MODELADO Y SIMULACION APLICADOS A LA PLANEACION

FECHA	DURACION	TEMA	PROFESOR
Marzo 26	16 a 21 h	INTRODUCCION AL MODELADO	DR. VICTOR GEREZ GREISER
" 27	9 a 13 h	INTRODUCCION A LA SIMULACION DE SISTEMAS SISTEMAS CONTINUOS, SISTEMAS DISCRETOS	DR. VICTOR GEREZ GREISER
" 27	14 a 17 h	GENERACION DE NUMEROS ALEATORIOS GENERACION DE VARIABLES ALEATORIAS	ING. ARMANDO TORRES FENTANES
Abril 2	16 a 17:30 h	PROCESOS ESTOCASTICOS PROGRAMACION DINAMICA	ING. ARMANDO TORRES FENTANES
" 2	17:30 a 21 h	PROGRAMACION LINEAL	ING. SALVADORA GONZALEZ G.
" 3	9 a 13 h	LENGUAJES DE SIMULACION	M. EN C. JOSE RUIZ ASCENCIO
" 3	14 a 17 h	POBLACION, CONTAMINACION, INVERSION DE CAPITAL, PRODUCCION AGRICOLA, ENFOCADA A LA PROBLEMATICA NACIONAL, MESA REDONDA	ING. MARCO AURELIO TORRES HERRERA
" 9	16 a 21 h	JUEGOS DE SIMULACION	M. EN C. MARCIAL PORTILLA ROBERTSON
" 10	9 a 17 h	JUEGOS DE SIMULACION	M. EN C. MARCIAL PORTILLA ROBERTSON Y OTROS.

'edcs.

centro de educación continua
división de estudios superiores
facultad de ingeniería, unam

MODELADO Y SIMULACION APLICADOS A LA PLANEACION

Población, Contaminación, Inversión de Capital Producción Agrícola, Enfocada a la Problemática Nacional.

Ing. Marco Aurelio Torres Herrera

Palacio de Minería
Tacuba 5, primer piso. México 1, D. F.
Tels.: 521-40-23 521-73-35 5123-123

B | Ecology of the Preindustrial City—Old and New

12 | Mexico City: Its Growth and Configuration, 1345–1960

NORMAN S. HAYNER

For almost six hundred years Mexico City grew slowly. Most of that time residential desirability declined with distance from the central plaza. But in recent years under the influence of rapid growth in population, many new industries, and some improvement in the means of transportation, the metropolis seems to be shifting toward a basic structure similar to that of large cities north of the Border. Yet it retains certain important differences.

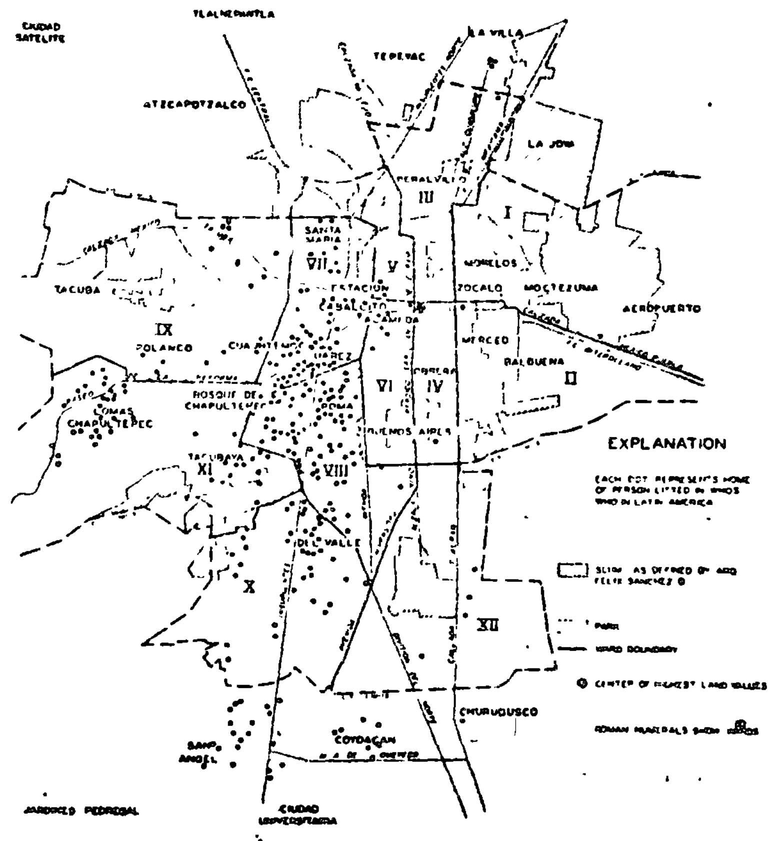
Anglo-American cities usually develop their worst slums in a zone just outside the central business district. As business expands outward, land values for commercial purposes rise but homes deteriorate and rents go down. Better residential areas are most frequently located a considerable distance from the center. Until recent decades, Latin American cities have grown very slowly over a long period of time. In a city that is not growing there is naturally no "zone in transition." The central business district is not expanding into surrounding residential areas. Where this is true it is more desirable to have a home within easy walking distance of the central square. Less favored sites for homes tend to be farther away, and the least desirable on the outskirts. Ordinarily the band plays for a *serenata* in the plaza two evenings a week. For smaller cities like Oaxaca or Querétaro, this public square is still the social center. But Mexico City today shows an interesting shift from this older pattern.

Since 1900, when its population was only 345,000, the capital's central business district has expanded outward, creating a "horseshoe" of high-land-value slums immediately to the north and east (see map, p. 167). Then too,

SOURCE: Norman S. Hayner in collaboration with Una Middleton Hayner, *New Patterns in Old Mexico* (New Haven, Conn.: College and University Press, 1966), pp. 51–64 and Map V.

one of its best residential areas (the Pedregal [illegible]) is located a full ten miles southwest of the Zócalo. In spite of these facts, its tendency to develop a circle of low-land-value slums on the outer edge of the metropolitan area has persisted for more than 440 years. In this respect it is similar to other large Latin American cities but different from the larger cities north of the Border. A 1957 United Nations *Report on the World Social Situation*

Mexico City—Homes of Intellectuals and the Poor



concludes that in Latin American cities "peripheral slums are frequently displaced and pushed farther out by the expansion of the city proper."

Aztec Settlement to Modern Metropolis

To understand the present configuration of Mexico's capital, it is helpful to think in terms of four major periods in its development. First there was the ancient Aztec city of Tenochtitlán (1345–1521). Then came the Spanish colonial city (1521–1821) founded by Hernán Cortés and his followers. With independence came a century of French influence (1821–1920). The present city combines a rich heritage from the past with an increasing infiltration of ideas from the United States.

Most archeologists agree that Tenochtitlán was founded about 1345 on islands in the salt sea of Texcoco. The name "México" was at that time used for the high valley in which Tenochtitlán was located. In the beginning, this Aztec settlement was a small village of reed huts with thatched roofs. By 1398 the earliest stone houses were built. When Cortés first saw Tenochtitlán and adjoining Tlaltelolco (1519), it was reputedly a city of more than five hundred thousand people, perhaps larger than any other in the world. It had narrow canals as in Venice and three main avenues two spear lengths in width. The pink stone dwellings of the nobles included courtyards with fountains, birds, and flowers. An aqueduct brought fresh water to the Aztec capital. Since these structures were almost completely destroyed by the conquistadors, few vestiges of the ancient city remain.

[The early growth of Mexico City may be understood as follows.] The central area, a block roughly one mile on each side, is the section planned for occupation by the Spaniards in 1521, but actually not used until 1524. During the period when the city streets were being reconstructed in the form of a grid, the seat of government, the home of Cortés, and that of his captain Alvarado, were in Coyoacán, a suburb just south of the present city limits. It is significant that even at this early date the native population was largely accommodated outside the limits of the Spanish city, their humble huts "scattered without order—as is the ancient custom among them."

During the next three centuries Mexico City grew slowly from perhaps thirty thousand, after the destruction of Tenochtitlán, to more than one hundred thousand. At the beginning of the nineteenth century it was again the largest city in the Western Hemisphere. A century later the capital had grown to more than three hundred thousand; by 1921, at the end of the revolution, its population had passed the six hundred thousand mark, finally exceeding the size Tenochtitlán is alleged to have reached four centuries earlier.

Growth during these four centuries has been primarily westward. During this long period the area occupied by dwellings expanded only one-half mile to the south and about a mile east and north, but three and one-

half miles to the west. Until 1903 further expansion eastward was blocked by Lake Texcoco. At that time this lake was partially drained by a gigantic canal and tunnel project, but the establishment of new residential neighborhoods to the east was still discouraged by the alkaline character of the reclaimed soil. During the major portion of these four centuries, the least desirable areas for residence were those beyond an easy walking distance from the Zócalo and the Alameda.

Throughout the colonial stage in its development, Spanish influence was of course dominant. The official language, the Roman Catholic church, the barrios, the siesta, the patio, paintings, public administration, were all heritages from Spain. Buildings in the older part of the city, whether governmental, ecclesiastical, educational, or residential, are still predominantly Spanish in architecture.

The oldest official panorama map of Mexico City, dated 1737, shows the largest and best residences in the center, the smallest and poorest on the periphery. Cortés had ordered the Indians to move out of the center and had divided up the more desirable section among his retainers. The house used by Cortés himself is still to be seen near the Zócalo. Canals came as far as the Zócalo from the east and almost connected with the oldest plaza in the Americas from the west.

After Mexico gained its political independence from Spain in 1821, Spanish cultural patterns continued to be important. Of the other European nations, probably the dominant influence through the next century came from France. During this period, French was the preferred foreign language in the schools. Up to the 1950's, it still was in Oaxaca's Institute of Arts and Sciences. It was not until after the Revolution of 1910–21 that English came to lead other foreign languages in the metropolis. Maximilian, emperor of Mexico from 1863 to 1867, designed a Boulevarde Imperiale patterned after the Champs-Elysées of Paris. This magnificent avenue—later renamed the Paseo de la Reforma—extends as a fourteen-lane, tree lined boulevard from the equestrian statue of Charles IV of Spain* about two miles southwest to Diana the Huntress at the entrance to Chapultepec Park. In the Díaz regime (1876–1910) many pretentious *palacios* were constructed along the Reforma. It is significant that the two older *colonias* (neighborhoods) north of the Reforma, Santa María (1869) and San Rafael (1891), are predominantly Spanish in architecture, whereas in the newer colonias south of the Reforma, Juárez (1902) and Roma (1906), the homes are distinctly French in style—many recently replaced, however, by modern commercial and residential buildings.

Before the Spaniards came the capital city was Indian; during the next four centuries it was predominantly Latin; recently it has been moving toward a fusion of these two elements supplemented by a growing influx of ideas and artifacts from the United States. Since the opening of the

* A statue affectionately referred to as the Caballito (Little Horse).

Mexico Laredo Highway (1932), followed by construction of three other highways from Mexico City to the Border, increasing streams of American tourists have poured into the capital. World War II accentuated the flow. Many travelers, who could not visit Europe, turned to Mexico. By 1960 the tourist trade was generating 23 per cent of Mexico's foreign exchange earnings. At the same time, *norteamericanos* not only had invested a billion dollars in Mexico's expanding industries but also had provided much of the technical know how for their development.

The impact of this North American influence may be seen both in the business center and in the newer colonias to the south and southwest. Many of the office buildings of the central business district and along the Reforma are now as modern as those in large cities north of the Rio Grande. In fact, Mexican engineers have recently overcome the handicap imposed by the spongy lake bottom on which the city is built. As a result, by 1961 many skyscrapers of more than ten stories had been completed and numerous others were under construction. Beginning with the 44 story Latin American Tower just south of the center of highest land values, these tall buildings tend to form a row extending west along Juárez and southwest along the Reforma. Such a development spells centralization.

Another influence from the United States and an index to decentralization of services, combined with centralization in control, is to be seen in the well organized *supermercados* located in better residential districts, such as the Lomas, Polanco, Anzures, Condesa, Roma, and Del Valle. These supermarkets sell a wide variety of both Mexican and foreign groceries. As in similar institutions north of the Rio Grande, prices are marked for every item, carts are available to carry purchases, and everything is checked over and paid for on departure. In the spring of 1961 the Mexico City supermarkets celebrated their fifteenth anniversary. None of the supermarkets, however, had parking lots for automobiles. In fact, at that time the writer saw no automobile oriented shopping center of the type that has developed in the United States.

Externally, the city's upper class residential district, the Lomas de Chapultepec, is very similar to certain sections of Los Angeles. All houses in the area must have gardens extending around the outside in repudiation of the patio of the Spanish-style home.

There is another way in which the Mexican capital can be compared with Los Angeles, and that is in its recent growth. In 1930 the population of Mexico City was over a million and that of the Federal District (comparable from a legal standpoint to the District of Columbia in the United States) about 1,250,000. By 1960 the city had almost reached 3,750,000, and the Federal District was approaching five million. In the period 1930–60 the population of the Republic doubled, that of the capital tripled; that of the Federal District quadrupled. The Mexico City "metropolitan area" as determined by International Urban Research had a population of 2,960,120 in 1950 and 4,816,393 in 1960—an increase of 63 per cent. This growth rate

is faster than the 54 per cent increase for the Los Angeles-Long Beach metropolitan area during that decade and almost as rapid as the estimated 67 per cent increase for the metropolitan area of São Paulo, Brazil.

Heavy in-migration to the capital, and to cities like Guadalajara (with 734,346 persons in 1960) and Monterrey (601,085), has been stimulated by such factors as the persistent low real incomes among peasants, and industrial developments fostered by the government's policy of "Mexico for the Mexicans." Up to 1926, for example, all makes of cars were imported. Beginning in that year automobile companies willing to assemble their cars in Mexico were favored. By 1961 importation of expensive cars, such as Cadillacs, was stopped while corporations willing to use a large proportion of parts manufactured in Mexico were encouraged. It is probable that eventually all new cars purchased by Mexicans will be manufactured within the Republic.

Along with the growth in Mexico City's population, there have been improvements in transportation, but nothing comparable to the subways or commuter trains of New York or Chicago. This fact has prevented the star-shaped spread of the metropolis along lines of fast transit. But wherever burgeoning cities are found, transportation has difficulty in keeping up. Two-fifths of the passenger automobiles of Mexico are registered in the Federal District—three times its quota in proportion to population, but still only one privately owned car registered as of 1958 for every 36 individuals (1960). During the past decade improvements have been made in the highways leading into the metropolis and in arterials within and around the city. With the exception of jitneys which operate along major avenues for a flat one peso charge per person and the slightly more expensive taxis, the use of passenger automobiles is largely limited to the middle and upper classes. For the masses, transportation is by streetcar or, increasingly, by the clumsy, crowded "ubiquitous bus." And the buses, which do seem to go everywhere, are neither rapid nor dependable.

The expansion of the city, at first largely to the south and southwest and more recently in all directions, has pulled the business district westward along the Avenida Juarez and southwest along the Reforma. The old French-style houses in the northern part of Colonia Juarez and the "palaces" along the northeastern end of the Reforma have been replaced by hotels, apartment houses, governmental and commercial establishments including automobile agencies. Apartments have increased greatly in number. In 1941 the huge Arch of the Revolution, on an extension westward of the Avenida Juarez, stood alone among vacant lots, now it is surrounded by apartment and office buildings. The area near the Caballito . . . is perhaps the clearest example of the process so common in North American cities in which a neighborhood of individual homes is "invaded" by apartments or commercial enterprises. Old buildings in this area have, in fact, been sold for the value of the land.

A similar process of invasion and succession has been taking place during the past two decades along Avenida de los Insurgentes [illegible] the economy. The success of Sears Roebuck de México in Colonia Roma, the congestion and parking difficulties for a major department store still located near the Zócalo, and the fact that most of its upper- and middle-class clients lived to the southwest, encouraged the Palacio de Hierro to establish a large branch a few blocks west of Roma. Changes such as these have resulted in sharp increases in land values in the areas invaded.

As is true to a greater or lesser extent for many capitals in progressing countries, and demonstrably so for Latin America, the metropolitan area of Mexico City had in 1960 six times the population of the metro area for the next largest Mexican city, Guadalajara. From this standpoint the capital ranked ninth in a list of 39 such areas having an estimated population of [illegible] one million in 1955 and may, therefore, be described as a "primate [illegible]." This growing metropolis is the dominant center of the political, business, and intellectual life of the Republic. Here is the largest center of manufacturing, the focus for transportation facilities, the financial hub of the Republic, and here are the managerial headquarters for many enterprises. It [illegible] its leading professionals from all the states and territories. In the [illegible] of Hubert Herring it "devours the leadership of the country. Every [illegible] politician, doctor and lawyer nurses the ambition to live in the capital—[illegible] the states of their leadership."

[illegible] on personages listed in *Who's Who in Latin America* indicate the [illegible] to which this was true in 1946. The occupations represented most [illegible] among these *intelectuales*, in order of frequency, were: lawyer, engineer, writer, physician, army officer, businessman, professor, painter, newspaperman, and banker. Although only 148, or 18 per cent, of the 808 listed for the whole of Mexico were born in the Federal District, 672 or 82 per cent lived there, a ratio of two to nine. It is assumed that the ratio between the number of distinguished persons born in a given state and the number who live there provides a rough measure of the relative drawing power of the Federal District as compared with the state. This ratio was twenty-one [illegible] for Oaxaca and eight to one for Puebla; but two to one for Jalisco where Guadalajara is located and four to three for Nuevo León, where Monterrey is the capital.

The distribution of Mexico City and local newspapers serves also as a rough measure of the extent to which the metropolis actually dominates the social and economic life of the country. Where the number of Mexico City papers drops below that of local papers it may be assumed that the capital has ceased to be dominant. If this index is adequate, the capital's social and economic pre-eminence is felt most on the Central Mesa with a substantial share of the western part of this great plateau controlled by Guadalajara.

In addition to the natural attraction which the metropolis offers, even to people from Guadalajara, there is the insecurity created by the governmental policy of expropriation of agricultural lands. In 1937 under President

Lázaro Cárdenas this reached the [illegible] (twelve and a half million acres). Whatever one's [illegible] opinion on this fundamental question, it has produced an insecurity among landowning classes that has caused many to migrate to the city.

Palaces and Slums

The best available index to the ecological structure of the metropolis proved to be land-value gradients. Estimated commercial land values for 1943 and 1948 . . . were based on actual sales, offers, or demands. A map prepared by Professor Edmundo Flores of the National University, using what are presented as "approximate commercial values" for 1958, shows a similar pattern with the most notable increases along the Reforma. The center of highest values is occupied by the Guardiola Building on San Juan de Letrán between Cinco de Mayo and Madero avenues. Values decline slightly as one moves east from this building to the Zócalo. Westward on Avenida Juárez and southwestward along the Reforma, values remain high as far as the intersection with Avenida Insurgentes. They are slightly lower from Insurgentes to Chapultepec Park. In general, values drop as one moves north or south from this Zócalo-Caballito-Chapultepec Park axis with the longest continuation of high values south along Insurgentes. The center of population in Mexico City gradually shifted from the Zócalo southwest so that by 1940 it reached La Garita at the intersection of Bucareli and Avenida Chapultepec (southeast corner of Ward VII). A panorama of the city as it was in 1856 shows this intersection three or four blocks beyond the built-up portions.

As the city grew, factors determining the southwestward movement of the middle and upper classes included the more fertile soil, higher elevations, and greater scenic beauty to the southwest, railroad yards little more than a mile to the north and northwest of the center (moved three miles northwest in 1958), city sewers that flow eastward and then, without covers, northward on a natural gradient, and prevailing winds from the northeast which, just before the rains season starts, stir up alkaline dust storms from the dried up bottom of Lake Texcoco northeast of the capital. This shift of people, together with the fact that the streets are wider, has helped to make Avenida Juárez and the Reforma more important than Avenida Madero, long the stronghold of real estate values.

Two phenomena seem to be correlated roughly with socioeconomic status in the metropolis. For one of these, the sex ratio, the correlation is negative, for the other, the number of distinguished persons, the correlation is positive. As the number of men per one hundred women decreases, the socioeconomic status of an area, within certain limits, increases. Two wards (*cuarteles*) with high average financial standing had in 1940 a sex ratio of 68.5 males to 100 females (76.5 in 1950). These wards (VII and VIII) include the prosperous [illegible] north and south of the Reforma. The ratio for the city as a whole was 83.3 (88.4 in 1960). In the three poorest wards (I, II, and IX), which include the Morelos, Merced, and Tacuba neighborhoods, the percentage stood at 90.6 (95.3 in 1960). This difference seems to be due to the larger number of women servants in the wealthier districts. Interestingly enough, when one studies the very wealthy Lomas, the sex ratio rises again. Chauffeurs and gardners have been added to the servant group. The fact that between 1940 and 1960 the sex ratio increased 1.6 times as much in the rich wards (8.0) as for the city as a whole (5.1) suggests that the higher wages offered by factories make it more difficult now to re- ... female help in the homes.

In contrast to large Anglo-American cities like Chicago, this Mexican city contains no area of homeless men. Women and children share with ... life in the worst slums. This is probably best explained in terms of ... mores. Women put up with more in Mexico.

The spots on [the] map ... show the "homes" of distinguished per... as revealed by the above mentioned study of *Who's Who in Latin America*. Cases where only office addresses were given are not included on the map. It will be noted that Colonia Juárez, the Lomas, and Del Valle, with about thirty personages each, have the largest number in *Who's Who*. ... Angel, Roma, Hipodromo (west of Roma), and Cuauhtémoc came ... with about fifteen each. The low number in Polanco (3) and Nueva Anzures (4) is to be accounted for by the newness of these colonias and the predominance of the *nouveaux riches*. The north-south avenue, which is named Guerrero on the north and Cuauhtémoc on the south, divides the land area occupied by dwellings and the population of the metropolitan area approximately in half, and yet there were only 22 spots east of this line as against 225 west. In fact, using the same line as [the] eastern boundary and an imaginary extension westward of Avenida Juárez as [the] northern, the southwest sector of the metropolitan area contained 202, or more than four-fifths, of these distinguished persons.

An interesting housing map prepared by the National Urban Mortgage Bank showed for 1932 the exact distribution of various types of housing in Mexico City. *Vecindades* and other types of homes for workers were most frequently present in the congested areas of "Old Mexico" north, east, and south of the Zócalo (Morelos, La Merced, Obrera), whereas west of the north-south line of Guerrero-Cuauhtémoc mentioned above there was a preponderance of residencias and very few vecindades. In 1947 the National Urban Mortgage Bank continued its studies of Mexico City's housing problem with an investigation by architect Felix Sánchez B. The slum areas on [the] map ... are based on this report. One fifth of the land area of the city (1946) was covered by these slum zones, and one-fourth of the estimated population—about half a million persons—lived in them. One hundred and thirty thousand individuals lived in dwellings whose destruction was recommended.

As was mentioned earlier, when the boundary of the Spanish city was established in 1521 the huts of the Indians were built outside. Architect Sánchez points out that this hodgepodge of jacales outside the Spanish city was the beginning of the present high land value slums. Areas of greatest density of population—from 1,000 to 2,500 persons to the acre—and some of the worst present day slums form a "horseshoe" around the north and east sides of "México Viejo," the older, central part of the city.

Northeast of the Merced district is the neighborhood called Moctezuma studied by the author in 1948. Here in 1961 the pressures from the expanding city could be seen. An earlier population of manual laborers (*obreros*) had been replaced by white-collar workers (*empleados*) who could pay higher rents. This increase in rentals, however, has made it necessary for many newly married couples to live in the home of the husband's father. Schools that operated two shifts in 1948 had four in 1961.

One answer to these problems is the construction now in process (1963), on the northern edge of the inner "horseshoe," of the largest housing project in Latin America. This project extends from Insurgentes Norte on the west to a proposed prolongation of the Reforma on the east—about one and a half miles. It will average three-eighths of a mile in width. Extensions northward of Guerrero and of San Juan de Letrán will divide it into three semi-independent units. Here, eventually, in buildings that are two, three, and seven stories high, ninety thousand people will be housed. Markets, playgrounds, and schools will be included. Apartments will range in size from one to four bedrooms and will rent for 12 to 40 dollars per month. They are planned for workers who earn from 32 to 96 dollars monthly. The construction is being financed by the same government-supported institution that has been making some of the housing studies—the National Urban Mortgage Bank. The location was made available by the moving of railroad freight yards (nationally owned) three miles to the northwest. It gets its hyphenated name, Nonoalco-Tlaltelolco, from two ancient communities that once occupied the site. A nine-level pyramid discovered here convinces archeologists that Tlatelolco (*sic*) was at least four hundred years older than Tenochtitlán.

At some distance outside the "horseshoe" Sánchez found a broken circle of low land-value slums. These slums seem to develop in vacant areas between or peripheral to established communities. Such clusters may be initiated by so-called "parachutists," squatters who just "fall" into these open spaces. In the beginning at least, these aggregations of makeshift huts and substandard houses lack transportation, lighting, water, and sewage disposal. Eventually such services tend to come in and the shantytown achieves the status of a "proletarian community."

If for the Federal District data on recorded offenders against the law consistently covered the geographical distribution of their homes rather than merely the place where the crime was committed, probably they would fail to show the same degree of concentration in a transitional belt near the center as in North American cities. In addition to the outer circle of slum zones outlined on [the] map . . . smaller slums are often to be found near the best residential neighborhoods. In some instances these have been started by a few people where poorly paid quarry workers and their families lived.

About three fourths of Sánchez' half million slum dwellers lived east of the Guerrero-Cuauhtemoc line. These poverty stricken people were for the most part crowded into the older, more congested sections or into the new proletarian additions to the east and northeast. In "Old Mexico" every sidewalk and every entrance to the numerous vecindades seem to be teeming with humanity. Due perhaps to better facilities than in the spot that is called home, eating and even sleeping on the street are commonplace. The other one fourth of the city's slum dwellers lived in the Tacubaya, Tacuba, and Pro-hogar (north of Santa María) zones. But between Tacubaya and [illegible] the magnificent residences of the Lomas rival anything in Hollywood.

Comparison of air photos of the metropolis and its vicinity for 1936 and [illegible] plus field observations of 1941 and 1961 indicate a large increase in homes for workers in the area north of Morelos outside the city limits, in the flat lands northeast of La Villa extending into the State of Mexico, and [illegible] the industrial suburbs of Atzcapotzalco and Tlalnepantla to the north[illegible]. New [illegible] colonias have sprung up to the east along the Puebla [illegible]. [illegible] the southeast in the Ixtapalapa delegation. A 1956 report [illegible] *Colonias Proletarias* locates three hundred such areas and concludes that [illegible] the end of 1955 they covered 30 per cent of the total area of the city. These neighborhoods make an almost complete circle around the outer part of the city with a two mile break on the west and another two mile break on the south. There is also an increase in homes, some of them palatial, in new subdivisions west and southwest of the city's legal boundaries.

Between 1940 and 1950 the tier of delegations in the Federal District immediately outside the political city grew four times as rapidly as the city [illegible], and from 1950 to 1960 eight times as rapidly. The remaining delegations in the Federal District, farther from the city, grew in the earlier decade a little less rapidly than the political city but between 1950 and 1960 2.4 times as fast. Actually, the four central wards of Mexico City declined 4.6 per cent between 1950 and 1960. This decrease at the center is, of course, to be found in other large cities. In other words, the Mexico City metropolitan region is growing most rapidly on the fringe of its built-up area and not in spatially independent suburban towns.

. . . . .

In conclusion, the following observations have been emphasized: (1) the slow and more recently the rapid growth of Mexico City; (2) the shift in basic configuration from the plaza-centered structure of the older Mexican cities toward certain characteristics similar to Anglo-American urbanized areas, including a "zone in transition"; and (3) certain differences between

Mexico City and the latter. The [illegible] of [illegible] residential suburbs [illegible], the tendency for low land value slums to [illegible] on the periphery, and the greater tendency to be the political, business, and intellectual center for an entire country are features of the Mexican capital which differ sharply from the ecological patterns presented by larger cities north of the Border. Reflecting as they do distinctive aspects of modern Latin American family and community life, these structural differences are apparently characteristic also of most of the larger urban aggregations south of Mexico.

13 | The Ecology of Bangalore, India: An East-West Comparison

NOEL P. GIST

Ecological research in American cities has revealed rather striking uniformities of ecological patterning, sufficient at least for certain tentative theories concerning urban ecology. It would be a fallacy, however, to assume that these theories are necessarily applicable to societies that differ strikingly from the United States in history, stage of technological development, socio-economic organization, and cultural interests.

Observations made in a few Latin American cities, for example, reveal ecological patterns quite different from those characteristic of many American cities.[1] The classical urban pattern in these cities may be summarized as follows: high status and high-income residents live near a central plaza, which is the social and institutional heart of the community; low-status and low-income residents locate near the periphery; economic establishments tend to be dispersed throughout the city, rather than highly centralized; and suburban growth from residential decentralization is limited.

Ecological segregation in one form or another appears to be almost universal, but the particular form in which such segregation occurs is highly variable, and changes in segregative patterns are affected by broad ideological, political, economic, and technological changes. Probably all communities

[1] See Theodore Caplow, "The Social Ecology of Guatemala City," *Social Forces*, 28 (December 1949), pp. 113–33.

Source: *Social Forces*, XXXV, 4 (May 1957), 356–359, 361, 363–365. Reprinted by permission of the publisher.

2 Introduction to METROPOLIS

METROPOLIS is available in both computer operated and hand operated versions; they are quite similar, although minor differences do exist. This description illustrates, basically, the computer version; however, all materials in the hand game closely mimic these materials. The participant will have no difficulty in establishing the necessary correlations.

To introduce you, we will discuss, in sequence:

- the basic model,
- the roles played,
- information available to you,
- (cycle 1) the starting environment,
- how to complete your decision forms,
- (cycle 2) the results of a typical cycle,
- (cycle 3) your first played cycle.

A. The Basic Model

METROPOLIS is an urban financial management game which focuses on the capital improvement program (CIP) aspect of local government as a major factor in urban development patterns. The environment in which players make their decisions is an abstraction of a moderate-sized (pop. about 215,000) metropolitan area, the size and wealth of which will change with time, dependent on both player decisions and exogenous influences which are outside the control of the players. The city is divided into three "wards" with fairly homogeneous populations in each, although significant differences in population characteristics exist between the wards. By compressing time, so that an hour long decision cycle represents one year of the community's life, the game allows participants to experiment with different strategies and forces them to live with the consequences of their decisions.

Since these decisions affect [illegible] the initiation of given projects [illegible] provided at the beginning of the [illegible] community [illegible] is intertwined with the development of a capital improvements program. [illegible] project needs are suggested by the [illegible] tributed at the beginning of each cycle [illegible] by any of the players, it is up to the participants assuming the roles of Politician, Administrator, or Speculator (an optional role available in the hand operated game exists for a School Board Representative) to reconcile the conflicts [illegible] interests they represent and [illegible] the city's resources.

METROPOLIS is, in essence, several minor games linked together, allowing simultaneous play of the parent game. The [illegible] roles of (1) the Administrators [illegible] recommend the financial plan [illegible] improvement program for the city's future, (2) the three Politicians (one for each political ward) who must make financial decisions in a [illegible] each year as well as deciding which specific projects are actually to [illegible], and (3) the real estate Speculators, who attempt to influence the location of various projects to maximize the return from their investments.

The Administrators, who correspond to the city manager and the various department heads responsible to him, are responsible for planning a capital improvement program for the Politicians' consideration in the following cycle of the game. *They are, thus working a year ahead of the other roles and are oriented to [illegible] the solution of [illegible] problems.* The Administrators are presumably seeking [illegible] satisfaction [illegible] their job properly. They would tend to be motivated by professional standards in making their recommendations, except as these might be tempered by their awareness of the political situation and the responsiveness of the population to particular issues. Financial inducements do relate to this occupation, both in the normal and legitimate form of salary (and raises in salary) and in the less frequent and illegal form of bribes and favors, but these are considered to be the lesser of the two motivations. As a consequence, the medium of exchange of the Administrator is represented by "points" of satisfaction. These points rise and fall with the condition of the city and the Administrators' ability to estimate future needs and resources and to influence other players to take "accepted" courses of action.

The Politicians are primarily presumed to operate under a different motivation—namely, that of obtaining and holding political power (which can in secondary ways be converted to economic reward, a phenomenon which is not introduced directly into the mechanics of the game). This is operationally defined as a set of probabilities for reelection in each of the three major wards.

By manipulation of favors and by general performance in office (particularly in terms of their capability to meet critical demands of the population), the level of these probabilities is within the control of the politician. Campaign contributions, a legitimate and recurrent form of bribe, can also be accepted by the Politician and converted to improved probabilities in a given locale by applying these funds to some appropriate form of campaigning.

Politicians initiate capital improvement projects in response to needs and wants as expressed by their constituents or as determined by the Administrator. The Politicians must decide on the degree of manipulation of the tax rate needed to support projects and satisfy constituents, as well as to determine what projects to undertake. Projects may be supported in part through nontax revenues which are dependent upon the area's economic health and population expansion. The Politician's skill in working with these factors determines in large part his chance of reelection, for which he must stand every other cycle.

Finally, the real estate speculator is presumed to be primarily motivated by the incentive to make a profit on his investment, although this may take the form of short run interest or perhaps a more enlightened long run interest which banks on a growing profit curve from a community which becomes increasingly more prosperous through time.

The speculator is a real estate operator who deals in options on land—*he is in and out of the market within the year*, either profiting on land, whereon he takes up the option (then reselling it), or losing his initial investment. The job of the speculator is to invest his cash in one or more of the three wards and in one or more of three land use categories (residential, commercial, or industrial). The possibility of a return on his investment is dependent upon the actions of the other players as they resolve community issues and institute public capital improvements, as well as upon a random factor representing the idiosyncracies of the real estate market.

B. Linking the Subgames

These three roles are linked together mechanically through the device of the capital improvement program which each bears some influence on and, in turn, is influenced by. This is illustrated schematically in Figure 2. The linkages shown are merely suggestive of the types of interrelations which exist in reality; the major significance lies in the realization that these function as a system. Various pressures are eventually balanced, and the resulting decisions, both at the level of each subgame and at the level of the community, are influenced in diverse ways, one by the other.

At this point, one significant difference between METROPOLIS and the typical business game must be emphasized. Whereas management games employ teams which operate competitively, *METROPOLIS*

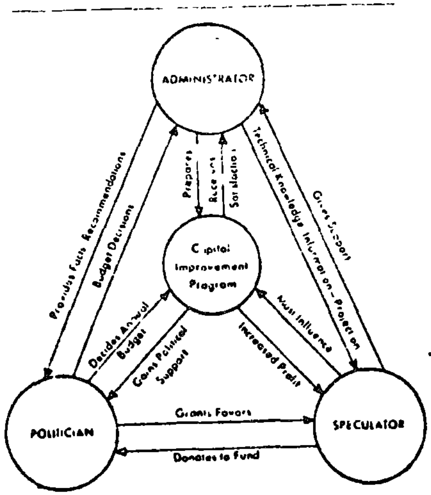


Figure 2 FUNCTIONAL INTERACTION DIAGRAM (Basic Game)

is essentially noncompetitive. In the jargon of game theory, it is a three player "non zero sum" game, that is, one player's gains are not necessarily at all or player's loss. In fact, intelligent cooperation (tempered with a concern for the rewards of each individual's game) can lead to a substantially higher payoff for all teams. This is the essence of what is being taught.

The subgames are linked in another way. In addition to their individual decisions, each role, as a representative of the power elite of the community, must cast its vote for one of the three alternative solutions to public issues presented in the Public Opinion Poll. Usually at least one of these three issues requires some kind of capital improvement project; others are statements of social policy or are "noise" issues which complicate decision making. Two out of three votes for a particular alternative determine the outcome of the issue and, for those issues involving projects, commit the Politician to budget the project in the following cycle. Issues may be postponed for one year if the three roles cannot arrive at consensus; the second time the issue comes up, however, it must be supported or rejected once and for all.

Figure 3 illustrates the linkages of the roles through time, and illustrates the dynamics of the program sequence.

Each cycle of the game begins with the distribution of the "Citizen's Gazette." This newspaper contains information about state and national conditions as well as the need for capital improvements in the various areas of town. Players are also given computer output from the immediately preceding cycle and the new Public Opinion Poll to vote on. After the votes on the issues have been completed, each role makes its own specialized decisions, which are then submitted to the computer for processing, before the next cycle begins. In a standard run of METROPOLIS, six or seven cycles should be completed before a critique session is held to discuss the events of the game.

The mechanics of the game are such as to permit the functioning of each of the static relationships as illustrated in Table 1 (i.e., two way communication among the three roles and two way influence between each role and the capital improvement program).

C. The Roles

Within the highly abstracted community of METROPOLIS, users [illegible] a game one of three roles: Politician, [illegible] or Speculator [illegible] in the land [illegible] game. [illegible] optional role of School Board [illegible] annual decisions [illegible] program and the [illegible] community [illegible] based community [illegible] versions of their real [illegible]

(1) POLITICIAN

The Politician is somewhat analogous to the real world city councilman [illegible] constituency to form part of the city's [illegible]

Table 1 METROPOLIS INFLUENCE LINKAGES

Player	Props Used	Decisions Required	Influences: Administrator Influenced [illegible]	Politician [illegible]	Speculator [illegible]
Administrator	Citizens Gazette Projects Recommended by Department Heads Past Budget Record of Projects Completed Issue Vote	Capital Improvement Program Projected Revenue (Projects by Ward Location)		Issue Vote C.I.P. [illegible] Projects [illegible] [illegible]	[illegible]
Politician	Capital Improvement Program Revenue Form Ward Probabilities Citizens Gazette	Project Expenditures Tax Rate (Projects by Ward Plus Budget Location)	Issue Vote Tax Rate Budgeted Projects		Issue Vote Tax Rate Budgeted [illegible] by [illegible]
Speculator	Ward Probabilities, Revenue [illegible] Citizens Gazette Land Odds Issue Vote	Investment by Ward by Land Use	[illegible] Ward [illegible] Existing	[illegible]	

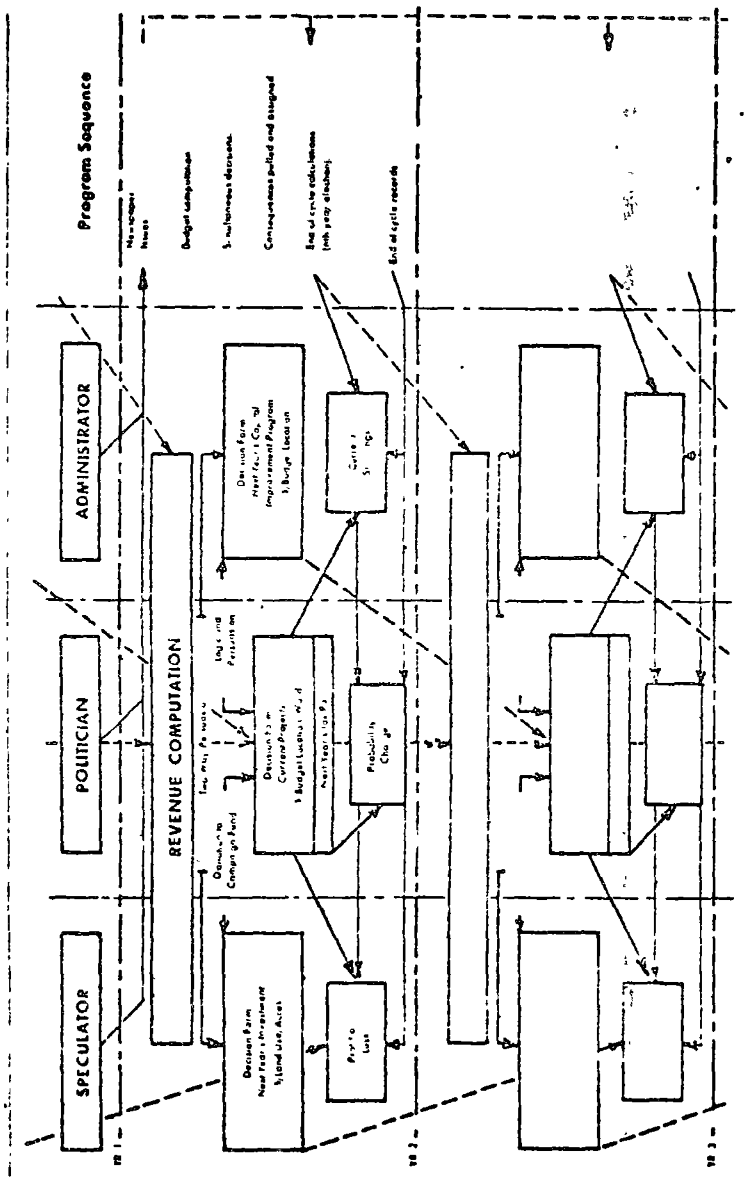


Figure 3 "METROPOLIS" SCHEMATIC

There should, in METROPOLIS, be three Politicians, so that one may represent each of the wards in the city. The primary function of the Politicians is to budget capital improvement projects, taking advice on what to build from the Administrator and from demands by the population as found in the newspaper. Although the Politicians may set the tax rate for the city, they should be aware that they share tax revenues with the school system, over which they exercise no control. The schools initially consume approximately 63% of all city revenue. Of the remaining funds, a constant 88% must be spent on city operating expenses, such as police and fire, public works maintenance, and administrative overhead. The amount of discretionary funds the Politicians have available to pay for capital projects is rather small and frequently insufficient to meet the demands of the people for facilities. Thus the Politician will have to make value judgments about which demands to meet and which to ignore. A general overview of the financial situation that has prevailed over the past decade is shown in Tables 2 and 3.

TABLE 2 Tax and Other Revenue

Year	Real Estate Tax	Total Revenue	Other Revenue	% Nontax Revenue
1960	$ 4 290	$1[illegible]	$ 5 8[illegible]0	57 8
1961	4 771	11 0[illegible]	6 239	56 7
1962	5,527	12 4[illegible]	6 [illegible]81	55 5
1963	6 0[illegible]3	13 786	7 775	56 4
1964	7 240	14 949	7 709	51 6
1965	8 692	16 129	7 957	49 3
1966	8 692	17 164	8,472	49 4
1967	8 931	17 665	8 734	49 4
1968	10 374	18 848	8 474	45 0
1969	11 681	23 043	11 367	49 3
1970	11,959	23 263	11,304	48 6

TABLE 3 Assessed Value and Tax Rate

Year	Assessed Value in 000's: Real	Total	% Increase	Tax Rate/1000: City	School	Total	% Increase
1960	$111 400	$161 200		17 5	1[illegible] 0	27 5	
1961	116 700	174 600	8 4	18 7	13 2	31 9	16 0
1962	122 200	1[illegible]9 500	6 6	18 7	12 9	31 6	−1 0
1963	125 600	19[illegible] 600	3 6	18 7	13 1	3[illegible] 9	16 8
1964	1[illegible] 900	202 8[illegible]0	3 0	16 7	21 5	4[illegible] 2	5 9
1965	14[illegible] 500	216 0[illegible]	6 5	17 5	22 6	40 1	−0 2
1966	157 100	232 900	7 8	18 5	19 6	38 1	−5 0
1967	1[illegible]7 400	256 300	10 0	18 5	21 7	40 2	5 5
1968	170 900	262 600	2 5	18 5	24 4	42 9	6 7
1969	18[illegible] 5[illegible]0	277 900	5 8	16 5	24 8	43 3	0 9
1970	1[illegible]	283 600	2 1	18 5	28 5	47 0	8 6
1971	207 900	291 000	2 6	18 5	29 1	47 6	1 3

The Politicians must also cast a vote for each issue which [illegible] cycle's Public Opinion Poll. Some of these issues require installation of capital facilities. If the vote of all roles favors a project, that project must be carried in the budget the following year and as many more years as it takes to complete construction. (The number of years [illegible] project is found in the Project [illegible].) [illegible] obvious [illegible] of the Public Opinion Poll outcome, the Politicians [illegible] to the outcome often differently from one ward to another. Each Politician [illegible] depending upon the outcome of the vote [illegible]. The Politicians may [illegible] to bargain among themselves in order to reach a [illegible] decision on a Public Opinion Poll; they may also bargain with one or both of the other two roles [illegible] to get the necessary two out of three votes which finally decide an issue. [illegible] in which the optional role of School Board is [illegible] a simple majority [illegible] required. In the case of a [illegible] Politician whose ward is affected by the [illegible] outcome [illegible].

The Politicians will have to [illegible] own [illegible] and what measures might be used to assess progress toward these goals. An obvious goal [illegible] is simply reelection; measures of success are the points accumulated [illegible] to ensure reelection. Other goals are possible [illegible]. For example, a Politician might [illegible] by the disadvantaged residents of ward [illegible] compensating through greater public expenditures. A Politician might measure his success by [illegible] influence he is able to exert on other players, how often the Public Opinion Poll reflects his views or how much of the public purse is spent in his ward. He might have totally nonpersonal goals, such as a rapid rate of growth for the city or equity of public expenditures among the three wards, or he might have almost completely personal goals, such as remaining in office or swaying other players. Players should be encouraged to formulate some set of goals for themselves before play is begun.

(2) ADMINISTRATOR

The Administrator has perhaps the most difficult role in METROPOLIS. *He has no power to spend money or to set policy.* He must vote in the Public Opinion Poll, as the other roles must, but his primary responsibility is to make recommendations of capital improvement projects which he wants the Politicians to budget in the next cycle. *He is working a year ahead of the other players*, trying to estimate the revenues which will be available to the city in the future and to respond to needs for facilities which will arise over time. The Administrator is assisted in his work by the provision of the "Crystal Ball Edition" of the newspaper (local news one year in advance). Like the Politician, the Administrator will have to make some sort of value judgments about which projects to include in his recommendations and which to exclude, since it is unlikely that future revenues will be sufficient to meet all new needs.

Probably the most difficult part of the Administrator's task (but the most important) is the estimation of revenues for the following year. These revenues depend on a number of variables: the community's future population growth rate, the economic health of the community, measured by per capita assessed value, the Politicians' decisions on tax rate which will generate the next year's revenues, and finally, the changes which may occur in nonproperty tax revenues. Obviously, the problem is one of great uncertainty. The aspect of the problem most susceptible to manipulation is the decision on tax rate, but even that is outside the Administrator's actual control. Some help in this difficult task may be found for the Administrator by identifying trends in revenue as reflected in the wall charts, particularly in Wall Chart 3 showing the revenue and total discretionary funds. In addition, a history of tax rates and assessed values is provided in Chapter 4, "The METROPOLIS Community."

The Administrator's rewards in life are less tangible than those of the Politicians and Speculators. To indicate a rough measure of his success, the Administrator is given points for some of his activities and for some of the same results which fall within his field of interest, although the only control he has over these is his ability to persuade others to take a "proper" course. The Administrator may try to influence votes on the Public Opinion Poll to favor alternative solutions which he feels are desirable. He may try to influence the Politicians' setting of taxes to ensure sufficient funds to carry out his proposals, and he may try to get the Politician to budget the projects he has recommended. His goals may be similar to some of those outlined for the Politicians or they may be decidedly different. In any event, the player of the Administrator's role is encouraged to define some set of goals for himself before play begins.

(3) SPECULATOR

The Speculator plays the role of the land opportunist. He buys options on land which he hopes or thinks will appreciate significantly in value. *He always returns to a pure cash state at the end of each year,* since his options are either taken up and *the land sold at a profit* or they are allowed to lapse and *the investment is lost.* On the basis of a simulated "land market" the computer will complete the operation which the Speculator begins with his initial investment in options. Each option is bought at 10% of the real value of the land.

In spending his money to purchase options, the Speculator will have to decide in which of the three wards land speculation is likely to be most profitable, *keeping a weather eye on the issues in the Public Opinion Poll which have the greatest influence on land profits.* He will try to influence the vote of other roles so the outcome of the poll will be favorable to his interests; failing to achieve this, he may at least try to figure out how the vote went in order to avoid losing investments. In addition to deciding *where* (which ward) to invest, the Speculator must spread his investments among either residential, commercial, or industrial land use categories; thus, there are nine possible investment categories from which he can select. *The choice of which land use to select is governed entirely and exclusively by a small game of chance.* The only variables which come into play are the odds represented in Table 4.

TABLE 4 Speculator's Bonus

LAND USE	% Pay Off	Odds	Dice Roll Required
Residential	10%	12/18	5, 6, 7, 8, 9,
Commercial	20%	5/18	3, 4, 10, 11
Industrial	50%	1/18	2, 12,

Thus, if the Speculator selects residential property in a given ward, he can expect a payoff two out of three times (cycles) *but* it will only amount to a 10% bonus. Conversely, investment in industry will yield a much more handsome payoff (50%), but can be expected only rarely. This payoff is always positive or zero, *never negative.* It represents in crude fashion the bonus often obtained by the Speculator in conjunction with an unusually lucrative property sale. The Speculator can, of course, spread his investment equally among all nine land use ward choices if he prefers, but this will be suboptimal.

The Speculator has an additional decision option which is not open to other players—that is, to give campaign contributions to the Politicians. These contributions must be in increments of $5,000 each, such contributions adding one point to the lowest ranking Politician's chances of reelection (in the hand game they may be assigned to a particular politician).

Profits and rate of increase of profit are obvious measures of the Speculator's success as the role has been defined for him. However, individual players in this role, as in the others, may find other measures

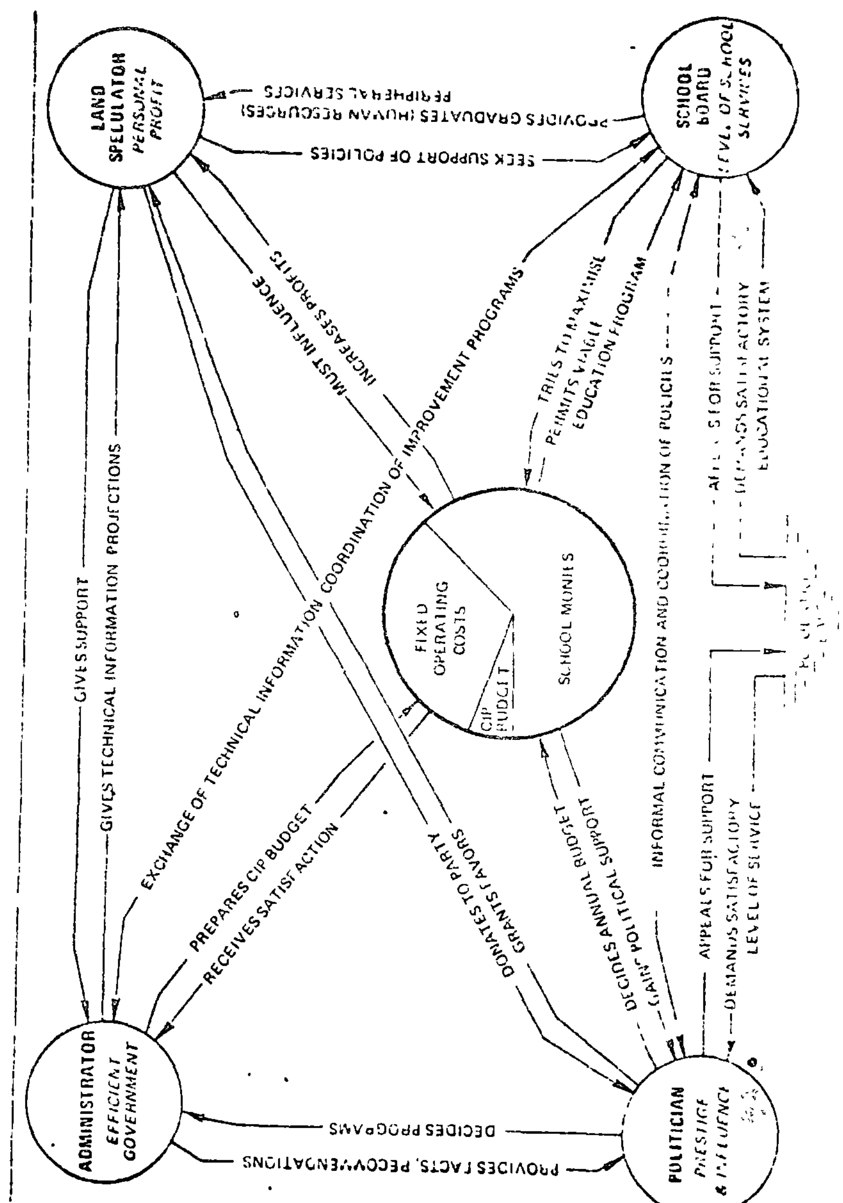


Figure 4 FUNCTIONAL INTER... ETC

of satisfaction. Political "clout" represented by influencing decisions or ensuring [illegible] reelection of a "good guy". Politician is esteemed by some players. Participating in a harmonious cooperative venture, on the one hand, or the head-on resolution of conflict on the other may become a Speculator's goal. Speculator players may be encouraged to design desirable end states other than the accumulation of wealth.

(4) SCHOOL BOARD (hand operated game only)

Often, during the play of METROPOLIS in the three player version, the most knowledgeable players object to the arbitrary allocation of tax money for education (School Board). They argue correctly that the location of schools in a growing community strongly affects the decisions of both the Administrators and Politicians, as well as the developers. Although School Boards are traditionally autonomous, their introduction, even in highly abstracted form, often improves the dynamics of play.

The function of the School Board is limited to two items: raising funds and estimating school needs for the next year. Funds for capital improvements are raised through bond issues on the Public Opinion Poll. The School Board must work with the other power groups to secure their support. The estimation of school needs is based on population increases and estimates of future per capita costs for education. The role of the School Board is a greatly simplified form of its real world equivalent; however, it adds some complexity and realism to the game. The School Board decisions affect all roles directly or indirectly. Its expenditures compete for money available to the city for capital improvements.

Materials available to the players of the School Board role include a diagram showing the relationships with other roles (Functional Interaction Diagram—Multirole, Figure 4), a history of school expenditures in the city (Table 5), a chart showing school expenditures as a percentage of total city revenue (Figure 5), a table showing projected per capita ex-

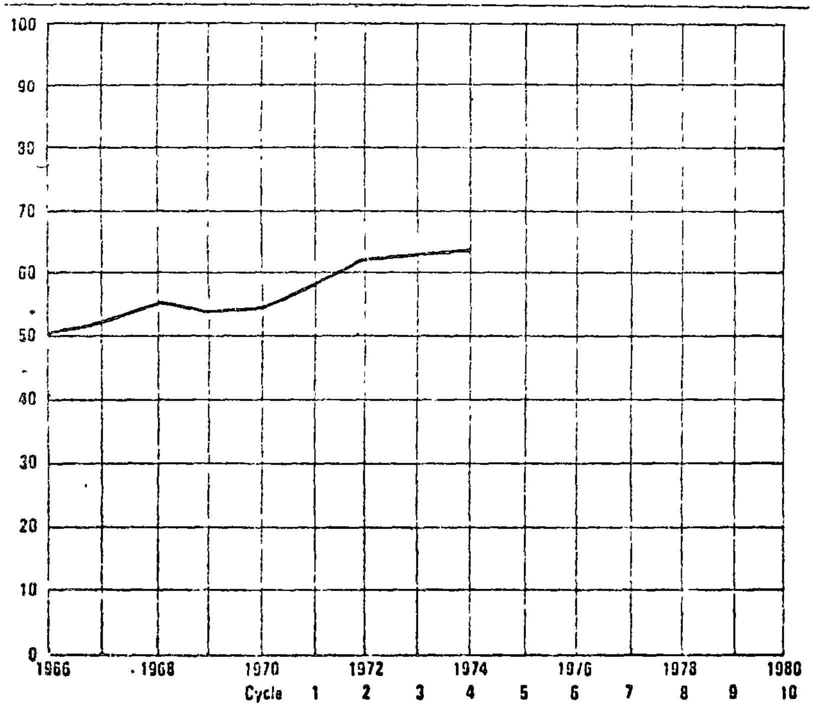


Figure 5 SCHOOL BOARD EXPENDITURES (as a percentage of total METROPOLIS revenue)

TABLE 4. METROPOLIS School Expenditures (recent history)

Year	Population (000's)	School Revenue (000's)	Per Capita School Cost	Cost Per School Child*	Per Cent City Revenue
1960	155	$ 3 [illegible]	$24 0	$130 0	39 2
1961	169	4 134	26 2	144 0	40 5
1962	173	4 [illegible]	23 6	[illegible]	40 0
1963	177	5 492	31 0	[illegible]	39 8
1964	180	6 652	37 0	201 3	44 5
1965	185	7 666	41 4	225 0	47 5
1966	190	8 807	46 4	250 4	51 4
1967	195	9 474	48 6	264 0	52 8
1968	200	10 105	50 5	278 6	54 6
1969	205	12 475	61 0	332 0	54 2
1970	210	13 656	60 2	353 0	67 0
1971	215	15,239	63 2	335 0	62 3

* The average ratio of school children to total population is assumed to be 18.4%.

penditures (Table 6), and, finally, a wall chart entitled "Public Opinion Poll Bond Issues," which indicates the major improvements already programmed (Wall Chart 11). The final chart also serves as a record of total special assessments for a given year. Rewards for the School Board are a result of the favorable action taken on Public Opinion Poll issues, which are usually bonds to finance capital improvements. Penalties are a result of the lack of ability to accurately estimate population and the percentage of available funds needed to supply the schools.

A penalty for too rapid growth is charged against the community through the mechanism of an extra percentage being added to the [illegible] fund for schools, to compensate for the added capital outlays needed in times of rapid growth. This is achieved through an expansion factor, which is the population growth divided by five thousand. If the factor is less than one, it is ignored. If one or more, it is added to

TABLE 6 Projected School Expenditures (national per capita estimates)

Estimates by Federal Education Agency of future public school expenditures

Year	Cost
1970	$360
1971	420
1972	460
1973	490
1974	520
1975	550
1976	600
1977	650
1978	710
1979	760
1980	850

NOTE Historically METROPOLIS per capita school expenditures have equaled or exceeded the national average.

the percentage present as the share of [illegible] tax that accrues to the schools.

The School Board [illegible] most [illegible] of a [illegible] The educator's primary [illegible] [illegible] public [illegible] school system [illegible] in office.

To achieve his [illegible] aware of existing [illegible] the projected [illegible] and alert for [illegible] [illegible]

The [illegible] education should [illegible] which may be of use in calculating future [illegible] population (to be posted [illegible] on Wall Chart [illegible]).

D Initial Information

At the beginning of the game players receive general information which helps them understand both the community as a whole and their specific role in it.

(1) PROJECT LIST (Form 14)

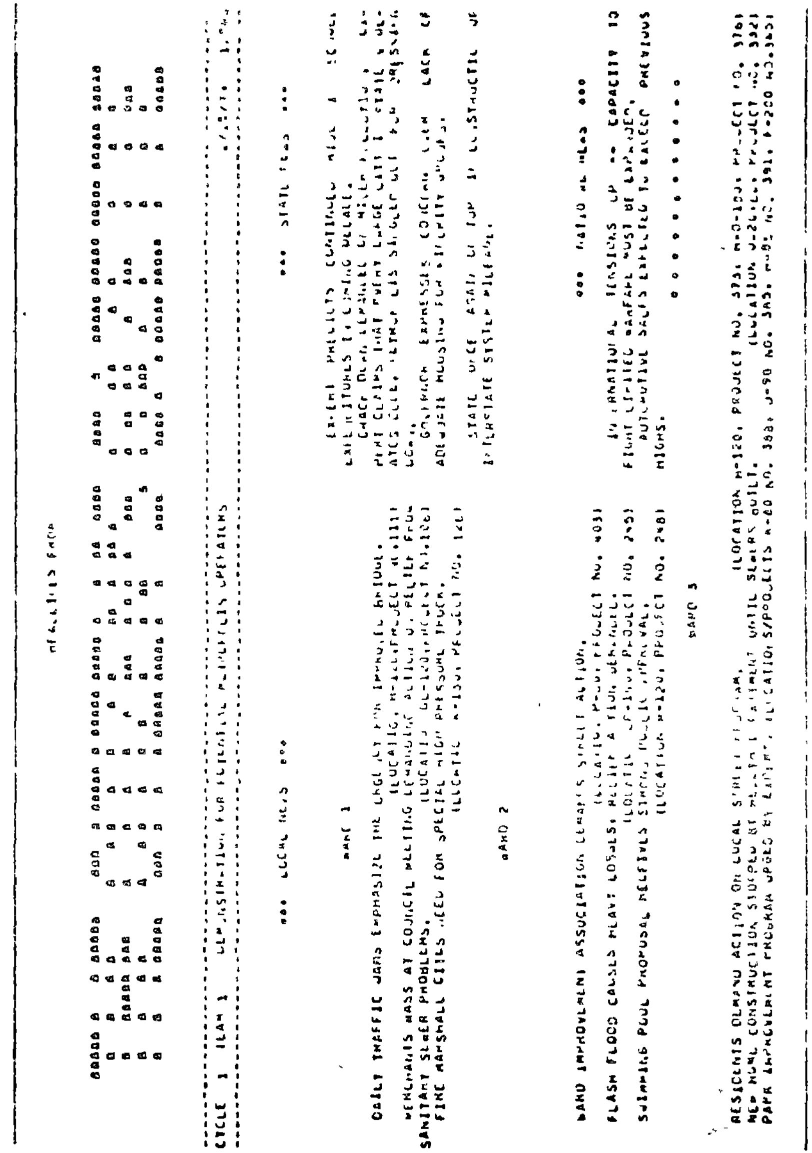
The Project List is invaluable to all players [illegible] the identity number, the description, [illegible] be affected, the cost and the number of years to complete construction and operation for all projects which players may initiate in METROPOLIS. (The Project List is in the appendix, at the back of the manual. Occasionally a player may desire a project not found on the list, in which case the operator may choose to add it to the Project List. Such modifications, however, depend on the time constraints during the operation of the game.)

(2) NEWSPAPER

The newspaper, Figure 6, highlights specific problems and needs within the urban community and suggests specific projects which would eliminate these needs. In addition, the newspaper makes note of state and national trends, particularly in the economy, that may have an effect on the city of METROPOLIS. A special newspaper, the "Crystal Ball Edition," Figure 7, is available to the Administrator. It behooves all players, then, to give careful consideration to these items as they appear.

(3) PUBLIC OPINION POLL

The Public Opinion Poll, Figure 8, elicits players' opinions on particular issues, many of which entail the initiation of a capital improvement project. Since the results of the poll may have different consequences for each of the players, voting on the issues may be accompanied by considerable negotiation, as players attempt to gain a majority vote for their choice of alternatives. A simple majority vote determines the outcome of any given issue. Issues may be postponed for one cycle by a majority of players voting for alternative 2, but upon their second appearance, they must be either rejected or approved.



CYCLE 1 TEAM 1 DEMONSTRATION FOR POTENTIAL METROPOLIS OPERATORS [illegible]

*** LOCAL NEWS ***

WARD 1

DAILY TRAFFIC JAMS EMPHASIZE THE URGENCY FOR IMPROVED BRIDGE.
(LOCATION [illegible], PROJECT NO. [illegible])
MERCHANTS MASS AT COUNCIL MEETING DEMANDING ACTION ON RELIEF FROM SANITARY SEWER PROBLEMS. (LOCATION [illegible]-120, PROJECT NO. [illegible])
FIRE MARSHALL CITES NEED FOR SPECIAL HIGH PRESSURE TRUCK.
(LOCATION [illegible]-150, PROJECT NO. 126)

WARD 2

WARD IMPROVEMENT ASSOCIATION DEMANDS STREET ACTION.
(LOCATION [illegible], PROJECT NO. 403)
FLASH FLOOD CAUSES HEAVY LOSSES, RELIEF ACTION DEMANDED.
(LOCATION [illegible], PROJECT NO. 245)
SWIMMING POOL PROPOSAL RECEIVES STRONG PUBLIC APPROVAL.
(LOCATION [illegible]-120, PROJECT NO. 248)

WARD 3

RESIDENTS DEMAND ACTION ON LOCAL STREET PROGRAM. (LOCATION [illegible]-120, PROJECT NO. 375, [illegible], PROJECT NO. 376)
NEW HOME CONSTRUCTION STOPPED BY HEALTH DEPARTMENT UNTIL SEWERS BUILT. (LOCATION [illegible], PROJECT NO. 332)
PARK IMPROVEMENT PROGRAM URGED BY EXPERT. (LOCATIONS/PROJECTS [illegible]-80 NO. 388, [illegible]-90 NO. 385, [illegible] NO. 391, [illegible]-200 NO. 365)

*** STATE NEWS ***

EXPERT PREDICTS CONTINUED RISE [illegible] SCHOOL EXPENDITURES IN COMING DECADE.
[illegible] DEAN [illegible] OF [illegible], EXPERT CLAIMS THAT EVERY LARGE CITY IN STATE [illegible] [illegible] CITY, [illegible] FOR [illegible].
GOVERNOR EXPRESSES CONCERN OVER LACK OF ADEQUATE HOUSING FOR MINORITY GROUPS.
STATE ONCE AGAIN AT TOP IN CONSTRUCTION OF INTERSTATE SYSTEM MILEAGE.

*** NATIONAL NEWS ***

INTERNATIONAL TENSIONS UP — CAPACITY TO FIGHT LIMITED WARFARE MUST BE EXPANDED.
AUTOMOTIVE SALES EXPECTED TO EXCEED PREVIOUS HIGHS.

Figure 6

HEADLINES FOR

[illegible]

CYCLE 2 TEAM 1 DEMONSTRATION FOR POTENTIAL METRO FOLIS LOCATIONS [illegible] 1 -00

*** LOCAL NEWS ***

WARD 1

CIVIC CENTER IMPROVEMENTS DECLARED URGENT -- CONVENTION FACILITIES EXPANSION A "MUST". (LOCATION J-110, PROJECT NO. 402)

CHAMBER OF COMMERCE DEMANDS IMPROVED STREET SYSTEMS. (LOCATION J-140, PROJECT NO. 401)

PARK SERVICE BUILDING DESTROYED BY FIRE -- IMMEDIATE REPLACEMENT SOUGHT. (LOCATION K-120, PROJECT NO. 124)

[illegible] ADMINISTRATOR, SPECIAL [illegible] CRYSTAL BALL EDITION [illegible]

WARD 2

SEWER PROBLEMS PLAGUE HOMEOWNERS -- RAW SEWAGE FLOWS IN DITCHES. (LOCATION W-70,140, PROJECT NO. 256)

NORTH END RESIDENTS DEMAND PARK. (LOCATION D-105, PROJECT NO. 251)

WARD 3

ANGRY PARENTS DEMAND SIDEWALKS AFTER THIRD CHILD IS INJURED. (LOCATION I-220, PROJECT NO. 451)

"CONTEMPORARY ESTATES" DEVELOPMENT HALTED BY SEWER LACK. (LOCATION F-[illegible], PROJECT NO. [illegible])

LAKEVIEW ESTATE, LAST MAJOR LAKE SITE AVAILABLE, OFFERED TO CITY AT GENEROUS PRICE. (LOCATION F-270, PROJECT NO. [illegible])

Figure 7

PUBLIC OPINION POLL

CYCLE 1

PLAYER

ADMINISTRATOR [ ] POLITICIAN [ ] SPECULATOR [ ]

ISSUE 1-1 NEW CITY HALL (PROJECT 501, LOCATION WARD 1, I-110)

THIS EFFICIENTLY DESIGNED NEW STRUCTURE IS PLANNED AS AN INTEGRAL PART OF THE CBD RENEWAL PROJECT. PRESENT CITY HALL IS OVER A CENTURY OLD. OFFICES ARE SCATTERED IN OLD BUILDINGS IN DOWNTOWN AREA.

ALTERNATIVES

1. SUPPORT CONSTRUCTION (PROJECT 501) [ ]
2. POSTPONE AND RECONSIDER [ ]
3. OPPOSE CONSTRUCTION [ ]

ISSUE 1-2 CIVIL RIGHTS COMMISSION

BLACK LEADERS, SUPPORTED BY THE UNIVERSITY COMMUNITY HAVE HELD SEVERAL DEMONSTRATIONS, SEEKING THE FORMATION OF AN EFFECTIVE COMMISSION TO INSURE FAIR HOUSING PRACTICES, EQUAL EMPLOYMENT, AND FAIR POLICE TREATMENT.

ALTERNATIVES

1. FAVOR FORMATION OF THE COMMISSION [ ]
2. POSTPONE AND RECONSIDER [ ]
3. NO COMMENT (PREFER TO AVOID THE ISSUE) [ ]

ISSUE 1-3 INTERSTATE HIGHWAY

STATE HIGHWAY DEPARTMENT DESIRES FORMAL APPROVAL OF AN INTERSTATE CONNECTOR ROUTE PLANNED FOR METROPOLIS.

(LOCATION K 100-160)

ALTERNATIVES

1. FAVOR THE ROUTE [ ]
2. POSTPONE AND RECONSIDER [ ]
3. OPPOSE THE ROUTE [ ]

Figure 8

3 Playing METROPOLIS

A CYCLE 1 (description of the output)

The output given to the players each cycle describes the results of decisions they made in the previous cycle. Much of this output is specifically reserved for a given role, other information available to the players is more general, describing the current community status.

(1) PUBLIC OPINION POLL RESULTS (Figure 9)

On this first page of the output, the outcomes of votes on the last cycle's Public Opinion Poll are given. The outcomes of each issue are determined by a two out of three vote of the roles represented in the game. If each role votes for a different alternative, the issue is automatically assumed to be postponed. A vote to *approve* a project forces that project into the budget in the cycle following the one in which the issue was resolved by approving the project (that is, an issue resolved in the cycle 1 poll makes budgeting of the project mandatory in cycle 2). For each issue presented, the following data is provided:

(a) *Project location* gives the map coordinates at which the required project will be located. More than two coordinates appearing here indicates the project is linear, extending between two points on a line. (Examples: The new city hall project is located at I 110; the interstate connector route is located *along* coordinate K, extending from coordinate 100 to coordinate 160.)

(b) *Local cost* refers to the total project cost which must be financed from local funds. If no value appears here, the project is funded from state and federal sources, requiring no local contribution. (Example: Interstate connector route.)

(c) *Years* indicates the number of years a locally funded project must be carried in the budget. Local cost divided by years indicates the annual payments which must be met. (Example: The new city hall will take 6 years to construct; annual cost is $70,000.)

(d) *Administrator's reward or penalty*, which may be either positive (reward) or negative (penalty), indicates the view of the Administrator's professional colleagues regarding the *outcome* of the issue vote. (Examples: New city hall and expressways are traditionally considered "good" things; the formation of civil rights commissions is traditionally not seen as part of the Administrator's responsibility and is therefore less highly valued by the professional societies.)

(e) *Politician's reward or penalty*, which may also be either positive or negative for each Politician, indicates the views of each ward constituency toward the issue outcome. Since population characteristics vary from ward to ward, different Politicians will fare differently on the outcomes of the various issues. (Examples: City image is important to ward 2 and ward 3 residents, so the new city hall is favorably viewed by them; it neither aids nor hurts ward 1 residents, who thus have no reaction. The connector route, on the other hand, wipes out a large quantity of low cost housing in ward 1, assists commuters in ward 3, and provides construction jobs for ward 2 residents, resulting in different reactions to the outcome from different wards.)

For both Administrator and Politicians, the individual team's vote does not lead to rewards or penalties, only the voted outcome of the issue. Administrators' reward or penalty points for issue outcome range from +10 to −10. Politicians' points range from +2 to −2, Speculators' range from −5 to +30.

METROPOLIS - VI ... URBAN GAMING SIMULATION
REVISION 11 OF APRIL 1973 ... FOR IBM 1130

ENVIRONMENTAL SIMULATION LABORATORY, UNIVERSITY OF MICHIGAN
PROF. RICHARD D. DUKE, DIRECTOR

CYCLE 1
YEAR 8

METROPOLIS CYCLE 1 -- DEMONSTRATION WITH PRESET DECISIONS

DATE 9/ 1/73
TIME 12-01

PUBLIC OPINION POLL RESULTS

CYCLE..ISSUE...ACTION....	(a) PROJECT ...LOCATION......	(b) LOCAL COST..	(c) YEARS..........	COMMENT..............	(d) PENALTY OF ADMINISTRATOR	(e) REWARD POLITICIAN RANK 1	2	3
1 1 APPROVED	I-110	820000.	6	NEW CITY HALL PROPOSAL	5	0	1	1
1 2 REJECTED	NONE	0.	0	CIVIL RIGHTS COMMISSION	0	-2	1	-1
1 3 APPROVED	K-100,160	0.	0	INTERSTATE CONNECTOR ROUTE	5	-2	1	1

(f) CALCULATED GROWTH INDEX = 1

(g) GROWTH INDICES, NEXT FIVE CYCLES 2 1 2 1 1

Figure 9

(2) OTHER INFORMATION

(f) Calculated growth index is the percentage population growth which occurred during the immediately preceding cycle (that is, the cycle from which the output is produced).

(g) Growth indices, next five cycles are estimated population growth rates, in percentages, for the next five years, based on the decision pattern represented by the issue decisions. Some decisions are growth *inducing*, such as major construction programs or industrial attraction, while others may be growth *retarding*, such as rejection of industrial opportunity. The estimated population growth for the next cycle is particularly useful to the Administrator as he seeks to estimate future revenues and to the School Board representative, who uses it to determine future school age population sizes.

(3) OUTPUT FOR EACH ROLE

(a) Politicians (Figure 10)

(1) *City council budgets capital improvements.* The first table received by the Politicians lists the one year projects which were budgeted by the Politicians giving the project number, ward, and map location of the project, the budget category (streets, utilities, recreation, or miscellaneous), project title, and annual cost.

(2) *Budget multiyear projects.* The next table lists the multiyear projects carried in the budget. The total of *all* projects' annual costs is found in *total CIP budget this year.* Projects found in the second table will automatically be carried for the number of years indicated under ***years to run.*** Politicians can calculate how many more years they will have to carry a project *by subtracting 1* from the years to run indicated. Administrators can calculate how many more years they will have to be careful to consider each multiyear project in their recommendations *by subtracting 2* from years to run.

(3) *Recommended projects not budgeted by the Politician* lists all projects which appeared in the preceding cycle's Administrator's recommendations (in cycle 1, for example, recommendations made in cycle 0 for budgeting in cycle 1) but which do not appear in the above two tables. Politicians may budget any projects they choose and need not follow the Administrator's advice. However, since constituents are assumed to know what has been recommended, some may respond unfavorably to failure to carry out the recommendations for their wards with adverse impact on the Politicians standing for reelection.

(4) *Spending schedule* (Figure 11) ... shows ... total CIP ... ward ... cycle 1 that $170,000 ... in ward 1, $500,000 in ward 2, and only $170,000 in ward 3. ... inequities may have an adverse effect on the Politician whose ward is least favored or a positive effect on the Politician whose ward is highly favored.

(5) *Tax rate for next cycle* will show any change made by the Politician and is applied to assessed value to generate tax revenue for the following cycle (in the cycle 1 output, this rate produces the revenues for cycle 2, as shown on the third page of the output).

(6) *Unspent funds of this year* are added to the money available to spend the next year (e.g., cycle 2). Conversely, deficits are subtracted from the money available for next year.

(7) *Reward or penalty points* indicate the three Politicians' chances of being reelected. Total points for an individual Politician may never exceed 6 (although particular activities may gain more than this) nor can they total less than 0. Since election is based on the rolling of a die, each Politician must roll the number shown as his standing *or less* in order to be reelected.

(8) *Public Opinion Poll* sums the points for or against the issue outcomes reported on the first page.

(9) *Recommended projects ignored* can only be negative, reflecting the failure of the Politicians to budget projects the constituents of a ward have been led to expect will be carried.

(10) *Budget inequities* can result in either positive or negative points (as explained above), resulting from unbalanced expenditures in the wards.

(11) *Taxes* may be changed by small amounts (1 mill or less) without having an effect on the Politician's standing. Larger *increases* in taxes may result in negative points.

(12) *Campaign contributions* made by the Speculator in $5,000 increments add points to the lowest ranking Politician: 1 point for every $5,000 contribution. (Since the maximum number of points a Politician may have is 6, the Speculator can ensure reelection of all three Politicians by contributing $90,000 ($5,000 x 6 points x 3 Politicians); however, economic reality makes this very unlikely.)

(b) Administrators (Figure 12) In addition to the general information described earlier, the Administra-

METROPOLIS ... DATE 5/ 1/73
TIME 12-01

P O L I T I C I A N S

(1) CITY COUNCIL BUDGETS CAPITAL IMPROVEMENT......

THE FOLLOWING PROJECTS HAVE BEEN APPROVED FOR THIS YEAR

PROJ NUM	WARD	LOCATION	BUDGET CATEGORY	DESCRIPTION	YEARS TO RUN	COST PER YEAR
			BUDGETED THIS-YEAR-ONLY PROJECTS			
111	1	H-120	STREETS	BRIDGE IMPROVEMENT	1	$ 200000.
106	1	GL-120	UTILITIES	SANITARY SEWERS	1	$ 200000.
245	2	LF-140	UTILITIES	STORM SEWER	1	$ 200000.
248	2	M-120	RECREATION	PARKS - RECREATION	1	$ 150000.
376	3	HO-100	STREETS	SECONDARY THOROFARE	1	$ 50000.
388	3	K-80	RECREATION	PARKS - RECREATION	1	$ 40000.
371	3	H-85	RECREATION	PARKS - RECREATION	1	$ 40000.
385	3	J-90	RECREATION	PARKS - RECREATION	1	$ 40000.
			TOTAL			$ 920000.
			(2) BUDGETED MULTI-YEAR PROJECTS			
403	2	P-80	STREETS	LOCAL STREETS	3	$ 150000.
501	1	I-110	MISCELLAN.	NEW CITY HALL	6	$ 70000.
			TOTAL			$ 22[illegible].
			TOTAL C I P BUDGET THIS YEAR			$1140000.

(3) RECOMMENDED PROJECTS NOT BUDGETED BY THE POLITICIAN

PROJ NUM	WARD	LOCATION	BUDGET CATEGORY	DESCRIPTION	YEARS TO RUN	COST PER YEAR
126	1	K-130	MISCELLAN.	FIRE PROTECTION	1	$ 20000.
373	3	H-210	STREETS	LOCAL STREETS	1	$ 50000.
362	3	J-20,60	UTILITIES	SANITARY SEWERS	1	$ 200000.
365	3	F-200	RECREATION	PARKS - RECREATION	1	$ 40000.
			TOTAL			$ 310000.

Figure 10

(4) SPENDING SCHEDULE

	WARD 1	WARD 2	WARD 3	TOTALS
STREETS	200000.	150000.	50000.	400000.
UTILITIES	200000.	200000.	0.	400000.
RECREATION	0.	150000.	120000.	270000.
MISCELLANEOUS	70000.	0.	0.	70000.
TOTALS	470000.	500000.	170000.	1140000.

(5) TAX RATE FOR NEXT CYCLE 48.0 MILLS

(6) UNSPENT FUNDS OF THIS YEAR PLUS INTEREST, $ 62400. HAVE BEEN ADDED TO CYCLE 2

(7) REWARD OR PENALTY POINTS

	WARD 1	WARD 2	WARD 3
POLITICIANS STANDING, END OF CYCLE 0	5	2	6
(8) PUBLIC OPINION POLL	-4	3	1
(9) RECOMMENDED PROJECTS IGNORED	0	0	-1
(10) BUDGET INEQUITIES	0	1	-2
(11) TAXES	0	0	0
(13) CAMPAIGN CONTRIBUTIONS, IF ANY	0	0	0
POLITICIANS STANDING, END OF CYCLE 1	1	6	4

Figure 11

... familiarize himself with the output shown ... as well as the explanatory notes ...

... The Administrator's standing, given at the ... different from the Politicians' ... there is no upper limit ... points he may have, although in a ... it is unusual for ... to exceed seventy.

... Reward points reflect the Administrator's ... Politicians budget the projects ... recommended for the last cycle. Some correc... is assured as inevitable beyond ... $100,000 carried earns a credit of 1 reward ...

... Over- or under-expenditure penalty, which ... reflects a significant disparity ... Administrator's recommendations and ... of money the Politicians have available ... projects at the time the recommendations ... the budget. Each percentage ... is a negative "reward point." *Under ... of available revenues is as serious a mistake* ... that in either of these cases ... recommendations are not of ... the Politicians. There will ... in assessing this penalty, i.e., ... for cycle 2 budgeting and up ... much less than the available funds ... cycle 2, the penalty will be ... in the cycle 2 output when the revenues are ... even though the error was actually made ... Administrator's cycle 1 estimate.

... *Public Opinion Poll points*, as for each Poli... from the first page of the output.

... *Administrator's recommended projects for ... year* (Figure 12), shows the Administrator deci... projects for the Politicians to consider in their ... The total of the annual costs of ... recommended CIP budget for next ... compared to available funds in calculating ... under-expenditure penalty which will be ... in the next cycle. *These Administrator's recommendations are particularly important for the Politicians* to look at in the upcoming cycle.

(6) *Breakdown of budgeted schedule* does the same thing for projects recommended as the spending schedule does for the projects actually budgeted.

(c) Speculators (Figure 13): The Speculator should be familiar with the general output described earlier, as well as his own output, described below.

(1) ... *growth factors* ... for the ... cycle ... Factors are shown for all nine possible investment categories ... residential, commercial, and industrial land in each of the three wards.

(2) *Public Opinion Poll.* Like Politicians and Administrators, the Speculator may ... the outcome of the Public Opinion Poll ... of the poll which would tend to have a positive impact on land value, such as major public construction or rezoning to a higher use, will result in a positive increment to the Speculator's investment return. On the other hand, decisions of the opinion poll which tend to make the land market shaky, such as reassessment, can have a negative impact. The Public Opinion Poll has the greatest impact on profits and is the single factor which is to some degree within the Speculator's control. The Speculator must estimate the impact of each issue on his likelihood of returns and may try to learn how the vote went before making his investment decisions. *The Public Opinion Poll affects the Speculator's investments only in the cycle in which a decision is reached on an issue.*

(3) *Dice roll* is a random feature introduced into the Speculator's profit picture, reflecting the remaining uncertainty of the real estate market. The chances of hitting on the dice roll are twelve out of eighteen for residential land, five out of eighteen for commercial land and one out of eighteen for industrial land. The windfall gains from random features are in inverse order: ten percent for residential land, twenty percent for commercial land, and fifty percent for industrial. *Dice roll is always positive or zero*, it is never a negative value.

(4) *Budget allocation* shows the percentage increase (1 percent to 7 percent) in land prices for each ward resulting from public improvements. This factor accounts for a minor portion of the Speculator's returns—usually about one half of his initial ten percent outlay for option purchase.

(5) *Totals 1, 2, and 3 above.*

(6) *Current cycle investments* recapitulates the Speculator's decisions on investment in each of the nine possible investment categories.

(7) *Return* 5 times 6 is the Speculator's payoff, in each investment category (e.g., ward, residential investment of $5,000 returns a 34.7% payoff).

CYCLE 1
YEAR 2

METROPOLIS CYCLE 1 -- DEMONSTRATION WITH PRESET DECISIONS

DATE 5/ 1/73
TIME 12-01

A D M I N I S T R A T O R S

(1) ADMINISTRATORS STANDING AT END OF PRECEDING CYCLE = 20

(2) REWARD POINTS = 6
(3) OVER-, UNDER-EXPENDITURE PENALTY = -10
(4) PUBLIC OPINION POLL POINTS = 10

ADMINISTRATORS CURRENT STANDING = 26

(5) ADMINISTRATORS RECOMMENDED PROJECTS FOR NEXT YEAR

PROJ NUM	WARD	LOCATION	BUDGET CATEGORY	DESCRIPTION	YEARS TO RUN	COST PER YEAR
129	1	K-120	RECREATION	PARKS - RECREATION	1	$ 20000.
231	2	D-105	RECREATION	PARKS - RECREATION	1	$ 100000.
368	3	FJ-65	UTILITIES	SANITARY SEWERS	1	$ 100000.
366	3	F-270	RECREATION	PARKS - RECREATION	1	$ 50000.
401	1	J-140	STREETS	LOCAL STREETS	3	$ 150000.
402	1	J-110	MISCELLAN.	CIVIC CENTER	4	$ 50000.
403	2	P-80	STREETS	LOCAL STREETS	2	$ 150000.
901	1	I-110	MISCELLAN.	NEW CITY HALL	5	$ 70000.
908	ALL	E-20	UTILITIES	SEWAGE TRTMT PLANT	6	$ 200000. PER WARD

RECOMMENDED C I P BUDGET FOR NEXT YEAR $1290000.

(6) BREAKDOWN OF BUDGETED SCHEDULE

	WARD 1	WARD 2	WARD 3	TOTALS
STREETS	150000.	150000.	0.	300000.
UTILITIES	200000.	200000.	300000.	700000.
RECREATION	25000.	100000.	50000.	170000.
MISCELLANEOUS	120000.	0.	0.	120000.
TOTALS	495000.	450000.	350000.	1290000.

Figure 12

CYCLE 1 METROPOLIS CYCLE 1 -- DEMONSTRATION WITH PRESET DECISIONS DATE 5/ 1/73
TEAM 1 TIME 12-01

SPECULATORS STANDING

(1) SPECULATORS GROWTH FACTORS

WARD. LAND USE	1 RES	1 COM	1 IND	2 RES	2 COM	2 IND	3 RES	3 COM	3 IND
(2) OPINION POLL	0.200	0.200	0.200	0.000	0.000	0.000	0.000	0.000	0.000
(3) DICE ROLL	0.100	0.000	0.000	0.100	0.000	0.000	0.100	0.000	0.000
(4) BUDGET ALLOCATION	0.047	0.047	0.047	0.050	0.050	0.050	0.017	0.017	0.017
(5) Total	347								

(6) CURRENT CYCLE INVESTMENTS

(7) WARD	RESIDENTIAL	COMMERCIAL	INDUSTRIAL	TOTAL
1	5000.	5000.	5000.	15000.
2	10000.	0.	5000.	15000.
3	5000.	10000.	0.	15000.
TOTL	20000.	15000.	10000.	45000.

TOTAL OF PREVIOUS INVESTMENTS

WARD	RESIDENTIAL	COMMERCIAL	INDUSTRIAL	TOTAL
1	0.	0.	0.	0.
2	0.	0.	0.	0.
3	0.	0.	0.	0.
TOTL	0.	0.	0.	0.

(8) NET WORTH AT END OF CYCLE 0 $ 50000.

(9) - AMOUNT INVESTED THIS CYCLE $ 45000.
(10) - CONTRIBUTED TO POLITICIAN $ 0.

(11) = CASH HELD IN RESERVE $ 5000.
(12) + 4 PERCENT INTEREST ON CASH RESERVE $ 200.
(13) + INVESTMENT RETURN $ 67099.

(14) NET WORTH AT END OF CYCLE 1 $ 72299.

Figure 13

(8) *Net worth at end of cycle 0* [illegible] total [illegible] Speculator has available to spend in cycle 1

(9) *Amount invested this cycle (cycle 1)* sums the investments in the nine possible investment categories (Figure 13, box 6)

(10) *Contributed to Politicians* shows the total of campaign contributions in increments of $5,000

(11) *Cash held in reserve* is what is left of net worth after investment and campaign contributions. This may be negative if the Speculator chooses to take out a "loan" by spending more money than he has (cost 20% per year), limited in quantity by operator decision

(12) *Percentage of interest on cash reserve* Interest on the cash reserve is four percent

(13) *Investment return* is calculated for each investment category by summing the growth factors pertaining to that category (top of page) and multiplying the total by ten (since options are purchased at ten percent of the total cost) and multiplying this new total by the investment in that category (Example: For residential land in ward 1 in cycle 1, .347 x 10 x $5,000 = return on residential land investment in ward 1.) When this has been done for each of the nine categories the results are summed to get total investment return

(14) *Net worth at end of cycle 1* is the amount of cash the Speculator has to play with in Cycle 2 (11 + 12 + 13 = $72,299). In the cycle 2 output, this figure appears in place of *net worth at end of cycle 0*

(d) Infrastructure accumulation (Figure 14). This page shows the *cumulative increase of programmed projects* which indicates the total capital expenditures for each ward as the game progresses. The *factor* for each ward is one one hundred thousandth of the *average* cycle expenditures for that ward and may be used to measure the long term equity of public investment decisions across wards

B. CYCLE 2 (starting with the budget)

(1) REVENUE FOR CYCLE 2 (Figure 14)

This page shows the generation of the money available to the Politicians to spend in cycle 2

(a) *Population* is the count for the current year

(b) *Per capita assessed value* is total assessed value (c) divided by population [illegible]

(c) *Total assessed value* [illegible] city

(d) *Tax rate* is set by the Politicians

(e) *Total city income from property tax* is found by [illegible] the mill tax rate [illegible] by the Politicians to the assessed value. (One mill = $1 per $1,000 of assessed value, or .001 times assessed value.)

(f) *Revenue from nonproperty tax sources* is income from sales tax, licenses, fines, and federal and state rebates. It is initially at 100% of the property tax revenue, but will vary in future cycles

(g) *Total city income* is divided between the school system, which takes an increasingly larger share, and the city

(h) *Income allocated to schools* is spent by an autonomous agency, the School Board (which may be simulated in the computer or an actual role in the hand game)

(i) *Net city income* is mostly spent on ongoing programs and services over which the Politicians have no discretionary control (e.g., paying firemen and policemen, maintenance of property, and the like), leaving only twelve percent for expenditure on capital projects. This twelve percent of net city income is allocated to the street fund (twenty percent of j), utilities fund (thirty percent of j), recreation fund (ten percent of j), and the general fund (forty percent of j)

(j) *Total discretionary funds* (the figure the Administrators must try to estimate a year ahead in recommending projects), are currently available to spend; however, the street fund, sewer fund and recreation fund are "earmarked" to be spent only on projects of the same category

The breakout of total discretionary funds among the four budget categories (k-m, n) reflects the earmarking of funds for particular types of projects and should not be arbitrarily violated. *General* monies may be used to supplement the minimum amounts shown for streets, utilities, and recreation. (Politicians should keep in mind that funds committed for multiyear projects are often already meeting at least part of the obligation to one or more of these budget categories.)

(o) *Committed funds* are the total of multiyear projects already committed. When this amount is subtracted from total discretionary funds, the residual is the net discretionary funds which are available this cycle to be committed to new projects

(p) *Funds available for new projects* are available to the Politician, including any which are required by the last cycle's Public Opinion Poll.

CYCLE 1
TEAM 1

CUMULATIVE INCREASE OF PRIVATE ... PROJECTS

CYCLE...WARD 1........FACTOR...WARD 2.......FACTOR...WARD 3.......FACTOR

1	470000.	4.700	500000.	5.100	170000.	1.700

REVENUE FOR CYCLE 2 ← Note Cycle 2 begins now!

(a)	POPULATION OF METROPOLIS RISES TO	214200.	
(b)	PER CAPITA ASSESSED VALUE= $	1350.	
(c)	TOTAL ASSESSED VALUE OF METROPOLIS= $	289170048.	
(d)	TAX RATE IN MILLS=	48.8	
(e)	TOTAL CITY INCOME FROM PROPERTY TAX= $	14111494.	
(f)	+ NON-TAX REVENUE= $	14111494.	WHICH IS 100. PERCENT OF PROPERTY TAX REVENUE
(g)	= TOTAL CITY INCOME= $	28222988.	
(h)	- INCOME ALLOCATED TO SCHOOLS= $	17035704.	WHICH IS 63.0 PERCENT OF TOTAL CITY INCOME
(i)	= NET CITY INCOME= $	10414286.	OF WHICH 12 PERCENT ARE YOUR
(j)	TOTAL DISCRETIONARY FUNDS= $	1249713.	
	+ UNSPENT FUNDS FROM CYCLE 1= $	62400.	
	= NET DISCRETIONARY FUNDS= $	1312113.	
(o) -	FUNDS COMMITTED FOR MULTI-YEAR PROJECTS= $	220000.	
(p) =	FUNDS AVAILABLE FOR NEW PROJECTS= $	1092113.	

DISCRETIONARY FUND ALLOCATION

BUDGET CATEGORY	NET DISC. FUNDS	COMMITTED FOR MULTI-YEAR	AVAILABLE DISC. FUNDS
(k) STREETS	$ 262422.	$ 150000.	$ 112422.
(l) UTILITIES	$ 393634.	$ 0.	$ 393634.
(m) RECREATION	$ 131211.	$ 0.	$ 131211.
(n) MISCELLAN.	$ 524845.	$ 70000.	$ 454845.
TOTAL	$ 1,312 112		

Figure 14

(2) COMPLETING THE DECISION FORMS

... completed by way of ... processed through the computer ... Chapter III, section ... Before completing the ... in any given cycle, the ... the newspaper (Figure 15) ... read the Crystall Ball Edition (Figure 16), and complete the Public Opinion Poll (Figure 17).

(a) Politician. The Politicians should complete their form in the numbered sequence.

(1) Check the results of the Public Opinion Poll to ... were selected during the previous cycle. In this cycle, only the approval of the city hall ... the initiation of a project. The Politicians ... therefore check the 500 series (issue projects) ... list, obtain the number of the city hall project (number 501) and write this number, with ... figure under the miscellaneous category on ... form. In this cycle, the ... the mandatory city hall project ... discussed the opinion poll voting for ... (other players) planned for the ... treatment plant project by ... 503 on the form as well.

(2) In completing their decision form (Figure 18) the Politicians should check their output to see ... projects are in operation. These must be listed ... until the project is complete. In this ... project 403 is a continuing project (please ... projects in 400 or 500 ranges are multi-year projects) and are listed at the top of the Politicians' form. The Politicians next check with the Administrator to find out which projects he recommends for this cycle (these are found on the Administrator's output). They find that the only multi-year projects recommended that they have not already initiated are numbers 401 and 402. They also note that these projects ... ward 1, where the Politicians are weak ... choose to budget these projects.

(3) The Administrators have also recommended four single-year projects which are spread over the wards ... to balance a budget which has so far ... in both these terms, listing these projects makes good political sense, so the Politicians add them to their budget.

(4) Because the Politicians are still insecure and because this year's budget is well within the city's discretionary fund allotment, they decide to leave the millage rate at 42 ... In future cycles, the Politicians may find that the funds required to support their budget are not available, in which case cutbacks will be necessary. Should this ... the Politicians ... among the ... and among the three wards.

(b) Administrator. In completing his form (Figure 19) the Administrator must estimate city revenue a year in advance. In order to do this, he should first check this cycle's total revenue, and the newspaper for information on what economic trends are currently being ... (A glance at Wall Chart F, showing city revenue, will indicate the recent trend.) This will give him some idea of the type of budget he should anticipate. (An increased budget? A gradually dropping one?) For the change percentage, the Administrator must, to a large extent, rely on his own perception of the economic state.

The Administrator should complete his form following the numbered sequence.

(1) Estimate the total budget projected to be available next time and allocate this to the four budget categories.

(2) List all projects required by the Public Opinion Poll. This will require a guess (or an investigation) to reveal how the Public Opinion Poll was decided in this cycle (and therefore affects next cycle).

(3) List all multi-year projects previously carried by the Politician, but not yet expired.

(4) List all projects suggested by the Crystal Ball Edition of the newspaper.

(5) Make a trial total of 2 + 3 + 4.

(6) Delete projects from 4 if the trial total (5) exceeds the estimated budget (1), add projects if it falls short.

(c) Speculator. In this introductory cycle (Figure 20), the Speculator has invested fairly conservatively. His consultations with the Politicians revealed that the bulk of the projects initiated this cycle affected wards 1 and 2. He has, therefore, invested nothing in Ward 3.

He takes a $10,000 option on residential land in ward 1, knowing that residential investments have the greatest probability of return. He also invests $20,000 on residential land and $10,000 on commercial land in ward 2. The greater amount in ward 2 is prompted by the Public Opinion Poll issue of a new interstate highway which the Speculator believes will pass, which should produce a marked increase in land values.

```
HEADLINES FROM

CYCLE 2  TEAM 1          METROPOLIS  CYCLE 1 -- DEMONSTRATION WITH PRESET DECISIONS          9/ 1/73   1.-.1

          *** LOCAL NEWS ***                                   *** STATE NEWS ***

               WARD 1                                   PROPERTY ASSESMENT PRACTICES HIT. LEGISLATORS
                                                      AUTHORIZE STUDY OF LOCAL PRACTICES.
   CIVIC CENTER IMPROVEMENTS DECLARED URGENT -- CONVENTION FACILI-     SCHOOL COSTS CONTINUE TO MOUNT -- NO RELIEF
TIES EXPANSION A *MUST*.         (LOCATION J-110, PROJECT NO. 402)  IN SIGHT.
   CHAMBER OF COMMERCE DEMANDS IMPROVED STREET SYSTEMS.              GOVERNOR ADDRESSES CONFERENCE ON PROBLEMS OF
                                 (LOCATION J-140, PROJECT NO. 401)  AGED. CITES NEED FOR MORE PUBLICLY SUPPORTED
   PARK SERVICE BUILDING DESTROYED BY FIRE -- IMMEDIATE REPLACEMENT HOUSING.
SOUGHT.                          (LOCATION K-120, PROJECT NO. 124)
                                                                        *** NATIONAL NEWS ***
               WARD 2
                                                                   BULLISH STOCK MARKET CONTINUES. PRESIDENT OF
   SEWER PROBLEMS PLAGUE HOMEOWNERS -- RAW SEWAGE FLOWS IN DITCHES. EXCHANGE SAYS FEARS OF RECESSION UNWARRANTED.
                             (LOCATION Q-T3-140, PROJECT NO. 256)     AUTO SALES EXCEED ALL PREVIOUS PEAKS.
   NORTH END RESIDENTS DEMAND PARK.                                    EMPLOYMENT REACHES NEW HIGH AS HIGH RATIO OF
                                 (LOCATION D-105, PROJECT NO. 231)  UNEMPLOYED PERSISTS.

                                           WARD 3                        * * * * * * * * * * * *

   ANGRY PARENTS DEMAND SIDEWALKS AFTER THIRD CHILD IS INJURED.                    (LOCATION I-220, PROJECT NO. 3[illegible])
   *CONTEMPORARY ESTATES* DEVELOPMENT H[illegible] BY SEWER LACK.                  (LOCATION FJ-65, PROJECT N[illegible]. 3[illegible])
   LAKEVIEW ESTATE, LAST MAJOR LAKE SITE AVAILABLE, OFFERED TO CITY AT GENEROUS PRICE. (LOCATION F-270, PROJECT NO. 246)
```

Figure 15

```
HEADLINES FROM

CYCLE 3  TEAM 1          METROPOLIS  CYCLE 1 -- DEMONSTRATION WITH PRESET DECISIONS          9/ 1/73   1[illegible]

          *** LOCAL NEWS ***

               WARD 1                                    *0*0*0*0*0*0*0*0*0*0*0*0*
                                                         0* ADMINISTRATOR'S SPECIAL *0
   BLACK LEADER DEPLORES CONDITION OF LOCAL PARK. IMPROVEMENTS  *0*     CRYSTAL  BALL      *0*
SOUGHT.                          (LOCATION H-112, PROJECT NO. 109)  0*       EDITION          *0
                                                         *0*0*0*0*0*0*0*0*0*0*0*0*
               WARD 2

   DEVELOPMENT STYMIED NEAR INTERCHANGE -- SEWER EXTENSION REQUIRED.
                            (LOCATION A-E-123, PROJECT NO. 228)
   JUDGE CITES INCREASE IN DELINQUENCY. URGES EXPANSION OF RECREA-
TIONAL ACTIVITIES.               (LOCATION H-110, PROJECT NO. 247)

               WARD 3

   UNIVERSITY TRAFFIC SUFFERING DAILY JAMS AS ENROLLMENT BREAKS ALL
RECORDS. PRESIDENT SAYS RELIEF THROUGH BRIDGE IMPROVEMENT A *MUST*.
                                 (LOCATION K-225, PROJECT NO. 390)
   SUMMER CAMP PROJECT SOUGHT AS MEANS TO RELIEVE JUVENILE PROBLEMS.
                                 (LOCATION F-275, PROJECT NO. 367)
   SCHOOL BUS SUBMERGED IN FLASH FLOOD -- HERO RESCUES 12 TOTS AS
DRIVER PERISHES.                 (LOCATION G-70, PROJECT NO. 369)
```

Figure 16

PUBLIC OPINION POLL — CYCLE 2

PLAYER: ADMINISTRATOR [ ] POLITICIAN [ ] SPECULATOR [ ]

ISSUE 2-1 PRIMARY THOROUGHFARE CONSTRUCTION

EXCESSIVE EAST-WEST TRAFFIC IN SOUTH END DEMANDS RELIEF BY NEW CONSTRUCTION. AS AN ALTERNATIVE, A ONE-WAY PAIR CAN BE CONSTRUCTED FOR TEN PERCENT OF NEW CONSTRUCTION COSTS.

ALTERNATIVES
1. FAVOR NEW ROUTE [ ]
2. POSTPONE AND RECONSIDER [ ]
3. FAVOR 1-WAY PAIR [ ]

ISSUE 2-2 CITY INCOME TAX

THE PROPOSED ONE PERCENT CITY INCOME TAX WOULD INFLUENCE ALL THOSE WORKING OR LIVING IN THE CITY, AND INCOME WOULD BE USED TO COMPENSATE FOR LOSSES FROM REDUCED PROPERTY TAXES.

ALTERNATIVES
1. FAVOR INCOME TAX [ ]
2. POSTPONE AND RECONSIDER [ ]
3. OPPOSE THE TAX [ ]

ISSUE 2-3 SEWAGE TREATMENT PLANT

(PROJECT NO. 508, ALL WARDS, LOCATION E-20)

MAJOR EXPANSION OF SEWAGE TREATMENT FACILITIES, DESIGNED TO ACCOMODATE GROWTH FOR THE NEXT DECADE. CURRENT PLANT HAS BEEN OPERATING IN EXCESS OF CAPACITY DURING DAILY PEAKS, REQUIRING BY-PASSING OF RAW SEWAGE. STATE THREATENS SUIT IF CONSTRUCTION DELAYED. ($200,000/YEAR FOR 6 YEARS)

ALTERNATIVES
1. SUPPORT PROJECT NO. 508 [ ]
2. POSTPONE AND RECONSIDER [ ]
3. REJECT PROJECT NO. 508 [ ]

Figure 17

FORM NO. 3, CYCLE 2 — POLITICIAN'S DECISION FORM — DATE 9/ 3/73

PROJECT NUMBER	BUDGET ALLOCATION A STREETS	B UTILITIES	C RECREAT., N	D MISCELLANEOUS
1. LIST ALL PROJECTS REQUIRED BY THE PUBLIC OPINION POLL				
A. 501				70,000
B. 508		600,000		
C.				
2. LIST ALL OTHER MULTIPLE YEAR PROJECTS STILL IN EFFECT				
A. 401	150,000			
B. 402				50,000
C. 403	150,000			
D.				
E.				
3. LIST ADMINISTRATOR'S RECOMMENDATIONS AS POSSIBLE				
A. 124			20,000	
B. 231			100,000	
C. 366			50,000	
D. 368		100,000		
E.				
F.				
G.				
H.				
I.				
J.				
K.				
4. TRIAL TOTAL (1 + 2 + 3)	300,000	700,000	170,000	120,000
5. BUDGET AVAILABLE	262,422	393,634	131,211	524,845
6. SURPLUS OR DEFICIT (5 - 4)	-37,578	-306,366	-38,789	404,845
7. TOTAL SURPLUS $ 22,112 OR TOTAL DEFICIT $ (6A + 6B + 6C + 6D)				
8. TAX RATE TO BE USED NEXT YEAR (CYCLE 3) 48.8 MILLS				

Figure 18

FORM NO. 4, CYCLE 2 — ADMINISTRATOR'S DECISION FORM — DATE 9/1/73

PROJECT NUMBERS	A STREETS (20 PCT OF 1-E)	B UTILITIES (30 PCT OF 1-E)	C RECREATION (10 PCT OF 1-E)	D MISCELLANEOUS (10 PCT OF 1-E)	E TOTAL (A + B + C + D)
	ESTIMATED BUDGET ALLOCATION				
1. ESTIMATE	350,000	700,000	150,000	140,000	1,340,000
2. LIST ALL PROJECTS REQUIRED BY THE PUBLIC OPINION POLL					
A. 501				70,000	70,000
B. 508		600,000			600,000
3. LIST ALL OF THE MULTIPLE YEAR PROJECTS RECOMMENDED					
A. 401	150,000				150,000
B. 402				50,000	50,000
C. 403	150,000				150,000
D.					
E.					
4. LIST ALL PROJECTS FROM THE CRYSTAL BALL EDITION (OR OTHERS, AS APPROPRIATE)					
A. 109			40,000		40,000
B. 228		50,000			50,000
C. 241			20,000		20,000
D. 357			50,000		50,000
E. 369		50,000			50,000
F. 565			40,000		40,000
G. 126				20,000	20,000
H. 373	50,000				50,000
I.					
J.					
K.					
5. TRIAL TOTAL (2+3+4)	350,000	700,000	150,000	140,000	1,340,000

ADD ALL PROJECTS FOR LINES 2, 3 AND 4. IF THE TRIAL TOTAL EXCEEDS THE BUDGET (LINE 1) [illegible] SOME PROJECTS FROM THE CRYSTAL BALL [illegible].

NOTE -- ONLY [illegible] IS MANDATORY. OTHERS ARE FOR DETAILED BUDGET PLANNING.

Figure 19

The Speculator's Decision Form (Figure 20) should be completed in numbered sequence:

(1) Transfer net worth (current cash assets) from the output.

(2) Subtract any cash you do not wish to invest (market uncertainty? hidden campaign funds?)

(3) Subtract any publicly acknowledged campaign funds.

(4) Total option cost is the money you have available to invest this cycle.

(5) Allocate option cost to ward(s) on the basis of your speculative judgment as to where the largest increases in land value will occur (remember the real test here is to successfully anticipate the outcome of the Public Opinion Poll currently being [illegible]).

(6) Ward by ward (as you have chosen to invest in them), allocate the available option cost by land use (remember that only the chance payoff discussed earlier affects this decision).

(d) School Board In completing this decision form (Table 7), the numbered sequence should be followed.

TABLE 7 School Board Decision Form

SCHOOL BOARD DECISION FORM FORM [illegible] CYCLE 2

(Estimation of revenues for next cycle, #3)

IMPORTANT. Follow the numbered steps in order when completing the form

1	Estimate the city population next cycle (from Wall Chart No. 9)	[illegible]
2	Standard ratio of school children	X .184
3	Total estimated school population (Line 1 times Line 2)	41,4[illegible]
4	Estimate the cost per school child	
	a National per capita estimate	$ 490
	b (1) Actual cost last year (Form 7, Line 2)	$ 17,808,704
	*(2) School population last cycle	40,450
	(3) Actual per/capita last cycle (1÷2) (Line 4b1 divided by Line 4b2)	440
	c Estimate actual per/capita next cycle (Professional judgement required, will probably be between 4a and 4b3)	$ 465
5.	Total estimated school expenditure for next cycle (#3) (Line 3 times Line 4c)	$ 19,251,000
6	Expected percentage change	
	a Estimated cost next year (#3) (Line 5 above)	$ 19,251,000
	b Actual cost this year (#2) (Line 4b1 above)	$ 17,808,704
	c Difference (Line 6a less Line 6b)	$ 1,442,296
	d Per cent change (Line 6c divided by Line 6b)	$ 8%

*Population, as shown on Wall Chart #9, multiplied by .184

FORM NO. 5- CYCLE 2 — SPECULATOR'S DECISION FORM — DATE 9/ 1 /75

1. NET WORTH (TRANSFER FROM OUTPUT) $ 72,299

2. CASH (LIST CASH TO BE KEPT ON HAND) $ (-) 32,299

SUB-TOTAL $ 40,000

3. CAMPAIGN CONTRIBUTIONS (MULTIPLES OF $5,000) $ (-) -0-

4. TOTAL OPTION COST (LINE 1 MINUS LINES 2 + 3) NET $ 40,000

5. ALLOCATE OPTION COST (FROM LINE 4) TO WARDS

	ALLOCATED OPTION COST
WARD 1	$ 10,000
WARD 2	$ 30,000
WARD 3	$ -0-

6. ALLOCATE OPTION COST TO LAND USE (THE $ AMOUNT ALLOCATED IN STEP 5, DIVIDED INTO LAND USE CATEGORIES)

LAND USE	WARD 1	WARD 2	WARD 3
RESIDENTIAL	10,000	20,000	
COMMERCIAL		10,000	
INDUSTRIAL			
ALLOCATED OPTION COST FROM STEP 5	10,000	30,000	

Figure 20

(1) The School Board representative must as... of the ... METROPOLIS ... in the next cycle. In order to do this, he should first note the current population (on the last page of the computer output or on Wall Chart 9) and the growth index located on the Public Opinion Poll ... of Wall Chart 1. If this growth figure appears stable he may count on approximately the same percentage of increase in population from year to year. Included are data ... which will help him compute these percentages (see Tables 5 and 6, Figure 5)

(2) In this sample cycle the representative has noted a fairly steady increase in population over the past several years. Since the growth index looks fairly stable, he has assumed a similar increase during this cycle and has noted this on line 1 of his form (from a current population of approximately 220,000 to an estimated 225,000)

(3) Multiplying this by 18.4, the standard ratio of school children to population, he estimates a total of 41,400 children enrolled in school next year.

(4) One final task remains for the educator: to estimate the cost of education per pupil. He turns to Table 5 and discovers that the per capita cost of education has fluctuated widely during the past decade. His next source of information are the Politicians, who tell him the city is currently enjoying a time of comparative prosperity and ... he can plan on a fairly large increase in the money available per pupil. The representative finally turns to the newspaper and reads predictions of rising school costs throughout the country. After review, he selects a per capita cost of $465. The total estimated cost is $19,251,000; this represents an estimated increase of about 8% for the year.

C CYCLE 3 (complete results of cycle 2 decisions)

(1) DECISION RESULTS FROM CYCLE 2

Having made the decisions described in the preceding section, players received output detailing the results of their activities (see Figures 21-26).

(2) YOUR FIRST PLAYED CYCLE (filling out the forms, Figures 27-32)

Once you have looked over the results of the sample decisions found in the preceding section, you should be ready to consider the decisions you will make for cycle 3. A Public Opinion Poll form and decision forms for all roles are provided later, in addition to a copy of the newspaper. Before you go to class (or to the game environment), complete each of these decision forms using the preceding "cycle 2 output" as a basis for your decisions. This set of decisions will familiarize you with the forms and test your understanding of the mechanics of the game.

These decisions ... not be used in actual play, although they ... you in ... the team you join for actual play.

Remember that the forms will appear slightly different *if the hand operated version is being used.* Nonetheless, the information is the same, and cycle 3 is precompleted on the hand forms (only) to get you started.

You may wish to read Chapter IV before completing these forms since it describes the METROPOLIS community more fully.

```
M E T R O P O L I S - VI ........ URBAN GAMING SIMULATION
REVISION 11 OF APRIL 1973 ... FOR IBM 1130

ENVIRONMENTAL SIMULATION LABORATORY, UNIVERSITY OF MICHIGAN
PROF. RICHARD D. DUKE, DIRECTOR

CYCLE  2        METROPOLIS  CYCLE 2 -- DEMONSTRATION USING PRESET DECISIONS        DATE  5/ 1/73
YEAR   1                                                                            TIME    12-01


                                 PUBLIC OPINION POLL RESULTS
                                                                                   PENALTY OR REWARD
                                                                              ADMINISTRATOR   POLITICIAN
                  PROJECT     LOCAL                                                             WARD
CYCLE.ISSUE..ACTION....LOCATION......COST..YEARS...........COMMENT............        ...        1...2...3

  2     1   APPROVED   THOROFARE        0.    0   PRIMARY THOROFARE CONSTRUCTION     5          0  -2   0

  2     2   REJECTED   NONE             0.    0   TAX REVISION, CITY INCOME TAX      0          0   0   0

  2     3   APPROVED   E-20       3600000.    6   METRO AREA SEWAGE TRTMT PLANT     10          1   1   1


     CALCULATED GROWTH INDEX =   3

     GROWTH INDICES, NEXT FIVE CYCLES        2    2    2    2    2
```

Figure 21

```
CYCLE  2        METROPOLIS  CYCLE 2 -- DEMONSTRATION USING PRESET DECISIONS        DATE  5/ 1/73
YEAR   1                                                                            TIME    12-01

                                  P O L I T I C I A N S

                          CITY COUNCIL BUDGETS CAPITAL IMPROVEMENT.....

                        THE FOLLOWING PROJECTS HAVE BEEN APPROVED FOR THIS YEAR

          PROJ                          BUDGET                                    YEARS      COST PER
          NUM....WARD...LOCATION......CATEGORY...........DESCRIPTION.........TO RUN.....   YEAR

                              BUDGETED THIS-YEAR-ONLY PROJECTS

          124       1   K-120        RECREATION       PARKS - RECREATION        1         $  20000.
          231       2   D-105        RECREATION       PARKS - RECREATION        1         $ 100000.
          368       3   FJ-65        UTILITIES        SANITARY SEWERS           1         $ 100000.
          366       3   F-270        RECREATION       PARKS - RECREATION        1         $  50000.

                                         TOTAL                                            $ 270000.

                              BUDGETED MULTI-YEAR PROJECTS

          401       1   J-140        STREETS          LOCAL STREETS             3         $ 150000.
          402       1   J-110        MISCELLAN.       CIVIC CENTER              4         $  50000.
          508     ALL   E-20         UTILITIES        SEWAGE TRTMT PLANT        6         $ 200000. PER WARD
          403       2   P-80         STREETS          LOCAL STREETS             2         $ 150000.
          301       1   I-110        MISCELLAN.       NEW CITY HALL             5         $  70000.

                                         TOTAL                                            $1020000.

                        TOTAL  C I P  BUDGET THIS YEAR                                    $1290000.


                        RECOMMENDED PROJECTS NOT BUDGETED BY THE POLITICIAN

                                         TOTAL                                            $      0.
```

Figure 22

SPENDING SCHEDULE

	WARD 1	WARD 2	WARD 3	TOTALS
STREETS	150000.	150000.	0.	300000.
UTILITIES	200000.	200000.	300000.	700000.
RECREATION	20000.	100000.	50000.	170000.
MISCELLANEOUS	120000.	0.	0.	120000.
TOTALS	490000.	450000.	350000.	1290000.

TAX RATE FOR NEXT CYCLE 48.8 MILLS

UNSPENT FUNDS OF THIS YEAR PLUS INTEREST, $ 2,997. HAVE BEEN ADDED TO CYCLE 3

REWARD OR PENALTY POINTS

	WARD 1	WARD 2	WARD 3
POLITICIANS STANDING, END OF CYCLE 1	1	6	4
PUBLIC OPINION POLL	1	-1	1
RECOMMENDED PROJECTS IGNORED	0	0	0
BUDGET INEQUITIES	0	0	0
TAXES	0	0	0
CAMPAIGN CONTRIBUTIONS, IF ANY	0	0	0
POLITICIANS STANDING, END OF CYCLE 2	2	5	5

Figure 23

CYCLE 2
YEAR 1

METROPOLIS CYCLE 2 -- DEMONSTRATION USING PRESET DECISIONS

DATE 5/ 1/75
TIME 12-01

A D M I N I S T R A T O R S

ADMINISTRATORS STANDING AT END OF PRECEDING CYCLE = 26

REWARD POINTS = 7
OVER-, UNDER-EXPENDITURE PENALTY = 0
PUBLIC OPINION POLL POINTS = 15

ADMINISTRATORS CURRENT STANDING = 48

ADMINISTRATORS RECOMMENDED PROJECTS FOR NEXT YEAR

PROJ NUM	WARD	LOCATION	BUDGET CATEGORY	DESCRIPTION	YEARS TO RUN	COST PER YEAR
109	1	H-112	RECREATION	PARKS - RECREATION	1	$ 40000.
126	1	K-130	MISCELLAN.	FIRE PROTECTION	1	$ 20000.
228	2	AE-120	UTILITIES	SANITARY SEWERS	1	$ 50000.
247	2	M-110	RECREATION	PARKS - RECREATION	1	$ 20000.
373	3	H-210	STREETS	LOCAL STREETS	1	$ 50000.
369	3	G-70	UTILITIES	STORM SEWER	1	$ 50000.
365	3	F-200	RECREATION	PARKS - RECREATION	1	$ 40000.
367	3	F-275	RECREATION	PARKS - RECREATION	1	$ 50000.
401	1	J-140	STREETS	LOCAL STREETS	2	$ 150000.
402	1	J-110	MISCELLAN.	CIVIC CENTER	3	$ 50000.
403	2	P-80	STREETS	LOCAL STREETS	1	$ 150000.
501	1	I-110	MISCELLAN.	NEW CITY HALL	4	$ 70000.
508	ALL	E-20	UTILITIES	SEWAGE TRTMT PLANT	5	$ 200000. PER WARD

RECOMMENDED C I P BUDGET FOR NEXT YEAR $1340000.

BREAKDOWN OF BUDGETED SCHEDULE

	WARD 1	WARD 2	WARD 3	TOTALS
STREETS	150000.	150000.	50000.	350000.
UTILITIES	200000.	250000.	250000.	700000.
RECREATION	40000.	20000.	90000.	150000.
MISCELLANEOUS	140000.	0.	0.	140000.
TOTALS	530000.	420000.	390000.	1340000.

Figure 24

CYCLE 2 METROPOLIS CYCLE 2 -- DEMONSTRATION USING PRESET DECISIONS DATE 5/ 1/73
YEAR 1 TIME 12-01

SPECULATORS STANDING

SPECULATORS GROWTH FACTORS

WARD. LAND USE	1 RES	1 COM	1 IND	2 RES	2 COM	2 IND	3 RES	3 COM	3 IND
OPINION POLL	0.000	0.000	0.000	0.100	0.100	0.100	0.000	0.000	0.000
DICE ROLL	0.100	0.000	0.000	0.100	0.200	0.000	0.100	0.000	0.000
BUDGET ALLOCATION	0.049	0.049	0.049	0.045	0.045	0.045	0.035	0.035	0.035

CURRENT CYCLE INVESTMENTS

WARD	RESIDENTIAL	COMMERCIAL	INDUSTRIAL	TOTAL
1	10000.	0.	0.	10000.
2	20000.	10000.	0.	30000.
3	0.	0.	0.	0.
TOTL	30000.	10000.	0.	40000.

TOTAL OF PREVIOUS INVESTMENTS

WARD	RESIDENTIAL	COMMERCIAL	INDUSTRIAL	TOTAL
1	5000.	5000.	5000.	15000.
2	10000.	0.	5000.	15000.
3	5000.	10000.	0.	15000.
TOTL	20000.	15000.	10000.	45000.

NET WORTH AT END OF CYCLE 1 $ 72299.

- AMOUNT INVESTED THIS CYCLE $ 40000.
- CONTRIBUTED TO POLITICIAN $ 0.

= CASH HELD IN RESERVE $ 32299.
+ 4 PERCENT INTEREST ON CASH RESERVE $ 1291.
+ INVESTMENT RETURN $ 98399.

NET WORTH AT END OF CYCLE 2 $ 131991.

Figure 25

CYCLE 2 METROPOLIS CYCLE 2 -- DEMONSTRATION USING PRESET DECISIONS DATE 5/ 1/73
YEAR 1 TIME 12-01

METROPOLIS INFRA-STRUCTURE ACCUMULATIONS

CUMULATIVE INCREASE OF PROGRAMMED PROJECTS

CYCLE	WARD 1	FACTOR	WARD 2	FACTOR	WARD 3	FACTOR
1	470000.	4.700	500000.	5.000	170000.	1.700
2	960000.	4.800	950000.	4.750	520000.	2.600

<u>REVENUE FOR CYCLE 3</u> ←— Note!

POPULATION OF METROPOLIS RISES TO 219700.

PER CAPITA ASSESSED VALUE= $ 1390.

TOTAL ASSESSED VALUE OF METROPOLIS= $ 305383040.

TAX RATE IN MILLS= 48.8

TOTAL CITY INCOME FROM PROPERTY TAX= $ 14902688.

+ NON-TAX REVENUE= $ 14902688. WHICH IS 100. PERCENT OF PROPERTY TAX REVENUE

= TOTAL CITY INCOME= $ 29805376.

- INCOME ALLOCATED TO SCHOOLS= $ 18986020. WHICH IS 63.6 PERCENT OF TOTAL CITY INCOME

= NET CITY INCOME= $ 10819356. OF WHICH 12 PERCENT ARE YOUR

TOTAL DISCRETIONARY FUNDS= $ 1298322.

+ UNSPENT FUNDS FROM CYCLE 2= $ 22997.

= NET DISCRETIONARY FUNDS= $ 1321320.

- FUNDS COMMITTED FOR MULTI-YEAR PROJECTS= $ 1020000.

= FUNDS AVAILABLE FOR NEW PROJECTS= $ 301320.

DISCRETIONARY FUND ALLOCATION

BUDGET CATEGORY	NET DISC. FUNDS	COMMITTED FOR MULTI-YEAR	AVAILABLE DISC. FUNDS
STREETS	$ 264264.	$ 300000.	$ -35736.
UTILITIES	$ 396396.	$ 600000.	$ -203604.
RECREATION	$ 132132.	$ 0.	$ 132132.
MISCELLAN.	$ 528528.	$ 120000.	$ 408528.

Figure 26

HEADLINES FROM

CYCLE 3 TEAM 1 METROPOLIS CYCLE 2 -- DEMONSTRATION USING PRESET DECISIONS 9/ 1/73 12-01

*** LOCAL NEWS ***

WARD 1

BLACK LEADER DEPLORES CONDITION OF LOCAL PARK. IMPROVEMENTS SOUGHT. (LOCATION H-112, PROJECT NO. 109)

WARD 2

DEVELOPMENT STYMIED NEAR INTERCHANGE -- SEWER EXTENSION REQUIRED. (LOCATION F-125, PROJECT NO. 228)

JUDGE CITES INCREASE IN DELINQUENCY. URGES EXPANSION OF RECREATIONAL ACTIVITIES. (LOCATION H-110, PROJECT NO. 247)

WARD 3

UNIVERSITY TRAFFIC SUFFERING DAILY JAMS AS ENROLLMENT BREAKS ALL RECORDS. PRESIDENT SAYS RELIEF THROUGH BRIDGE IMPROVEMENT A 'MUST'. (LOCATION K-225, PROJECT NO. 390)

SUMMER CAMP PROJECT SOUGHT AS MEANS TO RELIEVE JUVENILE PROBLEMS. (LOCATION F-275, PROJECT NO. 367)

SCHOOL BUS SUBMERGED IN FLASH FLOOD -- HERO RESCUES 12 TOTS AS DRIVER PERISHES. (LOCATION G-70, PROJECT NO. 369)

*** STATE NEWS ***

LEGISLATURE STRUGGLES WITH HOME RULE LEGISLATION. REVISED INCORPORATION STATUTES ARE THE LIKELY OUTCOME.

STATE FIRE MARSHALL INVESTIGATES NURSING HOME FIRE IN METROPOLIS IN WHICH FIVE DIE.

LEGISLATURE PASSES LEGISLATION ENABLING METROPOLIS TO FINANCE NEW STORM SEWER RELIEF PROGRAM.

GOVERNOR SIGNS AGED BILL.

*** NATIONAL NEWS ***

PRESIDENT'S COUNCIL OF ECONOMIC ADVISORS PREDICTS BIGGEST ECONOMIC SURGE. RECESSION TALK BRANDED AS PARTISAN EFFORT TO DISCREDIT ADMINISTRATION.

CONFLICTING NATIONAL SURVEYS ON FUTURE OF AUTOMOTIVE SALES. PRODUCTION CONTINUES AT ALL TIME HIGH.

UNEMPLOYMENT DROPS SLIGHTLY. FIRST TIME IN TWO YEARS.

Figure 27

HEADLINES FROM

CYCLE 4 TEAM 1 METROPOLIS CYCLE 2 -- DEMONSTRATION USING PRESET DECISIONS 9/ 1/73 12-01

*** LOCAL NEWS ***

WARD 1

PARK IMPROVEMENTS SOUGHT BY NEIGHBORHOOD GROUP -- FACILITIES FOR AGED URGENTLY NEEDED. (LOCATION H-115, PROJECT NO. 110)

WARD 2

COMPLETION OF EXPRESSWAY LINK CAUSES MAMMOTH JAM - - MOTORISTS DELAYED FOR HOURS. (LOCATION E-90,165, PROJECT NO. 233)

CIVIL DEFENSE DIRECTOR JOINS NATIONAL GUARD IN SUPPORT OF ARMORY. (LOCATION H-150, PROJECT NO. 243)

ARBORETUM PARK HAS HIGHEST ATTENDENCE - - IMPROVEMENTS REQUIRED. (LOCATION H-150, PROJECT NO. 250)

WARD 3

LARGEST COUNCIL MEETING AS RESIDENTS PROTEST STREET CONDITIONS. (LOCATION F-80, PROJECT NO. 361)

RECENTLY ANNEXED AREA NEEDS FIRE-POLICE BUILDING. INSURANCE RATES GO UP UNTIL SITUATION REMEDIED. (LOCATION J-80, PROJECT NO. 383)

'CONTEMPORARY ESTATES' DEVELOPER URGES COUNCIL TO COMPLETE GOLF COURSE. COMMUNITY BUILDING AND COURSE URGENTLY NEEDED. (LOCATION H-70, PROJECT NO. 370)

*0*0*0*0*0*0*0*0*0*0*0*0*0*0*
0* ADMINISTRATOR'S SPECIAL *0
0 CRYSTAL BALL *0*
0* EDITION *0
*0*0*0*0*0*0*0*0*0*0*0*0*0*0*

Figure 28

CYCLE 3

PUBLIC OPINION POLL

PLAYER	ADMINISTRATOR I I	POLITICIAN I I	SPECULATOR I I

ISSUE 3-1 PROPERTY REASSESSMENT

THE CITY HAS NOT BEEN REASSESSED IN A UNIFORM MANNER SINCE 1948, IN VIOLATION OF A STATUTE WHICH REQUIRES CHANGES EACH FIVE YEARS. THE CITY ADMINISTRATION HAS MADE A STRONG PLEA FOR A COMPLETE STUDY, PARTICULAR EMPHASIS TO BE PLACED ON COMMERCIAL STRUCTURES AND NEW HOUSING.

ALTERNATIVES
1. FAVOR THE REASSESSMENT I I
2. POSTPONE AND RECONSIDER I I
3. OPPOSE THE REASSESSMENT I I

ISSUE 3-2 SENIOR CITIZENS

A RECENT WEEK - LONG CONFERENCE AT THE UNIVERSITY HAS RECOMMENDED FORMATION OF A "COMMISSION ON PROBLEMS OF THE AGED"

ALTERNATIVES
1. FAVOR THE COMMISSION I I
2. POSTPONE AND RECONSIDER I I
3. OPPOSE THE COMMISSION I I

ISSUE 3-3 HOUSING FOR THE AGED

(PROJECT NO. 505, WARD 5, LOCATION J-70)

LOW COST HOUSING FOR THE AGED, WITH SPECIALLY DESIGNED FACILITIES. WOULD PROVIDE FOR THE MOST URGENT SECTOR OF THE MARKET. WOULD COST $150,000 PER YEAR FOR TWO YEARS.

ALTERNATIVES
1. FAVOR PROJECT NO. 505 I I
2. POSTPONE AND RECONSIDER I I
3. OPPOSE PROJECT NO. 505 I I

Figure 29

FORM NO. 3, CYCLE 3 — POLITICIAN'S DECISION FORM — DATE 5/ 1/73

PROJECT NUMBER	BUDGET ALLOCATION A STREETS	B UTILITIES	C RECREATION	D MISCELLANEOUS
1. LIST ALL PROJECTS REQUIRED BY THE PUBLIC OPINION POLL				
A.				
B.				
C.				
2. LIST ALL OTHER MULTIPLE YEAR PROJECTS STILL IN EFFECT				
A.				
B.				
C.				
D.				
E.				
3. LIST ADMINISTRATOR'S RECOMMENDATION, AS POSSIBLE				
A.				
B.				
C.				
D.				
E.				
F.				
G.				
H.				
I.				
J.				
K.				
4. TRIAL TOTAL (1 + 2 + 3)				
5. BUDGET AVAILABLE				
6. SURPLUS OR DEFICIT (5 - 4)				

7. TOTAL SURPLUS $ OR TOTAL DEFICIT $ (6A + 6B + 6C + 6D)

8. TAX RATE TO BE USED NEXT YEAR (CYCLE 4) MILLS

Figure 30

FORM NO. 4, CYCLE 3 — [illegible] DECISION FORM — DATE 9/ 1/73

PROJECT NUMBERS	A STREETS ([illegible] PCT OF 1-E)	B UTILITIES ([illegible] PCT OF 1-E)	C RECREATION ([illegible] PCT OF 1-E)	D MISCELLANEOUS ([illegible] PCT OF 1-E)	E TOTAL (A + B + C + D)
	ESTIMATED BUDGET ALLOCATION				
1. ESTIMATE					
2. LIST ALL PROJECTS REQUIRED BY THE PUBLIC OPINION POLL					
A.					
B.					
3. LIST ALL OTHER MULTIPLE YEAR PROJECTS RECOMMENDED					
A.					
B.					
C.					
D.					
E.					
4. LIST ALL PROJECTS FROM THE CRYSTAL BALL EDITION					
A.					
B.					
C.					
D.					
E.					
F.					
G.					
H.					
I.					
J.					
K.					
5. TRIAL TOTAL (2+3+4)					

ADD ALL PROJECTS FROM LINES 2, 3 AND 4. IF THE TRIAL TOTAL EXCEEDS THE BUDGET (LINE 1) YOU MUST ELIMINATE SOME PROJECTS FROM THE CRYSTAL BALL (LINE 4).

NOTE -- ONLY COL. E IS MANDATORY. OTHERS ARE FOR DETAILED BUDGET PLANNING.

Figure 31

FORM NO. 5, CYCLE 3 — SPECULATOR'S DECISION FORM — DATE 9/ 1/73

1. NET WORTH (TRANSFER FROM OUTPUT) $ ________

2. CASH (LIST CASH TO BE KEPT ON HAND) $ (-) ________

SUB-TOTAL $ ________

3. CAMPAIGN CONTRIBUTIONS (MULTIPLES OF $5,000) $ (-) ________

4. TOTAL OPTION COST (LINE 1 MINUS LINES 2 + 3) NET $ ________

5. ALLOCATE OPTION COST (FROM LINE 4) TO WARDS

	ALLOCATED OPTION COST
WARD 1	$
WARD 2	$
WARD 3	$

6. ALLOCATE OPTION COST TO LAND USE (THE $ AMOUNT ALLOCATED IN STEP 5, DIVIDED INTO LAND USE CATEGORIES)

LAND USE	WARD 1	WARD 2	WARD 3
RESIDENTIAL			
COMMERCIAL			
INDUSTRIAL			
ALLOCATED OPTION COST FROM STEP 5			

Figure 32

4 The METROPOLIS Community

A. Brief History of METROPOLIS

Founded in the early nineteenth century, METROPOLIS was initially a lumbering and agricultural center. In mid century, the state capitol was located in METROPOLIS, lending impetus to the industrial boom of the late nineteenth century. By the turn of the century, METROPOLIS had a well established automotive industry, which continues as the dominant industry of the community. The state university, established for over one hundred years, is now one of the largest in the country. Its location in ward 3 (Figure 34) strongly influences development in this part of the community. The university is the third largest employer in METROPOLIS, ranking after state government and a large automobile manufacturer. METROPOLIS is located in a highly industrialized midwestern state, having a growth rate above that of the national average. The economy of the state is, however, heavily dependent on the fortunes of the automotive world. METROPOLIS is a progressive community with good transportation, including air service, rail, pipeline, and highways, including two interstate routes. Summer temperatures average about 70 degrees, winter about 25 degrees, rainfall averages about 30 inches.

The community has a history of planning dating back to the early 1930s when a master plan was prepared. This was reviewed and a new plan developed just prior to World War II. A completely new master plan based on extensive study of the community has recently been prepared by a consultant of international reputation; the local planning staff is keeping the various studies up to date.

Increasing congestion on the streets, deterioration and obsolescence of housing, commercial and industrial facilities, declining downtown business, attrition of tax base, housing shortages, social malaise, air and water pollution, and incompatible and conflicting land uses are part of the legacy of past rapid growth and inadequate commitment to long range planning. Remedial programs have begun but will need to be expanded and sustained indefinitely into the foreseeable future.

The technical capacity to deal effectively with these major urban problems is already available. The greatest roadblocks are defining and obtaining consensus on major community issues and adequately funded programs to implement the necessary improvement programs.

B General Characteristics

The tables below give specific data from census reports and other information sources for METROPOLIS. Of particular importance to players of METROPOLIS is the summary of specific characteristics (Table 8).

At the time the players assume responsibility for managing the gamed city, the city has a total urban population (SMSA) of approximately 214 000. This population is quite evenly divided among the three wards, each of which is represented by one Politician. The ward boundaries were drawn (for gaming purposes) to ensure this roughly equal distribution of population as well as to keep the population characteristics for each ward homogeneous. A map showing land use and ward boundaries is located at the back of this manual (Figure 34).

Ward 1 is the oldest part of the city and has the highest population density. This ward also contains virtually all of the Blacks in the metropolitan area (approximately sixteen percent of the ward's popula

Table [illegible] METROPOLIS—General Population Characteristics

Characteristic	Ward 1	Total 2	Total 3	Metropolis Total
Total Population	70 134	71 676	71 734	214 004
White	67 515	70 956	63 4??	201 972
Population in Institution	1 011	811	13 628	16 070
Persons Per Household	2.9	3.4	3.2	3.2
Number Married Couples	14 660	17 337	14 705	46 762
School Enrollment	16,?24	19 035	33 762	60 292
Kindergarten	1 446	2 010	1 310	4 76?
Elementary	5 ?90	11 946	8 456	30 ?64
High School	3 67?	3 640	2 ?94	10 134
College	1,6??	1 129	?? 702	35 660
Median School Years Completed	10.6	11.8	14.1	12.1
Residence Same in 1955	30 772	3? 335	20 793	74 972
Median Family Income	$ 5 ?02	$ 8 751	$13 744	$ 9 460
Married Population	31 237	35 677	29 7?0	96 664
Separated Widowed Divorced	8 656	4 660	3 095	17 402
Labor Force	16 4?3	18 094	18,518	55,093
[illegible] Employed	1 076	929	633	2 638
Percent Unemployed	5.6	4.9	3.4	4.7
[illegible]	5 445	5 421	5,?60	16 126
[illegible]	29 162	27 065	27 461	83 629
[illegible] Construction	5?9	1 652	1 066	4 376
[illegible] Manufacturing	7 ?07	8 164	3 620	19 092
[illegible] Trans Equipment	4 775	5 44?	2 021	12 243
Total Housing Units	25 85?	21 5?2	18 795	66 164
Housing Units Owner Occupied	12 779	17,816	11 702	42 298
Sound Housing Units	2? ?0	19 175	17 915	58 353
Deteriorated Housing Units	4,?7	1 953	754	6 679
Dilapidated Housing Units	4?2	460	123	994
Median Number Rooms	4.8	5.1	5.1	5.0
Median Number Persons	2.4	2.9	3.1	2.8
Households With No Automobiles	5 736	1 806	428	8 030
Median Value of Housing Units	$12 075	$17 400	$29,350	$ 23,159

[illegible] non-white), as well as many elderly and retired [illegible] family income of ward 1 residents is [illegible] than for the city as a whole, and [illegible] employment rate is quite high. Nearly four [illegible] of ward 1 residents are in the category [illegible] widowed, or divorced", for the city as [illegible] percent of the population is so [illegible] percent of the housing [illegible] are deteriorating or dilapidated, [illegible] number have no automobiles. Nearly [illegible] residents are renters.

The city's central business district, the state capitol, and the area's largest employer, an auto assembly plant, are all located in ward 1. Public facilities in the ward are old, deteriorating, and overused.

Ward 2 is split into two distinct geographic areas, one at the top and one at the bottom of the map. It is predominately blue collar with some lower level white-collar workers. Family size is larger in ward 2, on the average, than in the city as a whole, and this is reflected in the number of school children: excluding college enrollments, nearly half the school children for the whole city are found in this ward. Housing units are in good condition though of comparatively low value, and are overwhelmingly occupied by their owners. Median family income is just about the same in ward 2 as in the city as a whole. The population tends to be concerned about the level of property taxes and to have a low opinion of governmental services—low taxes are frequently preferable to a high level of services. On racial matters, ward 2 residents tend to be very conservative.

Ward 3 is also split on the map, half lying in the western portion of the city and the other half in the eastern portion. The state university consumes a huge piece of land in the eastern part of ward 3 and has forced development to go around it (note the large number of college age residents, Figure 33). All

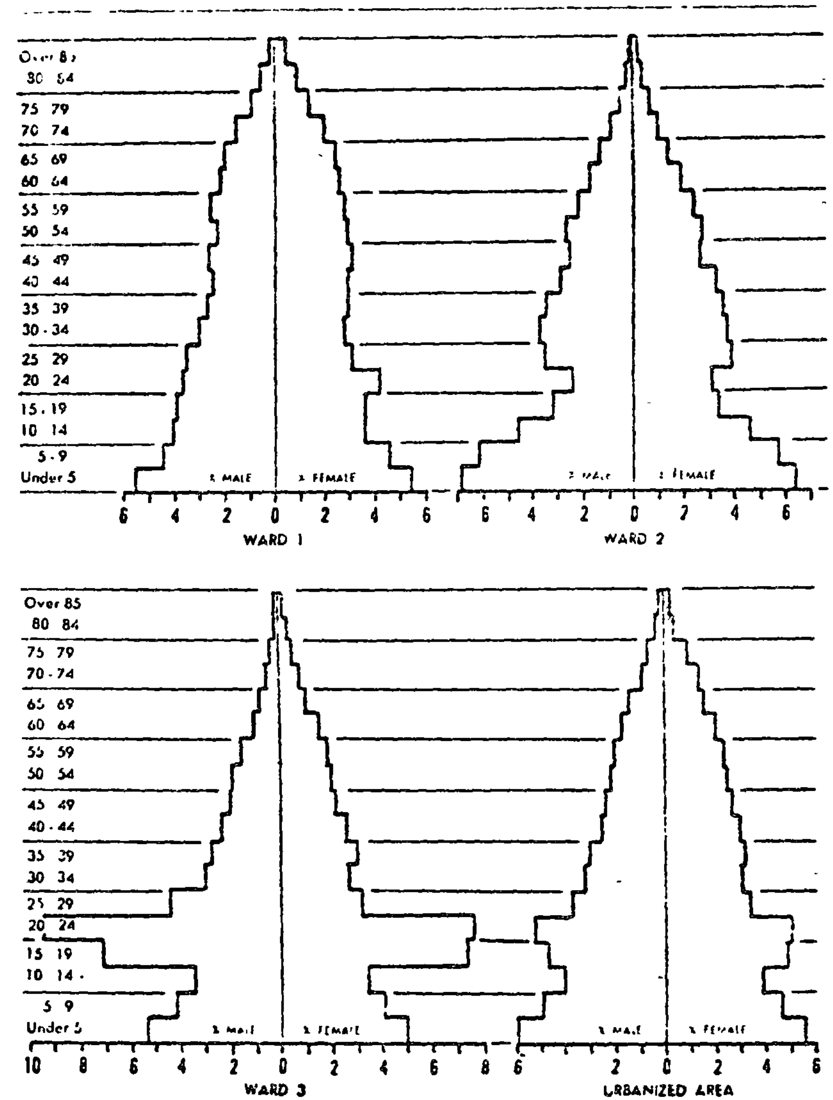


Figure 33 POPULATION AGE BY SEX, AS PERCENT OF UNIT

[illegible] of the [illegible] Ward 3 [illegible] of [illegible] percentage of [illegible] higher than average [illegible] very low percentage of [illegible] Median family income is [illegible] significantly higher than overall and the unemployment rate, appropriately, considerably [illegible] Occupations of Ward 3 residents are primarily in professional and managerial categories, and the [illegible] educational level includes more than two [illegible] of college. Ward 3 residents [illegible] concerned [illegible], also demand a high level of government services and tend to be liberal on civil rights [illegible] which do not appear to cost them much.

Population

METROPOLIS' population has increased rapidly [illegible] to the present with the exception of [illegible] decade, 1930-1940, when the [illegible] remained virtually constant, increasing [illegible] about one half of one percent. In the following decades—1940-1960—population increased at [illegible] faster than the nation during the [illegible] slightly slower during the 1950s and [illegible]. The 1960-1970 population trends in the [illegible] generally followed the same [illegible] to most [illegible] centers during the [illegible]. These trends include:

- —Metropolitan area population increased more [illegible] than the nation: 22% compared with 18.5%.
- —Central city population increased only 17% during the same period. Nearly 70% of metropolitan population growth occurred in the adjacent fringe.
- —Suburban population consists chiefly of middle- and higher income white families, both in [illegible] and those moving out of the central city.
- —Nonwhite population in the central city more than doubled during the decade as a result of high birth rates and immigration. Nonwhites made up 6.5% of the city's population in 1967 compared with 3.3% in 1960. By 1970, nonwhites made up 9.0% of the city's population.
- —Females increased faster than males—due chiefly to an out-migration of males—in the twenty to thirty year age group. Also, the concentration of [illegible] in the METROPOLIS area [illegible] for young females from [illegible].
- —The population [illegible] has been [illegible] to a greater proportion of persons under 45. The [illegible] percentage increases were in the very young (under 15) and in the very old (70 and over) age groups.

Present demographic trends, coupled with a favorable economic outlook, indicate that METROPOLIS' population should increase by 72% over the next 25 years. However, future population of the city is largely dependent on future annexation policies. Significant factors influencing population growth are listed below:

- —If present annexation policies are continued into the future, city population is anticipated to increase 45-55% by 1990.
- —Household formations are expected to increase [illegible] due to the relatively young age structure and continued "undoubling" of households resulting from rising income, stricter code enforcement, continued demolitions, and conversions of older [illegible] residences to other uses.
- —Average size of households is expected to drop [illegible], from 3.17 in 1960 to 3.11 by 1990, due to [illegible] family planning among the lower and middle income groups, and increased apartment construction.
- —The proportion of the population in the dependent age group under 15 and over 65 is expected to increase steadily over the projection period.
- —Nonwhite population in the city is anticipated to [illegible] over the next two decades, increasing from 12,000 to about 36,000.

D Education

In 1960, the median school years completed in the METROPOLIS area was 12.0. The city was slightly below the regional average (11.0 [illegible]). By 1970, the median school years completed in the region had risen to 12.3. In 1970, 12% of the [illegible] years of age and over had less than an eighth grade education.

The demands of new and more complex technology will require more formal education, and the average number of school years completed in the area will continue to rise to 14.0 years by 1990. College enrollments will increase sharply in the early 1970s as the [illegible] reach college age.

E Economic Base

Manufacturing, distribution, government, and education are the major components of the economy. These activities provide the main economic support for the area's population, generating a little more than half the employment and most of the "basic" income in the metropolitan area.

The city is a major regional job center. About 84,000 jobs are located within the city boundaries, of which about 30,000 are held by commuters living outside the city limits. It is the largest source of [illegible] in the local economy.

Manufacturing, employing 36,600, provides about 30% of metropolitan area jobs and about 43% of the city's total employment. Nearly 90% of the metropolitan area manufacturing jobs are located in the city. Motor vehicles, a high wage industry, provides nearly 70% of the total manufacturing jobs in the metropolitan area.

Government is the second largest employer in the metropolitan area. State government alone employs about 16,000, including university employees. In the city, government and retail trade are about equal in employment size, each employing about one third the number in manufacturing.

The economy as a whole can be typified as a high output, highly mechanized, highly skilled, high income, [illegible] market oriented economy. Most of the [illegible] are leaders in technological advancement. Skill requirements have progressively increased, while requirements for lower-skilled occupations have decreased.

The labor force is drawn from a wide area. About 39% of the jobs are held by commuters to the city. Resident employment gains have been mostly in service, government, and professional occupations. [illegible] manufacturing jobs gained have been [illegible] commuters; resident employment in manufacturing has declined in recent years. Generally, city resident employment gains have lagged behind city population growth, indicating that nonparticipants in the labor force are concentrating in the city. Resident employment gains have been greatest in the [illegible] lower wage activities, while commuters [illegible] jobs in higher wage activities, reflecting the tendency of the more affluent to move to the suburbs.

The three primary employers of METROPOLIS are the university, the automobile assembly plant and the state government. These three "basic" industries in turn foster supporting industries whose economic health is dependent upon the "Big Three." The university is a rapidly growing "industry" which is not dissimilar in economic characteristics to a large research and development company which is [illegible] highly [illegible] technical personnel. State government tends to be a highly stable, slowly growing employer, not particularly subject to business cycle fluctuations. The automobile plant, on the other hand, is typical of most heavy manufacturing and fluctuates considerably with the business cycle, growing rapidly or declining sharply.

Of the approximately 81,000 jobs in the city, 54,000 are held by city residents. Because of its balanced economy, METROPOLIS has [illegible] one of the lowest unemployment rates in the state and country: 4.7% in 1970. Minority groups in the city account for more than 50% of the present unemployment although they represent only 12% of the population.

Over the years, METROPOLIS has acquired a combination of resources, facilities, skills, management and organizational talents, and industrial location factors which ensure the area's participation in a broad range of national economic activity. Economic ties to other industrial areas already are well established. Substantial growth is forecast in all major components of the economic base. Past growth in employment, output, and income has been faster in the METROPOLIS area than in the state as a whole, the region, or the nation. The major factors influencing [illegible] are:

- —Potential for continued growth and diversity in manufacturing has been enhanced by recent transportation improvements, particularly the Interstate Highway System.
- —The area has been gaining prominence as a regional distribution center. The area's central location in the state and completion of the interstate freeway network promise an even greater role as a regional trade center.
- —The growth of state government will continue as more services are demanded by the growing regional population and as state and federal government programs are expanded to meet those demands.
- —Facilities for higher education will continue to expand to meet an increasing demand for highly trained personnel.

Trends since 1960 indicate that changes in resident employment, class of worker, and occupation indicate a loss from the city of the "middle class population." Despite this outmigration, the city has retained the majority of the jobs in the region. Most of the recent growth in employment opportunities has resulted [illegible] manufacturing, [illegible] with [illegible] The [illegible] city [illegible] of the city. [illegible] extension of municipal [illegible] of the [illegible] city [illegible] on [illegible] for all components of the economic base. The [illegible] segments of the economy will provide 60 [illegible] 1970, an increase of 4 6 [illegible] employment is [illegible] increase, the proportion of industrial jobs [illegible] will decrease as a result of improved technology and automation.

[illegible]ncome

[illegible] historically has had higher incomes than [illegible] in the [illegible] 1970 the area ranked [illegible] in the United States with an average [illegible] $9,409 per household. Per [illegible] was $2,997 for the city.

[illegible] the result [illegible] industries [illegible] Per [illegible] Median [illegible] expected to increase 60 [illegible] $1,500 [illegible] $17 [illegible]

[illegible] the planning commission indicates [illegible] expenditures for such "basics" as food, [illegible] and beverages are likely to be proportionately [illegible] while the amount of income devoted to [illegible] care, travel, recreation, housing, transportation, and education will probably increase.

G. Housing

[illegible] past five years, [illegible] economic conditions have [illegible] in a rapid expansion of job opportunities [illegible] of families to the city, [illegible] in the lower income groups. At the same [illegible] of factory expansions, construction of the [illegible] Highway System, and expansion of the [illegible] capitol complex have resulted in an increased rate of housing [illegible] predominantly those of lower [illegible]. The combination of these factors plus a high household formation rate has resulted in an extremely tight supply of low- and moderate-income housing. Overcrowding has thus occurred (6% of the present supply), and a marked deterioration of housing [illegible] because [illegible] house [illegible] to [illegible] of public housing [illegible] has helped [illegible] indicate that additional housing for low- and moderate-income groups [illegible] the future. A high [illegible] of the city's population resides in separate [illegible] (Most of the [illegible] area [illegible] units are around the [illegible] of the university.) [illegible] of households has generally increased [illegible] population growth, indicating a higher [illegible] and some "undoubling" of multifamily housing as a result of rising incomes.

The next decade will [illegible] demand for new housing. The planning commission says that this will be due largely to an increase in per capita incomes and a marked increase in young persons reaching marriageable age." As the figures indicate, the greatest amount of growth will take place in the area surrounding the city.

The commission projects a need for over 70,000 additional dwelling units [illegible] by 1990. It states that the single family house will be preferred by most [illegible] that [illegible] of more young adult [illegible] in [illegible] society, individual financial limitations and increasing land and development costs will increase the demand for townhouses, apartments, and other forms of multiple unit development." Most of this development [illegible] place outside the city.

METROPOLIS is typical of most midwestern communities in the sense that 7[illegible] of the city's households [illegible] single family homes and approximately 60% of METROPOLIS families own their own homes. However, homeownership is declining because of a marked increase in multifamily unit construction since 1964. This trend is the result of an increase in smaller households made up of younger persons—a product of the postwar "baby boom"—and elderly persons—a product of early retirement and increased longevity. The bulk of these households are in the lower middle income ranges. In addition, there is a sizeable lower-income population in the city, including minority groups and students whose housing demand can be met most readily by multiunit construction.

H. Trends—Direction of Future Growth

The completion of the Interstate Highway System will strongly influence the direction of future growth [illegible] economic activity can be expected to develop near or adjacent to major interchanges on the freeway network.

The [illegible] interest [illegible] system and [illegible] have already fostered the growth of [illegible] communities surrounding METROPOLIS and these [illegible] continue to draw increasing numbers of METROPOLIS employees and particularly those in the middle income groups.

The growth of suburban communities has resulted in the decentralization of commercial [illegible] activities and a decline in retail activity in METROPOLIS central business district. Two regional shopping centers now being built to the east and west will serve as a nucleus for future residential development. Redevelopment efforts under way in the downtown area, concentration of state governmental facilities, and continued expansion of a community [illegible] in the urban core should help arrest the decline in the central business district. Soil and drainage conditions and the economics of development [illegible] that new [illegible] to the east, south [illegible] of the [illegible] complex.

I. Municipal Finance

A history of city revenues and expenditures is shown in Tables 9 and 10. Over the past decade the school system has increased its demand from approximately 50% of all revenues to a current figure exceeding 60%. This trend is expected to continue.

Revenues derived from the property tax over the past decade for both the school system and the municipal budget have increased from 40% to 50% of the total revenue. This places a very severe burden on the property holder. In addition, there has been a consistent program to reassess all property upward to reflect true market value. During this time the tax rate has also risen sharply from 27.5 mills to 47.6 mills.

TABLE 9 City Budget Expenditures[a]

Year	Schools $	Schools %	Streets $	Streets %	Sewers $	Sewers %	Police Fire $	Police Fire %	Recreation $	Recreation %	Admin $	Admin %	Misc[b] $	Misc[b] %	Total Expenditures $
1960	5 601	55.1	974	9.5	743	7.3	1 130	11.1	400	3.9	34[illegible]	3.4	956	9.7	[illegible]
1961	5 708	52.6	1 [illegible]	11.8	768	7.1	1 269	11.5	471	4.2	[illegible]	3.5	[illegible]	9.[illegible]	11 [illegible]
1962	6 591	53.1	1 393	11.2	810	6.5	1 4[illegible]2	11.6	472	3.8	431	3.4	1 [illegible]	10.4	12 [illegible]
1963	6 769	49.0	1 579	11.4	[illegible]	[illegible]	1 591	11.5	611	4.4	47[illegible]	3.4	[illegible]784	[illegible]	13 [illegible]
1964	7,503	50.7	1 509	10.6	897	6.0	1,757	11.7	628	4.6	562	3.9	1 [illegible]	[illegible]	14 [illegible]
1965	[illegible] 563	53.1	1 635	10.1	913	5.7	[illegible] 846	[illegible]	628	3.9	[illegible]	3.4	[illegible]	[illegible]	[illegible]
1966	9 [illegible]08	55.4	1 521	[illegible]	767	4.5	1 903	11.[illegible]	603	3.5	561	[illegible]	2 301	13.3	17 [illegible]
1967	9 628	54.5	1 501	8.5	1 244	7.0	2 13[illegible]	12.1	7[illegible]0	4.4	[illegible]47	3.7	1 743	9.8	17 [illegible]
1968	10 724	56.9	1 395	7.4	1 440	7.6	2 3[illegible]	12.3	90[illegible]	4.8	[illegible]	4.3	1 2[illegible]	6.7	[illegible]
1969	14 689	64.0	1,498	6.5	1 274	5.5	[illegible]	10.4	849	3.7	95[illegible]	4.1	1 [illegible]	5.2	23 [illegible]
1970	14 355	61.7	1 695	7.3	1 156	5.0	2 526	10.9	963	4.1	[illegible]	4.2	1 [illegible]	6.8	23 [illegible]
1971	15,043	62.1	1 420	5.9	1 424	5.9	2 563	10.6	1 012	4.2	1 015	4.2	1 755	7.1	24 [illegible]

a In thousands of dollars

b Includes building fund reserve, airport, garage, parking, and election funds

TABLE 10 City Budget Revenue[a]

Year	Schools: Property Tax $	Property Tax %	Schools: Other[b] $	Other[b] %	General Fund: Property Tax $	Property Tax %	State Aid[c] $	State Aid[c] %	Utilities $	Utilities %	Garbage $	Garbage %	Misc[d] $	Misc[d] %	Total $
1960	1 945	19.1	2,037	20.0	2 345	23.1	814	8.0	697	6.9	55	.5	2 26[illegible]	22.3	10 160
1961	1 966	17.0	2,466	22.4	2 603	25.5	917	8.3	746	6.8	62	.6	2 048	18.6	11 010
1962	2 301	18.5	2 635	21.2	3 266	26.3	1 015	8.2	760	6.1	65	.5	2 3[illegible]	19.1	12 [illegible]
1963	2 466	17.0	3,026	21.9	3 547	25.7	1 262	9.2	1 115	8.1	[illegible]	.7	2 [illegible]	16.6	13 [illegible]
1964	3 565	23.8	3 087	20.7	3 675	24.6	1 414	9.5	1 130	7.6	91	.6	1 987	13.3	14 949
1965	4 383	27.2	3 281	20.4	3 789	23.5	1 423	8.8	1 136	7.4	91	.6	1 964	12.2	16 1[illegible]
1966	4 918	28.7	3 889	22.7	3 774	22.0	1 4[illegible]	8.7	1 242	7.2	91	.5	1,752	10.2	17 164
1967	4 626	26.2	4 848	27.4	4 305	24.4	1 6[illegible]	9.6	1 420	8.4	141	.8	572	3.2	17 66[illegible]
1968	5 635	30.5	4 470	23.7	4 739	25.1	1 656	9.0	1 393	7.4	[illegible]	.6	833	4.5	18 648
1969	6 627	29.[illegible]	5 648	24.5	4 654	21.1	1 6[illegible]	7.2	1 386	6.0	120	.5	2 5[illegible]	11.1	23 0[illegible]
1970	6 821	29.2	5 835	25.1	5 1[illegible]	22.1	1,811	7.6	1,3[illegible]	5.7	125	.5	2 1[illegible]7	9.4	23 [illegible]
1971	8 1[illegible]	33.6	6 006	25.[illegible]	5,241	21.6	1,835	7.6	1,426	5.9	200	.8	1,261	6.3	24 737

a In thousands of dollars

b Contains various state aids, federal funds, fees, etc

c Includes sales tax, intangibles tax, and weight and gas tax

d Includes departmental revenues, licenses, fees, rentals, and miscellaneous

Appendix

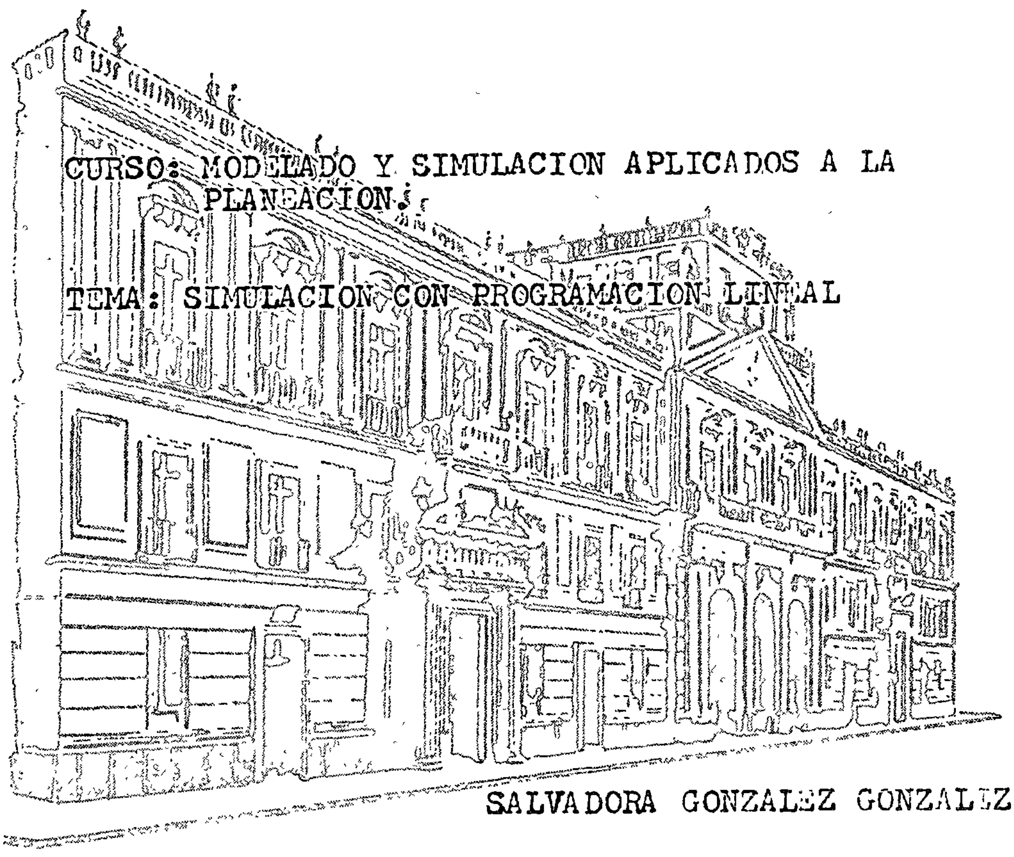
Project List

Single Year Projects

Proj. #	Map Location	Description	Budget Category	Actual Cost
Ward 1				
101	G 120	local streets & parking imp	streets	50 000
102	G 120, 140	storm sewers main & imp	sewers	100 000
103	G 135	residential alley closings	streets	50 000
104	GH 120	sanitary sewers commercial	sewers	50 000
105	GJ 125	corner entries for major routes	streets	50 000
106	GL 120	sanitary sewers interceptors	sewers	200 000
107	H 110	local streets resurfacing	streets	200 000
108	H 110, 140	thoroughfare widen & improve	streets	[illegible]
109	H 112	picnic equipment	recreation	40 000
110	H 115	parks general improvement	recreation	43 000
111	H 120	expand bridge capacity	streets	200 000
112	H 125	fire protection service truck	misc	50 [illegible]
113	H 130	park shelter & improvement	recreation	[illegible]
114	I 105	playground equipment	recreation	40 000
115	I 120, 160	[illegible] widen & improve	streets	50 [illegible]
116	I 130	improve main & relief lines	sewer	100 000
117	J 115	storm sewer comm. & indus.	sewers	50 000
118	J 122	riverbank improvements	recreation	20 000
119	J 138	street trees	recreation	40 [illegible]
120	J 143	parking improvement	recreation	40 000
121	J 150	gutter & sidewalk curb impr	streets	100 000
122	J 153	playground equipment	recreation	4[illegible]
123	J 160	sanitary sewers residential	sewers	50 [illegible]
124	K 120	service building	recreation	2[illegible]
125	K 120, 150	storm sewer interceptor	sewers	200 [illegible]
126	K 130	fire protection small truck	misc	[illegible]
127	K 150	storm sewer residential	sewers	50 000
Ward 2				
228	AE 120	sanitary sewers residential	sewers	50 000
229	AG 160	storm sewers improvement	sewers	100 000
230	C 60	cemetery improvements	recreation	40 000
231	D 105	land acquisition & dev	recreation	10[illegible]
232	D 110	road resurfacing	streets	200 000
233	E 90, 105	widen & improve thoroughfare	streets	100 000
234	E 100, 160	sanitary sewer [illegible]	sewers	200 000
235	E 110	general improvement	recreation	40 000
236	E 123	service & facilities park	recreation	50 000
237	E 133	bridge expansion	streets	200 000
238	F 85	playground development	recreation	40 000
239	F 160	storm sewers residential	sewers	50 000
240	G 100	street trees	recreation	43 [illegible]
241	G 105	playground equipment	recreation	40 000
242	G 150	parking improvement	recreation	40 000
243	H 150	new armory	misc	200 000
244	L 110	residential alley closings	streets	50 000
245	LP 140	storm sewers interceptors	sewers	200 000
246	LQ 120	corner for major routes	streets	50 000
247	M 110	playfield	recreation	20 000
248	M 120	swimming pool	recreation	150 000
249	M 140	fencing park	recreation	[illegible]
250	M 153	arboretum plantings	recreation	[illegible]
251	N 80, 160	widen & improve road	streets	50 000
252	N 100	sanitary sewers comm. & indus.	sewers	50 000
253	N 110	parking commercial areas	streets	50 000
254	O 145	park shelter improvement	recreation	40 000

centro de educación continua
división de estudios superiores
facultad de ingeniería, unam

CENTRO DE EDUCACION CONTINUA



CURSO: MODELADO Y SIMULACION APLICADOS A LA PLANEACION.

TEMA: SIMULACION CON PROGRAMACION LINEAL

SALVADORA GONZALEZ GONZALEZ

ABRIL DE 1976.

Palacio de Minería
Tacuba 5, primer piso. México 1, D. F.
Tels: 521-40-23 521-73-35 5123-123

INDICE

INTRODUCCION

Hasta ahora se han analizado los modelos de teoría de espera o "colas", modelos de inventarios, modelos estadísticos, etc. Vamos a analizar en este tema a la Programación lineal ya que es una de las técnicas mas importantes que se han desarrollado dentro de la programación matemática para la administración científica.

En el siguiente esquema se pueden apreciar los distintos problemas existentes en programación matemática (P.M.).

DIMENSION EN EL TIEMPO — ELEMENTOS DEL PROBLEMA — INTERRELACIONES FUNCIONALES — VARIABLES O ACTIVIDADES

- P.M.
  - DISCRETA (finito)
    - DETERMINISTICA (si se conocen exactamente)
      - LINEAL
        - CONTINUAS
        - ENTERAS
        - MIXTAS
      - NO LINEAL
        - GENERAL
        - CONVEXA
        - CUADRATICA
        - GEOMETRICA
        - CONTROL OPTIMO
    - ESTOCASTICA (si solo se conocen sus propiedades)
  - CONTINUA (infinito)
    - DETERMINISTICA
    - ESTOCASTICA

Nuestro estudio quedará enfocado al problema de programación matemática discreta, determinística y lineal. Algunos de los problemas que pueden ser resueltos con los métodos de cómputo de Programación Lineal son:

1.- Distribución y envío de productos desde determinados puntos de origen a varios destinos, de tal manera - de satisfacer una demanda, determinando de donde y - cuánto se debe de enviar a cada punto, de tal forma de minimizar el costo total de transportación.

2.- Estudios de distribución de múltiples plantas o centros de producción de algún producto en común, de tal forma de descentralizar la producción y minimizar el costo total de producción-distribución en el sistema.

3.- Planeación de la producción de una planta con demanda estacional para minimizar los costos totales de fabricación, basándose en la capacidad de planta instalada, pronósticos de ventas, costos de inventarios. y costos de fabricación.

4.- Asignación de recursos. Cuando no hay suficientes recursos o equipo para llevar a cabo cada actividad en la - forma más eficiente, o cuando el proceso de producción se compone de varias actividades diferentes y existen varias alternativas en la manera de llevarlas a cabo, es posible determinar la combinación óptima entre las actividades y - recursos disponibles.

5.- Mezclas de productos. Escoger de un conjunto de ingredientes de distintas propiedades y costos, las cantidades que nos proporcione un producto con ciertas características y al mínimo costo posible.

6.- Análisis de tráfico, enfocado a la sincronización de semáforos para optimizar el tránsito de vehículos

7.- Evaluación de puestos para determinar los pesos relativos de cada factor involucrado, eliminando así la utilización de la correlación múltiple, y así poder asignar a cada puesto, la persona que le rinda óptimos resultados a la empresa.

II CONCEPTO GENERAL DE PROGRAMACION MATEMATICA

Para poder aplicar algún método de programación matemática es necesario elaborar un modelo matemático que represente cada uno de los elementos del sistema, que a saber son:

MODELO MATEMATICO	SISTEMA ECONOMICO
i) Parámetros o constantes	i) Bienes o recursos (requerimiento de los mismos en forma cuantificada.
ii) Variables de decisión	ii) Actividades económicas
iii) Funciones matemáticas de restricciones	iii) Interrelaciones entre las distintas actividades y el consumo o producción de recursos

iv) Funcíon objetivo a optimizar (maximizar o minimizar.)	iv) Objetivo económico a satisfacer.

quedando el planteamiento del problema establecido como:

$$\text{OPTIMIZAR } Z = f_0(x)$$

sujeto a las restricciones $f_i(x) \geq b_i$

$f_j(x) = b_j$

$f_k(x) \leq b_k$

Las actividades las vamos a representar por variables x_1, x_2, x_3, .., x_n que van a formar un vector en el espacio euclidiano $x \in E^n$.

Las relaciones funcionales entre las diversas variables pueden ser de tres tipos:

- la función debe ser mayor

$$f_i(x) > b_i \qquad i = 1, \ldots, p$$

- la función debe ser igual

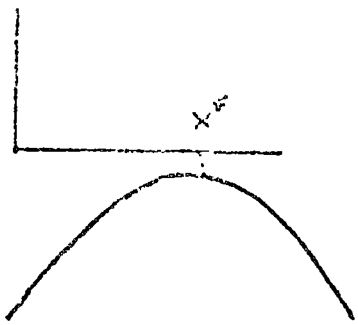
$$g_j(x) = b_j \qquad i = p-1, \ldots, \alpha$$

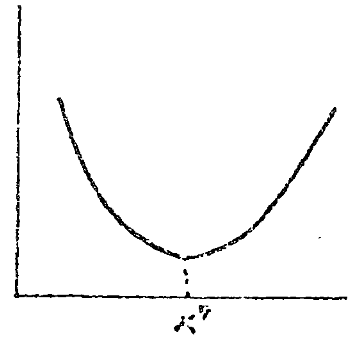
- la función debe ser menor

$$h_k(x) < b_k \qquad i = \alpha, \ldots, m$$

La función objetivo es la medida única de la valuación de actividades. Esta función debe ser escalar.

Los problemas de maximización y minimización son equivalentes como se puede apreciar en las gráficas, pero generalmente se tratan con el de minimización





$-\text{MAX} \; -f(x) = -f(x^*)$ $\qquad$ $\text{Min} \; f(x) = f(x^*)$

La Programación Lineal, como su nombre lo indica, trata de resolver problemas en que tanto las funciones que representan las interrelaciones entre las distintas actividades como las que describen las componentes de producción o consumo son lineales, es decir, mantienen estricta proporcionalidad.

En algunas ocasiones, cuando la condición real del problema no presenta esta linealidad, es posible obtener alguna aproximación al mismo para poder aplicar esta metodología. En caso contrario, se debe aplicar alguno de los algoritmos conocidos de programación discreta determinística no lineal.

III TEORIA DE LA PROGRAMACION LINEAL

Una función es lineal, si para todo par de valores x_1, x_2 y todo par de escalares α_1, α_2 se cumple lo siguiente:

$$f(\alpha_1 x_1 + \alpha_2 x_2) = \alpha_1 f(x_1) + \alpha_2 f(x_2)$$

es decir, cumple con las características de:

a) Aditividad : Esta propiedad presupone que la medida total de efectividad y la utilizacíon de recursos resultante de la operacíon conjunta de actividades debe igualar las sumas respectivas de estas cantidades resultantes de la operacíon individual de las actividades.

$$f (x_1 + x_2) = f (x_1) + f (x_2)$$

b) Proporcionalidad: En el modelo se exige que tanto la funcíon objetivo como la utilizacíón de los recursos sean proporcionales al nivel de actividad.

c) No negatividad: Resulta obvio que el nivel de las actividades debe ser siempre mayor o igual que cero.

Otras dos características que simplifican enormenente la solución del problema son:

d) Divisibilidad: debido a esta propiedad, nos es permitido obtener y aceptar niveles de actividad fraccionarios.

e) Determinismo: esta caractrística hace que las cantidades que intervienen en el problema en forma de coeficientes de la función objetivo y de restricciones al nivel de las actividades permanezcan constantes.

El planteamiento general del problema de programación lineal es:

$$\text{MAX ó MIN} \quad Z = C_1 X_1 + C_2 X_2 + \dots + C_n X_n$$

siendo Z la función objetivo a optimizar, sujeta a las siguientes restricciones:

$$
\begin{array}{l}
A_{11} X_1 + A_{12} X_2 + \cdots\cdots + A_{1n} X_n \leq B_1 \\
A_{21} X_1 + A_{22} X_2 + \cdots\cdots + A_{2n} X_n \leq B_2 \\
\vdots \\
A_{m1} X_1 + A_{m2} X_2 + \cdots\cdots + A_{mn} X_n \leq B_m
\end{array}
$$

La interpretación del modelo matemático es la siguiente: Se consideran "n" actividades que están compitiendo por la obtención de ciertos recursos escasos, en donde X_j representa la intensidad que tomará la actividad "j". El incremento que se obtendrá en el valor de la función objetivo por cada unidad de X_j que se incremente, se representa por C_j . Asi, C_j puede representar el costo de producir un artículo "j".

B_i es la cantidad del recurso "i" con que se cuenta para surtir las "n" actividades. A_{ij} es la cantidad del recurso "i" que es utilizada por cada unidad de actividad "j".

Las desigualdades de las restricciones indican que la suma de las cantidades del recurso escaso "i" utilizado en las "n" actividades deberá ser menor o igual que la cantidad del mismo de que se dispone.

Para que sea factible que el problema tenga mas de una solución es necesario que $m < n$, en donde 'm' es el número de restricciones y n es el conjunto de índices de las actividades o variables. Una condición para que las restricciones sean no redundantes es que el rango de $|A| = m$.

Las suposiciones que se hacen en un problema real para resolverlo por programación lineal son:

1.- Los costos marginales son constantes

2.- Se trata con mercados puramente competitivos (la utilidad o costo de cualquier producto es proporcional a la cantidad producida del mismo). Se excluyen los casos de monopolio y oligopolio.

3.- Existe un número finito de actividades.

4.- El problema siempre se considera a corto plazo porque las facilidades de producción y demás relaciones se consideran fijas. De lo contario habría que introducir la variable tiempo pues la estructura del problema varía con él.

III i) INTERPRETACION ALGEBRAICA

Hasta ahora sólo se ha tratado el problema de programación lineal en su forma general. Vamos a describir brevemente las formulaciones equivalentes del mismo.

FROMA ESTANDAR.

Mediante ciertas transformaciones el problema se puede expresar en función de variables no negativas que satisfagan un conjunto de ecuaciones lineales y minimicen una forma lineal. Tal formulación es la utilizada por el Método Primal Simplex o de Dantzing

$$\text{MAX} \quad Z = \sum_{i=1}^{n} C_i \; X_i$$

$$\text{s.a.} \quad \sum_{j=1}^{n} A_{ij} \; X_j = D_i \; ; \quad i = 1, \ldots, m$$

$$X_j \geq 0$$

a) Si en el problema original la restricción i-ésima es de la forma $a_{i1}x_1 + a_{i2}x_2 + \dots + a_{in}x_n \geq d_i$, se introduce una variable de holgura para transformarla en igualdad. O sea, toda inecuación de la forma

$$A\,\bar{X} \geq \bar{D} \quad ó \quad A\,\bar{X} \leq \bar{D}$$

se puede substitur por las relaciones

$$A_i\,\bar{X} - x_i = d_i \quad ó$$

$$A_i\,\bar{X} + x_i = d_i$$

b) Si la variable i-ésima X_i está no restringuida puede sustituirse por dos variables no negativas

$$X_i = X_i^+ - X_i^-$$

FORMA CANONICA

Si el problema de programación se expresa en función de variables no negativas que satisfacen un conjunto de restricciones de la forma ($\geq$) se dice que está en forma canónica.

$$\text{MIN } Z = \sum_{i=1}^{n} C_i \quad X_i$$

$$\text{s.a.} \quad \sum_{j=1}^{n} A_{ij} \quad X_j \geq D_i \quad ; \; i = 1,\dots,m$$

$$X_j \geq 0$$

FORMA MIXTA

Es aquella en que las restricciones pueden ser tanto igualdedes o desigualdades y se representa como:

$$\text{MIN } \; Z = \sum_{i=1}^{n} C_i \quad X_i$$

$$\text{s.a.} \quad \sum_{j=1}^{n} A_{ij}\,X_j = D_i \quad ; \quad i = 1, \dots, p$$

$$\sum_{j=1}^{n} A_{ij}\,X_j \geq D_i \quad ; \quad i = p-1, \dots, m$$

$$X_j \geq 0$$

III ii) INTERPRETACION GRAFICA

Para problemas sencillos, con dos actividades como máximo , se puede mostrar gráficamente como las restricciones limitan la posible solución y como la función objetivo determina la solución óptima del problema.

EJEMPLO 1

Consideremos el sistema formado por:

$$MAX \quad Z = X_1 + X_2$$

sujeto a

$$-X_1 + X_2 = 1$$

$$X_1 + 2X_2 = 11$$

$$3X_1 + X_2 = 18$$

$$X_1 , X_2 > 0$$

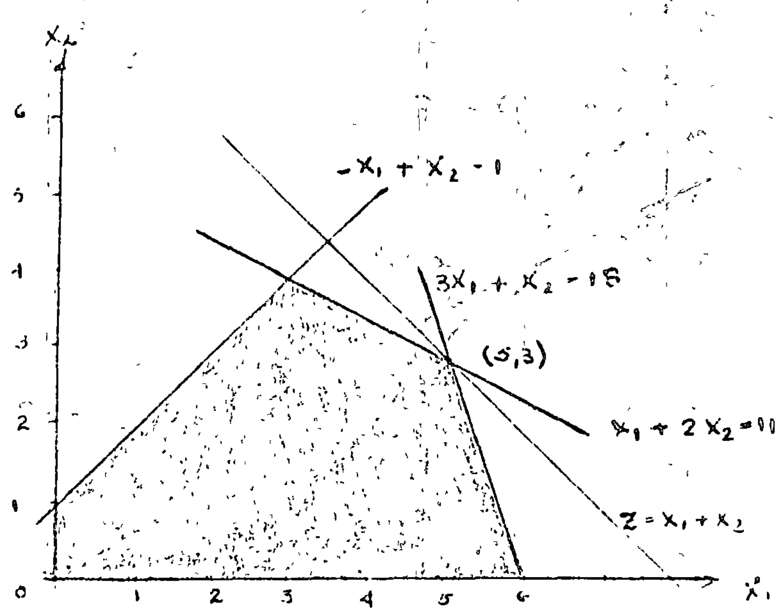
La solución de este sistema es :

$$X_1 = 5$$

$$X_2 = 3$$

$$Z = 8$$

y gráficamente se representa:



EJEMPLO 2

Una compañía produce dos artículos diferentes: X , Y , que pueden ser procesados en dos máquinas distintas: A o B . El tiempo de proceso para cada artículo en cada máquina - es:

Producto	Máquina A	Máquina B
X	2 Hrs	3 Hrs
Y	4 Hrs	2 Hrs

El período semanal de utilización es de 80 Hrs para la máquina A y de 60 Hrs para la B. Se desea saber cuál es la política de producción que maximiza las ganancias totales, si la ganancia unitaria del producto X es 60 y la del producto Y es de $ 50.

$$\text{MAX} \quad Z = 60X + 50Y$$

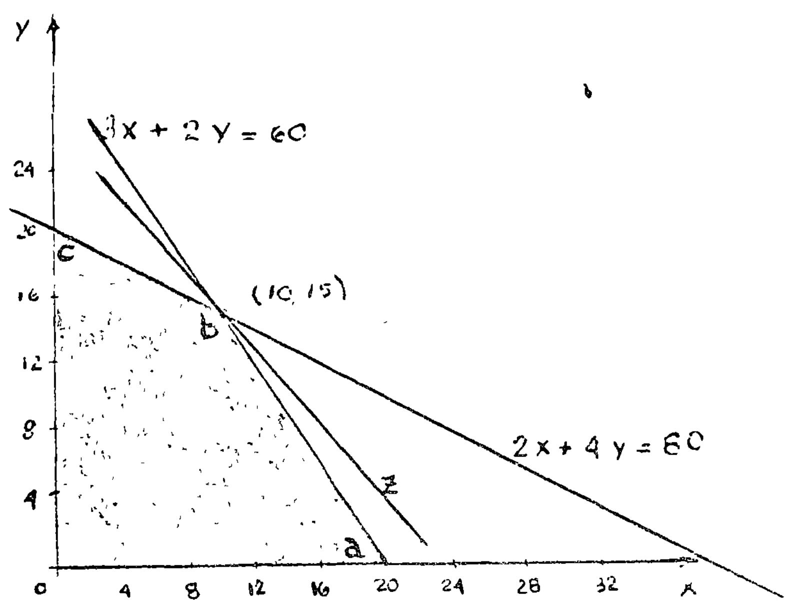
$$\text{s.a.} \quad 2X + 4Y \leq 80$$

$$3X + 2Y \leq 60$$

$$X, Y \geq 0$$

La solución es : Z = 1350 ; X = 10 ; Y = 15

que en forma gráfica queda:



Tratemos ahora de resolver el ejemplo anterior algebráicamente. Para esto haremos la transformación del problema a la forma estándar agregando las respectivas variables de holgura.

$$\text{Max } Z = 60X + 50Y \qquad (1)$$

s.a.

$$2X + 4Y + Za = 80 \qquad (2)$$

$$3X + 2Y + Zb = 60 \qquad (3)$$

Resolviendo este sistema de ecuaciones se tiene:

$$X = \frac{60 - 2Y - Za}{3} \qquad (4)$$

Despejando (2) y sustituyendo el valor de X :

$$Za = 80 - 2\left(20 - \frac{2Y}{3} - \frac{Za}{3}\right) - 4Y \qquad (5)$$

Si fijamos a Y y a Za valores de cero obtenemos

$$X = 20$$

$$Za = 40$$

$$Z = 60\,(20) = 1200$$

Estos valores corresponden al punto 'a' de la gráfica. Esto desde luego, no corresponde al máximo valor de Z.

Ahora, despejemos a Y de la ecuación (5) y reacomodando da:

$$Y = 15 - \frac{1}{4}Zb - \frac{3}{8}Za$$

que al sustituir en (4) nos da:

$$X = 10 + \frac{1}{4}Za - \frac{1}{2}Zb$$

Dando valores de cero a las variables de holgura Za y Zb

$$X = 10 \qquad Y = 15$$

$$Z = 60\,(10) + 50\,(15) = 1350$$

valor que corresponde al punto 'b' de la figura.

EJEMPLO 3

Considérese el caso de un carpintero que produce mesas y sillas y desea saber que cantidad de ellas deberá producir con el fin de mazimizar sus utilidades. Cada mesa consume 5 m^2 de madera de caoba y 4 metros de tablón de pino de 5 x 5 cm; cada silla requiere 2 m^2 de caoba y 2 metros de pino. El tiempo requerido para fabricar una mesa es de 9 horas-hombre mientras que fabricar una silla demanda 18 horas-hombre. El carpintero dispone de 60 m^2 de caoba y de 72 metros de tablón de pino, así como de 180 horas-hombre para surtir un pedido. Para simplificar supondremos que un pedido abarca todas las sillas y mesas que pueden producirse. La utilidad que obtiene por la venta de cada mesa es de $ 100.00, mientras que la venta de cada silla reditua $ 80.00.

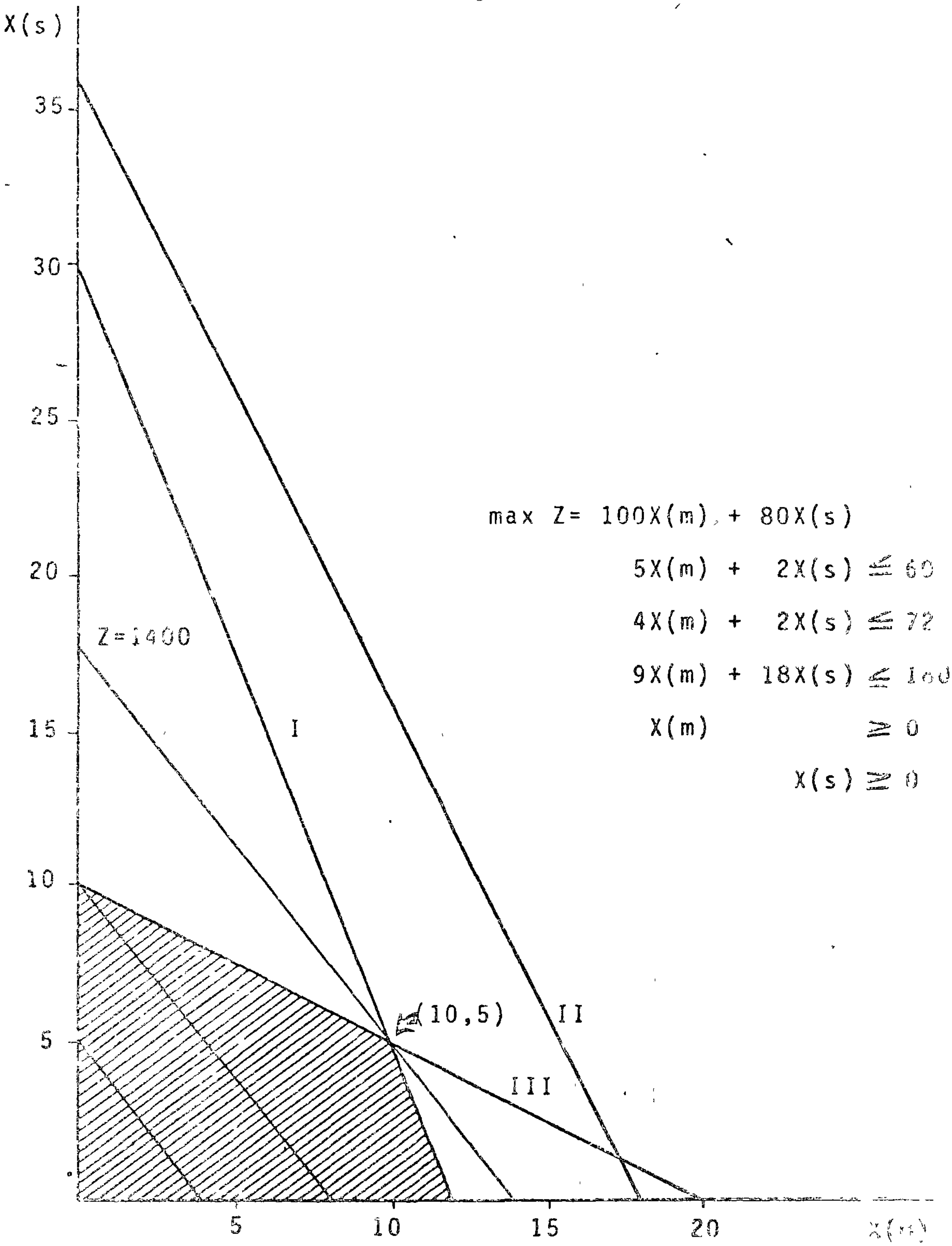
El modelo matemático para el problema anterior es esl sig:

$$\text{MAX} \quad Z = 100 \quad X(m) + 80 \quad X(s)$$

$$\begin{aligned} \text{sujeta a:} \quad & 5 \; X(m) + 2 \; X(s) \leq 60 \\ & 4 \; X(m) + 2 \; X(s) \leq 72 \\ & 9 \; X(m) + 18 \; X(s) \leq 180 \\ & X(m) , \; X(s) \geq 0 \end{aligned}$$

en donde Z representa la utilidad total, X(m) el número de mesas a producir y X(s) el número de sillas. La primera restricción se refiere a la cantidad de caoba disponible, la segunda a la de pino, la tercera a la de horas-hombre y la última fija la no negatividad de las cantidades a producir.

A continuación se presenta la solución gráfica del problema:



Analizando los ejemplos anteriores, observamos que hay una región obscura en la gráfica de la solución. A esta región se le denomina región factible, pues dentro de ella se encuentran los posibles puntos de solución.

IV MÉTODO SIMPLEX

En realidad los problemas a que se enfrontan las empresas constan de mas de dos actividades en competencia por otros tantos recursos. Para la solución de estos problemas es que se han desarrollado ciertos algoritmos entre los que se puede mencionar el Método Simplex. La forma de proceder de ésto método es básicamente:

a) Localizar un vértice como punto de partida (generalmente el origen.)

b) Examinar las aristas del vértice para determinar si, al moverse por una de ellas hasta el siguiente vértice adyacente, se aumenta el valor de Z. Si aumenta se utiliza el paso (c), en caso contario, el vértice en el cual estamos situados hace máximo el valor de Z.

c) Se escoge una de las aristas a lo largo de la cual aumenta el valor de Z, y se sigue sobre ella hasta alcanzar el siguiente vértice adyacente.

d) Se repiten los pasos (b) y (c) hasta que el valor de Z no pueda ya aumentarse.

Con este método se trata de sistematizar y simplificar la forma de solucionar un problema. Comparando lo anterior con la solución algebráica del ejemplo 2 (pag 13) , vemos que el procedimiento es el mismo que ya conocíamos.

Las propiedades de los problemas de programación lineal que permiten aplicar el método Simplex son.

a) Si existe una solución óptima del problema, esta debe ser una solución básica factible.; se les llama soluciones básicas factibles a los vértices del polígono qeu delimita la región factible. Si existen varias soluciones óptimas, entonces por lo menos dos de ellas deben ser soluciones básicas factibles.

b) Existe un número finito de soluciones básicas factibles. Recordemos que cada solución básica factible es la solución simultánea de un sistema de "n" de m+n ecuaciones (restricciones del problema). El número de diferentes combinaciones de m+n ecuaciones tomadas de 'n' a la vez es:

$$\frac{(m+n)!}{m!\, n!}$$

el cual es un número finito. A su vez, este número es el límite superior del número de posibles soluciones básicas factibles. Así en el ejemplo 3 (pag 14), m = 3 y n = 2, por lo tanto hay 10 diferentes sistemas de dos ecuaciones, pero solo 5 soluciones básicas factibles. Nótese que el sistema de ecuaciones consta de m+n renglones debido a que se deben introducir siempre las restricciones que hagan positivas o nulas las distintas actividades o variables.

c) Si la solución básica factible es mejor que (o al menos tan buena como), todas las soluciones básicas factibles adyacentes, entonces es mejor que todas las demás soluciones básicas factibles. Esta propiedad nos proporciona una prueba conveniente de si una solución básica factible es óptima sin tener que enumerar todas las soluciones posibles.

La solución obtenida al resolver el sistema de ecuaciones para m variables en términos de las restantes n-m variables, asignándoles un valor de cero a esta últimas se conoce como una solución básica; si el valor de cada una de las m variables es mayor o igual que cero, tendremos una solución básica factible; si el valor de cada una de las m variables es estrictamente mayor que cero, tendremos una solución factible básica no degenerada. Las m variables escogidas se denominan básicas o variables en la base; las n-m restantes se conocen como variables no básicas.

El método Simplex parte siempre de una forma Estándar de un conjunto de igualdades. Estas se pueden obtener de las desigualdades introduciendo tantas variables de holgura como desigualdades tengamos en las restricciones.

La solución básica factible en la etapa 't' está relacionada con la de la etapa 't+1' de manera que:una de las variables no básicas en la etapa 't' toma un valor mayor o igual a cero en la etapa 't+1' y se denomina variable de entrada; para compensar, una de las variables básicas en la etapa 't' se hace cero en 't+1' y se denomina variable de salida. Las otras variables no básicas de valor cero se mantienen en cero. Las otras variables básicas no nulas, en general siguen siendo diferentes de cero , aunque su valor puede variar.
Lo anterior es equivalente a pasar de un vértice adyacente del conjunto de soluciones factibles a otro que incremente el valor de Z.

El criterio para determinar que variable deberá entrar a la base, es escoger aquella variable no básica que cumpla con:

$$|z_k - c_k| = \text{Max} \quad |z_j - c_j|.$$

La selección de la variable de salida se determina, en general, por la selección de la variable de entrada junto con las restricciones. Se escogerá como X(L) a aquella variable básica cuyo valór se haga negativo primero cuando el valor de la variable de entrada X(k) se incremente.

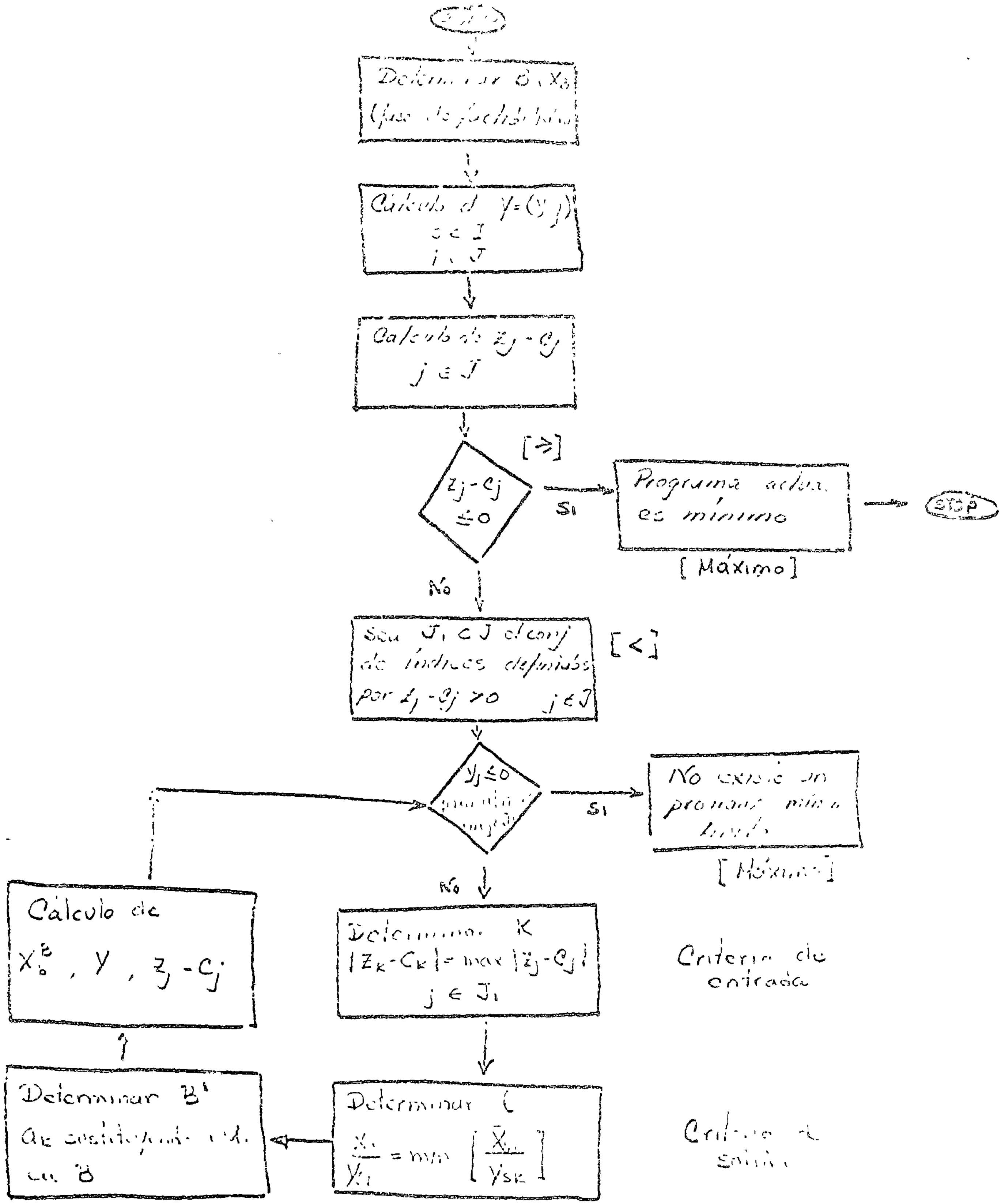
$$\frac{X_1}{Y_{1k}} \quad \text{Min} \quad \frac{X_s}{Y_{sk}}$$

A continuación es necesario determinar la nueva solución básica factible. Para lograrlo es necesario resolver las variables básicas en términos de las variables no básicas, es decir, formar un sistema canónico para la nueva base y utilizar el método de Gauss-Jordan haciendo cero todas las variables no básicas.

Para probar la optimalidad de la solución obtenida, se revisa la función objetivo (estando ésta en función exclusivamente de las variables no básicas) y se analiza si el valor de Z puede ser incrementado aumentando el valor de alguna de las variables no básicas. O sea, si todos los coeficientes son positivos, la solución es óptima, en caso contarios hay que volver a iterar encontrando la nuevas variables de entrada y salida.

A continuación se presenta el diagrama de flujo del Método Simplex y un listado del método así como algunos problemas resueltos.

Algoritmo Primal Simplex



```
COMPILE METHOD/SIMPLE FORTRAN
DATA
FILE  5=MATE,UNIT=READER
FILE  6=LINE,UNIT=PRINT
C      METODO SIMPLEX PARA RESOLVER PROBLEMAS DE PROGRAMACION LINEAL
C
C      LIMITES EN DIMENSION DEL PROGRAMA
C           MAX.NUMERO DE VARIABLES, INCLUYENDO VARIABLES DE HOLGURA= 84
C           MAX.NUMERO DE ECUACIONES INCLUYENDO FUNCION OBJETIVO=34
C
C      SE DEBE FORMAR UNA MATRIZ DE ORDEN (M+1,N) DONDE EL PRIMER RENGLON
C      CORRESPONDE A LOS COEFICIENTES DE LA FUNCION OBJETIVO Y EL ULTIMO
C      RENGLON SE COMPONE DE ELEMENTOS NULOS. DICHA MATRIZ SE REPRESENTA
C      CON LA VARIABLE A
C
C      EL PROGRAMA MINIMIZA LA FUNCION OBJETIVO, LO CUAL DEBE TOMARSE EN
C      CUENTA PARA PROBLEMAS DE MAXIMIZACION
C
C      SE DEBE PROPORCIONAR LA COLUMNA DE LAS PRIMERAS VARIABLES QUE DAN
C      UNA SOLUCIN BASICA
C
       DIMENSION A(35,85),W(35),L(35)
C
C      LECTURA DE
C           II=NUMERO TOTAL DE LAS ECUACIONES DADAS, INCLUYENDO LA FUNCION
C              OBJETIVO
C           JJ=NUMERO TOTAL DE COLUMNAS DE LA MATRIZ A.
C
       DATA IREAD,IWRITE/5,6/
       WRITE(IWRITE,2)
     1 FORMAT(18I4)
     2 FORMAT(1H1)
   108 READ(IREAD,1) II,JJ
       IF(II.EQ.0) CALL EXIT
       WRITE(IWRITE,2)
       III=II + 1
       DO 10 I=1,III
       W(I)=0.
    10 L(I)=0
C
C      LECTURA DE LOS ELEMENTOS DE LA MATRIZ RENGLON POR R
C
       DO 533 I=1,III
   533 READ(IREAD,4) (A(I,J),J=1,JJ)
     4 FORMAT(7F10.4)
C
C      LECTURA DE LA COLUMNA DE LAS VARIABLES QUE FORMARAN LA   RA SOL
C      BASICA, EN GENERAL SE TRATARA DE LAS VARIABLES DE     ARTIFI
C      CIALES
C
       READ(IREAD,1) (L(I),I=2,II)
C
C      INICIO
C
       KKK=0
C
C      BUSQUEDA DE RENGLONES EN LOS QUE NO HAY VARIABLES DE
C
    22 I=1
    23 I=I+1
       IF(I.GE.III) GO TO 40
```

```
      IF(L(I).EQ.0) GO TO 25
C
C     EVALUA
C          NVO.ULTIMO RENGLON= ULTIMO RENGLON - RENGLON SIN VARIABLE DE
C                                                              HOLGURA.
C
   25 DO 27 J=1,JJ
      IF(A(I,J).EQ.0.0) GO TO 27
   26 A(III,J)=A(III,J) -A(I,J)
   27 CONTINUE
      GO TO 23
C
C     BUSQUEDA DE LA COLUMNA EN LA CUAL EXISTE EL COEFICIENTE MAS NEGATI
C     VO EN LA F.OBJETIVO(PRIMER RENGLON) O EN EL ULTIMO RENGLON (FORMA
C     P)
C
   40 K=III
   44 J=0
      W(K)=0.
      L(K)=0
   42 J=J+1
      IF(J.GE.JJ) GO TO 45
      IF((A(K,J).GE.0.).OR.(W(K).LE.A(K,J))) GO TO 42
   47 W(K)=A(K,J)
      L(K)=J
      GO TO 42
C
C     SE PRUEBA SI L(K)=0, EN DICHO CASO TODOS LOS COEFICIENTES SON MAYO
C     RES O IGUALES A CERO. SE MANDA A DIRECCION 62 PARA UN EXAMEN MAS
C     EXHAUSTIVO
C
   45 IF(L(K).EQ.0) GO TO 62
C
C     BUSQUEDA DE COLUMNA PIVOTE
C
   46 KJ=L(K)
C
C     SE PRUEBA CADA ENTRADA DE LA COLUMNA PIVOTE PARA VER SI ES POSITI-
C     VA, EN CASO DE SERLO IR A DIRECCION 121 PARA EVALUAR EL COCIENTE
C     QUE SIRVE PARA DETERMINAR QUE VARIABLE SALE DE LA BASE
C
      DO 120 I=2,II
      IF(A(I,KJ).GT.0.) GO TO 121
  120 CONTINUE
C
C     SI LAS ENTRADAS SON NULAS O NEGATIVAS EL PROBLEMA NO ES ACOTADO
C
      WRITE(IWRITE,130)
  130 FORMAT(5X,'NO ACOTADO')
      GO TO 70
C
C     DETERMINACION DEL COCIENTE QUE INDICA EL NUEVO ELEMENTO PIVOTE
C
  121 I=1
      JK=0
   50 I=I+1
      IF(I.GT.II) GO TO 56
      IF(A(I,KJ).LE.0.) GO TO 50
   51 X=A(I,JJ)/A(I,KJ)
      IF(JK.EQ.0) GO TO 53
      IF(X.GE.XMIN) GO TO 50
```

```
   53 XMIN=X
      JK=I
      GO TO 50
C
C     LA SIG. PREPOSICION INDICA EL ELEMENTO PIVOTE
C
   56 X=A(JK,KJ)
      L(JK)=KJ
C
C     TRANSFORMACION DE LA MATRIZ A , PIVOTEANDO SOBRE EL ELEMENTO X
C
      DO 57 I=1,III
   57 W(I)=A(I,KJ)
      IJ=JK-1
      DO 59 I=1,IJ
      DO 59 J=1,JJ
      IF((A(JK,J).EQ.0.).OR.(W(I).EQ.0.)) GO TO 59
  580 A(I,J)=A(I,J) - W(I)*(A(JK,J)/X)
   59 CONTINUE
      IJ=JK+1
      DO 61 I=IJ,III
      DO 61 J=1,JJ
      IF((A(JK,J).EQ.0.).OR.(W(I).EQ.0.)) GO TO 61
  600 A(I,J)=A(I,J) - W(I)*(A(JK,J)/X)
   61 CONTINUE
      DO 205 J=1,JJ
  205 A(JK,J)=A(JK,J)/X
      KKK=KKK +1
      WRITE(IWRITE,105) KKK,A(K,JJ),L(JK)
  105 FORMAT(1X,I4,6X,F15.2,10X,I4)
      GO TO 44
C
C     SE OBSERVA SI TODOS LOS TERMINOS DEL PRIMER RENGLON SON MAYORES O
C     IGUALES A CERO, EN DICHO CASO LAS VARIABLES QUE ESTAN EN LA BASE
C     SON LA SOLUCION OPTIMA
C
   62 IF(K.LE.1) GO TO 70
   63 IJ=JJ-1
C
C     SE OBSERVA SI LOS ELEMENTOS DEL ULTIMO RENGLON ESTAN CERCANOS A
C     CERO; SI UNO O MAS DE ELLOS SON MAYORES DE 0.0001, EL PROBLEMA
C     NO TIENE SOLUCION
C
      DO 65 J=1,IJ
      IF(A(K,J).GT.0.0001) GO TO 66
   65 CONTINUE
      WRITE(IWRITE,103)
  103 FORMAT(5X,'TIENE SOLUCION')
      WRITE(IWRITE,101)
  101 FORMAT(1X,'ITERACION       FUNCION OBJ.   NUEVA VAR. BASICA.')
C
C     SI LOS ELEMENTOS DEL ULTIMO RENGLON SON MENORES QUE 0.0001 , SE
C     LES IGUALA A CERO
C
      DO 140 J=1,JJ
  140 A(III,J)=0.
C
C     EN CASO DE PROBLEMAS SIN VARIABLES ARTIFICIALES, SE DEFINE K=1
C
      K=1
      KKK=0
```

```
      GO TO 44
C
C     IMPR-SION DE RESULTADOS
C
   66 WRITE(IWRITE,6)
    6 FORMAT(5X,'NO TIENE SOLUCION')
   70 WRITE(IWRITE,8) A(1,JJ)
    8 FORMAT(///,5X,'FUNCION OBJETIVO',F20.8,/)
      WRITE(IWRITE,7)
    7 FORMAT(1X,'VARIABLE              VALOR')
      DO 71 I=2,II
   71 WRITE(IWRITE,5) L(I),A(I,JJ)
    5 FORMAT(1X,I4,F20.8)
C
C     IMPRESION DE LA MATRIZ FINAL
C
      WRITE(IWRITE,100)
  100 FORMAT(////,1X,'MATRIZ FINAL')
      DO 78 I=1,III
      WRITE(IWRITE,150) I
  150 FORMAT(//,35X,'RENGLON ',I2,/)
   78 WRITE(IWRITE,4) (A(I,J),J=1,JJ)
      GO TO 108
      END
    3   8
  2.0       -3.0

 2.0       -1.0      -1.0      -1.0      1.0
 3.0
 1.0       -1.0      1.0                                 -1.0     1.0
 2.0
 0.0
 0.0
    5   7
    4   6
-10.0      -11.0
   3.0       4.0     1.0                          9.0
   5.0       2.0               1.0                8.0
   1.0      -2.0                         1.0      1.0
 0.0
    3   4   5
    4   8
 2.0       -3.0      6.0

 3.0       -4.0      -6.0      1.0
 2.0
 2.0        1.0      2.0                 -1.0     1.0
 11.0
 1.0        3.0      -2.0                                  1.0
 5.0
 0.0
 0.0
    4   6   7
    4   7
-10.0      -11.0
   3.0       4.0     1.0                                    9.0
   5.0       2.0                -1.0            1.0         8.0
   1.0      -2.0                         1.0                1.0
   0.0
```

EJEMPLO 4

Considere el problema del ejemplo 3 (pag 14) y resolvámoslo utilizando el Método Simplex

1)

$$
\begin{aligned}
Z - 100X(m) - 80X(s) \qquad\qquad\qquad\qquad &= 0 \\
5X(m) + 2X(s) + X(1) \qquad\qquad &= 60 \\
4X(m) + 2X(s) \qquad + X(2) \qquad &= 72 \\
9X(m) + 18X(s) \qquad\qquad + X(3) &= 180
\end{aligned}
$$

variables básicas:

$$
Z = 0,\quad X(1) = 60,\quad X(2) = 72,\quad X(3) = 180
$$

variables no básicas:

$$
X(m) = 0,\quad X(s) = 0
$$

2) $C'(e) = \min C'(j) = -100 = C(m)$

por lo tanto, seleccionamos a X(m) como variable de entrada.

3)

$$
\begin{aligned}
X(1) &= 60 - 5X(m) - 2X(s) \\
X(2) &= 72 - 4X(m) - 2X(s) \\
X(3) &= 180 - 9X(m) - 18X(s)
\end{aligned}
$$

vemos que, al incrementar el valor de X(m), la primera variable básica en hacerse negativa es X(1); por lo tanto, seleccionamos a ésta como variable de salida.

4)

$$
\begin{aligned}
Z \qquad - 40X(s) + 20X(1) \qquad\qquad &= 1200 \\
X(m) + 0.4X(s) + 0.2X(1) \qquad\qquad &= 12 \\
0.4X(s) - 0.8X(1) + X(2) \qquad &= 24 \\
14.4X(s) - 1.8X(1) \qquad + X(3) &= 72
\end{aligned}
$$

5) la nueva solución es ahora:

variables básicas: Z = 1200

X(m) = 12

X(2) = 24

X(3) = 72

variables no básicas: X(1) = 0

X(s) = 0

como el coeficiente de X(s) es aún negativo, la solución aún no es óptima. Volvemos a aplicar el método.

2) C'(e) = c(s), X(e) = X(s)

3) X(m) = 12 - 0.4X(s) - 0.2X(1)

X(2) = 24 - 0.4X(s) - 0.8X(1)

X(3) = 72 -14.4X(s) - 1.8X(1)

la nueva variable de salida es X(3)

4)

Z		+ 25X(1)		+ 2.776X(3)	=	1400
	X(m)	+ 0.15X(1)		- 0.023X(3)	=	10
		- 0.85X(1)	+ X(2)	- 0.023X(3)	=	22
	X(s)	- 0.125X(1)		+ 0.069X(3)	=	5

5) variables básicas: Z = 1400

X(m) = 10

X(s) = 5

X(2) = 22

variables no básicas: X(1) = 0

X(3) = 0

Analizando la función objetivo, vemos que la solución es óptima, ya que los coeficientes de las variables no básicas en que está expresada son positivos (con las variables en el primer miembro).

Por lo tanto, si el carpintero desea hacer máximo su utilidad, deberá fabricar 10 mesas y 5 sillas.

EJEMPLO 5

Considerando el problema del ejemplo 2 (pag 12) cuya formulación matemática es:

$$MAX \; Z = 60 X + 50 Y$$

$$\text{sujeta a:} \quad 2X + 4Y + Za = 80$$

$$3X + 2Y + Zb = 60$$

$$X, Y \geq 0$$

y tabulando los resultados, la tabla inicial del Simplex quedará formada por:

		60	50	0	0
vb	b	X	Y	Za	Zb
Za	80	2	4	1	0
Zb	60	3*	2	0	1

Las variables de la base son X_B (Za, Zb)

Los costos asociados a la base C_B (0 , 0)

La variable que entra a la base será:

$$Z_k - C_k \quad \text{Max} \quad Z_j - C_j$$

o sea, Max de: (80 x 0   60 x 0) - 0   0

(2 x 0   3 x 0) -60   = 60

(4 x 0   2 x 0) -50   = 50

(1 x 0   0 x 0) - 0   0

(0 x 0   1 x 0) - 0   0

por lo que la variable que entra a la base es X

La variable que sale de la base será:

$$\frac{X_1}{Y_{1k}} \quad \text{Min} \quad \frac{X_s}{Y_{sk}}$$

o sea, $\frac{80}{2} = 40$ $\frac{60}{3} = 20$ ← mínimo

por lo que la variable que sale es Zb

a) SE PROCEDE AHORA A SUSTITUIR EN θ, A_i POR A_k, ESTA COL., EL RENGLON DE LA VARIABLE QUE SALE, SE DIVIDE ENTRE EL VALOR DEL PIVOTE Y EL RESTO DE LA MATRIZ SE CALCULA

$$\text{NUEVO NUMERO} = \text{NUMERO ANTERIOR} - \frac{(\text{NUMERO DEL RENGLON}) \times (\text{NUMERO COLUMNA})}{\text{PIVOTE}}$$

PARA NUESTRO PROBLEMA EL PRIMER RENGLÓN QUEDARA:

$$\text{NUEVO NUMERO} = 80 - \frac{60 \times 2}{3} = 40$$

$$= 2 - \frac{3 \times 2}{3} = 0$$

$$= 4 - \frac{2 \times 2}{3} = 2\tfrac{2}{3}$$

$$= 1 - \frac{0 \times 2}{3} = 1$$

$$= 0 - \frac{1 \times 2}{3} = -\tfrac{2}{3}$$

Y EL TERCER RENGLON:

$$\text{NUEVO NUMERO} = -60 - \frac{3 \times (-60)}{3} = 0$$

$$= -50 - \frac{2 \times (-60)}{3} = -10$$

$$= 0 - \frac{0 \times (-60)}{3} = 0$$

$$= 0 - \frac{1 \times (-60)}{3} = 20$$

COMPLETANDO CON ESTO LA PRIMERA ITERACION

II MATRIZ.

θ	Zb	Z	X	Y	Z_3	Z_4
0	Z_3	40	0	2 2/3 *	1	-2/3
60	X	20	1	2/3	0	1/3
		1200	0	-10	0	20

ESTE RESULTADO [illegible]
ALGEBRAICO, ES DECIR, NOS ENCONTRAMOS EN EL PUNTO 'A'

$$x = 20 \qquad y = 0$$

$$z_a = 40 \qquad z_b = 0$$

ANALIZANDO LOS $z_j - c_j$ PARA VER SI YA SE HA LLEGADO A LA SOLUCIÓN OPTIMA, NOS ENCONTRAMOS QUE TODAVIA HAY ALGUNO ≤ 0, POR LO QUE PROCEDEMOS A ITERAR DE NUEVO.

LA VARIABLE A ENTRAR A LA BASE SERÁ 'y', EN TANTO QUE LA QUE SALE ES 'za'.

SIGUIENDO EL PROCEDIMIENTO ANTERIOR OBTENEMOS

MATRIZ III

θ	vb	b	x	y	za	zb
50	y	15	0	1	3/5	-1/4
60	x	10	1	0	-1/4	1/2
		1350	0	0	3¾	17½

AHORA TENEMOS QUE TODAS LAS $z_j - c_j \leq 0$, POR LO QUE SE HA LLEGADO A LA SOLUCIÓN OPTIMA, MISMA QUE COINCIDE CON EL RESULTADO ANTERIOR

$$Z = 1350$$

$$x = 10 \qquad z_a = z_b = [illegible]$$

$$y = 15$$

Otro problema que nos ayudará a entender el desarrollo [illegible] Simplex, así como otra utilización de la Programación Lineal, es el que se presenta a continuación:

EJEMPLO 6

El departamento de compras de un Compañía se encuentra realizando un estudio para determinar la conveniencia de comprar las máquinas A, B y C en diversas cantidades, sabiendo que la utiliza ción de cada unidad habrá de reportar una utilidad de 2,3 y 1 millones de pesos respectivamente. El primer tipo de máquina requiere de un operario en forma permanente para su manejo y las máquinas B y C de dos y tres operarios cada unidad. La máquina A requiere de 2 m^2 y 2000 unidades de energía. La B, 3 m^3 y la misma cantidad de energía. La C igual área que A pero la mitad de energía. El área total de que dispone la empresa para la colocación de maquinaria es de 30 m^2 y debido a las políticas del departamento de personal no se podrá disponer de mas de 18 empleados. Además solo se ha autorizado la utilización de 36,000 unidades de energía. Se sabe que de no hacerse la compra de maquinaria, se haría una inversión para una reforma administrativa que incrementaría las ganancias en 29 millones. Cual de las políticas recomendaría ?

Máquinas	A	B	C
Ganancia	2	3	1
Operarios	1	2	3
Area	2	3	2
Energia	2000	2000	1000

Formulación matemática del problema:

$$\text{Max} \quad Z = 2X_1 + 3X_2 + X_3$$

$$\begin{aligned} \text{sujeto a} \quad X_1 + 2X_2 + 3X_3 + X_4 \qquad\qquad &= 18 \\ 2X_1 + 3X_2 + 2X_3 \qquad + X_5 \qquad &= 30 \\ 2X_1 + 2X_2 + X_3 \qquad\qquad + X_6 &= 36 \end{aligned}$$

θ	v.b.	$\bar{b}$	x_1	x_2	x_3	x_4	x_5	x_6
9	x_4	18	1	2 *	3	1	0	0
10	x_5	30	2	3	2	0	1	0
18	x_6	36	2	2	1	0	0	1
	$z_j - c_j$	0	-2	-3 ↑	1	0	0	0

18	x_2	9	.5	1	1.5	.5	0	0
6	x_5	3	.5 *	0	-2.5	-1.5	1	0
18	x_6	18	1	0	-2	-1	0	1
	$z_j - c_j$	27	-.5 ↑	0	3.5	1.5	0	0

	x_2	6	0	1	4	2	1	0
	x_1	6	1	0	-5	-3	2	0
	x_6	12	0	0	3	2	-2	1
	$z_j - c_j$	30	0	0	1	0	1	0

Z_{max} = 30 millones con:

$x_1 = 6$

$x_2 = 6$

$x_3 = 0$

Se recomienda comprar la maquinaria pues la ganancia supera a la causada por la reforma administrativa

EJEMPLO 7

Demostrar por el metodo SIMPLEX que el siguiente problema no tiene solucion factible

$$\text{Max } Z = 2X_1 + 3X_2 + 5X_3$$

s.a.

$$3X_1 + 10X_2 + 5X_3 \leq 15$$
$$33X_1 - 10X_2 + 9X_3 \leq 33$$
$$X_1 + 2X_2 + X_3 \geq 4$$

$X_i \geq 0$ para toda i

θ	v. b.	$\bar{b}$	X_1	X_2	X_3	X_4	X_5	X_6
1.5	X_4	15	3	10*	5	1	0	0
3.7	X_5	33	33	-10	9	0	1	0
2	∘	4	1	2	1	0	0	-1
	$Z_j^{**} - C_j^{**}$	-4	-1	-2	-1	0	0	1
	$Z_j - C_j$	0	-2	-3	-5	0	0	0

5	X_2	1.5	.3	1	.5	.1	0	0
1.3	X_5	48	36*	0	14	1	1	0
2.5	∘	1	.4	0	0	-.2	0	-1
	$Z_j^{**} - C_j^{**}$	-1	-.4	0	0	.2	0	1
	$Z_j - C_j$	4.5	-1.1	0	1	.3	0	0

	X_2	1.1	0	1	.4	.01	-.010	0
	X_1	1.3	1	0	.4	.03	.03	0
	∘	1.5	0	0	-.16	-.21	-.01	1
	$Z_j^{**} - C_j^{**}$	-1.5	0	0	.16	.21	.01	-1
	$Z_j - C_j$	0						

No existe solucion factible pues en la columna de las $Z_j - C_j$ todos los valores no fueron ceros o negativos

ADEMAS DEL METODO PRIMAL DEL SIMPLEX, SE HAN [illegible] OTROS METODOS, ALGUNOS DE ELLOS MODIFICACIONES DEL PRIMAL SIMPLEX. ENTRE OTROS, PODEMOS CITAR, EL METODO DUAL, LA FORMA [illegible] GRAFICA, METODO DE LAS DOS FASES, METODO DE LA GRAN "M", SIMPLEX REVISADO, ETC

DENTRO DE ESTE CURSO, SOLO SE ANALIZARA [illegible]

IV SIMPLEX REVISADO.-

PARA UTILIZAR ESTE METODO ES NECESARIO QUE LA FUNCION LINEAL A OPTIMIZAR, SE AÑADA AL SISTEMA DE LAS M ECUACIONES DE RESTRICCIÓN, FORMANDO ASÍ UN SISTEMA AUMENTADO.

CONTINUANDO CON EL PROBLEMA ANTERIOR TENEMOS:

$$\text{MAX } Z = 60x + 50y$$

$$\text{s.a} \quad 2x + 4y + Z_a = 80$$

$$3x + 2y + Z_b = 60$$

Haciendo $X_0 = Z$ se tiene

$$X_0 - 60x + 50y = 0$$

LAS MATRICES DE CALCULO DE LA FORMA REVISADA SON TOTALMENTE DISTINTAS A LAS DEL SIMPEX ORDINARIO, PUES UTILIZAN EN SUS CASILLAS LAS MATRICES:

$$\hat{A} = \begin{bmatrix} 1 & -C \\ 0 & A \end{bmatrix} \qquad \hat{B} = \begin{bmatrix} 1 & -C_B \\ 0 & B \end{bmatrix} \qquad \hat{B}^{-1} = \begin{bmatrix} 1 & C_B B^{-1} \\ 0 & B^{-1} \end{bmatrix}$$

PARA DETERMINAR LA j-esima COLUMNA DE LA MATRIZ SE TIENE

$$\hat{y}_j = \hat{B}^{-1}\hat{a}_j = \begin{bmatrix} 1 & C_B B^{-1} \\ 0 & B^{-1} \end{bmatrix} \begin{bmatrix} -C_j \\ A_j \end{bmatrix} = \begin{bmatrix} z_j - C_j \\ y_j \end{bmatrix}$$

EJEMPLO 8

Resolvamos el problema planteado en el ejemplo 2 y resuelto por el Método Simplex en el ejemplo 5 (pag. 27)

EN EL SIMPLEX [illegible] TABLAS, LA PRIMERA DENOMINADA TABLA [illegible] LAS VARIABLES QUE SALDRAN DE LA BASE Y ASIMISMO NOS INDICARA EL FIN DE NUESTRO PROBLEMA LO QUE SE PRESENTARA CUANDO $(Z_j - C_j)_i \geq 0$. LA SEGUNDA TABLA ES EN SI LA METODOLOGIA, QUE NOS PERMITIRA LLEGAR A LA SOLUCION OPTIMA DEL PROBLEMA PLANTEADO. LAS TABLAS SE FORMULAN COMO SIGUE

C	60	50	0	0
b	X	Y	Z_A	Z_B
80	2	4	1	0
60	3	2	0	1
$Z_j - C_j$	-60 ↑	-50		

ESTA PRIMERA TABLA COMO SE VE CONTIENE TODOS LOS DATOS PROPORCIONADOS POR EL PROBLEMA

LA MATRIZ DE NUESTRAS VARIABLES BASICAS SERA:

$$\hat{B} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \text{Y SU} \quad \hat{B}^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

LA SEGUNDA TABLA CONTENDRA COMO VALORES INICIALES A $\hat{B}^{-1}$

	β_1	β_2	X_B	Y_K	θ
$C_B \hat{B}^{-1}$					
Z_A	1	0			
Z_B	0	1			

LOS VALORES PARA LAS COLUMNAS DE X_B Y Y_K SERAN

PARA X_B LOS TERMINOS INDEPENDIENTES DE LAS RESTRICCIONES EN EL ORDEN EN QUE SE COLOCARON EN LA PRIMERA COLUMNA, DE $Z_j - C_j$ DE LA PRIMER TABLA SE ESCOGE EL MAS NEGATIVO QUE NOS INDICARA LA COLUMNA PIVOTE LA QUE SE TRASLADARA A LA COLUMNA Y_K, EL MENOR VALOR POSITIVO OBTENIDO POR EL COCIENTE $\frac{X_B}{Y_i}$ NOS INDICARA EL PIVOTE PARA LA COLUMNA Y_K.

$C_B B^{-1}$	[illegible]	[illegible]	[illegible]		
Z_A	0	0	80	[illegible]	40
Z_B	0	1	60	5*	20 ←
$C_B B^{-1}$	0	20	1200	-5	
Y	1	$-2/3$	40	$8/3$*	15 ←
Z_b	0	$1/3$	20	$2/3$	30
$C_B B^{-1}$	$30/8$	$70/4$	1350		
Y	$3/8$	$-1/4$	15		
X	$-1/4$	$1/6$	10		

EL PIVOTE DIVIDIRA A TODOS LOS ELEMENTOS DE Z_B HASTA X_B. PARA COMPLETAR EL CUADRO SE TRABAJARA CON EL RENGLON PIVOTE EL CUAL ES EL QUE RESULTO DE DIVIDIR Z_B/PIVOTE, CON ESTO YA ENCONTRAMOS VALORES PARA $C_B B^{-1}$ LOS QUE NO EXISTIAN EN EL PRIMER CUADRO DE LA 2a. TABLA, EVALUAMOS LOS $z_j - c_j$ PARA LA PRIMERA TABLA Y SI TODAVIA EXISTE ALGUN VALOR NEGATIVO ESTE NOS INDICARA CUAL SERA LA VARIABLE A CAMBIAR Y DICHO VALOR NEGATIVO SE ASIGNARA A LA INTERSECCION DE $C_B B^{-1}$ CON Y_i. LOS VALORES RESTANTES DE Y_B ESTARAN DADOS POR [ORIGINAL] 2a TABLA. SE OPERA COMO YA SE HABIA ESPECIFICADO ANTERIORMENTE, EL PROBLEMA TERMINARA CUANDO $(z_j - c_j)_i \geq 0$ Y LA SOLUCION SE ENCONTRARÁ EN LA COLUMNA X_B.

SOLUCION $\begin{cases} Y = 15 \\ X = 10 \\ Z = 1350 \end{cases}$ SOLUCION QUE CONCORDARA CON EL PUNTO B DE LA GRAFICA.

A continuación se presenta el planteamiento matemático de un problema que se tratará de resolver por el método Simplex Revisado.

EJEMPLO

Obtener la solucion por el metodo SIMPLEX REVISADO

$$\text{Max } Z = 4x_1 + 6x_2 + 2x_3$$

s.a.

$$750x_1 + 1500x_2 + 2250x_3 \leq 40500 \qquad x_1 = x_{11} + x_{12} + x_{13}$$

$$1500x_1 + 1500x_2 + 750x_3 \leq 27000 \qquad x_2 = x_{21} + x_{22} + x_{23}$$

$$20x_1 + 30x_2 + 20x_3 \leq 300 \qquad x_3 = x_{31} + x_{32} + x_{33}$$

Reduciendo

$$\text{Max } Z = 2x_1 + 3x_2 + x_3$$

s.a.

$$x_1 + 2x_2 + 3x_3 \leq 54$$

$$2x_1 + 2x_2 + x_3 \leq 36$$

$$2x_1 + 3x_2 + 2x_3 \leq 30$$

C	2	3	1	0	0	0
$\bar{b}$	x_1	x_2	x_3	x_4	x_5	x_6
54	1	2	3	1	0	0
30	2	3	2	0	1	0
36	2	2 ↑	1	0	0	1
$Z_j - C_j$	-2	-3	-1			
$Z_j - C_j$	0	0	1	0	1	0
	β_1	β_2	β_3	X_B	Y_k	θ
$C_B \beta^{-1}$	0	0	0		-3	
x_4	1	0	0	54	2	27
x_5	0	1	0	30	3*	10
x_6	0	0	1	36	2	18
$C_B \beta^{-1}$	0	1	0	30		
x_4	1	-2/3	0	34		
x_2	0	1/3	0	10		
x_6	0	-2/3	1	16		

Z_{max} = 60 millones

con: $x_1 = x_3 = 0$

$x_2 = 10$

El valor de X_2 10 indica que la actividad 2 deberá tomar ese valor para maximizar Z.

Existen varios tipos de problemas de programación lineal que se presentan frecuentemente en una variedad de contextos en la práctica, los cuales requieren una gran cantidad de restricciones y variables, por lo cual una aplicación del método Simplex puede tomar demasiado tiempo por computadora. Debido a esto ha sido posible desarrollar versiones especiales del método simplex que permiten ahorros considerables de aritmética aprovechando la estructura especial de estos problemas. A continuación se presentará el planteamiento gerenal de dos de estos problemas.: el de Transporte y el problema de asignación.

EL PROBLEMA DEL TRANSPORTE

El clásico problema de transportación surge cuando se debe determinar un plan óptimo de embarque que:

a) Se origina en fuentes de suministro o almacenes donde existe disponible un determinado inventario fijo de un artículo.

b) Son mandados directamente a su destino final en donde existe la necesidad de una cantidad fija del artículo.

c) Se agotan los inventarios y satisfacen las demandas, o sea, la demanda total iguala la oferta total.

d) El costo debe satisfacer una función objetiva lineal, o sea que el costo del embarque es directamente proporcional a la cantidad enviada de cada producto.

El problema del transporte lo formuló por vez primera en 1941 Frank L. Hitchcock en el artículo " The distribution of a product from several sources to numerous localities ".

Para describir el modelo general para el problema del transporte, requerimos emplear términos mucho menos específicos que los que usamos para describir los componentes del problema de programación lineal general. Se tienen 'm' almacenes con un inventario fijo $a(i)$, donde $i = 1, 2, \ldots, m$; además, existen 'n' destinos con una demanda $b(j)$, donde $j = 1, 2, \ldots, n$. Al número de unidades que van del almacén 'i' al destino 'j' se le llama $X(i\ j)$.

De la definición del problema:

$$a_1 + a_2 + \ldots + a_m = b_1 + b_2 + \ldots + b_n$$

El problema se plantea de la siguiente forma:

$$\begin{aligned}
X(11) + X(12) + \ldots + X(1n) &= a(1) \\
X(21) + X(22) + \ldots + X(2n) &= a(2) \\
X(m1) + X(m2) + \ldots + X(mn) &= a(m) \\
\\
X(11) + X(21) + \ldots + X(m1) &= b(1) \\
X(12) + X(22) + \ldots + X(m2) &= b(2) \\
X(1n) + X(2n) + \ldots + X(mn) &= b(n)
\end{aligned}$$

y la función objetivo será:

$$\begin{aligned}
\min Z = {} & C(11)X(11)+C(12)X(12)+\ldots+C(1n)X(1n)+ \\
& C(2n)X(2n)+\ldots+C(m1)X(m1)+C(m2)X(m2)+\ldots+ \\
& C(mn)X(mn)
\end{aligned}$$

Del planteamiento del problema sobresalen dos características: los coeficientes de las restricciones son nulos o unitarios y sólo aparecen igualdades.

Cuando se tiene que la oferta total es mayor que la demanda total, se introduce un destino ficticio, el cuál representa la cantidad de unidades que se quedan en bodega; en caso contrario, se crean orígenes ficticios, y lo que sale de éstos es la demanda insatisfecha.

Otra propiedad del problema es que si el número de unidades en cada almacén y el número de unidades que demanda cada destino son enteros, entonces la solución es entera. En la figura 1 se presenta la forma tabular del problema, con el fin de facilitar el cálculo.

Un conjunto de entradas en un arreglo rectangular se dice que ocupan posiciones independientes si es imposible formar círculos cerrados recorriendo dichas entradas. Las condiciones para que una solución básica factible no degenerada exista son las siguientes:

a) Que satisfagan las resticciones.

b) Que haya exactamente m+n-1 entradas mayores que cero.

c) Que las entradas se encuentren en posiciones independientes.

La solución del problema del transporte siempre parte de una solución básica factible. Existen varios métodos para obtener una solución básica factible inicial, siendo los más comunes el de la esquina noroeste y el de Vogel.

Método de la esquina noroeste: Como candidato para la primera variable básica escójase cualquier X(ij) y hágase su valor tan grande como las restricciones renglón y columna lo permitan, es decir, iguále a X(ij) con el mínimo de a(i) y b(j).

FIGURA # 4 : FORMA TABULAR DEL PROBLEMA DE TRANSPORTE

d e s t i n o

o r i g e n	1	2	j	n	
1	X(11) C(11)	X(12) C(12)	X(1j) C(1j)	X(1n) C(1n)	a(1)
2	X(21) C(21)	X(22) C(22)	X(2j) C(2j)	X(2n) C(2n)	a(2)
i	X(i1) C(i1)	X(i2) C(i2)	X(ij) C(ij)	X(in) C(in)	a(i)
m	X(m1) C(m1)	X(m2) C(m2)	X(mj) C(mj)	X(mn) C(mn)	a(m)
	b(1)	b(2)	b(j)	b(n)	

Caso 1 : a(i) menor que b(j). En este caso, a todas las demás variables en el renglón 'i' se les asigna el valor cero y serán variables no básicas; elimínese el renglón 'i' y reduzca el valor de b(j) a b(j) -a(i) y proceda en forma similar para evaluar una variable en el arreglo rectangular reducido compuesto de los 'm-1' renglones y 'n' columnas restantes.

Caso 2 : a(i) mayor que b(j). En este caso se elimina la columna 'j' y a(i) se reemplaza por a(i) -b(j), se procede con el arreglo rectangular reducido en forma similar a la descrita en el caso 1.

Caso 3 : a(i)=b(j). Elimínese ya sea el renglón o la columna, pero no ambos; si varias columnas pero sólo un renglón quedan en el arreglo reducido, entonces elimínese la columna 'j'; en caso contrario, procédase al revés.

Continúe en esta forma paso a paso, alejándose de la esquina noroeste, hasta que finalmente obtenga un valor para la celda sureste.

METODO DE VOGEL:

a) Se calculan las diferencias en cada renglón y columna (diferencia entre el costo menor y el que le sigue en magnitud).

b) Seleccione la columna o renglón con la mayor diferencia; si hay empate, la decisión es arbitraria.

c) Se le asigna el número mayor posible de unidades a la celda con el menor costo de esa columna o renglón.

d) Se cancela la columna o renglón con demanda u oferta igualada y se ajusta la no agotada.

e) Se calculan los nuevos diferenciales y se repite el proceso hasta agotar la tabla.

Sin entrar en la justificación matemática del método, a continuación hago un bosquejo del mismo, para después resolver un ejemplo ilustrativo.

a) Se plantea el problema en forma tabular, añadiendo almacenes o destinos ficticios, según se requiera.

b) Se obtiene una solución básica factible por alguno de los métodos descritos, señalando las casillas de las variables básicas por medio de una diagonal.

c) Si llamamos $u(i)$ al múltiplo del renglón original 'i' que ha sido restado (directa o indirectamente) del renglón cero original por el método simplex, y $v(j)$ al múltiplo del renglón original $n+j$ que ha sido restado (directa o indirectamente) del renglón cero original por el método simplex, entonces:

$C(rs)-U(r)-V(s) = 0$ para toda variable básica.

$C'(ij)=C(ij)-U(i)-V(j)$ Para toda variable no básica.

Se tienen 'm' $u(i)$ y 'n' $v(j)$; sin embargo se puede demostrar que el rango del sistema de ecuaciones original es $(m+n-1)$, por lo tanto, podemos asignar un valor arbitrario a alguno de estos valores. Convencionalmente, este valor es cero y se le asigna a $u(1)$. A partir de ésta se obtienen los valores para las demás 'u' y 'v', colocándolos en la parte inferior derecha de las casillas de oferta y demanda. Una vez obtenidos éstos, se calcula C'(ij) para las variables no básicas del arreglo, apuntando los respectivos valores en la esquina superior derecha de cada casilla.

d) Se selecciona como variable de entrada a aquélla cuya C'(ij) es más negativa; se traza un "círculo" entre las variables básicas a partir de la variable de entrada.

e) Se le asigna un valor 't' a la nueva variable básica; con el fin de que se sigan cumpliendo las restricciones de oferta y demanda, se le suma o resta este valor de 't' a las variables de los vértices del "círculo".

f) Se selecciona como variable de salida a aquélla que se hace negativa primero al incrementarse el valor de 't'.

g) Se le asigna a 't' el valor de la variable de salida y se forma una nueva tabla con la nueva solución básica factible.

h) Conviene probar tres condiciones al final de cada interación: que el valor de la función objetivo (expresada sólo en función de variables básicas como siempre) ha disminuido y que las restricciones de oferta y demanda se cumplen.

i) Se sabe que se ha llegado a la solución óptima cuando todos los valores de C'(ij) son positivos.

En el caso de que se llegue a una solución degenerada, es decir, que haya dos variables básicas que se anulen al asignar su valor a '0', entonces se hace cero cualquiera de ellas y se le da un valor 'E' (supuestamente pequeñísimo) a la otra, con lo cuál se puede seguir trabajando en forma normal.

Ejemplo 10 Una compañía de motores tiene cinco clientes a los cuáles les manda motores desde tres diferentes almacenes. Los cinco clientes se identifican como J, K, L, M, N y los tres almacenes son P, Q, R. Los costos de los almacenes son idénticos, de aquí que los costos pertinentes sean los costos variables de embarque a cada cliente. La tabla de costos es la siguiente:

	J	K	L	M	N
P	20	10	15	10	8
Q	15	30	5	12	14
R	18	15	20	7	19

Los requerimientos de los clientes son los siguientes:

J	-	100	unidades
K	-	70	" "
L	-	140	" "
M	-	100	" "
N	-	90	" "

Las unidades disponibles en cada almacén son las siguientes:

P	-	200	unidades
Q	-	300	" "
R	-	150	" "

Para este problema la función objetivo es:

min Z = 20X(PJ)+ 13X(PK)+ 15X(PL)+ 12X(PM)+ 0X(PN)
+ 15X(QJ)+ 30X(QK)+ 9X(QL)+ 12X(QM)+ 14X(QN)
+ 13X(RJ)+ 15X(RK)+ 20X(RL)+ 7X(RM)+ 14X(RN)

sujeto a las siguientes restricciones:

X(PJ)+X(PK)+X(PL)+X(PM)+X(PN)	≤	200
X(QJ)+X(QK)+X(QL)+X(QM)+X(QN)	≤	300
X(RJ)+X(RK)+X(RL)+X(RM)+X(RN)	≤	150
X(PJ)+X(QJ)+X(RJ)	=	100
X(PK)+X(QK)+X(RK)	=	70
X(PL)+X(QL)+X(RL)	=	140
X(PM)+X(QM)+X(RM)	=	180
X(PN)+X(QN)+X(RN)	=	90

Nótese que, con objeto de cumplir con la restricción de igualdad entre oferta y demanda, es necesario añadir en este caso un destino ficticio. En la figura 2 presente la solución de este ejemplo, obteniendo la solución básica factible inicial con el método de Vogel.

Supóngase que no fuera posible por alguna razón el envío de motores de alguno de los almacenes a alguno de los destinos. En este caso el modelo tabular se construye dando un valor muy grande 'M' (mayor que cualquier otro costo) al costo del envío y se resuelve el problema en forma normal.

EL PROBLEMA DE ASIGNACION

El problema de asignación es el caso más sencillo de programación lineal: aquél en el cuál los recursos se asignan a las actividades uno para cada una. Así, cada recurso se asigna en forma única a una actividad. Hay un costo C(ij) asociado a cada recurso 'i' asigna-

FIGURA # : SOLUCION AL EJEMPLO DEL PROBLEMA DE TRANSPORTE

a) Planteamiento del problema.

	J	K	L	M	N	F	a
P	20	10	15	10	8	0	200
Q	15	30	5	12	14	0	300
R	18	15	20	7	19	0	[illegible]
b	100	70	140	180	90	70	650

FIGURA # : CONTINUACION

b) Solución básica factible inicial:

	J	K	L	M	N	F	a
P	10 20	70 10	15	30 10	90 8	0	200
Q	90 15	30	140 5	12	14	70 0	300
R	18	15	20	150 7	19	0	150
b	100	70	140	180	90	70	650

c) Cálculo de u(i) y v(j), y de las variables de entrada y de salida (d,e,f)

	J	K	L	M	N	F	a
P	10-t 20	70 10	5 15	30 10	90 8	t -5 0	200 0
Q	90+t 15	25 30	140 5	7 12	11 14	70-t 0	300 -5
R	1 18	8 15	13 20	150 7	14 19	-2 0	150 -3
b	100 20	70 10	140 10	180 10	90 8	70 5	650

variable de entrada: X(PF)

variable de salida : X(PJ)

t = 10

FIGURA # : CONTINUACION

h) Obtencion de la nueva solución y prueba de su optimalidad (repetición de 'C')

	J	K	L	M	N	F	a
P	5 20	70 10	10 15	30 10	90 8	10 0	200 0
Q	100 15	20 30	140 5	2 12	6 14	60 0	300 0
R	6 18	8 15	18 20	150 7	14 19	3 0	150 -3
b	100 15	70 10	140 5	180 10	90 8	70 0	650

Puesto que todas las C'(ij) son positivas, la solución obtenida es óptima. El costo mínimo de transporte se logra abasteciendo a los clientes de la siguiente forma:

X(PK) = 70

X(PM) = 30

X(PN) = 90

X(QJ) = 100

X(QL) = 140

X(RM) = 150

quedando 10 motores en el almacén P y 60 en el Q.

do a cada actividad 'j', de tal forma que el objetivo es determinar las asignaciones de manera que el costo sea mínimo.

El problema de la asignación es un caso especial del modelo de transporte en el que existe una matriz cuadrada (m=n) con cada una de las restricciones de disponibilidad y cada uno de los requerimientos iguales a la unidad, es decir, a(i) = b(j) = 1 para toda 'i' y toda 'j'. A X(ij) se le da el valor de uno si el recurso 'i' se asigna a la actividad 'j' y el valor de cero en caso contrario.

El modelo matemático del problema es:

min Z = C(11)X(11)+C(12)X(12)+ ... + C(1n)X(1n)+
C(21)X(21)+C(22)X(22)+ ... + C(2n)X(2n)+
...+C(n1)X(n1)+C(n2)X(n2)+ ... + C(nn)X(nn)

sujeto a las siguientes restricciones:

X(i1)+X(i2)+ ... +X(in) = 1 para i= 1,2,....,n
X(1j)+X(2j)+ ... +X(nj) = 1 para j= 1,2,....,n

El método de solución se basa en el hecho de que si en un problema de asignación sumamos o restamos una constante a cada elemento de un renglón (o columna) de la matriz de efectividad, entonces una asignación que hace mínima la efectividad total en una matriz también minimizará la efectividad total de la otra matriz.

Un conjunto de ceros se dice independiente si no existen dos o más ceros del conjunto considerado en la misma columna o en el mismo renglón.

La técnica de solución consiste en reducir la matriz de costos hasta encontrar un conjunto de 'n' ceros independientes, uno en cada renglón y en cada columna. Este conjunto, no necesariamente único, proporciona una solución óptima para el problema de asignación considerado. A continuación doy una descripción general del método para luego presentar un problema ilustrativo.

a) A todos los elementos del renglón 'i' se les resta el elemento más pequeño del renglón; se repite desde i= 1 hasta i=n.

b) Si aún no se ha obtenido una matriz que posee cuando menos un cero en cada renglón y cuando menos un cero en cada columna, procedemos a restar el elemento mínimo en cada columna que aún no tenga ceros.

c) 1.- Examine los renglones sucesivamente hasta que se encuentre uno de ellos que tenga exactamente un cero no marcado; márquelo con un cuadrito, ya que ahí se efectuará una asignación. Marque con una 'X' todos los otros ceros en la misma columna para indicar que no se pueden usar para hacer otras asignaciones.

2.- Examine a continuación las columnas para encontrar una que tenga exactamente un cero no marcado; márquelo con un cuadrito. Marque con 'X' todos los demás ceros no marcados en el renglón correspondiente.

3.- Repita los pasos uno y dos sucesivamente hasta que una de dos situaciones ocurra:

(A) Ya no hay ceros sin marcar.

(B) Los ceros que quedan sin marcar son cuando menos dos en cada renglón o columna.

Si lo que resulta es el caso A, tenemos una asignación máxima, que puede no ser completa.

Si resulta el caso I podemos usar el tanteo o tanteos para localizar una asignación máxima.

Sea una asignación máxima obtenida de la situación A o a partir de la situación B. Podemos decir que si se tiene una asignación en cada renglón esta asignación máxima es una solución completa del problema original y hemos terminado (caso I). Si no se tiene una asignación en cada renglón nos enfrentamos con el problema de modificar la matriz de efectividad mediante sumas o restas (caso II).

d) Este paso sólo es necesario cuando llegamos al caso II del paso 'C', ya que entonces hemos llegado a una asignación máxima (obtenida de la situación A o de la B) la cuál no constituye una asignación completa. Para agregar ceros adicionales se usan las siguientes reglas. Empezando con la asignación máxima obtenida:

1.- Se marcan todos los renglones en los que no se ha hecho una asignación.

2.- Se marcan las columnas que no han sido marcadas y que tienen ceros asignados o no en los renglones marcados.

3.- Se marcan los renglones que aún no lo están y que tienen asignaciones en las columnas marcadas.

4.- Se repiten los pasos dos y tres hasta que termina la cadena de marcas.

5.- Se trazan líneas a través de todos los renglones no marcados y a través de todas las columnas marcadas. Deberemos obtener tantas líneas como asignaciones teníamos.

4.- Se analizan los elementos que no tienen líneas que pasan por ellos, y se resta el menor de ellos de todos los elementos de la matriz que no tienen una línea que pase por ellos. Se suma además este elemento a cada uno de los localizados en la intersección de dos líneas. Se dejan los elementos sobrantes de la matriz sin cambio, olvidando las asignaciones anteriores se vuelve al paso '3'.

Si se da el caso de que un recurso no pueda ser asignado a una actividad específica, se le da a esta asignación un costo muy grande 'M' y se trabaja en forma normal.

De la misma forma, si se tiene una matriz de efectividad rectangular, es decir, si se tienen más recursos que actividades o viceversa, se agrega uno o varios recursos (o actividades) ficticios y se da el valor de cero al costo de sus respectivas asignaciones; el problema así planteado puede resolverse por el método descrito; la asignación o asignaciones ficticias nos indican cuál de los recursos (o actividades) se queda sin ser asignado.

<u>Ejemplo II</u>. En una fábrica de aparatos eléctricos se tienen tres máquinas, dos de las cuales pueden realizar tres trabajos diferentes (uno a la vez) y una cuatro. La máquina 1 puede emplearse para los trabajos A, B, C y D, la 2 para los trabajos A, B y D, y la 3 para los trabajos A, B y C. Debido a diferencias en las máquinas y en la habilidad de sus operarios, los costos de realizar estas tareas varían de acuerdo con la máquina. Estos se presentan en la siguiente tabla.

[illegible]	M(1)	M(2)	M(3)
A	10	15	12
B	8	12	15
C	15	-	13
D	12	15	-

Se requiere asignar los trabajos a las máquinas de tal forma que el costo total sea mínimo. La solución al problema se presenta en la figura 3

FIGURA # 3 : SOLUCION AL EJEMPLO DE ASIGNACION

	M(1)	M(2)	M(3)	M(F)
A	10	16	12	0
B	8	12	15	0
C	15	M	13	0
D	12	15	M	0

b)

	M(1)	M(2)	M(3)	M(F)
A	2	4	0	0
B	0	0	3	0
C	7	M	1	0
D	4	3	M	0

c)

	M(1)	M(2)	M(3)	M(F)
A	2	4	[0]	Ø
B	[0]	Ø	3	Ø
C	7	M	1	[0]
D	4	3	M	Ø

d)

	M(1)	M(2)	M(3)	M(F)
A	2	4	0	1
B	0	0	3	1
C	6	M	0	0
D	3	2	M	0

FIGURA # 3 : CONTINUACION

c)

	M(1)	M(2)	M(3)	M(F)
A	2	4	[0]	1
B	[0]	Ø	3	1
C	6	M	Ø	[0]
D	3	2	M	Ø

d)

	M(1)	M(2)	M(3)	M(F)
A	0	2	0	1
B	0	0	5	3
C	4	M	0	0
D	1	0	M	0

c)

	M(1)	M(2)	M(3)	M(F)
A	[0]	2	Ø	1
B	Ø	[0]	5	3
C	4	M	[0]	Ø
D	1	Ø	M	[0]

Esta última constituye una asignación completa. Por lo tanto, se obtendrá el costo mínimo si se asigna el trabajo a las máquinas de la siguiente forma:

máquina 1 - trabajo A

máquina 2 - trabajo B

máquina 3 - trabajo C

Bibliografía

- Ackoff and Sasieni "Fundamentals of Operations Research"
  New York, Wiley 1968

- Dantzing 63 "Linear Programming and Extensions"
  Princeton Press, 1963

- Gass, SI "Linear Programming - Methods and Applications"
  New York, McGraw Hill, 1958

- Hadley "Linear Programming", Reading Mass
  Addison Wesley, 1962

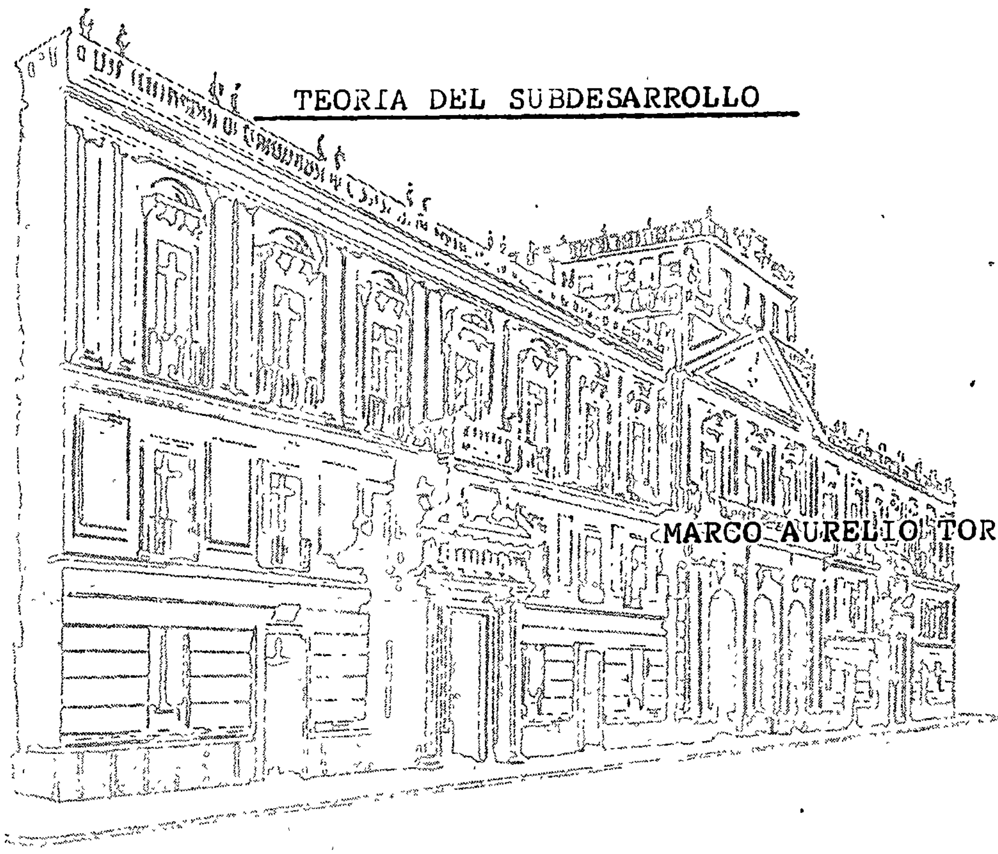
- Hillier and Lieberman "Introduction to Operations Research"
  Holden Day, 1968

- Jauffred, Moreno, Acosta "Métodos de Optimización"
  Representaciones y Servicios de Ingeniería

- Simonnard, M. "Linear Programming"
  Paris, Dunod, 1962.

centro de educación continua
división de estudios superiores
facultad de ingeniería, unam

MODELADO Y SIMULACION APLICADOS A LA PLANEACION



TEORIA DEL SUBDESARROLLO

MARCO AURELIO TORRES H.

Ciudad Universitaria, 1976.

Palacio de Minería
Tacuba 5, primer piso. México 1, D. F.
Tels: 521-40-23 521-73-35 5123-123

CAPITULO II

E L S U B D E S A R R O L L O .-

2.1.- GENERALIDADES .-

No tiene sentido estudiar los recursos de un país -ni sus necesidades tampoco- si no se conoce de antemano el marco económico en el que ese país se desarrolla.

Brasil tiene muchos más recursos naturales que Japón (el cual no tiene ninguno, salvo la pesca) pero Brasil es una nación pobre, subdesarrollada, y el Japón, en cambio, marcha a la cabeza de los países de más rápido desarrollo económico.

Así pues, el estudio de los recursos y necesidades de un país, por sí solo, no arroja ninguna luz sobre la riqueza o la penuria de una nación, ni puede dar pábulo, en consecuencia, a ninguna política económica que la guíe razonablemente en los avatares de su desarrollo.

Es un hecho evidente, que las naciones de la tierra son unas ricas y otras pobres, pero aun cuando pueda parecer extraño, no existe continuidad entre los diversos grados de penuria o abundancia de las naciones : cuando son ricas, lo son con opulencia y cuando son pobres, el hambre y la miseria agobian a una gran parte de su pueblo. No existen términos medios.

Pero falta aún señalar otro rasgo -quizás el fundamental- común a las naciones pobres : su permanencia, su persistencia en el estado de estrechez que las aflige; no sólo son pobres, sino porfiadas en su miseria. México, que nació indigente en su independencia, no ha dejado de padecer escasez global y privaciones y carencias esenciales a través de su historia; y en el último tercio del siglo XX -más de ciento cincuenta años después- se encuentra en las mismas condiciones de necesidad y de miseria, pese a su plata, a sus mares, a su amplio territorio, a sus minerales y petróleo. ¿ Cuál es la causa ? .

Al conjunto de países que ostentan la pobreza como su esencial característica, se les ha denominado con muy diversos adjetivos : atrasados, no desarrollados, dependientes o "en vías de desarrollo"; pero, con mayor o menor eufemismo, todos los calificativos entrañan : exiguo producto por cabeza, incremento demográfico explosivo, desempleo agudo, desnutrición endémica, escasez generalizada de viviendas y analfabetismo considerable de su población.

Todos los países pobres, con sus enormes necesidades comunes, fueron agrupados, por el sociólogo francés Alfred Sauvy, bajo la denominación de "tercer mundo". (*) Pero bien pronto esa designación resultó insuficiente, pues a partir del año de 1973, el precio del petróleo -por mucho tiempo casi constante- quintuplicó su valor en el lapso de 3 años, provocando cambios tan profundos en las economías de las naciones subdesarrolladas que no contaban con ese recurso, que se ha tenido que echar mano de otro ordinal para agruparlas : así quedó constituído el "cuarto mundo", sinónimo de pobreza y desesperanza.

Pero es indispensable destacar, que ESA DIVISION TAN AGUDA Y TAN TAJANTE ENTRE NACIONES POBRES Y RICAS, NO SE HABIA PRESENTADO NUNCA, ANTES DEL SIGLO XVII. El nivel de vida de los ahora países industrializados, era por aquel entonces igual -y en algunos casos inferior- al de muchos de los actualmente subdesarrollados. "Entre la Francia de Luis XIV, la Inglaterra de Guillermo III, la Prusia de Federico I y la Rusia de Pedro el Grande, por un lado, y la India de Aurangzeb y la China de Kiang-Hi, por otro (para no hablar más que de Europa y de las dos principales potencias de Asia) había profundas diferencias entre sus estructuras sociales y religiosas; también había diferencias climáticas importantes; pero, TOMADAS EN CONJUNTO, ES MUY DIFICIL DETERMINAR CUAL DE ESTOS DOS GRUPOS DE SOCIEDADES HABIA ALCANZADO, EN ESA EPOCA, UN NIVEL DE DESARROLLO ECONOMICO MAS AVANZADO; CUAL DE LOS DOS TENIA UN NIVEL DE VIDA MEJOR. (...) Tal ausencia de significativa diferencia entre los niveles de desarrollo económico de las diferentes sociedades no primitivas, era una constante histórica desde hacía algunos milenios. (6)

Así pues, la brecha que separa a las naciones ricas de las pobres y atrasadas, no existía antes del siglo XVII.

¿Qué ocurrió en esa época, de tan singular importancia que logró trastrocar el orden económico que se había conservado durante miles de

---

(*).- El "primer mundo" abarcaría entonces a las naciones ricas capitalistas de economía de mercado y el "segundo mundo" comprendería, a su vez, a las naciones socialistas o de economía centralizada.

(6).- Bairoch, Paul.- "El Tercer Mundo en la Encrucijada".- Alianza Editorial, Madrid, 1973. p. 8 .

años? ¿Qué fenómeno fue capaz de producir tan hondas modificaciones en la economía de las naciones, que no lograron en su turno histórico ni el "modo de producción" (*) esclavista del mundo antiguo ni el feudal de la edad media?

A menudo los cambios históricos se gestan durante siglos, pero las convulsiones sociales que provocan pueden hacer erupción violenta y dejar constancia de su paso en fecha fija, en general trágica, a manera de jalones que indican el rumbo de la historia; tales fueron las revoluciones francesa y rusa. Pero en otras ocasiones los cambios se realizan con menos aspavientos, sin alardes de guerras y batallas, sin repique de campanas, casi imperceptibles, pero afectan profundamente las costumbres y la vida de las naciones; así llegó al mundo el modo de producción capitalista.

El siglo XVIII fue testigo de una de las revoluciones más trascendentales que ha vivido la humanidad; fue una revolucion lenta, larga, y en cierta medida pacífica, pero sus consecuencias fueron más importantes que si se hubieran librado mil batallas. Esa revolución tuvo su asiento en Inglaterra y en honor de su rasgo más característico, que hizo su aparición por primera vez en el mundo, se la llamó " LA REVOLUCION INDUSTRIAL " .

La Revolución Industrial comprende aproximadamente el lapso entre 1760 y 1830, en el que aparecieron "una serie de inventos técnicos que iban a modificar las condiciones de producción en varias ramas de la industria". (7) En 1763 silbó la primera máquina de vapor (nervio de la revolución industrial) y en 1776 apareció la primera edición de la "Riqueza de las Naciones" de Adam Smith. El capitalismo y la economía fueron coetáneos.

(*).- Entiendo por "MODO DE PRODUCCION", la acción recíproca característica entre "las fuerzas productivas" y las "relaciones de producción" en un proceso productivo determinado.
Defino "FUERZAS PRODUCTIVAS" como el conjunto de los "medios de producción" (edificios, herramientas, máquinas) y la "fuerza de trabajo" (conjunto de energías físicas y espirituales del hombre, que le permiten producir los bienes materiales).
Llamo "RELACIONES DE PRODUCCION", a las relaciones sociales determinadas que los hombres contraen en el proceso de producción de los bienes materiales. Comprende: las formas de propiedad sobre los medios de producción y las formas de distribución del producto, como consecuencia de la propiedad de los medios de producción (Véase: "Manual de Economía Política". Academia de Ciencias de URSS.- 2a. Ed. Grijalbo, México. pp. 2 y sig).

(7).- Bairoch, P.- "Revolución Industrial y Subdesarrollo". Ed. Siglo-XXI, México, 1967. p. 12 .

Probablemente ningún otro acontecimiento histórico (salvo quizás las revoluciones francesa y rusa) tuvo mayor trascendencia para la humanidad, que la revolución industrial del siglo XVIII. Gracias a ella, terminan las verdaderas hecatombes que periódicamente provocaban las pestes que asolaron a Europa; por su influencia, mejora radicalmente la alimentación y la higiene de muchos seres humanos y como consecuencia directa, por primera vez en la historia, la población logra un incremento firmemente sostenido. A su amparo, o por su causa, varias naciones lograron, en breve lapso, un desarrollo industrial impresionante : Inglaterra, ya lo dijimos, hacia 1830; Francia y Estados Unidos, hacia 1860; Alemania en 1870; Suecia en 1890; Japón en 1905; Rusia en 1918 y Canadá en 1920. (8)

Pero también es verdad que quizás la consecuencia más importante para la historia universal, de esa revolución industrial, "fue el establecimiento del dominio del globo por parte de unos cuantos regímenes occidentales (especialmente el inglés) sin paralelo en la historia. Ante los mercaderes, las máquinas de vapor, los barcos y los cañones de occidente, los viejos imperios y civilizaciones del mundo se derrumbaban y capitulaban. La India se convirtió en provincia administrada por procónsules británicos; los Estados Islámicos fueron sacudidos por terribles crisis; Africa quedó abierta a la conquista directa. Incluso el gran imperio chino se vio obligado, en 1839-1842, a abrir sus fronteras a la explotación occidental. En 1848, nada se oponía a la conquista, por occidente, de todos los territorios que los gobiernos y comerciantes consideraban conveniente ocupar. (9)

Así pues, a partir de la Revolución Industrial se inició la aguda diferencia entre países ricos y naciones pobres; en ese período se origina también, ese peculiar y complicado fenómeno económico y social que llamamos subdesarrollo. Pero no nace solo; la Revolución Industrial alimenta y al fin hace triunfar, a la estructura económica que ha hecho posible la existencia misma de nuestro mundo contemporáneo : hacia 1830, EL CAPITALISMO consigue imponerse como el "modo de producción" dominante en todo el orbe y su influencia hará más difícil aún, la superación de la miseria en las naciones del tercer mundo.

La Revolución Industrial dividió, pues, en dos grandes grupos, a las naciones de la tierra : las que lograron industrializarse y fue-

---

(8).- Rostow, W. W.- "The Stages of Economic Growth". Cambridge University Press. U.S.A. 1967. p. XII.

(9).- Hobsbawm, Eric J.- " Las Revoluciones Burguesas ". Ed. Guadarrama, Madrid. 1971, p. 59.

ron productoras de manufacturas que necesitaban mercados cada vez más amplios que las consumieran, y las otras, las que permanecieron como simples productoras de materias primas, subordinadas a las necesidades económicas de las primeras, siendo a la vez mercado para sus artículos y almacén y fuente de productos primarios para la elaboración de sus manufacturas. Así se dividió el mundo en naciones desarrolladas, industrializadas, y países pobres, dependientes, subdesarrollados. La explotación colonial capitalista, arranca también de esa época.

De acuerdo pues con su nacimiento y su evolución, EL "SUBDESARROLLO" ES UN FENOMENO EN SI MISMO; nació como término opuesto al capitalismo exportador de mercancías. No es pues una "etapa" en el proceso continuo del desarrollo. Las naciones pobres, subdesarrolladas, tienen una estructura económica y social que las diferencia profundamente de las industrializadas y que se formó como "una resultante de las relaciones que existieran históricamente y perduran actualmente entre ambos grupos de países". (10)

Habida cuenta, por otra parte, de que el subdesarrollo lo padecen un conjunto de naciones con muy diferentes grados de pobreza como lo indica la tabla número 1 de la página 13, lo que resulta en diferencias aún más agudas entre sus problemas sociales y económicos fundamentales conviene a todas luces elegir, para el análisis de este fenómeno, un grupo de países en los que el subdesarrollo se manifieste con todos sus rasgos esenciales, con sus características en su más avanzada madurez, considerando que "la anatomía del hombre es una clave para la anatomía del mono". (11) Las naciones latinoamericanas, y desde luego México, satisfacen el requisito.

Así pues, el subdesarrollo, particularmente en América Latina, es una consecuencia de la industrialización, primero de Europa y en seguida de los Estados Unidos.

Es menester insistir, que desarrollo y subdesarrollo son dos facetas del mismo proceso de expansión del capitalismo occidental que se instala en el siglo XIX y que establece, por primera vez en la historia, la DIVISION INTERNACIONAL DEL TRABAJO, agrupando a las naciones de la tierra, como ya se explicó, en industrializadas productoras de mercancías, y en subdesarrolladas, que aportan a las primeras, materias

---

(10).- Sunkel, O. y Paz, Pedro.- "El Subdesarrollo Latinoamericano y la Teoría del Desarrollo" Ed. Siglo XXI; México, 1971, p. 25 .

(11).- Karl Marx.- "Introducción General a la Crítica de la Economía Política".- Ed. Pasado y Presente. Argentina, 1972, p. 26 .

P A I S	Total (millones)	P O B L A C I O N crecimiento anual (%)	Proyección al año 2000	menor de 15 años (%)	Producto per cápita (dólares)
Estados Unidos	214	0.9	264	25	5 590
Canadá	23	1.3	32	27	4 440
Francia	53	0.9	62	24	3 620
Alemania Oc.	62	0.3	66	22	3 390
Japón	111	1.3	133	24	2 320
Italia	55	0.5	61	24	1 960
Rusia	255	0.9	310	36	1 400
España	35	1.0	45	27	1 210
SUBDESARROLLADAS.					
Argentina	25	1.3	33	28	1 290
Chile	10	1.8	15	36	800
México	59	3.5	136	46	800
Brasil	110	2.8	213	42	530
Cuba	10	2.0	15	38	510
China	823	1.7	1126	24	160
India	613	2.4	1059	42	110
Bangladesh	74	1.7	144	46	70

Fuente : World Population Data Sheet 1975; Population Reference Bureau. U.S.A.

TABLA No. 1 CARACTERISTICAS DE UN GRUPO SELECTO DE NACIONES.-

primas y productos básicos, para el consumo de su industria y de sus pueblos.

Desarrollo y subdesarrollo constituyen una contradicción dialéctica, y se suponen y condicionan mutuamente, creando un sistema interpendiente; pero, MIENTRAS LAS NACIONES AVANZADAS ENCUENTRAN EN SU GRAN INDUSTRIA, EL APOYO NECESARIO PARA SU DESARROLLO ECONOMICO AUTOSOSTENIDO, LAS SUBDESARROLLADAS, DEBIDO A LAS LIMITACIONES ESTRUCTURALES DE SU CRECIMIENTO, SE VUELVEN DEPENDIENTES Y DOMINADAS. Esto explica una de las características básicas del subdesarrollo, ya apuntada : la persistencia de todas las naciones atrasadas, a permanecer en la pobreza, y demuestra, sin lugar a dudas, el error fundamental en que incurrió Carlos Marx, al afirmar que : "Los países industrialmente más desarrollados no hacen más que poner delante de los países menos progresivos e espejo de su propio porvenir". (12)

La subordinación de las naciones pobres del planeta se expresa de diversas maneras, pero encuentra su manifestación más aguda en el comercio internacional, donde son víctimas del sistema de dominación, mediante los llamados "TERMINOS DE INTERCAMBIO", (*) los cuales siempre sufren deterioro para las naciones subdesarrolladas, pero, aún cuando no fuese así, el comercio internacional, por sí mismo, ya entraña una permanente injusticia para las naciones más atrasadas, puesto que una hora de trabajo socialmente necesario en los países industrializados, se hace valer mucho más que una hora de trabajo en los países productores de materias primas, puesto que : "Como los productos se intercambian a su precio de producción, los países en los cuales la composición orgánica (**) es más baja, no obtienen a cambio del producto

---

(12).- "El Capital".- Ed. Fondo de Cultura Económica, México, 1972, Tomo I p. XIV.

(*).- Se entiende por "términos de intercambio" la relación entre los precios de exportación y los de importación con base en un año determinado.
Se dice que los términos de intercambio mejoran, cuando los precios de exportación suben más rápidamente (o bajan menos rápidamente) que los de importación, o cuando los precios de exportación suben, mientras bajan los de importación. Cuando esto ocurre, una nación está en capacidad de obtener un mayor volumen de importaciones por una cantidad dada de exportaciones.
Hay deterioro en los términos de intercambio, cuando ocurre lo contrario.
Los términos de intercambio deben medirse por los precios en moneda extranjera; también pueden ser medidos por precios en moneda local, siempre que los factores de conversión sean los mismos para exportación e importación.

(**).- Llámase "composición orgánica del capital" al cociente entre el "capital constante" (valor de los medios de producción) y el "capital variable" (valor de la fuerza de trabajo, suma global de los salarios). Véase "El Capital".- F.C.E. 1972, Tomo I p. 517

de una hora de trabajo nacional (...) nada más que los productos que han costado menos de una hora de trabajo socialmente necesario en los países en que la composición orgánica del capital es más elevada".--- (13) "En cuanto a los salarios reales, están determinados por razones sociológicas e históricas, lo que permite introducir el supuesto de que el valor de la fuerza de trabajo se mantiene a nivel de subsistencia en los países dependientes, mientras que se multiplica por veinte o por treinta en los centros imperialistas". (14)

Este es el principal medio y no las inversiones directas, por el cual las naciones centrales industrializadas, succionan el capital de las naciones periféricas dependientes, y explica también la persistencia de la pobreza en las naciones subdesarrolladas y explotadas.

---

(13).- Bettelheim, Charles.- "Imperialismo y Comercio Internacional" Ed. Pasado y Presente, Argentina, 1972, p. 34.

(14).- Ibidem.- p. XI.-

A la luz y en el marco del subdesarrollo ya definido en páginas anteriores, se estudiará cada una de las características apuntadas, tratando en conjunto aquéllas (como el producto per cápita y la explosión demográfica) que sean notoriamente interdependientes.

Una simple ojeada a la tabla número 1 de la página 13, nos revela que las naciones pobres, subdesarrolladas, de escaso producto per cápita, sacando fuerza de flaqueza, logran los incrementos de población más altos, mientras que las naciones ricas, industrializadas, se distinguen, a su vez, por aumentos en su población muy modestos, en general inferiores a la unidad.

Y aquí se antojaría preguntar si las naciones actualmente desarrolladas no padecieron, en su época, los mismos problemas económicos y sociales que ahora afligen a las naciones pobres del planeta; en una palabra, si las naciones ricas estuvieron alguna vez hundidas en el subdesarrollo.

De los diez rasgos característicos del subdesarrollo ya apuntados, muchos les fueron comunes -durante su despegue- a las naciones ahora desarrolladas. En el siglo XIX, cuando casi todas ellas lograron industrializarse, su producto por cabeza era muy pequeño y una gran parte de su población se dedicaba -desde antaño- a las labores agrícolas, pues "hacia 1688, la agricultura inglesa ocupaba alrededor del 75% de las familias". (15) Sufrieron también la inflación (quizás la más severa se presentó cuando el oro y la plata llegaron de América, a partir de la segunda mitad del siglo XVI) porque "de la disparidad entre los precios y los salarios resulta un aumento de las ganancias, tanto mayor cuanto más acentuada es la separación entre esos dos factores (...) las ganancias se multiplicaron por cuatro aproximadamente entre 1740 y 1790". (16) Además, el hambre también -como ahora- era crónica y la mendicidad fue brutalmente castigada, con frecuencia en nombre de la propia caridad cristiana, como en la aplicación de la famosa " ley de pobres " de Inglaterra, promulgada en 1572 con el santo designio "de hacer desaparecer la clase peligrosa de los mendigos profesionales, que había adquirido, a mediados del siglo XVI, un desarrollo terrible". (17) Y en cuanto al analfabetismo, que ahora priva en tantas naciones atrasadas, era no solamente más agudo antes de la Revolución Industrial, sino universalmente difundido; la persona que sabía leer constituía una extraña excepción.

---

(15).- Bairoch P.- "Revolución Industrial y Subdesarrollo". Ed. Siglo XXI, México, 1967, p. 292.

(16).- Ibid. p. 38.

(17).- Mantoux, P. "La Revolución Industrial en el Siglo XVIII". Ed. Aguilar, Madrid, 1962, p. 426.

...

¿ Qué característica queda pues, de las naciones subdesarrolladas, que no fuera común a las naciones industrializadas, en el momento del "despegue" en el siglo XIX ?

De nuevo la tabla de la página 13 nos lo muestra en seguida, pues salta a la vista que las naciones subdesarrolladas tienen, en general, un elevado, explosivo incremento anual de su población, en promedio superior al 2% anual, mientras que "antes de su despegue, los países actualmente desarrollados conocieron una progresión demográfica próxima al 0.5% al año, y esa proporción pasó a 0.8% aproximadamente, en el curso de los sesenta primeros años de su despegue. POR OTRA PARTE, HASTA EL PRESENTE NINGUN PAIS INICIO SU DESARROLLO CON UNA TASA DE CRECIMIENTO DE LA POBLACION SUPERIOR AL 1.1%. Pero los países subdesarrollados conocen una progresión demográfica del 2.2% anual por término medio" (18)

Parece pues -y se demostrará sin lugar a dudas más adelante- que el excesivo incremento de la población es un freno insuperable para salir del subdesarrollo.

Y otra vez aquí se antoja preguntar si, aun aceptando que la explosión demográfica es un obstáculo definitivo para el desarrollo económico de una nación, ¿ es el único ?

Por poco avisados que fuésemos, la historia inmediatamente nos señalaría, en la época actual, la presencia de un grupo de naciones poderosas, con una gran industria consumidora de materias primas y ávida de mercados para sus productos y una enorme acumulación de capital, que por su propio volumen, necesita ser exportado. Las naciones que se desarrollaron en el siglo XIX, lo hicieron libremente, sin la presencia ominosa del imperialismo. Y en esto radica el error -inconcebible dado su talento- de Carlos Marx, como ya señalamos en la página 14, cuando afirma (vale la pena repetirlo) que "Los países industrialmente más desarrollados no hacen más que poner delante de los países menos progresivos el espejo de su propio porvenir". Cuando Inglaterra se industrializó, no existía la amenaza de Alemania, ni de Francia, ni de los Estados Unidos; no se había establecido la "división internacional del trabajo" ni se empleaban tampoco los caritativos "términos del intercambio". En verdad, Carlos Marx poco se preocupó de las naciones atrasadas, y los marxistas -para no ser menos- tampoco han podido, hasta la fecha, elaborar una teoría consistente acerca del subdesarrollo. Lo único serio que se ha hecho, se le debe, por entero, a la "Comisión Económica para la América Latina" (CEPAL) y gran parte de las ideas anteriores, aquí expuestas, han encontrado en ella su fuente y su respaldo.

---

(18).- Bairoch, P.- Op. cit. p. 226.

Resulta pues evidente, que los países subdesarrollados -CUYA CARACTERISTICA PRINCIPAL, EL ESCASO PRODUCTO PER CAPITA, se señala tácitamente en la tabla de la página 13 y se destaca en la página 15- se enfrentan, de hecho, a dos obstáculos fundamentales que se oponen a su desarrollo: LA EXPLOSION DEMOGRAFICA -factor interno, responsabilidad nuestra- y LA ACCION DEL IMPERIALISMO -amenaza externa, empeño ecuménico de las naciones atrasadas por escapar a su influencia-; y queda bien claro que ninguno de los dos fenómenos estuvo presente en el siglo XIX, cuando las naciones, ahora tan opulentas, lograron el gran impulso que las sacó de su pobreza. El subdesarrollo es pues un fenómeno característico y singular, que no constituye -pese a Marx- una etapa más en el proceso continuo del desarrollo.

2.2.- LA TEORIA DE LA DEPENDENCIA.

La presencia del imperialismo, que acabamos de destacar, ha llevado a varios autores latinoamericanos a plantear las relaciones económicas entre los países industrializados y las naciones de América Latina, en un marco de dominación fundamental de los primeros sobre las segundas, que se ha denominado "Dependencia", la cual se define formalmente como "una relación bilateral y asimétrica (...) que se caracteriza por el hecho de que un cambio en la unidad dominante, resulta invariablemente en un cambio en la unidad dominada, mientras que un cambio similar en la última, afecta poco o nada a la primera" (19)

Esa "teoría de la dependencia latinoamericana" -pues la aplican fundamentalmente a esa zona- considera al subdesarrollo como un producto EXCLUSIVO del imperialismo internacional, subordinándolo completamente, en consecuencia, al destino de las naciones industrializadas : "La revolución industrial, que dará inicio a la creación de la gran industria, corresponde en América Latina a la independencia política que, conquistada en las primeras décadas del siglo XIX, hará surgir, con base en la nervadura demográfica y administrativa tejida durante la colonia, a un conjunto de países que entran a gravitar en torno a Inglaterra (...) es a partir de entonces que se configura la dependencia, entendida como una relación de subordinación entre naciones formalmente independientes, en cuyo marco las relaciones de producción de las naciones subordinadas son modificadas o recreadas para asegurar la producción ampliada de la dependencia". (20)

Sin embargo, aquí conviene aclarar que la América Latina sólo se incorpora a la economía mundial, hasta el año de 1870; el período comprendido desde 1820 a 1870 se caracterizó por una serie de violentas convulsiones políticas en la zona, que hubieran hecho imposible el establecimiento de cualquiera relación comercial estable. En México, este fenómeno destaca claramente en el marco de contínuas y sucesivas revueltas y asonadas. "Durante sus primeros cincuenta años de independencia, los asuntos de México fueron dirigidos por más de cincuenta gobiernos, como son treinta diferentes hombres (SIC) actuando como presidentes" (21) Además, difícilmente podría hablarse de actividad comercial y ni siquiera de integración nacional, en un país como México (y el res

---

(19).- Sagasti, Francisco R.- "Política Tecnológica y Desarrollo Económico" Secretaría de Relaciones Exteriores, México, 1975.- p. 20.

(20).- Marini, Ruy Mauro.- "Dialéctica de la Dependencia". Ed. Era, México, 1973. pp. 17 y 18.

(21).- Hansen, Roger D.- "La política del desarrollo mexicano". Ed. Siglo XXI, México, 1971, p. 20.

to de América Latina no era excepción) que "en 1820 tan sólo poseía tres caminos que pudieran llamarse carreteras, e incluso ésos estaban muy deteriorados y para 1860, después de veintitrés años de esfuerzos, México tan sólo poseía veinticuatro kilómetros de vías férreas utilizables". (22)

Así pues, el imperialismo no pudo condicionar al subdesarrollo de América Latina antes de 1870, por dos razones : la primera, ya apuntada, por imposibilidad física; la segunda, no de menor monta, se refiere a que el imperialismo, hacia 1870, apenas hacía su aparición en el mundo y no iba a madurar sino hasta 1914, durante la primera guerra imperialista, en la que las naciones poderosas, sujetas entre las contradicciones de la "repartición económica del mundo", desembocaron finalmente en la "repartición territorial del planeta", como lo describió -mitad padeciéndolo, mitad adivinándolo- hacia 1916, el propio Lenin. (23)

Pero cabe asimismo advertir, que si el imperialismo -y su consecuencia inmediata, la "dependencia"- no pudo influir en Latinoamérica ni en nuestro país, desde 1870, es indudable que lo hizo después, puesto que ya se muestra en plena madurez histórica en los albores de nuestro siglo XX. La única cuestión a debatir sería si la "dependencia" -heraldo del imperialismo- es el único factor que define y condiciona al subdesarrollo, como lo sostienen casi todos los escritores marxistas que se ocupan de los problemas económicos de Latinoamérica. Parece esa posición un tanto intransigente, pues resulta inadecuada para explicar, tanto la explosión demográfica de los países subdesarrollados, como la política de "sustitución de importaciones" por éstos emprendida, la cual tiene un innegable carácter nacionalista.

2.3.- LAS ETAPAS DEL SUBDESARROLLO LATINOAMERICANO.

América Latina, con su siglo y medio de independencia política, es la zona del planeta -con los Balcanes quizás- donde el subdesarrollo ha madurado; en su historia económica se distinguen tres etapas bien diferenciadas.

PRIMERA ETAPA : (1822-1874) caracterizada, como ya se apuntó, por constantes convulsiones políticas, que desanimaron la inversión extranjera : "Es de destacar esta paralización de las inversiones británicas

(22).- Ibid. p. 21.

(23).- Véase: "El Imperialismo, fase superior del capitalismo". por V. I. Lenin, en "Obras Escogidas en Tres Tomos" Ed. en Lenguas Extranjeras, Moscú, 1960. Tomo I. p. 719 y siguientes.

en los países latinoamericanos, sobre todo en contraste con el capital inglés invertido en países situados fuera de la región, que aproximadamente se duplicó entre 1825 y 1850". (24)

SEGUNDA ETAPA : (1874-1929) que representó la edad de oro del capital extranjero, aprovechando la integración del subcontinente a la economía mundial.

En México se inició con el arranque de la época porfirista, en cuyo período "mientras la población creció a una tasa anual de 1.4 por ciento, la correspondiente tasa del producto nacional bruto fue aproximadamente de 2.7 por ciento". (25)

Las condiciones que prevalecían en Europa a finales del siglo XIX, oscurecidas por la amenaza de guerra entre los países de esa región y la necesidad de alimentos para la creciente población urbana de la Gran Bretaña, "indujeron a los capitalistas británicos a apartarse de Europa y a concentrar sus esfuerzos en las zonas de la periferia". (26)

Las grandes inversiones que recibió en esa época América Latina y las intensas relaciones comerciales que sostuvo con el extranjero, caracterizaron la actividad económica de la zona en lo que se llamó el "modelo de crecimiento hacia afuera", en atención a la importancia fundamental que en ese período tuvo el sector externo de las principales naciones latinoamericanas.

En México, la inversión extranjera alcanzó cifras extraordinarias y "es indudable que durante los años porfiristas el capital extranjero fluyó hacia el país en cantidades proporcionalmente mucho mayores -en relación con el capital nacional y los recursos naturales y humanos de México- que el volumen de capital europeo que entró a los Estados Unidos durante la etapa de su desarrollo más intensivo". (27)

Esa inversión extranjera, amén de lanzar a México al mercado mundial, creó un pujante sector externo y logró que "entre 1877 y 1910 el valor de las exportaciones mexicanas se elevara en más de 600 por ciento en términos reales". (28)

---

(24).- Naciones Unidas.- "El Financiamiento Externo de América Latina" Nueva York, 1964, p. 2.
(25).- Rosenzweig, Fernando.- "El Desarrollo Económico de México".- El Trimestre Económico.- F.C.E. México, No. 127, p. 405.
(26).- Naciones Unidas.- Op. Cit. p. 3.
(27).- Hansen, Roger D.- Op. Cit. p. 26.- Citando a Ernesto Fernández Hurtado en "Private Enterprise and Government in Mexico".
(28).- Ibid. p. 24.

Alrededor de Porfirio Díaz se organizó el grupo llamado de "los científicos", que no era otra cosa sino una "Oligarquía Moderna Exportadora-Importadora" con Limantour a la cabeza y la que, por su actividad económica característica, era de corte necesariamente imperialista.

La "Oligarquía Moderna Exportadora-Importadora", estaba obligada a impedir la creación de aranceles aduanales que gravaran los productos del comercio exterior y por ello iba a entrar en aguda contradicción con la Burguesía Industrial que se formaba principalmente en el norte del país, de carácter nacionalista por su propia actividad económica y urgida de barreras aduanales, que protegieran sus productos de la competencia extranjera.

Esa violenta contradicción iba a enfrentar a las dos burguesías mexicanas, el 20 de noviembre de 1910.

TERCERA ETAPA : (1929 a 1945). El año de 1929 señala el principio de la más grande y desastroza crisis económica que haya azotado al mundo hasta el presente. América Latina se vió afectada profundamente al caer casi a cero sus exportaciones y, al mismo tiempo, "las entradas de capital extranjero en la región se redujeron a un nivel insignificante". (29) Esto condujo a la desaparición del modelo de "crecimiento hacia afuera", que dependía de las exportaciones y obligó, a los países latinoamericanos, a iniciar el modelo de "crecimiento hacia dentro", fundado en una paulatina industrialización, mediante lo que se ha dado en llamar "LA SUSTITUCION DE IMPORTACIONES" y que consiste en dejar de importar sucesivamente determinados artículos, principiando por los de consumo inmediato y terminando por los bienes de capital, para producirlos en el propio país.

La segunda guerra mundial favoreció notablemente la política de sustitución de importaciones, pues los Estados Unidos se vieron obligados a importar materias primas de latinoamérica, para abastecer su industria de guerra y satisfacer las necesidades alimenticias de su población y a la vez, fueron incapaces de exportar manufacturas ni maquinaria, pues su industria estaba totalmente comprometida en el esfuerzo bélico que sostenían. De esa manera, varios países latinoamericanos (México entre ellos) vieron sus exportaciones aumentar más rápidamente que sus importaciones, "generando en consecuencia excedentes comerciales que les permitieron reembolsar parte de la deuda exterior y aún comprar empresas extranjeras radicadas en la región". (30)

Una cuarta época, desde la segunda guerra hasta nuestros días, se considera ya, para su estudio en el siguiente capítulo, concretamente dedicada a México.

---

(29).- Naciones Unidas.- Op. cit. p. 23.

(30).- Ibid. p. 23.

CAPITULO III

EL SUBDESARROLLO MEXICANO.

3.1.- INTRODUCCION

Un país es pobre, porque no cuenta con suficiente capital para que el producto de su economía, logre un ingreso por habitante suficientemente alto, para sostener un "nivel de vida" de su población por encima de los mínimos requeridos por el bienestar moderno.

Y un país permanecerá hundido en la pobreza, si no logra acumular capital con suficiente rapidez, para alcanzar un ingreso mínimo por habitante, compatible con el bienestar moderno mencionado.

A diferencia de los países industrializados, las naciones subdesarrolladas carecen, en general, de la dotación de capital suficiente para generar el empleo que necesita su fuerza de trabajo (*). En consecuencia, la desocupación alcanza a menudo valores exagerados. La capacidad productiva de estos países, escasa y poco eficiente, no sólo tendría que utilizarse plenamente, sino además, incrementarse con rapidez y la única manera de conseguirlo es a través de una considerable inversión (**), que aumente sustancialmente el valor del producto nacional.

(*).- "FUERZA DE TRABAJO" de un país, es el número de hombres mayores de 14 y menores de 65 años, más la tercera parte de las mujeres comprendidas entre esas edades.

"POBLACION ECONOMICAMENTE ACTIVA" (PEA) está formada por el contingente que suministra efectivamente trabajo para la producción de bienes y servicios, incluyendo los empleados, los trabajadores por cuenta propia los asalariados, tanto los empleados como los desocupados en un momento determinado.

(**).- El término "inversión" debe comprender todo tipo de desembolso efectuado en el momento actual, con el propósito de aumentar el ingreso futuro, ya se trate de maquinaria o conocimientos tecnológicos productivos, caminos transitables o centros de planificación familiar. No se olvide que la inversión (neta) mide el incremento del capital social.
Es fundamental recordar que LA INVERSION TIENE UN EFECTO " DUAL " : por una parte, genera ingreso, vía el efecto multiplicador, y por la otra, eleva la capacidad productiva. La construcción de una nueva fábrica aumenta la capacidad de producción y genera, a la vez, un ingreso. (Véase "Crecimiento y Ocupación", Domar, Evsey D.- El Trimestre Económico.- F.C.E., México, No. 90, p. 191.

... 24

Pero en una economía mixta subdesarrollada como la de México, el problema de invertir suele tropezar con obstáculos frecuentemente insuperables, pues en primer lugar, puede acontecer que la inversión privada no alcance el nivel necesario que se haya calculado, o lo que es peor, que no se ajuste a las modalidades que exige una economía planificada. El Gobierno puede suplir lo que le falte a la inversión privada (aun a trueque de aumentar su déficit y su deuda externa) pero, a su vez, tiene que regular y distribuir esa inversión, de acuerdo con la planeación previa de la economía, lo que significa frecuentemente, una profunda intervención estatal en los dominios de la iniciativa privada, con las subsecuentes reacciones y resistencias que en nuestro país han sido tan evidentes : "La estatización de la economía lleva a una pérdida de la libertad, ya que si al poder político y moral del Estado se aumenta (SIC) el poder económico, no nos hagamos, eso es una dictadura". (31) "Las declaraciones del Secretario del Patrimonio Nacional, respecto a que el Estado debe participar más en la actividad empresarial, son poco afortunadas, porque mediatizan o detienen la nueva inversión". (32)

Además de la inversión en la industria, es indispensable, en segundo lugar, obtener un desarrollo adecuado del sector agrario, con el fin de lograr una oferta suficiente, tanto de alimentos básicos, como de insumos (*) para la industria nacional y productos de exportación; pero son bien claras las dificultades con que se tropieza para lograrlo -aun cuando se haya realizado la reforma agraria- pues por una parte, las inversiones del gobierno en ese sector -pese a la ley federal de aguas (**)-contribuyen a monopolizar aún más la producción agrícola, toda vez que, de acuerdo con las leyes de la acumulación de capital, se desata una intensa competencia entre productores, que se resuelve siempre a favor de los que emplean métodos más avanzados de producción, mediante maquinaria moderna, insecticidas, y semillas mejoradas. De otra parte se encuentran los productores pobres : ejidatarios y parvifundistas, sin recursos de capital, sin poder emplear la tecnología adecuada, pero multiplicándose incansablemente a través de familias numerosas que presionan sobre la ya escasa tierra, subdividiéndola cada vez más, atomizándola a tal punto, que la unidad de medida agraria, la hectárea, carece ya de sentido en su aplicación y se la ha sustituído por el "surco", que mide con mayor propiedad su miseria.

---

(31).- Jorge Sánchez Mejorada, presidente de la Confederación de Cámaras Industriales, en declaraciones a la prensa nacional.- Excelsior, marzo 1. de 1976.

(32).- José Luis Ordóñez, presidente de la Cámara Nacional de Comercio de la Ciudad de México, en declaraciones a la prensa nacional.- Excelsior, marzo 18 de 1976.

(*).- INSUMO, es cualquier bien o servicio que contribuye a la producción de un producto.

(**).- Promulgada el 26 de enero de 19_2, y limita la propiedad de la tierra con riego, a 20 hectáreas.

Quedan aún por considerar los completamente marginados; los que no tienen ni tierra ni empleo completo; los subempleados que trabajan unos cuantos días al año y se alimentan en la misma proporción, escenificando las invasiones de tierras que tanto alteran la tranquilidad del campo mexicano. Varios investigadores han estimado que los campesinos subempleados alcanzaban, hacia 1970, cuatro millones de personas (33) (34).

Las características del agro mexicano que se acaban de apuntar, han polarizado al sector agrícola en dos grandes secciones : una moderna, dinámica y altamente productiva, y otra tradicional, estancada y con producción apenas de subsistencia.

Esa contradicción en la estructura agrícola, se refleja en la historia de su producto, el cual ha ido perdiendo importancia en los últimos años, ya que, mientras en 1960 contribuyó con el 9.8% en la formación del producto interno bruto, en 1965 sólo alcanzó el 9.4% y en 1969 únicamente contribuyó con el 7.2% del total (35).

Su evolución productiva resulta aún menos alentadora, ya que si bien es cierto que, entre 1935 y 1967, sostuvo un incremento medio anual del 4.4%, en el lapso de 1960 a 1969 apenas logró un desarrollo promedio del 3.4% y en el último período de 1965 a 1969, únicamente alcanzó el 0.2% anual (36). Por otra parte, en el año de 1969 el producto agrícola sufrió un decremento (- 2.1%) con respecto al año anterior, y en 1970 aumentó en 2%, alcanzando en 1971 únicamente un 2.7% anual, "cifra superior al promedio de los cinco años anteriores" (37). El año de 1972 fue peor aún, pues el producto sufrió un nuevo decremento (- 1.4%) avanzando el 1.7% en 1973, descendiendo al 0.9% en 1974 y obteniendo sólo el 1.0% en 1975, a pesar de las cuantiosas inversiones públicas en el sector primario (38).

Resulta pues evidente, que cuando menos desde el año de 1967 a la fecha, el deterioro de la agricultura ha sido notable y los incrementos de su producto han resultado ya inferiores al de la población (3.5% anual, sostenido y con tendencia a elevarse).

---

(33).- Orive Alba, Adolfo.- "La Irrigación en México". Ed. Grijalbo, México, 1970, p. 236.
(34).- González Salazar, Gloria.- "Problemas de la Mano de Obra en México". Ed. UNAM.- México, 1971, p. 58.
(35).- "México, 1970".- Banco Nacional de Comercio Exterior.- p. 53.
(36). Ibídem.- p. 53.
(37).- Banco de México, S. A.- Informe Anual 1971, p. 18.
(38).- Banco de México, S. A.- Informes Anuales 1972, 1973, 1974 y 1975.

En ese panorama desolador, es difícil entender que "de 1964 a - - 1969, México exportó 5.4 millones de toneladas de maíz, con valor de 303 millones de dólares; 1.8 millones de toneladas de trigo, con valor de - 115 millones de dólares y 339 mil toneladas de frijol, con valor de 48 millones de dólares" (39). Y como los precios internacionales eran inferiores a los de garantía de nuestro país, se exportaba además, con pérdida.

De ahí que el gobierno mexicano desalentara la producción de cereales básicos -vía los precios de garantía que permanecieron congelados, excepto el del trigo, que incluso disminuyó- en favor de cultivos más - complicados, pero a la vez más redituables, como el de jitomate, azúcar, algodón, fresa, tabaco y frutas. Los cultivos tradicionales, por consecuencia, fueron relegados a las zonas de temporal (40).

De esta guisa pronto se contrajo la producción de cereales básicos, la que acusó su primer déficit en el año de 1971, coincidiendo, para - - nuestra desgracia, con un aumento notable de sus precios, en el mercado internacional. En esa coyuntura, México se vió obligado a importar cereales (ahora caros) por 833 millones de dólares, entre 1971 y 1974 (41).

Para remediar esta indeseable situación, el gobierno resolvió aumentar los precios de garantía del maíz y del trigo, a la vez que canalizaba cuantiosos recursos hacia el sector primario. Como consecuencia, - "se presentaron aumentos importantes en el volumen de la producción de granos, semillas oleaginosas y algunos otros productos" (42).

Pero aun cuando la producción de cereales se haya casi recuperado, es necesaria una planeación integral del sector agrícola, toda vez que el cultivo de los granos básicos es el más elemental que existe y añade escaso "valor agregado" (*), empleando mucho menos mano de obra que el algodón, la caña de azúcar o las legumbres, pues "mientras un avío para

---

(39).- Banco Nacional de México.- "Carta Mensual" Marzo de 1976, p. 2.
(40).- Ibid. p. 2.
(41).- Ibid. p. 3.
(42).- Banco de México.- "Informe Anual 1975", p. 35.
(*).- "VALOR AGREGADO" en una etapa de un proceso productivo, es la diferencia entre el valor del producto generado en esa etapa (vía volumen por precio) menos la suma de los valores de los insumos (bienes y servicios) que proceden de otros procesos productivos. Así por ej. : Si una fábrica produce 100 mil pares de calzado al año, los cuales vende a $150.00 el par, habrá producido 15 millones de pesos en zapatos; y si la suma de los costos de las pieles, clavos, hilo y tacones, alcanza los 9 millones de pesos, se dice que el "valor agregado" por esa etapa productiva, fue de 6 millones de pesos.

granos básicos fluctúa entre 3 mil y 3 mil quinientos pesos por hectárea, un avío para algodón es 4 veces mayor y uno para jitomate, 12 veces mayor. Esto da idea del dinero que deja de circular, al sustituirse una hectárea de ellos, por cereales" (43); y queda todavía en favor de los primeros, sus mejores perspectivas de exportación.

Finalmente, y aun cuando aquí se soslaya el problema fundamental de la escasez de la tierra ante una demanda de la población con crecimiento explosivo, conviene, en cambio, advertir, la política "agrarista" de casi todos los regímenes posteriores a 1915, cuyo celo revolucionario los llevó a repartir, indistintamente, tierras y aguas y bosques y montañas, acumulando un área repartida -como lo indica la tabla número 2- casi tres veces mayor de la propiamente laborable, que llega apenas, en nuestro país, a los 30 millones de hectáreas (44).

Presidentes	períodos	hectáreas repartidas (miles)
1. De Carranza a Abelardo Rodríguez.	1915-1934	11 03[illegible]
2.- Lázaro Cárdenas.	1935-1940	20 13[illegible]
3.- Avila Camacho.	1941-1946	5 97[illegible]
4.- Alemán Valdez.	1947-1952	5 [illegible]
5.- Ruiz Cortínez.	1953-1958	5 77[illegible]
6.- López Mateos.	1959-1964	9 02[illegible]
7.- Díaz Ordaz.	1965-1970	23 05[illegible]
8.- Echeverría.	hasta 1972	6 94[illegible]
	Suma :	87 37[illegible]

Fuente : "Informe de Labores 1971-1972" Departamento de Asuntos Agrarios y Colonización. México, 1972.

En : "Problemas Económicos de México". Diego G. López Rosado. Ed. UNAM, 1975, p. 131.

Tabla No. 2.- LA REPARTICION DE LA TIERRA EN MEXICO.

---

(43).- Banco Nacional de México, S. A.- Op. cit. p. 4.

(44).- Orive Alba, Adolfo.- Op. cit. p. 54.

Se ha intentado -en esta introducción al subdesarrollo mexicano- describir, así sea someramente, los avatares que sufre la inversión del gobierno en nuestro país, en su afán de aumentar el valor del producto nacional; si la intención es buena -y parece que lo es- no podemos pronunciarnos, en cambio, de la misma manera, acerca de lo atinado de los medios para conseguirlo.

Los hechos manifiestan una contínua baja porcentual de la inversión privada, la cual fue, al iniciarse el sexenio del presidente Echeverría, del 62.5% contra el 37.5% del gobierno; en el año de 1971, la proporción aumentó al 64.7%, por 35.3% de la inversión pública; en 1972 principió a contraerse la inversión privada, la cual alcanzó el 57% contra el 43% del Estado; en 1973 persistió la misma tendencia, con el 53% correspondiente a la iniciativa privada y el 47% al sector oficial; para 1974, los porcentajes de inversión casi se igualaron : 50.2% la privada y 49.8% la pública; pero en el año de 1975, los porcentajes, por primera vez en la historia se invirtieron, correspondiendo únicamente el 49.6% a la iniciativa privada, contra el 50.4% del sector público.

Se estima que en el año de 1975, la inversión fija bruta en México fue de 110 923 millones de pesos (perteneciendo 54 976 millones al sector privado y 55 947 millones al Gobierno) la cual representó sólo un 2.2% de aumento sobre la correspondiente a 1974, en cuyo año la inversión fue 8.8% mayor que la de 1973, la que a su vez se incrementó en el 12.2% sobre la del año anterior (45).

Los números anteriores ponen de manifiesto que no sólo se redujo, en este sexenio, el volumen de la inversión privada, sino que también la inversión total sufrió una contínua contracción, lo cual se refleja, a su vez, en los bajos incrementos del producto nacional, el cual apenas alcanzó, en el año de 1975, un aumento modesto entre el 3.8 y el 4.2% (46) lo cual no ocurría (salvo el año de 1971) desde hace veinticinco años (47).

---

(45).- Declaraciones del "Centro de Estudios Económicos del Sector Privado" a la prensa nacional; "Novedades", marzo 28 de 1976, primera plana.

(46).- Banco de México.- "Informe Anual 1975" p. 21.

(47).- Nacional Financiera, S.A.- "La Economía Mexicana en Cifras, 1972" pp. 30, 31, 32, 33 y 34.

3.2.- LA EXPLOSION DEMOGRAFICA MEXICANA.

3.21.- LA MAGNITUD UNIVERSAL DEL FENOMENO.

A semejanza del capitalismo y el subdesarrollo, el aumento cada vez más acelerado de la población, tuvo su origen en la revolución industrial, cuyas primeras consecuencias se reflejaron en profundos cambios en las costumbres de los habitantes de los países que esa revolución afectó : se establecieron las tres comidas al día, se generalizó el uso de los cubiertos en la mesa y se adoptaron nuevas medidas de higiene que, junto a descubrimientos médicos como la vacuna antivariólica ideada por Jenner a principios del siglo XVIII, contribuyeron a disminuir, paulatinamente, las tasas de mortalidad, especialmente de los recién nacidos. Sin embargo, el descenso de la mortalidad fue acompañado, casi inmediatamente, por una caída semejante en las tasas de natalidad, con el resultado final de un crecimiento relativamente lento, de la población en su conjunto.

Pero en la segunda mitad del siglo XIX el panorama cambia radicalmente; los pueblos de Asia y de América Latina, que a través de los años habían conservado casi estable el número de sus habitantes, se incorporan al comercio mundial y hacen suyas gran parte de las modernas costumbres de higiene y principian a disfrutar de los nuevos descubrimientos médicos. Pero, a diferencia de las naciones europeas, disfrutaron del descenso de sus tasas de mortalidad, SIN ABATIR PROPORCIONALMENTE SU COCIENTE DE NATALIDAD, toda vez que recibieron más o menos súbitamente los beneficios de la civilización, pero no se incorporaron masivamente a ella, quedando una gran proporción de sus pueblos, ajena a la conciencia de su responsabilidad social. La consecuencia inmediata fue una verdadera explosión en el incremento demográfico, el cual necesitó un millón de años -desde la aparición del "homo habilis" hasta el nacimiento de Cristo- para alcanzar los 250 millones de seres humanos sobre la tierra y 500 años más, a la caída de Roma, para llegar a los 290 millones de habitantes y fue necesario que transcurrieran, de nuevo, más de mil años, hasta 1750, en los albores de la Revolución Industrial, para alcanzar los 630 millones de personas, que correspondía a un incremento promedio, de 224 mil habitantes por año

Pero todo el panorama principió a cambiar a partir de la Revolución Industrial y se tornó explosivo hacia la segunda mitad del siglo XIX, y así, en 1840 la población mundial llegó a los mil millones y para 1915 alcanzó los 1750 millones, consiguiendo exceder los 2 200 millones en 1940, llegando a los 3700 millones a mediados de 1971 y finalmente, a los 3967 millones en 1975 con un incremento anual del 1.9%(48).

(48).- Population Reference Bureau.- "World Population Data Sheet 1975" Washington, 1975.

Con esa tasa, se tienen 4 042 millones de habitantes en 1976 y considerando el actual cociente de natalidad (31.5 al millar) se debe aceptar que diariamente nacen 348 800 niños, es decir, unos 14 500 cada hora. Pero para el año 2000, a sólo 24 años del presente, la humanidad alcanzará los 6 500 millones de personas y, de continuar las actuales tendencias, estarán naciendo, para esas fechas, unos 23 400 niños por hora.

Con los datos anteriores se dibujó la gráfica de la página 32 que ilustra la evolución del fenómeno y señala la tendencia asintótica de la curva. con respecto a las ordenadas.

3.22.- LA MAGNITUD NACIONAL DE LA EXPLOSION DEMOGRAFICA.

México, con una tasa del 3.5% anual, ostenta, entre las 139 naciones que pertenecen a la ONU, el más alto incremento demográfico del mundo (49).

Tal como se señaló para el conjunto de las naciones del tercer mundo, nuestro país ha mantenido sus altas "tasas brutas de natalidad" (*) (45.0 en 1960 y 44.0 en 1970) y en cambio, ha visto descender su "tasa bruta de mortalidad" (**) ya de suyo baja en 1960 (11.8) a un valor todavía menor (9.0 en 1970). En tal virtud, la tasa de crecimiento de la población, que resulta de la diferencia entre ambas tasas de natalidad y mortalidad, alcanza, como ya se había expresado, un valor de 3.5% anual, la más alta del mundo (50).

Con la tasa bruta de natalidad prácticamente constante y una tasa de mortalidad en continuo descenso, la evolución del incremento demográfico en nuestro país ha presentado, históricamente, los siguientes valores : (51)

---

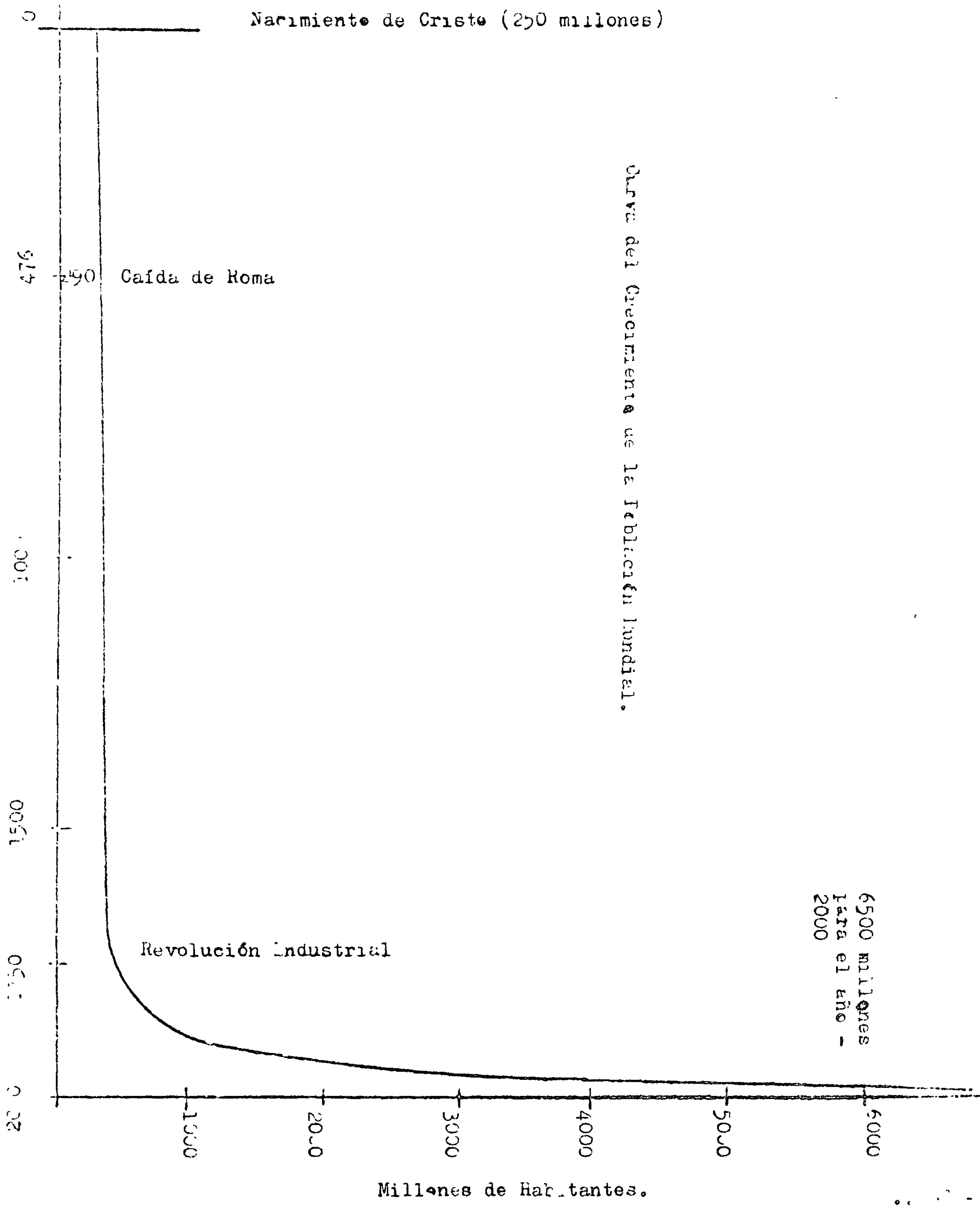
(49).- Ibidem. Data Sheet 1975.

(*).- TASA BRUTA DE NATALIDAD.- Número anual de nacidos vivos por cada mil habitantes.

(**).- TASA BRUTA DE MORTALIDAD.- Número anual de defunciones por cada mil habitantes.

(50).- CEPAL.- "Población y Desarrollo en América Latina". F.C.E., México, 1975, p. 74.

(51).- Ibidem. p. 73.


Nacimiento de Cristo (250 millones)
0
476
490
Caída de Roma
Curva del Crecimiento de la Población Mundial.
1000
1500
1750
Revolución Industrial
6500 millones para el año - 2000
2000
1000
2000
3000
4000
5000
6000
Millones de Habitantes.

Periodo	Tasa de incremento demográfico (%)	Periodo	Tasa de incremento demográfico (%)
1920-1925	0.95	1945-1950	3.12
1925-1930	1.76	1950-1955	2.94
1930-1935	1.75	1955-1960	3.20
1935-1940	1.84	1960-1965	3.45
1940-1945	2.88	1965-1970	3.50

Fuente : CEPAL.- "Población y Desarrollo en América Latina".

Tabla No. 3.- TASAS DE INCREMENTO DEMOGRAFICO PARA LA REPUBLICA MEXICANA.

Es fácil advertir que LA VERDADERA EXPLOSION DEMOGRAFICA SE INICIA A PARTIR DE 1940, coincidiendo con el arranque de la industrialización acelerada del país y con el incremento sostenido -por más de 30 años- de la acumulación de capital y del producto nacional (este último con 6.3% anual en promedio en el período) (52) todos los cuales, dieron pábulo al llamado "milagro mexicano" y explicaría, de paso, el porqué ese milagro no nos sacó de la pobreza.

Lo más destacado en las funciones de crecimiento de la población, es que, siendo exponenciales, SE DUPLICAN A INTERVALOS CONSTANTES cuya magnitud depende del valor de la tasa anual del incremento. Tienen la forma matemática del interés compuesto y pueden expresarse de la siguiente manera :

$$Pn = Po\,(1 + k)^n \qquad (1)$$

en la cual :

Pn : población en el año "n".
Po : población en el año base.
k : tasa anual de incremento de la población.

---

(52).- Nacional Financiera.- Op. cit. pp. 30 a 34.

México tiene actualmente (1976), 59 millones de habitantes y nacen cada año, dos y medio millones de niños; 6 850 diariamente (53). La población se duplica cada 20 años (quince menos que la población mundial) y si permanecen las tendencias actuales (y pocas esperanzas hay de cambiarlas, pero, aun cuando se alteraran, no modificarían sustancialmente el fenómeno en el mediano plazo) la población mexicana alcanzará, los siguientes valores, a partir del censo de 1970.

Año	Población (millones)	Año	Población (millones)
1970	48.38	1995	114.33
1975	57.46	2000	135.79
1976	59.47	2005	161.28
1980	68.24	2010	191.55
1985	81.05	2011	198.25
1990	96.27		

Fuente : SIC. Censo de 1970 y proyecciones del autor.

Tabla No. 4.- PROYECCION DE LA POBLACION MEXICANA

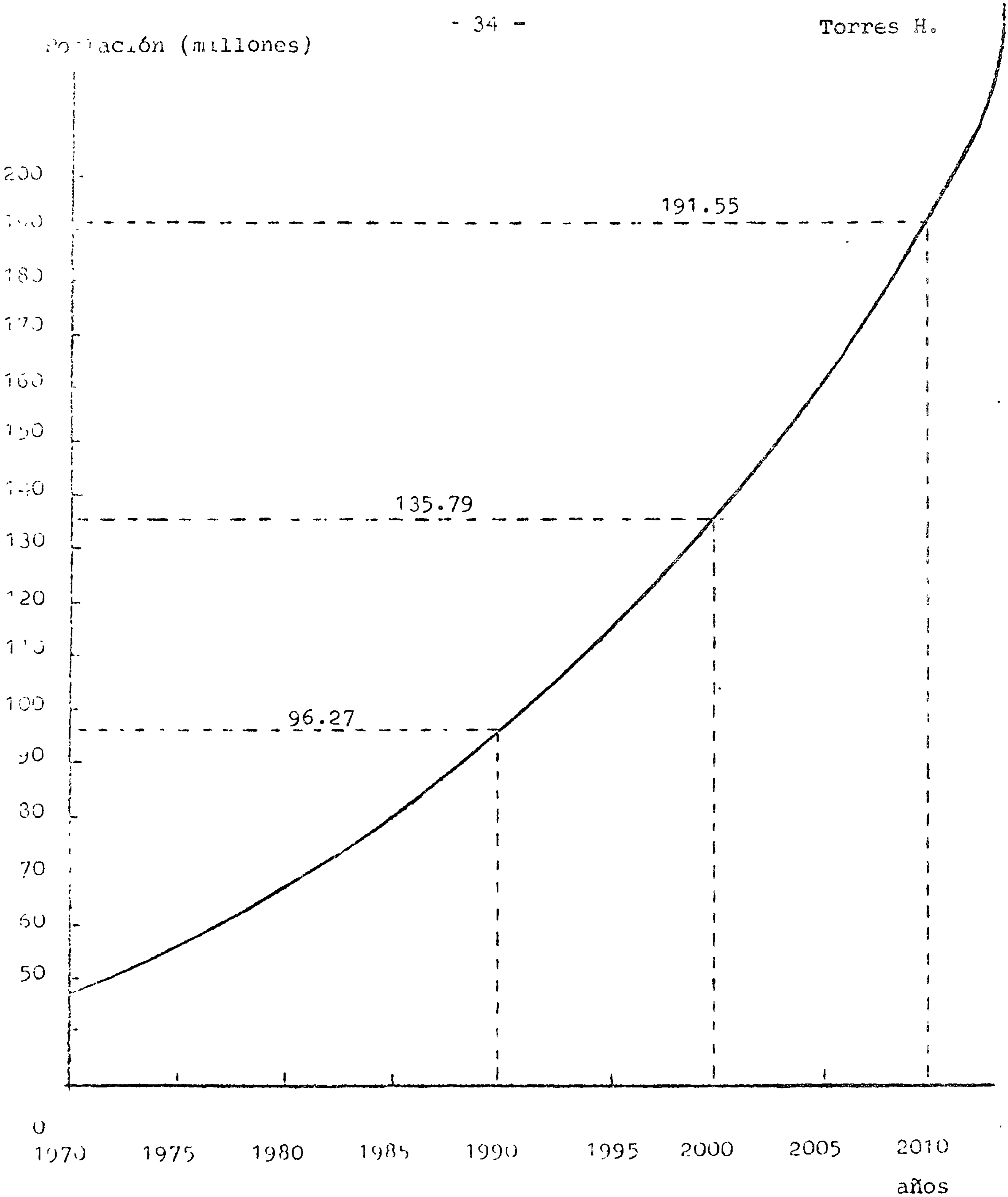
Con los datos de la tabla número 4, se dibujó la curva de la página 34.

Como no es lógico suponer que la población puede crecer indefinidamente (aunque sólo se restringiera en atención al cupo físico) es a todas luces recomendable establecer un límite razonable para su desarrollo. Quizás pudiera parecer adecuada una población más de 4 veces mayor que la de 1970; es decir, unos 200 millones de mexicanos. Pues bien, a pesar de que esa cantidad de ciudadanos nos pueda parecer, por ahora, exagerada, la población del país llegará a los 136 millones al finalizar el presente siglo y SOLO ONCE AÑOS DESPUES, en el año 2011, la población aumentará en 84 millones de personas y alcanzará bruscamente el límite propuesto.

---

(53).- Leal, Luisa María.- "Explosión Demográfica y Nuestra Responsabilidad Presente".- Folleto editado por la Secretaría de Gobernación, México, 1974, p. 2.

... 34



CURVA DE POBLACION PARA LA REPUBLICA MEXICANA .

Se consideró un incremento k = 3.5% anual.

... 35

Pero si no se señalara ningún límite, cuando menos conviene tener conciencia de la amenaza que se avecina, pues la población mexicana dentro de 74 años, en el año 2050 sería de 758 millones y aumentaría, cada año, en casi 27 millones de personas, TRECE VECES MAS APRISA QUE LA EXPLOSION ACTUAL.

EL AREA METROPOLITANA DE LA CIUDAD DE MEXICO.

Especial atención merece el "Area Metropolitana de la Ciudad de México" (*), tanto porque en ella vivimos, cuanto porque es una de las zonas urbanas con más rápida expansión en el mundo. Si México en su conjunto ostenta el primer lugar de la tierra por su crecimiento demográfico del 3.5% anual, su zona metropolitana capital desborda cualquiera competencia, pues casi lo duplica con el ¡ 5.7% — — anual ! (53). Esto quiere decir que SU POBLACION SE DUPLICA CADA 12 AÑOS y con ella, si se quiere mantener solamente el nivel de vida actual, se tendrían que duplicar a su vez, el área que actualmente ocupa, el ancho de sus calles; el servicio de transportes y viaductos; sus escuelas, bibliotecas, centros de servicio social, parques, servicio de agua y de energía eléctrica. ¡ Todos ellos duplicados en el lapso de dos períodos presidenciales !

Con esa expansión tan violenta resulta difícil -si no imposible- siquiera planear -no ya realizar- las obras de infraestructura urbana que demanda la población y es por eso tan frecuente comprobar el azoro de las autoridades municipales, ante los problemas que continuamente los desbordan.

El Area Metropolitana de la Ciudad de México alojaba, en 1970, una población de 8.621 millones de personas, lo que representaba el 17.82% de los habitantes totales del país y el 50% de la actividad industrial (54). Con el incremento demográfico tan extraordinario ya citado, de 7.5% anual, el Area Metropolitana daba cabida, a principios de 1975, a 11.375 millones de residentes, igualando al -

(*).- El "AREA METROPOLITANA DE LA CIUDAD DE MEXICO" comprende el Distrito Federal menos Milpa Alta y diez municipios del Estado de México, a saber : Atizapán de Zaragoza, Cuautitlán, Chimalhuacán, Ecatepec, Huixquilucan, La Paz, Naucalpan, Netzahualcóyotl, Tlalnepantla y Tultitlán.

(53).- Leal, Luisa María.- Op. cit. p. 3.

(54).- Gleason Galicia, Rubén.- "Demografía y Repercusiones Sociales". México, 1972, p. 18.- Ejemplar mimeografiado de circulación restringida.- El autor era, a la sazón, Director General de Estadística.

área metropolitana de la Ciudad de Nueva York, la cual contaba, a mediados de ese mismo año, con 11.5 millones de personas (55).

Para fines del presente siglo y considerando el mismo crecimiento que actualmente padece, el Area Metropolitana de la Ciudad de México alojará una población de 45.5 millones de personas, cantidad casi igual a la totalidad de los habitantes del país en 1970. Pero, por peligrosa y falta de sentido que esta enorme concentración demográfica pueda parecer, el absurdo engendrado por nuestra irresponsabilidad ante este ingente problema, se hace más patente si se recuerda que esa población habita a 2 300 metros sobre el nivel del mar y que, para dotarla de agua potable, sería necesario un caudal de 111 metros cúbicos por segundo, en el mejor de los casos, ya considerando una buena parte del consumo, ahorrado mediante la recirculación del líquido, o gracias al tratamiento primario de las aguas negras. Para este último fin, se deberá relocalizar la industria a lo largo del gran canal o sobre el trazo de la salida del drenaje profundo. Pero aún así, los 111 metros cúbicos por segundo, que se traerían de los ríos Cutzamala, Tecolutla y Amacuzac, deberán bombearse hasta la altura de la Ciudad de México, lo que supone contar con una capacidad eléctrica de 2 millones de kilovatios, correspondiente al 25% de toda la capacidad eléctrica actualmente instalada en el país.

Puede ser ya demasiado tarde para contener la macrocefalia de la Ciudad de México, pero si no se modifican sus actuales tendencias, para el año 2030, a sólo 54 años de la fecha actual -y por lo mismo, dentro de las expectativas de vida de gran parte de la juventud de hoy- la población del Area Metropolitana de la Ciudad de México alcanzará la cifra de 240 millones de personas y aumentará a razón de 14 millones por año.

Las cifras anteriores pueden no impresionar a nadie, pero indican la causa de casi todos los problemas que padece la capital, los cuales además, se agudizan con el transcurso del tiempo, toda vez que su solución demandaría inversiones tan cuantiosas, que desbordan las posibilidades del erario nacional.

3.23.- LA EXPLOSION DEMOGRAFICA, FRENO AL DESARROLLO ECONOMICO.

Quizás en ninguna otra forma se manifieste el agobio demográfico con tanta claridad, como a través del concepto de "TASA DE DESA

(55).- Midwest Research Institute; informe en la revista "Time", septiembre 29 de 1975, p. 25.

RROLLO" (incremento del producto per cápita ya establecido en la página 5 y el cual mide, con regular exactitud, el grado de enriquecimiento de una nación.)

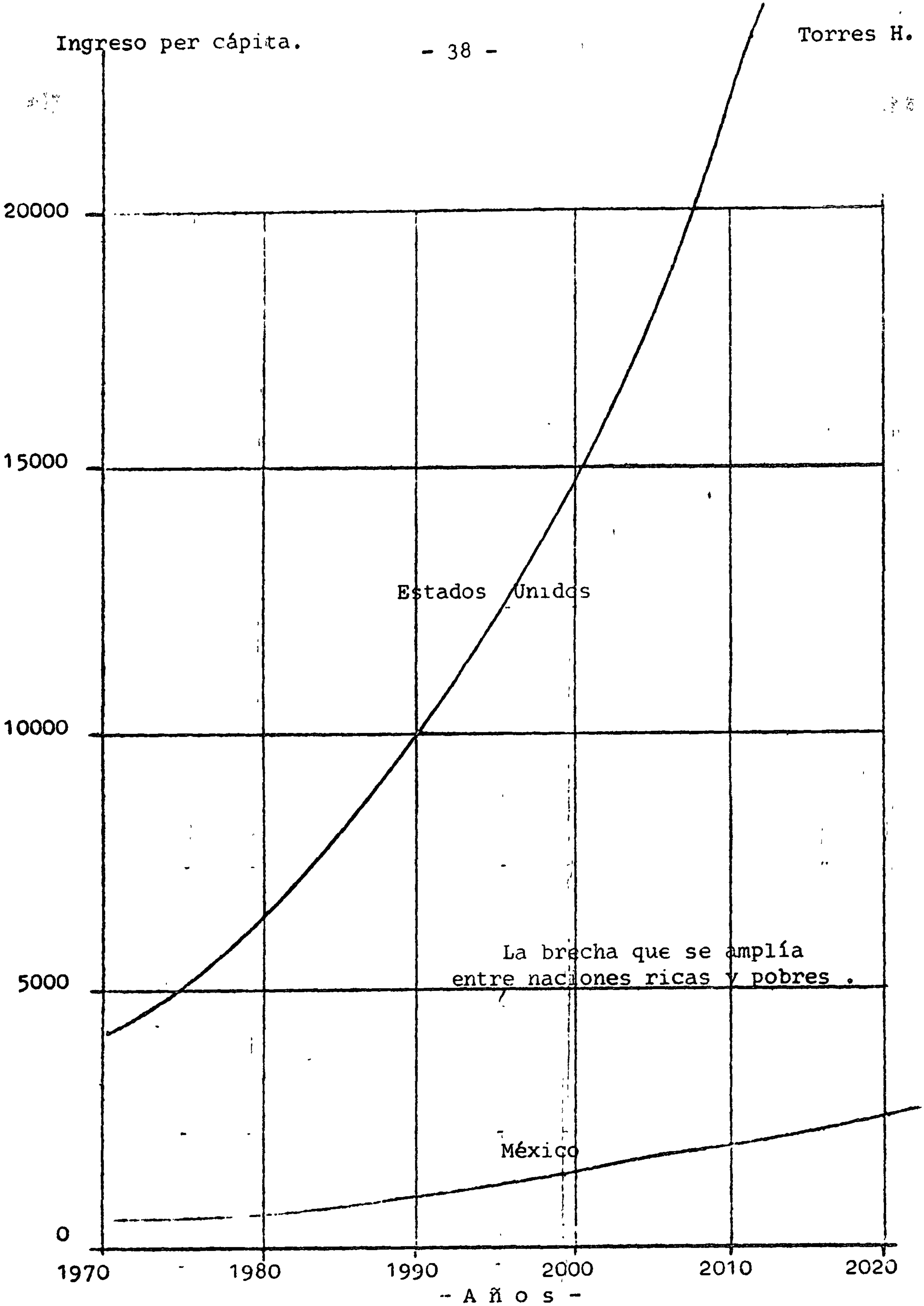
La expresión matemática de ese concepto, desarrollada anteriormente, es la siguiente :

TASA DE DESARROLLO = Incremento anual del (PNB) MENOS incremento anual de la población.

No la población, pero sí su incremento anual (es menester insistir en esto) se sustrae, se resta al aumento del nivel de vida, de enriquecimiento, que haya logrado una nación, a través del esfuerzo, en conjunto, de su pueblo.

La nación mexicana -como ya se apuntó- se ha distinguido por haber logrado -a lo largo de un periodo de 30 años- un incremento del 6.3% de su producto nacional, en promedio. Sin embargo, por culpa de su explosión demográfica desbordante (también sostenida con singular tesón) el incremento real de su producto per cápita, no ha excedido del 2.8% anual. Las naciones industrializadas, por su parte, consiguen incrementos de su producto menos impresionantes que nuestro país, pero como sus aumentos de población se mantienen, en general, por debajo del 1%, fácilmente obtienen valores elevados para su tasa de desarrollo y en esa forma, se sustenta el tan conocido fenómeno de la amplitud, cada vez mayor, que separa el ingreso de los países ricos, del correspondiente de los subdesarrollados. Un buen ejemplo (y clásico además) lo forma el incremento del ingreso per cápita de México y los Estados Unidos, el cual, para los años de 1968 a 1970 creció 5.0% en promedio para los Estados Unidos (56) mientras que para México, como ya se advirtió, puede tomarse el valor medio de 6.3%. De esa manera y restando el incremento de la población (0.9% para los E.U. y 3.5% para México) se obtiene una tasa de desarrollo de 4.1% para el primero y 2.8% para el último. Es decir, que aunque la economía general del país ha crecido más aprisa en México, por el peso de la explosión demográfica la posición se invierte y los Estados Unidos se desarrollan, en realidad, más rápidamente. Partiendo de un ingreso per cápita de 4 294 dólares para los Estados Unidos en 1970 y de 632 dólares para México en la misma fecha (57) y suponiendo que se mantenga la tendencia que señalan los incrementos citados (más difícil de cumplir para México, por lo elevado que resulta un incremento del 6.3% anual del producto en el largo plazo) las proyecciones del ingreso per cápita serían como lo ilustra la gráfica de la página 38, de la cual

(56).- Statistical Year book 1972.- Naciones Unidas; p. 622
(57).- Ibidem. p. 622.


Ingreso per cápita.
20000
15000
10000
5000
0
1970
1980
1990
2000
2010
2020
- A ñ o s -
Estados Unidos
La brecha que se amplía
entre naciones ricas y pobres .
México

se deduce que, si bien en 1970 la diferencia del ingreso per cápita entre los Estados Unidos y México, fue de 3.662 dólares, para el año de 1984 será de 5 585 y para el fin de siglo, alcanzará la cantidad de 12 885 dólares.

No hay que olvidar que riqueza y pobreza son términos contradictorios que se suponen el uno al otro; carece de sentido ser rico si no existen indigentes y es evidente que muchas de las naciones subdesarrolladas de la actualidad, tienen un nivel de vida muy superior al que imperaba en las naciones que realizaron, en el siglo XIX, la revolución industrial.

No debe pues sorprender, que al ampliarse, cada vez más, la brecha entre naciones ricas y pobres, los más distinguidos economistas de las grandes metrópolis (Estados Unidos e Inglaterra) manifiesten su euforia por haber vencido, en sus pueblos, la escasez; y "así, por la primera vez desde su creación, el hombre se encarará con su real, su permanente problema : cómo usar su independencia de las preocupaciones económicas, cómo llenar sus ocios, que le habrán conquistado la ciencia y el interés compuesto, para vivir prudente, agradablemente y bien" (58). "Este es un pasaje de profundo y profético contenido (...) Estamos ahora en la meta. En los 40 años transcurridos desde que Keynes escribió lo anterior, nuestro producto per cápita ha crecido, casi exactamente, a la tasa del 2% anual que Keynes había profetizado" (59).

Y ahora resta, sobre este tema, hacer una última aclaración: EL FRENO AL DESARROLLO ECONOMICO RADICA EN EL INCREMENTO DE LA POBLACION NO EN SU DENSIDAD. Japón (283 habitantes por kilómetro cuadrado) no tiene problema demográfico, ni lo tiene Bélgica (317 habitantes por kilómetro cuadrado) (60) pero en cambio sí lo tiene, y agudo, México, el cual, con sólo 30 habitantes por kilómetro cuadrado padece desnutrición en el 50% de su pueblo.

Un país tiene derecho a procrear únicamente el número de niños a quienes pueda alimentar, dar vivienda decorosa, educación eficiente, y a su tiempo, un empleo bien remunerado; menos de eso y principia la escalada de la miseria.

---

(58).- Keynes, J. M.- "Economic Possibilities for Our Grandchildren" Citado por Samuelson, Paul A.- en "Economics". Mc Graw-Hill Co., 1970, p. 776.
(59).- Samuelson, Paul A.- "Economics".- Mc Graw-Hill Co. 1970, p. 777
(60).- Statistical Year book 1972.- United Nations.- pp. 70 y 71.

centro de educación continua
división de estudios superiores
facultad de ingeniería, unam

MODELADO Y SIMULACION APLICADOS A LA PLANEACION

MODELADO Y SIMULACION DE INVENTARIOS

MARCIAL PORTILLA ROBERTSON

MARZO DE 1976.

Palacio de Minería
Tacuba 5, primer piso. México 1, D. F.
Tels.: 521-40-23 521-73-35 5123-123

MODELADO Y SIMULACION DE INVENTARIOS

1. Modelo del Sistema de Inventarios

No es difícil encontrarse Compañías Manufactureras, en las cuales el 25 % o más de total de su capital lo - tienen invertido en inventarios. Lockheed Aircraft Co. en Diciembre 1969 tenía $ 500,000,000 (us dls) en in-- ventarios, General Electric también en Diciembre de 1969 tenía $ 1,482,000,000 en inventarios, General Motors, en la misma facha tenía $ 3,700,000,000 en inventarios. - Hoy en día los directivos de empresas están conscientes del problema que representan los inventarios. En esta parte del curso, veremos como se plantea un modelo general de inventarios, un modelo para estimar el lote económico en los inventarios, así como dos programas de computadora que simulan lo anterior.

Es importante que los modelos de los sistemas de inventarios reflejen el incremento de los costos asociados a las diferentes políticas. Los siguientes costos afectan el modelo de inventarios:

Costo que Depende del Número de Lotes.

Al decidir el ~~el~~ tamaño o número de lotes a comprar, hay que recordar que existen gastos fijos internos de oficina, que permanecen constantes, es decir, estos -

gastos no varian con el tamaño del lote pedido, una gran parte de los costos totales de las operaciones de compra son costos fijos.

Costos de Producción.

Existen en los modelos de inventarios costos de producción que están ligados con el inventario, como son - costos de movimiento de materiales, limpieza y organización de los mismos. Además de estos existen otros cos-- tos que deben reflejarse en el modelo de inventarios como pu eden ser pago de tiempo extra, fluctuaciones en -- los costos de producción, costos efectuados en el balance del inventario.

Costos de Almacenamiento y Movimiento de Materiales.

Estos costos resultan también importantes en el modelo, pues mover materiales dentro y fuera del almacén "cuesta", además de otros costos como seguros de materiales almacenados, impuestos, renta,mermas y costos de capital.

Costos de"Falta de Material".

Este costo es extremadamente importante en el modelo. Este costo nunca aparece en los records, o balances del inventario: Por Ejemplo, sea la falta de algún material que ocaciona el paro total o parcial de la produc--

ción. La pérdida de algún cliente el cual no se pudo satisfacer. Lotes incompletos que no pueden salir al mercado, etc. En el segundo ejemplo de este curso se dará enfasís a este costo.

Costos de Capital.

El invertir en inventarios, resulta ser un incremento de costos de importancia en el diseño de modelos de inventarios.

Este costo sería el valor de la unitario de los materiales, el tiempo que los materiales estan en inventario, y la tasa de interés.

Objetivos de la Dirección o la Administración.

El objetivo de la dirección es miniminizar los costos descritos con anterioridad.

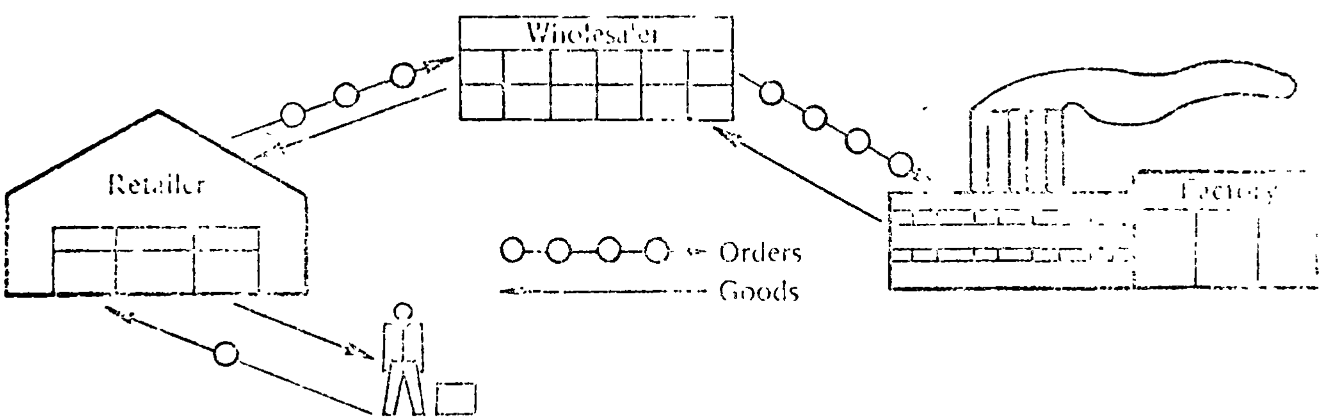
El costo en sistemas Producción-Distribución, es generalmente distribuido desde la entrada de la materia prima, a través de los pasos intermedios, hasta la venta. Por lo que si la dirección quiere obtener mayores ganancias y/o menor pecio de venta al público, tendrá que seguir una política de inventarios de acuerdo a ciertos objetivos.

Empezaremos el analisís de inventarios desde un punto de vista simple, a través de un ejemplo. Complicaremos la situación en el segundo ejemplo a tratar, y al final daremos 2 programas de computadora, en los cuales se simulan las situaciones de los ejemplos vistos. El primer caso a tratar es el modelo de un sistema de inventarios con una estructura simple.

El propósito de un inventario es proveer una separación en tiempo o localidad entre bienes de producción y bienes de consumo. En las economías modernas centros de producción especializada, con grande vólumen es de -- bienes de producción.

Los sistemas inventarios en nuestra economía funcionan de la siguiente manera: El mayorista provee una - separación en tiempo entre el menudeo y la fábrica. De la misma manera en la mayoría de los casos la fábrica -- provee un desacoplamiento en localidad, debido a que los productos se distribuyen en diferentes áreas geográficas.

El propósito de este ejemplo es ilustrar la naturaleza dinámica de un sistema de inventarios fábrica Mayo-- reo-Menudeo. Se utiliza un programa de computadora para simular el modelo. El usuario podrá en el modelo cambiar la política de inventarios en el menudeo y mayoreo, para controlar el sistema global.



I) MODELO FABRICA MAYOREO-MENUDEO

1. La función del menudeo es la siguiente (ver fig. 1)

   a) Toma órdenes de los clientes.

   b) Provee al cliente con bienes tomados del inventario (de los anaqueles).

   c) Reordena bienes del mayorista

   d) Recibe la entrega del mayorista.

2. La función del mayorista es semejante a la del menudeo, excepto en el cliente del mayorista es el menudeo y existe un retrazo en tiempo entre la orнena y entrega de bienes. El mayorista debe:

   a) Recibir órdenes del menudeo

   b) Mandar bienes del almacén del mayoreo al menudeo

   c) Reornenar de la fábrica.

   d) Recibir entregas de la fábrica.

3. Finalmente la fábrica debe:

   a) Producir bienes en cierto vólumen o cantidad

   b) Cambiar el vólumen de producción, según sea la demanda del mayoreo.

II) FORMULAS PARA EL MODELO MENUDEO

Las fórmulas aquí expuestas, es la interpretación matemática de lo escrito anteriormente.

Ventas de Menudeo.- Estas ventas estan controladas por el cliente, para este ejemplo vamos a considerar que las ventas a los clientes es de 100 unidades por semana.

Entradas de Bienes de Menudeo.- Son la cantidad de unidades recibidas (que manda el mayorista), cada lunes por la mañana y que fueron ordenadas el Viernes de la semana ante-anterior (10 días)

Nivel del Inventario de Menudeo.- Son el número de unidades que se tienen el viernes a las 12:00 hrs. al hacer el balance, este nivel varía com 0 10 muestra la fig. 2.

nivel del inventario = nivel anterior + entradas - ventas

Ordenes del Menudeo.- Estas ordenes son pedidas al mayorista cada Viernes por la tarde, después del balance, bajo la siguiente política.

Ordenes de menudeo = Ventas menudeo + (100-Nivel del inventario)

Por ejemplo suponga que en una semana "normal" se tuvo lo siguiente:

Ventas Menudeo = 100

Entrada de Bienes = 100

Nivel de Inventario = 100+100-100=100

Ordenes de Menudeo = 100+ (100-100) = 100

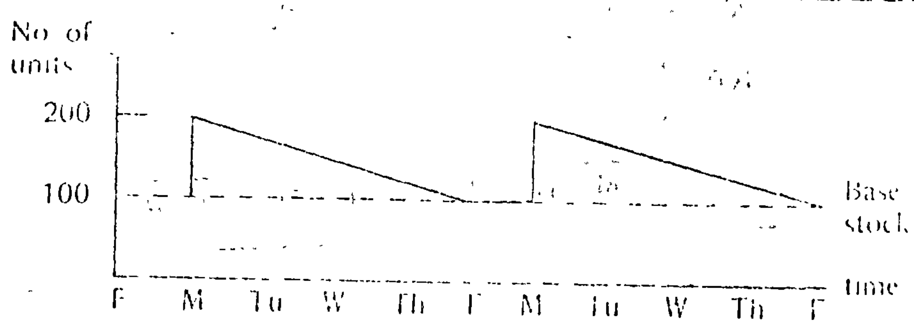
En una semana en la cual las ventas aumentan se tiene:

Ventas menudeo = 100

Entrada de bienes = 100

Nivel de Inventario = 100+100-110=90

Orden de Menudeo = 110 + (100-90) = 120



III FORMULAS PARA EL MODELO DE MAYORISTA

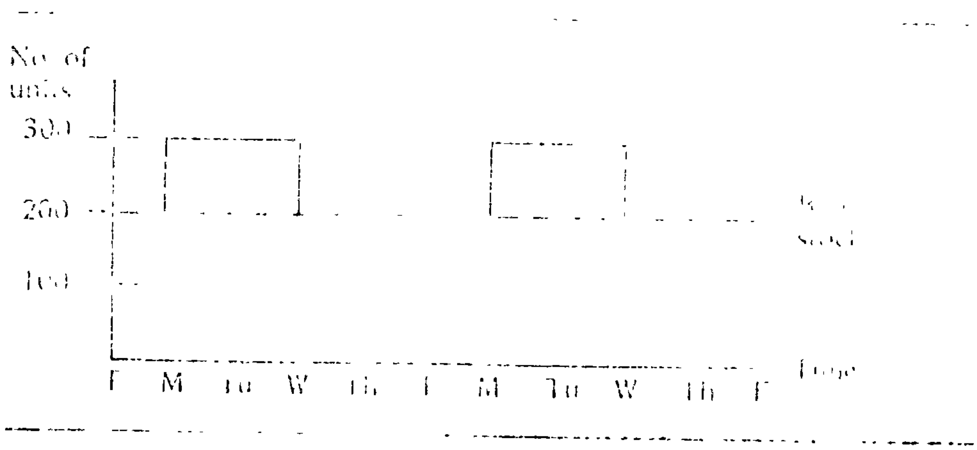
Envío de Materiales, Mayoreo.- Estas salen cada Jueves, de acuerdo a las órdenes de compra del menudeo, hechas el Viernes anterior, y es entregada el Lunes, (recibo de materiales).

Envío de Mayoreo = Ordenes Menudeo (Viernes anterior)

Recibo de Materiales Mayoreo.- Es la producción de la fábrica de la semana anterior y es recibida el Lunes por la mañana.

Recibo de Materiales = Producción de la Fábrica (semana anterior)

Nivel de Inventario Mayoreo.- Es el número de unidades después del balance del Viernes por la tarde, este nivel varía como se muestra en la fig. 3



La expresión siguiente nos determina el nivel del inventario el Viernes por la tarde.

Nivel de Inventario Mayoreo= Nivel Anterior+Llegadas-Salidas

Ordenes de Mayoreo.- Estas órdenes se mandan a la fábrica cada Viernes por la tarde, una vez hecho el inventario. La fábrica, requiere cambiar su vólumen de producción, osea que 2 semanas pasan hasta que el mayorista recibe su órden. La política a seguir es ordenar para la semana en curso + 200 unidades estos es:

Ordenes de Mayoreo = Envíos Mayoreo + (200-Nivel del inventario)

Sea por ejemplo en una semana "normal" se tendrá:

Envíos Mayoreos = 100

Recibo Mayoreo = 100

Nivel Inventario Mayoreo = 200+100-100=200

Ordenes de Mayoreo = 100 (+200-200) =100

En una semana en que los envíos "bajan"

Envios mayoreo = 90

Recibo Mayoreo = 100

Nivel Inventario Mayoreo=200+100-9=210

Ordenes de Mayoreo = 90 + (200-210) =80

Notemos que en presente modelo, el mayorista solo provee a un vendedor de menudeo, obviamente es una simplificación, pero creemos que los asistentes al curso no tendrán ninguna dificultad, para generalizar los resultados.

Vólumen de Producción de la Fábrica.

En este modelo la fábrica no mantiene inventario, la fábrica produce cierto vólumen, según sean las órdenes del mayoreo; sin embargo hay un retrazo de una semana, para -- que la fábrica cambie su vólumen de producción. Esto lo - podemos ver en la siguiente figura.

Week number	*Wholesale order*	*Factory rate*	*Wholesale receipts*
1	100	100	100
2	120	100	100
3	80	100	100
4	100	120	100
5	100	80	120
6	100	100	80

Política a Seguir.

La regla normal de compra es la siguiente: "ordene lo que vendió esta semana mas (o menos) otra cantidad para que el inventario tenga su nivel normal"

Por Ejemplo: si las ventas de menudeo son 100 en las semanas 1 y 2 y aumenta a 110 en las semanas 4,5,6 y 7 los resultados pueden observarse en la figura 5, y son calculados de la siguiente manera.

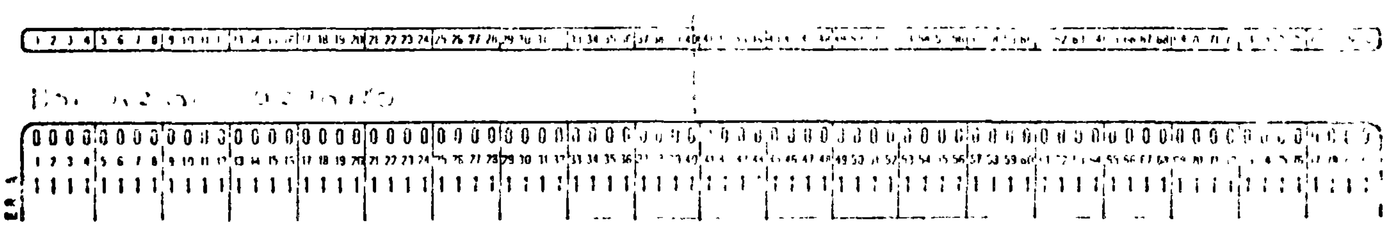
Ordenes Menudeo = Venta de la Semana + (100- Nivel de Inventarios)

Las órdenes de menudeo en la quinta semana son 120 unidades, osea que:

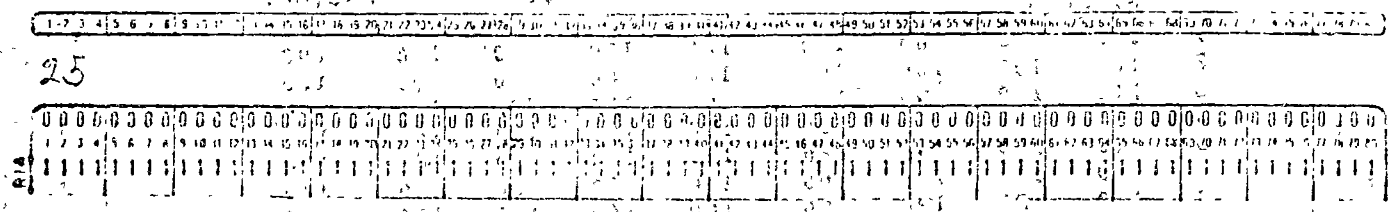
Ordenes Menudeo = 110 + (100 - 90) = 120

Entrada de datos en la Computadora.

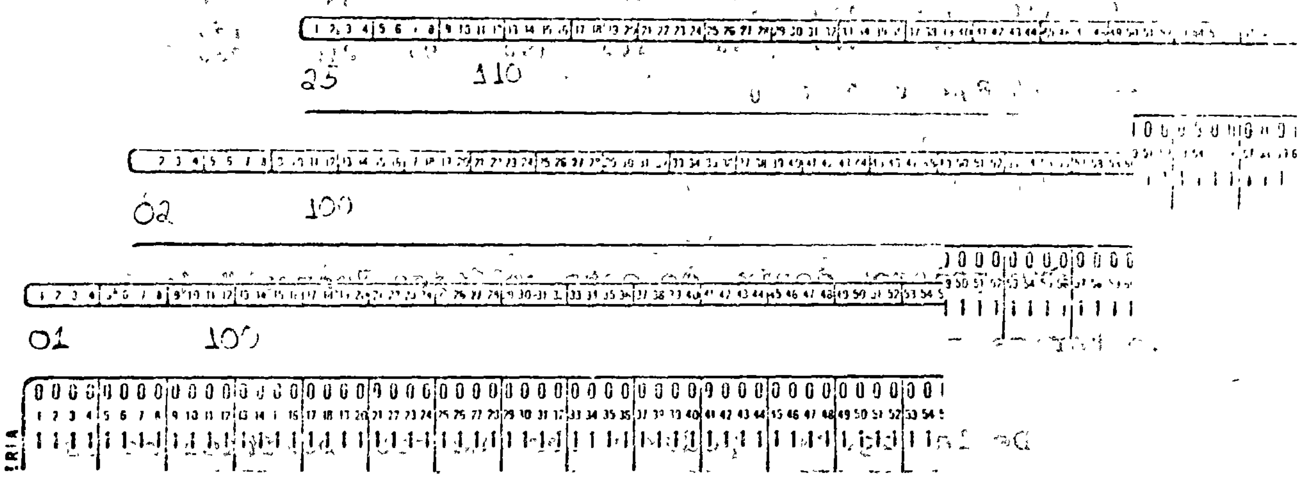
1a. Tarjeta (hasta 40 caracteres, empezando de la columna 1)



En la 2a. Tarjeta pondremos en las columnas 1 y 2 el número de semanas que se analizan.



De la 3a. Tarjeta (y 24 más) pondremos en las primeras 2 columnas el número de la semana, y en las columnas 11, 12 y 13 la venta en menudeo en esa semana



los resultados arrojados por la computadora se ven en la figura 7, en caso en que los datos sean erróneos, la computadora imprimirá el mensaje "algo esta mal en los datos"

PROGRAM INPUTS FOR EXERCISE ONE BY ROY HARRIS

WEEK	RETAIL				WHOLESALE				FACTORY
NO.	SALES	REC	INV	ORDER	SHIP	REC	INV	ORDER	RATE
1	100	100	100	100	100	100	200	100	100
2	100	100	100	100	100	100	200	100	100
3	110	100	90	120	100	100	200	100	[illegible]
4	110	100	80	120	120	[illegible]	190	140	[illegible]
5	110	120	90	120	[illegible]	100	150	180	[illegible]
6	110	130	110	100	120	100	130	150	[illegible]
7	110	120	120	90	100	140	170	130	180
8	110	100	110	100	90	100	260	30	150
9	110	90	90	120	100	190	350	0	130
10	110	100	80	130	120	130	360	0	30
11	110	120	90	120	130	30	250	70	0
12	110	130	110	100	120	0	140	160	0
13	110	120	120	90	100	0	40	200	70
14	110	100	110	100	90	70	20	270	180
15	110	90	90	1?0	100	170	100	200	250
16	110	100	80	130	120	200	240	80	270
17	110	120	90	120	130	270	300	0	200
18	110	130	110	100	120	200	460	0	60
19	110	120	120	90	100	80	440	0	0
20	110	100	110	100	90	0	350	0	0
21	110	90	90	120	100	0	250	50	0
22	110	100	80	130	120	0	150	190	0
23	110	120	90	120	130	0	0	330	50
24	110	130	110	100	120	50	0	320	190
25	110	120	120	90	100	190	90	210	330

25 WEEKS RUN 0 0 0 0

Que podemos decir de esta política "normal" de inventarios.

De la figura 7 podemos ver que esta política no es nada buena, puesto que una fluctuación del 10 %, en las ventas de menudeo trae consigo una fluctuación incontrolable en las ventas de mayoreo y vólumenes de producción. Observemos que en la semana 25 el menudeo no ha estabilizado su inventario 100 unidades, el mayorista tampoco, y el vólumen de la fábrica va a empezar a fluctuar, o sea que hay que controlar el vólumen de reórdenes.

CONTROL DE REORDENES

En esta sección consideramos el problema de controlar la fluctuación en un sistema de inventarios, cambiando las políticas del menudeo y del mayoreo. El concepto básico es el de cargar un amortiguado Esto se implementa cambiando la política de reorden haciendo recrecer la cantidad a surtir. La nueva política y el resultado de la computadora se muestran a continuación.

La política que habíamos seguido (con malos resultados) para el pedido de bienes en el menudeo era:

Ordenes de Menudeo = Ventas Menudeo + (100-nivel inventario).

Esta política parece a primera vista razonable, pero resulta ser miope, puesto que supone

a) Las ventas de la próxima semana serán las mismas que las ventas de la presente semana.

b) El inventario es resurtido de inmediato.

La primera suposición implica cierto riesgo para cualquier sistema, y la segunda resulta falsa en el modelo descrito.

Una manera de corregir lo anterior, es decir, amortiguar las oscilaciones (ver fig 7) es la siguiente

Ordenes Menudeo = Ventas Menudeo + (100-inventario) (A%)

En donde si A es 50 %, quiere decir que tratamos de corregir únicamente la mitad de la diferencia , veamos esto -- con un ejemplo en donde las ventas fueron de 110 unidades.

Política Vieja	Política Nueva
Orden Menudeo = 110 + (100-90)	Orden Menudeo = 100 +
= 120	(100 - 90) x .05 = 115

Si las ventas bajan tendremos

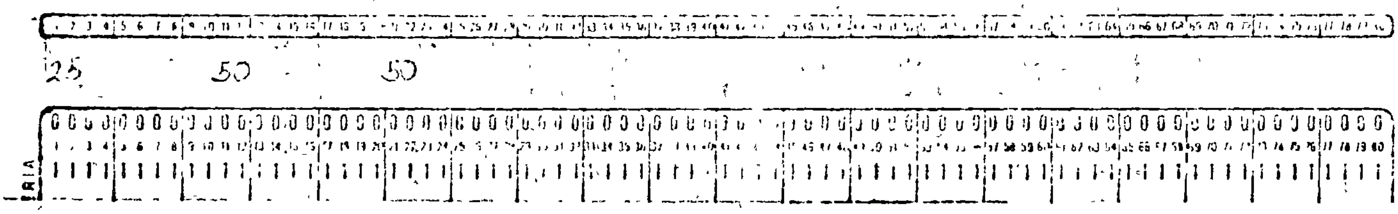
Política Vieja	Política Nueva
Orden Menudeo = 90+ (100-110)	Orden Menudeo = 90+ (100-110)
=90-10=80	x.05 = 85

El efecto de esta política es que únicamente reacciona parcialmente al cambio. El mayorista actuará de una manera semejante incluyendo un porcentaje B (B%) en la fórmula - de órdenes mayoreo.

La nueva política se implementa en el programa de la -- computadora de la siguiente manera:

en la 2a. Tarjeta, además del número de semanas - incluímos los porcentajes A y B y estos se perforan en las columnas 11-12 y 21-22 respectivamente, en caso de no utilizar estos porcentajes que pueden variar desde 1 % hasta 99%

la computadora tomará default 100 % para A y B.



los resultados obtenidos los podemos ver en las figuras siguientes.

EXERCISE TWO BY ROY HARRIS

25	50	50
01	100	
02	100	
03	110	
04	110	
05	110	
06	110	
07	110	
08	110	
09	110	
10	110	
11	110	
12	110	
13	110	
14	110	
15	110	
16	11	
17	11	
18	110	
19	11	
20	110	
21	11	
22	11	
23	11	
24	11	
25	11	

[illegible]

WEEK NO.	RETAIL SALES	REC	INV	ORDER	WHOLESALE SHIP	REC	INV	ORDER	FACTORY REC
1	100	100	100	100	100	100	200	100	[illegible]
2	100	100	100	100	100	100	200	[illegible]	[illegible]
3	120	100	90	115	100	100	200	103	100
4	120	100	80	[illegible]	[illegible]	100	[illegible]	[illegible]	[illegible]
5	110	115	85	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
6	110	120	95	113	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
7	110	115	[illegible]	[illegible]	113	[illegible]	[illegible]	[illegible]	[illegible]
8	110	113	105	106	109	138	166	116	[illegible]
9	110	107	104	105	103	144	223	94	[illegible]
10	110	[illegible]	101	109	108	134	270	84	[illegible]
11	110	105	99	110	105	116	254	87	[illegible]
12	110	107	99	111	110	96	270	91	[illegible]
13	110	110	99	110	111	84	216	[illegible]	82
14	110	111	100	110	110	82	185	[illegible]	90
15	110	110	100	110	110	97	165	[illegible]	[illegible]
16	110	110	100	110	110	104	159	13	116
17	110	110	100	110	110	115	167	120	127
18	110	110	100	110	110	127	185	118	130
19	110	110	100	110	110	130	205	107	126
20	110	110	100	110	110	126	221	99	[illegible]
21	110	110	100	110	110	118	229	96	117
22	110	110	100	110	110	107	226	97	99
23	110	110	100	110	110	99	216	102	96
24	110	110	100	110	110	96	201	103	97
25	110	110	100	110	110	97	188	116	102

25 WEEKS RUN 50 50 0 0

En esta Sección considera un control en el tiempo (control de adelanto en el tiempo).

El concepto básico es cambiar el tiempo requerido (adelanto) para que el sistema responda a los cambios. Fste concepto se implementa cambiando los tiempos de entrega del mayorista, así como el tiempo de cambio en los vólumenes de pro-- ducción.

CONCEPTO DE ADELANTO DE TIEMPO

Dentro de cirscunstancias normales los 2 adelantos de -- tiempo en el sistema son 1.- la órden y el recibo de bienes del menudeo y 2.- entre la órden y el recibo de bienes de la fábrica, y esto es:

ADELANTO NORMAL DE TIEMPO EN EL MENUDEO

Ordenado en	Entregado en
1a. semana Viernes	3a. semana Lunes.

ADELANTO NORMAL DE ~~ADELANTO~~ TIEMPO EN EL MAYOREO

Ordenado en	Cambio Vólumen	Bienes Entregados
1a. semana Viernes	semana 3	Lunes 4a. semana

Como notamos la fábrica toma 7 semanas para responder a un cambio en el menudeo.

Veamos que sucede si aminoramos el tiempo de adelanto cambiando la política.

REDUCCION DE TIEMPO EN EL MENUDEO

Ordenado en	Entregado en
Viernes 1a. semana	Lunes 2a. semana

Similarmente el tiempo de adelanto de mayoreo puede - cambiarse si la fábrica cambia su vólumen de producción sin tener una semana de retrazo y si la fábrica hace los envíos en el fin de la semana.

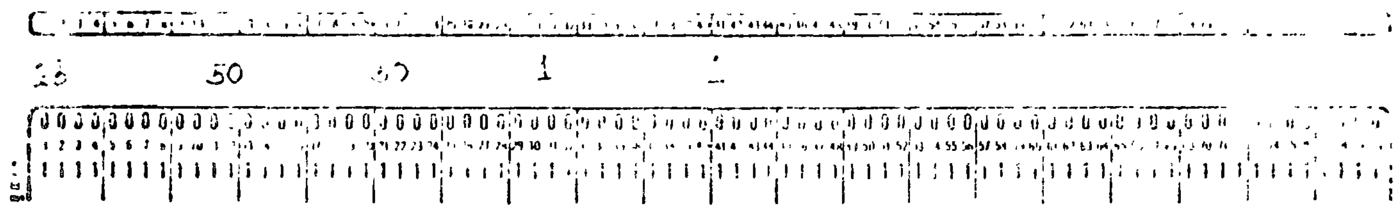
REDUCCION DE TIEMPO DE ADELANTO DEL MAYORISTA

Ordenado en	Cambio	Entregado en
Viernes 1a. semana	semana dos	tercera semana

En la computadora se hace de la siguiente manera:

El adelanto en semanas, para el menudeo se perfora en la columna 31 y para el mayoreo en la 41.

La tarjeta de control se verá así:



los resultados obtenidos se muestran en las siguientes figuras.

```
EXERCISE THREE BY ROY HARRIS
25          50          50          1          1
01          100
02          100
03          110
04          110
05          110
06          110
07          110
08          110
09          110
10          110
11          11
12          11
13          110
14          110
15          110
16          110
17          110
18          110
19          11
20          110
21          110
22          110
23          110
24          110
25          110
```

MODELO PARA ESTABLECER UNA POLITICA

DE REORDENES EN INVENTARIOS

El tópico de este modelo será el de resurtir inventarios, el modelo aunque fundamental establece el tamaño del lote económico. Primeramente se desarrollará el modelo básico, después introducirá el descuento en el precio en la compra; al final se introduciran el efecto debido a la falta de material, así como limitaciones fisícas del almacén y sus efectos en el inventario.

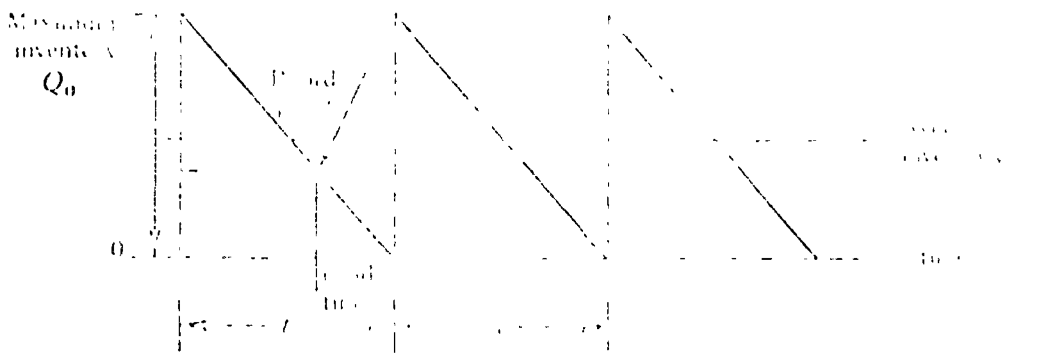
La única base que trataremos y bajo la cual estimaremos nuestro inventario es la económica. Existen costos asociados al tamaño (o vólumen) del inventario, impuestos, seguros, interés en el capital, etc. Existen también costos en que los cuales se incurre cada vez que se compra, (por ejemplo papelería, movimiento de materiales, etc.)

A través de este ejemplo veremos los efectos al cambiar las ordenes económicas.

LOTE ECONOMICO

En esta sección se introducen las bases para establecer el lote económico, así como los datos que hay que dar a la computadora y los resultados de la misma.

La primera suposición en el modelo, es que el consumo del inventario es constante a través del tiempo, y que se --



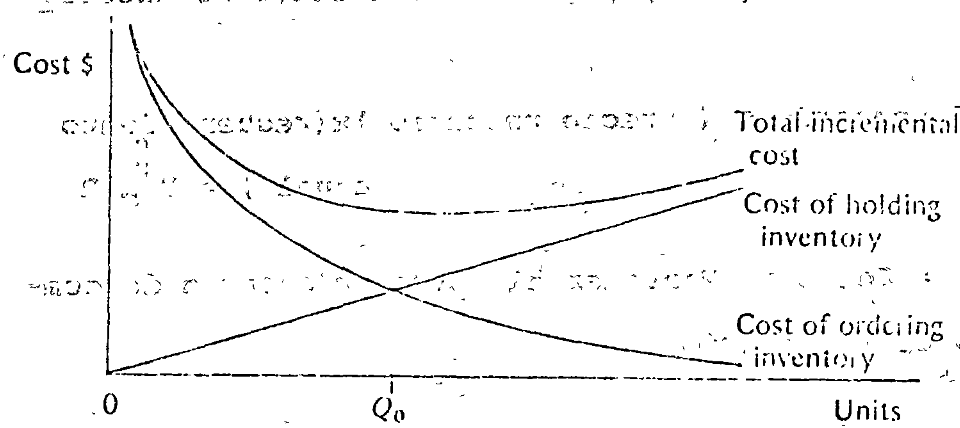
puede resurtir el inventario rápidamente, la cantidad de bienes en el inventario en cualquier tiempo puede verse en la figura 1.

COSTO DEL INVENTARIO

La base para determinar el nivel del inventario es la comparación entre costo de mantener un inventario y el costo de no mantener un inventario. El costo de mantener un inventario resulta ovbio, aunque no tanto el de mantenerlo aunque en este modelo se supone que el inventario se surte instantaneamente, parece a primera vista que no existe penalización si no se mantiene un inventario, este sería el caso del amacasa la cual va 3 veces a la tienda para cada comida que tiene que hacer, obviamente hay un costo al - adquirir bienes (además del plan del bien en si).

El punto a tratar es el de establecer un modelo cuantiativo que nos determine el vólumen del inventario a lo largo del tiempo. Tendremos básicamente 2 costos, el primero de ellos será el de mantener un inventario, y el segundo de ellos será el costo que se incurre al ordenar bienes (o productos), estas dos curvas las podemos ver en la figura 2, la tercer curva que aparece en la parte superior de la figura es el costo total incremental que se sostiene de sumar las 2 curvas anteriores.

Costo Total Incremental = Costo de Mantener el Inventario + Costo de Compra.



Costo de Mantener el Inventario = Promedio Inventario x(Costo Unitario de mantener el inventario por un año)

$$= \left(\frac{Q}{2}\right) \times (P \times Fh)$$

donde Q = cantidad ordenada

P = Precio unitario

Fh = costo anual unitario dado como porcentaje del precio unitario

Costo de Compra = (número de órdenes hechas por año) x (costo de cada órden)

$$= \left(\frac{R}{Q} \right) \times Cp$$

Donde:

R = unidades requeridas anualmente, (nivel de demanda)

Cp = Costo que se incurre al hacer cada compra

Este costo incluye (papelería, movimiento de materiales, etc)

Costo del Inventario = (precio unitario)x(requerimiento anual) = P x R

Costo total = Costo de Mantener el Inventario+costo de compra + costo de inventario

$$\text{Costo Total} = \frac{QxPxFh}{2} + \frac{RxCp}{Q} + PxR$$

$$\text{Costo total incremental} = \frac{QxPxFh}{2} + \frac{RxCp}{Q}$$

El punto económico es donde las curvas del costo de mantener el inventario y costo de compra se cortan, a este punto de intersección le llamamos Q.

$$\frac{Q}{2} \; (P \times Fh) = \frac{R}{Q} \; Cp.$$

$$Q \; (Q) \; (P \times FH) = 2R \times Cp$$

$$Q^2 = \frac{2RCp}{P \times FH}$$

$$Q = \sqrt{\frac{2R\ Cp}{PxFH}}$$

Otra manera de obtener el mismo resultado es igualando (1) a cero y derivando respecto a Q.

EJEMPLO

Se requiere determinar el lote económico dado

R = 1600 unidades (Requerimiento anual)

Cp = $ 5.00 (costo de una compra)

P = $ 1.00 (costo unitario del producto)

PH = 0.10 (costo unitario, mantener inventario)

$$Q = \sqrt{\frac{2x\ 1600 \times 5.00}{1.00 \times 0.1}} = 400 \text{ unidades}$$

$$\text{Costo Total (TC)} = \frac{400x1.00x0.1}{2} + \frac{1600x5.00}{400} + 1.00x1600$$

$$TC = \$\ 1640$$

Hasta ahora el modelo resulta bastante simple (y los calculos), pero mas adelante se analizarán varías alternativas.

PROBLEMA RESUELTO CON COMPUTADORA

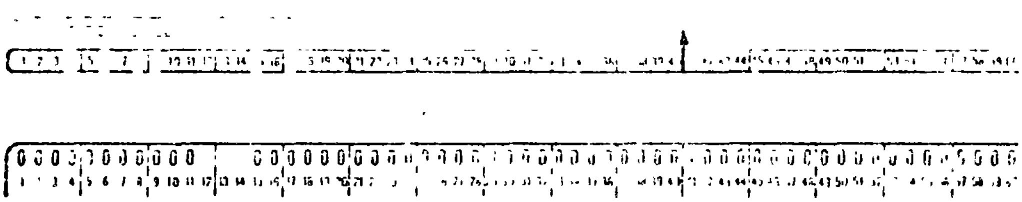
Resolvamos ahora el mismo problema utilizando el programa que se incluye.

Este programa es aplicable a sistemas de inventarios en los cuales la orden de compra y la cantidad son fijas, todas las cantidades son expresadas en anualidades, en este caso R. (requerimiento anual), se calcula a través del año, o sea, que hay que convertir las entradas de días, semanas o meses a la misma unidad de tiempo.

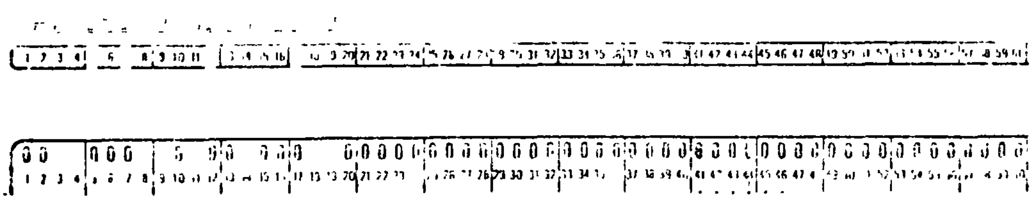
Este programa requiere además que costo de mantener un inventario, se exprese como porcentaje del valor unitario del inventario.

Se necesitan solamente dos tarjetas en este modelo.

La primera:



La segunda de datos:



La solución será

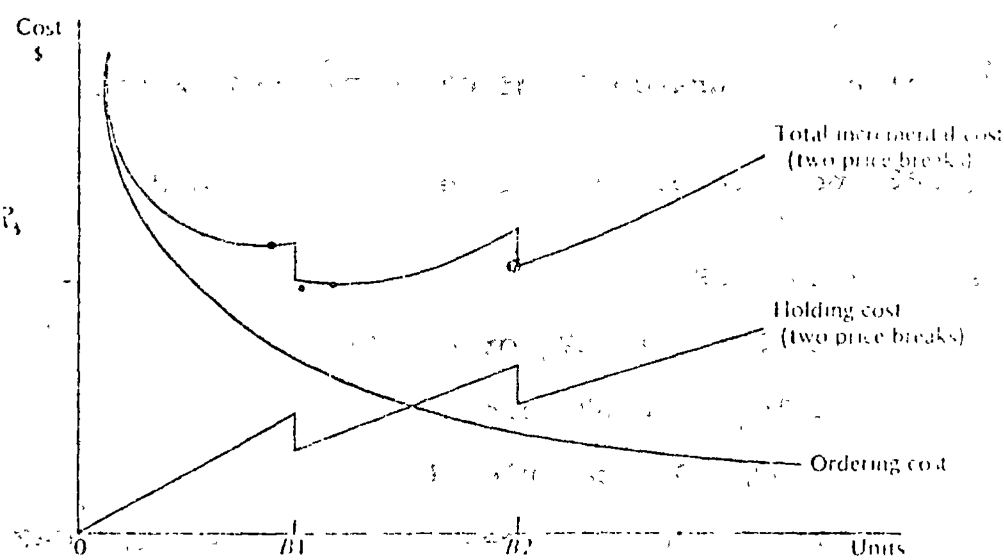
```
PROGRAM LOG FOR SABCARD LOQ PROBLEM ONE
INPUT DATA IS
    R      CP     FH     P1     CS     B1     P2     B2     P3     V
  1000   5.00    .10   1.00     0      0      0      0      0      0

ANALYSIS RESULTS ARE
 OPTIMUM ORDER QUANTITY IS                              400.00
 AT A PRICE PER ITEM OF                                   1.00
 YIELDING A TOTAL INVENTORY COST OF                    1040.00
 WHERE THE NUMBER OF ORDER CYCLES PER YEAR IS             4.00
```

DESCUENTOS

El modelo pasado no incluye descuentos en el precio debido a cantidades comprobadas, sin embargo dichos descuentos existen en la realidad.

El costo total incremental (TIC) se verá afectado por dichos descuentos, tal como se ve en la figura 5.



El camino a seguir para determinar el lote optimo consiste el examinar la curva en cada punto de descuento, y puntos aledaños, veamos esto con el siguiente ejemplo:

Ejemplo Dos

El proveedor examina su política de ventas y ofrece los siguientes descuentos. Si se ordena un lote de tamaño B1 ($Q_{B1} = 300$) el precio unitario será de 0.90 (P_2) si se ordena una cantidad B2 ($Q_{B2} = 2000$) el precio -- unitario será de 0.80 (P_3)

<u>Solución:</u> (con dos descuentos)

Primero calcule Q3 usando P3, si es mas grande que - QB2, ordene Q3, si es menos que QB2 no es una solución factible.

Segundo calcule Q2 usando P2 si Q2 > Q B2, ordene QB2

Si Q2 < QB2 pero mayor que QB1 ie QB1 < Q2 < QB2

Compare TC2 con TCB2

Si TC 2 > TC B2, ordene QB2

Si TC 2 < TC B2 ordene Q2

Si Q2 < QB1 calcule Q1

Si Q1 > QB1 entonces compare TC B1 con TCB2

Si TCB1 > TC B2 ordene QB2

Si TC B1 < TC B2 ordene QB1

Si $Q_1 < Q_{B1}$, compare TC_1 con TC_{B1} y con TC_{B2}

Ordene la cantidad correspondiente al mínimo costo total

$$Q_3 = \sqrt{\frac{2(1600)\ (5.00)}{0.80\ (\ 0.10\ )}} = 447.2 \text{ (usando } P_3 \text{)}$$

Como $Q_3 \not< Q_{B2}$ calcule Q_2 usando P_2

$$Q_2 = \sqrt{\frac{2\ (1600)\ (5.00)}{0.90\ (\ 1.10\ )}} = 421.6 \text{ (usando } P_2 \text{)}$$

Como Q_2 es menor que Q_{B2} (2000) pero mayor que Q_{B1} (300) compramos TC_2 con TC_{B2}

$$TC_2 = \frac{(\ 0.90\ )\ (0.10)\ (421.6)}{2} + \frac{1600(5.00)}{421.6} + 1600\ (.90)$$

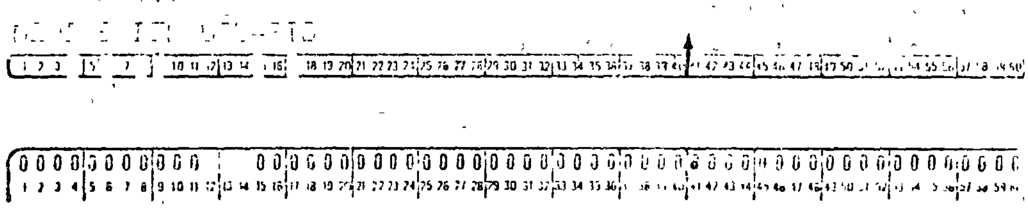
$$= \$\ 1477.98$$

$$TC_{B2} = \frac{0.80\ (\ 0.10\ )\ (300)}{2} + \frac{1600\ (5.00\ )}{2000} + 1600\ (0.80)$$

$$= 1364.00$$

Vemos que $TC_{B2} < Tc_2$ por lo que tenemos que ordenar cantidades B2 (Q_{B2} = 2000 a \$ 0.80/unidad)

<u>En la Computadora</u>



La segunda tarjeta sería.

COLUMNA	FORMATO	DATO
1-5	F5.0	Requerimiento anual
6-10	F5.0	Costo de Compra
11-15	F5.0	Costo de Mantener el Inventario
16-20	F5.0	Precio unitario
21-25-	F5.0	Costo de alta de material
26-30	F5.0	Cantidad minima de compra-1er.descuento
31-35	F5.0	Precio unitario 1er. descuento
36-40	F5.0	Cant. minima de compra-2o. descuento
41-45	F5.0	Precio unitario-segundo descuento

y los resultados serán:

```
PROGRAM EOQ FOR [illegible] EOQ PROBLEM TWO, PRICE DISCOUNTS
INPUT DATA IS [illegible]
      R     CP     CH     P1      CS     Q1     P2     Q2     P3
   1600   5.00    .10   1.00      0    300    .90   2000   .[illegible]

ANALYSIS RESULTS [illegible]
 OPTIMUM ORDER QUANTITY IS                                  2000.[illegible]
 AT A PRICE PER ITEM OF                                       .[illegible]
 YIELDING A TOTAL INVENTORY COST OF                         130[illegible]
 WHERE THE NUMBER OF ORDER CYCLES PER YEAR IS                 .[illegible]
```

COSTOS DEBIDOS A LA FALTA DE MATERIA EN INVENTARIO

Estas son debidas a que una orden no puede surtirse debido a la falta de material en inventario, y estos costos generalmente son:

A) posible baja en las ventas

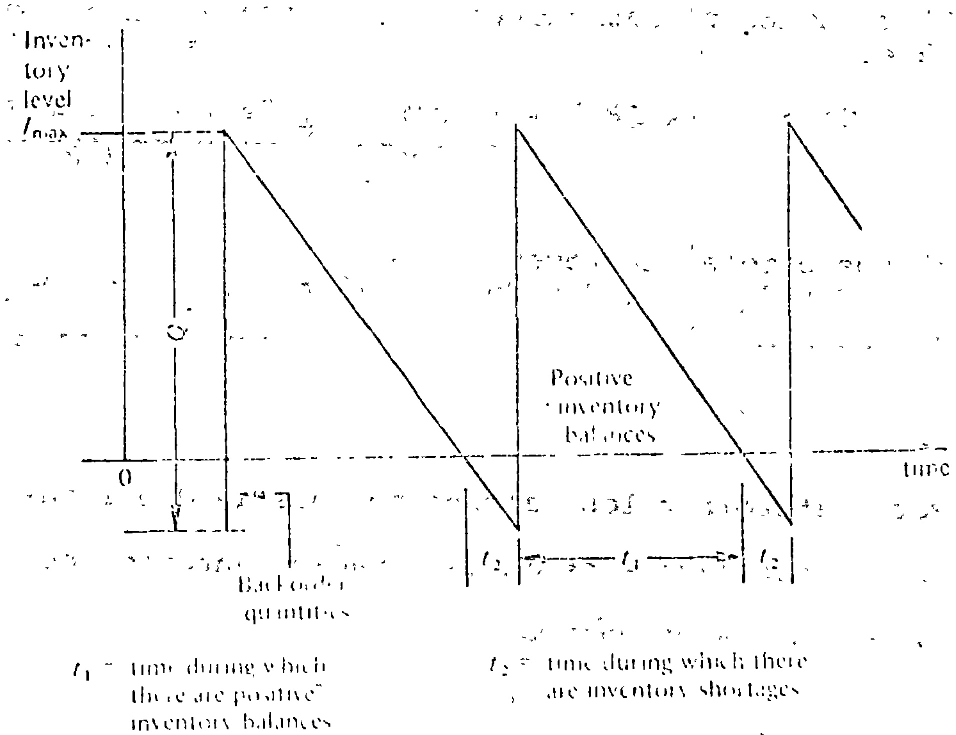
b) posible pérdida de clientes

c) no poder satisfacer pedidos urgentes

d) En caso de producción parar algún equipo debido a la falta de piezas para su reparación, etc.

e) tener lotes incompletos

Un inventario en el cual falta material se verá de la siguiente forma:



Las Fórmulas Básicas serán:

1. Cp-Costo que se incurre al hacer una compra (no cambia)

2. $$\frac{(P \times FH)\ I_{max}}{2} t_1$$

t_1 = Costo de mantener un inventario positivo

t_1 = tiempo en el cual se tiene un balance positivo

como

$$t_1 = \frac{I\ max}{R}$$

Imax - Nivel máximo de inventario

Podemos Escribir

$$\frac{(P \times FH)\ I^2 max}{2\ R}$$

3. $$Cs \times \frac{(Q - I\ max)^2}{2\ R}$$

Donde Cs= Costo por falta de Material.

El costo total incremental se obtiene en un ciclo t1 + t2 es

$$Cp + (P \times FH)\ \frac{(I^2 max)}{2\ R} + \frac{Cs\ (Q - I\ max)^2}{2\ R}$$

El costo total incremental será:

$$TIC = \frac{R \times C_p}{Q} + (P \times FH)\ \frac{(I^2 max)}{2Q}\ \frac{Cs\ (Q - Imax)^2}{2Q}$$

Para determinar los valores optimos de Q e I max, tomaremos derivados parciales de la ecuación anterior con respecto a Q, e I max y obtenemos:

$$Q = \sqrt{\frac{2RCp}{P \times FH}} \times \sqrt{\frac{(P \times FH) + Cs}{Cs}}$$

$$I\ max = \sqrt{\frac{2RCp}{P \times FH}} \times \sqrt{\frac{Cs}{(P \times FH) + Cs}}$$

$$TIC = \sqrt{2(P \times FH)\ RCp} \times \sqrt{\frac{Cs}{(P \times FH) + Cs}}$$

En la Computadora:

Lo único que hay que hacer en las columnas 21-25 es perforar el costo de falta de material, para el caso que se ha venido utilizando si Cs = $ 0.30.

Los resultados serán:

```
PROGRAM EOQ FOR BIGGARD EOQ PROBLEM THREE, SHORTAGE COST
INPUT DATA IS [illegible]
     R      CP      FH     P1      CS      H1      P2      H2    P3     W
  1000    5.00     .10   1.00     .30       0       0       0     0     0

ANALYSIS RESULTS ARE ****
 OPTIMUM ORDER QUANTITY IS                                   461.[illegible]
 WITH OPTIMUM INVENTORY OF                                   3[illegible]
 AT A PRICE PER ITEM OF                                      1.[illegible]
 YIELDING A TOTAL INVENTORY COST OF                          163[illegible]
 WHERE THE NUMBER OF ORDER CYCLES PER YEAR IS                [illegible].[illegible]6
```

centro de educación continua
división de estudios superiores
facultad de ingeniería, unam

MODELADO Y SIMULACION APLICADOS A LA PLANEACION

GENERACION DE VARIABLES ALEATORIAS

M. en C. JORGE RIVERA BENITEZ

Palacio de Minería
Tacuba 5, primer piso. México 1, D. F.
Tels.: 521-40-23 521-73-35 5123-123

INTRODUCCION

1. VARIABLES ALEATORIAS

2. FUNCIONES DE DENSIDAD DE PROBABILIDAD

   2.1 Funciones de densidad de probabilidad discretas: uniforme, binomial, Poisson

   2.2 Funciones de densidad de probabilidad contínuos: uniforme, exponencial, normal.

3. VALOR ESPERADO.

4. GENERACION DE VARIABLES ALEATORIAS.

5. METODO DE MONTECARLO.

APENDICES

A.1 SUMARIO DE ALGUNAS FUNCIONES DE DENSIDAD DE PROBABILIDAD CONTINUAS Y DISCRETAS.

A.2 MEDIDAS DE TENDENCIA CENTRAL Y DISPERSION, HISTOGRAMAS, PROGRAMAS DE COMPUTADORA. EJEMPLOS.

A.3 BONDAD DE AJUSTE DE DATOS EMPIRICOS A DISTRIBUCIONES UNIFORME, NORMAL, EXPONENCIAL Y POISSON PROGRAMA DE COMPUTADORA. EJEMPLOS.

A.4 MODELOS DE RIESGO A TRAVES DE ARBOLES DE DECISION Y ANALISIS DE BAYES. PROGRAMA DE COMPUTADORA. EJEMPLOS.

A.5 MODELOS DE INCERTIDUMBRE: METODOS DE LAPLACE, HURWICZ, SAVAGE Y WALD PROGRAMA DE COMPUTADORA. EJEMPLOS.

FENOMENOS ALEATORIOS

Los diferentes fenómenos que se presentan en el mundo real, ya sean físicos, biológicos, económicos, sociológicos, etc., se pueden clasificar dentro de dos grandes grupos: los fenómenos determinísticos y los fenómenos aleatorios * . A este último grupo de fenómenos pertenecen la mayoría de los fenómenos mencionados.

Un fenómeno determinístico es aquel que se caracteriza por la propiedad de que al realizarlo bajo condiciones similares se obtienen los mismos resultados. Algunos ejemplos de fenómenos determinísticos son:

1. Observar el color de una bola extraída de una que contiene solamente bolas blancas. Al ocurrir este fenómeno siempre observaremos una bola blanca.

2. Determinar el resultado de la suma de dos números pares. Por teoría de los números sabemos que si sumamos dos números pares, la suma de ellos es par, por lo tanto el resultado observado siempre seró par cada vez que realicemos el experimento de sumar dos números pares.

A diferencia de los fenómenos determinísticos, los fenómenos aleatorios se caracterizan por el hecho de que al realizarlos, bajo condiciones similares, nunca se observa que todos los resultados sean iguales sino solo se advierte una frecuencia de ocurrencia de cada uno de ellos. A esta frecuencia de ocurrencia se le llama regularidad estadística. Se acostumbra cuantificar a la regularidad estadística de cada uno de los resultados por un número entre cero y uno, en forma tal que representa la frecuencia de ocurrencia del resultado considerado. Entre los numerosos fenómenos que son fenómenos aleatorios se encuentran los siguientes:

---

*También son llamados fenómenos al azar, probabilísticos ó casuales.

1. Observar la cara de la moneda, al tirarla desde una altura determinada.

   En este fenómeno cada vez que tiremos la moneda podemos tener dos posibles resultados: águila ó sol.

2. Observar el número obtenido al tirar un dado.

   Los resultados posibles son: 1,2,3,4,5,6.

3. Registrar el número de accidentes de tránsito en la intersección de dos avenidas, entre las 7 y 8 de la mañana. Los posibles resultados serían : 0,1,2,3, ...

4. Registrar la demanda semanal de un artículo.

5. Observa diariamente el precio de un comestible.

6. Analizar el contenido de niquel en cierto tipo de acero.

7. El número de cuentas bacterianas en un producto alimenticio.

8. El número de defectos encontrados en la fabricación de un medidor de presión.

9. Determinar el número de personas que llegan a un cierto servicio como un supermercado, ó un aeropuerto, etc.

10. Observar el tiempo de vida de una máquina, un problema de interés asociado a este fenómeno aleatorio es determinar el tiempo óptimo para reemplazar la máquina.

PROBABILIDAD.

El estudio de los fenómenos determinísticos se realiza en los propios campos de los que surgen, así, si la temperatura de ebullición de una substancia se considera determinístico entonces el estudio de la temperatura y de sus implicaciones que tenga será dentro de la física ó físicoquímica.

El objetivo de la probabilidad es el estudio de los fenómenos aleatorios con el fin de predecir su comportamiento en un tiempo futuro y así poder elegir la acción más apropiada para hacer frente a las implicaciones que tenga el fenómeno aleatorio bajo estudio, así por ejemplo, si el fenómeno aleatorio estudiado es el número de personas que arriban a un cierto servicio, nos interesa conocer su comportamiento futuro ó incluso presente para después decidir qué número de servidores son necesarios para atender esa demanda de servicios. Si el fenómeno aleatorio de interés es observar el porcentaje de niquel de un envío de materia previa nos interesa conocer el número de muestras que debemos tomar y después tener un criterio para decidir si este producto cumple con la calidad especificada en el contrato de compra-venta.

Entonces con la ayuda de probabilidad conoceremos el comportamiento del fenómeno aleatorio pero no las implicaciones del sistema del que forma parte. Por lo tanto en un sistema complejo en el que están involucrados uno ó varios fenómenos generalmente aleatorios, el objetivo del analista de sistemas no es solo conocer el fenómeno aleatorios, el objetivo del analista de sistemas no es solo conocer el fenómeno aleatorio sino las implicaciones que tiene al tomar una decisión para resolver el sistema. Por ejemplo, considere que el sistema de interés es un sistema de inventarios en el que debemos decidir cuando surtir nuestro almacén y cuanto ordenar para que el nivel de inventarios en el almacén sea tal que minimice los costos de inventarios. En este sistema interviene un fenómeno determinístico ó aleatorio (generalmente aleatorio) que es la demanda, el cual forma parte del sistema y es un factor importante en la solución del sistema. Por lo tanto, el analista de sistema le interesa la probabilidad solo como una herramienta para conocer las propiedades probabilísticas de su demanda, pero su papel principal será encontrar un modelo matemático que represente su sistema de inventarios y la solución del mismo.

Por esta razon uno de los objetivos de estas notas es dar el conocimiento prác- tico necesario que requiere un análista de sistema en la modelación y en particular cuando el modelo elegido es un modelo de simulación. También se presentará el método de Montecarlo como un método de simulación de variables aleatorias independientes con una función de probabilidad preespecificada.

La teoría de probabilidad nos presenta medios para representar y conocer un fenómeno aleatorio. Sin embargo una vez estudiado las características del fenómeno, y posiblemente habiendo elegido alguna función de probabilidad que proporciona la frecuencia de ocurrencia de cada uno de los posibles resultados, será necesario estimar los parámetros de la función de probabilidad elegido. Este problema de estimación corresponde al campo de la Estadística.

ESTADISTICA.

En general la estadística se divide en dos ramas : la estadística descriptiva y la inferencia estadística. La estadística descriptiva comprende técnicas de agrupación, representación y calculo de ciertas medidas de interés de un conjunto de datos obtenidos al observar un fenómeno aleatorio. Algunas medidas básicas son la media, variancia, Etc. En el apéndice A.2 se presentan estos conceptos, así como un programa de computadora para su cálculo. La inferencia estadística proporciona técnicas que en base a un conjunto de datos experimentales proporcionan las características desconocidas de un fenómeno aleatorio. Algunos problemas concernientes, a la inferencia estadística son : estimación de parametros de la función de distribución que sigue un fenómeno aleatorio, pruebas de hipótesis, regresión, análisis de variancia, control de calidad, Etc. Uno de los problemas importantes es el llamado Bondad de Ajuste, el cual es un criterio para determinar si un conjunto de --

datos sigue una función de probabilidad preespecificada. En el apéndice A.3 se presenta la técnica de Bondad de Ajuste de datos empíricos a distribuciones uniforme, normal, exponencial y de Poisson, así como un programa de computadora para llevarlo a cabo.

En los apéndices A.4 y A.5 se presentan algunos modelos de riesgo y de incertidumbre, respectivamente, con el fin de presentar aspectos probabilísticos en la formulación de algún tipo de modelos, usando un modelo distinto al de simulación.

1. VARIABLES ALEATORIAS

DEFINICION 1.1 El espacio muestra de un fenómeno aleatorio es el conjunto de todos los posibles resultados que se pueden obtener al realizar el fenómeno. El espacio muestra se indicará por Ω. Otra notación que se acostumbra usar para el espacio muestra es S. Los resultados, o sea los elementos de Ω, se indicarán por ω.

EJEMPLOS.

1. Si el fenómeno aleatorio consiste en tirar una moneda desde una altura determinada, entonces los posibles resultados son aguila (que será abreviada por a) y sol (abreviado s), ie.,

$$\Omega = \{\omega : \omega = a, s\} = \{a, s\}$$

2. Si observamos el número obtenido al tirar un dado entonces su espacio muestra es

$$\Omega = \{\omega : \omega = 1,2,3,4,5,6\}$$

3. Si el fenómeno aleatorio consiste en observar el número de accidentes de transito en la intersección de dos avenidas durante las 7 y 8 de la mañana entonces

$$\Omega = \{\omega : \omega = 0,1,2,3,\ldots\}$$

4. El espacio muestra correspondiente a la demanda semanal de un artículo podría ser

$$\Omega = \{\omega : \omega = 0,1,2,3,\ldots\}$$

o en caso de que la región tuviera 1000 habitantes, el espacio muestra sería

$$\Omega = \{\omega : \omega = 0,1,2,3,\ldots,1000\}$$

5. Para el precio de un artículo determinado, el espacio muestra seria

$$\Omega = \{\omega : \omega \text{ es un número real positivo o cero}\}$$

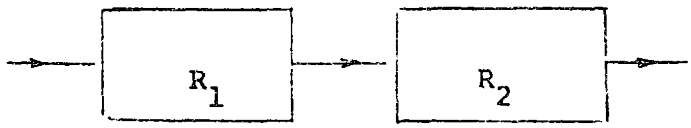
$$\Omega = \{\omega : \omega \geq 0\}$$

6. Si observamos el tiempo de vida de una máquina, éste podría ser cualquier numero real positivo, ie.,

$$\Omega = \{\omega : \omega \geq 0\}$$

El posible resultado $\omega = 0$, significa que la máquina estaba defectuosa cuando fue adquirida.

7. Considere el diagrama que muestra dos reactores de etileno y cloro-etileno conectados en serie.



Suponga que cada reactor puede operar o no operar en un tiempo determinado, y que el fenómeno aleatorio de interés es observar las condiciones en que se encuentra el sistema formado por los reactores en serie.
Defina Z_1 como una variable que nos representa el estado del reactor 1, y Z_2 otra variable que representa el estado del reactor 2. Si definimos Z_i por

$$Z_i = \begin{cases} 0 \text{ si el reactor i no opera} \\ 1 \text{ si el reactor i opera} \end{cases} \qquad (i=1,2)$$

entonces el estado del sistema esta dado por el par $\omega=(Z_1,Z_2)$, y el espacio muestra será

$$\Omega = \{(Z_1,Z_2): (0,0),\ (0,1),\ (1,0),\ (1,1\ \}$$
$$\Omega = \{(0,0),\ (0,1),\ (1,0),\ (1,1)\}$$
$$\Omega = \{\ \omega^1\ ,\ \omega^2,\ \omega^3,\ \omega^4\}$$

donde $\omega^1=(0,0)$, $\omega^2=(0,1)$, $\omega^3=(1,0)$ y $\omega^4=(1,1)$ son puntos de un espacio bidimensional.

DEFINICION 1.2 Sea Ω el espacio muestra de un experimento. Un evento del experimento, es cualquier subconjunto de Ω. Los eventos se indicarán por letras latinas mayusculas.
NOTA. Se dice que el evento A ocurre si cualquier elemento de A ocurre, ie., si $A=\{1,2,3\}$ es un subconjunto de un espacio muestra y al observar un experimento se obtiene $\omega=2$, entonces el evento A ocurre, o sea que para que el evento A ocurra no se necesita que cada uno de los elementos de A occurra.
EJEMPLOS.
1. Para el ejemplo 1 mencionado anteriormente, con $\Omega=\{a,s\}$, los subconjuntos

$$A=\{a\}, \qquad B=\{s\}, \qquad C=\emptyset, \qquad D=\Omega$$

son eventos. El subconjunto $A=\{a\}$, representa el evento de obtener un aguila. El subconjunto $B=\{s\}$, representa el evento de obtener un sol. El subconjunto $C=\emptyset$, el vacio, representa el evento de que moneda desaparece o que caiga de canto, o que ruede indefinidamente. El subconjunto propio $D=\Omega$, llamado el evento seguro, representa la seguridad de que el experimento (o fenómeno aleatorio) se llevo a cabo.

2. Algunos eventos del ejemplo de la tirada de un dado, con espacio muestra $\Omega=\{1,2,\ldots,6\}$, son

$$A=\{1,3,5\}, \qquad B=\{2.4,6\}, \qquad C=\{3,4,5,6\}$$

El subconjunto A representa el evento de obtener un número par, B el evento de obtener un número par, C obtener un número mayor que dos, etc.

3. Para el sistema de reactores mostrado en el ejemplo 7, defina los eventos A y B como sigue

A es el evento de que el sistema de reactores opere

B es el evento de que al menos uno de los reactores opere

Exprese A y B como subconjuntos de Ω, dando sus elementos. Encuentre A^c y B^c.

SOLUCION.

$$A=\{(1,1)\}, \qquad B=\{(1,0),\ (0,1),\ (1,1)\}$$

$A^c=\{(0,1),\ (1,0),\ (0,0)\}$ representa que el sistema no funciona, ya que están en serie

$B^c=\{(0,0)\}$ representa que ninguno de los reactores funciona

DEFINICION 1.3 Se dice que dos eventos son mutuamente exclusivos si no ocurren simultaneamente, ie., si su intersección es el vacio.

EJEMPLO. Sea $\Omega=\{1,2,3,4,5,6\}$ el espacio muestra de la tirada de un dado. Sea A el evento correspondiente a aquellas tiradas que muestren un número menor o igual a 4, sea B que muestren un número mayor de 3, y sea C el evento equivalente a obtener el número 5. Entonces,

$$A=\{1,2,3,4\}, \qquad B=\{4,5,6\}, \qquad C=\{5\}$$

Los eventos A y B no son mutuamente exclusivos ya que pueden ocurrir simultáneamente, porque el elemento 4 es común a ambos conjuntos, $A \cap B = \{4\} \neq \phi$

Para los eventos A y C, se tiene que $A \cap C = \phi$, por lo tanto son mutuamente exclusivos. B y C no son mutuamente exclusivos porque $B \cap C = \{5\} = \phi$.

DEFINICION 1.4 Se dice que dos eventos elementales w_i y w_j son igualmente probables si se espera que cada uno de ellos ocurra con igual frecuencia cuando se repite el experimento un gran número de veces, es decir, si existe el mismo número de oportunidades para que ocurra cada uno de ellos.

EJEMPLO. Si $\Omega = \{a, s\}$ es el espacio muestra el juego de volados, entonces $w_1 = a$ y $w_2 = s$ son igualmente probables ya que una moneda solo tiene dos cantos marcados: uno para águila y otro para sol. Por lo tanto, el número de oportunidades para obtener sol - (ó sea el número de veces que está acuñado el sol en la moneda) es igual al número de -- oportunidades para obtener águila.

DEFINICION 1.5 (DEFINICION CLASICA DE PROBABILIDAD). Sea Ω el espacio muestra de un fenómeno aleatorio que puede ocurrir en n caminos mutuamente exclusivos e igualmente probables. Si el evento A puede ocurrir en n_A de esos n caminos entonces la probabilidad del evento A, indicada por P(A), se define por

$$P(A) = \frac{n_A}{n}$$

EJEMPLO. Sea $\Omega = \{1,2,3,4,5,6\}$ el espacio muestra en la tirada de un dado. Si - $A = \{1,3,5\}$ es el evento de obtener un número par entonces $n_A = 3$ y $n = 6$, po lo tanto.

$$P(A) = \frac{3}{6} = \frac{1}{2}$$

Si $B = \{1,4\}$ entonces

$$P(B) = \frac{2}{6} = \frac{1}{3}$$

NOTA. La definición clásica de probabilidad está restringida a determinado tipo de fenómenos aleatorios que reuna los condiciones dadas en la definición. Las desventajas que tiene son:

i. Solo se aplica a espacios muestra finitos

ii. Define probabilidad en términos del concepto de eventos elementales igualmente probables, ie hay una redundancia al definir probabilidad en términos de la misma palabra.

iii. Está restringida a problemas con eventos elementales igualmente probables.

DEFINICION 1.6 (DEFINICION FRECUENCIAL DE PROBABILIDAD). La probabilidad del evento A, se define como el límite de la frecuencia relativa del evento A cuando el número de veces que se repite el fenómeno tiende a infinito, ie. si $f_A(N)$ es la frecuencia de ocurrencia del evento A cuando el esperimento se repite N veces, ó sea la frecuencia relativa de A es $f_A(N) / N$, entonces

$$P(A) = \lim_{N \to \infty} \frac{f_A(N)}{N}$$

NOTAS.

1. La definición frecuencial es también llamada una definición aposteriori, porque su determinación se hace después de realizado el experimento.

2. Las desventajas de esta definición son:

   i) Es difícil saber cual es el límite de esta frecuencia relativa, ya que esta frecuencia es empírica y no analítica.

   ii) Es necesaria la experimentación para conocer la frecuencia relativa.

EJEMPLO. Considere el juego de volados en el que deseamos conocer la probabilidad de obtener un águila. Defina

$$Z_N = \begin{cases} 0 & \text{si no ocurre águila en el N-ésimo volado} \\ 1 & \text{si ocurre águila en el N-ésimo volado} \end{cases}$$

y defina A como el evento de obtener un águila, $A = \{a\}$. Suponga que al tirar 30 veces un volado se obtuvo los resultados siguientes:

Repetición N	Z_N	$f_A(N)$	$f_A(N)/N$
1	1	1	1/1
2	1	2	2/2
3	0	2	2/3
4	1	3	3/4
5	0	3	3/5
6	0	3	3/6
7	1	4	4/7
8	0	4	4/8
9	0	4	4/9
10	1	5	5/10
11	1	6	6/11
12	0	6	6/12
13	0	6	6/13
14	0	6	6/14
15	1	7	7/15
16	1	8	8/16
17	1	9	9/17
18	1	10	10/18
19	0	10	10/19
20	0	10	10/20
21	0	10	10/21
22	1	11	11/22
23	1	12	12/23
24	1	13	13/24
25	0	13	13/25
26	0	13	13/26
27	1	14	14/27
28	0	15	15/28
29	1	16	16/29
30	1	17	17/30

Si se traza la frecuencia $f_A(N)/N$ contra N se puede "intuir" que el límite de $f_A(N)/N$ es 1/2, sin embargo es necesario repetir muchas veces el experimento para poder tener una idea a que número tiende esta frecuencia.

DEFINICION 1.7 (DEFINICION AXIOMATICA DE PROBABILIDAD). Sea Ω un conjunto arbitrario que representa el espacio muestra de un fenómeno aleatorio. Una función probabilidad es una función que va de una colección de subconjuntos de Ω al intervalo $[0,1]$, en forma tal que satisface los siguientes axiomas.

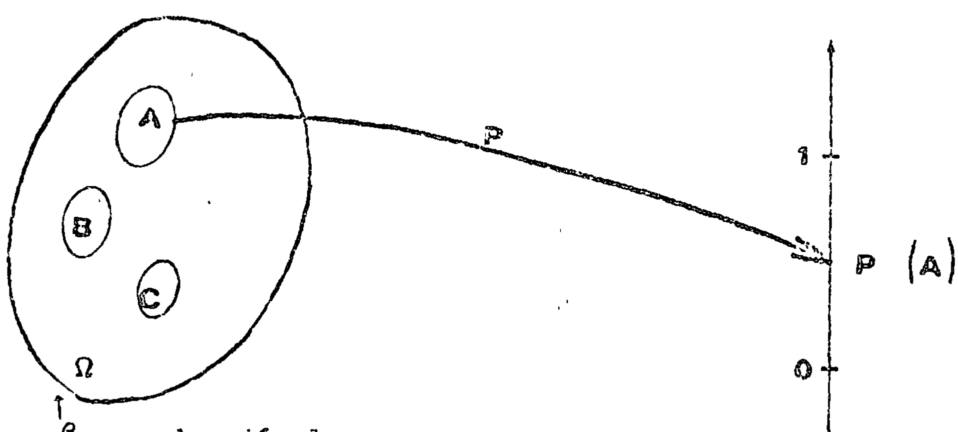
i) $P(A) \geq 0$ para cada A de la colección de subconjuntos de Ω

ii) $P(\Omega) = 1$

iii) $P(A_1 \cup A_2 \cup \ldots) = P(A_1) + P(A_2) + \ldots$
Si $A_1, A_2, \ldots$ es una colección de subconjuntos de Ω, mutuamente exclusivos, ie $A_i \cap A_j = \phi$ para toda $i \neq j$.

NOTAS

1. La representación de la función de probabilidad, usando la representación de función en teoria de conjuntos, es



β = colección de subconjuntos de Ω

2. El número P(A), asociado al elemento A de B, es llamado la probabilidad del evento A. Este número P(A) debe satisfacer los axiomas de la definición.

TEOREMA 1.8 Sea Ω un espacio muestra y P una función de probabilidad asociada a una colección β de subconjuntos de Ω. Se afirma:

i) $P(A) = 1 - P(A^c)$

ii) $P(\emptyset) = 0$

iii) Si $A \subseteq B$ entonces $P(A) \leq P(B)$

iv) $0 \leq P(A) \leq 1$ para cada $A \in \beta$

v) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

vi) $P(A \Delta B) = P(A) + P(B) - 2P(A \cap B)$

donde $A \Delta B$ es la diferencia simétrica de A y B, la cual se define por $A \Delta B = (A-B) \cup (B-A)$.

DEMOSTRACION. (Ver Mood, Parzen, ó Hogg dados en las referencias).

La siguiente definición de función solo tiene como objeto recordar conceptos fundamentales que serán necesarios para definir variable aleatoria.

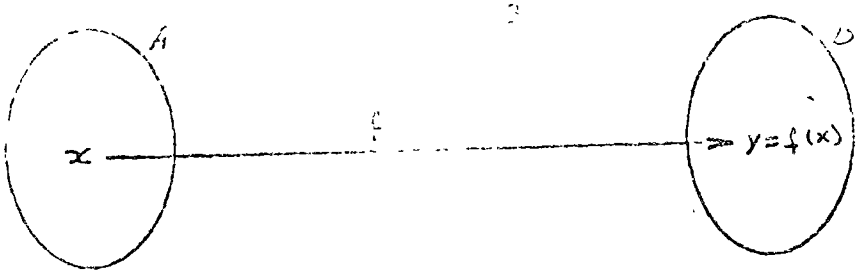
DEFINICION 1.9 Sean A y B dos conjuntos preespecificados. Si a cada elemento x de A le está asociado uno y solo un elemento y de B, a través de una regla f, entonces se dice que f es una función de A a B. Al conjunto A se le llama el dominio de la función f, y al conjunto B se le llama el contradominio de f. Si a un elemento x de A le corresponde el elemento. Y de B, se dice que y es la imagen de x bajo la regla f, y se escribe $y = f(x)$. El conjunto de todas las imágenes de los elementos de A, se llama el rango de f y se indica por rng (f), ie.

$$\text{rng}(f) = \{ y: \ y = f(x) \ \text{para alguna } x \in A \}$$

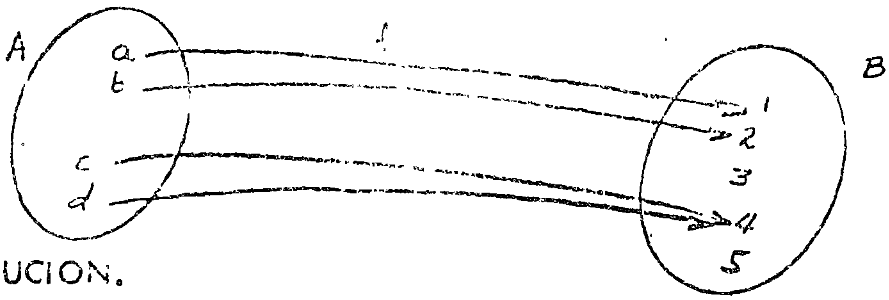
NOTAS:

1. Si una función f tiene como dominio al conjunto A y contradominio el conjunto B, entonces se escribe $f: \ A \rightarrow B$.

2. La representación de una función f, $f: \ A \rightarrow B$, en teoría de conjuntos es



EJEMPLO. Si f, f: A → B, es la función mostrada en la figura de abajo, dé el dominio, el contradominio y el rango de f.



SOLUCION.

dominio de f = $\{a, b, c, d\}$

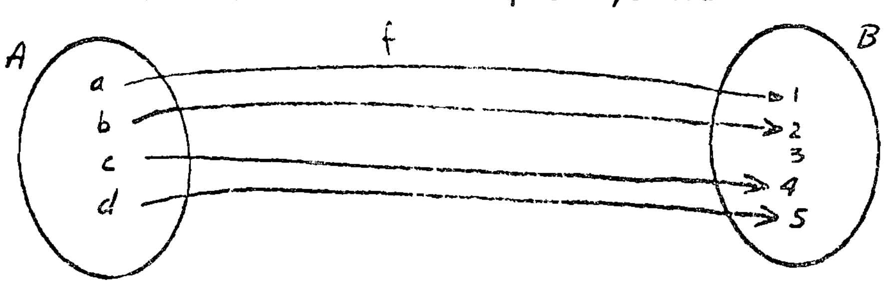
contradominio de f = $\{1, 2, 3, 4, 5\}$

ranfo de f = rng (f) = $\{1, 2, 4\}$

DEFINICION 1.10 Se dice que una función f, f: A→B, es inyectiva si para cada par de elementos x_1 y x_2 del dominio les corresponden distintos elementos y_1 y y_2 respectivamente, del contradominio, ie f es inyectiva si para todo

$$x_1 \neq x_2 \qquad y_1 = f(x_1) \neq y_2 = f(x_2).$$

EJEMPLO. La función mostrada abajo es inyectiva



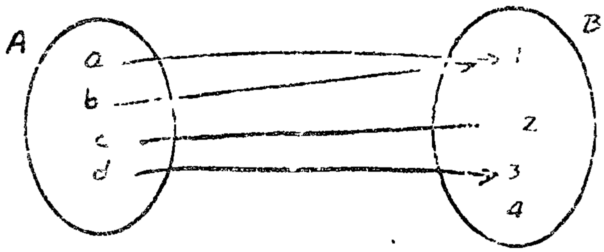
observe en este ejemplo que:

dom (f) = $\{a, b, c, d\}$ = A

contradom (f) = $\{1,2,3,4,5\}$ = B

rng (f) = $\{1,2,4,5\}$

EJEMPLO. La función siguiente

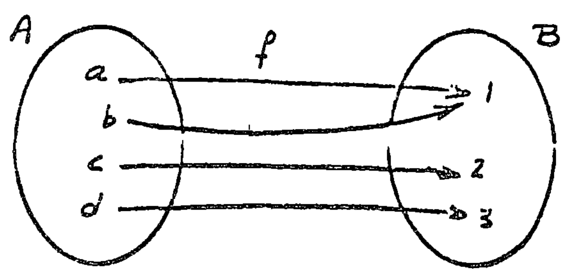


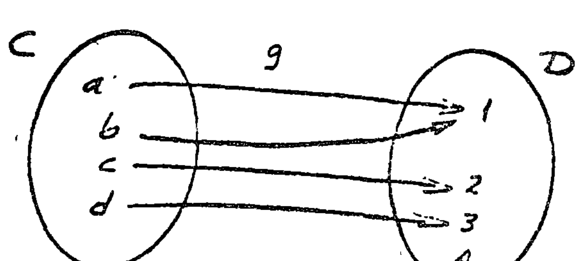
no es inyectiva porque existe un par, a y b, de elementos de A que tienen la misma imagen, ie.,

$$f(a) = 1 = f(b)$$

DEFINICION 1.11. Se dice que una función f, f: A→B, es suprayectiva si el rango de f es igual a su contradominio.

EJEMPLOS. Considere las funciones f, f: A → B, y g, g: C → D, mostradas a continuación.





Se observa que contradom (f) = $\{1,2,3\}$ = B es igual al rng (f) = $\{1,2,3\}$. Por lo tanto f es suprayectiva. Para la función g, se tiene que

$$\text{contradom } (g) = \{1,2,3,4\}$$

es distinto que

$$rng(g) = \{1, 2, 3\}$$

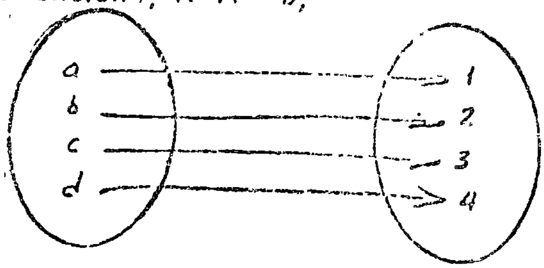
Por lo tanto g no es suprayectiva

DEFINICION 1.12 Se dice que una función f, f: $A \to B$, es biyectiva si es inyectivo y suprayectiva.

NOTA. Si una función es biyectiva también se acostumbra decir que es uno a cero.

EJEMPLOS.

1. La función f, f: $A \to B$,

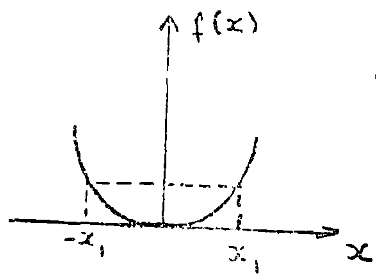


es biyectiva ya que es inyectiva y suprayectiva.

2. La función f, f: $A \to B$, con

   $A = R$; $B = R$, R es el conjunto de números reales

   $f(x) = x^2$



no es inyectiva ya que para todo x_1 y $x_2 = -x_1$ tiene que $f(x_1) = x_1^2$, y $f(x_2) = x_2^2 = (-x_1)^2 = x_1^2$ son iguales. Por lo tanto, no es biyectiva.

3. La función f, f: $A \to B$, con

   $A = R$, $B = R$

   $f(x) = 2x + 4$

   es inyectiva y suprayectiva. Por lo tanto, es la biyectiva.

DEFINICION 1.13. Sea f, f: A → B, una función biyectiva dada. La función inversa de f, indicada por f^{-1}, es una función cuyo dominio es B y su contradominio A en forma tal que si x es la imagen de y bajo la regla f^{-1} (ie., $x = f^{-1}(y)$) entonces se satisface $y = f(x)$.

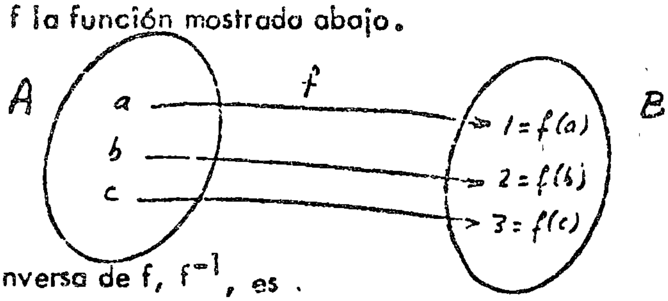
NOTAS:

1. Si f: A→B, es una función dada y f^{-1} es su inversa entonces se escribe f^{-1}: B→A

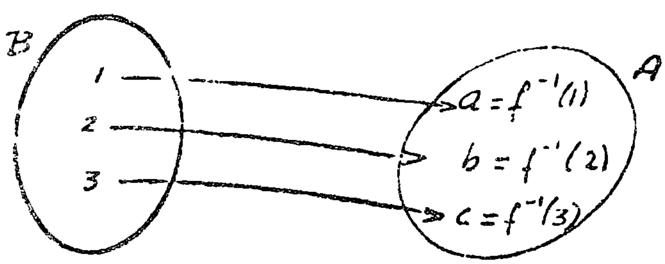
2. Para que la inversa de f se defina es necesario que f sea biyectiva.

EJEMPLOS

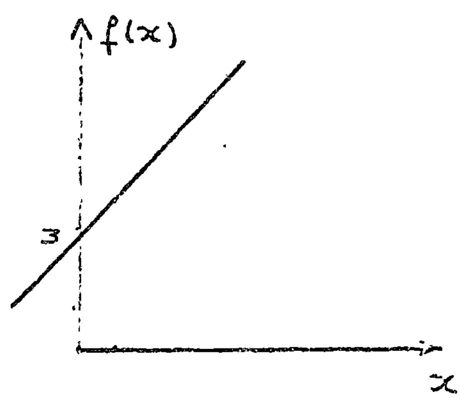
1. Sea f la función mostrado abajo.



La inversa de f, f^{-1}, es .

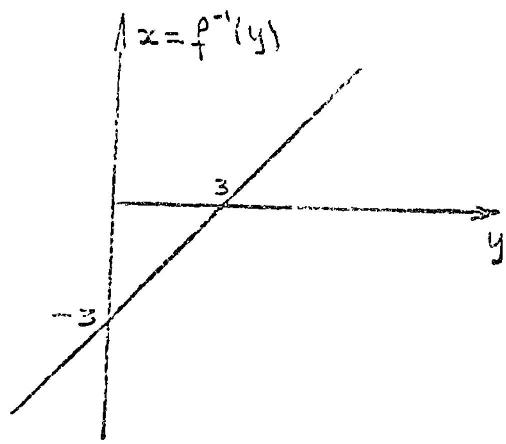


2. Sea $y = f(x) = x + 3$, f: R R. Determine su inversa

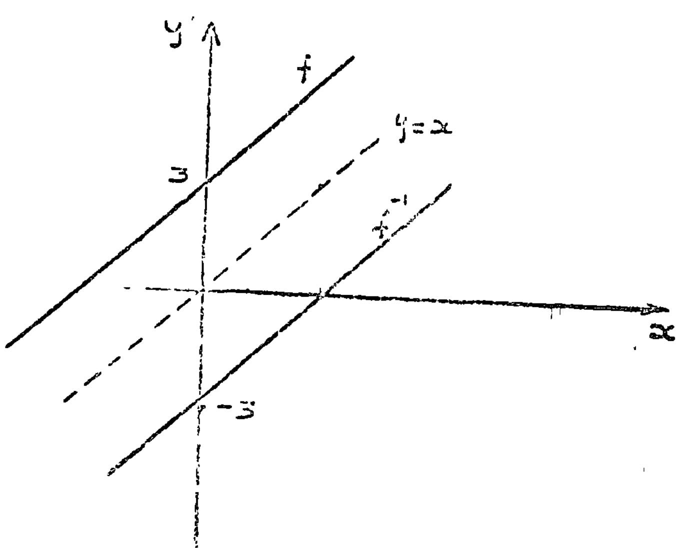


Si $y = x + 3$ entonces $x = y - 3$. Por lo tanto, la función inversa f^{-1}, está dada por

$$x = f^{-1}(y) = y - 3$$



En la función inversa $x = f^{-1}(y) = y + 3$, los elementos del cominio están indicados por y, y los elementos de controdominio por x. Sin embargo, debido a la costrumbre que se tiene de indicar a los elementos del dominio por x y a los del contradominio por y, podemos rebautizar estos elementos en la función inversa $x = f^{-1}(y) = y + 3$ escribiendo $y = f^{-1}(x) = x + 3$. Con esta convención, las gráficas de f y f^{-1}, en un mismo sistema de coordenadas aparece abajo

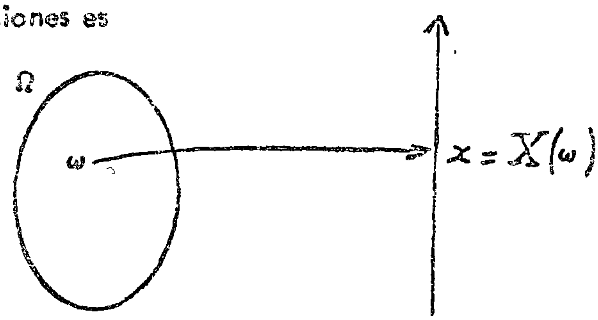


NOTA; Observe que f^{-1} es la imagen de f con respecto a la línea $y = x$.

DEFINICION 1.14 Sea Ω un espacio muestra arbitrario, y sea P una función de probabilidad asociada a una clase de subconjuntos de Ω. Una variable aleatoria, indicada por X, es una función cuyo dominio es Ω y su contradominio los reales.

NOTAS

1. La variable aleatoria (v.a.) X, $X : \Omega \longrightarrow R$, representada en diagrama de funciones es



2. Observe que existe una contradicción entre el nombre de variable aleatoria y su definición, ya que en el nombre se habla de una variable y en su definición se dice que es una función. Sin embargo esta ambiguedad se sigue manteniendo por motivos históricos.

3. Otra observación importante es con respecto a los símbolos usados para una variable aleatoria. La variable aleatoria se indica por letras mayúsculas, por ejemplo X. Este símbolo X debe entenderse como la regla que asocia elementos de Ω con elementos de R, de acuerdo con el carácter de función que tiene X. Por lo tanto, la notación

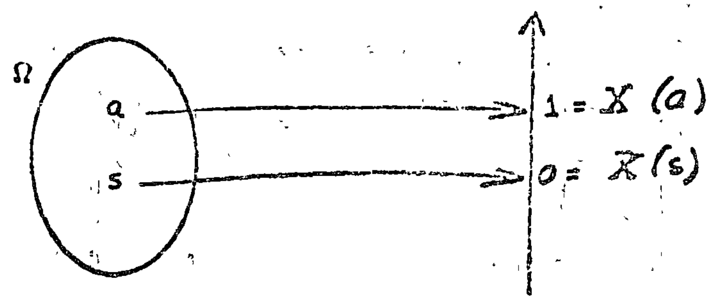
$$x = X(\omega)$$

Significa que x es el valor que toma la función X cuando se evalua en el punto ω, o que x es la imagen de ω bajo la regla X. Esta observación se refuerza con la notación

$$X(\cdot)$$

la cual significa que X es una función, y que X (·) corresponde al valor que toma la función cuando se evalua en un punto cualquiera del dominio. El punto que aparece entre paréntesis representa cualquier elemento del dominio, y es llamado el argumento de la función X.

EJEMPLO. Para el fenómeno de la tirada de una moneda con espacio muestra $\Omega = \{ a, s \}$, se podrá definir la siguiente función.



DEFINICION 1.15. Se dice que una variable aleatoria X es discreta si los valores x que puede tomar la función X son discretos, y se dice que es continua si los valores que toma son continuos.

NOTA. Una definición más apropiada de variable aleatoria discreta y continua involucra el concepto de conjuntos finitos infinitos, denumerables y contables. Estos conceptos se dan a continuación.

1. Se dice que un conjunto A es finito si existe un número entero M, $M < \infty$, tal que se puede encontrar una función uno a uno entre el conjunto A y el conjunto $\{1, 2, \dots, M\}$, y se dice que A es infinito si A no es finito.

   Ejemplos.

   i) El conjunto $\{a, b, c, d\}$ es finito ya que existe un número entero M, $M = 4 < \infty$, tal que existe una función f entre A y $\{1, 2, 3, 4\}$, por ejemplo la mostrada en la figura.

   ii) El conjunto $A = \{2, 4, 6, 8, \dots\}$ no es finito ya que no es posible encontrar un número entero M, $M < \infty$, tal que exista una función uno a uno entre A y $\{1, 2, \dots, M\}$. Por lo tanto A es infinito.

   iii) El conjunto $A = \{x : 0 \leq x \leq 1\}$ no es finito.

2. Se dice que un conjunto A es denumerable si existe una función uno a uno entre A y el conjunto N de números naturales; $N = \{n : n = 1, 2, 3, \dots\}$.

   EJEMPLOS

   i) El conjunto $A = \{x : x = 2, 4, 6, \dots\}$ es denumerable ya que existe una función f, $f(x) = \frac{x}{2}$, entre A y N, o sea para cada x de A existe un n de N, a través de la función f: $n = \frac{x}{2}$

   ii) El conjunto $A = \{x : 0 \leq x \leq 1\}$ no es denumerable.

3. Se dice que un conjunto A es contable si es finito o es infinito denumerable. Se dice que A es no contable si es infinito no denumerable. Entonces un conjunto A puede clasificarse de acuerdo a la siguiente caracterización esquemática:

$$\text{Conjunto} \begin{cases} \text{Finito} \\ \text{Infinito} \begin{cases} \text{Denumerable} \\ \text{No denumerable} \end{cases} \end{cases}$$

Finito y Denumerable: contables

No denumerable: No contable

DEFINICION 1.16. Se dice que la variable aleatoria es discreta si $rng(X)$ es un conjunto contable, y es continua si $rng(X)$ es un conjunto no contable.

2. FUNCIONES DE DENSIDAD DE PROBABILIDAD Y FUNCIONES DE DISTRIBUCION DE PROBABILIDAD

DEFINICION 2.1. Se dice que $f_X(x)$ es una función de densidad de probabilidad discreta de la v.a. X, si

i) $f_X(x) \geq 0$ para todo x

ii) $\sum_{x \in \Omega} f(x) = 1$

iii) Si A es un evento, entonces

$$P(A) = \sum_{x \in A} f(x)$$

DEFINICION 2.2 Se dice que $f_X(x)$ es una función de densidad de probabilidad continua si

i) $f_X(x) \geq 0$ para todo x.

ii) $\int_{x \in \Omega} f_X(x)\,dx = 1$

iii) Si A es un evento, entonces

$$P(A) = \int_{x \in A} f_X(x)\,dx$$

DEFINICION 2.3 La función de distribución de probabilidad (acumulada) de una v.a. X, indicada por $F_X(x)$, se define por

$$F_X(x) = P\{X \leq x\}.$$

NOTAS 1. Otra notación para $F_X(x)$ es $F(x)$.

2. Si X es una v.a. discreta entonces

$$F_X(x) = \sum_{s \leq x} f_X(s)$$

3. Si X es una v.a. continua entonces

$$F_X(x) = \int_{-\infty}^{x} f_X(s)\,ds.$$

3. GENERACION DE VARIABLES ALEATORIAS CON FUNCION DE DENSIDAD PREESPECIFICADA.

RESULTADOS PRELIMINARES.

Dada una v.a. X con función de distribución de probabilidad $F_X(x)$, es útil considerar a la propia función $F_X(x)$ como otra v.a. Para entender esto observe

i) X es una función $X : \Omega \to R$, por definición de X.

ii) $x = X(\omega)$ es el valor que toma la función X para un valor prefijado ω. Sin embargo, si ω no es prefijado entonces $X(\cdot)$, es una v.a., ya que el resultado ω es aleatorio. Por lo dicho la notación $x = X(\omega)$ significa que x es el valor de X en un valor prefijado ω, y la notación $X(\cdot)$ será un valor aleatorio de acuerdo al resultado aleatorio ω que pueda ocurrir.

iii) Si $U = F[X(\cdot)]$ entonces U es una v.a., $U \; \Omega \to R$, y puede escribirse $U(\cdot) = F[X(\cdot)]$

iv) Si ω es un valor prefijado entonces

$x = X(\omega)$ es un valor de X en ω.

$u = U(\omega)$ es un valor de U en ω, o sea

$u = U(\omega) = F[X(\omega)] = F(x)$

Por lo tanto, si $u = F(x)$ entonces se entiende que u es un valor de la v.a $U(\cdot) = F[X(\cdot)]$, lo cual se escribirá sólo por $U = F[X]$

-0

NOTACION. Si $X_1, \ldots, X_m$ son v.a. independientes con función de densidad común, entonces se acostumbra decir que $X_1, \ldots, X_m$ tienen la función de densidad de $F_{X_1}(x)$, lo cual significa que $F_{X_1}(x) = F_{X_2}(x) = \ldots = F_{X_m}(x)$.

Otra notación es: si $X_1, \ldots, X_m$ tienen función densidad común $F_X(x)$, se entiende que

$$F_{X_1}(x) = F_{X_2}(x) = \cdots = F_{X_m}(x) = F_X(x)$$

PROPOSICION 3.1 Si $X_1, \ldots, X_m$ son v.a. independientes idénticamente distribuidas con función de distribución común $F_X(x)$, entonces las v.a.

$$U_1(\cdot) = F[X_1(\cdot)], \ldots, U_n(\cdot) = F[X_m(\cdot)]$$

o sea

$$U_1 = F_X(X_1), \ldots, U_n = F_X(X_m)$$

también son v.a. independientes distribuidas idénticamente.

PROPOSICION 3.2 Si $X_1, \ldots, X_m$ son v.a. continuas, independientes e idénticamente distribuidas con función de distribución $F_X(x)$ entonces las v.a.

$$U_1 = F_X(X_1), \ldots, U_m = F_X(X_m)$$

son independientes, idénticamente distribuidas con función de distribución uniforme $F_U(u)$ en el intervalo $[0, 1]$.

DEMOSTRACION

$$F_{U_i}(u) = P\{U_i \leq u\}$$
$$= P\{F_X(X_i) \leq u\}$$
$$= P\{X_i \leq F_X^{-1}(u)\}$$ ya que X_i es continua y la función $F_X(\cdot)$ es no decreciente.
$$= F_X(F_X^{-1}(u)) = u$$

∴ $F_{U_i}(u) = u$, ∴ esta función de distribución corresponde a la función de distribución uniforme en el intervalo $[0,1]$, para cada función $U_i = F_X(X_i)$

TEOREMA 3.2 Sean $X_1, \ldots, X_m$ v.a. independientes con función de distribución común $F_X(x)$. Si $u_1, \ldots, u_m$ son valores de las v.a. $U_1 = F_X(X_1), \ldots, U_n = F_X(X_m)$, las cuales tienen distribución uniforme en el intervalo $[0,1]$; Entonces los valores $x_1, \ldots, x_m$ encontrados por

$$x_1 = F_X^{-1}(u_1), \ldots, \quad x_m = F_X^{-1}(u_m)$$

corresponden a valores de v.a. $X_1, \ldots, X_m$ con distribución común $F_X(x)$.

DEMOSTRACION

Demostraremos que las variables aleatorias X_i, definidas por

$$X_i = F_X^{-1}(U_i)$$

tienen una distribución común $F_X(x)$

$$F_{X_i}(x) = P\{X_i \le x\}$$
$$= P\{F_X^{-1}(U_i) \le x\}$$
$$= P\{U_i \le F_X(x)\}$$
$$= F_U(F_X(x))$$
$$= F_X(x) \quad \square$$

por que U tiene una distribución uniforme (Recuerde qui $F_U(u) = u$).

NOTA. El teorema anterior nos da un procedimiento para encontrar valores $x_1, \ldots, x_m$ de v.a. $X_1, \ldots, X_m$ que tengan una distribución $F_X(x)$ preespecificada. Este procedimiento consiste en los siguientes pasos.

PASO 1: Encuentre la función inversa $F_X^{-1}(x)$ de la función $F_X(x)$ preespecificada, i.e si $u = F(x)$ entonces despeje a x de esta ecuación para expresarla en terminos de u, o sea encuentre

$$x = F^{-1}(u)$$

PASO 2: Encuentre valores aleatorios $u_1, u_2, \ldots, u_m$ de una distribución uniforme en el intervalo $[0, 1]$.

PASO 3 Para los valores $u_1, \ldots, u_m$ encontrados en el paso 2, encuentre los valores $x_1, \ldots, x_m$ usando la expresión derivada en el paso 1, i.e.

$$x_1 = F^{-1}(u_1), \ldots, x_m = F^{-1}(u_m)$$

EJEMPLO Encuentre 5 observaciones aleatorias $x_1, \ldots, x_5$ de v.a. $X_1, \ldots, X_5$ con función de densidad dada por

$$f_X(x) = \begin{cases} 2x & \text{si } 0 \leq x \leq 1 \\ 0 & \text{en otro caso} \end{cases}$$

SOLUCION. Primero encontraremos la función de distribución $F_X(x)$ y luego seguiremos los pasos de la nota anterior

$$F_X(x) = P\{X \leq x\} = \int_{-\infty}^{x}$$

Caso $x < 0$:

$$F_X(x) = \int_{-\infty}^{x} 0\,dx = 0$$

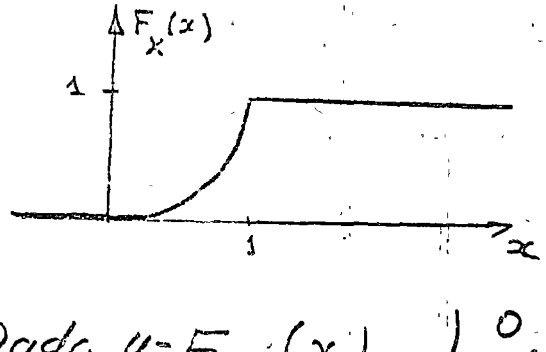
Caso $0 < x < 1$:

$$F_X(x) = \int_{-\infty}^{0} f_X(x)\,dx + \int_{0}^{x} 2s\,ds = 2\,\frac{s^2}{2}\Big]_0^x = x^2 \quad \text{si } 0 < x < 1$$

Caso $x \geq 1$

$$F_X(x) = \int_0^1 2s\,ds + \int_1^x 0\,ds = 1$$

$$\therefore F_X(x) = \begin{cases} 0 & \text{si } x \leq 0 \\ x^2 & \text{si } 0 < x < 1 \\ 1 & \text{si } x \geq 1 \end{cases}$$



PASO 1. Dada $u = F_X(x) = \begin{cases} 0 & \text{si } x \leq 0 \\ x^2 & \text{si } 0 < x < 1 \\ 1 & \text{si } x \geq 1 \end{cases}$

entonces para $0 \leq u \leq 1$; $u = F(x) = x^2$, implica que

$$x = \sqrt{u}$$

observe que sólo se toma la raíz positiva porque la variable x sólo toma valores positivos

PASO 2 y 3. Para los números $u_1, \ldots, u_5$ que aparecen en la tabla (encontrados de cualquier tabla de numeros aleatorios distribuidos uniformemente) los valores $x_1, \ldots, x_5$ se encuentran por la formula anterior.

u_i	$x_i = F^{-1}(u_i) = \sqrt{u_i}$
.096	0.310
.569	0.754
.665	0.815
.764	0.874
.842	0.918

REFERENCIAS

1. Freiberger, W., and Grenander, U., A SHORT COURSE IN COMPUTATIONAL PROBABILITY AND STATISTICS, Springer-Verlag, 1971.

2. Gridgement, N. T., GEOMETRIC PROBABILITY AND THE NUMBER π, Scripta Mathematica, 25, p. 183-195, 1960.

3. Gross, D., and Harris, C. M., FUNDAMENTALS OF QUEUEING THEORY, Wiley, 1974.

4. Harris, R. D., Maggard, M. J., and Lesoo, W. G., COMPUTERS MODEL IN OPERATIONS RESEARCH, Harper and Row, 1974.

5. Hillier, F. S., and Lieberman, G. J., INTRODUCTION TO OPERATIONS RESEARCH, 2nd. ed., Holden Day, 1974.

6. Hogg, R. V., and Craig, A. T., INTRODUCTION TO MATHEMATICAL STATISTICS, MacMillan, 1970.

7. Kendall, M. G., and Moran, P. A. P., GEOMETRICAL PROBABILITY, Griffin, 1963, p. 70-71.

8. Meier, R. C., Newell, W. T., and Pazer, H. L., SIMULATION IN BUSINESS AND ECONOMICS, Prentice Hall, 1969.

9. Mood, A. M., Graybill, F. A., and Boes, D. C., INTRODUCTION TO THE THEORY OF STATISTICS, 3th. ed., Mc Graw Hill, 1974.

10. Neuts, M., PROBABILITY, Allyn and Bacon, 1973.

11. Parzen, E., MODERN PROBABILITY THEORY AND ITS APPLICATIONS, Wiley-Toppan, 1960.

12. Wagner, H. M., PRINCIPLES OF OPERATIONS RESEARCH, 2nd. ed., Prentice Hall, 1975.

TABLE 3A

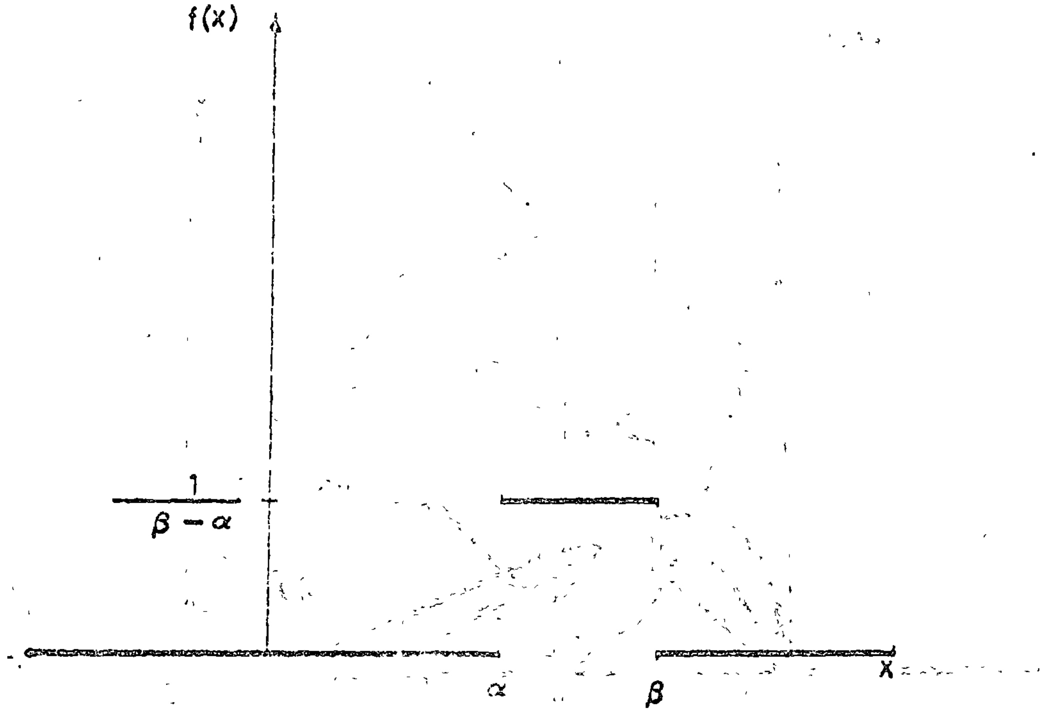
SOME FREQUENTLY ENCOUNTERED DISCRETE PROBABILITY LAWS WITH THEIR MOMENTS AND GENERATING FUNCTIONS

Probability Law	Parameters	Probability Mass Function $p(\cdot)$	Mean $m = E[x]$	Variance $\sigma^2 = E[x^2] - E^2[x]$
Bernoulli	$0 \le p \le 1$	$p(x) = p \quad x = 1$ $= q \quad x = 0$ $= 0$ otherwise	p	pq
Binomial	$n = 1, 2, \cdots$ $0 \le p \le 1$	$p(x) = \binom{n}{x} p^x q^{n-x} \quad x = 0, 1, 2, \cdots, n$ $= 0$ otherwise	np	npq
Poisson	$\lambda > 0$	$p(x) = e^{-\lambda} \frac{\lambda^x}{x!} \quad x = 0, 1, 2, \cdots$ $= 0$ otherwise	λ	λ
Geometric	$0 \le p \le 1$	$p(x) = pq^{x-1} \quad x = 1, 2, \cdots$ $= 0$ otherwise	$\frac{1}{p}$	$\frac{q}{p^2}$
Negative binomial	$r > 0$ $0 \le p \le 1$	$p(x) = \binom{r + x - 1}{x} p^r q^x$ $= \binom{-r}{x} p^r (-q)^x, \quad x = 0, 1, 2, \cdots$ $= 0$ otherwise	$\frac{rq}{p} = rP$ if $P = \frac{q}{p}$	$\frac{rq}{p^2} = rPQ$ if $Q = \frac{1}{p}$
Hypergeometric	$N = 1, 2, \cdots$ $n = 1, 2, \cdots, N$ $p = 0, \frac{1}{N}, \frac{2}{N}, \cdots, 1$	$p(x) = \frac{\binom{Np}{x}\binom{Nq}{n-x}}{\binom{N}{n}} \quad x = 0, 1, \cdots, n$ $= 0$ otherwise	np	$npq\left(\frac{N-n}{N-1}\right)$

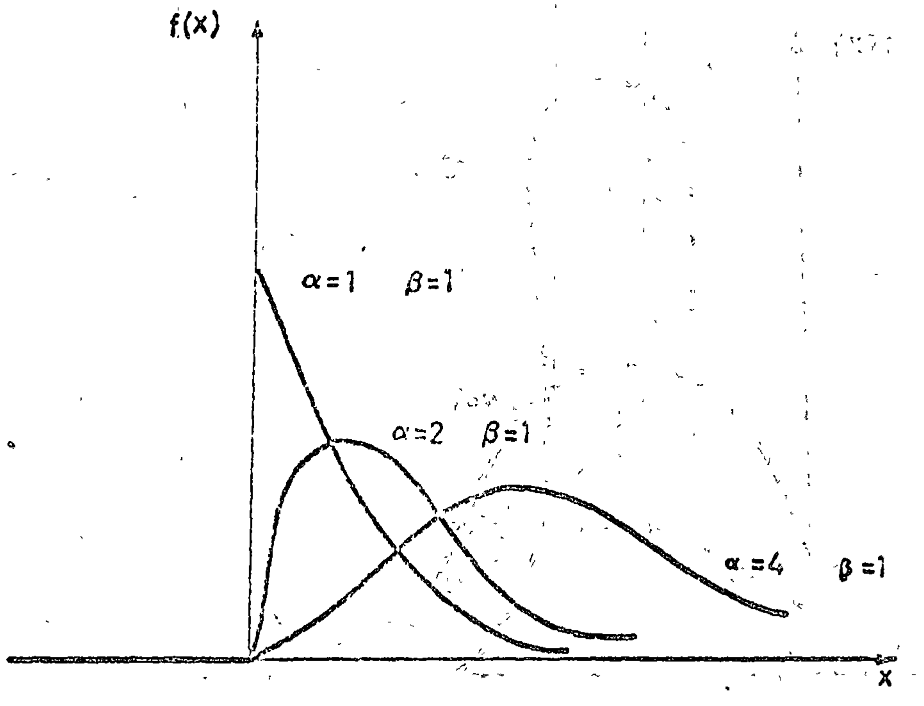
TABLE 3B

SOME FREQUENTLY ENCOUNTERED CONTINUOUS PROBABILITY LAWS WITH THEIR MOMENTS AND GENERATING FUNCTIONS

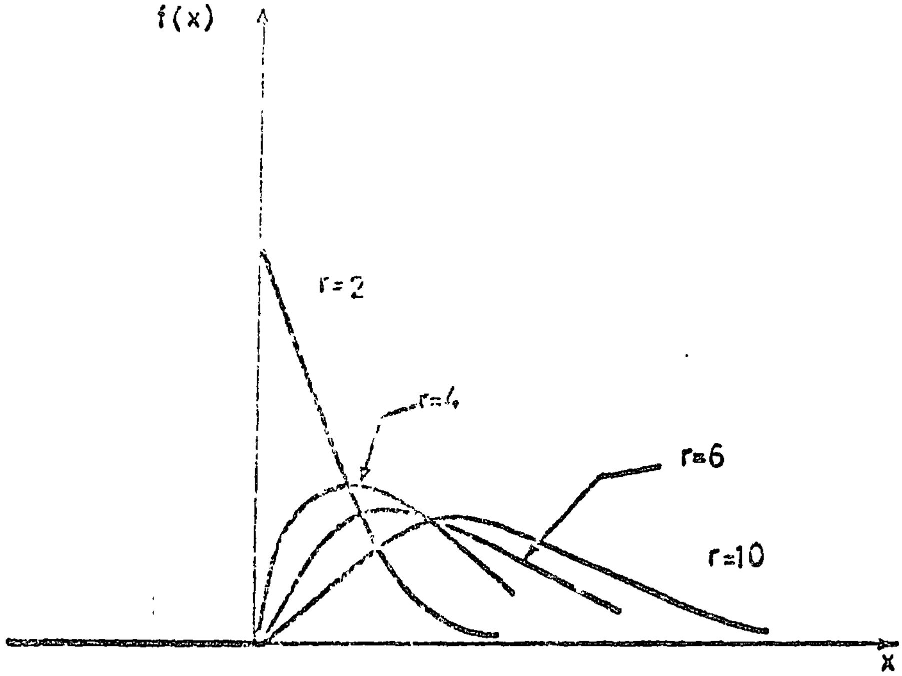
Probability Law	Parameters	Probability Density Function $f(\cdot)$	Mean $m = E[x]$	Variance $\sigma^2 = E[x^2] - E^2(x)$
Uniform over interval a to b	$-\infty < a < b < \infty$	$f(x) = \frac{1}{b-a} \quad a < x < b$ $= 0$ otherwise	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
Normal	$-\infty < m < \infty$ $\sigma > 0$	$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-m}{\sigma}\right)^2}$	m	σ^2
Exponential	$\lambda > 0$	$f(x) = \lambda e^{-\lambda x} \quad x > 0$ $= 0$ otherwise	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$
Gamma	$r > 0$	$f(x) = \frac{\lambda}{\Gamma(r)} (\lambda x)^{r-1} e^{-\lambda x} \quad x > 0$ $= 0$ otherwise	$\frac{r}{\lambda}$	$\frac{r}{\lambda^2}$



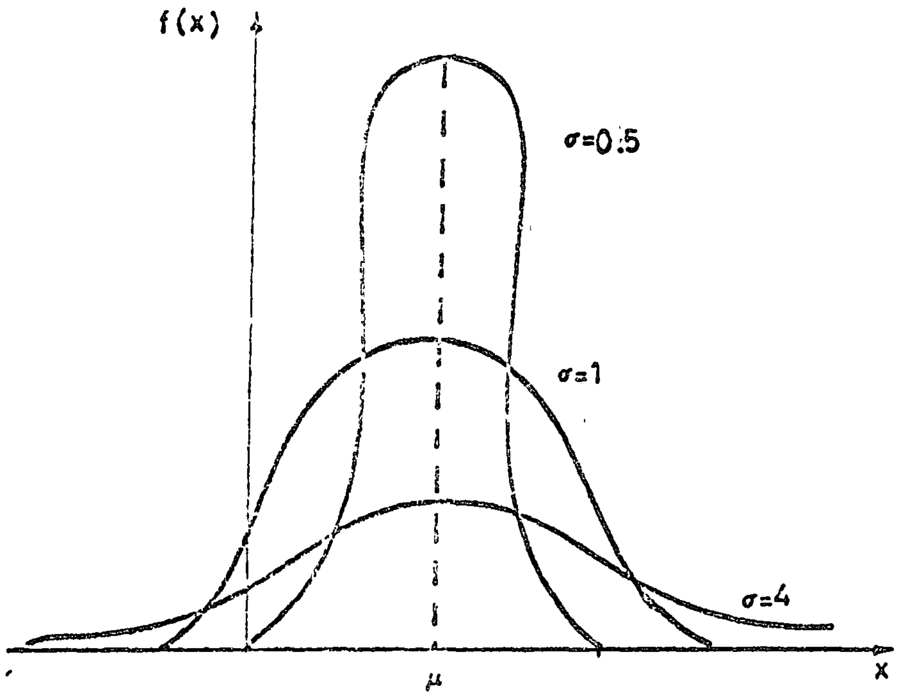
1.- Funcion de densidad de probabilidad uniforme de parametros α y β



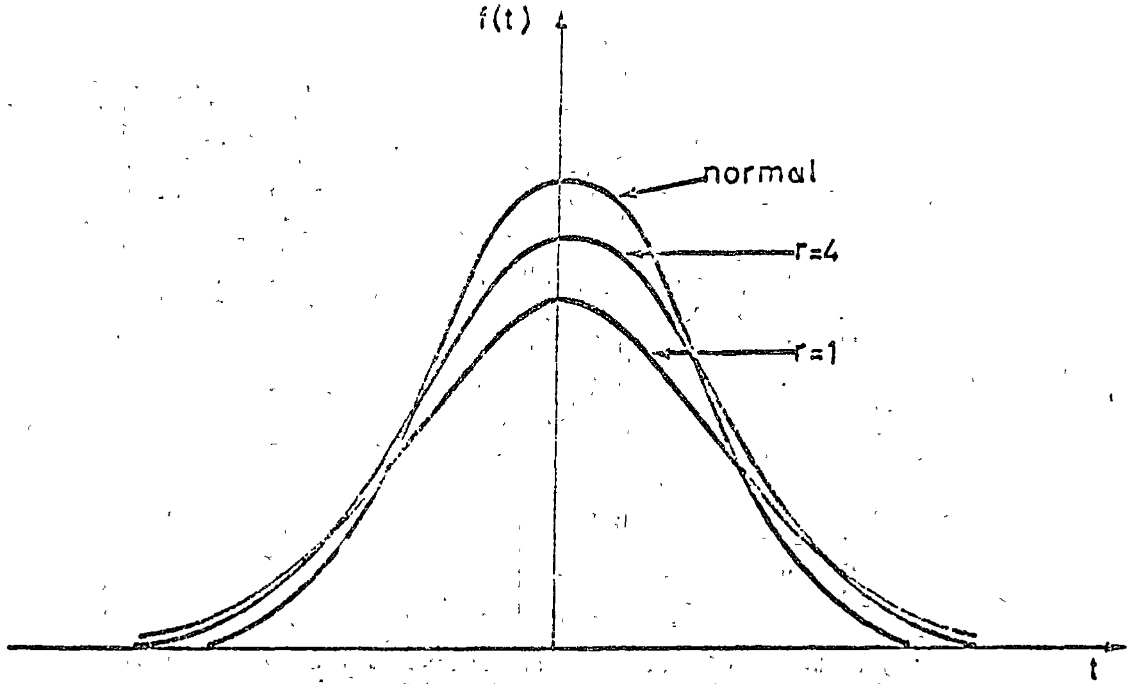
2.- Funcion de densidad de probabilidad gamma de parametros α y β



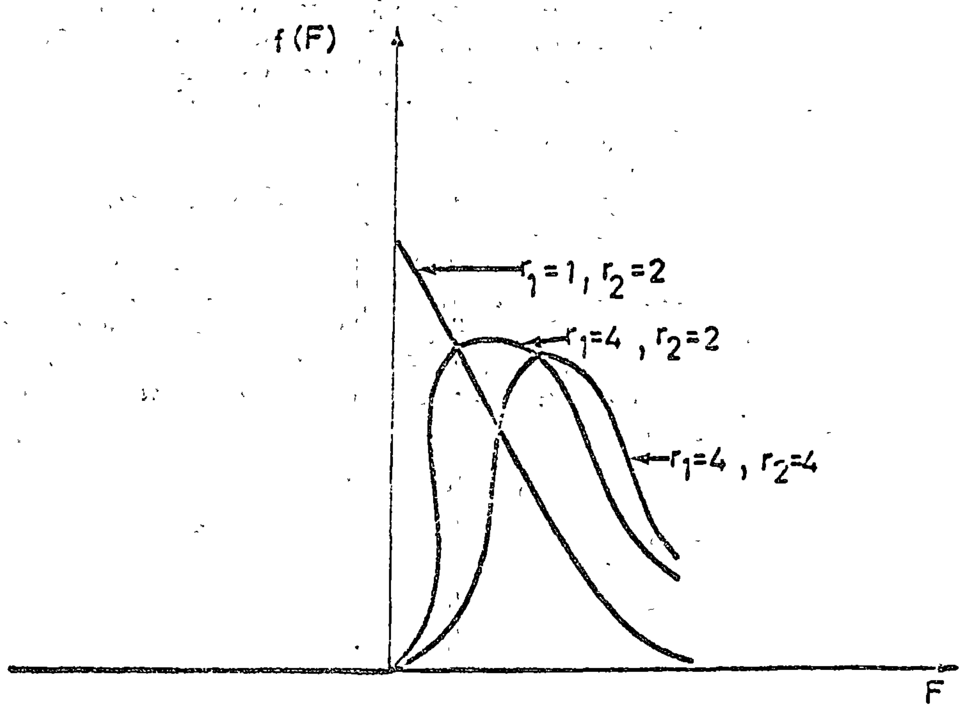
3.- Funcion de densidad de probabilidad ji cuadrada de parametro r



4.- Funcion de densidad de probabilidad normal de parametros μ, σ



5-Funcion de densidad de probabilidad de student con parametro



6.-Funcion de densidad de probabilidad F con parametros r_1 y r_2

Table 15.1 Table of Random Digits[†]

09656	96657	[illegible]	49222	[illegible]	10145	48455	23505	90430	04180
24712	55799	60857	73479	73581	17360	30406	05842	72044	90764
07202	96341	23699	76171	79126	04512	15426	15980	88898	06358
84575	46820	54063	43919	46769	05379	70682	43081	66171	38942
38144	87037	46626	70529	27918	31191	98668	33482	43998	75733
48048	56713	01986	[illegible]	69600	91609	65374	22928	09704	59343
41936	58566	31276	19952	01352	18834	99596	09302	20087	19063
73391	94006	03822	81845	76158	41352	40596	14325	27020	17546
57580	08954	73554	28698	29022	11568	35668	59906	39557	27217
92646	41113	91411	56215	69302	86419	61224	41936	56939	27816
07118	12707	35622	81485	73354	49800	60805	05048	28898	60933
57842	57831	24130	75408	83784	64307	91620	40810	06539	70367
65078	44981	81009	33697	98324	46928	34198	96032	98426	77488
04294	96120	67629	55265	26248	40602	25566	12520	89785	93932
48381	06807	43775	09708	73199	53406	02910	83292	59249	18597
00459	62045	19249	67095	22752	24636	16965	91836	00582	46721
38824	81681	33323	64086	55970	04849	24819	20749	51711	86173
91465	22232	02907	01050	07121	53536	71070	26916	47620	01619
50874	00807	77751	73952	03073	69063	16894	85570	81746	07568
26644	75871	15618	50310	72610	66205	82640	86205	73453	90232

[†] Reproduced with permission from The Rand Corporation, *A Million Random Digits with 100,000 Normal Deviates*. Copyright, The Free Press, Glencoe, Ill., 1955, top of p. 182.

bers or not.[1] Basically the requirements are that each successive number in the sequence must have an equal probability of taking on any one of the possible values, and it must be statistically independent of the other numbers in the sequence. In other words, the numbers need to be random observations from a (discretized) *uniform distribution*.

If a digital computer is to be used for executing the simulation, the random numbers it needs could be fed into the computer from one of the available tables. (In fact, the Rand table already is available on punched cards.) However, it is more common to have the computer itself generate the random numbers. There are a number of methods for doing this, of which the most popular are the *congruential methods* (additive, multiplicative, and mixed). The *mixed congruential method* has become probably the most widely used in recent years, so we shall focus on this approach.

The mixed congruential method generates a *sequence* of random numbers by always calculating the next random number from the last one obtained, given an initial random number x_0 (called the *seed*), which may be obtained from some published source such as the Rand table. In particular, it calculates the $(n+1)$st random number x_{n+1} from the nth random number x_n by using the recurrence relation

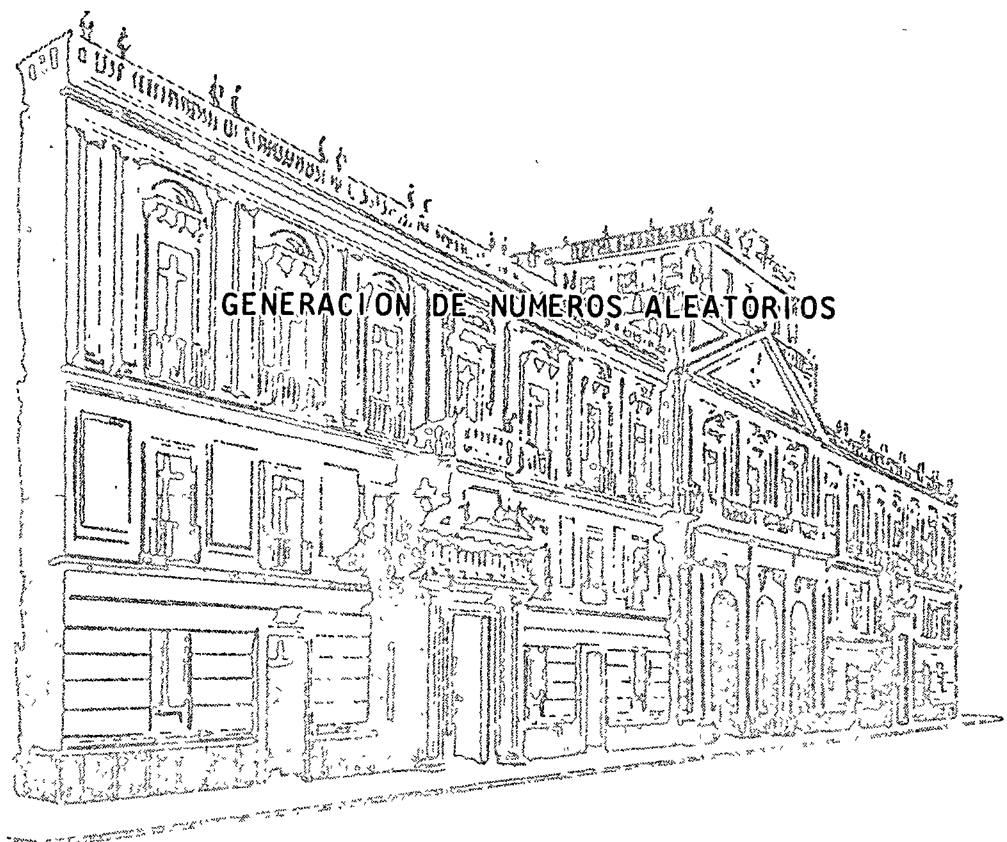
$$x_{n+1} \equiv (ax_n + c)(\text{modulo } m),$$

[1] The interested reader is referred to Selected References 4 and 6 for a description of these tests and for more details about the generation of random numbers.

centro de educación continua
división de estudios superiores
facultad de ingeniería, unam

MODELADO Y SIMULACION APLICADOS A LA PLANEACION

GENERACION DE NUMEROS ALEATORIOS



ING. ARMANDO TORRES FENTANES

Palacio de Minería
Tacuba 5, primer piso. México 1, D. F.
Tels: 521-40-23 521-73-35 5123-123

CURSO: JUEGOS DE SIMULACION

TEMA: GENERACION DE NUMEROS ALEATORIOS

I N D I C E

1. Introducción

En la vida diaria se presentan dos tipos de modelos de acuerdo a las variables involucradas en los procesos: modelos determinísticos y modelos estocásticos. Un proceso estocástico es aquél cuyas actividades se comportan aleatoriamente y en donde la secuencia de eventos no es conocida.

Una variable que representa la salida en un proceso aleatorio se denomina variable aleatoria o estocástica (v.a.) y tiene como característica el que no se pueda determinar con anterioridad su secuencia de posibles valores; sin embargo, se debe conocer el rango de valores que puede adoptar y la probabilidad de que adquiera dichos valores.

En muchos sistemas las variables de interés muestran un comportamiento aleatorio, por lo que al modelar y simular dichos sistemas es necesario incluir en alguna forma la generación de las v.a. para poder llevar a cabo la simulación.

El presente curso se enfocará exclusivamente a la obtención de v.a. con funciones densidad de probabilidad (f.d.p.) uniformes, es decir, variables aleatorias uniformemente distribuidas. Por uniformemente distribuidas se entiende, que cada punto del rango de posibles valores tiene igual probabilidad de salir, esto es:

$$f(x) = \begin{cases} 1 & a \leq x \leq b \\ 0 & \forall \text{ otro caso} \end{cases} \quad (1.1)$$

Nos limitaremos al rango de valores $[0,1]$, dado que es posible proyectar dichos valores en cualquier otro intervalo y las variables en ese nuevo intervalo también serán uniformemente distribuidas. Si se buscan v.a. unif. distribuidas en el intervalo $[A, B]$ y la v.a. generada (X_i) está en el intervalo $[0,1]$, entonces la v.a. dentro del rango de valores deseado estará dada por:

$$Y_i = A + (B - A) X_i \quad (1.2)$$

En procesos de simulación, usualmente se requiere el obtener secuencias repetibles de v.a. a fin de poder comparar y comprobar resultados, dicho tipo de números aleatorios se conocen como números ó variables pseudoaleatorias. Los requisitos que deben cumplir dichas variables son: distribución uniforme e independencia estadística.

Entre algunas de las posibles aplicaciones de v.a. uniformemente distribuidas tenemos:

- obtención de v.a. con diversas f.d.p.
- simulaciones mediante el Método de Monte Carlo
- obtención de funciones densidad de probabilidad (f.d.p.) a partir de la función densidad acumulada (f.d.a.)
- Simulaciones en teoría de colas.

Al final de estas notas aparece una serie de programas para computadora digital binaria con algunas de las posibles aplicaciones de v.a. uniformemente distribuidas. Dichos programas son : A1, A2, A3, A4.

2. Métodos para generar números aleatorios

Para la generación de variables aleatorias se han desarrollado 4 enfoques diferentes:

- métodos manuales
- tablas de números aleatorios
- métodos para computadoras analógicas
- métodos para computadoras digitales.

En la actualidad los dos primeros se han relegado al aspecto didáctico; el tercero, dado el auge de las computadoras digitales así como la decadencia de las computadoras analógicas y otras características, ha caído en desuso. De lo antes dicho se concluye que el método de mayor auge y desarrollo así como de utilidad práctica es el aplicable a computadoras digitales.

Para que un método de generación de números aleatorios sea considerado como satisfactorio deberá proporcionar secuencias de números con las siguientes características:

- uniformemente distribuida
- independencia estadística
- secuencias reproducibles
- cadenas de números suficientemente grandes
- alta velocidad para generar v.a.
- empleo mínimo de memoria en la máquina a utilizar.

3. Métodos manuales

Son los más sencillos y menos usados de los métodos existentes para generar números aleatorios dado que son muy lentos para la generación de v.a.; además de que es imposible reproducir las secuencias de números, lo cual en ocasiones es indispensable para propósitos de simulación. Algunas de las técnicas empleadas son: extracción de cartas de una baraja, tirar monedas al aire, arrojar varios dados sobre una mesa, etc.. En general el empleo de estos métodos es de carácter didáctico.

4. Tablas de números aleatorios

Es común el encontrar tablas de números aleatorios en libros de probabilidad y estadística; el empleo de dichas tablas está destinado casi exclusivamente a pruebas de escritorio para alguna simulación dada. Para la creación de estas tablas es necesario generar con anterioridad dichos números y la dimensión de las mismas es elástica en virtud de que se pueden utilizar diversos criterios para la selección de los números a emplear.

La ventaja de estas tablas es la facilidad para reproducir secuencias de v.a., sin embargo, el proceso de obtención de estos números aleatorios es lento y a pesar de la elasticidad de las tablas se presentan casos en que son insuficientes para los requerimientos de un problema en particular. La tabla A1 que aparece al final de estas notas es un ejemplo de tablas de números aleatorios uniformemente distribuidos.

5. Métodos para computadoras analógicas

En estos métodos la generación de números aleatorios depende de algún proceso aleatorio, por lo que las variables así generadas son realmente aleatorias y no pseudoaleatorias.

Algunos de los procesos aleatorios empleados para la generación de dichos números son: el comportamiento de la corriente eléctrica, generación de ruido gaussiano, recepción de señales aleatorias mediante una antena acoplada a la computadora, etc.

Las secuencias producidas no se pueden repetir y generalmente los procesos de simulación se llevan a cabo en computadoras digitales, lo cual implicaría el empleo de un transductor analógico digital en caso de emplearse este método para la generación de las v.a. requeridas. Lo anterior es poco práctico y antieconómico dado que existen métodos especiales para generar v.a. en computadoras digitales.

6. Métodos para computadoras digitales

En la simulación de sistemas mediante el empleo de una computadora digital, la alimentación de las v.a. requeridas para dicho efecto se puede realizar en tres formas:

- alimentación externa mediante una cinta magnética que tenga grabada una secuencia de v.a., lo cual equivale a una tabla de números aleatorios. Desventajas: lentitud y dimensiones limitadas.
- empleo de algún proceso aleatorio analógico y la transducción de dicho proceso para obtener secuencias de dígitos. Desventajas: secuencias no reproducibles y falta de control sobre el proceso.
- generación de números pseudo aleatorios mediante algún método ó proceso que permita obtener secuencias suficientemente largas y estadísticamente independientes. Este método reduce los inconvenientes de los dos anteriores.

Dentro de la última forma de generar números aleatorios existe un método que ha tenido gran aceptación: el método congruencial ó método del residuo. Este método genera secuencias de variables pseudo aleatorias, reproducibles y estadísticamente independientes.

El método está basado en la fórmula congruencial.

$$X_{n+1} = \left[A X_n + C \right] \text{mod } M^{*} \qquad (6.1)$$

para poder arrancar el proceso se requiere dar un valor inicial X_0 que se denomina "semilla" del proceso y los parámetros (A, C, M) se deben escoger apropiadamente para tener una secuencia suficientemente grande así como una correlación estadística lo más cercana a cero.

Los números obtenidos mediante dicho proceso son números enteros comprendidos en el rango de valores [0,M] ; para la obtención de variables aleatorias uniformemente distribuidas (R_n) comprendidas en el intervalo [0,1] se emplea la relación:

$$R_n = \frac{X_n}{M} \qquad (6.2)$$

Para la generación de v.a. mediante la fórmula congruencial se han desarrollado tres métodos diferentes, siendo el objeto de cada uno de ellos el generar secuencias de máximo período en un tiempo mínimo; por máximo período se entiende el poder generar secuencias compuestas de M números diferentes si se está empleando modM. Dichos métodos son:

a) Congruencia lineal aditiva
b) Congruencia lineal multiplicativa
c) Congruencia lineal mixta.

A continuación se describe cada uno de ellos.

---

* Z mod M = residuo de $\frac{Z}{M}$; donde Z, M y el residuo son números enteros.

6.1 Método de Congruencia lineal aditiva

Para iniciar este método se requiere proporcionar k valores iniciales (enteros positivos) y la fórmula de recurrencia está dada por:

$$X_{n+1} = (X_n + X_{n-k}) \bmod M \qquad (6.3)$$

Este método es el único que produce secuencias con períodos mayores a M, sin embargo, se requiere que "k" sea grande para que tenga un buen comportamiento estadístico la secuencia generada; pruebas estadísticas han demostrado que el menor valor de "k" para tener secuencias aceptables es 16.

Este método es de los menos empleados.

6.2 Método de congruencia lineal multiplicativa

La fórmula de recurrencia para este método está dada por:

$$X_{n+1} = (AX_n) \bmod M \qquad (6.4)$$

Este método es el más empleado en computadoras digitales debido a la rapidez con que se pueden generar los números aleatorios y además dichas secuencias presentan buenas propiedades estadísticas si se hace una selección adecuada de los parámetros A, X_0, M. Su inconveniente es que no proporciona períodos completos, por lo que es necesario escoger M suficientemente grande a fin de que la secuencia generada sea lo más larga posible.

El entero "M" debe cumplir la siguiente relación:

$$M = p^b \qquad (6.5)$$

donde:

p es el número de dígitos del sistema numérico empleado en la computadora (decimal ó binario)

b es el número de bits para una palabra de computadora.

De acuerdo a lo anterior, la ecuación (6.4) se transforma en:

$$X_{n+1} = (A X_n) \mod p^b \qquad (6.6)$$

De acuerdo al sistema numérico empleado debe ser la selección de los parámetros (A, b), dado que el proceso interno en la computadora se realiza mediante truncamientos de las operaciones efectuadas.

En computadoras binarias se tendrá:

$$M = 2^b \qquad (6.7)$$

donde "b" es el número de bits por palabra de la computadora empleada.

De acuerdo a la teoría de números enteros, el máximo período a obtener en este caso será:

$$T = 2^{b-2} \qquad (6.8)$$

El parámetro "A" deberá ser un número primo relativo a "M" y además non. Para que lo anterior se cumpla dicho parámetro debe satisfacer la relación:

$$A = 8t \pm 3 \qquad (6.9)$$

donde t es cualquier entero positivo. Además para eliminar hasta donde sea posible la correlación entre los números aleatorios generados, el parámetro "A" deberá estar muy próximo a:

$$A \cong 2^{b/2} \qquad (6.10)$$

El valor inicial ó semilla del proceso deberá ser primo relativo a "M", lo cual se cumple si X_0 es un número entero non y positivo.

Los pasos a seguir para generar una secuencia de números aleatorios mediante este método son:

(1) Seleccionar la cantidad de números aleatorios a generar (N).

(2) Escoger el valor inicial ó semilla (X_0). Debe ser non y entero positivo.

(3) Seleccionar el parámetro "A" en forma tal que cumpla:

$$\text{i)}\quad A = 8t \pm 3$$

$$\text{ii)}\quad A \cong 2^{b/2}$$

(4) Generar el número aleatorio X_n mediante:

$$X_n = (AX_{n-1}) \bmod M$$

(5) Obtener la v.a. uniformemente distribuida (R_n) correspondiente a X_n:

$$R_n = \frac{X_n}{M}$$

(6) Regresar al paso (4) y así sucesivamente hasta obtener las "N" v.a.

El programa A5, que aparece al final de estas notas, sirve para determinar v.a. uniformemente distribuidas en el intervalo $[0,1]$ empleando una computadora digital binaria.

En <u>computadoras decimales</u> se tendrá:

$$M = 10^b = 2^b\, 5^b \qquad (6.11)$$

donde "b" representa el número de dígitos decimales por palabra de computadora. De acuerdo a la teoría de números enteros el período máximo será:

$$T = m.c.m. \left[2^{b-2},\ 4 \times 5^{b-1} \right]$$

$$T = 5 \times 10^{b-2},\quad b \geq 5 \qquad (6.12)$$

El parámetro "A" deberá ser primo relativo a "M", non y de orden $\left[5^{b-1} \text{ ó } 2 \times 5^{b-1} \text{ ó } 4 \times 5^{b-1} \right] \bmod 5^b$. Lo anterior se cumple si:

$$A = 200t \pm p \qquad (6.13)$$

donde:

- t es cualquier entero positivo
- p puede tener cualquiera de los siguientes 16 valores:

  3, 11, 13, 19, 21, 27, 29, 37, 53, 59, 61, 67, 69, 77, 83, 91.

Además para poder obtener correlación de primer orden casi nula se requiere:

$$A \cong 10^{b/2} \qquad (6.14)$$

La semilla del proceso (X_0) debe ser primo relativo a 10^b, lo cual es equivalente a que sea un número entero, non, no divisible entre 5.

Los pasos a seguir para la generación de números aleatorios con este método son:

(1) Seleccionar la cantidad de números aleatorios a generar (N).

(2) Escoger el valor inicial ó semilla (X_0); el cual deberá ser entero, non y no divisible entre 5.

(3) Seleccionar el parámetro "A" en forma tal que:

i) $A = 200t \pm p$ { t : entero positivo; p : 3, 11, 13, 19, 21, 27, 29, 37, 53, 59, 61, 67, 69, 77, 83, 91

ii) $A \cong 10^{b/2}$

(4) Generar el número aleatorio X_n mediante·

$$X_n = (A\ X_{n-1}) \bmod M$$

(5) Obtener la v.a. uniformemente distribuida (R_n) correspondiente a X_n :

$$R_n = \frac{X_n}{M}$$

(6) Regresar al paso (4) y así sucesivamente hasta obtener las "N" v.a.

6.3 Método de congruencia mixta

Este método elimina algunas de las desventajas estadísticas del método anterior (Vgr.: correlaciones de segundo y tercer orden), a expensas de una velocidad menor en la generación de los números aleatorios. La fórmula de recurrencia para este método es:

$$X_{n+1} = (AX_n + C) \bmod M \qquad (6.15)$$

Además, el método presenta la ventaja de dar períodos completos de "M" números diferentes.

Similarmente al caso de la congruencia lineal multiplicativa, el módulo "M" debe cumplir la relación:

$$M = p^b \qquad (6.16)$$

donde:

p representa el número de dígitos del sistema de numeración empleado (decimal ó binario)

b representa el número de bits por palabra de computadora.

Para lograr períodos completos se requiere:

$$\left.\begin{array}{l} C \text{ sea primo relativo a } M \\ A \equiv 1 \ (\text{mod } Z), \text{ si } Z \text{ es factor primo de } M \\ A \equiv 1 \ (\text{mod } 4), \text{ si } 4 \text{ es factor de } M^{*} \end{array}\right\} \qquad (6.16)$$

En computadoras binarias se tendrá:

$$M = 2^b \qquad (6.17)$$

donde "b" es el número de bits por palabra de computadora. El período que se obtendrá en este caso con una adecuada selección de parámetros será:

$$T = M = 2^b \qquad (6.18)$$

Para lograrlo se requiere que "C" sea primo relativo a "M", lo cual se satisface si es número entero, non y positivo. El parámetro "A" para cumplir la relación (6.17) deberá obtenerse mediante:

$$A = 2^S + 1, \ S \geq 2 \text{ y entero} \qquad (6.18)$$

además, como en el método anterior, se requerirá para buenas propiedades estadísticas de la secuencia que:

$$A \geq 2^{b/2} \qquad (6.19)$$

lo cual implica que:

$$A = 2^{b/2} + 1 \quad , \ \frac{b}{2} \text{ entero} \qquad (6.20)$$

La ecuación anterior es una condición necesaria pero no suficiente.

La semilla del método (X_0) puede ser cualquier número entero, non y positivo; su elección tiene muy poca influencia en las propiedades estadísticas de la secuencia.

Los pasos a seguir para generar una secuencia de v.a. es:

---

* $D \equiv E$ mod M implica que $\frac{D-E}{M}$ da residuo nulo.

(1) Seleccionar cantidad de números aleatorios a generar (N).

(2) Escoger el valor inicial (X_o) y la constante aditiva (C) en forma tal que sean enteros positivos nones.

(3) Seleccionar el parámetro "A" mediante:

$$A = 2^{b/2} + 1$$

(4) Generar el número aleatorio X_n, empleando la relación:

$$X_n = (AX_{n-1} + C) \bmod M$$

(5) Obtener la v.a. uniformemente distribuida (R_n) correspondiente al valor X_n:

$$R_n = \frac{X_n}{M}$$

(6) Regresar al inciso (4) y así sucesivamente hasta obtener los "N" valores aleatorios.

En <u>computadoras decimales</u> se tendrá:

$$M = 10^b \qquad (6.21)$$

donde "b" es el número de dígitos decimales por palabra de computadora.

Para obtener período completo se requiere que "C" sea entero non, positivo, no divisible entre cinco. El multiplicador "A" debe satisfacer la relación:

$$A \equiv 1 \pmod{20} \qquad (6.22)$$

lo que equivale a :

$$A = 10^s + 1, \; S > 1 \text{ y entero} \qquad (6.23)$$

además se requerirá para tener buenas propiedades estadísticas en la secuencia que:

$$A = 10^{b/2} \qquad (6.24)$$

por lo que se tendrá:

$$A = 10^{b/2} + 1, \ \frac{b}{2} \text{ entero} \qquad (6.25)$$

La ecuación anterior es una condición necesaria pero no suficiente para garantizar una adecuada calidad estadística de la secuencia.

Para la semilla (X_0), es suficiente que sea un número entero, non, positivo y no divisible entre cinco.

Los pasos a seguir para generar v.a. mediante este método en computadora digital decimal son:

1. Seleccionar la cantidad de v.a. que se desea generar (N).
2. Escoger el valor inicial (X_0) y la constante aditiva (c) en forma tal que sean enteros positivos, nones y no divisibles entre cinco.
3. Seleccionar el parámetro "A" en la siguiente forma:

   $$A = 10^{b/2} + 1, \ \frac{b}{2} \text{ entero}$$

4. Generar el número aleatorio X_n mediante :

   $$X_n = (AX_{n-1} + C) \bmod M$$

5. Evaluar la variable aleatoria uniformemente distribuida (R_n) correspondiente al valor X_n:

   $$R_n = \frac{X_n}{M}$$

6. Regresar al inciso 4 y así sucesivamente hasta obtener las "N" variables aleatorias.

BIBLIOGRAFIA

1. Carnahan B., Luther H.A., Wilkes O., "Applied Numerical - Methods", New York, John Wiley and Sons Co., 1969.

2. Gordon Geoffrey, "Systems Simulation", New Jersey, Prentice Hall, 1969.

3. Hammersley J.M. and Handscomb D.C., "Monte Carlo Methods", New York, John Wiley and Sons Co., 1964.

4. Hamming R., "Numerical Methods for Scientists and Engineers", Tokyo, Mc Graw Hill, 1962.

5. Knuth D.F., "The Art of Computer Programming", Reading Mass., Adisson Wesley, 1971.

6. Naylor, Thomas H., Balintfy J., Burdick D., Kong Chu, "Computer Simulation Techniques", New York, John Wiley and Sons - Co., 1966.

7. RAND Corporation, "A Million Randon Digits With 100,000 - - Normal Deviates", Glencoe Ill., The Free Press, 1955.

T A B L A   A1

TABLA DE NUMEROS ALEATORIOS

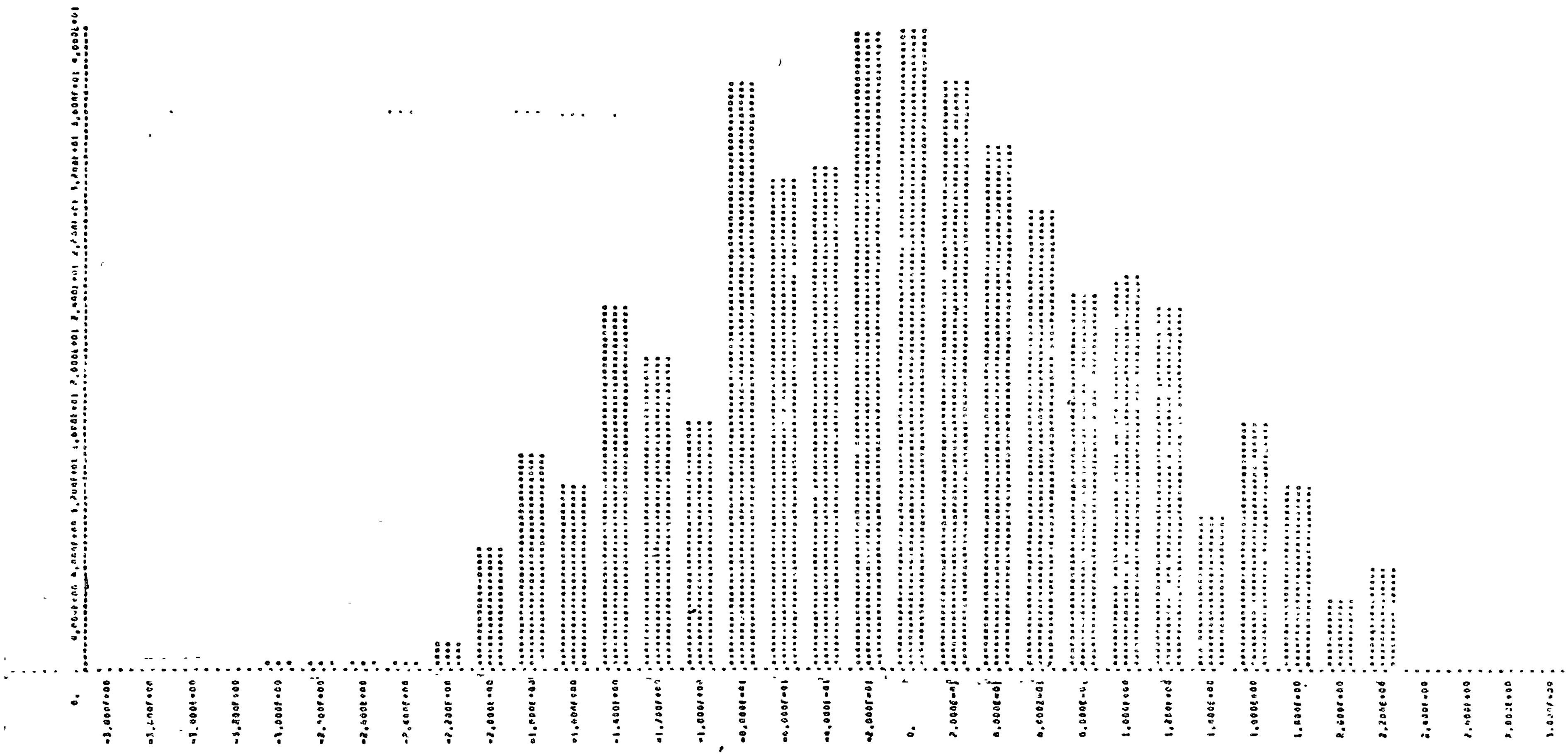
0.354	0.761	0.006	0.892	0.317	0.420	0.751	0.342	0.926	0.490
0.417	0.926	0.534	0.012	0.809	0.389	0.063	0.243	0.[illegible]20	0.408
0.742	0.020	0.166	0.177	0.069	0.297	0.394	0.438	0.480	0.603
0.277	0.911	0.177	0.753	0.274	0.817	0.571	0.280	0.350	0.083
0.330	0.616	0.552	0.765	0.555	0.065	0.342	0.422	0.747	0.314
0.241	0.566	0.294	0.645	0.395	0.379	0.911	0.406	0.420	0.973
0.125	0.870	0.966	0.457	0.113	0.639	0.472	0.477	0.463	0.599
0.426	0.232	0.202	0.678	0.230	0.696	0.232	0.257	0.386	0.373
0.683	0.507	0.652	0.998	0.034	0.757	0.520	0.504	0.791	0.236
0.331	0.932	0.413	0.744	0.145	0.234	0.694	0.515	0.467	0.555
0.476	0.602	0.836	0.175	0.461	0.258	0.308	0.235	0.019	0.991
0.827	0.290	0.933	0.875	0.583	0.987	0.551	0.761	0.470	0.377
0.144	0.729	0.930	0.955	0.924	0.995	0.524	0.051	0.427	0.006
0.854	0.081	0.056	0.134	0.525	0.009	0.846	0.710	0.210	0.568
0.813	0.641	0.015	0.952	0.104	0.983	0.744	0.199	0.042	0.156
0.897	0.951	0.546	0.454	0.591	0.098	0.848	0.590	0.663	0.614
0.425	0.822	0.966	0.002	0.998	0.114	0.759	0.871	0.053	0.366
0.942	0.636	0.850	0.508	0.035	0.698	0.981	0.612	0.346	0.259
0.700	0.796	0.935	0.628	0.849	0.811	0.280	0.955	0.927	0.649
0.235	0.352	0.057	0.492	0.895	0.763	0.838	0.492	0.544	0.204
0.878	0.746	0.708	0.210	0.590	0.771	0.965	0.254	0.172	0.465
0.113	0.328	0.102	0.337	0.970	0.595	0.328	0.230	0.069	0.742
0.319	0.399	0.292	0.245	0.582	0.065	0.502	0.809	0.744	0.995
0.089	0.300	0.358	0.744	0.426	0.968	0.612	0.438	0.147	0.561
0.750	0.863	0.301	0.686	0.605	0.691	0.405	0.149	0.816	0.378

```
C     PROGRAMA PARA OBTENER V.A. A PARTIR DE SU FUNCION DE PROBABILIDAD
C     ACUMULADA POR EL METODO DE LA TRANSFORMADA INVERSA.
C     EL SIGNIFICADO DE LAS VARIABLES USADAS ES
C     N=NUMERO DE PARTES EN QUE SE DISCRETIZA LA CURVA DE PROBABILIDAD A
C     CUMULADA, M=CANTIDAD DE V.A. UNIFORMEMENTE DISTRIBUIDAS QUE SE USA
C     RA PARA EFECTUAR LA PROYECCION, V=SEMILLA DE LAS V.A. UNIFORMEMEN-
C     TE DISTRIBUIDAS, F=ALTURAS DE LA CURVA DE PROBABILIDAD ACUMULADA,
C     X=ABSCISAS DE LA MISMA, NX=NUMERO DE VECES QUE LA PROYECCION CAE
C     SOBRE LA ABSCISA X, R=V.A.UNIFORMEMENTE DISTRIBUIDAS, IPOS=RENGLO-
C     NES QUE COMPONEN LA SUBTABLA DE BUSQUEDA BINARIA, MD2=ELEMENTO CEN
C     TRAL DE LA MISMA
      DIMENSION X(40),F(40),NX(40),IPOS(40),R(600)
C     LECTURA E IMPRESION DE DATOS
    1 READ(5,100) N,M,V
      IF(N) 2,2,3
    2 CALL EXIT
    3 READ(5,150) (X(I),I=1,N)
      READ(5,150) (F(I),I=1,N)
      WRITE(6,200)
      DO 4 I=1,N
      WRITE(6,250) X(I),F(I)
    4 NX(I)=0
C     GENERACION DE V.A. UNIFORMEMENTE DISTRIBUIDAS
      DO 17 I=1,M
   17 R(I)=RANDOM(V)
C     APLICACION DEL METODO DE LA TRANSFORMADA INVERSA
      DO 15 I=1,M
      NUM=N
      IPOS(1)=1
      IPOS(N)=N
      MD2=N/2
    5 IF(R(I)-F(MD2)) 6,7,8
    6 LS=IPOS(1)
      LI=MD2
      GO TO 9
    7 NX(MD2)=NX(MD2) + 1
      GO TO 15
    8 LS=MD2
      LI=IPOS(NUM)
    9 LDIF=LS-LI
      IF(LDIF + 1) 13,10,10
   10 DIF=(F(LS)+F(LI))/2.0
      IF(R(I)-DIF) 11,12,12
   11 NX(LS)=NX(LS)+1
      GO TO 15
   12 NX(LI)=NX(LI)+1
      GO TO 15
   13 NUM=0
      DO 14 K=LS,LI
      NUM=NUM + 1
   14 IPOS(NUM)=K
      MD2=(IPOS(1)+IPOS(NUM))/2
      GO TO 5
   15 CONTINUE
      IMPRESION DE RESULTADOS
      WRITE(6,300)
      DO 16 I=1,N
   16 WRITE(6,350) X(I),NX(I)
      LLAMADO DE SUBRUTINA PARA GRAFICAR
```

```
      GO TO 1
C     FORMATOS DE LECTURA E IMPRESION
  100 FORMAT(2I5,F10.0)
  150 FORMAT(7F10.0)
  200 FORMAT(1H1,4(/),40X, LA FUNCION DE PROBABILIDAD ACUMULADA ES ,//,40
     17X, X ,22X, F(X) ,/)
  250 FORMAT(/,40X,2(1PE15.8,10X))
  300 FORMAT(4(/),46X, LOS RESULTADOS OBTENIDOS SON ,//,42X, VARIABLE AL
     1EATORIA(X) ,5X, FRECUENCIA ,/)
  350 FORMAT(/,45X,1PE15.8,10X,I5)
      END
      SUBROUTINE HISTOG(N,X,NX)
C     SUBRUTINA PARA OBTENER HISTOGRAMAS
C     N=NUMERO DE PUNTOS MUESTRALES, X=ABSCISAS DE CADA PUNTO, NX=ALTURA
C     DE CADA PUNTO
      DIMENSION X(40),NX(40),ORDIN(11),KAXIS(101)
      DATA ISTAR,IBLAN,IPOIN,IDASH/1H*,1H ,1H.,1H-/
   60 FORMAT(1H1)
   61 FORMAT(////,6X,11(1PE10.3))
   62 FORMAT(12X,101A1)
   63 FORMAT(1X,1PE10.3,1X,101A1)
      WRITE(6,60)
      NMAX=NX(1)
      DO 2 I=2,N
      IF(NX(I)-NMAX) 2,2,1
    1 NMAX=NX(I)
    2 CONTINUE
      PASO=FLOAT(NMAX)/10.0
      ORDIN(1)=0.0
      ORDIN(11)=FLOAT(NMAX)
      DO 3 I=2,10
    3 ORDIN(I)=PASO + ORDIN(I-1)
      DO 4 I=1,101
    4 KAXIS(I)=IDASH
      DO 5 I=1,101,10
    5 KAXIS(I)=ISTAR
      WRITE(6,61) ORDIN
      WRITE(6,62) KAXIS
      DO 18 I=1,N
      KAXIS(1)=IPOIN
      L=(NX(I)*101)/NMAX
      IF(L-2) 6,8,8
    6 DO 7 K=2,101
    7 KAXIS(K)=IBLAN
      GO TO 11
    8 DO 9 K=2,L
    9 KAXIS(K)=ISTAR
      IF(L-101) 19,11,11
   19 LP1=L+1
      DO 10 K=LP1,101
   10 KAXIS(K)=IBLAN
   11 DO 14 K=1,3
      IF(K-2) 13,12,13
   12 WRITE(6,63) X(I),KAXIS
      GO TO 14
   13 WRITE(6,62) KAXIS
   14 CONTINUE
      IF(L-2) 17,15,15
   15 DO 16 K=2,L
   16 KAXIS(K)=IBLAN
```

```
17 WRITE(6,62) KAXIS                                                        09NAI121
18 CONTINUE                                                                 09NAI122
   RETURN                                                                   09NAI123
   END                                                                      09NAI124
```

```
****************************************************************************************
1234567890123456789012345678901234567890123456789012345678901234567890123456789O
*********************  LOS DATOS PARA ESTE PROBLEMA SON  *********************
  39   500   251.37
 -3.8      -3.6      -3.4      -3.2      -3.0      -2.8      -2.6
 -2.4      -2.2      -2.0      -1.8      -1.6      -1.4      -1.2
 -1.0      -0.8      -0.6      -0.4      -0.2      0.0       0.2
 0.4       0.6       0.8       1.0       1.2       1.4       1.6
 1.8       2.0       2.2       2.4       2.6       2.8       3.0
 3.2       3.4       3.6       3.8
 .0        .00016    .00034    .00069    .00135    .00256    .00466    .00820
 .00820    .0130     .02275    .03593    .0548     .08076    .11507
 .15866    .21186    .27425    .34458    .42074    .5        .57926
 .65542    .72575    .78814    .84134    .88493    .91924    .9452
 .96407    .97725    .987      .9918     .99534    .99744    .99865
 .99931    .99966    .99984    1.0
**********  FINALIZAN LOS DATOS CON UNA TARJETA EN BLANCO  *******************
```



RESULTADOS DEL PROGRAMA A1. Se dió como dato los valores dis-
cretizados de la f.d.a. de una v.a. gaussiana

(A)

P R O G R A M A   A2

```
C       PROGRAMA PARA GENERAR V.A. CON F.D.P. EXPONENCIAL POR EL METODO DE
C       LA TRANSFORMADA INVERSA
C       EL SIGNIFICADO DE LAS VARIABLES USADAS ES
C       DASH=VARIABLE ALFANUMERICA PARA IMPRESION, N=CANTIDAD DE V.A. QUE
C       SE DESEA GENERAR, V=SEMILLA PARA GENERAR V.A. UNIFORMEMENTE DISTRI
C       BUIDAS, T=V.A. CON F.D.P. EXPONENCIAL, TMED=MEDIA DE T
        DIMENSION T(50),DASH(50)
        DO 1 I=1,50
      1 DASH(I)=1H-
C       LECTURA DE DATOS
      2 READ(5,50) N,TMED,V
        IF(N) 3,3,4
      3 CALL EXIT
C       METODO DE LA TRANSFORMADA INVERSA
      4 DO 5 I=1,N
        R=RANDOM(V)
      5 T(I)=-TMED*ALOG(R)
C       IMPRESION DE RESULTADOS
        WRITE(6,100) TMED,DASH
        DO 6 I=1,N
      6 WRITE(6,150) T(I)
        GO TO 2
C       FORMATOS DE LECTURA E IMPRESION
     50 FORMAT(I2,2F10.0)
    100 FORMAT(1H1,4(/),40X, TABLA DE V.A. CON F.D.P. EXPONENCIAL Y ,/,40X
       1, MEDIA=  ,1PE12.5,/,34X,50A1,/)
    150 FORMAT(/,50X,1PE15.8)
        END
```

```
********************************************************************************
12345678901234567890123456789012345678901234567890123456789012345678901234567890
**********************  LOS DATOS PARA ESTE PROBLEMA SON  **********************
30  2.0         39.335                                                        30

***********  FINALIZAN LOS DATOS CON UNA TARJETA EN BLANCO  *******************
```

RESULTADOS DEL PROGRAMA A2

TABLA DE V.A. CON F.D.P. EXPONENCIAL Y
MEDIA= 2.00000E+00

4.09618746E+00

2.67646863E+00

3.52867571E+00

4.11561594E+00

3.87982626E+00

3.93948294E+00

2.74078861E-01

4.20815313E+00

4.63682184E+00

1.47563109E+00

1.77074952E+00

1.38129353E-01

4.61659283E+00

1.81852675E+00

9.28588654E-01

1.37579539E+00

1.04866204E+00

3.22997946E+00

5.13171831E-01

5.02136215E+00

6.84689330E-01

2.89702449E-02

2.99067203E+00

1.48542146E+00

4.22960238E-03

7.43135958E+00

P R O G R A M A A3

```
C       PROGRAMA PARA GENERAR V.A. GAUSSIANAS NORMALIZADAS POR EL METODO
C       POLAR
C       EL SIGNIFICADO DE LAS VARIABLES USADAS ES
C       N=CANTIDAD DE V.A. QUE SE DESEA GENERAR, V Y W=SEMILLAS PARA GENE-
C       RAR DOS V.A. UNIFORMEMENTE DISTRIBUIDAS E INDEPENDIENTES, X1 Y X2=
C       V.A. GAUSSIANAS NORMALIZADAS
        DIMENSION X1(50),X2(50)
C       LECTURA DE DATOS
      1 READ(5,50) N,V,W
        IF(N) 2,2,3
      2 CALL EXIT
C       METODO POLAR
      3 DO 4 I=1,N
        X1(I)=SQRT(-2.0*ALOG(RANDOM(V)))*COS(2.0*3.14159*RANDOM(W))
      4 X2(I)=SQRT(-2.0*ALOG(RANDOM(V)))*SIN(2.0*3.14159*RANDOM(W))
C       IMPRESION DE RESULTADOS
        WRITE(6,100)
        DO 5 I=1,N
      5 WRITE(6,150) X1(I),X2(I)
        GO TO 1
C       FORMATOS DE LECTURA E IMPRESION
     50 FORMAT(I2,2F10.0)
    100 FORMAT(1H1,4(/),30X, TABLA DE V.A. GAUSSIANAS GENERADAS POR EL MET
       1ODO POLAR ,//,39X, X1(USANDO COSENO) ,9X, X2(USANDO SENO) ,/)
    150 FORMAT(/,40X,2(1PE15.8,10X))
        END

*******************************************************************************
12345678901234567890123456789012345678901234567890123456789012345678901234567890
***********************    LOS DATOS PARA ESTE PROBLEMA SON    ****************
30  98.798      63.435                                                     27

**********   FINALIZAN LOS DATOS CON UNA TARJETA EN BLANCO   ******************
```

RESULTADOS DEL PROGRAMA A3

TABLA DE V.A. GAUSSIANAS GENERADAS POR EL METODO POLAR

X1(USANDO COSENO)	X2(USANDO SENO)
3.96491860E-01	-2.80382365E-01
-2.36134780E-02	-1.75703973E+00
4.02427375E-01	-8.72292458E-02
1.86969163E-01	5.69764744E-02
9.87021896E-01	-1.90288618E-01
1.39000379E-01	1.20919806E-01
-4.22973226E-01	2.59285727E-02
1.52950927E+00	3.72777853E-01
-6.00923363E-02	-5.35781180E-01
-1.86221988E-02	8.23131415E-01
-6.92983242E-01	1.53917926E+00
-2.93306123E-01	1.60081915E+00
-6.55419272E-01	-8.80235572E-01
2.12915344E+00	-6.72302142E-01
1.02014606E+00	3.73158016E-01
3.12874634E-01	1.11108406E+00
8.80172399E-02	1.31814998E+00
1.02489845E+00	1.33549912E+00
1.63578434E-01	3.03852843E-01
5.14986850E-01	6.26174240E-01
1.26881937E+00	1.57643494E+00
-1.08072134E+00	3.88624337E-01
-6.45314805E-01	-4.90349495E-01
-2.44339573E-01	3.10051118E-01
9.70039370E-01	-2.52003627E-01
9.76475323E-01	2.33053098E+00

```
C      PROGRAMA PARA SIMULAR UN MODELO DE CANALES MULTIPLES (GASOLINERA)
C      EL SIGNIFICADO DE LAS VARIABLES USADAS ES
C      NA=TOTAL DE AUTOMOVILES QUE LLEGARAN A LA GASOLINERA EN TODO EL PE
C      RIODO DE TIEMPO, N=CANTIDAD DE BOMBAS, LIM=MAXIMO NUMERO DE COCHES
C      POR BOMBA, MTA=MEDIA DEL TIEMPO ESPERADO DE LLEGADAS, MTS Y VTS=ME
C      DIA Y DESVIACION ESTANDAR DEL TIEMPO DE SERVICIO, NCOLA=NUMERO DE
C      VEHICULOS QUE HAY EN CADA BOMBA, TO Y TES=TIEMPO OCIOSO Y DE ESPE-
C      RA PARA CADA BOMBA, TTB=TIEMPO TOTAL DE ESPERA PARA CADA BOMBA,
C      TCOLA=TIEMPO DE ESPERA PARA CADA COCHE DE LA COLA, T=TIEMPO ACUMU-
C      LADO DE LLEGADAS, TTO Y TTES=TIEMPOS TOTALES DE OCIO Y ESPERA, NR=
C      NUMERO DE VEHICULOS RECHAZADOS, TA=TIEMPO DE LLEGADAS, TS=TIEMPO
C      DE SERVICIO, ETTO=TIEMPO OCIOSO ESPERADO, ETTES=TIEMPO DE SERVICIO
C      ESPERADO, POR=PORCENTAJE DE COCHES RECHAZADOS, L=BOMBA CON MENOR
C      NUMERO DE VEHICULOS, J=BOMBA CON EL MENOR TIEMPO TOTAL DE ESPERA
       REAL MTA,MTS
       DIMENSION NCOLA(30),TO(30),TES(30),TTB(30),V1(30),V2(30),TS(30,200
      1),TCOLA(30,10),TA(200)
       DATA A/119.78/
       DATA V1/23.96,55.82,75.56,74.68,85.8,48.4,38.2,10.11,89.34,95.25,4
      14.66,61.17,57.9,87.58,58.67,34.42,43.38,62.84,19.38,45.86,71.85,59
      22.12,20.27,32.46,50.58,43.69,84.53,10.48,69.3,16.61/
       DATA V2/79.84,85.79,24.86,78.8,88.72,86.45,32.0,65.99,72.59,97.69,
      146.29,85.5,84.48,86.1,18.3,80.84,51.45,41.79,43.24,25.96,89.38,27.
      268,42.33,48.77,71.82,39.1,17.51,81.07,89.34,70.54/
C      LECTURA DE DATOS
     1 READ(5,100) NA,N,LIM,MTA,MTS,VTS
       IF(NA) 2,2,3
     2 CALL EXIT
C      INICIO DE TIEMPO
     3 DO 4 I=1,N
       NCOLA(I)=0
       TO(I)=0.0
       TES(I)=0.0
       TTB(I)=0.0
       DO 15 K=1,LIM
    15 TCOLA(I,K)=0.0
     4 CONTINUE
       T=0.0
       TTES=0.0
       TTO=0.0
       NR=0
       NCOLA(1)=1
C      GENERACION DE LOS TIEMPOS DE LLEGADA Y SERVICIO
       DO 22 I=1,NA
    22 TA(I)=-MTA*ALOG(RANDOM(A))
       DO 23 I=1,N
       C=V1(I)
       D=V2(I)
       DO 23 K=1,NA
       TS(I,K)=VTS*SQRT(-2.0*ALOG(RANDOM(C)))*COS(2.0*3.14159*RANDOM(D))
      1 + MTS
    23 CONTINUE
       IF(N.EQ.1) GO TO 6
       DO 5 I=2,N
       T=T+TA(I)
       TO(I)=T
       TTO=TTO + TO(I)
     5 NCOLA(I)=1
     6 NUM=N
       DO 7 I=1,N
```

```
      TTB(I)=TO(I) + TS(I,1)
    7 TCOLA(I,1)=TTB(I)
    8 NUM=NUM + 1
      IF(NUM.GT.NA) GO TO 18
      T=T+TA(NUM)
      DO 17 I=1,N
      M=NCOLA(I)
      IF(M.EQ.0) GO TO 17
      DO 16 K=1,M
      IF(T.LE.TCOLA(I,K)) GO TO 16
      NCOLA(I)=NCOLA(I) - 1
   16 CONTINUE
      L=M-NCOLA(I)
      IF(L) 17,17,20
   20 M=NCOLA(I)
      DO 21 J=1,M
   21 TCOLA(I,J)=TCOLA(I,J+L)
   17 CONTINUE
      L=1
      J=1
      NCOMI=NCOLA(1)
      TTBMI=TTB(1)
      IF(N.EQ.1) GO TO 12
      DO 11 I=2,N
      IF(NCOMI.LE.NCOLA(I)) GO TO 10
      NCOMI=NCOLA (I)
      L=I
   10 IF(TTBMI.LE.TTB(I)) GO TO 11
      TTBMI=TTB(I)
      J=I
   11 CONTINUE
   12 DIF=T - TTB(J)
      IF(DIF.GT.0.0) GO TO 14
      IF(NCOMI.GE.LIM) GO TO 13
      DIF=T-TTB(L)
      TES(L)=-DIF
      TO(L)=0.0
      J=L
      GO TO 9
   13 NR=NR+1
      GO TO 8
   14 TO(J)=DIF
      TES(J)=0.0
    9 NCOLA(J)=NCOLA(J) + 1
      M=NCOLA(J)
      TTES=TTES + TES(J)
      TTO=TTO + TO(J)
      TTB(J)=T + TES(J) + TS(J,NUM)
      TCOLA(J,M)=TTB(J)
      GO TO 8
   18 ANA=NA
      ETTO=TTO/ANA
      ETTES=TTES/ANA
      POR=NA-NR
      POR=POR/ANA
      IMPRESION DE RESULTADOS
      WRITE(6,150) N,NA,T,NR,POR,ETTO,ETTES
      IF(POR.LT.0.95) GO TO 19
      GO TO 1
   19 IF(N.GE.30) GO TO 1
      N=N + 1
```

```
      GO TO 3                                                                  13SIM12
C     FORMATOS DE LECTURA E IMPRESION                                          13SIM12
  100 FORMAT(I5,I2,I3,3F10.0)                                                  13SIM124
  150 FORMAT(5(/),40X, LOS VALORES OBTENIDOS SON ,///,40X, NUMERO DE BOM13SIM125
     1BAS=  ,I2,//,40X, TOTAL DE VEHICULOS= ,I5,//,40X, TIEMPO TOTAL DE 13SIM12
     2LLEGADAS=  ,1PE15.8,//,40X, VEHICULOS RECHAZADOS=  ,I5,//,40X, POR13SIM127
     3CENTAJE DE VEHICULOS ATENDIDOS=  ,E11.4,//,40X, TIEMPO OCIOSO ESPER13SIM128
     4ADO=  ,1PE12.5,//,40X, TIEMPO DE ESPERA ESPERADO=  ,1PE12.5)           13SIM12
      END                                                                      13SIM130

*****************************************************************************************
1234567890123456789012345678901234567890123456789012345678901234567890123456789
*********************  LOS DATOS PARA ESTE PROBLEMA SON  ***********************
   90 1  4     2.0       5.0        2.0
**********  FINALIZAN LOS DATOS CON UNA TARJETA EN BLANCO  *******************
```

RESULTADOS DEL PROGRAMA A4

LOS VALORES OBTENIDOS SON

NUMERO DE BOMBAS= 1

TOTAL DE VEHICULOS= 90

TIEMPO TOTAL DE LLEGADAS= 1.52891918E+02

VEHICULOS RECHAZADOS= 55

PORCENTAJE DE VEHICULOS ATENDIDOS= 3.8889E-01

TIEMPO OCIOSO ESPERADO= 1.51602E-02

TIEMPO DE ESPERA ESPERADO= 4.53043E+00

LOS VALORES OBTENIDOS SON

NUMERO DE BOMBAS= 2

TOTAL DE VEHICULOS= 90

TIEMPO TOTAL DE LLEGADAS= 1.91604037E+02

VEHICULOS RECHAZADOS= 10

PORCENTAJE DE VEHICULOS ATENDIDOS= 8.8889E-01

TIEMPO OCIOSO ESPERADO= 6.16666E-02

TIEMPO DE ESPERA ESPERADO= 6.90198E+00

LOS VALORES OBTENIDOS SON

NUMERO DE BOMBAS= 3

TOTAL DE VEHICULOS= 90

TIEMPO TOTAL DE LLEGADAS= 1.45268346E+02

VEHICULOS RECHAZADOS= 1

PORCENTAJE DE VEHICULOS ATENDIDOS= 9.8889E-01

TIEMPO OCIOSO ESPERADO= 5.05192E-01

TIEMPO DE ESPERA ESPERADO= 4.94212E+00

P R O G R A M A   A5

```
C     PROGRAMA PARA GENERAR NUMEROS ALEATORIOS POR EL METODO DE CONGRUEN
C     CIA LINEAL MULTIPLICATIVA
C     EL SIGNIFICADO DE LAS VARIABLES UDADAS ES
C     N=CANTIDAD DE NUMEROS ALEATORIOS QUE SE DESEA, A=CONSTANTE MULTI-
C     PLICATIVA, X(1)=SEMILLA PARA GENERAR LOS NUMEROS ALEATORIOS, M=MO-
C     DULO, X=NUMEROS ALEATORIOS, R=NUMEROS ALEATORIOS DISTRIBUIDOS UNI-
C     FORMEMENTE
      INTEGER A,X(100)
      DIMENSION R(100)
      M=2**16
C     LECTURA E IMPRESION DE LOS PARAMETROS
    1 READ(5,100) N,A,X(1)
      IF(N) 2,2,3
    2 CALL EXIT
    3 WRITE(6,150) X(1),A
      N=N+3
C     VERIFICANDO SI ES ADECUADO EL PARAMETRO A
      L=A/8
      L=-L*8 + A
      IF(L-3) 4,5,4
    4 IF(L-5) 1,5,1
C     METODO DE LA CONGRUENCIA LINEAL
    5 DO 6 I=2,N
      L=A*X(I-1)
      X(I)=L/M
      X(I)=-X(I)*M + L
    6 R(I)=FLOAT(X(I))/FLOAT(M)
C     IMPRESION DE RESULTADOS
      WRITE(6,200)
      DO 7 I=3,N
    7 WRITE(6,250) X(I),R(I)
      GO TO 1
C     FORMATOS DE LECTURA E IMPRESION
  100 FORMAT(3I5)
  150 FORMAT(1H1,4(/),37X, GENERACION DE NUMEROS ALEATORIOS POR EL METODO
     1O ,//,37X, CONGRUENCIAL LINEAL, USANDO COMO PARAMETROS ,//,37X, X0
     2=  ,I7,//,37X, A=  ,I7)
  200 FORMAT(4(/),45X, TABLA DE NUMEROS ALEATORIOS ,//,50X, X ,16X, R ,/
     1)
  250 FORMAT(/,47X,I6,10X,F9.6)
      END
```

```
********************************************************************************
12345678901234567890123456789012345678901234567890123456789012345678901234567890
**********************  LOS DATOS PARA ESTE PROBLEMA SON  **********************
   50  251    13

**********  FINALIZAN LOS DATOS CON UNA TARJETA EN BLANCO  ********************
```

RESULTADOS DEL PROGRAMA A5

GENERACION DE NUMEROS ALEATORIOS POR EL METODO CONGRUENCIAL LINEAL, USANDO COMO PARAMETROS

X0= 13

A= 251

TABLA DE NUMEROS ALEATORIOS

X	R
32581	0.497147
51367	0.783798
48061	0.733353
4687	0.071518
62325	0.951004
46007	0.702011
13421	0.204788
26335	0.401840
56485	0.861893
21959	0.335068
6685	0.102005
39535	0.603256
27349	0.417313
48855	0.745468
7373	0.112503
15615	0.238266
52741	0.804764
65255	0.995712
60541	0.923782
56975	0.869370

5 MEAN

is a statistical analysis program that computes measures of central tendency and measures of dispersion and prints frequency histograms.

The purpose of statistical analysis is to make generalizations about counted items. Counting consists of assigning numbers to things. All distinguishable entities, tangible or intangible, identical or different, can be counted. The assigned numbers form an abstract representation of the entities that were counted. The abstracted numbers are useful because they can be manipulated according to the agreed rules of mathematics. Sometimes the manipulation yields useful generalizations about the abstract numbers which can be transferred back to the original entities that were counted.

The manipulation of the numbers does not depend on the origin of the count, only on the rules of mathematical statistics. However, there are no firm rules for the transfer of the generalities back to the entities originally counted. This transfer depends on the honesty, goodwill, and judgment of the person who makes the transfer. The old saying is, "Statistics don't lie, only statisticians."

The moral of this short digression into the theory of numbers is this. The analyst who wants to use statistical generalities must not forget the humble circumstances of the original entities and the circumstances of the count.

As an example, consider the set of 30 two-digit numbers in Figure 5-1.

FIGURE 5-1

A SET OF 30 NUMBERS

80, 85, 56, 85, 81, 99, 73, 68, 69, 86, 67, 80, 86, 95, 95,
75, 86, 68, 81, 57, 63, 97, 98, 91, 83, 84, 84, 77, 67, 94

Without any knowledge of the entities represented by these numbers, we can make the following generalizations.

There are 30, two-digit number entities.
There are 29 commas in the set.
The digit "eight" occurs 16 times.
The smallest two-digit number is 56.
The largest two-digit number is 99.
The arithmetic mean is 81.0.

All these generalizations about the set of numbers are correct, according to the rules of mathematical statistics. Also, they may not be very interesting. The circumstances of the original entities and the circumstances of the count are what give the number set meaning and interest.

As a matter of fact, the set of numbers represents a count of units produced in a particular procedure during an 8-hour shift by 30 workers producing the same product on the same day. Therefore, we have a "population" of 30 workers. There is no significance attached to the order in which the workers' output was selected to be counted. The type of entity counted was the same for each worker The count for other workers on other days is not included in the set. The exact time of day for the production of each unit has been lost in the counting procedure.

This knowledge about the circumstances of the count allows us to be very selective in the statistical procedures applied and the generalizations that are transferred back to the original entities. We do care, for example, that the set includes 30 two-digit numbers for 30 workers. We do not care that the digit "eight" occurs 16 times or that it takes 29 commas to separate 30 items. We do care that the lowest output is 56 units and the highest is 99 units. Let us turn to some statistical methods that can be applied to the numbers generated by the count.

RANKING AND FREQUENCY DISTRIBUTIONS

A simple and sometimes useful statistical method is to rank the numbers from lowest to highest. Figure 5-2 ranks the worker unit output count from lowest to highest. This sorting process helps facilitate finding the range, the low and high values, and the median, as defined later. Program MEAN ranks numbers from low to high by a simple sorting process.

FIGURE 5-2

30 NUMBERS RANKED FROM LOWEST TO HIGHEST

56, 57, 63, 67, 67, 68, 69, 73, 75, 77, 80, 80, 81,
81, 83, 84, 84, 85, 85, 86, 86, 86, 88, 91, 94, 95,
95, 97, 98, 99

A quantity of data such as Figure 5-2 is often much easier to represent in a frequency distribution. The construction of a frequency distribution from raw data involves defining the class interval in which several number items will be grouped. The class interval size becomes an important consideration because an improper selection can distort the appearance of the data. For instance, the selection of an extremely large class interval would group all 30 numbers into a single group and would reveal very little about the distribution of production levels. Likewise, the selection of a very small class interval would produce a distribution in which many classes would contain few or no numbers. When forming distributions, it is *helpful* to choose an odd value for the interval width.

For example, in the distribution of production levels for the workers, a class interval of nine was chosen, with the midpoints falling on the integer values 59, 68, 77, 86, and 95. This will ensure that any individual number will fall within a class and not on a border between two classes.

Table 5-1 divides the 30 numbers into five class intervals, each nine units wide. The count of the numbers within each class interval is known as the frequency. Thus a *frequency distribution* is a "count" of the original count within class intervals. This is a further abstraction from the original entities. It is useful, however, because it summarizes many numbers.

TABLE 5-1

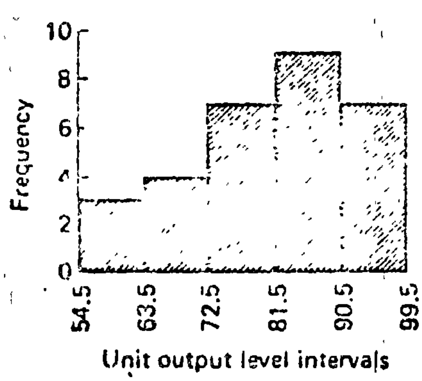
30 NUMBERS DIVIDED INTO FIVE CLASS INTERVALS

Class interval	Count of numbers in interval
54.5-63.499	3
63.5-72.499	4
72.5-81.499	7
81.5-90.499	9
90.5-99.499	7
	30

Frequency distributions are often represented graphically in histograms, frequency polygons, and ogives. *Histograms*, commonly called bar charts or bar graphs, use vertical or horizontal bars of varying length to represent data frequencies within each class interval. The bars are placed side by side because there are no gaps between the upper limit of a class and the lower limit of the next higher class. Figure 5-3 is a histogram of the distribution of production levels.

FIGURE 5-3

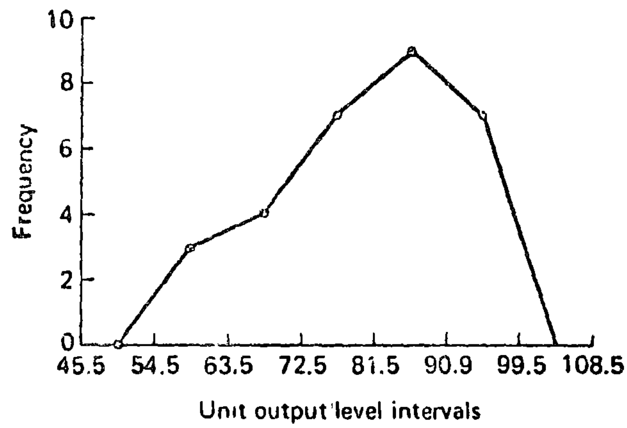
HISTOGRAM OF THE 30 NUMBERS



Frequency polygons are produced by connecting the midpoints of adjacent classes with straight lines. At both ends of the distribution, the lines are connected to the baseline at the midpoints of the vacant classes bordering each side of the distribution. Figure 5-4 is the frequency polygon for the distribution of production levels.

FIGURE 5-4

FREQUENCY POLYGON FOR THE 30 NUMBERS

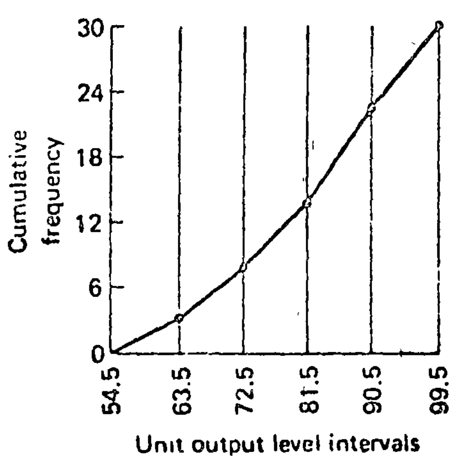


Ogives, which are cumulative frequency distributions, are used to show the number of data observations that fall either above or below a particular value. An ogive for the distribution of production levels might serve to indicate the total number of workers who worked below a particular level (see Figure 5-5). As a further illustration, the cumulative frequency scale can be shown as a percentage, with 30 items set at 100 percent, 15 items at 50 percent, and so on, for the entire scale. This normalization process results in a *relative frequency scale.*

Program MEAN produces horizontal frequency polygons with a cumulative frequency ogive superimposed.

FIGURE 5-5

CUMULATIVE FREQUENCY PLOT FOR THE 30 NUMBERS



MEASURES OF CENTRAL TENDENCY

Measures of central tendency attempt to summarize a large amount of number items by a single number. The most commonly used measure of central tendency is the *arithmetic mean*—the sum of the numbers in the population divided by the size of the population.

In mathematical form this is expressed as

$$\mu = \frac{\Sigma X}{N} \tag{5.1}$$

where μ = arithmetic mean
X = a number
N = population size

For the workers' output, the arithmetic mean is computed by summing the numbers in Figure 5-1 and dividing by 30. Thus we have

$$\mu = \frac{\Sigma X}{N} = \frac{80 + 85 + \cdots + 94}{30}$$

$$\mu = \frac{2430}{30}$$

$$\mu = 81.0 \tag{5.2}$$

The arithmetic mean is widely used because it can summarize a large amount of data in a single number. One can say, "The output per worker is 81 units," forgetting all about the differences among the 30 workers. This type of statement is fine as long as we want only a rough estimate of the circumstances of the original count.

Another measure of central tendency is the *median*, the value of the middle item in a ranked or sorted list of number items. Let N be the count of items in the list (population size). Then when N is odd we write

$$\text{median is } \frac{N+1}{2} \text{ item in the list}$$

and when N is even

$$\text{median is } \frac{(N/2) \text{ item} + (N/2 + 1) \text{ item}}{2}$$

The 30 numbers for the output levels are ranked in Figure 5-2. Since 30 is even, the second formula must be used:

$$\text{median} = \frac{(30/2) \text{ item} + (30/2 + 1) \text{ item}}{2}$$

$$= \frac{15\text{th item} + 16\text{th item}}{2}$$

$$= \frac{83 + 84}{2}$$

$$= 83.5 \tag{5.3}$$

Whether this median is a better or worse estimate of central tendency for the data depends on the biases of the users.

Still another measure of central tendency is the mode. The *mode* of a list of numbers is the value of the number that occurs most often. The mode of the 30 numbers of Figure 5-2 is 86, which occurs three times. Whether one wishes to make a strong statement about central tendency based on three occurrences out of 30 is left to the users of the statistic.

MEASURES OF ABSOLUTE DISPERSION

Central tendency measurements aim to describe, with one value, all the values in a set of numbers. It may also be helpful to know about the dispersion of elements in the distribution—that is, how highly concentrated or widely scattered are the observations.

The most common measures of dispersion are the range, the average deviation, the variance, and the standard deviation. Each of these statistical measures of dispersion is discussed in turn, and all are computed by program MEAN.

The *range* of a distribution has two definitions. According to one, *range* is the difference between the highest and lowest values in a distribution. According to the second, *range* is the lowest and highest values themselves. For the 30 numbers of Figure 5-1, the range is defined either as 99 minus 56 which is 43 units, or with the statement "the range is from 56 to 99."

The *average deviation* is the arithmetic mean of *absolute* differences between each item and any measure of central tendency. When the arithmetic mean of the number set is used as the measure of central tendency, the formula is

$$AD = \frac{\Sigma|X - \mu|}{N} \qquad (5.4)$$

where AD = average deviation
μ = arithmetic mean
X = a number item
N = count of numbers

For the numbers of Figure 5-1, the average deviation is

$$\begin{aligned} AD &= \frac{|80 - 81| + |85 - 81| + \cdots + |94 - 81|}{30} \\ &= \frac{1 + 4 + \cdots + 13}{30} \\ &= \frac{280}{30} \\ &= 9.3333 \end{aligned} \qquad (5.5)$$

The *variance* is the arithmetic mean of the squared difference between each number and the measure of central tendency. The measure of central tendency used is always the arithmetic mean.

$$\sigma^2 = \frac{\Sigma (X - \mu)^2}{N} \qquad (5.6)$$

where σ^2 is the variance.

Again, for the numbers for the factory output, we write

$$\sigma^2 = \frac{(80-81)^2 + (85-81)^2 + \cdots + (94-81)^2}{30}$$
$$= \frac{-1^2 + 4^2 + \cdots + 13^2}{30}$$
$$= \frac{1 + 16 + \cdots + 169}{30}$$
$$= \frac{4050.00}{30}$$
$$= 135.20 \qquad (5.7)$$

Finally, a common measure of dispersion is the *standard deviation*. The standard deviation (6σ or SD) is simply the square root of the *variance*:

$$\sigma = \sqrt{\sigma^2} \qquad (5.8)$$

For the 30 numbers, the standard deviation is

$$\sigma = \sqrt{135.2}$$
$$= 11.6278 \qquad (5.9)$$

Two additional measures of dispersion are sometimes used and are included in MEAN. These are skewness and kurtosis. *Skewness* is a measure of the direction of dispersion from the arithmetic mean. *Kurtosis* is a measure of the flatness or peakedness of the distribution. The computation of skewness and kurtosis requires the intermediate computation of the second, third, and fourth moments, as follows.

$$M_2 = \frac{\Sigma (X-\mu)^2}{N} \qquad (5.10)$$

$$M_3 = \frac{\Sigma (X-\mu)^3}{N} \qquad (5.11)$$

$$M_4 = \frac{\Sigma (X-\mu)^4}{N} \qquad (5.12)$$

where M_2 = second moment (same as the variance)
M_3 = third moment
M_4 = fourth moment

Skewness and kurtosis are then computed using these intermediate computations as follows:

$$\text{skewness} = \frac{(M_3)^2}{(M_2)^3} \qquad (5.13)$$

$$\text{kurtosis} = \frac{M_4}{(M_2)^2} \qquad (5.14)$$

For the 30 numbers, we have

$M_2 = 135.20$
$M_3 = -701.80$
$M_4 = 44{,}451.60$

$$\text{skewness} = \frac{(-701.80)^2}{(135.20)^3} = 0.199 \qquad (5.15)$$

$$\text{kurtosis} = \frac{44{,}451.60}{(135.2)^2} = 2.43 \qquad (5.16)$$

A symmetrical distribution has a skewness value of zero. The small skewness value of 0.199 indicates that the distribution of numbers about the mean of 81 is fairly even. The sign of the third moment is negative, indicating that the small amount of skewness that does exist is toward the lower values.

The "normal" distribution has a kurtosis of 3. The value of 2.43 for kurtosis of the 30 numbers reveals what we already knew by casual inspection and the mode computation—namely, the distribution of 30 numbers is somewhat flatter than the normal distribution.

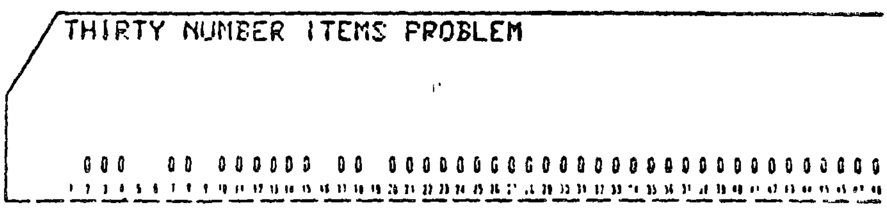
PROGRAM MEAN

We turn now to program MEAN, which will perform all these computations for measures of central tendency and of dispersion.

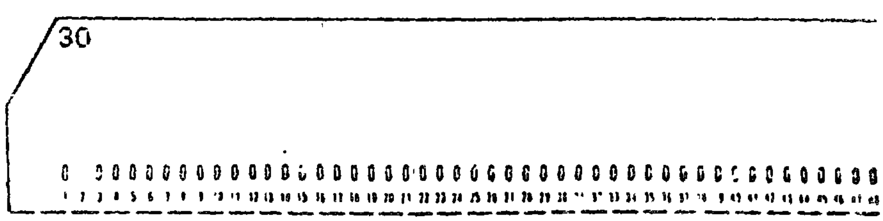
Computer input—program MEAN

We first describe the required input data for the use of program MEAN, taking as our example problem the 30 numbers discussed in the preceding section.

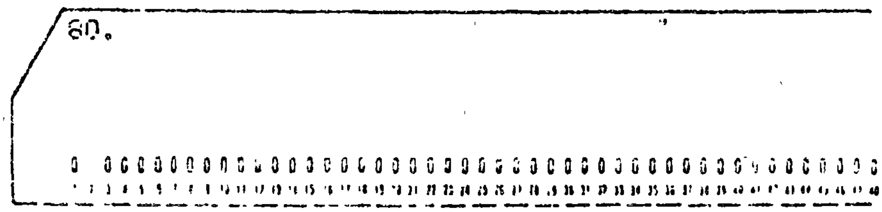
User name card is the first card in the data deck. It is used to identify the maker of the analysis or any other data heading information. It is free field in that any information may be keypunched in columns 1–40.



Data control card, the second card in the data deck, contains the number of data cards to be read in, keypunched in columns 1–2. For the 30-number problem, this card reads "30" in columns 1–2, since 30 numbers are to be read in, one data item per card. The last section contains instructions for specification of the reading of several data items on each keypunch card.

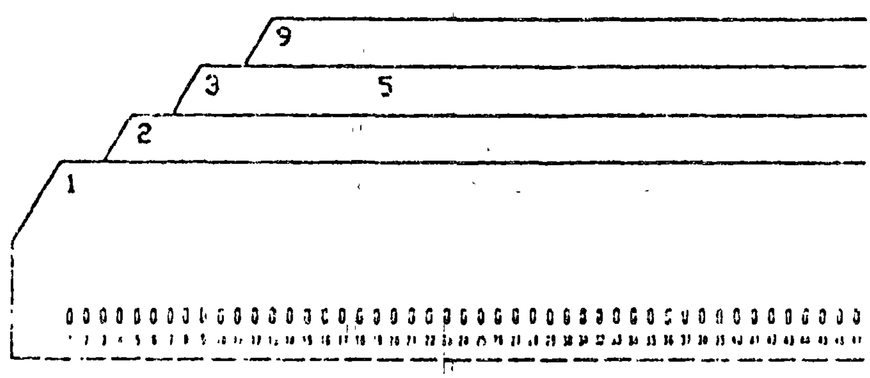


Data cards are the third type of card in the input data deck. There are to be 30 data cards to correspond to the number specified by the data control card. For the 30-number problem, every number is keypunched beginning in column 1, with the decimal point included. The last section contains instructions for reading several data items on each keypunch card. The first data card is shown below:



Analysis control card is the final type of card in the input deck for program MEAN. It specifies the type of statistical analysis to be performed on the input data by program MEAN. The type of analysis is controlled by a code keypunched into column 1.

For the 30-number problem, all three types of analysis will be requested. Code "1" keypunched into column 1 requests the computation of measures of central tendency. Code "2" keypunched into column 1 on the second card requests the computation of measures of dispersion. Code "3" keypunched into column 1 of the third card requests a histogram to be printed. The number of class intervals, in this case 5, must be specified in column 11 of the histogram analysis control card. Any number of analysis control cards may be placed one after the other in any order desired. The analysis control card with code "9" terminates these analysis requests.



Termination card with STOP in columns 1–4 is used to terminate all input data.

The complete computer input listing for the 30-number problem is shown in Figure 5-6.

Computer output—program MEAN

The three-part computer output returned by program MEAN is presented in Figure 5-7. The first part of the computer output (Figure 5-7*a*) is a printout of the user name card, the data control card, and the input data exactly as it was read in by program MEAN. This printout allows the user to check the data for keypunch errors and for data accuracy.

FIGURE 5-6

COMPUTER INPUT—30-NUMBER PROBLEM

```
THIRTY NUMBER ITEMS PROBLEM
30
60.
85.
56.
65.
81.
99.
73.
68.
69.
86.
67.
80.
86.
95.
95.
75.
86.
88.
81.
57.
63.
97.
98.
91.
83.
84.
84.
77.
67.
94.
1
2
3          5
9
STOP
```

The second part of the output returned by program MEAN (Figure 5-7*b*) is a printout of a sort from lowest value to highest value. This sort is performed as an aid to computation by program MEAN and as a convenience to the user. The sort indicates that the smallest number is 56 and the largest number is 99.

The third and final part of the output returned by program MEAN (Figure 5-7*c*) is the analysis results requested on the analysis control card.

The first part of the results, the computation of the measures of central tendency, is preceded by the user name card printout and by the heading MEASURES OF CENTRAL TENDENCY (1). The "(1)" is a reminder that this analysis was requested by an analysis control card with a "1" keypunched in the first column. The values shown for mean, median, and mode are the same as were previously computed by hand.

The second part of the results was requested by the second analysis control card with a "2" punched in the first column. The measures of dispersion appearing on this part of the printout are the same as those on the hand-computed values except for roundoff error or the computation of fewer significant digits. The amount of computation required for these statistics makes program MEAN a handy computational tool.

The heading FREQUENCY COUNT (3) WITH (5) INTERVALS is a reminder that the third analysis control card requested a frequency count and a histogram of the 30 numbers. Program MEAN sets up the frequency

FIGURE 5-7a

COMPUTER PRINTOUT—30-NUMBER PROBLEM, PART ONE

```
PROGRAM MEAN FOR THIRTY NUMBER ITEMS PROBLEM

DATA CONTROL CARD REQUESTS
30 DATA CARDS TO BE READ IN
 1 DATA ITEMS PER CARD

******INPUT DATA AS READ******
     89.0000
     85.0000
     56.0000
     85.0000
     63.0000
     99.0000
     73.0000
     68.0000
     69.0000
     86.0000
     67.0000
     80.0000
     55.0000
     95.0000
     95.0000
     75.0000
     86.0000
     80.0000
     81.0000
     57.0000
     63.0000
     97.0000
     98.0000
     91.0000
     83.0000
     54.0000
     84.0000
     77.0000
     67.0000
     94.0000
```

FIGURE 5-7b

COMPUTER PRINTOUT—30-NUMBER PROBLEM, PART TWO

```
*****SORTED INPUT DATA*****
     56.0000
     57.0000
     63.0000
     67.0000
     67.0000
     68.0000
     69.0000
     73.0000
     75.0000
     77.0000
     80.0000
     80.0000
     81.0000
     81.0000
     83.0000
     84.0000
     84.0000
     85.0000
     85.0000
     86.0000
     85.0000
     86.0000
     88.0000
     91.0000
     94.0000
     95.0000
     95.0000
     97.0000
     98.0000
     99.0000
```

FIGURE 5-7c COMPUTER PRINTOUT–30-NUMBER PROBLEM, PART THREE

```
MEAN RESULTS FOR THIRTY NUMBER ITEMS PROBLEM
MEASURES OF CENTRAL TENDENCY (1)

MEAN      =    81.0000
MEDIAN    =    83.5000
MODE(1)   =    86.0000

MEAN RESULTS FOR THIRTY NUMBER ITEMS PROBLEM
MEASURES OF DISPERSION (2)

AVE DEVIATION =       9.3333
STD DEVIATION =      11.6276
VARIANCE      =     135.2000
THIRD MOMENT  =    -701.6000
FOURTH MOMENT =   44451.6000
SKEWNESS      =       .1493
KURTOSIS      =      2.^318

MEAN RESULTS FOR THIRTY NUMBER ITEMS PROBLEM
FREQUENCY COUNT (3) WITH (5) INTERVALS

oooooFREQUENCY TABLEoooooo
      FROM       UP TO    COUNT
   56.0000     64.6000       3
   64.6000     73.2000       5
   73.2000     81.8000       6
   81.8000     90.4000       9
   90.4000     99.0000       7

MID POINToooooooooooooooooooooHISTOGRAMoooooooooooooooooooooo
     60.30 ooooo
     68.90 oooooooo-----
     77.50 oooooooooo-------------
     86.10 ooooooooooooooo-----------------------
     94.70 oooooooooooo--------------------------------------
           0   10   20   30   40   50   60   70   80   90  100
           (o) IS RELATIVE AND (-) IS CUMULATIVE PERCENTAGES
```

table by dividing the range of the data into equal intervals. Data items that fall precisely on the edge of a class interval go into the next higher class interval. The class intervals shown on the computer printout and the resultant frequency counts differ slightly from the previous hand computations. This discrepancy illustrates that the computer program must follow a set procedure, whereas hand computations may always choose computationally convenient intervals.

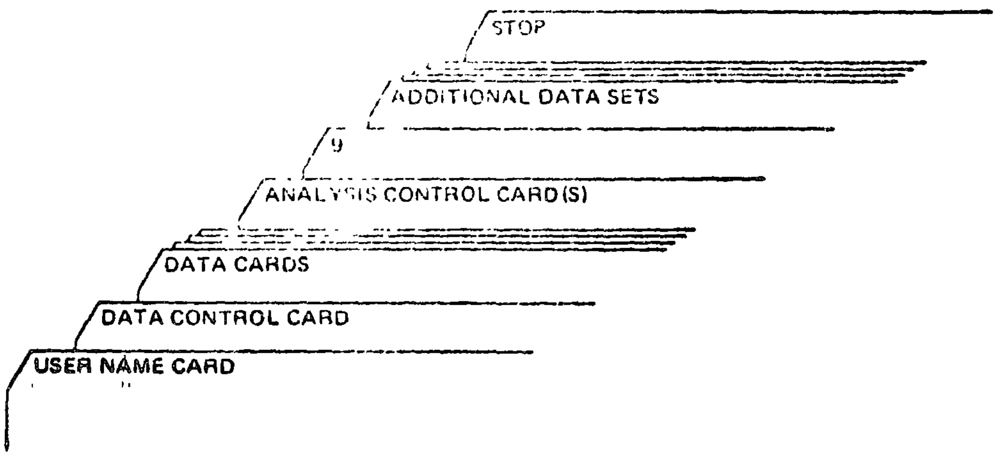
The last part of Figure 5-7*c* is a histogram of the same data in the frequency table. The midpoint of the class interval is given, along with the percentages of data items falling into the class interval. The cumulative histogram is "laid on its right side" for the convenience of the computer programmer and for faster computer printouts.

The histogram allows a visual estimate of the type of frequency distribution represented by the data. Like the other computation performed by program MEAN, the histogram is intended as a convenience to the data analyst who needs the first estimates about the characteristics of his basic data.

References

1. Chao, L. L., *Statistics: Methods and Analysis*. New York: McGraw-Hill, 1969.
2. Chow, Ya-lun, *Statistical Analysis*. New York: Holt, Rinehart & Winston, 1969.
3. Clelland, R. S., et al., *Basic Statistics with Business Applications*. New York: Wiley, 1966.
4. Duncan, A. J., *Quality Control and Industrial Statistics*, 3rd ed. Homewood, Ill.: Irwin, 1965.
5. Hamburg, Morris. *Statistical Analysis for Decision Making*. New York: Harcourt Brace Jovanovich, 1972.
6. Harnett, D. L., *Introduction to Statistical Methods*. Reading, Mass.: Addison-Wesley, 1970.
7. Hillier, Frederick S., and Gerald J. Lieberman, *Introduction to Operations Research*. San Francisco: Holden-Day, 1967.
8. Miller, I., and J. E. Freund, *Probability and Statistics for Engineers*. Englewood Cliffs, N.J.: Prentice-Hall, 1965.
9. Richmond, S. B., *Statistical Analysis*, New York: Ronald, 1964.
10. Stockton, J. R., *Business and Economics Statistics*, 3rd ed. Cincinnati, Ohio: Southwestern, 1966.

MEAN DATA STRUCTURE



Card	Format	Data
User name card	(10A4)	Any identification
Data control card	(F2.0, 8X, I1)	No. data cards, No. items (1 to 6) per card
Data cards	(6F10.0)	Data items (1 to 6 per card)
Analysis control card	I1, 9X, I1	Code, number of intervals 0 = Stop run, error 1 = Central tendency 2 = Dispersion 3 = Histogram 4-9 = Return for complete new data set

```
      PROGRAM MEAN
C     PROGRAM MEAN DOES GENERAL ... DISPERSION AND HISTOGRAM
C     ROY E HARRIS, JUNE 1972
C     THIS VERSION FOR CDC 3100
      DIMENSION ILPHA(10), ICOM(9), IC(9), ILINE(50), DATA(600), XHIGH(
     19), XMID(9), XMOD(9), XLOW(9)
      COMMON/DATA/IDATA(4)
      DATA (IDATA=1H*,1H-,1H ,4HSTOP)
      ISTOP=IDATA(4)
      NIN=60
      NOUT=61
C     READ AND WRITE USER NAME CARD
1     READ (NIN,39) ILPHA
      WRITE (NOUT,40) ILPHA
      IF (ILPHA(1).EQ.ISTOP) GO TO 38
C     READ CHECK AND WRITE OUT DATA CONTROL CARD
      READ (NIN,41) XNC,NF
      NC=XNC
      IF (NC.EQ.0) GO TO 37
      IF (NF.NE.0) GO TO 2
      NF=1
2     IF (NF.GT.8) GO TO 37
      WRITE (NOUT,42)
      WRITE (NOUT,43) NC
      WRITE (NOUT,44) NF
      WRITE (NOUT,45)
C     READ INPUT DATA CARDS
      IS=0
      IE=0
      DO 3 J=1,NC
      IS=IE+1
      IE=IS+NF-1
      READ (NIN,46) (DATA(I),I=IS,IE)
3     WRITE (NOUT,48) (DATA(I),I=IS,IE)
C     SORT INPUT DATA
      N=NC*NF
      K=N-1
      DO 4 I=1,K
      M=I+1
      DO 4 J=M,N
      IF (DATA(I).LE.DATA(J)) GO TO 4
      TEMP=DATA(I)
      DATA (I)=DATA(J)
      DATA (J)=TEMP
4     CONTINUE
C     WRITE OUT SORTED INPUT DATA
      WRITE (NOUT,47)
      IS=0
      IE=0
      DO 5 J=1,NC
      IS=IE+1
      IE=IS+NF-1
5     WRITE (NOUT,48) (DATA(I),I=IS,IE)
C     READ ANALYSIS CONTROL CARD
6     READ (NIN,49) ICODE,INT
      IF (ICODE.EQ.0) GO TO 36
      IF (ICODE.GT.3) GO TO 1
      GO TO (7,21,26), ICODE
C     COMPUTE THE MEAN
7     WRITE (NOUT,50) ILPHA
      SUM=0.
      DO 8 I=1,N
8     SUM=SUM+DATA(I)
      XN=N
      AVE=SUM/XN
```

```
C       COMPUTE THE MEDIAN
        IF (N.GE.2) GO TO 9
        XMED=0.
        GO TO 12
9       N2=N/2
        N3=N2+1
        IF (N-((N/2)*2)) 11,10,11
10      XMED=(DATA(N2)+DATA(N3))/2.
        GO TO 12
11      XMED=DATA(N2)
C       COMPUTE THE MODE(IF MORE THAN 3 MODES GIVE UP)
12      N1=N+1
        DATA (N1)=999999.
        MAX=1
        NS=0
        NT=1
        DET=DATA(1)
        DO 18 I=2,N1
        IF (DET.EQ.DATA(I)) GO TO 17
        IF (NT-MAX) 16,14,13
13      XMOD(1)=DET
        MAX=NT
        NS=1
        GO TO 16
14      NS=NS+1
        IF (NS.LT.4) GO TO 15
        NS=4
15      XMOD(NS)=DET
16      DET=DATA(I)
        NT=1
        GO TO 18
17      NT=NT+1
18      CONTINUE
C       WRITE OUT CENTRAL TENDENCY RESULTS
        WRITE (NOUT,51) ICODE
        WRITE (NOUT,52) AVE
        WRITE (NOUT,53) XMED
        IF (NS.LT.4) GO TO 19
        WRITE (NOUT,54)
        GO TO 6
19      DO 20 I=1,NS
20      WRITE (NOUT,55) I,XMOD(I)
C       RETURN FOR NEXT ANALYSIS CONTROL CARD
        GO TO 6
C       COMPUTE MEASURES OF DISPERSION
21      WRITE (NOUT,50) ILPHA
        XM2=0.
        XM3=0.
        XM4=0.
        XM1=0.
        SUM=0.
        DO 22 I=1,N
22      SUM=SUM+DATA(I)
        XN=N
        AVE=SUM/XN
        DO 23 I=1,N
        TA=ABS(DATA(I)-AVE)
        T1=DATA(I)-AVE
        T2=T1*T1
        T3=T2*T1
        T4=T3*T1
        XM1=XM1+TA
        XM2=XM2+T2
        XM3=XM3+T3
23      XM4=XM4+T4
        XN=N
```

```
      XM2=XM2/XN
      STDV=SQRT(XM2)
      XM1=XM1/XN
      XM3=XM3/XN
      XM4=XM4/XN
      IF (XM2.LT..000001) GO TO 24
      SKEW=(XM3*XM3)/(XM2*XM2*XM2)
      XKURT=XM4/(XM2*XM2)
      GO TO 25
24    SKEW=0.
      XKURT=0.
C     WRITE OUT DISPERSION RESULTS
25    WRITE (NOUT,56) ICODE
      WRITE (NOUT,57) XM1
      WRITE (NOUT,58) STDV
      WRITE (NOUT,59) XM2
      WRITE (NOUT,60) XM3
      WRITE (NOUT,61) XM4
      WRITE (NOUT,62) SKEW
      WRITE (NOUT,63) XKURT
C     RETURN FOR NEXT CONTROL CARD
      GO TO 6
C     COMPUTE FREQUENCY COUNTS AND HISTOGRAM
26    WRITE (NOUT,50) ILPHA
      WRITE (NOUT,54) ICODE,INT
      IF (INT.EQ.0) GO TO 36
      XINT=INT
      WIDE=(DATA(N)-DATA(1))/XINT
      ISTR=IDATA(1)
      IBAR=IDATA(2)
      IBLK=IDATA(3)
      DO 27 I=1,INT
27    ICT(I)=0
      DO 28 I=1,N
      J=((DATA(I)-DATA(1))/WIDE)+1.
      IF (J.LT.INT) GO TO 28
      J=INT
28    ICT(J)=ICT(J)+1
C     COMPUTE HISTOGRAM INTERVALS
      ISUM=0
      DO 29 I=1,INT
      ISUM=ISUM+ICT(I)
      ICUM(I)=ISUM
      XI=I
      XLOW(I)=DATA(1)+((XI-1.)*WIDE)
      XHIGH(I)=DATA(1)+(XI*WIDE)
29    XMID(I)=XLOW(I)+WIDE/2.
C     WRITE OUT FREQUENCY TABLE
      WRITE (NOUT,65)
      WRITE (NOUT,66)
      DO 30 I=1,INT
30    WRITE (NOUT,67) XLOW(I),XHIGH(I),ICT(I)
C     COMPUTE AND WRITE HISTOGRAM
      WRITE (NOUT,68)
      DO 35 I=1,INT
      KBAR=(ICT(I)*50)/N
      KBAR1=KBAR+1
      JBAR=(ICUM(I)*50)/N
      DO 31 J=1,50
31    ILINE(J)=IBLK
      IF (KBAR.EQ.0) GO TO 33
      DO 32 J=1,KBAR
32    ILINE(J)=ISTR
33    IF (JBAR.EQ.KBAR) GO TO 35
      DO 34 J=KBAR1,JBAR
34    ILINE(J)=IBAR
```

```
35    WRITE (NOUT,69) XMID(I),(ILIFF(J),J=1,50)
      WRITE (NOUT,70)
      WRITE (NOUT,71)
C     RETURN FOR NEXT CONTROL CARD
      GO TO 6
C     THIS SECITON PRINTS OUT ERROR MESSAGES
36    WRITE (NOUT,72)
      GO TO 38
37    WRITE (NOUT,73)
      WRITE (NOUT,74) NC,NF
38    WRITE (NOUT,75)
C
39    FORMAT (20A4)
40    FORMAT (19H1PROGRAM MEAN FOR ,1,A4)
41    FORMAT (F2.0,8X,I1)
42    FORMAT (27H0DATA CONTROL CARD REQUESTS)
43    FORMAT (1X,I2,25H DATA CARDS TO BE READ IN)
44    FORMAT (1X,I2,20H DATA ITEMS PER CARD)
45    FORMAT (29H0*****INPUT DATA AS READ*****)
46    FORMAT (6(F5.0,5X))
47    FORMAT (28H0*****SORTED INPUT DATA*****)
48    FORMAT (1X,6F10.4)
49    FORMAT (I1,9X,I1)
50    FORMAT (14H0MEAN RESULTS FOR ,10A4)
51    FORMAT (31H MEASURES OF CENTRAL TENDENCY (,I1,1H))
52    FORMAT (10H0MEAN     =,F11.4)
53    FORMAT (10H MEDIAN   =,F11.4)
54    FORMAT (20H MODE DOES NOT EXIST)
55    FORMAT (6H MODE(,I1,3H) =,F11.4)
56    FORMAT (25H MEASURES OF DISPERSION (,I1,1H))
57    FORMAT (16H0AVE DEVIATION =,F12.4)
58    FORMAT (16H STD DEVIATION =,F12.4)
59    FORMAT (16H VARIANCE      =,F12.4)
60    FORMAT (16H THIRD MOMENT  =,F12.4)
61    FORMAT (16H FOURTH MOMENT =,F12.4)
62    FORMAT (16H SKEWNESS      =,F12.4)
63    FORMAT (16H KURTOSIS      =,F12.4)
64    FORMAT (18H FREQUENCY COUNT (,I1,8H) WITH (,I1,11H) INTERVALS)
65    FORMAT (20H0*******FREQUENCY TABLE*******)
66    FORMAT (29H       FROM      UP TO    COUNT)
67    FORMAT (1X,2F10.4,I8)
68    FORMAT (61H0MID POINT*********************HISTOGRAM***************
     1******)
69    FORMAT (1X,F9.2,1X,50A1)
70    FORMAT (11X,50H0  10   20   30   40   50   60   70   80   90  100)
71    FORMAT (11X,50H(*) IS RELATIVE AND (-) IS CUMMULATIVE PERCENTAGES)
72    FORMAT (31H0 ZERO IN ANALYSIS CONTROL CARD)
73    FORMAT (27H0ERROR IN DATA CONTROL CARD)
74    FORMAT (14H CARD READS ((,I2,8X,I1,2H)))
75    FORMAT (27H0MEAN PROCESSING TERMINATED)
      END
```

Exercise 6 FIT

Is a statistical analysis program that will fit empirical data to the uniform, normal, exponential, and Poisson distributions.

Curve fitting helps us to determine whether an empirically derived set of numbers can be described by some theoretical distribution. If it can, there are immediate large gains in the ease of mathematical manipulations, in the number of models that might be used, and in the number of generalizations that can be made.

Most operations research models depend on the empirical data fitting the available theoretical distribution. For example, the simple Erlang single-channel, single-phase queueing model is based on an assumption of a Poisson arrival process and a negative exponential service process. If the real-world structure fits the model, the number in the queue and the waiting time may be found with a few simple computations.

Program FIT is designed to help the analyst determine whether the empirical data fits the assumption of the model. If there is a "good" fit, the analyst may proceed with confidence.

The data used to illustrate the curve fitting procedure is the count of the output level of 30 factory workers on a particular day.

For curve fitting to a theoretical distribution, the raw data of Figure 6-1 must be grouped into class intervals and the count of occurrences in each class interval must be tabulated. This task has already been accomplished on the 30 numbers by the use of program MEAN. (See Exercise 5.)

The selection of the "proper" number of class intervals and the "proper" width of each class interval is part of the art of statistical analysis. For

FIGURE 6-1

OUTPUT LEVEL OF 30 WORKERS ON A PARTICULAR DAY

56, 57, 63, 67, 67, 66, 69, 73, 75, 77, 80, 80, 81,
81 83, 84, 85, 86, 86, 86, 88, 91, 94, 95, 95, 97,
98, 99,

example, the range of the 30 num[illegible] 43. If this range were extended to 45, with a choice of five class intervals, each would be a "neat" nine units wide. Also, if the lower range boundary were set at 54.5 instead of the lowest actual value of 56, each class interval would fall on a "neat" whole integer. Such adjustments are made at the discretion of the analyst.

In the "neat" frequency table of Table 6–1 the range has been extended to 45 units and the lowest class interval boundary has been set at 54.50. A distribution in this form can be "fitted" to a theoretical distribution, and program FIT can perform the fitting task after the data has been grouped in a frequency table such as Table 6–1. The first type of theoretical distribution found in program FIT is the uniform distribution.

TABLE 6-1 30 NUMBERS GROUPED IN FREQUENCY TABLE

Class interval From	Up to	Count of numbers within this group
54.50	63.50	3
63 50	72.50	4
72.50	81 50	7
81.50	90.50	9
90.50	99.50	7

UNIFORM DISTRIBUTION

The uniform (or rectangular) distribution has equal frequency counts for each class interval. The density function can be written

$$f(x) = \frac{1}{r} \text{ for } x_{min} \leq x \leq x_{max} \tag{6.1}$$

where r is $x_{max} - x_{min}$.

Graphically, this can be seen as

The mean and standard deviations of the distribution are

$$\mu = x_{min} + \frac{r}{2} \tag{6.2a}$$

or

$$\mu = \frac{x_{min} + x_{max}}{2}$$

and $$\sigma = \sqrt{\frac{r^2}{12}} \tag{6.2b}$$

For the production output example, we have

$$x_{max} = 99$$
$$x_{min} = 56$$

where $r = 43$

$$\mu = 56 + \frac{43}{2} \quad \text{and} \quad \sigma = \sqrt{\frac{43^2}{12}}$$
$$= 77.5 \qquad\qquad = 12.4 \tag{6.3}$$

The 30 numbers have already been tabulated into five class intervals from 54.50 to 99.50 (Table 6-1). This arbitrary widening of the range from 43 to 45 and assignment to five class intervals is for the convenience of the analyst. If the data were drawn from this wider population, the mean would be

$$\mu = 54.50 + \frac{45}{2}$$
$$= 78.0 \tag{6.4}$$

and the standard deviation would be

$$\sigma = \sqrt{\frac{45^2}{12}}$$
$$= 12.9 \tag{6.5}$$

These are slight adjustments in the data

To find the expected probability in each class interval, we must integrate the density function over the class interval:

$$F(X) = \int_{x_1}^{x_2} \frac{1}{r} dx \tag{6.6}$$

After integrating this simple expression, we have

$$F(X) = x_2 \left(\frac{1}{r}\right) - x_1 \left(\frac{1}{r}\right) \tag{6.7}$$

For the first class interval, from 54.5 to 63.50, the frequency count becomes

$$F(X) = 63.50 \left(\frac{1}{45}\right) - 54.5 \left(\frac{1}{45}\right)$$
$$= (9)\left(\frac{1}{45}\right)$$
$$= 0.20 \tag{6.8}$$

We could have arrived at this conclusion more simply by observing that in five class intervals of equal size, one-fifth of the population (0.2) should fall into each frequency class.

The frequency count in each class interval can be found by multiplying the probability times the total population size:

$$\text{frequency count in each class} = \text{expected probability from density function} \times \text{population size} \tag{6.9}$$

For the uniform distribution probability just computed, the expected count is

$$\begin{aligned} \text{frequency count in each class} &= 0.2 \times 30 \\ &= 6 \end{aligned} \tag{6.10}$$

CHI-SQUARE TEST OF SIGNIFICANCE

The basic issue in curve fitting is to determine whether the actual frequency count in each class interval is the "same" as the theoretical frequency count. The problem is determining an acceptable measure of "same," since we do not mean exactly the "same" and would be very suspicious of any empirical data that followed precisely a theoretical distribution.

The most commonly used measure of "sameness" i.e., goodness of fit, is the *chi-square test*. *Chi-square* is a frequency distribution of the expected variation between observed and theoretical frequency counts in an honest experiment. To use the chi-square test, we perform the following computation:

$$\chi^2 = \sum \frac{(f_o - f_e)^2}{f_e} \tag{6.11}$$

where χ^2 = chi-square statistic
f_o = observed frequency count
f_e = theoretically expected frequency count

The summation is over all the class intervals.

The test for the 30 numbers fitted to the uniform distribution is performed as follows

f_o	f_e	$(f_o - f_e)^2$	$(f_o - f_e)^2/(f_e)$
3	6	9	1.5000
4	6	4	0.6667
7	6	1	0.1667
9	6	9	1.5000
7	6	1	0.1667
			χ^2 = 4.0001

The final step in this test is to determine whether to accept the hypothesis that the 30 numbers were drawn from (can be approximated by) a uniform distribution. This step takes the analyst out of the realm of straight computation into the realm of judgment and inference about the items he is counting. Program FIT does not make any judgments. It simply computes the summed value for chi-square as shown above. The analyst may consult any of the standard texts given in the references for the procedure and caveats of making this judgment.

NORMAL DISTRIBUTION

The most widely discussed distribution is the *normal distribution*. The density function is given by:

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-\mu)^2/2\sigma^2}, \quad -\infty < x < \infty \tag{6.12}$$

where π = a constant (3.1416)
e = a constant (2.7183)
σ = standard deviation
μ = arithmetic mean

This form of the normal distribution is not used in ordinary computations. The *standard* form is obtained and tabled by the following substitutions:

$$\mu = 0$$
$$\sigma = 1$$
$$Z = \frac{x - \mu}{\sigma} \tag{6.13}$$

The resultant *standard* form of the normal distribution is:

$$\phi(Z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}, \quad -\infty < Z < \infty \tag{6.14}$$

No equation exists for the cumulative form for the normal distribution, since the density function cannot be integrated. However, an approximation to the cumulative standard form of the normal distribution may be obtained by use of a power series expansion (1):

$$\text{Prob}(Z) = \frac{1}{2} + \frac{1}{\sqrt{2\pi}} \sum_{n=0}^{n=\infty} \frac{(-1)^n z^{2n+1}}{n!\, 2^n (2n+1)} \tag{6.15}$$

where Prob(Z) is the cumulative probability for positive values of Z.

The normal distribution is symmetrical about the mean. That is,

$$\phi\left(\frac{\mu + x}{\sigma}\right) = \phi\left(\frac{\mu - x}{\sigma}\right) \tag{6.16}$$

Thus negative values of Z are found by 1 − Prob(|Z|). The series expansion is used by program FIT to compute an approximation to the cumulative area under the standard normal curve by summing until the next term is arbitrarily small.

For the example data of Table 6-1, we know that the first class interval is from 54.50 to 63.50, and three numbers out of the total of 30 fall in this class interval. We want the theoretical count in this interval if the distribution is normal. This can be demonstrated by use of the actual mean and the actual standard deviation of the tabled items as the "most likely" estimate of the theoretical mean and the theoretical standard deviation of a normal distribution. (Program FIT also allows the user to specify any values for the mean and the standard deviation, and to obtain a fit to the resulting theoretical distribution.)

The mean of the 30 numbers of Table 6-1 is obtained by the summing of the count in each class interval multiplied by the midpoint of the interval. That is:

$$\mu = \frac{\Sigma \text{ midpoint} \cdot \text{count}}{\Sigma \text{ count}} \tag{6.17}$$

From	Up to	Midpoint	Count	Midcount
54.5	63.5	59	3	177
63.5	72.5	68	4	272
72.5	81.5	77	7	539
81.5	90.5	86	9	774
90.5	99.5	95	7	665
			Σ30	Σ2,427

$$\mu = \frac{2427}{30}$$
$$= 80.90$$

The standard deviation for grouped data is obtained from

$$\sigma = \sqrt{\frac{\Sigma \text{ count} \cdot (\text{midpoint} - \text{mean})^2}{\Sigma \text{ count}}}$$

$$= \sqrt{\frac{3 \cdot (59 - 80.9)^2 + 4 \cdot (68 - 80.9)^2 + \cdots + 7 \cdot (95 - 80.9)^2}{30}}$$

$$= \sqrt{\frac{3836.70}{30}}$$

$$= 11.31 \tag{6.18}$$

These values for the mean and the standard deviation can serve as an estimate of the mean and the standard deviation of a theoretical normal distribution. The estimate of the proportion of the total population falling into each class interval requires the computation of the cumulative probability at each end of the class interval.

The standard form of the normal distribution is used, thus Z must be computed for each class end point. For the first class interval, we have

$$Z = \frac{x - \mu}{\sigma} \tag{6.19a}$$

$$Z \text{ (lower limit)} = \frac{54.5 - 80.9}{11.31}$$
$$= -2.3342 \tag{6.19b}$$

$$Z \text{ (upper limit)} = \frac{63.5 - 80.9}{11.31}$$
$$= -1.5380 \tag{6.19c}$$

Since both values of Z are negative, it will be necessary to compute the Prob(Z) for the absolute value $|Z|$ and to subtract the result from 1, as previously outlined. The power series expansion for the probability of the standardized Z at the lower limit is

$$\text{Prob}(Z) = \frac{1}{2} + \frac{1}{\sqrt{2\pi}} \sum_{n=0}^{n=\infty} \frac{(-1)^n}{n!\, 2^n} \frac{Z^{2n+1}}{(2n+1)} \tag{6.20}$$

The expansion of the first four of the summed terms is

$$\sum_{n=0}^{4} (\cdot) = Z - \frac{1}{6} Z^3 + \frac{1}{40} Z^5 - \frac{1}{336} Z^7 + \cdots \tag{6.21}$$

and for Z = 2.3342 at the lower limit, the expansion yields

$$\sum_{n=0}^{4} (\cdot) = 2.3342175 - 2.1196916 + 1.7323936 - 1.0487855 \tag{6.22}$$

At this point, discretion is the better part of valor, and the reader will be asked to trust the authors' computer to properly continue the expansion until the expansion terms are smaller than the roundoff error. The series expansion result is approximately 1.2286. The complete expression for the probability of |Z| = 2.3342 is

$$\begin{aligned} \text{Prob}(|2.3342|) &= .5 + \frac{1.2286}{\sqrt{2\pi}} \\ &= .99009 \end{aligned} \tag{6.23}$$

Since Z is negative, Prob(Z) = 1 − Prob(|Z|), or

$$\text{Prob}(-2.334) = .0099 \tag{6.24}$$

which is the cumulative probability for $-\infty < Z < -2.334$ or, in absolute terms, for $-\infty \le X < 54.5$ for a population mean of 80.9 and a standard deviation of 11.31. The cumulative probability at the upper bound, Z = −1.53846, is obtained in the same manner. Sparing the reader a repeat of the tedious computation of the expansion terms, we simply write

$$\text{Prob}(-1.53846) = .0618 \tag{6.25}$$

An estimate of the probability within the class interval is the difference between these two cumulative probability values:

$$\begin{aligned} \text{Prob}(-2.3342 \le Z \le -1.53846) &= (.0618 - .0099) \\ &= .0527 \end{aligned} \tag{6.26}$$

which is the same as

$$\text{Prob}(54.5 \le X \le 63.5)$$

The final computation is for the theoretical count within the class interval. This count is the total population count times the probability for the class. Since there are 30 numbers in the population, we have

$$\text{class interval count} = 30 \times 0.0527 = 1.58 \tag{6.27}$$

All these computations are required to determine that the theoretical frequency count for the class interval of 54.5 to 63.50 is 1.58 items.

The observed count in this class interval is 3 data items. After the theoretical counts have been determined for the other class intervals, the chi-square goodness of fit may be performed, or the analyst may choose simply to graph and "eyeball" the goodness of fit. Program FIT computes the chi-square values and prints out a histogram of the observed versus theoretical frequency counts for the user.

NEGATIVE EXPONENTIAL DISTRIBUTION

The negative exponential distribution is often used to describe the duration of service times. The probability density function of this distribution is given by the following formula:

$$f(X) = \frac{1}{\mu} e^{-x/\mu}, \quad \text{for } x \geq 0 \tag{6.28}$$

where μ is the mean.

In this distribution, the mean and the standard deviation are the same; that is, $\sigma = \mu$. The cumulative form of the distribution is given by

$$F(X) = \int_0^X \frac{1}{\mu} e^{-x/\mu} dx \tag{6.29}$$

which reduces, after integration, to

$$F(X) = 1 - e^{-x/\mu} \tag{6.30}$$

This form of the negative exponential distribution is used by program FIT.

The interpretation of $F(X)$ as applied to a service center in queueing theory is as follows:

μ is the mean service duration in time units per event
$1/\mu$ is the mean service rate in events per time interval
$F(X) = 1 - e^{-x/\mu}$ is the probability that a given customer will take X time units or fewer to complete service when the mean service duration μ is known.

The computation of the theoretical frequency counts resembles the computation for the normal distribution, except that it is an easier process. The negative exponential distribution is limited in its applications because the value of X must be zero or greater.

The computation of the theoretical frequency counts proceeds as follows from the cumulative form of the equation. For the first class interval we write

$$F(X \leq 54.50) = 1 - e^{-54.5/80.9} = .490 \tag{6.31a}$$

$$F(X \leq 63.50) = 1 - e^{-63.5/80.9} = .545 \tag{6.31b}$$

$$F(54.5 \leq X \leq 63.5) = .545 - .490 = .055 \tag{6.31c}$$

The expected count is

= 30 X .055
= 1.65

The theoretical frequency count for the other class intervals is computed in the same manner.

POISSON DISTRIBUTION

The Poisson distribution is a discrete distribution that is useful in queueing theory and in statistical sampling theory. The Poisson function assumes that X is a discrete, positive, random variable. Its probability function is given by

$$\text{Prob}(X = k) = f(x) = \frac{e^{-\mu}\mu^k}{k!} \quad \text{for } k = 0, 1, 2, 3, \ldots \tag{6.32}$$

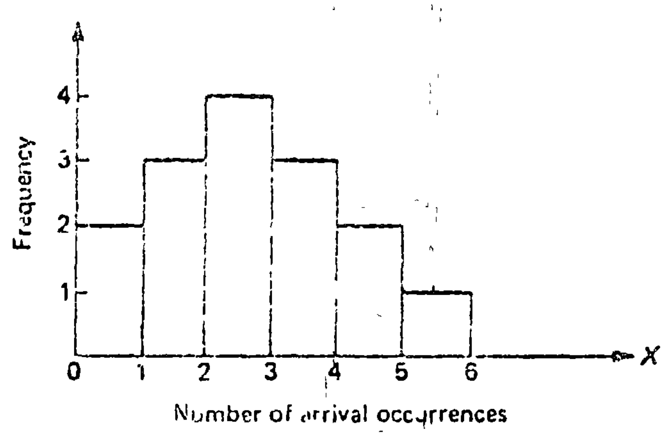
where Prob($X = k$) is the probability that k discrete events will occur and μ is the mean of the distribution.

In this distribution, the mean and the variance are the same. That is, $\mu = \sigma^2$

Since the Poisson distribution is discrete, it is not necessary to integrate to determine the area under the curve. The probability function can be used to compute directly the probability that k events will occur. Thus the probability function in Equation 6.32 is used directly in program FIT.

The interpretation of Prob($X = k$) as applied to the arrivals of a queueing system can be seen in Figure 6-2, where the X axis contains the number of arrivals observed during a preset constant time interval, such as one hour. The frequency is the count of the number of observed hours during which that number of arrivals occurred. When the mean is known, the Poisson equation can be used to compute the probability that 0, 1, 2, 3, ... arrivals will occur during the preset time interval.

FIGURE 6-2 TYPICAL GRAPH OF QUEUE ARRIVALS



In program FIT, the computation of the theoretical Poisson frequency count takes place directly from the Poisson function. The midpoint of the class interval, rounded to the nearest whole integer, is used as the approximate value of k. For the first class interval, we have midpoint for (54.5 - 63.5) = 59, thus

$$\begin{aligned}\text{Prob}(Y = 59) &= \frac{(e^{-80.9})(80.9^{59})}{59!}\\ &= \frac{3.682 \times 10^{111}}{(1.3628 \times 10^{34})(1.3868 \times 10^{80})}\\ &= .0019741271\end{aligned}$$

For a total population of 30 units, the expected count is

$$\begin{aligned}&= .001974 \times 30\\ &= .059\end{aligned}$$

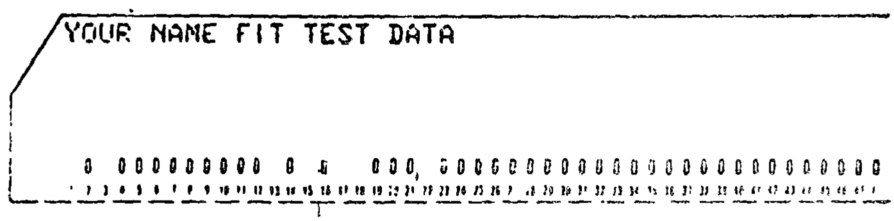
The counts for the other class intervals can be computed in a similar manner. The next section presents the input data cards needed to use Program FIT for performing these computations.

PROGRAM FIT

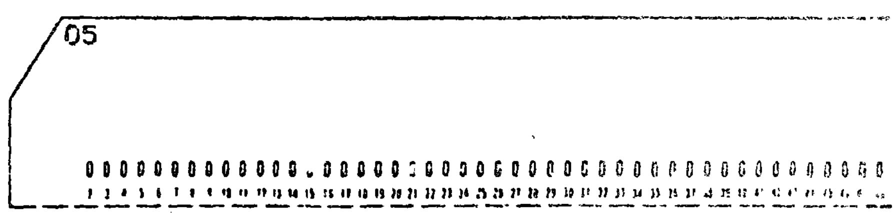
Computer input—FIT test data

In our presentation of the input data cards for program FIT, the 30-number problem of the previous section is used to demonstrate how FIT can be employed in the performance of curve-fitting computations.

User name card is the first card in the input data deck. Any identification information desired can be keypunched into the first 40 columns.



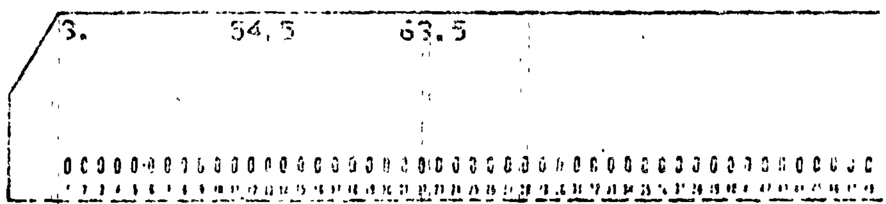
Data control card, the second input card in the input data deck, specifies the number of class intervals in the frequency distribution to be read in, one class interval on each of the following cards. Columns 1, 2 are keypunched with the appropriate number of class intervals. For the 30-number data, five class intervals are used as in Table 6-1.



Interval data cards are the next set of cards in the input data deck. They contain the information about each class interval. Since program FIT does not accept raw input data, the data must be grouped into class intervals and frequency counts completed before curve fitting is begun.

The frequency count is keypunched in columns 1–5. The beginning point of the class interval (left-hand side) is keypunched in columns 11-15, and the end point in columns 21-25.

The first interval data card for the data of Table 6–1 is shown below.



(As an option on the interval data card, the user may specify only the midpoint of the interval in columns 11–15. Program FIT will then compute the end points halfway between the midpoints.)

Analysis control card, the last type of card in the input data deck, is used to specify the type of theoretical distribution to be fit to the actual distribution of the data. A code keypunched into columns 1–4 of this card controls the type of analysis. The appropriate codes are:

Code	Theoretical distribution
0 or blank	Error, stop run
1	Uniform
2	Normal
3	Negative exponential
4	Poisson
5-9	Return for next data set

The analysis control cards can be stacked one after another as many times as desired. The analysis control code "9" is used to stop the analysis and return for a new complete set of input data.

Termination card, with STOP keypunched in columns 1–4, is used to terminate all data input.

In the ordinary procedure, program FIT computes the population count, the mean, and the standard deviation from the frequency distribution that is input on the interval data cards. However, the user can elect to override these computations and fit the distribution to any population size, mean, and variance by specifying these items on the analysis control card. If the user specifies any one of these items, however, he must specify them all. (See the last section for the appropriate input and format for each of the theoretical distributions.

The complete input data deck (Figure 6–3) contains the data from Table 6–1. Code "1" requests a fit to the uniform distribution. Code "9" terminates the analysis control cards, and the STOP card terminates all data processing.

FIGURE 6-3

COMPUTER INPUT—FIT TEST DATA

```
YOUR NAME FIT TEST DATA
05
3.          54.5        63.5
4.          63.5        72.5
7.          72.5        81.5
9.          81.5        90.5
7.          90.5        99.5
1
9
STOP
```

Computer output—FIT test data

We now present and discuss the output returned from the input data for the 30-number problem shown in Figure 6-3.

The input data printout from program FIT is the first part of the computer printout (see Figure 6-4). The first line is a printout of the user name card. The second line states that 5 data cards for five class intervals are to be read in as specified on the input data control card. These two lines are followed by a printout of the interval data cards exactly as they were read by the computer. This printout allows the user to verify the input data.

FIGURE 6-4

COMPUTER OUTPUT—FIT TEST DATA

```
PROGRAM FIT FOR YOUR NAME FIT TEST DATA

     5 DATA CARDS (INTERVALS) TO BE READ IN
COUNT        ******INTERVAL******
    3          54.5000     63.5000
    4          63.5000     72.5000
    7          72.5000     81.5000
    9          81.5000     90.5000
    7          90.5000     99.5000

**********EVALUATED INPUT DATA**********
 COUNT           BEGIN     MIDPOINT        END
    3          54.5000     59.0000     63.5000
    4          63.5000     68.0000     72.5000
    7          72.5000     77.0000     81.5000
    9          81.5000     86.0000     90.5000
    7          90.5000     95.0000     99.5000
```

The remainder of Figure 6-4 is a second printout of the input data after it has been evaluated by program FIT. The input data is sorted from the lowest to the highest class interval. The midpoint of the class interval is determined. (If the input data is midpoints, the end points are computed.) The class interval end points are evaluated to make sure the lower end of the interval is smaller than the upper end. If there are any problems with the input data, an error message will be printed and the computer processing will be stopped.

As an example of the curve-fitting results, the first line in Figure 6-5 is a printout of the user name card; this serves as a reminder of the input data associated with the curve-fitting result. The next three lines consist of the header for the printout of the analysis control card, the analysis control

FIGURE 6-5

COMPUTER OUTPUT—FIT TEST DATA

```
FIT RESULTS FOR YOUR NAME. FIT TEST DATA

******ANALYSIS CONTROL CARD READS*********
(1                 0           0          0)
UNIFORM DISTRIBUTION (CODE1)

              RANGE       45.0000
               MEAN       77.0000
           VARIANCE      108.7500
  TOTAL POPULATION            30.

******INTERVAL*****  OBSERVED  EXPECTED CHISQUARE
   54.5000   63.5000        3    6.0000    1.5000
   63.5000   72.5000        4    6.0000     .6667
   72.5000   81.5000        7    6.0000     .1667
   81.5000   90.5000        9    6.0000    1.5000
   90.5000   99.5000        7    6.0000     .1667

   TOTAL CHISQUARE        6.0000
DEGREES OF FREEDOM             4
     WARNING,CHISQUARE CELL COUNT FIVE OR LESS***

  MIDPOINT.....*...20.....*...40.....*...60.....*...80.....*...100
   59.0000-----     X
   68.0000------    X
   77.0000----------X-
   86.0000----------X-----
   95.0000----------X-
```

card, and a reminder of the type of analysis requested by the code keypunched in column 1. The zeros appearing on the line with the analysis control card indicate these control fields are blank and that FIT is to set the population count and the distribution parameters from the input data for the class intervals and count. The user should consult the last section for the correct format for specifying the theoretical distribution parameters on the analysis control card.

The next line on the analysis result printout gives the parameters to be used by program FIT in determining the theoretical distribution. For the uniform distribution shown in Figure 6-5, the appropriate parameters are range, mean, variance, and population count. If these parameters are not specifically keypunched on the analysis control card, they will be computed by program FIT.

The midsection of the analysis result printout is the table of class intervals, their observed frequency counts, the theoretically expected frequency counts, the chi-square value for that class interval, and finally, the total sum of the chi-square values. In Figure 6-5, the theoretical count for the uniform distribution is "6.0000," which is the same value previously computed by hand.

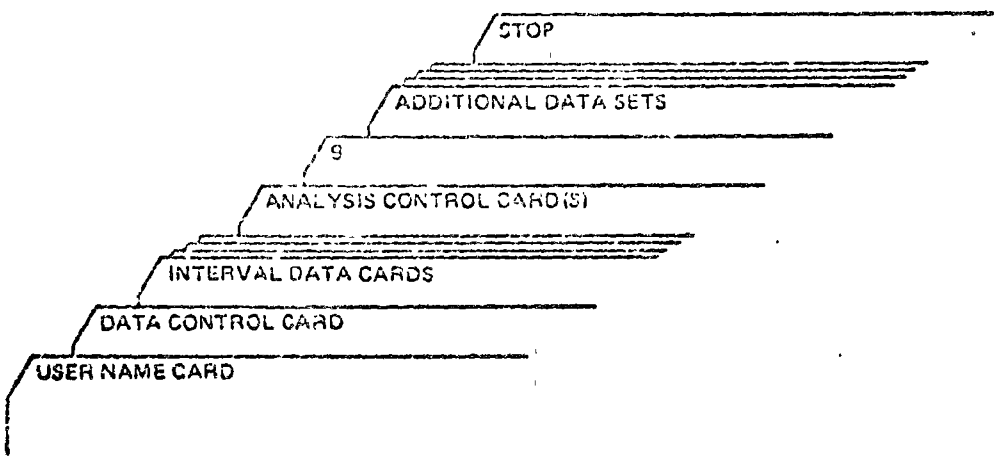
The appropriate degrees of freedom for the chi-square test are printed with the total chi square. The user must complete the chi-square test for himself based on his own confidence level and a table of chi-square values.

The final part of the analysis result printout is a histogram of the observed and the theoretical count in each class interval. The class interval is designated by its midpoint. The horizontal dashed line (——) is the percentage of the total population observed within the class interval. The single (X) marks the percentage of the total population theoretically expected in the class interval. This histogram is intended as a handy visual check of the goodness of fit to the theoretical distribution.

References

1. Abramowitz, M., and I. A. Stegun, *Handbook of Mathematical Functions*, National Bureau of Standards, Applied Mathematics Series 55. Washington, D.C.: Government Printing Office, 1964.
2. Cramer, H., *The Elements of Probability Theory and Some of its Applications.* New York: Wiley, 1955.
3. Derman, C., and M. Klein, *Probability and Statistical Inference for Engineers.* New York: Oxford University Press, 1959.
4. Freund, J. E., *Mathematical Statistics.* Englewood Cliffs, N.J.: Prentice-Hall, 1962.
5. Harnett, D. L., *Introduction to Statistical Methods.* Reading, Mass.: Addison-Wesley, 1970.
6. Hillier, Frederick S., and Gerald J. Lieberman, *Introduction to Operations Research.* San Francisco: Holden-Day, 1967.
7. Hogg, R., and A. Craig, *Introduction to Mathematical Statistics*, 2nd ed., New York: Macmillan, 1965.
8. Lindgren, B. W., *Statistical Theory.* New York: Macmillan, 1960.
9. Mood, A. M., and F. A Graybill, *Introduction to the Theory of Statistics*, 2nd ed. New York: McGraw-Hill, 1963.
10. Parzen, E., *Modern Probability Theory and its Applications.* New York: Wiley, 1960.
11. Schlaifer, R., *Probability and Statistics for Business Decisions.* New York: McGraw-Hill, 1959.
12. Taha, Hamdy, A., *Operations Research, An Introduction.* New York: Macmillan, 1971.
13. Wagner, Harvey M., *Principles of Operations Research.* Englewood Cliffs, N.J.: Prentice-Hall, 1969.

FIT DATA STRUCTURE



Card	Format	Items
User name	(10A4)	Identification information
Data control	(F2.0)	Number of interval data cards
Interval data card	(3(F.0, 5X))	Count, begin interval, end interval or count and midpoint
Analysis control	(11,9X, 3(F5.0, 5X)	Code, population parameters (shown below)

	Code	Optional population parameters		
	cc1	cc11-15	cc21-25	cc31-35
Stop run, error	0			
Uniform	1	Count	Low value	High value
Normal	2	Count	Mean	s.d.
Exponential	3	Count	Mean	
Poisson	4	Count	Mean	
More data sets	5-9			

```
PROGRAM FIT
PROGRAM FIT DOES CURVE FITTI.G TO GROUPED DATA
AVAILABLE DISTRIBUTIONS ARE UNIFORM
                            NORMAL
                            EXPONENTIAL
                            POISSON
ROY D HARRIS  JUNE 1972
THIS VERSION FOR CDC 3100
COMMON ILPHA(10),FREQ(100),BEGIN(100),END(100),XMID(100),XXP(100),
1ILINE(50),XN,NOUT,NG
COMMON/DATA/IDATA(4)
DATA (IDATA=1H-,1HX,1H ,4HSTOP)
DATA STATEMENT ALSO FOUND IN SUBROUTINE HISTO
ISTOP=IDATA(4)
NIN=60
NOUT=61
READ AND WRITE USER NAME CARD
READ (NIN,52) ILPHA
WRITE (NOUT,53) ILPHA
IF (ILPHA(1).EQ.ISTOP) GO TO 51
READ,CHECK,AND WRITE DATA CONTROL CARD
READ (NIN,54) XNG
NG=XNG
NG1=NG-1
WRITE (NOUT,55) NG
IF (NG.LT.3) GO TO 45
READ AND PRINT INPUT DATA
WRITE (NOUT,56)
DO 2 I=1,NG
READ (NIN,57) FREQ(I),BEGIN(I),END(I)
WRITE (NOUT,58) FREQ(I),BEGIN(I),END(I)
SORT DATA LOWEST TO HIGHEST INTERVAL
DO 4 I=1,NG1
J=I+1
DO 4 K=J,NG
IF (BEGIN(I)-BEGIN(K)) 4,4,3
X1=FREQ(I)
X2=BEGIN(I)
X3=END(I)
FREQ(I)=FREQ(K)
BEGIN(I)=BEGIN(K)
END(I)=END(K)
FREQ(K)=X1
BEGIN(K)=X2
END(K)=X3
CONTINUE
CHECK OUT INPUT DATA
SUM=0.
DO 5 I=1,NG
SUM=SUM+END(I)
IF (SUM.GT..0001) GO TO 7
COMPUTE BEGIN AND OF INTERVALS WHEN MID POINT GIVEN
XMID(1)=BEGIN(1)
BEGIN(1)=XMID(1)-(BEGIN(2)-BEGIN(1)/2.)
END(1)=XMID(1)+(BEGIN(2)-BEGIN(1)/2.)
DO 6 I=2,NG1
I1=I+1
I2=I-1
XMID(I)=BEGIN(I)
BEGIN(I)=-((XMID(I)-XMID(I2))/2.)+XMID(I)
END(I)=((BEGIN(I1)-XMID(I))/2.)+XMID(I)
XINV=(BEGIN(NG)-XMID(NG1))/2.
XMID(NG)=BEGIN(NG)
BEGIN(NG)=XMID(NG)-XINV
END(NG)=XMID(NG)+XINV
```

```
      GO TO 9
C     COMPUTE MID POINT WHEN BEGIN AND END GIVEN
7     DO 8 I=1,NG
8     XMID(I)=BEGIN(I)+((END(I)-BEGIN(I))/2.)
C     CHECK FOR OVERLAP AND PRINT OUT DATA
9     WRITE (NOUT,59)
      WRITE (NOUT,60)
      NG2=NG+1
      BEGIN(NG2)=999999.
      DO 10 I=1,NG
      IF (END(I).LT.BEGIN(I+1)) GO TO 50
10    WRITE (NOUT,61) FREQ(I),BEGIN(I),XMID(I),END(I)
C     READ ANALYSIS CONTROL CARD
11    READ (NIN,62) ICODE,C1,C2,C3
      WRITE (NOUT,63) ILPHA
      WRITE (NOUT,64)
      WRITE (NOUT,65) ICODE,C1,C2,C3
C     CHECK ANALYSIS CONTROL
      IF (ICODE.EQ.0) GO TO 45
      IF (ICODE.GE.5) GO TO 1
C     BRANCH TO DISTRIBUTION REQUESTED
      KFLAG=0
      GO TO (12,19,29,36), ICODE
C
C     UNIFORM DISTRURUTION SECTION
12    WRITE (NOUT,66)
      IF (C1.LT..0001) GO TO 13
C     RANGE AND BEGIN POINT SUPPLIED
      R=C3-C2
      AVE=C2+(R/2.)
      XN=C1
      GO TO 15
C     RANGE AND BEGIN POINT COMPUTED FROM DATA
13    R=END(NG)-BEGIN(1)
      AVE=BEGIN(1)+R/2.
      XN=0
      DO 14 I=1,NG
14    XN=XN+FREQ(I)
15    VAR=(R*R)/12.
C     PRINT OUT RESULTS
      WRITE (NOUT,67) R
      WRITE (NOUT,68) AVE
      WRITE (NOUT,69) VAR
      WRITE (NOUT,71) XN
      IF (R.LT..0001) GO TO 46
      IF (XN.LT..0001) GO TO 47
      WRITE (NOUT,72)
C     COMPUTE EXPECTED VALUE,CHISQUARE,AND PRINTOUT
      CHIS=0.
      DO 16 I=1,NG
      XXP(I)=((END(I)-BEGIN(I))/R)*XN
      CHI=((FREQ(I)-XXP(I))*(FREQ(I)-XXP(I)))/XXP(I)
      CHIS=CHIS+CHI
      IF (FREQ(I).GT.5.) GO TO 16
      KFLAG=9
16    WRITE (NOUT,74) BEGIN(I),END(I),FREQ(I),XXP(I),CHI
      WRITE (NOUT,75) CHIS
      WRITE (NOUT,76) NG1
      IF (KFLAG.EQ.0) GO TO 17
      WRITE (NOUT,73)
C     END OF UNIFORM DISTRIBUTION SECTION
17    CALL HISTO
C     RETURN FOR NEXT CONTROL CARD
      GO TO 11
C
C     NORMAL DISTRIBUTION SECTION
```

```
18    WRITE (NOUT,77)
      IF (C1.GT..0001) GO TO 21
C     MEAN VARIANCE COMPUTED
      SUM=0
      XN=0
      DO 19 I=1,NG
      SUM=SUM+FREQ(I)*XMID(I)
19    XN=XN+FREQ(I)
      AVE=SUM/XN
      VAR=0
      DO 20 I=1,NG
      T1=XMID(I)-AVE
20    VAR=VAR+(FREQ(I)*T1*T1)
      VAR=VAR/XN
      STDV=SQRT(VAR)
      GO TO 22
C     MEAN,STANDARD DEVIATION AND POPULATION SUPPLIED
21    AVE=C2
      STDV=C3
      XN=C1
C     PRINT OUT MEAN ETC
22    WRITE (NOUT,68) AVE
      WRITE (NOUT,70) STDV
      WRITE (NOUT,71) XN
      IF (STDV.LT..0001) GO TO 48
      IF (XN.LT..0001) GO TO 47
      WRITE (NOUT,72)
C     COMPUTE PROBABILITY FOR EACH CLASS INTERVAL
C     COMPUTE CHISQUARE AND PRINTOUT
      CHIS=0.
      DO 27 I=1,NG
      Z1=(BEGIN(I)-AVE)/STDV
      IF (Z1.LT.0.) GO TO 23
      AZ1=XNOR(Z1)
      GO TO 24
23    AZ1=1.-XNOR(-Z1)
24    Z2=(END(I)-AVE)/STDV
      IF (Z2.LT.0.) GO TO 25
      AZ2=XNOR(Z2)
      GO TO 26
25    AZ2=1.-XNOR(-Z2)
26    XXP(I)=(AZ2-AZ1)*XN
      CHI=((FREQ(I)-XXP(I))*(FREQ(I)-XXP(I)))/XXP(I)
      CHIS=CHIS+CHI
      IF (FREQ(I).GT.5) GO TO 27
      KFLAG=9
27    WRITE (NOUT,74) BEGIN(I),END(I),FREQ(I),XXP(I),CHI
      WRITE (NOUT,75) CHIS
      WRITE (NOUT,76) NG1
      IF (KFLAG.EQ.0) GO TO 28
      WRITE (NOUT,73)
C     END OF NORMAL DISTRIBUTION SECTION
28    CALL HISTO
C     RETURN FOR NEXT CONTROL CARD
      GO TO 11
C
C     NEGATIVE EXPONENTIAL DISTRIBUTION SECTION
29    WRITE (NOUT,78)
      IF (C1.GT.0.0001) GO TO 32
C     MEAN AND VARIANCE COMPUTED
      SUM=0
      XN=0
      DO 30 I=1,NG
      SUM=SUM+FREQ(I)*XMID(I)
30    XN=XN+FREQ(I)
      AVE=SUM/XN
```

```
      VAR=0
      DO 31 I=1,NG
      T1=XMID(I)-AVE
31    VAR=VAR+(FREQ(I)*T1*T1)
      VAR=VAR/AN
      GO TO 33
C     MEAN,AND POPULATION SUPPLIED
32    AVE=C2
      VAR=C2
      XN=C1
C     PRINT OUT MEAN AND VARIANCE
33    WRITE (NOUT,68) AVE
      WRITE (NOUT,69) VAR
      WRITE (NOUT,71) XN
      IF (AVE.LE.0.) GO TO 49
      IF (XN.LE.0.) GO TO 47
      WRITE (NOUT,72)
C     COMPUTE EXPECTED VALUE,CHISQUARE AND PRINTOUT
      CHIS=0.
      BEG1=BEGIN(1)
      BEGIN(1)=0.0
      DO 34 I=1,NG
      XT1=BEGIN(I)/AVE
      XT2=END(I)/AVE
      P1=EXP(-XT1)
      P2=EXP(-XT2)
      XXP(I)=(P1-P2)*XN
      CHI=((FREQ(I)-XXP(I))*(FREQ(I)-XXP(I)))/XXP(I)
      CHIS=CHIS+CHI
      IF (FREQ(I).GT.5) GO TO 34
      KFLAG=9
34    WRITE (NOUT,74) BEGIN(I),END(I),FREQ(I),XXP(I),CHI
      BEGIN(1)=BEG1
      WRITE (NOUT,75) CHIS
      WRITE (NOUT,76) NG1
      IF (KFLAG.EQ.0) GO TO 35
      WRITE (NOUT,73)
C     END OF EXPONENTIAL DISTRIBUTION SECTION
35    CALL HISTO
      GO TO 11
C
C     POISSON DISTRIBUTION SECTION
36    WRITE (NOUT,79)
      IF (C1.GT..0001) GO TO 39
C     MEAN AND VARIANCE COMPUTED
      SUM=0
      XN=0
      DO 37 I=1,NG
      SUM=SUM+FREQ(I)*XMID(I)
37    XN=XN+FREQ(I)
      AVE=SUM/AN
      VAR=0
      DO 38 I=1,NG
      T1=XMID(I)-AVE
38    VAR=VAR+(FREQ(I)*T1*T1)
      VAR=VAR/AN
      GO TO 40
C     MEAN AND POPULATION SUPPLIED
39    AVE=C2
      VAR=C2
      XN=C1
C     PRINT OUT MEAN AND VARIANCE
40    WRITE (NOUT,68) AVE
      WRITE (NOUT,69) VAR
      WRITE (NOUT,71) XN
      IF (AVE.LE.0.) GO TO 49
```

```
      IF (XN.LT.0.01) GO TO 47
      WRITE (NOUT,80)
C     COMPUTE EXPECTED FREQUENCY USING MID POINT OF CLASS
C     ROUNDED TO NEXT HIGHEST INTEGER
      CHIS=0.
      DO 43 I=1,NG
      NF=XMID(I)+0.5
      XXP(I)=1./EXP(AVE)
      IF (NF.LT.1) GO TO 42
      DO 41 J=1,NF
41    XXP(I)=XXP(I)*(AVE/J)
42    XXP(I)=XXP(I)*XN
      CHI=((FREQ(I)-XXP(I))*(FREQ(I)-XXP(I)))/XXP(I)
      CHIS=CHIS+CHI
      IF (FREQ(I).GT.5.) GO TO 43
      KFLAG=9
43    WRITE (NOUT,81) XMID(I),FREQ(I),XXP(I),CHI
      WRITE (NOUT,75) CHIS
      WRITE (NOUT,76) NGI
      IF (KFLAG.EQ.0) GO TO 44
      WRITE (NOUT,73)
C     END OF POISSON SECTION
44    CALL HISTO
C     RETURN FOR NEXT CONTROL CARD
      GO TO 11
C
C     BRANCH TO THIS SECTION WHEN ERROR
45    WRITE (NOUT,82)
      GO TO 51
46    WRITE (NOUT,83)
      GO TO 51
47    WRITE (NOUT,84)
      GO TO 51
48    WRITE (NOUT,85)
49    WRITE (NOUT,86)
      GO TO 51
50    WRITE (NOUT,87)
      WRITE (NOUT,61) FREQ(I),BEGIN(I),END(I)
51    WRITE (NOUT,88)
C
52    FORMAT (10A4)
53    FORMAT (17H1PROGRAM FIT FOR ,10A4)
54    FORMAT (F2.0)
55    FORMAT (1H0,I5,37H DATA CARDS (INTERVALS) TO BE READ IN)
56    FORMAT (31H COUNT          ******INTERVAL******)
57    FORMAT (3(F5.0,5X))
58    FORMAT (1X,F6.0,4X,F10.4,F10.4)
59    FORMAT (41H0**********EVALUATED INPUT DATA**********)
60    FORMAT (41H  COUNT           BEGIN  MIDPOINT        END)
61    FORMAT (1X,F6.0,4X,3(F10.4))
62    FORMAT (I1,9X,3(F5.0,5X))
63    FORMAT (17H1FIT RESULTS FOR ,10A4)
64    FORMAT (43H0******ANALYSIS CONTROL CARD READS**********)
65    FORMAT (2H (,I1,9X,3(F10.4),1H))
66    FORMAT (29H UNIFORM DISTRIBUTION (CODE1))
67    FORMAT (20H0                RANGE ,F12.4)
68    FORMAT (20H                  MEAN ,F12.4)
69    FORMAT (20H              VARIANCE ,F12.4)
70    FORMAT (20H STANDARD DEVIATION ,F12.4)
71    FORMAT (20H    TOTAL POPULATION ,F13.0)
72    FORMAT (51H0*******INTERVAL*****  OBSERVED  EXPECTED CHISQUARE)
73    FORMAT (50H        WARNING,CHISQUARE CELL COUNT FIVE OR LESS***)
74    FORMAT (1X,2F10.4,F10.0,2F10.4)
75    FORMAT (20H0   TOTAL CHISQUARE ,F12.4)
76    FORMAT (20H DEGREES OF FREEDOM ,10X,I2)
77    FORMAT (29H NORMAL DISTRIBUTION (CODE 2))
```

```
78    FORMAT (..H NEGATIVE EXPONENTIAL DISTRIBUTION (CODE 3))
79    FORMAT (3.H POISSON DISTRIBUTION (CODE 4))
80    FORMAT (51H0.............MIDPOINT  OBSERVED  EXPECTED CHISQUARE)
81    FORMAT (15X,F6.0,9X,F6.0,2F10.2)
82    FORMAT (27H0ERROR IN DATA CONTROL CARD)
83    FORMAT (30H0RANGE EQUAL ZERO, NO ANALYSIS)
84    FORMAT (35H0POPULATION EQUAL ZERO, NO ANALYSIS)
85    FORMAT (37H0STANDARD DEVIATION ZERO, NO ANALYSIS)
86    FORMAT (28H0MEAN EQUAL ZERO, NO ANALYSIS)
87    FORMAT (31H0ERROR IN DATA CARD SHOWN BELOW)
88    FORMAT (23H0PROGRAM FIT TERMINATED)
      END

      FUNCTION XNOR (Z)
C     FIGURES AREA UNDER STANDARD NORMAL CURVE
C     FROM LEFT TAIL TO Z
C     BY USE OF A POWER SERIES EXPANSION
      SUM=Z
      TERM=Z
      ZZ=Z*Z/2.
      XI=1.
1     XII=XI+XI
      TERM=-TERM*ZZ*(XII-1.)/XI/(XII+1.)
      IF (ABS(TERM).LT.0.00005) GO TO 2
      SUM=SUM+TERM
      XI=XI+1.
      GO TO 1
2     XNOR=0.5+SUM/2.506628
      RETURN
      END

      SUBROUTINE HISTO
C     PRINTS OUT HISTOGRAM
      COMMON ILPHA(10),FREQ(100),BEGIN(100),END(100),XMID(100),XXP(100),
     1ILINE(50),XN,NOUT,NG
      COMMON/DATA/IDATA(4)
      DATA (IDATA=1H*,1HX,1H ,4HSTOP)
      IBAR=IDATA(1)
      IXXX=IDATA(2)
      IBLK=IDATA(3)
      WRITE (NOUT,5)
      DO 4 I=1,NG
      DO 1 J=1,50
1     ILINE(J)=IBLK
      JBAR=(FREQ(I)/XN)*50.
      KBAR=(XXP(I)/XN)*50.
      IF (JBAR.LT.1) GO TO 3
      DO 2 K=1,JBAR
2     ILINE(K)=IBAR
3     IF (KBAR.LT.1) GO TO 4
      ILINE(KBAR)=IXXX
4     WRITE (NOUT,6) XMID(I),(ILINE(L),L=1,50)
C     THIS SUBROUTINE PRINTS OUT ACTUAL VERSUS EXPECTED HISTOGRAM
      RETURN
C
C
5     FORMAT (61H0 MIDPOINT.....*....20.....*....40.....*....60.....*....80....
     1*...100)
6     FORMAT (1X,F10.4,50A1)
      END
```

Exercise 7 RISK

Is a model for solving problems structured as decision trees considering risk. Present value and Bayesian analysis may be included.

Momentous personal decisions such as whether or who to marry are seldom the subject of intense formal analysis. Trivial decisions such as what to wear today don't matter enough to require formal analysis. Between the trivial and the momentous, however, is a wide range of potential application of formal decision analysis.

The decision model presented in this exercise is applicable when there is some knowledge of possible outcomes and some knowledge about the probabilities of these outcomes. Also presented is a formal method for updating the probabilities via the Bayes formula as more information about the outcomes is discovered.

The real difficulty in applying RISK or any other formal decision model lies in the generation of the alternatives. The analyst who wishes to use the model must search for the alternatives and gather the required information about each one. Such research is expensive. Also there is always the nagging fear that a feasible alternative, perhaps the best one, has been overlooked.

As an example of a formal decision process, we present an oil well drilling problem that can be solved with the aid of the computer model RISK. The example problem is adapted from *Decision Analysis* by Howard Raiffa (5).

STRUCTURING DECISION TREES

We begin with an elementary decision problem under risk which can be solved by use of a decision tree. The problem is solved first by hand computation and then by use of Program RISK.

The oil wildcatter problem

An oil wildcatter must decide whether to drill at a given site before his option expires. He is uncertain whether the well will be dry, wet, or soaking. He is also uncertain of his exact payoff, although from his experience on other wells, he guesses that the payoff will fit the following scale:

Outcome	Payoff
Dry	$0
Wet	$120,000
Soaking	$270,000

He is also unsure about the cost of drilling, although his best estimate is about $70,000 for this rather shallow well. Moreover his long-run experience in the industry tells him that the probability of drilling a dry hole is .5 a wet hole .3, and a soaking hole .2. For a price, the wildcatter can take seismic soundings to try to improve these odds in his favor. Before considering this complication, however, let us analyze the problem in its simpler form.

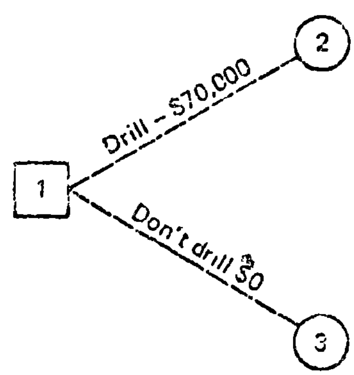
Development of the decision tree

A decision tree is one way to describe formally the alternatives and possible outcomes in a problem. There are two types of branches in the decision tree, a "decision" branch and an "outcome" branch.

The decision branch is under the control of the decision maker. This type of branch is illustrated in Figure 7-1 for the decision of the oil wildcatter. The upper branch represents a decision to drill at a cost of $70,000. The lower branch represents the decision not to drill.

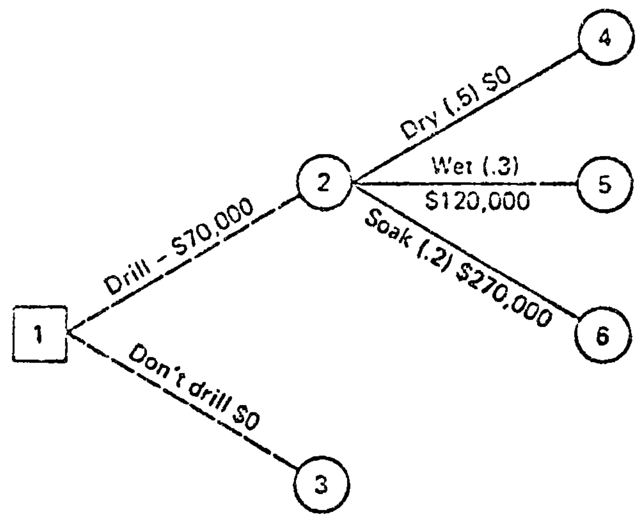
FIGURE 7-1

RAIFFA'S WILDCATTER DECISION BRANCHES



The "outcome" branches represent the consequences of the decision. These outcomes are not under the direct control of the decision maker - in this case, the wildcatter. The possible outcomes are to hit a dry hole, a wet hole, or a soaking hole, as shown in Figure 7-2. Obviously, no outcome follows a decision not to drill. The three outcome branches in Figure 7-2 are attached to the drill decision branch with their probability of occurrence and the associated "payoff" indicated, as well. These three outcomes are assumed to be both exhaustive and exclusive. It should be obvious from the physical appearance of Figure 7-2 why this type of analysis is called a "decision tree." The "node" numbers on each branch end help identify the branches on the tree.

FIGURE 7-2 RAIFFA'S WILDCATTER OUTCOME BRANCHES



Expected value The analysis of decisions incorporating risk are based on the concept of "expected value." Although the expected value is seldom an actual outcome for the decision maker, it can be used to measure the relative value of several outcomes when the probabilities and the payoffs are known.

The formula for computing the expected value is as follows:

expected value = outcome payoff X probability of outcome

For the decision problem of the wildcatter (Figure 7-2), the total expected value at node 2 is the sum of the outcomes which branch from node 2.

Expected value (dry)	=	(.5) X $0	=	$0
Expected value (wet)	=	(.3) X $120,000	=	$36,000
Expected value (soak)	=	(.2) X $270,000	=	$54,000
Total expected value at node 2			=	$90,000

These outcomes are assumed to be exhaustive, hence the probabilities must add to 1. The outcomes are also assumed to be exclusive; thus it is proper to sum the expected values over all the outcomes. Since the drilling operations costs $70,000, the expected value for a drill decision at node 1 is:

Expected value drill (node 2)	$90,000
Cost of drill decision (branch 1–2)	-70,000
Expected value (branch 1–2) drill decision	$20,000

Note that the wildcatter's payoff will never be exactly $20,000. It will be:

-$70,000 if the hole is dry
$50,000 net after drilling expenses if the hole is wet
$200,000 net after drilling expenses if the hole is soaking

The lower decision (branch 1–3) for the don't drill decision costs zero and produces zero income. Given these two alternatives, the logical choice for the wildcatter is to go ahead and drill, since the drill decision has a positive expected value. If the wildcatter cannot afford to lose $70,000 he is in the wrong game.

PROGRAM RISK

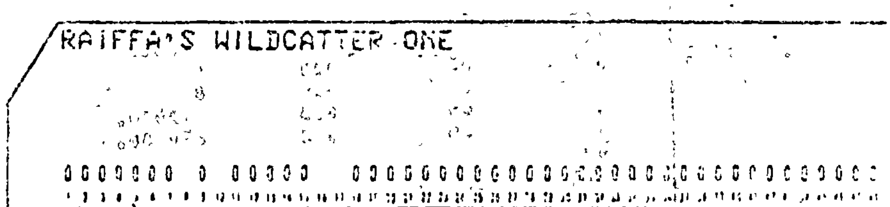
Before demonstrating how program RISK is used to solve the wildcatter problem, let us state some rules for structuring decision trees that must be followed:

1. Each node number must be unique to that node.
2. The node number at the right-hand end of branch must be larger than the left-hand node number.
3. Each decision must have at least two choices (and two branches).
4. A decision branch is identified as such by being assigned a probability of zero.
5. An outcome branch must have a positive probability.
6. The sum of probabilities of all outcome branches from a common node must equal one.

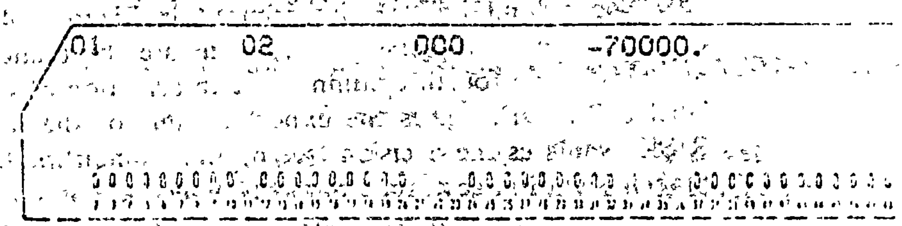
Computer Input—Raiffa's Wildcatter One

The user of program RISK should draw and label the decision tree following these rules before preparing input-data cards.

User name card is the first card in the data deck. Any identification desired may be punched in columns 1–40.

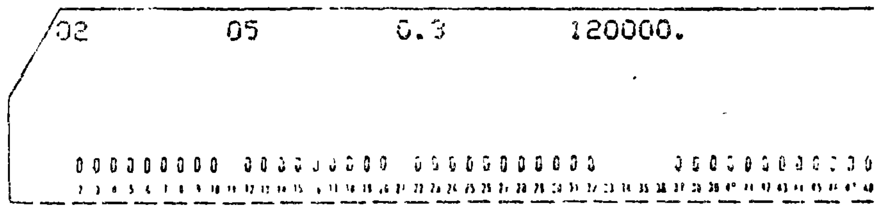


Branch data card contains all the necessary information about each branch in the decision tree, one card per branch. This card contains the left-hand node number for the start of the branch in columns 1–2 and the right-hand node number for the end of the branch in columns 11–12. The probability for the branch is keypunched, with decimal, in columns 21–30. The payoff is keypunched in columns 31–40, with decimal. The first branch data card from Figure 7-2 (below) is for the branch 01–02, which is the "drill" decision.



The probability in columns 21–30 is "000." which designates this a decision branch. The "payoff" is the "-70000," which is the cost of this decision.

Another example is the branch data card for the outcome branch 02–05. This card specifies a probability of "0.3" and a payoff of "120,000." for the outcome of a wet hole.



Final data card must contain "99" in columns 1–2. This card signals that all the branches in the tree have been specified and that the evaluation of the decision tree may begin.

Termination card is the last card in the data deck and contains STOP in columns 1–4.

The complete input data deck for the decision tree of Figure 7-2 appears in Figure 7-3. The reader should validate for himself that each of the six rules for specifying decision trees for program RISK have been followed in this input data.

FIGURE 7-3

COMPUTER INPUT—RAIFFA'S WILDCATTER ONE

```
RAIFFA'S WILDCATTER ONE
01       02       000      -70000.
01       03       000      0
02       04       0.5      0
02       05       0.3      120000.
02       06       0.2      270000.
99
STOP
```

Computer output—Raiffa's Wildcatter One

In Figure 7–4, the printout returned for the input data, the first line is a printout of the user name card. The second line is a header indicating that the input data has been sorted. Sorting facilitates the evaluation of the decision tree by program RISK.

The computer printout also contains a column labeled PERIODS and a column labeled DISCOUNT RATE. These columns are used when the time value of the payoff is considered. In the current problem they are not specified and do not affect the evaluation of the tree.

The second half of the printout in Figure 7-4 shows the evaluated decision tree. The first two columns are the branch identification node numbers. The fourth column is the information of interest. This column, labeled PAYOFF, gives the expected value of the return for that branch. RISK evaluates the decision tree by first computing the expected value for branches with largest node numbers, which are on the right-hand side of the tree. These expected values are summed and moved to the left to the parent branch. The values shown under PAYOFF are the sum of the expected values for the branch and for all branches attached to it to the right.

FIGURE 7-4 COMPUTER OUTPUT—RAIFFA'S WILDCATTER ONE

```
PROGRAM RISK FOR RAIFFA'S WILDCATTER ONE

*************SORTED INPUT DATA**************
LEFT    RIGHT                                 DISCOUNT
NODE    NODE    PROB     PAYOFF    PERIODS    RATE
 1       2         0     -70000        0        0
 1       3         0          0        0        0
 2       4      .5000         0        0        0
 2       5      .3000    120000        0        0
 2       6      .2000    270000        0        0

RISK RESULTS FOR RAIFFA'S WILDCATTER ONE

************EVALUATED DECISION TREE***********
LEFT    RIGHT                                 DISCOUNT
NODE    NODE    PROB     PAYOFF    PERIODS    RATE
 1       2     DECIDE     20000        0        0
 1       3     DECIDE         0        0        0
 2       4      .5000         0        0        0
 2       5      .3000     36000        0        0
 2       6      .2000     54000        0        0

PREFERRED DECISIONS ARE
FROM    TO     EXP. VALUE
 1       2        20000
```

The expected value may be visualized as the accumulated sums of the expected value as the tree is evaluated by RISK, moving from the outer branches on the right, back toward the origin at the left of the tree. In Figure 7-4 these values are the same as those previously computed by hand.

The last line of the printout is a reminder of the preferred decision—in this case, is to go ahead and drill the wildcat well because of the positive $20,000 expected value associated with the drill decision.

EXPECTED VALUE OF PERFECT INFORMATION

This section considers the effect of obtaining improved information about the probabilities involved in the decision tree. It is still assumed that there is risk involved; but the decision maker wants improved information about the size of these risks. We will study the effect of a seismic experiment on the wildcatter's decision.

Wildcatter seismic experiment

At a cost of $10,000, the wildcatter could order a seismic study that would help determine the underlying geological structure at the site of his lease. The study would indicate whether the terrain below has no structure (that's bad), an open structure (that's so-so) or a closed structure (that's good). The experts have kindly listed for him the proportions of dry, wet, and soaking wells which occur in the geological structure types just named (see Table 7-1).

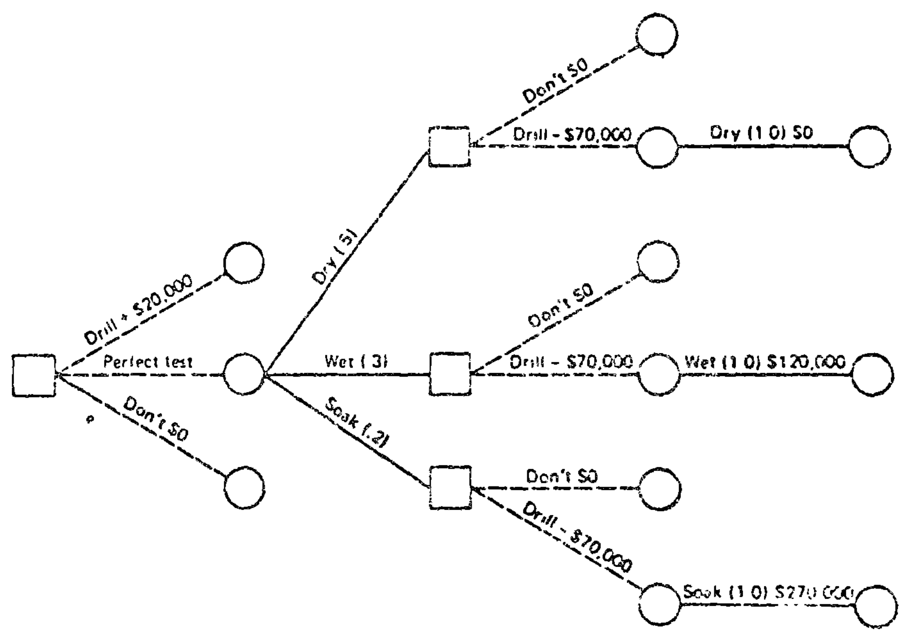
TABLE 7-1 OUTCOME PROBABILITIES AFTER SEISMIC TEST

Proportion of Wells that are:	Geological structure: No S	Open S	Closed S
Dry	0.6	0.3	0.1
Wet	0.3	0.4	0.3
Soaking	0.1	0.4	0.5

The information in Table 7-1 will allow the wildcatter to update his probabilities when he has learned the seismic outcome. Before we use this information, let us consider what it would be worth to be positive of the drilling outcome ahead of time.

Perfect information Perfect information (a priori) analysis allows us to determine what it would be worth to have perfect information about the outcomes. In this case we are considering a mythical test that would say with certainty that the well would be dry, wet, or soaking. The decision tree for this situation is given in Figure 7-5.

FIGURE 7-5 WILDCATTER'S "PERFECT" SEISMIC TEST TREES



In the tree of Figure 7-5 the expected value for the drill-now decision is already known to be $20,000 (from the evaluation of Figure 7-2). The choice to take the "perfect" seismic test is the alternative of current interest, and its complete decision tree is shown. We take first the outcomes from the "perfect" test to be the same as the long-run outcomes to find dry (.5), wet (.3), and soaking (.2). After each outcome we must still decide to drill or not drill.

Faced with the dry outcome from the perfect test, the rational analyst would choose not to drill, rather than spend $70,000 to drill a hole which is certain (probability 1.0) to be dry. The rational choice is "don't" and the expected value is

$.5 \times \$0 = \$0$

For the wet outcome, the rational choice is to drill at a cost of $70,000 to achieve the sure $120,000. The expected value is the net $50,000 times the probability of this result from the perfect test:

$.3 \times \$50,000 = \$15,000$

The soaking outcome expected value is similarly

$.2 \times \$200,000 = \$40,000$

Thus the expected value for the perfect test tree is the total of these outcomes:

Dry	$0
Wet	$15,000
Soaking	$40,000
Total worth of perfect test	$65,000

Without any test, the expected value is $20,000. With the perfect test, the expected value rises to $65,000. Therefore, we should be willing to pay up to $45,000 for such a test. If our mythical perfect test cost more than $45,000, we would do better to simply drill blindly. This type of analysis can be used to value information and to put an upper limit on experimentation for additional information.

BAYESIAN REVISION OF PROBABILITIES

The analysis just performed indicates that perfect information has some worth. However, seismic outcome probabilities available to us from Table 7-1 are not "perfect." They can be used to revise the probabilities after the seismic test, but they do not guarantee outcomes. Here we demonstrate how the probabilities may be revised after the seismic test via the Bayes theorem.

Bayes based his formula on the multiplicative law of probability of conditional, independent events, and his formula can be stated as follows:

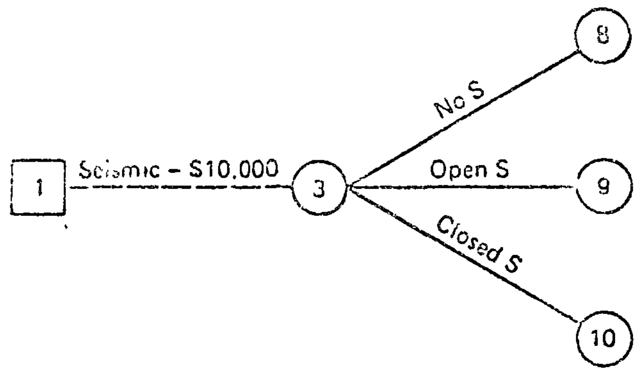
$$\text{Prob}(B/A) = \frac{\text{Prob}(A/B) \cdot \text{Prob}(B)}{\text{Prob}(A)} \qquad (7\ 2)$$

In prose form, this theorem gives the probability that event B *will* occur, assuming that we know that event A *has* occurred. This equals the probability that event A occurs given that event B occurs, times the probability that B will occur (indepent of A), and divided by the probability of A.

Since Bayes first presented this formula in the mid-eighteenth century, it has caused a basic split among statisticians. It has also caused some confusion to students of statistics. Both these effects are due to the apparent contradiction that the "conditional" probability of an event can be found if we know the "reverse" conditional probability. This is similar to saying we need to know what will happen to predict the outcome!

Actually, we do predict the outcome by considering all the possibilities. Let us define event B as the end result of drilling the well: dry, wet, or soaking. Event A is defined as the result indicated by the seismic analysis. Figure 7-6 illustrates event A.

FIGURE 7-6 SEISMIC EXPERIMENT OUTCOME STATES



Independent of the seismic outcome is the actual amount of oil in the field. The seismic test will not change this condition, but the amount of oil does affect the outcome of the seismic. To illustrate how we can use the seismic to revise probabilities through the Bayes formula, let us assume that there is no oil, (event B, Dry) and that the seismic indicates no structure (event A, No S). The Bayes equation reads:

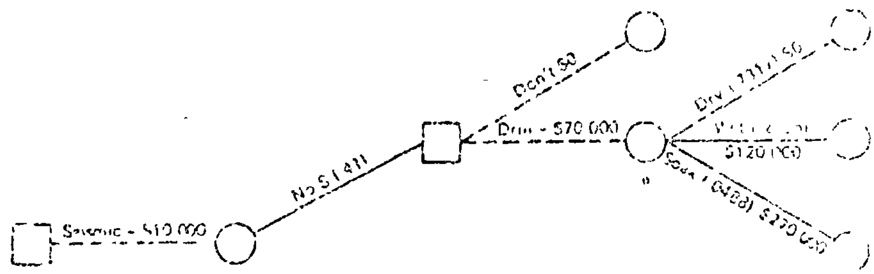
> *The probability that there is no oil, given that the seismic indicates no structure [Prob(B/A)], equals the probability that the seismic shows no structures, given that there is no oil [Prob(A/B)], times the probability that there is no oil [Prob(B)], divided by probability that the seismic will show no structure [Prob(A)].*

Some of the probabilities are presently known. The probability of no oil (dry) is .5. The probability that the seismic shows no structure, if there is no oil, is .6 (Table 7-1). The unconditional probability that the seismic will show no structure is not known, but it can be found by considering all possible end results. This situation is portrayed in Figure 7-7, and the calculations are shown in Table 7-2, where the column labeled "Relative" gives the numerator of Bayes's formula. The sum of the probabilities in this column is the unconditional probability that the seismic will show no structure Prob (A) and is equal to .41. That is, regardless of how much oil is present, the seismic will show no structure 41 percent of the time. Now we can revise our probabilities of finding oil.

$$\begin{aligned} \text{Prob(Dry No S)} &= \frac{\text{Prob(No S Dry) Prob(Dry)}}{\text{Prob(No S)}} \\ &= \frac{(.6)(.5)}{(.41)} \\ &= .7317 \end{aligned} \tag{7.3}$$

FIGURE 7-7

DECISION TREE FOR BAYESIAN UPDATE OF NO S SEISMIC RESULT



These new estimates appear in the "Absolute" column in Table 7-2 and also are given on the branches from the drill decision in Figure 7-7.

TABLE 7-2

COMPUTATION OF BAYESIAN REVISION FOR NO S SEISMIC RESULT

State	No structure (Event A)		Posterior probabilities	
(Event B)	Prob (*A*/*B*)	Prob (*B*)	Relative	Absolute [Prob(*B*/*A*)]
Dry	.6	.5	= .30	.7317
Wet	.3	.3	= .09	.2195
Soaking	.1	.2	= .02	.0488
			.41	
			Prob (*A*)	

Note how the Bayes update changes the expectation about a dry hole following a No S seismic result. The probability is now .7317 of dry—up from .5 on a blind drilling. The change in "soaking" is even more dramatic—it is now .0488, down from .2, or some five times more unlikely.

The probability of the No S seismic result has also been obtained as part of the Bayes formula revision. The sum of the relative posterior probabilities is .41. Since these are all the No S results possible, this is also the proportion of No S seismic results from all possible seismic experiments. Thus seismic outcome probability is shown on branch 3-6.

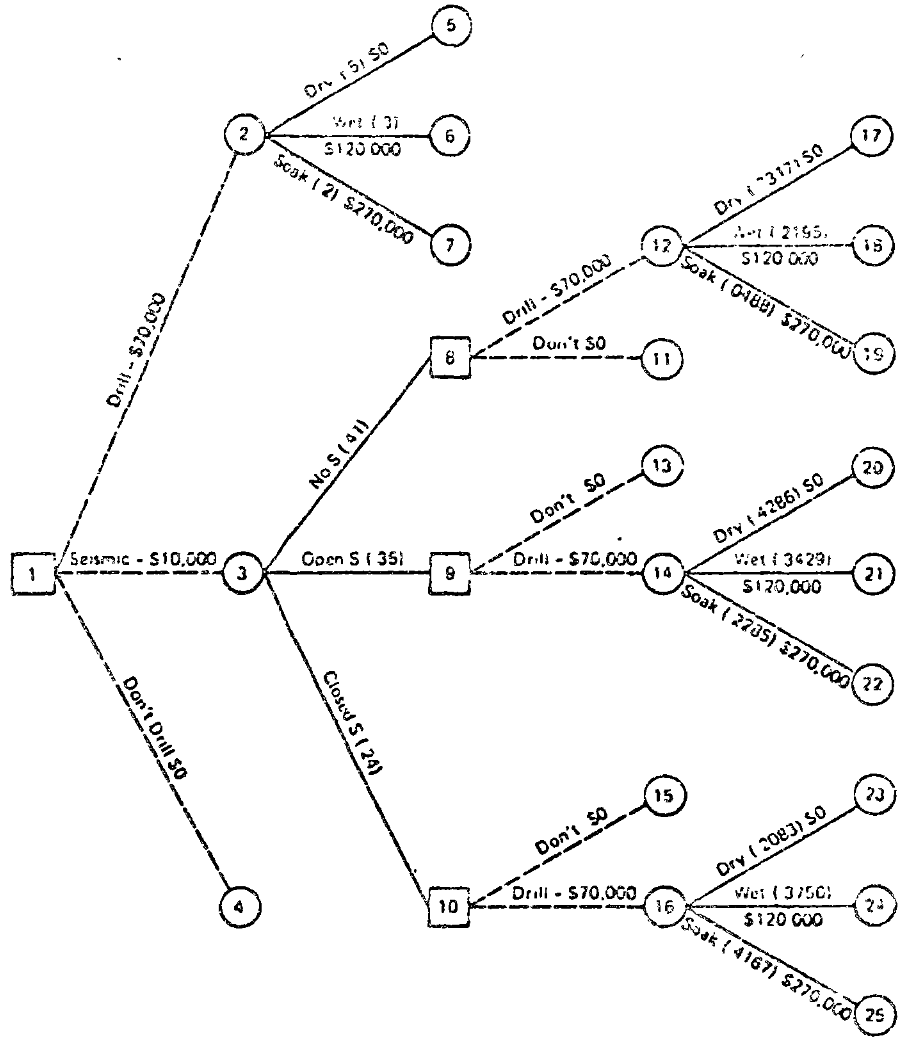
The complete decision tree, which includes a seismic experiment, is presented in Figure 7-8. The Bayes revisions, following the Open S and Closed S results are obtained in the same manner as for No S in Table 7-2. This tree can be solved to determine whether the seismic experiment is worthwhile.

Computer input—Raiffa's Wildcatter Two

Although the decision tree of Figure 7-8 can be solved by hand, program RISK can be used for this and even larger trees. The input for the program is the same as that previously described. There are:

User name card (Anything in columns 1–40).
Branch data cards, one per branch, including node numbers, probability, and payoff

FIGURE 7-8 DECISION TREE WITH SEISMIC EXPERIMENT FOR RAIFFA'S WILDCATTER TWO



Final data card with "99" in columns 1–2 to signal the end of the branch data cards.

Termination card, with STOP in columns 1–4.

Figure 7-9 shows the complete input data deck for the wildcatter problem with the seismic experiment.

Computer output-Raiffa's Wildcatter Two

The output returned for Wildcatter Two appears in Figures 7-10 and 7-11. Figure 7-10 is the printout of the input data after the sort on the node numbers.

The PAYOFF column of Figure 7-11 yields the answers for the expected values of the outcomes and decision branches. These should be read from the bottom up, which corresponds to working back through the tree from right to left.

FIGURE 7-9 COMPUTER INPUT—RAIFFA'S WILDCATTER TWO

```
RAIFFAS WILDCATTER TWO
01       02       000       -70000.
01       03       000       -10000.
01       04       000       0
02       05       0.5       0
02       06       0.3       120000.
02       07       0.2       270000.
03       08       0.41      0
03       09       0.35      0
03       10       0.24      0
08       12       000       -70000.
08       11       000       0
09       13       000       0
09       14       000       -70000.
10       15       000       0
10       16       000       -70000.
12       17       0.7317    0
12       18       0.2195    120000.
12       19       0.0488    270000.
14       20       0.4286    0
14       21       0.3429    120000.
14       22       0.2285    270000.
16       23       0.2083    0
16       24       0.3750    120000.
16       25       0.4167    270000.
99
STOP
```

FIGURE 7-10 COMPUTER OUTPUT—RAIFFA'S WILDCATTER TWO, PAGE ONE

PROGRAM RISK FOR RAIFFAS WILDCATTER TWO

**************SORTED INPUT DATA**************

LEFT NODE	RIGHT NODE	PROB	PAYOFF	PERIODS	DISCOUNT RATE
1	2	0	-70000	0	0
1	3	0	-10000	0	0
1	4	0	0	0	0
2	5	.5000	0	0	0
2	6	.3000	120000	0	0
2	7	.2000	270000	0	0
3	8	.4100	0	0	0
3	9	.3500	0	0	0
3	10	.2400	0	0	0
8	11	0	0	0	0
8	12	0	-70000	0	0
9	13	0	0	0	0
9	14	0	-70000	0	0
10	15	0	0	0	0
10	16	0	-70000	0	0
12	17	.7317	0	0	0
12	18	.2195	120000	0	0
12	19	.0488	270000	0	0
14	20	.4286	0	0	0
14	21	.3429	120000	0	0
14	22	.2285	270000	0	0
16	23	.2083	0	0	0
16	24	.3750	120000	0	0
16	25	.4167	270000	0	0

FIGURE 7-11 COMPUTER OUTPUT—RAIFFA'S WILDCATTER TWO, PAGE TWO

```
RISK RESULTS FOR RAIFFA'S WILDCATTER TWO

°°°°°°°°°°°°°EVALUATED DECISION TREE°°°°°°°°°°°°
LEFT   RIGHT                                    DISCOUNT
NODE   NODE     PROB    PAYOFF    PERIODS        RATE
  1      2     DECIDE    20000       0             0
  1      3     DECIDE    22497       0             0
  1      4     DECIDE        0       0             0
  2      5      .5000        0       0             0
  2      6      .3000    36000       0             0
  2      7      .2000    54000       0             0
  3      8      .4100        0       0             0
  3      9      .3500    11495       0             0
  3     10      .2400    21002       0             0
  8     11     DECIDE        0       0             0
  8     12     DECIDE   -30484       0             0
  9     13     DECIDE        0       0             0
  9     14     DECIDE    32843       0             0
 10     15     DECIDE        0       0             0
 10     16     DECIDE    87509       0             0
 12     17      .7317        0       0             0
 12     18      .2195    26340       0             0
 12     19      .1488    13176       0             0
 14     20      .4286        0       0             0
 14     21      .3429    41148       0             0
 14     22      .2285    61695       0             0
 16     23      .2083        0       0             0
 16     24      .3750    45000       0             0
 16     25      .4167   112509       0             0

PREFERRED DECISIONS ARE
FROM    TO    EXP. VALUE
  1      3      22497
  8     11          0
  9     14      32843
 10     16      87509
```

Note first that the outcome branches from node 16 have "payoffs" of $112,509, $45,000, and $0. These are the expected payoffs after a Closed S seismic result and a drill decision. When the drilling cost has been subtracted, the sum of these values if $87,509 as shown on branch 10–16. The preferred decision is to drill if the seismic result is Closed S. However, the probability of a Closed S result is only .24; thus this decision is reduced in expected value of $21,002, as shown on branch 3–10.

The outcome branches from node 14 can be evaluated in a similar manner. The drill decision after an Open S result has an expected value of $102,843 (total of branches 14–20, 14–21, 14–22), which reduces to $32,843 after the drilling cost is subtracted on branch 9–14. The expected value of this Open S seismic outcome is further reduced to $11,495 on branch 3–9, since it only has a .35 probability.

The seismic outcome of No S leads to a decision not to drill. The probability of hitting oil is too low to support the drilling cost, and branch 8–11 (don't drill) is selected here. The potential expected loss is -$30,484 as shown on the other decision branch 8–12.

Finally, we want to determine whether the seismic experiment can support its cost of $10,000. The expected value at node 3 must be evaluated to determine this. For branch 3–8 the expected value is $0, reflecting the decision not to drill following a No S outcome. The expected value of branch 3–9 is $11,495 for the Open S drill decision; for branch 3–10 it is $21,002 for the Closed S drill decision. This sum of $0 + $11,495 + $21,002 yields $32,497 before and $22,497 after the seismic test cost has been subtracted. The value of $22,497 is larger than the "blind drill" value of $20,000. Therefore, the seismic test did indeed pay off and is the preferred decision. Note once again, however, that the wildcatter will not receive the $22,497. He will lose $10,000 for a No S seismic result. He will lose $80,000 for a good test but a dry hole. Following a good test, he may make $40,000 net for a wet well or $190,000 net on a soaking well.

PRESENT VALUE OF PAYOFFS

When payoffs occur over time, it is common practice to take into account the time value of money. This valuation weighs less heavily payoffs or outlays that are scheduled to occur at some future time.

The formula used to discount to the present a single payment to be received n years from now and discounted at interest rate i is as follows:

$$\text{present value, single payment} = \text{future payment} \times \frac{1}{(1+i)^n} \quad (7.4)$$

If a uniform series of payments is made over time (an annuity), a different formula must be used.

$$\text{present value annuity} = \text{future payment} \times \frac{(1+i)^n - 1}{i(1+i)^n} \quad (7.5)$$

where n = number of years uniform payment to be made
i = interest rate

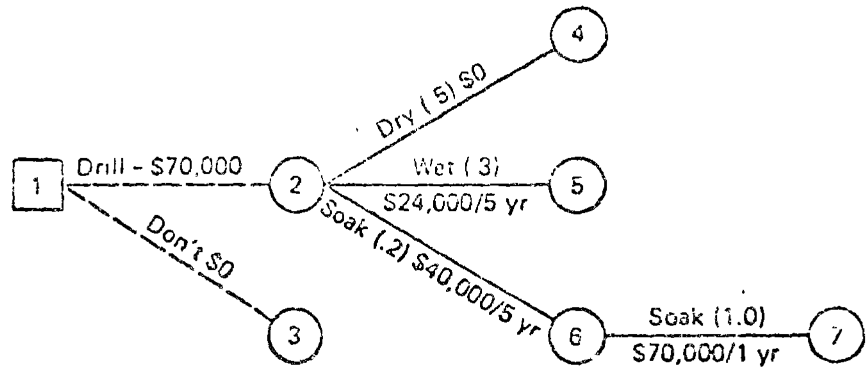
In program RISK, the time value computations are performed before the expected value computation. That is, the payoffs are first treated as if they were certain to occur, and the appropriate discounting is performed. The discounted values then are used as the appropriate payoff for the expected value computation.

In the analysis of the wildcatter's problem, the time value of money was ignored. The drilling cost and the payoffs were treated as if both had occurred in the present. Program RISK can be used to perform the present value computations when the appropriate number of years and the interest rate are supplied.

To illustrate, suppose that the wildcatter's drilling cost are incurred at once, which is most likely, but the payoffs will be spread over a number of years. Suppose, for example, that a wet well will pay off in equal annual payments of $24,000 spread over the next 5 years. The payoff on the soaking well is more complicated. The payoff is $40,000 per year for the first 5 years and $70,000 paid in the sixth year. We will recompute the simple drill decision using an interest rate of 10 percent, which is what the wildcatter figures his money is worth.

Figure 7–12 is a redrawing of the original decision tree of Figure 7–2 with the payoffs spread over time. Note that the payoffs have been changed and the appropriate time spans added. The payoff for the soaking well has an additional branch to accommodate the change in the payoff during the sixth year. This additional payoff has a probability of 1.0, since it is sure to occur once its preceding branch has occurred. The payoffs with positive numbers of years are assumed to be annuity payoffs in RISK.

FIGURE 7-12 PAYOFFS SPREAD OVER TIME—RAIFFA'S WILDCATTER THREE



Computer input—Raiffa's Wildcatter Three

The input data deck for Program RISK always includes the user name card, one branch data card per branch, the final data cards, and the termination card. This section presents the additional information that may be input on the branch data cards when discounting is involved in the decision.

Branch data card with annuities includes the following data items.

Card columns	Format	Item
1-2	(Right justified)	Left node number
11-12	(Right justified)	Right node number
21-30	(Decimal)	Probability (zero for decision branch)
31-40	(Decimal)	Payoff
41-42	(Right justified)	Number of periods for payoff
51-60	(Decimal)	Discount rate

When the number of periods is present, an interest rate must be included. The payoffs are assumed to be equal payments (annuities) over the number of periods specified. When the periods and discount rate are not specified, the payoff is assumed to take place at once with no discounting.

The complete input data deck for Wildcatter Three appears in Figure 7-13, with the specified periods and discount rates.

FIGURE 7-13 COMPUTER INPUT—RAIFFA'S WILDCATTER THREE

```
RAIFFA'S  WILDCATTER  THREE
01        02        000       -70000.
01        03        000       0
02        040       0.5       0
02        05        0.3       24000.    5.        0.10
02        06        0.2       40000.    5.        0.10
06        07        1.0       70000.    1.        0.10
99
STOP
```

FIGURE 7-14 COMPUTER OUTPUT—RAIFFA'S WILDCATTER THREE

PROGRAM RISK FOR RAIFFA'S WILDCATTER THREE

INPUT DATA

LEFT NODE	RIGHT NODE	PROB	PAYOFF	PERIODS	DISCOUNT RATE
1	2	0	[illegible]	0	0
[illegible]	[illegible]	0	0	0	0
[illegible]	[illegible]	[illegible]	0	0	0
[illegible]	[illegible]	[illegible]	[illegible]	5	.100
[illegible]	[illegible]	[illegible]	[illegible]	5	[illegible]
[illegible]	7	[illegible]	70000		.100

RISK RESULTS FOR RAIFFA'S WILDCATTER THREE

EVALUATED DECISION TREE

LEFT NODE	RIGHT NODE	PROB	PAYOFF	PERIODS	DISCOUNT RATE
1	2	DECIDE	[illegible]	0	0
1	3	DECIDE	0	0	0
2	4	.5000	0	0	0
2	5	.3000	27294	5	.100
2	6	.2000	38229	5	.100
6	7	1.0000	63635	1	.100

PREFERRED DECISIONS ARE

FROM	TO	EXP. VALUE
1	3	0

Computer output—Raiffa's Wildcatter Three

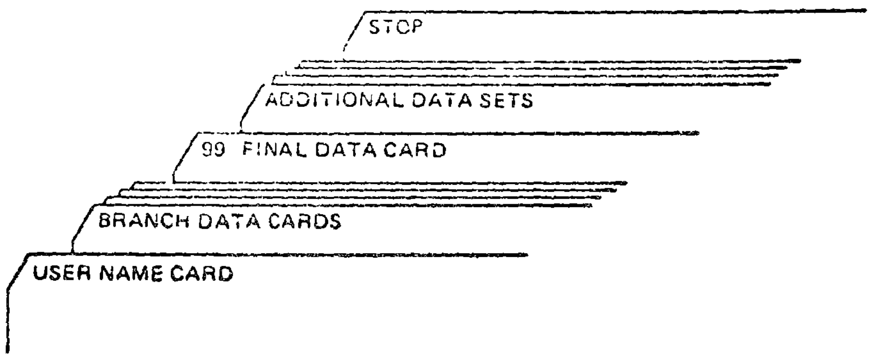
The output returned for Wildcatter Three is shown in Figure 7-14. The expected present worths in the payoff column are lower than those for the original problem, Wildcatter One. In fact, the discounting is so severe that the decision is switched to "don't drill" for the wildcatter.

It is left for the interested user to determine whether the seismic experiment decision also will change when the time value of money is considered.

References

1. Chernoff, H., and L. Moses, *Elementary Decision Theory.* New York: Wiley, 1959.
2. Luce, R. Duncan, and Howard Raiffa, *Games and Decisions.* New York: Wiley, 1957.
3. Miller, David W. and Martin K. Starr, *Executive Decisions and Operations Research,* 2nd ed. Englewood Cliffs, N.J.: Prentice-Hall, 1969.
4. Morris, William T., *Management Science, A Bayesian Introduction.* Englewood Cliffs, N.J.: Prentice-Hall, 1968.
5. Raiffa, Howard, *Decision Analysis.* Reading, Mass.: Addison-Wesley, 1968.
6. Riggs, James L., *Economic Decision Models.* New York: McGraw-Hill, 1968.
7. Thierauf, Robert J., and Richard A. Grosse, *Decision Making Through Operations Research.* New York: Wiley, 1970.
8. Williams, J. D., *The Compleat Strategyst,* rev. ed. New York: McGraw-Hill, 1966.

RISK DATA STRUCTURE



BRANCH DATA CARD

Card columns	Format	Item
1, 2	I2	Left node number
11, 12	I2	Right node number
21-30	F10.0	Probability
31-40	F10.0	Payoff
41, 42	F2.0	Periods
51-60	F10.0	Discount rate

```
      PROGRAM RISK
C     DECISION TREE ANALYSIS
C     ROY D HARRIS FEB 1972
C     THIS VERSION FOR CDC 3100
C     NFR LEFT NODE
C     NTO RIGHT NODE
C     PROB PROBABILITY ACROSS BRANCH
C     VAL INITIAL PAYOFF OF BRANCH
C     VAL ANNUITY VALUE OF BRANCH AFTER ANNUITY CALCULATION
C     YEAR PERIODS ACROSS BRANCH (OPTIONAL)
C     RATE INTEREST RATE ACROSS BRANCH (OPTIONAL)
C     SUM ACCUMULATED EXPECTED VALUE OF BRANCH
C     I MAIN DO LOOP INDEX
C     J INTERNAL DO LOOP INDEX
C     N NUMBER OF BRANCHES
C     TEMPORARY STORAGE VARIABLES INCLUDE
C     NX,NY,TP,TV,TY,TR,PV,PVA,STEMP,JTEMP,NN,II,JJ,K
C
      DIMENSION ILPHA(10), NFR(100), NTO(100), PROB(100), VAL(100), YEAR
     1(100), RATE(100), SUM(100)
      COMMON/DATA/IEND
      DATA (IEND = 4HSTOP)
      ISTOP=IEND
      MI=60
      MO=61
C     READ AND PRINT STUDENT NAME CARD
    1 READ (MI,47) ILPHA
      WRITE (MO,48) ILPHA
      IF (ILPHA(1).EQ.ISTOP) GO TO 42
C     INITIALIZE
      DO 2 I=1,100
    2 SUM(I)=0.0
C
C     READ INPUT DATA
      DO 3 I=1,100
      READ (MI,49) NFR(I),NTO(I),PROB(I),VAL(I),YEAR(I),RATE(I)
C     CHECK FOR LAST DATA CARD
      IF (NFR(I).GT.98) GO TO 4
    3 CONTINUE
    4 N=I-1
C
      DO 7 I=1,N
C     SORT INPUT DATA (FROM NODE---MAJOR) (TO NODE---MINOR)
      NN=N-I
      DO 7 J=1,NN
      IF (NFR(J)-NFR(J+1)) 7,5,6
C     MINOR SORT
    5 IF (NTO(J)-NTO(J+1)) 7,40,6
C     SHIFT ALL DATA
    6 NX=NFR(J)
      NY=NTO(J)
      TP=PROB(J)
      TV=VAL(J)
      TY=YEAR(J)
      TR=RATE(J)
      NFR(J)=NFR(J+1)
      NTO(J)=NTO(J+1)
      PROB(J)=PROB(J+1)
      VAL(J)=VAL(J+1)
      YEAR(J)=YEAR(J+1)
      RATE(J)=RATE(J+1)
      NFR(J+1)=NX
      NTO(J+1)=NY
      PROB(J+1)=TP
      VAL(J+1)=TV
```

```
      YEAR(J+1)=T5
      RATE(J+1)=T6
      CONTINUE
C
C     PRINT SORTED INPUT DATA
      WRITE (MU,50)
      WRITE (MU,51)
      WRITE (MU,52)
      DO 8 I=1,N
    8 WRITE (MU,53) NFR(I),NTO(I),PROB(I),VAL(I),YEAR(I),RATE(I)
C
C     COMPUTE ANNUITY VALUE ON EACH BRANCH  STORE IN VAL(I)
      DO 10 I=1,N
      IF (YEAR(I)-1.0) 10,10,9
    9 PV=(1+RATE(I))**YEAR(I)
      PVA=(PV-1.0)/(RATE(I)*PV)
      VAL(I)=VAL(I)*PVA
C     MOVE ANNUITY VALUE TO RIGHT NODE
      VAL(I)=(VAL(I)*PV)
   10 CONTINUE
C
C     EDIT PROBABILITIES
C     INITIALIZE
      J=1
      PTEMP=0.0
      NFRT=NFR(J)
C     CHECK FOR OUT OF DATA CONDITION
   11 IF (N-J) 14,12,12
C     CHECK IF SAME TO-NODE NUMBER
   12 IF (NFR(J)-NFRT) 40,13,14
C     SUM PROBABILITIES ON SAME TO-NODES
   13 PTEMP=PTEMP+PROB(J)
      J=J+1
      GO TO 11
C     CHECK IF PROBABILITIES ADD TO 1 OR ZERO
   14 IF (ABS(PTEMP-1.0)-.00001) 16,16,15
   15 IF (PTEMP-0.0) 41,16,41
C     RESET FOR NEXT GROUP OF NODES
   16 CONTINUE
      PTEMP=PROB(J)
      NFRT=NFR(J)
      J=J+1
C     CHECK FOR OUT OF DATA
      IF (N-J) 17,11,11
C     END OF PROBABILITY EDIT
   17 CONTINUE
C
C     START OF MAIN DO LOOP TO EVALUATE TREE
      K=1
   18 CONTINUE
      I=(N+1)-K
C     I INDEXES FROM N DOWN TO 1
C     CHECK TYPE OF BRANCH
      IF (PROB(I)-0.0) 23,23,19
C
C     PROBABILITY BRANCH EVALUATION
C     COMPUTE PRESENT VALUE
   19 PV=(1+RATE(I))**YEAR(I)
      SUM(I)=(SUM(I)+VAL(I))/PV
C     COMPUTE EXPECTED VALUE
      SUM(I)=SUM(I)*PROB(I)
C     FIND ADJOINING BRANCH AND MOVE SUM
      J=I
   20 J=J-1
      IF (J-0) 22,22,21
   21 IF (NFR(I)-NTO(J)) 20,22,20
```

```
   22 SUM(J)=SUM(J)+SUM(I)
      GO TO 33
C     END OF CHANCE BRANCH EVALUATION
C
C     START DECISION BRANCH EVALUATION
C     COMPUTE PRESENT VALUE OF FIRST BRANCH
   23 PV=(1.+RATE(I))**YEAR(I)
      SUM(I)=(SUM(I)+VAL(I))/PV
      STEMP=SUM(I)
      NFRT=NFR(I)
      JTEMP=I
C     EVALUATE OTHER BRANCHES FOR THIS DECISION
      J=I
   24 J=J-1
      IF (J-0) 28,28,25
   25 IF (NFR(J)-NFR(J)) 40,26,28
   26 PV=(1.+RATE(J))**YEAR(J)
      SUM(J)=(SUM(J)+VAL(J))/PV
      IF (SUM(J)-STEMP) 24,24,27
   27 STEMP=SUM(J)
      JTEMP=J
      GO TO 24
C     ALL OF THIS DECISION BRANCHES CHECKED
C     FLAG WINNING DECISION
   28 PROB(JTEMP)=-99.0
C     RESET MAIN DO LOOP INDEX K
      K=N-J
C     CHECK IF MOVE SUM POSSIBLE
      IF (N-K) 34,34,29
C     FIND ADJOINING BRANCH AND MOVE WINNER SUM
   29 J=(N+1)-K
   30 J=J-1
      IF (J-0) 32,32,31
   31 IF (NFRT-NTO(J)) 30,32,30
   32 SUM(J)=SUM(J)+STEMP
C     END OF DECIDE BRANCH EVALUATION
C     END OF MAIN DO LOOP
   33 K=K+1
      IF (K.LE.N) GO TO 18
   34 CONTINUE
C
C     PRINT EVALUATED DECISION TREE
      WRITE (MO,43) ILPHA
      WRITE (MO,54)
      WRITE (MO,51)
      WRITE (MO,52)
      DO 37 I=1,N
      IF (PROB(I)-0.0) 35,35,36
   35 WRITE (MO,55) NFR(I),NTO(I),SUM(I),YEAR(I),RATE(I)
      GO TO 37
   36 WRITE (MO,53) NFR(I),NTO(I),PROB(I),SUM(I),YEAR(I),RATE(I)
   37 CONTINUE
C
C     PRINT PREFERRED DECISIONS
      WRITE (MO,56)
      WRITE (MO,57)
      J=1
      DO 39 J=1,N
      IF (PROB(J)-(-99.0)) 39,38,39
   38 WRITE (MO,58) NFR(J),NTO(J),SUM(J)
   39 CONTINUE
C     RETURN FOR NEXT SET OF DATA
      GO TO 1
C
C     ERROR MESSAGE SECTION
   60 WRITE (MO,59) NFR(J),NTO(J)
```

```
      WRITE (MU,60)
      GO TO 42
41    WRITE (MU,44)
      WRITE (MU,45) NFR(J),NTO(J)
42    WRITE (MU,46)
C
43    FORMAT (18H1RISK RESULTS FOR ,10A4)
44    FORMAT (35H0ERROR IN PROBABILITY SPECIFICATION)
45    FORMAT (24H CHECK BRANCHES JUST BEFORE ,I2,4H TO ,I2)
46    FORMAT (26H0PROGRAM RISK TERMINATED)
47    FORMAT (10A4)
48    FORMAT (19H1PROGRAM RISK FOR ,10A4)
49    FORMAT (I2,8X,I2,8X,F10.0,F10.0,F20.0,8X,F10.0)
50    FORMAT (47H0***************SORTED INPUT DATA**************)
51    FORMAT (47H LEFT    RIGHT                          DISCOUNT)
52    FORMAT (47H NODE    NODE    PROB  PAYOFF  PERIODS   RATE  )
53    FORMAT (1X,I2,5X,I2,4X,F6.4,F9.0,4X,F3.0,4X,F5.3)
54    FORMAT (47H0************EVALUATED DECISION TREE***********)
55    FORMAT (1X,I2,5X,I2,4X,6HDECIDE,F9.0,4X,F3.0,4X,F5.3)
56    FORMAT (24H0PREFERRED DECISIONS ARE)
57    FORMAT (24H FROM    TO    EXP. VALUE)
58    FORMAT (2H  ,I2,4X,I2,3X,F10.0)
59    FORMAT (22H0ERROR IN BRANCH FROM ,I2,4H TO ,I2)
60    FORMAT (16H CHECK NODE NUMBERS)
      END
```

Exercise 8 UNCERT

Is a model for decision making under uncertainty which incorporates the methods of Laplace, Hurwicz, Savage, and Wald.

Part of the human experience involves the making of choices among various alternatives. When we awake each day, we must decide what to wear, what to eat, and what to do, if anything. Most decisions affect only the individual involved and those close to him. However, the decisions made by the king, the royal astrologers, the general, the ambassador, the president, and others with real power may have a wide impact.

This exercise deals with the process of making decisions. More specifically, it is concerned with a set of formal models that can be applied when the future state of the world is considered to be uncertain. The development of the data for these models and obtaining a solution requires some time, effort, and energy. Therefore, these models will probably be more useful to the royal astrologer, the general, or the president than to someone making a personal decision.

The first section discusses four decision methods that are applicable when the future is uncertain. The methods are illustrated on a simple problem. The following section introduces program UNCERT, which will do the computations required. The last section presents a larger, hopefully more realistic problem and the computer program solution.

This model is intended for the systems analyst who is interested in structuring alternatives and outcomes using formal methods.

DECISION METHODS UNDER UNCERTAINTY

One popular method of classifying decision situations has the following categories.

Certainty—*A decision is made with the prior knowledge of what will be the* actual *outcome because of this choice.*

Risk—*There are several possible outcomes for each strategy selected, and the probabilities of these outcomes are known.*

Uncertainty—*Each decision has a set of consequences, but there are no known probabilities associated with these outcomes.*

The choice among alternative pieces of equipment represents a decision under certainty. It is usually assumed that the cost and the performance of the equipment is known. The decision problem reduces to determining the equipment available and the cost and output of each item. The decision method is to choose the equipment that is most cost-effective. The assumption of certainty in cost and performance may be dubious, however, as the owner of a "lemon" will be glad to remind you.

Decisions involving risk assume that the outcomes occur with a probability that is outside the control of the decision maker. In all betting games, the probabilities of each outcome are known or estimated by the players. Formal decision models take advantage of these known or assumed probabilities. There are known probabilities of being killed in an auto accident, drawing an inside straight flush in poker, succeeding in a new business, living past age 90, and so on, and these can be incorporated into formal decision models.

Probability judgments are based on a knowledge of the structure of the situation, as in card games, or on past experience (e.g., the prior performance of a sports team). Decisions under uncertainty occur when the probabilities are not known. Examples of situations under uncertainty are the introduction of a new class of new products, the launching of a new experiment, or the undertaking of a unique one-time endeavor.

THE DECISION MATRIX

The basic structure, on which all formal decision models depend, is that there are A_n alternatives with S_m possible end states for each alternative. These are represented by the decision matrix in Equation 8–1.

$$\begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_n \end{array} \begin{array}{c} \begin{array}{cccc} S_1 & S_2 & \ldots & S_m \end{array} \\ \begin{bmatrix} OC_{11} & OC_{12} & \ldots & OC_{1m} \\ OC_{21} & OC_{22} & \ldots & OC_{2m} \\ \vdots & \vdots & & \vdots \\ OC_{n1} & OC_{n2} & \ldots & OC_{nm} \end{bmatrix} \end{array} \qquad (8.1)$$

A decision matrix

The outcomes are labeled OC and these outcomes form the body of the matrix. For example, OC_{23} is the outcome when alternative 2 is chosen and the end state turns out to be S_3.

The development of the matrix implies that all the relevant alternatives are included. The matrix development also implies that all possible future states are included and properly classified into mutually exclusive states. Furthermore, each of the outcome values OC must be generated for the decision matrix. This implies that the proper data can be obtained, which may be impossible.

It is fairly obvious that the development of the decision matrix is the most difficult part of the decision process. It is also the process that "haunts" the decision maker. What if a viable alternative is omitted? What if a possible end state is forgotten? Has the best alternative or the worse possible end state been omitted?

Implicit in the development of the decision matrix is the assumption that the "better" outcome from a "better" decision process will pay the costs of the analysis. Thus formal decision analysis is usually reserved for decisions of some importance. In these decisions, the avoidance of disaster or the possible gain of large returns more than compensates for the cost of the information and cost of the decision analysis.

The use of the decision methods in this exercise assumes that the decision matrix can be generated for the problem under consideration by the analyst. It remains for us to choose from among the alternatives. Four methods for making this selection follow.

The Laplace equality method

The Laplace criterion treats all end states S_j as equally likely, thereby converting the decision into one of risk, with each outcome having the same probability. This method is also known as the "principle of insufficient reason" because the decision maker, without any evidence to the contrary, assumes that each future outcome is equally likely. This method is attributed to Pierre Simon, Marquis de Laplace.

Having assumed equal likelihood of occurrence, the decision maker multiplies each outcome by $1/m$, where m is the number of end states, and finds the total value of each alternative A_n by summing these products across each row. He then maximizes his expectations by selecting the alternative A_i with the largest expected value. Of course in cost minimization problems, the smallest expected value is chosen.

A simple example of this method can be demonstrated through the use of a 2 X 2 profit matrix, in Equation 8.2a.

$$\begin{array}{c} \\ A_1 \\ A_2 \end{array} \begin{array}{c} \begin{array}{cc} S_1 & S_2 \end{array} \\ \begin{bmatrix} 5 & 0 \\ 2 & 2 \end{bmatrix} \end{array} \tag{8.2a}$$

Profit matrix

The formula for the Laplace method states that each element of the matrix is divided by the number of end states, m. Thus we have

$$\begin{array}{c} \\ A_1 \\ A_2 \end{array} \begin{array}{c} \begin{array}{cc} S_1 & S_2 \end{array} \\ \begin{bmatrix} 5/2 & 0/2 \\ 2/2 & 2/2 \end{bmatrix} \end{array} \tag{8.2b}$$

Next the totals of each row are determined;

$$A_1: \ 5/2 + 0/2 = 5/2$$
$$A_2: \ 2/2 + 2/2 = 4/2$$

In this case, alternative A_1 with the value of 5/2 is slightly better than alternative A_2, with the value of 4/2. This method is used by the "rational" analyst who believes that each end state is equally likely to happen because he has no insight into the probability of occurrence of any of the individual states.

The Wald maximin method

The "maximin" principle is often used by the pessimist because it selects the alternative A_i that is the maximum across all alternatives of the minimum outcome of each alternative. This is written as $\max_i \min_j OC_{ij}$. This method, which is attributed to Abraham Wald, is used in "maximimization" problems. If the decision is one of minimization the minimax principle is applied. There is also a "maximax" principle, but the maximax results can be derived through the use of the Hurwicz principle, to be discussed next.

The maximin method for the decision matrix of Equation 8.2a is simply the maximum of the row minima. The row minima are shown below in parentheses.

$$\begin{array}{c} \\ A_1 \\ A_2 \end{array} \begin{array}{c} \begin{array}{cc} S_1 & S_2 \end{array} \\ \begin{bmatrix} 5 & (0) \\ (2) & (2) \end{bmatrix} \end{array} \tag{8.2c}$$

Both outcomes on row 2 are equal to "2" and are tied for row minima.

A_1: 0
A_2: 2

The choice is the maximum of these two values; thus A_2 is chosen. The conservative analyst would use this method in selecting his alternative. The method follows the old wisdom that a bird in hand is worth two in the bush. Or in this case, two birds in hand are worth five in the bush.

The Hurwicz optimism method

The adventuristic or slightly optimistic person would probably use the Hurwicz principle for decision making. The principle states. "Select an index of optimism α, such that $0 \leqslant \alpha \leqslant 1$. For each A_i, compute $\alpha[\max_j OC_{ij}] + (1 - \alpha)[\min_j OC_{ij}]$; then select the alternative that has the maximum of these quantities." This method is attributed to Leonid Hurwicz.

In the extreme case of optimism, $\alpha = 1$, and this gives the same solution as maximax. In the case of extreme pessimism, $\alpha = 0$, which equals the minimax principle. The decision maker simply takes the maximum values of each alternative, multiplies by α, and adds this quantity to the minimum value of each alternative multiplied by $(1 - \alpha)$. The maximum of these products for each alternative A_i is then selected.

In applying the Hurwicz method, we assume that the decision maker has some "hope" for the likelihood of the maximum or minimum values. He applies this "feeling" to the decision matrix by assuming that the minimum value will occur if the maximum value does not occur.

This method can be applied to the decision matrix of Equation 8.2a if an α value is chosen. To illustrate, we say arbitrarily that $\alpha = 0.6$.

(1) For alternative A_1:

$$\begin{aligned} &(\alpha)\ \text{maximum row value} + (1 - \alpha)\ \text{minimum row value} \\ &= (0.6)(5) + (1 - 0.6)(0) \\ &= 3.0 + 0 \\ &= 3.0 \end{aligned}$$

(2) For alternative A_2:

$$= (0.6)(2) + (1 - 0.6)(2)$$
$$= 1.2 + 0.8$$
$$= 2.0$$

The choice is between

A_1 : 3.0
A_2 : 2.0

The alternative with the larger value, A_1, would be chosen.

The Savage regret method

The last method to be considered, the Savage regret method, uses the premise that the analyst wishes to minimize his regret if he makes the wrong choice. A new matrix needs to be formed, called the regret matrix. This matrix determines the disappointment the analyst would incur if he selected the wrong strategy. The regret matrix is computed by subtracting the maximum value in each column from each other element in that column. This method is attributed to Leonard J. Savage.

Again using the decision matrix from Equation 8.2a, the regret matrix can be illustrated. The column for the first state S_1 is

S_1
5
2

The largest or maximum value is 5, and every value in the column is subtracted from 5, yielding

	S_1 (original)		S_1 (regret)
A_1	(5 − 5)	=	0
A_2	(2 − 5)	=	−3

The "regret" indicates that the decision maker will "lose" the possibility of gaining three extra units of profit if he chooses A_2. If he chooses A_1, he has a chance to get the entire five units of profit if S_1 prevails, thus the zero "regret" for alternative A_1.

The regret column for state S_2 is

S_2 (original)		S_2 (regret)
(0 − 2)	=	−2
(2 − 2)	=	0

The entire regret matrix is

$$\begin{array}{cc} & \begin{array}{cc} S_1 & S_2 \end{array} \\ \begin{array}{c} A_1 \\ A_2 \end{array} & \begin{bmatrix} 0 & -2 \\ -3 & 0 \end{bmatrix} \end{array} \qquad (8.2d)$$

The decision maker must now minimize the maximum regret between the two alternatives, A_1 and A_2. The maximum regret for A_1 is -2 and the maximum regret for A_2 is -3. Thus the choice is between

$$\begin{array}{ll} A_1 & -2 \\ A_2 & -3 \end{array}$$

Since the minimum value is chosen, A_1 is the preferred decision.

This method might be used by the decision analyst who is a bad loser. That is, the decision maker does not want to let the big payoff get away. He tries to minimize the disappointment between the choice of alternatives and the eventual outcome.

The four methods have now been illustrated as applied to the decision matrix of Equation 8.2a. Three of the methods—Laplace, Hurwicz, and Savage—indicate that alternative A_1 should be chosen. The Wald "maximin" method, which is a conservative approach, selected alternative A_2. The decision analyst of course must choose for himself which method to employ, based on his own psychological bias as a decision maker.

We next present program UNCERT, which will perform the necessary computation on the decision matrix for each of the four methods.

PROGRAM UNCERT

The basic input to program UNCERT is a decision matrix. A sample decision matrix is followed by the input deck for this problem. The resultant output is shown and discussed in the last subsection.

Test problem: profit maximization

The decision matrix for a problem with five alternatives and four possible outcomes appears in Equation 8.3. The value for each outcome is the profit that will be realized by the analyst. The most attractive alternative is OC_{32} with a profit of 26 units, the least attractive is OC_{31} with a profit of only 5 units.

$$\begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{array} \begin{array}{c} \begin{array}{cccc} S_1 & S_2 & S_3 & S_4 \end{array} \\ \begin{bmatrix} 18 & 18 & 10 & 14 \\ 14 & 14 & 14 & 14 \\ 5 & 26 & 10 & 10 \\ 14 & 22 & 10 & 10 \\ 10 & 12 & 12 & 10 \end{bmatrix} \end{array} \qquad (8.3)$$

Test problem profit maximization matrix

The decision matrix is assumed to be both accurate and complete. It is also implied that the probabilities associated with each outcome state S_j are not known. If these assumptions hold, it is possible to use the decision methods under uncertainty on the matrix.

To apply the Laplace method to the matrix in Equation 8.3, we must first divide each element in the matrix by the number of possible outcome states. Then each row is summed. This is shown in Equation 8.4, with the right-hand column equal to the sum of the rows.

	S_1	S_2	S_3	S_4	(Sum)
A_1	18/4	18/4	10/4	14/4	60/4
A_2	14/4	14/4	14/4	14/4	56/4
A_3	5/4	26/4	10/4	10/4	51/4
A_4	14/4	22/4	10/4	10/4	56/4
A_5	10/4	12/4	12/4	10/4	44/4

(8.4)

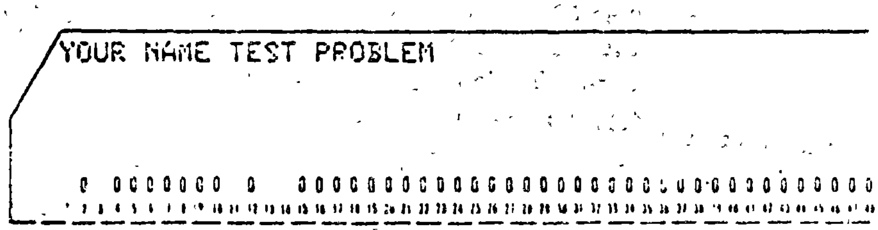
Test problem Laplace matrix

Although the problem is not too difficult to solve by hand, it is solved here by program UNCERT to introduce the reader to the use of UNCERT on decision problems. The computer program becomes advantageous when there are larger matrices to be solved and several types of analysis to be performed.

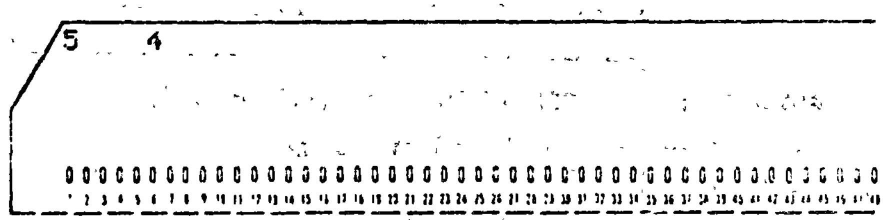
Computer input—test problem

This section presents the input data deck sequence and the formats for using program UNCERT, but showing only the data input required for solving the decision matrix of Equation 8.3 using the Laplace method. Other program features are illustrated in the next section, and a complete listing of all program features and options is found in the UNCERT Input Data section.

User name card is the first card in the input data deck. It can contain any identifying information keypunched into the first 40 columns.



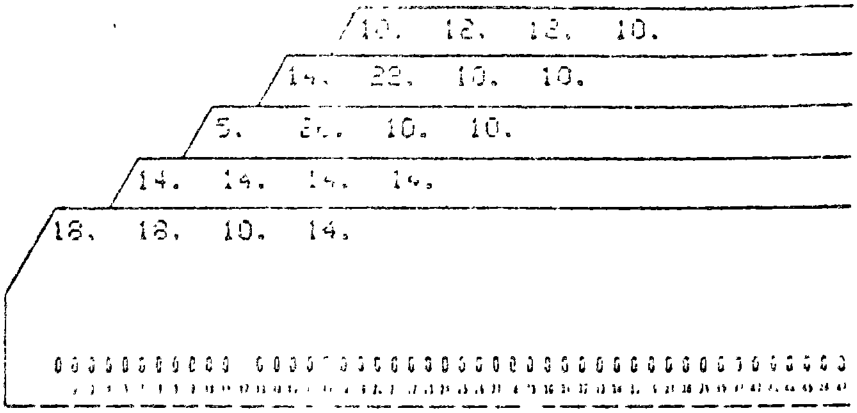
Matrix control card is the second card in the input data deck. The number of alternatives (rows) is keypunched in column 1. The number of states (columns) is keypunched in column 6. The program will accept from two to nine rows and columns in the matrix. The current problem has five rows (alternatives) and four columns (states).



Matrix data cards are the next cards in the input data deck, one data card for each row in the matrix. This card contains all the outcomes for each state of a single alternative.

The data is keypunched beginning in column 1, 6, 11, 16, 21, . . . ; decimals must be punched. Each card must contain enough outcomes for the number of states specified on the matrix control card.

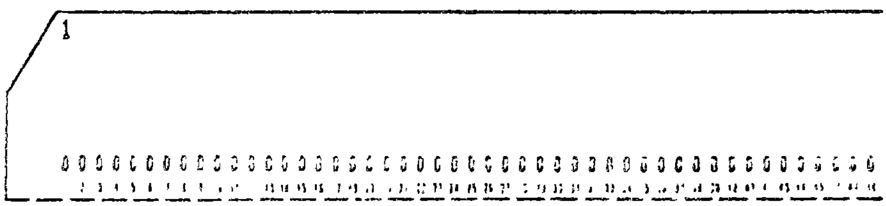
There are five data cards for the matrix of equation 8.3.



Matrix data cards for equation 8.3

Method control card is the next card in the input data deck. This card specifies which decision method is to be applied to the matrix that has just been read in. One method control card is required for each decision method to be applied to the data. Additional method control cards may be added, one after another, until all the desired decision methods have been specified.

For the present problem, we require only one method control card with "1" keypunched into column 1. The "1" is a code specifying the Laplace equality method. Code "2" is for the Wald maximin method. Code "3" is the Hurwicz optimism method, and code "4" is the Savage regret method. Code "9" completes the method cards for the data set and allows additional complete data sets to be read in.



Termination card contains STOP in columns 1–4.

The complete input data deck appears in Figure 8-1, each line representing one keypunched data card.

FIGURE 8-1

COMPUTER INPUT—TEST PROBLEM

```
YOUR NAME TEST PROBLEM
5    4
18.  18.  10.  14.
14.  14.  14.  14.
5.   26.  10.  10.
14.  22.  10.  10.
10.  12.  12.  10.
1
9
STOP
```

Computer output—test problem The top portion of Figure 8-2 is a printout of the input data for program UNCERT. The lower or output portion contains the method control card and the analysis results. The printout of the method control card shows a "1" in column 1, which is the request for the Laplace type of decision analysis. The row-by-row results of the Laplace analysis appear. These are the same answers as derived by hand in Equation 8.4.

FIGURE 8-2 COMPUTER OUTPUT—TEST PROBLEM

```
PROGRAM UNCERT FOR YOUR NAME TEST PROBLEM

******INPUT DATA IS*******
(5 X 4) UNCERTAINTY MATRIX
    18      16      10      14
    14      14      14      14
     5      26      10      10
    14      22      10      10
    10      12      12      10


****METHOD CONTROL CARD****
  (1     0          0))

******LAPLACE METHOD******
***     PROFIT VALUES        ***
ROW  1    VALUE =         15.00
ROW  2    VALUE =         14.00
ROW  3    VALUE =         12.75
ROW  4    VALUE =         14.00
ROW  5    VALUE =         11.00

******BEST DECISION*******
***     ALTERNATIVE  1     ***
***     VALUE =      15    ***
***************************

******WORST DECISION******
***     ALTERNATIVE  5     ***
***     VALUE =      11    ***
***************************
```

Finally, program UNCERT selects the best and worst decisions by the Laplace method and prints the expected value of these decisions. Since profit is desired in this problem, alternative 1, with an expected value of 15, is the preferred decision.

The determination of the best and worse alternatives using the other decision methods is left to the user of UNCERT.

CURRENCY EXCHANGE PROBLEM

The currency exchange problem adapted from Miller and Starr (8), can be stated as follows:

You are working as a consultant in a foreign country where the exchange rate is 100 units of the foreign currency for $1. You will be there for five months and your expenses are fixed at 50,000 units of the foreign currency per month. A sky-rocketing inflation is temporarily in check while the government is attempting to negotiate a large loan. If the government gets the loan, the effect will be to lower the exchange rate by 10 percent. If the loan is refused, the exchange rate will increase by 20 percent. In addition, a general strike has been called. If this strike is successful, the government will be forced to take some economic measures that will increase the exchange rate by 15 percent. If the *strike fails, the rate will decrease by 10 percent. It can be assumed that the two events in equation (loan and strike) are independent.*

The first step in the solution of the problem is to formulate the outcome matrix. At least four states of nature are indicated.

S_1: OK loan, strike fails, rate drops to 81.0

S_2: OK loan, strike succeeds, rate increase to 103.5.

S_3: No loan, strike fails, rate increase to 108.0.

S_4: No loan, strike succeeds, rate increase to 138.0.

Although the analyst might also want to consider the possibilities of a delay in both the loan and the strike or holding at the status quo, these states are ignored in the present analysis.

The following strategies illustrate some of the alternatives available to the analyst.

A_1: Convert enough dollars, immediately, to meet all five months' expenses.

A_2: Wait until one of the four states has occurred, meanwhile holding dollars.

A_3: Hedge by converting half the amount now and holding half in dollars.

The outcome matrix can now be formulated. The outcome value for each item in the matrix is the cost in dollars if that alternative is chosen and that state of nature occurs. Equation 8.5, the outcome matrix, will serve as the basis for evaluating the best decision under each of the decision methods.

$$
\begin{array}{c}
\\ A_1 \\ A_2 \\ A_3
\end{array}
\begin{array}{c}
\begin{array}{cccc} S_1 & S_2 & S_3 & S_4 \end{array} \\
\begin{bmatrix}
2500 & 2500 & 2500 & 2500 \\
3086 & 2415 & 2315 & 1812 \\
2793 & 2458 & 2407 & 2156
\end{bmatrix}
\end{array}
\qquad (8.5)
$$

Outcome matrix—currency exchange problem

Computer input—currency exchange problem

The sequence of input data cards for UNCERT, by now familiar, is as follows:

User name card (one card).

Matrix control card (one card).

Matrix data cards (one per alternative).

Method control card (one per method desired).

Examining the input data deck for the currency exchange problem (Figure 8-3), note that there are four method control cards following the matrix data cards and preceding the "9" card. The code in column 1 of these four cards calls for the four decision methods to be run on the outcome matrix, one after the other. The "1" keypunched in column 6 of the method control cards indicates that the outcome matrix is for cost data. For maximization, this column contains a "0" or is left blank. Thus UNCERT is instructed to treat the outcome matrix as a cost matrix. The minimum cost, rather than the maximum profit, alternative is to be chosen. Finally, the .6 keypunched starting in column 11 of the code "3" (Hurwicz) method control card is the α value, to be chosen by the analyst. The α value can be varied in the same computer run to test the sensitivity of the preferred decision simply by adding code "3" cards with different αs. The "9" terminates the method control cards and the STOP card terminates all data input.

FIGURE 8-3

COMPUTER INPUT—CURRENCY EXCHANGE PROBLEM

```
CURRENCY EXCHANGE PROBLEM
3    4
2500,2500,2500,2500,
3086,2415,2315,1812,
2793,2458,2407,2156,
1    1
2    1
3    1    .6
4    1
9
STOP
```

Computer output—currency exchange problem

Pages one and two of the computer output for the currency exchange problem (Figure 8-4) contains the printout of the input data and the results for the Laplace and Wald methods, and the results for the Hurwicz and Savage decision methods, respectively.

FIGURE 8-4a

COMPUTER OUTPUT—CURRENCY EXCHANGE PROBLEM, PAGE ONE

```
PROGRAM UNCERT FOR CURRENCY EXCHANGE PROBLEM

******INPUT DATA IS*******
(3 X 4) UNCERTAINTY MATRIX
  2500  2500  2500  2500
  3086  2415  2315  1812
  2793  2458  2407  2156


 ***METHOD CONTROL CARD****
   ((1     1         0))

******LAPLACE METHOD******
***     COST VALUES      ***
ROW  1   VALUE =   2500.00
ROW  2   VALUE =   2407.00
ROW  3   VALUE =   2453.50

******BEST DECISION*******
***    ALTERNATIVE  2    ***
***    VALUE =   2407    ***
**************************

******WORST DECISION******
***    ALTERNATIVE  1    ***
***    VALUE =   2500    ***
**************************

 ***METHOD CONTROL CARD****
   ((2     1         0))
*******WALD METHOD*******
***     COST VALUES      ***
ROW  1   VALUE =   2500.00
ROW  2   VALUE =   3086.00
ROW  3   VALUE =   2793.00

******BEST DECISION*******
***    ALTERNATIVE  1    ***
***    VALUE =   2500    ***
**************************

******WORST DECISION******
***    ALTERNATIVE  2    ***
***    VALUE =   3086    ***
**************************
```

FIGURE 8-4b

COMPUTER OUTPUT—CURRENCY EXCHANGE PROBLEM, PAGE TWO

```
***METHOD CONTROL CARD*****
  ((3      1        .6000))
******HURWICZ METHOD*******
***    ALPHA =  .6000    ***
***     COST VALUES      ***
ROW  1    VALUE =   2500.00
ROW  2    VALUE =   2321.60
ROW  3    VALUE =   2410.00

******BEST DECISION*******
***   ALTERNATIVE  2   ***
***   VALUE =   2322   ***
**************************

******WORST DECISION******
***   ALTERNATIVE  1   ***
***   VALUE =   2500   ***
**************************

***METHOD CONTROL CARD*****
  ((4      1         0))
******SAVAGE METHOD*******
******REGRET MATRIX*******
     0     85    185    600
   586      0      0      0
   243     43     92    344
***     COST VALUES     ***
ROW  1    VALUE =    688.00
ROW  2    VALUE =    586.00
ROW  3    VALUE =    344.00

******BEST DECISION*******
***   ALTERNATIVE  3   ***
***   VALUE =    344   ***
**************************

******WORST DECISION******
***   ALTERNATIVE  1   ***
***   VALUE =    688   ***
**************************
```

Clearly, the suggested choice in this problem depends on the method. The method, in turn, depends on the needs, desires, and feelings of the decision maker.

TABLE 8-1

SUMMARY OF RESULTS CURRENCY EXCHANGE PROBLEM

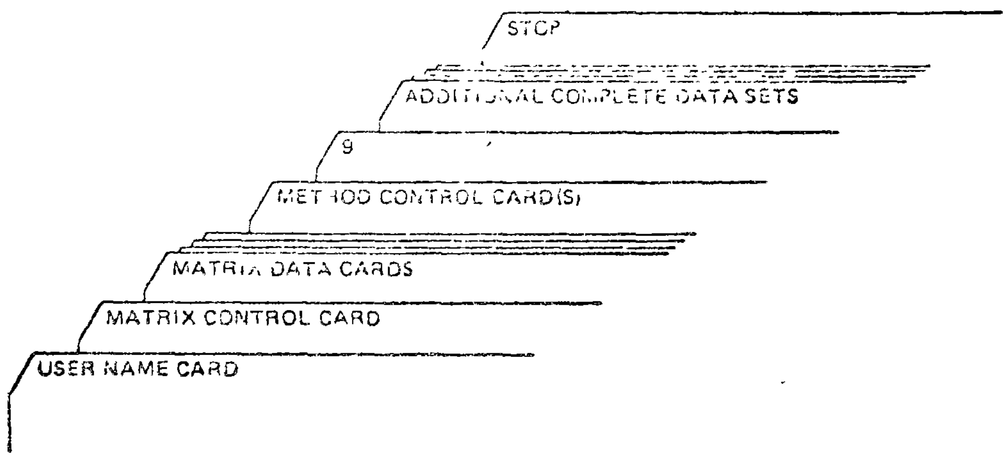
Decision		Method: Laplace	Wald	Hurwicz	Savage
A_1:	Convert now	Worst	Best	Worst	Worst
A_2:	Wait and see	Best	Worst	Best	—
A_3:	Hedge	—	—	—	Best

From Table 8-1, which summarizes his choices, we realize that the pessimism of the Wald minimax method is in clear conflict here, with the advice of the other three methods. The Laplace equality and the Hurwicz optimism methods agree that it is best to wait and see. The Savage regret-avoider prefers to hedge a bit rather than commit himself entirely to the wait-and-see posture. The resolution of these conflicting positions falls, of course, directly back on the decision maker.

References

1. Bierman, H., Jr., et al., *Quantitative Analysis for Business Decisions.* Homewood, Ill.: Irwin, 1965.
2. Borch, Karl, *The Economics of Uncertainty.* Princeton, N.J.: Princeton University Press, 1968.
3. Borch, Karl, and Jan Mossin, *Risk and Uncertainty.* New York: Macmillan, 1968.
4. Chernoff, H., and L. Moses, *Elementary Decision Theory.* New York: Wiley, 1959.
5. Grawoig, Dennis E., *Decision Mathematics.* New York: McGraw-Hill, 1967.
6. King, William R., *Quantitative Analysis for Marketing Management.* New York: McGraw-Hill, 1967.
7. Luce, R. Duncan, and Howard Raiffa, *Games and Decisions.* New York: Wiley, 1957.
8. Miller, David W., and Martin K. Starr, *Executive Decisions and Operations Research,* 2nd ed. Englewood Cliffs, N.J.: Prentice-Hall, 1969.
9. Morris, William T., *Engineering Economy.* Homewood, Ill.: Irwin, 1960.
10. Morris, William T., *The Analysis of Management Decisions.* Homewood, Ill.: Irwin, 1964.
11. Riggs, James L., *Economic Decision Models.* New York: McGraw-Hill, 1968.
12. Schlaifer, R., *Analysis of Decisions Under Uncertainty.* New York: McGraw-Hill, 1969.
13. Williams, J. D., *The Compleat Strategyst,* rev. ed. New York: McGraw-Hill, 1966.

UNCERT DATA STRUCTURE



USER NAME CARD

Item	Column	Format	Explanation
NAME	1-40	10A4	User identification

MATRIX CONTROL CARD

Item	Column	Format	Explanation
NROW	1	I1	Number of rows in outcome matrix
NCOL	6	I1	Number of columns in outcome matrix

MATRIX DATA CARDS

Item	Column	Format	Explanation
OM(I,J)	1-45	9F5.0	Values on one row of the outcome matrix. One to nine values as specified by NCOL. One to nine cards as specified by NROW.

METHOD CONTROL CARD

Item	Column	Format	Explanation
NMET	1	I1	Code for method control 0 = Stop the program, error 1 = Laplace–Equality 2 = Wald–Maximin 3 = Hurwicz–Optimism 4 = Savage–Regret 5-9 = Return for another complete set of data
NPC	6	I1	Control use of values in OM(I,J) 0 = Values treated as profit 1-9 = Values treated as cost
ALPHA	11-15	F5.4	Alpha value for Hurwicz method (NMET = 3). Must be equal to or less than one.

```
      PROGRAM UNCERT
C     ROY D HARRIS AUGUST 1971
C     THIS VERSION FOR CDC 3100
C     THIS PROGRAM COMPUTES FOUR METHODS
C     FOR DECISION ANALYSIS UNDER UNCERTAINTY
C     DICTIONARY OF VARIABLES
C     NROW = NUMBER OF ROWS IN OUTCOME MATRIX
C     NCOL = NUMBER OF COLUMNS IN OUTCOME MATRIX
C     OM(I,J) = OUTCOME MATRIX
C     NMET = CODE FOR DECISION METHOD
C          1 = LAPLACE-EQUALITY
C          2 = WALD-MAXIMIN
C          3 = HURWICZ-OPTIMISM
C          4 = SAVAGE-REGRET
C     NPC = CODE FOR PROFIT OR COST IN OM(I,J)
C          0 = PROFIT DATA IN OM(I,J)
C          1-9 = COST DATA IN OM(I,J)
C     ALPHA = ALPHA VALUE FOR HURWICZ METHOD
C     RM(I,J) = REGRET MATRIX FOR SAVAGE METHOD
C     AB(I) = VECTOR OF LARGEST ROW VALUES
C     AS(I)
C     BN = BIGGEST NUMBER IN SOME SET
C     SN = SMALLEST NUMBER IN SOME SET
C     IBN,ISN = ROW LOCATION OF BN AND SN
C
      COMMON NIN,NOUT,OM(9,9),RM(9,9),AB(9),AS(9)
      COMMON NROW,NCOL,NMET,NPC,ALPHA,BN,SN,IBN,ISN
      DIMENSION NAME(10)
      COMMON /DATA/ IEND
      DATA (IEND=4HSTOP)
      ISTOP=IEND
      NIN=60
      NOUT=61
C
C     READ AND WRITE USER NAME CARD
1     READ (NIN,13) NAME
      WRITE (NOUT,14) NAME
      IF (NAME(1).EQ.ISTOP) GO TO 11
C
C     READ AND  RITE MATRIX CONTROL CARD
      READ (NIN,15) NROW,NCOL
      WRITE (NOUT,16)
      WRITE (NOUT,17) NROW,NCOL
      IF (NROW.EQ.0) GO TO 8
      IF (NCOL.EQ.0) GO TO 8
C
C     READ AND PRINT OUTCOME MATRIX DATA CARDS
      DO 2 I=1,NROW
      READ (NIN,18) (OM(I,J),J=1,NCOL)
2     WRITE (NOUT,19) (OM(I,J),J=1,NCOL)
C
C     READ AND PRINT METHOD CONTROL CARD
3     READ (NIN,20) NMET,NPC,ALPHA
      WRITE (NOUT,12)
      WRITE (NOUT,21)
      WRITE (NOUT,22) NMET,NPC,ALPHA
      IF (NMET.GT.4) GO TO 1
      IF (NMET.LT.1) GO TO 10
      IF (ALPHA.GT.1.0) GO TO 9
C
C     CALL SUBROUTINE FOR DECISION METHOD
      GO TO (4,5,6,7), NMET
4     CALL ALAP
      CALL OUT
      GO TO 3
```

```
5     CALL AMAX
      CALL OUT
      GO TO 3
6     CALL AMIN
      CALL OUT
      GO TO 3
7     CALL ASAV
      CALL OUT
      GO TO 3
C
C     BRANCH TO HERE WHEN ERRORS IN INPUT DATA
8     WRITE (NOUT,23)
      GO TO 11
9     WRITE (NOUT,24)
      GO TO 11
10    WRITE (NOUT,25) NMET
11    WRITE (NOUT,26)
C
C
12    FORMAT (1H0)
13    FORMAT (1 A4)
14    FORMAT (2 H1PROGRAM UNCERT FOR ,10A4)
15    FORMAT (I1,4X,I1)
16    FORMAT (27H0******INPUT DATA IS******)
17    FORMAT (2H (,I1,3H X ,I1,20H) UNCERTAINTY MATRIX)
18    FORMAT (9F5.0)
19    FORMAT (1X,9F6.0)
20    FORMAT (I1,4X,I1,4X,F5.4)
21    FORMAT (28H0 ***METHOD CONTROL CARD****)
22    FORMAT (6H     ((,I1,6X,I1,4X,F5.4,2H)))
23    FORMAT (27H0NO VALUE FOR ROW OR COLUMN)
24    FORMAT (22H0ALPHA VALUE INCORRECT)
25    FORMAT (17H0WRONG CODE VALUE,1X,I1)
26    FORMAT (26H0PROGRAM UNCERT TERMINATED)
      END

      SUBROUTINE ALAP
C     THIS SUBROUTINE COMPUTES VALUE OF EACH
C     ALTERNATIVE BY LAPLACE-EQUALITY METHOD
      COMMON NIN, NOUT, OM(9, 9), RM(9, 9), AB(9), AS(9)
      COMMON NROW, NCOL, NMET, NPC, ALPHA, BN, SN, IBN, ISN
      WRITE (NOUT,7)
C
C     FIND VALUE OF EACH ROW IN THE MATRIX
      DO 2 I = 1, NROW
      SUM = 0.0
      DO 1 J = 1, NCOL
      A = NCOL
1     SUM = SUM+OM(I, J)/A
      AS(I) = SUM
2     AB(I) = SUM
C
C     PRINT OUT ROW VALUES
      IF (NPC .NE. 0) GO TO 4
      WRITE (NOUT,8)
      DO 3 I = 1, NROW
3     WRITE (NOUT,10) I, AB(I)
      GO TO 6
4     WRITE (NOUT,9)
      DO 5 I = 1, NROW
5     WRITE (NOUT,10) I, AS(I)
6     CONTINUE
C
7     FORMAT  (27H0******LAPLACE METHOD******)
8     FORMAT  (27H ***     PROFIT VALUES     ***)
9     FORMAT  (27H ***      COST VALUES      ***)
```

```
 10   FORMAT  (6H ROW  ,I1,10H   VALUE =,F10.2)
      RETURN
      END

      SUBROUTINE AMAX
C     THIS SUBROUTINE FINDS SMALLEST OR LARGEST VALUE
C     OF EACH ROW BY WALD MAXIMIN METHOD
      COMMON NIN, NOUT, OM(9, 9), RM(9, 9), AB(9), AS(9)
      COMMON NROW, NCOL, NMET, NPC, ALPHA, BN, SN, IBN, ISN
      WRITE (NOUT,8)
C
C     FIND THE LARGEST AND SMALLEST VALUE ON EACH ROW
      DO 3 I = 1, NROW
      BN = OM(I, 1)
      SN = OM(I, 1)
      DO 2 J = 1, NCOL
      IF (OM(I, J) .LT. BN) GO TO 1
      BN = OM(I, J)
 1    IF (OM(I, J) .GT. SN) GO TO 2
      SN = OM(I, J)
 2    AB(I) = SN
      AS(I) = BN
 3    CONTINUE
C
C     PRINT OUT ROW VALUES FOR PROFIT OR COST
      IF (NPC .NE. 0) GO TO 5
      WRITE (NOUT,9)
      DO 4 I = 1, NROW
 4    WRITE (NOUT,11) I, AB(I)
      GO TO 7
 5    WRITE (NOUT,10)
      DO 6 I = 1, NROW
 6    WRITE (NOUT,11) I, AS(I)
 7    CONTINUE
C
 8    FORMAT  (27H *******WALD METHOD********)
 9    FORMAT  (27H ***    PROFIT VALUES     ***)
 10   FORMAT  (27H ***     COST VALUES      ***)
 11   FORMAT  (6H ROW  ,I1,10H   VALUE =,F10.2)
      RETURN
      END

      SUBROUTINE AHUR
C     THIS SUBROUTINE COMPUTES THE VALUE OF EACH
C     ALTERNATIVE BY THE METHOD OF HURWICZ
      COMMON NIN, NOUT, OM(9, 9), RM(9, 9), AB(9), AS(9)
      COMMON NROW, NCOL, NMET, NPC, ALPHA, BN, SN, IBN, ISN
      WRITE (NOUT,8)
      WRITE (NOUT,9) ALPHA
C
C     FIND LARGEST AND SMALLEST VALUE ON EACH ROW
      DO 3 I = 1, NROW
      BN = OM(I, 1)
      SN = OM(I, 1)
      DO 2 J = 1, NCOL
      IF (OM(I, J) .LT. BN) GO TO 1
      BN = OM(I, J)
 1    IF (OM(I, J) .GT. SN) GO TO 2
      SN = OM(I, J)
 2    CONTINUE
C     AB(I) CONTAINS HURWICZ VALUE  FOR PROFIT MATRIX
C     AS(I) CONTAINS HURWICZ VALUES FOR COST MATRIX
      AB(I) = (ALPHA*BN)+((1.0-ALPHA)*SN)
 3    AS(I) = ((1.0-ALPHA)*BN)+(ALPHA*SN)
C
C     PRINT OUT ROW VALUES FOR PROFIT OR COST
```

```
      IF (NPC .NE. 0) GO TO 5
      WRITE (NOUT,11)
      DO 4 I = 1, NCOL
4     WRITE (NOUT,12) I, AS(I)
      GO TO 7
5     WRITE (NOUT,10)
      DO 6 I = 1, NROW
6     WRITE (NOUT,12) I, AS(I)
7     CONTINUE
C
8     FORMAT  (27H ******HURWICZ METHOD******)
9     FORMAT  (15H ***   ALPHA = ,F6.4,3H   ***)
10    FORMAT  (27H ***   PROFIT VALUES    ***)
11    FORMAT  (27H ***    COST VALUES     ***)
12    FORMAT  (6H ROW  ,I1,10H    VALUE =,F10.2)
      RETURN
      END

      SUBROUTINE ASAV
C     THIS SUBROUTINE COMPUTES THE VALUE OF EACH
C     ALTERNATIVE BY THE SAVAGE REGRET METHOD
      COMMON NIN, NOUT, OM(9, 9), RM(9, 9), AB(9), AS(9)
      COMMON NROW, NCOL, NMET, NPC, ALPHA, BN, SN, IBN, ISN
      WRITE (NOUT,19)
      IF (NPC .NE. 0) GO TO 5
C
C     COMPUTE THE REGRET MATRIX ASSUMING PROFIT
      DO 2 J = 1, NCOL
      BN = OM(1, J)
      DO 1 I = 1, NROW
      IF (OM(I, J) .LT. BN) GO TO 1
      BN = OM(I, J)
1     CONTINUE
2     AB(J) = BN
      DO 3 J = 1, NCOL
      DO 3 I = 1, NROW
3     RM(I, J) = ABS(AB(J)-OM(I, J))
C
C     PRINT OUT THE REGRET MATRIX
      WRITE (NOUT,16)
      DO 4 I = 1, NROW
4     WRITE (NOUT,17) (RM(I, J), J = 1, NCOL)
      WRITE (NOUT,15)
      GO TO 10
C
C     COMPUTE THE REGRET MATRIX ASSUMING COST
5     DO 7 J = 1, NCOL
      SN = OM(1, J)
      DO 6 I = 1, NROW
      IF (OM(I, J) .GT. SN) GO TO 6
      SN = OM(I, J)
6     CONTINUE
7     AS(J) = SN
      DO 8 J = 1, NCOL
      DO 8 I = 1, NROW
8     RM(I, J) = ABS(AS(J)-OM(I, J))
C
C     PRINT OUT THE REGRET MATRIX
      WRITE (NOUT,16)
      DO 9 I = 1, NROW
9     WRITE (NOUT,17) (RM(I, J), J = 1, NCOL)
      WRITE (NOUT,18)
C
C     FIND THE MOST REGRET ON EACH ROW
10    DO 12 I = 1, NROW
      BN = RM(I, 1)
```

```
      DO 11 J = 1, NCOL
      IF (RM(I, J) .LT. BN) GO TO 11
      BN = RM(I, J)
 11   AS(I) = BN
 12   CONTINUE
C
C     PRINT OUT LARGEST ROW VALUES
      DO 13 I = 1, NROW
 13   WRITE (NOUT,19) I, AS(I)
C
 14   FORMAT  (27H ******SAVAGE METHOD*******)
 15   FORMAT  (27H ***    PROFIT VALUES     ***)
 16   FORMAT  (27H *****REGRET MATRIX*******)
 17   FORMAT  (1X,9F6.0)
 18   FORMAT  (27H ***     COST VALUES      ***)
 19   FORMAT  (6H ROW  ,I1,10H   VALUE =,F10.2)
      RETURN
      END

      SUBROUTINE OUT
C     THIS SUBROUTINE COMPUTES BEST DECISION AND PRINTS IT
      COMMON NIN, NOUT, OM(9, 9), RM(9, 9), AB(9), AS(9)
      COMMON NROW, NCOL, NMET, NPC, ALPHA, BN, SN, IBN, ISN
C
C     SWITCH TO CORRECT DECIDE SECTION
      IF (NMET .EQ. 4) GO TO 3
      IF (NPC .NE. 0) GO TO 3
C
C     THIS SECTION COMPUTES BEST DECISION ASSUMING PROFIT
C     (EXCEPT  FOR SAVAGE REGRET METHOD)
      IBN = 1
      ISN = 1
      BN = AB(1)
      SN = AB(1)
      DO 2 I = 1, NROW
      IF (AB(I) .LT. BN) GO TO 1
      BN = AB(I)
      IBN = I
 1    IF (AB(I) .GT. SN) GO TO 2
      SN = AB(I)
      ISN = I
 2    CONTINUE
      WRITE (NOUT,7)
      WRITE (NOUT,8) IBN
      WRITE (NOUT,10) BN
      WRITE (NOUT,11)
      WRITE (NOUT,9)
      WRITE (NOUT,8) ISN
      WRITE (NOUT,10) SN
      WRITE (NOUT,11)
      GO TO 6
C
C     THIS SECTION COMPUTES BEST DECISION ASSUMING COST
C     AND FOR SAVAGE REGRET METHOD PROFIT OR COST
 3    IBN = 1
      ISN = 1
      BN = AS(1)
      SN = AS(1)
      DO 5 I = 1, NROW
      IF (AS(I) .LT. BN) GO TO 4
      BN = AS(I)
      IBN = I
 4    IF (AS(I) .GT. SN) GO TO 5
      SN = AS(I)
      ISN = I
 5    CONTINUE
```

```
      WRITE (NOUT,7)
      WRITE (NOUT,8) ISN
      WRITE (NOUT,10) SN
      WRITE (NOUT,11)
      WRITE (NOUT,9)
      WRITE (NOUT,8) IDN
      WRITE (NOUT,1O) DN
      WRITE (NOUT,11)
6     CONTINUE
C
7     FORMAT  (27H0*******BEST DECISION*******)
8     FORMAT  (20H ***    ALTERNATIVE  ,I1,6H    ***)
9     FORMAT  (27H0******WORST DECISION******)
10    FORMAT  (14H ***    VALUE =,F7.0,6H    ***)
11    FORMAT  (27H **************************)
      RETURN
      ENU
```

centro de educación continua
división de estudios superiores
facultad de ingeniería, unam

MODELADO Y SIMULACION APLICADOS A LA PLANEACION

SIMULACION DE SISTEMAS DE INVENTARIOS

M. en C. MARCIAL PORTILLA R.

Palacio de Minería
Tacuba 5, primer piso. México 1, D. F.
Tels.: 521-40-23 521-73-35 5123-123

What is simulation?

by D. M. Bishop

By experimenting with models valuable insight into the behaviour of real systems can be obtained

For a long time it has been usual in the aircraft and shipbuilding industries to use scale models when designing new planes or ships. These have been used in wind-tunnels and water tanks respectively in order to discover how the full-scale model will behave when it is built, and to confirm those parts of the design which might be in doubt because of the errors inherent in the engineering design methods. The models were made as simple as possible consistent with getting reliable answers to specific questions, ie, they basically represented the external structure that had to pass through and interact with the air or water, and no detail which did not affect their motion was included.

These models are still used, but to some extent have been superseded by 'mathematical models'. This possibility has arisen because it is now often possible to solve the mathematical equations which describe the behaviour of the system with the help of a computer.

Whether we use a physical or a mathematical model, we are using a device (real or abstract) to represent the real system and to simulate its behaviour in some respect – hence the term 'simulation'.

Simulation is the process of using a model to obtain information about a real system which exists or is being designed. By experimenting on the model, valuable insight into the behaviour of the real system can be obtained. The benefits include improved design and performance without the expense, difficulties, and possibly hazards of experimenting on the real system.

SCALE MODELS

A scale model is a version of the real system which has been made larger or, more usual, smaller than the original, but behaves in all important respects in such a way that the relevant behaviour of the real system can be readily deduced. As already mentioned, this sort of model is widely used in the aerospace or shipbuilding industries, in the chemical industry pilot plants fall into this category. Scale models have the advantage that the qualitative equivalence to the real system is obvious. Quantitative relationships are more difficult to establish, and great care is required to ensure that the correct scaling factors are used, however, if one gets this right, results will be good, reliable and credible. A disadvantage is that the making and testing of the models can be expensive and time consuming. They are also of only limited application often to relatively simple situations.

MATHEMATICAL MODELS

In order to determine the behaviour of the system, it is described by a set of mathematical equations or statements, and these are solved to give the required answers.

For example, the length of a linear spring (see Fig 1) is given by the simple equation

$$T = kx \qquad \ldots(1)$$

where T represents the tension, x the extension of the spring, and k is a constant of the spring. If it is supported at the top and has a mass M suspended from the bottom, this can be rewritten

$$Mg = kx \qquad \ldots(2)$$

This implies that no motion is occurring and that the spring itself has no mass. If what we are interested in is how the mass moves when the system is disturbed and then left to sort itself out, we end up with a differential equation instead of an algebraic one, and this, together with a statement about its position and velocity immediately after it has been disturbed, will tell us how the mass moves from then on.

$$\frac{Md^2x}{dt^2} = Mg - kx \qquad \ldots(3)$$

$$x = x_0 \quad \text{at} \quad t = 0 \qquad \ldots(4)$$

$$\frac{dx}{dt} = x_0' \quad \text{at} \quad t = 0 \qquad \ldots(5)$$

This again is a considerable simplification as it implies that the mass is constrained to move vertically. It would be possible to write other equations which allowed the spring to have its own mass and to be non-linear, and allowed the mass to swing like a pendulum at the same time as moving up and down. It can be seen that equation (3) can be easily solved analytically, and that the position, velocity and acceleration of the mass can be deduced for all times. Solution of the more comprehensive equations is clearly more difficult, and at best tedious. If the system described above were extended to represent the suspension of a car as it proceeded along a rough road, the difficulties would be further exaggerated, and it would be only by using a computer as an aid that answers could be found.

The easiest way to solve multiple mixed algebraic and differential equations on a digital computer is to use a 'program package' specifically designed for the purpose – a continuous system simulation language (CSSL). The digital computer translates this into the correct sequence of operations, this being necessary because it works sequentially performing one calculation at a time, whereas the equations which must be solved describe the real world when many things happen simultaneously. A CSSL program looks very like the original equations, so that it is easily written and read by someone who understands mathematics. These languages recognise the concept of simultaneity by allowing the equations to be written in any order.

This technique has the following advantages.

1. Easily and quickly programmed.
2. Easily modified.
3. Programs can be stored for re-running for additional experiments and for development.

The disadvantages are:

1. It can take a long time to solve simultaneous equations. If many experiments have to be done this can be costly in computer time.
2. The results cannot easily (or cheaply) be displayed in a way which gives insight into the model and so of the system being simulated.

Mr Bishop is with the Control Development Group, Petrochemicals Division, Imperial Chemical Industries Limited.

Originally published in the July–August 1973 issue of *Data Processing*.

[illegible] phenomena [illegible] similar mathematical [illegible] by representing the system [illegible] system on which [illegible] the experiments are [illegible] can be more readily [illegible]. Generally the analogous system is electrical, and the model is described as an analogue simulation

If we look at a simple electrical system as an example; the equations which describe this are:

$$C\frac{d^2V}{dt^2} = \frac{E}{L} - \frac{V}{L} \qquad \ldots(6)$$

$$V = V_0 \quad \text{at} \quad t = 0 \qquad \ldots(7)$$

$$\frac{dV}{dt} = V_0' \quad \text{at} \quad t = 0 \qquad \ldots(8)$$

It will be seen that these are identical in form to (3), (4), (5) and that it is therefore possible to infer the behaviour of one system from measurements made on the other and the electrical system can be regarded as a model of the mechanical system

Now if we have a much more complex system, it becomes difficult and expensive to assemble the necessary electrical components from scratch, and modifications and experimentation become complicated. To overcome this difficulty, an analogue computer is used. This is a general-purpose computer

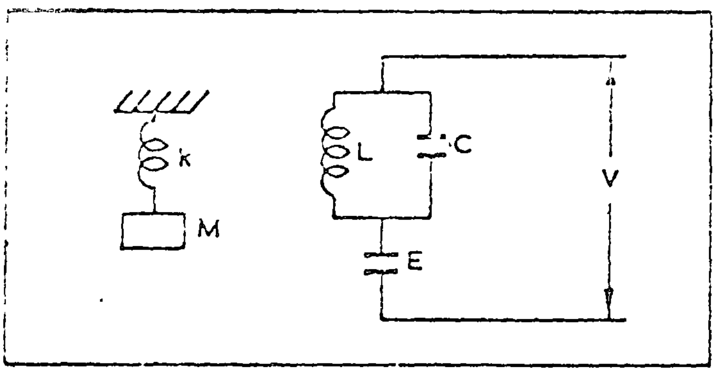


Fig 1 Fig 2

which provides all the necessary components, has a simple arrangement for connecting them together, organises the initial conditions for the running of the model, and provides the switches and variable components needed for the performing of experiments

The components are not as simple as the capacitor and inductor of Fig 2, but are easier to use as it is no longer necessary to think about them as electrical devices. They are units which perform mathematical operations such as summation, integration, multiplication and the generation of non-linear functions. The model is thus built up directly from the mathematical equations describing the system.

The behaviour of any part of the system can be observed and recorded using standard oscilloscopes or chart recorders, and so is presented to the user as it is happening.

The advantages of this technique are

1. The simultaneous real world is modelled by a simultaneous analogy, and the model may be operated faster or slower than the real system so that its behaviour can be assimilated by the operator
2. Experimental results can be displayed in a way which is immediately meaningful to the operator.
3. The ease of experimentation makes for rapid insight into the way in which the system works

The disadvantages are

1. Programming, ie, designing the interconnection scheme, is relatively slow

[illegible] [illegible] [illegible] by the [illegible] in the computer

HYBRID COMPUTER SIMULATION

Where any of the disadvantages of the digital or analogue computer make it difficult, or impossible, or impossibly expensive, to simulate a particular system, a hybrid computer may be capable of dealing with the situation very effectively. A hybrid computer consists of a general-purpose analogue computer and a general-purpose digital computer linked intimately in such a way that they form a single integrated computer system

For example, the long time taken by a digital computer to solve simultaneous equations, or to integrate some differential equations, can be avoided by performing these operations on the analogue computer. Those operations which require high accuracy may be performed digitally. The digital computer makes it possible to store programs, initial conditions, etc, this greatly simplifies the re-running of the problem at any time even if most of it is performed on the analogue part. The results of a purely digital simulation can be displayed conveniently via the analogue part.

WHY SIMULATE?

Simulation may be valuable in the following conditions

a. The real system has not yet been made, and it is necessary to find out in advance how it will behave. The objective here is to ensure that the proposed design will work properly and/or safely, or to suggest modifications which are necessary to achieve this objective. The value of simulation is directly related to the cost of complete or partial failure of the design, and can be very high in the chemical and oil industry, in aerospace, and in many others which may or may not as yet be familiar with the available techniques, for example, civil engineering
b. Experiments on the real system are too dangerous. This may apply to some chemical plants which have an explosion hazard, and where the objective of the experiment is, for example, to determine whether explosive conditions are reached
c. Experiments on the real system are too expensive. Testing systems to destruction, upsetting large chemical plants for experimental purposes, are two examples.
d. Experiments on the real system are impossible or inconvenient. It may be impossible to make essential measurements, or there may be excessive delay in installing the measuring system. The real system may be subject to multiple interacting disturbances which make it difficult to sort out the required effects.
e. Experiments on the real system might take too long. Where the time-scale of the system is days, weeks or years, experimentation on a model offers very real advantages in that many tests can be done in a short time. Examples of such systems are business and national economics

WHAT QUESTIONS CAN BE ANSWERED BY SIMULATION?

Some questions which can be answered from a simulation are:

a. Will a proposed control system (manual or automatic) work as intended? What are the relative merits of different control systems? Can the best system cope with the known disturbances, or should steps be taken to limit the disturbances in some way?
b. Will a plant be tripped before a hazardous situation is reached? Will a trip cause secondary problems which could be avoided?
c. Will the directional control system for a vehicle be stable and effective under all conditions?
d. What will be the effect of random breakdown [illegible]

e How does the system really work? Which model best describes observed behaviour? Does the current theory agree with experiment?
f How will a complex electrical/hydraulic/gaseous network behave in abnormal conditions?
g What will be the effect of continuing trends in price structure or of fiscal policy changes on the economic viability of a firm?
h What happens to the suspension of a vehicle going over very rough ground, what stresses are met, how much will the driver/passengers be shaken up? How effective is the damping?

HOW TO SET ABOUT IT

The following steps give a simple indication of what has to be done:

a Determine what questions must be answered.
b Decide whether simulation is the best approach.
c Develop the mathematical model and gather essential data.
d Translate model into computer programs.
e Decide whether the simulation agrees with the mathematics. If it does not, go back to d.
f Decide whether the simulation agrees with known facts about the real system. If it does not, go back to c.
g. Experiment on simulation.

INSYS

an inventory system model

computes inventory levels and factory output for a factory-wholesaler-retailer inventory system. Inventory replenishment and lead time policies may be changed in order to test the effect of these policies on the performance of the overall system.

The purpose of an inventory is to provide a separation in time or location between the production of goods and the consumption of goods. In our specialized economy a man is no longer his own butcher, baker, and candlestick-maker. Rather, we have production centers (factories) which are specialized, centrally located, and have high production rates. There is a great gain in production efficiency from this specialization, but it also requires a large increase in inventories to separate the centralized factory from the ultimate consumer. No longer do we follow the example of the little red hen who planted, reaped, milled, baked, and ate (without the help of the pig, cow, rabbit, or duck) her own loaf of bread.

The most common inventory system in our economy is the factory-wholesaler-retailer system. The wholesaler provides a time decoupling service between the factory and the retailer, in that he holds the factory output until ordered by the retailer. The wholesaler also provides a location decoupling, in that he generally ships goods over a wide geographic area. Similarly, the retailer provides a decoupling service between the wholesaler and the consumer, in that he maintains an inventory of goods on display for sale to customers.

The purpose of this exercise is to provide an illustration of the dynamic nature of the factory-wholesaler-retailer inventory system. A computer model is used to calculate week by week how the retail inventory, the wholesale inventory, and the factory output rate change in response to retail sales. The model user may make changes in retail and wholesale inventory policy in an attempt to control the overall inventory system.

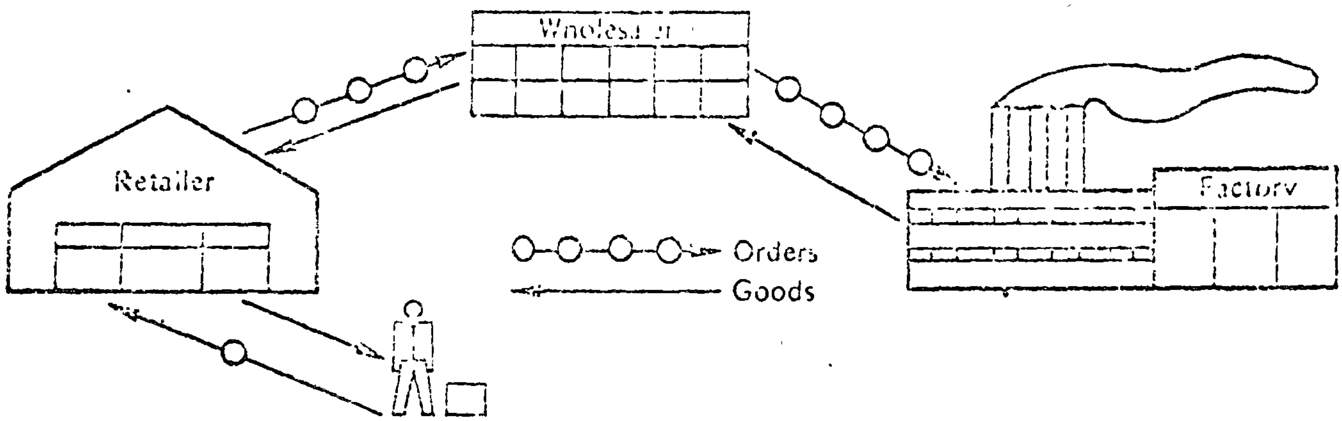


Figure 2-1 The factory wholesaler-retailer system.

Section 2.1 explains the normal inventory systems and the rules for maintaining inventory levels. The following sections present three illustrated computer problems for maintaining and controlling the inventory system.

2.1 FACTORY-WHOLESALER-RETAILER MODEL

The normal system for the production and distribution of goods in our economy is through the factory-wholesaler-retailer system. A visual conceptualization of this system is shown in Figure 2-1. The function of the retailer in this system is to

—take orders from customers
—deliver goods to customers from on-the-shelf inventory
—reorder goods from the wholesaler
—receive shipments from the wholesaler

The function of the wholesaler is similar to the retailer except that the wholesaler's customer is the retailer and there is a time lag between the ordering and the delivery of goods. The wholesaler must

—receive orders from the retailer
—ship goods from the warehouse inventory
—reorder goods from the factory
—receive shipments from the factory.

Finally, the factory must produce the goods which are ultimately sold to the customers. The factory may or may not hold inventories. In the current model the factory does not maintain any inventory so that its only functions are to

—produce goods at some rate
—change production rate as requested by wholesaler

The model just described is a simple abstraction of that which is found in the industrial system. Durable goods, such as appliances, more or less follow the system described. There are variations in that some large retailers order directly from the factory, or the factory may maintain a showroom and make direct retail sales. In other cases, the factory maintains large inventories and performs the function of the wholesaler. In all of these modifications, however, there is a dynamic interplay of sales with the inventories maintained and the factory rate as illustrated in this model.

Retailer Model Formulas

The parameters and formulas for the actual computer model of the retailer are presented in this section. These formulas are a mathematical statement of the verbal model description above. We also present some sample computations using the retail formulas.

Retail sales are controlled by the customer. They are part of the input to the program by the reader. Retail sales in the past have been about 100 units per week.

Retail receipts are the units received from the wholesaler each Monday morning that were ordered Friday one week (10 days) prior.

Retail inventory level is the number of units on hand Friday afternoon at the close of business. The inventory level varies through the week as shown in Figure 2-2. The formula for determining the inventory level is

Inventory level = prior inventory level + retail receipts − retail sales

Retail orders are placed with the wholesaler each Friday afternoon after determining the inventory level. The order policy is to order the retail sales for the week plus or minus enough units to return the base stock level to 100 units. Thus

Retail order = retail sales + (100 − inventory level)

The effects of these formulas on inventory level and the retail order can be seen in the following sample computation.

In a normal week

Retail sales = 100
Retail receipts = 100
Retail inventory level = 100 + 100 − 100
= 100
Retail order = 100 + (100 − 100)
= 100

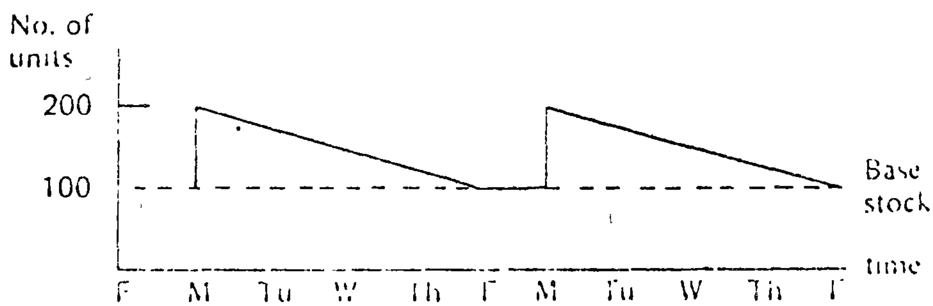


Figure 2-2 Retail inventory level.

In a week in which sales increase

Retail sales = 110
Retail receipts = 100
Retail inventory level = 100 + 100 − 110
= 90
Retail order = 110 + (100 − 90)
= 120

In a week in which sales decrease

Retail sales = 90
Retail receipts = 100
Retail inventory level = 100 + 100 − 90
= 110
Retail order = 90 + (100 − 110)
= 80

Wholesaler Model Formulas

The wholesaler's policies for maintaining inventory and reordering from the factory are similar to the retailer's policies. The formulas for the wholesaler and sample computations are now presented.

Wholesale shipments are dispatched each Wednesday from orders submitted by the retailer on the prior Friday. These orders arrive at the retailer's on the following Monday.

Wholesale shipments = retail order (prior week)

Wholesale receipts is the factory production of the previous week which is received each Monday morning.

Wholesale receipts = factory production (prior week)

Wholesale inventory level is the number of units on hand Friday afternoon at the close of business. The inventory level actually varies through the week as shown in Figure 2-3.

The formula for determining the Friday afternoon inventory level is as follows:

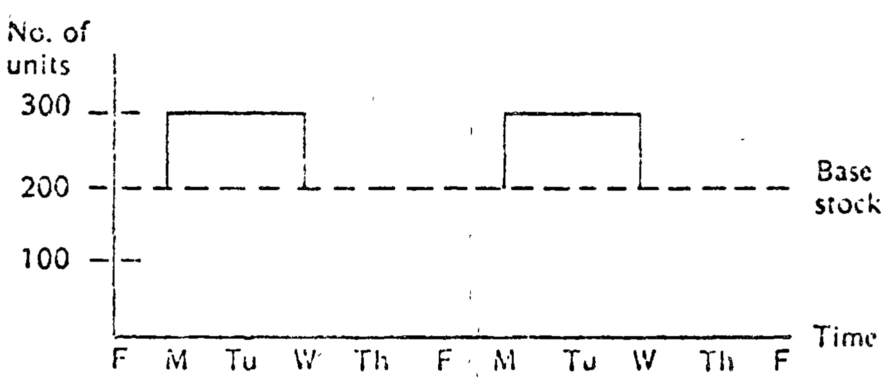


Figure 2-3 Wholesale inventory.

Wholesale inventory level = prior inventory level + wholesale receipts
− wholesale shipments

Wholesale orders are placed with the factory each Friday afternoon after taking inventory. The factory, however, requires a week to change the production rate, so two weeks pass before the wholesaler receives the actual order. The policy is to order the current week's shipments plus enough units to return the base stock to a normal level of 200 units. The formula is

Wholesale order = wholesale shipments
+ (200 − inventory level)

The effects of the wholesaler's policies can be seen in the following sample computation.

In a normal week:

Wholesale shipments = 100
Wholesale receipts = 100
Wholesale inventory level = 200 + 100 − 100
= 200
Wholesale order = 100 + (200 − 200)
= 100

In a week in which shipments increase

Wholesale shipments = 110
Wholesale receipts = 100
Wholesale inventory level = 200 + 100 − 110
= 190
Wholesale order = 110 + (200 − 190)
= 120

In a week in which shipments decrease

Wholesale shipments = 90
Wholesale receipts = 100
Wholesale inventory level = 200 + 100 − 90
= 210
Wholesale order = 90 + (200 − 210)
= 80

It should be noted that in the present simplified model, the wholesaler only services one retailer. This is an obvious oversimplification from the real world and allows the analysis to isolate the effect of a single retailer in the entire system.

Factory Production Rate

In this model, the factory maintains no inventory. The factory produces at the rate specified by the wholesale order. There is, however, a one-week delay when changing the production rate and a one-week delay for shipping. The net effect is that the wholesaler receives the actual order two weeks after it is placed with the factory. Thus, for example, one might have the situation shown in Figure 2-4.

Week number	Wholesale order	Factory rate	Wholesale receipts
1	100	100	100
2	120	100	100
3	80	100	100
4	100	120	100
5	100	80	120
6	100	100	80

Figure 2-4 Changing factory production rate.

2.2 NORMAL INVENTORY POLICY

This section presents the results of following a normal inventory policy. By "normal" we mean that the retailer and wholesaler follow the rules described in the preceding sections. The most significant rule, which will be analyzed in detail later, is the reorder rule. The normal reorder rule which is followed in the current problem is

Order the current week's sales plus or minus enough to bring the base stock back to its normal level.

Following this reorder rule and the other inventory policies outlined in Section 2.1, one can compute over a number of weeks the inventory level and orders in response to retail sales. For example, if retail sales are 100 in weeks 1 and 2, then increase to 110 in weeks 4, 5, 6, and 7, results will occur as shown in Figure 2-5.

These results are arrived at by following the computation formulas given in the preceding section. For example, the formula for the retail order each week is as follows:

Retail order = weekly sales + (100 − inventory level)

The retail order in week 5 is 120 units, derived from the above formula as follows:

Retail order = 110 + (100 − 90)
= 120

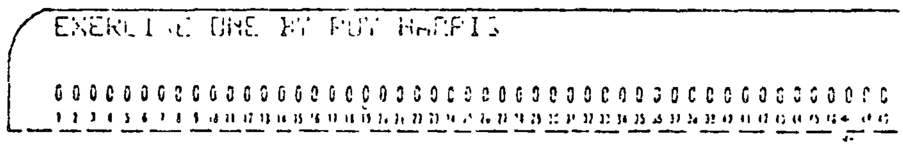
Week	Retail				Wholesale				Factory
No.	Sales	Rec	Inv	Order	Ship	Rec	Inv	Order	Rate
1	100	100	100	100	100	100	200	100	100
2	100	100	100	100	100	100	200	100	100
3	110	100	90	120	100	100	200	100	100
4	110	100	80	130	120	100	180	140	100
5	110	120	90	120	130	100	150	180	100
6	110	130	110	100	120	100	130	190	110
7	110	120	120	90	100	140	170	130	180

Figure 2-5 "Normal" inventory policy.

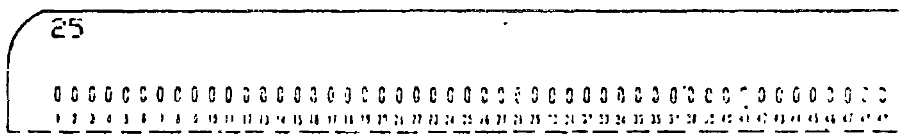
It is quite possible to compute the results by hand for as many weeks of operation as desired. Fortunately, the computer program will do the tedious calculations. We next consider briefly the data cards required to run the computer program and then we will analyze twenty-five weeks of operation by using the computer model.

Computer Input This section presents the data cards required as input to the computer program. The program listing is found in Section 2.6.

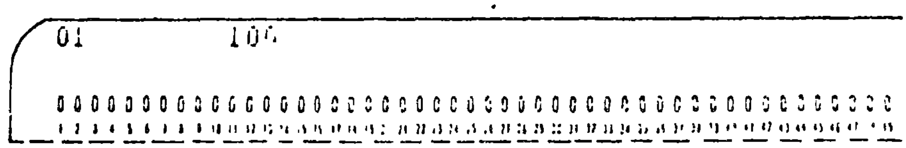
User name card is the first card in the data deck. It is used to identify who is making the analysis and any other identification desired. This card is ***free field***, in that any information may be keypunched in columns 1 to 40.



Control card is the second card in the data deck. This card contains the number of weeks to be analyzed, which is punched in columns 1 and 2.



Weekly sales cards contain the week number and the retail sales for that week. There is one card per week and the total number of cards must be exactly the number specified by the control card. The week number is keypunched in columns 1 and 2. The retail sales for the week are in columns 11, 12, and 13, *right justified* (that is, the last digit is always in column 13).



The complete data deck setup for problem one is shown in Figure 2-6. Each typewritten line in the figure corresponds to one keypunched data card.

Computer Output The computer printout resulting from the normal reorder policy is depicted in Figure 2-7. The first line in the computer output is a reproduction of the information keypunched on the first data card. The last line of the computer output is information from the second data card, the control card. The notation 25 WEEKS RUN is a reminder that the control card specified that 25 weeks of data were to be run. If there is too little weekly sales data or if the weekly sales data is out of order the computer will print the message SOMETHING WRONG WITH YOUR DATA and stop processing the program.

```
EXERCISE ONE BY ROY HARRIS
25
01          100
02          100
03          110
04          110
05          110
06          110
07          110
08          110
09          110
10          110
11          110
12          110
13          110
14          110
15          110
16          110
17          110
18          110
19          110
20          110
21          110
22          110
23          110
24          110
25          110
```

Figure 2-6 Computer input—normal policy.

PROGRAM INSYS FOR EXERCISE ONE BY ROY HARRIS

WEEK	--------RETAIL--------				******WHOLESALE******				FACTORY
NO.	SALES	REC	INV	ORDER	SHIP	REC	INV	ORDER	RATE
1	100	100	100	100	100	100	200	100	100
2	100	100	100	100	100	100	200	100	100
3	110	100	70	120	100	100	200	100	100
4	110	100	80	130	120	100	180	140	100
5	110	120	90	120	130	100	150	160	100
6	110	130	110	100	120	100	130	190	140
7	110	120	120	90	100	140	170	130	160
8	110	100	110	100	90	180	260	30	190
9	110	90	90	120	100	190	350	0	130
10	110	100	80	130	120	130	360	0	30
11	110	120	90	120	130	30	260	70	0
12	110	130	110	100	120	0	140	180	0
13	110	120	120	90	100	0	40	250	70
14	110	100	110	100	90	70	20	270	180
15	110	90	90	120	100	180	100	200	200
16	110	100	80	130	120	260	240	80	270
17	110	120	90	120	130	270	350	0	200
18	110	130	110	100	120	200	460	0	80
19	110	120	120	90	100	80	440	0	0
20	110	100	110	100	90	0	350	0	0
21	110	90	90	120	100	0	250	50	0
22	110	100	80	130	120	0	130	190	0
23	110	120	90	120	130	0	0	330	50
24	110	130	110	100	120	50	0	320	190
25	110	120	120	90	100	190	90	210	330

25 WEEKS RUN 0 0 0 0

Figure 2-7 Computer output—normal policy.

Analysis of the Normal Inventory Policy

It is quite evident that the so-called normal inventory policy is not a very smart policy. A simple increase in retail sales to a new level 10 percent higher than before has set off uncontrollable fluctuations in the wholesale inventory and in the factory rate. Even though the factory services only one wholesaler and one retailer these uncontrollable swings cause the factory to completely shut down by week 11. Negative inventories, orders, or factory rates are not allowed.

By week 25 the situation is still not in control. The retailer has not stabilized his inventory level back to 100 units, the wholesaler has not stabilized, and the factory is going from boom to bust. This cyclic behavior in the system is the result of the lead times in the system and the "blind" ordering policies of the retailer and the wholesaler. The next two sections consider some methods for bringing this situation under control.

2.3 CONTROLLING THE REPLENISHMENT RATE

This section considers the problem of controlling fluctuation in the inventory system through a change in the reorder policies of the retailer and the wholesaler. The basic concept applied is that of *dampening* the amplitude of change. This concept is implemented by changing the reorder policy to decrease the amount of replenishment of the base stock. The new policy, the computer output, and an analysis of the results are presented here.

The Replenishment Concept

According to the old policy, the reorder formula for the retailer is

Retail order = retail sales + (100 − inventory level)

This policy says in effect that the retailer wants to replenish the stock he has actually sold during the week. In addition, if sales are above or below the base stock level of 100 units he wants to maintain, he will order enough to bring the base stock to 100.

This policy appears reasonable enough but it is based on the rather nearsighted assumptions that

—future sales will be the same as this week's
—stock replenishment is instantaneous

The first assumption is obviously risky for almost any retail operation. The second assumption is obviously not fulfilled in the present system. The retailer orders on Friday, the goods are shipped on the next Wednesday and received the following Monday. Each Friday, the retailer orders enough "to bring the base stock back to normal" even though the goods he ordered the prior Friday to bring the base stock back to normal still have not arrived. When the order does arrive the retailer overreacts by ordering too little the next time. The net result, as seen in Section 2.2, is that the retailer is not able to stabilize his order or inventory level. Business cycles may be caused by just this kind of behavior.

One way to dampen the swings is to change the replenishment policy to specify that only a percentage of the base stock difference is to be ordered. Thus we change the formula to

Retail order = retail sales + (100 − inventory level) (40%)

If we set A at 50 percent and thus try to make up only one-half the difference, then we can compare the retail order when sales are up to 110 units.

Old Policy	*New Policy*
Retail order = 110 + (100 − 90)	Retail order = 110 + [(100 − 90) × 0.5]
= 110 + 10	= 110 + 5
= 120	= 115

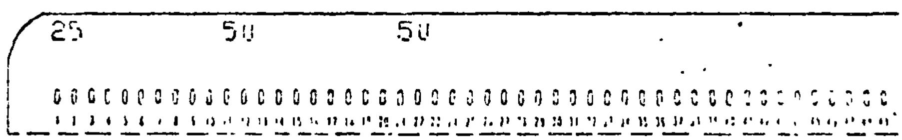
When sales are down to 90 units the result is

Old Policy	*New Policy*
Retail order = 90 + (100 − 110)	Retail order = 90 + [(100 − 110) × 0.5]
= 90 − 10	= 90 − 5
= 80	= 85

The overall effect of the new policy is that the retailer only partly reacts to increases or decreases in the base stock and allows some time for inventories to return to normal. The wholesaler may follow a similar policy in ordering from the factory by including B percent in the wholesale order formula.

Computer Input —Replenishment Rate

User name card remains unchanged. The new reorder policy is implemented by specifying on the *control card* the percentage value for A (retailer) in columns 11 and 12, and B (wholesaler) in columns 21 and 22.



If the field is left blank, the value for A or B is set to 100 percent. Otherwise, A or B may be set from 01 to 99 by the user.

Weekly sales cards are keypunched as in Section 2.2.

A complete data deck listing for the new policy is shown in Figure 2-8. The retailer and the wholesaler only try to make up 50 percent of the difference in base stock under the new policy.

Computer Output —Replenishment Rate

The computer printout for the new 50 percent reordering level policy which is generated from these data cards is shown in Figure 2-9. Note that the last line of the printout includes the input values for A and B specified in the control card.

Analysis of the Replenishment Rate Control

The overall result of the new reordering policy is a dramatic improvement in the performance of the inventory system.

Retail reorders match the new sales level within eleven weeks.
Wholesale reorders match the new sales level within twelve weeks.
Factory rate is not yet stable, but appears to be dampening out.

Most significantly, the system is no longer out of control, i.e., caught up in uncontrollable fluctuation. The fluctuations have been dampened out and the system stabilizes itself to the new sales level.

```
EXERCISE TWO BY ROY HARRIS
25          50        50
01          100
02          100
03          110
04          110
05          110
06          110
07          110
08          110
09          110
10          110
11          110
12          110
13          110
14          110
15          110
16          110
17          110
18          110
19          110
20          110
21          110
22          110
23          110
24          110
25          110
```

Figure 2-8 Computer input—replenishment rate

PROGRAM INSYS FOR EXERCISE TWO BY ROY HARRIS

WEEK	RETAIL				WHOLESALE				FACTORY
NO.	SALES	REC	INV	ORDER	SHIP	REC	INV	ORDER	RATE
1	100	100	100	100	100	100	200	100	100
2	100	100	100	100	100	100	200	100	100
3	110	100	90	115	100	100	200	100	100
4	110	100	80	120	115	100	185	123	100
5	110	115	85	118	120	100	165	134	100
6	110	120	95	113	118	100	148	144	123
7	110	118	103	109	113	123	158	134	138
8	110	113	105	108	109	138	186	116	144
9	110	109	104	108	108	144	223	95	134
10	110	108	101	109	108	134	248	84	116
11	110	108	99	110	109	116	254	82	96
12	110	109	99	111	110	96	240	91	84
13	110	110	99	110	111	84	214	104	82
14	110	111	100	110	110	82	185	118	90
15	110	110	100	110	110	90	165	127	104
16	110	110	100	110	110	104	159	13[illegible]	118
17	110	110	100	110	110	118	167	120	127
18	110	110	100	110	110	127	185	118	130
19	110	110	100	110	110	130	205	107	120
20	110	110	100	110	110	126	221	99	115
21	110	110	100	110	110	118	229	96	107
22	110	110	100	110	110	107	226	97	99
23	110	110	100	110	110	99	216	102	96
24	110	110	100	110	110	96	201	104	97
25	110	110	100	110	110	97	188	116	102

25 WEEKS RUN 50 50 0 0

Figure 2-9 Computer output—replenishment rate

However, all is still not perfect. There is still a long time lag before the factory "catches on" to the new rate. Moreover, a simple 10 percent increase in sales still causes a 20 percent change in the wholesale shipments and a 41 percent change in the factory rate. Section 2.4 considers additional control measures for bringing the inventory system under even tighter control.

2.4 CONTROL OF LEAD TIME

This section considers the problem of controlling fluctuations in the inventory system through a decrease in the lead time between the order and the receipt of goods. The basic concept is to change the lead time required to respond to changes in the system. This concept is implemented by testing the effect of faster delivery from the wholesaler and faster changeover to a new production rate by the factory.

Lead Time Concept

Under the "normal" system setup the two basic lead times in the system were (1) between the order and receipt of goods by the retailer, and (2) between the order and receipt of goods from the factory. These were as follows:

Retailer normal lead time

Order on	Delivered on
Friday week 1	Monday week 3

Wholesaler normal lead time

Order on	Change rate	Deliver goods
Friday week 1	Week 3	Monday week 4

The effects of these lead times are clearly seen in Section 2.2 when the retailer reorders every Friday to make up goods that have previously been ordered. In effect, he makes a double reorder for the same goods. In addition, the factory takes seven weeks to begin to respond to a change in retail sales.

In the current example we will consider the effects of decreasing this lead time. The new policy is to work the wholesaler on Saturday in order to deliver Friday-afternoon orders the very next Monday. Thus

Retailer decreased lead time

Order on	Delivered on
Friday week 1	Monday week 2

Similarly the lead time for the wholesaler may be changed if the factory can shift to a new production rate without a week lag and if the factory ships over the weekend

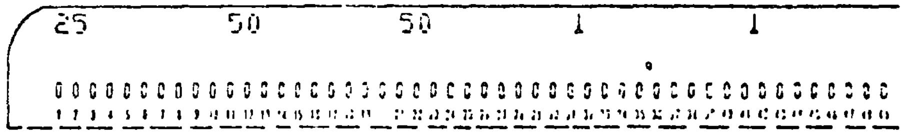
Wholesaler decreased lead time

Order on	Change rate	Delivered in
Friday week 1	week 2	week 3

Computer Input—Lead Times

The *user name card* is the same as that in Section 2.2. The change in the lead times is implemented in the computer model by two fields in the *control card*. A speedup of one week in the wholesaler deliveries is accomplished by placing a 1 in column 31 of the control card. A speedup of one week in the changeover of the factory is accomplished by placing a 1 in column 41 of the control card.

The control card now reads



The *weekly sales cards* retain the same format.

The complete data deck setup to test the effects of the decreased lead time is shown in Figure 2-10.

Computer Output

The computer printout with lead time decreased is shown in Figure 2-11.

Analysis of Lead Time Control

The result of the new lead time policy is a further improvement in the overall performance of the inventory system.

Retail orders match the new sales rate within five weeks.

```
EXERCISE THREE BY ROY HARRIS
25       50       50       1       1
01       100
02       100
03       110
04       110
05       110
06       110
07       110
08       110
09       110
10       110
11       110
12       110
13       110
14       110
15       110
16       110
17       110
18       110
19       110
20       110
21       110
22       110
23       110
24       110
25       110
```

Figure 2-10 Computer input—lead times

PROGRAM INS'S FOR EXERCISE THREE BY ROY HARRIS

WEEK	----------RETAIL----------				----------WHOLESALE----------				FACTORY
NO.	SALES	REC	INV	ORDER	SHIP	REC	INV	ORDER	RATE
1	100	100	100	100	100	100	200	[illegible]	100
2	100	100	100	100	100	100	200	[illegible]	100
3	110	100	90	115	115	100	165	[illegible]	100
4	110	115	95	113	113	100	173	125	123
5	110	113	98	111	111	123	184	115	126
6	110	111	99	111	111	126	199	111	119
7	110	111	99	110	110	119	208	[illegible]	111
8	110	110	100	110	110	111	209	106	106
9	110	110	100	110	110	106	205	107	106
10	110	110	100	110	110	106	201	[illegible]	107
11	110	110	100	110	110	107	198	[illegible]	110
12	110	110	100	110	110	110	198	111	111
13	110	110	100	110	110	111	199	111	111
14	110	110	100	110	110	111	200	[illegible]	111
15	110	110	100	110	110	111	200	[illegible]	110
16	110	110	100	110	110	110	201	[illegible]	110
17	110	110	100	110	110	110	200	[illegible]	110
18	110	110	100	110	110	110	200	[illegible]	110
19	110	110	100	110	110	110	200	110	110
20	110	110	100	110	110	110	200	110	110
21	110	110	100	110	110	110	200	110	110
22	110	110	100	110	110	110	200	110	110
23	110	110	100	110	110	110	200	110	110
24	110	110	100	110	110	110	200	110	110
25	110	110	100	110	110	110	200	110	110

25 WEEKS RUN 50 50 1 1

Figure 2-11 Computer output—lead times

Wholesaler orders match the new sales rate within eight weeks
Factory rate is set to the new sales level in nine weeks.

In addition to cutting response lags down, there is less fluctuation in the inventory levels.

A simple increase in sales of 10 percent causes a 15 percent change in wholesale shipments, down from the prior 20 percent change. Also, the factory rate changes 20 percent, down from the prior 44 percent. Thus, in general, it may be said that the inventory system is now in better control.

There are, however, many complications one could add to the model before it would approximate the real world. For example, customers are never so kind as to provide such nice uniform retail sales. Hence, the user might want to try his hand at controlling the inventory system if retail sales were to randomly fluctuate between, say, 80 units and 120 units in any given week.

REFERENCES

Ansoff, H. I., and D. Slevin, "An Appreciation of Industrial Dynamics," *Management Science*, vol. 14, no. 7 (March 1968), pp. [illegible].

Buffa, E. S., *Modern Production Management*, New York: Wiley, 1969.

———, *Models for Production and Operations Management*, New York: Wiley, 1963.

Fetter, R. B., et al, *Decision Models for Inventory Management*, Homewood, Ill.; Irwin, 1961.

Forrester, J. W., *Industrial Dynamics*, Cambridge, Mass.: M.I.T., 1961.

———, "Industrial Dynamics--After the First Decade," *Management Science*, vol. 14, no. 7 (March 1968), pp. 398-415.

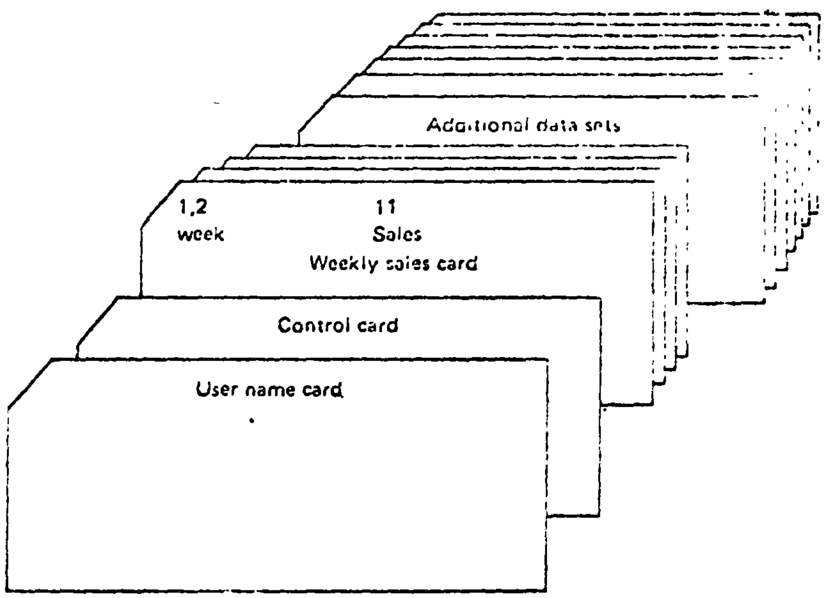
Gavett, J. W., *Production and Operations Management*, New York: Harcourt, Brace & World, 1968.

Meier, R. C., W. T. Newell, and H. L. Pazer, *Simulation in Business and Economics*, Englewood Cliffs, N.J.: Prentice Hall, 1969.

Olson, R. A., *Manufacturing Management: A Quantitative Approach*, Scranton, Pa.: International Textbook Co., 1968.

Plossl, G. W., et al, *Production and Inventory Control*, Englewood Cliffs, N.J.: Prentice-Hall, 1967.

2.5 INSYS DATA DECK STRUCTURE



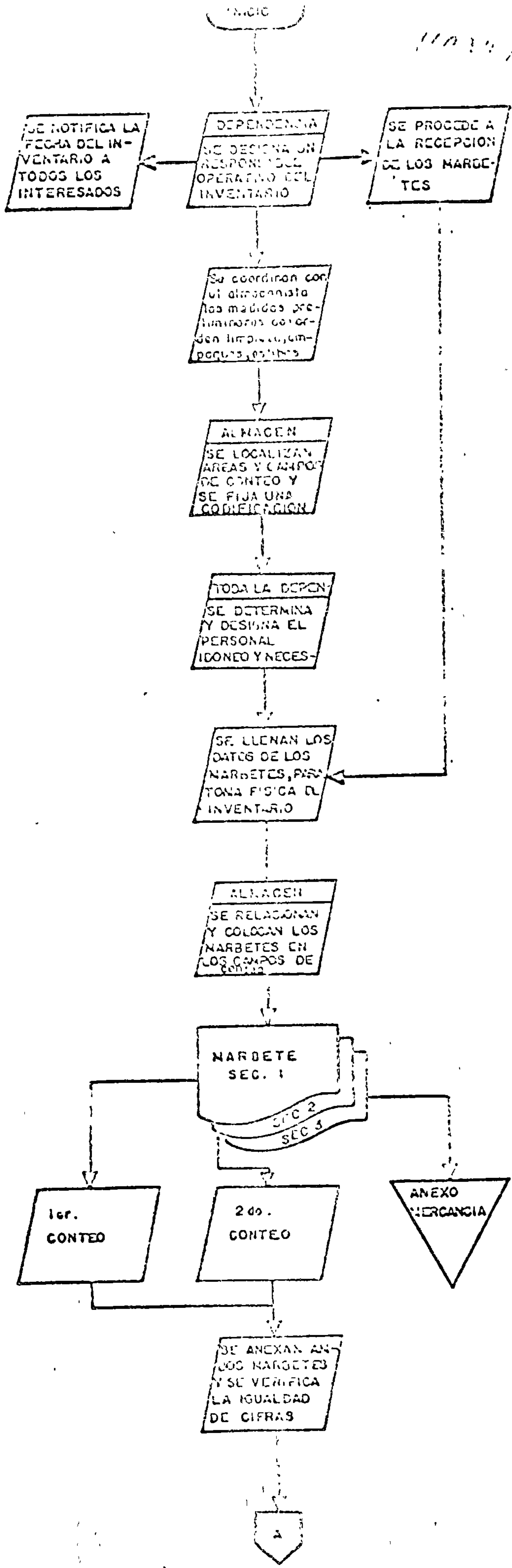
Control Card

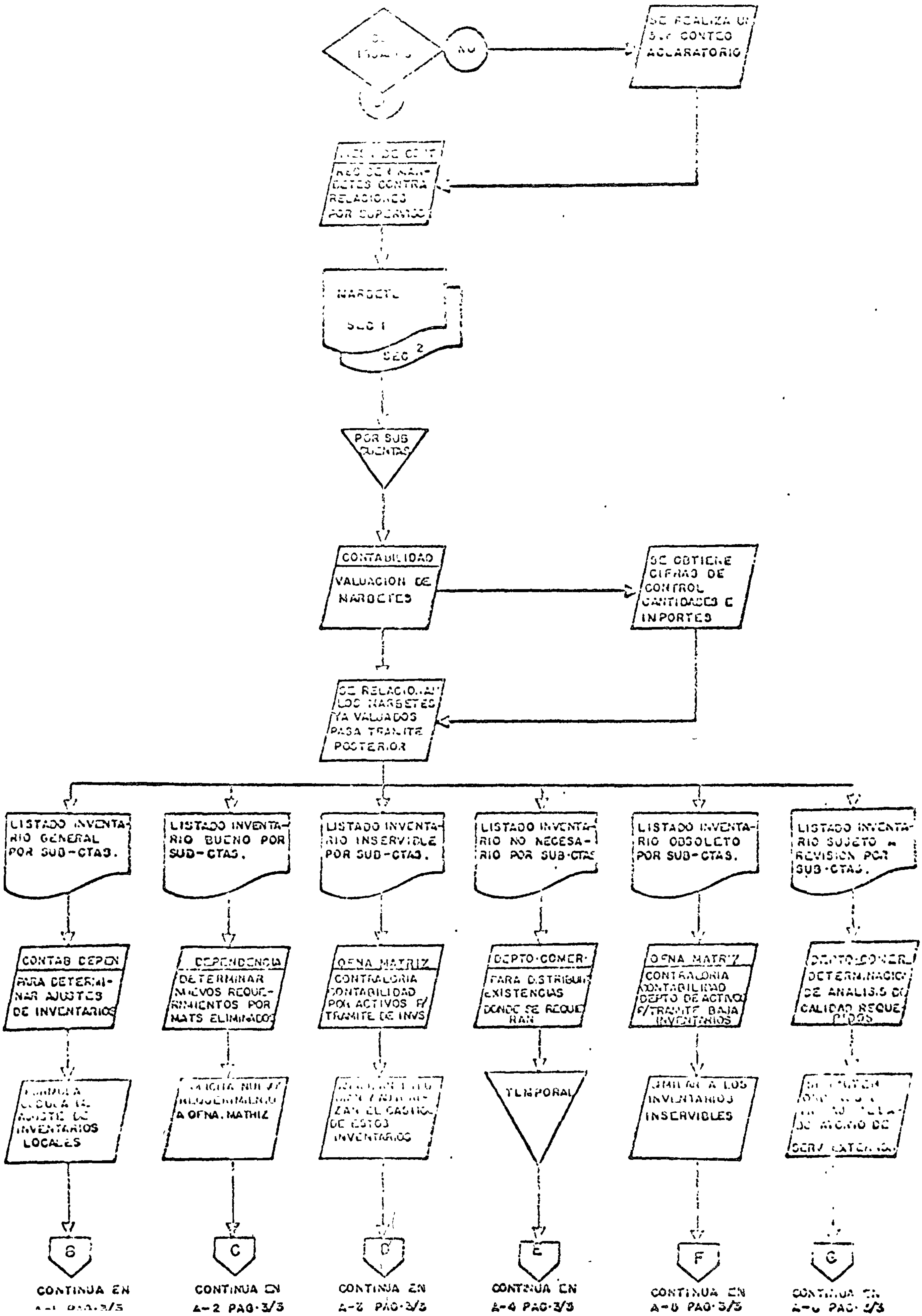
Card columns	*Format*	*Item*
1, 2	I 2	No. of weeks
11, 12	I 2	Percentage value for retailer
21, 22	I 2	Percentage value for wholesaler
31	I 1	Wholesaler lead time
41	I 1	Factory lead time

2.6 INSYS PROGRAM LISTING

```
C       PROGRAM INSYS
C       INVENTORY SYSTEM CONTROL MODEL
C       COPYRIGHT 1967 BY ROY D HARRIS
C       THIS VERSION FOR IBM 360
        DIMENSION ALPHA(10)
C
C       READ AND PRINT STUDENTS NAME CARD
        MI = 5
        MO = 6
  1     READ (MI,26) ALPHA
        WRITE (MO,27) ALPHA
C
C       READ CONTROL CARD AND INITIALIZE
        READ (MI,28) N, IR, IW, LW, LF
        IF (IR) 2,2,3
  2     A = 1.0
        GO TO 4
  3     A = IR/100.
  4     IF (IW) 5,5,6
  5     B = 1.0
        GO TO 7
  6     B = IW/100.
  7     RI = 100.
        RO = 100.
        WS = 100.
        WI = 200.
        WO2 = 100.
        WO1 = 100.
        FR = 100.
C
C       PRINT HEADINGS FOR WEEKLY OUTPUT
        WRITE (MO,29)
        WRITE (MO,30)
C
C       START OF DO LOOP THROUGH WEEKLY COMPUTATIONS
        DO 24 I = 1, N
C
C       READ AND CHECK DATA CARD CONTAINING WEEKLY SALES
        READ (MI,31) KWEEK, SALES
        IF (I-KWEEK) 8,9,8
  8     WRITE (MO,32)
        GO TO 25
C
C       COMPUTE RETAIL INVENTORY LEVEL AND ORDER
  9     RREC = WS
        RINV = RI+RREC-SALES
        IF (RINV) 10,10,11
 10     RINV = 0.0
 11     RORD = SALES+((100.-RINV)*A)
        IF (RORD) 12,12,13
 12     RORD = 0.0
C
C       SET WHOLESALE DELIVERY RATE FROM CONTROL CARD
 13     IF (LW-1) 15,14,15
 14     WSHIP = RORD
        GO TO 16
 15     WSHIP = RO
C
C       COMPUTE WHOLESALE INVENTORY LEVEL AND ORDER
 16     WREC = FR
        WINV = WI+WREC-WSHIP
        IF (WINV) 17,17,18
```

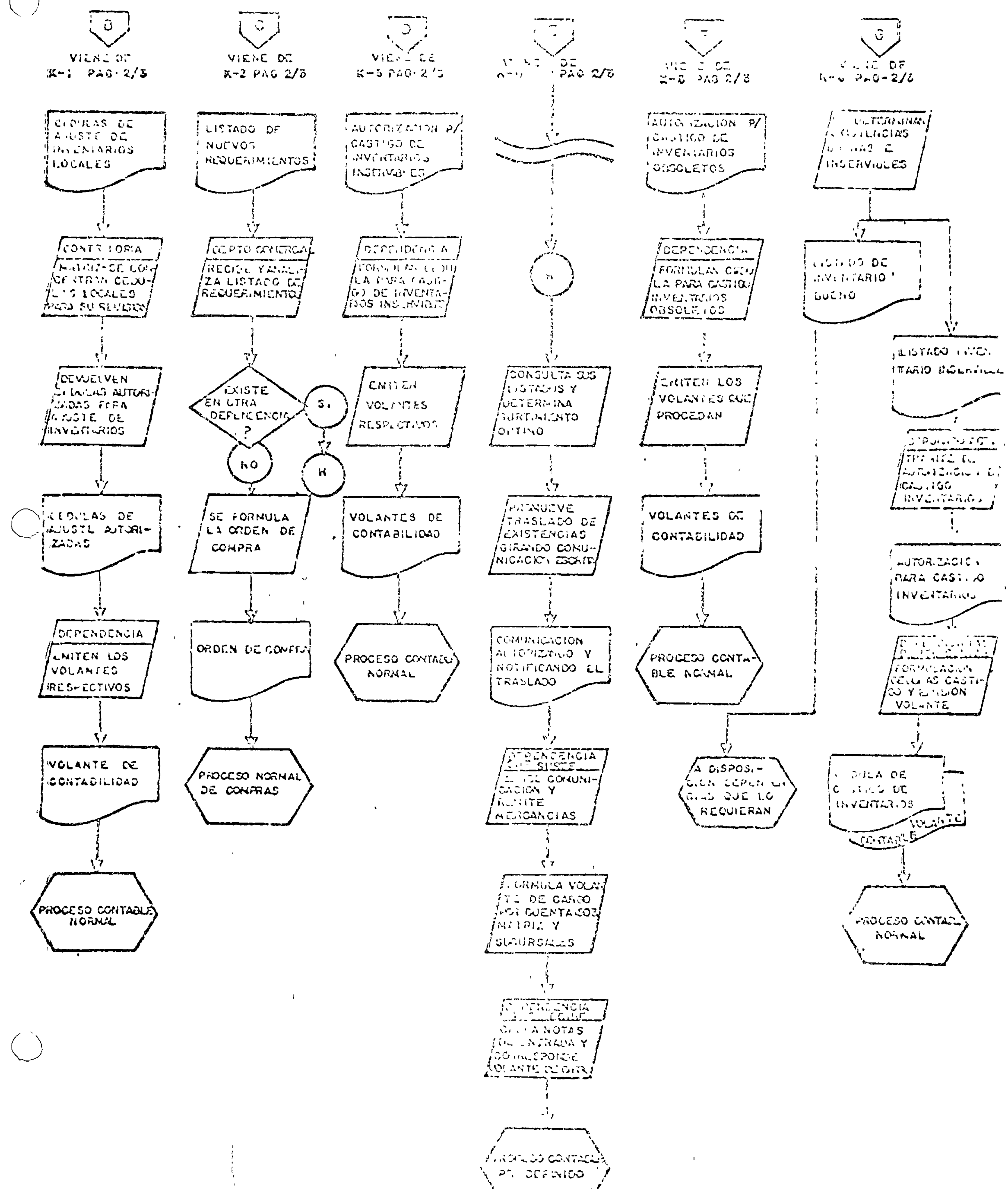
```
 17    WINV = 0.0
 18    WORD = WSHIP+((200.-WINV)*B)
       IF (WORD) 19,19,20
 19    WORD = 0.0
C
C      SET FACTORY RATE FROM CONTROL CARD
 20    IF (LF-1) 22,21,22
 21    FRATE = WO1
       GO TO 23
 22    FRATE = WO2
C
C      PRINT OUT CURRENT WEEK RESULTS
 23    WRITE (MO,33) I, SALES, RREC, RINV, RORD, WSHIP, WREC, WINV, WORD,
      1 FRATE
C      UPDATE ORDERING AND FACTORY RATE FOR NEXT WEEK
       RI = RINV
       RO = RORD
       WS = WSHIP
       WI = WINV
       WO2 = WO1
       WO1 = WORD
       FR = FRATE
 24    CONTINUE
C
C      PRINT CONTROL CARD VALUES
       WRITE (MO,34) N, IR, IW, LW, LF
 25    CONTINUE
       GO TO 1
C
C
C
 26    FORMAT    (10A6)
 27    FORMAT    (1H1,18HPROGRAM INSYS FOR  ,10A6)
 28    FORMAT    (I2,9X,I2,8X,I2,8X,I1,9X,I1)
 29    FORMAT    (30H0WEEK --------RETAIL---------
      1    30H******WHOLESALE******** FACTORY)
 30    FORMAT    (1H ,36HNO.  SALES REC   INV  ORDER   SHIP
      1    23HREC   INV  ORDER   RATE)
 31    FORMAT    (I2,8X,F3.0)
 32    FORMAT    (3 H SOMETHING WRONG WITH YOUR DATA)
 33    FORMAT    (1H ,I2,4F6.0,F7.0,3F6.0,F7.0)
 34    FORMAT    (1H0,I2,10H WEEKS RUN,4I3)
       END
```


INICIO
DEPENDENCIA
SE DESIGNA UN RESPONSABLE DEL OPERATIVO DEL INVENTARIO
SE NOTIFICA LA FECHA DEL INVENTARIO A TODOS LOS INTERESADOS
SE PROCEDE A LA RECEPCION DE LOS MARBETES
ALMACEN
SE LOCALIZAN AREAS Y CAMPOS DE CONTEO Y SE FIJA UNA CODIFICACION
TODA LA DEPEN
SE DETERMINA Y DESIGNA EL PERSONAL IDONEO Y NECES-
SE LLENAN LOS DATOS DE LOS MARBETES, PARA TOMA FISICA DEL INVENTARIO
ALMACEN
SE RELACIONAN Y COLOCAN LOS MARBETES EN LOS CAMPOS DE CONTEO
MARBETE SEC. 1
SEC 2
SEC 3
1er. CONTEO
2do. CONTEO
ANEXO MERCANCIA
SE ANEXAN AN- LOS MARBETES Y SE VERIFICA LA IGUALDAD DE CIFRAS
A



NO
CONTEO
ACLARATORIO
CONTRA
POR SUB CUENTAS
CONTABILIDAD
VALUACION DE MARBETES
SE OBTIENE CIFRAS DE CONTROL CANTIDADES E IMPORTES
SE RELACIONAN LOS MARBETES YA VALUADOS PASA TRAMITE POSTERIOR
LISTADO INVENTARIO GENERAL POR SUB-CTAS.
LISTADO INVENTARIO BUENO POR SUB-CTAS.
LISTADO INVENTARIO INSERVIBLE POR SUB-CTAS.
LISTADO INVENTARIO NO NECESARIO POR SUB-CTAS
LISTADO INVENTARIO OBSOLETO POR SUB-CTAS.
LISTADO INVENTARIO SUJETO A REVISION POR SUB-CTAS.
CONTAB DEPEN
PARA DETERMINAR AJUSTES DE INVENTARIOS
DEPENDENCIA
DETERMINAR NUEVOS REQUERIMIENTOS POR MATS ELIMINADOS
OFNA MATRIZ
CONTRALORIA CONTABILIDAD POR ACTIVOS F/ TRAMITE DE INVS
DEPTO COMER
PARA DISTRIBUIR EXISTENCIAS DONDE SE REQUIERAN
OFNA MATRIZ
CONTRALORIA CONTABILIDAD DEPTO DE ACTIVO F/TRAMITE BAJA INVENTARIOS
DEPTO COMER
DETERMINACION DE ANALISIS DE CALIDAD REQUERIDOS
AJUSTE DE INVENTARIOS LOCALES
A OFNA. MATRIZ
ZAN EL CASTIGO DE ESTOS INVENTARIOS
TEMPORAL
SIMILAR A LOS INVENTARIOS INSERVIBLES
B
C
D
E
F
G
CONTINUA EN
CONTINUA EN A-2 PAG·3/3
CONTINUA EN
CONTINUA EN A-4 PAG·3/3
CONTINUA EN
CONTINUA EN

DIAGRAMA DE FLUJO ... LA TOMA FISICA DE INVENTARIOS-
HOJA 3/3



EOQ

a model for inventory ordering policy

computes the most economical inventory order quantity under a variety of conditions, including price discounts, shortage cost, and storage limitations.

A primary purpose of inventories is to decouple production from consumption. Inventories are goods which may be used as a hedge against uncertainty in demand or as a buffer for production fluctuations.

The *replenishment* of inventories is the topic of this exercise. We describe an elementary, but fundamental, inventory replenishment model: the Economic Order Quantity (EOQ) model. In Section 3.1, the development of the basic EOQ model is given. In Section 3.2, the basic EOQ model is extended to include a real world phenomenon: quantity price discounts. In Section 3.3, the basic model is modified to incorporate shortage costs. Consideration of storage limitations and their effects upon the order quantity decision are the subject of Section 3.4.

The rational basis for deciding how much, if any, inventory to hold, and to order, is an economic basis. There are costs associated with holding inventory in stock, e.g., insurance, taxes, interest on capital, and so on. Conversely, there are costs related to not holding inventory, e.g., lost sales, frequent purchase orders, production delays, etc. There is also the cost of purchasing the replenishment for inventories, e.g., paperwork and material handling.

The intent of this exercise is to allow the user an opportunity to get a feel for the *effects* of changing parametric values in the basic economic order quantity formulas. Hence, the reader is encouraged to conduct *sensitivity analysis* on each parameter in the EOQ formula.

3.1 ECONOMIC ORDER QUANTITY

This section introduces the basic Economic Order Quantity (EOQ) model. It also describes in detail the data cards required for the accompanying computer program, and the computer output.

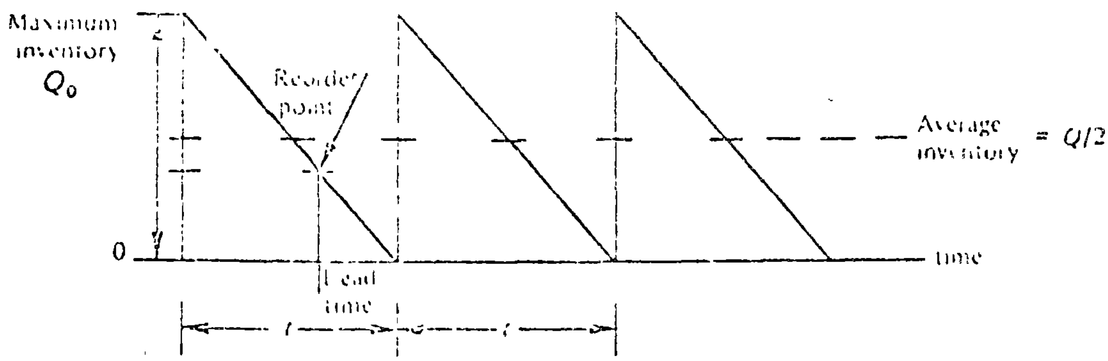


Figure 3-1 Inventory level and usage pattern for EOQ model.

The Economic Order Quantity Inventory Model

A basic assumption of the EOQ model is that the consumption of the inventory is constant over time and that it is possible to replenish the inventory on very short notice. The quantity in inventory at any point in time, for this circumstance, is shown in Figure 3-1. The basic EOQ model also assumes no quantity price discounts and no backorders.

In this "sawtooth" usage pattern the inventory is consumed over time until it is depleted. It is then instantly replenished (straight vertical line), and the usage continued.

Inventory Cost

The rational basis for determining inventory levels is to balance the cost of holding inventory against the cost of not holding inventory. In the consumption situation described above, the types of cost for holding the inventory are fairly obvious. Storage must be provided for the inventory and the inventory must be financed.

The cost of not holding inventory may not be so obvious. Since it was assumed that the inventory was easily and instantaneously obtainable, then there is no cost attached to being caught short. However, like a housewife who goes to the grocery store three times each day to purchase a single meal, there is a cost attached to procurement of the inventory. Not holding inventory may lead to very high procurement cost.

The issue in the economic order quantity model is to determine how much inventory to order each time. The cost of procurement per unit goes down if more is ordered each time, but the cost of holding inventory goes up. Figure 3-2 shows a graphic representation of these costs as they vary with the size of the order. The total incremental cost of inventory is shown as the top curve on the graph in the figure.

Total incremental cost = holding cost + ordering cost

The best inventory policy is to order the amount of inventory each time which yields the minimum total cost. This "correct quantity" to order is called the economic order quantity (EOQ).

The following definitions and variables will be used in deriving a mathematical expression for the EOQ.

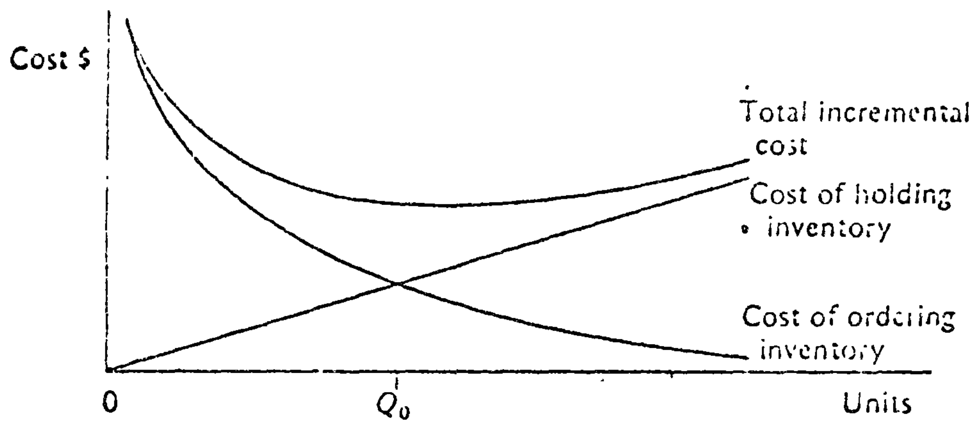


Figure 3-2 Costs of holding and ordering inventory.

Holding cost = (average inventory) X (unit inventory holding cost per year)
= $(Q/2) \times (P \times FH)$

where Q = quantity ordered
P = price per unit
FH = annual unit holding cost as percentage of the unit price

Order cost = (number of orders per year) X (cost per order)

$$= \left(\frac{R}{Q}\right) \times (Cp)$$

where R = annual requirements in units, level demand
Cp = procurement cost (includes costs of paperwork, handling, etc.)

Cost of inventory = (unit price) X (annual requirements)
= $(P) \times (R)$

Total cost = holding cost + order cost + cost of inventory

$$\text{Total cost} = \frac{Q \times P \times FH}{2} + \frac{R \times Cp}{Q} + P \times R$$

$$\text{Total incremental cost} = \frac{Q \times P \times FH}{2} + \frac{R \times Cp}{Q}$$

Solving for the economic order quantity Q_0 by algebra: the minimum point on the total incremental cost curve is where the inventory holding cost and the procurement cost curves intersect. Where they intersect they must be equal. Therefore, at Q_0:

$$\frac{Q}{2}\,(P \times FH) = \left(\frac{R}{Q}\right) Cp$$

Clearing denominators

$$Q(Q)(P \times FH) = 2(R)Cp$$

$$Q^2 = \frac{2R\,Cp}{P \times FH}$$

$$Q = \sqrt{\frac{2R\,Cp}{P \times FH}}$$

Solving for Q by calculus: the minimum point on the total incremental cost curve is where the first derivative equals zero. Taking the first derivative with respect to Q and setting it equal to zero:

$$0 = (P \times FH)/2 - (R/Q^2)Cp$$

$$\frac{(P \times FH)}{2} = \frac{R\,Cp}{Q^2}$$

$$Q^2 = \frac{2R\,Cp}{P \times FH}$$

$$Q = \sqrt{\frac{2R\,Cp}{P \times FH}}$$

Sample Problem One

It is a straightforward matter to find the economic order quantity (EOQ) and the total inventory cost (TC), when the values for R, Cp, P, and FH are known. For example, if

R = 1600 units (total annual usage)
Cp = \$5.00 (cost of one procurement)
P = \$1.00 (unit price of product)
FH = 0.10 (unit holding cost per year as percentage of price), then

$$Q = \sqrt{\frac{2 \times 1600 \times 5.00}{1.00 \times 0.1}}$$

$$Q = 400 \text{ units}$$

$$\text{Total cost } (TC) = \frac{400 \times 1.00 \times 0.1}{2} + \frac{1600 \times 5.00}{400} + 1.00 \times 1600$$

$$TC = 20.00 \quad + \quad 20.00 \quad + \quad 1600.00$$

$$TC = \$1640.00$$

This computation is not too tedious to do, if there is only one. However, when there are many alternatives to test and when more complicated formulas are required, then a computer program is a great computational aid. The next section introduces the data input and computer output for the simple example shown above. In following sections more complicated problems illustrating the use of the computer program will be presented.

Computer Input—Problem One

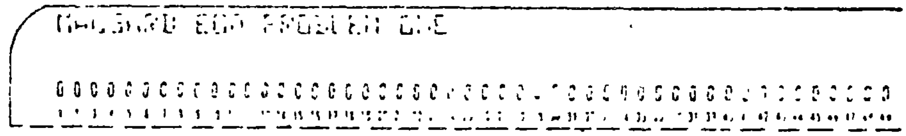
Before describing the data cards, several comments will be made pertaining to the program itself. The user should keep these comments in mind when using the program.

The program is applicable only to a fixed-order-quantity inventory system, and all quantities in the program are expressed in annual amounts or rates. In the case of R (annual inventory requirement), a level usage rate is assumed throughout the year. In order to convert the program for monthly or seasonal calculations one would have to adjust the inputs to the same time scale.

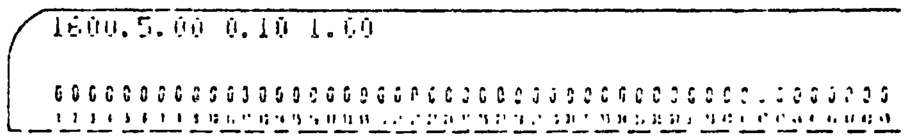
Although the figure available for inventory holding costs is often stated as an annual cost per unit, this program *requires* that holding costs be expressed as a percentage of the unit value of inventory.

To run the economic order quantity computer model, only two cards are required: (1) the user name card, and (2) the "data" card. When multiple problems are batched together, a new name card is required for each problem.

The *user name card* may contain any identifying information (such as the user's name) which is desired. This identifying information is keypunched in the first forty columns.



The *data card* contains the numerical data for the economic order quantity computation. Columns 1-5 contain the annual usage requirement, and the ordering cost is punched in columns 6-10. The holding cost, expressed as a percentage of the unit price, is keypunched in columns 11-15, and the unit price is punched in columns 16-20.



The user name card and the data card are the only two cards required. A listing of the two cards which produced the output shown in the next section is shown in Figure 3-3.

```
MAGCARD EOQ PROBLEM ONE
1600.5.00 0.10 1.00
```

Figure 3-3 Computer input—Problem One.

Computer Output—Problem One

The computer output (Figure 3-4) includes the identification information from the user name card and the information specified on the data card. Below this, the program prints out the quantities calculated in the program. These are (1) the optimum order quantity, (2) the total inventory cost, (3) the number of orders to be placed annually, and (4) the unit price at the order quantity determined.

3.2 PRICE DISCOUNTS

When discussing the economic order quantity model in Section 3.1, it was noted that the basic model assumes no quantity price discounts. However, the basic EOQ model may be extended to include price discounts

```
PROGRAM EOQ FOR MAGCARD EOQ PROBLEM ONE
INPUT DATA IS ***********
     R    CP    FH    P1    CS    B1    P2    B2    P3    W
  1600  5.00   .10  1.00     0     0     0     0     0    0

ANALYSIS RESULTS ARE ****
 OPTIMUM ORDER QUANTITY IS                              400.00
 AT A PRICE PER ITEM OF                                   1.00
 YIELDING A TOTAL INVENTORY COST OF                    1640.00
 WHERE THE NUMBER OF ORDER CYCLES PER YEAR IS             4.00
```

Figure 3-4 Computer output—Problem One.

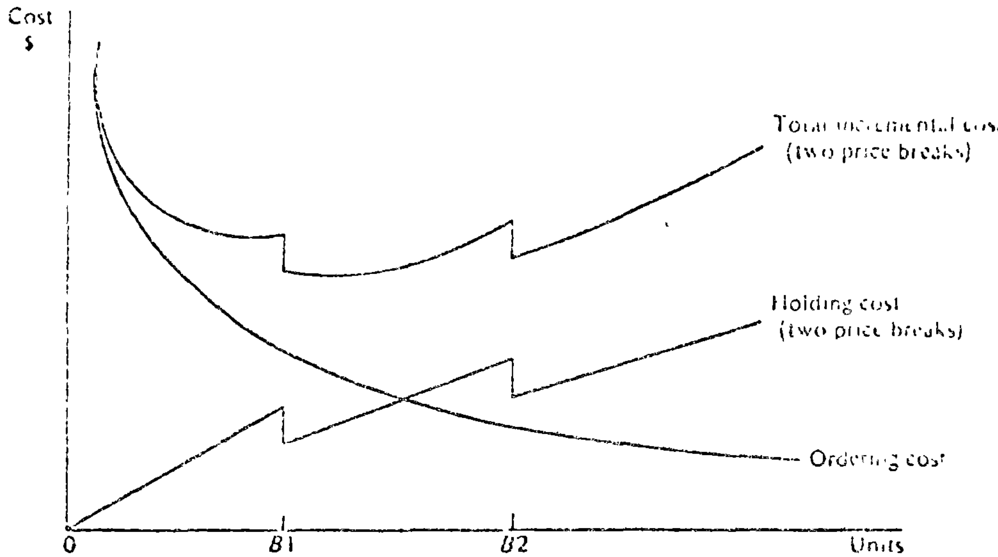


Figure 3-5 The effects of price discounts on the economic order quantity.

as input variables. Inasmuch as price discounts do happen in reality, the extension of the EOQ model to include price discounts will be the subject of this section.

The EOQ Model—With Price Discounts

Referring to Section 3.1, the user should note that in the derivation of the EOQ model the price per unit (P) affects the holding cost $(Q/2) \times (P \times F/H)$, but not the ordering costs. Nevertheless, if price discounts are introduced as variables, they will influence the total incremental costs (TIC). The effects of price discounts are graphically illustrated in Figure 3-5.

The addition of the quantity discounts to the economic order quantity model makes it somewhat more difficult to obtain a solution. It is not possible to find directly the lowest point on the Total Incremental Cost (TIC) curve shown in Figure 3-4. The general approach used is to investigate the TIC curve at each price break. In addition, the curve must be analyzed at different points near the price break giving the lowest TIC to see if an even better solution can be found. Problem Two illustrates this general search solution when price discounts are to be considered.

Sample Problem Two

The supplier has recently revised his pricing policies and now offers the following price discounts: If one orders in lot sizes of $B1$ (Q_{B1} = 500), the price will be \$0.90/unit (P_2); if one orders quantity $B2$ (Q_{B2} = 2000), the price will be \$0.80/unit (P_3).

Solution for EOQ with Two Price Breaks

First calculate Q_3 using P_3; if it is greater than Q_B, then order Q_3. If it is less than Q_B, then (using P_3) it is infeasible.

Next, calculate Q_2 using P_2. If $Q_2 > Q_{B2}$, then order Q_{B2}.

If Q_2 is less than Q_{B2} but greater than Q_{B1}, i.e., $Q_{B1} < Q_2 < Q_{B2}$, then compare TC_2 with TC_{B2}.

If $TC_2 > TC_{B2}$, then order Q_{B2}.

If $TC_2 < TC_{B2}$, then order Q_2.
If Q_2 is less than Q_{B1}, calculate Q_1.
If $Q_1 > Q_{B1}$, then compare TC_{B1} with TC_{B2}.
If $TC_{B1} > TC_{B2}$, then order Q_{B2}.
If $TC_{B1} < TC_{B2}$, then order Q_{B1}.
If Q_1 is less than Q_{B1}, then compare TC_1 with TC_{B1} with TC_{B2}. Order the quantity corresponding to the minimum total cost.

$$Q_3 = \sqrt{\frac{2(1600)(5.00)}{0.80(0.10)}} = 447.2 \text{ (using } P_3)$$

Since $Q_3 < Q_{B2}$, calculate Q_2 using P_2.

$$Q_2 = \sqrt{\frac{2(1600)(5.00)}{0.90(0.10)}} = 421.6 \text{ (using } P_2)$$

Since Q_2 is less than Q_{B2} (2000), but greater than Q_{B1} (300), we must compare TC_2 with TC_{B2}.

$$TC_2 = \frac{(0.90)(0.10)(421.6)}{2} + \frac{1600(5.00)}{421.6} + 1600(0.90)$$
$$= 18.99 + 18.99 + 1440$$
$$= \$1477.98$$

$$TC_{B2} = \frac{0.80(0.10)(2000)}{2} + \frac{1600(5.00)}{2000} + 1600(0.80)$$
$$= 80.00 + 4.00 + 1280.00$$
$$= \$1364.00$$

TC_2 is greater than TC_{B2}, therefore, order in quantities of $B2(Q_{B2} =$ 2000 units @ \$0.80/unit).

Computer Input—Problem Two

The *user name card* is the first card in the data deck. The *data card* for this example problem contains some additional information. Columns 1-20 are the same as described in Section 3.1. The minimum quantity that can be ordered to take advantage of the first price discount is punched in columns 26-30 and the unit price at the first price discount is punched in columns 31-35. Columns 36-40 and 41-45 contain the corresponding information for the second price discount. The data card is as shown:

```
1600.5.00 0.10 1.00 0    300. 0.90 2000.0.8
```

The complete computer input for this example problem is exhibited in Figure 3-6.

```
HAGGARD EOQ PROBLEM TWO, PRICE DISCOUNTS
1600.5.00 0.10 1.00 0    300. 0.90 2000.0.8
```

Figure 3-6. Computer input—Problem Two.

```
PROGRAM EOQ FOR MAGGARD EOQ PROBLEM TWO, PRICE DISCOUNTS
INPUT DATA IS ..........
     R      CP     FH     P1      CS      B1      P2      B2     P3      W
  1600   5.00    .10   1.00       0      300     .90   2000    .00      0
ANALYSIS RESULTS ARE....
 OPTIMUM ORDER QUANTITY IS                                  2000.00
 AT A PRICE PER ITEM OF                                         .00
 YIELDING A TOTAL INVENTORY COST OF                         1364.00
 WHERE THE NUMBER OF ORDER CYCLES PER YEAR IS                   .80
```

Figure 3-7 Computer output—Problem Two.

Computer Output—Problem Two The computer printout resulting from the above data is shown in Figure 3-7.

3.3 SHORTAGE COSTS

Just as it is true that in the real world quantity price discounts exist, it is also true that *backorders* are a reality. By allowing backorders we are saying that if an order cannot be filled at this time due to stock shortages, then as soon as inventory is available previously unfilled orders, i.e., backorders, will be the first orders to be filled. However, in an inventory system allowing for backorders (see Figure 3-8) a *shortage cost* is usually input relating to the backorder quantities. Generally, this shortage cost consists of costs due to (1) possible lost sales due to stockouts, (2) decreased customer satisfaction, (3) additional costs associated with rush shipments, and so on.

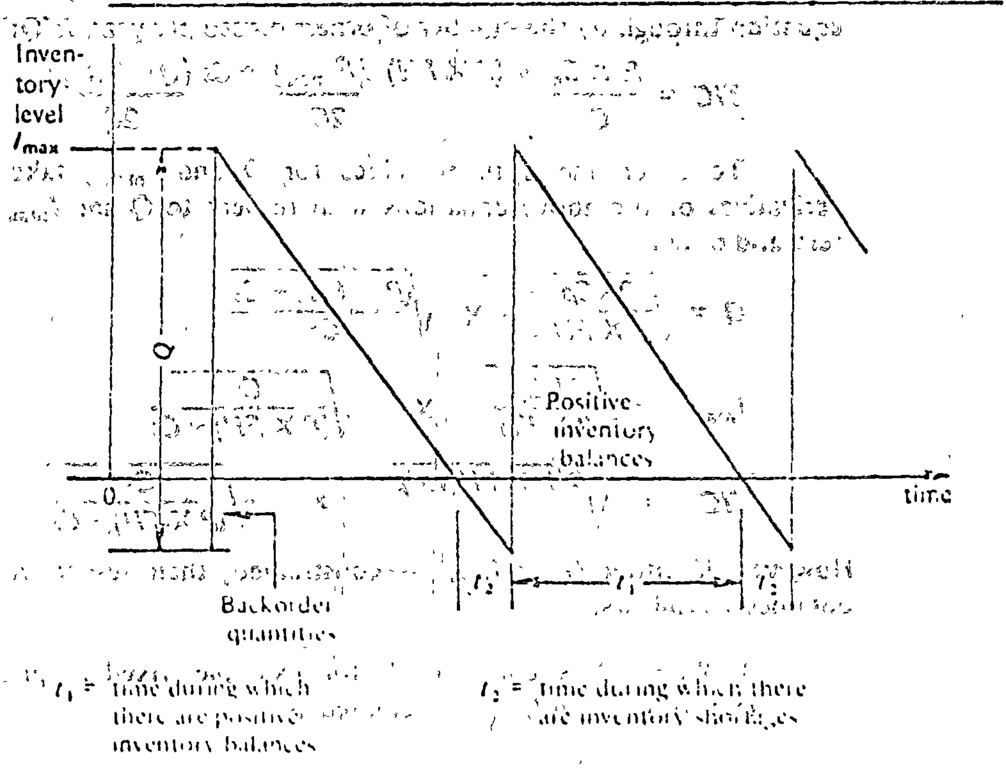


Figure 3-8 An inventory system with $(Q - I_{max})$ backorders allowed.

The basic EOQ formula may be modified to incorporate shortage costs as follows:

1. Cp = procurement cost per order (unchanged)
2. $\frac{(P \times FH)(I_{max})}{2} t_1$ = the holding cost of the *positive inventory balance* during time t_1 (t is on an annual basis, i.e., t = a fraction of a year). Since $t_1 = I_{max}/R$, this becomes:

$$\frac{(P \times FH)I^2_{max}}{2R}$$

3. $Cs \frac{(Q - I_{max})}{2} t_2$ = the shortage cost of t . backorders during time t_2. Since $t_2 = \frac{Q - I_{max}}{R}$, this becomes:

$$Cs \frac{(Q - I_{max})^2}{2R}$$

where I_{max} = maximum level of inventory and
Cs = shortage cost.

Hence, the total incremental cost for one cycle, $t_1 + t_2$, of an inventory system which allows backorders is

$$C_p + (P \times FH) \frac{(I^2_{max})}{2R} + Cs \frac{(Q - I_{max})^2}{2R}$$

The annual total incremental cost is now obtained by multiplying the above equation through by the number of orders placed per year, R/Q:

$$TIC = \frac{R \times C_p}{Q} + (P \times FH) \frac{(I^2_{max})}{2Q} + Cs \frac{(Q - I_{max})^2}{2Q}$$

To determine optimal values for Q and I_{max}, take the partial derivatives of the above equations with respect to Q and I_{max}, equate to zero and obtain

$$Q = \sqrt{\frac{2RCp}{P \times FH}} \times \sqrt{\frac{(P \times FH) + Cs}{Cs}}$$

$$I_{max} = \sqrt{\frac{2RCp}{P \times FH}} \times \sqrt{\frac{Cs}{(P \times FH) + Cs}}$$

$$TIC = \sqrt{2(P \times FH)RCp} \times \sqrt{\frac{Cs}{(P \times FH) + Cs}}$$

However, if either Q or I_{max} is constrained, their respective values are obtained as follows.

1. When Q is constrained, for example, fixed at price discount quantities, then I_{max} is calculated as

$$I_{max} = \frac{CsQ}{CH + CS}$$

2 When I_{max} is constrained, for example, limited by storage constraints (see next section), then Q is calculated as

$$Q = \sqrt{\frac{2CpR + (P \times FH)I_{max}^2 + CsI_{max}^2}{Cs}}$$

Sample Problem Three

This problem is basically the same as Sample Problem One except that shortage costs (Cs) are included. In this problem $Cs = \$0.30$.

$$Q = \sqrt{\frac{2(1600)\,5.00}{1.00\,(0.10)}} \times \sqrt{\frac{(1.00 \times 0.10) + 0.30}{0.30}} = 400(1.154)$$

$$= 461.88 \text{ units}$$

$$T/C = \sqrt{2(5.00)\,(1.00 \times .10)\,1600} \times \sqrt{\frac{0.30}{(1.00 \times 0.10) + 0.30}}$$

$$= 40(.866) = 34.64$$

$$TC = (1.00)\,(1600) + 34.64 = \$1634.64$$

The reader should note that the effect of including shortage costs is to increase the size of Q. This is because the annual inventory holding costs are smaller due to the smaller average inventory. In addition, the total incremental costs (T/C) are less than in the classical model because both holding costs and preparation costs are lower. (This may be verified by comparing the results of this analysis with those from Section 3.1.)

Computer Input—Problem Three

The data card for this example problem follows the same general form outlined previously. One additional data input is necessary. The shortage cost of \$0.30 is punched in columns 21-25.

```
1600.5.00 0.10 1.00 0.30
```

The complete computer input is shown in Figure 3-9.

```
MAGGARD EOQ PROBLEM THREE, SHORTAGE COST
1600.5.00 0.10 1.00 0.30
```

Figure 3-9 Computer input—Problem Three.

The computer printout, using this input, is shown in Figure 3-10.

```
PROGRAM EOQ FOR MAGGARD EOQ PROBLEM THREE, SHORTAGE COST
INPUT DATA IS = [illegible]
     R      CP      FH      P1      CS      H1      P2      H2      P3
  1600    5.00     .10    1.00     .30       0       0       0       0

ANALYSIS RESULTS ARE ****
 OPTIMUM ORDER QUANTITY IS                                    461.88
 WITH OPTIMUM INVENTORY OF                               [illegible]
 AT A PRICE PER ITEM OF                                         1.00
 YIELDING A TOTAL INVENTORY COST OF                          1634.64
 WHERE THE NUMBER OF ORDER CYCLES PER YEAR IS                   3.46
```

Figure 3-10 Computer output—Problem Three.

3.4 STORAGE LIMITATIONS

In this section, an additional constraint of maximum storage limitations, either in terms of available warehouse space or available capital, will be placed upon the basic EOQ model. This is intended to be an illustrative example of an additional type of constraint which may be (and in the real world is) imposed upon the basic EOQ model. By incorporating this added constraint the reader should get some additional insight into the problems that face management when determining what quantities to purchase from suppliers.

Sample Problem Four

This problem is basically the same as Sample Problem One except that the additional constraint of maximum warehouse space available (W) has been imposed.

In this problem, W = 100 units. From Sample Problem One, Q = 400 units, but since $W < Q$, the order quantity must be $Q = W$ = 100 units.

In this problem, the cost of the limited storage constraint is $TC_{prob1}^{*} - TC_W$.

$$TC_W = \frac{100\,(1.00)\,(0.10)}{2} + \frac{1600(5.00)}{100} + 1600(1.00)$$

$$= 5 + 80 + 1600 = \$1685.00$$

$$\text{Cost}_W = TC_1 - TC_W = \$1640.00 - \$1685.00 = \$45.00$$

Computer Input—Problem Four

The data card of the above example problem follows the same form as outlined previously, with one addition. The maximum warehouse space available, expressed in units, is keypunched in columns 46-50. The data card looks like this:

```
1600.5.00 0.10 1.00 0.    0.    0.    0.    0.    100.
```

The computer input for this example problem is shown in Figure 3-11.

```
MAGGARD EOQ PROBLEM FOUR, STORAGE LIMITS
1600.5.00 0.10 1.00 0.    0.    0.    0.    0.    100.
```

Figure 3-11 Computer input—Problem Four.

Computer Output—Problem Four

The computer output will indicate whether or not the warehouse constraint has had an effect on the economic order quantity. If an economic order quantity has been determined which exceeds W, the output will indicate that this has happened.

Furthermore, on the output will be a statement to the effect that, if the warehouse restrictions are operative, the order quantity determined may not be optimal. Suggestions are made for a method to determine the optimal quantity if this restraint is present. The computer output is shown in Figure 3-12.

```
PROGRAM FOR . OR HAGGARD EOQ PROBLEM FOUR, STORAGE LIMITS
INPUT DATA IS ********
      R     CP     FH     P1      CS      B1      P2      B2     P3      W
   1600   5.00    .10   1.00       0       0       0       0      0    100

ANALYSIS RESULTS ARE ****
 BEFORE THE WAREHOUSE STORAGE LIMITATION IS APPLIED
 OPTIMUM ORDER QUANTITY IS                                      400.00
 AT A PRICE PER ITEM OF                                           1.00
 YIELDING A TOTAL INVENTORY COST OF                            1640.00
 WHERE THE NUMBER OF ORDER CYCLES PER YEAR IS                     4.00

 THE ORDER QUANTITY IS LIMITED BY THE WAREHOUSE SPACE
 RESTRICTION AND IS NOT AT AN OPTIMUM. LOOSEN THE
 RESTRICTION AND RUN AGAIN OBSERVING THE EFFECT.

ANALYSIS RESULTS ARE ****
 AFTER THE WAREHOUSE STORAGE LIMITATION IS APPLIED
 OPTIMUM ORDER QUANTITY IS                                      100.00
 AT A PRICE PER ITEM OF                                           1.00
 YIELDING A TOTAL INVENTORY COST OF                            1685.00
 WHERE THE NUMBER OF ORDER CYCLES PER YEAR IS                    16.00
 THIS ORDER QUANTITY IS AT THE MAXIMUM WAREHOUSE CAPACITY
```

Figure 3-12 Computer output—Problem Four.

To conclude our discussion of economic order quantity models and, in particular this computer model, we would point out:

1. that the model is, in its present form, limited to only two price breaks.

2. the inclusion of shortage costs certainly complicates the storage limitation problem. In this model, when backorders and storage limitations are included in the same problem, the assumption is made that the backorders are instantaneously filled and that the storage limitation W is a constraint upon I_{max} and not upon Q. The user must remember that if I_{max} is constrained by W then neither Q nor I_{max} will be optimal.

3. this model will solve problems including one or all of the constraints previously described in a single problem. To appreciate this fact the reader may wish to solve the following problem manually and then by the use of the herein described EOQ model.

Data

R	Cp	FH	P_1	Cs	$B1$	P_2	$B2$	P_3	W
1600	5.00	0.10	1.00	0.30	300	0.90	500	0.80	350

REFERENCES

Bowman, E. H., and R. B. Fetter, *Analysis for Production and Operations Management*, 3d ed., Homewood, Ill.: Irwin, 1967.

Buffa, E. S., *Modern Production Management*, 3d ed., New York: Wiley, 1969.

——, *Operations Management: Problems and Models*, 2d ed., New York: Wiley, 1968.

——, *Production-Inventory Systems: Planning and Control*, Homewood, Ill.: Irwin, 1968.

Eilon, S., *Elements of Production Planning and Control*, New York: Macmillan, 1962.

Fabrycky, W. J., and P. E. Torgersen, *Operations Economy: Industrial Applications of Operations Research*, Englewood Cliffs, N.J.: Prentice-Hall, 1966.

Garrett, L. J., and M. Silver, *Production Management Analysis*, New York: Harcourt, Brace & World, 1966.

Hadley, G., and T. M. Whitin, *Analysis of Inventory Systems*, Englewood Cliffs, N.J.: Prentice Hall, 1963.

Hopeman, R. J., *Systems Analysis and Operations Management*, Columbus, Ohio: Merrill, 1969.

Magee, J. F., and D. M. Boodman, *Production Planning and Inventory Control*, 2d ed., New York: McGraw-Hill, 1967.

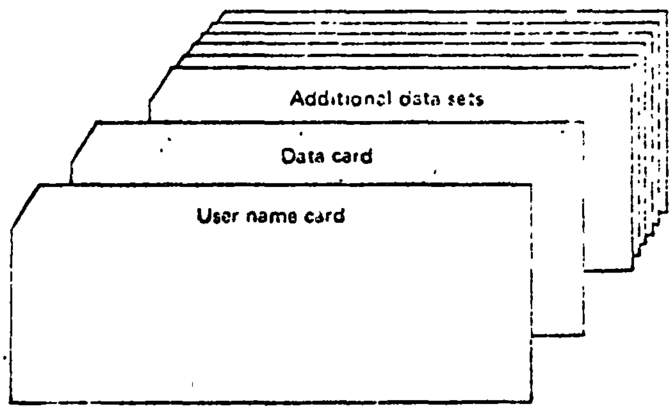
Naddor, E., *Inventory Systems*, New York: Wiley, 1966.

Olsen, R. A., *Manufacturing Management: A Quantitative Approach*, Scranton, Pa.: International Textbook Company, 1968.

Plossl, G. W., and O. W. Wright, *Production and Inventory Control*, Englewood Cliffs, N.J.: Prentice-Hall, 1967.

Riggs, J. L., *Production Systems: Planning, Analysis and Control*, New York: Wiley, 1970.

3.5 EOQ DATA DECK STRUCTURE



Data Card	*Card column*	*Format*	*Item*
	1-5	F5.0	annual usage requirement
	6-10	F5.0	ordering cost
	11-15	F5.0	holding cost
	16-20	F5.0	unit price
	21-25	F5.0	shortage cost
	26-30	F5.0	minimum order quantity—first price discount
	31-35	F5.0	unit price—first price discount
	36-40	F5.0	minimum order quantity—second price discount
	41-45	F5.0	unit price—second price discount
	46-50	F5.0	maximum warehouse space available

```
06704/7700   F O R T R A N   C O M P I L A T I O N   D A T E   d.c.

C*****************************************************************************
C*                                                                           *
C*                                                                           *
C*                       PROGRAMA IOCOE.                                     *
C*                                                                           *
C*          MODELO CUANTITATIVO DE ORDEN ECONOMICO.                          *
C*          PROGRAMA PREPARADO POR LA SECCION DE COMPUTACION                 *
C*          DE LA FACULTAD DE INGENIERIA DE LA ** U.N.A.M. **                *
C*          OCTUBRE DE  1974                                                 *
C*                                                                           *
C*                                                                           *
C*                                                                           *
C*              VARIABLES QUE INTERVIENEN EN EL PROGRAMA                     *
C*                                                                           *
C*     R= REQUERIMIENTO DEL USO ANUAL, NIVEL DE DEMANDA                      *
C*     CP= COSTO DE UNA ORDEN DE COMPRA.                                     *
C*     FH= COSTO DADO (SOSTENIDO) COMO PORCENTAJE DEL                        *
C*     PRECIO UNITARIO                                                       *
C*     P(1)= PRECIO POR UNIDAD ANTES DEL DESCUENTO, P(1)=P1                  *
C*     P(2)= PRECIO POR UNIDAD EN EL MOMENTO QUE OCURRE EL PRIMER            *
C*     DESCUENTO.                                                            *
C*     P(3)= PRECIO POR UNIDAD DESPUES DEL PRIMER DESCUENTO,                 *
C*     P(3)= P2                                                              *
C*     P(4)= PRECIO POR UNIDAD EN EL MOMENTO QUE OCURRE EL                   *
C*     SEGUNDO DESCUENTO.                                                    *
C*     P(5)= PRECIO POR UNIDAD DESPUES DEL SEGUNDO DESCUENTO,                *
C*     P(5)= P3                                                              *
C*     P(6)= RESTRICCION: PRECIO POR UNIDAD A LA CAPACIDAD                   *
C*     DE LA BODEGA.                                                         *
C*     ECOO= LOTE ECONOMICO A SER ORDENADO.                                  *
C*     B(1)= PUNTO DONDE OCURRE EL PRIMER DESCUENTO, B(1)=B1                 *
C*     B(2)= PUNTO DONDE OCURRE EL SEGUNDO DESCUENTO, B(2)=B2                *
C*     CS= COSTO EN CASO DE ESCASEZ, (DEFICIT).                              *
C*     W= ESPACIO MAXIMO DISPONIBLE EN LA BODEGA.                            *
C*     TCST= COSTO TOTAL DEL LOTE ECONOMICO.                                 *
C*     ON= NUMERO DE ORDENES POR AÑO DEL LOTE ECONOMICO.                     *
C*     Q(1)= LOTE ECONOMICO PARA P(1)                                        *
C*     Q(3)= LOTE ECONOMICO PARA P(3)                                        *
C*     Q(5)= LOTE ECONOMICO PARA P(5)                                        *
C*     Q(2)= B(1)                                                            *
C*     Q(4)= B(2)                                                            *
C*     Q(6)= W PARA EL CASO EN QUE CS=0                                      *
C*     Q(6)= LOTE OPTIMO PARA ENV=(6) = W CUANDO CS NO ES 0.                 *
C*     TC(I)= COSTO TOTAL.                                                   *
C*     TC(I)= COSTO TOTAL PARA Q(I) Y ENV(I) PARA I=1,6                      *
C*     ENV(I)= NIVEL ECONOMICO DE INVENTARIO PARA CUANDO EL LOTE             *
C*     ECONOMICO = Q(I)  I=1,5                                               *
C*     ENV(6)= W                                                             *
C*     ENVT= INVENTARIO OPTIMO CUANDO CS NO ES CERO.                         *
C*                                                                           *
C*                                                                           *
C*****************************************************************************
C
C
C
C      LAS TARJETAS DE DATOS SON DOS :
C      I) IDENTIFICACION, FORMATO  (10A4)
C      II) DATOS : R,CP,FH,P(1),CS,B(1),P(3),B(2),P(5),W, FORMATO (1F5.0
```

```
C
C     AL TERMINAR DE PROCESAR UN JUEGO DE DATOS
C     EL PROGRAMA REGRESA A LEER OTRO JUEGO.
C

      DIMENSION P(6),B(6),ENV(6),TC(6),R(3),ALPHA(10)
C
C     LEE Y ESCRIBE LA PRIMERA TARJETA DE DATOS
C
    1 READ 71,ALPHA

C
C     LEE Y ESCRIBE LA SEGUNDA TARJETA DE DATOS
C
      READ 72,R,CP,FH,P(1),CS,B(1),P(3),B(2),P(5),W
      PRINT 73,ALPHA

      PRINT 76,R,CP,FH,P(1),CS,B(1),P(3),B(2),P(5),W
C
C     CHEQUEO DE DATOS, SI  R=0, TERMINA EL PROGRAMA.
C
      IF (R) 19,100,2
    2 IF (CP) 19,19,3
    3 IF (FH) 19,19,4
    4 IF (P(1)) 19,19,5
    5 IF (B(1)) 19,14,6
    6 IF (P(3)) 19,19,7
    7 IF (B(2)) 19,16,8
    8 IF (B(2)-B(1)) 19,9,9
C
C     DOS PUNTOS DE CAMBIO DE PRECIO.
C
    9 NSEG=3
   10 IF (P(5)) 19,19,11
   11 IF (W) 19,12,13
   12 W=1.E25
   13 B(3)=W
      IF (CS) 19,20,20
C
C     NO HAY PUNTO DE CAMBIO DE PRECIO.
C
   14 NSEG=1
      B(1)=1.E25
      IF (P(3)) 19,15,19
   15 P(3)=P(1)
      IF (B(2)) 19,17,19
C
C     UN PUNTO EN EL CAMBIO DE PRECIO.
C
   16 NSEG=2
   17 B(2)=1.E25
      IF (P(5)) 19,18,19
   18 P(5)=P(3)
      GO TO 10
C
C     ERROR EN LOS DATOS
C
   19 PRINT 77
```

```
   20 P(4)=P(5)
      P(2)=P(5)
      IF (P(5)-P(3)) 22,22,21
   21 P(4)=P(3)
   22 IF (P(3)-P(1)) 24,24,23
   23 P(2)=P(1)
   24 P(6)=P(2)
      IF ([illegible]-P(1)) 28,30,25
   25 IF ([illegible]-P(2)) 27,29,26
   26 P(6)=P(5)
      GO TO 30
   27 P(6)=P(3)
      GO TO 30
   28 P(6)=P(1)
      GO TO 30
   29 P(6)=P(4)
   30 Q(2)=Q(1)
      Q(4)=Q(2)
      Q(6)=Q(3)
      IF (CS) 31,31,61
C
C     NO SE PERMITEN DEFICITS.
C
   31 DO 32 I=1,3
      J=2*I-1
C
C     CALCULA Q(I) I=1,3,5
C
   32 Q(J)=SQRT(2.*CP*R/(FH*P(J)))
      DO 35 I=1,6
      IF (Q(I)-1.E25) 33,34,33
C
C     CALCULA TC(I)  I=1,6
C
   33 TC(I)=(CP*R/Q(I)+P(I)*R+P(I)*FH*Q(I)/2.)
      GO TO 35
   34 TC(I)=1.E25
   35 CONTINUE
      DO 36 I=1,6
   36 FMV(I)=Q(I)
C
C     PRUEBA DE FACTIBILIDAD CON RESPECTO A LOS PUNTOS DE
C     CAMBIO DE PRECIO.
C
   37 IF (Q(1)-Q(2)) 39,39,38
   38 TC(1)=1.E25
   39 IF (Q(2)-Q(3)) 40,40,41
   40 IF (Q(3)-Q(4)) 42,42,41
   41 TC(3)=1.E25
   42 IF (Q(4)-Q(5)) 44,44,43
   43 TC(5)=1.E25
C
C     ENCUENTRE EL LOTE OPTIMO SIN DEFICIT.
C
   44 KFLB=1
      TCST=TC(1)
      FCUQ=Q(1)
      FMVT=FMV(1)
      DO 46 I=1,5
```

```
      ... I-TC(I)) 45,46,45
   45 TCST=TC(I)
      ...
      ...=...()
      ...V(I)
   46 ...
      ... ZECO...
C
C     IMPRIMA RESULTADOS ANTES DE LAS LIMITACIONES POR DEFICIT.
C
      PRINT 72
      (...-1.E25) 47,48,47
   47 ... 79
   48 PRINT ... ,ECON
      IF (CS) 49,50,49
   49 PRINT 90,ENVT
   50 PRINT 81,P(KFLB)
      PRINT 82,TCST
      PRINT 83,ON
      IF (...-1.E25) 51,1,51
C
C     PRUEBA DE FACTIBILIDAD CON RESPECTO A LAS LIMITACIONES DEL
C     DEFICIT.
C
   51 DO 53 I=1,S
      IF (ENV(I)-W) 53,53,52
   52 TC(I)=1.E25
   53 CONTINUE
      IF (TC(KFLB)-1.E25) 54,55,...
   54 PRINT 84
      GO TO 1
C
C     ENCUENTRE EL LOTE OPTIMO, CON LAS LIMITACIONES DEL DEFICIT.
C
   55 KFLB=1
      TCST=TC(1)
      ECON=Q(1)
      ENVT=ENV(1)
      DO 57 I=1,S
      IF (TCST-TC(I)) 57,57,56
   56 TCST=TC(I)
      ECON=Q(I)
      KFLB=I
      ENVT=ENV(I)
   57 CONTINUE
      ON=R/ECON
C
C     IMPRIMA LOS RESULTADOS DESPUES DE LAS LIMITACIONES DEL
C     DEFICIT
C
      PRINT 85
      PRINT 86
      PRINT 87
      PRINT 78
      PRINT 88
      PRINT 80,ECON
      IF (CS) 58,59,58
   58 PRINT 90,ENVT
   59 CONTINUE
      PRINT 81,P(KFLB)
```

```
      PRINT 82,TCST
      PRINT 83,ON
      IF (NFLG-6) 1,60,1
   60 PRINT 89
      GO TO 1
C
C     DEFICITS PERMITIDOS.
C
   61 ENV(6)=Q(6)
      DO 62 I=1,5,2
      ENV(I)=SQRT(2.*CP*R*CS/(FH*P(I)*(FH*P(I)+CS)))
   62 Q(I)=SQRT((2.*CP*R*(FH*P(I)+CS))/(FH*P(I)*CS))
      DO 65 I=1,2
      J=2*I
      IF (Q(J)-1.E25) 64,63,64
   63 ENV(J)=1.E25
      GO TO 65
   64 ENV(J)=(Q(J)*CS)/(P(J)*FH+CS)
   65 CONTINUE
      IF (ENV(6)-1.E25) 66,67,66
   66 Q(6)=SQRT((2.*CP*R+ENV(6)**2*(CS+P(6)*FH))/CS)
   67 DO 70 I=1,6
      IF (Q(I)-1.E25) 68,69,68
   68 TC(I)=(CP*R/Q(I)+FH*P(I)*ENV(I)**2/(2.*Q(I))+R*P(I)+(CS*(Q(I)-E
     1NV(I))**2)/(2.*Q(I)))
      GO TO 70
   69 TC(I)=1.E25
   70 CONTINUE
      GO TO 37
C
C
C     TABLA DE FORMATOS.
C
C
   71 FORMAT (10A4)
   72 FORMAT (11F5.0)
   73 FORMAT (42H1                ----------- PROGRAMA MOCOE PARA ,10A4,
     112H ----------///
     248H ********** LOS DATOS DE ENTRADA SON ********** //
     362H         R         CP         FH         P1         CS         B1         P2,
     427H         B2         P3         W )
   76 FORMAT (5X,F6.0,1X,9F9.2)
   77 FORMAT (49H0ERROR EN LOS DATOS DE ENTRADA, VERIFICA Y PRUEBA,
     110H OTRA VEZ )
   78 FORMAT (54H0*************** LOS RESULTADOS DEL ANALISIS SON *****,
     111H*********** )
   79 FORMAT (48H0ANTES DE APLICAR EL LIMITE DE ALMACENAMIENTO DE,
     111H LA BODEGA )
   80 FORMAT (54H LA CANTIDAD OPTIMA ORDENADA ES DE -------------------,
     117H-----------------,F10.2)
   81 FORMAT (54H AL PRECIO UNITARIO DE ------------------------------,
     117H-----------------,F10.2)
   82 FORMAT (54H OBTENIENDO UN COSTO TOTAL DE INVENTARIO DE ---------,
     117H-----------------,F10.2)
   83 FORMAT (54H DONDE EL NUMERO DE CICLOS DE ORDENES POR A  OS D   --,
     117H-----------------,F10.2)
   84 FORMAT (48H0EL LIMITE DE LA BODEGA NO TUVO EFECTO EN MOCOE )
   85 FORMAT (51H0LA CANTIDAD ORDENADA ESTA LIMITADA POR EL ESPACIO ,
     120H FISICO DE LA BODEGA )
   86 FORMAT (48H Y ESTA RESTRICCION NO ES OPTIMA. SE ELIMINA  )
```

```
 57 FORMAT (5.H RESTRICCION Y SI [illegible] EL PROGRAMA DE NUEVO, OBSERVE ,
   11OHEL EFECTO.)
 58 FORMAT (53H DESPUES DE HABER [illegible]ICADO LA [illegible] A LA [illegible] )
 59 FORMAT (52H ESTA ORDEN ESTA SUJETA A LA CAPACIDAD MAXIMA DE LA ,
   17HBODEGA)
 60 FORMAT (31H CON UN INVENTARIO OPTIMO DE : ,50X,F10.2)
C
C
 100 CONTINUE
     CALL EXIT
     END
```

---------- PROGRAMA MODOE PARA PROBLEMA CUATRO, CORRE PROGRAMA [illegible]

********** LOS DATOS DE ENTRADA SON **********

P	CP	[illegible]	P1	CS	D1	P2	Z
1000.	5.00	0.10	1.00	0.00	0.00	0.00	0.00

*************** LOS RESULTADOS DEL ANALISIS SON **************

ANTES DE APLICAR EL LIMITE DE ALMACENAMIENTO DE LA BODEGA

LA CANTIDAD OPTIMA ORDENADA ES DE -- [illegible]

AL PRECIO UNITARIO DE -- [illegible]

OBTENIENDO UN COSTO TOTAL DE INVENTARIO DE -- [illegible]

DONDE EL NUMERO DE CICLOS DE ORDENES POR AÑO ES DE -------------------- [illegible]

LA CANTIDAD ORDENADA ESTA LIMITADA POR EL ESPACIO FISICO DE LA BODEGA
Y ESTA RESTRICCION NO ES OPTIMA. SE ELIMINA LA
RESTRICCION Y SE CORRE EL PROGRAMA DE NUEVO, OBSERVE EL EFECTO.

*************** LOS RESULTADOS DEL ANALISIS SON **************

DESPUES DE HABER APLICADO LA LIMITACION A LA BODEGA

LA CANTIDAD OPTIMA ORDENADA ES DE -- [illegible]

AL PRECIO UNITARIO DE -- [illegible]

OBTENIENDO UN COSTO TOTAL DE INVENTARIO DE -- [illegible]

DONDE EL NUMERO DE CICLOS DE ORDENES POR AÑO ES DE -------------------- [illegible]

ESTA ORDEN ESTA SUJETA A LA CAPACIDAD MAXIMA DE LA BODEGA

---------- PROGRAMA [illegible] CIRCO, [illegible]

******** LOS DATOS DE ENTRADA SON **********

[illegible]	CP	F[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
0.	0.00	0.00	0.00	0.00	0.00	0.00	[illegible]	0.

chapter 8

ELEMENTARY INVENTORY MODELS

It is understandable that businessmen are concerned about the problem of inventories It is not uncommon for a manufacturing company to have 25 percent or more of its total invested capital tied up in inventories On December 31, 1969, the TRW Company had 26 percent of its assets in inventories and the Lockheed Aircraft Corporation had over $500,000,000, or about 39 percent of its assets represented by inventories The General Electric Company had nearly $1,482,000,000 and the General Motors Corporation more than $3,700,000,000 in inventories in December, 1969. Naturally, if good inventory management could change any of these totals by as much as even a few percent, we are talking about really big money

The current emphasis in management science began with the analysis of inventory systems. In 1915, F. W. Harris [9] developed the first economic lot size equation, and this was probably the beginning of the use of mathematical models to represent management problems In 1931, F. E. Raymond published his *Quantity and Economy in Manufacture* [14] in which he developed this idea much further, attempting to account for a wide variety of conditions In the postwar period, the management science literature has been filled with analyses of inventory and production control systems partly because of the great interest shown by the government and the military as well as the interest shown by such progressive companies as the Eastman Kodak Company, the Procter and Gamble Company, Johnson and Johnson, and many others

Management Objectives and Costs

It is important that models of inventory systems reflect true incremental costs associated with alternate plans or policies These costs represent "out-of-pocket" expenditures or foregone opportunities of profit Cost figures derived from the normal accounting records usually do not fit the requirements The following types of cost items are often incremental costs in inventory models: Costs depending on the number of lots, production costs, handling and storing costs, cost of shortages, and capital investment costs.

Costs Depending on Number of Lots. In deciding on purchased lot quantities, there are certain clerical costs of preparing purchase orders that are the same regardless of the quantity ordered These costs are important in deriving economic purchase quantities as we shall see, however, the cost figure used must be the true incremental cost of order preparation It is not correct to derive such a figure simply by dividing the total cost of [illegible] purchasing operation by the average number of purchase orders processed A large segment of the total costs of the purchasing operation are fixed regardless of the number of orders issued There is, however, a variable component, and this is the pertinent figure Quantity discounts and shipping costs are other factors which influence the quantity of materials purchased at one time and, therefore, influence the levels of material inventories A question parallel to the purchase quantity occurs within a production system in deciding the size of production orders, that is, the number of units to process at one time Here, the preparation costs are the incremental costs of preparing production orders, setting up machines, and controlling the flow of orders through the shop Intraplant material handling costs affect purchase lot quantities

Production Costs. Some of the components of production costs which have a bearing on inventory models such as set-up, change-over, and material handling costs, have been discussed in the preceding paragraph Certain other incremental costs, however, also have a direct bearing on inventory models For example, overtime premium and the incremental costs of production fluctuation, such as hiring, training [illegible] separation costs need to be balanced against the cost of carrying additional inventory [illegible] In this latter context, system inventories become an important part of the development of production-inventory programs which we cover in Chapter 13

Costs of Handling and Storing Inventory. There are certain incremental costs associated with the level of inventories They are represented by the costs of handling material in and out of inventory and storage costs, such as insurance, taxes, rent, obsolescence, spoilage, and capital costs. These incremental costs are commonly in proportion to inventory levels.

Cost of Shortages. An extremely important cost which never appears on accounting records is the cost of running out of stock. Such costs may appear in several ways. For example, within a production system a part shortage can cause idle labor on a production line or subsequent incremental labor cost to perform operations out of sequence, usually at higher than normal cost. There may be costs of avoiding shortages, such as expediting split lots. Shortage costs can be represented by profit foregone as when impatient customers take their business elsewhere. The realization of the importance of shortage costs raises the question, "What level of service is appropriate?"

Capital Costs. The opportunity cost of capital invested in inventory is an incremental cost of significance in designing inventory models. The cost figure itself is the product of inventory value per unit, the time that the unit is in inventory, and the appropriate interest rate. In general, the appropriate interest rate should reflect the opportunities for the investment of comparable funds within the organization, and, of course, it should not be lower than the cost of borrowed money. Since the funds are tied up in inventories, they cannot be used for the purchase of equipment, buildings, or other profit-producing investments. There is, therefore, an opportunity cost of having funds invested in inventory, and inventory models reflect this cost.

Management Objectives. The overall objective of management is to design policies and decision rules which view inventories in a "systems" context so that the broadly construed set of costs we have discussed generally are minimized. In a production-distribution system, the functions of inventories and their effects on costs are distributed throughout the system from raw material intake through all intermediate stages to the final point of sale. The result is that there are interactions between basic inventory policy and production planning, labor policy, production scheduling, facilities planning, customer service, etc. Although there are some operations which may be regarded as almost purely inventory situations, the most usual structure involves an interaction between what we think of as the limited inventory problem and many of the broad policies for operating the enterprise as a whole. We shall begin our analysis of inventory systems with the more limited and simple concepts and attempt to build a structure of concept and technique which tries to account for many of the interactions with the environment in which inventories exist.

The Classical Inventory Model

The classical inventory model assumes the highly idealized situation shown in Figure 1. Q units are ordered or manufactured at one time. The order is placed when the inventory level falls to a point where the normal usage would just use up the inventory within the fixed procurement lead time. The receipt of the order of lot size Q is perfectly timed so that at just the point in time when the inventory balance falls to zero, the order of size Q is received, the inventory balance is increased by Q units, and the cycle repeats. We will find this model useful in establishing the overall concepts with which we will be dealing. Let us establish the following list of symbols:

TIC = total incremental cost
TIC_0 = total incremental cost of an optimal solution
Q = lot size
Q_0 = optimal lot size
R = annual requirements in units
c_H = inventory holding cost per unit per year
c_P = preparation costs per order
c_S = shortage costs per unit per year
N_0 = number of orders or manufacturing runs per year for an optimal solution

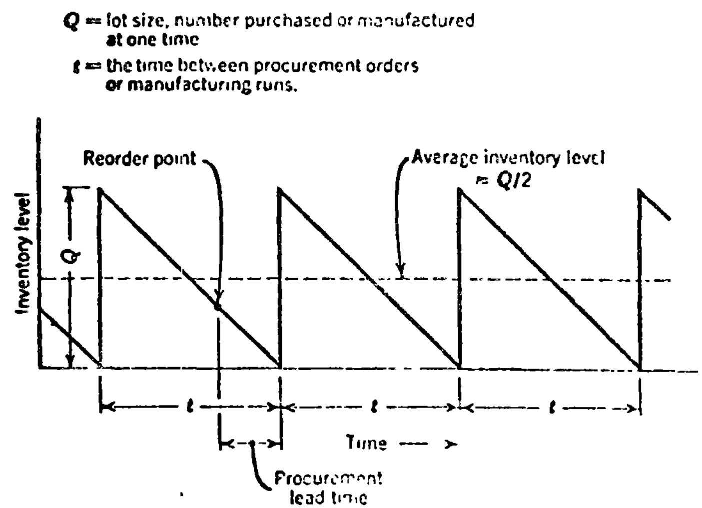


FIGURE 1. Graphic representation of inventory levels in the classical inventory model. Q = lot size, number purchased or manufactured at one time. t = the time between procurement orders or manufacturing runs. Lead time is less than order cycle time.

t = time between orders or manufacturing runs

t_0 = time between orders or manufacturing runs for an optimal solution

Objective. Our objective is to establish a mathematical model which expresses the relationship between Q, the variable under managerial control, and the incremental costs associated with the system. The incremental costs for the simple system we have defined are the costs associated with holding inventory and the costs associated with the procurement of an order of size Q. Therefore, the cost function we wish to minimize is:

$$TIC = \text{inventory holding costs} + \text{preparation costs}$$

We can see from Figure 1 that if Q is increased, the average inventory level, $Q/2$, will increase proportionately. If the inventory holding cost per unit per year is c_H, the annual incremental costs associated with inventory are

$$c_H \frac{Q}{2}$$

If the cost to hold a unit of inventory (interest costs, insurance, taxes, etc.) for a specific example was c_H = \$0.10, we could express the inventory holding cost function as $(0.10Q/2) = 0.05Q$. We could then plot this inventory holding cost function for different values of Q as we have done in Figure 2 curve (*a*).

Similarly, the annual preparation costs depend on the number of times that orders are placed per year and the cost to place an order. The number of orders required for an annual requirement of R will vary with the lot size Q of each order, or, R/Q. If it costs c_P to place an order, the annual preparation costs may be expressed as

$$c_P \frac{R}{Q}$$

If for a specific example, R = 1600 units per year, and c_P = \$5.00, we could express the annual preparation costs as $(5.00 \times 1600/Q) = 8000/Q$. As with inventory holding costs, we can plot the preparation costs for this example for different values of Q, as we have done in Figure 2 curve (*b*).

Figure 2 curve (*c*) shows a graphic model of cost versus lot size, showing the total incremental cost curve, the calculations for which are shown in Table I. Looking at either Table I or Figure 2 curve (*c*), we note that the minimum total incremental cost, TIC_0, occurs when 400 units are ordered

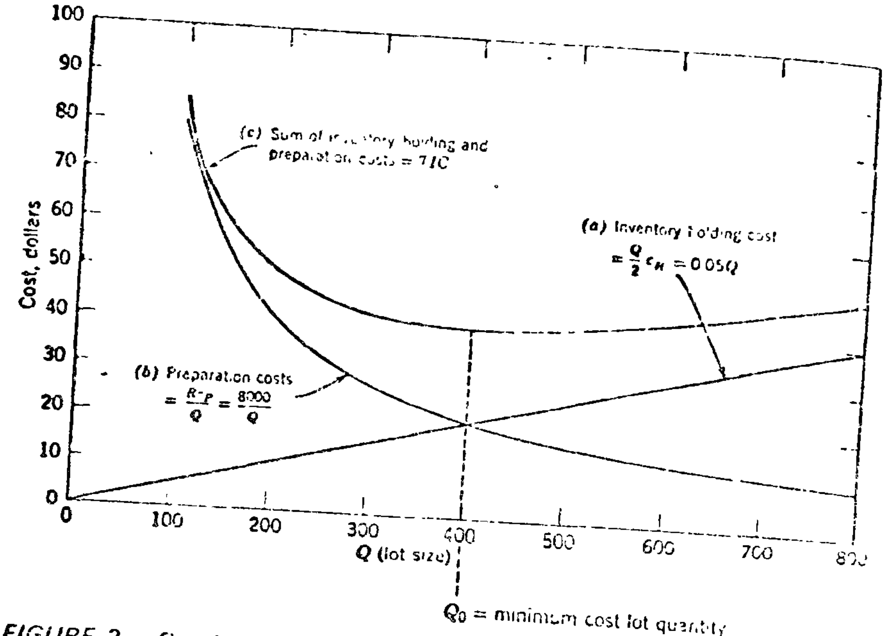


FIGURE 2 Graphic representation of classical inventory model. R = 1600 units per year, c_P = \$5.00, and c_H = \$0.10

TABLE I Computation of points for cost versus lot size for curves of Figure 2. R = 1600 units per year, c_P = \$5.00, and c_H = \$0.10

(1) Lot Size, Q	(2) Inventory Holding Cost $= \frac{Q}{2} \times c_H = 0.05Q$ (See Figure 2a)	(3) Preparation Costs $= \frac{Rc_P}{Q} = \frac{8000}{Q}$ (See Figure 2b)	(4) TIC = Sum of Columns (2) + (3) (See Figure 2c)
100	5.0	80.0	85.0
200	10.0	40.0	50.0
300	15.0	26.7	41.7
400 = Q_0	20.0	20.0	40.0
500	25.0	16.0	41.0
600	30.0	13.3	43.3
700	35.0	11.4	46.4

at one time. This is a solution for the specific data given, and we can see that the general form of the total incremental cost curve has a single minimum point. Note that though the intersection of the preparation and holding cost functions does correspond to the minimum point of the TIC function for this model, this is not generally true.

A General Solution. Regardless of the data used for specific examples, the general form of the curves are similar to those shown in Figure 2, and we can express the relationships in a completely general way,

$$TIC = \frac{c_H Q}{2} + \frac{c_P R}{Q} \qquad (1)$$

This is an equation for the total incremental cost curve, and we wish to find a general expression for Q_0, the lot size associated with the minimum point of the total incremental cost curve. Mathematically, this may be done by finding the value of Q for which the slope of the total incremental cost curve is zero. Using the calculus, the first differential of (1) with respect to Q is:

$$\frac{d(TIC)}{dQ} = \frac{c_H}{2} - \frac{c_P R}{Q^2} \qquad (2)$$

recalling that the rule for differentiation of a simple variable $x = ay^n$ is $dx/dy = nay^{n-1}$. For the first term of (1) the equivalent form which must be differentiated is $c_H Q^1/2$, where $c_H/2$ is equivalent to the constant, a. Therefore $d(TIC)/dQ = (1)(c_H/2)Q^{1-1}$, since $Q^{1-1} = Q^0 = 1$, $d(TIC)/dQ = c_H/2$.

Similarly, the equivalent form of the second term of (1) is

$$c_P R Q^{-1}$$

where $c_P R$ is equivalent to the constant, a. Therefore,

$$\frac{d(TIC)}{dQ} = (-1)(c_P R)Q^{-1-1} = -c_P R Q^{-2} = -\frac{c_P R}{Q^2}$$

The value of equation (2) is, in fact, the slope of the line tangent to the total incremental cost curve. We wish to know the value of Q when this slope is zero; therefore, we set (2) equal to zero, and solve for Q_0:

$$\frac{c_H}{2} - \frac{c_P R}{Q_0^2} = 0$$

$$Q_0^2 = \frac{2c_P R}{c_H}$$

and

$$Q_0 = \sqrt{2c_P R/c_H} \tag{3}$$

The cost of an optimal solution may be derived by substituting the value Q_0, in equation (1)

$$TIC_0 = \frac{c_H Q_0}{2} + c_P \frac{R}{Q_0}$$

$$= \frac{c_H}{2}\sqrt{2c_P R/c_H} + \frac{c_P R}{\sqrt{2c_P R/c_H}}$$

Combining the two terms with the common denominator

$$2\sqrt{2c_P R/c_H}$$

we have

$$TIC_0 = \frac{c_H \times \frac{2c_P R}{c_H} + 2c_P R}{2\sqrt{2c_P R/c_H}} = \frac{2c_P R}{\sqrt{2c_P R/c_H}}$$

and

$$\frac{\sqrt{(2c_P R)^2}}{\sqrt{2c_P R}} \cdot \sqrt{c_H} = \sqrt{2c_P c_H R} \tag{4}$$

The number of orders or manufacturing runs per year N_0 and the time between orders or manufacturing runs t_0 for an optimal solution are

$$N_0 = \frac{R}{Q_0} \tag{5}$$

$$t_0 = \frac{Q_0}{R} = \frac{1}{N_0} \tag{6}$$

If we substitute the values for R, c_P, and c_H used in our example, we obtain

$$Q_0 = \sqrt{2 \times 1600 \times \frac{5.00}{0.10}} = \sqrt{160{,}000} = 400 \text{ units}$$

$$TIC_0 = \sqrt{2 \times 5.00 \times 0.10 \times 1600} = \sqrt{1600} = \$40$$

$$N_0 = \frac{1600}{400} = 4 \text{ orders or manufacturing runs per year}$$

$$t_0 = \frac{1}{4} = 0.25 \text{ years between orders or runs}$$

An Inventory Model with Shortage Costs

If the assumption that back orders are zero in the classical model is relaxed we have the graphical structure of Figure 3. Our problem now is to determine the minimum cost order quantity when shortages are allowed at cost c_S. The inventory level rises to only I_{max} on the receipt of Q because the difference $Q - I_{max}$ is assumed to meet back orders instantaneously.

When shortage costs are accounted for, the classical model becomes slightly more general—the model represented by equation (3) being a special case. The rationale for derivation parallels that given for the simple

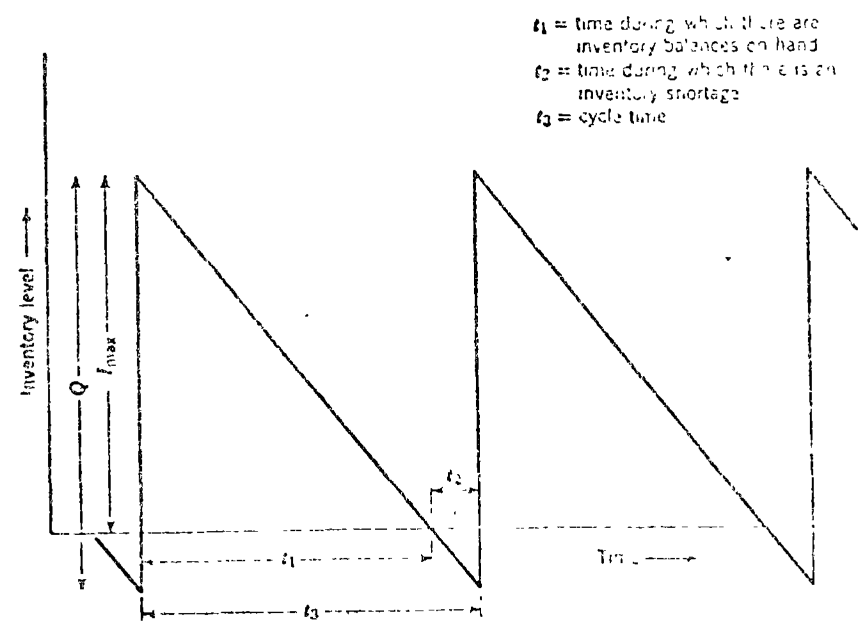


FIGURE 3 Idealized structure of inventory levels with back orders of $Q - I_{max}$ allowed

case, but it is somewhat more complex mathematically. Derivations may be found in references [3, 7], and the resulting formulas are

$$Q_0 = \sqrt{2c_P R/c_H} \cdot \sqrt{(c_H + c_S)/c_S} \quad (7)$$

$$TIC_0 = \sqrt{2c_P c_H R} \cdot \sqrt{c_S/(c_H + c_S)} \quad (8)$$

$$I_{max_0} = \sqrt{2c_P R/c_H} \cdot \sqrt{c_S/(c_H + c_S)} \quad (9)$$

Note that when comparing equations (7) and (8) with the comparable formulas (3) and (4), with shortages, Q_0 is increased by the factor $\sqrt{(c_H + c_S)/c_S}$, and TIC_0 is decreased by the factor $\sqrt{c_S/(c_H + c_S)}$. The influence of shortages, then, is dependent on the relative size of c_H and c_S. If c_H is large relative to c_S, the effect of shortages on Q_0 and TIC_0 is considerable, that is, Q_0 will be increased and TIC_0 decreased compared to equations (3) and (4). If, on the other hand, c_H is small relative to c_S, minor changes in Q_0 and TIC_0 will result. The net effect of shortages costs on Q_0 and TIC_0 may at first seem to be strange. Recognize, however, that when the model permits shortages, average holding costs are reduced because of smaller average inventory balances. This will result in a larger Q_0. For the shortage case, TIC_0 is smaller than when shortages are not included because both holding costs and annual preparation costs are somewhat lower. For example, if we consider shortages in the previous example where R = 1600 per year, c_P = \$5.00 per order, c_H = \$0.10 per unit per year, and in addition, c_S = \$0.50 per unit per year, we have the following results:

$$Q_0 = \sqrt{(2 \times 5.00 \times 1600)/0.10}\,\sqrt{(0.10 + 0.50)/0.50}$$
$$= 400\sqrt{1.2} = 400 \times 1.095 = 438 \text{ units}$$
$$TIC_0 = \sqrt{2 \times 5.00 \times 0.10 \times 1600}\,\sqrt{0.50/(0.10 + 0.50)}$$
$$= 40\sqrt{0.833} = 40 \times 0.912 = \$36.50.$$

The limiting values of c_S provide valuable insight. As c_S becomes infinitely large the factor in equation (7), $\sqrt{(c_H + c_S)/c_S}$, becomes 1 in the limit and we have the classical inventory model of equation (3). This corresponds to a policy of no shortages permitted. On the other hand if c_S is set at zero then the factor and therefore Q_0 becomes zero. This corresponds to a policy of [illegible] month supply [illegible] the basis of [illegible]

The Effect of Quantity Discounts

The basic economic lot size formula assumes a fixed price. When quantity discounts enter the picture, additional simple calculation will determine if there is a net advantage. As an illustration, assume the basic data of our previous example, that is, R = 1600 units per year, c_P = \$5.00 per order, and c_H = 10 percent per year. Recall that the economic order quantity was computed as 400 units. In addition, however, assume that the purchase prices are quoted as \$1.00 per unit in quantities below 800 and \$0.98 per unit in quantities above 800. If we buy in lots of 800 we save \$32 per year on the purchase price plus \$10 in order costs, since only two orders need to be placed per year to satisfy annual needs. This saving of \$42 per year must be greater than the additional inventory costs that would be incurred if the price discount is to be attractive. Under the 400 unit order size, average inventory is 200 units and inventory costs are 200 × 1.0 × 0.10 = \$20. If orders of 800 units were placed, the inventory costs would be 400 × 0.98 × 0.10 = \$39. There is a net gain of 42 − (39 − 20) = \$23 by ordering in lots of 800 instead of in lots of 400. If the vendor had a second price break of \$0.97 per unit for lots of 1600 or more, a similar analysis shows that the incremental inventory costs outweigh the incremental price and order savings, so that there is no net advantage in purchasing in lots of 1600. Table II summarizes the calculation for all three cases.

TABLE II. Incremental cost analysis to determine net advantage or disadvantage when price discounts are offered

R = 1600 Units per Year, c_P = \$5.00 per Order, c_H = 10 percent per Year	Lots of 400 Units, Price = \$1.00 per Unit	Lots of 800 Units, Price = \$0.98 per Unit	Lots of 1600 Units, Price = \$0.97 per Unit
Purchase cost of a year's supply (1600 units)	\$1600	\$1568	\$1552
Ordering cost (c_P = \$5.00 per order)	20	10	5
Inventory holding cost (avg. inv. × unit price adjustment × c_H)	20	39	74
	\$1640	\$1617	\$1631

Formal Models with Price Breaks. We may generalize our ideas about the effect of quantity discounts by examining a formal model which takes price breaks into account. Recall that the lot size equation (3) did not need to consider price or value of the item because for every value of Q considered, the price was the same, that is, price was not an incremental cost. Let us now consider a lot size model which includes the value of the item as a factor. To reflect this idea, the total incremental cost associated with such a system may be expressed as follows:

$$TIC = (\text{annual cost of placing orders}) + (\text{annual purchase cost of } R \text{ items}) + (\text{annual holding cost for inventory})$$

$$= c_P \frac{R}{Q} + kR + k\frac{Q}{2}F_H \qquad (10)$$

where k = cost or price per unit, and F_H = *fraction* of inventory value, representing inventory holding cost on an annual basis ($kF_H = c_H$).

Following the rationale developed previously, we seek the value of Q, Q_0, which minimizes this total incremental cost equation. This leads to

$$Q_0 = \sqrt{2c_P R/kF_H} \qquad (11)$$

$$TIC_0 = \sqrt{2c_P k F_H R} + kR \qquad (12)$$

The derivations of equations (11) and (12) parallel the derivations of equations (3) and (4).

We may now use formulas (11) and (12) in the analysis of inventory systems which involve a price break. For comparison, let us assume the data of Table II for the first price break at $b = 800$ units. Recall that in this example the price per unit below the break point was k_1 = \$1.00 and that above 800 units, the price was k_2 = \$0.98 per unit. To fit in with the present model, we will now express the inventory holding cost factor as F_H = 10 percent of inventory value. The other data remain the same, that is, R = 1600 units per year and c_P = \$5.00 per order.

In the logic of our analysis, let us first note that the total incremental cost curve TIC_2 will fall below the curve TIC_1. This is shown in Figure 4. The logical thing to do, then, is to calculate q_{2_0} to see if it falls within the range P_2 where the price k_2 = \$0.98 applies. Doing this we find that q_{2_0} = 404 units, using equation (11), which is less than the break point b = 800 units. Since 404 [illegible]

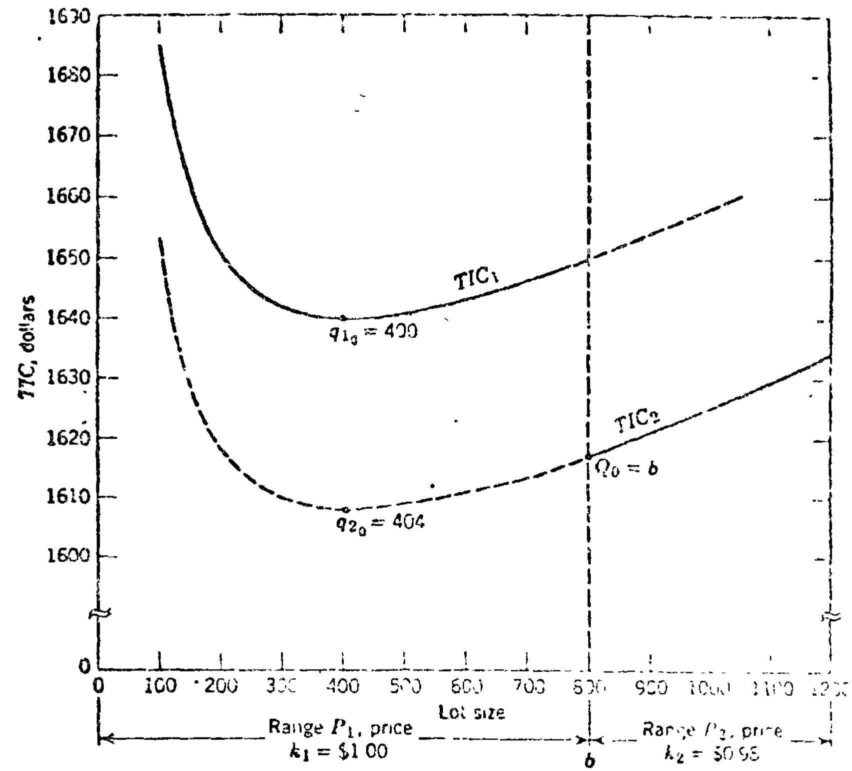


FIGURE 4 Total incremental cost curves for inventory model with one price break at b = 800 units. R = 1600 units per year, c_P = \$5.00, F_H = 10 percent of inventory value

curve, we know that the lowest possible cost of TIC_2 within the range where the price k_2 applies is at the lot size b = 800 units. If it had happened that q_{2_0} was in the range P_2, this would have determined immediately that the economic lot size for the system, Q_0, was the value calculated, q_{2_0}. Since this is not the case, however, we must continue our analysis to see if the minimum point on the curve TIC_1 is below TIC_2 at lot size b = 800 units. We may calculate TIC_{1_0} easily from equation (12) and its value is \$1640. Also, we may calculate TIC_b using equation (10), and this we find to be \$16[illegible]. The decision is now clear, $Q_0 = b = 800$, since TIC_2 at lot size b is less than TIC_{1_0}.

Compare these results with those obtained by the incremental cost analysis in Table II. This of course can be seen easily from the graph of Figure 4. Constructing the curves for each case would be laborious, however, compared to the simple computations required to come to [illegible] decision. Figure 5 shows a decision flow chart for an inventory model with one price break, indicating the flow of calculations and resulting decisions. In some instances, the final result is obtained with one calculation, as when q_{2_0} falls in the lot size range P_2 where the price k_2 is valid. Where this is not

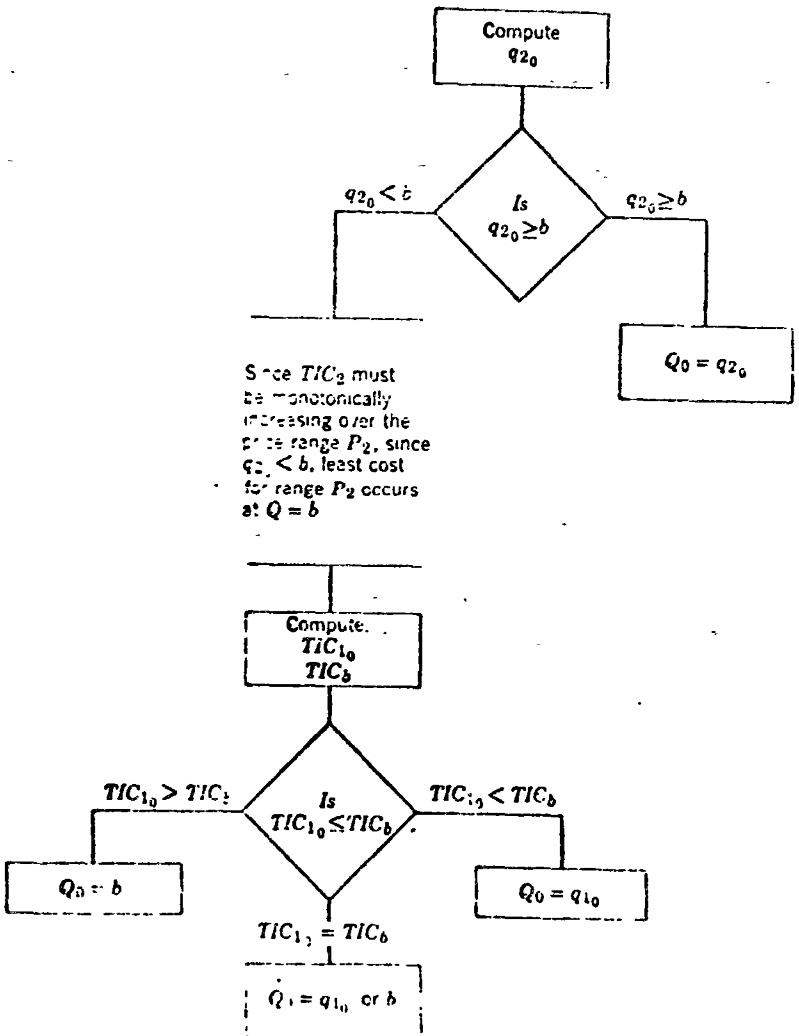


FIGURE 5 ... lot size ...

these cases, simple calculations for comparative total incremental cost yield a final result.

Using the same general rationale we can develop decision processes for inventory models with two or more price breaks. Also, models could be constructed for quantity discount situations that also took account of other factors, such as shortage costs and the value added into inventory through the accumulation of preparation costs [4, 8].

Determining the Length of Production Runs

Production order quantities and runs are based on the same general concepts as purchase order quantities, as we have noted previously. But the assumption that the order is received and placed into inventory all at one time is often not true in manufacturing runs. For many manufacturing situations the production of the total order quantity Q takes place over a period of time, and the parts go into inventory not in one large batch, but in smaller quantities as production continues. This results in an inventory pattern similar to Figure 6 when the run extends over a considerable period of time. When the run time is perhaps 30–60 percent of the total cycle time t shown in Figure 6, the effect on the average inventory of the system should be accounted for. Let r = daily usage rate and p = daily production rate--assuming, of course, that $p > r$. Other symbols remain as previously defined. During the production period t_p inventory is accumulating at the rate of the difference between production rate and usage rate, $p - r$. This rate of increase continues for the production period t_p, so that the peak inventory is $t_p(p - r)$, and the average inventory is

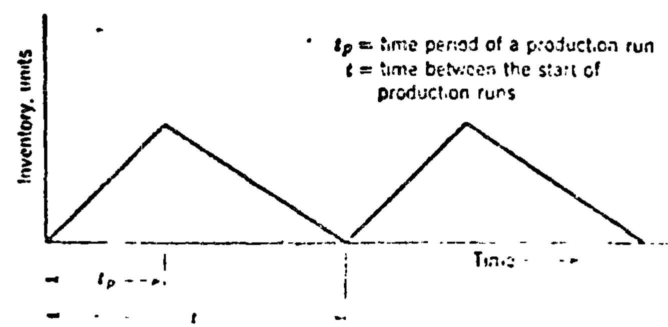


FIGURE 6 Diagram of inventory balance in relation to ...

and since n is the same for all products, the total annual setup cost is

$$n \sum_{i=1}^{m} c_{P_i} \tag{22}$$

The total incremental cost associated with the entire set of m products is then

$$TIC = \text{annual setup cost} + \text{annual inventory holding cost}$$

$$= n \sum_{i=1}^{m} c_{P_i} + \frac{1}{2n} \sum_{i=1}^{m} c_{H_i} R_i \left(1 - \frac{r_i}{p_i}\right) \tag{23}$$

Our objective is to determine the minimum of the TIC curve with respect to n, the number of production runs. The first derivative of TIC with respect to n which we set equal to zero is

$$\frac{d(TIC)}{dn} = \sum_{i=1}^{m} c_{P_i} - \frac{1}{2n^2} \sum_{i=1}^{m} c_{H_i} R_i \left(1 - \frac{r_i}{p_i}\right) = 0$$

and the optimal number of runs, N_0 is

$$N_0 = \sqrt{\frac{\sum_{i=1}^{m} c_{H_i} R_i (1 - r_i/p_i)}{2 \sum_{i=1}^{m} c_{P_i}}} \tag{24}$$

The total cost of an optimal solution is found by substituting N_0 for n in (23), or

$$TIC_0 = N_0 \sum_{i=1}^{m} c_{P_i} + \frac{1}{2N_0} \sum_{i=1}^{m} c_{H_i} R_i \left(1 - \frac{r_i}{p_i}\right)$$

Substituting the expression for N_0 shown in (24) and simplifying leads to

$$TIC_0 = \sqrt{2 \sum_{i=1}^{m} c_{P_i} \sum_{i=1}^{m} c_{H_i} R_i \left(1 - \frac{r_i}{p_i}\right)} \tag{25}$$

Where R_i [illegible] for the individual products, r_i [illegible] lent [illegible] for the [illegible] daily [illegible]

per unit, per year for the individual products, c_{P_i} = setup costs per [illegible] for the individual products, and m = the number of products.

Let us work out an example for the determination of the cycle length for the group of ten products shown in Table III, which shows the annual sales requirements, sales per production day, daily production rate, production days required, annual inventory holding costs, and setup costs

TABLE III. Sales, production, and cost data for ten products to be run on the same equipment

(1) Product Number	(2) Annual Sales, Units R_i	(3) Sales per Production Day (250 days per year) r_i	(4) Daily Production Rate p_i	(5) Production Days Required	(6) Annual Inventory Holding Cost c_{H_i}	(7) Setup Cost per Run c_{P_i}
1	10,000	40	250	40	$0.05	$ [illegible]
2	20,000	80	500	40	0.10	15
3	5,000	20	200	25	0.15	35
4	13,000	52	600	21.7	0.02	[illegible]
5	7,000	28	1000	7	0.30	25
6	8,000	32	800	10	0.40	37
7	15,000	60	500	30	0.02	42
8	17,000	68	500	34	0.05	50
9	3,000	12	200	15	0.35	10
10	1,000	4	125	8	0.10	12
				230.7		$252

Table IV then shows the calculation of the number of runs per year calculated by equation (24). The minimum cost number of cycles which results in four per year, each cycle lasting approximately 59 days and producing one-fourth of the sales requirements during each run. The total incremental cost from equation (25) is TIC_0 = $2420.

It is interesting to compare now the jointly determined number of [illegible] per year with the number that would have resulted had runs been [illegible] mined independently for each of the ten products. Table V summarizes these calculations. Note that products 4, 7, and 10 would have two or fewer runs per year, and products 2, 5, and 6 would have more than six runs per year. Magee [10] states a rule of thumb that if "the minimum cost number of runs for the product alone, for any one or more products [illegible] half the value for all products, the product is a possible can[illegible]

TABLE IV Determination of the number of runs, jointly, for ten products from equation (24)

(1) Product Number	(2) Ratio r_i/p_i Col 3/Col 4 from Table III	(3) $(1 - r_i/p_i)$	(4) $c_{H_i}R_i$ = Col 2 × Col. 6 from Table III	(5) $c_{H_i}R_i(1 - r_i/p_i)$ = Col 3 × Col. 4	(6) c_{P_i}, Col 7 from Table III
1	0.160	0.840	$ 500	$ 420	$ 20
2	0.160	0.840	2000	1,680	15
3	0.100	0.900	750	675	35
4	0.087	0.913	260	237	40
5	0.028	0.972	2100	2,041	25
6	0.040	0.960	3200	3,072	37
7	0.120	0.880	300	264	42
8	0.136	0.864	850	734	50
9	0.060	0.940	1050	987	16
10	0.032	0.968	100	97	12
				$10,207	$292

$$N_0 = \sqrt{\frac{10{,}207}{2 \times 292}} \approx 4 \text{ cycles per year}$$

for only occasional runs." Table V also summarizes the total incremental cost which would result if the number of runs for each product were determined independently. The figure of $1932 is $488 less than the total incremental cost figure of $2420 given by equation (25) when runs are determined jointly. The apparent cost saving through individual determination of production runs is, of course, illusory because it does not take account of congestion costs or possible shortage costs that might result from independent scheduling. On the other hand, at low shop loads the interferences and schedule conflicts should not appear with independent scheduling.

SUMMARY

The models devel[illegible] this chapter are meant to b[illegible] a general conceptual framework [illegible] of inventory systems. Althou[illegible]

TABLE V Calculation of runs, independently for each product from equation (17)

(1) Product Number	(2) $c_{H_i}R_i(1 - r_i/p_i)$ from Col 5, Table IV	(3) c_{P_i}, from Col 7, Table III	(4) $\frac{\text{Col 2}}{2 \times \text{Col. 3}}$	(5) $N_i = \sqrt{\text{Col 4}}$	(6) TIC_0
1	420	$20	10.5	3.2	$ 130
2	1680	15	56.0	7.5	224
3	675	35	9.7	3.1	21[illegible]
4	237	40	3.0	1.7	13[illegible]
5	2041	25	40.8	6.4	10[illegible]
6	3072	37	42.7	6.5	477
7	264	42	3.1	1.8	149
8	734	50	7.3	2.7	271
9	987	16	30.8	5.6	178
10	97	12	4.0	2.0	48
					$1932

may certainly be useful in some situations, they are not meant to be transplanted without modification into practical situations. Rather, they are meant to show some of the kinds of situations and factors that have been accounted for in simple inventory models. Actually, many more situations have been covered in the literature [2, 4, 6, 7, 8, 13, 15, 17, 19, [illegible]]. With a knowledge of the general functions of inventories, management objectives, and the nature of costs which enter inventory models, we are in a position to consider the influence of variability of demand and basic systems of control which take account of these risks, as well as the effects on inventory planning of production planning and seasonal sales patterns.

REVIEW QUESTIONS

1. What is the nature of costs affected by inventories? Outline them and discuss each.

2. What are the kinds of costs related to inventories but dependent on lot [illegible]? In a practical situation, how do we derive these costs?

3. What are management's objectives in designing inventory systems? In the classical inventory model, which of the variables are controllable and which are outside the control of management?

4. What is the general effect of shortage costs on lot sizes?

5. Why must the classical lot size formula be modified if we are attempting to take quantity discounts into account?

6. Outline the rationale for determining the minimum cost purchase quantity Q_0 when a price discount is involved.

7. How is the determination of a production run different from the determination of a purchase lot size?

8. How does the production run problem change when a number of products share the use of the same equipment on a cyclical basis? Is the problem the same when the operating level is somewhat below capacity?

PROBLEMS

1. Compute the optimal lot size, Q_0, when R = 10,000 units per year, c_P = \$5, and c_H = \$10 per unit per year.

2. What is the total incremental cost for the conditions of Problem 1?

3. How much would Q_0 change if our estimate of c_P was in error and was actually only \$4 in Problem 1? What would be the difference in actual total incremental costs between the two solutions?

4. How much would Q_0 change if our estimate of c_H was in error, being only \$8, in Problem 1? What would be the difference in actual total incremental costs between the two solutions?

5. What is the effect on Q_0 for Problem 1 if shortages cost c_S = \$1 per unit per year? What is the total incremental cost of this solution?

6. Suppose that shortages are very expensive, perhaps \$100 per unit per year. What is the answer to Problem 5?

7. Suppose that for Problem 1 a price discount is offered so that orders placed in quantities below 125 cost \$100 each but for orders of 125 or above this quantity the price is \$95 each. Inventory holding cost is now expressed as 10 percent of the value of the item. In what quantities should the items be purchased? Use the rationale of Figure 5.

8. Determine [illegible] of production runs for an item if R = 15,000 units per year, c_P = \$25, [illegible] per unit per year, and p = 100 units per working day. There are 250 working days [illegible]

9. Dete[illegible] for the [illegible] assuming 250 w[illegible]

Product Number	Annual Sales Units	Daily Production Rate	Annual Holding Cost per Unit	Setup Cost per Run
1	5,000	100	\$1.00	\$40
2	10,000	75	0.90	25
3	7,000	50	0.30	30
4	15,000	80	0.75	27
5	4,000	40	1.05	80
Total				\$202

10. Carson Manufacturing Co. finds ordering costs for its materials and supplies fall into three major categories depending on urgency and the ordering amount of follow up required. It therefore wishes to simplify its use of [illegible] (3) for use by ordering clerks. For class 1, 2, and 3 items ordering costs are respectively \$5, \$15, and \$40.

(*a*) Derive formulas for the three classes of items.

(*b*) Further examination shows that inventory carrying costs [illegible] at 18 percent of cost value for all items. Derive further simplified formulas for the three classes of items.

11. Carson Manufacturing Co. converted its entire ordering procedure to the EOQ basis described by problem 10. On examining one of the Class 3 items (c_P = \$40), however, they noted very high annual freight costs under the new policy. Freight costs have been \$200 per order under the EOQ policy and would cost only \$400 for a carload lot of 500 units. R = 5000 units per year, and the average value of the item is \$222.22. Should Carson order in carload lots?

REFERENCES

1. Brown, R. G., *Decision Rules for Inventory Management*, Holt, Rinehart and Winston, 1967.
2. Buchan, J., and E. Koenigsberg, *Scientific Inventory Management*, Prentice-Hall, Englewood Cliffs, N.J., 1963.
3. Buffa, E. S., and W. H. Taubert, *Production-Inventory Systems: Planning and Control*, Richard D. Irwin, Inc., Homewood, Ill., Rev. ed., 1972.
4. Churchman, C. W., R. L. Ackoff, and E. L. Arnoff, *Introduction to Operations Research*, John Wiley & Sons, New York, 1957.
5. Eilon, S., *Elements of Production Planning and Control*, Macmillan Company, New York, 1962.
6. Fetter, R. B., and W. C. Dalleck, *Decision Models for Inventory Management*, Richard D. Irwin, Homewood, Ill., 1961.

7. Hadley, G., and T. M. Whitin, *Analysis of Inventory Systems*, Prentice-Hall, Englewood Cliffs, N.J., 1963.
8. Hanssmann, F., *Operations Research in Production and Inventory Control*, John Wiley & Sons, New York, 1962.
9. Harris, F., *Operations and Cost* (Factory Management series), A. W. Shaw Company, Chicago, Ill., 1915, pp. 48-52.
10. Magee, J. F., and D. M. Boodman, *Production Planning and Inventory Control*, McGraw-Hill Book Company, New York, 2nd ed., 1967.
11. McMillan, C., and R. F. Gonzalez, *Systems Analysis - A Computer Approach To Decision Models*, Richard D. Irwin, Inc., Homewood, Ill., Rev. ed., 1968, Chap. 6.
12. Moore, F. G., and R. Jablonski, *Production Control*, McGraw-Hill Company, New York, 3rd ed., 1969.
13. Naddor, E., *Inventory Systems*, John Wiley & Sons, New York, 1966.
14. Raymond, F. E., *Quantity and Economy in Manufacture*, D. Van Nostrand Company, Princeton, N.J., 1931.
15. Starr, M. K., and D. W. Miller, *Inventory Control: Theory and Practice*, Prentice-Hall, Englewood Cliffs, N.J., 1962.
16. Thierauf, R. J., and R. A. Grosse, *Decision Making Through Operations Research*, John Wiley & Sons, Inc., New York, 1970, Chap. 7.
17. Veinott, A. F., "The Status of Mathematical Inventory Theory," *Management Science*, Vol. 12, No. 11, July 1966 (includes an exhaustive bibliography at end of paper).
18. Voris, W., *Production Control*, Richard D. Irwin, Homewood, Ill., 3rd ed., 1966.
19. Wagner, H. N., *Statistical Management of Inventory Systems*, John Wiley & Sons, New York, 1962.
20. Whitin, T. M., *The Theory of Inventory Management*, Princeton University Press, Princeton, N.J., 1953.

chapter 9

INVENTORY CONTROL SYSTEMS

Some of the major defects in the models developed in the previous chapter, so far as practical inventory policy is concerned, are the assumptions that requirements were known exactly and that the delivery of replenishment orders was perfectly timed. Also, those models did not place the inventory system in the context of the operating environment of the broader production-distribution system. In this chapter we shall attempt to introduce the idea of variability of demand and its influence on inventory policy, consider comparative systems for inventory control which take account of demand variability, and consider the impact of inventories on the problem of controlling production levels. In part V we shall develop models which focus on the impact of inventories in making aggregate production plans or programs, particularly in Chapter 13.

Variability of Demand

The source of our problem in dealing with variability of demands or requirements is focused on the lags inherent in the system for replenishment. If we could fill requirements immediately, there would be no problem. The elements of the problems caused by lags in the system were introduced in Chapter 7.

As we know, the demand for an item may vary considerably due to random causes, upward or downward trends in demand seasonal and cyclic variations. Figure 1 shows a sales curve which demonstrates three of these factors (cyclic variations dealing with the concept of the business cycle are beyond our scope). Let us begin with a consideration of expected random variation and how realistic inventory policy might take it into account. Figure 2 abstracts from Figure 1 just the random variations in sales from average expected levels. Such a distribution could be abstracted from sales records from which the trend and seasonal factors have been removed, through commonly known statistical procedures. The residual variation is then simply the variation due to chance causes, comparable in every way to expected random variation in any process.

Buffer Stocks. The variations in demand are absorbed through the provision of buffer stocks which must be maintained because of our inability to forecast random variations in demand of the type shown by Figure 2. The size of these planned extra inventories depends on the stability of demand in relation to our willingness to run out of stock. If we are determined almost never to run out of stock, these planned minimum balances must be very high. If service requirements permit stock runouts and back ordering, the safety stocks can be moderate. Figure 3 shows the general structure of inventory balance with a fixed-order quantity system.

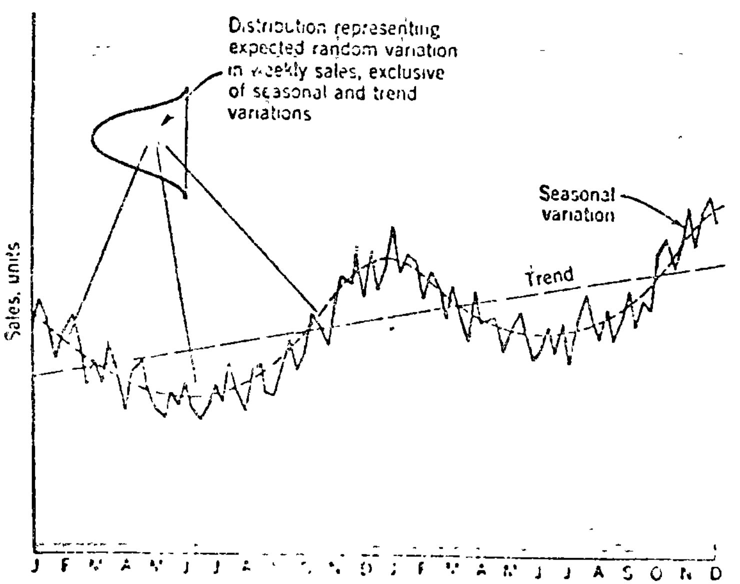


FIGURE 1 [illegible] in demand which [illegible] inventory [illegible]

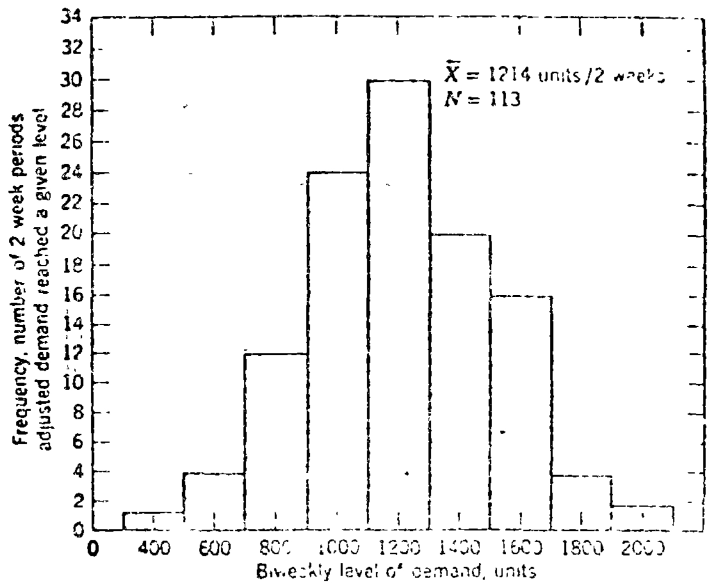


FIGURE 2 Distribution representing expected random variation in weekly sales, exclusive of seasonal and trend variations

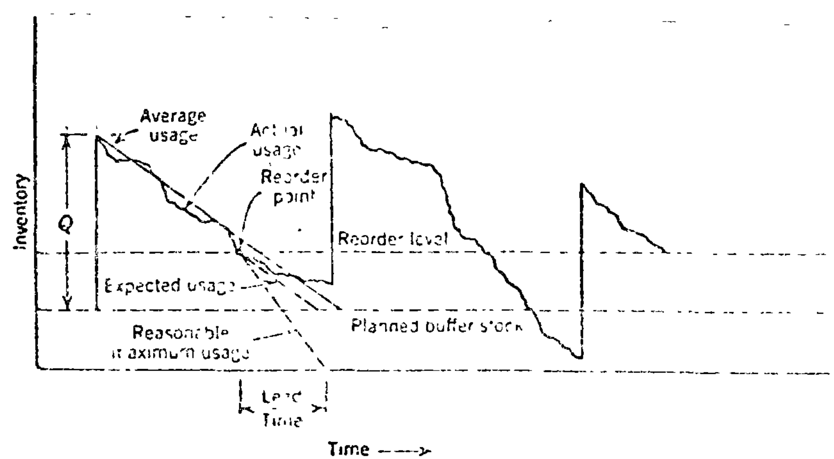


FIGURE 3 Structure of inventory balance for a fixed order quantity system [illegible] stocks to absorb fluctuations in demand and in supply time. The [illegible] level is set so that a reasonable figure for maximum usage would [illegible] inventory to zero during the lead time. [illegible] fixed quantity [illegible]

The buffer stock level is set so that inventory balances would be drawn down to zero during the constant lead time for supply, if we should experience near-maximum demand

The rational determination of buffer stocks, then, turns on a knowledge of the probability distribution of demand together with a decision regarding the risk of stock runout that we are willing to accept. To be most useful, the probability distribution of demand can be expressed in a form shown by Figure 4. Figure 4 was constructed from Figure 2, first, by plotting the number of periods that adjusted demand exceeded a given level, second, by establishing a percentage scale to represent a derived probability scale, and third, by idealizing the distribution as shown by the dashed curve of Figure 4. Since the approximate average two-week usage is 1214 units, and

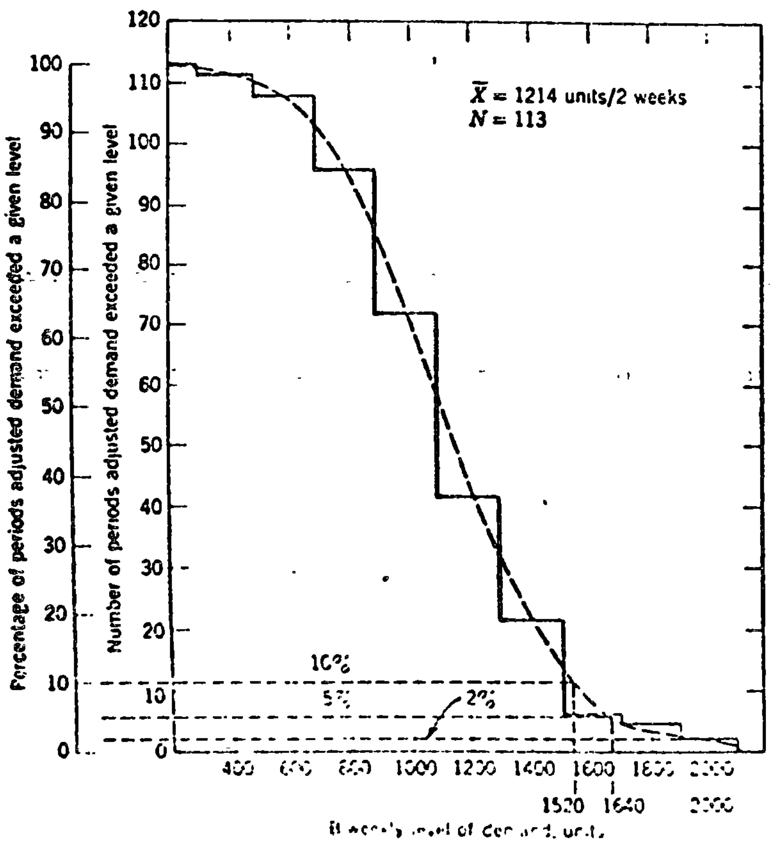


FIGURE 4 [illegible]

assuming a normal lead time of 2 weeks, we could be 90 percent sure of not running out of stock by having the 1520 units on hand when the replenishment order is placed The buffer stock is then 1520 - 1214 = 306 units. If we wish to be 95 percent sure of not running out of stock, the buffer stock must be 1640 - 1214 = 426 units. Similarly, to be sure that [illegible] only a 2 percent risk of running out of stock, the buffer stock [illegible] be increased to 768 units.

It is easy to see from the shape of the demand curve that, for high levels of protection, the buffer stock required goes up rapidly, [illegible] the cost of providing this assurance goes up. This is shown [illegible] the [illegible]lations in Table I where we have assumed the demand [illegible] of [illegible] 4, assigning a value of $50 to the item and inventory holding [illegible] 20 percent of value. The average inventory required to [illegible] expected maximum usage rates during the lead time of 2 weeks is calculated for the three service levels shown. To offer service at the 95 percent level instead of the 90 percent level requires an incremental $1200 per year, but to move to the 98 percent level of service from the 95 percent level requires an add[illegible] $3600 in inventory cost.

The demand curve, then, provides a rational basis for the determ[illegible] of buffer stock levels by helping to establish a reasonable maximum [illegible] rate during the lead time. To establish this rate, however, management must decide what risk of stock runout is acceptable. In some instances this must be a judgment, but where a cost of shortages can be realistically assigned, a simple incremental cost analysis can determine whether ad[illegible]tional protection is worthwhile. For example, for the data shown in Table I, there would be an incremental saving of $3156 in moving from the 90

TABLE I. Cost of providing the three levels of service shown in figure 4, when the item is valued at $50 each and inventory holding costs are 2[illegible]

	Service Level		
	90%	95%	9[illegible]
Expected maximum usage for 2-week replenishment time	1520	1640	2000
Buffer stock required	306	426	76[illegible]
Average inventory required for service level during replenish[illegible]ent period = ([illegible] − Buffer)/2 + Buffer	913	1033	1393
[illegible] of average inventory at $50 per unit	$45,650	$51,650	$6[illegible]
[illegible] cost at 20%	$ 9,130	$10,330	$13[illegible]

percent to the 95 percent level of service if the cost of a shortage was $1 each (1214 × 26 × 0.05 × 1.00). This incremental gain exceeds the incremental cost of $1200 shown in Table 1. On the other hand, to move from the 95 percent to the 98 percent level the incremental gain is only $950 whereas the incremental cost is $3600 as shown in Table 1. The 98 percent level of service is obviously too expensive in this instance.

In summary, we have a fairly general procedure. To determine buffer stocks, we must determine reasonable maximum usage rates during the lead time, and this requires the derivation of a demand distribution which reflects only the variation due to random fluctuations. Here, however, management must decide on a risk level for running out of stock, or if realistic shortage costs can be assigned, an incremental cost study can be made to determine the best risk level. If demand for the item is subject to seasonal variation or an upward or downward trend, the average of the distribution shifts, and it is necessary to reassess buffer stock level periodically. In such an instance, it would be better to express the demand distribution curve shown in Figure 4 in terms of deviations from expected mean values.

Practical Methods for Determining Buffer Stocks

The generalized methodology for setting buffer stocks when lead times are constant (just discussed) is too cumbersome for use in practical systems where large numbers of items may be involved. Computations are simplified considerably if we can justify the assumption that the demand distribution follows some particular mathematical function, such as the normal, Poisson, or negative exponential distributions. The general procedure is the same for all distributions: (*a*) determine the applicability of the normal, Poisson, or negative exponential distribution of demand *during lead time*, (*b*) establish a service level based on managerial policy or an assessment of the balance of costs, (*c*) define D_{max} during lead time based on the appropriate distribution and the service level, for example, if we have selected a service level of 10 percent then D_{max} is 1520 units in Figure 4, and (*d*) compute the required buffer stock from $B = D_{max} - \overline{D}$ where $\overline{D}$ is average demand and both D_{max} and $\overline{D}$ are based on the demand distribution over the constant lead time.

The three distributions have been found to be applicable in a number of situations at different stages in the supply-production-distribution system. For example, the normal distribution has been found to describe adequately many demand functions at the factory level, the Poisson distribution at the retail level, and the negative exponential distribution

at the wholesale and retail levels [4]. When both demand and lead time are variable the determination of buffer stocks is more complex. In this situation, we are faced with an interaction between fluctuating demand and fluctuating lead times, and there is no simple mathematical analysis. Nevertheless, buffer stocks can be determined through a process of Monte Carlo simulation as long as we have a knowledge of the demand and lead time distributions. Since the simulation methodology is being used in such a situation, the distributions need not be described by any of the standard mathematical ones. Detailed examples of inventory models with variable demand and lead time are developed in reference [11].

Basic Inventory Control Systems

In attempting to develop automatic control systems for inventories, it is necessary to take account of random fluctuations in demand as just discussed and actual shifts in average demand of either a seasonal or long-term nature. The variables of the system which can be manipulated by management to develop a control system are the size of the replenishment order, the frequency of replenishment orders, the frequency of review and forecast of usage levels, and the method of information feedback on which the reviews are based. Alternate inventory control systems blend these factors in somewhat different ways.

The Fixed-Order Quantity System. This system is diagrammed in Figure 3. The system has a reorder level set which allows the inventory level to be drawn down to the buffer stock level within the lead time if average usage rates are experienced. Replenishment orders are placed in a fixed predetermined amount (not necessarily the minimum cost quantity, Q_0) timed to be received at the end of the lead time. The maximum inventory level becomes the order quantity Q plus the buffer stock I_{min}. The average inventory expected is, then $I_{min} + Q/2$. Usage rates are reviewed periodically in an attempt to react to seasonal or long-term trends of the type shown in Figure 1. At the time of the periodic reviews, the order quantity and buffer stock levels may be changed to reflect the new conditions. Demand for an item is ordinarily taken from the subsequent operation. Assume that we are considering the can of the capacitor shown in Figure 4 of Chapter 3. The capacitor is made in three sizes of electrical capacity. The can which houses the capacitor, however, is identical for all three sizes.

Figure 5 shows the chain of demand for the can as reflected back through the series of stock points and manufacturing operations. Customer orders are placed at the warehouse which maintains an inventory with controls

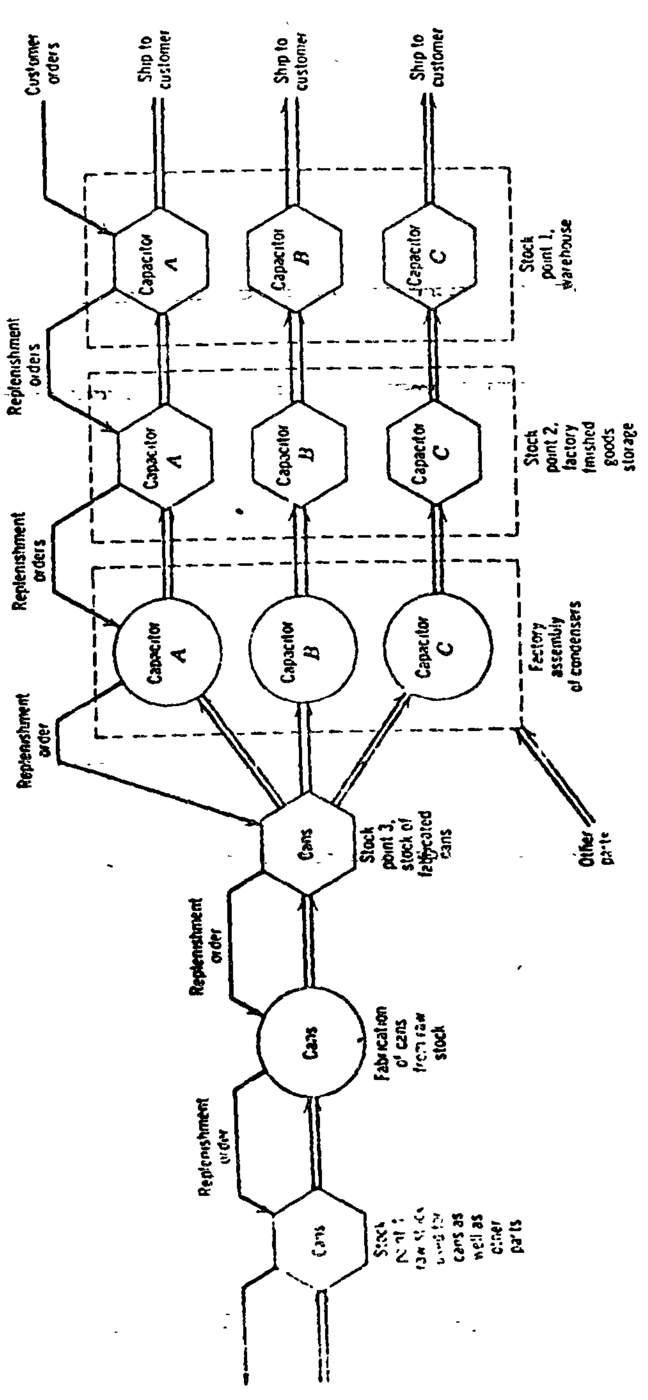


FIGURE 5. Chain of demand for the capacitor can, shown in Figure 4 of Chapter 3.

as described by Figure 3. When the warehouse inventory level falls to the reorder point, a replenishment order is sent to the factory, and the factory ships from its finished goods stock. When the finished goods inventory falls to a reorder point, however, a requisition is sent to the manufacturing department, and more condensers are assembled. To assemble the condensers, however, cans and other parts are requisitioned from stock point 3, a stock of fabricated cans. When the stock of fabricated cans falls to a reorder level, a shop order is written for a run of cans to be fabricated. The shop order requires raw stock which is drawn from stock point 4, raw material storage. When the inventory for the raw material falls to the reorder level, a purchase requisition is issued to vendors for replacement. Thus the demand for the capacitor can is reflected back in a chain involving 4 stock points and 2 factory operations. Figure 5 represents the structure of the information feedback system.

Fixed-reorder quantity systems are common where a perpetual inventory record is kept and with low-valued items such as nuts and bolts, where the inventory level is under rather continuous surveillance so that notice can be given when the reorder level is reached. One of the simplest methods for maintaining this close watch on inventory level is the use of the "two bin" system. In this system, the inventory is physically separated into two bins, one of which contains an amount equal to the reorder level. The balance of the stock is placed in the other bin, and day-to-day needs are drawn from it until it is empty. At that point it is obvious that the reorder level has been reached, and a stock requisition is issued. From that point on, stock is drawn from the second bin, which contains an amount equal to average usage over the lead time plus a buffer stock. When the stock is replenished by the receipt of the order, the physical segregation into two bins is made again and the cycle is repeated.

Fixed-reorder Cycle Systems. These systems focus control on a periodic basis, so that orders are placed weekly, monthly, or by some other cycle. The size of the order, however, is varied for each cycle to absorb the fluctuations in usage from period to period, as shown by Figure 6. The amount ordered covers normal usage during the procurement lead time plus the quantity necessary to replenish inventories to the level required for one cycle's usage plus buffer stock. This is, of course, the I_{max} level shown on Figure 6. Just as with lot size models, optimal relationships for the reorder cycle and I_{max} can be derived. See references [7, 13]. As with the fixed-quantity system, periodic reviews of usage rates are required to react to changes in the average usage rates of the type shown in Figure 1. Fixed-reorder cycle systems are prominent with higher valued items and where a large number of items are regularly ordered from the same vendor. With fixed-cycle ordering. Freight cost advantages can often be gained by grouping these orders together for shipment. The common information feedback system for fixed-cycle systems is diagrammed in Figure 5, based on a chain of demand.

The main operating difficulties with the fixed cycle system described lie in the time lags in the information chain, and the apparently irresistible temptation to outguess shifts in requirement rates. The shifts in usage rates are most often simply random shifts, and the buffer stock has been designed to absorb these variations. If we respond to these random shifts in requirements we will surely drive ourselves insane. Suppose we are ordering on a monthly cycle the fabrication of cans for stock point 3 of Figure 5. Average requirements have been 500 cans monthly, but last month's requirements jumped to 600 units. If we assume that this will be a continuing requirement, we might decide to place an order for the current month which not only replenishes the 600 units drawn, but adds another 100 units to build up inventory to meet the expected continuation of 600 units per month. This makes a total order of 700 units. Suppose, however, that last months increase was simply a random fluctuation, and in a true expression of the capriciousness of random processes, requirements for this period turn out to be only 300 units. We now have a 400-unit excess inventory, and we need place an order for only 100 units for the coming period to meet average requirements. The result is that the random variations in demand from 600 units to 300 units have been translated into variations in shop orders for cans ranging from 700 units to 100 units. Demand variability has been amplified, leading to severe problems on the production floor in attempting to accommodate these wide variations.

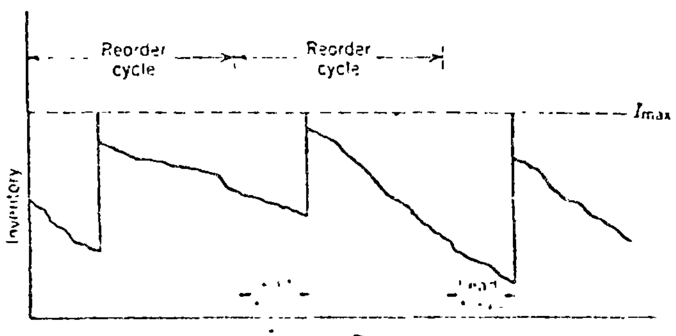


FIGURE 6 Fixed ... cycle system of control. An order is pl... inter als wh... on order plus the ...

The question of amplification of demand variability is of extreme importance in designing stable production-inventory control systems, and we shall consider it more carefully at a later point. The immediate question is, however, "How can we tell if a change in demand is merely a random fluctuation or a true shift in average requirements?" We have an obvious application of the principles of statistical control. Appropriate control limits could be established and requirements plotted in relation to the control limits. Variations in requirements that fall within the control limits may be ignored, since buffer stocks were designed to absorb them. When points fall outside the control limits the question may be raised whether planning figures for average requirements should be revised. Even then, adjustments in planning figures for requirements should be relatively modest, taking a wait and see attitude in order to avoid the costly results of fluctuations as in the situation described in the previous paragraph.

Control Theory Applied to Inventory Systems. Engineers have been interested in the design of automatic control systems, and the result has been the development of concepts and systems of control which have been

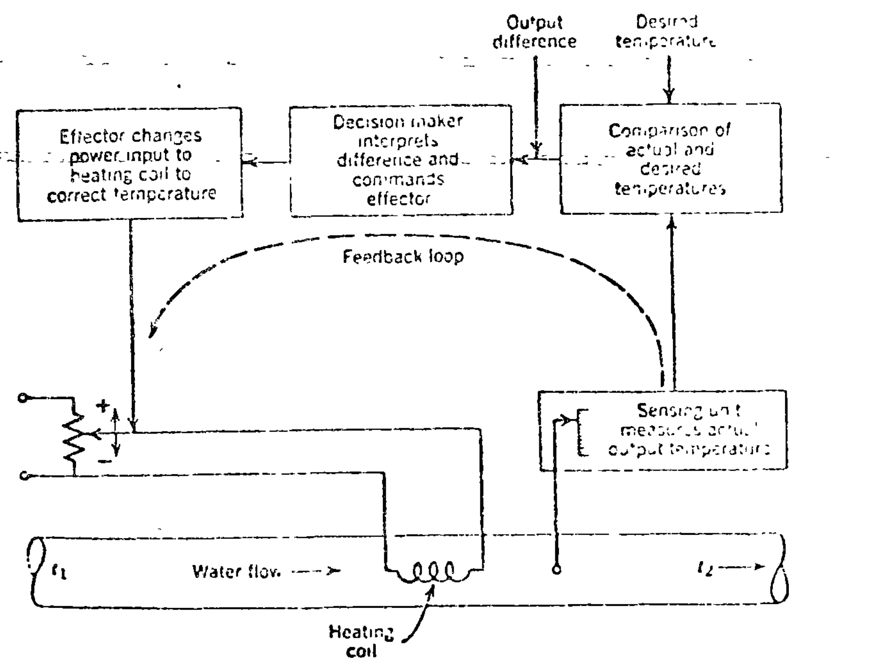


FIGURE 7 Diagram of elements of an automatic system for maintaining a ... temperature of water flow. The basic components of the feedback ... are common to automatic control systems.

applied largely in automation and other physical systems. These self-correcting systems establish automatic control over some variable (a dimension, temperature, pressure, etc.) through a feedback loop. Conceptually, the feedback loop is comprised of some *sensing unit* which measures the output of the variable being controlled, a *comparator* which compares the actual output with the desired level, and a *decision maker* which interprets the error information and finally commands the *effector* to make a correction in the proper magnitude and direction so that output will meet standards. Figure 7 shows a schematic representation of the maintenance of the temperature of flowing water under automatic control.

Many management control problems can be viewed in the same conceptual framework. For example, Figure 8 shows a diagram for information feedback, for the control of inventories and production levels. The parallels between the physical system and the inventory system are direct. From the principles of process control, we can learn some basic control concepts of considerable value in controlling inventories. These concepts are related to time lags and their effect on the stability of the system. Let us see what actual dynamic effects we might expect from an inventory system which was originally stable and now is stimulated by a 10 percent step increase in retail sales, the new sales level remaining stable.

Forrester [8], using a dynamic simulation model of the system shown in Figure 9 demonstrates the dynamic effects dramatically. There are three

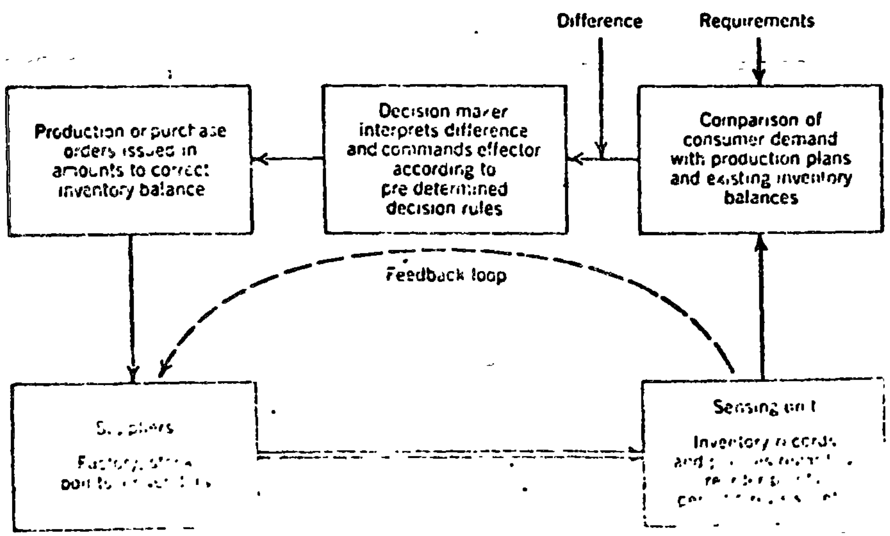


FIGURE 8. [illegible]

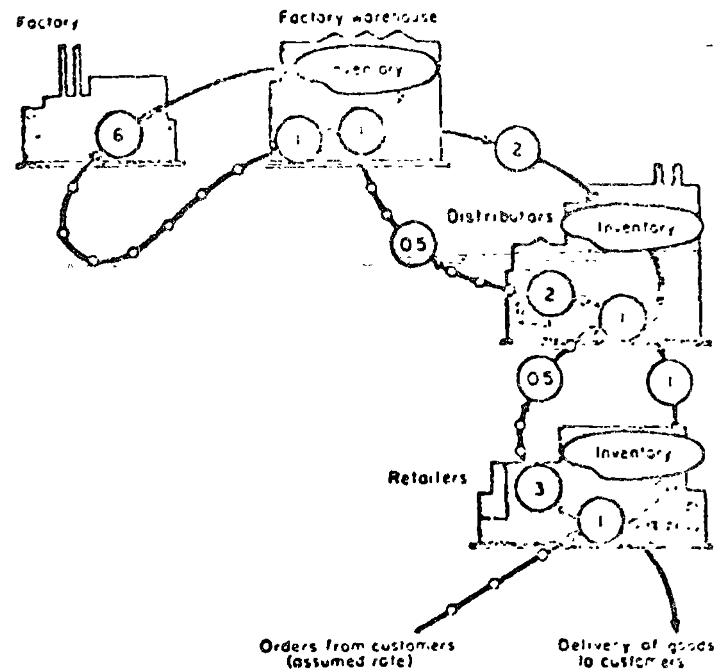


FIGURE 9. Structure of a production-distribution system. Solid lines represent physical flow, lines with dots represent information flow, and circled numbers represent time delays in weeks. From J. Forrester, Industrial Dynamics [8]

levels of inventories in the system: factory warehouses, distributors, and retailer. The circled lines show the flow of orders for goods from customers to retailers, retailers to distributors, distributors to factory warehouse, and finally from the warehouse as orders for the factory to produce. The solid lines show the flow of the physical goods between each of the levels of the structure in response to the orders. The circled numbers represent the time delays in weeks for each of the activities to take place. Figure 10 shows the effect of the 10 percent step increase in retail sales on inventories at the three levels, as well as on factory production output. Whereas the sales increase was simple and orderly, the response of the inventory and production system shows wild oscillations which increase in magnitude as we go up stream in the system from the retail level to the distributor, factory warehouse, and to the actual factory output. As we will demonstrate in Chapter 21, reducing the time lags in the system, for example, by eliminating the distributor level, or reducing the time for clerical delays will reduce considerably the magnitude of the fluctuations.

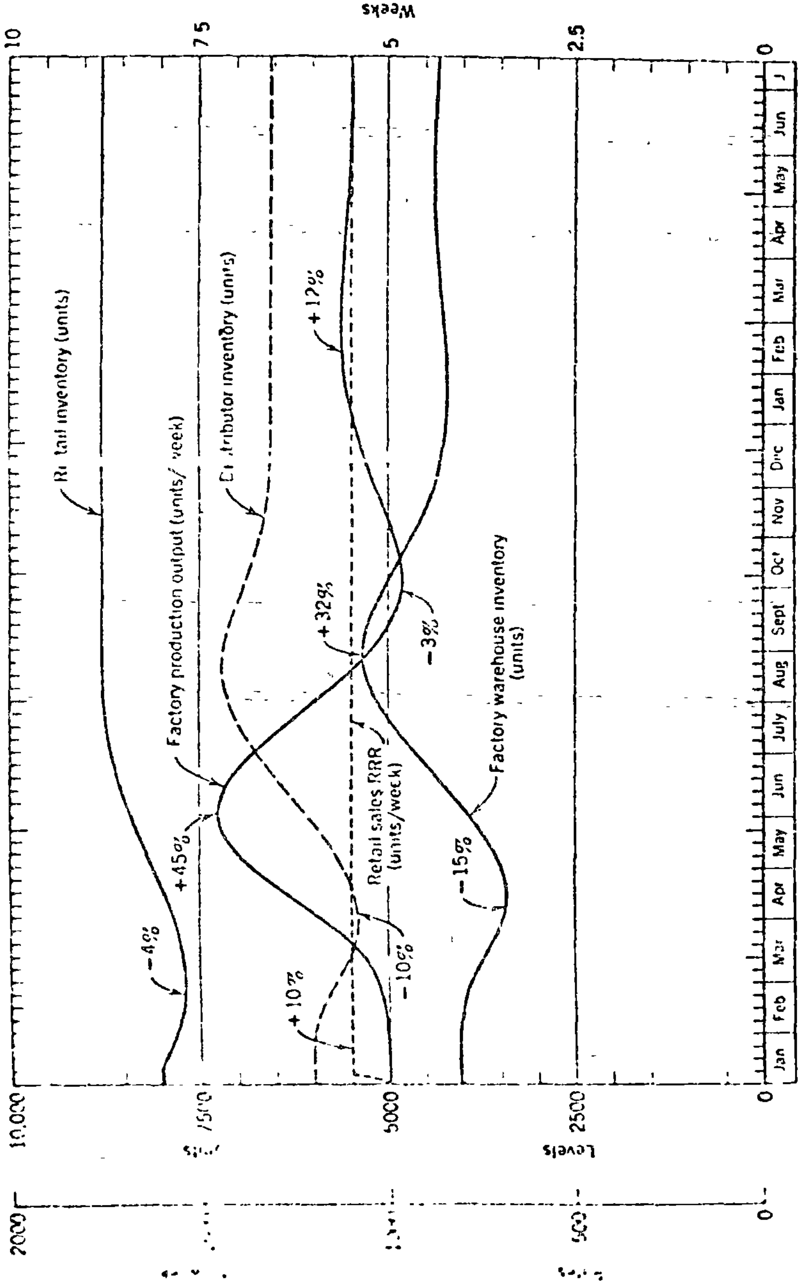


Fig. 10 Response of inventories at three levels and factory output to a step increase in retail sales of 10 … Adapted from J Forrester, Industrial Dynamics, [8]

More direct information feedback to the various stock points instead of through the chain of demand shown in Figure 5 will have important effects in stabilizing the entire system. We consider these dynamic effects more carefully and in greater detail in our Chapter 21 discussion of large-scale system simulation. At this point, however, a conclusion we might draw is that a more direct information feedback system similar to that shown in Figure 11 for the capacitor can production-inventory system will have a stabilizing effect so that no amplification of demand variability will take place at stock points up stream from the consumer inventory level. At each stock point in the system, then, we are working against actual consumer demand rather than against the secondary and tertiary effects of demand as reflected back through the chain. Reducing the lag in information flow has a stabilizing effect regardless of the inventory system used and would be appropriate for both the fixed quantity and fixed cycle systems.

Base Stock System. The base stock system [10] is a blend of the fixed quantity and fixed cycle systems which uses an information feedback system similar to that diagrammed in Figure 11. In this system stock levels are reviewed on a periodic basis but orders are placed only when inventories have fallen to a predetermined "reorder level." At this point an order is placed to replenish inventories to the "base stock" level, which is sufficient for buffer stock plus a fixed quantity calculated to cover current usage needs. Periodic reviews of current usage rates can result in upward or downward revisions in the base stock levels. The base stock system has the advantages of close control associated with the fixed cycle system which makes it possible to carry minimum buffer stocks. On the other hand, since replenishment orders are placed only when the reorder point has been reached, fewer orders, on the average, are placed so that order costs are comparable to those associated with the fixed quantity systems. Since all stock points are working against consumer demand, we do not have the amplification of demand variability at points up stream. Therefore, buffer stocks can be reduced even further, since the extreme levels of maximum demand are not experienced. Another result is a reduction in the cost of production fluctuations (hiring, separation, and training), since smaller production fluctuations are also associated with the type of information feedback system used.

In the sections just completed we tried to show the influence of variability of demand on inventory models, and the importance of the system as a whole. The important concept to carry over is that inventory models must take account of the environment … are operated and cannot be considered as an isolated problem

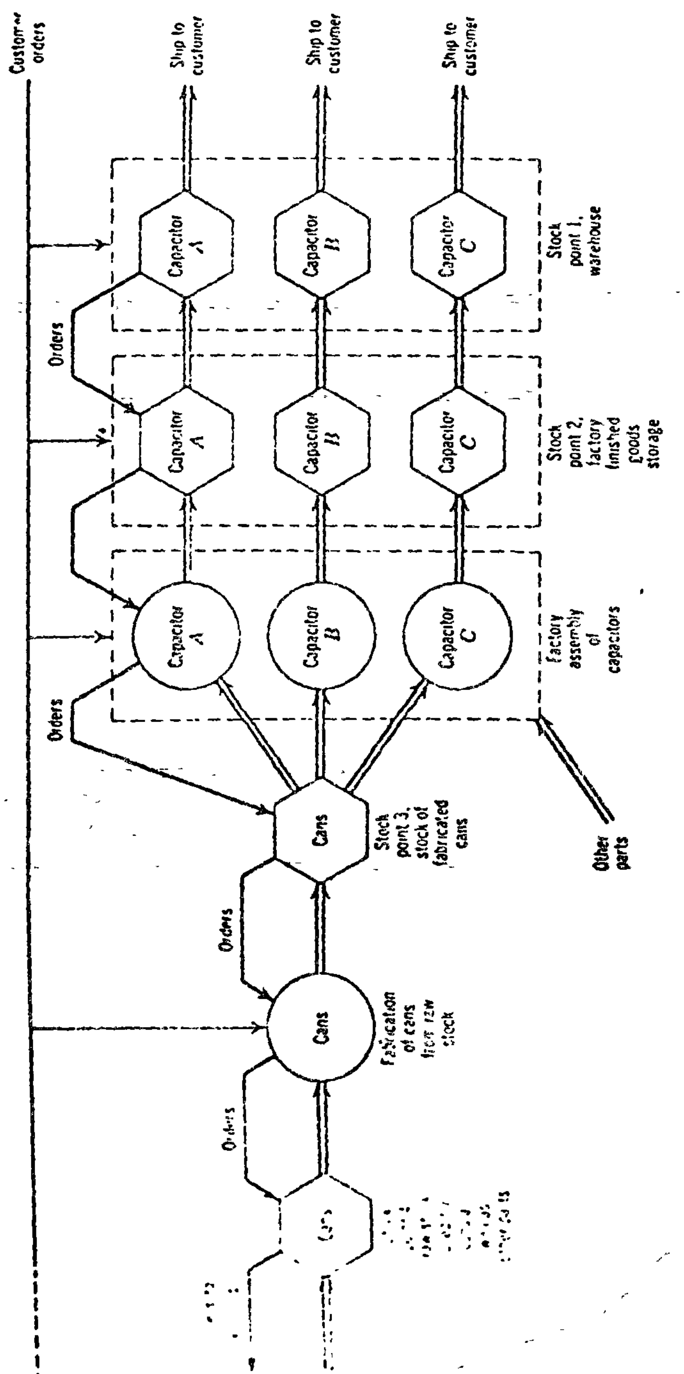


FIGURE 11 *Current demand from customers fed back directly to stock points and operations so that all links in the production-inventory chain work against current demand.*

We shall focus on the problem of operating a production-inventory system through the controlling of production levels. As we shall see, inventories play a major role

Controlling Production Levels

When a basic production-inventory program has been developed, the result is a schedule of planned production levels and inventory balances based on forecasts of requirements. As sales proceed, however, we must have some system for compensating for the differences between planned and actual requirements in order to maintain inventories at proper levels. If actual requirements exceed plans, we run the risk of running out of stock, with resulting poor customer service and possible additional costs related to shortages. If actual requirements are below expectations, inventories will build up with resulting high carrying costs. Therefore, a control plan is needed which adjusts production and inventory levels in keeping with sales experience. Such a control plan might be accomplished by constructing periodically a new production program that takes into account existing inventories by adjustments in the short-run levels of production.

Our objective in this control plan is *to increase or decrease production levels in the period ahead, proportional to differences between actual and forecast sales, by an amount that minimizes the incremental costs of inventories and fluctuations of production levels.* If the planning period is fairly short, this adjustment of levels would continuously correct inventory levels to be in keeping with present demand, thus preventing stock-outs or the buildup of excessive inventories because of changes in demand. The basic elements of this control plan are comparable to those described earlier in this chapter and illustrated by Figure 8. We wish to construct a feedback control system where information on desired levels of inventories (indicated by current requirements) is compared with actual inventories to determine an error function which is fed back and compared with information on planned production levels for the coming period. By some predetermined rule, the production level is then adjusted to compensate for the demand fluctuation and bring inventories into line.

Decision Rules for Controlling Production Levels. Let us first state an obvious kind of rule for controlling production levels as actual requirements vary from forecasted requirements. The rule we will use for introductory purposes is that when actual requirements deviate from forecasts we will add or subtract the difference as soon as possible to the amount produced in order to compensate for the variation from planned inventory levels. Let us illustrate with the forecast of requirements for 10 weeks shown in

TABLE II Calculation of production levels and inventories when the difference between forecasted and actual requirements is absorbed entirely by changes in production level, 2 weeks hence. Beginning inventory is 500 units

(1)	(2) Forecasted Requirements	(3) Planned Production	(4) Planned Inventories = Beginning Inventories + Col 3 − Col 2	(5) Actual Requirements	(6) Difference Between Actual and Forecasted Requirements, Col 5 − Col 2	(7) Actual Production Level = Planned Production + Difference Between Actual and Forecasted Requirements, 2 Weeks Ago, Col 3 + Col 6 for 2 Weeks Ago	(8) Actual Inventory Level = Beginning Inventory + Actual Production − Actual Requirements = Beginning Inventory + Col 7 − Col 5
0			500				500
1	590	600	510	595	5	600	505
2	590	600	520	430	−160	600	675
3	590	600	530	590	0	605	690
4	590	600	540	1000	410	440	130
5	600	600	540	50	−550	600	680
6	600	600	540	625	25	1010	1065
7	600	600	540	570	−30	50	545
8	600	600	540	575	−25	625	595
9	610	600	530	680	70	570	485
10	610	600	530	705	95	575	355
	5980	6000		5820		5675	

Average production rate = 567.5 units per week
Average inventory level = 573 units.

column 2 of Table II. Column 3 shows the planned production program for the product, and the planned inventories are easily calculated in column 4. As we would expect, however, actual requirements vary from forecast as shown in column 5 and the difference between actual and forecast requirements in column 6. The production lead time is 2 weeks, so that when a deviation from forecasted requirements occurs we can change production rate for the production period two weeks hence. Therefore, no change occurs from production plans in the first 2 weeks shown in column 7, but the third week's production reflects the shortage in planned requirements of five units. Similarly, the fourth week reflects the overage of 160 units which occurred in the second week, and so on. Actual inventory levels shown in column 8 are simply beginning inventory plus the amount produced during the week (column 7) less the actual requirements (column 5).

We see that this rule does indeed compensate for the variations, with a 2-week time lag, but at what cost? Actual production levels vary from 50 units to 1000 units per week in the short space of 10 weeks. But, notice that over the 10 weeks, actual total requirements were quite close to forecasted total requirements. Variation from forecast was largely week-to-week variation. As a matter of fact, the week-to-week variation reflects the random variations described in the demand distribution of Figure 4. In other words, it was variation that we should have expected to occur. Perhaps there is a better way to absorb this variation than by direct changes in the production level.

Let us test the idea just stated. Why not damp the effects of variation in actual requirements from forecast by changing production level by only 50 percent of the difference instead of 100 percent as we did previously. This is shown in Table III, under "50% Reaction." The original forecast of requirements and production and inventory plans are identical to those of Table II, but notice that violent swings in both production and inventory levels have been damped out considerably. Why not carry this idea farther? What happens with a 10 or 5 percent reaction rate? This is also shown in Table III with additional stabilizing factors in the form of simple heuristic rules. With the 10 percent reaction we have included the additional restriction that we will not respond to the variation from forecast at all unless 10 percent of the difference exceeds 10 units. In addition, with the 5 percent reaction we have included the 10 unit minimum and the restriction that larger changes in production level are made only in increments of 10 units. Therefore, if 5 percent of the difference is 27.5 units as it is in the fifth week, a change in production level of 30 units is made in the seventh week. Notice the results of progressively decreasing reaction rates in Table III. The results are more stable production and inventory levels. Also note, however, that average inventory levels have increased as reaction rate was decreased. The effect of reducing reaction rate could have been forecast. By using a

TABLE III Actual production and inventory levels when only 50 percent, 10 percent, or 5 percent of the difference between forecasted and actual requirements is absorbed by changes in production level [illegible] 2 weeks hence. Buffer stocks absorb the balance of the variation. Data for forecasted and actual requirements and planned production and inventory levels are shown in Table II

	50% Reaction		10% Reaction 10-Unit Minimum		5% Reaction 10-Unit Minimum Increments 10 Units	
Week	Actual Production Level	Actual Inventory Level	Actual Production Level	Actual Inventory Level	Actual Production Level	Actual Inventory Level
0	—	500	—	500	—	500
1	600	505	600	505	600	505
2	600	675	600	675	600	675
3	603	688	600	685	600	685
4	520	208	584	269	590	275
5	600	758	600	819	600	825
6	805	938	641	835	620	820
7	375	743	545	810	570	820
8	613	781	600	835	600	845
9	585	686	600	755	600	765
10	587	568	600	650	600	660
Average for 10 Weeks	589	655	597	684	598	688

relatively low reaction rate we are assuming that most deviations in actual requirements from forecasts are simply random deviations, so why become excited about them? If the deviation looks large, perhaps we should increase or decrease production rate *a little*, just in case it really marks the beginning of a trend. The question is, then, what should be the reaction rate for optimal cost performance? It is a good question, but it is slightly premature. Let us first discuss the general aspects of the decision rule and develop the ideas of reaction rates, review periods, and their interrelations.

Our decision rule really operates in the following context:

1. A long-term forecast of requirements on which is based a [illegible] production [illegible]
2. A [illegible] requ[illegible]
3. Based on this short-term review and forecast of requirements we can:
   a. Determine a production plan for these periods
   b. Set planned inventory levels for these periods
4. In the shortest-term planning period which is equal to the production lead time (the shortest notice used to change production levels in the period ahead), we can make a final adjustment in production level which takes account of the latest information we have regarding the comparison of actual and forecasted requirements.
5. The decision rule used is that production level in the immediate period ahead will be adjusted by some fraction k of the difference between actual and forecasted requirements for the current period.

In this context, we see that there are really two parameters we can manipulate to develop a model for the control of production levels. They are the value of k—the reaction rate—and the length of the review period mentioned in number 2 and 3 in the preceding outline. The importance of the reaction rate has already been discussed and demonstrated in the text material related to Tables II and III. In summary, k may take on values between the number 0 and 1.00, representing no reaction to deviations from forecasted requirements when $k = 0$, to 100 percent reaction and compensation when $k = 1.00$. In general terms, low values of k lead to stable production levels and relatively high buffer stock requirements, since variations from the plan must be absorbed by inventories. Conversely, high values of k lead to large production fluctuations and relatively low buffer stocks because variations from plan are absorbed by changing production levels. The significance of reaction rates in smoothing production rates is comparable to the smoothing constant α used in the exponentially smoothed forecasting methods discussed in Chapter 7.

The frequency of review also has a direct effect on both the [illegible] of production fluctuations and the size of needed buffer stocks. The reason is easy to see in relation to the general principle of process control which we discussed in connection with Figures 7 and 8. The longer the period between reviews, P, the greater the chance that forecasts of requirements may not reflect the most current trends. Therefore, it is more likely that relatively large differences between actual and forecasted requirements would accumulate. For a given value of k, longer review periods lead to both relatively large production fluctuations and buffer stocks in order to provide the needed compensation. Short periods between reviews, then, lead to closer [illegible] and relatively small production fluctuations and buffer stocks. [illegible] periods between reviews lead to looser control and larger [illegible] fluctuations and buffer stock requirements.

Determining k and P. Magee [10] derives two approximate formulas useful in solving the problem of determining the reaction rate k and the review period P for specific situations. He shows that the expected magnitude of production fluctuations is approximately proportional to

$$\sqrt{kP/(2-k)} \qquad (1)$$

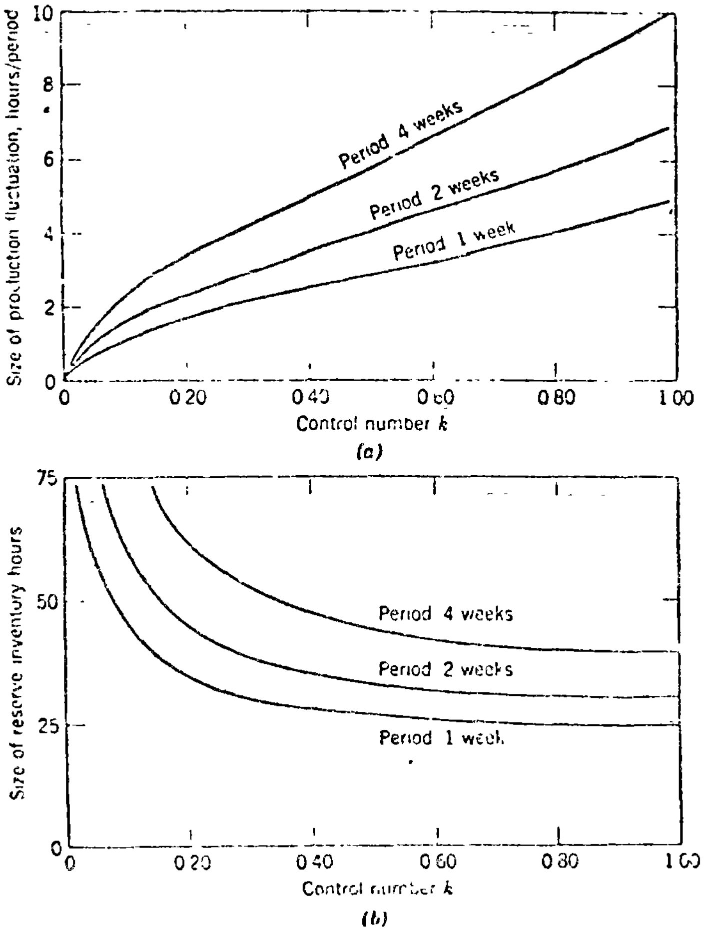


FIGURE 12 (a) Magnitude of production fluctuations versus control number and length of review period. (b) Reserve inventory required versus control number and length of review period. [illegible]

and that the required factory buffer stock will be approximately proportional to

$$\sqrt{[T(2k-k^2)+P]/(2k-k^2)} \qquad (2)$$

where T = production lead time, P = length of review period, and k = reaction rate in decimals.

The cost of production fluctuation, then, is proportional to (1) and the cost of buffer stocks are proportional to (2). Figure 12 shows the relationship of reaction rates and review period to the size of production fluctuations and reserve inventory requirements, expressed in equivalent hours. For a specific case, then, suppose that at k = 1.00 we experience a production fluctuation cost of \$5000 and a buffer stock cost of \$500, when the review period and production lead times are each 1 week. Using formulas (1) and (2), we can compute points for the curves shown in Figure 13 to find a value of k approximating 0.075 for minimum total incremental cost. Further similar calculations with different review periods would yield a combination of k and P which would minimize incremental costs for the entire system. Obviously, the right combination for a specific case like that shown in Figure 13 depends on the relative magnitudes of inventory carrying cost and the cost of production changes.

Let us summarize at this point some of the aspects of the control of inventories under uncertainty in a production-inventory system. Previously in this chapter we discussed systems for controlling inventories that

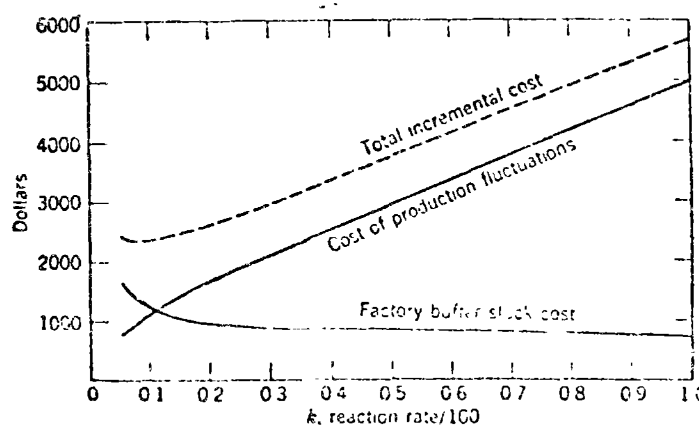


FIGURE 13 Relationship between incremental costs and k when the cost of production fluctuations and factory buffer stocks are \$5000 and \$500 respectively at k = 1. Review period and lead time are 1 week.

involved fixing the quantity ordered at one time letting the frequency of ordering vary, fixing the frequency of ordering letting the quantity ordered vary, and the base stock system which was a combination of the elements of the two different systems. Also, differences in the information feedback pattern and their effects were noted. In the operation of a production-inventory system we have noted that the cost of production fluctuations is also an important factor to take into account. By way of summary, let us now consider the overall comparison of systems of control.

A Comparative Example

Magee [10] relates a hypothetical case called the Hibernian Co. which compares operation and costs for different basic systems of production and inventory control. The example considers a company that manufactures and sells about 5000 small machines per year for $100 each. The factory supplies four warehouses located in strategic areas around the country, which in turn supply the customer. We shall show the calculated results for four alternate systems of control: an economical order quantity system, a two-week fixed reorder cycle system, a base stock system with a review period of 1 week and reaction rate of 100 per cent, and a base stock system with a 1-week review period but involving a production reaction rate of 5 percent.

Each of the four branches sold an average of 25 units per week, or 1300 units per year. This average rate was, of course, subject to considerable variation and Table IV shows distributions of demand at each of the four branches for 1-week periods, 2-week periods, etc. For example, at any given branch, sales would be expected to exceed 37 units per week only 1 percent of the time, 67 units per 2-week period 1 percent of the time, and so on. Requirements aggregated at the factory warehouse, reflecting demand from all four of the branches, are shown in Table V for eight different time groupings. Figure 14 shows the structure of the production-distribution system.

1. *Economical fixed reorder quantity system (EOQ)* Using an economical fixed reorder quantity system, we must analyze the requirements for buffer stocks, cycle stocks, transit stocks, and reordering costs for the branches, as well as buffer [illegible] cycle stocks, in-process inventory ordering costs, and the cost of production fluctuations at the factory and warehouse.

Branches. At each branch, the economical quantity to be ordered at one time [illegible] calculated if we know that c_p = $19 ($6 clerical cost, $13 cost of p[illegible], receiving, and stocking), R = 1300, and c_H = $5. Q_0 is then,

$$\sqrt{(2 \times 19 \times 1300)/5} = 100 \text{ units}$$

Therefore, each branch would place an order for 100 units each 4 weeks on the average, and the average *cycle stock* in each branch would be 100/2 = 50 units. The branch *buffer stock* is based on a 1 percent risk of running out of stock. Since the total

TABLE IV Distribution of demand at each of four branches by eight different time-period groupings

Percent of Periods Exceeding Levels Given	Units of Sales Period, Weeks							
	1	2	3	4	5	6	7	8
90	19	41	64	87	111	134	158	[illegible]
60	24	46	71	95	124	144	168	[illegible]
50	25	50	75	100	125	150	175	[illegible]
20	[illegible]	[illegible]	[illegible]	[illegible]	134	159	[illegible]	[illegible]
10	[illegible]	[illegible]	[illegible]	117	[illegible]	[illegible]	[illegible]	[illegible]
1	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

TABLE V Distribution of demand on factory warehouse from branches by eight different time period groupings

Percent of Periods Exceeding Levels Given	Units of Requirements in Period, Weeks							
	1	2	3	4	5	6	7	8
90	87	182	278	374	471	569	666	[illegible]
60	95	193	291	389	488	587	686	[illegible]
50	100	200	300	400	500	600	700	800
20	108	212	314	417	519	621	722	[illegible]
10	113	218	322	[illegible]	529	631	734	[illegible]
1	123	233	341	447	553	658	[illegible]	[illegible]

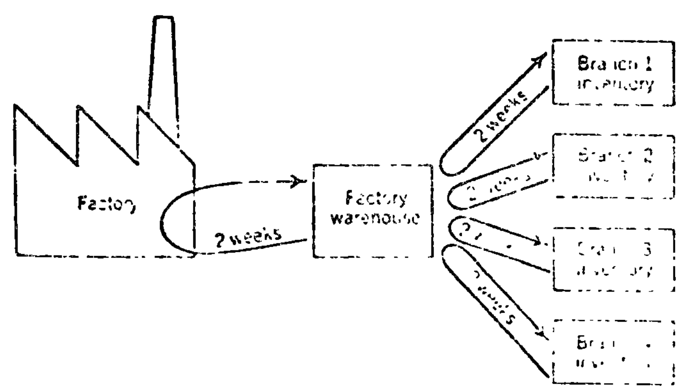


FIGURE 14 Structure of production-distribution system for Hibernian Co.

lead time was 2 weeks we can determine the reasonable maximum demand during that period from Table IV as 67 units. Since normal demand during the 2-week lead time would be 50 units the buffer stock is then the difference, or 17 units. Finally, the average *transit stock* is equal to the delivery time multiplied by the average demand rate, or 50 units. Average branch inventory is then as follows

Buffer stock,	$4 \times 17 =$	68 units
Cycle stock,	$4 \times 50 =$	200
Transit stock,	$4 \times 50 =$	200
		468 units

Since c_H = \$5 per unit per year this average inventory of 468 units has an annual cost of \$2340. Since each branch places an order once every 4 weeks on the average there are 52 orders per year from the four branches which cost \$19 each or a total annual reordering cost of \$988.

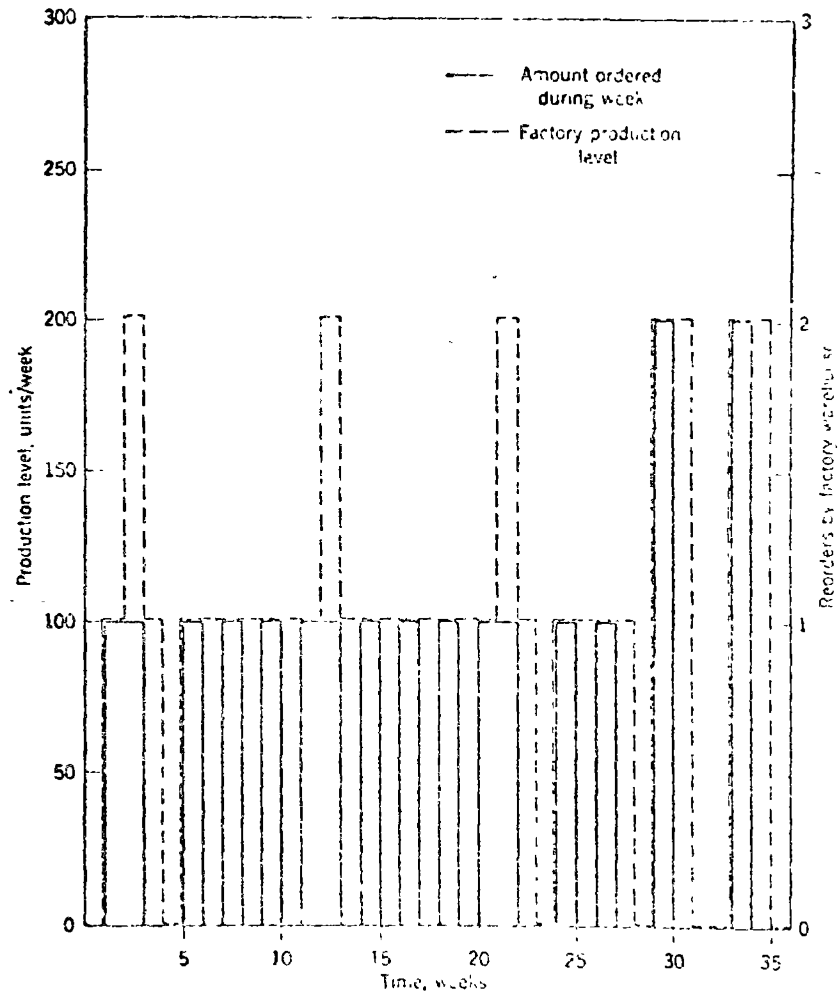
Factory Warehouse and Factory. The factory warehouse is of course, reflecting the aggregate demand from the four branches so that its economical order quantity reflects annual requirements R = 5200 units, and its own inventory holding and preparation costs of c_H = \$3.50, and c_P = \$13.50. Calculating Q_0 as before, we obtain Q_0 = 200 units. Maximum 2 week demand from the branches (using a 1 percent run-out risk criterion) under the economical reorder quantity system is 233 units, so that *factory warehouse buffer stocks* are set at 233 − 200 = 33 units. *Cycle stocks are* 200/2 = 100 units and *in-process inventories* in the factory average one-half the order quantity or 100 units. Total average inventory at the factory warehouse is therefore 233 units. On the average, 26 factory production orders per year must be issued at a cost of \$13.50 or \$351 per year. Table VI summarizes the inventory and ordering costs for the economical order quantity system. To this total we must add the *cost of production* fluctuations which occur with the economical order quantity system. Figure 15a shows a typical pattern of orders on the factory and the resulting factory production levels set. Note that very large fluctuations in production levels result and these fluctuations cost \$8500 per year.

TABLE VI Summary of incremental costs of economical order quantity system for Hiberman Co. from Magee [10]

Inventory cost	
Four branches	*\$ 2,340*
Factory	*816*
Reorder costs	
Four branches	*988*
Factory	*351*
Production fluctuations	[illegible]
	\$12,[illegible]

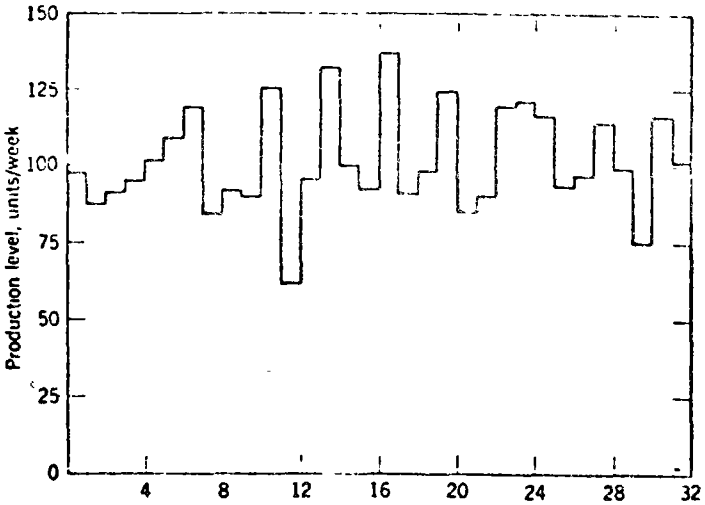
2 *Fixed Reorder Cycle Systems.*

Branches. Under the fixed reorder cycle system, each branch warehouse maintains its inventory sufficient to fill reasonable maximum demands during the review period

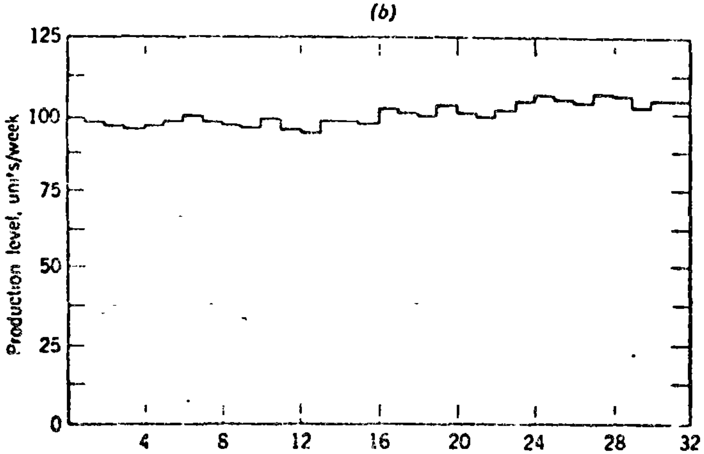


(a)

FIGURE 15 (a) Factory orders and production level, economic reorder quantity system (b) Production level fixed reorder cycle system (c) Production level basestock system reaction rate = 5 percent. Adapted by permission from [illegible] Planning, and Inventory Control *by J. F. Magee and D. M. Boodman, McGraw-Hill Book Company, 2nd ed., New York, copyright 1967*



(b)



(c)

FIGURE 15 (Continued)

plus the 2-week delivery time. We must first compare system costs for several different review periods to determine the appropriate length of review period. Table IX [illegible] the distribution [illegible] each branch warehouse for eight different [illegible] Therefore, we [illegible] requirements for each review [illegible] [illegible]

Table VII. Cycle stock would average one-half of the normal shipment. A shipment is made once each period so that the average amount [illegible] stock is 25 × the number of weeks in the period. Table VII shows the appropriate cycle stocks for each review period. Transit stock remains at 50 units for each review period. The number of orders placed varies inversely with the length of the review period that [illegible] 1-week period results in 52 orders per year at $19 per order or $988 per year. The branch ordering costs for the other periods are summarized in Table VII.

TABLE VII. Comparison of system costs for different lengths of review periods for the fixed reorder cycle system[a]

	Length of Review Period, Weeks					
	1	2	3	4	5	6
Branch warehouses, each branch						
Inventory						
Buffer stock	20	23	26	29	31	33
Cycle stock	12.5	25	37.5	50	62.5	75
Transit stock	50	50	50	50	50	50
Total	82.5	98	113.5	129	[illegible]	158
Annual inventory cost at $5	$ 412.5	$ 490	$ 567.5	$ [illegible]	$ 717.5	$ 790
Ordering cost	990	495	330	250	195	165
Total	$1402.5	$ 985	$ 897.5	$ [illegible]	$ 912.5	$ [illegible]
Total, four branches	$5610	$3940	$3590	$3580	$3650	$ [illegible]
Factory warehouse						
Buffer stock	41	47	53	58	62	67
Cycle stock	50	100	150	200	250	[illegible]
In-process stock	50	100	150	200	250	300
Total	141	247	353	458	562	667
Annual inventory cost at $3.50	$ 493	$ 865	$1235	$1630	$1967	$ 2 335
Ordering cost	700	350	235	175	140	120
Total factory warehouse cost	$1193	$119[illegible]	$1470	$180[illegible]	$2107	$ [illegible]
Cost of changing production levels	$1600	$2250	$27[illegible]0	$3[illegible]	$3[illegible]50	$ 3[illegible]
Total system costs	$8403	$73[illegible]	$7820	$8[illegible]	$9317	$10 175

[a] Modified from J. F. Magee and D. M. Boodman, Production Planning and Inventory Control, McGraw-Hill Book Company, New York, 2nd ed., 1967

SUMMARY

In this chapter we have tried to develop the importance of the factor of demand variability and its impact on inventory planning. In doing this, we have developed the rational determination of buffer stocks and discussed systems inventory control which take account of the resulting risks. In connection with these systems of inventory control, the concepts of process control and information feedback were introduced and the important effects of time lags shown.

In considering the problems posed by inventories, we are forced to consider several levels of planning covering different time spans. These are as follows:

1. *Long-range plans for plant capacity.* Plant capacity may be affected by seasonal peaks, and there are capital costs associated with this capacity. What combination of in-plant capacity, use of seasonal inventories, overtime, and subcontracting will minimize the combined capital costs, seasonal inventory costs, labor costs, production fluctuation costs, and extra costs of subcontracting? Is new capacity justified?
2. *Intermediate-range plans for a few months to a year* in advance, which attempt to determine for the expectations of sales what will be the best allocation of the resources of existing capacity. We are asking what combination of production within periods, size of work force, and seasonal inventories will minimize the combined costs of production fluctuation, seasonal inventory cost, labor costs, and extra subcontracting costs. We shall pay particular attention to this subject in Chapter 13.
3. *Short-range plans for the immediate period ahead.* Since actual requirements will change from forecasts, we must take a last look within the lead time to change production level, but neither can we change production levels capriciously because large costs can be involved, nor can we ignore what might develop into a huge inventory buildup. The result is that we need a control system that minimizes in the short range the cost of inventories and production fluctuations.
4. *In the shortest range of planning,* we need automatic decision rules that dispatch work to each and every workplace and machine. There is no time to ponder the question at this point. We must develop an automatic rule which operates quickly and [illegible] indicating [illegible] machine [illegible]. Here [illegible]

flow, such as those covered in Chapter 17, which will minimize inventory and idle labor costs while providing a high level of service to customers by completing their work on time.

Inventories have an important impact at all stages of planning and execution. The result is that we must view inventories in their multifaceted role in the broad system from raw material input, flow through the production-distribution system, and to the consumer. They cannot be examined in isolation with realism.

REVIEW QUESTIONS

1. What are the three kinds of variations which we might expect in sales [illegible] which result in variability of demand?
2. Why is it that we wish to abstract just the random variations due to chance causes from the total variation in demand curves from all causes for [illegible] determining buffer stocks?
3. How can we determine what stock runout level to use for a specific [illegible]
4. Describe each of the three inventory control systems which take [illegible] variability of demand which are described in this chapter.
5. What are the variables in inventory control systems that are subject to managerial control?
6. Which system has closer control over inventory levels, the fixed [illegible] quantity system or the fixed reorder cycle system?
7. What techniques may be applied to determine if an apparent change in demand is merely a random fluctuation or a true shift in average requirements?
8. Relate the general principles of process control to inventory control systems.
9. Describe the effects on retail inventories, distributor inventories, factory warehouse inventories, and on factory production levels when consumer demand changes, assuming the structure of a production-distribution system as shown in Figure 9.
10. What is the nature of our objective in controlling production levels?
11. Compare the expected results when a production control rule is used [illegible] reaction rates of 100, 50, 10, and 5 percent.
12. In controlling production levels, what are the two main variables that are under our control?
13. What is the general relationship between reaction rates and the frequency of adjustment of production levels? Which combinations produce high costs of production fluctuation? High costs of reserve inventories?
14. How can equations (1) and (2) help to determine the best reaction rates for [illegible]

15 Make a complete analysis of the four systems of control used in the Hiberman Co case, checking all calculations to show exactly where the different systems have relative advantages and disadvantages

PROBLEMS

1. Weekly demand for a product exclusive of seasonal and trend variations is represented by the empirical distribution given below What buffer stock would be required for the item to insure that one would not run out of stock more than 15 percent of the time? Five percent of the time? One percent of the time? Normal lead time is one-week

Weekly Demand, Units	Frequency, Number of Weeks Demand Reached a Given Level
0	0
20	2
30	5
40	10
50	9
60	20
70	30
80	25
90	18
100	17
110	10
120	8
130	6
140	3
150	2
Total	165

2 If the item for which data is given in Problem 1 has a unit value of $100, shortages costs of $10 each, and an annual inventory carrying cost of 25 percent of the average inventory value, which of the three levels of service would be most appropriate?

3 An organization is attempting to assess the cost of increasing its service level which is currently set at 80 percent Average demand during lead time is 18 units [illegible] described by the Poisson distribution Inventory [illegible] $10 per unit per year [illegible] the buffer inv[illegible] What are the [illegible]

the negative exponential distribution? The normal distribution with σ_D = 2 4 and 6 units?

4. Given a control number of 0 6, a decreased demand fluctuation of 600 units in the first period, and a forecasted production level of 15 000 units in the third period, what would be the revised production quantity set for period three? (Owing to lead times, it is not possible to adjust the production level for the second period.)

5. A company manufactures a single product for which the following table represents a schedule of forecasted and actual demand in units for one year

Month	Forecasted Demand	Actual Demand
Jan.	23,000	23,000
Feb.	24,000	25,000
Mar.	21,000	20,000
Apr.	23,000	22,000
May	20,000	22,000
June	19,000	24,000
July	17,000	22,000
Aug.	14,000	15,000
Sept.	8,000	6,000
Oct.	10,000	13,000
Nov.	9,000	10,000
Dec.	10,000	14,000
Total	198,000	216,000
Average	16,500	18,000

The initial inventory is 15,000 units. The desired ending inventory is 20,000 units The cost of storage is $1 per unit per month It costs $1000 to change production from zero to 3000 units and $3000 to change production from 3000 to 6000 units No change larger than 6000 units is possible in one period. Back orders are permitted at a cost of $5 per unit per period

(a) What is the best production plan for the forecasted demand if one wishes to minimize pertinent costs?

(b) Assuming that the year is over, what is the best production plan for the actual demand utilizing the benefit of hindsight?

(c) To correct for deviations in actual demand as compared to forecasted demand evaluate the choice of a control number of 0 25 versus one of 0.75 Assume that at the end of a month sufficient time exists to alter the planned production for the next month

(d) What would be the cost impact of these two control numbers if the following additional rules were formulated

(1) Determine the planned production change.

(2) Add or subtract the additional change due to the forecast error [illegible] the appropriate control number factor.

Introduction

During the last two decades there has been a natural and increasing application of computers to problems in the area of Operations and Logistics Management. We think that these are important problems and they span a very wide spectrum. Imagine, for example, trying to land men on the moon without the use of computers. In less spectacular but equally important applications, the computer aids management in coping with the incredible volume of records necessary for scheduling the production of automobiles, the shipping of coal, the management of inventories, and many other operating decisions. But these are not the only applications. There are several instances where the computer is a partner in the analysis process in the same sense as a slide rule for an engineer and a calculator for an accountant. The computer has been used to help design the layout of a plant, determine the configuration of an airport terminal, decide the location of branch warehouses, and schedule delivery trucks in a major metropolitan area. It is this latter role of the computer that we wish to emphasize in this book and we will outline how we intend to go about it in this section. Your task is described too.

The Nature of the Cases

It is important to point out, first and foremost, that each of these cases presents a management problem. We are interested in that problem and that is the focus of the cases. The computer is important only insofar as it helps us solve that problem. For this reason, we have already written the computer programs which you will use in analyzing the cases. It is our belief that, as managers, your most important computer-oriented task is to intelligently direct the use of the computer to help solve your problems. Therefore, it is up to you to ask the right questions and perform the right analysis of the data you get back from the computer. All the computer does for you, in most of these cases, is answer your question, "What if I used__________?", by simulating the situation the company faces. You must supply the thought that makes the question a good one.

Having the [illegible] from learning something about the techniques of analysis that are [illegible] available. In fact, these are often a prerequisite to asking the right questions. Therefore, not all of the cases in this book are computer augmented. Some cases are used to illustrate various techniques of analysis or to acquaint you with the nature of the computer program that is used to augment another case with a similar management problem. Some of the main cases in the text are designed to introduce you to the technique of simulation, which provides the basis for many of the computer programs that you will use. We will also provide you with an example of simulation in this section.

One important word of advice is necessary before getting on with the example. It does not pay to work with the computer programs without first devoting some time to deciding how they can best be used. This is a very hard discipline to learn, but it pays handsome dividends. In many cases, there are simply too many alternatives to be considered to try to look at them all. You must first decide those alternatives which are really useful to consider. In other cases you will need to know something more about the problem than what you get from the computer to arrive at useful solutions, and the thought that goes into deciding how to use the computer capability often provides you with the necessary insights into the problem. We will illustrate the value of pre-analysis in this section. It is time to turn to an example of the type of situation you might meet in one of the cases.

An Example Situation

We will use an example situation to illustrate simulation and the approach to case analysis using the computer programs. For this situation, suppose that you are the manager of a receiving dock and raw material warehouse for a sugar refinery in the Northwest. You receive, during the production season, loads of sugar beets that arrive in trucks leased by the company. You are planning the manpower requirements for the firm's receiving and storage operations. One of your decisions involves determining the number of men to be assigned to unload incoming shipments at the plant's receiving dock. This dock can accommodate only one truck at a time. Other trucks, waiting to be unloaded, park in the plant driveway. A diagram showing the access to the receiving dock is presented in Figure 1.

The supervisor of the receiving operations has prepared some data to help in deciding the size of the crew needed to unload the trucks. The trucks leased by the company cost $12 an hour. The manager is responsible for this cost while the trucks are waiting to be unloaded (for simplicity, we will assume that he is not responsible for the time during which the truck is being unloaded). The trucks are all the same size and their loads are very nearly equal. From past experience the supervisor knows that the time required to unload each truck is one-half-hour. The labor cost for this operation is $3 per hour. The supervisor indicated, however, that the elapsed time between the arrival of successive trucks varies somewhat. In a sample of 100 trucks the supervisor found that the time between truck arrivals had the pattern shown in Table 1.

Figure 1
Receiving Dock Access Layout

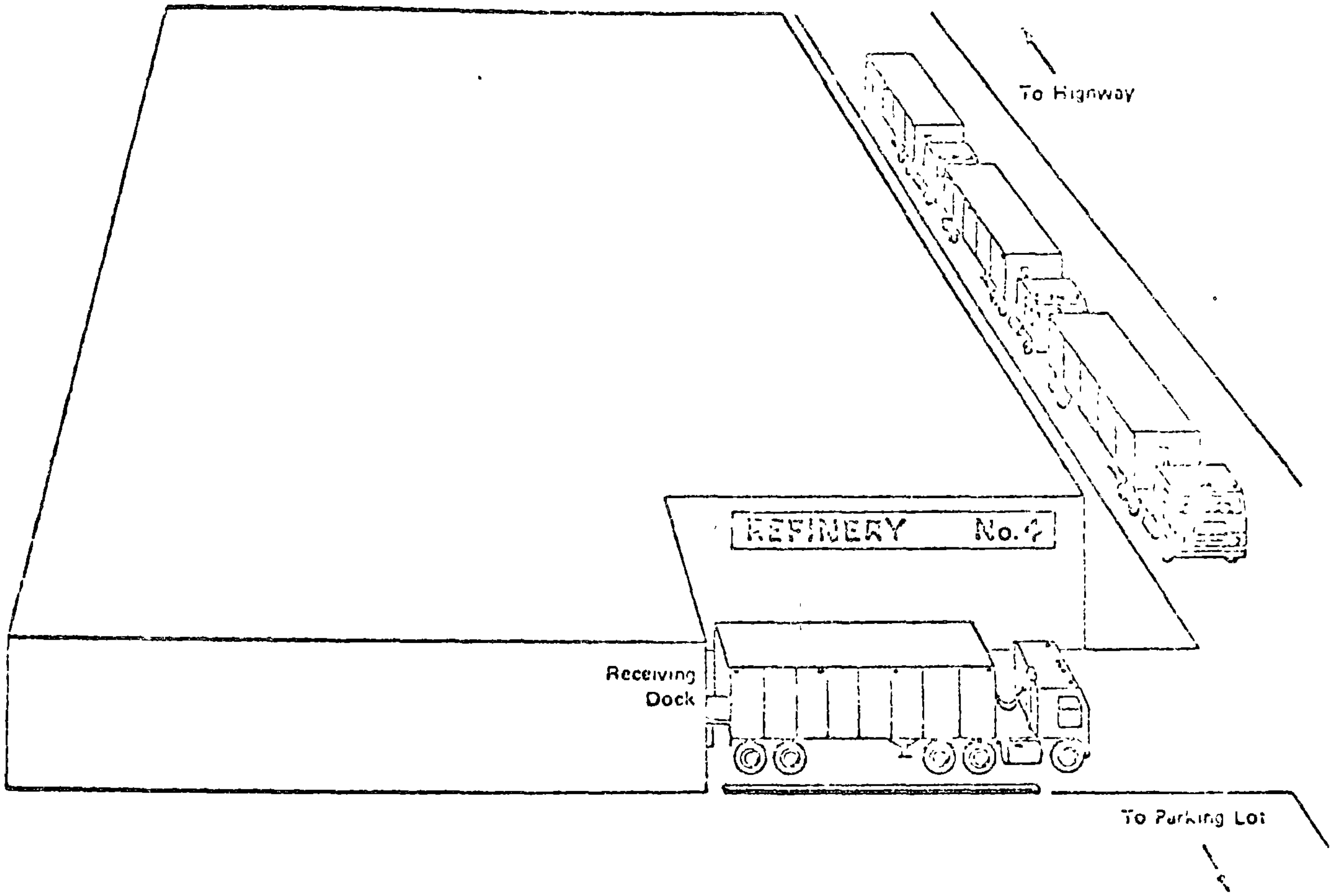


Table 1
A Summary of Observed Truck Arrivals

Time Between Successive Truck Arrivals (in minutes)	Number of Trucks
12	20
15	50
20	30
	100

At this point you should be able to define the basic trade-off that the manager faces as he makes the crew size decision: the balance between the firm's labor cost and truck rental costs. Increasing the size of the unloading crew, for example, will increase the firm's labor costs, but it will also reduce the firm's truck rental costs by speeding up the unloading process and

decreasing the [illegible] system c
determining [illegible] ,
trucks and operating [illegible] computer can
help you to analyze

A computer program [illegible] enable you to evaluate a large number of possible [illegible] the one with the lowest overall cost. Such a computer [illegible] perform a simulation of the unloading facility over [illegible] keeping track of the arrival and departure of trucks as [illegible]. That is, it would replicate the physical process [illegible] collapsing several months of operating experience [illegible] seconds of computer time. The output of the program could [illegible] a summary of [illegible] measures such as: 1) the average length of time trucks spend waiting [illegible] and being unloaded, 2) the amount of time the unloading crew is idle, and the total operating cost for the [illegible] the trucks.

One way of deciding the crew [illegible] is to use blindly the computer program to evaluate a wide range of possible crew sizes, say from 1 to 100 men, and sort through the output to find the alternative having the lowest cost. This approach would probably require more overall time and effort on your part than is necessary, would not help you to learn very much about the problem, and would not be useful in improving your analytical skills. In the following section we will use the crew size problem to illustrate the approach that we recommend for analyzing the cases in this book. This approach will help you make intelligent use of the computer programs and advance your analytical skills.

Some Analysis Before Using the Computer

The approach that we suggest in preparing solutions for the cases in this book involves defining the problem, developing a method for solving the problem, and carefully preparing a plan for using the computer program. For the manager in the sugar refinery, we have already defined the basic problem and discussed a solution procedure. Let's now see how a little time spent in analyzing the problem before turning to use the computer program will contribute to your understanding of the problem and reduce the effort required to reach a solution.

To see why it is not worthwhile to evaluate blindly a large number of alternatives, consider the data describing the time between successive truck arrivals shown in Table 1. Trucks arrive either 12, 15, or 20 minutes apart. Another way of stating this is that trucks arrive at the rate of 3, 4, or 5 trucks per hour. Since each truck requires one man-hour to unload, this means that 3, 4, or 5 man-hours of work may be received at the dock in a given hour. The maximum amount of work that could arrive during any hour is 5 man-hours; therefore a 5-man crew could unload the trucks without incurring any truck waiting time. We can now eliminate the evaluation of any crew size exceeding 5 men since labor costs [illegible] increase with no offsetting decrease in truck waiting costs. We [illegible] eliminate some alternatives by noting that an average of 3.8 trucks [illegible] per hour. This means that more than 3 man-hours of work [illegible] average hour so we need not evaluate crew sizes with 1, 2, or 3 men. [illegible] of less than 4 men would not be able to keep up with the arrival of trucks, and so the number of trucks waiting to be unloaded would become progressively larger as time passes.

Thus far, the analysis has indicated exactly which alternatives we need to evaluate: crew sizes of 4 and 5 men. We still don't need to turn to the computer, however. Since there will be no truck waiting time with a crew of 5 men, we can evaluate that alternative directly. The cost of a 5-man crew is $15 per hour and this, since idle truck costs are zero, is the total hourly cost of the 5-man crew alternative. We need the computer only to investigate one crew size, that of 4 men. If the computer results indicate that the combined costs of labor and idle trucks is less than $15 per hour for the 4-man crew, we would choose this alternative. Further refining the problem definition, we see that the computer program is only needed to estimate the truck idle time cost for the 4-man crew. More specifically, we would really like to know whether the idle truck time cost for the 4-man crew exceeds $3 per hour, the cost of adding the fifth crew member.

Actually, we have selected a very simple situation to illustrate the value of analyzing the case prior to using the computer program. The cases in this book generally involve many more alternatives which are worthwhile evaluating, and the pre-analysis may lead you to a range or starting point rather than exactly what must be evaluated. Our example does illustrate that, by performing some analysis, you can limit the number of alternatives that must be evaluated and save yourself a great deal of time. Therefore, in analyzing each case, you should end up with a carefully developed plan for conducting your analysis with the computer program.

How the Simulation Works

Most of the cases in this book involve a simulation analysis to estimate important process parameters or to compare alternative operating policies. In some cases you will be asked to perform the simulation manually, while in others the computer program performs this function. Where appropriate, we have provided simulation forms to assist you with the mechanics of conducting a manual simulation. An understanding of how a simulation works will help you in analyzing the cases which follow. To illustrate this method, let us return to the crew size problem.

Our analysis so far has shown us that the truck idle time resulting from the use of a 4-man crew is the important factor in deciding between the 4- and 5-man alternatives. To estimate this factor, we will conduct a simulation of the unloading facility, using a 4-man crew, and measure the amount of idle truck time incurred during a 3-hour period. This is what the computer will do for you in many of the cases.

Generating Truck Arrivals. The first step in conducting a simulation analysis for the crew size problem involves generating a sequence of truck arrivals that would appear to be typical of those observed by the supervisor at the truck dock. To accomplish this we shall use the data describing the time between truck arrivals presented in Table 1. In replicating the pattern of truck arrivals observed by the supervisor, we shall put together a sequence of arrivals in which the times between successive arrivals are: 12 minutes approximately 20% of the time, 15 minutes approximately 50% of the time, and

20 [illegible] the truck arrival time distribution [illegible] Table 2.

One method of generating the desired sequence of [illegible] arrivals is to place ten chips in a bowl. Two of the chips are marked [illegible] 12, five chips have a 15 marked on them, and the remaining three chips are marked with a 20. By randomly drawing a chip, [illegible] the interarrival time number, and replacing the chip in the bowl, we can simulate a sequence of truck arrivals that would appear to be similar to those observed by the supervisor.

A second method, which is better suited for the computer, is to associate one or more single-digit numbers with each of the truck interarrival times as is shown in Table 2. In this case, for example, the numbers 0 and 1 are associated with a 12-minute interarrival time. Therefore, if we were to draw chips randomly from a bowl that contains 10 chips numbered from 0 to 9, we could use this table to associate the numbers with the correct interarrival time. For example, if chip number 5 were drawn, it would be associated with a 15-minute interarrival time.

Table 2
Time Between Successive Truck Arrivals

Time (in minutes)	Probability	Cumulative Probability	Random Numbers
12	.2	.2	0-1
15	.5	.7	2-6
20	.3	1.0	7-9

As you probably suspect, we would like to do away with the bowl altogether. This can be done by using a random number table such as the one appearing in Appendix E of this book. The numbers in the random number table were generated so that each digit between 0 and 9 has an equal probability of being selected. Thus, instead of drawing one-digit numbers randomly from a bowl, we can simply look up a sequence of random numbers in the table and use them to develop a series of random truck arrivals. To use the random number table, simply read down a column or across a row taking each digit in order. Then use Table 2 to find the time until the next truck arrives.

Conducting the Simulation. Once a sequence of randomly selected times has been prepared, the operation of the unloading facility can be simulated. The data shown in Table 3 provide an illustration of how this is done. Here the arrival time for each truck is determined by adding the time from the random sequence of times between trucks to the arrival time of the last truck. For example, the first two random numbers were 8 and 7, both of which correspond to a time of 20 minutes. The first truck would then arrive at 8:20 in the morning and the second 20 minutes later or at 8:40 in the morning. Since the unloading time per truck is 15 minutes with a crew of 4 men, neither of these trucks must wait to be unloaded. The third truck in Table 3, however, arrives before the second truck is unloaded and must wait for 3 minutes.

The simulation results indicate that for this three-hour simulation the total truck idle time cost was $5.20 or $1.67/hour. The labor cost for a four-man crew is $12.00/hour, bringing the total hourly cost up to $13.67. Since the cost for a five-man crew was $15.[illegible] per hour, we would select the four-

Table [illegible]
Four-Man Crew Simulation

Truck Number	Random Number	Time* Between Trucks	Truck Arrival Time*	Time* Unloading Begins	Unloading Time*	Truck Departure Time*	Truck Waiting Time*	Truck Waiting Cost
1	8	20	20	20	15	35	0	0
2	7	20	40	40	15	55	0	0
3	0	12	52	55	15	70	3	$.60
4	8	20	72	72	15	87	0	0
5	6	15	87	87	15	102	0	0
6	6	15	102	102	15	117	0	0
7	0	12	114	117	15	132	3	.60
8	3	15	129	132	15	147	3	.60
9	1	12	141	147	15	162	6	1.20
10	9	20	161	162	15	177	1	.20
							16	$3.20

*Time is in minutes.

man crew alternative on the basis of cost. But the analysis of the case may not stop there.

Using the Simulation Results. Clearly, one must use the results of a simulation study with some care. We shall not treat the topic of analyzing simulation experiment data in detail, but rather point out two important considerations. First, estimates of a process parameter, such as truck idle time, will be affected by the length of the simulation run. Longer simulation runs will provide a better replication of the actual operation of the system being studied and an improved parameter estimate. But this requires more computations to be made. In analyzing some of the cases in this book, you will have to decide the simulation run length that you believe to be appropriate for the problem being studied.

A second consideration in these cases concerns the use of the other data that may be available or which come from the analysis. You may want to relate some of the qualitative data to the results of the computer runs before making a final choice. Another factor you might consider is the sensitivity of the results to changes in the random numbers chosen or the cost parameters. Your analysis may lead you to conclude that the computer approach is wrong for the problem and that some other technique would be more appropriate for the situation being studied. All of these factors should be considered in making your final recommendation. Thus, the cases in this book are designed to provide you with an opportunity to develop your analytical and decision-making skills — both of which are critical to the practicing manager.

A Note on Using the Computer Programs

Don't panic! You do not need to know how to program, but you will have to learn to use a key punch machine and we've included Appendix A to help those of you who may not be familiar with the key punch. Your instructor will provide you with the procedure and control cards necessary to use the programs. In addition, you will need to prepare data cards for each program. We have tried to keep the amount of data required to run each

The Gaming Company (A)

Late one Friday afternoon, George Heller was trying to formulate an approach to his job as finished goods warehouse manager for the Gaming Company. He had taken the job right after the company's two-week summer vacation. Just prior to that he had finished his second year in the M.B.A. program at State University. While looking for a job, during the M.B.A. program, he had specifically sought out smaller companies which he felt would provide him greater opportunities to make major contributions and to apply the training he had received. As finished goods warehouse manager at the Gaming Company, he had the responsibility for managing the inventories of the company's entire line of parlor games. He decided to study in detail a representative product, The Big Game, as the basis for his plans.

Background

One of the important elements of communication in the company was the Monday morning management meeting. During this time, the key people of the company got together and discussed current problems, production plans, and new product ideas. It was during these meetings that Mr. Heller was to place replenishment orders for products which were getting low in inventory. These orders were given directly to the production manager, Roger Blake.

The Gaming Company's production process was quite simple. The production manager received all replenishment orders at the Monday meeting and forwarded them to a nearby printing company where the game boards were printed during the early part of the week. During the latter part of the week and sometimes on the weekend, the games were completed, assembled, and boxed at the Gaming Company plant using part-time help from a local junior college. The completed games were packed in cases and transferred to the finished goods warehouse on Monday morning of the following week. Although rare, the assembly operations sometimes continued into the early part of the next week. When this was going to happen the production manager always finished at least part of the replenishment order for each of the games and made it a point to deliver them on Monday morning. The remainder of the orders were delivered as they were finished during the week.

In discussing the situation with the production manager, Mr. Heller was assured that, at least for the foreseeable future, there would be no limitations on production capacity. Mr. Blake stated that this meant that no replenishment orders would be turned down even though in some weeks this might mean more split deliveries to the finished goods warehouse. He again assured Mr. Heller that at least part of each replenishment order would be in the warehouse on the Monday morning following the management meeting in which the order was placed.

In his discussion with other key people in the company, Mr. Heller found that when the company didn't have enough inventory to fill a customer's order, the amount by which they were short was lost. That is, the company was not able to back order the shortage and its customers apparently filled their requirements with competitive products. The customers would accept partial shipments, however.

In discussing the finished goods inventory with other officers, Mr. Heller found that space was not a critical problem. The finished goods warehouse had been designed with space for expansion into new product lines should the company so desire. Capital, however, was a continual problem for the company because of a rapid growth in the product line. Mr. Heller felt he could use the Friday night inventory balance to determine the inventory level for capital investment purposes. An advantage of this was that the Friday night balance was already available to him and he wouldn't have to go to the expense of developing more information.

The Big Game

Mr. Heller turned his attention to The Big Game as a representative company product. His predecessor had left him no information on the management of the inventories, but there were two years of demand history for The Big Game. This is reproduced in Exhibit 1. In discussing The Big Game with the salesmen, Mr. Heller found that it was a relatively stable item in the company's product line and that it had no seasonal sales peaks. The salesmen were in agreement that conditions in the current year will not be different from those of past years and past demand will be a good indication of what to expect in the future.

In reviewing the costs of The Big Game, Mr. Heller found that the printing company charged a fixed amount of $9 for each order to cover the costs of setting up their presses and delivering the finished game boards to the company. There were no comparable fixed costs for the assembly of the completed games at the plant. The management of the company had estimated that it cost $1 per case for each case short on a customer's order. This represented not only the loss of profit on that case, but also some measure of the goodwill that was lost.

An estimate of the opportunity cost of capital and direct costs of carrying inventory had been made, and for The Big Game this amounted to $.10 per week per case. Since he had decided that the Friday night inventory was the relevant inventory, Mr. Heller decided to use that as the inventory level against which he would assess the $.10 cost. There was a balance of 43 cases of The Big Game in inventory and he hadn't placed a replenishment order in the last management meeting. He next turned his attention to investigating different methods for managing the inventories and to see if he needed to place an order on the following Monday.

Simulation

Mr. Heller thought one approach to the evaluation of different alternatives for managing the inventories would be simulation. He devised a sheet on which he could evaluate different alternatives by hand. An example of one of his evaluations is presented in Exhibit 2. In compiling this example, Mr. Heller used the sales history from Exhibit 1 as a representative demand sequence. Additional evaluation sheets are provided in Exhibit 3.

Exhibit 1
The Gaming Company (A)
Past Demand for The Big Game
(Cases per week)

Week	Two Years Ago	Last Year
1	25	19
2	18	26
3	21	17
4	[illegible]	18
5	[illegible]	25
6	17	17
7	19	17
8	18	17
9	20	22
10	19	22
11	25	18
12	24	17
13	20	19
14	19	18
15	19	19
16	17	16
17	16	22
18	25	20
19	[illegible]	17
20	19	22
21	[illegible]	26
22	18	17
23	19	17
24	19	18
25	19	22
26	22	19
27	19	26
28	26	22
29	18	28
30	18	28
31	19	18
32	21	17
33	18	16
34	19	16
35	20	17
36	18	16
37	19	17
38	25	21
39	23	26
40	18	24
41	17	18
42	17	18
43	23	19
44	20	23
45	19	18
46	17	25
47	27	23
48	20	18
49	28	16
50*	22	17

* The Gaming Company took a two-week vacation each summer.

Exhibit 2
The Gaming Company (A)
Evaluation Sheet for The Big Game

Week Number	1	2	3	4	5	6	7	8	9	10
Monday Morning Inventory	43[1]	53[4]	35	14	100[6]	79	62	43	25	40
Week's Demand (from Exhibit 1)	25	18	21	21	21	17	19	18	20	19
Friday Night Inventory	18	35	14	-7	79	62	43	25	5	21
Number Ordered	35	0	0	100	0	0	0	0	35	0
Setup Costs ($9 per order)	9.00[2]	0	0	9.00	0	0	0	0	9.00	0
Inventory Costs ($.1 per case)	1.80[3]	3.50	1.40	0	7.90	6.20	4.30	2.50	.50	2.10
Shortage Costs ($1 per case)	0	0	0	7.00[5]	0	0	0	0	0	0
Total Costs for Week	10.80	3.50	1.40	16.00	7.90	6.20	4.30	2.50	9.50	2.10
Cumulative Cost from Last Week		10.80	14.30	15.70	31.70	39.60	45.80	50.10	52.60	[illegible]
Cumulative Costs to Date	10.80	14.30	15.70	31.70	39.60	45.80	50.10	52.60	62.10	[illegible]

1. Opening balance as of next Monday
2. Cost of placing the order for 35 cases
3. Friday night inventory at $.10 per case
4. Friday night inventory of 18 plus order of 35
5. Short 7 cases at $1 per case
6. The order of 100 only ... the 7 cases short are not back ordered

DIRECTORIO DE ASISTENTES AL CURSO MODELADO Y SIMULACION APLICADOS A LA PLANEACION (26 Y 27 DE MARZO 2,3,9 Y 10 DE ABRIL DE - 1976)

NOMBRE Y DIRECCION	EMPRESA Y DIRECCION
1. ING. ANGEL ALCEDA HERNANDEZ Paseo del Faisán 192 Lomas Verdes Sección I México, D. F. Tel: 5-72-79-65	DIRECCION GENERAL DE INGENIERIA DE TRANSITO Y TRANSPORTE DEL DISTRITO FEDERAL Puente de Alvarado 84-1er. Piso México, D. F. Tel: 5-35-86-84
2. LILIA ALVAREZ MONTALVO Calzada de Tacubaya No. 39-9 Col. Condesa México 11, D. F. Tel: 5-25-37-87	SECRETARIA DE LA PRESIDENCIA Palma No. 40 México 1, D. F. Tel: 5-85-52-22 Ext. 3
3. LIC. JOAQUIN AVALOS ZAVALA Malintein 124-1 Col. Estrella México 14, D. F. Tel: 5-77-15-53	SECRETARIA DE RECURSOS HIDRAULICOS Viena 20-302 Col. Juárez México 6, D. F. Tel: 5-92-39-82
4. ING. JORGE A. CASTILLO OJEDA Candelaria Patos M-3 E-H-D-13 Col. Merced México, D. F.	
5. LIC. SERGIO ENRIQUE CASTRO PEÑA Nicolás San Juan 112-B Col. Narvarte México 12, D. F. Tel: 5-36-51-43	FIDEICOMISO CAMPAÑA NACIONAL CONTRA LA GARRAPATA Constituyentes 161-4o. Piso Col. Chapultepec México 11, D. F. Tel: 2-77-77-11 Ext.127
6. LIC. DANIEL DARDON MONZON Av. Cuauhtémoc 1103-4 Col. Vértiz Narvarte México 13, D. F. Tel: 5-59-57-24	ECONOMIA, UNAM Ciudad Universitaria México 20, D. F. Tel: 5-48-99-97
7. ING. JESUS PATRICIO DIAZ FRIAS Maximo Rojas No. 18 Col. P.E. Calles México 17, D. F. Tel: 5-47-37-03	PETROLEOS MEXICANOS Av. Marina Nacional No. 329 México 17, D. F. Tel: 5-45-74-60-2938

DIRECTORIO DE ASISTENTES AL CURSO MODELADO Y SIMULACION APLICADOS A LA PLANEACION (26 Y 27 DE MARZO 2,3 9 Y 10 DE ABRIL DE - 1976)

NOMBRE Y DIRECCION	EMPRESA Y DIRECCION
8. ING. JORGE FRIAS AVILA Merida 246-A Letra "C" Col. Roma México 7, D. F.	SECRETARIA DEL PATRIMONIO NACIONAL Insurgentes Sur No. 552 Col. Roma México 7, D. F. Tel: 5-64-12-82
9. ING. ALFONSO GARCIA ROMO Sotero Prieto No. 2 Circuito Científicos Cd. Satélite Edo. de México Tel: 5-72-53-09	ALTOS HORNOS DE MEXICO, S. A. Plaza de la República 43-2o.Piso México 1. D. F. Tel: 5-66-12-33
10. GUILLERMO GOODE M. Av. Taxqueña No. 1239-4 Campestre Churubusco México 21, D. F.	SECRETARIA DE COMUNICACIONES Y TRANSPORTES DIRECCION GENERAL DE PLANEACION Xola y Av. Universidad México 12, D. F. Tel: 5-30-34-82
11. ING. OSCAR LOZANO MARTINEZ Av. Universidad No. 1810 Edificio N-7 Col. Oxtapulco México 20, D. F. Tel: 5-48-19-65	PETROLEOS MEXICANOS Av. Marina Nacional No. 329 Col. Anáhuac México 17, D. F. Tel: 5-45-74-60
12. ING. FRANCISCO MEDINA MORA Almendros No. 70 Lomas de San Mateo Edo. de México	PETROLEOS MEXICANOS Marina Nacional No. 329 Col. Anáhuac México 17, D. F. Tel: 5-45-74-60 Ext.2890
13. ING. CUAUHTEMOC A. MUZQUIZ O. Corina 117-D-4 Col. del Carmen, Coyoacán México 21, D. F. Tel: 5-44-32-37	SECRETARIA DE OBRAS PUBLICAS Xola y Av. Universidad Col. Narvarte México 12, D. F. Tel: 519-39-42

DIRECTORIO DE ASISTENTES AL CURSO MODELADO Y SIMULACION APLICADOS A LA PLANEACION (26 Y 27 DE MARZO 2,3,9 Y 10 DE ABRIL DE - 1976)

NOMBRE Y DIRECCION	EMPRESA Y DIRECCION
14. ING. ALFONSO NAVA JAIMES Cerro San Francisco No. 44 Col. Campestre Churubusco México 21, D. F. Tel: 5-49-63-69	PETROLEOS MEXICANOS Av. Marina Nacional No. 329 Col. Anáhuac México 17, D. F. Tel: 5-31-61-70
15. ING. VIRGILIO OLVERA RODRIGUEZ Nonoalco 153-E-110 Unidad Nonoalco México 3, D. F. Tel: 5-83-22-73	PETROLEOS MEXICANOS Av. Marina Nacional No.329 Col. Anáhuac México D. F. Tel: 5-45-74-60 Ext.2122
16. ARMANDO ORTEGA MARQUEZ Casas Grandes 240 Desp.403 Col. Narvarte México 12, D. F.	DIRECCION GENERAL DE ADMINISTRACION FISCAL REGIONAL DE LA S.H.C.P. Lorenzo Boturini 52-6o. Piso México, D. F. Tel: 5-78-54-00 Ext. 198
17. J. JESUS PALACIOS RANGEL Calle 6 No.307 Fracc. Ind. Aviación San Luis Potosi, S. L. P. Tel: 3-14-61	ESCUELA DE ECONOMIA UNIVERSIDAD AUTONOMA DE SAN LUIS POTOSI Lado Poniente Parque Juan H. Sánchez San Luis Potosi, S. L. P. Tel: 3-12-32
18. FERNANDO PEREZ CASTRO Edificio 81-A-402 Unidad Cuitlahuac México 16, D. F. Tel: 5-56-85-35	PETROLEOS MEXICANOS Av. Marina Nacional No. 329 Col. Anáhuac México 17, D. F. Tel:545-74-60 Ext.2122
19. ING. PEDRO PRIETO VILLARREAL Amates No. 38 Sección Colina J. de San Mateo Edo. de México Tel: 3-73-09-37	PETROLEOS MEXICANOS Av. Marina Nacional No. 329-12o.P. Col. Anáhuac México 17, D. F. Tel: 5-45-74-60 Ext.2220

DIRECTORIO DE ASISTENTES AL CURSO MODELADO Y SIMULACION APLICADOS A LA PLANEACION (26 Y 27 DE MARZO 2.3.9 Y 10 DE ABRIL DE 1976)

NOMBRE Y DIRECCION	EMPRESA Y DIRECCION
20. ING. BENIEL G. RAMIREZ TELLO R. Rio Usumacinta No. 1279 González Gallo Guadalajara, Jal.	TESORERIA GENERAL DEL ESTADO DE JALISCO Carretera Saltillo Guadalajara, Jal. Tel: 23-39-57
21. ING. FELIX REYES PEÑA 4a. de Guayaquil No. 13 Las Américas Edo. de México Tel: 5-60-57-14	SECRETARIA DEL PATRIMONIO NACIONAL Insurgentes Sur 552 - 1er. Piso México, D. F. Tel: 5-64-13-58
22. ING. ERNESTO RIVAS MERINO México, D. F.	SECRETARIA DE OBRAS PUBLICAS Xola y Av. Universidad México 12 D. F.
23. MA. DEL CARMEN RODRIGUEZ FERNANDEZ Calle 7 No. 85 Col. Independencia México 13, D. F. Tel: 5-32-12-41	SECRETARIA DEL PATRIMONIO NACIONAL Insurgentes Sur No. 550 México, D. F. Tel: 5-64-12-82
24. ING. HECTOR SANTACRUZ TORRES Av. Universidad No. 1900 E-50-103 Col. Oxtopulco México 20, D. F. Tel: 5-50-04-17	SECRETARIA DE RECURSOS HIDRAULICOS Viena No. 20 Despacho 302 Col. Juárez México 6, D. F. Tel: 5-92-39-82
25. LIC. SAMUEL DE JESUS SEGOVIA P. Las Huertas No. 107-502 Col. del Valle México 12, D. F.	CAMPAÑA NACIONAL CONTRA LA GARRAPATA S.A.G. Av. Constituyentes No. 161-4o.Piso México 11, D. F. Tel: 2-77-77-11 Ext. 123
26. ING. RUBEN VAZQUEZ RIOS Angel Maria Garibay 110 Toluca, México Tel: 4-08-47	SERVICIOS COORDINADOS DE SALUD PUBLICA DEL EDO. DE MEXICO Independencia Ote. 1009 Toluca, México Tel: 5-19-25

DIRECTORIO DE ASISTENTES AL CURSO MODELADO Y SIMULACION APLICADOS A LA PLANEACION (26 Y 27 DE MARZO 2,3,9 Y 10 DE ABRIL DE 1976)

NOMBRE Y DIRECCION	EMPRESA Y DIRECCION
27. ING. SAUL TRACONIS RAMOS Copilco 76 Edif. A-10 Depto.601 San Angel México 20, D. F.	SECRETARIA DE RECURSOS HIDRAULICOS Paseo de la Reforma No. 107-8o.Piso México 4 D. F. Tel: 5-66-06-88 Ext. 120