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# COMPORTAMIENTO DE PILOTES SOMETIDOS A CARGA LATERAL

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# BEAM ON ELASTIC FOUNDATION

## Differential Equation

$$\frac{d^2y}{dx^2} = \frac{M}{EI}$$ Basic D.E. for flexure

$$\frac{d^4y}{dx^4} = \frac{p}{EI}$$ Assuming EI is constant

$$p = -E_s y$$

$$\beta^4 = \frac{E_s}{4EI}$$

$$\frac{d^4y}{dx^4} + 4\beta^4 y = 0$$

## Solution of Differential Equation

$$(D^4 + 4\beta^4)y = 0$$

$$m^4 + 4\beta^4 = 0$$

$$m_1 = -m_3 = \beta(1 + i)$$

$$m_2 = -m_4 = \beta(-1 + i)$$

$$y = e^{\beta x}(C_1 \cos \beta x + C_2 \sin \beta x) + e^{-\beta x}(C_3 \cos \beta x + C_4 \sin \beta x)$$

Derivatives

$$\frac{dv}{dx} = \beta e^{\beta x} ( C_1 \text{ Cos } \beta x + C_2 \text{ Sin } \beta x - C_1 \text{ Sin } \beta x + C_2 \text{ Cos } \beta x) + \beta e^{-\beta x}$$

$$(-C_3 \text{ Cos } \beta x - C_4 \text{ Sin } \beta x - C_3 \text{ Sin } \beta x + C_4 \text{ Cos } \beta x)$$

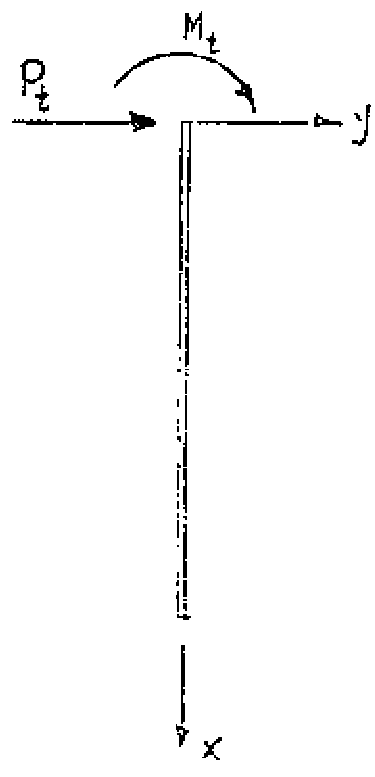
$$\frac{d^2y}{dx^2} = 2 \beta^2 e^{\beta x} (C_2 \text{ Cos } \beta x - C_1 \text{ Sin } \beta x) + 2 \beta^2 e^{-\beta x} (C_3 \text{ Sin } \beta x - C_4 \text{ Cos } \beta x)$$

$$\frac{d^3y}{dx^3} = 2 \beta^3 e^{\beta x} (C_2 \text{ Cos } \beta x - C_1 \text{ Sin } \beta x - C_2 \text{ Sin } \beta x - C_1 \text{ Cos } \beta x)$$

$$+ 2 \beta^3 e^{-\beta x} (-C_3 \text{ Sin } \beta x + C_4 \text{ Cos } \beta x + C_3 \text{ Cos } \beta x + C_4 \text{ Sin } \beta x)$$

$$\frac{d^4y}{dx^4} = 4 \beta^4 e^{\beta x} (-C_2 \text{ Sin } \beta x - C_1 \text{ Cos } \beta x) + 4 \beta^4 e^{-\beta x} (-C_3 \text{ Cos } \beta x - C_4 \text{ Sin } \beta x)$$

Pile of Infinite Length




- Boundary Conditions

at $x = 0$, $\frac{d^2y}{dx^2} = \frac{M_t}{EI}$

$$\frac{d^3y}{dx^3} = \frac{P_t}{EI}$$

at $x$ = very large, deflections must be small

Pile of Infinite Length (Continued)

For large x

$C_1 = C_2 = 0$

Applying $\frac{d^2y}{dx^2} = \frac{M_t}{EI}$

$$C_4 = -\frac{M_t}{2EI\beta^2}$$

Applying $\frac{d^3y}{dx^3} = \frac{P_t}{EI}$

$$C_3 + C_4 = \frac{P_t}{2EI\beta^3}$$

With the determination of the coefficients substitutions can be made and relevant equations derived as shown following.

Timoshenko says the "long" pile solution is satisfactory where $\beta L \geq 4$.

Substituting

$$y = \frac{e^{-\beta x}}{2EI\beta^2}\left[\frac{P_t}{\beta}\cos\beta x + M_t(\cos\beta x - \sin\beta x)\right]$$

$$S = -e^{-\beta x}\left[\frac{2P_t\beta^2}{E_s}(\sin\beta x + \cos\beta x) + \frac{M_t}{EI\beta}\cos\beta x\right]$$

$$M = e^{-\beta x}\left[\frac{P_t}{\beta}\sin\beta x + M_t(\sin\beta x + \cos\beta x)\right]$$

$$V = e^{-\beta x}\left[P_t(\cos\beta x - \sin\beta x) - 2M_t\beta\sin\beta x\right]$$

$$p = 2\beta e^{-\beta x}\left[-P_t\cos\beta x - M_t\beta(\cos\beta x - \sin\beta x)\right].$$

It is convenient to define some functions which make it easier to write the above equations. These are:

$$A_1 = e^{-\beta x} (\cos \beta x + \sin \beta x)$$

$$B_1 = e^{-\beta x} (\cos \beta x - \sin \beta x)$$

$$C_1 = e^{-\beta x} \cos \beta x$$

$$D_1 = e^{-\beta x} \sin \beta x$$

Using these functions, the above equations become:

$$y = \frac{2P_t \beta}{E_s} C_1 + \frac{M_t}{2EI\beta^2} B_1$$

$$S = \frac{-2P_t \beta^2}{E_s} A_1 - \frac{M_t}{EI\beta} C_1$$

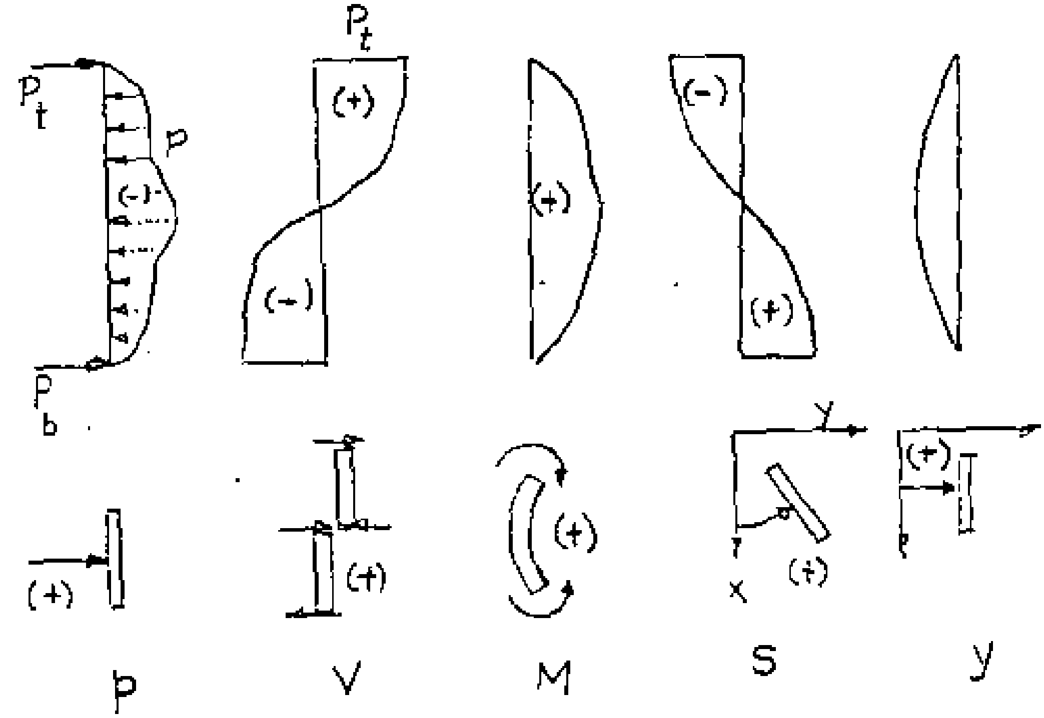
$$M = \frac{P_t}{\beta} D_1 + M_t A_1$$

$$V = P_t B_o - 2M_t \beta D_1$$

$$p = -2P_t \beta C_1 - 2M_t \beta^2 B_1$$

A table giving values for $A_1$, $B_1$, $C_1$, and $D_1$, is shown on page 11.

Sign Conventions



## Pile of Finite Length

Boundary Conditions

at $x = 0$ $\qquad M = M_t$ or $\frac{d^2y}{dx^2} = \frac{M_t}{EI}$ (A)

$\qquad V = P_t$ or $\frac{d^3y}{dx^3} = \frac{P_t}{EI}$ (B)

at $x = L$ $\qquad M = 0$ or $\frac{d^2y}{dx^2} = 0$ (C)

$\qquad V = 0$ or $\frac{d^3y}{dx^3} = 0$ (D)

Applying B.C. (A)

$$\frac{M_t}{EI} = 2\beta^2 (C_2 - C_4) \tag{1}$$

Applying B.C. (B)

$$\frac{P_t}{EI} = 2\beta^3 (C_2 - C_1) + 2\beta^3 (C_4 + C_3) \tag{2}$$

$$= 2\beta^3 (-C_1 + C_2 + C_3 + C_4) \tag{3}$$

Applying B.C. (C)

$$0 = 2\beta^2 e^{\beta L} (C_2 \text{ Cos } \beta L - C_1 \text{ Sin } \beta L) + 2\beta^2 e^{-\beta L} (C_3 \text{ Sin } \beta L - C_4 \text{ Cos } \beta L) \tag{4}$$

Pile of Finite Length (Continued)

Applying B.C. (D)

$$0 = 2\beta^3 e^{\beta L}(C_2 \cos\beta L - C_1 \sin\beta L - C_2 \sin\beta L - C_1 \cos\beta L)$$

$$+ 2\beta^3 e^{-\beta L}(-C_3 \sin\beta L + C_4 \cos\beta L + C_3 \cos\beta L + C_4 \sin\beta L)$$

$$C_2 - C_4 = A_1 \qquad (6)$$

(See Table of Symbols at end of this section)

$$-C_1 + C_2 + C_3 + C_4 = A_2 \qquad (7)$$

$$A_3C_2 - A_4C_1 + A_6C_3 - A_5C_4 = 0 \qquad (8)$$

$$A_7C_2 - A_8C_1 - A_8C_2 - A_7C_1 - A_{10}C_3 + A_9C_4 + A_9C_3 + A_{10}C_4 = 0 \qquad (9)$$

Rewriting (9) $-A_{11}C_1 + A_{12}C_2 + A_{13}C_3 + A_{14}C_4 = 0$ (10)

Eqs. 6, 7, 8 and 10 are the 4 equations to be solved

From Eq. 6

$$C_4 = C_2 - A_1 \qquad (11)$$

Substituting $C_4$ from Eq. (11) into Eqs. 7, 8, and 10.

Eq. 7 becomes

$$-C_1 + 2C_2 + C_3 = A_{15} \qquad (13)$$

Pile of Finite Length (Continued)

Eq. 8 becomes

$$-A_4C_1 + A_3C_2 + A_6C_3 - A_5C_2 + A_5A_1 = 0 \quad (14)$$

or

$$-A_4C_1 + A_{16}C_2 + A_6C_3 = A_{17} \quad (15)$$

Eq. 10 becomes

$$-A_{11}C_1 + A_{12}C_2 + A_{13}C_3 + A_{14}C_2 - A_1A_{14} = 0 \quad (16)$$

or

$$-A_{11}C_1 + A_{18}C_2 + A_{13}C_3 = A_{19} \quad (17)$$

Eqs. 13, 15, and 17 are now the 3 equations to be solved.

Eq. 13 becomes

$$C_1 = 2\ C_2 + C_3 - A_{15} \quad (18)$$

Sub. Eq. 18 into Eq. 15

$$-2\ A_4C_2 - A_4C_3 + A_4A_{15} + A_{16}C_2 + A_6C_3 = A_{17} \quad (19)$$

or

$$A_{20}C_2 + A_{21}C_3 = A_{22} \quad (21)$$

Sub. Eq. 18 into Eq. 17

$$-2\ A_{11}C_2 - A_{11}C_3 + A_{11}A_{15} + A_{18}C_2 + A_{13}C_3 = A_{19} \quad (22)$$

Pile of Finite Length (Continued)

or

$$A_{23}C_2 + A_{24}C_3 = A_{25} \qquad (24)$$

Eqs. 21 and 24 are now the 2 equations to be solved.

Solving for $C_2$ from Eq. 21

$$C_2 = \frac{A_{22}}{A_{20}} - \frac{A_{21}}{A_{20}} C_3 \qquad (25)$$

Sub. Eq. 25 into Eq. 24

$$\frac{A_{23}A_{22}}{A_{20}} - \frac{A_{23}A_{11}}{A_{20}} C_3 + A_{24}C_3 = A_{25} \qquad (26)$$

$$C_3 \left(- \frac{A_{21}A_{23}}{A_{20}} + A_{24}\right) = A_{25} - \frac{A_{22}A_{23}}{A_{20}} \qquad (27)$$

and

$$A_{27}C_3 = A_{26} \qquad (28)$$

Finally

$$C_3 = \frac{A_{26}}{A_{27}} \qquad (29)$$

This completes the derivation. See pages 10 and 11 for Table of Symbols.

Pile of Finite Length (Continued)

Table of Symbols

$A_1 = \dfrac{M_t}{2\beta^2 EI}$

$A_2 = \dfrac{V_t}{2\beta^3 EI}$

$A_3 = 2\beta^2 e^{\beta L} \text{Cos } \beta L$

$A_4 = 2\beta^2 e^{\beta L} \text{Sin } \beta L$

$A_5 = 2\beta^2 e^{-\beta L} \text{Cos } \beta L$

$A_6 = 2\beta^2 e^{-\beta L} \text{Sin } \beta L$

$A_7 = \beta A_3$

$A_8 = \beta A_4$

$A_9 = \beta A_5$

$A_{10} = \beta A_6$

$A_{11} = A_7 + A_8$

$A_{12} = A_7 - A_8$

$A_{13} = A_9 - A_{10}$

$A_{14} = A_9 + A_{10}$

$A_{15} = A_1 + A_2$

$A_{16} = A_3 - A_5$

$A_{17} = -A_1 A_5$

$A_{18} = A_{12} + A_{14}$

$A_{19} = A_1 A_{14}$

$A_{20} = -2A_4 + A_{16}$

$A_{21} = -A_4 + A_6$

$A_{22} = A_{17} - A_4 A_{15}$

$A_{23} = -2A_{11} + A_{13}$

$A_{24} = -A_{11} + A_{13}$

Table of Symbols (Continued)

$$A_{25} = A_{19} - A_{11}A_{15}$$

$$A_{26} = A_{25} - \frac{A_{22}A_{23}}{A_{20}}$$

$$A_{27} = -\frac{A_{21}A_{23}}{A_{20}} + A_{24}$$

Table of Functions for Pile of Infinite Length

| $\beta x$ | $A_1$ | $B_1$ | $C_1$ | $D_1$ | $\beta x$ | $A_1$ | $B_1$ | $C_1$ | $D_1$ |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 1.0000 | 1.0000 | 1.0000 | 0 | 2.4 | -0.0056 | -0.1282 | -0.0669 | 0.0613 |
| 0.1 | 0.9907 | 0.8100 | 0.9003 | 0.0903 | 2.6 | -0.0254 | -0.1019 | -0.0636 | 0.0383 |
| 0.2 | 0.9651 | 0.6398 | 0.8024 | 0.1627 | 2.8 | -0.0369 | -0.0777 | -0.0573 | 0.0204 |
| 0.3 | 0.9267 | 0.4888 | 0.7077 | 0.2189 | 3.2 | -0.0431 | -0.0383 | -0.0407 | -0.0024 |
| 0.4 | 0.8784 | 0.3564 | 0.6174 | 0.2610 | 3.6 | -0.0366 | -0.0124 | -0.0245 | -0.0121 |
| 0.5 | 0.8231 | 0.2415 | 0.5323 | 0.2908 | 4.0 | -0.0258 | 0.0019 | -0.0120 | -0.0139 |
| 0.6 | 0.7628 | 0.1431 | 0.4530 | 0.3099 | 4.4 | -0.0155 | 0.0079 | -0.0038 | -0.0117 |
| 0.7 | 0.6997 | 0.0599 | 0.3798 | 0.3199 | 4.8 | -0.0075 | 0.0089 | 0.0007 | -0.0082 |
| 0.8 | 0.6354 | -0.0093 | 0.3131 | 0.3223 | 5.2 | -0.0023 | 0.0075 | 0.0026 | -0.0049 |
| 0.9 | 0.5712 | -0.0657 | 0.2527 | 0.3185 | 5.6 | 0.0005 | 0.0052 | 0.0029 | -0.0023 |
| 1.0 | 0.5083 | -0.1108 | 0.1988 | 0.3096 | 6.0 | 0.0017 | 0.0031 | 0.0024 | -0.0007 |
| 1.1 | 0.4476 | -0.1457 | 0.1510 | 0.2967 | 6.4 | 0.0018 | 0.0015 | 0.0017 | 0.0003 |
| 1.2 | 0.3899 | -0.1716 | 0.1091 | 0.2807 | 6.8 | 0.0015 | 0.0004 | 0.0010 | 0.0006 |
| 1.3 | 0.3355 | -0.1897 | 0.0729 | 0.2626 | 7.2 | 0.0015 | -0.00014 | 0.00045 | 0.00060 |
| 1.4 | 0.2849 | -0.2011 | 0.0419 | 0.2430 | 7.6 | 0.00061 | -0.00036 | 0.00012 | 0.00049 |
| 1.5 | 0.2384 | -0.2068 | 0.0158 | 0.2226 | 8.0 | 0.00028 | -0.00038 | -0.0005 | 0.00033 |
| 1.6 | 0.1959 | -0.2077 | -0.0059 | 0.2018 | 8.4 | 0.00007 | -0.00031 | -0.00012 | 0.00019 |
| 1.7 | 0.1576 | -0.2047 | -0.0235 | 0.1812 | 8.8 | -0.00003 | -0.00021 | -0.00012 | 0.00009 |
| 1.8 | 0.1234 | -0.1985 | -0.0376 | 0.1610 | 9.2 | -0.00008 | -0.00012 | -0.00010 | 0.00002 |
| 1.9 | 0.0932 | -0.1899 | -0.0484 | 0.1415 | 9.6 | -0.00008 | -0.00005 | -0.00007 | -0.00001 |
| 2.0 | 0.0667 | -0.1794 | -0.0563 | 0.1230 | 10.0 | -0.00006 | -0.00001 | -0.00004 | -0.00002 |
| 2.2 | 0.0244 | -0.1548 | -0.0652 | 0.0895 | | | | | |

# LATERALLY LOADED PILES

by

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## LATERALLY-LOADED PILES

Piles must often support substantial lateral loads as well as axial loads. Examples of pile-supported structures which must support large lateral loads include marine structures, large advertising signs, and transmission towers.

While axially loaded piles may be designed satisfactorily by simple static methods, the design procedure for laterally loaded piles is more complex, involving the solution of a fourth-order differential equation. The solution must insure that conditions of equilibrium and compatibility are satisfied. The problem of laterally-loaded piles is further complicated because of the nonlinear soil response.

The differential equation to be solved, as derived from conventional beam theory (Hetenyi, 1946), is:

$$EI \frac{d^4y}{dx^4} + P_x \frac{d^2y}{dx^2} - p = 0 \quad (1)$$

where

$EI$ = flexural rigidity of pile

$y$ = deflection of pile

$x$ = length along pile

$P_x$ = axial load

$p$ = soil reaction per unit length.

Equation 1 may be solved very conveniently by a digital computer (Reese & Manoliu, 1973); however, non-dimensional methods may sometimes be employed to yield an acceptable solution (Matlock and Reese, 1961). Both solution methods give all the necessary design information including the moment, deflection, and shear at desired lengths along the pile.

SOIL RESPONSE

The behavior of the soil surrounding the laterally loaded pile is often described in terms of p-y curves, which relate the soil resistance to the pile deflection at various depths below the surface. A set of typical p-y curves are shown in Fig. 1. In general, these curves are non-linear and depend on several parameters, including depth, shearing strength of the soil, and number of load cycles.

For convenience in solving Eq. 1, a secant modulus of soil reaction, $E_s$, is often used.

$$E_s = p/y. \tag{2}$$

As may be understood from Fig. 1, $E_s$ can vary in an arbitrary manner with depth and with deflection; however, $E_s$ is often assumed to vary linearly with depth, i.e., $E_s = kx$. It is recommended that p-y curves be constructed as shown below and that the values of soil modulus be determined from those curves. Four experiments on full-sized, instrumented piles have led to methods for constructing p-y curves for the following cases: soft clays below the water surface, stiff clays below the water surface, stiff clays above the water table, and sands.

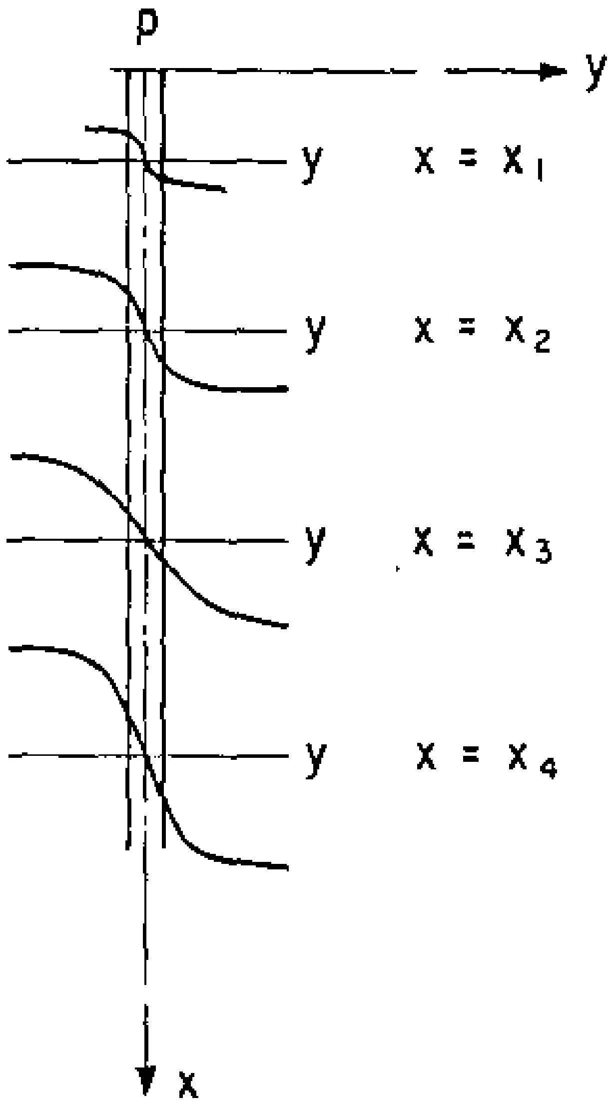



Fig. 1. Set of "p-y" Curves

p-y CURVES FOR SOFT CLAYS BELOW WATER SURFACE

Matlock (1970) presented procedures for developing p-y curves for soft clays below the water surface for two loading conditions: short-term static and cyclic. Those procedures, somewhat simplified are summarized below.

Short-Term Static Loading

1. Obtain the best possible estimate of the variation of shear strength and effective unit weight with depth. Also obtain the value of $\epsilon_{50}$, the strain corresponding to one-half the maximum principal stress difference. If no values of $\epsilon_{50}$ are available, typical values suggested by Skempton (1951) are given below in Table 1.

Table 1

| Consistency of Clay | $\epsilon_{50}$ |
|---|---|
| Soft | 0.020 |
| Medium | 0.010 |
| Stiff | 0.005 |

2. Compute the ultimate soil resistance per unit length of shaft, $p_u$, using the smaller of the values given by the equations below.

$$p_u = [3 + \frac{\gamma}{c} x + \frac{0.5}{b} x] \, cb \qquad (3)$$

$$p_u = 9\ cb \tag{4}$$

where

$\gamma$ = average effective unit weight from ground surface to p-y curve,

x = depth from ground surface to p-y curve,

c = shear strength at depth x,

b = width of pile.

The value of $p_u$ is computed at each depth where a p-y curve is desired, based on the shear strength at that depth.

3. Compute the deflection, $y_{50}$, at one-half the ultimate soil resistance from the following equation:

$$y_{50} = 2.5\ \epsilon_{50}\ b \tag{5}$$

4. Points describing the p-y curve are now computed from the relationship below.

$$p/p_u = 0.5\ (y/y_{50})^{1/3} \tag{6}$$

The value of p remains constant beyond $y=8y_{50}$

Cyclic Loading

1. Construct the p-y curve in the same manner as for short-term static loading for values of p less than 0.72 $p_u$.

2. Solve Equations 3 and 4 simultaneously to find the depth, $x_r$, where the transition occurs. If the unit weight and shear strength are constant in the upper zone, then

$$x_r = \frac{6cb}{(\gamma b + 0.5c)} \quad (7)$$

3. If the depth to the p-y curve is greater than or equal to $x_r$, then p is equal to 0.72 $p_u$ for all values of y greater than $3y_{50}$.
4. If the depth to the p-y curve is less than $x_r$, then the value of p decreases from 0.72 $p_u$ at $y = 3y_{50}$ to the value given by the expression below at $y = 15y_{50}$.

$$p = 0.72\ p_u\ (\frac{x}{x_r}) \quad (8)$$

The value of p remains constant beyond $y = 15y_{50}$.

p-y CURVES FOR STIFF CLAYS ABOVE THE WATER TABLE

Reese and Welch (1975) presented procedures for developing p-y curves for stiff clays above the water table. Methods are given for both short-term static and cyclic loadings.

Short-Term Static Loading

1. Obtain the best possible estimate of the variation of shear strength and effective unit weight with depth. Also obtain the value of $\varepsilon_{50}$, the strain corresponding to one-half the maximum

principal stress difference. If no value of $\varepsilon_{50}$ is available, use a value from Table 1, the larger value being more conservative.

2. Compute the ultimate soil resistance per unit length of shaft, $p_u$, using the smaller of the values given by Eqs. 3 and 4 (In the use of Eq. 3, the shear strength is taken as the average from the ground surface to the depth being considered).
3. Compute the deflection, $y_{50}$, at one-half the ultimate soil resistance from Eq. 5.
4. Points describing the p-y curve may be computed from the relationship below.

$$\frac{p}{p_u} = 0.5 \left(\frac{y}{y_{50}}\right)^{1/4} \qquad (9)$$

5. Beyond $y = 16y_{50}$, p is equal to $p_u$ for all values of y.

Cyclic Loading

1. Determine the p-y curve for short-term static loading by the procedure previously given.
2. Determine the number of times the design lateral load will be applied to the shaft.
3. For several values of $p/p_u$ obtain the value of C, the parameter describing the effect of repeated loading on deformation, from a relationship developed by laboratory tests, or in the absence of tests, from the following equation.

$$C = 9.6 \left(\frac{p}{p_u}\right)^4 \tag{10}$$

4. At the values of p corresponding to the values of $p/p_u$ selected in step 3, compute new values of y for cyclic loading from the following equation.

$$y_c = y_s + y_{50} \cdot C \cdot \log N \tag{11}$$

where

$y_c$ = deflection under N cycles of load

$y_s$ = deflection under short-term static load,

$y_{50}$ = deflection under short-term static load at one-half the ultimate resistance,

N = number of cycles of load application.

5. The $p$-$y_c$ curve defines the soil response after N cycles of load.

p-y CURVES FOR STIFF CLAYS BELOW WATER SURFACE

Reese et. al. (1975) present methods for obtaining p-y curves for stiff clays below the water surface. Methods are given below for both short-term static and cyclic loadings.

Short-Term Static Loading

1. Obtain values for undrained soil shear strengths c, soil submerged unit weight γ, and pile diameter b from ground surface to depth H.

2. Compute the average undrained soil shear strength $c_a$ over the depth H.
3. Use the following equations for computing soil resistance at depth H:

   a. Ultimate soil resistance near ground surface,

$$p_{c1} = 2\ c_a\ b + \gamma'\ b\ H + 2.83\ c_a\ H \tag{12}$$

   b. Ultimate soil resistance well below the ground surface,

$$p_{c2} = 11\ c\ b \tag{13}$$

   Use the smaller of Eq. 12 or 13 for the value of $p_c$.
4. Choose the appropriate value of A from Fig. 2 for the particular non-dimensional depth.
5. Establish the initial straight line portion of the p-y curve,

$$p = k\ x\ y \tag{14}$$

   Use the appropriate value of k from Table 2.
6. Compute the following:

$$y_c = \varepsilon_c\ b \tag{15}$$

   Use an appropriate value of $\varepsilon_c$ from Table 3.

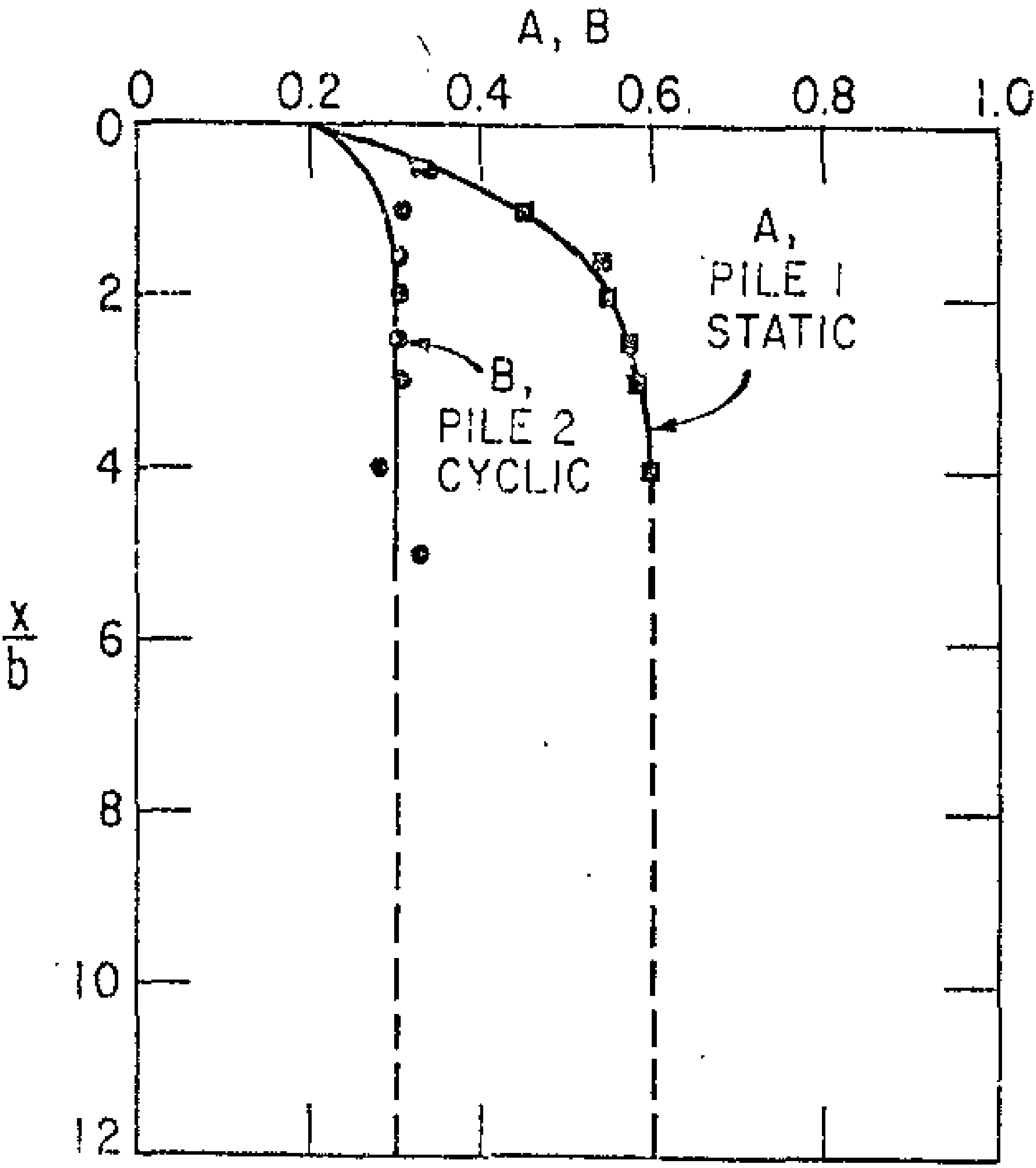



Fig. 2. Values of Constants A and B

TABLE 2

RECOMMENDED VALUES OF k FOR STIFF CLAYS

| | Average Undrained Shear Strength* ton/ft$^2$ | | |
|---|---|---|---|
| | 0.5-1 | 1-2 | 2.4 |
| $k_s$ (Static) lb/in$^3$ | 500 | 1000 | 2000 |
| $k_c$ (Cyclic) lb/in$^3$ | 200 | 400 | 800 |

*The average shear strength should be computed from the shear strength of the soil to a depth of 5 pile diameters.

TABLE 3

REPRESENTATIVE VALUES OF $\epsilon_c$ FOR STIFF CLAYS

| | Average Undrained Shear Strength ton/ft$^2$ | | |
|---|---|---|---|
| | 0.5-1 | 1-2 | 2-4 |
| $\epsilon_c$ (in./in.) | 0.007 | 0.005 | 0.004 |

7. Establish the first parabolic portion of the p-y curve,

$$p = 0.5\ p_c\ (\frac{y}{y_c})^{0.5} \qquad (16)$$

Intersection with Eq. 14 $\leq y \leq A\ y_c$ (if there is no intersection, Eq. 16 controls).

8. Establish the second parabolic portion of the p-y curve,

$$p = 0.5\ p_c\ (\frac{y}{y_c})^{0.5} - 0.055\ p_c\ (\frac{y - A\ y_c}{A\ y_c})^{1.25},$$

$$A\ y_c \leq y \leq 6\ A\ y_c \qquad (17)$$

9. Establish the next straight-line portion of the p-y curve,

$$p = 0.5\ p_c\ (6A)^{0.5} - 0.411\ p_c - \frac{0.0625}{y_c}\ p_c\ (y - 6\ A\ y_c),\ 6\ A\ y_c \leq y \leq 18\ A\ y_c$$

$$(18)$$

10. Establish the final straight-line portion of the p-y curve,

$$p = 0.5\ p_c\ (6A)^{0.5} - 0.411\ p_c - 0.75\ p_c\ A,\ 18\ A\ y_c \leq y \qquad (19)$$

Cyclic Loading

1. Steps 1, 2, 3, and 5 are the same as for the static case.

4. Choose the appropriate value of B from Fig. 2 for the particular non-dimensional depth.

6. Compute the following:

$$y_c = \varepsilon_c b \quad (20)$$

$$y_p = 4.1 A y_c \quad (21)$$

Use an appropriate value of $\varepsilon_c$ from Table 3.

7. Establish the parabolic portion of the p-y curve,

$$p = B p_c \left[1 - \left|\frac{y - 0.45 y_p}{0.45 y_p}\right|^{2.5}\right] \quad (22)$$

Intersection with Eq. 14 $\leq y \leq 0.6\ y_p$ (if there is no intersection, Eq. 22 controls).

8. Establish the next straight-line portion of the p-y curve,

$$p = 0.936 B p_c - \frac{0.085}{y_c} p_c (y - 0.6 y_p), \; 0.6 y_p \leq y \leq 1.8 y_p \quad (23)$$

9. Establish the final straight-line portion of the p-y curve,

$$p = 0.936 B p_c - \frac{0.102}{y_c} p_c y_p, \; 1.8 y_p \leq y \quad (24)$$

## p-y CURVES FOR SAND, SHORT-TERM STATIC AND CYCLIC LOADING

Reese, Cox, and Koop (1974) presented procedures for developing p-y curves for sand. Methods are given for both short-term static and for cyclic loadings.

1. Obtain values for significant soil properties and pile dimensions, $\phi$, $\gamma$, and b.
2. Make the following preliminary computations for use in the equations for computing soil resistance.

$$\alpha = \frac{\phi}{2},\ \beta = 45 + \frac{\phi}{2},\ K_o = 0.4,\ \text{and}\ K_a = \tan^2\left(45 - \frac{\phi}{2}\right)$$

3. Use the following equations for computing soil resistance:

a. Ultimate resistance near ground surface,

$$p_{ct} = \gamma H \left[\frac{K_o H \tan\phi \sin\beta}{\tan(\beta - \phi)\cos\alpha} + \frac{\tan\beta}{\tan(\beta - \phi)}(b + H \tan\beta \tan\alpha) + K_o H \tan\beta(\tan\phi \sin\beta - \tan\alpha) - K_a b\right] \quad (25)$$

b. Ultimate resistance well below the ground surface,

$$p_{cd} = K_a b\, \gamma H (\tan^8\beta - 1) + K_o b\, \gamma H \tan\phi \tan^4\beta \quad (26)$$

4. Find the intersection, $x_t$, of the equations for the ultimate soil resistance near the ground surface and the ultimate soil resistance well below the ground surface. Above this depth use Eq. 25. Below this depth use Eq. 26.

5. Select a depth at which a p-y curve is desired.

6. Establish $y_u$ as 3b/80. Compute $p_u$ by the following equation:

$$p_u = Ap_c \tag{27}$$

Use the appropriate value of A from Fig. 3, for the particular non-dimensional depth, and for either the static or cyclic case. Use the appropriate equation for $p_c$, Eq. 26 or Eq. 26 by referring to the computation in step 4.

7. Establish $y_m$ as b/60. Compute $p_m$ by the following equation:

$$p_m = Bp_c \tag{28}$$

Use the appropriate value of B from Fig. 4 for the particular non-dimensional depth, and for either the static or cyclic case. Use the appropriate equation for $p_c$.

8. Establish the slope of the initial portion of the p-y curve by selecting the appropriate value of k from Table 3a or 3b.

9. Select the following parabola to be fitted between points k and m.

$$p = Cy^{1/n} \tag{29}$$

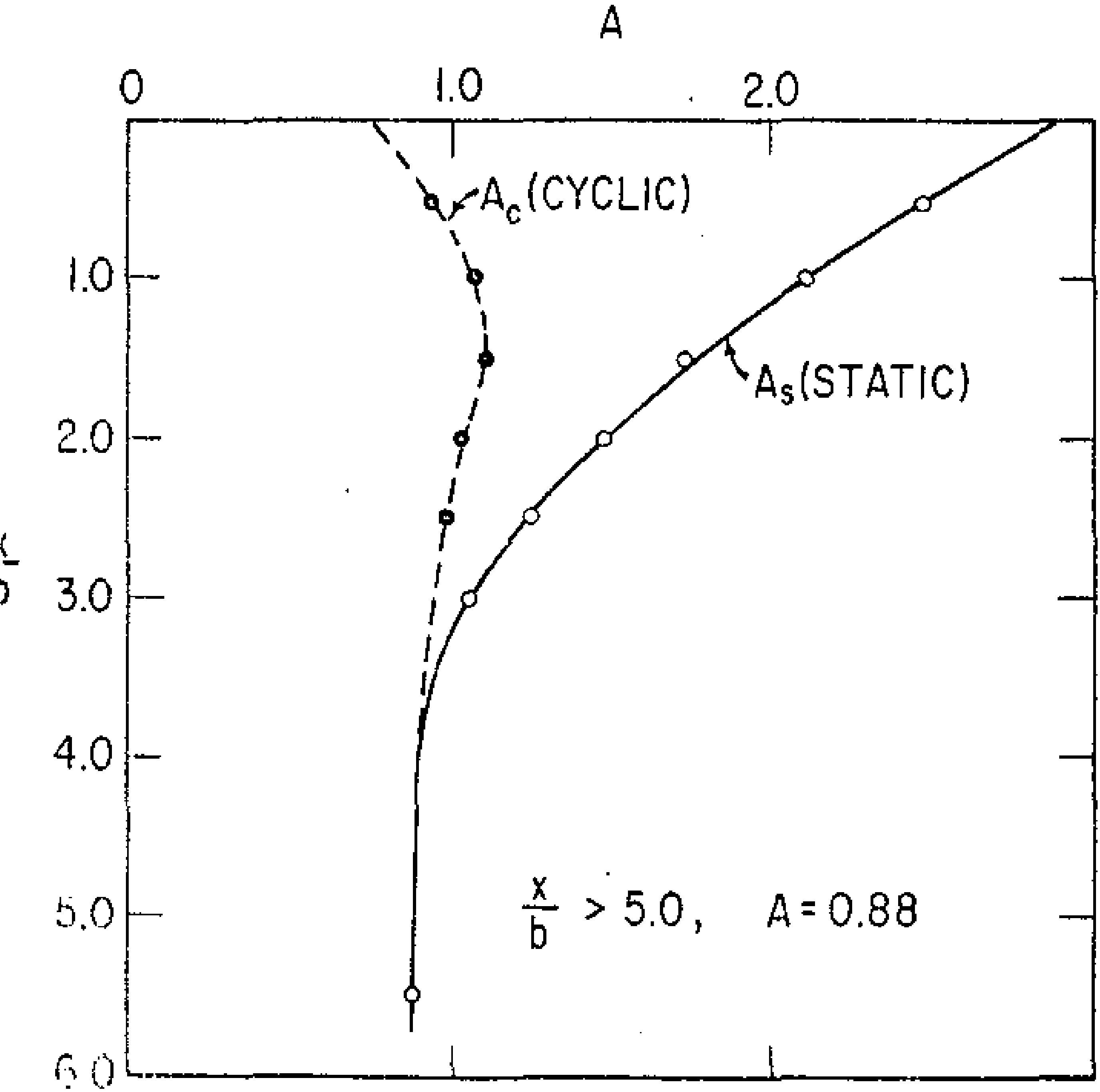



Fig. 3. Non-dimensional Coefficient A for Ultimate Soil Resistance Versus Depth.

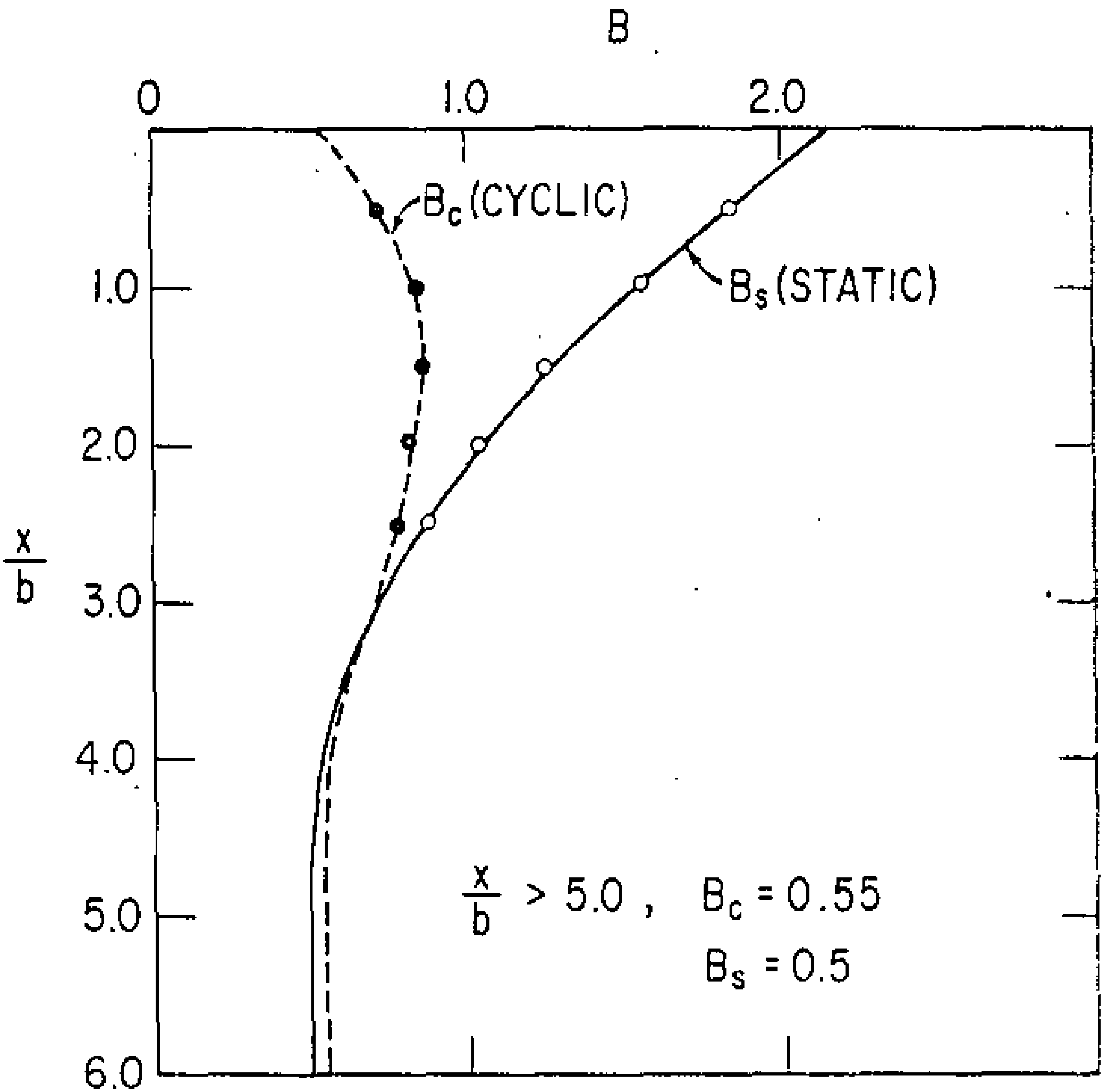



Fig. 4. Non-dimensional Coefficient B for Soil Resistance Versus Depth.

Table 3a

Recommended Values of k For Submerged Sand

| Relative Density | Loose | Medium | Dense |
|---|---|---|---|
| Recommended k $(lb/in^3)$ | 20 | 60 | 125 |

Table 3b

Recommended Values of k For Sand Above Water Table

| Relative Density | Loose | Medium | Dense |
|---|---|---|---|
| Recommended k $(lb/in^3)$ | 25 | 90 | 225 |

10. Fit the parabola between points k and m as follows:

a. Get slope of line between points m and u by,

$$m = \frac{p_u - p_m}{y_u - y_m} \tag{30}$$

b. Obtain the power of the parabolic section by,

$$n = \frac{p_m}{m y_m} \tag{31}$$

c. Obtain the coefficient C as follows:

$$C = \frac{p_m}{y_m^{1/n}} \tag{32}$$

d. Determine point k as,

$$y_k = \left(\frac{C}{kx}\right)^{n/n-1} \tag{33}$$

e. Compute appropriate number of points on the parabola by using Eq. 29.

This completes the development of the p-y curve for the desired depth. Any number of curves can be developed by repeating the steps above for each depth desired.

COMMENTS ON PROCEDURES FOR COMPUTING p-y CURVES

Each of the four procedures described above is based on experimental studies using full-sized, instrumented piles. In each case, p-y curves were derived from experimental results and were employed in developing the recommended procedures. Furthermore, theories for the behavior of soils under stress were employed as fully as possible in developing the recommendations. As a final step in the development, the recommendations for p-y curves were used to make predictions of pile behavior to compare with behavior observed in the experiments. In all cases the comparisons were excellent.

On the basis of the above comments, the designer may employ the p-y recommendations with reasonable confidence; however, only a limited number of experiments have been performed. The user of the design method described herein should read the reports on the experimental studies and may wish to perform full-scale experiments, perhaps on uninstrumented piles, to confirm pile designs for important projects.

A further comment should be made to the effect that the recommendations for p-y curves do not deal with the cases of sustained loading and of seismic loading.

SOLUTION PROCEDURE

An iterative procedure, using non-dimensional coefficients, is recommended for the solution of Eq. 1 for cases where there is no axial load and where the pile stiffness is constant. The solution procedure is described below for three sets of boundary conditions at the top of the pile: 1) pile head free to rotate, 2) pile head fixed against rotation, 3) pile head restrained against rotation. These boundary conditions, along with the sign convention used in the following solutions, are shown in Fig. 5.

Case I. Pile Head Free To Rotate

1. Construct p-y curves at various depths by procedures recommended above, with the spacing between p-y curves being closer near the ground surface than near the bottom of the pile.
2. Assume a value of T, the relative stiffness factor. The relative stiffness factor is given as:

$$T = \sqrt[5]{EI/k} \quad . \tag{34}$$

where

EI = flexural rigidity of pile,

k = constant relating the secant modulus of soil reaction to depth ($E_s = kx$).

3. Compute the depth coefficient, $z_{max}$, as follows:

$$z_{max} = \frac{x_{max}}{T} \tag{35}$$

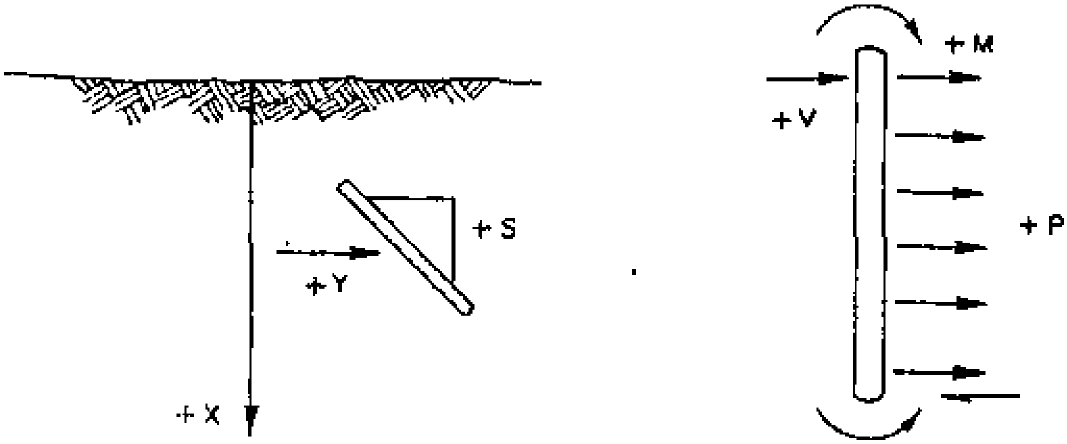
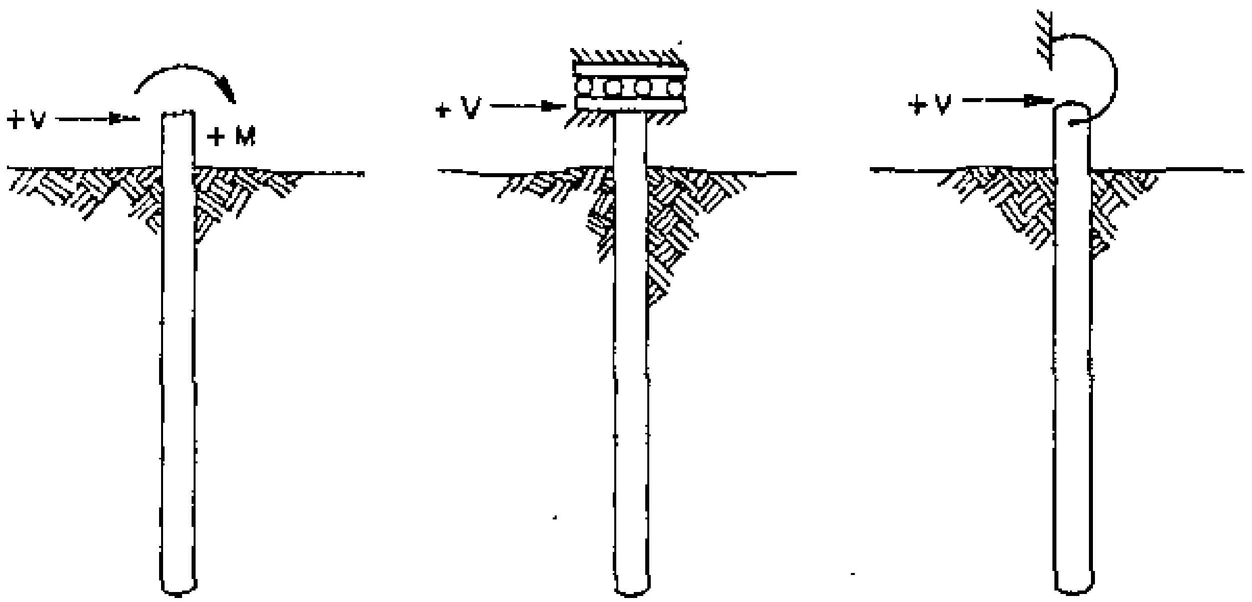



Fig. 5. (a) Sign Convention (After Matlock and Reese, 1961)




Case I. Pile Head Free to Rotate

Case II. Pile Head Fixed Against Rotation

Case III. Pile Head Restricted Against Ro

(b) Boundary Conditions Considered

4. Compute the deflection, y, at each depth along the pile where a p-y curve is available by using the following equation:

$$y = A_y \frac{P_T T^3}{EI} + B_y \frac{M_T T^2}{EI} \tag{36}$$

where

$A_y$ = deflection coefficient, found in Fig. 6,

$P_T$ = shear at top of pile,

$T$ = relative stiffness factor,

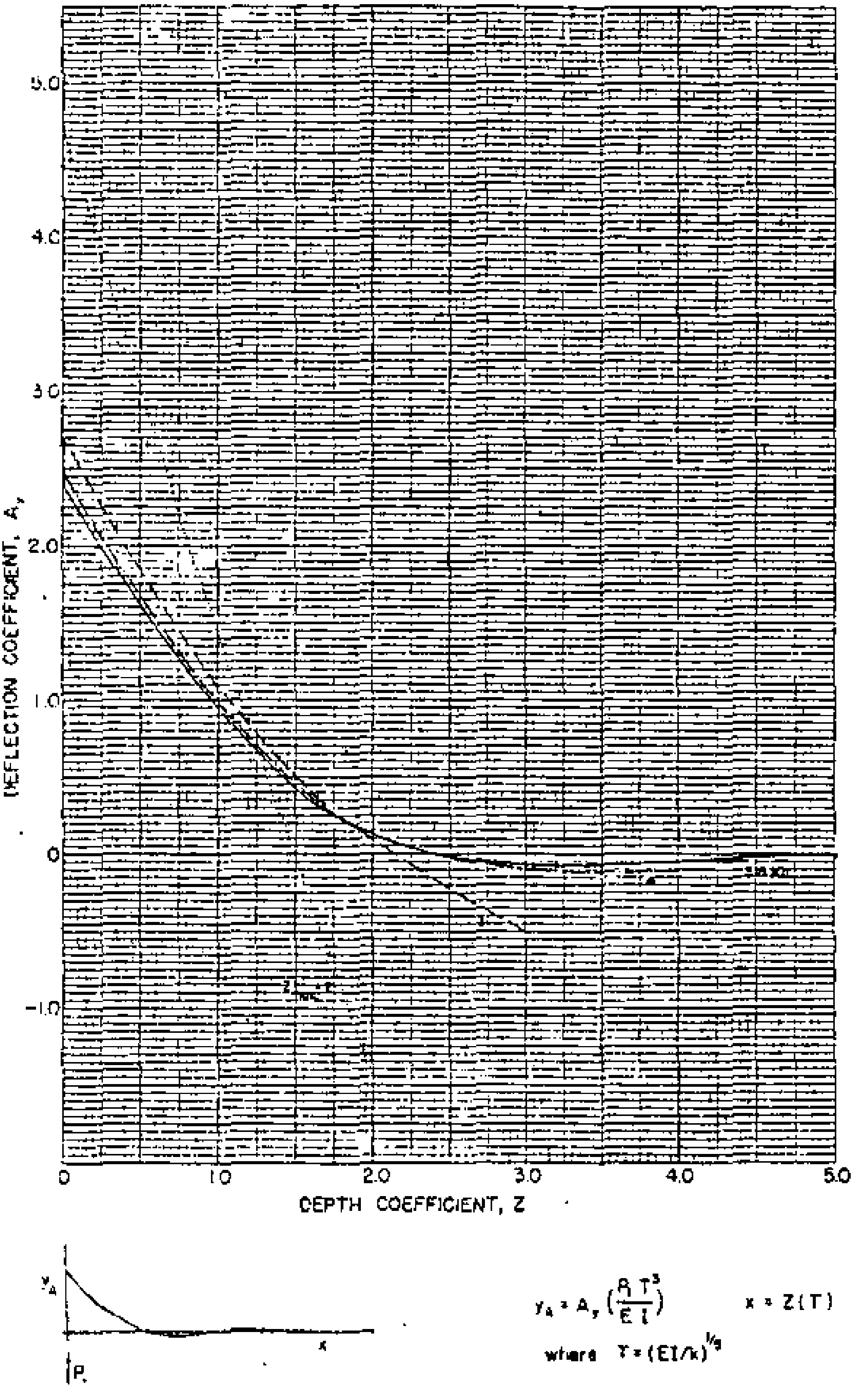
$B_y$ = deflection coefficient, found in Fig. 7,

$M_T$ = moment at top of pile

EI = flexural rigidity of pile.

The particular curves to be employed in getting the $A_y$ and $B_y$ coefficients depend on the value of $z_{max}$ computed in step 3.

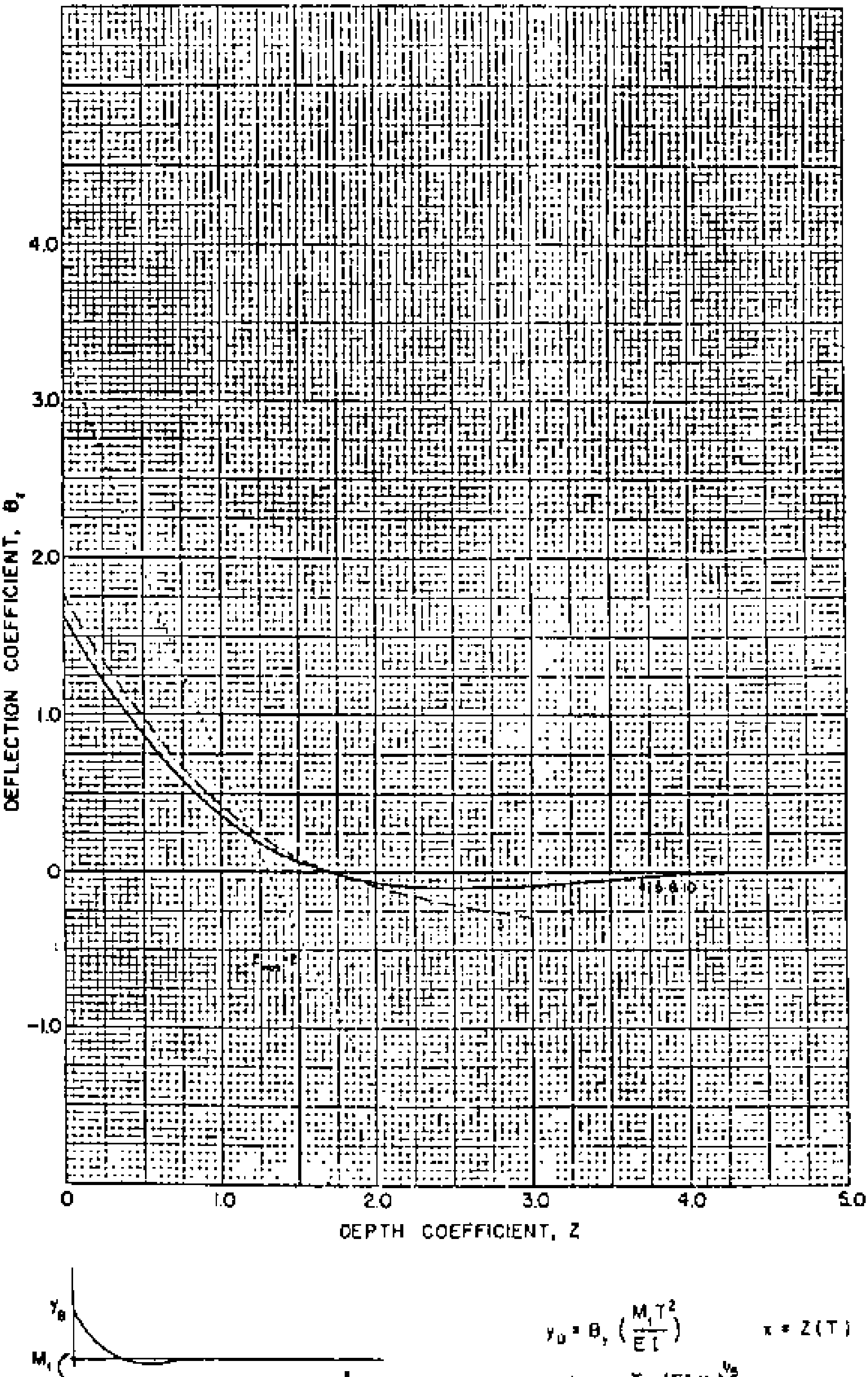
5. From a p-y curve, select the value of soil resistance, p, that corresponds to the pile deflection value, y, at the depth of the p-y curve. Repeat this procedure for every p-y curve that is available.
6. Compute a secant modulus of soil reaction, $E_s$, using Eq. 2. Plot the $E_s$ values versus depth.
7. From the $E_s$ vs. depth plot in step 6, compute the constant, k, which relates $E_s$ to depth ($k = E_s/x$). Give more weight to the $E_s$ values near the ground surface.




$$y_A = A_y \left(\frac{P_t T^3}{E I}\right) \qquad x = Z(T)$$

where $T = (EI/k)^{1/5}$

Fig. 6. Pile deflection produced by lateral load at mud line.




$$y_B = B_y \left(\frac{M_t T^2}{EI}\right) \qquad x = Z(T)$$

where $T = (EI/k)^{1/5}$

Fig. 7. Pile deflection produced by moment applied at mud line.

8. Compute a value of the relative stiffness factor, T, from the value of k found in step 8. Compare this value of T to the value of T assumed in step 2. Repeat steps 2 through 9 using the new value of T each time until the assumed value of T equals the calculated value of T.

9. When the iterative procedure has been completed, the values of deflection along the pile are known from step 4 of the final iteration. Values of soil reaction may be computed from the basic expression: $p = E_s y$. Values of slope, moment, and shear along the pile can be found by using the following equations.

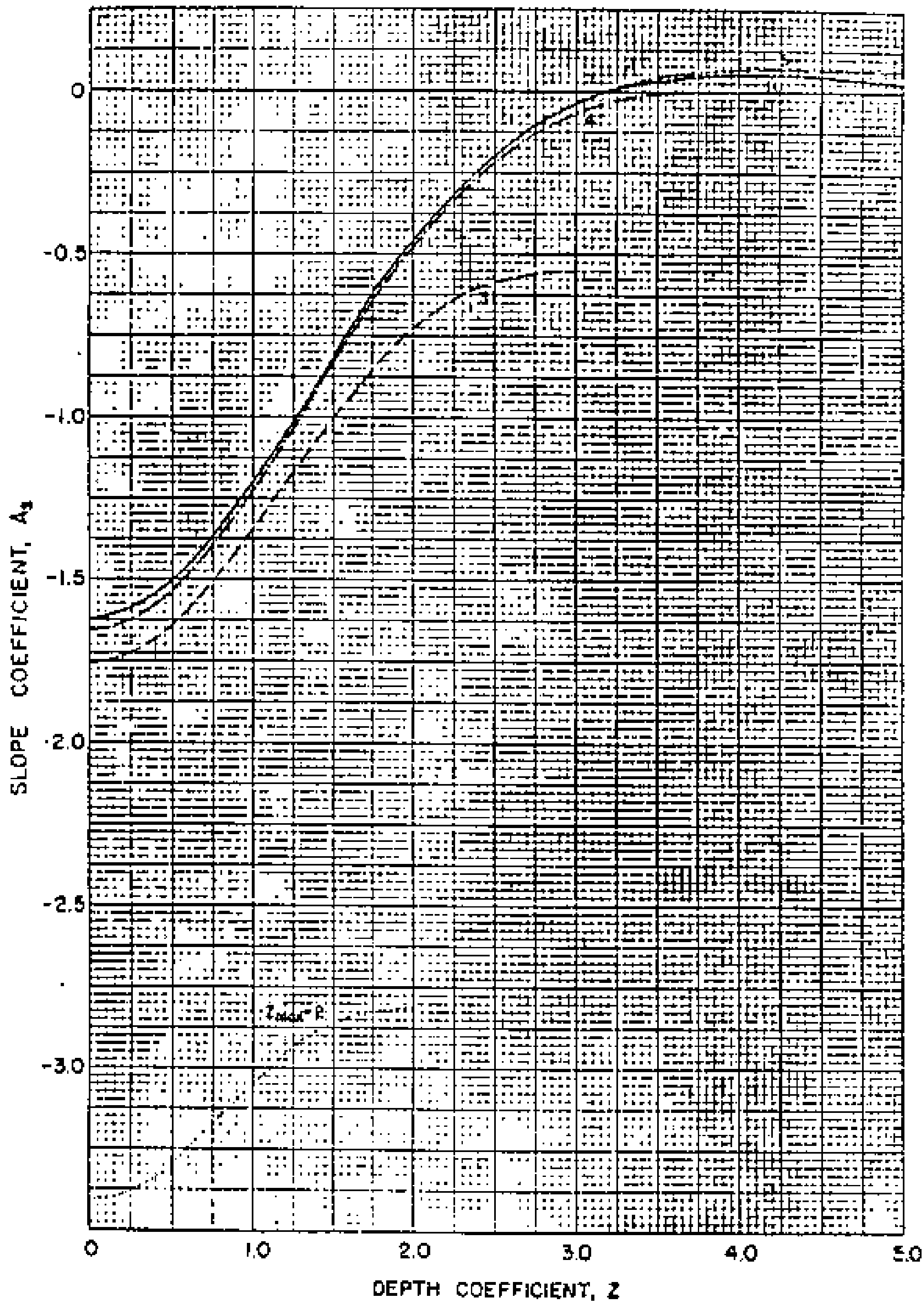
$$S = A_s \frac{P_t T^2}{EI} + B_s \frac{M_t T}{EI} \tag{37}$$

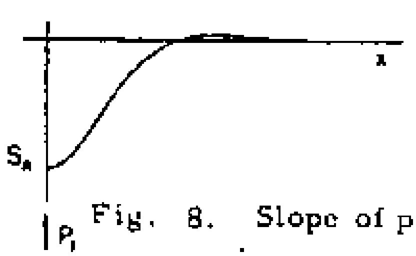
$$M = A_m P_t T + B_M M_t \tag{38}$$

and

$$V = A_v P_t + B_v \frac{M_t}{T} \tag{39}$$

The appropriate coefficients to be used in the above equations may be obtained from Figs. 6 through 13.

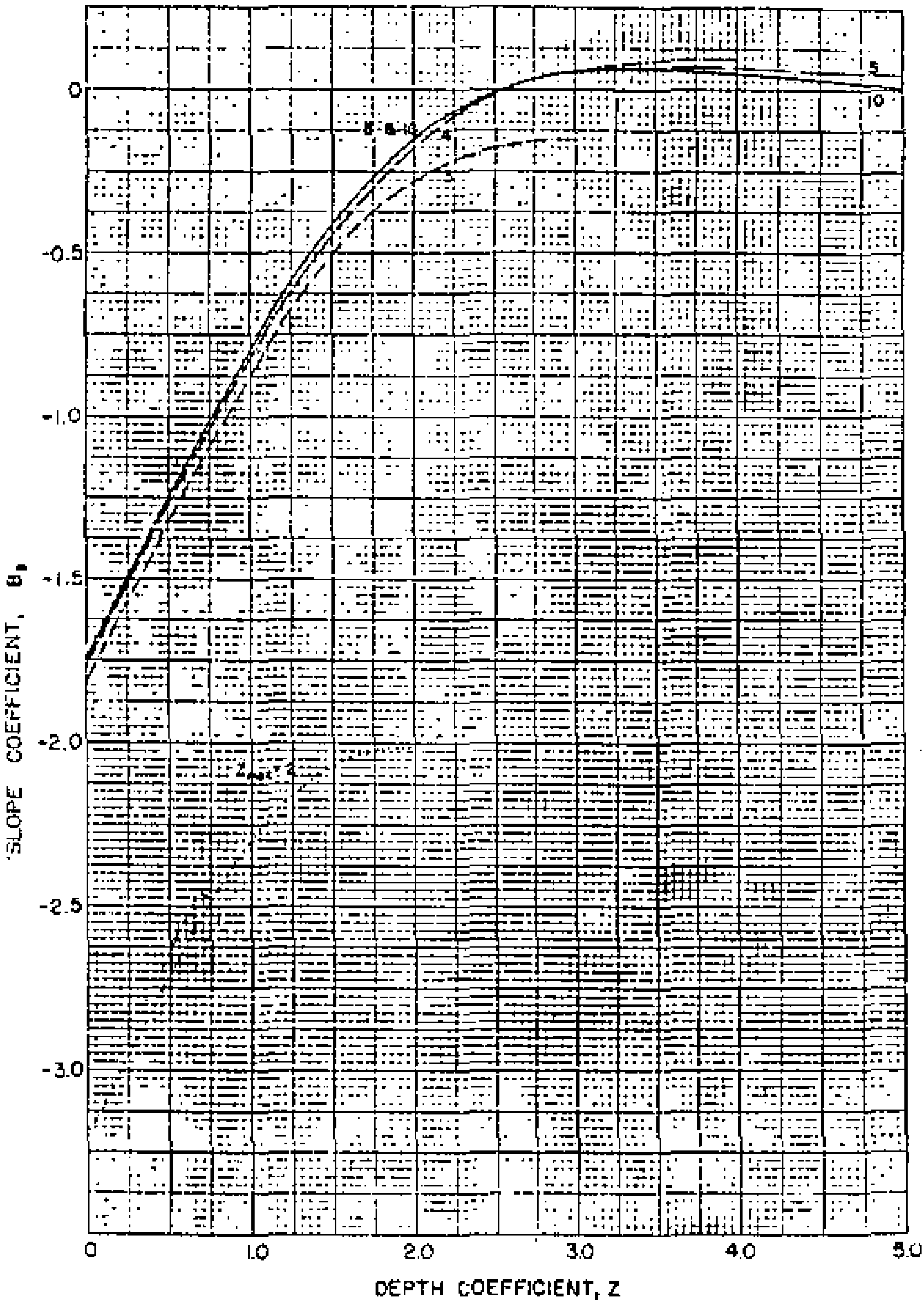


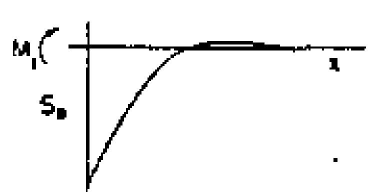




$$S_A = A_s\left(\frac{P_t T^2}{EI}\right) \qquad x = Z(T)$$

where $T = (EI/k)^{1/4}$

Fig. 8. Slope of pile caused by lateral load at mud line.




$$S_s = B_s\left(\frac{M_t T}{E I}\right) \qquad x = Z(T)$$

where $T = (EI/k)^{1/5}$

Fig. 9. Slope of pile caused by moment applied at the mud line.

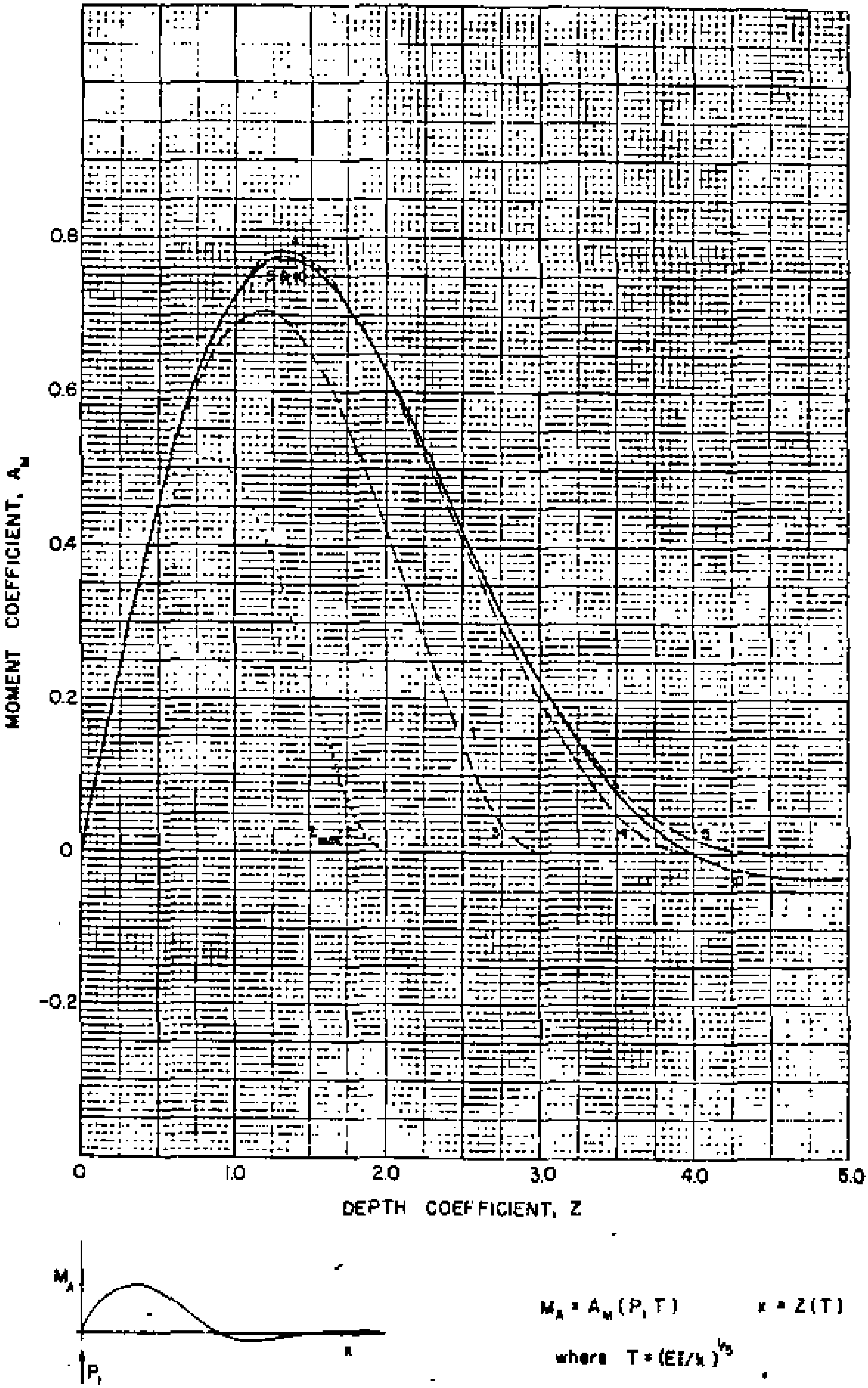
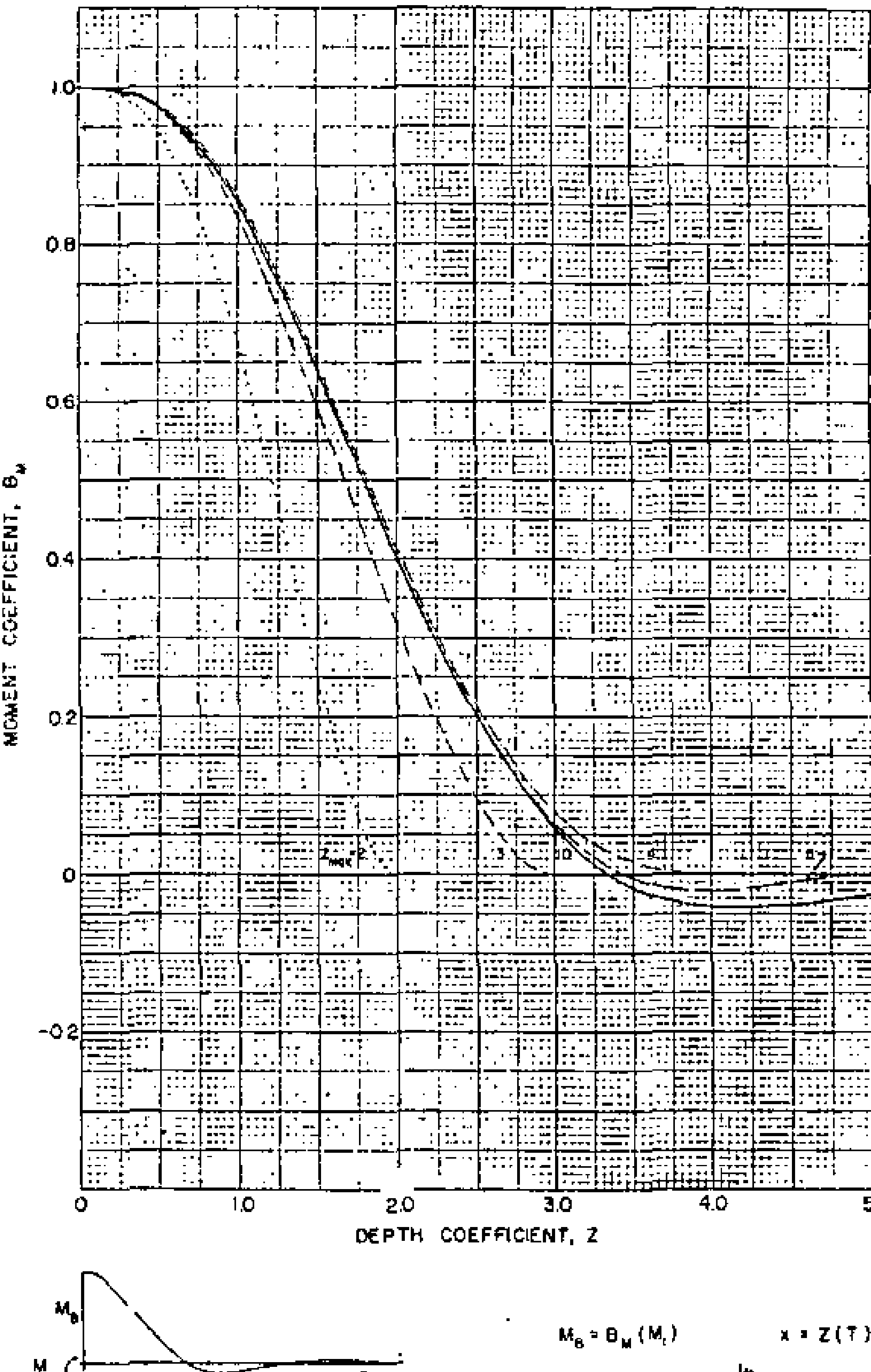



Fig. 10. Bending moment produced by lateral load at mud line.




$M_B = B_M (M_t)$ $x = Z(T)$

where $T = (EI/k)^{1/5}$

Fig. 11. Bending moment produced by moment applied at mud line.

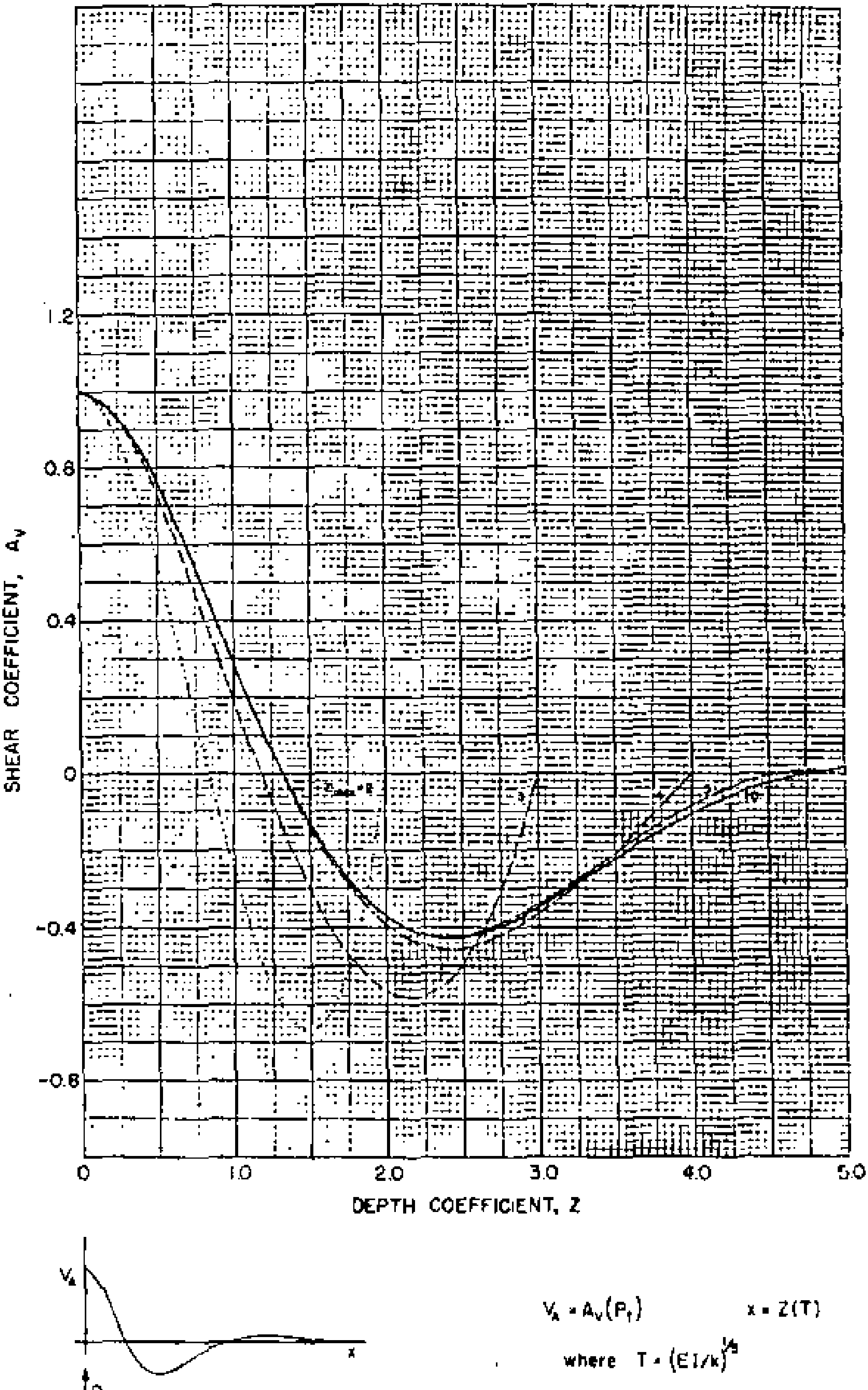
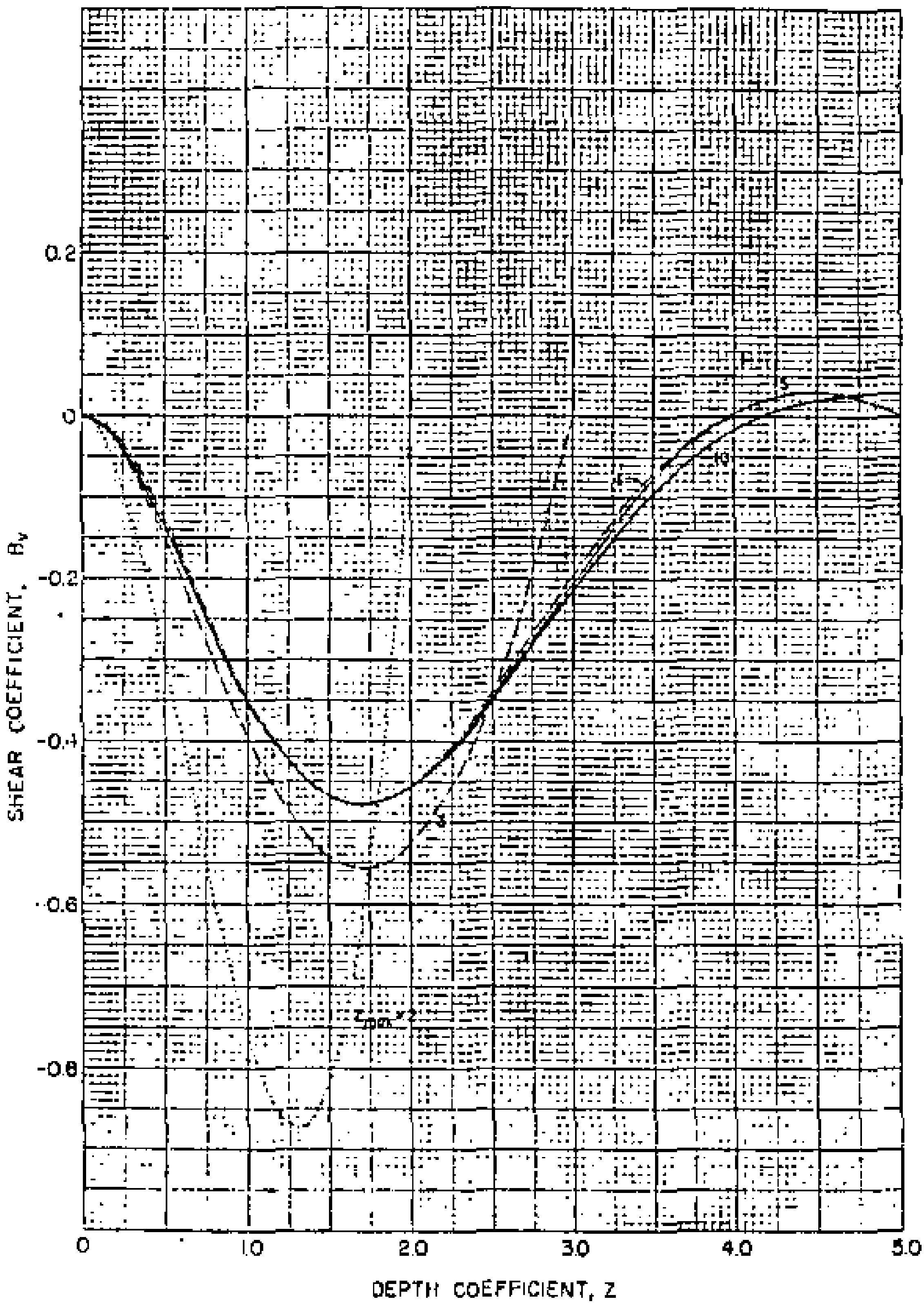
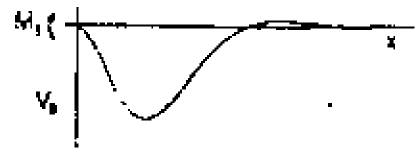



Fig. 12. Shear produced by lateral load at the mud line.




$V_s = B_v\left(\frac{M_t}{T}\right)$ $\quad x = Z(T)$

where $T = (EI/k)^{1/5}$

Fig. 13. Shear produced by moment applied at the mud line.

Case II. Pile Head Fixed Against Rotation

Case II may be used to obtain a solution for the case where the superstructure translates under load but does not rotate and where the superstructure is very, very stiff in relation to the pile.

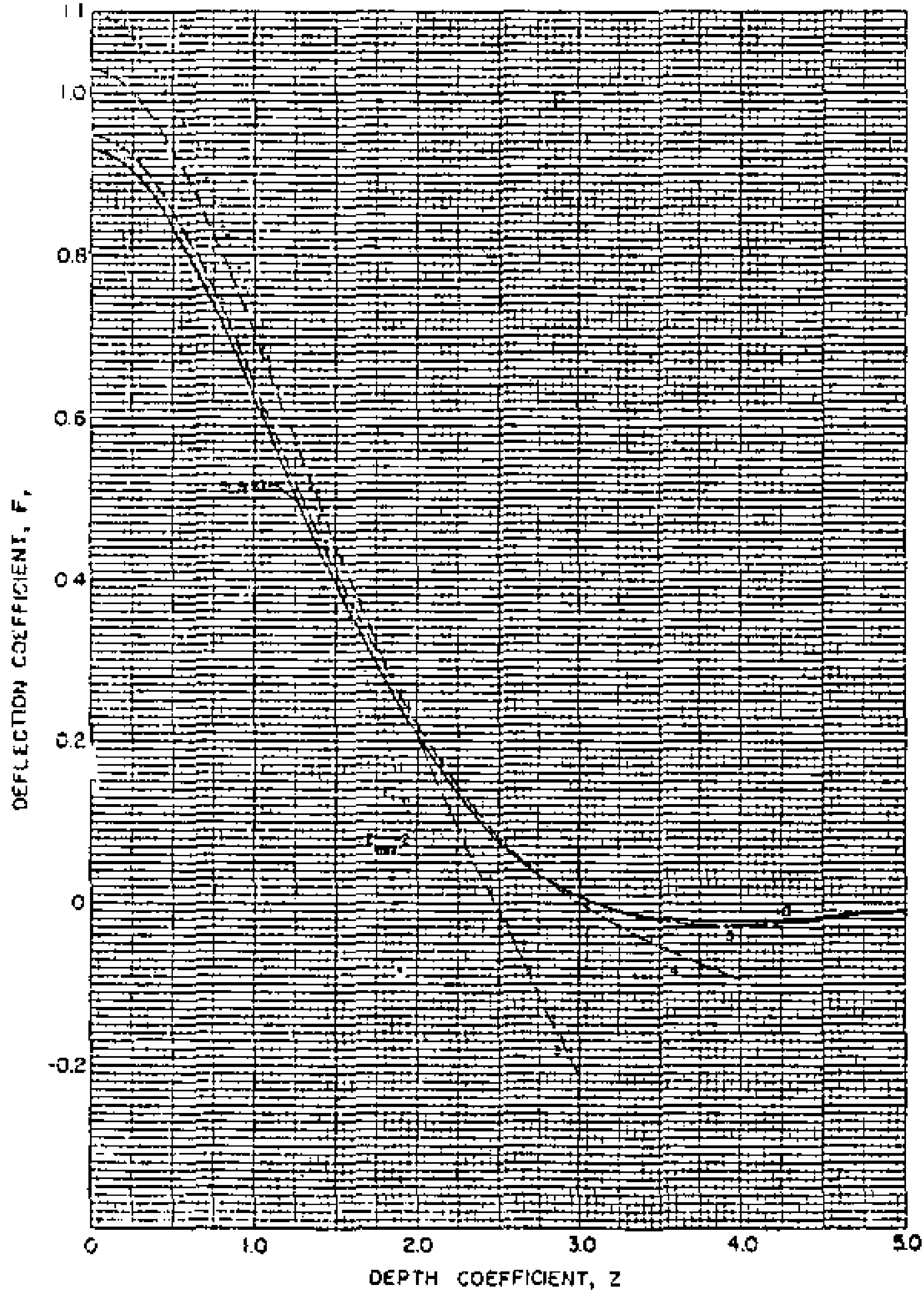
1. Perform steps 1, 2, and 3 of the solution procedure as for free-head piles, Case I.
2. Compute the deflection, y, at each depth along the pile where a p-y curve is available by using the following equation:

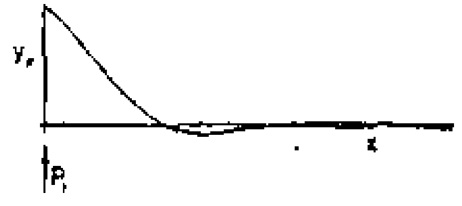
$$y_F = F_y \frac{P_t T^3}{EI} . \qquad (40)$$

   The deflection coefficients, $F_y$, may be found by entering Fig. 14 with the appropriate value of $z_{max}$.
3. The solution proceeds in steps similar to those of steps 5 through 8 for the free-head case.
4. Compute the moment at the top of the pile, $M_T$, from the following equation:

$$M_t = F_{Mt} P_t T . \qquad (41)$$

   The value of $F_{Mt}$ may be found by entering Table 4 with the appropriate value of $z_{max}$.




$$y_F = F_y \left( \frac{P_t T^3}{E I} \right) \qquad x = Z(T)$$

where $T = (EI/k)^{1/5}$

Fig. 14. Deflection of pile fixed against rotation at mud line.

Table 4

Moment Coefficients at Top of Pile for Fixed-Head Case

| $z_{max}$ | $F_{Mt}$ |
|---|---|
| 2 | -1.06 |
| 3 | -0.97 |
| 4 | -0.93 |
| 5 and above | -0.93 |

5. Compute values of slope, moment, shear, and soil reaction along the pile by following the procedure in step 9 for the free-head pile.

Case III. Pile Head Restrained Against Rotation

Case III may be used to obtain a solution for the case where the superstructure translates under load but does not rotate.

1. Perform steps 1, 2, 3 of the solution procedure for free head piles, Case I.
2. Obtain the value of the spring stiffness, $k_\theta$, of the pile-superstructure system. The spring stiffness is defined as follows:

$$k_\theta = \frac{M_t}{S_t} \qquad (42)$$

where

$M_t$ = moment at top of pile,

$S_t$ = slope at top of pile.

3. Compute the slope at the top of pile, $S_t$, as follows:

$$S_t = A_{st} \frac{P_T T^2}{EI} + B_{st} \frac{M_T T}{EI} \tag{43}$$

where

$A_{st}$ = slope coefficient, found in Fig. 8,

$B_{st}$ = slope coefficient, found in Fig. 9.

4. Solve equations 42 and 43 for the moment at the top of the pile, $M_t$.
5. Perform steps 1 through 9 of the solution procedure for free head piles, Case I.

This completes the solution of the laterally loaded pile problem for three sets of boundary conditions. The solution gives values of deflection, slope moment, and shear, and soil reaction as a function of depth. To illustrate the solution procedures, an example problem is presented.

EXAMPLE PROBLEM

## Problem Statement

Find the deflection, moment and shear as a function of depth along a pile that is free to rotate and is subjected to a horizontal force and a moment. The p-y curves are to be constructed at 2, 4, 8, 16, and 40 ft. The soil is a stiff clay above the water table. Other data for the problem are shown below.

$P_t$ = 35,000 lb

$M_t$ = 3.02 X $10^7$ in-lb.

L = 40 ft.

b = 2 ft.

EI = 7.39 X $10^{10}$ lbs-in.$^2$

c = 1000 psf

$\gamma$ = 110 pcf

N = 1000 cycles

## Solution

The solution will proceed in the step-by-step manner as described for Case I.

1. Construct p-y curves.

Assume $\epsilon_{50}$ = 0.01 in the absence of stress-strain curves. Compute $p_u$ as the smaller of the values from Eqs. 3 and 4 for depth of 0, 24, 48, 96, 144, 192, and 288 in.

Compute $y_{50}$ from Eq. 5 and compute points on the p-y curves for short-term static loading using Eq. 9.

Compute y values for cyclic loading by use of Eq. 11.

The results of the computations are shown in the Table 5.

Table 5

Computed p-y curves

| Depth, in. | | 0 | 24 | 48 | 96 | 144 | 192 | 288 |
|---|---|---|---|---|---|---|---|---|
| $y_{static}$ | $y_{cyclic}$ | | | | p, lb/in | | | |
| 0.000 | 0.000 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0.001 | 0.003 | 51 | 63 | 75 | 99 | 123 | 147 | 152 |
| 0.015 | 0.04 | 100 | 123 | 147 | 195 | 243 | 291 | 299 |
| 0.24 | 0.67 | 199 | 247 | 294 | 390 | 485 | 580 | 596 |
| 0.60 | 1.68 | 250 | 310 | 370 | 490 | 610 | 730 | 750 |
| 1.24 | 3.48 | 300 | 372 | 444 | 588 | 731 | 875 | 899 |
| 2.50 | 7.00 | 357 | 443 | 529 | 700 | 872 | 1043 | 1072 |
| 5.00 | 14.00 | 425 | 527 | 629 | 833 | 1036 | 1240 | 1274 |
| 9.60 | 26.88 | 500 | 620 | 740 | 980 | 1220 | 1460 | 1500 |

(The p-$y_{cyclic}$ curves are plotted in Fig. 15).

2. Assume T: T = 126 in.

3. Compute $z_{max}$: $z_{max} = \frac{x_{max}}{T} = \frac{40(12)}{126} = 3.81$.

4. Compute the deflection, y, at depths of 0, 2, 4, 8, 12, 16, and 24 ft. Using Eq. 36 (Use Figs. 6 and 7; the computations are tabulated in the following table.

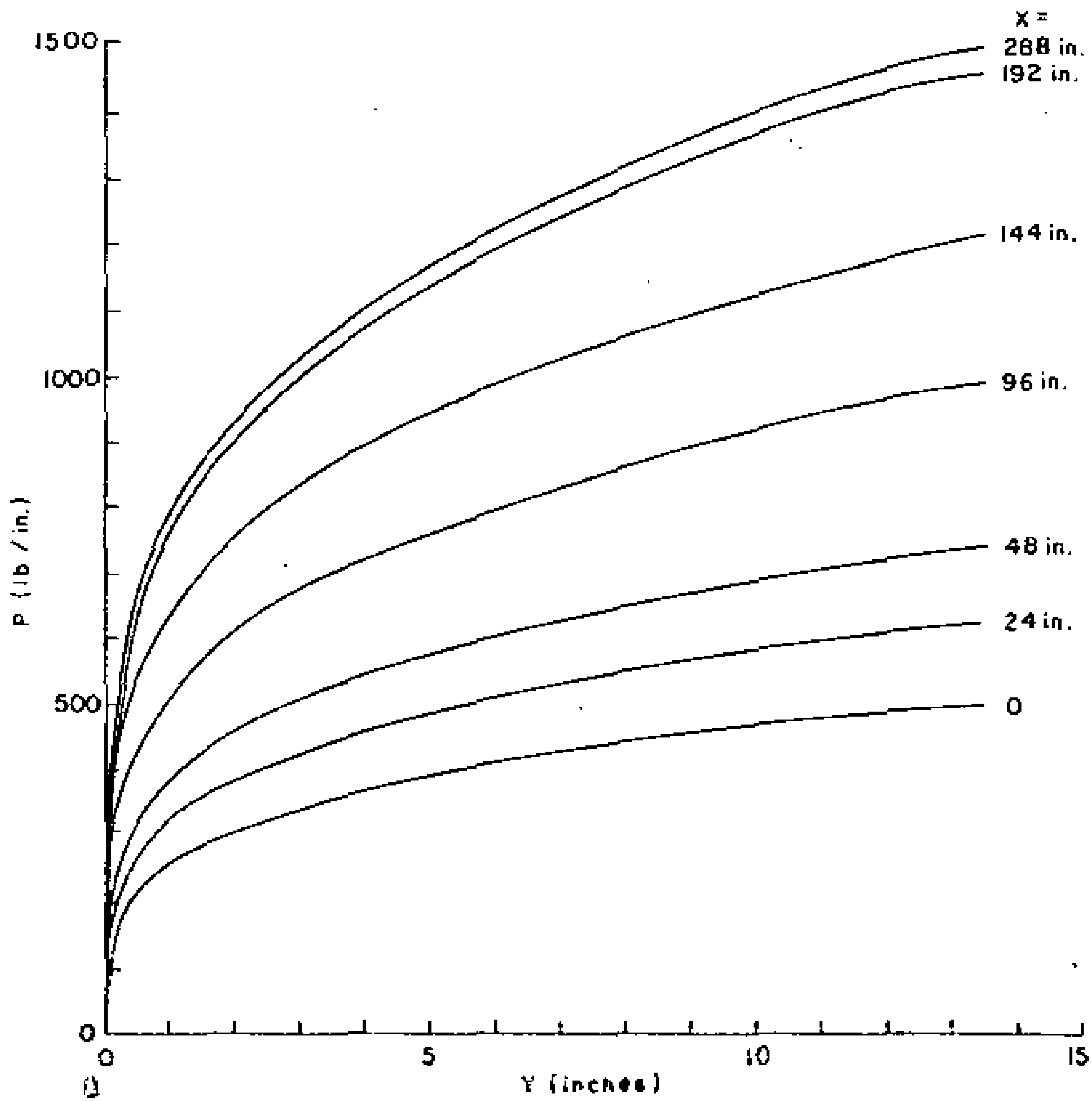



Fig. 15. Plot of p-y Curves for example problem, stiff clay above water table - cyclic loading

Table 6

Computed Deflections

| x, in. | z | $A_y$ | $y_A$, in. | $B_y$ | $y_B$, in. | y, in. |
|---|---|---|---|---|---|---|
| 0 | 0 | 2.50 | 2.16 | 1.65 | 10.71 | 12.87 |
| 24 | 0.19 | 2.20 | 1.91 | 1.30 | 8.43 | 10.34 |
| 48 | 0.38 | 1.85 | 1.60 | 1.0 | 6.49 | 8.09 |
| 96 | 0.76 | 1.35 | 1.17 | 0.60 | 5.89 | 5.06 |
| 144 | 1.14 | 0.98 | 0.85 | 0.25 | 1.62 | 2.47 |
| 192 | 1.52 | 0.45 | 0.39 | 0.07 | 0.45 | 0.84 |
| 288 | 2.29 | ---- | ---- | ---- | ---- | ---- |
| 480 | 3.81 | 0.20 | 0.11 | -0.10 | -0.65 | -0.82 |

5. From the set of p-y curves (Fig. 15), the values of p are selected, corresponding to the y values computed in step 4. (See tabulation in step 6.)

6. Compute the $E_s$ value at each depth (see following Table 7).

Table 7

Computed Soil Modulus Values

| x, in. | y, in. | p, lb/in. | $E_s$, psi |
|---|---|---|---|
| 0 | 12.87 | -420 | 33 |
| 24 | 10.34 | -470 | 47 |
| 48 | 8.09 | -540 | 67 |
| 96 | 5.06 | -642 | 127 |
| 144 | 2.47 | -670 | 271 |
| 192 | 0.84 | -600 | 714 |
| 288 | ---- | ---- | --- |
| 480 | -0.82 | +710 | 866 |

7. A plot of $E_s$ vs. depth is shown in Fig. 16. The k value is:

$$k = E_s/x = \frac{1000}{605} = 1.65 \text{ lb./in.}^3$$

8. Compute T:

$$T = \sqrt[5]{EI/k} = \sqrt[5]{\frac{7.39 \times 10^{10}}{1.65}} = 136 \text{ in.} \neq 126$$

This completes the first iteration of the solution procedure. Before proceeding to the next iteration, the results thus far will be examined for guidance with regard to further computations.

It is evident from Fig. 16 that $E_s = kx$ is not a good representation of the variation of the soil modulus with depth. A straight line, passing through the origin, does not fit the plotted points. However, the solution will proceed by use of the non-dimensional curves in order to gain an approximate idea of the final design.

Figure 16 also reveals that the solution has not been found because the k that was tried is not equal to the k that was obtained. Further, it would appear that the value of T is likely to be greater than 135.

With a value of T of 126, the $z_{max}$ was 3.81 and that value at convergence will be even smaller. A study of Figs. 6 and 7 shows that the bottom of the pile is deflecting for $z_{max}$ values of less than 5. Therefore, the pile length probably should be increased to insure lateral stability. The further computations will be performed with a length of 60 ft.

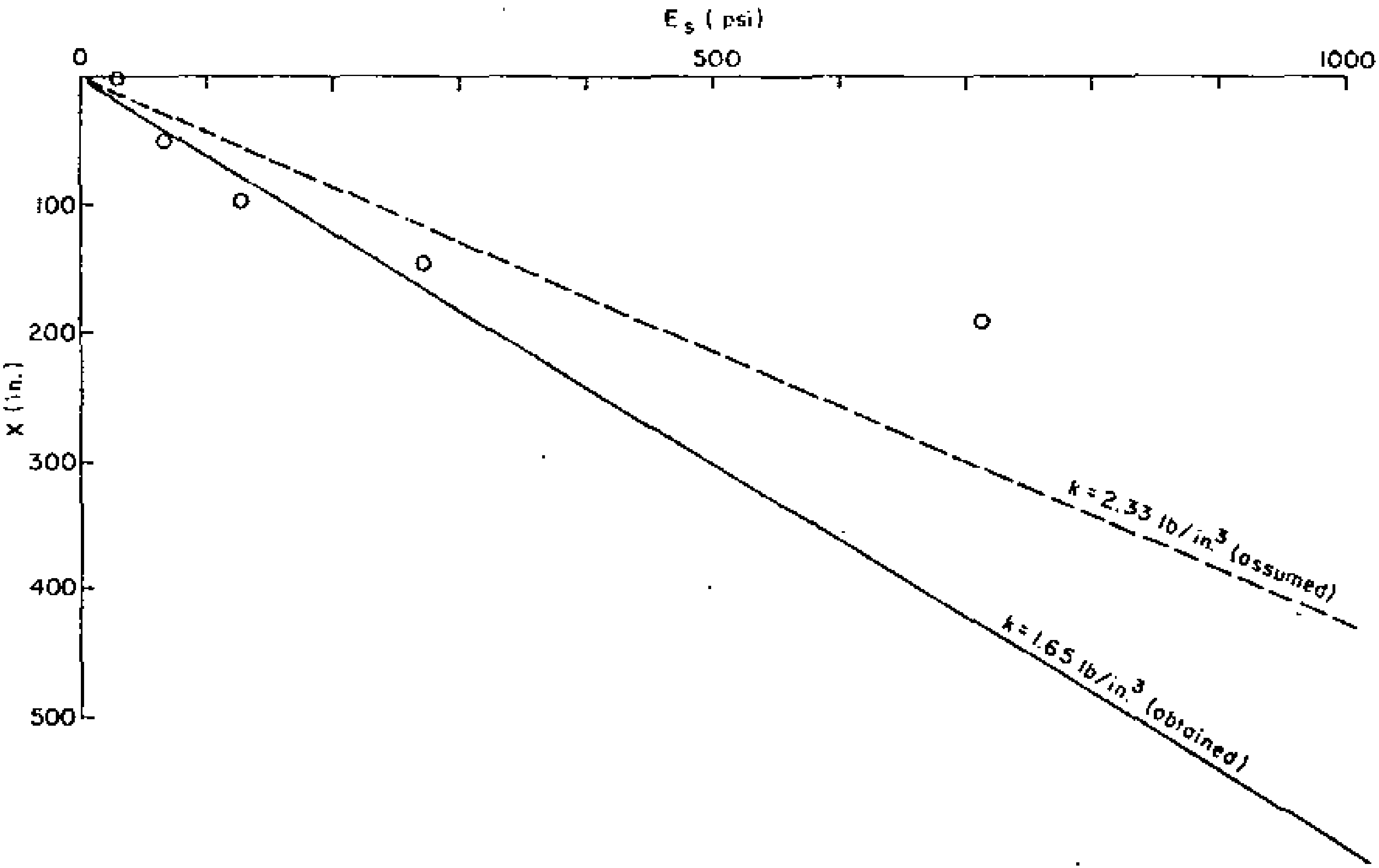



Fig. 16. Plot of $E_s$ versus X for example problem
First interaction

Even though the length of the pile is increased, the deflection at the groundline will be considerable. An evident way to decrease the deflection is to increase the stiffness of the pile; however, for this problem it will be assumed that the deflection is not a problem.

The iterations are continued but with a pile with length increased to 60 ft; convergence was achieved with $T = 143$ in.

9. Compute the values of moment and shear using Eqs. 38 and 39 (see Fig. 17). Also shown in Fig. 17 are plots of the moment and shear diagrams from a computer solution of the example problem (see next section). As may be seen, excellent agreement is found between the computer solutions and the non-dimensional solutions.

## ANALYSIS BY COMPUTER PROGRAM

While the non-dimensional method described above is satisfactory for many problems, some laterally-loaded piles can be analyzed more efficiently by means of a computer program. The program recommended for problems of laterally-loaded piles is COM622 (Reese and Manoliu, 1973).

The program uses successive difference-equation computations, based on repeated reference to the p-y curves, to determine at increments along the pile the values of soil modulus which insures both compatibility and equilibrium for the soil, the pile, and the superstructure. Some of the advantages of using COM622 are:

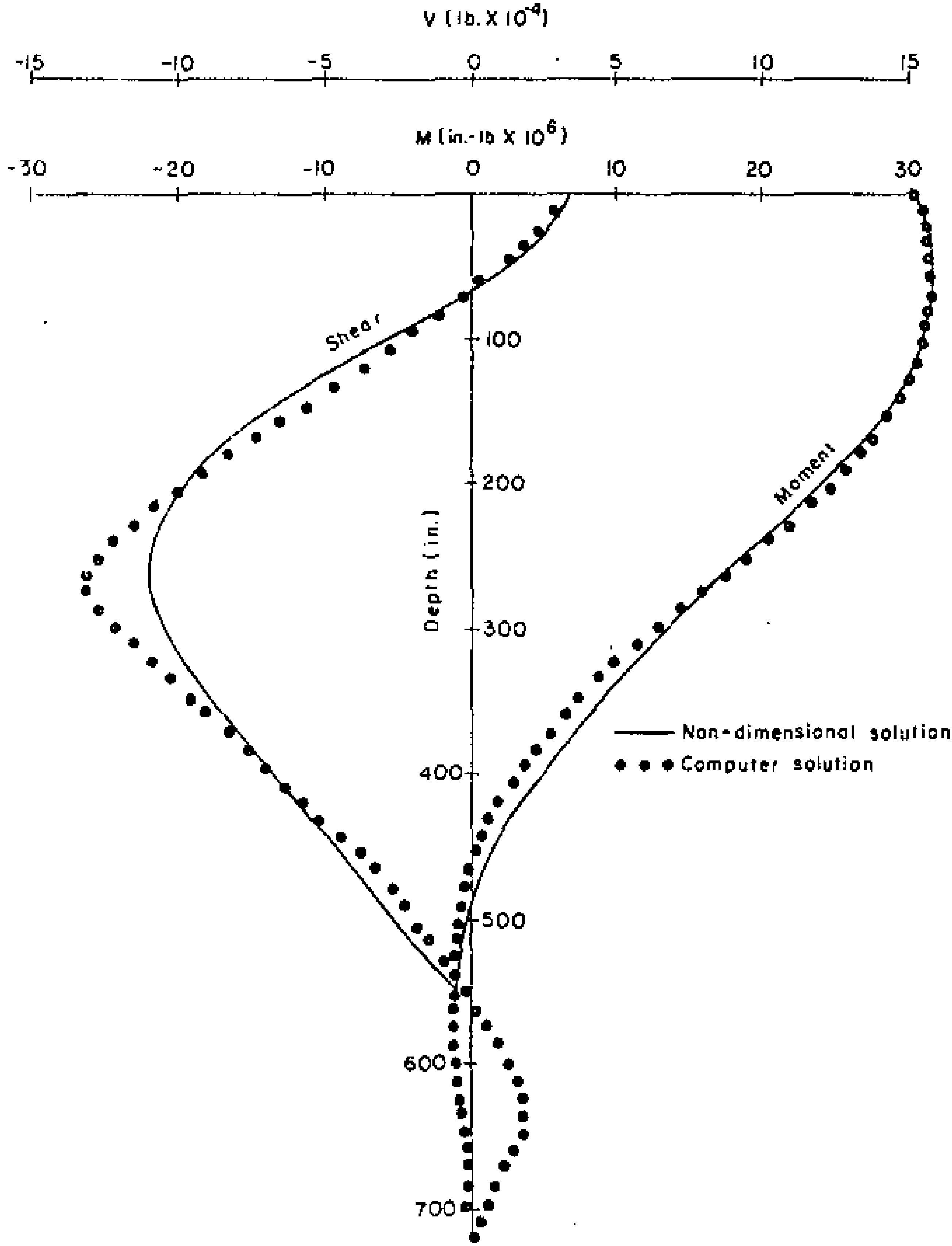



Fig. 17. Plot of moment and shear diagrams for example problem

1. Step changes in the flexural stiffness of the pile may be introduced at any depth.
2. The pile length may be changed as desired.
3. The p-y data may be introduced in several convenient ways.
4. The boundary conditions at the top of the pile may be specified as the lateral load and: a) the moment, b) slope, c) the rotational spring constant (moment/slope). In addition, an axial load may be specified.

The computer program may be obtained at the Computing Center, University of Bolder, Colorado.

CONCLUDING REMARKS

A number of methods have been suggested in technical literature for the analysis of piles under lateral load but the method presented above has the principal advantage of satisfying fully the principles of mechanics. The only limitation in the method is in regard to the meagerness of the data concerning the prediction of the p-y curves. However, as more information becomes available on the behavior of the soil around a pile under lateral loading, that information can be put to use in making analyses.

In general, the computer program is favored as an analytical method. Problems can be created more exactly, parameters can be varied to improve the ability of the engineer to make decisions of pile geometry, and the computations are inexpensive. However, the non-dimensional method should be employed on almost every occasion as a check of the computer solution or to give preliminary design information.

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## DERIVATION OF DIFFERENCE EQUATIONS

# DERIVATION OF DIFFERENCE EQUATIONS

## Basic Expression for $y_m$

The basic differential equation for flexure of a beam for small deflections is

$$EI \frac{d^2y}{dx^2} = M \qquad \text{A1}$$

When differentiated twice Eq A1 becomes

$$\frac{d^2M}{dx^2} = p \qquad \text{A2}$$

Where p = distributed soil reaction along the pile

For convenience in writing

$$R = EI \qquad \text{A3}$$

At this point it is convenient to show derivatives in difference form. Referring to the notation in Fig 1,

$$\left.\frac{dy}{dx}\right|_m = \frac{y_{m-1} - y_{m+1}}{2h} \qquad \text{A4}$$

$$\left.\frac{d^2y}{dx^2}\right|_m = \frac{y_{m-1} - 2y_m + y_{m+1}}{h^2} \qquad \text{A5}$$

$$\left.\frac{d^3y}{dx^3}\right|_m = \frac{y_{m-2} - 2y_{m-1} + 2y_{m+1} - y_{m+2}}{2h^3} \qquad \text{A6}$$

$$\left.\frac{d^4y}{dx^4}\right|_m = \frac{y_{m-2} - 4y_{m-1} + 6y_m - 4y_{m+1} + y_{m+2}}{h^4} \qquad \text{A7}$$

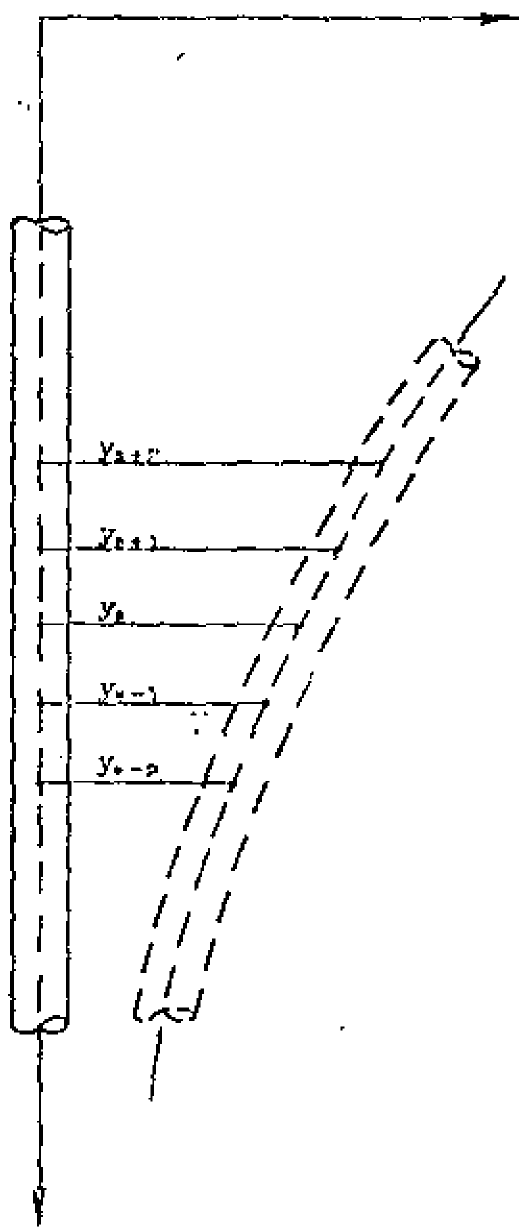

Fig. 11 Ruprosentation of deflectod pile.

In difference form, Eq A2 may be written as follows:

$$\frac{R_{m-1}\frac{(y_{m-2}\ -2y_{m-1}\ +y_m)}{h^2} \quad -R_m\frac{(y_{m-1}\ -2y_m\ +y_{m+1})}{h^2}}{h^2} \quad -$$

$$\frac{R_m\frac{(y_{m-1}\ -2y_m\ +y_{m+1})}{h^2} \quad - \quad R_{m+1}\frac{(y_m\ -2y_{m+1}\ +y_{m+2})}{h^2}}{h^2}$$

$$= \quad p_m \qquad \text{A8}$$

Eq A8 becomes

$$y_{m-2}\ (R_{m-1})\ -2y_{m-1}(R_{m-1}\ +R_m)\quad +y_m(R_{m-1}\ +4R_m\ +R_{m+1})$$

$$-2y_{m+1}\ (R_m\ +R_{m+1})\quad +y_{m+2}\ (R_{m+1})\quad =\quad p_m h^4 \qquad \text{A9}$$

Substitute

$$p_m = -k_m y_m \qquad \text{A10}$$

The following general expression is obtained.

$$y_{m-2}\ (R_{m-1})\ -2y_{m-1}\ (R_{m-1}\ +R_m)$$

$$+y_m\ (R_{m-1}\ +4R_m\ +R_{m+1}\ +k_m\ h^4)$$

$$-2y_{m+1}\ (R_m\ +R_{m+1})\ +y_{m+2}\ (R_{m+1})\quad =\quad 0 \qquad \text{A11}$$

The two boundary conditions at the bottom of the pile are that the moment and the shear are zero. Using the notation in Fig 2, these two boundary conditions are defined in Eqs A12 and A13.

$$y_{-1}\ -2y_o\ +y_1\quad =\quad 0 \qquad \text{A12}$$

$$y_{-2}\ -2y_{-1}\ +2y_1\ -y_2\quad =\quad 0 \qquad \text{A13}$$

Writing Eq A11 about Point 0

$$y_{-2}\ (R_{-1}) + y_{-1}\ (-2R_{-1} - 2R_o) + y_o\ (R_{-1} + 4R_o + R_1 + k_o\ h^4)$$

$$+ y_1\ (-2R_o - 2R_1) + y_2\ (R_1) = 0 \qquad \text{A14}$$

From Eq A13

$$y_{-2} = 2y_{-1} - 2y_1 + y_2 \qquad \text{A15}$$

Substituting expression in Eq A15 for $y_{-2}$ in Eq A14

$$y_{-1}\ (-2R_o) + y_o\ (R_{-1} + 4R_o + R_1 + k_o\ h^2)$$

$$+ y_1\ (-2R_o - 2R_1 - 2R_{-1}) + y_2\ (R_1 + R_{-1}) = 0 \qquad \text{A16}$$

From Eq A12

$$y_{-1} = 2y_o - y_1 \qquad \text{A17}$$

Substituting expression in Eq A17 for $y_{-1}$ in Eq A16

$$y_o\ (R_{-1} + R_1 + k_o\ h^4) + y_1\ (-2R_1 - 2R_{-1}) + y_2\ (R_1 + R_{-1}) = 0 \qquad \text{A18}$$

Putting Eq A18 into the form

$$y_o = a_o y_1 - b_o y_2 \qquad \text{A19}$$

And further setting

$$R_{-1} = R_o \qquad \text{A20}$$

The following expressions for $a_o$ and $b_o$ result

$$a_o = \frac{2R_o + 2R_1}{R_o + R_1 + k_o h^4} \qquad \text{A21}$$

$$b_o = \frac{R_o + R_1}{R_o + R_1 + k_o h^4} \qquad \text{A22}$$

The next step in the derivation is to write Eq A11 about Point 1

$$y_{-1} (R_o) + y_o (-2R_o - 2R_1) + y_1 (R_o + 4R_1 + R_2 + k_1 h^4)$$

$$+ y_2 (-2R_1 - 2R_2) + y_3 (R_2) = 0 \qquad \text{A23}$$

Using Eq A17

$$y_o (-2R_1) + y_1 (4R_1 + R_2 + k_1 h^4) + y_2 (-2R_1 - 2R_2)$$

$$+ y_3 (R_2) = 0 \qquad \text{A24}$$

Using Eq A19

$$y_1 (4R_1 + R_2 - 2a_o R_1 + k_1 h^4) + y_2 (-2R_1 - 2R_2 + 2b_o R_1)$$

$$+ y_3 (R_2) = 0 \qquad \text{A25}$$

Putting Eq A25 into the form

$$y_1 = a_1 y_2 - b_1 y_3 \qquad \text{A26}$$

The following expressions for $a_1$ and $b_1$ result

$$a_1 = \frac{2R_1 - 2b_oR_1 + 2R_2}{4R_1 - 2a_oR_1 + R_2 + k_1h^4} \qquad \text{A27}$$

$$b_1 = \frac{R_2}{4R_1 - 2a_oR_1 + R_2 + k_1h^4} \qquad \text{A28}$$

The derivation is continued by writing Eq A11 about Point 2.

$$y_o(R_1) + y_1(-2R_1 - 2R_2) + y_2(R_1 + 4R_2 + R_3 + k_2 h^4)$$

$$+ y_3 (-2R_2 - 2R_3) + y_4(R_3) = 0 \qquad \text{A29}$$

Using Eq A19

$$y_1(-2R_1 - 2R_2 + a_oR_1) + y_2(R_1 + 4R_2 + R_3 + k_2h^4 - b_oR_1)$$

$$+ y_3(-2R_2 - 2R_3) + y_4 (R_3) = 0 \qquad \text{A30}$$

Using Eq A26

$$y_2(R_1 + 4R_2 + R_3 - b_oR_1 - 2a_1R_1 - 2a_1R_2 + a_oa_1R_1 + k_2h^4)$$

$$+ y_3(-2R_2 - 2R_3 + 2b_1R_1 + 2b_1R_2 - a_ob_1R_1) + y_4(R_3) = 0 \qquad \text{A31}$$

Putting Eq A31 into the form

$$y_2 = a_2 y_3 - b_2 y_4 \qquad \text{A32}$$

the following expressions for $a_2$ and $b_2$ result

$$a_2 = \frac{-2b_1R_1 + a_o b_1 R_1 - 2b_1R_2 + 2R_2 + 2R_3}{R_1 - 2a_1R_1 - b_oR_1 + a_o a_1 R_1 + 4R_2 - 2a_1R_2 + R_3 + k_2 h^4} \qquad \text{A33}$$

$$b_2 = \frac{R_3}{R_1 - 2a_1R_1 - b_oR_1 + a_o a_1 R_1 + 4R_2 - 2a_1R_2 + R_3 + k_2 h^4} \qquad \text{A34}$$

By continuing to write Eq A11 about successive points along the pile and by making the substitutions indicated, general expressions emerge for y and for the coefficients as follows:

$$y_m = a_m y_{m+1} - b_m y_{m+2} \qquad \text{A35}$$

$$a_m = \frac{-2b_{m-1}R_{m-1} + a_{m-2}b_{m-1}R_{m-1} + 2R_m - 2b_{m-1}R_m + 2R_{m+1}}{C_m} \qquad \text{A36}$$

$$b_m = \frac{R_{m+1}}{C_m} \qquad \text{A37}$$

$$C_m = R_{m-1} - 2a_{m-1}R_{m-1} - b_{m-2}R_{m-1} + a_{m-2}a_{m-1}R_{m-1} + 4R_m$$

$$- 2a_{m-1}R_m + R_{m+1} + k_m h^4 \qquad \text{A38}$$

The general expressions shown in Eqs A35 through A38 can be used for computing values of y for points along the pile except Points 0 and 1, where Eqs A19 through A22 and A26 through A28 must be used.

The use of these expressions for y for computing the deflection along a pile requires that some deflections be known. Boundary conditions at the top of the pile are employed in deriving the necessary expressions for deflection.

## Expressions for Boundary Conditions $p_t$ and $M_t$

For convenience in writing, the following equalities are established.

$$J_2 = \frac{M_t h^2}{R_t} \qquad \text{A39}$$

$$J_3 = \frac{2P_t h^3}{R_t} \qquad \text{A40}$$

The difference expressions for moment and shear (see Eqs A5 and A6) are:

$$M_m = \frac{R_m}{h^2} (y_{m-1} - 2y_m + y_{m+1}) \qquad \text{A41}$$

$$V_m = \frac{R_m}{2h^3} (y_{m-2} - 2y_{m-1} + 2y_{m+1} - y_{m+2}) \qquad \text{A42}$$

Using the notation shown in Fig 3 for the subscripts and using the above four equations, the equations which express the desired boundary conditions at the top of the pile are:

$$y_{t-1} - 2y_t + y_{t+1} = J_2 \qquad \text{A43}$$

$$y_{t-2} - 2y_{t-1} + 2y_{t+1} - y_{t+2} = J_3 \qquad \text{A44}$$

Eq A35 may be used to obtain the following expressions:

$$y_t = a_t y_{t+1} - b_t y_{t+2} \qquad \text{A45}$$

$$y_{t-1} = a_{t-1} y_t - b_{t-1} y_{t+1} \qquad \text{A46}$$

$$y_{t-2} = a_{t-2} y_{t-1} - b_{t-2}\, y_t \qquad \text{A47}$$

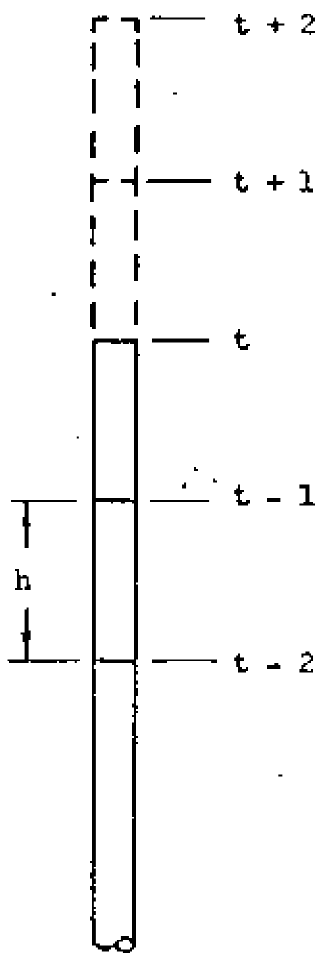


Fig. A3 Points at top of pile.

Substituting expression for $y_{t-1}$ from Eq A46 into Eq A43 the following expression is obtained:

$$-G_1 y_t + G_2 y_{t+1} = J_2 \qquad \text{A48}$$

where

$$G_1 = 2 - a_{t-1} \qquad \text{A49}$$

$$G_2 = 1 - b'_{t-1} \qquad \text{A50}$$

Substituting expressions for $y_{t-1}$ and $y_{t-2}$ from Eqs A46 and A47 into Eq A44 the following expression is obtained.

$$y_t \; (a_{t-1}a_{t-2} - 2a_{t-1} - b_{t-2}) + y_{t+1} \; (-a_{t-2}b_{t-1} + 2b_{t-1} + 2)$$

$$-y_{t+2} = J_3 \qquad \text{A51}$$

Solving Eqs A45, A48 and A51 for $y_t$, the following expression is obtained.

$$y_t = \frac{J_2G_3 + J_3G_2}{G_2G_5 - G_1G_3} \qquad \text{A52}$$

where

$$G_3 = \frac{a_t}{b_t} - 2.0 - 2.0b_{t-1} + b_{t-1}a_{t-2} \qquad \text{A53}$$

and

$$G_5 = \frac{1}{b_t} - b_{t-2} - 2a_{t-1} + a_{t-1}a_{t-2} \qquad \text{A54}$$

From Eq A48

$$y_{t+1} = \frac{y_tG_1 + J_2}{G_2} \qquad \text{A55}$$

From Eq A45

$$y_{t+2} = \frac{a_ty_{t+1} - y_t}{b_t} \qquad \text{A56}$$

Eqs A54, A55 and A56 are the desired expressions.

Expressions for Boundary Conditions $P_t$ and $S_t$

For this derivation it is convenient to define the following equality:

$$J_1 = 2hS_t \qquad \text{A57}$$

The difference equation for slope is:

$$S_m = \frac{1}{2h}\,(y_{m-1} - y_{m+1}) \qquad \text{A58}$$

Using the previous notation, the equation which represents the boundary condition for slope at the top of the pile is:

$$y_{t-1} - y_{t+1} = J_1 \qquad \text{A59}$$

In addition to making use of Eq A59 it is necessary to use Eqs A44, A45, A46, and A47 which were used in the previous derivation. Combining Eqs A46 and A59 to eliminate $y_{t-1}$, the following expression is obtained:

$$a_{t-1}y_t - G_4 y_{t+1} = J_1 \qquad \text{A60}$$

where

$$G_4 = 1 + b_{t-1} \qquad \text{A61}$$

Proceeding as previously Eqs A45, A51 and A60 are solved simultaneously for $y_t$, as follows:

$$y_t = \frac{J_3G_4 - J_1G_3}{G_4G_5 - G_3a_{t-1}} \qquad \text{A62}$$

Solving Eq A60 for $y_{t+1}$ :

$$y_{t+1} = \frac{y_t a_{t-1} - J_1}{G_4} \qquad \text{A63}$$

The expression for $y_{t+2}$ is the same as previously derived in Eq A56. This completes the derivation of the desired expression for these boundary conditions.

Expressions for Boundary Conditions $P_t$ and $M_t/S_t$

For this derivation it is convenient to define the following equality:

$$J_4 = \frac{h}{2R_t} \frac{M_t}{S_t} \qquad \text{A64}$$

Then, the boundary condition for $M_t/S_t$ is expressed as follows:

$$\frac{y_{t-1} - 2y_t + y_{t+1}}{y_{t-1} - y_{t+1}} = J_4 \qquad \text{A65}$$

In addition to making use of Eq A65, it is necessary to use Eqs A44, A45, A46, and A47 as previously. Combining equations A46 and A65 to eliminate $y_{t-1}$, the following expression is obtained:

$$y_t \left(G_1 + J_4 a_{t-1}\right) - y_{t+1}\left(G_2 + J_4 G_4\right) = 0 \qquad \text{A66}$$

Now solving Eqs A45, A51 and A66 simultaneously for $y_t$:

$$y_t = \frac{-J_3\left(G_2 + J_4 G_4\right)}{G_3\left(G_1 + J_4 a_{t-1}\right) - G_5 \left(G_2 + J_4 G_4\right)} \qquad \text{A67}$$

Solving Eq A66 for $y_{t+1}$:

$$y_{t+1} = \frac{y_t \left(G_1 + J_4 a_{t-1}\right)}{G_2 + J_4 G_4} \qquad \text{A68}$$

The expression for $y_{t+2}$ is the same as previously derived in Eq A56. This completes the derivation of the desired expressions for these boundary conditions.

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3B/14

# Foundation Analysis of Offshore Pile Supported Structures

## Analyse des fondations des constructions sur pieux en mer

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**Summary**

Where lateral loads on pile-supported structures are significant, the critical factor for determining the size of the piles is frequently the portion of the stress resulting from bending moment. While the analysis of the vertical load capacity of a pile can proceed by conventional methods, the analysis for lateral load is more difficult, requiring the solution of a fourth-order differential equation. Complicating the rational solution of the problem is that static equilibrium must be maintained and compatibility must be achieved between the behavior of the superstructure, the foundation piling and the supporting soil.

Two rational methods are presented for analyzing piles under lateral loads, one a hand solution and the other requiring a digital computer. By iterative procedures, each of the methods achieves compatibility between an inelastic soil and an elastic pile which is elastically restrained by the superstructure. The soil stiffness constants are adjusted for each trial in accordance with predicted force-deformation relations for the soil.

Example problems are solved by each of the methods and the results are compared. Charts and tables are included and computation procedures are shown for the hand solution. The digital computer solution is rigorous and more adaptable but the hand solution is indicated to be satisfactory for many problems.

**Sommaire**

Quand les charges latérales qui pèsent sur des structures supportées par des pieux sont importantes, le facteur critique pour déterminer les dimensions de ces pieux est fréquemment la composante de la contrainte résultant du moment de flexion. Alors que l'analyse de la capacité de charge verticale d'un pieu peut être effectué par les méthodes ordinaires, l'analyse de la capacité de charge latérale est plus complexe et exige pour sa solution une équation différentielle du quatrième ordre. L'équilibre statique, qui doit être maintenu, et la condition de compatibilité entre le comportement de la structure, les pieux de fondation et le sol qui les supporte, qui doit être satisfaite complique encore la solution rationnelle du problème.

L'auteur propose deux méthodes pour analyser le comportement des pieux soumis à des charges latérales, l'une ne comportant que des calculs pouvant se faire à la main, l'autre exigeant l'emploi d'un calculateur électronique. L'une et l'autre permettent par tâtonnements d'arriver à la compatibilité entre un sol non élastique et une pile élastique soumise à l'action de la superstructure. Les constantes de rigidité du sol sont choisies pour chaque essai en accord avec les relations force-déformation prévues pour le sol.

A titre d'exemple l'auteur a résolu des problèmes par chacune de ces deux méthodes et en a comparé les résultats. Des courbes et des tables sont incluses et les méthodes de calcul sont indiquées pour la solution manuelle. La solution par calculateur est rigoureuse et plus adaptable, mais la solution manuelle est déjà satisfaisante pour beaucoup de problèmes.

### Introduction

Offshore structures have been erected in many parts of the world for the production of oil and for many other purposes. Although a wide variety of structural forms and concepts has been employed most of the structures are supported by piles.

The design of offshore structures involves consideration of unusually large ratios of lateral to vertical load, particularly in areas subject to severe storms. While the analysis of the foundation for vertical load capacity follows conventional procedures, the lateral-load analysis poses a more complex problem. Since combined flexural and axial stresses are used to determine required pile sizes and since the flexural stresses are usually the major factor, bending moments in the piles must be reliably predicted. This requires that the interaction between the structure and the foundation elements be rationally analyzed.

The successful application of a rational method of lateral-load analysis depends upon the availability of detailed information concerning soil properties. It is especially important to have accurate soils information very near the ground surface, at depths less than 10 to 20 pile diameters. Offshore soil borings, usually made from a floating vessel, are expensive but are indispensable to the analysis and ultimate safety of any offshore structure.

In order to solve the problem of a laterally loaded pile, it is necessary to predict the lateral soil resistance along the pile as a function of deflection of the pile. This problem has been discussed by TERZAGHI (1955) and MCCLELLAND and FOCHT (1958), and has been the subject of extensive research (MATLOCK and RIPPERGER, 1956 and 1958). A potentially useful concept has been presented by SKEMPTON (1951).

*Rational Methods of Analysis*—For rational solutions of structure-soil interaction problems it is necessary that conditions of both static equilibrium and compatibility of deformations be achieved simultaneously for all parts of the system. In the case of laterally loaded pile-supported structures, it is usually possible to treat the structure and the piles as linearly elastic components, but in general the mechanical characteristics of the soil are very non-linear. Solutions may be obtained by repeated elastic-theory computations, with soil stiffness values adjusted after each iteration.

*An Example of Lateral-Load Analysis*—As an example of the type of soil-pile-structure interaction problems which can be solved, a typical offshore structure is shown in Fig. 1.

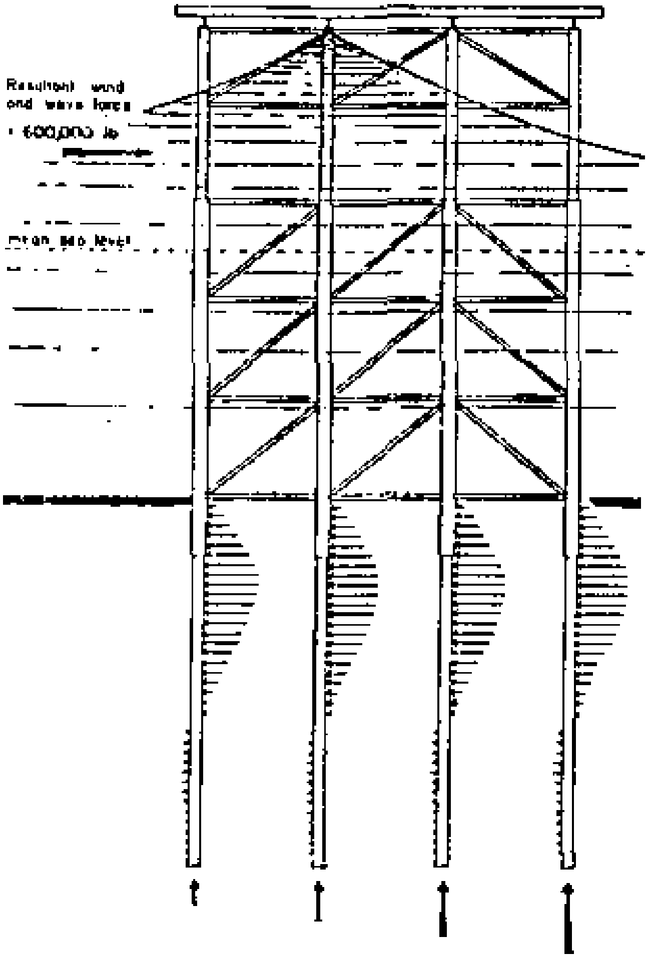



Fig. 1 Lateral forces applied to an offshore structure.
Forces latérales agissant sur une structure en mer.

The specific problem considered is that of solving for the bending moments in the portion of the structural system which lies beneath the soil surface. In erecting such a structure, a prefabricated welded-pipe framework or "jacket" is set in place on the ocean bottom and pipe piles are driven through the vertical members of the jacket.

The primary solution will consist of finding the set of elastic deflections of the pile (including the short jacket-leg extension) which simultaneously will satisfy (1) non-linear resistance-deformation relations which are predicted for the soil, (2) the elastic bending properties of the piles, and (3) the angular stiffness of the upper structure at the pile-to-structure connection.

The elastic elements of the problem are described in Fig. 2. The annular space between the pile and the jacket column is assumed to be grouted so that the two members will bend as a composite section. This is frequently but not always done in actual practice.

The elastic angular restraint provided by the portion of the structure above the soil many be analyzed by determining the moment required to produce a unit value of rotation at the connection. This value, and the imposed lateral load, constitute the boundary conditions for this particular problem. For the example, the elastic angular restraint $M_t/S_t$ = 6176 × 10⁹ in.-lb/radian and the lateral load $P_t$ = 150,000 lb per pile.

The force-deformation characteristics of the soil are described by a set of predicted "$p$-$y$" curves, such as are shown for the example problem in Fig. 3. Such curves may be developed from soil test data by methods previously mentioned. Since solutions for the interaction problem rely on repeated applications of elastic theory, a secant modulus of soil reaction $E_s$ is required which is defined as

$$E_s = \frac{-p}{y} \qquad \ldots . \quad (1)$$

This modulus is essentially only a computation device which varies with both depth and pile deflection. It is not a unique soil property.

The differential equation for a beam, from conventional theory, is

$$EI\frac{d^4y}{dx^4} = p \qquad \ldots . \quad (2)$$

Combining Equations 1 and 2, the general differential equation for the laterally loaded pile is

$$\frac{d^4y}{dx^4} + \frac{E_s}{EI}\,y = 0 \qquad \ldots . \quad (3)$$

Two methods of solution will be described for the example problem, by which the correct set of $E_s$ values are found and deflections and bending moments are computed.

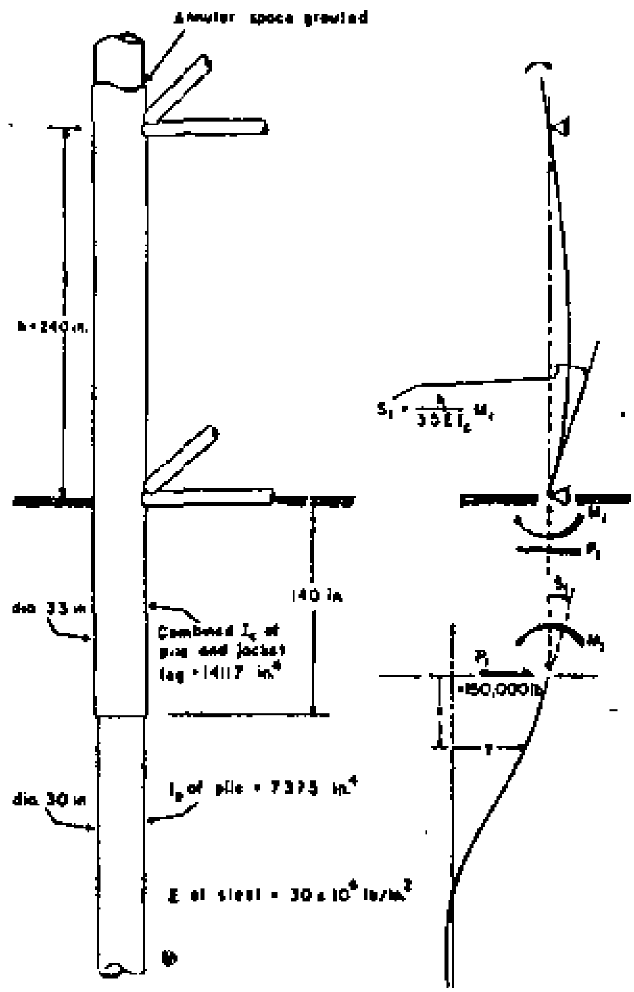



Fig. 2 The superstructure and the pile, considered as elastic elements of the problem.
La superstructure et la pile, considérées comme les éléments élastiques du problème.

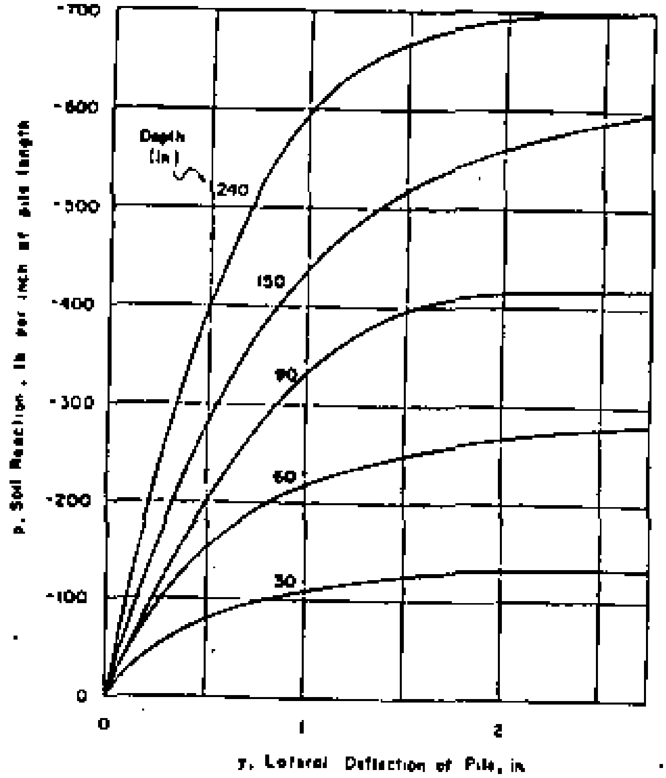



Fig. 3 Typical resistance-deflection curves predicted for the soil at various depths.

Courbe typique de variation de résistance prévue pour le sol à diverses profondeurs.

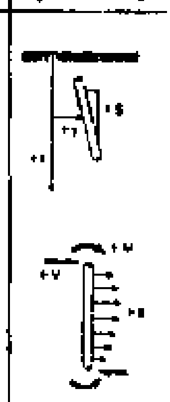
Table 1

Coefficients and Equations for Long Piles, $E_s = kx$

| $Z$ | $A_y$ | $A_s$ | $A_m$ | $A_v$ | $A_p$ |
|---|---|---|---|---|---|
| 0.0 | 2.435 | -1.623 | 0.000 | 1.000 | 0.000 |
| 0.1 | 2.273 | -1.618 | 0.100 | 0.989 | -0.227 |
| 0.2 | 2.112 | -1.603 | 0.198 | 0.956 | -0.422 |
| 0.3 | 1.952 | -1.578 | 0.291 | 0.906 | -0.586 |
| 0.4 | 1.796 | -1.545 | 0.379 | 0.840 | -0.718 |
| 0.5 | 1.644 | -1.503 | 0.459 | 0.764 | -0.822 |
| 0.6 | 1.496 | -1.454 | 0.532 | 0.677 | -0.897 |
| 0.7 | 1.353 | -1.397 | 0.595 | 0.585 | -0.947 |
| 0.8 | 1.216 | -1.335 | 0.649 | 0.489 | -0.973 |
| 0.9 | 1.086 | -1.268 | 0.693 | 0.392 | -0.977 |
| 1.0 | 0.962 | -1.197 | 0.727 | 0.295 | -0.962 |
| 1.2 | 0.738 | -1.047 | 0.767 | 0.109 | -0.885 |
| 1.4 | 0.544 | -0.893 | 0.772 | -0.056 | -0.761 |
| 1.6 | 0.381 | -0.741 | 0.746 | -0.193 | -0.609 |
| 1.8 | 0.247 | -0.596 | 0.696 | -0.298 | -0.445 |
| 2.0 | 0.142 | -0.464 | 0.628 | -0.371 | -0.283 |
| 3.0 | -0.075 | -0.040 | 0.225 | -0.349 | 0.226 |
| 4.0 | -0.050 | 0.052 | 0.000 | -0.106 | 0.201 |
| 5.0 | -0.009 | 0.025 | -0.033 | 0.013 | 0.046 |

| $Z$ | $B_y$ | $B_s$ | $B_m$ | $B_v$ | $B_p$ |
|---|---|---|---|---|---|
| 0.0 | 1.623 | -1.750 | 1.000 | 0.000 | 0.000 |
| 0.1 | 1.453 | -1.650 | 1.000 | -0.007 | -0.145 |
| 0.2 | 1.293 | -1.550 | 0.999 | -0.028 | -0.259 |
| 0.3 | 1.143 | -1.450 | 0.994 | -0.058 | -0.343 |
| 0.4 | 1.003 | -1.351 | 0.987 | -0.095 | -0.401 |
| 0.5 | 0.873 | -1.253 | 0.976 | -0.137 | -0.436 |
| 0.6 | 0.752 | -1.156 | 0.960 | -0.181 | -0.451 |
| 0.7 | 0.642 | -1.061 | 0.939 | -0.226 | -0.449 |
| 0.8 | 0.540 | -0.968 | 0.914 | -0.270 | -0.432 |
| 0.9 | 0.448 | -0.878 | 0.885 | -0.312 | -0.403 |
| 1.0 | 0.364 | -0.792 | 0.852 | -0.350 | -0.364 |
| 1.2 | 0.223 | -0.629 | 0.775 | -0.414 | -0.268 |
| 1.4 | 0.112 | -0.482 | 0.668 | -0.456 | -0.157 |
| 1.6 | 0.029 | -0.354 | 0.594 | -0.477 | -0.047 |
| 1.8 | -0.030 | -0.245 | 0.498 | -0.476 | 0.054 |
| 2.0 | -0.070 | -0.155 | 0.404 | -0.456 | 0.140 |
| 3.0 | -0.089 | 0.057 | 0.059 | -0.311 | 0.268 |
| 4.0 | -0.028 | 0.049 | -0.042 | 0.017 | 0.112 |
| 5.0 | 0.000 | 0.011 | -0.026 | 0.029 | -0.002 |

| Term | Equation | Sign Convention |
|---|---|---|
| Depth | $x = ZT$ | |
| Deflection | $y = A_y \frac{P_t T^3}{EI} + B_y \frac{M_t T^2}{EI}$ | |
| Slope | $S = A_s \frac{P_t T^2}{EI} + B_s \frac{M_t T}{EI}$ | |
| Moment | $M = A_m P_t T + B_m M_t$ | |
| Shear | $V = A_v P_t + B_v \frac{M_t}{T}$ | |
| Soil Reaction | $p = A_p \frac{P_t}{T} + B_p \frac{M_t}{T^2}$ | |




*Hand Solution* — Non-dimensional solutions may be developed for any fixed form of variation of $E_s$ with depth (MATLOCK and REESE, 1960). One set of such solutions, for $E_s$ proportional to depth $x$, or $E_s = kx$, is available (REESE and MATLOCK, 1956). In this set of solutions, the deflection y of the pile, at any depth $x$, is

$$y = A_y \frac{P_t T^3}{EI} + B_y \frac{M_t T^2}{EI} \quad \ldots\ (4)$$

where $EI$ is the flexural rigidity of the pile, where $T$ is the relative stiffness factor, defined by

$$T^5 = \frac{EI}{k} \quad \ldots\ (5)$$

and where $P_t$ and $M_t$ are as shown in Fig. 2. $A_y$ and $B_y$ are non-dimensional coefficients for deflection due to shear and deflection due to moment, respectively. They are functions of only a depth coefficient $Z$ which is equal to $x/T$. Coefficients for the case of very long piles, and appropriate equations and sign conventions, are given in Table 1.

It is convenient to define an additional set of non-dimensional deflection coefficients by rearranging Equation 4 as follows,

$$y = C_y \frac{P_t T^3}{EI} \quad \ldots\ (6)$$

where, at any depth coefficient $Z$,

$$C_y = A_y + \frac{M_t}{P_t T} B_y$$

Depending on the angular restraint provided by the structure, values of $M_t/P_t T$ will range from zero for the pinned-end case to -0·93 for the case where the structure prevents any rotation of the pile head. Values of $C_y$ are given by the curves in Fig. 4.

To begin the solution of the example problem it is necessary to assume, temporarily at least, that the form of soil modulus variation $E_s = kx$ will be a satisfactory approximation of the actual final $E_s$ variation. Also, available non-dimensional solutions are limited to a pile of constant bending stiffness. For the example hand solution the pile stiffness will be assumed equal to that of the combined pile and jacket leg. The effect of this assumption will be considered subsequently.

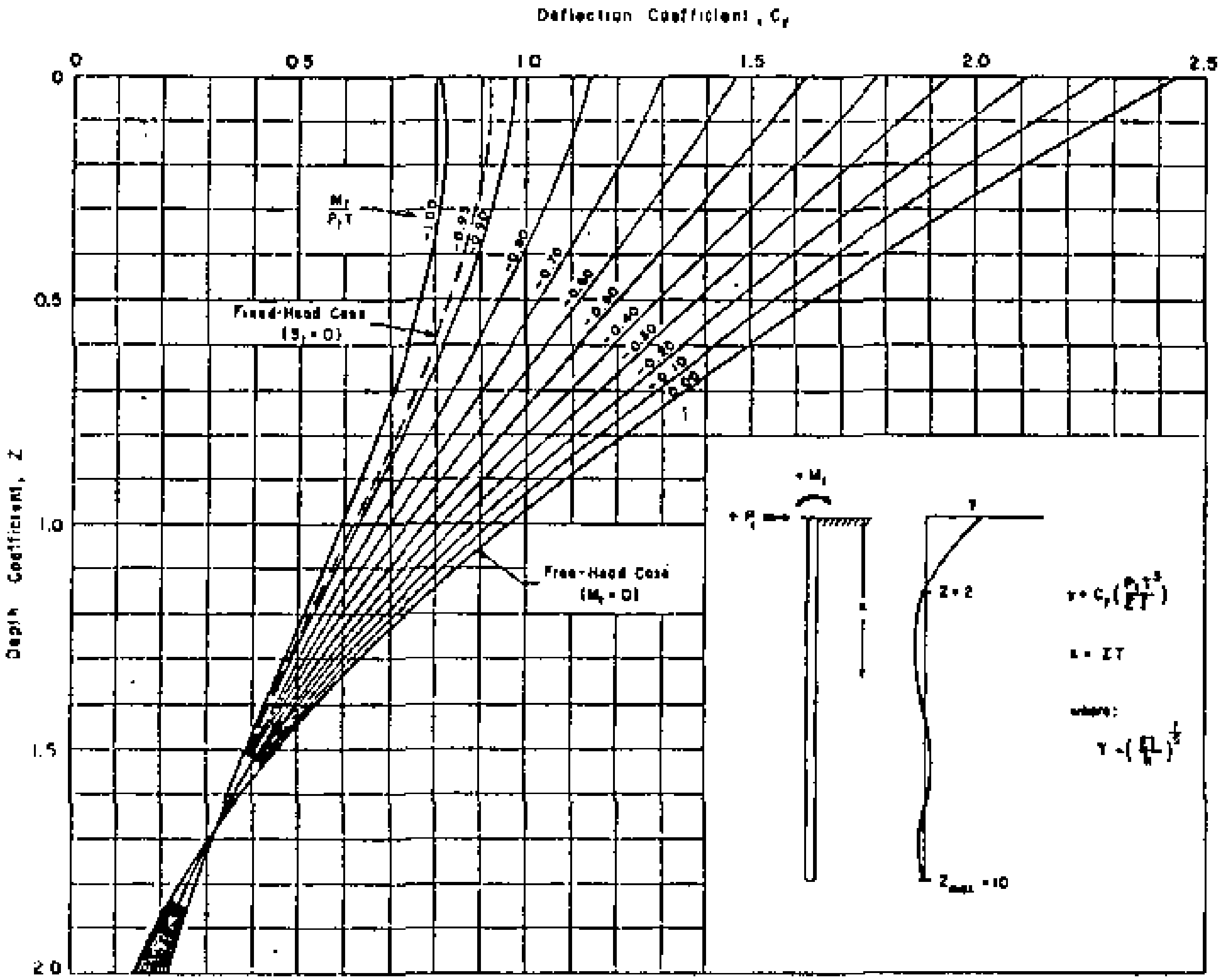



Fig. 4 Non-dimensional coefficients for lateral deflection of a pile, assuming soil modulus proportional to depth, or $E_s = kx$.

Coefficients sans dimensions pour déplacement lateral d'une pile avec comme hypothèse le module du sol proportionnel à la profondeur, ou $E_s = kx$.

The slope at the top of the pile is

$$S_t = A_{s_t} \frac{P_t T^2}{EI_t} + B_{s_t} \frac{M_t T}{EI_t} \qquad \ldots\ (7)$$

where the subscript $_t$ indicates values at $Z = 0$. The relation between $M_t$ and $S_t$ from Fig. 2 is

$$S_t = \frac{h}{3.5\ EI_t} M_t \qquad \ldots\ (8)$$

Combining Equations 7 and 8, and rearranging,

$$\frac{M_t}{P_t T} = \frac{A_{s_t}\ T}{\frac{h}{3.5} - B_{s_t} T} = \frac{-1.623\ T}{\frac{240}{3.5} + 1.750\ T} = -\frac{T}{42.25 + 1.078\ T} \qquad \ldots\ (9)$$

Since the relative stiffness factor $T$ depends on the coefficient of soil modulus variation $k$ and this quantity in turn depends on non-linear soil resistance characteristics, the solution must proceed by a process of repeated trial and adjustment of values of $T$ (or $k$) until the deflection and resistance patterns of the pile are made to agree as closely as possible with the resistance-deflection ($p$-$y$) relations previously estimated for the soil and shown in Fig. 3. Even though the final set of secant soil moduli ($E_s = p/y$) may not vary in a perfectly linear fashion with depth, proper fitting of $E_s = kx$ will usually produce satisfactory solutions. (See Matlock and Reese 1960.)

For the first trial, $T$ will be assumed equal to 200 in. From Equation 9, the corresponding value of $M_t/P_t T$ is -0·776. For this value of $M_t/P_t T$, values of $C_y$ are interpolated from Fig. 4 and are given in Table 2 at depths corresponding to the positions of the several $p$-$y$ curves of Fig. 3. Values of deflection $y$ are then computed at each depth. By reference to Fig. 3, values of soil resistance $p$ are obtained, and soil modulus values $E_s$ are computed. This is similar to the method of McClelland and Focht (1958).

## Table 2

### Sample Computations for First Trial

| Depth | Depth Coefficient | Deflection Coefficient | Deflection | Soil Resistance | Soil Modulus |
|---|---|---|---|---|---|
| $x$ | $Z$ | $C_y$ | $y$ | $p$ | $E_s$ |
| | $= \frac{x}{T}$ | from Fig. 4 | $= C_y \frac{P_t T^3}{EI_c}$ | from Fig. 3 | $= \frac{-p}{y}$ |
| in. | -- | -- | in. | lb/in. | lb/in.$^2$ |
| 30 | 0.15 | 1.13 | 3.20 | -132 | 41 |
| 60 | 0.30 | 1.06 | 3.00 | -285 | 95 |
| 90 | 0.45 | 0.99 | 2.81 | -420 | 149 |
| 150 | 0.75 | 0.82 | 2.32 | -578 | 249 |
| 240 | 1.20 | 0.57 | 1.62 | -675 | 416 |

Values of soil modulus from the first trial are plotted versus depth as shown in Fig. 5. A straight line through the origin is fitted to the points, with more weight being given to points at depths less than $x = 0{\cdot}5T$ than at greater depths. For this straight line, the coefficient of soil modulus variation resulting from the first trial is computed as

$$k = \frac{E_s}{x} = 1{\cdot}6 \text{ lb/in.}^3 \qquad \dots\dots (10)$$

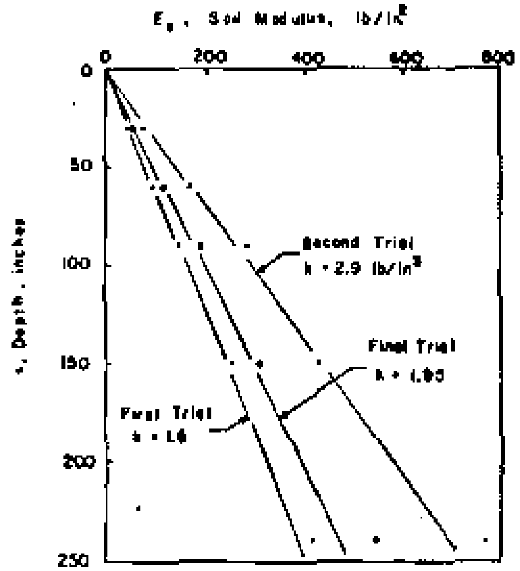



Fig. 5 Trial plots of soil modulus values. The first trial corresponds to computations in Table 2.

Courbes correspondant à diverses valeurs du module du sol. La première correspond au calcul figurant · Table 2.

The corresponding value of the relative stiffness factor is

$$T_{(obtained)} = \sqrt[5]{\frac{EI_c}{k}} = 194 \text{ in.} \qquad \dots\dots (11)$$

If the value of $T_{(obtained)}$ were equal to the value of $T_{(tried)}$, the trial and error process would have been completed. To facilitate additional estimating and to reach closure with a minimum of trials, a plot of $T$-values is used, as shown in Fig. 6. Two trials will usually allow interpolation for the final value of $T$. A final set of computations for $E_s$ values is then made as a check.

Computation of values of bending moment along the pile (and of deflection, slope, shear and soil reaction, if desired) are made by application of the equations and non-dimensional coefficients given in Table 1.

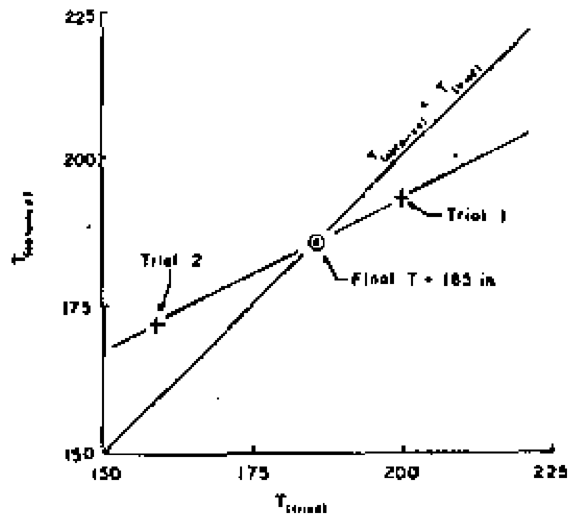



Fig. 6 Interpolation for final value of relative stiffness factor $T$.
Interpolation pour la valeur finale du facteur de rigidité relative $T$.

The moment curve from the hand solution of the example problem is given as a solid curve in Fig. 7. Subsequent comparisons with more rigorous computer solutions will demonstrate the effects of the simplifying assumptions used in the above solution. These assumptions are (1) that $E_s = kx$ is a satisfactory approximation of the real variation of $E_s$ values, and (2) that the use of a constant value for the flexural stiffness of the pile does not introduce excessive error.

*Computer Solutions*—Where unusual variations in soil resistance are encountered and where it is desirable to consider properly any changes in flexural stiffness of the pile, the use of a digital computer is a practical necessity.

The difference-equation method offers a convenient means of solving the problem of the laterally loaded pile (GLESER, 1953) (REESE and GINZBARG, 1960). A program has been developed for the IBM 650 computer, which provides the following principal features.

(1) Step-changes in flexural stiffness may be introduced at any depths.

(2) Length of the pile may be varied as desired.

(3) The soil *p-y* data may be introduced in several ways, including a simple numerical tabulation to define a set of individual *p-y* curves of any form and of any variation with depth.

(4) Various combinations of boundary conditions may be introduced, including lateral load $P_t$ and either (a) moment $M_t$, (b) slope $S_t$, or (c) moment divided by slope, $M_t/S_t$. (The last form is used with the example problem.)

(5) By successive elastic-theory difference-equation computations, based on repeated reference to the soil *p-y* data, the computer will determine, independently at each increment along the pile, the value of soil modulus which represents the proper compatibility and equilibrium conditions for the soil, the pile, and the superstructure.

*Comparison of Solutions*—The results of three computer solutions are shown by the dashed curves in Fig. 7 and may be compared with the hand solution previously described.

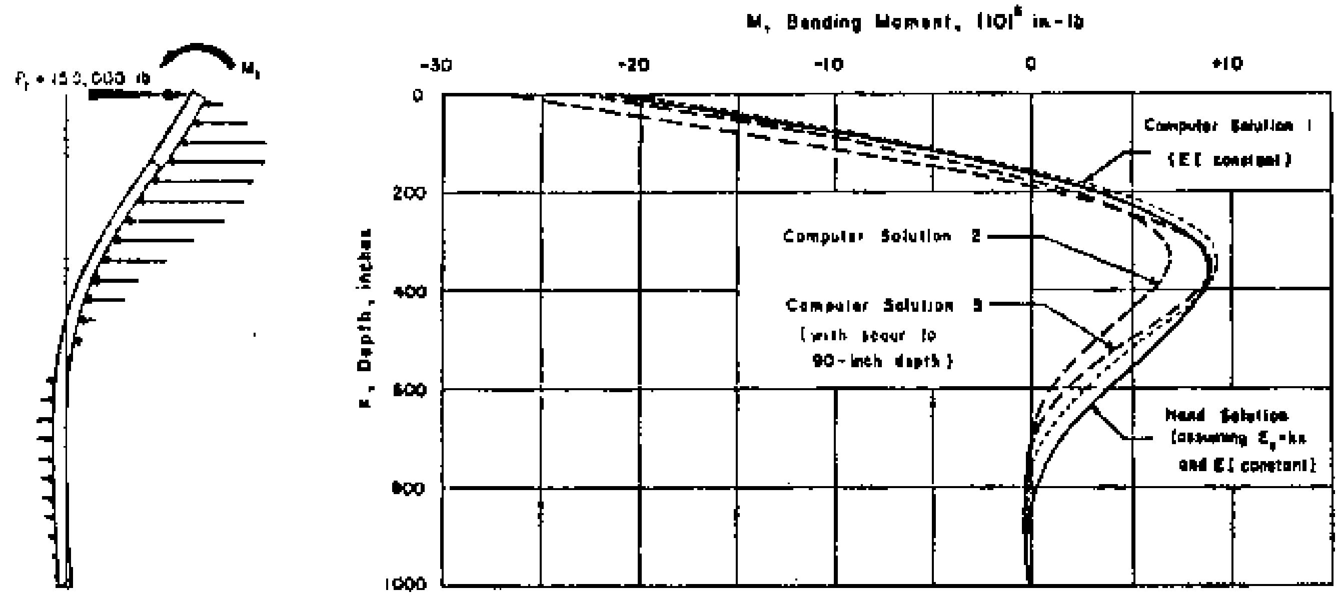



Fig. 7 Comparison of moment curves for Hand Solution (assuming constant flexural stiffness $EI_p$ of pile, and soil modulus $E_s = kx$) with (a) Computer Solution 1 (same as Hand Solution except $E_s$ varies as required), (b) Computer Solution 2 (the correct solution), and (c) Computer Solution 3 (a correct solution with soil resistance eliminated by scour to a depth of 90 inches).

Fig. 7 Comparaison des courbes des moments pour la solution manuelle (supposant une rigidité à la flexion du pieu constante $EI_p$ et le module du sol $E_s = kx$) avec, (a) solution du calculateur (identique à la solution manuelle à l'exception des conditions de variation de $E_s$), (b) solution 2 au calculateur (solution correcte), et (c) solution 3 au calculateur (solution correcte où la résistance du sol est éliminée par le creusement jusqu'à une profondeur de 90 inches).

To test the validity of the assumption that $E_s = kx$, Computer Solution 1 was performed on exactly the same basis as the hand solution except that the computer was allowed to seek the proper values of soil moduli independently at each depth. The flexural stiffness of the pile was held constant. While good agreement with the previous hand solution is obtained at points of maximum negative and maximum positive moment, some divergence is noted at greater depths.

The actual variation of flexural stiffness at the end of the jacket-leg extension was added to the conditions of the problem for Computer Solution 2. The differences between the results of Computer Solutions 1 and 2 represent the net change due to the step-change in flexural stiffness of the pile.

Scour of the ocean bottom around offshore structures is a matter of considerable importance to the safety of the structures. For Computer Solution 3 the input conditions of Computer Solution 2 were modified by eliminating completely the soil resistance to a depth of about eight feet. This modification would be difficult to introduce into a hand solution but is easily done with the computer by simple changes in the input data which describe the soil characteristics. An appreciable increase in both maximum negative and maximum positive bending moments may be noted.

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# TABLES OF NON-DIMENSIONAL SOLUTIONS FOR LATERALLY LOADED PILES

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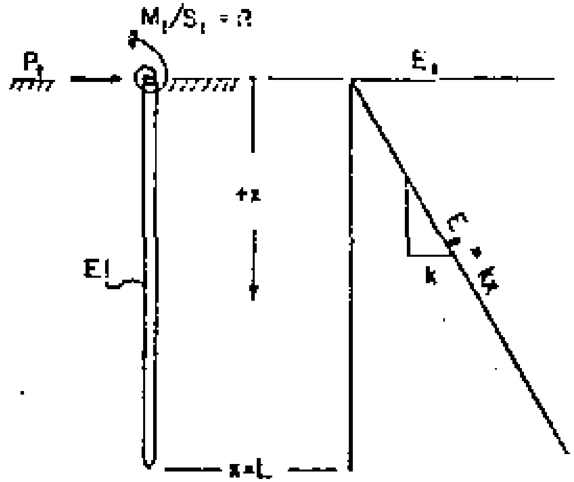
Matlock, Hudson, and Lymon C. Reese, "Non-Dimensional Solutions for Laterally Loaded Piles, with Soil Modulus Assumed Proportionate to Depth," Proceedings of the Eighth Texas Conference on Soil Mechanics and Foundation Engineering, Special Publication No. 29, Bureau of Engineering Research, The University of Texas, Austin, 1956, 41 pp.

Matlock, Hudson, and Lymon C. Reese, "Foundation Analysis of Offshore Pile-Supported Structures," Proceedings, Fifth International Conference, Internation Society of Soil Mechanics and Foundation Engineering, 3B/14, Paris, July 1961, pp. 91-97.

Matlock, Hudson, and Lymon C. Reese, "Generalized Solutions for Laterally Loaded Piles," Transactions, American Society of Civil Engineers, Paper No. 3370, Vol. 127, Part I, 1962, pp. 1220-1269.




$T = (EI/k)^{1/5}$

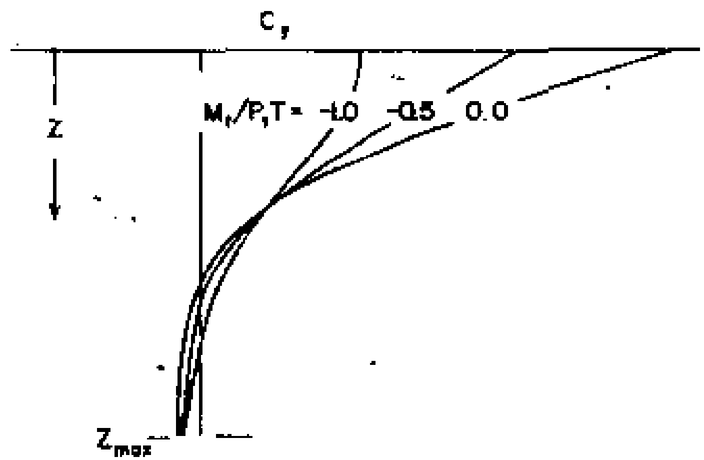
$x = ZT$

$Z_{max} = L/T$

$M_t/P_tT = \frac{A_{s_t}}{(EI/RT) - B_{s_t}}$

where $A_{s_t}$ and $B_{s_t}$ are coefficients for slo
at the top of the pile (Z = 0), whic
are obtained from Appendix A.2.

Deflection $y = C_y(P_tT^3/EI)$




Appendix A.1. Deflection Coefficients $C_y$ for Elastic Piles, $E_s = kx$, $Z_{max}$ varied from 2.2 to 10.

| Z | $M_t/P_tT$ -1.0 | -0.9 | -0.8 | -0.7 | -0.6 | -0.5 | -0.4 | -0.3 | -0.2 | -0.1 | 0.0 | Z |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| .0 | 0.812 | 0.975 | 1.137 | 1.299 | 1.461 | 1.624 | 1.786 | 1.948 | 2.111 | 2.273 | 2.435 | .0 |
| .1 | 0.820 | 0.965 | 1.111 | 1.256 | 1.401 | 1.546 | 1.692 | 1.837 | 1.982 | 2.128 | 2.273 | .1 |
| .2 | 0.819 | 0.948 | 1.077 | 1.207 | 1.336 | 1.465 | 1.594 | 1.724 | 1.853 | 1.982 | 2.112 | .2 |
| .3 | 0.809 | 0.924 | 1.038 | 1.152 | 1.267 | 1.381 | 1.495 | 1.609 | 1.724 | 1.838 | 1.952 | .3 |
| .4 | 0.793 | 0.893 | 0.994 | 1.094 | 1.194 | 1.294 | 1.395 | 1.495 | 1.595 | 1.696 | 1.796 | .4 |
| .5 | 0.771 | 0.858 | 0.945 | 1.032 | 1.120 | 1.207 | 1.294 | 1.381 | 1.469 | 1.556 | 1.643 | .5 |
| .6 | 0.743 | 0.818 | 0.893 | 0.969 | 1.044 | 1.119 | 1.194 | 1.270 | 1.345 | 1.420 | 1.495 | .6 |
| .7 | 0.711 | 0.775 | 0.839 | 0.904 | 0.968 | 1.032 | 1.096 | 1.160 | 1.224 | 1.288 | 1.353 | .7 |
| .8 | 0.676 | 0.730 | 0.784 | 0.838 | 0.892 | 0.946 | 1.000 | 1.054 | 1.108 | 1.162 | 1.216 | .8 |
| .9 | 0.638 | 0.683 | 0.727 | 0.772 | 0.817 | 0.862 | 0.907 | 0.951 | 0.996 | 1.041 | 1.086 | .9 |
| 1.0 | 0.598 | 0.634 | 0.671 | 0.707 | 0.744 | 0.780 | 0.817 | 0.853 | 0.889 | 0.926 | 0.962 | 1.0 |
| 1.2 | 0.515 | 0.537 | 0.559 | 0.582 | 0.604 | 0.626 | 0.649 | 0.671 | 0.788 | 0.817 | 0.738 | 1.2 |
| 1.4 | 0.432 | 0.443 | 0.454 | 0.465 | 0.476 | 0.488 | 0.499 | 0.510 | 0.521 | 0.533 | 0.544 | 1.4 |
| 1.6 | 0.351 | 0.354 | 0.357 | 0.360 | 0.363 | 0.366 | 0.369 | 0.372 | 0.375 | 0.378 | 0.381 | 1.6 |
| 1.8 | 0.277 | 0.274 | 0.271 | 0.268 | 0.265 | 0.262 | 0.259 | 0.256 | 0.253 | 0.250 | 0.247 | 1.8 |
| 2.0 | 0.211 | 0.204 | 0.197 | 0.190 | 0.183 | 0.176 | 0.169 | 0.162 | 0.155 | 0.148 | 0.141 | 2.0 |
| 2.5 | 0.085 | 0.075 | 0.064 | 0.054 | 0.043 | 0.033 | 0.022 | 0.012 | 0.001 | 0.009- | 0.020- | 2.5 |
| 3.0 | 0.014 | 0.005 | 0.004- | 0.013- | 0.022- | 0.031- | 0.040- | 0.048- | 0.057- | 0.066- | 0.075- | 3.0 |
| 3.5 | 0.016- | 0.022- | 0.028- | 0.034- | 0.039- | 0.045- | 0.051- | 0.056- | 0.062- | 0.068- | 0.074- | 3.5 |
| 4.0 | 0.022- | 0.025- | 0.028- | 0.031- | 0.033- | 0.036- | 0.039- | 0.042- | 0.045- | 0.047- | 0.050- | 4.0 |
| 4.5 | 0.017- | 0.018- | 0.019- | 0.020- | 0.021- | 0.022- | 0.022- | 0.023- | 0.024- | 0.025- | 0.026- | 4.5 |
| 5.0 | 0.010- | 0.010- | 0.010- | 0.010- | 0.009- | 0.009- | 0.009- | 0.009- | 0.009- | 0.009- | 0.009- | 5.0 |
| 10.0 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 10.0 |

Table A.1.1. Deflection Coefficients $C_y$ for Elastic Piles, $E_s$ = kx, $Z_{max}$ = 10.0

| Z | $M_t/P_tT$ -1.0 | -0.9 | -0.8 | -0.7 | -0.6 | -0.5 | -0.4 | -0.3 | -0.2 | -0.1 | 0.0 | Z |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| .0 | 0.821 | 0.984 | 1.146 | 1.308 | 1.471 | 1.633 | 1.796 | 1.958 | 2.121 | 2.283 | 2.445 | .0 |
| .1 | 0.829 | 0.974 | 1.120 | 1.265 | 1.411 | 1.556 | 1.702 | 1.847 | 1.992 | 2.138 | 2.283 | .1 |
| .2 | 0.828 | 0.957 | 1.087 | 1.216 | 1.346 | 1.475 | 1.604 | 1.734 | 1.863 | 1.992 | 2.122 | .2 |
| .3 | 0.819 | 0.933 | 1.048 | 1.162 | 1.276 | 1.391 | 1.505 | 1.619 | 1.734 | 1.848 | 1.962 | .3 |
| .4 | 0.803 | 0.903 | 1.004 | 1.104 | 1.204 | 1.305 | 1.405 | 1.505 | 1.605 | 1.706 | 1.806 | .4 |
| .5 | 0.781 | 0.868 | 0.955 | 1.043 | 1.130 | 1.217 | 1.304 | 1.392 | 1.479 | 1.566 | 1.653 | .5 |
| .6 | 0.754 | 0.829 | 0.904 | 0.979 | 1.054 | 1.129 | 1.205 | 1.280 | 1.355 | 1.430 | 1.505 | .6 |
| .7 | 0.722 | 0.786 | 0.850 | 0.914 | 0.978 | 1.042 | 1.106 | 1.170 | 1.234 | 1.299 | 1.363 | .7 |
| .8 | 0.687 | 0.741 | 0.795 | 0.849 | 0.902 | 0.956 | 1.010 | 1.064 | 1.118 | 1.172 | 1.226 | .8 |
| .9 | 0.649 | 0.694 | 0.738 | 0.783 | 0.828 | 0.872 | 0.917 | 0.961 | 1.006 | 1.051 | 1.095 | .9 |
| 1.0 | 0.609 | 0.646 | 0.682 | 0.718 | 0.754 | 0.791 | 0.827 | 0.863 | 0.899 | 0.936 | 0.972 | 1.0 |
| 1.2 | 0.527 | 0.549 | 0.571 | 0.593 | 0.615 | 0.637 | 0.659 | 0.681 | 0.703 | 0.725 | 0.747 | 1.2 |
| 1.4 | 0.444 | 0.455 | 0.466 | 0.476 | 0.487 | 0.498 | 0.509 | 0.520 | 0.531 | 0.542 | 0.552 | 1.4 |
| 1.6 | 0.364 | 0.366 | 0.369 | 0.371 | 0.374 | 0.376 | 0.379 | 0.381 | 0.384 | 0.386 | 0.388 | 1.6 |
| 1.8 | 0.289 | 0.286 | 0.282 | 0.279 | 0.275 | 0.272 | 0.268 | 0.265 | 0.261 | 0.258 | 0.254 | 1.8 |
| 2.0 | 0.222 | 0.215 | 0.207 | 0.200 | 0.192 | 0.185 | 0.177 | 0.170 | 0.162 | 0.154 | 0.147 | 2.0 |
| 2.5 | 0.091 | 0.080 | 0.069 | 0.058 | 0.047 | 0.035 | 0.024 | 0.013 | 0.002 | 0.009- | 0.020- | 2.5 |
| 3.0 | 0.007 | 0.002- | 0.012- | 0.021- | 0.031- | 0.040- | 0.050- | 0.059- | 0.069- | 0.078- | 0.088- | 3.0 |
| 3.5 | 0.048- | 0.054- | 0.059- | 0.065- | 0.071- | 0.076- | 0.082- | 0.088- | 0.094- | 0.099- | 0.105- | 3.5 |
| 4.0 | 0.093- | 0.094- | 0.096- | 0.097- | 0.099- | 0.100- | 0.102- | 0.103- | 0.105- | 0.106- | 0.108- | 4.0 |

**Table A.1.2.** Deflection Coefficients $C_y$ for Elastic Piles, $E_s = kx$, $Z_{max} = 4.0$

| Z | $M_t/P_tT$ -1.0 | -0.9 | -0.8 | -0.7 | -0.6 | -0.5 | -0.4 | -0.3 | -0.2 | -0.1 | 0.0 | Z |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| .0 | 0.857 | 1.020 | 1.184 | 1.348 | 1.511 | 1.675 | 1.839 | 2.002 | 2.166 | 2.330 | 2.493 | .0 |
| .1 | 0.864 | 1.010 | 1.157 | 1.304 | 1.450 | 1.597 | 1.743 | 1.890 | 2.037 | 2.183 | 2.330 | .1 |
| .2 | 0.862 | 0.992 | 1.123 | 1.253 | 1.384 | 1.515 | 1.645 | 1.776 | 1.906 | 2.037 | 2.167 | .2 |
| .3 | 0.852 | 0.967 | 1.083 | 1.198 | 1.314 | 1.429 | 1.545 | 1.660 | 1.776 | 1.891 | 2.007 | .3 |
| .4 | 0.835 | 0.936 | 1.038 | 1.139 | 1.240 | 1.342 | 1.443 | 1.545 | 1.646 | 1.748 | 1.849 | .4 |
| .5 | 0.812 | 0.900 | 0.988 | 1.077 | 1.165 | 1.253 | 1.342 | 1.430 | 1.518 | 1.607 | 1.695 | .5 |
| .6 | 0.783 | 0.859 | 0.936 | 1.012 | 1.088 | 1.164 | 1.241 | 1.317 | 1.393 | 1.470 | 1.546 | .6 |
| .7 | 0.751 | 0.816 | 0.881 | 0.946 | 1.011 | 1.076 | 1.141 | 1.206 | 1.271 | 1.337 | 1.402 | .7 |
| .8 | 0.714 | 0.769 | 0.824 | 0.879 | 0.934 | 0.989 | 1.044 | 1.099 | 1.154 | 1.209 | 1.263 | .8 |
| .9 | 0.676 | 0.721 | 0.767 | 0.812 | 0.858 | 0.904 | 0.949 | 0.995 | 1.040 | 1.086 | 1.132 | .9 |
| 1.0 | 0.635 | 0.672 | 0.709 | 0.746 | 0.784 | 0.821 | 0.858 | 0.895 | 0.932 | 0.970 | 1.007 | 1.0 |
| 1.2 | 0.550 | 0.573 | 0.595 | 0.618 | 0.641 | 0.664 | 0.687 | 0.710 | 0.733 | 0.756 | 0.778 | 1.2 |
| 1.4 | 0.464 | 0.475 | 0.487 | 0.498 | 0.510 | 0.522 | 0.533 | 0.545 | 0.557 | 0.568 | 0.580 | 1.4 |
| 1.6 | 0.380 | 0.383 | 0.386 | 0.389 | 0.392 | 0.395 | 0.398 | 0.401 | 0.405 | 0.408 | 0.411 | 1.6 |
| 1.8 | 0.301 | 0.298 | 0.295 | 0.292 | 0.289 | 0.285 | 0.282 | 0.279 | 0.276 | 0.273 | 0.270 | 1.8 |
| 2.0 | 0.228 | 0.221 | 0.214 | 0.206 | 0.199 | 0.192 | 0.184 | 0.177 | 0.169 | 0.162 | 0.155 | 2.0 |
| 2.5 | 0.076 | 0.064 | 0.052 | 0.040 | 0.028 | 0.016 | 0.004 | 0.008- | 0.020- | 0.032- | 0.044- | 2.5 |
| 3.0 | 0.044- | 0.056- | 0.068- | 0.080- | 0.092- | 0.104- | 0.116- | 0.128- | 0.140- | 0.152- | 0.164- | 3.0 |
| 3.5 | 0.152- | 0.162- | 0.173- | 0.183- | 0.194- | 0.205- | 0.215- | 0.226- | 0.236- | 0.247- | 0.257- | 3.5 |

Table A.1.3. Deflection Coefficients $C_y$ for Elastic Piles, $E_s = kx$, $Z_{max} = 3.5$

| Z | $M_t/P_tT$ | | | | | | | | | | | Z |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | -1.0 | -0.9 | -0.8 | -0.7 | -0.6 | -0.5 | -0.4 | -0.3 | -0.2 | -0.1 | 0.0 | |
| .0 | 0.967 | 1.143 | 1.318 | 1.494 | 1.670 | 1.845 | 2.021 | 2.197 | 2.372 | 2.548 | 2.723 | .0 |
| .1 | 0.968 | 1.126 | 1.284 | 1.442 | 1.600 | 1.758 | 1.916 | 2.074 | 2.232 | 2.390 | 2.548 | .1 |
| .2 | 0.961 | 1.102 | 1.243 | 1.385 | 1.526 | 1.667 | 1.808 | 1.950 | 2.091 | 2.232 | 2.373 | .2 |
| .3 | 0.945 | 1.071 | 1.196 | 1.322 | 1.447 | 1.573 | 1.699 | 1.824 | 1.950 | 2.075 | 2.201 | .3 |
| .4 | 0.922 | 1.033 | 1.144 | 1.255 | 1.366 | 1.477 | 1.588 | 1.699 | 1.809 | 1.920 | 2.031 | .4 |
| .5 | 0.893 | 0.991 | 1.088 | 1.185 | 1.282 | 1.379 | 1.477 | 1.574 | 1.671 | 1.768 | 1.865 | .5 |
| .6 | 0.859 | 0.944 | 1.028 | 1.113 | 1.197 | 1.282 | 1.366 | 1.451 | 1.535 | 1.619 | 1.704 | .6 |
| .7 | 0.821 | 0.894 | 0.966 | 1.039 | 1.112 | 1.184 | 1.257 | 1.330 | 1.402 | 1.475 | 1.548 | .7 |
| .8 | 0.779 | 0.841 | 0.903 | 0.965 | 1.026 | 1.088 | 1.150 | 1.212 | 1.274 | 1.335 | 1.397 | .8 |
| .9 | 0.734 | 0.786 | 0.838 | 0.890 | 0.942 | 0.994 | 1.046 | 1.098 | 1.149 | 1.201 | 1.253 | .9 |
| 1.0 | 0.688 | 0.730 | 0.773 | 0.816 | 0.859 | 0.902 | 0.944 | 0.987 | 1.030 | 1.073 | 1.116 | 1.0 |
| 1.2 | 0.590 | 0.617 | 0.644 | 0.671 | 0.699 | 0.726 | 0.753 | 0.780 | 0.807 | 0.834 | 0.861 | 1.2 |
| 1.4 | 0.491 | 0.505 | 0.520 | 0.534 | 0.548 | 0.563 | 0.577 | 0.592 | 0.606 | 0.621 | 0.635 | 1.4 |
| 1.6 | 0.393 | 0.397 | 0.401 | 0.406 | 0.410 | 0.414 | 0.419 | 0.423 | 0.427 | 0.431 | 0.436 | 1.6 |
| 1.8 | 0.298 | 0.294 | 0.290 | 0.287 | 0.283 | 0.279 | 0.276 | 0.272 | 0.268 | 0.265 | 0.261 | 1.8 |
| 2.0 | 0.207 | 0.197 | 0.187 | 0.177 | 0.167 | 0.157 | 0.147 | 0.137 | 0.127 | 0.118 | 0.108 | 2.0 |
| 2.5 | 0.004- | 0.025- | 0.045- | 0.066- | 0.087- | 0.108- | 0.129- | 0.150- | 0.170- | 0.191- | 0.212- | 2.5 |
| 3.0 | 0.202- | 0.231- | 0.260- | 0.289- | 0.319- | 0.348- | 0.377- | 0.406- | 0.435- | 0.464- | 0.493- | 3.0 |

**Table A.1.4. Deflection Coefficients $C_y$ for Elastic Piles, $E_s = kx$, $Z_{max} = 3.0$**

| Z | M_t/P_tT -1.0 | -0.9 | -0.8 | -0.7 | -0.6 | -0.5 | -0.4 | -0.3 | -0.2 | -0.1 | 0.0 | Z |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| .0 | 1.027 | 1.213 | 1.398 | 1.584 | 1.770 | 1.956 | 2.142 | 2.328 | 2.514 | 2.700 | 2.886 | .0 |
| .1 | 1.024 | 1.192 | 1.359 | 1.527 | 1.695 | 1.862 | 2.030 | 2.197 | 2.365 | 2.532 | 2.700 | .1 |
| .2 | 1.013 | 1.163 | 1.313 | 1.463 | 1.614 | 1.764 | 1.914 | 2.064 | 2.215 | 2.365 | 2.515 | .2 |
| .3 | 0.993 | 1.127 | 1.261 | 1.395 | 1.529 | 1.663 | 1.797 | 1.931 | 2.065 | 2.198 | 2.332 | .3 |
| .4 | 0.967 | 1.085 | 1.204 | 1.322 | 1.441 | 1.560 | 1.678 | 1.797 | 1.915 | 2.034 | 2.152 | .4 |
| .5 | 0.934 | 1.038 | 1.142 | 1.247 | 1.351 | 1.455 | 1.559 | 1.664 | 1.768 | 1.872 | 1.976 | .5 |
| .6 | 0.896 | 0.987 | 1.078 | 1.169 | 1.260 | 1.350 | 1.441 | 1.532 | 1.623 | 1.714 | 1.804 | .6 |
| .7 | 0.854 | 0.932 | 1.011 | 1.089 | 1.168 | 1.246 | 1.324 | 1.403 | 1.481 | 1.559 | 1.638 | .7 |
| .8 | 0.808 | 0.875 | 0.942 | 1.009 | 1.076 | 1.143 | 1.209 | 1.276 | 1.343 | 1.410 | 1.477 | .8 |
| .9 | 0.760 | 0.816 | 0.872 | 0.928 | 0.984 | 1.041 | 1.097 | 1.153 | 1.210 | 1.266 | 1.322 | .9 |
| 1.0 | 0.709 | 0.755 | 0.802 | 0.848 | 0.895 | 0.941 | 0.988 | 1.034 | 1.081 | 1.127 | 1.174 | 1.0 |
| 1.2 | 0.603 | 0.633 | 0.662 | 0.691 | 0.721 | 0.750 | 0.780 | 0.809 | 0.838 | 0.868 | 0.897 | 1.2 |
| 1.4 | 0.495 | 0.510 | 0.526 | 0.541 | 0.556 | 0.571 | 0.586 | 0.601 | 0.617 | 0.632 | 0.647 | 1.4 |
| 1.6 | 0.388 | 0.391 | 0.395 | 0.398 | 0.401 | 0.405 | 0.408 | 0.412 | 0.415 | 0.419 | 0.422 | 1.6 |
| 1.8 | 0.282 | 0.276 | 0.270 | 0.263 | 0.257 | 0.251 | 0.245 | 0.238 | 0.232 | 0.226 | 0.219 | 1.8 |
| 2.0 | 0.180 | 0.165 | 0.151 | 0.136 | 0.122 | 0.107 | 0.093 | 0.078 | 0.064 | 0.049 | 0.035 | 2.0 |
| 2.5 | 0.066- | 0.097- | 0.128- | 0.159- | 0.191- | 0.222- | 0.253- | 0.284- | 0.316- | 0.347- | 0.378- | 2.5 |
| 2.8 | 0.209- | 0.250- | 0.290- | 0.330- | 0.371- | 0.411- | 0.452- | 0.492- | 0.532- | 0.573- | 0.613- | 2.8 |

Table A.1.5. Deflection Coefficients $C_y$ for Elastic Piles, $E_s = kx$, $Z_{max} = 2.8$

| Z | $M_t/P_tT$ -1.0 | -0.9 | -0.8 | -0.7 | -0.6 | -0.5 | -0.4 | -0.3 | -0.2 | -0.1 | 0.0 | Z |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| .0 | 1.115 | 1.320 | 1.526 | 1.731 | 1.936 | 2.141 | 2.346 | 2.551 | 2.757 | 2.962 | 3.167 | .0 |
| .1 | 1.107 | 1.292 | 1.478 | 1.663 | 1.849 | 2.034 | 2.220 | 2.405 | 2.591 | 2.776 | 2.962 | .1 |
| .2 | 1.089 | 1.256 | 1.423 | 1.590 | 1.757 | 1.924 | 2.090 | 2.257 | 2.424 | 2.591 | 2.758 | .2 |
| .3 | 1.064 | 1.213 | 1.362 | 1.511 | 1.661 | 1.810 | 1.959 | 2.108 | 2.257 | 2.407 | 2.556 | .3 |
| .4 | 1.031 | 1.164 | 1.296 | 1.429 | 1.561 | 1.694 | 1.826 | 1.959 | 2.091 | 2.224 | 2.357 | .4 |
| .5 | 0.992 | 1.109 | 1.226 | 1.343 | 1.460 | 1.577 | 1.694 | 1.810 | 1.927 | 2.044 | 2.161 | .5 |
| .6 | 0.949 | 1.051 | 1.153 | 1.255 | 1.357 | 1.459 | 1.561 | 1.664 | 1.766 | 1.868 | 1.970 | .6 |
| .7 | 0.900 | 0.989 | 1.077 | 1.165 | 1.254 | 1.342 | 1.431 | 1.519 | 1.607 | 1.696 | 1.784 | .7 |
| .8 | 0.848 | 0.924 | 1.000 | 1.075 | 1.151 | 1.226 | 1.302 | 1.377 | 1.453 | 1.528 | 1.604 | .8 |
| .9 | 0.794 | 0.857 | 0.921 | 0.984 | 1.048 | 1.112 | 1.175 | 1.239 | 1.302 | 1.366 | 1.429 | .9 |
| 1.0 | 0.737 | 0.789 | 0.842 | 0.894 | 0.947 | 0.999 | 1.051 | 1.104 | 1.156 | 1.209 | 1.261 | 1.0 |
| 1.2 | 0.619 | 0.651 | 0.684 | 0.716 | 0.749 | 0.781 | 0.814 | 0.847 | 0.879 | 0.912 | 0.944 | 1.2 |
| 1.4 | 0.498 | 0.513 | 0.529 | 0.544 | 0.559 | 0.575 | 0.590 | 0.606 | 0.621 | 0.637 | 0.652 | 1.4 |
| 1.6 | 0.376 | 0.377 | 0.378 | 0.378 | 0.379 | 0.380 | 0.381 | 0.381 | 0.382 | 0.383 | 0.383 | 1.6 |
| 1.8 | 0.256 | 0.244 | 0.232 | 0.220 | 0.207 | 0.195 | 0.183 | 0.170 | 0.158 | 0.146 | 0.134 | 1.8 |
| 2.0 | 0.138 | 0.114 | 0.090 | 0.066 | 0.042 | 0.018 | 0.006- | 0.030- | 0.054- | 0.078- | 0.102- | 2.0 |
| 2.5 | 0.151- | 0.201- | 0.252- | 0.303- | 0.353- | 0.404- | 0.455- | 0.505- | 0.556- | 0.606- | 0.657- | 2.5 |
| 2.6 | 0.208- | 0.264- | 0.320- | 0.376- | 0.431- | 0.487- | 0.543- | 0.599- | 0.655- | 0.711- | 0.766- | 2.6 |

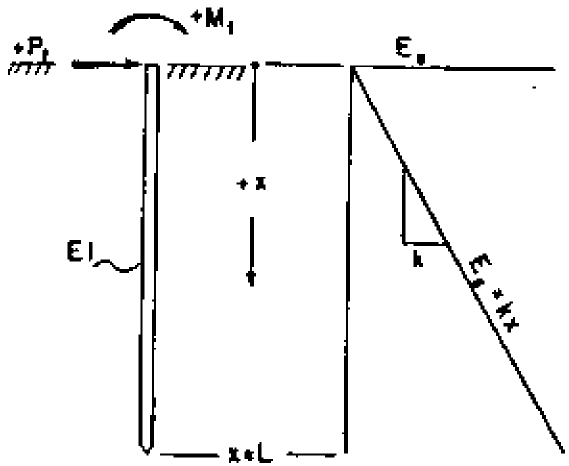
**Table A.1.6. Deflection Coefficients $C_y$ for Elastic Piles, $E_s = kx$, $Z_{max} = 2.6$**

| Z | $M_t/P_tT$ -1.0 | -0.9 | -0.8 | -0.7 | -0.6 | -0.5 | -0.4 | -0.3 | -0.2 | -0.1 | 0.0 | Z |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| .0 | 1.201 | 1.434 | 1.667 | 1.900 | 2.133 | 2.366 | 2.599 | 2.832 | 3.065 | 3.299 | 3.532 | .0 |
| .1 | 1.186 | 1.397 | 1.608 | 1.820 | 2.031 | 2.242 | 2.454 | 2.665 | 2.876 | 3.087 | 3.299 | .1 |
| .2 | 1.162 | 1.352 | 1.543 | 1.733 | 1.924 | 2.114 | 2.305 | 2.495 | 2.686 | 2.876 | 3.067 | .2 |
| .3 | 1.130 | 1.301 | 1.471 | 1.642 | 1.813 | 1.983 | 2.154 | 2.325 | 2.495 | 2.666 | 2.837 | .3 |
| .4 | 1.091 | 1.243 | 1.395 | 1.547 | 1.698 | 1.850 | 2.002 | 2.154 | 2.306 | 2.458 | 2.610 | .4 |
| .5 | 1.046 | 1.180 | 1.314 | 1.448 | 1.582 | 1.716 | 1.850 | 1.984 | 2.118 | 2.252 | 2.386 | .5 |
| .6 | 0.995 | 1.113 | 1.230 | 1.347 | 1.464 | 1.581 | 1.698 | 1.816 | 1.933 | 2.050 | 2.167 | .6 |
| .7 | 0.941 | 1.042 | 1.143 | 1.244 | 1.346 | 1.447 | 1.548 | 1.649 | 1.751 | 1.852 | 1.953 | .7 |
| .8 | 0.882 | 0.968 | 1.055 | 1.141 | 1.227 | 1.313 | 1.400 | 1.486 | 1.572 | 1.658 | 1.744 | .8 |
| .9 | 0.821 | 0.893 | 0.965 | 1.037 | 1.109 | 1.181 | 1.253 | 1.325 | 1.397 | 1.470 | 1.542 | .9 |
| 1.0 | 0.757 | 0.816 | 0.875 | 0.933 | 0.992 | 1.051 | 1.110 | 1.168 | 1.227 | 1.286 | 1.345 | 1.0 |
| 1.2 | 0.625 | 0.660 | 0.694 | 0.728 | 0.763 | 0.797 | 0.832 | 0.866 | 0.900 | 0.935 | 0.969 | 1.2 |
| 1.4 | 0.490 | 0.503 | 0.515 | 0.528 | 0.540 | 0.553 | 0.566 | 0.578 | 0.591 | 0.604 | 0.616 | 1.4 |
| 1.6 | 0.354 | 0.347 | 0.340 | 0.333 | 0.325 | 0.318 | 0.311 | 0.304 | 0.297 | 0.290 | 0.283 | 1.6 |
| 1.8 | 0.218 | 0.193 | 0.168 | 0.142 | 0.117 | 0.092 | 0.067 | 0.041 | 0.016 | 0.009- | 0.034- | 1.8 |
| 2.0 | 0.083 | 0.041 | 0.002- | 0.044- | 0.087- | 0.129- | 0.172- | 0.214- | 0.257- | 0.299- | 0.342- | 2.0 |
| 2.4 | 0.185- | 0.261- | 0.337- | 0.412- | 0.488- | 0.564- | 0.640- | 0.716- | 0.791- | 0.867- | 0.943- | 2.4 |

Table A.1.7. Deflection Coefficients $C_y$ for Elastic Piles, $E_s = kx$, $Z_{max} = 2.4$

| Z | $M_t/P_tT$ | | | | | | | | | | | Z |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | -1.0 | -0.9 | -0.8 | -0.7 | -0.6 | -0.5 | -0.4 | -0.3 | -0.2 | -0.1 | 0.0 | |
| .0 | 1.268 | 1.543 | 1.817 | 2.091 | 2.365 | 2.640 | 2.914 | 3.188 | 3.462 | 3.737 | 4.011 | .0 |
| .1 | 1.247 | 1.496 | 1.745 | 1.994 | 2.243 | 2.492 | 2.741 | 2.990 | 3.239 | 3.488 | 3.737 | .1 |
| .2 | 1.218 | 1.442 | 1.667 | 1.891 | 2.116 | 2.341 | 2.565 | 2.790 | 3.014 | 3.239 | 3.464 | .2 |
| .3 | 1.180 | 1.381 | 1.582 | 1.783 | 1.985 | 2.186 | 2.387 | 2.589 | 2.790 | 2.991 | 3.193 | .3 |
| .4 | 1.135 | 1.314 | 1.493 | 1.672 | 1.850 | 2.029 | 2.208 | 2.387 | 2.566 | 2.745 | 2.924 | .4 |
| .5 | 1.084 | 1.241 | 1.399 | 1.556 | 1.714 | 1.871 | 2.029 | 2.187 | 2.344 | 2.502 | 2.659 | .5 |
| .6 | 1.027 | 1.164 | 1.302 | 1.439 | 1.576 | 1.713 | 1.850 | 1.987 | 2.125 | 2.262 | 2.399 | .6 |
| .7 | 0.966 | 1.084 | 1.202 | 1.320 | 1.437 | 1.555 | 1.673 | 1.790 | 1.908 | 2.026 | 2.143 | .7 |
| .8 | 0.902 | 1.001 | 1.100 | 1.199 | 1.298 | 1.397 | 1.497 | 1.596 | 1.695 | 1.794 | 1.893 | .8 |
| .9 | 0.835 | 0.916 | 0.997 | 1.079 | 1.160 | 1.241 | 1.323 | 1.404 | 1.485 | 1.567 | 1.648 | .9 |
| 1.0 | 0.765 | 0.829 | 0.894 | 0.958 | 1.022 | 1.087 | 1.151 | 1.215 | 1.280 | 1.344 | 1.409 | 1.0 |
| 1.2 | 0.620 | 0.653 | 0.686 | 0.718 | 0.751 | 0.783 | 0.816 | 0.848 | 0.881 | 0.914 | 0.946 | 1.2 |
| 1.4 | 0.472 | 0.475 | 0.478 | 0.482 | 0.485 | 0.488 | 0.491 | 0.494 | 0.497 | 0.500 | 0.504 | 1.4 |
| 1.6 | 0.322 | 0.298 | 0.273 | 0.249 | 0.224 | 0.200 | 0.175 | 0.151 | 0.126 | 0.102 | 0.077 | 1.6 |
| 1.8 | 0.172 | 0.121 | 0.070 | 0.019 | 0.032- | 0.083- | 0.134- | 0.185- | 0.236- | 0.287- | 0.338- | 1.8 |
| 2.0 | 0.022 | 0.055- | 0.132- | 0.209- | 0.286- | 0.363- | 0.440- | 0.517- | 0.594- | 0.671- | 0.748- | 2.0 |
| 2.2 | 0.127- | 0.230- | 0.333- | 0.436- | 0.539- | 0.641- | 0.744- | 0.847- | 0.950- | 1.053- | 1.155- | 2.2 |

**Table A.1.8. Deflection Coefficients $C_y$ for Elastic Piles, $E_s = kx$, $Z_{max} = 2.2$**

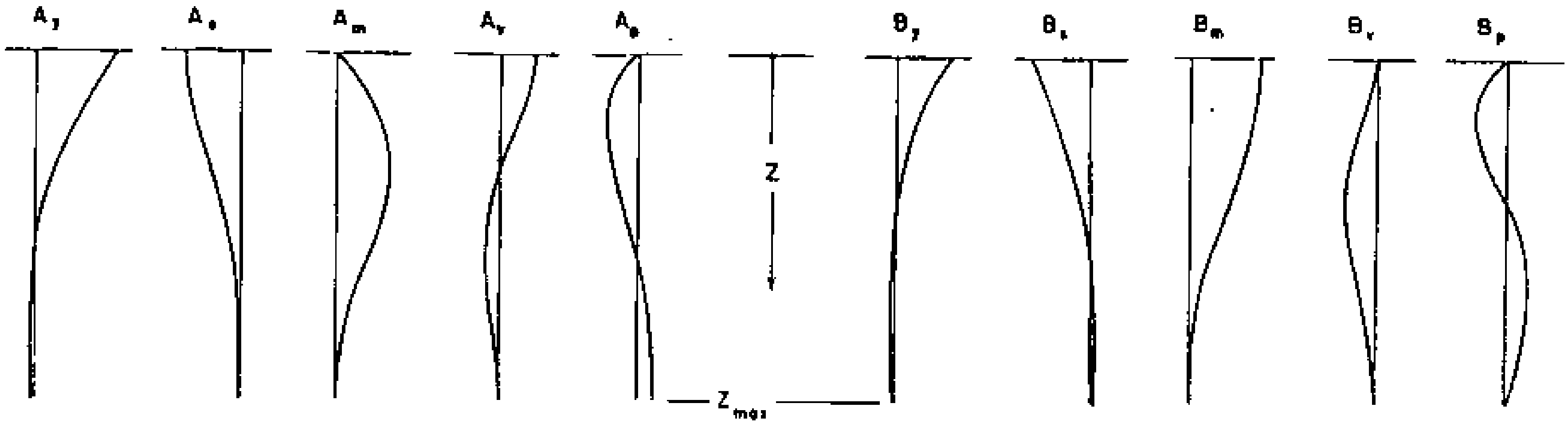



$$T = (EI/k)^{1/5}$$

$$x = ZT$$

$$Z_{max} = L/T$$

| | | | | | |
|---|---|---|---|---|---|
| Deflection | $y$ | = | $(P_t T^3/EI) A_y$ | + | $(M_t T^2/EI) B_y$ |
| Slope | $S$ | = | $(P_t T^2/EI) A_s$ | + | $(M_t T/EI) B_s$ |
| Moment | $M$ | = | $(P_t T) A_m$ | + | $(M_t) B_m$ |
| Shear | $V$ | = | $(P_t) A_v$ | + | $(M_t/T) B_v$ |
| Soil reaction | $p$ | = | $(P_t/T) A_p$ | + | $(M_t/T^2) B_p$ |




Appendix A.2. Elastic Pile Solutions, $E_s = kx$, $Z_{max}$ varied from 2.2 to 10.

| Z | $A_Y$ | $A_S$ | $A_M$ | $A_V$ | $A_P$ | $B_Y$ | $B_S$ | $B_M$ | $B_V$ | $B_P$ | Z |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | 2.435 | 1.623- | 0.000 | 1.000 | 0.000 | 1.623 | 1.749- | 1.000 | 0.000 | 0.000 | 0.0 |
| 0.1 | 2.273 | 1.618- | 0.100 | 0.988 | 0.227- | 1.453 | 1.649- | 1.000 | 0.007- | 0.145- | 0.1 |
| 0.2 | 2.112 | 1.603- | 0.198 | 0.956 | 0.422- | 1.293 | 1.549- | 0.999 | 0.028- | 0.259- | 0.2 |
| 0.3 | 1.952 | 1.578- | 0.291 | 0.906 | 0.596- | 1.143 | 1.450- | 0.994 | 0.058- | 0.343- | 0.3 |
| 0.4 | 1.796 | 1.545- | 0.379 | 0.840 | 0.718- | 1.003 | 1.351- | 0.987 | 0.095- | 0.401- | 0.4 |
| 0.5 | 1.643 | 1.503- | 0.459 | 0.763 | 0.822- | 0.873 | 1.253- | 0.976 | 0.137- | 0.436- | 0.5 |
| 0.6 | 1.495 | 1.453- | 0.531 | 0.677 | 0.897- | 0.752 | 1.156- | 0.960 | 0.181- | 0.451- | 0.6 |
| 0.7 | 1.353 | 1.397- | 0.595 | 0.585 | 0.947- | 0.641 | 1.061- | 0.939 | 0.226- | 0.449- | 0.7 |
| 0.8 | 1.216 | 1.335- | 0.649 | 0.489 | 0.973- | 0.540 | 0.968- | 0.914 | 0.270- | 0.432- | 0.8 |
| 0.9 | 1.086 | 1.266- | 0.693 | 0.392 | 0.977- | 0.448 | 0.878- | 0.885 | 0.312- | 0.403- | 0.9 |
| 1.0 | 0.962 | 1.197- | 0.727 | 0.295 | 0.952- | 0.364 | 0.791- | 0.852 | 0.350- | 0.364- | 1.0 |
| 1.2 | 0.738 | 1.047- | 0.767 | 0.109 | 0.885- | 0.223 | 0.628- | 0.775 | 0.414- | 0.267- | 1.2 |
| 1.4 | 0.544 | 0.893- | 0.772 | 0.056- | 0.761- | 0.112 | 0.482- | 0.688 | 0.456- | 0.157- | 1.4 |
| 1.6 | 0.381 | 0.741- | 0.746 | 0.193- | 0.609- | 0.029 | 0.354- | 0.594 | 0.477- | 0.046- | 1.6 |
| 1.8 | 0.247 | 0.596- | 0.696 | 0.299- | 0.445- | 0.030- | 0.245- | 0.498 | 0.476- | 0.055 | 1.8 |
| 2.0 | 0.141 | 0.464- | 0.628 | 0.371- | 0.293- | 0.070- | 0.155- | 0.404 | 0.456- | 0.140 | 2.0 |
| 2.5 | 0.020- | 0.200- | 0.422 | 0.424- | 0.049 | 0.105- | 0.006- | 0.200 | 0.350- | 0.263 | 2.5 |
| 3.0 | 0.075- | 0.040- | 0.225 | 0.349- | 0.226 | 0.089- | 0.057 | 0.059 | 0.213- | 0.268 | 3.0 |
| 3.5 | 0.074- | 0.034 | 0.081 | 0.223- | 0.257 | 0.057- | 0.065 | 0.016- | 0.095- | 0.200 | 3.5 |
| 4.0 | 0.050- | 0.052 | 0.000 | 0.106- | 0.201 | 0.028- | 0.049 | 0.042- | 0.017- | 0.113 | 4.0 |
| 4.5 | 0.026- | 0.042 | 0.032- | 0.027- | 0.117 | 0.009- | 0.028 | 0.039- | 0.021 | 0.041 | 4.5 |
| 5.0 | 0.009- | 0.025 | 0.033- | 0.013 | 0.046 | 0.000 | 0.011 | 0.026- | 0.029 | 0.002 | 5.0 |
| 10.0 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 10.0 |

**Table A.2.1. A and B Coefficients for Elastic Piles, $E_s = kx$, $Z_{max} = 10.0$**

| Z | $A_y$ | $A_s$ | $A_M$ | $A_v$ | $A_p$ | $B_y$ | $B_s$ | $B_M$ | $B_v$ | $B_p$ | Z |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | 2.445 | 1.624- | 0.000 | 1.000 | 0.000 | 1.624 | 1.753- | 1.000 | 0.000 | 0.000 | 0.0 |
| 0.1 | 2.283 | 1.619- | 0.100 | 0.991 | 0.228- | 1.454 | 1.653- | 1.000 | 0.006- | 0.145- | 0.1 |
| 0.2 | 2.122 | 1.604- | 0.198 | 0.959 | 0.424- | 1.294 | 1.553- | 0.998 | 0.026- | 0.259- | 0.2 |
| 0.3 | 1.962 | 1.580- | 0.291 | 0.906 | 0.589- | 1.143 | 1.453- | 0.994 | 0.057- | 0.343- | 0.3 |
| 0.4 | 1.806 | 1.546- | 0.378 | 0.841 | 0.722- | 1.003 | 1.354- | 0.987 | 0.094- | 0.401- | 0.4 |
| 0.5 | 1.653 | 1.505- | 0.459 | 0.762 | 0.827- | 0.873 | 1.256- | 0.975 | 0.137- | 0.436- | 0.5 |
| 0.6 | 1.505 | 1.455- | 0.531 | 0.676 | 0.903- | 0.752 | 1.159- | 0.960 | 0.180- | 0.451- | 0.6 |
| 0.7 | 1.363 | 1.399- | 0.594 | 0.583 | 0.954- | 0.641 | 1.064- | 0.939 | 0.226- | 0.448- | 0.7 |
| 0.8 | 1.226 | 1.337- | 0.647 | 0.486 | 0.981- | 0.539 | 0.972- | 0.914 | 0.270- | 0.431- | 0.8 |
| 0.9 | 1.095 | 1.270- | 0.691 | 0.387 | 0.986- | 0.446 | 0.882- | 0.885 | 0.312- | 0.402- | 0.9 |
| 1.0 | 0.972 | 1.199- | 0.725 | 0.290 | 0.972- | 0.362 | 0.795- | 0.852 | 0.350- | 0.362- | 1.0 |
| 1.2 | 0.747 | 1.049- | 0.764 | 0.102 | 0.896- | 0.220 | 0.632- | 0.775 | 0.413- | 0.264- | 1.2 |
| 1.4 | 0.552 | 0.896- | 0.767 | 0.066- | 0.773- | 0.109 | 0.485- | 0.688 | 0.455- | 0.152- | 1.4 |
| 1.6 | 0.388 | 0.745- | 0.739 | 0.206- | 0.622- | 0.025 | 0.357- | 0.595 | 0.474- | 0.040- | 1.6 |
| 1.8 | 0.254 | 0.602- | 0.686 | 0.314- | 0.457- | 0.035- | 0.248- | 0.500 | 0.471- | 0.064 | 1.8 |
| 2.0 | 0.147 | 0.471- | 0.616 | 0.389- | 0.294- | 0.075- | 0.157- | 0.407 | 0.450- | 0.151 | 2.0 |
| 2.5 | 0.020- | 0.217- | 0.400 | 0.445- | 0.051 | 0.112- | 0.005- | 0.208 | 0.336- | 0.280 | 2.5 |
| 3.0 | 0.088- | 0.070- | 0.194 | 0.361- | 0.263 | 0.095- | 0.063 | 0.076 | 0.191- | 0.284 | 3.0 |
| 3.5 | 0.105- | 0.012- | 0.051 | 0.200- | 0.358 | 0.057- | 0.083 | 0.014 | 0.067- | 0.200 | 3.5 |
| 4.0 | 0.108- | 0.003- | 0.000 | 0.000 | 0.431 | 0.015- | 0.085 | 0.000 | 0.000 | 0.060 | 4.0 |

Table A.2.2. A and B Coefficients for Elastic Piles, $E_s = kx$, $Z_{max} = 4.0$

| Z | $A_y$ | $A_s$ | $A_M$ | $A_V$ | $A_P$ | $B_y$ | $B_s$ | $B_M$ | $B_V$ | $B_P$ | Z |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | 2.493 | 1.636- | 0.000 | 1.000 | 0.000 | 1.636 | 1.755- | 1.000 | 0.000 | 0.000 | 0.0 |
| 0.1 | 2.330 | 1.632- | 0.100 | 0.988 | 0.233- | 1.466 | 1.655- | 1.000 | 0.008- | 0.147- | 0.1 |
| 0.2 | 2.167 | 1.617- | 0.197 | 0.956 | 0.433- | 1.305 | 1.555- | 0.998 | 0.027- | 0.261- | 0.2 |
| 0.3 | 2.007 | 1.592- | 0.290 | 0.900 | 0.602- | 1.155 | 1.456- | 0.994 | 0.061- | 0.346- | 0.3 |
| 0.4 | 1.849 | 1.559- | 0.377 | 0.832 | 0.740- | 1.014 | 1.357- | 0.986 | 0.098- | 0.406- | 0.4 |
| 0.5 | 1.695 | 1.517- | 0.456 | 0.754 | 0.848- | 0.884 | 1.259- | 0.974 | 0.140- | 0.442- | 0.5 |
| 0.6 | 1.546 | 1.468- | 0.527 | 0.664 | 0.927- | 0.763 | 1.162- | 0.958 | 0.186- | 0.458- | 0.6 |
| 0.7 | 1.402 | 1.412- | 0.589 | 0.567 | 0.981- | 0.651 | 1.067- | 0.937 | 0.232- | 0.456- | 0.7 |
| 0.8 | 1.263 | 1.351- | 0.641 | 0.467 | 1.011- | 0.549 | 0.975- | 0.911 | 0.277- | 0.439- | 0.8 |
| 0.9 | 1.132 | 1.284- | 0.682 | 0.365 | 1.019- | 0.456 | 0.885- | 0.882 | 0.320- | 0.410- | 0.9 |
| 1.0 | 1.007 | 1.214- | 0.714 | 0.264 | 1.007- | 0.372 | 0.799- | 0.848 | 0.359- | 0.372- | 1.0 |
| 1.2 | 0.778 | 1.068- | 0.747 | 0.068 | 0.934- | 0.229 | 0.637- | 0.769 | 0.424- | 0.274- | 1.2 |
| 1.4 | 0.580 | 0.918- | 0.742 | 0.108- | 0.812- | 0.116 | 0.492- | 0.679 | 0.468- | 0.163- | 1.4 |
| 1.6 | 0.411 | 0.773- | 0.705 | 0.256- | 0.657- | 0.031 | 0.365- | 0.583 | 0.489- | 0.049- | 1.6 |
| 1.8 | 0.270 | 0.638- | 0.642 | 0.370- | 0.486- | 0.031- | 0.259- | 0.485 | 0.488- | 0.056 | 1.8 |
| 2.0 | 0.155 | 0.518- | 0.559 | 0.449- | 0.309- | 0.074- | 0.171- | 0.389 | 0.468- | 0.148 | 2.0 |
| 2.5 | 0.044- | 0.298- | 0.314 | 0.497- | 0.110 | 0.120- | 0.031- | 0.181 | 0.351- | 0.300 | 2.5 |
| 3.0 | 0.164- | 0.199- | 0.095 | 0.346- | 0.492 | 0.120- | 0.022 | 0.046 | 0.184- | 0.359 | 3.0 |
| 3.5 | 0.257- | 0.183- | 0.000 | 0.000 | 0.901 | 0.106- | 0.030 | 0.000 | 0.000 | 0.370 | 3.5 |

Table A.2.3. A and B Coefficients for Elastic Piles, $E_s = kx$, $Z_{max} = 3.5$

| Z | $A_y$ | $A_s$ | $A_M$ | $A_V$ | $A_P$ | $B_y$ | $B_s$ | $B_M$ | $B_V$ | $B_P$ | Z |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | 2.723 | 1.756- | 0.000 | 1.000 | 0.000 | 1.756 | 1.818- | 1.000 | 0.000 | 0.000 | 0.0 |
| 0.1 | 2.548 | 1.751- | 0.100 | 0.987 | 0.255- | 1.579 | 1.718- | 1.000 | 0.008- | 0.158- | 0.1 |
| 0.2 | 2.373 | 1.736- | 0.197 | 0.950 | 0.475- | 1.413 | 1.618- | 0.998 | 0.030- | 0.283- | 0.2 |
| 0.3 | 2.201 | 1.712- | 0.289 | 0.893 | 0.660- | 1.256 | 1.519- | 0.993 | 0.064- | 0.377- | 0.3 |
| 0.4 | 2.031 | 1.679- | 0.375 | 0.818 | 0.812- | 1.109 | 1.420- | 0.985 | 0.105- | 0.444- | 0.4 |
| 0.5 | 1.865 | 1.637- | 0.453 | 0.731 | 0.933- | 0.972 | 1.322- | 0.972 | 0.152- | 0.486- | 0.5 |
| 0.6 | 1.704 | 1.589- | 0.521 | 0.635 | 1.022- | 0.845 | 1.225- | 0.955 | 0.201- | 0.507- | 0.6 |
| 0.7 | 1.548 | 1.534- | 0.579 | 0.527 | 1.083- | 0.727 | 1.131- | 0.932 | 0.253- | 0.509- | 0.7 |
| 0.8 | 1.397 | 1.473- | 0.626 | 0.417 | 1.118- | 0.618 | 1.039- | 0.904 | 0.302- | 0.495- | 0.8 |
| 0.9 | 1.253 | 1.409- | 0.662 | 0.304 | 1.128- | 0.519 | 0.950- | 0.871 | 0.352- | 0.467- | 0.9 |
| 1.0 | 1.116 | 1.341- | 0.687 | 0.192 | 1.116- | 0.428 | 0.865- | 0.834 | 0.396- | 0.428- | 1.0 |
| 1.2 | 0.861 | 1.201- | 0.704 | 0.025- | 1.034- | 0.271 | 0.707- | 0.747 | 0.472- | 0.325- | 1.2 |
| 1.4 | 0.635 | 1.063- | 0.679 | 0.218- | 0.889- | 0.144 | 0.567- | 0.647 | 0.525- | 0.202- | 1.4 |
| 1.6 | 0.436 | 0.932- | 0.618 | 0.378- | 0.697- | 0.043 | 0.449- | 0.538 | 0.552- | 0.069- | 1.6 |
| 1.8 | 0.261 | 0.817- | 0.530 | 0.495- | 0.470- | 0.037- | 0.352- | 0.427 | 0.553- | 0.066 | 1.8 |
| 2.0 | 0.108 | 0.722- | 0.423 | 0.564- | 0.215- | 0.099- | 0.278- | 0.319 | 0.526- | 0.198 | 2.0 |
| 2.5 | 0.212- | 0.581- | 0.143 | 0.492- | 0.530 | 0.208- | 0.178- | 0.094 | 0.346- | 0.521 | 2.5 |
| 3.0 | 0.493- | 0.556- | 0.000 | 0.000 | 1.480 | 0.291- | 0.162- | 0.000 | 0.000 | 0.874 | 3.0 |

Table A.2.4. A and B Coefficients for Elastic Piles, $E_s = kx$, $Z_{max} = 3.0$

| Z | $A_y$ | $A_s$ | $A_m$ | $A_v$ | $A_p$ | $B_y$ | $B_s$ | $B_m$ | $B_v$ | $B_p$ | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | 2.886 | 1.859- | 0.000 | 1.000 | 0.000 | 1.859 | 1.883- | 1.000 | 0.000 | 0.000 | 0.0 |
| 0.1 | 2.700- | 1.854- | 0.100 | 0.987 | 0.270- | 1.676 | 1.783- | 1.000 | 0.008- | 0.168- | 0.1 |
| 0.2 | 2.515 | 1.839- | 0.197 | 0.946 | 0.503- | 1.502 | 1.683- | 0.998 | 0.033- | 0.300- | 0.2 |
| 0.3 | 2.332 | 1.815- | 0.288 | 0.884 | 0.700- | 1.339 | 1.584- | 0.993 | 0.069- | 0.402- | 0.3 |
| 0.4 | 2.152 | 1.782- | 0.373 | 0.804 | 0.861- | 1.186 | 1.485- | 0.984 | 0.114- | 0.474- | 0.4 |
| 0.5 | 1.976 | 1.741- | 0.449 | 0.710 | 0.988- | 1.042 | 1.387- | 0.970 | 0.164- | 0.521- | 0.5 |
| 0.6 | 1.804 | 1.692- | 0.515 | 0.605 | 1.083- | 0.908 | 1.291- | 0.951 | 0.219- | 0.545- | 0.6 |
| 0.7 | 1.638 | 1.638- | 0.570 | 0.493 | 1.146- | 0.784 | 1.197- | 0.926 | 0.274- | 0.549- | 0.7 |
| 0.8 | 1.477 | 1.579- | 0.613 | 0.376 | 1.182- | 0.669 | 1.106- | 0.896 | 0.328- | 0.535- | 0.8 |
| 0.9 | 1.322 | 1.516- | 0.645 | 0.258 | 1.190- | 0.563 | 1.018- | 0.861 | 0.380- | 0.506- | 0.9 |
| 1.0 | 1.174 | 1.450- | 0.665 | 0.138 | 1.174- | 0.465 | 0.934- | 0.820 | 0.430- | 0.465- | 1.0 |
| 1.2 | 0.897 | 1.316- | 0.669 | 0.090- | 1.077- | 0.294 | 0.779- | 0.725 | 0.513- | 0.353- | 1.2 |
| 1.4 | 0.647 | 1.186- | 0.631 | 0.289- | 0.906- | 0.152 | 0.645- | 0.617 | 0.569- | 0.213- | 1.4 |
| 1.6 | 0.422 | 1.066- | 0.556 | 0.449- | 0.676- | 0.035 | 0.533- | 0.500 | 0.596- | 0.055- | 1.6 |
| 1.8 | 0.219 | 0.965- | 0.455 | 0.556- | 0.395- | 0.063- | 0.445- | 0.380 | 0.591- | 0.113 | 1.8 |
| 2.0 | 0.035 | 0.886- | 0.337 | 0.603- | 0.070- | 0.145- | 0.381- | 0.265 | 0.551- | 0.290 | 2.0 |
| 2.5 | 0.378- | 0.789- | 0.066 | 0.397- | 0.945 | 0.313- | 0.308- | 0.046 | 0.286- | 0.781 | 2.5 |
| 2.8 | 0.613- | 0.782- | 0.000 | 0.000 | 1.717 | 0.404- | 0.304- | 0.000 | 0.000 | 1.131 | 2.8 |

**Table A.2.5. A and B Coefficients for Elastic Piles, $E_s = kx$, $Z_{max} = 2.8$**

| Z | $A_y$ | $A_s$ | $A_M$ | $A_v$ | $A_p$ | $B_y$ | $B_s$ | $B_M$ | $B_v$ | $B_p$ | Z |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | 3.167 | 2.052- | 0.000 | 1.000 | 0.000 | 2.052 | 2.016- | 1.000 | 0.000 | 0.000 | 0.0 |
| 0.1 | 2.962 | 2.047- | 0.100 | 0.987 | 0.296- | 1.855 | 1.916- | 1.000 | 0.008- | 0.186- | 0.1 |
| 0.2 | 2.758 | 2.032- | 0.197 | 0.944 | 0.552- | 1.669 | 1.816- | 0.998 | 0.035- | 0.334- | 0.2 |
| 0.3 | 2.556 | 2.008- | 0.288 | 0.879 | 0.767- | 1.492 | 1.716- | 0.993 | 0.073- | 0.448- | 0.3 |
| 0.4 | 2.357 | 1.975- | 0.372 | 0.793 | 0.943- | 1.325 | 1.617- | 0.983 | 0.122- | 0.539- | 0.4 |
| 0.5 | 2.161 | 1.934- | 0.446 | 0.691 | 1.081- | 1.169 | 1.520- | 0.968 | 0.179- | 0.584- | 0.5 |
| 0.6 | 1.970 | 1.886- | 0.510 | 0.578 | 1.182- | 1.021 | 1.424- | 0.947 | 0.238- | 0.613- | 0.6 |
| 0.7 | 1.784 | 1.832- | 0.561 | 0.455 | 1.249- | 0.884 | 1.331- | 0.920 | 0.301- | 0.619- | 0.7 |
| 0.8 | 1.604 | 1.774- | 0.601 | 0.329 | 1.283- | 0.755 | 1.240- | 0.887 | 0.362- | 0.604- | 0.8 |
| 0.9 | 1.429 | 1.712- | 0.627 | 0.200 | 1.286- | 0.636 | 1.153- | 0.848 | 0.421- | 0.572- | 0.9 |
| 1.0 | 1.261 | 1.649- | 0.641 | 0.072 | 1.261- | 0.524 | 1.071- | 0.803 | 0.476- | 0.524- | 1.0 |
| 1.2 | 0.944 | 1.521- | 0.630 | 0.170- | 1.133- | 0.326 | 0.921- | 0.698 | 0.569- | 0.391- | 1.2 |
| 1.4 | 0.652 | 1.409- | 0.575 | 0.376- | 0.913- | 0.155 | 0.793- | 0.577 | 0.630- | 0.217- | 1.4 |
| 1.6 | 0.383 | 1.294- | 0.483 | 0.531- | 0.613- | 0.007 | 0.690- | 0.448 | 0.653- | 0.011- | 1.6 |
| 1.8 | 0.134 | 1.208- | 0.367 | 0.618- | 0.240- | 0.123- | 0.613- | 0.319 | 0.633- | 0.221 | 1.8 |
| 2.0 | 0.102- | 1.148- | 0.241 | 0.622- | 0.203 | 0.240- | 0.562- | 0.198 | 0.563- | 0.480 | 2.0 |
| 2.5 | 0.657- | 1.094- | 0.010 | 0.182- | 1.642 | 0.506- | 0.519- | 0.007 | 0.136- | 1.266 | 2.5 |
| 2.6 | 0.766- | 1.094- | 0.000 | 0.000 | 1.993 | 0.558- | 0.519- | 0.000 | 0.000 | 1.451 | 2.6 |

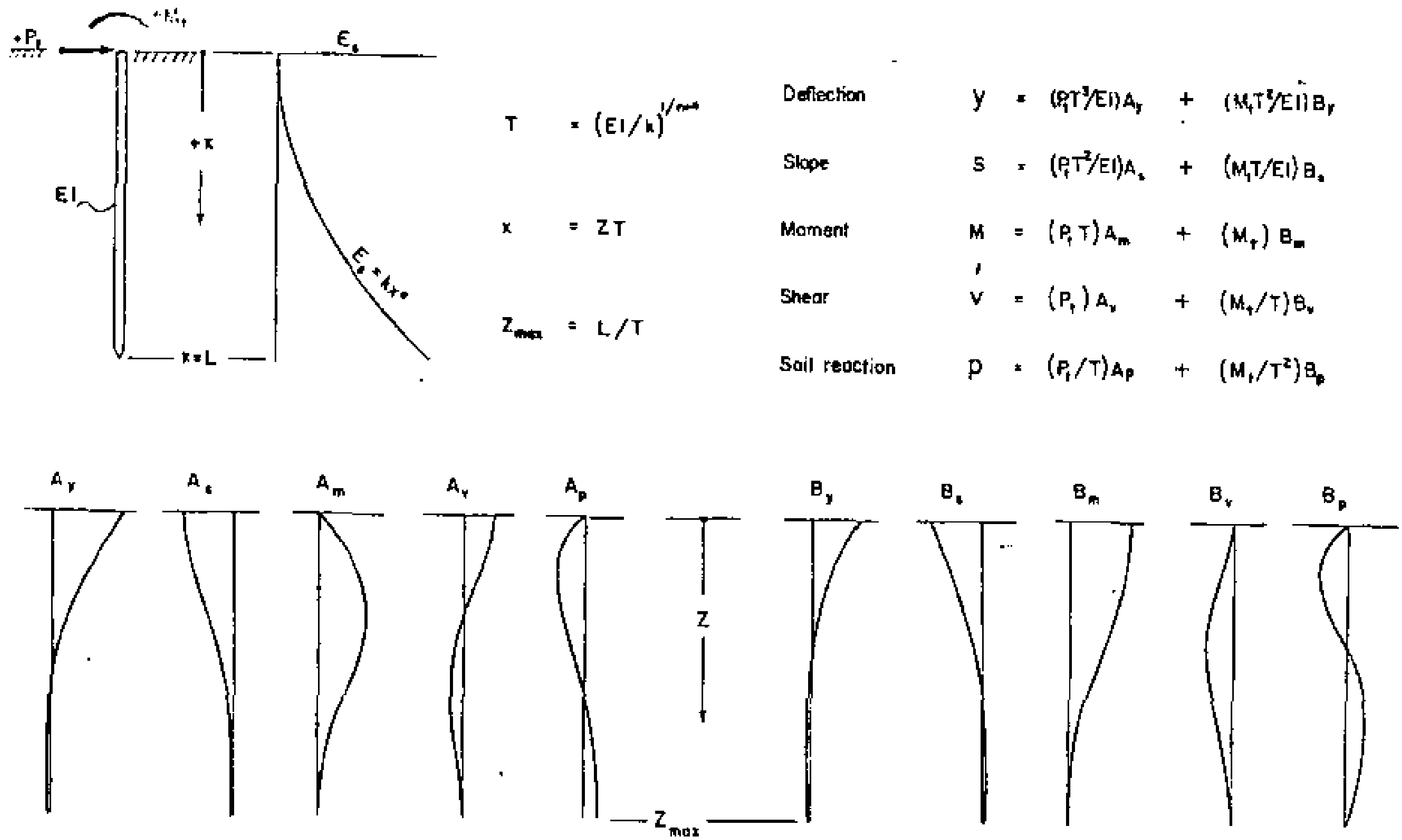
Table A.2.6. A and B Coefficients for Elastic Piles, $E_s = kx$, $Z_{max} = 2.6$

| Z | $A_Y$ | $A_S$ | $A_M$ | $A_V$ | $A_P$ | $B_Y$ | $B_S$ | $B_M$ | $B_V$ | $B_P$ | Z |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | 3.532 | 2.331- | 0.000 | 1.000 | 0.000 | 2.331 | 2.230- | 1.000 | 0.000 | 0.000 | 0.0 |
| 0.1 | 3.299 | 2.326- | 0.100 | 0.986 | 0.330- | 2.113 | 2.130- | 1.000 | 0.009- | 0.211- | 0.1 |
| 0.2 | 3.067 | 2.311- | 0.196 | 0.938 | 0.613- | 1.905 | 2.030- | 0.998 | 0.040- | 0.381- | 0.2 |
| 0.3 | 2.837 | 2.287- | 0.287 | 0.865 | 0.851- | 1.707 | 1.930- | 0.992 | 0.083- | 0.512- | 0.3 |
| 0.4 | 2.610 | 2.254- | 0.369 | 0.769 | 1.044- | 1.519 | 1.832- | 0.981 | 0.141- | 0.607- | 0.4 |
| 0.5 | 2.386 | 2.213- | 0.440 | 0.655 | 1.193- | 1.340 | 1.734- | 0.963 | 0.206- | 0.670- | 0.5 |
| 0.6 | 2.167 | 2.166- | 0.500 | 0.532 | 1.300- | 1.172 | 1.639- | 0.939 | 0.274- | 0.703- | 0.6 |
| 0.7 | 1.953 | 2.114- | 0.546 | 0.398 | 1.367- | 1.013 | 1.547- | 0.908 | 0.344- | 0.709- | 0.7 |
| 0.8 | 1.744 | 2.057- | 0.579 | 0.260 | 1.396- | 0.862 | 1.458- | 0.871 | 0.414- | 0.690- | 0.8 |
| 0.9 | 1.542 | 1.999- | 0.598 | 0.120 | 1.387- | 0.721 | 1.373- | 0.826 | 0.482- | 0.649- | 0.9 |
| 1.0 | 1.345 | 1.938- | 0.603 | 0.016- | 1.345- | 0.588 | 1.293- | 0.774 | 0.543- | 0.588- | 1.0 |
| 1.2 | 0.969 | 1.820- | 0.574 | 0.270- | 1.163- | 0.344 | 1.150- | 0.655 | 0.645- | 0.413- | 1.2 |
| 1.4 | 0.616 | 1.712- | 0.498 | 0.474- | 0.863- | 0.126 | 1.032- | 0.519 | 0.704- | 0.177- | 1.4 |
| 1.6 | 0.283 | 1.623- | 0.389 | 0.607- | 0.453- | 0.071- | 0.942- | 0.376 | 0.712- | 0.113 | 1.6 |
| 1.8 | 0.034- | 1.558- | 0.261 | 0.648- | 0.062 | 0.252- | 0.881- | 0.238 | 0.656- | 0.454 | 1.8 |
| 2.0 | 0.342- | 1.518- | 0.137 | 0.575- | 0.683 | 0.425- | 0.846- | 0.119 | 0.526- | 0.849 | 2.0 |
| 2.4 | 0.943- | 1.498- | 0.000 | 0.000 | 2.263 | 0.758- | 0.829- | 0.000 | 0.000 | 1.819 | 2.4 |

Table A.2.7. A and B Coefficients for Elastic Piles, $E_s = kx$, $Z_{max} = 2.4$

| Z | $A_Y$ | $A_S$ | $A_M$ | $A_V$ | $A_P$ | $B_Y$ | $B_S$ | $B_M$ | $B_V$ | $B_P$ | Z |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | 4.011 | 2.743- | 0.000 | 1.000 | 0.000 | 2.743 | 2.583- | 1.000 | 0.000 | 0.000 | 0.0 |
| 0.1 | 3.737 | 2.738- | 0.099 | 0.980 | 0.374- | 2.489 | 2.483- | 1.000 | 0.014- | 0.249- | 0.1 |
| 0.2 | 3.464 | 2.723- | 0.195 | 0.926 | 0.693- | 2.246 | 2.383- | 0.997 | 0.048- | 0.449- | 0.2 |
| 0.3 | 3.193 | 2.699- | 0.284 | 0.840 | 0.958- | 2.013 | 2.283- | 0.989 | 0.104- | 0.604- | 0.3 |
| 0.4 | 2.924 | 2.667- | 0.363 | 0.733 | 1.170- | 1.789 | 2.185- | 0.976 | 0.170- | 0.716- | 0.4 |
| 0.5 | 2.659 | 2.627- | 0.430 | 0.609 | 1.330- | 1.576 | 2.089- | 0.955 | 0.246- | 0.788- | 0.5 |
| 0.6 | 2.399 | 2.581- | 0.484 | 0.468 | 1.439- | 1.372 | 1.994- | 0.927 | 0.326- | 0.823- | 0.6 |
| 0.7 | 2.143 | 2.531- | 0.523 | 0.318 | 1.500- | 1.177 | 1.904- | 0.890 | 0.411- | 0.824- | 0.7 |
| 0.8 | 1.893 | 2.477- | 0.548 | 0.168 | 1.514- | 0.991 | 1.817- | 0.845 | 0.491- | 0.793- | 0.8 |
| 0.9 | 1.648 | 2.422- | 0.557 | 0.019 | 1.483- | 0.813 | 1.735- | 0.792 | 0.568- | 0.732- | 0.9 |
| 1.0 | 1.409 | 2.366- | 0.552 | 0.126- | 1.409- | 0.644 | 1.659- | 0.731 | 0.636- | 0.644- | 1.0 |
| 1.2 | 0.946 | 2.260- | 0.500 | 0.384- | 1.135- | 0.326 | 1.526- | 0.593 | 0.742- | 0.391- | 1.2 |
| 1.4 | 0.504 | 2.169- | 0.403 | 0.570- | 0.705- | 0.031 | 1.423- | 0.438 | 0.787- | 0.044- | 1.4 |
| 1.6 | 0.077 | 2.101- | 0.278 | 0.656- | 0.123- | 0.245- | 1.351- | 0.283 | 0.753- | 0.392 | 1.6 |
| 1.8 | 0.338- | 2.059- | 0.148 | 0.610- | 0.609 | 0.511- | 1.309- | 0.143 | 0.624- | 0.919 | 1.8 |
| 2.0 | 0.748- | 2.040- | 0.044 | 0.401- | 1.496 | 0.770- | 1.291- | 0.041 | 0.378- | 1.541 | 2.0 |
| 2.2 | 1.155- | 2.037- | 0.000 | 0.000 | 2.542 | 1.028- | 1.288- | 0.000 | 0.000 | 2.262 | 2.2 |

Table A.2.8. A and B Coefficients for Elastic Piles, $E_s = kx$, $Z_{max} = 2.2$




Appendix B.1. A and B Coefficients for Elastic Piles, $Z_{max} = 10$, $E_s = kx^n$, n varied from 0.0 to 4.

| Z | $A_Y$ | $A_S$ | $A_M$ | $A_V$ | $A_P$ | $B_Y$ | $B_S$ | $B_M$ | $B_V$ | $B_P$ | Z |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | 1.412 | 0.998- | 0.000 | 1.000 | 1.412- | 0.998 | 1.412- | 1.000 | 0.000 | 0.998- | 0.0 |
| 0.1 | 1.318 | 0.993- | 0.093 | 0.864 | 1.313- | 0.861 | 1.313- | 0.995 | 0.093- | 0.861- | 0.1 |
| 0.2 | 1.214 | 0.980- | 0.173 | 0.738 | 1.214- | 0.735 | 1.214- | 0.981 | 0.173- | 0.735- | 0.2 |
| 0.3 | 1.117 | 0.959- | 0.240 | 0.621 | 1.117- | 0.618 | 1.117- | 0.960 | 0.240- | 0.618- | 0.3 |
| 0.4 | 1.022 | 0.932- | 0.297 | 0.514 | 1.022- | 0.512 | 1.022- | 0.933 | 0.297- | 0.512- | 0.4 |
| 0.5 | 0.930 | 0.900- | 0.343 | 0.416 | 0.930- | 0.414 | 0.930- | 0.901 | 0.343- | 0.414- | 0.5 |
| 0.6 | 0.842 | 0.864- | 0.380 | 0.328 | 0.842- | 0.326 | 0.842- | 0.865 | 0.380- | 0.326- | 0.6 |
| 0.7 | 0.758 | 0.824- | 0.409 | 0.248 | 0.758- | 0.246 | 0.758- | 0.825 | 0.409- | 0.246- | 0.7 |
| 0.8 | 0.677 | 0.783- | 0.430 | 0.176 | 0.677- | 0.174 | 0.677- | 0.783 | 0.430- | 0.174- | 0.8 |
| 0.9 | 0.601 | 0.739- | 0.444 | 0.112 | 0.601- | 0.110 | 0.601- | 0.739 | 0.444- | 0.110- | 0.9 |
| 1.0 | 0.529 | 0.694- | 0.452 | 0.056 | 0.529- | 0.054 | 0.529- | 0.694 | 0.452- | 0.054- | 1.0 |
| 1.2 | 0.400 | 0.603- | 0.453 | 0.037- | 0.400- | 0.039- | 0.400- | 0.603 | 0.453- | 0.039 | 1.2 |
| 1.4 | 0.288 | 0.514- | 0.438 | 0.106- | 0.288- | 0.107- | 0.288- | 0.514 | 0.438- | 0.107 | 1.4 |
| 1.6 | 0.194 | 0.429- | 0.412 | 0.154- | 0.194- | 0.155- | 0.194- | 0.428 | 0.412- | 0.155 | 1.6 |
| 1.8 | 0.116 | 0.350- | 0.378 | 0.185- | 0.116- | 0.185- | 0.116- | 0.349 | 0.378- | 0.185 | 1.8 |
| 2.0 | 0.054 | 0.278- | 0.339 | 0.201- | 0.054- | 0.202- | 0.054- | 0.277 | 0.339- | 0.202 | 2.0 |
| 2.5 | 0.047- | 0.134- | 0.236 | 0.200- | 0.047 | 0.201- | 0.047 | 0.134 | 0.236- | 0.201 | 2.5 |
| 3.0 | 0.088- | 0.040- | 0.144 | 0.164- | 0.088 | 0.165- | 0.088 | 0.039 | 0.144- | 0.165 | 3.0 |
| 3.5 | 0.093- | 0.014 | 0.074 | 0.118- | 0.093 | 0.118- | 0.093 | 0.014- | 0.074- | 0.118 | 3.5 |
| 4.0 | 0.079- | 0.038 | 0.026 | 0.074- | 0.079 | 0.074- | 0.079 | 0.038- | 0.026- | 0.074 | 4.0 |
| 4.5 | 0.059- | 0.043 | 0.002- | 0.040- | 0.059 | 0.040- | 0.059 | 0.043- | 0.002 | 0.040 | 4.5 |
| 5.0 | 0.038- | 0.038 | 0.016- | 0.016- | 0.038 | 0.016- | 0.038 | 0.038- | 0.016 | 0.016 | 5.0 |
| 10.0 | 0.000 | 0.002- | 0.000 | 0.000 | 0.000 | 0.002- | 0.003- | 0.000 | 0.000 | 0.002 | 10.0 |

Table B.1.1. A and B Coefficients for Elastic Piles, $E_s = k$, $Z_{max} = 10$

| Z | $A_Y$ | $A_S$ | $A_M$ | $A_V$ | $A_P$ | $B_Y$ | $B_S$ | $B_M$ | $B_V$ | $B_P$ | Z |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | 1.794 | 1.236- | 0.000 | 1.000 | 0.000 | 1.236 | 1.550- | 1.000 | 0.000 | 0.000 | 0.0 |
| 0.1 | 1.670 | 1.231- | 0.100 | 0.953 | 0.939- | 1.086 | 1.450- | 1.000 | 0.031- | 0.611- | 0.1 |
| 0.2 | 1.548 | 1.217- | 0.191 | 0.854 | 1.035- | 0.946 | 1.350- | 0.994 | 0.093- | 0.633- | 0.2 |
| 0.3 | 1.427 | 1.193- | 0.271 | 0.750 | 1.056- | 0.816 | 1.252- | 0.981 | 0.154- | 0.604- | 0.3 |
| 0.4 | 1.309 | 1.163- | 0.341 | 0.645 | 1.041- | 0.696 | 1.154- | 0.963 | 0.212- | 0.553- | 0.4 |
| 0.5 | 1.194 | 1.126- | 0.400 | 0.543 | 1.004- | 0.585 | 1.059- | 0.939 | 0.265- | 0.492- | 0.5 |
| 0.6 | 1.084 | 1.083- | 0.449 | 0.445 | 0.954- | 0.484 | 0.967- | 0.910 | 0.311- | 0.426- | 0.6 |
| 0.7 | 0.978 | 1.036- | 0.489 | 0.353 | 0.894- | 0.392 | 0.878- | 0.877 | 0.350- | 0.358- | 0.7 |
| 0.8 | 0.877 | 0.986- | 0.520 | 0.266 | 0.829- | 0.308 | 0.792- | 0.840 | 0.382- | 0.292- | 0.8 |
| 0.9 | 0.781 | 0.933- | 0.542 | 0.187 | 0.760- | 0.233 | 0.710- | 0.800 | 0.408- | 0.227- | 0.9 |
| 1.0 | 0.690 | 0.878- | 0.557 | 0.114 | 0.690- | 0.166 | 0.632- | 0.759 | 0.428- | 0.166- | 1.0 |
| 1.2 | 0.526 | 0.765- | 0.567 | 0.010- | 0.550- | 0.055 | 0.489- | 0.670 | 0.450- | 0.057- | 1.2 |
| 1.4 | 0.384 | 0.653- | 0.555 | 0.106- | 0.417- | 0.030- | 0.364- | 0.580 | 0.452- | 0.033 | 1.4 |
| 1.6 | 0.264 | 0.545- | 0.526 | 0.177- | 0.297- | 0.092- | 0.257- | 0.490 | 0.438- | 0.103 | 1.6 |
| 1.8 | 0.165 | 0.444- | 0.485 | 0.226- | 0.192- | 0.134- | 0.167- | 0.405 | 0.412- | 0.155 | 1.8 |
| 2.0 | 0.086 | 0.352- | 0.436 | 0.255- | 0.103- | 0.159- | 0.094- | 0.326 | 0.378- | 0.190 | 2.0 |
| 2.5 | 0.040- | 0.167- | 0.303 | 0.265- | 0.051 | 0.173- | 0.025 | 0.162 | 0.273- | 0.218 | 2.5 |
| 3.0 | 0.091- | 0.047- | 0.180 | 0.219- | 0.120 | 0.145- | 0.077 | 0.052 | 0.169- | 0.191 | 3.0 |
| 3.5 | 0.096- | 0.018 | 0.087 | 0.155- | 0.131 | 0.103- | 0.086 | 0.010- | 0.086- | 0.141 | 3.5 |
| 4.0 | 0.079- | 0.045 | 0.025 | 0.093- | 0.111 | 0.063- | 0.073 | 0.038- | 0.029- | 0.088 | 4.0 |
| 4.5 | 0.055- | 0.048 | 0.009- | 0.045- | 0.080 | 0.031- | 0.052 | 0.043- | 0.004 | 0.045 | 4.5 |
| 5.0 | 0.032- | 0.040 | 0.023- | 0.013- | 0.048 | 0.011- | 0.032 | 0.037- | 0.019 | 0.016 | 5.0 |
| 10.0 | 0.001- | 0.002- | 0.000 | 0.000 | 0.002 | 0.001- | 0.002- | 0.000 | 0.000 | 0.003 | 10.0 |

Table B.1.2. A and B Coefficients for Elastic Piles, $E_s = kx^{0.25}$, $Z_{max} = 10$

| Z | $A_Y$ | $A_S$ | $A_M$ | $A_V$ | $A_P$ | $B_Y$ | $B_S$ | $B_M$ | $B_V$ | $B_P$ | Z |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | 2.041 | 1.384- | 0.000 | 1.000 | 0.000 | 1.384 | 1.626- | 1.000 | 0.000 | 0.000 | 0.0 |
| 0.1 | 1.903 | 1.379- | 0.100 | 0.970 | 0.602- | 1.226 | 1.526- | 1.000 | 0.020- | 0.388- | 0.1 |
| 0.2 | 1.766 | 1.364- | 0.194 | 0.900 | 0.790- | 1.078 | 1.426- | 0.996 | 0.063- | 0.482- | 0.2 |
| 0.3 | 1.630 | 1.340- | 0.280 | 0.816 | 0.893- | 0.941 | 1.327- | 0.987 | 0.113- | 0.515- | 0.3 |
| 0.4 | 1.498 | 1.308- | 0.357 | 0.724 | 0.947- | 0.813 | 1.229- | 0.974 | 0.164- | 0.514- | 0.4 |
| 0.5 | 1.368 | 1.269- | 0.425 | 0.628 | 0.968- | 0.695 | 1.133- | 0.955 | 0.215- | 0.491- | 0.5 |
| 0.6 | 1.244 | 1.224- | 0.483 | 0.532 | 0.963- | 0.586 | 1.039- | 0.931 | 0.262- | 0.454- | 0.6 |
| 0.7 | 1.124 | 1.173- | 0.531 | 0.436 | 0.940- | 0.487 | 0.947- | 0.902 | 0.305- | 0.408- | 0.7 |
| 0.8 | 1.009 | 1.118- | 0.570 | 0.344 | 0.902- | 0.397 | 0.858- | 0.870 | 0.343- | 0.355- | 0.8 |
| 0.9 | 0.900 | 1.060- | 0.600 | 0.256 | 0.863- | 0.316 | 0.773- | 0.833 | 0.376- | 0.302- | 0.9 |
| 1.0 | 0.797 | 0.999- | 0.621 | 0.173 | 0.797- | 0.242 | 0.692- | 0.794 | 0.403- | 0.242- | 1.0 |
| 1.2 | 0.610 | 0.872- | 0.641 | 0.026 | 0.668- | 0.119 | 0.541- | 0.709 | 0.441- | 0.131- | 1.2 |
| 1.4 | 0.448 | 0.744- | 0.633 | 0.094- | 0.530- | 0.025 | 0.408- | 0.619 | 0.456- | 0.030- | 1.4 |
| 1.6 | 0.312 | 0.620- | 0.605 | 0.186- | 0.395- | 0.045- | 0.294- | 0.528 | 0.454- | 0.057 | 1.6 |
| 1.8 | 0.200 | 0.504- | 0.560 | 0.252- | 0.268- | 0.093- | 0.197- | 0.439 | 0.435- | 0.125 | 1.8 |
| 2.0 | 0.110 | 0.397- | 0.505 | 0.294- | 0.156- | 0.125- | 0.118- | 0.354 | 0.405- | 0.175 | 2.0 |
| 2.5 | 0.032- | 0.184- | 0.348 | 0.316- | 0.050 | 0.147- | 0.013 | 0.177 | 0.299- | 0.232 | 2.5 |
| 3.0 | 0.086- | 0.048- | 0.201 | 0.263- | 0.149 | 0.124- | 0.069 | 0.057 | 0.185- | 0.214 | 3.0 |
| 3.5 | 0.090- | 0.023 | 0.089 | 0.181- | 0.168 | 0.085- | 0.078 | 0.011- | 0.091- | 0.160 | 3.5 |
| 4.0 | 0.070- | 0.049 | 0.019 | 0.103- | 0.140 | 0.049- | 0.064 | 0.039- | 0.027- | 0.098 | 4.0 |
| 4.5 | 0.045- | 0.048 | 0.016- | 0.044- | 0.095 | 0.022- | 0.043 | 0.043- | 0.009 | 0.047 | 4.5 |
| 5.0 | 0.024- | 0.036 | 0.028- | 0.007- | 0.053 | 0.005- | 0.024 | 0.034- | 0.023 | 0.012 | 5.0 |
| 10.0 | 0.001- | 0.001- | 0.000 | 0.000 | 0.002 | 0.001- | 0.000 | 0.000 | 0.000 | 0.002 | 10.0 |

Table B.1.3. A and B Coefficients for Elastic Piles, $E_s = kx^{0.5}$, $Z_{max} = 10$

| Z | $A_y$ | $A_s$ | $A_M$ | $A_v$ | $A_p$ | $B_y$ | $B_s$ | $B_M$ | $B_v$ | $B_p$ | Z |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | 2.435 | 1.623- | 0.000 | 1.000 | 0.000 | 1.623 | 1.749- | 1.000 | 0.000 | 0.000 | 0.0 |
| 0.1 | 2.273 | 1.618- | 0.100 | 0.988 | 0.227- | 1.453 | 1.649- | 1.000 | 0.007- | 0.145- | 0.1 |
| 0.2 | 2.112 | 1.603- | 0.198 | 0.956 | 0.422- | 1.293 | 1.549- | 0.999 | 0.028- | 0.259- | 0.2 |
| 0.3 | 1.952 | 1.578- | 0.291 | 0.906 | 0.586- | 1.143 | 1.450- | 0.994 | 0.058- | 0.343- | 0.3 |
| 0.4 | 1.796 | 1.545- | 0.379 | 0.840 | 0.718- | 1.003 | 1.351- | 0.987 | 0.095- | 0.401- | 0.4 |
| 0.5 | 1.643 | 1.503- | 0.459 | 0.763 | 0.822- | 0.873 | 1.253- | 0.976 | 0.137- | 0.436- | 0.5 |
| 0.6 | 1.495 | 1.453- | 0.531 | 0.677 | 0.897- | 0.752 | 1.156- | 0.960 | 0.181- | 0.451- | 0.6 |
| 0.7 | 1.353 | 1.397- | 0.595 | 0.585 | 0.947- | 0.641 | 1.061- | 0.939 | 0.226- | 0.449- | 0.7 |
| 0.8 | 1.216 | 1.335- | 0.649 | 0.489 | 0.973- | 0.540 | 0.968- | 0.914 | 0.270- | 0.432- | 0.8 |
| 0.9 | 1.086 | 1.268- | 0.693 | 0.392 | 0.977- | 0.448 | 0.870- | 0.885 | 0.312- | 0.403- | 0.9 |
| 1.0 | 0.962 | 1.197- | 0.727 | 0.295 | 0.962- | 0.364 | 0.791- | 0.852 | 0.350- | 0.364- | 1.0 |
| 1.2 | 0.738 | 1.047- | 0.767 | 0.109 | 0.885- | 0.223 | 0.678- | 0.775 | 0.414- | 0.267- | 1.2 |
| 1.4 | 0.544 | 0.893- | 0.772 | 0.056- | 0.761- | 0.112 | 0.482- | 0.688 | 0.456- | 0.157- | 1.4 |
| 1.6 | 0.381 | 0.741- | 0.746 | 0.193- | 0.609- | 0.029 | 0.354- | 0.594 | 0.477- | 0.046- | 1.6 |
| 1.8 | 0.247 | 0.596- | 0.696 | 0.299- | 0.445- | 0.030- | 0.245- | 0.498 | 0.476- | 0.055 | 1.8 |
| 2.0 | 0.141 | 0.464- | 0.628 | 0.371- | 0.293- | 0.070- | 0.155- | 0.404 | 0.456- | 0.140 | 2.0 |
| 2.5 | 0.020- | 0.200- | 0.422 | 0.424- | 0.049 | 0.105- | 0.006- | 0.200 | 0.350- | 0.263 | 2.5 |
| 3.0 | 0.075- | 0.040- | 0.225 | 0.349- | 0.226 | 0.089- | 0.057 | 0.059 | 0.213- | 0.268 | 3.0 |
| 3.5 | 0.074- | 0.034 | 0.081 | 0.223- | 0.257 | 0.057- | 0.065 | 0.016- | 0.095- | 0.200 | 3.5 |
| 4.0 | 0.050- | 0.052 | 0.000 | 0.106- | 0.201 | 0.028- | 0.049 | 0.042- | 0.017- | 0.113 | 4.0 |
| 4.5 | 0.026- | 0.042 | 0.032- | 0.027- | 0.117 | 0.009- | 0.028 | 0.039- | 0.021 | 0.041 | 4.5 |
| 5.0 | 0.009- | 0.025 | 0.033- | 0.013 | 0.046 | 0.000 | 0.011 | 0.026- | 0.029 | 0.002 | 5.0 |
| 10.0 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 10.0 |

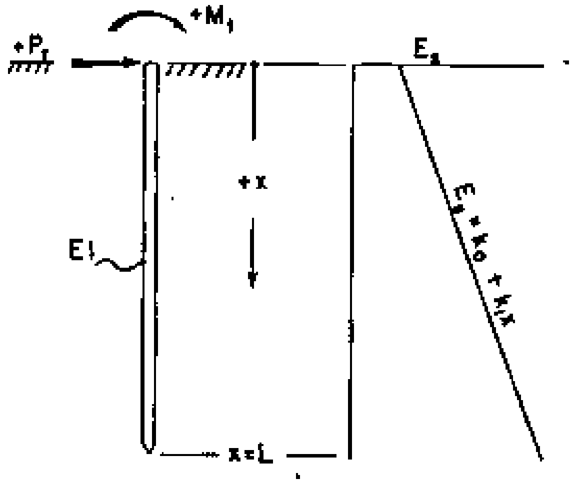
Table B.1.4. A and B Coefficients for Elastic Piles, $E_s = kx$, $Z_{max} = 10.0$

| Z | $A_Y$ | $A_S$ | $A_M$ | $A_V$ | $A_P$ | $B_Y$ | $B_S$ | $B_M$ | $B_V$ | $B_P$ | Z |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | 2.818 | 1.882- | 0.000 | 1.000 | 0.000 | 1.882 | 1.886- | 1.000 | 0.000 | 0.000 | 0.0 |
| 0.1 | 2.630 | 1.877- | 0.100 | 0.999 | 0.026- | 1.698 | 1.786- | 1.000 | 0.001- | 0.017- | 0.1 |
| 0.2 | 2.443 | 1.862- | 0.200 | 0.993 | 0.098- | 1.525 | 1.686- | 1.000 | 0.005- | 0.061- | 0.2 |
| 0.3 | 2.258 | 1.837- | 0.299 | 0.977 | 0.203- | 1.361 | 1.586- | 0.999 | 0.014- | 0.122- | 0.3 |
| 0.4 | 2.075 | 1.802- | 0.395 | 0.951 | 0.332- | 1.207 | 1.487- | 0.997 | 0.030- | 0.193- | 0.4 |
| 0.5 | 1.897 | 1.758- | 0.489 | 0.910 | 0.474- | 1.064 | 1.387- | 0.993 | 0.053- | 0.266- | 0.5 |
| 0.6 | 1.724 | 1.705- | 0.577 | 0.856 | 0.621- | 0.930 | 1.288- | 0.987 | 0.083- | 0.335- | 0.6 |
| 0.7 | 1.556 | 1.643- | 0.660 | 0.786 | 0.762- | 0.806 | 1.190- | 0.977 | 0.119- | 0.395- | 0.7 |
| 0.8 | 1.395 | 1.573- | 0.735 | 0.704 | 0.893- | 0.692 | 1.093- | 0.963 | 0.161- | 0.443- | 0.8 |
| 0.9 | 1.241 | 1.497- | 0.800 | 0.609 | 1.006- | 0.587 | 0.998- | 0.944 | 0.207- | 0.476- | 0.9 |
| 1.0 | 1.096 | 1.414- | 0.856 | 0.504 | 1.096- | 0.492 | 0.904- | 0.921 | 0.255- | 0.492- | 1.0 |
| 1.2 | 0.831 | 1.234- | 0.934 | 0.273 | 1.196- | 0.330 | 0.726- | 0.860 | 0.353- | 0.475- | 1.2 |
| 1.4 | 0.603 | 1.044- | 0.965 | 0.034 | 1.181- | 0.201 | 0.562- | 0.781 | 0.441- | 0.395- | 1.4 |
| 1.6 | 0.413 | 0.852- | 0.949 | 0.191- | 1.058- | 0.104 | 0.415- | 0.685 | 0.507- | 0.267- | 1.6 |
| 1.8 | 0.262 | 0.667- | 0.890 | 0.383- | 0.848- | 0.034 | 0.288- | 0.579 | 0.545- | 0.111- | 1.8 |
| 2.0 | 0.146 | 0.498- | 0.798 | 0.526- | 0.582- | 0.012- | 0.184- | 0.469 | 0.551- | 0.049 | 2.0 |
| 2.5 | 0.015- | 0.174- | 0.490 | 0.643- | 0.096 | 0.056- | 0.015- | 0.213 | 0.443- | 0.353 | 2.5 |
| 3.0 | 0.053- | 0.004- | 0.200 | 0.482- | 0.480 | 0.045- | 0.044 | 0.041 | 0.242- | 0.409 | 3.0 |
| 3.5 | 0.039- | 0.046 | 0.022 | 0.230- | 0.476 | 0.022- | 0.042 | 0.033- | 0.067- | 0.271 | 3.5 |
| 4.0 | 0.017- | 0.038 | 0.041- | 0.042- | 0.264 | 0.006- | 0.022 | 0.040- | 0.023 | 0.095 | 4.0 |
| 4.5 | 0.003- | 0.017 | 0.038- | 0.036 | 0.061 | 0.001 | 0.006 | 0.022- | 0.040 | 0.014- | 4.5 |
| 5.0 | 0.001 | 0.003 | 0.017- | 0.039 | 0.034- | 0.002 | 0.001- | 0.006- | 0.023 | 0.040- | 5.0 |
| 10.0 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 10.0 |

Table B.1.5. A and B Coefficients for Elastic Piles, $E_s = kx^2$, $Z_{max} = 10$

| Z | $A_y$ | $A_s$ | $A_M$ | $A_V$ | $A_P$ | $B_y$ | $B_s$ | $B_M$ | $B_V$ | $B_P$ | Z |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | 2.754 | 1.940- | 0.000 | 1.000 | 0.000 | 1.940 | 1.932- | 1.000 | 0.000 | 0.000 | 0.0 |
| 0.1 | 2.560 | 1.935- | 0.100 | 1.000 | 0.000- | 1.752 | 1.832- | 1.000 | 0.000 | 0.000- | 0.1 |
| 0.2 | 2.367 | 1.920- | 0.200 | 1.000 | 0.004- | 1.573 | 1.732- | 1.000 | 0.000 | 0.003- | 0.2 |
| 0.3 | 2.176 | 1.895- | 0.300 | 0.998 | 0.018- | 1.405 | 1.633- | 1.000 | 0.001- | 0.011- | 0.3 |
| 0.4 | 1.988 | 1.860- | 0.400 | 0.995 | 0.051- | 1.247 | 1.533- | 1.000 | 0.003- | 0.032- | 0.4 |
| 0.5 | 1.804 | 1.815- | 0.499 | 0.987 | 0.113- | 1.099 | 1.433- | 0.999 | 0.008- | 0.069- | 0.5 |
| 0.6 | 1.625 | 1.760- | 0.597 | 0.970 | 0.211- | 0.960 | 1.333- | 0.998 | 0.018- | 0.124- | 0.6 |
| 0.7 | 1.452 | 1.696- | 0.693 | 0.942 | 0.349- | 0.832 | 1.233- | 0.996 | 0.034- | 0.200- | 0.7 |
| 0.8 | 1.286 | 1.622- | 0.785 | 0.899 | 0.527- | 0.714 | 1.134- | 0.991 | 0.059- | 0.292- | 0.8 |
| 0.9 | 1.127 | 1.539- | 0.873 | 0.835 | 0.740- | 0.605 | 1.035- | 0.984 | 0.094- | 0.397- | 0.9 |
| 1.0 | 0.978 | 1.448- | 0.952 | 0.749 | 0.978- | 0.507 | 0.937- | 0.973 | 0.139- | 0.507- | 1.0 |
| 1.2 | 0.708 | 1.244- | 1.080 | 0.504 | 1.468- | 0.339 | 0.746- | 0.934 | 0.260- | 0.702- | 1.2 |
| 1.4 | 0.481 | 1.020- | 1.150 | 0.170 | 1.849- | 0.208 | 0.566- | 0.867 | 0.412- | 0.799- | 1.4 |
| 1.6 | 0.300 | 0.789- | 1.146 | 0.216- | 1.968- | 0.112 | 0.402- | 0.769 | 0.568- | 0.731- | 1.6 |
| 1.8 | 0.165 | 0.568- | 1.064 | 0.615- | 2.227- | 0.046 | 0.261- | 0.641 | 0.698- | 0.621- | 1.8 |
| 2.0 | 0.072 | 0.370- | 0.904 | 0.932- | 1.153- | 0.006 | 0.147- | 0.493 | 0.764- | 0.096- | 2.0 |
| 2.5 | 0.021- | 0.049- | 0.373 | 1.009- | 0.808 | 0.021- | 0.007 | 0.141 | 0.559- | 0.819 | 2.5 |
| 3.0 | 0.016- | 0.034 | 0.012 | 0.390- | 1.319 | 0.009- | 0.026 | 0.029- | 0.130- | 0.716 | 3.0 |
| 3.5 | 0.002- | 0.016 | 0.051- | 0.048 | 0.365 | 0.001- | 0.007 | 0.031- | 0.063 | 0.078 | 3.5 |
| 4.0 | 0.001 | 0.000 | 0.011- | 0.063 | 0.156- | 0.000 | 0.001- | 0.003- | 0.032 | 0.114- | 4.0 |
| 4.5 | 0.000 | 0.001- | 0.002 | 0.002 | 0.056- | 0.000 | 0.000 | 0.002 | 0.002- | 0.020- | 4.5 |
| 5.0 | 0.000 | 0.000 | 0.001 | 0.004- | 0.012 | 0.000 | 0.000 | 0.000 | 0.002- | 0.008 | 5.0 |
| 10.0 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 10.0 |

Table B.1.6. A and B Coefficients for Elastic Piles, $E_s = kx^4$, $Z_{max} = 10$

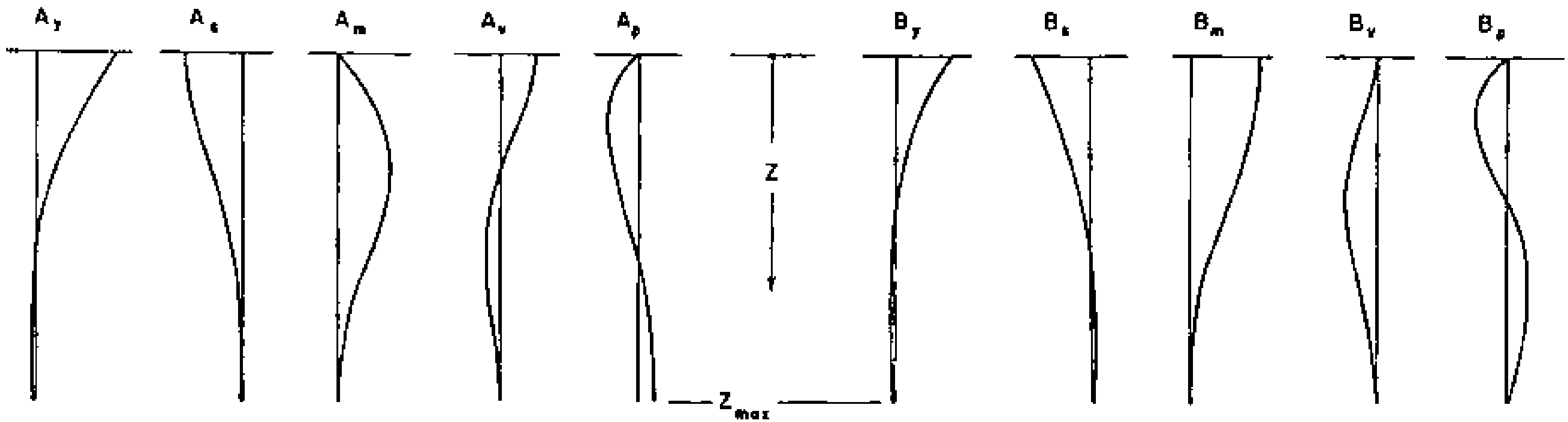



$T = (EI/k_1)^{1/5}$

$x = ZT$

$Z_{max} = L/T$

| | | | | |
|---|---|---|---|---|
| Deflection | $y$ | $=$ | $(P_tT^3/EI)A_y$ | $+ \ (M_tT^2/EI)B_y$ |
| Slope | $S$ | $=$ | $(P_tT^2/EI)A_s$ | $+ \ (M_tT/EI)B_s$ |
| Moment | $M$ | $=$ | $(P_tT)A_m$ | $+ \ (M_t)B_m$ |
| Shear | $V$ | $=$ | $(P_t)A_v$ | $+ \ (M_t/T)B_v$ |
| Soil reaction | $p$ | $=$ | $(P_t/T)A_p$ | $+ \ (M_t/T^2)B_p$ |




Appendix B.2. A and B Coefficients for Elastic Piles, $Z_{max} = 10$, $E_s = k_o + k_1x$, $k_o/k_1T$ varied from 0.1 to 5.

| Z | $A_Y$ | $A_S$ | $A_M$ | $A_V$ | $A_P$ | $B_Y$ | $B_S$ | $B_M$ | $B_V$ | $B_P$ | Z |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | 2.151 | 1.468- | 0.000 | 1.000 | 0.215- | 1.468 | 1.660- | 1.000 | 0.000 | 0.147- | 0.0 |
| 0.1 | 2.004 | 1.463- | 0.099 | 0.969 | 0.401- | 1.307 | 1.560- | 0.999 | 0.021- | 0.261- | 0.1 |
| 0.2 | 1.858 | 1.449- | 0.194 | 0.921 | 0.557- | 1.156 | 1.460- | 0.996 | 0.051- | 0.347- | 0.2 |
| 0.3 | 1.714 | 1.425- | 0.283 | 0.859 | 0.686- | 1.015 | 1.361- | 0.989 | 0.089- | 0.406- | 0.3 |
| 0.4 | 1.573 | 1.393- | 0.366 | 0.786 | 0.786- | 0.884 | 1.263- | 0.978 | 0.131- | 0.442- | 0.4 |
| 0.5 | 1.435 | 1.352- | 0.440 | 0.703 | 0.861- | 0.763 | 1.166- | 0.963 | 0.176- | 0.458- | 0.5 |
| 0.6 | 1.302 | 1.305- | 0.506 | 0.615 | 0.912- | 0.651 | 1.070- | 0.943 | 0.222- | 0.456- | 0.6 |
| 0.7 | 1.174 | 1.251- | 0.563 | 0.522 | 0.940- | 0.549 | 0.977- | 0.919 | 0.266- | 0.439- | 0.7 |
| 0.8 | 1.052 | 1.193- | 0.611 | 0.428 | 0.947- | 0.456 | 0.887- | 0.890 | 0.309- | 0.410- | 0.8 |
| 0.9 | 0.936 | 1.130- | 0.649 | 0.333 | 0.936- | 0.371 | 0.800- | 0.857 | 0.348- | 0.371- | 0.9 |
| 1.0 | 0.826 | 1.063- | 0.677 | 0.241 | 0.909- | 0.296 | 0.716- | 0.820 | 0.383- | 0.325- | 1.0 |
| 1.2 | 0.627 | 0.925- | 0.708 | 0.068 | 0.815- | 0.168 | 0.560- | 0.738 | 0.437- | 0.219- | 1.2 |
| 1.4 | 0.456 | 0.783- | 0.706 | 0.082- | 0.685- | 0.071 | 0.421- | 0.646 | 0.470- | 0.106- | 1.4 |
| 1.6 | 0.314 | 0.644- | 0.676 | 0.204- | 0.534- | 0.001- | 0.302- | 0.551 | 0.480- | 0.002 | 1.6 |
| 1.8 | 0.198 | 0.514- | 0.626 | 0.295- | 0.377- | 0.051- | 0.201- | 0.455 | 0.470- | 0.097 | 1.8 |
| 2.0 | 0.108 | 0.395- | 0.560 | 0.355- | 0.226- | 0.082- | 0.119- | 0.364 | 0.443- | 0.173 | 2.0 |
| 2.5 | 0.027- | 0.263- | 0.367 | 0.389- | 0.071 | 0.105- | 0.012 | 0.169 | 0.326- | 0.273 | 2.5 |
| 3.0 | 0.071- | 0.026- | 0.189 | 0.310- | 0.219 | 0.084- | 0.061 | 0.041 | 0.189- | 0.260 | 3.0 |
| 3.5 | 0.065- | 0.035 | 0.063 | 0.192- | 0.235 | 0.051- | 0.063 | 0.024- | 0.077- | 0.184 | 3.5 |
| 4.0 | 0.043- | 0.047 | 0.006- | 0.087- | 0.177 | 0.024- | 0.045 | 0.043- | 0.007- | 0.097 | 4.0 |
| 4.5 | 0.022- | 0.037 | 0.030- | 0.018- | 0.099 | 0.007- | 0.024 | 0.037- | 0.024 | 0.030 | 4.5 |
| 5.0 | 0.007- | 0.021 | 0.030- | 0.015 | 0.036 | 0.001 | 0.009 | 0.023- | 0.029 | 0.007- | 5.0 |
| 10.0 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 10.0 |

**Table B.2.1. A and B Coefficients for Elastic Piles, $E_s = k_0 + k_1 x$ where $k_0/k_1 T = 0.1$, $Z_{max} = 10$**

| Z | $A_Y$ | $A_S$ | $A_M$ | $A_V$ | $A_P$ | $B_Y$ | $B_S$ | $B_M$ | $B_V$ | $B_P$ | Z |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | 1.930 | 1.348- | 0.000 | 1.000 | 0.386- | 1.348 | 1.589- | 1.000 | 0.000 | 0.270- | 0.0 |
| 0.1 | 1.796 | 1.343- | 0.098 | 0.954 | 0.539- | 1.194 | 1.489- | 0.999 | 0.031- | 0.358- | 0.1 |
| 0.2 | 1.662 | 1.329- | 0.191 | 0.894 | 0.665- | 1.050 | 1.390- | 0.994 | 0.070- | 0.420- | 0.2 |
| 0.3 | 1.530 | 1.305- | 0.277 | 0.822 | 0.765- | 0.916 | 1.291- | 0.985 | 0.114- | 0.458- | 0.3 |
| 0.4 | 1.401 | 1.274- | 0.355 | 0.742 | 0.840- | 0.792 | 1.193- | 0.971 | 0.161- | 0.475- | 0.4 |
| 0.5 | 1.275 | 1.235- | 0.425 | 0.655 | 0.893- | 0.678 | 1.097- | 0.952 | 0.208- | 0.474- | 0.5 |
| 0.6 | 1.154 | 1.189- | 0.486 | 0.565 | 0.923- | 0.573 | 1.003- | 0.929 | 0.255- | 0.458- | 0.6 |
| 0.7 | 1.037 | 1.138- | 0.538 | 0.472 | 0.933- | 0.477 | 0.911- | 0.902 | 0.299- | 0.430- | 0.7 |
| 0.8 | 0.926 | 1.082- | 0.581 | 0.379 | 0.926- | 0.391 | 0.823- | 0.869 | 0.340- | 0.391- | 0.8 |
| 0.9 | 0.821 | 1.022- | 0.614 | 0.287 | 0.903- | 0.313 | 0.737- | 0.833 | 0.377- | 0.344- | 0.9 |
| 1.0 | 0.722 | 0.960- | 0.638 | 0.199 | 0.866- | 0.243 | 0.656- | 0.794 | 0.409- | 0.292- | 1.0 |
| 1.2 | 0.543 | 0.829- | 0.661 | 0.036 | 0.760- | 0.127 | 0.506- | 0.707 | 0.456- | 0.178- | 1.2 |
| 1.4 | 0.390 | 0.698- | 0.654 | 0.103- | 0.624- | 0.040 | 0.374- | 0.613 | 0.480- | 0.064- | 1.4 |
| 1.6 | 0.263 | 0.570- | 0.621 | 0.213- | 0.474- | 0.023- | 0.261- | 0.516 | 0.482- | 0.042 | 1.6 |
| 1.8 | 0.162 | 0.450- | 0.570 | 0.292- | 0.323- | 0.065- | 0.167- | 0.421 | 0.465- | 0.131 | 1.8 |
| 2.0 | 0.083 | 0.343- | 0.506 | 0.343- | 0.182- | 0.091- | 0.092- | 0.331 | 0.431- | 0.200 | 2.0 |
| 2.5 | 0.033- | 0.135- | 0.324 | 0.360- | 0.088 | 0.104- | 0.024 | 0.145 | 0.306- | 0.280 | 2.5 |
| 3.0 | 0.066- | 0.015- | 0.161 | 0.279- | 0.213 | 0.079- | 0.064 | 0.026 | 0.170- | 0.253 | 3.0 |
| 3.5 | 0.059- | 0.036 | 0.049 | 0.168- | 0.217 | 0.046- | 0.061 | 0.030- | 0.063- | 0.171 | 3.5 |
| 4.0 | 0.038- | 0.044 | 0.009- | 0.073- | 0.158 | 0.020- | 0.042 | 0.044- | 0.001 | 0.085 | 4.0 |
| 4.5 | 0.018- | 0.033 | 0.029- | 0.012- | 0.085 | 0.005- | 0.021 | 0.035- | 0.026 | 0.022 | 4.5 |
| 5.0 | 0.005- | 0.018 | 0.027- | 0.015 | 0.029 | 0.002 | 0.007 | 0.021- | 0.028 | 0.011- | 5.0 |
| 10.0 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 10.0 |

Table B.2.2. A and B Coefficients for Elastic Piles, $E_s = k_0 + k_1 x$ where $k_0/k_1 T = 0.2$, $Z_{max} = 10$

| Z | $A_Y$ | $A_S$ | $A_M$ | $A_V$ | $A_P$ | $B_Y$ | $B_S$ | $B_M$ | $B_V$ | $B_P$ | Z |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | 1.490 | 1.104- | 0.000 | 1.000 | 0.745- | 1.104 | 1.439- | 1.000 | 0.000 | 0.552- | 0.0 |
| 0.1 | 1.380 | 1.099- | 0.096 | 0.921 | 0.828- | 0.965 | 1.339- | 0.997 | 0.057- | 0.579- | 0.1 |
| 0.2 | 1.270 | 1.085- | 0.184 | 0.836 | 0.889- | 0.836 | 1.240- | 0.989 | 0.115- | 0.585- | 0.2 |
| 0.3 | 1.163 | 1.063- | 0.263 | 0.745 | 0.930- | 0.717 | 1.142- | 0.974 | 0.173- | 0.574- | 0.3 |
| 0.4 | 1.058 | 1.033- | 0.333 | 0.651 | 0.952- | 0.608 | 1.046- | 0.954 | 0.229- | 0.547- | 0.4 |
| 0.5 | 0.956 | 0.997- | 0.394 | 0.555 | 0.956- | 0.508 | 0.951- | 0.929 | 0.281- | 0.508- | 0.5 |
| 0.6 | 0.858 | 0.955- | 0.444 | 0.460 | 0.944- | 0.417 | 0.860- | 0.898 | 0.330- | 0.459- | 0.6 |
| 0.7 | 0.765 | 0.908- | 0.486 | 0.367 | 0.918- | 0.336 | 0.772- | 0.863 | 0.373- | 0.403- | 0.7 |
| 0.8 | 0.677 | 0.858- | 0.518 | 0.277 | 0.880- | 0.263 | 0.688- | 0.823 | 0.410- | 0.342- | 0.8 |
| 0.9 | 0.593 | 0.805- | 0.541 | 0.192 | 0.831- | 0.198 | 0.608- | 0.781 | 0.441- | 0.278- | 0.9 |
| 1.0 | 0.516 | 0.750- | 0.556 | 0.112 | 0.773- | 0.141 | 0.532- | 0.735 | 0.466- | 0.212- | 1.0 |
| 1.2 | 0.377 | 0.638- | 0.564 | 0.030- | 0.640- | 0.049 | 0.394- | 0.638 | 0.495- | 0.084- | 1.2 |
| 1.4 | 0.260 | 0.527- | 0.545 | 0.144- | 0.495- | 0.017- | 0.277- | 0.538 | 0.500- | 0.033 | 1.4 |
| 1.6 | 0.166 | 0.421- | 0.508 | 0.228- | 0.348- | 0.062- | 0.179- | 0.439 | 0.483- | 0.131 | 1.6 |
| 1.8 | 0.091 | 0.325- | 0.456 | 0.284- | 0.210- | 0.090- | 0.100- | 0.346 | 0.449- | 0.207 | 1.8 |
| 2.0 | 0.035 | 0.240- | 0.395 | 0.313- | 0.088- | 0.103- | 0.040- | 0.260 | 0.403- | 0.259 | 2.0 |
| 2.5 | 0.041- | 0.082- | 0.238 | 0.298- | 0.124 | 0.098- | 0.046 | 0.094 | 0.259- | 0.295 | 2.5 |
| 3.0 | 0.058- | 0.003 | 0.108 | 0.212- | 0.203 | 0.068- | 0.066 | 0.000 | 0.124- | 0.238 | 3.0 |
| 3.5 | 0.047- | 0.035 | 0.027 | 0.126- | 0.187 | 0.037- | 0.055 | 0.037- | 0.040- | 0.147 | 3.5 |
| 4.0 | 0.028- | 0.036 | 0.015- | 0.048- | 0.125 | 0.014- | 0.035 | 0.042- | 0.012 | 0.064 | 4.0 |
| 4.5 | 0.012- | 0.025 | 0.026- | 0.002- | 0.061 | 0.002- | 0.016 | 0.031- | 0.029 | 0.009 | 4.5 |
| 5.0 | 0.003- | 0.013 | 0.022- | 0.016 | 0.015 | 0.003 | 0.004 | 0.017- | 0.026 | 0.016- | 5.0 |
| 10.0 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 10.0 |

**Table B.2.3. A and B Coefficients for Elastic Piles, $E_s = k_0 + k_1 x$ where $k_0/k_1 T = 0.5$, $Z_{max} = 10$**

| Z | $A_Y$ | $A_S$ | $A_M$ | $A_V$ | $A_P$ | $B_Y$ | $B_S$ | $B_M$ | $B_V$ | $B_P$ | Z |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | 1.097 | 0.879- | 0.000 | 1.000 | 1.097- | 0.879 | 1.290- | 1.000 | 0.000 | 0.879- | 0.0 |
| 0.1 | 1.010 | 0.874- | 0.095 | 0.890 | 1.111- | 0.755 | 1.190- | 0.996 | 0.086- | 0.830- | 0.1 |
| 0.2 | 0.923 | 0.861- | 0.178 | 0.779 | 1.107- | 0.641 | 1.091- | 0.983 | 0.165- | 0.769- | 0.2 |
| 0.3 | 0.837 | 0.839- | 0.250 | 0.669 | 1.089- | 0.537 | 0.994- | 0.963 | 0.239- | 0.698- | 0.3 |
| 0.4 | 0.755 | 0.811- | 0.312 | 0.562 | 1.057- | 0.442 | 0.899- | 0.935 | 0.305- | 0.619- | 0.4 |
| 0.5 | 0.675 | 0.777- | 0.363 | 0.458 | 1.013- | 0.357 | 0.807- | 0.902 | 0.362- | 0.535- | 0.5 |
| 0.6 | 0.599 | 0.739- | 0.403 | 0.360 | 0.959- | 0.281 | 0.719- | 0.863 | 0.411- | 0.449- | 0.6 |
| 0.7 | 0.527 | 0.697- | 0.435 | 0.267 | 0.897- | 0.213 | 0.635- | 0.819 | 0.452- | 0.362- | 0.7 |
| 0.8 | 0.460 | 0.653- | 0.457 | 0.181 | 0.828- | 0.153 | 0.556- | 0.772 | 0.484- | 0.276- | 0.8 |
| 0.9 | 0.397 | 0.606- | 0.471 | 0.102 | 0.754- | 0.102 | 0.481- | 0.723 | 0.507- | 0.193- | 0.9 |
| 1.0 | 0.339 | 0.559- | 0.477 | 0.030 | 0.677- | 0.057 | 0.411- | 0.671 | 0.523- | 0.115- | 1.0 |
| 1.2 | 0.236 | 0.464- | 0.470 | 0.090- | 0.520- | 0.012- | 0.288- | 0.565 | 0.531- | 0.026 | 1.2 |
| 1.4 | 0.153 | 0.372- | 0.443 | 0.178- | 0.367- | 0.059- | 0.185- | 0.460 | 0.514- | 0.141 | 1.4 |
| 1.6 | 0.087 | 0.288- | 0.400 | 0.238- | 0.227- | 0.087- | 0.103- | 0.360 | 0.477- | 0.226 | 1.6 |
| 1.8 | 0.037 | 0.213- | 0.349 | 0.270- | 0.105- | 0.101- | 0.040- | 0.269 | 0.426- | 0.283 | 1.8 |
| 2.0 | 0.001 | 0.149- | 0.293 | 0.281- | 0.004- | 0.104- | 0.005 | 0.190 | 0.366- | 0.312 | 2.0 |
| 2.5 | 0.042- | 0.036- | 0.160 | 0.240- | 0.146 | 0.084- | 0.061 | 0.047 | 0.210- | 0.295 | 2.5 |
| 3.0 | 0.044- | 0.017 | 0.060 | 0.156- | 0.177 | 0.051- | 0.064 | 0.025- | 0.083- | 0.205 | 3.0 |
| 3.5 | 0.031- | 0.031 | 0.003 | 0.075- | 0.139 | 0.023- | 0.045 | 0.045- | 0.006- | 0.105 | 3.5 |
| 4.0 | 0.016- | 0.026 | 0.019- | 0.020- | 0.081 | 0.006- | 0.024 | 0.038- | 0.026 | 0.031 | 4.0 |
| 4.5 | 0.006- | 0.016 | 0.021- | 0.008 | 0.031 | 0.002 | 0.009 | 0.023- | 0.030 | 0.009- | 4.5 |
| 5.0 | 0.000 | 0.007 | 0.014- | 0.015 | 0.002 | 0.004 | 0.000 | 0.010- | 0.022 | 0.022- | 5.0 |
| 10.0 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 10.0 |

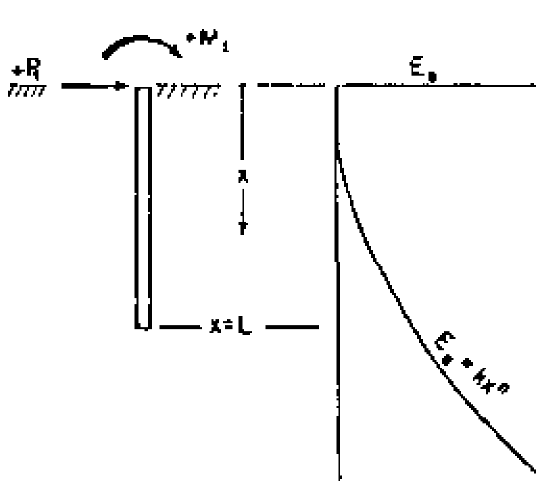
Table B.2.4. A and B Coefficients for Elastic Piles, $E_s = k_0 + k_1 x$ where $k_0/k_1 T = 1.0$, $Z_{max} = 10$

| Z | $A_Y$ | $A_S$ | $A_M$ | $A_V$ | $A_P$ | $B_Y$ | $B_S$ | $B_M$ | $B_V$ | $B_P$ | Z |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | 0.742 | 0.663- | 0.000 | 1.000 | 1.483- | 0.663 | 1.130- | 1.000 | 0.000 | 1.327- | 0.0 |
| 0.1 | 0.675 | 0.659- | 0.093 | 0.855 | 1.418- | 0.555 | 1.030- | 0.993 | 0.125- | 1.166- | 0.1 |
| 0.2 | 0.610 | 0.646- | 0.171 | 0.717 | 1.342- | 0.457 | 0.932- | 0.975 | 0.233- | 1.006- | 0.2 |
| 0.3 | 0.546 | 0.625- | 0.236 | 0.587 | 1.256- | 0.369 | 0.836- | 0.947 | 0.326- | 0.849- | 0.3 |
| 0.4 | 0.485 | 0.599- | 0.288 | 0.466 | 1.163- | 0.290 | 0.743- | 0.910 | 0.403- | 0.697- | 0.4 |
| 0.5 | 0.426 | 0.568- | 0.329 | 0.355 | 1.066- | 0.220 | 0.654- | 0.866 | 0.466- | 0.551- | 0.5 |
| 0.6 | 0.371 | 0.534- | 0.359 | 0.253 | 0.965- | 0.159 | 0.570- | 0.817 | 0.514- | 0.414- | 0.6 |
| 0.7 | 0.320 | 0.497- | 0.380 | 0.162 | 0.863- | 0.107 | 0.491- | 0.763 | 0.549- | 0.288- | 0.7 |
| 0.8 | 0.272 | 0.458- | 0.392 | 0.080 | 0.761- | 0.061 | 0.417- | 0.707 | 0.572- | 0.171- | 0.8 |
| 0.9 | 0.228 | 0.419- | 0.396 | 0.009 | 0.661- | 0.023 | 0.350- | 0.649 | 0.584- | 0.067- | 0.9 |
| 1.0 | 0.188 | 0.379- | 0.394 | 0.052- | 0.564- | 0.009- | 0.288- | 0.590 | 0.586- | 0.026 | 1.0 |
| 1.2 | 0.120 | 0.302- | 0.373 | 0.146- | 0.384- | 0.055- | 0.181- | 0.474 | 0.565- | 0.176 | 1.2 |
| 1.4 | 0.067 | 0.231- | 0.337 | 0.207- | 0.227- | 0.082- | 0.097- | 0.365 | 0.519- | 0.280 | 1.4 |
| 1.6 | 0.027 | 0.169- | 0.291 | 0.239- | 0.097- | 0.095- | 0.034- | 0.267 | 0.456- | 0.342 | 1.6 |
| 1.8 | 0.001- | 0.115- | 0.242 | 0.248- | 0.004 | 0.097- | 0.011 | 0.183 | 0.385- | 0.369 | 1.8 |
| 2.0 | 0.020- | 0.072- | 0.193 | 0.240- | 0.078 | 0.092- | 0.040 | 0.114 | 0.311- | 0.366 | 2.0 |
| 2.5 | 0.036- | 0.003- | 0.087 | 0.175- | 0.161 | 0.063- | 0.065 | 0.001 | 0.146- | 0.282 | 2.5 |
| 3.0 | 0.029- | 0.022 | 0.020 | 0.095- | 0.147 | 0.032- | 0.053 | 0.041- | 0.035- | 0.160 | 3.0 |
| 3.5 | 0.017- | 0.024 | 0.011- | 0.034- | 0.095 | 0.011- | 0.031 | 0.043- | 0.018 | 0.060 | 3.5 |
| 4.0 | 0.007- | 0.016 | 0.018- | 0.000 | 0.043 | 0.000 | 0.013 | 0.029- | 0.032 | 0.001 | 4.0 |
| 4.5 | 0.001- | 0.008 | 0.014- | 0.012 | 0.009 | 0.003 | 0.002 | 0.014- | 0.026 | 0.021- | 4.5 |
| 5.0 | 0.001 | 0.002 | 0.008- | 0.012 | 0.007- | 0.003 | 0.002- | 0.004- | 0.014 | 0.022- | 5.0 |
| 10.0 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 10.0 |

**Table B.2.5. A and B Coefficients for Elastic Piles, $E_s = k_0 + k_1 x$ where $k_0/k_1T = 2.0$, $Z_{max} = 10$**

| Z | $A_Y$ | $A_S$ | $A_M$ | $A_V$ | $A_P$ | $B_Y$ | $B_S$ | $B_M$ | $B_V$ | $B_P$ | Z |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | 0.404 | 0.435- | 0.000 | 1.000 | 2.019- | 0.435 | 0.925- | 1.000 | 0.000 | 2.177- | 0.0 |
| 0.1 | 0.360 | 0.431- | 0.090 | 0.807 | 1.837- | 0.348 | 0.825- | 0.989 | 0.198- | 1.775- | 0.1 |
| 0.2 | 0.318 | 0.418- | 0.161 | 0.633 | 1.651- | 0.270 | 0.728- | 0.960 | 0.357- | 1.406- | 0.2 |
| 0.3 | 0.277 | 0.399- | 0.216 | 0.477 | 1.465- | 0.202 | 0.634- | 0.918 | 0.481- | 1.073- | 0.3 |
| 0.4 | 0.238 | 0.376- | 0.257 | 0.340 | 1.283- | 0.144 | 0.545- | 0.864 | 0.573- | 0.775- | 0.4 |
| 0.5 | 0.201 | 0.349- | 0.284 | 0.220 | 1.107- | 0.093 | 0.461- | 0.803 | 0.637- | 0.514- | 0.5 |
| 0.6 | 0.168 | 0.319- | 0.301 | 0.118 | 0.940- | 0.051 | 0.384- | 0.737 | 0.677- | 0.287- | 0.6 |
| 0.7 | 0.137 | 0.289- | 0.308 | 0.031 | 0.783- | 0.017 | 0.314- | 0.668 | 0.697- | 0.094- | 0.7 |
| 0.8 | 0.110 | 0.258- | 0.307 | 0.040- | 0.638- | 0.012- | 0.251- | 0.598 | 0.698- | 0.067 | 0.8 |
| 0.9 | 0.086 | 0.228- | 0.300 | 0.097- | 0.506- | 0.034- | 0.195- | 0.528 | 0.685- | 0.198 | 0.9 |
| 1.0 | 0.064 | 0.199- | 0.288 | 0.142- | 0.387- | 0.050- | 0.145- | 0.461 | 0.660- | 0.303 | 1.0 |
| 1.2 | 0.030 | 0.144- | 0.253 | 0.198- | 0.188- | 0.071- | 0.066- | 0.336 | 0.584- | 0.440 | 1.2 |
| 1.4 | 0.006 | 0.098- | 0.210 | 0.221- | 0.041- | 0.078- | 0.010- | 0.228 | 0.490- | 0.499 | 1.4 |
| 1.6 | 0.009- | 0.060- | 0.166 | 0.218- | 0.061 | 0.076- | 0.027 | 0.140 | 0.389- | 0.500 | 1.6 |
| 1.8 | 0.018- | 0.032- | 0.124 | 0.199- | 0.124 | 0.068- | 0.048 | 0.072 | 0.293- | 0.462 | 1.8 |
| 2.0 | 0.022- | 0.011- | 0.086 | 0.171- | 0.156 | 0.057- | 0.057 | 0.022 | 0.206- | 0.401 | 2.0 |
| 2.5 | 0.020- | 0.015 | 0.021 | 0.091- | 0.149 | 0.029- | 0.050 | 0.038- | 0.051- | 0.218 | 2.5 |
| 3.0 | 0.011- | 0.017 | 0.008- | 0.030- | 0.091 | 0.009- | 0.029 | 0.043- | 0.019 | 0.074 | 3.0 |
| 3.5 | 0.004- | 0.011 | 0.014- | 0.001 | 0.037 | 0.000 | 0.011 | 0.028- | 0.034 | 0.002- | 3.5 |
| 4.0 | 0.001- | 0.005 | 0.010- | 0.010 | 0.005 | 0.003 | 0.001 | 0.012- | 0.026 | 0.026- | 4.0 |
| 4.5 | 0.001 | 0.001 | 0.005- | 0.009 | 0.007- | 0.002 | 0.002- | 0.002- | 0.013 | 0.023- | 4.5 |
| 5.0 | 0.001 | 0.001- | 0.001- | 0.005 | 0.008- | 0.001 | 0.002- | 0.001 | 0.004 | 0.013- | 5.0 |
| 10.0 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 10.0 |

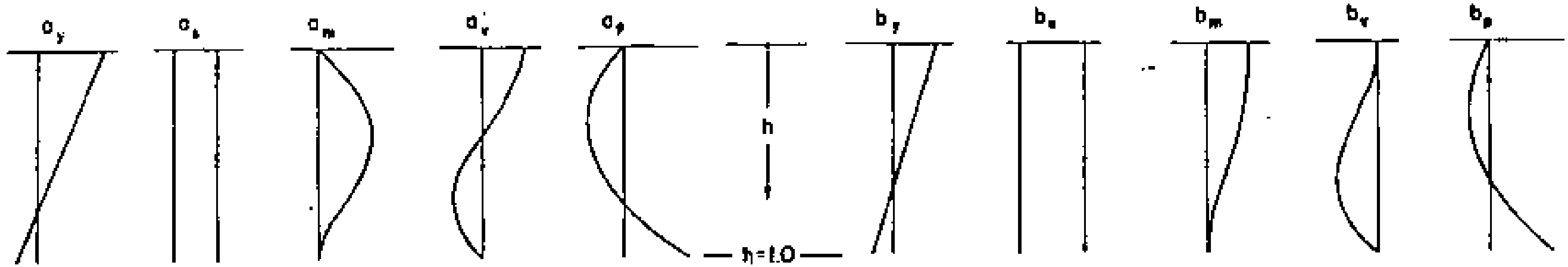
Table B.2.6. A and B Coefficients for Elastic Piles, $E_s = k_0 + k_1 x$ where $k_0/k_1 T = 5.0$, $Z_{max} = 10$




$J = kL^n$

$h = x/L$

| | | | |
|---|---|---|---|
| Deflection | $y$ | $= (P_t/JL)a_y$ | $+ (M_t/JL^2)b_y$ |
| Slope | $S$ | $= (P_t/JL^2)a_s$ | $+ (M_t/JL^3)b_s$ |
| Moment | $M$ | $= (P_t L)a_m$ | $+ (M_t)b_m$ |
| Shear | $V$ | $= (P_t)a_v$ | $+ (M_t/L)b_v$ |
| Soil reaction | $p$ | $= (P_t/L)a_p$ | $+ (M_t/L^2)b_p$ |




Appendix C.1. a and b Coefficients for Rigid Piles, $E_s = kx^n$, n varied from 0.0 to 4.

| h | $a_Y$ | $a_S$ | $a_M$ | $a_V$ | $a_P$ | $b_Y$ | $b_S$ | $b_M$ | $b_V$ | $b_P$ | h |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.00 | 4.000 | 6.000- | 0.000 | 1.000 | 4.000- | 6.000 | 12.000- | 1.000 | 0.000 | 6.000- | 0.00 |
| 0.05 | 3.700 | 6.000- | 0.045 | 0.808 | 3.700- | 5.400 | 12.000- | 0.993 | 0.285- | 5.400- | 0.05 |
| 0.10 | 3.400 | 6.000- | 0.075 | 0.623 | 3.400- | 4.800 | 12.000- | 0.963 | 0.555- | 4.800- | 0.10 |
| 0.15 | 3.100 | 6.000- | 0.108 | 0.468 | 3.100- | 4.200 | 12.000- | 0.939 | 0.765- | 4.200- | 0.15 |
| 0.20 | 2.800 | 6.000- | 0.128 | 0.320 | 2.800- | 3.600 | 12.000- | 0.896 | 0.960- | 3.600- | 0.20 |
| 0.25 | 2.500 | 6.000- | 0.141 | 0.188 | 2.500- | 3.000 | 12.000- | 0.844 | 1.125- | 3.000- | 0.25 |
| 0.30 | 2.200 | 6.000- | 0.147 | 0.070 | 2.200- | 2.400 | 12.000- | 0.784 | 1.260- | 2.400- | 0.30 |
| 0.35 | 1.900 | 6.000- | 0.148 | 0.032- | 1.900- | 1.800 | 12.000- | 0.718 | 1.365- | 1.800- | 0.35 |
| 0.40 | 1.600 | 6.000- | 0.144 | 0.120- | 1.600- | 1.200 | 12.000- | 0.648 | 1.440- | 1.200- | 0.40 |
| 0.45 | 1.300 | 6.000- | 0.136 | 0.192- | 1.300- | 0.600 | 12.000- | 0.575 | 1.485- | 0.600- | 0.45 |
| 0.50 | 1.000 | 6.000- | 0.125 | 0.250- | 1.000- | 0.000 | 12.000- | 0.500 | 1.500- | 0.000- | 0.50 |
| 0.55 | 0.700 | 6.000- | 0.111 | 0.293- | 0.700- | 0.600- | 12.000- | 0.425 | 1.485- | 0.600 | 0.55 |
| 0.60 | 0.400 | 6.000- | 0.096 | 0.320- | 0.400- | 1.200- | 12.000- | 0.352 | 1.440- | 1.200 | 0.60 |
| 0.65 | 0.100 | 6.000- | 0.080 | 0.333- | 0.100- | 1.800- | 12.000- | 0.282 | 1.365- | 1.800 | 0.65 |
| 0.70 | 0.200- | 6.000- | 0.063 | 0.330- | 0.200 | 2.400- | 12.000- | 0.216 | 1.260- | 2.400 | 0.70 |
| 0.75 | 0.500- | 6.000- | 0.047 | 0.313- | 0.500 | 3.000- | 12.000- | 0.156 | 1.125- | 3.000 | 0.75 |
| 0.80 | 0.800- | 6.000- | 0.032 | 0.280- | 0.800 | 3.600- | 12.000- | 0.104 | 0.960- | 3.600 | 0.80 |
| 0.85 | 1.100- | 6.000- | 0.019 | 0.233- | 1.100 | 4.200- | 12.000- | 0.061 | 0.765- | 4.200 | 0.85 |
| 0.90 | 1.400- | 6.000- | 0.009 | 0.170- | 1.400 | 4.800- | 12.000- | 0.028 | 0.540- | 4.800 | 0.90 |
| 0.95 | 1.700- | 6.000- | 0.002 | 0.093- | 1.700 | 5.400- | 12.000- | 0.007 | 0.285- | 5.400 | 0.95 |
| 1.00 | 2.000- | 6.000- | 0.000 | 0.000 | 2.000 | 6.000- | 12.000- | 0.000- | 0.000- | 6.000 | 1.00 |

Table C.1.1. a and b Coefficients for Rigid Piles, $E_s = k$

| h | $a_T$ | $a_S$ | $a_M$ | $a_V$ | $a_P$ | $b_T$ | $b_S$ | $b_M$ | $b_V$ | $b_P$ | h |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.00 | 6.398 | 9.242- | 0.000 | 1.000 | 0.000 | 9.242 | 16.600- | 1.000 | 0.000 | 0.000 | 0.00 |
| 0.05 | 5.936 | 9.242- | 0.048 | 0.895 | 2.807- | 8.412 | 16.600- | 0.997 | 0.150- | 3.978- | 0.05 |
| 0.10 | 5.474 | 9.242- | 0.081 | 0.702 | 3.078- | 7.582 | 16.600- | 0.973 | 0.424- | 4.264- | 0.10 |
| 0.15 | 5.012 | 9.242- | 0.122 | 0.591 | 3.119- | 6.752 | 16.600- | 0.961 | 0.571- | 4.202- | 0.15 |
| 0.20 | 4.550 | 9.242- | 0.148 | 0.436 | 3.043- | 5.922 | 16.600- | 0.927 | 0.776- | 3.960- | 0.20 |
| 0.25 | 4.088 | 9.242- | 0.166 | 0.288 | 2.890- | 5.092 | 16.600- | 0.884 | 0.965- | 3.601- | 0.25 |
| 0.30 | 3.626 | 9.242- | 0.177 | 0.148 | 2.683- | 4.262 | 16.600- | 0.831 | 1.134- | 3.154- | 0.30 |
| 0.35 | 3.164 | 9.242- | 0.181 | 0.020 | 2.433- | 3.432 | 16.600- | 0.771 | 1.279- | 2.640- | 0.35 |
| 0.40 | 2.701 | 9.242- | 0.179 | 0.095- | 2.148- | 2.602 | 16.600- | 0.704 | 1.397- | 2.069- | 0.40 |
| 0.45 | 2.239 | 9.242- | 0.172 | 0.194- | 1.834- | 1.772 | 16.600- | 0.632 | 1.485- | 1.451- | 0.45 |
| 0.50 | 1.777 | 9.242- | 0.160 | 0.278- | 1.494- | 0.942 | 16.600- | 0.556 | 1.542- | 0.792- | 0.50 |
| 0.55 | 1.315 | 9.242- | 0.145 | 0.343- | 1.133- | 0.112 | 16.600- | 0.478 | 1.564- | 0.097- | 0.55 |
| 0.60 | 0.853 | 9.242- | 0.126 | 0.390- | 0.751- | 0.718- | 16.600- | 0.400 | 1.551- | 0.632 | 0.60 |
| 0.65 | 0.391 | 9.242- | 0.106 | 0.418- | 0.351- | 1.548- | 16.600- | 0.324 | 1.500- | 1.390 | 0.65 |
| 0.70 | 0.071- | 9.242- | 0.085 | 0.425- | 0.065 | 2.378- | 16.600- | 0.251 | 1.411- | 2.175 | 0.70 |
| 0.75 | 0.533- | 9.242- | 0.064 | 0.411- | 0.496 | 3.208- | 16.600- | 0.183 | 1.282- | 2.985 | 0.75 |
| 0.80 | 0.995- | 9.242- | 0.044 | 0.375- | 0.941 | 4.038- | 16.600- | 0.123 | 1.112- | 3.819 | 0.80 |
| 0.85 | 1.457- | 9.242- | 0.026 | 0.317- | 1.399 | 4.868- | 16.600- | 0.073 | 0.900- | 4.674 | 0.85 |
| 0.90 | 1.920- | 9.242- | 0.013 | 0.235- | 1.870 | 5.698- | 16.600- | 0.034 | 0.645- | 5.550 | 0.90 |
| 0.95 | 2.382- | 9.242- | 0.003 | 0.130- | 2.351 | 6.528- | 16.600- | 0.009 | 0.345- | 6.444 | 0.95 |
| 1.00 | 2.844- | 9.242- | 0.000- | 0.000- | 2.844 | 7.358- | 16.600- | 0.000- | 0.000- | 7.358 | 1.00 |

Table C.1.2. a and b Coefficients for Rigid Piles, $E_s = kx^{0.25}$

| h | $a_Y$ | $a_S$ | $a_M$ | $a_V$ | $a_P$ | $b_Y$ | $b_S$ | $b_M$ | $b_V$ | $b_P$ | h |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.00 | 9.404 | 13.165- | 0.000 | 1.000 | 0.000 | 13.165 | 21.931- | 1.000 | 0.000 | 0.000 | 0.00 |
| 0.05 | 8.745 | 13.165- | 0.049 | 0.936 | 1.956- | 12.069 | 21.931- | 0.998 | 0.089- | 2.699- | 0.05 |
| 0.10 | 8.087 | 13.165- | 0.087 | 0.782 | 2.557- | 10.972 | 21.931- | 0.982 | 0.302- | 3.470- | 0.10 |
| 0.15 | 7.429 | 13.165- | 0.131 | 0.685 | 2.877- | 9.875 | 21.931- | 0.973 | 0.429- | 3.825- | 0.15 |
| 0.20 | 6.771 | 13.165- | 0.161 | 0.537 | 3.028- | 8.779 | 21.931- | 0.947 | 0.624- | 3.926- | 0.20 |
| 0.25 | 6.112 | 13.165- | 0.184 | 0.384 | 3.056- | 7.682 | 21.931- | 0.911 | 0.819- | 3.841- | 0.25 |
| 0.30 | 5.454 | 13.165- | 0.200 | 0.233 | 2.987- | 6.586 | 21.931- | 0.865 | 1.005- | 3.607- | 0.30 |
| 0.35 | 4.796 | 13.165- | 0.208 | 0.087 | 2.837- | 5.489 | 21.931- | 0.811 | 1.177- | 3.247- | 0.35 |
| 0.40 | 4.138 | 13.165- | 0.208 | 0.050- | 2.617- | 4.393 | 21.931- | 0.748 | 1.328- | 2.778- | 0.40 |
| 0.45 | 3.479 | 13.165- | 0.203 | 0.174- | 2.334- | 3.296 | 21.931- | 0.678 | 1.454- | 2.211- | 0.45 |
| 0.50 | 2.821 | 13.165- | 0.191 | 0.283- | 1.995- | 2.200 | 21.931- | 0.603 | 1.548- | 1.555- | 0.50 |
| 0.55 | 2.163 | 13.165- | 0.175 | 0.373- | 1.604- | 1.103 | 21.931- | 0.524 | 1.608- | 0.818- | 0.55 |
| 0.60 | 1.505 | 13.165- | 0.154 | 0.442- | 1.165- | 0.006 | 21.931- | 0.443 | 1.629- | 0.005- | 0.60 |
| 0.65 | 0.846 | 13.165- | 0.131 | 0.488- | 0.682- | 1.090- | 21.931- | 0.362 | 1.607- | 0.879 | 0.65 |
| 0.70 | 0.188 | 13.165- | 0.106 | 0.510- | 0.157- | 2.187- | 21.931- | 0.283 | 1.540- | 1.830 | 0.70 |
| 0.75 | 0.470- | 13.165- | 0.080 | 0.504- | 0.407 | 3.283- | 21.931- | 0.209 | 1.423- | 2.843 | 0.75 |
| 0.80 | 1.128- | 13.165- | 0.056 | 0.468- | 1.009 | 4.380- | 21.931- | 0.142 | 1.254- | 3.917 | 0.80 |
| 0.85 | 1.787- | 13.165- | 0.034 | 0.402- | 1.647 | 5.476- | 21.931- | 0.084 | 1.030- | 5.049 | 0.85 |
| 0.90 | 2.445- | 13.165- | 0.016 | 0.303- | 2.320 | 6.573- | 21.931- | 0.040 | 0.748- | 6.236 | 0.90 |
| 0.95 | 3.103- | 13.165- | 0.004 | 0.170- | 3.025 | 7.670- | 21.931- | 0.010 | 0.406- | 7.475 | 0.95 |
| 1.00 | 3.762- | 13.165- | 0.000 | 0.000 | 3.762 | 8.766- | 21.931- | 0.000 | 0.000- | 8.766 | 1.00 |

Table C.1.3. a and b Coefficients for Rigid Piles, $E_s = kx^{0.5}$ .

| h | $a_Y$ | $a_S$ | $a_M$ | $a_V$ | $a_P$ | $b_Y$ | $b_S$ | $b_M$ | $b_V$ | $b_P$ | h |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.00 | 18.000 | 24.000- | 0.000 | 1.000 | 0.000 | 24.000 | 36.000- | 1.000 | 0.000 | 0.000 | 0.00 |
| 0.05 | 16.800 | 24.000- | 0.050 | 0.979 | 0.840- | 22.200 | 36.000- | 1.000 | 0.029- | 1.110- | 0.05 |
| 0.10 | 15.600 | 24.000- | 0.094 | 0.895 | 1.560- | 20.400 | 36.000- | 0.992 | 0.139- | 2.040- | 0.10 |
| 0.15 | 14.400 | 24.000- | 0.141 | 0.825 | 2.160- | 18.600 | 36.000- | 0.988 | 0.230- | 2.790- | 0.15 |
| 0.20 | 13.200 | 24.000- | 0.179 | 0.704 | 2.640- | 16.800 | 36.000- | 0.973 | 0.384- | 3.360- | 0.20 |
| 0.25 | 12.000 | 24.000- | 0.211 | 0.562 | 3.000- | 15.000 | 36.000- | 0.949 | 0.562- | 3.750- | 0.25 |
| 0.30 | 10.800 | 24.000- | 0.235 | 0.406 | 3.240- | 13.200 | 36.000- | 0.916 | 0.756- | 3.960- | 0.30 |
| 0.35 | 9.600 | 24.000- | 0.251 | 0.241 | 3.360- | 11.400 | 36.000- | 0.874 | 0.955- | 3.990- | 0.35 |
| 0.40 | 8.400 | 24.000- | 0.259 | 0.072 | 3.360- | 9.600 | 36.000- | 0.821 | 1.152- | 3.840- | 0.40 |
| 0.45 | 7.200 | 24.000- | 0.259 | 0.093- | 3.240- | 7.800 | 36.000- | 0.759 | 1.336- | 3.510- | 0.45 |
| 0.50 | 6.000 | 24.000- | 0.250 | 0.250- | 3.000- | 6.000 | 36.000- | 0.687 | 1.500- | 3.000- | 0.50 |
| 0.55 | 4.800 | 24.000- | 0.234 | 0.391- | 2.640- | 4.200 | 36.000- | 0.609 | 1.633- | 2.310- | 0.55 |
| 0.60 | 3.600 | 24.000- | 0.211 | 0.512- | 2.160- | 2.400 | 36.000- | 0.525 | 1.728- | 1.440- | 0.60 |
| 0.65 | 2.400 | 24.000- | 0.183 | 0.605- | 1.560- | 0.600 | 36.000- | 0.437 | 1.774- | 0.390- | 0.65 |
| 0.70 | 1.200 | 24.000- | 0.151 | 0.666- | 0.840- | 1.200- | 36.000- | 0.348 | 1.764- | 0.840 | 0.70 |
| 0.75 | 0.000 | 24.000- | 0.117 | 0.688- | 0.000 | 3.000- | 36.000- | 0.262 | 1.687- | 2.250 | 0.75 |
| 0.80 | 1.200- | 24.000- | 0.083 | 0.664- | 0.960 | 4.800- | 36.000- | 0.181 | 1.536- | 3.840 | 0.80 |
| 0.85 | 2.400- | 24.000- | 0.052 | 0.590- | 2.040 | 6.600- | 36.000- | 0.110 | 1.301- | 5.610 | 0.85 |
| 0.90 | 3.600- | 24.000- | 0.025 | 0.458- | 3.240 | 8.400- | 36.000- | 0.052 | 0.972- | 7.560 | 0.90 |
| 0.95 | 4.800- | 24.000- | 0.007 | 0.264- | 4.560 | 10.200- | 36.000- | 0.014 | 0.542- | 9.690 | 0.95 |
| 1.00 | 6.000- | 24.000- | 0.000 | 0.000 | 6.000 | 12.000- | 36.000- | 0.000 | 0.000 | 12.000 | 1.00 |

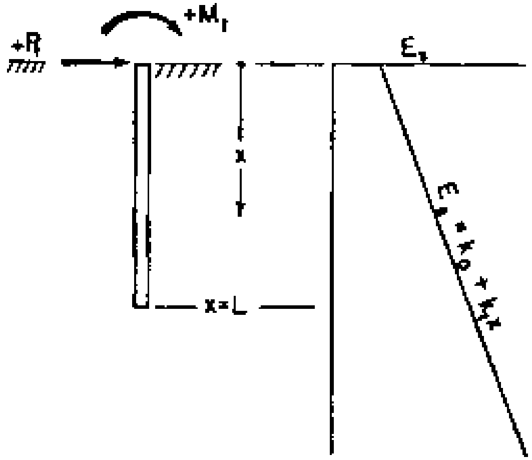
Table C.1.4. a and b Coefficients for Rigid Piles, $E_s = kx$

| h | $a_Y$ | $a_S$ | $a_M$ | $a_V$ | $a_P$ | $b_Y$ | $b_S$ | $b_M$ | $b_V$ | $b_P$ | h |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.00 | 48.000 | 60.000- | 0.000 | 1.000 | 0.000 | 60.000 | 80.000- | 1.000 | 0.000 | 0.000 | 0.00 |
| 0.05 | 45.000 | 60.000- | 0.050 | 0.998 | 0.112- | 56.000 | 80.000- | 1.000 | 0.002- | 0.140- | 0.05 |
| 0.10 | 42.000 | 60.000- | 0.099 | 0.981 | 0.420- | 52.000 | 80.000- | 0.999 | 0.024- | 0.520- | 0.10 |
| 0.15 | 39.000 | 60.000- | 0.148 | 0.954 | 0.877- | 48.000 | 80.000- | 0.998 | 0.057- | 1.080- | 0.15 |
| 0.20 | 36.000 | 60.000- | 0.195 | 0.896 | 1.440- | 44.000 | 80.000- | 0.993 | 0.128- | 1.760- | 0.20 |
| 0.25 | 33.000 | 60.000- | 0.237 | 0.809 | 2.062- | 40.000 | 80.000- | 0.984 | 0.234- | 2.500- | 0.25 |
| 0.30 | 30.000 | 60.000- | 0.275 | 0.690 | 2.700- | 36.000 | 80.000- | 0.969 | 0.378- | 3.240- | 0.30 |
| 0.35 | 27.000 | 60.000- | 0.306 | 0.539 | 3.307- | 32.000 | 80.000- | 0.946 | 0.557- | 3.920- | 0.35 |
| 0.40 | 24.000 | 60.000- | 0.328 | 0.360 | 3.840- | 28.000 | 80.000- | 0.913 | 0.768- | 4.480- | 0.40 |
| 0.45 | 21.000 | 60.000- | 0.341 | 0.157 | 4.252- | 24.000 | 80.000- | 0.869 | 1.002- | 4.860- | 0.45 |
| 0.50 | 18.000 | 60.000- | 0.344 | 0.062- | 4.500- | 20.000 | 80.000- | 0.812 | 1.250- | 5.000- | 0.50 |
| 0.55 | 15.000 | 60.000- | 0.335 | 0.289- | 4.537- | 16.000 | 80.000- | 0.744 | 1.497- | 4.840- | 0.55 |
| 0.60 | 12.000 | 60.000- | 0.315 | 0.512- | 4.320- | 12.000 | 80.000- | 0.663 | 1.728- | 4.320- | 0.60 |
| 0.65 | 09.000 | 60.000- | 0.284 | 0.716- | 3.802- | 08.000 | 80.000- | 0.572 | 1.922- | 3.380- | 0.65 |
| 0.70 | 06.000 | 60.000- | 0.244 | 0.886- | 2.940- | 04.000 | 80.000- | 0.472 | 2.058- | 1.960- | 0.70 |
| 0.75 | 03.000 | 60.000- | 0.196 | 1.004- | 1.688- | 00.000 | 80.000- | 0.367 | 2.109- | 0.000- | 0.75 |
| 0.80 | 00.000 | 60.000- | 0.145 | 1.048- | 0.000- | 04.000- | 80.000- | 0.263 | 2.048- | 2.560 | 0.80 |
| 0.85 | 03.000- | 60.000- | 0.093 | 0.996- | 2.167 | 08.000- | 80.000- | 0.165 | 1.842- | 5.780 | 0.85 |
| 0.90 | 06.000- | 60.000- | 0.047 | 0.822- | 4.860 | 12.000- | 80.000- | 0.081 | 1.458- | 9.720 | 0.90 |
| 0.95 | 09.000- | 60.000- | 0.013 | 0.500- | 8.122 | 16.000- | 80.000- | 0.023 | 0.857- | 14.440 | 0.95 |
| 1.00 | 12.000- | 60.000- | 0.000 | 0.000 | 12.000 | 20.000- | 80.000- | 0.000 | 0.000 | 20.000 | 1.00 |

Table C.1.5. a and b Coefficients for Rigid Piles, $E_s = kx^2$

| h | $a_Y$ | $a_S$ | $a_M$ | $a_V$ | $a_P$ | $b_Y$ | $b_S$ | $b_M$ | $b_V$ | $b_P$ | h |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.00 | 179.998 | 209.997- | 0.000 | 1.000 | 0.000 | 209.997 | 251.996- | 1.000 | 0.000 | 0.000 | 0.00 |
| 0.05 | 169.498 | 209.997- | 0.050 | 1.000 | 0.001- | 197.397 | 251.996- | 1.000 | 0.000- | 0.001- | 0.05 |
| 0.10 | 158.998 | 209.997- | 0.100 | 1.000 | 0.016- | 184.797 | 251.996- | 1.000 | 0.000- | 0.018- | 0.10 |
| 0.15 | 148.498 | 209.997- | 0.150 | 0.998 | 0.075- | 172.198 | 251.996- | 1.000 | 0.003- | 0.087- | 0.15 |
| 0.20 | 137.998 | 209.997- | 0.200 | 0.991 | 0.221- | 159.598 | 251.996- | 1.000 | 0.011- | 0.255- | 0.20 |
| 0.25 | 127.498 | 209.997- | 0.249 | 0.973 | 0.498- | 146.998 | 251.996- | 0.999 | 0.031- | 0.574- | 0.25 |
| 0.30 | 116.998 | 209.997- | 0.297 | 0.938 | 0.948- | 134.398 | 251.996- | 0.996 | 0.071- | 1.089- | 0.30 |
| 0.35 | 106.499 | 209.997- | 0.342 | 0.875 | 1.598- | 121.798 | 251.996- | 0.991 | 0.143- | 1.828- | 0.35 |
| 0.40 | 95.999 | 209.997- | 0.384 | 0.775 | 2.458- | 109.198 | 251.996- | 0.981 | 0.258- | 2.795- | 0.40 |
| 0.45 | 85.499 | 209.997- | 0.419 | 0.626 | 3.506- | 96.599 | 251.996- | 0.964 | 0.426- | 3.961- | 0.45 |
| 0.50 | 74.999 | 209.997- | 0.445 | 0.422 | 4.687- | 83.999 | 251.996- | 0.938 | 0.656- | 5.250- | 0.50 |
| 0.55 | 64.499 | 209.997- | 0.460 | 0.157 | 5.902- | 71.399 | 251.996- | 0.898 | 0.951- | 6.533- | 0.55 |
| 0.60 | 53.999 | 209.997- | 0.460 | 0.166- | 6.998- | 58.799 | 251.996- | 0.841 | 1.306- | 7.620- | 0.60 |
| 0.65 | 43.500 | 209.997- | 0.443 | 0.537- | 7.765- | 46.199 | 251.996- | 0.766 | 1.706- | 8.247- | 0.65 |
| 0.70 | 33.000 | 209.997- | 0.406 | 0.933- | 7.923- | 33.600 | 251.996- | 0.671 | 2.118- | 8.067- | 0.70 |
| 0.75 | 22.500 | 209.997- | 0.350 | 1.314- | 7.119- | 21.000 | 251.996- | 0.555 | 2.492- | 6.644- | 0.75 |
| 0.80 | 12.000 | 209.997- | 0.276 | 1.621- | 4.915- | 08.400 | 251.996- | 0.423 | 2.752- | 3.441- | 0.80 |
| 0.85 | 01.500 | 209.997- | 0.190 | 1.773- | 0.783- | 04.200- | 251.996- | 0.283 | 2.795- | 2.192 | 0.85 |
| 0.90 | 09.000- | 209.997- | 0.103 | 1.657- | 5.905 | 16.800- | 251.996- | 0.150 | 2.480- | 11.022 | 0.90 |
| 0.95 | 19.500- | 209.997- | 0.031 | 1.128- | 15.882 | 29.399- | 251.996- | 0.044 | 1.625- | 23.946 | 0.95 |
| 1.00 | 29.999- | 209.997- | 0.000 | 0.000- | 29.999 | 41.999- | 251.996- | 0.000 | 0.000 | 41.999 | 1.00 |

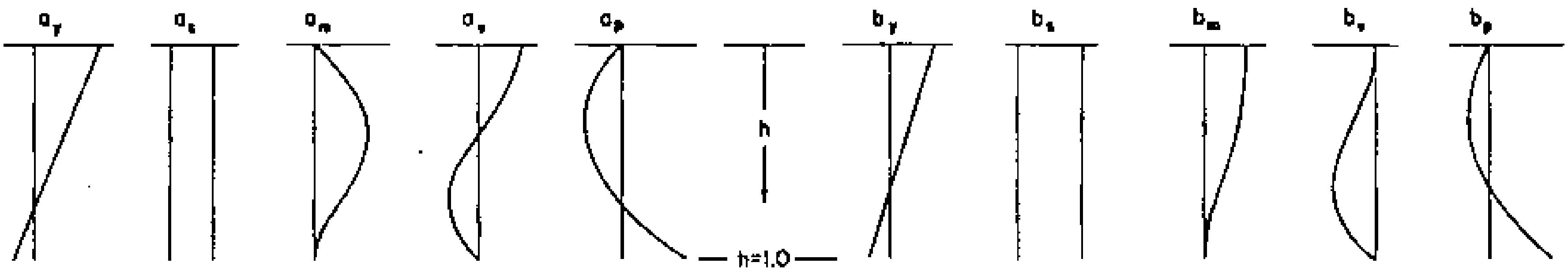
Table C.1.6. a and b Coefficients for Rigid Piles, $E_s = kx^4$




$J = k_1 L$

$h = x/L$

| | | |
|---|---|---|
| Deflection | $y$ | $= (P_t/JL)a_y + (M_t/JL^2)b_y$ |
| Slope | $S$ | $= (P_t/JL^2)a_s + (M_t/JL^3)b_s$ |
| Moment | $M$ | $= (P_t L)a_m + (M_t)b_m$ |
| Shear | $V$ | $= (P_t)a_v + (M_t/L)b_v$ . |
| Soil reaction | $p$ | $= (P_t/L)a_p + (M_t/L^2)b_p$ |




Appendix C.2. a and b Coefficients for Rigid Piles, $E_s = k_o + k_1 x$, $k_o/k_1 L$ varied from 0.1 to 5.

| h | $a_Y$ | $a_S$ | $a_M$ | $a_V$ | $a_P$ | $b_Y$ | $b_S$ | $b_M$ | $b_V$ | $b_P$ | h |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.00 | 12.289 | 16.627- | 0.000 | 1.000 | 1.229- | 16.627 | 26.024- | 1.000 | 0.000 | 1.663- | 0.00 |
| 0.05 | 11.458 | 16.627- | 0.048 | 0.926 | 1.719- | 15.325 | 26.024- | 0.998 | 0.100- | 2.299- | 0.05 |
| 0.10 | 10.627 | 16.627- | 0.089 | 0.812 | 2.125- | 14.024 | 26.024- | 0.985 | 0.253- | 2.805- | 0.10 |
| 0.15 | 09.795 | 16.627- | 0.131 | 0.715 | 2.449- | 12.723 | 26.024- | 0.975 | 0.378- | 3.181- | 0.15 |
| 0.20 | 08.964 | 16.627- | 0.163 | 0.586 | 2.689- | 11.422 | 26.024- | 0.952 | 0.544- | 3.427- | 0.20 |
| 0.25 | 08.133 | 16.627- | 0.189 | 0.447 | 2.846- | 10.120 | 26.024- | 0.920 | 0.718- | 3.542- | 0.25 |
| 0.30 | 07.301 | 16.627- | 0.208 | 0.303 | 2.920- | 08.819 | 26.024- | 0.880 | 0.896- | 3.528- | 0.30 |
| 0.35 | 06.470 | 16.627- | 0.220 | 0.157 | 2.911- | 07.518 | 26.024- | 0.830 | 1.069- | 3.383- | 0.35 |
| 0.40 | 05.639 | 16.627- | 0.224 | 0.013 | 2.819- | 06.217 | 26.024- | 0.773 | 1.232- | 3.108- | 0.40 |
| 0.45 | 04.807 | 16.627- | 0.221 | 0.124- | 2.644- | 04.916 | 26.024- | 0.708 | 1.378- | 2.704- | 0.45 |
| 0.50 | 03.976 | 16.627- | 0.212 | 0.250- | 2.386- | 03.614 | 26.024- | 0.636 | 1.500- | 2.169- | 0.50 |
| 0.55 | 03.145 | 16.627- | 0.196 | 0.361- | 2.044- | 02.313 | 26.024- | 0.558 | 1.592- | 1.504- | 0.55 |
| 0.60 | 02.313 | 16.627- | 0.176 | 0.453- | 1.619- | 01.012 | 26.024- | 0.477 | 1.648- | 0.708- | 0.60 |
| 0.65 | 01.482 | 16.627- | 0.151 | 0.522- | 1.111- | 00.289- | 26.024- | 0.394 | 1.661- | 0.217 | 0.65 |
| 0.70 | 00.651 | 16.627- | 0.124 | 0.563- | 0.520- | 01.590- | 26.024- | 0.312 | 1.624- | 1.272 | 0.70 |
| 0.75 | 00.181- | 16.627- | 0.096 | 0.572- | 0.154 | 02.892- | 26.024- | 0.232 | 1.532- | 2.458 | 0.75 |
| 0.80 | 01.012- | 16.627- | 0.067 | 0.546- | 0.911 | 04.193- | 26.024- | 0.160 | 1.376- | 3.773 | 0.80 |
| 0.85 | 01.843- | 16.627- | 0.042 | 0.480- | 1.751 | 05.494- | 26.024- | 0.096 | 1.152- | 5.219 | 0.85 |
| 0.90 | 02.675- | 16.627- | 0.020 | 0.370- | 2.675 | 06.795- | 26.024- | 0.046 | 0.852- | 6.795 | 0.90 |
| 0.95 | 03.506- | 16.627- | 0.006 | 0.211- | 3.681 | 08.096- | 26.024- | 0.012 | 0.470- | 8.501 | 0.95 |
| 1.00 | 04.337- | 16.627- | 0.000 | 0.000 | 4.771 | 09.398- | 26.024- | 0.000 | 0.000- | 10.337 | 1.00 |

Table C.2.1. a and b Coefficients for Rigid Piles. $E_s = k_0 + k_1 x$ where $k_0/k_1 L = 0.1$

| h | $a_Y$ | $a_S$ | $a_M$ | $a_V$ | $a_P$ | $b_Y$ | $b_S$ | $b_M$ | $b_V$ | $b_P$ | h |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.00 | 9.344 | 12.787- | 0.000 | 1.000 | 1.869- | 12.787 | 20.656- | 1.000 | 0.000 | 2.557- | 0.00 |
| 0.05 | 8.705 | 12.787- | 0.048 | 0.899 | 2.176- | 11.754 | 20.656- | 0.997 | 0.138- | 2.939- | 0.05 |
| 0.10 | 8.066 | 12.787- | 0.086 | 0.768 | 2.420- | 10.721 | 20.656- | 0.980 | 0.314- | 3.216- | 0.10 |
| 0.15 | 7.426 | 12.787- | 0.126 | 0.658 | 2.599- | 9.689 | 20.656- | 0.967 | 0.458- | 3.391- | 0.15 |
| 0.20 | 6.787 | 12.787- | 0.155 | 0.525 | 2.715- | 8.656 | 20.656- | 0.940 | 0.630- | 3.462- | 0.20 |
| 0.25 | 6.148 | 12.787- | 0.178 | 0.387 | 2.766- | 7.623 | 20.656- | 0.904 | 0.802- | 3.430- | 0.25 |
| 0.30 | 5.508 | 12.787- | 0.194 | 0.249 | 2.754- | 6.590 | 20.656- | 0.860 | 0.971- | 3.295- | 0.30 |
| 0.35 | 4.869 | 12.787- | 0.203 | 0.113 | 2.678- | 5.557 | 20.656- | 0.807 | 1.130- | 3.057- | 0.35 |
| 0.40 | 4.230 | 12.787- | 0.205 | 0.018- | 2.538- | 4.525 | 20.656- | 0.747 | 1.275- | 2.715- | 0.40 |
| 0.45 | 3.590 | 12.787- | 0.201 | 0.140- | 2.334- | 3.492 | 20.656- | 0.680 | 1.400- | 2.270- | 0.45 |
| 0.50 | 2.951 | 12.787- | 0.192 | 0.250- | 2.066- | 2.459 | 20.656- | 0.608 | 1.500- | 1.721- | 0.50 |
| 0.55 | 2.311 | 12.787- | 0.177 | 0.345- | 1.734- | 1.426 | 20.656- | 0.531 | 1.570- | 1.070- | 0.55 |
| 0.60 | 1.672 | 12.787- | 0.157 | 0.422- | 1.338- | 0.393 | 20.656- | 0.451 | 1.605- | 0.315- | 0.60 |
| 0.65 | 1.033 | 12.787- | 0.135 | 0.478- | 0.878- | 0.639- | 20.656- | 0.371 | 1.600- | 0.543 | 0.65 |
| 0.70 | 0.393 | 12.787- | 0.110 | 0.509- | 0.354- | 1.672- | 20.656- | 0.292 | 1.549- | 1.505 | 0.70 |
| 0.75 | 0.246- | 12.787- | 0.084 | 0.512- | 0.234 | 2.705- | 20.656- | 0.217 | 1.448- | 2.570 | 0.75 |
| 0.80 | 0.885- | 12.787- | 0.059 | 0.485- | 0.885 | 3.738- | 20.656- | 0.148 | 1.290- | 3.738 | 0.80 |
| 0.85 | 1.525- | 12.787- | 0.036 | 0.423- | 1.601 | 4.770- | 20.656- | 0.089 | 1.072- | 5.009 | 0.85 |
| 0.90 | 2.164- | 12.787- | 0.018 | 0.323- | 2.380 | 5.803- | 20.656- | 0.042 | 0.788- | 6.384 | 0.90 |
| 0.95 | 2.803- | 12.787- | 0.005 | 0.184- | 3.224 | 6.836- | 20.656- | 0.011 | 0.432- | 7.861 | 0.95 |
| 1.00 | 3.443- | 12.787- | 0.000 | 0.000 | 4.131 | 7.869- | 20.656- | 0.000 | 0.000 | 9.443 | 1.00 |

Table C.2.2. a and b Coefficients for Rigid Piles, $E_s = k_0 + k_1 x$ where $k_0/k_1 L = 0.2$

| h | $a_Y$ | $a_S$ | $a_M$ | $a_V$ | $a_P$ | $b_Y$ | $b_S$ | $b_M$ | $b_V$ | $b_P$ | h |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.00 | 5.455 | 7.636- | 0.000 | 1.000 | 2.727- | 7.636 | 13.091- | 1.000 | 0.000 | 3.818- | 0.00 |
| 0.05 | 5.073 | 7.636- | 0.047 | 0.862 | 2.790- | 6.982 | 13.091- | 0.995 | 0.192- | 3.840- | 0.05 |
| 0.10 | 4.691 | 7.636- | 0.082 | 0.710 | 2.815- | 6.327 | 13.091- | 0.974 | 0.401- | 3.796- | 0.10 |
| 0.15 | 4.309 | 7.636- | 0.119 | 0.581 | 2.801- | 5.673 | 13.091- | 0.957 | 0.570- | 3.687- | 0.15 |
| 0.20 | 3.927 | 7.636- | 0.144 | 0.442 | 2.749- | 5.018 | 13.091- | 0.924 | 0.751- | 3.513- | 0.20 |
| 0.25 | 3.545 | 7.636- | 0.163 | 0.307 | 2.659- | 4.364 | 13.091- | 0.882 | 0.920- | 3.273- | 0.25 |
| 0.30 | 3.164 | 7.636- | 0.175 | 0.177 | 2.531- | 3.709 | 13.091- | 0.832 | 1.077- | 2.967- | 0.30 |
| 0.35 | 2.782 | 7.636- | 0.181 | 0.054 | 2.365- | 3.055 | 13.091- | 0.775 | 1.216- | 2.596- | 0.35 |
| 0.40 | 2.400 | 7.636- | 0.181 | 0.059- | 2.160- | 2.400 | 13.091- | 0.711 | 1.335- | 2.160- | 0.40 |
| 0.45 | 2.018 | 7.636- | 0.175 | 0.161- | 1.917- | 1.745 | 13.091- | 0.642 | 1.431- | 1.658- | 0.45 |
| 0.50 | 1.636 | 7.636- | 0.165 | 0.250- | 1.636- | 1.091 | 13.091- | 0.568 | 1.500- | 1.091- | 0.50 |
| 0.55 | 1.255 | 7.636- | 0.150 | 0.324- | 1.317- | 0.436 | 13.091- | 0.492 | 1.539- | 0.458- | 0.55 |
| 0.60 | 0.873 | 7.636- | 0.133 | 0.381- | 0.960- | 0.218- | 13.091- | 0.415 | 1.545- | 0.240 | 0.60 |
| 0.65 | 0.491 | 7.636- | 0.113 | 0.419- | 0.565- | 0.873- | 13.091- | 0.338 | 1.514- | 1.004 | 0.65 |
| 0.70 | 0.109 | 7.636- | 0.091 | 0.437- | 0.131- | 1.527- | 13.091- | 0.264 | 1.443- | 1.833 | 0.70 |
| 0.75 | 0.273- | 7.636- | 0.069 | 0.432- | 0.341 | 2.182- | 13.091- | 0.195 | 1.330- | 2.727 | 0.75 |
| 0.80 | 0.655- | 7.636- | 0.048 | 0.402- | 0.851 | 2.836- | 13.091- | 0.132 | 1.169- | 3.687 | 0.80 |
| 0.85 | 1.036- | 7.636- | 0.029 | 0.346- | 1.399 | 3.491- | 13.091- | 0.078 | 0.960- | 4.713 | 0.85 |
| 0.90 | 1.418- | 7.636- | 0.014 | 0.262- | 1.985 | 4.145- | 13.091- | 0.037 | 0.697- | 5.804 | 0.90 |
| 0.95 | 1.800- | 7.636- | 0.004 | 0.147- | 2.610 | 4.800- | 13.091- | 0.010 | 0.378- | 6.960 | 0.95 |
| 1.00 | 2.182- | 7.636- | 0.000 | 0.000 | 3.273 | 5.455- | 13.091- | 0.000 | 0.000- | 8.182 | 1.00 |

Table C.2.3. a and b Coefficients for Rigid Piles, $E_s = k_0 + k_1 x$ where $k_0/k_1 L = 0.5$

| h | $a_Y$ | $a_S$ | $a_M$ | $a_V$ | $a_P$ | $b_Y$ | $b_S$ | $b_M$ | $b_V$ | $b_P$ | h |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.00 | 3.231 | 4.615- | 0.000 | 1.000 | 3.231- | 4.615 | 8.308- | 1.000 | 0.000 | 4.615- | 0.00 |
| 0.05 | 3.000 | 4.615- | 0.046 | 0.840 | 3.150- | 4.200 | 8.308- | 0.994 | 0.226- | 4.410- | 0.05 |
| 0.10 | 2.769 | 4.615- | 0.079 | 0.675 | 3.046- | 3.785 | 8.308- | 0.970 | 0.457- | 4.163- | 0.10 |
| 0.15 | 2.538 | 4.615- | 0.115 | 0.536 | 2.919- | 3.369 | 8.308- | 0.951 | 0.641- | 3.875- | 0.15 |
| 0.20 | 2.308 | 4.615- | 0.138 | 0.394 | 2.769- | 2.954 | 8.308- | 0.914 | 0.827- | 3.545- | 0.20 |
| 0.25 | 2.077 | 4.615- | 0.154 | 0.260 | 2.596- | 2.538 | 8.308- | 0.868 | 0.995- | 3.173- | 0.25 |
| 0.30 | 1.846 | 4.615- | 0.164 | 0.135 | 2.400- | 2.123 | 8.308- | 0.815 | 1.144- | 2.760- | 0.30 |
| 0.35 | 1.615 | 4.615- | 0.168 | 0.020 | 2.181- | 1.708 | 8.308- | 0.754 | 1.271- | 2.305- | 0.35 |
| 0.40 | 1.385 | 4.615- | 0.166 | 0.083- | 1.938- | 1.292 | 8.308- | 0.688 | 1.374- | 1.809- | 0.40 |
| 0.45 | 1.154 | 4.615- | 0.160 | 0.173- | 1.673- | 0.877 | 8.308- | 0.617 | 1.451- | 1.272- | 0.45 |
| 0.50 | 0.923 | 4.615- | 0.149 | 0.250- | 1.385- | 0.462 | 8.308- | 0.543 | 1.500- | 0.692- | 0.50 |
| 0.55 | 0.692 | 4.615- | 0.135 | 0.312- | 1.073- | 0.046 | 8.308- | 0.468 | 1.519- | 0.072- | 0.55 |
| 0.60 | 0.462 | 4.615- | 0.118 | 0.357- | 0.738- | 0.369- | 8.308- | 0.392 | 1.506- | 0.591 | 0.60 |
| 0.65 | 0.231 | 4.615- | 0.100 | 0.385- | 0.381- | 0.785- | 8.308- | 0.318 | 1.459- | 1.295 | 0.65 |
| 0.70 | 0.000- | 4.615- | 0.080 | 0.395- | 0.000 | 1.200- | 8.308- | 0.247 | 1.376- | 2.040 | 0.70 |
| 0.75 | 0.231- | 4.615- | 0.060 | 0.385- | 0.404 | 1.615- | 8.308- | 0.181 | 1.255- | 2.827 | 0.75 |
| 0.80 | 0.462- | 4.615- | 0.042 | 0.354- | 0.831 | 2.031- | 8.308- | 0.122 | 1.093- | 3.655 | 0.80 |
| 0.85 | 0.692- | 4.615- | 0.025 | 0.301- | 1.281 | 2.446- | 8.308- | 0.072 | 0.889- | 4.525 | 0.85 |
| 0.90 | 0.923- | 4.615- | 0.012 | 0.225- | 1.754 | 2.862- | 8.308- | 0.034 | 0.640- | 5.437 | 0.90 |
| 0.95 | 1.154- | 4.615- | 0.003 | 0.125- | 2.250 | 3.277- | 8.308- | 0.009 | 0.344- | 6.390 | 0.95 |
| 1.00 | 1.385- | 4.615- | 0.000 | 0.000 | 2.769 | 3.692- | 8.308- | 0.000- | 0.000- | 7.385 | 1.00 |

Table C.2.4. a and b Coefficients for Rigid Piles, $E_s = k_0 + k_1 x$ where $k_0/k_1 L = 1.0$

| h | $a_Y$ | $a_S$ | $a_M$ | $a_V$ | $a_P$ | $b_Y$ | $b_S$ | $b_M$ | $b_V$ | $b_P$ | h |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.00 | 1.784 | 2.595- | 0.000 | 1.000 | 3.568- | 2.595 | 4.865- | 1.000 | 0.000 | 5.189- | 0.00 |
| 0.05 | 1.654 | 2.595- | 0.046 | 0.826 | 3.391- | 2.351 | 4.865- | 0.994 | 0.250- | 4.820- | 0.05 |
| 0.10 | 1.524 | 2.595- | 0.078 | 0.652 | 3.201- | 2.108 | 4.865- | 0.967 | 0.497- | 4.427- | 0.10 |
| 0.15 | 1.395 | 2.595- | 0.112 | 0.506 | 2.998- | 1.865 | 4.865- | 0.946 | 0.693- | 4.009- | 0.15 |
| 0.20 | 1.265 | 2.595- | 0.134 | 0.362 | 2.783- | 1.622 | 4.865- | 0.906 | 0.882- | 3.568- | 0.20 |
| 0.25 | 1.135 | 2.595- | 0.148 | 0.228 | 2.554- | 1.378 | 4.865- | 0.858 | 1.049- | 3.101- | 0.25 |
| 0.30 | 1.005 | 2.595- | 0.157 | 0.106 | 2.312- | 1.135 | 4.865- | 0.802 | 1.192- | 2.611- | 0.30 |
| 0.35 | 0.876 | 2.595- | 0.159 | 0.003- | 2.058- | 0.892 | 4.865- | 0.739 | 1.310- | 2.096- | 0.35 |
| 0.40 | 0.746 | 2.595- | 0.156 | 0.099- | 1.790- | 0.649 | 4.865- | 0.671 | 1.401- | 1.557- | 0.40 |
| 0.45 | 0.616 | 2.595- | 0.149 | 0.182- | 1.510- | 0.405 | 4.865- | 0.600 | 1.465- | 0.993- | 0.45 |
| 0.50 | 0.486 | 2.595- | 0.139 | 0.250- | 1.216- | 0.162 | 4.865- | 0.525 | 1.500- | 0.405- | 0.50 |
| 0.55 | 0.357 | 2.595- | 0.125 | 0.303- | 0.910- | 0.081- | 4.865- | 0.450 | 1.505- | 0.207 | 0.55 |
| 0.60 | 0.227 | 2.595- | 0.108 | 0.341- | 0.590- | 0.324- | 4.865- | 0.375 | 1.479- | 0.843 | 0.60 |
| 0.65 | 0.097 | 2.595- | 0.091 | 0.362- | 0.258- | 0.568- | 4.865- | 0.303 | 1.420- | 1.504 | 0.65 |
| 0.70 | 0.032- | 2.595- | 0.073 | 0.366- | 0.088 | 0.811- | 4.865- | 0.234 | 1.328- | 2.189 | 0.70 |
| 0.75 | 0.162- | 2.595- | 0.054 | 0.353- | 0.446 | 1.054- | 4.865- | 0.171 | 1.201- | 2.899 | 0.75 |
| 0.80 | 0.292- | 2.595- | 0.038 | 0.322- | 0.817 | 1.297- | 4.865- | 0.114 | 1.038- | 3.632 | 0.80 |
| 0.85 | 0.422- | 2.595- | 0.023 | 0.271- | 1.202 | 1.541- | 4.865- | 0.067 | 0.837- | 4.391 | 0.85 |
| 0.90 | 0.551- | 2.595- | 0.011 | 0.201- | 1.599 | 1.784- | 4.865- | 0.031 | 0.598- | 5.173 | 0.90 |
| 0.95 | 0.681- | 2.595- | 0.003 | 0.111- | 2.009 | 2.027- | 4.865- | 0.008 | 0.320- | 5.980 | 0.95 |
| 1.00 | 0.811- | 2.595- | 0.000 | 0.000 | 2.432 | 2.270- | 4.865- | 0.000 | 0.000- | 6.811 | 1.00 |

Table C.2.5. a and b Coefficients for Rigid Piles, $E_s = k_0 + k_1 x$ where $k_0/k_1 L = 2.0$

| h | $a_Y$ | $a_S$ | $a_M$ | $a_V$ | $a_P$ | $b_Y$ | $b_S$ | $b_M$ | $b_V$ | $b_P$ | h |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.00 | 0.762 | 1.127- | 0.000 | 1.000 | 3.812- | 1.127 | 2.188- | 1.000 | 0.000 | 5.635- | 0.00 |
| 0.05 | 0.706 | 1.127- | 0.045 | 0.816 | 3.566- | 1.018 | 2.188- | 0.993 | 0.269- | 5.139- | 0.05 |
| 0.10 | 0.650 | 1.127- | 0.076 | 0.635 | 3.314- | 0.908 | 2.188- | 0.965 | 0.529- | 4.632- | 0.10 |
| 0.15 | 0.593 | 1.127- | 0.110 | 0.484 | 3.056- | 0.799 | 2.188- | 0.942 | 0.732- | 4.114- | 0.15 |
| 0.20 | 0.537 | 1.127- | 0.130 | 0.338 | 2.792- | 0.690 | 2.188- | 0.901 | 0.925- | 3.585- | 0.20 |
| 0.25 | 0.481 | 1.127- | 0.144 | 0.205 | 2.523- | 0.580 | 2.188- | 0.850 | 1.091- | 3.046- | 0.25 |
| 0.30 | 0.424 | 1.127- | 0.151 | 0.086 | 2.249- | 0.471 | 2.188- | 0.792 | 1.229- | 2.495- | 0.30 |
| 0.35 | 0.368 | 1.127- | 0.153 | 0.020- | 1.969- | 0.361 | 2.188- | 0.728 | 1.340- | 1.933- | 0.35 |
| 0.40 | 0.312 | 1.127- | 0.149 | 0.111- | 1.683- | 0.252 | 2.188- | 0.659 | 1.422- | 1.360- | 0.40 |
| 0.45 | 0.255 | 1.127- | 0.142 | 0.188- | 1.391- | 0.143 | 2.188- | 0.586 | 1.476- | 0.777- | 0.45 |
| 0.50 | 0.199 | 1.127- | 0.131 | 0.250- | 1.094- | 0.033 | 2.188- | 0.511 | 1.500- | 0.182- | 0.50 |
| 0.55 | 0.143 | 1.127- | 0.117 | 0.297- | 0.791- | 0.076- | 2.188- | 0.436 | 1.494- | 0.423 | 0.55 |
| 0.60 | 0.086 | 1.127- | 0.101 | 0.329- | 0.483- | 0.186- | 2.188- | 0.363 | 1.458- | 1.040 | 0.60 |
| 0.65 | 0.030 | 1.127- | 0.084 | 0.345- | 0.169- | 0.295- | 2.188- | 0.291 | 1.390- | 1.667 | 0.65 |
| 0.70 | 0.027- | 1.127- | 0.067 | 0.346- | 0.151 | 0.404- | 2.188- | 0.224 | 1.291- | 2.305 | 0.70 |
| 0.75 | 0.083- | 1.127- | 0.050 | 0.330- | 0.477 | 0.514- | 2.188- | 0.163 | 1.159- | 2.954 | 0.75 |
| 0.80 | 0.139- | 1.127- | 0.034 | 0.298- | 0.808 | 0.623- | 2.188- | 0.109 | 0.995- | 3.615 | 0.80 |
| 0.85 | 0.196- | 1.127- | 0.021 | 0.249- | 1.144 | 0.733- | 2.188- | 0.064 | 0.798- | 4.286 | 0.85 |
| 0.90 | 0.252- | 1.127- | 0.010 | 0.184- | 1.486 | 0.842- | 2.188- | 0.029 | 0.566- | 4.968 | 0.90 |
| 0.95 | 0.308- | 1.127- | 0.003 | 0.101- | 1.834 | 0.951- | 2.188- | 0.008 | 0.301- | 5.661 | 0.95 |
| 1.00 | 0.365- | 1.127- | 0.000- | 0.000 | 2.188 | 1.061- | 2.188- | 0.000- | 0.000- | 6.365 | 1.00 |

Table C.2.6. a and b Coefficients for Rigid Piles, $E_s = k_0 + k_1 x$ where $k_0/k_1 L = 5.0$

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# COMPORTAMIENTO DE PILOTES SOMETIDOS A CARGAS LATERALES

## SOIL RESPONSE

AGOSTO, 1979.

# II. SOIL RESPONSE

## INTRODUCTION

As discussed in the previous section, the response of a soil around a laterally loaded pile can be characterized by a set of p-y curves which show the soil resistance when the pile is deflected a distance y. A typical family of p-y curves is shown in Fig. 5(a).

The form of the p-y relationship depends on many factors, such as soil properties, depth, the geometry of the pile, and the state of stress and strain throughout the affected soil zone. In addition, the p-y relation will be affected by the type of loading, i.e., whether it is short-term static, long-term static, short-term cyclic, long-term cyclic, or dynamic.

The characteristic shape of a p-y curve, plotted in the first quadrant for convenience, is shown in Fig. 5(b). As may be noted in the figure, the initial portion of the curve is essentially a straight line, defined by the modulus $E_{si}$, and indicates a linear elastic behavior of soil. Also as shown in the figure, the soil resistance p attains a limiting value, defined as the ultimate soil resistance $p_u$. In most of the practical design problems, deflections over a significant portion of the length of the pile are greater than that at the limit of the straight-line portion of the p-y curves. Therefore, the soil modulus $E_s$ is a nonlinear function of deflection and depth.

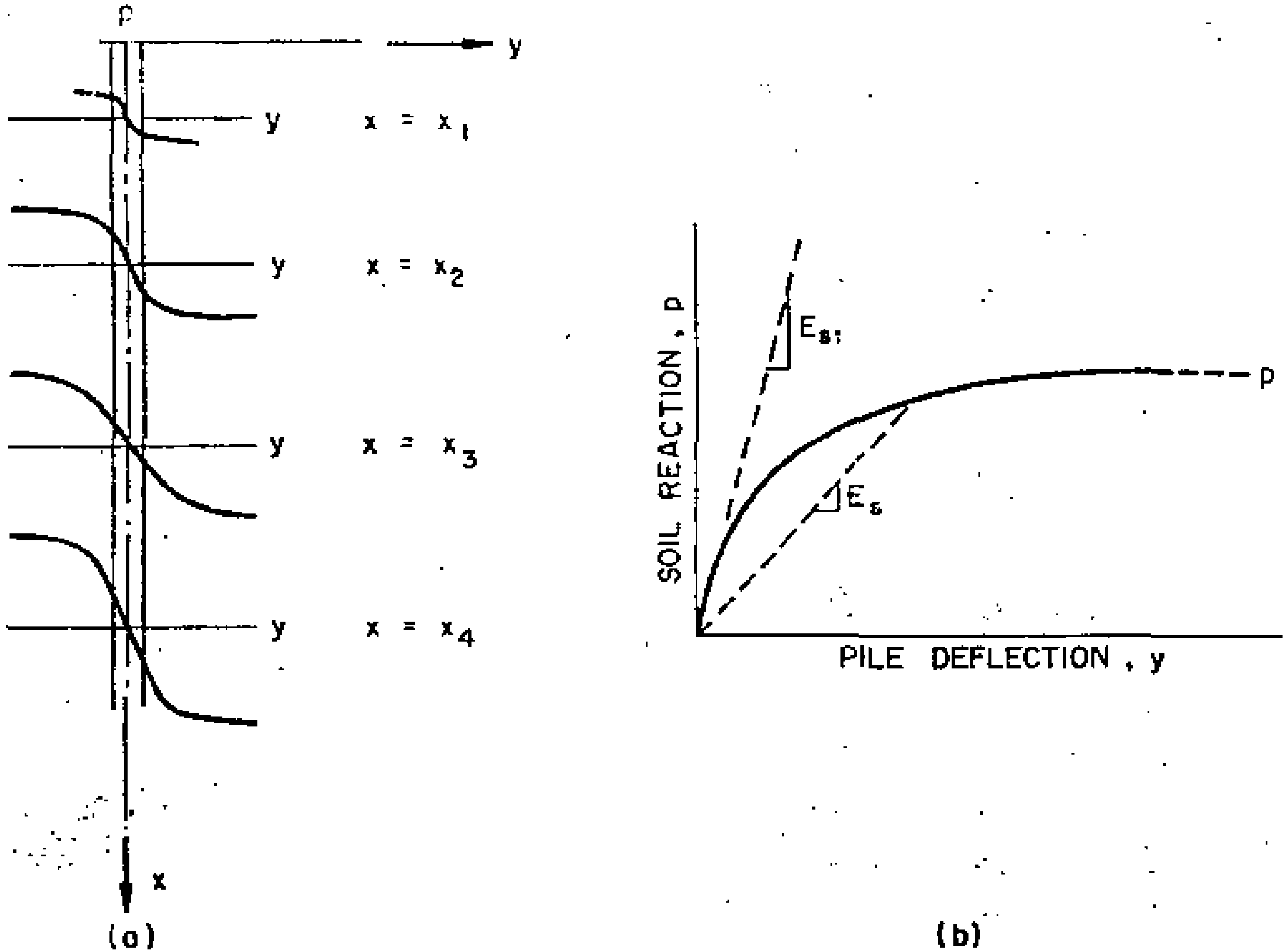


Fig. 5. Typical p-y Curves

(a) Family of curves.
(b) Characteristic shape of p-y curve.

The use of methods of analysis for soils to predict an entire p-y curve is not possible, but there are analytical methods for predicting the initial portion and final portion of the curves as discussed below.

## ANALYTICAL BASIS FOR INITIAL PORTION OF p-y CURVE

### Sand

The initial portion of the p-y curve has been discussed by Parker and Reese (1972), based on work by Terzaghi (1955). Terzaghi uses the following equation as the basis for his recommendations.

$$y = \frac{\rho D}{E_m} I_y \tag{4}$$

where

$\rho$ = unit pressure

D = diameter of pile

$I_y$ = influence coefficient

$E_m$ = modulus of elasticity of sand

Equation 4 is derived from a theory-of-elasticity solution for a line of pressure $\rho$ acting on an elastic layer of thickness 3D. From the elastic solution, a value of 1.35 is found for the influence factor $I_y$. The following equation results by substituting for $I_y$ and noting that p is equal to $\rho D$.

$$p = \frac{y\,E_m}{1.35} \tag{5}$$

Thus, the modulus of the initial portion of a p-y curve can be computed from the value of the modulus of elasticity of the sand. The $E_m$ of sand can be found from laboratory stress-strain curves. If no laboratory data are available, $E_m$ can be estimated by using an expression suggested by Terzaghi. As would be expected, $E_m$ increases linearly with depth for a homogeneous sand with a constant effective unit weight (Eq. 6).

$$E_m = J\bar{\gamma}x \tag{6}$$

where

$J$ = nondimensional coefficient

$\bar{\gamma}$ = effective unit weight of sand

$x$ = depth below ground surface

Substitution of Eq. (6) into Eq. (5) leads to

$$p = \frac{yJ\bar{\gamma}x}{1.35} \tag{7}$$

Rearringing the terms yields Eq. 8.

$$\frac{p}{y} = \frac{J\bar{\gamma}x}{1.35} \qquad (8)$$

Or,

$$-E_s = \frac{J\bar{\gamma}x}{1.35} \qquad (9)$$

The expression $-E_s = kx$ has often been used (Reese and Matlock, 1956) in solutions of the laterally loaded pile problem. The value of k in Eq. 9 is $J\bar{\gamma}/1.35$.

The values proposed by Terzaghi for a pile 1 ft wide are reproduced in the table below.

Table 1

| Relative Density of sand | Loose | Medium | Dense |
|---|---|---|---|
| Range of values of J | 100-300 | 300-1000 | 1000-2000 |
| Adopted values of J | 200 | 600 | 1500 |
| Dry or moist sand, values of k, $tons/ft^3$ | 7 | 21 | 56 |
| $(lbs/in^3)$ | (8.1) | (24.3) | (64.8) |
| Submerged sand, values of k, $tons/ft^3$ | 4 | 14 | 34 |
| $(lbs/in^3)$ | (4.6) | (16.2) | (39.4) |

The factor k is utilized in the criteria for deriving p-y curves for sands below the water table. These criteria will be discussed later in this section. Although Terzaghi proposed values of k for a pile 1 ft wide, the values are applicable to a pile of any width. Computations using the equations of elasticity for a linearly elastic material reveal that the soil modulus ($E_s$) is independent of beam width or pile diameter. Thus, no change in k or J is required to apply the formula to piles of different diameters.

Undrained Clay

In his paper on subgrade reaction, Terzaghi (1955) also proposes values of the coefficient of subgrade reaction for stiff to very hard clays. Terzaghi assumed the soil modulus of stiff clays to be constant with depth. He gave values in terms of a coefficient of subgrade reaction as follows.

$$E_s = k_h b = \bar{k}_{s1}(1\ \mathrm{ft})/1.5 \qquad (10)$$

where

$E_s$ = modulus of soil reaction as defined in Eq. (3).

$k_h$ = coefficient of horizontal subgrade reaction

$\bar{k}_{s1}$ = basic value of coefficient of vertical subgrade reaction

b = width of pile

Terzaghi proposed the values of $\overline{k_{s1}}$ given in Table 2. The values are limited to contact pressures which do not exceed one-half of the ultimate unit bearing capacity of the clay.

Table 2. Values of $\overline{k_{s1}}$ for Calculating Soil Modulus ($E_s$) Values

| Consistency of clay | Stiff | Very stiff | Hard |
|---|---|---|---|
| Values of $q_u$ (tsf) | 1 - 2 | 2 - 4 | > 4 |
| Range of $\overline{k_{s1}}$, tons/ft$^3$ | 50-100 | 100-200 | >200 |
| Proposed values | 75 | 150 | 300 |

## ANALYTICAL BASIS FOR ULTIMATE RESISTANCE

### Sand

The analytical basis for the ultimate resistance of sand with regard to laterally loaded piles can be divided into two types of failure (Parker and Reese, 1971): a deep or flow-around failure and a wedge-type failure near the surface. The flow-around failure will occur at depths where the overburden pressure is sufficient to confine the soil movement to horizontal planes. Near the surface where overburden pressures are small, the soil may fail as a wedge with the wedge moving both horizontally and vertically. The two types of failure are discussed in more detail below.

Deep or flow-around failure. At some point below the ground surface the soil fails by flowing around the pile. Based on this type of soil failure an expression will be derived for the ultimate lateral soil resistance well below the ground surface. The derivation is taken from Reese, Cox, and Koop (1974).

The behavior of the sand as it moves horizontally around the pile is quite indeterminate. The simplifying assumptions which are made in order to obtain an approximate solution to the problem are illustrated in Fig. 6. A sketch of a section through the pile is shown in the upper portion of the figure. The simple blocks for computation of limiting stresses in soils lead to ease in computation and are probably rigorous enough for the purpose. The sketch indicates the flow of soil from the front, around one side, and to the back of the pile. The flow will occur, of course, around each side

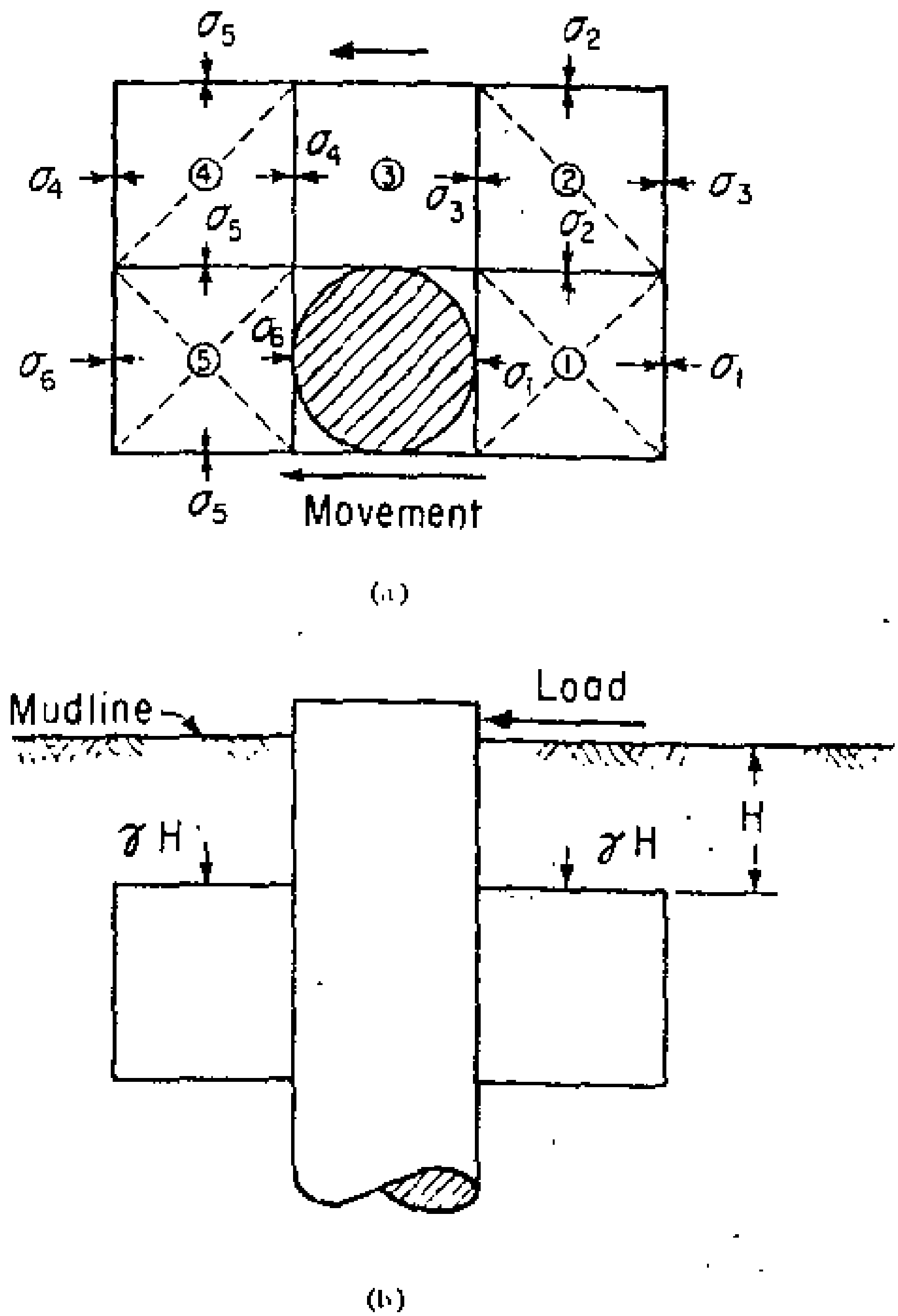


Fig. 6. Assumed Mode of Soil Failure by Lateral Flow Around the Pile. (a) Section through the pile. (b) Elevation of the pile.

but for simplicity only one side is shown. As the pile moves in the direction indicated, stresses of sufficient magnitude develop in block 5 to put that block into the failure condition. The possible failure surfaces which develop are indicated by the dotted lines. Stresses are in turn transmitted through block 4 and on around to block 1, such that the soil flows from the front to the rear of the pile. The movement involves the simultaneous failure of four of the five prismatic blocks and the sliding of block 3.

The representation of the failure stresses is shown on the Mohr-Coulomb diagram, Fig. 7. There is a lack of rigor in the assumption of the failure surfaces shown in Fig. 6. The Mohr-Coulomb theory would indicate that the failure surface should not occur at an angle of 45 degrees; however, this lack of rigor is thought not to be inconsistent with the overall approach to this part of the problem.

The computation of the stress at the front of the pile can proceed without difficuly if a value is assigned to the stress $\sigma_1$. It is reasoned that the value of $\sigma_1$ could not be less than the minimum active earth pressure. If block 1 is considered, a stress for $\sigma_1$ of less than minimum active earth pressure would imply that the soil could slump and fill the space behind the pile as the pile moved forward. However, this behavior was expressly eliminated since it is assumed that the soil will move in a horizontal direction.

Block 3 is assumed to move intact in a direction opposite to the motion of the pile. The stress $\sigma_4$ is assumed to be larger than $\sigma_3$ by an amount sufficient to cause sliding of this particular block. If it is assumed that at-rest earth pressure is acting on the outside of block 3 and that there is no friction between that block and the pile,

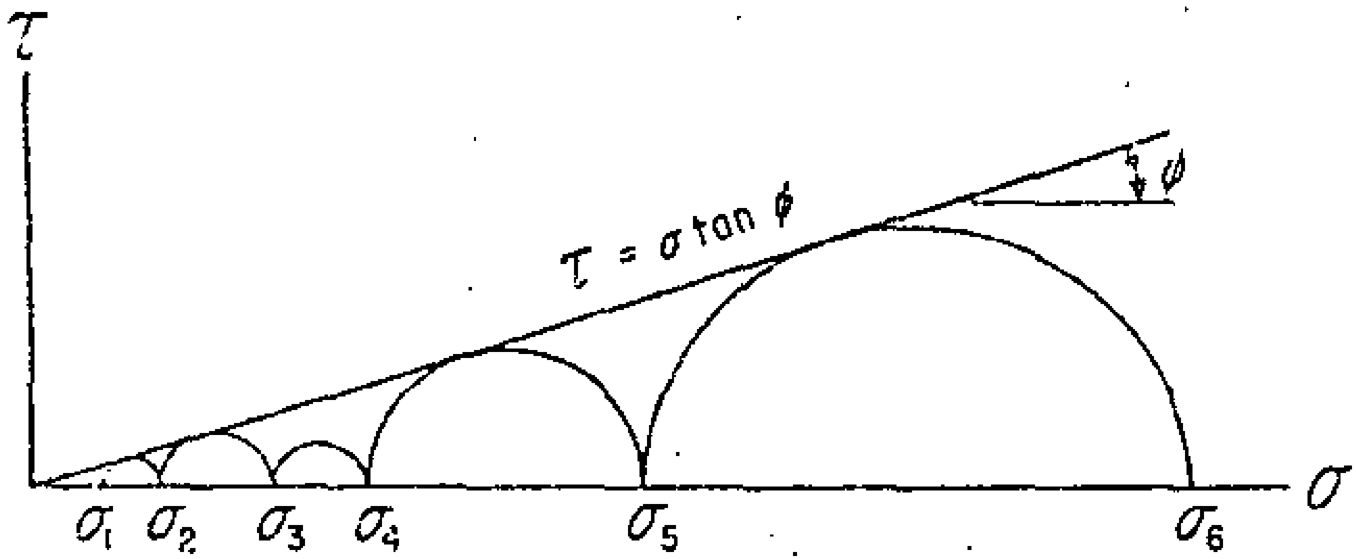


Fig. 7. Mohr-Coulomb Diagram Representing States of Stress of Soil Flowing Around a Pile.

the derivation can proceed as shown in the following equations.

$$\sigma_1 = K_a \bar{\gamma} H \tag{11}$$

where

$K_a$ = Rankine active pressure coefficient

$\bar{\gamma}$ = Effective unit weight, the unit weight which would be used in calculating effective overburden pressures.

$$\sigma_2 = \sigma_1 \; K_p = K_a \bar{\gamma} H K_p \tag{12}$$

where

$K_p$ = Rankine passive pressure coefficient

$$\sigma_3 = \sigma_2 \; K_p = K_a \bar{\gamma} H K_p^2 \tag{13}$$

$$\sigma_4 = \sigma_3 + K_o \bar{\gamma} H \tan \phi = K_a \bar{\gamma} H K_p^2 + K_o \bar{\gamma} H \tan \phi \tag{14}$$

where

$K$ = at rest pressure coefficient

$\phi$ = angle of internal friction

$$\sigma_5 = \sigma_4 \; K_p = K_a \bar{\gamma} H K^3 + K_o \bar{\gamma} H \tan \phi \; K_p \tag{15}$$

and

$$\sigma_6 = \sigma_5 \; K_p = K_a \bar{\gamma} H K_p^4 + K_o \bar{\gamma} H \tan \phi \; K_p^2 \tag{16}$$

Therefore, the limiting or ultimate soil resistance may be computed from Eqs. 16 and 16 such that

$$p_{cd} = K_a b \; \bar{\gamma} \; H \; (K_p^4 - 1) + K_o b \; \bar{\gamma} \; H \tan \phi \; K_p^2 \tag{17}$$

where $p_{cd}$ is the ultimate soil resistance per unit length of pile of width b. All terms in Eq. 17 are defined with the exception of $K_o$.

Various values of $K_o$ have been suggested, but it is normally assumed that $K_o$ is approximately 0.4 for loose sands and 0.5 for dense sands.

Near-ground-surface failure. Theoretical solutions for the ultimate soil resistance near the ground surface have been presented previously (Bowman, 956; Reese, 1962; Reese, Cox, and Koop, 1974). These solutions were based on the assumption that at ultimate load a wedge of sand was developed near the ground surface. The shape of the wedge was assumed to be as shown in Fig. 8.

Equations for the ultimate soil resistance based on the model shown in Fig. 8 will be derived by procedures similar to those used previously. (Reese et al, 1974) The same theoretical assumption will still be made that failure surfaces such as those shown in Fig. 8 are planes rather than curves. This assumption is not incompatible with the overall approach to the problem.

The force, $F_a$, resulting from active earth pressure on the back of the pile may be estimated by Rankine's theory as follows:

$$F_a = 1/2\, K_a\, \bar{\gamma}\, bH^2 \qquad (18)$$

This force acts in the direction of movement of the pile.

The forces resisting movement are computed from the assumed wedge shown in Fig. 8. In developing an expression for the resisting forces, the forces against the vertical side planes, ADE and BCF, and against the slanted plane, ABFE, will be considered seperately.

The force normal to planes ADE and BCF, which are subjected to at-rest earth pressure, is

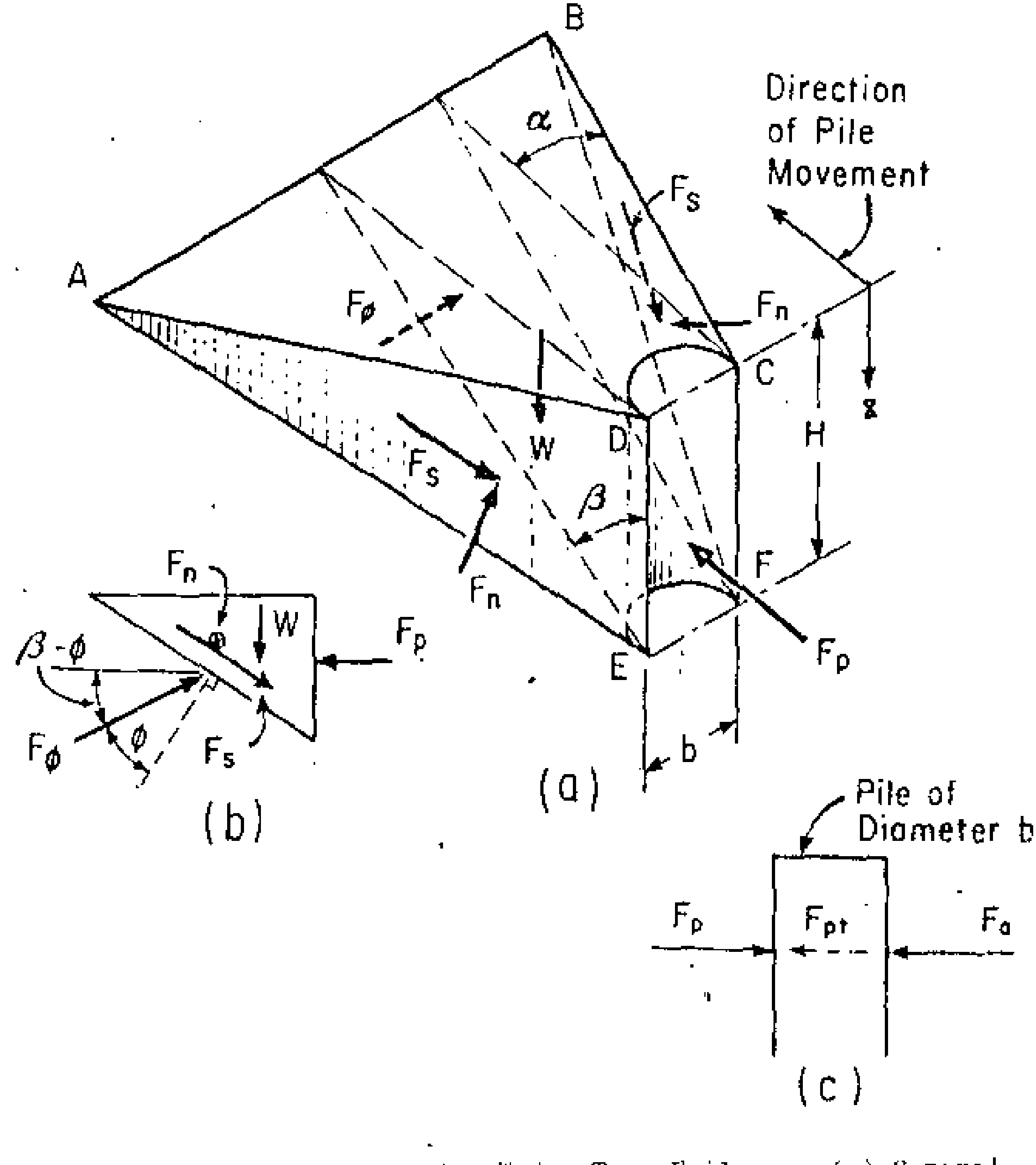


Fig. 8. Assumed Passive Wedge-Type Failure. (a) General shape of wedge. (b) Forces on wedge. (c) Forces on pile.

$$F_n = \int_0^H \sigma_n \, dA$$

where

$\sigma_n$ = normal stress on planes ADE and BCF.

From the Mohr-Coulomb diagram in Fig. 9, the normal stress on the failure planes ADE and BCF, is

$$\sigma_n = K_o \bar{\gamma} x \qquad (20)$$

where

x = depth measured from ground surface.

The expression for the differential area, dA, of these planes is,

$$dA = (H-x) \sec \alpha \tan \beta \, dx \qquad (21)$$

where

$\alpha$ and $\beta$ are defined in Fig. 8.

Substituting the expressions from Eq. 20 and 21 into Eq. 19 and integrating gives,

$$F_n = \frac{H^3 K_o \gamma \tan \beta}{6 \cos \alpha} \qquad (22)$$

The force, $F_s$, due to shear stress on planes ADE and BCF is given by the expression,

$$F_s = \int_0^H \sigma_n \tan \phi \, dA \qquad (23)$$

Substituting the expression from Eqs. 20 and 21 into the above equation and integrating gives,

$$F_s = \frac{H^3 K_o \bar{\gamma} \tan \phi \tan \beta}{6 \cos \alpha} \qquad (24)$$

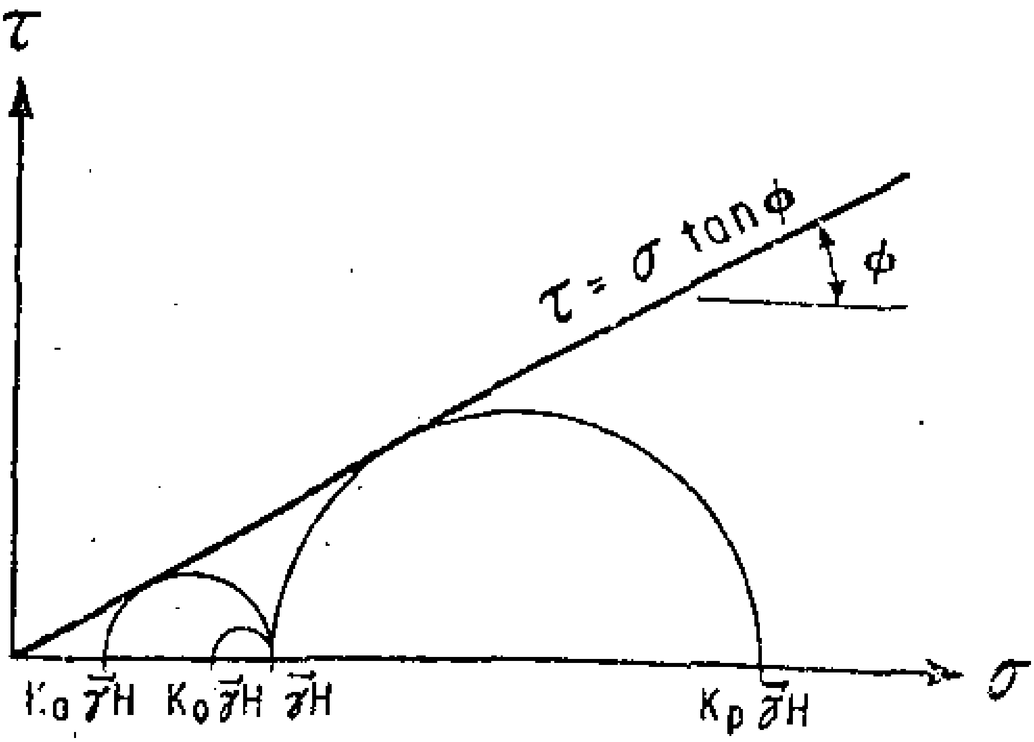


Fig. 9. Möhr-Coulomb Diagram Representing States of Stress for Various Earth Pressure Conditions.

Summing forces on the wedge (Fig. 8b) in the horizontal direction yields,

$$F_p = \cos(\beta-\phi)\, F_\phi + 2\cos\alpha \sin\beta\, F_s - 2\sin\alpha\, F_n \tag{25}$$

Summing forces also in the vertical direction,

$$F_\phi = \frac{1}{\sin(\beta-\phi)} (2\cos\beta\, F_s + W) \tag{26}$$

where, W, the weight of the soil wedge is,

$$W = \bar{\gamma} H^2 \tan\beta \left[ \frac{b}{2} + \frac{H}{3}\tan\beta\tan\alpha \right] \tag{27}$$

Substituting Eq. 26 into Eq. 25 gives,

$$F_p = \frac{\cos(\beta-\phi)}{\sin(\beta-\phi)} (2\cos\beta\, F_s + W) + 2\cos\alpha\sin\beta\, F_s - 2\sin\alpha\, F_n \tag{28}$$

Substituting the expressions from Eqs. 22, 24, and 27 into Eq. 28 gives,

$$F_p = \bar{\gamma} H^2 \left[ \frac{K_o H \tan\phi \sin\beta}{3\tan(\beta-\phi)\cos\alpha} + \frac{\tan\beta}{\tan(\beta-\phi)} \left( \frac{b}{2} + \frac{H}{3}\tan\beta\tan\alpha \right) + \frac{K_o H \tan\beta}{3} (\tan\phi\sin\beta - \tan\alpha) \right] \tag{29}$$

The total ultimate lateral resistance of the pile section is given by

$$F_{pt} = F_p - F_a \tag{30}$$

Substituting the expressions from Eqs. 18 and 29 into the above expression for the total soil resistance from the ground surface to the depth H gives,

$$F_{pt} = \bar{\gamma} H^2 \left[ \frac{K_o H \tan\phi \sin\beta}{3\tan(\beta-\phi)\cos\alpha} + \frac{\tan\beta}{\tan(\beta-\phi)} \left( \frac{b}{2} + \frac{H}{3}\tan\beta\tan\alpha \right) + \frac{K_o H \tan\beta}{3} (\tan\phi\sin\beta - \tan\alpha) - \frac{K_a b}{2} \right] \tag{31}$$

However, the ultimate soil resistance per unit length of pile is desired for use in the p-y curve construction. This soil resistance per unit length of pile at any depth may be found by differentiating Eq. 31 with respect to depth H, giving

$$p_{ct} = \frac{dF_{pt}}{dH} = \bar{\gamma} H \left[ \frac{K_o H \tan\phi \sin\beta}{\tan(\beta-\phi)\cos\alpha} + \frac{\tan\beta}{\tan(\beta-\phi)}(b + H\tan\alpha) + K_o H \tan\beta(\tan\phi\sin\beta - \tan\alpha) - K_a b \right] \quad (32)$$

Equation 32 is based on the assumption that stresses on plane CDEF in Fig. 8 act in the horizontal direction only; the frictional force between plane CDEF and the face of the pile was neglected. This is a conservative assumption.

The values of the parameters in Eq. 32 can be determined from theory and experimental data. Values for $K_o$ have been discussed previously. The angle $\beta$ is approximated by the following equation:

$$\beta = 45^o + \phi/2 . \quad (33)$$

Values of the angle $\alpha$ have been determined from results of model tests with a small, flat plate in sand. From these model tests, Bowman (1958) states that $\alpha$ is probably a function of the void ratio of the sand, with values ranging from $\phi/3$ to $\phi/2$ for loose sand to $\phi$ for dense sand.

Undrained Clay

The ultimate resistance of an undrained clay can also be divided into two different types of failure, as discussed below.

Deep or flow-around failure. The model for this failure, shown in Fig. 10 is similar to the deep failure model for sand. The derivation that follows is taken from Reese (1958).

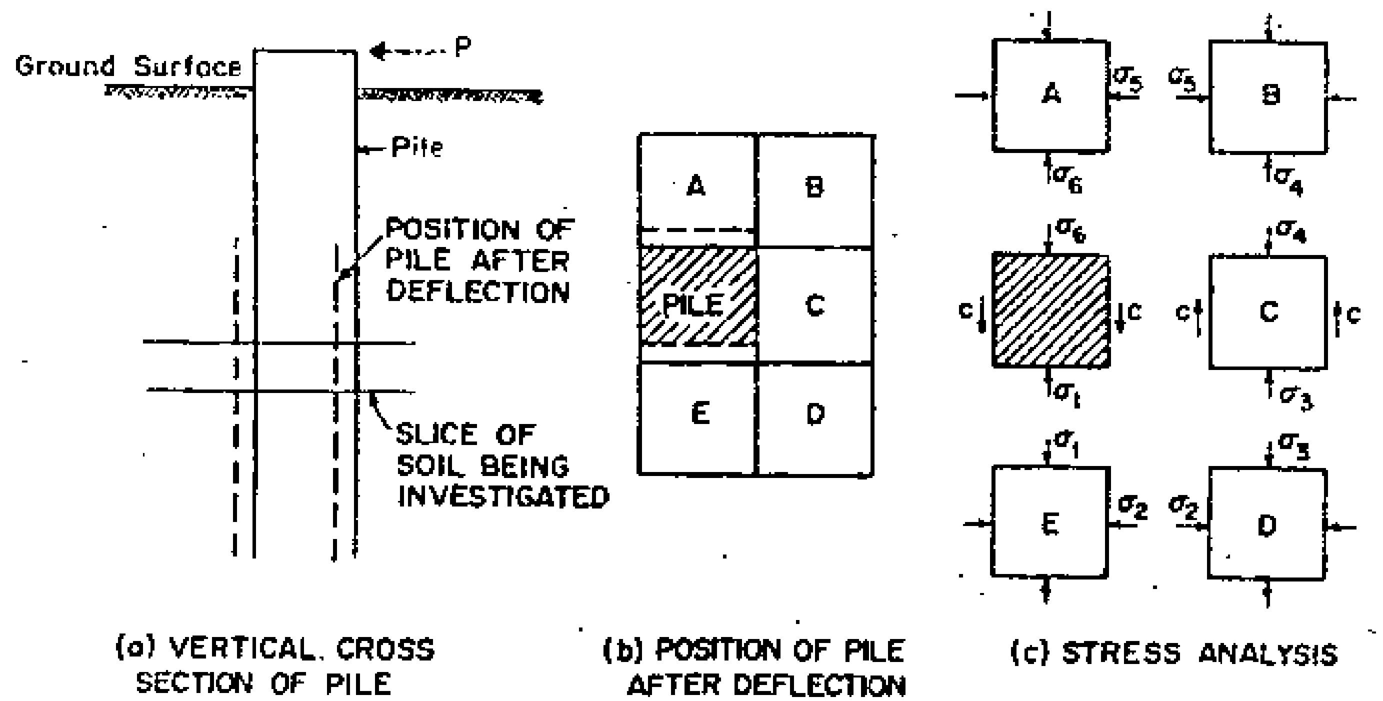


Fig. 10. Failure Mechanism for Flow-Around Ultimate Resistance for Clay

First, it is assumed that the stress, $\sigma_1$, acting on the pile block is zero. Then, if the shear strength, s, of the soil is assumed to be defined by s = c, (as determined in an unconsolidated-undrained triaxial test or vane test) the stress $\sigma_2$ will be 2c. This process is repeated to obtain the following values of the stresses.

$$\sigma_3 = 4c \qquad (34)$$

$$\sigma_4 = 6c \qquad (35)$$

$$\sigma_5 = 8c \qquad (36)$$

$$\sigma_6 = 10c \qquad (37)$$

By considering equilibrium of the pile, the following equation results.

$$P_{ULT} = (\sigma_6 + 2c - \sigma_1)\ b = 12\ cb \qquad (38)$$

Near-ground-surface failure. Figure 11 shows a failure wedge assumed for a near-surface failure for stiff clay. The forces shown in Fig. 11 are defined below.

$$F_1 = 1/2\ \bar{\gamma} bH^2 \tan\theta \qquad (39)$$

where

$\bar{\gamma}$ = effective unit weight of clay, the unit weight which would be used in calculating effective overburden pressures

$$F_2 = cbH \sec\theta \qquad (40)$$

$$F_3 = 1/2\ cH^2 \tan\theta \qquad (41)$$

$$F_4 = 1/2\ cH^2 \tan\theta \qquad (42)$$

$$F_5 = \alpha cbH \qquad (43)$$

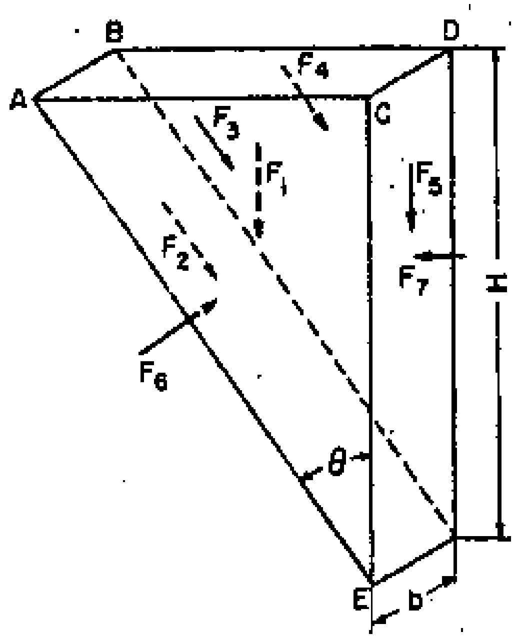


Fig. 11. Failure Mechanism for Wedge Failure for Clay

where

$\alpha$ = reduction factor, similar to that employed for computing side resistance of pile in axial loading.

All other quantities are indicated in Fig. 11.

Summing forces in the vertical direction yields the following,

$$F_6 = 1/2\ \bar{\gamma} b H^2 \sec\theta + \alpha c b H \csc\theta + c b H \csc\theta + c H^2 \qquad (44)$$

Summing forces in the horizontal direction results in:

$$F_7 = 1/2\ \bar{\gamma} b H^2 + \alpha c b H \cot\theta + 2\ c b H \sec\theta \csc\theta + c H^2 \sec\theta \qquad (45)$$

The ultimate soil resistance can be found by differenting Eq. (45) with respect to H.

$$P_{ULT} = b\ (\bar{\gamma} H + \alpha c \cot\theta + 2\ c \cot\theta + 2\ \frac{cH \sec\theta}{b}) \qquad (46)$$

If the vertical resistance along the length of the pile is ignored and $\theta$ is assumed to be 45°, the following equation results,

$$P_u = b\ (\bar{\gamma} H + 2c + 3\ c\ \frac{H}{b}) \qquad (47)$$

where

$$J = 2.83 = 2 \sec 45^{\circ}$$

According to Eq. 47, the ultimate resistance at the surface is 2c. If the pile is assumed to be a square with its sides equal to the diameter with soil shear acting against the sides also, the ultimate resistance will increase to 4c. In correlating Eq. 47 to experimental results, an ultimate resistance value at the surface of 3c is usually assumed.

## EXPERIMENTAL TECHNIQUES FOR OBTAINING p-y CURVES

There are several methods of obtaining p-y relations from laterally loaded pile experiments. Some of these methods are discussed below.

### Direct Measurement

If a test is performed on a laterally loaded pile, it would be theoretically possible to instrument the pile so that values of p and y could be obtained directly. To measure pressure, gages would be required that would be sensitive to both shear stress and normal stress. For measuring the deflection a hollow pile instrumented with scales readable by some type of optical system would be necessary. While such equipment probably does not exceed present technology, the procedure would be quite expensive and has not been used to date (Reese and Cox, 1968).

### Experimental Moment Curves

Another method of obtaining p-y curves from a lateral load test is to instrument the pile with strain gages at various points along the pile.

The strain gages readings can be converted into moment values by use of calibration curves that are obtained by applying known values of moment. With values of experimental moment at various points along the length of the pile are known, the deflections at each depth can be calculated from the following equation.

$$y = \iint \frac{M}{EI}\, dx \qquad (48)$$

where

EI = stiffness of the pile

M = measured moment.

The double integration can be accomplished by numerical procedures or by integrating a polynomial which has been fitted to the data by least squares techniques.

The resistance per unit length values can be determined from the measured moments by double differentiation as follows.

$$p = \frac{d^2M}{dx^2} \qquad (49)$$

Again, the double differentiation can be performed by numerical or xact procedures. However, extremely accurate moment values are needed for this procedure. To avoid the need for such accuracy, the resistance values are often obtained by nondimensional coefficients as discussed below.

After the p and y values have been obtained, they are cross plotted at each depth to yield a family of p-y curves.

Nondimensional Methods

The method of obtaining p-y curves by the use of nondimensional coefficients, described in Reese and Cox (1968), involves of course the use of a lateral load test. However, no costly instrumentation is necessary. While the p-y curves obtained from nondimensional methods are not as accurate as those obtained from fully instrumented tests, good and sometimes very good p-y curves can be obtained.

The test procedure requires that the deflection and slope at the groundline be measured for each applied moment and lateral load. Then, nondimensional solutions (to be discussed) are generated using different assumed variations of the soil modulus with depth until the solutions agree with the measured values of deflection and slope at the groundline. Agreement between the nondimensional solutions and the measured deflection and slope values indicate the proper variation of soil modulus has been found. This correct soil modulus is used to solve the differential equation by difference methods to obtain the deflection as a function of depth. Thus, both the soil modulus and deflection are known along the length of the pile. The value of resistance at desired depths can then be computed.

The procedure described above is repeated for various values of applied load and moment to generate a family of p-y curves.

RECOMMENDATIONS FOR CALCULATING p-y CURVES

Because of the complex nature of the soil response around a laterally loaded pile, as mentioned previously no purely analytical procedure can be used to predict p-y curves. Instead, the results of experiments on full-scale load tests are employed as the basis of the prediction of p-y curves. Experimental results are compared with analytical expressions for the initial slope and for the ultimate resistance of the p-y curve (see derivations at the beginning of this section). Empirical coefficients are used to obtain agreement between experiment and theory. Transition portions of the p-y curve, as well as the effects of cyclic loading, are also described empirically. While a particular set of recommendations for p-y curves will have some basis in theory, the recommendations are strongly related to the experiments on which they are based. That fact should be considered carefully in using the curves for design.

The recommendations for calculating p-y curves for four soil conditions, soft clays below the water surface, sands, and stiff clays above the below the water surface, are given below along with a brief description of the relevant experiments..

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