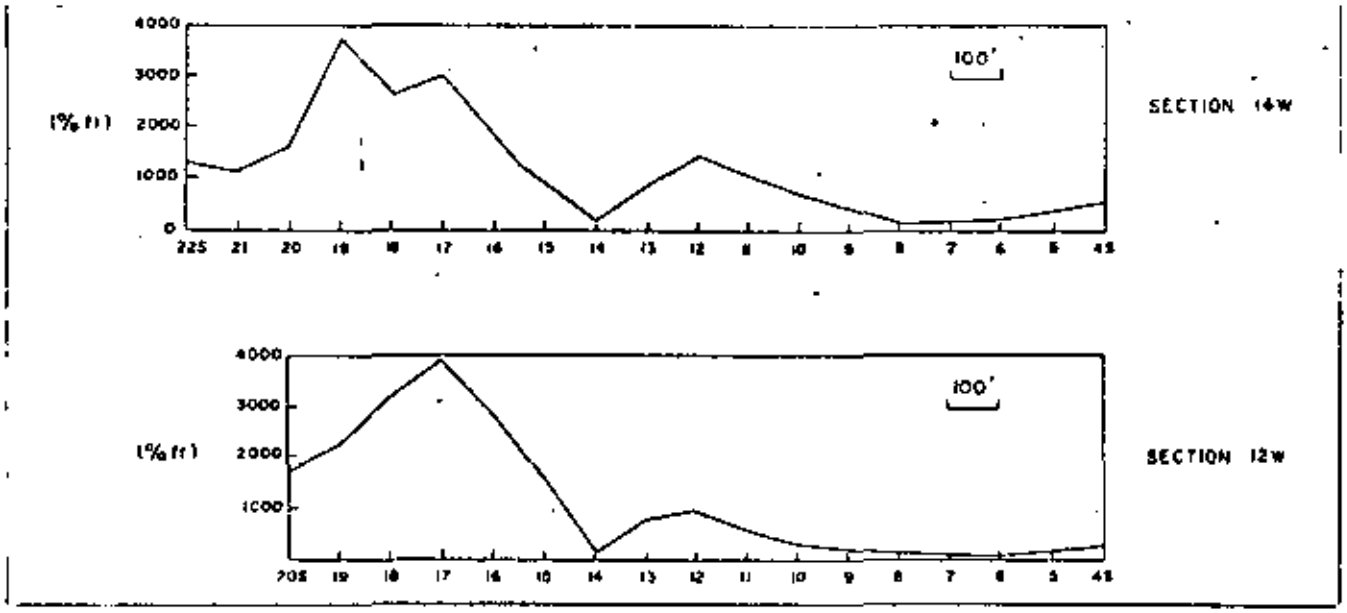

FIGURE 7 — Zn-Pb accumulation profiles along selected sections.

grams are computed directly from values of accumulation at mesh points. Several variograms have to be defined along different directions in order to check the presence of anisotropy in the spatial correlation structure. Pairs of samples are grouped in classes of distance (step Δ h) and direction (step $\Delta\ \theta$). As the smallest distance between two samples is 100 ft, Δ h will be given this value. To precisely define the features of the variograms, angular regularization is progressively reduced from $\Delta\ \theta = 180$ degrees to $\Delta\ \theta = 10$ degrees as the number of directions investigated increases (Fig. 6). Details on the procedure are given in David (1974).

The conclusions of that variography are as follows:

— The variograms display a discontinuity at the origin, the "nugget effect", which is characteristic of microstructures of sizes smaller than 100 ft. The amplitude of the nugget effect reaches approximately 2.10^3 (% ft)^2, one-tenth of the total variance.

— A transition structure with a range of about 400 ft can be visually defined on most of the variograms. The corresponding sill, $C = 8.10^3$ (% ft)^2, is far below the variance level.

— The monotonous growing of some variograms beyond the variance level suggests the presence of a non-stationary drift at the scale of the deposit. This statement is supported by the study of the function $E[Z(x + h) - Z(x)]$ or the profiles of accumulation along selected cross sections (Fig. 7). They show a regular increase of accumulation toward the south-southwest.

— As the N-S variograms seem to grow more rapidly toward the transition sill than the E-W ones, anisotropy may be suspected, but the effect of the non-stationary drift may influence the shape of the different curves.

A simplified spherical model can be derived for the whole deposit which fairly incorporates the previously defined features:

$\gamma(h) = C_0 + C\ (1.5\ h/A - 0.5\ (h/A)^3)$ ..... for $h \leq A$

$\gamma(h) = C_0 + C$ ..... for $h > A$

with $C_0 = 2.10^3$, $C = 8.10^3$ and $A = 400$ ft.

Nevertheless, the hypothesis of a stationary drift at the scale of the deposit is a rough approximation. As we are dealing with blocks of relatively small size, we can reduce the dimensions of the field U and compute experimental variograms from data confined to domains of that size.

We will later compare the effect of the different hypotheses on the final results.

Experimental Variograms on Vicinities of Restricted Size

A 400-ft-diameter circle, U, is moved across the deposit and a quadratic drift is assumed for $m(\underset{\sim}{x})$ in such vicinities.

$f_1(\underset{\sim}{x}) = x$, $f_2(\underset{\sim}{x}) = y$, $f_3(\underset{\sim}{x}) = x^2$, $f_4(\underset{\sim}{x}) = xy$, $f_5(\underset{\sim}{x}) = y^2$; x and y are coordinates of $\underset{\sim}{x}$.

The a_l coefficients of the quadratic drift are appraised by an unbiased estimator which is optimal when the underlying variogram is isotropic and linear. Such an evaluation leads to the expression of $m^*(x)$ on U and the definition at every data point of an estimated residual:

$$Y^*(\underset{\sim}{x}) = Z(\underset{\sim}{x}) - m^*(\underset{\sim}{x})$$

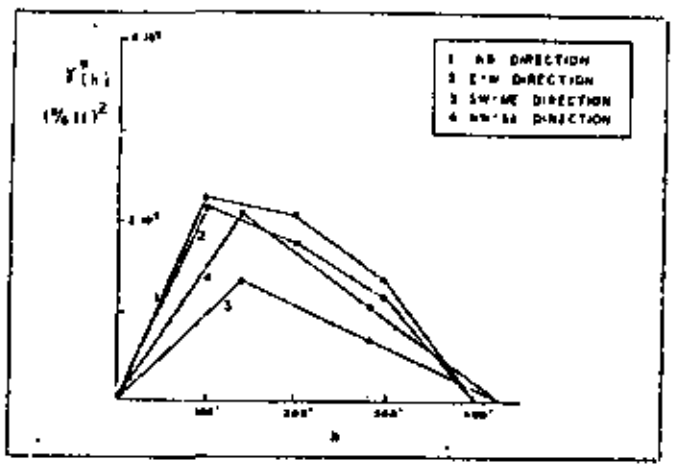

FIGURE 8 — Experimental semivariograms of estimated residuals for quadratic drift and 400-ft neighbourhoods.

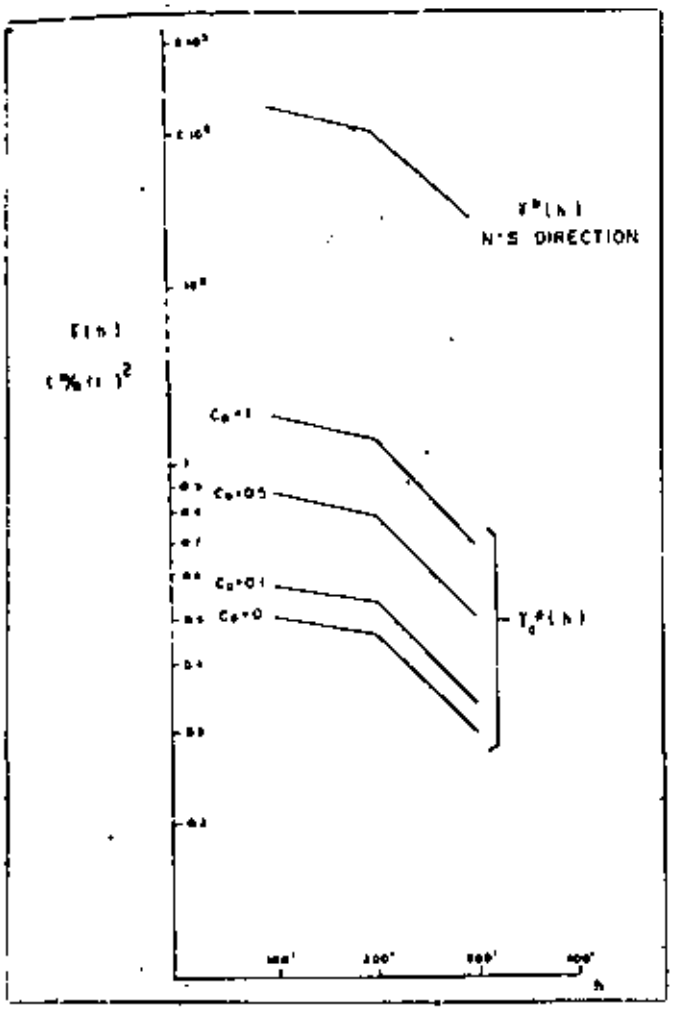

FIGURE 9 — Comparison of an experimental variogram of residuals $\gamma^*(h)$ with several models $\gamma_0^*(h)$.

The biased experimental variogram $\gamma^*(h)$ is computed from the residuals $Y^*(x)$ located in the same $\underline{U}$ and along four directions: N-S, E-W, NW-SE, SW-NE (Fig. 8). The experimental variograms are compared to various biased variograms computed with the same kind of estimator $m^*(x)$ and corresponding to several models of underlying variograms. The comparison is made on logarithmic paper. As the underlying variogram $\gamma(h)$ can be written as $\gamma(h) = \omega\,\gamma_0(h)$, parameter ω is defined in this semilogarithmic system of coordinates by the translation which brings $\gamma^*(h)$ on $\gamma_0^*(h)$. In the present case, $\gamma^*(h)$ is defined only for three points along directions N-S and E-W and two points along directions NW-SE and NE-SW on 400-ft intervals [$\gamma^*(o)$ is undefined and $\gamma^*(400)$ is always 0]. This lack of information about $\gamma^*(h)$ leads to the search of very simple models of $\gamma_0(h)$ defined by a small number of parameters. On the other hand, we have to keep in mind the general features of the spatial correlation previously defined for the whole deposit. If we assume that the transition structure with a 400-ft range is an important structural characteristic, then, by reducing $\underline{U}$ to 400-ft intervals, we keep essentially within the linear part of the variogram. All these reasons guide our research toward models of the form:

$$\gamma_0(h) = C_0 + \frac{h}{}$$

— and Figure 9 portrays the corresponding biased variograms $\gamma_0^*(h)$ on logarithmic paper with different values for C_0 as well as the experimental variogram $\gamma^*(h)$ along the N-S direction.

The best fit is obtained for:

$$\gamma_0(h) = 0.5 + h$$

with

$$\omega = \frac{.227\ 10^8}{0.875} = 2.6\ 10^8$$

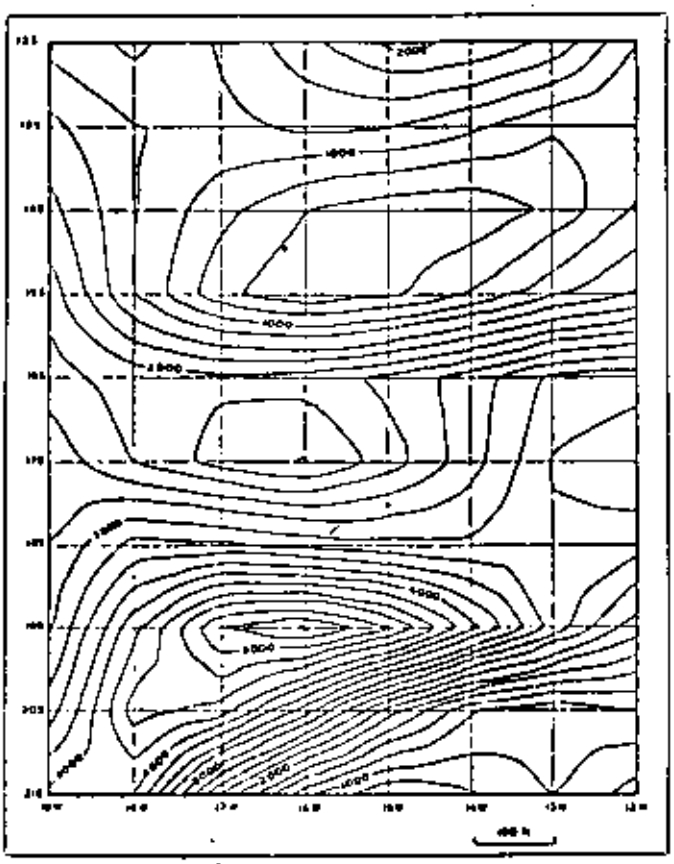

FIGURE 10 — Drift accumulation at Tara — $\underline{U}$ = 400 ft.

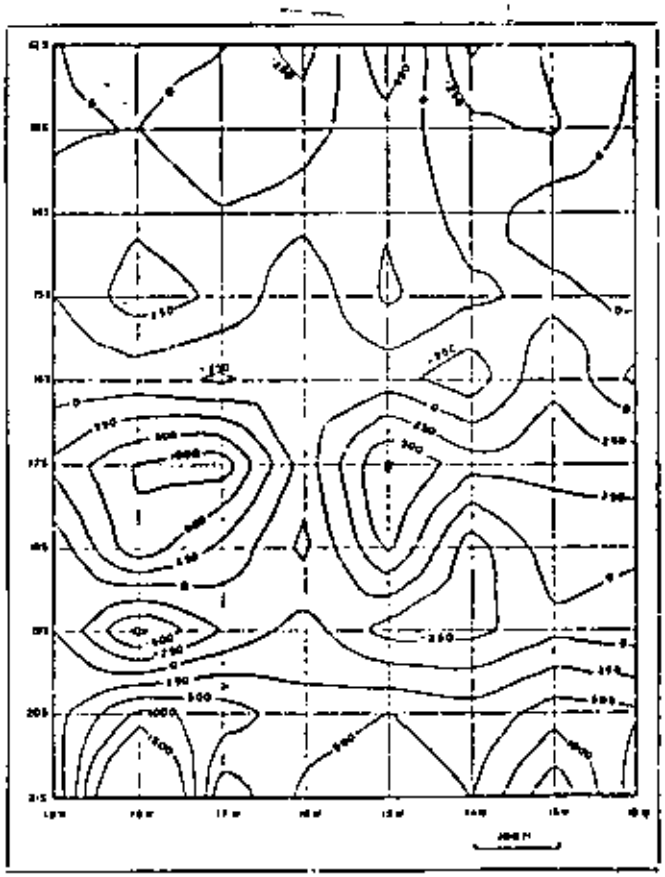

FIGURE 11 — Residuals accumulation at Tara — $\underline{U}$ = 400 ft.

Thus, the model for the underlying variogram $\gamma(h)$ in that direction is taken as:

$$\gamma(h) = 1.3\ 10^5 + 2.6\ 10^4\ h \ldots\ldots (h \text{ in } 100 \text{ ft})$$

The same operation can be repeated for experimental biased variograms along other directions, but no significant anisotropy can be detected. Therefore, on 400-ft-diameter circular vicinities, with a quadratic drift, the variogram is an isotropic linear scheme.

Structural Information in the Drift

In order to visualize the results of the series of operations performed, maps of the drift and residuals are given, on Figure 10 and 11, for the south and richer part of the deposit. At mesh points where we lack information, punctual accumulation has been kriged using the preceding variogram model. The drift at the mesh points has been evaluated by the linear estimator $m^*(x)$ which is optimal (lowest estimation variance) with that variogram. In the intrinsic hypothesis, as we are dealing with increments of accumulation rather than accumulation itself, the drift is defined, except for a constant, so that $m^*(x_o)$ where x_o = [-19W, - 21S] has been arbitrarily assigned the value of accumulation for that hole.

As shown by the map in Figure 10, with the reduction of $\underline{U}$-to intervals 400 ft in length and the assumption of a quadratic trend on these intervals, the drift takes into account two kinds of structural variations: the progressive growing of accumulation toward the south and southwest, and periodical large structures of elliptical profile and elongated along the E-W direction. The second kind of structure suggests a lenticular although continuous mineralization, trending E-W. Moreover, the elliptic form of these structures confirms the anisotropy suspected on the experimental variograms computed on the whole deposit.

A better picture of such structures is given when $\underline{U}$ is enlarged to intervals 700 ft in length. A map of the quadratic drift on these intervals is produced on Figure 12. It has been estimated by a similar procedure and the corresponding experimental variogram is found to fit a spherical model with a nugget effect of C_o = 1.7 10^5, a sill of $C_o + C = 10^6$ and a range of A = 400 ft.

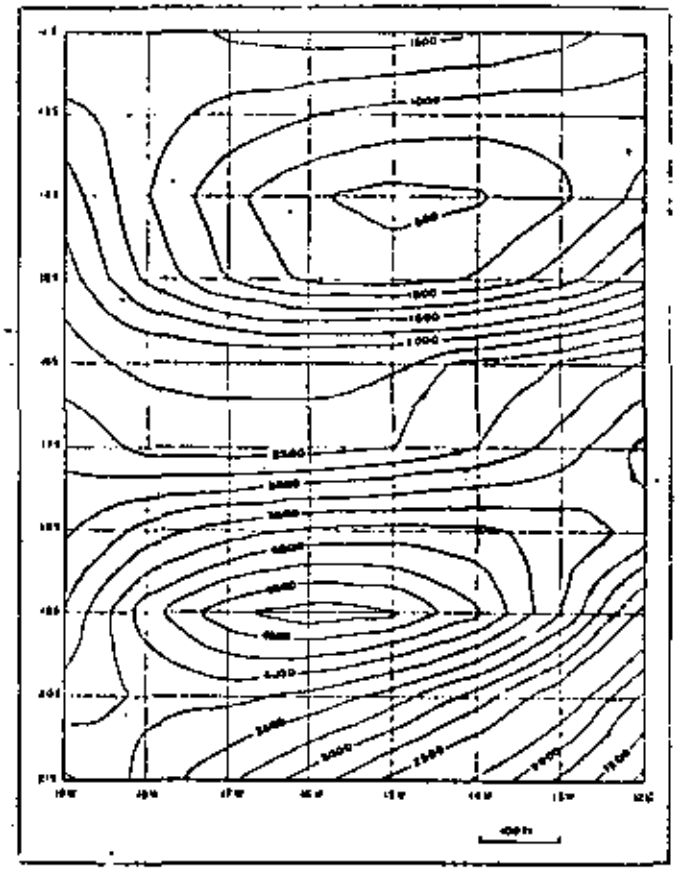

FIGURE 12 — Drift accumulation at Tera — $\underline{U}$ = 700 ft.

Navan Deposit — Kriging

In order to compare the way variogram models affect the kriging procedure, the average accumulation in 50- by 50-ft blocks is estimated. Two typical situations are investigated, and we call Model 1, 2 and 3 respectively:

— the linear model with a quadratic drift on 400-ft intervals;
— the spherical model with a quadratic drift on 700-ft intervals;
— the spherical model with a constant drift on the whole deposit.

Kriging blocks from complete and balanced punctual information

Kriged estimates of 36 blocks are shown on Figure 13. All the surrounding diamond drill holes (DDH) have been used to infer the average value of accumulation in each block, and two models (1 and 3) are tested. For the majority of blocks, the two posted values (universal kriging with model 1 and normal kriging with model 3) are strikingly similar, despite the difference between assumptions made on the underlying R.F.

For two particular blocks, kriging weights of surrounding samples are reproduced on Figure 14. In both cases, the nearest four samples around the block held 85.8% of the inference and a quite strong screen effect is reported by the small weight ascribed to samples of the second aureole. Thus, inference of block accumulation is realized within intervals no more than 100 ft long, corresponding to the elementary spacing between samples. On such intervals of reduced length, the weighting procedure does not depend on the specific form of the variogram or the drift on much larger domains. Due to the regular sampling pattern, the weights of the four nearest DDH are more or less inversely proportional to their distance to the center of the block.

Some discrepancy appears between the kriging variances more sensitive to the model parameters and form. In the case where there is an additional uncertainty on the drift (universal kriging with Model 1), one should expect a greater variance. The reverse situation which occurs here comes from different values of the parameters (nugget effect and slope at the origin of the variogram) for the two models.

As a conclusion, in quite ideal cases (low nugget effect, dense and regular information, small blocks), there is no point to make sophisticated assumptions concerning the behaviour of the drift and normal kriging will give approximately the same result as any conventional interpolation method based on the relative distances between samples and blocks.

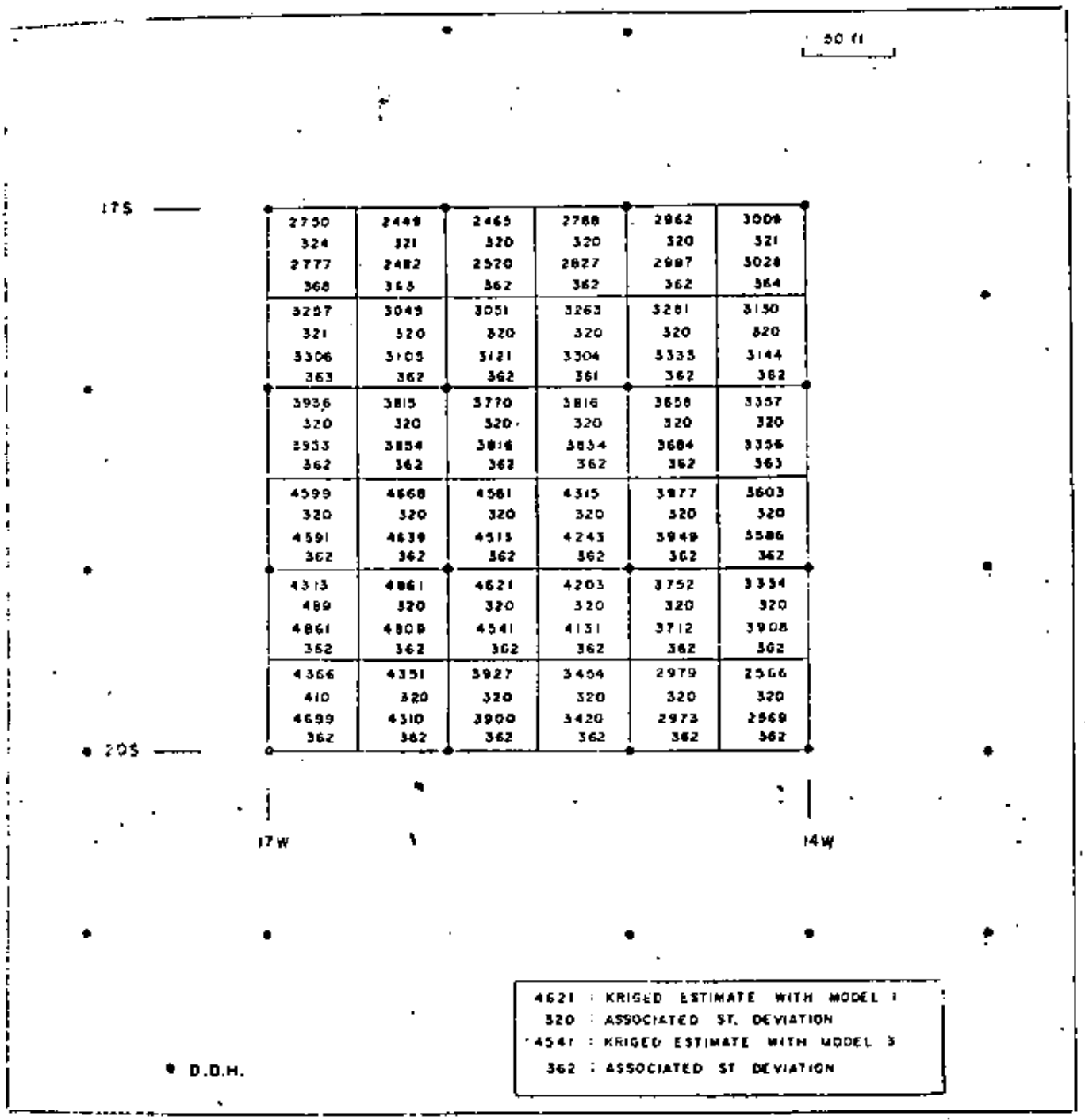

FIGURE 13 — Kriged estimates of the Zn-Pb accumulation in 50- by 50-ft blocks.

Kriging blocks from truncated and unbalanced information

The real advantage of universal kriging and the associated structural study will appear in the particular kriging situation reported on Figure 15a. Accumulation is kriged for two rows of 50- by 50-ft blocks supposed to be found at the margin of two informative sets of punctual samples, respectively called north and south. Such extrapolations are generally made when the grade of blocks in a pit has to be assessed from grades in mined-out blocks. Necessarily, all the information is concentrated on the mined-out side.

For each block, four kriging estimates are computed:

— by universal kriging with Model 2 from the north set (Fig. 15b);
— by normal kriging with Model 3 from the same set (Fig. 15b);
— by universal kriging with Model 2 from the south set (Fig. 15c).
— by normal kriging with Model 3 from the same set (Fig. 15c).

A table of the differences between north and south estimates for both procedures is also given (Fig. 15d).

The divergence between normal and universal kriging appears more clearly in the present situation. With normal kriging, south estimates are constantly higher than north ones. This result is not surprising. Referring to the drift map of Figure 12, the kriged zone appears to be an intermediate one before a richer lens on the south. If kriging is done

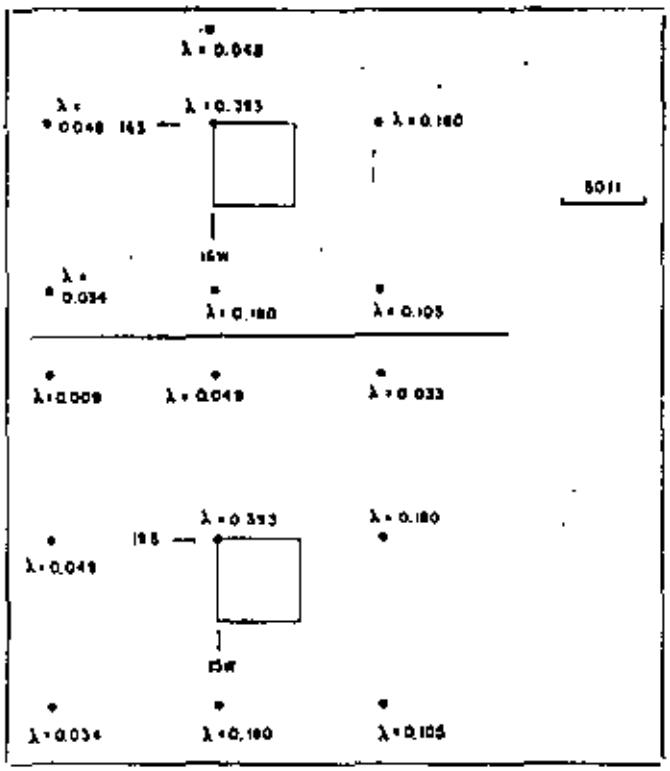

FIGURE 14 — Weighting coefficients for neighbour DDH in kriging Zn-Pb accumulation for two blocks (Model 1).

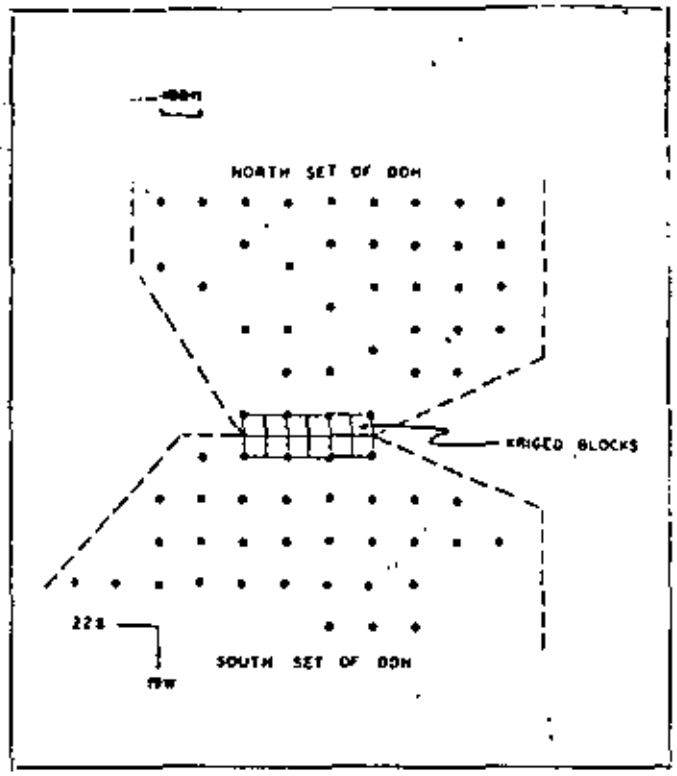

FIGURE 15(a) — Definition of the sample sets and the kriged blocks.

15b

KRIGING FROM THE NORTH

2713	2497	2570	2927	3207	3410
497	462	455	457	465	488
2325	2142	2205	2503	2718	2824
439	425	423	423	425	435
3324	3268	3376	3633	3915	4184
675	613	790	795	818	869
2275	2221	2277	2446	2605	2695
600	585	581	582	586	599

18S

17W

15c

KRIGING FROM THE SOUTH

2809	2844	2937	3052	3111	3089
744	727	714	702	689	667
3022	2978	2994	3041	3003	2970
581	578	579	576	571	560
3355	3252	3289	3430	3387	3192
447	443	440	437	432	424
3399	3275	3278	3384	3310	3127
422	422	422	421	419	415

18S

17W

15d

DIFFERENCE BETWEEN THE TWO KRIGING ESTIMATES (5'N)

+96	+347	+361	+125	−96	−321
+697	+836	+789	+538	+284	+146
+31	−14	−87	−203	−518	−992
+1124	+1054	+1002	+938	+705	+432

18S

17W

AVERAGE DIFFERENCE (MODEL 2) : +105

AVERAGE DIFFERENCE (MODEL 3) : +712

FIGURE 15 — Two ways of extrapolating Zn-Pb accumulation in 50- by 50-ft blocks by kriging (Models 2 and 3).

from the north, a drift is assumed to remain constant and estimates will be underevaluated, whereas, from the south, they will be overevaluated. On the contrary, universal kriging takes into account the trend expressed in each set of samples. As a consequence, the differences between both estimates are more symmetrical and their order of magnitude is considerably reduced. This also means that the kriging variance is higher, as the uncertainty concerning the form of the drift increases considerably with such unbalanced information.

As a conclusion, universal kriging could prove its advantages in particular situations where extrapolation is performed rather than interpolation. It provides non-biased estimates, but generally higher estimation variances — there is some trade-off between exactitude (non-bias) and precision (low variance) to accommodate (Marechal and Ugarte, 1972).

Conclusion

The volume of preliminary structural studies involved in simple or universal kriging may dissuade the mining engineer or geologist, who will probably favour simpler methods of estimation or geostatistical inference. Nevertheless, this preparatory work is far from being useless and by itself allows us to obtain a good insight into the spatial correlation structure and continuity of the mineralization in the deposit. The geological characteristics are expressed in mathematical terms (variogram, covariance ...) in order to adjust the estimation procedure to what nature has produced. Therefore, the resulting estimation procedure is a made-to-measure one for every deposit and any sample-block configuration. Moreover, in any situation, the estimate does not pretend to be exact and the amount of confidence that can be attached to it is quantified by the kriging variance. Finally, the principles of the method insure that this error is the minimal that can be expected taking the available information into account.

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Geological Reconnaissance to Exploitation — A Decade of Applied Geostatistics

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Abstract

The last ten years have seen considerable developments in geostatistics and other applications of the Theory of Regionalized Variables initiated by G. Matheron. Various successful applications have been achieved, mainly in the mineral industry, but also in other fields, such as bathymetry, gravimetry, meteorology, forest surveys and hydrogeology. At the same time, the theory has been developed, its fundamental hypotheses have been widened, allowing the tackling of more and more sophisticated problems, and original new techniques have been designed and successfully applied, mainly Universal Kriging and conditional simulations.

A few presentations of geostatistics have been published in English (see ref. [2], [5], [24]). However, it is the opinion of the author that they are somewhat outdated by recent developments of the theory and its applications. This article is therefore intended to give a more up-to-date presentation of geostatistics, stressing its highly operational aspects.

Chapter 1 considers the main type-problems that could occur during the lifetime of a mine, from the original geological survey to mine development. Chapter 2 presents briefly the basic language of geostatistics — the probabilistic language of random functions. Chapter 3 shows how geostatistics has given a homogeneous and operational answer to each type-problem.

Initially, geostatistics was defined as the practical application of the Theory of Regionalized Variables to the recognition and estimation of mineral deposits. The very general character of the formal theory, as well as the numerous applications outside the mining field (see ref. [9], [13], [42], [43], [45]), leads us to redefine geostatistics as the "Practice of the Theory of Regionalized Variables."

This practice stems from a sound knowledge of the theory (which, we must emphasize, is quite accessible with a minimum knowledge of classical statistics), but requires as well: a clear physical knowledge of the phenomenon studied; and common sense in the art of making approximations in adapting the theory.

The best forestry geostatistician is primarily a forestry engineer; the best mining geostatistician is, in the first place, a geologist, then a mining engineer and finally an ore-dresser or chemist. Their common tool is the probabilistic language of the Theory of the Regionalized Variables.

This article will not present the theory in an abstract manner, but rather indicate how this theoretical tool permits the formalization and resolution of some typical mining problems. It will be noticed throughout the article that a number of these mining type-problems are easily transposed to various fields other than mining (petroleum, geology, geophysics, ecology, meteorology, etc.).

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Keywords: Geostatistics, Regionalized variables, Kriging, Reserves estimation, Geological exploration, Ore reserves, Variances.

Typical Problems in Mining Evaluation

LET US consider a deposit from its first geological reconnaissance to its day-to-day exploitation.

FIRST GEOLOGICAL RECONNAISSANCE

In a province, or in a favourable metallogenic zone, one attempts to reveal existing mineral deposits, proceeding as follows.

(1) — Generally, at the time of the first large-scale reconnaissance, the initiation of drilling is commonly guided either by preliminary studies (geochemical, geophysical) or by geological considerations or hypotheses.

(2) — In some cases, an initial systematic blind reconnaissance is made, and it becomes a matter of counting the number of positive drill holes.

At the end of this first reconnaissance, it is a question of interpreting the results. What group or alignment of positive drill holes is indicative of a mineral deposit? What is the extent of its surface or volume? The answers to these questions are as follows.

(1) — In the case of a first preferential geological reconnaissance, the geologist must reconsider his preliminary hypotheses and transform them where necessary. Geostatistics cannot intervene!

(2) — In the case of a blind but systematic reconnaissance, the reply depends on the respective competences of the geologist and the geostatistician. In fact, guided by the geologist, the geostatistician can build an *a-priori* model of the position or dispersion of possible deposits. Adopting this model, decision criteria can be deduced as to what group of positive drill holes is significant. From the *a-priori* model and from the first reconnaissance data, one could also deduce the first estimators of mineralized resources with estimation variances.

Let us emphasize, however, that these first estimators are likely to be probabilistic or strongly dependent on the *a-priori* geological model adopted. They are valid only to the same extent as the model itself. This fact is, in particular, a characteristic of all the blind reconnaissance models.

In practice, models of the deterministic type would be preferable when extrapolations of neighbouring deposits or precise geological knowledge permit the development of a plausible model (see, for example, Koch and Link, 1972).[34]

SYSTEMATIC EVALUATIONS OF KNOWN DEPOSITS

The deposit being localized, it is now a question of estimating its various characteristics with the help of more and more detailed and sequential knowledge. Schematically, one must furnish: a global estimation of resources in situ, ore tonnage, metal tonnage, grade of impurities, etc., and a local estimation of reserves.

These estimations must have corresponding variances that quantify their confidence intervals. The study of these estimation variances involves the advance calculation of the gain in information attributable to factors such as supplementary drilling investment or the definition of the drilling pattern required to attain a fixed precision.

At a later stage, certain selection criteria of resources in situ are defined, and the problem is then to study these selected resources or "reserves":

local estimations of panels or selected zones;
influence of selection criteria on reserves, calculation of tonnage curve selection criteria, e.g. —

tonnage curve { cut-off grade / minimum mining width / size of selection unit

Although the difference between the two notions of "resources in situ" and "selected reserves" is generally clear, few authors realize the serious estimation biases that are introduced by the fact that a selection is rarely based on the true grade, which is generally unknown, but is most commonly based on an estimated grade, this estimation changing with the level of knowledge. In fact, actual exploitation will provide more and more detailed information, i.e. a different and better estimated grade than that available at the time of the reserves study.

If these biases are not corrected, one arrives irrevocably at the situation[31][39][44], in which the proportion of high-grade material effectively recovered in the mine will be overestimated. The results could be financially catastrophic. The principal advantage of geostatistics is to have formalized rigorously the problem of selection and to have proposed some simple unbiased reserve estimators (Kriging and Selection).

CHARACTERIZATION OF THE RECOVERED ORE

Generally, it is not enough to evaluate the recoverable ore tonnage and its average grade. For ore dressing purposes, it is often necessary to know the distribution of the ore characteristics; e.g., variation of the average mine grade through a production period; variation of the overburden ratio within a certain mining zone; distribution of various grades, impurities, etc. The choice of the mining tool, the necessity and the volume of a blending station, and the flexibility of the mill may be highly conditioned by these variables. They refer to the notion of *dispersion variance*, which must be clearly distinguished from the previous notion of *estimation variance* (see Appendix).

In an estimation problem, an attempt is made to form the best possible estimator (Z^*) of the true unknown characteristic (Z); that is the estimator (Z^*) minimizing the estimation variance $\sigma_E^2 = E\{(Z - Z^*)^2\}$, or the expected value of the squared error considered as a random variable. In the problem of mineral beneficiation, however, the ore-dresser will be more interested in the variation of the ore grades that he is going to receive.

$Z_v(x)$ being the grade of mining unit v located at point x, and Z_V being the average grade in production zone V, corresponding to one day, 1 month or 1 year, the ore-dresser will want to know the dispersion of grades $[Z_v(x)]$ within the production zone (V), this dispersion being characterized by the so-called "dispersion variance"[1]:

$$D^2(v/V) = E\left\{\frac{1}{V}\int_V (Z_v(x) - Z_V)^2\, dx\right\}$$

and he may try to diminish it by judging the size (v) of the selection unit as the criterion of selection itself.

The second principal purpose of geostatistics is to permit the calculation or simulation of these dispersion variances.

OPTIMIZATION OF PRODUCTION

The deposit is now placed in production. This will provide very precise day-to-day information (for example, from sampling in individual drill or blast holes). From day to day, the previous estimations can then be improved, and from the ultimate base an optimal scheme of production or selection is obtained. A typical problem could be in a certain defined zone, which selection will be effective in maximizing ore recovery, the quality of this ore being imposed in a certain interval with a defined risk?

We have thus made a rapid chronological survey of the main type-problems posed during the history of a mineral deposit. It is now a question of showing how geostatistics gives a practical answer to these type-problems.

Geostatistical Language

CONCEPT OF THE REGIONALIZED VARIABLE

A quantified mineralization phenomenon is characterized by the development in space (or time) of certain quantities that we will call "Regionalized Variables" (in short, Re.V.) Some examples are:

ore grade in 3-dimensional space;
thickness of a subhorizontal sedimentary unit (2 dimensions);
metal price, varying with time (1 dimension).

From a mathematical point of view, a Re.V. is a $y(x)$ function of a point with coordinate x — in 3-dimensions, a $y(x_1,x_2,x_3)$ function of a point with coordinates x_1,x_2,x_3. The variation of this function in space is often very irregular — the saw-tooth curve traced from known values still underestimates the real variations of the phenomenon (see Fig. 1). A direct study of the mathematical function is thus excluded. However, a certain structure is hidden under this chaotic appearance; in spite of local fluctuations, there exists in nearly all deposits:

(1) rich and poor zones — a drill hole positioned in a rich zone is, on the average, richer than a drill hole positioned in a poor zone — the $y(x)$ value depends on the position x;
(2) phenomena of progressive enrichment or impoverishment.

A true formalization must then take into account these two apparently contradictory characteristics of the Re.V.: (a) the disordered or random character, which suggests recourse to the notion of random variables; and (b) the structured character particular to each "regionalization" y(x).

The language that allows us to take both the random and the structured aspects of the Re.V. into account is the probabilistic language of *Random Functions*. This probabilistic conceptualization of reality y(x) is useful only if it permits us to attain our objectives: characterization of the structure and homogeneous formalization and solution of the problem-types cited.

CONCEPT OF RANDOM FUNCTIONS

A random variable is a variable that can take a certain number of values with a certain law of probability. Thus, the result of one throw of an unbiased die is a random variable that can take six values (1 to 6), each with an equal probability (one sixth). For example, if a particular throw gives 5, we would say that 5 is one particular realization of the random variable "result of throwing the die".

Similarly, let us consider the grade $y(x_i) = 1.8\%$ Cu at a precise point x_i. We say that this grade is a particular realization of a certain random variable $Y(x_i)$ positioned at point x_i. Consequently, the group of grades $y(x)$ for $x \in$ Deposit, i.e. the regionalized variable $y(x)$, is one particular realization of all the random variables $\{Y(x), x \in \text{Deposit}\}$. This infinite group of random variables is called a random function and is denoted $Y(x)$.

The expression "random function" contains the double aspect — randomness and structure — of a regionalized variable:

locally $Y(x_i)$ is a random variable;

but $Y(x)$ is a random function, i.e. the two random variables, $Y(x_i)$ and $Y(x_i+h)$, are linked by correlation.

PROBABILISTIC INFERENCE AND STATIONARITY

Therefore, a regionalized variable is interpreted as one particular realization $y(x)$ of a certain random function $Y(x)$. However, the knowledge of a single realization $y(x)$ limited to a certain number of drill holes $\{x_i \in \text{Information}\}$ does not permit determination of the law of $Y(x)$. Similarly, the knowledge of a single outcome, 5, in throwing a die does not demonstrate the probability law for the die: is the die true? Several throws must be made to know that. Similarly, we must have several realizations, $y_1(x)$, $y_2(x)$, . . . $y_n(x)$, of our random function to infer the law, or at least the part of the law that is of interest. Now, we have at our disposal a single limited realization: $\{y(x_i), x_i \in \text{Information}\}$. Here the hypothesis of stationarity intervenes:

In practice, the regionalized variable $y(x)$ studied very commonly repeats itself in space. Two values $y(x_i)$ and $y(x_i+h)$ can be considered as two different realizations of the same random variable, $Y(x_i)$ for example. It is this hypothesis that justifies the estimation of the histogram of the random variable $Y(x)$ by the experimental dispersion of available data $\{y(x_i), x_i \in \text{Information}\}$.

Geostatistics requires a weaker hypothesis. Interest is focused not on the values themselves, but only on their differences, $[y(x) - y(x+h)]$, and it is the stationarity of these increments that geostatistics requires. Thus, two pairs of data, $[y(x_i)—y(x_i+h)]$ and $[y(x_j)—y(x_j+h)]$, are considered as two different realizations of the same random increment $[Y(x)—Y(x+h)]$. This stationary hypothesis, limited to the increments, is called the *Intrinsic Hypothesis*. The available data then permit estimation of the moment of order 2, i.e., the variogram:

$$2\gamma(h) = E\{[Y(x+\vec{h}) - Y(x)]^2\}$$

which can be estimated by the mean value of the quadratic differences $[y(x+h)—y(x)]^2$. Still, geostatistics does not demand that the intrinsic hypothesis be verified throughout the deposit, D, i.e. for any vector $\vec{h}$. For calculations, it is sufficient if the intrinsic hypothesis prevails over short domains ($|h| <$ limit) throughout the deposit.

Let us note that non-stationarity of differences or non-stationarity in general is due to two main causes.

(1) — Existence of a progressive drift of the characteristics of the vein (trend). On the average, $y(x)$ decreases as one goes from A to B: $E\{Y(x)\} = m(x)$. It is sufficient then to extract this trend or moment of order 1, which is either functional or random, and to study the stationary residual $R(x)$, which has its own spatial autocorrelation:

$$Y(x) = m(x) + R(x)$$

(2) — A profound heterogeneity of mineralization between two or more parts of the same deposit. It is imperative in this case to distinguish the different types of mineralization and to study them separately.

Let us point out that the variogram tool constructed from rough available data in general reveals and distinguishes between these two cases of non-stationarity.

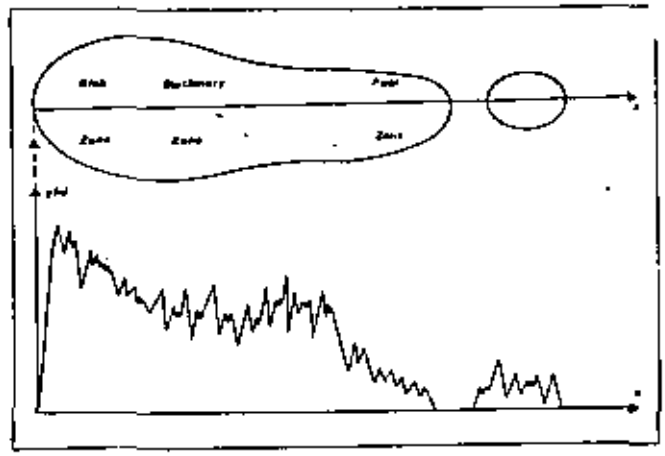


FIGURE 1 — Random and structured aspect of a Re.V.

Problem-Types and Geostatistics

Let us return to the problem-types of mining exploration and evaluation outlined previously, and indicate briefly how the language of random functions permits expression of these problems and how the variogram tool can be used to solve them.

INITIAL GEOLOGICAL EXPLORATION

We have already noted that in this first phase of reconnaissance, in the absence of systematic and quantitative information, the direction of the geological reconnaissance as well as the interpretation of results depends principally on the *a-priori* model adopted

— i.e., the genetic model of the geologist, the deterministic or probabilistic model of the location of the deposits, their size, dispersion, etc. It is then important to know whether geostatistics can supply a purely geometric response that is perfectly general and devoid of any probabilistic hypothesis. Surface estimation or volume estimation by a systematic grid or regular density of samples are such problems (Fig. 2).

In such problems, the regionalized variable of interest is of the "hit-or-miss" variety:

$$k(x) = \begin{cases} 1 \text{ if } x \in \text{Deposit} \\ 0 \text{ if not} \end{cases}$$

and we are interested in estimating the integral $S = \int_{-\infty}^{+\infty} k(x)\,dx$. Geostatistics defines the structural function or "geometric covariogram" as:

$$K(h) = \int_{-\infty}^{+\infty} k(x)\,k(x+h)\,dx$$

Note that: $S = K(0)$

The real unknown surface, S, is estimated by the combination of n surfaces of influence of the n positive (hit) drill holes; i.e., by

$$S^*(x_0) = a \sum_{p=-\infty}^{+\infty} k(x_0 + pa)$$ written unidimensionally

a: grid unit surface — x_0: origin of the grid

It can be seen that the estimator, S^*, depends on the position of the origin, x_0, of the data grid. Consequently, the estimation variance (a) of S by S^*, i.e. the mean quadratic error, is written:

$$\sigma_E^2(a) = \frac{1}{a} \int |S^*(x_0) - S|^2\,dx_0$$

(a: grid unit)

The two terms $S^*(x_0)$ and S, under this integral, are expressed as a function of the variable k(x). On developing this integral, one finds [see also ref. (40) p. 21]:

$$\sigma_E^2(a) = a \sum_{p=-\infty}^{+\infty} K(pa) - \int_{-\infty}^{+\infty} K(h)\,dh$$

The estimator S^* is unbiased; the grid is expressly uniquely with the structural function K(h). However this function is as unknown as is the real surface S. However, whatever the form and extension of it may be, all geometric covariograms, K(h), have a linear slope at the origin (h = 0), which can be estimated for each surface S^*. This general and purely geometric property allows us to know the first term of the limited expansion, near the origin ($a \to 0$), of the estimation variance. Thus, in a 2-dimensional surface estimation problem, the following first-order approximation formula is used:

$$\frac{\sigma_E^2}{S^2} = \frac{1}{n^2}\left[\frac{1}{6} N_2 + 0.06\,\frac{N_1^2}{N_2}\right] + \ldots \quad (N_2 \leq N_1),$$

This gives the relative estimation variance, which is useful when $n \geq 10$, where n = number of positive drill holes, and $2N_1$ and $2N_2$ are the numbers of elements parallel to the two sides of the grid (a_1, a_2) that constitute the contour of the combination S^* of the n positive zones of influence (example given in Fig. 2).

For volume estimation (three dimensions) by a parallelopipedic grid or several two-dimensional grids, an equally simple first-order approximation formula is available [see ref. (40), p. 30].

When a deposit is localized, the question is now to evaluate and characterize it.

STRUCTURAL ANALYSIS

With the hypothesis of local stationarity of differences $[Y(x+h) - Y(x)]$, it is possible to estimate the variogram $2\gamma(h)$, defined as the expected mean square of differences:

$$2\gamma(h) = E\{|Y(x+h) - Y(x)|^2\}$$

It is important to see that this unidimensional expression can be generalized to the n-dimensional space. In the common 3-dimensional space, h represents a vector (h_1, h_2, h_3), and the condensed notation $\gamma(h)$ represents the vectorial function: $\gamma(h_1, h_2, h_3) = \gamma(|h|, \alpha)$, α being the direction of the vector $\vec{h}$ and $|h|$ its modulus.

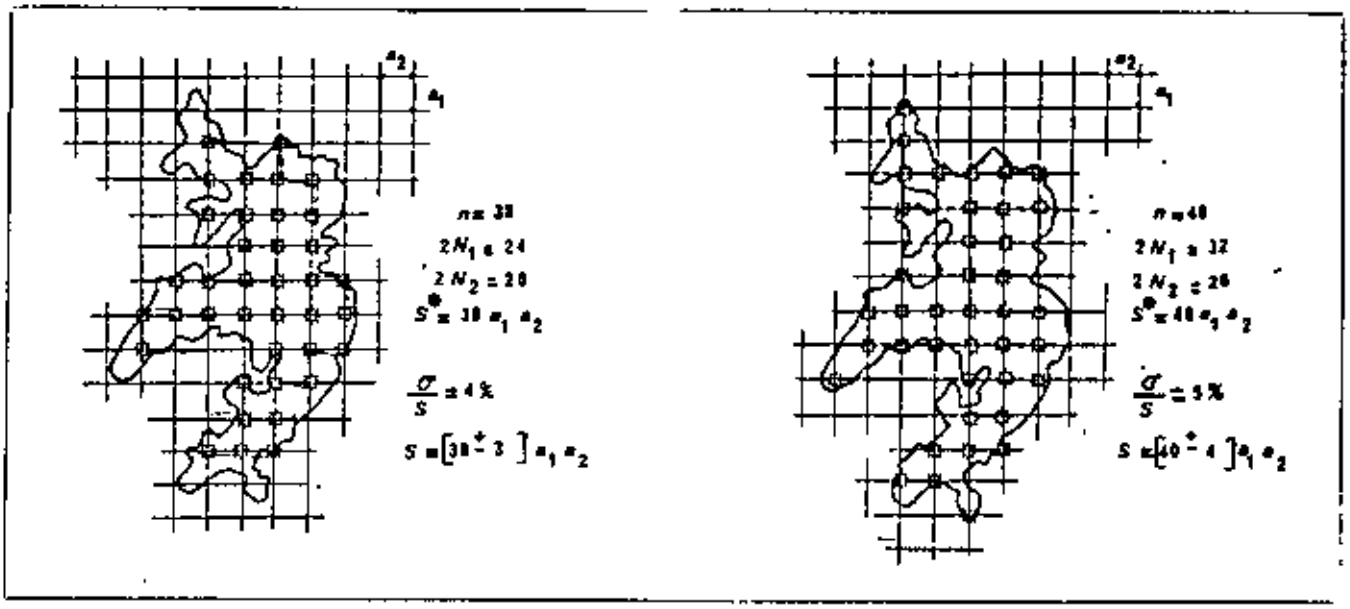


FIGURE 2 — Estimation of surface with two distinct origins.

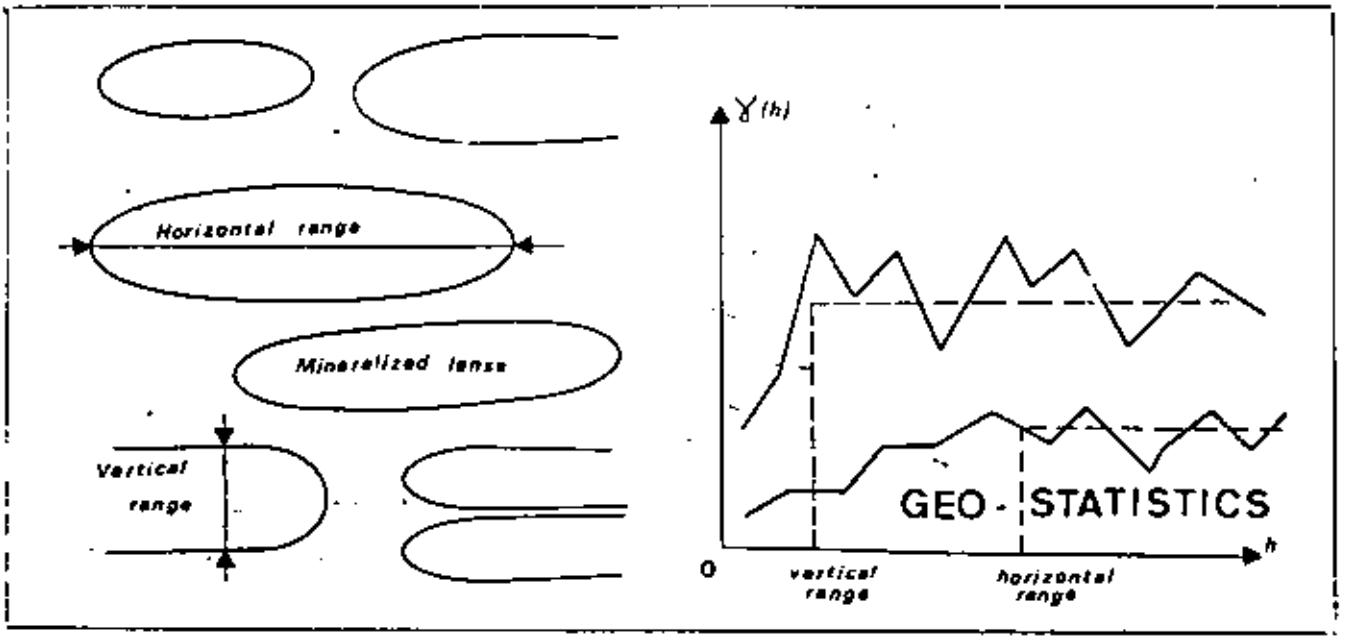


FIGURE 3 — Variogram of a structural function.

Zone of Influence — Let us fix the direction α. In general, the variogram increases with the modulus $|h|$; the values taken at two distinct points are generally more and more different in mean value as the points become farther apart (see Fig. 3).

The nature of this crescent-shaped pattern gives a precise meaning to the traditional notion of zone of sample influence. It often happens that this crescent pattern flattens beyond a certain distance, which we will call the range; i.e., $\gamma(h)$ becomes constant. When the mean variation of the grades of two samples no longer depends on the distance between them, the state of independence of samples postulated by statistics without spatial correlation is attained. It is not only the sill of such a variogram, i.e. practically the variance of dispersion of sample grades, that characterizes our phenomenon, but the entire curve $\gamma(h)$, especially the part — $|h| <$ range a — within the zone of influence.

It is essential to note that geostatistics proves the classical results of statistics without spatial correlations as particular cases in making: $\gamma(h) =$ Constant A $|h| \neq 0$.

Anisotropies — Let us consider the direction α of the vector $\vec{h}$. If directional anisotropies are suspected, the variogram ought to be constructed along several directions in space. Studies of the variations of $\gamma(h)$ when the direction α is varied permit the recognition of anisotropies and their characterization[11]. In such a particular case, one may finally arrive at a model of the type $\gamma(h_1, h_2, h_3) = \gamma_1(\sqrt{h_1^2 + h_2^2}) + \gamma_2(|h_3|)$, the 3-dimensional variogram being the sum of a horizontal isotropic structure γ_1 and a vertical structure γ_2.

Local Stationarity — Very often, it is not necessary in practice to know the variogram for all distances $|h|$. In general, it is sufficient to know its behaviour at the origin, then its growth (with or without sill) up to a certain distance ($|h| = b$) useful for making estimations. One is then content with local stationarity of the differences $[Y(x+h)-Y(x)]$ within the sliding neighbourhood of extent b. In particular, an effect of drift (non-stationarity) that would be apparent only beyond distance b is unimportant [for a theoretical justification see ref. (40), p. 142].

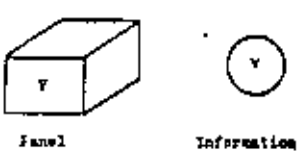
Support — The experimental variograms, as well as the data from which they have been calculated, are based on a certain support; for instance, a segment of core of fixed diameter and length. Changing the support corresponds to a simple formalism that allows geostatistics to deduce γ_p (panels) from the experimental γ_c (cores), or γ (punctual) from the same γ_c [see ref. (40), p. 60]. It is easy to point out the *extreme* practical importance of this formalism: a deposit is not exploited by cores, and it is quite evident that the laws of correlations and dispersions between panels of thousands of tons have nothing in common with corresponding laws relating to cores of several pounds. This example of common sense does not seem to have been understood by authors who pretend to perform selections or estimations of selected reserves only from the properties of dispersion of their available samples. We shall return to this important point (see "Estimation of Reserves").

ESTIMATION OF RESOURCES (IN SITU)

It is clear that the estimation of global resources of a deposit depends on:

(1) — available information, not only on quantity but also on relative positions — a regular systematic grid does not give the same information as does a preferential or random grid;

(2) — the underlying structure of the studied characteristics — for example, the thickness of a mineralized zone will often have a character of continuity more favourable for estimation than the corresponding variables of metal accumulation or mean grade.



When estimating the mean grade Y_V of a panel of volume V by the mean Y_v of available surrounding data, the error committed, $Y_V - Y_v$ appears as a random variable for which the variance can be calculated

$$\sigma_E^2 = E[(Y_V - Y_v)^2]$$

This variance is independent of the particular numerical values of the information used. It is expressed solely with the help of the punctual variogram 2γ for the variable considered [ref. (40), p. 64]:

$$\sigma_E^2 = 2\bar{\gamma}(V, v) - \bar{\gamma}(V, V) - \bar{\gamma}(v, v)$$

$\bar{\gamma}(V,v)$ designates the mean value of $\gamma(h) = \gamma(MM')$ when the two extremities M and M' of the vector $\vec{h}$ describe, independently from each other and respectively, the two sets V and v. The mean values $\bar{\gamma}$ are calculated either directly if the two sets V and v are of simple geometry, or by numerical integration with the aid of a computer.

Note that the mere writing of this formula takes into account four essential, and moreover intuitive, facts that condition all estimations. The quality of an estimation depends on:

(1) geometry of the volume to be estimated — $\bar{\gamma}(V,V)$;

(2) relative distances between panels to be estimated and information used for estimation — term $\bar{\gamma}(V,v)$;

(3) geometry of the information itself — term $\bar{\gamma}(v,v)$; and

(4) degree of regularity of the phenomenon studied — utilization of the structural tool γ.

Let us point out that, in the particular case of spatial independence, $\gamma(h)$ = Constant for $|h| > 0$; this general formula corresponds to the variances in $\frac{\sigma^2}{N}$ of classical statistics.

Composition of Terms — In a global estimation — V representing the entire deposit or a significant part of it and v a group of data with different supports (drill cuttings, channel samplings along drifts, etc.) — the preceding exact formula quickly becomes highly complex. Approximation formulae are then defined, calling on the principle of the composition of elementary estimators. Thus, in the estimation of a mass by levels (drifts and channel samples), it is possible to distinguish and then recombine:

(1) the estimation of each drift from its channel samples;
(2) the estimation of each level from its drifts, supposed to be perfectly known; and
(3) finally, the estimation of each slice of the mass from intermediate levels (or two extreme levels) supposed to be perfectly known.

In a global estimation, it would serve no useful purpose to increase the number of drift samples if the number of drifts and also the number of levels required to explore the orebody are not increased correspondingly. Knowing the price per foot of workings and the price of sampling, this composition of terms of estimation permits the best distribution of an exploration budget involving various types of workings, sampling, drilling, etc.

Let us emphasize that the calculation of an estimation variance only requires knowledge of the structural function (γ) and the geometries of data. Consequently, it can be calculated prior to actually realizing the data. Thus, the improvement in knowledge and the cost of this improvement can be compared before the additional work is actually undertaken. Several possible exploration schemes could even be compared.

Kriging (see Fig. 4) — In particular, the possibility of calculating the estimation variance of all estimators permits the definition of the best linear estimator that can be deduced from any given data. The formal procedure is called "Kriging" [see ref. (40), (3), (10) and (9)].

Let us give an example: an estimate of panel P is required. Available information consists of drill holes both inside and outside P. In our example, we assume that we have one segment of interior core (S_i) and seven segments of exterior cores (S_{e_1}, .. S_{e_7}). What weight must be given to each piece of information to define the best estimator of the mean unknown grade (Z) of panel P? Kriging furnishes the estimator Z^*, a linear combination of the eight available data:

$$Z^* = \lambda_i S_i + \sum_{j=1}^{7} \lambda_j S_{e_j}$$

This estimator, when it is interpreted as a random function, has the following characteristics:

(1) it is unbiased, i.e. the mean error is zero — $E\{Z-Z^*\} = 0$;
(2) it is optimal, i.e. the weights are determined in such a way that the estimation variance $E\{[Z-Z^*]^2\}$ will be minimal.

The system of kriging that permits determination of the weights, λ, as well as providing a corresponding minimum estimation variance, is a system of linear equations involving only integrals of the function γ.

In our example, and in the hypothesis of structural isotropy, kriging reconfirms:

(1) the evident symmetries, such as $\lambda_{e_1} = \lambda_{e_2}$ and $\lambda_{e_3} = \lambda_{e_6}$;
(2) the inequalities, such as $\lambda_i \geq \lambda_{e_j}$ for any j, and $\lambda_{e_4} \leq \lambda_{e_5}$; S_{e_4} forms a screen to the influence of S_{e_5}; $\lambda_{e_7} > \lambda_{e_4}$, and S_{e_4} has transferred part of its influence to nearby data S_{e_3}, S_{e_5}, S_{e_6}.

The importance of phenomena such as "screen effect" and "transfer of influence" depends on the regularity of the regionalization; i.e., the regularity of the variogram 2γ.

Kriging procedures permit programming (by day, month or year according to the size of the estimated panels) of mine production, and facilitates the selection problem; i.e. the definition of reserves.

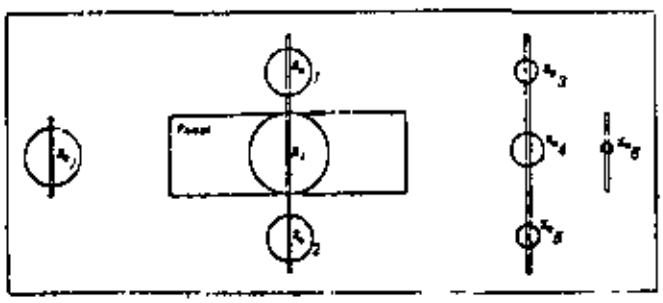


FIGURE 4 — Kriging best linear estimator.

ESTIMATION OF RESERVES (SELECTION)[10]

The total in-situ resources of a deposit can rarely be exploited in a non-selective manner. Various possible selections must be studied in order to judge the feasibility of working a deposit or part thereof, or to decide on the best method of exploitation. Interest is then focused on these selected reserves. In order to tackle the study of selections, the comprehension of a preliminary essential notion is required: *reality is unknown, and an estimator does not behave like reality.*

It is well known that the grade of a core (Y_c) is not the grade (Z_P) of the panel that contains it: in particular, dispersion of grades (Y_c) of cores in the deposit D is much greater than the dispersion of grades (Z_P) of panels in the same deposit. Thus, in a gold deposit, the grade (Y_c) can vary from 0 to 100% (nugget), but the mean grade Z_P of a panel of several cubic meters will vary a great deal less; for example, between 10^{-7} and 8×10^{-6} (0,1 to 8 g/ton). Although centered on the same mean, $E\{Y_c\} = E\{Z_P\} = m$ = mean grade of the deposit D, the two dispersion laws of Y_c and Z_P are different; they are linked by *Krige's additivity relationship*, found experimentally by Krige in South Africa and formalized by Matheron:

$$D^2(c/D) = D^2(c/P) + D^2(P/D)$$

The dispersion variance of grades Y_c of cores of fixed volume within the deposit D is equal to the dispersion variance of these grades (Y_c) of cores within a panel P *plus* the dispersion variance of grades Z_P of panels P within the deposit D (see Fig. 5 and Appendix).

Thus, the mean grade $E(Z_P) = m$ of panels P can be estimated, and the dispersion $D^2(P/D)$ of these grades can be determined either:

— by a formal geostatistical calculation

$$D^2(P/D) = \bar{\gamma}(D, D) - \bar{\gamma}(P, P)$$

[see ref. (40)]

— or with the aid of Krige's relationship in estimating experimentally the two terms $D^2(c/D)$ and $D^2(c/P)$. However, nothing is known about the type of the dispersion law of Z.

Certain authors adapt particular theoretical laws (lognormal, gamma, etc.) to the experimental histograms of available core grades Y_c; then, postulating a principle of preservation of law, they assume that the law for panel grades Z is of the same type. In particular, the postulate of preservation of lognormality is very common. In fact, in a great many low-grade deposits (Ag, Cu, Ni, U), the experimental dispersion of core grades is approximated closely enough by a lognormal law. However, there is no theoretical justification to the postulate of preservation of lognormality; on the contrary, numerous experimental examples refute it. The grades (Z) of panels often present a law more or less similar to a truncated normal distribution (see Fig. 6). Y_c, available grade of core, is not the true unknown grade (Z) of panel P; likewise, any estimator (Z^*) formed from the information $\{Y_c(x_i),\ x_i \in \text{Information}\}$ is not the true unknown grade Z.

If estimator Z^* is unbiased, we have by definition:

$$E\{Z^*\} = E\{Z\} = m$$

The two laws of dispersion of Z and Z^*, although centered on the same mean, are different in type of laws and in variances (see Fig. 6). What of the two dispersion variances?

If and only if the estimator Z^* is the optimal unbiased estimator of Kriging, Z_K^*, the "*dispersion relationship*" can be demonstrated with the minimum estimation variance σ_K^2:

$D^2(Z) = D^2(Z_K^*) + \sigma_K^2 \sim B$ (under stationary hypothesis)

$D^2(Z) = D^2(P/D)$: dispersion variance of true panel grades within D

$D^2(Z_K^*) = D^2(Z_K^*/D)$: dispersion variance of kriged panel grades within D

$\sigma_K^2 = E\{|Z - Z_K^*|^2\}$: estimation variance of Z by Z_K^*

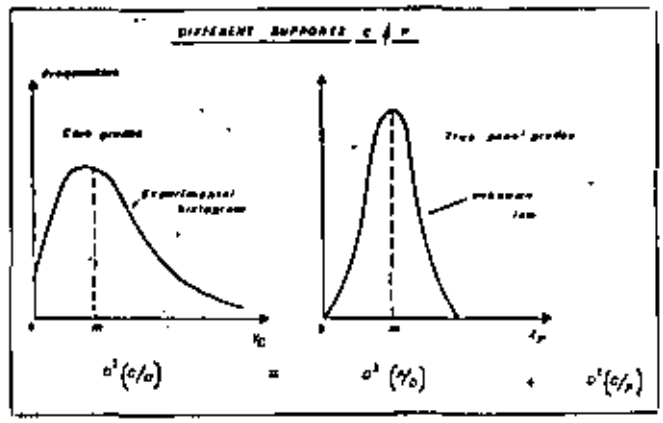


FIGURE 5 — Krige's additivity relationship.

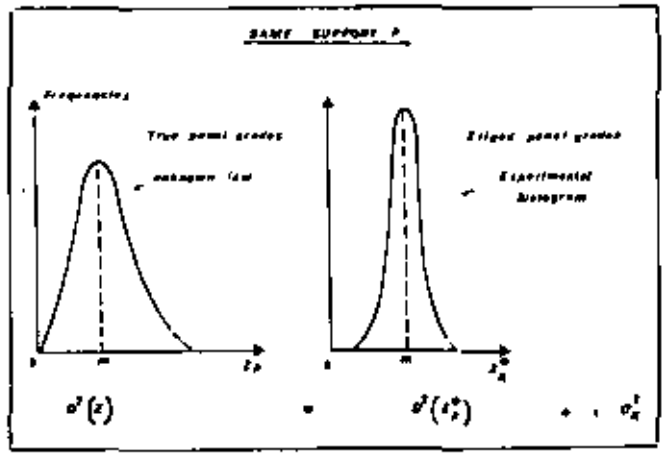


FIGURE 6 — Dispersion relationship.

B is a positive corrective term that can be neglected when the true mean value (m) is fairly well estimated (see Fig. 6 and also the Appendix). Then:

$$D^2(Z) = D^2(Z_K^*) + \sigma_K^2 \text{ and } D^2(Z) \geq D^2(Z_K^*)$$

Therefore, it can be seen that the available experimental dispersion $D^2(Z_K^*)$ differs from the true dispersion $D^2(Z)$, and all the more as the estimation of Z by Z_K^* becomes poorer, i.e., as σ_K^2 becomes larger.

Regarding types of dispersion laws, if a theoretical law can be adapted to the available experimental histogram of estimators Z_K^*, still nothing is known of the dispersion law of the true values of Z.

It is essential to see that the dispersion law of the estimators (Z_K^*) evolves with the level of information. As this information becomes more and more complete, Z_K^* is elaborated more and more precisely by kriging with, σ_K^2 decreasing, i.e. the dispersion $D^2(Z_K^*)$ tends to approach the true dispersion $D^2(Z)$ closer and closer. When a level of information is attained, such that the relative difference

$$\frac{D^2(Z) - D^2(Z_K^*)}{D^2(Z)} = \frac{\sigma_K^2}{D^2(Z)}$$

is less than 20%, it could be justifiable in practice to hypothesize the preservation of the type of dispersion law between Z_K^* and Z. Note that the two laws (Z_K^* and Z) refer to the same support, the volume of panel P.

Having noted these differences among Y_c, Z and Z_K^*, let us return to the specific problem of selection. Take as an example a selection by cut-off grade g_c: each panel P of defined geometry is retained if its mean grade is higher than g_c (see Fig. 7). Which mean grade of P is important?

Not the true unknown grade Z, and not the grade Z_i^* estimated from actual information[1], even if this estimation is the optimal kriged Z_K^*, but the ultimate estimated grade Z_v^* available at the moment of the effective selection. It is at this ultimate instant that the decision will be made whether or not to exploit panel P on the criterion that its estimator Z_v^* is higher or lower than the cut-off grade g_c. Therefore, if at the instant[1] it is wished to estimate available reserves with a cut-off g_c on defined units P, the single dispersion $D^2(Z_v^*)$ must be considered, (see Fig. 7) and not $D^2(Z)$ or $D^2(Z_i^*)$, or still less $D^2(Y_v)$.

Experimentally, however, only the dispersion law fitted to Z_i^* is known. How can we obtain a useful dispersion law for Z_v^* ? We have to evaluate the ultimate available information at the moment of effective selection and deduce the coresponding kriging variance σ_K^2 (we have seen that calculation of σ_K^2 can precede actual realization of the ultimate data). The dispersion variance $D^2(Z)$ of true grades is determined either with the aid of Krige's relation or directly with the aid of the variogram. The dispersion relationship then provides:

$$D^2(Z_v^*) = D^2(Z) - \sigma_K^2$$

If $D^2(Z_i^*)$ is not too different from $D^2(Z_v^*)$, the justifiable hypothesis of conservation of type of dispersion law between Z_i^* and Z_v^* can be made. The sought-after theoretical dispersion law of Z_v^* is then described in mean $E\{Z_v^*\} = E\{Z_i^*\}$, variance $D^2(Z_v^*)$ and type of law. The cut-off grade (g_c) is then applied to this law $D^2(Z_v^*)$ to evaluate the proportion of panel P that the effective selection will conserve. Similar selections are repeated for various hypotheses, different selection parameters (thickness, stripping ratio, impurities, grades, etc.), different levels of ultimate available information, different panel sizes, etc. Comparison of results obtained from these various selection hypotheses allows a judgment to be made of the feasibility of exploiting a deposit. A decision can then be made as to the type of selection required and, if this selection is imposed *a priori*, the ultimate information required for optimization can be determined.

We emphasize the rigorous formalization that geostatistics brings to this difficult problem of estimation of selected reserves. In such problems, rigor is essential, its omission resulting in possible serious biases (systematic overestimation) to reserve estimations.

STUDY OF FLUCTUATIONS AND CONDITIONAL SIMULATIONS

One final problem remains, that of the study of characteristic fluctuations of exploited reserves. The fluctuation of grades (y) at every scale (day — month —year) is of direct interest to the millman or the chemist. Similarly, fluctuations in thickness of cover concern the mining engineer dealing with ore extraction. For such problems, a perfect knowledge of reality would be required, i.e. the knowledge of the real "grade surface" $y(x)$ at every location ($x \in$ Deposit). For lack of such perfect knowledge, geostatistics proposes a surface, $y_s(x)$, that

— meets the true surface $y(x)$ at each location (x_i) of experimental data;

$$y_s(x_i) = y(x_i) \quad \forall\, i \in \text{Information}$$

— presents the same characteristics of fluctuation as the real surface (see Fig. 8); and

— in the case of stationarity, simulated values $y_s(x)$ can be such that they reproduce the histogram of experimental data $\{y(x_i),\ i \in I\}$.

Surface $y_s(x)$ is called the "conditional simulation" of reality $y(x)$. In probabilistic terms, reality $y(x)$ and simulation $y_s(x)$ are two different realizations of the *same* random function $Y(x)$, $Y(x)$ being characterized by its first two moments (mean and variogram) estimated from the experimental data $\{y(x_i),\ i \in I\}$. Reality and simulation can be considered as two variants of the *same* mineralized phenomenon $Y(x)$.

Simulation $y_s(x)$ is made on a regular grid as dense as desired, the only limitation being that of the computer. For instance, given a 100- by 100-m grid of available data, a simulation can be realized on a grid as fine as 5 by 5 m, or even finer if desired. To this new simulated — and quasi-perfectly known — reality, one can apply a production scheme (overburden workings, selection, haulage, storage, etc. and study the consequences of this scheme in the mill, the fluctuations of grades at the mill entry, the homogenization of a storage bin or the necessity of exploiting several workings at once.

We must stress the fact that a simulation $y_s(x)$ is not an estimator $y^*(x)$. On the average, an estimator $y^*(x)$ must be as close as possible to the unknown real

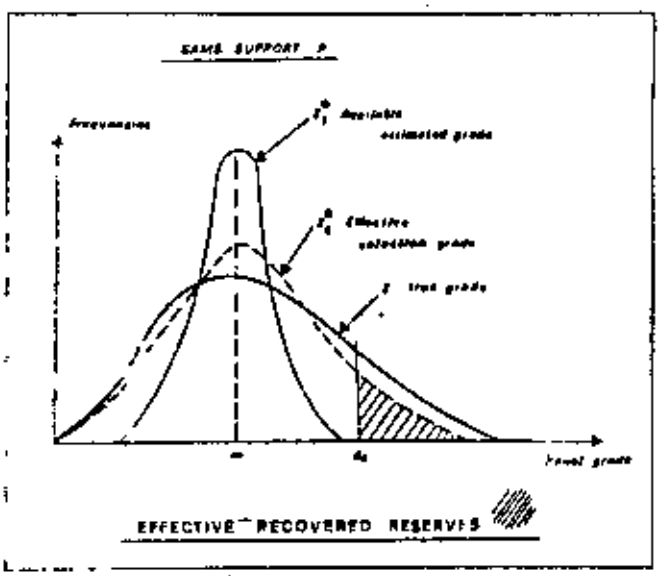


FIGURE 7 — Effective recovered reserves.

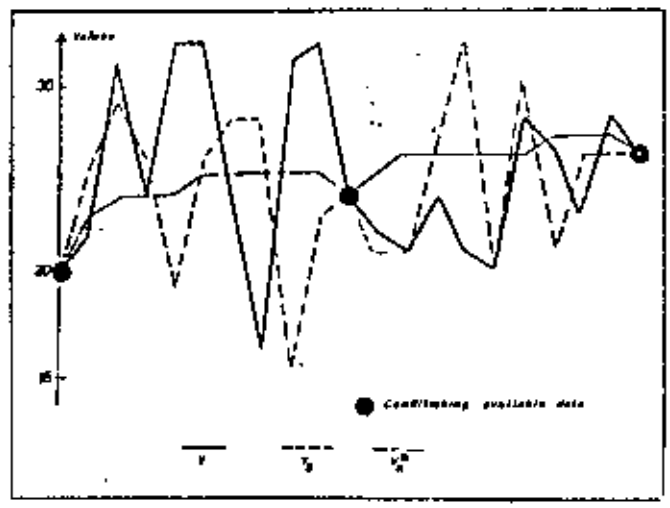


FIGURE 8 — Reality - Simulation - Kriging.

value $y(x)$; i.e. the estimation variance or quadratic mean error should be minimized:

$E\{[Y(x)-Y^*(x)]^2\}$ minimal, the estimator y^* is then a kriging.

However, this estimator $y^*(x)$, although optimal, no longer presents the same structural characteristics as does reality $y(x)$. In particular, the estimated phenomenon $y^*(x)$ is much smoother than reality $y(x)$ (see Fig. 8): $D^2(Y^*) \leq D^2(Y)$ in terms of dispersion-variances.

On the contrary, the simulation $y_s(x)$ does not pretend to be an optimal estimator of reality $y(x)$, but simply presents the same structural characteristics as this reality. In particular, the dispersion of simulated values is equal to the dispersion of real values:

$$D^2(Y_s) = D^2(Y)$$

In estimation problems, we are interested in estimation variances that we attempt to minimize (kriging). In fluctuation problems, we are interested in dispersion variances that we attempt to identify (conditional simulations).

NOTE: The technique of Conditional Simulation was originated by G. Matheron and the Centre de Morphologie Mathématique (see ref. [12], 1972 and [41], 1973). A detailed presentation of Conditional Simulations is given in [22]. Applications of these techniques are given in [18], [1] and [7].

COREGIONALIZATIONS[40]

Until now, we were interested in a single regionalized variable, for example the grade $y(x)$ at point x; we characterized the autocorrelation of $y(x)$ by the structural variogram function:

$$2\gamma(h) = E\{[Y(x+h) - Y(x)]^2\}$$

However, the formalism of all preceding results can be generalized, and two different regionalized variables can be simultaneously studied; for instance, the Pb grades $y_1(x)$ and the Zn grades $y_2(x)$. Each one of these two variables possesses its own autocorrelation variogram function:

$$2\gamma^1(h) = E\{[Y_1(x+h) - Y_1(x)]^2\} \qquad \text{for Pb}$$

$$2\gamma^2(h) = E\{[Y^2(x+h) - Y_2(x)]^2\} \qquad \text{for Zn}$$

However, we know that some cross-correlation exists between grades of Pb and Zn. We will thus define and estimate the cross-variogram for Pb and Zn:

$$2\gamma_{12}(h) = E\{[Y_1(x+h)-Y_1(x)]\,[Y_2(x+h)-Y_2(x)]\}$$

which characterizes the spatial cross-correlation between the differences of the two variables lead and zinc. If classical authors admit the existence of local correlation ($h = 0$) between the two variables $y_1(x)$ and $y_2(x)$,

$$\rho_{12} = \frac{E\{Y_1(x)\cdot Y_2(x)\}}{\sqrt{E\{Y_1^2(x)\}\cdot E\{Y^2(x)\}}} \quad \text{coefficient of correlation,}$$

they ignore in general the spatial correlation between $y_1(x)$ and $y_2(x)$, just as they ignore the spatial autocorrelation of variables $y_1(x)$ and $y_2(x)$.

Knowledge of the matrix $\begin{bmatrix}\gamma_1 & \gamma_{12}\\ \gamma_{21} & \gamma_2\end{bmatrix}$ or more generally with n variables $\{\gamma_{ij}\}$ $i, j = 1, n$ permits cross co-estimations. For example:

(1) the estimation of an unknown Pb grade $y_1(x)$ at location x with the aid of neighbouring grades of both Pb and Zn;

(2) In the case of a heterogeneous reconnaissance, the estimation of a single variable with the aid of two different groups of information — there are two types of Fe analyses: $y_1(x_i)$, precise core grades, and $y_2(x_i)$, imprecise grades from blast-hole cuttings.

APPENDIX — The Two Notions of Dispersion Variance and Estimation Variance

Let us consider a deposit D composed of five panels (x_1 to x_5), and give the practical calculations of the two types of variances.

P / x_1		i	true $Z(x_i)$	estim. $Z^*(x_i)$	$(Z-m)^2$	$(Z^*-m)^2$	$(Z-Z^*)^2$
		1	5	6	0	1	1
x_2	x_4	2	7	6	4	1	1
		3	6	4	1	1	4
		4	2	4	9	1	4
x_3	x_5	5	5	5	0	0	0
		Sum	25	25	14	4	10

$$m = m^* = \frac{25}{5} = 5$$

mean value

Dispersion variance of true values	Dispersion variance of estimated values
$D^2(P/D) \sim \frac{1}{5} \times$	$D^2(P^*/D) \sim \frac{1}{5} \times$
$\Sigma\,[Z(x_i)-m]^2 = 2.8$	$\Sigma\,[Z^*(x_i)-m^*] = 0.8$

Unbiased estimation: $\frac{1}{5}\Sigma\,[Z(x_i) - Z^*(x_i)] = 0$

Estimation variance: $\sigma_E^2 \sim \frac{1}{5}\Sigma\,[Z(x_i)-Z^*(x_i)]^2 = 2$

Note: the dispersion relationship: $D^2(P/D) = D^2(P^*/D) + \sigma_E^2$ the estimator smoothes reality: $D^2(P^*/D) < D^2(P/D)$

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