centro de educación continua
división de estudios superiores
facultad de ingeniería, unam

A LOS ASISTENTES A LOS CURSOS DEL CENTRO DE EDUCACION CONTINUA

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Con objeto de mejorar los servicios que el Centro de Educación Continua ofrece, es importante que todos los asistentes llenen y entreguen su hoja de inscripción con los datos que se les solicitan al iniciarse el curso.

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DIVISION DE ESTUDIOS SUPERIORES
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Programa de actividades para el segundo semestre de 1976

Exámenes de admisión: 10, 11 y 12 de mayo

Inscripciones: 31 de mayo al 4 de junio

Iniciación de clases: 7 de junio

Requisitos de admisión

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c) Presentar, dentro del período de inscripciones arriba mencionado, la documentación que se indica en el folleto de Actividades Académicas 1975 de la DESFI

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"POR MI RAZA HABLARA EL ESPIRITU"
Cd. Universitaria, febrero 3. 1976

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CENTRO DE EDUCACION CONTINUA
DIVISION DE ESTUDIOS SUPERIORES

PROGRAMACION Y MODELOS DE INGENIERIA AMBIENTAL
ABRIL, 1976

HORARIO PARA LAS MAÑANAS

Día / Hora	Lun 26	Mar 27	Mier 28	Jue 29	Vier 30
9-10	Introducción a la Computación. Lenguaje Fortran (Cuevas)	Proposiciones GOTO, IF (Cuevas)	Variables con subíndices (Cuevas)	Subprogramas FUNCTION (Cuevas)	Tópicos avanzados de programación
10-11	Constantes, variables y expresiones aritméticas (Cuevas)	Proposición DO (Cuevas)	Variables con subíndices (Cuevas)	Subprogramas SUBROUTINE (Cuevas)	Análisis de redes de abastecimiento de agua potable
11-12	Instrucciones de entrada y salida de información (Cuevas)	Ejemplos (Cuevas)	Modelo de transporte	Ejemplos (Cuevas)	Programación Lineal (Cuevas)
12-13	Taller (Codificación y Perforación) (Cuevas)	Taller (Codificación y Perforación) (Cuevas)	de Desechos Sólidos (Zepeda)	Taller (Codificación y Perforación) (Grupo)	Determinación de parámetros (Bonilla)

CENTRO DE EDUCACION CONTINUA
DIVISION DE ESTUDIOS SUPERIORES

PROGRAMACION Y MODELOS DE INGENIERIA AMBIENTAL
HORARIO PARA LAS TARDES
ABRIL, 1976

Día / Hora	LUNES 26	MARTES 27	MIERCOLES 28	JUEVES 29	VIERNES 30
14-15 15-16	Modelos de dispersión de contaminación atmosférica. (Canter)	Modelos de contaminación bacteriana del agua. (Canter)	Análisis de redes de abastecimiento de agua. Hardy - Cross (Reid)	Modelos para abastecimiento de agua y modelos para sistemas de alcantarillado. (Reid)	Modelos generales de sistemas para manejo de la calidad del agua.
16-17 17-18	Modelos de propagación y exposición al ruido. (Canter)	Modelos de contaminación témica. (Canter)	Modelos de calidad del agua. Streeter Phelps (Reid)	Modelo para sistemas de drenaje. (Reid)	(Reid)

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[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

PROGRAMACION Y MODELOS DE INGENIERIA AMBIENTAL

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C O N T E N I D O

1.- INTRODUCCION A LA COMPUTACION ELECTRONICA

Todos los dispositivos y la organización tan compleja que en conjunto forman una computadora y que hacen posible la realización de una serie de funciones y trabajos, que sencillamente son increibles, se basan en un hecho esencial: una computadora puede ejecutar ciertas operaciones muy simples, pero extremadamente rápido.

Una computadora común puede sumar un millón de números en un segundo; las computadoras realmente rápidas pueden hacer esto mejor.

Las computadoras pueden hacer otros pases elementarios de algoritmos aritméticos (por ejemplo: multiplicaciones, comparaciones numéricas, transferencias de control), a velocidades comparables. De esta forma, se puede entender como ejecutan algoritmos complicados en milésimas de segundo.

El fin que se ha perseguido en la organización de una computadora, es aprovechar efectivamente y eficientemente su velocidad de computación. Para visualizar esto, piense en una calculadora electrónica de escritorio, basta apoyar una tecla para que en milésimas de segundo obtengamos el resultado de una operación de multiplicación; pero perdimos más tiempo en darle a la calculadora los números a multiplicar.

Esto mismo se presenta en una computadora y el problema ha sido la organización de ésta, así que se aproveche su gran velocidad de computación.

Las primeras computadoras tenían tres elementos principales. Estos eran un procesador central, una memoria y un lenguaje de máquina.

El procesador central es esencialmente una calculadora rápida. La memoria es el lugar donde la computadora puede leer y escribir números o instrucciones. El lenguaje de máquina es un lenguaje en el que se dan instrucciones a la computadora. Así en lugar de apoyar la tecla de multiplicación, se usa la instrucción multiplicar; y en lugar de darle manualmente los números, se puede escribir "toma el número escrito en tal lugar de la memoria y colócalo en la unidad aritmética". Ya que la computadora ejecuta las instrucciones que se le dan, tiene otro elemento muy importante, la unidad de control. Su función es interpretar las instrucciones, ejecutarlas y controlar todo el proceso.

Los pasos básicos para obtener la solución de un problema, en este tipo de computadoras eran:

1) Preparar las instrucciones.
2) Almacenar las instrucciones y los datos en la memoria de la computadora
3) Dar la orden de iniciar la ejecución del

programa.

4) Ejecutar el programa.

5) Obtener los resultados de la memoria de la computadora.

En el caso de la computadora IBM 650, una de las primeras, las instrucciones y datos eran perforados en tarjetas y almacenados en la memoria de la computadora a través de una lectora. Una vez que éstos se encontraban almacenados, la unidad de control se colocaba en la localización de la primera instrucción, esto se hacia usando la consola; y el operador esperaba hasta que la computadora terminaba el proceso. Ya que el procesador central opera extremadamente rápido y la velocidad de la unidad de control y memoria son comparables, la velocidad con que se ejecutaban algoritmos era extremadamente rápida.

La organización de este tipo de computadoras fue a principios de 1950 y durante esta década fue el único tipo o modelo disponible. A finales de esta década, se vió que ésta no era la forma de usar eficientemente la velocidad del procesador central, por las siguientes razones:

1) Se requería de mucho tiempo para la preparación de las instrucciones del programa (escribirlas correctamente en lenguaje de máquina).

2) Se requería de un tiempo relativamente grande en almacenar las instrucciones y datos en la memoria de la computadora.

3) Se requería de un tiempo relativamente grande, para iniciar la ejecución del programa (colocar la unidad de control, apoyar el botón de inicio, etc.).

4) Se requería de un tiempo grande para obtener los resultados de la memoria de la computadora.

Dos cambios en la organización de las computadoras, se hacen para tratar de eliminar estas dificultades. Uno de ellos es la introducción de lenguajes de alto nivel, que ayudan a aumentar la eficiencia en la preparación de programas. El segundo cambio es la introduccion de un sistema operativo, junto con memorias auxiliares de entrada y salida.

El primer lenguaje de alto nivel, usado extensamente fue el FORTRAN, cuyo nombre viene de "FORMULA TRANSLATOR". El FORTRAN original no fue tan versatil como el que se dispone en la actualidad.

El uso de lenguajes de alto nivel requiere del uso de programas que hagan la traducción a lenguaje de máquina.

Los programas que hacen esta traducción se llaman compiladores. Ellos leen un programa escrito en lenguaje de alto nivel y elaboran un programa equivalente en lenguaje de máquina.

La introducción de lenguajes de alto nivel da lugar a la introducción de un "sistema" que opere a la computadora. Este sistema está formado por un conjunto de programas y algoritmos, que incluyen los algoritmos para la traducción de lenguajes, algoritmos para catalogar y ordenar el trabajo de la computadora, programas que inicien y terminen trabajos, programas para enviar los resultados al exterior, etc.

La forma en que se ejecutan trabajos en este tipo de computadoras, es el siguiente: cada trabajo es leido, registrado y almacenado en una memoria auxiliar de la computadora, y espera su turno para ser ejecutado. La computadora una vez que termina un trabajo, va al registro y busca el nombre del siguiente programa por ejecutar, lo trae a su memoria principal y lo ejecuta.

Los resultados son enviados a otra memoria auxiliar y posteriormente a la impresora.

La computadora dispone ahora de una serie de dispositivos de entrada y salida, éstos se muestran esquemáticamente en la Figura 1.

Este tipo de organización dá como resultado una operación eficiente de la computadora. El procesador cen-

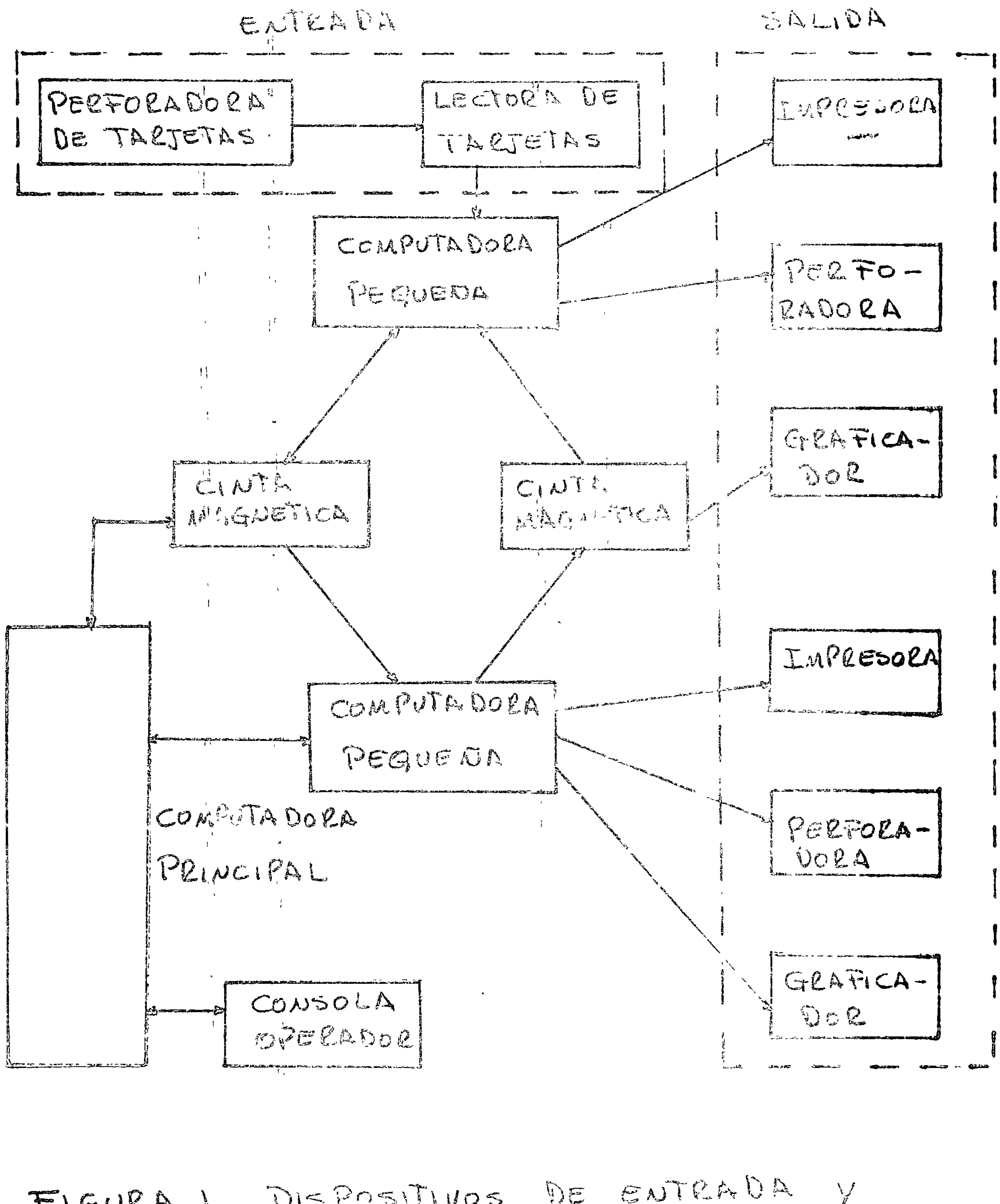


FIGURA 1. DISPOSITIVOS DE ENTRADA Y SALIDA DE UNA COMPUTADORA.

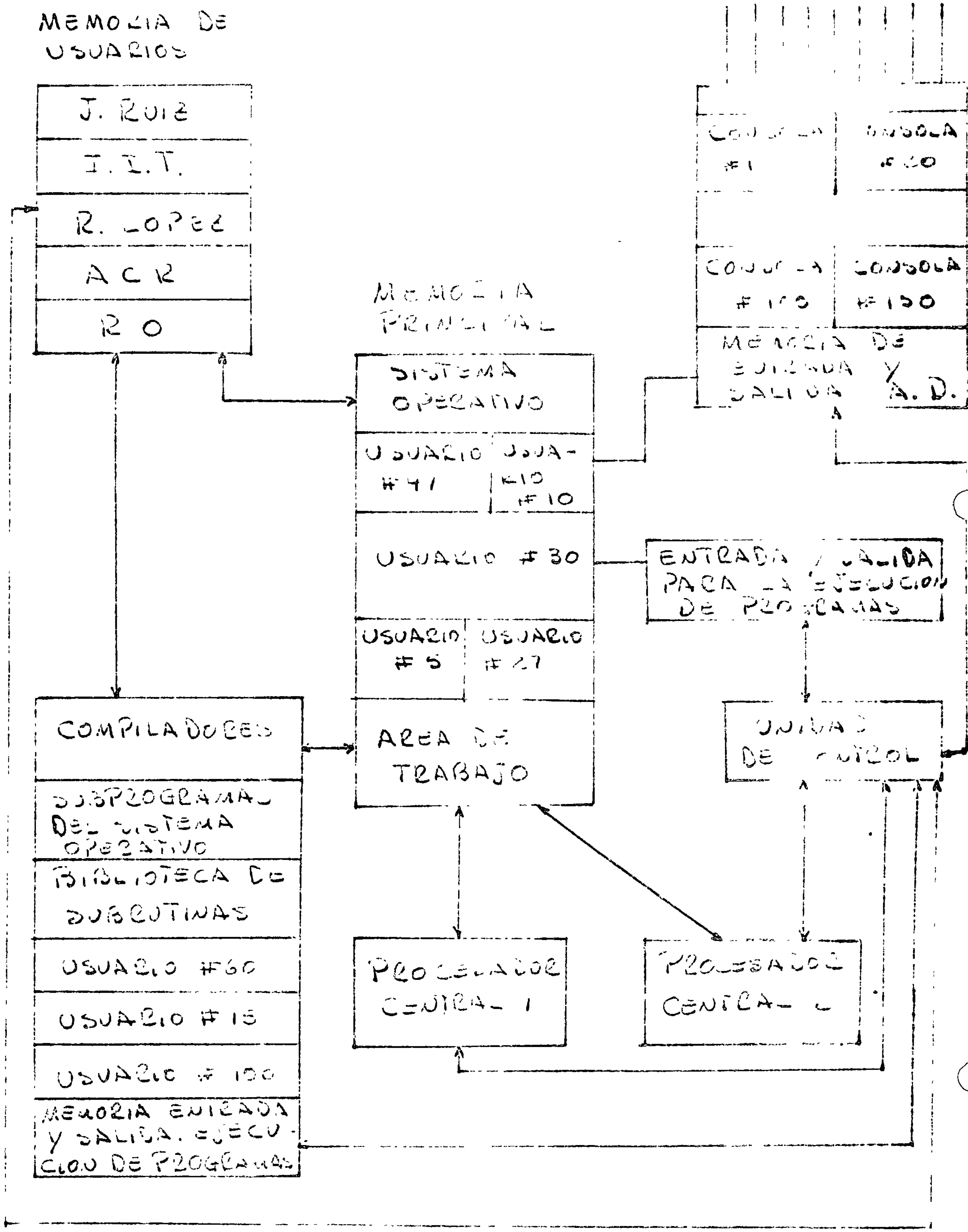
tral se encuentra casi constantemente trabajando a toda su velocidad; por otro lado, el uso de compiladores y del sistema operativo, introduce una gran cantidad de trabajo a la computadora.

Aunque esta forma de organización es eficiente desde el punto de vista de la computadora, no lo es desde el punto de vista de los usuarios. El problema que se presenta es el siguiente: la computadora ejecuta sólo un programa simultáneamente; supongamos que un usuario desea correr un programa que es pequeño, tiene que esperar su turno para que éste sea ejecutado, este tiempo de espera en que deja el programa y obtiene los resultados, depende de muchos factores y varía desde unas cuantas horas hasta días.

Para solucionar este problema, una nueva organización de la computadora aparece a mediados de la década de 1960. Este tipo de organización se llama tiempo compartido, es extremadamente complicada, en la Figura 2 se muestra esquemáticamente.

Para apreciar esta complejidad, se analizarán brevemente sus componentes principales:

Memoria de entrada y salida para acceso directo.- Cada consola tiene asignada un área de memoria (durante su operación) y la información enviada es almacenada temporalmente, hasta que se lleve a cabo al


MEMORIA DE USUARIOS
J. RUIZ
I.I.T.
R. LOPEZ
ACR
RO
MEMORIA PRINCIPAL
SISTEMA OPERATIVO
USUARIO #41
USUARIO #10
USUARIO #30
USUARIO #5
USUARIO #27
AREA DE TRABAJO
CONSOLA #1
CONSOLA #20
CONSOLA #100
CONSOLA #150
MEMORIA DE ENTRADA Y SALIDA
A.D.
ENTRADA Y SALIDA PARA LA EJECUCION DE PROGRAMAS
UNIDAD DE CONTROL
COMPILADORES
SUBPROGRAMAS DEL SISTEMA OPERATIVO
BIBLIOTECA DE SUBRUTINAS
USUARIO #60
USUARIO #15
USUARIO #100
MEMORIA ENTRADA Y SALIDA. EJECUCION DE PROGRAMAS
PROCESADOR CENTRAL 1
PROCESADOR CENTRAL 2

gún procesamiento.

Este dispositivo es una computadora pequeña, de propósito especial que puede realizar procesamientos muy simples; por ejemplo: verificar cada proposición que es enviada, si es correcta de acuerdo con el lenguaje que está usando. Cuando se tienen una serie de proposiciones y el usuario quiere enviarlas a procesar, este dispositivo envía estas proposiciones a la computadora principal; y cuando ésta envía los resultados, los pone en la forma adecuada para enviarlas posteriormente a la consola.

Memoria de entrada y salida para la ejecución de programas.- Esta parte del sistema almacena los programas que debe ejecutar la computadora, así como los que ya han sido ejecutados y serán enviados a impresión.

Aunque se tengan operando simultáneamente un gran número de consolas, a la computadora le sobra tiempo y puede ejecutar programas que se encuentran en lista de espera.

Memoria principal.- Esta es el área principal de trabajo de la computadora. En ella se encuentra el sistema operativo.

Cada uno de los programas que en un momento dado se encuentra en ejecución, tienen una parte de esta memoria asignada para su uso.

Los programas almacenados en la memoria principal son ejecutados casi simultáneamente, es decir, el sistema operativo va recorriendo todos los trabajos y en cada uno de ellos ejecuta una parte del programa. Si la computadora tiene más de un procesador central, la ejecución de los trabajos es prácticamente simultánea.

Memoria Auxiliar.- Esta memoria tiene las funciones mencionadas previamente, en adición almacena los trabajos que han sido parcialmente ejecutados. Ya que la computadora va ejecutando todos los trabajos simultáneamente, los resultados parciales de cada programa así como la información referente, son almacenados en esta memoria auxiliar.

Cada vez que la computadora va a ejecutar una parte de un programa, obtiene de esta memoria la información necesaria para proseguir la ejecución del programa.

Memoria de usuarios.- En esta memoria los usuarios pueden guardar sus programas por periodos largos de tiempo (días, meses). En esta memoria se guardan programas que han sido compilados, archivos de datos, etc.

Unidad central y procesadores centrales.- La unidad central recibe instrucciones del sistema operativo y decide la forma en que se deben ejecutar. Los procesadores centrales hacen las operaciones aritméticas, comparaciones, etc.

El querer describir con un poco más de detalle la organización actual de una computadora, se sale de los objetivos de este curso. Por otro lado, desde el punto de vista de usuarios, nos interesa más conocer en detalle un lenguaje de alto nivel.

2.- ESC[illegible] FORTRAN

En la discusión del uso de este lenguaje se considerará, por simplicidad, que la forma de darle instrucciones y datos a la computadora, es a través de tarjetas de codificación; sin embargo, se puede generalizar a cualquier otro dispositivo de entrada (disco, cinta magnética).

Los programas fuente FORTRAN se escriben normalmente en tarjetas de codificación, en las columnas 7-72 inclusive.

Para fines de referencia, se pueden enumerar las proposiciones del programa, hasta 5 dígitos, enteros y sin signo.

Normalmente cada proposición ocupa un solo renglón en la forma de codificación; si por alguna razón no es suficiente un renglón, se puede continuar en renglones subsecuentes, y se permite hasta un máximo de 19 renglones contínuos, que se identifican escribiendo un carácter que no sea blanco en la columna 6.

Se pueden insertar comentarios en el programa, que aunque se imprimen en el listado de salida, son ignorados por el compilador; estos comentarios se identifican por una letra c en la columna 1.

Las columnas 73-80 son ignoradas por el compilador, se usan generalmente para enumerar secuencialmente las tarjetas del programa, o para escribir comentarios.

Existen 5 tipos de proposiciones en FORTRAN:

1) Proposiciones de asignación, que corresponden a las proposiciones aritméticas.

2) Proposiciones de entrada y salida de información.

3) Proposiciones de control, que corresponden a las instrucciones de comparación y ramificación.

4) Proposiciones de especificación, éstas no dan lugar a instrucciones en el programa objeto, pero sirven para informar al compilador dónde y cómo se almacenarán los datos en la memoria.

5) Proposiciones para uso de subprogramas,

3.- PROPOSICIONES DE ASIGNACION

3.1. Constantes y variables

Una constante entera FORTRAN es cualquier número escrito sin punto decimal; los números negativos pueden ir precedidos del signo menos y un entero que no vaya precedido del signo negativo, se considera positivo.

Una constante real en FORTRAN, es cualquier número escrito con punto decimal. Se puede escribir una constante real usando notación exponencial, para lo cual se usa la letra E seguida de una constante entera, así por ejemplo:

$$6.5E + 6 = (6.5 \cdot 10^{6})$$

$$4.8E - 10 = (4.8 \cdot 10^{-10})$$

Un número al que se refiere por medio de un nombre, en vez de su valor es una variable.

Los nombres que se asignan a las variables, pueden estar formados por una serie de caracteres (excepto el blanco), máximo 6, con la restricción de que el primero debe ser alfabético.

Si el primer carácter de una variable es I, J, K, L, M ó N, la variable representa una cantidad entera. Si la variable empieza con cualquier otro carácter alfabé tico, representa una cantidad real.

Esta convención se ignora si se especifica al principio del programa qué variables son reales y cuáles enteras; para esto se usan las proposiciones:

INTEGER $u_1, u_2, u_3, u_4....$

REAL $v_1, v_2, v_3, v_4....$

Cada variable toma el tipo especificado, independientemente de la primera letra.

3.2. Expresiones Aritméticas

Una expresión aritmética es cualquier secuencia de constantes, variables y funciones, separadas por símbolos de operación (+, -, etc.) que en conjunto tienen un significado matemático.

Se tienen 5 operaciones en FORTRAN:

- + para indicar suma.
- - para indicar resta.
- * para indicar multiplicación.
- / para indicar división.
- ** para indicar exponenciación.

Al escribir expresiones aritméticas se deben ob servar las siguientes reglas:

1) No se puede tener dos símbolos de operación consecutivos. Así A/-B no está permitido, pero A/(-B) sí es válido.

2) Los paréntesis deben ser usados para indicar grupos, tal como se hace en notación matemática ordinaria, así $(x+y)^2$ se escribe como (x+y)**2.

3) Cualquier expresión puede ser elevada a una potencia que es una cantidad entera positiva o negativa, pero sólamente una expresión real puede ser elevada a una potencia real.

4) En general, los programas serán más eficientes si no se tienen variables enteras y reales entremezcladas en una expresión aritmética; la única excepción es en la operación de exponenciación. Por ejemplo, al calcular la expresión A + I el compilador debe insertar instrucciones para convertir primero la variable I a la forma de punto flotante, por lo que deben ahorrarse estas conversiones.

Así X + 5.0 es preferible que X + 5 y

x*3.0 es mejor que x*3; de la misma forma x**3 es más rápido que x**3.0.

3.3. Funciones matemáticas

En FORTRAN se usan ciertas funciones matemáticas. La siguiente tabla muestra el nombre de cada una de ellas y la función que representa:

FUNCION MATEMATICA	NOMBRE EN FORTRAN
Exponencial	EXP
Logaritmo natural	ALOG
Logaritmo común	ALOG 10
Seno de un ángulo en radianes	SIN
Coseno de un ángulo en radianes	COS
Tangente hiperbólica	TANH
Raíz cuadrada	SQRT
Angulo-tangente, en radianes	ATAN
Valor absoluto	ABS

Los elementos básicos del lenguaje FORTRAN que se han tratado hasta ahora, tienen muchas aplicaciones al escribir programas. La más importante es el cálculo de un nuevo valor de una variable, el que se hace con una proposición de asignación.

Su fórmula general es a = b, en la cual a es el nombre de la variable escrito sin signo, y b es cualquier expresión aritmética.

Una proposición de asignación es una orden para calcular el valor de la expresión de la derecha y proporcionar ese valor a la variable nombrada a la izquierda.

La proposición x = y*SIN (R), es una orden para realizar el producto de la variable y por el valor de la función SIN(R); y reemplazar el valor de x con el resultado de este producto. El valor anterior de x se pierde

Otro ejemplo de una proposición de asignación es la siguiente: N = N+1, que significa reemplazar el valor de la variable N por su antiguo valor más 1.

3.4. Variables de precisión doble, complejas y lógicas

En algunos casos se requieren resolver problemas con gran precisión, es decir, se desea tener el mayor número de dígitos posible. Esto se puede hacer a través de variables de doble precisión.

Generalmente las variables reales son de precisión sencilla, las variables de doble precisión tienen dos veces la precisión de una variable real, o un poco más; esto depende de la computadora. En términos de palabra de memoria, una variable de precisión doble requiere dos palabras de memoria.

La proposición para declarar una variable de doble precisión tiene la forma:

DOUBLE PRECISION $v_1, v_2, v_3 \ldots$

donde v_1, v_2, v_3 son los nombres de las variables.

Una constante de precisión doble se escribe como una constante en forma exponencial, pero con una D en lugar de una E. Las siguientes son constantes de doble precisión:

1.5 DO

5.6 D-10 = $(5.6 \cdot 10^{-10})$

1.56568 DO

Las variables de precisión sencilla dan la exactitud que se requiere en la solución de un gran número de problemas. Sólo en casos muy especiales se usan variables de doble precisión.

En notación matemática los números complejos son pares ordenados de números (a,b), generalmente denotados por la expresión a+ib, donde i representa la raíz cuadrada de -1.

Los números complejos se declaran con la proposición:

COMPLEX v_1, v_2...

donde v_1, v_2 son nombres de variables.

Una variable lógica es aquélla que sólo puede tener los valores "verdadero" y "falso".

Una constante lógica es cualquiera de las siguientes dos:

.TRUE.

.FALSE.

Una proposición de asignación lógica tiene la forma:

a = b

en la cual a es una variable lógica y b una expresión lógica. Por ejemplo:

L = .TRUE.

M = .FALSE.

Una expresión lógica se forma con la combinación de operadores lógicos y operadores de relación.

Los operadores lógicos se muestran en la siguiente tabla.

OPERADOR	FUNCION
.AND.	Y
.OR.	O
.NOT.	NO

Los operadores de relación están definidos como:

OPERADOR	SIGNIFICADO
.LT.	menor que
.LE.	menor o igual que
.EQ.	igual que
.NE.	no igual a
.GT.	mayor que
.GE.	mayor o igual que

Como ejemplo considere las siguientes proposiciones:

A = X.LT.P.OR.T.GT.TMAX

B = M.GT.30. AND.N.GT.20

La variable A será verdadera si X es menor que P, ó T es mayor que TMAX; de la misma forma B será verdadera si M es mayor que 30 y N mayor que 20.

Uno de los usos más importantes de estas variables, es en la proposición IF lógica, que se verá más adelante.

4.- PROPOSICIONES DE ENTRADA Y SALIDA DE INFORMACION

Las proposiciones de entrada y salida, son las que permiten al usuario dar a la computadora los datos necesarios para la ejecución de algún programa en particular; y por otro lado, permiten que la computadora imprima los resultados del programa, en la forma que desee el usuario.

Para leer una tarjeta en FORTRAN se utilizan las proposiciones READ y FORMAT, las cuales están relacionadas entre sí. Una proposición de lectura puede ser:

```
    READ(5,10) N, QSAL, QENT
10  FORMAT(I5, 2F10.2)
```

En la proposición READ el número 5 se refiere a la unidad de entrada, de donde se leen las tarjetas; es decir, la lectora de tarjetas. Si no se definen al principio del programa archivos de disco ó cinta, el compilador considera el número 5, como la lectora de tarjetas.

El número 10 se refiere al número de la proposición FORMAT, en donde se indica a la computadora la forma en que vienen los datos que va a leer.

De esta forma la computadora asigna a cada una de las variables que aparecen en la proposición READ, el valor que lee en la tarjeta de datos.

Se tienen diferentes tipos de especificación para la proposición FORMAT, los más importantes son:

a) Especificación I

Esta se usa cuando la variable que se va a leer es una variable entera, su forma es:

Iw

donde w especifica el número total de caracteres de campo, incluyendo el signo.

Por ejemplo, la proposición I5 permite leer los siguientes números:

30

-1214

12501

b) Especificación F

La especificación de decimal fijo tiene la forma:

Fw.d

en donde w se refiere al número total de caracteres incluyendo signo y punto decimal. El número d representa el numero de caracteres después del punto decimal. Así por ejemplo la proposición:

FORMAT(F10.3, F15.5)

significará que el campo correspondiente a la primera variable, será de 10 caracteres con tres cifras decimales; para la segunda variable, se tendrá un campo de 15 caracteres, con 5 cifras decimales.

c) Especificación E

La forma de esta especificación es:

Ew.d

donde E especifica la conversión entre un valor real interno y un número externo escrito con un exponente. El número total de caracteres es w, incluyendo signo, punto decimal y exponente. El número de decimales después del punto es d.

d) Especificación A

Esta especificación tiene la forma:

Aw

y permite la lectura de cualquier carácter alfanumérico. Normalmente w no es mayor que el número de bytes, 2, 4 u 8, que ocupa la variable.

e) Especificación D

Esta se utiliza con los números de doble precisión y se perfora la letra D en lugar de la letra E.

La impresión en FORTRAN se hace a través de las proposiciones WRITE y FORMAT.

Una proposición de impresión puede ser:

```
      WRITE(6,50) N, QENT, QSAL
   50 FORMAT("1", ///, 10X, "NO. DE CELDA",
       I5, 10X, "GASTO DE ENTRADA", F10.3,
       10X, "GASTO DE SALIDA", F10.3).
```

En la proposición WRITE, si no se especifican archivos, el número 6 se refiere a la unidad de impresión del sistema.

El número 50 se refiere al número de la proposición FORMAT, en donde se indica a la computadora la forma en que se quiere la impresión de las variables.

Las formas de especificación de la proposición FORMAT discutidas para la lectura de datos, son también aplicables para el caso de impresión.

En adición se tienen otras proposiciones muy útiles para la impresión de resultados. Considere la proposicion FORMAT escrita arriba.

La primera instrucción ("1"), le indica a la computadora que debe saltar una hoja, esto es que empezará a escribir en una hoja nueva.

La siguiente instrucción (///), es la encargada del control vertical de impresión. Cada uno de los caracteres /, le indica a la computadora saltar un renglón antes de escribir; en este caso saltará tres renglones.

A continuación se da la instrucción encargada del espaciamiento horizontal, tiene la forma general wX, en donde w indica el número de espacios horizontales que se deben dejar antes de escribir. En este caso se dejarán 10 espacios en blanco a partir del margen izquierdo de la hoja.

Los encabezados que se quieran imprimir, se escriben entre comillas; así en éste ejemplo se imprimirá el letrero NO. DE CELDA.

La siguiente instrucción indica que se escribirá un número entero, con un máximo de 5 caracteres; después se dejarán 10 espacios en blanco y se imprimirá el letrero GASTO DE ENTRADA. A continuación se escribirá un número real (el correspondiente a la variable QENT), con un máximo de 10 caracteres y 3 decimales. Nuevamente se dejarán 10 espacios en blanco, y se imprime GASTO DE SALIDA; y el valor de la variable QSAL.

El número máximo de caracteres que se pueden imprimir en un renglón es 132. El primer carácter, a partir del margen izquierdo de la hoja, se usa para el control de la impresora y no puede ser usado para impresión.

Estas instrucciones de entrada y salida se entenderán mejor analizando los ejemplos de programas que se presentan más adelante.

5.- PROPOSICIONES DE CONTROL

Un programa se compila y se ejecuta siguiendo la secuencia en que se ha escrito. En todos los programas el control empieza con la primera proposición, después de la cual el control pasa a la siguiente y así sucesivamente, a menos que una proposición de control altere este orden.

Se discutirán a continuación las principales proposiciones de control.

5.1. Proposición GO TO

Esta proposición ocasiona una transferencia incondicional del control, tiene la forma:

GO TO n

en donde n es el número de una proposición.

Así GO TO 50, hace que la proposición 50 se ejecute, y que las proposiciones que le siguen se ejecuten secuencialmente.

5.2. Proposición calculada GO TO

En este caso, la transferencia de control está condicionada al valor de una variable entera. La forma ge neral de esta proposición es:

$$GO\ TO\ (n_1,\ n_2,\ n_3 \ldots n_n),\ I$$

donde I es una variable entera positiva y diferente de cero; n_1, n_2, n_3 son números de proposiciones que existen dentro del programa.

Considere la proposición:

GO TO (10, 50, 80, 100), NI

si NI es igual a 1, el control se transfiere a la proposición 10; si es igual a 2 a la proposición 50; si es igual a 3 a la proposición 80; etc.

5.3. Proposición aritmética IF

Esta proposición tiene la forma:

$$IF(A)\ n_1,\ n_2,\ n_3$$

en este caso la computadora calcula el valor de la expresión A. Si A es menor que cero, el control se transfiere a la proposición n_1; si es igual a cero a n_2; y si es mayor que cero a n_3.

Así en el ejemplo siguiente:

IF(N-30) 10, 20, 20

si N es menor que 30, el control pasa a la proposición 10; y si es mayor o igual que 30, a la proposición 20.

Note que la expresión A puede ser real o entera.

5.4. Proposición lógica IF

La proposición tiene la forma general:

IF(B) C

en donde B es una expresión lógica (ver sección 3.4), y C cualquier otra expresión, con excepción de las proposiciones IF y DO.

La forma en que trabaja esta proposición es la siguiente: si la expresión lógica es verdadera, se ejecuta entonces la proposición C y el control pasa a la siguiente proposición. Si la proposición C es GO TO n, el control pasará a la proposición n.

Así por ejemplo:

```
IF(N.LE.30) GO TO 10
IF(X.GT.5.0) DCA = 20.0*X
```

En el primer caso el control pasará a la proposición 10 si la variable N es menor o igual a 30.

En el segundo caso si la variable X es mayor o igual que 5.0, la variable DCA tomará el valor 20.0*X; y el control pasa a la siguiente proposición.

5.5. Proposición iterativa DO

Esta proposición es la más poderosa y posiblemente la más usada en FORTRAN. Tiene la forma siguiente:

$$DO\ n\ I = m_1, m_2, m_3$$

en donde n es el número de una proposición, I es una variable entera y m_1, m_2, m_3 son constantes enteras positivas mayores que cero.

La proposición DO es un comando para ejecutar todas las proposiciones que siguen al DO, hasta e incluyendo la proposición numerada con n. Esta ejecución se realiza iterativamente, primero con $I = m_1$, después con $I = m_1 + m_3$, posteriormente con $I = m_1 + 2m_3$ y así sucesivamente hasta el valor mayor de $I \leq m_2$.

Considere el siguiente ejemplo:

```
       L = 0
 DO 10 I = 1, 9, 2
       L = L + I
 10    M = I*L
```

En este caso el valor de L y M para la primera iteración serán L = 1 y M = 1; en la segunda iteración I = 3, L = 4 y M = 12; para la tercera iteración I = 5, L = 9 y M = 45; etc. En la última iteración I = 9, L = 25 y M = 225.

Si en la proposición DO m_3 es igual a 1, se puede escribir como:

$$DO\ n\ I = m_1, m_2$$

es decir, en este caso el incremento de I será de una unidad en una unidad.

Se tienen las siguientes reglas para el uso de la proposición DO.

1) La primera proposición en el recorrido de un DO, debe ser una proposición que pueda ejecutarse; esto excluye las proposiciones tales como DIMENSION, FORMAT, etc.; estas proposiciones de especificación proporcionan información al compilador acerca del programa; pero no dan lugar a la ejecución de algún cálculo.

2) La última proposición en el recorrido de un DO, no puede ser una proposición GO TO ó IF.

La proposición CONTINUE, es una proposición simulada que no produce funciones en el programa. y que se usa para ser la última proposición en el recorrido de un DO. Así por ejemplo:

```
      DO 30 I = /, 30
      F = X*VS+VS*Y-U
      DF = X+Y
      VS = VS-F/DF
   30 CONTINUE
```

3) No se permite ninguna proposición durante el recorrido de un DO, que redefina el valor del índice o de los parámetros m_1, m_2, m_3.

4) Es posible tener iteraciones DO dentro de otras iguales y, en este caso, el recorrido del DO más interior debe estar completamente dentro del recorrido del DO exterior.

   No hay límite para el número de iteraciones DO permisibles una dentro de la otra.

5) No se permite ninguna transferencia de control dentro del recorrido de un DO exterior, al recorrido de un DO interior.

Existen un sinnúmero de aplicaciones de esta proposición, que es imposible describirlas. El uso y dominio de esta proposición se logra con la experiencia, analizando y escribiendo programas.

6.- E J E M P L O S

6.1. Programa para calcular las características hidráulicas de un canal de sección circular

Usando la ecuación de Maning, la velocidad en un canal está dado por la ecuación:

$$V = \frac{1}{n} R^{2/3} S^{1/2}$$

donde:

V velocidad (m/seg).

n coef. de rugosidad

R - radio hidráulico

S - pendiente

El radio hidráulico está definido por:

$$R = \frac{A}{P}$$

donde:

A - área del canal

P - perímetro mojado

Si el canal está lleno, se tienen las siguientes relaciones:

1

$$P = \pi . D$$

$$R = D/4$$

$$V = \frac{1}{n} R^{2/3} S^{1/2}$$

$$Q = A.V$$

Considerando la figura, se pueden demostrar las siguientes relaciones:

$$y = \frac{1}{2} - \frac{1}{2} \cos \frac{\theta}{2} \quad . D$$

$$p = \frac{P.\theta}{2\pi}$$

$$r = R. \quad 1 - \frac{SEN(\theta)}{2p.D}$$

$$v = V. \quad \frac{r}{R}^{2/3}$$

$$a = \frac{p.r}{P.R} A$$

$$q = \frac{v.a}{V.A} Q$$

A continuación se muestra el listado del programa que calcula las características hidráulicas de un canal de sección circular, usando las ecuaciones descritas. El programa es sencillo y no requiere de una explicación detallada.

CARACTERISTICAS DEL CANAL

DIAMETRO (M) 2.000

COEFICIENTE DE RUGOSIDAD 0.0300

PENDIENTE DEL CANAL 0.003

ALTURA DEL AGUA (M)	PERIMETRO MOJADO (M)	RADIO HIDRAULICO (M)	VELOCIDAD (M/SEG)	AREA SECCION (M**2)	GASTO (M**3/SEG)
0.0038	0.1745	0.3756	1.3776	0.0655	0.0903
0.0152	0.3490	0.3775	1.3822	0.1318	0.1821
0.0341	0.5235	0.3806	1.3898	0.1993	0.2769
0.0603	0.6980	0.3849	1.4002	0.2687	0.3762
0.0937	0.8725	0.3903	1.4131	0.3405	0.4812
0.1339	1.0470	0.3966	1.4284	0.4153	0.5932
0.1808	1.2215	0.4038	1.4457	0.4933	0.7132
0.2339	1.3960	0.4118	1.4646	0.5749	0.8420
0.2928	1.5705	0.4204	1.4849	0.6603	0.9804
0.3571	1.7450	0.4295	1.5061	0.7494	1.1287
0.4263	1.9195	0.4388	1.5279	0.8423	1.2869
0.4998	2.0940	0.4483	1.5498	0.9387	1.4548
0.5772	2.2685	0.4578	1.5716	1.0385	1.6320
0.6578	2.4430	0.4671	1.5928	1.1411	1.8175

6.2. Programa para calcular la velocidad de sedimentación de partículas discretas

La velocidad de sedimentación de una partícula está dada por la ecuación:

$$V_S = \left(\frac{4}{3}\frac{g}{C_a}(De - 1)\,D\right)^{1/2} \qquad (1)$$

donde:

- V_S - velocidad de sedimentación (cm/seg).
- g - aceleración de la gravedad (cm/seg^2).
- De - densidad de la partícula (gr/cm^3).
- D - diámetro de la partícula (cm)
- C_a - coeficiente de arrastre.

El coeficiente de arrastre se calcula usando las ecuaciones:

$$C_a = \frac{24}{R} \qquad \text{Si } R \leq 0.5 \qquad (2)$$

$$C_a = \frac{24}{R} + \frac{3}{R^{1/2}} + 0.34 \qquad \text{Si } 0.5 < R < 1000 \qquad (3)$$

$$C_a = 0.4 \qquad \text{Si } R \geq 1000 \qquad (4)$$

donde R es el número de Reynolds definido como:

$$R = \frac{V_S \cdot D}{V_C} \qquad (5)$$

donde V_C es la viscosidad cinemática definida como:

```
C     CHCSC= PROGRAMA PARA CALCULAR LAS CARACTERISTICAS
C            HIDRAULICAS DE UN CANAL DE SECCION CIRCULAR.
C     SIMBOLOS USADOS
C     AI=AREA DEL CANAL LLENO
C     ANG=ANGULO CORRESPONDIENTE A LA ALTURA Y DEL AGUA EN EL CANAL
C     CR=COEF. DE RUGOSIDAD
C     D=DIAMETRO DEL CANAL
C     PI=PERIMETRO MOJADO DEL CANAL LLENO
C     PJ=PERIMETRO MOJADO
C     QI=GASTO DEL CANAL LLENO
C     QJ=GASTO
C     RI=RADIO HIDRAULICO DEL CANAL LLENO
C     RJ=RADIO HIDRAULICO
C     S=PENDIENTE DEL CANAL
C     VI=VELOCIDAD DEL AGUA EN EL CANAL LLENO
C     VJ=VELOCIDAD DEL AGUA
C     Y=ALTURA DEL AGUA EN EL CANAL
C     LEER CARACTERISTICAS DE LA SECCION
   10 READ(5,20) D, CR, S
   20 FORMAT(3F10.0)
C     NO. DE INTERVALOS, VALOR DEL INTERVALO
      READ(5,30) NI, DA
   30 FORMAT(I5,F10.3)
      WRITE(6,40) D, CR, S
   40 FORMAT('1',10X,'CARACTERISTICAS DEL CANAL',//,10X,
     1'DIAMETRO (M)',F10.3,//,10X,'COEFICIENTE DE RUGOSIDAD',F10.4,
     2//,10X,'PENDIENTE DEL CANAL',F10.3)
C     CARACTERISTICAS CANAL LLENO
      AI=0.7854*D*D
      RI=D/4.
      PI=3.1416*D
      VI=((D/2.0)**0.6666)*(SQRT(S))/CR
      QI=VI*AI
      WRITE(6,50)
   50 FORMAT(///,5X,'ALTURA DEL AGUA (M)',3X,'PERIMETRO MOJADO (M)',
     1 3X,'RADIO HIDRAULICO (M)',3X,'VELOCIDAD (M/SEG)',3X,
     2 'AREA SECCION (M**2)',3X,'GASTO (M**3/SEG)',///)
      ANG=0.0
      DO 70 I=1, NI
      ANG=ANG+DA
      Y=(0.5-0.5*COS(ANG/2.0))*D
      RJ=PI*ANG/6.2832
      RJ=RI*(1.0-SIN(ANG)/(2.0*PJ/D))
      VJ=VI*((RJ/RI)**0.666)
      AJ=PJ*RJ*AI/(PI*RI)
      QJ=VJ*AJ*QI/(VI*AI)
      WRITE(6,60) Y, PJ, RJ, VJ, AJ, QJ
   60 FORMAT(//,5X,F10.4,16X,F10.4,13X,F10.4,12X,F10.4,
     112X,F10.4,12X,F10.4)
   70 CONTINUE
      GO TO 10
      END
```

$$V_C = \frac{0.7806}{1.8 \cdot T + 42} \qquad (6)$$

donde T es la temperatura (°C)

Sustituyendo la ecuación (2) en la ecuación (1) se obtiene:

$$V_S = \frac{g(De-1)D^2}{18\ V_C} \qquad R \leq 0.5 \qquad (7)$$

Sustituyendo la ecuación (4) en la ecuación (1):

$$V_S = \left[\frac{4g.(De-1).D}{1.2}\right]^{1/2} \qquad R \geq 1000.0 \qquad (8)$$

y sustituyendo la ecuación (3) en la ecuación (1):

$$0.34\ V_S^2 + \frac{3\ V_C^{1/2}}{D^{1/2}}\ V_S^{3/2} + \frac{24\ V_C}{D}\ V_S$$

$$- \frac{4}{3}\ g\ (De-1) \cdot D = 0 \qquad (9)$$

$$0.5 < R < 1000.0$$

Esta ecuación se puede escribir como:

$$F = X.V_S^2 + Y \cdot V_S^{3/2} + Z \cdot V_S + U = 0 \qquad (10)$$

donde:

$$X = 0.34$$

$$Y = 3\ \left(\frac{V_C}{D}\right)^{1/2}$$

$$Z = \frac{24 \cdot V_C}{D}$$

$$U = -\frac{4}{3}\, g\, (De-1) \cdot D$$

Para obtener la solución de la ecuación (10), se usará el método de Newton-Ramphson. La derivada de la función F con respecto a V_S es

$$\frac{dF}{dV_S} = 2 \cdot X \cdot V_S - \frac{3}{2} \cdot Y \cdot V_S^{1/2} + Z \tag{11}$$

y el valor de V_S en la $k+1$ iteración es:

$$V_S^{k+1} = V_S^{k} - \frac{F(V_S^{k})}{F'(V_S^{k})} \tag{12}$$

A continuación se describirá el programa **VELSED**, que calcula la velocidad de sedimentación de una partícula; sus partes principales son:

1) Leer el diámetro de la partícula (D), su densidad (DEN), la temperatura (T). Calcular la viscosidad cinemática (VC), usando la ecuación (6).

2) Calcular la velocidad de sedimentación (V_S), usando la ecuación (7). Verificar si el número de Reynolds es menor o igual que 0.5, si éste es el caso termina el proceso; en caso contrario:

3) Calcular la velocidad de sedimentación usando la ecuación (4).

   Verificar que el número de Reynolds sea mayor o igual que 1000.0, si éste es el caso termina el proceso; en caso contrario;

4) Calcular la velocidad de sedimentación, resolviendo la ecuación (10).

   Se tiene un máximo de 30 iteraciones para encontrar el valor de V_s que satisfaga la ecuación (10), con un criterio de error. Si no encuentra la solución, imprime un mensaje y termina el proceso.

5) Imprimir los resultados.

```
      [illegible]
C     VELSED= PROGRAMA PARA CALCULAR LA VELOCIDAD DE SEDIMENTACION
C                DE UNA PARTICULA DISCRETA
C     SIMBOLOS USADOS
C     D=DIAMETRO DE LA PARTICULA
C     DEN=DENSIDAD DE LA PARTICULA
C     R=NO. DE REYNOLDS
C     T=TEMPERATURA
C     VC=VISCOSIDAD CINEMATICA
C     VS=VELOCIDAD DE SEDIMENTACION
C     LEER DIAMETRO DE LA PARTICULA, SU DENSIDAD Y LA TEMPERATURA
   10 READ (5,20) D, DEN, T
   20 FORMAT(3F10.4)
C     CALCULAR VISCOSIDAD CINEMATICA
      VC=[illegible]
C     VELOC. DE SEDIMENTACION SI R MENOR O IGUAL QUE [illegible]
      VS=[illegible]*(DEN-1.0)*D*D/VC
      R=VS*D/VC
      IF(R.[illegible]) [illegible]
C     VEL[illegible] SEDIMENTACION SI R MAYOR [illegible] QUE [illegible]
      VS=[illegible](DEN-1.0)[illegible]
      R=VS*D/VC
      IF(R.GT.1000.0) GO TO 50
C     VELOC. DE SEDIMENTACION SI R MAYOR QUE 0.5 Y MENOR QUE 1000.0
C     CALCULAR COEFICIENTES ECUACION POR RESOLVER
      X=0.34
      Y=3.[illegible]*SQRT(VC/D)
      Z=2[illegible]*VC/D
      U=1[illegible]*(DEN-1.0)*D
C     METODO ITERATIVO
      COMP=0.0
      VS=10.0
      DO 30 I=1,[illegible]
      F=X[illegible]
      IF(ABS(F).LT.[illegible]) GO TO 50
      DF=2.[illegible]
      VS=VS-F/DF
      IF(ABS(COMP-VS).GT.0.00001) GO TO 25
      VS=VS/2.0
   25 COMP=VS
      IF(VS.LT.0.0) VS=-VS
   30 CONTINUE
      WRITE(6,40)
   40 FORMAT(//,1OX,'NO SE ENCONTRO SOLUCION [illegible]',F10.4)
      CONTINUE
   50 WRITE(6,60)D, DEN, T, VS
   60 FORMAT(//,[illegible],'DIAMETRO DE LA PARTICULA [illegible]
     1'DENSIDAD (GR./CC.)',F10.4,//,[illegible]),F8.3,//,
     2[illegible],'VELOCIDAD DE SEDIMENTACION (CM./SEG.)',F[illegible])
      GO TO 10
      END
```

DIAMETRO DE LA PARTICULA 0.100

DENSIDAD (GR./CC.) 2.650

TEMPERATURA (O C) 4.000

VELOCIDAD DE SEDIMENTACION (CM./SEG.) 15.602250

DIAMETRO DE LA PARTICULA [illegible]

DENSIDAD (GR./CC.) 2.650

TEMPERATURA (O C) 4.000

VELOCIDAD DE SEDIMENTACION (CM./SEG.) [illegible]

DIAMETRO DE LA PARTICULA [illegible]

DENSIDAD (GR./CC.) 2.650

TEMPERATURA (O C) 4.00

VELOCIDAD DE SEDIMENTACION (CM./SEG.) 8.081072

DIAMETRO DE LA PARTICULA [illegible]

DENSIDAD (GR./CC.) 2.650

TEMPERATURA (O C) 4.000

VELOCIDAD DE SEDIMENTACION (CM./SEG.) 6.597718

DIAMETRO DE LA PARTICULA [illegible]

DENSIDAD (GR./CC.) 2.650

TEMPERATURA (O C) 4.000

[illegible]

DIAMETRO DE LA PARTICULA [illegible]

DENSIDAD (GR./CC.) 2.650

TEMPERATURA (O C) 4.000

VELOCIDAD DE SEDIMENTACION (CM./SEG.) [illegible]

DIAMETRO DE LA PARTICULA [illegible]

DENSIDAD (GR./CC.) 2.650

[illegible]

VELOCIDAD DE SEDIMENTACION (CM./SEG.) [illegible]

DIAMETRO DE LA PARTICULA [illegible]

DENSIDAD (GR./CC.) 2.650

TEMPERATURA (0 C) [illegible]

VELOCIDAD DE SEDIMENTACION (CM./SEG.) 2.3409[illegible]

DIAMETRO DE LA PARTICULA [illegible]

DENSIDAD (GR./CC.) [illegible]

TEMPERATURA (0 C) [illegible]

VELOCIDAD DE SEDIMENTACION (CM./SEG.) [illegible]

TAREA

1) El gasto de abastecimiento de agua de una población está dada por la ecuación:

$$Q_a = 204 \cdot P^{1.25}$$

El gasto contra incendios por la ecuación:

$$Q_i = 3.8607 \cdot P^{1/2} \cdot (1.0 - 0.01 \cdot P^{1/2})$$

El coeficiente de Harmon como:

$$C = \frac{18 + P^{1/2}}{4 + P^{1/2}}$$

donde P es la población en millares de habitantes.

Hacer un programa que calcule los gastos de abastecimiento, contra incendios y el coeficiente de Harmon. Suponga que la población varía entre 10,000 y 200,000 habitantes y que los cálculos se quieren en incrementos de 10,000 habitantes.

2) Usando la ecuación de Hazen-Williams, la resistencia de un tubo está dada por la ecuación:

$$R = \frac{1.131 \cdot 10^9 \, X}{D^{4.872} \cdot C^{1.852}}$$

donde.

X longitud (m)

D - diâmetro (mm)

C - coeficiente de Hazen-Williams

Hacer un programa que calcule la resistencia de un tubo; la longitud y el diámetro del tubo se leen, el coeficiente de Hazen-Williams deberá variar entre 80.0 y 160.0, en intervalos de 20.0.

3) La media de un conjunto de números se define como:

$$\bar{X} = \frac{\sum_{i=1}^{N} X_i}{N}$$

la varianza como:

$$S^2 = \frac{N \sum_{i=1}^{N} (X_i^2) - (\Sigma X_i)^2}{N (N-1)}$$

y la desviación estandar:

$$S = (S^2)^{1/2}$$

Hacer un programa que calcule la media y la desviación estandar de una serie de datos.

7.- PROPOSICIONES DE ESPECIFICACION

Hasta ahora se han visto proposiciones que permiten hacer un gran número de programas sencillos; sin embargo, se tiene serias limitaciones si se quieren hacer programas más complicados.

Suponga, por ejemplo, que se tiene una serie grande de datos que se van a usar repetidamente en la ejecución de un programa. Con las instrucciones que se han visto hasta ahora, este problema se podría resolver leyendo los datos cada vez que se necesiten. El uso de variables con índice permite reservar una parte de la memoria de la computadora para guardar estos datos y usarlos repetidamente.

A las variables con índice se les llama normalmente arreglos y a sus cantidades individuales elementos. Para visualizar la forma en que la computadora almacena información usando arreglos, piense en una matriz. En una matriz cada uno de sus elementos representa un número; de la misma forma en un arreglo la computadora reserva una o varias palabras, para cada uno de los elementos del arreglo. Por ejemplo, el arreglo A(30,30), se puede visualizar como una matriz de 30 x 30 elementos.

7.1. Proposición DIMENSION

Esta proposición sirve para indicar al compilador los arreglos que se van a usar y la dimensión de cada uno de ellos. Generalmente esta proposición aparece al principio del programa.

Esta proposición tiene la forma:

DIMENSION V_1, V_2, ..., V_n

en donde V_i representa el nombre del arreglo, seguido por paréntesis que encierran una o más constantes enteras, positivas y mayores que cero; que indican la dimensión del arreglo.

Por ejemplo:

DIMENSION A(20,10), DATOS([illegible])

Para estas variables se sigue la misma convención, que para las variables [illegible]; es decir, si se empieza con la letra I, J, K, L, M o N, se considera que todos sus elementos son enteros; en caso contrario, sus elementos serán [illegible] numéricos. El nombre de cada variable debe empezar con un carácter alfabético y su longitud no debe ser mayor de [illegible] caracteres.

Una vez que se ha dimensionado una variable con índice, ésta sólo se puede hacer referencia a ella, en el programa, a través de índices. Por ejemplo, suponga que se quieren leer datos para el arreglo A(20,10); esto se hace de la siguiente forma:

```
      DO 10 I = 1,20
      DO 10 J = 1,10
      READ (5,20) A (I,J)
20    FORMAT (F10.2)
10    CONTINUE
```

A través de los dos DO iterativos se va recorriendo todo el arreglo; y en cada uno de sus elementos se va almacenando un número.

Considere ahora el siguiente ejemplo que hace la suma de dos matrices:

```
      DO 30 I = 1,N
      DO 30 J = 1,M
30    C(I,J) = A(I,J) + B(I,J)
```

Sería muy laborioso y de poca utilidad describir un gran número de ejemplos ilustrativos sobre el uso de arreglos. Nuevamente el hacer programas de computadora es lo que permite conocer y usar las variables con índice.

8.- E J E M P L O S

8.1. Programa para resolver un sistema de ecuaciones lineales, usando el método de Gauss-Jordan

Suponga que se quiere resolver el sistema de ecuaciones:

$$2X_1 - 7X_2 + 4X_3 = 9$$

$$X_1 + 9X_2 - 6X_3 = 1$$

$$-3X_1 + 8X_2 + 5X_3 = 6$$

Se define la matriz aumentada como:

2	-7	4	9	1	0	0
1	9	-6	1	0	1	0
-3	8	5	6	0	0	1

El método consiste en normalizar el primer renglón, dividiéndolo por el elemento pivote 2; después reducir los elementos restantes de la primera columna a cero, restando el primer renglón (normalizado) del segundo renglón; y restando del tercer renglón el primer renglón (normalizado), multiplicado por -3.

1	-7/2	2	9/2	1/2	0	0
0	25/2	-8	-7/2	-1/2	1	0
0	-5/2	11	39/2	3/2	0	1

Ahora se normaliza el segundo renglón dividiéndolo por 25/2, y se reducen a cero los elementos restantes de la segunda columna, multiplicando el segundo renglón (normalizado) por -7/2 y restándolo del primer renglón. De la misma forma se multiplica el segundo renglón (normalizado) por -5/2 y se resta del tercer renglón.

1	0	-6/25	88/25	9/25	7/25	0
0	1	-16/25	-7/25	-1/25	2/25	0
0	0	47/5	94/5	7/5	1/5	1

Procediendo de la misma forma se obtiene finalmente:

1	0	0	4	93/235	67/235	6/235
0	1	0	1	13/235	22/235	16/235
0	0	1	2	7/47	1/47	5/47

La solución del sistema de ecuaciones es: $X_1 = 4$, $X_2 = 1$, $X_3 = 2$; y la matriz resultante es la matriz inversa asociada.

El listado del programa que se presenta a continuación, resuelve un sistema de ecuaciones lineales, siguiendo el método descrito. Es sencillo y no requiere mayor descripción.

```
C
C      [illegible]
C      DR-[illegible]
C      DT-[illegible]
C      H(I,J)=ALTURA DEL NIVEL [illegible] INTERVALO
C               DE TIEMPO I, EN EL INTERVALO DE DISTANCIA J
C      HO=ALTURA DEL NIVEL ESTATICO [illegible] INICIAR EL BOMBEO
C      NR=NO. DE INCREMENTOS DE DISTANCIA
C      NT=NO. DE INCREMENTOS DE TIEMPO
C      Q=GASTO DE BOMBEO DEL POZO
C      R(I)=DISTANCIA (M), CORRESPONDIENTE AL INTERVALO I
C      S=COEF. DE ALMACENAMIENTO
C      T(I)=TIEMPO (DIAS) CORRESPONDIENTE AL INTERVALO I
C      TR=TRANSMISIVIDAD DEL ACUIFERO

       DIMENSION T(50), R(50), H(50,50)
C      LEER NO. INCREMENTOS DE TIEMPO, DISTANCIA, Y SUS VALORES
   10 READ(5,20) NT, NR, DT, DR

   20 FORMAT(2I[illegible],2F[illegible])
C      LEER CARACTERISTICAS DEL ACUIFERO, [illegible]
       READ(5,30) TR, S, HO, Q
   30 FORMAT(F[illegible],2,F10.6,F[illegible],2)
       WRITE(6,40) TR, S, HO, Q

   40 FORMAT(//,10X,'TRANSMISIVI[illegible]',F10.2,
     1//,10X,'COEFICIENTE DE ALMACENAMIENTO',F[illegible],//,10X,
     2'ALTURA INICIAL DEL NIVEL ESTATICO [illegible]',//,10X,
     3'GASTO DE BOMBEO(L**3/T)',F10.2)
C      CALCULAR INTERVALOS DE TIEMPO Y DE DISTANCIA
       T(1)=DT
       R(1)=DR
       DO 50 I=2, NT
   50 T(I)=T(I-1)+DT
       DO 60 I=2, NR
   60 R(I)=R(I-1)+DR
C      CALCULAR ABATIMIENTOS
       C=-Q*24.0/(4.0*3.1416*TR)
       DO 90 I=1, NT
       DO 90 J=1, NR
C      CALCULAR FUNCION U
       U=S*R(J)*R(J)/(4.0*TR*T(I))
       F=1.0
       SUMA=0.0
C      CALCULAR SERIE
       DO 70 K=1, 50
       F=F*K
       FI=((-1.0)**K)*(U**K)/(K*F)
       SUMA=SUMA+FI
       IF(ABS(FI).LE.0.0001) GO TO [illegible]
   70 CONTINUE
   80 H(I,J)=[illegible](-0.5772-ALOG(U)+SUMA)
   90 CONTINUE
C      IMPRESION DE RESULTADOS
       L1=(NR-1)/15+1
       DO 1[illegible] I=1, L1
       L2=[illegible](L-[illegible])
```

MATRIZ ORIGINAL

2.000	-7.000	4.000	0.000	1.000	0.000	0.000
1.000	9.000	-6.000	1.000	0.000	1.000	0.000
-3.000	6.000	5.000	6.000	0.000	0.000	1.000

MATRIZ INVERSA

1.00	0.00	0.00	4.00	0.40	0.29	0.03
0.00	1.00	0.00	1.00	0.06	0.09	0.07
0.00	0.00	1.00	2.00	0.15	0.02	0.11

8.2. Programa para calcular el abatimiento del nivel freático de un pozo con simetría radial

La ecuación que describe el abatimiento del nivel freático de un pozo con simetría radial, cuya profundidad abarca todo el espesor del acuífero, es:

$$h = h_o - \frac{Q}{4\pi T} \left(-0.5772 - \ln (u) + u - \frac{u^2}{2\cdot 2!} + \frac{u^3}{3\cdot 3!} - \frac{u^4}{4\cdot 4!} + \cdots \right) \quad (1)$$

donde:

h_o - altura del nivel freático (m) antes de iniciar el bombeo

Q - gasto de bombeo del pozo (m^3/diám.)

$$u - \frac{r^2 . S}{4 \cdot T \cdot t} \quad (2)$$

r - distancia del centro del pozo (m)

S - coeficiente de almacenamiento

T - coeficiente de transmisividad (m^2/diám.)

t - tiempo (días)

El programa que se describirá a continuación, calcula la altura del nivel freático, a diferentes distancias del pozo y a diferentes intervalos de tiempo. Sus partes principales son:

1) Leer el número de intervalos de tiempo

(NT), de distancia (NR) y el valor de cada uno de ellos (DT, DR).

Leer el coeficiente de transmisividad (TR), el coeficiente de almacenamiento (S), la altura inicial del nivel freático (HO), el gasto de bombeo (Q).

2) Calcular el abatimiento del nivel freático.

   Para cada uno de los tiempos T(I), y de las distancias R(I):

   a) Calcular la función U, definida por la ecuación (2).

   b) Calcular la serie asociada a la ecuación (1).

   c) Calcular la altura del nivel freático (H(I,J)), usando la ecuación (1).

3) Imprimir los resultados.

[illegible]

```
C     POZOSIPA- PROGRAMA PARA CALCULAR EL ABATIMIENTO DEL
C               NIVEL ESTATICO DURANTE EL BOMBEO DE UN POZO
C             CON SIMBOLOS DADOS
C     SIMBOLOS USADOS
C     DR=DURACION INCREMENTO DISTANCIA
C     DT=DURACION INCREMENTO DE TIEMPO
C     H(I,J)=ALTURA DEL NIVEL ESTATICO (M), PARA EL INTERVALO
C           DE TIEMPO I, EN EL INTERVALO DE DISTANCIA J
C     H0=ALTURA DEL NIVEL ESTATICO, ANTES DE INICIAR EL BOMBEO
C     NR=NO. DE INCREMENTOS DE DISTANCIA
C     NT=NO. DE INCREMENTOS DE TIEMPO
C     Q=GASTO DE BOMBEO DEL POZO
C     R(I)=DISTANCIA (M), CORRESPONDIENTE AL INTERVALO I
C     S=COEF. DE ALMACENAMIENTO
C     T(I)=TIEMPO (DIAS) CORRESPONDIENTE AL INTERVALO I
C     TR=TRANSMISIVIDAD DEL ACUIFERO

      DIMENSION T(50), R(50), H(50,50)
C     LEER NO. INCREMENTOS DE TIEMPO, DISTANCIA, Y SUS VALORES
   10 READ(5,20) NT, NR, DT, DR

   20 FORMAT(2I5,2F10.2)
C     LEER CARACTERISTICAS DEL ACUIFERO, GASTO DE BOMBEO
      READ(5,30) TR, S, H0, Q
   30 FORMAT(F10.2,F10.6,2F10.2)
      WRITE(6,40) TR, S, H0, Q

   40 FORMAT(////,10X,'TRASMISIVIDAD DEL ACUIFERO (M**2/DIA)',F10.2,
     1//,10X,'COEFICIENTE DE ALMACENAMIENTO',F10.6,//,10X,
     2'ALTURA INICIAL DEL NIVEL ESTATICO (M)',F10.2,//,10X,
     3'GASTO DE BOMBEO(M**3/DIA)',F10.2)
C     CALCULAR INTERVALOS DE TIEMPO Y DE DISTANCIA
      T(1)=DT
      R(1)=DR
      DO 50 I=2, NT
   50 T(I)=T(I-1)+DT
      DO 60 I=2, NR
   60 R(I)=R(I-1)+DR
C     CALCULAR ABATIMIENTOS
      C=-Q*24.0/(4.0*3.1416*TR)
      DO 90 I=1, NT
      DO 90 J=1, NR
C     CALCULAR FUNCION U
      U=S*R(J)*R(J)/(4.0*TR*T(I))
      F=1.0
      SUMA=0.0
C     CALCULAR SERIE
      DO 70 K=1, 50
      F=F*K
      FI=((-1.0)**K)*(U**K)/(K*F)
      SUMA=SUMA+FI
      IF(ABS(FI/SUMA).LE.0.0001) GO TO 80
   70 CONTINUE
   80 H(I,J)=H0+C*(-0.5772-ALOG(U)+SUMA)
   90 CONTINUE
C     IMPRESION DE RESULTADOS
      L1=(NR-1)/10+1
      DO 100 L=1, L1
      L2=10*(L-1)
```

```
  110 [illegible]
  120 FORMAT(/ [illegible]
      WRITE(6,130) [illegible]
  130 FORMAT(20X,10E10.3)
      DO 140 I=1, NT
  140 WRITE(6,150) T(I), [illegible](I,J), J=L0, [illegible])
  150 FORMAT(/,5X,F10.2,5X,10E15.2)
  160 CONTINUE
      GO TO 10
      END
```

TRANSMISIVIDAD DEL ACUIFERO EN M2/DIA 5000.00

COEFICIENTE DE ALMACENAMIENTO 0.000010

ALTURA INICIAL DEL NIVEL FREATICO (M) 50.00

GASTO DE BOMBEO(M**3/HR) 150.00

TIEMPO (DIAS)	DISTANCIA (M)					
	20.000	40.000	60.000	80.000	100.000	120.000
1.00	48.63	48.76	48.84	48.90	48.94	48.97
2.00	48.56	48.70	48.77	48.83	48.87	48.91
3.00	48.53	48.66	48.74	48.79	48.83	48.87
4.00	48.50	48.63	48.71	48.76	48.81	48.84
5.00	48.48	48.61	48.69	48.74	48.78	48.82
6.00	48.46	48.59	48.67	48.72	48.77	48.80
7.00	48.45	48.58	48.65	48.71	48.75	48.79
8.00	48.43	48.56	48.64	48.70	48.74	48.77
9.00	48.42	48.55	48.63	48.69	48.73	48.76
10.00	48.41	48.54	48.62	48.68	48.72	48.75

TIEMPO (DIAS)	DISTANCIA (M)					
	220.000	240.000	260.000	280.000	300.000	320.000
1.00	49.09	49.11	49.12	49.13	49.15	49.16
2.00	49.02	49.04	49.05	49.07	49.08	49.09
3.00	48.98	49.00	49.02	49.03	49.04	49.06
4.00	48.96	48.97	48.99	49.00	49.02	49.03
5.00	48.94	48.95	48.97	48.98	48.99	49.01
6.00	48.92	48.93	48.95	48.96	48.98	48.99
7.00	48.90	48.92	48.93	48.95	48.96	48.97
8.00	48.89	48.91	48.92	48.94	48.95	48.96

T A R E A

Las ecuaciones normales para la aproximación de un conjunto de puntos $(X_1,Y_1), (X_2,Y_2) \ldots (X_n,Y_n)$, a una cuna de la forma:

$$Y = a_0 + a_1X + a_2X^2$$

están dadas por:

$$a_0N + a_1\Sigma X + a_2\Sigma X^2 = \Sigma Y$$
$$a_0\Sigma X + a_1\Sigma X^2 + a_2\Sigma X^3 = \Sigma X\cdot Y$$
$$a_0\Sigma X^2 + a_1\Sigma X^3 + a_2\Sigma X^4 = \Sigma X^2\cdot Y$$

Hacer un programa que lea las coordenadas de un conjunto de N puntos, y calcule los coeficientes de las ecuaciones de las ecuaciones normales. Estos coeficientes se guardarán en la matriz A(3,7).

9.- PROPOSICIONES PARA USO DE SUBPROGRAMAS

En la elaboración de programas, muy a menudo es necesario realizar conjuntos de cálculo repetidamente, o en diferentes partes de un programa; suponga por ejemplo la velocidad de sedimentación de una partícula. Sería muy laborioso escribir este algoritmo en cada una de las partes del programa, donde se requiere. Es más eficiente escribir un subprograma que calcule únicamente la velocidad de sedimentación y llamarlo en el programa principal cada vez que se necesite.

En programas grandes, el uso de subprogramas hace que sean más fáciles de entender y de seguir.

9.1. Subprograma FUNCTION

Un subprograma FUNCTION difiere de las funciones vistas anteriormente, en que puede usarse más de una proposición para escribir la función. Su forma general es:

```
FUNCTION nombre (a1, a2, a3... )
    ════
proposiciones
    ════
RETURN
    ════
END
```

A las funciones pueden dárseles nombres, bajo las mismas condiciones que las variables regulares. Las funciones tienen argumentos $a_1, a_2, \ldots a_n$; cuando menos uno de los argumentos no debe ser una variable con índice, el nombre de otra función o de otro subprograma.

Un subprograma FUNCTION puede tener cualquier número de proposiciones; pero la última ejecutada debe ser RETURN, que transfiere el control nuevamente, al programa principal. La proposición END debe ser la última y aún cuando puede haber varias RETURN, debe existir una sola END.

Como ejemplo considere la siguiente función que calcula el abatimiento del nivel freático de un pozo, visto en la sección anterior:

```
      FUNCTION HI(C,U)
      F = 1.0
      SUMA = 0.0
      DO 10 K = 1,50
      F = F*K
      FI = ((-1.0*K)*(U**K)/(K*F)
      SUMA = SUMA + FI
      IF(ABS(FI/SUMA).LE.0.0001) GO TO 20
   10 CONTINUE
   20 HI = C*(-0.5772-ALOG(U) + SUMA)
      RETURN
      END
```

La altura del nivel freático, en el programa principal se calcula como:

H(I,J) = HO + HI(C,U)

9.2. Subprogramas SUBROUTINE

Los subprogramas FUNCTION pueden tener muchos argumentos, pero siempre un resultado explícito. En el caso de que se desee más de un resultado, se puede usar el subprograma SUBROUTINE que tiene la forma:

```
SUBROUTINE nombre (a1, a2, a3... )
    ===
proposiciones
    ===
RETURN
    ===
END
```

La proposición RETURN debe ser la última de las proposiciones ejecutadas y regresa al programa principal que lo llama; END debe ser físicamente la última proposición en el subprograma.

Como en el subprograma FUNCTION, el subprograma SUBROUTINE es un programa separado del principal y, por lo tanto todos los nombres de las variables son locales, esto es, se definen sólamente para el subprograma y no tienen conexión alguna con cualquier variable del programa principal.

Es válido que un subprograma no tenga argumentos, pero si existen, éstos siguen las mismas reglas que en los subprogramas FUNCTION.

Explícitamente, la subrutina no da ningún resultado, pero implícitamente se pueden obtener tantos resultados como se deseen; por ejemplo, las proposiciones:

```
SUBROUTINE FNH (X,Y,Z,W)
Z = X+Y
W = X-Y
RETURN
END
```

tomarán las variables X e Y como entrada y producirán al regreso las variables Z y W.

En el programa principal, para especificar las variables implicadas, se usa la proposición CALL, que tiene la forma siguiente:

CALL nombre (a_1, a_2, a_3...)

en donde nombre es el nombre de la subrutina y a_1, a_2, a_3 ... son los argumentos de la subrutina.

Si en una subrutina se usan arreglos es necesario especificar la dimensión de cada uno de ellos; obviamente la dimensión de cada arreglo debe ser congruente con la dimensión de los arreglos del programa principal.

En el siguiente ejemplo se muestra una subrutina que hace la suma de dos matrices:

```
      SUBROUTINE SUMA (A,B,C)
      DIMENSION A(10,10), B(10,10), C(10,10)
      DO 10 I = 1,10
      DO 10 J = 1,10
   10 C(I,J) = A(I,J) + B(I,J)
      RETURN
      END
```

Un programa es más eficiente si necesita menor tiempo de computadora para su ejecución y menor cantidad de memoria. Si en un programa uno o varios arreglos van a ser comunes a uno o varios subprogramas, no tiene objeto reservar memoria en cada subprograma para cada uno de los arreglos; es mejor reservar un área de memoria común para todos los subprogramas.

Esto se hace a través de la proposición COMMON, que tiene la forma:

$$\text{COMMON} \quad v_1, v_2 \ldots$$

en la que v_i representa ya sea un arreglo o una variable.

Así por ejemplo:

```
SUBROUTINE MAGON
COMMON A, B, N, NI
DIMENSION A(20,20), B(30), N(10,20)
```

en este caso A, B, N representan arreglos y NJ una variable.

Ya que se reserva un área común de memoria, es necesario que en el programa principal y en cada subprograma, al definir la proposición COMMON se tenga el mismo orden en las variables comunes definidas; de otra forma lo que se tiene es una confusión de archivos.

10.- E J E M P L O S

10.1. Programa para calcular las características de un filtro de arena

El espesor requerido para un filtro de arena está dado por la ecuación:

$$E_R = \frac{2.54 \cdot C_E \cdot V_{CT}}{V_{C10}} \cdot \frac{1}{\sum_{i=1}^{N} \frac{Pi}{Di^{5/3}}} \qquad (1)$$

donde:

- E_R - espesor requerido (cm)
- C_E - constante de espesor
- V_{CT} - viscosidad cinemática a T°C
- V_{C10} - viscosidad cinemática a 10°C
- Pi - fracción de partículas de cada diámetro
- Di - diámetro medio de las mallas en donde se retuvo la fracción Pi (cm).

La pérdida de carga está dada por la ecuación:

$$H = \frac{0.178 \cdot E_R \cdot (VO)^2 \cdot F_f}{g \cdot C_p^{\ 4}} \sum_{i=1}^{N} C_a \cdot \frac{Pi}{Di} \qquad (2)$$

donde:

- H - pérdida de carga (cm)
- VO - velocidad de operación (cm/seg)

F_f - factor de forma

C_a - coeficiente de arrastre

g - aceleración de la gravedad (cm/seg^2)

C_p - coeficiente de porosidad

El espesor expandido está dado por la ecuación:

$$E_e = (1-CP) \cdot E_R \cdot \sum_{i=1}^{N} \frac{Pi}{1-\left(\frac{V\ell}{V_{Si}}\right)^{0.22}} \qquad (3)$$

donde:

E_e - espesor expandido (cm)

$V\ell$ - velocidad de lavado (cm/seg)

V_{Si} - velocidad de sedimentación en la capa i (cm/seg).

Las partes principales del programa que calcula estas características son:

1) Leer la información necesaria.

2) Calcular el espesor requerido.

   a) Calcular la viscosidad cinemática (VC).

   b) Calcular el espesor requerido usando la ecuación (1).

3) Calcular la pérdida de carga

   a) Calcular el coeficiente de arrastre (CA).

b) Calcular la pérdida de carga usando la ecuación (2).

4) Calcular el espesor expandido.

a) Calcular la velocidad de sedimentación (VS), llamando a la subrutina VELSED.

b) Calcular cada término de la sumatoria de la ecuación (3).

c) Calcular el espesor expandido usando la ecuación (3).

```
C      [illegible] CARACTERISTICAS
C      [illegible]
C      [illegible]
C      [illegible]
C      [illegible] ESPESOR
C      [illegible]
C      [illegible] DE LAS [illegible]
C      [illegible] MATERIAL
C      [illegible]
C      [illegible]
C      [illegible]
C      [illegible] CARGA
C      [illegible] CAPAS
C      P(I)=[illegible] PARTICULAS [illegible]
C      [illegible] GRUPOS
C      T=[illegible] TEMPERATURA
C      V=VISCOSIDAD CINEMATICA
C      VL=VELOCIDAD DE LAVADO
C      VO=VELOCIDAD DE OPERACION

      DIMENSION D(20), P(20)
C      LEE VELOCIDAD DE OPERACION, [illegible], DENSIDAD [illegible]
C      TEMPERATURA
   10 READ (5,20) VO, VL, DEN, T
   20 FORMAT ([illegible])
C      LEE [illegible] CAPAS, CONST. [illegible] FACTOR [illegible]
C      C[illegible] POROSIDAD
      READ (5,30) N, CE, FF, CP
   30 FORMAT (I5,3F10.4)
C      LEE DIAMETRO MEDIO Y FRACCION DE PARTICULAS PARA CADA [illegible]
      READ (5,20) (D(I), P(I), I=1,N)
C      IMPRIME INFORMACION LEIDA
      WRITE (6,40) VO, VL, DEN, T, CE, FF, CP
   40 FORMAT (/////,10X,'VELOCIDAD DE OPERACION (CM/SEG)',F10.4,//,
     110X,'VELOCIDAD DE LAVADO (CM/SEG)',F10.4,//,
     2[illegible]DENSIDAD DEL MATERIAL (GR/CM**3)',F10.4,//,
     310X,'TEMPERATURA (O C)',F10.4,//,
     4[illegible]CONSTANTE DE ESPESOR',F1[illegible],//,10X,'FACTOR DE FOR[illegible]',F10.4,
     5//,10X,'COEF. DE POROSIDAD',F[illegible],///,
     6[illegible],'CAPA',10X,'DIAMETRO (CM)',[illegible],'FRACCION',//)
      [illegible] I=1,N
   50 WRITE (6,60) I, D(I), P(I)
   60 FORMAT (7X,I5,12X,F10.4,5X,F[illegible])
C      [illegible] ESPESOR TEORICO
C      [illegible] VISCOSIDAD CINEMATICA
      V=[illegible]/(1.0+[illegible]+42.[illegible])
      [illegible]=0.
      [illegible] I=1,N
      [illegible]+P(I)/[illegible](I)**[illegible].6667)
      [illegible]+VL/(0.0[illegible]+SU)
      [illegible]
      [illegible]/[illegible]
   7[illegible] FORMAT(//,[illegible]X,'ESPESOR TEORICO (C[illegible]',F10.4)
      [illegible]
      [illegible]
```

```
      WRITE(6,80) ER
   80 FORMAT(//,10X,'ESPESOR FLOCULADO (CM)',F10.4)
C     CALCULAR PERDIDA DE CARGA
C     COEF. DE ARRASTRE
   90 SUMA=0.0
      V=VO/VC
      DO 120 I=1, N
      R=V*D(I)
C     VERIFICAR NO. DE REYNOLDS
      IF(R.LT.1000.0) GO TO 100
      CA=0.4
      GO TO 120
  100 IF(R.GT.0.5) GO TO 110
      CA=24.0/R
      GO TO 120
  110 CA=24.0/R+3.0/SQRT(R)+0.34
  120 SUMA=SUMA+CA*P(I)/D(I)
      H=0.178*ER*VO*VO*F*SUMA/(981.0*(CP**4))
      WRITE(6,130) H
  130 FORMAT(//,10X,'PERDIDA DE CARGA (CM)',F10.4)
C     CALCULAR ESPESOR EXPANDIDO
      SUMA=0.0
      DO 140 I=1, N
C     CALCULAR RELACION DE POROSIDAD DE LA CAPA FILTRANTE
C      EXPANDIDA
C     VELOCIDAD DE SEDIMENTACION
      DI=D(I)
      CALL VELSED(VC,DI,DEN,VS)
  140 SUMA=SUMA+P(I)/(1.0-(VI/VS)**0.22)
      EEX=(1.0-CP)*SUMA
      EEX=EEX*ER
      WRITE(6,150) EEX
  150 FORMAT(//,10X,'ESPESOR EXPANDIDO (CM)',F10.4)
      GO TO 10
      END
```

```
      SUBROUTINE VELSED(VC,D,DEN,VS)
C     SUBRUTINA VELSED.- CALCULA LA VELOCIDAD DE SEDIMENTACION
C     VELOCIDAD DE SEDIMENTACION SI R MENOR O IGUAL QUE 0.5
      VS=54.5*(DEN-1.0)*D*D/VC
      R=VS*D/VC
      IF(R.LE.0.5) GO TO 50
C     VELOCIDAD DE SEDIMENTACION SI R MAYOR O IGUAL QUE 1000.0
      VS=SQRT(3270.0*(DEN-1.0)*D)
      R=VS*D/VC
      IF(R.GE.1000.0) GO TO 50
C     VELOCIDAD DE SEDIMENTACION SI R MAYOR QUE 0.5 Y MENOR QUE 1000.0
C     CALCULAR COEFICIENTES ECUACION POR RESOLVER
      X=0.34
      Y=3.0*SQRT(VC/D)
      Z=24.0*VC/D
      U=1308.0*(DEN-1.0)*D
C     METODO ITERATIVO
      CORR=0.0
      VS=10.0
      DO 30 I=1, 30
      F=X*VS*VS+Y*(VS**1.5)+Z*VS-U
      IF(ABS(F).LT.0.0001) GO TO 50
      DF=2.0*X*VS+1.5*Y*SQRT(VS)+Z
      VS=VS-F/DF
      IF(ABS(CORR-VS).GT.0.00001) GO TO 10
      VS=VS/2.0
   10 CORR=VS
      IF(VS.LT.0.0) VS=-VS
   30 CONTINUE
      WRITE(6,40) F
   40 FORMAT(//,10X,'NO SE ENCONTRO SOLUCION, DESBALANCE',F10.5)
      CALL EXIT
   50 RETURN
      END
```

VELOCIDAD DE OPERACION (CM/SEG) 0.1350

VELOCIDAD DE LAVADO (CM/SEG) 1.0160

DENSIDAD DEL MATERIAL (G/CM**3) 2.6500

TEMPERATURA (O C) 4.0000

CONSTANTE DE ESPESOR 790.0000

FACTOR DE FORMA 0.000[illegible]

COEF. DE [illegible] 0.4100

CAPA	DIAMETRO (CM)	FRACCION
1	0.1000	0.0092
2	0.0700	0.0471
3	0.0540	0.1467
4	0.0460	0.1790
5	0.0380	0.1750
6	0.0320	0.1930
7	0.0270	0.1[illegible]0
8	0.0230	0.0710
9	0.0160	0.0200

ESPESOR TEORICO (CM) 0.9540

ESPESOR RECOMENDADO (CM) 75.2000

PERDIDA DE CARGA (CM) 123.6413

ESPESOR EXPANDIDO (CM) [illegible].9597

11.- TOPICOS AVANZADOS DE PROGRAMACION

11.1. Aplicaciones de programación lineal

En un centro de cómputo no sólo se tiene un compilador FORTRAN, o compiladores para otros lenguajes. Se tiene también un gran número de programas de biblioteca que están disponibles al usuario.

Estos programas de biblioteca pueden realizar análisis estadísticos, resolver problemas de optimización, calcular o resolver funciones y problemas matemáticos complejos, etc., etc.

La gran ventaja que presentan es que no es necesario conocer el algoritmo que da la solución, o la teoría en que se basa; sino únicamente las instrucciones para su uso.

Como ejemplo se verá a continuación el uso de programas de biblioteca para la solución de problemas de programación lineal.

Un problema de programación lineal consiste en encontrar los valores de las variables $X_1, X_2, X_3 \dots X_n$ tales que minimizan la función objetivo:

$$Z = C_1X_1 + C_2X_2 + \dots C_nX_n$$

donde C_j son constantes; sujeta a las restricciones:

$$a_{11}X_1 + a_{12}X_2 + \dots a_{1n}X_n = b_1$$
$$a_{21}X_1 + a_{22}X_2 + \dots a_{2n}X_n = b_2$$
$$\vdots$$
$$a_{m1}X_1 + a_{m2}X_2 + \dots a_{mn}X_n = b_m$$
$$X_i \geq 0 \qquad i = 1,2\dots n$$

El método usado para su solución es el algoritmo Simplex. El estudiar este algoritmo está fuera de los objetivos de este curso; sin embargo, se analizarán algunos problemas resueltos con ayuda de la computadora.

a) Satisfacer la demanda de agua de una población al costo mínimo

Kuwait, un país pequeño en el océano Indico, tiene como fuentes de abastecimiento de agua, el océano, un poco de agua solobre y agua de pozos. En la siguiente tabla se muestran estos recursos y su costo.

	CANTIDAD	COSTO ($)
Agua dulce	10 MGD	0.15/1000
Agua solobre	30 "	0.42/1000
Agua de mar	sin límite	1.0/1000
Agua tratada	80% de las necesidades municipales	0.30/1000

La demanda de agua de Kuwait es:

riego	70 MGD
municipal	80 MGD

En este caso la función objetivo a minimizar será:

$$Z = 0.15X_1 + 0.42X_2 + 1.0X_3 + 0.30X_4$$

Sujeta a las restricciones:

$$X_1 \leq 10.0$$
$$X_2 \leq 30.0$$
$$X_3 \leq 64.0$$
$$X_1 + X_2 + X_3 + X_4 = 150.0$$
$$X_4 \leq 70.0$$

donde:

X_1 - cantidad de agua dulce

X_2 - cantidad de agua solobre

X_3 - cantidad de agua de mar

X_4 - cantidad de agua tratada

Generalmente a la computadora se le da este tipo de información, usando renglones y columnas:

	COL1	COL2	COL3	COL4		REST
FOB	0.15	0.42	1.0	0.3		
REN1	1.0				$\leq$	10.0
REN2		1.0			$\leq$	30.0
REN3				1.0	$\leq$	64.0
REN4	1.0	1.0	1.0	1.0	$\leq$	150.0
REN5				1.0	$\leq$	70.0

En las siguientes hojas se muestran los resultados obtenidos usando el programa de biblioteca ALPS-1, de la Compañía Burroughs.

La solución del problema es: $X_1 = 10$; $X_2 = 30$; $X_3 = 46$; $X_4 = 64$.

b) Diseño óptimo de redes de distribución

Uno de los métodos más bonitos para el diseño de redes de distribución, es el desarrollado por Karmelli et al[2]. Este método transforma el problema de diseño de una red, en un problema de programación lineal.

Las siguientes hojas son copias del Capítulo 3 del trabajo desarrollado por Cuevas[3], en donde se utiliza el método de Karmelli.

ITER.	OBJECTIVE	ELECT.	ENTER.	MIN RED. COST	EXIT.	PIVOT RATIO	R.H.S.	ARTIFICIALS	ADJ	ATRAN
3	7.92999999E 01	FOB	COL2	0.00000000E 00	REN2	30.00000000	REST	0	0	16

ACTIVITY	CURRENT VALUE	LINE COUNT	CONSTRAINT NAME	TRANSFORMATION VECTOR	PRICING VECTOR
ARTFCL	-79.29999991	1	FOB	0.58000000	1.00000000
ARTFCL	0.00000000	2	INFEAS	-0.00000000	0.00000000
COL1	10.00000000	3	+REN1	-0.00000000	0.85000000
COL2	30.00000000	4	+REN2	1.00000000	0.58000000
COL4	64.00000000	5	+REN3	-0.00000000	0.70000000
COL3	46.00000000	6	REN4	-1.00000000	-1.00000000
REN5	6.00000000	7	+REN5	-0.00000000	0.00000000

VARIABLE NAME	VALUE	LINE COUNT	REDUCED COST	POSITION IN BASIS
COL1	10.00000000	1	-0.00000000	3
COL2	30.00000000	2	-0.00000000	4
COL3	46.00000000	3	-0.00000000	6
COL4	61.00000000	4	0.00000000	5
REN1	0.00000000	5	0.85000000	0
REN2	0.00000000	6	0.58000000	0
REN3	0.00000000	7	0.70000000	0
REN5	5.00000000	8	0.00000000	7

MAXIMUM ERROR ON ROW 6 6.28643E-09. SUM 8.64384E-09

MAXIMUM ERROR ON COL 6 -7.27596E-11. SUM 1.16415E-10

*****<[BASIS]>****** TIMES: PROC. = 3.2 ELAPSED = 9.5

*****<[INPUT]>****** TIMES: PROC. = 3.4 ELAPSED = 120.2

COST RANGING CONTROL CARD

S-E-N-S-I-T-I-V-I-T-Y A-N-A-L-Y-S-I-S

VARIABLE NAME	BASIS STATUS	ACTUAL COST	LOWER RANGE	ENTERING	EXITING	UPPER RANGE	ENTERING	EXITING
COL1	IN	0.15000	OPEN			1.00000	REN1	COL1
COL2	IN	0.42000	OPEN			1.00000	REN2	COL2
COL4	IN	0.30000	OPEN			1.00000	REN3	COL4
COL3	IN	1.00000	0.42000	REN2	COL2	OPEN		
REN5	IN	0.00000	-0.70000	REN3	COL4	OPEN		

** - SOLUTION IS UNBOUNDED FOR PARAMETER VALUES ' PRESENT PARAMETER

CONCLUDE CONTROL CARD

END OF RUN PROB

ITER	OBJECTIVE	FUNCT.	ENTER.	MIN RED. COST	EXIT.	PIVOT RATIO	R.H.S.	ARTIFICIALS	#DJ	#TRAN
1	1.05200000E 02	FOB	COL4	-7.00000001E-01	REN3	64.00000000	REST	0	3	10
2	9.66999999E 01	FOB	COL1	-8.50000002E-01	REN1	10.00000000	REST	0	2	13
3	7.92999999E 01	FOB	COL2	-5.80000000E-01	REN2	30.00000000	REST	0	1	16

*****<LE/BH#2_1>***** TIMES: PROC. = 2.8 ELAPSED = 8.9

EXECUTION TIME = 3 SEC. I/O TIME = 1 SEC. APR. 15, 1976 TIME: 1611 HOURS

```
PROB    25   PS                 WITH  A L P S-1 FOR B6700
                                ********************************

                                         ******< INPUT  >******  TIMES:    PROC. =     0.3   ELAPSED =      0.4
HEADING      CONTROL CARD
APLICACIONES DE PROGRAMACION LINEAL
READ DATA    CONTROL CARD
CARD
CARD
THE PROBLEM HAS    7 ROWS AND    3 COLUMNS.   16 MATRIX ENTRIES AND    0 R.H.S.
THE PROBLEM HAS    7 ROWS AND    3 COLUMNS.   21 MATRIX ENTRIES AND    1 R.H.S.
START PHASE  CONTROL CARD
FOB        BEST
                                         ******< INVERT >******  TIMES:    PROC. =     0.7   ELAPSED =      1.5
                    REINVERTING AFTER     0-TH ITERATION.     0 TRANSFORMATIONS WITH      0 ENTRIES.
                    NUMBER OF ENTRIES IN BASIS PRESENTED TO INVERSION:       6
                                         ******< E/INV  >******  TIMES:    PROC. =     2.7   ELAPSED =      7.8
COMPUTE      CONTROL CARD
FOB        BEST
                                         ******< E/INP  >******  TIMES:    PROC. =     2.7   ELAPSED =      7.9
```

Estos resultados se pueden comparar con los obtenidos usando programación lineal; en todos los casos el uso de programación lineal permite hacer un diseño más económico.

3.3. APLICACION DE PROGRAMACION LINEAL

3.3.1. Descripción del método

Este método tiene dos partes principales:

1) Transformación y representación del problema de diseño de una red, en un problema de programación lineal.

2) Solución del problema de programación lineal.

Para la solución de un problema de programación lineal, se dispone de métodos bien establecidos (método Simplex, Simplex modificado, etc.).

Esta segunda parte del método no será discutida en este trabajo.

1) Transformación y representación del problema de diseño de una red, en un problema de programación lineal

Considere una red con las características dadas en la primera parte de este capítulo. Suponga que para cada sección de la red, se tienen ND_i tubos de diámetro diferente;

estos se pueden representar por la matriz D_{ij}, donde para la sección i se tienen $j = 1, 2, \ldots ND_i$ diámetros diferentes disponibles.

Asociado a cada diámetro D_{ij} se tiene el costo anual por unidad de longitud del tubo correspondiente, C_{ij}; definido por la ecuación (3.5).

El costo anual de operación del sistema, por metro de agua de presión, es:

$$C = \frac{Q \cdot t \cdot CE}{270.0 \cdot E} \tag{3.19}$$

donde:

Q - descarga total del sistema (m^3/h)

t - tiempo anual de riego (h)

CE - costo de la energía (H.P. - h)

E - eficiencia de la bomba

Los diámetros seleccionados y la presión de la bomba deben minimizar la ecuación:

$$Z = \sum_{i=1}^{NS} \sum_{j=1}^{ND_i} C_{ij} \, x_{ij} + C \cdot P_0 \tag{3.20}$$

donde:

C_{ij} - costo anual por unidad de longitud del tubo cuyo diámetro es j, para la sección i.

x_{ij} - longitud del tubo cuyo diámetro es j, para la sección i.

NS - número de secciones de la red.

C - costo anual de operación, definido por la ecuación (3.19).

P_0 - presión de la bomba.

Para cada sección la suma de las longitudes x_{ij}, de los diámetros seleccionados, debe ser igual a su longitud, L_i, esto es:

$$\sum_{j=1}^{ND_i} x_{ij} = L_i \qquad (3.21)$$

$$i = 1, 2, \ldots NS$$

El flujo en cada sección es conocido, así que el gradiente de pérdida de potencial para cada diámetro disponible, usando la ecuación de Hazen-Williams es:

$$J_{ij} = 1.131 \times 10^9 \left(\frac{Q_i}{HZ_{ij}}\right)^{1.852} D_{ij}^{-4.872} \qquad (3.22)$$

donde:

J_{ij} - gradiente de pérdida de potencial (m/m).

Q_i - flujo en la sección i (m^3/h)

D_{ij} - diámetro disponible j (mm)

HZ_{ij} - coeficiente de Hazen-Williams para el diámetro D_{ij}

La pérdida total de potencial para cada sección está dada por:

$$\sum_{j=1}^{ND_i} J_{ij} \cdot x_{ij} \qquad (3.23)$$

$$i = 1, 2, \ldots NS$$

En algunos nodos de la red el potencial debe ser mayor o igual que un potencial mínimo; esta restricción se expresa por la ecuación:

$$H_0 - \sum_{\ell} \sum_{j=1}^{ND_\ell} J_{\ell,j} \cdot x_{\ell,j} \leq H_{Kmin} \qquad (3.24)$$

donde:

- H_0 - potencial de la bomba (presión más elevación)
- ℓ - diferentes secciones que conectan el nodo k, con el nodo donde se enduentra la bomba
- $J_{\ell,j}$ - gradiente de pérdida de potencial, para el diámetro disponible j, correspondiente a la sección ℓ.
- $x_{\ell,j}$ - longitud del tubo de diámetro j, correspondiente a la sección ℓ.
- H_{Kmin} - potencial mínimo requerido en el nodo k.

El potencial de la bomba se puede expresar como:

$$H_0 = P_0 + E_0 \tag{3.25}$$

donde:

P_0 - presión de la bomba

E_0 - elevación de la bomba

La ecuación (3.24) se puede escribir entonces, como:

$$\Sigma \sum_{j=1}^{ND_\ell} J_{\ell,j} \cdot x_{\ell,j} - P_0 \leq E_0 - H_{Kmin} \tag{3.26}$$

La última restricción es:

$$x_{ij} \geq 0 \tag{3.27}$$

$$i = 1, 2, \ldots NS$$

$$j = 1, 2, \ldots ND_i$$

En resumen, el diseño de una red usando este método consiste en encontrar los valores de x_{ij} y P_0 que minimicen la función:

$$Z = \sum_{i=1}^{NS} \sum_{j=1}^{ND_i} C_{ij} \cdot x_{ij} + C \cdot P_0$$

Sujeta a las restricciones:

$$\sum_{j=1}^{ND_i} x_{ij} = L_i \qquad (3.28)$$

$$i = 1, 2, \ldots NS$$

$$\sum_{\ell} \sum_{j=1}^{ND_\ell} J_{\ell,j} \cdot x_{\ell,j} - P_0 \leq E_0 - H_{Kmin}$$

$$x_{ij} \geq 0 \qquad i = 1, 2, \ldots NS$$

$$j = 1, 2, \ldots ND_i$$

Este conjunto de ecuaciones representa un problema de programación lineal; una función objetivo a minimizar, sujeta a un conjunto de restricciones.

Para ilustrar la forma de las ecuaciones (3.28), considere la red simple mostrada en la Figura 3.3.

Usando la ecuación (3.19), el costo anual de operación por metro de agua de presión, es:

$$C = \frac{420.0 \times 2500.0 \times 0.20}{270.0 \times 0.9} = \$864.20 \text{ mex.} \qquad (3.29)$$

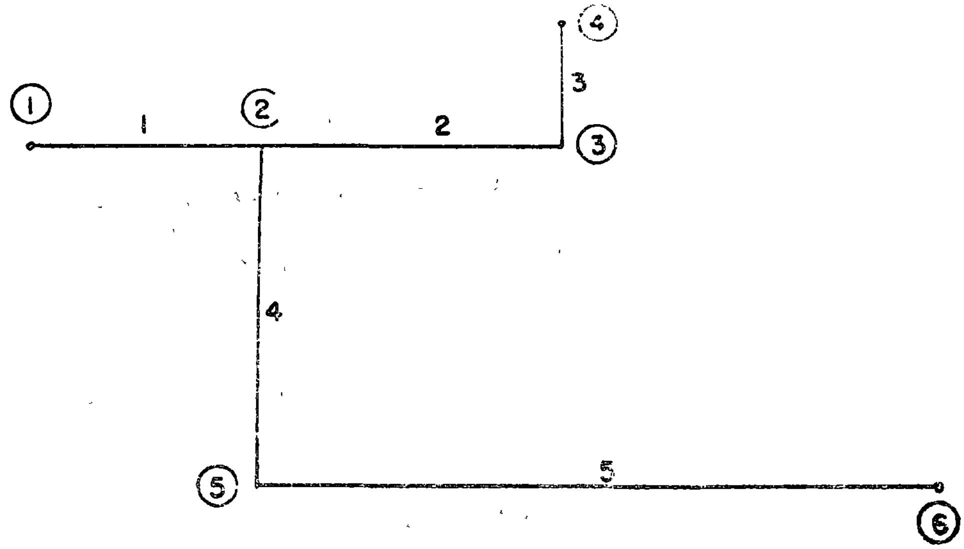
La función objetivo a minimizar es:

$$Z = C_{11}\, x_{11} + C_{12}\, x_{12} + C_{21}\, x_{21} + C_{22}\, x_{22} + \ldots + C_{52}\, x_{52} + 864.20 \times P_0 \qquad (3.30)$$

donde C_{ij} son los costos anuales por unidad de longitud, para cada uno de los diámetros disponibles; así

$$C_{21} = 99.00 \times 0.14 = \$13.68 \text{ mex.}$$

Las restricciones son:



SECCION	LONGITUD (m.)	FLUJO (m3/ hr.)	DIAMETROS DISPONIBLES (mm.)	COSTO ($ Mex/m.)	Gradiente de pérdida de potencial (m/m)
1	600.0	420.0	300.0	172.60	0.007
			250.0	133.30	0.017
2	800.0	240.0	200.0	99.00	0.017
			150.0	70.10	0.071
3	360.0	120.0	200.0	99.00	0.005
			150.0	70.10	0.020
4	900.0	180.0	200.0	99.00	0.010
			150.0	70.10	0.042
5	1800.0	80.0	200.0	99.00	0.002
			150.0	70.10	0.009

NODO	POTENCIAL MINIMO REQUERIDO (m.)
3	274.0
4	270.0
5	287.0
6	317.0

Elevación de la bomba, (nodo 1) : 240 cm.
Descarga de la bomba : 420.0 m^3/hr.
Eficiencia de la bomba : 0.90
Tiempo anual de riego : 2500.0 hr.
Costo de la energía, (por H.P.-hr.) : 0.20 $ MEX.

fig. 3.3 : RED DE DISTRIBUCION SIMPLE

Donde el gradiente de pérdida de potencial para cada diámetro es dado en la Figura 3.3; así $K_{21} = 0.017$, y de la misma forma para los demás diámetros.

Se puede ver que aún en el caso de redes simples, el número de variables y ecuaciones tiende a ser grande, lo que significa más tiempo y memoria de computadora.

En algunos casos es posible reducir el número de ecuaciones. Considere la Figura 3.4, que muestra el mínimo potencial requerido en una parte de la red mostrada en la Figura 3.3.

Ya que el gradiente de pérdida de potencial decrece en la dirección del flujo, se puede ver que si el potencial mínimo en el nodo 6 se satisface, el potencial mínimo en el nodo 5 también se satisface. Entonces la ecuación correspondiente a esta restricción se puede eliminar.

3.3.2. Descripción del programa de computadora

El programa que se discutirá a continuación, tiene dos pasos principales; en el primero, el problema de diseño de una red es transformado en un problema de programación lineal. Las ecuaciones correspondientes son escritas en un archivo temporal, en la forma requerida por el programa de biblioteca MPS/360 (Mathematical Programing System).

En el segundo paso, la computadora resuelve el pro-

$$
\begin{array}{lcr}
x_{11} + x_{12} & = & 600 \\
x_{21} + x_{22} & = & 800 \\
x_{31} + x_{32} & = & 360 \\
x_{41} + x_{42} & = & 900 \\
x_{51} + x_{52} & = & 1800 \\
J_{11}\,x_{11} + J_{12}\,x_{12} + J_{21}\,x_{21} + J_{22}\,x_{22} - P_0 & \leq & -42 \\
J_{11}\,x_{11} + J_{12}\,x_{12} + J_{21}\,x_{21} + J_{22}\,x_{22} + J_{31}\,x_{31} + J_{32}\,x_{32} - P_0 & \leq & -40 \\
J_{11}\,x_{11} + J_{12}\,x_{12} + J_{41}\,x_{41} + J_{42}\,x_{42} - P_0 & \leq & -39 \\
J_{11}\,x_{11} + J_{12}\,x_{12} + J_{41}\,x_{41} + J_{42}\,x_{42} + J_{51}\,x_{51} + J_{52}\,x_{52} - P_0 & \leq & -44
\end{array}
$$

$$
\text{Y} \quad x_{ij} \geq 0 \qquad i = 1, 2, \ldots 5 \quad j = 1, 2 \qquad (3.31)
$$

blema de programación lineal usando el programa MPS.

En consecuencia, el programa que se discutirá puede ser usado sólo en sistemas que tengan el programa de biblioteca MPS de I.B.M.

Es útil describir brevemente la forma de dar la información necesaria al programa MPS.

Suponga que el problema por resolver es el mostrado por las ecuaciones (3.30) y (3.31). La forma de indicarle al programa qué variables son desconocidas y sus respectivos coeficientes, es por columnas y renglones:

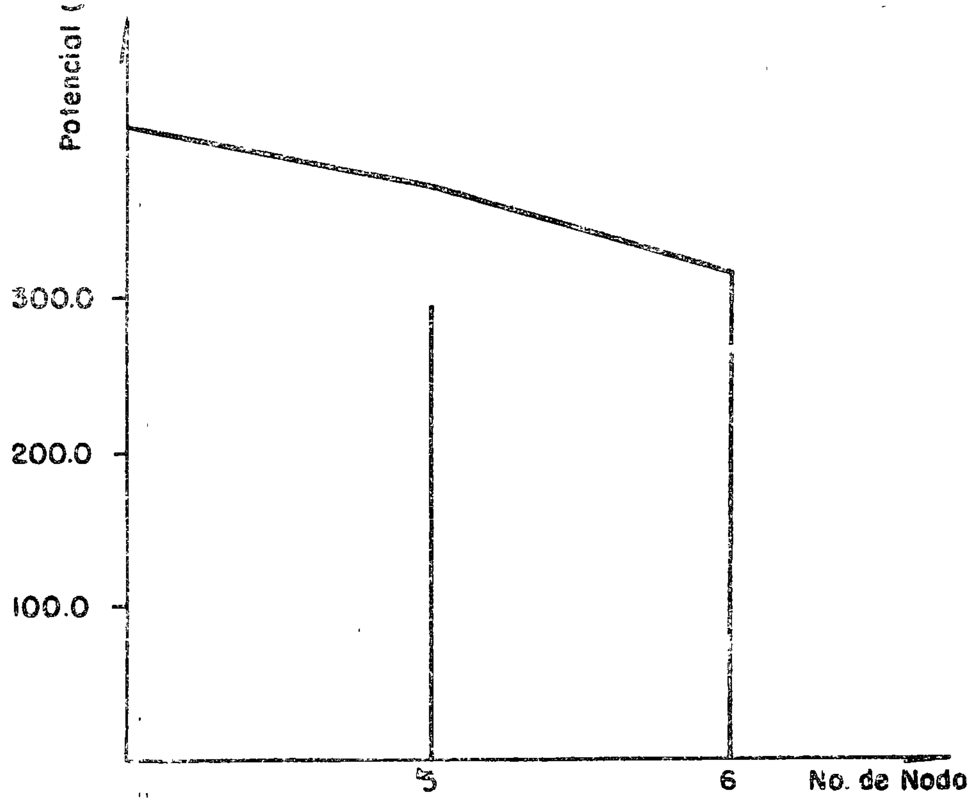


fig. 3.4 : POTENCIAL MINIMO REQUERIDO EN LOS NODOS 5 y 6, DE LA RED MOSTRADA EN LA FIG. 3.3

	1 COL	2 COL	3 COL	4 COL	5 COL	6 COL	7 COL	8 COL	9 COL	10 COL	11 COL	REST
COSTOS	C_{11}	C_{12}	C_{21}	C_{22}	C_{31}	C_{32}	C_{41}	C_{42}	C_{51}	C_{52}	864.20	
1 REN	1.0	1.0										500.0
2 REN			1.0	1.0								800.0
3 REN					1.0	1.0						360.0
4 REN							1.0	1.0				900.0
5 REN									1.0	1.0		1800.0
6 REN	J_{11}	J_{12}	J_{21}	J_{22}							-1.0	-34.0
7 REN	J_{11}	J_{12}	J_{21}	J_{22}	J_{31}	J_{32}					-1.0	-30.0
8 REN	J_{11}	J_{12}					J_{41}	J_{42}			-1.0	-47.0
9 REN	J_{11}	J_{12}					J_{41}	J_{42}	J_{51}	J_{52}	-1.0	-77.0

Entonces la columna 4 (4 COL), representará la variable x_{22}; su coeficiente en la función objetivo es C_{22}, en el renglón 2 (2 REN) es 1.0 y en los renglones 6 y 7 (6 REN y 7 REN) es J_{22}.

La primera información que se debe dar, es el signo de equivalencia de cada renglón. La función objetivo tiene la letra N; la letra E es usada en el caso de igualdad; y L si es menor o igual.

Para el ejemplo que se está considerando:

N	COSTOS
E	1 REN
E	2 REN
E	4 REN
E	5 REN
L	6 REN
L	7 REN
L	8 REN
L	9 REN

A continuación se dan los coeficientes de las variables desconocidas, para cada columna y cada renglón:

1 COL	COSTOS	C_{11}
1 COL	1 REN	1.0
1 COL	6 REN	J_{11}
1 COL	7 REN	J_{11}
1 COL	8 REN	J_{11}
1 COL	9 REN	J_{11}
2 COL	COSTOS	C_{12}
2 COL	6 REN	J_{12}
2 COL	7 REN	J_{12}
-----	-----	-----
11 COL	9 REN	-1.0

Si el coeficiente es igual a cero, no es necesario escribirlo.

Por último, las cantidades del lado derecho de la ecuación son escritas, éstas corresponden a la columna REST:

REST	1 REN	600.0
REST	2 REN	800.0
-----	-----	-----
REST	9 REN	-44.0

A la salida el programa da el tipo de solución obtenida (óptima, no factible); el valor de la función objetivo, y el valor de las variables desconocidas usando la notación de columnas.

Las partes principales del programa son:

1) Calcular el coeficiente de cada una de las variables desconocidas.

2) Eliminar las restricciones redundantes.

3) Generar un código para la interpretación de los resultados del programa MPS.

4) Escribir la información necesaria para el programa MPS, en un archivo temporal.

1) Calcular el coeficiente de cada una de las variables desconocidas

a) Leer el número total de nodos (ND)* y secciones (NS) de la red; el número de nodos donde se tiene la restricción de satisfacer un potencial mínimo (NR).

El nodo de referencia(NDR) es el nodo donde se encuentra la bomba.

b) Para cada sección de la red, leer los nodos que la conectan (N1(I)), N2(I)); el número de diámetros diferentes disponibles (NDIS(I)).

Leer cada diámetro disponible (D(I,J)); su coefi

* El símbolo dentro del paréntesis, representa el nombre de la variable, en el programa.

ciente de Hazen-Williams (HZ(I,J)); su costo por unidad de longitud (C(I,J)).

Leer la longitud de la sección (AL(I)); y su flujo (Q(I)).

c) Calcular el gradiente de pérdida de potencial, usando la ecuación (3.22), para los diámetros disponibles de cada sección (GR(I,J)).

d) Leer el nodo (NH(I)) y su mínimo potencial requerido (H(I)).

e) Leer el factor de recuperación del capital (CRF); la descarga de la bomba (QB); su eficiencia (EB); el costo de la energía (CE); tiempo anual de riego (T); elevación de la bomba (ELEVB).

f) Calcular el costo de operación de la bomba, definido por la ecuación (3.19), (COP).

2) Eliminar las restricciones redundantes

Para cada uno de los nodos donde se tiene la restricción de satisfacer un potencial mínimo:

a) Llamar a la subrutina ICSYN, que encuentra las secciones y nodos que conectan el nodo en consideración, con el nodo de referencia (IM(I) , JM(I)).

b) Se compara el mínimo potencial requerido en el nodo en consideración, con cada potencial mínimo de lo nodos del vector JM(I); si este último es menor que el primero, se elimina la restricción (HRES(I) = 0.).

3) Generar un código para la interpretación de los resultados del programa MPS.

a) A cada diámetro disponible de cada sección, se le asocia un número de columna.

Se imprime esta información.

4) Escribir la información necesaria para el programa MPS, en un archivo temporal

a) Escribir la información necesaria para iniciar el programa (nombre del programa, definición de la función objetivo, las restricciones, minimizar la función objetivo, solución primitiva).

b) Escribir el signo de equivalencia de cada renglón; primero el correspondiente a la función objetivo, N; después los correspondientes a igualdad, E; y después los correspondientes a desigualdad, L.

c) Para cada columna escribir el coeficiente correspondiente a la función objetivo y a cada renglón. Si el coeficiente es igual a cero, no se escribe.

d) Escribir los coeficientes de la columna correspondiente a la presión de la bomba.

e) Escribir la columna correspondiente a las restricciones:

1) Para cada renglón con el signo de igualdad, la longitud de la sección correspondiente, es escrita.

2) Para cada restricción de potencial mínimo diferente de cero (NRESI(I)) se cambia su signo y se le suma la elevación de la bomba (ELEVB). Se escribe el resultado en el renglón correspondiente.

Subrutina ICSYN

Esta subrutina permite encontrar las secciones y nodos que conectan el nodo con restricción de potencia mínimo, dado en el programa principal, con el nodo de referencia (NDR).

Las secciones son almacenadas en el vector IM(I) y los nodos en JM(I).

El algoritmo usado es el siguiente: comenzando con el nodo dado en el programa principal, encuentra el número de secciones que están conectadas a él, usando la matriz N(I,J). Escoge una de estas secciones y usando la misma matriz, encuentra el otro nodo que la conecta; si este último es el nodo de referencia, regresa al programa principal, de otra forma se tienen dos posibilidades:

a) El nodo tiene más de una sección conectada a él, entonces escoge una de estas secciones y continúa el algoritmo.

b) El nodo tiene sólo una sección conectada a él, lo que significa que no hay más nodos a donde saltar; entonces regresa al último nodo anterior, escoge otra sección diferente y sigue el algoritmo.

Puede suceder que la computadora, después de buscar por todos los caminos posibles el nodo de referencia, no lo encuentre y regresa al nodo inicial; en este caso imprime un mensaje diciendo que existe un error en la topología de la red.

3.3.3. Ejemplos

1) Diseño de una red simple

La Tabla 3.10 muestra el diseño óptimo obtenido usando programación lineal, para la red mostrada en la Figura 3.3.

2) Comparación con el método ICE

La Tabla 3.11 muestra el diseño obtenido para la red mostrada en la Figura 3.2.

En este caso, el número de variables por determinar fue de 24, en casos como este, la disponibilidad de una com-

TABLA 3.10. DISEÑO OPTIMO USANDO PROGRAMACION LINEAL PARA LA RED MOSTRADA EN LA FIGURA 3.3.

SECCION	LONGITUD (m)	FLUJO (m^3/h)	DIAMETRO SELECCIONADO (mm)	LONGITUD SELECCIONADA (m)
1	600.00	420.0	300.0	600.0
			250.0	
2	800.0	240.0	200.0	66.47
			150.0	733.53
3	360.0	120.0	200.0	
			150.0	360.0
4	900.0	180.0	200.0	900.0
			150.0	
5	1800.0	80.0	200.0	1800.0
			150.0	

TIEMPO ANUAL DE RIEGO: 2500.0 h

EFICIENCIA DE LA BOMBA: 0.90.

COSTO DE LA ENERGIA (POR H.P.-HORA): 0.20 $MEX.

FACTOR DE RECUPERACION DEL CAPITAL: 0.136

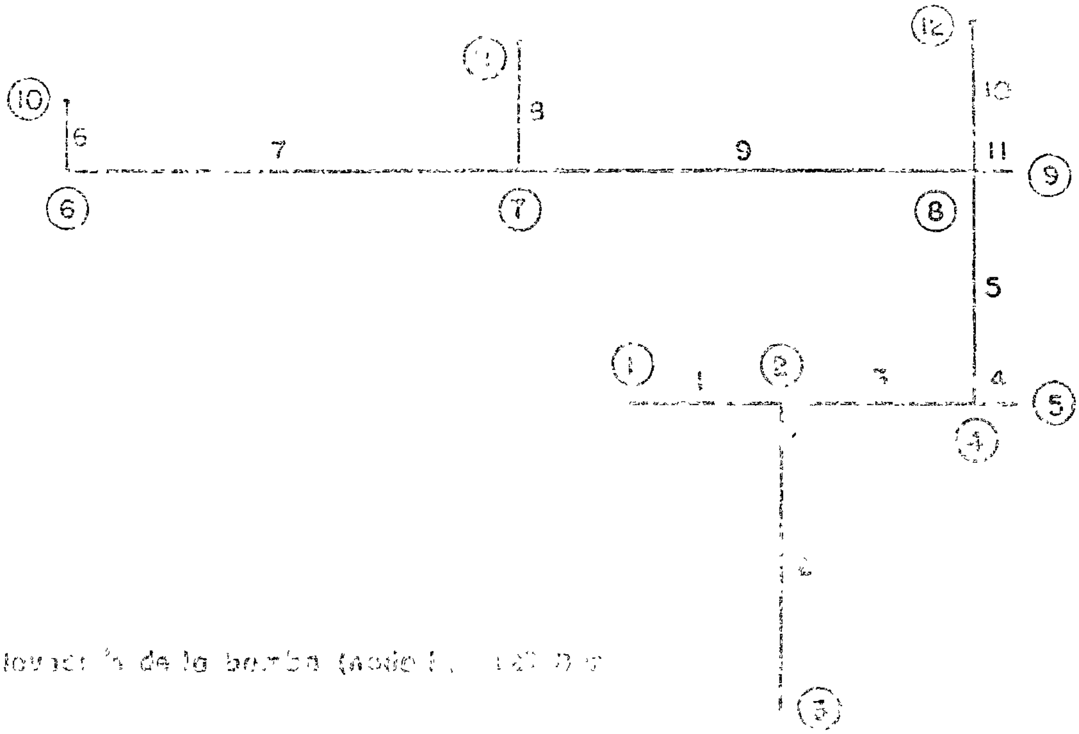
PRESION EN LA BOMBA: 95.46 m.

COSTO TOTAL ANUAL DE LOS TUBOS: 61737.97 $MEX.

COSTO ANUAL DE OPERACION: 82496.53 $MEX.

COSTO TOTAL ANUAL: 144234.50 $MEX.

EL FACTOR CRC SE CALCULO SUPONIENDO UN INTERES DE 14% Y UNA VIDA DE 30 AÑOS.



Elevación de la bomba (nodo 1) [illegible]

SECCION	LONGITUD (m)	FLUJO (m /hr)
1	400.0	510.0
2	800.0	130.0
3	500.0	390.0
4	100.0	100.0
5	600.0	290.0
6	200.0	50.0
7	1200.0	50.0
8	350.0	60.0
9	1200.0	110.0
10	400.0	80.0
11	1000	100.0

NODO	POTENCIAL MINIMO REQUERIDO (m.)
3	164.0
5	172.0
9	155.0
10	159.0
11	159.0
12	163.0

DIAMETROS DISPONIBLES (mm)	COSTO INICIAL ($ MEX / m)
100.0	41.09
150.0	70.10
200.0	93.00
250.0	113.30
300.0	172.50
350.0	232.70

fig. 3.2 : RED DE DISTRIBUCION PARA UN SISTEMA DE RIEGO POR ASPERSION

TABLA 3.11. DISEÑO OPTIMO USANDO PROGRAMACION LINEAL PARA LA RED MOSTRADA EN LA FIGURA 3.2.

SECCION	LONGITUD (m)	FLUJO (m^3/h)	DIAMETROS DISPONIBLES (mm)	LONGITUD SELECCIONADA (m)
1	[illegible]	[illegible]	350.0 300.0 250.0	400.0
2	[illegible]	[illegible]	250.0 [illegible]	145.49 654.51
3	[illegible]	390.0	[illegible] [illegible]	500.0
4	100.0	[illegible]	200.0 150.0	100.0
5	600.0	290.0	300.0 250.0 200.0	600.0
6	200.0	50.0	150.0 100.0	193.16 6.84
7	1200.0	[illegible]	150.0 100.0	1200.0
8	350.0	60.0	150.0 100.0	[illegible]
9	1200.0	110.0	200.0 150.0	762.62 437.38
10	400.0	80.0	150.0 100.0	313.97 86.03
11	100.0	100.0	200.0 150.0	100.0

TIEMPO ANUAL DE RIEGO: 2500.0 h.
EFICIENCIA DE LA BOMBA: 0.90
COSTO DE LA ENERGIA (POR H.P.-HORA): 0.20 $MEX.
FACTOR DE RECUPERACION DEL CAPITAL: 0.136.
PRESION EN LA BOMBA: 58.4 m.
COSTO TOTAL ANUAL DE LOS TUBOS: 77061.59 $MEX.
COSTO ANUAL DE OPERACION: 624[illegible] $MEX
COSTO TOTAL ANUAL: 139557.95 $MEX

EL FACTOR CRF SE CALCULO SUPONIENDO UN INTERES DE 14.0% Y UNA VIDA DE 30 AÑOS.

putadora es necesaria.

Comparando este diseño con el obtenido usando el método ICET (sección 3.2.4), se puede ver que la presión de la bomba, usando el método de programación lineal, es de 1.14 m mayor; lo que da un aumento en el costo de operación de $1214.85. Sin embargo, este método selecciona, en un mayor número de secciones de la red, combinaciones de tubo de diámetros diferentes, lo que da un ahorro de $7696.47 en el costo total de los tubos.

En consecuencia, el uso de programación lineal da un diseño con un ahorro, en el costo total del sistema de $6582.00.

Las Tablas 3.12 y 3.13, muestran el diseño obtenido para la red mostrada en la Figura 3.1, usando programación lineal, para diferentes costos de la energía.

Comparando este diseño con el obtenido usando el método ICET (sección 3.2.4), se puede ver que la presión en la bomba es un poco mayor en todos los casos, pero que el costo total anual es siempre menor. Sin embargo, esta diferencia no es mayor del 5% del costo total, la cual puede ser considerada pequeña; así que se puede decir que el diseño obtenido usando el método ICET, se encuentra muy cerca del óptimo económico. De cualquier forma, es mejor usar el método de programación lineal, ya que el diseño obtenido es siempre el óptimo económico.

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PROGRAMACION Y MODELOS DE INGENIERIA AMBIENTAL

MODELOS DE CONTAMINACION BACTERIANA DEL AGUA

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PREDICTION AND ASSESSMENT OF

IMPACTS ON THE NOISE ENVIRONMENT

by

L. W. Canter*

I. Introduction

A. Examples of projects causing noise impacts

1. Construction of power plants, highways, airports and pipelines.

2. Operation of airports, highways and compressor stations.

B. Basic steps in prediction and assessment of noise impacts

1. Identify the noise levels for the alternatives for a project need during both construction and operational phases. This may involve a literature review, analysis of other EIS's on similar types of alternatives, or field measurements at existing installations of similar types.

2. Determine the existing noise levels for the project area. May involve field measurements or determination of land uses in the area. Note any unique noise sources in the area, or any unique places where noise levels must be minimized.

3. Procure applicable noise standards and criteria.

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4. Predict the anticipated noise levels (microscale impact) in the area of the project for each alternative during both the construction and operational phases. Compare the predicted levels with the applicable standards or criteria.

5. If standards or criteria are exceeded, consider noise control methods.

II. Basic Information on Noise

A. Definition

1. Noise is unwanted sound. It is sound in the wrong place at the wrong time.

2. Any sound which is undesirable because it interferes with speech and hearing, or is intense enough to damage hearing, or is otherwise annoying. (1)

3. Noise is discordant sound resulting from non-periodic vibrations in air. (2)

4. The definition of noise as unwanted sound implies that it has an adverse effect on human beings and their environment, including land, structures, and domestic animals. Noise also affects natural wildlife and ecological systems.

B. The Nature of Sound (3)

1. Sound is a mechanical energy from a vibrating surface, transmitted by a cyclic series of compressions and rarefactions of the molecules of the material through which it passes.

2. Sound is transmitted through gases, liquids and solids.

3. A vibrating source producing sound has some total power, and the sound results in a sound pressure which alternately rises to a maximum pressure of compression and drops to a minimum pressure of rarefaction.

4. Values of sound power or sound pressure do not provide a practical unit for sound or noise measurement for two reasons:

   (a) There is a tremendous range of sound power and sound pressures produced. Expressed in microbars, one-millionth of 1 atmosphere of pressure, the range is from 0.0002 microbar (μbar), the minimum sound pressure of a healthy young human ear can detect, to 10,000 μbars for peak noises within 100 ft. from large jet and rocket propulsion devices.

   (b) Our ears do not respond linearly to increases in sound pressure. The nonlinear response is essentially logarithmic. The human ear can discern without pain sounds ranging from a threshold to sounds 10^{12} times as intense.(2)

5. The number of compressions and rarefactions of the air molecule density in a unit of time associated with a sound wave is described as its frequency. The unit of time is usually one second, and the term "Hertz" (after an early investigator of the physics of sound) is used to designate the number of cycles per second. Again, the human ear and that of most animals has a wide range of response. Humans can identify sounds with frequencies from about 16 Hz to 20,000 Hz. (2)

C. Sound and Noise Measurement (3)

1. The measurement needs are met by a term, sound pressure level (SPL), expressed as a logarithmic ratio to a reference level and stated in a dimensionless unit of power, the decibel (dB). The reference level is 0.0002 μbar, the threshold of human hearing.

$$SPL = 20 \log_{10} \left(\frac{P}{Po}\right)$$

where SPL = sound pressure level expressed in dB

P = sound pressure (μbar)

Po = reference pressure (0.0002 μbar)

2. Some example calculations

At P = 0.0002,

$$dB = 20 \log \frac{0.0002}{0.0002} = 20 \log 1 = 20\ (0) = 0$$

At P = 0.2

$$dB = 20 \log \frac{0.2}{0.0002} = 20 \log 1{,}000 = 20\ (3) = 60$$

At P = 20

$$dB = 20 \log \frac{20}{0.0002} = 20 \log 100{,}000 = 20\ (5) = 100$$

At P = 20,000

$$dB = 20 \log \frac{20{,}000}{0.0002} = 20 \log 100{,}000{,}000 = 20\ (8) = 160$$

3. Table 1 contains a summary of various sound pressures and the corresponding decibel levels, with examples of recognized sources of noise being cited.

4. As the SPL-decibel scale is logarithmic, decibel values are not additive. For example, and SPL of 74 dB from one source superimposed on one of 75 dB does not result in 149 dB.

Table 1: THE DECIBEL SCALE OF SPL, WITH SOUND PRESSURES IN MICROBARS, AND RECOGNIZED SOURCES OF NOISE IN OUR DAILY EXPERIENCES

Sound Pressure μbar	dB	Example
0.0002	0	Threshold of Hearing
0.00063	10	
0.002	20	Studio for sound pictures
0.0063	30	Studio for speech broadcasting
0.02	40	Very Quiet room
0.063	50	Residence
0.2	60	Conventional speech
0.63	70	Street traffic at 100 ft.
1.0	74	Passing automobile at 20 feet
2.0	80	Light trucks at 20 ft.
6.3	90	Subway at 20 ft.
20	100	Looms in textile mill
63	110	Loud motorcycle at 20 ft.
200	120	Peak level from rock and roll band
2,000	140	Jet plane on the ground at 20 ft.

An SPL of 77.6 dB results. To determine the total effect, it is necessary to convert decibel readings to intensity ratios, then reconvert the new sum back to a decibel value. To aid in this process, Table 2 is provided for determining the cumulative decibel values of two or more known observations on individual sources. The value in the difference column in Table 2 is always added to the highest of the two decibel values being handled.

5. In most noise considerations, the A - weighted sound level is used. This level is explained as follows: The ear does not respond equally to sounds of all frequencies, but is less efficient at low and high frequencies than it is at medium or speech range frequencies. Thus, to obtain a single number representing the sound level of a noise containing a wide range of frequencies in a manner representative of the ear's response, it is necessary to reduce, or weight, the effects of the low and high frequencies with respect to the medium frequencies. The resultant sound level is said to be A-weighted, and the units are dB. A popular method of indicating the units, dBA, is frequently used. The A-weighted sound level is also called the noise level. Sound level meters have an A-weighting network for measuring A-weighted sound level.

D. Some General Facts on Noise Abatement

1. The "Noise Control Act of 1972" is the basic Federal legislation for noise emissions from a broad range of sources. (4)

Table 2: DETERMINING THE CUMULATIVE DECIBEL SPL WHEN THE DIFFERENCES BETWEEN TWO OR MORE LEVELS ARE KNOWN

Difference between levels, dB	No. of dB to be added to higher level
0	3.0
1	2.6
2	2.1
3	1.8
4	1.5
5	1.2
6	1.0
7	0.8
8	0.6
10	0.4
12	0.3
14	0.2
16	0.1

2. The basic needs for the Noise Control Act of 1972 were cited in the Act as follows:

(1) that inadequately controlled noise presents a growing danger to the health and welfare of the Nation's population, particularly in urban areas;

(2) that the major sources of noise include transportation vehicles and equipment, machinery, appliances, and other products in commerce; and

(3) that, while primary responsibility for control of noise rests with State and local governments, Federal action is essential to deal with major noise sources in commerce control of which require national uniformity of treatment.

3. The policy of the U.S. is to promote an environment for all Americans free from noise that jeopardizes their health or welfare.

4. Predictions indicate that the noise in our environment is increasing by as much as 1 dB per year, or 10 dB per decade.

(1) Some reasons for these increases include:

(a) growth in number of miles of urban freeway.

(b) increase in commercial air traffic, and shift in aircraft from propeller-type to jet-type.

(c) increase in construction activity.

(d) increase in noisy devices such as power lawnmowers and motorcycles.

5. An estimated 16 million persons are presently exposed to aircraft noise levels with effects ranging from moderate to very severe. (5)

6. Hearing losses in the overall population are estimated in Table 3. (1)

III. Anticipated Noise Levels (Step 1)

A. Construction Equipment and Operations (1)

1. Construction site categories can be considered to be comprised of four major types:

   (a) Domestic housing - including residences for one to several families.

   (b) Nonresidential buildings - including offices, public buildings, hotels, hospitals, schools.

   (c) Industrial - including industrial buildings, religious and recreational centers, stores, service and repair facilities.

   (d) Public works - including roads, streets, water mains, sewers.

   Noise from construction of such major civil works as dams and bridges affects relatively few people (other than those employed at or near such construction sites).

2. Noise from a construction site varies as to the particular operation in progress; there are five consecutive phases:

   (a) Ground clearing - including demolition and removal of prior structures, trees, rocks.

TABLE 3: Hearing Loss (Moderate to Profound) in U.S.

Age Range	Population Totals (in thousands)	Loss of Hearing Totals (thousands)	Noise-Associated Hearing Loss (thousands)
0-5	17,000	850	?
5-10	20,000	1,000- 1,400	*200
10-18	32,500	650- 975	**150
18-65	113,000	2,260	2,000 (Approx)
Over 65	20,000	4,000	400-600
TOTALS	202,500	8,700-11,135	2,750-2,950

* Most common cause is explosions from toy caps (20% sensory-neural hearing loss).

** Firearms and toy caps (based on approximately 20% sensory-neural hearing loss).

(b) Excavation.

(c) Placing foundations - including reconditioning old roadbeds, compacting trench floors.

(d) Erection - including framing, placing of walls, floors, windows, pipe installation.

(e) Finishing - including filling, paving, cleanup.

3. Table 4 shows typical energy equivalent noise levels at construction sites. Energy equivalent noise level (Leq) refers to the equivalent steady noise level which in a stated period of time would contain the same noise energy as the time-varying noise during the same time period. (6) The maximum levels range from 77 to 89 dBA for all categories and have an average value of approximately 85 dBA. The minimum values for all categories have a wider range, extending from 65 to 88 dBA, and have an average value of 78 dBA. The table also shows that the initial ground clearing and excavation phases generally are the noisiest, that the intermediate foundation placement and erection phases are somewhat quieter, and that the final finishing phase tends to produce considerable noise annoyance.

4. Noise levels observed 50 ft. from construction equipment are shown in Table 5. These levels range from 72 to 96 dBA for earthmoving equipment, from 75 to 88 dBA for materials handling equipment, and from 70 to 87 dBA for stationary equipment.

B. Examples of Noise Levels from Project Operation

1. Examples to be considered include highway vehicles, aircraft,

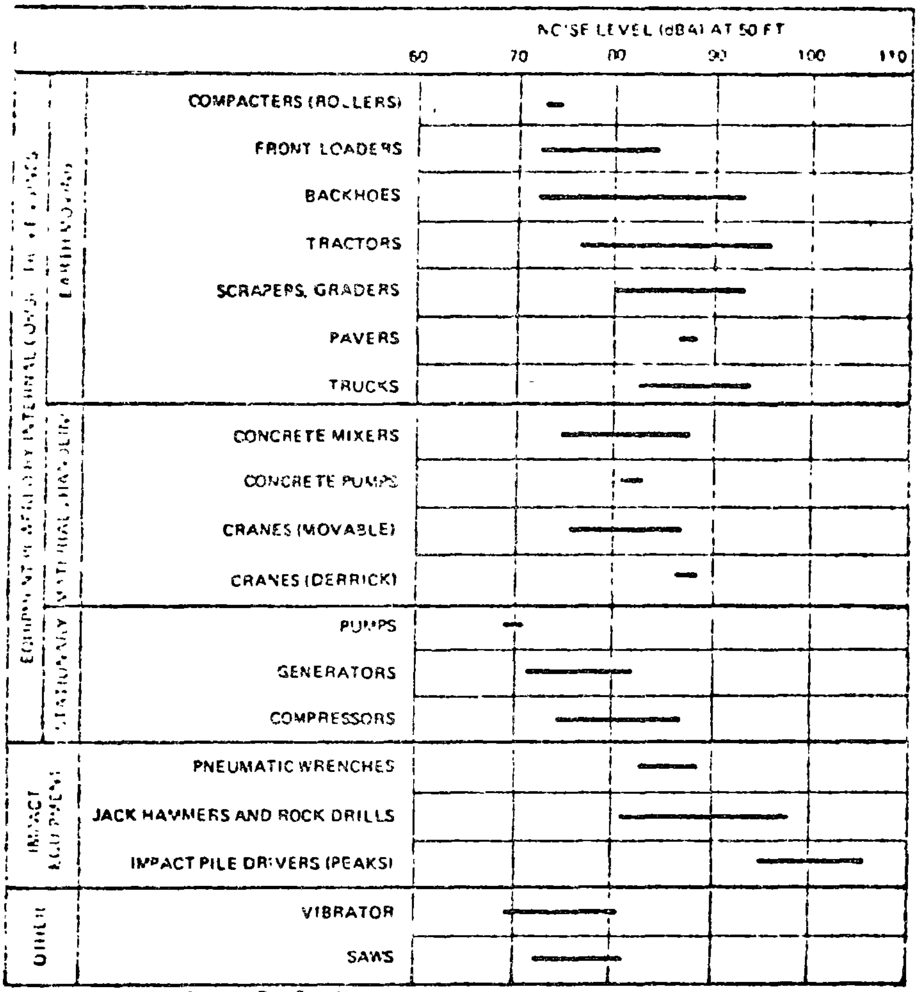
TABLE 4: Typical Ranges of Energy Equivalent Noise Levels, L_{eq} in dBA, at Construction Sites

	Domestic Housing		Office Building, Hotel, Hospital, School, Public Works		Industrial Parking Garage, Religious Amusement and Recreations, Store, Service Station		Public Works Roads & Highways, Sewers, and Trenches	
	I	II	I	II	I	II	I	II
Ground Clearing	83	83	84	84	84	83	84	84
Exaavation	88	75	89	79	89	71	88	78
Foundations	81	81	78	78	77	77	88	88
Erection	81	65	87	75	84	72	79	78
Finishing	88	72	89	75	89	74	84	84

I - All pertinent equipment present at site.

II - Minimum required equipment present at site.

TABLE 5: Construction Equipment Noise Ranges



Note Based on Limited Available Data Samples rail systems, recreation vehicles, internal combustion engines, industrial machinery, building equipment, and home appliances.

2. The noise levels produced by highway vehicles can be attributed to the following three major noise generating systems: (7)

   (a) rolling stock; tires and gearing

   (b) propulsion system: engine and related accessories

   (c) aerodynamic and body

3. The noise levels produced by highway vehicles are generally dependent upon vehicle speed, as illustrated for a number of different vehicle types in Figure 1.

4. General Characteristics of highway vehicles are shown in Figure 2. (7)

5. General characteristics of commercial aircraft are shown in Figure 3; V/STOL aircraft in Figure 4; and general aviation aircraft in Figure 5.

6. General characteristics of rail systems are shown in Figure 6.

7. General characteristics of recreation vehicles are shown in Figure 7.

8. General characteristics of devices powered by internal combustion engines are shown in Figure 8.

9. Typical ranges of noise levels from industrial machinery, equipment and processes are shown in Table 6.

10. Typical ranges of building equipment noise levels are in Table 7.

11. Typical noise levels of home appliances are in Table 8.

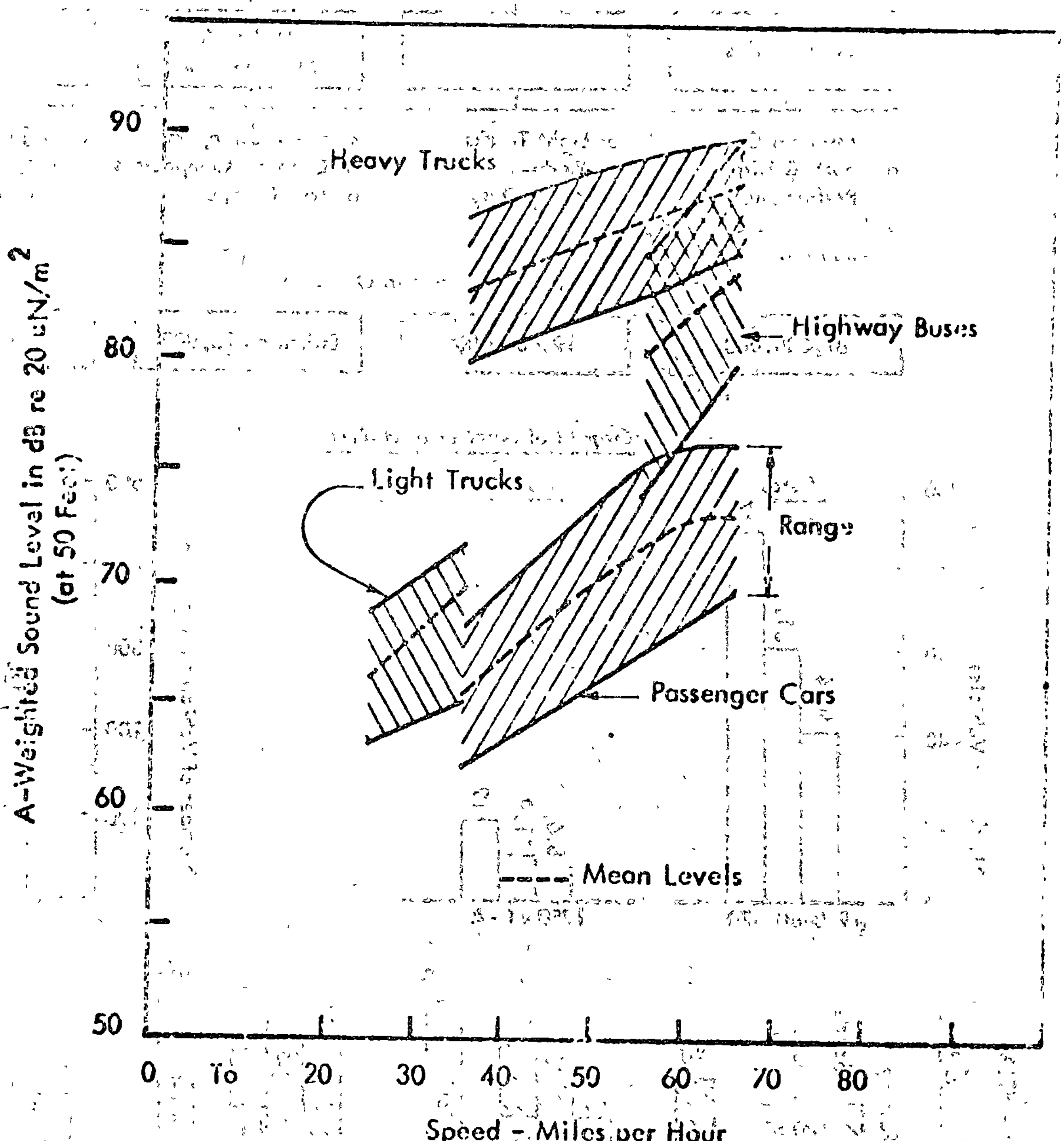


Figure 1: Single Vehicle Noise Output as a Function of Vehicle Speed

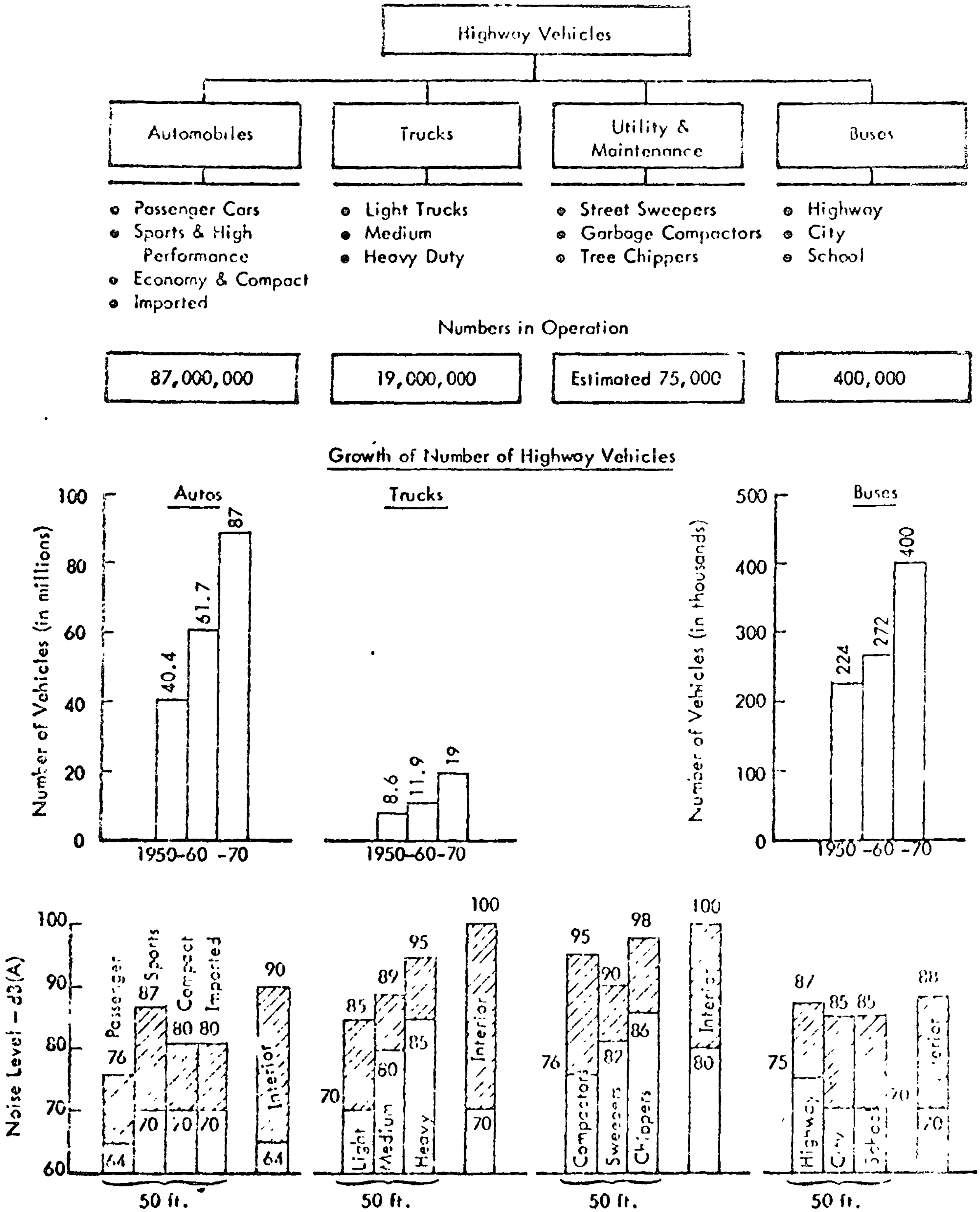


Figure 2: Characteristics of Highway Vehicles

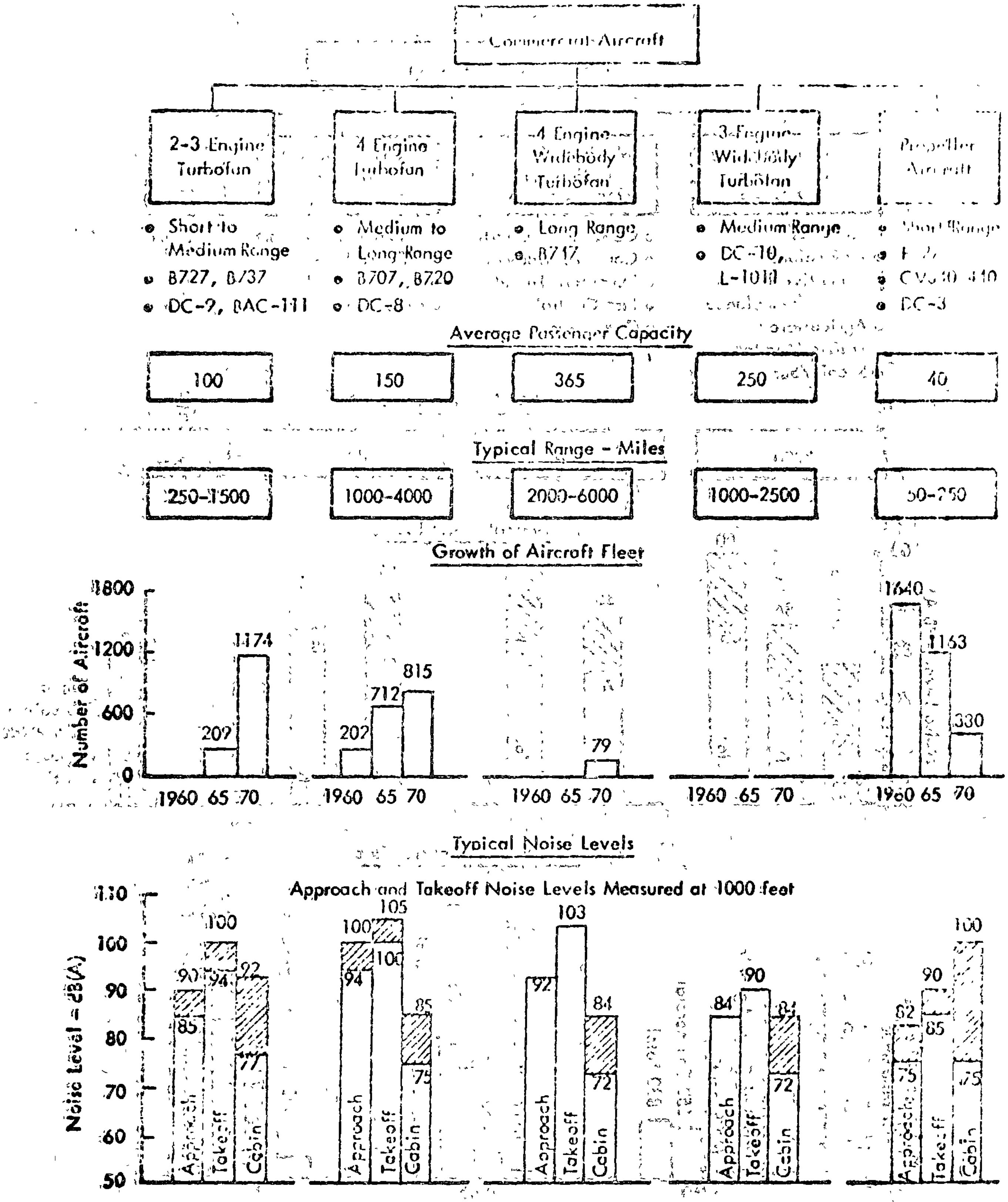


Figure 3: Characterisitcs of Commerical Aircraft

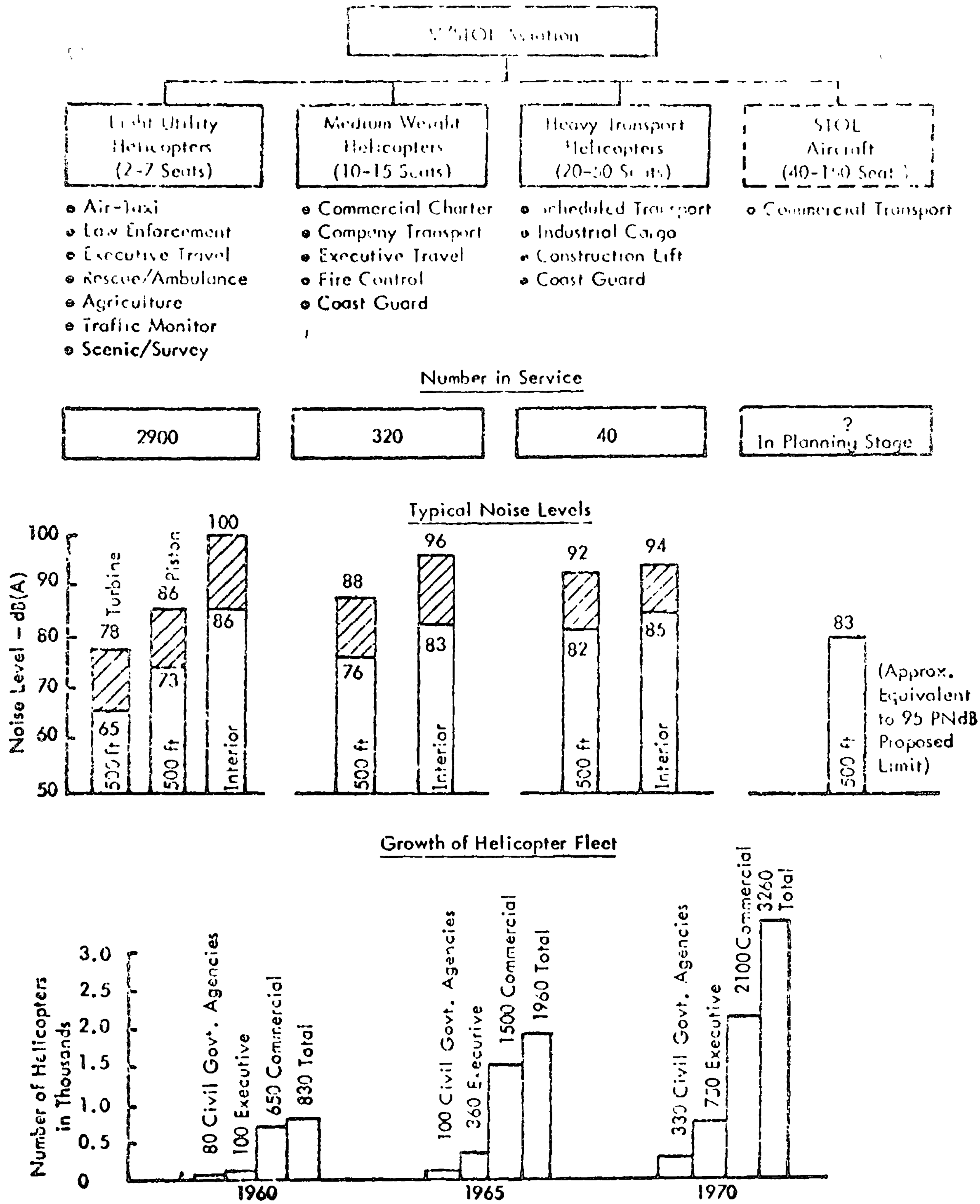


Figure 4: Characteristics of V/STOL Aircraft

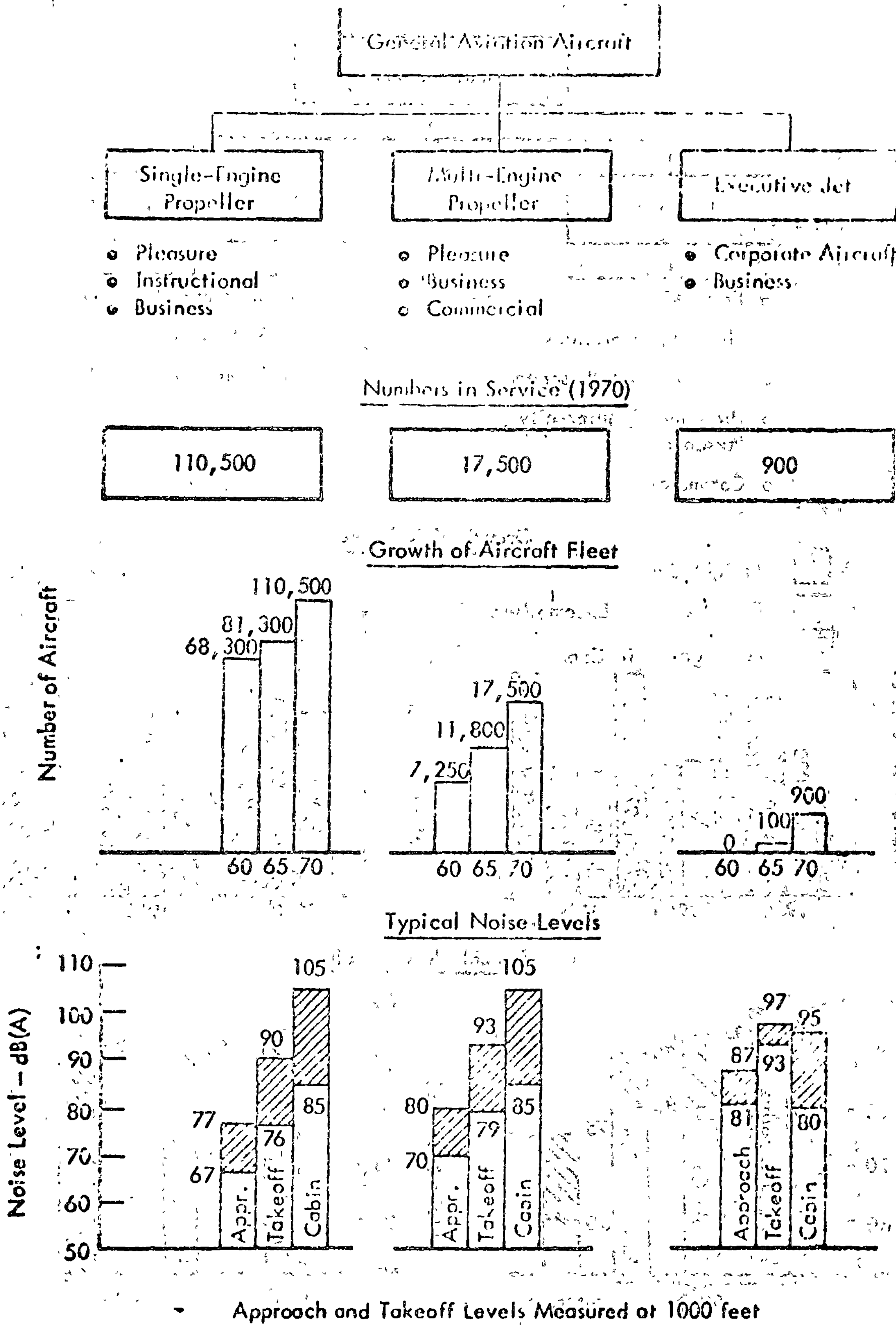


Figure 5: Characteristics of General Aviation Aircraft

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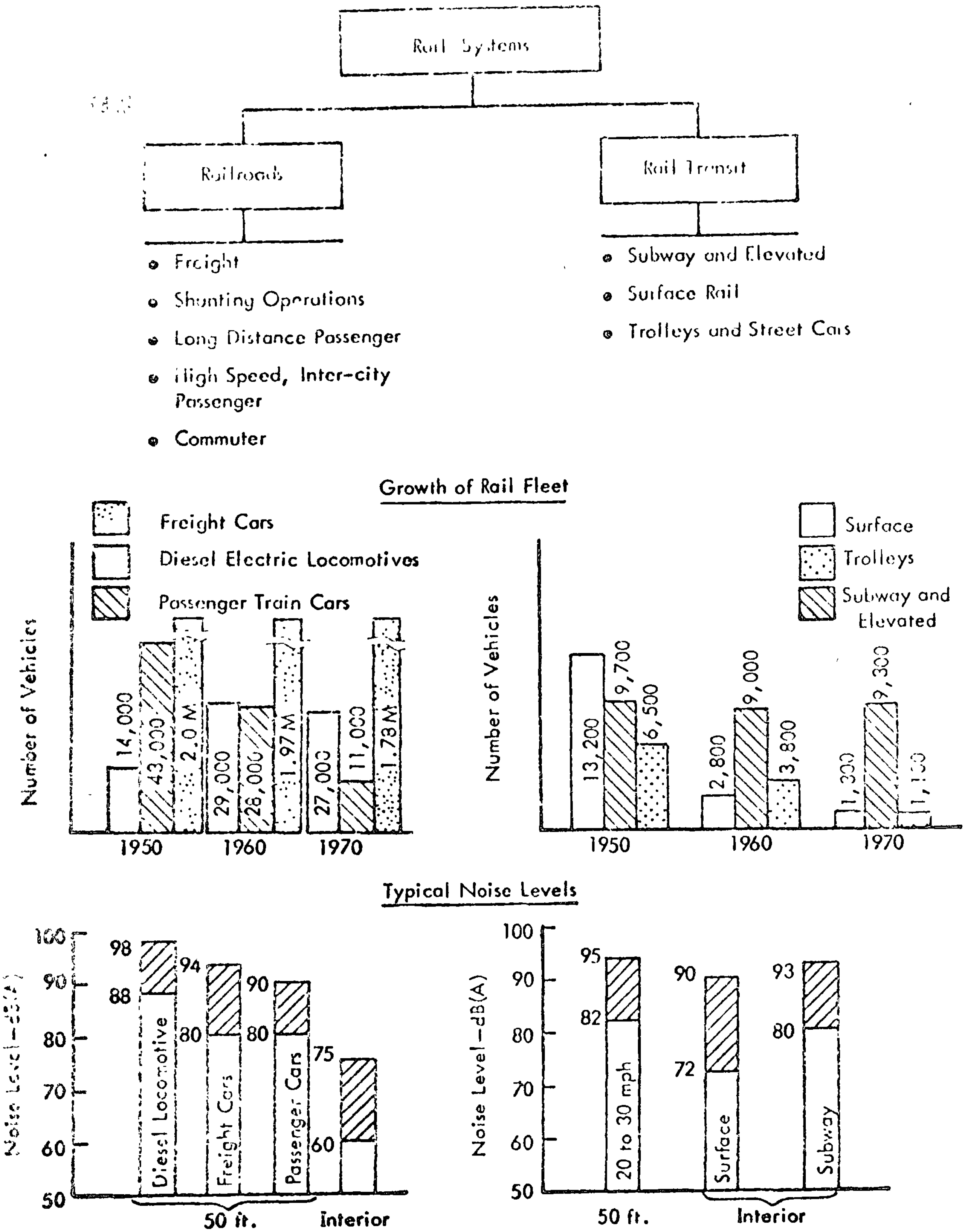


Figure 6: Characteristics of Rail Systems

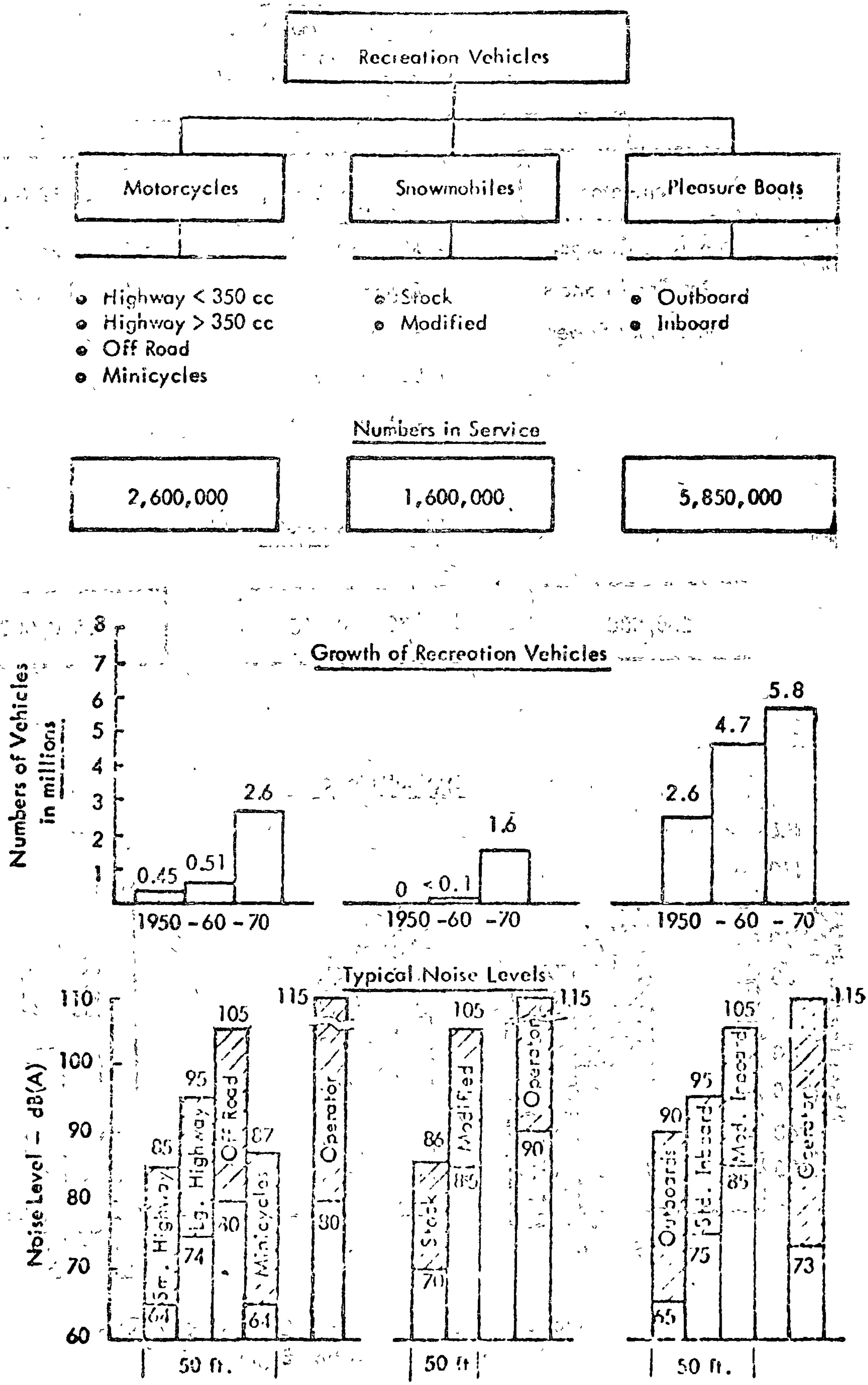


Figure 7: Characteristics of Recreation Vehicles

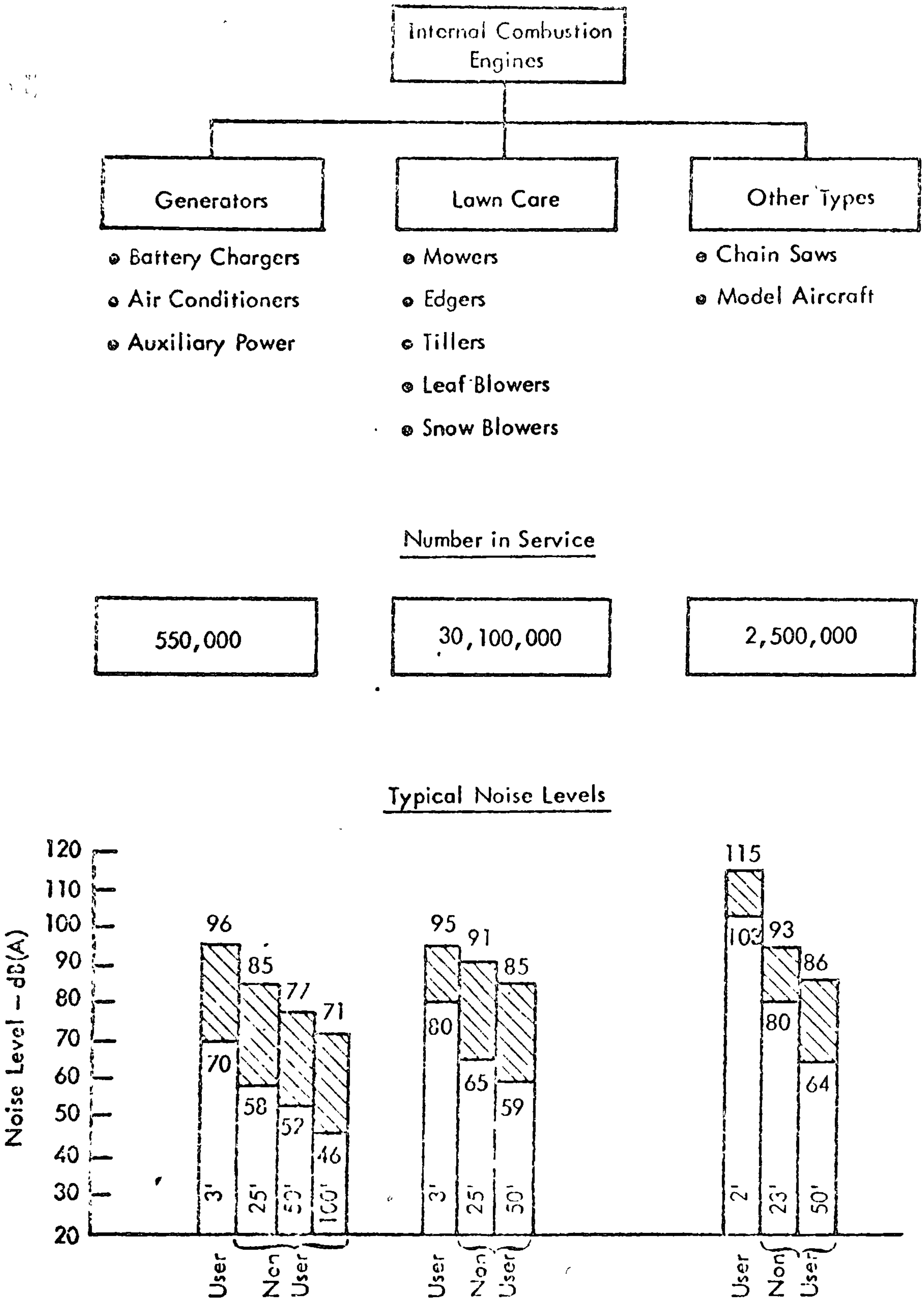
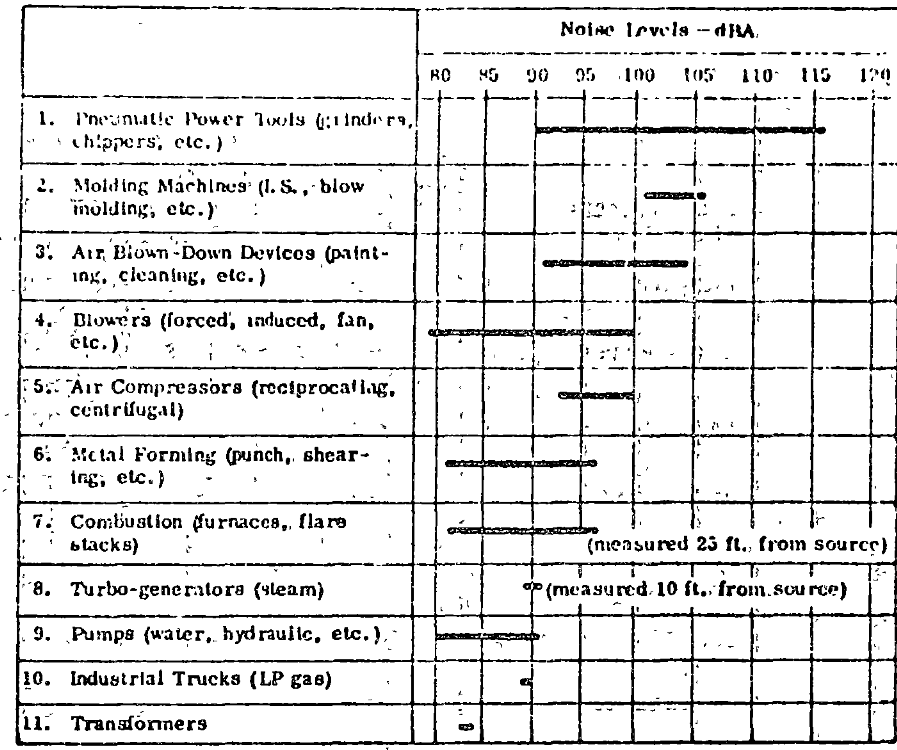


Figure 8: Characteristics of Devices Powered by Internal Combustion Engines

TABLE 6: Range of Industrial Machinery, Equipment, and Process Noise Levels*



*Measured at operator positions, except for 7 and 8.

TABLE 7: Range of Building Equipment Noise Levels to Which People Are Exposed

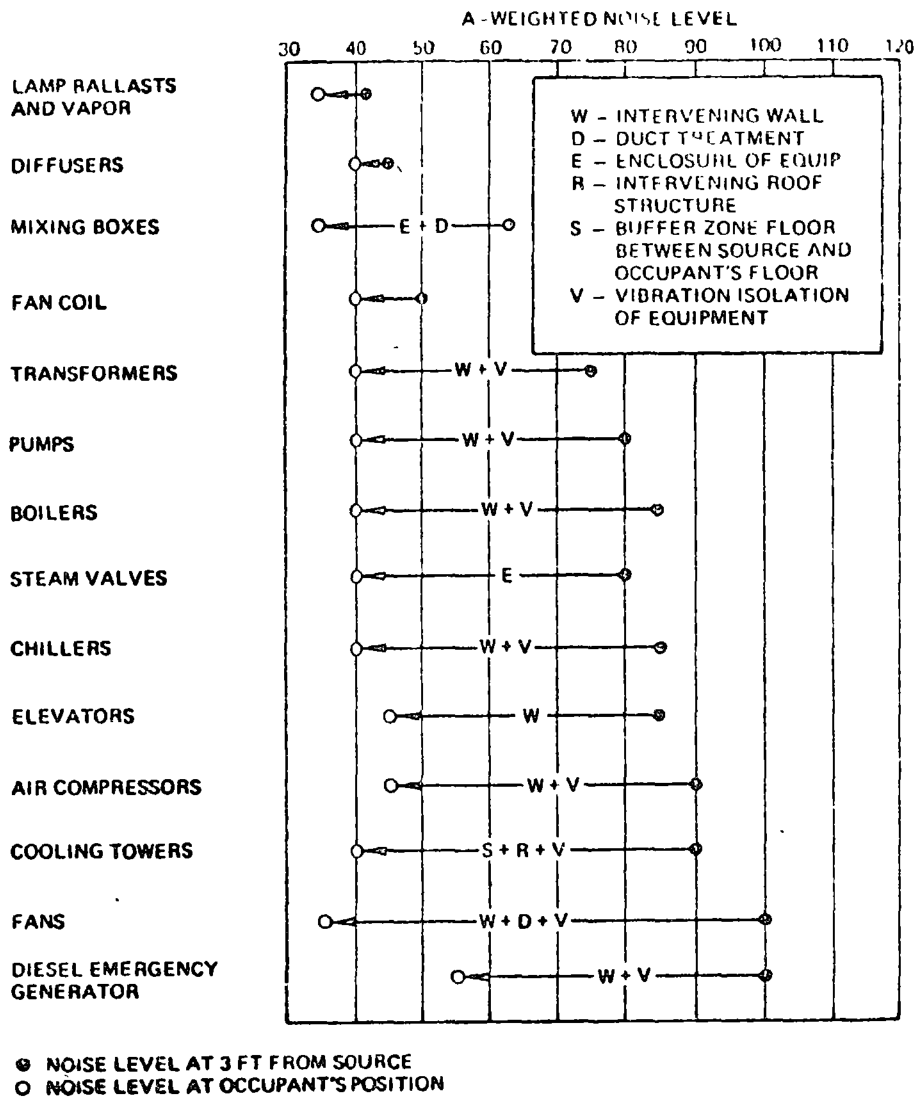


TABLE 8: Noise Levels of Home Appliances and Building Equipment Adjusted for Location of Exposure (IN dBA)

Noise Source	Level of Operator Exposure*	Level of Exposure** of People in Other Rooms
Group I: Quiet Major Equipment and Appliances		
Refrigerator	40	32
Freezer	41	33
Electric Heater	44	37
Humidifier	50	43
Floor Fan	51	44
Dehumidifier	52	45
Window Fan	54	47
Clothes Dryer	55	48
Air Conditioner	55	48
Group II: Quiet Equipment and Small Appliances		
Hair Clipper	60	40
Clothes Washer	60	52
Stove Hood Exhaust Fan	61	53
Electric Toothbrush	62	42
Water Closet	62	54
Dishwasher	64	56
Electric Can Opener	64	56
Food Mixer	65	57
Hair Dryer	66	51
Faucet	66	51
Vacuum Cleaner	67	60
Electric Knife	68	60
Group III: Noisy Small Appliances		
Electric Knife Sharpener	70	62
Sewing Machine	70	62
Oral Lavage	72	62
Food Blender	73	65
Electric Shaver	75	52
Electric Lawn Mower	75	55
Food Disposal (Grinder)	76	68
Group IV: Noisy Electric Tools		
Electric Edger and Trimmer	81	61
Hedge Clippers	84	64
Home Shop Tools	85	75

*Termed "primary exposure"
**Termed "secondary exposure"

32[illegible]

IV. Existing Noise Levels (Step 2)

A. Some Typical Outdoor Values

1. The outdoor daytime residual noise level in a wilderness, such as the Grand Canyon rim, is of the order of 16 dB(A), on the farm it is of the order of 30 to 35 dB(A), and in the city it is of the order of 60 to 75 dB(A). (8)

2. Noise levels for the urban population are shown in Table 9. (6)

   L_{dn} = day-night noise level

3. Outdoor day-night sound levels are shown in Figure 9 for various locations. (6) Figure 10 shows the national population as a function of exterior day-night sound levels. (6)

B. Some Typical Indoor Values --- see Table 10. (6)

V. Effects of Noise (Basis for Step 3)

A. General Comments (3)

Information on effects of noise is best for hearing loss due to noise at work. Other effects of occupational noise, except speech intelligibility interferences, are less certain. These are changes in psychological and physiological states, including annoyance and sleep interruptions. The last two are principal complaints against community and aircraft noise. Property damage by actual vibrational or boom destruction and by depreciation because noise paths and patterns impinge on the property is known, and is to some degree measurable and predictable. Effects on animals seem to have been studied very little. These effects are of concern for wildlife around airports and along highways, and for fish and wildlife in

TABLE 9: Estimated Percentage of Urban Population (134 Million) Residing In Areas With Various Day-Night Noise Levels Together With Customary Qualitative Description Of The Area

Description	Typical Range L_{dn} in dB	Average L_{dn} in dB	Estimated Percentage of Urban Population	Average Census Tract Population Density, Number of People Per Square Mile
Quiet Suburban Residential	48-52	50	12	630
Normal Suburban Residential	53-57	55	21	2,000
Urban Residential	58-62	60	28	6,300
Noisy Urban Residential	63-67	65	19	20,000
Very Noisy Urban Residential	68-72	70	7	63,000

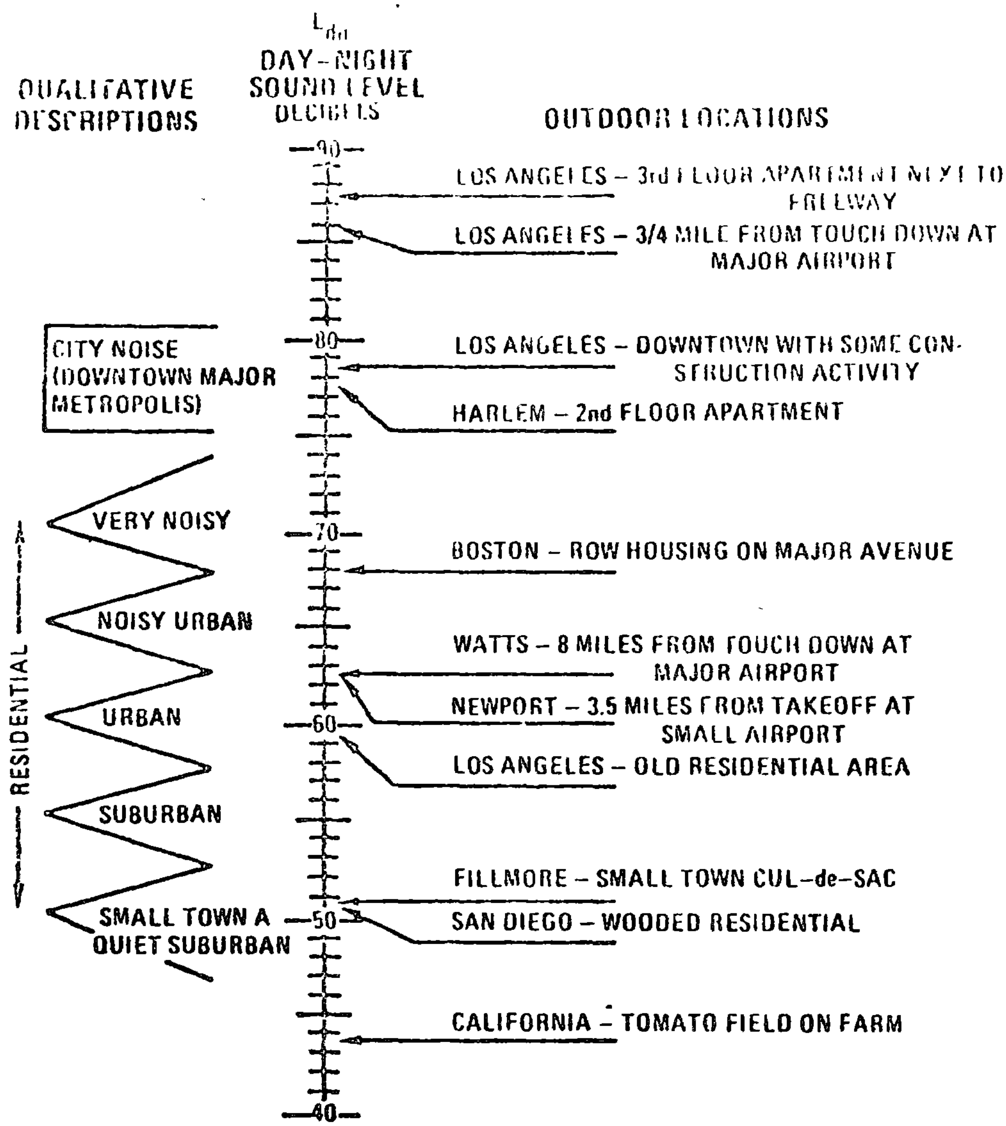


Figure 9: Outdoor Day-Night Sound Level in dB (re 20 micropascals) at Various Locations[4]

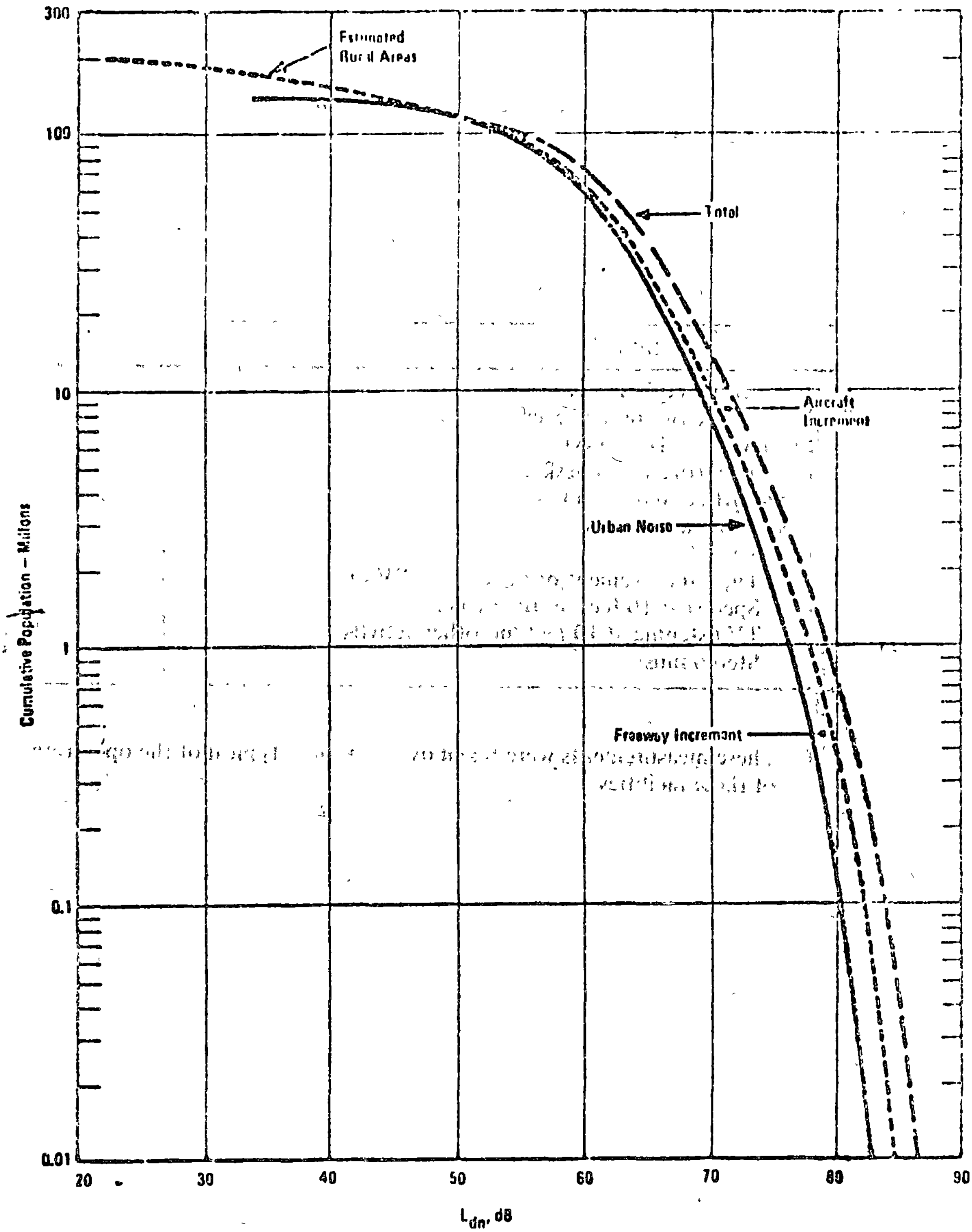


Figure 10: Residential Noise Environment of the National Population As a Function of Exterior Day-Night Average Sound Level[B-5]

TABLE 10: Equivalent Sound Levels in Decibels Normally Occurring Inside Various Places [6]

SPACE	L_{eq}(+)
Small Store (1-5 clerks)	60
Large Store (more than 5 clerks)	65
Small Office (1-2 desks)	58
Medium Office (3-10 desks)	63
Large Office (more than 10 desks)	67
Miscellaneous Business	63
Residences	
Typical movement of people - no TV or radio	40-45
Speech at 10 feet, normal voice	55
TV listening at 10 feet, no other activity	55-60
Stereo music	50-70

(+) These measurements were taken over durations typical of the operation of these facilities.

the pathways of sonic boom. In the first instances habitats may be lost, but the creatures have a chance to migrate and to reestablish beyond the reach of the noise. If there are bad immediate effects on those in the sonic boom paths, there is no escape time.

B. Principles of Hearing (2)

1. Figure 11 shows the functional diagram of the human ear.

2. Sound is generated by a source producing vibrations (sound waves) that may travel through any media and which, in air, actuate the hearing mechanisms of humans and animals. These vibrations set in motion the ear drum and small bones or ossicles of the middle ear as shown in the schematic drawing in Figure 11. The motion of the ossicles, in turn, produces vibrations in the fluid in the inner ear's sensory organ, the cochlea. The vibrations are then transduced into nerve impulses by sensory hair cells and transmitted to the brain, where they are perceived as sound or, depending upon circumstances, as noise.

C. Hearing Changes and Losses

1. There are two types of hearing changes caused by noise exposure. Temporary threshold shift (TTS) is the lessened ability to hear weak auditory signals, from which there is recovery in a matter of hours and at most in 2 to 4 weeks. Noise-induced permanent threshold shift (NIPTS) is a loss from which there is no recovery. The relations between the two are not clear.

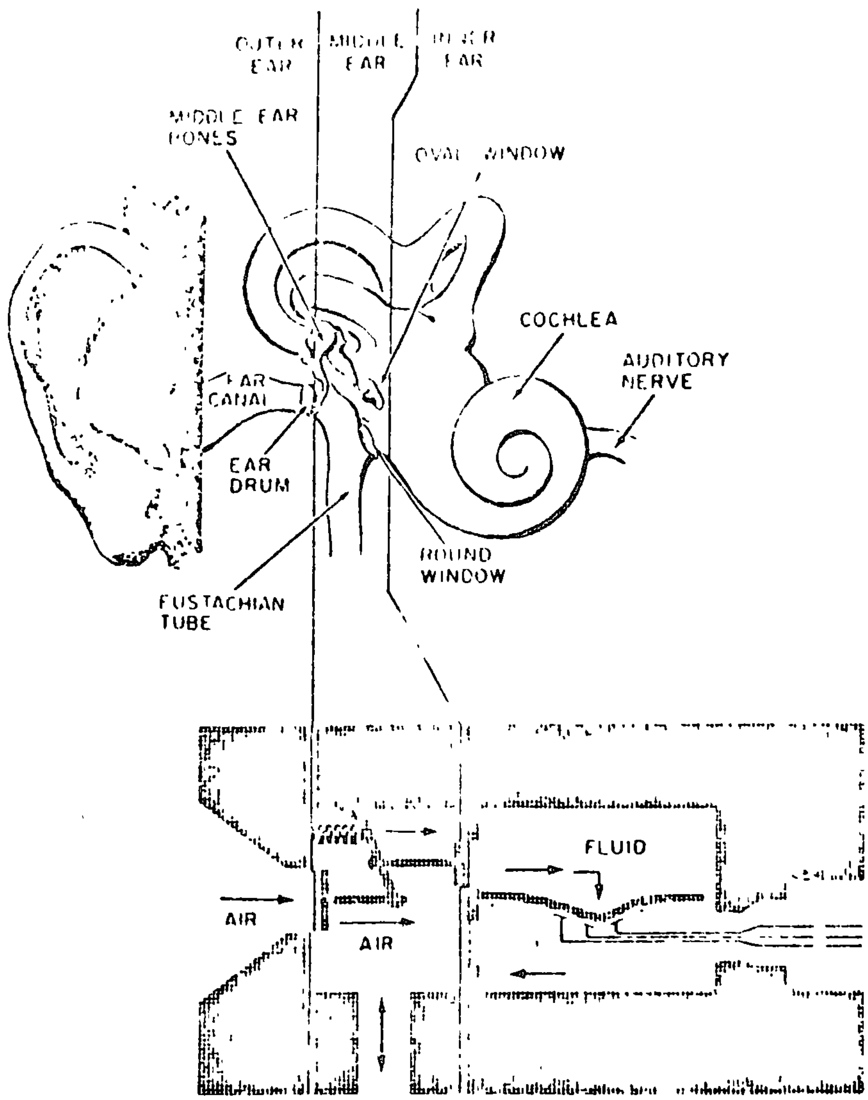


Figure 11: Functional Diagram of Ear

2. For both conditions higher SPLs for long time periods increase severity. By its nature TTS has been easier to study. Some things are known about TTS is that it increases linearly with the average noise level, from about 80 dB up to 130 dB., It is proportional to the fraction of time that the noise is present; therefore steady noise is the major offender.

3. From extended observations several things can be said about NIPTS, a form of deafness.

   (a) Exposures of 8 h/day for several years, to SPLs above 105 dBA are sure to produce NIPTS in a normal unprotected ear. The A in dBA refers to the A scale of measurement, with this scale approximating the frequency response of the human ear.

   (b) The first and most severe NIPTS is at frequencies in the neighborhood of 4,000 Hertz (Hz). Hertz is the unit describing frequencies in cycles per second. The ear transmits sound to the brain best at frequencies between 1,000 and 4,000 Hz.

   (c) If there is going to be partial recovery of the loss, that is, if part of the loss is TTS, almost all such recovery will occur in 2 weeks. There will be some added recovery in a month. Single event injury, as a gun shot near the ear, may show recovery up to 2 months.

   (d) Noise-induced permanent threshold shift is not progressive after the person is moved from the noise. Neither is a noise-damaged ear more susceptible to further injury than a normal ear.

(e) Regular exposure to moderate noise does not make the ear more resistant to occasional exposures to high-intensity noise. The ear does not toughen.

(f) Susceptible individuals cannot be identified before they suffer hearing losses. Monitoring audiometry detects early NIPTS before it becomes severe.

(g) After onset, further NIPTS cannot be avoided except by reducing the noise exposure. There is no way to restore loss from NIPTS.

(h) In the occupational setting, NIPTS will appear in almost all men exposed 8 h/day to broadband noise above 105 dBA. It will appear in about 50 percent of those exposed similarly to a level of 95 dBA. It will not appear in anyone at a level below 80 dBA.

D. Interference with Speech Communication

1. Interference with speech communication by noise impedes our activities and understanding of one another at work, in the home, and in the general social scene.

2. With the increase of the speed and power of machines in manufacture, construction, office work, on the highways, and in the home, the interference noise has become all pervasive.

3. See Figure 12 for visual portrayal of speech interference.

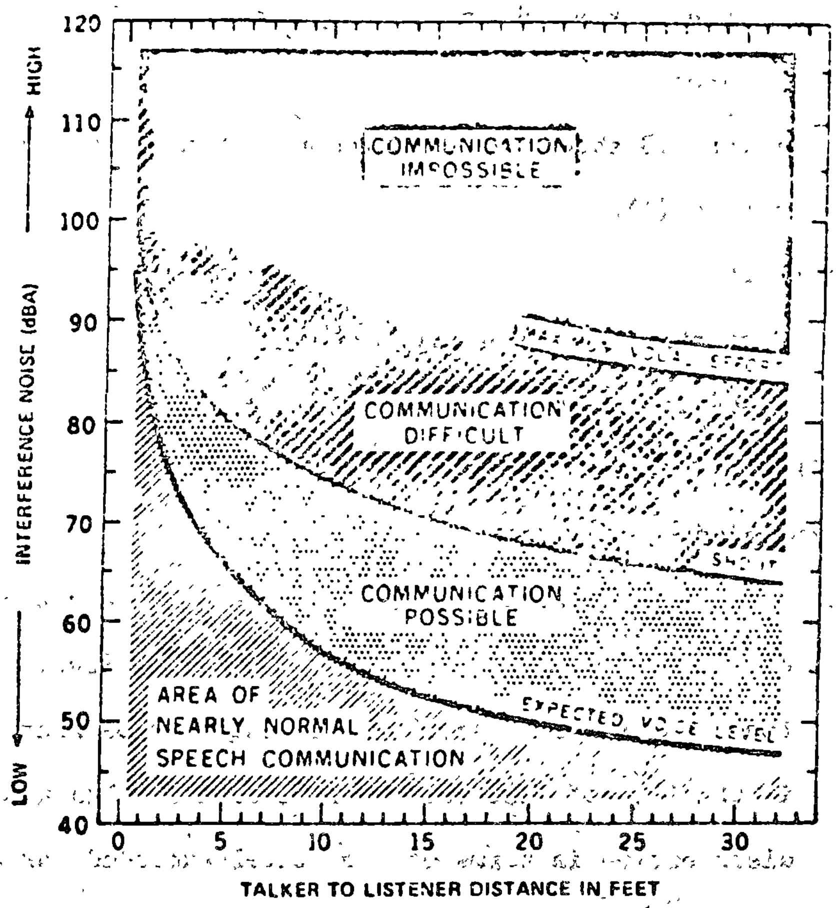


Figure 12: Speech Interference Levels

E. Annoyance

1. To venture into the annoyance effects of noise is to encounter the subjective response of people to noise head-on.

2. Figure 13 shows typical community responses to noise levels (1).

F. Other Effects

1. Disruption of sleep and rest.

2. Reduction in work performance.

VI. Noise Standards and Criteria (Step 3)

A. Terminology

1. A statistical analysis of the noise level gives the percentage or total time that the value of the noise level is found between any two set limits. Such data can be presented directly in the form of histograms, or be used to obtain a cumulative distribution in terms of the "level exceeded for a stated percentage of time". For the sample statistical distribution of Table 11, the noise level exceeds 60 dB(A) for 1 percent of the hour, 55 dB(A) for 10 percent of the hour, 50 dB(A) for 50 percent of the hour, and 45 dB(A) for 90 percent of the hour. These noise levels are abbreviated symbolically as L_1, L_{10}, L_{50} and L_{90}, respectively. (8)

2. Definitions (1)

Community Noise Equivalent Level - Community Noise Equivalent Level (CNEL) is a scale which takes account of all the A-weighted acoustic energy received at a point, from all noise events causing

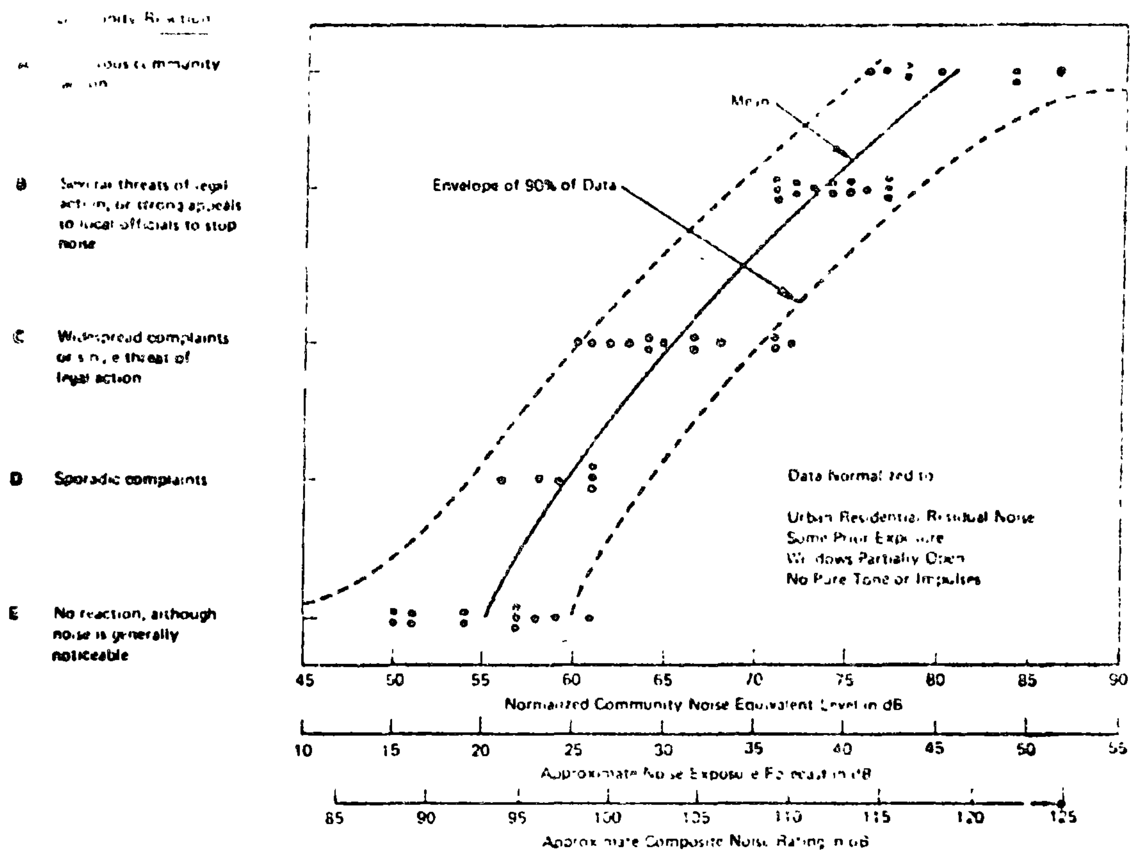


Figure 13: Community Reaction to Intrusive Noises of Many Types as a Function of the Normalized Community Noise Equivalent Level

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TABLE 11: Example of Statistical Distribution of Outdoor Noise Analyzed in Intervals of 5 dB Widths

Interval in dB(A)	Percent of Total Time	Cumulative Percent of Total Time
61 through 65	1	1
56 through 60	9	10
51 through 55	40	50
46 through 50	40	90
41 through 45	10	100

noise levels above some prescribed value. Weighting factors are included which place greater importance upon noise events occurring during the evening hours (7:00 p.m. to 10:00 p.m.) and even greater importance upon noise events at night (10:00 p.m. to 6:00 a.m.).

Composite Noise Rating - Composite noise rating (CNR) is a scale which takes account of the totality of all aircraft operations at an airport in quanitifying the total aircraft noise environment. It was the earliest method for evaluating compatible land use around airports and is still in wide use by the Department of Defense in predicting noise environments around military airfields. Basically, to calculate a CNR value one begins with a measure of the maximum noise magnitude from each aircraft flyby and adds weighting factors which sum the cumulative effect of all flights. The scale used to describe individual noise events is perceived noise level (in PNdB), the term accounting for number of flights is $10 \log_{10} N$ (where N is the number of flight operations), and each night operation counts as much as 10 daytime operations. Very approximately, the noise exposure level at a point expressed in the CNR scale will be numerically 35-37 dB higher than if expressed in the CNEL scale.

Effective Perceived Noise Level (EPNL) - A physical measure designed to estimate the effective "noisiness" of a single noise event, usually an aircraft fly-over; it is derived from instantaneous Perceived Noise Level (PNL) values by applying corrections for pure tones and for the duration of the noise.

Noise Exposure Forecast - Noise exposure forecast (NEF) is a scale (analogous to CNEL and CNR) which has been used by the federal government in land use planning guides for use in connection with airports.

In the NEF scale, the basic measure of magnitude for individual noise events is the effective perceived noise level (EPNL), in units of EPNdB. This magnitude measure includes the effect of duration per event. The terms accounting for number of flights and for weighting by time period are the same as in the CNR scale. Very approximately, the noise exposure level at a point expressed in the NEF scale will be numerically about 33 dB lower than if expressed in the CNEL scale.

Noise and Number Index (NNI) - A measure based on Perceived Noise Level, and with weighting factors added to account for the number of noise events, and used (in some European countries) for rating the noise environment near airports.

Noise Pollution Level (L_{NP} or NPL) - A measure of the total community noise, postulated to be applicable to both traffic noise and aircraft noise. It is computed from the "energy average" of the noise level and the standard deviation of the time-varying noise level.

NOYS - A unit used in the calculation of perceived noise level.

Perceived Noise Level (PNL) - A quantity expressed in decibels that provides a subjective assessment of the perceived "noisiness" of aircraft noise. The units of Perceived Noise Level are Perceived Noise Decibels, PNdB.

Speech-Interference Level (SIL) - A calculated quantity providing a guide to the interfering effect of a noise on reception of speech communication. The speech-interence level is the arithmetic average of the octave-band sound-pressure levels of the interfering noise in the most important part of the speech frequency range. The levels in the three octave-frequency bands centered at 500, 1000, and 2000 Hz are commonly averaged to determine the speech-interence level. Numerically, the magnitudes of aircraft sounds in the Speech-Interference Level scale are approxima ly 18 to 22 dB less than the same sounds in the Perceived Noise Level Scale in PNdB, depending on the spectrum of the sound.

3. See Table 12 for a general comparison of CNR, NEF, and CNEL.

B. Criteria for Protection of Public Health and Welfare (6)

1. The phrase "health and welfare" is defined as complete physical, mental and social well-being and not merely the absence of disease and infirmity.

2. See Table 13 for criteria.

C. Design Objectives --- see Table 14.

D. Example Standards for Federal Highway Administration (9)

1. Table 15 portrays the design noise/land use relationships.

2. The exterior noise levels apply to outdoor areas which have regular human use and in which a lowered noise level would be of benefit. These design noise level values are to be applied

TABLE 12: Factors Considered in Each of Three Methods Used for Describing the Intrusion of Aircraft Noise into the Community

Factor	Composite Noise Rating (CNR)	Noise Exposure Forecast (NEF)	Community Noise Equivalent Level (CNEL)
Basic measure of single event noise magnitude	Maximum perceived noise level	Tone-corrected perceived noise level	A-weighted noise level
Measure of duration of individual single event	None	Energy integration	Energy integration
Time periods during day	Daytime (7 AM-10 PM) Nighttime (10 PM-7 AM)		Daytime (7 AM-7 PM) Evening (7 PM-10 PM) Nighttime (10 PM-7 AM)
Approximate weighting added to noise of single event which occurs in indicated period	Daytime 0 dB Nighttime 12 dB		Daytime 0 dB Evening 5 dB Nighttime 10 dB
Number (N) of identical events in time period	10 log N		10 log N
Summation of contributions	Logarithmic		Logarithmic

TABLE 13: Yearly Average* Equivalent Sound Levels Identified as Requisite to Protect the Public Health and Welfare with an Adequate Margin of Safety

	Measure	Indoor Activity Interference	Indoor Hearing Loss Consideration	To Protect Against Both Effects (b)	Outdoor Activity Interference	Outdoor Hearing Loss Consideration	To Protect Against Both Effects (b)
Residential with Outside Space and Farm Residences	L_{dn}	45		45	55		55
	$L_{eq(24)}$		70			70	
Residential with No Outside Space	L_{dn}	45		45			
	$L_{eq(24)}$		70				
Commercial	$L_{eq(24)}$	(a)	70	70(c)	(a)	70	70(c)
Inside Transportation	$L_{eq(24)}$	(a)	70	(a)			
Industrial	$L_{eq(24)}$(d)	(a)	70	70(c)	(a)	70	70(c)
Hospitals	L_{dn}	45		45	55		55
	$L_{eq(24)}$		70			70	
Educational	$L_{eq(24)}$	45		45	55		55
	$L_{eq(24)}$(d)		70			70	
Recreational Areas	$L_{eq(24)}$	(a)	70	70(c)	(a)	70	70(c)
Farm Land and General Unpopulated Land	$L_{eq(24)}$				(a)	70	70(c)

Code

a. Since different types of activities appear to be associated with different levels, identification of a maximum level for activity interference [illegible] be difficult except in those circumstances where speech communication is a critical activity.
b. Based on lowest level.
c. Based only on hearing loss.
d. An $L_{eq(8)}$ of 75 dB may be identified in these situations so long as the exposure over the remaining 16 hours per day is low enough to result in a negligible contribution to the 24-hour average, i.e., no greater than an L_{eq} of 60 dB.

Note: Explanation of identified level for hearing loss: The exposure period which results in hearing loss at the identified level is a period of 40 years.

*Refers to energy rather than arithmetic averages.

TABLE 14: Design Objectives for Indoor A-Weighted Sound Levels in Rooms with Various Uses

Type or use of space	Approximate A weighted sound level (dBA)
Concert halls, opera houses, recital halls	21 to 30
Large auditoriums, large drama theaters, churches (for excellent listening conditions)	Not above 30
Broadcast, television and recording studios	Not above 34
Small auditoriums, small theaters, small churches, music rehearsal rooms, large meeting and conference rooms (for good listening)	Not above 42
Bedrooms, sleeping quarters, hospitals residences, apartments, hotels, motels (for sleeping, resting, relaxing)	34 to 47
Private or semiprivate offices, small conference rooms, classrooms, libraries, etc. (for good listening conditions)	38 to 47
Living rooms and similar spaces in dwellings (for conversing or listening to radio and television)	38 to 47
Large offices, reception areas, retail shops and stores, cafeterias, restaurants, etc. (moderately good listening)	42 to 52
Lobbies, laboratory work spaces, drafting and engineering rooms, general secretarial areas (for fair listening conditions)	47 to 56
Light maintenance shops, office and computer equipment rooms, kitchens, laundries (moderately fair listening conditions)	52 to 61
Shops, garages, power-plant control rooms, etc. (for just-acceptable speech and telephone communication)	56 to 66

*As recommended by an acoustical engineering firm on the basis of experience with acceptability limits exhibited by the users of the rooms.[16]

TABLE 15: Design Noise Level/Land Use Relationships

Land Use Category	Design Noise Level - L_{10}	Description of Land Use Category
A	60dBA (Exterior)	Tracts of lands in which serenity and quiet are of extraordinary significance and serve an important public need, and where the preservation of those qualities is essential if the area is to continue to serve its intended purpose. Such areas could include amphitheaters, particular parks or portions of parks, or open spaces which are dedicated or recognized by appropriate local officials for activities requiring special qualities of serenity and quiet.
B	70 dBA (Exterior)	Residences, motels, hotels, public meeting rooms, schools, churches, libraries, hospitals, picnic areas, recreation areas, playgrounds, active sports areas, and parks.
C	75 dBA (Exterior)	Developed lands, properties or activities not included in categories A and B above.
D	--	For requirements on undeveloped lands see paragraphs 5a(5) and (6), this PPM.
E	55 dBA (Interior)	Residences, motels, hotels, public meeting rooms, schools, churches, libraries, hospitals and auditoriums.

at those points within the sphere of human activity (at approximate ear level height) where outdoor activities actually occur. The values do not apply to an entire tract upon which the activity is based, but only to that portion in which the activity occurs. The noise level values need not be applied to areas having limited human use or where lowered noise levels would produce little benefit. Such areas would include but not be limited to junkyards, industrial areas, railroad yards, parking lots, and storage yards.

3. The interior design noise level in Category E applies to indoor activities for those situations where no exterior noise sensitive land use or activity is identified. The interior design noise level in Category E may also be considered as a basis for noise abatement measures in special situations when, in the judgment of FHWA, such consideration is in the best public interest. In the absence of noise insulating values for specific structures, interior noise level predictions may be estimated from the predicted outdoor noise level by using the following noise reduction factors:

Building Type	Window Condition	Noise Reduction Due to Exterior of the structure	Corresponding Highest Exterior Noise Level Which Would Achieve an Interior Design Noise Level of 55 dBA
All	Open	10 dB	65 dBA
Light Frame	Ordinary Sash Closed	20	75
	With Storm Windows	25	80
Masonry	Single Glazed	25	80
Masonry	Double Glazed	35	90

Noise reduction factors higher than those shown above may be used when field measurements of the structure in question indicate that a higher value is justified. In determining whether to use open or closed windows, the choice should be governed by the normal condition of the windows. That is, any building having year round air treatment should be treated as the colsed window case. Buildings not having air conditioning in warm and hot climates and which have open windows a substantial amount of time should be treated as the open window case.

E. Example Standards for HUD (10) --- see Table 16.

VII. Prediction of Noise Levles

A. General Model (10)

1. Sound travels through the air in waves, with characteristics of frequency (cycles per second or Hertz) and wave length.

2. If a sound were created at a point, a system of spherical waves would propagate from that point outward through the air at a speed of 1100 feet per second, with the first wave making an ever-increasing sphere with time. As the wave spreads, the height of the wave or the intensity of the sound at any given point must diminish as the fixed amount of energy is spread over an increasing surface area of the sphere. This phenomenon is known as geometric attenuation of the sound.

3. For point-source propagation

$$\text{sound level}_1 - \text{sound level}_2 = 20 \log \frac{r_2}{r_1}$$

where the sound level at station one minus the sound level at station two is equal to twenty times the log of the ratio

TABLE 16: Residential Noise Level Guidelines

(Note: Measurements and projections of noise exposures are made at appropriate heights above site boundaries)

GENERAL EXTERNAL EXPOSURES - dB(A)	AIRPORT ENVIRONS - NEF ZONE
UNACCEPTABLE	
Exceeds 80 dB(A) 60 minutes per 24 hours Exceeds 75 dB(A) 8 hours per 24 hours	greater than 40
(Exceptions are strongly discouraged and require a 102(2)C environmental statement and the Secretary's approval)	
DISCRETIONARY-NORMALLY UNACCEPTABLE	
Exceeds 65 dB(A) 8 hours per 24 hours Loud repetitive sounds on site	between 30 and 40
(Approvals require noise attenuation measures, the Regional Administrator's concurrence and a 102(2)C environmental statement)	
DISCRETIONARY-NORMALLY ACCEPTABLE	
Does not exceed 65 dB(A) more than 8 hours per 24 hours	less than 30
ACCEPTABLE	
Does not exceed 45 dB(A) more than 30 minutes per 24 hours	less than 30

of the radii. This means that for every doubling of distance, the sound level will decrease by 6 decibels. In other words, if station one were at a distance of 50 feet from the point source and, if station two were a distance of 100 feet from the point source, the sound level measured at point two would be 6 dB less than the sound level measured at point one. This is called the inverse-square law. This kind of relationship holds true for single vehicles and aircraft when sound is propagating in free air, either from the airplane to the ground in a complete spherical sense, or in the case of an automobile on the ground when the propagation field is only half a sphere. See Figure 14.

4. Line-source propagation --- When a number of vehicles are lined up and constitue a continuous stream of noise sources, the situation is no longer characterized by a spherical or hemispherical spreading of the sound, but rather the reinforcement by the line of point-sources makes the propagation field like a cylinder or half a cylinder. In this case the equation is as follows:

For Line-Source Propagation:

$$\text{sound level}_1 - \text{sound level}_2 = 10 \log \frac{r_2}{r_1}$$

Thus, the decrease in sound for each doubling of distance from a line source is only 3 decibels. When we consider noise levels from busy highways, we will essentially consider the noise source of the highway as an infinite line source and a 3 dB per

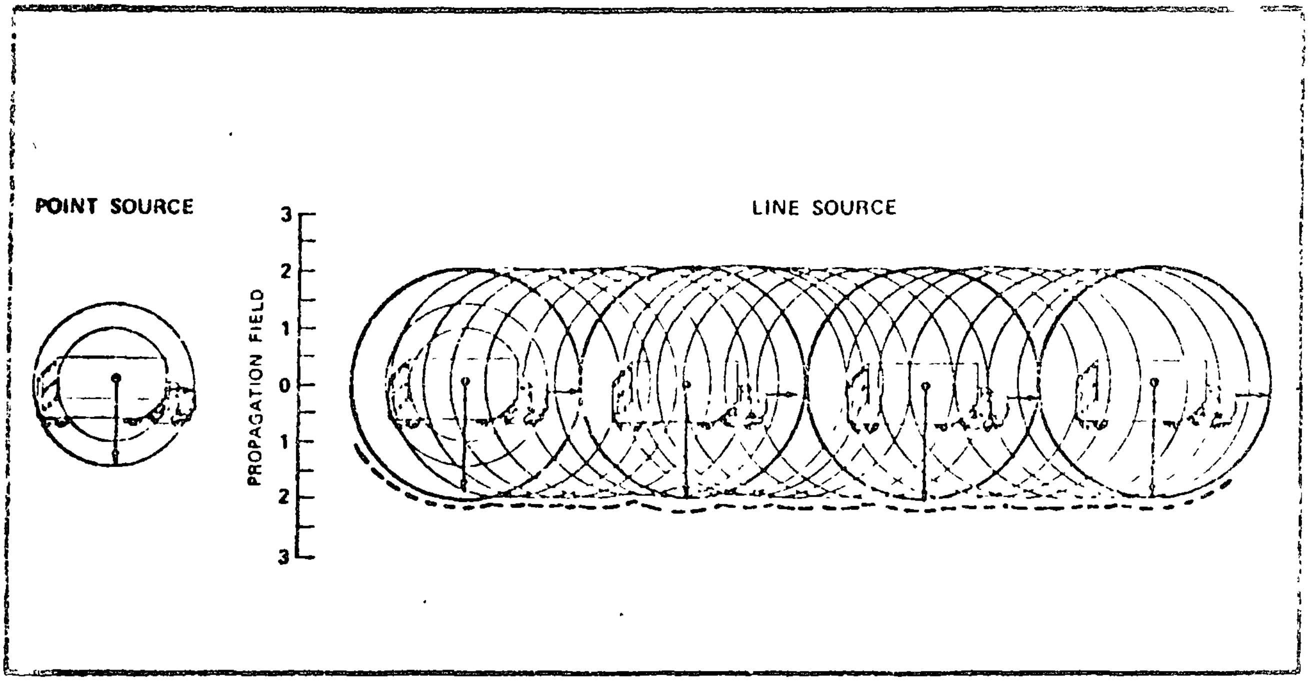

Figure 14: Sound Propagation Comparison

doubling of distance propagation rate will be utilized. However, when we consider a situation such as the propagation of noise from a multi-car transit train, the 3 dB per doubling of distance propagation rate will be applicable to approximately a distance of 3/10 of the length of the train or a finite line source. Beyond that distance, a 6 dB per doubling of distance attenuation rate should be applied.

B. Aircraft Noise Exposure Contour Model (11)

1. This model is based on the physical characteristics of the runways and flights using the facility.

2. The program provides the user with three output options. The first is the Contour Output, which provides a list of coordinates that give the location of a specified Noise Exposure Level. Also given is the area enclosed by these points. The second output, the Grid Output, provides the Noise Exposure Level for each set of coordinates specified by the user. Finally, the Diagnostic Output provides a subtotal, the Effective Perceived Noise Level, which includes the present flight condidered plus all previous flights. This output also gives a complete description of the present flight being considered, including such other information as aircraft type, percent thrust, and elevation.

3. The outline of the model, including input requirements, output options, and flexibility, is shown in Table 17.

TABLE 17: Aircraft Noise Exposure Contour Model Outline

A. Input Requirements of Model

1. Number of runways, and for each runway:

   (a) Runway coordinates - Cartesian coordinates of runway end points;

   (b) Distance from the end of runway to the start of takeoff roll;

   (c) Distance from the end of runway to the touchdown point; and

   (d) Number of flights on each runway (a flight is a given aircraft type on a given flight path), and for each flight:

      (1) Type of flight - takeoff or landing;

      (2) Type of aircraft;

      (3) Number of operations; and

      (4) Number of segments composing each operation, and for each segment:

         a) Segment length;

         b) Flight path climb angle;

         c) Thrust level of aircraft; and

         d) Radius of curved segment.

B. Output Options of Model

1. Contour Output

   (a) Required inputs:

      (1) Noise exposure (NE) level;

      (2) Tolerance of NE Level; and

      (3) Estimate of X and Y coordinates of location of first point with given NE level.

   (b) Output:

      (1) X and Y coordinates of locations of specified NE level; and

      (2) Area enclosed by points on line connecting the prescribed NE level.

2. Grid output
   (a) Required inputs.
      (1) X and Y coordinates of the starting point on grid;
      (2) Increments for X and Y; and
      (3) Number of points on X and Y axis.
   (b) Output:
      (1) NEF levels for each set of coordinates specified.
3. Diagnostic output
   (a) Required inputs:
      Sames as one required for grid output.
   (b) Output:
      (1) Flight number;
      (2) Aircraft type;
      (3) Slant range to observer;
      (4) Elevation angle to observer;
      (5) Percent thrust;
      (6) EPNL (Effective Perceived Noise Level of aircraft at slant range H);
      (7) Corrections for
         a) Shielding;
         b) Excess ground attenuation; and
         c) Number of operations.
      (8) Net EPNL; and
      (9) Subtotal EPNL - includes present flight plus all preceding ones.

C. Flexibility of Model
   1. Has been converted for use on the IBM 360/195. (The original program was written for the CDC 6400.)
   2. Program has been designed to generate NEF levels, but it can easily be modified to produce
      (a) NEF:

$$NEF = EPNL + 10 \log NOPS - 88$$

(NOPS = number of observations)

      (b) NEF:

$$NOPS = NOPS_{day} + 16.67\ NOPS_{night}$$

3. Program presently has:

(a) 24 noise curves EPNL vs Slant Height - one curve for takeoff and one for landing for each of 12 aircraft types;

(b) 6 thrust correction curves - EPNL vs Percent Thrust; and

(c) 2 excess ground attenuation curves EPNL vs Distance.

4. Finally, new relationships developed for both new and existing aircraft may be easily incorporated into the program as data statements.

C. Highway Noise Prediction Models

1. The Federal Highway Administration PPM 90-2 (9) stated that two highway noise prediction methods were acceptable:

   (a) The method in the National Cooperative Highway Research Program Report 117 (12).

   (b) The method in the Department of Transportation, Transportation Systems Center Report DOT-TSC-FHWA-72-1. (13)

2. A model developed at Argonne National Laboratory was published in 1973 (11). Basically, the model requires characteristics of the highway segments as input. The characteristics include a description of the traffic using the highway (the speeds and volumes of both automobiles and trucks), the physical dimensions of the facility (the elevation, depression, grades and surface types), and the aspects of the environment bordering the facility that have an effect on the noise levels (the landscaping, structures, and barriers). Fundamentally, the model calculates a noise level at a particular point along the highway and a perpendicular distance away from the highway. Once this noise level is calculated, the model moves outward an incremental distance away from the facility and calculates another noise level. This process is repeated until the model reaches a maximum prescribed distance away from the highway. At this point, the model moves farther down the highway and calculates another group of noise levels. This is repeated until the model has covered the entire length of the highway. The model prints out a contour map of noise levels at given distances from the facility over

the entire length of the facility.

(a) Figure 15 shows a schematic diagram for the use of the model. The fiftieth percentile noise level is that noise level that will be exceeded 50% of the time.

(b) Two options for estimating highway noise are available to the user of this model. The first allows the user to predict noise levels at various desired heights above ground level. This may be of special importance in an area that is composed of multi-story apartments and commerical sturctures. The second option allows the user to calculate the predicted noise levels in the fiftieth or tenth percentile noise levels. The tenth percentile noise level is that level that will be exceeded 10% of the time. The fiftieth percentile level is useful as it provides what may be termed the median noise level. On the other hand, the tenth percentile noise level will also be required as many noise standards are beginning to incorporate this level.

VIII. Noise Control Principles (Basis for Step 5)

A. Reduction of Vibrating Sources (3)

Noise is produced by an aerodynamic disturbance such as air moving in a duct, discharging from a pneumatic tool,and being pushed about along the surfaces of speeding cars and trucks, beat about by propellors, or squeezed and thermally expanded through jet engines. Or noise is generated by the vibration of structures purposefully set in motion, as in internal-combustion engine or the shuttle of a

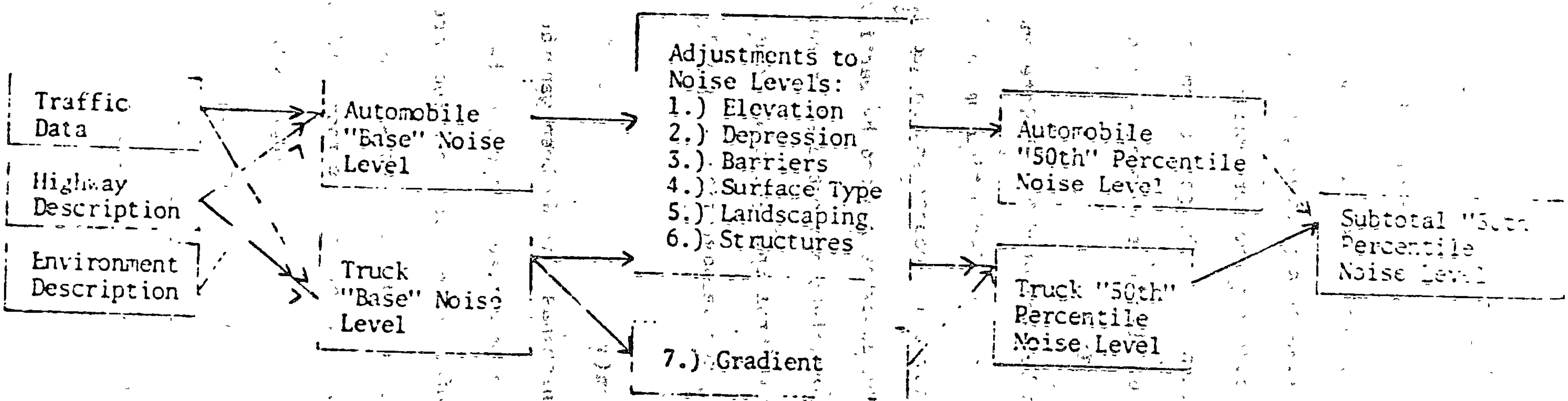

Figure 15: Determination of noise levels

loom. Noise is also produced by a surface that is vibrating as it is connected to the moving parts of machinery, such as a fan housing or a mounting of a punch press or a packaging machine. Control at the source then depends on altering the aerodynamic characteristics of the vibrating air by dimensional changes, by smoothing its flow to reduce turbulence, and by absorbent materials along its path.

B. Enclosure of the Source (3)

The escape of noise can be prevented by complete enclosure of the noise-maker. With provision for heat dissipation, motors and production machines can be put on vibration mounts and housed in sound-absorbent materials. Escape can be somewhat reduced by partial enclosure. Abosorbent baffles at air inlets and outlets can reduce the escape of fan noise. Mufflers control the escape of exhaust air and gas noise partly by altering the aerodynamics and partly by absorbing existing vibrations.

C. Attenuation by Absorption(3)

The behavior of noise which has already been generated and which has escaped into a room can be modified. Acoustic characteristics which influence the behavior of emitted noise in a room are the absorption coefficients of surfaces exposed to the noise; the reverberation time which depends on the noise source and the room; and the transmission losses through the walls, floor, and ceiling. Acoustically, reverberation time of a room is the period required for any sound to decrease by 60 dB after the source is cut off and absorption takes place. This technique of control depends on reducing the noise level by improving the absorption characteristics of the room. Modification and wall and ceiling qualities is the principle procedure, and floors as well where change does not interfer with floor serviceability. Application of this approach is a demanding task for a skilled, experienced acoustic control expert.

IX. Noise Control Practices (Step 5)

A. Industrial Noise (3)

The methods of noise control in the United States are well formulated for controlling industrial noise. The principles embrace plant planning; substitution of quieter equipment, processes, and materials; reduction at the source and reduction by transmission by air.

B. Subsonic Aircraft Noise Abatement (10)

The following lists some of the current noise abatement techniques, procedures, and other alternatives to counter subsonic aircraft noise sources:

1. Aircraft Design or Modification

New quiet engine designs with high bypass ratios and low velocity nets.

Acoustically treated nacelles and ducts

Exhaust silencers for reciprocating and turboshaft engines

Noise suppression for on-board auxiliary power units

Rotor and propeller aerodynamics for reduced noise

Noise suppression for mechanical components such as helicopter gear boxes

Vehicle aerodynamics to allow for steeper ascent and descent, and/or reduction in time required for ascent/descent

2. Aircraft Operations

Restrict operations by type of aircraft, number of operations or time of day

Power cutback on takeoff or steep climb-out depending on situation

Steep or multi-segment approach depending on situation

Preferential runway assignments

3. Aircraft Mainenance

Restrict engine "runups" during ground maintenance operations

Maintenance of additonal hardware for noise suppression (i.e., treated nacelles or auxiliary-power-unit silencers

4. Aircraft Route Location

Avoid noise sensitive areas in new route assignments

Modify existing routes to avoid noise sensitive areas

Utilize noise-insensitive areas for ascent and descent paths

5. Landscape Architecture

Shield airport surroundings from noise resulting from aircraft ground operations and surface vehicle operations

6. Acoustic Insulation

Insulation of dwellings against aircraft noise

Insulation of commercial structures against aircraft noise

7. Land Use

Control by zoning authorities for compatible land use

C. Highway Noise Abatement (10)

1. Three options for noise reduction are:

(a) Man-constructed barriers to obstruct or dissipate sound emissions

(b) Elevated or depressed highway through grading

(c) Absorption effects of landscaping (trees, bushes, shrubs, etc.)

2. Constructing Barriers.

A rigid (fairly massive) barrier can be an effective means to reduce noise from highways depending upon the relative heights of the barrier, the noise source, and the affected area, as well as the horizontal distance between the source and the barrier and between the barrier and the noise-affected area.

3. Elevated or Depressed Highways.

Often a highway in an urban area can be built above or below the surrounding property. Such differences in grade provide some shielding of traffic noise and can reduce the noise levels at adjacent properties.

4. Effects of Planting.

Planting adjacent to a highway produces little physical reduction in noise level unless it is very dense and of significant depth.

5. Some other noise control measures for highways:

(a) Limitations on allowable grades.

(b) Road surface repairs

(c) Route locations planned to insure maximum separation between roadway and existing noise sensitive areas and to make maximum use of shielding provided by natural barriers

(d) Provide for compatible use of land adjacent to highways

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PROGRAMACION Y MODELOS DE INGENIERIA AMBIENTAL

MODELOS DE DISPERSION DE CONTAMINACION ATMOSFERICA

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ABRIL DE 1976

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PREDICTION AND ASSESSMENT OF IMPACTS ON THE AIR ENVIRONMENT

One of the major impacts from many actions is on the air quality in the vicinity of the project area. Construction of highways, airports, dams, waterways, power plants, industrial parks, apartment houses, and pipelines generate construction dusts and exhaust emissions from construction equipment. The operation of airports and power plants, and the use of highways and industrial parks also cause emission of gaseous and particulate air pollutants. This chapter is addressed to data needs and associated technology for predicting and assessing the impact of a proposed action on air quality.

BASIC STEPS FOR PREDICTION AND ASSESSMENT

The basic steps associated with prediction of changes in air quality, and assessment of the impact of these changes are as follows:

1. Identify air pollutants emitted from the alternatives under consideration for meeting a given need.

2. Describe or determine the existing air quality levels in the area. If possible, present historical trends in air quality. Examine the frequency distribution, and the median and mean concentrations for each gaseous or particulate air pollutant which has an ambient air quality standard.

3. Determine the air pollution dispersion potential for the area; this can be accomplished by aggregating information on seasonal or monthly variations of mean mixing depth, inversion heights, wind speeds, high air pollution potential, and episode-days. Historical records of air pollution episodes in the area should be described.

4. Summarize the basic meteorological data for the area; this should include monthly summaries of precipitation, temperature, wind speed and direction, solar radiation and other parameters if deemed appropriate. Included in this information should be the monthly, seasonal, and annual wind roses. Any unique meteorological phenomenon which have occurred in the area should be identified.

5. Procure ambient air quality standards and emissions standards, if they are relevant. Consideration should be given to the time schedule required for meeting these standards.

6. Summarize emission inventory data for the smallest applicable scale of the region, and include the regional emission inventory. Identify the major point sources of air pollution in the area and indicate the quantities of pollutants emitted as well as the specific location of these point sources relative to the sites of the alternatives under study.

7. Determine the mesoscale impact due to construction and operation of each alternative. This can be accomplished by calculating the estimated annual quantity of air pollutants from each alternative and determining the percentage increase in the regional and local emission inventory for each pollutant emitted. Particular attention should be addressed to increases in Priority I or Priority II pollutants (these priorities will be defined later).

8. Calculate the ground level concentrations of air pollutants from the alternatives under varied meteorological conditions. Develop isopleths of concentration in the vicinity of the sources of emission. In order to determine the microscale impact, compare the calculated air quality levels with the applicable ambient air standards. The proportion between the calculated levels and the applicable ambient air standards should be considered. If emissions standards are applicable to the alternatives, then the emission standards for the action should be considered in relationship to the anticipated emission levels.

9. If ambient air or emission standards are exceeded by the proposed action, consider mitigation or control measures in order to minimize the air quality impact.

The above nine steps are directed toward determining the air quality impacts of alternatives and a proposed action on the mesoscale and microscale levels. The mesoscale level assessment is oriented to the contribution of the proposed action to area and regional emission inventories. The microscale level assessment is oriented to a comparison of calculated concentration levels of air pollutants at specific locations to applicable ambient air quality standards. Both levels of impact assessment are necessary in order to adequately address the air quality impacts associated with proposed actions. Steps 2-6 should be summarized in the environmental setting, and the remainder in the environmental impact section.

The organization of this chapter is primarily oriented to the nine steps identified above. However, prior to the step-by-step discussion and analysis, some brief information on basic air pollution considerations is presented.

BASIC INFORMATION ON AIR POLLUTION

Air pollution maybe defined as (1):

> "the presence in the outdoor atmosphere of one or more contaminants such as dust, fumes, gas, mist, odor, smoke, or vapor in quantities, of characteristics, and of duration, such as to be injurious to human, plant or animal life or to property, or which unreasonably interferes with the comfortable enjoyment of life and property."

Several key ideas are embodied in the above definition of air pollution, namely; (1) the focus is on the outdoor atmosphere and does not include the industrial working environment, (2) air pollution may be caused by single contaminant gases or particulates, or combinations of these contaminants, (3) the concentration or quantity of material is a basic determinant in causing air pollutant effects, (4) the time of exposure, or the persistence of a given concentration level of a pollutant is also a basic determinant in the effects of air pollution, and (5) the effects of air pollutants can occur on living things, inanimane objects, and the aesthetic features of an area.

There are a number of significant dates in the history of air pollution occurrences. (2) One of the first recorded dates occurred about 1300, when King Edward I issued a proclamation prohibiting the use of sea coal during sessions of the English Parliament. In 1866 the first paper on the health effects of air pollution was presented, and in 1875, cattle deaths in London were shown to be due to an air pollution episode. In the early 1900's the term "smoke-fog" began to be utilized, with this term being shortened to "smog" which has become a synanym for air pollution. Some major air pollution episodes occurred in the Meuse Valley in Belgium in 1930, in Donora, Pennsylvania in 1948, and in London, England in 1952. In 1955 the first U.S. legislation dealing with air pollution was passed, and in 1963 the Clean Air Act was created. The Clean Air Act was amended in 1967, 1970 and 1974. (3)

The sources of air pollution can be categorized according to type, that is, whether natural or manmade; by number and spatial distribution; or by type of emissions such as gases and particulates. The number and spatial distribution category includes single or point sources, area or non-point sources, and line sources.

The two major classes of gaseous air pollutants are inorganic gases and organic vapors. Examples of widely-occurring inorganic gases include sulfur dioxide, oxides of nitrogen, carbon monoxide, and hydrogen sulfide; and organic vapors include hydrocarbons, mercaptans, alcohols, ketones, and esters. Organic vapors are generally localized pollutants. Secondary gaseous air pollutants resulting from photochemical reactions include oxidants, with the primary component being ozone. Sulfur, nitrogen, and carbon-containing inorganic gases can be oxidized in the atmosphere to their most oxidized form and then be combined with water vapor to create acidic mists such as sulfuric acid, nitric acid, and carbonic acid.

Particulate air pollutants are any dispersed matter, solid or liquid, in which the individual aggregates are larger than single small molecules (about 0.0002 μ in diameter), but smaller than about 500 microns. Particles persist in the air from a few seconds to several months. (4) Particulate matter is basically divided into two broad categories dependent upon the sampling technique. Total suspended particulates are those materials that can be filtered from the atmosphere through the use of a high-volume air sampler. Settleable solids or "fallout" or dustfall refers to those materials that are deposited by gravity into a dustfall sampler over a period of one month. The most used particulate measurement for air quality control is the total suspended particulates.

One of the primary concerns is the effect of air pollutants on aesthetics, economic viability, safety, personal discomfort and health. (5) The aesthetic effects include loss of clarity of the atmosphere due to the presense of particulates and/or photochemical smog, and the presense of objectionable odors primarily associated with gases such as ammonia and sulfur-containing mercaptans. Economic losses attributable to air pollution include the soiling effect of particulates; damage to vegetation and crops resulting from exposure to excessive concentrations of gases such as sulfur dioxide, oxides of nitrogen and ozone; damage to livestock associated with exposure to flourine; and deterioration of exposed materials by a variety of air pollutants. Materials deterioration includes corrosion of metals by sulfur dioxide, weathering of stone by acidic mists, darkening of lead-based white paint by hydrogen sulfide,accellerated cracking of rubber by ozone, and deterioration of fabrics such as nylons by sulfur dioxide. Safety hazards associated with air pollution result primarily from decreased visibility, and they can become of major concern in conjunction with airport operations as well as ground transportation. Personal discomfort is associated with eye irritation from photochemical oxidants and irritation to individuals with respiratory difficulties from a variety of pollutant forms. Actual health hazards may result from air pollution, with an example being the short-term affects of carbon monoxide in urban areas characterized by heavy traffic. There is some evidence regarding accute illness and even death resulting from air pollution, while substantive data regarding long-term effects of exposures to lower concentrations of air pollutants is minimal. Tables 1 and 2 contain a summary of the results of a 1968 survey of air pollution damages. (6)

IDENTIFICATION OF AIR POLLUTANTS (STEP 1)

The first step associated with prediction and assessment of air quality impacts involve identification of the type and quantities of air pollutants emitted from the construction and operation of each alternative under consideration for a proposed action. One approach for identifying the anticipated air pollutants is to review other environmental statements prepared on projects of similar type. Perhaps the best approach is to utilize emission factors arranged according to man's various activities. An emission factor is the statistical average of the rate at which a pollutant is released to the atmosphere as a result of some activity, such as

Table 1: NATIONAL COSTS OF POLLUTION DAMAGE, BY POLLUTANTS, 1968

($-billion)

Effects (loss category)	SO_x	Part.	Oxidant	NO_x	Total
Residential property	2.808	2.392	-	-	5.200
Materials	2.202	0.691	1.127	0.732	4.752
Health	3.272	2.788	-	-	6.060
Vegetation	0.013	0.007	0.060	0.040	0.120
Total	8.295	5.878	1.187	0.772	$16.132

Table 2: NATIONAL COSTS OF POLLUTION DAMAGE,

BY SOURCE AND EFFECT, 1968

($ billion)

Effects	Stationary source fuel combustion	Transportation	Industrial processes	Solid waste	Miscellaneous	Total
Residential property	2.802	0.156	1.248	0.104	0.884	5.200
Materials	1.853	1.093	0.808	0.143	0.855	4.752
Health	3.281	0.197	1.458	0.119	1.005	6.060
Vegetation	0.047	0.028	0.020	0.004	0.021	0.120
Total	7.983	1.474	3.534	0.370	2.765	$16.132

production by industry or combustion, divided by the activity. (7) Examples of construction phase emission factors are shown in Table 3 (8) for asphalt plants and Table 4 (9) for concrete batching plants. To give an idea of the accuracy of the factors presented for a specific process, each process is ranked as "A," "B," "C," "D," or "E." For a process with an "A" ranking, the emission factor is considered excellent, i.e., based on field measurements of a large number of sources. A process ranked "B" is considered above average, i.e., based on a limited number of field measurements. A ranking of "C" is considered average; "D," below average; and "E," poor. (7) Examples of operational phase emission factors are shown in Table 5 (10) for incinerators, Table 6 (11) for zinc smelters and Table 7 (12) for aircraft. The primary thing to note from Tables 3-7 is that information is provided, at various levels of reliability, to enable the calculation of the total quantity of air pollution anticipated from the given activity. This information is basic to the prediction of the mesoscale air quality impact of the alternatives for a proposed action.

DESCRIPTION OF EXISTING AIR QUALITY LEVELS (STEP 2)

The second step in the process is to assemble information on the existing air quality levels in the area of the project, particularly for those anticipated air pollutants to be emitted from the construction and operational phases of the project. Sources of information include the relevant county and State air pollution control agency, and private industries in the area that might be maintaining an air quality monitoring program for their particular interests. One of the best sources of information is the SAROAD System of the Environmental Protection Agency, with

Table 3: PARTICULATE EMISSION FACTORS

FOR ASPHALTIC CONCRETE PLANTS

EMISSION FACTOR RATING: A

	Emissions	
Type of Control	lb/ton	kg/MT
Uncontrolled[a]	45.0	22.5
Precleaner	15.0	7.5
High-efficiency cyclone	1.7	0.85
Spray tower	0.4	0.20
Multiple centrifugal scrubger	0.3	0.15
Baffle spray tower	0.3	0.15
Orifice-type scrubber	0.04	0.02
Baghouse[b]	0.1	0.05

[a] Almost all plants have at least a precleaner following the rotary dryer.

[b] Emissions for a properly designed, installed, operated, and maintained collecter can be as low as 0.005 to 0.020 lb/ton (0.0025 to 0.010 kg/MT).

Table 4: PARTICULATE EMISSION FACTORS FOR CONCRETE BATCHING[a]

EMISSION FACTOR RATING: C

Concrete batching	Emission lb/yd^3 of concrete	Emission kg/m^3 of concrete
Uncontrolled	0.2	0.12
Good control	0.02	0.012

[a]One cubic yard of concrete weighs 4000 pounds (1 m^3 = 2400 kg). The cement content varies with the type of concrete mixed, but 735 pounds of cement per yard (436 kg/m^3) may be used as a typical value.

Table 5: EMISSION FACTORS FOR REFUSE INCINERATORS WITHOUT CONTROLS[a]

EMISSION FACTOR RATING: A

Incinerator type	Particulates lb/ton	Particulates kg/MT	Sulfur oxides[b] lb/ton	Sulfur oxides[b] kg/MT	Carbon monoxide lb/ton	Carbon monoxide kg/MT	Hydrocarbons[c] lb/ton	Hydrocarbons[c] kg/MT	Nitrogen oxides[d] lb/ton	Nitrogen oxides[d] kg/MT
Municipal										
Multiple chamber, uncontrolled	30	15	2.5	1.25	35	17.5	1.5	0.75	3	1.5
With settling chamber and water spray system	14	7	2.5	1.25	35	17.5	1.5	0.75	3	1.5
Industrial/commercial										
Multiple chamber	7	3.5	2.5	1.25	10	5	3	1.5	3	1.5
Single chamber	15	7.5	2.5	1.25	20	10	15	7.5	2	1
Trench										
Wood	13	6.5	0.1	0.05	NA[e]	NA	NA	NA	4	2
Rubber tires	138	69	NA	NA	NA	NA	NA	NA	NA	NA
Municipal refuse	37	18.5	2.5[h]	1.25	NA	NA	NA	NA	NA	NA
Controlled air	1.4	0.7	1.5	0.75	Neg	Neg	Neg	Neg	10	5
Flue-fed single chamber	30	15	0.5	0.25	20	10	15	7.5	3	1.5
Flue-fed (modified)	6	3	0.5	0.25	10	5	3	1.5	10	5
Domestic single chamber										
Without primary burner	35	17.5	0.5	0.25	300	150	100	50	1	0.5
With primary burner	7	3.5	0.5	0.25	Neg	Neg	2	1	2	1
Pathological	8	4	Neg	Neg	Neg	Neg	Neg	Neg	3	1.5

[a] Average factors given based on EPA procedures for incinerator stack testing.

[b] Expressed as sulfur dioxide

[c] Expressed as methane

[d] Expressed as nitrogen dioxide

[e] Not available

Table 6: EMISSION FACTORS FOR PRIMARY ZINC SMELTING WITHOUT CONTROLS[a]

EMISSION FACTOR RATING: B

Type of operation	Particulates		Sulfur oxides	
	lb/ton	kg/MT	lb/ton	kg/MT
Roasting (multiple-hearth)	120	60	1100	550
Sintering	90	45	b	b
Horizontal retorts	8	4	-	-
Vertical retorts	100	50	-	-
Electrolytic process	3	1.5	-	-

[a] Approximately 2 unit weights of concentrated ore are required to produce 1 unit weight of zinc metal. Emission factors expressed as units per unit weight of concentrated or produced.

[b] Included in SO_2 losses from roasting.

Table 7: EMISSION FACTORS PER AIRCRAFT LANDING-TAKEOFF CYCLE

(lb/engine and kg/engine)

EMISSION FACTOR RATING: B

Aircraft	Solid particulates		Sulfur oxides[a]		Carbon monoxide		Hydrocarbons		Nitrogen oxides (NO_x as NO_2)	
	lb	kg	lb	kg	lb	kg	lb	kg	lb	kg
Jumbo jet	1.30	0.59	1.82	0.83	46.8	21.2	12.2	5.5	31.4	14.2
Long range jet	1.21	0.55	1.56	0.71	47.4	21.5	41.2	18.7	7.9	3.6
Medium range jet	0.41	0.19	1.01	0.46	17.0	7.71	4.9	2.2	10.2	4.6
Air carrier turboprop	1.1	0.49	0.40	0.18	6.6	3.0	2.9	1.3	2.5	1.1
Business jet	0.11	0.05	0.37	0.17	15.8	7.17	3.6	1.6	1.6	0.73
General aviation turboprop	0.20	0.09	0.18	0.08	3.1	1.4	1.1	0.5	1.2	0.54
General aviation piston	0.02	0.01	0.014	0.006	12.2	5.5	0.40	0.18	0.047	0.021
Piston transport	0.56	0.25	0.28	0.13	304.0	138.0	40.7	18.5	0.40	0.18
Helicopter	0.25	0.11	0.18	0.08	5.7	2.6	0.52	0.24	0.57	0.26
Military transport	1.1	0.49	0.41	0.19	5.7	2.6	2.7	1.2	2.2	1.0
Military jet	0.31	0.14	0.76	0.35	15.1	6.85	9.93	4.5	3.29	1.49
Military piston[b]	0.28	0.13	0.14	0.04	152.0	69.0	20.4	9.3	0.20	0.09

[a] Based on 0.05 percent sulfur content fuel.

[b] Engine emissions based on Pratt & Whitney R-2800 engine scaled down two times.

the anacronym SAROAD meaning Storage And Retrieval Of Aerometric Data. (13) A typical print-out of information from the SAROAD System is shown in Table 8(14), with data presented based on a frequency distribution as well as various maximum and minimum values and statistical parameters. If possible, it is desirable to examine the complete history of air quality for the sampling stations in the particular locale. In conjunction with utilizing this information, one must carefully describe the characteristics of the sampling site and in any unique factors about the sampling site itself. Some information relevant in this step could possibly be procurred from the Air Quality Implementation Plan encompassing the area, or special studies that have been conducted such as tranportation control strategy studies for various air quality regions. (3) Graphical presentation of information of this type if useful, particularly if there appear to be trends either upward or downward in the air quality levels of any of the air pollutants.

Information on existing air quality levels in the area will be useful as a baseline for establishing the impact of the particular proposed action.

DETERMINATION OF AIR POLLUTION DISPERSION POTENTIAL (STEP 3)

The next step is to gather information on the general characteristics of the area with regard to air pollution dispersion. This information will be useful in evaluating the mesoscale impact of the proposed action. There are several useful parameters including mixing height, inversion height, annual wind speeds, high air pollution potential advisories, and episode-days.

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TABLE 8: 1973 SUSPENDED PARTICULATE CONCENTRATIONS (μg/m^3)

REGION - CITY	SITE NUMBER	YEAR	MINIMUM VALUE	FREQUENCY DISTRIBUTION (PERCENTILE) (% OF VALUES LESS THAN THE STATED ONE)							HIGHEST VALUE REACHED/EXCEEDED MORE THAN ONE TIME	MAXIMUM VALUE	TOTAL NO. OBSERVED	ARITHMETIC STANDARD DEVIATION	ARITHMETIC MEAN	GEOMETRIC STANDARD DEVIATION	GEOMETRIC MEAN	CHANGE IN GEOMETRIC MEAN (1972 - 1973)
				10%	30%	50%	70%	90%	95%	99%								
Region 184		73	11	79	106	124	149	183	195	233	233	329	53	[illegible]	132.2	1.6	122.1	[illegible]
OKLA. CITY	022	72	43	92	118	150	172	245	321	487		487	74	77.1	152.3	1.6	[illegible]	
		73	24	38	58	67	81	99	126	140	140	171	110	25.4	71.[illegible]	1.4	67.5	12.0
KINGFISHER	030	72	7	24	46	60	77	122	165	207		207	54	41.2	67.7	2.0	55.5	
		73	8	20	32	41	51	69	77	96	96	147	112	21.0	44.0	1.6	[illegible]	1.5
NORMAN	040	72	1	20	33	42	52	65	73	94		105	109	18.2	42.[illegible]	1.8	[illegible]	
		73	2	30	39	46	64	81	85	113	108	113	50	22.2	52.2	1.8	[illegible]	- 4.1
MOORE	044	72	6	27	44	49	62	106	117	130		130	57	28.5	57.1	1.7	50.2	
		73	11	25	32	39	46	61	75	85	85	119	81	16.8	42.0	1.5	39.2	- 1.9
SHAWNEE	052	72	1	26	32	45	55	69	82	110		110	64	19.0	46.0	1.8	41.1	
		73	7	12	18	25	39	53	99	226	99	226	40	39.1	37.[illegible]	2.1	23.2	- 3.3
ALEX	063	72	10	15	25	38	59	84	95	141		141	60	30.6	45.5	2.0	[illegible]	
		73	9	31	42	54	63	88	92	212	103	212	35	33.1	58.7	1.7	[illegible]	- 0.3
PURCELL	072	72	26	32	45	50	57	106	116	125		125	47	25.7	57.0	1.5	[illegible]	
		73	11	15	31	47	58	77	79	113	113	654	53	86.5	57.5	2.0	[illegible]	-10.3
CHANDLER	080	72	11	27	43	58	70	83	101	140		140	54	[illegible]	[illegible]	1.6	[illegible]	
		73	8	23	43	59	71	84	97	122	107	[illegible]	47	[illegible]	[illegible]	1.8	[illegible]	[illegible]
GUTHRIE	090	[illegible]	17	34	50	60	68	95	117	235		[illegible]	[illegible]	[illegible]	[illegible]	1.5	[illegible]	
		73	14	21	43	55	73	100	118	172	172	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
RENO	100	72	10	22	48	62	86	124	[illegible]	166		[illegible]	46	[illegible]	[illegible]	2.0	[illegible]	

The first general indicator of air pollution dispersion potential is mixing height, which can be defined as the vertical distance available above the earth's surface at a given location and at a given time period for the mixing of pollutants. The mixing heights vary daily, seasonally, and with the topography. (15) Figures 1 (16) and 2 (17) indicate the mean annual morning and afternoon mixing heights for the entire United States. Information on this parameter, as well as essentially all of the parameters in this step, can be obtained through Climatological offices in states, or the National Oceanographic and Atmospheric Administration.

Inversions occur when temperature increases with height above the earth's surface. (18) Inversions typically are present during the night or early morning hours due to the heating and cooling pattern at the earth's surface. In general, inversions are most frequent during the fall season of the year than during any other season. One of the characteristics of inversions is that they are often accompanied by wind speeds less than seven miles per hour, thus they often represent time periods when there is limited horizonal and vertical dispersion. The maximum inversion height in most areas of the country is limited to about five hundred meters above the earth's surface. Figure No. 3 (19) contains seasonal maps of the percentage of total hours of the occurrence of inversions or isothermal conditions below five hundred feet during the winter and the summer.

Another indicator of air pollution dispersion potential is the high pollution potential advisory (HAPPA). A high air pollution potential advisory is issued following the occurrence of limiting dispersion conditions over a 36-hour period and covering an area of approximately 75,000

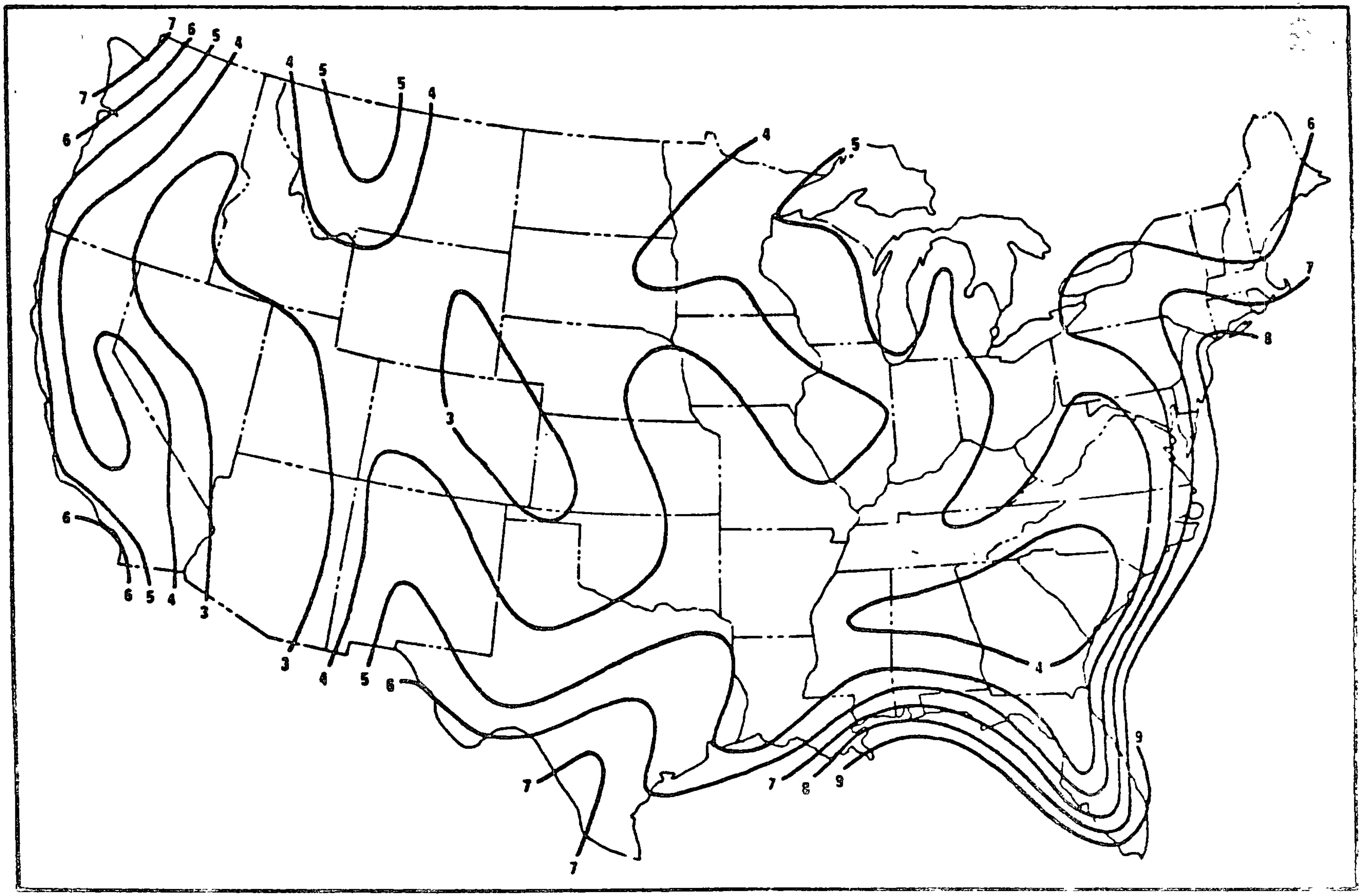


Figure 1. Isopleths (m x 10^2) of mean annual morning mixing heights

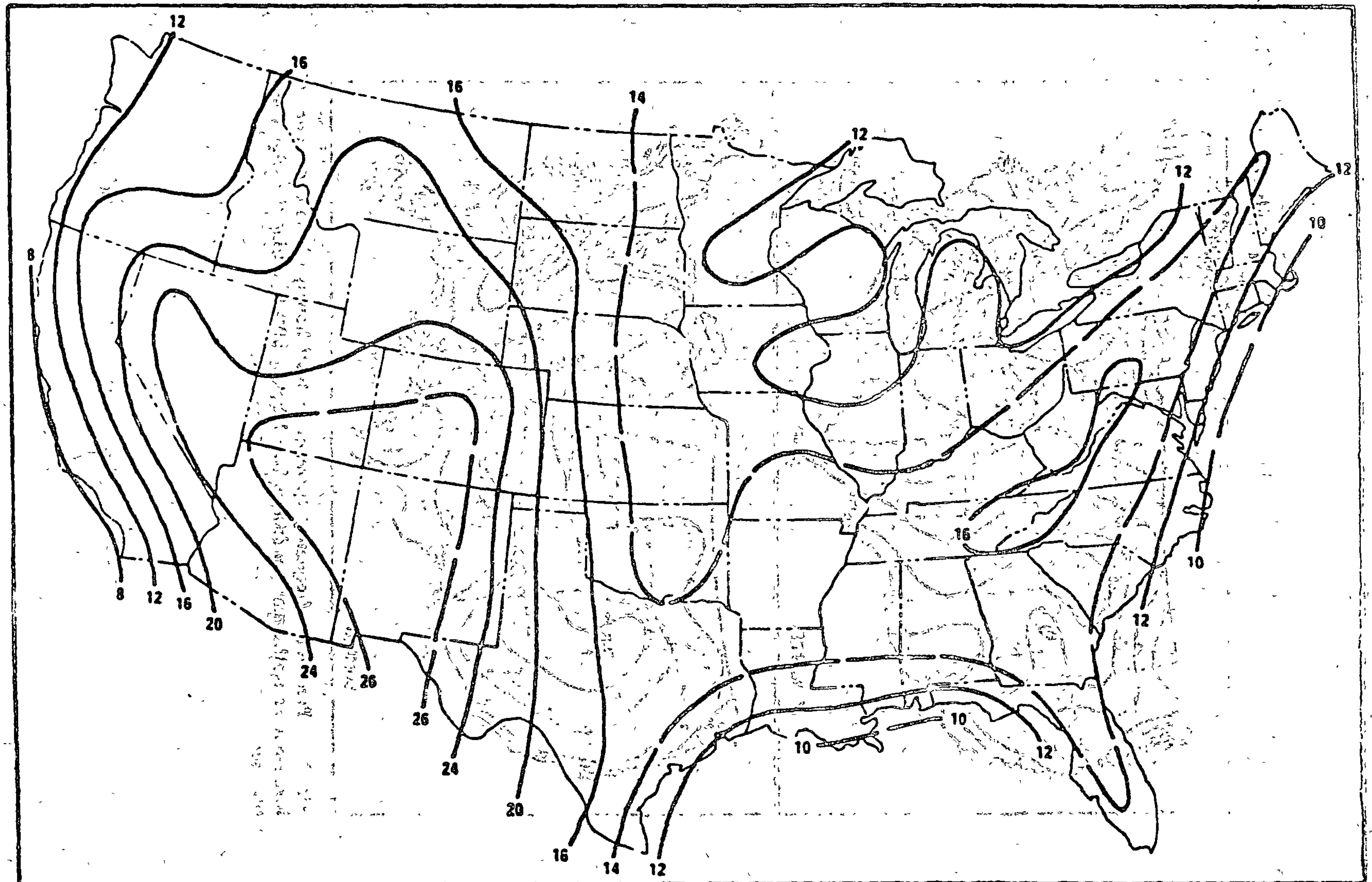


Figure 2: Isopleths (m x 10^2) of mean annual afternoon mixing heights

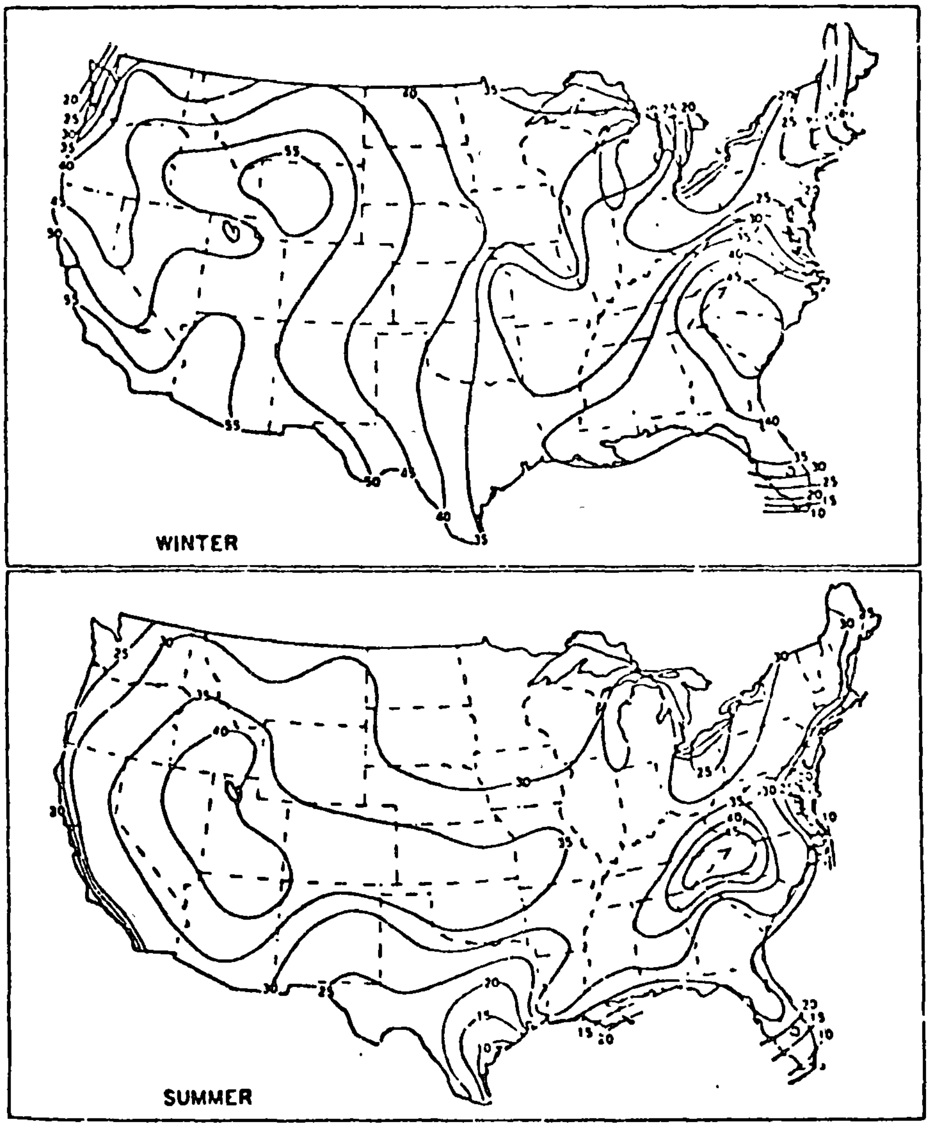


Fig. 3. Percentage frequency (percent of total hours) of the occurrence of inversions or isothermal conditions based below 500 ft during the winter and summer.

square miles (20). The conditions are: the morning urban mixing height is equal to or less than five hundred meters, the morning wind speed is equal to or less than four meters per second, the afternoon wind speed is equal to or less than four meters per second, and the afternoon ventilation rate (which is wind speed times mixing height) is equal to or less than 6,000 square meters per second. Figure No. 4 (21) contains a map of forecast-days of high air pollution potential in the United States.

Another air pollution advisory is termed episode-days, with the criteria required for defining an episode-day varying for mixing height, average wind speed in the mixing layer, degree of precipitation, and time period of persistence. Figure No. 5 (22) indicates isopleths of the total number of episode-days in the United States in a five-year period for certain criteria.

Another general indicator of air pollution dispersion potential is the mean annual wind speed at a given location. Figures 6 (23) and 7 (24) represent isopleths of the mean annual wind speed averaged through the morning and afternoon mixing layers, respectively, in the United States. Some areas are characterized by mean annual wind speeds as low as 3 meters per second in the morning, whereas others are as high as 9 meters per second in the afternoon.

One other thing that should be considered is the historical records of air pollution episodes in the area. Information of this type has been compiled, and if previous air pollution episodes have occurred, these need to be documented in the environmental impact statement. (25).

ASSEMBLAGE OF BASIC METEOROLOGICAL DATA (STEP 4)

Information which is associated with general air pollution dispersion

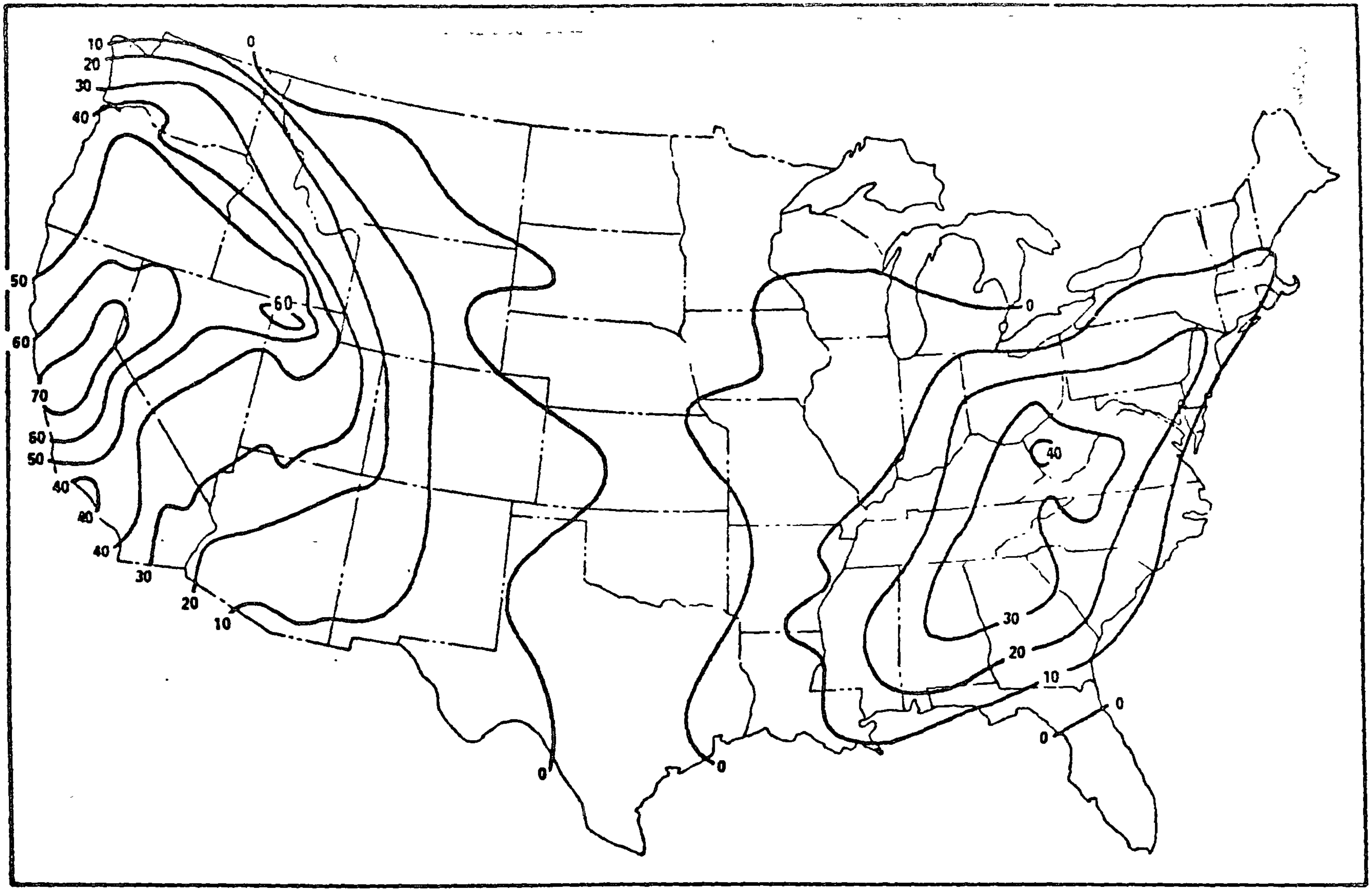


Figure 4: Isopleths of total number of forecast-days of high meteorological potential for air pollution in a 5-year period. Data are based on forecasts issued since the program began, 1 August 1960 and 1 October 1963 for eastern and western parts of the United States, respectively, through 3 April 1970.

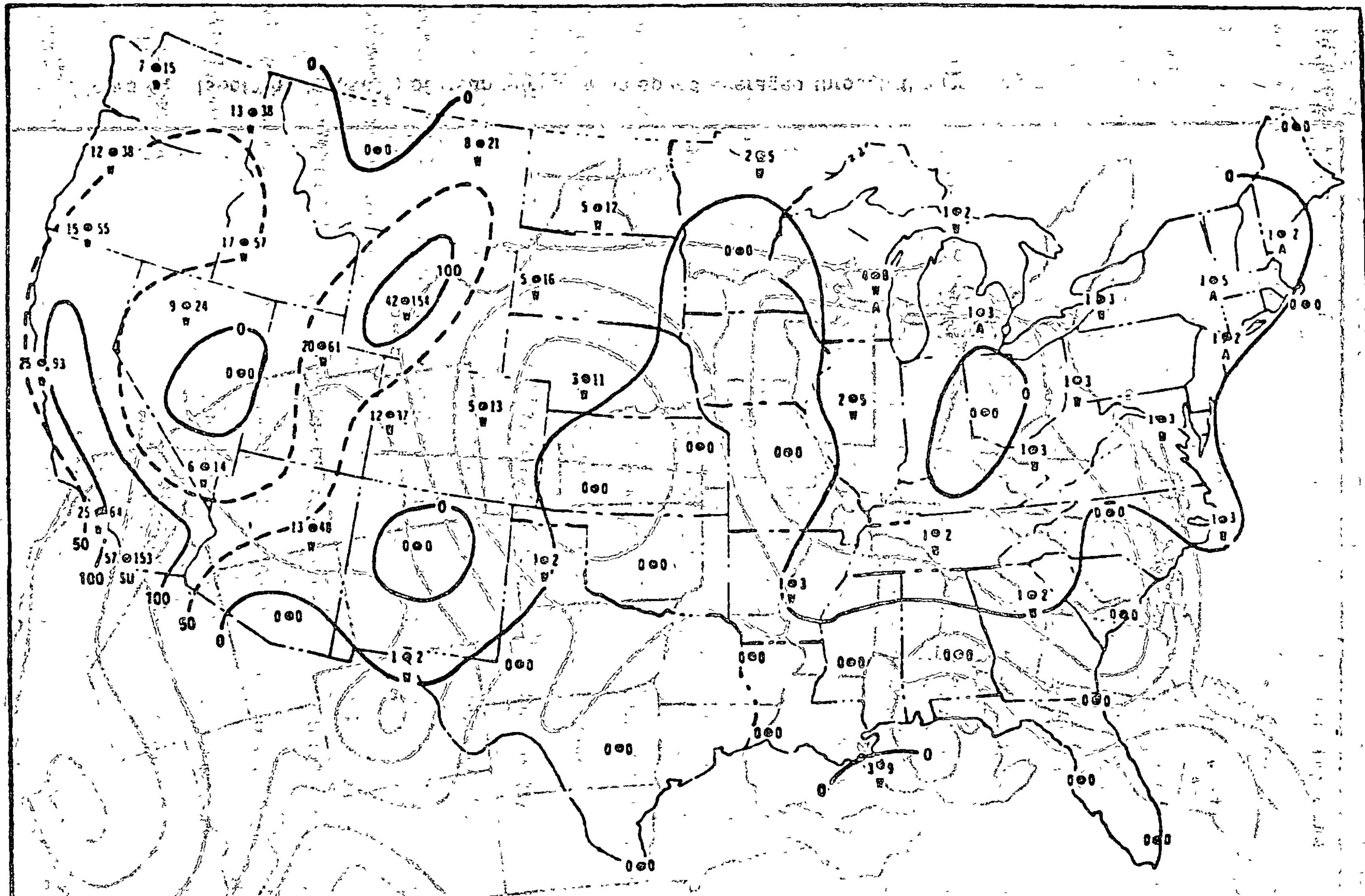


Figure 5: Isopleths of total number of episode-days in 5 years with mixing heights $\leq$ 500 m, wind speeds $\leq$ 4.0 m sec^{-1}, and no significant precipitation - - for episodes lasting at least 2 days. Numerals on left and right give total number of episodes and episode-days, respectively. Season with greatest number of episode-days indicated as W (winter), SP (spring), SU (summer), or A (autumn).

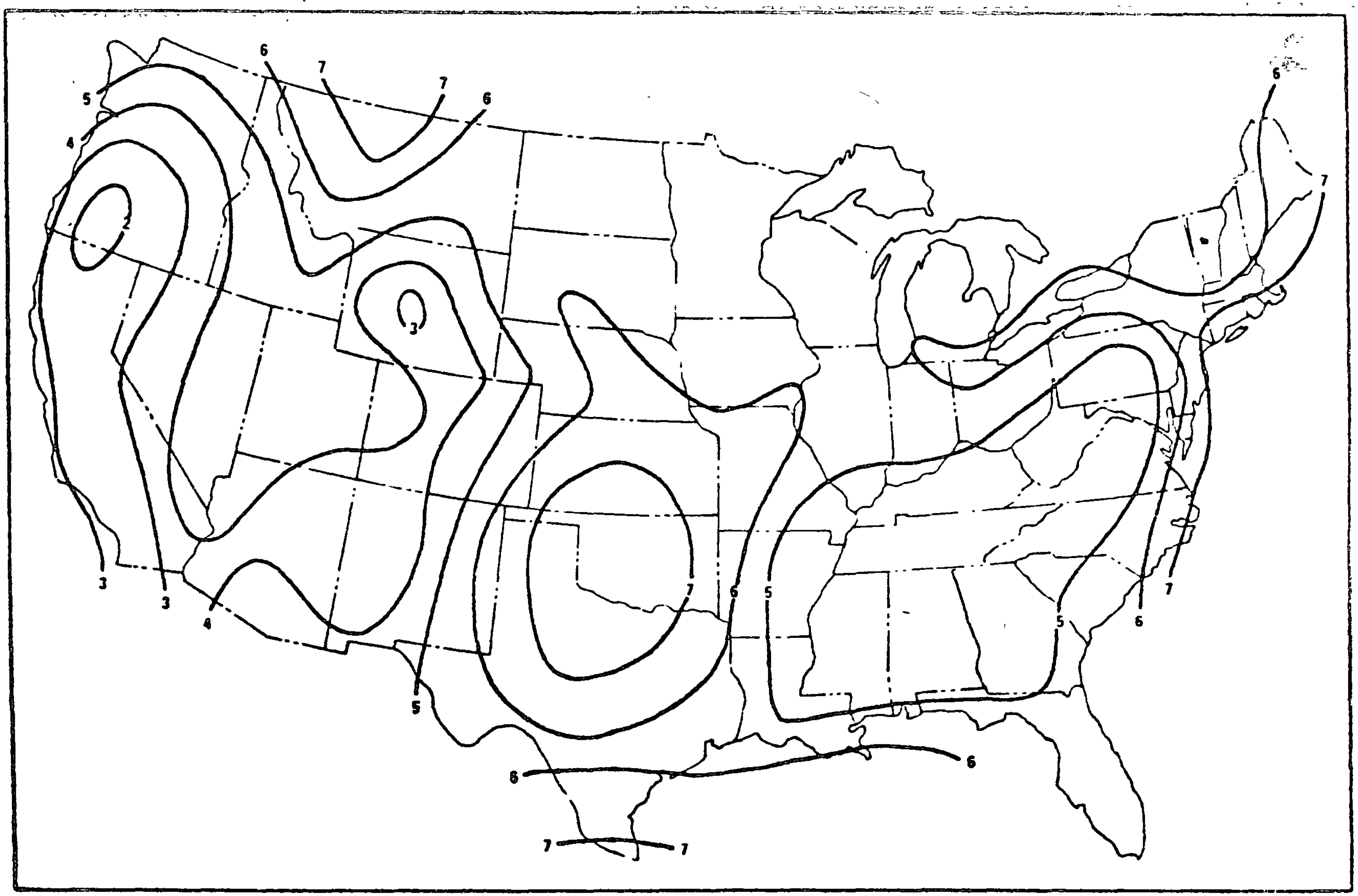


Figure 6: Isopleths (m sec^{-1}) of mean annual wind speed averaged through the morning m... layer

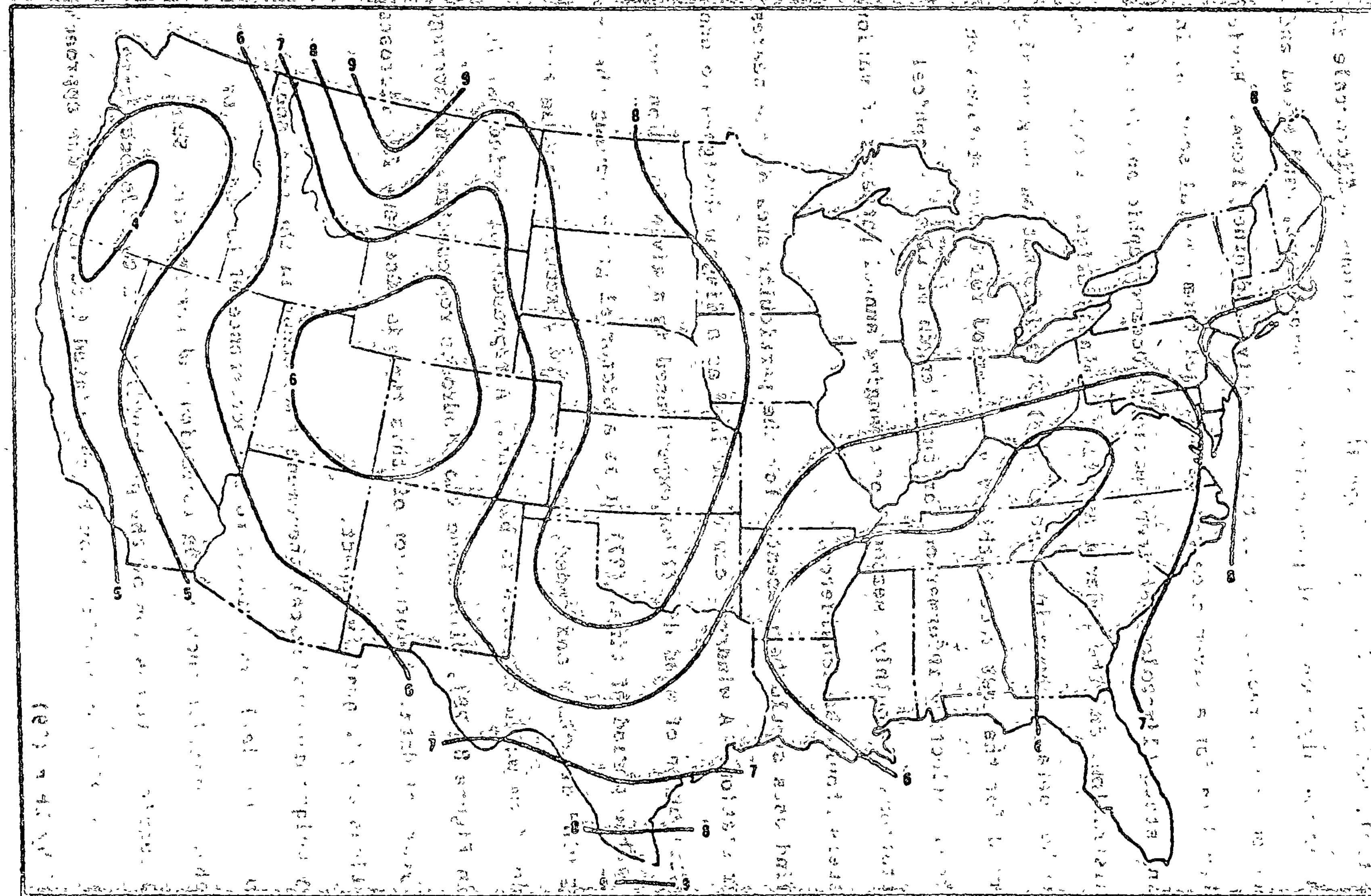


Figure 7: Isopleths (m sec^{-1}) of mean annual wind speed averaged through the afternoon mixing layer (Figure 6).

potential, but more relevant to specific calculations of microscale impact, includes monthly records of precipitation, temperature, wind speed and direction, solar radiation, relative humidity, and other items. Basic weather data for a given area can be obtained from several sources, including state Climatology offices, National Oceanographic and Atmospheric Administration, and the Federal Aviation Administration. A typical record of weather data is shown in Table 9. (26) This information is generally presented for the most recent thirty-year period for those stations which have been collecting information for that length of time. Graphical presentations of monthly, seasonal or changing annual patterns of various parameters should be considered.

Windroses should be presented for the particular area or the nearest weather station. A windrose can be defined as a diagram designed to show the distribution of wind direction experienced in a given location over a considerable period of time. (27) It is a pictorial graph showing the prevailing wind direction and speed, with the wind direction shown as the direction from which the wind is blowing. A representative windrose is shown in Figure 8 (28), with these two windroses representing the averages of ten years of data. Information of this type is necessary for microscale calculations of air quality impact.

Any unique meteorological phenomena that occur in the area should be noted, particularly as related to the occurrence of tornadoes, or unique characteristics such as fog formation and persistence. Some agencies require a discussion of the probability of a tornado occurring in an area, an this probability can be calculated from use of the approach suggested by Thom (29).

TABLE 9: SUMMARY OF CLIMATOLOGICAL DATA IN NEW ORLEANS

LATITUDE 29° 59.2' N
LONGITUDE 90° 15.3' W
ELEVATION (ground) 3 Feet

NEW ORLEANS, LOUISIANA
INTERNATIONAL AIRPORT

Month	Temperature: Normal: Daily maximum	Daily minimum	Monthly	Extremes: Record highest	Year	Record lowest	Year	Normal degree days
(a)	5	5	(b)	22		22	✓	(b)
J	64.4	44.8	54.6	83	1957	16	1963*	363
F	65.7	47.5	57.1	86	1948	19	1951	258
M	71.6	51.6	61.4	87	1955	26	1968	192
A	77.7	58.1	67.9	91	1948	38	1962	39
M	[illegible]	[illegible]	[illegible]	[illegible]	1953*	[illegible]	[illegible]	[illegible]
J	89.6	70.5	80.1	[illegible]	1954	55	[illegible]	0
J	90.6	74.0	81.6	[illegible]	1951	60	1967	0
A	90.7	73.[illegible]	81.9	100	1951	60	1968	0
S	87.[illegible]	69.3	78.3	97	1954*	42	1967	0
O	80.[illegible]	60.5	70.4	92	1962*	35	1968	19
N	70.3	49.8	60.0	86	1951	28	1968*	192
D	65.3	45.5	55.4	83	1951	17	1962	322
YR	78.2	59.0	68.6	100	JUN. 1954*	14	JAN. 1963*	1385

Month	Precipitation: Normal total	Maximum monthly	Year	Minimum monthly	Year	Maximum in 24 hrs.	Year	Snow, Sleet: Mean total	Maximum monthly	Year	Maximum in 24 hrs.	Year
(a)	(b)	22		22		22		22	22		22	
J	3.84	12.62	1966	0.56	1968	4.77	1955	T	T	1963*	T	1963*
F	3.99	10.56	1959	1.02	1962	5.60	1961	0.1	2.0	1958	2.0	1958
M	5.34	19.09	1948	0.24	1955	7.87	1948	T	T	1959	T	1959
A	4.55	8.75	1949	0.33	1965	4.35	1953	0.0	0.0		0.0	
M	4.35	14.33	1959	0.99	1949	9.86	1959	0.0	0.0		0.0	
J	4.43	8.87	1962	1.12	1954	4.19	1959	0.0	0.0		0.0	
J	6.72	11.46	1956	3.45	1951	6.30	1966	0.0	0.0		0.0	
A	5.34	11.77	1955	2.00	1952	2.48	1957	0.0	0.0		0.0	
S	5.03	13.53	1948	0.24	1953	5.46	1957	0.0	0.0		0.0	
O	2.84	6.45	1959	0.00	1952	2.38	1960	0.0	0.0		0.0	
N	3.34	14.59	1967	0.21	1949	6.38	1953	T	T	1950	T	1950
D	4.10	10.77	1967	1.46	1958	3.94	1952	0.1	2.7	1963	2.7	1963
YR	53.90	19.09	MAR. 1948	0.00	OCT. 1952	9.86	MAY 1959	0.2	2.7	DEC. 1963	2.7	DEC. 1963

Month	Relative humidity (Standard time used CENTRAL): Mid	6 AM	Noon	6 PM	Wind: Mean hourly speed	Prevailing direction	Fastest mile: Speed	Direction	Year	Pct. of possible sunshine	Mean sky cover, sunrise to sunset	Mean number of days: Sunrise to sunset: Clear	Partly cloudy	Cloudy	Precipitation .01 inch or more	Snow, Sleet 1.0 inch or more	Thunderstorms	Heavy fog	Temperatures: Max. 90° and above	Max. 32° and below	Min. 32° and below	Min. 0° and below
(a)	20	20	20	20	20		9	9			20	20	20	20	20	20	20	20	22	22	22	[illegible]
J	84	86	67	73	9.6		33	28	1966		6.6	7	8	16	10	0	2	7	0	[illegible]	[illegible]	[illegible]
F	83	86	64	69	10.3		39	15	1966		6.1	8	6	14	10	*	2	5	0	[illegible]	[illegible]	[illegible]
M	83	85	60	65	10.2		33	15	1964		6.1	7	8	14	9	0	3	5	0	[illegible]	[illegible]	[illegible]
A	85	88	60	66	9.7		35	07	1960		5.7	8	11	11	7	0	5	2	0	[illegible]	[illegible]	[illegible]
M	87	89	60	65	8.4		[illegible]	23	1962		5.1	11	11	9	8	0	6	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
J	87	90	63	68	7.0		38	27	1962		5.0	10	13	7	13	0	10	*	17	0	[illegible]	[illegible]
J	89	90	65	73	6.4		[illegible]	25	1967*		6.1	5	16	10	15	[illegible]	16	*	23	[illegible]	[illegible]	[illegible]
A	89	91	66	73	6.2		35	20	1952		5.6	[illegible]	16	9	13	[illegible]	13	*	17	[illegible]	[illegible]	[illegible]
S	87	89	65	74	7.6		67	[illegible]	1965		5.2	[illegible]	10	9	9	0	7	*	10	[illegible]	[illegible]	[illegible]
O	84	87	58	72	7.7		60	[illegible]	1956		4.6	16	8	7	6	0	2	2	[illegible]	[illegible]	[illegible]	[illegible]
N	83	86	59	73	9.0		29	[illegible]	1968		5.0	11	9	10	6	0	1	5	0	[illegible]	[illegible]	[illegible]
D	83	86	67	75	9.3		31	32	1960		6.1	8	8	15	10	*	2	5	0	[illegible]	[illegible]	[illegible]
YR	85	88	63	71	8.5		67	09	SEP. 1965		5.6	113	122	130	112	*	69	32	69	[illegible]	[illegible]	[illegible]

Means and extremes in the above table are from the existing or comparable location(s). Annual extremes have been exceeded at other locations as follows. Highest temperature 102 in June 1954 and earlier; lowest temperature 7 in February 1899, maximum monthly precipitation 25.11 in October 1937, maximum precipitation in 24 hours 14.01 in April 1927; maximum monthly snowfall 8.2 in February 1895; maximum snowfall in 24 hours 8.2 in February 1895.

(a) Length of record, years.
(b) [illegible] normals (1931-1960).
* Less than one half.
\+ Also on earlier dates, months or years.
T Trace, an amount too small to measure.
Below zero temperatures are preceded by a minus sign.

Unless otherwise indicated, dimensional units used in this bulletin are: temperature in degrees F.; precipitation, including snowfall, in inches; wind movement in miles per hour; and relative humidity in percent. Degree day totals are the sums of the negative departures of average daily temperatures from 65° F. Sleet was included in snowfall totals beginning with July 1948. Heavy fog reduces visibility to 1/4 mile or less.

Sky cover is expressed in a range of 0 for no clouds or obscuring phenomena to 10 for complete sky cover. The number of clear days is based on average cloudiness 0-3; partly cloudy days 4-7, and cloudy days 8-10 tenths.

& Figures instead of letters in a direction column (a), are [illegible] in tens of degrees from true North, i.e., 09 East, 18 South, 27 West, 36 North and 00 Calm. Resultant wind is the vector sum of wind directions and speeds divided by the number of observations. If figures appear in a direction column under "Fastest mile" the corresponding speeds are fastest observed 1-minute values.

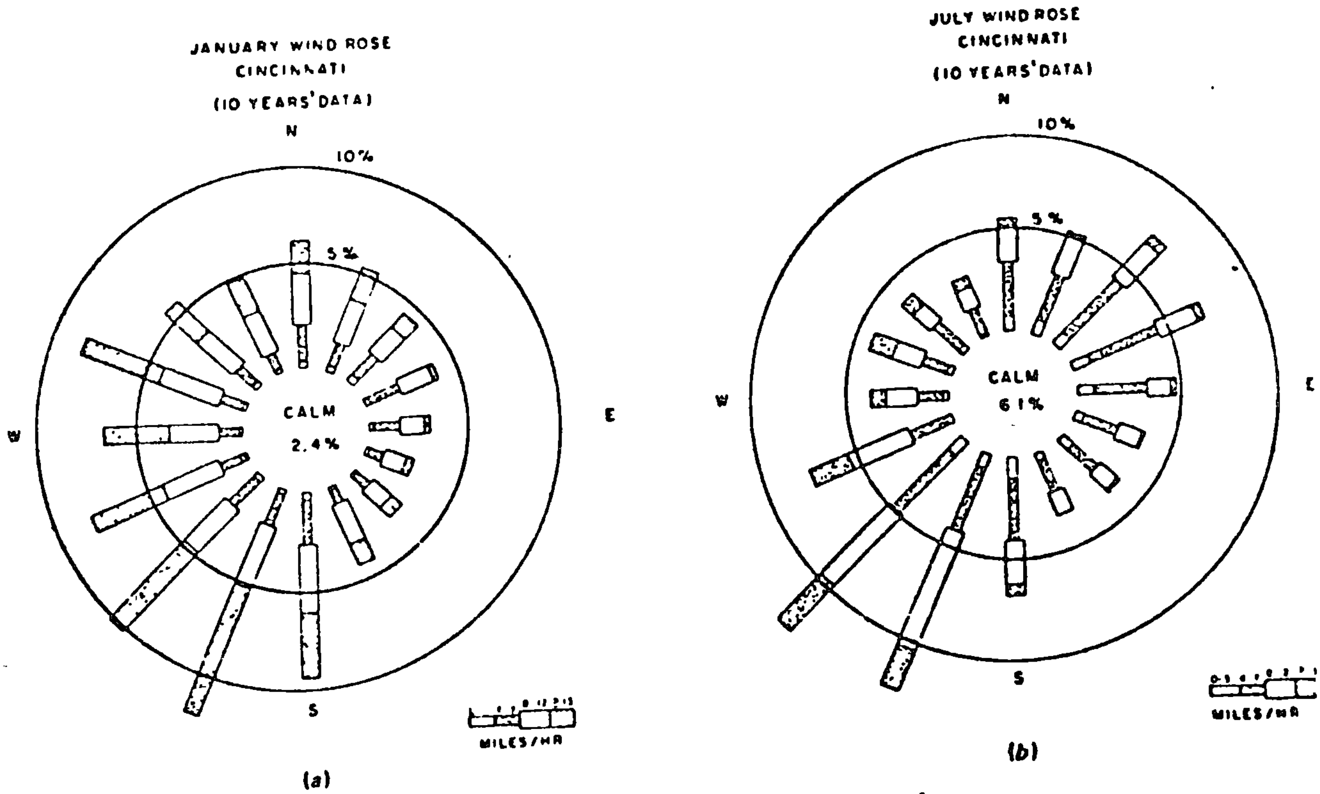


Fig. 8: January and July wind roses, Cincinnati. The monthly distributions of wind direction and wind speed are summarized on polar diagrams. The positions of the spokes show the direction from which the wind was blowing, the length of the segments indicate the percentage of the speeds in various groups.

PRESENTATION OF AIR QUALITY STANDARDS (STEP 5)

One of the major points of concern in assessment of air quality impacts is the question of whether or not air quality standards will be exceeded. Two types of standards are relevant; ambient air standards and emission standards. Ambient air quality standards are those that apply to the general ambient atmosphere, whereas emission standards are related to pollutant materials which can be emitted from a source into the ambient atmosphere.

The Federal Clean Air Act of 1970 established a program for the creation of air quality standards. (3) A summary of the ambient air standards is contained in Table 10. (30) Primary standards are those oriented to the protection of public health, whereas secondary standards are oriented to the protection of public welfare. The criteria for the primary standards are shown in Table 11. (31) Table 12 (32) contains some representative new source performance standards for various types of point sources of emission. These new source performance standards are emission standards for new point sources contructed following the passage of the Clean Air Act of 1970.

Every state has air quality standards at least as stringent as those at the national level, and some states have standards that are more stringent. If the state or local standards do not match the national standards, this should be noted and both levels of standards presented. Where there are time schedules required for meeting standards, these schedules should be discussed.

TABLE 10: NATIONAL AMBIENT AIR QUALITY STANDARDS

Pollutant	Primary	Secondary
Particulate Matter		
Annual geometric mean	75	60
Maximum 24-hour concentration*	260	150
Sulfur Oxides		
Annual arithmetic mean	80 (.03 ppm)	60 (.02 ppm)
Maximum 24-hour concentration*	365 (.14 ppm)	260 (.1 ppm)
Maximum 3-hour concentration*		1,300 (.5 ppm)
Carbon Monoxide		
Maximum 8-hour concentration*	10 (9 ppm)	
Maximum 1-hour concentration*	40 (35 ppm)	same as primary
Photochemical Oxidants		
Maximum 1-hour concentration*	160 (.08 ppm)	same as primary
Hydrocarbons		
Maximum 3-hour (6-9 am) concentration*	160 (.24 ppm)	same as primary
Nitrogen Oxides		
Annual arithmetic mean	100 (.05 ppm)	same as primary

(All measurements are expressed in micrograms per cubic meter ($\mu g/m^3$) except for those for carbon monoxide, which are expressed in milligrams per cubic meter (mg/m^3). Equivalent measurements in parts per million (ppm) are given for the gaseous pollutants.)
*Not to be exceeded more than once a year.

Table 11: NATIONAL AIR QUALITY CRITERIA AND NATIONAL AIR QUALITY STANDARDS IN THE UNITED STATES, 1971

Pollutant	Adverse health effects observed at these concentrations	Air quality standard, protective for health
Particulate matter	80 $\mu g/m^3$, annual mean.	75 $\mu g/m^3$, annual geometric mean. 260 $\mu g/m^3$, max. 24-h value may occur once each year.
Sulfur dioxide	115 $\mu g/m^3$, annual mean.	80 $\mu g/m^3$, annual arithmetic mean.
	300 $\mu g/m^3$, 24-h av. for 3-4 days	365 $\mu g/m^3$, max. 24-h value may occur once each year.
Carbon monoxide	12-17 mg/m^3 for 8 h produces concn of 2-2.5% carboxyhemoglobin.	10 mg/m^3, max. 8-h value may occur once each year.
	35 mg/m^3 for 8 h produces concn. of 5% carboxyhemoglobin.	40 mg/m^3, max. 1-h value may occur once each year.
Photochemical oxidants	130 $\mu g/m^3$ hourly av. impaired performance of student athletes. 200 $\mu g/m^3$ instantaneous level increased eye irritation. 490 $\mu g/m^3$ peaks with 300 $\mu g/m^3$ hourly av. increased asthma attacks.	160 $\mu g/m^3$, max. 1-h value may occur once each year.
Hydrocarbons	With nonmethane hydrocarbon, 200 $\mu g/m^3$ in 3 h, 6-9 A.M., produced (2-4 h later) photochemical oxidant of up to 200 $\mu g/m^3$ that lasted 1 h. By extrapolation downward, conc. of HC of 100 $\mu g/m^3$ can produce lowest injurious level of photochemical oxidant.	160 $\mu g/m^3$, max. level that may occur 6-9 A.M. once each year.
Oxides of nitrogen	118-156 $\mu g/m^3$, 24-h mean over 6 mo produced increase in acute bronchitis in infants and schoolchildren; this av. associated with a 24-h max. of 284 $\mu g/m^3$. 117-205 $\mu g/m^3$, 24-h mean over 6 mo and mean suspended nitrate level of 3.8 $\mu g/m^3$ or more produced increased respiratory disease in family groups.	100 $\mu g/m^3$, annual arithmetic mean.

TABLE 12: New source performance standards

Source category	Emission
1. Fossil fuel-fired steam generators (250 million Btu/hr heat input or greater)	
Particulates	0.1 lb/10^6 Btu (maximum 2-hr average)
Sulfur dioxide	
oil-fired	0.80 lb/10^6 Btu
coal fired	1.2 lb/10^6 Btu (maximum 2-hr average)
Nitrogen oxides	
gas-fired	0.20 lb/10^6 Btu
oil-fired	0 30 lb/10^6 Btu
coal-fired	0.70 lb/10^6 Btu (maximum 2-hr average expressed as NO_2)
Visible emissions	not to exceed 20% opacity, except that for 2 min in any 1-hr, emissions may be as great as 40% opacity
2. Incinerators	
Particulates	0.08 grains/scf
3. Nitric acid plants	
Nitrogen oxides	3 lb/ton acid
Visible emissions	< 10% opacity
4. Sulfuric acid plants	
Sulfur dioxide	4 lb/ton acid
Acid mist	0.15 lb/ton acid
Visible emissions	< 10% opacity
5. Portland cement plants	
Particulates	
kilns	0.3 lb/ton feed
clinker coolers	0.1 lb/ton feed
Visible emissions	
kilns	10% opacity
others	< 10% opacity

EMISSION INVENTORY (STEP 6)

An emission inventory is the compilation of the quantities of air pollutants from all sources in a defined area entering the air in a given time period. The time period that is typically utilized in an emission inventory is one year, and the areas are usually associated with a county or perhaps with a multi-county region. (33) Typical emission inventory data contained in a state air quality implementation plan is shown in Table 13. (34).

It should be noted that emission inventory information is useful for general air quality management and trend analysis. It does not include consideration of atmospheric reactions and the damage associated with a given weight of pollutant.

Another thing that should be done with emission inventory information is to identify major point sources of air pollution in the area. A point source emits 25 tons per year or more of air pollutants. Figure 9 (35) shows the major point sources of air pollution in southwestern Oklahoma. For each point source that might be located close to the area of a proposed action, information should be procured on the nature of the source, the pollutants, and the quantities of the emissions.

CALCULATION OF MESOSCALE IMPACT (STEP 7)

This step involves calculation of the estimated annual quantity of air pollutants to be emitted from the construction and operational phases of each alternative for the proposed action. This can be done based on emission factors and information regarding the size or type of activity. The calculated quantities of air pollutants should then be compared with

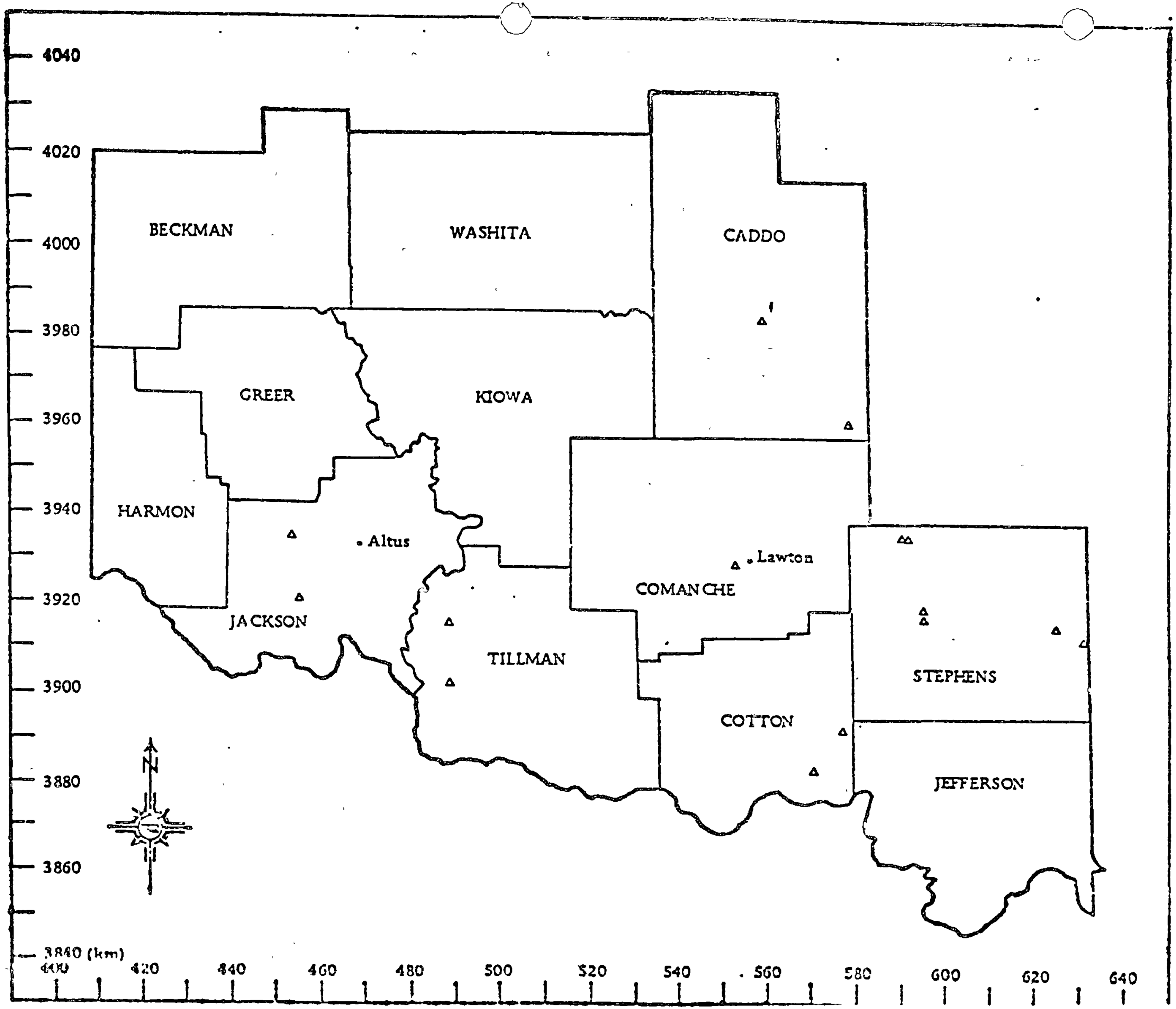
TABLE 13 EMISSIONS INVENTORY SUMMARY

For the State of Oklahoma

Southwestern Oklahoma Intrastate Air Quality Control Region 189

Data Representative of Calendar Year 1970

Source Category	Tons of Pollutant/Year					Fuel Etc.	
	Part	SO_2	CO	HC	NO_x	Quantity	Units
I. Fuel Combustion							
A. Residential Fuel - Area Source							
2. Distillate Oil	8	60	4	2	10	1597	10^3 gal
3. Natural Gas	82	3	86	35	324	8641	10^6 CF
6. Total	90	63	90	37	334		
B. Comm. - Instl. and Ind.							
1. b. (Bituminous) Coal - Point Source	0	4	0	0	3	0	tons
3. a. Distillate Oil - Area Source	64	309	0	0	279	8591	10^3 gal
b. Distillate Oil - Point Source	0	0	0	13	0	0	-
4. a. Residual Oil - Area Source	15	217	0	2	41	1363	10^3 gal
b. Residual Oil - Point Source	0	0	0	0	0	0	-
5. a. Natural Gas - Area Source	56	2	30	74	379	6002	10^6 CF
b. Natural Gas - Point Source	161	6	4	374	11695	18693	10^6 CF
LPG 8. a. Other (Specify) - Area Source	72	53	75	29	235	23483	10^3 gal
b. Other (Specify) - Point Source	8	0	0	15	6474		
9. Total	376	591	109	507	19106		
C. Steam - Electric Power Plant							
2. Bituminous Coal	0	0	0	0	0	0	-
3. Distillate Oil	0	0	0	0	0	0	-
4. Residual Oil	0	0	0	0	0	0	-
5. Natural Gas	276	11	7	735	7165	36744	10^6 CF
7. Total	276	11	7	735	7165		
D. Total Fuel Combustion	742	665	206	1279	26605		
II. Process Losses							
A. Area Sources	0	0	0	30	0		
B. Point Sources	47079	2579	2323	2323	309		
III. Solid Waste Disposal							
A. Incineration							
2. Municipal, Etc. - Point Source	0	0	0	0	0	0	-
B. Open Burning							
1. a. Onsite - Area Source	2238	140	11889	4196	839	279749	tons
D. Total Solid Waste Disposal	2238	140	11889	4196	839		
IV. Transportation - Area Source							
A. 1. Motor Vehicles - Gasoline	335	203	72553	13584	11475	1014335	10^3 gal
2. Motor Vehicles - Diesel	122	219	1584	317	1658	9750	10^3 gal
B. Off-Highway Fuel Usage	78	327	59	27	169	20087	10^3 gal
C. Aircraft	555	202	7528	1875	226		
D. Railroad	43	112	121	86	129	3444	10^3 gal
E. Vessels	0	0	0	0	0	0	-
F. Gasoline Handling Evap. Losses	0	0	0	1669	0		
G. Other (Specify) Petroleum Storage Loss	0	0	0	25088	0		
H. Total Transportation	1133	1063	81845	42646	13657		
V. Miscellaneous - Area Sources							
B. Other (Specify)	0	0	0	294	0	294	tons
C. Total Miscellaneous	0	0	0	294	0	294	tons
VI. Grand Total							
A. Area Source	47524	1847	93929	47308	14761		
B. Point Source	3668	2600	2334	3460	25646		
C. Total	51192	4447	96263	50768	40410		



Δ - Point Sources Emitting 25 or More Tons per Year

Figure 9: Southwestern Oklahoma AQCR, Region 189

regional or local emission inventory information, and percentage increases should be calculated based on the proposed action.

Attention should be focused on any Priority I or Priority II air pollutants. Priority I represent those pollutants whose levels in the ambient air violate primary standards, Priority II indicates that the levels are at or near primary standards, and Priority III indicates that the levels are better than primary standards. This particular approach is based on the concept that a given percentage increase in a pollutant is more important for a Priority I or Priority II pollutant than for a Priority III pollutant. Table 14 (36) shows the priority classifications in Oklahoma. (37)

MICROSCALE IMPACT DETERMINATION (STEP 8)

The next step is to calculate ground level concentrations of air pollutants to be anticipated from each alternative for the proposed action during both the construction and operational phases. It is beyond the scope of this presentation to derive the various mathematical models for gaseous and particulate dispersion. This section will be oriented to the simple presentation of several mathematical models for use in microscale impact calculations, and the identification of reference sources which will lead to a more detailed study for an individual project.

The two basic factors that influence movement of pollutants from their point of origin to some other location are horizonal wind speed and direction, and the vertical temperature structure of the atmosphere. These two parameters influence the vertical and horizontal motion of pollutants

TABLE 14: STATE OF OKLAHOMA AQCR PRIORITY CLASSIFICATIONS

AQCR	1970 Population	Priority				
		Particulates	SO_2	CO	NO_2	Oxidants (HC)
Central Oklahoma (184)	779,518	I	III	III	III	I
Northeastern Oklahoma (186)	771,412	I	III	III	III	I
Southeastern Oklahoma (188)	305,750	I	III	III	III	III
North Central Oklahoma (185)	171,970	III	III	III	III	III
Southwestern Oklahoma (189)	284,279	III	III	III	III	III
Northwestern Oklahoma (187)	123,876	III	III	III	III	III
Fort Smith Interstate (017)	93,822	II	III	III	III	III
Shreveport-Texarkana-Tyler Interstate (022)	28,642	II	III	III	III	III

released into the atmosphere. The influence of these two parameters can be combined into a term called atmospheric stability, with representative values shown in Table 15. (38) The class A category indicates the greatest amount of spreading in the most unstable atmospheric conditions, where the Class F category indicates the least amount of spreading in the most stable atmospheric conditions. Each of the mathematical models for prediction of microscale impact involves the use of a stability classification.

The first model can be utilized to calculate ground level concentrations from an elevated point source. The mathematical model is as follows (38):

$$C_{x,y,o} = \frac{Q}{\pi \sigma_y \sigma_z \bar{u}} e^{-\left[\frac{H^2}{2\sigma_z^2} + \frac{Y^2}{2\sigma_y^2}\right]}$$

C = ground level concentration (μg/sec)

Q = release rate from stack (μg/sec)

σ_y = crosswind standard deviation (m); function of stability classification and X

σ_z = vertical standard deviation (m); function of stability classification and X.

$\bar{u}$ = mean wind speed (m/sec)

H = effective stack height (m)

x,y = downwind and crosswind distances, respectively (m).

The vertical standard deviation and crosswind standard deviation is a function of atmospheric stability and the distance downwind for which

TABLE 15: KEY TO STABILITY CATEGORIES

Surface Wind Speed at Ten Meters Height m/sec	*Insolation Stability Classes* — *Day* — *Strong (1)*	*Moderate (2)*	*Slight (3)*	*Night* — *Thinly Overcast or >½ Cloud (4)*	*Clear to <½ Cloud*
>2 (4.5mph)	A	A-B	B	-	-
2-3 (4.5-6.7)	A-B	B	C	E	F
3-5 (6.7-11)	B	B-C	C	D	E
5-6 (11-13.5)	C	C-D	D	D	D
>6 (>13.5mph)	C	D	D	D	D

(1) Sun>60° above horizontal; sunny summer afternoon; very convective
(2) Summer day with few broken clouds
(3) Sunny fall afternoon; summer day with broken low clouds; or summer day with sun from 15-35° with clear sky
(4) Winter day

Insolation = amount of sunshine

Class A indicates greatest amount of spreading and most unstable atmospheric conditions and Class F indicates least spreading and most stable atmospheric conditions.

the calculation is being made. Figures 10 and 11 contain summaries of these dispersion coefficients in the vertical and horizontal direction, respectively. (39) An example problem utilizing this model is shown as follows:

$Q = 10^6$ μg/sec

$\bar{U}$ = 1.0 meters/sec

H = 30 meters

Property line is at 1000 m, find ground level concentration at property line under most stable condition (class F).

From Figures 10 and 11,

σ_y = 35 m.

σ_z = 14 m.

$$C_{1000,0,0} = \frac{10^6}{(3.14)\ (1)\ (35)\ (14)}\ e^{-\left(\frac{(30)^2}{2\ (14)^2}\right)}$$

$$= 65\ \mu g/M^3$$

It is possible to directly calculate the maximum gaseous ground-level concentration from an elevated point source. (38) The location X of the maximum ground concentration will occur approximately where $\sigma_z = \frac{H}{2}$ for a given stability condition. The maximum concentration can be calculated from:

$$C_{x,o,o_{Max}} = \frac{0.117\ Q}{\bar{U}\ \sigma_y\ \sigma_z}$$

σ_y and σ_z for given stability condition and distance x

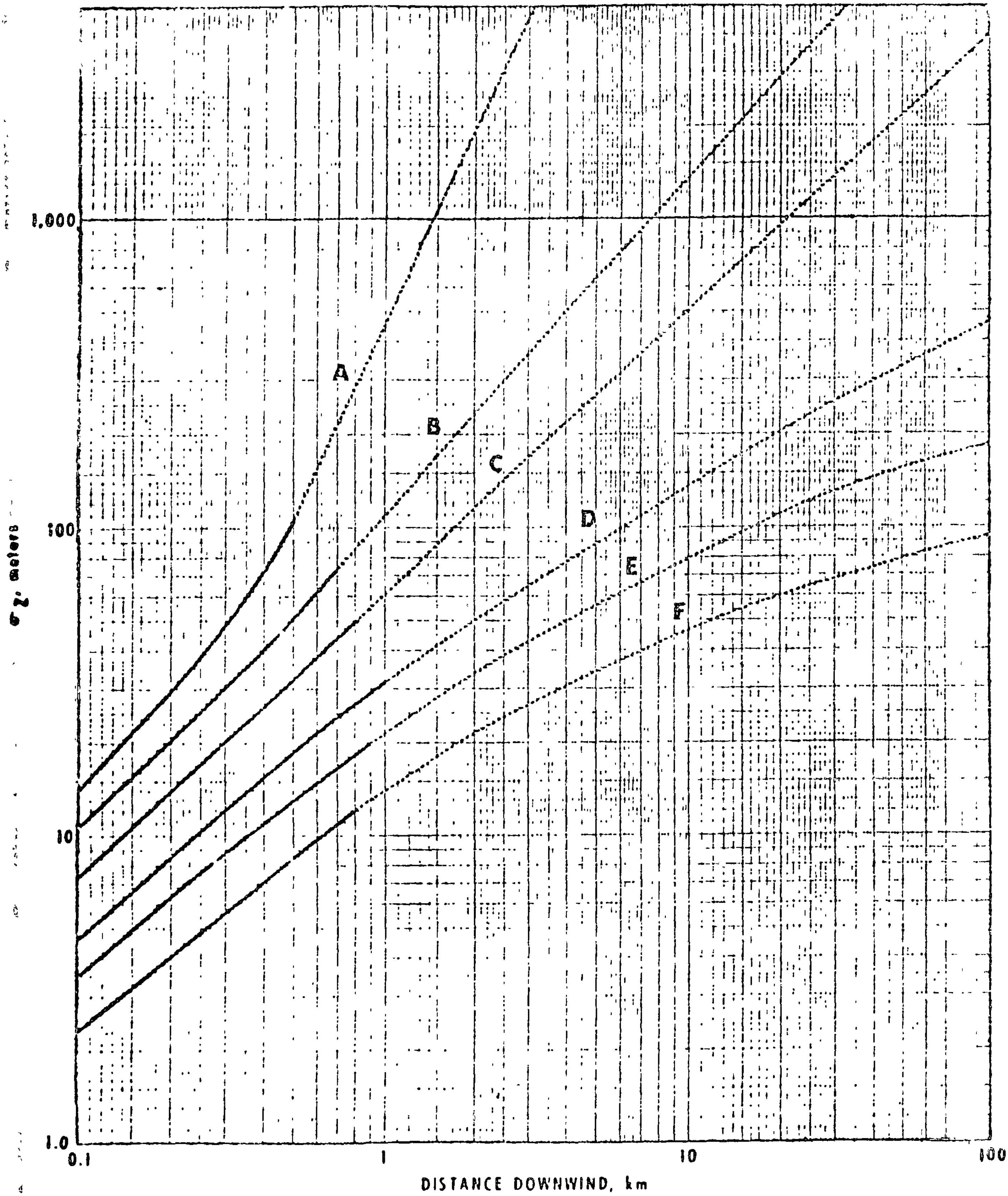


Figure 10: Vertical dispersion coefficient as a function of downwind distance from the source.

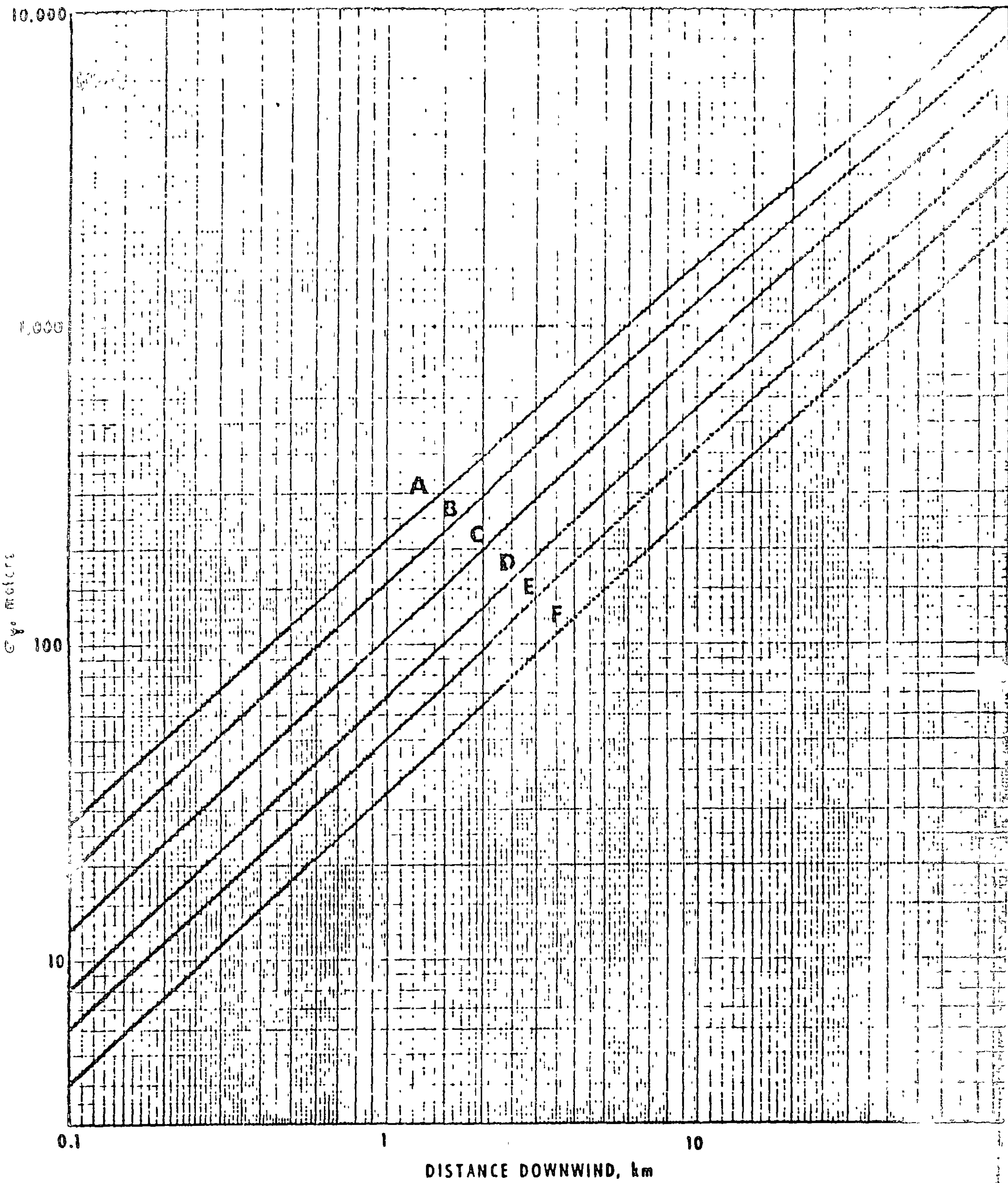


Figure 11 : Horizontal dispersion coefficient as a function of downwind distance from the source

Effective stack height is equal to the actual stack height plus any rise of the plume that occurs as it leaves the stack. There are two basic reasons for plume rise; the momentum effect due to the vertical velocity of the gas leaving the stack, and the buoyancy effect which is related to warm stack gases tending to rise in a cooler surrounding atmosphere. The Holland equation as follows can be utilized to calculate plume rise when the vertical temperature gradient is equal to the adiabatic lapse rate (38):

$$\Delta h = \frac{VeD}{\bar{u}} \left(1.5 + 2.68 \times 10^{-3} \, P \, \frac{\Delta T}{T_s} \, D \right)$$

Δh = plume rise above stack (meters)

Ve = stack exit velocity (m/sec)

D = stack internal diameter (meters)

P = atmospheric pressure (mb)

(1 atm. = 1013 mb)

T_s = stack gas temperature (^{o}k)

ΔT = T_s - ambient temp (^{o}k)

The adiabatic lapse rate describes the rate of cooling with lifting (or heating upon descent) of a parcel of air with no heat exchange. The adiabatic lapse rate is - 5.4 ^{o}F/1000 ft. If the actual or environmental lapse rate is greater than the adiabatic rate, say - 8^{o}F/1000 ft., then the Δh above should be multiplied by 1.2; if the environmental lapse rate is less than the adiabatic value, then the Δh should be multiplied by 0.8.

The concentration of gases from ground level point sources can be calculated on the basis of the following equation (38):

$$C_{x,y,z} = \frac{Q}{\pi \bar{u} \sigma_y \sigma_z} e^{-\left[\frac{y^2}{2\sigma_y^2} + \frac{z^2}{2\sigma_z^2}\right]}$$

The concentrations of gases from ground level area sources or line sources can be calculated utilizing the following mathematical model (38):

$$C_{x,o,o,} = \frac{Q}{\pi \bar{u}(\sigma_y^2 + \sigma_{y_o}^2)^{1/2} \sigma_z}$$

where

σ_{y_o} = ¼ of emission width (m)

The ground level center line concentrations of particulates from an elevated source can be calculated using the following model (38):

$$C_{x,o,o} = \frac{PQ}{\bar{u} \sigma_y \sigma_z} e^{-\left[\frac{1}{2} \frac{B^2}{\sigma^2}\right]}$$

P = wt. fraction of effluent in a particular size range.

$$B = H - \frac{V_s X}{100 \bar{u}}$$

V_s = Stokes terminal settling velocity for particles in size range (cm/sec), determine from Stokes law as follows:

$$V_s = \frac{2r_p^2 \, g \, \rho_p}{9 \, \mu a}$$

where

r_p = particle radius (cm)

g = gravitational constant (980 cm/sec^2)

ρ_p = particle density (gm/cm^3)

μ_a = air viscosity (gm/cm/sec^2)

Utilization of the basic ground level concentration model above requires the selection of several P values and calculation of the resultant concentration. Summation of these concentrations over the range of particle sizes anticipated from the point source will yield the total anticipated ground level concentrations.

The mathematical models shown above are basically those oriented to point sources for gases and particulates, and areas sources for gases. (40) Refinement of these models in terms of use for highway impact statements have been made, and they are summarized elsewhere. (41) Numerous other mathematical models have been developed for calculation of the dispersion of air pollution from aircraft, and urban transport and reactions of pollutants (42,43,44,45).

Utilization of mathematical models will allow the development of isopleths of equal concentration around the source of emission for various types of air pollutants. These isopleths should be calculated with concern toward the frequency of occurrence based on the meteorological data assembled earlier. The calculated ground level concentrations at various positions in the project area should then be compared to the applicable ambient air standards. If the calculated ground level concentrations are less than the standards, then fractional proportions should be indicated to present the degree of safety. If the calculated ground level concentrations are greater than the ambient air quality standards, then control measures or abatement strategies must be developed. If there are other sources of air pollution in the area of the proposed action that would significantly contribute to the anticipated ground level concentrations from the proposed action, these should also be included in the calculations so as to examine the possible additive or even potentiating effect of various air pollutant sources.

In addition to these determinations, it is necessary to examine the emissions from each alternative for the proposed action in light of applicable emission standards. It is presumed that the proposed action will be in compliance with the emission standards, and this point should be discussed as to the extent of compliance involved in the proposed action.

ABATEMENT STRATEGIES (STEP 9)

If it is determined what ambient air standards are exceeded by the proposed action, then abatement strategies or control measures should be presented. Excellent references are available to describe various control technologies which can be utilized for various gaseous and particulate emissions (46,47,48,49,50,51,52,53,54), and it is beyond the scope of this particular presentation to present details on control methodologies.

SUMMARY

In summary, this chapter is oriented to the types of information and practical steps that can be utilized in predicting the impacts of a given proposed action both in terms of the mesoscale viewpoint as well as the microscale viewpoint. Suggestions are provided for assessment or interpretation of the predicted impacts.

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PREDICTION OF BACTERIAL SELF-PURFICATION IN STREAMS

by
L.W. Canter*

In order to assess the potential environmental impact of a proposed action, it may be necessary to predict bacterial self-purification in streams. This discussion is oriented to the principles of bacterial self-purification in streams as well as the mathematical techniques available for describing these phenomena.

I. Introduction

A. Bacterial Self-Purification

1. Definition --- the decrease of bacteria of all types, and especially those of fecal origin, as a function of flow distance or flow time in a river.

2. Bacteria may starve to death, be devoured by predators, or be otherwise inactivated.

B. Classic Work --- early work by Streeter and his co-workers in Cincinnati on the Ohio River. (1920's)

C. Pathogenic Microorganisms

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1. Special public health concern with the disposal and ultimate fate of microbial wastes contained in municipal sewage, especially those of intestinal origin that cause waterborne diseases. These are principally pathogenic bacteria and viruses, and the cysts and ova of parasitic worms.

2. Examples of pathogenic bacteria and their associated diseases.

   (a) Salmonella typhi --- typhoid fever

   (b) Shigella and Salmonella organisms --- dysentery and diarrhea, respectively

   (c) Vibrio comma --- cholera

   (d) Mycobacterium tuberculosis --- tuberculosis

3. Example of amoebas --- Entamoeba histolytica is known to have caused epidemics of amoebic dysentery.

4. Viruses have been known to cause infectious hepatitis.

D. Indicator Organisms and Enumeration

1. Coliforms (non-pathogenic bacteria) and fecal coliforms are used as indicator organisms.

2. Values reported as "most probable number" (MPN).

E. Per Capita Contribution and Seasonal Variation

1. Coliform generation rate

   200×10^9/capita/day

2. Typical seasonal variations are shown in Table 1. Whether these variations are traceable to actual multiplication of intestinal bacteria within the sewers or to a greater per capita discharge of these organisms in the summer months cannot be stated.

F. General Principles of Self-Purification

Table 1: Expected Seasonal Pattern of Monthly Average Coliform Contribution Based on an Annual Average of 200 Billion per Capita per Day

Month	Monthly Average (Percentage of Annual Average)	Monthly Average Coliform Contribution per Capita per Day ($X10^9$)
January	45	90
February	46	92
March	53	106
April	65	130
May	92	184
June	135	270
July	172	344
August	182	364
September	165	330
October	110	220
November	80	160
December	55	110
Annual average	100	200

1. Some general conclusions of Streeter's work are as follows:

   (a) The maximum density of coliforms occurs at some distance downstream of a source of pollution

   (b) based upon the maximum numbers of organisms, the percentage of surviving cells downstream follows a curve with a slightly decreasing slope when a log linear plot is applied

   (c) the variance of single observations rapidly increases with a diminishing of the absolute numbers of organisms

2. Representative killing times for the first 90% of enteric bacteria in river water are shown in Table 2.

3. Killing rates of bacteria in natural streams depend largely on the hydraulic and biological characteristics of the stream. In shallow rivers, where a large ratio of wetted surface to water flow exists, and a considerable amount of fixed biomass is normally present, rate constants for the initial part of the death rate curves (e.g., until T_{90}) of coliforms might be found which are 20 or more times higher than reported from the Ohio River (see examples in Table 2).

4. Increasing temperature enhances bacterial disappearance considerably.

5. The influence of the organic pollution load on the persistence of coliforms or other fecal bacteria in a river is controversial. The multiplication of pathogenic organisms in natural waters is certainly negligible (sediments very rich in fecal matter may be an exception). Dissolved organic pollutants

Table 2: Killing Times for the First 90% (T_{90}) of Enteric Bacteria in River Water

Organism	Medium	Temp., °C	T_{90}, hours
			60
E. coli	Ohio River	Summer	47
		Winter	51
	Missouri River	Winter	115
	Tennessee River	Summer	53
	Sacramento River	Summer	32
	Cumberland River	Summer	10
	Glatt River	Summer/winter	2.1[a]
S. typhi	Thames River	0	172
		5	108
		10	77

[a] Small shallow streams.

probably indirectly increase the death rate by favoring the growth of secondary consumers (bacteria feeders).

6. Systematic studies on the removal of viruses in rivers are practically nonexistent.

II. Prediction of bacterial Self-Purification

A. Chick's Law

1. Bacteria die at a constant rate; that is, a given percentage of the residual population dies during each successive time unit.

2. Basic relationship

$$\frac{dB}{dt} = KB$$

This integrates to:

$$\log_{10} \frac{B}{B_o} = -Kt$$

where B = residual after any time t (day)

B_o = initial number of bacteria

K = reaction rate or death rate (day^{-1})

B/B_o = proportion of organisms that survive

$1 - B/B_o$ = proportion of organisms that die

B. Factors That Modify Chick's Law in a Stream

1. Temperature --- increase in temperature increases the death rate. See Figure 1.

2. Acidity and alkalinity --- both acidity and alkalinity increase the bacterial death rate, but under stream conditions the specific contribution of pH is not definable except when there is pronounced deviation from neutrality.

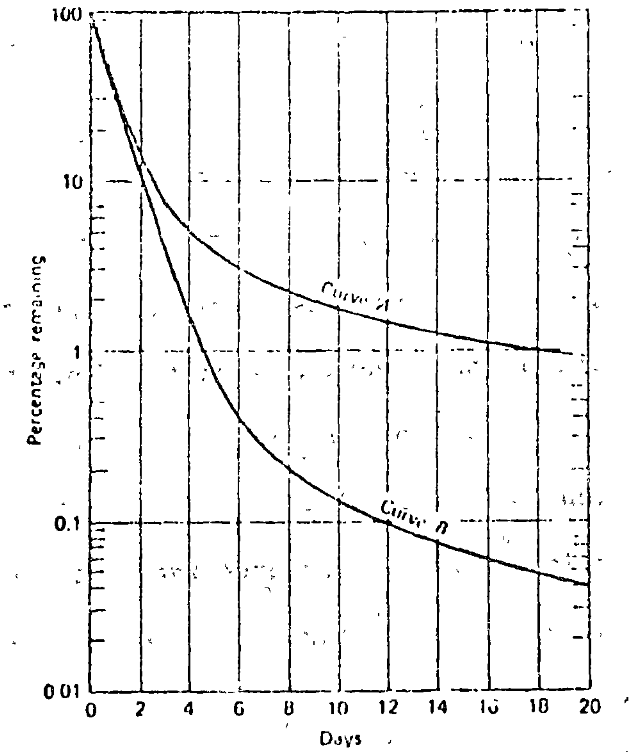

Figure [redacted] 1 Death rate curves of *E. coli* in the Ohio River. Curve *A*, cool-weather conditions, curve *B*, warm-weather conditions.

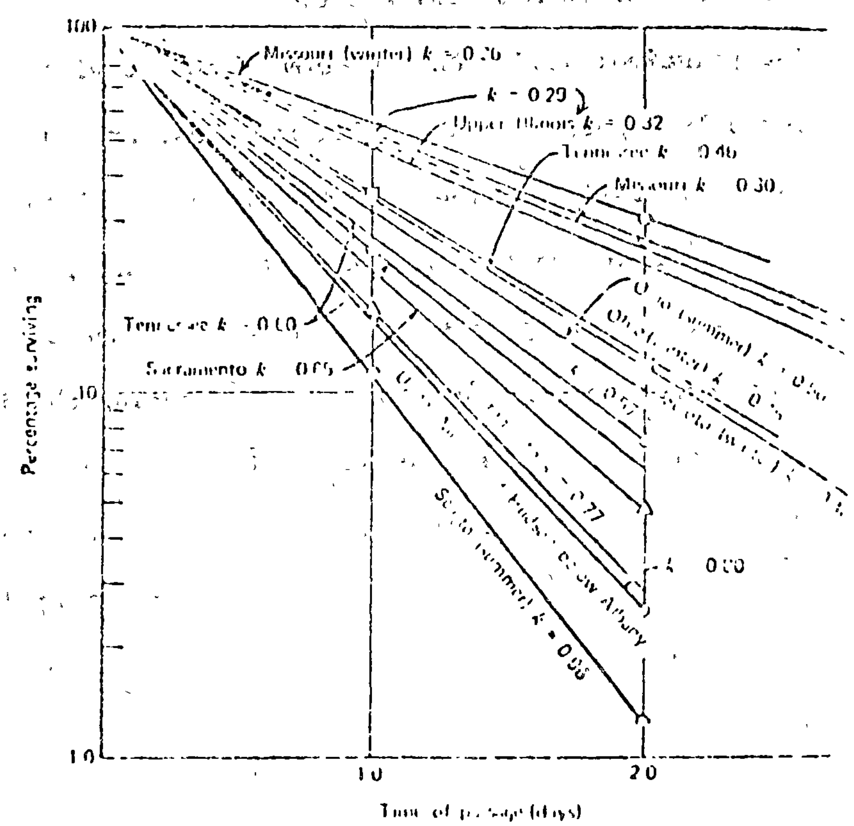

Figure [redacted] 2 Observed coliform bacteria death rate in the stream environment

3. Nutrients --- not much is known about the affect of nutrients on bacterial self-purification.

4. Sedimentation and adsorption --- both increase the death rate.

5. Presence of competitive life --- the natural biological life of polluted streams is much too rugged for the survival of organisms whose normal habitat is the shelter of the intestinal tract of man and other warm-blooded animals. In some instances coliform bacteria appear to multiply to some extent immediately after discharge into the stream.

6. These modifying factors can cause two basic changes to Chick's Law:

   (a) deviation in form

   (b) variations in observed death rates

C. Deviation in the Form of Chick's Law

1. Two principal deviations in form of the survival curve are commonly reported: an initial lag phase preceding onset of the logarithmic decline, and an afterphase of apparent decreasing death rate. In some instances during the lag phase an apparent multiplication in bacterial numbers is observed. This phenomenon is dealt with by considering the initial bacterial load as the peak and shifting the orgin of the time scale in determining the death rate to the position of the peak. The afterphase of apparent decline in death rate is shown when the semilog plot bends upward at the end of the depletion curve.

2. Mathematical formulation of a two-stage equation.

$$B = B_o(10^{-kt}) + B'_o(10^{-k't})$$

This calculation is predicated on the assumption that the residual number of bacteria at any time consists of two fractions, one resulting from the application of a death rate constant k to an initial population B_o and another from similar application of the rate k' to the initial population B'_o.

3. Phelps described some representative values for the Ohio River for the two-stage equation as shown in Table 3.

4. A review of Figure 1 and Table 3 indicates that the survival curve deviates substantially f om the straight-line Chick form. The coliform group (in summer) is composed of the large fraction of 99.51 percent with a half-life of 0.64 day, and a small fraction of 0.49 percent of mcre resistant organisms with a half-life of over 5 days. For practical purposes this means that 99.51 percent of the coliform bacteria dieoff can be represented by the simple Chick straight-line form. Less than 0.5 percent are involved in the apparent deviation refelected in the upward turn of the survival curve.

D. Observed Death Rates

1. Some typical death rate K values are shown in Table 4.

2. The K values for large rivers are 0.5 ± 0.15 day^{-1}; for moderate size rivers the values are 0.8 ± 0.2 day^{-1}.

3. Some death rates are shown in Figure 2. It will be noted from Figure 2 that for summer conditions only a small percentage of coliforms survives after 2 days' time of passage.

Table 3: Two-Stage Equation for Bacterial Self-Purification on the Ohio River

Parameter	Warm Weather	Cool Weather
B_o (percent)	99.51	97
k (day)	0.467	0.506
Half-life (day)	0.64	0.59
B'_o (percent)	0.49	3.0
k' (day)	0.0581	0.026
Half-life (days)	5.16	11.5

Table 4: Coliform Death Rates k Observed in Rivers

Chick's Law: $\log (B/B_o) = -kt$

River	Reaction Rate k(day^{-1}) Warm Weather	Reaction Rate k(day^{-1}) Cool Weather	Authority for Survey Data	Remarks
Ohio	0.50	0.45	Frost, Streeter et. al.	Generalized results of analysis of extensive data
Upper Illinois	0.90 0.67	0.32 0129	Hoskins et. al.	1-day decline 2-day decline
Scioto	0.96	0.46	Kehr et. al.	
Hudson	0.80		Hall, Riddick, Phelps	Freshwater reach below Albany
Upper Miami	0.80		Velz, Gannon, Kinney	Mean through reach above Dayton
Tennessee	0.46		Kittrell	1- and 2-day declines, below Knoxville
Tennessee	0.60 0.57		Kittrell	1-day decline 2-day decline (below Knoxville)
Sacramento	0.77 0.65		Kittrell	1-day decline 2-day decline (below Sacremento)
Missouri		0.30 0.26	Kittrell	1-day decline 2-day decline (below Kansas City)

For a large stream (k = 0.5) only 10 percent survive after 2 days; for a moderate-size stream (k = 0.8) less than 3 percent survive after 2 days. However, although these survival percentages are small, it must be remembered that, applied to the summer peak coliform bacteria loading, these declines may result in large absolute number of survivors, which, converted to concentrations in the diluting streamflow, may infringe the required criteria of water quality.

<u>Selected References for Bacterial Self-Purification</u>

1. Phelps, Earle B., <u>Stream Sanitation</u>, John Wiley and Sons, Inc., New York, 1944, pp. 201-221.

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PREDICTION OF THE IMPACTS OF THERMAL DISCHARGES ON RECEIVING STREAMS

L.W. Canter*

Mathematical models of varying degrees of complexity have been developed to determine the fate and persistence of heat in quiescent waters, flowing streams, estuaries, and the ocean. The ability to predict water temperatures accurately is necessary in order to determine the thermal impact of:

1. Proposed waste heat discharges.
2. Changes in the hydraulic characteristics of a water body or stream--for example, due to the construction of a dam with its resulting flow regulation.
3. Releases of water from stratified reservoirs with multilevel outlets.
4. Unusual meteorological conditions.

The following discussion presents a basic approach which can be used to solve temperature prediction problems. The material for this discussion is from two basic references (1,2).

1. Basic Principles

A. Heat Transport

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1. There are two heat transport mechanisms which occur in water--advection and dispersion or turbulent mixing. Advection is the transport of heat by the motion of a mass of water and is accomplished through ordinary streamflow, utilization of a discharge stream's kinetic energy, or water movement due to density gradients.

2. Turbulent mixing or dispersion causes heat interchange through eddy diffusion or molecular diffusion. Eddy diffusion occurs under turbulent flow, which depends on fluid velocity and channel characteristics. Mixing results from the action of small fluid masses known as eddies, which are random both in size and orientation. Molecular diffusion is that resulting from random motion of molecules. Its influence is much less than that from turbulent mixing.

B. Heat Exchange

1. Heat exchange which takes place between the water surface and the atmosphere is made up of seven mechanisms.

2. The mechanisms which are independent of water temperature are:

   Q_s = Incoming short-wave solar radiation (400 to 2800 BTU/ft^2/day).

   Q_a = Incoming long-wave atmospheric radiation (2400 to 3200 BTU/ft^2/day).

   Q_{sr}, Q_{ar} = Portions of both short-wave and long-wave radiation which are reflected or scattered by the water surface (40 to 200 and 70 to 120 BTU/ft^2/day, respectively).

3. The mechanisms of heat transfer which are dependent on the surface water temperature include the following:

Q_{br} = long-wave back radiation from the water to the atmosphere (2400 to 3600 BTU/ft^2/day). It is proportional to the fourth power of the absolute water surface temperature (AT_s), i.e., $Q_{br} \propto (AT_s)^4$.

Q_c = Heat exchange due to conduction-convection (-300 to +400 BTU/ft^2/day), which is proportional to the wind speed (W) and the difference between water temperature (T_s) and air temperature (T_a), i.e., $Q_c \propto W(T_s - T_a)$. A positive Q_c indicates an energy loss.

Q_e = Heat loss due to evaporation (2000 to 8000 BTU/ft^2/day), which is proportional to the product of wind speed (W) and the difference between the water vapor pressure in saturated air at the water temperature (e_s) and the water vapor pressure in the overlying air (e_a), i.e., $Q_e \propto W(e_s - e_a)$. If $e_a > e_s$, condensation takes place and the water body gains energy.

4. The algebraic sum of these surface heat exchange parameters is equal to the net rate of surface heat exchange. Equilibrium temperature is reached when this sum is zero.

II. Temperature Prediction

A. Macro Models

1. Macro models are those which describe or predict temperature regimes in a complete river or river system, lake or reservoir,

estuary, or coastal area. Such models combine heat transfer mechanisms and water movement on a continual basis with respect to time.

2. A macro model maintains an energy budget for the water body under consideration, i.e., it maintains a heat balance of both the internal heat exchange and the heat transfer at the water surface. This heat budget may be expressed as follows: (Rate of Heat In)-(Rate of Heat Out)+(Rate of Heat Storage)+ (Rate of Heat Exchange at Water Surface)=0

3. The rate at which heat flows into and out of the water body is determined from the flow rates and temperatures of inflowing and outflowing water. These rates require evaluation of the heat transfer mechanisms in water to define the motion of heat entrained water masses. The rate of heat storage is determined from the temperature and volume of the water body in consecutive time periods. The rate of heat exchange at the water surface is the algebraic sum of all the water-atmosphere heat exchange rates.

B. Example Problem

A power plant releases 6.41×10^9 BTU/hr. If the flow in the receiving stream is 3500 cfs, determine the temperature changes in the stream downstream from the power plant.

Solution

A. Compute the temperature rise in the stream, assuming complete mixing.

Given a design flow in the stream of 3500 cfs. which in terms of lb/hr is:

$$Q = 3500 \text{ cfs}) \ (62.4 \text{ lb/ft}^3)(3600 \text{ sec/hr})$$

$$Q = 7.86 \times 10^8 \text{ lb/hr}$$

Since 1 BTU will raise the temperature of 1 lb. of water 1 °F,

$$\Delta T_r = \Delta T \text{ in river} = \frac{(6.41 \times 10^9 \text{ BTU/hr})}{(7.86 \times 10^8 \text{ lb/hr})(1 \text{ BTU/lb } ^oF}$$

$$\Delta T_r = 8.2 ^oF$$

B. Equation for computing downstream temperatures.

Downstream temperatures are computed by assuming exponential temperature decay. This concept is presented mathematically as:

$$\frac{dH}{dt} = -K(T - E)$$

where $\frac{dH}{dt}$ = net rate of water surface heat exchange (BTU ft^{-2} day^{-1})

K = energy exchange coefficient (BTU ft^{-2} day^{-1} $^{o}F^{-1}$)

T = water surface temperature (^{o}F)

E = equilibrium temperature (^{o}F)

For a well-mixed stream, this equation can be written as:

$$\rho C_p yU \frac{\partial T}{\partial x} = -K(T_x - E)$$

where ρ = water density (62.4 lb ft^{-3})

C_p = specific heat of water (1 BTU lb^{-1} $^{o}F^{-1}$)

y = mean stream depth (ft)

U = mean stream velocity (ft day^{-1})

$\frac{\partial T}{\partial x}$ = longitudinal temperature gradient (^{o}F ft^{-1})

x = downstream distance (ft)

Define T_o = temperature at x = o; then

$$T_x = (T_o - E)e^{\left(\frac{-Kx}{\rho C_p yU}\right)} + E$$

By defining $\alpha = \frac{-Kx}{\rho C_p yU}$; then

$$T_x = (T_o - E)e^{\alpha} + E$$

C. Meteorologic Data

The following data are used in determining K and E:

	For K	For E			
Time Period (6 hr intervals)	Wind Speed (W) (mph)	Net Radiation Input (H_R) (BTU ft^{-2} hr^{-1})	Air Temp (T_a) (°F)	Relative Humidity (%)	Water Vapor Pressure of Ambient Air (e_a) (mm Hg)
Midnight - 6 am	4.0	120	65	40	6.3
6 am - Noon	12.0	299	75	30	6.7
Noon - 6 pm	12.0	320	85	20	6.2
6 pm - Midnight	6.0	130	70	35	6.6
DAILY AVERAGE	8.5	215	74	--	6.5

D. Determination of K

The energy exchange coefficient is computed using the following equation:

$$K = [15.7 + (0.26 + \beta)(bW)]$$

where W = wind speed (mph)

b = experimental evaporation coefficient (a value of 15 is typical.

β = proportionality coefficient [See following table]

Range of T (°F)	β (mm Hg °F^{-1})
50 to 60	0.405
60 to 70	0.555
70 to 80	0.744
80 to 90	0.990

Thus, for an average daily value of K, using W = 8.5 mph:

$$K = (15.7 + [0.26 + \beta] [(15)(8.5)])$$

Using appropriate values of β for two ranges of E:

E (°F)	K(BTU ft^{-2} day^{-1} $°F^{-1}$)
60 to 70	120
70 to 80	144

E. Determination of E

The equilibrium temperature is reached when the rate of change of energy at the water surface equals zero. Edinger and Geyer (2) present a method for computing E. The method involves assuming a likely 10°F temperature range for E and by using the appropriate value for K and the given meteorological data, computing a value for E. If the computed value of E falls within the assumed range, the process is complete. However, if the computed value of E falls outside the assumed range, another range must be assumed and the process repeated until E falls within the proper limits. Thus, E is computed by a trial and error method.

For the stated meteorological conditions and computed values of K, we can determine a daily average E by the following seven steps

Step 1. Assumed range of E = 70 to 80°F

Step 2. Compute F(K) for use in step 6:

$$F(K) = \frac{K - 15.7}{K}$$

As computed for an E range of 70 to 80°F, K = 144 BTU ft^{-2} day^{-1} $°F^{-1}$

$$\therefore F(K) = \frac{144 - 15.7}{144} = 0.891$$

Step 3. Compute E_1 for use in step 6:

$$E_1 = \frac{H_R - 1801}{K}$$

From the meteorologic data table, H_R = 215 BTU ft^{-2} hr^{-1}

or in terms of days, H_R = 5160 BTU ft^{-2} day^{-1}

$$\therefore E_1 = \frac{5160 - 1801}{144} = 23.3$$

Step 4. Compute E_2 for use in step 6:

$$E_2 = \frac{(0.26)\ (T_a)}{(0.26 + \beta)}$$

From the meteorologic data table, T_a = 74°F, and from the table of E range vs. β, β = 0.744

$$\therefore E_2 = \frac{(0.26)\ (74)}{(0.26 + 0.744)} = 19.2$$

Step 5. Compute E_3 for use in step 6:

$$E_3 = \frac{e_a - C(\beta)}{(0.26 + \beta)}$$

From the meteorologic data table, e_a = 6.5 mm Hg. C(β) is related to ranges of E as follows:

Range of E (°F)	C(β) (mm Hg)
50 to 60	-11.22
60 to 70	-20.15
70 to 80	-33.30
80 to 90	-53.33

Thus for an E range of 70 to 80^{o}F, $C(\beta) = -33.3$

$$\therefore E_3 = \frac{6.5 - (-33.3)}{(0.26 + 0.744)} = 39.6$$

Step 6. Compute M for use in step 7:

$$M = E_1 + \Gamma(K)\ (E_2 + E_3)$$

$$M = 23.3 + (0.891)\ (19.2 + 39.6) = 75.7$$

Step 7. Compute E using the following relationship:

$$M = E + \frac{0.051E^2}{K}$$

Inserting M and K and setting up a quadratic equation gives:

$$E^2 \left(\frac{0.051}{144}\right) + E - 75.7 = 0$$

$$\therefore 0.000354E^2 + E - 75.7 = 0$$

Solving this equation using the quadratic formula gives:

$$E = \frac{-1 \pm [1 - (4)\ (0.000354)\ (-75.7)]^{1/2}}{2\ (0.000354)}$$

$$E = \frac{-1 \pm (1.10719)^{1/2}}{0.000708} = \frac{-1 \pm (1.05223)}{0.000708}$$

Rejecting the negative value gives:

$$E = \frac{0.05223}{0.000708} = \underline{73.8^{o}F}$$ (This value is acceptable because it falls within the assumed range of 70 to 80^{o}F.)

F. Compute Average Stream Velocity

$Q = 3500$ cfs

Given an average cross section 800 feet wide and 5 feet deep:

$$U = \frac{3500\ ft^3\ sec^{-1}}{(800\ ft)\ (5\ ft)} = 0.875\ ft/sec = \underline{75,600\ ft/day}$$

G. Evaluation of α

$$\alpha = \frac{-Kx}{\rho C_p y U}$$

For x' in miles: $\alpha = \frac{-(144)\ (5280)x'}{(62.4)(1)(5)(75,600)}$

$\alpha = -0.0322x'$

H. Solve for $T_{x'}$ for $x' = 10, 20, \ldots .50$ miles

Assume unheated river temperature = 7[illegible]

$\therefore\ T_o = 74^oF + \Delta T_R = 74^oF + 8.2^oF = 82.2^oF$

$$T_{x'} = (T_o - E)e^{-0.0322x'} + E$$

For $x' = 10$ miles

$$T_{x'} = (82.2 - 73.8)e^{-(0.0322)(10)} + 73.8$$

$$T_{x'} = (8.4)e^{-0.322} + 73.8$$

$$T_{x'} = (8.4)\ (0.725) + 73.8 = 79.9^oF$$

For $x' = 20$ miles

Use same value of α and replace T_o by $T_{x'}$ for $x' = 10$ miles:

$$T_{x'} = (79.9 - 73.8)\ (0.725) + 73.8 = 73.2^oF$$

For $x' = 30, 40, 50$ miles

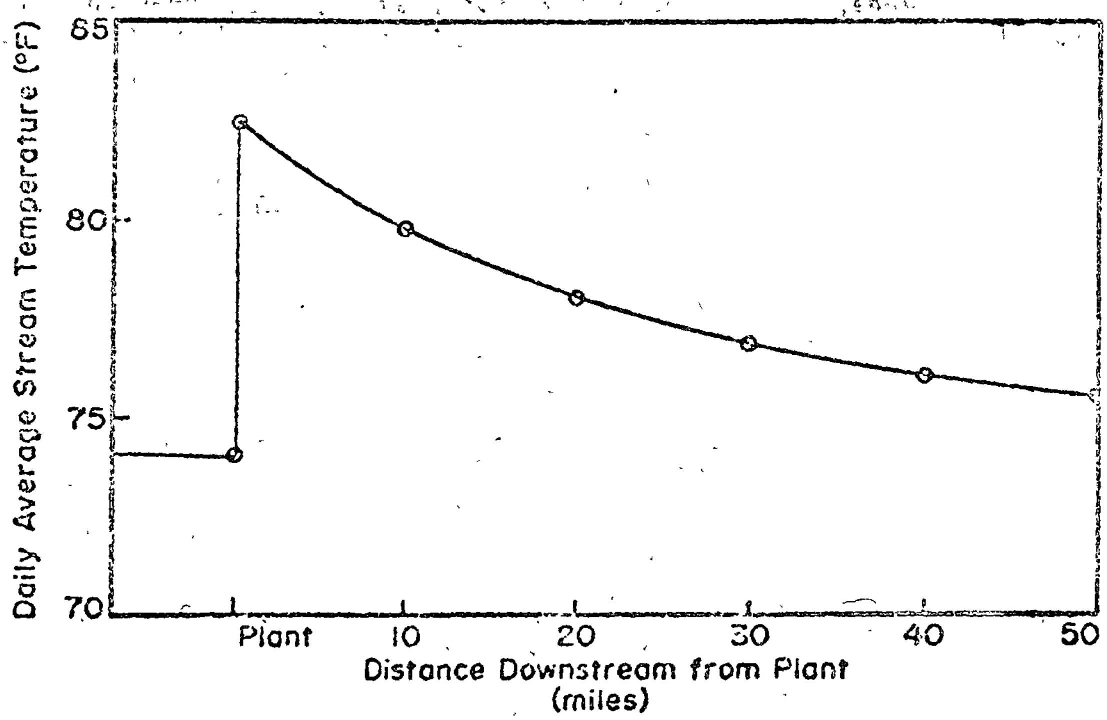
Following the same procedure:

30 miles, $T_{x'} = (78.2 - 73.8)\ (0.725) + 73.8 = 77.0^oF$

40 miles, $T_{x'} = 76.1^oF$

50 miles, $T_{x'} = 75.5^oF$

1. These values represent the exponential temperature decay which is graphically shown on the following plot:



2. This graph presents an idealized picture of the downstream temperatures, since the computations were based on average daily conditions, and thus no diurnal effect is evident. It also assumes that the weather data on which K and E are based are indicative of conditions along the 50-mile stretch of the river. In addition, no tributary inflows or heated discharges are accounted for in the 50 miles.

Selected References

1. "Industrial Waste Guide on Thermal Pollution:, Federal Water Pollution Control Administration, U. S. Department of Interior, September, 1968, 112 pages.

2. Edinger, J. E., and Geyer, J. C., "Heat Exchange in the Environment", Publication No. 65-902, Edison Electric Institute, New York New York, June, 1965.

GEOFISICA INTERNACIONAL

REVISTA DE LA UNION GEOFISICA MEXICANA, AUSPICIADA POR EL INSTITUTO DE GEOFISICA DE LA UNIVERSIDAD NACIONAL AUTONOMA DE MEXICO

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THEORETICAL STUDY OF THERMAL POLLUTION

RAUL ZALTZMAN* AND
GEORGE W. REID**

RESUMEN

Aproximadamente el 80% de toda el agua usada en la industria de los Estados Unidos lo es en operaciones de enfriamiento. Como resultado, la carga termal (calor) de los lagos y corrientes recipientes es más pesada cerca de los complejos industriales. Cuando la cantidad de calor en tales áreas comienza a desequilibrar el proceso acuático o a afectar las especies acuáticas, el resultado es la contaminación termal. Estimaciones teóricas sobre el exceso futuro de la producción del calor apuntan la necesidad de obtener más información en este problema vital.

Este estudio se refiere a los efectos de la contaminación termal y a los procesos termales en corrientes y lagos. Su objeto fue el de contribuir a la comprensión teorética de los procesos involucrados para que puedan desarrollarse planteamientos con criterio sólido. Con tal fin, un modelo matemático del proceso de contaminacion fue desarrollado y evaluado.

ABSTRACT

Approximately 80% of all water used by United States industry is in cooling operations. As a result, the thermal load (heat) of receiving streams and lakes is heaviest near industrial complexes. When the quantity of heat in such areas begins to impair aquatic processes or kill

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aquatic species, the result is thermal pollution. Projected estimates on future excess heat production point up the need for more information on this vital problem.

This study concerns the effects of thermal pollution and thermal processes in streams and lakes. The objective was to contribute to the theoretical understanding of the processes involved in order that sound design criteria could be developed. Toward that end, a mathematical model of the pollution process was developed and evaluated.

INTRODUCTION

Investigation conducted at the University of Oklahoma for the U.S. Senate Select Committee on Water Resources on Low Flow Augmentation considered only the efects of dissolved oxygen (D. O.) (Reid, G. W., 1960).

However, the Select Committee studies indicated the severity of the thermal pollution problem and that more work should be done in this area.

The addition of a thermal load (heat) to a stream or body of water in quantities detrimental to beneficial activity constitutes thermal pollution. Aproximately 80% of all water used by United States industry is in cooling operations, with the result that discharge of large quantities of heat into the streams and lakes is commonplace. Projecting this situation into the future, industry is expected to double its excess heat production each decade until at least 1980. This in turn will create a greatly increased water demand — from 74 bgd in 1954 to an estimated 260 bgd for 1980. (Richardson, B., 1931).

The necessary requirement for temperature as a major factor of the physical environment of surface waters must be that it allow a level of activity commensurate with maintaining desirable aquatic species in acceptable numbers. Allowable temperatures are not within one specific range — they fluctuate with season and other natural environmental factors. Present knowledge (or the lack of it) of nature's processes for maintaining a balance of life indicates that there can be no simple statement on thermal requirements of surface water.

Apparently the only feasible remedy to this situation is to develop specific and sound engineering design provisions to minimize thermal pollution from industrial sites. The objective of this study was to contribute to the theoretical understanding of such designs by deriving a mathematical model of the pollution process.

THERMAL POLLUTION

Effects of Thermal Pollution

The impact of thermal pollution on water conditions is exemplified by at least four distinct reactions

(1) death or displacement of aquatic species
(2) environmental modification
(3) activity reduction
(4) impairment of stream self-purification

Lethal temperature levels among different species of fish vary markedly, in some instances as much as 31°F (Brett, J R., 1960)

For example, with fish the critical limit for *Carassius auratus* is 106°F, while *Oncorhynchus gorbusha* cannot endure temperatures above 75°F Stream temperatures which consistently exceed 70°F favor warm-water fishes rather than cold-water species such as the Salmonoids, (Belding, D L 1928) Increases in temperature of 2-3°F above the 70°F level have been observed to cause depopulation of *Salmonoids* to less than 10% of the total fish population Minnows, suckers, and other warm-water species gradually replaced them under this condition (Tarzwell, M C. and A R Gaufin, 1953)

While the change of temperature may be within the thermal tolerance of the particular fish, it may make *environmental conditions* unfavorable for essential food organisms and for certain developmental stages of fish life -- it is known that the eggs of some daphnia have to be chilled or frozen before they hatch Or the higher temperatures may favor competitors or predators of desirable species. Many other organisms go through resting periods or specific stages of development during certain seasons Furthermore, certain diatoms are abundant only, at temperatures below 50°F. Thus, by elevating the temperature of a stream, the biota distribution may be changed and the entire food chain seriously disrupted (Tarzwell, M C., 1957).

The observation was made in 1947 that water temperature affects *aquatic activity* through the process of metabolism in the life forms (Fry, F. E J , 1947). Experiments were made to determine the effects of temperature and cruising speed of the *sockeye* to demonstrate this relationship (Brett, J. R., *et al* 1958) The maximum sustaned swimming speeds of *sockeyes* at 10°C and 19°C are approximately equal. However, because of increased metabolic demand at the higher temperature, energy reserves of the fish were found to be exhausted 1 ½ to 2 times as quickly at 20°C (Brett, J. R., *et al*, 1958). From these studies it is apparent that the rate of activity is directly related to the water temperature, and to the difference between resting metabolism and active metabolism. Activity reduction aspects may be extended to man who also experiences undesirable effects at higher temperatures. For example, the use of a stream for recreational purposes is reduced by excessive heat.

Bacterial growth rate in streams is maximum at 86°F. As a consequence, higher stream temperatures decrease the *self-purification capacity* of the stream. Addition of heat to surface waters increases the possibility of septic conditions and tends to make the water unsuitable for industrial reuse or for drinking purposes without expensive treatment.

The dissolved oxygen content of water, as noted in the introduction, is a function of temperature, ranging from for example, 10 ppm at 59°F to 7.6 ppm at 86°F. This relationship has been noted in self-purification studies as well as the bacterial growth rate. It should be stressed thath these conditions relate to equilibrium conditions and rarely occur in vivo.

The development of a theoretical formulation will give a broad idea of the assimilation capacity of thermal waste in a certain point of a stream thereby providing design criteria to help prevent pollution in an increasingly critical area.

ANALYSIS OF THE THERMAL BUDGET IN A BODY OF WATER

Study of thermal processes in water must begin by accounting for all significant factors. After this has been done, the *nature of functioning* can be analyzed and evaluated as performed below (Bowen, I. S., 1926, Richardson, B., 1931):

A. The continuty expression of the conservation of energy can be stated as:

$$H_e + H_h = H_v \quad H_i + H_s \quad H_r \quad H_b \qquad (1)$$

where

H_e = heat required for evaporation
H_h = heat loss due to conduction to the atmoshphere
H_v = net energy advected
H_i = increase in energy stored in the body of water
H_s = incident short wave radiation
H_r = reflected short wave radiation
H_b = net energy loss throuth exhange of long wave radiation

or if we evaluate these terms as a function of Insolation (Richardson, B., 1931), we find that Insolation is a positive quantity during the day, but at night is equal to zero. Evaporation is always positive, while the other terms (convection, sensible heat, conduction and radiation) may be either positive or negative. Expressed simbolically this is,

$$L \cdot E \cdot R + L \cdot E = I - S - C - B_r \tag{2}$$

L = latent heat of vaporization of water (585.4 at 20°C)
E = quantity of water evaporated in cm of depth per day
R = ratio of convection evaporation (Bowen's ratio)
I = radiant power per unit area of earth surface from sun, calories per minute
S = detectable heat measured by warming or cooling of water, calories/day
C = conduction of heath through the walls of the container
B_r = radiation from water, calories/per sq cm per day.

B. Effects of evaporation

From equation (2), heat loss by evaporation is:

$$H_{loss} = L \cdot E + L \cdot E \cdot R = EL(1 + R) \text{ with} \tag{3}$$

$$R = \frac{0.46\,(T_0 - T_a)\,P}{(P_1 - P_2)\,760} \tag{4}$$

where

T_o = temperature of water °C

T_a = temperature of air °F

P_1 = vapor pressure of sat. vapor @ T_o – mmHg

P_2 = partial pressure of actual vapor in air – mmHg

P = atmospheric pressure

The effect of wind in evaporation has been disregarded because no detectable error results from neglecting it (Bowen, I. S., 1926).

So far, no mention has been made of the artificial heating of the stream, but with it the value of R will change. As the values for E are known for the initial temperature (T_0) of the water, the value of R is good only for this condition. Thus when an increase of temperature in the stream has changed R To R', the geometric mean value of R is to be used, and equation (3) will be

$$H_{loss} = EL(1 + \sqrt{RR'}) \qquad (5)$$

C. Capacity of thermal assimilation:

where using a consistent set of units,

Q_o = initial flow

Q_1 = final flow

ΔQ = added flow (waste flow)

$\frac{\Delta Q}{Q_o}$ = ratio of dilution

T_o = initial temperature

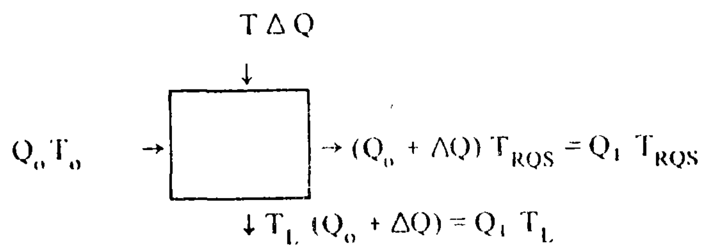
T_{RQS} = temperature of river quality standards, or final

ΔT_Q = allowable temperature change, or $T_{RQS} - T_o$

ΔT_w = allowable temperature difference between added flow and T_{RQS}.

T_L = temperature loss
T = temperature od added flow
A = unit surface area of stream
V_S = subsidiance velocity, or Q/A
L = heat of vaporization of water at given temperature $1/2\,(t_o - t_{RQS})$

we have.



from eq (5)

$$Q_1 T_L = ELA(1 + \sqrt{RR'}) \tag{6}$$

if

$$A = \frac{Q + \Delta Q}{V_S}, \text{and } K = (1 + \sqrt{RR'})\, EL$$

$$Q_1\, T_L = K\left(\frac{Q_o + \Delta Q}{V_S}\right) \tag{7}$$

then:

$$(Q_o + \Delta Q)T_{RQS} = T_o Q_o + T\Delta Q \quad K\left(\frac{Q_o + \Delta Q}{V_S}\right) \tag{8}$$

$$\Delta Q(T \quad T_{RQS}) = Q_o(T_{RQS} \quad T_o) + K\left(\frac{Q_o + \Delta Q}{V_S}\right) \tag{9}$$

if

$$\Delta T_W = T - T_{RQS}, \; \& \; \Delta T_o = T_{RQS} - T_o:$$

$$\Delta Q\,(\Delta T_w \quad K/V_S) = Q_o\,(\Delta T_Q + K/V_S) \tag{10}$$

then

$$\frac{Q_o}{\Delta Q} = \frac{\Delta T_w + K/V_s}{\Delta T_Q + K/V_s} \qquad (11)$$

and finally if $C = K/V_s$

$$\frac{Q_o}{\Delta Q} = \frac{\Delta T_w + C}{\Delta T_Q + C} \qquad (12)$$

Equation (12) them may be considered a working expression for calculating the effects on water thermal conditions from the addition of wastes having any given characteristics. For more information on the inter-relation of the variables in practical dynamic operation see Appendix A. To illustrate futher the use of Equation (12) see Appendix B which shows the determination of the thermal addition capacity of the Ohio Basin from data obtained from the Select Committee studies. (Reid, G. W., 1960). The solution is made graphically from Figures 1 and 2 (Appendix C). Figure 1 and 2 were developed from data presented in Appendix D.

RECOMMENDATIONS FOR ENGINEERING CRITERIA

In order to provide a reference basis of acceptable temperature ranges, it is recommended that aquatic species be allowed to have the benefit of at least 3/4 of their optimun total activity capability (or work capacity at most favorable temperature). This provision must take into account the living requirements of desirable species of the specific water body. For example, the adverse effects of a 3-5°F temperature rise on trout forbid the discharge of significant amounts of heat into a stream in such a manner that a temperature block is created.

It is further recommended that close attention be paid in the design of facilities to careful studies of the species which are intended to inhabit the water. Under no circumstances should peak endurance temperatures prevail for more than 8 hours of each 24 hours period. Fishways must be provided when these aceptable conditions cannot otherwise be met.

In conclusion, the authors hope that this report will stimulate interest in this important problem and that this interest will lead to the collection of urgently needed field data. The acceptance or rejection of the theoretical considerations made for this study should provide the insights needed to help solve this "fast growing problem."

APPENDICES

Appendix A — Analysis of the Variables in Equation 12

Appendix B — Dilution Water Requirements of the Ohio Basin

Appendix C — Figure 1 Determination of the Corrected Heat of Vaporization

Figure 2 Nomograph for Solution of Equation 12

Appendix D — Table 1 Sequence of the Solution of Equation 12

Table 2 Dilution Water Requirements for the Basins of the United States

APPENDIX A

ANALYSIS OF THE VARIABLES IN EQUATION No. 12

Equation 12 as stated says.

$$\frac{Q_0}{\Delta Q} = \frac{\Delta T_w}{\Delta T_0} \cdot \frac{C}{C}$$

where.

$$C = \frac{K}{V_S}$$

in therms of depth "d" we have.

$$V_s = \frac{vd}{l}$$

where·

v = velocity of the stream (in units of distance per day).
d = average depth of the stream.
l = total distance traveled by the water in one day.

Then the dimensiones of V_S for a given period of time are:

$$(v = \frac{L}{\theta}\ ;\ d = L_1;\ l = L)$$

$$V_S = \frac{L/\theta \cdot L_1}{L} = \frac{L_1}{\theta} = d/\text{time} = d_\theta$$

where:

time = unit,

or in other words,

The loading rate in our study is equal to the average depth per unit of time (one day).

Therefore:

$$C = \frac{K}{d_\theta} \quad \ldots\ldots\ldots\ldots\ldots \text{"A"*}.$$

* Expression "A" will give the formula (12) in terms of the average depth of the stream in the section in study

WORKING INTER-RELATION OF VARIABLES

	INCREASE	DECREASE	ΔT_w	K/E	C	ΔT_o	Q/ΔQ
T	X		Inc	Inc	Inc.	N/C	Inc
T_{ROS}		X	Inc	N/C	N/C	Dec.	Dec
T_{ROS}	X	-	Inc	N/C	N/C	Dec.	Dec.
Depth	X	–	N/C	N/C	Dec.	N/C	Dec,
Depth	-	X	N/C	N/C	Inc.	N/C	Inc.

Inc = Increase, Dec.= Decrease, N/C= No Change

APENDIX B

DILUTION WATER REQUIREMENTS OF THE OHIO BASIN

1. A graphical solution of the equation (12) can be made by using Figure 1 and Figure 2 (Appendix C).

2. *Required Data:*

T_o = Average seasonal temperature of the water in the basin (for the warmest).
T_{RQS} = Maximum temperature allowable of the water in the basin.
T = Average temperature of the waste water.
T_{air} = Average temperature of the air in the layer in contact with the body of water.
E = Average evaporation in the basin (Yearly average will be good).
d = Average depth of the basin.

3. *Available Data:*

T_o – in print 29. = 75°F pág. 6 (Reid G. W., 1960)

$T_{RQS} - T_o = 5.4°F = \Delta T_Q$ in °C = 3°C (Assumed 9°F)
T = 95°F Assumed temperature of the waste water. (To be given by the plants).
$T - T_{RQS} = \Delta T_w$ in °C = ΔT_Q in °F/1.8 = ΔT_Q in °C
d -- in print. 29. pg. 6 (Reid, G. W., 1960)
E -- in print 13. Class A pan evaporation (Reid G. W., 1960)

4. *Solution for the Ohio Basin:*

a. Find T T_{air} = 20°F
b. Go with this value of 20°F to Figure (1) and find: K/E = 846
c. Go with this value of 846 to the nomogran (Figure 2) and solve for E = 45 inches/year, drawing a straight line through the pivot line, then draw another straight line. . .
d. . . . from the point in the pivot line through d = 4' and find a value for C (8 hour period) = 0.73.
e. In equation (12),

$$\frac{Q_o}{\Delta Q} = \frac{(\Delta T_w - C)}{(\Delta T_Q + C)}$$

we have that:

$$T_o = 75°F \qquad C = 0.73$$

$$\Delta T_w = T - T_{RQS}$$

$$T_{RQS} = T_o + 5.4 = 75.0 + 5.4 = 80.4$$

$$\therefore \Delta T_w = 95 - 80.4 = \underline{14.6°F} = 8.0°C$$

$$\Delta T_Q = T_{RQS} - T_o = 80.4 - 75.0 = 5.4°F = \underline{3°C}$$

$$\frac{Q_o}{\Delta Q} = \frac{8 - 0.73}{3 + 0.73} = \frac{7.27}{3.73} = 1.92$$

and from Table (1):

$$\Delta Q_{low-1980} = 19,121$$

$$\therefore Q_o = \Delta Q(1.92) = 19,121 \times 1.92 = \underline{\underline{36,712.32}}$$

APPENDIX C

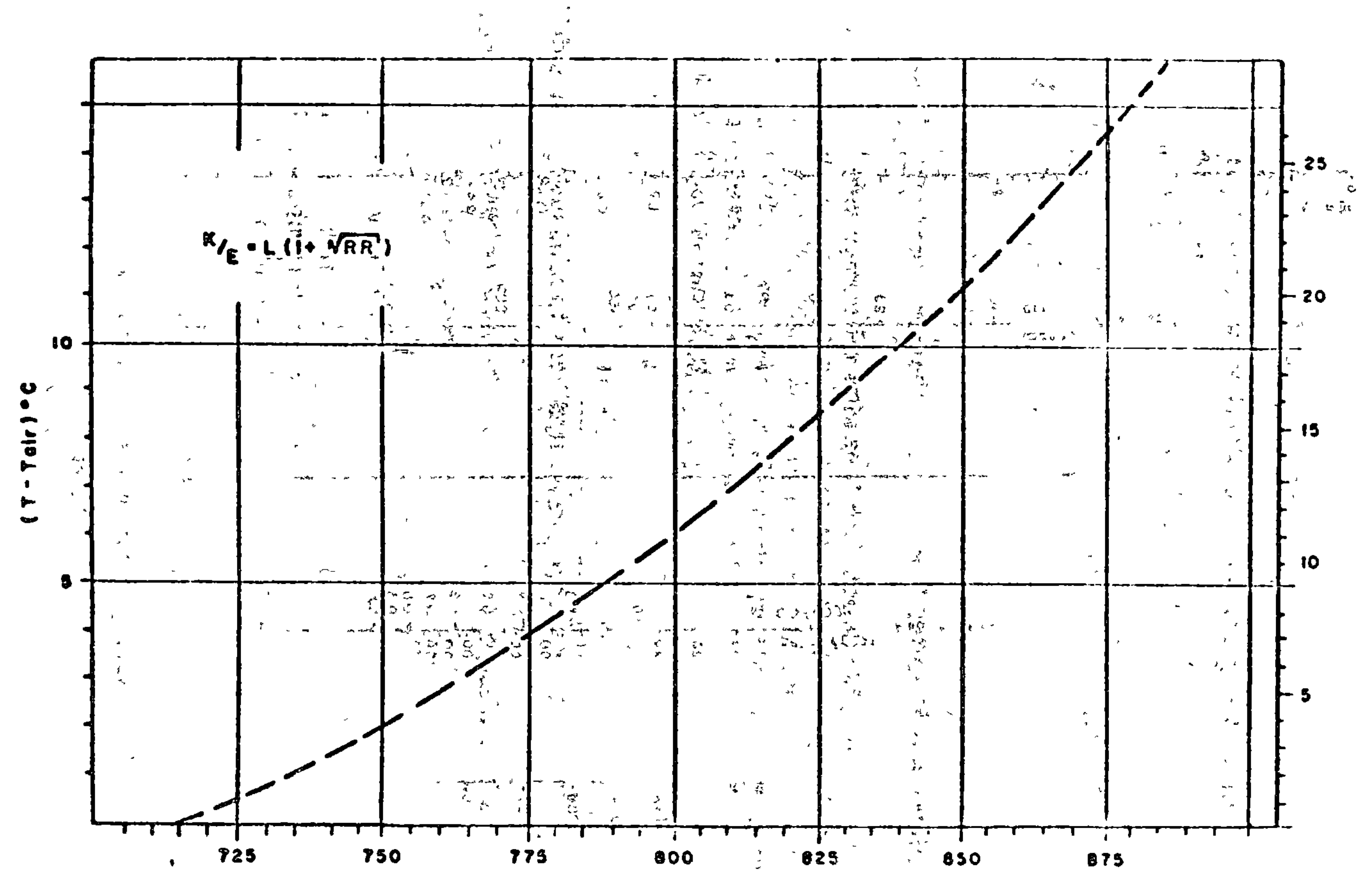
Figure 1 Determination of the Corrected Heat of Vaporization

Figure 2 Nomograph for Solution of Equation 12

APPENDIX D

Table 1 Sequence of the Solution of Equation 12

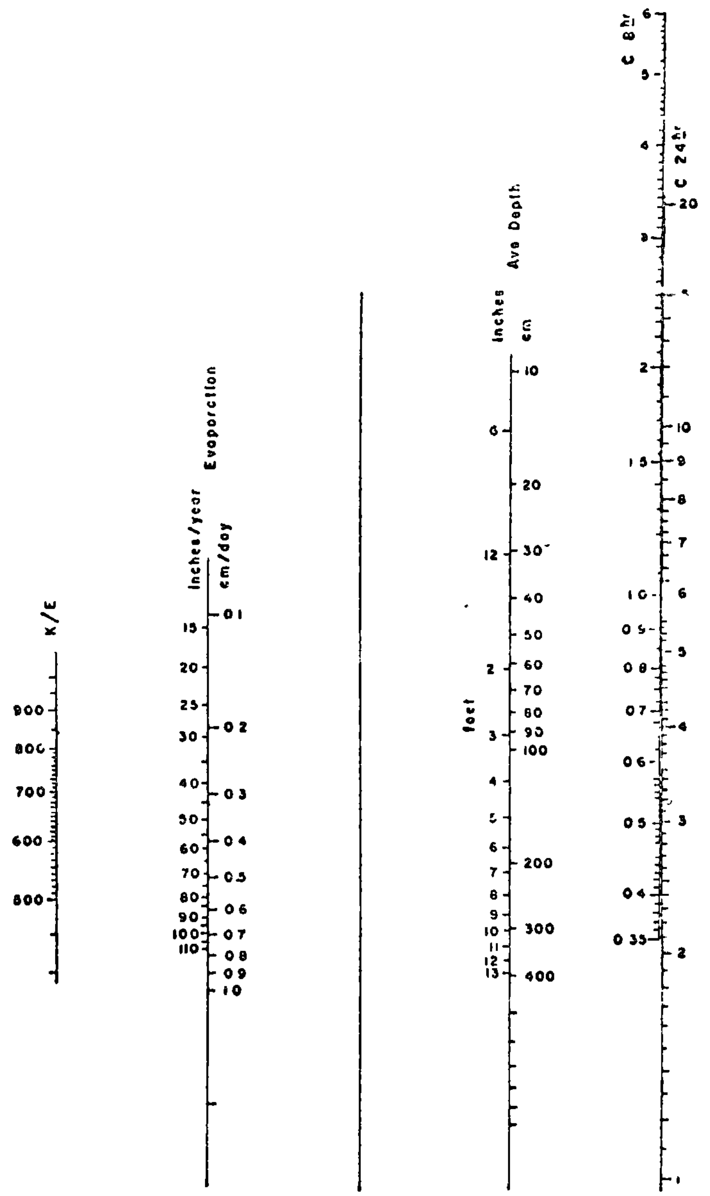
Table 2 Dilution Water Requirements for the Basin of the United States



DETERMINATION OF THE CORRECTED HEAT OF VAPORIZATION

VALUES OF K/E

FIGURE I



NOMOGRAPH FOR SOLUTION OF EQUATION 12

FIGURE 2

TABLE 1

SEQUENCE OF THE SOLUTION OF FORMULA 12

ASSUME: $T_o = T_{air}$, $T = 100$, $T_{RQS} - T_o = 5.4°F$

BASINS (NUMBER)	av T_o	T_{RQS}	$T-T_{air}$	K/I	I (in/year)	av depth z (feet)	$T-T_{RQS}$
1	70	75.4	25.0	869	30	4.0	19.6
2	70	75.4	25.0	869	40	3.0	19.6
3	68	73.4	27.0	877	35	3.0	21.6
4	70	75.4	25.0	869	35	3.0	19.6
5	73	78.4	22.0	856	45	5.0	16.6
6	75	78.4	20.0	846	45	4.0	14.6
7	73	78.4	22.0	856	48	3.0	16.6
8	78	83.4	17.0	830	50	4.0	11.6
9	83	88.4	12.0	805	57	5.0	6.6
10	74	79.4	21.0	851	44	3.0	15.6
11	80	85.4	15.0	821	56	4.0	9.6
12	68	73.4	27.0	877	60	3.0	21.6
13	74	79.4	21.0	851	55	3.5	15.6
14	78	83.4	17.0	830	91	2.0	11.6
15	82	87.4	13.0	811	60	3.5	7.6
16	85	90.4	10.0	791	85	4.0	4.6
17	83	88.4	12.0	805	90	1.5	6.6
18	80	85.4	15.0	821	80	2.0	9.6
19	75	80.4	20.0	846	70	1.0	14.6
20	65	70.4	30.0	889	40	4.0	24.6
21	68	73.4	27.0	877	70	4.0	21.6
22	83	88.4	17.0	830	110	0.5	11.6

= 95°F C	°F $T_{RQS}-T_o$	°C $T-T_{RQS}$	°C $T_{RQS}-T_o$	$(T-T_{RQS})$ C	$(T_{RQS}-T_o)+C$	$\frac{(T-T_{RQS})-C}{(T_{RQS}-T_o)+C}$
0.50	5.4	10.8	3	10.30	3.50	2.80
0.90	5.4	10.8	3	10.30	3.90	2.64
0.80	5.4	12.1	3	11.30	3.80	2.97
0.79	5.4	10.8	3	10.01	3.79	2.64
0.59	5.4	9.2	3	8.61	3.59	2.41
0.73	5.4	8.0	3	7.27	3.73	1.92
1.07	5.4	9.2	3	8.13	4.07	2.00
0.80	5.4	5.9	3	5.10	3.80	1.34
0.70	5.4	3.7	3	3.00	3.70	0.81
0.98	5.4	8.6	3	7.62	3.98	1.91
0.87	5.4	5.3	3	4.43	3.87	1.14
1.00	5.4	12.1	3	11.10	4.00	2.77
1.05	5.4	8.6	3	7.55	4.05	1.86
2.90	5.4	5.9	3	3.00	5.90	0.51
1.05	5.4	4.2	3	3.15	4.05	0.77
1.37	5.4	2.6	3	1.23	4.37	0.28
3.80	5.4	3.7	3	− 0.10	6.80	−0 −
2.49	5.4	5.3	3	2.81	5.49	0.51
4.50	5.4	8.0	3	3.50	7.50	0.46
0.68	5.4	13.7	3	13.02	3.68	3.54
1.27	5.4	12.6	3	11.33	4.27	2.65
10.01	5.4	5.9	3	− 4.11	13.01	− 0-

TABLE 2

DILUTION WATER REQUIREMENTS FOR THE BASINS OF THE UNITED STATES[1]

BASINS (NUMBER)	TOTAL WASTE WATER (M. G. D.)				
	1980			2000	
	LOW	MEDIUM	HIGH	LOW	MEDIUM
1 New England	8,911	12,219	21,507	10,893	18,819
2 Delaware Hudson	19,807	27,607	47,974	23,584	44,090
3 Eastern Great Lakes	11,354	15,862	27,628	13,612	25,708
4 Western Great Lakes	21,082	29,032	50,721	26,322	48,558
5 Chesapeake Bay	15,963	22,951	39,917	20,089	39,404
6 Ohio	19,121	26,922	47,225	23,409	44,418
7 Cumberland	40,515	57,119	100,406	47,845	90,737
8 Tennessee	155	239	364	589	1,096
9 Southeast	7,511	10,583	18,454	11,321	21,436
10 Upper Mississippi	12,126	16,664	29,452	14,265	26,072
11 Lower Mississippi	3,410	4,826	8,295	4,300	8,230
12 Upper Missouri	5,370	7,353	12,998	6,665	12,106
13 Lower Missouri	1,367	1,872	3,311	2,072	3,762
14 Upper Arkansas & Red	4,026	5,526	9,711	4,652	8,507
15 Lower Ark. Red, & White	2,978	4,026	7,067	3,352	6,208
16 Western Gulf	17,740	25,721	42,812	26,225	53,242
17 Rio Grande & Pecos	1,336	1,838	3,241	1,572	2,865
18 Colorado	2,629	3,598	6,398	4,480	8,133
19 Great Basin	1,366	1,887	3,346	1,572	2,895
20 Pacific Northwest	3,909	5,409	9,388	10,320	18,876
21 Central Pacific	10,181	13,926	24,542	14,380	26,092
22 South Pacific	8,298	11,330	19,852	10,565	19,093
TOTALS	219,155	306,510	534,669	282,084	531,347

1 Source Senate Select Committee and University of New Mexico Studies on Resources for the

2 Whenever heated water is to be dumped in the stream it is recommended to raise the weir to a

	$\frac{Q_o}{\Delta Q}$	DILUTION WATER REQUIREMENTS (M.G.D.)					
		1980			2000		
HIGH	(Eq 12)	LOW	MEDIUM	HIGH	LOW	MEDIUM	HIGH
44,627	2.80	24,950	34,213	60,219	30,500	55,493	124,955
99,083	2.64	52,290	72,882	126,651	62,261	116,397	261,579
58,210	2.97	33,721	47,110	82,055	40,427	76,352	172,883
109,190	2.64	55,656	76,644	133,903	69,490	128,193	288,261
90,437	2.41	38,470	55,311	96,199	48,414	94,963	217,953
101,141	1.92	36,712	51,690	90,729	44,945	85,282	194,190
206,972	2.00	81,030	114,238	200,812	95,690	181,474	413,944
2,459	1.34	207	320	487	789	1,468	3,295
48,799	0.81	7,511	10,583	18,454	11,321	21,436	48,799
58,965	1.91	23,160	31,828	56,253	27,246	49,797	112,623
18,397	1.14	3,887	5,501	9,456	4,902	9,382	20,972
27,286	2.77	14,874	20,367	36,004	18,462	33,533	75,582
8,479	1.86	2,542	3,481	6,158	3,853	6,997	15,770
19,193	0.51	4,026	5,526	9,771	4,652	8,507	19,193
14,012	0.77	2,978	4,026	7,067	3,352	6,208	14,012
119,442	0.28	17,740	25,721	42,812	26,225	53,242	119,442
6,425	- 0	1,336	1,838	3,241	1,572	2,865	6,425[2]
18,407	0.51	2,629	3,598	6,398	4,480	8,133	18,407
6,567	0.46	1,366	1,887	3,346	1,572	2,895	6,567
42,585	3.54	13,837	19,147	33,233	36,532	66,821	150,750
58,693	2.65	21,989	36,903	65,036	38,107	69,143	155,536
42,591	- 0	8,298	11,330	19,852	10,565	19,093	42,591[2]
1,201,960	35.93	449,209	634,144	1,108,136	585,357	1,097,674	2,483,729

Future.

minimum of 21 (total depth of water)

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Assessment of the Impact of Organic Discharges on the Dissolved Oxygen Resources of a Receiving Stream

by

L.W. Canter*

One of the most common problems of stream pollution is that resulting from oxygen deficiency caused by organic waste discharges. Dissolved oxygen in a stream is deficient when the actual concentration is less than the saturation concentration. Nemerow and Velz have recently published excellent books which describe the myriad considerations involved in stream pollution analysis (1,2). This discussion represents only a cursory presentation of dissolved oxygen relationships in streams.

I. Dissolved Oxygen

A. Dissolved oxygen is one of the primary chemical parameters used to describe water quality. Dissolved oxygen is basic to the maintenance and promotion of aquatic flora and fauna.

B. Water quality standards have been established for dissolved oxygen; some examples from Oklahoma standards are (3):

1. General standards

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a. The instream numerical criteria limits shall be maintained at all times with the exception of when the flow is equal to or less than the seven-day, two-year low flow value or times when the flow rate is not significant or discernable by the naked eye. The numerical criteria limits apply at all times to lakes and reservoirs unless otherwise exempted.

b. Dissolved Oxygen: The dissolved oxygen concentration shall not be less than 5 mg/l for all warm waters, and 6 mg/l for those waters designated as small-mouth bass or trout fisheries. Under extreme conditions, the diurnal variations may cause the dissolved oxygen concentration to be as much as 1 mg/l below the above values for short periods (not to exceed 8 hours) during any 24-hour period provided that the water quality is favorable in all other respects.

2. Special Standards - Return Flow Streams

Dissolved Oxygen: The dissolved oxygen content of a return flow stream shall not be less that 2 mg/l. The dissolved oxygen concentration just above the point where the flow of the stream combines with the flow of a stream of higher designated use shall not be less than 3 mg/l.

3. Special Standards - Mixing Zones and Zones of Passage

a. Mixing Zones: Except as indicated below, mixing zones shall be no larger than one-fourth (¼) the cross sectional area of the stream or no more than one-fourth (¼) the volume of flow, whichever is most restrictive. The remaining portion of the stream's cross section or flow shall constitute a zone of passage for free swimming and drifting organisms. Where more than one effluent enters a stream and the mixing zones would overlap, the combined mixing zones shall not exceed the one-fourth (¼) value described above. The mixing zone shall begin at the point of discharge and extend downstream to the point of complete mixing.

Special mixing zones shall be designated where return flows exceed one-fourth (¼) of the combined stream and return flows.

Mixing zones in lakes shall be designated on a case by case basis.

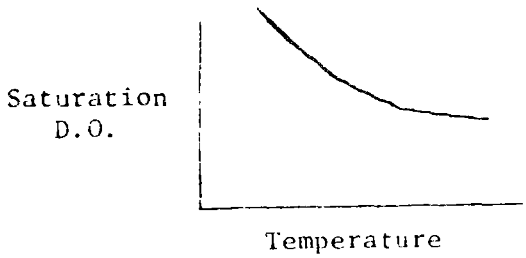
b. Dissolved Oxygen: The dissolved oxygen shall not be less than 2 mg/l within the mixing zone.

C. Saturation Dissolved Oxygen

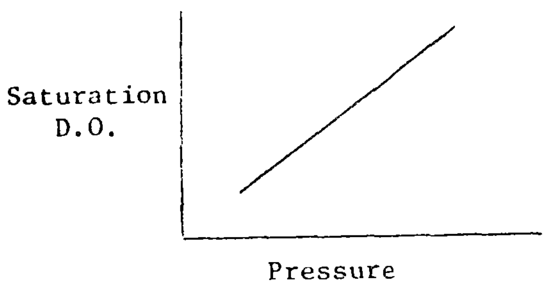
1. If water is saturated with dissolved oxygen, it means that the water contains 100% of the dissolved oxygen it could contain under specified conditions of temperature, pressure and salt content.

2

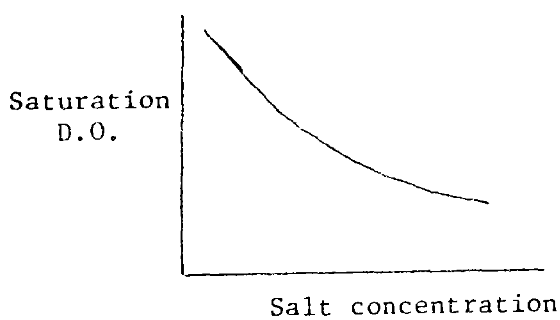
2. Influence of temperature on the saturation concentration of dissolved oxygen



3. Influence of pressure on the saturation concentration of oxygen



4. Influence of salt content on the saturation concentration of oxygen, mainly on conern in sea water.



5. The actual concentration of dissolved oxygen in polluted water is generally less than the saturation concentration.

Deficit = D = saturation concentration (Cs) - actual concentration

Example: Temperature of 25°C, normal atmospheric pressure and non-sea water; if the measured D.O. is 2.0 mg/l, what is the deficit?

C_s = 8.18 mg/l

D = 8.18 - 2.0 = 6.18 mg/l

II. Oxygen Relationships in Streams

A. Basic Forces

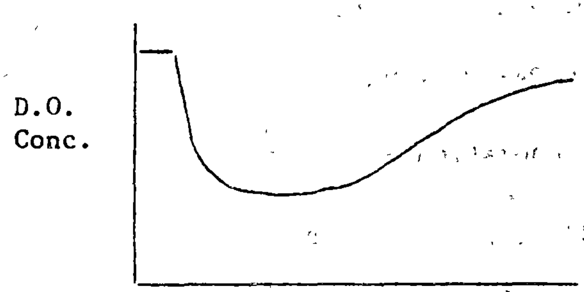
1. There is a demand for oxygen exerted by bacterial decomposition of organics; shown in simplified equation which follows:

$$\text{Organics} + O_2 \xrightarrow{\text{bacteria}} CO_2 + H_2O$$

2. There is a supply of oxygen from natural reaeration.

3. Other forces are oxygen demand from bottom deposits and oxygen supply from photosynthesis.

4. The basic forces of organic oxygen demand and natural reaeration yield what is called an oxygen sag curve.



B. Streeter-Phelps Equation

1. Originally developed in 1925.

2. Assumptions for oxygen sag curve

2

a. BOD decrease dye only to bacterial oxidation

b. No benthal O_2 demand

c. No photosyntnetic effect

d. Reoxygenation by reaeration

3. Basic differential equation

$$\frac{dD}{dt} = \text{oxygen demand} - \text{oxygen supply}$$

$$= K_1L - K_2D$$

where

K_1 = rate of oxygen use by bacteria

L = biochemical oxygen demand

K_2 = rate of reaeration

4. Solution of the basic differential equation yields the following:

$$D = \frac{K_1La}{K_2-K_1}\left(10^{-K_1t} - 10^{-K_2t}\right) + Da\ 10^{-K_2t}$$

D = D.O. deficit at time t

= sat. conc. - actual conc.

K_1 = coef. of deoxygenation

K_2 = coef. of reaeration

La = initial BOD (ultimate) (in stream)

Da = initial deficit (in stream)

5. K_1 = coef. of deoxygenation

determined in BOD test at 20°C

$$K_{1(T)} = K_{1(20)}\ (1.047)^{T-20}$$

T = temp. (°C)

6. $K_2 = \phi$(surface exposure, depth, turbulence, velocity)

- Tables or Calculations -

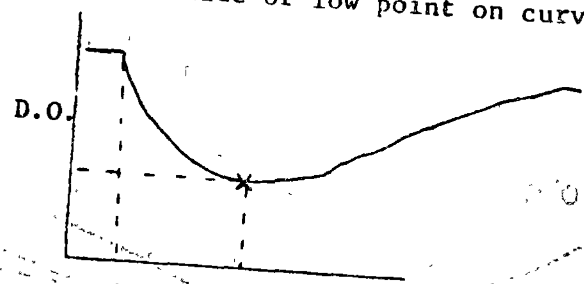
$$K_{2(T)} = K_{2(20)} \ (1.016)^{T-20}$$

7. $La_{(T)} = La_{(20)} \ (0.02T + 0.6)$

8. Influence of temperature on the oxygen sag curve (see Fig.1)

9. Critical Conditions

Location and value of low point on curve.



a. Critical time

$$t_c = \frac{1}{K_2-K_1} \log_{10}\left(\frac{K_1La-K_2Da+K_1Da}{K_1La} \frac{K_2}{K_1}\right)$$

b. Critical deficit

$$Dc = \frac{K_1}{K_2} La \ 10^{-K_1tc}$$

10. Maximum Permissable BOD Load

Rearrange Streeter-Phelps as follows:

$$\log La = \log Dc + \left[1 + \frac{K_1}{K_2-K_1}\left(1 - \frac{Da}{Dc}\right)^{0.418}\right] \log \frac{K_2}{K_1}$$

La = ultimate BOD (mg/l)

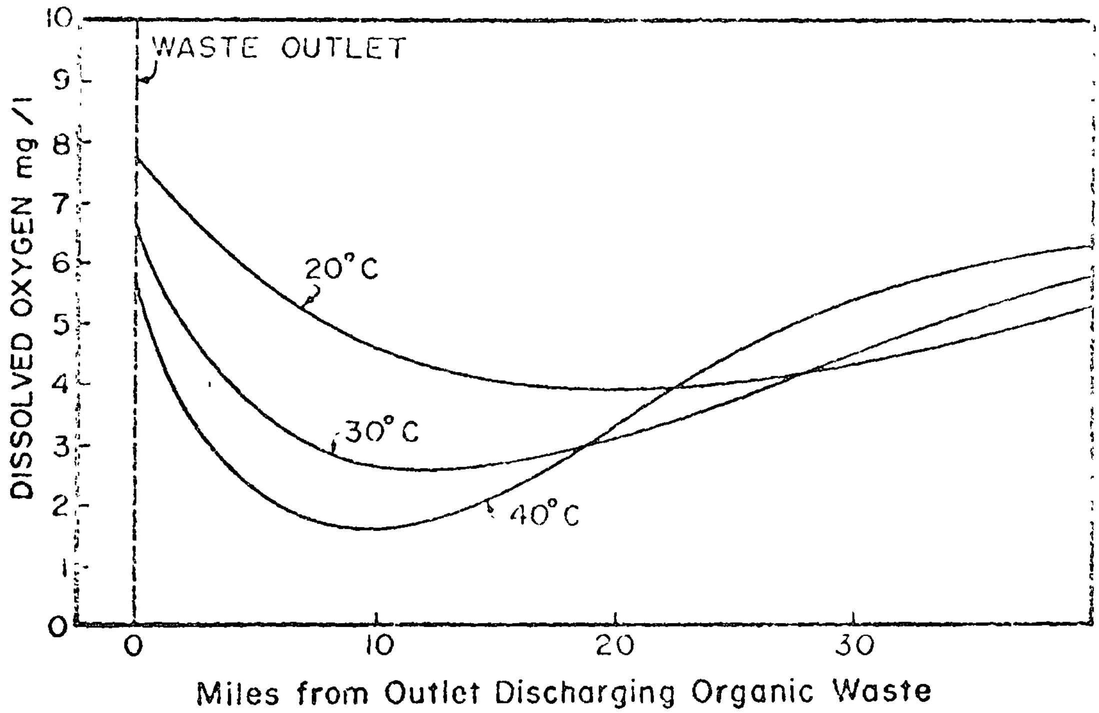
Dc = allowable deficit = saturation DO conc. - required DO conc. in standard

Da = initial deficit

FIGURE 1 (26)

RELATION BETWEEN TEMPERATURE AND OXYGEN PROFILE

(After La Berge)



K_1 = BOD reaction rate

K_2 = reaeration rate

C. Thomas modification of Streeter-Phelps equation

1. It is known that stream deoxygenation (Kd) values differ from K_1.

2. Deoxygenation is stream by oxidation.

$$\frac{dD}{dt} = K_D Lt$$

D = oxygen deficit

Kd = coef. of deoxygenation, not equal to K_1 in BOD bottles in lab, includes only deoxygenation by oxidation in stream

Lt = oxidizable organic matter remaining at any time t

$= La\ 10^{-K_1 t}$

La = initial oxidizable organic matter at t=o (in stream)

K_1 = rate of oxidation from laboratory

t = time

3. Combining above the rate of deoxygenation may be expressed as follows:

$$\frac{dD}{dt} = K_D L_a\ 10^{-K_1 t}$$

This equation expresses the rate of deoxygenation in terms of the coefficient of deoxygenation (K_D) and the laboratory rate of oxidation (K_1).

4. The practical considerations to be answered not is whether the rate of oxidation determined in the laboratory (K_1) is identical to the rate of organic (BOD) removal determined in the actual receiving water.

K_r = rate of BOD removal in the stream itself

$K_r = K_3 + K_1$

If the answer is negative then the above equation should be expressed as:

$$\frac{dD}{dt} = K_D \, La \, 10^{-K_r t}$$

5. Environmental factors that affect the rate of oxidation

Condition	Probable Relationship between K_r and K_1
a) Turbulence	$K_r > K_1$ (BOD removal in stream>BOD removal in bottle)
b) Biological Growth on Stream bed	$K_r > K_1$
c) Immediate O_2 demand	$K_r > K_1$
d) Nutrient deficiency in stream	$K_r < K_1$
e) Toxic conditions	$K_r < K_1$

6. Environmental factors affecting the rate of organic material removal in the receiving stream but not necessarily the rate of oxidation.

Condition	Probable Relationship between K_r and K_1
Sedimentation and Flocculation taking away	$K_r > K_1$
Scour adding	$K_r < K_1$
Volatilization taking away	$K_r > K_1$

7. Overall relationship between K_r, K_d and K_1

a. If the reactions in the stream were identical to the reactions in the laboratory then

$K_1 = K_r = K_D$

b. If BOD removal in the stream was by oxidation only but the rate was increased by turbulence or slime growths on the stream bottom then it may be assumed that

$K_r > K_1$ but $K_D = K_r$

c. If BOD removal in the stream was accomplished by a combination of oxidation and sedimentation, scour, volatilization then it may be assumed that

$K_r > K_1$ but $K_D = K_1$

8. Thomas equ integrates to

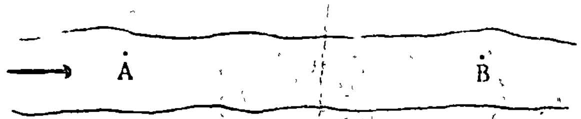
$$D = \frac{K_D La}{K_2 - K_r} \left[10^{-K_r t} - 10^{-K_2 t} \right] + Da10^{-K_2 t}$$

9. Determine K_r

$$K_r = \frac{1}{t} \log \frac{L_A}{L_B}$$

K_r is rate of BOD removal, not necessarily just oxidation.

t = time of flow between stations A and B.



L_A L_B

L_A = BOD loading at sta. A (lb/day)

L_B = " " " B (")

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2. Velz, Clarence J., Applied Stream Sanitation, Wiley-Interscience, John Wiley and Sons, Inc., New York, 1970, 619 pages.

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PROGRAMACION Y MODELOS DE INGENIERIA AMBIENTAL

Método de Diferencias para la Resolución de Ecuaciones Trascendentales

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METODO DE DIFERENCIAS PARA LA RESOLUCION DE ECUACIONES TRASCENDENTES

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Contenido:

PONENCIA

METODO DE DIFERENCIAS PARA LA RESOLUCION DE ECUACIONES TRASCENDENTES

Por el Dr. Ubaldo Bonilla D.

0.- RESUMEN

En diversas ramas de la ingeniería, economía, etc., el uso de ecuaciones trascendentes como modelos que deben ajustarse a conjuntos de datos relativos a dos o más variables, es muy común. El problema consiste en determinar los parámetros del modelo a partir de los datos. Cuando el modelo puede transformarse, por medio de funciones simples, a un modelo lineal cuyos parámetros son combinaciones de los parámetros del modelo original, el problema se reduce a un problema convencional de estadística; tal es el caso por ejemplo - del modelo geométrico de crecimiento de población. En muchos cssos, sin embargo, este tipo de transformaciones conduce a modelos lineales en los cuales los parámetros aparecen dentro de argumentos que son combinaciones de los mismos parámetros y una o más variables, lo cual hace prácticamente imposible la resolución matemática del problema; tales son los casos, por ejemplo, del modelo logístico de crecimiento de población, y de la ecuación de Theis para flujo hidráulico insta

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ble hacia pozos. En este trabajo, se presenta un método de diferencias, matemáticamente exacto, que se aplica en forma general a la resolución de ecuaciones trascendentes.

METODO DE DIFERENCIAS PARA LA RESOLUCION DE ECUACIONES TRASCENDENTES

1.- Conceptos generales

Puede considerarse que un modelo es la representación cuantitativa o matemática de un fenómeno. A un fenómeno particular pueden corresponder varios modelos, y el mismo modelo puede ser aplicado para representar diversos fenómenos; por ejemplo, la descomposición anaeróbica de la materia orgánica puede ser descrita por un modelo logístico o por un modelo autocatalítico, y el modelo logístico puede ser usado, además, para describir el crecimiento de diversos tipos de población.

Los parámetros de un modelo son las constantes envueltas en la relación matemática que describe a un fenómeno en particular. Las variables del modelo representan las distintas entidades del fenómeno y el modelo al fenómeno en total.

En diversas ramas de la ciencia se presenta frecuentemente el problema de determinación de los parámetros de un modelo o ecuación matemática. Por ejemplo, en química puede ser de interés determinar la rapidez con que procede una cierta reacción; en física resulta a veces necesario determinar la rapidez con que un gas se disuelve en agua; en demografía puede ser requerido determinar el límite de saturación de una población en crecimiento; y en un programa de conservación de recursos puede ser necesario determinar los parámetros que rigen la autopurificación de una cierta corriente.

En todos los ejemplos citados, así como en muchos otros casos, la determinación de los parámetros del modelo usado para representar cada fenómeno en particular, se basa en conjuntos de observaciones de laboratorio o de campo, según sea el caso.

Para determinar los parámetros de un modelo, es práctica generalizada, cuando resulta posible, linearizar el modelo por medio de transformaciones funcionales simples; en tal caso, los parámetros de la forma lineal resultante pueden ser estimados, para obtener de ellos los parámetros del modelo original. Por ejemplo, en el caso del modelo geométrico

$$y = a \exp. (bx) \qquad (1)$$

la linearización se efectúa tomando logaritmos para obtener

$$\ln y = \ln a + b x \qquad (2)$$

La aplicación del método de mínimos cuadrados o cualquier otro método gráfico o analítico al conjunto de valores (x, ln y) observados, permite determinar los valores de lna y b. El valor de a se obtiene, obviamente, tomando el antilogaritmo de ln a.

En muchos casos, sin embargo, la forma linearizada del modelo no presenta los parámetros en forma explícita. Por ejemplo si en la ecuación de primer orden

$$y = a\,(1 - e^{-bx}) \tag{3}$$

se toman logaritmos, se obtiene

$$\ln\,(a - y) = \ln a - b\,x \tag{4}$$

resultando imposible determinar los parámetros ln a, b, a menos que se proceda por tanteos.

En este trabajo, se denominan ecuaciones trascendentes, a aquellas que presentan sus parámetros envueltos en formas exponenciales, logarítmicas o trigonométricas, y trascendentes explícitas a aquellas en que parámetros de la forma lineal correspondientes son explícitas (como en el caso de la ecuación 1), e implícitas en caso contrario (como en el caso de la ecuación 2).

La determinación de los parámetros de algunas ecuaciones trascendentes implícitas se ha logrado mediante la aplicación de métodos especiales, siendo una característica general de estos el uso, en alguna etapa del proceso de cálculo, de gráficas o escalas especialmente diseñadas. Obviamente, estos métodos carecen de generalidad y solo pueden aplicarse a los modelos para los cuales han sido desarrollados, y además están limitados por los rangos cubiertos en las gráficas correspondientes. Entre estos métodos son dignos de citarse el de Thomas-Snow[1] para resolver los parámetros de la ecuación de la reacción de primer orden; el de Verhulst[1], correspondiente al modelo logístico; y el de C. V. Theis[3], usado para determinar las constantes de formación de acuíferos en flujo inestable.

En este trabajo, se presenta un método de diferencias finitas, matemáticamente exacto, que puede ser aplicado en forma general a la determinación de parámetros de modelos trascendentes. Para su presentación se han seleccionado dos modelos; el de la reacción de primer orden, y el de flujo de agua a pozos en régimen inestable, ya que para ellos existen métodos especiales que permiten la comparación de resultados, tanto en la precisión como en el grado de dificultad, con respecto al método de diferencias. En la referencia 4 aparece un buen número de ejemplos de aplicación del método de diferencias.

2.- Determinación de los parámetros del modelo lineal

Una vez que una ecuación trascendente ha sido reducida a su forma lineal, los parámetros de ésta deben ser determinados. Para ello existen diversos métodos que se aplican de acuerdo con la precisión deseada y la naturaleza de los datos disponibles.

El método de mínimos cuadrados es el más favorecido debido a sus múltiples ventajas estadísticas. Si el modelo lineal está dado por

$$Y = I + P X \tag{5}$$

los estimadores de P e I están dados por

$$P = \frac{n \Sigma XY - \Sigma X \Sigma Y}{n \Sigma X^2 - (\Sigma X)^2} \tag{6}$$

$$I = \frac{\Sigma Y}{n} - P \frac{\Sigma X}{n} \tag{7}$$

donde:

X, Y - variables

n - número de parejas de datos

I - intersección al origen

P - pendiente

El método de promedios consiste simplemente en dividir el conjunto de n parejas de datos en dos grupos conteniendo k y n-k datos respectivamente, para obtener dos ecuaciones simultáneas

$$\sum_{1}^{k} Y = k\, I + P \sum_{1}^{k} X \qquad (8)$$

$$\sum_{n-k}^{n} Y = (n - k)\, I + P \sum_{n-k}^{n} X \qquad (9)$$

cuya resolución permite determinar los valores de P e I.

El método gráfico es el método más usado en la resolución de problemas de ingeniería. El procedimiento consiste en graficar los datos, y en pasar una recta por el promedio $(\bar{X}, \bar{Y})$, de manera que se obtenga, a ojo, el mejor ajuste. Los valores de P e I se miden directamente en la gráfica.

3.- Reacción de primer orden

Este modelo corresponde a la ecuación

$$\frac{dy}{dx} = k\,(L - y) \qquad (10)$$

expresión que integrada considerando y = 0 para x = 0 conduce a:

$$y = L\,(1 - \exp.\,k\,x) \qquad (11)$$

Este modelo tiene múltiples aplicaciones en el estudio de diversos fenómenos, entre los que pueden mencionarse la demanda bioquímica de oxígeno en la estabilización aeróbica de la materia orgánica, la rapidez de estabilización de embalses de agua, procesos radiológicos, crecimiento limitado de población, etc.

Método de momentos de Thomas - Snow

Este método se basa en el uso de gráficas que relacionan los primeros dos momentos de la ecuación 11 con los parámetros k y L. En una gráfica aparece una serie de curvas $(\Sigma y/\Sigma yx)$ vs. k y en la otra, una familia de curvas $\Sigma (y/L)$ vs. k.

Después de calcular los primeros dos momentos del conjunto de pares de datos, k se determina usando la primera gráfica, y usando este valor se determina $\Sigma\, y/L$ mediante la segunda gráfica, y de aquí, finalmente L. Obviamente cada familia de curvas en las gráficas corresponde a una secuencia determinada de valores de x, a un cierto rango de valores de los parámetros, y al sistema de unidades inherente al fenómeno bajo estudio; estos inconvenientes limitan el uso de las gráficas a fenómenos específicos. Su aplicación se ha hecho exclusivamente al estudio de la demanda bioquímica de oxígeno en muestras de agua, y en otros casos se sigue usando el método de tanteos para resolver la forma lineal logarítmica (ecuación 4) del modelo.

4.- Ecuación de Theis

La ecuación que muestra la relación entre el abatimiento de los niveles h, de la superficie piezométrica de un pozo, a una distancia T de su centro y al tiempo x después de iniciado el bombeo, está dado por la ecuación diferencial parcial:

$$\frac{\partial^2 h}{\partial r^2} + \frac{1}{r}\frac{\partial h}{\partial r} = \frac{S}{T}\frac{\partial h}{\partial x} \qquad (12)$$

C. V. Theis desarrolló una solución a esta ecuación, basada en una analogía con el flujo de calor a un sumidero. Esta solución se expresa:

$$h_o - h = \frac{Q}{4\,\pi T}\left(-0.5772 - l_n\, u + u - \frac{u^2}{2.2!} + \frac{u^3}{3.3!} - \dots\right) \qquad (13)$$

$$u = \frac{r^2 S'}{4\,T\,x} \qquad (14)$$

donde Q es el gasto de bombeo en el pozo, T es un parámetro denominado coeficiente de transmisibilidad, y S' es un parámetro denominado coeficiente de almacenamiento. El coeficiente de transmisibilidad representa la cantidad de agua que pueda fluir por unidad de ancho de acuífero por unidad de tiempo, cuando el gradiente piezométrico es unitario, y el coeficiente de almacenamiento la cantidad de agua liberada por una columna de acuífero de área recta unitaria, cuando el nivel piezométrico desciende una unidad.

Método de superposición de Theis

Se han desarrollado diversos métodos para determinar los parámetros de la ecuación de Theis. Se considera que de entre ellos el método de superposición debido al propio Theis es el más exacto.

Llamando W(u) a la serie infinita escrita dentro del paréntesis de la ecuación 13, y haciendo Q/4 T = C_1, se obtiene:

$$h_0 - h = C_1 W(u) \qquad (15)$$

Haciendo S'/4 T = C_2, la ecuación 14 puede escribirse

$$u = C_2 \frac{r^2}{x} \qquad (16)$$

Tomando logaritmos, en las ecuaciones 15 y 16, sumando y rearreglando, queda la expresión

$$\ln u - \ln (W(u) = \ln C_1 C_2 + \ln \frac{r^2}{x} - \ln (h - h_o) \qquad (17)$$

ecuación en la que se observa que las curvas resultantes de gráficas U vs W(u) y r^2/x vs $(h - h_o)$ en papel log-log, tienen exactamente la misma forma.

Para resolver el problema particular, la curva típica (u, W(u)) dibujada en papel transparente, se superpone a la curva $(r^2/x, (h - h_o))$, tratando de hacer coincidir tantos puntos de ambas curvas como sea posible ó, tomando un punto común a ambas curvas, se leen los valores r^2/x, $h - h_o$, u, y W(u) en las escalas respectivas; la substitución de estos valores en las ecuaciones 13 y 14 permiten determinar los valores de T y S'.

5.- Método de diferencias

Se sabe por el cálculo elemental, que para cualquier función

$$y = f (x) \qquad (17)$$

a cada incremento Δx de la variable independiente corresponde un incremento Δy en la variable dependiente, que los valores asumidos por Δx dependen solamente del dominio de la función y no del orden de magnitud, y que f(x) puede ser cualquier clase de función, continua o discreta.

En base a lo anterior, el método de diferencias para la determinación de parámetros de ecuaciones trascendentes consiste simplemente en

- asignar un incremento a la variable dependiente para obtener la forma

$$f(x) + \Delta y = f(x + \Delta x) \qquad (18)$$

- establecer el tipo de incremento que debe darse a una de las variables, por ejemplo Δx = const., $x = kn$, etc., para que una relación entre las expresiones 17 y 18 resulte en una ecuación con parámetros explícitos.

5.1.- Reacción de primer orden

La ecuación de la reacción de primer orden puede escribirse

$$L - y_x = L \exp. (-kx) \qquad (19)$$

Dando incrementos Δx, Δy en la expresión anterior se obtiene:

$$L - Y_{x + \Delta x} = L \exp(-k(x + \Delta x)) \qquad (20)$$

Dividiendo la ecuación 20 entre la ecuación (19) da como resultado

$$(L - Y_{x + \Delta x}) / (L - Y_x) = \exp. (-k \Delta x)$$

de donde se obtiene finalmente:

$$Y_{x+\Delta x} = L\,(1 - \exp(-k\,\Delta x)) + \exp(-k\Delta x)\,Y_x \qquad (21)$$

que es la ecuación de una recta aritmética cuando Δx se toma constante.

5.2.- Ecuación de Theis

La ecuación 13, de Theis, puede escribirse

$$h_o - h = a\left(-0.5772 - \ln\frac{c}{x} + \frac{c}{x} - \frac{c^2}{2.2!\,x^2} + \ldots\right) \qquad (22)$$

donde:

$$a = \frac{Q}{4\pi T} \qquad (23)$$

$$c = \frac{r^2\,S'}{4\,T} \qquad (24)$$

dando incrementos a x y h, de la ecuación 22 se obtiene:

$$h_o - (h + \Delta h) = a\left(-0.5772 - \ln\frac{c}{x+\Delta x} + \frac{c}{x+\Delta x} - \frac{c^2}{2.2!(x+\Delta x)^2} + \ldots\right) \qquad (25)$$

Substrayendo la ecuación 25 de la ecuación 22 se obtiene

$$\Delta h = a\left(\ln\frac{x+\Delta x}{x} + \frac{c}{x+\Delta x} - \frac{c}{x} - \frac{c^2}{2.2!(x+\Delta x)^2} + \frac{c^2}{2.2!\,\Delta x^2} + \ldots\right)$$

de manera que, haciendo $\Delta x = x$

$$\Delta h = a \ln 2 - \frac{c\,a}{2}\;\frac{1}{x} + \frac{3\,c^{a}\,a}{2.2!\;4}\;\frac{1}{x^{2}} + \cdots \qquad (26)$$

Si "u" es pequeño, como es generalmente el caso, que es u a-forma lineal,

$$\Delta h = a \ln 2 - \frac{c\,a}{2}\;\frac{1}{x} \qquad (27)$$

que es la ecuación de una recta en el plano cuando se consideran las parejas $(1/x, \Delta h)$.

6.- Ejemplos

Los ejemplos que se presentan en seguida, están tomados de la bibliografía que aparece al final de este trabajo. El propósito es comparar los resultados obtenidos de la aplicación de los métodos tradicionales con los que se obtienen cuando se aplica el método de diferencias. El método más eficiente en la resolución de un problema particular, es aquel que muestra la menor suma de errores estadísticos

$$V = \Sigma\,(y - y')^{2} \qquad (28)$$

donde y representa a valores observados y "y'" los valores calculados correspondientes.

Cada uno de los ejemplos se presenta resuelto en dos formas, gráficamente para demostrar la simplicidad del método de diferencias, y mediante la técnica de mínimos cuadrados, con objeto de comparación de resultados.

Las operaciones analíticas se efectuaron con una calculadora Hewlet-Packard HP 35 sin efectuar redondeo de cifras durante el proceso, por lo que al leer los cuadros de cálculos se ruega al lector tener esto en cuenta.

6.1.- Ecuación de primer orden

Dadas las lecturas de demanda bioquímica de oxígeno (D. B. O) obtenidas en una muestra de aguas negras, determinar los parámetros k y L de la ecuación 11, usada como modelo para representar este fenómeno.

Tiempo x, días	0	1	2	3	4	5
DBO y, mg/l	0	82	112	153	163	176

Solución gráfica:

De acuerdo con la ecuación 21, se grafican las parejas (0, 82), (82, 112), (112, 153), (153, 163), (163, 176).

En la figura 1 aparecen estos puntos y la recta de ajuste que se hace pasar por el promedio. Mediante medición directa se obtiene:

pendiente P = 0.571

Intersección al origen I = 79

De la ecuación 21 se obtiene, ya que $\Delta x = 1$

$$k = -\ln P \qquad (29)$$

$$= I/(1 - P) \qquad (30)$$

Por tanto:

$$k = -\ln 0.571 \qquad k = 0.560 \text{ días}^{-1}$$

$$L = 79/(1-0.579) \qquad L = 184 \text{ mg/l}$$

Mínimos cuadrados:

Las operaciones para determinar P e I por el método de mínimos se muestran en la tabla 1. La substitución de los resultados que aparecen en el renglón de sumas, en las fórmulas 6 y 7 conducen a:

$$P = \frac{5(79947) - (510)(686)}{5(69246) - (510)^2} \; ; \; P = 0.579$$

$$I = \frac{686}{5} - \frac{510}{5}(0.579) \; ; \; I = 78.1$$

Finalmente, aplicando las ecuaciones 29 y 30, se obtiene

$$k = -\ln 0.579 \; ; \qquad k = 0.543 \text{ días}^{-1}$$

$$L = 78.1/(1 - 0.579) \qquad L = 185.6 \text{ mg/l}$$

Comparación de resultados

En la tabla 2 aparecen los cálculos para determinar la suma de errores cuadráticos tanto si se usan los parámetros obtenidos con el método de diferencias y mínimos cuadrados ($k = 0.543$, $L = 185.6$) como usando el método tradicional de momentos ($k = 0.523$, $L = 188$) según se reportan en la referencia 1.

Puede observarse que el método de diferencias propuesto resulta más eficiente en la solución de este problema en particular.

5.2.- Ecuación de Theis

Los abatimientos del nivel piezométrico en un pozo de observación situado a 200 ft. de un pozo bombeado a razón de 500 gal/min, así como los tiempos contados a partir de la iniciación del bombeo, aparecen en las dos primeras columnas del cuadro 3. Determinar los valores de las constantes de formación T y S'.

Solución gráfica:

En la columna 1 de la tabla 3 se observa que los datos de tiempo satisfacen la relación $\Delta x = x$. Por lo tanto puede aplicarse la ecuación 27, graficando los valores $1/x$ (columna 3) contra Δh (columna 4). Obsérvese que a partir de t = 8 min los datos carecen de precisión, por lo cual se desechan. En la figura 2 aparecen graficadas las parejas (1, 0.33), (0.5, 0.37), (0.25, 0.39) y (0.33, 0.38). La recta de ajuste pasa por el punto medio de estos valores. De esta figura resulta, por medición directa:

$$P = -0.0795 \text{ ft} - \text{min}$$

$$I = 0.409 \text{ ft.}$$

De las fórmulas 23, 24 y 27 se obtiene:

$$T = 0.055 \frac{Q}{I} \quad (31)$$

$$S' = -5.544 \frac{TP}{Ir^2} \quad (32)$$

De manera que:

$$T = 0.055 \quad \frac{500 \quad \text{gal} \quad 60(24)\text{min}}{0.409 \quad \text{min} \quad \text{ft} \quad \text{día}}$$

$$T = 97250 \text{ gal/ft-día}$$

$$S' = -5.544 \quad \frac{97250\ (-0.0795)}{0.409\ (200)^2 (7.48)(60)(24)}$$

$$S' = 2.432 \times 10^{-4}$$

Mínimos cuadrados:

En el cuadro 3 se muestran las operaciones para determinar los argumentos necesarios para calcular P e I usando las fórmulas 6 y 7, de donde se obtiene:

$$P = \frac{4\ (0.7392) - 2.0833\ (1.47)}{4\ (1.4236) - (2.0833)^2} \quad ; \quad P = -\ 0.780 \text{ ft-min}$$

$$I = \frac{1.47}{4} + (0.0780)\ \frac{2.0833}{4} \quad ; \quad I = 0.4081 \text{ ft.}$$

Aplicando las fórmulas 31 y 32 se obtiene:

$$T = 0.055\ \frac{500\ (24)(60)}{0.4081} \quad ; \quad T = 97030 \text{ gal/ft-día}$$

$$S' = 5.544\ \frac{97030\ (0.078)}{0.4081\ (200)^2\ (7.48)(24)(60)}$$

$$S' = 2.386 \times 10^{-4}$$

Comparación de resultados

En la tabla 4 aparecen los cálculos para determinar la suma de errores cuadráticos usando los parámetros obtenidos con el método de diferencias (T = 97,030 gal/ft-día, S' = 2.386), y la suma de errores cuadráticos usando los parámetros reportados en la referencia 3, (T = 103000 gal/ft-día, $S' = 1.98 \times 10^{-4}$).

Obsérvese que al método de diferencias corresponde la menor suma de errores cuadráticos.

7.- Ventajas del método de diferencias

Las principales ventajas del método de diferencias sobre los métodos tradicionales usados para estimar parámetros de -

ecuaciones trascendentes implícitas, derivan de la precisión y simplicidad de su base matemática, esto es, del concepto de modelo lineal que permite soluciones analíticas, o soluciones gráficas simples usando escalas aritméticas. Este hecho permite completa flexibilidad en la selección del método de solución al modelo lineal, así como en los medos de cálculo que se prefieran (calculadora, computadora, etc.)

Una ventaja definitiva del método de diferencias es la posibilidad inmediata de aplicar la teoría estadística a los estimadores obtenidos de esta manera. Por ejemplo, se pueden obtener fácilmente límites de confianza para los valores de "k" en casos similares al problema de D.B.O presentado en este trabajo. También, es interesante señalar que el método de diferencias vence las restricciones impuestas a los métodos tradicionales en relación con el sistema de unidades usado en la solución de un problema particular y con el rango de variación de los parámetros envueltos.

El método de diferencias permite, en muchos casos, determinar si un modelo preasignado corresponde a una serie de datos. Por ejemplo, si una serie de datos que pertenecen a una serie de observaciones de D. B. O se grafican de acuerdo con la forma lineal 21 y la curva resultante no es una recta, evidentemente ocurre una de dos situaciones: o los datos no se ajustan al fenómeno (aguas negras frescas o muy viejas), o los experimentos de laboratorio no se efectúan correctamente.

Un problema encontrado frecuentemente consiste en determinar cuantos datos son necesarios y con qué periodicidad deben colectarse para resolver un problema particular. La primera parte de esta cuestión depende de la precisión deseada y es resuelto por la teoría estadística cuya aplicación es favorecida cuando se usa el método de diferencias. La segunda parte depende del modelo en si mismo; por ejemplo, si se usa el método de diferencias para determinar las constantes de formación de un acuífero, resulta evidente que es antieconómico medir niveles de la superficie piezométrica cuando no pueda detectarse ninguna diferencia entre dos lecturas consecutivas. Los intervalos entre mediciones no deben ser necesariamente constantes, sino seguir el patrón dictado por el propio modelo. Los datos son más valiosos en donde la curvatura del modelo cambia más rápidamente. La práctica de tratar de obtener datos en largos periodos de tiempo, además de ser an tieconómica puede introducir errores en la determinación de los parámetros del modelo correspondiente.

Finalmente, como se observa en los ejemplos presentados, el método de diferencias es más eficiente, tiene un nivel de dificultad más bajo que los métodos tradicionales, y sobre todo, tiene naturaleza general.

8.-Figuras.

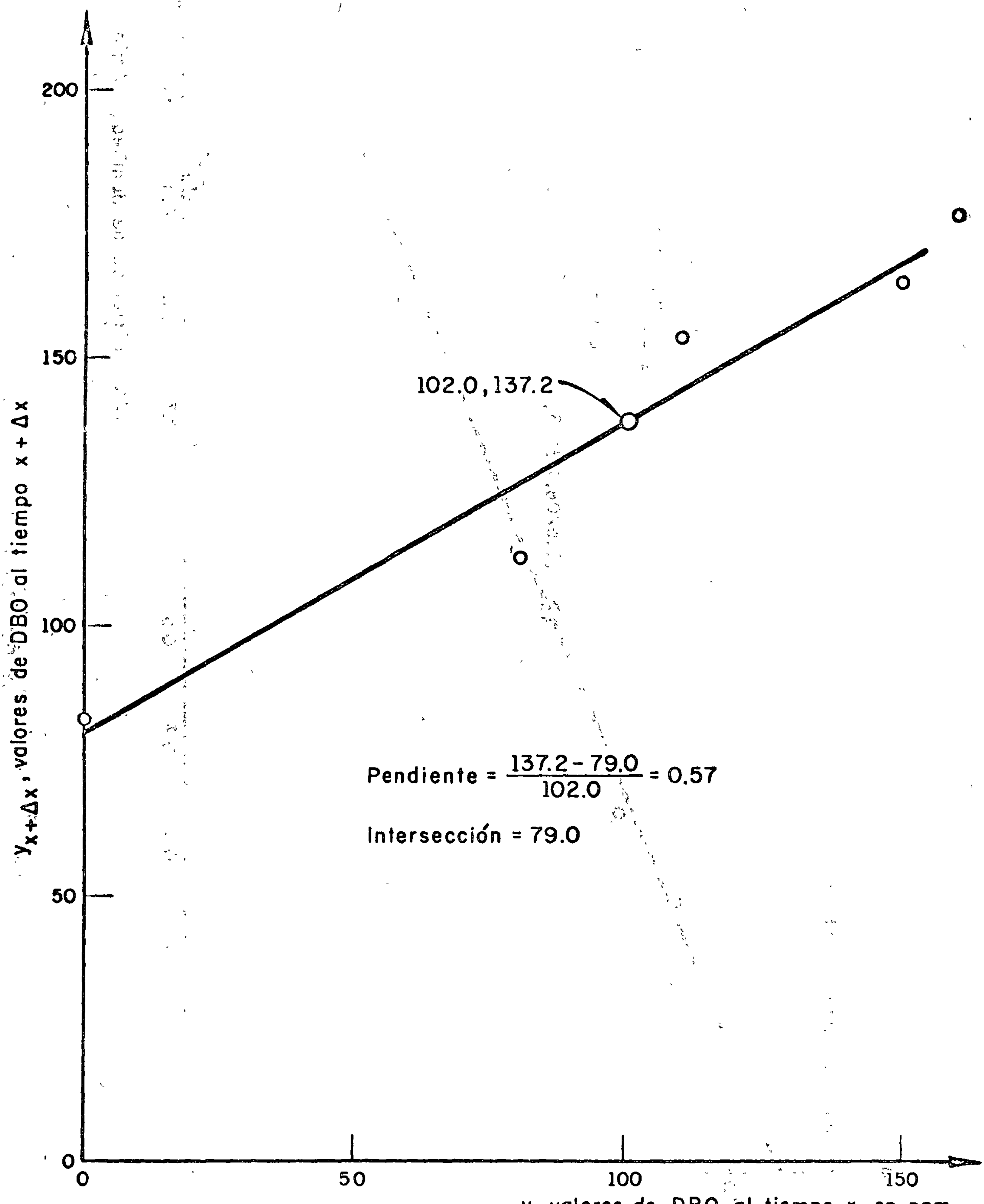

Fig 1. Determinación de los parámetros de la ecuación de la D.B.O

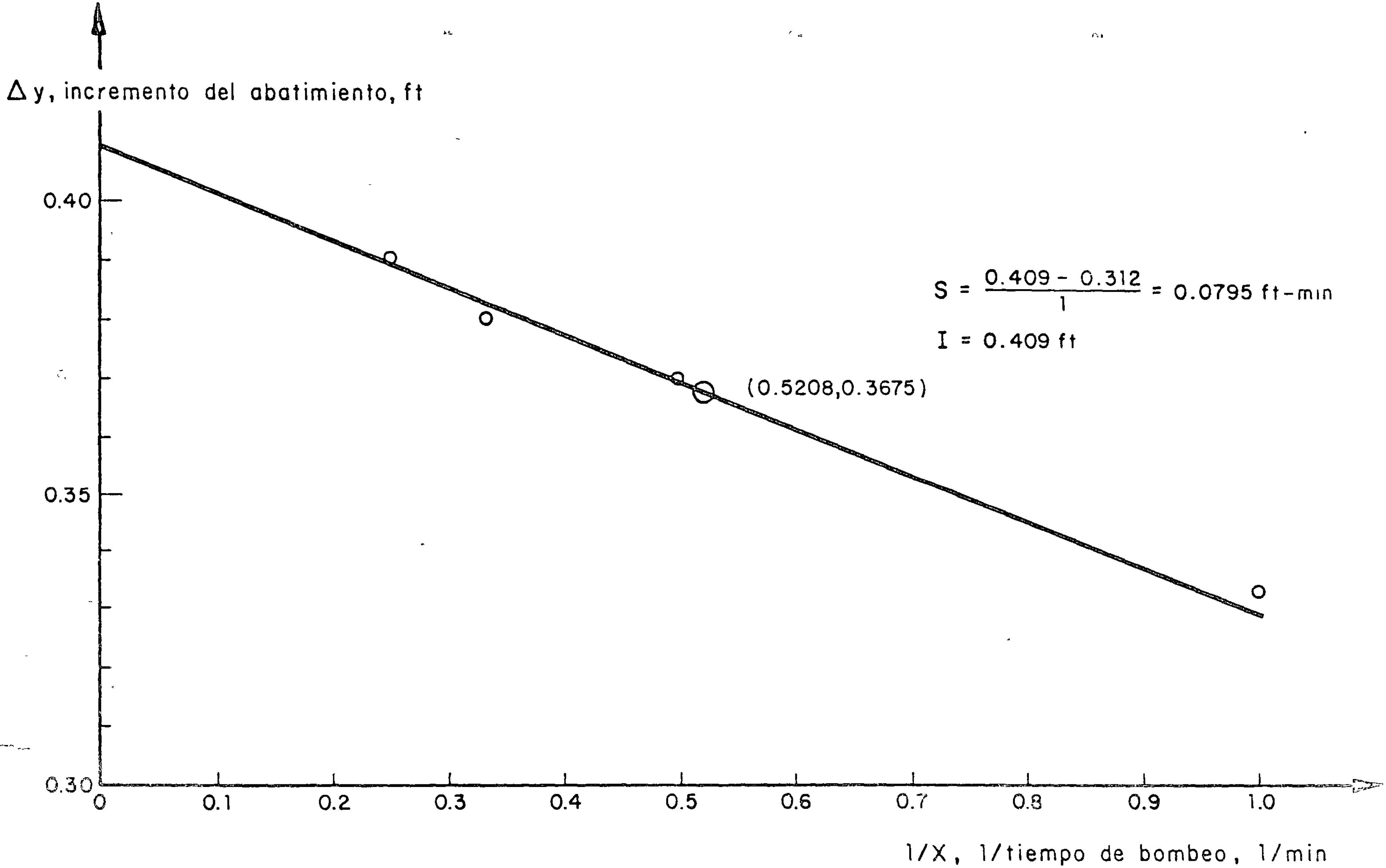

Fig. 2 Determinación de las constantes de formación en flujos a pozos

9.-Tablas.

TABLA 1.

SOLUCION A LA ECUACION DE DB.O

Datos		Cálculos		
Tiempo, Días (1); x	BOD, mg/l (2); y=X	(3); $y_{x+\Delta x}$ =Y	(4); XY	(5); X^2
0	0	82	0	0
1	82	112	9184	6724
2	112	153	17136	12544
3	153	163	24939	23409
4	163	176	28688	26569
5	176	-	-	-
Σ	510	686	79947	69246

TABLA 2.

EFICIENCIAS DEL METODO DE DIFERENCIA Y DEL METODO DE MOMENTOS: ECUACION DE LA DBO.

Datos		Metodos de Diferencias			Metodos de Momentos		
(1), x	(2), y Observedos	(3), y' Calc.	(4), (y-y') Error	(5), $(y-y')^2$ $Error^2$	(6), y' Calc.	(7), (y-y') Error	(8), (y-y') Error
0	0	0	0	0	0	0	0
1	82	78	14	196	77	15	225
2	112	123	-11	121	122	-10	100
3	153	149	4	16	149	4	16
4	163	164	-1	1	165	-2	4
5	176	174	2	4	174	2	4
Variancia		-	-	338	-	-	349

TABLA 3.

CALCULOS PARA LA SOLUCION DE LA ECUACION DE THEIS.

Datos		Cálculos			
Tiempo, min. (1), x	abatimiento, ft. (2), (h_0-h), y	(3), $\frac{1}{x}=X$	(4), $\Delta y=Y$	(5), XY	(6), X^2
1	0.66	1	0.33	0.[illegible]3	1
2	0.99	0.5	0.37	0.185	[illegible]
4	1.36	0.25	0.39	0.0975	[illegible]
8	1.75		0.39		
16	2.14		0.38		
32	2.52				
3	1.21	0.333	0.38	0.1267	[illegible]
6	1.59				
	Σ	2.0833	1.47	0.739[illegible]	

TABLA 4

EFICIENCIA DE LOS METODOS DE DIFERENCIA Y SUPERPOSICION DE CURVAS EN LA SOLUCION DE LA ECUACION DE THEIS.

Datos		Metodos de diferencias			Metodos de Theis		
(1), x Observados	(2), y	(3), y' Calc.	(4), y-y' Error	(5), $(y-y')^2$ $Error^2$	(6), y' Calc.	(7), y-y' Error	(8), $(y-y')^2$ $Error^2$
1	0.66	0.60	0.60	0.0036	0.85	-0.19	0.0361
2	0.99	0.96	0.03	0.0009	1.00	-0.01	0.0001
3	1.21	1.26	-0.05	0.0025	1.20	0.01	0.0001
4	1.36	1.30	0.06	0.0036	1.36	0.00	0.0000
8	1.75	1.70	0.05	0.0025	1.73	0.02	0.0004
16	2.14	1.12	0.02	0.0004	2.13	0.01	0.0001
32	2.51	2.50	0.01	0.0001	2.49	0.02	0.0004
64	2.91	2.91	0.00	0.0000	2.87	0.04	0.0016
Variancias				0.0136			0.0388

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PROGRAMACION Y MODELOS DE INGENIERIA AMBIENTAL

Aplicación del Modelo del Transporte al Manejo de las Basuras

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APLICACION DEL MODELO DEL TRANSPORTE AL MANEJO DE LAS BASURAS.

por: Francisco Zepeda P.

El crecimiento explisivo de ciudades en México, tanto en área como en población, hace que la planeación de un sistema de limpia por los métodos empíricos tradicionales sea obsoleta. Se requiere del auxilio de los modelos matemáticos para minimizar los costos de transporte sobre todo en las tres urbes mas grandes del país; D.F., Monterrey y Guadalajara.

El modelo del transporte trata con un problema típico de la programación lineal en el cual el objetivo es transportar un material específico desde varios "orígenes" hasta varios "destinos" con elcosto global mínimo. Se necesita conocer la cantidad de material que produce cada origen y lo que demanda o puede aceptar cada destino, así como los costos unitarios de transporte del material de los orígenes a los diferentes destinos. En el caso de un sistema de limpia se debe conocer la cantidad de basura por recolectar en cada sector de la ciudad, y la cantidad de basura que puede aceptar cada sitio de disposción en función de la capacidad de manejo de la maquinaria existente, tractores en el caso de los rellenos sanitarios, y capacidad instalada en el caso de incineradores o plantas industrializadoras. Los costos unitarios de transporte de cualquier origen a cualquier destino se encuentra dividiendo la distancia por recorrer entre la velocidad promedio y multiplicando por el costo horario del vehículo.

El problema del transporte es un caso particular del problema general de la programación lineal, que puede manejarse mediante el método "simplex", sin embargo, se cuenta con el "algoritmo del transporte que es mucho mas eficiente para el caso. Se tiene como en todos los casos una función lineal objetivo, un conjunto de restricciones lineales estructurales y otro de restricciones de no negatividad.

Pero antes de exponer estas relaciones, coloquemos el cuadro típico que se usa para la solución del algoritmo:

chapter 10

The Transportation Model

10.1 INTRODUCTION

The transportation model deals with a special class of linear-programming problems in which the objective is to "transport" a *single commodity* from various "origins" to different "destinations" at a minimum total cost.* The total supply available at the origins and the total quantity demanded by the destinations are given in the statement of the problem. Also given is the cost of shipping a unit of goods from a known origin to a known destination. As in the linear-programming problems discussed in previous chapters, all relationships are assumed to be linear.

With information about the total capacities of the origins, the total requirements of the destinations, and the shipping cost per unit of goods for available shipping routes, the transportation model is used to determine the optimum shipping program(s) resulting in minimum total shipping costs.

In so far as the transportation problem is a special case of the general linear-programming problem, it can always be solved by the simplex method. However, the *transportation algorithm,* which we shall develop in later sections of this chapter, provides a much more efficient method of handling such a problem. Let us now turn our attention to delineating the relationship between the general linear-programming problem and the transportation problem.

* Of course, if the payoff measure is of the "profit" variety, the objective will be to maximize total payoff.

10.2 TRANSPORTATION PROBLEM —A SPECIAL CASE

Having solved a general linear-programming problem by various methods in previous chapters, we observe again that such a problem always consists essentially of three components: (1) a linear objective function, (2) a set of linear structural constraints, and (3) a set of nonnegativity constraints. Let us illustrate these three components.

1 Linear Objective Function

Every linear-programming problem has as its objective the maximization or minimization of a linear objective function. This function is usually of the form

$$F(X) = \sum_{j=1}^{n} c_j x_j \qquad j = 1, 2, \ldots, n$$

where x_j = set of *structural variables;* these variables represent competing candidates or activities

c_j = set of so-called "price coefficients"; in the problem, c_j's are coefficients of structural variables in the objective function

A typical linear objective function involving n variables can be written as follows:

$$F(X) = c_1x_1 + c_2x_2 + \cdots + c_nx_n$$

2 Linear Structural Constraints

Every linear-programming problem contains a set of linear constraints. They embody the technical specifications and resource capacities of the problem structure and are therefore called *structural* constraints. These constraints are of the form

$$\sum_{j=1}^{n} \sum_{i=1}^{m} a_{ij}x_j \leq b_i \qquad \begin{cases} i = 1, 2, \ldots, m \\ j = 1, 2, \ldots, n \end{cases}$$

where the a_{ij}'s are a set of *structural coefficients* reflecting the technical specifications of the problem, and they appear as coefficients of the

structural variables in the structural constraints. The b_i's are a set of *constants* reflecting the maximum resource capacities or minimum resource requirements.

An expanded form of the linear structural constraints is given below:*

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \leq b_1$$
$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \leq b_2$$
$$\cdots\cdots\cdots\cdots\cdots\cdots$$
$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \leq b_m$$

3 Nonnegativity Constraints

The structural variables, slack variables, and artificial slack variables of all linear-programming problems are restricted to nonnegative values. This is accomplished by imposing nonnegativity constraints of the form

$$x_j \geq 0 \qquad j = 1, 2, \ldots, n$$

If we let x_j's denote the structural variables, S_i's denote the slack variables, and A_i's denote the artificial slack variables, we may write these constraints as follows:

$$x_1 \geq 0 \qquad x_2 \geq 0 \qquad \cdots \qquad x_n \geq 0$$
$$S_1 \geq 0 \qquad S_2 \geq 0 \qquad \cdots \qquad S_m \geq 0$$
$$A_1 \geq 0 \qquad A_2 \geq 0 \qquad \cdots \qquad A_m \geq 0$$

Two remarks must be made at this time in reference to the general linear-programming problem. First, the structural coefficients a_{ij} are not restricted to any particular value or values. For example, a particular a_{ij} may be specified to have a value of 10, 20, 1, or 0. Second, no restrictions are imposed regarding the homogeneity of units among the various inequalities representing the structural constraints. Of the given constraints, some may refer to available capacities of machines performing different kinds of operations, while others may specify different types of, say, chemical characteristics. In other words, the units of any one

* The structural constraints in their original form can, of course, be simple equalities or inequalities of the "less than or equal to" or "greater than or equal to" type.

constraint may not be the same as those of the other constraints and hence may not be interchangeable with the units of any other constraint. We can illustrate this by referring back to the vitamin problem of Chapter 7, in which the two structural constraints were concerned with different types of vitamins.

The transportation problem, in comparison with the general linear-programming problem, restricts the values that can be assigned to the structural coefficients and limits the constraints to only one type of units. In particular, the general linear-programming problem can be reduced to what is called a *transportation problem* if (1) the a_{ij}'s (coefficients of the structural variables in the constraints) are restricted to the values 0 and 1 and (2) there exists a homogeneity of units among the constraints.

The transportation problem and the general linear-programming problem can be compared by examining the simplex tableau (Figure 10.1) constructed from the transportation problem given in Table 10.1.

Let us now formulate a typical transportation problem involving three origins and four destinations.

10.3 THE PROBLEM

A manufacturing concern has three plants located in three different cities, all producing the same product. The total supply potential of the firm is absorbed by four large customers. Let us identify the three plants as O_1, O_2, and O_3, and the customers as D_1, D_2, D_3, and D_4. The relevant data on plant capacities, destination requirements, and shipping costs for individual shipping routes are recorded, in general terms, in Table 10.1.

As shown in the table, the matrix of our transportation problem has three rows and four columns and hence is *not* a square matrix. This emphasizes the point that in a transportation problem a given origin can simultaneously supply goods to more than one destination. As we shall see later, the so-called "assignment model" is restricted to a square matrix in the sense that one origin cannot simultaneously associate with more than one destination in the assignment problem.

Notice that the first subscript in each symbol used in Table 10.1 refers to the specific origin, and the second subscript to the particular destination. For example, c_{12} is the cost of shipping 1 unit of goods from origin O_1 to destination D_2, and the variable x_{34} is the quantity to be shipped from origin O_3 to destination D_4. Origin capacities and destination requirements are given along the outside (rims) of Table 10.1 and are usually referred to as *rim requirements*. Our problem is to choose that

strategy (a particular program of shipping) which will satisfy the rim requirements at a minimum total cost.

Analysis of the Problem

The transportation problem given above, like the general linear-programming problem, consists of three components. First, we can formulate a linear objective function which is to be minimized. This function will represent the total shipping cost of all the goods to be sent from the origins to the destinations. Second, we can write a set of linear structural constraints. Of the seven constraints of this problem, three (one for

Table 10.1

Origin	Destination				Origin capacity per time period
	D_1	D_2	D_3	D_4	
O_1	c_{11} x_{11}	c_{12} x_{12}	c_{13} x_{13}	c_{14} x_{14}	$\leq b_1$
O_2	c_{21} x_{21}	c_{22} x_{22}	c_{23} x_{23}	c_{24} x_{24}	$\leq b_2$
O_3	c_{31} x_{31}	c_{32} x_{32}	c_{33} x_{33}	c_{34} x_{34}	$\leq b_3$
Destination requirement per time period	d_1	d_2	d_3	d_4	

c_{ij} = cost of shipping a unit of goods from ith origin to jth destination
x_{ij} = number of units to be shipped from ith origin to jth destination

Assume

$$\Sigma b_i = \Sigma d_j$$

That is, total origin capacities equal total destination requirements, and $i = 1, 2, 3$;

each row) will give the relationships between the origin capacities and the goods to be received by different destinations. These are called *capacity* constraints. The other four constraints (one for each column) will specify the relationships between destination requirements and the goods to be shipped from different origins. These are called *requirement* constraints. Third, we can specify a set of nonnegativity constraints for the structural variables x_{ij}. They will state that no negative shipment is permitted. The general correspondence between a typical linear-programming problem and the transportation problem is thus complete.

The three component parts of our transportation problem are given below:

Minimize

$$F(X) = c_{11}x_{11} + c_{12}x_{12} + c_{13}x_{13} + c_{14}x_{14} + c_{21}x_{21} + c_{22}x_{22} + c_{23}x_{23} + c_{24}x_{24} + c_{31}x_{31} + c_{32}x_{32} + c_{33}x_{33} + c_{34}x_{34}$$

subject to

$$\begin{array}{lr}
x_{11} + x_{12} + x_{13} + x_{14} = b_1 & (1)\\
x_{21} + x_{22} + x_{23} + x_{24} = b_2 & (2)\\
x_{31} + x_{32} + x_{33} + x_{34} = b_3 & (3)\\
x_{11} + x_{21} + x_{31} = d_1 & (4)\\
x_{12} + x_{22} + x_{32} = d_2 & (5)\\
x_{13} + x_{23} + x_{33} = d_3 & (6)\\
x_{14} + x_{24} + x_{34} = d_4 & (7)
\end{array}$$

and $x_{ij} \geq 0$; $i = 1, 2, 3$; $j = 1, 2, 3, 4$.

10.4 BUILDING A SIMPLEX TABLEAU FOR THE TRANSPORTATION PROBLEM

Since Equations (1) through (3) refer to origin capacities, we can think of these as inequalities of the "less than or equal to" type asserting the fact that different origins cannot produce more than their respective capacities. For purposes of construction of the simplex tableau, therefore, we may modify Equations (1) through (3) with the addition of slack variables S_1, S_2, and S_3, respectively. The cost coefficients of these slack variables are, of course, zero. Variables S_1, S_2, and S_3 represent

On the other hand, we should consider Equations (4) through (7) as strict equations or inequalities of the "greater than or equal to" type. Long-run interests of the company require that it be willing to supply exactly that quantity of goods which is specified by each customer (or perhaps more, if circumstances demand). For purposes of construction of the first simplex tableau, therefore, Equations (4) through (7) may be modified with the addition of nonnegative artificial slack variables A_1, A_2, A_3, and A_4, respectively.* The cost coefficient of each of these artificial slack variables is obviously M.

Program	Cost per unit	Quantity	c_{11} x_{11}	c_{12} x_{12}	c_{13} x_{13}	c_{14} x_{14}	c_{21} x_{21}	c_{22} x_{22}	c_{23} x_{23}	c_{24} x_{24}	c_{31} x_{31}	c_{32} x_{32}	c_{33} x_{33}	c_{34} x_{34}	0 S_1	0 S_2	0 S_3	M A_1	M A_2	M A_3	M A_4
S_1	0	b_1	1	1	1	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
S_2	0	b_2	0	0	0	0	1	1	1	1	0	0	0	0	0	1	0	0	0	0	0
S_3	0	b_3	0	0	0	0	0	0	0	0	1	1	1	1	0	0	1	0	0	0	0
A_1	M	d_1	1	0	0	0	1	0	0	0	1	0	0	0	0	0	0	1	0	0	0
A_2	M	d_2	0	1	0	0	0	1	0	0	0	1	0	0	0	0	0	0	1	0	0
A_3	M	d_3	0	0	1	0	0	0	1	0	0	0	1	0	0	0	0	0	0	1	0
A_4	M	d_4	0	0	0	1	0	0	0	1	0	0	0	1	0	0	0	0	0	0	1

Figure 10.1

With this completed, the data of our problem can be reflected in the first simplex tableau, as shown in Figure 10.1. We note that all entries in the initial simplex tableau are either 1 or 0. The 1s appear in the form of scattered rows and slanted diagonals. Further, each column, except those in the identity part of the tableau, represents a column vector in which two elements are 1 and the rest are zero.† Whenever the initial simplex tableau appears in a form such as that of Figure 10.1, the linear-programming problem can be classified as a transportation problem.

The assignment of specific numerical values to origin capacities (b_1, b_2, b_3), destination requirements (d_1, d_2, d_3, d_4), and cost coefficients (c_{ij}'s) in Table 10.1 would give us a concrete transportation problem whose *initial* solution by the simplex method would be given by Figure 10.1. Further iterations following the rules of the simplex method would, no doubt, yield the optimum solution to this problem. This would be a rather lengthy process and would not add anything new to our knowledge of the simplex method. Fortunately, however, a simple and routine method of solving such problems has been developed. It is fittingly called the *transportation model.* Whenever a given linear-programming problem can be placed in the transportation framework, it is far simpler to solve it by the transportation method than by the simplex method. Before we describe and develop the transportation method, let us comment on certain characteristics of the transportation problem and its solution.

* We assume exact equality in the original formulations of the requirement constraints.

† This particular characteristic of the transportation problem forms the basis of the so-called "modified-distribution method" of solving such problems (see Section 10.8).

First, a little reflection will show that for the transportation problem of Table 10.1 only six rather than seven structural constraints need be specified. In view of the fact that the sum of the origin capacities equals the sum of the destination requirements ($\Sigma b_i = \Sigma d_j$), any solution satisfying six of the seven constraints will automatically satisfy the last constraint. In general, therefore, if m represents the number of rows and n represents the number of columns in a given transportation problem, we can state the problem completely with $m + n - 1$ equations. This means that one of the rows of the simplex tableau in Figure 10.1 represents a redundant constraint and, hence, can be deleted. This also means that a basic feasible solution of a transportation problem has only $m + n - 1$ positive components.

Second, if origin capacities equal destination requirements, it is always possible to design an initial basic feasible solution in such a manner that the rim requirements are satisfied. This can be accomplished either by inspection or by following certain formal methods for making the initial allocation. Three such methods, the so-called "northwest-corner" rule, the "Vogel's approximation method," and the "inspection" method, will be described later.

10.5 APPROACH OF THE TRANSPORTATION METHOD

The transportation method consists of three basic steps. The first step involves making the initial shipping assignment in such a manner that a basic feasible solution is obtained. This means that $m + n - 1$ cells (routes) of the transportation matrix are used for shipping purposes.

The cells having the shipping assignment will be called *occupied* cells, while the remaining cells of the transportation matrix will be referred to as *empty* cells.

The purpose of the second step is to determine the opportunity costs associated with the empty cells. The opportunity costs of the empty cells can be calculated individually for each cell or simultaneously for the whole matrix. If the opportunity costs of all the empty cells are nonpositive, we can be confident that an optimum solution has been obtained.* On the other hand, if even a single empty cell has a positive opportunity cost, we proceed to step 3.

The third step involves determining a new and better basic feasible solution. Once this new basic feasible solution has been obtained, we repeat steps 2 and 3 until an optimum solution has been designed.

The remaining sections of this chapter are devoted to illustrating the development and application of the above-mentioned approach to the solution of a given transportation problem.

10.6 METHODS OF MAKING THE INITIAL ASSIGNMENT

The first step in the transportation method, as stated above, consists in making an initial assignment in such a manner that a basic feasible solution (number of occupied cells equals $m + n - 1$) is obtained. Various methods of making such an assignment are available. We shall discuss three such methods in considering the transportation problem in Table 10.2.

Northwest-corner Rule

According to this rule, the first allocation is made to the cell occupying the upper left-hand (northwest) corner of the matrix. Further, this allocation is of such a magnitude that either the origin capacity of the first row is exhausted or the destination requirement of the first column is satisfied or both. If the origin capacity of row 1 is exhausted first, we move down the first column and make another allocation which either exhausts the origin capacity of row 2 or satisfies the remaining destination

* The transportation problem falls under the category of decision making under *certainty*. Hence, the optimum solution must not be associated with positive opportunity costs.

requirement of column 1. On the other hand, if the first allocation completely satisfies the destination requirement of column 1, we move to the right in row 1 and make a second allocation which either exhausts the remaining capacity of row 1 or satisfies the destination requirement of column 2, and so on. In this manner, starting from the upper left-hand corner of the given transportation matrix, satisfying the individual destination requirements, and exhausting the origin capacities *one at a time*,

Table 10.2

Origin	Destination					Origin capacity per time period
	D_1	D_2	D_3	D_4	D_5	
O_1	12	4	9	5	9	55
O_2	8	1	6	6	7	45
O_3	1	12	4	7	7	30
O_4	10	15	6	9	1	50
Destination requirement per time period	40	20	50	30	40	

we move toward the lower right-hand corner until all the rim requirements are satisfied. It should be noted that when we follow the northwest-corner rule we pay no attention to the relative costs of the different routes while making the first assignment.

For the transportation problem of Table 10.2, application of the northwest-corner rule dictates that we first "load" or "fill" cell O_1D_1, which lies in the upper left-hand (northwest) corner. The product requirement of D_1 is 40 units, and the capacity of O_1 is 55 units; the lower of these two numbers, that is, 40, is placed in cell O_1D_1. This means that the requirement of D_1 is fully satisfied, but we still have 15 units (55 − 40) of unused capacity at O_1. Thus we move to the right of cell O_1D_1 in the first row.

At this stage we note that the destination requirement of column D_2 is 20 units. Knowing that 15 units of capacity O_1 are still unused, we route all 15 units to destination D_2 (place 15 in cell O_1D_2). This completely exhausts the capacity O_1, but column D_2 still needs 5 units (20 − 15) to satisfy its requirement. Thus, we move down column D_2 and supply these 5 units from capacity O_2 (place 5 in cell O_2D_2). This leaves 40 units of unused capacity at O_2; these are routed to D_3 (place 40 in cell

Table 10.3 *Initial Assignment by Northwest-corner Rule*

Origin	Destination					Total
	D_1	D_2	D_3	D_4	D_5	
O_1	12 (40)	4 (15)	9	5	9	55
O_2	8	1 (5)	6 (40)	6	7	45
O_3	1	12	4 (10)	7 (20)	7	30
O_4	10	15	6	9 (10)	1 (40)	50
Total	40	20	50	30	40	180 / 180

O_2D_3). The remaining requirement of 10 units (50 − 40) for D_3 is supplied from O_3 (place 10 in cell O_3D_3). This leaves 20 units of unused capacity at O_3; these are routed to D_4 (place 20 in cell O_3D_4). The remaining requirement of 10 units (30 − 20) for D_4 is supplied from O_4 (place 10 in cell O_4D_4). We are now left with 40 units of unused capacity at O_4; these are finally routed to D_5 (place 40 in cell O_4D_5). The entire table has now been loaded, resulting in the initial program given in Table 10.3. The circled numbers in the table give the number of units shipped from a particular origin to a certain destination. The cells in which these circled numbers are entered are our occupied cells. The rest of the cells are the empty cells.

It is to be observed that the number of occupied cells is

$$m + n - 1 = 4 + 5 - 1 = 8$$
$$= (\text{number of rows} + \text{number of columns} - 1)$$

The solution at this stage is therefore not degenerate.*

The total cost of this assignment is

$$1{,}095 =$$
$$40 \times 12 + 15 \times 4 + 5 \times 1 + 40 \times 6 + 10 \times 4 + 20 \times 7 + 10 \times 9 + 40 \times 1$$

It should be noted that the last allocation (cell O_4D_5) *simultaneously* satisfied the requirement of column D_5 and exhausted the capacity of O_4. This is the "normal" situation in the last allocation made by the northwest-corner rule; if this occurs *only* in the last allocation, we can be certain of having a basic feasible solution. However, if any allocation previous to the last allocation happens to be such that it simultaneously satisfies the requirement of some destination and exhausts the capacity of some origin, then the number of occupied cells will be less than $m + n - 1$. This will mean that we have a degenerate basic feasible solution.† The reader should try to make an initial assignment in Table 10.2 by following the northwest-corner rule after having changed the destination requirements of D_2 and D_3 to 15 and 55 units, respectively.

Vogel's Approximation Method (VAM)

According to this method, a *difference column* and a *difference row* representing the difference between the costs of the *two cheapest routes* for each origin and destination are computed. Each individual difference can be thought of as a penalty‡ for not using the cheapest route. After all such penalty ratings have been computed for the given data, the highest difference or penalty rating is identified. Then the lowest-cost cell in

* A basic feasible solution for a transportation problem requires only $m + n - 1$ positive components. Thus, whenever a transportation program has $m + n - 1$ occupied cells, the solution is not degenerate.

† A method for resolving degeneracy in transportation problems is discussed later in this chapter (see Section 10.13).

‡ An-Min Chung, "Linear Programming," p. 248, Charles E. Merrill Books, Inc., Columbus, Ohio, 1963.

Table 10.4a

	D_1	D_2	D_3	D_4	D_5	Difference column or penalty
O_1	12	4	9	5	9	1
O_2	8	1	6	6	7	5
O_3	1	12	4	7	7	3
O_4	10	15	6	9	1	5
Difference row or penalty	7 ✓	3	2	1	6	

	D_1	D_2	D_3	D_4	D_5	Capacity
O_1	12	4	9	5	9	55
O_2	8	1	6	6	7	45
O_3	1 (10)	12	4	7	7	~~10~~ 0
O_4	10	15	6	9	1	50
Requirement	~~20~~ 10	20	50	30	40	

Table 10.4b

	D_1	D_2	D_3	D_4	D_5	Difference column or penalty
O_1	12	4	9	5	9	1
O_2	8	1	6	6	7	5
O_4	10	15	6	9	1	5
Difference row or penalty	2	3	0	1	6 ✓	

	D_1	D_2	D_3	D_4	D_5	Capacity
O_1	12	4	9	5	9	55
O_2	8	1	6	6	7	45
O_4	10	15	6	9	1 (40)	~~50~~ 10
Requirement	10	20	50	30	~~40~~ 0	

Table 10.4c

	D_1	D_2	D_3	D_4	Difference column or penalty
O_1	12	4	9	5	1
O_2	8	1	6	6	5 ✓
O_4	10	15	6	9	3
Difference row or penalty	2	3	0	1	

	D_1	D_2	D_3	D_4	Capacity
O_1	12	4	9	5	55
O_2	8	1 (20)	6	6	~~45~~ 25
O_4	10	15	6	9	10
Requirement	10	~~20~~ 0	50	30	

that row or column in which the highest penalty rating* was placed is the cell to which the first assignment is made. This assignment either exhausts the capacity of an origin or meets the requirement of a destina-

* Should there be a tie for highest penalty rating or difference value, we can arbitrarily choose one to break the tie. Although rules for breaking ties are available, it is usually easier simply to pick one of the tied columns or rows for making the allocation. See N. V. Reinfeld and William R. Vogel, "Mathematical Programming," chap. 4, Prentice-Hall, Inc., Englewood Cliffs, N.J., 1958.

Table 10.4d

	D_1	D_3	D_4	Difference column or penalty
O_1	12	9	5	4 ✓
O_2	8	6	6	0
O_4	10	6	9	3
Difference row or penalty	2	0	1	

	D_1	D_3	D_4	Capacity
O_1	12	9	5 (30)	~~55~~ 25
O_2	8	6	6	25
O_4	10	6	9	10
Requirement	10	50	~~30~~ 0	

Table 10.4e

	D_1	D_3	Difference column or penalty
O_1	12	9	3
O_2	8	6	2
O_4	10	6	4 ✓
Difference row or penalty	2	0	

	D_1	D_3	Capacity
O_1	12	9	25
O_2	8	6	25
O_4	10	6 (10)	~~10~~ 0
Requirement	10	~~50~~ 40	

Table 10.4f

	D_1	D_3	Difference column or penalty
O_1	12	9	3
O_2	8	6	2
Difference row or penalty	4 ✓	3	

	D_1	D_3	Capacity
O_1	12	9	25
O_2	8 (10)	6	~~25~~ 15
Requirement	~~10~~ 0	40	

Table 10.4g

	D_3	Capacity
O_1	9 (25)	25
O_2	6 (15)	15
Requirement	40	

tion or both. The particular row or column which has been thus satisfied is removed from the transportation matrix. The process is then repeated until an initial assignment using $m + n - 1$ routes has been obtained. This approach has the disadvantage of necessitating some computational work before the initial program is obtained, but it usually results in the attainment of the optimal program in fewer iterations than are required when the initial program is obtained by using the northwest-corner rule.

Table 10.5. *Initial Assignment by VAM*

Origin	Destination D_1	D_2	D_3	D_4	D_5	Total
O_1	[12]	[4]	[9] (25)	[5] (30)	[9]	55
O_2	[8] (10)	[1] (20)	[6] (15)	[6]	[7]	45
O_3	[1] (30)	[12]	[4]	[7]	[7]	30
O_4	[10]	[15]	[6] (10)	[9]	[1] (40)	50
Total	40	20	50	30	40	180 / 180

The mechanics for obtaining the initial assignment for the transportation problem of Table 10.2 by Vogel's approximation method (VAM) is illustrated in Tables 10.4*a* through 10.4*g*. In Table 10.4*a*, the highest difference or penalty rating is 7, and this falls under column D_1. The first allocation, therefore, must be made to that cell in column D_1 which has the lowest shipping cost. Since cell O_3D_1 has the lowest shipping cost in that column, we now compare the capacity of O_3 (30 units) with the requirement of D_1 (40 units). The lower of the two numbers, that is, 30, is placed in cell O_3D_1. This means that the capacity of O_3 has been fully utilized, and row O_3 can be removed temporarily from the transportation matrix. Column D_1, however, cannot be removed, since we still need 10 units to satisfy its requirements fully.

We now have arrived at Table 10.4*b*. The process of computing penalties is repeated, and in Table 10.4*b* we note that the highest penalty falls under column D_5. We therefore make an assignment in the lowest-cost cell of column D_5. This assignment (place 40 in cell O_4D_5) is such that column D_5 can now be removed from the matrix, and we proceed to Table 10.4*c*. By repeating the same process in Tables 10.4*c* to 10.4*g*, we finally obtain the assignment in Table 10.5. Observe that the number of occupied cells is $m + n - 1 = 4 + 5 - 1 = 8$. The initial solution is therefore a basic feasible solution, and the problem at this stage is not degenerate. The total cost of this assignment is \$695—considerably less than the total cost associated with the initial program obtained via the northwest-corner rule.

Initial Assignment by Inspection

One can, no doubt, make an initial assignment in a transportation problem simply by inspection and judgment. This is, needless to say, not a formal method of obtaining an initial assignment. However, for transportation problems of small dimensions, it has the advantage of speed. The first allocation is made to that cell whose shipping cost per unit is lowest. This lowest-cost cell is loaded or filled as much as possible in view of the origin capacity of its row and the destination requirement of its column. Then we move to the next lowest-cost cell and make an allocation in view of the remaining capacity and requirement of its row and column, and so on. Should there be a tie for lowest-cost cell during any allocation, we can exercise "judgment" in breaking the tie or we can arbitrarily choose a cell for allocation. The total number of allocations, of course, must be such that a basic feasible solution ($m + n - 1$ occupied cells) is obtained. Let us illustrate the inspection method for the transportation problem of Table 10.2. We note that cells O_2D_2, O_3D_1, and O_4D_5 each have a shipping cost of \$1 per unit. Thus there is a tie for the first allocation. We arbitrarily choose cell O_3D_1 for the first allocation and route 30 units from O_3 to D_1.* This means that the capacity of O_3 is fully utilized (cross off row O_3 with a light pencil). For the second allocation, we observe that there is a tie between cells O_2D_2 and O_4D_5. We arbitrarily choose O_4D_5 and ship 40 units through this route. This completely satisfies the requirement of column D_5 (cross off column D_5 with a light pencil). For the third allocation, we note that cell O_2D_2 is

* In this case, one could easily have exercised judgment in terms of the VAM penalty ratings associated with the tied cells. As a matter of fact, one should exercise such a judgment in all stages of the inspection method.

now the lowest-cost cell and, therefore, we ship 20 units through this route (place 20 in cell O_2D_2 and cross off column D_2). Of those remaining, cell O_1D_4 has the lowest cost, and we route 30 units through O_1D_4 (place 30 in cell O_1D_4 and cross off column D_4). Next, we observe that there is a tie between O_2D_3 and O_4D_3 for the fifth allocation. We arbitrarily choose cell O_4D_3 and ship 10 units (the remaining capacity of O_4) through this route (place 10 in cell O_4D_3 and cross off row O_4). Of those remaining, cell O_2D_3 has the lowest cost, and we route 25 units (the remaining capacity of row O_2) through cell O_2D_3 (place 25 in cell O_2D_3 and cross off row O_2). We are now left with 25 units at O_1, while D_1 and D_3 still require 10 and 15 units, respectively. Hence, we route 10 units through O_1D_1 and 15 units through O_1D_3. All the rim requirements have now been satisfied, and we have the initial assignment in Table 10.6. The dotted lines crossing the cost squares (c_{ij}'s) have been numbered to show the order in which different rows and columns were crossed off, as an aid to making the initial assignment by inspection.

Table 10.6 *Initial Assignment by Inspection*

Origin	Destination D_1	D_2	D_3	D_4	D_5	Total	
O_1	[12] (10)	[4]	[9] (15)	[5] (30)	[9]	~~55~~ 25	
O_2	[8]	[1] (20)	[6] (25)	[6]	[7]	~~45~~ ~~25~~ 0	6th
O_3	[1] (30)	[12]	[4]	[7]	[7]	~~30~~ 0	1st
O_4	[10]	[15]	[6] (10)	[9]	[1] (40)	~~50~~ ~~10~~ 0	5th
Total	~~40~~ ~~10~~ 0	~~20~~ 0	~~50~~ ~~40~~ ~~15~~ 0	~~30~~ 0	~~40~~ 0		
	8th	3d	7th	4th	2d		

It is to be observed that the number of occupied cells is 8 (that is, $m + n - 1$), and thus we have a basic feasible solution. The total cost of this assignment is \$705. It should be compared with the total costs associated with the initial solutions obtained by the northwest-corner rule (\$1,095) and Vogel's approximation method (\$695).

It will be recalled that the approach of the transportation method is based on three steps: (1) making the initial assignment in order to obtain a basic feasible solution, (2) determining the opportunity costs of the empty cells, and (3) designing a better basic feasible solution (provided step 2 indicates that the program can be improved) and repeating steps 2 and 3 until an optimal solution has been obtained. The application of the first step has now been illustrated in connection with the transportation problem of Table 10.2. Next we shall show the application of steps 2 and 3 to complete our illustration of the transportation method. There are, however, two methods of carrying out steps 2 and 3. One is called the *steppingstone* method, whereas the other is referred to as the *modified-distribution* method. We shall first discuss the steppingstone method.

10.7 STEPPINGSTONE METHOD FOR OBTAINING AN OPTIMAL SOLUTION

To illustrate the steppingstone method, we shall first solve the very simple transportation problem given in Table 10.7. The method will then be used in deriving the optimum solution to our problem of Table 10.2. The purpose of solving the simple problem of Table 10.7 is to

Table 10.7

Origin	Destination D_1	D_2	Origin capacity per time period
O_1	[2]	[2]	1,000
O_2	[1]	[2]	600
Destination requirement per time period	900	700	1,600 / 1,600

familiarize the reader with the terminology and rationale of the stepping-stone method.

Following the northwest-corner rule, we obtain the initial program in Table 10.8. The circled numbers within the body of the matrix refer to the specific allocations of the first program. This program calls for shipping 900 units from O_1 to D_1, 100 units from O_1 to D_2, and 600 units from O_2 to D_2. Obviously, this program satisfies all the rim requirements. Note further that the number of occupied cells is 3, which is 1 less than

Table 10.8

Origin	Destination		Total
	D_1	D_2	
O_1	[2] (900)	[2] (100)	1,000
O_2	[1]	[2] (600)	600
Total	900	700	

the sum of the numbers of rows and columns. In other words, the number of occupied cells in this program equals $m + n - 1$.* Thus we have a basic feasible solution.

* As we have established previously, only $m + n - 1$ equations are needed to state a transportation problem. The present problem can be stated with the following equations:

$$x_{11} + x_{12} = 1{,}000$$

$$x_{21} + x_{22} = 600$$

$$x_{11} + x_{21} = 900$$

Naturally, letting $x_{21} = 0$ gives a solution in which $x_{11} = 900$, $x_{22} = 600$, and $x_{12} =$

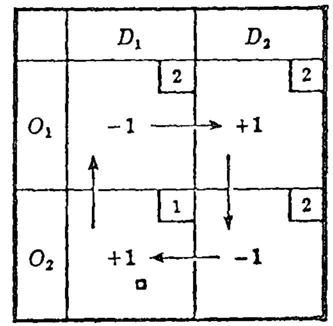
Determining the Opportunity Cost of the Empty Cells

Is the above program an optimal program? To answer this question, we must apply step 2, that is, determine the opportunity costs of the empty cells. In so far as the transportation model involves decision making under certainty, we know that an optimal solution must not incur any positive opportunity cost. Thus, to determine whether any positive opportunity cost is associated with a given program, we must test the empty cells (cells representing routes not used in the given program) of the transportation matrix for the presence or absence of opportunity cost. The absence of positive opportunity costs in *all empty cells* will indicate that an optimal solution has been obtained. If, on the other

Table 10.9

Take 1 unit out of O_2D_2: -1
Add 1 unit to O_2D_1: $+1$
Take 1 unit out of O_1D_1: -1
Add 1 unit to O_1D_2: $+1$

$\Longrightarrow$



hand, even a single empty cell has a positive opportunity cost, the given program is not the optimal program and, hence, should be revised.*

Let us examine our first program in view of the above discussion. Since cell O_2D_1 in this program is empty, we wish to determine whether or not there is an opportunity cost associated with it. This is accomplished by shifting 1 unit of goods to cell O_2D_1, making other shifts necessary to satisfy the rim requirements, and then finding the cost consequence of these changes. Let us shift 1 unit from cell O_2D_2 to cell O_2D_1. This shift will necessitate the changes noted in Table 10.9 in order to keep the rim requirements satisfied. These changes are associated with the following cost consequence or cost change:

$$-2 + 1 - 2 + 2 = -1 \text{ dollar}$$

* It will be recalled that the test for optimality in the simplex method was also based on the concept of opportunity cost.

Since the shifting of 1 unit to O_2D_1 yields a negative cost change, it is obviously a desirable shift. The fact that the transfer of 1 unit to cell O_2D_1 resulted in a net cost change of -1 dollar indicates that the opportunity cost of *not* including cell O_2D_1 in the first program is $+1$ dollar per unit of shipment. The empty cell O_2D_1 must, therefore, be included in a new and improved program.

Revision of the Given Program

Having discovered that the opportunity cost of the empty cell O_2D_1 is positive, we must next obtain a new basic feasible solution. This is done

Table 10.10

First program

	D_1	D_2
O_1	[2] − 900	[2] + 100
O_2	[1] +	[2] − 600

Revised program (with just 1 unit shifted to O_2D_1)

	D_1	D_2
O_1	[2] 899	[2] 101
O_2	[1] 1	[2] 599

by designing a new improved program in which cell O_2D_1 is included in the shipping strategy. Let us make the improvement by shifting just 1 unit from cell O_2D_2 to cell O_2D_1. The revised program is given in Table 10.10. The shift of 1 unit from cell O_2D_2 to cell O_2D_1 means that we are left with 599 units in cell O_2D_2. This change, it should be noted, has not violated the capacity constraints of either row 1 or row 2. But what about the requirement constraints of columns 1 and 2? With the above change, we now have 1 unit in O_2D_1, 900 units in O_1D_1, 599 units in O_2D_2, and 100 units in O_1D_2. In other words, column 1 has 901 units (one more unit than the requirement of column 1), and column 2 has 699 units (one less unit than the requirement of column 2). Clearly, this situation can be remedied by shifting 1 unit from cell O_1D_1 to cell

O_1D_2, a change which will simultaneously satisfy the row and column requirements.

The revised program, with 1 unit shifted to cell O_2D_1, is shown in Table 10.10. The change in the program effected by the introduction of 1 unit to cell O_2D_1, as we established earlier, reduces the total shipping cost by $1. In so far as we gain this advantage each time a unit is shifted to cell O_2D_1, we must shift to cell O_2D_1 as many units as possible. As the *closed loop* (plus and minus signs connected by arrows) of Table 10 10 shows, we cannot shift more than 600 units to O_2D_1, for the allocation of more than 600 units to cell O_2D_1 would certainly violate the capacity constraint of row O_2.

Table 10.11

	D_1	D_2	Total
O_1	[2] 300	[2] 700	1,000
O_2	[1] 600	[2]	600
Total	900	700	

Our second program (a better basic feasible solution), the result of the above discussion, appears in Table 10.11. Is this the optimal allocation? The answer to this question can be obtained by testing the opportunity cost of cell O_2D_2, which is now the only empty cell. The answer is in the affirmative, since the opportunity cost of cell O_2D_2 is not positive. This can be verified by shifting 1 unit to cell O_2D_2 and noting that the net cost consequence of such a shift is $+1$ dollar $(+2 - 2 + 2 - 1)$. The opportunity cost, being the negative of the corresponding net cost change, is therefore negative. Hence, the assignment of Table 10.11 gives an optimal solution with a total shipping cost of $2,600. No other program for this problem can result in a lower total shipping cost.

Let us recapitulate briefly the method of attack followed in solving this transportation problem. First, we designed a basic feasible solu-

tion by following the northwest-corner rule for making an initial assignment.*

Second, having obtained a basic feasible solution, we proceeded to determine the opportunity cost of the empty cell in order to determine whether the first program was an optimal program. The method employed to determine the opportunity cost of the empty cell consisted in (1) drawing a closed loop which passed through the empty cell and the adjacent occupied cells with proper plus and minus signs at the corners of the loop,† (2) shifting 1 unit of goods to the empty cell (accomplished by the addition of 1 unit to all those cells in which fell a plus sign of the closed loop and by the subtraction of 1 unit from all cells in which fell a minus sign), (3) determining the net cost change associated with shifting 1 unit to the empty cell, and (4) taking the negative of the net cost change in (3) to find the opportunity cost of the empty cell. In the simple transportation problem of Table 10.7, we had to find the opportunity cost of only one empty cell. In a problem of larger dimensions, the opportunity costs of all the empty cells must be determined by this procedure. The point to emphasize here is that a separate closed loop must be established for every empty cell (in the steppingstone method) before the opportunity costs of the empty cells can be determined.

Third, after ascertaining that the opportunity cost of the empty cell was positive, we changed the initial program by filling the empty cell

* In most cases, the initial assignment made by following the northwest-corner rule will be such that the number of occupied or filled cells equals $m + n - 1$, where m is the number of rows and n is the number of columns. When this happens, we have a basic feasible solution. If the number of occupied cells in the initial assignment is less than $m + n - 1$, the problem is said to be degenerate at the very beginning. This type of degeneracy, as well as the degeneracy occurring during the solution stages, can easily be resolved by judicious placement of a small number epsilon in the empty cell(s). The epsilon is to be placed in that cell(s) which will help complete the different loops for all the empty cells. An illustrative example is given in a later section (see Section 10.13).

† While tracing this closed loop, one should start with the empty cell being evaluated and draw an arrow from that empty cell to an occupied cell in the same row or column. Then, a plus sign is placed in the empty cell, and a negative sign in the occupied cell to which the arrow was drawn. Next, one moves horizontally or vertically (never diagonally) to another occupied cell, and so on, until one is back to the original empty cell. At each turn of the loop, plus and minus signs are placed alternately. Further, there is the important restriction that there are *exactly one positive* terminal and *exactly one negative* terminal in any row or column through which the loop happens to pass. Obviously, this restriction is imposed to ensure that the rim requirement will not be violated when the units are shifted (to obtain a new program) along this closed loop. Mechanically, this implies that during the tracing of the closed loop right-angle turns must be made *only* at the occupied cells. The starting point of a closed loop is identified by the symbol □ in this book.

(O_2D_1) as much as possible in view of the rim requirements.* The revision of the given program was guided by the plus and minus signs of the closed loop. The smallest of the numbers in the cells in which minus signs of the closed loop appeared (600) gave the total number of units to be shifted to the empty cell. The shifting was accomplished by adding this number (600) to all the cells containing the plus signs of the loop and subtracting it from all the cells containing the minus signs of the loop. These changes gave us our new basic feasible solution.

Finally, we tested the empty cell (O_2D_2) of the second program and found that its opportunity cost was not positive. We therefore came to the conclusion that an optimal solution to our problem had been obtained.

The procedure described above forms the core of the steppingstone method. Although the transportation problem that we solved was represented by only a 2×2 matrix (Table 10.7), the steppingstone method may be applied to any $m \times n$ matrix.

Suppose that our initial assignment in a 4×5 (4 rows and 5 columns) transportation problem results in 8 occupied cells and 12 empty cells. In order to test the optimality of this program and, then, to revise it, we must calculate the opportunity cost of each of the 12 empty cells. If we discover that the initial program can be improved, we revise the program by including that empty cell whose opportunity cost is highest. Note that, regardless of the number of empty cells having positive opportunity costs, only one cell at a time is included in the new program.

We shall now apply the steppingstone method to the problem of Table 10.2.

Step 1 Obtain an Initial Basic Feasible Solution

An initial basic feasible solution for a given transportation problem may be obtained by following the northwest-corner rule, by the application of Vogel's approximation method, or simply by inspection. It will be recalled that for the transportation problem of Table 10.2 we obtained three different basic feasible solutions, given in Tables 10.3, 10.5, and 10.6. Of these, let us take the basic feasible solution of Table 10.6 (initial assignment by inspection) as the starting point for obtaining the

* The fact that the opportunity cost of even a single cell is positive indicates that an optimum solution has not been obtained and that the given program must be revised to obtain a better basic feasible solution. Normally, the improved program will include that empty cell whose opportunity cost is highest. Since in this problem we had only one empty cell O_2D_1, the new program included that cell.

Table 10.12 *First Program*

Origin	Destination					Total
	D_1	D_2	D_3	D_4	D_5	
O_1	(10) / 12	4	(15) / 9	(30) / 5	9	55
O_2	8	(20) / 1	(25) / 6	6	7	45
O_3	(30) / 1	12	4	7	7	30
O_4	10	15	(10) / 6	9	(40) / 1	50
Total	40	20	50	30	40	

optimum solution by the steppingstone method. The data of Table 10.6 are reproduced in Table 10.12.

Check on Step 1

Since the number of occupied cells in this program equals $m + n - 1$, that is, $4 + 5 - 1 = 8$, this is indeed a basic feasible solution.

Step 2 Determine the Opportunity Costs of the Empty Cells

We repeat: in the steppingstone method a separate closed loop with proper plus and minus signs must be completed for *each* of the empty cells *before* the respective opportunity costs can be calculated.* Since our first program has a total of 12 empty cells, 12 different closed loops must be drawn. The opportunity cost associated with each empty cell is calculated in Table 10.13. An examination of these opportunity costs

* The reader should firmly grasp both the logic and the technique utilized in drawing these closed loops. Note the difference between the closed loop for cell O_1D_2 and that for O_3D_2.

Table 10.13 *Calculation of the Opportunity Costs for Table 10.6 and Indicated Changes*

Empty cell	Closed loop	Net cost change	Opportunity cost	Action
O_1D_2	$+O_1D_2 - O_1D_3 + O_2D_3 - O_2D_2$	$+4 - 9 + 6 - 1 = 0$	0	Indifferent
O_1D_5	$+O_1D_5 - O_4D_5 + O_4D_3 - O_1D_3$	$+9 - 1 + 6 - 9 = +5$	−5	Do not include in next program
O_2D_1	$+O_2D_1 - O_1D_1 + O_1D_3 - O_2D_3$	$+8 - 12 + 9 - 6 = -1$	+1	Candidate for inclusion in next program
O_2D_4	$+O_2D_4 - O_1D_4 + O_1D_3 - O_2D_3$	$+6 - 5 + 9 - 6 = +4$	−4	Do not include in next program
O_2D_5	$+O_2D_5 - O_2D_3 + O_4D_3 - O_4D_5$	$+7 - 6 + 6 - 1 = +6$	−6	Do not include in next program
O_3D_2	$+O_3D_2 - O_3D_1 + O_1D_1 - O_1D_3 + O_2D_3 - O_2D_2$	$+12 - 1 + 12 - 9 + 6 - 1 = +19$	−19	Do not include in next program
O_3D_3	$+O_3D_3 - O_1D_3 + O_1D_1 - O_3D_1$	$+4 - 9 + 12 - 1 = +6$	−6	Do not include in next program
O_3D_4	$+O_3D_4 - O_1D_4 + O_1D_1 - O_3D_1$	$+7 - 5 + 12 - 1 = +13$	−13	Do not include in next program
O_3D_5	$+O_3D_5 - O_4D_5 + O_4D_3 - O_1D_3 + O_1D_1 - O_3D_1$	$+7 - 1 + 6 - 9 + 12 - 1 = +14$	−14	Do not include in next program
O_4D_1	$+O_4D_1 - O_1D_1 + O_1D_3 - O_4D_3$	$+10 - 12 + 9 - 6 = +1$	−1	Do not include in next program
O_4D_2	$+O_4D_2 - O_2D_2 + O_2D_3 - O_4D_3$	$+15 - 1 + 6 - 6 = +14$	−14	Do not include in next program
O_4D_4	$+O_4D_4 - O_4D_3 + O_1D_3 - O_1D_4$	$+9 - 6 + 9 - 5 = +7$	−7	Do not include in next program

shows that cell O_2D_1 is the only cell with a positive opportunity cost. Hence, cell O_2D_1 must be included in our next program.

A comment about the opportunity cost of zero (for empty cell O_1D_2) is in order. An opportunity cost of zero associated with a particular empty cell at any stage of the problem solution indicates that if this cell is included in the next program the total cost of the new program will be the same as that of the current program. Thus we list "Indifferent" in the "Action" column of Table 10.13.

Step 3 Revising a Given Program

Having ascertained that a given program is not an optimal program (one or more empty cells have positive opportunity cost), we next revise the

Table 10.14

First program

	D_1	D_2	D_3	D_4	D_5
O_1	12 − (10)	4	9 + (15)	5 (30)	9
O_2	8 +	1 (20)	6 − (25)	6	7
O_3	1 (30)	12	4	7	7
O_4	10	15	6 (10)	9	1 (40)

Revised program

	D_1	D_2	D_3	D_4	D_5
O_1	12	4	9 (25)	5 (30)	9
O_2	8 (10)	1 (20)	6 (15)	6	7
O_3	1 (30)	12	4	7	7
O_4	10	15	6 (10)	9	1 (40)

given program to obtain a new basic feasible solution. The revised program must include that empty cell of the current program whose opportunity cost is highest. No choice is necessary here, since cell O_2D_1 is the only empty cell having a positive opportunity cost.

The revision of the first program is guided by the closed loop of the empty cell to be included (in this case cell O_2D_1) and is as shown in Table 10.14. Since 10 is the smallest number in a negative cell in the closed loop, it is added to the cells containing plus signs and subtracted from the cells containing minus signs.

The next question is: Does our revised program represent an optimal solution? To answer this question, we have to repeat step 2, as discussed

previously. Should the result of step 2 indicate a nonoptimal solution, we would repeat step 3, namely, obtain another basic feasible solution. In other words, after the initial basic feasible solution has been obtained, the optimal solution is derived by the repeated application of steps 2 and 3. Determination of the opportunity costs of all the empty cells of the revised program (Table 10.14) will reveal that an optimal solution has indeed been derived. The reader is encouraged to verify this by the application of step 2 to Table 10.14.

10.8 MODIFIED-DISTRIBUTION METHOD FOR DERIVING AN OPTIMAL SOLUTION (MODI)

The main difference between the steppingstone method and the modified-distribution method (MODI) for solving transportation problems concerns the stage of the problem solution at which the closed loop(s) is drawn. In the steppingstone method, the closed loops for all the empty cells are drawn *before* their respective opportunity costs can be calculated. The empty cell to be included in the next program is then identified as that having the highest opportunity cost. In other words, the procedure for calculating the opportunity costs of the empty cells is dependent on the tracing of the closed loops.

In the modified-distribution method, however, the opportunity costs of all the empty cells are calculated and the highest opportunity cost is identified before any closed loop is drawn. As a matter of fact, in the modified-distribution method we need draw *only one* closed loop *after* the highest-opportunity-cost cell has been identified. Thus, the procedure for calculating the opportunity costs of the empty cells in MODI is independent of the tracing of the loops.

We shall illustrate the mechanics and rationale of the modified-distribution method by solving the simple transportation problem of Table 10.7, the data of which are reproduced in Table 10.15. The initial assignment made by following the northwest-corner rule is given in Table 10.16.

Determining the Opportunity Costs of the Empty Cells

Having solved this problem by the steppingstone method, we are aware of the fact that the transfer of 1 unit to cell O_2D_1 (a closed loop for the empty cell was established in Table 10.9 *before* this transfer was made) results in a net cost change of −1 dollar. This, of course, means that the

opportunity cost of *not* including cell O_2D_1 in the first program is +1 dollar per unit of goods. Another method of reaching the same conclusion is via the determination of what may be called the *implied cost* of an empty cell. The implied cost of an empty cell sets an upper limit (in view of the existing program*) beyond which the inclusion of this cell in a new program is not an advantageous proposition. Let us explain in connection with our transportation problem.

Table 10.15

Origin	Destination D_1	Destination D_2	Origin capacity per time period
O_1	[2]	[2]	1,000
O_2	[1]	[2]	600
Destination requirement per time period	900	700	

Table 10.16 *Initial Basic Feasible Solution*

	D_1	D_2
O_1	[2] (900)	[2] (100)
O_2	[1]	[2] (600)

* The implied cost of a given empty cell can change from one program to another, since the implied cost is indicative of the *relative* advantage or disadvantage of not using a given cell in a particular program.

In this case, cell O_2D_1 is the only empty cell. One way to find its implied cost is by drawing a closed loop and determining the net cost consequence of shifting 1 unit of goods into O_2D_1. Ignoring, for the time being, the actual shipping cost per unit via route O_2D_1, we may calculate the net cost consequence of shifting 1 unit of goods into O_2D_1 as

$$O_2D_1 - O_1D_1 + O_1D_2 - O_2D_2 = O_2D_1 - 2 + 2 - 2 = O_2D_1 - 2$$

Whatever the actual shipping cost per unit via cell O_2D_1, it is obvious that the above shift is desirable only if the net cost change $(O_2D_1 - 2)$ is negative. It will be negative so long as the actual cost of O_2D_1 is less than 2. The calculated upper limit for the actual cost of cell O_2D_1 (in the existing program), beyond which the inclusion of this cell is not an advantageous proposition, is therefore 2. In other words, if the actual shipping cost via cell O_2D_1 is greater than \$2 per unit, the shift is not desirable. On the other hand, if the actual shipping cost is less than \$2 per unit, the shift is desirable and cell O_2D_1 should be included in the next program.

The implied cost of the empty cell O_2D_1 therefore is \$2 per unit.

Also, as we noted earlier, the negative of the net cost change involved in shifting 1 unit of goods to an empty cell gives the opportunity cost associated with the empty cell. For cell O_2D_1,

$$\text{Opportunity cost} = -(\text{net cost change}) = -(O_2D_1 - 2) = 2 - O_2D_1$$

where O_2D_1 is the *actual* cost of shipment per unit via cell O_2D_1. But, as we have just calculated, the implied cost of not using cell O_2D_1 is \$2 per unit. Hence,

$$\text{Opportunity cost} = \text{implied cost} - \text{actual cost}$$

Substituting the actual shipping cost via cell O_2D_1 (\$1) and the calculated implied cost of cell O_2D_1 in the above expression, we find that the opportunity cost (of cell O_2D_1) is $2 - 1 = +1$ dollar. This is the same value of opportunity cost (for cell O_2D_1) that we found earlier by a direct observation of the net cost consequence associated with shifting 1 unit of goods into cell O_2D_1. This equivalence holds for *any* empty cell, and we state again the general relationship:

$$\text{Opportunity cost} = \text{implied cost} - \text{actual cost}$$

Although we have now succeeded in determining the opportunity cost of an empty cell by developing the concept of implied cost, it has been

possible to do so only by first drawing a closed loop. The next logical question is Can we somehow determine the implied cost of an empty cell without first drawing the closed loop? Should we find this to be possible, we would establish the main framework for the MODI method, for then we could subtract the actual cost of the empty cell from its calculated implied cost and thus determine its opportunity cost without first drawing the closed loop.*

In this and the following paragraphs we shall develop a method for determining the implied costs of empty cells without drawing their respective loops Let us refer back to the initial basic feasible solution of Table 10.16. In this program we have three occupied cells. In linear-programming terms, this means that three (x_{11}, x_{12}, x_{22}) of the four variables are basis variables. It will be recalled from the simplex method that the opportunity cost (represented by the numbers in the net-evaluation row) of any variable comprising the basis is zero. Similarly, it can be shown in the case of the transportation problem that the opportunity cost of each of the occupied cells (cells containing the basis variables) is zero In other words, if the basis variables are not going to be changed, then the hypothetical introduction and removal of 1 unit in any occupied cell will not result in any net cost change. Now, if we assign a complete set of row numbers (to be placed at the extreme right-hand side of the table containing a given program) and a complete set of column numbers (to be placed at the bottom of the table) in such a way that the shipping cost per unit of *each* of the occupied cells equals the sum of its row and column numbers, we shall satisfy the condition that the opportunity cost of each occupied cell be zero.† Further, since the sum of the row and column numbers of any occupied cell equals the cost of that cell (a basis variable), the sum of the row and column numbers corresponding to each empty cell (nonbasis routes) gives the implied cost of that empty cell.

* As the reader will recall, it is this feature that distinguishes the MODI method from the steppingstone method.

† The transportation problem, if fed into the first simplex tableau, consists of column vectors representing structural and other variables (see Figure 10.1) In each column vector representing a structural variable, two of the entries are 1, and the rest are 0. It is this special property of the transportation problem which makes it quite easy to find a new basis by the MODI method The MODI method guides us to a new basis after all the empty cells of the transportation matrix have been "evaluated" simultaneously. A set of row numbers u_i and a set of column numbers v_j are chosen so that the opportunity cost of *each* cell is given by $u_i + v_j - c_{ij}$, where c_{ij} is the actual shipping cost per unit of the cell falling in ith row and jth column. *Thus, if we choose u_i and v_j such that for all the occupied cells (basis routes) $c_{ij} = u_i + v_j$, we satisfy the requirement that the opportunity cost of each occupied cell is zero* For the empty cells (nonbasis routes), opportunity cost is given by $u_i + v_j - c_{ij}$

The implied cost of *any* empty cell, therefore, is given by

$$\text{Implied cost} = \text{row number} + \text{column number} = u_i + v_j$$

Thus, by the assignment of row and column numbers, we can calculate the implied cost of each empty cell without drawing a closed loop. We must now tackle the problem of assigning these row and column numbers.

For each occupied cell, we have to choose u_i (row number) and v_j (column number) such that c_{ij} (the actual shipping cost per unit in the occupied cell) equals the sum of u_i and v_j. For the occupied cell falling in row 1 and column 1, for example, u_1 and v_1 are chosen such that $c_{11} = u_1 + v_1$. Similarly, for cell O_1D_2 we must chose u_1 and v_2 such that $c_{12} = u_1 + v_2$. This process must be carried out for all the occupied cells But it should be realized that although a basic feasible solution for a transportation problem consists of $m + n - 1$ variables (in other words, there are $m + n - 1$ occupied cells), we must assign $m + n$ values to obtain a complete set of row and column numbers. Hence, to determine all the row and column numbers, one *arbitrary* number, serving as either a row or a column number, must be chosen. Once one row number or column number has been chosen arbitrarily, the rest of the row and column numbers can be determined by the relationship $c_{ij} = u_i + v_j$. This relationship, as stated earlier, *must* hold for all the *occupied* cells. In so far as any arbitrary number can be chosen to represent one of the u_i's or v_j's, we shall follow the practice of making u_1 take the value zero. This completes the description of the procedure for determining the row and column numbers. The actual numbers for our example are given in Table 10.17. If we arbitrarily choose a value of zero for u_1, our next question is. What value must be given to v_1 so that $c_{11} = u_1 + v_1$ or $2 = 0 + v_1$? Obviously, v_1 must take a value of 2. Next, we ask: What value must be given to v_2 so that $c_{12} = u_1 + v_2$ or $2 = 0 + v_2$? The value of v_2 must be 2. Again, what value must be given to u_2 so that $c_{22} = u_2 + v_2$ or $2 = u_2 + 2$? Obviously, $u_2 = 0$. By first assigning an arbitrary value to u_1 and then posing a series of questions, we have determined all the row and column numbers.

Let us now calculate the opportunity cost for the empty cell O_2D_1. The opportunity cost of an empty cell, as stated earlier, is given by implied cost − actual cost, that is, by $(u_i + v_j) - c_{ij}$. For cell O_2D_1, therefore, the opportunity cost is $u_2 + v_1 - c_{21} = 0 + 2 - 1 = +1$ dollar. The answer, of course, is the same as that obtained by the long method. In so far as the opportunity cost of the empty cell O_2D_1 is positive, this is not an optimum program and hence must be revised.

Before [illegible] the above program, let us summarize the role of the

row and column numbers. In so far as the row and column numbers are assigned in such a manner that the *actual* cost of every occupied cell equals the sum of its row and column numbers, the sum of the row and column numbers of each empty cell gives the implied cost of that empty

Table 10.17

Origin	Destination D_1	Destination D_2	Row number
O_1	[2] (900)	[2] (100)	0
O_2	[1]	[2] (600)	0
Column number	2	2	

Table 10.18

Implied cost		Actual cost	Action
$u_i + v_j$	$>$	c_{ij}	A better program can be designed by including this cell in the solution
$u_i + v_j$	$=$	c_{ij}	Indifferent; however, an alternative program with the same total cost and including this cell can be designed
$u_i + v_j$	$<$	c_{ij}	Do not include this cell in the program

cell (unused route). If the implied cost of the empty cell is less than its actual cost, this route should be left out of our shipping program. If, on the other hand, the implied cost ($u_i + v_j$) of an empty cell is more than its actual cost (c_{ij}), then this route would be a candidate for inclusion in our next program. In summary, to evaluate and improve a given program in which the objective is to *minimize* a given function, the rules given in Table 10 18 apply.[4] For a transportation problem in which the

objective is to *maximize* a given function, the signs of the inequalities given in the table must be reversed to establish the guidelines for action.

Let us now return to our problem.

Revising a Given Program

The last step in the MODI method is exactly the same as the corresponding step in the steppingstone method. Having identified the empty cell to be included in the next program (the cell with the highest opportunity cost), we draw a closed loop for this cell. The new basic feasible solution is then derived by shifting into the empty cell the maximum possible

Table 10.19

First program

	D_1	D_2
O_1	[2] − (900)	[2] + (100)
O_2	[1] + □	[2] − (600)

Revised program

	D_1	D_2
O_1	[2] (300)	[2] (700)
O_2	[1] (600)	[2]

number of units without violating the rim requirements. The revised program is given in Table 10 19. To determine if the revised program is an optimal program, we must determine the opportunity cost of the empty cell O_2D_2. This is illustrated in Table 10 20. From the table, we see that

$$\textit{Implied} \text{ cost of cell } O_2D_2 = u_2 + v_2 = -1 + 2 = +1$$

$$\text{Actual cost of cell } O_2D_2 = +2$$

Hence

$$\text{Opportunity cost of empty cell } O_2D_2 = \text{implied cost} - \text{actual cost} = +1 - 2 = -1$$

In so far as the opportunity cost of the only empty cell is nonpositive, no

improvement in the present program is possible. This program, then, is the optimal program.

We shall now apply the modified-distribution method to the problem of Table 10.2.

Table 10.20

Origin	Destination D_1	D_2	Row number
O_1	2 / (300)	2 / (700)	0
O_2	1 / (600)	2	−1
Column number	2	2	

Step 1 Obtaining an Initial Basic Feasible Solution

As discussed earlier, an initial basic feasible solution for a given transportation problem may be obtained by following the northwest-corner rule, by the application of Vogel's approximation method, or simply by inspection. Table 10.21 reproduces the basic feasible solution of Table 10.6 (initial assignment by inspection), which we shall take as a starting point for obtaining the optimal solution by MODI. In so far as the number of occupied cells in this program equals $m + n - 1$, that is, $4 + 5 - 1 = 8$, this is indeed a basic feasible solution.

Step 2 Determining the Opportunity Costs of the Empty Cells

To determine the opportunity costs of the empty cells by the MODI method, we must first determine the implied costs of the empty cells by assigning a complete set of row and column numbers. This is shown in Table 10.22. The uncircled numbers in the matrix represent the implied

Table 10.21 *First Program*

Origin	Destination D_1	D_2	D_3	D_4	D_5	Total
O_1	12 / (10)	4	9 / (15)	5 / (30)	9	55
O_2	8	1 / (20)	6 / (25)	6	7	45
O_3	1 / (30)	12	4	7	7	30
O_4	10	15	6 / (10)	9	1 / (40)	50
Total	40	20	50	30	40	

Table 10.22

	D_1	D_2	D_3	D_4	D_5	Row number
O_1	12 / (10)	4 / 4	9 / (15)	5 / (30)	9 / 4	0
O_2	8 / 9	1 / (20)	6 / (25)	6 / 2	7 / 1	−3
O_3	1 / (30)	12 / −7	4 / −2	7 / −6	7 / −7	−11
O_4	10 / 9	15 / 1	6 / (10)	9 / 2	1 / (40)	−3
Column number	12	4	9	5	4	

costs of the empty cells. A comparison of the implied and actual costs of each empty cell shows that only cell O_2D_1 has a positive opportunity cost of +1 dollar. For cell O_2D_1, opportunity cost = implied cost − actual cost = 9 − 8 = +1. A similar calculation for cell O_1D_2 shows that its opportunity cost is zero. The opportunity costs for the rest of the empty cells are negative.

Having identified the presence of positive opportunity cost, we know that this program is not an optimum program. Hence it must be revised to include that empty cell which has the highest opportunity cost (in this case cell O_2D_1).

Step 3 Revising the Given Program

The revision of the given program is guided by a closed loop drawn for the empty cell which is to be included in the next program. The loop for

Table 10.23

	D_1	D_2	D_3	D_4	D_5	Row number
O_1	[12] 11	[4] 4	[9] (25)	[5] (30)	[9] 4	0
O_2	[8] (10)	[1] (20)	[6] (15)	[6] 2	[7] 1	−3
O_3	[1] (30)	[12] −6	[4] −1	[7] −5	[7] −6	−10
O_4	[10] 8	[15] 1	[6] (10)	[9] 2	[1] (40)	−3
Column number	11	4	9	5	4	

cell O_2D_1 is drawn and the program is revised in exactly the same manner as shown in Table 10.14. The revised program is then tested for optimality (by the application of step 2), as shown in Table 10.23. A comparison of the uncircled numbers (representing the implied costs) in the

empty cells and the respective actual costs shows that no empty cell has a positive opportunity cost.* Hence, this is an optimal solution.

10.9 PROCEDURE SUMMARY FOR THE MODIFIED-DISTRIBUTION METHOD (MINIMIZATION CASE)

Step 1 Obtain a Basic Feasible Solution

An initial basic feasible solution for a given transportation problem may be obtained by following the northwest-corner rule, by the application of Vogel's approximation method, or by simple inspection.

Test for step 1. A basic feasible solution must include shipments covering $m + n - 1$ cells. That is, the number of occupied cells (basis variables) is 1 less than the number of rows and columns in the transportation matrix.

If the number of occupied cells in the initial solution is more than $m + n - 1$, there is a computational error which can easily be corrected by rechecking the data. If the number of occupied cells is less than $m + n - 1$, this is a degenerate solution. To resolve degeneracy, add one or more epsilons to some "suitable" empty cells so that the number of occupied cells becomes equal to $m + n - 1$.†

Step 2 Determine the Opportunity Costs of the Empty Cells (Opportunity Cost = Implied Cost − Actual Cost)

a. *Determine a complete set of row and column numbers (values).* When, in a given program, the number of occupied cells equals $m + n - 1$, proceed to assign row and column numbers (values) in such a manner that, for *each occupied cell*, the relationship $c_{ij} = u_i + v_j$ holds. To start, a value of zero can be assigned to any row having an occupied cell. The rest of the row and column numbers can then be determined by making sure that, for each occupied cell, $c_{ij} = u_i + v_j$. In other words, for each occupied cell, the actual shipping cost per unit should equal the sum of its row and column values.
b. *Calculate the implied costs of the empty cells.* Once all the row and column values have been assigned, the implied cost of a given empty

* The fact that empty cell O_1D_2 has an opportunity cost of zero means that an alternative program which will include cell O_1D_2 and have the same total shipping cost as this program can be designed.

† Degeneracy in transportation problems is illustrated in Section 10.13.

cell can be calculated as follows:

Implied cost = row value + column value

c. *Determine the opportunity costs of the empty cells.* The opportunity cost of an empty cell is determined by subtracting the actual cost of the empty cell from its implied cost In other words, opportunity cost, for each cell, is given by

Opportunity cost = $u_i + v_j - c_{ij}$

If the opportunity costs of all the empty cells are nonpositive, an optimal solution has been obtained. If, on the other hand, even a single cell has a positive opportunity cost, a better program can be designed. Thus, step 2 serves as a test for optimality.

Step 3 Design an Improved Program

Design a new program such that the empty cell having the largest opportunity cost (in the program to be revised) is included in the solution. This is accomplished in the following manner:

a Draw a loop of horizontal and vertical arrows in such a manner that it starts from the empty cell to be filled, passes to an occupied cell in the same row or column as the empty cell, and then, making a series of alternate horizontal and vertical turns through *occupied* cells, returns to the original empty cell.

b Place a plus sign (+) in the empty cell to be filled. Then, alternately, place minus signs (−) and plus signs (+) at the beginnings and ends of the connecting links of the loop.

c Examine those occupied cells in which the minus signs have been placed. Of these, the cell having the least number of units is vacated by transferring these units to the empty cell This is accomplished by adding the same amount to all cells having plus signs and subtracting it from all cells having minus signs. The improved program should have the same number of occupied cells as the preceding program. If the number of occupied cells in the improved program is less than that of the preceding program, the problem becomes degenerate. In such a case, add epsilon(s) to some *recently vacated* cell(s) such that the number of occupied cells again equals $m + n - 1$.*

* See Section 10.13.

Step 4

Repeat steps 2 and 3 until a program is achieved in which each empty cell has an opportunity-cost value which is either zero or negative. This program will be the optimal program.

10.10 MODIFIED-DISTRIBUTION METHOD (MAXIMIZATION CASE)

Except for one transformation, a transportation problem in which the objective is to maximize a given function can be solved by the MODI algorithm as presented above. The transformation is made by subtracting all the c_{ij}'s from the highest c_{ij} (profit) of the given transportation matrix. The transformed c_{ij}'s give us the *relative costs*, and the problem then becomes a minimization problem. Once an optimal solution to this transformed minimization problem has been found, the value of the objective function can be calculated by inserting the original values of the c_{ij}'s for those routes which form the basis (occupied cells) of the optimal solution.

10.11 BALANCING THE GIVEN TRANSPORTATION PROBLEM

To solve a given transportation problem by the step-by-step procedure given in Section 10.9, we must establish equality between the total capacities of the origins and the total requirements of the destinations. Three cases can arise.

Case 1 $\Sigma b_i = \Sigma d_j$

In this case, the total capacity of the origins equals the total requirement of the destinations. The problem can be arranged in the form of a matrix, along with the relevant cost data, and the transportation algorithm may be applied directly to obtain a solution.

Case 2 $\Sigma b_i > \Sigma d_j$

In this case, the total capacity of the origins exceeds the total requirement of the destinations. A "dummy" destination can be added to the matrix

to absorb the excess capacity. The cost of shipping from each origin to this dummy destination is assumed to be zero. The adding of a dummy destination establishes equality between the total origin capacities and total destination requirements. The problem is then amenable to solution by the transportation algorithm.

Illustrative Example

Table 10.24 gives both the unbalanced and balanced forms of a transportation problem in which the total given capacity of the origins exceeds the total given requirement of the destinations ($\Sigma b_i > \Sigma d_j$). In practice, the optimal solution identifies the particular origin at which the excess capacity should be left idle.

Table 10.24

Unbalanced form

	D_1	D_2	D_3	Origin capacity
O_1	5	3	2	200
O_2	6	4	1	400
Destination requirement	200	200	150	

Balanced form

	D_1	D_2	D_3	Dummy	Origin capacity
O_1	5	3	2	0	200
O_2	6	4	1	0	400
Destination requirement	200	200	150	50	

Case 3 $\Sigma b_i < \Sigma d_j$

In this case, the total capacity of the origins is less than the total requirement of the destinations. A dummy origin can be added to the transportation matrix to meet the excess demand. The cost of shipping from the dummy origin to each destination is assumed to be zero. The adding of a dummy origin in this case establishes the equality between the total capacity of the origins and the total requirement of the destinations.*

* The reader will observe that the role of the dummy column or dummy row containing dummy variables in a transportation problem is parallel to the role of the slack

Illustrative Example

Table 10.25 gives both the unbalanced and balanced forms of a transportation problem in which the total given capacity of the origins is less than the total requirement of the destinations ($\Sigma b_i < \Sigma d_j$). In practice, the optimal solution identifies the particular destination whose requirement cannot be fully satisfied.

In the initial assignment for a transportation problem which has been balanced by the addition of a dummy origin or a dummy destination, only the last necessary allocations should be made to the dummy cells.

Table 10.25

Unbalanced form

	D_1	D_2	D_3	Origin capacity
O_1	5	3	2	200
O_2	6	4	1	400
Destination requirement	300	200	150	

Balanced form

	D_1	D_2	D_3	Origin capacity
O_1	5	3	2	[illegible]
O_2	6	4	1	[illegible]
Dummy	0	0	0	50
Destination requirement	300	200	150	

This procedure, in general, will result in fewer iterations before an optimal solution is derived.

10.12 SIMPLEX TRANSLATION OF THE TRANSPORTATION METHOD

Some important observations can be made regarding parallels in the general transportation model and the simplex method. First, the role of the dummy variables in the transportation problem is similar to the role of the slack variables in the general linear-programming problem. Second, the occupied cells and empty cells of the transportation program correspond, respectively, to the basis variables and nonbasis variables of the simplex tableau.

Third, the revision of a given transportation program is parallel to the process of obtaining a new basis in the simplex method... Let us

explain this point further. A given transportation program, it will be recalled, is improved by filling or including one empty cell (that having the highest opportunity cost) at a time. In this process, all the units from *at least one* cell are removed. Thus, a new cell is filled and becomes an occupied cell, and at least one of the previously occupied cells joins the category of empty cells. The total number of occupied cells, therefore, can either remain constant (only one previously occupied cell becomes an empty cell) or decrease (more than one of the previously occupied cells become empty cells) from one program to the next. If the number of occupied cells remains the same from one program to the next, the process is similar to a simplex iteration in which one new (nonbasis) variable is introduced into the solution to remove one of the basis variables currently in the solution. Of course, in this case we obtain a new basic feasible solution. If, on the other hand, the process of filling one empty cell results in the simultaneous vacating of two or more of the currently occupied cells, the transportation problem becomes degenerate. This latter situation, as the reader will observe, is parallel to the simplex iteration in which the introduction of one new (nonbasis) variable removes, simultaneously, two or more of the current basis variables—here, too, the problem becomes degenerate.

10.13 DEGENERACY IN TRANSPORTATION PROBLEMS

It was established earlier that a basic feasible solution for a transportation problem consists of $m + n - 1$ basis variables. This means that the number of occupied cells in a given transportation program is 1 less than the number of rows and columns in the transportation matrix. Whenever the number of occupied cells is less than $m + n - 1$, the transportation problem is said to be degenerate.

Degeneracy in transportation problems can develop in two ways. First, the problem may become degenerate when the initial program is designed via one of the initial-assignment methods discussed earlier. To resolve degeneracy in this case, we can allocate an extremely small amount of goods (close to zero) to one or more of the empty cells,* so that the number of occupied cells becomes $m + n - 1$. The cell containing this extremely small allocation is, of course, considered to be an occupied cell.

* This extremely small amount, represented by epsilon, ϵ, may be allocated to any empty cell subject to the condition that this will make possible the determination of a unique set of row and column numbers.

In linear-programming literature, this extremely small amount is usually denoted by the Greek letter ϵ (epsilon). The amount ϵ is assumed to be so small that its addition to or subtraction from a given number does not change that number. For example, $50 + \epsilon = 50$, and $200 - \epsilon = 200$. Of course, if ϵ is subtracted from itself, the result is assumed to be zero; that is, $\epsilon - \epsilon = 0$.

The development of degeneracy during the initial assignment and its resolution will be illustrated with the transportation problem of Table 10.26.

Table 10.26 *Data for the Transportation Problem*

Origin	Destination			Origin capacity
	D_1	D_2	D_3	
O_1	2	1	2	20
O_2	3	4	1	40
Destination requirement	20	15	25	

Second, the transportation problem may become degenerate during the solution stages. This happens when the inclusion of the most favorable empty cell (the cell having the highest opportunity cost) results in the simultaneous vacating of two or more of the currently occupied cells. To resolve degeneracy in this case, we allocate ϵ to one or more of the recently vacated cells, so that the number of occupied cells in the new program is $m + n - 1$. This type of degeneracy and its resolution will be illustrated with the transportation problem of Table 10.30.

Case 1 Degeneracy during the Initial Assignment

Following the northwest-corner rule, we obtain the initial assignment given in Table 10.27. Note that the number of occupied cells in this

program is 3, which does not equal $m + n - 1$. Hence, the problem is degenerate at the very beginning, and no attempt to assign row and column numbers to Table 10.27 will succeed. However, we can resolve this degeneracy by the addition of epsilon to any of the empty cells. In so far as this is a minimization problem, we allocate epsilon to the lowest-cost cell O_1D_2 (see Table 10.28). With this modification, the number of occupied cells equals 4, that is, $m + n - 1$. Hence, it is now possible to assign a unique set of row and column numbers in order to apply the MODI method (see Table 10.28). It is clear that the implied cost of cell O_2D_1 is 5 ($u_2 + v_1 = 2 + 3 = 5$), whereas its actual cost is 3. Hence,

Table 10.27 *Initial Assignment by Northwest-corner Rule (a Degenerate Solution)*

	D_1	D_2	D_3	Total
O_1	[2] (20)	[1]	[2]	20
O_2	[3]	[4] (15)	[1] (25)	40
Total	20	15	25	

the opportunity cost of cell O_2D_1 is $5 - 3 = +2$. Thus, the program in Table 10.28 is not optimal.

The revision of the program is shown in Table 10.29. The closed loop of the table shows that at most 15 units can be shifted to cell O_2D_1. This necessitates the subtraction of 15 units from O_1D_1 and from O_2D_2 and the addition of 15 units to O_1D_2. In so far as $15 + \epsilon = 15$, cell O_1D_2 has been assigned a total of 15 units by this shifting process. Also, note that the revised program is not a degenerate solution. If we test the revised program for optimality, we find that it represents an optimal solution to the given problem. The reader can immediately verify this by assigning a set of row and column numbers to the matrix representing the revised program and then calculating the opportunity costs of its empty cells.

When degeneracy developed in the initial assignment for the above transportation problem (Table 10.27), we noted that the addition of ϵ to

any of the empty cells enabled us to determine a unique set of row and column numbers. This is usually true whenever the initial assignment is made by following the northwest-corner rule. However, when the initial assignment is made by another method, such as by inspection, one cannot add epsilon to just any empty cell. Instead, ϵ must be added to one of

Table 10.28*

	D_1	D_2	D_3	Row number
O_1	[2] (20)	[1] (ϵ)	[2]	0
O_2	[3]	[4] (15)	[1] (25)	3
Column number	2	1	−2	

* Initial basic feasible solution (after the addition of ϵ to cell O_1D_2) and its row and column numbers.

Table 10.29

First program

	D_1	D_2	D_3	Total
O_1	[2] − (20)	[1] + (ϵ)	[2]	20
O_2	[3] + □	[4] − (15)	[1] (25)	40
Total	20	15	25	

Revised program

	D_1	D_2	D_3	Total
O_1	[2] (5)	[1] (15)	[2]	20
O_2	[3] (15)	[4]	[1] (25)	40
Total	20	15	25	

those empty cells which will make possible the determination of a unique set of row and column numbers. Let us illustrate this point by considering one particular degenerate solution of the transportation problem of Table 10.30.

Clearly, the initial assignment, by inspection, gives a degenerate solution, since the number of occupied cells is less than $m + n - 1$. To resolve this degenerate solution which has developed at the very start,

we must add ϵ to one of the empty cells. But we must choose this empty cell with careful judgment, for if we make any one of the empty cells O_2D_2, O_4D_1, and O_5D_1 an occupied cell by the addition of ϵ we shall not be able to assign a unique set of row and column numbers. On the other hand, the addition of ϵ to any one of the cells O_1D_1, O_1D_2, O_2D_3, O_3D_3, O_4D_3, and O_5D_3 will enable us to resolve the degeneracy and allow us to determine a unique set of row and column numbers. The task of verifying these statements is left to the reader.

Table 10.30

Data for the transportation problem

	D_1	D_2	D_3	Total
O_1	[2]	[4]	[1]	40
O_2	[6]	[3]	[2]	50
O_3	[4]	[5]	[6]	20
O_4	[3]	[2]	[1]	30
O_5	[5]	[2]	[5]	10
Total	50	60	40	

An initial assignment by inspection

	D_1	D_2	D_3	Total
O_1	[2]	[4]	[1] (40)	40
O_2	[6] (30)	[3] (20)	[2]	50
O_3	[4] (20)	[5]	[6]	20
O_4	[3]	[2] (30)	[1]	30
O_5	[5]	[2] (10)	[5]	10
Total	50	60	40	

Once a unique set of row and column numbers has been determined, the various steps of the transportation algorithm can be applied in a routine manner to obtain an optimal solution.

Case 2 Degeneracy during the Solution Stages

Given in Table 10.31*a* and *b* are a transportation problem and an initial assignment derived by following the northwest-corner rule. The solution represented by Table 10.31*b* is a basic feasible solution. Should we assign a set of row and column numbers to the empty cells of this

program, we would discover that there are several empty cells (including cell O_1D_3) having positive opportunity costs. Let us decide, arbitrarily, to include cell O_1D_3 in a new program. This necessitates shifting 10 units to cell O_1D_3 as guided by the closed loop shown in Table 10.31*b*. The resulting program, given in Table 10.31*c*, is a degenerate solution.

Table 10.31*a* *Data for the Transportation Problem*

	D_1	D_2	D_3	D_4	D_5	Total
O_1	[4]	[3]	[1]	[2]	[6]	40
O_2	[5]	[2]	[3]	[4]	[5]	30
O_3	[3]	[5]	[6]	[3]	[2]	20
O_4	[2]	[4]	[4]	[5]	[3]	10
Total	30	30	15	20	5	

Table 10.31*b* *Initial Assignment by Northwest-corner Rule*

	D_1	D_2	D_3	D_4	D_5	Total
O_1	[4] (30)	[3] − (10)	[1] + □	[2]	[6]	40
O_2	[5]	[2] + (20)	[3] − (10)	[4]	[5]	30
O_3	[3]	[5]	[6] (5)	[3] (15)	[2]	20
O_4	[2]	[4]	[4]	[5] (5)	[3] (5)	10
Total	30	30	15	20	5	

Table 10.31c *Program 2*

	D_1	D_2	D_3	D_4	D_5	Total
O_1	[4] (30)	[3]	[1] (10)	[2]	[6]	40
O_2	[5]	[2] (30)	[3]	[4]	[5]	30
O_3	[3]	[5]	[6] (5)	[3] (15)	[2]	20
O_4	[2]	[4]	[4]	[5] (5)	[3] (5)	10
Total	30	30	15	20	5	

Since the degeneracy has developed during the solution stages, we should resolve it by adding ϵ to one of the recently vacated cells, i.e., cell O_1D_2 or cell O_2D_3. The reader should verify that, in this case, a set of row and column values can be determined *only* if epsilon is added to one of the empty cells O_1D_2, O_3D_2, O_4D_2, O_2D_1, O_2D_3, O_2D_4, or O_2D_5.

In so far as this is a minimization problem, we should add epsilon to that recently vacated cell which has the lowest shipping cost per unit. There being a tie in this case between cell O_1D_2 and cell O_2D_3, we arbitrarily decide to add ϵ to cell O_1D_2. This enables us to assign a unique set of row and column values to program 2, calculate the opportunity costs of the empty cells, and identify the most favorable empty cell to be included in the next program (see Table 10.31*e*).

The solution represented by program 3 is, again, degenerate. We therefore add another epsilon to cell O_4D_4 because, of the two cells recently vacated (O_3D_3 and O_4D_4), cell O_4D_4 has the lower shipping cost per unit.* This means that we now have two cells (O_1D_2 and O_4D_4 in Table 10.31*f*) having an assignment of epsilon. This modification enables

Table 10.31*d* · *Revised Program 2*

	D	D	D_3	D_4	D_5	Row number
O	[4] [illegible]	[illegible] ε	[1] − 10 +	[2]	[illegible]	0
O_2	[illegible]	[illegible] [illegible]	[3]	[4]	[5]	-1
O_3	[illegible]	[illegible]	[6] − 5	[3] 15 +	[2]	5
O_4	[illegible] □ +	[4]	[4]	[5] 5 −	[3] 5	7
Column number	4	3	1	-2	-4	

Table 10.31*e* *Program 3*

	D_1	D_2	D_3	D_4	D_5	Row number
O_1	[4] 25	[3] ε	[1] 15	[2]	[6]	
O_2	[5]	[2] 30	[3]	[4]	[5]	
O_3	[3]	[5]	[6]	[3] 20	[2]	
O_4	[2] 5	[4]	[4]	[5]	[3] 5	
Column number						

us to continue the application of the transportation algorithm. The assignment of row and column numbers to program 3 is shown in Table 10.31*f*. The calculation of the opportunity costs of the empty cells of program 3 will show that cell O_1D_4 is the most favorable cell. Hence, we draw a closed loop for cell O_1D_4 and include this cell in our next program (see Table 10.31*g*). Note that program 4 is essentially the same as program 3, except that one of the epsilons has been shifted from cell O_4D_4 (in program 3) to cell O_1D_4 (in program 4). This illustrates the procedure by which we handle the shifting process when the most negative amount shown in the negative terminals of the closed loop is epsilon (see the closed loop $O_1D_4 - O_4D_4 + O_4D_1 - O_1D_1$ in the revised program 3).

Calculation of the opportunity costs of the empty cells of program 4

* The reader should verify this and other details which have not been explicitly stated in the solution of this problem.

will reveal that cell O_3D_5 is the most favorable cell. Hence, we draw a closed loop for cell O_3D_5 and include this cell in our next program (see Table 10.31*h*). Notice that, as a result of the inclusion of cell O_3D_5 in program 5, cell O_1D_4 has become a normally occupied cell. That is, instead of having an assignment of ϵ, as was the case in program 4, cell

Table 10.31*f* *Revised Program 3*

	D_1	D_2	D_3	D_4	D_5	Row number
O_1	[4] − 25	[3] ε	[1] 15	[2] + □	[6]	0
O_2	[5]	[2] 30	[3]	[4]	[5]	-1
O_3	[3]	[5]	[6]	[3] 20	[2]	-4
O_4	[2] + 5	[4]	[4]	[5] − ε	[3] 5	-2
Column number	4	3	1	7	5	

Table 10.31*g* *Program 4*

	D_1	D_2	D_3	D_4	D_5	Row number
O_1	[4] 25	[3] ε	[1] 15	[2] ε	[6]	
O_2	[5]	[2] 30	[3]	[4]	[5]	
O_3	[3]	[5]	[6]	[3] 20	[2]	
O_4	[2] 5	[4]	[4]	[5]	[3] 5	
Column number						

Table 10.31*h* *Revised Program 4*

	D_1	D_2	D_3	D_4	D_5	Row number
O_1	[4] − 25	[3] ε	[1] 15	[2] + ε	[6]	0
O_2	[5]	[2] 30	[3]	[4]	[5]	-1
O_3	[3]	[5]	[6]	[3] − 20	[2] + □	1
O_4	[2] + 5	[4]	[4]	[5]	[3] − 5	-2
Column number	4	3	1	2	5	

Table 10.31*i* *Program 5*

	D_1	D_2	D_3	D_4	D_5	Row number
O_1	[4] 20	[3] ε	[1] 15	[2] 5	[6]	
O_2	[5]	[2] 30	[3]	[4]	[5]	
O_3	[3]	[5]	[6]	[3] 15	[2] 5	
O_4	[2] 10	[4]	[4]	[5]	[3]	
Column number						

O_1D_4 now has an assignment of 5 units. The epsilons disappear during the solution stages of any transportation problem whose optimal program is a basic feasible solution (see Tables 10.27 through 10.29). However, if the optimal program of a given transportation problem is not a basic feasible solution, one or more epsilons will remain in the optimal program (see Tables 10.31*e* through 10.31*i*). In the latter case, we simply disregard the routes in which the epsilons appear.

A test of optimality applied to program 5 will show that it is not an optimal program. We therefore must design a new and better program (see Table 10.31j). The reader should verify the details of the derivation of program 6.

If we test program 6 for optimality, we shall find that it represents an optimal solution. The assignment of ϵ to cell O_1D_2 can now be disregarded, which means that no goods will be shipped from origin O_1 to destination D_2. It is to be observed that program 6 is an example of an optimal solution which is not a basic feasible solution.

Table 10.31j *Revised Program 5*

	D_1	D_2	D_3	D_4	D_5	Row number
O_1	[4] (20)	[3] (ε)	[1] (15)	[2] (5)	[6]	0
O_2	[5]	[2] (30)	[3]	[4]	[5]	−1
O_3	[3] ε	[5]	[6]	[3] (15)	[2] (5)	1
O_4	[2] (10)	[4]	[4]	[5]	[3]	−2
Column number	4	3	1	2	1	

Table 10.31k *Program 6*

	D_1	D_2	D_3	D_4	D_5	Row number
O_1	[4] (5)	[3] (ε)	[1] (15)	[2] (20)	[6]	
O_2	[5]	[2] (30)	[3]	[4]	[5]	
O_3	[3] (15)	[5]	[6]	[3]	[2] (5)	
O_4	[2] (10)	[4]	[4]	[5]	[3]	
Column number						

10.14 ALTERNATIVE OPTIMAL SOLUTIONS TO TRANSPORTATION PROBLEMS

An optimal solution to a given transportation problem is not always a unique solution. The existence of more than one optimal solution for a transportation problem can be determined by examining the opportunity costs of the empty cells in the optimal program designed by following the transportation algorithm. If there is any empty cell having an opportunity cost of zero in the optimal program, another optimal program with the same total shipping cost as the first can always be designed. The second optimal program is obtained by revising the first program so as to include the zero-opportunity-cost cell. Let us illustrate this by considering the transportation problem of Table 10.2, for which the optimal solution in Table 10.23 was obtained. Along with a unique set of row and column numbers, the optimal solution of Table 10.23 is reproduced in Table 10.32.

The uncircled numbers in the matrix represent the implied costs of the

empty cells. A quick visual check reveals that the opportunity cost of the empty cell O_1D_2 is zero.* Hence, the total shipping cost of a new program including cell O_1D_2 will be the same as the total shipping cost of

Table 10.32 *Matrix A (First Optimal Program)*

	D_1	D_2	D_3	D_4	D_5	Row number
O_1	[12] 11	[4] 4	[9] (25)	[5] (30)	[9] 4	0
O_2	[8] (10)	[1] (20)	[6] (15)	[6] 2	[7] 1	−3
O_3	[1] (30)	[12] −6	[4] −1	[7] −5	[7] −6	−10
O_4	[10] 8	[15] 1	[6] (10)	[9] 2	[1] (40)	−2
Column number	11	4	9	5	4	

Table 10.33

Matrix A (first optimum program)

	D_1	D_2	D_3	D_4	D_5	Total
O_1	[12]	[4] 0	[9] (25)	[5] (30)	[9]	55
O_2	[8] (10)	[1] (20)	[6] (15)	[6]	[7]	45
O_3	[1] (30)	[12]	[4]	[7]	[7]	30
O_4	[10]	[15]	[6] (10)	[9]	[1] (40)	50
Total	40	20	50	30	40	

Matrix B (second optimum program)

	D_1	D_2	D_3	D_4	D_5	Total
O_1	[12]	[4] (20)	[9] (5)	[5] (30)	[9]	55
O_2	[8] (10)	[1]	[6] (35)	[6]	[7]	45
O_3	[1] (30)	[12]	[4]	[7]	[7]	30
O_4	[10]	[15]	[6] (10)	[9]	[1] (40)	50
Total	40	20	50	30	40	

the present (first optimal) program. The revision of the first optimal program is illustrated in Table 10.33. The total shipping cost in each of the two programs is $695, as the reader can verify.

* Opportunity cost = implied cost − actual cost. For cell O_1D_2, the implied cost equals 4 and the actual cost equals 4. Thus the opportunity cost of cell O_1D_2 is zero.

Once the existence of two alternative optimal programs is established, an infinite number of other alternative optimal programs can be derived. The following relationship governs the derivation of these alternative programs:

$$\text{Derived program} = dA + (1 - d)B$$

where A = matrix representing first optimal program
B = matrix representing second optimal program
d = any positive fraction less than 1

Let us illustrate the application of the above formula by considering the two optimal programs given in Table 10.33. Assume that $d = \frac{2}{5}$. Then

$$\text{Derived program} = \tfrac{2}{5}A + \tfrac{3}{5}B$$

Thus, we multiply each assignment in matrix A by $\frac{2}{5}$, multiply each assignment in matrix B by $\frac{3}{5}$, and add the corresponding elements (circled numbers representing allocations) of the two matrices. The

Table 10.34 *Derived Program (Third Alternative Solution)*

	D_1	D_2	D_3	D_4	D_5	Total
O_1	[12]	[4] 0+12=(12)	[9] 10+3=(13)	[5] 12+18=(30)	[9]	55
O_2	[8] 4+6=(10)	[1] 8+0=(8)	[6] 6+21=(27)	[6]	[7]	45
O_3	[1] 12+18=(30)	[12]	[4]	[7]	[7]	30
O_4	[10]	[15]	[6] 4+6=(10)	[9]	[1] 16+24=(40)	50
	40	20	50	30	40	

result is the derived program given in Table 10.34. The total shipping cost of the derived program, as can easily be verified from Table 10.34, is the same ($695) as that of the first two optimal programs. The derived program fully satisfies the rim requirements. However, the number of occupied cells in the derived program is 9, as compared with 8 in the first two optimal programs. This means that our derived program is a feasible solution, but not a basic feasible solution.

In so far as we can let d be any positive fraction, it is obvious that an infinite number of derived solutions can be obtained so long as two alternative optimal solutions can be identified.

In terms of practical significance, the possibility of designing alternative solutions gives valuable flexibility to the decision maker. It should also be realized that an examination of the opportunity costs of the empty cells (of the optimal program) enables us to identify solutions in descending order of preference in terms of total shipping cost.

GENERALIZED SYSTEM MODELS FOR QUALITY CONTROL

George W. Reid

THE DESIGN OF WATER QUALITY MANAGEMENT PROJECTS WITH INADEQUATE DATA

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University of Oklahoma

One of the increasingly important elements in the design of water resource projects is, of course, the management of quality and a technology that was almost purely hydrological and hydraulic is now being expanded to include what might be classed as the environmental and ecological impact areas and systems. So, it is no longer sufficient to understand the interrelationships, flows and transports, but to this must be added the impacts on the living and nonliving water, and peripherral environments; with a need to develop ecological models or more specifically, water quality models. Unfortunately, there is rarely adequate data to properly describe these interrelationships. The methodology used for hydrological studies involving inadequate data such as the transfer of observed points to points of interest; short term intense studies; or use of simulatinn techniques, can and are being used in quality management modeling. Perhaps more basic is an understanding of data requirements, using the system approach, the sequence of events are (1) problem formulation, (2) symbolic modeling, (3) data collection, (4) analysis and (5) design. (See Figure 1) Frequently, the order is changed, particularly the entire process will start with available data.

The complexities, of course, arise due to the fact that the processes associated with water quality management: hydraulic, hydrological, chemical, biological and ecological -- are extremely and imperfectly understood. So, that is a complex reality, with a great many variables on which there is available very poor measures and which themselves interrelate in ways very inadequately understood -- must be measured and appropriately related to be useful. Certainly, one recognizes the superiority of an explicit quantifiable data and models over intuitive models and hunches. The alternatives to such a model, based on partial knowledge, is a mental model, based on the mixture of incomplete information and intuition similar to those controlling most political decisions. A mathematical model deals with the same incomplete information available to an intuitive model, but through organization of information from many different sources into a closed loop at last analyses is permitted and data needs studied.

Problem Formulation: To arrive at a water resource project design, the number of variables is enormous, and they are mostly nonlinear. The structure of the system is more hierarchical than functional, and many of the parameters and variables are unquantified at present, certainly those associated with ecology. Nonetheless, to some degree, a merging of disciplines and the increased use of the system approach has been taking place in the study of water systems, and it is not just a matter of collecting data and figuring out what one has.

If one looks at the type of models being postulated for the design of water quality systems today, it will be seen (Figure 2) that they fall within a spectrum ranging from erudite mathematical models at one end of the spectrum to scenarios at the other. In the first case, the mathematical models may be rigorously developed in a mathematical sense, but all too often are of little use in describing a real complex system in inadequate data. On the other hand, the scenario model -- little data, numerour ideas -- may accurately depict the significant elements of the real system, but it is of little use to the engineer-planer because he cannot manipulate it or quantify it.

The target one should try to hit is a reasonable and useable balance between the poles of intuition and selecting hard data. One would like to be able to use the mathematical rigor of the physical scientist and, at the same time, give equal weight to the hueristic insight of the social scientist. The result would be a useable model for a system design. So, perhaps, or certainly, for planning purposes, one is dealing with the lowest level of quantification that allows good estimates and the lowest level of complexity which gives a reasonable picture of the real world system with the hope of expounding in both directions.

The application of mathematical modeling techniques to water quality management can significantly aid the decision-makers to arrive at better decisions. Thus, modeling provides relevant facts and alternatives, the decision-maker chooses the strategy. Operational models are still primitive, primarily because of the probilistic or random nature of the physical processes involved in waste diffusion. One is sometimes inclined to be skeptical of the value of increasing model sophistication which often seems to have progressed much further than our understanding of the complex real world situation; all models currently proposed in the literature have enormous data requirements, which far exceed the data usually available, and which, for the most part, must be derived from actual measurement. Many parameters in the more sophisticated models are simply not known in actual situations.

The water quality management problem requires:

1. The cause and effect relationship between pollution from any source and the present deteriorated quality of water in the estuary.
2. Forecasting variation of water quality due to the natural and man-made causes.
3. Methods of optimal management, including treatment and flow regulation to control the quality in the estuary for municipal, industrial, agricultural, fisheries, recreation and wild life propagation.
4. Chemical, biological, hydrological, hydraulic, at the same time, same place, and same accuracy.

Models. In modeling there is always a certain incompatibility between concepts of substance ans generality; data representativeness of the real world. The aim, of course, is to provide through an idealized abstraction an approximate behavior of the system which always in a compromise between simplicity and reality. Water quality models can be used to simulate, describe and predict, and programming leading to optimization of design. Programming which leads to policy requires an explicit set of objectives, or an objective function to maximize benefits or minimize costs. Simulation does not require explicit results. So, simulations are misunderstood, if one expects to use the numerical projections and values.

Using numbers is wrong if it leaves the impression that design projections are in any way predictions of the future. It is helpful, not as a prediction but to get one to realize how short-sighted -- how present-oriented -- images of the future ordinarily are, but extrapolation of present trends is a time-honored way of looking into the future. Most people intuitively and correctly reject extrapolations -- the point is that it provides indications of the system's behavioral tendencies and as an analysis of current trends, of their influence on each other, and of their possible outcomes.

Models may be classified usefully by areal extent into national, regional and local . At the highest, or national level, data is necessary for broad planning purposes, such as to determine an overall level of water pollution, to determine the total investment necessary for pollution abatement, to determine national policies and to project the problems into the future. At the second highest level, the regional level, all of the above information is necessary, plus the particular information needs for the region. The third, local level, consists usually of checking the operation of waste treatment plants to insure compliance with regulations and statutes. Thus, due to the different requirements and objectives, a data program which may be optimal at one level, is usually far from optimal at some other level. Unless a clear objective has been set, there is no guarantee that all critical bits and bytes of information are collected, and that the gathering of useless data is minimized. Similar calssification classification can be made with relation to time.

Hypothetical attempts to describe the intricate relationships between nutrients, phytoplankton, zooplankton, fish, detritus, bacteria and man-induced waste loads have resulted in a great variety of models. One of the first developed, classical Streeter-Phelps equation, describes adequately the deoxygenation and reoxygenation in the river. The familiar form of the oxygen sag equation is:

$$D = \frac{kL_o}{r-k}\left(e^{-k_1 t} - e^{-k_2 t} + D_o e^{-k_2 t}\right)\left(\frac{k_1}{k_2} - \frac{L_o}{k_1}\right) \qquad (1)$$

where: D = oxygen dificit at time t
D_o = oxygen deficit at time zero
L_o = BOD at time zero
t = time (distance) in days
k_1 = deoxygenation coefficient
k_2 = reoxygenation coefficient

This equation has been expanded to provide for evection and diffusion; algae growth, benthal deposits, etc., into, inreality, impossible data requirements. The basic need is for models somewhere between two poles that are built using existing data and as such can be responsive to the needs of the action agencies. It is in this realm in which the author has developed a series of water quality models. The projects being modeled generally are of such a nature that the ultimate realization will occur long after the departure of the designers, and as such direct validation procedures are impossible, necessitating some form of internal validation or internal integrity. The problem is one of using what information is available for a 50-100 year future, and doing it in such a fashion that it is not so elegant that it becomes a classroom make-believe world. The essential thread in the author's methodology is that of recognizing the complexity of a problem and drawing on a combination of OR techniques, deterministic techniques, as well as imperical, phenomological, and analytical methods. River system models respond to organized pollution in modes.

There are suggested six categories of stream responses: biodegradable, nutritional, bacterial, solids, persistant of slowly degradable chemicals and thermal. The response of a given stream to these categories can be formulated; or the reverse, given an instream criteria (RQS), allowable effluent quality can be calculated. The specific criteria now can be grouped under response headings; for nutritional, one might select N, P, N/P, or AGP, etc. If primary treatment is established as a lower constraint on the effluent, the solids criteria can be deleted; and further, if a public health constraint on toxic and bacterial levels can be exercised, four rather than six responses can now be used leaving a four-by-four matrix to be examined.

TABLE I

	Municipal	Industrial	Agricultural	Recreational
Biodegradable	Controlled by D. O. levels			
Nutritional	Controlled by N and P levels			
Thermal	Controlled by Temperature increases			
Persistent Chemical	Controlled by Salt	CCE's or ABS, etc.		

So, a response/use matrix, changing with time will set goals; based on a matrix such as the one in Table 1 and alternative socio-operated projections. A linking technical basin model can be built and operated to provide the optimal use of water resources, and of necessary treatments; or in planning for future population increases and the concomitant increased use of water, it is possible to build mathematical models depicting the optimum treatments and stream flows necessary to meet the RQS. The one-to-one input-output relationships for the four catagories of waste discharges follows with the Low Flow Augmentation FA), associated with each treatment level (TL_i), will be Q_L, Q_N, Q_P and Q_T. This is a terminal flow in MCD. TL_i is a fraction where i refers to BOD, N and P.

BIODEGRADABLE MODEL (L)

$$Q_L = \frac{Y}{\varepsilon} + (1-Y)\ \frac{PE \text{ or } P}{C_s - RQS_{DO}}\ A\ (P) \tag{2}$$

(1) where:

Y = Fraction of total population in SMA's

ε = Efficiency term, Point Load/Uniform Load

PE = Population Equivalent in millions

P = Percentage discharge to river, expressed as a fraction, Decision Variable = (1-TL)

C_s = DO saturation level @ given temperature

$A = \frac{942,900}{k_2 \frac{nL}{V}}$ relates to stream characteristics

where n is essentially the number of reoxygenized volumes, k_2 the reaeration constant, L the reach, V the velocity -- these values will change as the stream itself is subject to management.

ACCELERATED EUTROPHICATION MODEL (N)

$$Q_N = \frac{Z \cdot P}{F_N \cdot RQS_N} \quad (1-TL_N-1.44\ (1-TL_L) \quad (TL_L 3250) \qquad (3)$$

$$Q_P = \frac{Z \cdot P}{F_P \cdot RQS} \quad (1-TL_P) - .27\ (1-TL_L) \quad (TL_L\ 1080) \qquad (4)$$

where:

Q_P or Q_N = Nutritional Dilution Required, MGD

Z = Relative portion impounded and effected by RQS_{AGP} level

P = Population, millions

TL_P or TL_N = Phosphorus or Nitrogen removal level expressed as a decimal

F_N or F_P = BOD/P Ratio divided by optimum combining ratio

TL_N = BOD removal level expressed as a decimal

RQS_P or RQS_N = Acceptable level, RQS determined by RQS_{AGP}

THERMAL MODEL (T)

$$Q_T = \frac{\Delta T_W - C}{\Delta T_Q + C} \cdot \Delta Q \qquad (5)$$

Q_T = Thermal Dilution Required, MGD

ΔT_W = Allowable temperature difference between added flow and RQS_t ($t-RQS_t$)

ΔT_Q = Allowable temperature change ($RQS_T - T_o$)

. C = Ratio of K/V_x when K = Geometric mean for Bowmen's ratio and V =

subsidance velocity
ΔQ = Waste Flow, MGD

CONSERVED OR PERSISTENT SHEMICAL MODEL (C)

$$Q_C = \frac{W_C \times \Delta Q}{RQS_C} \quad (6)$$

These models, though cast in terms of dilution requirements, can be altered, given a diluted level to provide permissible loadings. The models (2) thru (6) are based on organized (sewered) pollution. Models for storm drainings or dispersed pollution have also developed such as:

DISPERSED POLLUTION MODEL (D)

$$Y_2 = 4.8 + 0.0827X_2 + 0.489X_8 \quad (7)$$

where Y_3 is BOD

$$Y_5 = 2.36 - 0.188 \ln X + .310 \ln X_{10} \quad (8)$$

where Y_5 is ON and Y_6 is PO_x in

$$Y_6 = 2.90 + .00003X_1 - .0001X_3 - .0137X_8 - .741X_{11} \quad (9)$$

and X_1 = population

X_2 = population density

X_3 = number of households

X_8 = commercial establishments

X_{10} = streets

X_{11} = environmental index

Models (2--9) can be used to relate waste inputs to stream responses under varying municipal stream characteristics and against varying goals (RQS). Many technical models are available to project flows (Q), and other stream characteristics k_2, L, V, etc. but a final model is needed for evaluation of the effects of the rural upstream watershed programs on downstream runoff to complete the set. Such a model was developed for the Congress in 1969.[1,2]

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1 For details of model development see, THE OUTLOOK FOR WATER, Wollman and Bonem, The John Hopkins Press, Baltimore & London, 1971, Appendix C., p. 203.

2 This was a special consultative report to the Secretary of the Interior, October, 1967.

UPSTREAM USE MODEL (U)

$$Y = -16 + X_1 X_3 - 137 X_2 \qquad (10)$$

Where:

Y = percentage of normal runoff

X_1 = percentage of normal precipitation

X_2 = percentage of watersheds controlled by hydraulic structures

X_3 = annual above one inch precipitation

In these equation, the simple Phelps equation (1) has been reduced to:

$$dL = \frac{k_2}{k_1} dO = f\ dO \qquad (11)$$

That is to say, the load equals the capacity. Distribution factors are added, load is put in terms of people, PE's, etc. This is useable. On the other hand and by way of contrast, O'Connor uses a one dimensional, differential equation, first involving:

$$\frac{\partial D}{\partial t} = \frac{1}{A} \frac{\partial}{\partial x} (QC) - kdL - k_n L^n + ka\ (C_s - C) -$$

$$S_B\ (X_1 t) + P(X_1 t) - R(X_1 t) + C_r (\frac{\partial Q}{\partial X} / A) \qquad (12)$$

Expanding this to three dimension, (x, y, z,) would require:

$$\frac{\partial O}{\partial t} = \frac{E_x \partial^2 O}{\partial x^2} + \frac{E_y \partial^2 O}{\partial y^2} + \frac{E_z \partial^2 O}{\partial x^2} -$$

$$\frac{U_x \partial O}{\partial x} - \frac{U_y \partial O}{\partial y} - \frac{U_z \partial O}{\partial x} - k_i c\ , \text{etc.} \qquad (13)$$

Also the evaluation of E's, U, K_i, etc. in terms of velocity, solar energy, depth, turbidity, etc. [3]

The effectiveness of models is, of course, acceptance. Actually, very few models have been used. Limitations of applying them to "real" systems are rooted in many factors, most related to data inadequacies; the acquisition of proper data, adjustment of non-homogenity, or inconsistency, to name a few.

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[3] SYSTEMS ANALYSIS AND WATER QUALITY, Thoman, Environmental Science Service, New York, 1972.

Every model, or system, is always embedded in a larger system in space or time, so one is limited to selection of a free body cut and exogenously determined parameters. Finally, serious factors, mostly associated with social values cannot, at present, be quantified.

An efficient use of models thus, argues for different models to answer different question. For example, one for sediments, one for social costs, etc. The system process is iterative and continues while the models are refined and until satisfactory results are obtained.

The flow of information for all the mested models eventually leads to the decision process. Forward and feedback information flows take place between models until the alternative selection and information developed is accepted for decision-making. As illustrated, there is no attempt to "hang" all models together. More important, different levels of data, can be used in each mode, providing homogenity in each model.

DATA

The data must support the models. Some of the questions for which answers are needed are, goals, include,:

1. What significant parameters of water quality should be measured, for an alert system, for treatment plant control, for a quality forecasting system, for a river management system?
2. What should be the periodicity or time interval in collecting specific data?
3. What are the cross correlations of these parameters?
4. Are there any synergistic relationships between the parameters?
5. What is being accomplished to develop instrumentation that can gage quantitatively those essential parameters, such as BOD, that are not being measured automatically at the present time?

So, there are all sorts of data, much of it redundant. One needs a model to discover needs, costs, etc. The process is shown graphically in Figure 3.

Data has a cost, collection and deferral of decisions.

The quantity of information collected should be increased so long as the present value of the investment opportunity (or cost savings if this is the use to which the information is put) is increased by more than the cost of the information.

The expected value of a decision will be low with little data available, but will rise with more data available. With little data available, the solution often would be overstated (resulting in unused capacity) or understated (resulting in lost opportunity), thus reducing the expected present value of the opportunity. For small enough quantities of data, the expected value will be negative.

The conclusion that the decision take place when the cost of getting one more year of information is equal to the resulting increase in expected present value. The cost of getting one more year of data is made up of two elements, the outlay during the during the year to get the data, k, and interest on the expected present value of the opportunity one would experience if a year of waiting is not included. That is, if $V(t)$ is the basic function, one should not wait until its rate of increase, $V'(t)$, is equal to $[rV(t) + k]$, where r is the rate of discount (the rate of return on investment).

Several conclusions are evident. First, it never will pay to wait for "complete" information. Second, an extremely important element of the problem is the cost coming from postponement of the stream of net revenues from the decision. This factor means it does not pay to accumulate data until the increment in expected value is equal to the annual cost of the data.

Experience in the United States has resulted in the common utilization of only eight water quality parameters that are thought to satisfy the requirements of reliability, accuracy, and low maintenance. These parameters are dissolved oxygen, pH, turbidity, conductivity, temperature, ORP, solar radiation intensity and chlorides. Time sequence is important. Parameters needed today may not be the correct ones tomorrow.

FIGURE 4

TIME SCHEDULE FOR WATER POLLUTION ABATEMENT

Time	Secondary Treatment	BOD Eff	N&P Eff	TDS Eff	Thermal Eff
1960	x				
1970	x	x			
1980	x	x	x		
1990	x	x	x	x	
2000	x	x	x	x	x
Criteria	Fish Kills Water Treatment Problems		Eutrophication	Reuse	Recycle

Figure 4 suggest a water pollution abatement time scale; that is, the standard will be upgraded with time, and the resource must be used within these constraints.

One is still concerned with the frequency with which data should be collected, the optimum locations of collection, the provisions for data storage and the resources for analysis of the data. The use of a short-term survey approach or establishment of a minimal number of permanent stateions. An analysis of historical data will yield insight into those parameters which require continuous analysis because of significant fluctuations and help to identify those locations which best identify changing conditions in the receiving water.

In contrast to the monitoring of a single point over a long period, studies can be concentrated over shorter times but more intensive. There is a question of manual collection versus continuous, automatic recording. All parameters of interest can be determined on a continuous basis and the results transmitted to a central storage facility, while water quality parameters that can be economically and dependently measured in the field are still somewhat limited.

CONCLUSIONS

Briefly, models to illucidate design parameters should be built with available data in mind. By a process of separating and nesting, sub-models can overcome inconsistencies. If goals are precisely stated as to function, various parameters can be represented by what is available. The author has developed a series of models using very general data, leaving a latitude of alternative data items to define a parameter. Data has a cost, collection and opportunity or decision errors also cost. If inadequacies continue, short-term intensive studies are justified, either now or backward, for example, point reviews can be used. Manual systems can be replaced by automatic monitors; all eight suggested parameters handled by electrodes. Generally speaking, however, automatic monitors tend to provide more data than are needed, because noone dares to turn these expensive machines off or set the sampling interval to such a time interval that meaningful deviations can be recorded.

One never has adequate data, nor can one afford to wait for it. So, models must be made using every device available, recognizing that the final result will still involve uncertainty and risks, and require judgement -- the only defense against inadequate data.

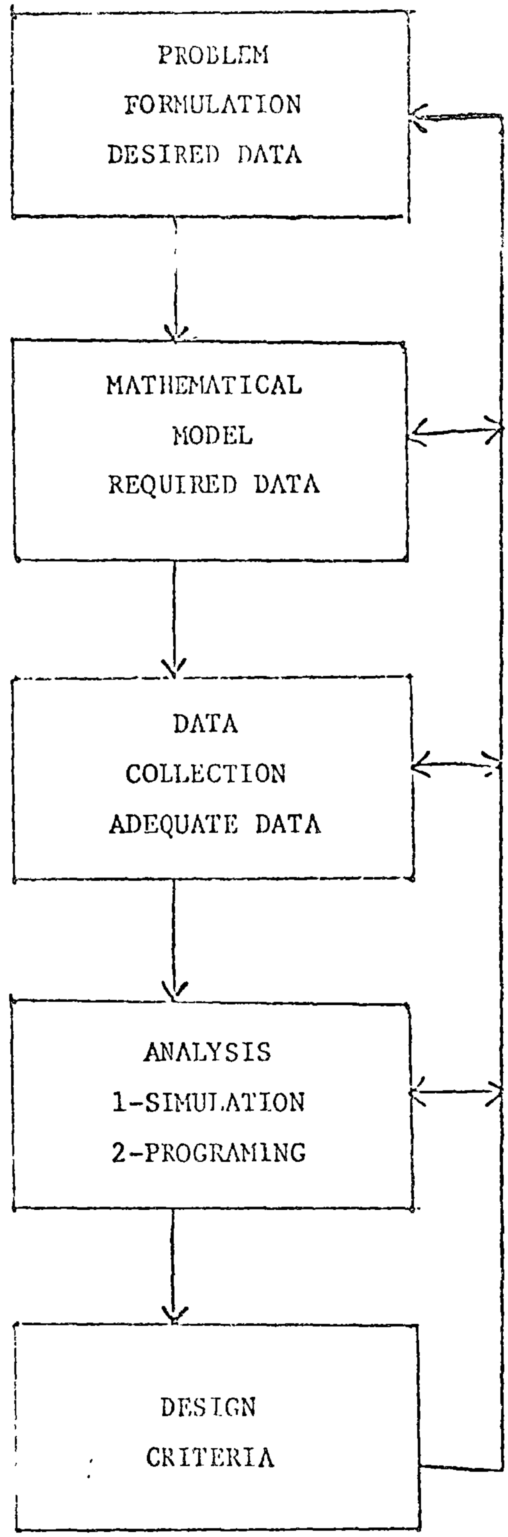
PROBLEM
FORMULATION
DESIRED DATA
MATHEMATICAL
MODEL
REQUIRED DATA
DATA
COLLECTION
ADEQUATE DATA
ANALYSIS
1-SIMULATION
2-PROGRAMING
DESIGN
CRITERIA

FIG. ONE

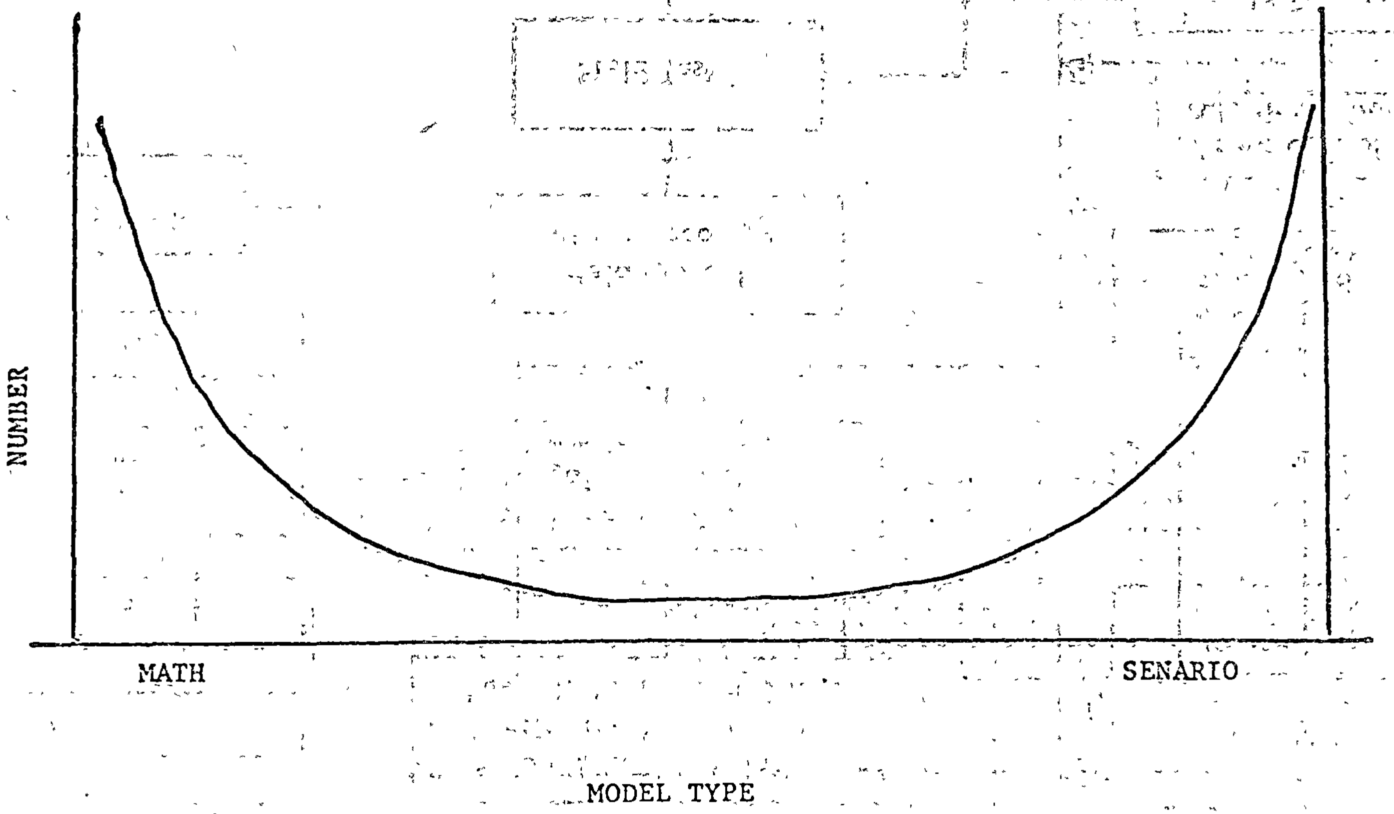

Figure 2.

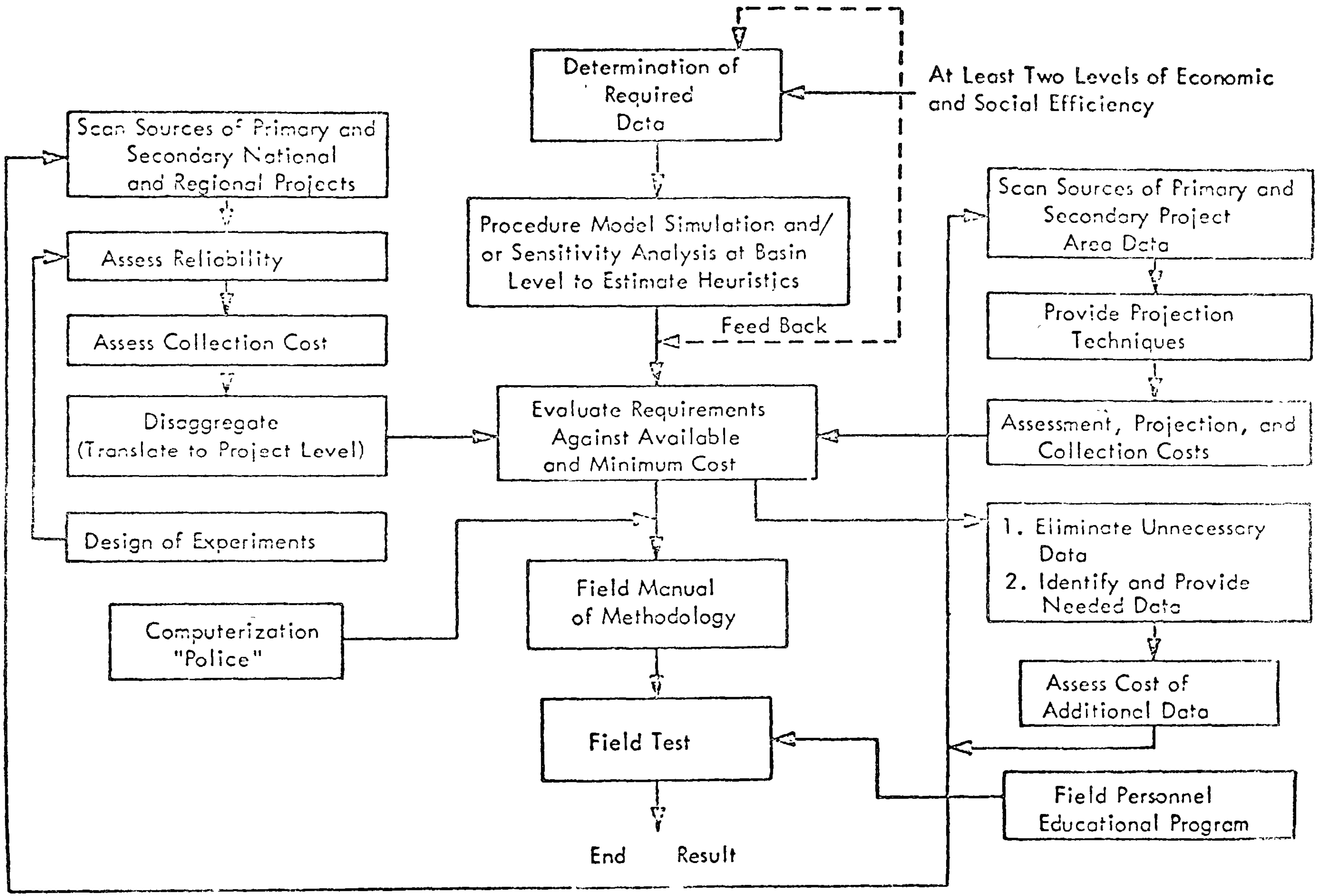

Figure 3. Data Needs Management Model

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DRAINAGE MODEL

George W. Reid

CHAPTER 10. THE DRAINAGE MODEL

10.1 INTRODUCTION

The general purpose of the drainage submodel is to develop a method that can be used to provide indicators concerning storm runoff drainage in an urban environment. More specifically, the submodel is aimed at obtaining pollutant load indicators.

The present study is based on several previous studies. A recent study by the AVCO Economics System Corporation (1) relating to pollution activities and urban land use was aimed at developing techniques for estimating types and concentrations of contaminants found in run-off from storm water. These techniques require a basic knowledge of land use, street area, sanitary conditions, impervious cover, and the average amount of rainfall for each season for the area under study. Although land-use allocation is not sufficiently sensitive to explain the variability of storm water contaminant concentrations, the techniques developed do provide an estimate.

The mathematical models that resulted from this project are of linear form. Although each equation contains from one to five independent variables that require careful projection and estimation, the results are reliable and reasonable. However, data were not available to permit development of regression equations based on the detailed variation of land-use and population cohorts required for the current research. The measurements of storm water quality and quantity based on the approach used to develop the mathematical models along with land-use tabulation would permit a timely and feasible development of output equations for all urban areas.

Another study (2), performed at the University of Oklahoma subsequent to the one just described developed methods for predicting dispersed pollutional storm water loads to receiving streams from land-use activity cohorts. Three alternate methods were used. The original methodology was developed with unrestricted cohorts.

The study yielded the average expected pollutional load per season. This is done by estimating the amount of seasonal storm water run-off by rational methods and multiplying these volumes by the average seasonal pollutant concentrations (using the appropriate conversion factors). The product gives each pollutional parameter in pounds per season.

The study employed linear regression equations, with more significant input independent variables, to obtain average seasonal concentrations or loads. This procedure requires extensive information of land use and the sanitary conditions of the area, and it is uniquely applicable to urban areas similar to Tulsa, Oklahoma.

The latest study in the pollution sequence (3) has been an effort to relate urban storm water pollution to the activity of the land, environmental factors, and precipitation on six candidate cities in the United States similar in terrain and activity to Tulsa, Oklahoma. One of the major concerns was to envision measures to control pollution rather than construction of facilities to treat or dispose of it. The study also dealt with appraisal of the sources of pollution from the various land activities to provide a means of control of storm water pollution by regulation of land use and urban planning.

Regression analysis was also used to determine urban pollutant concentrations in that study. The average annual loads for storm water pollution were obtained by multiplying the average concentrations by the approximate annual rainfall in pounds per acre per year.

The study pointed out that:

(1) The greatest amount of pollutant from the test areas resulted from wash out of materials which were deposited on the areas, and from the erosion of drainage channels caused by great amounts of run-off.

(2) The season with the most run-off has the most pollutants.

(3) Functional relationships can be established between storm water pollution parameters and variables grouped in either the precipitation or land-use classification to obtain a first order estimate of the average pollutant concentrations at other geographical locations. These techniques provide a procedure for looking at the impact of urban storm water pollutional loads on the receiving streams and also planning for control.

(4) Land surface characteristics with the strongest relationships to storm water pollutant concentrations are environmental conditions, geomorphic characteristics which affect drainage, and degree of development or urbanization.

(5) The environmental index (Table 10-1), which reflects the general sanitary conditions of the sites, is a good prediction variable for bacterial pollution parameters.

(6) BOD concentrations decrease with increasing flow during the rising limb of the run-off hydrograph. The amounts of BOD contained in the flow increased with run-off rates because the time rate of flow increased at a greater rate than the BOD concentration decreased.

(7) There is an increase in pollution produced per unit area of commercial and industrial use if the daily number of people who visit the area is high.

(8) In the residential section, the amount of pollution produced per unit area increases with the population density.

The research efforts culminating in the reports cited above had the common goal of investigating the urban area drainage problem. The practical purpose has been to enable the engineer and city planner to assess the need for urban drainage systems in the near future and to do so by looking at the appropriate environmental factors and the correlation between these factors. In these studies, land-use classifications, along with land condition and precipitation, have been used as input functions to generate pollution loads.

10.2 MODEL DESCRIPTION

The drainage submodel developed for this study was based on the above-mentioned studies. Regression analysis is used as the principal technique for predicting urban storm run-off drainage pollutional loads. However, to relate this submodel to other submodels developed in this study, only the cohorst of population, land use, and precipitation were used as inputs for prediction of pollutional loads. Although the restricted cohort inputs tend to decrease the accuracy of the estimates by regression equations, (due to the decrease in the number of independent variables used), in many cases reasonable estimates and qualified magnitudes of pollution loads were obtained.

Regression equations for predicting the quality of storm water run-off were established by correlating the measurements of surface characteristics of urban watersheds with associated measures of storm water pollutants for various test areas in Tulsa, Oklahoma. The four pollutants items being studied are total coliform, COD, total solids, and soluble orthophosphates. The variables used and their symbols and units are listed in Table 10-1. The regression equations developed for predicting the average concentrations of the four different pollutants are stated in the following section.

TABLE 10-1. Symbols And Units For Parameters Utilized in Regression Models.

Symbol	Item	Units
Dependent Variables		
M_1	Total coliform[a]	thousands/100 ml
M_2	COD[b]	mg/1
M_3	Soluble othophosphate[b]	mg/1
M_4	Total solids[b]	mg/1
Independent Variables		
D_1	Area of watershed	acres
D_2	Length of main stream	ft
D_3	Length to center of area	ft
D_4	Fall of drainage area	ft
D_5	Form factor*	dimensionless
X_1	Population	number
X_2	Environmental Index (EI)**	dimensionless
X_3	Arterial streets	acres/acre
X_4	Arterial streets	miles/acre
X_5	Other streets	acres/acre
X_6	Other streets	miles/acre
X_7	Residential density	people/res. acre
X_8	Residential density	people/acre
X_9	Length of main covered storm sewer	miles
X_{10}	Covered sewer/total length	ratio

Symbol	Item	Units
X_{11}	Arterial streets	%
X_{12}	Other streets	%
X_{13}	Commercial land	%
X_{14}	Industrial land	%
X_{15}	Unused space	%

a Geometric mean by events.

b Arithmetic mean by events.

* Form factor (D_5): An indicator of the drainage characteristics of a watershed by the following equation:

$$\text{Form factor} = \frac{43{,}560\ A}{(L_c)^2}$$

where, A = area of watershed in acres

L_c = length to center of area in feet

The form factor itself is dimensionless.

** Environmental Index (X_2; EI): An indicator of the general environmental condition of an urban watershed. The Environmental Index of a watershed is calculated by the following procedure:

First, the environmental quality of an urban watershed is assumed to be a function of three factors: the housing condition, the vacant lot condition, and the total number of environmental deficiencies (refuse, old autos, privies, etc.)

The second assumption is that, in defining an equation to represent the environmental-condition of a watershed, the number of deficiencies should be weighed more heavily than the housing conditions, and the housing conditions should be weighed more heavily than the vacant lot conditions. Stated in equation form:

$$EI = \frac{2A + B + 3C}{6}$$

TABLE 10-1 (Continued)

where $A = \frac{\text{Total Housing Structures}}{G + 2F + 3P}$ (A = Housing Index)

Note: G = number of good houses
F = number of fair houses
P = number of poor houses

$B = \frac{\text{Total Vacant Lots}}{G + 2F + 3P}$

Note: G = number of good vacant lots
F = number of fair vacant lots
P = number of poor vacant lots

$C = \frac{\text{Total Structures - Total Defiencies}}{\text{Total Structures}}$

Note: Total deficiencies can be defined as the sum of refuse, burners, rubble, lumber, old autos, poor sheds, livestock, poultry, and privies.

The necessary items, including the different types of environmental deficiencies and the number of houses and vacant lots in each classification, are defined as set forth in the manual "Community Block Survey and Socioeconomic Stratification" published by the U.S. Public Health Service.

Factors A and B in the above equation may vary from a minimum of 0.33 to a maximum of 1.00; factor C may vary from a negative number to 1.00. The Environmenial Index, when the necessary substitutions are made, may vary from a negative number to a maximum of 1.00. An EI of 1.00 denotes an area with all good houses, all good vacant lots, and no parcel deficiencies.

The EI calculated in this manner does not take into consideration a number of other factors which, if used, would result in a more refined indicator of the general environmental conditions of an area. Such items include air pollution, parks, noise level, and traffic volume.

10.2.1 Concentration of Coliforms

The independent variables used to develop regression equations for the concentration of total coliforms are those shown in the following functions:

$$\text{Concentration of total coliforms} = f(X_1, X_6, X_7, X_9, X_{10}, X_{12}, X_{13}, X_{15}, D_5)$$

The best regression equations for total coliforms together with their coefficients of correlation (R) and F values (99 percent significance level) are as follows:

for combined land use area:

$$M_1 = 430 - 363\,(X_1)$$
$$R = -0.856$$
$$F = 35.76$$

for residential area:

$$M_1 = 521 - 427\,(X_1)$$
$$R = -0.383$$
$$F = 17.75$$

for commercial and industrial area:

$$M_1 = 372 - 329\,(X_1)$$
$$R = -0.930$$
$$F = 25.66$$

10.2.2 Concentration of COD

$$\text{Concentration of COD} = f(X_1, X_4, X_7, X_9, X_{10}, X_{11}, X_{12}, X_{13}, X_{14}, X_{15}, D_2, D_3, D_5)$$

The best regression equations for COD together with their coefficients of correlation (R) and F values are as follows:

for combined land use area:

$$M_2 = 71 - 45.4\,(X_1) + 2.61\,(X_{11}) + 0.00619\,(D_2)$$
$$R = 0.839$$
$$F = 6.32$$

for residential area:

$$M_2 = 69 - 74.71\,(X_1) + 3.68\,(X_{11}) + 0.0105\,(D_2)$$
$$R = 0.971$$
$$F = 16.55$$

for commercial and industrial area:

$$M_2 = 152 - 52.2\,(X_1) + 1.72\,(X_{11}) + 21.7\,(D_5)$$
$$R = 0.862$$
$$F = 2.89$$

10.2.3 Concentration of Total Solids

Concentration of total solids = $f\,(X_1, X_3, X_4, X_7, X_9, X_{10}, X_{11}, X_{12}, X_{13}, X_{14}, X_{15}, D_1, D_4)$

The best regression equations for total solids and their coefficients of correlation (R) and F values are as follows:

for combined land use area:

$$M_4 = 2.12 + 50.2\,(X_{15})$$
$$R = 0.733$$
$$F = 15.13$$

for residential area:

$$M_4 = -139 - 15.4\,(X_{10}) + 16.0\,(X_{12}) + 2.57\,(D_4)$$
$$R = 0.765$$
$$F = 1.41$$

for commercial and industrial area:

$$M_4 = 162 + 67\,(X_{15})$$
$$R = 0.839$$
$$F = 9.51$$

10.2.4 Concentration of Soluble Orthophospates

Concentration of Soluble Orthophosphates = $f\,(X_1, X_3, X_4, X_5, X_7, X_{10}, X_{11}, X_{12}, X_{13}, X_{14}, X_{15}, D_4, D_5)$.

The best equations for soluble orthophosphates and their coefficients of correlation (R) and F values are as follows:

for combined land use area:

$$M_3 = 0.62 + 0.08\,(X_{15})$$
$$R = 0.788$$
$$F = 21.66$$

for residential area:

$$M_3 = 0.66 - 0.0011 (X_{11}) + 0.0645 (X_{15})$$
$$R = 0.817$$
$$F = 4.01$$

for commercial and industrial area:

$$M_3 = 1.35 + 0.0252 (X_{14}) + 0.0622 (X_{15})$$
$$R = 0.882$$
$$F = 5.26$$

10.3 VALIDATION

By the very nature of the methodology used in developing the model, the accuracy of the predicted pollutant concentrations is limited by a number of factors related to the characteristics of the individual urban areas themselves. For example, the models are based on the estimation of dispersed rather than localized pollution sources. The existence of localized sources of pollutants within the watershed being studied would, of course, result in higher concentrations than would be predicted by the models.

The models, derived from the University of Oklahoma study, are based on the assumption that land-use classifications are among the most important factors influencing pollutant concentrations. When the effects of a number of additional watershed characteristics were assessed in the AVCO study, it was found that drainage characteristics were often far more important than land-use classifications alone. Finally, the research into the effects of climatological characteristics undertaken during the recent AVCO study had demonstrated that the effects of climate may, in turn, be more significant than either land use or drainage. In any event, storm water pollution is related to a vast number of factors, including climate, land use, environment, and drainage characteristics, as well as many other factors which have not yet been fully considered. Future studies to identify the influence of regional differences in precipitation, temperature, geology, and other important factors on the amounts of storm water pollution should modify the models to make their application more valid in regions distant from the Tulsa area.

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WATER SUPPLY, SEWERAGE

MODEL

George W. Reid

The water system of the model is very extensive and has many sub-systems: resources, withdrawal, consumption, quality, transmission, treatment, storage, and distribution. The sewage system also deals with many sub-systems: return flows (both quantity and quality), collection, treatment, discharge, receiving streams, and quality criteria. The main sub-systems of the solid waste system are waste generation, collection, and disposal.

The information system consists of a series of categorical inventories or data collection procedures. Preliminary to the collection of any data, an attempt must be made to identify the relevant data items needed for the completion of the planning model. These identifications are, of course, subject to refinement as the construction of the planning model proceeds. Briefly, inventories are included for:

1. Demographic variables
2. Areal land use
3. Economic factors
4. Existing water and sewer system facilities
5. Existing resources
6. Water use patterns
7. Cost data
8. Financial resources
9. Codes, ordinances, and statutes.
10. Socio-political factors

The interrelationship between these sectors and systems is shown in Figure 8-1. The information system is not shown separately because it deals with all the other sub-systems.

The model components and their general sequential arrangement are indicated in Figure 8-2. The first step is to conduct the required inventories and assemble the information system. Next, the projections of the sector parameters must be made. Then, based on these projections and a set of unit-use factors to be developed, estimates of the total water system requirements, sewage loads, and solid waste generation can be determined. Also, comprehensive regional development goals must be evaluated and, if necessary, modifications of the plan may be made to meet these goals.

As in the development of the other sub-models, the water supply, sewerage, and solid waste sub-models also use outputs from the population model as the initial inputs. However, a special demand model was developed for the water supply, sewage, and solid waste sectors of the urban system. Based on this demand model, a water network sub-model for the water supply system and a sewer network model for the sewage system were

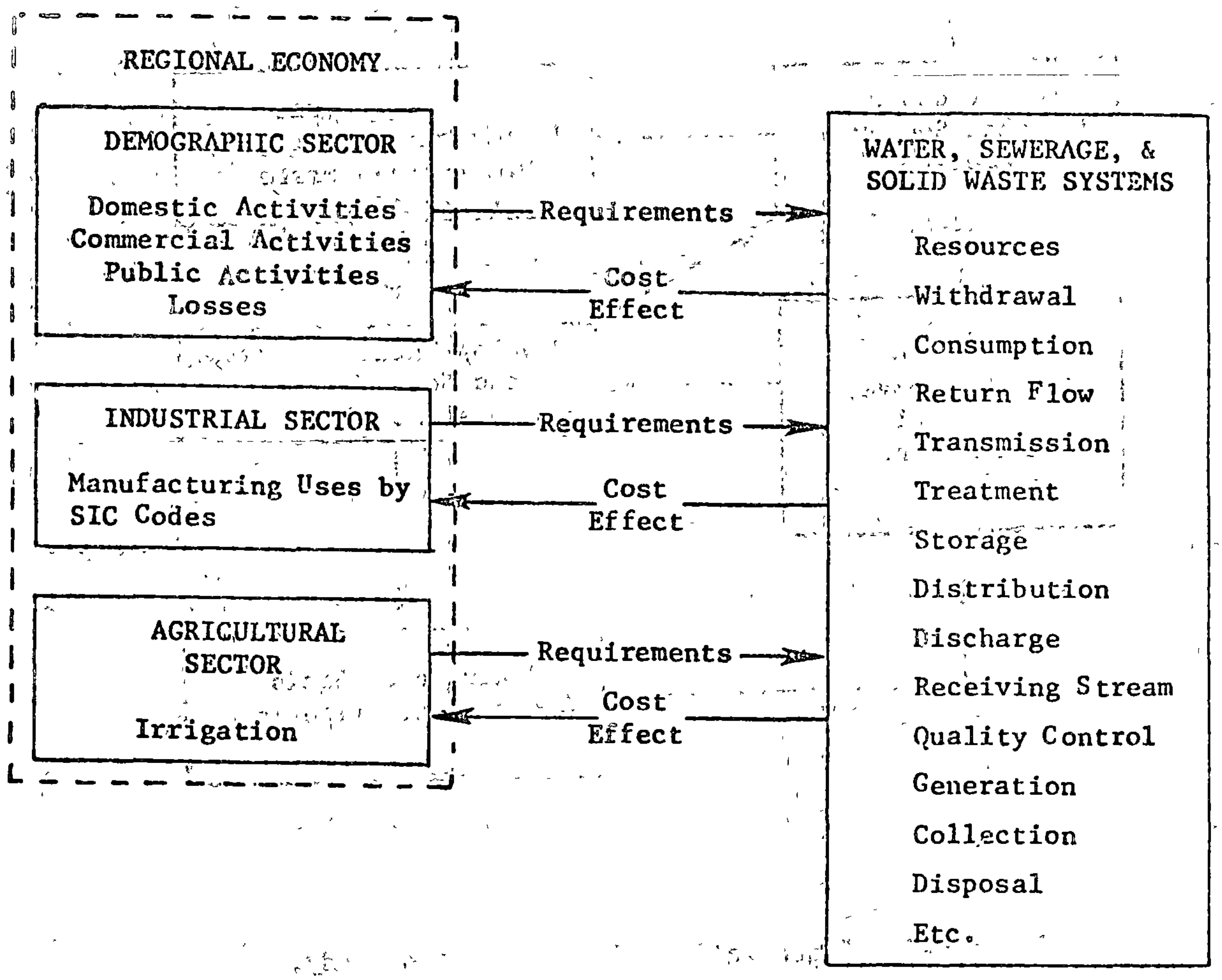


Figure 8-1. Interrelationships of Planning Model Sub-systems.

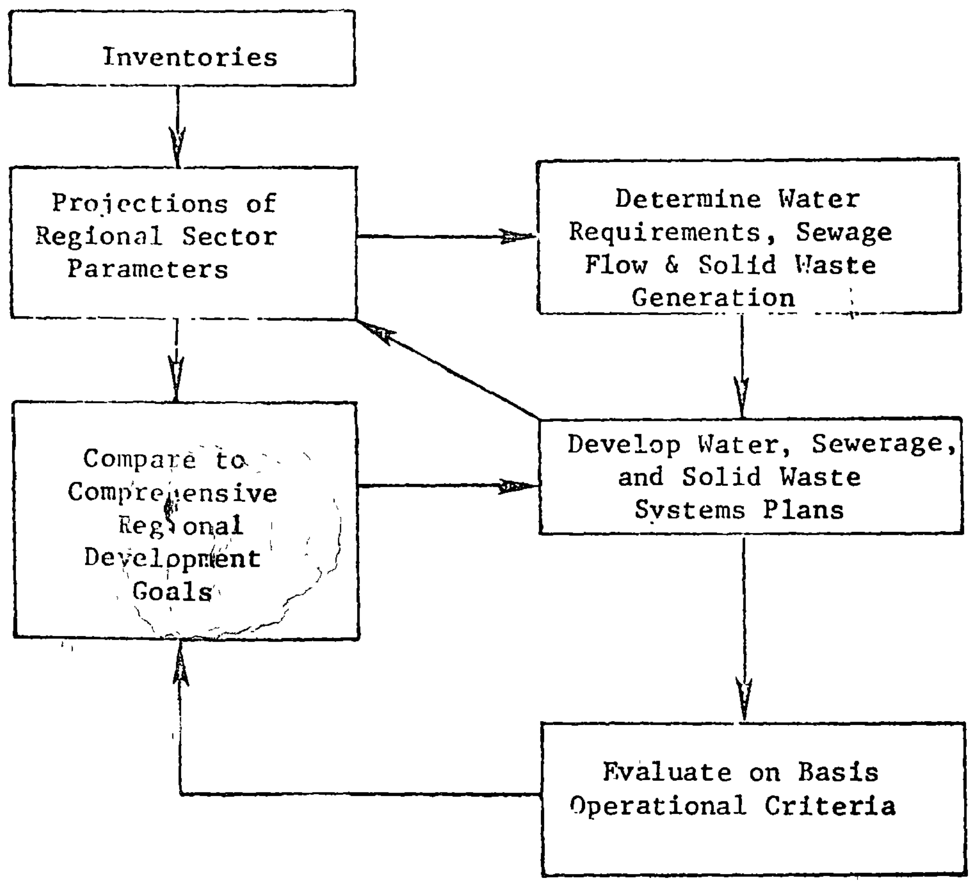


Figure 8-2. Organization of the Planning Study.

developed. All these sub-models, the demand model, the water network model, and the sewer network model, are discussed separately in the following sections. The solid waste sub-model is not completely developed at present; thus, only the generation component is discussed. However, a detailed solid waste model can be developed by using format similar to the format used for the water supply and sewage systems.

8.2 DEMAND SUB-MODEL

As stated earlier, the demand model described in this section was developed especially for the water supply and sewerage sub-models. This model can be termed as a sub-model of a sub-model. Actually, the demand model supplies information about the water demands and sewage outputs for the study region based on current and projected data accumulated from the output from the population model and other resources.

The model uses selected technical coefficients that were developed or acquired from other studies (see References 1, 2, 3). These coefficients are then applied to the data files to acquire the water and sewer outputs. The model is one of the final steps toward the development of the water and sewer plan for the region. It gives not only the future requirements of the study area but also the incremental increases these areas have. This allows the user to see the actual increase that the existing system can handle or the amount at which the system will be over its capacity if it is already at or near capacity. Applying this model, the user can gain an adequate perspective of the water and sewer network.

8.2.1. Model Description

The demand sub-model is a data acquisition model. Although the program is not extremely complicated, the data requirements are extremely tedious to obtain. This model has a great deal of flexibility built into the routines. The development of a model that gives the user this degree of flexibility is time consuming but extremely rewarding. The fact that a planning agency can easily explore all facets of the possible world without going through great data changes or even reprogramming and still have a comprehensive model is advantageous.

The demand sub-model is basically an application of technical coefficients to derive the water and sewage requirements and the accounting of these requirements to the different study area for output. The model will not only give the water and sewage requirements for any particular study year but from one study period to another. This capability, coupled with good editing sub-routines, allows the agency to look at any size or particular area by study years and/or incremental changes.

The data is supplied to the model for each statistical area unit (SAU). An SAU is usually selected as a one-square-mile area. An SAU can be as small as the type of detail needed, but it cannot be more than two or three square miles in area. The SAU's for rural areas can be larger, but if development is a possibility within the time frame of the study, they should be reduced to the size that they will be after development. If possible, the boundaries used in defining other types of areas (e.g., political jurisdiction, watershed, census tract, etc.) should be used in defining SAU's. Each SAU is given a code as are the following types of areas:

1. Political jurisdictions
2. Watersheds
3. Water treatment plants
4. Storage systems
5. Waste treatment plants
6. Receiving streams
7. Water sources

An example of coding of streams and watersheds is shown in Figure 8-3. It is suggested that all areas except watershed and receiving stream be coded numerically in sequence. In other words, start with one and numerically allocate each succeeding number to each area till they are all accounted for. Each new plant or jurisdiction will be assigned the next number in its area.

The demand sub-model is run for each time increment needed by the study. These data files are duplicated as far as the area codings are concerned. The base year (1970 for this study) contains the inventory data for the region. The data files for the future years are developed from the projected data. These files must be built using the same areas that existed in the previous files but can have new SAU's in addition to the old ones.

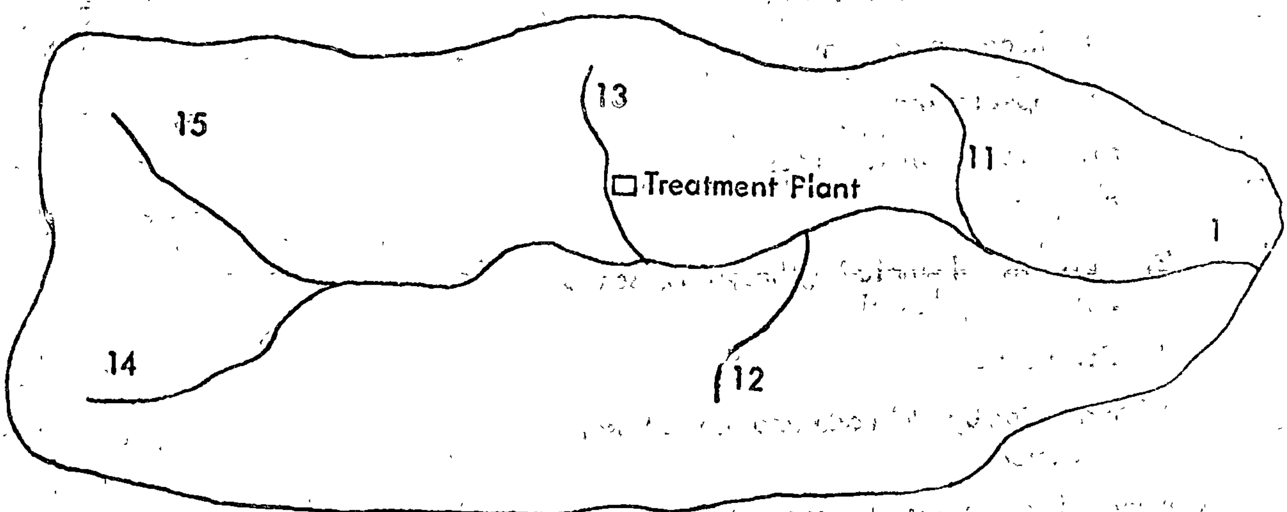


Figure 8-3. Sample Coding (Watershed 1 and treatment plant on receiving stream 13).

The model with all its sub-routines can be found in Reference (41). The outputs of the demand model are by the following categories:

1. Water requirements by:
   a. Political jurisdiction
   b. Source of supply
   c. Water treatment plant
   d. Water storage system
   e. Special area
2. Sewage loads by:
   a. Political jurisdiction
   b. Watershed
   c. Sewage treatment plant
   d. Receiving stream
   e. Special area

Each category must be broken into:

1. Domestic
2. Institutional (including hospitals, school and military bases)
3. Commercial
4. Industrial by SIC code and special user irrigation

The demand sub-model has the capability of looking at several special areas made up of selected SAU's independently or within the general study. This allows the consideration of several alternatives at one time to see which special area is more suited for certain goals or objectives.

The sub-model also has the capability of handling special users of water. These are the users that fit in one of the assignment areas but do not have water usage that fits the linear equation for computing it. These areas can be handled individually which relieves the model of complicated functions for water usage or sewage return flow.

The use of this model will greatly reduce the process of computing water and sewage demands for large metropolitan areas. The model also allows the user a large degree of freedom for exploring the alternatives for the future worlds.

8.3 WATER SUPPLY NETWORK MODEL

8.3.1 Network Formulation

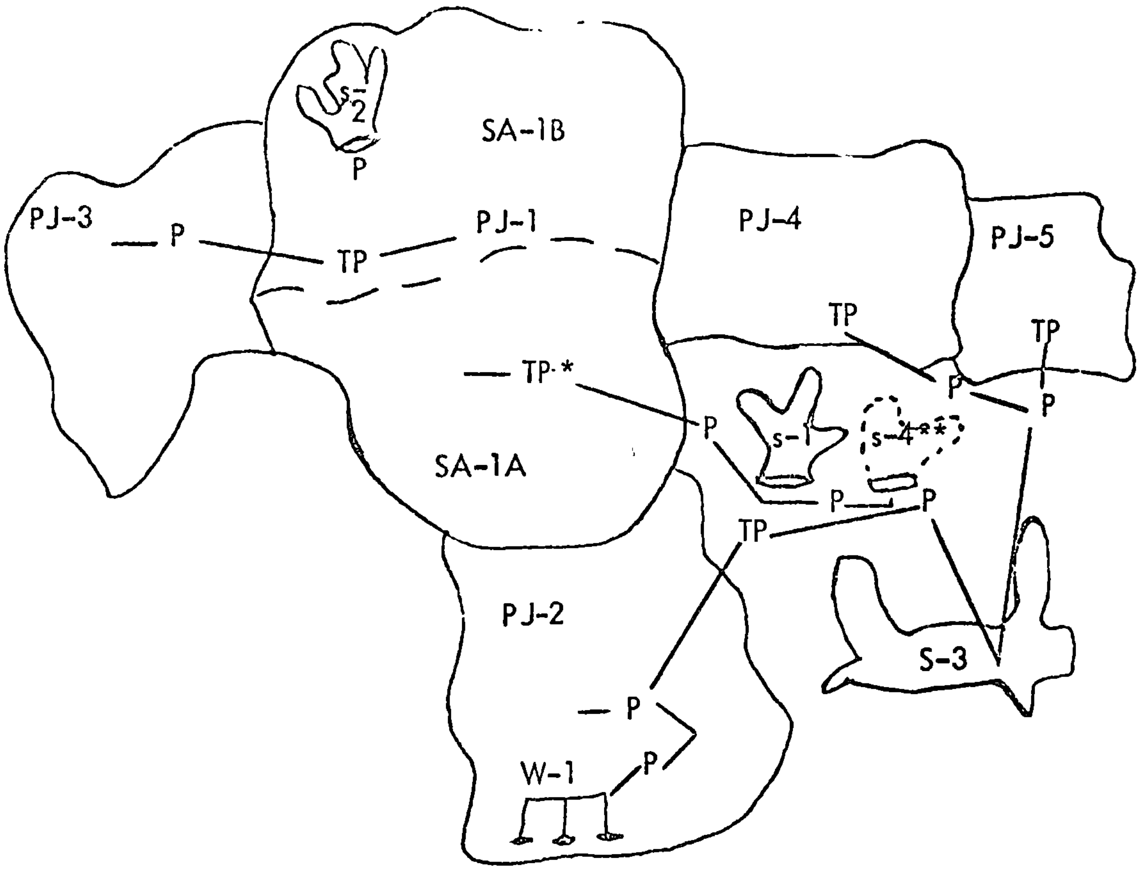
The basic structure of the water supply sub-model is the regional network. This network may vary greatly with each region but will be structured by political and corporate jurisdictions, sources, pipelines, treatment plants, storage facilities, etc. The region may be made up of several communities and metropolitan areas. Many of these political jurisdictions will have independent networks and some, mainly the metropolitan areas, will probably have an interconnected water system with several sources and treatment plants.

The main objective is to formulate these varied networks into one system for the whole region. This does not imply that the whole region should be made into one interconnected network, although this usually is a desirable goal. It does imply that the whole region has to be formulated as one problem and evaluated as a complete system.

The network consists of all sources that are available to the region, although some of these sources may be some distance from this region. It also includes the necessary raw and fresh water storage facilities, treatment plants, and connecting lines that provided the transportation links for this water. It does not include the local networks, those inside of the corporate limits, that distribute the water to the user. These are evaluated by using some special procedures (See Appendix B, Water, Sewer, and Storm Drainage Micro Area Requirements in Reference (41).) An example of a metropolitan water supply system formulation is shown in Figure 8-4.

The first phase in the network formulation is to form the current network using the inventory data. This network is then evaluated against the projected demand for the specified time span (usually a five-year interval) to determine the future requirements. These requirements are then compared with the existing and programmed capacities. By using this information, the actual network to be considered in the final network analysis is determined. The procedure used to determine this actual network is described below.

The existing facilities were evaluated in the inventory and analysis phase of the study. (The forms used to collect this data are shown in Reference (41).) This data, plus the inventory of the sources and major pipelines (both existing and those that are programmed within a five-year time interval), provide the existing network as shown



PJ - Political Jurisdictions

S - Water Source Surface

W- Water Source Sub-surface

SA- Special Area of Political Jurisdiction

TP - Treatment Plant

* - Programmed for Expansion

** - Programmed New Facility

Figure 8-4. Metropolitan Water Supply System Formulation.

in Figure 8-4. As shown in the sample existing network of Figure 8-4, the sources, treatment plants, and pipelines for the existing and programmed network are identified. Each political jurisdiction that receives its water supply from a particular source is noted, and if it is from separate sources, the political jurisdiction is divided into special areas. Also identified are the capabilities, cost, and liabilities of each portion of the existing network. At this point in the study procedure, the existing network has been evaluated and identified.

The demand model is then run with this existing network data to validate the model for the study area. If it is not correct for any area, then the technical coefficients and equations from the demand model are revalidated. The result is an evaluation of the first specified time span for the study area.

The demand model is set for the network by coding each SAU to its proper group (political jurisdiction, sources, special area, treatment plant, etc.) that coincides to the existing network. The demand model is then run for the next time interval for each alternative under consideration. The future land use of the existing area is coded into the same land use procedure as the current land use. The land use to be developed is added to the existing scheme as visualized by the planner and is entered as a special area. This is done to preclude having to re-evaluate this area if it becomes incompatible with the existing network. This is also done for each of the future alternatives that the planner wishes to explore. The demand model's option of evaluating the increase in water demand should also be run. This gives the increases and new requirements as a separate output which makes evaluation of these networks easier.

After the run of the demand model and the inventory, outputs will be used to evaluate the existing and programmed networks for the specified time intervals. Three components of the network are evaluated.

1. The first step is to examine, for each source of water by each of the user codes, if the existing and programmed sources take care of their respective users. The sources that can meet their future requirements are noted and their excesses in capacity are evaluated. The sources that cannot meet future requirements are examined next, and the reason for the deficiency is evaluated. Has it reached or been over its capacity because of growth of the old users alone or because of

growth and new development? If it is because of new development, then this new area is examined as a special area requiring a new source. The old area is then checked to see it if can be handled by the old source. The deficiency or excess is noted and recorded.

The source data is then compiled for the study area. The excess of water by each source is evaluated first. The controlling agency is contacted to determine if the excess is available for use in other areas. If it is being held in reserve and is not available, then it is removed from the excess roles. If it is available, then the cost per million gallons, amount available, and duration of the availability are determined. The above procedure includes those sources which have already been programmed for completion prior to the end of this five-year interval.

2. The second step is to evaluate the treatment facilities and their capabilities. The evaluation procedure is very similar to that for the sources evaluation as far as identifying the excesses and the deficiencies. The exceptions to the above procedures are the evaluations of the treatment plants. Each plant that has a deficiency must be examined individually. Consideration of expanding the present plant to a capability that would take care of the needed water supply is given first. The decision is based on the current condition of that facility. The expansion of a facility versus the construction of a new one must be carefully weighed if the latter is also being considered.

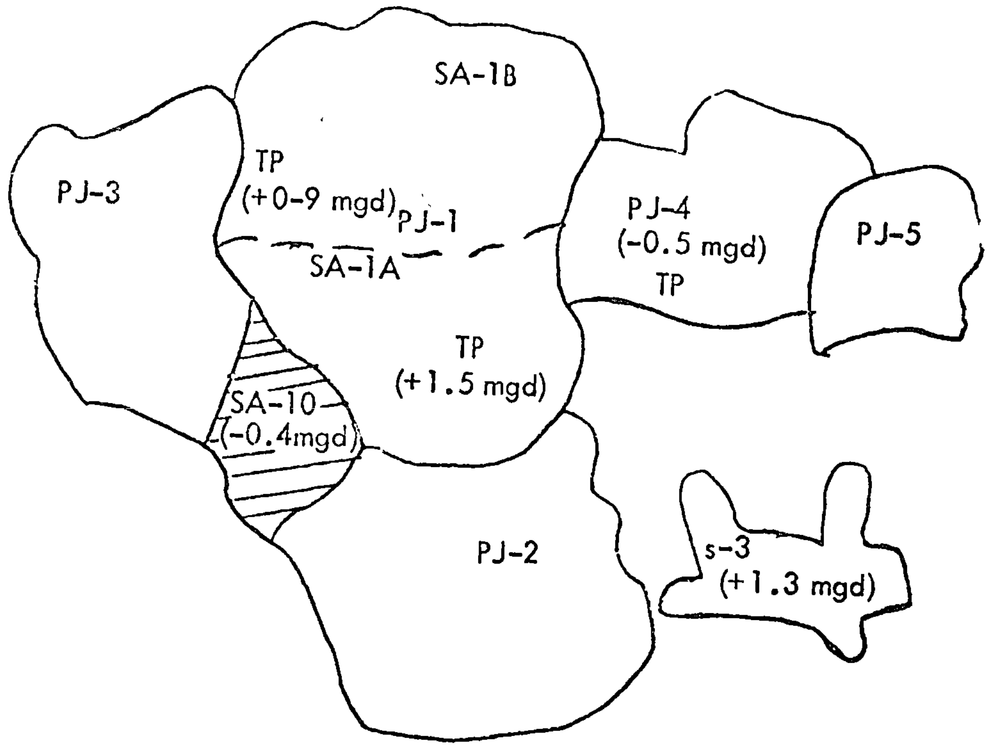
3. The final step is the evaluation of the pumping and pipeline facilities that interconnect the sources, treatment plants, and user networks. Again, these facilities are evaluated for their excesses, deficiencies, and availabilities. The procedure is the same as that described above for the sources and treatment plants. The completion of this phase concludes the information needed for the network formulation.

The next phase of the network formulation is to set this data down on a map on a tabulation that can be easily understood (see Figure 8-4). This then gives the planning agency its first real look at the future requirements. Above all else, it has reduced the problem to the actual network that needs to be evaluated. Rather than a maze of plants, pipelines, pumping stations, etc., the planner now has the problem in a form that can be easily explained to the decision-maker.

An example of a future alternative under consideration is shown in Figure 8-5. The study area has a new political jurisdiction. The deficiencies have been identified and noted. The facilities that do not have adequate capacity are also shown. These then become the requirements for the time interval under study.

The process is repeated for each new time increment until the complete study period has been evaluated. This procedure gives the planner an incremental analysis of the excesses and deficiencies of the study area for the "desired" alternative. The accumulation of the future data for the formulation of this desired network is now complete. If there is more than one "desired world" to be analyzed, then the process is repeated for each alternative.

Grouping this data into individual categories for each "desired world" facilitates its presentation to the decision maker for the selection of the network alternatives that will be analyzed in the final plan selection. An individual data sheet for each political jurisdiction, source, treatment plant, and storage facility gives a much clearer picture of the excesses and deficiencies, especially when accompanied by each individual mapping of categories. Table 8-1 shows a sample data sheet of the City of Norman, Oklahoma. (Norman is used as an example because it is one political jurisdiction in a large metropolitan area). This sample data sheet shows that Norman has an adequate groundwater supply of exceptional quality; the near-future water requirements can easily be met by the development of approximately six new wells. Norman's water treatment plant, on the other hand, will be inadequate shortly before 1985. The treatment plant in this city is new and can be easily expanded to double its present capacity of six mgd (million gallons per day). Since the groundwater supply requires no treatment and is added directly to the water network and Lake Thunderbird's capacity is only 8.55 mgd, then only 2.55 mgd needs to be treated. This can be accomplished by increasing the capacity of



Excess

1. Source 3 - 1.3 mgd
2. TP (SA-1A) - 1.5 mgd*
3. TP (SA-1B) - 0.9 mgd*

Deficiencies

1. TP (PJ-4) -0.5 mgd
2. SA-16-0.4 mgd

* Available for PJ-1 only

New Special Area

Figure 8-5. Future Requirements for the Water Supply Network Model.

Table 8-1. A Sample Data Sheet of Water Excesses and Deficiences.

WATER - EXCESSES AND DEFICIENCIES FOR NORMAN					
	Time Intervals				
	1970	1975	1980	1985	1990
POPULATION	52,117	59,500	68,000	76,500	87,000
Water usage-GPCD***	118	120	124	129	137
Water usage-MGD	6.15	7.14	8.43	9.87	11.92
Industrial Water Usage - MGD	1.65	7.42	3.81	5.47	7.12
Total Usage MGD	7.80	9.56	12.24	25.34	19.04
SOURCE					
Thunderbird Lake AVG - MGD	8.55	8.55	8.55	8.55	8.55
Ground Supply* AVG - MGD	30 wells** 9.00	30 wells** 9.00	30 wells** 9.00	30 wells** 9.00	30 wells** 9.00
Total MGD	17.55	17.55	17.55	17.55	17.55
Excess/ Deficiences	9.75	7.99	5.31	2.21	-1.49
Treatment plant - MGD	6.0	6.0	6.0	6.0	6.0
Excess/ Deficiences	up to 6.0	up to 5.44	up to 2.76	-6.34	-2.55

* Ground supply requires no treatment other than chlorination

** Avg. yield= 0.24 MGD/well

*** Maximum daily demand

the plant by only 50 percent. This procedure gives a good picture of the water requirements and possible solutions for the political jurisdiction of the City of Norman. The process is repeated for all other groupings being analyzed.

Many of the water networks in a metropolitan area are independent; i.e., not interconnected. The interconnection of all the networks into a regional system that serves the metropolitan area is a desirable goal and greatly helps the study area in meeting future water needs, as well as providing for emergency flows. This goal is usually difficult to meet due to the political and socio-economic nature of the system.

The formulative network allows the planner to develop a future plan for much, if not all, of the networks based on a simple cost analysis of the few alternatives of each network without using any computerized network model. Due to the nature of these network models, as much of the analysis of the future alternatives should be accomplished by this procedure as possible.

The network formulation and the initial evaluation allow a complete understanding of the networks, their requirements, and the alternatives that are feasible for the final selection of the alternatives.

The final selection of the alternatives is done by the network analysis through a computerized program, Fulkerson's Out-of-Kilter Algorithm, which will be described later. In the network analysis, the cost data of each component of the network is needed and the cost functions will be discussed next.

8.3.2 Cost Analysis

The network analysis consists of the employment of the different types of cost functions that are incurred in the development of water supplies. Basically, cost functions can be categorized into four components:

1. Water source costs for either surface or ground water, including costs for reservoirs, stream diversions, and well fields.

2. Transmission costs, including costs for pumping stations and pipelines used to convey the water from its source to the area of use.

3. Treatment costs, including costs for raw water storage, treatment plants, and pumping plants.

4. Distribution costs, including costs for pumping stations, storage tanks, and water mains.

In this study, each of these costs has been analyzed and estimated. In general, the costs are broken into capital expenditures and operation and maintenance costs. Capital expenditures include costs for engineering design, land and right-of-way, water rights, construction, administration, and financing. Operation and maintenance costs include labor, material, administration and overheads, chemicals, and power. In some cases, chemical and/or power costs are shown separately.

Capital costs are presented as equivalent annual costs using an interest rate of six percent and a period of twenty-five years. Operation and maintenance costs are presented as annual costs. Both are presented in 1970 dollars. Adjustment to a new base year is accomplished by use of the Engineering News-Record Building Cost Index 5 (7) for the southwest region (Dallas).

The cost data was obtained from previous studies of generalized costs for water supply systems by the Tulsa Metropolitan Area Planning Commission (8), Black and Veatch (9), and Dawes (10).

The cost estimating procedures used here are valid only for making preliminary comparisons and serve only to measure costs to a degree which will assist in evaluating planning alternatives. Cost estimates derived from these procedures should not be used in actual facilities design because they cannot take the place of detailed engineering estimates for specific projects. Cost equations are valid for facilities based on use rates from 0.1 to 100 million gallons per day. For use rates in excess of 100 mgd, proportionate increases in cost estimates are suggested (9).

The cost estimating procedures applicable to this model are described below. All costs given are unit annual costs. To obtain the total annual costs, the unit annual costs must be multiplied by a design capacity variable. Design capacities of future facilities are always intended to be the capacities required based on water requirements at the end of the design period, i.e., the long-range forecasts.

8.3.2.1 Water Source Costs

Unit capital costs for impounding reservoirs, including intake and pumping station, are given by

$$C_R = 74.2\ X_R^{-.38} \quad ....(9)$$

where

C_R = annual unit cost of impounding reservoirs in thousands of dollars per billion gallons.

X_R = design capacity of reservoir in billions of gallons.

The minimum design capacity of future reservoirs will be the capacity capable of supplying the total average daily water requirements for all users of the reservoir.

For well development, the equivalent annual cost is $2,780 per mgd capacity (9). This figure includes the development of the entire well field and should be equal to the maximum daily requirement of the user.

Natural supplies, such as lakes and rivers, require only an intake and pumping station. The capital costs for these facilities are given by

$$C_S = 3.95\ X_S^{-.178} \quad ....(9)$$

where

C_S = equivalent annual unit cost in thousands of dollars per mgd.

X_S = design capacity in mgd.

The design capacity is based on the maximum daily water requirement of the user.

Operation and maintenance costs, exclusive of pumping power, are $7.75 per million gallons produced (9), regardless of source. To obtain an annual production, multiply the average daily use by 365. Power costs are $5.25 per million gallons produced per 100 feet of head (9). Head requirements for wells are taken at 400 feet, and 100 feet of head is required for surface supplies. Again, a multiplier of 365 should be used to get annual production.

Finally there may be a water rights cost associated with each individual source. This cost should be ascertained separately by a review of legal agreements and local practices. The cost will generally be expressed in dollars per million gallons used, where the amount of total use is 365 times the average daily use.

8.3.2.2 Transmission Costs

Equivalent annual cost for capital investment in pipelines is given by

$$C_p = 41.3\, X_p^{-.49} \quad \text{.....................(8)}$$

where

C_p = equivalent annual cost for pipelines in thousands of dollars per mile per mgd.

X_s = design capacity in mgd.

Pipeline design capacity is based on the maximum daily water requirement of the user. The use of this equation for estimating pipeline costs requires an estimate of pipeline distance in miles. This is generally taken as the straight line distance between source intake point and the water treatment plant or discharge point.

Not included in the above capital costs is the cost of right-of-way for pipelines. An average cost figure for right-of-way is $3200 per mile (9). Amortizing this and reducing it to an equivalent annual cost yields $247 per mile per year. This is a fixed cost and should not be included in this equation because it is independent of design capacity.

Annual operation and maintenance costs for pipelines can be expressed as

$$A_p = 1.32\, X_p'^{-.49} \quad \text{......................(8)}$$

where

A_p = annual operation and maintenance cost in thousands of dollars per mgd of flow.

X_p' = pipeline utilization level in mgd.

In this instance, the annual operating level, not the design capacity, determines costs.

Pumping station costs depend on the number of pumping stations located along the pipeline. To obtain this number, both the available head and friction losses must be taken into account. Friction losses are assumed to be 4 feet per 1,000 feet of pipe. Available head is the difference in elevation between the intake and discharge point.

Positive head, by convention, means that the intake is higher than the discharge point. Assuming

h_f = elevation difference between intake and discharge points in feet and

d = distance between intake and discharge points in thousands of feet,

then, if $h_f - 4d \geq 0$, the head is sufficient to overcome friction losses and gravity flow will suffice (i.e., no pumping stations are needed).
If $h_f - 4d < 0$, the number of pumping stations required is

$$n = \left| \frac{h_f - 4d}{400} \right|$$

rounded to the next higher whole number.

The unit capital cost for each pumping station is given by

$$C_n = 6.65\, X_p^{-.314} \quad \ldots\ldots\ldots\ldots\ldots\ldots (9)$$

where

C_n = equivalent annual unit cost of pumping stations in thousands of dollars per station per mgd.

X_p = design capacity of pipeline.

Annual operation and maintenance costs for pumping stations are given by

$$A_n = 2.12\, X_p'^{\,-.314} \quad \ldots\ldots\ldots\ldots\ldots\ldots (9)$$

where

A_n = annual operation and maintenance cost in thousands of dollars per station per mgd of flow.

X_p' = pipeline flow level in mgd.

In addition to the operation and maintenance costs, the cost of pumping power must be included. As stated, pumping power is priced at \$5.37 million gallons of flow per hundred feet of head. The head requirements will be $\left| h_f - 4d \right|$ as defined above where $h_f - 4d < 0$. The annual flow is 365 X_p'.

8.3.2.3 Treatment Costs

To assure a reliable supply of water, raw water storage at the discharge end of the pipeline may be provided.

The capital cost for raw water storage is

$$C_{rs} = 1.55\, X_{rs}^{-.201} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

where

C_{rs} = equivalent annual unit costs for raw water storage in thousands of dollars per million gallons.

X_{rs} = raw water storage design capacity in millions of gallons.

The design capacity for reliable supply should be ten times the average daily requirement. For pipelines of less than five miles length, this capacity can be reduced proportionately.

The operation and maintenance costs for raw water storage are

$$A_{rs} = 0.10\, X_{rs}^{-.201} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

where

A_{rs} = annual operation and maintenance cost in thousands of dollars per million gallons.

Treatment plant costs include the costs of the treatment plant and treated water pumping plant. Unit capital costs are given by

$$C_T = 25.6\, X_T^{-.257} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

where

C_T = equivalent annual unit cost of treatment plant in thousands of dollars per mgd.

X_T = design capacity of treatment plant in mgd.

The design capacity is based on the maximum daily water requirement of the user.

Operation and maintenance costs of the treatment plant, exclusive of chemical and power costs, are given by

$$A_T = 7.25\, X'^{\,-.257}_T \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

where

A_T = annual operation and maintenance of treatment plant in thousands of dollars per mgd.

X'_T = operating level of plant in mgd.

The operating level of the treatment plant is based on the average daily requirements for the year of operation.

Chemical costs vary widely depending on the quality of the source water. Therefore, these costs should be determined individually for each source. This can most easily be done by preparing a schedule showing costs versus water quality by type of use. These costs should be given in dollars per million gallons treated where the total amount of treated water will be 365 X'_t.

8.3.2.4 Distribution Costs

Treated water storage requires a capital investment of

$$C_{ts} = 14.3\ X_{ts}^{-.274} \quad \ldots\ldots\ldots\ldots\ldots\ldots (9)$$

where

C_{ts} = equivalent annual unit cost for treated water storage in thousands of dollars per million gallons.

X_{ts} = design capacity of treated water storage facilities in millions of gallons.

The design capacity is estimated as 25 percent of the maximum daily use. Operation and maintenance costs for treated water storage are given by

$$A_{ts} = 1.80\, X_{ts}^{-.274} \quad \ldots\ldots\ldots\ldots\ldots\ldots (9)$$

where

A_{ts} = annual operation and maintenance costs in thousands of dollars per million gallons.

The distribution system network costs can be estimated at $800,000 to $1,000,000 per square mile of development. Distribution pumping power requirements assume a head of 250 feet; thus, the power costs are $14.50 per million gallons of flow, and the total flow is 365 times the average daily flow.

8.3.2.5 Total Costs

Using the above cost data, the annual total of any water supply for any use can be estimated in 1970 dollars. Since each system will have its own special requirements, no generalized total cost equations will be attempted. For example, one town may develop a surface supply requiring treatment while an industry may develop its own well water source requiring no treatment. For each identifiable future water use, an individual total annual cost can be developed by the above-described procedures.

The cost data shown here demonstrate the effect of economies of scale on water system development. As the size of the system increases, the level of service is improved and the unit cost of providing that service is reduced--a fact verified by the negative exponents on design capacity terms in the various unit cost equations. Water systems have long lives and require large capital investments, two factors that make consideration of scale economies imperative.

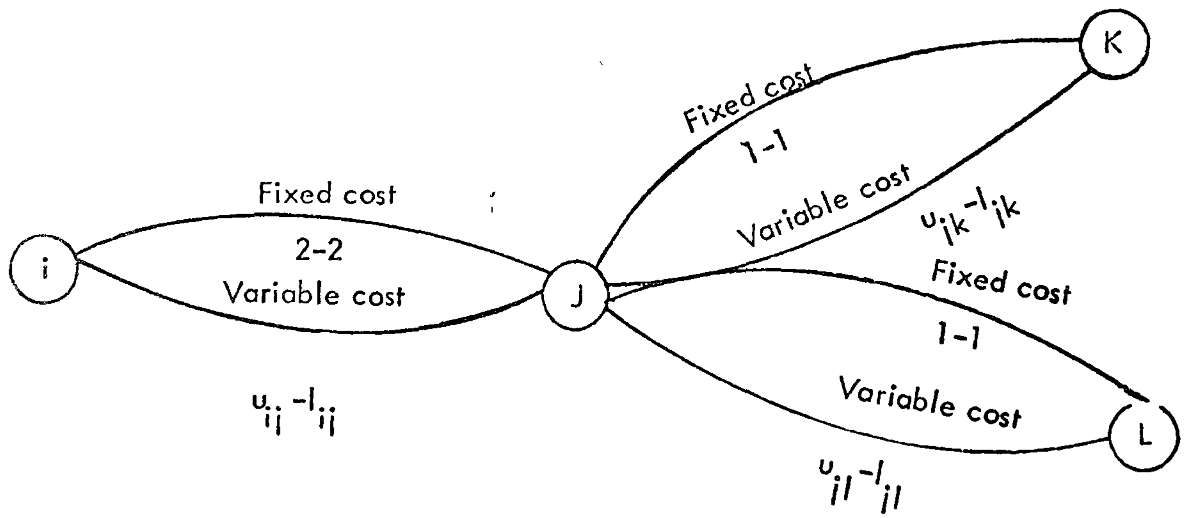
With the total costs of each alternative in the independent networks now derived, the decision as to the best alternative can now be made. This then concludes the analysis of independent networks.

Future cost data can be acquired applying the Engineering News-Record Building Cost Index to the actual and derived cost data (7). There are other methods used for deriving and projecting cost data. No attempt will be made to suggest that one method is better than another in this report. The methods best understood and used by the planner should be applied. The method described here was the one used for this research.

8.3.3 Network Analysis

The network analysis of the model actually is the further evaluation or analysis of a set of alternatives of the network after the initial evaluation described in the network formulation section. In this analysis, the computerized model determines the useful permutations of the network. These permutations are then used with the possible networks.

assigned to an arc that connects the two nodes which denote the entrance and exit of that facility. Then a flow of one mgd is assigned as both the upper and lower limits forcing a constant flow. These "one mgd fake flows" have to be added to the "super source" link for each arc of fixed costs that are assigned to the network. They must also be balanced in the network starting with the "super sink," (which is a fictitious stream where all water goes) and working backwards to the "super source."



The variable costs, which are linear in this model, are then assigned to another arc that describes the facility, and the proper upper and lower bounds are designated. By using this technique, the cost functions can be closely approximated for each link.

By following this basic procedure, a very good flow and costing analysis can be run on any type of network. This process may even be enhanced by using new techniques such as those developed by H.A. Reeder and P.A. Jensen, who developed a program that uses a convex cost function (6). The capabilities of this technique are only limited by the versatility and imagination of the user. (The flow charts of the model are presented in Appendix D of reference (41)).

After completion of the initial network evaluation which reduces the number of networks to the alternatives that needed to be considered, the process of network analysis is begun. The first step is to identify all the independent networks and their alternatives. These are simple in nature and may be analyzed by standard engineering procedures.

All networks in this category are analyzed using a procedure that applies the ... ived cost functions to each of the possible alternatives. The application of these functions on the independent networks constitute the network analysis of the ... el.

In the network analysis there are certain primary steps that have to be made. ... the selection of sources and treatment plants can be modified depending on qual- ... and treatment required. The selection of sources and the required treatment can be ... ined in part to fit the network.

Second, the cost indebtedness of existing facilities is fully taken into account ... obsolescence of these same facilities.

Third, if alternatives are to be considered, then the feasible locations for these ... within the network must be known prior to a model run. By establishing ... inimum flow, certain constraints on the network can be exercised on the network ... considering proposed and existing facilities. A zero minimum flow is used to ... the feasibility of proposed links. Since the solution may indicate a zero ... a proposed link, which means that it is not economically feasible, the obso- ... or feasibility of each link can be determined. Also, by establishing a set ... political jurisdiction can be held to a certain level of service within the ... The reverse is also available when one wishes to examine the economics of ... one or more political constraints in favor of metropolitan source and treatment.

... the maximum flow or capacity can also be used to explore alternatives and ... resources. The maximum flow of each link can be set at the existing capa- ... link or the capacity after a planned expansion. The maximum flow can ... to control the desired loading of a natural resource without exceeding its ...

... ired data for running the model is fed into the model by each arc. The ... for each pair of nodes within the network. This procedure gives ... gaming capability when alternatives are being explored.

... step in the establishment of the data requirements of this model is to ... the network. The nodes are established for each facility within the network. The facilities, primarily pipelines, can be broken apart to fit SAU political jurisdictions or basins if desired for complete analysis. A detailed network is built using successive runs, and the network should be kept as simple as possible with each step (Figure 8-6).

The data requirements for the source and sink nodes with their connecting links will now have been satisfied. The next step is the assignment of the upper and lower bounds for the flow in each link. The bounds can be set anywhere from zero to 9999 mgd. If zero is used, the solution will be equal to or greater than zero. When establishing a fixed cost or when it is desired that a plant be used at least to its debt limit a minimum flow can be set. If a specific flow is desired, then the upper and lower bounds are both set equal to that flow.

Since there are no limits to the number of links entering or departing a node, although each node must have at least one link entering and one link departing, a full range of possibilities is available for each of the facilities. The fixed cost is set by assigning one mgd to both the upper and lower bounds of one of the links. This mgd is then added to all the fixed source links that feed it so that it does not affect the actual flow. This network is referred to as "fake flows." The variable flows can be assigned a cost per mgd, and the minimum and upper bounds assigned from 0 to 9999 mgd. The only constraint is that the upper limit must be greater than or equal to the lower bound. Zero cost arcs can be added to provide continuity and/or a certain disaggregation of arcs.

After the capacities have been assigned, the remaining data requirements are added to each of the links. This is the cost of that link in dollars per million gallons per day($/mgd). This cost data should be the actual cost data in all cases possible. If the actual cost data is not available, then it should be estimated by standardized procedures. If the variable cost is a linear function that depends on the size of the facility, as in treatment plants, then an estimate has to be made initially as to the size needed. A family of curves is developed based on the cost per flow (Figure 8-7). The slope of that particular curve is entered as the cost of that link. After the run, the link results are examined. If the results show that the plant is being used to full capacity, then

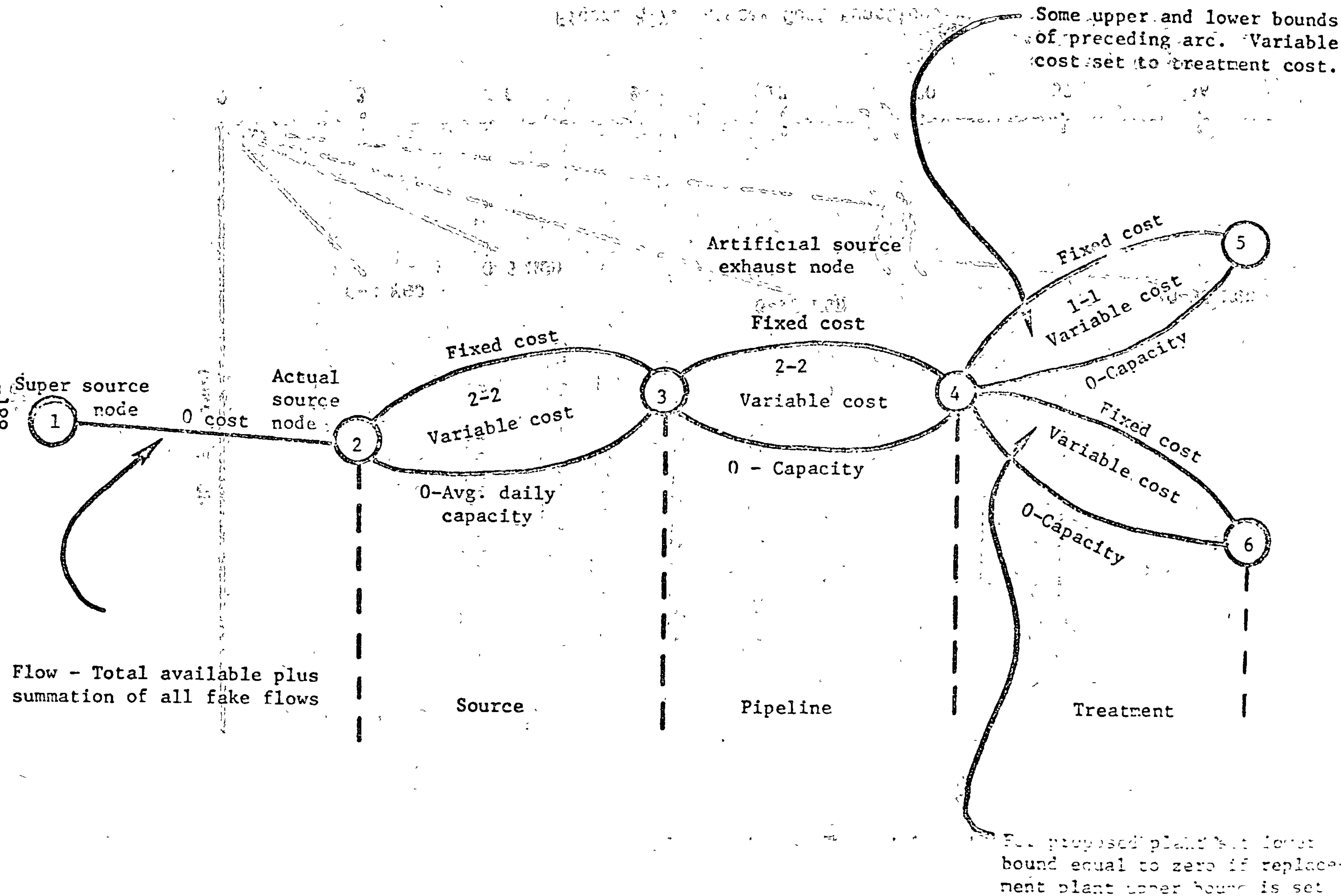


Figure 8-6. Example Flow Net.

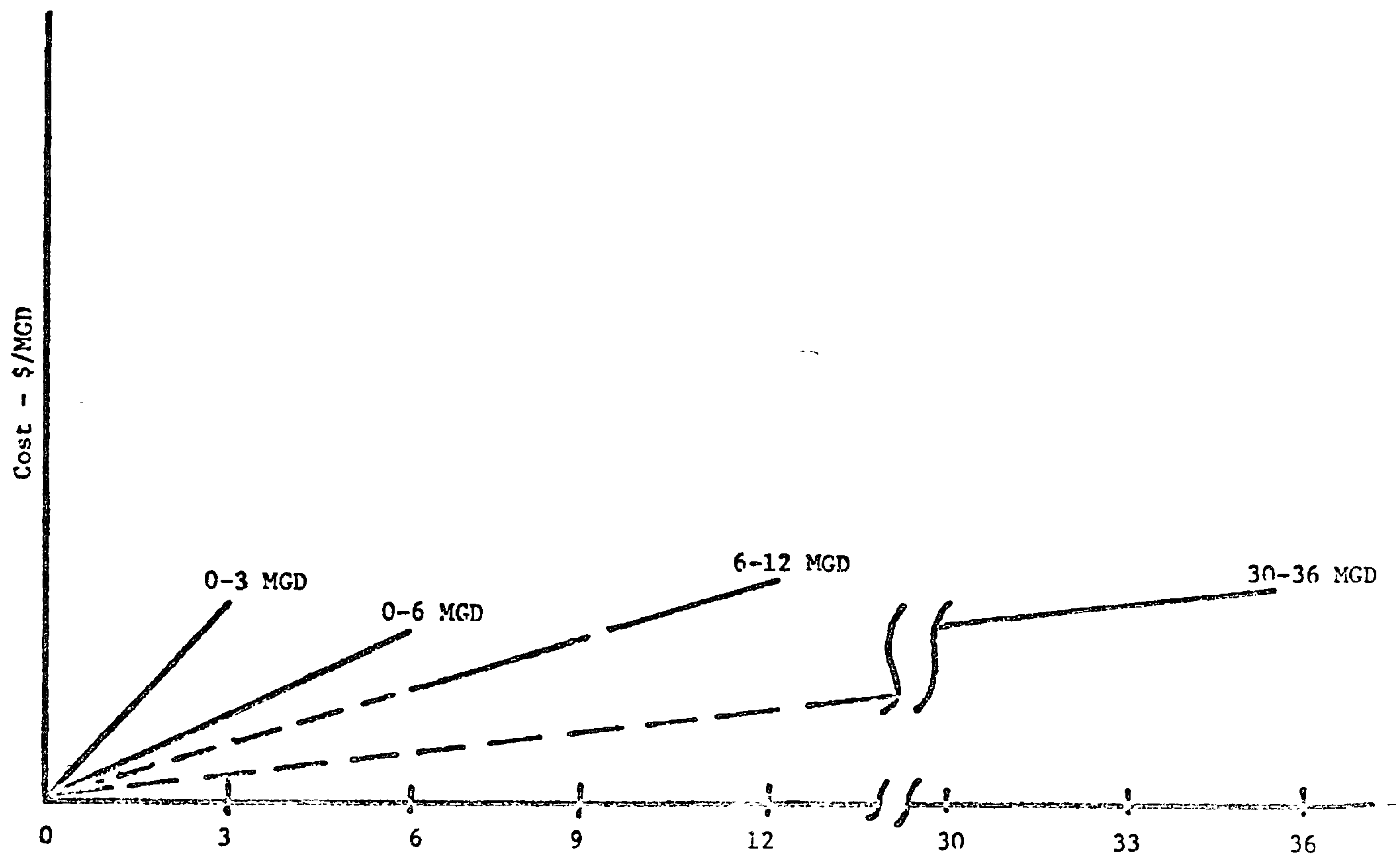


Figure 8-7. Linear Cost Functions.

. is desirable and a larger facility curve is used. If it is not being used to ... then a smaller facility curve can be tried.

... approach that has proven effective is to take the largest facility curve and ... cost and set the lower bounds equal to zero. After the first run, set the link cost equal ... facility curve slope that the results dictate. Then run the model again to verify the results.

... NETWORK MODEL

... structure of the sewer network model is basically the same as the water supply ... model. Therefore, only the philosophical and technical differences that ... network creates in the application of the model will be discussed in this ...

... previously discussed, the first phase is the isolation of the feasible independent ... the reduction of the problem to its simplest form. The first step is to re... network to the "real" network by identifying the infeasible alternatives and ... them from the network. This is a coarse screening done by the best qualified ... independent networks are then evaluated using engineering cost data ...

... interconnected networks or the alternate solutions that interconnect indepen... are evaluated by loading the system onto the computer and evaluating them ... network model. The flexibility built into the model for the water network anal... ... for the evaluation of the sewer system.

... Formulation

... network varies from the water network in that it is primarily a ... The use of pumps, pressurized lines, and lift stations are normally ... when absolutely necessary.

... network begins with a collector system in each small basin. These ... block collector networks are sized by using the technique described in ... Water, Sewer, and Storm Drainage Micro Area Requirements of References ... sewage then flows from the collector systems into the sewer mains which ... using the technique described in Appendix A of Reference (41).

The design of the collector systems that serve the small basins is the responsibility of the political jurisdictions involved. This study dealt only with the collection of the sewage from these small basins and political jurisdictions, its transportation to a system of treatment plants, and the discharge of the effluent into a receiving stream.

No attempt was made to develop a stream recovery model that examines in detail the effect that the effluent will have on the receiving streams. The network model does consider some of these effects by limiting the upper bounds of the arc that connects the outfall of a treatment plant to the receiving stream to a value of effluent that the stream can handle. This value is obtained by using methods described in Appendix E of Ref. (41). By controlling the amount of effluent, based on a specified level of treatment for the study area (in this case secondary) that can be discharged at different points, the quality of the receiving streams can be maintained. This can be made seasonal by changing these values based on the seasonal flows and cha acteristics of each receiving stream.

The sewerage model examines all these feasible alternatives for the region and optimizes them into a regional sewer network. It is also possible, if not probable, that the region is made up of several major basins of different characteristics that are not connected within the study area. These basins may be analyzed separately or by interconnecting two or more of the basins. There is also the capability of the planner to evaluate the alternatives of having one major treatment plant or any combination of smaller treatment plants.

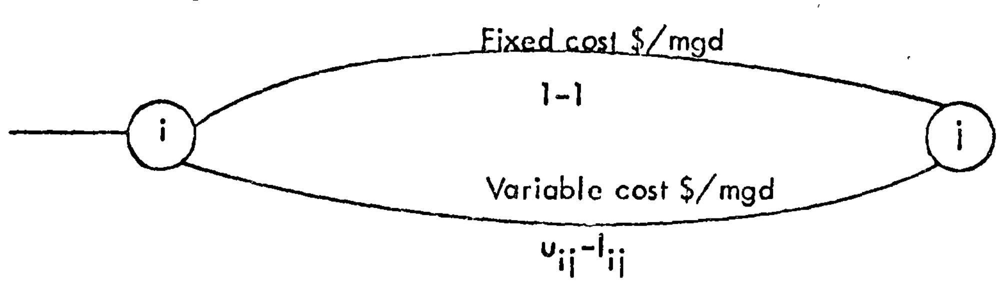
The procedure for formulating the sewer network is identical to that described in the water supply network model. The first step is to establish the current network from the inventory data. The only difference between the sewer networks and those for water is that the flow is reversed. The use of special areas, mainly basins, within each political area is greatly increased. This allows the demand portion of the model to work within the framework of gravity flow across corporate boundaries.

Literature review has failed to reveal any model that can effectively handle this phase of the problem on a general basis. A model that can be easily understood and run while not requiring larger computer capabilities than are generally available is desirable. A wide assortment of linear programs is available for a network analysis. The most successful algorithms in terms of the present requirements, are Fulkerson's Out-of-Kilter Algorithm and several of its variations (4,5,11). This method was developed into a program by R.J. Clasen (12). The basic description of this model can be reviewed in any of these publications if a detailed analysis is required.

The Clasen program was altered to handle both sewer and water networks. This was done to simplify the modeling requirements of this study and has proved to be adequate for planning purposes. It was then determined that the cost functions could be disaggregated and simplified to linear functions that would give good approximations. This allows a program to be run that is relatively simple and still indicates the feasibile alternatives to be analyzed in detail.

The network model is made up of a system of nodes and arcs that are interconnected by arc. Each node represents an intake or exhaust of some facilities. Depending on the degree of accuracy needed, computer capabilities and available cost data, this network can be as detailed as needed. An arc-node grouping can represent a complete treatment plant or each of the steps through the plant. The usual procedure is to simplify the network as much as possible, depending primarily on cost data, for the initial runs. When flows have been determined, then infeasible or undesirable permutations of the network can be removed and new detailed networks can be derived and run.

The model starts with a "super source," which is a fictitious source of water for the water sources that actually supply the network. These sources are the first series of nodes. Since each node can be interconnected with one or more arcs, the cost functions can be disaggregated by the user. This is accomplished by determining the fixed cost (the cost incurred by the planner no matter whether the facility is used or not) and then assigning a flow of one mgd to one arc. In other words, the fixed costs of a link in the network are



The identification of the real sewage network and its shortages and capabilities is handled in the same manner as for the water network. It is also advanced into the "desirable" worlds and evaluated as to their incremental capabilities, deficiencies, and availabilities.

At the conclusion of this procedure, the planner will have a complete understanding of the sewer networks, their requirements, and feasible alternatives for the region under study. The problem will have again been reduced to its simplest form and be ready for presentation to the decisionmaker for final selection of the alternatives for further analysis. Each choice will then be modeled for the selection of the best alternatives to be incorporated into the final plan.

8.4.2 Network Analysis

The model again varies with the type of network under consideration. If it is a simple independent network, the use of derived cost functions are used, but if it is a complicated independent or interconnected network, then the computer model is employed.

When the computer model is run, it is used in the same manner as that of the water network. The network is again made up of nodes and arcs. Each facility is represented by a node for the inlet and another node for outlet. If a fixed cost is encountered in this facility, it is represented by an arc with a fixed "fake flow," usually one mgd. If the conservation of flow changes this fake flow to another value, then the fixed cost is reduced proportionately. Another arc is used to represent that portion of the cost, above fixed cost, which varies linearly with flow. A system of cost lines are developed for each type of facility. These arcs and nodes with their derived cost functions can be used to depict accurately any type of facility. The use of these dual nodes and multi-arcs is limited only by the users' ability to depict each facility by its proper combination of arcs.

If the network under consideration is a simple independent network, it can be handled using cost functions similar (in many cases identical) to those used in the water network. The equations for transmission, pipeline, and right-of-way costs are the same. The remainder of the costs for the system are from references (10) and (13) to (16).

The data requirements for this model are the same as those required for the water network. The only difference is in the cost curves and functions used to provide the actual costs. It is obvious that these cost data are available from many sources and those that are used here are not considered as absolute. The planner should use those functions that he feels are most accurate. The functions that the author uses are obtained from those listed in references (10) and (13) to (16).

There were many runs made of different arrangements for the network. The model's full usefulness is limited only by the skill of the planner in depicting the system and alternatives in modeling nomenclature. Once a proper set of cost functions for their systems has been developed and projected [this study used the ENR cost index (7)], the optimization of the network will be obtained.

8.5 THE SOLID WASTE SUB-MODEL

The objective of the solid waste sub-model was to develop methods for predicting and managing solid waste generated in an urban area. The prediction is made by using parameters previously employed in the population model. The network model used for the water supply and sewerage system can be modified to deal with the management of solid waste generated in an urban area although the application of the network model for solid waste is not included in the present study. In other words, the present model only deals with the quantity of solid waste produced in an urban area. It does not consider the other components of the solid waste system.

Actually, the solid waste system in an urban area consists of three main components: generation, collection, and disposal. Solid waste generated in an urban area can be divided according to the sources of generation; e.g., from the commercial sector, industrial sector, residential sector, etc. Obviously, the collection component varies among different sectors of an urban area. Unless the characteristics of an urban area are known, it would be useless to develop a solid waste collection system for an urban area. The third component of a solid waste system, the disposal component, is also very much dependent on the local characteristics. This is due to the methods used for disposal; e.g., sanitary land fill, incineration, composting, etc. are dependent on the local needs and characteristics. Therefore, the solid waste model is concerned only with the volume of waste generated from different sectors of an urban area.

8.5.1 Description of the Sub-Model

The solid waste sub-model is an analytical formulation developed to compute the volume of solid wastes generated in an urban area. The model uses only parameters developed in the population model. In other words, the solid waste sub-model does not make direct predictions, it only uses outputs from the population model and several assumed constants to compute future solid waste generation.

In the development of the model, several assumptions were made. First, the amount of solid wastes generated in an urban area are assumed to be basically a function of the number and the characteristics of people residing in that area. Further, the amount of solid waste generated per person per day is assumed to increase at a constant rate. For example, the current national average increase of 2.5 percent per year was used in the present computation.

The main variables used in the model formulation have all been previously used in the population model. These variables are: population growth rate (person/year), work force in manufacturing (percent of total work force), and work force in service industries.

Other parameters needed for calculating future solid waste generation are current generation rate (lb/cap./day) and national solid waste generation annual increase rate (percent per year). The current generation rate usually can be obtained from the public works division of a city government.

The model computes the total solid waste generation in an urban area and also the fractions of those generated from the commercial, industrial, and residential sectors.

The total solid waste generated at time (T) can be computed by:

$$W_T = [P_o + \Delta P (T_P - T_o)] \quad [W_{GR} (1 + \Delta W_{GR} (T_P - T_o)]$$

where

W_T = Total solid waste (lbs/day).

W_{GR} = Waste generation rate (lbs/cap./day).

P_o = Present population.

ΔP = Annual population increase

T_o = Present year.

T_P = Prediction year.

ΔW_{GR} = Annual increase rate of solid waste.

From data collected from eleven cities, shown in Table 8-2, there is a positive correlation, which is shown in Figure 8-8. between the percent of total solid waste generated from the commercial and industrial sectors and the percent of work force in the manufacturing or the service industries. Hence, the percent of work force in the manufacturing or in the service industries was used as a parameter to compute the volume of waste produced in the commercial and industrial sectors. The formula derived is

$$W_{C,I} = W_T (M + S)$$

where

$W_{C,I}$ = Solid waste produced by the commercial and industrial sectors.

W_T = Total solid waste generated.

M = Percent of work force in manufacturing.

S = Percent of work force in service industry.

The volume of solid waste generated from the residential sector is computed by subtracting the solid waste produced by the commercial and industrial sectors as indicated by:

$$W_R = W_T - W_{C,I}$$

where

W_R = Solid waste produced by the residential sector.

W_T = Total solid waste.

$W_{C,I}$ = Solid waste produced by the commercial and industrial sectors.

In summary, these formulas can be used to predict the total solid waste produced in an urban area as well as the fraction of waste produced by the commercial, industrial, and residential sectors.

8.6 VALIDATION

The primary objective of this research project was to provide the average planning group with a usable model for regional water, sewerage, and solid waste planning networks. The model developed fulfills all the originally stated objectives for the water supply and

Table 8-2. Percent of Work Force in Manufacturing and Services Industries and Percent of Solid Waste Generated from the Commercial and Industrial Sectors.

City	Percent of Work Force			lbs/cap./day	
	MFG	Service	Total	COM & IND	Percent of Total
San Jose, Cal.	34.5	19.4	53.9	3.50	58.3
Hackensack, N. J.	38.7	14.7	53.4	3.70	53.6
Paterson, N. J.	38.7	14.7	53.4	2.89	54.0
Clifton, N. J.	38.7	14.7	53.4	3.78	58.6
Passiac, N. J.	38.7	14.7	53.4	3.31	54.7
Phoenix, Ariz.	24.3	16.2	40.5	2.20	44.0
Flint, Mich.	53.8	10.2	64.0	3.13	67.6
New Orleans	15.2	17.7	32.9	2.12	46.9
Los Angeles	30.4	18.2	48.6	3.20	48.3
Okla. City	14.0	14.7	28.7	1.40	26.9
Tulsa	24.6	16.1	40.7	1.50	31.3

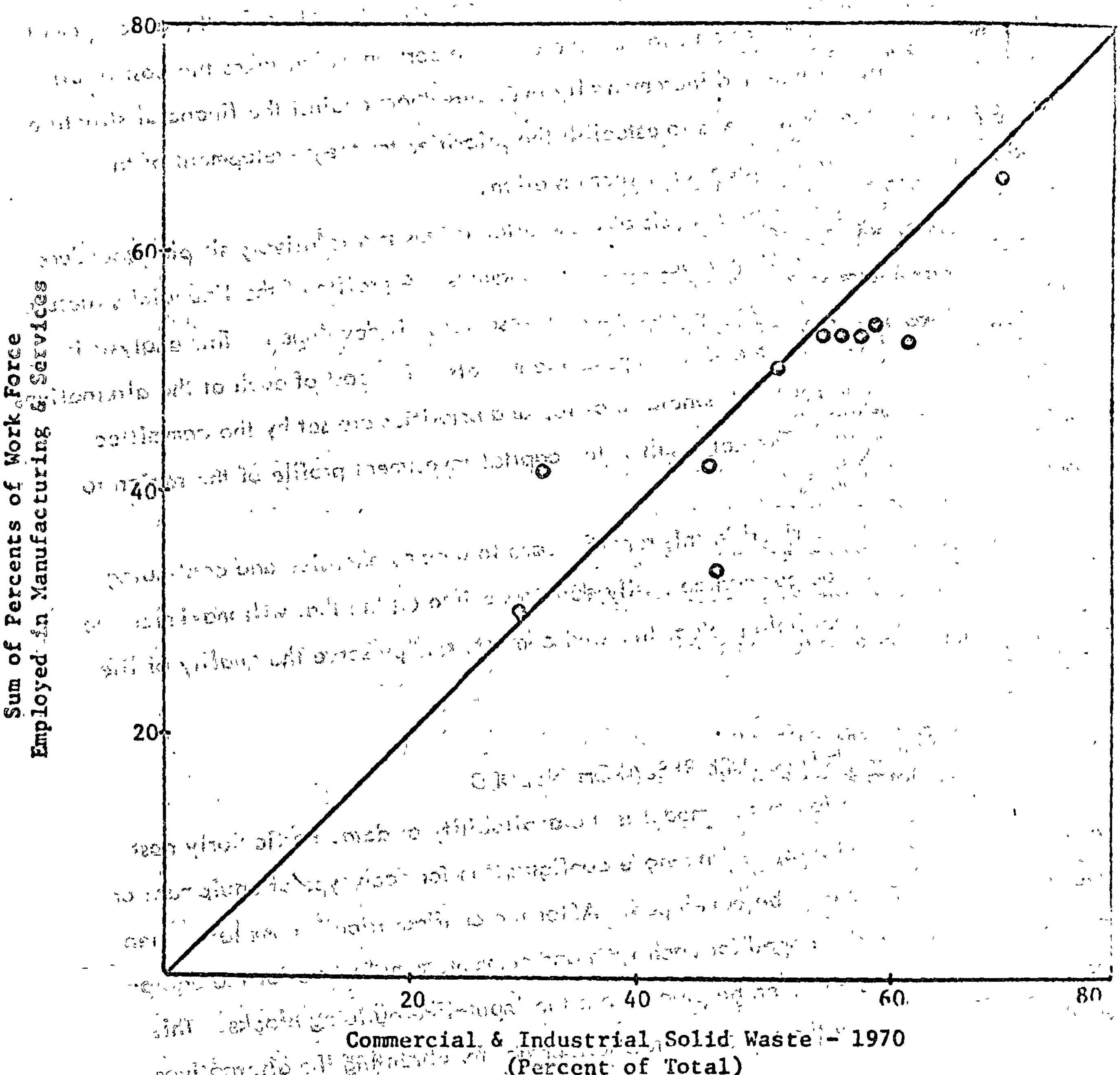


Figure 8-8. Relationship between Percent of Work Force in Manufacturing and Service Industrial and Percent of Solid Waste Generated from the Commercial and Industrial Sectors.

sewerage systems. For the solid waste system, the application of the network model to the system still needs to be done.

At this point, the model has completed all necessary information for the development of the final plan. There remains to be done only the portion which takes the cost of all the desirable alternatives and incrementally evaluates them against the financial structure of the region. The final step is to establish the priorities for the development of the alternatives that were selected for implementation.

The process of financial analysis of these alternatives is a relatively simple procedure that follows the standard techniques used in economics. A profile of the financial structure, including bond debt limits and future financial resources, is developed. This analysis is done incrementally, using the same steps as the models. The cost of each of the alternatives is then evaluated against the financial profile, and priorities are set by the committee of concerned agencies. The net result is the capital investment profile of the region to obtain the selected goals.

The process, as described in this report, leads to a comprehensive and continuing planning model. The output can be easily developed into a plan that will maximize the use of natural resources, help protect the environment, and preserve the quality of life that is desired.

8.7 LIMITATIONS AND FUTURE RESEARCH NEEDED

The primary limitation of this model is the availability of data, particularly cost functions. The simplest possible link-node configuration for each type of equipment or facility in the system should be developed. After the configuration is complete, then cost functions can be developed for each type and common manufacturer of the equipment. These packages can then be placed into the input-like building blocks. This would greatly facilitate the use of the network model for obtaining the alternatives to be considered in the final evaluation.

This model can very easily be expanded to include solid waste, stream control, air pollution, or transportation, to name but a few. For the solid waste system especially the network model can be used for the solid waste management analysis. However, this would have to be done in future research. In any case, the model should be developed to its full potential because the availability of a single technique for analysis of so many urban planning functions would be invaluable.

The only other limitation is, as always, the development of a usable and genera land use model, one that would bridge the tedious step from the population model to the demand model. What is needed is a model that evaluates old neighborhoods on an incremental basis and allocates the proper portion of the population model to them. It should also take the difference between two consecutive periods and compute the new neighborhoods and industrial areas needed to support this growth. The planner would then only have to intervene by allocating the different types of neighborhoods to the land before proceeding to the demand model. The development of these areas would then give the planner a model that could be used to depict completely the development of the region using a minimum of computerized models--a most desirable goal.

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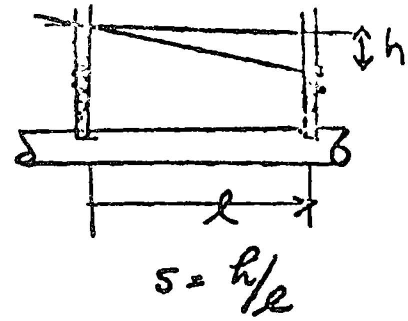
HARDY CROSS DISTRIBUTION SYSTEM ANALYSIS

George W. Reid



PART III

IV. Hydraulics of Pipe Systems

A. $h_f = f \dfrac{lv^2}{d2g}$

1. h_f is the head loss in feet of water.
2. l is the length of the pipe in feet.
3. d is the diameter of the pipe in feet.
4. v is the mean velocity of flow in feet per second.
5. f is a dimensionlèss friction factor

   a. $f = \dfrac{64}{R} = \dfrac{64\mu}{vd}$, where R is the Reynolds number and μ is the kinematic viscosity (for laminar flow; R - 2000)

   b. when R = 4000, f varies with R and $\dfrac{n}{d}$, where n is the absolute roughness (turbulent flow)

Temperature, C	0	5	10	15	20	25	30
μ, centipoise	1.792	1.519	1.310	1.146	1.011	0.8975	0.8039

B. The Hazen-Williams Formula (See Table Alignment Chart for Flow in Pipes, page)

1. Q = rate of discharge in mgd, gpd, gpm, or cfs as needed.
2. D = diameter of large circular conduits in fts.
3. d = diameter of small circular conduits, especially pipes, in inches.
4. v = average velocity of flow in fps.
5. $a = \dfrac{\pi d^2}{4}$ = cross-sectional area of conduit in square feet.
6. r = a (wetted perimeter) = D/4 = d/48 = hydraulic radius in feet.
7. $s = h_f/I$ = slope of hydraulic gradient, or loss of head
8. $v = C\, r^{0.63}\, s^{0.54} \times 0.001^{-0.04}$
9. $v = 0.55CD^{0.63}\, s^{0.54} = 0.115C^{0.63}\, s^{0.54}$
10. $Q\ (mgd) = 0.279\ CD^{2.63}\, s^{0.54}$
11. $Q\ (gpd) = 405\ Cd^{2.63}\, s^{0.54}$
12. $D = 1.53a^{0.38}\, r^{0.24}$, for identical values of C and s

C. Pipe Characteristics

1. in parallel: $Q_1 + Q_2 \ldots\ldots + Q_n = Q_t$ where Q_t is the total flow;

   $h_1 = h_2 = \ldots\ldots h_n$ where h is the head loss;

   $$\frac{1}{R_t^{0.54}} = \frac{1}{R_1^{0.54}} + \frac{1}{R_2^{0.54}} \text{ etc. } - \frac{1}{R_n^{0.54}}$$

   where R_t is the total resistance to flow

2. in series:

$Q_1 = Q_2 = \ldots\ldots = Q_n$; $h_1 + h_2 + \ldots\ldots + h_n = h_t$;

$R_t = R_1 + R_2 + \ldots\ldots + R_n$

3. An equivalent pipe is used to replace two or more pipes in series or in parallels and still have the same overall properties of the original pipes.

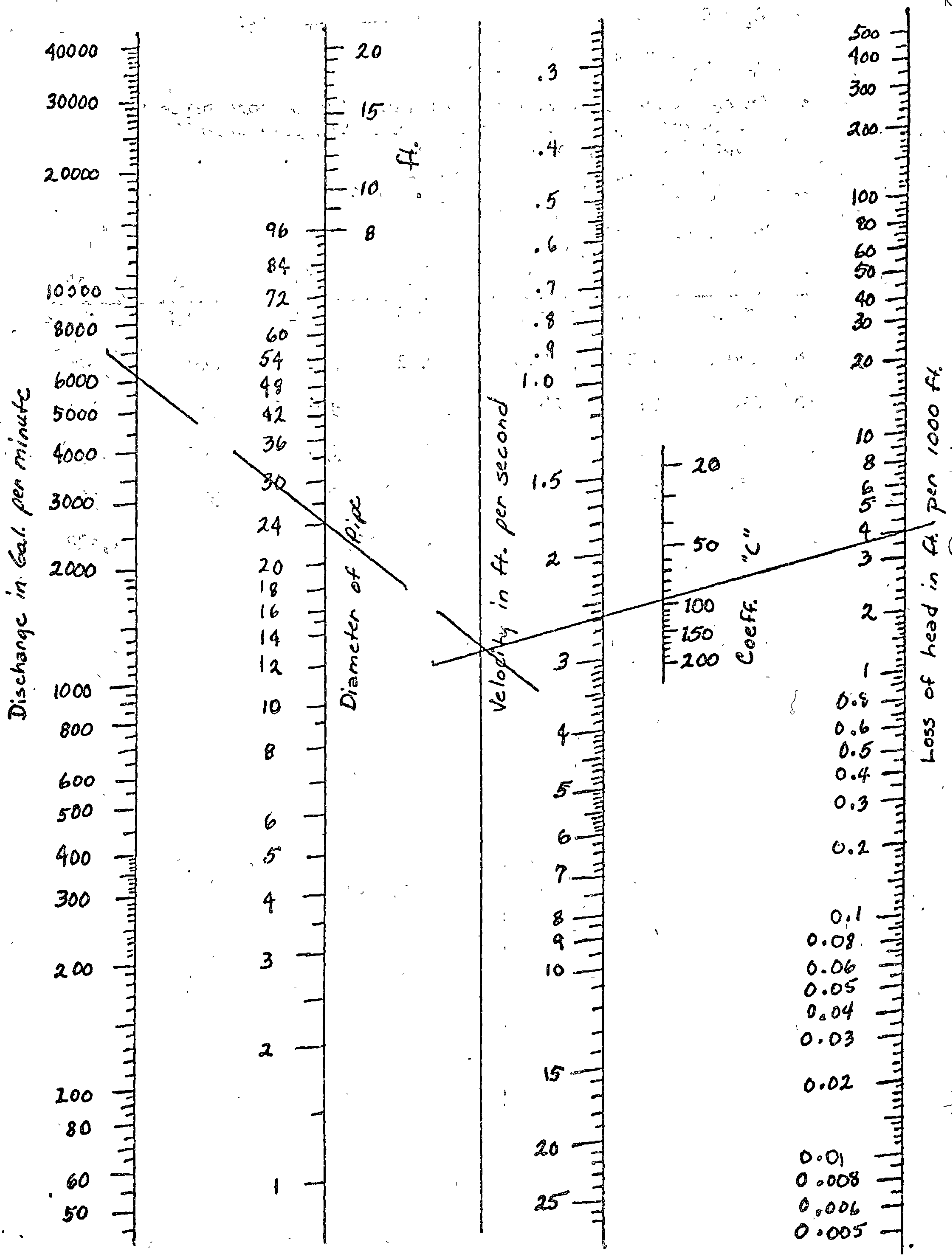
4. $Q = (Kh/L)^{0.54} = (\frac{1}{r} \times \frac{h}{L})^{0.54}$

Values of K follow:

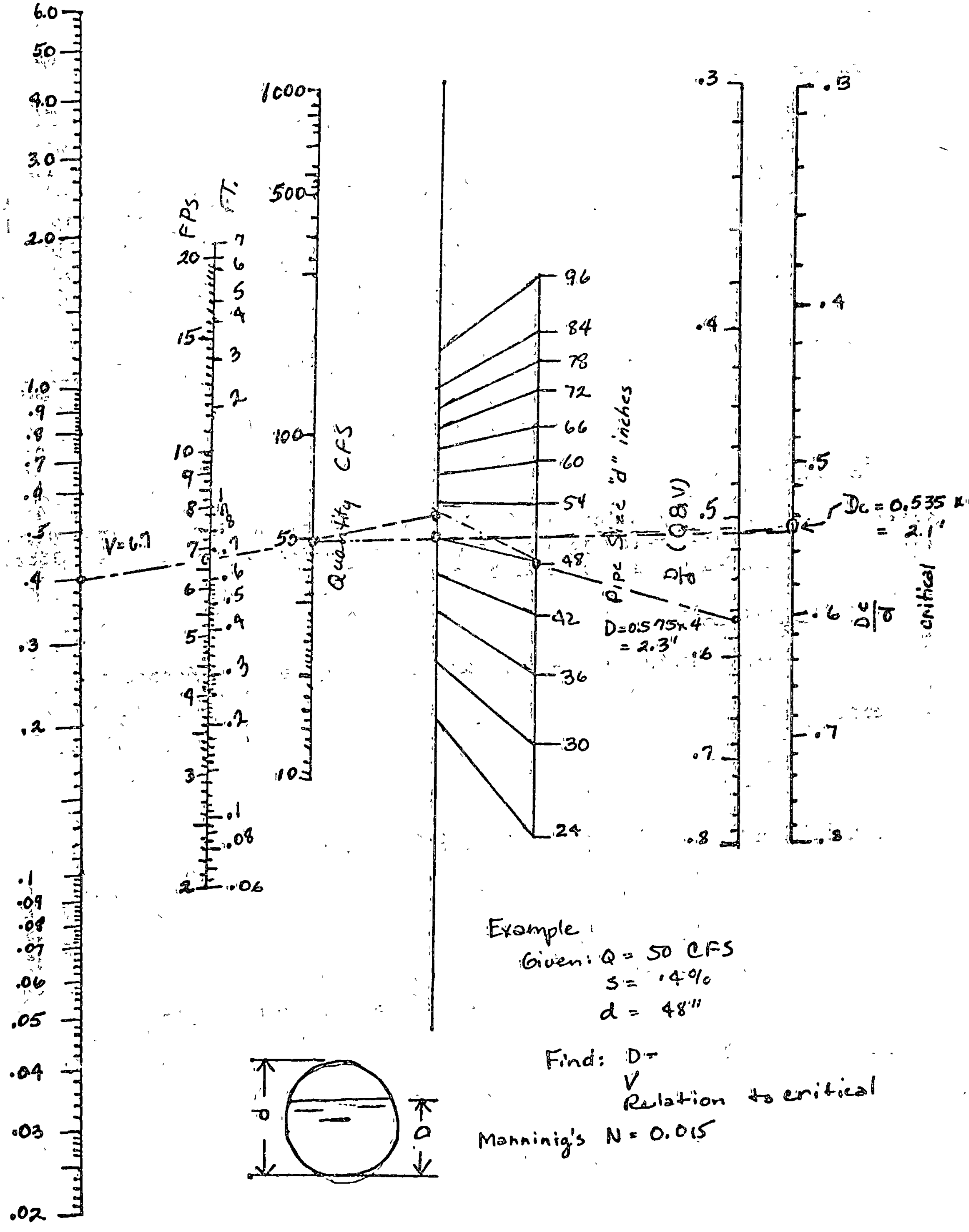
C	80	90	100	110	120	130	140
6"	60.2	67.6	75.4	82.8	90.4	97.8	105.2
8"	128.2	143.2	160.2	176.2	192.4	208	224
10"	230	259	288	317	346	374	404
12"	372	419	465	511	558	605	654
14"	562	632	703	773	843	913	983
16"	793	832	991	1,190	1,190	1,290	1,388

Alignment Chart for Flow in Pipes

Hazen-William Formula: $V = 1.318\, C R^{0.63} S^{0.54}$



CULVERT DESIGN



Example:
Given: Q = 50 CFS
S = .4%
d = 48"

Find: D =
V
Relation to critical

Manninig's N = 0.015

HAZEN-WILLIAMS PIPE FLOW FORMULA

Suggested Values of C

Cast Iron Pipe

Age of Pipe in Years	Diameter of pipe in inches: 4	6	8	10	12	16	24	36	60
0	130	130	130	130	130	130	130	130	130
10	106	108	109	110	110	111	112	112	112
20	88	92	94	96	97	98	99	99	100
30	75	80	83	85	86	87	89	90	91
40	64	71	74	76	78	79	81	83	84
50	56	63	67	69	71	73	75	76	78

Reinforced Concrete pipe and Bitumastic and Cement-lined pipe (also transite).

C - 140 remains there almost indefinitely. For safety say
C = 130 Best Probable, Worst 145, 120, 110

Steel Pipe

Lock - bar joints and welded joints - same as for cast-iron pipe.
Double-riveted joints - same as for cast-iron pipe 10 yrs. older.
Welded steel - same as 5 year old CIP.

Masonry Aqueducts

Very smooth sides - 125 reduced by roughness and deposits.

Sewers

Tile - 110
Brick - 100

Pipes of Uncertain Age and Miscellaneous Channels

Basis of general calculation - 100. From average experience.

Concrete Pipe and Tunnels (Wood Forms)

Best, Probable, Worst - 130, 120, 90
If slimed - 20 points, 15 points, and 10 points below.

Wood Stove Pipe

Best, Probable, Worst - 145, 120, 110

RESISTANT VALUES

Values of $r \times 10^5$ for 1000 feet of pipe based on the Hazen-Williams Formula where $h = rQ^n$ (Q in gpm)

Pipe Diameter, Inches	C = 90	100	110	120	130	140
4	300	248	208	177	153	133
6	41	33.7	28.4	24.2	20.9	18.2
8	10	8.4	7.0	6.0	5.2	4.5
10	3.4	2.8	2.4	2.0	1.7	1.5
12	1.5	1.2	1.0	.83	.71	.62
14	.66	.55	.46	.39	.34	.30
16	.35	.29	.24	.20	.18	.15
18	.20	.16	.14	.12	.10	.09
20	.12	.10	.08	.07	.06	.05
24	.049	.04	.03	.03	.02	.02
30	.016	.013	.011	.010	.008	.007
36	.0067	.0054	.0046	.0039	.0034	.0030

D. Minor Losses

Dia.	1	1½	2	2½	3	4	5	6	7
C	2.5	3.75	5	6.25	7.5	10	12.5	15	17.5

Dia.	8	9	10	11	12	15	16	18	20
	20	22.5	25	27.5	30	37.5	40	45	50

45° L - 0.26

90° L - 1.0

Tee - 2.0

Gate Valve - 0.37

Globe Valve - 3.33

Check Valve - 3.0

Disk Meter - 9.0

Piston Meter - 20.0

Turbine Meter - 8.3

Venturi - 2.66 - 5.66

$\frac{d_1}{d}$ - 0.5 -0.4

For example - 10 pipe - 300' long with 2-90° L, 1 gate, 1 check

and a venturi $\frac{d_1}{d}$ = 0.5 - the additional footage

2L - 25 ft. ea. = 50'

1 gate 25 x 0.37 = 9'

1 check 3 x 25 = 75

1 venturi =
25 x 2.66 = 66

200 ft + 300 ft. of pipe = 500 ft.

Allowable Leakage

$$Q = nd \left(\frac{P}{1.85} \right)^{\frac{1}{2}}$$

Q = g/h
P = psi
n = joints
d = diameter in inches

V. Collection and Distribution of Water
circle, section, Hardy Cross, network analyser, algebraic

A. Circle Method (Fire - flow)

1. Used to determine the number of fire streams available at any point in the water system (one fire stream is 250 gallons per minute)
2. Draw a circle with a radius of 500 feet and its center at the point of the fire
3. Count the number of pipes cut by the circle

4. Determine $h_f = \dfrac{\text{(pressure drop from main to hydrant) (2.31)}}{\text{distance from main to radius of circle}}$

5. Determine flow per pipe in gallons per minute; $Q = \frac{1}{r} \frac{h}{L}^{0.54}$

6. Multiply flow per pipe by the number of pipes cut to obtain total flow.

7. Divide total flow by 250 to obtain the number of fire streams available.

B. Hardy Cross Method of Analysis of Flow in a Pipe Network - Used for a micro solution.

1. $Q = Q_1 \pm \Delta Q$

   a. Q is the actual amount of water flowing, Q_1 is the assumed value, and ΔQ is the flow correction.

2. For any pipe system the sum of the head loss to any point in the system is the same regardless of the route travelled.

3. $\Delta Q = \dfrac{\sum R\, Q_1^{1.85}}{\sum 1.85\, R\, Q_1^{0.85}}$

4. Procedure:

   a. Assume any distribution of flow as to amount and direction
   b. Compute the head loss in each pipe $h = RQ^n$
   c. Compute the total head loss around each circuit

   Algebraically: $\sum h_1 = \sum R\, Q_1^{1.85}$

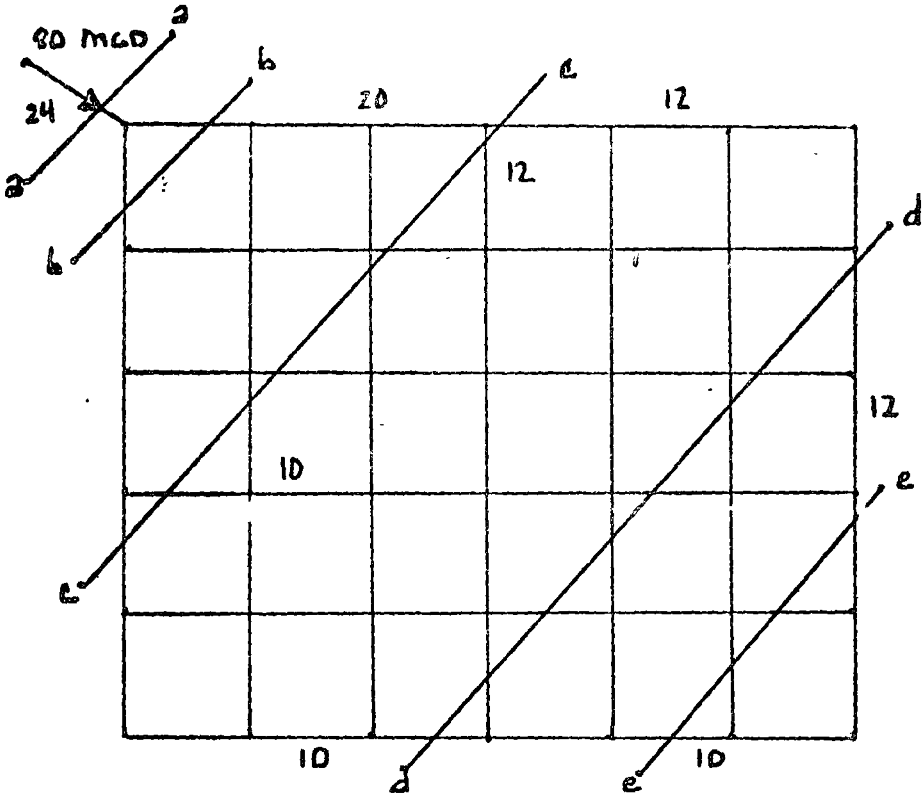
   d. Compute, without regard to sign, for the same circuit $\sum R\, 1.85\, Q_1^{0.85} = \sum \frac{h}{Q_1}$ per pipe

   e. $\Delta Q = \dfrac{\sum h_1 - \sum h_2}{1.85 \sum \frac{h}{Q_1}}$, $h_2 < h_1$

   f. ΔQ is added to the route having h_2, and is subtracted from the route having h_1.

   g. Usually a system can be balanced in three trials. $\Delta Q \simeq 1\%Q$

C. Method of Sections, Good as a macro solution.



Pipes 6" diameter unless noted; C = 100 for all pipes

Assume

1 - Est water Reqd beyond arbitrary sections

2 - Count cut pipes (aa = 1, bb = 2, cc = 8 and note size and distribution)

3 - Grade; 1-3 and S≅2-4

Solution

Sec a-a
Demand 8.0 MGD
Capacity 1-24" @ 2 = 6.0 MGD
Deficit 2.0 MGD
With no additional pipes a-a carries 8.0 @ 3.6

Sec b-b
Demand 8.0 MGD
Capacity 2 to 12" 1.8
Deficit 6.2
Try 2-20" 7.8
20" will 8.0 @ 2.4

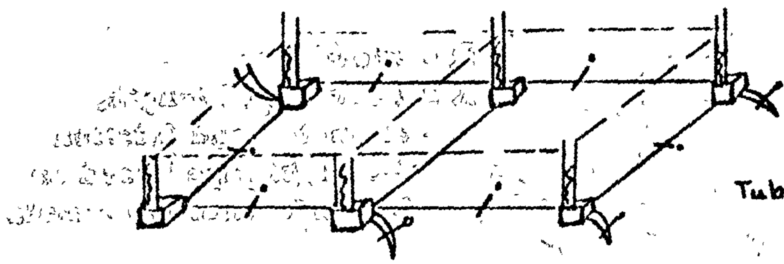
Sec c-c
Demand (reduced 1/8) 7.0 MGD

Existing pipes	2 to 12"	2.0	1 to 20	3.7
	1 to 8"	0.3	2 to 12	2.0
	5 to 6"	0.8	5 to 6	0.8
Capacity		3.1		6.5
		3.9		0.5

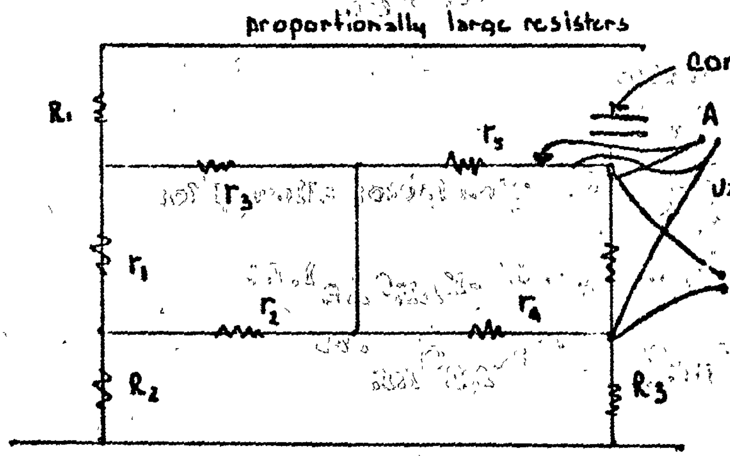
Change 1-12 to 20, 1-8 to 12. Add 1-10" - 0.6 and bring to capacity.
Continue thru (e-e)

D. Analogs

1. Hydraulic

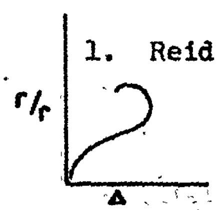


2. Electrical Network Analyser



$E = IR$

$H = RQ^n$

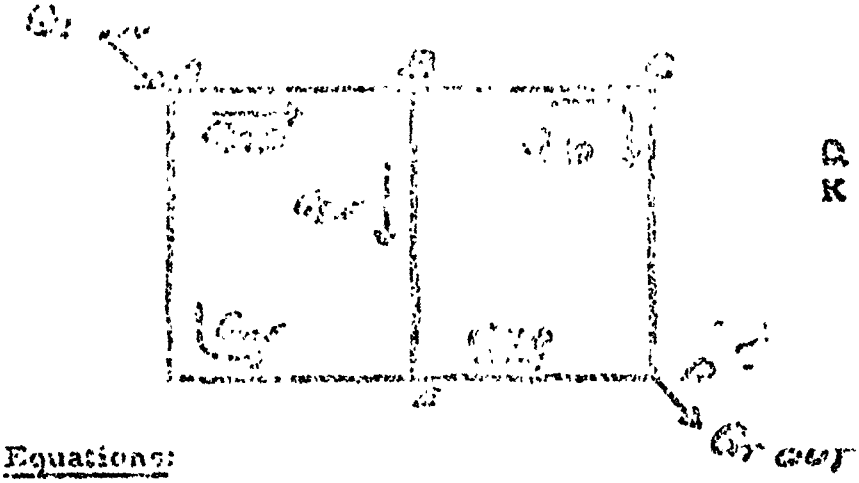


1. Reid - corrective curve

2. Hazen
$H = \left[RQ^{(h-1)}\right]Q$

3. McIlroy
$E = I^n R$

4. Reid - H balance



Q = Flow Rate
K = Function of length, diameter and friction factor of pipe based on Hazen Williams formula

Equations:

Diff = "Loss of head" factor

CF = Correction Factor

Δ CF = Maximum allowable correction factor allowed for acceptable solution

$Diff_1 = K_{AB}Q_{AB}^{1.85} + K_{BE}Q_{BE}^{1.85} - K_{AE}Q_{AE}^{1.85}$

$\Sigma KQm_1^{.85} = K_{AB}Q_{AB}^{.85} + K_{BE}Q_{BE}^{.85} + K_{AE}Q_{AE}^{.85}$

$$CF_1 = \frac{Diff_1(M)}{1.85 \cdot \Sigma KQm^{.85}}$$

$Q_{ABCORR} = Q_{AB} + CF_1$

$Q_{AECORR} = Q_{AE} - CF_1$

$Diff_2 = K_{BD}Q_{BD}^{1.85} - K_{BE}Q_{AE}^{1.85} - K_{ED}Q_{ED}^{1.85}$

$\Sigma KQm_2^{.85} = K_{BD}Q_{ED}^{.85} + K_{BE}Q_{BE}^{.85} + K_{ED}Q_{ED}^{.85}$

$$CF_2 = \frac{Diff_2(M)}{1.85 \times \Sigma KQm^{.85}}$$

$Q_{BDCORR} = Q_{BD} + CF_2$

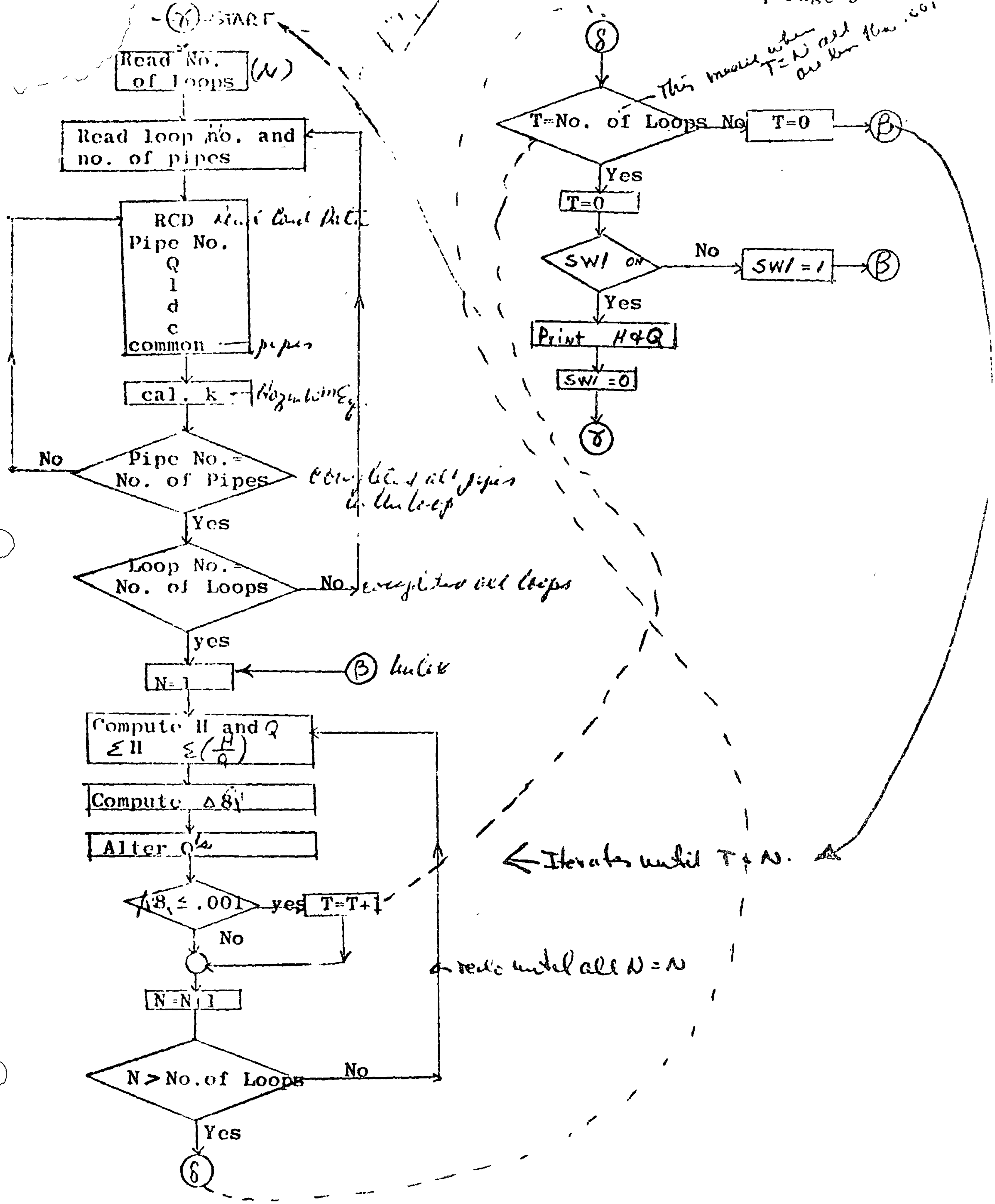
$Q_{BECORR} = Q_{BE} + CF_1 - CF_2$

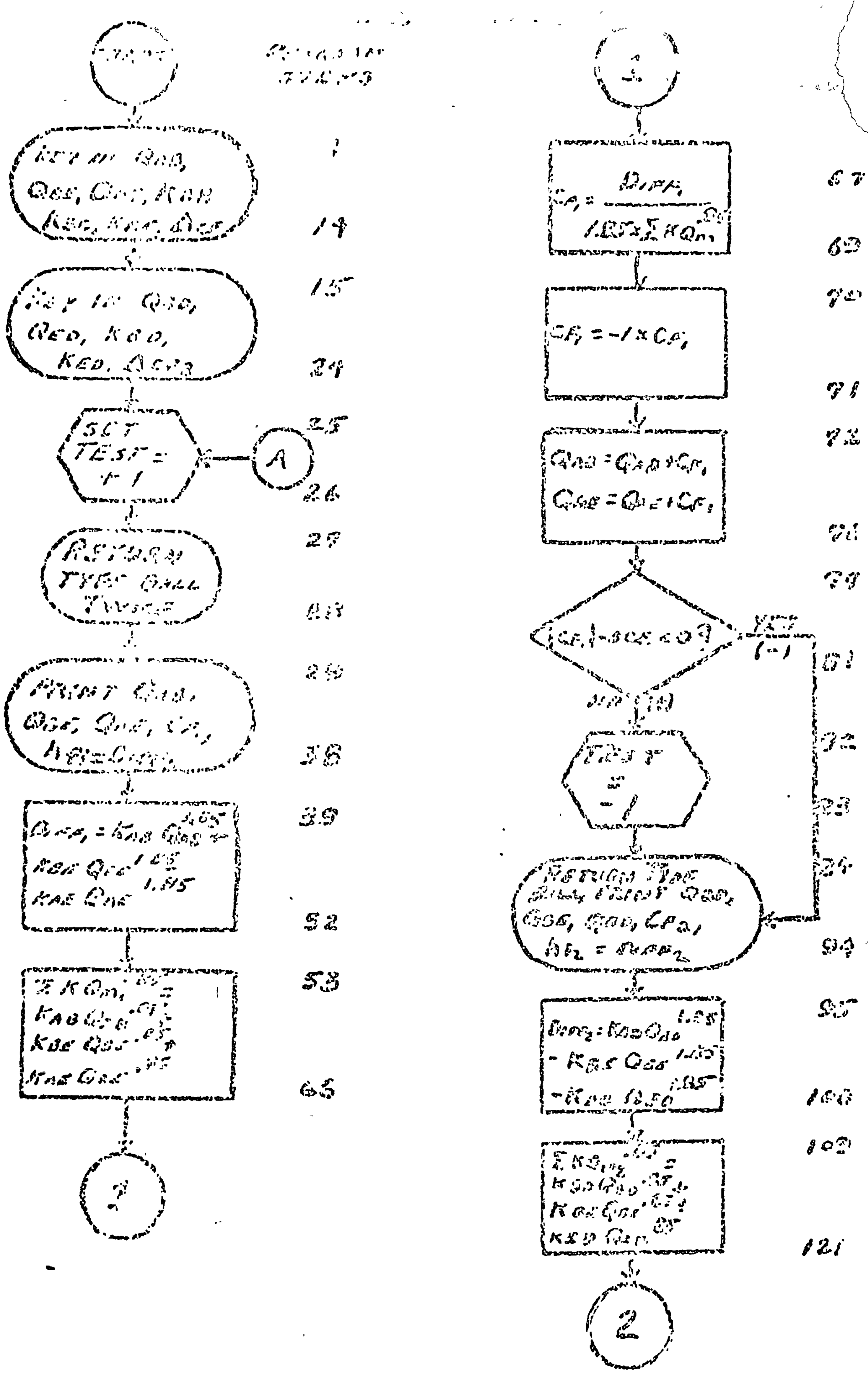
$Q_{EDCORR} = Q_{ED} - CF_2$

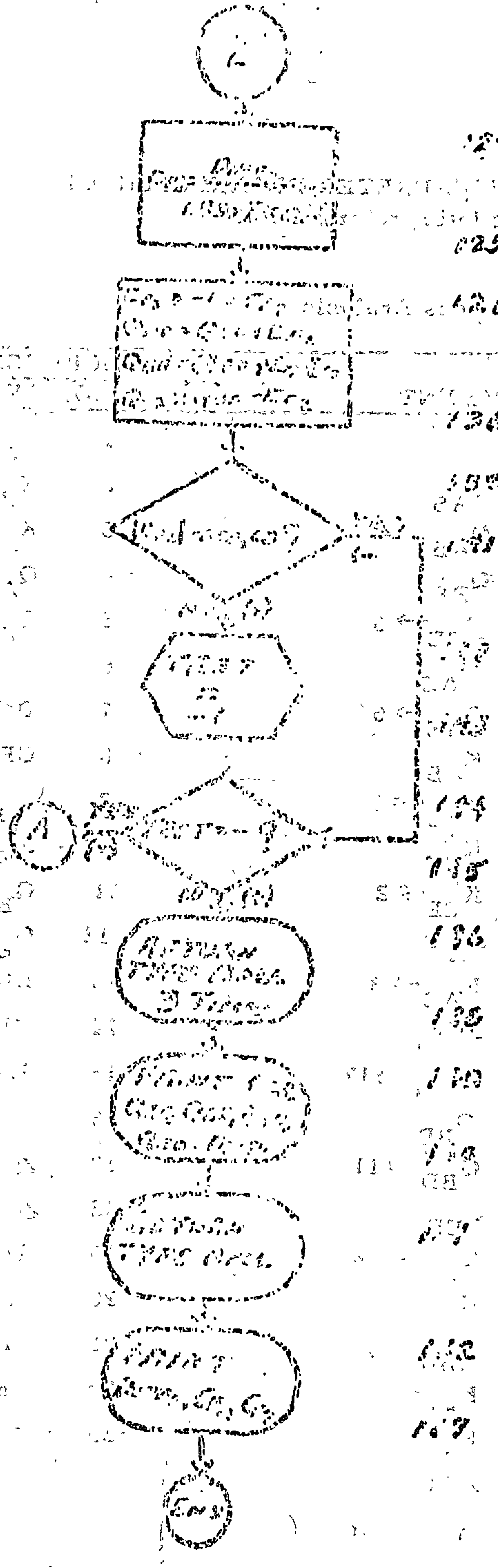
NOTE: The operation $Q^{1.85}$ is = to $e^{1.85 \log_e Q}$

$Q^{.85}$ is = to $e^{.85 \log_e Q}$

FLOW CHART


START
Read No. of Loops
Read loop no. and no. of pipes
RCD
Pipe No.
Q
l
d
c
common
cal. k
No
Pipe No. = No. of Pipes
Yes
Loop No. = No. of Loops
No
yes
N=1
Compute H and Q
Compute Δ8
Alter Q's
≤ .001
yes
T=T+1
No
N=N 1
N > No.of Loops
No
Yes
T=No. of Loops
No
T=0
Yes
T=0
SW1 ON
No
SW1 = 1
Yes
Print H&Q
SW1 = 0


SET TEST = +1
A
TEST = -1
2
1
14
15
24
25
26
27
29
39
52
53
65
67
70
71
72
121



Hardy Cross

PDS 1020 INTERPRETER PLANNING SHEET
Using Interpreter # PR5413

PROBLEM: Two Loop Hardy Cross Analysis

STEP NO.	OPERATION	COMMENT
	RETAIN 1+	
1	INV	Key in Q_{AB}
2	C 4 Go	$Q_{AB} \rightarrow 4$
3	INV	" Q_{BE}
4	C 5 Go	$Q_{BE} \rightarrow 5$
5	INV	" Q_{AE}
6	C 6 Go	$Q_{AE} \rightarrow 6$
7	INV	" K_{AB}
8	C 1 Go	$K_{AB} \rightarrow 1$
9	INV	" K_{BE}
10	C 2 Go	$K_{BE} \rightarrow 2$
11	INV	" K_{AE}
12	C 3 Go	$K_{AE} \rightarrow 3$
13	INV	" ΔCF_1
14	C 17 Go	$\Delta CF_1 \rightarrow 17$
15	INV	" Q_{BD}
16	C 11 Go	$Q_{BD} \rightarrow 11$
17	INV	" Q_{ED}
18	C 12 Go	$Q_{ED} \rightarrow 12$
19	INV	" K_{BD}
20	C 9 Go	$K_{BD} \rightarrow 9$
21	INV	" K_{ED}
22	C 10 Go	$K_{ED} \rightarrow 10$
23	INV	" ΔCF_2
24	C 18 Go	$\Delta CF_2 \rightarrow 18$

SCRATCHPAD ASSIGNMENT

NO.	SYMBOL
1	K_{AB}
2	K_{BE}
3	K_{AE}
4	Q_{AB}
5	Q_{BE}
6	Q_{AE}
7	$Diff_1$
8	CF_1
9	K_{BD}
10	K_{ED}
11	Q_{BD}
12	Q_{ED}
13	$Diff_2$
14	CF_2
15	1.85+
16	.85+
17	ΔCF_1
18	ΔCF_2
19	Test
20	1.+
21	.1-
22	Temp. Storage
23	$\Sigma KQm^{.85}$

Cross

	OPERATION	COMMENT
	L 20 Go	+1 → Arithmetic Unit
5	C 19 Go	+1 → (Test)
27	D +	Return Type Ball
28	D +	Return Type Ball
29	L 4 Go	Q_{AB} → A.U.
30	C +	Print Q_{AB}
31	L 5 Go	Q_{BE} → A.U.
32	C +	Print Q_{BE}
33	L 6 Go	Q_{AE} → A.U.
34	C +	Print Q_{AE}
35	L 8 Go	CF_1 → A.U.
36	C +	Print CF_1
37	L 7 Go	$Diff_1$ → A.U.
38	C +	Print $Diff_1$
39	L 6 Go	Q_{AE} → A.U.
40	Execute 2+	$Q_{AE}^{1.85}$
41	M 3 Go	$K_{AE}Q_{AE}^{1.85}$
42	C 22 Go	" → 22
43	L 5 Go	Q_{BE} → A.U.
44	Execute 2+	$Q_{BE}^{1.85}$
45	M 2 Go	$K_{BE}Q_{BE}^{1.85}$
46	S 22 Go	$K_{BE}Q_{BE}^{1.85} - K_{AE}Q_{AE}^{1.85}$
47	C 22 Go	" → 22
48	L 4 Go	Q_{AB} → A.U.
49	Execute 2+	$Q_{AB}^{1.85}$
50	M 1 Go	$K_{AB}Q_{AB}^{1.85}$
51	A 22 Go	$K_{AB}Q_{AB}^{1.85} + K_{BE}Q_{BE}^{1.85} - K_{AE}Q_{AE}^{1.85} = Diff_1$
52	C 7 Go	$Diff_1$ → 7
53	L 6 Go	Q_{AE} → A.U.
54	Execute 3+	$Q_{AE}^{.85}$
55	M 3 Go	$K_{AE}Q_{AE}^{.85}$

STEP NO.	OPERATION	COMMENT
57	L 5 Go	$Q_{BE} \rightarrow$ A.U.
58	Execute 3+	$Q_{BE}^{.85}$
59	M 2 Go	$K_{BE}Q_{BE}^{.85}$
60	A 22 Go	$K_{BE}Q_{BE}^{.85} + K_{AE}Q_{AE}^{.85}$
61	C 22 Go	" $\rightarrow$ 22
62	L 4 Go	$Q_{AB} \rightarrow$ A.U.
63	Execute 3+	$Q_{AB}^{.85}$
64	M 1 Go	$K_{AB}Q_{AB}^{.85}$
65	A 22 Go	$K_{AB}Q_{AB}^{.85} + K_{BE}Q_{BE}^{.85} + K_{AE}Q_{AE}^{.85} = \Sigma KQm^{.85}$
66	C 23 Go	$\Sigma KQm^{.85} \rightarrow 23$
67	L 7 Go	$Diff_1 \rightarrow$ A.U.
68	D 23 Go	$Diff_1 / \Sigma KQm^{.85}$
69	D 15 Go	$Diff_1 / 1.85 \times \Sigma KQm^{.85} = CF_1$
70	M 21 Go	$-1 \cdot CF_1 = CF_1$
71	C 8 Go	$CF_1 \rightarrow 8$
72	A 4 Go	$Q_{AB} + CF_1 = Q_{AB}$
73	C 4 Go	$Q_{AB} \rightarrow 4$
74	L 6 Go	$Q_{AE} \rightarrow$ A.U.
75	S 8 Go	$Q_{AE} - CF_1 = Q_{AE}$
76	C 6 Go	$Q_{AE} \rightarrow 6$
77	L 8 Go	CF_1
78	M 8 Go	CF_1^2
79	Sq. Rt.	$(CF_1^2)^{1/2} = \lvert CF_1 \rvert$
80	S 17 Go	$\lvert CF_1 \rvert - \Delta CF_1$
81	T/J 84 +	Is this Diff (-) or < 0?
82	L 21 Go	$-1 \rightarrow$ A.U.
83	C 19 Go	$-1 \rightarrow$ Test
84	D +	Return Type Ball
85	L 11 Co	$Q_{BD} \rightarrow$ A.U.
86	C +	Print Q_{BD}
87	L 5 Go	$Q \rightarrow$ A.U.