A los asistentes a los cursos del Centro de Educación Continua

La Facultad de Ingeniería, por conducto del Centro de Educación Continua, otorga constancia de asistencia a quienes cumplan con los requisitos establecidos para cada curso. Las personas que deseen que aparezca su título profesional precediendo a su nombre en el diploma, deberán entregar copia del mismo o de su cédula profesional a más tardar 15 días antes de la terminación del curso, en las oficinas del Centro, con la Sra. Sánchez.

El control de asistencia se efectuará al terminar la primera hora de cada día de clase, mediante listas especiales en las que los interesados anotarán personalmente su asistencia. Las ausencias serán computadas por las autoridades del Centro.

Se recomienda a los asistentes participar activamente con sus ideas y experiencias, pues los cursos que ofrece el Centro están planeados para que los profesores expongan una tesis, pero sobre todo para que coordinen las opiniones de todos los interesados constituyendo verdaderos seminarios.

Al finalizar el curso se hará una evaluación del mismo a través de un cuestionario diseñado para emitir juicios anónimos por parte de los asistentes.

Las personas comisionadas por alguna institución deberán pasar a inscribirse en las oficinas del Centro en la misma forma que los demás asistentes.

Ing. Melesio Gutiérrez

FACULTAD DE INGENIERIA

CENTRO DE EDUCACION CONTINUA

EVALUACION ECONOMICA Y METODOS PARA DECISION DE INVERSIONES EN LA INDUSTRIA MINERA

INTERES, EVALUACION DE PROYECTOS Y ANALISIS DE RIESGO

Septiembre, 1973.

LUZBEL NAPOLEON SOLORZANO

I N T R O D U C C I O N

Con el objeto de proporcionar una idea general sobre los aspectos fundamentales de este curso y a fin de describir de una manera sencilla y breve los factores e índices que se utilizan con mayor frecuencia en la evaluación de proyectos, se empleará como referencia una gráfica (Fig. 1) que representa el comportamiento económico típico de un proyecto de inversión. En la figura el eje vertical se utiliza para representar la ganancia acumulada y el horizontal para el tiempo.

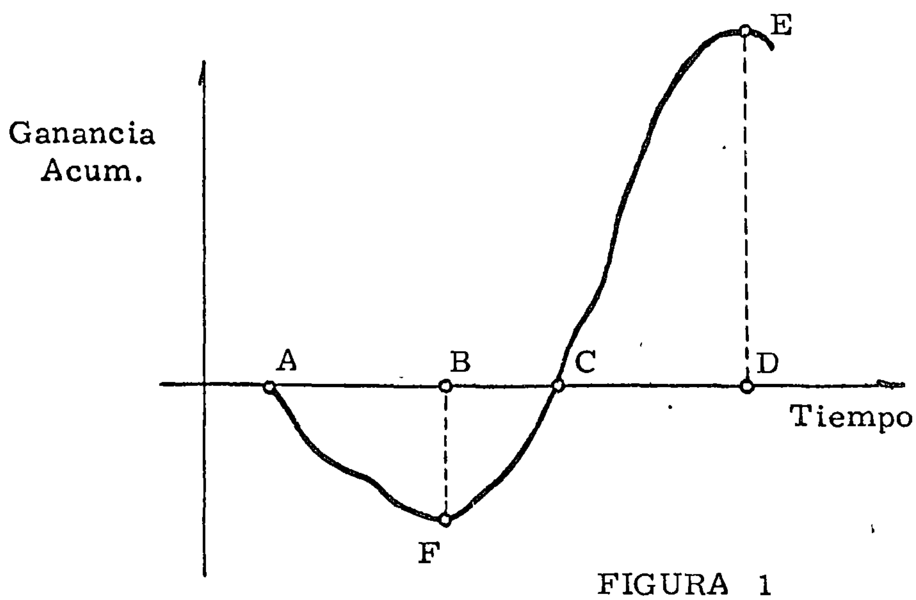


FIGURA 1

El período de inversión se cuenta desde que se realiza la primera erogación (punto A), hasta el momento en que el proyecto está listo para empezar a producir utilidades (punto B). Como se sabe la inversión inicial está constituída por diferentes conceptos, entre los que se pueden mencionar: estudio previo del proyecto, pruebas piloto, estudio definitivo, adquisición, transporte, seguros, instalación y prueba del equipo, compra de materiales y servicios e intereses durante la construcción. Se llama período de cancelación al tiempo en el cual el proyecto paga la inversión inicial y en su cálculo se supone que todos los ingresos se utilizan para amortizar la inversión, aunque en la mayoría de los proyectos el plan de financiamiento no coincide con esto. A partir del punto C se empiezan a obtener ganancias y el proyecto se abandona en el punto D.

El conocimiento de este esquema permite dar respuesta a una serie de preguntas que se generan en la iniciación de cualquier proyecto: ¿Se puede costear el proyecto? La respuesta está implícita en el segmento BF. ¿En cuánto tiempo se recuperará la inversión? Lo señala el segmento BC. ¿Cuánto se ganará y en qué tiempo? A esto dan respuesta los segmentos DE y CD, respectivamente.

Estos factores son necesarios pero no suficientes para jerarquizar las distintas alternativas que se presentan en un proyecto dado. Para enfatizar este hecho, considérese el siguiente ejemplo (Fig. 2).

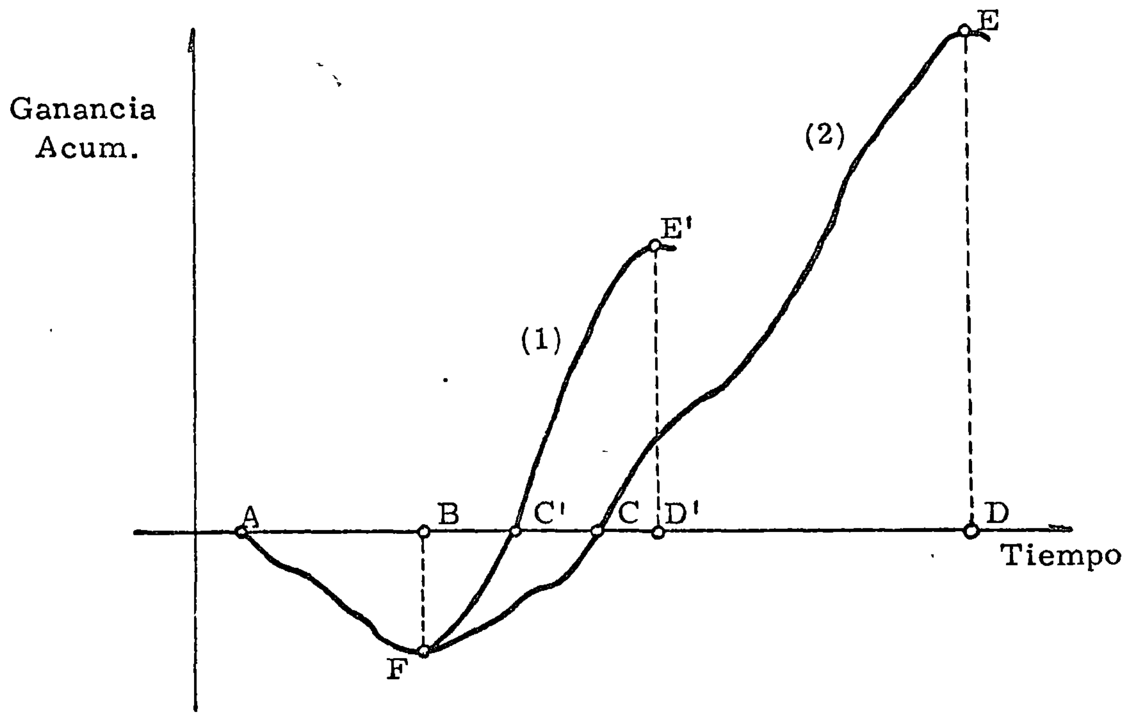


FIGURA 2

Se tiene un proyecto con alternativas que coinciden en el período de inversión y en la inversión inicial. ¿Cuál de las dos elegir? Si se escoge la alternativa 2, la ganancia es mayor pero se requiere más tiempo que en la alternativa 1 para obtenerla; por otro lado, con la alternativa 1 se recupera más pronto el capital invertido que en la alternativa 2. Surge así la necesidad de contar con una serie de factores e índices que señalen cuantita tivamente la bondad económica de los proyectos. Algunos de éstos son: TASA DE GANANCIA, PORCENTAJE DE GANANCIA y PERIODO DE CANCELACION.

En la actualidad, con el auxilio de las computadoras electrónicas, es posible determinar el valor exacto de los índices mencionados; pero como su cálculo está basado en estimaciones del comportamiento futuro de los factores que intervienen en el análisis económico, los valores únicos de tales elementos de juicio son menos definitivos a medida que el monto de las inversiones crece. La decisión no se toma en igual forma cuando se invierte una cantidad pequeña, cuya pérdida involucraría sólo algunos reajustes, que cuando se invierten cantidades de tal magnitud que pondrían en peligro la existencia de la empresa de no proporcionar un mínimo preestablecido de ganancias. Existe pues la necesidad de considerar la incertidumbre implicada en las predicciones, a fin de cuantificar el riesgo correspondiente.

Tradicionalmente se acostumbra tratar este problema efectuando predicciones conservadoras o considerando en los estudios económicos tasas de descuento elevadas; pero este procedimiento introduce el peligro de presentar no atractivas las buenas posibilidades de inversión. Así pues, para que la evaluación de un proyecto quede completa y sea confiable, es necesario que incluya un análisis de riesgo para cada una de las alternativas que se presenten.

Conviene señalar que una vez tomada la decisión de ejecutar un proyecto determinado, es necesario controlarlo para que se comporte de acuerdo con lo planeado. Esto significa que en cada una de las etapas de la vida económica se tendrán que resolver problemas de importancia cardinal, como por ejemplo durante el período de inversión, donde la empresa debe enfrentarse a cuestiones relacionadas con el financiamiento y la asignación óptima de recursos, para que las actividades sean desarrolladas al menor costo posible y con toda oportunidad. Estos mismos problemas y otros como los relacionados con el reemplazo de equipo y transporte de materiales, se presentan a su vez durante la vida productiva del proyecto.

Los temas esbozados constituyen el material de este curso, que se inicia a continuación.

Capítulo I

I N T E R E S

****

Se denomina INTERES al dinero que se paga por el uso de un capital prestado, o al dinero ganado por un capital invertido. La cantidad colocada a interés se llama PRINCIPAL y recibe el nombre de MONTO la suma del interés más el principal.

TASA DE INTERES i, es el cociente del interés I sobre el principal P para un período de tiempo t; esto es,

$$i = \frac{I/P}{t} \qquad (1.1)$$

El período de tiempo puede adquirir diferentes valores: un año, un semestre, un trimestre, etc. De la ecuación (1.1), la tasa de interés resulta una fracción por unidad de tiempo; pero se acostumbra expresarla en por ciento, sobre la unidad de tiempo. Una tasa de interés de 7% anual, significa que en un año se pagarán 7 pesos por cada 100 pesos de principal.

Durante siglos se ha discutido la justificación del interés. El punto de vista del prestamista señala que al desprenderse por algún tiempo de su dinero, deja de satisfacer algunas de sus necesidades personales. Al cederlo en préstamo, debe determinar la magnitud del interés a ganar cuantificando la probabilidad que existe de no recuperar su dinero, los gastos que origina la investigación de la solvencia del prestatario y la posible ganancia que no percibirá por desprenderse de su capital. El prestatario, por su parte, paga la suma adecuada por disfrutar anticipadamente de un capital, porque considera que con éste satisfará algunas de sus necesidades inmediatamente o conseguirá bienes de producción, con los cuales generará más dinero.

Independientemente de su justificación, el interés es una realidad en toda transacción económica y se debe tomar siempre en cuenta en cualquier estudio económico.

INTERES SIMPLE E INTERES COMPUESTO

Cuando el interés se paga al término de cada período de tiempo, permaneciendo constante el principal, se dice que tiene lugar una operación bajo INTERES SIMPLE. Si el interés se suma al principal al final de cada período de tiempo, el interés se convierte o COMPONE en principal y entonces se origina el INTERES COMPUESTO.

Si t es igual a uno, entonces de la ecuación (1.1):

$$I = iP$$

y el interés acumulado en n períodos de tiempo iguales es iPn, resultando el monto

$$S = P + iPn = P(1 + in) \qquad (1.2)$$

EJEMPLO 1.1 ¿Cuánto tiempo se necesita para que una cantidad de \$4,300 se transforme en \$5,000 al 8% de interés simple anual?

SOLUCION.- De la ecuación (1.2), haciendo P=4300, S=5000 e i=0.08, se tiene

$$n = \frac{(S/P) - 1}{i} = 2.03 \text{ años}$$

Obsérvese que en operaciones bajo interés simple, el interés es directamente proporcional al tiempo transcurrido. Cuando el período de tiempo es igual a un año y se desea calcular el interés para fracciones de éste, se acostumbra multiplicar iP por la fracción

$$\frac{\text{número de días}}{360 \text{ ó } 365}$$

A fin de obtener una expresión que permita calcular el interés compuesto, sea P el principal en el tiempo 0 y S el monto en el tiempo n. El interés al terminar el primer período es iP, de modo que el monto en ese tiempo es

$$P + iP = P(1 + i)$$

La expresión anterior se toma como un nuevo principal que, al finalizar el segundo período, aporta un interés P(1 + i)i, resultando el monto en el tiempo 2 igual a

$$P(1 + i) + P(1 + i)i = P(1 + i)^2$$

Generalizando este razonamiento se deduce que el monto en el tiempo n es

$$S = P(1 + i)^n \qquad (1.3)$$

El factor $(1 + i)^n$ se encuentra tabulado en el Apéndice para distintos valores de n e i, representándose por SP.

EJEMPLO 1.2 El monto actual de una inversión inicial de $1,200 realizada hace 5 años al 10% de interés simple anual, se deposita al 8% de interés compuesto anual. ¿Cuál será el monto total dentro de 7 años ?

SOLUCION.- El monto actual, de acuerdo con la ecuación (1.2) es

$$1200(1 + 5 \times 0.1) = 1800;$$

el monto dentro de 7 años será

$$1800(1.08)^7 = \$3,084.84$$

En lo sucesivo, a menos que se aclare lo contrario, cuando se hable de interés se entenderá que se trata de interés compuesto.

VALOR ACTUAL

Por medio del concepto de interés se observa que el valor del dinero varía con el tiempo. Por ejemplo, si se deposita la cantidad de $100 al 8% de interés anual, dentro de 2 años se transformará en $116.64; lo que quiere decir que $100 de ahora son $116.64 dentro de 2 años, o bien que el VALOR ACTUAL de \$116.64 es \$100. Para determinar el valor actual de \$116.64, se descontó el interés ganado durante 2 años; es decir, se dividió por $(1.08)^2$.

En general, si se desea obtener el valor actual P de una cantidad S disponible dentro de n períodos de tiempo, se debe dividir ésta por $(1 + i)^n$:

$$P = \frac{S}{(1 + i)^n} \qquad (1.4)$$

El factor $(1 + i)^{-n}$ se representa en el Apéndice por PS.

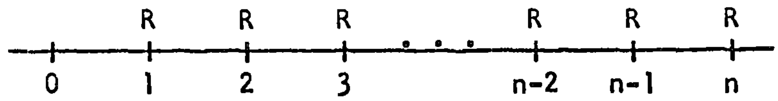
EJEMPLO 1.3 Una persona posee una letra de cambio por valor de \$3,450 que se vence dentro de 4 años. Decide negociar el documento a través de un banco cuya tasa de interés para este tipo de operaciones es de 8% anual. ¿Cuánto recibirá ahora por su letra ?

SOLUCION.- El banco paga por la letra de cambio el valor actual de ésta, o sea

$$3450 \times PS_{4,8\%} = \$2,535.75$$

SERIES DE PAGOS

Se da el nombre de SERIE DE PAGOS o de ANUALIDADES, a una distribución uniforme en el tiempo de cantidades iguales R. Por su aplicación tan frecuente, conviene tener expresiones para calcular el monto y el valor actual de estas series. A fin de obtenerlas, supóngase que se desea constituir un capital S, aportando cantidades



iguales R anualmente, empezando dentro de un año. La primera entrega, realizada en el año 1, ganará intereses durante n-1 períodos, para convertirse en

$$R(1 + i)^{n-1}$$

al terminar el año n. La segunda, al término del año 2, ganará intereses durante n-2 períodos y se convertirá, al finalizar el tiempo señalado, en

$$R(1 + i)^{n-2}$$

y así sucesivamente, resultando el monto:

$$S = R(1+i)^{n-1} + R(1+i)^{n-2} + \ldots + R(1+i) + R \qquad (1.5)$$

Multiplicando la ecuación anterior por (1+i) y restando (1.5) al resultado, de inmediato se despeja S:

$$S = R\,\frac{(1+i)^n - 1}{i} \qquad (1.6)$$

El factor de R en (1.6) se representa por SR en el Apéndice.

El valor actual P de la serie se puede càlcular sumando los valores actuales de cada entrega, como se indica a continuación:

$$P = R(1+i)^{-1} + R(1+i)^{-2} + \ldots + R(1+i)^{-n}$$

Pero, aprovechando la ecuación (1.6), que expresa el monto de la serie, se obtiene el valor buscado multiplicando S por $(1+i)^{-n}$, esto es,

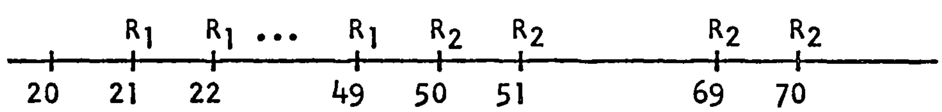
$$P = R\,\frac{(1+i)^n - 1}{i(1+i)^n} \qquad (1.7)$$

EJEMPLO 1.4 Una persona de 20 años desea tener un ingreso anual constante de \$36,000 desde la edad de 50 años, hasta la de 70. ¿Cuánto debe depositar anualmente, para lograr su propósito, empezando dentro de un año, en un fondo donde ganará intereses al 10% anual, suponiendo que cuando tenga 49 años la tasa aumentará a 12% anual?

SOLUCION.- Primero se debe determinar con la ecuación (1.7) el valor, en el año 49, de la serie de pagos R_2:

$$P_{49} = 36000\,(PR)_{21,12\%} = \$272,232$$

Este valor, considerado como el monto en el año 49 de la serie R_1,

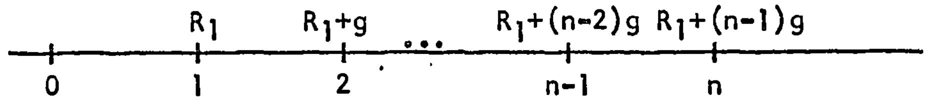


$R_1 =$ $R_2 = \$36,000$

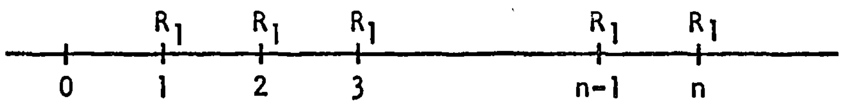
proporciona la solución del problema a través de la ecuación (1.6):

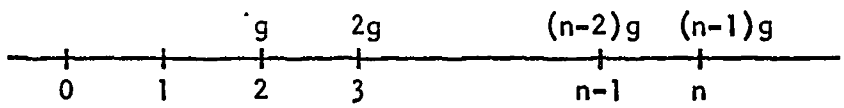
$$R_1 = 272,232\,(RS)_{29,10\%} = \$17,151$$

Con frecuencia se encuentran series de pagos que crecen o



decrecen en proporción aritmética. El monto de éstas se puede calcular considerando dos series, una de cantidades iguales R_1 y otra cu-





yo primer pago g se efectúa en el tiempo 2, cuyo segundo pago 2g se efectúa en el tiempo 3 y así sucesivamente hasta el penúltimo $(n-2)g$ en el tiempo n-1 y el último, $(n-1)g$, en el tiempo n. En estas condiciones, la expresión para el valor actual resulta

$$P = R_1 \frac{(1+i)^n - 1}{i(1+i)^n} + g(1+i)^{-2} + 2g(1+i)^{-3} + \ldots +$$

$$(n-1)g(1+i)^{-n} \qquad (1.8)$$

A fin de escribir (1.8) de una manera más adecuada, sea

$$M = g(1+i)^{-2} + 2g(1+i)^{-3} + \ldots + (n-1)g(1+i)^{-n} \qquad (1.9)$$

Si ahora se multiplica la ecuación anterior por $(1+i)$ y al resultado se le resta (1.9), queda

$$Mi = g(1+i)^{-1} + g(1+i)^{-2} + \ldots + g(1+i)^{-(n-1)} - (n-1)g(1+i)^{-n} \quad \text{--- (1.10)}$$

Multiplicando de nuevo por (1+i) y restando (1.10), se obtiene

$$Mi^2 = g\left[1 - n(1+i)^{-(n-1)} + n(1+i)^{-n} - (1+i)^{-n}\right]$$

de donde, multiplicando y dividiendo por $(1+i)^n$, resulta la espresión

$$M = \frac{g}{i(1+i)^n}\left[\frac{(1+i)^n - 1}{i} - n\right]$$

que sustituída en (1.8) produce finalmente el resultado buscado:

$$P = (R_1 + \frac{g}{i})\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right] - \frac{ng}{i(1+i)^n} \qquad (1.11)$$

El monto S de la serie se obtiene multiplicando (1.11) por $(1+i)^n$:

$$S = (R_1 + \frac{g}{i})\left[\frac{(1+i)^n - 1}{i}\right] - \frac{gn}{i} \qquad (1.12)$$

EJEMPLO 1.5 Un yacimiento petrolífero produce actualmente 35,000 metros cúbicos de aceite al año. Se estima que la producción declinará a razón de 2,500 metros cúbicos por año, durante los próximos 14 años. Si el precio del aceite es de $\$175/m^3$ durante los primeros 8 años y de $\$200/m^3$ de ese tiempo en adelante, ¿cuál es el ingreso total reducido a valor actual, para una tasa de interés de 8% anual?

SOLUCION.- Durante el primer año se tiene una producción de 35,000 metros cúbicos. Un cálculo más preciso incluiría los intereses ganados durante todo el año; pero para fines de ilustración del uso de la ecuación (1.11), se supondrá que todos los ingresos se obtienen instantáneamente, al final de cada año. Más adelante se estudiará el procedimiento que debe seguirse cuando el período de conversión de interés a capital, es distinto del especificado por la tasa de interés nominal i.

Puesto que existen dos diferentes precios del aceite, el problema debe resolverse en dos partes. El ingreso de los primeros 8 años se calcula con $R_1 = 35000 \times 175$, $g = -2500 \times 175$, $i = 0.08$ y $n = 8$, resultando mediante la aplicación de (1.11):

$$P_1 = \$34'353,783$$

El valor actual de los ingresos de los últimos 6 años se calcula empleando (1.11) para reducir al año 8 y (1.4) para reducir al año 0. Así,

$$P_2 = \$10'337,334$$

De donde finalmente

$$P = P_1 + P_2 = \$44'691,117$$

TASA DE INTERES EFECTIVA

Con mucha frecuencia los intereses no se capitalizan al finalizar cada período especificado por la tasa de interés nominal i, sino a intervalos de tiempo inferiores. Por ejemplo, a menudo se escucha la frase: "... 8% de interés anual, pagadero trimestralmente ...", lo cual significa que si se depositan \$100 al 8% anual, al término de cada trimestre se podrá disponer del interés correspondiente (\$2), pero que si se reinvierte, el monto al terminar el año es

$$S = P(1+\frac{i}{4})^4 = 100(1.02)^4 = \$108.24$$

De manera que la tasa de interés, calculada de acuerdo con la definición (1.1), resulta

$$\text{Tasa de Interés Efectiva, } j = \frac{\text{Interés}}{\text{Principal}}$$

$$= \frac{S - P}{P}$$

$$= 8.24\% \text{ anual.}$$

o sea 0.24% más que la tasa de interés nominal i.

En general, si el período que señala la tasa de interés nominal se divide en m partes iguales, la tasa de interés efectiva será

$$j = \frac{P(1+\frac{i}{m})^m - P}{P}$$

$$= (1+\frac{i}{m})^m - 1 \qquad (1.13)$$

Habiendo calculado la tasa de interés efectiva j, el monto al tiempo n se puede calcular fácilmente mediante la expresión

$$S = P(1+j)^n \qquad (1.14)$$

o bien, sustituyendo (1.13) en (1.14):

$$S = P(1+\frac{i}{m})^{mn} \qquad (1.15)$$

La tasa de interés efectiva se puede tabular para diferentes valores de m, n e i.

EJEMPLO 1.6 Se desea tener una tasa de interés efectiva de 6% anual. ¿Cuál debe ser la tasa de interés nominal si la composición es semestral?

SOLUCION.- Para j=0.06 y m=2, de la ecuación (1.13) resulta

$$i = m(\sqrt{j+1} - 1) = 5.91\% \text{ anual}$$

COMPOSICION CONTINUA

Como ya se indicó, para una tasa de interés nominal de 8% anual se tiene una tasa de interés efectiva de 8.24%, cuando la composición es trimestral. Aplicando el mismo procedimiento se obtiene j=8.16% para una composición semestral. Esto es, a medida que crece el número de períodos m, la tasa de interés efectiva aumenta. Si la composición es instantánea o CONTINUA, j tiene un límite, el cual se determina haciendo tender m a infinito en la expresión (1.13):

$$j = \lim_{m \to \infty} \left[(1+\frac{i}{m})^m - 1 \right]$$

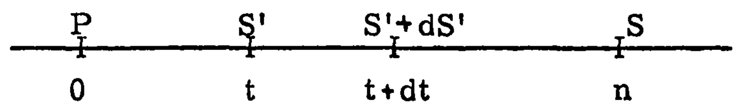
$$= \lim_{m \to \infty} \left[(1+\frac{i}{m})^{m/i} \right]^i - 1$$

$$= e^i - 1 \qquad (1.16)$$

El monto al término del período n se obtiene aplicando (1.16) a (1.14) y resulta

$$S = P e^{in} \tag{1.17}$$

La expresión (1.17) también se puede obtener como sigue: Si S' es el monto en un tiempo cualquiera t, el interés



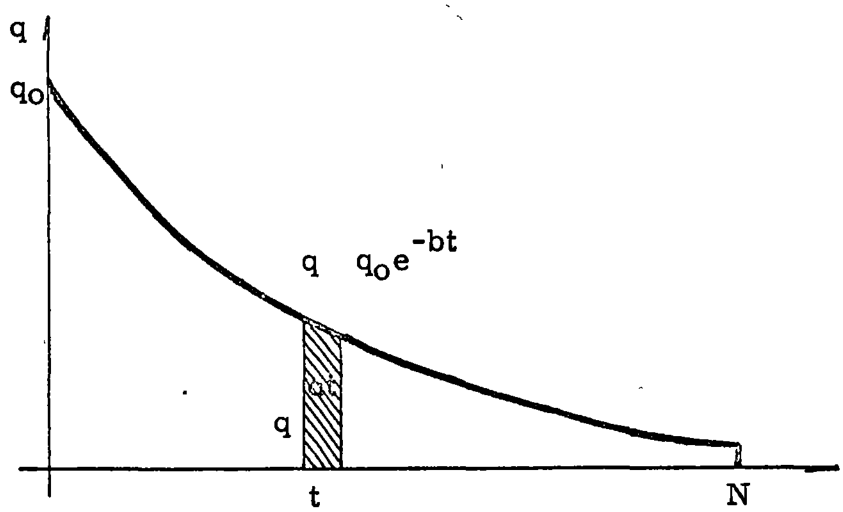
aportado por esta cantidad durante el período dt será

$$dS' = i\,S'\,dt$$

de donde, separando variables e integrando entre los límites $S' = P$ para $t = 0$ y $S' = S$ para $t = n$, resulta de inmediato (1.17).

EJEMPLO 1.7 Para ilustrar el uso del concepto de composición contínua, calcularemos la ganancia en un proyecto de perforación de un pozo petrolífero, cuyo costo inicial se estima en C pesos, y cuya producción, según los estudios técnicos correspondientes, se espera que decline exponencialmente. El ritmo de producción inicial es q_o (m^3/unidad de tiempo) y el límite económico de producción se encuentra en el tiempo N.

Si los ritmos de producción se dibujan en papel semilogarítmico, la gráfica será una recta de pendiente b, resultando por lo tanto su ecuación $q = q_o e^{-bt}$.



Considérese el volumen producido durante el período de tiempo dt, a un tiempo cualquiera t:

$$q\,dt$$

Este volumen, multiplicado por el precio neto del aceite (precio en el mercado menos costo unitario de operación y mantenimiento) u, da el ingreso neto durante el tiempo dt y su valor actual es

$$\frac{u\,q\,dt}{(1+j)^t}$$

o bien, de acuerdo con la ecuación (1.17):

$$\frac{u\,q\,dt}{e^{it}} = u\,q\,e^{-it} \qquad (1.18)$$

Integrando la ecuación (1.18) para obtener el ingreso total reducido a valor actual, queda

$$\int_0^N u\, q_o\, e^{-(b+i)t}\, dt = u\, q_o \frac{1 - e^{-(b+i)N}}{b+i}$$

donde se ha considerado u constante y se ha sustituído q por su expresión. Finalmente, la ganancia G resulta

$$G = u\, q_o \frac{1 - e^{-(b+i)N}}{(b+i)} - C \qquad (1.19)$$

En el capítulo siguiente, sobre Evaluación de Proyectos, se hará referencia a esta última expresión y se presentarán ejemplos numéricos.

El valor actual de una serie de pagos iguales en composición contínua resulta ser

$$P = Re^{-i} + Re^{-2i} + Re^{-3i} + \dots + Re^{-(n-1)i} + Re^{-ni} \qquad (1.20)$$

de donde, multiplicando ambos miembros por e^i y restando (1.20) queda

$$P = R \frac{1 - e^{-ni}}{e^i - 1} \qquad (1.21)$$

En forma análoga se obtiene el monto:

$$S = R \frac{e^{ni} - 1}{e^i - 1} \qquad (1.22)$$

EJEMPLO 1.8 ¿Cuál es el valor actual de $4,000 colocados en el año 13, al 6% anual compuesto contínuamente? ¿Compuesto anualmente?

SOLUCION.- Aplicando la ecuación (1.17) se tiene

$$P = S\ e^{-ni} = 4{,}000 \times 0.45841 = \$1{,}833.64$$

Para composición anual:

$$P = S(1 \quad i)^{-n} = 4{,}000 \times 0.46884 = \$1{,}875.36$$

EJEMPLO 1.9 En una inversión de $20,000 se desea tener una tasa de interés de 10% anual en composición contínua. ¿Cuántos años se requieren para recuperar el capital, a un ritmo de $2,500 por año? Resolver el mismo problema para composición anual.

SOLUCION.- De la ecuación (1.21), con $P = 20{,}000$ y $R = 2{,}500$, se tiene

$$\frac{20{,}000}{2{,}500} = 8 = \frac{1 - e^{-0.1n}}{e^{0.1} - 1}$$

de donde $n = 18.42$ años.

Para composición anual resulta $n = 16.89$ años.

EQUIVALENCIA

Usando los conceptos presentados en este capítulo, el lector está en condiciones de expresar, para una tasa de interés dada, series de pagos en términos de una sola cantidad actual o futura, o viceversa. Con este método puede comparar unas sumas con otras, aunque estén distribuídas arbitrariamente en el tiempo. Se dice que las cantidades P, S y las series R son equivalentes, cuando satisfacen las ecuaciones (1.2), (1.3), (1.4), (1.6), (1.7), (1.17) y similares.

El concepto de equivalencia se puede aprovechar cuando se desea comparar dos o más proyectos desde el punto de vista económico, tomando como base sus ingresos o sus costos.

EJEMPLO 1.10 Un equipo (A) tiene un costo inicial de \$16,000 y se estima que sus costos de operación y mantenimiento anuales serán: combustible, \$5,000; mantenimiento, \$1,100; mano de obra \$7,000; seguro, \$160 (1% costo inicial); supervisión \$1,500 y renta de espacio, \$500. Otro equipo (B) tiene un costo inicial de \$11,000 y los siguientes costos anuales: combustible, \$6,000; mantenimiento, \$1,150; mano de obra, \$7,500; seguro, \$110; supervisión \$1,600 y renta de espacio, \$475. Determinar cuál de los dos es más económico, suponiendo que ambos tienen una vida útil de 5 años y son capaces de realizar el mismo trabajo con igual eficiencia técnica.

SOLUCION.- El equipo más económico será el que tenga menor costo total. Los cálculos se pueden efectuar considerando cualquier tasa de interés.

$$C_A = 16{,}000 + 15{,}446\ (PR_{5,7\%}) = \$78{,}566$$

$$C_B = 11{,}000 + 16{,}835\ (PR_{5,7\%}) = \$80{,}024$$

El equipo A resulta entonces más económico.

EJEMPLO 1.11 Hallar la serie de pagos iguales R equivalente a la serie de pagos que crecen en proporción aritmética (página 8).

SOLUCION. - Las dos series son equivalentes si tienen el mismo valor actual; es decir, de las ecuaciones (1.7) y (1.11):

$$R \frac{(1+i)^n - 1}{i(1+i)^n} = R_1 \frac{(1+i)^n - 1}{i(1+i)^n} + \frac{g}{i(1+i)^n}\left[\frac{(1+i)^n - 1}{i} - n\right]$$

de donde

$$R = R_1 + g\left\{\frac{1}{i} - \frac{n}{i}\left[\frac{i}{(1+i)^n - 1}\right]\right\}$$

C a p í t u l o II
EVALUACION DE PROYECTOS
****

Al principiar el curso se hizo notar la necesidad de contar con una serie de factores e índices, que permitan evaluar económicamente las alternativas que se presenten en un proyecto de inversión dado. En este capítulo se estudiarán los índices de uso más frecuente: GANANCIA, TASA DE GANANCIA, PORCENTAJE DE GANANCIA SOBRE LA INVERSION, PERIODO DE CANCELACION y PERIODO DE RESTITUCION. Se dejará para un capítulo especial el estudio de la INVERSION INICIAL.

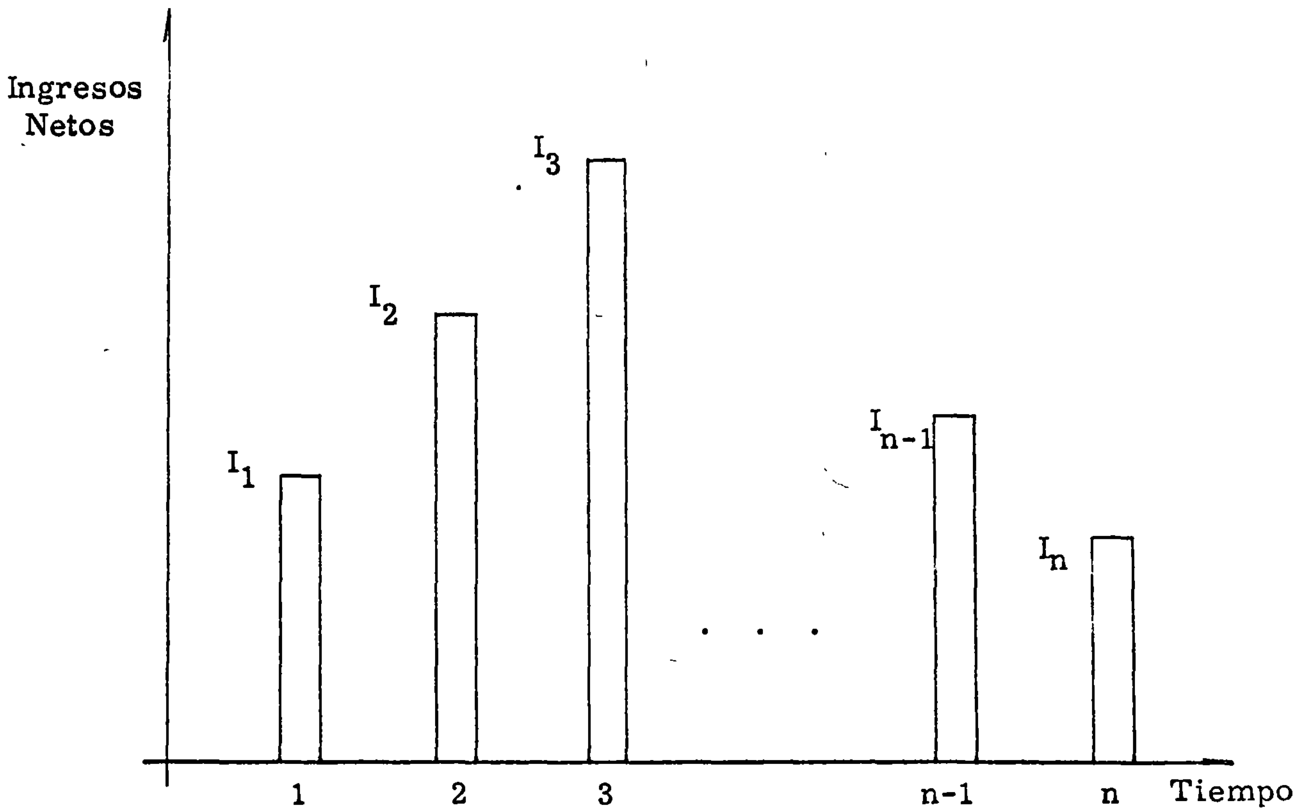
En su oportunidad se estudiarán con detalle algunos de los conceptos que sirven de base para comprender el significado de los índices citados. Por el momento basta señalar que los cuatro factores fundamentales de la evaluación de proyectos se representarán como sigue:

C, inversión inicial

I_k, ingreso neto (ingreso bruto menos costo de operación y mantenimiento) en el período de tiempo k

n, vida económica (vida útil) del proyecto

i, tasa de interés de oportunidad



GANANCIA

Se llama GANANCIA a la diferencia obtenida entre la suma de los ingresos netos reducidos a valor actual y la inversión inicial:

$$G = I_1(1+i)^{-1} + I_2(1+i)^{-2} + \ldots + I_n(1+i)^{-n} - C$$

o sea brevemente

$$G = \sum_{k=1}^{n} I_k(1+i)^{-k} - C \qquad (2.1)$$

EJEMPLO 2.1 En un proyecto se tiene una inversión inicial de $50,000 e ingresos netos constantes de $15,000 por año durante la vida del proyecto (6 años). Determinar la ganancia considerando una tasa de interés de oportunidad de 12% anual.

SOLUCION. - Por ser los ingresos constantes, el problema se puede resolver como una serie de pagos iguales:

$$G = 15{,}000(PR_{6,12\%}) - 50{,}000 = \$11{,}669.50$$

TASA DE GANANCIA

Para calcular la TASA DE GANANCIA o RENTABILIDAD de la inversión se ha usado durante mucho tiempo un método que proporciona resultados optimistas, por las consideraciones favorables que implica su determinación. Nos parece más adecuado el cálculo de la tasa de ganancia con un método que también presentaremos en esta sección. Como se verá, los resultados que se obtienen con la aplicación de este último son realistas, pero con tendencia a ser conservadores. Llamaremos al primer procedimiento Método

Optimista y al segundo Método Realista.

Método Realista

Para determinar la rentabilidad o tasa de ganancia por este método se supone que todos los ingresos se depositan en un fondo a medida que se van obteniendo y ganan intereses a la tasa de oportunidad i. Así, el primer ingreso I_1 se deposita en el tiempo 1 y gana intereses durante n-1 períodos; o sea que I_1 se convierte, al terminar la vida del proyecto, en

$$I_1(1+i)^{n-1}$$

El ingreso I_2, depositado en el tiempo 2, gana intereses durante n-2 períodos y se convierte en

$$I_2(1+i)^{n-2}$$

Generalizando este razonamiento, se concluye que al terminar el año n en el fondo se tendrá la cantidad

$$I_1(1+i)^{n-1} + I_2(1+i)^{n-2} + I_3(1+i)^{n-3} + \ldots + I_{n-1}(1+i) + I_n \tag{2.2}$$

Todo inversionista tiene siempre la oportunidad de invertir su dinero a la tasa i. De no invertir en el negocio en cuestión, su capital C se convertiría, al transcurrir n períodos de tiempo, en

$$C(1+i)^n \tag{2.3}$$

Para que el negocio sea atractivo, la cantidad (2.2) debe ser mayor que la cantidad (2.3); o sea que debe haber una tasa R mayor que i, que aplicada a (2.3) establezca la igualdad entre las dos expresiones:

$$C(1+R)^n = \sum_{k=1}^{n} I_k(1+i)^{n-k} \qquad (2.4)$$

De esta manera puede R interpretarse como la tasa de interés a la que el capital C invertido en el negocio ha ganado intereses. Es decir, R es la rentabilidad de la inversión. Despejando de (2.4) resulta finalmente la expresión para la tasa de ganancia

$$R = \sqrt[n]{\frac{\sum_{k=1}^{n} I_k(1+i)^{n-k}}{C}} - 1 \qquad (2.5)$$

EJEMPLO 2.2 Determinar la tasa de ganancia usando el método realista, con los datos del ejemplo 2.1.

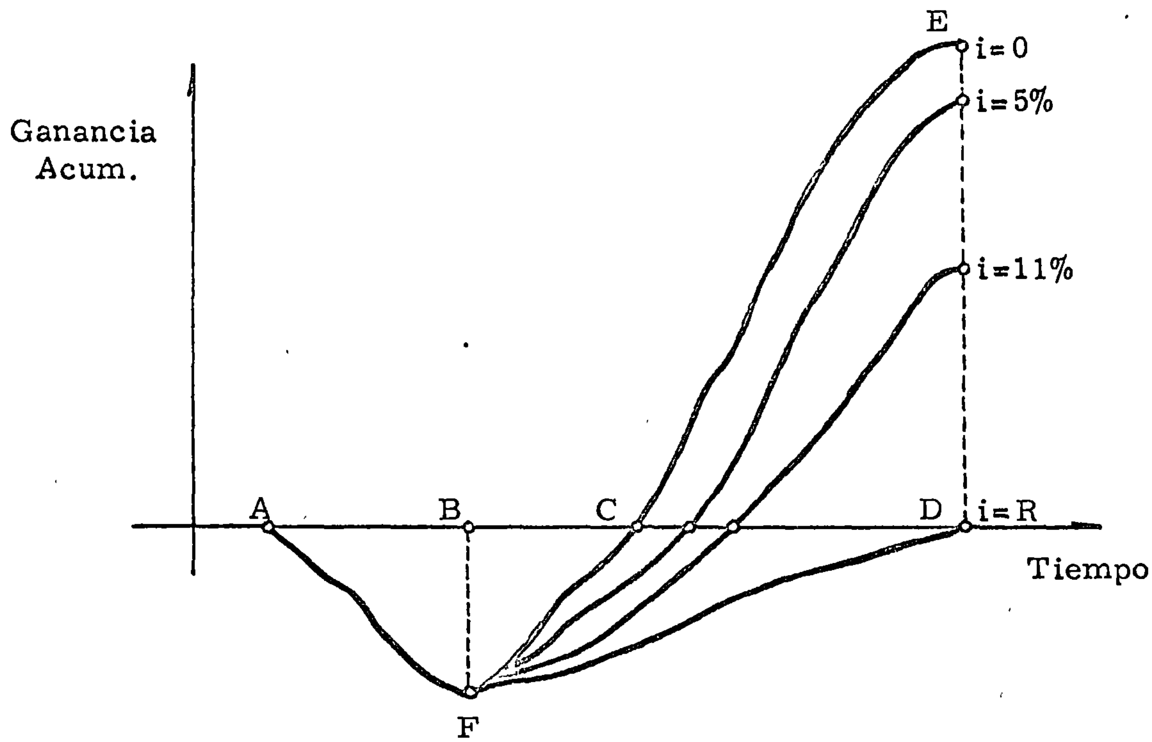
SOLUCION.- Como los ingresos son constantes, el problema se resuelve fácilmente:

$$R = \sqrt[6]{\frac{15{,}000\ (SR_{6,12\%})}{50{,}000}} - 1$$

$$= 16\%\ \text{anual}$$

Método Optimista

Tomando como referencia la figura 1 de la Introducción, se observa que si los ingresos netos se reducen a valor actual usando tasas de interés cada vez mayores, la ganancia acumulada



es cada vez menor, haciéndose cero a una tasa igual a R. De esta manera se interpreta a R como la tasa que aplicada a los ingresos para su reducción a valor actual, hace que la suma de los mismos sea igual a la inversión inicial. Puede interpretarse más fácilmente el significado de la tasa de ganancia con el método optimista, si se emplea un procedimiento análogo al usado para explicar el método realista. En efecto, supóngase que los ingresos netos I_k se reinvierten en el negocio a medida que se van obteniendo; así, al finalizar el tiempo n, los ingresos se convierten en

$$I_1(1+R)^{n-1} + I_2(1+R)^{n-2} + \ldots + I_n(1+R)^0 \qquad (2.6)$$

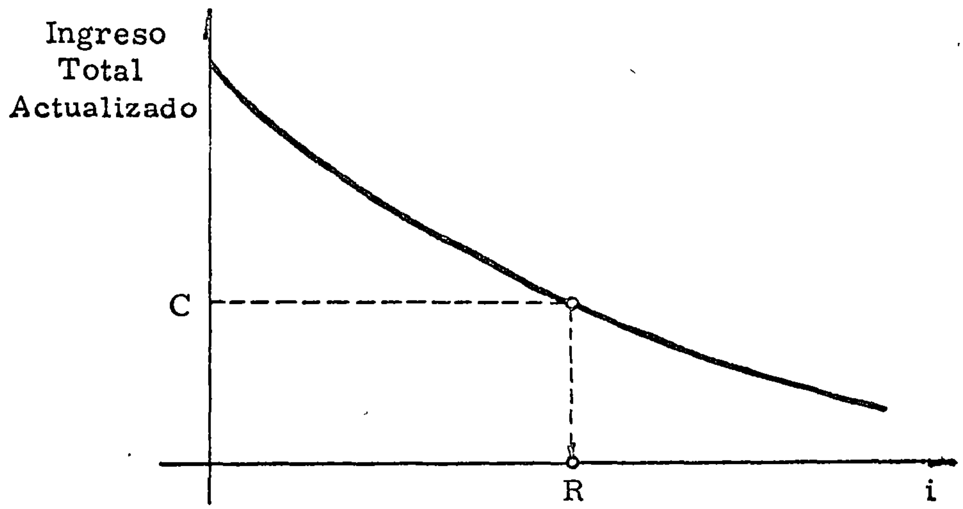
Al igualar (2.6) con la expresión $C(1+R)^n$ resulta la ecuación de grado n para R

$$C(1+R)^n = \sum_{k=1}^{n} I_k(1+R)^{n-k} \qquad (2.7)$$

Eliminando las consideraciones teóricas acerca de las soluciones de (2.7), es posible hallar un procedimiento de ensaye y error sencillo dividiendo (2.7) por $(1+R)^n$:

$$C = \frac{I_1}{(1+R)^1} + \frac{I_2}{(1+R)^2} + \frac{I_3}{(1+R)^3} + \ldots + \frac{I_n}{(1+R)^n} \qquad (2.8)$$

La expresión (2.8) indica que si los ingresos se reducen a valor actual utilizando la tasa de ganancia como tasa de descuento, la suma de los ingresos actualizados es la inversión inicial. Esta observación permite utilizar el método descrito gráficamente a continuación, que consiste en actualizar los ingresos bajo diferentes tasas, trazar la curva correspondiente y hallar el punto de intersección con la ordenada constante igual a la inversión inicial. La tasa correspondiente al punto de intersección, hace que la suma de los ingresos netos actualizados sea igual a C, equivaliendo por lo tanto a la tasa de ganancia.



No siempre es posible reinvertir los ingresos en el negocio a medida que se van obteniendo, por eso este método, que se basa en esa consideración, arroja resultados optimistas. Aparte de esta fuerte limitación, tiene la desventaja de requerir un procedimiento de ensaye y error para su determinación.

EJEMPLO 2.3 Determinar la tasa de ganancia con el método optimista, utilizando los datos del ejemplo 2.1.

SOLUCION. - Empleando primero la tasa de oportunidad, resulta la siguiente suma de ingresos actualizados:

$$15{,}000\,(PR_{6,12\%}) = \$61{,}669.50$$

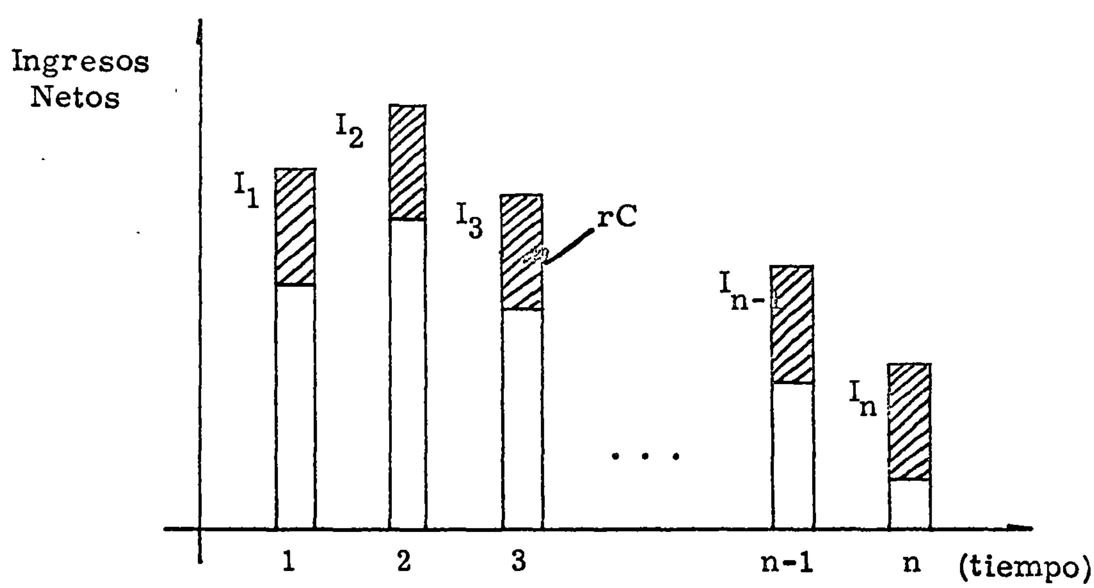
la cual es superior a la inversión inicial ($50,000).

Para i=14% resulta la cantidad de $58,329, también superior a $50,000.

Para i=17% la suma actualizada de los ingresos netos resulta de $53,838. Como para i=20% los ingresos netos reducidos a valor actual suman $49,883, puede decirse que aproximadamente la tasa de ganancia es R = 20%. Nótese que resultó superior a la calculada con el primer método.

PROCENTAJE DE GANANCIA SOBRE LA INVERSION

El PROCENTAJE DE GANANCIA SOBRE LA INVERSION indica la cantidad que con respecto a la inversión inicial, se puede considerar como ganancia por período; es decir, cuánto se debe tomar de los ingresos en cada período para dejar exclusivamente las cantidades que amortizarán la inversión.



Sea rC la cantidad constante que se extrae de los ingresos netos en cada período y que puede considerarse como ganancia. Después de esta disminución, de cada ingreso queda $I_k - rC$, para $k=1,2,\ldots,n$. Estas diferencias, reducidas a valor actual, deben entonces sumar exactamente la inversión inicial:

$$C=(I_1-rC)(1+i)^{-1}+(I_2-rC)(1+i)^{-2}+\ldots+(I_n-rC)(1+i)^{-n}$$

De esta expresión se despeja fácilmente r; de manera que considerando el Porcentaje de Ganancia sobre la Inversión (PGI) igual a 100r, queda:

$$PGI = \frac{i(1+i)^n}{(1+i)^n - 1} \; \frac{\sum_{k=1}^{n} I_k(1+i)^{-k} - C}{C} \; 100 \qquad (2.9)$$

EJEMPLO 2.4 Calcular el porcentaje de ganancia del proyecto citado en el ejemplo 2.1.

SOLUCION. - Aplicando la ecuación (2.9) y tomando en cuenta (2.1) resulta de inmediato

$$PGI = (RP_{6,12\%}) \frac{11{,}669.5}{50{,}000} \; 100 = 6.9\%$$

o sea que si de cada ingreso neto se extrae la cantidad de $3,450 (rC), con el resto se paga la inversión inicial.

PERIODO DE CANCELACION

Se denomina PERIODO DE CANCELACION al tiempo en el cual los ingresos netos reducidos a valor actual igualan la inversión inicial:

$$C = \sum_{k\ 1}^{Pc} I_k(1+i)^{-k} \qquad (2.10)$$

EJEMPLO 2.5 Determinar el período de cancelación del proyecto citado en el ejemplo 2.1.

SOLUCION. - El problema se puede resolver aplicando ecuaciones de series de pagos iguales:

$$50{,}000 = 15{,}000(PR_{Pc,12\%})$$

Si se busca en las Tablas de Interés para i=12% el valor de (PR) = 3.333, se obtiene aproximadamente Pc = 4.5 años.

Analizando los índices calculados, se puede tomar la decisión de invertir o no en el proyecto en cuestión. A reserva de discutir más adelante la relación entre el porcentaje de ganancia, la tasa de oportunidad y la tasa de ganancia, se deja al lector la tarea de meditar sobre los resultados obtenidos en los ejemplos: PGI = 7%, R=16% con i=12%. ¿Conviene invertir en este proyecto?

EJEMPLO 2.6 Si en un proyecto se tiene una inversión inicial de $250,000, una vida útil de 7 años y una tasa de oportunidad de 7.5% anual, determinar la ganancia, la tasa de ganancia, el porcentaje de ganancia y el período de cancelación. Utilizar los siguientes datos de ingresos brutos y costos de operación y mantenimiento:

AÑO	INGRESOS BRUTOS	COSTOS DE OPERACION Y MANTENIMIENTO
1	$ 50,000	$10,000
2	80,000	15,000
3	95,000	15,000
4	120,000	20,000
5	120,000	20,000
6	80,000	25,000
7	40,000	30,000

SOLUCION. - Para el cálculo de los índices se obtienen primero los ingresos netos, restando de los ingresos brutos los costos de operación y mantenimiento. Así,

$$G = 40000(.930) + 65000(.865) + 80000(.805) + 100000(.749) + 100000(.697) + 55000(.648) + 10000(.603) - 250{,}000$$

$$= \$343{,}027 - \$250{,}000 = \$92{,}027$$

$$R = \sqrt[7]{\frac{(343{,}027)(1.659)}{250{,}000}} - 1 = 12.5\% \text{ anual}$$

$$PGI = .1888 \frac{92,027}{250,000} \times 100 = 6.95\% \text{ anual}$$

Pc = 4.3 años

ACELERACION DE PROYECTOS

Cuando en un proyecto se efectúa una inversión adicional con el objeto de apresurar la obtención de las ganancias, se dice que tiene lugar una ACELERACION DEL PROYECTO. La rentabilidad de los proyectos acelerados o modificados se analiza de igual manera que los proyectos nuevos; pero ahora se necesita considerar simultáneamente los ingresos del proyecto modificado (Ia) y los ingresos del proyecto original (Io). Las expresiones (2.1), (2.5), (2.9) y (2.10) se pueden extender fácilmente para el cálculo de la ganancia adicional Ga, la tasa de ganancia adicional Ra y el porcentaje de ganancia adicional PGIa:

$$Ga = \sum_{k=1}^{n} (Ia - Io)_k (1 + i)^{-k} - Ca \qquad (2.11)$$

$$Ra = \sqrt[m]{\frac{(Ga + Ca)(1 + i)^n}{Ca}} - 1 \qquad (2.12)$$

$$PGIa = (RP_{i,m}) \frac{Ga}{Ca} (100) \qquad (2.13)$$

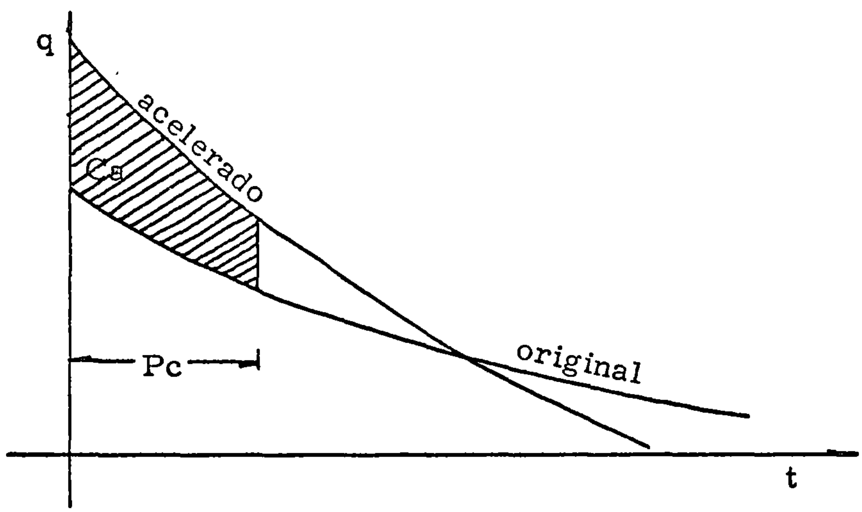
En los proyectos acelerados el período de cancelación y el período de restitución tienen significados diferentes:

PERIODO DE CANCELACION

Se llama PERIODO DE CANCELACION al tiempo en el cual la suma de las diferencias entre ingresos netos del proyecto acelerado y del proyecto original, es igual a la inversión adicional:

$$Ca = \sum_{k=1}^{Pc} (Ia - Io)_k (1 + i)^{-k} \qquad (2.14)$$

El período de cancelación se representa gráficamente a continuación, donde q representa el ritmo de ingresos y t el tiempo. El área asciurada representa la cantidad Ca.

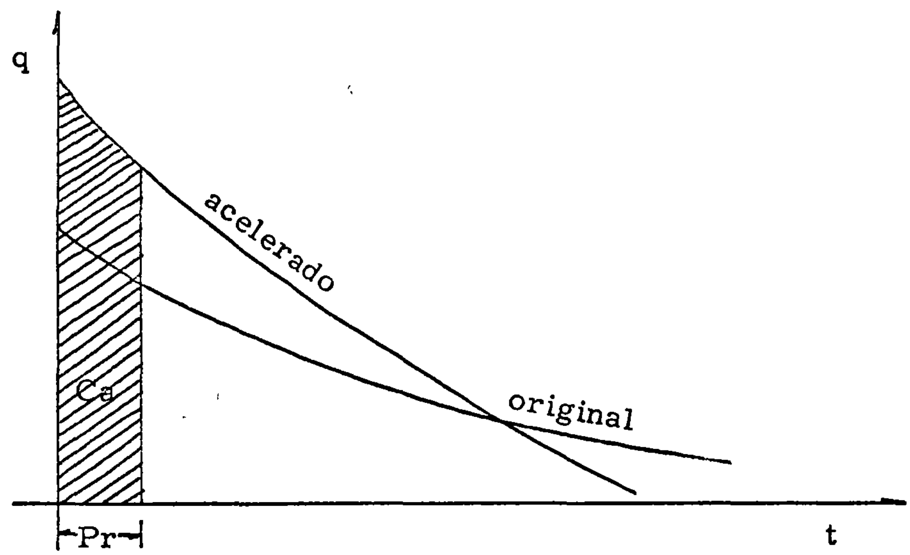


PERIODO DE RESTITUCION

El PERIODO DE RESTITUCION es el tiempo en el cual la suma de los ingresos netos actualizados del proyecto acelerado es igual a la inversión adicional:

$$Ca = \sum_{k=1}^{Pr} Ia_k (1+i)^{-k} \qquad (2.15)$$

Por supuesto, el período de restitución es inferior al período de cancelación. Este señala en qué tiempo queda cancelada la inversión adicional y aquél indica el tiempo en que la mencionada inversión se recupera.



EJEMPLO 2.7 Investigar la costeabilidad de la aceleración de un proyecto que tiene las siguientes características: Vida del proyecto original, 10 años; Vida del proyecto acelerado, 5 años; Ingresos netos del proyecto original, \$200,000/año; Ingresos netos del proyecto acelerado, \$400,000/año; Inversión adicional, \$185,000. Resolver el problema con tasas de oportunidad de 6 y 12%.

SOLUCION.- Como se trata de ingresos constantes, la solución es inmediata. Para $i=6\%$ anual:

$$Ga = (400{,}000 - 200{,}000)(PR)_{6\%,5} - 185{,}000$$

$$= \$657{,}460$$

$$R = \sqrt[5]{\frac{8.4246 \times 10^5 \, (SP)_{6\%,5}}{185{,}000}} - 1$$

$$= 43\% \text{ anual}$$

$$PGI = (RP)_{6\%,5} \, \frac{657{,}460}{185{,}000} \, 100$$

$$= 84.2\% \text{ anual}$$

$$Pc = 0$$

$$Pr = 0$$

Para una tasa de oportunidad de 12% anual, siguiendo el mismo procedimiento, resulta:

$$Ga = \$535{,}940$$

$$R = 47\% \text{ anual}$$

$$PGI = 80.4\% \text{ anual}$$

$$Pc = 0$$

$$Pr = 0$$

INGRESOS CONTINUOS

Los criterios de evaluación se pueden determinar fácilmente cuando los ingresos netos varían con continuidad y según cierta ley. Por ejemplo, en el caso de declinación exponencial, tomando en cuenta (1.19) y el razonamiento seguido en (2.4) para la tasa de ganancia, se tiene:

$$C\, e^{Rn} = u\, qo \; \frac{1 - e^{-(b+i)n}}{b+i} \; e^{in} \tag{2.16}$$

de donde

$$R = \ln \sqrt[n]{\frac{u\, q_o(e^{in} - e^{-bn})}{C(b+i)}} \tag{2.17}$$

Si n es grande entonces la tasa de ganancia se puede escribir como

$$R = i + \ln \sqrt[n]{\frac{uq_o}{C(b+i)}} \tag{2.18}$$

De (1.19), (1.21) y (2.9) el porcentaje de ganancia sobre la inversión resulta

$$PGI = \frac{e^{i} - 1}{1 - e^{-ni}} \left[\frac{u\, q_o}{C(b+i)} (1 - e^{-(b+i)n}) - 1 \right] 100 \tag{2.19}$$

pero si n es grande

$$PGI = (e^{i} - 1) \left[\frac{u\, q_o}{C(b\;\; i)} - 1 \right] 100 \tag{2.20}$$

El período de cancelación se obtiene fácilmente de (1.19) y (2.10):

$$Pc = -\frac{1}{b+i} \ln\left[1 - \frac{C(b+i)}{u\,q_o}\right] \qquad (2.21)$$

La declinación de la producción por período, d, significa que la producción en el período n es menor en la cantidad dq_{n-1} con respecto a la producción q_{n-1} del período anterior; esto es,

$$q_1 = q_o - q_o d = q_o(1 - d)$$

$$q_2 = q_1(1 - d)$$

$$\vdots$$

$$q_n = q_{n-1}(1 - d)$$

Tomando en cuenta ahora que $q = q_o e^{-bt}$, resulta una expresión útil para la determinación de b:

$$\frac{q_1}{q_o} = \frac{q_2}{q_1} = \ldots = \frac{q_n}{q_{n-1}} = e^{-b}$$

de donde

$$e^{-b} = 1 - d$$

o bien

$$b = \ln \frac{1}{1 - d} \qquad (2.22)$$

La expresión (1.19) para la ganancia se puede escribir también como

VALORES DE LA FUNCION $\phi(x) = \frac{1 - e^{-x}}{x}$

x	$\phi(x)$	x	$\phi(x)$	x	$\phi(x)$	x	$\phi(x)$
0.1	0.9516	3.1	0.3081	6.1	0.1635	9.1	0.1099
0.2	0.9063	3.2	0.2998	6.2	0.1609	9.2	0.1087
0.3	0.8639	3.3	0.2918	6.3	0.1584	9.3	0.1075
0.4	0.8242	3.4	0.2843	6.4	0.1560	9.4	0.1064
0.5	0.7869	3.5	0.2771	6.5	0.1536	9.5	0.1052
0.6	0.7519	3.6	0.2702	6.6	0.1513	9.6	0.1041
0.7	0.7191	3.7	0.2636	6.7	0.1491	9.7	0.1031
0.8	0.6883	3.8	0.2573	6.8	0.1469	9.8	0.1020
0.9	0.6593	3.9	0.2512	6.9	0.1447	9.9	0.1010
1.0	0.6321	4.0	0.2454	7.0	0.1427	10.0	0.0000
1.1	0.6065	4.1	0.2399	7.1	0.1407	10.1	0.0990
1.2	0.5823	4.2	0.2345	7.2	0.1387	10.2	0.0980
1.3	0.5596	4.3	0.2294	7.3	0.1369	10.3	0.0971
1.4	0.5382	4.4	0.2245	7.4	0.1350	10.4	0.0961
1.5	0.5179	4.5	0.2197	7.5	0.1332	10.5	0.0952
1.6	0.4988	4.6	0.2152	7.6	0.1315	10.6	0.0943
1.7	0 4808	4.7	0.2108	7.7	0.1298	10.7	0.0934
1.8	0.4637	4.8	0.2066	7.8	0.1281	10.8	0.0926
1.9	0.4476	4.9	0.2026	7.9	0.1265	10.9	0.0917
2.0	0.4323	5.0	0.1986	8.0	0.1249	11.0	0.0909
2.1	0.4179	5.1	0.1948	8.1	0.1234	11.1	0.0901
2.2	0.4042	5.2	0.1912	8.2	0.1219	11.2	0.0893
2.3	0.3912	5.3	0.1877	8.3	0.1204	11 3	0.0885
2.4	0.3788	5.4	0.1843	8.4	0.1190	11.4	0.0877
2.5	0.3672	5.5	0.1811	8.5	0.1176	11.5	0.0869
2.6	0 3560	5.6	0.1779	8.6	0.1162	11.6	0.0862
2.7	0.3455	5.7	0.1748	8.7	0.1149	11.7	0.0854
2.8	0.3354	5.8	0.1719	8.8	0.1136	11.8	0.0847
2.9	0.3258	5.9	0.1690	8.9	0.1123	11.9	0.0840
3.0	0.3167	6.0	0.1662	9.0	0.1111	12.0	0.0833
12.1	0.0826	14.1	0.0709	16.1	0.0621	18.1	0.0552
12.2	0.0819	14.2	0.0704	16.2	0.0617	18.2	0.0549
12.3	0.0813	14.3	0.0699	16.3	0.0613	18.3	0.0546
12.4	0.0806	14.4	0.0694	16.4	0.0610	18.4	0.0543
12.5	0.0800	14.5	0.0689	16.5	0.0606	18.5	0.0543
12.6	0.0793	14.6	0.0685	16.6	0.0602	18.6	0.0540
12.7	0.0787	14.7	0.0680	16.7	0.0599	18.7	0.0537
12.8	0.0781	14.8	0.0675	16.8	0.0595	18.8	0 0532
12.9	0.0775	14.9	0.0671	16.9	0.0592	18.9	0.0529
13.0	0.0769	15.0	0.0666	17.0	0.0588	19.0	0.0526

$$G = uq_oN \frac{1 - e^{-(b+i)N}}{(b+i)N} - C \qquad (2.23)$$

Este cambio tiene la ventaja de permitir tabular diferentes valores de la función

$$\phi(x) = \frac{1 - e^{-x}}{x} \qquad (2.24)$$

donde $x = (b+i)N$ y facilitar el cálculo numérico de G.

Desde luego para determinar la tasa de ganancia por el método optimista, resulta muy útil la tabulación de $\phi(x)$. En este caso el procedimiento de cálculo se inicia con la suposición de una tasa de ganancia R y sucesivamente se va corrigiendo el valor, hasta lograr satisfacer la función

$$Ce^{RN} = uq_oN \frac{1 - e^{-(b+R)N}}{(b+R)N} \qquad (2.25)$$

EJEMPLO 2.8 Un pozo petrolífero de exploración tuvo un ritmo de producción inicial de 64 metros cúbicos diarios, con una declinación aparente de 15% anual. Se estima que con 120 millones de pesos se pueden perforar 30 pozos más, 5 de los cuales saldrán secos, construyéndose con esta misma inversión las instalaciones necesarias para la recolección de los hidrocarburos. Si el precio neto del aceite es de $\$90/m^3$, la vida productiva es de

15 años y la tasa de oportunidad es de 7% anual, calcular:

a) Ganancia

b) Tasa de Ganancia

c) Porcentaje de Ganancia

d) Período de Cancelación

SOLUCION. - Tomando como unidad de tiempo 1 año, el ritmo de producción inicial de los 25 pozos productores resulta de

$$q_o = 64 \times 365 \times 25 = 834{,}000 \text{ m}^3/\text{año}$$

De (2.22), (2.23) y (2.24) la ganancia se calcula como sigue:

$$b = \ln \frac{1}{1 - 0.15} = 0.163$$

$$x = (0.163 + 0.07) = 3.495$$

De las Tablas para $\phi(x)$ resulta

$$\phi(x) = 0.277$$

de manera que

$$G = 90 \times 834{,}000 \times 15 \times 0.277 - 120'000{,}000$$

$$= \$\ 192'000{,}000$$

Para la tasa de ganancia por el método realista, calculando parcialmente los factores de (2.17), se tiene:

$$\frac{u q_o}{C(b+i)} = 2.68$$

$$e^{0.07 \times 15} = 2.86$$

$$e^{-0.163 \times 15} = 0.0863$$

$$R = \ln \sqrt[15]{2.68 \times 2.7777} = 13.45\%$$

Si n es grande, entonces de (2.18):

$$R = 0.07 + \ln \sqrt[15]{2.68} = 13.55\%$$

El porcentaje de ganancia, de acuerdo con (2.20), resulta

$$PGI = (e^{0.07} - 1)(2.68 - 1)\, 100 = 12.2\%$$

El período de cancelación, finalmente, es

$$Pc = -\frac{1}{0.233} \ln(1 - \frac{1}{2.68}) = 2 \text{ años}$$

Capítulo III

ANALISIS DE RIESGO

Los métodos usuales de evaluación de proyectos conducen al cálculo de una serie de factores e íncides que señalan el comportamiento económico de las diferentes alternativas que se presentan. En la actualidad, con el auxilio de las computadoras electrónicas, es posible determinar rápidamente y con mínimo error en el cálculo la Inversión Inicial, la Tasa de Ganancia, la Ganancia, el Porcentaje de Ganancia sobre la Inversión, el Período de Cancelación y el Período de Restitución, por complejos que sean los proyectos.

El cálculo de los índices anteriores está basado principalmente en estimaciones del comportamiento futuro de los factores que intervienen en cada uno de ellos. El modelo utilizado para efectuar este estudio puede dar resultados confiables, pero en general los valores que arroja son inciertos en mayor o menor grado. El comportamiento de los costos de operación y mantenimiento, por ejemplo, puede seguir una tendencia conocida en los años futuros, pero está sujeto a cambios que no se pueden prever con exactitud. La magnitud de las tasas de interés y la extensión de la vida económica de los equipos, son también ejemplos de factores que intervienen en el análisis económico en forma relevante y que sin embargo no se conocen de manera precisa cuando se estudia la implantación de un proyecto.

Tradicionalmente se acostumbra tratar el problema de la incertidumbre en la información, efectuando predicciones conservadoras, o considerando en los estudios económicos tasas de descuento elevadas. Sin embargo, con la aplicación de estos procedimientos, algunas de las buenas posibilidades de inversión pueden parecer no atractivas.

Con frecuencia se consideran también sólo los valores medios de los diferentes factores, teniendo presentes sus niveles de confianza. Pero la confiabilidad, aunque sea alta individualmente, puede resultar muy baja al combinar aleatoriamente los diferentes elementos. Por lo tanto, este procedimiento tampoco es adecuado.

En el momento de tomar una decisión, el ejecutivo debe tener una idea precisa del riesgo involucrado. El estudio que con este objeto realice debe contestar por lo menos las siguientes preguntas:

1) ¿Cuál es el riesgo de no recuperar la inversión?

2) ¿Cuál es la vida económica más probable del proyecto?

3) ¿Cuál es el riesgo de que las ganancias se obtengan a una tasa inferior a la de oportunidad?

4) ¿Cuál es la confiabilidad de obtener un ingreso específico?

La respuesta a estas preguntas resulta más importante a medida que el monto de las inversiones crece: La decisión no se toma en igual forma cuando se invierte una cantidad pequeña, cuya pérdida involucraría sólo algunos reajustes, que cuando se invierten sumas que por su magnitud pondrían en peligro la existencia de la empresa si no proporcionaran un mínimo preestablecido de ganancias. Es necesario, pues, considerar la incertidumbre implicada en las predicciones, a fin de cuantificar el riesgo correspondiente en que se incurre.

Un método más adecuado que los anteriores, pero que sólo señala cualitativamente el efecto de la incertidumbre, es el análisis de sensibilidad. Este consiste en efectuar varias veces el análisis económico, haciendo variar dentro de rangos definidos los factores relevantes del proyecto. En el análisis de sensibilidad se considera que todos los valores dentro de los rangos dados, tienen la misma probabilidad de ocurrencia. Así, en los análisis sucesivos, se van tomando al azar diferentes combinaciones de ingresos, costos, tasas, etc., para calcular los índices de evaluación. Después de efectuar el número adecuado de simulaciones y obtener los histogramas correspondientes, se puede saber la influencia que tiene cada uno de los factores en los resultados finales del estudio. Esta técnica de muestreo y simulación se conoce con el nombre de Método de Monte Carlo.

El procedimiento que se describe aquí para calcular el riesgo en proyectos de inversión, es bastante parecido al análisis de sensibilidad. Utiliza el método de Monte Carlo para asignar valores al azar y efectuar las diferentes simulaciones, las cuales se cuentan por cientos o por miles. Sin embargo, a diferencia de la técnica anterior, a cada uno de los factores relevantes se asigna una distribución de probabilidad dentro de los rangos especificados. De esta manera se logra una aproximación más realista del problema.

Al terminar el proceso de simulación, se agrupan y clasifican convenientemente los conjuntos de valores obtenidos para cada uno de los índices considerados. En este momento es posible determinar el valor esperado y el riesgo.

En la parte final del estudio se clasifican las alternativas en base a un análisis de su comportamiento más probable y de su riesgo. Se seleccionan las que presentan menor riesgo para diferentes valores del índice considerado y de éstas, el ejecutivo escoge la que satisface más ampliamente el concepto de utilidad de su empresa.

EJEMPLO INTRODUCTORIO

Antes de describir con detalle el procedimiento que se empleará para analizar el riesgo, estudiaremos un ejemplo ilustrativo del método de Monte Carlo.

Supongamos que los estudios de un negocio en perspectiva arrojan los siguientes valores de inversión, tasa de interés, vida económica e ingresos netos: $C = \$100{,}000$, $i = 6\%$ anual, $n = 4$ años, $I_1 = \$20{,}000$, $I_2 = \$30{,}000$, $I_3 = \$40{,}000$, $I_4 =$

$10,000 con una probabilidad de 5%

$20,000 con una probabilidad de 10%

$30,000 con una probabilidad de 15%

$40,000 con una probabilidad de 25%

$50,000 con una probabilidad de 30%

$60,000 con una probabilidad de 10%

$70,000 con una probabilidad de 5%

Se desea determinar el valor esperado $\overline{G}$ de la ganancia y el riesgo de no recuperar la inversión.

Por tratarse de un problema muy sencillo, en donde la única variable aleatoria es el ingreso en el año 4, el cálculo de la ganancia esperada se puede efectuar determinando primero el valor más probable de I_4:

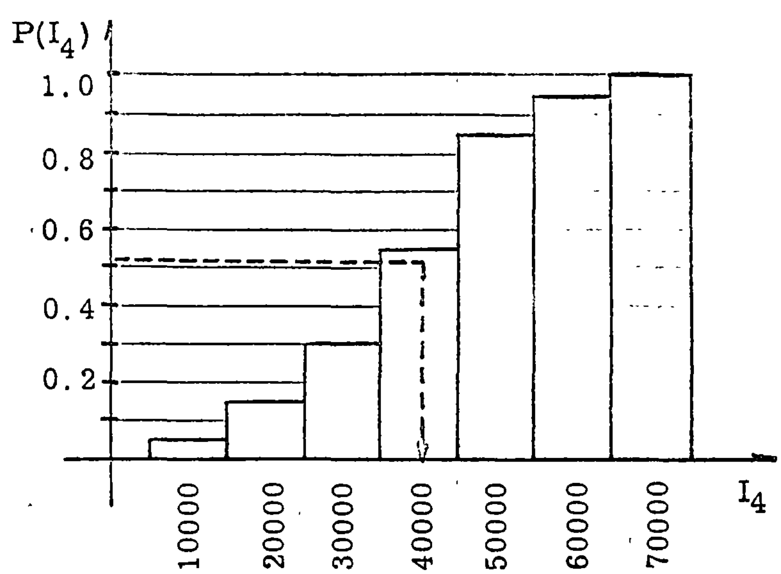
$$\bar{I}_4 = 10,000 \times 0.05 + 20,000 \times 0.10 + 30,000 \times 0.15$$
$$+ 40,000 \times 0.25 + 50,000 \times 0.30 + 60,000 \times 0.10$$
$$+ 70,000 \times 0.05$$
$$= \$41,500$$

Así,

$$\bar{G} = 20,000 \times 0.94 + 30,000 \times 0.89 + 40,000 \times 0.84$$
$$+ 41,500 \times 0.79 - 100,000$$
$$= \$11,885$$

La determinación de la ganancia esperada por el método de simulación de Monte Carlo se lleva a cabo de la siguiente manera:

1) Con la información sobre las probabilidades de ocurrencia de I_4 se construye un histograma de probabilidades acumuladas:



2) Con una tabla de números aleatorios se obtiene un número aleatorio normalizado; es decir, un número entre 0 y 1. (En el ejemplo se toman números de 2 cifras decimales en dirección vertical). Sea el primero igual a 0.51; entonces, pasando a la gráfica anterior con P= 0.51 se determina un valor para el ingreso igual a $40,000. De esta manera, la ganancia resulta

$$G_1 = 20{,}000 \times 0.94 + 30{,}000 \times 0.89 + 40{,}000 \times 0.84$$
$$+ 40{,}000 \times 0.79 - 100{,}000$$
$$= 79{,}100 + 40{,}000 \times 0.79 - 100{,}000$$
$$= \$10{,}700$$

Repitiendo el paso 2 se tiene por ejemplo el número aleatorio 0.24, y de la gráfica resulta $I_4 = 30{,}000$. Así,

$$G_2 = 79{,}100 + 30{,}000 \times 0.79 - 100{,}000$$
$$= \$2{,}800$$

Con estas dos simulaciones, la ganancia esperada es

$$\bar{G} = \tfrac{1}{2}(G_1 + G_2) = \$6{,}750$$

De una tercera simulación resulta:

$$G_3 = 79{,}100 + 40{,}000 \times 0.79 - 100{,}000$$
$$= \$10{,}700$$

$$\bar{G} = (10{,}700 + 2{,}800 + 10{,}700)/3 = \$8{,}066$$

Si se repite el paso 2 muchas veces se obtienen los siguientes resultados:

G_1 = 10,700	$\bar{G}$ = 10,700
G_2 = 2,800	$\bar{G}$ = 6,750
G_3 = 10,700	$\bar{G}$ = 8,066
G_4 = 2,800	$\bar{G}$ = 6,750
G_5 = - 13,000	$\bar{G}$ = 2,800
G_6 = 18,600	$\bar{G}$ = 5,433
G_7 = -.5,100	$\bar{G}$ = 3,928
G_8 = - 5,100	$\bar{G}$ = 2,800
G_9 = 2,800	$\bar{G}$ = 2,800
G_{10} = 26,500	$\bar{G}$ = 5,170
G_{11} = 10,700	$\bar{G}$ = 5,672
G_{12} = - 5,100	$\bar{G}$ = 4,775
G_{13} = 2,800	$\bar{G}$ = 4,623
G_{14} = 25,600	$\bar{G}$ = 6,121
G_{15} = 10,700	$\bar{G}$ = 6,426
G_{16} = 25,600	$\bar{G}$ = 7,625
G_{17} = - 5,100	$\bar{G}$ = 6,876
G_{18} = 10,700	$\bar{G}$ = 7,088
G_{19} = 25,600	$\bar{G}$ = 8,063
G_{20} = 10,700	$\bar{G}$ = 8,195
G_{21} = 25,600	$\bar{G}$ = 9,023
G_{22} = - 13,000	$\bar{G}$ = 7,982

$G_{23} = 26,500$	$\bar{G} = 8,780$
$G_{24} = 25,600$	$\bar{G} = 9,487$
$G_{25} = 25,600$	$\bar{G} = 10,132$
$G_{26} = 26,500$	$\bar{G} = 10,761$
$G_{27} = 25,600$	$\bar{G} = 11,311$
$G_{28} = 10,700$	$\bar{G} = 11,289$
$G_{29} = 25,600$	$\bar{G} = 11,782$
$G_{30} = -5,100$	$\bar{G} = 11,220$
$G_{31} = 10,700$	$\bar{G} = 11,203$
$G_{32} = -5,100$	$\bar{G} = 10,693$
$G_{33} = 34,400$	$\bar{G} = 11,412$
$G_{34} = 2,800$	$\bar{G} = 11,158$
$G_{35} = 10,700$	$\bar{G} = 11,145$
$G_{36} = 10,700$	$\bar{G} = 11,133$
$G_{37} = 2,800$	$\bar{G} = 10,908$
$G_{38} = 25,600$	$\bar{G} = 11,294$
$G_{39} = 2,800$	$\bar{G} = 11,076$
$G_{40} = 2,800$	$\bar{G} = 10,870$
$G_{41} = 2,800$	$\bar{G} = 10,673$
$G_{42} = 10,700$	$\bar{G} = 10,673$
$G_{43} = 34,400$	$\bar{G} = 11,225$
$G_{44} = 25,600$	$\bar{G} = 11,552$
$G_{45} = 25,600$	$\bar{G} = 11,863$

$G_{46} = 25,600$ $\bar{G} = 12,163$

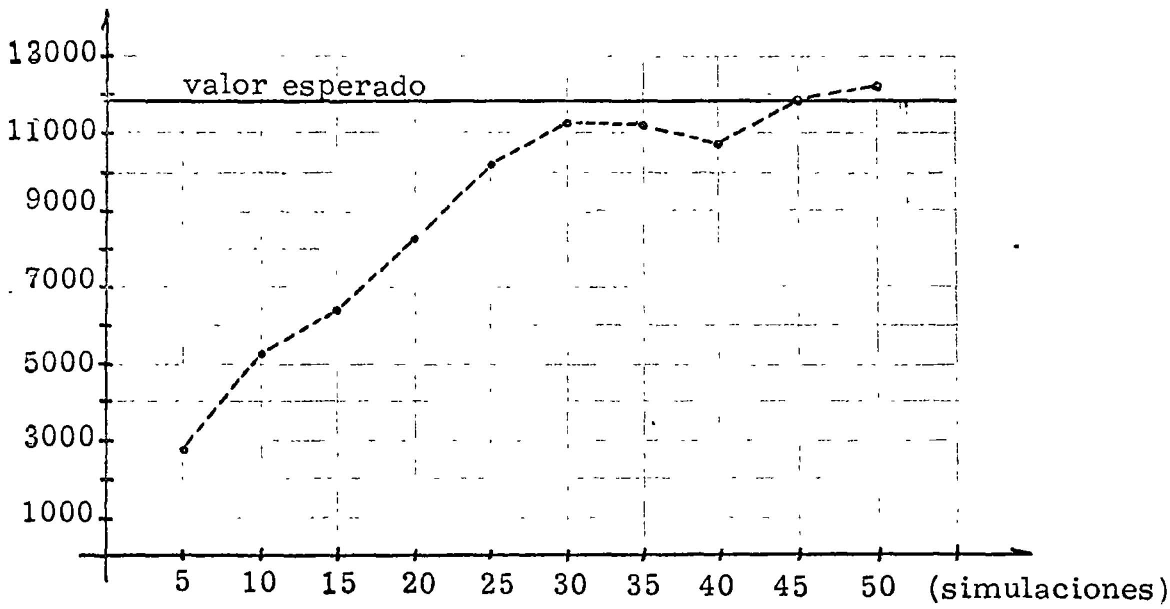
$G_{47} = -5,100$ $\bar{G} = 11,797$

$G_{48} = 25,600$ $\bar{G} = 12,081$

$G_{49} = 2,800$ $\bar{G} = 11,894$

$G_{50} = 25,600$ $\bar{G} = 12,168$

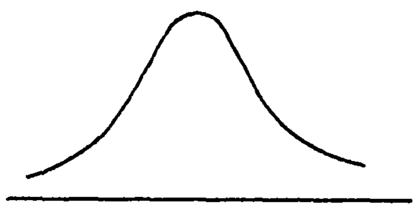
Graficando los resultados obtenidos después de cada 5 simulaciones, se puede observar una tendencia hacia el valor esperado exacto.



El riesgo de no recuperar la inversión se cuantifica fácilmente, observando que, de las 50 simulaciones, en 9 ocasiones resultó negativa la ganancia; es decir, el riesgo de perder es

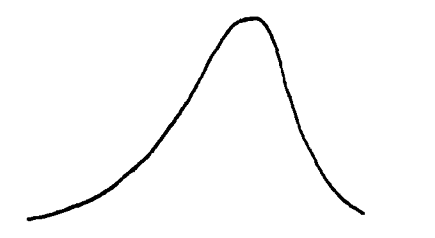
PROCEDIMIENTO DE CALCULO

Sean C, i, I_k y n las variables aleatorias, cuyas distribuciones de probabilidad, ya sea en forma discreta o contínua, se conocen:



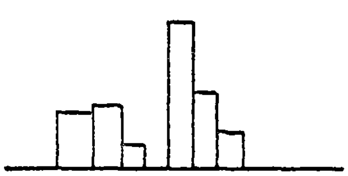
C

i



I_1

I_2



n

$$9/50 = 18\%$$

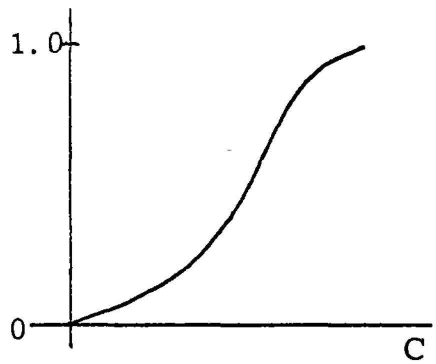
Un valor más exacto se puede obtener aumentando el número de simulaciones; pero como la sencillez del ejemplo lo permite, ese riesgo se puede obtener como sigue:

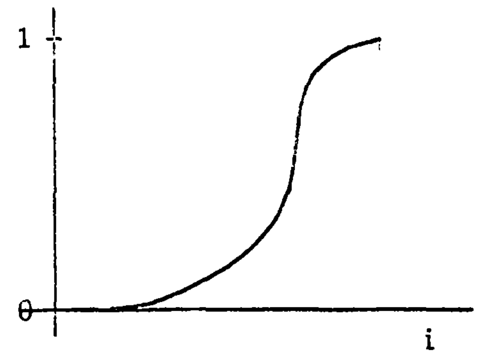
Si I_4	= 10,000	entonces	G = -13,000
	= 20,000		= -5,100
	= 30,000		= 2,800
	= 40,000		= 10,700
	= 50,000		= 25,600
	= 60,000		= 26,500
	= 70,000		= 34,400

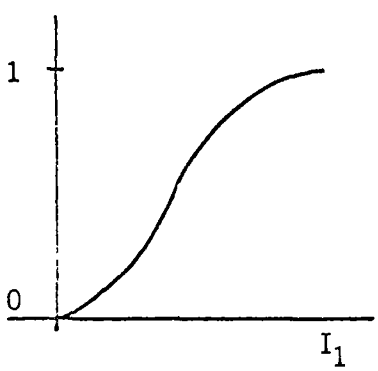
Ahora, tomando en cuenta las probabilidades de ocurrencia, podemos afirmar que de cada 100 veces, 5 veces la ganancia será igual a -13,000 y 10 veces igual a -5,100. Es decir, sólamente 15 veces resultará negativa. Por lo tanto, el riesgo de no recuperar la inversión resulta de 15%.

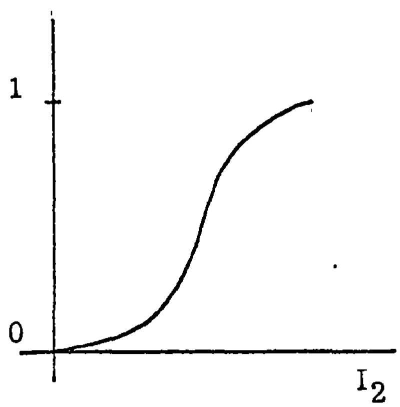
Cuando todos los elementos que intervienen en el cálculo de los índices son variables aleatorias, es más complicado efectuar el cálculo directamente que por medio de la simulación de Monte Carlo. A continuación se describe una posible secuencia de cálculos para el caso más general.

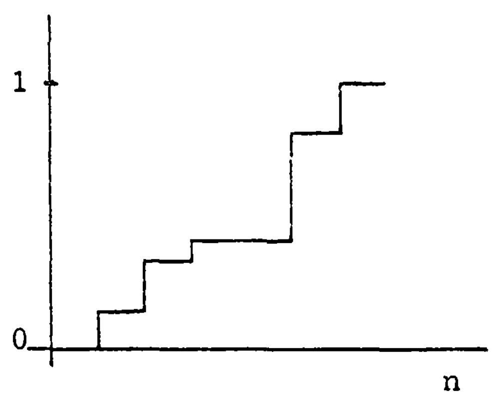
De estas distribuciones se obtienen las curvas de probabilidad acumulada:











Una vez arreglada así la información, el procedimiento de cálculo puede ser el siguiente:

1) Obtener un número aleatorio normalizado y determinar un valor para i.

2) Obtener un número aleatorio normalizado y determinar un valor para C.

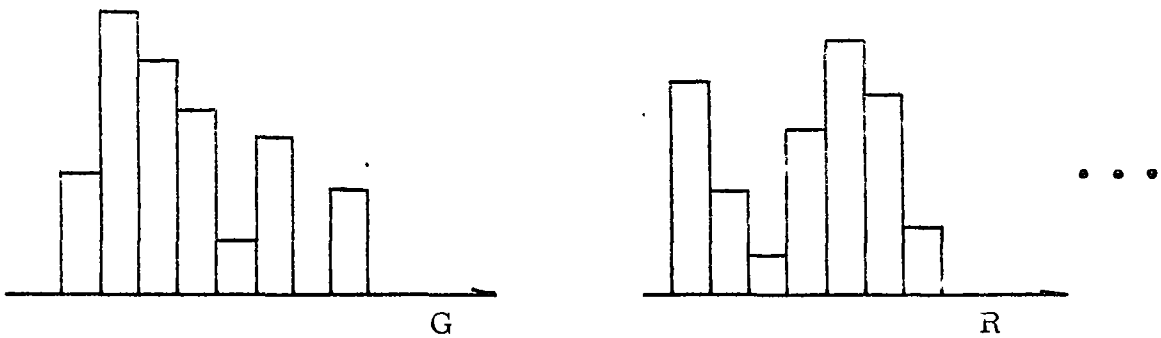
3) Obtener un número aleatorio normalizado y determinar un valor para n.

4) Obtener n números aleatorios normalizados y determinar los n ingresos correspondientes en las respectivas curvas de probabilidad acumulada.

5) Efectuar el cálculo de los índices (Ganancia, Tasa de Ganancia, etc.).

6) Repetir los pasos del 1 al 5 en un número m de veces prefijado (50,100,1000, etc.) y pasar al siguiente punto.

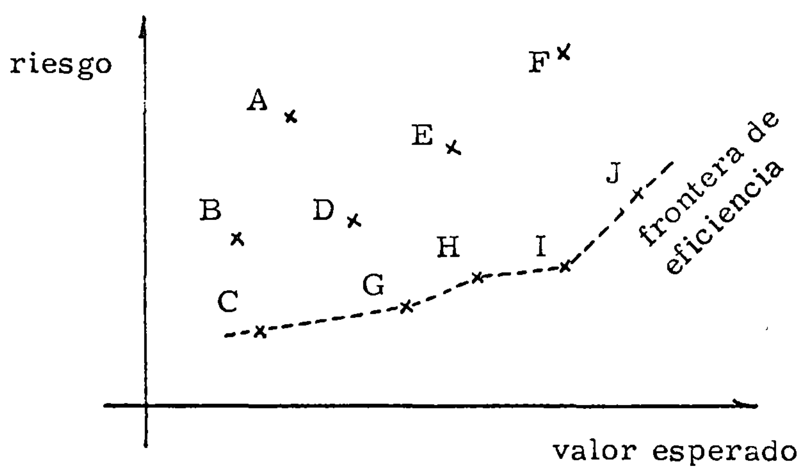
7) Ordenar los m valores calculados de los índices y clasificarlos para construir un histograma para cada uno:



8) Determinar la media (valor esperado), la variancia, la desviación estándar y el coeficiente de variación (riesgo).

9) Seguir el procedimiento desde 1 hasta 8 para las demás alternativas y continuar en el siguiente punto.

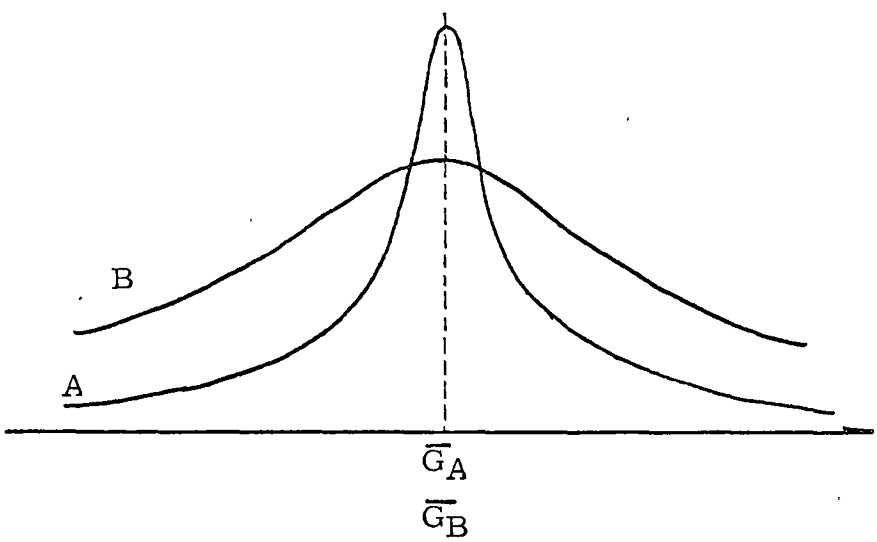
10) Dibujar una gráfica de riesgo contra valor esperado para cada uno de los índices, señalando con puntos las diferentes alternativas y marcar la frontera de eficiencia.



11) Determinar de los histogramas correspondientes y para cada alternativa, el riesgo de no recuperar la inversión, el riesgo de obtener ganancias a una tasa inferior a la de oportunidad, etc.

A continuación se ofrece una aclaración pertinente sobre los puntos del 8 al 10.

Supongamos que las alternativas A y B proporcionaran para la ganancia las distribuciones que siguen:



Las dos tienen el mismo valor esperado, pero se advierte que una es mejor que la otra. En efecto, es más probable que se cumpla G_A porque los valores en esta distribución están menos dispersos que en B. Mientras más grande sea la dispersión de los valores, menos probable será que se cumpla el valor esperado.

Para calcular la dispersión en una distribución de probabilidad, existen varios índices, siendo los más usados la variancia (v), la desviación estándar (s) y el coeficiente de variación (c), los cuales se calculan como sigue:

$$v = \sum_{j=1}^{m} (X_j - \bar{X})^2 P(X_j) \qquad (3.1)$$

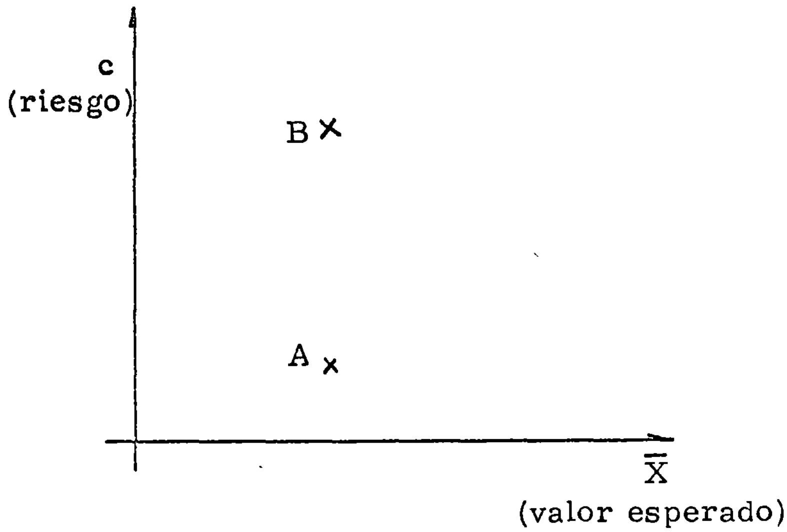
$$s = v^{\frac{1}{2}} \qquad (3.2)$$

$$c = s/\bar{X} \qquad (3.3)$$

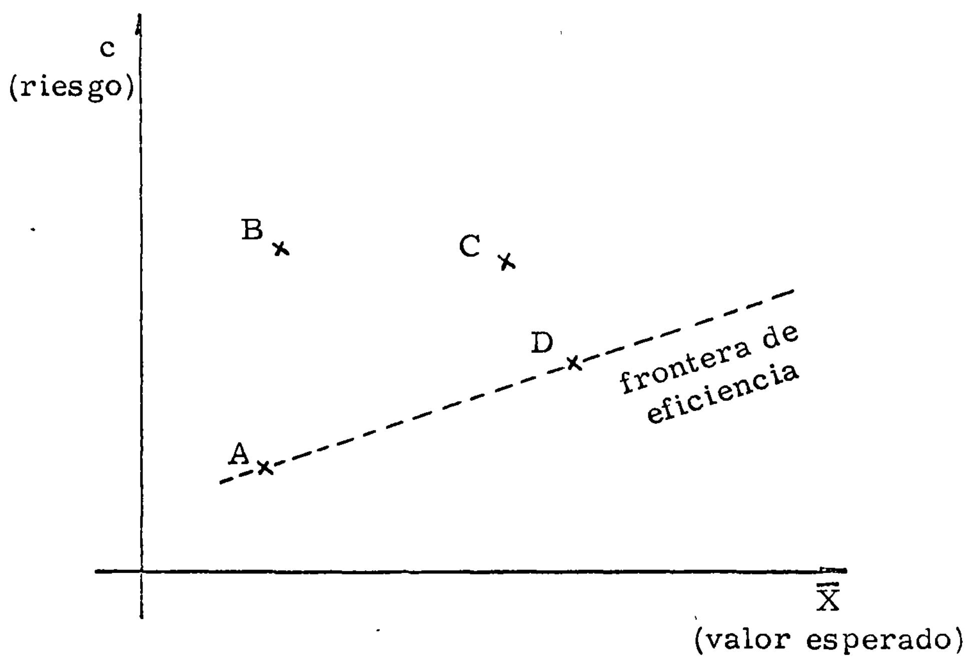
$$\bar{X} = \sum_{j=1}^{m} X_j P(X_j) \qquad (3.4)$$

donde las X_j constituyen el conjunto de eventos, $\bar{X}$ la media y $P(X_j)$ la probabilidad del evento X_j.

En el caso que estamos analizando, puesto que $\bar{G}_A$ y $\bar{G}_B$ son iguales, entonces c_A es menor que c_B. Dibujados estos valores en una gráfica de c contra $\bar{X}$, es evidente la superioridad de la alternativa A con respecto a B: Con el mismo valor esperado de ganancia, la alternativa A tiene menor riesgo.



En el caso de tener cuatro alternativas ubicadas en la gráfica como se señala abajo, es evidente que la atención se fijará



en las alternativas A y D, por ofrecer menor riesgo. Un ejecutivo podrá seleccionar la alternativa A (menor ganancia), si no desea riesgos altos; o bien, la alternativa D, si considera que la ganancia que esta última ofrece merece la pena de correr un riesgo mayor.

El estudio del riesgo termina con la demarcación de la frontera de eficiencia. La selección de una alternativa, entre las que se encuentren en esta línea, será congruente con el concepto de utilidad de las empresas.

Para terminar, es conveniente hacer notar que se ha considerado la distribución de ingresos de un período independiente de la distribución del período anterior. Si esto no se cumple, el problema debe ser tratado con probabilidades condicionales.

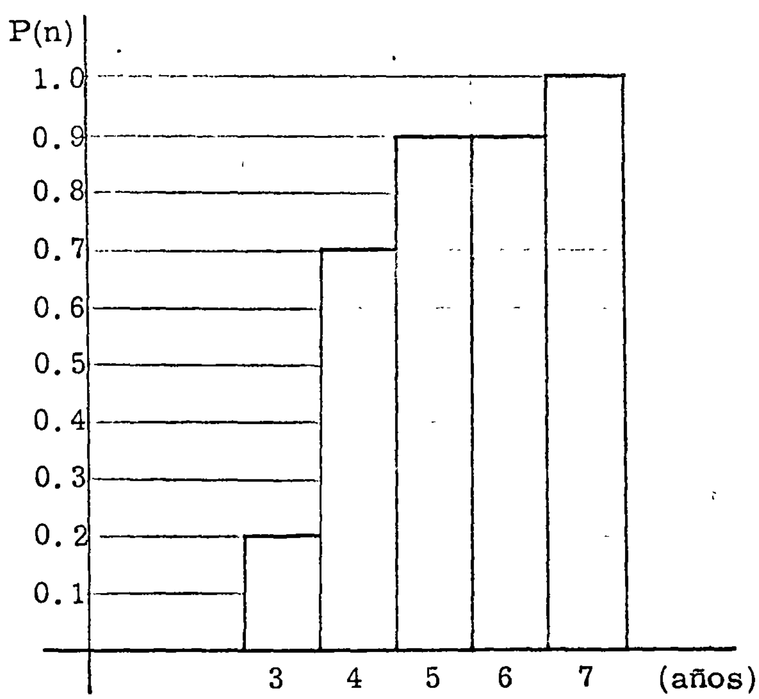
EJEMPLO 3.1 Las investigaciones previas de un proyecto de inversión señalan los siguientes valores posibles para la vida económica: 3,4,3,7,5,4,4,4,5,y 4 años. La predicción de los ingresos netos señala que estos serán constantes de tres en tres años, pudiendo ocurrir cualquiera de los valores que siguen:

AÑOS	INGRESO NETO ANUAL	PROBABILIDAD
1-3	$20,000	0.10
	$40,000	0.20
	$60,000	0.30
	$80,000	0.40
4-6	$20,000	0.20
	$40,000	0.30
	$60,000	0.30

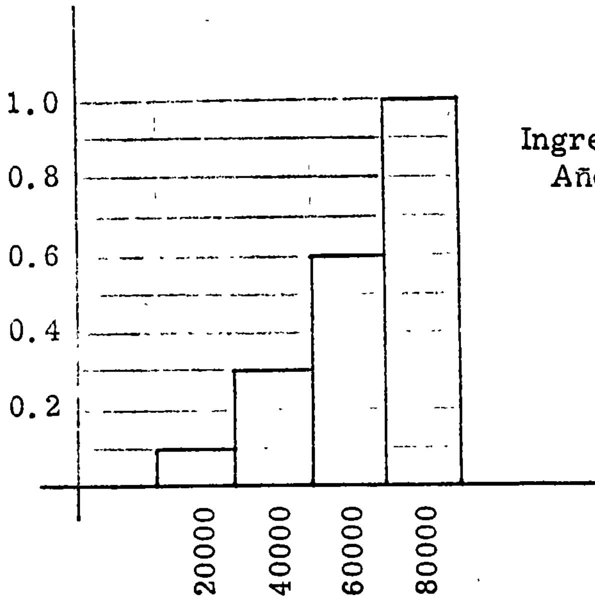
	$80,000	0.20
7=θ	$20,000	0.40
	$40,000	0.30
	$60,000	0.20
	$80,000	0.10

Efectuar un análisis sobre el riesgo de este proyecto, suponiendo que la inversión es exactamente de $200,000 y la tasa de interés vale siempre 7.5% anual.

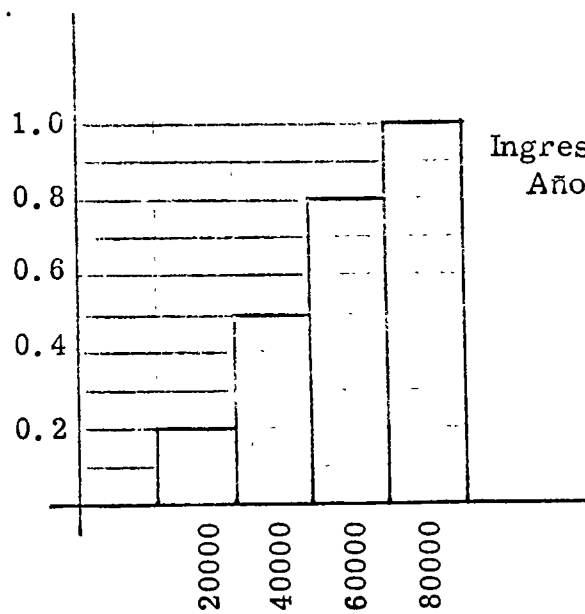
SOLUCION.- De la información disponible se construyen los siguientes histogramas de probabilidades acumuladas:



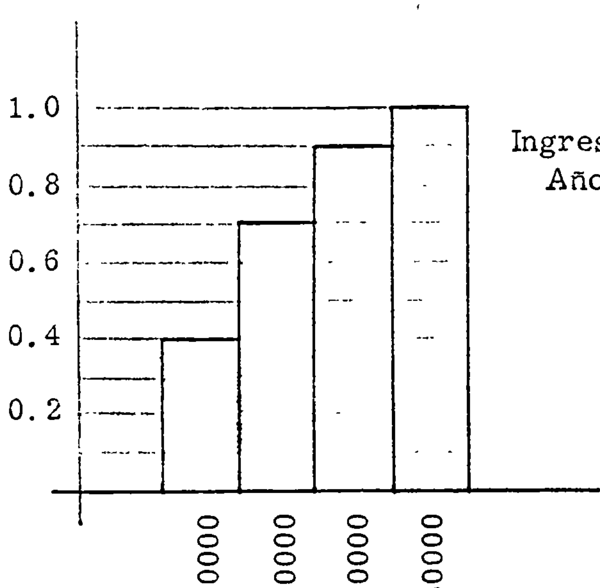
VIDA ECONOMICA



Ingresos Netos
Años 1-3



Ingresos Netos
Años 4-6



Ingresos Netos
Años 7-9

Los cálculos pueden llevarse a cabo siguiendo la tabla de números aleatorios horizontalmente, empezando por ejemplo con 0.51. De esta manera, pasando al histograma de vida económica resulta $n = 4$ años, por lo que a continuación se obtienen 2 números aleatorios normalizados más: 0.77, 0.27. El primero se utiliza en el primer histograma de ingresos y el segundo número en el histograma siguiente:

$$I_1 = I_2 = I_3 = \$80,000$$

$$I_4 = \$40,000$$

De donde

$$G_1 = 80,000 \times 2.60 + 40,000 \times 0.75 - 200,000$$

$$= \$38,000$$

$$R_1 = (238,000 \times 1.34/200,000)^{1/4} - 1 = 12.4\%$$

Para la segunda simulación, el número aleatorio resulta ser 0.46, o sea que $n = 4$ años. Los siguientes dos números aleatorios son 0.40 y 0.42, de manera que

$$I_1 = I_2 = I_3 = \$60,000$$

$$I_4 = \$40,000$$

y, en consecuencia:

$$G_2 = 60,000 \times 2.60 + 40,000 \times 0.75 - 200,000 = -\$14,000$$

$$R_2 = (186,000 \times 1.34/200,000)^{1/4} - 1 = 5.6\%$$

Capítulo IV

C O S T O S

****

Los costos constituyen un factor importante en todo análisis económico y deben ser estudiados sistemáticamente. Con el objeto de clasificarlos lógica y provechosamente, es necesario estudiar antes, en forma general, el movimiento de valores económicos en las empresas industriales.

FLUJO DE VALORES ECONOMICOS

Siguiendo el método de Rautenstrauch y Villers, los renglones principales en que se pueden clasificar los negocios, son: 1), manufactura; 2), agricultura; 3), explotación de recursos naturales; 4), construcción; 5), transportes; 6), comercio y 7), otros. De éstos, el más completo por abarcar todos los aspectos es el primero y se utiliza como base para la presentación de los temas de este capítulo.

Los valores económicos mencionados, se refieren a equipo, materiales, servicios, mercancía, dinero en efectivo y documentos; todos ellos expresados en términos de la unidad de medida llamada peso, que se supondrá constante. Se habla de flujo porque los valores cambian de una forma a otra, siguiendo a grandes rasgos, aunque no necesariamente en el orden indicado, las siguientes etapas, que van desde que se inicia un negocio hasta que se distribuyen los productos:

1) Hechura de formas impresas de contratos o convenios (bonos, acciones).
2) Cambio de certificados por dinero en efectivo o crédito.
3) Adquisición de equipo, materiales y servicios.
4) Manufactura de productos.
5) Cambio de productos por dinero o crédito.

Entre los puntos anteriores se distinguen cuatro fases, en las que los valores van tomando diferentes formas, a saber: 1), Financiamiento; 2), Compra de Equipo, Materiales y Servicios; 3), Manufactura; 4), Distribución y Venta.

Supóngase un negocio que, según investigaciones previas, requiere de un capital inicial de $500,000, el cual deberá reunirse mediante la venta de bonos y acciones al público. Una vez realizado el primer punto de la secuela indicada, se inicia la fase de conversión de valores por financiamiento.

Los bonos y las acciones son contratos que proporcionan a sus poseedores, ciertos derechos sobre los activos fijos de la empresa y sobre las ganancias, respectivamente. Cuando el dinero se obtiene a través de la venta de bonos, la empresa tiene que pagar periódicamente el interés especificado y, en caso de no cumplir, los poseedores de bonos satisfacen sus derechos, embargando y vendiendo los activos correspondientes.

Hay acciones preferentes y acciones comunes; con las primeras, la compañía está obligada a pagar un interés fijo periódicamente y, con las segundas, no. Una forma eficiente de financiamiento puede ser la emisión de acciones comunes, con las que no hay demandas fijas sobre las ganancias, ni riesgos en los derechos de propiedad y la eliminación de sociedades financieras como medio para vender los certificados al público.

Siguiendo con el ejemplo, la primera fase se podría expresar por medio de la ecuación

BONOS ACCIONES $500,000

La siguiente fase puede consistir en la compra de una fábrica ($300,000), materiales ($50,000), servicios de dirección, mano de obra, etc. ($50,000), teniendo lugar una conversión representada por la ecuación

$500,000 =

FABRICA ($300,000)
MATERIALES ($50,000)
SERVICIOS ($50,000)
EFECTIVO EN CAJA ($100,000)

En esta segunda etapa se resuelven los problemas relacionados con la compra económica de terrenos, los cuales deben presentar ventajas sobre transportes, distribución de edificios, abastecimiento de agua y energía eléctrica, localización con respecto a mercados, adquisición de materias primas, etc.; con la compra económica de equipo, el cual debe trabajar eficientemente y a bajo costo; con la adquisición de los materiales más adecuados y con la obtención de los mejores servicios.

Después de operar el negocio durante cierto tiempo, los materiales se han incorporado al producto, cargándose al costo de éste el valor de aquéllos; los salarios del personal, el costo de energía, seguros, impuestos, etc., también están incluídos en el producto, así como parte del activo fijo, mediante el proceso llamado DEPRECIACION. Supóngase tener en esta etapa la siguiente situación:

FABRICA ($300,000)		FABRICA ($290,000)
MATERIALES ($50,000)	=	PRODUCTOS ($110,000)
SERVICIOS ($50,000)		EFECTIVO ($100,000)
EFECTIVO ($100,000)		

La fase de manufactura involucra problemas de almacenamiento y movimiento de materiales, volumen adecuado de fabricación, contabilidad, etc.

Por último, la fase de distribución y venta, se desarrolla mediante el empleo de publicistas, vendedores, equipos de transporte, etc., teniendo como ejemplo la transformación siguiente:

FABRICA ($290,000)		FABRICA ($290,000)
PRODUCTOS ($110,000)	=	PRODUCTOS ($160,000)
EFECTIVO ($100,000)		EFECTIVO ($50,000)

En este momento los productos deben ser cambiados por dinero o crédito, ofreciéndolos a un precio superior a $160,000, con el objeto de que exista una ganancia. Los problemas de esta fase tienen relación con la necesidad de crear un deseo de posesión de los productos, y de contar con un sistema eficiente para la distribución de los mismos.

Otros de los negocios mencionados tienen, en términos generales, la siguientes características:

La agricultura consiste en el cultivo o labranza de la tierra, mediante el empleo de equipo (tractores, etc.), materiales (abonos, etc.) y servicios (mano de obra, etc.). El financiamiento de este negocio por lo general se realiza a través de hipotecas sobre la tierra y otros bienes.

Las empresas que explotan los recursos naturales (petróleo, cobre, etc.), pueden financiarse por la venta de valores. Sus actividades principales consisten en la explotación, tratamiento y venta de los recursos mencionados, empleando equipo, materiales y servicios de diversos tipos.

CLASIFICACION DE COSTOS

De la exposición anterior resulta evidente que para determinar el costo de un producto, se deben considerar el consumo de materiales, los sueldos al personal y pago de otros servicios, así como la depreciación de maquinaria y edificios. El costo unitario se determina tomando en cuenta, además, el número de unidades producidas.

Por lo general, los cálculos no son tan simples, pues las empresas grandes y aún las de tamaño moderado, presentar serios problemas de contabilidad de costos. En la actualidad, los sistemas contables facilitan la determinación de costos unitarios y permiten medir la eficiencia administrativa. Para alcanzar este objetivo, se necesita una clasificación adecuada, como la siguiente:

1) Costos de Fabricación
2) Costos de Venta
3) Costos de Administración

La clasificación anterior tiene, como ya se indicó, gran importancia desde el punto de vista de la contabilidad de las empresas; sin embargo, para fines de análisis económico, conviene más emplear los conceptos de

1) Inversión Inicial
2) Costos de Operación y Mantenimiento Fijos
3) Costos de Operación y Mantenimiento Variables

COSTOS DE FABRICACION

Estos costos se originan en la tercera fase del movimiento de valores económicos, en donde el flujo se realiza a través de tres canales principales: equipo, materiales y servicios. Los más importantes renglones de que constan, son:

MATERIALES.- Los materiales pueden ser directos o indirectos. Los primeros son aquellos que llegan a formar parte del producto, como por ejemplo el tabique en las casas, el hule en las llantas, etc.; los últimos son los que se consumen o utilizan durante las operaciones de la fábrica y que no están asociados a un proceso o producto específico, pero que son necesarios para el buen funcionamiento del sistema. El costo de los materiales se determina tomando en cuenta su precio y, en ocasiones, los gastos de transporte, seguros, almacenamiento, etc.

MANO DE OBRA. - Esta puede ser directa o indirecta. La directa se refiere a los salarios de los trabajadores adscritos a operaciones de producción específicas. La mano de obra indirecta es la empleada en departamentos de servicio.

DEPRECIACION. - Es el costo cargado a los productos, que corresponde a la pérdida de valor de los activos fijos.

ENERGIA. - Incluye costos sobre salarios, combustible, seguros y depreciación, como si se tratara de una industria independiente.

IMPUESTOS. - Lo que se paga al gobierno por este concepto.

SEGUROS. - Primas contra incendio, robo, etc.

<u>COSTOS DE VENTA</u>

Estos costos incluyen gastos por salarios (vendedores, etc.), comisiones, publicidad, exposiciones, convenciones, oficinas, salas de exposición, almacenes, asociaciones mercantiles, deudas incobrables, etc.

COSTOS DE ADMINISTRACION

En la administración de las empresas se tienen gastos por salarios (funcionarios y empleados administrativos), equipo de oficina (operación y mantenimiento, incluyendo depreciación, como si se tratara de una unidad independiente), materiales, gastos de viaje, gastos legales y financieros, y otros servicios (teléfono, telégrafo, correo, etc.).

INVERSION INICIAL

De acuerdo con el Manual de Proyectos de Desarrollo Económico publicado por la ONU, la inversión inicial está constituída por el conjunto de bienes que no son motivo de transacciones frecuentes. En este concepto se deben considerar los siguientes rubros:

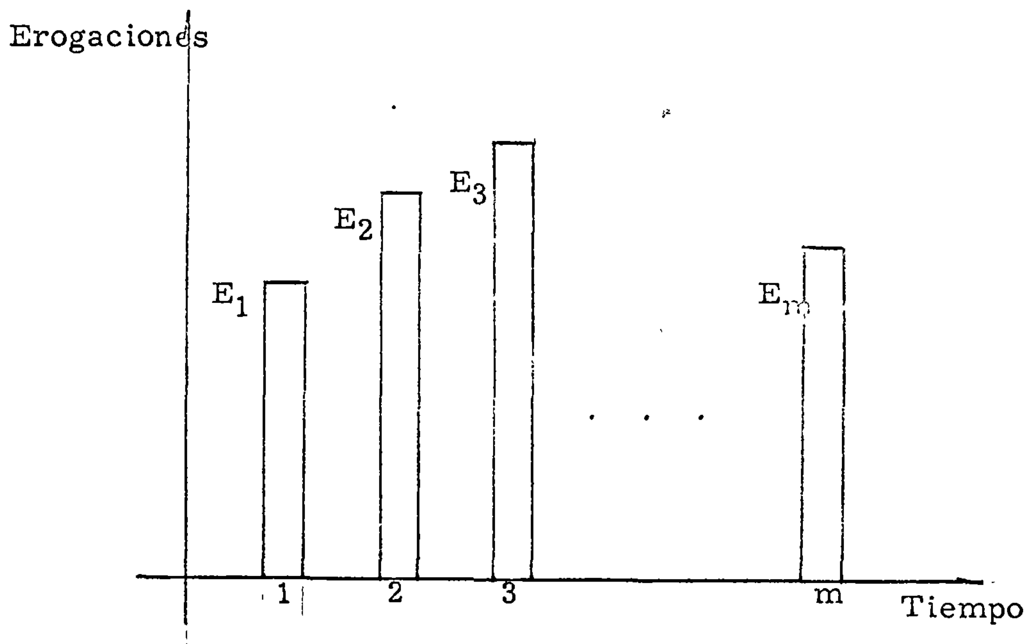
1) Costo de las investigaciones previas y estudio definitivo del proyecto.

2) Costo del equipo, edificios y otras instalaciones. Se debe incluir precio de adquisición, transporte, seguros y almacenamiento.

3) Instalación y puesta en marcha.

4) Intereses durante la construcción.

Sean E_1, E_2, . . , E_m las erogaciones realizadas en los tiempos 1, 2, . . . , m, respectivamente. Para cuantificar la inversión total desde que se realiza el primer gasto (tiempo 1), hasta que el proyecto está en condiciones de empezar a rendir frutos (tiempo m),



se deben sumar todas las erogaciones con sus intereses correspondientes. Si C es la inversión inicial e i la tasa de interés, entonces

$$C = E_1(1+i)^{m-1} + E_2(1+i)^{m-2} + \ldots + E_{m-1}(1+i) + E_m$$

$$= \sum_{k=1}^{m} E_k (1+i)^{m-k} \qquad (4.1)$$

EJEMPLO 4.1 En un proyecto de recuperación secundaria por inyección de agua, se tiene el siguiente calendario de inversiones:

AÑO 1 - Prueba piloto (acondicionamiento de pozos, planta de tratamiento, estudio preliminar), $2,500,000.

AÑO 2. - Operación de la prueba piloto, $1,000,000.

AÑO 3. - Operación de la prueba piloto y estudio definitivo, $1,500,000.

AÑO 4. - Acondicionamiento de los pozos inyectores, planta de tratamiento y red de distribución, $16,000,000.

Determinar la inversión inicial total, para una tasa de interés de 7% anual.

SOLUCION. - Suponiendo que las operaciones se efectúan en los tiempos indicados, de manera que el proyecto empiece a funcionar completamente en el año 4, entonces, de acuerdo con la ecuación (4.1):

$$C = 2{,}500{,}000(1.07)^3 + 1{,}000{,}000(1.07)^2 + 1{,}500{,}000(1.07)^1$$
$$+16{,}000{,}000$$
$$= \$21{,}812{,}500$$

COSTOS DE OPERACION Y MANTENIMIENTO FIJOS

Estos costos permanecen aproximadamente constantes, independientemente de la producción, tales como: depreciación, seguros, gastos financieros, ingeniería, algunas refacciones, rentas, gastos administrativos generales y de distribución y venta.

COSTOS DE OPERACION Y MANTENIMIENTO VARIABLES

Estos costos varían directamente con la producción. Entre éstos se pueden considerar las erogaciones por materiales directos, mano de obra directa, servicios, servicios varios (vapor, agua, refrigeración), refacciones diversas, supervisión directa, patentes, pérdidas.

DEPRECIACION

Con el transcurso del tiempo, el valor de los activos fijos generalmente disminuye. Esta pérdida de valor se denomina depreciación y cabe distinguir dos aspectos, a saber: Depreciación Física y Depreciación Funcional.

La depreciación física se debe al deterioro ocasionado por la acción de los elementos (corrosión, acción bacteriológica, descomposición química, etc.) y al desgaste debido al uso (impactos, vibración, etc.).

La depreciación funcional es de carácter puramente económico. Un equipo puede no estar depreciado físicamente, pero si ha aparecido otro que realice el mismo trabajo a menor costo, puede llegar a sustituirse, haciéndose OBSOLETO. Los automóviles proporcionan un buen ejemplo de este tipo de depreciación, que en ocasiones se sustituyen no por deficiencia técnica sino por capricho de los compradores, que se ven atraídos por los modelos nuevos. Un equipo puede sustituírse también debido a cambios en la demanda. Este tipo de depreciación se suele llamar OBSOLESCENCIA.

Para estimar la depreciación existen varios procedimientos, tales como: 1) Método de Línea Recta; 2), Método de Porcentaje Constante; 3), Método del Fondo de Amortización y 4), Método de Suma de Años. En la descripción de éstos se empleará la siguiente nomenclatura:

P, Costo Inicial o Inversión Inicial

L, Valor de Rescate

n, Vida Económica

D, Depreciación por Período

i, Tasa de Interés

METODO DE LINEA RECTA. - Con este método, que no toma en cuenta el interés, la depreciación por período es constante y se determina mediante la ecuación

$$D = \frac{P - L}{n} \qquad (4.2)$$

EJEMPLO 4.2 Calcular la depreciación por período para P = \$6,000, L = \$1,000, n = 5 años, usando el método de línea recta.

SOLUCION.- De acuerdo con la ecuación (4.2), la depreciación anual es constante en los 5 años e igual a \$1,000. En la Tabla que sigue se señala la recuperación de capital (depreciación) y la forma en que éste se va amortizando.

AÑO	CAPITAL RECUPERADO POR PERIODO	CAPITAL INVERTIDO AL PRINCIPIAR EL PERIODO
1	1,000	5,000
2	1,000	4,000
3	1,000	3,000
4	1,000	2,000
5	1,000	1,000
	5,000	

METODO DE PORCENTAJE CONSTANTE.- A diferencia del método anterior, en éste la depreciación por período es variable, siendo mayor al principio y menor al final de la vida económica. La razón es que para cada período se obtiene la recuperación correspondiente, multiplicando una fracción constante por el capital invertido, como sigue: Sea d la fracción constante, entonces la depreciación correspondiente al primer período es Pd y el capital invertido en ese tiempo es

$$P - Pd = P(1 - d)$$

La depreciación durante el segundo período es $P(1 - d)d$ y el capital invertido al terminar el mismo período es

$$P(1 - d) - P(1 - d)d = P(1 - d)^2$$

Generalizando este razonamiento resulta que al terminar el período n se tiene invertido el capital $P(1 - d)^n$, que por definición es el valor de rescate L; de este hecho resulta finalmente

$$d = 1 - \sqrt[n]{\frac{L}{P}} \tag{4.3}$$

EJEMPLO 4.3 Calcular la depreciación por período por el método de porcentaje constante, empleando los datos del ejemplo anterior.

SOLUCION. - De la ecuación (4.3), resulta

$$d = 1 - (1/6)^{1/5} = 30.1\%$$

de donde

$$D_1 = 0.301 \times 6{,}000 = 1{,}806$$

$$D_2 = 0.301 \times (6{,}000 - 1{,}806) = 1{,}262$$

etcétera, resultando los valores que aparecen en la Tabla que sigue.

AÑO	CAPITAL RECUPERADO POR PERIODO	CAPITAL INVERTIDO AL PRINCIPIAR EL PERIODO
1	1,806	4,194
2	1,262	2,932
3	884	2,048
4	617	1,431
5	431	1,000
	5,000	

METODO DEL FONDO DE AMORTIZACION. - En los dos métodos anteriores no se ha especificado si el capital que se va recuperando gana o no intereses; sin embargo, en el método del fondo de amortización, la depreciación está constituída por el capital separado directamente de los ingresos y por los intereses ganados en el fondo. Así, la suma que debe depositarse por período es

$$a = (P - L) \frac{i}{(1+i)^n - 1} \tag{4.4}$$

EJEMPLO 4.4 Calcular la depreciación por período, con los datos del problema 4.2 e $i = 7\%$ anual, por el método del fondo de amortización.

SOLUCION. - De la ecuación (4.4)

$$a = (6{,}000 - 1{,}000)(0.17389) = 869$$

de donde

$$D_1 = \$869$$

Para el segundo período se cuenta con la cantidad de $869 de los ingresos, y con 0. 07 x 869 = $61, producto del fondo, de manera que

$$D_2 = 869 + 61 = \$930$$

Los otros valores aparecen en la Tabla que sigue.

AÑO	CAPITAL RECUPERADO POR PERIODO	CAPITAL INVERTIDO AL PRINCIPIAR EL PERIODO
1	869	5,131
2	930	4,201
3	995	3,206
4	1,066	2,140
5	1,140	1,000
	5,000	

METODO DE SUMA DE AÑOS. - Con este método la depreciación por período se determina mediante la expresión

$$D_j = (P - L) \frac{n - j + 1}{1+2+. . .+n} \qquad (4.5)$$

EJEMPLO 4.5 Con los datos del ejemplo 4.2, la depreciación por período por el método de suma de años es

$$D_1 = 5{,}000\ (5/15) = \$1{,}667$$

$$D_2 = 5{,}000\ (4/15) = \$1{,}333$$

$$D_3 = 5{,}000\ (3/15) = \$1{,}000$$

$$D_4 = 5{,}000\ (2/15) = \$667$$

$$D_5 = 5{,}000\ (1/15) = \$333$$

METODO EQUIVALENTE.- Cuando se hace intervenir el interés, se puede demostrar que todos los métodos de depreciación son equivalentes. Como ilustración considérese el ejemplo anterior con $i = 7\%$ anual.

AÑO	DEPRECIACION POR PERIODO	CAPITAL INVERTIDO EN EL PER.	INTERES SOBRE CAP. INV	COLUMNAS (2) (4)
1	1,667	4,333	420	2,087
2	1,333	3,000	303	1,636
3	1,000	2,000	210	1,210
4	667	1,333	140	807
5	333	1,000	93	426
	5,000			

El primer valor que aparece en la columna 4 corresponde al interés durante un año de la inversión inicial; esto es, 0.07 x 6000. El segundo valor de esa misma columna, es el 7% del capital invertido durante el segundo año: 0.07 x 4333. Así sucesivamente, se obtienen las otras cifras.

La suma de los valores actuales de las cantidades que aparecen en la última columna, es

$$2087(1.07)^{-1} + 1636(1.07)^{-2} + 1210(1.07)^{-3} + 807(1.07)^{-4} + 426(1.07)^{-5} = \$5,286.53$$

y esta cantidad es igual al valor actual del costo real del equipo, o sea

$$P - L(1+i)^{-n} = \$5,287$$

Siguiendo este procedimiento en los ejemplos de los otros métodos, se llega al mismo resultado. Por lo tanto, para fines de análisis económico, la depreciación por período resulta constante e igual a

$$D = \left[P - \frac{L}{(1+i)^n}\right] \frac{i(1+i)^n}{(1+i)^n - 1}$$

$$= (P - L)\frac{i(1+i)^n}{(1+i)^n - 1} + Li \qquad (4.6)$$

En resumen, si A, B, C, . . . , N son las depreciaciones en los períodos 1, 2, . . . , n, independientemente del método de depreciación seleccionado, de acuerdo con los ejemplos anteriores debe tenerse que

$$A + B + C + . \; . \; . + N = P - L$$

Si se construye una tabla general que señale la depreciación, la inversión y los interes, se demuestra que la suma de los valores actuales de las cantidades de la última columna, satisfacen la ecuación (4.6).

TABLAS DE INTERES. COMPOSICION DISCRETA

TASA DE INTERES = 0.25 POR CIENTO

N	SP	PS	SR	RS	PR	RP
1	1.0025	0.9975	0.9998	1.0002	0.9973	1.0027
2	1.0050	0.9950	2.0018	0.4996	1.9918	0.5021
3	1.0075	0.9925	3.0065	0.3326	2.9841	0.3351
4	1.0100	0.9901	4.0135	0.2492	3.9736	0.2517
5	1.0126	0.9876	5.0232	0.1991	4.9609	0.2016
6	1.0151	0.9851	6.0351	0.1657	5.9454	0.1682
7	1.0176	0.9827	7.0501	0.1418	6.9280	0.1443
8	1.0202	0.9802	8.0671	0.1240	7.9076	0.1265
9	1.0227	0.9778	9.0866	0.1101	8.8848	0.1126
10	1.0253	0.9753	10.1089	0.0989	9.8597	0.1014
11	1.0278	0.9729	11.1338	0.0898	10.8323	0.0923
12	1.0304	0.9705	12.1614	0.0822	11.8026	0.0847
13	1.0330	0.9681	13.1913	0.0758	12.7701	0.0783
14	1.0356	0.9657	14.2239	0.0703	13.7354	0.0728
15	1.0382	0.9633	15.2591	0.0655	14.6983	0.0680
16	1.0407	0.9609	16.2966	0.0614	15.6586	0.0639
17	1.0433	0.9585	17.3368	0.0577	16.6165	0.0602
18	1.0460	0.9561	18.3796	0.0544	17.5722	0.0569
19	1.0486	0.9537	19.4256	0.0515	18.5258	0.0540
20	1.0512	0.9513	20.4734	0.0488	19.4764	0.0513
21	1.0538	0.9489	21.5242	0.0465	20.4250	0.0490
22	1.0564	0.9466	22.5774	0.0443	21.3710	0.0468
23	1.0591	0.9442	23.6340	0.0423	22.3154	0.0448
24	1.0617	0.9419	24.6921	0.0405	23.2563	0.0430
25	1.0644	0.9395	25.7533	0.0388	24.1953	0.0413
26	1.0670	0.9372	26.8171	0.0373	25.1320	0.0398
27	1.0697	0.9348	27.8840	0.0359	26.0667	0.0384
28	1.0724	0.9325	28.9535	0.0345	26.9991	0.0370
29	1.0751	0.9302	30.0254	0.0333	27.9288	0.0358
30	1.0778	0.9279	31.0999	0.0322	28.8561	0.0347
35	1.0913	0.9163	36.5133	0.0274	33.4588	0.0299
40	1.1050	0.9050	41.9946	0.0238	38.0043	0.0263
45	1.1189	0.8938	47.5449	0.0210	42.4936	0.0235
50	1.1329	0.8827	53.1646	0.0188	46.9270	0.0213
55	1.1472	0.8717	58.8557	0.0170	51.3060	0.0195
60	1.1616	0.8609	64.6177	0.0155	55.6302	0.0180
65	1.1761	0.8502	70.4514	0.0142	59.9004	0.0167
70	1.1909	0.8397	76.3592	0.0131	64.1182	0.0156
75	1.2059	0.8293	82.3409	0.0121	68.2835	0.0146
80	1.2210	0.8190	88.3982	0.0113	72.3974	0.0138
85	1.2363	0.8088	94.5313	0.0106	76.4602	0.0131
90	1.2519	0.7988	100.7408	0.0099	80.4721	0.0124
95	1.2676	0.7889	107.0292	0.0093	84.4349	0.0118
100	1.2835	0.7791	113.3958	0.0088	88.3480	0.0113

TABLAS DE INTERES. COMPOSICION DISCRETA

TASA DE INTERES = 0.50 POR CIENTO

N	SP	PS	SR	RS	PR	RP
1	1.0050	0.9950	0.9999	1.0001	0.9950	1.0051
2	1.0100	0.9901	2.0048	0.4988	1.9849	0.5038
3	1.0151	0.9851	3.0147	0.3317	2.9699	0.3367
4	1.0202	0.9802	4.0297	0.2482	3.9501	0.2532
5	1.0253	0.9754	5.0497	0.1980	4.9253	0.2030
6	1.0304	0.9705	6.0748	0.1646	5.8957	0.1696
7	1.0355	0.9657	7.1051	0.1407	6.8613	0.1457
8	1.0407	0.9609	8.1405	0.1228	7.8221	0.1278
9	1.0459	0.9561	9.1811	0.1089	8.7781	0.1139
10	1.0511	0.9513	10.2268	0.0978	9.7292	0.1028
11	1.0564	0.9466	11.2778	0.0887	10.6758	0.0937
12	1.0617	0.9419	12.3342	0.0811	11.6177	0.0861
13	1.0670	0.9372	13.3956	0.0747	12.5546	0.0797
14	1.0723	0.9326	14.4625	0.0691	13.4871	0.0741
15	1.0777	0.9279	15.5347	0.0644	14.4149	0.0694
16	1.0831	0.9233	16.6122	0.0602	15.3382	0.0652
17	1.0885	0.9187	17.6952	0.0565	16.2567	0.0615
18	1.0939	0.9141	18.7836	0.0532	17.1708	0.0582
19	1.0994	0.9096	19.8774	0.0503	18.0803	0.0553
20	1.1049	0.9051	20.9767	0.0477	18.9853	0.0527
21	1.1104	0.9006	22.0813	0.0453	19.8857	0.0503
22	1.1160	0.8961	23.1915	0.0431	20.7815	0.0481
23	1.1215	0.8916	24.3074	0.0411	21.6731	0.0461
24	1.1272	0.8872	25.4288	0.0393	22.5602	0.0443
25	1.1328	0.8828	26.5558	0.0377	23.4428	0.0427
26	1.1385	0.8784	27.6885	0.0361	24.3212	0.0411
27	1.1441	0.8740	28.8269	0.0347	25.1951	0.0397
28	1.1499	0.8697	29.9710	0.0334	26.0648	0.0384
29	1.1556	0.8653	31.1205	0.0321	26.9298	0.0371
30	1.1614	0.8610	32.2761	0.0310	27.7908	0.0360
35	1.1907	0.8398	38.1407	0.0262	32.0317	0.0312
40	1.2208	0.8191	44.1533	0.0226	36.1680	0.0276
45	1.2516	0.7990	50.3178	0.0199	40.2025	0.0249
50	1.2832	0.7793	56.6380	0.0177	44.1377	0.0227
55	1.3156	0.7601	63.1177	0.0158	47.9759	0.0208
60	1.3488	0.7414	69.7614	0.0143	51.7198	0.0193
65	1.3829	0.7231	76.5722	0.0131	55.3711	0.0181
70	1.4178	0.7053	83.5553	0.0120	58.9327	0.0170
75	1.4536	0.6879	90.7147	0.0110	62.4065	0.0160
80	1.4903	0.6710	98.0549	0.0102	65.7948	0.0152
85	1.5279	0.6545	105.5805	0.0095	69.0997	0.0145
90	1.5665	0.6384	113.2961	0.0088	72.3232	0.0138
95	1.6061	0.6226	121.2066	0.0083	75.4673	0.0133
100	1.6466	0.6073	129.3167	0.0077	78.5340	0.0127

TABLAS DE INTERES. COMPOSICION DISCRETA

TASA DE INTERES = 0.75 POR CIENTO

N	SP	PS	SR	RS	PR	RP
1	1.0075	0.9926	0.9999	1.0001	0.9924	1.0076
2	1.0151	0.9852	2.0071	0.4982	1.9774	0.5057
3	1.0227	0.9778	3.0220	0.3309	2.9550	0.3384
4	1.0303	0.9706	4.0444	0.2473	3.9253	0.2548
5	1.0381	0.9633	5.0745	0.1971	4.8885	0.2046
6	1.0458	0.9562	6.1123	0.1636	5.8444	0.1711
7	1.0537	0.9490	7.1581	0.1397	6.7934	0.1472
8	1.0616	0.9420	8.2114	0.1218	7.7350	0.1293
9	1.0696	0.9350	9.2728	0.1078	8.6698	0.1153
10	1.0776	0.9280	10.3421	0.0967	9.5976	0.1042
11	1.0857	0.9211	11.4194	0.0876	10.5185	0.0951
12	1.0938	0.9142	12.5049	0.0800	11.4326	0.0875
13	1.1020	0.9074	13.5984	0.0735	12.3398	0.0810
14	1.1103	0.9007	14.7000	0.0680	13.2402	0.0755
15	1.1186	0.8940	15.8103	0.0632	14.1342	0.0708
16	1.1270	0.8873	16.9284	0.0591	15.0211	0.0666
17	1.1354	0.8807	18.0552	0.0554	15.9017	0.0629
18	1.1439	0.8742	19.1903	0.0521	16.7757	0.0596
19	1.1525	0.8677	20.3340	0.0492	17.6431	0.0567
20	1.1612	0.8612	21.4862	0.0465	18.5041	0.0540
21	1.1699	0.8548	22.6472	0.0442	19.3588	0.0517
22	1.1786	0.8484	23.8167	0.0420	20.2069	0.0495
23	1.1875	0.8421	24.9953	0.0400	21.0491	0.0475
24	1.1964	0.8359	26.1824	0.0382	21.8846	0.0457
25	1.2054	0.8296	27.3786	0.0365	22.7141	0.0440
26	1.2144	0.8235	28.5835	0.0350	23.5373	0.0425
27	1.2235	0.8173	29.7977	0.0336	24.3544	0.0411
28	1.2327	0.8112	31.0209	0.0322	25.1655	0.0397
29	1.2419	0.8052	32.2534	0.0310	25.9706	0.0385
30	1.2512	0.7992	33.4949	0.0299	26.7696	0.0374
35	1.2989	0.7699	39.8443	0.0251	30.6764	0.0326
40	1.3483	0.7417	46.4352	0.0215	34.4400	0.0290
45	1.3996	0.7145	53.2772	0.0188	38.0657	0.0263
50	1.4529	0.6883	60.3792	0.0166	41.5583	0.0241
55	1.5082	0.6631	67.7519	0.0148	44.9230	0.0223
60	1.5656	0.6387	75.4048	0.0133	48.1641	0.0208
65	1.6252	0.6153	83.3493	0.0120	51.2865	0.0195
70	1.6870	0.5928	91.5961	0.0109	54.2944	0.0184
75	1.7512	0.5710	100.1567	0.0100	57.1920	0.0175
80	1.8179	0.5501	109.0432	0.0092	59.9834	0.0167
85	1.8871	0.5299	118.2679	0.0085	62.6725	0.0160
90	1.9589	0.5105	127.8437	0.0078	65.2629	0.0153
95	2.0335	0.4918	137.7843	0.0073	67.7584	0.0148
100	2.1109	0.4737	148.1027	0.0068	70.1623	0.0143

TABLAS DE INTERES. COMPOSICION DISCRETA

TASA DE INTERES = 1.00 POR CIENTO

N	SP	PS	SR	RS	PR	RP
1	1.0100	0.9901	0.9999	1.0001	0.9900	1.0101
2	1.0201	0.9803	2.0098	0.4976	1.9702	0.5076
3	1.0303	0.9706	3.0298	0.3301	2.9407	0.3401
4	1.0406	0.9610	4.0599	0.2463	3.9015	0.2563
5	1.0510	0.9515	5.1004	0.1961	4.8529	0.2061
6	1.0615	0.9420	6.1513	0.1626	5.7948	0.1726
7	1.0721	0.9327	7.2127	0.1386	6.7275	0.1486
8	1.0829	0.9235	8.2847	0.1207	7.6508	0.1307
9	1.0937	0.9143	9.3674	0.1068	8.5650	0.1168
10	1.1046	0.9053	10.4610	0.0956	9.4702	0.1056
11	1.1157	0.8963	11.5654	0.0865	10.3664	0.0965
12	1.1268	0.8875	12.6810	0.0789	11.2538	0.0889
13	1.1381	0.8787	13.8076	0.0724	12.1323	0.0824
14	1.1495	0.8700	14.9456	0.0669	13.0022	0.0769
15	1.1610	0.8614	16.0949	0.0621	13.8634	0.0721
16	1.1726	0.8528	17.2557	0.0580	14.7162	0.0680
17	1.1843	0.8444	18.4282	0.0543	15.5605	0.0643
18	1.1961	0.8360	19.6124	0.0510	16.3964	0.0610
19	1.2081	0.8277	20.8083	0.0481	17.2240	0.0581
20	1.2202	0.8196	22.0163	0.0454	18.0435	0.0554
21	1.2324	0.8114	23.2363	0.0430	18.8548	0.0530
22	1.2447	0.8034	24.4685	0.0409	19.6581	0.0509
23	1.2572	0.7954	25.7131	0.0389	20.4535	0.0489
24	1.2697	0.7876	26.9700	0.0371	21.2409	0.0471
25	1.2824	0.7798	28.2397	0.0354	22.0206	0.0454
26	1.2952	0.7721	29.5220	0.0339	22.7926	0.0439
27	1.3082	0.7644	30.8170	0.0324	23.5569	0.0425
28	1.3213	0.7568	32.1250	0.0311	24.3136	0.0411
29	1.3345	0.7494	33.4461	0.0299	25.0629	0.0399
30	1.3478	0.7419	34.7804	0.0288	25.8047	0.0388
35	1.4166	0.7059	41.6550	0.0240	29.4053	0.0340
40	1.4888	0.6717	48.8800	0.0205	32.8310	0.0305
45	1.5648	0.6391	56.4736	0.0177	36.0904	0.0277
50	1.6446	0.6081	64.4548	0.0155	39.1918	0.0255
55	1.7285	0.5785	72.8428	0.0137	42.1425	0.0237
60	1.8167	0.5505	81.6586	0.0122	44.9501	0.0222
65	1.9093	0.5237	90.9244	0.0110	47.6214	0.0210
70	2.0067	0.4983	100.6625	0.0099	50.1631	0.0199
75	2.1091	0.4741	110.8975	0.0090	52.5814	0.0190
80	2.2166	0.4511	121.6546	0.0082	54.8824	0.0182
85	2.3297	0.4292	132.9603	0.0075	57.0716	0.0175
90	2.4485	0.4084	144.8428	0.0069	59.1547	0.0169
95	2.5734	0.3886	157.3312	0.0064	61.1366	0.0164
100	2.7047	0.3697	170.4568	0.0059	63.0224	0.0159

TABLAS DE INTERES. COMPOSICION DISCRETA

TASA DE INTERES = 1.25 POR CIENTO

N	SP	PS	SR	RS	PR	RP
1	1.0125	0.9877	1.0000	1.0000	0.9876	1.0125
2	1.0252	0.9755	2.0124	0.4969	1.9630	0.5094
3	1.0380	0.9634	3.0375	0.3292	2.9264	0.3417
4	1.0509	0.9515	4.0754	0.2454	3.8778	0.2579
5	1.0641	0.9398	5.1262	0.1951	4.8175	0.2076
6	1.0774	0.9282	6.1902	0.1615	5.7456	0.1740
7	1.0909	0.9167	7.2676	0.1376	6.6623	0.1501
8	1.1045	0.9054	8.3583	0.1196	7.5676	0.1321
9	1.1183	0.8942	9.4627	0.1057	8.4618	0.1182
10	1.1323	0.8832	10.5810	0.0945	9.3449	0.1070
11	1.1464	0.8723	11.7132	0.0854	10.2171	0.0979
12	1.1608	0.8615	12.8595	0.0778	11.0785	0.0903
13	1.1753	0.8509	14.0202	0.0713	11.9294	0.0838
14	1.1900	0.8404	15.1953	0.0658	12.7696	0.0783
15	1.2048	0.8300	16.3852	0.0610	13.5996	0.0735
16	1.2199	0.8197	17.5899	0.0569	14.4193	0.0694
17	1.2351	0.8096	18.8098	0.0532	15.2288	0.0657
18	1.2506	0.7996	20.0448	0.0499	16.0284	0.0624
19	1.2662	0.7898	21.2953	0.0470	16.8181	0.0595
20	1.2820	0.7800	22.5614	0.0443	17.5980	0.0568
21	1.2981	0.7704	23.8433	0.0419	18.3683	0.0544
22	1.3143	0.7609	25.1413	0.0398	19.1292	0.0523
23	1.3307	0.7515	26.4555	0.0378	19.8806	0.0503
24	1.3474	0.7422	27.7861	0.0360	20.6227	0.0485
25	1.3642	0.7330	29.1334	0.0343	21.3557	0.0468
26	1.3812	0.7240	30.4975	0.0328	22.0797	0.0453
27	1.3985	0.7150	31.8786	0.0314	22.7946	0.0439
28	1.4160	0.7062	33.2771	0.0301	23.5008	0.0426
29	1.4337	0.6975	34.6930	0.0288	24.1983	0.0413
30	1.4516	0.6889	36.1265	0.0277	24.8871	0.0402
35	1.5446	0.6474	43.5678	0.0230	28.2058	0.0355
40	1.6436	0.6084	51.4860	0.0194	31.3246	0.0319
45	1.7490	0.5718	59.9116	0.0167	34.2557	0.0292
50	1.8610	0.5373	68.8770	0.0145	37.0101	0.0270
55	1.9803	0.5050	78.4171	0.0128	39.5988	0.0253
60	2.1072	0.4746	88.5685	0.0113	42.0315	0.0238
65	2.2422	0.4460	99.3705	0.0101	44.3177	0.0226
70	2.3859	0.4191	110.8644	0.0090	46.4662	0.0215
75	2.5388	0.3939	123.0953	0.0081	48.4854	0.0206
80	2.7015	0.3702	136.1096	0.0073	50.3829	0.0198
85	2.8746	0.3479	149.9581	0.0067	52.1661	0.0192
90	3.0588	0.3269	164.6941	0.0061	53.8420	0.0186
95	3.2549	0.3072	180.3742	0.0055	55.4169	0.0180
100	3.4634	0.2887	197.0594	0.0051	56.8970	0.0176

TABLAS DE INTERES. COMPOSICION DISCRETA

TASA DE INTERES = 1.50 POR CIENTO

N	SP	PS	SR	RS	PR	RP
1	1.0150	0.9852	0.9999	1.0001	0.9852	1.0151
2	1.0302	0.9707	2.0148	0.4963	1.9557	0.5113
3	1.0457	0.9563	3.0450	0.3284	2.9120	0.3434
4	1.0614	0.9422	4.0905	0.2445	3.8540	0.2595
5	1.0773	0.9283	5.1518	0.1941	4.7822	0.2091
6	1.0934	0.9145	6.2289	0.1605	5.6966	0.1755
7	1.1098	0.9010	7.3223	0.1366	6.5976	0.1516
8	1.1265	0.8877	8.4320	0.1186	7.4852	0.1336
9	1.1434	0.8746	9.5584	0.1046	8.3597	0.1196
10	1.1605	0.8617	10.7016	0.0934	9.2213	0.1084
11	1.1779	0.8489	11.8621	0.0843	10.0701	0.0993
12	1.1956	0.8364	13.0398	0.0767	10.9064	0.0917
13	1.2135	0.8240	14.2354	0.0702	11.7304	0.0852
14	1.2317	0.8119	15.4487	0.0647	12.5421	0.0797
15	1.2502	0.7999	16.6804	0.0600	13.3419	0.0750
16	1.2690	0.7880	17.9304	0.0558	14.1298	0.0708
17	1.2880	0.7764	19.1993	0.0521	14.9062	0.0671
18	1.3073	0.7649	20.4872	0.0488	15.6710	0.0638
19	1.3269	0.7536	21.7944	0.0459	16.4245	0.0609
20	1.3468	0.7425	23.1211	0.0433	17.1669	0.0583
21	1.3670	0.7315	24.4679	0.0409	17.8984	0.0559
22	1.3876	0.7207	25.8348	0.0387	18.6189	0.0537
23	1.4084	0.7100	27.2222	0.0367	19.3289	0.0517
24	1.4295	0.6996	28.6304	0.0349	20.0284	0.0499
25	1.4509	0.6892	30.0598	0.0333	20.7175	0.0483
26	1.4727	0.6790	31.5105	0.0317	21.3965	0.0467
27	1.4948	0.6690	32.9831	0.0303	22.0654	0.0453
28	1.5172	0.6591	34.4777	0.0290	22.7244	0.0440
29	1.5400	0.6494	35.9948	0.0278	23.3738	0.0428
30	1.5631	0.6398	37.5345	0.0266	24.0134	0.0416
35	1.6839	0.5939	45.5870	0.0219	27.0729	0.0369
40	1.8140	0.5513	54.2618	0.0184	29.9129	0.0334
45	1.9542	0.5117	63.6070	0.0157	32.5492	0.0307
50	2.1052	0.4750	73.6743	0.0136	34.9963	0.0286
55	2.2679	0.4409	84.5198	0.0118	37.2679	0.0268
60	2.4432	0.4093	96.2033	0.0104	39.3765	0.0254
65	2.6320	0.3799	108.7899	0.0092	41.3339	0.0242
70	2.8354	0.3527	122.3490	0.0082	43.1508	0.0232
75	3.0545	0.3274	136.9563	0.0073	44.8374	0.0223
80	3.2906	0.3039	152.6921	0.0065	46.4030	0.0216
85	3.5449	0.2821	169.6443	0.0059	47.8563	0.0209
90	3.8188	0.2619	187.9063	0.0053	49.2053	0.0203
95	4.1139	0.2431	207.5799	0.0048	50.4576	0.0198
100	4.4319	0.2256	228.7735	0.0044	51.6201	0.0194

TABLAS DE INTERES. COMPOSICION DISCRETA

TASA DE INTERES = 1.75 POR CIENTO

N	SP	PS	SR	RS	PR	RP
1	1.0175	0.9828	1.0000	1.0000	0.9828	1.0175
2	1.0353	0.9659	2.0174	0.4957	1.9486	0.5132
3	1.0534	0.9493	3.0527	0.3276	2.8978	0.3451
4	1.0719	0.9330	4.1061	0.2435	3.8308	0.2610
5	1.0906	0.9169	5.1779	0.1931	4.7476	0.2106
6	1.1097	0.9011	6.2685	0.1595	5.6488	0.1770
7	1.1291	0.8856	7.3781	0.1355	6.5343	0.1530
8	1.1489	0.8704	8.5072	0.1175	7.4048	0.1350
9	1.1690	0.8554	9.6561	0.1036	8.2602	0.1211
10	1.1895	0.8407	10.8250	0.0924	9.1008	0.1099
11	1.2103	0.8263	12.0144	0.0832	9.9270	0.1007
12	1.2315	0.8121	13.2247	0.0756	10.7391	0.0931
13	1.2530	0.7981	14.4560	0.0692	11.5371	0.0867
14	1.2749	0.7844	15.7090	0.0637	12.3215	0.0812
15	1.2972	0.7709	16.9838	0.0589	13.0923	0.0764
16	1.3199	0.7576	18.2811	0.0547	13.8499	0.0722
17	1.3430	0.7446	19.6010	0.0510	14.5944	0.0685
18	1.3665	0.7318	20.9440	0.0477	15.3262	0.0652
19	1.3905	0.7192	22.3104	0.0448	16.0453	0.0623
20	1.4148	0.7068	23.7008	0.0422	16.7521	0.0597
21	1.4396	0.6947	25.1155	0.0398	17.4467	0.0573
22	1.4648	0.6827	26.5551	0.0377	18.1294	0.0552
23	1.4904	0.6710	28.0197	0.0357	18.8003	0.0532
24	1.5165	0.6594	29.5102	0.0339	19.4598	0.0514
25	1.5430	0.6481	31.0266	0.0322	20.1078	0.0497
26	1.5700	0.6369	32.5695	0.0307	20.7447	0.0482
27	1.5975	0.6260	34.1393	0.0293	21.3707	0.0468
28	1.6254	0.6152	35.7369	0.0280	21.9859	0.0455
29	1.6539	0.6046	37.3622	0.0268	22.5905	0.0443
30	1.6828	0.5942	39.0160	0.0256	23.1847	0.0431
35	1.8353	0.5449	47.7295	0.0210	26.0059	0.0385
40	2.0017	0.4996	57.2328	0.0175	28.5928	0.0350
45	2.1830	0.4581	67.5970	0.0148	30.9646	0.0323
50	2.3809	0.4200	78.9007	0.0127	33.1394	0.0302
55	2.5966	0.3851	91.2284	0.0110	35.1335	0.0285
60	2.8319	0.3531	104.6737	0.0096	36.9619	0.0271
65	3.0886	0.3238	119.3371	0.0084	38.6384	0.0259
70	3.3685	0.2969	135.3293	0.0074	40.1755	0.0249
75	3.6737	0.2722	152.7706	0.0065	41.5850	0.0240
80	4.0066	0.2496	171.7929	0.0058	42.8773	0.0233
85	4.3697	0.2288	192.5385	0.0052	44.0623	0.0227
90	4.7657	0.2098	215.1645	0.0046	45.1488	0.0221
95	5.1975	0.1924	239.8403	0.0042	46.1450	0.0217
100	5.6685	0.1764	266.7529	0.0037	47.0584	0.0213

TABLAS DE INTERES. COMPOSICION DISCRETA

TASA DE INTERES = 2.00 POR CIENTO

N	SP	PS	SR	RS	PR	RP
1	1.0200	0.9804	1.0000	1.0000	0.9804	1.0200
2	1.0404	0.9612	2.0200	0.4951	1.9415	0.5151
3	1.0612	0.9423	3.0603	0.3268	2.8838	0.3468
4	1.0824	0.9238	4.1215	0.2426	3.8076	0.2626
5	1.1041	0.9057	5.2039	0.1922	4.7133	0.2122
6	1.1262	0.8880	6.3080	0.1585	5.6013	0.1785
7	1.1487	0.8706	7.4341	0.1345	6.4718	0.1545
8	1.1717	0.8535	8.5827	0.1165	7.3252	0.1365
9	1.1951	0.8367	9.7544	0.1025	8.1619	0.1225
10	1.2190	0.8203	10.9494	0.0913	8.9822	0.1113
11	1.2434	0.8043	12.1684	0.0822	9.7865	0.1022
12	1.2683	0.7885	13.4117	0.0746	10.5749	0.0946
13	1.2936	0.7730	14.6799	0.0681	11.3479	0.0881
14	1.3195	0.7579	15.9735	0.0626	12.1058	0.0826
15	1.3459	0.7430	17.2930	0.0578	12.8488	0.0778
16	1.3728	0.7284	18.6388	0.0537	13.5772	0.0737
17	1.4003	0.7142	20.0115	0.0500	14.2913	0.0700
18	1.4283	0.7001	21.4118	0.0467	14.9914	0.0667
19	1.4568	0.6864	22.8400	0.0438	15.6778	0.0638
20	1.4860	0.6730	24.2968	0.0412	16.3507	0.0612
21	1.5157	0.6598	25.7827	0.0388	17.0105	0.0588
22	1.5460	0.6468	27.2984	0.0366	17.6573	0.0566
23	1.5769	0.6341	28.8443	0.0347	18.2914	0.0547
24	1.6085	0.6217	30.4212	0.0329	18.9131	0.0529
25	1.6406	0.6095	32.0296	0.0312	19.5226	0.0512
26	1.6735	0.5976	33.6702	0.0297	20.1201	0.0497
27	1.7069	0.5858	35.3436	0.0283	20.7060	0.0483
28	1.7411	0.5744	37.0505	0.0270	21.2803	0.0470
29	1.7759	0.5631	38.7915	0.0258	21.8434	0.0458
30	1.8114	0.5521	40.5673	0.0247	22.3954	0.0447
35	2.0000	0.5000	49.9937	0.0200	24.9974	0.0400
40	2.2081	0.4529	60.4012	0.0166	27.3541	0.0366
45	2.4380	0.4102	71.8919	0.0139	29.4886	0.0339
50	2.6917	0.3715	84.5787	0.0118	31.4219	0.0318
55	2.9719	0.3365	98.5861	0.0101	33.1730	0.0301
60	3.2812	0.3048	114.0514	0.0088	34.7590	0.0288
65	3.6227	0.2760	131.1264	0.0076	36.1954	0.0276
70	3.9998	0.2500	149.9787	0.0067	37.4964	0.0267
75	4.4161	0.2264	170.7932	0.0059	38.6748	0.0259
80	4.8758	0.2051	193.7741	0.0052	39.7421	0.0252
85	5.3833	0.1858	219.1470	0.0046	40.7088	0.0246
90	5.9436	0.1682	247.1610	0.0040	41.5843	0.0240
95	6.5623	0.1524	278.0908	0.0036	42.3773	0.0236
100	7.2453	0.1380	312.2400	0.0032	43.0956	0.0232

TABLAS DE INTERES. COMPOSICION DISCRETA

TASA DE INTERES = 2.25 POR CIENTO

N	SP	PS	SR	RS	PR	RP
1	1.0225	0.9780	1.0000	1.0000	0.9780	1.0225
2	1.0455	0.9565	2.0224	0.4945	1.9344	0.5170
3	1.0690	0.9354	3.0678	0.3260	2.8697	0.3485
4	1.0931	0.9148	4.1368	0.2417	3.7845	0.2642
5	1.1177	0.8947	5.2298	0.1912	4.6792	0.2137
6	1.1428	0.8750	6.3474	0.1575	5.5541	0.1800
7	1.1685	0.8558	7.4902	0.1335	6.4098	0.1560
8	1.1948	0.8369	8.6586	0.1155	7.2467	0.1380
9	1.2217	0.8185	9.8534	0.1015	8.0652	0.1240
10	1.2492	0.8005	11.0751	0.0903	8.8657	0.1128
11	1.2773	0.7829	12.3242	0.0811	9.6485	0.1036
12	1.3061	0.7657	13.6014	0.0735	10.4141	0.0960
13	1.3354	0.7488	14.9074	0.0671	11.1629	0.0896
14	1.3655	0.7323	16.2427	0.0616	11.8952	0.0841
15	1.3962	0.7162	17.6081	0.0568	12.6113	0.0793
16	1.4276	0.7005	19.0042	0.0526	13.3117	0.0751
17	1.4598	0.6850	20.4318	0.0489	13.9968	0.0714
18	1.4926	0.6700	21.8914	0.0457	14.6667	0.0682
19	1.5262	0.6552	23.3839	0.0428	15.3219	0.0653
20	1.5605	0.6408	24.9100	0.0401	15.9626	0.0626
21	1.5956	0.6267	26.4705	0.0378	16.5893	0.0603
22	1.6315	0.6129	28.0660	0.0356	17.2022	0.0581
23	1.6682	0.5994	29.6974	0.0337	17.8016	0.0562
24	1.7058	0.5862	31.3655	0.0319	18.3878	0.0544
25	1.7442	0.5733	33.0712	0.0302	18.9611	0.0527
26	1.7834	0.5607	34.8153	0.0287	19.5218	0.0512
27	1.8235	0.5484	36.5986	0.0273	20.0701	0.0498
28	1.8646	0.5363	38.4220	0.0260	20.6064	0.0485
29	1.9065	0.5245	40.2865	0.0248	21.1309	0.0473
30	1.9494	0.5130	42.1928	0.0237	21.6438	0.0462
35	2.1788	0.4590	52.3878	0.0191	24.0441	0.0416
40	2.4352	0.4106	63.7825	0.0157	26.1917	0.0382
45	2.7218	0.3674	76.5184	0.0131	28.1131	0.0356
50	3.0421	0.3287	90.7527	0.0110	29.8323	0.0335
55	3.4001	0.2941	106.6622	0.0094	31.3704	0.0319
60	3.8002	0.2631	124.4440	0.0080	32.7466	0.0305
65	4.2474	0.2354	144.3183	0.0069	33.9779	0.0294
70	4.7473	0.2106	166.5314	0.0060	35.0795	0.0285
75	5.3059	0.1885	191.3586	0.0052	36.0652	0.0277
80	5.9303	0.1686	219.1073	0.0046	36.9471	0.0271
85	6.6282	0.1509	250.1217	0.0040	37.7361	0.0265
90	7.4082	0.1350	284.7854	0.0035	38.4420	0.0260
95	8.2800	0.1208	323.5283	0.0031	39.0736	0.0256
100	9.2544	0.1081	366.8313	0.0027	39.6387	0.0252

TABLAS DE INTERES. COMPOSICION DISCRETA

TASA DE INTERES = 2.50 POR CIENTO

N	SP	PS	SR	RS	PR	RP
1	1.0250	0.9756	1.0000	1.0000	0.9756	1.0250
2	1.0506	0.9518	2.0249	0.4938	1.9274	0.5188
3	1.0769	0.9286	3.0755	0.3251	2.8559	0.3501
4	1.1038	0.9059	4.1524	0.2408	3.7618	0.2658
5	1.1314	0.8838	5.2562	0.1903	4.6457	0.2153
6	1.1597	0.8623	6.3875	0.1566	5.5079	0.1816
7	1.1887	0.8413	7.5472	0.1325	6.3491	0.1575
8	1.2184	0.8207	8.7358	0.1145	7.1698	0.1395
9	1.2489	0.8007	9.9542	0.1005	7.9705	0.1255
10	1.2801	0.7812	11.2030	0.0893	8.7517	0.1143
11	1.3121	0.7621	12.4831	0.0801	9.5138	0.1051
12	1.3449	0.7435	13.7951	0.0725	10.2573	0.0975
13	1.3785	0.7254	15.1400	0.0661	10.9827	0.0911
14	1.4130	0.7077	16.5185	0.0605	11.6904	0.0855
15	1.4483	0.6905	17.9314	0.0558	12.3808	0.0808
16	1.4845	0.6736	19.3797	0.0516	13.0544	0.0766
17	1.5216	0.6572	20.8642	0.0479	13.7116	0.0729
18	1.5597	0.6412	22.3858	0.0447	14.3527	0.0697
19	1.5987	0.6255	23.9454	0.0418	14.9782	0.0668
20	1.6387	0.6103	25.5440	0.0391	15.5884	0.0642
21	1.6796	0.5954	27.1826	0.0368	16.1838	0.0618
22	1.7216	0.5809	28.8621	0.0346	16.7646	0.0596
23	1.7647	0.5667	30.5837	0.0327	17.3313	0.0577
24	1.8088	0.5529	32.3483	0.0309	17.8841	0.0559
25	1.8540	0.5394	34.1570	0.0293	18.4235	0.0543
26	1.9003	0.5262	36.0109	0.0278	18.9497	0.0528
27	1.9479	0.5134	37.9112	0.0264	19.4630	0.0514
28	1.9966	0.5009	39.8590	0.0251	19.9639	0.0501
29	2.0465	0.4886	41.8554	0.0239	20.4525	0.0489
30	2.0976	0.4767	43.9019	0.0228	20.9292	0.0478
35	2.3733	0.4214	54.9273	0.0182	23.1439	0.0432
40	2.6852	0.3724	67.4016	0.0148	25.1014	0.0398
45	3.0380	0.3292	81.5153	0.0123	26.8315	0.0373
50	3.4373	0.2909	97.4837	0.0103	28.3607	0.0353
55	3.8890	0.2571	115.5506	0.0087	29.7122	0.0337
60	4.4001	0.2273	135.9917	0.0074	30.9068	0.0324
65	4.9783	0.2009	159.1191	0.0063	31.9626	0.0313
70	5.6325	0.1775	185.2857	0.0054	32.8958	0.0304
75	6.3727	0.1569	214.8911	0.0047	33.7205	0.0297
80	7.2102	0.1387	248.3868	0.0040	34.4495	0.0290
85	8.1577	0.1226	286.2844	0.0035	35.0939	0.0285
90	9.2297	0.1083	329.1624	0.0030	35.6633	0.0280
95	10.4426	0.0958	377.6750	0.0026	36.1667	0.0276
100	11.8149	0.0846	432.5630	0.0023	36.6115	0.0273

TABLAS DE INTERES. COMPOSICION DISCRETA

TASA DE INTERES = 2.75 POR CIENTO

N	SP	PS	SR	RS	PR	RP
1	1.0275	0.9732	1.0000	1.0000	0.9732	1.0275
2	1.0558	0.9472	2.0275	0.4932	1.9204	0.5207
3	1.0848	0.9218	3.0832	0.3243	2.8422	0.3518
4	1.1146	0.8972	4.1680	0.2399	3.7394	0.2674
5	1.1453	0.8731	5.2826	0.1893	4.6125	0.2168
6	1.1768	0.8498	6.4279	0.1556	5.4623	0.1831
7	1.2091	0.8270	7.6047	0.1315	6.2893	0.1590
8	1.2424	0.8049	8.8138	0.1135	7.0942	0.1410
9	1.2766	0.7834	10.0562	0.0994	7.8775	0.1269
10	1.3117	0.7624	11.3327	0.0882	8.6399	0.1157
11	1.3477	0.7420	12.6444	0.0791	9.3819	0.1066
12	1.3848	0.7221	13.9921	0.0715	10.1040	0.0990
13	1.4229	0.7028	15.3769	0.0650	10.8068	0.0925
14	1.4620	0.6840	16.7998	0.0595	11.4907	0.0870
15	1.5022	0.6657	18.2619	0.0548	12.1564	0.0823
16	1.5436	0.6479	19.7641	0.0506	12.8043	0.0781
17	1.5860	0.6305	21.3076	0.0469	13.4348	0.0744
18	1.6296	0.6136	22.8936	0.0437	14.0484	0.0712
19	1.6744	0.5972	24.5232	0.0408	14.6456	0.0683
20	1.7205	0.5812	26.1976	0.0382	15.2268	0.0657
21	1.7678	0.5657	27.9181	0.0358	15.7925	0.0633
22	1.8164	0.5505	29.6859	0.0337	16.3430	0.0612
23	1.8664	0.5358	31.5023	0.0317	16.8788	0.0592
24	1.9177	0.5215	33.3687	0.0300	17.4003	0.0575
25	1.9705	0.5075	35.2864	0.0283	17.9077	0.0558
26	2.0246	0.4939	37.2568	0.0268	18.4016	0.0543
27	2.0803	0.4807	39.2814	0.0255	18.8823	0.0530
28	2.1375	0.4678	41.3617	0.0242	19.3502	0.0517
29	2.1963	0.4553	43.4992	0.0230	19.8055	0.0505
30	2.2567	0.4431	45.6955	0.0219	20.2486	0.0494
35	2.5846	0.3869	57.6170	0.0174	22.2925	0.0449
40	2.9601	0.3378	71.2704	0.0140	24.0771	0.0415
45	3.3901	0.2950	86.9075	0.0115	25.6353	0.0390
50	3.8827	0.2576	104.8165	0.0095	26.9959	0.0370
55	4.4468	0.2249	125.3272	0.0080	28.1839	0.0355
60	5.0928	0.1964	148.8178	0.0067	29.2211	0.0342
65	5.8327	0.1714	175.7214	0.0057	30.1268	0.0332
70	6.6801	0.1497	206.5335	0.0048	30.9176	0.0323
75	7.6506	0.1307	241.8221	0.0041	31.6081	0.0316
80	8.7622	0.1141	282.2375	0.0035	32.2110	0.0310
85	10.0351	0.0996	328.5244	0.0030	32.7374	0.0305
90	11.4931	0.0870	381.5366	0.0026	33.1970	0.0301
95	13.1629	0.0760	442.2505	0.0023	33.5984	0.0298
100	15.0752	0.0663	511.7852	0.0020	33.9488	0.0295

TABLAS DE INTERES. COMPOSICION DISCRETA

TASA DE INTERES = 3.00 POR CIENTO

N	SP	PS	SR	RS	PR	RP
1	1.0300	0.9709	1.0000	1.0000	0.9708	1.0300
2	1.0609	0.9426	2.0299	0.4926	1.9134	0.5226
3	1.0927	0.9151	3.0908	0.3235	2.8285	0.3535
4	1.1255	0.8885	4.1835	0.2390	3.7169	0.2690
5	1.1593	0.8626	5.3089	0.1884	4.5795	0.2184
6	1.1941	0.8375	6.4682	0.1546	5.4169	0.1846
7	1.2299	0.8131	7.6622	0.1305	6.2300	0.1605
8	1.2668	0.7894	8.8920	0.1125	7.0194	0.1425
9	1.3048	0.7664	10.1587	0.0984	7.7858	0.1284
10	1.3439	0.7441	11.4635	0.0872	8.5298	0.1172
11	1.3843	0.7224	12.8073	0.0781	9.2522	0.1081
12	1.4258	0.7014	14.1915	0.0705	9.9535	0.1005
13	1.4686	0.6809	15.6173	0.0640	10.6344	0.0940
14	1.5126	0.6611	17.0857	0.0585	11.2955	0.0885
15	1.5580	0.6419	18.5983	0.0538	11.9373	0.0838
16	1.6047	0.6232	20.1562	0.0496	12.5605	0.0796
17	1.6529	0.6050	21.7609	0.0460	13.1655	0.0760
18	1.7025	0.5874	23.4137	0.0427	13.7528	0.0727
19	1.7535	0.5703	25.1161	0.0398	14.3231	0.0698
20	1.8062	0.5537	26.8696	0.0372	14.8767	0.0672
21	1.8603	0.5375	28.6757	0.0349	15.4142	0.0649
22	1.9162	0.5219	30.5359	0.0327	15.9361	0.0628
23	1.9736	0.5067	32.4520	0.0308	16.4427	0.0608
24	2.0328	0.4919	34.4256	0.0290	16.9346	0.0591
25	2.0938	0.4776	36.4583	0.0274	17.4122	0.0574
26	2.1567	0.4637	38.5520	0.0259	17.8759	0.0559
27	2.2214	0.4502	40.7086	0.0246	18.3260	0.0546
28	2.2880	0.4371	42.9299	0.0233	18.7631	0.0533
29	2.3566	0.4243	45.2178	0.0221	19.1874	0.0521
30	2.4273	0.4120	47.5743	0.0210	19.5993	0.0510
35	2.8140	0.3554	60.4609	0.0165	21.4860	0.0465
40	3.2622	0.3065	75.4002	0.0133	23.1134	0.0433
45	3.7818	0.2644	92.7189	0.0108	24.5172	0.0408
50	4.3842	0.2281	112.7961	0.0089	25.7281	0.0389
55	5.0825	0.1968	136.0712	0.0073	26.7727	0.0374
60	5.8920	0.1697	163.0536	0.0061	27.6737	0.0361
65	6.8305	0.1464	194.3340	0.0051	28.4510	0.0351
70	7.9184	0.1263	230.5964	0.0043	29.1214	0.0343
75	9.1797	0.1089	272.6348	0.0037	29.6998	0.0337
80	10.6418	0.0940	321.3691	0.0031	30.1986	0.0331
85	12.3369	0.0811	377.8660	0.0026	30.6290	0.0326
90	14.3019	0.0699	443.3613	0.0023	31.0002	0.0323
95	16.5799	0.0603	519.2888	0.0019	31.3204	0.0319
100	19.2208	0.0520	607.3108	0.0016	31.5966	0.0316

TABLAS DE INTERES. COMPOSICION DISCRETA

TASA DE INTERES = 3.50 POR CIENTO

N	SP	PS	SR	RS	PR	RP
1	1.0350	0.9662	1.0000	1.0000	0.9662	1.0350
2	1.0712	0.9335	2.0349	0.4914	1.8996	0.5264
3	1.1087	0.9019	3.1061	0.3219	2.8015	0.3569
4	1.1475	0.8714	4.2148	0.2373	3.6729	0.2723
5	1.1877	0.8420	5.3623	0.1865	4.5149	0.2215
6	1.2293	0.8135	6.5499	0.1527	5.3283	0.1877
7	1.2723	0.7860	7.7791	0.1285	6.1143	0.1636
8	1.3168	0.7594	9.0514	0.1105	6.8736	0.1455
9	1.3629	0.7337	10.3681	0.0964	7.6073	0.1315
10	1.4106	0.7089	11.7310	0.0852	8.3162	0.1202
11	1.4600	0.6849	13.1416	0.0761	9.0011	0.1111
12	1.5111	0.6618	14.6015	0.0685	9.6629	0.1035
13	1.5640	0.6394	16.1125	0.0621	10.3022	0.0971
14	1.6187	0.6178	17.6764	0.0566	10.9200	0.0916
15	1.6754	0.5969	19.2951	0.0518	11.5169	0.0868
16	1.7340	0.5767	20.9704	0.0477	12.0935	0.0827
17	1.7947	0.5572	22.7043	0.0440	12.6507	0.0790
18	1.8575	0.5383	24.4990	0.0408	13.1890	0.0758
19	1.9225	0.5201	26.3564	0.0379	13.7092	0.0729
20	1.9898	0.5026	28.2789	0.0354	14.2117	0.0704
21	2.0595	0.4856	30.2686	0.0330	14.6972	0.0680
22	2.1316	0.4691	32.3280	0.0309	15.1664	0.0659
23	2.2062	0.4533	34.4595	0.0290	15.6196	0.0640
24	2.2834	0.4379	36.6656	0.0273	16.0575	0.0623
25	2.3633	0.4231	38.9489	0.0257	16.4806	0.0607
26	2.4460	0.4088	41.3121	0.0242	16.8895	0.0592
27	2.5316	0.3950	43.7580	0.0229	17.2844	0.0579
28	2.6203	0.3816	46.2896	0.0216	17.6661	0.0566
29	2.7120	0.3687	48.9097	0.0204	18.0348	0.0554
30	2.8069	0.3563	51.6216	0.0194	18.3910	0.0544
35	3.3337	0.3000	66.6729	0.0150	19.9995	0.0500
40	3.9594	0.2526	84.5491	0.0118	21.3538	0.0468
45	4.7026	0.2126	105.7807	0.0095	22.4941	0.0445
50	5.5852	0.1790	130.9973	0.0076	23.4542	0.0426
55	6.6336	0.1507	160.9469	0.0062	24.2625	0.0412
60	7.8786	0.1269	196.5175	0.0051	24.9431	0.0401
65	9.3574	0.1069	238.7649	0.0042	25.5162	0.0392
70	11.1137	0.0900	288.9417	0.0035	25.9987	0.0385
75	13.1997	0.0758	348.5361	0.0029	26.4049	0.0379
80	15.6771	0.0638	419.3162	0.0024	26.7470	0.0374
85	18.6196	0.0537	503.3806	0.0020	27.0349	0.0370
90	22.1144	0.0452	603.2244	0.0017	27.2774	0.0367
95	26.2651	0.0381	721.8076	0.0014	27.4816	0.0364
100	31.1949	0.0321	862.6482	0.0012	27.6535	0.0362

TABLAS DE INTERES. COMPOSICION DISCRETA

TASA DE INTERES = 4.00 POR CIENTO

N	SP	PS	SR	RS	PR	RP
1	1.0400	0.9615	1.0000	1.0000	0.9615	1.0400
2	1.0816	0.9246	2.0399	0.4902	1.8860	0.5302
3	1.1249	0.8890	3.1215	0.3204	2.7750	0.3604
4	1.1699	0.8548	4.2463	0.2355	3.6298	0.2755
5	1.2167	0.8219	5.4161	0.1846	4.4516	0.2246
6	1.2653	0.7903	6.6327	0.1508	5.2419	0.1908
7	1.3159	0.7599	7.8980	0.1266	6.0018	0.1666
8	1.3686	0.7307	9.2139	0.1085	6.7325	0.1485
9	1.4233	0.7026	10.5825	0.0945	7.4350	0.1345
10	1.4803	0.6756	12.0057	0.0833	8.1105	0.1233
11	1.5395	0.6496	13.4859	0.0742	8.7601	0.1142
12	1.6011	0.6246	15.0254	0.0666	9.3847	0.1066
13	1.6651	0.6006	16.6263	0.0601	9.9852	0.1001
14	1.7317	0.5775	18.2914	0.0547	10.5626	0.0947
15	1.8010	0.5553	20.0230	0.0499	11.1179	0.0899
16	1.8730	0.5339	21.8239	0.0458	11.6518	0.0858
17	1.9479	0.5134	23.6969	0.0422	12.1651	0.0822
18	2.0259	0.4936	25.6448	0.0390	12.6587	0.0790
19	2.1069	0.4746	27.6705	0.0361	13.1333	0.0761
20	2.1912	0.4564	29.7774	0.0336	13.5897	0.0736
21	2.2788	0.4388	31.9684	0.0313	14.0285	0.0713
22	2.3700	0.4219	34.2472	0.0292	14.4504	0.0692
23	2.4648	0.4057	36.6171	0.0273	14.8561	0.0673
24	2.5634	0.3901	39.0818	0.0256	15.2462	0.0656
25	2.6659	0.3751	41.6451	0.0240	15.6213	0.0640
26	2.7726	0.3607	44.3109	0.0226	15.9820	0.0626
27	2.8835	0.3468	47.0833	0.0212	16.3287	0.0612
28	2.9988	0.3335	49.9667	0.0200	16.6622	0.0600
29	3.1188	0.3206	52.9654	0.0189	16.9828	0.0589
30	3.2435	0.3083	56.0840	0.0178	17.2911	0.0578
35	3.9463	0.2534	73.6514	0.0136	18.6636	0.0536
40	4.8013	0.2083	95.0248	0.0105	19.7917	0.0505
45	5.8415	0.1712	121.0291	0.0083	20.7188	0.0483
50	7.1071	0.1407	152.6674	0.0066	21.4809	0.0466
55	8.6470	0.1156	191.1606	0.0052	22.1073	0.0452
60	10.5204	0.0951	237.9936	0.0042	22.6221	0.0442
65	12.7998	0.0781	294.9734	0.0034	23.0452	0.0434
70	15.5730	0.0642	364.2986	0.0027	23.3930	0.0427
75	18.9470	0.0528	448.6438	0.0022	23.6789	0.0422
80	23.0521	0.0434	551.2632	0.0018	23.9138	0.0418
85	28.0465	0.0357	676.1160	0.0015	24.1069	0.0415
90	34.1231	0.0293	828.0200	0.0012	24.2657	0.0412
95	41.5163	0.0241	1012.8357	0.0010	24.3961	0.0410
100	50.5112	0.0198	1237.6929	0.0008	24.5034	0.0408

TABLAS DE INTERES. COMPOSICION DISCRETA

TASA DE INTERES = 4.50 POR CIENTO

N	SP	PS	SR	RS	PR	RP
1	1.0450	0.9569	1.0000	1.0000	0.9569	1.0450
2	1.0920	0.9157	2.0450	0.4890	1.8726	0.5340
3	1.1412	0.8763	3.1370	0.3188	2.7489	0.3638
4	1.1925	0.8386	4.2782	0.2337	3.5875	0.2787
5	1.2462	0.8024	5.4707	0.1828	4.3899	0.2278
6	1.3023	0.7679	6.7169	0.1489	5.1578	0.1939
7	1.3609	0.7348	8.0191	0.1247	5.8926	0.1697
8	1.4221	0.7032	9.3800	0.1066	6.5958	0.1516
9	1.4861	0.6729	10.8021	0.0926	7.2686	0.1376
10	1.5530	0.6439	12.2882	0.0814	7.9125	0.1264
11	1.6229	0.6162	13.8412	0.0722	8.5287	0.1173
12	1.6959	0.5896	15.4641	0.0647	9.1184	0.1097
13	1.7723	0.5643	17.1600	0.0583	9.6826	0.1033
14	1.8520	0.5400	18.9323	0.0528	10.2226	0.0978
15	1.9354	0.5167	20.7842	0.0481	10.7393	0.0931
16	2.0224	0.4944	22.7196	0.0440	11.2337	0.0890
17	2.1135	0.4732	24.7420	0.0404	11.7069	0.0854
18	2.2086	0.4528	26.8555	0.0372	12.1596	0.0822
19	2.3080	0.4333	29.0640	0.0344	12.5929	0.0794
20	2.4118	0.4146	31.3720	0.0319	13.0075	0.0769
21	2.5204	0.3968	33.7838	0.0296	13.4043	0.0746
22	2.6338	0.3797	36.3041	0.0275	13.7840	0.0725
23	2.7523	0.3633	38.9379	0.0257	14.1473	0.0707
24	2.8762	0.3477	41.6902	0.0240	14.4950	0.0690
25	3.0056	0.3327	44.5663	0.0224	14.8277	0.0674
26	3.1409	0.3184	47.5719	0.0210	15.1461	0.0660
27	3.2822	0.3047	50.7127	0.0197	15.4507	0.0647
28	3.4299	0.2916	53.9950	0.0185	15.7423	0.0635
29	3.5843	0.2790	57.4249	0.0174	16.0213	0.0624
30	3.7456	0.2670	61.0092	0.0164	16.2882	0.0614
35	4.6677	0.2142	81.5001	0.0123	17.4603	0.0573
40	5.8169	0.1719	107.0359	0.0093	18.4007	0.0543
45	7.2490	0.1379	138.8586	0.0072	19.1554	0.0522
50	9.0337	0.1107	178.5158	0.0056	19.7610	0.0506
55	11.2578	0.0888	227.9365	0.0044	20.2469	0.0494
60	14.0295	0.0713	289.5244	0.0035	20.6369	0.0485
65	17.4835	0.0572	366.2751	0.0027	20.9498	0.0477
70	21.7879	0.0459	461.9216	0.0022	21.2009	0.0472
75	27.1520	0.0368	581.1162	0.0017	21.4024	0.0467
80	33.8367	0.0296	729.6560	0.0014	21.5640	0.0464
85	42.1672	0.0237	914.7666	0.0011	21.6938	0.0461
90	52.5486	0.0190	1145.4492	0.0009	21.7979	0.0459
95	65.4859	0.0153	1432.9263	0.0007	21.8814	0.0457
100	81.6085	0.0123	1791.1809	0.0006	21.9485	0.0456

TABLAS DE INTERES. COMPOSICION DISCRETA

TASA DE INTERES = 5.00 POR CIENTO

N	SP	PS	SR	RS	PR	RP
1	1.0500	0.9524	1.0000	1.0000	0.9524	1.0500
2	1.1025	0.9070	2.0500	0.4878	1.8594	0.5378
3	1.1576	0.8638	3.1525	0.3172	2.7232	0.3672
4	1.2155	0.8227	4.3101	0.2320	3.5459	0.2820
5	1.2763	0.7835	5.5256	0.1810	4.3294	0.2310
6	1.3401	0.7462	6.8019	0.1470	5.0756	0.1970
7	1.4071	0.7107	8.1420	0.1228	5.7863	0.1728
8	1.4775	0.6768	9.5491	0.1047	6.4631	0.1547
9	1.5514	0.6446	11.0265	0.0907	7.1077	0.1407
10	1.6289	0.6139	12.5779	0.0795	7.7215	0.1295
11	1.7104	0.5847	14.2068	0.0704	8.3062	0.1204
12	1.7959	0.5568	15.9172	0.0628	8.8630	0.1128
13	1.8857	0.5303	17.7130	0.0565	9.3933	0.1065
14	1.9800	0.5051	19.5987	0.0510	9.8984	0.1010
15	2.0790	0.4810	21.5787	0.0463	10.3793	0.0963
16	2.1830	0.4581	23.6576	0.0423	10.8374	0.0923
17	2.2921	0.4363	25.8406	0.0387	11.2737	0.0887
18	2.4067	0.4155	28.1327	0.0355	11.6892	0.0855
19	2.5271	0.3957	30.5394	0.0327	12.0849	0.0827
20	2.6534	0.3769	33.0664	0.0302	12.4618	0.0802
21	2.7861	0.3589	35.7198	0.0280	12.8207	0.0780
22	2.9254	0.3418	38.5058	0.0260	13.1625	0.0760
23	3.0717	0.3256	41.4312	0.0241	13.4881	0.0741
24	3.2253	0.3101	44.5028	0.0225	13.7981	0.0725
25	3.3866	0.2953	47.7281	0.0210	14.0934	0.0710
26	3.5559	0.2812	51.1146	0.0196	14.3746	0.0696
27	3.7337	0.2678	54.6704	0.0183	14.6425	0.0683
28	3.9204	0.2551	58.4041	0.0171	14.8975	0.0671
29	4.1164	0.2429	62.3245	0.0160	15.1405	0.0660
30	4.3222	0.2314	66.4408	0.0151	15.3718	0.0651
35	5.5165	0.1813	90.3238	0.0111	16.3735	0.0611
40	7.0407	0.1420	120.8055	0.0083	17.1583	0.0583
45	8.9860	0.1113	159.7094	0.0063	17.7732	0.0563
50	11.4688	0.0872	209.3620	0.0048	18.2550	0.0548
55	14.6375	0.0683	272.7334	0.0037	18.6325	0.0537
60	18.6818	0.0535	353.6143	0.0028	18.9282	0.0528
65	23.8436	0.0419	456.8425	0.0022	19.1600	0.0522
70	30.4315	0.0329	588.5920	0.0017	19.3416	0.0517
75	38.8396	0.0257	756.7439	0.0013	19.4838	0.0513
80	49.5708	0.0202	971.3552	0.0010	19.5953	0.0510
85	63.2672	0.0158	1245.2639	0.0008	19.6826	0.0508
90	80.7476	0.0124	1594.8511	0.0006	19.7511	0.0506
95	103.0580	0.0097	2041.0295	0.0005	19.8047	0.0505
100	131.5325	0.0076	2610.4836	0.0004	19.8467	0.0504

TABLAS DE INTERES. COMPOSICION DISCRETA

TASA DE INTERES = 5.50 POR CIENTO

N	SP	PS	SR	RS	PR	RP
1	1.0550	0.9479	1.0000	1.0000	0.9479	1.0550
2	1.1130	0.8984	2.0550	0.4866	1.8463	0.5416
3	1.1743	0.8516	3.1680	0.3157	2.6979	0.3707
4	1.2388	0.8072	4.3422	0.2303	3.5051	0.2853
5	1.3070	0.7651	5.5811	0.1792	4.2702	0.2342
6	1.3789	0.7252	6.8880	0.1452	4.9954	0.2002
7	1.4547	0.6874	8.2669	0.1210	5.6829	0.1760
8	1.5347	0.6516	9.7216	0.1029	6.3344	0.1579
9	1.6191	0.6176	11.2563	0.0888	6.9520	0.1438
10	1.7082	0.5854	12.8754	0.0777	7.5375	0.1327
11	1.8021	0.5549	14.5836	0.0686	8.0923	0.1236
12	1.9013	0.5260	16.3857	0.0610	8.6183	0.1160
13	2.0058	0.4985	18.2869	0.0547	9.1168	0.1097
14	2.1162	0.4726	20.2928	0.0493	9.5894	0.1043
15	2.2326	0.4479	22.4089	0.0446	10.0373	0.0996
16	2.3554	0.4246	24.6414	0.0406	10.4619	0.0956
17	2.4849	0.4024	26.9968	0.0370	10.8643	0.0920
18	2.6216	0.3814	29.4817	0.0339	11.2457	0.0889
19	2.7658	0.3616	32.1033	0.0311	11.6073	0.0862
20	2.9179	0.3427	34.8690	0.0287	11.9500	0.0837
21	3.0784	0.3248	37.7869	0.0265	12.2748	0.0815
22	3.2477	0.3079	40.8653	0.0245	12.5827	0.0795
23	3.4264	0.2919	44.1130	0.0227	12.8746	0.0777
24	3.6148	0.2766	47.5393	0.0210	13.1512	0.0760
25	3.8136	0.2622	51.1541	0.0195	13.4134	0.0746
26	4.0234	0.2485	54.9677	0.0182	13.6620	0.0732
27	4.2447	0.2356	58.9911	0.0170	13.8976	0.0720
28	4.4782	0.2233	63.2357	0.0158	14.1209	0.0708
29	4.7245	0.2117	67.7138	0.0148	14.3325	0.0698
30	4.9844	0.2006	72.4383	0.0138	14.5331	0.0688
35	6.5144	0.1535	100.2563	0.0100	15.3899	0.0650
40	8.5142	0.1175	136.6137	0.0073	16.0454	0.0623
45	11.1279	0.0899	184.1321	0.0054	16.5469	0.0604
50	14.5439	0.0688	246.2376	0.0041	16.9306	0.0591
55	19.0085	0.0526	327.4080	0.0031	17.2243	0.0581
60	24.8437	0.0403	433.4958	0.0023	17.4489	0.0573
65	32.4702	0.0308	572.1499	0.0017	17.6208	0.0568
70	42.4378	0.0236	753.3684	0.0013	17.7523	0.0563
75	55.4653	0.0180	990.2166	0.0010	17.8529	0.0560
80	72.4918	0.0138	1299.7712	0.0008	17.9299	0.0558
85	94.7451	0.0106	1704.3525	0.0006	17.9888	0.0556
90	123.8298	0.0081	2233.1328	0.0004	18.0339	0.0555
95	161.8428	0.0062	2924.2354	0.0003	18.0684	0.0553
100	211.5248	0.0047	3827.4893	0.0003	18.0947	0.0553

TABLAS DE INTERES. COMPOSICION DISCRETA

TASA DE INTERES = 6.00 POR CIENTO

N	SP	PS	SR	RS	PR	RP
1	1.0600	0.9434	1.0000	1.0000	0.9434	1.0600
2	1.1236	0.8900	2.0600	0.4854	1.8334	0.5454
3	1.1910	0.8396	3.1836	0.3141	2.6730	0.3741
4	1.2625	0.7921	4.3746	0.2286	3.4651	0.2886
5	1.3382	0.7472	5.6371	0.1774	4.2123	0.2374
6	1.4185	0.7049	6.9753	0.1434	4.9172	0.2034
7	1.5037	0.6650	8.3938	0.1191	5.5823	0.1791
8	1.5939	0.6274	9.8975	0.1010	6.2097	0.1610
9	1.6895	0.5919	11.4913	0.0870	6.8015	0.1470
10	1.7909	0.5584	13.1808	0.0759	7.3599	0.1359
11	1.8984	0.5268	14.9717	0.0668	7.8867	0.1268
12	2.0123	0.4970	16.8700	0.0593	8.3836	0.1193
13	2.1330	0.4688	18.8823	0.0530	8.8524	0.1130
14	2.2610	0.4423	21.0153	0.0476	9.2947	0.1076
15	2.3967	0.4172	23.2763	0.0430	9.7120	0.1030
16	2.5405	0.3936	25.6729	0.0390	10.1056	0.0990
17	2.6929	0.3713	28.2133	0.0354	10.4769	0.0954
18	2.8545	0.3503	30.9062	0.0324	10.8273	0.0924
19	3.0258	0.3305	33.7607	0.0296	11.1577	0.0896
20	3.2073	0.3118	36.7864	0.0272	11.4695	0.0872
21	3.3998	0.2941	39.9937	0.0250	11.7637	0.0850
22	3.6038	0.2775	43.3934	0.0230	12.0412	0.0830
23	3.8200	0.2618	46.9971	0.0213	12.3029	0.0813
24	4.0492	0.2470	50.8170	0.0197	12.5499	0.0797
25	4.2922	0.2330	54.8662	0.0182	12.7829	0.0782
26	4.5497	0.2198	59.1583	0.0169	13.0027	0.0769
27	4.8227	0.2074	63.7080	0.0157	13.2100	0.0757
28	5.1121	0.1956	68.5306	0.0146	13.4056	0.0746
29	5.4188	0.1845	73.6427	0.0136	13.5902	0.0736
30	5.7440	0.1741	79.0614	0.0126	13.7643	0.0727
35	7.6868	0.1301	111.4406	0.0090	14.4976	0.0690
40	10.2869	0.0972	154.7718	0.0065	15.0456	0.0665
45	13.7663	0.0726	212.7595	0.0047	15.4551	0.0647
50	18.4227	0.0543	290.3608	0.0034	15.7611	0.0634
55	24.6541	0.0406	394.2112	0.0025	15.9897	0.0625
60	32.9932	0.0303	533.1875	0.0019	16.1606	0.0619
65	44.1529	0.0226	719.1726	0.0014	16.2882	0.0614
70	59.0874	0.0169	968.0654	0.0010	16.3836	0.0610
75	79.0734	0.0126	1301.1455	0.0008	16.4549	0.0608
80	105.8194	0.0095	1746.8865	0.0006	16.5082	0.0606
85	141.6123	0.0071	2343.3984	0.0004	16.5480	0.0604
90	189.5117	0.0053	3141.6753	0.0003	16.5777	0.0603
95	253.6131	0.0039	4209.9648	0.0002	16.6000	0.0602
100	339.3960	0.0029	5639.5977	0.0002	16.6166	0.0602

TABLAS DE INTERES. COMPOSICION DISCRETA

TASA DE INTERES = 6.50 POR CIENTO

N	SP	PS	SR	RS	PR	RP
1	1.0650	0.9390	1.0000	1.0000	0.9390	1.0650
2	1.1342	0.8817	2.0650	0.4843	1.8206	0.5493
3	1.2080	0.8278	3.1992	0.3126	2.6484	0.3776
4	1.2865	0.7773	4.4071	0.2269	3.4257	0.2919
5	1.3701	0.7299	5.6936	0.1756	4.1556	0.2406
6	1.4592	0.6853	7.0637	0.1416	4.8409	0.2066
7	1.5540	0.6435	8.5228	0.1173	5.4844	0.1823
8	1.6550	0.6042	10.0769	0.0992	6.0886	0.1642
9	1.7626	0.5673	11.7319	0.0852	6.6559	0.1502
10	1.8772	0.5327	13.4945	0.0741	7.1886	0.1391
11	1.9992	0.5002	15.3716	0.0651	7.6888	0.1301
12	2.1292	0.4697	17.3708	0.0576	8.1585	0.1226
13	2.2676	0.4410	19.4999	0.0513	8.5995	0.1163
14	2.4150	0.4141	21.7675	0.0459	9.0136	0.1109
15	2.5719	0.3888	24.1824	0.0414	9.4024	0.1064
16	2.7391	0.3651	26.7543	0.0374	9.7675	0.1024
17	2.9172	0.3428	29.4935	0.0339	10.1102	0.0989
18	3.1068	0.3219	32.4106	0.0309	10.4321	0.0959
19	3.3088	0.3022	35.5174	0.0282	10.7343	0.0932
20	3.5238	0.2838	38.8261	0.0258	11.0181	0.0908
21	3.7529	0.2665	42.3499	0.0236	11.2846	0.0886
22	3.9969	0.2502	46.1027	0.0217	11.5348	0.0867
23	4.2567	0.2349	50.0996	0.0200	11.7697	0.0850
24	4.5334	0.2206	54.3562	0.0184	11.9903	0.0834
25	4.8280	0.2071	58.8895	0.0170	12.1974	0.0820
26	5.1419	0.1945	63.7175	0.0157	12.3919	0.0807
27	5.4761	0.1826	68.8593	0.0145	12.5745	0.0795
28	5.8321	0.1715	74.3353	0.0135	12.7459	0.0785
29	6.2112	0.1610	80.1673	0.0125	12.9069	0.0775
30	6.6149	0.1512	86.3784	0.0116	13.0581	0.0766
35	9.0631	0.1103	124.0413	0.0081	13.6863	0.0731
40	12.4175	0.0805	175.6432	0.0057	14.1448	0.0707
45	17.0132	0.0588	246.3431	0.0041	14.4795	0.0691
50	23.3099	0.0429	343.2100	0.0029	14.7238	0.0679
55	31.9371	0.0313	475.9275	0.0021	14.9020	0.0671
60	43.7572	0.0229	657.7649	0.0015	15.0322	0.0665
65	59.9520	0.0167	906.9016	0.0011	15.1271	0.0661
70	82.1406	0.0122	1248.2441	0.0008	15.1964	0.0658
75	112.5413	0.0089	1715.9209	0.0006	15.2470	0.0656
80	154.1935	0.0065	2356.6870	0.0004	15.2840	0.0654
85	211.2614	0.0047	3234.6040	0.0003	15.3109	0.0653
90	289.4504	0.0035	4437.4414	0.0002	15.3306	0.0652
95	396.5776	0.0025	6085.4570	0.0002	15.3449	0.0652
100	543.3535	0.0018	8343.4180	0.0001	15.3554	0.0651

TABLAS DE INTERES. COMPOSICION DISCRETA

TASA DE INTERES = 7.00 POR CIENTO

N	SP	PS	SR	RS	PR	RP
1	1.0700	0.9346	1.0000	1.0000	0.9346	1.0700
2	1.1449	0.8734	2.0700	0.4831	1.8080	0.5531
3	1.2251	0.8163	3.2149	0.3111	2.6243	0.3811
4	1.3108	0.7629	4.4399	0.2252	3.3872	0.2952
5	1.4026	0.7130	5.7507	0.1739	4.1001	0.2439
6	1.5008	0.6663	7.1533	0.1398	4.7664	0.2098
7	1.6058	0.6227	8.6540	0.1156	5.3892	0.1856
8	1.7182	0.5820	10.2598	0.0975	5.9712	0.1675
9	1.8385	0.5439	11.9780	0.0835	6.5151	0.1535
10	1.9672	0.5083	13.8165	0.0724	7.0234	0.1424
11	2.1049	0.4751	15.7837	0.0634	7.4985	0.1334
12	2.2523	0.4440	17.8886	0.0559	7.9425	0.1259
13	2.4099	0.4149	20.1408	0.0497	8.3574	0.1197
14	2.5786	0.3878	22.5508	0.0443	8.7452	0.1143
15	2.7592	0.3624	25.1294	0.0398	9.1076	0.1098
16	2.9523	0.3387	27.8885	0.0359	9.4464	0.1059
17	3.1590	0.3166	30.8408	0.0324	9.7629	0.1024
18	3.3801	0.2958	33.9997	0.0294	10.0588	0.0994
19	3.6167	0.2765	37.3798	0.0268	10.3352	0.0968
20	3.8699	0.2584	40.9965	0.0244	10.5936	0.0944
21	4.1408	0.2415	44.8663	0.0223	10.8351	0.0923
22	4.4307	0.2257	49.0071	0.0204	11.0608	0.0904
23	4.7409	0.2109	53.4378	0.0187	11.2718	0.0887
24	5.0727	0.1971	58.1786	0.0172	11.4689	0.0872
25	5.4278	0.1842	63.2513	0.0158	11.6531	0.0858
26	5.8078	0.1722	68.6799	0.0146	11.8253	0.0846
27	6.2144	0.1609	74.4868	0.0134	11.9862	0.0834
28	6.6494	0.1504	80.7011	0.0124	12.1366	0.0824
29	7.1149	0.1406	87.3504	0.0114	12.2772	0.0815
30	7.6129	0.1314	94.4652	0.0106	12.4085	0.0806
35	10.6777	0.0937	138.2450	0.0072	12.9471	0.0772
40	14.9762	0.0668	199.6495	0.0050	13.3311	0.0750
45	21.0053	0.0476	285.7734	0.0035	13.6049	0.0735
50	29.4614	0.0339	406.5686	0.0025	13.8000	0.0725
55	41.3218	0.0242	575.9927	0.0017	13.9392	0.0717
60	57.9568	0.0173	813.6228	0.0012	14.0384	0.0712
65	81.2887	0.0123	1146.9163	0.0009	14.1092	0.0709
70	114.0132	0.0088	1614.3838	0.0006	14.1596	0.0706
75	159.9118	0.0063	2270.0405	0.0004	14.1956	0.0704
80	224.2879	0.0045	3189.6487	0.0003	14.2212	0.0703
85	314.5801	0.0032	4479.4609	0.0002	14.2395	0.0702
90	441.2212	0.0023	6288.5195	0.0002	14.2525	0.0702
95	618.8450	0.0016	8825.8594	0.0001	14.2618	0.0701
100	867.9756	0.0012	12384.6680	0.0001	14.2685	0.0701

TABLAS DE INTERES. COMPOSICION DISCRETA

TASA DE INTERES = 7.50 POR CIENTO

N	SP	PS	SR	RS	PR	RP
1	1.0750	0.9302	1.0000	1.0000	0.9302	1.0750
2	1.1556	0.8653	2.0750	0.4819	1.7955	0.5569
3	1.2423	0.8050	3.2306	0.3095	2.6005	0.3845
4	1.3355	0.7488	4.4729	0.2236	3.3493	0.2986
5	1.4356	0.6965	5.8083	0.1722	4.0458	0.2472
6	1.5433	0.6480	7.2440	0.1380	4.6937	0.2130
7	1.6591	0.6027	8.7873	0.1138	5.2965	0.1888
8	1.7835	0.5607	10.4463	0.0957	5.8572	0.1707
9	1.9173	0.5216	12.2298	0.0818	6.3787	0.1568
10	2.0611	0.4852	14.1471	0.0707	6.8639	0.1457
11	2.2157	0.4513	16.2081	0.0617	7.3152	0.1367
12	2.3819	0.4198	18.4238	0.0543	7.7350	0.1293
13	2.5605	0.3905	20.8056	0.0481	8.1256	0.1231
14	2.7526	0.3633	23.3661	0.0428	8.4889	0.1178
15	2.9590	0.3380	26.1186	0.0383	8.8268	0.1133
16	3.1809	0.3144	29.0776	0.0344	9.1412	0.1094
17	3.4195	0.2924	32.2585	0.0310	9.4336	0.1060
18	3.6760	0.2720	35.6780	0.0280	9.7057	0.1030
19	3.9517	0.2531	39.3539	0.0254	9.9587	0.1004
20	4.2481	0.2354	43.3055	0.0231	10.1941	0.0981
21	4.5667	0.2190	47.5536	0.0210	10.4131	0.0960
22	4.9092	0.2037	52.1202	0.0192	10.6168	0.0942
23	5.2774	0.1895	57.0294	0.0175	10.8063	0.0925
24	5.6733	0.1763	62.3068	0.0160	10.9825	0.0911
25	6.0988	0.1640	67.9800	0.0147	11.1465	0.0897
26	6.5562	0.1525	74.0786	0.0135	11.2990	0.0885
27	7.0479	0.1419	80.6347	0.0124	11.4409	0.0874
28	7.5766	0.1320	87.6826	0.0114	11.5729	0.0864
29	8.1448	0.1228	95.2590	0.0105	11.6957	0.0855
30	8.7557	0.1142	103.4038	0.0097	11.8099	0.0847
35	12.5701	0.0796	154.2599	0.0065	12.2719	0.0815
40	18.0463	0.0554	227.2715	0.0044	12.5938	0.0794
45	25.9082	0.0386	332.0906	0.0030	12.8180	0.0780
50	37.1951	0.0269	482.5745	0.0021	12.9742	0.0771
55	53.3991	0.0187	698.6160	0.0014	13.0829	0.0764
60	76.6624	0.0130	1008.7766	0.0010	13.1587	0.0760
65	110.0604	0.0091	1454.0591	0.0007	13.2115	0.0757
70	158.0081	0.0063	2093.3271	0.0005	13.2482	0.0755
75	226.8443	0.0044	3011.0923	0.0003	13.2738	0.0753
80	325.6689	0.0031	4328.6797	0.0002	13.2917	0.0752
85	467.5464	0.0021	6220.2773	0.0002	13.3041	0.0752
90	671.2332	0.0015	8935.9531	0.0001	13.3127	0.0751
95	963.6560	0.0010	12834.7109	0.0001	13.3188	0.0751
100	1383.4727	0.0007	18431.9609	0.0001	13.3230	0.0751

TABLAS DE INTERES. COMPOSICION DISCRETA

TASA DE INTERES = 8.00 POR CIENTO

N	SP	PS	SR	RS	PR	RP
1	1.0800	0.9259	1.0000	1.0000	0.9259	1.0800
2	1.1664	0.8573	2.0800	0.4808	1.7832	0.5608
3	1.2597	0.7938	3.2464	0.3080	2.5771	0.3880
4	1.3605	0.7350	4.5061	0.2219	3.3121	0.3019
5	1.4694	0.6806	5.8666	0.1705	3.9926	0.2505
6	1.5869	0.6302	7.3359	0.1363	4.6228	0.2163
7	1.7139	0.5835	8.9228	0.1121	5.2063	0.1921
8	1.8510	0.5403	10.6366	0.0940	5.7465	0.1740
9	1.9991	0.5002	12.4876	0.0801	6.2467	0.1601
10	2.1590	0.4632	14.4866	0.0690	6.7099	0.1490
11	2.3317	0.4289	16.6456	0.0601	7.1388	0.1401
12	2.5183	0.3971	18.9773	0.0527	7.5359	0.1327
13	2.7197	0.3677	21.4955	0.0465	7.9035	0.1265
14	2.9373	0.3404	24.2152	0.0413	8.2440	0.1213
15	3.1723	0.3152	27.1525	0.0368	8.5592	0.1168
16	3.4261	0.2919	30.3248	0.0330	8.8511	0.1130
17	3.7002	0.2703	33.7508	0.0296	9.1213	0.1096
18	3.9962	0.2502	37.4510	0.0267	9.3716	0.1067
19	4.3160	0.2317	41.4472	0.0241	9.6033	0.1041
20	4.6612	0.2145	45.7631	0.0219	9.8178	0.1019
21	5.0342	0.1986	50.4243	0.0198	10.0164	0.0998
22	5.4369	0.1839	55.4584	0.0180	10.2004	0.0980
23	5.8719	0.1703	60.8952	0.0164	10.3707	0.0964
24	6.3416	0.1577	66.7670	0.0150	10.5283	0.0950
25	6.8490	0.1460	73.1086	0.0137	10.6743	0.0937
26	7.3969	0.1352	79.9576	0.0125	10.8095	0.0925
27	7.9887	0.1252	87.3544	0.0114	10.9347	0.0915
28	8.6279	0.1159	95.3430	0.0105	11.0506	0.0905
29	9.3181	0.1073	103.9707	0.0096	11.1579	0.0896
30	10.0636	0.0994	113.2888	0.0088	11.2573	0.0888
35	14.7869	0.0676	172.3275	0.0058	11.6540	0.0858
40	21.7272	0.0460	259.0759	0.0039	11.9240	0.0839
45	31.9249	0.0313	386.5398	0.0026	12.1078	0.0826
50	46.9088	0.0213	573.8296	0.0017	12.2329	0.0817
55	68.9255	0.0145	849.0234	0.0012	12.3180	0.0812
60	101.2758	0.0099	1253.3799	0.0008	12.3759	0.0808
65	148.8096	0.0067	1847.5212	0.0005	12.4153	0.0805
70	218.6535	0.0046	2720.5239	0.0004	12.4422	0.0804
75	321.2783	0.0031	4003.2649	0.0002	12.4604	0.0803
80	472.0708	0.0021	5888.0703	0.0002	12.4729	0.0802
85	693.6375	0.0014	8657.5039	0.0001	12.4813	0.0801
90	1019.1970	0.0010	12726.7813	0.0001	12.4871	0.0801
95	1497.5576	0.0007	18705.9688	0.0001	12.4910	0.0801
100	2200.4377	0.0005	27491.5000	0.0000	12.4937	0.0800

TABLAS DE INTERES. COMPOSICION DISCRETA

TASA DE INTERES = 8.50 POR CIENTO

N	SP	PS	SR	RS	PR	RP
1	1.0850	0.9217	1.0000	1.0000	0.9216	1.0850
2	1.1772	0.8494	2.0850	0.4796	1.7711	0.5646
3	1.2773	0.7829	3.2622	0.3065	2.5540	0.3915
4	1.3859	0.7216	4.5395	0.2203	3.2755	0.3053
5	1.5037	0.6650	5.9254	0.1688	3.9406	0.2538
6	1.6315	0.6129	7.4290	0.1346	4.5535	0.2196
7	1.7702	0.5649	9.0605	0.1104	5.1184	0.1954
8	1.9207	0.5207	10.8306	0.0923	5.6390	0.1773
9	2.0839	0.4799	12.7513	0.0784	6.1189	0.1634
10	2.2611	0.4423	14.8352	0.0674	6.5612	0.1524
11	2.4533	0.4076	17.0962	0.0585	6.9688	0.1435
12	2.6618	0.3757	19.5494	0.0512	7.3445	0.1362
13	2.8881	0.3463	22.2112	0.0450	7.6907	0.1300
14	3.1335	0.3191	25.0992	0.0398	8.0098	0.1248
15	3.3999	0.2941	28.2327	0.0354	8.3040	0.1204
16	3.6889	0.2711	31.6326	0.0316	8.5750	0.1166
17	4.0025	0.2498	35.3214	0.0283	8.8249	0.1133
18	4.3427	0.2303	39.3239	0.0254	9.0552	0.1104
19	4.7118	0.2122	43.6665	0.0229	9.2674	0.1079
20	5.1124	0.1956	48.3783	0.0207	9.4630	0.1057
21	5.5469	0.1803	53.4906	0.0187	9.6433	0.1037
22	6.0185	0.1662	59.0375	0.0169	9.8094	0.1019
23	6.5300	0.1531	65.0559	0.0154	9.9626	0.1004
24	7.0851	0.1411	71.5858	0.0140	10.1037	0.0990
25	7.6874	0.1301	78.6709	0.0127	10.2338	0.0977
26	8.3408	0.1199	86.3582	0.0116	10.3537	0.0966
27	9.0498	0.1105	94.6989	0.0106	10.4642	0.0956
28	9.8191	0.1018	103.7486	0.0096	10.5660	0.0946
29	10.6538	0.0939	113.5676	0.0088	10.6599	0.0938
30	11.5594	0.0865	124.2212	0.0081	10.7464	0.0931
35	17.3816	0.0575	192.7143	0.0052	11.0873	0.0902
40	26.1363	0.0383	295.7061	0.0034	11.3140	0.0884
45	39.3007	0.0254	450.5728	0.0022	11.4648	0.0872
50	59.0957	0.0169	683.4429	0.0015	11.5650	0.0865
55	88.8611	0.0113	1033.6057	0.0010	11.6317	0.0860
60	133.6186	0.0075	1560.1375	0.0006	11.6760	0.0856
65	200.9198	0.0050	2351.8745	0.0004	11.7055	0.0854
70	302.1191	0.0033	3542.3926	0.0003	11.7252	0.0853
75	454.2908	0.0022	5332.5508	0.0002	11.7382	0.0852
80	683.1079	0.0015	8024.3789	0.0001	11.7469	0.0851
85	1027.1765	0.0010	12072.0313	0.0001	11.7526	0.0851
90	1544.5452	0.0006	18158.4023	0.0001	11.7565	0.0851
95	2322.5037	0.0004	27310.3750	0.0000	11.7590	0.0850
100	3492.3015	0.0003	41071.9844	0.0000	11.7607	0.0850

TABLAS DE INTERES. COMPOSICION DISCRETA

TASA DE INTERES = 9.00 POR CIENTO

N	SP	PS	SR	RS	PR	RP
1	1.0900	0.9174	1.0000	1.0000	0.9174	1.0900
2	1.1881	0.8417	2.0900	0.4785	1.7591	0.5685
3	1.2950	0.7722	3.2781	0.3051	2.5313	0.3951
4	1.4116	0.7084	4.5731	0.2187	3.2397	0.3087
5	1.5387	0.6499	5.9847	0.1671	3.8896	0.2571
6	1.6771	0.5963	7.5233	0.1329	4.4858	0.2229
7	1.8281	0.5470	9.2004	0.1087	5.0328	0.1987
8	1.9926	0.5019	11.0285	0.0907	5.5347	0.1807
9	2.1720	0.4604	13.0211	0.0768	5.9951	0.1668
10	2.3674	0.4224	15.1930	0.0658	6.4175	0.1558
11	2.5805	0.3875	17.5604	0.0569	6.8050	0.1470
12	2.8128	0.3555	20.1409	0.0497	7.1605	0.1397
13	3.0659	0.3262	22.9537	0.0436	7.4867	0.1336
14	3.3419	0.2992	26.0196	0.0384	7.7859	0.1284
15	3.6427	0.2745	29.3614	0.0341	8.0604	0.1241
16	3.9705	0.2519	33.0041	0.0303	8.3123	0.1203
17	4.3279	0.2311	36.9746	0.0270	8.5433	0.1171
18	4.7174	0.2120	41.3024	0.0242	8.7553	0.1142
19	5.1420	0.1945	46.0197	0.0217	8.9498	0.1117
20	5.6048	0.1784	51.1617	0.0195	9.1282	0.1096
21	6.1092	0.1637	56.7664	0.0176	9.2919	0.1076
22	6.6591	0.1502	62.8756	0.0159	9.4421	0.1059
23	7.2584	0.1378	69.5346	0.0144	9.5798	0.1044
24	7.9117	0.1264	76.7930	0.0130	9.7062	0.1030
25	8.6238	0.1160	84.7047	0.0118	9.8222	0.1018
26	9.4000	0.1064	93.3284	0.0107	9.9286	0.1007
27	10.2460	0.0976	102.7282	0.0097	10.0262	0.0997
28	11.1682	0.0895	112.9742	0.0089	10.1157	0.0989
29	12.1734	0.0821	124.1423	0.0081	10.1978	0.0981
30	13.2690	0.0754	136.3155	0.0073	10.2732	0.0973
35	20.4164	0.0490	215.7265	0.0046	10.5663	0.0946
40	31.4137	0.0318	337.9121	0.0030	10.7569	0.0930
45	48.3346	0.0207	525.9133	0.0019	10.8807	0.0919
50	74.3701	0.0134	815.1816	0.0012	10.9611	0.0912
55	114.4295	0.0087	1260.2634	0.0008	11.0134	0.0908
60	176.0671	0.0057	1945.0896	0.0005	11.0474	0.0905
65	270.9055	0.0037	2998.7961	0.0003	11.0695	0.0903
70	416.8289	0.0024	4620.0820	0.0002	11.0839	0.0902
75	641.3533	0.0016	7114.6680	0.0001	11.0932	0.0901
80	986.8188	0.0010	10952.9766	0.0001	11.0993	0.0901
85	1518.3691	0.0007	16858.7891	0.0001	11.1032	0.0901
90	2336.2383	0.0004	25945.7578	0.0000	11.1058	0.0900
95	3594.6514	0.0003	39927.4063	0.0000	11.1075	0.0900
100	5530.9102	0.0002	61440.2891	0.0000	11.1085	0.0900

TABLAS DE INTERES. COMPOSICION DISCRETA

TASA DE INTERES = 10.00 POR CIENTO

N	SP	PS	SR	RS	PR	RP
1	1.1000	0.9091	1.0000	1.0000	0.9091	1.1000
2	1.2100	0.8264	2.1000	0.4762	1.7355	0.5762
3	1.3310	0.7513	3.3100	0.3021	2.4868	0.4021
4	1.4641	0.6830	4.6410	0.2155	3.1698	0.3155
5	1.6105	0.6209	6.1051	0.1638	3.7907	0.2638
6	1.7716	0.5645	7.7156	0.1296	4.3552	0.2296
7	1.9488	0.5131	9.4872	0.1054	4.8683	0.2054
8	2.1436	0.4665	11.4359	0.0874	5.3348	0.1874
9	2.3580	0.4241	13.5795	0.0736	5.7589	0.1736
10	2.5938	0.3855	15.9375	0.0627	6.1444	0.1627
11	2.8532	0.3505	18.5313	0.0540	6.4949	0.1540
12	3.1386	0.3186	21.3845	0.0468	6.8135	0.1468
13	3.4524	0.2897	24.5230	0.0408	7.1031	0.1408
14	3.7977	0.2633	27.9754	0.0357	7.3665	0.1358
15	4.1775	0.2394	31.7730	0.0315	7.6058	0.1315
16	4.5952	0.2176	35.9505	0.0278	7.8235	0.1278
17	5.0548	0.1978	40.5456	0.0247	8.0213	0.1247
18	5.5603	0.1798	45.6003	0.0219	8.2011	0.1219
19	6.1163	0.1635	51.1605	0.0195	8.3646	0.1196
20	6.7280	0.1486	57.2768	0.0175	8.5133	0.1175
21	7.4008	0.1351	64.0046	0.0156	8.6484	0.1156
22	8.1409	0.1228	71.4053	0.0140	8.7712	0.1140
23	8.9550	0.1117	79.5461	0.0126	8.8829	0.1126
24	9.8505	0.1015	88.5010	0.0113	8.9844	0.1113
25	10.8356	0.0923	98.3514	0.0102	9.0767	0.1102
26	11.9192	0.0839	109.1869	0.0092	9.1606	0.1092
27	13.1112	0.0763	121.1060	0.0083	9.2369	0.1083
28	14.4223	0.0693	134.2170	0.0075	9.3062	0.1075
29	15.8646	0.0630	148.6392	0.0067	9.3692	0.1067
30	17.4511	0.0573	164.5035	0.0061	9.4265	0.1061
35	28.1057	0.0356	271.0439	0.0037	9.6437	0.1037
40	45.2653	0.0221	442.6316	0.0023	9.7786	0.1023
45	72.9014	0.0137	718.9797	0.0014	9.8624	0.1014
50	117.4105	0.0085	1164.0488	0.0009	9.9144	0.1009
55	189.0938	0.0053	1880.8481	0.0005	9.9466	0.1005
60	304.5425	0.0033	3035.2793	0.0003	9.9667	0.1003
65	490.4768	0.0020	4894.5313	0.0002	9.9791	0.1002
70	789.9309	0.0013	7888.9297	0.0001	9.9869	0.1001
75	1272.2129	0.0008	12711.5195	0.0001	9.9917	0.1001
80	2048.9458	0.0005	20478.4766	0.0000	9.9946	0.1001
85	3299.9041	0.0003	32987.4570	0.0000	9.9965	0.1000
90	5314.6172	0.0002	53133.6250	0.0000	9.9976	0.1000
95	8559.3789	0.0001	85579.6875	0.0000	9.9984	0.1000
100	13785.1953	0.0001	137835.3125	0.0000	9.9988	0.1000

TABLAS DE INTERES. COMPOSICION DISCRETA

TASA DE INTERES = 11.00 POR CIENTO

N	SP	PS	SR	RS	PR	RP
1	1.1100	0.9009	1.0000	1.0000	0.9009	1.1100
2	1.2321	0.8116	2.1100	0.4739	1.7125	0.5839
3	1.3676	0.7312	3.3421	0.2992	2.4437	0.4092
4	1.5181	0.6587	4.7097	0.2123	3.1024	0.3223
5	1.6851	0.5934	6.2278	0.1606	3.6958	0.2706
6	1.8705	0.5346	7.9129	0.1264	4.2305	0.2364
7	2.0762	0.4816	9.7833	0.1022	4.7121	0.2122
8	2.3046	0.4339	11.8595	0.0843	5.1460	0.1943
9	2.5581	0.3909	14.1641	0.0706	5.5369	0.1806
10	2.8395	0.3522	16.7222	0.0598	5.8891	0.1698
11	3.1519	0.3173	19.5617	0.0511	6.2063	0.1611
12	3.4986	0.2858	22.7135	0.0440	6.4922	0.1540
13	3.8835	0.2575	26.2121	0.0382	6.7497	0.1482
14	4.3107	0.2320	30.0955	0.0332	6.9817	0.1432
15	4.7848	0.2090	34.4061	0.0291	7.1906	0.1391
16	5.3112	0.1883	39.1909	0.0255	7.3789	0.1355
17	5.8955	0.1696	44.5021	0.0225	7.5485	0.1325
18	6.5440	0.1528	50.3975	0.0198	7.7014	0.1298
19	7.2638	0.1377	56.9414	0.0176	7.8390	0.1276
20	8.0629	0.1240	64.2052	0.0156	7.9630	0.1256
21	8.9493	0.1117	72.2680	0.0138	8.0748	0.1238
22	9.9344	0.1007	81.2177	0.0123	8.1754	0.1223
23	11.0272	0.0907	91.1520	0.0110	8.2661	0.1210
24	12.2402	0.0817	102.1791	0.0098	8.3478	0.1198
25	13.5867	0.0736	114.4192	0.0087	8.4214	0.1187
26	15.0813	0.0663	128.0057	0.0078	8.4877	0.1178
27	16.7403	0.0597	143.0869	0.0070	8.5475	0.1170
28	18.5817	0.0538	159.8268	0.0063	8.6013	0.1163
29	20.6258	0.0485	178.4084	0.0056	8.6498	0.1156
30	22.8947	0.0437	199.0341	0.0050	8.6934	0.1150
35	38.5797	0.0259	341.6179	0.0029	8.8549	0.1129
40	65.0101	0.0154	581.8840	0.0017	8.9507	0.1117
45	109.5478	0.0091	986.7537	0.0010	9.0075	0.1110
50	184.5977	0.0054	1668.9951	0.0006	9.0413	0.1106
55	311.0635	0.0032	2818.6316	0.0004	9.0613	0.1104
60	524.1687	0.0019	4755.8633	0.0002	9.0732	0.1102
65	883.2717	0.0011	8020.2891	0.0001	9.0802	0.1101
70	1488.3906	0.0007	13521.1211	0.0001	9.0844	0.1101
75	2508.0701	0.0004	22790.5156	0.0000	9.0869	0.1100
80	4226.3164	0.0002	38410.2344	0.0000	9.0883	0.1100
85	7121.7148	0.0001	64730.8516	0.0000	9.0892	0.1100
90	12000.7227	0.0001	109083.4375	0.0000	9.0897	0.1100
95	20222.2734	0.0000	183821.4375	0.0000	9.0900	0.1100
100	34076.3164	0.0000	309761.6250	0.0000	9.0902	0.1100

TABLAS DE INTERES. COMPOSICION DISCRETA

TASA DE INTERES = 12.00 POR CIENTO

N	SP	PS	SR	RS	PR	RP
1	1.1200	0.8929	1.0000	1.0000	0.8929	1.1200
2	1.2544	0.7972	2.1200	0.4717	1.6900	0.5917
3	1.4049	0.7118	3.3744	0.2963	2.4018	0.4164
4	1.5735	0.6355	4.7793	0.2092	3.0373	0.3292
5	1.7624	0.5674	6.3529	0.1574	3.6047	0.2774
6	1.9739	0.5066	8.1152	0.1232	4.1113	0.2432
7	2.2107	0.4523	10.0891	0.0991	4.5637	0.2191
8	2.4760	0.4039	12.2998	0.0813	4.9675	0.2013
9	2.7732	0.3606	14.7758	0.0677	5.3281	0.1877
10	3.1060	0.3220	17.5490	0.0570	5.6501	0.1770
11	3.4787	0.2875	20.6549	0.0484	5.9375	0.1684
12	3.8962	0.2567	24.1336	0.0414	6.1942	0.1614
13	4.3637	0.2292	28.0297	0.0357	6.4234	0.1557
14	4.8874	0.2046	32.3934	0.0309	6.6280	0.1509
15	5.4739	0.1827	37.2807	0.0268	6.8107	0.1468
16	6.1308	0.1631	42.7546	0.0234	6.9738	0.1434
17	6.8665	0.1456	48.8853	0.0205	7.1194	0.1405
18	7.6905	0.1300	55.7517	0.0179	7.2494	0.1379
19	8.6134	0.1161	63.4422	0.0158	7.3655	0.1358
20	9.6470	0.1037	72.0555	0.0139	7.4692	0.1339
21	10.8047	0.0926	81.7025	0.0122	7.5617	0.1322
22	12.1013	0.0826	92.5071	0.0108	7.6444	0.1308
23	13.5535	0.0738	104.6084	0.0096	7.7182	0.1296
24	15.1800	0.0659	118.1618	0.0085	7.7840	0.1285
25	17.0017	0.0588	133.3416	0.0075	7.8429	0.1275
26	19.0420	0.0525	150.3432	0.0067	7.8954	0.1267
27	21.3271	0.0469	169.3850	0.0059	7.9423	0.1259
28	23.8864	0.0419	190.7119	0.0052	7.9841	0.1252
29	26.7529	0.0374	214.5983	0.0047	8.0215	0.1247
30	29.9634	0.0334	241.3510	0.0041	8.0549	0.1241
35	52.8067	0.0189	431.7039	0.0023	8.1752	0.1223
40	93.0652	0.0107	767.1770	0.0013	8.2434	0.1213
45	164.0158	0.0061	1358.4070	0.0007	8.2822	0.1207
50	289.0574	0.0035	2400.3752	0.0004	8.3042	0.1204
55	509.4277	0.0020	4236.7148	0.0002	8.3166	0.1202
60	897.8022	0.0011	7473.0313	0.0001	8.3237	0.1201
65	1582.2654	0.0006	13176.6445	0.0001	8.3277	0.1201
70	2788.5454	0.0004	23228.5508	0.0000	8.3300	0.1200
75	4914.4609	0.0002	40943.7539	0.0000	8.3313	0.1200
80	8661.1250	0.0001	72164.5625	0.0000	8.3320	0.1200
85	15264.1602	0.0001	127187.5625	0.0000	8.3324	0.1200
90	26901.1719	0.0000	224158.5000	0.0000	8.3327	0.1200
95	47409.9805	0.0000	395057.9375	0.0000	8.3328	0.1200
100	83554.1250	0.0000	696246.2500	0.0000	8.3329	0.1200

TABLAS DE INTERES. COMPOSICION DISCRETA

TASA DE INTERES = 13.00 POR CIENTO

N	SP	PS	SR	RS	PR	RP
1	1.1300	0.8850	1.0000	1.0000	0.8849	1.1300
2	1.2769	0.7831	2.1300	0.4695	1.6681	0.5995
3	1.4429	0.6930	3.4069	0.2935	2.3611	0.4235
4	1.6305	0.6133	4.8498	0.2062	2.9744	0.3362
5	1.8425	0.5427	6.4803	0.1543	3.5172	0.2843
6	2.0820	0.4803	8.3228	0.1202	3.9975	0.2502
7	2.3527	0.4250	10.4048	0.0961	4.4225	0.2261
8	2.6585	0.3761	12.7574	0.0784	4.7987	0.2084
9	3.0042	0.3329	15.4159	0.0649	5.1315	0.1949
10	3.3947	0.2946	18.4201	0.0543	5.4261	0.1843
11	3.8360	0.2607	21.8148	0.0458	5.6868	0.1758
12	4.3347	0.2307	25.6508	0.0390	5.9175	0.1690
13	4.8983	0.2042	29.9855	0.0333	6.1216	0.1634
14	5.5351	0.1807	34.8837	0.0287	6.3023	0.1587
15	6.2547	0.1599	40.4188	0.0247	6.4622	0.1547
16	7.0678	0.1415	46.6734	0.0214	6.6037	0.1514
17	7.9866	0.1252	53.7412	0.0186	6.7289	0.1486
18	9.0249	0.1108	61.7278	0.0162	6.8397	0.1462
19	10.1982	0.0981	70.7527	0.0141	6.9377	0.1441
20	11.5240	0.0868	80.9509	0.0124	7.0245	0.1424
21	13.0222	0.0768	92.4748	0.0108	7.1013	0.1408
22	14.7152	0.0680	105.4970	0.0095	7.1693	0.1395
23	16.6282	0.0601	120.2121	0.0083	7.2294	0.1383
24	18.7899	0.0532	136.8402	0.0073	7.2826	0.1373
25	21.2327	0.0471	155.6300	0.0064	7.3297	0.1364
26	23.9931	0.0417	176.8627	0.0057	7.3714	0.1357
27	27.1123	0.0369	200.8557	0.0050	7.4083	0.1350
28	30.6370	0.0326	227.9678	0.0044	7.4409	0.1344
29	34.6200	0.0289	258.6045	0.0039	7.4698	0.1339
30	39.1207	0.0256	293.2244	0.0034	7.4954	0.1334
35	72.0789	0.0139	546.7385	0.0018	7.5853	0.1318
40	132.8035	0.0075	1013.8315	0.0010	7.6341	0.1310
45	244.6868	0.0041	1874.4375	0.0005	7.6606	0.1305
50	450.8289	0.0022	3460.0808	0.0003	7.6749	0.1303
55	830.6401	0.0012	6381.5859	0.0002	7.6827	0.1302
60	1530.4324	0.0007	11764.3828	0.0001	7.6870	0.1301
65	2819.7815	0.0004	21682.0469	0.0000	7.6893	0.1301
70	5195.3711	0.0002	39955.0703	0.0000	7.6905	0.1300
75	9572.3320	0.0001	73622.6250	0.0000	7.6912	0.1300
80	17636.7656	0.0001	135654.1875	0.0000	7.6916	0.1300
85	32495.2656	0.0000	249945.6875	0.0000	7.6918	0.1300
90	59871.6563	0.0000	460524.6875	0.0000	7.6919	0.1300
95	110311.8750	0.0000	848510.5625	0.0000	7.6919	0.1300
100	203246.7500	0.0000	1563365.0000	0.0000	7.6920	0.1300

TABLAS DE INTERES. COMPOSICION DISCRETA

TASA DE INTERES = 14.00 POR CIENTO

N	SP	PS	SR	RS	PR	RP
1	1.1400	0.8772	1.0000	1.0000	0.8772	1.1400
2	1.2996	0.7695	2.1400	0.4673	1.6466	0.6073
3	1.4816	0.6750	3.4396	0.2907	2.3216	0.4307
4	1.6890	0.5921	4.9212	0.2032	2.9137	0.3432
5	1.9255	0.5194	6.6101	0.1513	3.4330	0.2913
6	2.1950	0.4556	8.5356	0.1172	3.8886	0.2572
7	2.5023	0.3996	10.7306	0.0932	4.2882	0.2332
8	2.8527	0.3505	13.2329	0.0756	4.6388	0.2156
9	3.2521	0.3075	16.0856	0.0622	4.9463	0.2022
10	3.7074	0.2697	19.3376	0.0517	5.2160	0.1917
11	4.2264	0.2366	23.0450	0.0434	5.4526	0.1834
12	4.8181	0.2075	27.2714	0.0367	5.6601	0.1767
13	5.4927	0.1821	32.0895	0.0312	5.8422	0.1712
14	6.2617	0.1597	37.5822	0.0266	6.0019	0.1666
15	7.1384	0.1401	43.8439	0.0228	6.1420	0.1628
16	8.1378	0.1229	50.9823	0.0196	6.2649	0.1596
17	9.2771	0.1078	59.1200	0.0169	6.3727	0.1569
18	10.5760	0.0946	68.3971	0.0146	6.4672	0.1546
19	12.0567	0.0829	78.9730	0.0127	6.5502	0.1527
20	13.7446	0.0728	91.0296	0.0110	6.6229	0.1510
21	15.6690	0.0638	104.7742	0.0095	6.6867	0.1495
22	17.8627	0.0560	120.4430	0.0083	6.7427	0.1483
23	20.3635	0.0491	138.3056	0.0072	6.7918	0.1472
24	23.2145	0.0431	158.6691	0.0063	6.8349	0.1463
25	26.4647	0.0378	181.8834	0.0055	6.8727	0.1455
26	30.1699	0.0331	208.3480	0.0048	6.9058	0.1448
27	34.3938	0.0291	238.5177	0.0042	6.9349	0.1442
28	39.2091	0.0255	272.9114	0.0037	6.9604	0.1437
29	44.6985	0.0224	312.1201	0.0032	6.9828	0.1432
30	50.9566	0.0196	356.8186	0.0028	7.0024	0.1428
35	98.1145	0.0102	693.6482	0.0014	7.0698	0.1414
40	188.9151	0.0053	1342.1985	0.0007	7.1048	0.1408
45	363.7476	0.0027	2590.9526	0.0004	7.1229	0.1404
50	700.3796	0.0014	4995.3711	0.0002	7.1324	0.1402
55	1348.5488	0.0007	9624.9727	0.0001	7.1373	0.1401
60	2596.5708	0.0004	18539.0664	0.0001	7.1398	0.1401
65	4999.5742	0.0002	35702.7031	0.0000	7.1412	0.1400
70	9626.4570	0.0001	68750.5625	0.0000	7.1418	0.1400
75	18535.3086	0.0001	132382.6875	0.0000	7.1422	0.1400
80	35688.8984	0.0000	254903.5625	0.0000	7.1424	0.1400
85	68717.3750	0.0000	490812.0625	0.0000	7.1425	0.1400
90	132312.1250	0.0000	945042.5000	0.0000	7.1425	0.1400
95	254761.1250	0.0000	1819644.0000	0.0000	7.1425	0.1400
100	490531.0625	0.0000	3503649.0000	0.0000	7.1426	0.1400

TABLAS DE INTERES. COMPOSICION DISCRETA

TASA DE INTERES = 15.00 POR CIENTO

N	SP	PS	SR	RS	PR	RP
1	1.1500	0.8696	1.0000	1.0000	0.8696	1.1500
2	1.3225	0.7561	2.1500	0.4651	1.6257	0.6151
3	1.5209	0.6575	3.4725	0.2880	2.2832	0.4380
4	1.7490	0.5717	4.9934	0.2003	2.8549	0.3503
5	2.0114	0.4972	6.7424	0.1483	3.3521	0.2983
6	2.3131	0.4323	8.7538	0.1142	3.7844	0.2642
7	2.6601	0.3759	11.0669	0.0904	4.1603	0.2404
8	3.0591	0.3269	13.7270	0.0728	4.4872	0.2229
9	3.5180	0.2843	16.7861	0.0596	4.7715	0.2096
10	4.0457	0.2472	20.3041	0.0493	5.0187	0.1993
11	4.6526	0.2149	24.3498	0.0411	5.2336	0.1911
12	5.3505	0.1869	29.0024	0.0345	5.4205	0.1845
13	6.1531	0.1625	34.3529	0.0291	5.5830	0.1791
14	7.0761	0.1413	40.5060	0.0247	5.7243	0.1747
15	8.1376	0.1229	47.5821	0.0210	5.8472	0.1710
16	9.3583	0.1069	55.7197	0.0179	5.9541	0.1680
17	10.7621	0.0929	65.0779	0.0154	6.0470	0.1654
18	12.3764	0.0808	75.8399	0.0132	6.1278	0.1632
19	14.2329	0.0703	88.2163	0.0113	6.1980	0.1613
20	16.3680	0.0611	102.4491	0.0098	6.2591	0.1598
21	18.8232	0.0531	118.8170	0.0084	6.3123	0.1584
22	21.6468	0.0462	137.6402	0.0073	6.3585	0.1573
23	24.8939	0.0402	159.2869	0.0063	6.3986	0.1563
24	28.6281	0.0349	184.1807	0.0054	6.4336	0.1554
25	32.9225	0.0304	212.8088	0.0047	6.4639	0.1547
26	37.8611	0.0264	245.7311	0.0041	6.4903	0.1541
27	43.5404	0.0230	283.5920	0.0035	6.5133	0.1535
28	50.0717	0.0200	327.1321	0.0031	6.5333	0.1531
29	57.5827	0.0174	377.2036	0.0027	6.5506	0.1527
30	66.2204	0.0151	434.7859	0.0023	6.5657	0.1523
35	133.1957	0.0075	881.2712	0.0011	6.6164	0.1511
40	267.9097	0.0037	1779.3308	0.0006	6.6415	0.1506
45	538.8740	0.0019	3585.6919	0.0003	6.6540	0.1503
50	1083.8918	0.0009	7219.0039	0.0001	6.6603	0.1501
55	2180.1406	0.0005	14527.0547	0.0001	6.6634	0.1501
60	4385.1328	0.0002	29226.4531	0.0000	6.6649	0.1500
65	8820.2656	0.0001	58792.8906	0.0000	6.6657	0.1500
70	17741.0820	0.0001	118262.7500	0.0000	6.6660	0.1500
75	35684.4375	0.0000	237880.6250	0.0000	6.6662	0.1500
80	71775.6875	0.0000	478479.8750	0.0000	6.6663	0.1500
85	144369.6250	0.0000	962421.3125	0.0000	6.6664	0.1500
90	290385.2500	0.0000	1935822.0000	0.0000	6.6664	0.1500
95	584081.0625	0.0000	3893720.0000	0.0000	6.6664	0.1500
100	1174821.0000	0.0000	7831838.0000	0.0000	6.6664	0.1500

CURSO INTENSIVO SOBRE EVALUACION ECONOMICA Y METODOS PARA DECISION DE INVERSIONES EN LA INDUSTRIA MINERA

(Septiembre 10 al 21 de 1973)

ECONOMIA

MINERIA

Y

ECONOMIA MINERA

POR

ING. DAVID GOMEZ RUIZ

CENTRO DE EDUCACION CONTINUA, FACULTAD DE INGENIERIA
U. N. A. M.

México, D. F., septiembre de 1973

MINERIA, ECONOMIA, Y ECONOMIA MINERA

I.- INTRODUCCION

La Historia contemporánea nos enseña, que del suministro adecuado de metales, minerales, y energéticos, y del aprovechamiento racional de los mismos, depende el poderío económico y político de las naciones.

El objetivo esencial de estas líneas, es construir una perspectiva de conjunto o visión panorámica, de la Ingenieria Minera, la Economía, y la Economía Minera.

A todos nosotros nos interesa identificar, entender, y ayudar a resolver, mediante la ayuda de las tres disciplinas antes mencionadas, los problemas de carácter técnico, económico, político y social, que confronta la industria minero-metalúrgica nacional.

II.- INGENIERIA DE MINAS Y METALURGIA

La ingeniería es una combinación de ciencia y arte; mediante dicha combinación, los materiales y las fuerzas de la naturaleza se aprovechan en beneficio de la humanidad, en la forma más económica y eficiente posibles. Si queremos proporcionar empleo a todo el que lo busca, vivir en hogares confortables, estar libres de epidemias e inundaciones, disfrutar de transportación rápida, gozar de programas de radio y televisión; en fin, si aspiramos a constituir una comunidad civilizada, feliz, saludable y próspera, necesitamos del concurso de los hombres de ciencia y de los técnicos; quienes no podrán hacerlo todo, por supuesto, ya que se necesitará la cooperación de los demás profesionistas; pero sin el trabajo de los ingenieros, faltarían los fundamentos materiales requeridos, para alcanzar los objetivos antes mencionados, y el mundo soñado por los humanistas, se reduciría a un mundo de puras teorías y palabras.

El uso y aprovechamiento de metales, minerales, y energéticos a través de la historia, desde la Edad de Piedra hasta la actual Era Atómica, ha sido de importancia primordial para el desarrollo y progreso del género humano. En efecto, el cobre es la materia prima esencial para la industria eléctrica; el acero, para las industrias mecánica y de la construcción; el azufre para la química, y las fosforitas para la fabricación de fertilizantes. Además, petróleo y gas, uranio y carbón, oro y plata, plomo y zinc, manganeso y tungsteno, cromo y níquel, aluminio y magnesio, estaño y mercurio, y muchos otros metales y minerales, son materiales indispensables en la civilización moderna.

Por otra parte, y debido al aumento de población y al incremento de sus niveles de vida, la humanidad está consumiendo cantidades crecientes de metales, minerales y energéticos, que deberán ser localizados y extraídos por geólogos y por ingenieros de minas, respectivamente.

Es decir, la tarea o función primordial del ingeniero de minas y metalurgista consiste en "producir" suficientes materias primas de origen mineral, y convertirlas en materiales útiles para la humanidad, mediante el uso de métodos y procedimientos que arrojen máxima economía, seguridad, eficiencia y productividad.

Por consiguiente, las asignaturas básicas en la formación y en el ejercicio profesional del ingeniero de minas y metalurgista, son matemáticas, física y química; topografía, geología, explotación de minas, preparación mecánica de minerales, metalurgia, economía minera, y administración; y cada dia aumenta la aplicación a la minería, de la ingeniería industrial, investigación de operaciones, computación electrónica, y mecánica de rocas.

Según es del conocimiento de todos ustedes, el ingeniero de minas lleva los adelantos de la civilización hasta los lugares en donde se localicen los criaderos minerales, que por lo regular son regiones remotas e inhóspitas; y en dichas regiones apartadas, se crean empleos, se extraen las riquezas del subsuelo, y se convierten zonas improductivas y pobres, en verdaderos emporios de actividad económica.

III.- ECONOMIA

Según dijimos anteriormente, el ingeniero de minas y metalurgista enfoca principalmente su atención y actividades, hacia la producción de metales, minerales y energéticos; y el "economista" concentra su atención, en la producción, consumo, distribución y financiamiento de toda clase de "bienes y servicios", desde un punto de vista eminentemente social.

El hecho de que las necesidades humanas sean ilimitadas, y los recursos para satisfacerlas limitados, constituye la piedra angular de la ciencia económica. En efecto, el economista correlaciona los recursos de que dispone una empresa o una nación, con sus respectivas necesidades por satisfacer, a fin de optimizar el uso de dichos recursos. Es decir, el economista escoge el mejor entre todos los usos posibles de un recurso, y el ingeniero lo pone en acción.

El economista adquiere sus conocimientos académicos básicos, mediante el estudio concienzudo de las asignaturas siguientes:

1. Teoría económica.
2. Teoría de los Ciclos Económicos.
3. Historia del Pensamiento Económico, y Estudio de los Diversos Sistemas Económicos.
4. Comercio Internacional.

5. Teoría Monetaria y del Crédito.
6. Economía Industrial.
7. Métodos Cuantitativos Usados en la Investigación Económica.
8. Política Económica.
9. Teoría del Desarrollo Económico y Planificación.
10. Finanzas Públicas.

Demos un vistazo general, a algunas de estas disciplinas de la Ciencia Económica.

1. Teoría Económica.-

Comprende la micro y la macro economía.

La Microeconomía estudia y analiza concienzudamente, conceptos tales como demanda, oferta, precios, minimización de costos, maximización de utilidades, y estructura y funcionamiento de los mercados; y la Macroeconomía estudia la Contabilidad Social o Nacional, así como el Nivel General de precios, de empleo, y de ingresos, y la distribución de estos ingresos, entre los diversos factores de la producción.

Es decir, la Microeconomía estudia los problemas económicos a nivel de empresa, y sienta las bases económicas de la Administración de Empresas; y la Macroeconomía estudia los problemas económicos a nivel nacional, y sienta las bases económicas de la Administración Pública.

2. Teoría de los Ciclos Económicos.-

Describe y analiza las "fluctuaciones periódicas" que sufren los sistemas económicos capitalistas, fluctuaciones que se detectan mediante "indicadores económicos" de importancia estratégica. También deduce las causas que originan estos ciclos económicos, e indica la manera de predecirlos y la forma de controlarlos, mediante la aplicación de políticas monetarias y fiscales anticíclicas. Con fundamento en esta disciplina se elaboran los "pronósticos económicos".

3. Doctrinas y Sistemas Económicos.-

La "Historia del Pensamiento Económico" discute las doctrinas económicas sustentadas por los mas conspicuos economistas, entre los cuales citaremos a Platón, Aristóteles, Santo Tomás de Aquino, Adam Smith, John Stuart Mill, David Ricardo, Carlos Marx, John Maynard Keynes, y Joseph Schumpeter. En "Sistemas Económicos" se analizan la estructura, el funcionamiento, las ventajas y las desventajas de sistemas económico-políticos tales como el esclavismo, feudalismo, mercantilismo, liberalismo, capitalismo, imperialismo, socialismo, y comunismo.

4. Comercio Internacional.-

Comercio es el movimiento de mercancias de las manos de los productores a las de los consumidores. Este flujo de mercancias es tan importante como la producción de las mismas. En general, las naciones deben exportar aquellas mercancias que puedan producir con las mayores ventajas, e importar aquellas ótras que puedan producir con las mayores desventajas. Es decir, un pais debe concentrar sus energias y destinar sus recursos a la producción de bienes, en relación con los cuales tenga la mayor "ventaja comparativa". Procediendo en esta forma, el comercio internacional redundará en beneficio de todos los paises participantes, los cuales podrán obtener mayor provecho de sus recursos naturales y de su trabajo.

5. Métodos Cuantitativos de Investigación Económica.-

Comprenden métodos modernos y muy sofisticados de correlación estadística, econometría, modelos matemáticos, técnicas del insumo producto, investigación de operaciones, simulación con computadoras, y elaboración de pronósticos económicos.

6. Política Económica.-

Es el conjunto de principios y reglas que guian el pensamiento y la acción, de las personas responsables de alcanzar ciertos objetivos económicos prefijados. Toda política sirve de "guia", para poder pensar y actuar correctamente dentro de cierto campo de actividades, y en consonancia con los objetivos por alcanzar.

7. Desarrollo Económico y Planificación.-

La Teoría del Desarrollo Económico tiene como objetivo esencial, elaborar una estrategia eficaz que permita a los paises subdesarrollados, romper el círculo vicioso de su pobreza, e iniciar su despegue económico. Esto último será posible, cuando los paises subdesarrollados, que constituyen el 70% de la población mundial, aprendan a sacar el máximo provecho de todos los recursos naturales, humanos, de capital y tecnológicos, que se requieren para su desarrollo.

La Planificación Económica se está convirtiendo en el instrumento más eficaz, para promover el desarrollo económico de los paises subdesarrollados. En efecto, la Macroplanificación estima las inversiones requeridas en cada sector de la economía de un pais, con fundamento en su tasa deseada de crecimiento anual; y la Microplanificación estima las inversiones requeridas en cada proyecto específico.

El economista está mejor preparado para la macro-planificación y las finanzas públicas, y el ingeniero para la micro-planificación, y la ejecución de proyectos concretos y específicos.

IV.- ECONOMIA MINERA

La Economía Minera resultó de aplicar, a la resolución de los problemas económicos de la minería, los principios, conceptos, técnicas, y procedimientos desarrollados por los economistas. En efecto, podemos citar las siguientes aplicaciones prácticas de la Economía Minera, para resolver los problemas económicos de la Minería.

1. La "Teoría Económica" se utiliza para hacer el análisis de la demanda, de la oferta, y de los precios de los metales, minerales, y energéticos; y para estimar las tendencias futuras de dicha demanda, oferta y precios, tánto en el mercado nacional cómo en el mundial.

2. Debido a nuestra proximidad geográfica con el "Coloso del Norte", los ciclos económicos o movimientos ondulatorios de las actividades industriales norteamericanas, ocasionan notables expansiones o contracciones en nuestras actividades mineras. Por este motivo, conviene vigilar constantemente los "indicadores o barómetros económicos", que anuncian anticipadamente y en forma aproximada, los períodos de auge y de depresión del sistema económico norteamericano, y por ende, de nuestro propio sistema económico. Esta disciplina de los "Ciclos Económicos", es de importancia capital para poder elaborar "pronósticos económicos" y "políticas económicas".

3. Con fundamento en el estudio de las "Doctrinas y Sistemas Económicos", se pueden entender y analizar mejor, las normas que regulan las relaciones entre el Estado y la Industria Minero-Metalúrgica, ya que dichas normas dependen del tipo de régimen político-económico existente en cada pais.

4. En virtud de que los criaderos minerales no están repartidos uniformemente entre los diversos paises, el comercio internacional de metales, minerales y energéticos, remedia la escasez de los mismos, en los paises que padecen dicha escasez. Por otra parte, las exportaciones de metales, minerales y energéticos, han servido para financiar las primeras etapas del desarrollo económico, de los paises con abundantes recursos minerales. Sin embargo, sólo mediante su industrialización, y la integración vertical y horizontal, puede un pais obtener el máximo provecho de sus recursos naturales no renovables.

5. Con fundamento en los "Métodos Estadísticos" se recaban, analizan, e interpretan cifras numéricas, referentes a producción, consumo, precios, exportaciones e importaciones de metales y minerales. Estos datos estadísticos permiten extraer valiosas informaciones económicas, y estimar proyecciones futuras de dichas variables económicas. Entre

los métodos cuantitativos modernos, está aumentando el uso de modelos económicos, investigación de operaciones, y computación electrónica.

6. Con fundamento en la "Teoría del Desarrollo Económico" y en las técnicas de "Planificación", se establecen las políticas e inversiones más adecuadas, que permitan a una empresa y al pais en general, sacar el máximo provecho de la explotación de sus recursos naturales no renovables. Con tal fin, este Centro de Educación Continua y la Cámara Minera de México, han organizado el Curso Intensivo que hoy se inicia, y que se intitula: "Evaluación Económica y Métodos para Decisión de Inversiones en la Industria Minera".

V.- RESUMEN

Resumiendo, la finalidad práctica de la Tecnología Minera, es resolver los problemas técnicos relacionados con la exploración, explotación, y beneficio de metales y minerales. Los objetivos prácticos de la Ciencia Económica, son analizar la producción, consumo, distribución, y financiamiento de toda clase de "bienes y servicios", desde el punto de vista del bienestar social. La finalidad práctica de la Economía Minera, es atacar los problemas económicos relacionados con la producción, el consumo, la distribución local, y el consumo mundial de metales, minerales y energéticos, incluyendo las inversiones respectivas.

No debemos pasar por alto la importancia del "Derecho Minero", que emergiendo de la "Constitución", es el conjunto de normas jurídicas que regulan la propiedad y el aprovechamiento de los recursos naturales no renovables de un pais. Por otra parte, la "Política Minera" deberá sustentarse en el Derecho Minero, y deberá ser congruente con los principios de la tecnología y economía mineras.

VI.- CONCLUSIONES Y RECOMENDACIONES

1. La profunda preocupación de la Economía por el "bienestar de todo un grupo social", bienestar que depende en gran parte del uso adecuado de sus recursos, convierte a la Economía en una ciencia eminentemente social.

2. Por consiguiente, se recomienda el estudio de la Ciencia Económica, a todos aquellos ingenieros que tengan que tomar decisiones relacionadas con la planeación, desarrollo, uso y administración de recursos naturales, y que tengan que dar preferencia al bienestar de toda la comunidad.

3. La "Ingeniería" y la "Economía" tienen en común lo siguiente:

   a) Primeramente, ingenieros y economistas hablan con mucha frecuencia de maximizar, minimizar, y optimizar ciertas variables económicas. Por este motivo, ambos profesionistas deberán tener en común una sólida preparación matemática. En efecto, tanto en el campo de la ingeniería como en el de la economía, está aumentando el uso de la investigación de operaciones, programación lineal y dinámica, y simulación con computadoras.

b) En segundo lugar, economistas e ingenieros comparten su interés en el estudio de criterios para invertir, en el análisis de tasas de utilidad sobre la inversión, en el cálculo de "valor presente", y en la técnica del"flujo de caja".

c) En tercero y último lugar, economistas e ingenieros se enfrentan con mucha frecuencia, a eventos riesgosos o inciertos. Por este motivo, ambos profesionistas deberán estudiar "Métodos Estadísticos", "Teoría de las Probabilidades", y "Procesos Estocásticos".

Terminaré diciendo, que una adecuada "Política Minera", inspi rada y basada en nuestra Constitución, y congruente con los principios cambiantes y dinámicos de la "Tecnología y Economía Mineras", permitirá al Estado y a las empresas privadas, obtener el "mayor bien para el mayor número", de los recursos naturales no renovables de nuestro Pais.

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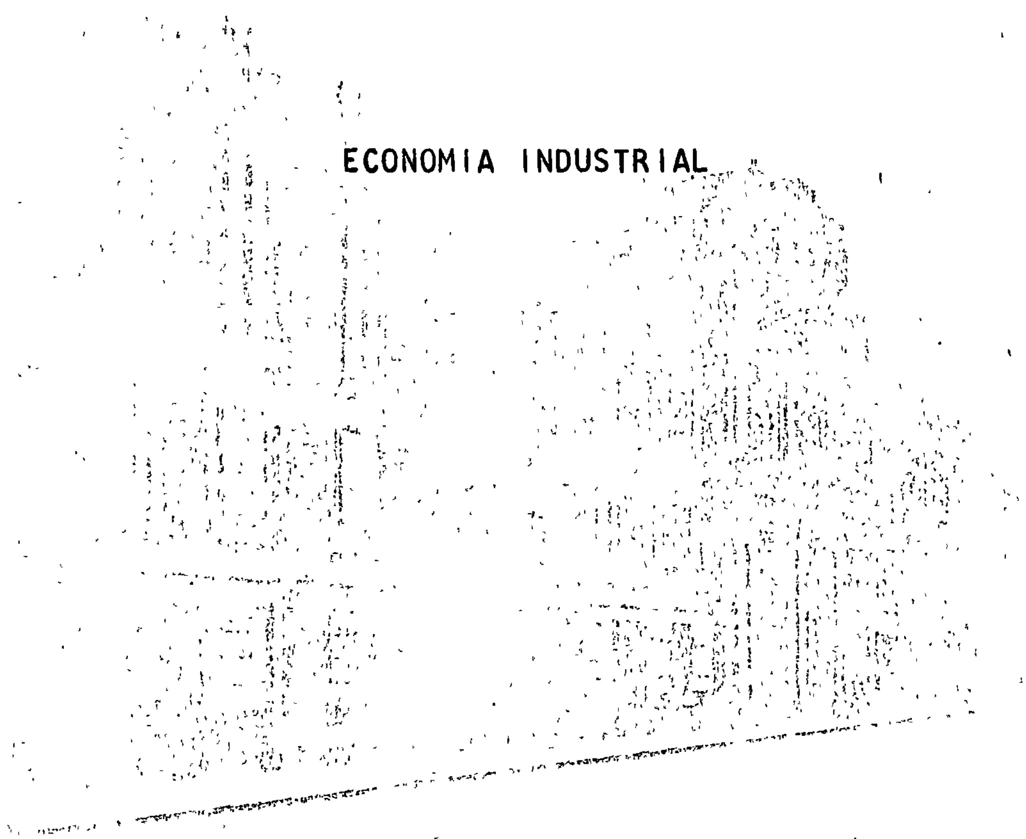
EVALUACION ECONOMICA Y METODOS PARA DECISION DE INVERSIONES DE LA INDUSTRIA MINERA

ECONOMIA INDUSTRIAL



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ECONOMIA INDUSTRIAL.

Por: Lic. Alberto Sepúlveda Coria.

Despertar la inquietud sobre este aspecto fundamental de la Ciencia Económica, es el objetivo de este trabajo.

Todos sabemos que el país se ha fijado como meta, dentro de su política de desarrollo, la industrialización; para lograrla, se requiere que las industrias nacionales operen en la forma más eficiente posible, lo que puede alcanzarse aplicando los conocimientos de esta rama de la economía.

En virtud de que este campo es sumamente amplio, en esta ocasión sólo se abordarán los aspectos que se juzgaron más sobresalientes, los cuales se mencionan a continuación:

Definición. Factores de la Producción. Estudio de Mercado. Localización Industrial. Inversión y Financiamiento. Organización y Control Industrial.

En relación con lo primeramente citado, debe indicarse que: "Economía Industrial es la ciencia de las leyes que rigen la producción y el intercambio de los bienes y servicios producidos por la industria".

Ahora bien, ese enunciado no sería lo suficientemente explícito, si no se señala lo que se entiende por industria, que se le define como "La actividad sistemática del hombre cuya finalidad es obtener bienes y servicios para satisfacer las necesidades de la colectividad.

FACTORES DE LA PRODUCCION.

Una de las funciones más importantes dentro de una empresa consiste en agrupar y organizar cada uno de los componentes que intervienen en el proceso de la producción, los cuales son los siguientes:

I) Elementos naturales.

II) Trabajo.

III) Capital.

IV) Organización.

En el primer factor se incluye la tierra y sus elementos, - tanto los que se encuentran en la superficie como en el subsuelo; - los animales y las plantas; el agua con los depósitos y corrientes que forma; el aire; la luz, el calor del sol; el clima y otros.

El trabajo es la aplicación de la fuerza del hombre por medio de la cual transforma y adapta los objetos de la naturaleza incorporándoles utilidad.

De acuerdo con la clase de actividad que se realiza, el trabajo se puede clasificar en la forma siguiente:

1) Intelectual. Es aquél en que el hombre pone en práctica su inteligencia o conjunto de conocimientos.

2) Corporal. El que requiere del empleo de la fuerza física.

3) Inventivo. Es el que crea nuevos bienes.

4) Imitativo. Se realiza tomando como base los bienes ya elaborados por el trabajo inventivo.

5) Calificado. El que se apoya en técnicas avanzadas y que por lo tanto exige un cierto grado de especialización.

6) No Calificado. Es el que utiliza procedimientos muy elementales para su ejecución.

Con respecto al capital, está constituído por la maquinaria, equipo, muebles, enseres, terrenos, edificios, etc., y se divide en fijo y circulante.

La organización es una función derivada de la actividad coordinadora de la Dirección o Gerencia y de la que se hablará con más detalle en capítulo por separado.

ESTUDIO DE MERCADO.

Se realiza con el propósito de "estimar la cuantía de los bienes o servicios provenientes de una nueva unidad de producción que la comunidad estaría dispuesta a adquirir a determinados precios". 1/

1/ Naciones Unidas, Manual de Proyectos de Desarrollo Económico. Op. Cit., Pág. 18.

El estudio debe de satisfacer las siguientes preguntas: 2/

A) ¿El producto está en aptitud de llegar físicamente al consumidor?

B) ¿Existen consumidores potenciales con capacidad de -- compra para la cantidad y calidad de la producción planeada?

C) ¿Hay razones suficientes para suponer la preferencia del consumidor por el producto planeado y poca sustituibilidad o desventaja, por parte de otros artículos de uso semejante?

El estudio de mercado está integrado por dos etapas sucesivas y complementarias. La primera, puede llamársele "preliminar"- en el cual se determina e individualiza el producto deseado, se -- cuantifica su demanda y se analizan las condiciones de competencia.

Este trabajo es rápido y relativamente superficial, ya que cuando se trata de estudios calificados, todo movimiento se traduce en un gasto que bien puede no ser justificable, y que en esta primera etapa que, aunque ligera, debe de ser suficiente para mostrarnos con claridad las posibilidades y limitaciones de la producción de un artículo, en forma que pueda el capitalista decidir si le conviene - o no entrar al negocio, e invertir en él cantidades mayores para investigaciones calificadas y profundas como son los llamados estudios "definitivos".

2/ Tratado de Economía Industrial, Gral. Antonio Rojas García. Manuales Universitarios. U.N.A.M. 1964.

En el Preliminar, se analizan básicamente los siguientes renglones:

1. Determinación del producto.

Es indispensable tener conciencia clara y precisa de las características que tiene el artículo que se quiere producir, para lo cual se puede recurrir tanto a un análisis de la competencia, a revistas especializadas, o al consejo técnico, si se trata de un producto ya conocido en el mercado.

En caso de ser una innovación, la asesoría de un técnico especialista es indispensable para determinar las especificaciones o características que definen e individualizan al producto y conocer con ello los fines precisos a que se destina. También se deberá de investigar quienes lo usan, cómo, y la distribución geográfica de su consumo. A la vez, el proceso de industrialización trae consigo el establecimiento de normas técnicas de calidad, las cuales también deben de ser consideradas en el estudio.

2. Cuantificación global de la demanda.

Es elemental cuando se quiere establecer una empresa, el saber la cantidad del artículo que se producirá y que el mercado puede absorber, tanto para cuantificar los recursos financieros necesarios, como para determinar el tamaño apropiado de la planta y la capacidad aprovechable.

Este cálculo se puede lograr, recopilando estadísticas sobre producción, importaciones, exportaciones y cambios en las existencias que nos permitan determinar la cuantía del mercado por medio del concepto llamado "consumo aparente" 1/. Estos datos deben de ir acompañados de series de precios que comprendan las cotizaciones de los tres niveles más importantes en los cuales se realizan las transacciones, que son:

A) En el origen (L.A.B. planta)

B) Con el distribuidor mayorista, y

c) El precio en el mercado consumidor.

Es por medio de este cálculo por el cual se aprecia si la magnitud de la demanda justifica o no una nueva empresa y si ésta será de una capacidad aconsejable.

Sin embargo, no hay que perder nunca de vista que la cuantificación de la demanda actual, así como el conocimiento de las condiciones en que ésta se satisface, sólo muestran la situación y potencia actual del mercado y que el elemento realmente importante para la futura empresa productora es el análisis del dinamismo de la demanda, así como los niveles que en lo futuro alcanzará la misma, y a los cuales se dirigirá la nueva producción.

3. Empresas competitivas.

Es aconsejable el informarse en la forma más veraz posi-

1/ Ver Manual de Proyectos de Desarrollo Económico. Pág. 20.

ble, de las condiciones de operación de los negocios con los que se va a competir (costos, capacidades, aprovechamiento, etc.), así como también de sus redes de distribución y comercialización del producto, dado que ello nos marca la pauta probable que ha de seguir nuestra empresa.

4. Precios probables.

El conocimiento de ellos, es quizá uno de los elementos más valiosos con que se cuenta cuando queremos pronosticar el éxito o fracaso de una empresa.

El problema de los precios reviste varios aspectos ya sea que se trate de un artículo conocido en el mercado, o de un producto nuevo. A la vez, el precio como indicador económico tiene una doble función: una nos indica la cantidad máxima que obtendremos por un artículo y a la cual debemos ajustar nuestros cálculos si queremos obtener utilidad y ventas; la otra función del precio es la de representar, por el contrario, la cantidad mínima por la cual podremos vender nuestro artículo en el mercado.

La combinación de ambos conceptos, indicará el punto óptimo en relación con el mercado y con la empresa. Es decir, la producción de competencia, requiere precios competitivos si se trata de un artículo existente en el mercado. Cuando el producto es nuevo, se necesita el consejo de un experto para poder determinar su precio, ya que un gran número de productos originales han sido lamentablemente devaluados por sus creadores, en tanto que a otros les han puesto precios tan exagerados, que les han cerrado magníficos mercados en perspectiva.

5. Tendencias sociales y económicas.

Siempre que se proyecte una nueva empresa, se deberá de tomar en cuenta las tendencias y el dinamismo económico de la sociedad, especialmente referidas al sector y a la región directamente involucrados. Es así como podría ser más aconsejable el instalar una industria en una población chica, pero con un gran dinamismo económico, que en otra mayor pero que se encuentra ya estancada en su proceso de desarrollo, máxime en países como México en los cuales es relativamente más sencillo establecerse en la provincia que en el Distrito Federal.

6. Métodos de comercialización.

Este es un aspecto de cuyo análisis a priori, generalmente se olvidan los nuevos industriales, originándose por tal causa serias consecuencias en su liquidez y en su capital de trabajo, lo cual ha causado ya innumerables fracasos.

Por ello es recomendable el estudiar previamente las probables redes de distribución al mayoreo y al menudeo, que incluye comisionistas, vendedores, representantes, almacenajes, conservación, entrega, abastecimientos, existencias, etc. También el aspecto fi

nanciero de las ventas es aconsejable conocer, ya sea que se reali-
cen al contado, a crédito o por intercambio.

Una vez conocidos todos los renglones anotados y si resultan favorables para invertir en un negocio, se procede a efectuar el Estudio Definitivo del Mercado, el cual se compone básicamente de los mismos renglones que el "preliminar", sólo que analizados con mayor profundidad y detalle, debiendo proporcionar los criterios para determinar la capacidad productiva que se ha de instalar en la nueva unidad, los probables niveles de ingreso durante la vida útil del proyecto, así como las necesidades financieras del mismo.

LOCALIZACION INDUSTRIAL.

En general, han sido muchos los pensadores que se han ocupado del tema, pero es hasta la aparición de la obra de Weber intitulada "La Industria Manufacturera en el Sistema Económico", cuando se ponen los cimientos de la teoría moderna de la localización.

En la actualidad, podemos considerar que esa teoría es el soporte técnico de la economía industrial desde un punto de vista económico.

Weber agrupa a los factores de localización en dos tipos, los primarios que incluye facilidades de transporte, ubicación de las materias primas, energéticos, así como mercados; y los secundarios que comprende mano de obra y los denominados de aglomeración y desaglomeración.

Para determinar el lugar exacto de la localización, Weber recurrió a lo que llamó "figura localizadora", que puede ser un triángulo o un polígono de fuerzas en equilibrio, formado por las distancias entre los lugares en donde se localizan las materias primas, energéticos y los mercados.

El problema concreto es determinar en un plano las coordenadas del punto en que se cruzan tres vectores concurrentes que están en equilibrio y cuyas líneas de acción deben pasar forzosamente por tres puntos fijos.

Principales Factores Localizadores.

Se encontrará con frecuencia que la posibilidad técnica de un proyecto depende mucho de su localización, pues suelen existir diferencias considerables en las disponibilidades, calidad y costo de los medios productivos, según la ubicación de la planta, Hay proyectos para los que se han tenido en cuenta todo y que, sin embargo, fracasan por que se establecen en un sitio distinto de donde se hizo el estudio y en el que las condiciones son menos favorables.

La elección del lugar preciso donde ha de iniciarse una industria, está influída por numerosos factores; algunos de ellos son de carácter puramente subjetivo, otros actúan por simple inercia y otros más por efectos del azar. Sin embargo, un análisis cuidadoso del conjunto de fuerzas que determinan la localización de la industria, revela que continuamente sobre el ánimo del empresario, están actuando una serie de consideraciones de carácter económico que son, en la mayor parte de los casos, la base de la elección. Todo esto tiende a reflejarse en diferencias de costos y precios, condicionando por tanto, la rentabilidad de la inversión.

Hay industrias atadas de modo tan manifiesto a un lugar dado y sujetas a la acción de fuerzas tan potentes, que cualquiera otra influencia de carácter fiscal o de cualquier otro tipo, será virtualmente inoperante. Pero hay otro grupo de industrias en el cual las ventajas relativas de operar en uno o en otro lugar son más bien pequeñas; pudiendo operar prácticamente con igual economía en distintos lugares. Es éste, el grupo de industrias llamadas "móviles", 1/ y son ellas las que ofrecen un campo más propicio para cualquier instrumento de política que pretende modificar la localización existente.

El problema de la localización se suele abordar en dos etapas: en la primera, se decide la zona general en que se instalará la empresa y en la segunda, se elige el punto preciso, considerando ya todos los elementos en detalle. Estos elementos son comúnmente llamados "factores locacionales".

Para citar algunos ejemplos de localización, debe señalarse que las industrias que se ubican cerca de las materias primas lo --

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1/ Cristóbal Lara Beautel. Investigación Económica. Vol. XI. "La movilidad de las Industrias Mexicanas".

hacen porque en la producción hay pérdida considerable de peso a -- causa de desperdicio.

Operaciones como la fundición de metales y la transformación inicial de la mayoría de las materias primas, -pertenecen a esta categoría-, ya que GRAN PARTE ~~la mayoría~~ de ellas tal como se extraen, tienen una gran proporción de desperdicio que resulta económico suprimir antes de transportarlas.

A la inversa, la orientación de las industrias hacia los mercados, obedece a una ganancia de peso en la elaboración, debido a que los costos de transporte por Ton/Km. de productos acabados, son más altos que los de materias primas. Ejemplo de esto se tiene en la - manufactura de ácido sulfúrico, fábricas de hierro y elaboración de refrescos.

INVERSION Y FINANCIAMIENTO.

Si se analiza someramente el programa de inversión de la empresa en operación o del proyecto en estudio, puede decirse con cierta seguridad el grado de meticulosidad con que opera la primera o ha sido proyectada la segunda. La elaboración de este programa sirve - para diferenciar a la empresa cuyo futuro se presenta halagador (en el caso de tener un programa de inversión bien estructurado) y por - el contrario podrá apreciarse la incertidumbre con que se operará -- una empresa al no contar con ese valiosísimo instrumento financiero.

Es decir, para llevar a cabo correctamente un proyecto de - inversión, es necesario establecer los montos de capital disponible y a la vez el destino que se dará al mismo.

El problema de financiamiento de una empresa y los montos de capital que se requieran, están en función directa del tamaño de la planta y de la capacidad aprovechada; es decir, no sería útil ni -- aconsejable realizar todos los estudios tendientes a la instalación de una unidad productora, si no se cuenta con los suficientes recursos para llevar a cabo su construcción.

En lo tocante al equilibrio que deben de guardar los fondos provenientes de fuentes internas y externas, se tendrá que estudiar en el caso de estas últimas, la capacidad de pago de la empresa para evitar caer en la insolvencia.

Considerando las fuentes internas, es importante calcular - la rentabilidad que la utilización de capitales propios tendrá con la operación de la planta, ya que ésta, en el caso de las inversiones lucrativas, norma principalmente la decisión de llevar a cabo o no, los proyectos industriales estudiados.

En resumen, el programa de financiamiento del proyecto debe de describir con claridad las fuentes y montos de capital a los cuales se tiene acceso, puesto que gran parte de los estudios que sobre promoción se realizan y que no llegan a cristalizarse en empresas productoras, se debe a la falta de financiamiento o previsión - en sus fuentes de capital.

Los recursos para el financiamiento de una empresa, provienen generalmente de dos fuentes:

1. Internas, que en el caso de un proyecto se circunscriben a la aportación que de capital propio hagan los accionistas o propietarios de la misma y cuando se trata de una empresa en operación, podemos añadir a esta fuente, los renglones de utilidades no distribuídas y reservas de capital.

2. Externas, constituídas por el mercado de capitales y -- por el sistema bancario.

Por lo regular, las fuentes internas de capital se utilizan para inversiones cuya rotación sea a corto o mediano plazo y las externas, (acciones y bonos), se dejen para necesidades de capital a largo plazo. Las características que estos créditos pueden adquirir, serán siempre distintas y acordes con los elementos que individualicen a cada proyecto.

Sobre las posibilidades de financiamiento, quisiéramos apuntar aquí otras fuentes que comúnmente se subestiman por desconocimiento - o por apatía. Estas son: a) el inmenso potencial económico de que - dispondrían países como México si lograran canalizar en forma productiva, parte del gasto superfluo y atesoramiento de las clases adineradas, y b) fuentes extranjeras.

Respecto a lo indicado en primer término, se comparte la opinión con Hirschman 1/ al señalar que en países como México "el desarrollo se ve frenado principalmente por el problema de canalizar los ahorros existentes o potenciales hacia las oportunidades productivas disponibles". Este hecho se puede observar fácilmente en la llamada "paradoja bancaria", que nos muestra cómo por un lado, se da una falta angustiosa de recursos crediticios para impulsar la industria y demás actividades productivas, y por otro lado, existe una plétora de recursos crediticios existentes en los bancos de depósito.

A lo anterior, debe agregarse que en los países subdesarrollados, bastaría que las clases con altos ingresos renunciaran tan sólo a una parte de sus consumos excesivos, para que la inversión mejorara sustancialmente, acelerándose con ello el ritmo de crecimiento económico, sin sacrificar los consumos de las clases con ingresos medianos y bajos que, en muchos casos se debaten en verdaderos límites de subsistencia.

Entre las fuentes extranjeras de financiamiento a las cuales han recurrido empresas importantes del país, pueden citarse; al Banco Interamericano de Desarrollo, Banco Internacional de Reconstrucción y Fomento, Banco Mundial y otros.

1/ Albert O. Hirschman. La Estrategia del Desarrollo Económico. F.C.E. 1961.

ORGANIZACION.

Es una función derivada de la actividad coordinadora de la - Dirección o Gerencia, cuyo propósito es disponer, proponer o colocar sistemática y racionalmente, los factores productivos de manera que cada uno, en forma armónica, se combine buscando los mayores beneficios económicos o el mayor aprovechamiento.

La organización se basa en la división del trabajo, la especialización y la simplificación.

Dadas las múltiples funciones propias de una empresa, es necesario separar los trabajos o tareas a desarrollar, así se tiene - que una sola persona no puede llevar al cabo con eficiencia las labores de fabricación, compras, ventas, suministros, cobros.

Debido a lo anterior, en la empresa se distribuye el trabajo entre varios colaboradores.

Cuando se trata de firmas que deben manufacturar artículos que requieren ciertos conocimientos técnicos, es obligado recurrir a especialistas (Ingenieros Químicos, Contadores, etc.) A lo anterior puede denominársele especialización.

La simplificación tiene lugar o hace su aparición en la industria, cuando ésta ha crecido o se ha desarrollado en forma tal - que las secciones o departamentos tienen que fusionarse en otras - secciones o departamentos.

Ahora bien, para formar o establecer la estructura en la organización, se requiere:

1o. Clasificar o separar las actividades, para las que sean homogéneas, agruparlas en secciones o departamentos análogos.

2o. Delegar autoridad a cada uno de los miembros de las secciones y departamentos, lo cual implica el señalamiento de sus deberes y responsabilidades.

3o. Establecer los niveles de autoridad de los diversos componentes de la organización, los cuales se determinan en función de los deberes y de las responsabilidades de cada una de las personas que intervienen en la empresa.

4o. El paso siguiente, consiste en determinar las líneas de comunicación de cada una de las secciones internamente, y de todas las secciones entre sí y las líneas de comunicación entre los diferentes individuos de la organización.

Las comunicaciones en una empresa deben seguir determinando cauces y regirse por el principio de autoridad. Esto es necesario para evitar fricciones entre los miembros componentes del NEGOCIO ~~la empresa~~.

Las líneas de comunicación tienen por objeto, transmitir de los niveles superiores de autoridad a los niveles inferiores: las discusiones, los planes a desarrollar y cualquier orden o instrucción concreta.

Cada empresa debe buscar su propia organización y diseñarla conforme a sus condiciones específicas. Los tipos de organización más conocidos y difundidos son: lineal o militar, funcional, capataz funcional y de línea y plana mayor.

De estos 4 tipos, el más sencillo que se aplica a empresas pequeñas donde el proceso es muy simple, es el lineal y el más complicado es el de linea y plana mayor que es el que aplican los complejos industriales o las industrias integradas. Para citar dos ejemplos, debe señalarse las fábricas de automóviles y las que producen máquinas de escribir y calculadoras.

Es lógico suponer que a medida que una empresa crece en su tamaño y se diversifica, obligadamente tiene que revisar el tipo de organización para adecuarla a sus nuevas necesidades. De ahí que ésta sea dinámica y los gerentes o directores tengan que estar pendientes de que el tipo sea el más adecuado, con el propósito de obtener el máximo beneficio de los factores productivos.

CONTROL INDUSTRIAL.

En la dirección de una empresa, el control organizado de la producción es necesario para conseguir el éxito, los métodos implantados deberán ser funcionales y adaptarse de manera definida a la fábrica de que se trate.

El control industrial es la técnica necesaria para poner en marcha los planes de producción, observando y registrando al mismo tiempo los progresos alcanzados, de manera que se mantenga una comparación contínua entre lo planeado y los resultados reales.

Es así como todas las empresas tienen que recurrir a los elementos de control general de sus movimientos, los cuales constituyen un valioso auxiliar para su labor de supervisión; los de mayor importancia son: el análisis de los estados financieros, la determinación del punto de equilibrio y el análisis factorial.

Análisis de Estados Financieros.- Se puede decir que es el mejor indicador de la marcha de la empresa, ya que muestra las características y limitaciones del negocio.

Aunque la sola cifra de utilidad que arrojan dichos documentos, nos puede dar una idea aproximada de la situación general de la empresa, se deben de tomar en cuenta los demás conceptos considerados en estos estados, formando con ellos relaciones aritméticas que revelen todas las fases y enfoques del desarrollo del negocio. Estas relaciones se conocen con el nombre de "razones financieras" y deben elaborarse en los distintos niveles económicos, para poder conocer no sólo la situación interna de la empresa, sino también la relación que guarda su actividad con los diversos grupos productivos de una sociedad.

Cuando se utilicen razones financieras, debe tenerse presente que éstas al igual que los estados financieros, son únicamente parte de la información contable de la empresa y que no representan soluciones por sí mismas.

Los estados financieros más importantes para toda empresa, son los siguientes:

Balance General, Estado de Pérdidas y Ganancias, Origen y aplicación de recursos, Costos de producción y Estado de Movimiento de Fondos.

Los estados financieros cumplen mejor su objetivo de información cuando son entendibles fácilmente.

Punto de Equilibrio.- Otro instrumento de gran utilidad para el control y la comprensión de los problemas de la empresa, es el análisis de la gráfica del punto de equilibrio.

Esta gráfica puede mostrarnos en sus ejes cartesianos, diferentes elementos representativos de la situación de la empresa. Entre los elementos que pueden representarse, están los montos correspondientes a: ingresos brutos, costos totales, ventas, capacidad de la planta.

La aplicación del punto de equilibrio a varias secciones de una empresa es de gran importancia en los negocios industriales que tengan fábricas en diferentes lugares y deseen controlar por separado cada unidad productora.

Además, en una fábrica pueden hacerse diagramas del punto de equilibrio económico para los diferentes artículos que se elaboran, lo que permitirá fijar en un momento dado, la contribución de cada producto en las pérdidas o ganancias de la unidad productora.

Cuando se desea hacer una gráfica para un período futuro, las cantidades que forman los ingresos y los gastos para estos fines, deben obtenerse de un presupuesto de ingresos y egresos. Asimismo, para efectos de proyección, el análisis de la gráfica del punto de equilibrio es de gran utilidad, puesto que refleja claramente las variaciones que experimentará el presupuesto y el costo unitario de producción al cambiar el porcentaje de aprovechamiento de la capacidad instalada de la planta.

En síntesis, se puede decir que los elementos integrantes de este instrumento de control, son:

a) La producción o las ventas.
b) Los costos fijos.
c) Los costos variables.

Este método nos permite determinar con exactitud el volumen de producción, costos y ventas en el cual la empresa obtiene su punto de equilibrio, o sea, el nivel en donde los ingresos son iguales a los costos.

Análisis Factorial.- Las deficiencias en la operación de -- una empresa pueden ser ocasionadas por diferentes causas, entre las que se pueden citar las siguientes: insuficiente financiamiento, -- procesos de producción inadecuados, ventas reducidas, defectos en el control financiero o contable de las operaciones, así como las influencias adversas que sobre la empresa ejerce el medio ambiente.

Por lo tanto, el análisis factorial consiste en determinar las fallas y lograr el control de la empresa, mediante el estudio - de todos aquellos aspectos que son de vital importancia en el funcionamiento de la misma y a los que se les llama "Factores de Operación".

Todos ellos son considerados como los principales constituyentes de las operaciones de la empresa y hacia ellos debe de concentrar su atención la administración general de la misma.

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TONNAGE-GRADE ESTIMATION FOR MINERAL DEPOSITS AND THE ASSESSMENT OF ORE RESERVES

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INTRODUCTION

The objective of this paper is to review statistical and economic principles relating to the estimation of tonnage and grade for mineral deposits and the assessment of ore reserves. Emphasis is placed on developing a broad appreciation of the topic in terms of information requirements, techniques of analysis, and application of results. Each mineral deposit is geologically unique and, thus, has special requirements for estimating tonnage and grade parameters. This paper focuses on fundamental concepts which are generally applicable rather than on detailed practice for specific deposit conditions.*

Mine development and mine operating decision problems differ from those of other industrial situations because of the conditions imposed by the geological environment, particularly the mineral deposit conditions. The fixed size of mineral deposits results, inevitably, in their exhaustion and effects the determination of mine capacity. Variations in grade within mineral deposits give rise to cut-off grade decisions. These mineral deposit conditions are determined by the estimation of tonnage and grade and the assessment of ore reserves.

DEFINITION OF ORE RESERVES

Standard practices have been developed for the definition of ore and the classification of ore reserves. "Ore" is usually defined as a natural aggregation of minerals, the tonnage and grade of which make extraction commercially profitable. The classification of ore reserves -- typically into "proven", "probable", and "possible" categories -- is based on the amount and type of sampling which is carried out and on the distances over which sample values are projected. Such traditional definitions have been the subject of much debate for many years. This reflects the difficulty of developing standards which are simple, practical, and generally applicable over a wide range of mineral deposit conditions.

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* A large number of case studies have been published, including those in the proceedings volume of the CIM conference on ore reserve estimation and grade control (15).

The definition of ore reserves for estimating purposes should embody two distinct dimensions -- a geological dimension reflecting the level and reliability of sampling information, and an economic dimension reflecting the relationship which exists between cost and revenue. A tonnage-grade classification for mineral deposits based on these properties is shown in Figure 1. *

The level of sampling information is a function of the amount of exploration and development work which has been completed. The unknown part of the mineral deposit exists but has yet to be discovered. The known part of the deposit, including past production, has been discovered and at least partially delineated. Tonnage and grade parameters for the known part of the deposit may be estimated with differing degrees of reliability depending on the level of sampling information.

The economic dimension subdivides the mineral deposit into economic and uneconomic categories based on the relationship between cost and revenue in different parts of the deposit. The economic margin is reflected in cut-off grade which is (at least conceptually) determined by the equation of marginal cost and marginal revenue.**

Thus, ore reserves are defined as the known, economic, unexploited part of the mineral deposit. Tonnage and grade estimates for ore reserves are classified -- for example, into proven, probable, and possible categories -- to reflect differing reliability and confidence of estimates resulting from differing levels of sampling information.

A tonnage-grade classification as shown in Figure 1 is fixed for only a particular point in time. Changing geological and economic conditions over time shift the ore reserve boundaries. Mine production exhausts ore reserves. Exploration results in the discovery and delineation of unknown economic resources and the "proving-up" of known ore reserves.

Changes in revenue result from short-term fluctuations and long-term trends in supply-demand-price conditions for the mineral commodities produced. Changes in cost occur as the mine moves from development into

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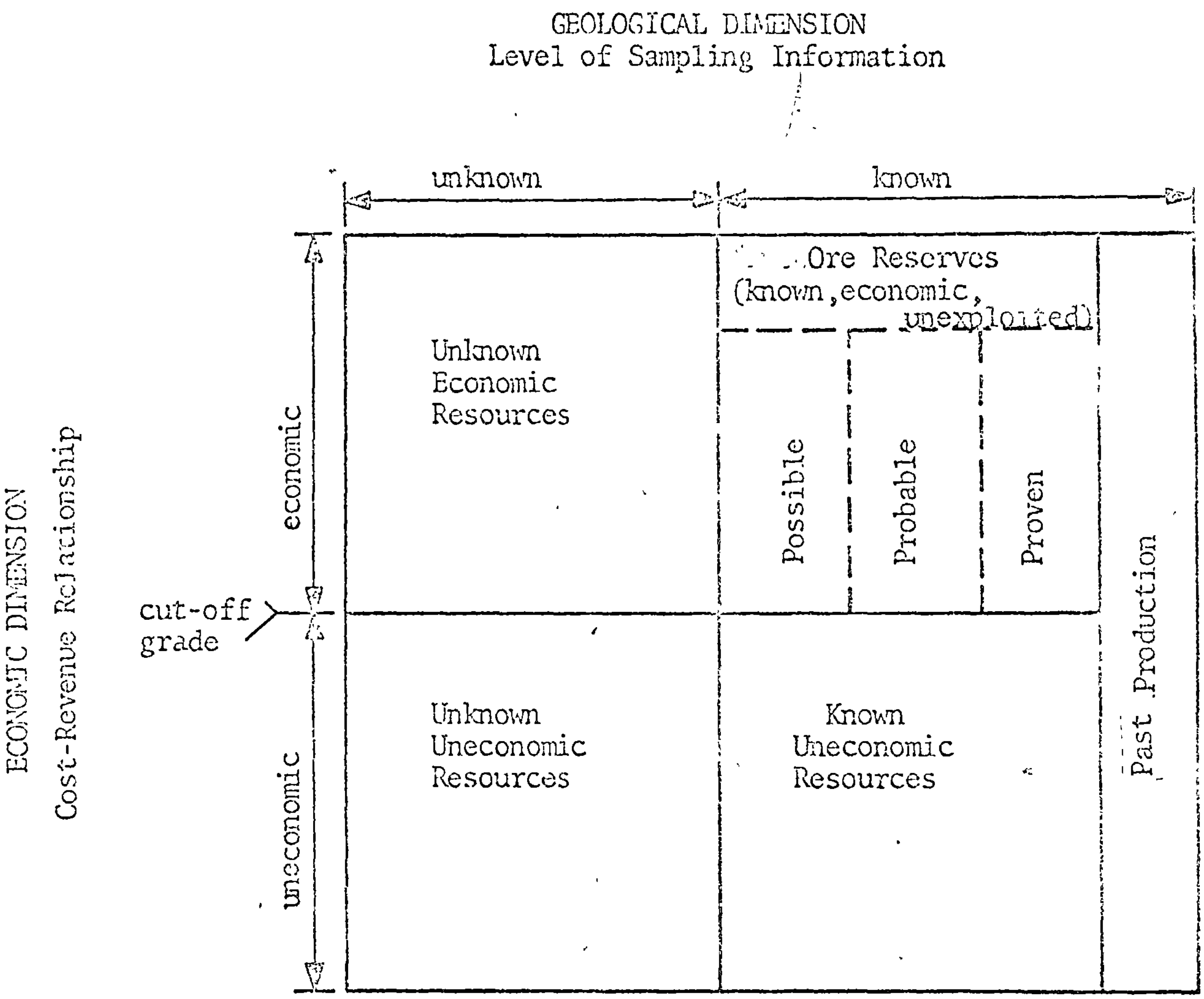
* Comprehensive treatments of this subject are given by Brooks (2) and Zwartendyk (14).

** Practically, due to uncertainties and changing economic conditions over time, cut-off grade is not usually directly determinable, particularly for mine development decision purposes.

Figure 1

MINERAL DEPOSIT TONNAGE-GRADE CLASSIFICATION



production, and with advances in mining and processing technology, changes in productivity, depth, and depletion. Favourable changes in the cost-revenue relationship reduce cut-off grade and thereby convert known uneconomic resources to ore reserves.

Favourable changes in both geological and economic conditions are required for the conversion of unknown uneconomic resources to ore reserves.

Ore reserve definition must be dynamic, reflecting the cumulative effect of changes in geological and economic conditions over time. The two-dimensional concept of ore reserves outlined above provides guidelines for the estimation of ore reserves.

Ore reserve estimates are required to assist with the analysis of mine development and mine operating decisions. Such decisions concern choice of the optimum economic alternative from a number of technically feasible alternatives. To determine the optimum economic alternative, each technically feasible alternative must be evaluated. Decision parameters usually include selection of the optimum cut-off grade and ore reserves, and, therefore, ore reserve estimates are required for each alternative considered. These estimates are facilitated if, initially, overall geological estimates of tonnage and grade for known resources are made. These geological estimates establish a base for the assessment of ore reserves for each technically feasible alternative. The specific conditions assumed for each alternative are imposed on the geological estimates to assess the tonnage and grade characteristics of ore reserves. Geological estimates are only a function of the level of information available, and, thus, are fundamental to assessments which embody differing sets of economic conditions.

The tonnage and grade characteristics of ore reserves are initially estimated in terms of content within the mineral deposit. For use in economic evaluation, such estimates must be converted to a "recoverable" basis to assess the tonnages and grades which will actually be mined and recovered. This requires estimates of mining and milling recovery, and dilution, based on knowledge of the geological environment, and the mining and processing methods to be employed.

GEOLOGICAL ENVIRONMENT

The mode of origin of a mineral deposit influences its tonnage and grade characteristics, and therefore, should have an important bearing on sampling strategy and the application of techniques for tonnage and grade estimation.

The occurrence of mineral deposits is controlled by the geological environment. Mineral deposits can be host or wall rock controlled, structurally controlled, or climatically and topographically controlled. Most deposits are formed as the result of combinations of these factors. Usually the host rock provides the regional constraint for mineralization, and structural features the local constraint. Thus, mineralization is not usually random but rather has regional trends and local variations depending on these geological features.

The tonnage and grade of ore reserves is assessed by imposing an assumed cut-off grade on overall geological estimates. The significance of variations in cut-off depends on geological features of the mineral deposit, particularly the relationship which exists between tonnage and grade. For this purpose, Lacy (7) has classified mineral deposits into three categories.

(1) High-grade, relatively small to moderate-sized deposits with high unit value and generally a wide margin of operating profit. This type of deposit is normally worked with a low capital investment and small ore reserves. It is common for a sharp boundary to exist between ore and waste; ore reserves are high-grade and adjacent material contains essentially no values. Variations in cut-off grade will normally have little effect on ore reserves and mine life as grades within the deposit will be considerably above the economic cut-off grade. This is reflected in the high profit margins which characterize the mining of this type of deposit. Tonnage rather than grade is usually the critical decision variable because the deposit limits are erratic and hard to define, ore reserves are small, and economic criteria for investment are sensitive to mine life and mine capacity.

(2) Low grade, bulk deposits of a size and shape that lend themselves to low-cost, bulk mining methods. This type of deposit is mined with a low profit margin and a high capital investment, based on high manpower productivity and large ore reserves. These deposits are characterized by a dispersed distribution of mineral values. Ore boundaries are determined through careful analysis of contained values. The shape and size of ore reserves will be altered considerably by minor fluctuations in cut-off grade. Grade rather than tonnage will be the critical decision variable. Grade trends within the deposit will be important because they may present an opportunity for mining above the average grade of ore reserves in the early years of mine life. This has an important influence on economic criteria for investment.

(3) Disseminated deposits with dispersed values, having a relatively small and concentrated enriched zone capping the deposit. The enriched zone has the characteristics of the high-grade type of deposit while the unenriched primary zone is similar to the low-grade bulk deposits. Variations in cut-off grade will have little effect on mining the enriched zone, but economic and technological conditions which lower the cut-off grade to concentrations contained in the primary zone will result in a large increase in ore reserves. Porphyry copper mines in the southwestern United States provide a good example of this type of deposit. In the early development of the porphyry coppers only the blanket of enriched ore capping the top of the deposit could be profitably mined. Values in the enriched blanket averaged 1.0% to 1.5% copper. It was not until economic and technological conditions were such that cut-off grade dropped below 0.7% copper that these deposits underwent tremendous expansion as ore boundaries extended into the unenriched material.

STATISTICAL ANALYSIS FOR TONNAGE AND GRADE ESTIMATION

Classical Statistical techniques find practical application in the analysis of sample values for purposes of estimating tonnage and grade parameters. Statistical analysis also provides a conceptual insight into mineral deposit characteristics, estimating techniques, and the application of results.

Relevant deposit characteristics include all attributes of the deposit which influence the estimation of tonnage and grade, e.g. individual mineral and metal assays, specific gravity, and thickness. The natural variability of each characteristic is defined by the probability distribution of its population and is estimated by its sample frequency distribution. Estimating errors are a function of the degree of variability but can be controlled by the sample size.

Preparation of a sample frequency distribution involves dividing the sample values into class intervals and plotting the number of occurrences (frequency) as a function of the class interval as shown in Figure 2. Sample frequency distributions, constructed for all relevant deposit characteristics, provide the starting point for the estimation of tonnage and grade parameters.

The shape of a sample frequency distribution is not fixed but will vary depending on the sample size selected. A small-size sample unit will reflect a larger varaition between sample values than will a larger-size sample unit. However, when a sampling unit has been selected, the parameters of the probability distribution of the population being sampled are fixed.

The two most important paramaters of the probability distributions for relevant deposit characteristics are the mean (expected) value and the standard deviation. Estimates of these parameters are made from their sample frequency distributions.

The mean value estimate($\overline{X}$) may be determined by adding all sample values and dividing by the sample size (N):

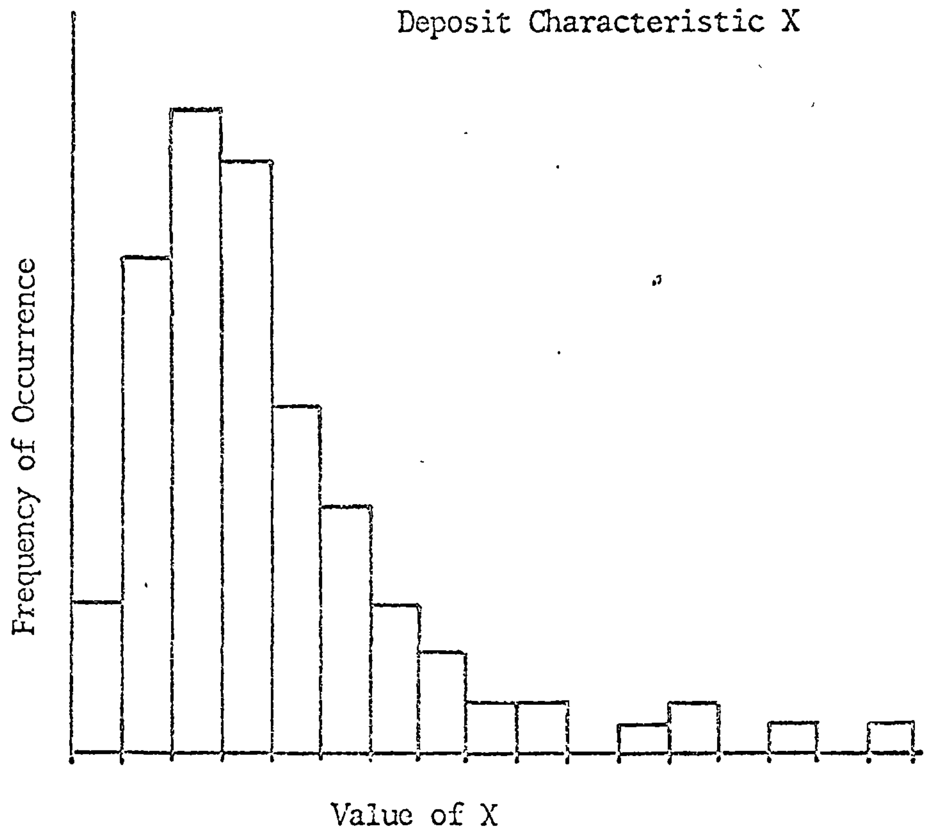
$$\overline{X} = \frac{\sum X}{N}$$

Using the sample frequency distribution, a close approximation of the mean value is determined by multiplying the midpoint of each class interval (mp) by the frequency of occurrence in the class interval (f), taking the summation of these products for all class intervals, and dividing by the sample size:

$$\overline{X} = \frac{\sum[(f)\ (mp)]}{N}$$

Figure 2

SAMPLE FREQUENCY DISTRIBUTION



The reliability of the mean value estimate is a function of both the degree of variability of the deposit characteristic and the sample size.

Variability is estimated by the standard deviation of the sample frequency distribution. The standard deviation is defined as the square root of the mean of the squares of the deviations of values (d) about the mean value. The standard deviation of the sample (S) is an estimator of the standard deviation of the population. Thus:

$$S = \sqrt{\frac{\sum (X - \bar{X})^2}{N}} = \sqrt{\frac{\sum d^2}{N}}$$

Using the sample frequency distribution, S is approximated by the following expression:

$$S = \sqrt{\frac{\sum f\ (mp)^2}{N} - \left[\frac{\sum mp}{N}\right]^2}$$

The standard deviation estimate is used to assess the reliability of the mean value estimate. The reliability of the mean value estimate is measured by the "standard error of the mean."*

$$S_{\bar{X}} = \frac{S}{\sqrt{N - 1}}$$

where,

$S_{\bar{X}}$ = standard error of the mean,

S = standard deviation of the sample frequency distribution,

N = sample size.

Thus, the standard error of the mean varies directly as the variability of a deposit characteristic as reflected by its estimated standard deviation, and inversely as the square root of the sample size.

The standard error of the mean defines the possible distribution of

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* Providing the sample data are random. Randomness assumptions and tests are discussed below.

actual mean values about the mean value estimate. For reasonably large sample sizes, this mean value distribution will approximate the normal distribution, even if the sample frequency distribution is skewed.

The standard error of the mean is used to estimate confidence limits for the mean value estimate. These limits reflect the reliability of sampling and the risk or uncertainty associated with the mean value estimate.

The mean and standard error of the mean estimates for tonnage and grade parameters are used to determine the economic sampling limit and to derive ore reserve estimates for the economic evaluation of alternatives.

The use of statistical analysis, as outlined above, requires that the sample data are random, i.e. individual sample values are essentially independent of one another. Random data is a fundamental requirement for determining the reliability of sampling results and expressing reliability as confidence limits about the mean or expected value estimate. For data that are not random, there is no known means of quantitatively expressing reliability or confidence.

Hazen describes a number of statistical tests which may be applied to sample data to determine whether or not the randomness assumption is met (5). Nearest neighbour, equal expectancy, and poisson distribution tests may be used to assess the randomness of drill hole locations. Generally, systematic drill hole patterns satisfy randomness criteria. The mean-square-successive-difference test is used to assess the randomness of successive sample values within a drill hole. Randomness in this third dimension may be controlled by adjusting the sample size used, e.g. there will be greater independence or randomness between successive sample values if 10-foot sample lengths are used as compared to 5-foot lengths.

Grade trends within a mineral deposit are non-random and complicate the estimation of grade parameters. Trend is the change in mean grade which occurs over some distance or some area which has been sampled. Trend is caused by the host rock and structural features of the geological environment.

Trend may be a sharp change in grade across a structural boundary, or it may be a gradual change, either increasing or decreasing in assay value. However, the change must be persistent or continuous over some distance in contrast to the rapid or erratic changes over short distances which are associated with random variation.

All deposits have trend; that is, the grade changes either abruptly or gradually from uneconomic mineralization to high-grade ore. Grade trends within a deposit do not alter overall deposit parameters, but these within-deposit characteristics influence subpopulation parameters, such as the grade of a high-grade zone within the deposit. An example of grade trends is shown in Figure 3.

The best approach for sampling under conditions where trend is present is to subdivide or stratify the total deposit into zones of essentially no trend. The manner in which a deposit is stratified should be based on a knowledge of the geological environment as outlined in the preceeding section. For many deposits grade is sufficiently uniform over large volumes that blocks may be stratified according to grade, and assay data can be used to estimate grade parameters for these stratified blocks of substantially uniform grade or no trend.

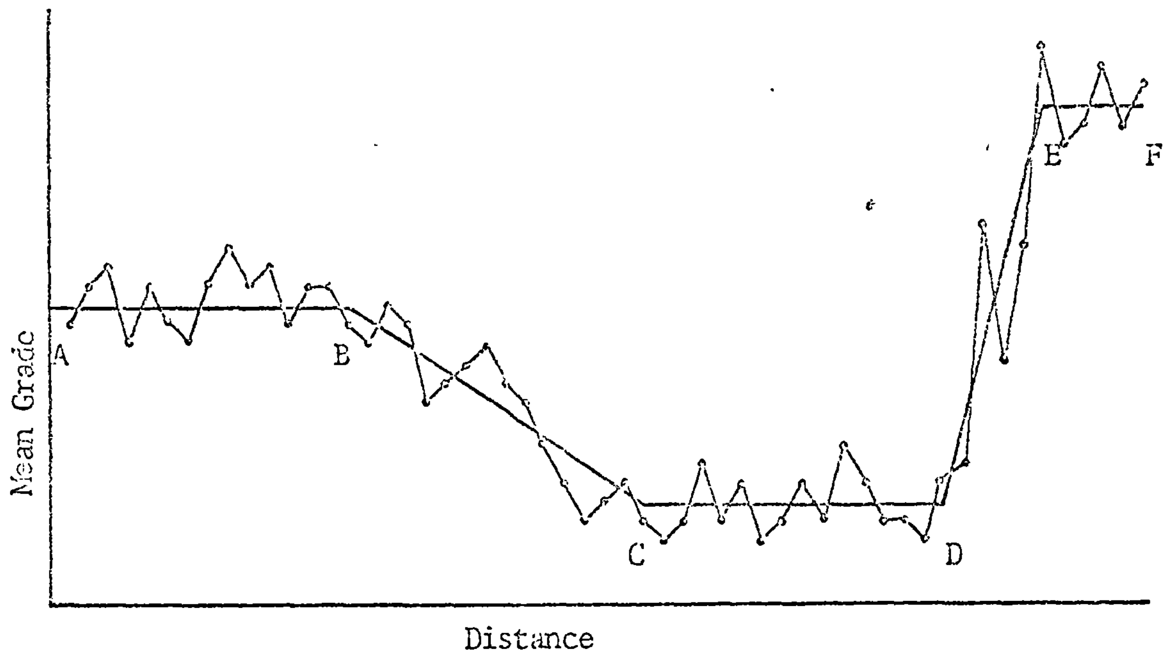
As discussed in the preceeding section, grade is the critical decision variable for most deposits of economic significance. Grade parameters are estimated from their assay frequency distributions. The mathematical representation (fitting of a curve) to an assay frequency distribution is of practical value when knowledge of the population must be inferred from the sample distribution.

It is generally recognized that frequency distributions of low-grade assays are positively skewed, medium-grade assay frequency distributions are normally distributed, and high-grade assay frequency distributions are negatively skewed. The majority of assay distributions are low-grade and positively skewed. The lognormal distribution, an example of which is shown in Figure 4, provides a useful approximation of low-grade assay distributions. For a lognormal distribution, the assay values are not in themselves normally distributed, but if grade intervals are expressed in terms of logarithms, a frequency distribution of the logarithm values of assays within the logarithmic grade intervals becomes a normal distribution. Such a transformation is useful because there is a large amount of statistical theory and test information developed on the basis of the normal curve.

Assay frequency distributions, based on a fixed sample size, also provide an estimate of the tonnage-grade distribution within a mineral deposit. Given an overall estimate of deposit size, assay frequencies may be

Figure 3

GRADE TRENDS



• sample assays, plotted by location

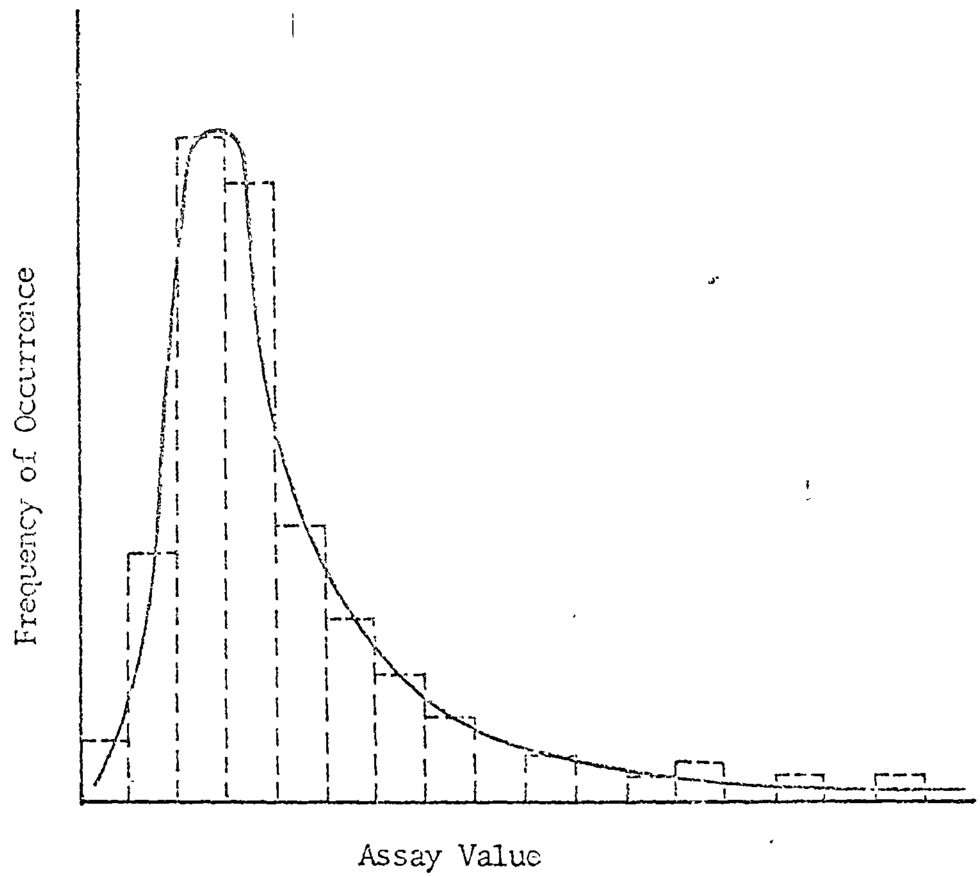
A-B, C-D, E-F: no trend, random fluctuations of grade about mean

B-C trend, gradual decrease in grade accompanied by random fluctuations in grade about decreasing grade trend.

D-E trend, abrupt increase in grade accompanied by random fluctuations in grade about increasing grade trend.

Figure 4

THE LOGNORMAL DISTRIBUTION



applied to estimate the tonnage within each grade interval. Knowledge of the tonnage-grade relationship is useful when specific sets of economic conditions are imposed on geological assessments of tonnage and grade to derive estimates of ore reserves.

If attention is focused on grade estimates above some economically significant level of mineralization, the exponential distribution may be used to estimate the tonnage-grade relationship within a deposit. Using the exponential tonnage-grade relationship, as formulated and tested by Lasky (8) and Musgrove (12), the tonnage of ore increases geometrically as the grade declines arithmetically. Figure 5 shows the exponential distribution of ore reserves and metal content as functions of cut-off grade and average grade of ore reserves. These curves illustrate the following exponential relationships:

$$R(G) = Ae^{-G/K}$$

$$M(G) = KAe^{-G/K}(1 + G/K)$$

$$Gavg(G) = M(G)/R(G) = K + G$$

where G = cut-off grade;

R(G) = tonnage of ore reserves above cut-off grade G;

M(G) = mineral content in ore reserve above cut-off grade G;

Gavg(G) = average grade of ore reserves above cut-off grade G;

A = total deposit tonnage, i.e. R(0) = A;

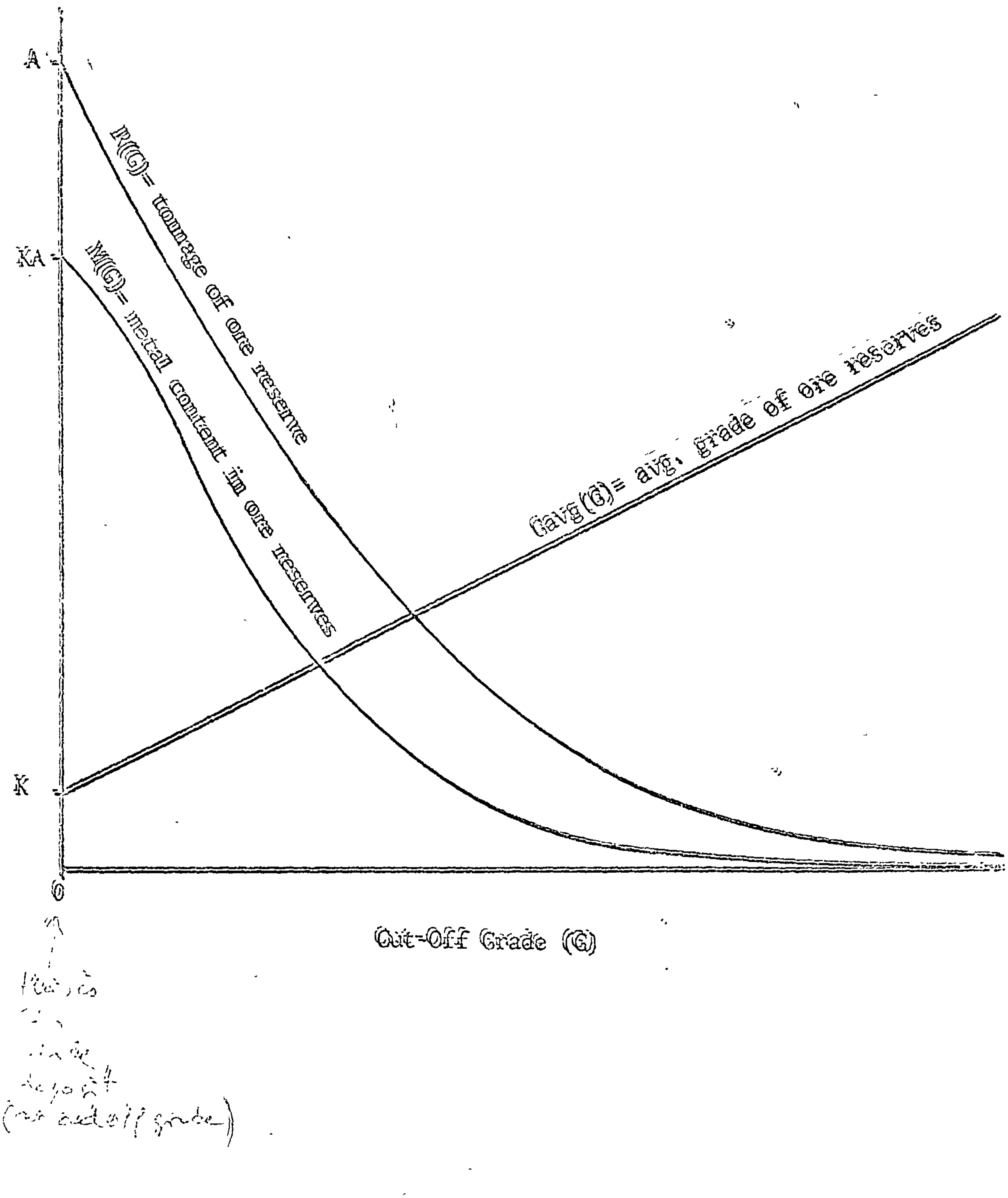
K = average grade of total deposit, i.e. M(0)/R(0) = KA/A.

The following table illustrates the application of the exponential tonnage-grade relationship for a hypothetical copper deposit containing 200 million tons (A) with an average grade of 0.5 percent copper (K).

Cut-Off Grade (% Cu)	Ore Reserves (tons)	Metal Content (tons)	Average Grade (% Cu)
0.00	200,000,000	1,000,000	0.50
0.50	73,580,000	735,800	1.00
1.00	27,070,000	406,000	1.50
1.50	9,960,000	199,100	2.00
2.00	3,660,000	91,600	2.50
2.50	1,350,000	40,400	3.00
3.00	496,000	17,350	3.50
3.50	182,000	7,290	4.00
4.00	67,000	3,020	4.50

Figure 5

EXPONENTIAL TONNAGE-GRADE RELATIONSHIP



ECONOMIC CRITERIA FOR MINE DEVELOPMENT AND OPERATING DECISIONS

Tonnage and grade estimates are made to assist economic evaluation of technically feasible alternatives for selection of the optimum economic alternative. The economic criteria which are used for evaluation purposes prescribe the types of ore reserve estimates required. Economic criteria for two types of decision problem are described here -- criteria for the mine development investment decision and criteria for mine operating decisions.

Mine Development Decisions *

The mine development decision is made on completion of exploration to determine whether or not a mineral deposit should be developed to production and, if so, how. This involves the selection of an optimum mine development alternative from a number of technically feasible alternatives and the comparison of this optimum with those of other available investment opportunities. If the optimum is sufficiently attractive, the deposit is developed to production.

On the basis of available information and expertise, ore reserve, revenue and cost estimates are made for each technically feasible alterntive. Economic evaluation techniques are applied to these estimates to assess the profitability characteristics of each alternative. These assessments provide support for the investment decision.

Ore reserve, revenue and cost estimates are based on limited information. Thus, profitability for decision purposes is more realistically expressed as "expected profitability". Associated with expected profitability is a degree of risk which reflects the reliability of information at the time of evaluation. To compare and select alternatives, both the expected value and the risk of profitability should be evaluated. To do this, estimates of expected value and uncertainty are required for each of the input variables, including ore reserves. For this purpose, estimates of the mean (expected) value and the standard error of

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* The analysis framework summarized here is based on a more detailed treatment of the subject (9).

the mean are made for relevant tonnage and grade characteristics.

In addition to information which can be evaluated in terms of expected profitability and risk, intangible information must also be considered in the decision process. "Unknown economic resources" may constitute an important intangible.

Evaluation of expected profitability is based on single-point expected value estimates for each of the input variables. The Monte Carlo method simulates the risk of a profitability outcome by means of random sampling. This technique is based on the estimation of probability distributions to reflect the uncertainty of each of the input variables. The result of the simulation is an estimated probability distribution of the profitability outcome about its expected value. The critical component of this distribution is the lower confidence limit, reflecting the insurance of some minimum level of profitability.

Cash flow and time value are the basic concepts used to assess profitability. Net present value may be used to compare the profitability characteristics of alternatives given a fixed level of investment (e.g. fixed capacity). To estimate net present value, positive and negative annual cash flows are discounted at a predetermined interest rate representing the cost of capital, and the discounted values are summed. The rate of return, one of the most widely used profitability criteria, is the discount rate which equates the present value of negative cash flows with the present value of positive cash flows.

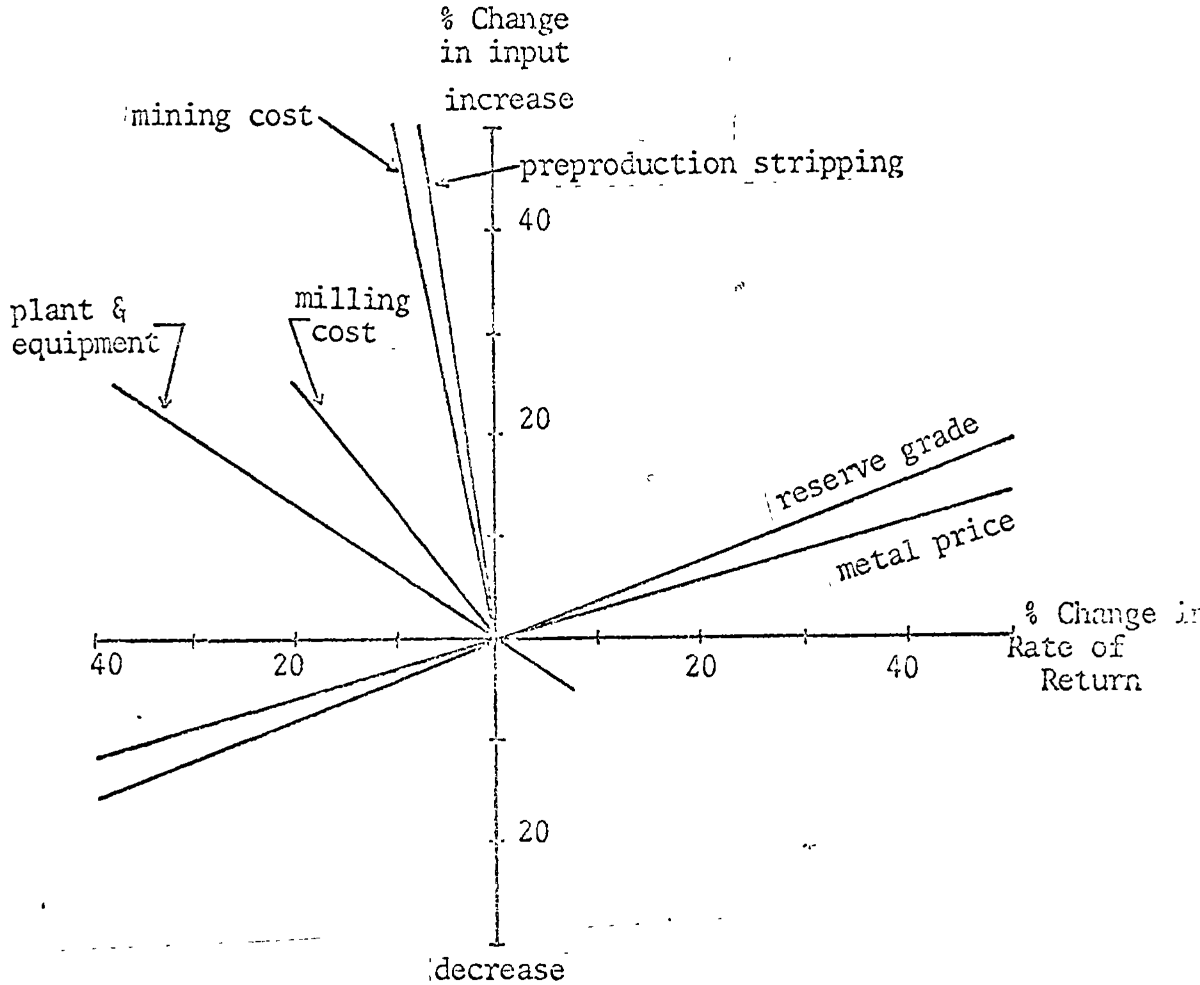
The selection of the optimum mine development alternative will depend on the economic criteria evaluated as well as on the size and risk preference of the mining organization. On the one hand, the mining organization, motivated by the desire for profitable investment, prefers a development alternative with a higher expected profitability to one with less. On the other hand, it is averse to risk and, therefore prefers a development alternative with less risk to one with more. In economic terms, the organization will select the optimum alternative on the basis of maximizing its "utility".

Sensitivity analysis can be used to determine the relative importance of ore reserve input estimates to the profitability outcome. A porphyry copper example, developed by McCrea (10), is shown in Figure 6. It can be seen that rate of return is very sensitive to possible changes in grade.

Figure 6

PORPHYRY COPPER MINE DEVELOPMENT

SENSITIVITY OF PROFITABILITY TO CHANGE IN INPUT VARIABLES



Expected Value Conditions

Capacity: 25,000 tpd
Avg. Grade: 0.4% Cu
Metal Price: $0.50 per pound
Rate of Return: 15.5%

Source: McCrea (10)

#10

In general, grade and metal price are the two most important input variables in evaluating the economics of mine development. Thus, estimation of the expected (mean) value and reliability of grade parameters is of critical importance to mine development investment decisions.

Mine Operating Decisions

For the mining operation, investment in exploration and mine development is sunk, and decisions are of a tactical nature. Operational decisions embody such elements as cost control, production planning, productivity, the development and implementation of new mining and processing technology, and equipment replacement decisions. The effectiveness of these policies will be reflected in cost trends which in turn will determine changes in cut-off grade, ore reserves, and mine life. A primary objective of the mining organization with respect to a mining operation is maximization of total profit (cash flow) over mine life. This objective will be achieved by minimizing economic cut-off grade, and maximizing ore reserves and mine life.

The cut-off grade is the minimum grade which can be economically mined. It is the "extensive margin of cultivation" discussed in classical economic theory. For operational conditions where the primary economic criteria is maximization of total profit over mine life, cut-off grade is determined by the equation of marginal cost and marginal revenue. At the margin, grade is just sufficient to offset cost, i.e.

$$MC = MR = PGR/(1 + D)$$

where, MC = cost of mining and processing marginal ton or ore, $;

MR = revenue realized from marginal ton of ore, $;

P = net price of metal recovered, $ per ton;

G = cut-off grade, percentage content of mineral product in marginal ton of ore;

R = mill recovery factor (metal recovered/metal content of ore reserves);

D = dilution factor (waste mined/ore mined).

Thus, cut-off grade varies directly with the cost of producing a saleable mineral product and inversely with the price received for the product. As discussed in a preceeding section, the significance of variations in cut-off grade for operational decisions depends on the geological parameters of the deposit, particularly the relationship which exists between tonnage and grade.

Operational decisions relating to ore reserve estimation fall into the category of "grade control". The time domain for this type of decision varies from the daily control required to minimize grade variability in mill feed, to production planning on quarterly, annual, and five-year bases.

THE ECONOMIC SAMPLING LIMIT

The objective of mineral deposit sampling is to provide a data base for the tonnage and grade estimates required for the economic evaluation of alternatives. The costs of sampling are justified by the benefits which result from improving the reliability of the profitability criteria evaluated.

Tonnage and grade estimates are made to assist with mine development and mine operating decisions. A prior decision concerns the level of investment in sampling which is economically justified. This decision defines the economic sampling limit.

The reliability of tonnage and grade estimates is improved by increasing the sample size, thereby reducing the estimated standard error of the mean and the spread of confidence limits about the mean value estimate. However, the benefits from increased sampling are subject to diminishing returns, i.e. as sample size is increased, the standard error of the mean is reduced but at a decreasing rate. At some point, marginal benefits from sampling investment just balance marginal costs. This point defines the economic sampling limit.

The economic benefit from increased sampling is the reduction of economic risk. A reduction in the standard error of the mean for ore reserve parameters reduces the standard deviation of the profitability outcome. This has the effect of increasing the lower limit of profitability. At the same time, costs are associated with increased sampling -- the direct cost of the increased drilling or underground exploration required, as well as the time cost of delaying the mine development or mine operating decision. The time cost is the reduction in the present value of benefits which would result from the decision. These monetary and time costs reduce expected profitability and lower the entire profitability distribution. This has the effect of reducing the lower limit of profitability. Initially, benefits will exceed costs and the lower limit of profitability will be increased by further sampling. Economic risk will be minimized at the level of sampling where marginal benefits are equated with marginal costs. Sampling beyond this point will result in increased economic risk, i.e. a reduction in the lower limit of profitability. Figure 7 illustrates the determination of the economic sampling limit based on the objective of minimizing economic risk. *

ORE RESERVE ESTIMATION FOR EVALUATING THE ECONOMICS OF MINE DEVELOPMENT

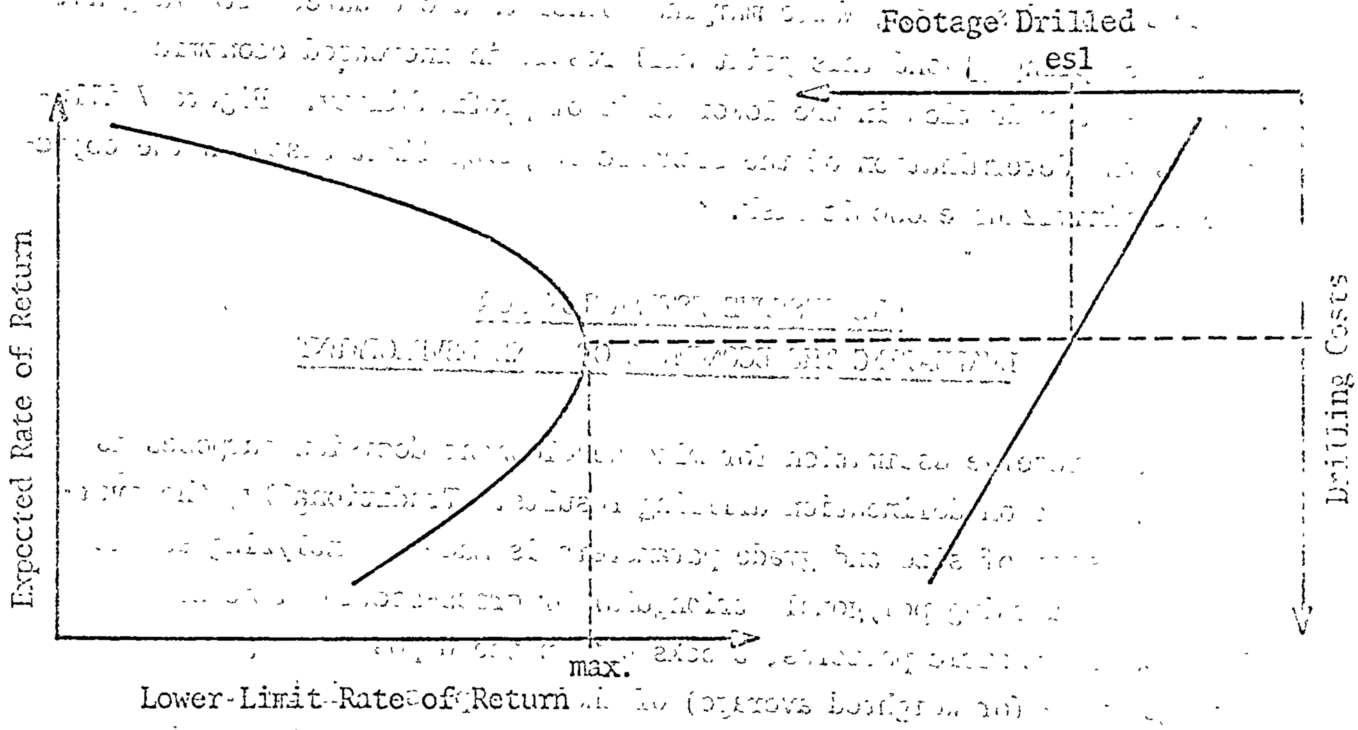
Ore reserve estimation for mine development decision purposes is normally based on delineation drilling results. Traditionally, the overall estimation of size and grade parameters is made by analyzing sample drilling data using polygonal, triangular or cross-sectional methods. Using such geometric patterns, blocks within the deposit are given the average grade (or weighted average) of their samples. Choice of a particular geometric pattern is governed by the shape and attitude of the deposit and the drilling pattern which has been used for delineation.

The polygonal method of computation assumes that the influence of a sample or the grade given by the sample extends halfway to the next sample. Use of the method involves construction of polygons about each of the drill holes. The first step in the construction is to draw lines joining each drill hole to every other adjacent drill hole. This forms a series of triangles; and, if perpendiculars are erected from the mid-points of each of the sides of a triangle, they will meet at a common point. By joining these common points found for each of the triangles around a drill hole, a polygon will be constructed about each drill hole. The area of the polygon is com-

* This objective may not always be considered desirable. For example, in eastern European countries mineral deposits are fully explored before a development decision is taken. This sacrifices the economic benefits of more immediate development in the interests of encouraging the formulation of a more satisfactory long-term plan.

Figure 7

THE ECONOMIC SAMPLING LIMIT



esl: economic sampling limit

puted and the thickness of the deposit within each hole is estimated. The average grade within each hole is assigned to the tonnage estimated on the basis of polygonal area and thickness. The tonnages of all polygons are summed and a weighted average grade computed to provide tonnage and mean grade estimates for the mineral deposit.

Computing size and grade by the triangular method involves constructing triangles with drill holes at the corners. Lines drawn from each corner of a triangle to the midpoints of the opposite sides intersect at a point that becomes the common apex of three smaller equal-area triangles, each of which has one side of the original triangle as the base. The average grade of the point of intersection, which is assigned to the triangular area, may be computed by weighting the average grade of each hole inversely as the distance of each hole from the common point of intersection. An average thickness is computed in a similar manner. The tonnages associated with all triangles are summed and a weighted average grade computed to provide tonnage and mean grade estimates for the mineral deposit.

Computation of size and grade using the cross-sectional method involves preparation of several sections of the deposit and plotting either the actual intersections or the projections of drill holes on these sections. Computations of volumes and grades can be made in various ways. For example, computations of volume can be made by determining the average area of adjacent sections and multiplying by the distance between them. This volume is the then assigned the average grade of the adjacent sections. The tonnages associated with all such segments are summed and a weighted average grade computed to provide tonnage and mean grade estimates for the mineral deposit.

The polygonal, triangular and cross-sectional methods used for estimating tonnage and mean grade for a mineral deposit assume that the area of influence of an assay in any one drill hole is a function of the distance to any other adjacent drill hole. In using these methods, no consideration is given to the mineralization which actually exists between the drill holes. The methods are a function of geometry, which simplifies the calculations. They are not a function of the mineralization that the methods propose to predict.

These traditional geometric methods have two major shortcomings:

(1) Generally, the area of influence given to individual drill hole assays far exceeds their actual area of influence. Drill hole spacing is typically several hundred feet. Although the actual area of influence depends on mineral deposit conditions, statistical tests on a number of deposits have shown that the radius of influence of a sample grade may be as low as one foot and rarely exceeds 100 feet. Thus, the apparent precision of the geometric method in assigning drill hole assays to large areas is fallacious. The actual mean grade contained within such an area may be quite unrelated to the drill hole assays obtained. However, to the extent that results are used only to estimate overall deposit parameters this is not a problem in itself although the different weightings assigned to assays depending on relative distances between holes is incorrect and results in overall bias.

(2) The geometric methods do not provide a measure of confidence in the overall tonnage and mean grade estimates. A knowledge of the risk associated with these estimates, particularly grade, is important for evaluating the economics of mine development. Use of the geometric methods precludes risk analysis and the assessment of risk associated with profitability criteria. This limitation is not, however, inherent in the data. Rather, it occurs because these methods fail to fully utilize the data.

The application of statistical analysis does not have the above limitations. Using statistical analysis, individual sample values are not assigned to a particular part of the deposit. Rather, they are treated as random samples from the deposit as a whole and in this manner are used to provide overall tonnage and grade estimates. Estimates are made for the mean value, standard deviation, and standard error of the mean. This gives a basis for assessing not only the expected or mean values of the deposit parameters, but also the reliability or confidence of those estimates. Thus, there is a mechanism for evaluating the economics of controlling risk through additional sampling as well as providing the type of probabilistic estimates required to assess the risk of mine development alternatives.

As outlined in a preceeding section, the existence of trend within a deposit is non-random. However, if trend is known to exist, the deposit can be stratified into grade zones or other natural geological subdivisions to provide essentially random sets of data and permit the application of statistical analysis. This technique reduces error in estimating the expected grade of the total deposit being sampled and provides a more detailed basis for mine development planning. Some previous knowledge of the deposit is required to stratify the samples in this manner. Thus, it can usually only be used in the final stages of delineation sampling programs.

The following example, concerning the Maggie Canyon manganese deposit in Arizona, was developed by Hazen (4) to compare tonnage and grade estimates by each of four methods -- polygonal, triangular, cross-sectional, and statistical analysis.

Four separate exploration projects were carried out at Maggie Canyon between 1936 and 1950. In total, 42 surface drill holes intersected the deposit, comprising 406 five-foot samples. The deposit was sub-divided into three zones for estimating purposes -- south, southeast, and north. The following size and mean grade estimates were obtained.

	Method of Estimation			
Zone	Polygonal	Triangular	Cross-Sectional	Statistical Analysis
South				
size-mill. tons	7.2	7.3	7.0	7.5
grade-% Mn	5.5	5.5	5.3	5.9
Southeast				
size-mill. tons	10.6	10.5	11.0	10.1
grade-% Mn	4.2	4.4	4.0	4.1
North				
size-mill. tons	9.6	9.8	7.1	8.5
grade-% Mn	6.7	6.3	5.3	6.5
Total				
size-mill. tons	27.4	27.6	25.1	26.1
grade-% Mn	5.4	5.3	4.7	5.4

Results show close agreement for the grade estimates obtained using the polygonal, triangular and statistical analysis methods. The cross-sectional method results show the largest variation.

The application of statistical analysis to grade estimation for the total Maggie Canyon deposit is shown in Figure 8. The advantage of statistical analysis is that the reliability or confidence of the mean value estimate may be assessed. For example, using the Maggie Canyon case:

$$N = 406$$
$$\bar{X} = 5.43\%$$
$$S = 4.41\%$$
$$S_{\bar{X}} = 4.41/\sqrt{406} = 0.22\%$$

If 95% confidence limits are desired, a table of areas under the normal curve shows that these limits will lie ± 1.97 standard errors from the mean value estimate, i.e. $5.43 \pm (1.97)\ (0.22)$, or 5.0% and 5.8%. Thus, there is 90% confidence that the mean grade of the total deposit will actually be between 5.0% and 5.8% manganese.

Given size and grade estimates for the overall deposit, ore reserves are derived for each alternative on the basis of assumed cut-off grade conditions. The cut-off grade associated with the optimum economic alternative will be used in practice. Since the economic evaluations generally involve complex interrelationships among variables and changing economic conditions over time, it is difficult to develop rules to guide initial cut-off grade assumptions. Nevertheless, it has been shown by Lane (9), Blackwell (1), and Taylor (13), that, generally, economic benefits result from sequencing mining so that cut-off grade is gradually reduced over mine life.

The optimization of cut-off grade is shown by Blackwell for a hypothetical open-pit copper mine development. The example assumes fixed mine and mill capacities and does not consider uncertainty. Thus, the net present value criterion may be utilized. The assumed mine development conditions are as follows:

Figure 8

STATISTICAL ANALYSIS - MAGGIE CANYON MANGANESE DEPOSIT

Sample frequency distribution and computations for 406 sample manganese assays from 42 surface drill holes within deposit limits, assuming 2-percent grade intervals.

Grade interval (GI)	Frequency (f)	Average grade weighted (percent)	Midpoint of grade interval (mp)	Frequency times midpoint (f x mp)	Frequency times midpoint squared (f x (mp)²)	Frequency times average grade
0.00-1.99.......	69	1.39	1.00	69	69	95.91
2.00-3.99.......	130	2.80	3.00	390	1,170	364.00
4.00-5.99.......	79	4.84	5.00	395	1,975	382.36
6.00-7.99.......	42	6.85	7.00	294	2,058	287.70
8.00-9.99.......	35	8.95	9.00	315	2,835	313.25
10.00-11.99......	15	10.77	11.00	165	1,815	161.55
12.00-13.99......	19	12.84	13.00	247	3,211	243.96
14.00-15.99......	5	14.71	15.00	75	1,125	73.55
16.00-17.99......	4	16.59	17.00	68	1,156	66.36
18.00-19.99......	1	19.96	19.00	19	361	19.96
20.00-21.99......	3	20.88	21.00	63	1,323	62.64
22.00-23.99......	2	23.05	23.00	46	1,058	46.10
24.00-25.99......	0	0	25.00	0	0	0
26.00-27.99......	1	26.35	27.00	27	729	26.35
30.00-31.99......	1	30.70	31.00	31	961	30.70
	406	5.32		2,204	19,846	2,174.39

mean grade estimate $= \bar{X} = \frac{\sum X}{N} = \frac{\sum f \text{ (avg. grade)}}{N} = \frac{2,174.39}{406} = \underline{5.36\%}$

approximate mean grade estimate $= \bar{X} = \frac{\sum (f)(mp)}{N} = \frac{2,204}{406} = \underline{5.43\%}$

approximate standard deviation estimate $= S = \sqrt{\frac{\sum (f)(mp)^2}{N} - \left[\frac{\sum mp}{N}\right]^2} = \sqrt{\frac{19,846}{406} - \left[\frac{2,204}{406}\right]^2}$

$= \underline{4.41\%}$

Source: Hazen (4)

Mine Capacity : 40 mill. tpy
Mill Capacity : 15 mill. tpy
Mill Recovery : 85%
Unit Pit Costs per Ton Mined: $0.25
Unit Mill Costs per Ton Ore : $0.50
Selling Costs per Ton Copper: $50.
Copper Price : $0.39 per pound recovered

Tonnage and grade are specified by tonnage estimates over a range of grade categories. Economic results for assumed cut-off grade strategies are shown below:

Cut-Off Grade (%Cu)		Net Present Value (mill. $)	Total Cash Flow (mill. $)	Mine Life (yrs.)
Fixed:	0.2	255	910	25
	0.3	276	860	21
	0.4	294	819	18
	0.5	287	717	15
	0.6	236	548	15
Variable:	optimum	305	797	16

From the results it can be seen that the optimum variable cut-off grade strategy has an advantage of $9 million over the highest present value obtainable by a fixed cut-off grade strategy. With this strategy, the cut-off grade is reduced from 0.54% Cu in year 1 to 0.17% Cu at the end of mine life.

ORE RESERVE ESTIMATION FOR OPERATIONAL DECISIONS

Ore reserve estimation for operational decisions must be much more detailed than for mine development decision purposes. These decisions involve tonnage and grade control issues. The planning horizon for these decisions varies from one day to several years. The following types of grade control problem may be considered:

- daily grade control of mine production to minimize grade variability in mill fee, thereby optimizing mill efficiency and recovery;

- cut-off grade decisions relating to closing of old working places and new stope development;
- production planning, including cut-off grade considerations, on quarterly, annual and 5-year bases;
- control and economic limitation of dilution.

From the mill operations point of view it is highly desirable that the grade of ore treated be as uniform as possible. Even with on-stream analysis and computerized control wide fluctuations are difficult to handle and result in loss of efficiency, reduced recovery, and increased costs. Ore blending procedures in the mine should be directed towards minimizing these short-term variations.

Nevertheless, beyond some point, the cost of ore blending procedures becomes prohibitive. This means that mines can not produce the same grade day after day. Grades will vary. However, over longer-term periods the average grade mined should be close to planned grade. This is required because of market commitments for concentrate sales and because of the needs imposed by financial planning for sources and uses of funds within the mining organization.

In some mining methods a certain amount of dilution is inevitable. In other cases it constitutes bad mining practice. Dilution should be classified into the following categories:

(i) dilution with mineralized material means that the dilutant may carry recoverable mineral which may pay for its recovery;

(ii) dilution with material without mineral content is a more serious matter because this material will remove mineral from the ore in the recovery circuit.

For dilution control the greatest threats are hanging wall and footwall conditions. In many cases, the footwall is faulted, consisting of friable poorly mineralized rock which will be recovered in the mining cycle. Little can be done to eliminate dilution of this type. Dilution by hanging wall failure is time dependent and structurally controlled. It can therefore be avoided if the danger is recognized. All other forms of dilution generally result from insufficient sampling information and bad mining practices. Dilution control requires a knowledge of the amount of dilution which can be economically accomodated and a flexibility in mining practice to adapt to unexpected changes in deposit conditions.

For purposes of grade control, tonnage and grade estimates are required for blocks associated with individual stopes and mining faces, and for stope-size blocks along development headings. The basis for these estimates is detailed stope and development samples, supplemented by mine car and skip sampling programmes.

The application of statistical analysis for tonnage and grade control purposes is much more difficult than for the mine development situation previously outlined. For grade control, a small block must be assigned a grade based on a relatively large number of closely-spaced samples. In this case, individual sample values are not random and cannot be assumed independent of each other. Thus, the methods of statistical analysis previously discussed are not applicable.

For statistical analysis of tonnage and grade in operating situations, a number of geostatistical techniques, primarily developed in France and South Africa, may find practical application. The concepts of regionalized variables, extension variance, and the variogram, as described by David (3), may be applied to determine optimal weights to assign to individual samples to obtain the best estimator for valuation of blocks. Krige (6) has applied correlation and regression analysis to the selective mining of gold ores in South Africa. A regression equation was employed to estimate the unknown face value as a function of the face values of approaching stope panel faces. These techniques provide the statistical bases for predicting grade contour surfaces for grade control purposes.

Given tonnage and grade estimates for individual working blocks within the deposit, economic conditions and criteria must be applied for decision purposes. Economic conditions include operating costs, dilution and recovery factors, market demand, price, and government policy considerations. Estimates of cost, recovery, and dilution conditions are required on individual stope and individual working face bases for meaningful control. Thus, a detailed mine costing system, structured for decision rather than accounting purposes, is a prerequisite.

The economic criteria applied for tonnage and grade control decisions are relatively simple. Generally, marginal analysis may be used. Since investment in mine capacity is sunk, time value and profitability are less important than in the mine development decision situation. Maximization of total profit is a primary objective. This will be achieved by mining to the economic margin. However, over the longer-term planning horizon, economic benefits may be realized from sequencing operations so that cut-off grade is reduced over time. The desirability of mining to the true economic margin will increase as the deposit becomes depleted.

The determination of cut-off grade and ore reserves for a mining operation may be strongly influenced by government policy considerations. Mineral conservation policy may require mining to the average grade of ore reserves as defined by the true economic margin. The level and structure of mining royalties and taxation may act to raise cut-off grade. These effects are illustrated by a hypothetical example.

A copper deposit (based on an earlier example) is assumed, containing 200 million tons at an average grade of 0.5 per cent copper. The tonnage-grade relationship is exponential. A mine capacity of 3 million tons of ore per year has been installed at a capital cost of $37.5 million. Other operating parameters are as follows:

operating cost : $3.60 per ton or ore mined
recovery factor: 80%
dilution : nil .
price : $750 per ton of copper recovered in concentrate.

Three levels of mining taxation (T) are considered over the mine's productive life: $50 million, $70 million, and $90 million.

The example compares the optimization of the mining operation at each level of taxation for two forms of taxation:

profit tax — t_p, % of taxable profit; operating and capital costs are the only allowable deductions.

value of production tax — t_{vp}, % of revenue.

It is assumed that the objective of the mining organization is to maximize total profit over mine life. Time value is neglected in the interest of simplicity.

Consider the $50 million level of taxation.

(1) Profit Tax

$$G = \frac{MC}{PR} = \frac{3.60}{750(.8)} = 0.60\% \text{ copper}$$

- applying the exponential tonnage-grade distribution as described above with A = 200 million tons, K = 0.5%, G = 0.6%:

ore reserves = 60,239,000 tons
copper content = 662,600 tons
avg. grade of ore reserves = 1.10% copper

Total Revenue =	662,600 (.8)(750) =	$397,578,000
Operating Cost =	60,239,000 (3.60) =	216,860,000
Operating Profit		180,718,000
Capital Cost		37,500,000
Taxable Profit		143,218,000
Mining Tax (T)		50,000,000
Total Profit (Cash Flow)		93,218,000

$$\text{tax rate} = t_p = \frac{50,000,000}{143,218,000} = 34.9\%$$

$$\text{mine life} = \frac{60,239,000}{3,000,000} = 20.1 \text{ years}$$

(2) Value of Production Tax

$$G = \frac{MC + t_{vp}(PRG)}{PR} = \frac{.006}{(1 - t_{vp})} \qquad (1)$$

$$T = t_{vp} \text{ (copper content of ore reserves) (PR)} = \$50,000,000$$

$$\text{copper content of ore reserves} = \frac{50,000,000}{600\ t_{vp}} \qquad (2)$$

- solutions for G, t_{vp} and copper content of ore reserves are obtained by solving expressions (1) and (2) simultaneously with the exponential tonnage-grade relationship:

cut-off grade = 0.70% copper
ore reserves = 49,319,000 tons
copper content = 591,800 tons
avg. grade of ore reserves = 1.20% copper

Total Revenue =	591,800 (.8)(750) =	$355,100,000
Mining Tax (T)		50,000,000
Operating Cost =	49,319,000 (3.60) =	177,550,000
Capital Cost		37,500,000
Total Profit (Cash Flow)		90,050,000

$$\text{tax rate} = t_{vp} = \frac{50,000,000}{355,100,000} = 14.2\%$$

$$\text{mine life} = \frac{49,319,000}{3,000,000} = 16.4 \text{ years}$$

The complete set of results for this example is presented in the following table.

	T = $50 mil.		T = $70 mil.		T = $90 mil.	
Parameter	t_p	t_{vp}	t_p	t_{vp}	t_p	t_{vp}
cut-off grade: % Cu	.60	.70	.60	.765	.60	.865
ore reserves: mil. tons	60.2	49.3	60.2	43.5	60.2	35.8
copper content: thou.tons	663	592	663	548	663	485
total revenue: mil.$	397.6	335.1	397.6	328.8	397.6	291.0
operating cost: mil.$	216.9	177.5	216.9	156.6	216.9	128.9
capital cost: mil.$	37.5	37.5	37.5	37.5	37.5	37.5
total profit: mil.$	93.2	90.0	73.2	64.7	53.2	34.6
tax rate: %	34.9	14.2	34.9	21.3	34.9	30.9
mine life: years	20.1	16.4	20.1	14.5	20.1	11.9

The example illustrates the differing effects of profit and vlaue of production forms of mining taxation on cut-off grade and ore reserves for three levels of taxation. Obviously, such government policy considerations have an important impact on operational cut-off grade decisions.

SUMMARY

The paper reviews statistical and economic principles relating to the estimation of tonnage and grade for mineral deposits and the assessment of ore reserves.

The definition of ore reserves for estimating purposes should embody distinct geological and economic dimensions. Geological estimates of tonnage and grade for the known part of a mineral deposit establish a base to assess ore reserves given various sets of economic conditions.

A knowledge of the geological environment is essential for the specification of grade trends within a deposit and for understanding the relative importance of tonnage and grade parameters and the significance of variations in cut-off grade.

Statistical analysis of sample data provides expected value estimates and confidence limits for tonnage and grade parameters. These types of estimate are required to evaluate expected profitability and risk for mine development alternatives. Economic criteria for mine operating decisions are based on marginal analysis.

Sampling improves the reliability of estimates for economic criteria. However, the benefits from increased sampling are subject to diminishing returns and, at some point, the marginal benefits from improved reliability are just sufficient to balance the marginal monetary and time costs incurred. This defines the economic sampling limit.

Traditional geometric techniques for tonnage and grade estimation do not provide a measure of confidence in the mean value estimates. While statistical analysis does not have this limitation, problems associated with non-random data and grade trends have to be overcome for realistic application. The optimization of cut-off grade for mine development is usually achieved by sequencing mining so that cut-off grade is gradually reduced over mine life.

Detailed tonnage and grade estimates based on stope and development sampling programmes are required for tonnage and grade control at operating mines. Geostatistical techniques utilizing correlation and regression analysis may be usefully applied for these purposes. Detailed economic conditions are imposed on the tonnage and grade estimates obtained to define ore reserves for economic analysis of mine planning and operational control alternatives. Government mineral policies, particularly those relating to royalty payments and taxation, may have important effects on operational cut-off grade and ore reserve estimates.

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THE ECONOMICS OF A FUND RESOURCE WITH PARTICULAR REFERENCE TO MINING

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A source of confusion in attempts to apply economic theory to mining has been the failure to distinguish clearly between the rate of recovery of the mineral product and the completeness of extraction of the total mineral occurring in the ground. The distinctinction applies in principle to any "fund" resource as opposed to a "flow" resource and should be applicable in varying degree in situations involving both stocks and flows of wealth in combination. In this paper certain modifications of the economic theory in question are discussed and some implications for policy are suggested.

I. *Diminishing Returns in Mining*

Ricardo recognized that mines are of differing quality and that attempts to increase the mineral production of an economy would eventually encounter diminishing returns in the necessity of recourse to poorer mines, and that this extensive application of capital and labor would give rise to a rent of mines.[1] He did not clarify, however, the manner in which diminishing returns would arise as capital and labor are applied more intensively to a given mine. On this latter point subsequent economists have disagreed.

Three opposing views on the operation of the law of diminishing returns with the intensive application of capital and labor to mines appear in the literature: (1) that diminishing returns is illustrated by the increasing difficulty of extracting the mineral product as the mine is depleted; (2) that the law in its normal technical sense does not apply to mines; and (3) that it is illustrated by the decreasing effectiveness of more intensive application of capital and labor in increasing the *rate* of working a deposit. Only the last of these is acceptable and this

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[1] David Ricardo, *The Principles of Political Economy and Taxation*, Ch. 3. *Cf.* Adam Smith, *The Wealth of Nations*, Bk. 1, Ch. 11, Pt. 2.

with reservation.[2] As Gray has indicated,[3] the average cost per ton of mining all the coal from a deposit might be expected to fall at first and then rise as the proposed rate of extraction is increased. One particular rate will yield a minimum average cost per ton. Gray then shows that the necessity of discounting future earnings at the interest rate to arrive at present value justifies a rate of extraction greater than that yielding minimum average expense per ton. Thus in this third view, diminishing returns appears not simply in the differing physical yield with more intensive utilization, as it normally does, but in the time distribution of a given total physical yield. The manner in which this total yield of a mine is also subject to economic decision is not discussed except for those cases in which ore may be left unmined for the sake of speed.

Serious misconceptions are likely to result from this one-sidedness because the total physical yield from a mine is neither fixed nor naturally determined. At least half the problem is neglected when only the rate of mining is considered and, except in so far as the rate directly affects the total yield, very little or nothing can be said about the extremely important decisions concerning grades of ore mined, completeness of mining, and per cent extraction of the valuable product from the mined ore.

II. *Rate of Recovery and Level of Recovery Defined*

My purpose is to show that two distinct kinds of choice are made in operating a mine; one, between alternative rates of production from the particular deposit or part of a deposit, here called the *rate of recovery*, and the other between alternative total amounts of mineral to be extracted from the particular deposit or part of a deposit (alternative fractions of an absolute total assumed to be present in the ground), here called the *level of recovery*. It is the second of these choices that has been neglected in the application of economic theory to mining.[4]

[2] Diminishing returns in the first view is used in a nontechnical sense, for it is said to arise not in the choice between alternative ratios of variable factors applied to a fixed factor (the mine in a given state of depletion) but in the time-related and unavoidable changes in the mine itself. See, e.g., F. W. Taussig, *Principles of Economics*, 4th ed. (New York, 1939), pp. 138-39. The second view appears in Alfred Marshall's reference to a mine as "nature's reservoir . . . if one man could pump it out in ten days, ten men could pump it out in one day; . . ." in *Principles of Economics*, 8th ed. (London, 1920), p. 167. Unhappily the simple relation does not hold.

[3] L. C. Gray, "Rent Under the Assumption of Exhaustibility," *Quart. Jour. Econ.*, May, 1914, XXVIII, 471-72. See also J. Orchard, "The Rent of Mineral Lands," *Quart. Jour. Econ.*, Feb., 1922, XXXVI, 310-33, W. A. Roberts, "Diminishing Returns in the Mining Industry," *Jour. Land and Pub. Utility Econ.*, 1939, XV, 21-29; H. C. Hoover, "The Economic Ratio of Treatment Capacity to Ore Reserves," *Engineering and Mining Jour.*, Mar., 1904, LXXVII, 475-76.

[4] The neglect may derive in part from a concept held by some that mineral deposits are more or less sharply delimited, and from the emphasis given to the rate of mining by some

Although decisions as to the rate and the level of recovery commonly influence each other, there is no inherent relationship between the two variables. Thus, to consider one possibility, by selectively mining the best ore, a high rate of recovery may be achieved at the expense of a lower level of recovery. "Gutting" or "picking the eyes out" refers to extreme forms of selective mining. Or, instead, a high level of recovery may be achieved while the rate of recovery is held constant or actually declines. This happened on the Rand following the rise in the price of gold in 1933 and the imposition of the excess profits duty tax. The tax so favored high-recovery, low-profit producers over low-recovery, high-profit producers that the latter were encouraged to mine much more low-grade ore, with the total result that the life of Rand mines as a whole was greatly increased while the yearly production of gold was not. Or thirdly, as is the case when metal prices rise without an offsetting rise in supply costs, it may become worth while either to mine lower-grade ore or more tons per day or both. Thus where mass mining of low-grade ores has successfully replaced selective mining of restricted high-grade portions, both the level and the rate of recovery have been raised by increasing the capital investment in the mine and recovery plant. Admittedly, such large-scale methods are not practical unless the deposit is susceptible to rapid mining, and to this extent the level is dependent on the rate, but there is still a wide range of rates at which such combinations of large-scale plants and ore bodies can be worked beyond the minimum rate necessary for mass mining.

Analysis of the level of recovery is extremely important, for it frequently happens that low-grade or high-cost ore can be mined at a profit only if it is removed and treated along with the neighboring high-grade ore. The low-grade could not bear, by itself, the costs of development and mining, not to mention the costs of rehabilitation of the mine should this be necessary after the better ore has been removed. Of course to remove every ton of potential ore from the mine or to recover

mining engineers. Coal mining has commonly been chosen as the type example for analysis and because coal is a bedded deposit, the quantity of mineable coal in a given seam beneath a given land surface does in fact tend to be less variable than, say, the ore in a disseminated copper deposit. Nevertheless there is a wide range of choice as to the amount of coal finally recovered (60 to 80 per cent of the total coal in the ground in American mines against 90 to 95 per cent in some European mines) and the deposit is by no means a "reservoir" of predetermined capacity. Considerable attention has been given to the level of recovery in the petroleum industry but the analysis proceeds through the rate of recovery and the close relationship between cumulative yield and rate. Rarely discussed but increasingly important are the purely level-of-recovery decisions surrounding tight, lean, or [illegible] sands, low-grade crude oils and gases, small traps, deep horizons, and similar conditions. S. V. Ciriacy-Wantrup in *Resource Conservation Economics and Policies* (Berkeley and Los Angeles, 1952), pp. 31, 74-75, 209-12, does distinguish between "cumulative use" and "rate of use" and discusses "cumulative use" as it relates to "rate of use" but turns his attention almost entirely to the latter.

100 per cent of the valuable contents from the ore would be privately and socially unjustifiable for it would entail the waste of other factors of production. Furthermore it is not always true that unmined low-grade ore and the metal remaining in incompletely treated tailings will be lost forever. Some placer gold deposits in California and elsewhere have been reworked several times; the Greater Butte Project is converting several million tons of formerly submarginal ore and rejected waste into mineable ore,[5] and some mines in the Tri-State district maintain their completed stopes in a workable condition in the belief that future demands may warrant the recovery of remaining low-grade ore.

III. *Cost Terminology*

In the following analysis costs of production include all the costs of discovering, purchasing or otherwise acquiring the right to mine, of developing and equipping, of mining and extracting the valuable product, and of marketing it. An unusually large proportion of the costs in mining are sunk costs not affected by the rate or the level of production in the short run. Other costs such as those of pumping, the minimum amount of management, obsolescence and interest are "fixed charges" in the customary sense because they must be carried at a more or less fixed rate per day if the mine is to produce at all. Their total, therefore, is directly proportional to the time required to exhaust the ore body. Still other costs may at times decrease and at other times increase on a per ton basis as either the rate of recovery or the level of recovery is raised. In grouping all these into fixed and variable costs, it is necessary to refer to costs as a total for the expected life of the mine or part of a mine (*i.e.*, unit cost times the total output to be obtained from the mine) rather than to total cost per unit of time (*i.e.*, unit cost times output per unit of time), as is ordinarily done, because we are dealing with a body of ore and both the rate at which it is mined and the total number of units of output to be obtained from the mine can be changed independently. The term *fixed cost* (*FC*) as used here, therefore, refers to costs which remain constant as a total for the life of the mine in spite of wide variations in *either* the rate or the level of recovery. They include mainly the investment of capital in acquiring the property, in developing and equipping the mine and in the recovery plant. Average fixed cost per unit of product is decreased by increasing the level of recovery but not by increasing the rate of recovery. All other costs are variable costs. Costs which vary as a total for the life of the mine with respect to both the rate and the level of recovery are termed

[5] J. B. Hattl, "Greater Butte M... Output Nears 10,000-ton Mark," *Engineering and Mining Jour.*, Apr., 1953, CLIV, ...

rate-level-variable costs (VC_{RL}). Among these are ordinary depreciation on plant and equipment, most direct labor and material costs and the "fixed charges" mentioned above. In general a rate-level-variable cost will vary by different amounts and perhaps in opposite directions (*e.g.*, pumping) with a given proportionate change in the rate and in the level of recovery. The two remaining categories are minor. *Level-variable costs* (VC_L) vary as a total for the life of the mine with the level of recovery but remain fixed with changes in the rate of recovery and include, for example, royalties on the gross product and costs of underground surveying, sampling and geology. *Rate-variable costs* (VC_R) vary as a total with the rate of recovery only.

Neither costs nor revenues are discounted to present value in the analysis. Instead, the resulting alternative profit streams are compared in the light of the presumed time preference of the recipient. This is in line with mining practice and it avoids the very cumbersome calculations required if alternative costs and revenues are discounted. Unless stated otherwise, product price and factor costs are assumed to remain constant not only currently but for the life of the mine.

IV. *Alternative Rates of Recovery: Case 1*

In a gravel deposit of approximately uniform quality and accessibility throughout, or in an equally uniform placer gold deposit, or even in a single underground block of ore of fairly uniform grade and accessibility, it is highly practical to set some limits to the grade and tonnage of material to be mined and to treat these limits as fixed quantities in further calculations. Where the grade or quality of the material is quite variable, it is still practical to set such limits, provided that a reasonable knowledge of the deposit or portion of a deposit is to be had; and such limits are commonly set. Even where the accessibility and therefore the cost of mining also vary within the deposit, as from level to level or from wide veins to narrow veins, it is still practical to plan on a particular total tonnage composed of a particular grade or grades of ore, all more or less well known according to the geological character of the deposit and the amount of exploration done on it. The per cent of extraction of the valuable product from the mined ore may likewise be set, at least roughly, through the selection of a particular kind of recovery plant. And all this is commonly done. In total it all amounts to fixing the level of recovery for the deposit or portion of the deposit.

As in all productive activities, attempts to increase the rate of output from a given plant or piece of land (the ore body) tend to encounter increasing returns per unit of expense at very low rates of output, and diminishing returns at higher rates. Decreasing costs result from a better coordination of men, materials, and machines as plant capa...

approached, as in the usual case, and from a reduction of total "fixed charges" through shortening the life of the mine. Total fixed costs are not spread because the total product is fixed. The economies of speed are opposed and eventually outweighed by diseconomies of overcrowding and the decreasing advantages or the impossibility of further enlarging mine openings or of adding to the number of working faces or transfer openings in the given ore body, of using more labor at each working face, more blast holes per round, more powder per hole and so on, or of further speeding up the drawing, hoisting, and transportation of broken rock. With very rapid extraction the cost of superintendence may also rise. The aggregate effect can be shown graphically as in

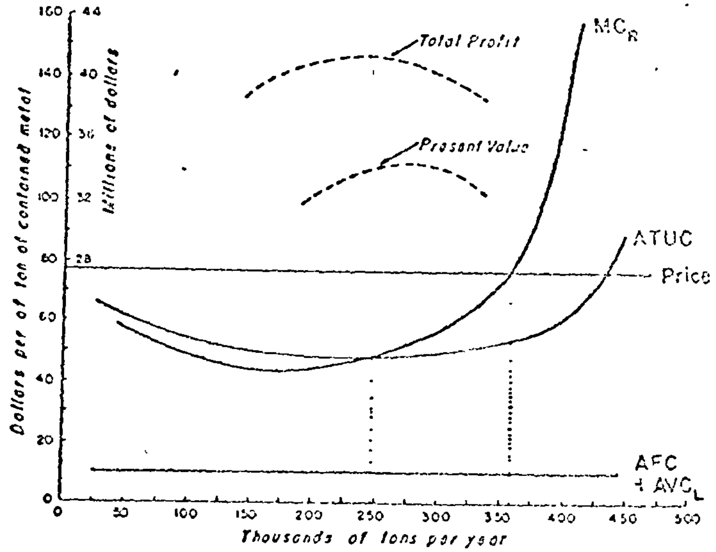


FIG. 1. ALTERNATE RATES OF RECOVERY: CASE 1

Figure 1 with the usual cautions regarding the smoothness and continuity of the curves. Since the total product is fixed and since fixed costs and level-variable costs are constant as a total for the life of the ore body, average fixed costs (*AFC*) and average level-variable costs (AVC_L) appear on the graph as a horizontal line. The average total unit cost curve (*ATUC*) is U-shaped reflecting the effects of economies and diseconomies. In practice there may be only a few alternative rates owing to the lumpiness of one or more of the variable factors, and cost-yield relations may be known over only a limited range and may not be strictly comparable. Nevertheless, estimates of the sort indicated here are made, approximately or precisely as the known facts allow.[6]

[6] The examples illustrated in Figures 1 and 2 are hypothetical, but to simulate reality as much as possible, particularly with regard to costs, total tonnages, and rates of recovery, the data are generalized from real examples.

Assuming pure competition, the largest *total* profit (or the smallest total loss if product price falls below the least cost per unit) on the stock of ore, irrespective of its distribution in time, is obtained by mining at the least-cost rate (250,000 tons per year in the example). The highest current rate of profit per unit of time, however, is obtained at the rate of recovery where marginal cost (MC_R) equals marginal revenue (*MR*) (360,000 tons per year in the example). The former "optimum" might be valid where a deposit is very small or where a discontinuous or intermittent demand precludes continuous operation at a higher than least-cost rate. The latter "optimum" is undoubtedly favored when rapid return of capital is required, but the total profits will have suffered by the change to a higher than least-cost rate. Since income in the future will be discounted at a rate of interest at least equal to that obtainable on a safe investment, and will in fact be discounted at a rate commensurate with the time preference of the investor-recipient, the true optimum rate of recovery is that which yields maximum present value. Ordinarily this falls between the least-cost rate and the rate at which profit per unit of time is maximized. It can be lower than the least-cost rate only if time discount is negative as might be the case, for example, if ore reserves are held as insurance against unfavorable developments elsewhere. It will be higher for higher assumed levels of recovery; and with a very high rate of discount, as when market risks are high, the optimum rate will rise toward the upper limit where marginal cost equals marginal revenue. It could not exceed this limit since both the life of the mine and the current rate of profit fall beyond this point.[7]

Except for gross maladjustments of mill capacity to ore reserves, the optimum rate of recovery in the short run is likely to be close to designed mill capacity. In many mines it is profitable to work the mill at greater than designed capacity but the upward range of possible rates

[7] Let d = discount rate, R = rate of recovery, L = level of recovery, n = life $= L/R$, TP = total profit, PV = present value.

Assuming simple interest; $PV = \frac{TP}{1+d(n)} = \frac{(AR - ATUC)L}{1+d(L/R)}$. As d approaches zero, PV approaches TP and is maximized at the least-cost rate. For positive d, PV is less than TP but increases at rates above the least-cost rate up to the point where the increasingly rapid rise in *ATUC* and the decreasingly rapid fall in n prohibits further gains. The rise in R is more effective when d and n are large and, hence, the optimum rate is higher.

In practice the income from a mine is usually treated as an annuity of A dollars per year at compound interest and n years in length. Hence, $PV = A\frac{(1+d)^n - 1}{(1+d)^n d}$, or according to the Hoskold formula $PV = \frac{A}{\frac{d}{(1+d)^n - 1} + d'}$ (where d is the "speculative rate" supposedly required by the investor and d' is the "safe rate"). See R. D. Parks, *Examination and Valuation of Mineral Property*, 3rd ed. (Cambridge, 1949), Pt. 2.

cannot be very large without additions to the plant itself. On the other hand, the structural and mineralogical characteristics of the ore body and the capacity of the underground plant governs the rate at which ore can be developed and prepared for mining. This in turn governs the number of stopes that can be kept in operation and thus limits the rate of recovery.

Most of the discussion by mining engineers on the optimum rate of working mines has been addressed to a longer-run yet recurrent problem of adjusting plant capacity to ore reserves. Even at moderate rates of discount, ore reserves that lengthen the life of the mine beyond thirty or forty years add little or nothing to its present value and usually it will pay to increase plant capacity before reserves are this large. Most, but not all costs become variable in this situation. Level-variable costs and costs that are more dependent upon land area than upon ore reserves or plant size, such as the costs of surface surveying will remain fixed. The cost of pumping may remain a "fixed charge" and continue to decline as a total with speed. On the other hand, the increasingly larger amounts of capital and labor must be applied to a given body of ore and the larger capital redemption charges must be borne by a given amount of product. Other advantages and disadvantages of larger scale are well known. The net result, an increase in short-run fixed costs offset by a decline in short-run variable costs up to a point of diminishing returns, can be represented by a set of longer-run cost curves in the traditional way. The three longer-run optima will be related to these longer-run curves in the same way as the short-run optima are related to the short-run curves just described although it should be remembered in calculating the present-value optimum that the time required to build a new plant may be longer for larger plants.

V. *Alternative Levels of Recovery: Case 2*

The assumption of a fixed ore supply now can be removed and attention given to the much neglected "level of recovery." Factors determining cut-off grade, workable limits of ore, and completeness of extraction will be examined, assuming at first that the rate of recovery is constant.

Grade of Ore. Other things being equal, the lower the grade or the poorer the quality of the ore,[8] the higher will be the cost of recovery of the valuable product. In most mineral deposits there are all gradations between waste rock and ore, and even in those where boundaries

[8] In metallic ores the term "grade" refers to the per cent metal content or the metal content per ton of rock, and the term 'quality" refers to such characteristics as mineral composition, presence of deleterious impurities, grain size, grain boundary relations, hardness, and larger textural features, all of which influence the costs of treating the ore and recovering the valuable product.

between waste and ore are sharp and clear, there can still be found wide variations in grade and quality of the ore material. To the extent that there is a choice of the grade of ore to be mined, there is also a choice of the total tonnage and accordingly of the total product recovered; the lower the allowable grade, the higher the tonnage. The proportionate increase in tonnage with a given proportionate fall in allowable grade will be different in different deposits and will depend on the relative amounts and distribution of higher and lower grades of ore. Thus in a few deposits with highly concentrated and sharply defined zones of mineralization the choice of grade may be quite insignificant, whereas in others it may be the major decision.[9] Fixing the cut-off grade in deposits of this latter sort may require several calculations of alternative tonnages and grades on the basis of different assumptions as to mineable limits. Simply by repeating the cut-and-try technique, a final acceptable estimate is reached.

Workability. Equally important with grade is the workability of the ore which is measured by the cost of physical removal of the rock. Accessibility from mine openings, thickness and regularity of the ore zone, hardness and toughness of the ore, presence of interfering structures such as faults or folds, weak ground which may require that pillars of ore be left or timber placed for support or which may cause dilution of ore with waste rock during mining, the amount of overburden, and several other factors all affect the workability of the ore, and must be considered when deciding what ore to take and what to leave. Of these variables, thickness may be as significant as grade and for low-grade ore bodies it may be the controlling factor. Costs are high in narrow stopes partly because the amount of ore broken per foot of hole drilled is less than in wide stopes. But frequently the largest economy from mining thicker ore bodies is a simple spread of more or less fixed preparatory costs. In almost every case the decrease in cost per foot of added width becomes less at very large widths and it may be reversed.[10]

[9] It has long been known that in the large copper deposits of the western United States the amount of ore available increases irregularly as the acceptable grade is lowered. At the Climax Molybdenum property according to W. J. Coulter, F S McNicholas, A D Storke, "History and Trend of Mining at Climax," *Min. and Met.* June, 1946, XXVII, 305: "Neither the footwall nor the hanging wall of the ore body have sharp definite limitations but values gradually grade off. On the footwall this gradation normally extends 25 to 50 feet whereas on the hanging wall values between 0.4 per cent and 0.2 per cent MoS_2 may extend as far as 200 feet beyond the present established economic limit."

[10] For details including cost curves see, C. F. Jackson and J. H. Hedges, *Metal Mining Practice,* U. S. Bur. Mines, Bull. 419 (Washington, 1939), p. 298; F. W. MacLennan, *Miami Copper Company Method of Mining Low-Grade Orebody,* Am. Inst. Min. & Met. Engrs, Tech. Pub No. 14 (1930), pp. 42-43; J. R. Finlay, *The Cost of Mining* (New York, 1920), p 105, and W. J. Busschau, *The Theory of Gold Supply* (London, 1936), pp. 93-95

The Blending Concept—"Controlled Mining." Variations in the grade and in the workability of ore may go hand in hand or they may partly compensate each other. However, ores of many different grades and many different costs, but sufficiently similar in other qualities to be amenable to the same treatment process can be mixed or blended to produce a mill-feed of uniform grade. This balancing process, sometimes known as "controlled mining," is an almost universal necessity in metal mining and is frequently applicable to nonmetallics as well. Blending often permits profitable recovery of otherwise submarginal ore. The low-grade or high-cost ore may be sufficiently near higher-grade ores that the same mine workings can be used for both, or it may be immediately adjacent to the high-grade so that advantage can be taken of the nonproportional rise in mining costs with greater thickness, or it may be "sweetened" by adding high-grade ore from a relatively small zone in the deposit merely to obtain the advantages of large-scale mining and processing.[11] In such cases and in general, unless separate mining or separate mining and recovery is both possible and necessary, it is incorrect to assume that each grade or block of ore must always stand or fall according to its own qualities. The questions of whether present value considerations justify the neglect of low-grade ore, and whether ore which remains submarginal in spite of its proximity to higher-grade ore should be mined are discussed in more detail below.

Completeness of Extraction. Complete removal of all available ore from the mine or complete extraction of all the valuable product from the mined ore is never practical. Costs per unit recovered rise almost continuously and usually with increasing steepness as attempts are made to increase the per cent extracted. It may be more profitable to recover only most of the mineral content cheaply than to recover nearly all of it at a high cost per unit. In the short run, with the recovery plant given, the per cent extraction of metal will depend to some extent on the grade of ore itself. Also the mining method usually limits the recovery of ore in the mine. Moreover the choice of the mining method is itself influenced by the grade of ore and the distribution of ore minerals.[12]

Cost-Yield Relations. Now, combining these variables, the economics and diseconomies of a higher level of recovery (at a given rate of recovery) may be viewed as a whole. Fixed costs and rate-variable costs, since their total is independent of the tonnage mined, are spread over the larger product at higher levels of recovery. Each development

[11] *Cf.* E. G. Lawford, "How Much of the Vein is Ore?" *Engineering and Mining Jour.*, Jan., 1928, CXXV, 54-55

[12] See Jackson and Hedges, *op. cit.*, pp. 282-91, for a comparison of stoping methods; pp. 305-11 for selection of the stoping method, especially pp. 306-7 respecting grade and distribution of ore minerals; and p. 385ff for economic factors in ore dressing.

heading, haulageway, transfer, chute, raise or shaft can be made to handle more material during its useful life. The acceptance of lower grades of ore may permit advantageous increases in the size of mine openings, particularly in the widths of stopes or of pits, which may result in greater efficiency and lower costs up to the point where problems of support and handling materials over large distances outweigh the advantages. In longer-run situations with the plant variable, relatively cheap systems of mining such as block-caving, diamond-drill stoping, even open pit operation might be substituted for costly underground selective mining. Parts of the underground plant—shafts, main haulageways, pumping and ventilation equipment—may not need to be augmented greatly to accommodate a much higher level of recovery at least within wide limits and, even with the plant variable, a few costs including rate-variable costs may remain fixed. Instead of only one kind of treatment process it may be practical to have modifications for special varieties of ore, or to recover minor by-products that in a small operation could not be saved profitably. Unlike the longer-run modification of Case 1, the added capital costs here are carried by a larger total product. These and other economies of larger scale, if they operate at all, sooner or later will be offset by diseconomies of mining lower-grade and less accessible ore and of attempting to extract larger proportions of the product from the mined ore. At very large scales the advantages of a very large, highly specialized management may tend to break down and in longer-run situations the greater delay usually required for construction of larger plants will increase the interest on borrowed capital and decrease present value.

The hypothetical short-run case illustrated in Figure 2 assumes an irregular ore body containing an available maximum of 150,000 tons of metal which has a price of $14 per ton. Ore boundaries are gradational and a variety of grades of ore and subore occur in a roughly zonal arrangement. There are numerous choices, then, between "high-grading" certain very rich portions and nonselectively mining the whole ore body. Extractions of, say, 75 to 90 per cent of the metal in the ore are feasible. It is estimated that regardless of the tonnage recovered, a total fixed cost of $100,000 will be necessary for equipment and primary development. Again this is total fixed cost for the life of the ore body. Note that the abscissa represents amounts rather than rates of output and the units are tons of total product at alternative levels of recovery. Marginal cost (MC_L) and marginal revenue, therefore, are calculated with level of recovery rather than rate of recovery as the independent variable.

In this case, with the price given and assumed constant for the life of the ore body, profits per unit of product are again largest where average total unit costs are a minimum (at a total recovery of about 57,000 tons in the example). Since the rate of recovery is assumed to be the same for alternative levels, this is also the level at which current rate of profit per unit of time is maximized. But total profits are largest where marginal cost equals marginal revenue (at a total recovery of about 87,000 tons in the example). This latter is the profit optimum if time distribution of earnings can be neglected, *i.e.*, if the discount rate is zero, or if the difference in the time required to mine alternative

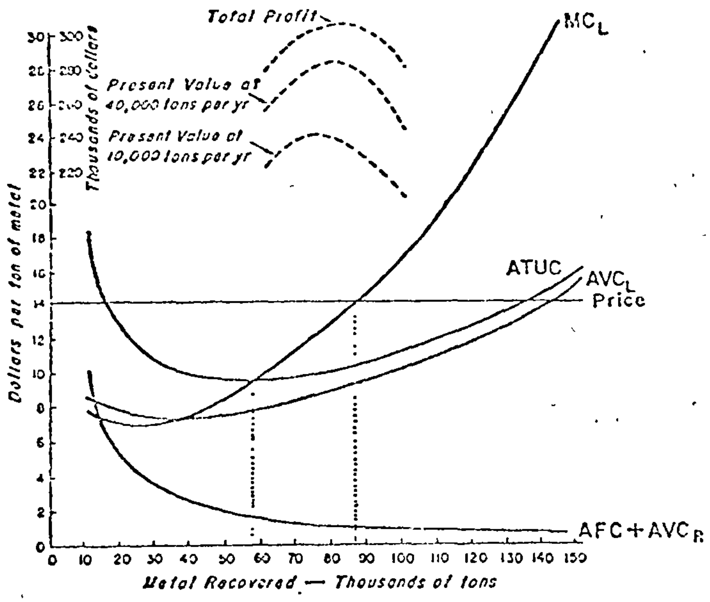


FIG. 2. ALTERNATE LEVELS OF RECOVERY: CASE 2

amounts in the relevant range is not great. Again the general optimum is that which yields maximum present value and, as in Case 1, if the discount rate is positive this must lie between the least-cost level and the level at which marginal cost equals marginal revenue. It will approach the least-cost level with very high discount rates but cannot fall below it since both the life of the mine and the current rate of profit decline below this level. It can fall above the upper level only if the discount rate is negative since above this level the miner would be sacrificing total returns for a longer life. For a particular discount rate, the optimum lies closer to the upper limit when the assumed rate of recovery is higher because the extra time required to mine at a higher level and the consequent loss through time discount is less at high rates of recovery than at low rates.

VI. *Jointly Alternative Rates and Levels of Recovery: Case 3*

The analysis so far has been restricted to alternative rates at given levels of recovery and alternative levels at given rates of recovery. Since most variable costs vary with both the rate and the level of recovery a change in either rate or level will alter the position and shape of the average total unit cost curve with respect to the other. If for no other reason, therefore, the optimum rate of recovery can be expected to change as the level is changed and vice versa, although the changes need not be proportional nor even in the same direction.[13] In addition, even if the average total unit cost curve with respect to one variable were unaffected by changes in the other variable, the present-value optimum, as shown, in Cases 1 and 2 respectively, would tend to be at slightly higher rates for higher given levels of recovery and slightly higher levels for higher given rates of recovery.

A combined rate-level optimum can be calculated either by comparing alternative levels of recovery each at its optimum rate of recovery or by comparing alternative rates of recovery each at its optimum level, but it is usually more realistic to follow the former procedure. This is partly because, in looking ahead, much less is likely to be known about the ore body than about costs at various rates of recovery for given grades of ore and given sizes and locations of ore bodies. Possible levels fall roughly into only a few alternatives. Also the rate of recovery is frequently related to the level of recovery through technicalities of production. In some deposits containing metal sulphides, for example, too slow a rate may allow partial oxidation of the ore, thereby adversely affecting mill recovery, or even resulting in underground fires. Or overly rapid mining may result in excessive loss, through overbreak and dilution of the ore.[14] The opposite order of analysis is necessary for captive mines and in short-run situations where a treatment or fabricating plant

[13] Long-run situations in which the optimum rate changes in the same direction as the level and vice versa are rather common. Thus a larger plant to raise the rate of recovery may became practical only if large tonnages of low-grade ore are accepted and, likewise large-scale methods usually result in rapid extraction. On the other hand, short-run situations commonly arise in which any shift in the rate or level would encourage a shift in the opposite direction in the other variable. For example, with a mill working at or near its capacity rate, any significant decline in the grade of ore mined is likely to lower the optimum rate of metal or concentrate production. Such an inverse relationship between rates and levels reflects physical or financial restrictions in the particular mine and is more likely to arise, therefore, as higher and higher alternative rates and levels are considered. In a somewhat unusual sense this inversion might be thought of as another form of diminishing returns in mining.

[14] A typical example of the technical interdependence of rate and level is given by T. Ertl in "Planning for the Optimum Rate of Draw in Cave Mining," *Engineering and Mining Jour.*, July, 1942, CXLIII, 56.

must be kept operating at or near a certain scale. Here the mine is forced to maintain a given minimum rate of recovery and the level is adjusted within this limit.

As in Cases 1 and 2, the maximum-present-value optimum calculated, say, by comparing alternative levels each at its maximum-present-value rate, ordinarily occurs between the particular optima yielding maximum current rate of profit and maximum total profit.[15] It approaches the former as the discount rate approaches infinity; the latter as the discount rate approaches zero. Within these limits it will occur at higher levels and faster rates if the optimum rate of recovery increases as the level is raised, but could reach the level at which total profit is maximized only if the rate of recovery were to follow the level so closely that the life of the deposit would be the same at all levels, which is very unlikely if not impossible.

Optima under the assumptions of Case 3 are not necessarily at higher rates and levels than under the assumptions of Cases 1 and 2 respectively. Thus in a mine in which physical or financial restrictions are such that the optimum level falls as the rate of recovery is raised over the relevant range (footnote 14), the assumption of a fixed rate of recovery below the optimum rate justifies a (Case 2) level of recovery higher than the (Case 3) optimum level and vice versa. Removal of the restrictions which cause the inverse relationship between changing rates and levels, however, would bring about still higher optimum rates and levels of recovery. If, on the other hand, rates and levels tend to rise together over the range of choice, physical or financial conditions which fix the rate of recovery below the optimum rate justify a (Case 2) level of recovery below the (Case 3) optimum level. In other words, such restrictions as undercapitalization and insufficient plant may promote overly selective mining.

Relations between the variables and the three optima can be shown most easily on a three-dimensional block diagram. In Figure 3 rate and level are along the two mutually perpendicular horizontal axes and cost or price is along the common vertical axis. Price under the assumption of pure competition is represented by a horizontal plane and *ATUC* by a curved surface roughly the shape of an ellipsoidal basin. Figures 1 and 2, then are perpendicular planes through the solid figure parallel to the rate axis and parallel to the level axis respectively. (The right

[15] Maximum total profit is obtained at the least-cost rate and a level where MC_L (with respect to level) equals MR. Maximum current rate of profit occurs at a rate of recovery where MC_R (with respect to rate) equals MR and at a least-cost level or slightly above or below this level if the particular optimum rate of recovery rises or falls respectively with increasing level.

end of the block is a Case 1 diagram.) In calculating combined rate-level optima by comparing alternative levels each solution is in terms of a vertical graph (though not necessarily a plane surface) passing obliquely through the solid figure in such a way that the rate of recovery is always at the required optimum for each level of recovery. The graph used to find maximum total profit by comparing alternative levels at least-cost rates would pass through the trough of the *ATUC* surface in Figure 3. The graph used to find the maximum current rate of profit by comparing alternative levels at rates where MC_R equals MR would

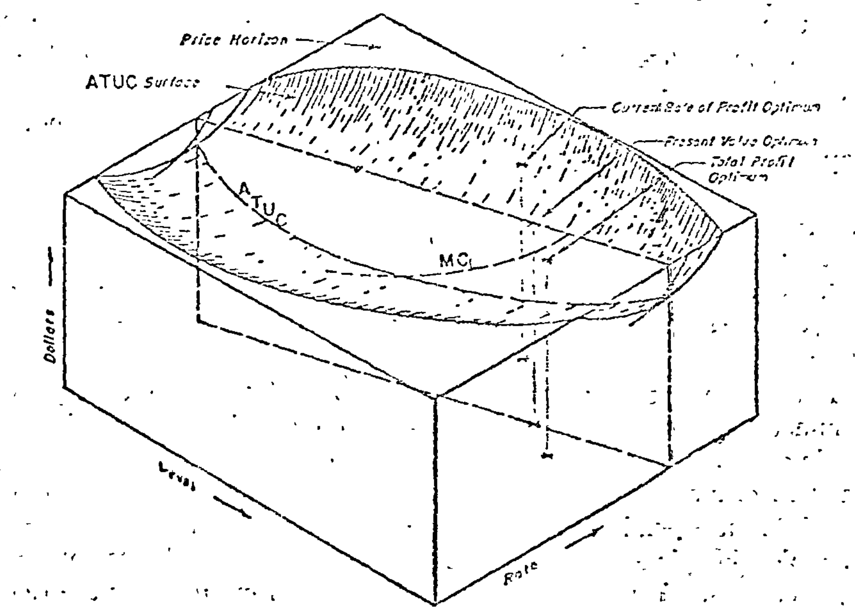


FIG. 3. THE RELATIONSHIP BETWEEN THE RATE AND THE LEVEL OF RECOVERY

be higher on the side of the basin away from the observer and the graph used to find maximum present value (dashed lines in the figure) would lie between these two. Figure 3 assumes that the optimum rate rises with rising level.

VII. *Uncertainty and Its Effect on the Level of Recovery*

The analysis has assumed that reasonably good working estimates can be made of the ore supply under consideration. Because the ore supply is seldom fully known either in grade or amount in the early stages of mine operation, analyses of this sort cannot usually be applied to the whole deposit. Usually some exploration proceeds hand in hand with exploitation, and mine operators are forced to build their plants in a patchwork fashion. Piecemeal development is also favored by the fact that ownership capital is likely to become more available and interest rates more favorable as the mine proves up substantial reserves of ore.[16]

In other words, because uncertainty beclouds estimates of ore supply, costs and market, miners are forced, just as economists are, to adopt a series of static solutions in a problem which is inherently dynamic. Of the two dynamic factors most unique to mining, namely depletion and discovery, discovery is the least amenable to prediction. The mere expectation of future ore discoveries may modify the calculations. In general, the more likely the occurrence of additional ore and the larger the likely occurrence, the greater will be the tendency to build a plant larger than the optimum for the proven ore supply.[17] Putting it another way, analyses of the type discussed above may include prospective ore, but only in amounts reduced in proportion to the uncertainty of its occurrence. Thus, by limiting the application of capital and thereby preventing full attainment of the economies of scale, uncertainty tends to increase costs and to decrease the total recovery from mineral deposits. Risks of factor supply and of the market may have a similar effect.

But to what extent does uncertainty also inhibit fuller recovery from a mineral deposit through its influence on the discount rate? Some mining engineers have argued that very large amounts of ore, perhaps one-half or more of the otherwise profitable ore, might more profitably be left in the ground because of time discount. Hotchkiss and Parks, for example, show in a hypothetical case that with Hoskold rates of 8 and 4 per cent (see footnote 8), the present value of mining only the richer

[16] An example of this step-by-step development, so common in mining, is the Climax Molybdenum property. According to Coulter, McNicholas and Storke, *op. cit*, pp. 303-7 and 317-19, prior to operation of the original 200-ton mill, proved plus probable ore reserves were estimated at 100,000 tons containing about 1 per cent MoS_2. By 1924, 400,000 tons had been removed and new ore bodies had been found. "For the first time a clearer picture of the structural features of the mineralization was developed." Shrinkage stope mining was introduced. "By 1930 the demands for molybdenum had so increased that the mine was producing 1500 tons per day." Over 100 million tons of low-grade ore had been found and new mining methods were developed which yielded greater efficiency, better recovery, and lower costs. By 1946, 43,000,000 tons of ore had been mined. During all this time, improvements, adaptations, and additions were being made to the mill to increase capacity as both the known ore reserves and the demand for molybdenum increased. In January, 1944, concentrator capacity was 20,000 tons per day.

[17] Operators of newly discovered mines in parts of northwestern Quebec frequently adopt a rule of thumb for treatment capacity which is based on experience at neighboring mines regarding continuity of ore in depth. Ore reserves are estimated for the upper few hundred feet of the mine which have been explored prior to building a mill and are expressed as the number of tons of ore per vertical foot of deposit. Daily mill capacity is then estimated at about forty per cent of the "tons per vertical foot."

one-half of a 4,000,000 ton ore body at $1.50 per ton profit in 20 yea[rs] exceeds the present value of profits from mining the whole 4,000,0[00] tons at $1.00 per ton profit over 40 years by about 20 per cent.[18] Cas[es] of this type are built upon three assumptions. First, plant capacity [is] small relative to total ore supply, and the optimum rate of mining do[es] not increase appreciably with higher levels of recovery. If, in t[he] Hotchkiss and Parks example, plant capacity were four times the ra[te] suggested or if the optimum rate of recovery were 150,000 tons per ye[ar] for the larger tonnage as compared to 100,000 tons per year for t[he] smaller, the advantage claimed for mining only the 2,000,000 to[ns] would disappear. The second assumption is that the kind of mining a[d]vocated will substantially increase the profit margin on certain hig[h-]grade portions of the ore body. Whether this is true will depend upon t[he] relative amounts and the distribution of high- and low-grade ore and [no] general rule as to its validity can be given. The third assumption is t[hat] a relatively high rate of discount should be applied to future earnin[gs] from a mine; commonly 8 to 10 and as high as 25 per cent is reco[m]mended. The traditional argument in favor of this assumption is th[at] mining risks are considerably higher, in general, than those in m[ost] other industries. But the highest risks of mining are strikingly as[so]ciated with prospecting for and developing new mines and the over-risk of mining should not necessarily be applied to an operating mi[ne] with substantial proved reserves. Furthermore, representing risk by [a] rate implies that the amount of risk is dependent on the time o[ver] which it operates, or that the likelihood of an unfavorable event [in]creases directly with time. The risks of mechanical breakdown [of] market, of factor supply or of social or political change are generally [of] this sort but these operate similarly in nearly all industries. The ri[sks] of mining that are large and unique are the geological uncertain[ties] surrounding any attempt to predict grades and amounts beyond [ex]posed faces. These are not time-dependent and they are not redu[ced] by shortening the life of the mine. On the contrary, many examples [can] be cited in which rapid recovery of only the rich ore would have [in]hibited thorough exploration and full development of the deposit. La

[18] W O Hotchkiss and R. D. Parks, *Total Profits vs Present Value in Mining*, Am. Min. & Met. Engrs, Tech Pub. No. 708 (Feb., 1936). The incomplete data pro[vided] suggest that the 4,000,000 ton alternative is close to the level at which marginal [cost] equals marginal revenue and that the present value optimum under the assumed condi[tions] would be slightly above the 2,000,000 ton level. See also, E. S. Berry, "Present Val[ue,] Its Relation to Ore Reserves, Plant Capacity and Grade of Ore," *Min. & Met.*, [...] 192?, III, 11; H. E. McKinstry, *Mining Geology* (New York, 1948), p. 474. F[or a] more detailed statement [of] the argument presented here see D. Carlisle, "Maximum [...] Recovery through M[ining] High-Grade and Low-Grade Ore Together Is Economi[cally] Sound," *Can. Inst. Min. & Met. Bull*, Jan., 1953, CXLVI, 21-27.

geological uncertainties do not justify a high discount rate in calculating optimum rates and levels of recovery. In fact, the hope of finding additional ore is one of the strongest incentives for raising the level of recovery and thus extending the life of known reserves.

Several institutional forces are likely to contribute further to the desire for maximum recovery, even to the extent that the level may be raised beyond the total-profit optimum. Among these are the advantages of a long-life stable property in producing a more desirable community, attracting a better class of worker, and in acting as a base for exploration and for holding staff and labor available for developing any new acquisitions. Large ore reserves serve as insurance against periods of unsuccessful exploration. If, through vertical integration, a smelter, or chemical or fabricating plant is dependent upon a stable output from the mine, conservation and maximum total output is a logical goal. The interests of management in the technical aspects of production and in the stability of their own and their employees' jobs act in the same direction. So also may concern for the national welfare. High income and excess profits taxes may discourage large profits in any one year.

On the other hand, there is one important kind of risk that can result in overly selective mining, the risk of price. For some mines, notably for relatively small producers of coal, oil, several nonmetallic and some metallic minerals, the risks of market may far exceed those of ore supply. The greater the likelihood that the market or the price may not last, as with war markets, with cutthroat competition, or with limited-term contracts, the more advantageous is rapid selective mining or gutting.

VIII. *Some Effects of Monopoly on Price and Rates and Levels of Recovery: Case 4*

Most minerals, and particularly such staple industrial minerals as iron, copper, lead, zinc and mercury, are producer rather than consumer goods and their consumption is more responsive to changes in general business conditions, therefore, than to changes in price. The assumption of perfect price elasticity in the above analysis is realistic enough, however, for gold mines, to some extent for mines which produce under a guaranteed price, and also for the great numbers of small mines contributing relatively minor amounts of metals such as lead, zinc, silver, or copper to a market dominated by a few large producers. It is not realistic for the dominant producers. Nor is it realistic for the larger part and sometimes almost the entire output of many industrial minerals which are produced under conditions of oligopoly or virtual monopoly.

Some of the effects of impure competition upon the rate and level of recovery are illustrated in Figure 4. Assuming a fixed level of recovery as in Case 1, the rate of recovery yielding the largest total profit on the stock of ore, neglecting time discount, is that at which the difference between average total unit cost and price is greatest, but this is not necessarily at the least-cost combination. It may be displaced to the left of the least-cost combination by an amount that will depend upon the shape and position of the average revenue curve relative to the average total unit cost curve (or it may be indeterminate). Similarly the rate of recovery yielding the maximum current rate of profit (dashed line) is lower than the rate at which marginal cost equals price. With positive discount rates the general optimum yielding maximum present

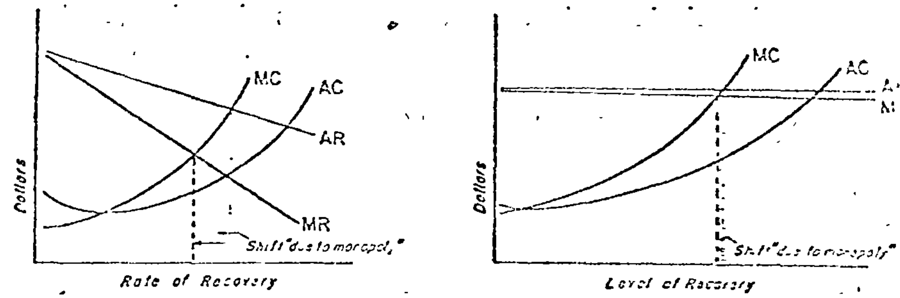


FIG. 4. A COMPARISON OF THE EFFECT OF MONOPOLY ON THE RATE AND THE LEVEL OF RECOVERY: CASE 4

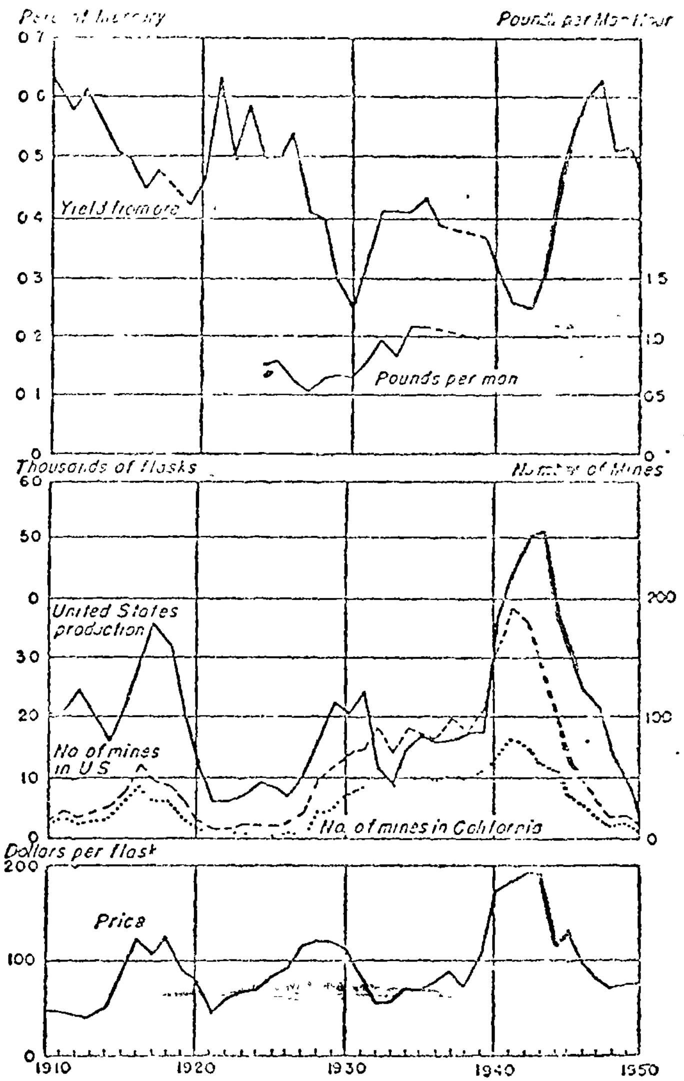
value again lies between the rate at which total profit is maximized and the rate at which marginal cost equals marginal revenue. Monopoly, as elsewhere then, unless it is more efficient than competition, would seem to promote lower rates of recovery and higher product price. It may be unwise, however, to apply this conclusion very rigorously or uniformly to the whole mining industry. Uncertainty and such dynamic factors as depletion and obsolescence, and particularly discovery and substitution may alter the effects of monopoly greatly. Production under conditions of declining cost relative to rate of recovery might result where competition is monopolistic and aggregate demand rather inelastic, as is probably the case for at least a part of the coal industry, and for several other nonmetallic industries. Or, as exemplified perhaps by the precious diamond industry, it might result from essentially pure monopoly and an aggregate demand that is thought to be highly inelastic and small relative to potential supply. A common situation in metal mining is that of a few workable deposits controlled by fewer producers selling a standardized product in an imperfect market; and without attempting

...her analysis here, it appears reasonable to believe that metal mines tend to be operated at rates of recovery above the least-cost combination at least in the short run. For most metal mines the uncertainty of ore supply is likely to be a much greater deterrent to operation at optimum rates of recovery, especially the long-run optimum, than is the monopoly-price effect.

The level of recovery, moreover, may be much less subject or nearly immune to the monopoly price effect. An increase in the level of recovery will affect price only in so far as it also increases the rate at which the mineral is supplied to the market, that is only in so far as the rate of recovery rises and falls with the level of recovery. Some reasons for expecting the optimum rate to increase and others for expecting it to decrease with the level were mentioned above. Under many circumstances, and particularly where only a part of a mine is under consideration, the level might be changed over a rather wide range without affecting the rate of recovery appreciably. In effect, then, demand may appear highly elastic relative to the level of recovery and the solution for optimum level under impure competition (Figure 4) would be substantially unchanged from Case 2. The present-value optimum would be at lower levels than the Case 3 solution because the reduced rate promoted by impure competition would intensify the effect of time discount; but at normal rates of interest the proportionate decline in optimum level from this secondary effect would be much less than the proportionate decline in rate of recovery (see Figure 2, for example). Furthermore, the ability of monopoly to hold price up may allow lower grades of ore to be mined than would be mined under competition.

IX. *Some Effects of Price Changes on Rates and Levels of Recovery and the Number of Mines*

One additional point illustrating the usefulness of a distinction between the rate and the level of recovery is worth noting. A change in price changes both the optimum rate and the optimum level of recovery but in practice the effect may be one-sided. As might be expected, the adjustment to lower grades under the stimulus of higher price is apparently much more difficult than the adjustment to higher grades when price falls. Moreover, rising price brings more and more formerly submarginal mines into production, and at the same time, because of the increasing steepness of cost curves, it encourages successively smaller increments in the level of recovery and usually in the rate of recovery in operating mines. It is rather common, in fact, for the number of mines to increase more rapidly than total output, many of the new mines being opened prematurely only to shut down after getting out



Source: Minerals Yearbook

FIG. 5. YIELD OF ORE, PRODUCTIVITY, PRODUCTION, NUMBER OF MINES AND PRICE FOR MERCURY

some of the higher-grade ore. The upward trend in total output tends to be continued then by the larger mines, both new and old, even as the number of mines decreases. Should price again fall, each plant erected under the stimulus of higher price either continues to produce at its own short-run optimum rate and level of recovery, thus contributing to overcapacity in the industry, or is forced to shut down and perhaps to allow flooding and collapse of mine workings and loss of ore. The record for mercury production (Figure 5) is particularly illustrative of these relations inasmuch as there have been large fluctuations in price, the geologic occurrence is such that production is sensitive to price, and the record for mercury has not been distorted by special premium prices for marginal producers. With each substantial fall in the price of mercury, operators have turned almost immediately to higher-grade portions of the deposits, shutting down low-grade stopes and reducing the total output of mercury. The opposite adjustment to lower grades of ore and higher rates of output has tended to lag behind price rise, partly because of rigidities in mining and partly because of the fear that high price will not last.

Before extending the theory much further it would be desirable to know more about the opinions and practices of mine operators. How widely some of the concepts outlined in the paper are recognized and used in approximating optimum conditions is not clear. Very little is written about determining optimum grades, for example, and what appears is contradictory. Undoubtedly in many operations the sole criterion for choice is maximum current rate of profit. Yet this is the least defensible in theory. In all cases the current-rate-of-profit optimum occurs at levels well below total-profit or present-value optima. In practice, however, the hope of more or better ore or of favorable price changes and the fear of the opposite, and several noneconomic factors may have as much influence on rates and levels of recovery as the choice between optima. Nevertheless, recognition of the two independent variables and the use of univariant (Cases 1 and 2) and bivariant (Case 3) models would appear to be most useful.

Evaluating the economics of mine development

By BRIAN W. MACKENZIE

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The following paper describes the use of economic evaluation techniques in making a mine development decision. These techniques are derived from capital investment theory. In selecting mine development alternatives, two things must be measured: expected profitability, involving the concepts of cash flow and time value; and uncertainty.

The mine development decision is made on completion of exploration to determine whether or not a mineral deposit should be developed to production and, if so, how. This involves the selection of an optimum mine development alternative from a number of technically feasible alternatives and the comparison of this optimum with those of other available investment opportunities. If the optimum is sufficiently attractive, the deposit is developed to production. The selection process is carried out within a framework of corporate objectives and resources.

The mine development decision is based on the balancing of two parameters — rate of production as reflected in mine capacity and cut-off grade. On the basis of available information and experience, detailed revenue and cost estimates are made for each feasible combination of mine capacity and cut-off grade. Economic evaluation techniques are applied to these estimates to reduce each alternative to economic criteria that will provide support for a sound investment decision.

The problem of determining optimum mine capacity and cut-off grade has been analyzed by mining engineers and economists for purposes of assisting with both corporate and mineral policy decisions (see reference list). Their work has generally assumed either that certainty conditions prevail with respect to reserve, cost and revenue inputs at the time of evaluation or that uncertainty exists but can be ignored. The optimum mine development alternative maximizes profitability.

In reality, the mine development decision is based on limited information. Ore reserves, mining costs and mineral markets are not assured, they are uncertain and are realized, to a greater or lesser extent, in future time periods. Therefore profitability for decision purposes should more realistically be expressed as an 'expected profitability'. Associated with expected profitability is a degree of uncertainty which reflects the reliability of information at the time of evaluation. For purposes of comparing and selecting mine development alternatives, both expected profitability and uncertainty should be evaluated.

In addition to information which can be evaluated in terms of expected profitability and uncertainty, intangible or nonquantifiable information must also be considered in the decision process. The avalanche at Granduc, the flooding of West Driefontein, and the nationalization of nickel laterite mining operations in Cuba, illustrate the importance of intangibles in mining. Thus, expected profitability and uncertainty criteria must be tempered with a judgement of intangibles in making a mine development decision. The overall decision process is outlined in Figure 1.

The objective of this paper is to describe economic evaluation techniques applicable to the mine development decision in sufficient detail that they can be clearly understood and realistically applied. The techniques are not new and are not difficult to grasp. They are derived from capital investment theory and are generally applicable to the evaluation of investment opportunities. The application of these techniques in the mineral industry has been somewhat retarded by a belief that mineral industry investment is sufficiently unique to necessitate the application of special evaluation formulae. This belief has no basis in fact.

Measuring expected profitability

Cash flow and time value are the basic concepts used in the measurement of expected profitability. They are embodied in the most widely used techniques — the present value ratio and the rate of return.

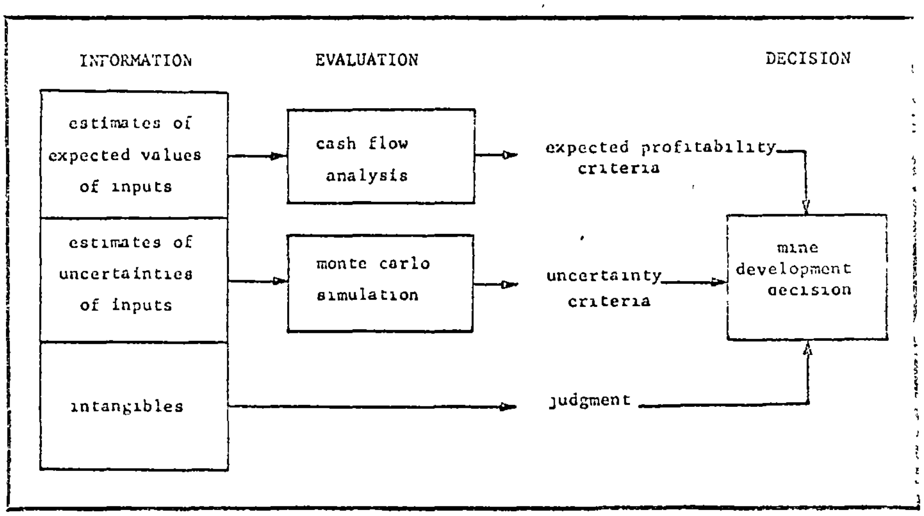
Cash flow

Cash flow measures the total income derived from an investment alternative per unit time after all real costs have been absorbed. Costs include both operating costs and capital expenditures.

An investment alternative may be viewed as a time distribution of cash flows as shown in Figure 2. In the preproduction period, capital is being invested and negative cash flows accrue. When production starts, positive cash flows commence as revenues exceed costs.

In the production period, cash flow is simply revenue or gross income per unit time minus all real costs — operating costs, tax payments and capital expenditures. However, the determination of cash flow is complicated by rules governing the derivation of tax payments. The following table illus-

Fig. 1 Outline of the overall decision process Expected profitability and uncertainty criteria must be tempered with a judgement of intangibles



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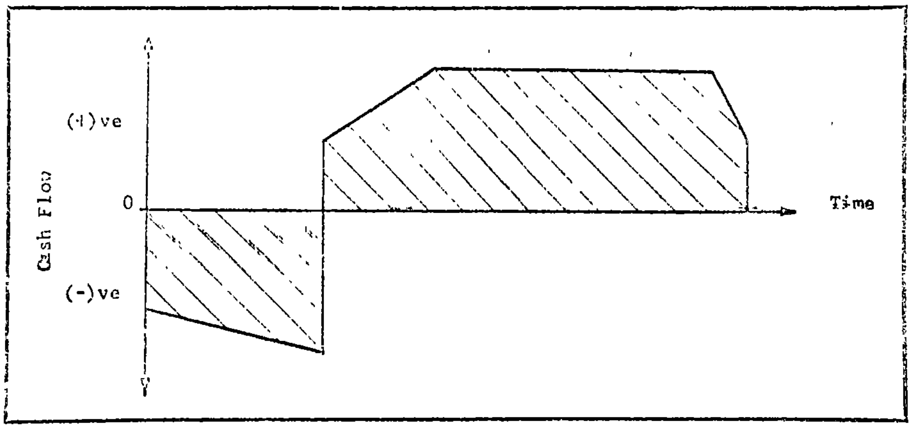


Fig. 2 An investment alternative may be viewed as a time distribution of cash flows In the pre-production period, capital is being invested and negative cash flows accrue When production starts, positive cash flows commence as revenues exceed costs

trates the determination of cash flow for a typical Canadian metal mine that is liable for both provincial mining tax and federal income tax.

1. Revenue
2. — *Operating Cost*
3. Net Income Before Allowances and Tax
4. — *Provincial Depreciation Allowance*
5. Net Income Before Processing Allowance
6. — *Provincial Processing Allowance*
7. Income for Provincial Mining Tax
8. Provincial Mining Tax

3. Net Income Before Allowances and Tax
9. — *Provincial Mining Tax Allowable*
10. Net Income Before Federal Depreciation Allowance
11. — *Federal Depreciation Allowance*
12. Net Income Before Federal Depletion Allowance
13. — *Federal Depletion Allowance*
14. Income for Federal Income Tax
15. Federal Income Tax

16. Capital Expenditures

17. Cash Flow (1-2-8-15-16)

The treatment of depreciation and other tax allowances is a common source of confusion in cash flow analysis. Depreciation is a permissible deduction for income tax purposes. It is not a real cost of the operation but, rather, is an allowance for capital expenditures that have previously been incurred. For cash flow purposes, these capital expenditures are accounted for in the time periods in which they are made.

After estimating cash flow for each year of mine life, a total cash flow may be summed. A total cash flow which is negative clearly indicates that the alternative is uneconomic. On the other hand, a positive cash flow does not necessarily indicate an 'economic' alternative although it does indicate that revenues exceed costs.

A detailed example of the determination of cash flow is given in the Appendix for a proposed metal mine in Ontario under existing federal and provincial legislation. Given estimates of expected revenues and costs, such calculations are primarily concerned with defining the legislative rules and following them in the determination of tax liability. The same procedure applies in other provinces and countries, and, for proposed federal tax changes.

Payout period

Payout period is the time required to pay back an investment from positive cash flows, measured from the start of production. Using this technique, the shorter the expected payout period, the more attractive the investment alternative.

The basic weakness of the payout period technique is that only part of the operating life of an alternative is considered. Payout measures return *of* investment rather than return *on* investment. Cash flows beyond the payout period are not considered. Yet these cash flows make an important contribution to expected profitability. For a mine development alternative of limited life, there is no fixed relationship between payout period and expected profitability. To illustrate this defect, consider two investment alternatives, A and B, requiring similar capital expenditures. Alternative A has an estimated payout period of 3 years and a productive life of 5 years Alternative B has an estimated payout period of 5 years and a productive life of 20 years. While alternative A is preferred on the basis of payout period, alternative B would normally be preferred on the basis of expected profitability.

A second weakness of the payout technique is its inability to account for differences in the rate at which an investment is returned over the payout period.

For these reasons payout period can only be looked on as a crude investment technique with little analytical power.

Time value

The estimation of expected annual cash flows is the starting point for the measurement of expected profitability. However, differences in the time distributions of cash flows among alternative investments must also be considered because money has a time value. For example, a return of $100 in year 1 may be reinvested and appreciate to $150 in year 5. In this sense a return of $100 in year 1 is equivalent to a return of $150 in year 5.

The value of money at different points in time depends on the interest rate, the amount of money invested, and the time period over which it is invested. If the interest on an investment is not repaid immediately, it is normal to think of this interest itself earning interest in subsequent periods. This is the concept of compound interest.

Using the compounding process, it is possible to link the present value of an investment with its future cash flow at any point in time, provided the rate of interest is known. Alternatively, if the present value of an investment and its future cash flow are known, the interest rate that will make the two equivalent can be determined.

In the evaluation of investment alternatives it is usual to determine the present value of a future cash flow. The conversion of a future cash flow to a present value is known as 'discounting'. The formula is:

$$PV = \frac{CF}{(1+i)^n},$$

where: PV = present value of a sum of money,
CF = future cash flow equivalent to present sum PV,
i = interest rate per interest period,
n = number of interest periods.

The interest rate is normally defined on an annual basis. This means that interest only accrues at the end of each year. For most practical purposes this is a reasonable approximation.

For example, the present value of a cash flow of $1,000,000 realized in 8 years using an interest or discount rate of 10% per year is $1,000,000/(1 + .10)^8 = $466,500. The present value factor, $1/(1 + i)^n$, can be obtained from standard discounting tables that are found in texts on mine valuation and engineering economics.

Present value ratio

The present value ratio requires a predetermined interest rate. Expected positive and negative cash flows are discounted at this rate and the annual present values are summed to obtain the present value of the investment alternative. If the investment alternatives being compared embody different capital investments it is incorrect to compare them on the basis of present value itself because a higher present value may result from a larger capital investment and, thus, present value does not necessarily reflect expected profitability. Therefore, present value is divided by the absolute present value of the negative cash flows (discounted investment) to determine the present value ratio.*

Present Value Ratio =

$$\frac{\sum_{x=1}^{n} \frac{CF_x}{(1+i)^x}}{\sum_{x=1}^{p} \frac{CF_x}{(1+i)^x}}$$

where: CF_x = annual cash flow in year x,
i = predetermined discount rate per year,
n = total life of alternative in years,
p = pre-production period in years.

Using this technique, the investment alternative with the highest present value ratio is preferred with respect to expected profitability (see Table I).

The following example illustrates the calculation of the present value ratio. A capital investment of $10,000, expended in year 1, is expected to generate positive cash flows of $3,000 in each of the following five years. The desired discount rate is 12% (see Table II)

The present value ratio is a cumbersome criterion because it embodies both the ratio itself and the predetermined discount rate. Its basic weakness is the need to select a discount rate. The discount rate is normally defined as the company's cost of capital or the opportunity cost of alternative investment opportunities in which corporate funds could be placed The opportunity cost concept requires that the alternatives have comparable uncertainties and that a stream of present and future investment opportunities be considered simultaneously at the present point in time. Obviously, this can only be done in an approximate way.

Often the selected discount rate is adjusted to account for the uncertainty of an investment alternative. More uncertain alternatives are thereby discounted at higher rates. This is a misleading and arbitrary way of allowing for uncertainty. It implies that the amount of uncertainty is dependent on the time over which it acts and that the likelihood of an unfavourable event increases directly with time. While the uncertainties of mechanical failure, shifts in market demand, and political change are generally of this type, geological uncertainties, often considered to be the most important in mining, are not. In any case, expected profitability and uncertainty are independent indicators of the worth of an investment alternative. Thus, they should be separately measured. To combine them in the evaluation process is to obscure the true basis for the investment decision.

TABLE I

Alternative	Present value i = 10%	Discounted investment i = 10%	PV ratio
A (preferred)	$2,000	$ 5,000	0.4
B	$2,000	$10,000	0.2
C	$3,000	$10,000	0.3

TABLE II

Year	Cash flow $	PV factor i = 12%	Present value $
1	−10,000	.8929	−8,930
2	3,000	.7972	2,390
3	3,000	.7118	2,130
4	3,000	.6355	1,910
5	3,000	.5674	1,700
6	3,000	.5067	1,520
			720

Present Value Ratio = 720/8,930 = .031

Rate of return

The rate of return is the discount rate that equates the present value of negative cash flows with the present value of positive cash flows, i.e. the discount rate that produces a zero present value.

Rate of Return = r,

where:

$$\sum_{x=p+1}^{n} \frac{CF_x}{(1+r)^x} + \sum_{x=1}^{p} \frac{CF_x}{(1+r)^x} = 0$$

Using this technique, the investment alternative with the highest rate of return is preferred on the basis of expected profitability.

The rate of return usually must be determined by trial and error method. Table III illustrates its calculation.

The attractiveness of the rate of return technique lies in its ability to reduce a stream of cash flows to a single measure that can be used to compare the expected profitability of investment alternatives The weakness of the concept is its implicit assumption that returns generated can be reinvested in opportunities offering a comparable rate of return. In many investment situations this may be a valid assumption but for companies engaged in relatively specialized projects, rate of return for individual investments may be highly variable.

*Alternatively, the present value ratio may be expressed as the ratio of the present value of the positive cash flows (rather than total present value) to the absolute present value of the negative cash flows. The two relationships result in equivalent expected profitability criteria.

Sensitivity analysis

Sensitivity analysis measures the effect of a given change in an input variable on profitability criteria. For example, sensitivity analysis may be used to determine the effect of a 10% decrease in price on rate of return for a particular investment alternative. In this way it is possible to define those input variables to which profitability is most sensitive. These strategic variables should then be given special attention in the investment decision process.

For example, consider the relative sensitivity of rate of return for a lead-zinc mine development to variations in power cost and combined metal price (see Table IV).

Obviously the forecasting of metal prices must be given considerable attention in the decision process. On the other hand, power cost is not a strategic variable.

Although sensitivity analysis gives some useful information, it does not measure the uncertainty of an investment alternative because it does not consider the probability of the given change in an input variable occurring and, thus, does not measure the probability of profitability departing from its expected value by the calculated amount. Sensitivity analysis only measures the relationship between an input variable and profitability. For example, to determine that a 20% decrease in price results in a 40% decrease in rate of return shows that rate of return is sensitive to price. But to use this information effectively in the investment decision, an estimate must be made of the probability of this event occurring. Is the probability .30, .01, or any other value? It is also necessary to know the cumulative effect on profitability of random variations in all the input variables. Thus, the usefulness of sensitivity analysis is limited.

TABLE III

Year	Cash flow : $	Trial rate = 15% PV factor	PV : $	Trial rate = 16% PV factor	PV : $
1	−10,000	.8696	−8,700	.8621	−8,620
2	3,000	.7561	2,270	.7432	2,230
3	3,000	.6575	1,970	.6407	1,920
4	3,000	.5718	1,720	.5523	1,660
5	3,000	.4972	1,490	.4761	1,430
6	3,000	.4323	1,300	.4104	1,230
			50		− 150

Rate of Return (by interpolation) = 15.25%

TABLE IV

Power cost mils	Rate of return %	Price ¢/lb	Rate of return %
6(−40%)	18.9	8.4(−40%)	3.3
8(−20%)	18.7	11.2(−20%)	10.8
10*	18.5	14.0*	18.5
12(+20%)	18.3	16.8(+20%)	25.9
14(+40%)	18.1	19.6(+40%)	33.0

*expected values

TABLE V

Cost inputs	Expected cost $/ton	Uncertainty estimate 90% Confidence interval	Standard deviation
Mining	4.00	±1.00	0.61
Milling	1.20	±0.10	0.06
Transportation.	0.60	±0.05	0.03
Mine overhead.	0.75	±0.20	0.12
Head office	0.40	±0.10	0.06
Total operating cost	6.95	?	?

Measuring uncertainty

The expected profitability criteria outlined above evaluate the expected result of a mine development alternative. The expected result is based on estimates of the expected values for all relevant problem inputs — ore reserves, mining costs, mineral markets, etc. These estimates are uncertain and, therefore, so is the derived expected profitability measure. Investment alternatives differ with respect to both expected profitability and uncertainty. Therefore, it is also necessary to measure their uncertainty. The uncertainty dimension is as critical a measure of the worth of an investment alternative as the expected profitability dimension.

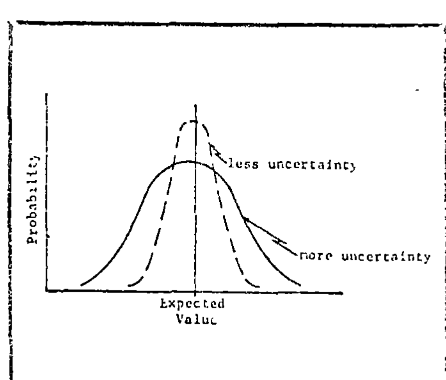
Uncertainty is difficult to measure quantitatively. It is useful to think of uncertainty as a probability distribution about an expected result. Flatter distributions indicate greater uncertainty, i.e. a greater chance that the actual result will depart farther from the expected result (see Figure 3). The estimated probability distributions of profitability criteria may be determined analytically or approximated using Monte Carlo simulation.

Analytical method

Given estimates of the expected value and uncertainty for the input variables in a decision problem, it is in some cases suitable to analytically evaluate the uncertainty of an economic outcome. For example, consider the estimates of operating cost components for a mining operation shown in Table V.

If the cost estimates are normally distributed and there is complete independence among them, the uncertainty of total operating cost (the economic outcome in this case) can be calculated as follows. A table of areas under the normal curve shows that limits for a 90% confidence interval lie 1.64 standard deviations above and below the expected value. This is used to calcu-

Fig. 3 Flatter distributions indicate greater uncertainty, i.e. a greater chance that the actual result will depart farther from the expected result



late the standard deviations of the cost components. The standard deviation of the total operating cost outcome is statistically defined as the square root of the sum of the squares of the input standard deviations

$$= \sqrt{(.61)^2 + (1.06)^2 + (.03)^2 + (.12)^2 + (.06)^2} = 0.68$$

Hence, the 90% confidence interval for total operating cost = \$6.95 ± 1.64 (.68) = \$6.95 ± 1.12. The limits provide a measure of the uncertainty of the cost outcome. Based on the estimates, there is 90% confidence that total operating cost will lie between \$5.83 and \$8.07.

This analytical procedure should be followed in any evaluation involving the addition and subtraction of input variables when normal distributions and independence are reasonable assumptions. However, for problems involving more complicated mathematical manipulations and skewed probability distributions, the application of analytical techniques is difficult. These circumstances generally include complete evaluations of the economics of mine development. In such cases the Monte Carlo simulation technique is usually applicable.

Monte Carlo simulation method

The Monte Carlo simulation method is a sampling procedure whereby realistic approximations of the uncertainty of profitability criteria are evaluated. Like the analytical method, it is based on the estimation of a probability distribution for each of the input variables in a decision problem. It is important to note that the result of the simulation — the estimated probability distribution of a profitability criterion about its expected value — is only as good as the input estimates. The input estimates are of necessity, subjective value judgements. In relation to the information usually available at the time a mine development is evaluated and the expertise that may be brought to bear on formulating reasonable estimates, it is generally recognized that the resulting uncertainty measure is better than ignoring uncertainty, assuming that it is equal between investment alternatives, or arbitrarily allowing for it in the discount rate.

Uncertainty is the result of variations in the value of an input variable which are caused by the action and interaction of many factors. This pattern of variations in the value of a variable is called 'random variation'. In the Monte Carlo method, random variation is simulated using 'random numbers'. Random numbers are numbers that have been generated in such a way that there is an equal probability of any digit appearing each time, regardless of any past appearances. Spinning a roulette wheel or throwing dice are possible ways of generating random numbers. Random numbers are used to obtain 'random samples' without actually taking a physical sample.

A simplified example is used to illustrate the Monte Carlo methodology. In actual practice the sampling procedure would parallel the cash flow and expected profitability calculations described in the previous section. Nevertheless, the simulation methodology is the same.

The example considers the application of the Monte Carlo simulation method to the evaluation of uncertainty for the profit of a proposed mine development. The mining operation would incur costs and the resulting mineral products would realize revenues. Costs and revenues are expressed in dollars per ton mined. In keeping with the simplified nature of the example, the profitability of the development alternative is measured in terms of profit per ton mined (profit = revenue − cost).

Estimates are made of the revenue and cost of the mine development alternative. These estimates are uncertain. For example, it cannot be said that revenue will be \$13/ton, cost \$9/ton and, therefore, profit will be \$4/ton. The following estimates are made.

1. The probability distributions for revenue and cost variables approximate normality about their expected values and are completely independent.*
2. Revenue is expected to be \$13/ton. It is 90% confident that the revenue outcome will be between \$10 and \$16/ton.
3. Cost is expected to be \$9/ton. It is 90% confident that the cost outcome will be between \$7 and \$11/ton.

The expected profit is obviously \$4/ton. To measure the uncertainty with which this expectation is held it is necessary to estimate the probability distribution of possible profit outcomes. The problem is illustrated in Figure 4.

The normal distribution is defined by two parameters — the expected or mean value (EV) and the standard deviation (S). The standard deviation describes the pattern of variation from the expected value. The estimate of expected value and the 90% confidence interval define the standard deviation for each input variable because as previously mentioned the properties of the normal distribution are such that,

Upper Limit = EV + 1.64S,
Lower Limit = EV − 1.64S.**
For the estimated revenue distribution,
S = (13-10)/1.64 = \$1.83/ton.
For the estimated cost distribution,
S = (11-9)/1.64 = \$1.22/ton.

It should be noted that the choice of a 90% confidence interval is arbitrary. Intervals of 80% to 95% are generally used. The interval chosen should be that which best suits the estimators in formulating subjective value judgments for the input variables. It should, therefore, define confidence limits that can be mentally grasped. Extreme values of an input distribution — the limits of a 99.9% confidence interval for example — are difficult if not impossible to comprehend.

The Monte Carlo method randomly samples values from each input distribution. The sample values are then

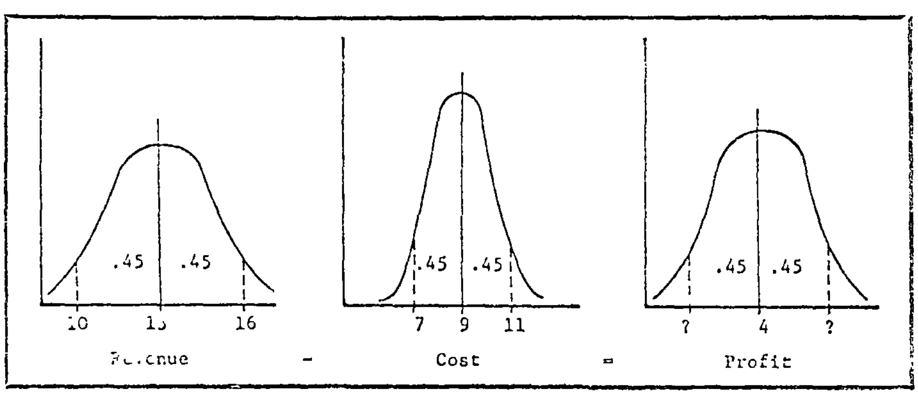


Fig. 4 To measure the uncertainty with which an expectation is held it is necessary to estimate the probability distribution of possible profit outcomes

*Revenue and cost variables usually exhibit probability distributions with a central tendency about their expected values, i.e. the probability of outcomes closer to the expected value is greater than farther away. The normal distribution is one such distribution. Furthermore, being a symmetrical distribution it is easy to work with. However, it requires the estimation of upper and lower limits for the input distributions that are equidistant from the mean. This is often not realistic. The split-normal distribution and the discrete distributions subsequently described do not have this limitation.

**Obtained from a table of areas under the normal curve.

combined to give a sample value for the profitability criterion (in the example, sample R — sample C = sample P). This process is repeated a large number of times. A sample size of at least thirty is required so that sampling errors do not seriously detract from the significance of the results. The sample values of the profitability criterion are used to estimate the expected value, standard deviation, and confidence limits of the profitability distribution. The steps involved in the analysis are listed below.

1. A cumulative probability function is determined. The general form of such a function for the normal distribution is shown in Figure 5. By using the function a cumulative probability value may be related to a deviation from the expected value. It should be noted that the curve is steepest around the expected value and is considerably flatter for points more than one standard deviation from the expected value.

If values between 0 and 1.000 are randomly selected for cumulative probabilities along the vertical axis, it is more probable that deviations closer to the expected value will be selected than deviations farther from the expected value. For example, the cumulative probability interval which corresponds to the range of –S to EV is approximately 0.16 to 0.50. In contrast, the interval which corresponds to the range of –2S to –S is approximately 0.02 to 0.16. Therefore, the probability of a sample value occurring in the simulation is the same as in the real world, provided the estimated input distributions realistically describe the actual probabilities.

2. Random values of deviations from the expected value (random normal deviates) may be obtained by randomly selecting cumulative probability values from a table of random numbers and determining the corresponding random normal deviates from Figure 5 (see Table VI).

Random normal deviates may be applied to any normal distribution. The availability of tables of random normal deviates eliminates the need for carrying out the mechanics of steps 1 and 2.

3. The random normal deviates are used to derive random normal samples for the input variables. For example, if the expected revenue is $13.00 and the

TABLE VI

Random no.	Random cumulative probability	Random normal deviate
61357	614	0 289
80545	805	0 861
07293	.073	−1 454

estimated standard deviation is $1.83, then the random normal samples corresponding to the random normal deviates derived in step 2 would be calculated as follows:

Random Normal Samples

$13.00 + 0.289\ (1.83) = \$13\ 53$
$13.00 + 0\ 861\ (1\ 83) = \$14\ 58$
$13.00 - 1\ 454\ (1\ 83) - \$10\ 34$

4. The process of selecting random normal samples is repeated for each input variable, so that thirty or more sets of random normal samples are generated. When it is assumed that the input variables are independent of one another, new sets of random deviates must be drawn for each input variable.

5. Each set of random normal samples is combined according to the model being used to determine a random normal profitability sample. The random normal profitability samples are used to estimate the expected value, standard deviation, and confidence limits of the profitability distribution.

The above methodology is applied to the simplified example of a mine development alternative. Assuming a sample size of thirty, a typical Monte Carlo simulation is shown in Table VII. The following estimates can be made from the simulation results.

1. Estimated expected profit
= 116.72/30 = $3.89/on.

This compares with the calculated expected profit of $4.00/ton. The error in the simulated result is due to the limited sample size.

2. Estimated standard deviation of profit

$$= \sqrt{\frac{\Sigma P^2}{n} - \left[\frac{\Sigma P}{n}\right]^2}$$

where n = sample size

$$= \sqrt{\frac{632}{30} - \left[\frac{117}{30}\right]^2}$$

$= \$2.43/\text{ton}.$

3. Estimated 90% confidence interval for profit.

Upper Limit = 4.00 + 1.64 (2.43) = $7.98/ton

Lower Limit = 4.00 − 1.64 (2.43) = $0.02/ton

The estimated confidence limits reflect the uncertainty with which expected profitability is held. The lower limit is critical because it defines a minimum level of profitability which can be insured with a given degree of confidence. In the example, there is a 95% chance of at least breaking even on the investment alternative.

The calculated confidence interval ± $3.98, compares with ± $5.00 that would be obtained by combining best possible and worst possible limits. A

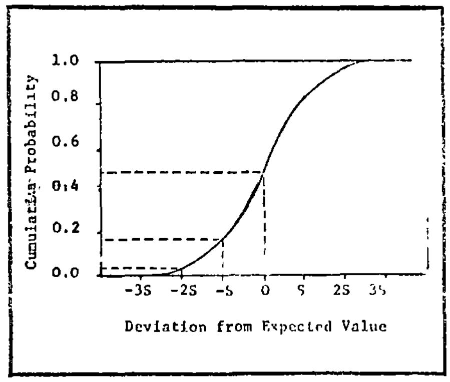


Fig. 5 By using a cumulative probability function, a cumulative probability value may be related to a deviation from the expected value

± $5.00 confidence interval would only be valid if the input variables were completely dependent with a high revenue sample dependent on a correspondingly low cost sample and vice versa. In the example, the revenue and cost inputs are considered to be completely independent. The rather close correspondence between the limits obtained under independent and dependent assumptions is due to the fact that in this simple example there are only two input variables In real world situations there may be fifteen or twenty input variables. More inputs increase the chance that individual sample deviations will cancel. This results in greater differences between the actual confidence interval and that obtained by combining best possible and worst possible limits.

The split-normal distribution

It is not usually realistic to apply the normal distribution to input variables. There is no reason why estimated upper and lower limits should be equidistant from the modal value.* For example, modal values for cost distributions tend to be closer to the lower limit than to the upper limit This implies a positively skewed probability distribution. Skewed distributions are difficult to work with. For simulation purposes it is convenient to approximate a skewed distribution by a split-normal distribution (see Figure 6). The split-normal distribution consists of the

*The mode is the value of a variable for which the probability of occurrence is a maximum, i e. the value corresponding to the peak of the probability distribution The expected value is the mean value of the probability distribution, the average value that would result from drawing many samples from the distribution. For symmetrical probability distributions, e g the normal distribution, the modal value and the expected value coincide. For skewed distributions, the two values do not coincide.

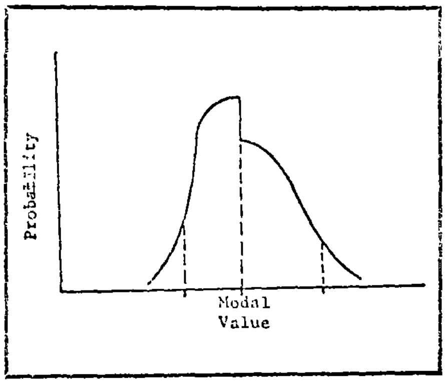


Fig. 6 For simulation purposes it is convenient to approximate a skewed distribution by a split-normal distribution

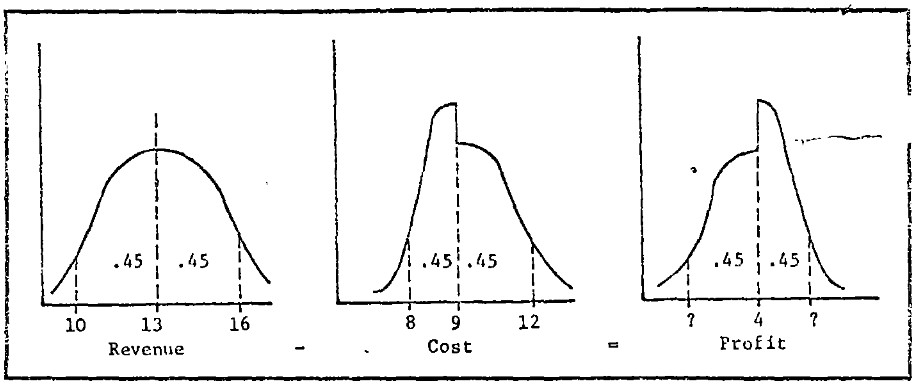


Fig 7 For purposes of Monte Carlo simulation, the split-normal distribution is treated in a similar way to the normal distribution

halves of two different normal distributions. The two normal distributions have the same expected value but different standard deviations. The expected value of the individual distributions corresponds to the modal value of the combined split-normal distribution. The expected value of the split-normal distribution occurs in the half of the distribution with the largest standard deviation.

For purposes of Monte Carlo simulation, the split-normal distribution is treated in a similar way to the normal distribution. The same cumulative probability function and table of random normal deviates are used. The only difference is that attention must be given to the half of an input distribution in which a random normal deviate is drawn so that the relevant standard deviation can be applied in determining the random normal sample value. Also, since the derived profitability distribution will also be split, estimates of its standard deviation must be made separately for each half of the distribution. A graphical example, similar to the one used for the normal distribution, is shown in Figure 7.

TABLE VII

	Revenue EV = 13.00, S = 1.83		Cost EV = 9.00, S = 1.22		Profit	
Sample	(1) Random-Normal Deviate	(2) Random R EV + S(1)	(3) Random Normal Deviate	(4) Random C EV + S(3)	(5) Random P (2) − (4)	P²
1	1 102	15 02	.393	9 48	5 54	31
2	.148	13 27	−.267	8 67	4 60	21
3	2.372	17 34	.195	9 24	8 10	66
4	− 145	12.73	.735	9 90	2 83	8
5	.104	13 19	1 925	11 35	1 84	3
6	1 419	15 60	.049	9 06	6.54	43
7	.069	13.13	.240	9 29	3 84	15
8	.797	14.46	.632	9 77	4 69	22
9	− 393	12 28	.939	10 14	2 14	5
10	− 874	11.40	.859	10 05	1 35	2
11	125	13 23	−1.389	7 31	5 92	35
12	−1 091	11 00	.270	9.33	1 67	3
13	2 304	17 22	1.517	10 85	6 37	41
14	−.961	11 24	− .935	7 83	3.41	12
15	−.783	11 57	−.804	8 02	3.55	13
16	.487	13 89	.331	9 40	4 49	20
17	− 299	12 45	026	9 03	3 42	12
18	1.831	16 35	− .844	7 97	8.38	70
19	.243	13 45	−.743	8 09	5.36	29
20	−2 181	9.00	2.261	11.76	−2.76	8
21	.154	13 28	558	9 68	3 60	13
22	−1 065	11 05	2 824	12.44	−1 39	2
23	1 083	14 98	2 380	11 90	3.08	9
24	.615	14 12	1 950	10 60	3 52	12
25	.178	13 26	− .661	8 19	5 07	26
26	−.507	12 07	.244	9 30	2 77	8
27	.362	13.66	1 049	10 28	3 38	11
28	775	14 42	− 585	8.29	6 13	37
29	818	14 50	− 873	7.94	6 56	43
30	014	13 03	.252	9 31	3 72	14
				Totals:	116 72	632

Discrete distributions

If information is sufficiently complete, discrete probability distributions of any shape may be estimated and used for the input variables. Probabilities must be estimated for each interval in the distribution. A graphical example is shown in Figure 8. A stepped cumulative probability function is derived from each input distribution for the purpose of drawing random samples. The result of the sampling procedure is an estimated discrete distribution for the profitability criterion.

Problems of realistic application

It should be obvious from the foregoing that the Monte Carlo simulation technique is statistically sound, mechanically simple and can accommodate any type of probability distribution. Problems of realistic application are associated with the information and estimation procedure that produces the required input estimates.

There is the point of view that uncertainty is embarrassing and should, therefore, be ignored. A single point estimate gives an impression of confidence and expertise that is not em-

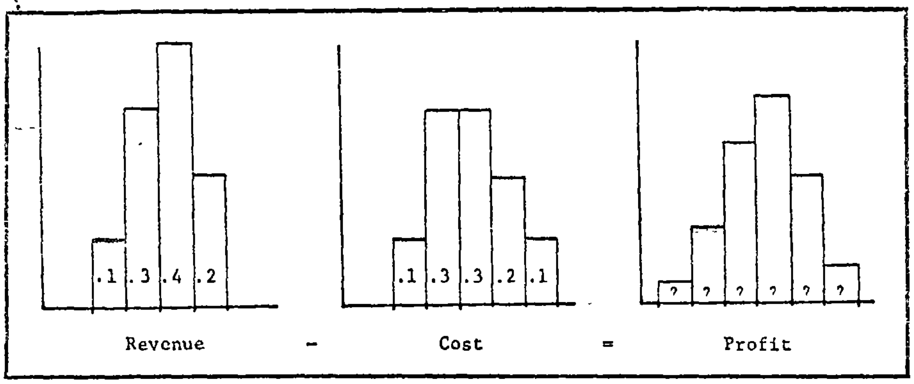


Fig. 8 A stepped cumulative probability function is derived from each input distribution for the purpose of drawing random samples

bodied in the admission that an outcome can vary within limits. Furthermore, rewards and retributions are sometimes such that safe conservative single point estimates are encouraged. This results in the evaluation of a lower confidence limit for profitability — one extreme point on the probability distribution of possible profitability outcomes. This type of evaluation will inevitably result in the rejection of uncertain but profitable investment opportunities.

Mine development decisions are based on limited information. Therefore, uncertainty is a fact of life. Both expectations and uncertainties should be measured wherever possible so that an estimate of the complete probability distribution of the profitability outcome can be evaluated. Expected value estimates should truly reflect expectations. Estimates of confidence limits should subjectively conform to the defined confidence interval. The averaging of estimates from a number of persons having expertise on particular aspects of the evaluation will improve the reliability of inputs and reduce bias. Intangible factors that cannot be quantified should nevertheless be given full consideration in the decision process. This type of evaluation provides a realistic basis for investment decisions.

The simple examples that have been presented to illustrate the Monte Carlo simulation technique assume that the input variables are completely independent. This is not always a reasonable assumption. For example, dilution and mining cost are dependent as are mill recovery and concentrate grade. In such cases, it is necessary to estimate the degree of dependence between input variables. Having done this, the dependencies are taken into account in the sampling procedure by having the random sample of a variable determine a restricted sampling range for any partially dependent variables.

Obviously there are investment situations where information and knowledge are sufficiently limited that the application of the Monte Carlo simulation technique is not meaningful. However, it is generally recognized that at the point in time when a decision is to be made on whether or not to invest large sums in the development of plant capacity, information should be sufficient to provide a basis for estimating both expected profitability and uncertainty. If this is not the case, further investment in information gathering and analysis may be warranted. The potential for uncertainty measurement is reflected by the fact that several Canadian mining companies are applying the Monte Carlo simulation technique, at least on an experimental basis, to mineral investment situations. It would be surprising if these efforts do not provide an improved basis for mine development decisions.

The optimum investment alternative

The economic evaluation of feasible mine development alternatives with respect to both expected profitability and uncertainty provides the basis for selecting the optimum alternative. On the one hand, the decision-maker, motivated by the desire for profitable investment, prefers a development alternative with a higher expected profitability to one with less On the other hand, he is averse to uncertainty and, therefore, prefers a development alternative with less uncertainty to one with more. Uncertainty aversion is due to a concern for the insurance of some minimum level of profitability with a given degree of confidence Thus, while uncertainty is measured by an estimated probability distribution about the expected value, its critical component is the lower confidence limit.

In special cases where one alternative is preferred to all others with respect to both expected profitability and uncertainty, the mine development decision is clear-cut. In the following example, alternative A is clearly preferred.

	A	B
Expected Rate of Return	25%	20%
95% Lower Confidence Limit	18%	15%

However, often one alternative will be preferred on the basis of expected profitability and another on the basis of uncertainty. The decision then becomes a matter of corporate preference. The preferred alternative for one mining company will not necessarily be preferred by another In the following example, the preferred alternative is not obvious.

	A	B
Expected Rate of Return	21%	26%
95% Lower Confidence Limit	15%	10%

Corporate preference depends on the interplay between the profit, growth and survival objectives of the company and the capital and skilled resources that it is able to direct towards their realization. A company's preference for investment alternatives may be conceptually expressed as a utility function. An example of a utility function is given in Figure 9, using rate of return as the profitability criterion and the 95% lower confidence limit as the uncertainty measure. The shape of the utility curves — convex towards the origin — indicates that a balancing of expected profitability and uncertainty is preferred to more extreme values of each. The mine development alternative with the highest utility is the optimum. Applying this particular utility function to the previous example, alternative B has a higher utility than alternative A and is, therefore, preferred.

The utility function does not replace judgment in the decision process, it simply formalizes the judgment so it can be consistently applied. Intangible factors that cannot be measured by quantitative evaluation techniques nor expressed in utility curves must nevertheless be given due attention in the mine development decision.

Conclusions

Traditional approaches to mine evaluation were based on the presumption that investment in mine development was in some sense unique and, therefore, required the use of special evaluation techniques. Today, economic evaluation techniques derived from capital investment theory and applicable to investment opportunities in general, are being usefully applied by mining companies to the mine development decision. The quantitative bases for the decision are the measurement of expected profitability and uncer-

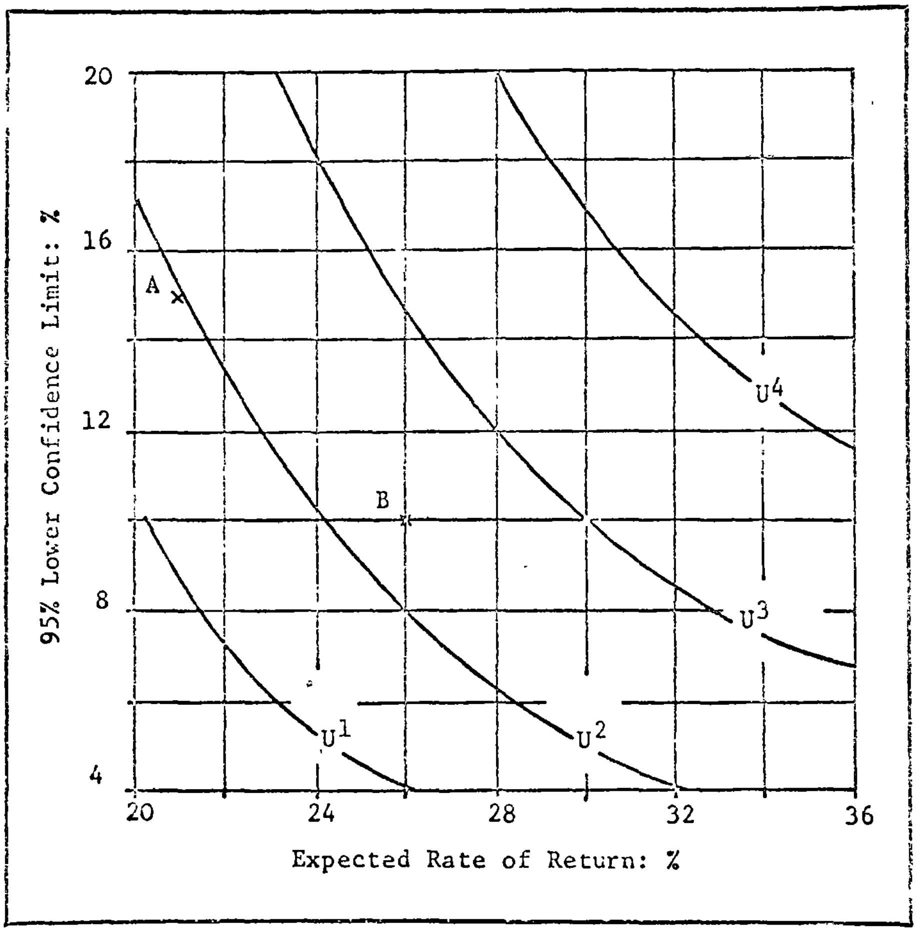


Fig. 9 A company's preference for investment alternatives may be conceptually expressed as a utility function

tainty. In this way the mining industry has been able to progress from the Hoskold formula syndrome to the use of modern investment techniques that permit a more informed mine development decision. □

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Appendix

The example shows the detailed calculation of cash flow and expected profitability criteria for the proposed development of a metallic mineral deposit in Ontario. A 1,000 ton-per-day mine capacity is being considered. Capital expenditures are expected to total $10 million over a two year pre-production period.*

*The capital cost is assumed to be evenly expended over the development period. For the purpose of determining federal and provincial depreciation allowances it is assumed that of the total expected capital expenditure, $3 million is for underground development and $7 million for plant and machinery. For the purpose of determining provincial processing allowance it is assumed that the capital cost of mill facilities is $2 million

The productive life of the mine is estimated to be 10 years with expected annual revenues and costs of $8 million and $3 million respectively. It is assumed that existing federal and provincial legislation is applicable.

It is first necessary to define existing Ontario and Canadian legislation as it relates to the determination of tax liability for the proposed development. Then annual cash flows and expected profitability criteria — rate of return, the present value at 12%, and payout period — can be calculated.

Existing Ontario Legislation

3. Net Income Before Allowances and Tax
4. — *Ontario Depreciation Allowance*

5. Net Income Before Processing Allowance
6. — *Ontario Processing Allowance*
7. Income for Ontario Mining Tax
8. Ontario Mining Tax

Depreciation Allowance — development expenditures at 10% straight line if the ore is processed to the smelter stage in Canada.
— other capital expenditures at 15% straight line.

Processing Allowance — rates applied to the total original cost of processing assets —
8% if ore concentrated
16% if ore smelted
20% if ore refined
— not to be less than 15% nor more than 65% of the net income before processing allowance.

Mining Tax — 15% of income for mining tax.

Existing Canadian Legislation

3. Net Income Before Allowances and Tax
9. — *Provincial Mining Tax Allowable*
10. Net Income Before Federal Depreciation Allowance
11. Federal Depreciation Allowance
12. Net Income Before Federal Depletion Allowance
13. — *Federal Depletion Allowance*
14. Income for Federal Income Tax
15. Federal Income Tax

Provincial Mining Tax Allowable — the lesser of:
(1) Provincial Mining Tax
(2) (NI Before Allowances and Tax — Depreciation Allowance — Processing Allowance) Provincial Tax Rate

where:
Depreciation Allowance — provincial depreciation allowance *excluding* allowance for development expenditures, i.e. excluding those expenditures allowable at 100% rate for federal tax purposes.

Processing Allowance — 8% of the original cost of processing assets (for mill, smelter, refinery)
— not to be less than 15% nor more than 65% of (NI Before Allowances and Tax — Depreciation Allowance).

Depreciation Allowance — at varying diminishing balance rates depending on the type of capital expenditure. For example:

	Rate
roads	4%
storage tanks	10%
oil or gas pipelines	6-20%
plant and machinery	30%
mine development — shafts, main haulage ways, stripping	100%

Depletion Allowance — 33 1/3% of net income before depletion allowance.

Three Year Tax Exempt Period — a mine is exempt from federal income tax for a period of three years after the start of production.

Federal Income Tax — applied to income for federal income tax.

21% on first $35,000
50% over $35,000

Cash flow and expected profitability calculation (thousands of dollars)

	1	2	3	4	5	6	7	8	9	10	11	12	Totals
1 Revenue			8,000	8,000	8,000	8,000	8,000	8,000	8,000	8,000	8,000	8,000	80,000
2. Operating Cost			3,000	3,000	3,000	3,000	3,000	3,000	3,000	3,000	3,000	3,000	30,000
3. Net Income Before Allowances & Tax			5,000	5,000	5,000	5,000	5,000	5,000	5,000	5,000	5,000	5,000	50,000
Ontario Mining Tax													
Depreciation Allowance at 10%			300	300	300	300	300	300	300	300	300	300	3,000
Depreciation Allowance at 15%			1,050	1,050	1,050	1,050	1,050	1,050	700				7,000
4. Ontario Depreciation Allowance			1,350	1,350	1,350	1,350	1,350	1,350	1,000	300	300	300	10,000
5 Net Income Before Processing Allowance			3,650	3,650	3,650	3,650	3,650	3,650	4,000	4,700	4,700	4,700	40,000
6 Ontario Processing Allowance			547	547	547	547	547	547	600	705	705	705	6 000
7 Income for Ontario Mining Tax			3,103	3,103	3,103	3,103	3,103	3,103	3,400	3,995	3,995	3,995	34 000
8. Ontario Mining Tax			465	465	465	465	465	465	510	600	600	600	5,100
Federal Income Tax													
9 Ontario Mining Tax Allowable						465	465	465	510	600	600	600	3,705
10. Net Income Before Federal Depreciation Allowance						4,535	4,535	4,535	4,490	4,400	4,400	4,400	31,295
Depreciation Allowance at 30%						2,100	1,470	1,029	720	504	353	247	6,423
Depreciation Allowance at 100%						2,435	565						3,000
11. Federal Depreciation Allowance						4,535	2,035	1,029	720	504	353	247	9,423
12. Net Income Before Federal Depletion Allowance							2,500	3,506	3,770	3,896	4,047	4,143	21,872
13 Federal Depletion Allowance							833	1,168	1,256	1,299	1,349	1 384	7 289
14 Income for Federal Income Tax							1,667	2,338	2,514	2,597	2,698	2,769	14,583
15 Federal Income Tax							823	1,158	1,246	1,289	1,339	1,374	7,229
16. Capital Expenditures	5,000	5,000											10,000
17 Cash Flow	−5,000	−5,000	4,535	4,535	4,535	4,535	3,712	3,376	3,244	3,111	3,001	3,025	27,671
DCF at 12%	−4,465	−3,986	3,228	2,882	2,573	2,297	1,679	1,364	1,170	1,002	880	777	9,401
DCF at 35%	−3,703	−2,743	1,843	1,365	1,011	749	454	306	218	155	113	83	−151
DCF at 34%	−3,732	−2,785	1,885	1,407	1,050	783	478	325	233	167	122	90	23

Rate of Return 34.1% Present Value Ratio at 12%: 1.11 Payout Period. 2.2 years

LOUIS Y. POULIQUEN

RISK ANALYSIS IN PROJECT APPRAISAL

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FOREWORD

I would like to explain *why* the World Bank Group does research work, and why it publishes it. We feel an obligation to look beyond the projects we help to finance toward the whole resource allocation of an economy, and the effectiveness of the use of those resources. Our major concern, in dealings with member countries, is that all scarce resources, including capital, skilled labor, enterprise and know-how, should be used to their best advantage. We want to see policies that encourage appropriate increases in the supply of savings, whether domestic or international. Finally, we are required by our Articles, as well as by inclination, to use objective economic criteria in all our judgments.

These are our preoccupations, and these, one way or another, are the subjects of most of our research work. Clearly, they are also the proper concern of anyone who is interested in promoting development, and so we seek to make our research papers widely available. In doing so, we have to take the risk of being misunderstood. Although these studies are published by the Bank, the views expressed and the methods explored should not necessarily be considered to represent the Bank's views or policies. Rather they are offered as a modest contribution to the great discussion on how to advance the economic development of the underdeveloped world.

Robert S. McNamara
President
International Bank for
Reconstruction and Development

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PREFACE

This paper is part of a continuing effort in the Bank to find ways to tackle the problem of uncertainty. It relates primarily to work in the Transportation and Public Utilities Projects Departments; part of it was prepared while the author was on a temporary assignment in the Economics Department.

The reader is probably already familiar with Shlomo Reutlinger's recent paper, *Techniques for Project Appraisal under Uncertainty* (World Bank Staff Occasional Paper No. 10). A deliberate effort has been made to focus this new paper on particular problems arising in Projects Departments' work. If the attempt has been successful, it is only because of the wholehearted participation of a great many staff members of the Projects Departments. This participation has sometimes taken the form of reasoned skepticism rather than immediate acceptance, but the challenge of the former has proved at least as useful as the encouragement of the latter.

It would take too long to mention by name all those who contributed to this paper. Special mention is deserved, however, by Messrs. Aldewereld, Chadenet and Baum for their full support in this enterprise, Mr. Jaycox for taking the initiative of using risk analysis in the appraisal of three of the four projects which constitute the basis of this paper, Messrs. Higginbottom, Jones, Scoffier and Soges for their sustained assistance in solving the problems of evaluating probability distributions and correlations, Mrs. Comer for her patience and efficiency in carrying out all the computer programming work, and Miss Snell, Miss Maguire and Mr. Latimer for editing the final drafts.

ANDREW M. KAMARCK
Director
Economics Department

I

INTRODUCTION

The material in this paper is drawn from the results of about a year's experimentation with risk analysis; conclusions, therefore, can only be tentative at this stage. The purpose of the paper is threefold. First, it describes for the general reader three case studies in the use of risk analysis in project appraisal which serve to illustrate different aspects of the practical problem. Secondly, it discusses and illustrates a number of methodological problems. Thirdly, it makes some general observations on the usefulness of the approach.

The subject is introduced in general terms in Chapter II. The methodology is explained in Chapter III, a case study of the Bank Group's earliest analysis, the Port of Mogadiscio project. Chapter IV outlines the economic appraisal of a Tanzanian section of the Tanzam highway in order to show how the probability analysis fits the framework and also describes how the same analysis was used to resolve a technical problem. The benefits from disaggregation are shown in the rather special case presented by the Great East Road pre-project study in Chapter V.

In the second part of the paper some of the problems met in the analysis are explored in more detail. Chapter VI is devoted to the correlation problem. The techniques used in obtaining probability distribution judgments from technical experts are described in Chapter VII. Chapter VIII comments on the time and money costs of computer use implied by the methods described, while Chapter IX discusses questions of sample size and other statistical questions. Chapter X summarizes four ways in which risk analysis is thought to be especially useful, and the general advantages of the method, with a repeated warning about the correlation problem.

II

RISK ANALYSIS AND THE SIMULATION APPROACH

Risk analysis is essentially a method of dealing with the problem of uncertainty. Uncertainty usually affects most of the variables which we combine to obtain a cost estimate, an economic rate of return or net present value, a financial return, or any of the other indicators which may be used to evaluate a project. Sometimes we deal with this uncertainty by combining values for all input variables, chosen in such a way that they yield a conservative estimate for the result of the analysis. In other cases we may select the best estimate value, that is, the value which we think is most likely to be achieved. Both these solutions imply a decision: the first to look at the project with a conservative eye, the second, to disregard the consequences of any variation around the best estimate value. Both can lead to biased decisions. For example, if we combine only conservative estimates of our variables, our final result is likely to be "overconservative." On the other hand, by using only best estimate values we fail to take into account that other values of the variables we combine might result in substantial variations in the final estimate; thus, by basing our decision on a single value of the decision variable, we may be taking more risk than we intend.

The purpose of risk analysis is to eliminate the need for restricting one's judgment to a single optimistic, pessimistic, or "best" evaluation, by carrying throughout the analysis a complete judgment on the possible range of each variable and on the likelihood of each value within this range. At each step of the analysis these judgments are combined at the same time as the variables themselves are combined. As a result, the product of the analysis is not just a single value of the decision variable, but a judgment on the possible range of the decision variable around this value, and a judgment on the likelihood of each value within this range.

These judgments take the form of probability distributions. That is to say, each possible value of each variable is associated with a number between 0 and 1, such that for each variable the sum of all these numbers, or probabilities, is equal to 1. These probabilities, which are called subjective probabilities because they represent some degree of subjective judgment,[1] follow all the rules of traditional probability theory. From a mathematical point of view, risk analysis, therefore, consists of aggregating probabilities. Of the various ways in which this can be done, the only one we refer to in this paper, and the one which seems best fitted to risk analysis, is the Monte Carlo simulation technique.

The idea underlying the Monte Carlo technique is simple. When we say that a project has a 30 percent chance of earning a 10 percent return, we mean that if we had a great number of similar projects we would expect about 30 percent of them to earn a 10 percent return. Conversely, if we had a great number of projects and if 30 percent of them earn a 10 percent return, we could say that the probability of a 10 percent return is 30 percent. Hence the simplest application of the Monte Carlo technique is to build a great number of projects with the characteristics of the one we are interested in, and see how many of them earn 10 percent, 15 percent, 20 percent, etc. In practice, the value of each of the uncertain variables is chosen by random selection, and the rate of return or some other decision variable is computed for the project defined by these values. The process is repeated many times and the results are statistically analyzed. The only difficulty is in making sure that the distribution of the values of each of the input variables, as it emerges from the random selection, is consistent with the distribution for that variable chosen for the analysis.[2] The technique will become clearer after description of the Mogadiscio and the Tanzam highway cases.

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[1] All "subjective" judgments that we are likely to obtain from experts are based on some sort of "objective" experience. For example, usually the past record of similar events leads the expert to attach more importance to one outcome than to another.

[2] The reader may wish to refer to James W. Butler, "Machine Sampling from Given Probability Distributions," *Symposium on Monte Carlo Methods* (John Wiley & Sons, Inc.) 1956.

III

THE PORT OF MOGADISCIO: A CASE STUDY

The risk analysis used to appraise this project was the first to be undertaken in the IBRD. Initially, a conventional cost-benefit analysis was used to appraise the project. A Bank appraisal mission consisting of an engineer, a financial analyst and an economist, in 1967 visited the existing lighterage port at Mogadiscio, Somalia, which the project would have replaced with a two-berth, deep-water port. But the conventional analysis, based on information the mission gathered and on a consultant's report, ran into serious difficulties in its effort to assess the economic justification of the project using best estimates of the variables. A sensitivity analysis, undertaken at this stage to pinpoint the most crucial elements of the project, narrowed the sources of uncertainty to seven variables. It was then decided that a risk analysis using probability distributions would be a useful tool to deal with these uncertainties, though such a risk analysis had not been undertaken before in the Bank and had not been anticipated at the time of the mission's visit to Somalia. The Bank might nowadays carry out this risk analysis slightly differently, but the general approach is thought to be correct and the later modifications would not change the decision about the economic justification.

The Project's Background

The project included the construction of a breakwater, two berths, two transit sheds, storage area and office accommodations. No dredging was neces-

sary since natural depth existed in the approach from the sea as well as at the site of the two proposed berths.

Traffic through the existing lighterage port of Mogadiscio for the period 1964 to 1966 averaged about 125,000 tons per year. It was expected that, in addition to generating some traffic, the construction of the new port would result in the diversion of about 85,000 tons per year of bananas which were exported through the port of Merca, about 50 miles south of Mogadiscio.

Economic Justification

The cost-benefit analysis takes into consideration, on the cost side, the capital cost of the project and, on the benefit side, three types of projected savings: (a) savings in cargo handling cost, (b) savings in reduction of damages, (c) savings in ship turnaround time. These savings are applied to the projections of future traffic, broken down into the normal growth of traffic that might be expected without the port, generated traffic, and diverted traffic (see below). The result of the analysis is an internal rate of return over the average life of the assets.

Cost of the project

The cost of the project has been estimated at $14.6 million. A cost breakdown is roughly as follows:

Breakwater	43 percent
Berth and storage area	25 percent
Engineering fees	8 percent
Auxiliary works	24 percent

The major single item is the breakwater. Among the auxiliary works, the biggest single item represents only 3 percent of the total cost.

Existing composition and the uncertain trends of traffic

Traffic through the port of Mogadiscio consisted primarily of imported goods and materials. Imports constituted, on the average, about 75 percent of total traffic at Mogadiscio, and the port handled over 45 percent of the country's total imports. There was no large bulk traffic through the port. The largest single category of import traffic was cereal grains, which usually accounted for about 25 percent of total imports. Otherwise import traffic consisted of small consignments of manufactured goods, machinery and raw materials. For export, the main commodity handled during the previous six had been charc   which until 1966 accounted for about 75 percent of exports. The only other exports of individual importance were live ani- skins and hides. Traffic through the lighterage port at Merca, south of idiscio, consisted almost entirely of bananas for export.

y cargo traffic through the port of Mogadiscio increased rapidly and steadily from about 102,000 tons in 1960 to nearly 164,000 tons in at an annual rate of 10 percent. But in 1966, total traffic fell to 110,000 almost to the 1960–61 level. Traffic since 1960 has been affected by a er of factors. The main reason why import tonnage doubled between and 1965 was the rapid growth of the population and of construction ties in the city of Mogadiscio during its first years as the capital of the d Republic. In 1964 and 1965, imports increased considerably due to lation against the imposition of strict import licensing to improve the ce of payments. Another factor in 1965 was a severe drought in 1964–65 caused food grain imports to double. In 1966 imports fell from 126,000 o 85,000 tons, as import restrictions were imposed and as inventories built 1964–65 were run down. Imports through Merca were small; they ised from about 5,500 tons to about 8,000 tons over six years, or at an ge rate of 5 percent per annum.

ie export tonnages handled have varied from year to year with the fluctua- of the charcoal trade, which the Government had been attempting to end der to retard land erosion. In June 1967 trade in charcoal was made l. A comparison of average export tonnages (excluding charcoal) of the d 1964–1966 with the 1960–1966 average indicated a growth rate of y 7 percent per annum for the six years. The export of bananas through a increased from about 49,000 tons in 1960 to about 62,000 tons in 1964 remained at about this level during 1965 and 1966. The growth of banana t tonnage over the six years had been about 4.5 percent per annum, but onnage growth rate understated trade in bananas because improved pack- echniques had reduced shipping by about 12 percent.

re traffic projections

ormal Traffic. The foreseeable long-term need for restrictions on imports probable slowdown of economic growth in the Mogadiscio area implied rate "best estimates" of future import traffic. Imports were projected ow at a rate of 3.5 percent per annum from a base of 106,000 tons, the ige of import tonnages over the 1964–1966 period. Exports of live als, hides and skins should continue to grow fairly rapidly but at a much er rate than in recent years, because these exports had already been ex- ed considerably and further increases were likely to be more difficult.

Export tonnages (excluding charcoal) were projected to grow   the rate of 6 percent per annum from the 1964–1966 average.

Generated Traffic. In view of the large unit savings to be achieved by constructing a deepwater port at Mogadiscio, a large proportion of which could be passed on to the Somalian consumer and producer, considerable ex- port and import traffic should be generated by the proposed project. Taking price elasticity of demand for this traffic as .08, generated traffic was estimated at 10,000 tons per annum by the third year of operations of the new port, in- creasing thereafter at the average growth rate indicated above for normal traffic.

Diverted Traffic. Once the new port was in operation, the banana exports and small import tonnages passing through Merca would be diverted to Mogadiscio. The banana production potential of the Genale/Scialambot area in the Merca hinterland, estimated at 100,000 tons, was to be realized within five years, according to current plans. The plan, however, was probably overly optimistic. The banana industry appeared to have a fairly bright future, but there were marketing problems which had not been met because of protection in the Italian market. Therefore it was assumed that banana exports would build up from the 1964–1966 average level of 61,000 tons to about 85,000 tons over the ten-year period 1967-1976.

Savings

Table 1 shows a year-by-year breakdown of the benefits from 1972 to 1978 based on the best estimate of each variable.

TABLE 1: Port of Mogadiscio: Estimated Benefits on the Basis of the Best Estimate of All the Variables

(US $'000, 1967 prices)

Type of Benefits	1972	1973	1974	1975	1976	1977	1978
Reduction in general cargo handling costs	296.4	308.4	320.8	333.8	347.3	361.5	375.9
Reduction in diverted traffic handling costs	136.6	145.2	154.1	163.4	168.6	170.3	170.7
Reduction of physical damage to cargo	656.0	678.7	704.7	735.0	761.5	790.0	[illegible]
Reduction in ship turnaround time	532.7	552.6	572.5	591.3	609.3	629.0	[illegible]
Savings from generated traffic	18.5	37.0	46.2	50.7	55.1	[illegible]	[illegible]
	1,640.2	1,721.9	1,798.3	1,874.2	1,941.8	2,010.4	2,069.5

[S]avings in Cargo Handling Cost. Lighterage is a rather inefficient opera[tion] since it requires double handling of the cargo. In addition, the transship[men]t between ship and lighter is hampered by the movements of both the ship [and] the lighter and is, therefore, much slower than loading or unloading on [a p]rotected quay. For the existing lighterage port the cargo handling costs—[divi]ded into a fixed and a variable component—were taken at their actual [val]ue after some adjustments to eliminate costs resulting from redundant labor [and] faulty work methods. For the proposed port fixed and variable costs were [com]puted from their components, which are essentially:

–for the variable costs: the number of men and the productivity of these men, i.e., the number of tons they can handle in an hour;

–for the fixed costs: the costs of maintenance (labor and materials), administrative staff and staff in warehouses and transit sheds.

[R]eduction in Damages. In the lighterage port, the necessity of handling [the] cargo twice, including one time at sea, means not only higher costs but high [dam]ages. Benefits resulting from a reduction of these damages have been [com]puted on the basis of the proportion (P) of the forecast tonnage (T) [wh]ich is expected to be saved through easier handling and of the value (V_c) of [the] average ton of cargo. Therefore, the resulting saving (S_D) is given by

$$S_D = P \times V_c \times T$$

[S]avings in Ship Turnaround Time. Savings were expected in ship turn[aro]und time because the higher productivity of labor anticipated in the new [por]t implied faster loading and unloading and less ship time in port per ton of [car]go. The savings formula essentially compares the observed number of ship-[day]s required to move a given tonnage (T) of cargo of each type (general [car]go, bananas) in the existing port, with the ship-days estimated to be required [un]der the improved conditions to move the same tonnage, less an allowance for [wa]iting time. The savings in ship-days are then put into monetary terms by [mu]ltiplying them by the value of a ship working day (V_s). The savings in [tur]naround time (S_T) for tonnage T is:

$$S_T = V_s \left[\frac{T}{P_S} - \frac{T}{P_L \times H} - W(T) \right]^1$$

[wh]ere P_S is the tonnage loaded or unloaded per ship per day, observed in exist[in]g conditions, P_L is the estimated cargo handling rate per hour under the [cha]nged conditions and H is the number of hours to be worked per day. The

[1] Simplied formula.

estimate of turnaround time derived from handling capacity has to be adjusted for the expectation that some ships may have to wait. A simple Poisson queuing model gave this waiting time (W) as a function of the total tonnage. The waiting time has then been allotted to each type of traffic proportionally to the share of this traffic in the overall traffic.

Projected savings in all three categories were considered functions of the traffic handled through the port, which was estimated year by year from the projected growth rates described earlier. The traffic demand was assumed to be linear and consequently unit savings for generated traffic were taken as one half of the unit savings for normal traffic. In the case of diverted traffic the benefits have been reduced by the cost of an increase of about 25 miles in the land transport of bananas. The traffic taken for the computation of the benefits is the real traffic in the port up to the time when the economic capacity of the two berths will be reached. Thereafter all benefits stay constant except those resulting from a sudden reduction in ship turnaround time[2] followed by a progressive increase again in ship waiting time. The economic capacity of the two berths was computed separately, using the queuing model referred to above.[3]

Shortcomings of the Analysis

We did not feel much confidence in our results. To arrive at the final 12.2 percent economic rate of return, we had used best estimates for each variable, but on some occasions, we had been obliged to resort to awkward ways of finding them (combining notions of both the mean and the mode, for example). Furthermore, the rate of return was based on highly uncertain data, and was interpreted under some rather optimistic assumptions about the variables.

On the traffic side, for example, the difficulties of accurately predicting traffic growth have already been indicated, even without taking into consideration the closure of the Suez Canal. Traffic in cereals, the major import commodity, fluctuated with the success of competing internal crops and with

[2] It was assumed that a third berth would be brought into operation as soon as the economic capacity of the two existing ones was reached.

[3] Since a detailed description of the financial analysis of this project would not add very much to the case, this part of our work will be described only briefly. On the basis of projected traffic, the operating revenues resulting from a given system of port charges (different for imports and exports) were estimated. By comparing these revenues with investment cost and operating expenses the rate of return was computed over the 10 year life of the project. This gave, with linear depreciation, the financial return on net fixed assets for the 5 years following the construction of the project. In addition, the converse problem was solved to determine a system of charges which would yield a 6 percent financial return over the life of the assets.

the [illegible] and therefore was difficult to forecast. Developments in other import traffic depended on regional development, which had shown no clear trend. Traffic in charcoal, the major export commodity, was ruled out by law, and it was hard to guess to what extent it would be replaced by increased livestock trade. Bananas, which could constitute about one-third of the traffic of the new port, might not be diverted, or they might not be able to compete when Italy's preferential treatment was abolished under the rules of the Treaty of Rome and thus might disappear as an export altogether.

Nor could the cost of the project be precisely estimated. The costing of the most expensive single item, the breakwater, was the most uncertain. The usual uncertainty about the quantity of construction material required was increased by uncertainty about contingency allowance for storm losses (the quantity of materials lost during construction because of rough seas). On the price side, the lack of detailed analysis on rock availability reduced the estimate of the unit cost of rock to a guess.

The number and nature of these and countless other project uncertainties are not new to anyone acquainted with project appraisal. It is therefore only necessary to add that the productivity of labor in African ports varies from about 5 to 12 tons per gang-hour, that we were not very sure of the value of an average ton of cargo nor of the value of a ship working day, and that our hypothesis on the reduction in cargo damage was made without much reliable data support.

Sensitivity Analysis

The conventional analysis had failed to give a satisfactory result using single best estimates. The most natural way to deal with this situation was to make a sensitivity analysis, in other words, to see what would happen if other values of the input data were substituted. Using the most unfavorable estimate for each variable, we obtained a 2 percent rate of return, which confirmed our suspicion that the project was risky. But how risky? Again, a natural approach to this question was to try to find out which variables were principally responsible for the variations of the rate of return.

For this purpose we examined each one of the 27 uncertain variables which appeared in our rate of return computations. We varied them, one at a time, holding all other variables at their best estimate value. We found the variation of the rate of return as the best estimate of each variable was replaced by the maximum value, the minimum value, and a value 10 percent above the best estimate. Table 2 shows the results we obtained for the economic rate of return. On the basis of this table, and a similar one for the financial rate of return, it appeared that the performance of the project was essentially ex-

TABLE 2: Port of Mogadiscio: Sensitivity Analysis

Parameter Varied (1)	Value Assigned (Somali shillings except where stated otherwise) Highest (2)	Best Estimate (3)	Lowest (4)	Internal Rate of Return, % Based on Highest Value (2) (5)	Lowest Value (4) (6)	Percentage Response to 10 percent Variation of Best Estimate[b] (7)
1 Cost of project ('000 sh.)	119,255	103,700	98,000	10.8	12.9	8.20
2 Gang productivity, tons per hour	12	10	5	14.2	7.3	4.10
3 Reduced damages, percentage	2	1.5	1	13.6	10.8	3.28
4 Ton cargo, average value	2,500	2,000	1,500	13.3	11.2	3.28
5 Unnecessary staff[a]	650	400	150	10.7	11.9	1.64
6 Road transport cost, per ton, bananas	37.5	25.4	12.5	11.5	13.0	0.82
7 Ship working day value, general cargo	7,868	6,423	4,988	12.5	11.9	1.64
8 Life of assets, years	60	40	20	12.3	10.9	0.82
9 Growth rate percent, imports	8	3	2	13	11.8	0.82
10 Avoided investment costs ('000 sh.)	4,285	2,800	0	12.4	11.9	0.82
11 Ship working day value, banana	12,780	10,650	8,520	12.3	12.1	0.82
12 Maximum banana traffic p.a. ('000 tons)	100		0	12.1	12.1	0.82
13 Growth rate percent, exports	11	6	3	12.5	12.1	0
14 Growth rate percent, bananas	8	4	0	12.2	12.6	0
15 Maintenance cost p.a. materials ('000 sh.)	900	780	600	12.1	12.4	0
16 Men in gang, general cargo	35	30	25	12.1	12.3	0
17 Men in gang, bananas	28	24	20	12.2	12.3	0
18 Men in transit sheds	115	102	85	12.2	12.3	0
19 Men during maintenance	115	100	85	12.2	12.3	0
20 Miscellaneous cost p.a. ('000 sh.)	360	300	270	12.2	12.2	0
21 Banana charge per ton[c]	27	18	9	12.2	12.2	0
22 Port charges per ton[c]	65	54	45	12.2	12.2	0
23 Ratio export/import charges[c]	.5	.19	1	12.2	12.2	0
24 Number of men in warehouses	36	32	29	12.2	12.2	0
25 Number of men in administration	95	82	65	12.2	12.2	0
26 Elasticity of traffic demand	0.2	0.08	0	12.2	12.2	0
27 Ship charges per call[c]	150	100	50	12.2	12.2	0

Note: Based on best estimates of parameters varied, the economic rate of return is 12.2 percent.

[a] This line assumes that this staff receives 3,000 shillings a year; in the rest of the analysis it was assumed that it received nothing.

[b] See page 10. It was assumed that the variations were reasonably well-behaved and that therefore the percentage response to increasing the best estimate by 10 percent was a good [illegible] of a 10 percent variation in either direction around the best estimate.

[c] These refer to the financial rate of return only.

plained by seven variables: (1) cost of the project, (2) productivity of labor, (3) value of an average ton of cargo, (4) percentage of the tonnage which would be saved through reduction of damages, (5) rate of growth of imports, (6) value of a ship working day, and (7) the life of the assets.

Above, it was explained that in this analysis we varied only one variable at a time. However, we made exceptions to this rule in the case of variables whose variation would, in the real world, very probably be correlated. For example, when we varied the productivity of a general cargo gang, we varied at the same time the productivity of a banana gang. These two variables both depend on the efficiency of the organization of the new port, and on how efficient port operators the Somalis will turn out to be. Therefore, they are likely to be correlated. There is no reason, however, why their variations should be completely interdependent, since, for example, the productivity of the general cargo gang also depends on the degree of unitization of the cargo, a factor which is unlikely to affect the productivity of the banana gang. In assuming, as we did, that the variations of these two variables were fully correlated we may have somewhat overestimated the sensitivity of the final result to the productivity of labor.

We used a more rigorous way of handling correlation in estimates of the number of persons required in the various operations of the port. The estimates of the number of men in gangs, transit sheds, warehouses, etc., are subject to uncertainty not only as to the exact numbers of men required to operate the port in the most efficient way, but also as to the Port Authority's efficiency in eliminating the redundant labor presently employed in the port. The first uncertainty is likely to affect the variables independently of one another since overestimation of the number of men needed in a banana gang needs not necessarily imply overestimation of the number of men needed in a transit shed. The second uncertainty, on the contrary, is likely to affect all the variables in the same direction. If the Port Authority does not manage to eliminate redundant personnel, it is likely that this personnel will be distributed among the various services of the port and so increase the cost of all of them.

We resolved this difficulty by creating an artificial variable, which we called "unnecessary staff," and which represents all redundant labor in the port. Then, rather than testing the sensitivity of the actual number of persons presently employed in each of the various services of the port, we tested separately the sensitivity of the theoretically most efficient number of persons required in each one of these services and the sensitivity of all unnecessary staff for the whole port. In sum, we tried to test the importance not so much of a variable *per se*, but rather of various sources of uncertainty.[4]

[4] The problem of correlation is discussed further in Chapter VI.

If, instead of isolating seven major sources of uncertainty, we had isolated only one, or eventually two, our task would have been completed. We could have concluded that if this determining variable were, say, less than a given value *a*, the project was very likely to be justified, and that if it were more than *a*, the project was very likely not to be justified. A simple evaluation of the likelihood that this variable was less than *a* would have been enough to give us an idea of the riskiness of the project. With two variables, our judgment would have been more difficult to put in words. We might, however, have been able to illustrate it with the help of a graph showing the limits within which the project would be justified.

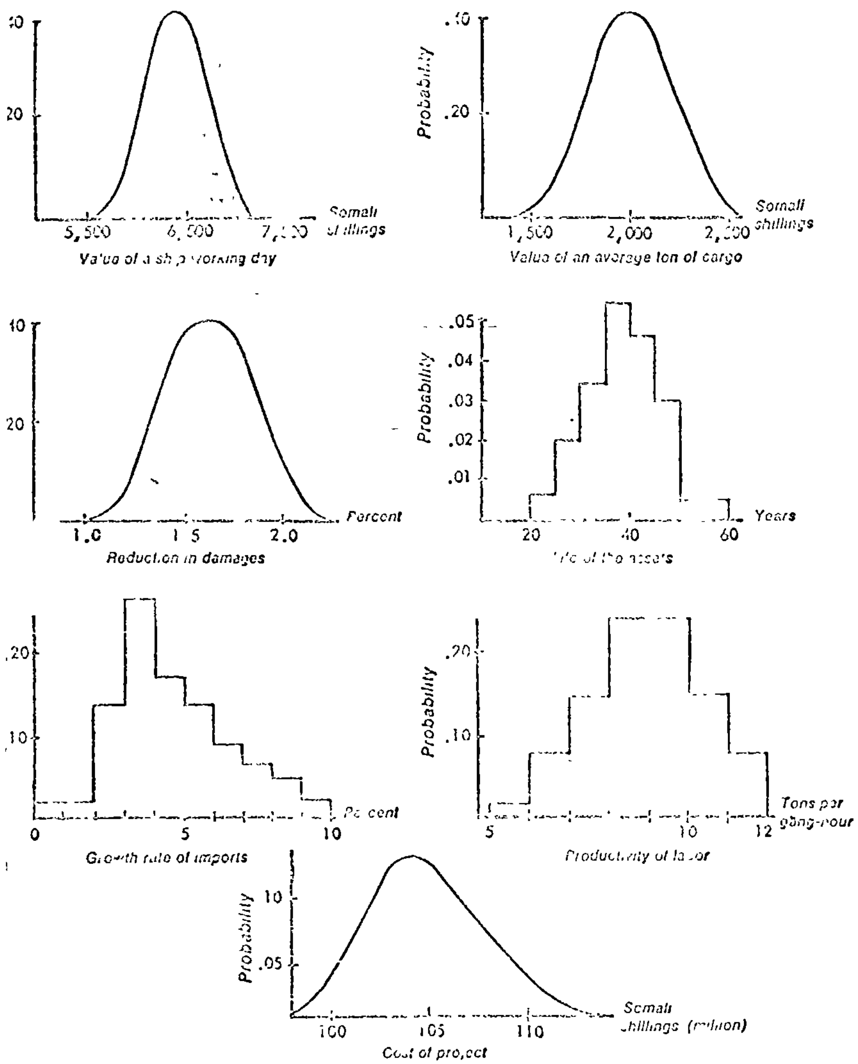
But with seven variables, such a task is impossible. One can find an infinity of combinations of the variables for which the project is justified, and an infinity of combinations for which it is not. Ironically, the more combinations of variables one tries, the less clear the picture of the project becomes. The only way to obtain an overall, synthetic picture of the project is to proceed with a probability analysis.

Probability Analysis

The first step of this risk analysis is to assign to each variable a probability distribution. Since we had found out that the variation of the rate of return was essentially explained by the variations of seven variables, we limited our analysis to these seven variables. The distributions were based essentially on subjective judgment. This did not raise any difficulty in practice and did not take much time, since we had limited ourselves to a small number of variables. The distributions we obtained are shown in Figure 1. They were obtained in essentially two ways.

The three normal distributions that we adopted for the value of a ship working day, the value of an average ton of cargo, and the percentage reduction in damages, and the chi-square distribution that we used for the cost of the project are the result of an approach which could be compared to the portrait method used to identify suspects. On the basis of limited information a portrait is drawn and subsequently modified until the informant is satisfied with it. Similarly, on the basis of limited information obtained from the appraiser, we chose among classical probability distributions one which seemed to fit the case. We drew it, indicated the corresponding probabilities for various intervals, and went back to the appraiser. He decided whether it was too skewed or whether an interval had too high a probability, and on the basis of this new information we modified it. We repeated this process until the appraiser was satisfied with the distribution.

The distributions we used for the life of the assets, the growth rate of im-



The area enclosed by a probability curve = 1

ure 1. Probability Distributions Used in the Simulation of Mogadiscio Port Project

ts and the productivity of labor, which we have called step rectangular ributions, were obtained with the somewhat more active participation of appraiser. Let us take for example the case of the productivity of labor. e steps by which the appraisal team set up its distribution are illustrated Figure 2. We first divided the total range of variation we had delineated the sensitivity analysis (5-12 tons per gang-hour) into two intervals:

5-10 and 10-12, and tried to assign a probability to each one of them. For this we used a trial and error approach based on the engineer's experience: 50%-50% gives too high a probability to the 10-12 interval, as does 60%-40% and 70%-30%, but 80%-20% does not seem to give enough; therefore, we tried 75%-25%. In other words, the appraisal team's judgment was best expressed quantitatively by saying that the probability of exceeding 10 tons per gang-hour is only one-third of the probability of getting a lower productivity of labor.

In the second step, we chose to subdivide the 5-10 interval into 5-8 and 8-10. Then following the same trial and error process we allocated a 30 percent probability to the 5-8 interval and a 45 percent probability to the 8-10. The sum of these two probabilities is, of course, equal to the 75 percent probability of the entire 5-10 range. In a third step we pushed this subdivision further and obtained the following distribution:

from	5 to 6	a 5	percent probability
"	6 to 7	a 10	percent "
"	7 to 8	a 15	percent "
"	8 to 9	a 22 5	percent "
"	9 to 10	a 22.5	percent "
"	10 to 11	a 15	percent "
"	11 to 12	a 10	percent "

Finally, in a fourth step we made some minor adjustments to give the distribution a final polish. For example, we found that compared to the probability of the 6-7 range, the probability of the 8-9 and 9-10 ranges was too low. We therefore raised them to 25 percent and decreased the 6-7 range probability to 8 percent, which in turn led us to decrease the probability of the 5-6 range to 3 percent. Similar considerations for the 10-12 range led us to the final distribution in Figure 2.

This approach and the portrait method for choosing the other distributions include an iterative interaction between quantitative and qualitative judgment. On the basis of a qualitative judgment one attempts to produce tentative figures. These figures in turn are translated back into qualitative judgment which is compared to the initial qualitative one. The figures are modified in light of the discrepancy, and the procedure is repeated until the qualitative judgment derived from the quantitative one fully agrees with the initial judgment.

The simulation

The simulation is by far the fastest and the easiest operation of the entire analysis. When, as in our case, computer help is used, the computer can be

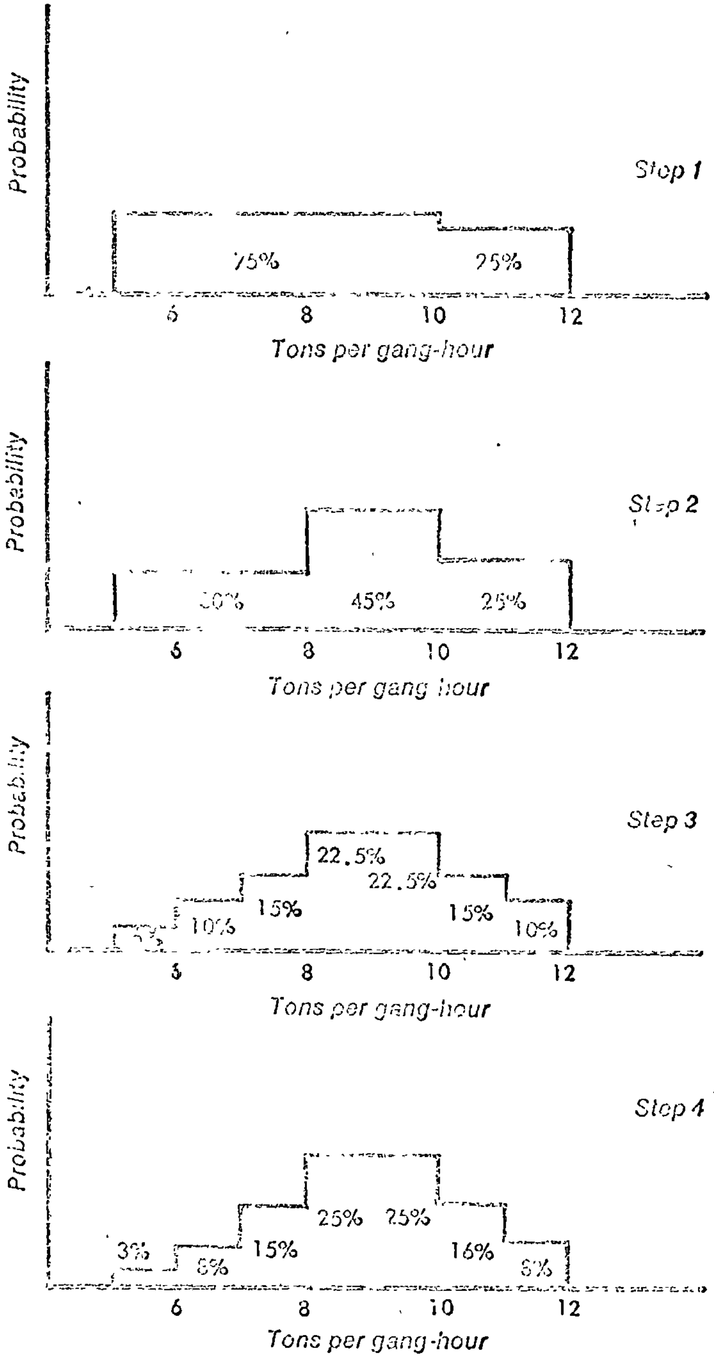


Figure 2. Steps in Establishing the Probability Distribution of the Productivity of Labor

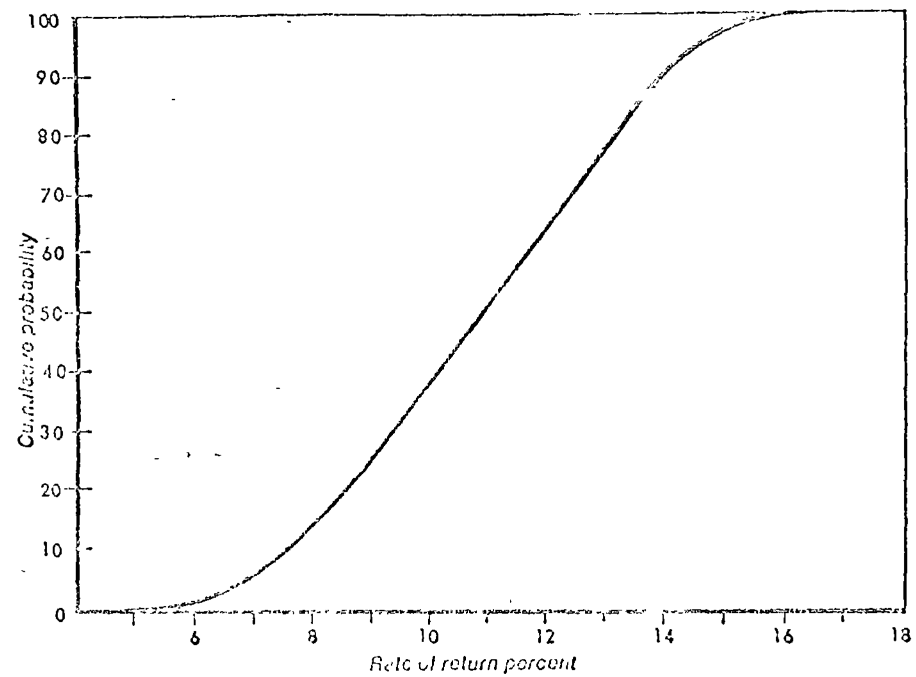


Figure 3. Mogadiscio Port Project: Cumulative Probability Distribution of the Economic Rate of Return

instructed to generate random values for each of the parameters varied in the analysis, to compute the rates of return, to repeat the process until enough values are obtained (300 times in the present case) and then to give the observed distribution of the result. In this case the computer was also used to draw the curve in Figure 3. The simulation is, therefore, an operation which requires no outside intervention and takes only a few minutes. Programming the computer to calculate rates of return is very simple; for random number generation, it is slightly more involved but still easily derived from the basic random number generators which exist in all computer libraries.

The results

The results for the economic rate of return are summarized by the cumulative probability distribution in Figure 3. It has a mean of 10.6 and standard deviation of 2.5. Along the x-axis are the rates of return and along the y-axis the probability that these rates of return will not be exceeded. For example, we find that there is a 99 percent probability that the rate of return will exceed 5 percent, a 94 percent chance of it exceeding 7 percent, and so on along the curve until we reach a 2 percent chance of exceeding a 15 percent rate of return. The curve can also be used to determine the probability that the rate of return will fall within a given range: we take the difference along the ordinate of the two extreme points of the range. For example, we find there is about a 40 percent chance that the rate of return will be between 10 percent and 13 percent. The figure also shows that the probability of getting a return inferior to 12.2 percent, the rate of return we obtained in the conventional analysis using best estimates for each variable, is 70 percent, but the probability of getting more is only 30 percent. So at first glance, the results of this risk analysis seem to indicate that doubts about the likelihood of the 12.2 rate of return were fully justified.

But, unlike the sensitivity analysis, the probability analysis gives us a complete picture of the project and enables quantification of project risk—not, of course, the "true" risk, but the risk as it appeared to the appraisal mission. The probability distribution of the rate of return summarizes this risk; one could say that it represents the complete judgment of the appraisal mission.

How to use this probability distribution in a scientific way could constitute an entire study on its own. In the absence of any such scientific criteria, we used the distribution in a very pragmatic way. We first saw that, while the sensitivity analysis had told us that the minimum rate of return was 2 percent, the chances of ever getting below 5 percent were so slim that it could be considered the minimum for all practical purposes. We then looked at the probability of getting less than 8 percent, since we thought 8 percent was a low but probably still acceptable value of the opportunity cost of capital in Somalia, and found it to be about 15 percent. We thought that this was acceptable, when combined with the information that the project had a better than even chance of earning more than 10 percent and nearly a 20 percent chance of earning more than 13 percent.

Our judgment was therefore arrived at by combining considerations of what the project could turn out to be at the extremes and the probabilities that this would happen, with a weighted estimation of how any unfavorable outcomes might be compensated by favorable ones. In this respect, the mean rate of return was particularly helpful. It indicated to us that on balance, we could expect the project to yield an 11 percent rate of return; we thought this was acceptable, especially since we did not have to fear any large variations around this value. On the basis of this simple analysis, we decided to recommend the project for financing.

We felt particularly free to make this recommendation because in presenting it, we were not just presenting our own difficult decision, we were also presenting to management all the information necessary to check this recommendation, and possibly to overrule it. If we had indicated in our appraisal report only the

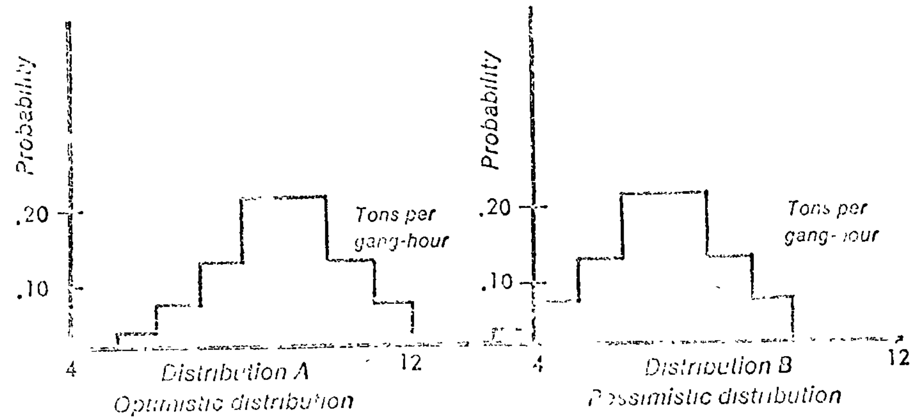


Figure 4. Mogadiscio Port Project: Shift of the Productivity of Labor Distribution

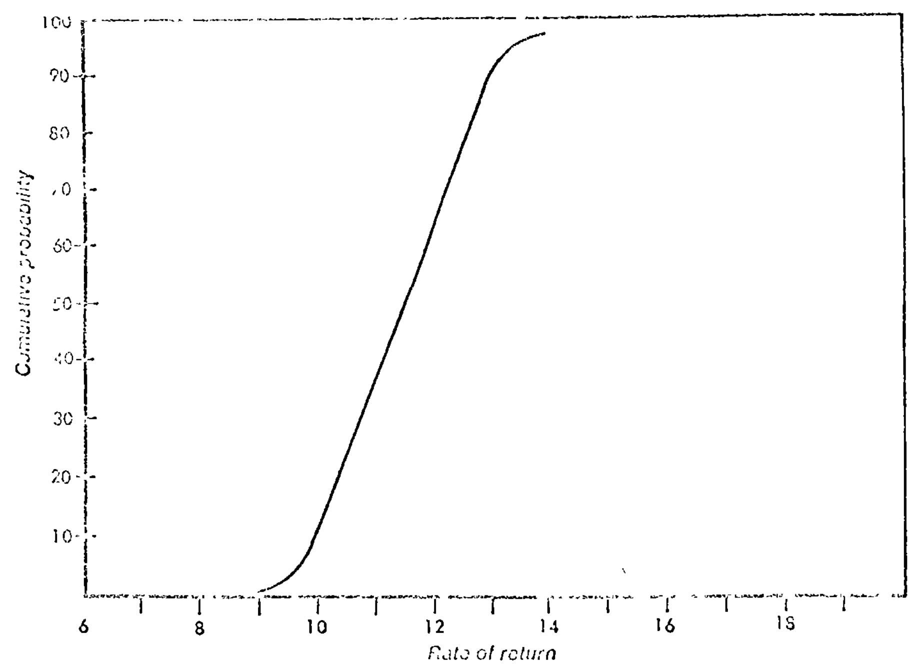


Figure 5. Mogadiscio Port Project: Cumulative Distribution of the Economic Rate of Return

12.2 percent rate of return found in our best estimates calculation, the situation would have been quite different. The decision-maker would have been acting in the dark. In fact, it would not have been possible for anybody but the team of appraisers to evaluate the risk of the project. We would have been recommending the financing of a project earning a 12.2 percent rate of return after having already decided that the risk of the project was acceptable—a dangerous mixing of analysis and decision-making.

Another important outcome of the risk analysis was that we found a way to reduce project risk. We were led to this finding by the results of a second sensitivity analysis which we carried out, this time on the entire probability distribution of each variable. We found that the productivity of labor had a much higher sensitivity in the risk analysis than in the first sensitivity analysis because we were considering its entire variation rather than its value at a single point. This higher sensitivity means that if our judgment about the probability distribution of the productivity of labor is too optimistic (i.e. in Figure 4, if the true distribution is B and not A), then the probability of having a rate of return inferior to 8 percent is no longer 15 but 30 percent (see also Figure 21). On the other hand, this assumption that the productivity of labor could be confined to the 9-10 tons per gang-hour range yields the distribution in Figure 5; the risk of the project has practically been eliminated. We therefore suggested that a consultant be engaged at an appropriate time to help organize cargo handling operations and that this be made part of the loan agreement. In this case, we were not only able to quantify the risk attached to the project, but also to find a feasible way to reduce it.

IV

THE TANZAM HIGHWAY: A CASE STUDY

The Project

The Tanzam highway is a 1170-mile-long highway from the Copper belt in Zambia to the port of Dar es Salaam in Tanzania. This route, most of which was a poor gravel road, became important after Zambia achieved independence in 1964 and after the Rhodesian unilateral declaration of independence in 1965 led to a UN embargo on traffic with Rhodesia. Compliance with the embargo requires Zambia to divert its seagoing import-export traffic from the existing main route across Rhodesia and Mozambique to a more expensive route via the Tanzam highway. Reconstruction to two-lane bituminous paved standard of the gravel or earth sections of the road from Kapiri Mposhi in Zambia to Morogoro in Tanzania (965 miles) was planned under various financing arrangements. The Bank group was asked to finance two sections in Zambia, of 122 miles and 235 miles respectively, and one in central Tanzania of 311 miles. It is this last project of 311 miles which is briefly presented here.

The road was not the only transport mode proposed to meet the Zambian demand for a new access to the sea: among other projects, a railway, to come into operation at an unknown date, was also proposed, and an oil pipeline was under construction. Uncertainty about the economic benefits therefore existed in the project from the start. Before describing the probability analysis, however, it is useful to outline the framework in which it fitted. The economic

TABLE 3: Tanzania, Highway Project: Estimated Vehicle Operating Costs on: I. Engineered Bitumen/Asphalt, II. Engineered Gravel, and III. Unimproved Earth. (1967–1968)

(*US cents per vehicle mile*)

Vehicle Category / Fuel Type / Road Type	Average Car Gasoline I	II	III	Pick-up Truck Gasoline I	II	III	Average Truck 7-Ton Capacity Diesel I	II	III
A. *Flat to Rolling Terrain*									
Fuel	1.11	1.24	1.38	1.62	1.68	1.61	1.66	2.26	2.68
Lubricants	.06	.08	.10	.08	.11	.15	[illegible]	[illegible]	.26
Maintenance									
Labor	.22	.29	.39	.26	.32	.55	.88	1.39	2.32
Parts	.44	.56	.82	.48	.69	1.10	1.58	2.41	[illegible]
Tires	.20	[illegible]	.68	.52	.62	1.20	1.18	2.66	6.05
Crew Wages	1.1[illegible]	1.[illegible]0	1.55	1.24	1.56	2.08	1.40	1.77	2.34
Interest	.78	.97	1.30	.6[illegible]	.79	1.05	[illegible]	.49	.66
Insurance	.57	.46	.61	.32	.40	.53	[illegible]	.43	.57
Depreciation	2.12	2.79	4.14	2.02	2.74	4.54	1.52	2.32	3.97
Total	6.42	8.19	11.27	6.97	8.91	13.01	9.26	13.95	22.85
B. *Rolling to Hilly Terrain*									
Total	6.65	8.58	11.80	7.41	9.41	13.90	9.75	15.05	25.02

Vehicle Category / Fuel Type / Road Type	Average Bus 50 Passengers Diesel I	II	III	Truck-trailer 3?-Ton Capacity Diesel I	II	III
A. *Flat to Rolling Terrain*						
Fuel	1.82	2.28	2.70	4.15	5.18	6.15
Lubricants	.15	.19	.26	.42	.50	.63
Maintenance						
Labor	.77	1.31	2.29	6.30	10.47	22.30
Parts	2.16	3.71	6.40			
Tires	.61	1.80	4.48	4.30	5.88	11.30
Crew Wages	1.77	2.31	2.80	1.95	3.55	6.30
Interest	.67	1.15	1.40	1.10	1.60	2.52
Insurance	.79	1.05	1.26	.40	.60	.90
Depreciation	4.[illegible]7	6.28	10.32	6.80	12.40	20.05
Total	13.11	20.11	31.91	25.42	40.18	70.15
B. *Rolling to Hilly Terrain*						
Total	14.49	[illegible]	31.43	26.81	43.49	76.65

Note Costs are net of taxes, license fees and other transfer payments. Costs of terminal operation, waybills, company administration, which are not directly affected by road improvement, are not included.

Sources Jan de Weille, *Quantification of Road User Savings*, Occasional Paper No 2. (East African prices), United Research Incorporated, Consultant to the Government of Tanzania, interviews with trucking firms in Zambia and Tanzania, mission estimates.

analysis was that normally employed in the Bank in the assessment of a road project.[1]

Estimates of costs were first established, with their probable years of occurrence. Shadow prices, sometimes used, were not required in this case, and the allowance generally made for contingencies was also unnecessary since a probability analysis was to be made. A date was set, to which the present value calculations relate. Separate analyses were made of each section of the project: in this case eight sections of the road were separately costed. Benefits and costs were assessed from the national angle. In this case separate calculations were made for Tanzania alone and for Tanzania and Zambia together.

The main quantifiable benefits were four:

a) Savings on operating costs because the road would be upgraded for existing traffic and for the normal increase of traffic which would have taken place even without the road. These were estimated by obtaining costs per mile per type of vehicle on the new road and subtracting similar costs on the old road. General comparative data were used for this purpose, particularly those in Occasional Paper No. 2, as amended by information obtained from the government of Tanzania and its consultants, and from interviews with trucking firms (Table 3). It was necessary to know for each road section the average daily traffic per vehicle type at the relevant date, and to assess the rate of normal traffic growth of each (Table 4).

TABLE 4: Tanzania, Highway Project: Estimated Tanzanian[a] Traffic on Project Road Sections

(*Average Daily Traffic 1967*)

Project Road Section	Cars	Trucks and Buses	Truck-Trailers	Total
Morogoro +20	120	165	20	305
Morogoro +20 – +38	80	130	20	230
Morogoro +38 – Mikumi	54	118	14	186
Mikumi-Mahenge	36	79	9	124
Mahenge-Kitonga-Iringa	58	120	10	188
Iringa-Sao Hill	62	89	6	157
Sao Hill-Makumbako	36	74	6	116
Makumbako-Iyayi	24	54	5	83
Annual Growth Rate	6%	8%	8%	

[a] Not including Zambian transit traffic. The best estimate for this was an ADT of 85.

b) Benefits from a shortening of the distance may be related directly to the annual trucking costs per vehicle-mile (as in Iran, in Occasional Paper No 7) or as here, to the operating costs of vehicles on the road (Table 5).

[1] The methodology is described in Occasional Papers Nos 2, 4 and 7 of this series.

alternative for the foreseeable future. It was assumed that Zambian seagoing traffic would not increase beyond its peak in 1971 and that it would decline by 50 percent annually from the completion of the railway. The distribution shows best guesses of the commencement of this decline.

Unit savings in vehicle operating costs due to improvement will be large. The distances between the major origin and destinations on this road are also large. Thus, a large absolute fall in the cost of most vehicle trips should stimulate significant economic activity in the high potential agricultural areas south and west of Iringa. High transport costs caused by poor roads and long distances have been a significant barrier to the development of this fertile, temperate highland.

Except for trucks, the operating costs (and thence the savings from an old to a new road) will be completely correlated; i.e., it is estimated that if the costs for cars are 15 percent higher than estimated from the general data (see above), they will be 15 percent higher for buses, truck trailers or special traffic. In the case of trucks, lack of accurate data on the average truck size contributed additional uncertainty.

Road maintenance cost savings (Item F) were computed according to the formula

$$\text{Maintenance cost per mile p a.} = a + b\,x$$

where x represents the projected average daily traffic, expressed in traffic units, and a and b are the parameters of a linear approximation of the road main- tenance cost *within the range of traffic covered by this project.*[3] This is com- puted for the road with and without the road improvement, and the cost savings constitute a benefit. The variable term G only was varied in the prob- ability analysis. It will also be noticed that the traffic unit, which in the case of the existing road was defined as a car, was changed to a truck in the case of the new road. In effect this implies that the passage of cars over the new (asphalt) road is estimated to have a negligible effect on the cost of the road's maintenance.

The final item H relates to the cost of pavement strengthening, estimated at $13,500 per mile (± 25 percent) and its timing. These are discussed below. Note that this cost is directly related to the corresponding benefits (the traffic use of the roads).

Results of the analysis

The overall results of the best estimates are shown in Table 7, and of the probability analysis in Figure 6. Table 7 sets out the rates of return based on the best estimate of each variable. In column A the expected benefits to the Zambian through traffic are included; in column B they are excluded. Com-

[3] See Figure 9.

parison of columns A and B shows that the rate of return is only moderately sensitive to future Zambian traffic level and duration. The overall rate of return to Tanzania is thus about 14 percent. In addition, Tanzania would probably receive a share of the savings likely to accrue in respect of the Zambian transit traffic. The rates of return for each section individually are generally satisfactory, even though from the Tanzanian point of view the return becomes marginal in the neighborhood of the Zambian border.

The probability analysis (Figure 6) indicates that while there is a high degree of uncertainty with respect to some of the major variables, there is a relatively small risk that the project will not yield a satisfactory return. The analysis showed that there is less than 5 percent probability that the project will earn less than a 10 percent return for Tanzania—i.e. more than 95 percent probability that the rate of return will be over 10 percent. The mean returns under probability analysis, 20.1 percent with the Zambian traffic and 15 percent without, are marginally higher than those found by a best estimate calculation, 19.6 percent and 13.9 percent, in Table 7.

The Problem of the Road Surface Thickness

The uncertainty about the traffic level for which the road was to be designed made the choice of an asphalt surface thickness difficult. Since the cost-benefit analysis was already in probability form, it was only necessary to feed in the alternative thicknesses of surface and their corresponding expected lives to the probability analysis, to evaluate which alternative was best.

The problem of optimization is one which is met frequently in Bank projects, and stems from the different perspectives of those who tend to seek the best, i.e. "safest" engineering solutions, and those who seek to optimize the use of scarce resources. In the Tanzanian case, two issues were at stake. The first was the strength of the road, i.e. the number of vehicle passes (of an equivalent standard 18 kip axle) for which the road should be designed. The lower the strength, the sooner the pavement was likely to wear out and to need replacement. The second issue was the manner in which the design strength should be achieved, through the base or through the pavement surface. An asphalt concrete surface of minimum (¾″) thickness and a thick base were theoretically equivalent to a 2″ asphalt concrete surface and a less thick base, but there was large uncertainty as to what the cost of laying a pavement of ¾″ would be. The second alternative was being pressed by the consultant though its initial cost was admittedly higher.[4]

[4] A third alternative, a double-seal bituminous surface course, as originally specified, was not considered because its costs appeared very slightly higher than the minimum thickness ¾″ asphalt concrete alternative.

A cost analysis was therefore conducted for various strengths and for the two extreme alternatives, with the relevant maintenance and replacement costs discounted to present value. The results are shown in the following table:

Expected (Mean) Present Value of Cost Per Mile at 10 Percent Discount Rate (US dollars)

Number of vehicle passes	125,000	150,000	175,000	200,000	250,000	300,000
Structural strength:						
¾″ asphalt concrete combination	27,335	27,714	28,434	29,109	30,317	31,557
2″ asphalt concrete combination	30,499	30,736	31,292	31,845	32,726	33,978

Note: Costs include initial construction cost for sub-base and surface course, maintenance costs and strengthening costs, less salvage value.

This exercise showed first that at all strengths, the ¾″ asphalt concrete combination was economically preferable to the 2″ one. The difference between the two was greatest at the minimum design strength of 125,000 passes, but this was rejected because the probability of the need for a strengthening as early as 1978/79 (regardless of surface course chosen) was felt to be too high. The small cost difference between this alternative and the 150,000 pass alternative appeared a reasonable cost to pay for the additional security on that score. It was estimated that the minimum cost acceptable solution thus identified at 150,000 passes strength would be on the average $1.3 million less expensive to build than the equivalent 2″ pavement, and about $750,000 less expensive on a present value basis at a 10 percent rate of discount.

However, quite properly, engineering consultants have a professionally high aversion to risk, and this was heightened in this case by their relative lack of experience with lesser thicknesses of asphalt concrete. Thus the engineer's judgment was that additional risk was attached to the ¾″ alternative. Even though the *mean* expected present value of cost had been shown to be less, this would not therefore be convincing if there were a *fair chance* of the cost of a less thick surface proving more expensive in the long run or of the cost difference being so small as not to be worth the trouble of optimizing.

Fortunately the probability distribution of Figure 7 shows that there was nil probability, according to the assumptions that had been technically agreed, that the 2″ alternative would be cheaper, and over an 80 percent chance that the extra cost would be only $2,000 per mile in present value. The mean expected extra cost was $3,024 per mile. The Bank suggestion therefore appeared worth considering.

The final outcome was a compromise. The Bank financed section as well as the other sections of the road were designed to a 175,000 passes strength and a 1½″ asphalt concrete thickness.

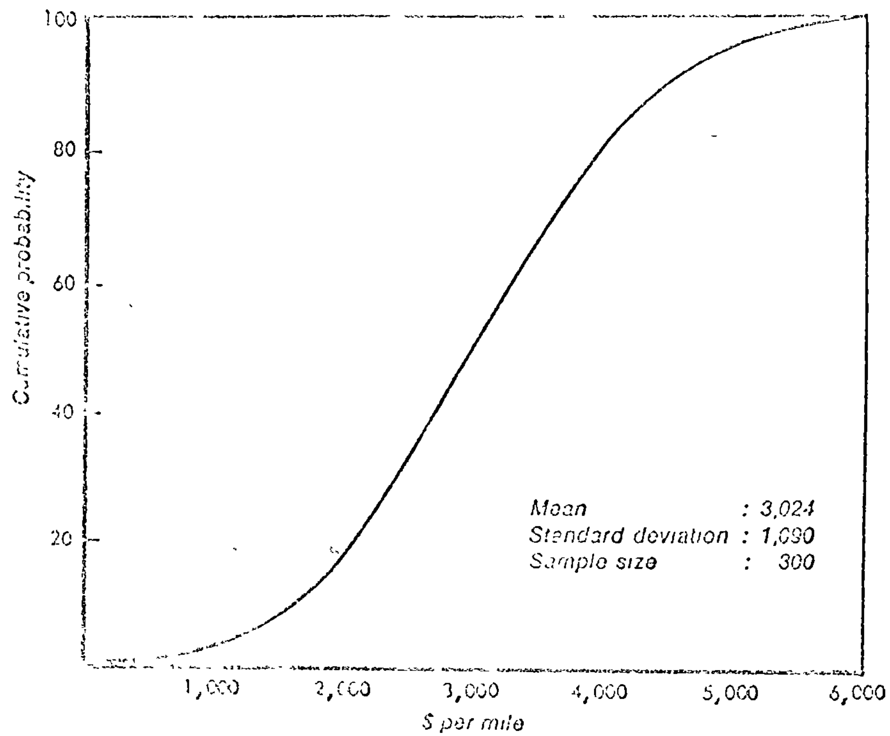


Figure 7. Probability of Cost Difference between ¾″ and 2″ Asphalt Concrete Surface Course Solutions

V

DISAGGREGATION

This chapter and the next discuss the problems arising from the choice of a level of aggregation and the evaluation of correlation, which create the major difficulties of risk analysis and whose solution is a critical condition for the validity of the results.

Aggregation

By level of aggregation we mean the degree of detail which the analysis encompasses. For example, in the cost of a road, the costs of land clearance, earthwork, base, sub-base, and pavement can be distinguished. The cost of the base can be further subdivided into the costs of extracting stones, crushing them, transporting them, and laying them, and each of these stages can also be broken down. Where to stop subdividing in order to make the best risk analysis is the aggregation problem.

Our experience indicates that risk analysis calls for more disaggregation than usually is used in Bank project appraisal and that the smaller the component, the easier it is to formulate a judgment, though there is clearly a limit to this rule. To illustrate we cite the case of the Great East Road.

Great East Road Project

The proposal to pave the 64-mile Luangwa-Nyimba section of the Great East Road, which leads from the line-of-rail in Zambia to Malawi, was presented to the Bank at the same time as that of upgrading the Tanzam highway to Tanzania. The existing traffic level was known to be low, and the traffic growth rate only normal. There was a 25 percent possibility that the road might be needed as a spare carrier for Zambian export-import traffic. In that case an additional ADT of about 35 was expected.

However the dominant uncertainty in this case was the cost of the project. The detailed engineering study had not been completed at the time of appraisal and estimates ranged widely from £1.5 million to £2.8 million. The higher cost would have entailed a negative return. But a decision had to be made whether to go ahead with the project, to postpone it or even to discard it.

Accordingly the opportunity was taken to employ risk analysis, in which the chief uncertainties revolved around the technical elements of cost. The fact-finding advantages of a probability analysis in such a case soon became apparent. When the consultant was asked to give a single cost estimate, he was unwilling to do so before completing more detailed engineering work, since the figure might have been quoted against him later. Also, not being familiar with this method, he was at first unable to give variations around an estimate.

The discussion then backtracked to subjects like cost of cement, thickness of base, amount of earthworks, etc., and the attitude of the consultant changed. He had designed many other roads and, on the basis of the preliminary soil survey, it was easy for him to guess what base would be needed on this particular road. He knew that economic considerations would permit only a minimum realignment of the existing road and consequently was able to form a judgment as to how much earthmoving work could be dispensed with—and so on. By dealing with the components separately—by disaggregating—it was possible to obtain a range of cost estimates and a full probability distribution over this range—which at first sight had appeared impossible. The full details of the information we obtained is given in Table 9, and there is further discussion of the analysis in Reutlinger's *"Techniques for Project Appraisal under Uncertainty,"* which is Occasional Paper No. 10 in this series.

The result of the analysis on very preliminary cost estimates indicated that if the proposed construction were to begin after the completion of engineering preparation, the mean expected rate of return would be less than 5 percent, and there was only a very low probability that the project would earn more than a 10 percent return. If construction started in 1974, the result was only slightly better. As a result it was agreed that the Bank should not at that time consider financing that part of the road, though it would be prepared to reconsider the project on receipt of further study.

Advantages of Disaggregation

Incomplete or inaccurate judgment often results from a lack of disaggregation. Vehicle operating costs and road maintenance costs in the risk analyses on

TABLE 9: Zambia, Great East Road: Hypotheses Made in the Probability Analysis of the Economic Rate of Return of the Proposed Bituminous Paving of the Luangwa-Nyimba Section

	Item	Best Estimates	Nature of Uncertainty	Probability Distribution
	Construction Costs	*(kwachas)*		
1	Pavement base	466,000	Price and quantity	Discrete: *Probability (percent)* — *Total Cost (kwachas)* 48 — 579,000 (6″ base) 52 — 466,000 (5″ base)
2	Sub-base and shoulders	311,150	Price, quantity and thickness of base required	(A) *Basic Cost*: Step rectangular distribution reflecting the uncertainty about the salvage value of the existing road *Probability (percent)* — *Cost within Range (kwachas)* 30 — 219,200–311,150 50 — 311,150–342,000 20 — 342,000–561,165 (B) *Final Cost*: Correlated to cost of pavement base (Item 1 above) (i) If cost of base is K 579,000 then: *Probability* — *Final Cost sub-base & shoulders* 42% — Equal to Basic Cost (2A) 51% — 80% of Basic Cost 7% — 60% Basic Cost (ii) If cost of base is K 477,000 then: 88% — Equal to Basic Cost 12% — 60% of Basic Cost
3	Earthworks	92,400	Price and quantity	Uniform between K 4[illegible],200 and K 92,400
4	Borrow materials	15,000	"	Triangular on the range K 10,000 to K 30,000
5	Others	1,023,000	Price	Triangular on range K 941,850–K 1,163,500
	Traffic Count in 1968	*Best Estimates (ADT)*		
6	Cars	41	Statistical error	Normal: Mean 41, Standard deviation 3.35
7	Trucks	23	Statistical error and number of tankers in actual traffic count	Triangular on the range 15–25
8	Buses	6	Statistical error	Normal. Mean 6, Standard deviation 1
9	Truck trailers	15	" "	Normal. Mean 15, Standard deviation 3.5
10	Special traffic (1971)	nil	Necessity to use Great East Road (as a safety valve) for import-export traffic	Discrete. *Probability (percent)* — *Traffic Level (ADT)* 25 — 35 75 — nil

[illegible]

	Item	Best Estimates	Nature of Uncertainty	Probability Distribution
	Traffic Growth	(annual rate) (percent)		
11	Cars	6	Forecasting Error	Uniform on range 4%–8%
12	Buses	6	" "	" " " 4%–8%
13	Trucks and truck trailers	8	" "	Uniform on range 6%–10%. Growth of truck and truck trailer traffic is fully correlated
14	Special Traffic:			
	Period 1968–1972:	0	—	—
	Period 1972 on.	−20	—	—
	Elasticity of Traffic Demand			
15	Cars	.75		
16	Trucks	.75		
17	Buses	0		
18	Truck trailers	.75		
19	Special traffic	0		
	Vehicle Operating Costs (per vehicle mile)	(kwachas)		
20	*Old Road*			
	Cars	[illegible]	Lack of data	Uniform on a −12% + 15% range; all fully correlated.[a] In addition operating cost of trucks is varied uniformly on a −5% +10% range to account for uncertainty on size of trucks.
	Trucks	[illegible]	Lack of data and size of trucks	
	Buses	[illegible]	Lack of data	
	Truck trailers	[illegible]	" " "	
	Special traffic	[illegible]	" " "	
21	*New Road*			
	Cars	0.0179		
	Trucks	0.0670		
	Buses	0.1034	—	[b]
	Truck trailers	[illegible]		
	Special traffic	0.140		
	Maintenance (Formula, $a + b$ (Traffic units per day) = Maintenance Costs per annum)			
22	*Old Road*	(kwachas)		
	Fixed term	[illegible] = a	Divergence of existing data	Uncertainty accounted for by taking variable term uniformly distributed between K 3 and K 5
	Variable term	3.6 = b		
23	*New Road*			
	Fixed term	600.0 = a	Divergence of existing data	Uncertainty accounted for by taking variable term uniformly distributed between K 1 and K 2.5
	Variable term	2.25 = b		
24	*Traffic units for maintenance calculations*			
	Cars	1		
	Trucks	2		
	Buses	2		
	Truck trailers	3		
	Special traffic	3		
	Others			
25	Life of road (years)	20	Incomplete analysis	Triangular on range 12–25 years
26	Length of road	64 miles		
27	Year construction starts	1969		
28	Construction time	2 years		
29	Distribution of cost over construction period:			
	1st year	50 percent		
	2nd year	50 percent		

[a] This distribution is artificial and is only geared at getting a correct distribution of the savings from the improvement of the road.

[b] Ignored because variation of savings is fully taken care of by variation of operating costs on old road.

ad projects provide examples. Lack of time prevented us from disaggregating id consequently we had great difficulty in choosing probability distributions hich reflected our uncertainties. Clearly, if we went into the details of the mponents of vehicle operating costs, the problem would become simpler cause we have had experience with fuel consumption, tire wear, depreciation, :. The same applies to road maintenance co ts because we have an idea of the nount of gravel necessary to maintain a gravel road, the number of gradings quired per year, etc., and we can also find ways to express the uncertainties sulting from the weather, the geometry of the road and other such factors hich are difficult to introduce into a maintenance formula. The choice of an propriate level of aggregation, therefore, appears to be an essential condition the expression of a clear judgment.

PART II

VI

CORRELATION

The Importance of Correlations

The example given in the previous chapter seems to indicate that, in many cases, the more disaggregation the better. Unfortunately there is a limit to disaggregation because of the problem raised by correlation. The problem was touched on in the sensitivity analysis of the Mogadiscio port project (Chapter III), where there was expected correlation between the productivity of a banana gang and the productivity of a general cargo gang, and correlation between the number of men employed in a transit shed and that in a warehouse. Correlated variables, or in simple terms, variables which are likely to vary together in a systematic way, appear in every project. However, an experienced professional may feel he is familiar with two separate variables and knows how they are related, without being able to describe their correlation—how their variations are related. Correlations are difficult to detect, and even more difficult to measure, but overlooking them may lead to a completely wrong interpretation in the analysis.

The Mogadiscio case gives an idea of the importance of correlations. We initially neglected the correlation between productivity of labor and port capacity. The probability that the project would earn less than 10 percent in this case was 15 percent. After we introduced this correlation, the probability rose to 40 percent, i.e. it almost tripled. In another project (a telecommunicais project in Malaya) the standard deviation of the rate-of-return distribution without allowing for correlation between two variables was about .35. With the appropriate allowance for correlation it rose to about 1.1. Since in this case the rate-of-return distribution was practically a normal distribution, this change meant that the probability that the rate of return would fall outside of a ±1 percent range in absolute value around the best estimate rose from 5 percent to about 40 percent. The consequences of mishandling the correlations are so serious that they can eventually lead to the wrong decision.

It is easy to understand the way correlation works. When independent variables are aggregated, the effect of the variation of one may be compensated by the variation of another one in an opposite direction. If they are positively related, the effect of the variation of one will always be aggravated by the variation of the others. If productivity of labor and capacity of the Mogadiscio port are independent of each other, a low value of the productivity of labor may be compensated by a high value of the capacity of the port. If they are positively correlated, as is likely to be the case, the effect of a low productivity of labor on the economic rate of return is heightened by the low capacity of the port, and the probability of getting a low rate of return will be higher than if they were independent. Correlations can also be negative, that is, the variables may systematically compensate each other. However, in the type of projects we have worked with this occurs less frequently than positive correlation.

Correlations are difficult to detect. The first reason is that they do not have to be taken into consideration in the single point estimate method and are therefore not familiar to most people. Let us take the port of Mogadiscio case. The engineer generally knows that in a port which operates normally we'll be able to expect that the productivity of labor will be around 10 tons per gang-hour and the capacity around 700 tons per linear yard of berth. He also knows from experience how much productivity can vary from port to port and under different situations, and likewise for port capacity. But he tends to think of the port as an organic whole, rather than to analyze its functions; to ask him how the variations of capacity relate to the variations of productivity is to ask the engineer a question he does not usually ask himself. Therefore it is equally difficult to get an answer, and if the question is not asked, there is a good chance the engineer will not notice and the correlation will be overlooked.

The second reason that correlations are difficult to detect is that they are often hidden. Some correlations, particularly those which relate to engineering specifications, are not too difficult to identify. For example, the strength of a road is given by the thickness of sub-base, base and pavement. The engineer's uncertainty about each of these three parameters is tied to his uncertainty

about the other two, and he will therefore recognize their correlation. Items 1 and 2 of Table 9 illustrate this point in the case of the Great East Road in Zambia, and show the hypotheses which grew out of discussion with the consulting engineer.

Other correlations are more difficult to spot and to assess. For example, vehicle operating costs are an important element in road projects. We usually make the distinction between operating costs for different types of vehicles and, *a priori*, if we have underestimated the operating cost of cars, there is no reason why we should also have underestimated the operating cost of trucks. However, if we have underestimated the operating costs of cars, it may be because we have overestimated the quality of the road. It is, therefore, likely that this will affect operating costs of all vehicles in the same direction. Item 20 of Table 9 shows that we decided to treat as fully correlated the operating costs for all vehicles except trucks (we were not sure of the composition of the truck fleet). Another example is the uncertainty about the amount of work required to build a road. If the road has been designed by one person and if this person has overestimated the amount of earthwork, is it because he has a systematic tendency to overestimate—in which case he will also probably have overestimated all the other elements of the road—or is it just by chance? We have assumed that the latter is more likely than the former, or at least will explain a greater part of the variations in the amount of earthwork, but the question is open for discussion.

The problem of correlations should therefore be approached with great care. However, its solution is not impossible and, while we may not yet have mastered it perfectly, the following points can serve as a guide.

Suggestions of Ways to Meet the Problem

Limitation of the disaggregation

To limit disaggregation is to solve the problem of correlation by eliminating it. If we work with the total cost of a road, we do not have to worry about the correlation between the cost of the base and the cost of the sub-base. The distribution we shall use for the cost will implicitly include this relation. However, as we have indicated, there is a limit to the level of aggregation which is feasible in obtaining probability distributions. Therefore the choice of the level of aggregation requires a trade-off between the advantages of clarity of judgment and of avoiding the hazards of disaggregation. It is a difficult choice and one often guided by the availability of time. Because we believe that the influence of correlations on the outcome of the analysis is more important than the influence of the shape of any particular distribution, we have usually opted for as little disaggregation as possible. The distribution of vehicle operating costs referred to earlier is a case in point. For the same reason, all the variables used in the case of the port of Mogadiscio project are rather highly aggregated.

Isolation of the sources of uncertainties

Limiting disaggregation can be considered only as an emergency measure in dealing with correlation. The advantage of risk analysis, after all, is that it permits disaggregation, and we want to retain this advantage. We found that it helps to think not so much in terms of disaggregating the technological components of the project, but in terms of disaggregating the sources of uncertainty. Let us again refer to the case of vehicle operating costs for the Great East Road in Zambia. To compute the rate of return we have to distinguish between the operating cost of cars, trucks, and truck trailers; this is a technological disaggregation. For the purpose of the risk analysis, we may ignore the technological distinctions *per se* and think in terms of the sources of uncertainty on these vehicle operating costs. We may distinguish three essential and independent sources of uncertainty: errors in the general data on which we based our estimation, errors in the way we have extrapolated these data to the particular case of the Great East Road,[1] and uncertainty about the average truck size. By so defining the uncertainties, it is easier to assess the correlations. We have treated as fully correlated for all types of vehicles the uncertainties resulting from the first two sources, because the data for all vehicle types originated from the same sources, and also because we thought the condition of the road would affect operating costs of all vehicles in the same way. But the third source of uncertainty affects only the operating cost of trucks. Variations of the vehicle operating costs resulting from this particular source of uncertainty should not be treated as correlated with others.

Applying the same line of reasoning to the analysis of the uncertainty on the cost of a project, it may be useful to distinguish between quantity uncertainty, unit cost uncertainty, and bidding uncertainty. In making this distinction in the case of the Tanzam highway, we considered all the unit cost uncertainties as independent and, except for the technological correlations, also all the quantity uncertainties. We then allowed for uncertainty about bidding on the total cost of the project, i.e. we assumed that this uncertainty would affect the cost of all the components of the project in the same direction. As a result, our

[1] Tables usually give vehicle operating cost for a typical earth road or a typical gravel road. The roads the Bank considers for financing, before they are improved, are always something of a cross between the two.

uncertainty about the actual unit price of each component of the project is explained to a very small extent by uncertainty about the economic unit cost, and to a much larger extent by the uncertainty about the outcome of the bidding process. When possible, this isolation of independent sources of uncertainty seems to be the easiest and most rigorous way of handling correlations, but in many cases we have also had to rely on the following approach.

The pessimistic-optimistic approach

Suppose we suspect some correlation between two variables but we cannot quantify its exact effect on the distribution of rates of return to a project which has tended to look favorable in the analysis to date. We can often reinforce our confidence about the project by examining a pessimistic view of the suspected correlation. If we find the project still acceptable, even in this light, then we can feel our confidence in the project has been justified. Conversely, if to date a project has tended not to look good we may be able to reassure ourselves that we are justified in rejecting it by examining an optimistic view of the effect of the suspected correlation. If the project still looks unfavorable, even with all the benefit of the doubt, our doubts are confirmed. Of course, if the reverse comes to light and the favorable project looks bad under pessimistic correlation assumptions or the unfavorable project looks good under optimistic assumptions, we must try some other route.

For each case, we have an example where this approach worked for us: in the first case, the Mogadiscio port project, and in the second, the Great East Road. In the Mogadiscio case, we were concerned with the correlation between productivity of labor and port capacity. The project had looked good to date, so we looked at the pessimistic case: the complete dependence of the two variables. Curve 1 of Figure 8 illustrates the result we obtained. Also indicated in the figure is the curve if the two variables were completely independent (Curve 2).[2] We know that the true curve (3) lies somewhere in between.

The decision to accept a project is based—among other things—on the probability of having more than a 10 percent return, if 10 percent represents the opportunity cost of capital. If we are sure in the Mogadiscio case that the project is acceptable under the assumption of complete dependence of one variable on another, it will be even more acceptable under the true assumptions. In the Mogadiscio case, 10 percent fell at A; this is still a favorable result. In the case of the Great East Road, a project which did not look good,

[2] The figure as here drawn and described applies to the correlation of the variables mentioned in the port of Mogadiscio case and the Great East Road case; it is not a universal diagram. In fact, the positions of the curves of complete dependence and independence as here drawn may be reversed in different cases or using different variables.

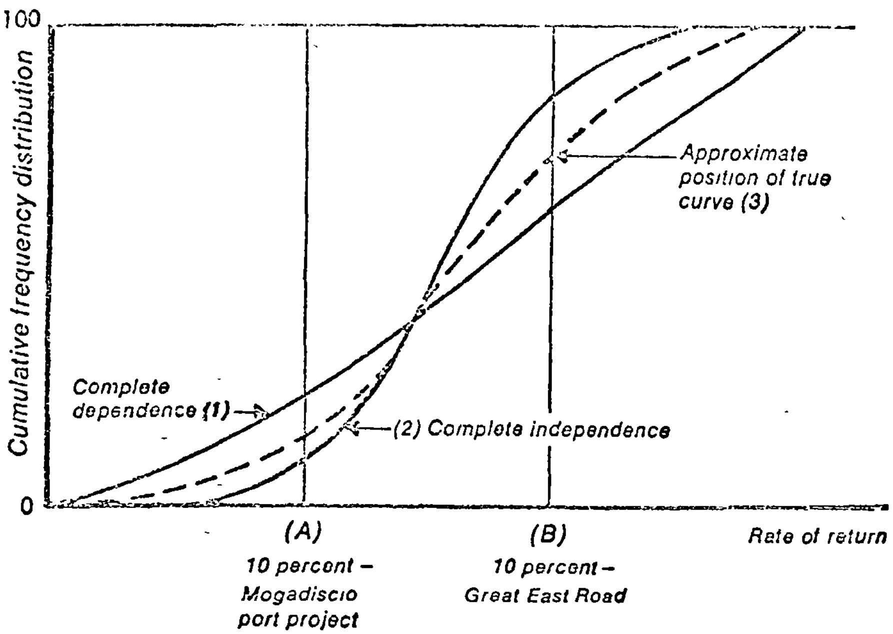


Figure 8. Cumulative Distribution Functions with and without Correlation of Variables

we were concerned with the correlation of vehicle operating costs. We made the optimistic assumption and got the same picture as in Figure 8, except that 10 percent fell at B, still a poor result. Thus, introducing a pessimistic correlation hypothesis has not damaged the Mogadiscio case; and introducing an optimistic correlation hypothesis has not helped the Great East Road case.

We mention this approach for what it may be worth. We have used it rather extensively because we did not have time to handle the problem more accurately and because it was the best method we could think of. It turned out to be practical because the models we were using were simple and it was easy to anticipate the consequences of various correlation assumptions. In the future, however, our models will undoubtedly become more and more complicated, making it difficult to say whether an assumption is more or less optimistic. Let us hope that at the same time we will improve our understanding of the correlation problem as it affects working practice and be able to handle it more rigorously.

Collection of more data

An essential step toward a better understanding of correlation is to make a serious effort to collect more data. We have noticed the great difficulty, and sometimes impossibility, of making subjective judgments about correlations. We have imputed this to a lack of experience on a problem which is new to us,

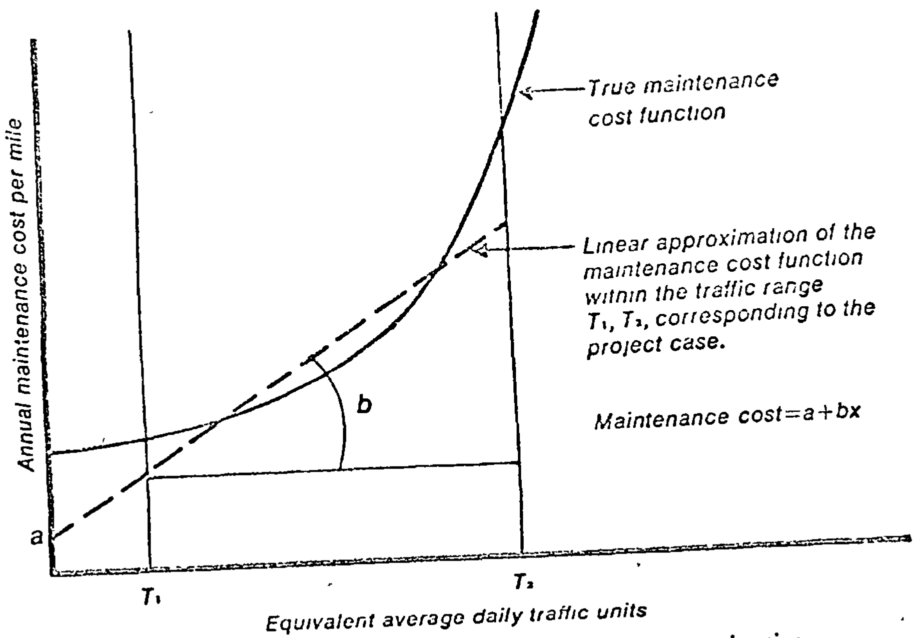


Figure 9. True Maintenance Function and Linear Approximation

and for which we must somehow develop a feeling. Whether we do it by way of statistical analysis or more empirically, we need data to acquire this feeling. In the case of the Mogadiscio project the availability of port data enables us to check a number of possible correlations. Besides the correlation between imports and exports, which we neglected because it was not sensitive, we suspected some correlation between the productivity of labor and GNP, or anything which would reflect improvement of living conditions in Somalia. Fortunately, at least for the sake of the mathematical analysis, available data indicated that there is none. In the case of roads we have not been so lucky. As an example, consider the case of the road maintenance cost (M), which we usually approximate by a linear function of the type:

$$M = a + bx$$

where x represents the projected average daily traffic (see figure 9). This formula is surrounded by uncertainties which affect both the constant coefficient a, and the variable coefficient b. The uncertainty about these two coefficients is likely to be correlated: if a is overestimated, b is probably underestimated, and conversely. In the absence of more reliable data on road maintenance cost this correlation has proven to be very difficult to assess. The only way to do better is to collect and analyze data, a relatively easy assignment in this case. In other cases it will be more difficult.

VII

THE CHOICE OF A PROBABILITY DISTRIBUTION

Choosing probability distributions for the variables is what seems to worry most people about risk analysis, possibly because they think it requires finding the true distribution of each variable. This is indeed quite impossible, though ways to improve the drawing-up of distributions through team evaluation and Bayesian approaches have been studied.[1] The aim of risk analysis is more modest. Risk analysis does not aim to give the exact true distribution of the rate of return, that is, the distribution we would obtain if we were omniscient rather than human beings, but rather the one which best represents the judgment of an appraisal team. Therefore, it is not a question of finding the true distributions of the input variables, but for each variable the distribution which best expresses the judgment of the appraiser. The distribution corresponding to a vague judgment will be as appropriate and useful as the one corresponding to a detailed judgment.

We have already mentioned in the Mogadiscio case two ways of obtaining probability distributions, one which we have called the portrait approach and the other which leads to the step rectangular distributions. Bank appraisal missions have now practically abandoned the portrait approach. When participating in this approach the appraiser tends to accept any smooth distribution. Possibly he is aesthetically influenced by the deceptively attractive appearance of the smooth curve, and impressed by the complicated formulas. His judgment

[1] See Robert L. Winkler, "The Consensus of Subjective Probability Distributions," in *Management Science*, Vol. 15, No. 2, October 1968, and its bibliography (19 articles).

seems to lose its sharpness, and in the end the approach means more work for fewer results. Therefore, as often as possible the second approach is used which leads to step rectangular distributions. Attempts have also been made to supplement it by using distributions which would fit cases in which not enough information is available to obtain a good step rectangular distribution, but in which information would be wasted by using a distribution which failed to discriminate between the likelihoods of any two values on a given range. The need felt by the appraisers for such distributions is a good illustration of what seems to be the main objective of the probability distribution choice, namely, to make use of all information available but not to require more information than is, in fact, available. The various distributions we shall now review, all of which have been used, are precisely geared to making the maximum use of available information.

The Step Rectangular Distribution

We have already described in detail in the Mogadiscio case how to obtain the distribution shown in Figure 10. This distribution is an attractive one for a number of reasons. In the first place it takes explicit advantage of the fact that the quantification of subjective probability judgments, in both theory and practice, is based on preference ranking. It also has the advantage that it can be drawn up by the appraiser himself. He has the freedom to choose whatever intervals he wants and to divide them into as many sub-intervals as he wants. This complete freedom of initiative, which he lacked in the case of the portrait approach, seems to help him considerably in the expression of his judgment.

In use, this distribution has proven astonishingly reliable: when the data generation process has been repeated for several distributions after a period of

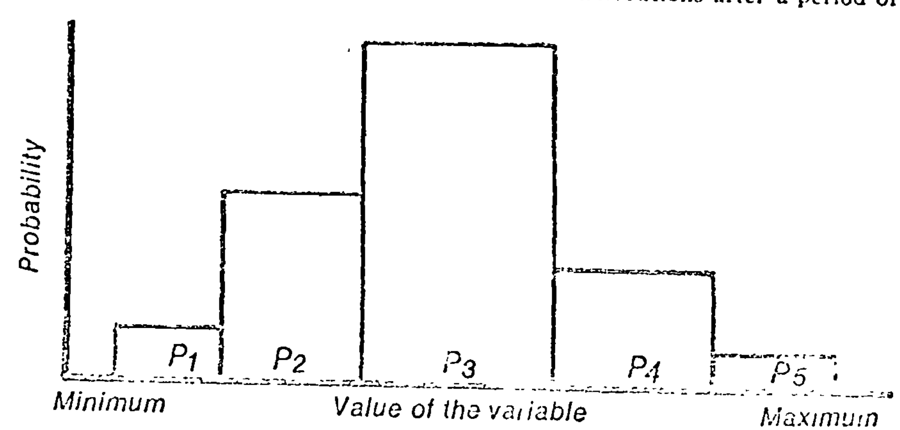


Figure 10. The Step Rectangular Distribution

ime, it has usually come up with the same or a very similar result. It is also distribution which fits well with the rule of using all the information available but not requiring more. If the appraiser thinks that he can express more ccurately a judgment he has just made, he can sub-divide intervals one step urther and create a more detailed distribution. If, on the contrary, he thinks hat he will be guessing to say that one value in an interval is more probable han another, he may stop sub-dividing. Furthermore, this distribution lends tself well to the final review and polishing, described as the fourth and last tep in the process described in Figure 2.

We have asked ourselves whether it is useful to smooth out this distribution efore beginning the risk analysis. In many cases a continuous distribution vould appear better fitted to the type of judgment we wish to express. But here are difficulties. The final outcome of our smoothing has to be a distribuion from which random numbers can be easily generated. But by trying to mprove the presentation of the judgment we want to simulate, we may end up vith a less useful distribution. First, it may be difficult to find a smooth listribution which is close to the step distribution. Even if we do find a coninuous distribution approximating well the one we start with, the improvenent we gain may not be worth the trouble we may run into in the generation f random numbers. All computer random number generators start from niformly distributed numbers; for the type of distribution we described in Figure 10 this generation is extremely simple. On the other hand, most other ypes of distribution require mathematical transformations which are often lifficult and usually time-consuming. Therefore, the extra accuracy which can e obtained through smoothing is usually not worth the supplementary work vhich it requires.

The Discrete Distribution

The discrete distribution is very similar to the step rectangular distribution. The only difference is that the probabilities P_1, P_2.... etc., of Figure 10, nstead of being assigned to a range, are assigned to one value only. This disribution is obtained in the same way and has the same properties as the step ectangular distribution. We have used it when the variables we were conidering were, by nature, discrete variables—for example, in the Tanzam highvay case, the year in which the Tanzam railway might come into operation.

The Uniform Distribution

From the point of view of information availability, the uniform distribution overs the opposite case from the step rectangular distribution. It is used where udgment is very vague and the appraiser is not able to differentiate between

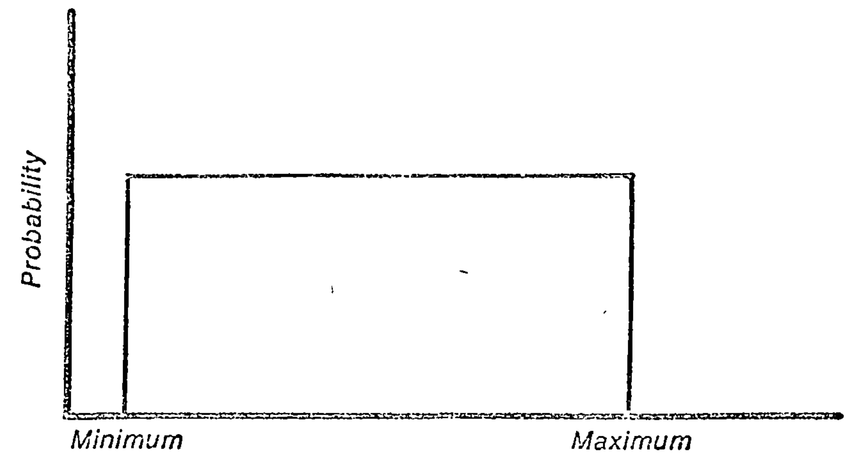


Figure 11. Uniform Distribution

any two values within the range of the variable. It is shown in Figure 11 and can be viewed as a particular case of the step rectangular distribution, with only one sub-range. As far as is possible, this distribution should be avoided. It is almost contradictory to suppose that a project with an equal chance of costing anywhere from \$10 million to \$15 million will under no circumstances cost \$9.9 million or \$15.5 million, which is what we assume when we say that the cost of a project is uniformly distributed between \$10 million and \$15 million. We have therefore used the uniform distribution only in the case of low sensitivity variables or whenever we wanted, to be on the safe side, to overestimate the probability of the extremes of the variables' range.

The Beta Distribution

The Beta distribution is the first distribution we tested to fill the gap between the step rectangular distribution, for which detailed information is needed, and the uniform distribution, for which minimal information is needed. Figure 12 shows its appearance. Use of the Beta distribution was suggested by the wide use made of the Beta distribution in the PERT system (Program Evaluation and Review Technique). The Beta distribution is entirely defined, if in addition to its range, one fixes two parameters. The literature on PERT[2] suggests use of the mode and a standard deviation

[2] See for example D. G. Malcom, J. H Roseboom, C. E. Clark and W. F. Fazar, "Application of a Technique for Research and Development Program Evaluation," *Operations Research*, Vol. 7, pp. 646–669, 1959.

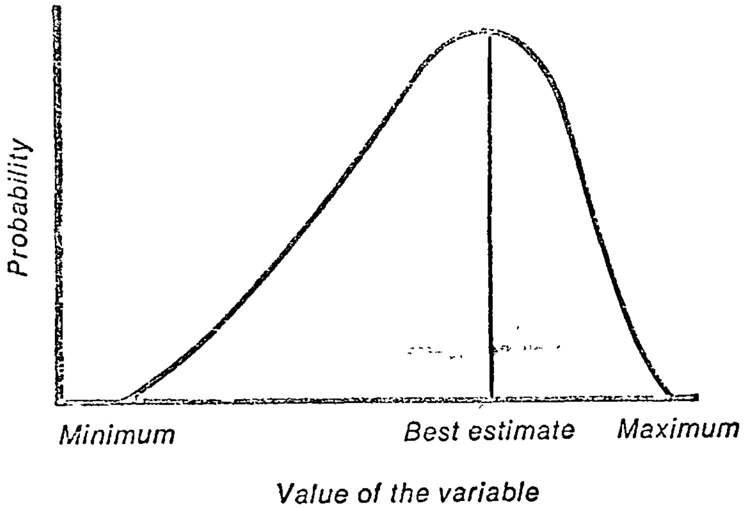


Figure 12. Beta Distribution

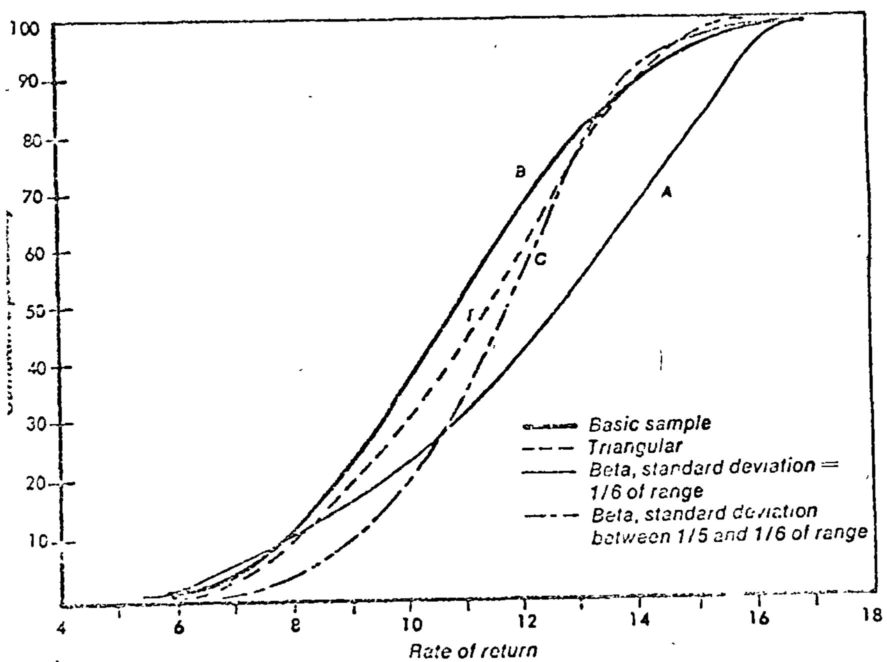


Figure 13. Mogadiscio Port Project: Substitution of Triangular and Two Different Beta Distributions for the Original Distribution of the Productivity of Labor

equal to ⅙ of the range. We have compared the results of substituting this distribution for the step rectangular distribution of the productivity of labor in the Mogadiscio case. The result, shown in Figure 13 (Curve A), indicates that this particular Beta distribution is a bad choice; it is far from being close to the basic sample distribution. We have not investigated further the use of the Beta distribution defined in this way because we think that it relies too much on the value assigned to the best estimate. In our experience the best estimate is not a reliable datum, and in practice it often is an imprecise mixture of the value with the highest probability and the mean.

To find a way to limit the influence of the best estimate, let us reconsider the case of the port of Mogadiscio. The step distribution in Figure 13 shows that all values between 8 tons per gang-hour and 10 tons per gang-hour have the same probability. The best estimate (m) could therefore be anything

TABLE 10: Degrees of Freedom of Selected Beta Distributions of Range 0–1

P = probability of getting less than the best estimate

P	Degrees of Freedom	m (best estimate) .55	.60	.65	.70	.75	.80	.85
	a	3.0	4.5	1.5	4.0			
.50	b	2.5	3.0	2.5	2.0	—		
	a	4.0	4.0	4.0				
.55	b	3.5	3.0	2.5				
	a	4.0	3.0	3.5	4.5	4.0	4.0	
.60	b	4.0	2.5	2.5	2.5	2.0	1.5	—
	a		4.0	4.0	4.0	3.5	3.5	
.65	b	—	3.5	3.0	2.5	2.0	1.5	—
	a		3.0	4.5	4.5	4.5	4.5	4.0
.70	b		3.0	3.5	3.0	2.5	2.0	1.5
	a				4.0	4.0	4.0	3.5
.75	b		—	—	3.0	2.5	2.0	1.5
	a						4.5	4.5
.80	b				—	—	2.5	2.0
	a							4.0
.85	b						—	2.0

The Beta distribution given in this table is such that:

1. $P = \frac{1}{B(a,b)} \int_0^m t^{a-1} (1-t)^{b-1} dt$

   with $B(a,b) = \int_0^1 t^{a-1} (1-t)^{b-1} dt$, accuracy on $P = \pm 0.02$.
2. The standard deviation is between ⅕ and ⅙.
3. The mode falls within the range $m - 0.10$ to $m + 0.05$.
4. The degrees of freedom a and b are such that $2a$ and $2b$ are integers.

etween the value we originally chose (10 tons per gang-hour) and a value nferior to this by almost 30 percent of the range. From Table 10 it will now e seen that if we try to keep the standard deviation of the Beta distribution between 1/5 and 1/6 of the range, a small shift in the mode will result in a substantial modification of the shape of the distribution. For example, let us look at the olumn $m = 0.70$. It indicates that there exist five different Beta distributions f range 0-1 with a standard deviation between 1/5 and 1/6 and a mode between 60 and .75. The first column shows that for these five distributions the probbility of exceeding .70 (or $1 - P$) varies from 50 percent to 25 percent. These ive distributions are, therefore, quite different. If we had decided to use a Beta istribution defined in this way for the productivity of labor in the Mogadiscio ase, this distribution would have given a probability for exceeding 10 tons per ang-hour of anywhere from 25 to 50 percent, depending on where we had ecided to place the mode.

This led to the idea of making the choice of the degrees of freedom of the eta distribution depend not only on estimates of the standard deviation and he mode of the distribution, but also on an estimate of *the probability that his mode will be exceeded.* Table 10 is designed for this purpose. Given a best stimate m and the probability $(1 - P)$ of exceeding m, it gives the degrees f freedom of a Beta distribution over the 0-1 range such that its mode will e between $m + 0.05$ and $m - 0.10$ and the standard deviation between 1/5 nd 1/6. In introducing this form of estimation, the appraisal team's idea was ot only to limit the importance of the best estimate but also to make use of nformation on the probability of exceeding the best estimate, which is often vailable. Figure 13, curve C, shows the result of introducing this information; it is clearly closer to the basic sample than the previous Beta distribution (curve A).

The Trapezoidal Distribution

This distribution is shown in Figure 14 below. It owes its appearance here o the experience of the lack of reliability of the best estimate, referred to in he previous section, and to the observation that it is often helpful to distinuish a smaller range around the best estimate within the total range. This maller range will often correspond to what may happen under normal circumstances as opposed to what may happen under extreme circumstances. For xample under normal circumstances a project may cost between −5 percent nd +10 percent of the estimated cost. Under unusual circumstances it may ost between −25 percent and +100 percent of the estimated cost. While the ppraiser may not be able to say that it is more likely that the project will xceed its cost by 10 percent than by 5 percent, he probably knows that +100 ercent is less likely than +25 percent. Therefore, while within the inner range

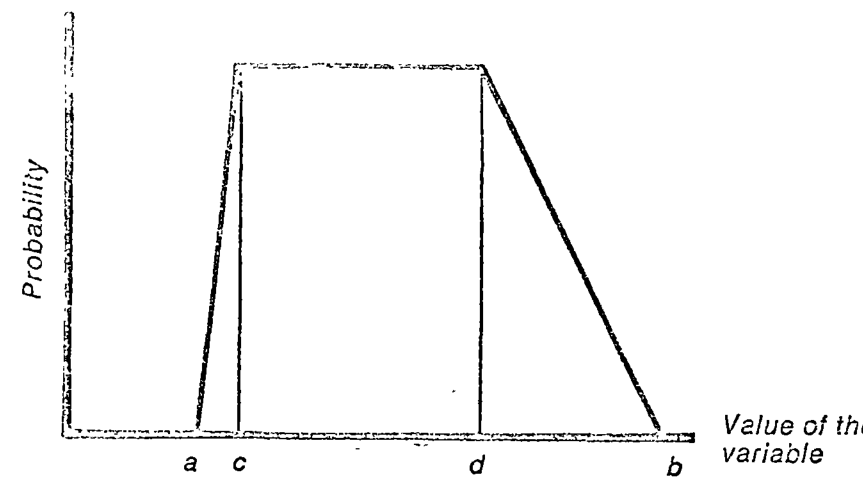


Figure 14. Trapezoidal Distribution

all values have the same probability, outside of this range and toward the outer limit these probabilities decrease. Use of this distribution indicates that it fits well a large class of subjective judgments.

The Triangular Distribution

This distribution, illustrated in Figure 15 below, is a particular case of the trapezoidal distribution and requires little comment. It simply reflects the fact that one is tempted to assign to a value close to the extreme of a range a lower probability than to a value close to the best estimate. It is only a convenient guess that this probability varies linearly from the value of the best estimate to the extreme value of the range—a guess which makes random number generation very easy. Surprisingly, especially if we consider the high sensitivity of this variable in the Mogadiscio analysis, curve T in Figure 13 shows that if we had substituted a triangular distribution for the basic sample distribution of labor productivity, we would have obtained a result remarkably close to our original result.

The Normal Distribution

In our admittedly limited experience, the normal distribution (see Figure 16) seems to be of little use with risk analysis variables.[3] Outside of its appearance in the Mogadiscio case as the result of a portrait approach, we have now abandoned it. We have used it only on one occasion when the availability of an

[3] However, normality may be a good assumption about the final rate of return distribution.

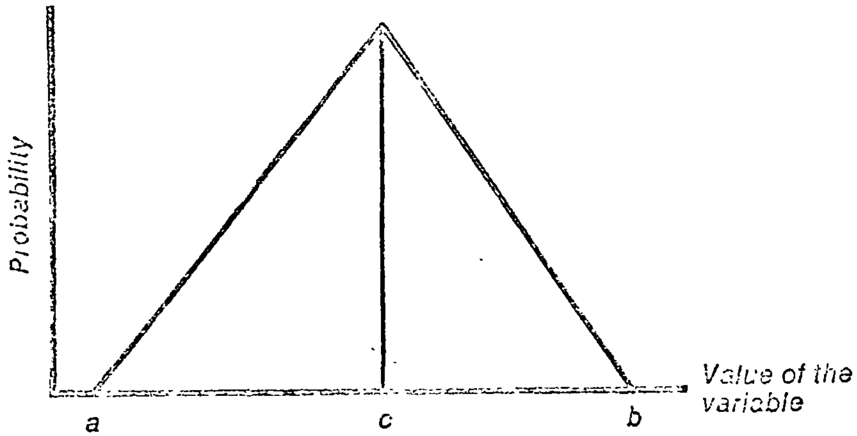


Figure 15. Triangular Distribution

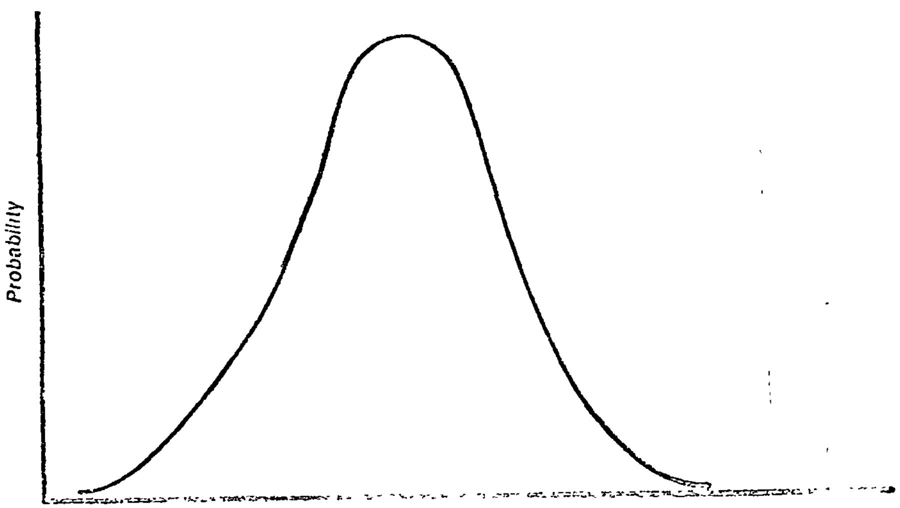


Figure 16. Normal Distribution

exceptional amount of data permitted a statistical analysis, and it turned out in the analysis that a normal distribution was an appropriate choice. But, except in rare cases, there is probably no justification for expressing a subjective judgment by a normal distribution. The variations we are trying to anticipate are the result of neither statistical errors nor random disturbances. Take the distribution of the value of an average ton of cargo in the port of Mogadiscio. Only a minor part of the uncertainty on this variable originated from the specificity of the sample of merchandise from which the mean was computed.

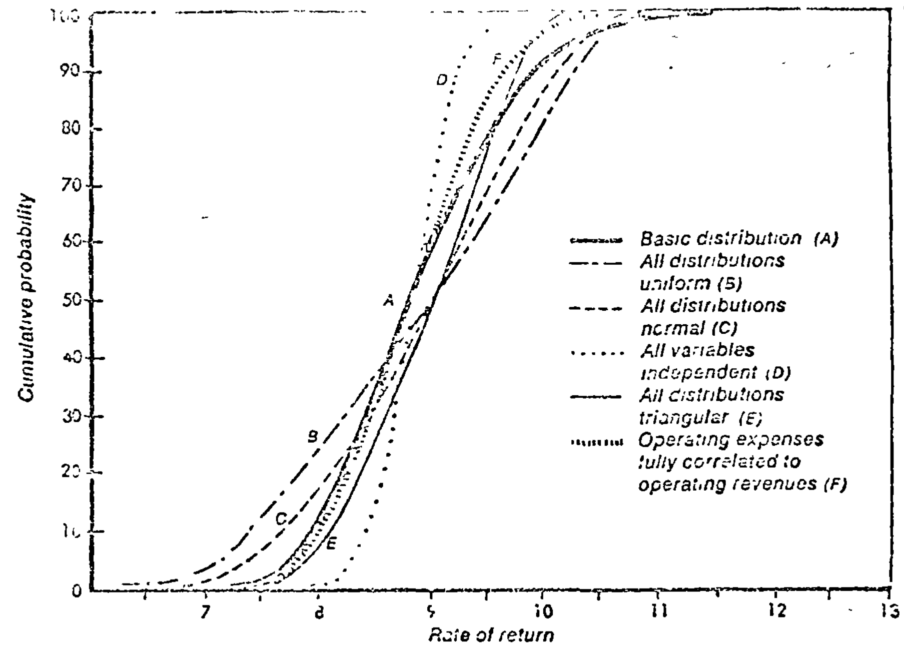


Figure 17. Telecommunications Project: Cumulative Distribution of the Rate of Return under Different Distribution and Correlation Hypotheses

The bulk of the uncertainty we wanted to represent resulted from three facts: (a) in 1966, Somalia imposed restrictions on its imports; thus their composition was likely to change over time; (b) by the time the port was constructed, livestock exports were likely to represent a greater share of the traffic; (c) the data on which the analysis was based were taken from customs statistics and were likely to be biased. There was nothing about these uncertainties which pointed to a normal distribution. They represented subjective doubts about the course of events, which may happen to be best represented by a skewed distribution, a bimodal one, or anything else.

The Comparative Importance of Correlations and Probability Distributions

We have emphasized the importance of correlation and pointed out that it constitutes a much more serious problem than the choice of the probability distributions. As an example, Figure 17 shows what would have happened in the case of another project[4] if we had:

a) replaced the distributions of uncertain variables with uniform distributions (curve B);

[4] The telecommunications project in Malaysia.

b) replaced all the distributions with normal distributions of the same mode[5] and same standard deviation as the original distributions (curve C);
c) replaced all the distributions by triangular distributions of the same mode[5] and same range as the original distributions (curve E);
d) kept the same distributions but considered all the variables independent (curve D); and
e) kept the same distributions for revenues and assumed that the operating expenses were fully correlated with the operating revenues (curve F).

The effect of a different assumption about the correlations (curve D) is as great as the greatest effect from changing the shape of the variable distributions (curve B, all distributions uniform). That curve B is so much less steep than the basic distribution is to be expected, because the uniform distribution exaggerates the probabilities of the extremes. Using normal distributions for all variables yields a curve less close to the basic distribution than using triangular distributions. This result confirmed experience in the Mogadiscio case, when the normal distribution was used for the profile approach; the use of the normal distribution does not appear appropriate when subjective judgment is involved. Curve F (operating expenses fully correlated to operating revenues) shows that adding a full correlation assumption need not flatten the distribution curve, as it did for vehicle operating costs in the Great East Road case, and in the case for the correlation of port capacity and gang productivity in the port of Mogadiscio, as shown in Figure 4 (curve 2). Adding a correlation assumption in the case we are now looking at has resulted in a curve steeper than the original or true distribution, because the two variables assumed to be correlated —operating expenses and operating revenues—pull on the rate of return in opposite directions. Correlation of their variation logically tends to increase the likelihood that the internal rate of return will fall close to its mode and to decrease the likelihood it will fall in the tails of the distribution, thus tipping the cumulative distribution curve to the vertical. In the case of port capacity and gang productivity in Mogadiscio, the two correlated variables pull the same way on the rate of return; correlation tends to increase the weight of the more extreme probabilities, thus flattening the curve. However, even this kind of reasoning about correlations should be applied with caution, because often the effect of correlation between two variables will be more complex and not at all obvious before the data are analyzed. This is especially true when more than two variables are correlated.

[5] Whenever the mode was undetermined, we chose the value originally given as best estimate.

VIII

MEANS AND COST OF RISK ANALYSIS

The financial cost and the time constraints involved in a risk analysis are important elements in the decision whether to undertake a risk analysis. There follow a few comments, which lead us to the conclusion that these constraints should not be allowed to limit the use of the method.

All the simulations we made have been carried out on a computer. An important step of our work has, therefore, consisted in the development of computer programs. In the case of Mogadiscio this took us about six months and in the case of the first road project about two months. The lower time cost in the latter case results partly from the fact that the model used for roads is conceptually simpler and more standard than the one developed for ports. It also results from the fact that we have been able to use for the roads some of the programs we had developed in the Mogadiscio case. Development of computer programs, therefore, appears to be an important capital investment which can speed up a risk analysis considerably. For example, we can now carry out a road risk analysis in a maximum of three calendar days, the actual programming time spent by the programmer being from one-half day to one day. Even in the case of the pavement cost analysis made for the Tanzam highway, which was a completely new exercise, we were able to use parts of the existing programs and the whole programming work did not take more than a calendar week.

It should be pointed out, however, that while we have tried to standardize r programs to the maximum, we found it best to rewrite each time the part aling with probabilities. The appraiser can, therefore, eliminate or introduce the probability analysis any variables he wants. He can also use for each riable any probability distribution, and have the variables correlated in any ay he wishes. This, of course, requires more programming work, but we ink that this is necessary to ensure a good risk analysis. Even though a road a road the world around, all road projects are different because the judgents attached to each of the elements of the project are different. Furtherore, these judgments may take different forms in the minds of different rsons. If, as we think, a good risk analysis depends essentially on how well ne is able to capture the appraiser's judgment, a very flexible framework is eeded which can adapt to any judgment and retain its integrity, no matter hat form it may take. There is, therefore, a limit to standardization. The ajor drawback to a lack of standardization does not seem to be the delay hich may follow in obtaining the results—three days or even a week is still n acceptable time for a risk analysis—but the greater possibility of errors.

A computer program is a delicate tool which, once it is tested, should be odified as little as possible. While the computer will not make any error in ne computation, it will not be able to detect any error in the logic of the proram unless it is instructed to do so. It is difficult enough to detect an error in rate of return and nearly impossible to detect an error in the probability istribution of this rate of return. The best way found so far to overcome this anger seems to be to present the program in a form which makes its checking s easy as possible. However, this does not seem good enough and we are ow thinking of introducing into our models built-in tests which will detect ossible anomalies in the results.

Outside of the programming work, which requires a programmer's time as ell as computer time—and the former may be very expensive, about $5,000 n the Mogadiscio case—the risk analysis proper is inexpensive. To give an dea of the order of magnitude of the cost, the Mogadiscio simulation takes bout 7 minutes on an IBM 7090 computer; on the same computer, the road imulation takes from 4 to 10 minutes depending upon the number of sections nto which the road is divided. At commercial rates the machine costs about 6 a minute. Exclusive of the program preparation, a risk analysis will cost rom $50 to $100 of computer time. It may become more expensive when the nodels become more sophisticated but, on the other hand, the cost of the program preparation should become cheaper as a result of more experience, better organization and the existence of a program library, which is already beginning to accumulate.[1]

[1] Robert Schlaifer, *Analysis of Decisions Under Uncertainty* (New York: McGraw-Hill, 1969); and "Computer Programs for a First Course Decision Under Uncertainty," Boston, Division of Research, Harvard Business School.

A last observation is that risk analysis does not seem to require any particular mathematical skill. Points which require some knowledge of mathematics or statistics, such as the sampling problem, once resolved for one project, are resolved for all. Difficulties might arise in getting a good feeling for the importance of the problem raised by correlations. But, here again, the mathematical treatment of correlations does not raise any problem, and as far as finding and quantifying correlation goes, practical knowledge of a project is more important to those in charge of the appraisal than is theoretical knowledge of the properties of correlations.

IX

MISCELLANEOUS TECHNICAL PROBLEMS

Sample Size

The choice of an appropriate sample size relates to the decision as to how many times the computer should repeat the computation of the rate of return, based each time on randomly generated values for the variables. This is a problem of statistics. The solution essentially focuses on:

a) the mean and standard deviation of the distribution; or
b) the probability of achieving a minimum return; or
c) the shape of the entire distribution.

More details on the first two approaches can be found in most statistics textbooks, and on the third one in the article on the subject by Feller.[1] Although, in the cases we have dealt with, the rate of return is nearly normally distributed and thus use of the first approach may be justified,[2] we were initially interested in the entire distribution and therefore based the choice of our sample size on the third approach, using samples of size 300. Kolmogorov's theorem then indicates that there is 95 percent probability that the maximum vertical distance between the true distribution and the distribution we obtain with this sample will be inferior to 8 percent. This sample size gives more

[1] W. Feller, on the Kolmogorov-Smirnov limit theorems for empirical distributions, *The Annals of Mathematics*, volume 19, no. 2, June 1948.
[2] See following section.

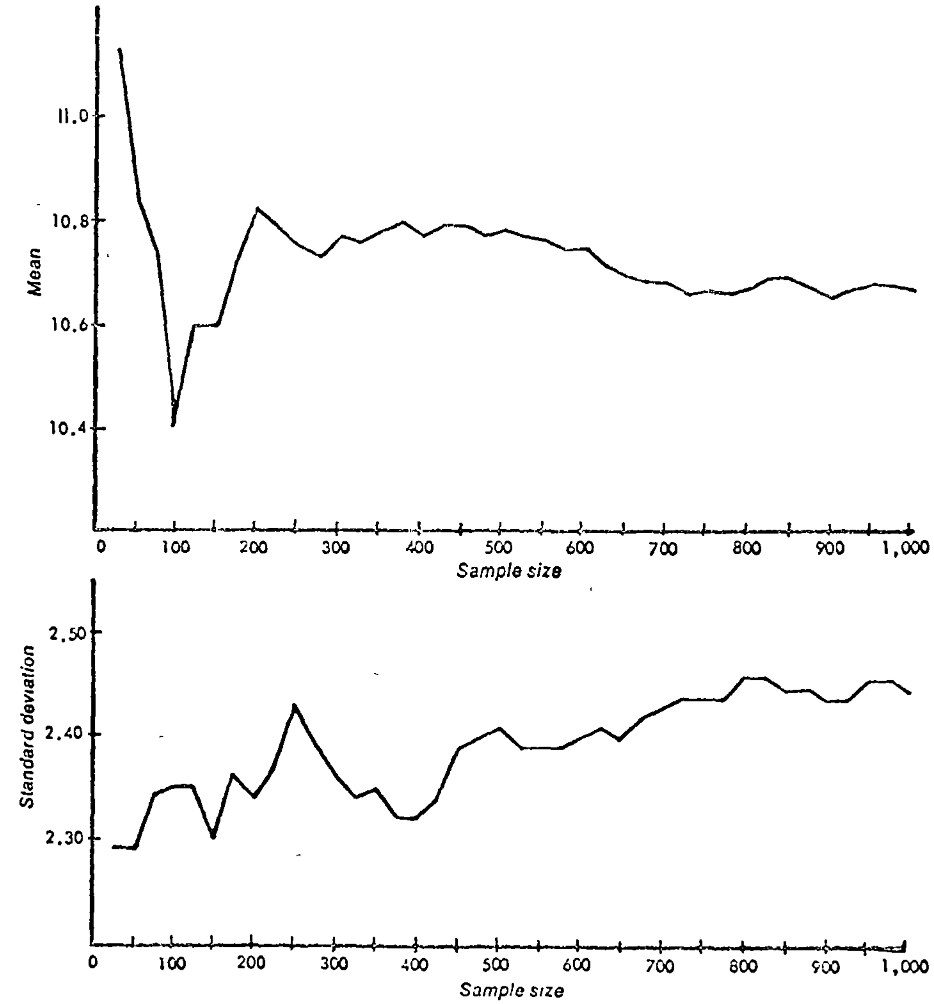


Figure 18. Mogadiscio Port Project: Variations of Mean and Standard Deviation According to Sample Size

accuracy than we need for the mean of the distribution and an acceptable accuracy for the standard deviation.

To illustrate the effect of sample sizes, we experimented with different sample sizes in the Mogadiscio case. Figure 18 shows what may happen to the mean and standard deviation when the sample size is increased to 1,000. In this exercise we increased the sample size by 25 at a time and computed, each time, the mean and standard deviation of the total sample. Even the smallest sample (25) gives a mean which differs from the mean for the largest samples by not more than 0.4 units or 4 percent, which is accurate enough for project appraisal. Therefore, a sample of size 50 would have yielded an acceptable estimate of the mean. From the results for the standard deviation, it is more difficult to make a similar judgment, but again, a sample of size 25 or 50 yields a standard deviation less than 7 percent different from the standard deviation for the largest samples. This is an acceptable result. Figure 19 shows the type of dispersion which can be expected with samples of size 100. With the sample of this size we have an 80 percent probability that the Kolmogorov's distance between sample distribution and the true distribution will not exceed 11 percent. This result seems to be confirmed by Figure 19 which—assuming that the distribution we obtained with a sample of size 1,000 is very close to the true distribution—indicates that only two out of the ten observed distributions of sample size 100 differ from the true distribution by more than 10 percent. The reader will of course realize that this is only a simplistic illustration of a complex statistical problem.

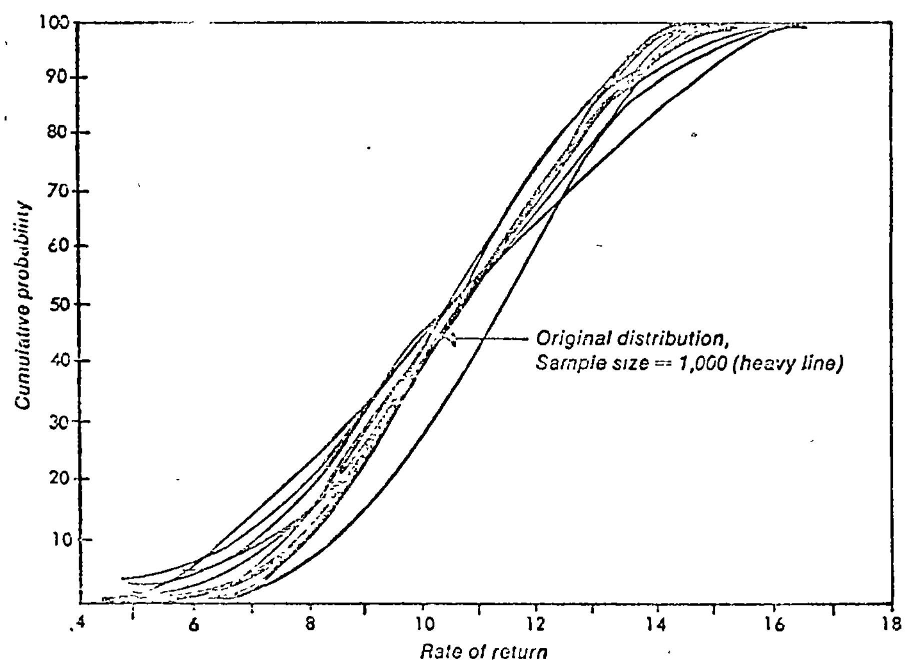


Figure 19. Mogadiscio Port Project: Ten Cumulative Distributions of the Rate of Return

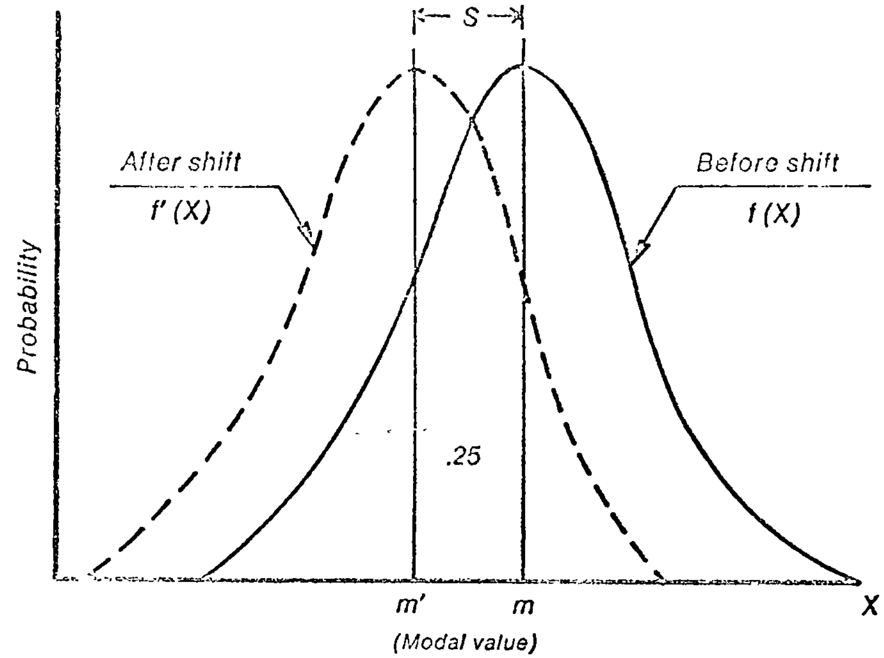


Figure 20. Quartile Shift of a Distribution

$$f'(X) = f(X + S)$$

where S is such that

$$\int_{m-S}^{m} f(X)\, dX = .25$$

and m is such that: $\frac{df(X)}{dX} = 0$

Normality of Rate of Return

Theoretical considerations indicate that, under certain conditions, the rate of return should follow a normal distribution.[3] If this were always the case, the distribution would be entirely defined by its mean and standard deviation, and our work would be somewhat simplified. In practice, we did obtain normal distributions in the cases we undertook. However, the conditions of applicability of the central limits theorem (sufficient in proving a distribution normal) were only partially fulfilled. The following table shows for the Mogadiscio case that the distribution obtained was very close to a normal distribution, with the mean standard deviation equal to the sample mean and standard deviation:

[3] F. S. Hillier, "The Derivation of Probabilistic Information for the Evaluation of Risky Investments," *Management Science*, April 1963.

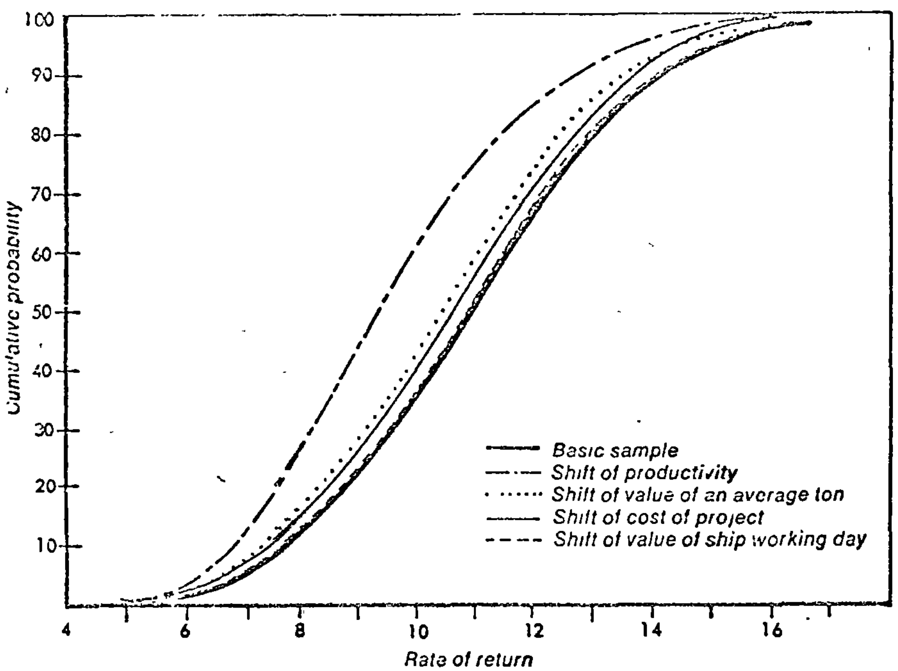


Figure 21. Mogadiscio Port Project: Effect of a Quartile Shift of Variable Distributions on Distribution of the Rate of Return

Cumulative probability of rate of return R being less or more than the rate shown	*Normal Distribution*	*Actual Results*
Prob. (R ≦ 6%)	3.5%	3%
Prob. (R ≦ 8%)	15%	15%
Prob. (R ≦ 10%)	41%	41%
Prob. (R ≧ 12%)	29%	30%
Prob. (R ≧ 14%)	9%	9%

Ve have made the same observation in the other cases of risk analysis and, hough the divergences were greater than in the Mogadiscio case, they were ever great enough to change the conclusion of the analysis. Though it cannot e demonstrated by the central limits theorem, normality may be a good ssumption after all.[4]

[4] This applies, however, to the rate of return, not to the individual variables, as we xplained earlier in Chapter VIII.

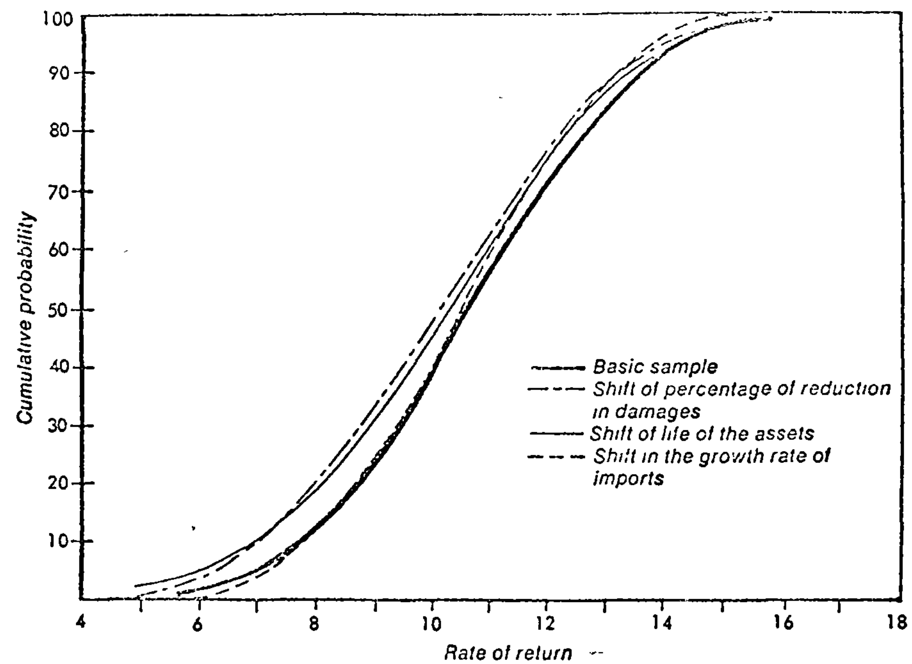


Figure 22. Mogadiscio Port Project: Effect of a Quartile Shift of Variable Distributions on Distribution of the Rate of Return

Sensitivity to Quartile Shifts

In order to discover the effect on the distribution of the rate of return of any possible variations in the distribution of the variables, we performed a quartile shift as defined in Figure 20 on each of the variable distributions used in the Mogadiscio port project (keeping the other variables unchanged) and compared the resulting rate of return distributions with the original.[5] The results are shown in Figures 21 and 22.

The additional information does not justify the expense of repeating the simulation for each variable. However, we believe that it may be possible to carry out this shift sensitivity analysis without actually repeating the simulation, just by extracting from the original sample a sub-sample in which one of the variables is distributed according to the modified distribution. This may lead to using somewhat bigger samples (say 500), but would make it possible to make the shift sensitivity analysis without using any extra computer time.

[5] This exercise was suggested by Mr. David Herz of McKinsey Inc.

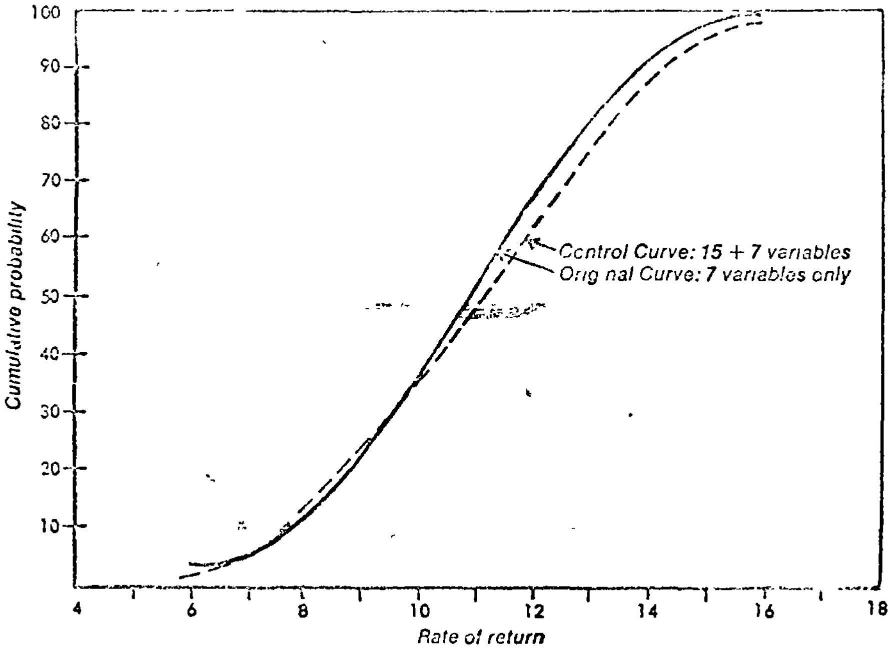


Figure 23. Mogadiscio Port Project. Effect of Including Low Sensitivity Variables on Distribution of the Rate of Return

Elimination of Low Sensitivity Variables

To simplify the simulations it was made a general rule that low sensitivity variables should be kept constant, as in the Mogadiscio case. The computer timesaving is negligible, but we think that this rule helped the appraiser focus on the important distributions of the analysis. In order to test the legitimacy of this approach, in the Mogadiscio case we made a simulation in which fifteen of the variables which were originally kept constant were varied randomly according to specified distributions as were the seven basic variables. Figure 23 shows that the result is very similar to our original result and indicates that this simplification is probably justified.

X

THE USEFULNESS OF RISK ANALYSIS

The major advantage of risk analysis is that it enables us to attack problem that we would otherwise avoid and to make decisions we would not otherwi feel competent to make. In cases like the Tanzam highway and the Mogadisc port project, where uncertainty is high, the appraisers would usually follo the procedure of calculating several rates of return under different assum tions, basing their overall decision about the project on these few calculatio and their best judgment, and presenting that unique and final rate of retu which most accurately reflects the sum total of their knowledge of the projec Without probability analysis, this is the best they can do: the best estima technique confines them to packing all the complexities of their understandi into a single number and then defending it as well as they can. With pro ability analysis, not only are the conclusions presented by the appraisers le limited, but the supporting material has all been quantified in easily comprehe sible, standardized form. This means that, whereas previously it might ha been recognized that some further information was needed in a particul area, with this kind of presentation it is usually possible to specify what ki of information is needed and how much difference it will make— that is, pro lems can be attacked which might otherwise have had to be passed over.

Special Advantages: Four Cases in which Risk is a Major Factor

Among the projects to which we have applied risk analysis there seem be four distinct kinds of problems in which uncertainty plays an importa role: whether to undertake a marginal project, how to handle a project with unusual uncertainties, how to settle on the best combination of specifications in a single project, and how to identify a project with only minimal information.

1. *Marginal Projects.* For some projects, like Mogadiscio port or the telecommunications project, the rate of return computed on the basis of the best estimate for each variable is very close to the estimated opportunity cost of capital. Then normal kinds of uncertainties about the value of the input variables are enough to turn a satisfactory rate of return into an unsatisfactory one. The decision to accept such a project implies judgments on the likelihood that the project will earn a satisfactory rate of return nonetheless and on the extreme ranges of possible results.

2. *Unusual Uncertainties.* For other projects like the Tanzam highway, despite a satisfactory rate of return based on the best estimate of each variable (say 13 percent to 18 percent), the uncertainty on some of the variables is so great that there is a distinct possibility that the project may not earn a satisfactory return. This kind of uncertainty is built into the project and cannot be eliminated or even reduced by any amount of additional study.

3. *Optimization of Project Specifications.* In many cases, the overall justification of the project has already been established at the identification or preappraisal stage. But the analysis of design standards, project timing, project phasing, and project size can only be done at the appraisal stage, and such analyses may lead to saving millions of dollars in project cost, as the Tanzam highway case has suggested. Specification analysis is basic to most Bank project work. In both the Great East Road case and the Mogadiscio case there were problems of timing and scale, though we have not described them explicitly here. Choice among alternatives on such specific issues is made particularly difficult by uncertainty.

In the choice between alternatives A and B shown in Figure 24, for example, uncertainty about data is not critical to the decision whether to go ahead with the project, for either alternative will return an adequate yield. But it may still be critical to the choice between alternatives.

In the neighborhood of S*, alternative A should be preferred to alternative B because it would yield a much higher return. On the other hand, in the neighborhood of S**, B should be preferred to A. The choice between A and B will therefore involve some estimation of the probability distribution of S, and if this choice also involves other uncertain variables it will very likely require a probability analysis.

4. *Project Identification.* The best example of project identification among our four examples is the Great East Road (Chapter V). Here no detailed

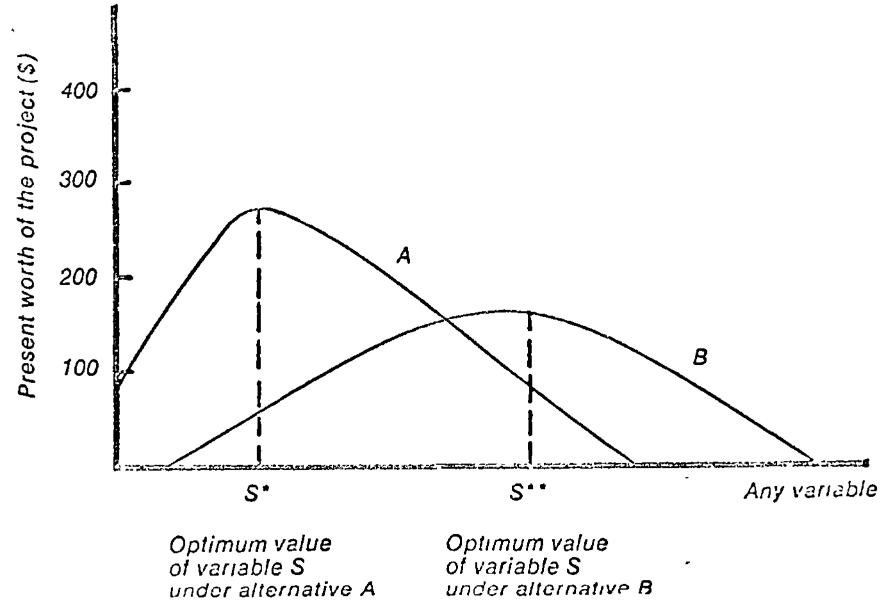


Figure 24. Choice between Two Alternatives, Both with Adequate Rates of Return

study had been made; only rough data estimates were available. Yet a decision had to be made to go ahead with the project, to postpone it, or discard it. The uncertainty is less elusive than that associated with marginal projects, since most of it could be eliminated through study. However, studies are expensive, take time, and even the decision to undertake a study requires careful analysis and involves a judgment on the possible outcome of the study.

The frequency of such cases

It is for these four classes of problems that risk analysis seems best designed. They seem also to represent the essential cases in which uncertainty has to be dealt with in one way or another. The first two classes, marginal projects and projects with unusual uncertainties, while they are not frequent, particularly among transportation projects, often raise critical issues because they involve important decisions. The third case (optimization) is much more frequent but less critical because in practice it is easy to bypass the issue by falling back on rules-of-thumb. It is hard enough to conceive a workable project under the usual uncertainties without also trying to optimize. But if, as our experiences suggest, risk analysis provides an efficient tool to handle the difficulties of optimization under uncertainty, we may be able to do it more often. Finally, in project identification, risk analysis can increase the scope for action. The more sophisticated project planning becomes, the longer in advance ecisions must be made and the more elements—all uncertain—intervene in hese decisions. The present need to improve decision-making processes at the dentification stage may be reflected in the present overwhelming number of project studies. In turn, more and better decisions at the project identification tage should have the effect of decreasing the number of studies and focusing hem on the most important issues.

Risk analysis might well find its major application in the optimization field. Consequently it may become a tool for the consultants even more than for the Bank. Once a project has been fully designed, it is often too late to optimize, but if, at the Bank's request, the optimization were to be made by the consultants or by the project designer, many improvements could probably be achieved at that stage.

General Advantages

Risk analysis requires only one set of computations, either by mental calculation or by computer, to obtain a complete picture of the project. To obtain a similarly adequate picture of the project using the conventional method and the identical computational aids,[1] one must repeat the entire computational process at least once. Therefore, even though a single rate of return can be calculated more quickly by the conventional method, in practice appraisers at the Bank seldom stop at the first rate of return obtained by conventional analysis. The final range of alternatives from which a decision is made can be calculated more quickly by using risk analysis. In the case of the Great East Road in Zambia, we obtained a result from the risk analysis no later than three days after the return of the mission from the field. By the usual Bank method, it would probably have taken two full working days to figure out the traffic, the cost of the project, the savings in vehicle operating cost, the savings in maintenance cost and, finally, the rate of return. Then we would have found that this rate of return was too low and did not correspond to the opinion we had of the project. We would then have repeated the operation, changing the value used for, say, the traffic level, savings in vehicle operating cost, or the cost of the project. In this particular case, it would have been easy to pick second and third values just as good as our first, because of the great uncertainty about most of the variables. This unsystematic sensitivity analysis might have required another two to five days. In the same time, using risk analysis, the final report on the project was already finished. In addition, it took only about one programmer-hour to rerun the program six months after

[1] Naturally, developing computer programs takes time and using them speeds up calculations, but these facts are true for any method and do not affect the validity of the argument here. See also Chapter VIII.

the original decision, when we received new information on the cost estimate. With faster computer turnaround time, the time to get the computer results could be reduced from three days to one day, which is the time it takes the programmer to prepare the probability part of the computer program.

Clarity of presentation

Another benefit of risk analysis is that it results in greater report clarity and thereby permits more people to make useful contributions to project appraisals. Our appraisal reports give the values of the elements used in the evaluation of a project. However, they very seldom give the judgment lying behind these values and when they do, it is always in qualitative form. As a consequence it is very difficult for anyone to discuss these judgments, and comments often focus more on the presentation than on the substance of a report. In some cases, a high degree of technicality creates a natural barrier to wide discussion. But often, as with the Tanzam highway, a discussion of the assumptions is both possible and desirable.

This transparency of the analysis, while serving to make discussion more effective, also seems to facilitate the adoption of recommendations. However, our experience is limited and it would be interesting to investigate this point further, in particular in connection with negotiations relating to technical questions.

Convenience

Because of its compactness, a probability distribution not only communicates information well, but also is very convenient to work with. It was surprising to find that after even a very little experience with risk analysis, it became easier and more natural to express a judgment in probabilistic terms than in terms of a best estimate or, indeed, of any other kind of estimate. We found that an expert consultant may be unwilling to commit himself to a single cost estimate before the completion of his study, but he may quite readily proffer a range of cost estimates and a full probability distribution over that range.

Rigor in analysis

Risk analysis both demands and permits the use of greater rigor in analysis. Risk analysis demands more rigor simply because it is a more systematic method. It permits more rigor than the single-estimate approach because even in a simple analysis it allows for more than one course of action. Paradoxically, it is always easier to be rigorous than to approximate.

Present Worth versus Rates of Return

In concluding this chapter it may be suggested that, for the purposes of risk analysis, in many cases present worth may be a better criterion than the internal rate of return. One difficulty in decision-making is the estimation of the opportunity cost of capital. This is often cited in the Bank as an argument in favor of using an internal rate of return rather than a present value. We need not know the opportunity cost of capital to compute the internal rate of return; it is used only at the last stage to decide whether the calculated rate of return is acceptable or not. Since in practice the rates of return obtained are often higher than the highest likely value of the opportunity cost of capital, the need to calculate the latter in detail does not then arise. However, if risk analysis is to be applied to marginal projects or to marginal components of a project, the decision as to the acceptability of the project will no longer be so obvious. Mogadiscio's port project, for example, implies the comparison of an internal rate of return, varying over a wide range, to an estimated opportunity cost of capital, also varying over a wide range. This is not easy. Under these circumstances it seems possible to consider the opportunity cost of capital as an uncertain variable similar in all respects to the other variables of our analysis, and to use a present worth approach. The probability of failure of the project would thus simply appear as the probability that the project has negative net present worth, and the decision as to its acceptability would be made very simple.

Summary and Conclusions

The overall conclusions are numerous, and many are, of course, still tentative. They can be summarized in the following four points:

(a) Risk analysis is a powerful technique which permits the use of a great deal of information which would otherwise be lost. It enables us to handle uncertainty not only about the viability of a marginal project, but also about the most appropriate design or phasing or size of a clearly acceptable project.

(b) Perhaps even more importantly, the entire framework of risk analysis provides a highly efficient medium of communication, a focus for evaluation and discussion, whether between one person and his superior, among the various members of a team, or possibly (looking toward the future) between consultants and the Bank or a borrower and the Bank.

(c) Risk analysis is in no sense a technique which replaces skilled judgment. On the contrary, it often requires the use of far more judgment than the traditional analysis. The technique cannot provide correct a[illegible]s on the basis of false assumptions.

(d) Despite the method's value, the treatment of correlations between variables remains a major problem. It is clear that results can be completely misleading if these correlations are not properly handled. This danger is not merely theoretical; there is apparently a systematic tendency to overlook correlations. It follows that risk analysis should be undertaken only with great caution.

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OPERATIONS RESEARCH AND REGIONAL MINERAL EXPLORATION

by D. P. Harris

This paper surveys a few of the quantitative exploration models that might be of interest to an explorationist seeking to apply methods of operations research to mineral exploration. A general development of each model is presented, and the model is evaluated relative to the exploration activity.

Finally, a philosophy of exploration and mineral resources is presented, and a possible exploration strategy incorporating concepts of some of these models consistent with this philosophy is proposed.

The high cost of conducting an effective exploration program has generated an increasing interest in methods of optimizing the allocation of the resources of a firm engaged in mineral exploration. As a step towards this, mathematical models have been formulated which are designed to quantify those variables of mineral exploration in such a manner that they can be integrated with all phases of the firm's operations so as to optimize its profit objective.

QUANTITATIVE MODELS

General: Exploration models express at least two concepts: mineral occurrence and effectiveness of search. In simplest terms, an exploration model might be formulated mathematically as follows:

$P(X) = F(X) \cdot G(X)$, where $P(X)$ = the unconditional probability of discovery of X deposits, $F(X)$ = the probability of occurrence of X deposits, and $G(X)$ = the probability of discovery of X deposits, conditional upon their existence in the search area.

With these general models as a basis of reference, some of the specific exploration models are examined below.

The Allais Model:

DEVELOPMENT – M. Allais[1] performed the first study in which a philosophy of exploration was formulated into a probability framework. His objective was to estimate the economic profitability of the exploration of the Algerian Sahara. Allais divided the Algerian Sahara into blocks of land (called cells) 10 km square and defined the random variable, X, as the number of mining districts per cell. Thus, $F(X)$ would describe the probability of 1, 2, or n mining districts occurring within any cell of the search area. Allais reasoned that his best information as to the form of the distribution, $F(X)$, was the number of mining districts per cell that had been found in well explored areas in the world, such as the Western States of the U.S. He found that the Poisson function produced a reasonably good fit to the distribution of districts in these explored areas:

$$F(X) = \frac{e^{-\lambda}\lambda^{X}}{X!},$$

where λ is a parameter, such as the average number of districts per cell. Thus, by determining the value

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of the parameter, λ, on the control areas, the probability of 1, 2, or n deposits occurring in a cell of the study area could be estimated.

Allais considered the search activity to consist of three separate stages: 1) the skimming stage, 2) the screening stage and 3) the development stage. Stage 1 consists of regional reconnaissance exploration such as photogeological mapping, general geological field work, and airborne geophysical surveys. The objective of this stage is to select those subareas where additional exploration should be undertaken. Stage 2 consists of detailed geological, geophysical, and geochemical surveying and exploration drilling and trenching. Those cells retained after the completion of Stage 2 are examined in great detail by drill holes and test shafts in Stage 3, with the objective of proving ore reserves. Allais thus defined $G(X)$, the search activity, by three probabilities: g_1, g_2, and g_3 where g_1 = the probability that a cell containing an economic deposit would be retained in Stage 1 for Stage 2, g_2 = the probability that a cell containing an economic deposit would be retained in Stage 2 for Stage 3, and g_3 = the probability that the cell would yield a producing property.

Allais found that if the 1,000,000-sq-km of the study area were explored, twenty economic deposits might be discovered. The net gain of this exploration was predicted to be 30 billion francs with a probability of 0.35 of realizing this profit and a probability of 0.65 of losing 20 billion francs. He also found that the success or failure of the venture was really dependent upon the discovery of the few large deposits expected in the area.

EVALUATION — Allais' work constituted a significant contribution, for it was the first quantitative analysis of the expected outcome of mineral exploration within a probability framework. He proposed mathematical concepts for the distributions of number of targets per cell and value of production of mining districts, and he treated the search activity as a probabilistic phenomenon.

A major contribution of Allais is his analogy of mineral exploration to the concept of "gambler's ruin." He showed that the probability of loss in a given venture decreases with the amount of capital invested and, assuming a constant cost per cell in the study area, the implication is that the larger the number of cells explored, the lower the probability of loss. Allais constructed the following table illustrating this phenomena for the Algerian Sahara:

T	1	2	4	10	15	20	40	100	150
P	.93	.95	.92	.81	.65	.42	.18	.12	.04

where T = billions of francs, P = probability of loss.

Thus, exploration must be on a large scale in order to be profitable, for only on a large scale can an explorationist be relatively sure that the initial area contains the few very large deposits that will make a venture a paying one. Relating to the proposed three-stage search effort, the concept of gambler's ruin requires emphasis upon the skimming of a large number of cells to select a subset of cells for Stage 2.

The Poisson function as a spatial occurrence model has been challenged by Slichter,[2] who proposed the exponential function as better describing the number of mines per cell. More recently, a thorough study of the spatial distribution of base and precious metal deposits in the Basin and Range Province of the Southwestern United States has been completed by the U.S. Bureau of Mines.[3] The desirability of using mining districts as the random variable in the spatial occurrence model is questionable, for, only occasionally does a mining district relate to a physiochemical phenomena. Oftentimes, political boundaries, topographic relief, or just plain whim led to the definition of a district. It may well be that anomalous mineral concentrations (assuming such can be defined) are distributed according to some well-known function. There is little confidence in the proposition that after introducing economic and political factors as defining criteria the resulting unit is distributed according to Poisson or any other mathematical law.

A geologist's reaction to this study is that the evaluation was performed without considering the geology of the area. This may not be a fair criticism of Allais' study, because Allais did not attempt to indicate where within the large area the important deposits could be found, but only that they probably existed.

Thus, it might be concluded that for the appraisal of a large unexplored area, a method such as that of Allais can give a general indication in probability terms of the economic outcome of exploration. It cannot, however, be used as a tool to guide exploration, for each cell (subdivision) has the same probability of containing 1, 2, or n deposits. Furthermore, after an initial stage of exploration, better decisions may be made in light of the acquired information; if this is not true, then the science of geology has no value in exploration and only serves to misidentify a random process as one that is Bayesian.

Koopman's Search Theory:

DEVELOPMENT — In 1956 and 1957, B.O. Koopman[4] published three papers entitled "The Theory of Search," in which he summarized search theory developed to that time. Fundamental in the theory of search propounded by Koopman is the concept of glimpse probability of instantaneous detection density; a glimpse is one reading on a detection device. The probability of detecting a target in n discrete glimpses is:

$P_{(n)} = 1 - (1 - g)^n$, where g = the probability of detection in one glimpse. If the search process is continuous, the probability of detection becomes a function of time:

$$P(t) = 1 - e^{-kt},$$

where t is the time interval. In terms of the effort expended per unit of volume, length, area, etc., the equation describing the conditional probability of detection can be written in the general form:

$$Q(w) = 1 - e^{-S(w)},$$

where $S(w)$ is some function defining the amount of effort expended at w.

The unconditional probability of detection of a target located in some interval $(w, w + dw)$ of the search area is:

$$p(w) = F(w) \cdot (1 - e^{-S(w)})\, dw,$$

where $F(w)\,dw$ = the probability that the target is in the interval $(w, w + dw)$. The total probability of detection is found by integrating $p(w)$ over the target area:

$$P(w) = \int p(w)\, dw$$

Generally, the total search effort is limited in some way, either by time, money, or some other constraint. Let T be the total search effort:

$$T = \int S(w)\, dw,\ S(w) \geq 0$$

Then, the optimizing problem is to find $S(w)$ that maximizes $P(w)$, given T. The solution that Koopman found for this model is $S(w) = \ln F(w) - K$, where the target lies somewhere on the w axis. The amount of search effort $S(w)$ allocated to a point w is determined by calculating the distance from $Y = K$ to $Y = \ln F(w)$, for all $F(w) \geq K$.

EVALUATION — Two basic concepts are expressed in Koopman's model:

1) The probability of discovery conditional upon the existence of a deposit at w is a function of the exploration effort at w. This probability of detection follows the exponential saturation law.

2) The optimal distribution of exploration effort requires the allocation of the search effort to those locations where the probability of occurrence of a target is the highest.

Relating now to the mineral exploration model, the Poisson function proposed by Allais for mining districts might substitute in the Koopman Model for $F(w)$, where no discriminating information is available. However, when geologic information is available, and it is assumed that such is meaningful information, the optimum distribution of search effort would require that those cells with the most favorable geology be given the most thorough searching. If the level of information concerning the location of the targets is zero, then each area or location is equally likely, and each cell is allocated the same search effort.

The Koopman Model provides a formal, very general framework of theory that can serve as a base for further development and refinement with special attention given to the circumstances of mineral occurrence and exploration. Jacques de Guenin[5] adapted Koopman's search model to oil exploration, substituting the Poisson probability function for the occurrence model and developing search allocation relationships pertinent to oil exploration.

The Engel Model:

DEVELOPMENT — J. Engel[6] developed an exploration model in which the binomial probability distribution served as the occurrence model. As with the Poisson, this model treats the occurrence of mineral deposits as a spatial phenomenon only. The search model consists of two stages, each searching the entire area for indicators. The observation of an indicator is considered a contact. False contacts are considered to be randomly distributed, while real contacts are not. Thus, the observation of a contact at the same point in both stages would be more likely to indicate a real contact than a false one. The ultimate goal after two stages of search is the detailed drilling of those areas where real contacts were observed in both passes. This model requires parameters of the number and value of the targets in the search area and the costs for the two stages of exploration.

The basic parameters for oil and gas exploration were estimated by Griffiths and Drew[7] and exploration was simulated on a computer. From these simulations it was determined that a profit could be made over a wide range of exploration conditions.

EVALUATION — The Engel model has some similarities to the Allais model in that it proposes a mathematical function (the binomial) as a spatial occurrence model and treats exploration as a multiple-stage search activity. However, where Allais viewed exploration as a process of successive elimination of unfavorable cells in the first two stages resulting in a select few cells to be explored in the third stage, Engel passes over the entire area twice before reducing the area for detailed exploration. This is not necessarily contradictory to some procedures in exploration, for a first pass might be general geological reconnaissance in which favorable indicators (contacts) might be recorded (some of which might be false), and a second pass might be airborne geophysical surveys. Thus, if one were considering the exploration of a large area, one might estimate from other similar areas the total number of deposits that might exist in the area and their distribution of values (much as Allais did in his study) and use these as parameters for the Engel Model. Other cost parameters could be estimated and the exploration of the area simulated to generate an estimate of the profit that might be realized under the conditions specified. Boundary conditions might also be tested to show what might be the worst and best of case. This could give some insight into the advisability of undertaking the exploration of that area.

The Drew Model:

DEVELOPMENT – L. J. Drew[8] developed a grid drilling exploration model which is based upon an equal probability occurrence model for oil and gas targets and upon the search for these targets (oil and gas fields) by drilling on a square grid system. The economic feasibility of the Drew Model can be expressed as follows:

$$R = f(V, A, S, O, T) \quad [1]$$

where

R = the profit (expected and/or absolute) realized by grid drilling of the search area,

V = the value of the targets,

A = the areal size of the targets,

S = the spatial distribution of the targets,

O = the orientation of the targets,

T = the cost of drilling.

The cost of drilling the search area is, for an area of a given size, a function of the number of holes drilled, and since these holes are drilled on a square grid system, the number of holes drilled, and therefore the cost of drilling (T), is a function of the grid spacing, G. For a uniform distribution of targets, the probability of discovery of a target is of course a function of the grid spacing. A grid spacing could be selected that would be small enough so that the probability of detection would approach 1.0. However, a high probability of discovery implies a high exploration cost because of the large number of holes required on a close grid spacing. The problem, then, is to maximize the expected value of the deposits discovered after deducting the cost of exploration. The maximization criteria can be expressed symbolically as follows:

$$E(R) = \sum_i p_i v_i - T, \quad [2]$$

where

p_i = probability of hitting the ith target

v_i = value of the ith target

T = cost of drilling the search area

The Drew Model requires three conditions (assumptions) for mathematical completeness:

1) The targets in the search area are elliptical in form, with parameters "a" (semimajor axis) and "b" (semiminor axis).

2) The centers of the targets are distributed throughout the intergrid area in a two-dimensional uniform distribution.

3) The distribution of the long axis of the targets is a uniform distribution.

For elliptical targets with a semimajor axis less than one half the length of the grid spacing, Drew proposed that the probability of hitting the target could be described as follows:

$$P\,(\text{Hit}) = \frac{\dfrac{\pi b^2}{4} + \int_b^a \tan^{-1}\left(\dfrac{b}{a}\sqrt{\dfrac{a^2 - b^2}{r^2 - b^2}}\right) r\,dr}{(G/2)^2} \quad [3]$$

where

a = length of the semimajor axis of the elliptical target

b = the length of the semiminor axis of the elliptical target

r = the radius of a circle with a center coincident with the center of the elliptical target

G = grid spacing

Drew then constructed a simulator for a computer around the maximizing criteria [2] in which probabilities are determined by [3] except for cases where the semimajor axis of the elliptical target is longer than one half the grid spacing. For such cases he defined and built into the simulator the necessary relationships to assess the probabilities. Thus, given the distributions of value, areal size (axial measurements), orientation, and density of the targets in an area and an average cost per drill hole, the Drew Model via computer simulation determines the grid spacing that maximizes the expected profits generated by drilling the entire search area.

Drew found that in applying the grid drilling model to each of 15 major oil-producing areas within the United States that a profit would have been realized. He found further that the optimum grid spacing for drilling the entire United States is 3.5 miles and the expected profit associated with this grid spacing is $65 billion.

The results of grid drilling were considered by Drew to be conservative for the following reasons:

1) No reserves were included.

2) No allowance was made for undiscovered fields.

3) Appendages were excluded in measuring field axes.

4) Some fields were excluded because of unavailability of production data.

EVALUATION – The major contribution of the Drew model is that it provides an analytical method of computing the probability of hitting elliptical targets as a function of the shape and size of the targets and the grid spacing. Secondarily, the model provides a framework by which the economic factors of cost and value of targets discovered can be related to the grid drilling search activity to determine the optimum grid spacing. The economic relationships defined and employed by Drew are general and provide a means of illustrating the model.

An explorationist dealing with the economics of exploration might well question the figures of profit computed by the model as an evaluation of grid drilling compared to conventional exploration, because development costs, as Drew acknowledged, were not considered in computing the profit figure.

Furthermore, as Drew states, a drill hole in the areal reflection of the field does not in actuality always constitute a discovery of a field. These criticisms do not detract from the theoretical probability constructs of Drew's model; they only serve to modify the economic appraisal.

Apart from adjustments that an explorationist might make in the economic framework, the model implies an exploration strategy for certain conditions that merits consideration. Nearly every experienced explorationist can point to cases where the best geological and geophysical information has produced a discovery ratio that might have been as good or better if drilling locations had been selected by throwing darts at a map of the area. Such cases may arise from two causes:

1) The area may be covered in a way that provides no meaningful geological information and no sound geological control for geophysical interpretation.

2) The experience of the geologist in other areas may have biased his interpretation of geologic information in a completely new area so that he overlooks deposits controlled by a different set of geological factors until an initial discovery is made, which may be after considerable exploration.

Applying grid drilling to such an area after the case might indeed prove to be more profitable. The problem arises in determining *a priori* that such a condition exists. Case (1) above may possibly be recognized, but case (2) may not. If the geologist realizes that he is dealing with Case (1) and decides to drill on a grid, he faces the problem of not knowing the distributions of target characteristics in a new area. One solution to this problem would be to employ distributions determined in other areas; however, this might introduce a bias in the analyses. To avoid such a bias in the evaluation of an unexplored area, Drew suggests an alternative procedure of drilling first on a very wide grid, say ten miles. Discoveries are then developed to a point where their total value can be estimated. Then, the grid is made smaller by drilling between the holes drilled on the large grid. Again, the results are evaluated and expressed in a ratio of additional value gained to cost of drilling. This process is repeated until the ratio reaches one. Any drilling past this stage will result in a loss of profit. The major problem with this approach is the fact that evaluation of the ratio is made after the drilling for that grid spacing is completed, and this may already have resulted in a ratio of less than one. Restriction to drilling on a square grid makes the initial grid spacing a critical factor in optimizing, because all subsequent grid divisions are dependent upon the initial spacing. Therefore, it may be possible only to suboptimize, contingent upon the initial spacing. Once this initial spacing is determined from the best information available — which would be the characteristics of targets in other areas — it might be improved upon by revisions made in the light of targets discovered at each successive drilling stage. The revised distributions would then reduce the probability of drilling on a grid spacing smaller than the optimum. They would not necessarily result in the optimum, however, because the final grid spacing would always depend on the initial spacing.

A final point to be evaluated is the equal search effort allotted to each part of the study area. Such a procedure implies that the geologic information is not meaningful. While it may be true that the geologist cannot predict the exact location of a target, geology should be useful at least in blocking out those portions of a study area that are more or less favorable for mineral occurrence, particularly if exploration is aimed at only one mineral group. Then, drilling may be eliminated in less favorable areas where geology indicates that deposits would not justify the expense of drilling on a grid spacing necessary to detect them.

A Multivariate Geologic Model (MG Model):

DEVELOPMENT — The models discussed above are exploration models relying on some form of spatial occurrence and search. However, as soon as any geological information is available on an area, a spatial occurrence model may no longer be the best estimator. The model developed by Harris[9] is an occurrence model which associates the probability of occurrence of some measure of mineral wealth with geology.

The basic postulates of the MG model are as follows:

$$V = Q(L, S, F, A)$$

$$P(V) = G(L, S, F, A, V)$$

where

V = a measure of mineral wealth
$P(V)$ = the probability of occurrence of V
L = age and type of rock
S = structural forms
F = rock fracturing
A = age of igneous activity and contact relationships

This occurrence model is formulated to consider the geological factors at the reconnaissance level that relate to the occurrence of base and precious metals. The reconnaissance level of information is held to be comparable to that information supplied by Allais' skimming stage of exploration, in which obviously unfavorable cells are eliminated and the favorable cells are retained to be investigated in greater detail. Models could be formulated in a similar manner for other mineral groups, such as oil and gas.

The four geologic factors defined in the above postulates were measured by 26 measurement variables of information available from geologic maps.

These 26 measurements were made for each of 243 cells (20 miles square) in a control area located in New Mexico and Arizona. The value of mineral wealth was determined by accumulating the annual production of all mining districts within a cell. Each of the 243 cells was assigned to one of six value groups according to the value of its cumulative production of base and precious metals:

Group	
1	0 to $10,000
2	$10,000 to $100,000
3	$100,000 to $1,000,000
4	$1,000,000 to $10,000,000
5	$10,000,000 to $100,000,000
6	$100,000,000 +

Multiple-discriminant analysis and classification analysis by Bayesian statistics and the multivariate normal probability function were used to define the relationship of probabilities to the geologic variables and mineral wealth:

$$P_{ig}(V_g \mid \overrightarrow{DSC_i}) = \frac{\dfrac{F_g}{|D_g|^{1/2}} \exp(-X_{ig}^2/2)}{\sum_j \left\{ \dfrac{F_j}{|D_j|^{1/2}} \exp(-X_{ij}^2/2) \right\}}$$

$$j = 1, 2, \ldots, g, \ldots, K$$
$$i = 1, 2, \ldots, n$$

where

V_g = the gth value group
$\overrightarrow{DSC_i}$ = the vector of r discriminant scores for the ith cell
P_{ig} = the probability of membership of the ith cell in the gth group
F_g = relative frequency of membership in the gth group in the mixed population
D_g = the dispersion matrix of the gth group
K = number of groups
n = number of cells
$X_{ig}^2 = M_{ig}^T D_g^{-1} M_{ig}$
$M_{ig} = (DSC_{i1} - \overline{DSC}_{1g}, DSC_{i2} - \overline{DSC}_{2g}, \ldots, DSC_{ir} - \overline{DSC}_{rg})$, where
M_{ig}^T = the transpose of the vector M_{ig}
DSC_{ik} = the discriminant score of the kth discriminant function for the ith cell, and
$\overline{DSC}_{kg}$ = the mean discriminant score of the kth discriminant function for the cells of the gth group

A study area consisting of most of the State of Utah was selected upon which to test the performance of this model as a predictive tool. For this evaluation, the six groups used in the control area were reduced to two, with mineral wealth from $0 to $1,000,000 classified as Group I and that above $1,000,000 as Group II. A 0.200 or greater probability of belonging to Group II was then chosen as the criterion for retaining any cell for further exploration. This resulted in the selection of 19 of the original 144 cells in Utah for further exploration. The 19 cells selected were identified by their known mineral production as follows: all five cells in the area with a value of $100,000,000 +; two of the four cells with a value of $10,000,000-$100,000,000; three of the eight cells with a value of $1,000,000-$10,000,000, and nine cells with a value of less than $1,000,000 (these are cells that should have been eliminated from the second stage of exploration).

Raising the probability criterion results in the retention of fewer cells with a value of less than $1,000,000, but only at the expense of losing some of those of high value. Conversely, lowering the criterion results in the retention of more of the cells with high value but at the cost of retaining more cells with a value of less than $1,000,000 (see Fig. 1). The problem lies in selecting the optimum de-

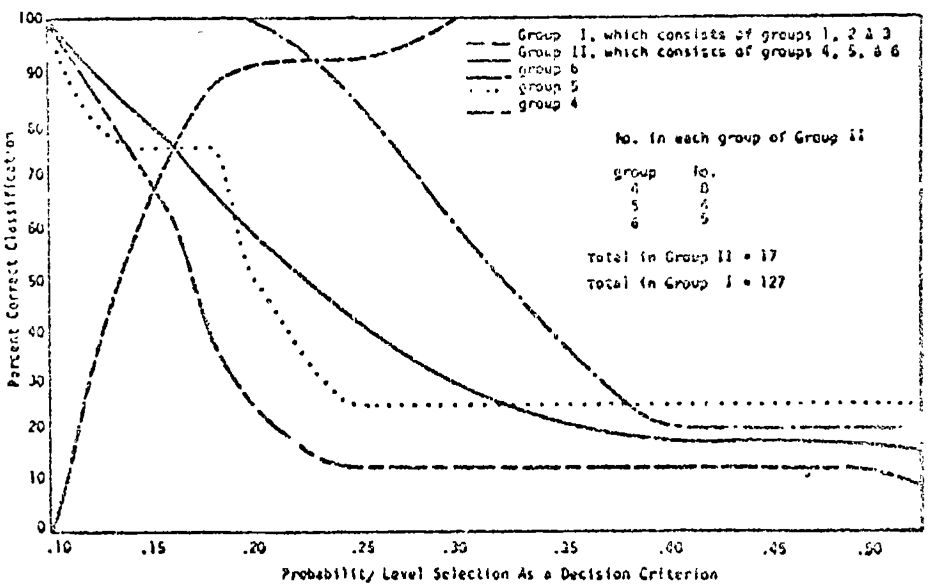

Fig. 1 — Analysis of skimming as a function of the decision criterion.

cision criterion, that is, selecting that level which balances the misallocation of exploration effort in the second stage to small value cells with the expected value of the large value cells retained. This is much like standard hypothesis testing in which for a given sample size the probability of a Type I error cannot be decreased without increasing the probability of Type II error. This emphasizes the importance stressed by Allais of beginning with a large area and performing an efficient and careful skimming of a large number of cells so as to be sure of retaining those few cells of large value.

EVALUATION – Simply stated, a methodology was developed for constructing an occurrence model that would generate probabilities of mineral wealth differing from cell to cell as the cells differ from each other in their geology. First, a sound conceptual model of the geologic factors that relate to mineral deposition must be formulated and then measurements that reflect these concepts must be sought. In this way, it is possible to progress from the conceptual framework to the quantitative one in which geology is treated as variables. Multiple discriminant analysis and classification analysis based upon Bayesian statistics and the multivariate normal probability distribution constitutes a two-phase probability model that associates three concepts: 1) probability, 2) mineral wealth (value), and 3) geology.

Unlike the spatial distributions of the foregoing exploration models, probability in this model relates to a measure of gross value for the cell rather than numbers of mining districts or elliptical targets. Thus, the target is a cell of specified mineral value; therefore, as an occurrence model, it cannot substitute directly for the spatial occurrence models of the foregoing exploration models.

This model requires a certain level of geological information, and in the absence of such information, it is no substitute for an analysis such as that performed by Allais or Drew. But, given geological information, the model allows the discrimination between cells on the basis of probability. Based upon multivariate statistical techniques, the use of this model as an aid to exploration of a new area requires statistical inference from some base area to the study area. This is at the same time a strength and a weakness of the model, for while it allows the use of prior information in the evaluation of an unknown, it requires certain conditions that must be met for such inference to be valid. One of these conditions is that either both areas must belong to the same geological province or those variables that describe the effects of change from one province to another must be included in the model. If the second of these solutions is employed, then the statistical base must be very wide and made up of cells of areas from different geological provinces.

Another aspect of this model that must be examined is the fact that it assumes the variables are distributed according to a multivariate normal distribution. Since the variables employed are likely to be crude measurement variables, their distributions may take on rather weird forms, some of which are far removed from the normal distribution. Severe departure from normality decreases significantly the accuracy of inference by the model. Therefore, when necessary, transformations should be made upon the raw variables to decrease their departure from normality. Unfortunately, some distributions cannot be adequately transformed.

The MG model performs quantitatively and explicitly what an explorationist performs intuitively; that is, it employs information from experience in other areas to analyze a study area. Then, where is the gain? First, the model performs the analysis quantitatively, generating probabilities, which are an explicit measure of uncertainty. Second, the multivariate models and the electronic computer allow a large set of variables to be analyzed simultaneously; this lessens the chance of overlooking meaningful variables or interactions of variables in the evaluation of the study area. Third, a statistical base can be developed upon several areas, each representing different geologic provinces. This feature combined with the capability of handling a large number of variables makes possible an extrapolation of experience with much less chance of bias than one made by any one decision maker or decision-making team.

Care should be taken in formulating the conceptual model, for the performance of the MG model can be no better than the information upon which it is defined. In the definition of the statistical bases upon the control areas, the discriminant analysis computes a statistic that can be tested. Thus, early in the analysis and prior to inference, it can be determined whether the relationships of geology to mineral occurrence are significant or due to random influences.

WHAT HAS BEEN LEARNED

In the foregoing section, some aspects of several analytical models that might be of value in applying operations research techniques to exploration were evaluated. Each of these models is quite different from the others, having been constructed to achieve different objectives. None of them is applicable to all exploration problems. What appears to be a fruitful area of research is the extension and integration of these models with applicable modifications and additions to a more complete exploration model.*

*A project has been initiated at Penn State which is aimed at simulating geology and mineral deposits on a computer so that a mineral system is available upon which any level of "signal-to-noise" ratio can be induced. Exploration models will be examined under varying conditions and their performances evaluated.

Four Points: Certain characteristics and philosophies can be gleaned from these models that should be reflected in further developments. One of these is the *necessity of exploring a large area initially*. This was first shown by Allais[1] and later emphasized by Harris.[9]

A second point to consider is that, given an area of equal probability of occurrence of mineral deposits and given characteristics of the target, *an analytical framework exists whereby the optimum grid spacing for the exploration of that area can be determined* (Drew's model).

Third, where the probabilities for the occurrence of a target vary from one area to another, *optimization of search requires that the greatest effort be devoted to those areas with the highest probability* (Koopman's model).

Fourth, *geologic information can be treated as variables and incorporated in an occurrence model that associates probability of mineral wealth with geology* (the MG model).

THE PROBLEM RE-EXAMINED

Traditional Exploration: Traditionally, mineral exploration has sought targets of one particular kind, such as petroleum, gold, copper, etc. The search for these targets has been conducted in a manner similar to that described by Allais, *i.e.*, by successive examination of indicators culminating in drilling and detailed exploration. Although explorationists did not deal explicitly with occurrence and search models, these concepts were implicit in their philosophy. The discovery of a deposit of base or precious metals was known to be more likely in areas in which igneous intrusives were exposed or near the surface. Decision criteria such as this were developed from long years of experience and supported by geologic theory. Many valuable deposits have been found by this traditional approach.

A Philosophy of Mineral Resources: A question to be considered in formulating an operations research model for mineral exploration is, "To what extent is it desirable that the model endorse the methodology of the traditional approach?" The answer to this question is quite complex and depends upon one's philosophy of exploration and mineral resources, which must consider and relate such things as the level of information available, economic parameters, and earth science. At the beginning of this paper, the exploration model was defined as consisting of two submodels: the occurrence and the search models. Drew, Allais, and Engel employ spatial models to describe the distribution of mineral targets. Such models assert that in exploration it makes sense to regard all cells as equally likely. This is certainly in contrast to the traditional approach. In fact, to a geoscientist experienced in mineral exploration, such a proposition may appear very naive. Certainly, for some small area such as a 20-mile-square cell, an oil or gas field is much more likely to be found in a cell located on the flanks of a sedimentary basin than in one situated in the middle of an igneous stock surrounded by high rank-metamorphic rocks. Although this is an extreme case, it serves to illustrate the point that areas are not equally favorable.

This argument that cells are not equally favorable loses strength as one (a) increases the size of the cells, (b) regards multiple targets or mineral groups, and (c) assumes that resources complementary to minerals are uniformly distributed. Specifically, the variance of value per cell decreases as the size of the cell increases. Thus, for cells of very large size, the distribution of values throughout the search area tends to uniformity. This tendency to uniformity becomes stronger when targets of all mineral commodities are considered (metallic, nonmetallic, and fuels), especially if each cell is considered to be equally endowed with resources and economic factors other than mineral. The equal distribution of complementary resources is important because of the high location value of some mineral products. Mineral resources are defined only within an economic framework. Under these very special conditions, it may be that some unit volume of the earth's crust may be approximately equal in value throughout a study area.

The conditions just outlined for a spatial occurrence model are highly restrictive and depart considerably from the "real world." First, mineral resources are defined only within an economic framework, and resources other than minerals are not equally distributed. Secondly, intense exploration of areas large enough to equalize mineral endowments defeats the objective of the search model — that is, to allocate the most effort to those areas most favorable so as to optimize some measure of profit. Further, firms have limited funds available and cannot explore areas of unrestricted size. Other criticisms of an economic nature can be leveled against the *intense* exploration of *very large areas* (such as grid drilling of the U.S.) One of these is that many of the resources located would lie dormant for many years because of saturated markets and the unequal distribution of the complementary resources. The capital expended in their location must be charged against them as an investment at some cost of capital, r, and compounded to the day that such resources are exploited. This would create a large cost for such deposits and make many of them uneconomic. This seems to be a limiting factor in the trend to initially explore large areas, as suggested by Allais. The point to be made here is that the distribution of mineral resources within the economic system by which such resources are valued *is nonuniform.*

Information and Decision Making: The foregoing conclusion does not of itself mean that spatial distributions have no use in exploration, for, although the occurrence model may be nonuniform, the geologist's decisions are based upon information available to him from the study area. Conditions may be such that he has no knowledge concerning the parameters of the occurrence model, and such that there are no geophysical or geological data on the area. In such a case, spatial models do provide for large areas an estimate of total resources contained. In most cases, however, the geologist does have geological and/or geophysical data on the study area but does not know the parameters of the occurrence model. Traditionally, the geologist has interpreted the information available on the study area as a function of his past experience (past experience constitutes his occurrence model). Here lies a basic problem, for the factors that control ore formation in one area may not be the same in another area. This fact has been responsible for passing over valuable deposits. At the same time, one must bear in mind that many valuable deposits have been found by this inductive geologic reasoning. The multivariate geological model combined with the computer provides a means of decreasing this bias due to extrapolation from one area to another, for the relationships between geology and mineral wealth in many different areas can be combined to form a general occurrence model which may contain most geologic factors. This may be especially true for geostatistical inference at the reconnaissance level, for the relationships are general and not so likely to vary from one area to another.

It is doubtful that terminal decisions, such as optimal grid spacing, made solely upon inference from other areas can, as a general rule, be optimum decisions. Decisions will probably be nonoptimal, whether the inference be made *via* spatial distributions concerning target characteristics (Drew's model) or *via* multivariate analysis of geological data (the MG model). A better strategy would allow for the modification of decision criteria (probabilities, distributions of target characteristics, etc.) as information is received from the study area. This implies a *multistage exploration strategy.* Such a strategy progressively, as each stage is completed, discounts the *a priori* decision criteria based upon previous experience by relationships observed in the study area itself. Thus, bias (if it is present) resulting from the initial extrapolation of previously observed geologic relationships is continually reduced.

A POSSIBLE STRATEGY

It may now be profitable to examine the models discussed previously in this paper to see how they might fit together within this philosophy to form an acceptable over-all strategy of exploration. For this purpose, it will be assumed that a large area is being searched for targets of one mineral group and that geologic and geophysical data are available on each of several well explored and developed areas. These areas can be pooled to give a broad statistical base upon which the relationships of mineral wealth to geology are defined. Furthermore, distributions of target characteristics can be developed for each value group.

The first step in the exploration of the new area would be the gathering of the reconnaissance geological and geophysical data. These data could then be processed through the multivariate geological models to generate probabilities. The drilling of the most valuable cell could then be simulated on a computer using the distributions of targets that characterize cells of that value in the control area. In such a simulation, drilling could represent all subsequent exploration and be valued accordingly. Given the pertinent distributions of target characteristics from the control areas for a similar cell, simulated drilling could provide an estimate of the optimum grid spacing and the associated rate of return for the discovery and exploitation of the deposits. If this rate of return meets the goals of management then the decision to explore this cell would be made.

In the actual exploration of the cell, a wide grid — some function of the optimum spacing estimated by the simulator — could first be drilled. In this way information would be acquired on the geology and deposits early in the search program. Any deposit discovered would be developed. Information on geology and target characteristics would be used to revise the *a priori* decision criterion. Successively smaller grid spacings would be made by intergrid drilling, and after the completion of each stage the decision criterion and pertinent distributions would be revised in light of the information gained until the optimum grid spacing is approached as closely as possible, given the initial grid spacing.

The process would be repeated on the next most valuable cell until with all available information obtained from the cells explored, the remaining cells appear to be unprofitable to explore as determined by the simulated drilling. The important feature of this strategy is that it is dynamic, adding new data from the explored areas to the initial *a priori* information as they are gained. Furthermore, the best area, as determined from the initial statistical inference and the simulator, is drilled first, at the lowest level of information, for the chance of misallocation here is less than for any other cell. As the decision becomes more critical, the reliability of the information base increases, thereby decreasing the likelihood of making a wrong decision.

EXTENSIONS

A strategy such as that outlined is only a beginning. An obvious improvement would be to make the problem multidimensional; that is, search for targets of several mineral groups (a total resource model). In addition, since some of the mineral groups (chiefly nonmetallic) have high place value, an economic structure introducing the relationship of the cell to consuming centers and transportation networks would be a desirable addition.* Finally, since firms have limited capital available for exploration, the amount of capital should be considered as an economic constraint.

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STRESS DISTRIBUTION IN SHORT COLUMNS

by J. E. Willson and N. K. Bohidar

An evaluation of internal stress distribution based on photoelastic studies and destructive testing of simulation models shows that pillar failure is a function of the internal stress distribution, and that stability can be changed by altering the internal stress pattern.

Induced stresses in three-dimensional epoxy disks subjected to uniaxial forces were evaluated by photoelastic studies. Stress profiles across different sections perpendicular to the loading direction revealed two maxima for each profile. Based on the stress distribution for the entire column, an attempt was made to explain pillar spalling and the hour glass pattern of failure in a mine pillar.

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For maximum stability and optimum design of mine pillars, it is necessary to consider the external forces, the nature and distribution of induced stresses and the characteristics of the pillar material as determined in the laboratory. This paper summarizes the research done by N. K. Bohidar[1] and Ben L. Seegmiller[2] as partial fulfilment of the requirement for Master of Science degree in Mining Engineering, University of Utah. It is an evaluation of a study to determine the stress distribution in short columns in order that regions of high stress concentration in mine pillars may be found and their effect on pillar failure be ascertained. The ratios of the dimensions of the columns studied are representative of those found in mines located in flat-bedded deposits.

The weight of the overburden generally constitutes the major external force. This force is calculated by taking into account specific weight and thickness of the cover. Determination of the distribution of this force on pillars may often require a complex analytical study.

There have been many investigations of the strength

Corporate Exploration Strategies

By BRIAN W. MACKENZIE*

SYNOPSIS

The mineral exploration environment has observable economic characteristics, the assessment of which provides guidelines for exploration planning in the mining company. The long-term survival of the mining company is directly dependent on successful exploration. Exploration planning contributes to successful exploration through the development of exploration strategies. The objective of exploration planning is to select exploration environments which will enable the mining company to realize best possible profit and corporate risk conditions. Thus, the formulation of exploration strategies is based on profit and corporate risk criteria. Strategic considerations include the selection of commodities as exploration targets, geographic concentration of exploration effort, the realization of acceptable survival conditions, and the structuring of an exploration programme to optimize the economic benefits of the exploration investment process.

INTRODUCTION

In recent years much progress has been made in analyzing the economics of mineral exploration. This has included the development of quantitative mineral occurrence and search models, assessments of the risks and returns of exploration, and descriptions of exploration philosophy. However, little direct attention has been given to the application of quantitative concepts to exploration planning in the mining company. This paper suggests a decision framework for exploration planning, developing economic criteria for exploration investment and applying these criteria to the formulation of exploration strategies.

Three assumptions are made:

(i) There is common ground between quantitative concepts and realistic applications and, thus, theoretical analysis can assist with the solution of practical problems.

(ii) The mining company of today and tomorrow requires a formal planning framework to contend effectively with changing economic conditions.

(iii) The mineral exploration environment possesses observable quantitative dimensions which can provide guidelines for the development of corporate exploration strategies.

The paper focuses on the relationship between mineral exploration and the mining company. Initially, the mineral exploration environment is analyzed to define observable and relevant economic characteristics. Then the role of exploration within the mining company is described to establish terms of reference for exploration planning. These provide bases for developing profit and corporate risk criteria for exploration investment which are then applied to the formulation of exploration strategies.

THE MINERAL EXPLORATION ENVIRONMENT

The mineral exploration environment comprises the distribution of undiscovered mineral deposits in nature and the exploration techniques and skills available for their discovery. Specific environments are defined by geological setting, deposit type, and geographical area.

In the exploration environment, search targets are successively narrowed and the level of detail of information increased by proceeding sequentially through a number of information-gathering investment stages. The techniques of applied geology constitute the search method. In general, five sequential exploration stages may be distinguished, namely, regional selection, area selection, area exploration, follow-up ground exploration, and detailed exploration. Exploration continues as long as the analysis of information at the end of each stage provides economic justification for further investment. Ultimately, economic mineral deposits are sufficiently well explored to permit a mine development decision to be made.

To provide a quantitative basis for exploration planning, assessments must be made of the economic characteristics of the exploration environment. For this purpose, three environmental parameters are of fundamental importance:

(i) *the cost associated with discovering a mineral deposit,*

(ii) *the environmental probability of discovering an economic mineral deposit*, and

(iii) *the return resulting from an economic discovery.*

The definition and measurement of these parameters presents many practical difficulties. In general, mineral exploration is characterized by a very low probability of economic discovery, and a very large return given an economic discovery relative to the exploration costs associated with discovering a deposit.

Exploration cost is the least difficult environmental parameter to assess. The costs of individual exploration techniques are well documented in the literature; see, for example, exploration cost break-downs in Morgan (1963), Peters (1969) and the CONSAD study (1969). Exploration cost will also depend on the particularities of both the exploration environment and the mining company. In this respect the mining company will have its own cost experience upon which to draw. Thus, costs can be estimated on a unit rate basis for each exploration technique. The costs of individual techniques together with a knowledge of the information responses they provide and the overall programme suitable for a specific exploration environment, provide the bases for assessing *the cost associated with discovering a mineral deposit.* For this purpose exploration discoveries could be defined as deposits where initial drilling indicates the possibility of mineralization and a mineable width, for example, anomalies which justify more than one drill hole. Discovery costs would include any drilling costs resulting directly from the initial intersection, for example, inclusive of the first few drill holes. Subsequent exploration costs, more closely associated with economic discoveries, would be charged against the returns resulting therefrom.

For example, consider the following hypothetical geophysical exploration programme for copper-zinc deposits in the Canadian Shield region. Initially, favourable 300-square mile areas are selected for airborne electromagnetic survey and flown with a quarter-mile spacing. Airborne survey costs average $30 per line mile. Following airborne work, a variable

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number of airborne anomalies are selected for investigation on the ground. In an area containing a large number of promising anomalies as many as 80 zones might be selected whereas in an area showing fewer responses only 40 anomalies might be selected; say, on the average, 60 anomalies are selected. Follow-up ground exploration includes property acquisition, line cutting, electromagnetic and magnetic surveys, and geological mapping, with costs averaging $4 000 per square mile. Follow-up grids average 0.2 square miles. On the average, one-third of the follow-up anomalies show favourable geophysical responses and geological settings, and are drilled. Of the anomalies drilled, 95 per cent are rejected after the first hole. The remainder give sufficient encouragement to justify an additional four holes on the average. Drilling averages 200 ft per hole and costs $10 per foot.

Airborne survey cost = 1 200 (30) = $36 000
Follow-up cost = 60 (0.2) (4 000) = $48 000
Drilling cost = 20 (200) (10) + 4 (4) (200) (10) = $72 000
Total discovery cost = $156 000
Number of deposits discovered = 1
Cost of an exploration discovery = $156 000.

The environmental probability and return parameters are more difficult to assess because the economic characteristics of undiscovered deposits are not known. However, these characteristics can be estimated either by using mineral occurrence models or by analyzing past exploration results.

Mineral occurrence predictions for this purpose should give the expected number of deposits in feasible tonnage-grade categories on a unit area basis. Given terrain and cover type, empirical revenue and cost functions can be used to simulate the economics of developing predicted deposits within each area. An example of this type of mineral occurrence model is the subjective probability approach developed by Harris, *et al* (1970) and applied in the Canadian northwest. Thus, *the environmental probability of discovering an economic mineral deposit* would be estimated by the ratio of the predicted number of undiscovered economic deposits (defined on the basis of minimum acceptable size and profitability criteria) to the predicted total number of undiscovered deposits. *The return resulting from an economic discovery* would be measured by the size and profitability distributions for the economic deposits.

For exploration environments with a reasonable history of exploration, the economic characteristics of undiscovered deposits can be estimated on the basis of past exploration results. Thus, *the environmental probability of discovering an economic mineral deposit* is estimated by the historical trend in the ratio of economic discoveries to total discoveries. Economic discoveries could be defined as those known deposits which would satisfy minimum acceptable size and profitability criteria if developed under present-day conditions. It may not be possible to measure the total number of past discoveries directly, but this parameter can be derived if total pre-discovery exploration expenditures and the cost associated with a single discovery can be estimated for a particular exploration environment. Roscoe (1970), on the basis of the work of Derry (1970), used this approach for the case of metallic mineral exploration in Canada during the period 1950 to 1970. His results indicate a decline in the probability of economic discovery from 0.01 in 1950 to 0.001 in 1970. *The return resulting from an economic discovery* would be assessed by the size and profitability distributions for past economic discoveries if developed under present-day conditions.

The economic characteristics of the exploration environment are changing constantly. The development of new exploration techniques, and the increasing depth and decreasing grade of undiscovered deposits alter the costs and information responses associated with the exploration process. Changes in market and cost conditions shift the probability and return parameters. Each geographical area, geological environment and deposit type will have its own unique characteristics. Thus, assessment of economic parameters for the exploration environment is required for each time and place.

THE RÔLE OF EXPLORATION WITHIN THE MINING COMPANY

The mining company is concerned with the development and operation of mines and, more importantly, with the strategic decisions which lie beyond current operations. The most important problem for most mining companies is mine replacement. The mine replacement problem is the result of the certainty of exhaustion for currently operating mines coupled with the very low probability which characterizes new economic discoveries. Thus, mineral exploration is fundamental to the long-term success of the mining company.

Mining company planning, as described by the author (1969), is based on three objectives. These are profit, survival and growth. Corporate resources consist of capital and managerial and technical skills. Corporate planning is concerned with the development of strategies to guide the investment of resources for the realization of objectives. Exploration strategies guide corporate investment in the exploration environment.

For the small mining company, the relationship between limited corporate resources and the low probability of economic discovery is dominant. To succeed, resources must be concentrated on the discovery of economic deposits. Success will result in the development and operation of an increasing number of mines with consequent increases in corporate resources. Thus, as the mining company grows, its mine replacement problem eases. At the same time, market opportunities and constraints become increasingly important. These factors result in a shift of emphasis from exploration towards markets. Forward processing and diversification will be encouraged. Nevertheless, there will be a continuing important role for exploration to support the company's processing and marketing activities and to the extent that exploration offers relatively attractive investment opportunities.

The large mining company has three basic alternatives, which are exploration, forward processing, and diversification, for investing resources to realize its profit, survival and growth objectives. The role of each alternative is a function of tradition, the balance sought between profit, survival and growth and the number and characteristics of investment opportunities in each area. For example, emphasis on profit may give a predominant role to exploration while emphasis on survival and growth may encourage processing and diversification.

The role of exploration within the mining company determines the level and proportion of total corporate resources which should be allocated to exploration. According to Morgan (1969), large mining companies generally spend between three and twenty per cent of their pre-tax cash flow on exploration and research, but the outlay by small mining companies, in efforts to survive, must often exceed this range. A survey of 11 base metal mining companies described by Ensign (1969) shows an average exploration expenditure of five per cent of pre-tax cash flow. Kruger (1969) quotes exploration expenditures as a proportion of gross value for selected mining companies during the period 1954 to 1[illegible] showing ranges of 0.1 to five per cent for small and medium size companies and 0.5 to three per cent for large companies. These surveys indicate that mining companies allocate from five to forty per cent of their post-tax cash flows to exploration investment.

Given an allocation of corporate resources to exploration based on growth alternatives, exploration planning should be based on profit and survival considerations. From a profit viewpoint, corporate exploration strategies will be acceptable only if they generate investment opportunities which satisfy a minimum expected profitability condition and a minimum size condition. The minimum expected profitability condition will be the corporate cost of capital as reflected in the opportunity cost of alternative investments. For example, Ensign (1969) states that the Copper Range Company has sufficient copper reserves at White Pine to expand current mine capacity. Their exploration investment is, therefore, directed towards discoveries which can offer a greater expected profitability than investment in the White Pine expansion. The minimum size condition ensures that exploration investment opportunities are sufficiently large to make a significant contribution to overall company performance. Thus, the minimum size condition will be related directly to company size. For example, the Copper Range Company accepts exploration targets only if they have the potential to make an annual contribution of at least 20 cents per share, equivalent to annual earnings of $400 000.

While exploration strategies are guided by profit criteria, they also are, in another important respect, strategies for corporate survival. Survival for the mining company is associated primarily with solving the problem of mine replacement. The long-term survival of the mining company is directly dependent on successful exploration. But if resources are limited and the probability of economic discovery very low, the mining company assumes the risk that it will expend resources without success. Corporate risk is measured by the relationship between exploration resources and the economic parameters of the exploration environment. Obviously, the importance of survival and corporate risk considerations is an inverse function of company size. Nevertheless, corporate exploration risk may still be significant for large companies under favourable profit conditions.

PROFIT CRITERIA

Long-term exploration decisions are based on expected profitability and size criteria. In more general terms, the profit potential of exploration investment is measured by expected value, that is, the average value that the exploration environment will yield in the long term, balancing the successes and failures of a large number of investments. For illustrative purposes:

$$EV = pR - C,$$

where EV = expected value,
p = probability of an economic discovery,
R = return resulting from an economic discovery,
C = cost of discovering a deposit.

If C comprises all exploration costs up to and including the first few drill hole intersections, then subsequent exploration costs associated with delineating deposits for the purpose of determining whether there is economic justification for development must be deducted in the assessment of R. R represents the net present value of an economic discovery; the difference between discounted positive cash flows and discounted investments (including exploration costs beyond C) using a rate which reflects the corporate cost of capital.

Expected value conditions will depend on specific environmental conditions as well as the particularities of the individual mining company, including the minimum acceptable size of exploration target. For example:

(i) small mining company
p = 0.02
C = $150 000
R = $10 000 000
EV = 0.02 (10 000 000) − 150 000 = $50 000

(ii) large mining company
p = 0.005
C = $150 000
R = $35 000 000
EV = 0.005 (35 000 000) − 150 000 = $25 000

The expected value concept has been advocated by Preston (1960), Grayson (1960), Brant (1968), and others, as a useful technique for assessing the favourability of exploration investment. The higher the expected value, the more attractive the investment. Usually an expected value greater than zero is regarded as a necessary condition for investment. However, in some cases a 'windfall' strategy may guide exploration. If R is very large in relation to C, a characteristic of exploration as well as sweepstakes, a mining company may undertake at least a limited number of investments, even if it knows that expected value is negative, because of the possibility of a spectacular return. Such a strategy is, of course, ultimately ruinous and can be justified only in the short term. If the mining company is lucky, and smart enough to quit when it is ahead, efforts under such conditions can be rewarding for some.

Expected value is a function of the three environmental parameters, the assessment of which has been described in a foregoing section. Since these parameters are usually estimated as frequency distributions to reflect variations in the value of economic discoveries and the multistage nature of the exploration investment process, the expected value function is modified as follows:

$$EV = \Sigma p_i R_i - \Sigma q_j C_j,$$

where $\Sigma p_i = p$
$\Sigma q_j = 1$

If sufficient information is available, the expected value concept may be refined to assess expected profitability and size. This is useful because these are the common criteria for exploration investment decisions. If expected profitability is measured by the present value ratio, investment and positive cash flow values are discounted to the present point in time using the corporate cost of capital and the ratio between discounted net return (expected value) and discounted investment (including both exploration and mine development expenditures) is calculated. Note that the discounting procedure has been embodied in the definition of R. Thus:

$$EP = \frac{EV}{pI + C} = \frac{\Sigma p_i R_i - \Sigma q_j C_j}{\Sigma p_i I_i + \Sigma q_j C_j},$$

where EP is the expected profitability as measured by the present value ratio, and I is the investment required to develop an economic discovery, discounted to a present value using a rate reflecting the corporate cost of capital.

The size criterion can be assessed by several parameters including R, average R per year, or I (reflecting mine capacity).

Economic discovery is defined by the minimum expected profitability and size limits which the company sets for exploration investment.

CORPORATE RISK CRITERIA

An important characteristic of the mineral exploration environment is the low probability associated with economic discovery. Under this condition the application of limited corporate funds does not insure the realization of expected value and exploration resources may be expended without success. The corporate risk associated with the realization of expected value introduces a survival element into the development of corporate exploration strategies. This risk may be quantified by applying the classical problem of the gambler's ruin.

The classical problem of the gambler's ruin concerns a gambler with limited capital who wagers against a 'house' with essentially unlimited resources. The gambler is ruined and the game terminates if at any point his capital balance falls to zero. Survival is the complement of ruin. The gambler survives if his capital balance is maintained above zero. The problem is to determine the gambler's probability of ruin.

If the mining company is to survive beyond the life of its current operations, it must search for and discover economic mineral deposits. This activity is analogous to a gambler wagering on a chance device. The mining company is the gambler, nature is the opponent, and exploration the chance device. The cost of discovering a deposit is the wager. Each discovery has a probability of success, in which case a return is realized by the company, and a probability of failure, in which case the company loses its wager. The company allocates some limited amount of capital to the search activity. What is the corporate risk associated with survival?

Allais (1957), Slichter (1960), and Brant (1967) consider the application of a special case of the gambler's ruin problem to exploration investment, the probability of zero successes in n successive wagers. However, the mining company may also be ruined after one or more successes. This broader concept of gambler's ruin applied to corporate risk assessment for the development of exploration strategies is shown in Fig. 1. It is assumed that a proportion 'e' of the return R resulting from an economic discovery is reinvested in exploration, equivalent to an amount $E = eR$.

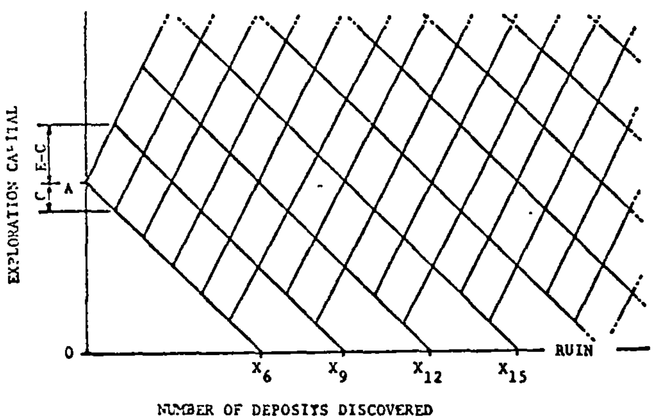

Fig. 1. Gambler's ruin in mineral exploration.

The mining company starts with exploration funds A. It invests in the discovery of a deposit. Exploration capital increases to $(A + E - C)$ with probability p or falls to $(A - C)$ with probability $(1 - p)$. The company then invests in a second discovery, conditional to funds being at one of these two points. Following this second investment, capital may be at one of three possible levels, that is, $(A + 2E - 2C)$ with probability p^2, $(A + E - 2C)$ with probability $2p\ (1 - p)$, and $(A - 2C)$ with probability $(1 - p)^2$. And so the investment process continues.

Each grid intersection represents a possible capital level. The grid is bounded by lines representing straight runs of success and failure. When the boundary representing failure in every investment reaches the zero capital level, the grid is bounded by the zero capital axis, an absorbing state which represents ruin of the mining company. To the right the grid remains open, implying that the company will invest in exploration indefinitely. The summation of the probabilities associated with the ruin points along the zero capital axis, each representing a unique number of successes, failures, and total investments, is the mining company's probability of ruin.

Uspensky (1937) developed a variation of the gambler's ruin problem which has been applied by the author (1968) to the mining company's survival conditions in the exploration environment. The derivation of the solution for this application is described in the Appendix.

Assuming that initial exploration funds are large in relation to discovery cost and that the expected value of costs and returns to exploration $(pE - C)$ is positive, then

$$Pr = \theta^A.$$

Here θ is that root of

$$p\theta^{E+1} - \theta + q = 0$$

for which $0 < \theta < 1$. In these expressions Pr is the probability of the mining company's being ruined, that is, $Pr = (1 - Ps)$ where Ps is the probability of survival. Also, A denotes the initial exploration funds of the mining company measured in discovery units, that is, number of discoveries afforded, $C = 1$ is the cost of an exploration discovery, E is the amount reinvested in exploration from each economic discovery measured in discovery units, $E = eR$, where R is the return to the mining company given an economic discovery, and e is the proportion reinvested in exploration. Finally, p is the probability of an economic discovery and q is the probability of uneconomic mineralization, $q = (1 - p)$.

These relationships provide the basis for estimating the corporate risk associated with exploration, given assessments of environmental parameters and corporate funds available for exploration.

For example:

p = 0.01
C = \$150 000
R = \$45 000 000
E = \$21 000 000
θ = 0.994 861 4

Exploration investment	*Probability of survival*
\$1 500 000	0.05
\$9 000 000	0.27
\$15 000 000	0.40
\$37 500 000	0.72
\$75 000 000	0.92
\$150 000 000	0.99

'WHAT TO LOOK FOR' STRATEGIES

In the long term, exploration investment is guided by long-term market trends for specific mineral commodities in relation to their resource potentials. The more favourable the relationship, the more attractive the commodity as an exploration target. A study by Booth (1971) determines relationships between the natural abundance of metals in the earth's crust, trends in annual production, and trends in price, for various metals over the past decade. Availability of a metal is measured as the product of natural abundance and price, and degree of exploitation by abundance divided by annual production. Metals having a relatively high availability factor and a relatively rapidly decreasing exploitation factor are regarded as the most attractive exploration targets, reflecting high natural abundance, high price and rapidly increasing demand. On this basis copper, nickel and tin ranked highest as attractive exploration targets. Although relative differences in exploration, production and processing costs for the metals are neglected, this type of analysis provides general guidelines in selecting commodities as exploration targets.

However, the selection of commodities for exploration also depends on the particularities of the individual mining company, such as profit and corporate risk criteria, forward processing requirements, product research and development activities, market control for specific commodities, and marketing, exploration and processing skills. For example, it is for both general commodity and specific corporate reasons that International Nickel has concentrated on nickel as an exploration target, that an aluminium company explores for fluorspar, or that a petroleum company explores for uranium.

Given that a particular commodity is acceptable in terms of corporate policy, the attractiveness of exploring for it depends on forecast trends in supply, demand and price. These trends are reflected in the R parameter, the return given an economic discovery. Thus, the long-term attractiveness of a mineral commodity is expressed in the expected value and expected profitability criteria. The higher the expected assessments, the more attractive the commodity as a target for exploration investment.

Long-term profit criteria will be the primary consideration for mining companies which are sufficiently large to ensure the realization of expected values. Smaller companies must also consider corporate risk conditions, selecting those commodities which offer relatively low discovery costs and a relatively high probability of economic discovery. By searching for this type of target, and accepting relatively small and marginal deposits as economic discoveries, the small company may have the best chance of survival.

'WHERE TO LOOK' STRATEGIES

Geographical concentration of exploration is based on geological concepts for the types of deposits sought as exploration targets. Good geological concepts are fundamental to exploration success and should be the focal point for geographical concentration. Geological concepts are a function of time, information and experience. The mining company with the best geological concepts will concentrate exploration in areas having the highest probability of economic discovery.

Sullivan (1968) and Michener (1970) have described the associations between particular types of mineral deposits and particular geological environments. Accordingly, the contact between rhyolites and andesites provides a favourable setting for copper-zinc deposits, nickel and asbestos deposits are associated with particular types of basic and ultrabasic igneous rocks, and copper-molybdenum deposits occur in porphyry stocks or in surrounding metamorphosed skarn zones.

Within the set of geologically favourable areas, geographical selection is based on mineral occurrence and economic and political factors. Michener (1969) provides a subjective assessment of these factors in a large number of countries for the purpose of ranking them in terms of overall exploration favourability. In this assessment, Australia, Canada, Mexico, South Africa, New Zealand and Spain have the best ratings while Liberia, Bolivia, Venezuela, Haiti and Nigeria have the lowest.

For area selection from geologically favourable regions within a particular country, attention must be given to both mineral resource potential and economic factors. At this level, mineral occurrence models may be of assistance in predicting area resource potentials. Economic differences between regions due to differences in local taxation, transportation facilities, regional development incentives, power costs and manpower considerations, must also be assessed.

As in the case of commodity selection, geographical concentration will also be based on corporate particularities, namely, traditional exploration regions, balance required between a stable political environment and a favourable geological environment, and corporate marketing considerations. For example, Hudson Bay Mining and Smelting, a regionally-based smelter company, has concentrated exploration in the greenstone belts of northwestern Manitoba. On the other hand, International Nickel's global exploration strategy, as described by Zurbrigg (1971), has major exploration programmes in most countries of the non-communist world with nickel potential including Canada, United States, Australia, Indonesia, Guatemala, New Caledonia and Southern Africa.

Individual corporate assessments of geographical favourability will differ markedly. Armstrong (1970) of Cominco views the Canadian Cordilleran and the Shield area of the Canadian Arctic as the most geologically favourable areas for exploration in a country which he prefers to the United States and Australia in the economics of exploration. Futterer (1970) has described Noranda's exploration philosophy which has resulted in international exploration on an expanding scale and the investment of 25 per cent of its total programme in Australia. Holmes (1971) advocates the metallogenic belts of Western Europe and the Middle East as exploration areas of traditional neglect and high potential.

Short-term survival considerations will also influence geographical selection for small mining companies. Small companies, perhaps a generation or two ahead of the economics, often lead the way in remote-area exploration. Small discoveries will be of significance to the small company and their high grade may overcome adverse economic factors associated with remote locations.

SURVIVAL STRATEGIES

Survival is a primary objective of the mining company. For purposes of exploration planning this objective may be expressed as a probability of survival confidence limit. The corporate risk criterion can be used to determine the combinations of environmental parameters and corporate exploration funds required to achieve the company's survival objective. For example, Fig. 2 shows the necessary conditions to attain a probability of survival objective of 0.90. The asymptotic values for individual curves have important implications for the mining company's survival strategy. They give critical values for the allocation of exploration funds, and the amount reinvested in exploration from each economic discovery, below which the corporate survival objective cannot be realized.

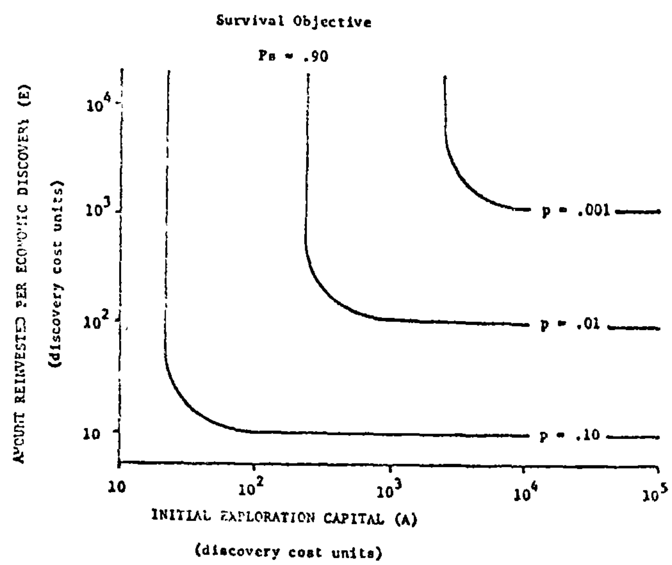

Fig. 2. Exploration survival conditions.

The mining company's survival strategy centres on one of the following questions:

(i) What environmental probability of an economic discovery should the company seek to fulfil its survival objective?

(ii) What changes in amount reinvested from each economic discovery would be required to balance a change in the probability of an economic discovery in order to maintain the company's survival position?

(iii) How much capital should the company allocate to exploration to realize its survival objective?

Answers to these questions may be obtained from the corporate risk functions as illustrated in Fig. 2. In general terms it is clear that low probabilities of economic discovery are manageable only for large companies, that is, as the probability of an economic discovery is reduced, there must be a compensating increase in return given an economic discovery and amount reinvested from each economic discovery to induce firms to participate; also, higher levels of exploration funds are required for lower probabilities of economic discovery.

To this point, it has been assumed that exploration is financed internally. When internal resources are insufficient to realize the company's survival objective, it may be suitable for the company to pool its resources together with other companies in joint ventures. Joint ventures are commonly used in mineral exploration, primarily as a survival strategy. It is a reasonable strategy provided that control is not diluted excessively.

The ratio of internal exploration funds to the total amount required to realize the company's survival objective represents the joint venture participation which the company should seek. For example:

p = 0.01
C = \$150 000
R = \$45 000 000
E = \$21 000 000
Ps objective = 0.90.

Internal exploration funds	*Required joint venture participation*
\$5 000 000	7%
\$25 000 000	35%
\$50 000 000	70%
\$75 000 000	100%

Many mining companies are not large enough for joint ventures to offer a reasonable alternative. These companies must seek other survival alternatives. At the limit the total return from economic discoveries can be reinvested in exploration. This helps only after the first economic discovery has been made. Alternatively, attention may be focussed on special exploration environments, such as commodities with very limited demand, surface deposits, and remote areas, where environmental parameters encourage survival. Also, the small mining company may invest in deposits which have been discovered and rejected by larger companies. These will usually be small, marginal deposits which do not meet the minimum acceptable conditions of the larger companies. In some instances fully-fledged economic discoveries which have not been perceived by the discoverers may be acquired.

'HOW TO LOOK' STRATEGIES

The fundamental issue in structuring an exploration programme is the balancing of available exploration funds between direct investment in discovering deposits and investment in the level and quality of skills in the exploration group. The larger the allocation to direct exploration investment, the larger the number of discoveries and the better the company's chances of realizing expectations. The benefits which result from increasing exploration group investment are in the selection of exploration environments with superior economic characteristics. This will result from better geological concepts, and more informed assessments of mineral resource potential and the economic characteristics of exploration environments. A more highly skilled exploration group will increase the company's probability of economic discovery. The optimum balancing of exploration funds will maximize the company's chances of realizing its exploration objectives.

Because a low probability of economic discovery is associated with each exploration investment, a sequential investment strategy is used to limit the cost of failure by breaking the exploration investment into a number of components each of which is associated with a particular sequential stage. The cost of discovering uneconomic deposits is reduced when these discoveries are determined to be uneconomic and are rejected before being fully explored. A three-stage sequential example is shown in Fig. 3. The cost associated with discovering and exploring a deposit fully is F. The boundary condition for continuing investment is assumed to be an expected value greater than zero. The boundary condition defines two zones; these are a zone of favourable expectations, in this case representing positive expected values, and a zone of unfavourable expectations representing negative expected values. Assessments of p, R and expected further exploration costs (xF) are made on completion of each stage. Exploration investment continues until either the assessment of expected value is negative or until the probability of an economic discovery justifies a mine development decision. The starting point for an investment, S, represents a fixed cost, $0.1F$, that is incurred in defining the exploration opportunity. Three failure paths for the exploration opportunity are shown. If exploration investment had been in a single stage, the cost of the three failures would have been $3.0\ F$. The three-stage sequential process has limited the cost of the failures to $1.6\ F$.

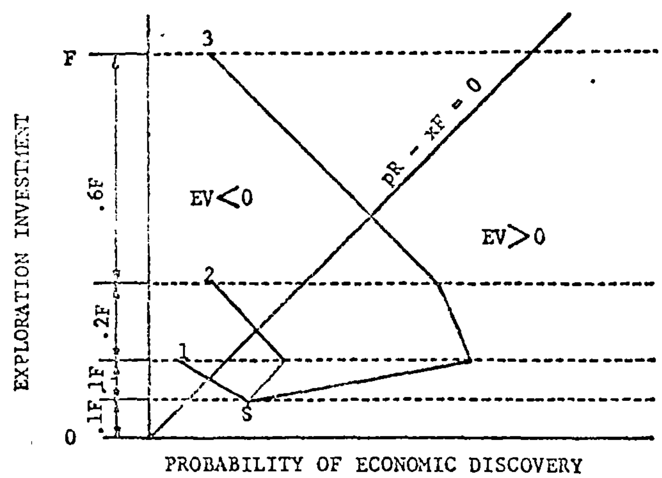

Fig. 3. Sequential exploration investment.

The benefits of the sequential process, in terms of limiting the exploration investment in uneconomic deposits, must be balanced against the opportunity cost associated with rejecting economic discoveries. It is this benefit-cost relationship which determines the optimum boundary condition for accepting and rejecting opportunities at the end of each sequential stage. The selectivity should be such that combined direct and opportunity costs are minimized. This is known as the problem of 'skimming' and it is of critical importance in determining the economics of exploration investment to the mining company.

In the sequential exploration process the prior probability distribution of the expected profitability of a discovery is relatively flat to begin with. On the other hand, the successive posterior distributions as more and more exploration has been completed are more and more sharply peaked or concentrated in a more limited range and, therefore, there is better and better information for deciding on acceptance or rejection and a basis for increasingly rigorous decision criteria. In the early stages, the expected profitability is much less sharply defined than it is later on and overly rigorous decision criteria will result in the rejection of economic discoveries. As deposits are carried through the sequential stages, the less promising are gradually eliminated as information is accumulated.

Skimming is used to minimize costs and, thereby, optimize the exploration investment process. It may not maximize the number of economic discoveries. Skimming should mean that a high proportion of the deposits fully explored are economic with the early elimination of a high proportion of total discoveries. The possibility that economic deposits are rejected is clearly increased as the number of deposits fully explored is reduced, since the nature of skimming involves accept-reject decisions on less than conclusive evidence.

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APPENDIX

Uspensky (1937) developed a variation of the gambler's ruin problem which has been applied by the author (1968) to the mining company's survival conditions in the exploration environment.

Two opponents, A and B, play a series of games, the probability of winning a single game being p and $q = (1 - p)$, respectively, each game ending in a loss for one of them. If the stakes in a single game are C for A and R for B, and the players' respective fortunes are A and B, what is the probability that one of the players will be ruined, in the sense that at a certain stage his fortune is less than a single game stake? No limit is set for the number of games.

Let P_x be the probability for A to be ruined by the lack of sufficient funds to set a full stake C when his fortune amounts to x and, consequently, his adversary's fortune is $A + B - x$.

Considering the result of the game immediately following the situation in which the fortune of A amounted to x, it is possible to establish an equation in finite differences that P_x must satisfy. If A wins this game, his fortune becomes $x + R$ and the probability of being ruined later is P_{x+R}. By the theorem of compound probability, the probability of this case is pP_{x+R}. But if A loses, his fortune becomes $x - C$ and the probability of being ruined later is P_{x-C}. The probability of this case is qP_{x-C}. Now, applying the theorem of total probability, P_x is a solution of the equation in finite differences:

$$P_x = pP_{x+R} + qP_{x-C} \quad \ldots\ldots\ldots\ldots (1)$$

To determine P_x completely, in addition to (1) there are two boundary conditions:

(i) if the fortune of A becomes less than C, he is ruined;

(ii) it is impossible for A to be ruined if the fortune of B falls below R.

Equation (1) is an ordinary equation in finite differences of the order $C + R$. It has particular solutions of the form θ^x, where θ is the root of the equation:

$$p\theta^{C+R} - \theta^{C} + q = 0 \quad \ldots\ldots\ldots\ldots (2)$$

Equation (2) has two positive roots, $\theta = 1$, and another which is either greater or smaller than unity, according to whether the expected value of the game to A is negative or positive, that is, $pR - C < 0$ or > 0. That is, the positive root of (2) different from unity is > 1 when single games are favourable to B and < 1 if they are favourable to A. In the case of equitable games, both positive roots coincide and $\theta = 1$ is a double root. All other roots are negative or imaginary.

The general solution leads to a complicated expression for P_x. However, simple lower and upper limits for P_x can be established that are close enough, provided the fortunes of the players are large in comparison with their stakes. Taking $x = a$, the following limits are obtained for the probability of ruin for player A, P_r:

$$\theta^{A}\,\frac{\theta^{B-R+1} - 1}{\theta^{A+B-R+1} - 1} \leqslant P_r \leqslant \theta^{A-C+1}\,\frac{\theta^{B} - 1}{\theta^{A+B-C+1} - 1} \quad \ldots (3)$$

Assuming that nature has essentially unlimited resources, that the mining company will invest in exploration indefinitely and that only a portion of the returns from economic discoveries will be reinvested in exploration, the mining company's probability of being ruined can be determined.

Let A be the mining company, B the nature, $C = 1$ the cost of an exploration discovery, R the return to the mining company given an economic discovery in cost units and E be the amount reinvested in exploration from each economic discovery measured in cost units, $E = cR$, where c is the proportion of R reinvested. Also, denote by p the probability of an economic discovery, by q the probability of uneconomic mineralization, $q = (1 - p)$, and by A the initial exploration funds of the mining company measured in cost units, that is, number of discoveries afforded. Let nature's resources $B = \infty$. Finally, EV' is the expected value of costs and returns to exploration in cost units, $EV' = pE - 1$, and Pr is the probability of the mining company being ruined, $Pr = (1 - Ps)$ where Ps is the probability of survival.

Substituting in (2):

$$p\theta^{E+1} - \theta + q = 0 \quad \ldots\ldots\ldots\ldots (4)$$

and in (3):

$$\theta^{A}\,\frac{\theta^{\infty-E+1} - 1}{\theta^{A+\infty-E+1} - 1} \leqslant P_r \leqslant \theta^{A}\,\frac{\theta^{\infty} - 1}{\theta^{A+\infty} - 1}$$

If EV' is positive for the mining company, then $\theta < 1$ and the above expression reduces to:

$$P_r = \theta^{A} \quad \ldots\ldots\ldots\ldots (5)$$

If EV' is equitable or negative for the mining company then $\theta \geqslant 1$ and:

$$P_r = 1. \quad \ldots\ldots\ldots\ldots (6)$$

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NOTES ON

THE EVALUATION OF EXPLORATION PROJECTS

Mineral Economics Course
Department of Mining Engineering
McGill University

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INTRODUCTION

In many respects, the evaluation of exploration projects is similar to the evaluation of other types of investments. Revenue and cost projections must be made and analyzed in a manner which yields superior economic decisions. The major difference is that, whereas most investments outside the exploration field can be based upon reasonably reliable estimates of the factors involved, the type of information available to assess an exploration venture is usually uncertain and incomplete, and the cost of improving the information is high. Consequently, the consideration of risk and uncertainty is an integral part of the evaluation of any exploration project. Furthermore, unlike industrial investments, the cost of analysis is a major consideration in the overall cost of exploration since it requires the frequent evaluation of relatively low value projects. Therefore, the analysis of exploration projects must be performed cheaply and quickly. The purpose of this paper is, firstly, to outline some of the prime considerations in arriving at a set of estimates and, secondly, to discuss methods of analyzing these estimates quickly, efficiently and, it is hoped, correctly.

TYPES OF PROJECTS

Exploration projects, in the broadest sense, can range from the selection of areas for regional reconnaissance through the processes of target search, target selection and target development to a full scale feasibility study which would be made prior to a production decision.

When developing a regional exploration programme the primary concern is in selecting geologically favourable areas. The traditional approach to this process has been mainly subjective, although certain farsighted geologists were attempting to quantify the effect of geological parameters as long ago as the late 1940's. In recent years considerable emphasis has been placed upon quantifying the effect of geological parameters and applying sophisticated statistical techniques to select favourable areas. Attempts have also been made to define favourable areas by transforming expert opinion into probabilities. The Department of Mineral Economics of The Pennsylvania State University has been the forerunner in both these efforts, largely under the direction of Dr. D. P. Harris. The interested reader should also refer to earlier work by Allais and Brant (see bibliography).

The application of conventional economic analysis at this stage is limited to defining the potential profitability for varying grade-tonnage-metal price relationships. The following types of graphs are equally useful to the exploration geologist and the corporate executive although there are inherent dangers in providing economic "bibles" to either of them:

- minimum grade and tonnage to be economic (possibly at various metal prices)

- profitability at various grades and metal prices with one or two different reserve tonnages

- profitability at various grades and tonnages with one or two metal prices.

- effect of ownership alternatives on the profitability of various types of deposits.

This latter item is particularly important in areas such as Canada where 100 per cent ownership of a property is the exception rather than the rule. Royalties based upon revenues can be particularly onerous on low profit margin, capital intensive mines, such as many of the bulk-type deposits in British Columbia.

As the exploration programme advances from regions to areas to specific targets the general guidelines mentioned above should be refined in accordance with the new information.

Once a target of possible economic significance has been found and some information obtained on the possible grade and tonnage, the potential of this specific target should be evaluated. Initially the evaluation will be very approximate, however, as work continues and the reliability of the grade-tonnage data improves, the degree of accuracy in other areas should be improved upon concurrently. This system of continuous review with increasing accuracy is essential if the correct decisions are to be made regarding the continuation of the exploration programme. It will also provide a sound data base in the fortunate event that the property is sufficiently attractive to warrant a full-scale feasibility study.

BASIC INFORMATION REQUIREMENTS

Prior to commencing even the most preliminary study, certain information will be required. The following listing outlines the factors which are of economic significance in most evaluations.

Mineralogical Factors

- What are the predominant minerals?
- What is their relative abundance, grain size and mineralogical complexity.
- Are there indications of the presence of clays, schists or other features which could affect the mining and milling of the ore?
- Metallurgical tests and comprehensive assays if available.

Geological and Locational Factors

- Location of deposit with respect to transportation, water and sources of power.
- A general description of the topography and the location of the deposit relative to the topography.
- The shape of the deposit.
- The range of possible grade-tonnage combinations which could exist in the deposit.

Marketing Factors

- Market availability.
- Market accessibility.
- Typical marketing arrangements.

PRELIMINARY CONSIDERATIONS

Once the basic information is obtained, the evaluator may proceed through the following steps:

1. Define the process. The basic geological information is usually sufficient to outline the recovery process in general terms including an estimate of the amount of grinding required to liberate the valuable minerals.

2. Select a mining method. The basic information provided by the geologist together with the estimated plant size should be sufficient information to select a mining method. In making this selection, it is not the specific method which is important but the dilution factors, the costs associated with it and the amount of total reserves that will be recoverable by the method.

3. Select a plant size. The initial selection of a plant size is based primarily upon the tonnage of the recoverable ore reserves; however, preliminary consideration should be given to the following possible constraints:

   - physical characteristics of the orebody
   - availability of markets
   - availability of capital
   - plant location
   - environmental factors
   - government incentives
   - company policy.

   It is usual for the first five constraints to limit the maximum size of a plant; however, they may, in certain circumstances, impose a minimum plant size. For instance, it is virtually impossible to sell iron ore unless there is a considerable amount for sale. Within the range set by these constraints, the optimal plant size will be determined by the grade and tonnage of the deposit together with the economic factors developed in the analysis. For initial calculations, select a plant size which will yield a mine life of 10-15 years without violating the above constraints. The effect of mine life on profitability can readily be determined once the evaluator has obtained a greater familiarity with the problem.

ESTIMATION OF REVENUE

Calculation of the potential revenue from a property must take into account the following factors which can be derived from the basic information discussed previously:

Factor	Considerations
Mill Feed Grade	Reserve grade, grade of dilution, amount of dilution (mining method)
Mill Recovery	Metal content, grain size, mineralogical complexity, method of recovery

Factor	Considerations
Concentrate Grade	Mineralogical complexity, grain size, composition of the ore
Sales Agreement	Typical selling agreements are sufficiently accurate for preliminary evaluations. Care must be taken to ensure that the agreement is representative of current conditions and the product is being produced. In certain cases the location of the buyer and seller, the buyer's facilities and his ability to handle the product should be considered.
Metal Price	The price for a commodity may either be set by a contract or based upon the free market price at the time of production. In the latter case, forecasting a price will be difficult and subject to wide variations. The normal procedure in most evaluations is either to use a range of metal prices or to investigate the sensitivity of profits to charges from a "most likely" price. If the current price is used, some comment should be made regarding the relationship between the current price and the long-term trend.
Concentrate Shipping	Overland shipping costs, whether by truck or rail, can usually be estimated with reasonable accuracy and are not subject to rapid change. Ocean freight rates are much more variable, subject to rapid change and may be negotiable.

A few years ago the mill feed grade and metal prices would have been considered to be the least certain components of a revenue estimate. Recent economic and monetary conditions have compounded the difficulty in estimating metal prices through floating exchange rates and fluctuations in tariff and non-tariff barriers. Concurrently there have been substantial increases in treatment charges as the world's smelters began to conform to more stringent environmental controls and rapid escalation of ocean freight charges. These factors may play a very significant part in the viability of a project; yet, an estimation of long term average figures is extremely difficult. Procedures for handling these uncertainties will be discussed later in this paper.

ESTIMATION OF OPERATING COSTS

As in other areas of the evaluation, the level of accuracy in estimating operating costs should be consistent with the accuracy of the entire study. Although there is a continuum from pure guesses to detailed engineering studies, the level of accuracy in operating cost estimates falls into four main groups:

1. The educated guess. The estimate at this level of accuracy could be simply a guess based on the evaluator's experience; however, it is more common for the evaluator to consider the actual historical costs at a specific property. This could be

carried further by considering average costs at properties which exhibited similar characteristics and plotting a graph of costs versus plant size. Unless the property being evaluated is very similar to another operation, it is improbable that the level of accuracy will be greater than plus or minus 20 per cent. There are two basic reasons for this. First, even if the two properties appear to be the same on the surface there are likely to be small differences which, in total, could make a significant difference in the cost. Second, it is very seldom that two mining companies report their costs on the same basis. Furthermore, much of the best data which could be available is from large operating companies with a number of mines, however, these companies seldom publish their detailed cost information. Therefore, the cost information which is available for this type of analysis is usually from one-mine operations and the smaller mining companies. This can lend a considerable bias to the results.

2. Historical - component costs. Rather than take the total costs for a particular operation or group of mines it is often possible to break these costs down into their components. This type of breakdown can lend considerably more accuracy and understanding to the analysis. At the first level of accuracy within this group one would, as a minimum, separate the costs into mining, milling and overheads. This can be extended further to breakdowns either by plant subsection or into groups such as operating labour, operating supplies, repair labour, repair supplies, and power. The most common procedure has been to estimate costs by plant subsection; however, it is often easier and more meaningful to estimate the operating labour, etc., for each major activity.

3. Engineering - component estimates. At this level of accuracy the historical cost base is abandoned and first attempts are made to perform a preliminary engineering estimate. This type of estimate would involve estimating the manpower requirements (staff and hourly paid), a rough layout of the mining operation and the plant, and preliminary assessment of equipment and operating supply requirements.

4. Detailed engineering studies. At this level of accuracy the estimate would be made after detailed engineering studies had been completed and the mine design finalized. At this point in time metallurgical testing will have been completed and it will be possible not only to estimate labour requirements quite accurately but also to estimate the supply costs with sufficient accuracy that the overall estimate should be within plus or minus 5 per cent to 10 per cent depending on the size and complexity of the project.

In most evaluations of exploration projects the fourth category is far too detailed. On the other hand, there is often sufficient information available to remove the estimate from the educated guess category and to perform the cost estimate using either the second or third level of accuracy.

In many instances, the amount of information available to the evaluator makes it mandatory to use a combination of the above methods. Frequently the evaluator has a reasonable amount of information about all the factors except one or two. The estimate should be carried out to a level of accuracy consistent with the overall information rather than the least accurate factor.

The construction of the operating cost estimate, as in the development of a revenue estimate, requires that the evaluator knows the relationships between the basic information and the factor being estimated. The following is a list of the factors which should be considered in estimating milling costs. The most important factors are underlined.

Operating Labour	= f	(type of process, plant size, number of products produced, wage rates including fringe benefits, complexity of ore, metal content, ore hardness, layout of plant)
Repair Labour	= f	(type of process, equipment size, wage rates, layout of plant)
Operating Supplies		
(i) Grinding steel cost	= f	(ore hardness, grain size, abrasiveness, location, cost of steel)
(ii) Reagent cost	= f	(metals recovered, metal content of ore, grain size, ore complexity, interfering minerals)
(iii) Other supplies	= f	(type of process, method of shipping product and disposal of tailings)
Repair Supplies	= f	(type of process, equipment size) As a rule of thumb, repair supply costs equal repair labour costs.
Power	= f	(ore hardness, grain size, power source, location, plant complexity.

A similar analysis may be performed for various types of mining methods. In general, mining costs lend themselves to analysis by activity rather than type of cost. For example, open pit costs could be subdivided into drilling, blasting, loading and hauling for both ore and waste. Underground mining costs are more difficult to estimate because the shape of the deposit, rock conditions and many unforeseen factors can affect development and production costs. Furthermore, the absolute size of mining costs in an underground mine means that this estimate is often the most important one in the entire estimate. An error of plus or minus 20 per cent on a $5.00 per ton mining cost has far greater impact on project economics than the same percentage error in milling costs of $1.00 per ton.

Another cost which must be considered is general overhead; that is, the costs incurred by the operation which are not directly related to the mining or milling of a ton of ore. This does not include plant supervision, maintenance, heating, ventilation and so on which are plant overheads and should be allocated directly to the operation concerned. It does include general administration expense, taxes, insurance, safety and a host of other items which cannot be realistically allocated to specific opeating codes. Because of the difficulty in estimating these costs, it is perhaps fortunate that they are usually a small portion of the total cost.

ESTIMATION OF CAPITAL COSTS

The estimation of the capital investment required for a mining project is one of the most difficult and most error-prone which the evaluator makes. The initial capital investment is usually composed of two parts: 1) the pre-production capital investment which is the amount of money needed to provide the physical facilities, and 2) the working capital investment which is a revolving fund of money kept available for plant operation. The principal characteristic of pre-production capital costs is that it is a one-time investment. A building, a piece of equipment or a complete plant needs to be paid for only once. When the item either wears out or becomes obsolete and has to be replaced it gives rise to a third kind of capital cost known as sustaining capital investment.

PRE-PRODUCTION CAPITAL

Except for the most preliminary studies, an estimate of the pre-production capital will require some degree of mine planning and plant design. For open pit mines without any unusual complexities, an estimate of the tonnage of waste to be removed prior to production, together with estimates of the tonnage of ore and waste to be moved during operations, is normally sufficient to determine equipment requirements for preliminary evaluation. Later evaluations should include scheduling of equipment. Equipment costs can be readily obtained from manufacturers. Mine development expense (pre-production stripping) is readily estimated using the operating costs for waste removal.

In order to estimate the capital requirements for an underground deposit it is usually necessary to make preliminary layouts of the main haulage routes and estimate both the approach to developing the deposit and the amount of development required to enable the deposit to sustain production at the required tonnage rate. The expected skill of the operating and managerial personnel along with the expected quality of maintenance work will play an important part in deriving both the capital and operating costs for an underground mining operation. In general there are more significant unknowns in an underground mine with the result that, although more time and expertise are applied to the estimates, their reliability will be less than for open pit operations. Because the cost of developing an underground mine is normally a larger percentage of the total cost than the development of a pit, the effect of the estimate on the viability of the project is more significant.

The other components of the capital cost estimate, namely the concentrator and ancillary facilities, will be discussed by other speakers. The estimation of plant costs is critical to most evaluations since the surface plant normally is a much larger component of the total investment than the mine. This is particularly true of deposits amenable to open pit mining. Estimates based upon the historical cost of similar sizes of plants can be in error by as much as plus or minus 50 per cent, although the error will be less if the plant is quite similar to newly constructed plants treating similar ores in similar locations. Since a potential error of this magnitude would make most evaluations meaningless, it is usually necessary to make a more detailed estimate. Developing a rough layout of the plant and sizing the major items of equipment allows one to apply cost-capacity relationships to the various components of the plant. Further improvement can be attained by obtaining actual costs from suppliers. The procedures used in making these estimates have been developed to a high degree of sophistication by the chemical industry and cost engineers in consulting firms. The interested reader should consult the reference material in the bibliography or any good text in cost engineering. It should be noted that, despite the implied accuracy of the procedures, the overall accuracy of the estimate is likely to be in the order of plus or minus 25 per cent. In order to attain greater accuracy reliable metallurgical results, detailed plant layouts, accurate equipment sizing and detailed cost estimates would be required.

In order to complete the estimates in the form required for cash flow analysis, it is also necessary to determine the capital cost allowance or depreciation classes and the timing of the pre-production expenditures. As a general rule, 10 per cent of the total investment will be spent in the first quarter of the construction period, 20 per cent in the second quarter, 30 per cent in the third and 40 per cent in the fourth. There will, of course, be large variations from this if lengthy construction items such as roads or railroads must be completed prior to building the plant.

Working Capital

The estimation of working capital is an important part of the evaluation since it represents funds which must be raised prior to production, yet it is often neglected in economic evaluations.

For financial analysis, working capital is defined as cash which must be advanced by the company to get the project started and meet subsequent obligations as they come due. Consequently, working capital is considered to consist of the following:

- cash laid out to purchase an inventory of operating supplies and spare parts prior to startup
- cash required to pay wages and other operating expenses in the period between startup and the first receipt of smelter settlements.

Stores inventory cost usually is assumed to be expended in the quarter immediately preceding the startup point. At the end of the life of the project, a proportion of stores will be recovered as cash, mostly by consumption in normal operations and partly by sale if the remaining inventory can be sold. There is little precedent for estimates of this kind but because working capital recovery occurs at the end of the project errors in these estimates are usually insignificant.

The working capital needed to finance operations until revenue is received requires an estimate of the time lag between start of production and payment of sales. Except in unusual circumstances, this outlay takes place within the first year of production. The sales time lag causes revenue to continue past the end of production until the last settlement is made. The overall effect is to shift revenue receipts down the cash flow scale relative to operating costs by the amount of the time lag.

Subsequent changes in working capital due to changes in the cost of the concentrate inventory may need determination if they significantly affect the estimate of operating profit.

Sustaining Capital

A part of the cost of operating a mine may be defined, for tax purposes, as capital expense. This includes such items as the replacement of equipment, a major underground development programme or deepening of a shaft. These factors would all be estimated separately from either the operating cost or the capital cost and would be called "sustaining capital." As the name implies this is the expenditure required later in the life of the mine to keep the operation going. Generally, sustaining capital expense would not start until the first major piece of equipment had worn out. This might be anywhere between the third and fifth year of operation. Furthermore, because this is a sustaining investment it should be terminated several years before the end of the mine life.

In most instances, the sustaining capital is a small but significant portion of the capital investment. The size of this investment depends to a large extent on the type of operation being considered. In open pit mines, the replacement of mobile equipment is a major sustaining capital expense and can amount to an appreciable portion of the operating profit.

TAX STRUCTURE

In order to complete the evaluation, information on tax structure and tax rates must be obtained. In countries which have a history of stable tax policies and where mining is an economically minor part of the economy, the present legislation is the best guideline to future taxes. Problems will occur in countries such as Canada where federal legislation has recently been changed but provincial legislation has yet to be adjusted. In other areas of the world, taxes may be negotiable either independently or in conjunction with transportation and power agreements. Since concessions are the rule rather than the exception in these areas, a common practice is to evaluate the property using the formal tax structure. If it is economic then it will certainly be economic at the negotiated rates. If uneconomic, alternative rates should be evaluated. Providing the property could be attractive at reasonable rates, discussions should be entered into with the State. Not only will it be to the company's advantage to keep them informed but also it will reduce the time required to finalize the tax agreement should a mine be found.

Because profitability is not overly sensitive to minor changes in tax rates, detailed tax calculations will not normally be required until the project reaches the feasibility stage. The intricacies of provincial or state taxes can frequently be estimated as a flat rate without introducing significant errors. Federal taxes, which are usually the largest tax payments, are frequently simple calculations so that accurate calculations can be made without excessive work being required. Tax calculations for feasibility studies should incorporate all the details of the tax structure so that the overall evaluation is as accurate as possible.

ANALYSIS

Once the components of the revenue and cost estimates have been obtained, the evaluator should work out several cash flow projections using the "most likely" estimates for all components except the most critical one, which is varied over a range. In most cases, the price of the product is varied not only because it often has the greatest effect on profitability but also because it is one of the most uncertain variables and is independent of the project. These preliminary calculations will provide the evaluator with a basic understanding of the problem and allow him to deduce whether the property is

- economic at all metal prices considered
- economic at certain metal prices
- uneconomic at all metal prices considered.

If the project falls into the last category the analysis is usually complete at this point although further calculations might be performed varying the second most important and uncertain variable, usually reserve grade. If it becomes economic under conditions of metal price and feed grade which have been stated a priori to be realistic, the project would move into the "possibly economic" category.

All projects which are potentially profitable should be analyzed further to determine the minimum conditions required for profitability and the sensitivity of profits to either errors or changes in the cost and revenue components. If the project is definitely profitable with all factors at their "most likely" points, the analysis of more favourable conditions should be confined to no more than the one or two most important parameters.

Management is far more interested in the factors which may reduce the rate of return from 15 per cent to 13 per cent than in the factors which increase it from 25 per cent to 30 per cent.

There is no firm methodology for the analysis of potentially profitable projects as each one is, in some way, unique and different; however, there are certain procedures which are either common to them or occur frequently enough to be considered as standard techniques. The remainder of this paper will briefly discuss some of these procedures.

Reduction of Possible Errors in Estimates

Each figure used in an economic evaluation is a forecast of future events and consequently cannot be made with certainty. If sufficient time and money are spent in deriving the estimate, the resulting figure may contain a low degree of error but uncertainty still exists as to what the actual figure will be under operating conditions. Preliminary studies, however, must be made promptly and inexpensively with less information, and consequently contain larger probable errors. One method of reducing the magnitude of error is to make unbiased estimates of the components and use these estimates to derive the required figure. For example, developing a smelter schedule and converting the deductions into cents per pound of metal at a certain concentrate grade and moisture content will yield a better estimate (other things being equal) than assuming a rule of thumb deduction to determine net smelter returns.

A superior approach is to estimate both the most likely value and the range of values for each component and then use these figures to determine the range for a major factor in the estimate. Although the range of the sum is not equal to the sum of the ranges it is readily determined and should be calculated since it can significantly improve the accuracy of the analysis. This approach, which is illustrated below, is most useful in developing capital and operating cost estimates.

For example, let us consider the following estimates which are assumed to be normally distributed and independent:

Mining costs	-	\$5.00/ton ± \$1.00 (i.e. ± 20%)
Milling costs	-	\$1.00/ton ± \$0.10 (i.e. ± 10%)
Overhead costs	-	\$1.00/ton ± \$0.30 (i.e. ± 30%)

Assume that the estimates have been made so that the costs fall within the above ranges 90 per cent of the time (Z = 1.64):

Cost Item	Range (1.64 Std. Dev.)	Std. Dev.
Mining	1.00	0.61
Milling	0.10	0.06
Overhead	0.30	0.18

The standard deviation of the total operating cost is

$$\sqrt{(0.61)^2 + (0.06)^2 + (0.18)^2}$$

$$= 0.65$$

The 90 per cent range is

$$\$7.00 \pm 1.64\ (0.65)$$
$$= \$7.00 \pm \$1.07$$

Therefore, the range of operating costs is \$7.00 ± \$1.10 (after rounding) rather than the \$7.00 ± \$1.40 which would have been obtained by simple addition. The range of probable error has been reduced from ± 20 per cent to ± 16 per cent. Using more components reduces the margin of error even further. For example, if there are 100 components of \$0.10 ± 0.05, the total is \$10.00 ± 0.50 (5%) rather than \$10.00 ± \$5.00 (50%). This procedure should be followed in any estimate involving the addition and subtraction of variables when normal distributions and independence are reasonable assumptions.

Components which must be multiplied can be handled using the equations:

$$U_{xy} = U_x U_y$$
$$\sigma_{xy} = \sqrt{(\sigma_x^2 + U_x^2)(\sigma_y^2 + U_y^2) - (U_x U_y)^2}$$
$$U_x = \text{mean value of } x$$
$$\sigma_x = \text{standard deviation of } x, \text{ etc.}$$

The entire approach illustrated above has been criticized because it assumes normal distributions when most things are not normally distributed. While the non-normal argument is a valid one, the size of error introduced into evaluation by assuming normal distribution will usually be minor relative to the overall risks and uncertainties. Detailed studies requiring greater accuracy and, presumably, based upon better estimates of range and distribution, should use Monte Carlo simulation to arrive at accuracies which are consistent with the overall study.

Changes in Plant Capacity

As mentioned earlier, the initial evaluation should be performed on the basis of a plant size which will yield a 10- to 15-year mine life for a certain reserve tonnage, usually the "most likely" one. The effect of moderate changes in reserves on the project's profitability can be determined by changing the mine life; however, large changes in the reserve tonnage would indicate a change in the assumed plant size.

Alternatively it may be desirable to change the plant size mine life relationship either as an independent exercise in the analysis or to obtain an optimum size for preliminary design purposes. In either instance, the evaluator will be changing costs and the timing of profits. Unless the amount of dilution (mining method) is changed, the revenue per ton will be constant.

Capital and operating cost estimates can be derived from the initial estimate by using size exponent factors. For example, the capital cost of a plant can be estimated from

$$\left(\frac{C_1}{C_2}\right) = \left(\frac{P_1}{P_2}\right)^x$$

where

C_1 = cost of Plant 1

C_2 = cost of Plant 2

P_1 = production rate of Plant 1

P_2 = production rate of Plant 2

x = exponential factor

The size of x for an entire plant is usually between 0.6 and 1.0 depending upon whether the expansion is accomplished by increasing the size of the equipment or by adding additional units of the same size. A factor of 0.7 is commonly used as an average. Because the size of the factor has a great impact on the resultant cost, the factor should be applied at the most detailed level possible. If a preliminary equipment list is available, the factors should be applied at this level.

The variation of plant operating costs is more complex and requires a breakdown into their components. The following exponents are generally applicable but will vary for the same reasons as the capital cost factors.

Item	Exponent
Mill Operating Costs	
Operating labour	0.4
Operating supplies	1.0
Repair labour	derived from capital costs
Repair supplies	derived from capital costs
Power	1.0
Working Capital	same as operating cost average
Overhead	0.4 (?)

Mine equipment costs per ton of production will not vary appreciably with moderate changes in tonnage. A factor of 0.8 to 1.0 is often used.

Preproduction development should be considered separately. The appropriate factor will depend on the shape of the deposit and the mining method. Mine operating costs should be divided into those direct costs which do not vary with the production rate and the indirect costs where a 0.4 exponent is applicable. When evaluating underground mines, the evaluator should continually review the feasibility and desirability of certain mining methods and the ability of the deposit to produce the required tonnage.

Sensitivity Analysis

The investigation of the effect of changes in one factor on the profitability of the project as a whole is called sensitivity analysis. Despite the many factors upon which the economic analysis is based, sensitivity is a simple exercise since profitability is only a function of mine life, operating profit, the amount and timing of the initial capital investment, and the tax structure. Since all the components of the revenues and operating costs can be resolved to a profit per ton figure, three cash flow calculations at various operating profit levels are sufficient to develop an operating profit vs profitability graph from which the sensitivity of these factors can be obtained. The capital costs or taxes can be handled in a similar manner.

Experience has shown that the sensitivity analysis and the answers to many other questions can best be handled if the profitability of the project is expressed in dollars rather than a payback period, d.c.f. rate of return or return on investment. Thus, present value is the preferred profitability criteria from the viewpoint of the analyst although the d.c.f. rate of return is also satisfactory. If necessary, the results may be transposed into a different format for management.

In the writer's opinion, a well conceived and well executed sensitivity analysis is an invaluable tool in evaluating any investment.

Simulation and Risk Analysis

Monte Carlo simulation is a technique for determining the expected variability in one factor on the basis of variability in other factors. The general approach is to describe the probability distributions of the component factors, sample the distributions in a random manner and calculate the result. If repeated a large number of times, a profitability distribution of the results can be plotted. Although extremely tedious and time-consuming to perform manually, simulation can be done quickly and cheaply on a computer and can handle any type of probability distribution or complex calculation. For example, a series of 200 cash flows with 10 variables can be produced in 3 to 4 minutes of computer time.

The speed and versatility of Monte Carlo simulation make it an invaluable analytical tool. The areas of application favoured by the writer are as follows:

- estimating the range of capital and/or operating costs from a large number of variables, i.e. using simulation in place of the procedure previously described which was based upon normal distributions

- estimating components of the revenue calculation which are computationally complex

- illustrating the range of potential profitability such as might be desirable to accompany a financing proposal.

The application of simulation in these areas, where the basic problems and decisions are well defined, will provide valuable information to both the analyst and management. The only qualification in its use is to ensure that dependent variables are properly represented in the simulation model.

Simulation models have also found application in the evaluation of potential profitability where there is considerable uncertainty. This type of study, commonly called risk analysis, can be valuable in analyzing certain types of problems but does not warrant the wide acclaim that it has received in technical literature. The limitations in using simulation as an analytical tool are as follows:

- simulation with a large number of variables cannot demonstrate the effect of any one variable adequately. Thus the "sensitivity analysis" aspect of the analysis is lost and the effect of critical variables may be masked by other factors
- there is a tendency to forget that the results of the simulation model are only as good as the design of the model and the input data. Failure to consider the dependence of certain variables while designing the model or using it for unsuitable applications can result in significant errors which are difficult to detect due to the masking effect of the variables. Similarly, incorrect input data cannot be readily identified.
- ranges will frequently be applied without a sound basis for the decision
- the output will frequently be accepted as correct without an adequate review of supporting data.

A further limitation, which is particularly prevalent in exploration projects, is that one or two factors (e.g. metal price and reserve grade) are of such great importance to profitability and have such great uncertainty that the simulation output may be too uncertain to be meaningful - for example, an output which indicates a rate of return of between zero and 50 per cent. One way around this problem is to assume that the most critical and uncertain variable is constant and run a number of simulations over a range of possible values. This approach is commonly used in exploration projects with the metal price being fixed for each run. Not only will the simulated profitability be in a narrower range for each case but also it removes the objection of the writer and others to assigning probabilities to a factor which is highly important, largely uncontrollable and, in many instances, unknown.

In conclusion, Monte Carlo simulation is a useful tool when properly applied to estimate risks. Its suitability to assess the degree of risk in profitability of a project depends upon the specific situation. Simulation cannot be considered as a substitute for sensitivity analysis.

Exploration Risks

The discussion thus far has been concerned with the evaluation of a deposit assuming that it exists. In most evaluations this condition is by no means certain. The determination of the value of the deposit prior to being certain that it exists requires that consideration be given to onward exploration costs and the probabilities of success at each stage. In general terms the problem may be stated as

$$EV = pR - C$$

where

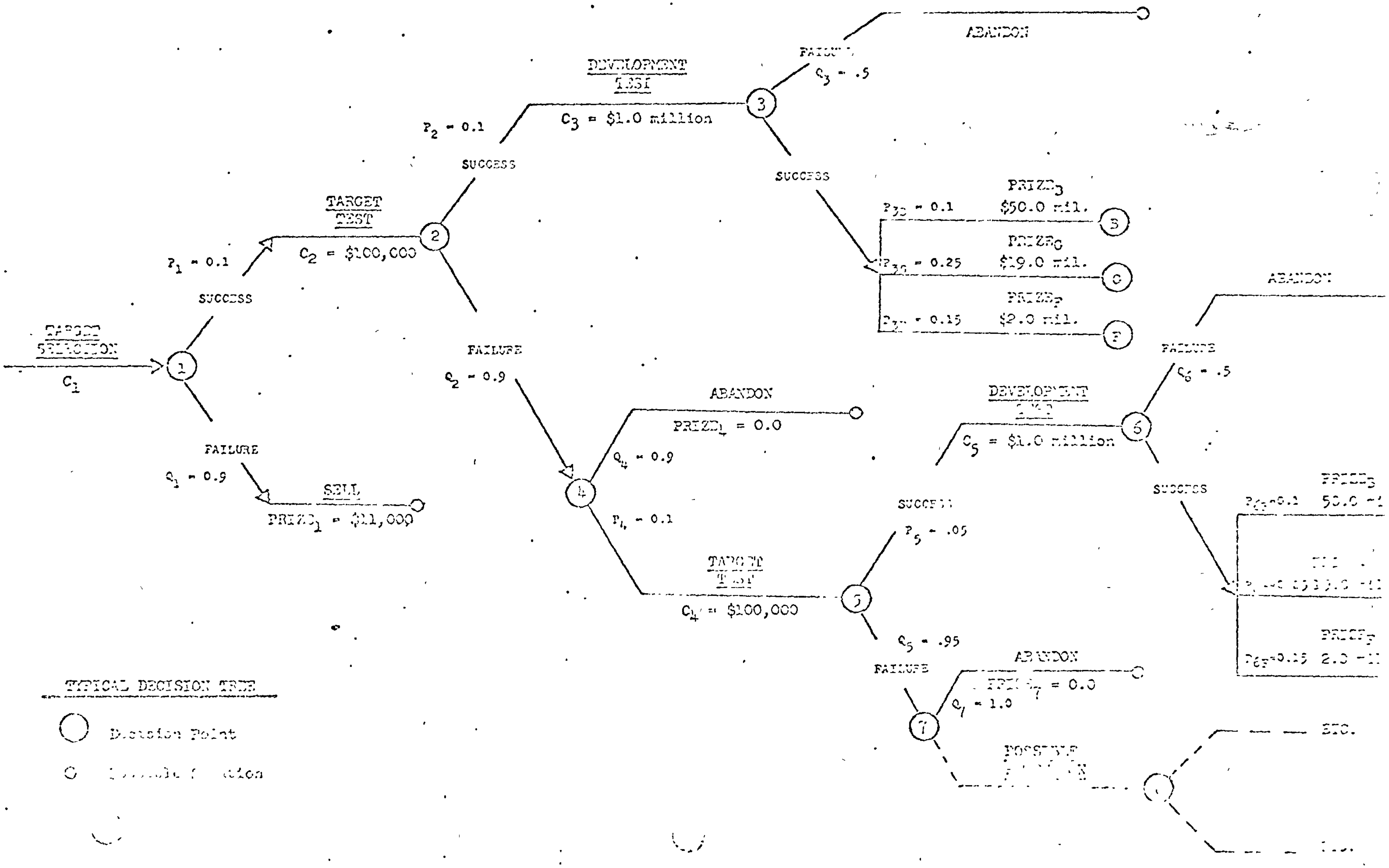
EV	=	expected value
p	=	probability of success
R	=	the present value of the net return given success in exploration
C	=	cost of onward exploration to the point of a production decision.

Since exploration is a sequential decision-making process and onward costs are only incurred if there is continued success, the equation becomes a more complex function. If the final "success" is defined in terms of what is anticipated to occur, "R" may be determined according to conventional evaluation techniques. The cost of each stage of exploration required to achieve success can also be estimated. The definition of success and the assignment of probabilities to the intermediate stages must be considered concurrently.

Unless the decision-maker has considerable experience in the specific geological environment, the definition of "success" and the estimation of probabilities will be highly subjective. Slichter, Brant and others have attempted to quantify the probability of success in certain regions where there is substantial historical information. The interested reader should consult the papers in the bibliography. In most instances there is neither the time nor the data to formulate quantitatively-based probability estimates; however, this does not mean that the exercise should be abandoned. It has been demonstrated in other industries that experienced personnel are good estimators of subjective probabilities and there is every reason to believe that the same is true of exploration managers. The exercise of thinking out the stages of a project and estimating probabilities will clarify one's understanding of the project and the economic implications of each step.

The approach described below is known as Network Analysis or Decision Tree Analysis and assumes that the probabilities are estimated with certainty. Another technique, Decision Theory, considers the reliability of the probability estimate but has not found extensive application due to the complexities of formulating and solving the problem.

An example of the Decision Tree approach is shown on the following page after it has been refined by eliminating the definitely unfeasible alternatives. Once the network has been developed and the costs, returns and probabilities filled in, the expected value at the start (node 1) is determined by calculating the expected values of the farthest



TYPICAL DECISION TREE

right nodes (3 and 6) and working back through the network to the start. The initial calculations are

$$\text{EV "6" or EV "3"} = \text{Prize B} \times P_B + \text{Prize G} \times P_G + \text{Prize F} \times P_F$$

$$= (0.1 \times 50.0) + (0.25 \times 19.0) + (0.15 \times 2.0)$$

$$= 10.0 \text{ million}$$

$$\text{EV "5"} = (\text{EV "6"} - C_5) \times P_5 + (\text{Prize } 7 \times Q_7 \times Q_5)$$

$$= (10.0 - 1.0) \times 0.5 + (0.0 \times 1.0 \times 0.95)$$

$$= \$450,000.$$

Continuing this back results in EV "1" = \$93,000, which can then be compared to the estimated expenditure for the target selection process, C_1. Although, theoretically, projects should only be accepted if the expected return exceeds the cost, the primary practical application of the technique is to rate projects and reject the totally hopeless ones.

In view of the potential errors in assigning probabilities, projects which are determined to have slightly negative expected values will have some chance of being positive. Changing the first probability estimate to an optimistic one will demonstrate the degree to which the first (high risk) stage affects the values. Furthermore, exploration, like horse racing, is a business for risk-takers. Betting on the occasional long shot in conjunction with a number of lower risk bets will yield greater rewards than the low risk bets alone.

SUMMARY AND CONCLUSIONS

The preceding notes have attempted to illustrate the information requirements and problem areas commonly encountered in the evaluation of exploration projects. General comments have been made on certain aspects of the analysis of these ventures and the suitability of commonly applied techniques. No presentation short of a textbook can describe in detail the factors which must be considered and nothing except experience can be adequate for performing these evaluations.

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A Decision-Making Theory of the Mining Firm

By

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INTRODUCTION

The mining firm, for the purpose of this study, is defined as a decision-making unit the primary functions of which are associated with the exploitation of mineral resources.

The purpose of this study is to outline relationships between the mining firm's objectives, its resources, and environmental conditions. Three objectives are considered: profit, survival and growth. Firm resources consist of capital and skill. The mining firm's environment has two components: an exploration environment in which it searches for new deposits and a market environment in which it sells mineral products. The environment provides both opportunities and constraints for the mining firm. How should it allocate resources within this environment to insure the realization of objectives? This is the essence of a decision-making theory of the mining firm.

PROBLEM SETTING

Theory is developed to facilitate the analysis of real-world problems. The theoretical framework required depends on the problem being analyzed. Different problems have different needs.

There are two basic approaches to the firm. One approach was developed in conventional economic theory, the other in decision-making theory. Both approaches appear valid, each in relation to the problem area for which it was designed.

The concept of the firm in conventional economic theory has proven to be a valuable tool for the analysis of public policy problems: market conditions, industrial organization, taxation, commercial policy, etc. Within this approach a concept of the mining firm has been developed for specific application to public policy problems in the mineral industry.*

The firm in conventional economic theory is intended to add up to an approximation of the aggregate behaviour of firms in a particular industry. It is not intended to provide a realistic representation of the decision-making process and objectives of the individual business firm, an approach that could be used by management to guide decisions within the firm. Thus, there is also the need for decision-making theory for application to this problem area. It is this decision-making approach that the present study is concerned with and, thus, it is closely tied to decision-making literature.

It is useful to make a distinction between two aspects of decision-making theory. One aspect considers decisions related to the development and operation of an individual investment opportunity whether it be an industrial plant or a mine. This aspect of decision-making theory is closely related to the concept of the firm in conventional economic theory, a firm that is essentially an investment opportunity pursued for a profit maximizing objective. With respect to the mine these decisions include the determination of optimum rate of output and optimum cut-off grade.** The mining firm in conventional economic theory while somewhat abstract and not primarily designed for decision-making purposes, has made an important contribution to mine decision-making. The second aspect of decision-making theory considers the business firm as a decision-making unit and is essentially concerned with strategic decisions that lie beyond current operations and individual investment opportunities. Decision-making theories of the firm stress realism in both the objectives of the firm and in the decision-making process that is used to allocate firm resources to areas of environmental opportunity.*** These problems are not considered in literature on the mining firm in conventional economic theory. This aspect of decision-making theory is the focal point of the present study.

If a decision-making theory of the firm has been developed, why make a special case of the mining firm? Existing decision-making theories are essentially concerned with interactions between firm resources and market conditions, the critical problem for most business firms. However,

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* See for example concepts of the mining firm developed by Gray(12), Hotelling(14), Herfindahl(13), Gordon(11), and Scott(20).

** See for example the concept of the mine developed by Carlisle(5).

*** Important contributions to the development of a decision-making theory of the firm have been made by Penrose(18), Simon(22), Drucker(8), Williamson(25), Cyert and March(7), Ansoff(2), Chandler(6), and Marris(16).

the critical concern of most mining firms is the relationship between firm resources and exploration conditions. The critical problem is mine replacement. A decision-making theory of the mining firm should provide a more relevant framework for decision-makers in the mining firm. This is the basic rationale of the present study.

The study is intended to be useful in two other ways. Traditionally, spokesmen within the mineral industry have claimed that mining is a unique business. This argument will be tested. Also, it will be possible to examine the relationship between concepts of the mining firm in conventional economic theory and in this decision-making theory. It is important to determine whether or not the concepts are consistent.

The significant departure in the decision-making framework of the mining firm compared with that of other business firms is with respect to the problem of mine replacement. The mining firm's problem of mine replacement is the result of the certainty of exhaustion for currently producing mines coupled with the uncertainty of making new discoveries.* It is a problem of suvival. The mining firm is forced to participate in the uncertain exploration environment and cannot survive unless successful there. Of course firms outside the mineral industry may have survival problems as well but they arise from market uncertainties and are more obscure. That is, market uncertainties may endanger a firms current operations but, by definition, they do not limit current operations with certainty. Thus, there is not necessarily the same urgency for the development of new operations. Also, there is an obvious difference in the direction of the survival problem. The mining firm's survival problem is in the direction of raw material supply rather than in the direction of product markets. This has important growth implications for the mining firm.

A decision-making theory of the mining firm may be subdivided into three components: profit, survival and growth. Profit is the basic objective of the firm and, thus, is an important determinant of investment decisions. Survival is a most critical decision-making component for most mining firms because of the problems associated with mine replacement. The profit and growth of the firm beyond current operations are contingent on its survival. As the mining firm grows, relationships between firm objectives, firm resources and environmental conditions change. This dynamic aspect of mining firm development is explained in terms of the growth component.

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* It has been suggested that the mine replacement problem is similar to the machinery replacement problem with the difference being in the degree of uncertainty. This is misleading. The machinery replacement decision under conditions of rapidly advancing technology presents a timing uncertainty. The problem is to decide when a piece of machinery should be replaced. A number of alternatives will be available. This is akin to the mining firm selecting between a number of deposits at the development and production stages. The mine replacement problem is different; it is an exploration problem concerned with the discovery of alternatives.

PROFIT

Of the mining firm's three decision-making components profit has traditionally been regarded as the most important.

For decision-making purposes the term 'profit' is somewhat inadequate. Decisions are based on limited information. The expected results of decisions are not assured; they are uncertain and are realized, to a greater or lesser extent, in future time periods. Therefore, profit for decision-making purposes should be expressed in probabilistic terms. The profit component is, more correctly, a 'profit expectation' component. Profit expectation is held with a degree of uncertainty that reflects the information available at the time of evaluation.

The mining firm in the short-run is circumscribed by current operations. Short-run decisions are of a tactical or operational nature and are made within an established plant capacity. The decisions involve scheduling, sequencing, grade control, machinery replacement, and a host of other problems, normally considered on a routine basis. Such decisions are based on the evaluation of profit expectations for a number of technically feasible alternatives. For this purpose, operations research and investment evaluation techniques have been applied to the mining situation.

The long-run decisions of the mining firm are of a strategic nature, extending beyond the firm's current operations. The decisions concern the structuring of the firm's exploration effort and the development of productive capacity. Profit expectation plays an important role in these decisions. The long-run profit expectation of the mining firm is largely determined by its performance in the exploration environment. As the firm grows, market conditions become increasingly important.

The techniques used to measure profit expectation depend on the nature of the investment that is being made. Firstly, exploration investment may be regarded as an investment in ' information gathering'.* At this stage, information is very incomplete and the quantification of profit expectation is difficult. Certain methods for evaluating profit expectation are suitable for application under these conditions. Secondly, the mining firm invests in the 'development of productive capacity'. Such investment decisions include the development-to-production decision for a mineral deposit, the decision to develop a forward processing opportunity and the decision to expand existing plant capacity. At the development stage information is reasonably complete and thus, the application of more precise methods for evaluating profit expectation is facilitated. The techniques that are applied are derived from capital investment theory.

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* Exploration investment is similar to investment in research and innovation activities. The objective in each case is to gather and direct information towards a 'discovery'.

Profit Expectation and Exploration Decisions

The long-run profit expectation of the mining firm is directly related to anticipated exploration success. The sensitivity of the mining firm's exploration investment decisions to profit expectation depends on the degree to which the firm is able to insure the realization of profit expectation. This depends on the relationship between firm resources and environmental uncertainty. In situations where resources are large in relation to uncertainty, strategies that increase profit expectation will be critical. In cases where resources are relatively small, uncertainty reducing strategies will also be important.

The exploration investment process can, for illustrative purposes, be characterized by

$$PE = pR - qC,$$

where PE = profit expectation,
p = probability of an exploration success (environmental uncertainty),
R = net return given an exploration success,
q = probability of an exploration failure = $(1-p)$,
C = cost of creating an exploration opportunity (investment).

Profit expectation is the average value that the exploration environment will yield in the long-run, balancing the successes and failures of a large number of investments. Usually a profit expectation greater than zero is regarded as a necessary condition for investment. (19,p.59). However, it is possible that in some cases a 'lottery strategy' may guide investment in mineral exploration. If the return given a success is very large in relation to a single investment, a condition normally fulfilled in exploration, an investor might undertake at least a limited number of investments, even if the profit expectation is negative, because of the possibility of a spectacular return. Since such a strategy is ultimately ruinous, it can only be pursued for a limited period when the investments made are not a critical portion of total income, or when the investor is desperate or ignorant. In the long-run, profit expectation must be greater than zero with higher profit expectations indicating more preferable investments.

Environmental uncertainty is measured by the probability of an exploration success. Success, in the context of the profit expectation equation, is the realization of the net return R. The net return is 'net' in the sense that the exploration investment, C, has been deducted. Realistically, net return should be expressed in discounted terms to account for time value. The cost of creating an exploration opportunity is the investment required to gather the information necessary to make the decision as to whether or not the opportunity will be a success.

The profit expectation equation has two important limitations. Firstly, it does not consider the sequential nature of the exploration process. An obvious consequence of the uncertain exploration environment is that of the many investment opportunities initially pursued, few ultimately prove successful. The sequential investment process limits the cost of unsuccessful opportunities by breaking down the investment in an exploration opportunity (C) into a number of smaller components each of which is associated with a particular sequential investment stage.

The cost of unsuccessful opportunities is reduced when they are determined to be unsuccessful before the whole investment has been expended. In determining the optimum number of stages for an exploration opportunity and the investment placed in each stage, benefits in terms of limiting the cost of failures must be balanced against evaluation costs and technical and information limitations. Generally the greater the uncertainty, the larger the number of sequential stages that are warranted.

Secondly, the variables in the profit expectation equation are assumed to have certain discrete values. In reality, the exploration environment is characterized by continuous distributions of possible values for each variable. The values used in the equation might be assumed to represent the modal values of such arrays. While these assumptions severely limit the applicability of the relationship for decision-making purposes, the profit expectation equation is, nevertheless, a useful concept for illustrating exploration conditions.

An important characteristic of the exploration environment is high uncertainty; i.e. the probability of success for individual exploration opportunities is low. Under these circumstances the mining firm with limited resources is unable to insure the realization of profit expectation. This introduces a survival element into the firm's exploration decisions. Thus, the mining firm's exploration strategy is guided by both profit expectation and survival considerations.

Profit Expectation and Development Decisions

Assuming that the mining firm's exploration programme has been successful, resulting in the discovery and detailed exploration of a number of mineral deposits, the firm must make decisions on whether or not particular deposits should be developed to production. Development decisions are made in two phases. An economically optimum alternative is selected from a number of technically feasible alternatives for each deposit. Then the economically optimum alternatives are compared and a number of them selected for development to production. Selection is constrained by the resources that the mining firm can draw on.

Development alternatives should be evaluated and compared on the basis of both profit expectation and uncertainty. Profit expectation is measured by the relationship between expected investment and returns. In general terms the decision paramaters are similar to those considered at the exploration stage. However, because information at the development stage is more complete, it is realistic and useful to employ more sophisticated economic evaluation techniques.

The measurement of profit expectation for a development alternative is based on the time distribution of expected investment and returns. Expected investment and returns are normally expressed as annual cash flow; returns minus investment for each year in the life of the alternative. Annual cash flow may be positive or negative depending on whether the firm on net balance is realizing a return or making an investment in a particular year. A compound interest rate is used to discount annual cash flows to a point in time, usually the present. This accounts for the time distribution of cash flows.

Two basic techniques are available for measuring the time valued profit expectation of a development alternative. One measure is the internal rate of return, the interest rate that equates discounted positive and negative cash flows. The alternative showing the highest internal rate of return has the highest profit expectation. The second measure is the net discounted cash flow ratio. An interest rate is assumed. Discounted positive and negative cash flows are determined. The net discounted cash flow is divided by the discounted negative cash flow (discounted investment) to obtain the net discounted cash flow ratio. The alternative showing the highest ratio has the highest profit expectation. Both evaluation techniques are widely used and there are intensive arguments in decision-making literature on their relative merits. The question of which of the two techniques to use is perhaps not so important as an awareness of the implicit assumptions that are 'built' into each.

The measuring of uncertainty for a development alternative may be in terms of an interest rate. This type of uncertainty measure may then be directly combined with the profit expectation measures which also embody interest rates. However, this technique is somewhat arbitrary. Some types of uncertainty, particularly geological uncertainty, are not time dependent and should not be measured by an interest rate. The Monte Carlo simulation technique, a more realistic method for measuring uncertainty, bounds profit expectation with a probability distribution which may be expressed as upper and lower confidence limits.

Uncertainty at the development stage is often sufficiently large to justify a sequential build up in the rate of production rather than initially installing the scale of plant that is optimal from a profit expectation viewpoint. Profit expectation must be balanced with uncertainty. If uncertainty at the development stage is high, an optimal economic balance between profit expectation and uncertainty may be achieved at a relatively low initial capacity. Also, development uncertainty may constrain initial capacity.by precluding access to external sources of capital. In any event, expansion only becomes justified as uncertainty is reduced.

SURVIVAL

The mining firm is unable to survive within the context of its currently producing mines. To survive it must successfully participate in the uncertain exploration environment. Under conditions of uncertainty, the application of limited firm resources cannot insure the realization of profit expectation and the mining firm may expend its resources without success. These conditions describe the survival problem of the mining firm.*

The application of survival concepts to a decision-making theory of the mining firm may be derived from two sources. Firstly, behavioural considerations of survival from decision-making theories of the firm may be related to the mining firm's survival problem.** Secondly, game theory may be applied.*** The game theory approach provides a useful framework for illustrating the relationships that exist between the mining firm's survival problems, its resources, and the exploration environment in which it searches. These three factors form the basis for developing survival strategies.

Survival and the Mining Firm

The classical probability problem of the gambler's ruin has been applied to the exploration strategy of the mining firm. Allais(1), Slichter(23) and Brant(4) all consider the application of a special case of the gambler's ruin problem to investment in mineral exploration, the probability of zero successes in n successive wagers. By this definition of ruin

$$Pr = q^n = (1-p)^n,$$

where Pr = probability of ruin = (1-probability of survival),
n = number of wagers,
p = probability of an exploration success (environmental uncertainty),
q = probability of an exploration failure = $(1-p)$.

Probability of ruin in this sense is the probability of a straight bad run streak broken by no successes. The mining firm may also be ruined after one or more exploration successes. In considering this larger probability of ruin, the net return given a success must be

* The exploration environment is characterized by the distribution of investment opportunities through a number of sequential stages of uncertainty. It may be worthwhile for the firm to buy into the exploration process at less uncertain stages and in this way improve its chances of survival. However, entry barriers increase as uncertainty is reduced. As firms see their chances of success increasing, they become increasingly reluctant to sell out to others. Therefore, the firm that does not participate in the more uncertain stages will have to pay a premium for its opportunities and/or accept residual opportunities with lower expectations.

** See, for example, survival considerations in the work of Winter(26), Marris(16,p.277), Drucker(9), and Galbraith(10,pp.166-178).

*** Survival concepts from game theory have been applied to insurance company strategy by Borch(3), dividend policy by Shubik(21), and problem solving strategy by Miller and Starr(17,pp.366-375).

considered in relation to the cost of creating individual exploration opportunities.

The classical problem of the gambler's ruin concerns a gambler with limited capital who wagers against a 'house' with essentially unlimited resources. The gambler is ruined and the game terminates if at any point his capital balance falls to zero. Survival is the complement of ruin. The gambler survives if his capital balance is maintained above zero. The problem is to determine the gambler's probability of ruin, Pr, which equals (1-Ps) where Ps is the gambler's probability of survival.

If the mining firm is to survive beyond the life of its current operations, it must search for and discover new mineral deposits. This activity is analogous to the gambler wagering on a chance device. The mining firm is the gambler, nature is the opponent, and exploration the chance device. The cost of creating an exploration opportunity is the wager. Each opportunity has a probability of success, in which case a return accrues to the firm, and a probability of failure, in which event the firm loses its wager. The firm applies some limited amount of capital to the search activity. What is the probability that it will survive?

A Survival Model

Uspensky develops a variation of the gambler's ruin problem that is suitable for application to the mining firm's survival problem. (24, pp. 143-147). Two opponents, A and B, play a series of games, the probability of winning a single game being p and q = (1-p) respectively, each game ending in a loss for one of them. If the stakes in a single game are C for A and R for B, and the players' respective fortunes are a and b, what is the probability that one of the players will be ruined, in the sense that at a certain stage his fortune is less than a single game stake? No limit is set for the number of games.

Assuming that nature has essentially unlimited resources and that the mining firm will create exploration opportunities indefinitely, what is the mining firm's probability of ruin?

Let: A = the mining firm,
- B = nature,
- C = 1 = stake of the mining firm = cost of creating an exploration opportunity,
- R = stake of nature measured in cost units = net return to the mining firm given an exploration success,
- p = probability of an exploration success (environmental uncertainty),
- q = probability of an exploration failure = (1-p),
- a = resources of the mining firm = capital allocated to exploration in terms of number of stakes (wagers) afforded,
- b = nature's resources,
- PE = profit expectation to the mining firm in cost units,
- Pr = probability of the mining firm being ruined = (1-Ps) where Ps is the probability of survival.

Assuming that the resources of the mining firm are large in comparison to its stake and that the profit expectation is positive for the mining firm then

$$Pr = \theta^{a},$$

where θ is the positive root <1 of

$$p\theta^{R+1} - \theta + q = 0.$$

Also, as previously described,

$$PE = pR - q.$$

Using the foregoing relationships, it is possible to hypothesize the survival conditions of the mining firm. Sample results are shown in Figures 1 to 3. An analysis of results from this model has important implications for the mining firm's exploration strategy.

Sensitivity of Probability of Ruin to Changes in Profit Expectation

Over a wide range of conditions, reducing profit expectation has little effect on increasing the firm's probability of ruin. The mining firm may be able to trade off profit expectation for a reduction in environmental uncertainty in order to reduce its probability of ruin to a tolerable level. If, for example, profit expectation decreases from 4.0 to 0.4, probability of success increases from .001 to .01, and capital is available for 250 wagers, then the firm's probability of ruin is reduced from 0.78 to 0.28.

Allocation of Financial Resources for Exploration

One factor determining probability of success is transferable between firms. It may be termed the 'theoretical probability of success', measuring the environmental conditions of mineral deposition in a region at any point of time in relation to the skills available for searching and finding deposits. This factor may be considered a theoretical maximum. The other factor is the skill of the individual mining firm relative to the skills available.

$$p = (\text{theoretical maximum } p)\ \frac{(\text{skill of firm})}{(\text{skills available})}$$

Within a given region, mining firms will encounter differing probabilities of success in their respective exploration programs depending on the skill each applies to the search.

An important factor in the exploration strategy of the mining firm is the allocation of financial resources for exploration between increasing number of wagers and increasing probability of success. The firm can allocate capital directly to individual exploration opportunities, and the more capital the firm applies, the greater the number of opportunities it can afford. On the other hand, the firm can apply financial resources to improving the skills available for its overall

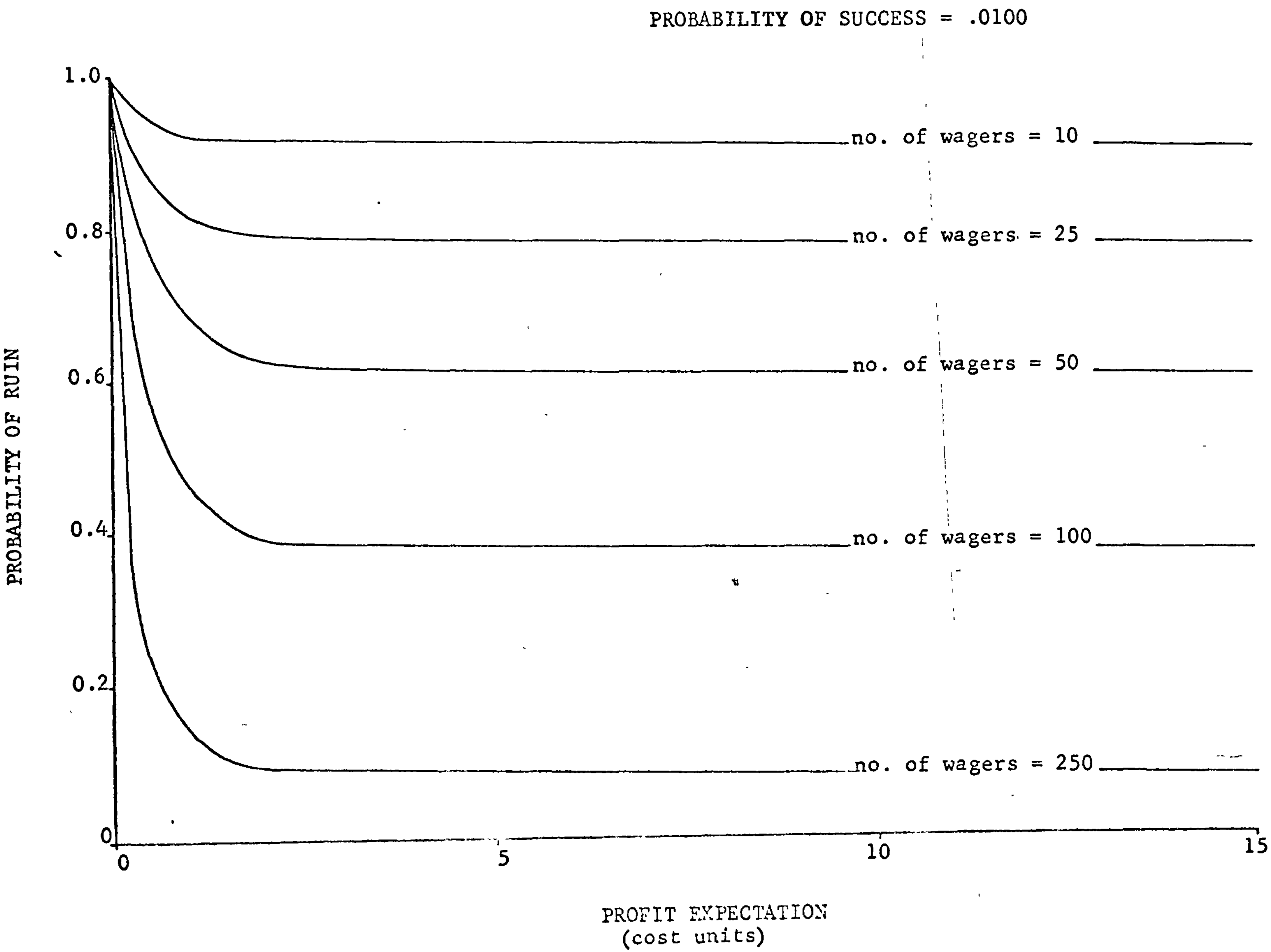

PROBABILITY OF SUCCESS = .0100
PROBABILITY OF RUIN
1.0
0.8
0.6
0.4
0.2
0
no. of wagers = 10
no. of wagers = 25
no. of wagers = 50
no. of wagers = 100
no. of wagers = 250
0
5
10
15
PROFIT EXPECTATION
(cost units)

Figure 2

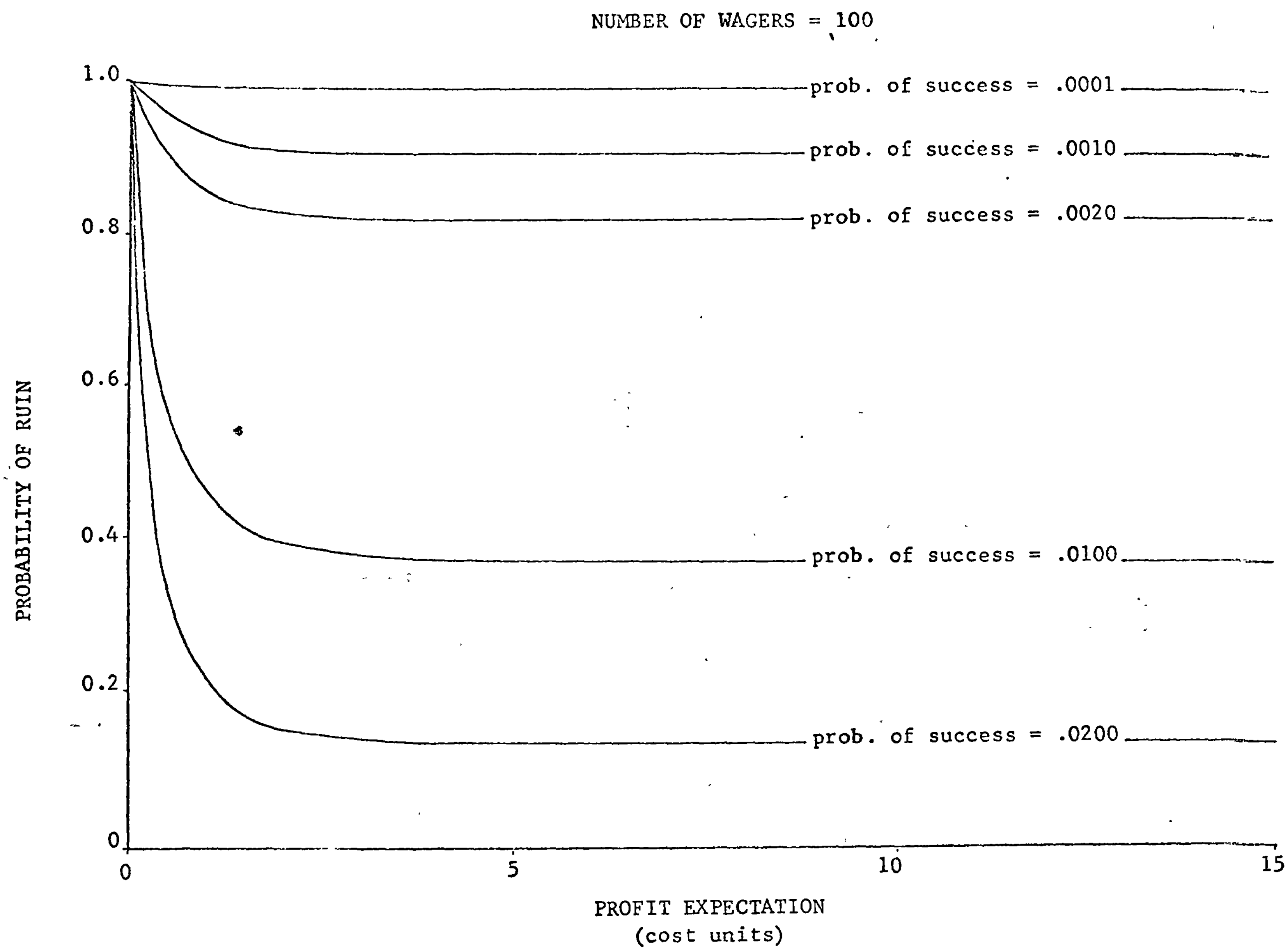
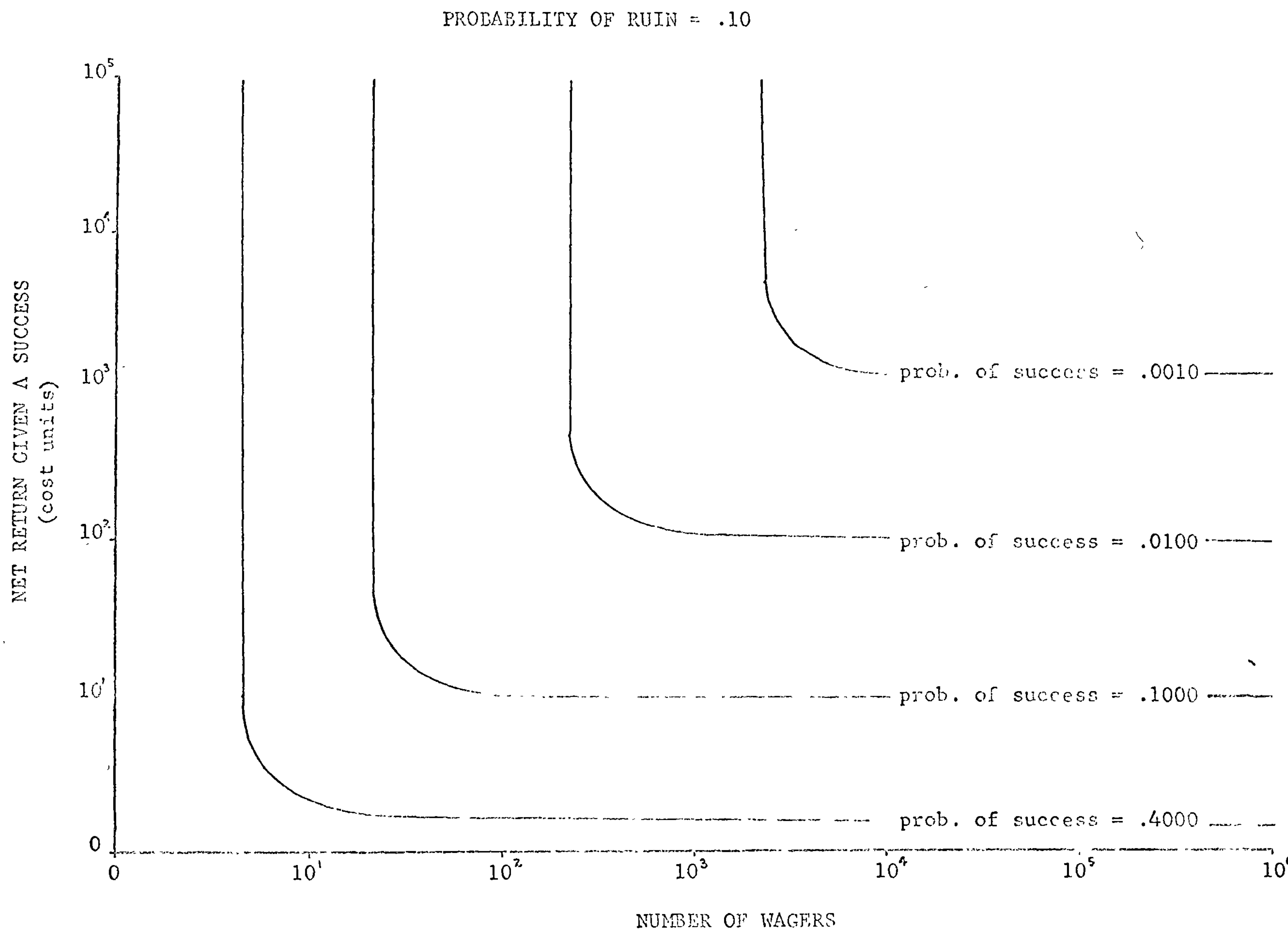

NUMBER OF WAGERS = 100
prob. of success = .0001
prob. of success = .0010
prob. of success = .0020
prob. of success = .0100
prob. of success = .0200
PROBABILITY OF RUIN
1.0
0.8
0.6
0.4
0.2
0
0
5
10
15
PROFIT EXPECTATION
(cost units)

Figure 1


PROBABILITY OF RUIN = .10
NET RETURN GIVEN A SUCCESS (cost units)
10^5
10^4
10^3
10^2
10^1
0
prob. of success = .0010
prob. of success = .0100
prob. of success = .1000
prob. of success = .4000
0
10^1
10^2
10^3
10^4
10^5
10^6
NUMBER OF WAGERS

exploration program and in this way increase probability of success. From a survival viewpoint, the optimum balancing of financial resources for exploration will minimize the mining firm's probability of ruin.

Probability of Ruin and Environmental Uncertainty

Variations in environmental uncertainty have a pronounced effect on the firm's survival problems (see Figure 1). As environmental uncertainty increases, the firm's decision-making process becomes increasingly sensitive to survival conditions. To the extent that the mining firm encounters relatively high environmental uncertainty, it has more serious survival problems than other firms. And mining firms that participate in the most uncertain stages of exploration have more intense survival problems than those that are able to buy in at later stages (assuming no entry barriers at less uncertain stages).

Probability of Ruin and Firm Size

The survival problem is directly related to the mining firm's resources and, thus, to firm size (see Figure 2). If probability of success in the exploration environment can be assumed in the order of .001 to .01, then survival will be a critical problem for most mining firms and an important decision-making problem for large mining firms.

The Mining Firm's Survival Strategy

The survival objective of the mining firm may be expressed in terms of a probability of survival confidence limit. For example, Figure 3 shows the combinations of the three exploration parameters -- environmental uncertainty, number of wagers, and net return given a success -- necessary to realize a probability of survival confidence limit of 0.90 (probability of ruin = 0.10).

The asymptotic values for individual curves have important implications for the mining firm's survival strategy:

1. Given environmental uncertainty and survival confidence, there is a net return value below which no amount of capital will suffice. This net return value represents a profit expectation of zero.

2. Given environmental uncertainty and survival confidence, there is a critical number of wagers below which no net return value, however high, will suffice.

Assuming that two of the three exploration parameters have been predetermined, the firm's survival strategy centers on one of the following questions:

1. What level of environmental uncertainty should the firm seek to fulfill its survival objective?

2. What changes in net return given a success would be required to balance a change in environmental uncertainty in order to maintain the firm's survival objective?

3. How much capital in terms of number of wagers should the firm direct to exploration activities to realize its survival objective?

Answers to the above questions may be obtained from the survival model. However, they represent theoretically optimum values and it should be realized that in practical terms they may not be attainable. Generalized answers are as follows:

1. High environmental uncertainty is most suitable for large firms. Small and medium size mining firms should ideally search in the less uncertain areas of exploration opportunity. Entry barriers and differences in uncertainty preference normally distort such a distribution of exploration opportunities.

2. As environmental uncertainty increases, there must be a compensating increase in net return given a success to induce firms to participate.

3. Larger amounts of capital and numbers of wagers are required under conditions of higher environmental uncertainty.

To this point it has been assumed that exploration is financed by individual firm resources. Firm resources may be insufficient to insure the realization of its survival objective or the cost of such a strategy may be prohibitive. If the mining firm does not consider survival by internal means feasible, then it will have to insure its survival externally, by pooling its resources with those of other firms. Joint ventures are commonly used in mineral exploration, primarily as a survival strategy.

The ratio of firm resources available internally to the resources required to attain the survival objective represents the joint venture participation that the mining firm should seek. The cost of such a strategy is the diluted control that the firm exercises over any resulting successes. Sample figures are given in the following table.

Pr	PE	p	R	Resources Available: no. of wagers	Resources Required: no. of wagers	Optimal Joint Venture Participation: %
.01	4.0	.0010	4999	100	4635	2
.01	4.0	.0010	4999	1000	4635	22
.01	4.0	.0100	499	100	461	22
.01	4.0	.0100	499	1000	461	100
.20	4.0	.0010	4999	100	1620	6
.20	4.0	.0010	4999	1000	1620	62
.20	4.0	.0100	499	100	161	62
.20	4.0	.0100	499	1000	161	100

Pr: probability of ruin.
PE: profit expectation.
p: probability of a success (environmental uncertainty).
R: net return given a success.

Summary

The limitations of the survival model are severe. The model does not take account of the sequential investment process for individual exploration opportunities. A sequential strategy eases the firm's survival conditions by limiting the cost of failures. Secondly, the model does not consider the possibility of a portfolio balancing strategy; the spreading of investment through a number of areas of environmental uncertainty. It is assumed that survival conditions are optimized by concentrating investment in one particular area of environmental uncertainty. The most important limitation of the model concerns the information available for exploration conditions. What are the statistical distributions of environmental uncertainty, cost of an exploration opportunity, and net return given a success, and what relationships actually exist between them?

Nevertheless, the survival model is useful in several ways. By quantifying the concept of survival, the model illustrates important relationships that exist between the mining firm's resources and the exploration environment in which it searches. These relationships provide a basis for survival strategies and for the use of a survival confidence limit as one component of the mining firm's decision-making process. To the extent that firm resources are limited and environmental uncertainty high, this concept is of fundamental importance to the mining firm.

GROWTH

Considering the uncertainty of the exploration environment, it is surprising to find large numbers of small independent mining firms. Only a small fraction of the 2,500 mines producing nonferrous metals in the United States in 1960 were controlled by large integrated mining firms. (19,p.21). There are several possible explanations.

There is relatively free entry into the early stages of mineral exploration. Individuals, joint ventures and small firms are free to participate, at least for a time. Most will be ruined before reaching the production stage. Others may produce from small or marginal deposits that may have been previously examined and rejected by larger mining firms. However, such deposits seldom offer sustained growth potential. If a deposit proves attractive, development by the small firm may be sufficiently limited to encourage it to sell control to a larger organization. For most small mining firms, long-run growth is precluded by a combination of environmental uncertainty and limited firm resources.

There are mining firms that are small only because they have not had time to grow. They will ultimately develop the necessary resources to overcome exploration uncertainty and insure long-run growth. A small but growing mining firm implies the availability of suitable investment opportunities; it presumes that large mining firms forego more than just marginal investment opportunities and/or that the small firm is able to find profitable opportunities not perceived by larger firms. The former condition results when the mineral industry is growing faster than the growth of the large firms, and thus, the number of investment opportunities is increasing at a faster rate than large firms can develop them. The latter condition results when a small mining firm is able to apply superior skills to a specialized opportunity level.

The long-run growth of the mining firm is constrained by its skills and financial resources.

Direction of Growth

The mining firm has three growth direction alternatives: horizontal integration within mining; forward integration into smelting, refining or fabricating functions; diversification into other areas of industrial activity. The firm's direction of growth is dependent on the cumulative effect of the internal and environmental forces which act on it. Environmental inducements include; growing demand or tighterning supply for particular products, technological advances, tax incentives, and special market and supply opportunities. Environmental barriers include entry barriers, a discriminatory tax structure, and a high level of uncertainty. Internal barriers to growth center on specific deficiencies in the firm's resources, e.g., lack of marketing skills. On the other hand, growth is encouraged by the presence of unused resources within the firm.

As the mining firm grows, changes in these forces and, therefore, changes in the firm's direction of growth appear to follow a predictable evolutionary pattern. The mining firm will initially follow a strategy of horizontal integration. As it grows, it will gradually shift its direction of growth towards forward integration. At a later stage of development the mining firm will begin to diversify. As the mining firm evolves, it is drawn increasingly closer to conventional business firm norms.

First Stage: Horizontal Integration

For the small mining firm, uncertainty is primarily associated with the exploration environment. If the firm is to survive and grow, it must be successful in discovering new deposits. To insure success, most of the small mining firm's resources must be directed to exploration strategies. Success results in a horizontally integrated mining firm.

The firm's exploration uncertainty (survival problem) is a function of firm size. Therefore, as the mining firm grows, continued growth can be insured by allocating a decreasing proportion of the firm's total resources to its exploration strategies. The residual resources may then be applied to other problem areas.

The small mining firm is able to vertically disintegrate from selling by passing on the market function to custom smelters or metal brokers. This enables the firm to concentrate its resources on the exploration environment. However, such a strategy limits the firm's ability to grow vertically to the extent that marketing skills are not developed within the firm. Yet, as the mining firm grows, there are increasing incentives for forward integration. Imperfect market conditions with respect to the degree of market control exercised by forward processors may increase the firm's market problems. Also, at some point the firm's mine and mill output of individual commodities will become sufficient to justify economical scales of a forward processing plant.

Second Stage: Forward Vertical Integration

As the mining firm grows, three changes usually occur: exploration uncertainty decreases, market uncertainty increases and the output of individual mineral products increases. These changes encourage forward vertical integration.

Vertical integration may only be effected gradually, in a number of stages over a period of time. The production of individual mineral commodities must be sufficient to support forward processing functions. Time is required to develop within the firm the necessary marketing skills and processing technology. Financial resources are required to develop the forward processing plant. Realization of these basic requirements renders forward vertical integration feasible.

In spite of increasing incentives for vertical integration, the mining firm is not likely to abandon its horizontal integration strategy. The depletion of existing mines will provide a continuing need for the discovery of new deposits. Furthermore, the mining firm will continue to invest in mineral exploration, where its skills are well developed, if there continue to be opportunities for profitable investment.

However, at some stage of growth the mining firm will begin to develop forward processing facilities. The firm becomes its own customer between the integrated functions. Vertical integration transfers the market problem in the direction of the manufactured product where product differentiation may give some market security. In pursuing such a strategy, the firm develops market skills and a technological base. Integration continues until the limit of profitable forward growth has been reached. A fully integrated mining firm embraces mining, milling, smelting, refining and fabricating functions.

Third Stage: Diversification

As the mining firm's direction of growth shifts forward, market skills and a technological base become increasingly important. These are the requisite characteristics for diversification. Whether such a strategy will be pursued depends on a number of factors. Diversification will be encouraged when: growth rates and profit expectations in other industrial sectors are greater than within the mineral industry; market uncertainties for mineral products are high and it is desirable to spread the market uncertainty of the firm as a whole; sufficient opportunities are not available within the mineral industry to efficienty utilize the mining firm's resources.

As the mining firm integrates vertically and diversifies, it is drawn increasingly closer to conventional business firm norms. The end result will not necessarily be a mining firm. Once the mining firm's special environmental conditions have been surmounted, its decision-making problems should approach those of other business firms.

Method of Growth

If a firm is able to realize its growth rate objective by internal building, then it will likely follow a strategy which will, other things being equal, allow the firm absolute control over its productive activities. The requirement for this method is that the skills and financial resources available within the firm for growth are, in relation to environmental conditions, sufficient to insure a desired rate of growth. In such circumstances the corporate entity provides a suitable vehicle for growth.

Merger is an important method of growth for large firms. It is a strategy that increases growth rate and facilitates diversification. In the mining industry, merger was important in the period around 1900 when emphasis was placed on the combination of existing producers into a single firm; and in the period prior to 1930 when strategies of integration and cartelization attempted to control marketing arrangements. (6,pp.40-42).

Since World War II, the joint venture has been the primary external method of growth for large mining firms. Such a method is concerned less with production and marketing, where merger remains the predominant form for inter-firm relationships, than with the exploration and development of specific mineral investment opportunities. Joint ventures are used to insure growth when envionmental uncertainty is high rather than to increase rate of growth.

Joint venture is a less severe form of consolidation than merger in the sense that each participant retains its own identity with respect to its currently producing operations and other long-run non-joint venture pursuits. Each firm continues to exist as a separate decision-making entity. The advantages of the joint venture method are with respect to the increased skills and financial resources that are brought to bear on environmental conditions. For the mining firm, joint ventures are normally used for horizontal integration. The firm with limited resources is thereby able to mount an effective exploration program, using expensive modern methods and participating in several opportunity areas.

THE DECISION-MAKING PROCESS

The decision-making process comprises a number of sequential steps. An objective function is specified in terms of a profit expectation, a survival confidence limit and a growth rate. Once objectives have been determined, strategies are developed for directing the firm's resources toward their realization. Strategies channel the firm's financial resources and skills to those types of investment opportunity for which it is best suited.

Exploration strategies strive for an optimum balancing of firm resources and exploration parameters. A sequential investment strategy is used to limit the cost of failures under uncertainty conditions. The mining firm must also develop strategies for the allocation of resources to exploration, the balancing of exploration capital and skills, optimum uncertainty reduction with respect to both the initial sequential stage and joint venture participation, and the allocation of exploration resources to the initial sequential stage.

The marketing strategies available to small mining firms are limited and are primarily associated with short-run market fluctuations. Market uncertainty becomes increasingly important and controllable as the mining firm grows and emphasis shifts from short-run to long-run market problems. Large mining firms are in a stronger position to cope with short-run market uncertainty. Also, the necessary resources may be available for the pursuit of long-run market strategies.

	Suitability For	
	Small Mining Firm	Large Mining Firm
Short-run Market Strategy		
Inventory	X	XX
Smelter Contract	XX	
Metal Broker	XX	
Long-run Market Strategy		
Product Research and Development		XX
Processing Research and Development	X	XX
Multi-product	X	XX
Market Control		XX

X: minor suitability.
XX: major suitability.

The strategies developed result in the creation of investment opportunities and the generation of information. Each opportunity is evaluated with respect to the firm's objectives. The alternatives showing the highest expectations of realizing firm objectives are selected and implemented within the constraints imposed by the limited resources of the firm. After implementation, actual results are compared with expectations. Significant departures in results from expectations will necessitate new decisions concerning particular operations and may lead to revisions in the overall decision-making process. The feedback of information and experience with respect to previously implemented opportunities will guide subsequent decisions. Thus, the firm's knowledge and experience are used to exercise continuous control over its operations.

In general terms, the decision-making process of the mining firm is similar to that of other business firms. What is different for the mining firm is the environmental conditions within which it functions. Because of these conditions, the sensitivity of the mining firm's decision-making process to individual firm objectives is quite different from that of other firms. Thus the need for understanding the mining firm's objective-resource-environment relationships and the development of suitable exploration and market strategies.

SUMMARY

The primary purpose of this study is to provide a more relevant framework for decision-making in the mining firm. Firstly, an explanation has been given of why existing theories do not provide an adequate representation of the objectives and decision-making process of the real-world mining firm. Secondly, a decision-making theory of the mining firm has been developed. Emphasis is placed on those aspects of mining firm decision-making that are significantly different from those of other business firms. The decision-making framework that has been developed is intended to foster an understanding of the environmental conditions within which the mining firm functions and of the strategies available to it for countering and exploiting its position.

The study is also intended to examine the traditional uniqueness claims of spokesmen within the mineral industry. In any theoretical analysis it is essential to draw on and relate to existing work. In considering the decision-making problems of the mining firm there is much to be learned from conventional economic theory and, more directly, from decision-making theory. The decision-making theory of the mining firm represents a special case within the larger body of decision-making theory. The problem of mine replacement, with its implications for mining firm survival and growth, provides the rationale for special attention. The sensitivity of the mining firm's decision-making process to mine replacement considerations is largely a function of firm size. If the mining firm grows, it is drawn increasingly closer to business firm norms. It would appear that the traditional uniqueness claims of the mineral industry are over-rated with respect to the decision-making problems of the large mining firm and with respect to public policy problems, to the extent that in these circumstances resources are large enough to insure mine replacement and investment is directly related to profit expectation.

Finally, the study is intended to examine the relationship between the mining firm in conventional economic theory and in decision-making theory, to determine whether or not the concepts are consistent. To the extent that the present analysis considers the decision-making <u>process</u> of the mining firm -- relationships between firm resources and environmental conditions, and resulting strategies -- it is compatible with both conventional economic theory and decision-making theory although its usefulness is largely in the decision-making problem area. To the extent that the present analysis considers the <u>motivation</u> of the mining firm -- the balancing of a number of at least partially conflicting objectives -- it is inconsistent with the 'profit expectation maximization' assumption of conventional economic theory. It has been shown that profit expectation maximization would be a ruinous objective for mining firms with limited resources and thus, profit expectation must be balanced with survival considerations. Again, survival sensitivity is a function of firm size. Large mining firms may have sufficient resources to insure the realization of profit expectations and if so, one would expect these firms to be profit expectation maximizers. Since large mining firms account for most of the output in the mineral industry, the profit expectation maximization assumption of conventional economic theory may very well be a reasonable approximation for public policy analyses. The present study is primarily concerned with the large numbers of small independant mining firms relatively less important from the mineral industry viewpoint. For these firms, profit expectation maximization is not a valid assumption.

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A continuación se presenta el resultado de 25 simulaciones, con cálculos de ganancia y tasa de ganancia:

GANANCIA($)	TASA DE GANANCIA(%)
-14,000	5.63
-36,000	1.73
7,500	8.05
-66,000	-4.10
144,800	16.15
23,000	10.55
-103,000	-11.88
-81,000	-5.45
16,000	9.74
42,500	11.08
-36,000	1.73
-96,000	-13.60
32,800	10.80
-103,000	-11.88
72,000	12.35
71,000	14.30
-68,000	1.31
8,000	8.60
-81,000	-5.45
1,000	7.70

71,000	14.30
38,000	14.35
105,000	15.80
9,500	6.50
-66,000	-4.10

Con el objeto de clasificar esta información, es conveniente realizar primero una ordenación de la misma por orden creciente:

GANANCIA ($)	TASA DE GANANCIA (%)
-103,000	-13,60
-103,000	-11,80
-96,000	-11.80
-81,000	-5.45
-81,000	-5.45
-68,000	-4.10
-66,000	-4.10
-66,000	1.31
-36,000	1.73
-36,000	1.73
-14,000	5.63
1,000	6.50
7,500	7.70

8,000	8.05
9,500	8.60
16,000	9.74
23,000	10.55
32,800	10.80
38,000	11.08
42,500	12,35
71,000	14.30
71,000	14.30
72,000	14.30
105,000	15.80
144,800	16.15

Para construir el histograma de la ganancia utilizaremos 5 clases, las cuales cubrirán el rango de valores (-103,000, 144,800). La amplitud de cada intervalo resulta entonces de

$$\frac{144,800 - (-103,000)}{5} = 49,560$$

o sea que los límites de clase serán

-103,000 y -103,000 + 49,560 = -53,440

-53,440 y -3,880

-3,880 y 45,680

45,680 y ~~105,240~~ 95,240

~~105,240~~ 95240 y 144,800

De esta manera las marcas de clase resultarán

$$\frac{-103,000 + (-53,440)}{2} = -78,220$$

-28,660

20,900

70,460

120,020

pudiéndose escribir entonces la siguiente tabulación o distribución de frecuencias:

INTERVALO	MARCA DE CLASE	FRECUENCIA ABSOLUTA	FRECUENCIA RELATIVA
-103000, -53440	-78,220	8	0.32
-53440, -3880	-28,660	3	0.12
-3880, 45680	20,900	9	0.36
46680, 105240	70,460	4	0.16
105240, 144800	120,020	1	0.04

Para determinar por ejemplo la frecuencia absoluta del primer intervalo, se cuentan las veces en que la ganancia cae entre -103,000 y -53,440, lo que resulta igual a 8 como se comprueba fácilmente en la lista ordenada de ganancias. En igual forma se procede con los demás intervalos. Obsérvese que la frecuencia relativa es igual a la frecuencia absoluta, entre el número total de simulaciones.

Así, el valor medio de la ganancia queda

$$\bar{G} = -78,220 \times 0.32 - 28,660 \times 0.12 + 20,900 \times 0.36$$
$$+ 75,460 \times 0.16 + 125,020 \times 0.04$$
$$= - \$3,830$$

resultando entonces la variancia de

$$v_G = (-74,390)^2(0.32) + (-24,830)^2(0.12) + (24,730)^2(0.36)$$
$$+ (79,290)^2(0.16) + (128,850)^2(0.04)$$
$$= 3709'100,000$$

la desviación estándar

$$s_G = 61,000$$

y por último el coeficiente de variación

$$c_G = -15.9$$

Para la tasa de ganancia, con un procedimiento semejante, se tiene:

Amplitud de los intervalos $= (16.15 + 13.60)/5 = 5.95$

INTERVALO	MARCA DE CLASE	FRECUENCIA ABSOLUTA	FRECUENCIA RELATIVA
-13.60, -7.65	-10.62	3	0.125
-7.65, -1.70	-4.67	4	0.16
-1.70, 4.25	2.55	3	0.125
4.25, 10.20	7.22	6	0.24
10.20, 16.15	13.17	9	0.36

$$R = -10.62 \times 0.125 - 4.67 \times 0.16 + 2.55 \times 0.125$$
$$+ 7.22 \times 0.24 + 13.17 \times 0.36$$
$$= 3.707\%$$

$$v_R = 71.57$$

$$s_R = 8.45$$

$$c_R = 2.28$$

Por último, la probabilidad de no recuperar la inversión es aproximadamente igual a

$$32\% + 12\% = 44\%$$

y la probabilidad de que la tasa de ganancia sea inferior a la de oportunidad es de

$$12.5\% + 16\% + 12.5\% + 24\% = 64\%$$

Table 20. Random Digits*

48867	33971	29678	13151	56614	19193	[illegible]	13252	11006	17173
32267	09746	00113	51336	36551	56310	[illegible]	53153	09741	61116
27435	03196	33877	35032	98054	48358	[illegible]	98862	67191	42221
55753	05256	51557	90119	10716	61589	[illegible]	37070	78318	02918
93142	50675	01507	41001	06365	77897	81566	99600	67985	49134
98658	86583	97433	10733	80195	62709	61357	66903	76730	79355
68216	91830	11218	50712	46878	87317	80515	31484	03195	14755
17901	30815	78360	78260	67866	42301	07293	61290	61301	01515
88124	21868	14942	25893	72695	56231	18918	72534	86737	77792
83464	36749	22336	50143	83576	19238	91730	39507	22717	94719
91310	99003	25704	55581	00729	22021	61319	66162	20933	67713
32739	38352	91256	77744	75080	01192	90984	63090	53087	41301
07751	66724	03290	56386	06070	67105	64219	18192	70178	84722
55228	61156	90180	97774	08055	01435	26999	42039	16589	06757
89013	51781	81116	21383	95569	97217	11137	36293	29967	16058
51828	81819	81038	89116	39192	89470	76331	56420	14527	34828
59783	85454	93327	06078	61924	07271	77563	92710	12183	12389
80267	47103	90556	16128	41190	07996	78154	17929	81586	67024
82919	44210	61607	93001	26314	26865	26714	43793	94937	28139
77019	77117	19166	14967	75521	49967	71065	09746	27881	01070
66225	61832	06212	40093	10800	76819	29929	18988	10888	40314
98534	12777	84601	56336	00034	85939	32138	09519	01855	40559
63175	70789	51345	43723	06995	11186	38615	56646	51320	39642
92362	73011	09115	78303	38801	58107	95366	17226	71626	78208
61831	44794	65079	97130	91289	73502	01857	68855	47015	06309
42502	01646	88493	48207	01283	16474	08864	68322	92451	19287
89733	86230	04903	55015	11811	98185	32014	84761	80926	11509
01336	66633	26015	66768	24816	00321	73118	15802	13549	41335
72623	56083	65799	88931	87274	19117	81897	90877	76172	52145
71004	68388	04090	35239	49379	01456	07642	68612	01026	13510
09383	51633	27684	47117	67584	42496	20703	68579	65883	10729
51771	92019	39791	60400	08585	60680	28811	09921	00520	73135
69796	30304	79836	20631	10713	00216	21979	35707	75283	39211
98417	33103	63418	90162	91615	21919	73609	26663	09780	30615
56150	18324	43011	02660	86574	86097	19399	21249	90380	91575
76199	75692	09063	72999	91672	69128	39016	15379	98150	09159
74978	98693	21433	31676	97603	48534	59205	66265	03561	83075
85769	92530	01107	53725	96963	19395	16193	51018	70333	12094
63819	65669	38960	71631	39650	39119	93707	61365	46302	26134
18892	43113	19619	43200	49613	50904	73502	19519	11667	53294
32855	17190	61587	80111	22827	38852	51952	17785	34952	94574
29135	96277	53583	92801	05027	19736	51918	66396	96547	00351
30211	67263	82064	11624	19826	17566	02176	79368	28831	02805
73514	00176	11638	01420	31850	41380	11613	06787	09011	88024
90895	93099	27850	29123	98693	71762	39928	35268	59359	20674
69719	90656	62186	50135	77015	29661	91698	56057	01388	33481
91982	81153	87162	28218	37921	21113	62673	81224	38972	92588
81136	01221	72790	04719	31914	95609	88695	60180	58790	12802
58515	80581	88112	65727	72121	10481	06001	13159	55324	[illegible]
20861	59161	75797	08928	68381	12616	97187	84803	92157	88847

* Reproduced with permission from the Rand Corporation, *A Million Random Numbers*, Free Press, Glencoe, [illegible] 1955.

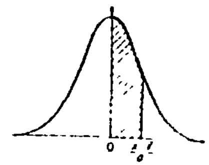
Table 21. Random Normal Numbers* (Gaussian deviates)

192	−.944	.101	.226	1.396	−1.030	−1.723	−.368	2.170	.393
148	−1.140	.192	−1.210	−.998	.573	.893	−.855	−2.209	−.267
572	1.353	−.900	−.554	−.343	.470	−1.033	−1.026	2.172	.195
145	.466	.854	−.282	−1.501	.431	−.060	.952	−.343	.735
104	.732	.601	−.016	−.266	1.372	−.925	−1.591	−2.004	1.925
1.419	−1.853	−.347	.155	−1.078	.623	−.024	.498	.466	.049
[illegible]	−.411	−.661	−.037	.703	.532	−.177	.395	−.278	.240
797	.488	−1.070	−.721	−1.112	−.976	−1.953	−.206	1.848	.632
3	−.351	.222	.557	−1.094	1.403	.173	−.113	.806	.939
574	−1.336	.523	.848	.304	−.202	−1.279	.501	.396	.859
125	−1.170	−.192	1.387	2.291	−.959	.090	1.031	.180	−1.389
1.691	−.649	−.514	−.232	−1.198	.822	.240	.951	−1.736	.270
2.304	.481	−.987	−1.222	.519	−1.056	.277	−.919	.148	1.517
961	2.057	−.546	−.896	.105	−.343	.696	.628	−.929	−.965
783	.854	−.139	1.087	.515	−.876	−.448	.485	.589	−.804
157	.557	.327	1.280	−1.731	−.339	.295	−.724	.720	.331
259	.979	−.924	−.649	.574	1.407	−.292	−.775	−.511	.026
1.631	−.937	−1.321	−1.734	1.677	−1.393	−1.187	−.079	−.181	−.844
213	.166	−1.330	1.078	−1.102	1.123	−.421	−.671	2.951	−.743
2.181	−1.854	−1.059	−.478	−1.119	.272	−.800	.841	−.061	2.261
154	−.333	1.011	−1.565	1.261	.776	1.130	1.552	−.563	.558
1.665	1.610	.163	.062	−.086	.021	1.633	1.788	.480	2.824
1.083	−.760	−.012	.183	.155	.676	−1.315	.067	.213	2.380
615	−.594	−.028	−.506	−.054	3.173	.817	.210	1.699	1.950
178	−.500	1.100	1.613	1.018	2.323	−.174	−.033	2.220	−.061
907	−1.273	.596	.690	−1.724	−1.689	.163	−.199	−.150	.244
862	−.588	−1.386	.072	.778	−.591	.365	.465	2.472	1.049
775	1.516	.217	−1.012	.778	.246	1.055	1.071	.417	−.585
818	.561	−1.024	2.105	−.868	.060	−.385	1.089	.017	−.873
014	.210	−.632	−.225	−.844	.448	1.651	1.423	.425	.252
1.236	−1.015	−1.628	.687	.983	−.810	−1.835	−1.861	1.327	−.408
567	−1.161	.010	−.853	.111	1.145	1.015	.056	.141	1.471
278	−1.783	.170	−.378	.705	−.054	1.098	.707	−.585	−.305
559	−.197	.688	−.268	−1.431	−.791	−.727	.958	.237	.092
1.249	.037	.497	.579	−.227	.860	.319	2.355	2.184	−1.744
915	−.164	−1.166	1.529	.008	.636	−1.080	−.688	2.414	−1.316
132	2.809	−1.918	−1.083	−.642	−.179	.339	.637	.063	−.679
156	−1.661	1.140	.295	1.086	−2.516	−.002	−.672	.205	−.039
538	−1.113	−.390	.165	−.160	.457	−1.307	.273	−.670	[illegible]
627	−.057	.742	−.149	−.801	1.702	−.316	−.073	.892	−1.181
023	.123	1.051	−.831	−.325	−.795	−1.129	−.287	.172	[illegible]
196	−1.457	1.060	.557	−.190	−.891	−.768	.282	−1.132	[illegible]
134	.577	−.332	−1.932	.220	.189	−1.621	.896	−.781	[illegible]
620	−.217	−.856	.605	.072	.520	1.922	−.181	−.266	[illegible]
1.405	1.065	1.350	1.353	−2.289	−1.003	.375	1.621	−1.126	[illegible]
178	−1.237	−.520	−.603	−1.615	−.358	.605	−.107	−2.579	−1.811
1.438	.101	−1.821	−.390	−.630	1.294	1.170	.991	[illegible]	[illegible]
1.768	−.175	−.150	.915	−.221	−.019	1.861	[illegible]	[illegible]	[illegible]
099	1.076	2.318	−1.550	.158	.147	−1.223	[illegible]	[illegible]	[illegible]
951	.252	−1.261	−.963	.221	[illegible]	−.395	[illegible]	1.379	[illegible]

* Reproduced with permission from the Rand Corporation, *A Million Random Numbers*, Free Press, Glencoe, Ill., 1955.

Table 18. Areas under the Normal Curve*

Fractional parts of the total area (1.000) under the normal curve between the mean and a perpendicular erected at various numbers of standard deviations from the mean, $(X - \bar{X})/\sigma$. To illustrate the use of the table, 39.065 per cent of the total area under the curve will lie between the mean and a perpendicular erected at a distance of 1.23σ from the mean. The values in the table represent the shaded area in the normal distribution shown at the right. Each figure in the body of the table is preceded by a decimal point.



$\frac{(X-\bar{X})}{\sigma}$	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	00000	00399	00798	01197	01595	01994	02392	02790	03188	03586
0.1	03983	04380	04776	05172	05567	05962	06356	06749	07142	07535
0.2	07926	08317	08706	09095	09483	09871	10257	10642	11026	11409
0.3	11791	12172	12552	12930	13307	13683	14058	14431	14803	15173
0.4	15554	15910	16276	16640	17003	17364	17724	18082	18439	18793
0.5	19146	19497	19847	20194	20540	20884	21226	21566	21904	22240
0.6	22575	22907	23237	23565	23891	24215	24537	24857	25175	25490
0.7	25804	26115	26424	26730	27035	27337	27637	27935	28230	28524
0.8	28814	29103	29389	29673	29955	30234	30511	30785	31057	31327
0.9	31594	31859	32121	32381	32639	32894	33147	33398	33646	33891
1.0	34134	34375	34614	34850	35083	35313	35543	35769	35993	36214
1.1	36433	36650	36864	37076	37286	37493	37698	37900	38100	38298
1.2	38493	38686	38877	39065	39251	39435	39617	39796	39973	40147
1.3	40320	40490	40658	40824	40988	41149	41308	41466	41621	41774
1.4	41924	42073	42220	42364	42507	42647	42786	42922	43056	43189
1.5	43319	43448	43574	43699	43822	43943	44062	44179	44295	44408
1.6	44520	44630	44738	44845	44950	45053	45154	45254	45352	45449
1.7	45543	45637	45728	45818	45907	45994	46080	46164	46246	46327
1.8	46407	46485	46562	46638	46712	46784	46856	46926	46995	47062
1.9	47128	47193	47257	47320	47381	47441	47500	47558	47615	47670
2.0	47725	47778	47831	47882	47932	47982	48030	48077	48124	48169
2.1	48214	48257	48300	48341	48382	48422	48461	48500	48537	48574
2.2	48610	48645	48679	48713	48745	48778	48809	48840	48870	48899
2.3	48928	48956	48983	49010	49036	49061	49086	49111	49134	49158
2.4	49180	49202	49224	49245	49266	49286	49305	49324	49343	49361
2.5	49379	49396	49413	49430	49446	49461	49477	49492	49506	49520
2.6	49534	49547	49560	49573	49585	49598	49609	49621	49632	49643
2.7	49653	49664	49674	49683	49693	49702	49711	49720	49728	49736
2.8	49744	49752	49760	49767	49774	49781	49788	49795	49801	49807
2.9	49813	49819	49825	49831	49836	49841	49846	49851	49856	49861
3.0	49865									
3.5	4997674									
4.0	4999683									
4.5	4999966									
5.0	4999997133									

* Adapted from F. C. Kent, *Elements of Statistics*, McGraw-Hill Book Company, Inc., New York, 1924.

RANDOM NUMBERS

51772	74640	42331	29044	46621	62898	93582	04186	19640	87056
24033	23491	83587	06568	21960	21387	76105	10863	97453	90581
45939	60173	52078	25424	11645	55870	56974	37428	93507	94271
30586	02133	75797	45406	31041	86707	12973	17169	88116	42187
03585	79353	81938	82322	96799	85659	36081	50884	14070	74950
64937	03355	95863	20790	65301	55189	00745	65253	11822	15804
15630	64759	51135	98527	62586	41889	25439	88036	24034	67283
09448	56301	57683	30277	94623	85418	68829	06652	41982	49159
21631	91157	77331	60710	52290	16835	48653	71590	16159	14676
91097	17480	29414	06829	87843	28195	27279	47152	35683	47280
50532	25496	95652	42457	73547	76552	50020	24819	52984	76168
07136	40876	79971	54195	25708	51817	36752	72484	94923	75936
27989	64728	10744	08396	56242	90985	28868	99431	50995	20507
85181	73949	36601	46253	00477	25234	09908	36574	72139	70185
54398	21154	97810	36764	32869	11785	55261	59009	38714	38723
65544	34371	09591	07839	58892	92843	72828	91341	84821	63886
08263	65952	85762	64236	39238	18776	84303	99247	46149	03229
39817	67906	48236	16057	81812	15815	63700	85915	19219	45943
62257	04077	79443	95203	02479	30763	92486	54083	23631	05825
53298	90276	62545	21944	16530	03878	07516	95715	02526	33537

centro de educación continua
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EVALUACION ECONOMICA Y METODOS PARA DECISION DE INVERSIONES DE LA INDUSTRIA MINERA

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CONDICIONES HISTORICAS DE LA PLANIFICACION.

Alcance y Significado de la Planificación Económica

La planificación económica está de moda. En todas partes se habla hoy día de ella y a menudo se la presenta incluso como una panacea, en los textos de economía, en los informes oficiales de los gobiernos, en las reuniones de hombres de negocios y en la literatura sobre el desarrollo económico y sobre el socialismo, se alude a la planificación a cada momento. Mas a pesar de las múltiples referencias que se hacen de ella, aún se está lejos de haber llegado a establecer con presición su alcance. Con frecuencia confunden la planeación y la programación; a veces se intercambian sin fundamento, términos tales como el de "planificación integral" y la llamada "planeación indicativa". En ocaciones se subraya el carácter flexible de esta última, insinuándose sin razón que la planeación "central" o "integral" debe por fuerza ser rígida o inflexible; y más comunmente se concibe la planificación económica como un mero instrumento técnico, -como lo es por ejemplo la programación matemática-, cuya aparición y posibilidades de empleo se supone que nada tienen que ver con el desarrollo histórico del sistema económico.

Se llega a tales extremos en el manejo del término y del concepto mismo de la planificación, que algunos suelen y gustan asimilar la planificación "democrática" a la que se intenta en los países capitalistas en tanto que consideran "totalitaria" y "antidemocrática" a la que es propia de los países socialistas; y aun hay autores que parecen creer que la planificación económica es simplemente una especulación mental que supone construir un modelo macroeconómico estático y establecer ciertas relaciones o coeficientes más o menos simples, que nada o poco tienen que ver con el proceso real del desarrollo, ni con la verdadera planificación.

LA PLANIFICACION Y EL DESARROLLO

Las opiniones anteriores adolecen a nuestro juicio de una falla o limitación fundamental: consideran a la planificación como algo abstracto, como algo desligado del desarrollo histórico de la sociedad, o en el mejor de los casos, como un mecanismo prácticamente mágico, que ha de ser capaz de acabar con la anarquía en el seno mismo de la anarquía. A ello equivale querer planificar el desarrollo en el marco de una economía de mercado, cuyo funcionamiento siga básicamente sujeto a las leyes del desarrollo capitalista.

Más adelante veremos hasta donde es posible - y bajo qué condiciones-, planificar en una economía de mercado; lo que por ahora deseamos subrayar es que la planificación no se produce al margen del desarrollo histórico, sino todo lo contrario. Entendida en su ascepción más más rigurosa, la planeación racional o integral es una fase del desarrollo; al igual que el mercado, es un categoría histórica, que supone la creación de condiciones objetivas que la hagan posible. Y ello es así, entre otras cosas porque planificar no consiste en hacer planes en el gabinete, en decidir en una oficina técnica o burocrática que la economía de un país ha de crecer tal o cual ritmo y su estructura modificarse de tal o cual manera, sino en lograrlo, en conseguir que el desarrollo se vuelva en gran medida un proceso deliberado, consciente, racional; y no simplemente en desearlo.

Por estas razones creemos que, para entender a fondo un curso teórico de planificación, es preciso empezar por recordar el marco histórico en que surge la planificación, entender lo que es el mercado y lo que han sido las diversas fases del desarrollo del capitalismo. - Si estas cuestiones fundamentales no se comprenden, es fácil caer en el error de creer que planificar significa simplemente hacer tablas de insumo-producto o combinar la relación marginal capital producto con otros coeficientes.

RACIONALIDAD ECONOMICA Y PLANIFICACION.

Es el tránsito hacia nuevas relaciones de producción basadas en el - control social de los medios productivos, lo que hace posible pasar de la etapa de la racionalidad de la empresa a la de racionalidad de la sociedad en conjunto.

La racionalidad social de la actividad económica requiere y supone - la supeditación de las empresas individuales a un fin social general como dice el doctor Lange requiere la coordinación de sus actividades y la integración de sus fines en torno a un objetivo común que encauza y dirige la actividad económica de la sociedad; y en esa coordinación consiste en buena parte la planificación. La necesidad de tal - coordinación se presente incluso bajo el capitalismo, en las formas de integración propias de los monopolios, pero tiene siempre, como - se ha señalado, un carácter restringido y antagónico.

En un sentido estricto, la planificación económica racional o integral sólo es posible bajo el socialismo, aunque mucho puede lograrse en un sistema en que un estado democrático y progresista tome a su - cargo la dirección del desarrollo económico y la defensa resuelta de los intereses mayoritarios de la población, poniéndolos por encima - de la empresa privada./La propiedad social de los medios de producción modifica esencialmente el carácter de la empresa y elimina el - fin de la ganancia máxima; subordina a la empresa al plan general y sustituye la obtención de la ganancia individual por fines más racionales que interesan al conjunto de la sociedad./

Lo que persigue la planificación no es la máxima utilidad de una o - varias empresas; lo que busca es maximizar el excedente, maximizar - el ingreso, maximizar y armonizar el desarrollo de toda la economía. El objetivo principal de un plan económico es lograr un aumento determinado del ingreso de la comunidad. Es un objetivo medible cuantificable y susceptible de alcanzarse mediante el empleo de ciertos - medios, como son un volumen determinado de inversiones de capital - distribuídas en la forma que más convenga para elevar la producción y obtener las metas previamente establecidas.

La ganancia juega aún en una economía planificada, un papel que no deja de tener importancia; pero que no constituye ya el móvil principal de la actividad económica sino un medio subordinado o al servicio de los objetivos del plan. La ganancia se convierte por lo tanto en un estímulo para el cumplimiento mismo del plan.

Esto pone de relieve que bajo un sistema de planificación existen fines u objetivos de diverso nivel, que se relacionan entre sí en una estructura jerárquica, a diferencia de lo que es propio del capitalismo, en que generalmente los fines de las empresas son paralelos e independientes unos de otors. Pero esa estructura jerárquica propia de una economía planificada, no surge de la nocha a la mañana, sino que se desenvuelve poco a poco, de acuerdo con cambios profundos en la estructura socio-económica.

La planificación no solo es difícil en un país subdesarrollado que se lanza resueltamente a conquistar un mayor bienestar y un crecimiento más rápido de su economía, sino que lo es incluso bajo el socialismo. En las fases iniciales de este sistema, en que aún están presentes ciertos rasgos capitalistas y semifeudales, la coordinación de los diferentes objetivos y la obtención de los mismos plantea serios problemas. Falta todavía experiencia, se carece de medios de información adecuados, las previsiones en que se descansa son defectuosas y la metodología de la planificación adolece de fallas que gradualmente se van superando. De ahí que mucho de lo que se hace, aun de lo que se hace mal en esas primeras etapas, constituye en rigor una necesidad y no meros errores que carezcan de toda justificación.

PRINCIPALES OBRAS CONSULTADAS

Maurice Dobb	Economía Política y Capitalismo
Oscar Lange	Political Economy
P.J.D. Wiles	The Political Economy of Comunism.

iniciadas, impulsar la reforma agraria y el desarrollo de las cooperativas. En la medida en que todo eso se ponga en marcha, la planificación hará posible evaluar y utilizar mejor los recursos existentes crear nuevas instituciones y mecanismos, ampliar y mejorar la información estadística y canalizar los recursos hacia las actividades productivas fundamentales.

Y otra condición del desarrollo y de la propia planificación, es que haya un clima democrático que permita a los principales sectores populares tener acceso a los órganos del poder. Sin una base democrática, la planificación fácilmente deriva en regimentación burocrática y en impulso a los privilegios y la desigualdad social y económica - Por eso la planificación, lejos de plantear principalmente problemas técnicos, entraña un proceso de transformación social y política que sólo puede llevarse a cabo con éxito si el pueblo llega a tener organización, capacidad y decisión para enfrentarse a los obstáculos que impiden o detienen el desarrollo.

INTENTOS DE PROGRAMACION Y PLANIFICACION EN AMERICA LATINA

En varios países de América Latina, al igual que en las naciones industriales de occidente, empezó a hablarse de la planificación económica desde los años treinta; pero fué en realidad, después de 1954 y sobre todo en 1960, cuando la programación económica comenzó a tener cierta importancia práctica.

En los años de la segunda guerra mundial, Latinoamérica logró acelerar su desarrollo económico. El mejoramiento de los precios de sus - exportaciones, la ausencia de las grandes naciones industriales del mercado exterior de numerosos productos e incluso del mercado interno de cada país latinoamericano, y la creciente intervención estatal que primero sirvió para hacer frente a la depresión y después para - fomentar el desarrollo económico, hicieron posible que se avanzara - en el proceso de industrialización, no obstante los desequilibrios, la inflación y la injusticia en el reparto del ingreso, que también fueron propios de esos años.

El aumento de la demanda interna y externa permitió aprovechar en mayor medida la capacidad de producción instalada y requirió de inversiones crecientes que a su vez contribuyeron a elevar la tasa de crecimiento del ingreso nacional. Entre 1942 y 1951, dicho ingreso creció a razón de 6% al año, aunque hubo una etapa de reajuste a partir de 1948, en la que el creciente desequilibrio de las balanzas de pagos se tradujo en devaluaciones y una inflación similar y a veces - aún más severa que la del período bélico.

Cuando la prosperidad prácticamente llegaba a su fin, en 1949-1950, la guerra de Corea abrió una nueva perspectiva de altos precios y - aumento de la demanda exterior. Pero tras de una breve mejoría, a - partir de 1954 el crecimiento latinoamericano perdió impulso, hasta caerse en muchos países en una situación de estancamiento. En efecto, los precios de las exportaciones bajaron sensiblemente entre - 1954-1956, la relación de intercambio se volvió más desfavorable, - las tasas de inversión dejó de crecer y en la década comprendida entre 1950 y 1960, sólo tres países del área lograron que su ingreso - por habitante aumentara más de 2% al año.

Frente a tal situación, los gobiernos latinoamericanos tendieron a obtener mayores recursos financieros en el exterior, principalmente a través de préstamos de Estados Unidos y de instituciones financieras internacionales; y a principios de 1961, cuando el impacto de la revolución cubana sobre el continente era mayor, ante el temor de que los pueblos de Latinoamérica trataran de resolver sus graves problemas económicos por vías revolucionarias, el presidente de Estados Unidos, John F. Kennedy, anunció el programa llamado "Alianza para el Progreso", en el que la acción conjunta de los países de América haría posible un gran desarrollo en la siguiente década. Unos meses más tarde, en la reunión al nivel de Ministros del Consejo Interamericano Económico y Social de la OEA, celebrada en Punta del Este, se aprobaron varios documentos en los que el programa anunciado por el mandatario norteamericano, tomaba cuerpo y empezaba a precisarse. Y al plantearse el problema del desarrollo económico latinoamericano, se señaló que tal desarrollo requeriría de la planificación.

En los años inmediatos anteriores, algunos países habían ya creado ciertos organismos que en parte tenían funciones de programación o planificación y elaborado programas regionales o sectoriales; pero fué a partir de la Conferencia de Punta del Este, cuando la Planificación pasó al primer plano en todas partes.

La concepción aprobada en esa Conferencia tuvo su base en un estudio previo, hecho por un grupo de expertos, que se elaboró precisamente para orientar las discuciones en materia de planificación. En él se reconoce que América Latina es pobre, económicamente subdesarrollada y que sólo podrá superar su atraso y mejorar en un plazo razonable el nivel de vida de sus habitantes, mediante la planificación económica o más bien elaborando programas de desarrollo sobre todo de corto y medio plazo.

El objetivo de esos programas debería ser lograr que el ingreso por habitante crezca a razón de 2.5% al año en la década 1961-1970; aumentar sustancialmente la productividad agrícola, pues la mayor parte de la población vive de la agricultura recurriendo para ello entre otras cosas a la reforma agraria; conseguir cierta estabilidad de precios, ya que la inflación es uno de los obstáculos al desarrollo latinoamericano; lograr un mejor reparto del ingreso nacional y mejorar también las condiciones de habitación, salubridad, educación etc.

Se señalaba, asímismo, que la planificación a largo plazo serviría para contar con un mejor diagnóstico de la situación latinoamericana cuantificar, jerarquizar y coordinar los objetivos fundamentales; obtener mayor ayuda financiera externa y armonizar los esfuerzos nacionales a escala continental. Finalmente, se recomendaba la formulación de planes de corto plazo, destinados sobre todo a aumentar las inversiones con fines sociales, así como la elaboración de proyecciones que podrían servir de base a los primeros intentos de planeación a largo plazo.

En cuanto a los planes de plazo medio, se subrayaba que debían esencialmente incluir:

1) Una estrategia adecuada, capaz de movilizar al potencial productivo y de superar los obstáculos al desarrollo;

2) Metas precisas en cuanto al ingreso, inversión, consumo, exportaciones, importaciones y producción agrícola e industrial, así como respecto a habitación popular, educación, salud pública, etc.
3) Cuantificación de los recursos disponibles, concretamente a disposición del Estado, y distribución o asignación de los mismos.
4) Medios para asegurar la realización del plan, que fundamentalmente consistían en ciertos mecanismos administrativos y en una política de aliento a la iniciativa privada, de la que procedería la mayor parte de la inversión.

Al amparo de esos principios, entre 1961, y 1962 se hicieron diversos intentos de planificación, elaborándose concretamente los llamados "planes de acción inmediata", que también habían sido recomendados por los técnicos de la Organización de Estados Americanos, en tanto se adquiría información y cierta experiencia para una planeación de más largo alcance. Sobre esos primeros avances en materia de planificación, el Comité o Nómina de los nueve, de la Alianza para el Progreso, considera lo siguiente:

1) Que no basta una estrategia general del desarrollo que comprende el manejo conjunto de la política monetaria, fiscal y de comercio exterior, sino que es preciso formular verdaderos planes económicos;
2) Que los planes o programas sectoriales deben ser parte de los planes nacionales; no así los de carácter regional, que pueden preceder o ser indispensables de los planes nacionales;
3) Que los intentos de planeación de los países latinoamericanos deben coordinarse;
4) Que en las presentes condiciones, resulta muy difícil y de escaso valor práctico hacer planes de largo plazo, porque falta información y porque se depende en gran medida del mercado exterior y sus vicisitudes, y faltan aún proyectos específicos bien estudiados y viables que sirvan de base a esos planes;
5) Que en los planes primeramente elaborados no se señala a menudo con precisión las actividades básicas hacia las cuales se pretende canalizar el grueso de los recursos, olvidándose al parecer que tales recursos, que en general son escasos, no deben dispersarse;
6) Que las tasas de crecimiento previstas sólo podrán alcanzarse si se estimula el ahorro privado y se cuenta con un buen número de proyectos y con planes sectoriales bien coordinados; que, en particular, es preciso alentar la inversión privada incluyendo la inversión extranjera;
7) Que mientras más amplia sea la movilización de los recursos internos, mayor debería ser la ayuda financiera exterior, y que para estimar la importancia de esa movilización deben tomarse en cuenta las reformas realizadas en cada país, la política que se adopte para lograr el consumo no crezca más de prisa, que el ingreso nacional, a fin de lograr tasas de inversión satisfactorias y la contribución que se obtenga de los grupos de altos ingresos;
8) Que las reformas estructurales, -de que se habló en Punta del Este- no son una condición previa para la planificación ni para la obtención de ayuda del exterior, Lo que debe tomarse en cuenta es la "voluntad" que exista en cada caso para promoverlas, pues conforme a la Carta de Punta del Este, las reformas deben realizarse, pero en condiciones distintas y a diferente ritmo en cada país de América Latina.

Conforme al régimen aprobado en Punta del Este, un buen número de países ha elaborado sus primeros planes, pues entre otras cosas no podrían fácilmente obtener préstamos del exterior sin cumplir el requisito de hacer planes de desarrollo y someterlos a la consideración de los Comités Ad Hoc, de la Alianza para el Progreso, que funciona dentro de la OEA. Estos comités, que según se ha dicho oficialmente, "tienen funciones similares a las de los jueces" y cuyas decisiones habrán de ir formando jurisprudencia, se encargan de evaluar los programas o planes presentados por cada país, teniendo en cuenta para tal evaluación los siguientes criterios;

1). Que sin perjuicio de obtener ayuda del exterior, se haga un esfuerzo por utilizar mejor los recursos propios;
2) Que se tienda a distribuir mejor el ingreso, aunque de inmediato no se logre mejorar el nivel de vida;
3) Que se dé bastante atención a las inversiones sociales, aun cuando su relación capital-producto sea alta;
4) Que la inversión social no sea residual, sino que derive de una estimación independiente de las principales necesidades por satisfacer;
5) Que el país en cuestión revise su política tributaria, combata el consumosuntuario y las inversiones superfluas, a fin de obtener mayor capacidad de financiamiento del desarrollo;
6) Que se reconozca un papel importante al financiamiento del exterior, a fin de no imponer a la población cargas excesivas que deriven de una capitalización interna demasiado rápida;
7) Que las inversiones del Estado se orienten de preferencia hacia fines sociales y actividades de infraestructura.
8) Que la política del gobierno tienda a alentar y a ganar la confianza de la empresa privada;
9) Que se cuide de coordinar los planes a escala interamericana.

De la mera enunciación de esos criterios, se pueden advertir varias de las limitaciones de que adolece la planificación económica en Latinoamérica. aún teniendo en cuenta que las condiciones de cada país son distintas, creemos que algunos rasgos y defectos que parecen ser comunes; son los siguientes:

1)En general, los planes o programas se elaboran en forma antidemocrática, normalmente por pequeños grupos de expertos y sin la participación realde los principales sectores populares en su proyección, formulación y cumplimiento.
2) Aún en aquellos casos en que se presentan como planes nacionales, los elavorados parecen ser esencialmente programas de inverción pública;
3)Con frecuencia no existen programas sectoriales, y en otros casos no están debidamente coordinados entre sí y con los programas regionales y el plan nacional;
4) La coordinación entre el sector público y privado suele ser muy defectuosa y en algunos casos practicamente inexistente;
5) El estado tiende a aceptar un papel supletorio, complementario, y en el fondo subordinado a las empresas privadas, entre las que a veces predominan enpresas extranjeras a las que nada interesa racionalizar o acelerar un desarrollo económico independiente;
6) El sector público es a menudo débil y carece de recursos materiales y sus inversiones solo o principalmente se canalizan hacia actividades de infraestructura o a campos sociales que no interesan a la empresa privada;

7) Paradojicamente, dadas las dificultades que entraña una politica de movilización y utilización más racional del potencial productivo interno, los planes suelen hacerse para obtener más fácilmente recursos financieros en el exterior;
8) En atención a esa politica, para conseguirla hayuda extranjera y el apoyo de los grandes empresarios nacionales, los gobiernos renuncian a tomar ciertas medidas que serían esenciales en cualquier intento serio de planificación, pero que a la vez lesionarían esos intereses;
9) En vez de partirse, por ejemplo, el reconocimiento de que el imperialismo es el principal obstaculo a un desarrollo independiente y una planificación medianamente racional, y de que es preciso romper la dependencia bajo la cual han vivido nuestros paises, se tiende a contemporizar con los intereses extranjeros, en el marco de una politica débil y derrotista, conforme a la cual nada puede hacerse con éxito frente a intereses tan poderosos;
10) Los hechos que exhiben esta dependencia son numerosos; en el campo de la planificación, concretamente, los planes que cada país elabora en ejercicio de su soberanía nacional, son despues inexplicablemente revisados por comités Ad Hoc de la OEA, y el dictamen de estos Comités es de gran importancia para las instituciones internacionales en su decición de otorgar o no los creditos necesarios para financiar parte de las inverciones que en ellos se proyectan;
11) Desde otro punto de vista, los esfuersos planificados y programados tienden a veces a condicionarse a la existencia de ciertas informaciones estadisticas -como por ejemplo matriçes de insumo-producto-, como si de ello dependiera esencialmente la posibilidad de planificar;
12) Casi todos los planes son meras declaraciones de intención que con frecuencia no se cumplen ni por el sector público ni por el privado, y respecto a los que no existen mecanismos eficaces que garanticen su adecuada revición y ejecución;
13) Como es común en los sistemas de programación, los planes no señalan propiamente lo que ha de hacerse, sino más bien los que se cree que pasara en el futuro si se realizan las proyecciones estadísticas que les sirven de base: en otras palabras, más que planeación economica en el sentido estricto, se trata de pronósticos o previsiones;
14) En general, se pierde de vista que lo decisivo en un plan económico es lograr aumentar el nivel de inverción económica y canalizarla del modo más productivoposible, cayéndose en posiciones según las cuales la iniciativa privada queda como motor del desarrollo, y las inverciones sociales, de escasa productividad en la mayor parte de los casos, como las principales. Esto puede explicarse en virtud de que la Alianza para el Progreso, a su vez, más que un plan de largo plazo para liberar a Latinoamérica del subdesarrollo, es un intento para acallar a corto plazo, la inconformidad en los pueblos latinoamericanos;
15) Las reformas o cambios de estructura, que en 1961 se reconoció en punta de Este que eran esenciales, y que en mayor o menor medida siguen requiriéndose en todos los países de Latinoamerica, se están dejando de lado o sustituyendo por ajustes institucionales secundarios, mientras se pone énfasis en la integración económica regional y en un creciente endeudamiento con el exterior, que seguramente no podrán ser la base de una política y de una planificación económica adecuadas, que permitan superar los obstaculos fundamentales al desarrollo económico independiente de America Latina.

INDICE.

II. El Concepto de Desarrollo.

A. El significado de la idea de desarrollo.

1) El caracter comparativo y relativo del concepto de desarrollo.

El concepto de desarrollo económico es sumamente reciente en la terminología económica. En su sentido actual no se la comienza a usar y a aceptar sino después de la segunda guerra mundial. Tiene sus antecedentes en las nociones de evolución y progreso, propias de la gran expansión de la economía europea en los siglos XVIII y XIX bajo el signo del liberalismo. Sus antecedentes más inmediatos son las nociones de industrialización y de crecimiento. Todos éstos conceptos están sin embargo, limitados a problemas y son el producto de situaciones diferentes de los que encierra el concepto que nos ocupa.

Es un hecho histórico irrefutable que las sociedades en que se han logrado niveles de vida más elevados y mayor igualdad de oportunidades sociales son aquellas que han pasado por una gran expansión de su producción industrial, por una revolución industrial y por la consiguiente transformación integral de su vida social. Es también un hecho histórico irrefutable que solo un pequeño número de países y una escasa proporción de la población mundial han pasado por esa revolución industrial y alcanzado elevar los niveles de vida, como EUA, Europa noroccidental y Oceanía que representan menos de una cuarta parte de la población mundial. El resto se debate en una miseria mayor de la que prevalecía en Europa noroccidental hace 200 años.

Así pues, para Osvaldo Sunkel, el problema del desarrollo económico es esencialmente el de éste contraste violento y brutal entre las condiciones de vida de que disfrutan los habitantes de unos pocos países en relación a las que soportan los restantes.

De éste modo, dice Osvaldo Sunkel, para comprender el fenómeno del subdesarrollo en que quedaron los países de la América Latina es necesario comprender el significado de la revolución industrial en Europa durante el siglo XIX y las condiciones que permitieron su propagación a Norteamérica y Oceanía. El contraste entre las condiciones prevalecientes en esas áreas y las existentes en América Latina nos explica así las razones originarias por las que la revolución industrial no pudo propagarse en su debida ocasión h'stórica al área latinoamericana.

Por otrolado, dice Osvaldo Sunkel, aunque cada día es más claro que industrialización y desarrollo económico no son sinónimos, no deja de ser cierto que la industrialización -en mayor o menor medida- está siempre asociada al desarrollo económico, y también lo están algunas de las secuelas típicas que la industrializaciónha traido consigo en los países hoy industrializados: la urbanización, la monetización de las transacciónes económicas, el trabajo asalariado, las sindicalización laseguridad social, la mayor independencia individual dentro de la sociedad, la reducción en el tamaño de las familias, el trabajo femenino remunerado, la elevación de los niveles de vida, la mayor igualdad de opertunidades sociales, económicas y políticas, etc.

2) La formación de una economía internacional y la incorporación de las áreas periféricas,

El siglo XIX en su conjunto, pero particularmente el último cuarto de ese siglo presenciaron una enorme expansión del comercio internacional y la formación de una economía internacional integrada.

En la segunda mitad del siglo la economía europea comenzó a establecer centros productores de materias primas y alimentos en sus colonias y otras áreas nuevas para alimentar la colosal expansión de su población y producción. Estos países nuevos recibieron población, capital y técnica en abundancia e iniciaron un veloz proceso de expansión, llegando algunos de ellos en el presente a tener elevados niveles de ingreso y economías industrializadas y diversificadas, y quedándose otros al margen de ese desarrollo. En estos últimos países, la formación de sectores especializados de exportación en los que se lograba elevados niveles de productividad a base de la utilización de procedimientos tecnológicos modernos, tuvo una influencia muy limitada sobre el resto de la actividad económica que continuó desenvolviéndose en la forma primitiva tradicional.

3) Las condiciones sociales del desarrollo económico.

El análisis de las condiciones sociales para el desarrollo económico tiene particular interés desde dos puntos de vista. En primer lugar, porque es esencial para la formulación de una interpretación del proceso de desarrollo, tanto en lo que se refiere a la comprensión de sus orígenes móviles, orientaciones y resultados, cuanto por lo que toca a la explicación o diagnóstico de la situación presente y las tendencias futuras que existen en forma inherente y potencial en la actual situación.

Por otra parte, la comprensión de los valores, actitudes, motivaciones, instituciones y sistemas de estratificación de una sociedad son indispensables para evaluar la viabilidad y eficacia de la política de desarrollo propuesta. La política económica consta de una serie de estímulos y castigos económices que se aplican a las unidades económicas para que orienten su conducta en ciertos sentidos. Pero esas respuestas o reacciones de conducta estan en gran medida determinadas por los factores sociológicos que se acaban de mencionar. Además, la política económica se ejecuta por medio de una maquinaria administrativa, cuya eficacia y eficiendia también depende en parte de consideraciones de ese tipO.

La cita que resume elocuentemente el problema de las condiciones sociales del desarrollo, es la que sigue: "cuando hablamos del desarrollo de una economía y de la transformación de una sociedad, es esencial analizar los modulos y las formas económicas y sociales preexistentes, pues ellos son los que se transforman. No es, por lo tanto, por puro interés académico que vale la pena conocer lo que pre-existe al desarrollo, puesto que seráen ese módulo tradicional donde se encontrará la materia prima social e histórica, la masa con que se moldeará el nuevo modelo. Y esa materia prima consiste, sobre todo, en un hombre, un tipo histórico de hombre, con el cual y a favor del cual se promueva el desarrollo. Ese hombre de tipo tradicional, su vida cotidiana concreta, sus costumbres, sus instituciones y valores, la estratificación y la estructura social que lo enmarca -todo eso debe analizarse y comprenderse- pues es con el tabajo, el esfuerzo, el entusiasmo, la fé, las virtudes y, tal vez principalmente las insuficiencias de ese hombre "pre desarrollado", que se desarrollará una economía y una sociedad nuevas". (Luis A. Costa Pinto: O Desenvolvimiento: Seus Procesos eseus obstaculos. CEPAL).

III. La Teoría del Desarrollo Económico.

A. Teoría y realidad económica.

Los principales problemas económicos y sociales que nos preocupan actualmente en la América Latina, señala Osvaldo Sunkel, -el lento ritmo de crecimiento, el rápido crecimiento demográfico, la aceleración del proceso de urbanización, el rezago en la producción agropecuaria, la creciente concentración de la riqueza y del ingreso, la persistente presencia y a veces agudización de condiciones sociales intolerables en grandes sectores de la población, la inestabilidad y estancamiento de las exportaciones, la inflación y el desempleo- conforman en su conjunto la base problemática de la gran cuestión del desarrollo económico de ésta región.

En el amplio panorama de la historia económica dichos problemas no son desconocidos, pues cada uno de ellos ha caracterizado con frecuencia periodos o épocas de diversos países o grupos de países. Sin embargo, lo que se encuentra con más frecuencia es cada uno de esos problemas -o un entrelazamiento de dos o tres de ellos cuando más- predominando claramente en cada situación y configurando así el énfasis especial dado a su análisis e interpretación. No es este el caso actual de América Latina. Lo que interesa subrayar en nuestro caso es precisamente que ese conjunto de fenómenos, continúa Sunkel, en su apreciación integral e interrelacionada, ha llegado a constituir en latinoamérica un problema sobre el cual se ha creado conciencia colectiva en los últimos años. La superación de ese conjunto de problemas ha llegado a considerarse así un requisito fundamental para llegar a cumplir con las aspiraciones de construir una sociedad moderna, dinámica y justa, cuya visión también es propia y única pues emerge de la totalidad de la situa ción actual de América Latina. Y ésto plantea la cuestión de cuales son las orientaciones de una política económica y social apropiada a esos fines.

Al enfocar la cuestión desde éste punto de vista amplio, de considerar la totalidad de la problemática del momento, se aprecia con claridad que se trata necesariamente de un problema único, de una situación histórica singular. En consecuencia, en la historia económica no puede haberse producido antes de ahora, cuando por fín hemos tomado -- conciencia de nuestra problemática y formulado nuestras aspiraciones, un desafío intelectual derivado de una realidad similar a la que perseguimos. Por ello no podemos encontrar tampoco en la historia del pensamiento económico teorías o doctrinas, y modelos de interpretación, - que logren satisfacer plemamente nuestros esfuerzos para lograr una adecuada comprensión de la realidad actual y que así sirvan de base para precisar las líneas directrices de la política económica y social que las condiciones presentes de la región y sus aspiraciones para el futuro exigen.

Las teoría económicas de que disponemos en la actualidad, y los principios de política derivados de ellas son el producto de desafíos históricos que otras sociedades encontraron en el camino de su evolución. En esa época y lugar se convirtieron en problemas de los que se tomó conciencia colectiva, y su superación consciente llegó a ser preocupación fundamental pues constituía condición necesaria para cumplir con los objetivos de tales sociedades o de sus grupos dirigentes, de ahi el surgimiento del pensamiento económico medioeval, el del mercantilismo, los clásicos, neoclásicos, keynesianos,etc.

A lo largo de éste proceso histórico de creación intelectual en el campo de la economía se ha constituido así un importanteaarsenal de instrumentos de análisis en función de los problemas que diversas sociedades enfrentaron en su proceso de evolución. Dichos problemas fueron sin embargo diferentes a los que enfrentan los países poco desarrollados de hoy, no solo porque el marco histórico, tecnológico y social es diverso, sino también porque las aspiraciones sociales y condiciones políticas actuales divergen frontalmente de las que prevalecían en el caso de aquellas experiencias históricas que alguna similitu pudieron haber tenido con la situación actual.

En consecuencia, si aceptamos que el gran problema que nos plantea la sociedad actual es el desarrollo económico, contadas las implicacióne sociales, políticas, institucionales y económicas que el concepto acarrea, vemos que dicho problema es nuevo en la conciencia social y en la realidad económica, y por consiguiente no puede haber una teoría del desarrollo económico ya formulada; en todo caso ella estaría siendo formulada ahora, históricamente es imposible que lo haya sido antes de ésta época.

Es claro que para construir dicha teoría no vamos sin embargo a deshechar todo lo que ya se ha avanzado en la evolución del pensamiento económico. Hay elementos de dichos pensamientos, dice Sunkel, que obviamente no tienen nada que ver con nuestro problema, y por consiguiente esos elementos habrá que deshecharlos para que no conduzcan por caminos errados. Pero hay muchos elementos de análisis perfectamente adecuados, dice, puesto que surgieron de preocupaciones similares a algunas de las que constituyen el problema del desarrollo o pueden ser adaptados al estudio de talesasuntos. Estos elementos de análisis deben ser utilizados, pero ellos no constituyen por sí solos una teoría del desarrollo. Deberán ser organizados de una manera diferente, junto con elementos de análisis nuevos que aún es preciso crear, para que sea posible formular una teoría del desarrollo económico que esté basada en las características del mundo subdesarrollado de hoy. Esta no es tarea fácil, señala Sunkel. No se trata simplemente de una creación intelectual genial. Ello no satisfaría porque aquellas características que la teoría debe reflejar no son aún suficientemente conocidas. Hay que superar todavía una etapa larga y dificil de investigación estadística de base para llegar a formular una teoría razonablemente adecuada, es decir, basada en supuestos realistas.

Como ha señalado Stark, "existen, en úlimo análisis, dos maneras de considerar la historia del pensamiento económico: una, es juzgarla como un contínuo adelanto del error a la verdad, o cuando menos de una visión confusa y parcial a una percepción clara e inteligible; la otra consiste en interpretar toda teoría particular expuesta en el pasado, como la cabal expresión y reflejo de las condiciones contemporáneas, entendiendola así en su orígen histórico y en su significado".

En síntesis, apunta Sunkel, los modelos o teorías de desarrollo, crecimiento o estagnación formulados por diversos autores o escuelas de pensamiento solo tienen pleno valor interpretativo y vigencia como fuente de normas de política económica cuando se les refiere al periodo -- histórico y a la realidad social de la cual forman parte. No obstante, por lo que se refiere a los instrumentos de análisis utilizados para describir, aprehender y clasificar los elementos constituyentes de un sistema económico, así como a las relaciones funcionales que suelen encontrarse entre dichos elementos y también en lo que respecta a los factores que en diversas circunstancias han determinado el proceso de desarrollo, no cabe duda que la evolución del pensamiento económico ha ido acumulando y destilando un bagage sustancial. Disponemos en efecto de un importante acervo de métodos descriptivos y analíticos, de una selec ción básica de relaciones funcionales características y de algún consenso sobre el conjunto de factores estratégicos en el proceso de crecimiento.

IV. La Contribución de la Teoría Económica al Análisis del Desarrollo, Según Osvaldo Sunkel.

A. Los instrumentos del análisis económico.

A lo largo de la evolución del pensamiento económico se ha ido definiendo y precisando el concepto analítico de "sistema económico", es decir, se ha ido identificando los principales elementos que intervienen en el proceso económico y las principales categorías en que se pueden clasificar o descomponer esos elementos, a la vez, al precisar esos elementos y categorías típicas del sistema económico, se han podido establecer las siguientes relaciones de funcionalidad que la observación sugiere que existen entre dichos elementos y categorías.

Para aclarar lo anterior con un ejemplo, podría trazarse a lo largo de la evolución del pensamiento económico el concepto de producción para ver como este concepto se ha ido precisando a fin de llegar a distinguir los diferentes elementos -recursos naturales, capital, trabajo y técnica- que constituyen los factores responsables de la producción, para ligar funcionalmente los recursos con la producción por medio del concepto de productividad, y para distinguir también entre diferentes categorías de producción -bienes y servicios, de capital y de consumo- e incluso para llegar a definiciones diversas del concepto de producción según se requiera para el análisis: producción intermedia, valor agregado o valor bruto de la producción.

La contabilidad nacional, concebida en forma amplia para incluir no sólo la contabilidad referente al flujo anual de producción de ingresos, agregada y por actividades, sino para cubrir también la contabilidad de los recursos productivos (particularmente el capital y la fuerza de trabajo) y la de los flujos financieros, así como la de los principales sectores de la actividad económica, constituye sin duda la expresión más acabada de ese instrumental analítico-descriptivo que la ciencia económica ha ido construyendo y que le permite dar a conocer la realidad económica a través de la medición de categorías significativas desde el punto de vista analítico.

De este manera no cabe duda que se ha llegado a un grado grande de perfección en cuanto al estudio de la "anátomía" del sistema económico, aún cuando por cierto es todavía inmenso el trabajo empírico que queda por hacer en los países poco desarrollados para llegar a una descripción medianamente adecuada de la realidad objetiva de tales países, expresada cuantitativamente.

Por otra parte, todavía falta mucho para la construcción de una teoría del desarrollo de los países poco desarrollados, es decir, para el conocimiento de la "fisiología" o funcionamiento del sistema. Los economistas han construido en el pasado modelos de funcionamiento y de crecimiento de la economía capitalista sobre la base para identificar como fundamentales en el proceso de crecimiento algunos de esos elementos y categorías del sistema económico a que hemos hecho referencia, y de suponer determinadas relaciones de funcionalidad entre ellos. Esos modelos, construidos como reflejo abstracto de una determinada época y sociedad, y de sus problemas de crecimiento, no incluyen necesariamente los mismos elementos que nosotros consideraríamos indispensables en toda explicación del desarrollo de nuestros países; por otra parte no supondrán necesariamente el tipo de relaciones funcionales que encontramos actualmente en nuestras economías subdesarrolladas.

Con todo, un examen retrospectivo de las ideas de los principales exponentes del pensamiento económico que se han ocupado del desarrollo nos permite identificar una serie de elementos que en casi todos ellos constituyen factores claves para la explicación del proceso. Por otra-parte, incluso es posible detectar algunas relaciones funcionales que también parecen persistir no obstante las diferencias de enfoque y de época. Lo que es más difícil encontrar es la definición precisa de esas relaciones funcionales a través de una función matemática explícita.

Con todo, será difícil que en la construcción del modelo de desarrollo de los países poco desarrollados se pueda dejar de tener en cuenta esos elementos y esas relaciones funcionales que la evolución del pensamiento económico nos ha dejado como herencia. Es obvio que habrá que reajustarlos a las realidades de la época y que habrá que introducir nuevos elementos y relaciones funcionales para corresponder a la exigencia de explicar la realidad actual.

B. Los principales factores del desarrollo.

Aunque el pensamiento neoclásico y el keynesiano y postkeynesiano lo ignoran casi completamente, los clásicos muestran en todos sus modelos una preocupación muy grande por el problema de la dotación de recursos, y particularmente de los recursos naturales, que en su caso significaba simplemente la tierra cultivable. No interesaba tanto la magnitud absoluta de tierra disponible, sino que ello intreresaba en relación al volumen de la población y en cuanto a su ritmo de crecimiento. En otras palabras, los clásicos destacaron el problema de la presión de la población sobre los recursos tanto en términos estáticos -la densidad de población- como en términos dinámicos -la capacidad de los recursos para sustentar un creciente nivel de vida.

En este último sentido destacaron el problema de la combinación y complementariedad de los factores productivos, señalando que si uno de ellos era fijo, ello imponía un límite al crecimiento por la operación de la ley de rendimientos decrecientes. Esta apreciación fue el orígen de la importancia que en todos los autores y escuelas de pensamiento se ha dado al fenómeno del avance de la técnica productiva.

El progreso técnico o las innovaciones técnicas han constituído un elemento clave en todas las teorías del desarrollo ya que a través de él se producen los incrementos en la productividad o potencialidad de los recursos productivos, hasta el extremo que el avance de la técnica puede llegar a reemplazar las deficiencias que un país pueda tener en alguno de sus recursos fundamentales.

La importancia que se ha asignado en el proceso de elevación de la productividad -que es el fenómeno que sustenta todo el proceso de elevación de los niveles de vida- al avance de la técnica, ha llevado naturalmente a destacar tres elementos que están íntimamente asociados al proceso de introducción de la técnica moderna: la acumulación de capital, la función del empresario innovador y el grado de adiestramiento y capacitación de la población.

La acumulación de capital ha venido a constituirse en realidad en la piedra angular de todo modelo moderno de crecimiento, no sólo porque es el vehículo de la innovación técnica -todo avance técnico se materializa en definitiva en un bien de capital o en un instrumento de producción concreto-, sino también porque en una economía moderna toda la producción se realiza por intermedio de dichos bienes de capital o instrumentos de producción. Por consiguiente, la acumulación de capital, y el nivel técnico que conlleva, constituyen la determinante de la productividad y de la capacidad productiva, particularmente en aquellos casos en que el factor relativamente escaso es precisamente el capital.

El aprovechamiento adecuado del capital está en parte determinado por las destrezas y capacidades de la mano de obra que lo instala y lo opera, así como por los que dirigen el proceso productivo. Esto ha llevado en la mayor parte de los autores a señalar la importancia de la educación como factor del desarrollo, aunque en general este factor no se haya incorporado explícitamente en sus modelos.

Por otra parte, el acento que se ha puesto en la acumulaciónde capital ha permitido destacar la importancia de los procesos de ahorro e inversión y de los individuos que en la comunidad realizan en último término las decisiones de invertir, es decir, las decisiones de acumulación y en consecuencia también las decisiones de innovación técnica.

Schumpeter es quien ha destacado con más vigor y en forma más clara las funciones preponderantes que cumple en el sistema capitalista el empresario innovador, el agente productivo fundamental de la comunidad. Sus tareas son evidentemente claves para el progreso económico desde el momento en que en él se resumen los aspectos relativos a la organización de la producción, a la asignación de los recursos entre usos alternativos y a la introducción de innovaciones técnicas.

La preocupación de los autores clásicos por los límites a que tendía el crecimiento del sistema capitalista llevó a destacar por una parte la importancia trascendental del progreso técnico, pero permitió también a algunos autores -particularmente Smith, Ricardo y Marx- comprender la significación que tiene la amplitud del mercado como factor que permite la acumulación y los incrementos de productividad.

Las consideraciones sobre la amplitud del mercado llevaron precisamente a los neoclásicos a hacer sus aportes más significativos a la teoría del desarrollo al destacar las economías internas y externas que podían obtenerse dentro de mercados suficientemente amplios y los efectos que ello tiene sobre la productividad del capital. Por otra parte, ello llevó también a destacar la importancia que desde este punto de vista tiene la distribución del ingreso y el nivel real de salarios, determinado en gran medida por la productividad y la producción agrícola, así como por el excedente de mano de obra.

Otro factor que aparece en todas las teorías del crecimiento es la población, sea simplemente para destacar el problema de la dotación relativa de recursos, para confrontar el crecimiento demográfico con la disponibilidad del recuso tierra, o para mostrar la influencia estimulante del crecimiento demográfico sobre la demanda.

C. Limitaciones e insuficiencias de las teorías del crecimiento.

El recuento de factores fundamentales del desarrollo que se destacan en la literatura sobre este tema, afirma Sunkel, muestra un conjunto de elementos que sin duda tendrán que estar presentes en todo intento de interpretación del proceso de desarrollo de los países poco desarrollados. Sin embargo, es obvio que la lista es incompleta, que ningún modelo construido sobre la base de los factores señalados podría explicar satisfactoriamente el subdesarrollo.

Una primera omisión absolutamente clave, señala Sunkel, es la que se refiere al comportamiento del estado o del sector público en el proceso de desarrollo, n'sólo en cuanto se refiere a su participación directa en las transacciones económicas sino también, y tal vez principalmente, por lo que atiende a su acción indirecta, a través de la política económica.

La política de desarrollo económico está de hecho totalmente ausente del pensamiento económico hasta que este problema se plantea en los países poco desarrollados. El conjunto de doctrinas que forman la política económica moderna -la política monetaria, la política fiscal, la política de salarios, etc.- es en su orígen enteramente ajena al problema del desarrollo; responde casi exclusivamente a los objetivos de la ocupación plena y de la estabilidad monetaria y cambiaria. Su aplicación al caso de los países poco desarrollados, señala Sunkel, sufre de dos deficiencias decisivas: en primer lugar, que los objetivos que tales políticas persiguen no coinciden necesariamente con los objetivos primordiales del desarrollo económico; en segundo ligar, que dichas políticas se derivan y son aplicaciones de modelos estáticos de equilibrio, de modo que no corresponden ni analíticamente al enfoque que requiere el problema del desarrollo, ni prácticamente a las condiciones-institucionales y estructurales que presentan las economías subdesarrolladas.

La realidad del mundo moderno, y particularmente de los países - poco desarrollados nos confronta con sistemas económicos mixtos, en -- los que coexisten una economía de mercado imperfecta con un sector estatizado bastante amplio. Esto plantea problemas concretos de política económica que la teoría del desarrollo económico ha desconocido por -- completo.

Un segundo elemento que está presente indisolublemente con el -- subdesarrollo actual, y que también es casi enteramente ajeno a las -- preocupaciones de los autores de la teoría económica anteriores, es el que se refiere a la influencia del comercio internacional y de los movimientos internacionales de capital en el desarrollo económico de los países poco desarrollados. Como se ha explicado anteriormente, apunta Sunkel, al cotejar las condiciones históricas en que se ha dado el desarrollo en dichos países, queda perfectamente claro que la incorporación del progreso técnico y del sistema capitalista moderno en éstos - países se llevó a cabo inicialmente por medio de la intervención extranjera y la exportación de alimentos y materias primas. El desarrollo de sectores especializados de exportación fué en todos éstos países el nucleo originario del proceso de desarrollo, y la forma en que el progreso técnico y las formas modernas de producción se propagaron al resto de la economía, así como la evolución del propio comercio exterior, -- han sido factores determinantes en la formación de las estructuras subdesarrolladas características de nuestros países.

Este tipo de estructuras características del subdesarrollo actual

-especialización en unos pocos productos primarios de exportación, discrepancias enormes de productividad entre sectores de la actividad económica, coexistencia de formas modernas y altamente capitalizadas de producción con estructuras productivas tradicionales y primitivas, -- mercados internos de productos y sobre todo de factores escasamente integrados, nivel medio de ingreso relativamente elevado en contraste - con una estructura productiva muy primitiva, etc. -no estan por cierto presentes en las teorías del desarrollo que hemos heredado. Aquellos m modelos fueron construídos con un grado muy elevado de agregación, suponiendo en el fondo un sistema económico relativamente homogeneo. Así, dice Sunkel, los modelos de explicación del subdesarrollo actual no se conciben en cambio sino en términos de la desagregación sectorial. Solo así, dice, es posible comprender el problema de la transformación - estructural que está planteado a los países subdesarrollados, y los -- consiguientes obstáculos, resistencias y trabas de todo orden que se - oponene a la propagación del progreso técnico a todos los sectores y - niveles de la actividad económica.

Hay finalmente otra característica, nos dice Sunkel, del mundo - sibdesarrollado de hoy que no podría dejar de tomarse en cuenta. Es el hecho de que los países subdesarrollados de hoy lo son porque coexisten con países desarrollados. Como se señalaba al definir el concepto de desarrollo, éste es esencialmente comparativo, no puede definirse sino en relación a los países desarrollados, El proceso histórico de formación de un centro mundial de países industrializados, rodeados de una periferia de países subdesarrollados, condiciona y enmarca el problema del subdesarrollo actual como enteramente diferente al que enfrentaron los países ahora desarrollados en periodos anteriores.

Este somero examen crítico de las aportaciones a la teoría del d desarrollo, dice Sunkel, nos revela en resumen las siguientes limitaciones: escasa consideración al papel del estado y de la política económica; insuficiente preocupación con la influencia del comercio exterior y la inversión extranjera en la estructuración de los países subdesarrollados; elaboración de modelos de desarrollo excesivamente agregados que no permite apreciar el proceso de transformación estructural inherente al desarrollo; insuficiente preocupación con las condiciones institucionales y las características del medio social como determinantes del grado de aprovechamiento de los recursos productivos, del funcionamiento de los mercados, de la promoción de los agentes innovadores, etc.; formas en que la existencia de países desarrollados condiciona el desarrollo de los países periféricos.

V. El Subdesarrollo Latinoamericano. Su Trasfondo Estructural.

1) Los problemas básicos.

Como los problemas solo pueden definirse en términos de objetivos, nos dice Sunkel, se comenzará por precisar los objetivos de la política de desarrollo en América Latina. Aunque la situación varía entre un país y otro, y de un periodo a otro, puede suponerse que los objetivos a largo plazo de la política de desarrollo de todos los países de la región son los siguientes: maximizar la tasa de crecimiento de la economía; mejorar la distribución del ingrso; aumentar las oportunidades de empleo; y alcanzar y mantener un grado de estabilidad financiera razonable. Así, nos dice Sunkel, la situación en América Latina medida con éstos patrones, es la siguiente.

La tasa de crecimiento.- La economía latinoamericana aumentó persistentemente su tasa de crecimiento desde mediados de la década de 1930 hasta principios de los años 50, gracias a la tendencia expansionista registrada por una economía mundial que se recuperaba de los efectos de la gran crisis y que entraba en el periodo de auge. Entre 1945 y 1955 se alcanzaron tasas de crecimiento bastante elevadas, aunque ésta tendencia se invirtió en el último decenio cuando la relación de intercambio se tornó desfavorable a los exportadores latinoamericanos de productos básicos. El efecto negativo de ésta tendencia sobre el ritmo de crecimiento del ingreso por habitante se ha agravado entre tanto debido a la fuerte aceleración que ha experimentado la expansión demográfica. La comparación de las series del PIB con el índice de la relación de intercambio tanto desde el punto de vista de las tendencias del desarrollo latinoamericano a largo plazo como asimismo del aumento brusco del PIB en 1964, revela claramente la influencia decisiva que ejerce en los --

mercados mundiales de los productos básicos exportados por América Latina en las condiciones de largo y de corto plazo de la economómía de la región. La significación de éste hecho se revela en toda su gravedad cuando se advierte que la tasa de crecimiento del PIB comienza a disminuir precisamente cuando empieza a aumentar el volumen de exportaciones con tanta rapidez que en 1964 ya superaba en 66% el promedio 1950/55.

Se podría aducir, dice Sunkel, que no se justifica hablar de la región en su conjunto. Sin embargo, pese a las grandes diferencias existentes entre los países de América Latina es legítimo hablar con éste grado de generalidad, en vista de su semejanza en importantes características estructurales. Tanto en los países grandes como en los pequeños, las economías que crecen con rapidez y las que son relativamente estacionarias, las naciones de ingreso por habitante mas alto y las de nivel bajo, en casi todos los casos la tasa de desarrollo acusa en la post-guerra una tendencia de largo plazo a la baja. Más aún, observa Sunkel, han llegado a un punto en que la tasa de incremento del nivel medio de vida de la población -medido por el PIB por habitante- ya no ofrece esperanzas de una mejora apreciable de las condiciones de vida en un lapso prúdencial. En el periodo 1955/61 solo tres países-Brasil, Perú y Venezuela- superaron la tasa media de crecimiento del PIB por habitante de América Latina, que fué de solo 1.4%.

Por otra parte, excepción hecha del Brasil, en todos los países se observa un aumento sustancial del PIB en 1964. Podría estarse iniciando otro periodo de rápida expansión, pero el auge bien podría llegar a su fin el año siguiente si los mercados de productos básicos -- vuelven a la "normalidad". En algunos como Argentina Chile y México, ésto obedece principalmente a la reanudación de políticas expansionis tas luego de algunos años de esfuerzos de estabilización en que se --aplicaron políticas restrictivas. Así, comenta el autor, éstos resultados favorables son el producto de factores circunstanciales y sería aventurado suponer que constituyen el inicio de otro periodo de auge secular.

A éste respecto, cabe obsrevar que mientras la tasa de desarrollo se vincula claramente con las condiciones externas -elemento al cual no le asignan mucha importancia las teorías del desarrollo convencionales- la tasa de formación de capital, a la cual suele atribuirse principalmente el desarrollo, no parece estar relacionad a en forma clara con el crecimiento.

Resumienndo, sería lícito concluir que: la tasa de desarrollo económico -medida por el crecimiento del PIB por habitante- ha decrecido seriamente durante los últimos 15 años; que ha llegado a niveles francamente bajos en la mayoría de los países; y que, pese al gran esfuerzo desplegado en materia de industrialización y desarrollo, sigue dependiendo a corto y a largo plazo, del comercio exterior, y especialmente de la relación de intercambio.

Oportunidades de empleo. Una de las características relevantes de los países latinoamericanos es la coexistencia en todos los sectores de la actividad económica de métodos muy avanzados y muy primitivos de producción. Como el volumen de empleo por unidad de producción es mucho menor cuando se emplea una tecnología moderna que una primitiva, las oportunidades de empleo tienden a crecer muy lentamante. Es lo que ocurre especialmente cuando las actividades modernas no dan lugar a un aumento neto de la capacidad productiva sino que sustituyen en cierta medida la producción de las actividades tradicionales de gran densidad de mano de obra. Esto, junto con una fuerza de trabajo que aumenta con rapidez, resulta en que el excedente de mano de obra bien puede estar aumentando en lugar de absorberse.

En resumen, la política de desarrollo de América Latina ha sido muy deficiente en lo que toda a la oportunidad de creación de empleos, y las perspectivas a largo plazo son francamente abismantes, El examen del problema general de crecimiento y empleo en Puerto Rico, que es una especie de caso piloto porque la mano de obra superflua puede emigrar a EUA, muestra que el empleo disminuyó entre 1950-60 en tanto que la economía se expandía muy rapidamente, pasando su ingreso per cápita de 269 dls, en 1940 a 673 em 1961.

Estabilidad. Este es uno de los problemas más conocidos de América Latina. La situación que reflejan los índices de precios no es - solo muy desfavorable en la mayoría de los países, sino que ha empeorado en algunos de ellos en los últimos años. Para frenar la creciente inflación muchos países de América Latina han ensayado en los últimos 10 años varios programas y medidas de estabilización, pero ninguna parece haber tenido éxito hasta ahora, sobre todo, debido a que no han sido lo suficientemente rigurosas y persistentes.

2) Problemas y objetivos del desarrollo.

Un examen superficial de los efectos de la política de desarrollo en América Latina es no obstante muy revelador. La tasa de crecimiento del PIB por habitante ha declinado apreciablemente desde comienzos de los años 50 en casi todos los países, incliyendo Brasil, México y Venezuela cuyas economías eran muy dinámicas. La enormen influencia que ejercen los factores externos sobre las fluctuaciones a largo y -- corto plazo hacen dudar de la eficacia real de la política de desarrollo. La distribución del ingreso acusa un mejoramiento en la posición relativa de la alta clase media, en desmedro de los grupos de bajos in gresos y en algunos casos del 10% de más altos ingresos. Uno de los problemas más graves que tendrían que encarar nuestras economías, es el - de las oportunidades de empleo, y en éste caso la situación no solo se deteriora, sino que ni siquiera se está considerando la aplicación de una política positiva. Por otra parte, se han aplicado varios programas, de estabilizaqión para solucionar los problemas monetarios. En los casos en que la inflación es grave, éstas medidas han fracasado sistemáticamente y con frecuencia han contribuido a intensificar las presiones inflacionistas y a provocar contaacciones económicas.

Es faćil concluir de todo lo anterior que en América Latinam los objetivos del desarrollo se han cumplido en forma muy deficiente. Hoy se espera que los países insuficientemente desarrollados crezcan con mucha rapidez, mejor en la distribución del ingreso, ofrezcan amplias oportunidades de empleo a su crecientefuerza de trabajo, y que todo -- ellà ocurra en condiciones de absoluta estabilidad financiera. Quizás sea exigir demasiado de países en que no existen las condicione indispensables ni para que funcione eficazmente el sistema económico ni para formular y aplicar una política eficaz de desarrollo.

3) Algunas características estructurales de las economías latinoamericanas.

Es dificil entender el fùncionamiento de las economías de la región y las peculàaridades de sus procesos, políticas y problemas de desarrolão, si no se los relaciona con los acontecimientos históricos de donde arrancan las principales características estructurales e institucionales de éstas economías.

Debe reconocerse claramente que cada país de la América Latina tiene sus propias características y conjunto de problemas, que son con secuencia de diferencias en el tamaño de los países, en su dotación de recursos natuzales, en las características técnicas de su población, - en su localización y en el nivel de industrialización alcanzada.

Sin embargo, algunos de los problemas y características sociales y económicas básicas son muy semejantes. Las raíses de esas características comunes se apoyan principalmente en el trasfondo colonial español y portugués, en la forma característica en que se incorporaron las economías latinoamaricanas a la economía internacional a fines del sigloXIX y principios del XX, y en el proceso de industrialización y divessificación de la economía que se inició en algunos países hace 4 decenios y que ahora está presente en casi toda la región.

A) La propiedad de la tierra y los recursos naturales.

Los factores iniciales que explican la concentación pronunciad a de la propiedad de la tierra en pocas manos deben buscarse en la historia remota de los gegímenes doloniales implantados por españoles y portugueses y en los sistemas de repartición de la tierra de los conquistadores. Durante el sigloXIX, surgió un factor adicional bajo la forma de consesiones públicas de tierras y recursos naturales para la formación de grandes plantaciones y haciendas ganaderas y la organización de actividades mineras de exportación, algunos de los cuales quedaron en propiedad de extranjeros. Estos sectores de exportación alcanzaron niveles altos de productividad al compartir la tecnología de la economía internacional de la que eran una avanzada. Los sectores tradicionales no se vieron muy afectados y solo sintieron su influjo y de reorganizaron las actividades relacionadas con el comercio internacional. Como consecuencia de éstos factores históricos, la gran concentación de la propiedad de la tierra sigue siendo una de las características de - América Latina, excepto en menor medida en países como México, Cuba y

Bolivia, que han experimentado cambios revolucionarios en el régimen de tenencia de la tierra.

B. Dependencia del comercio exterior e inestabilidad.

Una de las características más conocidas de la región es su extrema dependencia de la exportación de algunos pocos productos básicos, - de la que derivan una gran proporción de sus ingresos de divisas, aunque países como México, Argentina y Perú tienen una gama de exportaciones mucho mayor. Es bien sabido que los mercados mundiales de productos primarios son muy inestables y por tanto, que las entradas de divisas provenientes de sus exportaciones fluctuan tambien ampliamente. El efecto de éste hecho probablemente tienda a ser mayor en aquellos países que dependen de un número más limitado de productos de exportación. El elemento esencial de ésta vulnerabilidad a las fluctuaciones del comercio exterior no es el tamaño relativo del sector externo sino la estructura de las exportaciones e importaciones.

C. Escasez y orientación del capital social básico.

La especialización de las economías latinoamericanas en torno de sus actividades de exportación y la expansión dinámica de los mercados mundiales correspondientes desde fines del siglo XIX hasta 1929, permitió un incremento de los niveles de vida de los exportadores y de la poblaciones urbanas, cuyas necesidades crecientes se satisfacían primordialmente gracias al aumento de las importaciones. Al estancarse los mercados mundiales en 1929 y mientras las guerras detenían la corriente normal del importaciones, los países de la región diversificaron la estructura de su producción. Aquí, su falta de integración económica interna se hizo evidente, ya que no se habían creado las condiciones necesarias para establece nuevas fábricas,etc., estando orientadas éstas o condiciones, además, a satisfacer las necesidades del sector exportador. Lo mismo sucedía con el sistema financiero y el mercado local de capital era insignificante, todo lo cual determinaba el caracter "centrífugo" de la economía latinoamericana. Ahora, pese a las invesiones cuantiosas en capital social básico (infraestructura, etc.) que se han efectuado en nuestros países durante los últimos decenios, quedan aún muchas necesidades insatisfechas.

D. Capacitación y educación de la población.

La naturaleza fragmentaria de las economías especializadas de -exportación en América Latina ha dejado a un gran segmento de la población al margen de los sectores modernos de la economía. Esta es una de las razones principales de la limitación de la educación primaria a la mitad de los niños latinoamericanos entre 5 y 14 años. La educación secundaria es aún más deficiente, en la capacitación vocacional técnica, puesto que su utilización solo apareció cuando la estructura de la producción de las economías de la región iniciaron su proceso de cambio.

E. Distribución del Ingreso.

La distribución del ingreso en la región es extremadamente dispareja. Según un estudi de la CEPAL, las diferencias serían esencialmente las siguientes:

a) el los países dela región hay mayor concentración del ingreso en un reducido porcentaje de la población, que es el sector social que tiene ingresos más elevados, ya que una tercera parte de los ingresos se concentra en un 5% de la población.

b) los sectores sociales que se encuentran en las escalas inferiores y que representan la mitad de la población total, solo po... en América Latina el 16% del ingreso total.

c) el cotejo de los niveles correspondientes a los grupos de altos ingresos con el promedio también pone de manifiesto diferencias considerables. Así, en América Latina el índice correspondiente es 6.5 superior al promedio de la sociedad en su conjunto.

d) la comparación de los grupos de ingresos más bajos con el promedio de la región da un índice de 32% en América Latina.

e) ésta caracterización de la distribución del ingreso puede apreciarse en forma más notoria observando la diferencia entre los ingresos promedios de los dós extremos: en América Latina, el promedio superior es 20 veces más alto que el inferior.

4) Algunas tendencias a largo plazo del proceso de desarrollo en América Latina.

En economías que se han especializado en la producción de productos básicos para la exportación y en las cuales la economía interna ha permanecido en un estado más primitivo, la creciente variedad de bienes y servicios asociados a niveles de vida más altos se obtienen por medio de importaciones.

Si el mercado de exportación de tales países se deteriora o se estanca, las políticas a largo plazo que éstos podrían aplicar serían: ceñirse al objetivo de mantener el equilibrio del balance de pagos y ajustar el nivel de actividad a las tendencias del dicho balance o bien dar impulso a la producción de manufacturas, si la solución anterior resulta intolerable. Esto equivaldría a transformar la estructura de producción de la economía.

En América Latina, la elección parece haber sido determinada por la crisis mundial de 1929-33. Las economías más pequeñas y menos desarrolladas, y en especial los países centroamericanos y del Caribe, se vig ron forzados a aceptar éste impacto y a continuar con su patrón tradicional de crecimiento, una vez que el mercado internacional se normalizó.

En países como Argentina, Brasil y México, que a fines de los años 20 habían alcanzado ya un nivel relativamente alto de ingreso por habitante y donde se habían dado ya los pasos iniciales hacia el desarrollo de la manufactura, tuvo repercusiones severas y las presiones sociales y políticas hicieron imposible unproceso de ajuste similar. Estos países aislaron sus edonomías con medidas diversas de protección (aranceles, devaluaciones, etc.) y trataron de mantener el nivel del ingreso a través de una política de gastos públicos anticíclica. Todo ésto, determiró un gran cambio favorable a las manufactúras en los precios relativos, comenzandose a producir sustitutos de las importaciones.

La inestabilidad y falta de expansión de los mercados extranjeros durante el decenio de 1930, y la restricción de las importaciones por la guerra mundial, condujo por lo tanto a una política de desarrollo de las fuentes internas de oferta de bienes y servicios, y al mismo tiempo, creó un clima propicio a la creación y crecimiento de la industria manufacturera.

La instalación de industrias manufactureras de bienes de consumo en economías dedicadas principalmente a los productos básicos obliga a importar los bienes de capital e intermendios necesarios para establecer y explótar las fabricas. Por lo tanto, la importación de manufacturas terminadas se ha sustituido gr adualmente por la impoitación de bienes de capital y de los productos intermedios necesarios para producir esas manufacturas en el país. De esta manera, el proceso de industriälización ha significado, por un lado, un cmbio en la estructura de las impoertaciones, y por otro, unproceso de transformación de la estructura productiva.

De aquí que el rápido avance del sector indùstrial, tanto en términos absolutos como enrelación con los otros sectores de la economía, origíné en toda ella tensiones muy agudas. Este avance exige además -- que todos los sectores, las actividadesy la utilización de los recursos económicos cambien en direcciónes determinadas y con tasas determinadas para satisfacer requerimientos crecientes de mano de obra, recursos natuzales, capital y de una variedad de bienes y servicios, nacionales e importados. Si algunas actividades no reaccionan con presteza ante lo que se exige de ellas, no será posible manrener el equilibrio dinámicq de la ecônomía y si los estrangulamientos llegan a ser suficientemente graves, pueden detener del todo el proceso de crecimiento. Así, se observa que se tendrían que cumplir un gran númenro de condiciones nada fáciles para obtener un equilibrio que permita el crecimiento sostenido de la economía.

Por lo que se refiere a las condiciones internas, la más importante es el tamaño del mercado nacional para las industrias manufactureras que se establezcan. No se trata solo de que los ingresos por habitante sean bajos, sino que la pareja distribución del ingreso reduce las posiblidades de formar mercados de masa para la mayoría de las manufacturas. Por otra parte, la tecnoñogía moderna se ha creado en gran parte para tales mercados de masa, de modo que la existencia de mercados pequeños puede significar la utilización antieconómica de equipo técnicamente modérno. Esta cuestión es de fundamental importancia ya - que fija un límite a las posibilidades de industrialización de algunos países de la región si solo se consideran los mercados nacionales. Desde el punto de vista del equilibrio del proceso de crecimiento, el tamaño del mercado constituye un elemento decisivo ya que en gran parte define las posibilidades del proceso de industrialización. Por otra -

parte, las demandasque éste impone a otros sectores de la economía, si no son satisfechas, pueden convertirse a su vez en otras limitaciones al proceso de industrialización.

Paradar equilibrio a éste proceso es necesario lograr la expansión de la capacidad productiva en todos los sectores de la economía, lo cual significa naturalmente, un gran esfuerzo de inversión, tanto pública como privada. En particular, habría que superar varios de los límites que imponen al proceso de industrialización, el a traso de la producción y la productividad agrícolas, las dificultades de desempeñar con éxito las formidables tareas que el sector público ha de ejecutar ya que actualmente, para realizar todas las tareas que se presisan, cuenta con un aparato administrativo incompleto y anticuado, heredado de épocas cuando ninguna de sus nuevas funciones entraba en la esfera de competencia del gobierno; además, su base financiera ha quedado rezagada, y la estructura política de los países impide llevar adelante una reforma tributaria a fondo y establecer una administaación eficiente.

VI. Conclusiónes.

Para concluir, tenemos que para Osvaldo Sunkel, los principales problemas económicos y sociales que nos preocupan actualmente en la América Latina, conforman en su conjunto la base problemática de la grán cuestión del desarrollo económico de ésta región.

En el caso actual de América Latina, para Sunkel, lo que interesa subrayar es precisamente ese conjunto de fenómenos (el lento ritmo de crecimiento, el rápido crecimiento demográfico, la desigual distribución del ingreso, nuestra depe dencia externa,etc.), en su apreciación integral e interrelacionada, ha llegado a constituir en latinoamérica un problema sobre el cual se ha creado conciencia colectiva en los últimos años. La superación de ese conjunto de problemas ha llegado a considerarde un requisito fundamental para llegar a cumplir con las aspiraciones de construir una sociedad moderna, dinámica y justa.

Por lo que hace a las teorías del desarrollo tradicionales, podemos concluir que , en opinión de Sunkel, si aceptamos que el gran problema que nos plantea la sociedad actual es el desarrollo econñomico, con todas las implicaciones sociales, pòlíticas, institucionales y económicas que el co cepto acarrea, vemos que dicho problema es nuevo en la conciencia social y en la realidad económica, por lo que no puede haber una teoría del desarrollo económico ya formulada. Es a nuestro periodo histórico al que corresponde formularla.

La interpretación dada por Sunkel de las principales tendencias y problemas del desarrollo debe llevarse más lejos, a fin de analizar la realidad de asuntos tales como la debilidad del sector privado, las actitudes de los empresarios, las condiciones del mercado de capital, y otros aspectos que suelen estrar en el ámbito de la investigación eco nómica. Asimismo, también debe extenderse en otra direcciones para analizar procesos y problemas sociales y políticos, sin lo cual es muy poco lo que puede decirse de provecho en materia de política de desarrollo económico.

La interpretación dada por Sunkel, si bien en nuestra opinión es de las más acertadas, ha tenido sobre todo la finalidad de ilustrar el caracter sumamente complejo e interrelacionado de los problemas a que hace frente la política de desarrollo día a dia, y sobre todo examinar las relaciones de algunos problemas que suelen clasificarse como de -- corto plazo, -la inflación, los déficit presupuestarios, las devaluaciones, la escasez de alimentos, etc.- con algunos factores institucionales y estructurales de raíces muy hondas.

Si no se tienen en cuenta todos esos elementos señalados por Sunkel, es completamente imposible entender la extraordinaria persistencia de los problemas del subdesarrollo en nuestros países y su constante - reaparición en casi todos los países de la región.

Puede verse también sin dificultad que los elementos manejados por Sunkel, juegan un papel preponderante en la disminución a largo plazo de la tasa de desarrollo, en la persistencia de una distribución muy dispareja del ingreso, en el creciente problema de las oportunidades - de empleo, y en la inestabilidad que caracteriza a las economías latinoamericanas.

De éste modo, tenemos que esas características constituyen algunos de los obstaculos fundamentales para el logro de los objetivos de una política de desarrollo en América Latina.

Así, cuesta comprender, por consiguiente, como podrían resultar eficaces ya sea las políticas de desarrollo o las llamadas políticas de corto plazo, sino se abordan seriamente esas cuestiones.

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EVALUACION ECONOMICA Y METODOS PARA DECISION DE INVERSIONES

EN LA INDUSTRIA MINERA

TEORIA MONTEARIA Y DEL CREDITO

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TEORIA MONETARIA Y DEL CREDITO.

El título del tema a exponer es demasiado amplio, es más, en los planes de estudio en la carrera de licenciatura en Economía, constituyen el curso de un año completo.

Indudablemente, considerando el tiempo disponible, no se pretenderá exponer con amplitud este tema, sino que se tratará en cambio, de poner en relieve, a juicio del ponente cuales son los aspectos esenciales para obtener una imagen del mismo. Para el efecto, se ha dividido la plática en cinco apartados, a saber: la moneda, sistemas monetarios, sistema bancario y valor de la moneda, para terminar con política monetaria.

MONEDA.

La moneda es objeto de tanto estudio principalmente por que si fluctúa mucho puede ejercer presiones, sobre los precios internos, la ocupación, la balanza de pagos, y por consiguiente, en el nivel de las reservas de oro, divisas del Banco Central y el tipo de cambio.

Indudablemente, la moneda actual es resultado de una evolución, el origen de la misma se encuentra en el desarrollo del cambio, por lo tanto, debemos considerar a la Economía Natural como punto de partida.

Este tipo de economía, también denominada primitiva, se caracteriza, por una producción obtenida mediante el trabajo en común y --

los bienes que se obtienen se destinan a satisfacer las necesidades de la familia, es decir, su consumo se hace también en común; por lo tanto, no existe el cambio.

A medida que se van introduciendo la división de tareas dentro de las sociedades primitivas, se va generando un excedente, lo cual origina la primera forma del cambio, o sea el trueque o permuta, esto es, los productos y los servicios son cambiados directamente contra productos o servicios.

Este tipo de cambio adolece de varios inconvenientes, como lo son: dificultad de coincidencia en los deseos recíprocos de quienes truecan; dificultad de concordancia en el valor de los bienes susceptibles de trueque; carencia de una medida común de valor respecto a los bienes cambiados e imposibilidad de separar las operaciones simultáneas de trueque tanto en el tiempo como en el espacio.

El paso del trueque a las operaciones de compraventa (segunda forma del cambio), separa los diversos inconvenientes que se presentan en aquel. Este tipo de cambio se realiza por la aparición de mercancías-moneda, los cuales surgen como un suplemento que se da, a manera de compensación, cuando la coincidencia en la intensidad de los deseos de los co-cambistas no es perfecta.

Para cumplir esta función en forma adecuada, una mercancía debía llenar diversas condiciones; ser de consumo generalizado; de fácil conservación y estar afectada de alguna escasez.

Como ejemplos de estas mercancías-moneda, podemos señalar - los siguientes: anzuelos en Ceilán; cuchillo primitivo de cobre que se utilizó en China; semillas de cacao; canutos de pluma rellenos - con polvo de oro; granos de toda especie; bloques de té comprimido; ciertos tipos de tela; tabaco, etc. Posteriormente, en la Edad Media, los metales comunes fungieron como moneda; el hierro se utilizó entre los griegos y en el Japón; el plomo en Grecia y Roma; estaño en Inglaterra y México y el cobre entre los hebreros y romanos.

Los objetos que se utilizaron como moneda-mercancía carecían de uniformidad en sus características, por ello, los metales preciosos llegaron a constituir en todas las sociedades, los instrumentos de medida de valor y medios de pago predilectos debido a que ofrecían las siguientes ventajas: a) Tener un valor intrínseco considerable; b) Ser inalterables; c) De fácil transporte; d) Ser de calidad relativamente constante; e) Ser objeto de una demanda constante; f) Ser relativamente escaso y g) Ser perfectamente divisible.

En un principio para su utilización era necesario ensayarlos y pesarlos para determinar su valor en cada operación; posteriormente, se optó por grabar una marca en el lingote o barra, para testificar su calidad y su peso; tal señal debía emanar de personas que gozaran de la confianza general.

Del lingote se pasó a formas más adecuadas a su uso monetario, o sea, a la de un disco plano. Se supone que en el siglo --

III a.c. se acuñaron las primeras monedas romanas de oro y plata.

A medida que se generalizó el uso de la moneda metálica, -- llegó un momento en que los poderes públicos intervinieran para sancionar su uso. Así, el Estado marcó las monedas para certificar su peso y su calidad, otorgando a las mismas una consagración oficial, que dió un nuevo estímulo de su circulación y a su aceptación; a -- partir de ese momento, les confiere el curso legal, es decir, impone a los acreedores la obligación de recibirlos como pago de sus -- créditos.

El incremento de las operaciones comerciales, hacian necesarias mayores cantidades de metales preciosos y el transporte de estos metales iba siendo cada vez más difícil y además riesgoso; por tal motivo, desde el siglo XVIII los patrones metálicos sufren diversas crisis e incluso desaparecen en ocasiones cediendo su lugar a las llamadas monedas de papel.

Este tipo de monedas pueden mantener un grado variable de - dependencia respecto a los metales, así tenemos que la moneda de papel representativa, consiste en billetes cuyo valor está garantizado 100% en metales preciosos, en el instituto emisor; la moneda de papel fiduciario o sea billetes cuyo valor está respaldado sólo -- parcialmente en metales preciosos y que circulan en virtud a la -- confianza (fiducia) que tienen los particulares de que el instituto emisor se los reembolsará en metales preciosos; cuando deseen hacer los efectivos y el papel-moneda o billete de banco inconvertible, - el cual carece en forma absoluta de respaldo metálico, por lo que - la institución emisora no asume la responsabilidad de su reembolso.

Por último, aparece la forma más elaborada de moneda de -- crédito o sea la moneda escritural, la cual está constituída por - los depósitos a la vista captados por el sistema bancario; es decir, por las cifras consignadas por escrito -de ahí su nombre- en cuentas nominales, que se movilizan mediante el empleo de cheques.

Una vez visto rápidamente el desarrollo de la moneda, podemos determinar su definición de acuerdo a sus funciones, la cual - considera a la moneda como un bien sui-géneris, que sirve de intermediario en los cambios y llena además, la función de medida común del valor, de medio de pago y de acumulador de valor.

SISTEMAS MONETARIOS.

Las diversas especies monetarias que se han citado, se han combinado en proporciones diversas, constituyendo sistemas monetarios con determinadas características que los hacen diferentes --- entre sí. Estos sistemas monetarios, se definen como el conjunto de monedas existentes y concurrentemente empleadas en un momento - y en un lugar determinado.

Ahora bien, aún cuando un sistema monetario esté definido por la ley, cualquier situación de hecho puede modificarlo y resul tar en la práctica un sistema distinto, por tal motivo, la clasificación que se representa es necesariamente artifical y señala las características fundamentales de los mismos.

Sistemas Monetarios de Base Metálica.

Los sistemas monetarios de base metálica son los que reposan esencialmente en el empleo de oro, de la plata o en ambos metales a la vez; la clasificación tradicional de estos sistemas distingue el monometalismo-oro, monometalismo-plata y el bimetalismo.

El criterio para distinguir el monometalismo y el bimetalismo no reside en el hecho de que las monedas estén fabricadas de un sólo metal o en dos, sino en que uno de ellos es considerado como base del sistema, como el metal patrón, o bien, que los dos metales tienen ese carácter.

Se conocen tres variantes del monometalismo oro a saber: el patrón-oro puro; el patrón lingotes oro y el patrón de cambio oro y cuyas características fundamentales son las siguientes:

En el patrón oro-puro únicamente las monedas de oro tienen poder liberatorio ilimitado; sólo el oro es admitido libremente en la acuñación; la autoridad monetaria compra y vende el oro a precio fijo en cantidades ilimitadas; en el campo internacional la libertad de movimientos de oro es total (se presupone en esta característica, que el sistema funciona en más de un país); en el interior del país el oro circula libremente y las monedas hechas de otro metal, guardan una relación legal de cambio respecto a la moneda patrón y el billete de banco es convertible unidad por unidad en especies de oro.

En este sistema, la cantidad de dinero en circulación no puede incrementarse a capricho de las autoridades, puesto que es imposible aumentar artificialmente la cantidad de oro existente en un país. Este patrón desapareció a partir de 1931.

El patrón de lingotes oro se caracteriza por que la circulación está formada preferentemente por billetes de banco, cuya convertibilidad en oro se hace por sumas de consideración y en lingotes; las autoridades monetarias compran y venden oro en cantidades ilimitadas a un precio fijo; no existen restricciones a la importación o a la exportación de dicho metal y los lingotes de oro se destinan a pagos en el exterior o para atesoramiento. La finalidad de este sistema era economizar el oro.

En el patrón monometalismo plata, este metal es el único admitido de la libre acuñación; sólo las piezas de plata tienen poder liberatorio ilimitado; el billete de banco es convertible en plata y existe libertad de exportación e importación para este metal. Las monedas fraccionarias hechas de otros metales tienen una relación de cambio fija con las de la plata.

Bimetalismo, empleo simultáneo de oro y plata para patrones monetarios; éste puede ser perfecto o imperfecto.

El bimetalismo perfecto, se caracteriza por la libertad de acuñación para el oro y la plata; las monedas de ambos metales tienen poder liberatorio ilimitado; existe una relación legal de cambio fijo entre ambas monedas; el billete es convertible, unidad por unidad, en especies de oro o de plata y existe libertad de movimientos para ambos metales. El empleo de este sistema trajo dificultades, pues se reducían alternativamente a un monometalismo plata o - un monometalismo oro.

Por su parte, el bimetalismo imperfecto se caracteriza por que existe libertad de acuñación únicamente para el oro; las monedas de oro y ciertas de plata (las de mayor denominación) tienen - poder liberatorio ilimitado y la ley fija una relación de cambio - entre las monedas de oro y las de plata.

Por ser el sistema prevaleciente, dejamos al último en la explicación al patrón de cambio oro, el cual se estableció a partir de 1945.

Este sistema tiene las siguientes características:

La circulación monetaria está compuesta por billetes de -- banco; el banco de emisión está obligado a cambiar sus billetes en divisas oro a una tasa fija y compra y vende estas divisas a un --

precio fijo, se permite la libre importación y exportación de divisas oro. Los pagos al exterior se hacen primero con las monedas divisas y una vez agotadas con el oro.

En el sistema la relación entre el oro y la unidad monetaria se establece indirectamente por intermedio de una moneda convertible en oro. Este tipo de moneda denominado divisa, recayó en el dólar americano y la libra esterlina.

Esto es, los países consideran dentro de sus reservas oficiales los activos de oro, dólares y libras esterlinas.

La incorporación de divisas convertibles en oro como parte de las reservas auríferas en numerosos países, indudablemente produjo una economía de dicho metal en el mundo.

El inconveniente de este sistema, es de que si en un país con moneda convertible en oro decide devaluarla, impone pérdidas considerables a todos los países que la han incorporado a su reservas como base de su sistema monetario.

Por tal motivo, el sistema se ha visto presionado en los últimos años, en primer término, por la devaluación de la libra esterlina en 1967 y la menor confianza en el dólar americano a partir de 1968 sobre todo cuando oficialmente se suspendió la convertibilidad del dólar por oro para las tesorerías y bancos centrales. En efecto, actualmente algunos individuos y bancos centrales no están dispuestos a aceptar obligaciones a corto plazo de los Estados Unidos.

Para enfrentarse a tal situación, los países miembros del Fondo Monetario Internacional, a principios de 1970 asignaron un nuevo activo a las reservas metálicas internacionales denominado Derechos Especiales de Giro. Desde entonces muchos bancos centrales han registrado en sus libros, junto con sus tenencias en oro y divisas, este nuevo tipo de reserva.

Los D.E.G. tienen un valor inmutable en oro, fueron creados para complementar los activos de reserva existentes y pueden utilizarse con la misma facilidad que los demás activos de reserva.

Una vez terminado con los sistemas monetarios, podemos ver el sistema bancario.

SISTEMA BANCARIO.

La característica de un banco moderno, es la de servir de intermediario entre los que poseen capitales ociosos y no saben o no quieren encontrar empleos para ellos y aquellos que necesitan capitales para invertirlos en objetos productivos o para destinarlos a objetos de consumo.

El banco moderno cumple con tres grandes funciones a saber: a) Intermediación en el crédito; b) Intermediación en los pagos y c) administración de los capitales.

Comúnmente la banca se ha dividido en banca comercial o de depósito; banca de inversión y banca central. Esta división parte de que los bancos no pueden practicar indistintamente toda clase de operaciones, sino aquellos que la ley correspondiente les autoriza y que están en función de la naturaleza de sus recursos.

Así, los bancos de depósito, que operan con los depósitos que reciben del público, pueden conferir créditos a corto plazo, o sea, operan dentro del mercado monetario auxiliando a las empresas en sus necesidades de activo circulante; la banca de inversión dispone de mayores recursos propios, confieren créditos a largo plazo y actúan en el mercado de capitales, auxiliando a las empresas en sus necesidades de activo fijo. El crédito a plazo medio (1 a 5 - años), es común en los dos tipos de banca.

En lo que concierne al Banco Central, es el eje del sistema bancario en el país correspondiente y tiene como fin primordial ser contralor monetario y crediticio,para lo cual disfruta de una situación monopólica en la emisión de billetes.

Por lo que respecta a nuestro país, se cuenta con un sistema mixto, es decir, donde intervienen instituciones privadas y oficiales, y su estructura es como sigue:

I. Secretaría de Hacienda y Crédito Público.

II. Banco de México, S.A. (Banco Central).

III. Comisión Nacional Bancaria.

IV. Instituciones Nacionales de Crédito (Banca Oficial).

V. Instituciones Privadas.
Bancos de depósito; Bancos de ahorro; Sociedades Financieras; Sociedades de Crédito Hipotecario; Sociedades de Capitalización e Instituciones Fiduciarias.

VI. Organizaciones Auxiliares.
Almacenes de depósito; Uniones de Crédito; [illegible] de Compensación y Bolsas de Valores.

A continuación presentaremos las principales funciones de la Banca Central de México.

a) Emisión reglamentada de billetes en condiciones de monopolio; esta condición es necesaria debido a que con ello se obtiene uniformidad en la circulación de billetes; imprime flexibilidad necesaria a la cantidad de dinero en circulación, la cual debe adaptarse a condiciones económicas cambiantes.

En todo tiempo la Nación responderá del valor de los billetes y monedas que el Banco ponga en circulación para sostener el valor del peso. El Banco mantendrá en todo momento una reserva no inferior en caso alguno del 25% de la cantidad de billetes en circulación más las obligaciones a la vista, en moneda nacional, a cargo del Banco.

Tal reserva deberá estar compuesta por oro amonedado o en barras y divisas o cambio extranjero, por una suma nunca menor del 80% de la reserva y la cantidad restante por plata acuñada o en barras.

b) Banquero, agente y consejero del Gobierno. Esto es, - lleva las cuentas bancarias, de oficinas y empresas gubernamentales; también le proporciona las divisas que requiera para el servicio de la deuda externa o para adquisiciones en el exterior o a la inversa, le compra divisas que haya obtenido, sea por empréstitos externos u otros conceptos; actúa como agente financiero del - Gobierno Federal en las operaciones de crédito externo o interno y en la emisión y atención de empréstitos públicos y se encarga del servicio de tesorería del propio Gobierno, así como la emisión, - compra y venta de valores a cargo del Gobierno Federal.

c) Depositario de las reservas en efectivo de los bancos comerciales.

La centralización de las reservas en efectivo de los bancos comerciales en el Central se estableció por conveniencia, tradición o por ley y ha constituído al correr del tiempo un factor de gran importancia para regular la situación monetaria y crediticia de un país.

La legislación mexicana exige depósitos obligatorios (encaje legal) en el Banco de México, a los bancos de depósito, de ahorro, sociedades financieras y a las sociedades de crédito hipotecario.

Dicho depósito quedará sujeto a las reglas que dicte el --

Banco de México y en términos generales, puede variar de un 15% al 50% respecto a los depósitos y en casos excepcionales hasta el --- 100%.

d) Depositario de las reservas metálicas y de divisas del país.

Esta función está muy relacionada con los enunciados en el primer y tercer término.

e) Servir como Cámara de Compensación.

f) Redescuento de efectos comerciales y préstamos de última instancia.

El ejercicio de esta doble función ha dado lugar a que se - considere al Banco Central como "Banco de Bancos".

Para los bancos comerciales y otras instituciones de crédito, el redescuento constituye un procedimiento para obtener efectivo en forma inmediata a cambio de efectos comerciales.

g) El Control del Crédito.

En atención a que el volumen general del crédito y los cambios que experimenta, condicionan variaciones en la cantidad de dinero en circulación que pueden repercutir en el nivel general de - precios, los bancos centrales han intervenido para regular las actividades de las instituciones facultadas para crear u otorgar créditos que por ello constituyen un elemento preponderante dentro de la política monetaria del país en que operan.

Las medidas que generalmente han sido adoptadas por los - bancos centrales para controlar el crédito, aunque varían en efectividad en razón del medio y del momento en que se aplican, pueden ser de carácter directo e indirecto.

Dentro de las primeras se cuentan las siguientes: la imposición de depósitos obligatorios a los bancos comerciales; racionamiento del crédito por parte del Banco Central y política de -- crédito selectivo.

Como medidas indirectas, se consideran la manipulación de la tasa de redescuento; las operaciones de mercado abierto y el - empleo de la persuasión moral.

Por lo que respecta al país, el Banco Central dispone de - los instrumentos tradicionales de regulación; sin embargo, la política monetaria ha descansado fundamentalmente en el manejo de los requisitos de depósito obligatorio, en combinación con operaciones con valores y de crédito selectivo.

A través del manejo del depósito obligatorio y de una activa y constante campaña de persuadición moral, las autoridades han tratado de canalizar una proporción cada vez mayor de los recursos de la banca comercial hacia operaciones crediticias a mediano y a largo plazo, que permitan ampliar la capacidad productiva del -- país.

La inversión obligatoria en valores, tanto del Gobierno - Federal como de otros organismos públicos de fomento, ha permitido las operaciones con valores oficiales.

Asimismo, el depósito obligatorio se ha utilizado como -- instrumento con criterio selectivo; con tal fin, los bancos comerciales deben canalizar no menos del 70% de sus operaciones activas en financiamientos a la producción y no más del 30% al comercio.

VALOR DEL DINERO.

Uno de los aspectos sobresalientes de la presente plática, es el relacionado a las teorías que explican el valor del dinero.

Si bien el valor de las mercancías o servicios, puede medirse en dinero, el valor de éste no se mide en función de el mismo. En consecuencia, la expresión "Valor del dinero", debe entenderse como la cantidad de mercancías y servicios que la moneda -- permite obtener en los diversos mercados; en otros términos, el - valor de la moneda se expresa por su poder de compra.

Las teorías del valor del dinero, en consecuencia, se ocupan de precisar los elementos de los cuales depende el poder de - compra del dinero y el por qué de sus variaciones.

La historia muestra en múltiples casos que un incremento importante de la cantidad de moneda en circulación ha determinado una alza general de precios.

La explicación del valor del dinero, se lleva a efecto por varias teorías, una de las cuales se denomina Teoría Cuantitativa cuyo método de transacciones, veremos a continuación.

Esta teoría se auxilia de la fórmula del cambio y se expresa en los siguientes términos:

$$MV = P\,T \quad o \quad P = \frac{MV}{T}$$

En donde: M = cantidad dinero de todas clases, que está en circulación en cierto lugar durante determinado período; V = número de veces que se gasta cada unidad de M en adquisición de mercancías, servicios y valores, durante un determinado período (velocidad de circulación); T = volumen físico de mercancías, servicios y valores que se venden por dinero en una cierta región, durante un determinado período de tiempo y tantas veces como sean objeto de venta y, finalmente, P = precio medio de cada unidad de T durante un período determinado o en otros términos, el nivel general de precios.

La cantidad de dinero (M) se divide en dos renglones: el efectivo en circulación emitido por el Banco Central (Billetes) y los depósitos en los bancos que se retiran mediante cheques.

Esta oferta monetaria está determinada en primer término, por el volumen de la base monetaria; ésta a su vez está integrada por la reserva monetaria de oro, de plata y divisas; por otras cla ses de dinero acuñadas o emitidas por el Estado y por los créditos otorgados por el Banco Central.

Como segundo determinante de (M) es la elección de la comunidad respecto a las cantidades relativas de efectivo y de depósitos en cuenta de cheques que se desea mantener.

Otro factor determinante de la oferta de dinero, son las disposiciones que regulan la capacidad de operar a los bancos de depósito; esto es, la reglamentación que existe respecto a la proporción que los bancos deben mantener entre su capital y reservas y los depósitos que pueden recibir en cuenta de cheques, así como el encaje legal.

La velocidad de circulación (V) de la moneda -la intensidad de su uso- depende del com-portamiento de los individuos respecto a esa moneda.

Entre los principales factores que determinan V, debe mencionarse el estado de desarrollo del sistema financiero y grado en que lo utiliza la comunidad.

Las costumbres predominantes de la población respecto al ahorro y al consumo, constituyen, asimismo, un factor determinante de V.

Los sistemas de pago empleados por la comunidad influyen también en la magnitud de V.

El grado de confianza que prevalezca en la comunidad respecto a la evolución de los ingresos y los precios futuros tendrá repercusión sobre V.

El nivel de los precios no varía de acuerdo con la cantidad absoluta de gastos en términos del dinero, sino de acuerdo con la relación entre los gastos y el volumen físico de las cosas que se pueden comprar con dinero; por lo tanto, el tercer elemento determinante del valor del dinero es el número de transacciones comerciales que se puede hacer mediante dinero.

El volumen físico de mercancías, servicios y valores que se venden por dinero en cierta región durante un período determinado, o sea (T).

Este factor dependerá de la capacidad potencial para producir mercancías y servicios. Esta a su vez está condicionada por múltiples factores, entre los cuales cabe señalar los siguientes: cantidad y calidad de la población, factor trabajo; extensión y riqueza del territorio; existencia de capital y desarrollo de la técnica.

Como segundo elemento determinante, se encuentra el grado de empleo de los factores productivos; asimismo, las veces que las mercancías y valores son cambiados por dinero están modificando a (T).

Finalmente, la magnitud de T está influída por el grado en que se utiliza el trueque.

Señalados los principales factores que componen M, V y T que son los determinantes directos de P, se establece que los cambios en los precios pueden deberse a variaciones en uno o varios de sus determinantes directos o en todos ellos a la vez.

En resumen, se acepta que en términos generales, si el producto MV tiende a aumentar más de prisa que T, entonces P manifestará una tendencia secular al alza; por el contrario, si MV aumenta menos rápidamente que T, entonces P registrará una tendencia secular descendente.

POLITICA MONETARIA.

La política monetaria y crediticia opera específicamente a través de los siguientes factores: la oferta de crédito en relación a su demanda, el precio del crédito, el volumen del dinero y la liquidez general de la economía.

Los fines más importantes de una sana política monetaria consisten en evitar el agotamiento de las reservas, depreciación del tipo de cambio o restricción del comercio exterior, así como mantener el poder adquisitivo de la moneda dentro del país.

Los instrumentos de que se sirve la política monetaria para el logro de sus objetivos inmediatos, son: la política de redescuento; las operaciones de mercado abierto; el requisito de depósito legal en el Banco Central para los Bancos Comerciales; coeficientes de liquidez de la banca comercial y los controles selectivos del -- crédito.

De acuerdo con lo anterior y en atención a las característi cas que reviste la economía mexicana, el objetivo primordial de la política monetaria es coadyuvar al logro de desarrollo económico - del país al ritmo más acelerado posible, dentro de condiciones de equilibrio en la Balanza de Pagos, de estabilidad cambiaria y de - absoluta libre convertibilidad de la moneda nacional en otras mone das.

Para tal objetivo, el país cuenta con los órganos adecuados como lo es el sistema bancario mexicano y los instrumentos necesa- rios.

Estos últimos se encuentran en las funciones del Banco de México, reglamentadas por su ley orgánica y la aplicación de la - Ley General de Instituciones de Crédito.

De los instrumentos más utilizados, se pueden señalar: el pasivo exigible respecto al capital más reservas de capital; el de pósito obligatorio; controles directos del crédito; la tasa de re- descuento y operaciones de mercado abierto.

Los fenómenos económicos que se han venido desarrollando - en 1973, motivaron las declaraciones del C. Secretario de Hacienda, en junio del presente año, en el sentido de señalar los lineamien- tos de la política económica de la presente administración.

De los 16 puntos citados, el primero señala que se requiere organizar y estructurar, en forma coordinada y armoniosa, los esfuerzos nacionales para aumentar la producción agropecuaria, industrial y de servicios, adecuando para ello los recursos humanos y físicos con los financieros en un todo equilibrado que combata la inflación y el desperdicio.

Del punto 9 al 12, habla de la parte correspondiente a la política monetaria y del crédito y en resumen el objetivo, es el de proteger la capacidad real de compra de los sueldos y salarios. Para ello, se aplicará con firmeza, una política monetaria estrictamente anti-inflacionaria, limitando por una parte, la expansión injustificada del crédito bancario para actividades no productivas, al mismo tiempo canalizar selectivamente el crédito a las actividades productivas; todo esto aunado a una mayor canalización de recursos no inflacionarios al sector público.

Por lo tanto, serán objeto de acciones redobladas el sector agropecuario, la sustitución eficiente de importaciones, principalmente de bienes de capital, fomento de las exportaciones y expansión de la industria turística.

Elaborado por: Lic. Fernando Rentería Ponce.

METODOS CUANTITATIVOS DE INVESTIGACION ECONOMICA

Por: Lic. Rafael Herrera Balbuena.

Estos métodos han alcanzado en los últimos años, una gran importancia en el análisis de todo fenómeno económico. La estadística era practicada desde los tiempos más remotos de la civilización, hasta llegar a constituirse en nuestros días, en uno de los instrumentos más importantes que el economista emplea para resolver problemas económicos de todo tipo que se le presenten.

A través del tiempo se han elaborado diversas definiciones de lo que es la estadística, pero en esencia significan lo mismo, a modo de ejemplo se cita la siguiente definición:

"Es el estudio numérico de grupos o masas a través del estudio de las unidades que los componen, ya sea que estas unidades sean humanas o subhumanas, animadas o inanimadas".

Ahora bien, los fenómenos que deben tratarse estadísticamente, se pueden dividir en tres grandes grupos:

1) Los que pueden ser retenidos por la observación objetiva, a saber: a) los de carácter colectivo (población de un país, los salarios, etc.) b) los que se producen o manifiestan muy separados entre sí (temblores, epidemias, etc.) y c) los que se presentan con igual frecuencia, pero impresionando más unos que otros, por lo que se tiene un concepto errático de las veces que se producen (nacimientos hombres y mujeres, ganancias y pérdidas de un jugador, etc.)

2) Fenómenos de los cuales se posee cierta apreciación cuantitativa, pero se desconoce su intensidad cuantitativa (la mayoría de los matrimonios, el hombre es mayor en edad y estatura que la mujer, por lo que es necesario la estadística).

3) Los que pueden ser apreciados cuantitativamente, pero cometiendo errores graves por ejemplo: (la longitud de un cuerpo, con una apreciación demasiado burda).

ELABORACIONES DE ESTADISTICAS

Las operaciones sucesivas de toda elaboración estadística, comprenden cinco fases que son:

1) Plan de recopilación.
2) Recolección de los datos.
3) Crítica de los datos.
4) Recuento de los datos.
5) Presentación de los datos.

1. El Plan de Recopilación.

Lo primero que debe hacerse en un plan de recopilación es determinar en forma precisa el fenómeno que se trata de captar, lo mismo que las informaciones que pueden ser útiles para los estudios posteriores que del fenómeno quieran hacerse. Como segundo paso, conviene fijar las modalidades que deben tomarse en consideración para llevar a cabo tal recopilación.

En muchos planes de recopilación, se hace indispensable fijar límites para la captación de los fenómenos y los más importantes son los siguientes:

a) De precisión. Consisten en fijar hasta que grado de aproximación debe llevarse a cabo tal medida.

b) De espacio. Cuando es imposible la medida de un fenómeno en todo el campo de su desarrollo, se hace una selección de los lugares donde deba estudiarse dicho fenómeno.

c) De tiempo. Este límite se aplica generalmente, siempre que un fenómeno no se pueda estudiar durante todo el tiempo en que se manifiesta, es decir, cuando se consideran estáticos en el momento de registrarlos.

d) Por casos observados. Si por alguna causa se hace imposible el estudio de un fenómeno colectivo en todos los casos particulares que lo forman, se procede a la limitación de los casos que deben observarse.

e) De especialización. Consiste en captar únicamente aquellos fenómenos que influyen más directamente en el fenómeno que se estudia.

En muchos casos, por motivos de orden económico, por falta de personal o de tiempo, no es posible realizar las investigaciones completas, y en tales casos, se recurre al "Método representativo" o "Muestreo", que consiste en seleccionar una fracción únicamente-de las unidades del conjunto, de manera que esa parte represente al colectivo en sus características más sobresalientes, incluyendo las medidas de variabilidad.

Ahora bien, los procedimientos que pueden seguirse para la obtención de las unidades del colectivo que han de formar la muestra son:

a) Escoger cuidadosamente del universo las unidades del colectivo que han de formar la nuestra.

b) Obtener al azar las unidades que han de integrar la muestra.

c) Formar la muestra, combinando los dos procedimientos anteriores.

Las modalidades de la recopilación estadística forman la segunda parte del plan de recopilación, a saber:

1° Cómo debe ejecutarse la recopilación.

2° Cuándo o en qué momento debe llevarse a cabo.

3° Por quienes debe ejecutarse.

4° Con qué medios y con que instrumentos debe realizarse.

2. Recolección de Datos.

Después de estudiar el plan de recopilación, se pasa a la segunda fase, o sea a la recolección misma de los datos, que puede ser "directa" o "indirecta".

Una recolección es directa cuando se capta el fenómeno que se trata de conocer; e indirecta cuando tiene por objeto la captación de fenómenos que no tienen importancia en sí, pero que por medio de ellos se deduce el fenómeno que interesa conocer.

Las recopilaciones directas se dividen en "contínuas" "periódicas" y "ocasionales".

Desde otro punto de vista, las recolecciones se dividen en: completas, incompletas, de datos preliminares y de datos definitivos.

3. Crítica de los Datos.

Esta es una de las fases más importantes de toda elaboración estadística, pues de ella depende que los datos sean los más reales posible y que no existan evaciones.

La primera parte consiste en ver si los cuestionarios empleados para la recolección están contestadas todas las preguntas. Después, se observa si las contestaciones se hallan acordes con las preguntas hechas. Como paso siguiente hay que estudiar si las respuestas que el informante ha dado son lógicamente aceptables.

4. Recuento de los Datos.

Esta parte de la recopilación se puede dividir en dos fases:

a) Clasificación de los datos.
b) Recuento material de los datos.

La clasificación de los datos recolectados consiste en -- agruparlos según los caracteres que se manifiestan en el fenómeno de masa.

El recuento material de los datos se puede hacer de dos - maneras: por medio de procedimientos mecánicos, que son los más - comúnmente usados en la actualidad, o por medio de procedimientos manuales.

5. Presentación de los Datos.

Finalmente, obtenidos ya los datos de los fenómenos recopilados, se pasa a la presentación de aquéllos en forma de cuadros, tablas o gráficas.

ESTUDIO DE LOS FENOMENOS ESTADISTICOS

Un fenómeno estadístico puede estudiarse bajo los cinco aspectos siguientes:

1) Medir la intensidad de un fenómeno por ejemplo: (número de habitantes de un país, intensidad global) y - (estatura media de ellos, intensidad media).

2) Determinar las relaciones que existen entre las intensidadesd de varios fenómenos (verbigracia relación entre los nacimientos y población total de un país).

3) Determinar la distribución de un - fenómeno colectivo que puede ser - por:

   a) Caracteres propios del fenómeno.
   b) Distribución en el tiempo.
   c) Distribución en el espacio.

4) Determinar las relaciones que existen entre las distribuciones de varios fenómenos colectivos.

5) Determinar las relaciones que existen entre las modalidades de dos o más fenómenos.

En el primer caso, el procedimiento matemático para obtener la intensidad global de un fenómeno colectivo es sencillo, - pues basta con sumar las intensidades de los fenómenos particulares que forman el fenómeno de masa. En cambio para medir la intensidad media se requiere la metodología, estadística que com-- prende lo que a continuación se indica:

SERIES SIMPLES, DE FRECUENCIA Y DE CLASES Y FRECUENCIAS

Una "serie simple" es el conjunto de números que miden las variaciones de un carácter de los fenómenos particulares. Ejemplo:

Supongamos que se han medido las estaturas de 20 personas, cuyos resultados son los siguientes:

1.54 m.	1.61 m.	1.67 m.	1.72 m.
1.54 m.	1.61 m.	1.67 m.	1.81 m.
1.54 m.	1.61 m.	1.67 m.	1.81 m.
1.57 m.	1.61 m.	1.67 m.	1.81 m.
1.57 m.	1.67 m.	1.72 m.	1.85 m.

Se le denomina "serie de frecuencia", el número de veces - que un término se repite en una serie simple.

ESTATURA EN M.	No. DE INDIVIDUOS (frecuencias)
1.54	3
1.57	2
1.61	4
1.67	5
1.72	2
1.81	3
1.85	1
SUMA	20

Del cuadro anterior podemos deducir el siguiente:

ESTATURA EN M. (clases)	No. de INDIVIDUOS (frecuencias)
De 1.50 a - 1.60	5
De 1.60 a - 1.70	9
De 1.70 a - 1.80	2
De 1.80 a - 1.90	4
S U M A	20

El conjunto de los números de este cuadro,recibe el nombre de "Serie de Clases y Frecuencias", es decir, los que se encuentran limitados entre dos cifras.

Los números que limitan una clase se denominan "fronteras" de clase, y la diferencia entre la frontera superior y la inferior se llama "amplitud"

PROMEDIO ARITMETICO

1) Serie simple. El promedio aritmético de una serie simple, es igual a la suma de los valores de sus términos, dividida entre el número de éstos.

$$M = \frac{\Sigma a}{n} = \frac{\sum_{i=1}^{i=n} a_i}{n}$$

2) Serie de frecuencias. El promedio aritmético de una serie de frecuencias, es igual a la suma de los productos de cada término por su frecuencia respectiva, dividida entre la suma de las frecuencias.

$$M = \frac{\Sigma af}{\Sigma f}$$

3) Serie de clases y frecuencias. El promedio aritmético de una serie de clases y frecuencias, es igual a la suma de los productos de los puntos medios de cada clase por su frecuencia respectiva, dividida entre la suma de las frecuencias.

$$M = \frac{\Sigma P_m f}{\Sigma f}$$

PROMEDIO GEOMETRICO

1) Serie simple. Recibe el nombre de promedio geométrico de una serie simple un valor medio tal, que reemplazando cada término por ese valor, resulta un producto igual al producto de los -

valores de los términos de la serie dada.

$$Mg = \sqrt[n]{a_1 \times a_2 \times a_3 \times \cdots \times a_n}$$

$$\therefore \ Log\ Mg = \frac{\Sigma \log a}{n}$$

Lo que nos indica que el logarítmo del promedio geométrico de una serie simple es igual al promedio aritmético de los logarítmos de sus términos; o bien, el promedio geométrico de una serie simple, es el artilogarítmo del promedio aritmético de los logarítmos de los términos.

2) Serie de frecuencias. El promedio geométrico de una serie de frecuencia, es igual a la raíz que tiene como índice la suma de las frecuencias del producto de los términos elevados a sus respectivas frecuencias.

$$Mg = \sqrt[\Sigma f]{(a_1)^{f_1} (a_2)^{f_2} (a_3)^{f_3} \cdots (a_n)^{f_n}}$$

$$\therefore \ Log\ Mg = \frac{\Sigma f \log a}{\Sigma f}$$

Lo que nos indica que el logarítmo del promedio geométrico de una serie de frecuencias es igual a la suma de los productos del logarítmo de cada término por su frecuencia respectiva, dividida entre la suma de las frecuencias.

3) Serie de clases y frecuencias. Para el cálculo de una serie de clases y frecuencias, se procede en la misma forma que en el caso de una serie de frecuencias, considerando como valor de cada término el promedio geométrico de las fronteras de cada clase, o sea, el punto medio de cada clase.

EJEMPLOS

SERIE SIMPLE		SERIE DE FRECUENCIAS			SERIE DE CLASES Y FRECUENCIAS			
Estaturas en mts.		Estaturas en metros (a)	Nº Estud. (f)	(a)(f)	Clases	Puntos Medios (a)	Nº Estud. (f)	(a)(f)
1.54	1.67	1.54	3	4.62	De 1.50 a - 1.60	1.55	5	7.75
1.54	1.67	1.57	2	3.14				
1.54	1.67							
1.57	1.67	1.61	4	6.44	De 1.60 a - 1.70	1.65	9	14.85
1.57	1.72	1.67	5	8.35				
1.61	1.72							
1.61	1.81	1.72	2	3.44	De 1.70 a - 1.80	1.75	2	3.50
1.61	1.81	1.81	3	5.43				
1.61	1.81							
1.67	1.85	1.85	1	1.85	De 1.80 a - 1.90	1.85	4	7.40
SUMAS	33.27		20	33.27			20	33.50

PROMEDIO ARITMETICO

1) serie simple: $M = \frac{\Sigma a}{n} = \frac{33.27}{20} = \underline{1.66}$

2) Serie de frecuencias: $M = \frac{\Sigma fa}{\Sigma f} = \frac{33.27}{20} = \underline{1.66}$

3) Serie de clases y frecuencias: $M = \frac{\Sigma Pm f}{\Sigma f} = \frac{33.50}{20} = \underline{1.67}$

PROMEDIO GEOMETRICO

1) serie simple: $\text{Log } Mg = \frac{\Sigma \log a}{n} = \frac{4.4063}{20} = 0.2203$

$\text{antilog } 0.2203 = 1.66 \quad \therefore Mg = \underline{1.66}$

2) serie de frecuencias: $\text{Log } Mg = \frac{\Sigma f \text{ Log } a}{\Sigma f}$

$\text{Log } Mg = \frac{4.4063}{20} = 0.2203 \quad \therefore Mg = \underline{1.66}$

3) serie de clases y frecuencias: $\text{Log } Mg = \frac{\Sigma f \log Pm}{\Sigma f}$

$\text{Log } Mg = \frac{4.4638}{20} = 0.2232 \quad \therefore Mg = \underline{1.67}$

MEDIA ARMONICA

1) Serie simple. La media armónica de una serie, es un número tal que las sumas de las relaciones entre él y cada término - es igual al número de términos de la serie:

$$M_a = \frac{n}{\left(\frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3} + \cdots + \frac{1}{a_n}\right)} = \frac{n}{\sum \frac{1}{a}}$$

Como toda fracción es igual a la unidad dividida entre la misma fracción pero invertida, la igualdad anterior se puede escribir en la forma siguiente:

$$M_a = \frac{1}{\frac{\left(\frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3} + \cdots + \frac{1}{a_n}\right)}{n}} = \frac{1}{\frac{\sum \frac{1}{a}}{n}}$$

Lo que nos indica que la media armónica de una serie es -- igual al recíproco de la media armónica de los recíprocos de los - términos.

2) Serie de frecuencias. La media armónica de una serie de frecuencias es igual a la suma de las frecuencias, dividida entre la suma de las relaciones entre cada frecuencia y su término - respectivo.

$$M_a = \frac{\sum f}{\sum \frac{f}{a}}$$

3) Serie de clases y frecuencias. En este caso, se procede en la misma forma que en una serie de frecuencias, tomando - como valor de cada término el punto medio de cada clase.

EJEMPLOS:

1) Serie simple: $Ma = \frac{20}{12.0609} \therefore Ma = 1.66$

TERMINOS (a)	FRECUENCIAS (f)	$\frac{1}{a}$	$(f)(\frac{1}{a}) = \frac{f}{a}$
1.54	3	0.6493	1.9479
1.57	2	0.6369	1.2738
1.61	4	0.6211	2.4844
1.67	5	0.5988	2.9940
1.72	2	0.5814	1.1628
1.81	3	0.5525	1.6575
1.85	1	0.5405	0.5405
	20		12.0609

2) Serie de frecuencias: $Ma = \frac{20}{12.0609} = 1.66$

CLASES	PUNTO MEDIO (a)	(f)	$(\frac{f}{a})$
De 1.50 a 1.60	1.55	5	3.2258
De 1.60 a 1.70	1.65	9	5.4545
De 1.70 a 1.80	1.75	2	1.1428
De 1.80 a 1.90	1.85	4	2.1622
S U M A		20	11.9853

3) Serie de clases y frecuencias.

$$Ma = \frac{20}{11.9853} \therefore Ma = 1.67$$

MEDIANA

1) Serie simple. Mediana o valor central de una serie, es el valor del término central de toda serie ordenada conforme a los valores crecientes o decrecientes de sus términos.

Como la mediana es el término medio de una serie, se deduce que si el número de orden de la mediana se le suma el número de términos que le siguen o anteceden, se obtiene el número total de términos de la serie (N), como el número de términos que anteceden a la mediana es igual a su número de orden menos 1, y si representamos por (No) al número de orden tenemos:

$$N_o + (N_o - 1) = N$$

$$2N_o = N + 1 \qquad \therefore \; N_o = \frac{N+1}{2}$$

Lo que nos indica que el número de orden de la mediana de una serie ordenada es igual al número de términos de la serie más 1 dividida entre 2.

2) Serie de Clases. El número de orden que ocupa la mediana en una serie de frecuencias es igual a la suma de las frecuencias más 1 dividida entre 2.

3) Serie de Clases y Frecuencias. Es el mismo procedimiento que el caso anterior.

EJEMPLOS

SERIE SIMPLE Estaturas (metros)		SERIE DE FRECUENCIAS Estaturas (a)	No. de Estudiantes (f)
1.54	1.67	1.54	3
1.54	1.67		
1.54	1.67	1.57	2
1.57	1.67	1.61	4
1.57	1.72		
1.61	1.72	1.67	5
1.61	1.81	1.72	2
1.61	1.81		
1.61	1.81	1.81	3
1.67	1.85	1.85	1
S U M A S	20		20

1) Serie Simple:

$$N_o = \frac{20+1}{2} = \frac{21}{2} = 10.5 \quad \therefore Md = 1.67$$

2) Serie de Frecuencias:

$$N_o = \frac{20+1}{2} = \frac{21}{2} = 10.5$$

$$\therefore Md = 1.67$$

C L A S E S	(F)
De 1.50 a - 1.60	5
De 1.60 a - 1.70	9
De 1.70 a - 1.80	2
De 1.80 a - 1.90	4
	20

3) Serie de Clases y Frecuencias.

$$N_o = \frac{20+1}{2} = \frac{21}{2} = 10.5$$

Este último resultado, nos indica que la mediana de la - serie es el valor del término de número de orden 10.5; ahora bien, las frecuencias de la primera clase suma un total de 5, términos - por lo tanto, faltan 5.5 para llegar a 10.5 que debe ocupar la mediana, por lo que ésta se encontraría en la segunda frecuencia, es decir, en el 9. Para calcular la mediana exactamente se procede - de la siguiente forma:

$$Md = D + C\left(\frac{E-D}{B}\right)$$

$$Md = 1.60 + 5.5\left(\frac{1.70-1.60}{9}\right) = 1.60 + \frac{0.55}{9}$$

$$Md = 1.60 + 0.06 = \underline{\underline{1.66}}$$

Como podrá observarse de lo expuesto anteriormente, por cualquier método que se quiera utilizar para determinar cualquier promedio, siempre se llegará al mismo resultado. Existen otros métodos como la media arbitraria, media cuadrática, la moda, etc. que se los menciono nada más para su conocimiento.

NUMEROS INDICES

Algunos autores no distinguen entre un "número relativo" y un "número índice"; sin embargo, los relativos son por cientos que expresan el precio o la cantidad de un producto determinado en un tiempo dado, en comparación con su precio o cantidad en un año base. En tanto que un número índice es una medida del cambio medio - de un fenómeno dado, en el tiempo o en el espacio.

Con los números índices se puede comparar el costo de alimentos u otros costos de vida en una ciudad durante un año con los del año anterior, o se puede comparar la producción de acero durante un año determinado en una parte del país con la registrada en - otra parte. Aunque su aplicación principal es en negociossy fenómenos económicos, los números índices pueden aplicarse en otros campos, por ejemplo, en comparar la inteligencia de estudiantes de diversos puntos o de diferentes edades, etc.

Existen varias fórmulas para calcular los números índices, pero los más importantes son los de Laspeyres, de Paasche, la de Fisher y la de Marshall, las cuales son:

1) $$\frac{\sum P_1 Q_0}{\sum P_0 Q_0}$$ Fórmula de Laspeyres.

2) $$\frac{\sum P_1 Q_1}{\sum P_0 Q_1}$$ Fórmula de Paasche.

3) $$\sqrt{\frac{\sum P_1 Q_0}{\sum P_0 Q_0} \cdot \frac{\sum P_1 Q_1}{\sum P_0 Q_1}}$$ Fórmula ideal de Fisher.

4) $$\frac{\sum P_1 (Q_0 + Q_1)}{\sum P_0 (Q_0 + Q_1)}$$ Fórmula de Marshall.

En donde Po es el precio del año base, P_1 el precio del - año de estudio; Qo es la cantidad del año base y Q_1 la cantidad - del año de estudios.

Ejemplos, con los datos del cuadro siguente, calcular los índices de precios utilizando las 4 fórmulas anteriores.

ARTICULO	UNIDAD	AÑO BASE PRECIO Po	AÑO BASE CANTIDAD Qo	AÑO DE ESTUDIO PRECIO P1	AÑO DE ESTUDIO CANTIDAD Q1
Maíz	Kg.	o.30	150	0.32	140
Frijol	Kg.	0.50	80	0.45	90
Azúcar	Kg.	0.90	15	0.80	25
Café	Kg.	2.80	8	2.90	10
Arroz	Kg.	1.40	25	1.50	22
Carne	Kg.	1.50	38	1.45	34

1) $$\frac{\Sigma P_1 Q_0}{\Sigma P_0 Q_0} = \frac{211.80}{212.90} = 99.48$$ Laspeyres.

2) $$\frac{\Sigma P_1 Q_1}{\Sigma P_0 Q_1} = \frac{216.60}{219.30} = 98.77$$ Paasche.

3) $$\sqrt{\frac{\Sigma P_1 Q_0}{\Sigma P_0 Q_0} \cdot \frac{\Sigma P_1 Q_1}{\Sigma P_0 Q_1}} = \sqrt{\frac{211.80}{212.90} \cdot \frac{216.60}{219.30}} =$$

$$= \sqrt{9825.6396} = 99.12$$ Fisher.

4) $$\frac{\Sigma P_1 (Q_0 + Q_1)}{\Sigma P_0 (Q_0 + Q_1)} = \frac{\Sigma P_1 Q_0 + \Sigma P_1 Q_1}{\Sigma P_0 Q_0 + \Sigma P_0 Q_1} = \frac{211.80 + 216.60}{212.90 + 219.30} =$$

$$= \frac{428.50}{432.20} = 99.14$$ Marshall.

CURVAS DE AJUSTE

Relación entre variables. Muy a menudo se encuentra en la práctica que existe una relación entre dos o más variables. Por ejemplo, las circunferencias de los círculos dependen de sus radios, los pesos de la gente adulta depende en cierto modo de sus alturas. Se desea frecuentemente expresar esta relación mediante una ecuación matemática que ligue las variables.

Curvas de Ajuste. Para determinar una ecuación que relacione las variables, un primer paso es la recolección de datos que muestren los correspondientes valores de las variables consideradas.

El paso siguiente es representar los puntos (X_1,Y_1), (X_2,Y_2)..., (Xn,Yn) en un sistema de coordenadas rectangulares. El sistema de puntos resultantes se llama "diagrama de dispersión".

Con el diagrama de dispersión se puede representar una curva que se aproxime a los datos; tal curva se llama "Curva de aproximación". Ejemplo, en la primera figura, los datos se aproximan bien a una recta, por lo tanto, se dice que existe una "relación lineal", en tanto que en la segunda es una "relación no lineal".

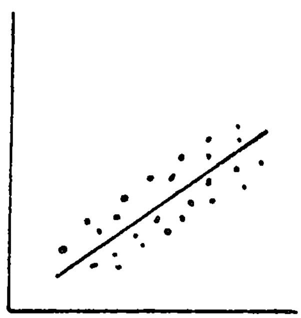


Fig 1

Fig 2

Ecuación de Curvas de Aproximación. Para que sirva de referencia se anotan a continuación varios tipos comunes de curvas de aproximación y sus ecuaciones. Todas las letras distintas a X e Y representan constantes; las variables X e Y se conocen a menudo como variable independiente y dependiente respectivamente, aunque estos papeles pueden intercambiarse.

1) $Y = A_0 + A_1 X$ ------------- Línea recta
2) $Y = A_0 + A_1 X + A_2 X^2$ --------- Parábola
3) $Y = A_0 + A_1 X + A_2 X^2 + A_3 X^3$ ------ Curva cúbica
4) $Y = A_0 + A_1 X + A_2 X^2 + A_3 X^3 + A_4 X^4$ -- Curva Cuadrática
5) $Y = A_0 + A_1 X + A_2 X^2 + \cdots + A_n X^n$ -- Curva de grado n

La línea recta. El tipo más sencillo de curva de aproximación es la línea recta, cuya ecuación puede escribirse.

$$Y = Ao + A_1 X \text{ ------ } (1)$$

Dados los puntos cualquiera (X_1, Y_1) y (X_2, Y_2) de la línea, las constantes Ao y A_1 puede ser determinadas; la ecuación de la línea resultante puede escribirse.

$$Y - Y_1 = \left(\frac{Y_2 - Y_1}{X_2 - X_1}\right)(X - X_1) \text{ ó } Y - Y_1 = m\,(X - X_1)$$

de donde $m = \dfrac{Y_2 - X_1}{X_2 - X_1}$ = Pendiente de la línea y representa el cambio de Y dividido por el correspondiente cambio de X.

Cuando la ecuación se escribe de la forma (1) la constante A_1 es igual a la pendiente m y la constante Ao se llama intersección de Y.

Recta de mínimos cuadrados. Este método hace mínima la suma de los cuadrados de las desviaciones entre los puntos dados y dicha curva.

La recta de aproximación de mínimos cuadrados del conjunto de puntos: (X_1,Y_1), (X_2,Y_2)...., (Xn,Yn) tiene la ecuación.

$$Y = Ao + A_1X$$

de donde las constantes Ao y A_1 se determinan mediante el sistema de ecuaciones.

$$\Sigma Y = AoN + A_1\Sigma X$$
$$\Sigma XY = Ao\Sigma X + A_1\Sigma X^2$$

Parábola de mínimos cuadrados. La parábola de aproximación de mínimos cuadrados a la serie de puntos (X_1,Y_1) (X_2,Y_2)...(Xn,Yn) tiene la ecuación.

$$Y = Ao + A_1X + A_2X^2$$

donde las constantes Ao, A_1 y A_2 se determinan mediante el sistema de ecuaciones.

$$\Sigma Y = A_oN + A_1\Sigma X + A_2\Sigma X^2$$
$$\Sigma XY = A_o\Sigma X + A_1\Sigma X^2 + A_2\Sigma X^3$$
$$\Sigma X^2Y = A_o\Sigma X^2 + A_1\Sigma X^3 + A_2\Sigma X^4$$

Todas las líneas o curvas de ajuste, tienen varios objetivos:

a) Describir la tendencia general mediante una línea más sencilla que la poligonal original.

b) Proporcionar una idea acerca del comportamiento futuro de la serie.

c) Obtener alguna información sobre el comportamiento de series similares.

Ejemplo: perecuar una recta que pase por el origen y entre los puntos A(1,1); B(2,3); C(3,2); D(4,5); E(5,4) y F(6,6), por el método de mínimos cuadrados.

X	Y	X^2	XY	Yc
1	1	1	1	1.30
2	3	4	6	2.18
3	2	9	6	3.06
4	5	16	20	3.94
5	4	25	20	4.82
6	6	36	36	5.70
21	21	91	89	21.00

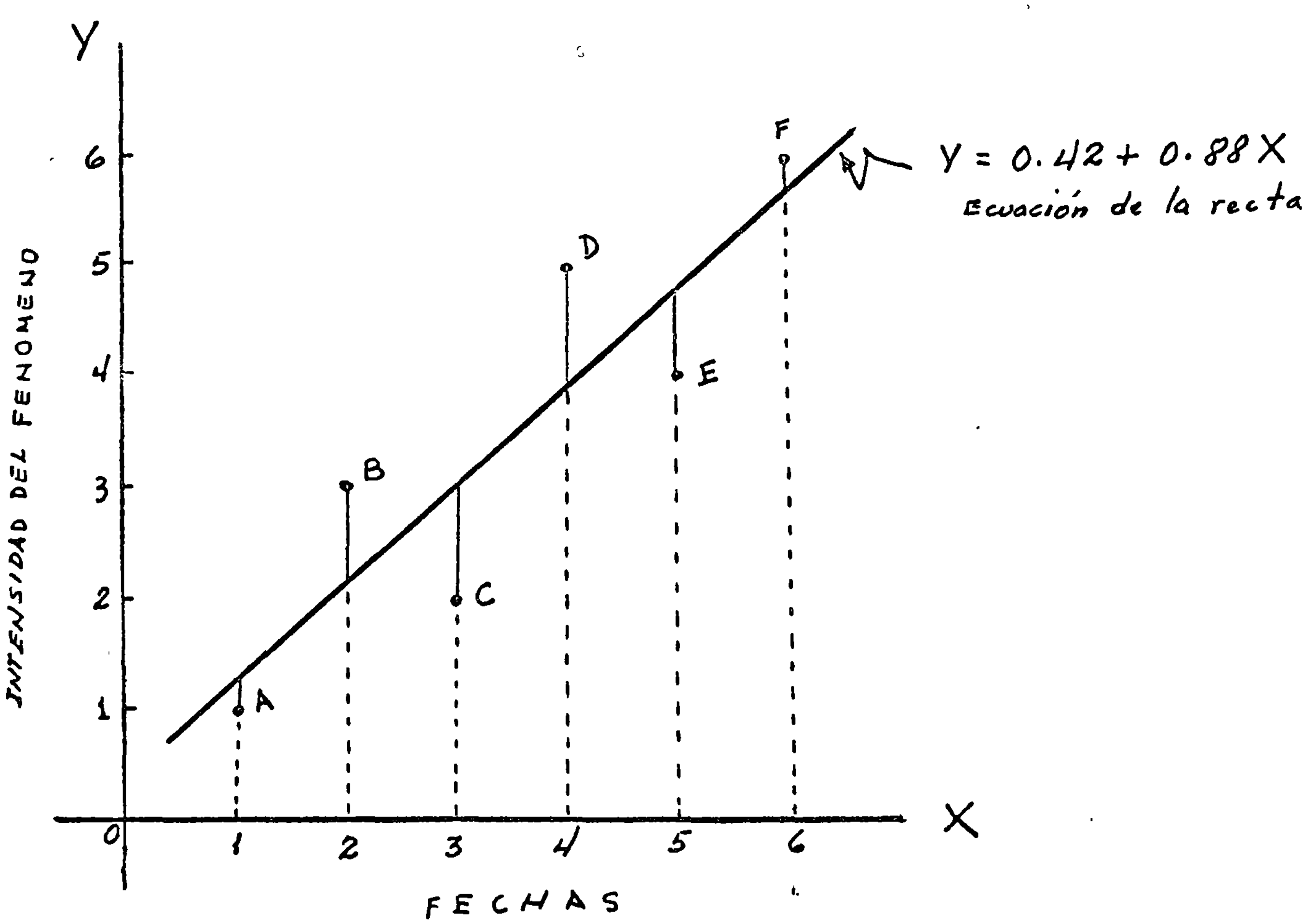
$$\Sigma Y = A_0 N + A_1 \Sigma X$$
$$\Sigma XY = A_0 \Sigma X + A_1 \Sigma X^2$$

$$21 = 6A_0 + 21A_1 \quad \therefore A_0 = \frac{21 - 21A_1}{6}$$

$$89 = 21\left(\frac{21-21A_1}{6}\right) + 91A_1 = 73.5 - 73.5A_1 + 91A_1$$

$$\therefore A_1 = \frac{15.5}{17.5} = 0.88$$

$$A_0 = \frac{2.56}{6} = 0.42$$



DESVIACION STANDARD

Una desviación standard o típica como también se le conoce, es un índice de variabilidad absoluta, que nos mide el grado de confiabilidad en una serie de distribución.

$$\sigma = \sqrt{\frac{\Sigma d^2 f}{\Sigma f}}$$

EJEMPLO:

(a)	f	(a)(f)	d	d^2	$(d^2)(f)$
1.54	3	4.62	- 0.12	0.0144	0.0432
1.57	2	3.14	- 0.09	0.0081	0.0162
1.61	4	6.44	- 0.05	0.0025	0.0100
1.67	5	8.35	0.01	0.0001	0.0005
1.72	2	3.44	0.06	0.0036	0.0072
1.81	3	5.43	0.15	0.0225	0.0675
	20	33.27			0.1807

$$M = \frac{33.27}{20} = 1.66$$

$$\sigma = \sqrt{\frac{0.1807}{20}} = 0.095$$

COEFICIENTE DE CORRELACION

Un coeficiente de correlación es la relación que existe entre dos fenómenos, es decir, la dependencia de uno con respecto al otro.

$$r = \frac{\Sigma X' Y'}{\sqrt{\Sigma X'^2 \Sigma Y'^2}}$$

X	Y	X'	Y'	Y^2	X'^2	X' Y'
3.5	3.4	- 0.7	- 0.4	0.49	0.16	0.28
4.1	3.5	- 0.1	- 0.3	0.01	0.09	0.03
4.1	3.9	- 0.1	0.1	0.01	0.01	- 0.01
4.4	3.8	0.2	0	0.04	0	0
4.9	4.4	0.7	0.6	0.49	0.36	0.42
21.1	18.9	0	0	1.04	0.62	0.72

$M_x = \frac{21.1}{5} = 4.2$ $\quad M_y = \frac{18.9}{5} = 3.8$

$\Sigma X'Y' = 0.72$

$\Sigma X'^2 = 1.04$

$\Sigma Y'^2 = 0.62$

$$r = \frac{0.72}{\sqrt{1.04 \times 0.62}} = \frac{0.72}{0.802} \quad \therefore r = 0.897$$

X = Consumo de carbón en millones de toneladas.

Y = Producción de acero en millones de toneladas.

Lo cual quiere decir que existe una gran correlación o dependencia entre el consumo de carbón y producción de acero en México.

EL MODELO DE INSUMO-PRODUCTO

En los últimos años ha sido manifiesto entre los economistas de todos los países del mundo, un interés creciente por las investigaciones que han venido efectuándose en materia de relaciones interindustriales. Ese interés aparejó una comprensible preocupación por examinar la utilidad que un modelo de insumo-producto podría prestar en el análisis de los problemas económicos.

El método de insumo-producto constituye en esencia un complemento de las cuentas nacionales. En el caso de éstas se está interesado en el resultado final de la actividad económica, prescindiendo de las transacciones que ha tenido lugar entre los distintos sectores productivos. En cambio, el examen de estas transacciones es lo que constituye el objetivo principal de un modelo de insumo-producto.

Examínese por ejemplo a través de un esquema muy sencillo, la forma en que se calcularía un producto bruto a precios de mercado. En tal caso, el problema consistiría básicamente en el cálculo del valor agregado por ramas de actividad económica, partiendo del valor bruto de la producción y deduciendo los pagos a otros sectores por concepto de compras de materias primas y productos intermedios o por otros servicios. La producción total de bienes y servicios se agrupa sólo en tres sectores: agricultura, industria y servicios.

CALCULO DEL VALOR AGREGADO

AGRICULTURA

Valor bruto de la producción		100
Menos: semillas	5	
abonos	10	
servicios	10	25
Valor agregado		75

INDUSTRIA

Valor bruto de la producción		150
Menos: Materias primas Agrícolas	30	
Productos intermedios manufacturados	40	
Servicios	10	80
Valor agregado.		70

SERVICIOS

Valor bruto de la producción	140
Menos: Productos intermedios manufacturados	10
Valor agregado.	130

Valor total y composición sectorial del producto bruto estaría dado por:

AGRICULTURA	75
INDUSTRIA	70
SERVICIOS	130
PRODUCTO BRUTO TOTAL	275

Lo anterior puede comprobarse clasificando la producción bruta de cada sector en la parte vendida a otros sectores (ventas o demanda intermedia) y la parte de la producción vendida fuera de los sectores productivos (ventas o demanda final) como bienes de consumo o inversión.

AGRICULTURA

Valor bruto de la producción		100
Ventas intermedias: a la propia agricultura (semillas)	5	
A la industria	30	35
Ventas finales.		65

INDUSTRIA

Valor bruto de la producción		150
Ventas Interm: A la agricultura (abonos)	10	
A la propia industria	40	
A servicios	10	60
Ventas finales.		90

SERVICOS

Valor bruto de la producción		140
Ventas Interm: A la agricultura	10	
A la industria	10	20
Ventas finales.		120

TOTAL VENTAS FINALES:

AGRICULTURA	65
INDUSTRIA	90
SERVICIOS	120
T O T A L	275

En el cálculo anterior fueron eliminadas las transacciones intersectoriales, por lo que en la matríz de insumo-producto, se incluye un registro completo de esas transacciones.

MATRIZ DE INSUMO-PRODUCTO

COMPOSICION DE INSUMOS (COMPRAS) ↓ / DISTRIBUCION DE LA PRODUCCION (VENTAS) →	AGRICULTURA	INDUSTRIA	SERVICIOS	TOTAL VENTAS A SECTORES PRODUCTIVOS	DEMANDA FINAL	VALOR BRUTO DE LA PRODUCCION
AGRICULTURA	5	30	-	35	65	100
INDUSTRIA	10	40	10	60	90	150
SERVICIOS	10	10	-	20	120	140
TOTAL INSUMOS	25	80	10	115		
VALOR AGREGADO (PRODUCTO BRUTO)	75	70	130		275	
VALOR BRUTO DE LA PRODUCCION	100	150	140			390

Como puede observarse, un cuadro de insumo-producto no constituye en último término sino un registro de todas las transacciones efectuadas en la economía durante un cierto período de tiempo, comprendiendo tanto las que han tenido lugar entre los sectores productivos como las ventas de demanda final.

Es evidente que un cuadro de esta naturaleza tendría un considerable interés en sí mismo, por las informaciones de tipo descriptivo que recoge en forma resumida: disponibilidad de los distintos tipos de productos clasificados por industrias de origen; distribución de la producción bruta entre bienes finales e intermedios; estructura de los costos de los diferentes sectores e independencia de los mismos; etc. Pero la mayor utilidad del modelo radica en que (al ofrecer una cuantificación de las inter-relaciones de los diversos sectores de la economía), permite examinar las repercuciones que sobre cada uno de esos sectores tendría una modificación cualquiera de la demanda final.

Para demostrar lo anterior, sería necesario en primer lugar, calcular una matríz de coeficientes técnicos (cantidad de insumo que se requiere para producir una unidad del producto); posteriormente, construir la matríz de Leontief que consisten en restar de una matríz unitaria dichos coeficientes técnicos. Como siguiente paso, se procedería a invertir esta matríz para calcular el valor bruto de la producción, y asímismo se procedería a estimar la cantidad de insumos que se requeriría para satisfacer el nuevo aumento en la demanda final de cualquier sector, para que finalmente se proceda a la construcción de la nueva matríz de insumo-producto.

Finalmente es conveniente señalar que el modelo que se ejemplificó, se refiere a una economía cerrada, es decir, no interviene el sector externo, lo único que se haría en este caso, sería agregar otra columna.