

DIRECTORIO DE ASISTENTES AL CURSO DE DISEÑO DE ESTRUCTURAS DE ACERO  
( DEL 4 DE SEPTIEMBRE AL 20 DE OCTUBRE DE 1972 )

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**NOTAS RELATIVAS A MIEMBROS SOMETIDOS A TENSION Y  
COMPRESION AXIAL.**

**Jose Luis Sánchez Martínez.**

Antes de estudiar métodos para el diseño de elementos estructurales de cualquier material es necesario conocer el comportamiento mecánico de ese material, para ello se recurre a menudo a gráficas esfuerzo-deformación que permiten conocer algunas de las más importantes características de ese comportamiento.

En el caso del acero estructural es muy conocida la gráfica  $\sigma$ - $\epsilon$  correspondiente a una probeta libre de esfuerzos residuales sometida a tensión (Ver figura N°. 1).

Menos conocida que la gráfica anterior pero quizá más importante por ser más real, es la gráfica que se obtiene cuando la probeta que se utiliza es un tramo de un perfil estructural real, por ejemplo una vigueta o una sección formada por varias placas soldadas (Ver. figura N°. 2).

Puede notarse que en el segundo caso, a diferencia del primero, el material no se conserva elástico hasta llegar al esfuerzo  $\sigma_y$  sino que para un valor del esfuerzo de aproximadamente  $\frac{\sigma_y}{2}$  la gráfica deja de ser recta y se convierte en una curva que se prolonga hasta  $\sigma_y$  valor a partir del cual la gráfica es una línea recta horizontal.

La diferencia entre las dos gráficas puede explicarse por el hecho de que en el segundo caso la probeta está sometida, antes de que cargas exteriores actúen sobre ella, a un estado de esfuerzos en equilibrio.

Dichos esfuerzos reciben el nombre de esfuerzos residuales y se pueden deber a varias causas, la más importante se encuentra en el proceso de fabricación del perfil.

Es bien sabido que para fabricar un perfil el acero que lo ha de formar se funde, se le da la forma requerida y luego se deja enfriar; al producirse este enfriamiento las partículas de acero se contraen, si esa contracción se efectuara libremente no se produciría ningún esfuerzo, sin embargo las distintas partes del perfil no se enfrían simultáneamente, en una vigueta por ejemplo, se enfrían primero los extremos de los patines y al hacerlos se contraen arrastrando al material adyacente aún en estado plástico, después se enfrían la parte central de los patines y el alma que tratan también de contraerse, pero esa contracción se ve parcialmente evitada por las partes ya endurecidas, esto da lugar a que las fibras de estas zonas queden con una longitud algo mayor que la que hubieran tenido de haberse enfriado libremente y por ello quedan sometidas a un esfuerzo inicial de tensión.

Una distribución de esfuerzos residuales típica para una vigueta se muestra en la figura N°. 2.

Es fácil mostrar por qué el efecto de estos esfuerzos residuales consiste en reducir el valor del esfuerzo en el límite de proporcionalidad del material.

En las figuras números 3 y 4 se hace esto para un caso simplificado.

## TENSIÓN AXIAL.

El problema de diseño de piezas de acero a tensión se reduce a seleccionar una sección con área suficiente para soportar la carga de diseño sin exceder el esfuerzo permisible a tensión.

El esfuerzo permisible se obtiene dividiendo el esfuerzo en el límite de fluencia entre un coeficiente de seguridad que frecuentemente se fija de 1.65 para estructuras para edificios pero que varía de acuerdo con las condiciones del problema.

Los miembros metálicos a tensión más comunes son varillas, cables o perfiles laminados, de estos el elemento más usado es el ángulo.

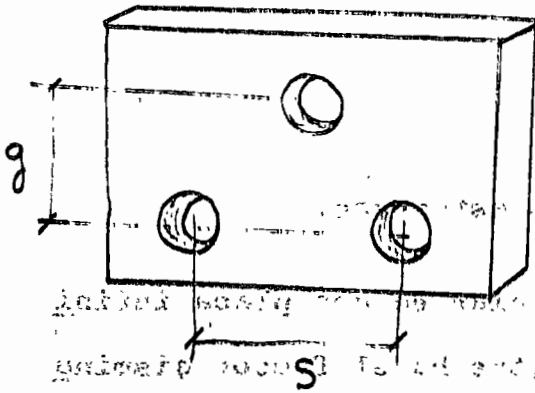
Cuando la unión del elemento a tensión con otras piezas es soldada, se puede considerar el área completa de la pieza para el cálculo de esfuerzos; cuando la conexión es remachada o atornillada el área necesaria debe ser mayor que la obtenida con la fórmula:  $\sigma = \frac{P}{A}$ , el área adicional es necesaria para compensar la presencia de los agujeros para los remaches o tornillos.

Cuando los agujeros están en una serie de líneas normales a la fuerza exterior es fácil determinar la sección que fallará y por tanto el área de huecos que deberá descontarse.

Sin embargo, para distribuciones de agujeros distintas no es tan simple determinar el área de falla que servirá de base para los cálculos de capacidad de carga y de determinación de esfuerzos.

El caso típico es el siguiente:





En este caso la reducción en área que se debe hacer puede ser mayor a la que corresponde a un solo hueco pero menor que la que correspondería a dos. Muchos investigadores han presentado ecuaciones para calcular el ancho neto, la más usada es la siguiente:

$$B_n = B - \sum \phi + \sum_{n=1}^3 \frac{S_n^2}{4g^2 n}$$

$$A_n = B_n \cdot t$$

La línea crítica es la que de menor ancho neto.

Quando se trate de ángulos, estos se desdoblarán idealmente para trabajarlos como con placas. (Ver. Figura N°. 5).

La presencia de agujeros en una pieza a tensión es causa

de la aparición en los puntos vecinos a ellos, de concentraciones de esfuerzos, es decir, de esfuerzos notablemente mayores al

esfuerzo promedio =  $\frac{P}{A}$ .

La determinación analítica de la distribución elástica

de tales esfuerzos es un problema complicado de la teoría de la

elasticidad, por ello para determinarlos se han utilizado métodos

experimentales que han permitido comprobar los resultados teóricos

en los casos en que se cuenta con ellos, y obtenerlos en las

ocasiones en que teóricamente no se han logrado obtener.

En general:

$$\sigma_{\max} = \sigma_{\text{prom}} \times K$$

$K$  = factor de concentración de esfuerzos.

Este factor para un hueco circular en una placa infinitamente larga es 3, si la placa es más estrecha el factor disminuye.

A pesar de lo anterior y en el caso del acero, el procedimiento de diseño común desprecia estas concentraciones de esfuerzos; la justificación de esto es que este material tiene un rango de comportamiento plástico muy amplio y por ello admite grandes deformaciones cuando se alcanza el esfuerzo en el límite de fluencia; por esto, cuando se llega al esfuerzo  $\sigma_y$  en los puntos más esforzados aumenta la deformación de toda la pieza, manteniéndose el esfuerzo  $\sigma_y$  en estos puntos y aumentando en todos los demás; cuando la falla sobreviene toda la sección está esforzada a  $\sigma_y$ .

De lo anterior puede concluirse que la resistencia máxima a tensión o resistencia última a tensión de una pieza de acero es simplemente el producto del área neta por el esfuerzo en el límite de fluencia del material.

Lo anterior es, sin embargo, solamente cierto para el caso en que la carga no fluctúa un gran número de veces entre límites muy diferentes como suele ocurrir en estructuras para puentes, en grúas o en torres, en estos casos la falla puede producirse por fatiga y la presencia de agujeros u otras causas de concen

tración de esfuerzos reduce notablemente la capacidad de la pieza. Lo mismo puede decirse en casos en que la estructura trabaja a temperaturas muy bajas o el acero tiene una composición química desfavorable, ya que entonces el acero puede perder en parte su ductilidad y presentar la falla conocida como frágil. Bajo estas circunstancias las concentraciones de esfuerzos son también causa de pérdida de resistencia del miembro.

Puede concluirse por lo tanto que en cualquier caso una buena práctica de diseño es limitar en lo posible las concentraciones de esfuerzos.

Las especificaciones A.I.S.C. recomiendan como esfuerzo permisible en el área neta de piezas a tensión:

$$F_t = 0.6 \sigma_y.$$

Debido al advenimiento de aceros de gran resistencia a la tensión en que está, en ocasiones, muy semejante a  $F_y$ , se indica en las últimas especificaciones como precaución adicional, no tomar un esfuerzo permisible mayor que la mitad del correspondiente a la resistencia a la tensión.

Del resultado de un gran número de investigaciones se ha concluido que el esfuerzo en secciones netas de agujeros para pasadores no debe ser mayor de  $0.45 F_y$ .

Se especifica también, de estudios de eficiencia de secciones netas, que esta no se tome nunca mayor de 85% del área de la sección total.

La relación de esbeltez en piezas a tensión se limita a 240 en miembros principales y a 300 en secundarios (esto es solo una recomendación de las normas y no se aplica a varillas).

### PIEZAS SUJETAS A COMPRESION AXIAL.

Sin duda el estudio de las piezas sujetas a compresión constituye uno de los temas más importantes de las estructuras de acero; nos encontramos por primera vez ante el fenómeno de pandeo, problema característico de este tipo de estructuras por aparecer en el diseño de la mayor parte de las piezas que las constituyen.

El elemento estructural más simple en que se presenta este fenómeno es la columna, pieza de eje recto sometida a compresión axial.

Consideremos una columna esbelta, de sección transversal constante, perfectamente recta y sometida a una carga axial creciente; la carga se puede aumentar hasta que al llegar a un determinado valor se presenta una flexión repentina alrededor de alguno de los ejes principales de inercia o una torsión alrededor del eje longitudinal de la pieza a la que sigue de inmediato el colapso. Si se calcula el esfuerzo que corresponde a la carga para la que se inició la deformación, se encuentra que dicho esfuerzo es menor al esfuerzo correspondiente al límite de fluencia del material, esto indica que la falla no ha sido un problema de resistencia sino de estabilidad.

A la carga para la que se ha iniciado la falla se le designa con el nombre de carga crítica y a la falla en si con el de falla por pandeo de la columna.

Para valores de la carga axial menores que el valor de la carga crítica la configuración recta de la columna es de equilibrio estable, si se somete a la columna a la acción de una pequeña fuerza normal al eje de la misma, la columna se flexiona ligeramente, pero cuando la fuerza se retira la columna regresa a su posición recta de equilibrio.

Para valores de la carga axial mayores que el de la carga crítica la configuración recta es también de equilibrio, pero el equilibrio en este caso es inestable, basta cualquier fuerza accidental, excentricidad ó falta de homogeneidad en el material para que la falla sobrevenga, la experiencia ha demostrado que no es posible sobrepasar la carga crítica o siquiera llegar a ella sin que sobrevenga la falla porque las imperfecciones de algún tipo son inevitables. (Ver figura C1).

La carga crítica corresponde a la transición de equilibrio estable a equilibrio inestable, esto es, corresponde a la condición de equilibrio indiferente, para ella son posibles una configuración recta de equilibrio y una configuración ligeramente deformada de equilibrio (se dice que ocurre una biformación de la posición de equilibrio) y en este hecho se basa su determinación. (Ver figura N°. C-2).

La determinación de la carga crítica para una pieza en estas condiciones fué realizada por primera vez en 1759 por Euler que fué el primer investigador que pensó que la capacidad de una columna podría estar controlada por un problema de inestabilidad, a Euler se deben los primeros estudios teóricos sobre el comportamiento de columnas.

El valor de la carga crítica para una pieza doblemente articulada es  $P_{cr} = \frac{\pi^2 EI}{L^2}$ , se puede obtener en forma similar al caso mostrado y se le ha llamado carga crítica fundamental o de Euler. En general  $P_{cr} = \frac{\pi^2 EI}{(kl)^2}$  (Ver. figura C-3).

La obtención de las fórmulas anteriores se basa en la hipótesis fundamental de que la columna se comporta elásticamente hasta la aparición del fenómeno de pandeo; por lo tanto dichas fórmulas no son válidas en columnas cortas en que se alcanza antes el esfuerzo correspondiente al límite de proporcionalidad que el esfuerzo crítico de pandeo.

Por lo anterior el rango de aplicación de la fórmula de Euler quedará limitado por la condición:

$$\sigma_{cr} = \sigma_{LP}$$

(Ver figura N°. C-4).

Muchas de las columnas que encuentran en la práctica tienen relaciones de esbeltez menores que la encontrada, es por ello importante estudiar el problema de pandeo en el rango de comportamiento enelástico del material.

Dicho problema fué atacado por primera vez por Engeser en 1889 en que publicó su teoría del módulo tangente, esta teoría se basa en la suposición de que para un determinado valor del esfuerzo, el esfuerzo crítico, es posible una configuración deformada de equilibrio y que la deformación es controlada por el módulo de elasticidad tangente correspondiente a ese esfuerzo en cuestión. Esta suposición implica la aplicación de la fórmula de Euler sustituyendo E por Et. (Ver figura C-4).

Dada la gráfica esfuerzo-deformación de un material, se puede obtener de ella el valor del módulo tangente para cualquier esfuerzo, si se supone que ese esfuerzo es el crítico para una columna determinada, se puede aplicar la fórmula: 
$$P_{cr} = \frac{\pi^2 Et}{(1/r)^2}$$
 y despejar el valor de  $1/r$  que corresponde al  $P_{cr}$  considerado.

Repetiendo esta operación se pueden tener los datos necesarios para trazar una gráfica  $P_{cr} = 1/r$  que puede usarse directamente para diseño.

La teoría del módulo tangente se ha deducido suponiendo que la columna se mantiene recta hasta que se llega a la carga crítica y que el módulo tangente es constante en toda la sección recta de la pieza, esta segunda suposición implica que no hay disminución de las deformaciones en el lado convexo de la pieza cuando se pasa de la configuración recta a la ligeramente deformada, esta suposición, por supuesto, no está justificada y esto se hizo ver casi inmediatamente después de la publicación de la teoría del módulo tangente.

El mismo Engesser en 1895 modificó su teoría presentando otra a la que llamó del módulo reducido o módulo doble y que tiene en cuenta el hecho de que en la sección recta de la pieza se tienen dos módulos de elasticidad distintos, el módulo tangente en la zona en que los esfuerzos aumentan al producirse la deformación y el módulo de elasticidad en la zona en que disminuye. (Ver. figura C-5).

Como para encontrar la carga crítica con la teoría del módulo reducido se ha utilizado el mismo criterio que para deducir la fórmula de Euler, se está aceptando que no es posible ningún estado deformado de la columna para cargas menores que  $P_{cr} = P_r$ .

La teoría del módulo reducido fué considerada correcta hasta que en 1947 Shanley llamó la atención sobre el hecho de que--suponer que la columna permanecía recta hasta  $P_r$  constituía una paradoja; en efecto para que la carga que exista sobre la columna sea mayor que la calculada con la teoría del módulo tangente es necesario que la pieza se deforme, pero  $P_t < P_r$  luego la columna no se puede mantener recta hasta  $P_r$ .

Shanley demostró que:

- a).-  $P_t$  es la máxima carga que se puede aplicar a la columna para que se mantenga recta.
- b).-  $P_T$  puede aumentarse aumentando simultáneamente la deformación hasta la carga de falla.
- c).- La carga de falla es solo ligeramente superior a  $P_T$ .

La aplicación de la teoría del módulo tangente al acero estructural permite trazar gráficas  $\sigma_{cr} - L/r$  utilizables para el diseño de columnas (Ver figura C-6).

La primera parte de la curva, para relaciones de esbeltez altas, se traza fácilmente utilizando la fórmula de Euler, para relaciones de esbeltez medias o bajas la forma de la curva depende de la distribución de esfuerzos residuales en la columna que es variable de caso en caso, se ha obtenido sin embargo una curva que se aproxima aceptablemente a la mayor parte de los casos prácticos.

Se ha supuesto que el material se comporta elásticamente hasta un esfuerzo igual a  $\sigma_y/2$ .



En algunas ocasiones el pandeo se inicia con una torsión de la pieza esto es, un giro alrededor de su eje longitudinal, y no con flexión alrededor del eje de menor momento de inercia de la sección transversal de la columna.

Aunque el caso es poco frecuente en columnas reales, puede tener importancia en algunos casos especiales de secciones abiertas delgadas.

La carga crítica para este tipo de pandeo se determina en una forma similar al pandeo debido a flexión. Una vez determinada se compara con la carga crítica debida a flexión y la más pequeña será la que ocurrirá primero y regirá por tanto el diseño.

La ecuación general de partida para determinar la carga crítica por torsión, equivalente a la de la elástica en el caso de pandeo por flexión, tiene la forma siguiente:

$$EK_1 \frac{d^3 \theta}{dz^3} - GK \frac{d\theta}{dz} = -T$$

K = cte. de torsión; para una sección compuesta de rectángulos

$$K = \frac{bt^3}{3}$$

K<sub>1</sub> = cte. de alabeo, es una propiedad de la sección, para una

vigueta:  $K_1 = \frac{I_y d^2}{4}$

Conocido el valor del momento torsionante T para un giro  $\beta$  se puede integrar esta ecuación y determinar los valores de la carga para los cuales la solución de la ecuación se cumple, esto es, para los que es posible una configuración ligeramente deformada de equilibrio.

Para una sección rectangular solo rige pandeo por torsión para relaciones  $\frac{L}{b} < 0.73$

Para un ángulo, una fórmula aproximada que da el pandeo por torsión es:

$$\sigma_{ct} = \frac{E}{2(1+\mu)} \left( \frac{t}{b} \right)^2$$

En el caso de una vigueta:

$$P_{cr_t} = \frac{A}{J} \left( GK + \frac{\pi^2 EK_1}{l^2} \right)$$

No rige para los perfiles usados en la práctica.

Las especificaciones de A.I.S.C. y las recomendaciones del Manual de Diseño de la C.F.E. presentan fórmulas de diseño obtenidas directamente de la curva de la figura C6.-

Así los esfuerzos permisibles están dados por la fórmula de Euler dividida entre un coeficiente de seguridad igual a

$$\frac{23}{12} \text{ y es válida hasta un valor de } kl/r = Cc = \sqrt{\frac{2\pi^2 E}{\sigma_y}}$$

$$\text{En este caso } F_a = \frac{12\pi^2 E}{23 (kl/r)^2}$$

para valores de  $\frac{1}{r} < Cc$

$$F_a \text{ C.S.} = \left[ 1 - \frac{\left(\frac{kl}{r}\right)^2}{\frac{4\pi^2 E}{\sigma_y}} \right] \sigma_y = \left[ 1 - \frac{\left(\frac{kl}{r}\right)^2}{2Cc^2} \right]$$

$$C.S. = \frac{5}{3} + \frac{3kl}{8Cc} - \frac{(kl/r)^2}{8Cc^3}$$

C.S. varía de  $\frac{5}{3}$  para  $\frac{1}{r} = 0$  hasta  $\frac{23}{12}$  para columnas largas.

Para miembros secundarios con  $\frac{1}{r} > 120$  se permite un factor de seguridad más liberal pero se especifica  $K = 1$  en todos los casos

$$F_{as} = \frac{F_a}{1.6 - \frac{1}{1200 r}}$$

Como resumen se puede proponer el siguiente método de diseño para piezas sometidas a compresión axial.

1.- Elegir el tipo de perfil que se usará para diseñar; lo pueden definir condiciones tales como: facilidad de conexión, forma estructural conveniente, requisitos arquitectónicos.

2.- Suponer un esfuerzo permisible aproximado y calcular el área aproximada necesaria (un valor usual para este tanteo es de  $1000 \text{ Kg/cm}^2$ .)

3.- Seleccionar un perfil con esta área.

4.- Calcular las relaciones  $\frac{Kl}{r}$ .

$Kl$  = longitud efectiva de pandeo.

$r$  = radio de giro de la sección en la dirección considerada.

5.- Calcular el esfuerzo permisible real en función de

$\frac{kl}{r}$  máximo.

6.- Calcular la capacidad de la pieza y compararla con la fuerza exterior.

Llamaremos  $r_x$  al radio de giro dado por  $\frac{I_x}{A}$

$I_x$  Momento de inercia respecto al eje x.

$K l_x$  Longitud efectiva de pandeo cuando el plano de pandeo es el plano y (pandeo alrededor del eje x).

$$r_y = \sqrt{\frac{I_y}{A}}$$

$I_y$  Momento de inercia respecto al eje y.

$K l_y$  Longitud efectiva de pandeo cuando el plano de pandeo es el plano x (Pandeo alrededor del eje y).

## PANDEO LOCAL

Salvo en casos excepcionales todos los perfiles estructurales de acero se componen de elementos planos delgados, des por ello necesario estudiar la posibilidad de una falla local por pandeo de estos elementos para cargas menores que las que se requerirían para hacer fallar a la columna como un conjunto. (Ver figura C7).

De hecho este problema se presenta si la relación ancho a espesor de las placas que constituyen el perfil excede ciertos valores.

Con objeto de definir las relaciones ancho a espesor límites, (aquellas a partir de las cuales ocurre pandeo local), es necesario considerar el comportamiento de placas sometidas a compresión axial.

Supongamos una placa delgada en que la fuerza axial que la comprime aumenta gradualmente, para un determinado valor de esta fuerza se inicia el pandeo de la placa pero, a diferencia de lo que ocurre con una columna, la iniciación del pandeo no significa, en general, el colapso inmediato de la placa, por el contrario, es necesario aumentar en forma notable la carga para que la falla sobrevenga.

Esto se debe al trabajo en dos sentidos de la placa, la deformación de las fibras longitudinales esta restringida por las transversales.

La resistencia de la placa más allá de la iniciación del fenómeno de pandeo depende de las características geométricas de la placa y de sus condiciones de apoyo. El caso mas común es el que se indica en la figura C8.

Como la longitud  $a$  es mucho mayor que la  $b$ , la influencia en el comportamiento de la placa de las condiciones de apoyo en los bordes  $b$  es despreciable, en cambio la influencia de las condiciones en los bordes  $a$  es fundamental. Estos apoyos pueden tener características muy variables, desde el apoyo empotrado hasta la ausencia total de apoyo.

Interesa, como en el caso de columnas, encontrar el valor del esfuerzo para el cual son posibles dos condiciones de equilibrio, esto es, el esfuerzo crítico correspondiente a la iniciación del pandeo.

Se obtiene el siguiente valor:

$$\sigma_{cr} = \frac{\pi^2 E \sqrt{Z}}{12(1-\mu^2)} \left(\frac{t}{b}\right)^2 K ; Z = \frac{Et}{E}$$

Que se considera válido para pandeo tanto en el rango elástico como en el inelástico.  $K$  depende de las condiciones de apoyo básicamente. (Ver figura 08)

La ecuación es aplicable directamente para pandeo elástico con  $Z = 1$

En el rango inelástico se puede usar la fórmula aproximada siguiente:

$$Z = \frac{(\sigma_y - \sigma_{cr}) \sigma_{cr}}{(\sigma_y - \sigma_{LP}) \sigma_{LP}}$$

Un criterio de diseño lógico contra pandeo local consistirá en conseguir que dicho pandeo no se presente antes que el pandeo de toda la columna. (ver figura 09)

El criterio de las especificaciones del A.I.S.C. es que aparezca para esfuerzos en la columna iguales a  $\sigma_y$ .

El problema de pandeo local tiene una influencia definitiva en el diseño de perfiles troquelados de lámina de calibre ligero. En perfiles laminados y en perfiles formados con placas el problema empieza a ser también importante pues cada vez se tiende más a la utilización de aceros de altas resistencias que conducen al uso de elementos delgados, de ahí que en las últimas especificaciones del A.I.S.C. se propongan por primera vez expresiones precisas para perfiles formados con este tipo de elementos.

Veamos primero algunos aspectos relativos al diseño de perfiles troquelados de lámina delgada, que son cubiertos por las especificaciones del A.I.S.I., y posteriormente las normas del A.I.S.C. relativas al mismo tema:



### Perfiles troquelados.

La aplicación de los perfiles de lámina de calibre ligero doblados en frío constituye una rama de las estructuras metálicas cuyo objeto es conseguir construcciones de gran ligereza aplicando los conocimientos más recientes sobre el comportamiento de placas de acero. Se han utilizado ampliamente complementando a los perfiles rolados en caliente en los casos en que estos, por ser más pesados serían poco económicos; muy a menudo ambos tipos de perfiles aparecen en la misma estructura llenando cada uno de ellos la función para la que es más adecuado.

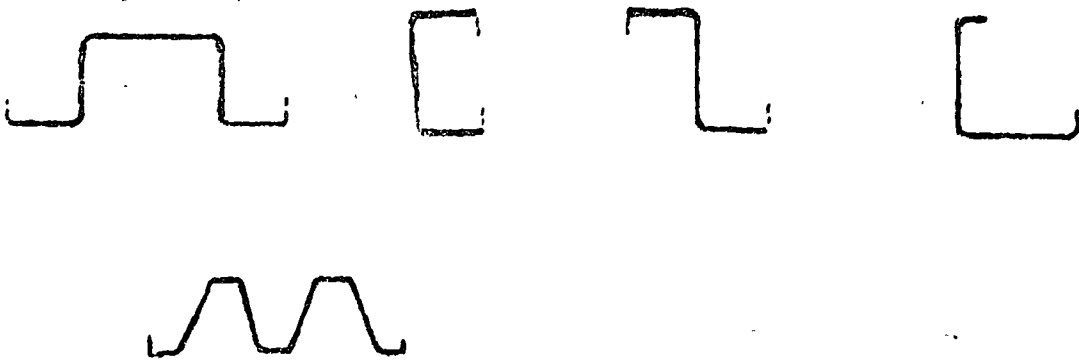
Los perfiles troquelados se obtienen doblando en frío láminas delgadas de acero estructural.

El espesor de las láminas varía usualmente de 3.4 mm (No. 10) a 0.38 mm (No. 28), aunque se pueden obtener perfiles doblando láminas hasta de  $\frac{1}{4}$ " de espesor.

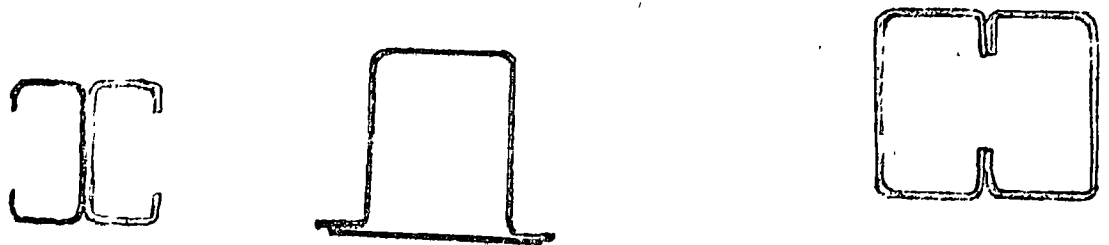
El acero utilizado para formar perfiles estructurales de este tipo tiene usualmente las características fijadas en las especificaciones A245, A303, A345 y A446 de la Sociedad Americana para la Prueba de Materiales (A.S.T.M.) dichas características aseguran los requisitos adecuados de resistencia, ductilidad y soldabilidad.

Una característica ventajosa de los perfiles de lámina de calibre ligero consiste en la facilidad con que se puede obtener una gran variedad de formas diseñadas especialmente para utilizar el material de la manera más efectiva.

A continuación se muestran algunas de las formas más usuales:

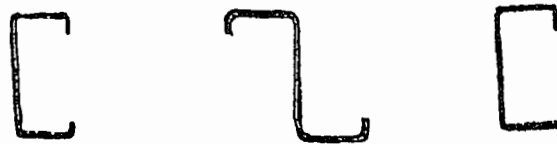


Se pueden obtener formas adicionales uniendo entre --  
si los perfiles simples, para ello se usa primordialmente la --  
soldadura de puntos.



En México existen varias empresas que elaboran per- -  
files formados en frio, principalmente la Cía. Fundidora de Fie

ro y Acero de Monterrey, S.A., que tiene en el mercado en calibres # 10 (3.416 mm), # 12 (2.667 mm) y # 14 (1.905 mm) las siguientes secciones.



y la Compañía de Manufacturas Metálicas de Monterrey, S.A. que tiene en calibres # 4 (5.7 mm), # 6 (4.93 mm), # 8 (4.18 mm), # 10 (3.416 mm) y # 12 los perfiles siguientes:



Aunque los perfiles ligeros se han usado en distintas formas por muchos años, los estudios teóricos y las especificaciones relativas a ellos son recientes. Las primeras investigaciones se iniciaron en la Universidad de Cornell en 1939 y basándose en ellas el Instituto Americano del Hierro y el Acero (A.I.S.I.) publicó en 1946 la primera edición de sus "Especificaciones para el diseño de perfiles de calibre ligero formados en frío", estas especificaciones acompañadas de un "Manual de Diseño" pusieron al alcance del ingeniero estructural ordinario el diseño de esos perfiles.

Las especificaciones del A.I.S.I. son para los perfiles troquelados los que la del A.I.S.C. son para los perfiles volados en caliente ordinarios.

La última edición de las especificaciones del - -

A.I.S.I. se publicó, acompañadas de un comentario completo de las mismas, en 1962.

Diferencias entre el diseño de perfiles troquelados y el de perfiles laminados.

Los principios básicos que se utilizan en el diseño de perfiles de calibre ligero son, desde luego, los mismos que se utilizan en el diseño de los perfiles convencionales rodados en caliente; sin embargo algunas características de los primeros hacen necesario dar especial atención a ciertos problemas que no siendo, en general, de gran importancia en perfiles pesados son, en los de calibre ligero, fundamentales.

a) PLANEO LOCAL.

Quizá el más importante de esos problemas sea el-

del pandeo local, esto es la aparición del pandeo en una zona limitada del perfil independiente de las condiciones de la pieza completa.

El hecho de que los perfiles troquelados estén formados generalmente por placas con una relación de ancho a espesor muy grande, hace que en ellos se inicie, para esfuerzos en general mucho menores que el correspondiente al límite de fluencia del material, el pandeo de alguna de las placas componentes de la sección. El comportamiento de dichas placas inmediatamente después de iniciado el pandeo local depende de sus condiciones de borde.

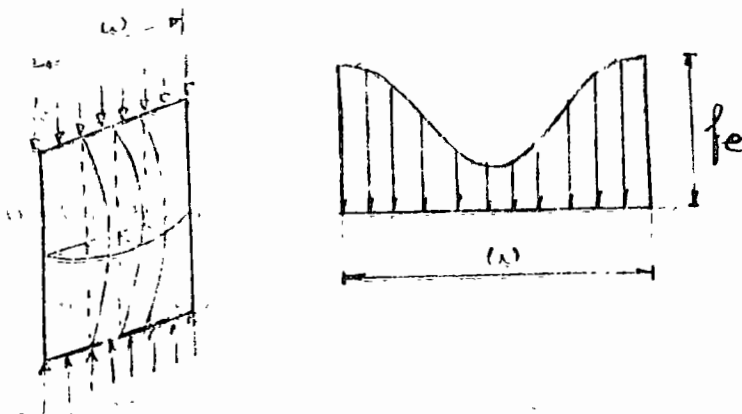
En placas apoyadas en sus bordes, a diferencia de lo que ocurre el caso de pandeo de columnas, la iniciación del pandeo no significa el colapso de la placa si no que -

puede, en muchos casos, aumentarse notablemente la carga para que sobrevenga la falla.

El aprovechamiento de esa resistencia posterior a la iniciación del pandeo es una característica importante en el diseño de perfiles de calibre ligero ya que si, como se procede generalmente al diseñar perfiles convencionales, se supusiera que la iniciación del pandeo es una condición de colapso, la utilidad de los perfiles ligeros se perdería en gran parte pues dicha condición se presenta en ellos para esfuerzos muy pequeños.

En la figura siguiente puede verse la forma en que una placa apoyada en sus lados se pandea bajo la acción de cargas que la comprimen, así como la distribución de esfuerzos en una sección transversal de la misma placa.





Se ha idealizado la placa con una retícula ortogonal de barras, al incrementarse la carga, las barras verticales tratarán de pandearse como si fueran columnas pero dicho pandeo se verá restringido por las barras horizontales unidas a los bordes fijos de la placa. Para un determinado valor de la carga la faja central se deforma y no es

capaz de tomar cargas adicionales, pero las fajas laterales si pueden hacerlo y cualquier incremento en la carga produce por ello, incrementos de esfuerzos en las fajas laterales pero no en la central.

La carga máxima se obtiene cuando el esfuerzo en las faja laterales es tal que la placa completa se deforma sin que sea posible un aumento adicional de carga; para columnas cortas el esfuerzo en las placas laterales coincide con el esfuerzo en el límite de fluencia del material.

Es necesario definir cuando se puede considerarse, en casos prácticos, que una placa está apoyada en dos de sus bordes en la forma idealizada en la figura anterior.

Se ha demostrado que la placa se comporta como si -

tuviera ese tipo de apoyo, cuando en cada uno de los bordes apoyados tiene un doblado con rigidez suficiente.

Es evidente que cuando la placa en consideración tiene apoyo solo en uno de sus bordes la resistencia posterior al pandeo que puede desarrollarse es mucho menor - que cuando el apoyo es en los dos; de hecho en este caso la resistencia posterior a la iniciación del pandeo para relaciones ancho a espesor menores de 30 se desprecia, y para relaciones mayores se considera en forma indirecta admitiendo un coeficiente de seguridad pequeño respecto a la iniciación del pandeo.

En el perfil en forma de canal que se muestra a continuación, el alma se puede considerar apoyada en sus dos bordes, su resistencia posterior al pandeo es -

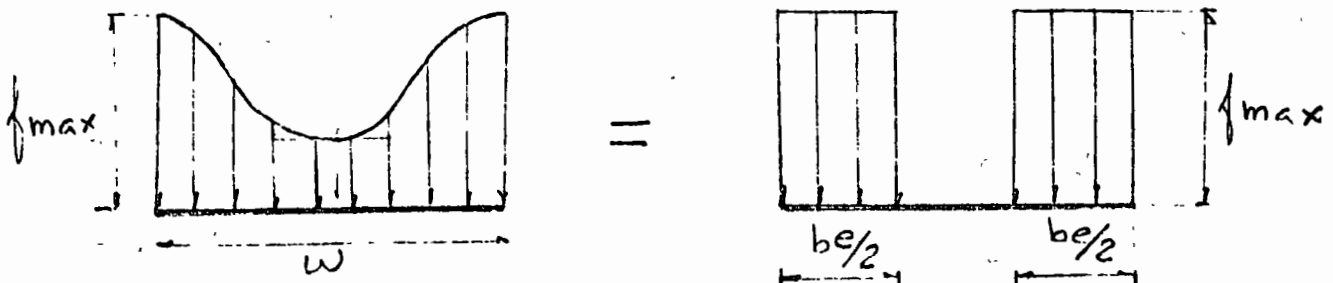
grande y se dice que está atiesada. Los patines tienen un solo apoyo, su resistencia posterior al anqueo es muy baja y se dice que no están atiesadas.



Para tener en cuenta, en el diseño de perfiles atiesados, la resistencia posterior al anqueo en una forma sencilla, se ha ideado el concepto de "ancho efectivo".

La fuerza total en una placa comprimida es igual al área bajo la curva que representa la distribución de esfuerzos en su sección transversal multiplicada por el espesor de la placa; se puede obtener la misma fuerza si se reemplaza la distribución real de esfuerzos por una distribución con esfuerzo constante e igual al máximo -

esfuerzo real pero distribuido sobre un ancho de la sección recta de la placa menor que el total. En la siguiente figura se muestran las dos distribuciones de esfuerzos equivalentes, en ellas  $w$  es el ancho total de la placa y  $b_e$  es el ancho efectivo.



Para una relación ancho a espesor dado si el esfuerzo de compresión es suficientemente pequeño no se iniciará el pandeo local y por lo tanto el ancho total y el ancho efectivo serán iguales, si el esfuerzo aumenta se iniciará el pandeo y el ancho efectivo disminuirá al aumentar el esfuerzo máximo en la placa.

Las especificaciones del A.I.S.I. (SEC. 2.3.1.1) proponen la fórmula siguiente para determinar el ancho efectivo de una placa.

$$\frac{b_e}{t} = 1.9 \sqrt{\frac{E}{f_{max}}} \left( 1 - \frac{0.475}{W/t} \sqrt{\frac{E}{f_{max}}} \right)$$

En el diseño de cualquier perfil troquelado en que exista esfuerzos de compresión es necesario calcular el ancho efectivo y con él todas las propiedades geométricas del mismo, con objeto de tener en cuenta la posible reducción en su capacidad de carga por pandeo local.

$b_e$  = Ancho efectivo de la placa.

$t$  = Espesor de la placa.

$W$  = Ancho total de la placa.

$E$  = Módulo de elasticidad del acero.

$f_{max}$  = Esfuerzo máximo en la placa.

En el hecho de que en perfiles troquelados sea necesario realizar ese cálculo, esto es, en el hecho de que se tenga en cuenta el fenómeno de pandeo local y la resistencia posterior a la iniciación del mismo, reside la diferencia más importante entre el diseño de este tipo de perfiles y el de los perfiles laminados en que el problema de pandeo local no existe.

#### b) C O L U M N A S .

El diseño de columnas y de piezas a flexocompresión de perfiles troquelados, una vez considerado el problema del pandeo local que como se ha dicho debe siempre revisarse, se realiza prácticamente en la misma forma que en el caso de perfiles laminados.

Se hace una distinción entre elementos no atiesados (placas - con apoyo longitudinal solo en un lado) y elementos atiesados (placas con dos apoyos longitudinales) ya que , como se ha -- visto, el comportamiento posterior al pandeo es completamente diferente en ambos casos, en el primero la resistencia posterior al pandeo es pequeña al contrario de la que sucede en el segundo caso.

Para perfiles compuestos de elementos no atiesados la fórmula fundamental utilizada para el diseño es la formula PL1. En la figura C12 se muestra gráficamente esta ecuación así - como la simplificación que de la misma se hace en las especificaciones. Dividiendo los valores del esfuerzo crítico entre un coeficiente de seguridad se obtiene la gráfica que da los esfuerzos permisibles de diseño, como se ve, el coeficiente - de seguridad es variable para tener en cuenta que a medida -- que la relación ancho - espesor aumenta la resistencia posterior al pandeo aumenta también. La ecuación PL1 da valores de los esfuerzos para los cuales el pandeo se inicia, pero para valores de la relación ancho-espesor de aproximadamente 30 y mayores la resistencia última de la placa empieza a ser notablemente mayor que la correspondiente a la iniciación del -- pandeo.

Para ángulos aislados y piezas formadas con placas de características similares a ellos se aceptan esfuerzos permisibles algo menores que para los demás perfiles, con ello se tiene - en cuenta el hecho de que estos perfiles se pandean por torsión para cargas menores que las que se predicen usando la ecuación PL1. En la figura C13 se presentan las fórmulas correspondientes.



Para perfiles atiesados se utiliza el concepto de ancho efectivo y para su determinación se recurre a la ecuación PL2, si bien, en las especificaciones del A.I.S.I. se hace la simplificación de sustituir  $\sigma_{max}$  por  $\sigma_y$ .

Los conceptos vistos se utilizan en el diseño de columnas compuestas de perfiles delgados y sometidas a compresión axial, modificando las fórmulas aplicables a perfiles comunes por medio de un coeficiente Q definido en la forma siguiente:

Para perfiles con elementos no atiesados  $Q_s = \frac{\sigma_p}{\sigma_a}$

Para perfiles con elementos atiesados  $Q_a = A_{ef}/A$

Para perfiles con elementos atiesados y no atiesados el coeficiente usado es  $Q_s \cdot Q_a$ .

Veanse las expresiones que proponen las especificaciones A.I.S.I. en la figura C13.

El criterio de las últimas especificaciones del A.I.S.C. es completamente similar al indicado. Se añade además una limitación a las dimensiones de perfiles en forma de canal y te -- para evitar el problema de pandeo por torsión que puede ser importante en este tipo de secciones.

Las fórmulas generales se presentan también en la figura C13.



MIEMBROS SUJETOS A FLEJO TENSION Y A  
FLEJO COMPRESION

Introducción.

Existen dos enfoques usuales hacia el diseño estructural en general. El primero, que es el más convencional se basa en el concepto de "esfuerzo permisible" y en el comportamiento elástico, y el segundo que parece ser más racional y está siendo gradualmente aceptado, se basa en el "diseño plástico" y en la carga última.

La carga permisible es una fracción de la resistencia última del miembro determinada sobre la base de un valor límite del esfuerzo máximo, llamado "esfuerzo permisible". Los esfuerzos permisibles están definidos en el código aplicable a cada estructura en particular. La magnitud del esfuerzo permisible es una fracción del esfuerzo de fluencia, y a la relación  $F_y/F_a$  se le llama "Factor de seguridad", donde  $F_y$  = esfuerzo de fluencia del acero y  $F_a$  = esfuerzo permisible. Este concepto de seguridad se basa en la suposición de que la iniciación del flujo plástico marca el límite de utilidad del miembro que se trate y que para obtener una seguridad adecuada, la carga permisible debe ser igual ó mayor que la carga de diseño calculada. La carga de diseño del miembro corresponde a las condiciones existentes bajo cargas de servicio, y se calcula usando la teoría elástica.

Este método de diseño, basado en cargas de servicio, comportamiento elástico y esfuerzos permisibles, es ampliamente aceptado porque se desarrolló como parte integral del análisis racional del A.I.S.C. (American Institute of Steel Construction), el cual en sus especificaciones han incluido muchas reglas empíricas para hacerlas prácticas. Su principal desventaja es que no suministra una capacidad uniforme de sobrecarga para todas las partes y tipos de estructuras, y

por lo mismo no son uniformemente económicos; sin embargo, por ser un método ampliamente probado, protege al público y al usuario, hasta cierto punto, de las faltas de criterio de diseñadores sin experiencia, que pudieran originar fallas estructurales si no se fijan ciertas restricciones.

El procedimiento de diseño plástico difiere del convencional de esfuerzos permisibles en tres aspectos importantes: a) se utilizan cargas últimas en vez de cargas de servicio, b) las fuerzas y los momentos en los miembros sometidos a cargas últimas se determinan sobre una base más realista, que incluye la acción inelástica, y c) los miembros se proporcionan de manera tal que su resistencia última exceda, ó cuando menos iguale a las fuerzas y momentos producidos por las cargas últimas.

Para determinar las cargas últimas se consideran las cargas vivas y muertas por separado, y se incrementa cada una de ellas según un factor distinto, para tomar en cuenta las condiciones de servicio más severas. Las cargas muertas, estimadas por medio de un diseño preliminar, no cambiarán probablemente durante la vida útil de la estructura; el factor de carga muerta debe tomar en cuenta solamente desviaciones menores sobre el valor estimado, debido a variaciones en la densidad de los materiales, las dimensiones de los elementos estructurales, en la naturaleza aproximada de la distribución supuesta en el análisis, y en algunas posibles ampliaciones futuras. Una variación del 20% en el valor estimado de las cargas muertas es suficiente en general, para tomar en cuenta esas posibilidades. Las cargas vivas, por otro lado, están sujetas a variaciones considerables; un aumento futuro, tal como un cambio en la naturaleza y densidad del tráfico sobre un puente, ó un cambio del tipo de ocupación de un edificio puede incrementarlas de manera apreciable. Debe considerarse también la posibilidad de

concentraciones locales; por ejemplo, la concentración de archiveros de acero en una cierta área de un piso de oficinas puede aumentar la carga viva hasta unos  $600 \text{ kg/cm}^2$ , siendo la carga promedio de diseño de  $250 \text{ kg/cm}^2$ . En algunos casos, pueden incluirse en el factor de carga ciertos efectos dinámicos ó de impácto; sin embargo, cuando estos efectos son de importancia principal, como en los soportes para un ascensor ó para maquinaria vibratoria pesada (cribas de agregados, grúas viajeras, malacates, etc), deben ser objeto de una valuación especial. Aunque no es necesario que el factor de carga viva tome en cuenta todas las condiciones posibles, sí debe considerar los sistemas de carga raros pero probables, que se pueden presentar en una estructura y traten de destruir su utilidad.

Generalmente un factor de carga comprendido entre 1.5 y 2.0 como mínimo, se considera adecuado para el caso de la carga viva; cuándo existen otras incertidumbres, se especifica otro valor más alto aún.

Otras cargas tales como las producidas por el viento y por el sismo, deben estimarse también e incrementarse por medio de un factor de carga adecuado, para ser utilizadas en el diseño último. Existen ciertas combinaciones de carga que pueden considerarse como críticas; por ejemplo, las Especificaciones AISC 1969 para el diseño plástico de Edificios, establecen que las cargas últimas mínimas deben ser 1.70 veces la suma de las cargas viva y muerta, para el caso de miembros en compresión axial simple, 1.85 veces la carga viva más la muerta, para marcos y pórticos continuos, y 1.40 veces la suma de las cargas viva, muerta y de viento ó de sismo, para cualquiera de los dos tipos anteriores.

El concepto de que la distribución de las cargas en estructuras estáticamente-

te indeterminadas está basado en la capacidad de carga máxima de los miembros, es determinante para la filosofía del diseño por carga última. Esto implica que los miembros y conexiones deben diseñarse y que su capacidad de carga debe determinarse antes de que quede definida la distribución de carga última.

Por ejemplo, en el caso de una viga atirantada (fig. 1), es posible proporcionar los miembros de la misma (tirante, viga en si y puntal) por cualquiera de los métodos siguientes:

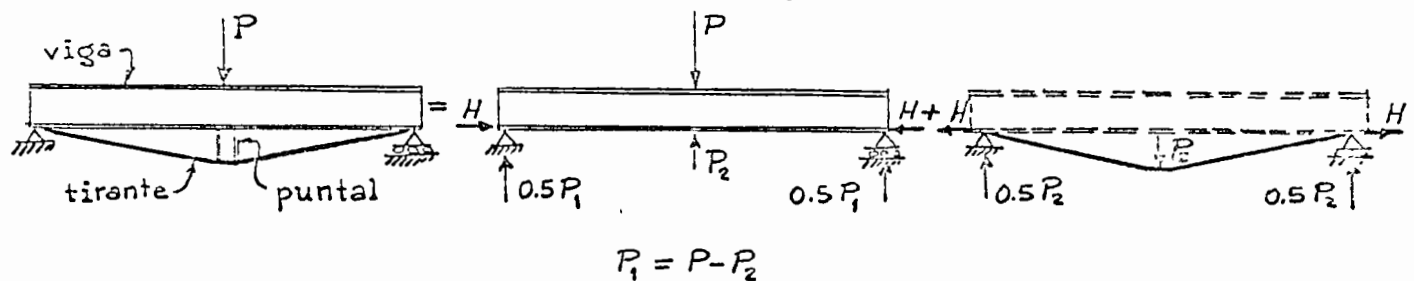


fig. 1.

a) Seleccionar, mas o menos arbitrariamente, un tamaño conveniente para la viga y determinar su capacidad de carga  $P_1'$ , trabajando sólo. El tirante tendrá así que soportar la carga  $P_2$ , que es el complemento de la carga última  $P'$ , es decir,  $P_2' = P' - P_1'$  y consecuentemente el tirante deberá proporcionarse para que ceda plásticamente a una carga igual o ligeramente mayor que  $P_2'$ .

b) Seleccionar primero el tirante y proporcionar después la viga para resistir el resto de la carga última, invirtiendo el proceso anterior.

c) Decidir la proporción de la carga última que debe soportar cada elemento; por ejemplo, proporcionar la viga y el tirante de manera que la capacidad de cada uno sea la mitad de esa carga (ó bien otras proporciones).

Este concepto de diseño ilustra claramente lo que el profesor Hardy Cross

llamó el concepto pragmático de la acción estructural... ó sea, que cuando se le dice a la estructura lo que debe hacer, esta tratará de hacerlo... Surgen ahora las preguntas: ¿Podrá hacerlo?, ¿podrá hacerlo eficientemente?, ¿en que forma el diseñador puede determinar como conseguir que lo haga?, - La viga atirantada escogida para ilustrar este concepto es una estructura relativamente simple, por lo que fué sencillo en este caso visualizar la naturaleza de la acción estructural hasta llegar<sup>a</sup> la falla. En estructuras más complejas puede ser muy difícil visualizar directamente esa acción hasta el colapso, y la determinación de la distribución de la carga última deja de ser más sencilla que el análisis estáticamente indeterminado convencional. Sin embargo, no es más complicada, aunque implica algunos conceptos y técnicas nuevas.

Después de que se ha verificado la seguridad de los miembros contra la falla bajo cargas últimas, deben revisarse para determinar su funcionamiento bajo las cargas de servicio. Esto incluye consideraciones de deformaciones, fatiga, respuesta dinámica, fluencia inicial y local y otras características que pueden tener influencia en el comportamiento funcional. Por ejemplo, con una relación grande de carga muerta a carga viva y un factor pequeño de carga viva, - el diseño puede quedar controlado por la limitación convencional de evitar el flujo plástico bajo condiciones normales de carga viva más carga muerta, en vez de que rija la capacidad de carga última. Deben de considerarse también los cambios de temperatura y los asentamientos en los apoyos en el grado en que afectan a los esfuerzos y deformaciones.

Aunque el diseño plástico es un método racional que tiene en cuenta el comportamiento inelástico de la estructura, no reemplazará a los demás métodos de análisis y diseño. El método tiene muchas ventajas que animan a usarlo, pero tiene también algunas limitaciones. Entre las ventajas cuentan: a) posibilidad de

determinar la capacidad de sobrecarga bajo condiciones de carga sencillas, b) uso eficiente del material, c) simplicidad de los cálculos del análisis plástico para estructuras reticulares sencillas, y d) diseño de detalles más económicos que reflejen el comportamiento plástico.

Se han mencionado ya varias limitaciones de la teoría plástica como un criterio para determinar la resistencia de las vigas, que consisten en suponer que las secciones planas antes de la deformación permanecen planas después de ella y en ignorar los esfuerzos residuales y las concentraciones locales de esfuerzos. Otras consideraciones importantes al valorar los criterios de resistencia plástica son las propiedades del material, los efectos de las fuerzas cortantes y normales, el pandeo lateral y local, el efecto del recubrimiento de concreto, las cargas repetidas y la posibilidad de fractura frágil. Los requisitos especiales de diseño que toma en cuenta estas consideraciones se discutirán con detalle en las sesiones relativas al diseño plástico.

Los miembros estructurales que se encuentran sometidos a una acción combinada de esfuerzos de flexión y esfuerzos axiales (tensión ó compresión) se presentan con sorprendente frecuencia en las estructuras reticulares. Por ejemplo, las vigas y traveses que llegan a conectarse a las columnas aún en la condición idealizada de articulación o apoyo libre, le producen a la columna cargas de compresión aplicadas excentricamente a su centroide en virtud del medio de conexión, que pueden ser un par de ángulos al alma, o una conexión de asiento. (figs. 2 y 3).

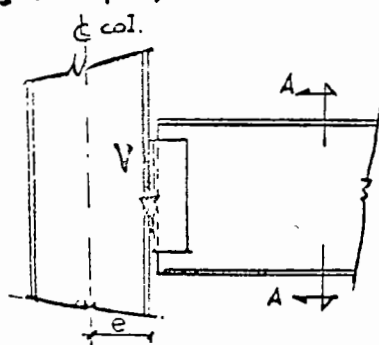


fig. 2

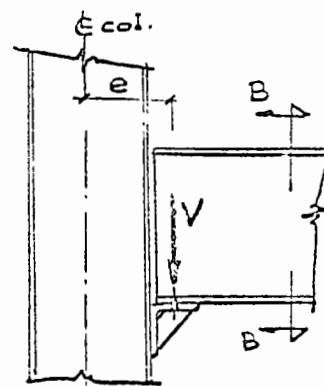
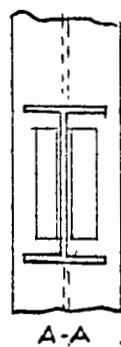
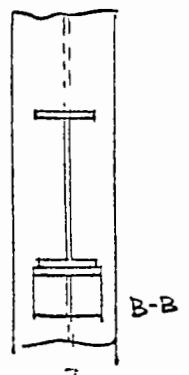


fig. 3





Otro caso muy frecuente es el que se presenta con los miembros de tensión que estando en posición horizontal se flexionan debido a su propio peso, o también el de cuerdas superiores de armaduras que reciben las descargas de los largueros de techo no solamente en los nudos de la armadura sino entre ellos, produciendoles una flexión adicional a una carga axial.

El comportamiento de los miembros flexotensionados es totalmente distinto al de los miembros flexocomprimidos, ya que mientras los primeros tienden a autoenderezarse, es decir, a reducir su deflexión transversal, y por lo tanto la excentricidad de la carga, los segundos tienden a incrementar dicha deflexión, aumentando con ello el brazo de palanca de la carga axial y consecuentemente de momento flexionante. (fig. 4); esto origina a su vez una mayor curvatura en el miembro que incrementa nuevamente la deflexión, ésta al momento flexionante, éste a la curvatura y así sucesivamente hasta que la pieza converge en su estado de equilibrio interno ó bien falla por alguno de los siguientes motivos.

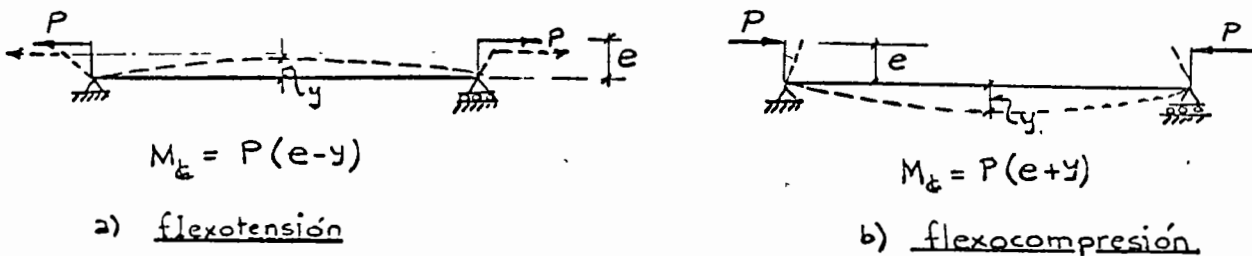


fig. 4

- Porque se exceda su capacidad para soportar momentos y fuerza axial combinados, formandose una articulación plástica donde el momento es máximo.
- Por inestabilidad en el plano de flexión, debida a un exceso de momento flexionante en dicho plano.

- c) Por pandeo lateral debido a flexotorsión,
- d) Por pandeo elástico o inelástico debido a compresión axial respecto a un eje de menor resistencia.
- e) Por pandeo local.

Todos los modos anteriores deben considerar si hay o no posibilidad de tener desplazamientos relativos entre los extremos de los miembros en estado.

En el caso de los miembros sometidos a flexión y tensión combinados, pueden llegar a fallar por:

- a) Tensión excesiva a través de la sección total del miembro (falla por tensión axial) o a través de la sección neta del miembro en sus extremos.
- b) Esfuerzos excesivos de flexión y tensión combinados en la zona de conexión (extremos).

A continuación comenzaremos a estudiar el comportamiento de los miembros sometidos a una combinación de carga axial (tensión o compresión) y flexión, sus criterios y diseño y proporcionamiento, y las especificaciones que los rigen.

## 2.- Miembros a flexo tensión

Uno de los casos más frecuentes que se tienen en las estructuras de acero con este tipo de miembros, es el de los ángulos simples a tensión. Como se sabe, este tipo de miembros se proporciona suponiendo que los esfuerzos de tensión se distribuyen uniformemente a través de su sección transversal, lo cual desde luego, no es cierto por la forma práctica en que estos ángulos se conectan en sus extremos (fig. 5).

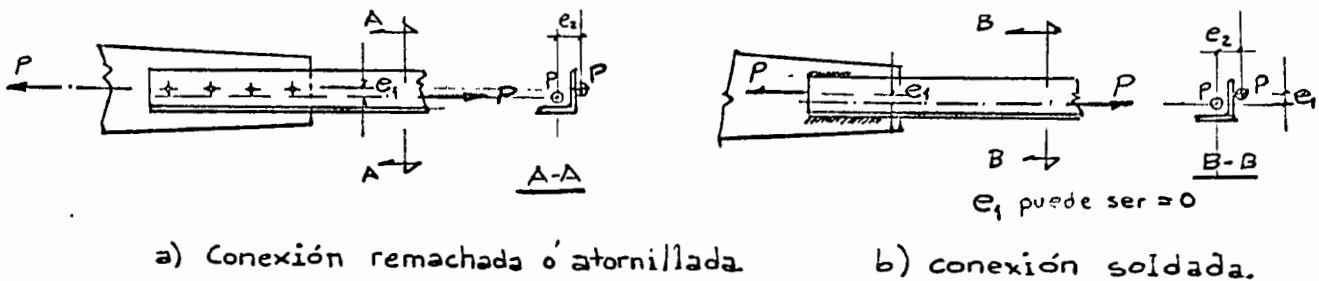


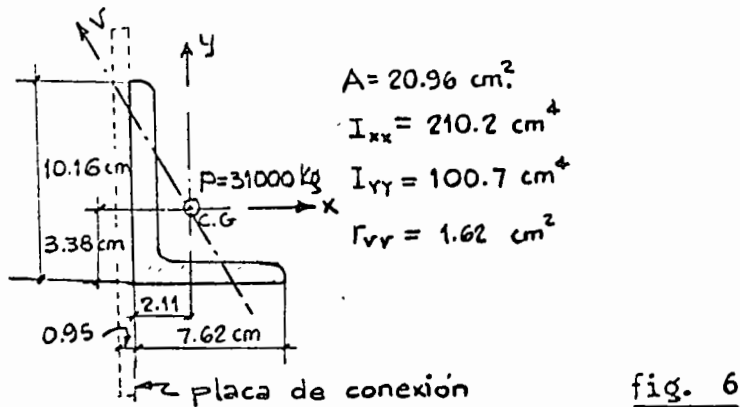
fig. 5

Los extremos de los ángulos se remachan, atornillan o sueldan a placas de conexión, normalmente en uno de sus patines, produciéndose con ésto en forma automática una excentricidad entre el punto de aplicación de la carga (centroide del ángulo) y el centro de resistencia de la conexión. Esta excentricidad puede existir en dos planos simultáneamente, produciéndose así flexión biaxial además de la tensión en el ángulo.

Para poder tener una idea de la magnitud de ésta flexión, analicemos el siguiente ejemplo:

Un ángulo simple de 101.6 x 76.2 x 12.7 mm. (L4"x3"x1/2") con extremos soldados a placas de conexión de 0.95 mm. (3/8") por su patín mayor, resiste una carga de tensión de 31000 Kg. Revísense los esfuerzos de tensión y de flexión en los extremos del ángulo, suponiendo que las

soldaduras están proporcionadas en forma tal que evitan la excentricidad, paralelamente al patín mayor. El acero del ángulo es del tipo ASTM-A-36. Ver Fig. 6.



Solución:

- a) esfuerzos de tensión: (suponiendo distribución uniforme de los esfuerzos).

$$f_t = \frac{31000}{20.96} = 1479 \text{ Kg/cm}^2 < 0.6 F_Y = 1520 \text{ Kg/cm}^2, \therefore \checkmark$$

- b) esfuerzos de flexión

$$\text{excentricidad } e_x = 2.11 + \frac{1}{2} \times 0.95 = 2.585 \text{ cm.}$$

$$M_{yy} = 31000 \times 2.585 = 80135 \text{ Kg-cm.}$$

$$f_{b_{xx}} = \frac{80135}{100.7} \times (10.16 - 3.38) = 5395 \text{ Kg/cm}^2 \gg F_Y = 2530 \text{ Kg/cm}^2$$

Se vé que los esfuerzos debidos a la flexión (tensión o compresión) son sumamente altos, y que aún sin superponer los a los de tensión axial producidos por la carga de 31000 Kg., exceden con mucho el esfuerzo de cedencia del material. Dichos esfuerzos, por supuesto no son reales, ya que no pueden exceder del esfuerzo de cedencia. Lo anterior implica que existe una plastificación del material cercano a las puntas de los patines no conectados del ángulo, en la zona de la conexión. Aparentemente este tipo de esfuerzos "locales" no son de gran importancia para el A.I.S.C., quien no considera hacer ninguna reducción en cuanto a los esfuerzos permisibles en este tipo de miembros; en cambio la AASHO establece que para considerar el efecto de la excentricidad en las conexiones, el área a considerar el ángulo es la del patín conectado más la mitad de la del patín no conectado. En

realidad lo que sucede es que los momentos que aparecen en los extremos del ángulo producen esfuerzos mayores que el de cedencia del material, plastificándose esa sección aún para valores chicos de la carga axial. Conforme la carga aumenta, los extremos solamente giran "orientándose" el ángulo en el sentido de la carga para disminuir la excentricidad original de la misma (Fig. 7) lo que hace que en las zonas centrales del miembro, ésta sea muy pequeña o casi nula, y que por lo mismo el efecto de la flexión en estas zonas sea prácticamente nulo, produciéndose distribuciones de esfuerzos muy cercanas a la uniforme. Cuando la carga en el miembro crece hasta que los esfuerzos en las zonas centrales del ángulo alcancen  $F_y$ , los extremos del ángulo no han fallado aún y siguen dentro del rango plástico. Se ha observado prácticamente que los ángulos fallan en las zonas cercanas a sus extremos soldados bajo cargas de tensión del orden del 82% del esfuerzo último de tensión del material, obtenido de una probeta de tensión del mismo material del ángulo.

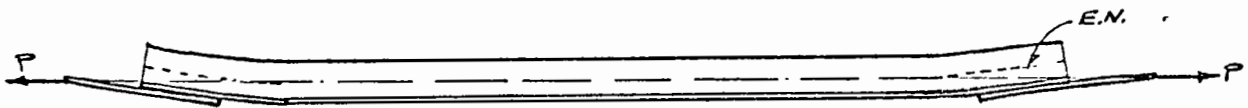


fig 7.

De lo anterior se concluye que la práctica recomendada por el A.I.S.C. de utilizar la sección total del ángulo se justifica por el hecho de que se puede alcanzar la carga de fluencia del ángulo sin menoscabo de la resistencia de los extremos, y que la pequeña deficiencia del miembro en su carga última respecto a la carga última teórica del material, resulta de poca importancia en las estructuras ductiles. Las provisiones del AASHO y del AREA para el diseño de estos elementos de un solo ángulo, consistentes en descontar del área total del ángulo, la mitad del área del patín no conectado, es una aproximación burda para tomar en cuenta la ineffectividad de ese patín para resistir carga dentro del rango elástico del material, lo cual es conservador para el caso de las estructuras cargadas -

estaticamente. El efecto de las excentricidades del tipo de las discutidas anteriormente puede ser de consecuencias en el caso de miembros sometidos a frecuentes ciclos de variaciones de carga (tensión y compresión), por el fenómeno de la fatiga y la fractura frágil. Todas las especificaciones, sin embargo, recomiendan evitar el uso de ángulos simples excepto para elementos secundarios.

El caso de otros tipos de elementos a flexotensión puede atacarse, estudiando el efecto combinado de los esfuerzos de tensión y de flexión, por medio de la fórmula de la escuadria, que para este caso sería:

$$f = \frac{P}{A} \pm \frac{M_u}{I_u} v \pm \frac{M_v}{I_v} u \quad \text{--- (1)}$$

donde:  $I_u$  e  $I_v$  son los momentos principales de inercia de la sección

$v$  y  $u$  las coordenadas del punto donde se valúa el esfuerzo  $f$  respecto a los ejes principales.

$M_u$  y  $M_v$  los momentos flexionantes respecto a los ejes principales  $U$  y  $V$  debidos a la carga axial  $P$  aplicada excentricamente.

Cuando la determinación de los ejes principales de inercia y los momentos flexionantes respecto a ellos sea problema tica, la ecuación (1) puede referirse a un par de ejes centroidales  $x$  e  $y$ , como sigue:

$$f = \frac{P}{A} \pm \frac{M_x}{I_{mx}} \left( y - \frac{I_{xy} X}{I_y} \right) \pm \frac{M_y}{I_{my}} \left( x - \frac{I_{xy} Y}{I_x} \right) \quad \text{--- (2)}$$

donde:

$$I_{mx} = \frac{I_x I_y - \overline{I_{xy}}^2}{I_y}, \quad I_{my} = \frac{I_x I_y - \overline{I_{xy}}^2}{I_x}$$

$$I_{xy} = \int xy (dA) = \text{producto de inercia de la sección}$$

$M_x$  y  $M_y$  son los momentos flexionantes respecto a los ejes centroidales  $x$  y  $y$  de la sección, producidas por la carga  $P$  aplicada excentricamente al centroide de la sección.

En el caso de que no se permita tener esfuerzos locales de cedencia bajo cargas de trabajo, se puede utilizar un enfoque racional del problema para encontrar la carga admisible, por medio de la fórmula de interacción propuesta por el AISC. En esencia, el método consiste en limitar la suma de las relaciones de los esfuerzos calculados axiales y de flexión, entre sus respectivos esfuerzos permisibles, a la unidad. Esto es:

$$\frac{f_t}{0.6 F_Y} + \frac{f_b}{F_b} \leq 1.0 \quad \text{-----} \textcircled{3}$$

donde:

$f_t = \frac{P}{A}$  es el esfuerzo axial calculado en el miembro

$0.6 F_Y =$  esfuerzo axial permisible

$f_b = \frac{M_u}{I_u} v + \frac{M_v}{I_v} u =$  esfuerzo de flexión biaxial calculado

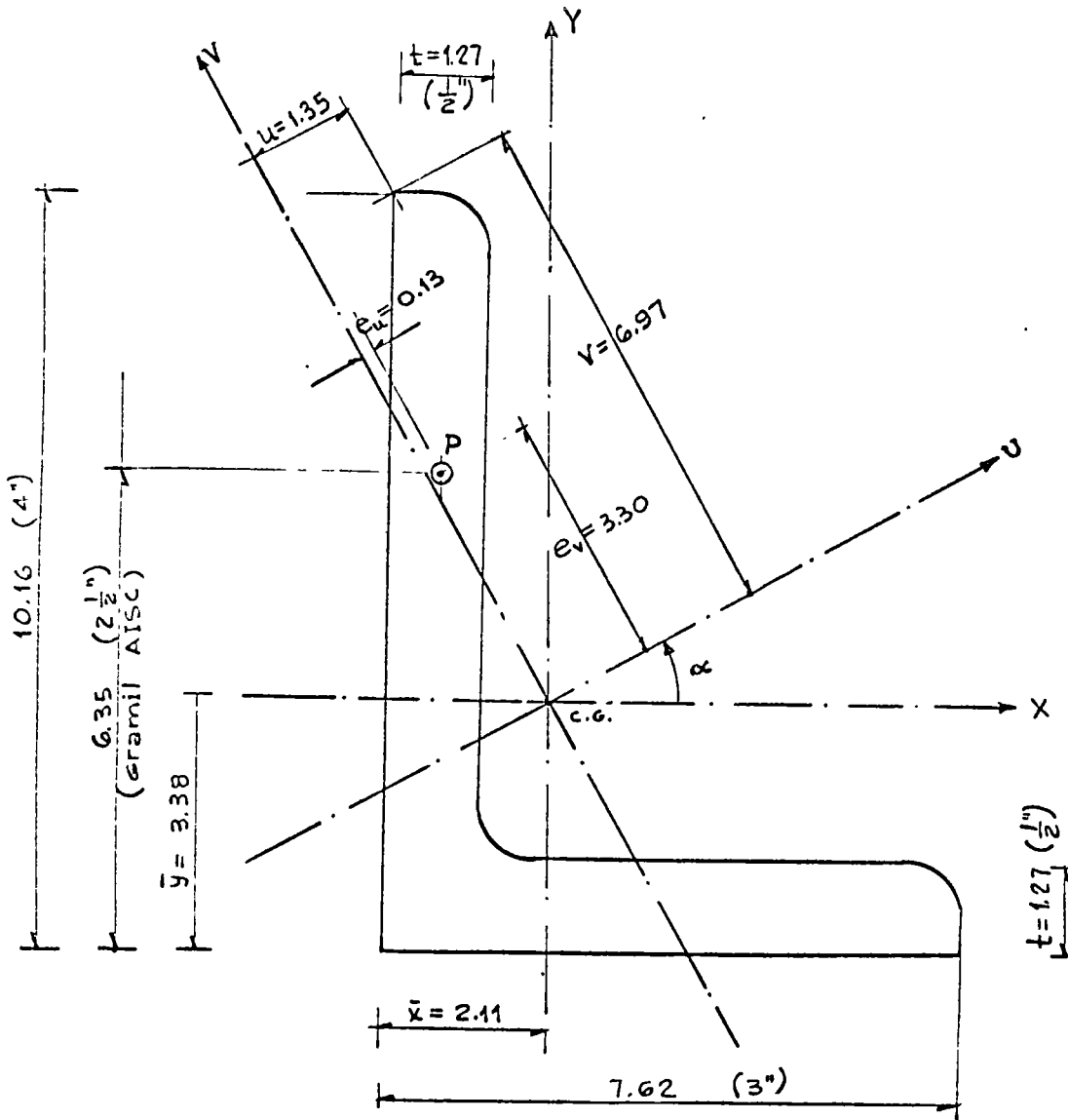
$F_b =$  Esfuerzo permisible a la tensión por flexión

El método de interacción no pretende ser una solución directa al problema de diseño, sino que es menester examinar varias secciones tentativas hasta encontrar aquella que satisfaga la ecuación (3).

Ilustremos ésto con otro ejemplo:

Sea el mismo ángulo de 101.6 x 76.2 x 12.7 mm. del ejemplo anterior, con la carga de tensión  $P$  aplicada en el punto de gramil del AISC (ver figura 8). Se pide obtener el máximo valor de la carga  $P$  para que no se excedan los esfuerzos combinados de los permisibles.

Solución: Calculamos gráficamente (por simplicidad) las excentricidades de la carga  $P$  respecto a los ejes principales  $U$  y  $V$ , así como también las coordenadas  $u$  y  $v$  del punto  $A$ , más esforzado



$$\tan \alpha = 0.543$$

fig. 8 (acotaciones en cm.) escala 1:1

$$M_u = 3.30 P$$

$$M_v = 0.13 P$$

$$I_v = 1.62^2 \times 20.96 = 55.01 \text{ cm}^4$$

$$I_u = I_x + I_y - I_v = 210.2 + 100.7 - 55.01 = 255.89 \text{ cm}^4$$

$$f_b = \frac{M_u}{I_u} v + \frac{M_v}{I_v} u = \frac{3.30 P}{255.89} \cdot 6.97 + \frac{0.13 P}{55.01} \cdot 1.35 = 0.0931 P \text{ Kg/cm}^2, \text{ si } P \text{ en Kg.}$$

$$f_t = \frac{P}{A} = \frac{P}{20.96} = 0.0477 P \text{ Kg/cm}^2, \text{ si } P \text{ en Kg.}$$

como  $F_b = 0.6 F_y$  (esf. perm. a la tensión por flexión, AISC 1.5.1.4.5)

$$\frac{f_t}{0.6 F_y} + \frac{f_b}{F_b} = \frac{f_t + f_b}{0.6 F_y} \leq 1.0 \quad \text{----- } \textcircled{3}$$



La fórmula de interacción (3) dá:

$$\frac{0.0477 P + 0.0931 P}{0.6 F_y} \leq 1.0 \quad \therefore P = \frac{0.6 F_y}{0.1408} = 4.261 F_y$$

Para un acero ASTM A-36,  $F_y = 2530 \text{ Kg/cm}^2$

$$\therefore P = 4.261 \times 2530 = 10781 \text{ Kg}$$

Se vé que esta carga es considerablemente menor que la carga admisible por el AISC para el caso de tensión pura, permitiendo plástificación en sus extremos y que es

$$P' = 0.6 \times 2530 \times 20.96 = 31859 \text{ Kg}$$

Para el caso de miembros en flexotensión distintos de los ángulos, podemos establecer los siguientes criterios.-

- a) Si la flexión se presenta exclusivamente en los extremos, por virtud de la excentricidad de la conexión, despréciase ésta permitiendo que ocurra una plastificación del elemento en la zona de la conexión, y diseñese el miembro a tensión pura. Las conexiones, sin embargo, deberán prever estos momentos extremos, y resistirlos adecuadamente (se verá en el capítulo de conexiones).
- b) Si la flexión es producto de cargas transversales al eje del elemento, y adquiere su valor máximo fuera de las conexiones extremas, valúese el esfuerzo máximo producido por la flexión y superpóngase con el de tensión axial utilizando para ello la fórmula de la escuadra (ecuación 1). El esfuerzo así valuado no deberá exceder el esfuerzo permisible a la tensión, dictado por las especificaciones empleadas (AISC, AREA, AASHO) en el caso de diseñar por esfuerzos permisibles; o bien la carga de colapso del elemento a la tensión pura ( $F_{ult} \times A$ ) deberá ser mayor que la carga de trabajo del elemento ( $f \cdot A$ ) multiplicado por su factor de carga.

Para valuar correctamente el efecto de la flexión combinada con la tensión, podemos utilizar la expresión aproximada para valuar la deflexión total al centro del claro,  $Y$ ,

$$Y = Y_0 \left( \frac{1}{1 + \frac{P}{P_e}} \right) \quad \text{-----} \quad (4)$$

donde:  $Y_0$  = deflexión al centro del claro producida por las cargas transversales, producida por los momentos extremos, o producida por ambos.

$P$  = Carga de tensión

$P_e$  = Cargacrítica de Euler en el plano de la flexión

El término entre paréntesis tiende a disminuir la deflexión transversal del miembro bajo la carga de tensión, y se puede llamar "factor de disminución" del momento al centro del claro.

El momento flexionante total al centro del del claro, será entonces

$$M = M_0 - \frac{PY_0}{1 + \frac{P}{P_e}} \quad \text{-----} \quad (5)$$

donde  $M_0$  es el momento flexionante inicial al centro del claro, producido por las cargas transversales, por los momentos extremos o por ambos.

Para el caso de un miembro recto inicialmente, sometido a la acción de fuerzas de tensión aplicadas excentricamente en sus extremos, a una distancia  $e$  del centroide del miembro (fig. 4), el esfuerzo máximo se presenta en los extremos, ya que en otros puntos intermedios la excentricidad se reduce por la tendencia del miembro a autoenderezarse, deflexionandose hacia la línea de acción de la fuerza.

- c) Por supuesto, si el efecto del momento flexionante es predominante, la falla del miembro puede ocurrir por pandeo lateral antes de que ocurra la falla por fluencia excesiva posterior a la plastificación en la zona de máximo esfuerzo combinado.

El AISC, establece que para el diseño de miembros a flexo-  
tensión, los esfuerzos combinados satisfagan la ecuación  
de interacción.

$$\frac{f_a}{0.6 F_y} + \frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_y} \leq 1.0 \quad \text{--- (6) (AISC 1.6-1b)}$$

Para las secciones compactadas  $F_b = 0.66 F_y$ , proporciona -  
un factor de seguridad de 1.67 a la cedencia de la fibra  
más alejada, para la secciones que tengan un factor de -  
forma de 1.10 o mayor. Para evitar el pandeo lateral, el  
AISC requiere también que el esfuerzo de compresión pro-  
ducido por la flexión,  $f_b$ , sólo, no exceda los valores -  
dados por su sección 1.5.1.4.

- d) Para utilizar el criterio de capacidad última de un ele-  
mento flexotensionado, haremos uso de la ecuación

$$\frac{P_0}{P_y} + \frac{M_0}{M_s} = 1 \quad \text{----- (7)}$$

lo cual nos permitirá determinar su resistencia en el caso  
de que no haya posibilidad de pandeo lateral por flexión,  
en función de su resistencia en dos condiciones de carga -  
simple : a) tensión axial, y b) flexión simple.

La ecuación (7) representa con precisión suficiente para -  
los fines de diseño de miembros metálicos de secciones -  
I, C, H, etc., tanto la combinación de elementos mecánicos  
para la que se alcanza el límite de comportamiento elástico  
como la combinación para la que se ocasiona la plastifica-  
ción total de la sección, aunque en este segundo caso es -  
ligeramente conservadora.

En la ecuación (7), las literales representan:

$P_0$  y  $M_0$ , los valores de la fuerza de tensión y el momento

flexionante que al actuar simultáneamente sobre la sección ocasionan la iniciación del flujo plástico en las fibras - exteriores de la sección, o bien, su plastificación completa.

$P_y = AF_y$ , fuerza de tensión axial que ocasionaría la plastificación total de la sección, si no existiese el momento flexionante.

$M_s$  = momento flexionante máximo que puede soportar la sección, en ausencia de fuerza normal y en puntos de soporte lateral, e igual a  $M_y = SF_y$ , cuando la sección no es compacta, y a  $M_p = XF_y$  cuando si lo es.

Como se mencionó antes, los valores  $P_o$ ,  $P_y$ ,  $M_o$  y  $M_s$  se refieren a la "falla", es decir, por un lado, a la capacidad total de carga bajo cargas combinadas ( $P_o$  y  $M_o$ ), y por el otro, a la capacidad última de carga bajo cargas de tensión axial  $P_y$  o momento flexionante  $M_s$ , separadamente. Comparativamente, el diseño elástico convencional (AISC) utiliza los esfuerzos calculados bajo cargas de trabajo (en vez de los esfuerzos a la carga de falla) y los compara con los esfuerzos permisibles (Ec. 3).

La ecuación (7) puede traducirse a los términos empleados en la ec. 3, en la siguiente forma: Dividiendo numerador y denominador del primer término entre el producto del área de la sección transversal multiplicada por el factor de seguridad F.S. y dividiendo numerador y denominador del segundo término entre el producto del módulo de sección por el factor de seguridad F.S.

Ejemplo:

Diseñar la cuerda inferior de una armadura para resistir las cargas mostradas en la Fig. 9. Por condiciones constructivas de la armadura dicha cuerda deberá tener la forma de [ ] , con dimensiones máximas de 30 cm. x 30 cm. El acero

a usar es ASTM A-242-68 ( $F_y = 50000 \text{ lb/pulg.}^2 = 3515 \text{ Kg/cm}^2$ ). Utilice el criterio de esfuerzos permisibles del AISC, despreciando la reducción de área por los agujeros de los tornillos de montaje, y el peso propio del miembro.

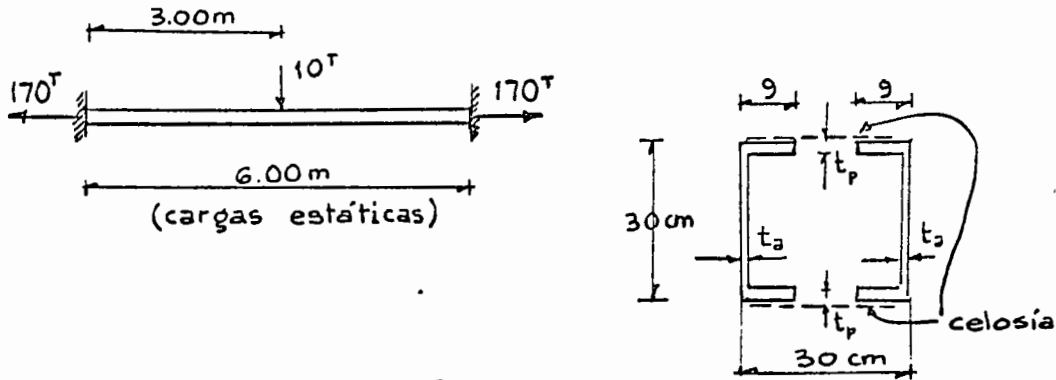


fig 9

Solución:

Diseño de acuerdo a las normas AISC.

Para proponer una sección tentativa, se supondrá un  $F_a = 0.4F_y$  (para tomar en cuenta el efecto de la flexión).

$$A_{req} \approx \frac{170000}{0.4 \times 3515} = 121 \text{ cm}^2$$

haciendo  $t_a = t_p$  para el primer tanteo,

$$A \doteq 2(30 + 2 \times 9)t_a = 96 t_a \quad \therefore t_a = \frac{121}{96} = 1.26 \text{ cm}$$

$$\text{Sea } t_a = t_p = 1.27 \text{ cm} = \frac{1}{2}'' \text{ (comercial)}$$

Revisión. (fig. 10)

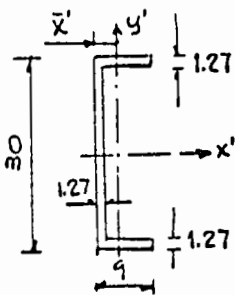


fig. 10

$$A = [4 \times 9 + 2(30 - 2 \times 1.27)] \times 1.27 = 115.47 \text{ cm}^2$$

$$I_{x'_c} = \frac{(30 - 1.27 \times 2)^3}{12} \times 1.27 + 2(9 \times 1.27) \left(\frac{30 - 1.27}{2}\right)^2 = 6908 \text{ cm}^4$$

$$I_{xx_{c1}} = 2 I_{x'_c} = 13816 \text{ cm}^4 \quad ; \quad r_{xx} = \sqrt{\frac{13816}{115.47}} = 10.94 \text{ cm}^4$$

$$S_{xx_{c1}} = \frac{13816}{15} = 921 \text{ cm}^3$$

$$\bar{x}' = \frac{27.46 \times 0.635 + 2 \times 9 \times 4.5}{27.46 + 18} = 2.17 \text{ cm}$$

$$\begin{aligned} \bar{I}_{y'} &= 27.46 \times 1.27 (2.17 - 0.64)^2 \times 2 + 2 \frac{9^3 \times 1.27}{12} + 2 \times 9 \times 1.27 (4.5 - 1.27)^2 = \\ &= 163.27 + 154.31 + 124.11 = 441.69 \text{ cm}^4 \end{aligned}$$

$$I_{yy_{c1}} = 2 \times 442 + 57.73 (15.0 - 2.17)^2 \times 2 = 884 + 19006 = 19860 \text{ cm}^4$$

$$r_{yy} = \sqrt{\frac{19860}{115.47}} = 13.13 \text{ cm}$$

Se ve que los radios de giro exactos que se acaban de obtener checan los valores aproximados de estos que se dan en la Tabla 1, y que para el presente caso son:

$$r_x \doteq 0.36 \times 30 = 10.8 \text{ cm.} \approx 10.94 \text{ cm.}$$

$$r_y \doteq 0.45 \times 30 = 13.5 \text{ cm.} \approx 13.13 \text{ cm.}$$

Por lo tanto, para cálculos estimativos utilizaremos los valores de la Tabla 1.

$$M_o = \frac{10 \times 6}{8} = 7.5 \text{ T-m} = 7.5 \times 10^5 \text{ Kg-cm.} \quad (\text{en el } \xi \text{ y en extremos})$$

Suponiendo  $K_x = K_y = 0.5$  (por la continuidad de la cuerda)

$$\frac{K_x l}{F_x} = \frac{0.5 \times 600}{10.94} = 27.4 \quad \therefore F_{e_x} = 13978 \text{ Kg/cm}^2 \quad (\text{tabla 6})$$

$$\therefore F_e = 13978 \times 115.47 = 1,614,086 \text{ Kg}$$

La deflexión inicial en el centro del claro, producida por la carga transversal de 10 toneladas, es:

$$Y_o \doteq \frac{PL^3}{192EI} = \frac{10 \times 10^3 \times 6^3 \times 10^6}{192 \times 2.1 \times 10^6 \times 13.816 \times 10^2} = 0.388 \text{ cm.}$$

y el momento flexionante total en el centro del claro:

$$M_{\xi} = 750000 - \frac{170000 \times 0.388}{1 + \frac{170,000}{1,614,086}} = 690324 \text{ Kg-cm}$$

$$f_{b_x} = \frac{690324}{921} = 749 \text{ Kg/cm}^2 \quad ; \quad f_a = \frac{170,000}{115.47} = 1472 \text{ Kg/cm}^2$$

$$F_{b_x} = 0.6 F_y = 2100 \text{ Kg/cm}^2 = F_{o \text{ tens}}$$

Aplicando la fórmula (3) en el  $\xi$ .

$$\frac{1472}{2109} + \frac{749}{2109} = 0.698 + 0.355 = 1.053 \approx 1.0$$

En los extremos:  $M = 750,000 \text{ Kg-cm}$

$$f_{b_x} = \frac{750,000}{921} = 814 \text{ Kg/cm}^2 \quad ; \quad f_a = 1472 \text{ Kg/cm}^2$$

$$F_{b_x} = 0.66 F_y \text{ (por no haber posibilidad de pandeo lateral)} = 2320 \text{ Kg/cm}^2$$

$$F_o = 2109 \text{ Kg/cm}^2$$

$$\therefore \text{ la fórmula (3) da } \frac{1472}{2109} + \frac{814}{2320} = 0.698 + 0.351 = 1.049 \approx 1.0$$

Por lo tanto se acepta la sección propuesta.

(No se diseñará la celosía). Nota: se sugiere al estudiante repetir el ejemplo con  $t_s = 0.95 \text{ cm}$  y  $t_p = 1.905 \text{ cm}$ .

Para ilustrar el método de capacidad última de un elemento - flexotensionado, estudiemos el mismo caso anterior, utilizando un factor de carga de 1.7 para las cargas axial y transversal

Tomando la sección del problema anterior como tentativa en este caso, y aplicando la ecuación (7).

$$\frac{P_o}{P_Y} + \frac{M_o}{M_s} = 1$$

$$P_Y = F_Y \cdot A = 3515 \times 115.47 = 405877 \text{ Kg}$$

$$P_o = 1.7 \times 170000 = 289000 \text{ Kg}$$

$$M_s = S \cdot F_Y = 921 \times 3515 = 3237315 \text{ Kg-cm (en el } \xi)$$

$$Q = 2 \times 9 \times 1.27 (15.0 - 0.64) + 2 (15 - 1.27) \times 1.27 \frac{(15.0 - 1.27)}{2} = 567.7 \text{ cm}^3$$

$$Z = 2Q = 1135.4 \text{ cm}^3$$

$$M_s = Z \cdot F_Y = 1135.4 \times 3515 = 3990931 \text{ Kg-cm (en los apoyos)}$$

$$M_o = 1.7 \times 690324 = 1173551 \text{ Kg-cm (en el } \xi)$$

$$M_o = 1.7 \times 750000 = 1275000 \text{ Kg-cm (en los apoyos)}$$

Revisando la sección en el centro del claro

$$\frac{289000}{405877} + \frac{1173551}{3237315} = 0.712 + 0.362 = 1.074 \approx 1.0 \therefore \checkmark$$

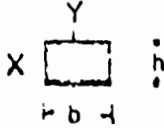
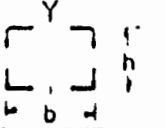
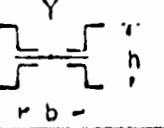
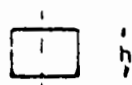

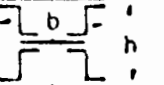

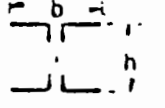
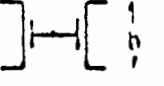

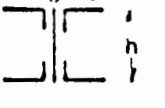

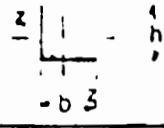
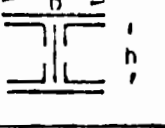
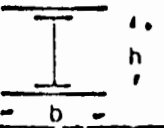
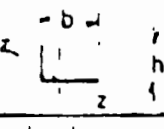
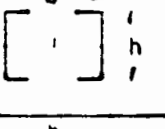
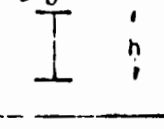
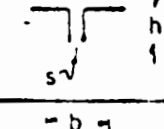
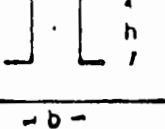
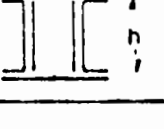
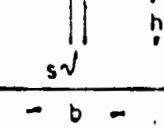
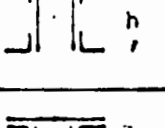
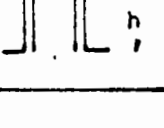
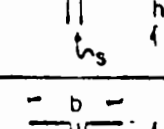
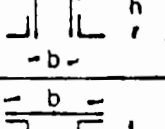
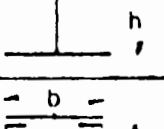
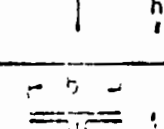
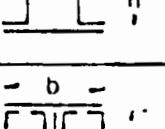
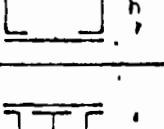
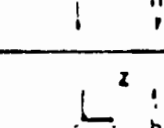
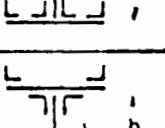
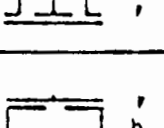
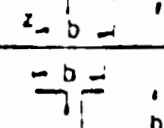
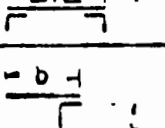
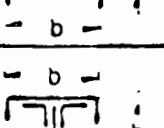



En los apoyos

$$\frac{289000}{405877} + \frac{1275000}{3990931} = 0.712 + 0.319 = 1.031 \approx 1.0 \therefore \checkmark$$

La sección propuesta se considera satisfactoria para un factor de carga de 1.7 y diseño último.

TABLA 1

# RADIO DE GIRO DE VARIAS SECCIONES

 $rx = 0.21h$ $ry = 0.21h$	 $rx = 0.42h$ $ry = 0.42b$	 $rx = 0.31h$ $ry = 0.48b$
 $rx = 0.40h$	 $ry = \text{igual que el de dos ángulos.}$	 $rx = 0.37h$ $ry = 0.28b$
 $rx = 0.25h$	 $rx = 0.42h$ $ry = \text{igual que el de dos ángulos.}$	 $rx = 0.31h$
 $r = \sqrt{\frac{H^2 + h^2}{16}}$ $r = 0.35Hm.$	 $rx = 0.39h$ $ry = 0.21b$	 $rx = 0.31h$
 $rx = 0.31h$ $ry = 0.31b$ $rz = 0.19h$	 $rx = 0.45h$ $ry = 0.235b$	 $rx = 0.40h$ $ry = 0.21b$
 $rx = 0.29h$ $ry = 0.32b$ $rz = 0.18 \frac{h+b}{2}$	 $rx = 0.36h$ $ry = 0.45b$	 $rx = 0.38h$ $ry = 0.22b$
 $rx = 0.31h$ $ry = 0.215b$ $= b(0.21 + 0.02s)$	 $rx = 0.36h$ $ry = 0.60b$	 $rx = 0.39h$
 $rx = 0.32h$ $ry = 0.21b$ $= b(0.19 + 0.02s)$	 $rx = 0.36h$ $ry = 0.53b$	 $rx = 0.35h$
 $rx = 0.29h$ $ry = 0.24b$ $= b(0.23 + 0.02s)$	 $rx = 0.39h$ $ry = 0.55b$	 $rx = 0.435h$ $ry = 0.25b$
 $rx = 0.30h$ $ry = 0.17b$	 $rx = 0.42h$ $ry = 0.32b$	 $rx = 0.42h$
 $rx = 0.25h$ $ry = 0.21b$	 $rx = 0.44h$ $ry = 0.28b$	 $rx = 0.42h$
 $rx = 0.21h$ $ry = 0.21b$ $rz = 0.19h$	 $rx = 0.50h$ $ry = 0.28b$	 $rx = 0.285h$ $ry = 0.37b$
 $rx = 0.38h$ $ry = 0.19b$	 $rx = 0.39h$ $ry = 0.21b$	 $rx = 0.42h$ $ry = 0.23b$



**Tabla 2**  
 $F_y = 2530 \text{ Kg/cm}^2 \text{ (36 ksi)}$   
**ESFUERZOS ADMISIBLES EN**  
**Kg/cm<sup>2</sup> PARA MIEMBROS**  
**EN COMPRESION**

Miembros Principales y Secundarios con $\frac{Kl}{r}$ no mayor de 120				Miembros Principales con $\frac{Kl}{r}$ de 121 a 200				Miembros Secundarios* con $l/r$ de 121 a 200					
$\frac{Kl}{r}$	$F_a$ Kg/ cm <sup>2</sup>	$\frac{Kl}{r}$	$F_a$ Kg/ cm <sup>2</sup>	$\frac{Kl}{r}$	$F_a$ Kg/ cm <sup>2</sup>	$\frac{Kl}{r}$	$F_a$ Kg/ cm <sup>2</sup>	$\frac{Kl}{r}$	$F_a$ Kg/ cm <sup>2</sup>	$\frac{Kl}{r}$	$F_a$ Kg/ cm <sup>2</sup>	$\frac{Kl}{r}$	$F_a$ Kg/ cm <sup>2</sup>
1	1516	41	1344	81	1072	121	713	161	405	121	716	161	510
2	1513	42	1338	82	1064	122	702	162	400	122	709	162	506
3	1510	43	1332	83	1056	123	693	163	395	123	703	163	503
4	1507	44	1326	84	1048	124	682	164	390	124	696	164	501
5	1504	45	1320	85	1040	125	671	165	386	125	689	165	498
6	1501	46	1315	86	1031	126	662	166	381	126	682	166	495
7	1498	47	1308	87	1024	127	651	167	376	127	674	167	492
8	1494	48	1303	88	1015	128	641	168	372	128	667	168	489
9	1491	49	1297	89	1007	129	631	169	368	129	661	169	487
10	1488	50	1290	90	998	130	622	170	364	130	654	170	484
11	1484	51	1284	91	991	131	612	171	359	131	648	171	482
12	1480	52	1278	92	982	132	603	172	355	132	641	172	480
13	1477	53	1271	93	973	133	593	173	351	133	635	173	477
14	1473	54	1265	94	965	134	585	174	347	134	629	174	475
15	1469	55	1259	95	956	135	576	175	343	135	623	175	473
16	1465	56	1252	96	948	136	567	176	339	136	617	176	471
17	1461	57	1245	97	939	137	560	177	335	137	612	177	469
18	1457	58	1239	98	930	138	551	178	331	138	606	178	467
19	1453	59	1233	99	921	139	543	179	328	139	600	179	465
20	1448	60	1226	100	913	140	536	180	324	140	596	180	463
21	1444	61	1218	101	903	141	528	181	321	141	590	181	461
22	1440	62	1212	102	894	142	521	182	317	142	585	182	459
23	1435	63	1205	103	885	143	513	183	314	143	580	183	458
24	1431	64	1198	104	877	144	506	184	310	144	575	184	456
25	1426	65	1191	105	867	145	499	185	307	145	571	185	454
26	1422	66	1184	106	858	146	493	186	304	146	566	186	453
27	1417	67	1177	107	849	147	486	187	300	147	562	187	451
28	1412	68	1170	108	840	148	480	188	297	148	558	188	450
29	1407	69	1162	109	830	149	473	189	294	149	553	189	449
30	1402	70	1155	110	821	150	467	190	291	150	549	190	447
31	1397	71	1148	111	811	151	461	191	288	151	545	191	446
32	1392	72	1140	112	802	152	454	192	285	152	541	192	445
33	1387	73	1133	113	792	153	449	193	282	153	537	193	444
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35	1377	75	1118	115	773	155	437	195	276	155	529	195	442
36	1371	76	1110	116	763	156	432	196	274	156	526	196	441
37	1365	77	1103	117	753	157	426	197	271	157	522	197	440
38	1360	78	1095	118	743	158	420	198	268	158	520	198	439
39	1355	79	1088	119	733	159	416	199	265	159	516	199	438
40	1349	80	1080	120	723	160	410	200	262	160	513	200	437

\*  $K = 1$  Para miembros secundarios.

TABLA 3  
 $F_y = 2950 \text{ Kg/cm}^2 \text{ (42 KSI)}$

MIEMBROS PRINCIPALES Y SECUNDARIOS KL/r MENOR DE 120			MIEMBROS PRINCIPALES KL/r 121 a 200				MIEMBROS SECUNDARIOS L/r 121 a 200						
$\frac{KL}{r}$	$\frac{KL}{r}$	$\frac{KL}{r}$	$\frac{KL}{r}$	$\frac{KL}{r}$	$\frac{KL}{r}$	$\frac{KL}{r}$	$\frac{KL}{r}$	$\frac{KL}{r}$	$\frac{KL}{r}$	$\frac{KL}{r}$	$\frac{KL}{r}$		
1	1768	41	1545	81	1190	121	717	161	405	121	721	161	510
2	1765	42	1538	82	1174	122	705	162	400	122	712	162	506
3	1761	43	1531	83	1169	123	694	163	395	123	705	163	503
4	1757	44	1523	84	1158	124	683	164	390	124	697	164	501
5	1754	45	1515	85	1147	125	672	165	386	125	689	165	498
6	1749	46	1507	86	1140	126	662	166	381	126	682	166	495
7	1745	47	1500	87	1126	127	651	167	376	127	674	167	492
8	1741	48	1492	88	1115	128	641	168	372	128	667	168	489
9	1737	49	1484	89	1105	129	631	169	368	129	661	169	487
10	1732	50	1476	90	1093	130	622	170	364	130	654	170	484
11	1728	51	1467	91	1082	131	612	171	359	131	648	171	482
12	1723	52	1460	92	1071	132	603	172	355	132	641	172	480
13	1718	53	1451	93	1060	133	593	173	351	133	635	173	477
14	1713	54	1443	94	1048	134	585	174	347	134	629	174	475
15	1708	55	1434	95	1037	135	576	175	343	135	623	175	473
16	1702	56	1426	96	1026	136	567	176	339	136	617	176	471
17	1698	57	1417	97	1015	137	560	177	335	137	612	177	469
18	1692	58	1408	98	1003	138	551	178	331	138	606	178	467
19	1681	59	1400	99	991	139	543	179	328	139	600	179	465
20	1682	60	1391	100	979	140	536	180	324	140	596	180	463
21	1676	61	1382	101	967	141	528	181	321	141	590	181	461
22	1671	62	1373	102	956	142	521	182	317	142	585	182	459
23	1665	63	1364	103	944	143	513	183	314	143	580	183	458
24	1659	64	1355	104	932	144	506	184	310	144	575	184	456
25	1653	65	1346	105	920	145	499	185	307	145	571	185	454
26	1647	66	1337	106	907	146	493	186	304	146	566	186	453
27	1640	67	1327	107	895	147	486	187	300	147	562	187	451
28	1634	68	1318	108	882	148	480	188	297	148	558	188	450
29	1626	69	1308	109	870	149	473	189	294	149	553	189	449
30	1621	70	1300	110	857	150	467	190	291	150	549	190	447
31	1615	71	1289	111	844	151	461	191	288	151	545	191	446
32	1609	72	1280	112	832	152	454	192	285	152	541	192	445
33	1602	73	1270	113	819	153	449	193	282	153	537	192	444
34	1595	74	1260	114	806	154	443	194	279	154	534	194	443
35	1588	75	1250	115	793	155	437	195	276	155	529	195	442
36	1581	76	1240	116	780	156	432	196	274	156	526	196	441
37	1574	77	1230	117	767	157	426	197	271	157	522	197	440
38	1567	78	1220	118	754	158	420	198	268	158	520	198	439
39	1560	79	1210	119	742	159	416	199	265	159	516	199	438
40	1552	80	1200	120	729	160	410	200	262	160	513	200	437

**TABLA 4**  
**Fy = 3160 Kg/cm<sup>2</sup> (45 KSI)**

MIEMBROS PRINCIPALES Y SECUNDARIOS KL/r MENOR DE 120					MIEMBROS PRINCIPALES KL/r 121 a 200				MIEMBROS SECUNDARIOS L/r 121 a 200				
$\frac{KL}{r}$		$\frac{KL}{r}$		$\frac{KL}{r}$		$\frac{KL}{r}$		$\frac{KL}{r}$		$\frac{KL}{r}$		$\frac{KL}{r}$	
1	1095	41	1645	81	1242	121	717	161	405	121	721	161	510
2	1891	42	1636	82	1231	122	705	162	400	122	712	162	506
3	1826	43	1628	83	1219	123	694	163	395	123	705	163	503
4	1832	44	1619	84	1207	124	683	164	390	124	697	164	501
5	1878	45	1610	85	1195	125	672	165	386	125	682	165	498
6	1873	46	1601	86	1183	126	662	166	381	126	682	166	495
7	1869	47	1595	87	1171	127	651	167	376	127	674	167	492
8	1864	48	1584	88	1159	128	641	168	372	128	667	168	489
9	1859	49	1575	89	1146	129	631	169	368	129	661	169	487
10	1854	50	1566	90	1132	130	622	170	364	130	654	170	484
11	1849	51	1527	91	1121	131	612	171	359	131	648	171	482
12	1844	52	1548	92	1109	132	603	172	355	132	641	172	480
13	1839	53	1538	93	1096	133	593	173	351	133	635	173	477
14	1833	54	1529	94	1080	134	585	174	347	134	629	174	475
15	1827	55	1519	95	1070	135	576	175	343	135	623	175	473
16	1822	56	1510	96	1057	136	567	176	339	136	617	176	471
17	1815	57	1500	97	1044	137	560	177	335	137	612	177	469
18	1810	58	1490	98	1031	138	551	178	331	138	606	178	467
19	1805	59	1480	99	1017	139	543	179	326	139	600	179	465
20	1798	60	1470	100	1004	140	534	180	324	140	596	180	463
21	1791	61	1460	101	991	141	528	181	321	141	590	181	461
22	1785	62	1450	102	977	142	521	182	317	142	585	182	459
23	1778	63	1440	103	964	143	513	183	314	143	580	183	458
24	1772	64	1430	104	950	144	506	184	310	144	575	184	456
25	1765	65	1420	105	937	145	499	185	307	145	571	185	454
26	1758	66	1409	106	922	146	493	186	304	146	566	186	453
27	1751	67	1398	107	908	147	486	187	300	147	562	187	451
28	1744	68	1388	108	894	148	480	188	297	148	558	188	450
29	1737	69	1377	109	880	149	473	189	294	149	553	189	449
30	1730	70	1366	110	866	150	467	190	291	150	549	190	447
31	1723	71	1356	111	851	151	461	191	288	151	545	191	446
32	1716	72	1344	112	837	152	454	192	285	152	541	192	445
33	1708	73	1334	113	822	153	449	193	282	153	537	193	444
34	1700	74	1324	114	808	154	443	194	279	154	534	194	443
35	1692	75	1311	115	794	155	437	195	276	155	529	195	442
36	1685	76	1309	116	780	156	432	196	274	156	526	196	441
37	1677	77	1289	117	767	157	426	197	271	157	522	197	440
38	1669	78	1278	118	754	158	420	198	268	158	520	198	439
39	1661	79	1266	119	742	159	416	199	265	159	516	199	438
40	1653	80	1254	120	729	160	410	200	262	160	513	200	437

TABLA 5  
 $F_y = 3515 \text{ Kg/cm}^2$  (50 KSI)

MIEMBROS PRINCIPALES Y SECUNDARIOS KL/r MENOR DE 120				MIEMBROS PRINCIPALES KL/r 121 a 200				MIEMBROS SECUNDARIOS L/r 121 a 200					
$\frac{KL}{r}$		$\frac{KL}{r}$		$\frac{KL}{r}$		$\frac{KL}{r}$		$\frac{KL}{r}$		$\frac{KL}{r}$			
1	2105	41	1806	81	1323	121	717	161	405	121	721	161	513
2	2100	42	1796	82	1308	122	705	162	400	122	712	162	506
3	2095	43	1786	83	1294	123	694	163	395	123	705	163	503
4	2090	44	1776	84	1280	124	683	164	390	124	697	164	501
5	2085	45	1765	85	1265	125	672	165	386	125	689	165	498
6	2080	46	1755	86	1251	126	662	166	381	126	682	166	495
7	2074	47	1744	87	1236	127	651	167	376	127	674	167	492
8	2069	48	1734	88	1221	128	641	168	372	128	667	168	489
9	2063	49	1723	89	1206	129	631	169	368	129	661	169	487
10	2057	50	1712	90	1191	130	622	170	364	130	654	170	484
11	2051	51	1701	91	1176	131	612	171	359	131	648	171	482
12	2045	52	1690	92	1160	132	603	172	355	132	641	172	480
13	2038	53	1679	93	1145	133	593	173	351	133	635	173	477
14	2032	54	1668	94	1129	134	585	174	347	134	629	174	475
15	2025	55	1656	95	1114	135	576	175	343	135	623	175	473
16	2019	56	1645	96	1098	136	567	176	339	136	617	176	471
17	2012	57	1633	97	1082	137	560	177	335	137	612	177	469
18	2005	58	1621	98	1067	138	551	178	331	138	606	178	467
19	1997	59	1609	99	1050	139	547	179	328	139	600	179	465
20	1990	60	1597	100	1034	140	536	180	324	140	596	180	463
21	1982	61	1585	101	1017	141	528	181	321	141	590	181	461
22	1974	62	1573	102	1001	142	521	182	317	142	585	182	459
23	1967	63	1561	103	984	143	513	183	314	143	580	183	458
24	1959	64	1548	104	968	144	506	184	310	144	575	184	456
25	1951	65	1536	105	951	145	499	185	307	145	571	185	454
26	1943	66	1524	106	934	146	493	186	304	146	566	186	453
27	1935	67	1511	107	917	147	486	187	300	147	562	187	451
28	1926	68	1498	108	900	148	480	188	297	148	558	188	450
29	1918	69	1485	109	884	149	473	189	294	149	553	189	449
30	1909	70	1472	110	868	150	467	190	291	150	549	190	447
31	1900	71	1459	111	852	151	461	191	288	151	545	191	446
32	1891	72	1446	112	837	152	454	192	285	152	541	192	445
33	1882	73	1433	113	822	153	449	193	282	153	537	193	444
34	1873	74	1420	114	808	154	443	194	279	154	534	194	443
35	1864	75	1405	115	794	155	437	195	276	155	529	195	442
36	1855	76	1392	116	780	156	432	196	274	156	526	196	441
37	1846	77	1379	117	767	157	426	197	271	157	522	197	440
38	1836	78	1365	118	754	158	420	198	268	158	520	198	439
39	1826	79	1351	119	742	159	416	199	265	159	516	199	438
40	1816	80	1337	120	729	160	410	200	262	160	513	200	437

Tabla 6

VALORES DE  $F'_c$  en Kg/cm<sup>2</sup> PARA  
ESFUERZOS COMBINADOS

(Para todo  $F_y$ )

$Kl_b$	$F'_c$	$Kl_b$	$F'_c$	$Kl_b$	$F'_c$	$Kl_b$	$F'_c$	$Kl_b$	$F'_c$	$Kl_b$	$F'_c$
$r_b$	Kg/cm <sup>2</sup>	$r_b$	Kg/cm <sup>2</sup>	$r_b$	Kg/cm <sup>2</sup>	$r_b$	Kg/cm <sup>2</sup>	$r_b$	Kg/cm <sup>2</sup>	$r_b$	Kg/cm <sup>2</sup>
21	23774	51	4031	81	1598	111	852	141	528	171	359
22	21662	52	3878	82	1559	112	837	142	521	172	355
23	19819	53	3733	83	1522	113	822	143	513	173	351
24	18203	54	3596	84	1486	114	808	144	506	174	347
25	16775	55	3466	85	1451	115	794	145	499	175	343
26	15509	56	3344	86	1417	116	780	146	493	176	339
27	14382	57	3227	87	1385	117	767	147	486	177	335
28	13373	58	3117	88	1354	118	754	148	480	178	331
29	12467	59	3012	89	1324	119	742	149	473	179	328
30	11649	60	2913	90	1294	120	729	150	467	180	324
31	10910	61	2817	91	1266	121	717	151	461	181	321
32	10239	62	2727	92	1239	122	705	152	454	182	317
33	9628	63	2642	93	1212	123	694	153	449	183	314
34	9069	64	2560	94	1187	124	683	154	443	184	310
35	8559	65	2481	95	1162	125	672	155	437	185	307
36	8090	66	2407	96	1138	126	662	156	432	186	304
37	7659	67	2336	97	1114	127	651	157	426	187	300
38	7261	68	2267	98	1092	128	641	158	420	188	297
39	6893	69	2202	99	1069	129	631	159	416	189	294
40	6553	70	2140	100	1048	130	622	160	410	190	291
41	6237	71	2080	101	1028	131	612	161	405	191	288
42	5943	72	2023	102	1008	132	603	162	400	192	285
43	5671	73	1968	103	989	133	593	163	395	193	282
44	5415	74	1915	104	965	134	585	164	390	194	279
45	5178	75	1864	105	951	135	576	165	386	195	276
46	4955	76	1815	106	934	136	567	166	381	196	274
47	4747	77	1768	107	915	137	560	167	377	197	270
48	4551	78	1723	108	900	138	551	168	372	198	268
49	4367	79	1680	109	884	139	543	169	368	199	265
50	4194	80	1638	110	868	140	536	170	364	200	262

$$F'_c = \frac{10'480,000}{\left(\frac{Kl_b}{r_b}\right)^2}$$

8-11 CARGAS CONCENTRADAS

Teóricamente, bajo el punto de aplicación de una carga concentrada, el esfuerzo en la fibra es infinito. En las estructuras reales no existen cargas concentradas, sino éstas se distribuyen sobre una longitud muy pequeña, a lo largo del claro (Fig. 8-31a). Estas cargas producen esfuerzos locales altos de aplastamiento,<sup>12</sup> y la distribución de los esfuerzos locales de flexión y de cortantes difiere considerablemente de los valores convencionales  $Mc/I$  y  $VQ/It$ . Para una viga de sección rectangular la distribución de esfuerzos correspondiente a una carga  $P$  localizada, se muestra en la figura 8-31b. Puede verse que los esfuerzos en una sección adyacente al punto de aplicación de la carga difieren considerablemente de la distribución convencional de esfuerzos; a una distancia aproximadamente igual al peralte de la viga, los esfuerzos reales se apegan muy bien a los determinados por la teoría convencional. Las concentraciones de esfuerzos debidas a cargas locales en vigas de sección "I" pueden disminuirse drásticamente mediante el uso de atesadores de carga, los cuales transmiten la carga concentrada al alma por medio de cortantes distribuidos a lo largo de su peralte.

Las combinaciones de esfuerzos cortantes, flexionantes y de aplasta-

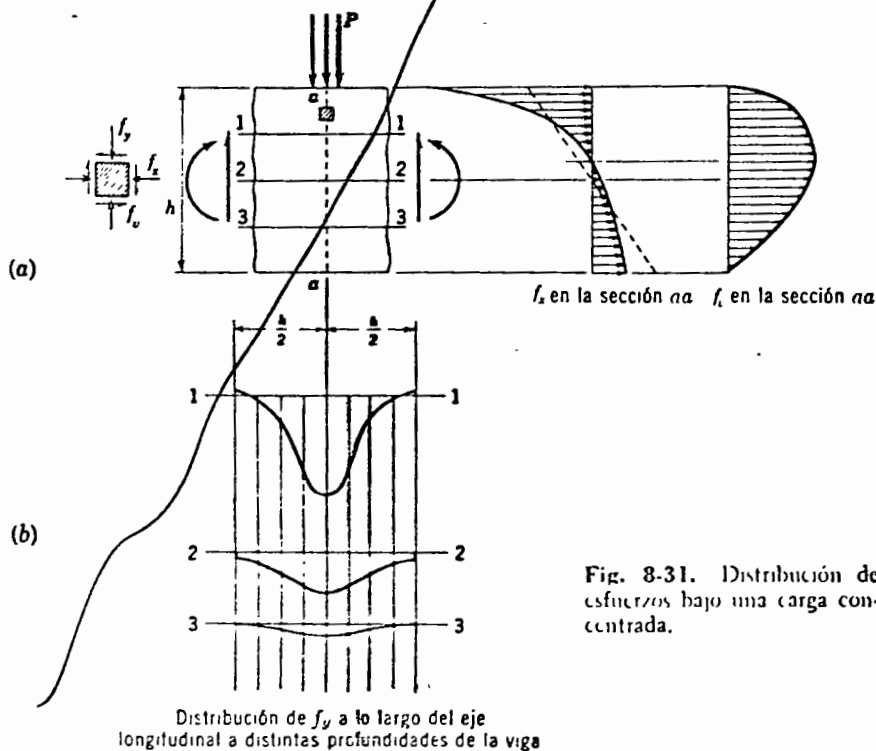


Fig. 8-31. Distribución de esfuerzos bajo una carga concentrada.

miento que ocurren en los puntos adyacentes a las cargas concentradas pueden aproximarse como se ilustra en la fig. 8-32. El criterio de Hencky-Von Mises permite definir con suficiente exactitud un factor de seguridad  $n$  contra la fluencia local del material de un elemento de placa sometido a esfuerzos normales y cortantes:

$$(f_x^2 + f_y^2 - f_x f_y + 3f_v^2)^{1/2} = \frac{f_y}{n}$$

donde  $f_v$  es la resistencia de fluencia por tensión del material de la placa. Los esfuerzos de flexión, de aplastamiento y cortantes en un elemento del alma varían dentro de la viga o trabe, y dependen del tipo de carga. En la Fig. 8-32 se ilustran varios casos típicos. Las magnitudes de los esfuerzos normales y cortantes no pueden determinarse con precisión en todos los casos, por los efectos desconocidos de la concentración de la carga y de la redistribución de esfuerzos en la postfluencia o en el postpandeo. Por tanto, solamente se pueden usar valores aproximados de estos esfuerzos para fines de diseño, con un factor de seguridad adecuado. En muchos casos un factor de seguridad de 1.5 puede ser suficiente, mientras que en otros, cuando la plasticidad reduce las concentraciones locales de esfuerzos sin daño permanente, es posible aceptar valores menores.

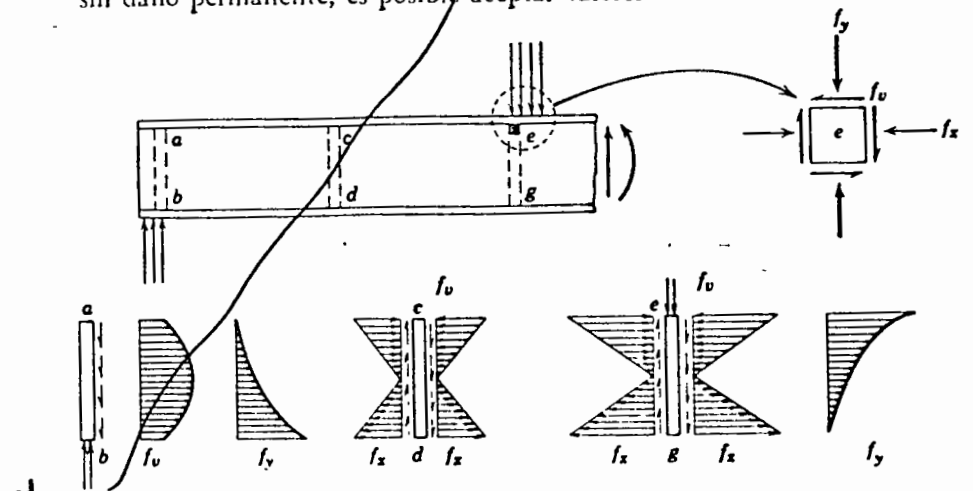


Fig. 8-32. Combinaciones de esfuerzos en vigas y traves.

8-12 FLEXIÓN Y CARGA AXIAL COMBINADOS — RANGO ELÁSTICO

La determinación precisa de los esfuerzos bajo flexión y carga axial combinados, en el rango elástico, se complica por el efecto de la deformación de la estructura y de la carga axial  $P$  sobre la magnitud del momento

flexionante  $M$ . En la Fig. 8-33, sea  $M_0$  el momento flexionante en cualquier punto a lo largo del miembro, debido a las cargas exteriores, despreciando el efecto de la deflexión del miembro. Si  $y$  es la deflexión eventual de un punto cualquiera, debida al efecto combinado de la flexión y la carga axial, y  $P$  es la magnitud de la carga de tensión (Fig. 8-33a), el momento flexionante real  $M$  en cualquier punto es

$$M = M_0 - PY \quad (8-46a)$$

y si  $P$  es la magnitud de la carga de compresión (Fig. 8-33b), el momento flexionante real  $M$  en cualquier punto es

$$M = M_0 + PY \quad (8-46b)$$

Puede deducirse de estas dos ecuaciones y de la Fig. 8-33 que el efecto de la carga de tensión es siempre disminuir el momento flexionante inicial  $M_0$ , y que el efecto de la compresión es incrementarlo. Es precisamente en esto donde radica la diferencia básica entre los miembros a tensión y los miembros a compresión: las cargas de tensión tienden a disminuir la flexión en el miembro y, por lo tanto, puede despreciarse el efecto de la deformación, quedando dentro del lado de la seguridad, mientras que las cargas de compresión tienden a aumentar la flexión, y el efecto de la de-

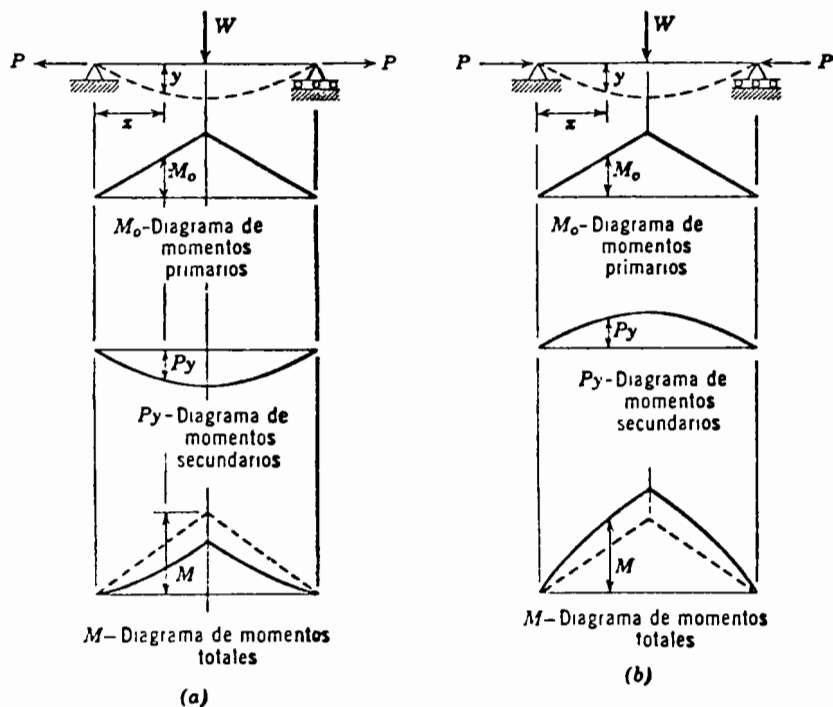


Fig. 8-33

flexión puede a menudo ser el factor crítico que determina la resistencia del miembro.

El momento  $M_0$  se evalúa fácilmente cuando se conoce la carga, pero en cambio el valor  $Py$  no puede determinarse tan fácilmente, ya que la deflexión  $y$  depende del momento  $M$ , el cual a su vez depende de la deflexión  $y$ .

Para resolver este problema se pueden utilizar dos métodos diferentes. Uno es el método numérico o método de aproximaciones sucesivas, y el otro implica la solución de una ecuación diferencial de la cual se puede eliminar una variable. A continuación se bosqueja el procedimiento numérico para determinar los momentos y las deflexiones:<sup>13</sup>

1. Supóngase una forma de la elástica deformada del miembro.

2. Substitúyanse los valores supuestos de  $y$  en la ecuación 8-46, obteniendo los primeros valores de tanteo de  $M$ .

3. Cálculense los valores de  $y$  para un número determinado de puntos a lo largo del miembro, basándose en los primeros valores de tanteo de  $M$  encontrados en el paso 2. Para estos cálculos se pueden utilizar los métodos convencionales, tales como el de las áreas de momentos, la doble integración de la ecuación de la elástica, o la "viga conjugada".

4. Compárense los valores de  $y$  obtenidos en el paso 3 con los supuestos en el 1. Si estos valores concuerdan con bastante aproximación, entonces la elástica supuesta originalmente es satisfactoria. Si, por el contrario, no concuerdan adecuadamente, el proceso deberá repetirse utilizando los valores de  $y$  obtenidos en 3 como la nueva elástica deformada supuesta. Se supone que en la carga axial,  $P$ , es menor que el valor crítico  $P_{cr}$ , de modo que el proceso de aproximaciones sucesivas converge necesariamente.

En casos simples, los momentos y las deflexiones pueden obtenerse mediante la solución de una ecuación diferencial deducida de la Ec. 8-46a o de la Ec. 8-46b, dependiendo esto de la dirección de la carga axial, tensión o compresión. Por ejemplo, cuando  $P$  es compresión, diferenciando  $M$  dos veces respecto a  $x$  en la Ec. 8-46b, se obtiene:

$$\frac{d^2M}{dx^2} = \frac{d^2M_0}{dx^2} + P \frac{d^2y}{dx^2} = f_1(w) + P \frac{d^2y}{dx^2} \quad (8-47)$$

donde  $f_1(w)$  es una función de la carga, igual a  $d^2M_0/dx^2$ . Para eliminar  $y$  de la Ec. 8-47, podemos utilizar la relación entre  $y$  y  $M$  que se basa en la teoría convencional de la flexión:

$$\frac{d^2y}{dx^2} = -\frac{M}{EI} \quad (8-48)$$

Sustituyendo la Ec. 8-48 en la Ec. 8-47, se obtiene la siguiente ecuación diferencial:

$$\frac{d^2M}{dx^2} + \frac{P}{EI} M = \frac{d^2M_0}{dx^2} + \frac{1}{J^2} M = f_1(w) \quad (8-49)$$

donde  $j = \sqrt{EI/P}$ . La solución de esta ecuación diferencial tiene como resultado la expresión siguiente:

$$M = C_1 \operatorname{sen} \frac{x}{j} + C_2 \cos \frac{x}{j} + f(w) \quad (8-50)$$

donde  $f(w)$  es una función de  $w$  tal que  $d^2f(w)/dx^2 = f_1(w)$ , y  $C_1$  y  $C_2$  son constantes numéricas que dependen de las condiciones de borde, es decir, del tipo de carga y del tipo de apoyos extremos.

Es posible demostrar que cuando  $P$  es tensión, la solución de la ecuación diferencial correspondiente tiene la forma

$$M = C_1' \operatorname{senh} \frac{x}{j} + C_2' \cosh \frac{x}{j} + f(w) \quad (8-51)$$

donde  $f(w)$  es la misma función que se definió anteriormente y  $C_1'$  y  $C_2'$  son constantes numéricas que dependen de la carga, de las características de la viga y de las condiciones de borde. Las ecuaciones 8-50 y 8-51 son válidas solamente para miembros de sección transversal constante. Para los miembros de sección variable, la solución de la ecuación diferencial se torna difícil y en ocasiones imposible, por lo cual es necesario utilizar el método de aproximaciones sucesivas para obtener una solución.

La deflexión de la viga puede determinarse fácilmente por medio de la Ec. 8-52:

$$y = \frac{M - M_o}{P} \quad (8-52)$$

Si se dibujan los diagramas de momentos  $M$  y  $M_o$ , la diferencia entre estas dos curvas es una medida de la deformación. La deflexión máxima  $y_{\max}$  ocurrirá donde esa diferencia de momentos sea máxima, y su posición puede encontrarse a menudo por observación. Si la localización de  $y_{\max}$  no es obvia, puede encontrarse analíticamente, diferenciando y respecto a  $x$  e igualando a cero: es decir,

$$\frac{dy}{dx} = \frac{1}{P} \left( \frac{dM}{dx} - \frac{dM_o}{dx} \right) = \frac{1}{P} (V - V_o) = 0 \quad (8-53)$$

Esto indica que  $V = V_o$  en el punto de flecha máxima, entonces, el punto de máxima deflexión puede localizarse si se trazan los diagramas de  $V$  y  $V_o$ .

Si en una sección cualquiera se conocen la carga axial y el momento flexionante debido a los efectos combinados de la excentricidad y las fuerzas transversales, la distribución de esfuerzos en esa sección puede definirse por medio de la ecuación

\* Para la definición de  $V$ , véase la Ec. 8-58.

$$f = \frac{P}{A} + \frac{My}{I} \quad (8-54)$$

Esta fórmula es válida solamente bajo las siguientes condiciones:

- La flexión tiene lugar con respecto a un eje principal.
- Una sección plana antes de la deformación permanece plana después de ella.
- Los esfuerzos están dentro del límite elástico.

Para una cierta condición de carga, la expresión general del momento flexionante está dada por la Ec. 8-50 o la 8-51. El momento máximo  $M_{\max}$  puede obtenerse trazando el diagrama de momento flexionante  $M$ ; es decir, valuando las condiciones de borde y de carga, y substituyendo estos valores de  $C_1$ ,  $C_2$ , y  $f(w)$  en la Ec. 8-50, calculando los valores de  $M$  correspondientes a los distintos valores de  $x$  y trazando estos valores. Con objeto de facilitar la solución de problemas sencillos, en la Tabla 8-1 se dan valores de  $M_{\max}$  para los casos en que  $P$  sea de tensión o de compresión. En la Tabla 8-2 se proporcionan las expresiones para  $C_1$ ,  $C_2$  y  $f(w)$  para varios otros casos de carga, con fuerza axial de compresión.

Si  $M$ , definida por la Ec. 8-50, es una función continua, el punto de momento máximo se puede obtener a partir de la condición de que  $(dM/dx) = 0$ :

$$\frac{dM}{dx} = \frac{1}{j} \left( C_1 \cos \frac{x}{j} - C_2 \operatorname{sen} \frac{x}{j} \right) + \frac{d}{dx} f(w) = 0 \quad (8-55)$$

De la tabla 8-2 se ve que para la mayoría de las condiciones de carga  $df(w)/dx = 0$ , y para esas condiciones  $M_{\max}$  ocurre en un punto  $x_m$  tal que

$$\tan \frac{x_m}{j} = \frac{C_1}{C_2} \quad (8-56)$$

Despejando a  $\operatorname{sen}(x_m/j)$  y a  $\cos(x_m/j)$  correspondientes a la Ec. 8-56 y substituyendo esos valores en la Ec. 8-50, se obtiene la siguiente ecuación:

$$M_{\max} = \sqrt{C_1^2 + C_2^2} \quad (8-57a)$$


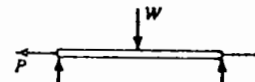
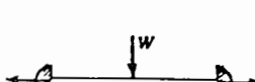
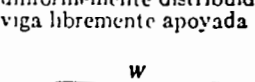
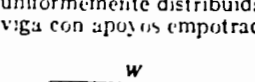
la cual es fácil de calcular con los valores de  $C_1$  y  $C_2$  dados en la tabla 8-2. Substituyendo los valores de  $M_{\max}$  en la Ec. 8-54, pueden calcularse los esfuerzos máximos  $f_{\max}$ , como

$$f_{\max} = \frac{P}{A} + \frac{M_{\max} c}{I} \quad (8-57b)$$



Tabla 8.1 Momentos máximos — Flexión y carga axial combinadas

- L = claro
- P = carga axial — tensión o compresión
- I = momento de inercia (constante)
- z = (PL<sup>2</sup>/4EI)<sup>1/2</sup>
- W = carga transversal total sobre la viga
- M<sub>o</sub> = momento aplicado en el extremo
- M<sub>r</sub> = momento en el apoyo de la viga
- M<sub>s</sub> = momento en el centro del claro de la viga

Condición de carga	Tensión	Compresión
1. Momentos extremos M <sub>o</sub> iguales y opuestos 	M <sub>e</sub> = M <sub>o</sub> M <sub>s</sub> = $\frac{M_o}{\cosh z}$	M <sub>e</sub> = M <sub>o</sub> M <sub>s</sub> = $\frac{M_o}{\cos z}$
2. Carga transversal W concentrada en el centro, viga libremente apoyada 	M <sub>e</sub> = 0 M <sub>s</sub> = $\frac{WL \tanh z}{4 z}$	M <sub>e</sub> = 0 M <sub>s</sub> = $\frac{WL \tan z}{4 z}$
3. Carga transversal W concentrada en el centro, viga con apoyos empotrados 	M <sub>e</sub> = $\frac{WL (\cosh z - 1)}{4 z \sinh z}$ M <sub>s</sub> = $\frac{WL}{4} \left( \frac{\tanh z}{z} - \frac{\cosh z - 1}{z \sinh z \cosh z} \right)$	M <sub>e</sub> = $\frac{WL (1 - \cos z)}{4 z \sin z}$ M <sub>s</sub> = $\frac{WL}{4} \left( \frac{\tan z}{z} - \frac{1 - \cos z}{z \sin z \cos z} \right)$
4. Carga transversal W uniformemente distribuida, viga libremente apoyada 	M <sub>e</sub> = 0 M <sub>s</sub> = $\frac{WL (\cosh z - 1)}{4 z^2 \cosh z}$	M <sub>e</sub> = 0 M <sub>s</sub> = $\frac{WL (1 - \cos z)}{4 z^2 \cos z}$
5. Carga transversal W uniformemente distribuida, viga con apoyos empotrados 	M <sub>e</sub> = $\frac{WL (z - \tanh z)}{4 z^2 \tanh z}$ M <sub>s</sub> = $\frac{WL (\operatorname{scnh} z - z)}{4 z^2 \operatorname{senh} z}$	M <sub>e</sub> = $\frac{WL (\tan z - z)}{4 z^2 \tan z}$ M <sub>s</sub> = $\frac{WL (z - \operatorname{sen} z)}{4 z^2 \operatorname{scn} z}$

Por lo general, los cortantes en vigas no gobiernan el diseño de los miembros; sin embargo, pueden calcularse de la siguiente manera: el cortante total real V difiere del cortante primario V<sub>o</sub> exactamente en la misma forma que el momento flexionante total real M difiere del momento primario M<sub>o</sub>. Si el cortante se define como la fuerza total en el plano de la sección normal a la curva elástica (Fig. 8-34), entonces, para una carga axial de compresión, diferenciando la Ec. 8-64b obtenemos

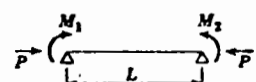
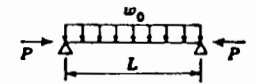
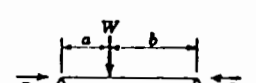
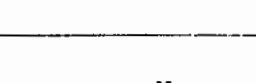
$$V = \frac{dM}{dx} = \frac{dM_o}{dx} + P \frac{dy}{dx} = V_o + P \frac{dy}{dx} \quad (8-58)$$

Esta ecuación indica que el cortante total es mayor que el cortante primario V<sub>o</sub> en una cantidad igual a la componente del cortante producida por P. Para ser precisos, el V<sub>o</sub> real es ligeramente menor que el cortante R<sub>1</sub>, que sería calculado por métodos convencionales como V<sub>o</sub> = R<sub>1</sub> cos θ. Cuando θ es pequeño, cos θ está muy próximo a la unidad, y V<sub>o</sub> se supone

Tabla 8-2 Coeficientes de momento — Flexión y carga axial combinadas

$$M = C_1 \operatorname{sen}(x/j) + C_2 \operatorname{cos}(x/j) + f(w)$$

$$j = \sqrt{EI/P}$$

Carga	C <sub>1</sub>	C <sub>2</sub>	f(w)
	$\frac{M_2 - M_1 \operatorname{cos}(L/j)}{\operatorname{sen}(L/j)}$	M <sub>1</sub>	0
	$\frac{w_0 j^2 [1 - \operatorname{cos}(L/j)]}{\operatorname{sen}(L/j)}$	w <sub>0</sub> j <sup>2</sup>	-w <sub>0</sub> j <sup>2</sup>
	x < a: $\frac{W j \operatorname{sen}(bj/j)}{\operatorname{sen}(L/j)}$	0	0
	x > a: $\frac{W j \operatorname{sen}(aj/j)}{\tan(L/j)}$	W j $\frac{\operatorname{sen} a}{j}$	0
	x < a: $-\frac{M_o \operatorname{cos}(bj/j)}{\operatorname{sen}(L/j)}$	0	0
	x > a: $-\frac{M_o \operatorname{cos}(aj/j)}{\operatorname{sen}(L/j)}$	M <sub>o</sub> $\frac{\operatorname{cos} a}{j}$	0

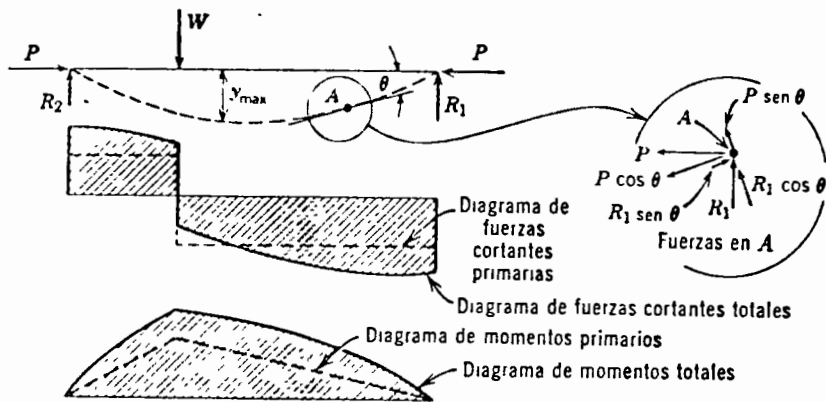


Fig. 8-34 Diagramas de cortante y de momentos para una viga-columna.

igual a \$R\_1\$, mientras que cuando \$P\$ es grande, \$P(dy/dx) = P \sin \theta\$ no puede despreciarse. Diferenciando la ecuación 8-50, obtenemos

$$V = \frac{dM}{dx} = \frac{1}{j} \left( C_1 \cos \frac{x}{j} - C_2 \sin \frac{x}{j} \right) + \frac{df(w)}{dx} \quad (8-59)$$

y, trazando la curva de \$V\$ contra \$x\$ en esta ecuación, puede obtenerse el diagrama total de cortantes para la viga-columna (Fig. 8-34). El esfuerzo cortante en cualquier punto de la viga-columna puede entonces calcularse en la forma convencional, como sigue:

$$f_v = \frac{VQ}{Ib} \quad (8-60)$$

La solución convencional del problema de la viga-columna, dada anteriormente, desprecia las deformaciones por cortante, de la misma manera que lo hace la teoría convencional de la flexión. Si se consideran dichas deformaciones por cortante, el problema se complica y su solución se torna más compleja; sin embargo, por lo general el efecto de las deformaciones por cortante no es de importancia.

Los cortantes, momentos, deformaciones y esfuerzos que se acaban de definir, no son funciones lineales de la carga transversal, en virtud de que dependen de la carga axial en una forma no lineal. Esto se hace evidente observando las soluciones de las ecuaciones diferenciales, las cuales involucran funciones hiperbólicas o trigonométricas (Ec. 8-50 u 8-51).

En un sistema estáticamente determinado, la respuesta a una carga transversal de flexión es lineal, para una carga axial determinada. Por tanto, para una combinación de cargas transversales, puede usarse el principio de superposición en la determinación de los valores de \$M\$ y \$V\$ y, por

consecuente, los valores de las deformaciones y los esfuerzos, siempre y cuando al estudiar los efectos de las cargas transversales se incluya la totalidad de la carga axial en la valuación del efecto de cada una de ellas, considerada individualmente.

Esta solución no es demasiado difícil, pero en cambio si requiere de cálculos tediosos, mientras que la solución aproximada que se presentará a continuación requiere una cantidad mucho menor de cálculos y proporciona resultados muy cercanos a los teóricos, siempre que la forma de la elástica pueda aproximarse por medio de una onda senoidal simple, o de una parábola. La deducción de esta ecuación aproximada se proporciona en los libros de texto,<sup>1</sup> y no se incluye aquí. Establece que la deformación en un punto cualquiera de una viga-columna se expresa aproximadamente por la fórmula

$$y = y_0 \frac{1}{1 \pm P/P_{cr}} \quad (8-61)$$

en donde \$y\_0\$ es la deflexión de la misma viga y en el mismo punto, calculada por los métodos convencionales elementales, despreciando el efecto de la carga axial \$P\$, y \$P\_{cr}\$ es la carga crítica de pandeo del miembro trabajando como columna, la cual queda definida por la fórmula de Euler

$$P_{cr} = \frac{\pi^2 EI}{L^2} \quad (8-62)$$

En esta ecuación se usa el signo *más* cuando la carga axial \$P\$ es tensión y el signo *menos* cuando \$P\$ es compresión.

### 8-13 FLEXIÓN Y CARGA AXIAL COMBINADOS — RANGO PLÁSTICO

Los experimentos indican que en el rango plástico una sección plana permanece plana también, pero que los esfuerzos ya no son proporcionales a las deformaciones unitarias, como tampoco la distribución de esfuerzos de flexión lo es a la distancia al eje neutro. Por consiguiente, la Ec. 8-48, así como las Ecs. 8-50 y 8-51, dejan de ser válidas, y las soluciones a las ecuaciones diferenciales descritas en la sección precedente no son aplicables a las vigas dentro del rango plástico. El método numérico<sup>12</sup> descrito en la pág. 335 es aplicable a este caso con la modificación de que la relación entre el momento y la curvatura, que se usa en el paso 2 para valuar la deformación, incluya el efecto del comportamiento inelástico del material y el de la carga axial. Esta condición hace prácticamente imposible el obtener una solución para la capacidad plástica de una sección sometida a flexión y carga axial combinadas mediante el uso de este método, sin la utilización de una computadora digital.

Un método conveniente y poderoso para determinar la resistencia de tales miembros es el de la interacción.<sup>14</sup> El criterio general para la falla se expresa mediante una relación funcional en términos de las relaciones de la carga real a la resistencia del miembro bajo carga axial pura o bajo flexión pura, como sigue:

$$\frac{P}{P_u} = f_1 \frac{M}{M_u} \quad \text{ó} \quad \frac{M}{M_u} = f_2 \frac{P}{P_u} \quad (8-63)$$

en donde  $P$  = carga axial real,  $M$  = momento flexionante máximo que actúa simultáneamente con  $P$ ,  $P_u$  = resistencia del miembro considerado, cuando está sometido a carga axial pura, y  $M_u$  = resistencia del miembro considerado, cuando se somete exclusivamente a flexión pura.

La deducción de este tipo de ecuación requiere el conocimiento de la relación esfuerzo-deformación unitaria (o su aproximación idealizada) del material, la definición de la capacidad en términos de ciertos esfuerzos o deformaciones unitarias límites, y la solución de dos condiciones de equilibrio para una determinada sección transversal; es decir,

$$\int f dA = P \quad \text{y} \quad \int f y dA = M \quad (8-64)$$

Las deducciones de las ecuaciones de interacción se ilustrarán para varios casos simples dentro del rango plástico y también para el caso elástico general.

1. *Caso elástico general.* Consideremos una sección sometida a carga axial  $P$  y a momento flexionante  $M$  respecto a su eje de simetría. Si la relación esfuerzo-deformación-unitaria es lineal,

$$f_m = \frac{P}{A} + \frac{M}{S} \quad (8-65)$$

en donde  $f_m$  es el esfuerzo máximo (límite), y  $A$  y  $S$  son el área de la sección transversal y su módulo de sección, respectivamente. Se puede reescribir la ecuación anterior como sigue:

$$1 = \frac{P}{A f_m} + \frac{M}{S f_m} \quad (8-66)$$

o bien

$$\frac{P}{P_u} + \frac{M}{M_u} = 1$$

donde  $P_u = A f_m$  y  $M_u = S f_m$  son los valores "últimos" o límites de la carga axial y del momento flexionante respectivamente, cuando actúan en forma separada.

2. *Sección rectangular -- Caso plástico ideal.* Considérese una sección rectangular sometida a las cargas  $P$  y  $M$ , como se muestra en la Fig. 8-35.

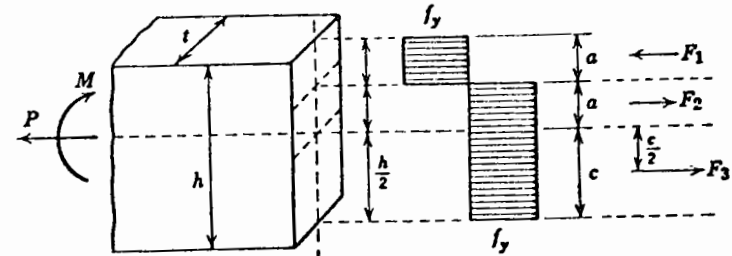


Fig. 8-35

Idealizando la relación-esfuerzo-deformación unitaria como rígida-plástica (idealmente plástica), la distribución de esfuerzos será la mostrada en la figura, con esfuerzo máximo  $f_m = f_y$ . Entonces, de las ecuaciones de equilibrio,

$$\int f dA = P:$$

$$F_1 - F_2 - F_3 + P = 0 \quad (8-67)$$

o puesto que  $F_1 = F_2 = f_y a t$

$$P = F_3 = f_y c t \quad (8-68)$$

$$\int f y dA = M:$$

$$F_1 \cdot a + F_3 \left( \frac{h}{2} - \frac{c}{2} \right) - M = 0 \quad (8-69)$$

o, puesto que  $\frac{1}{2}(h - c) = a$

$$f_y a^2 t + f_y c t a = M = f_y a t (a + c) \quad (8-70)$$

Para flexión pura,  $M_u = f_y (t h^2 / 4)$ , y para carga axial pura,  $P_u = f_y h t$ . Entonces:

$$\frac{P}{P_u} = \left( \frac{f_y \cdot c t}{f_y h t} \right) = \frac{c}{h} = \frac{h - 2a}{h} = 1 - 2 \frac{a}{h} \quad (8-71a)$$

o bien

$$\frac{a}{h} = \frac{1}{2} \left( 1 - \frac{P}{P_u} \right) \quad (8-71b)$$

Más aún,

$$\frac{M}{M_u} = \frac{f_y a t (a + c)}{f_y t h^2 (1)} = \frac{4a(a + c)}{h^2} \quad (8-72a)$$

ó

$$\frac{M}{M_u} = 4 \left( \frac{a}{h} \right) \left( \frac{h-a}{h} \right) = 4 \left( \frac{a}{h} \right) \left( 1 - \frac{a}{h} \right) \quad (8-72b)$$

Sustituyendo el valor de  $\frac{a}{h}$  obtenemos

$$\frac{M}{M_u} = 2 \left( 1 - \frac{P}{P_u} \right) \left( 1 - \frac{1}{2} + \frac{1}{2} \frac{P}{P_u} \right) = 1 - \left( \frac{P}{P_u} \right)^2 \quad (8-73a)$$

$$\frac{M}{M_u} + \left( \frac{P}{P_u} \right)^2 = 1 \quad (8-73b)$$

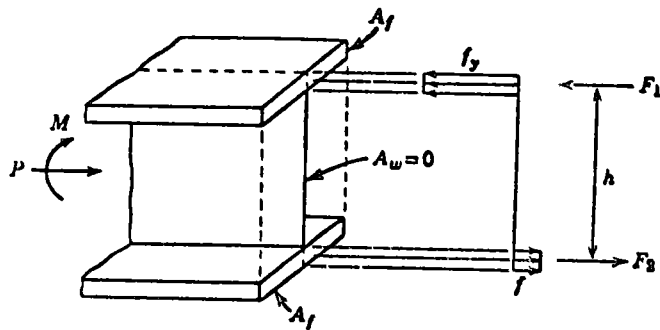


Fig. 8-36

3. *Sección WF ideal — Caso plástico ideal.* Una sección "ideal" tipo WF, aquí considerada, es tal, que el área del alma es despreciable comparada con las áreas de los patines. Esta sección no es necesariamente ideal desde un punto de vista práctico, sino que lo es solamente por el hecho de simplificar la solución del problema considerado aquí, con lo que sirve para ilustrar la influencia de la forma de la sección transversal en la forma de la ecuación de interacción. Considérese dicha sección idealizada, mostrada en la Fig. 8-36, sometida a un momento flexionante  $M$  y una carga axial  $P$ . Idealizando la relación esfuerzo-deformación unitaria como rígido plástico, y notando que las áreas de los patines  $A_f$  son iguales, y que el área del alma  $A_w = 0$ , se puede demostrar que  $f$  debe ser tensión y debe ser menor que  $f_y$ . Entonces, de las condiciones de equilibrio:

$$\int f dA = P:$$

$$F_1 - F_2 - P = 0 \quad (8-74a)$$

o bien

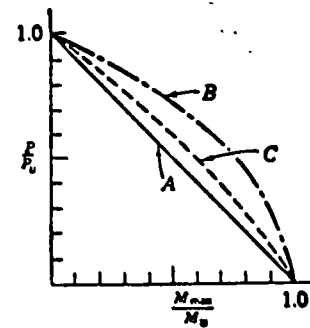
$$P = A_f(f_y - f) \quad (8-74b)$$

$$\int f y dA = M:$$

$$F_1 \cdot \frac{h}{2} + F_2 \cdot \frac{h}{2} - M = 0 \quad (8-75a)$$

o bien

$$M = \frac{h}{2} A_f(f_y + f) \quad (8-75b)$$



- (A) Ecuación lineal  
 $1 = P/P_u + M_{max}/M_u$
- (B) Ecuación plástica ideal  
 $1 = (P/P_u)^2 + M_{max}/M_u$
- (C) Perfil "I" con alma delgada

Fig. 8-37 Curvas de interacción, flexión y carga axial.

Para flexión pura,  $M_u = f_y A_f \cdot h$ , y para carga axial pura,  $P_u = 2A_f f_y$ . Entonces,

$$\frac{P}{P_u} = \frac{A_f(f_y - f)}{2A_f f_y} = \frac{1}{2} \left( 1 - \frac{f}{f_y} \right) \quad (8-76)$$

y

$$\frac{M}{M_u} = \frac{(h/2)A_f(f_y + f)}{hA_f f_y} = \frac{1}{2} \left( 1 + \frac{f}{f_y} \right) \quad (8-77)$$

Sumando

$$\frac{P}{P_u} + \frac{M}{M_u} = \frac{1}{2} \left( 1 - \frac{f}{f_y} + 1 + \frac{f}{f_y} \right) = 1 \quad (8-78a)$$

o bien

$$\frac{M}{M_u} + \frac{P}{P_u} = 1.0 \quad (8-78b)$$

En vista de que las secciones tipo WF reales, en las cuales  $A_w \neq 0$ , están entre la sección rectangular y la sección WF idealizada, la curva de interacción para dicho perfil real estará también entre las que representan a la sección rectangular y a la WF ideal.

Las ecuaciones de interacción correspondientes a los tres casos anteriores se encuentran trazadas en la Fig. 8-37.

15. Para la traba del Prob. 14, determinar el ángulo de torcimiento por pie de longitud.

16. Para las vigas mostradas en la Fig. P-16, y despreciando la resistencia del alma a la flexión, determinar (a) el esfuerzo normal máximo en los patines superior e inferior, (b) el esfuerzo cortante en el alma en el apoyo (sección A) y en el centro del claro (sección B), (c) la carga en el atisador del centro del claro, y (d), la variación del esfuerzo cortante en el alma entre las secciones A y B (trazar el diagrama).

17. Repetir las partes (a), (b) y (c) del Prob. 16, considerando la resistencia del alma a la flexión.

18. Para la viga de "mariposa" que se muestra en la Fig. P-18, y que soporta cortante y momento debidos a fuerzas laterales, determinar los esfuerzos normales máximos en los patines y el alma. Supóngase que la viga está articulada en el centro del claro y empotrada en los apoyos.

19. Determinar los esfuerzos en los patines y el alma de la viga del Prob. 18, cuando ésta soporta una carga concentrada vertical  $P = 80$  Kips en el centro del claro.

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## 9

# Pandeo de miembros prismáticos, marcos y placas

### 9-1 INTRODUCCIÓN

Los miembros esbeltos que trabajan a compresión fallan por inestabilidad (pandeo) cuando la carga axial alcanza un determinado valor crítico. Se dice que un sistema estructural o un miembro aislado es estable cuando vuelve a su estado original después de eliminar una pequeña acción perturbadora (fuerza o desplazamiento). Bajo ciertas condiciones, el sistema no puede alcanzar un estado de equilibrio, y la perturbación causa una deformación de magnitud indeterminada. Tal condición corresponde a un estado crítico del sistema y se conoce como pandeo.

Considérese un miembro esbelto en compresión, articulado en sus extremos y sujeto a una carga axial  $P$  y a otra transversal  $W$ , que actúa a la mitad de su longitud (Fig. 9-1). El comportamiento de esta viga-columna en el rango elástico se describe mediante las siguientes condiciones geométricas y de equilibrio, Ecs. 9-1 y 9-2.

$$M_x = \frac{W}{2} x + Py \quad (9-1)$$

y

$$M_x = -\frac{1}{\rho} EI = -EI \frac{d^2y}{dx^2} \quad (9-2)$$

La ecuación diferencial resultante es

$$EI \frac{d^2y}{dx^2} + Py + \frac{W}{2} x = 0 \quad (9-3)$$



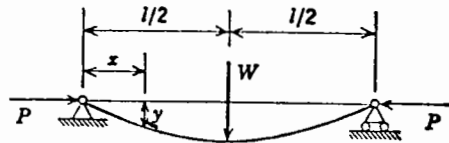


Fig. 9-1

y su solución es:

$$y = \frac{1}{2} \frac{W}{P} x \left[ \frac{\operatorname{sen} \alpha x}{\alpha x \cos(\alpha L/2)} - 1 \right] \quad (9-4)$$

donde  $\alpha = (P/EI)^{1/2}$ ,  $E$  es el módulo elástico del material, e  $I$  es el momento de inercia de la sección transversal.

Resulta evidente, de la ecuación 9-4, que cuando  $\cos(\alpha L/2)$  es cero, la deformación  $y$  es infinita, aun cuando  $W$  sea infinitesimalmente pequeña. Esta condición corresponde a:

$$\frac{\alpha L}{2} = \frac{\pi}{2}, \quad \text{ó} \quad \alpha = \left(\frac{P}{EI}\right)^{1/2} = \frac{\pi}{L}, \quad \text{ó} \quad P = P_{cr} = \frac{\pi^2 EI}{L^2} \quad (9-5)$$

En esta forma se obtiene la carga crítica de pandeo  $P_{cr}$  para un miembro esbelto en compresión. Este sencillo ejemplo puede utilizarse para ilustrar varios conceptos fundamentales respecto al pandeo o inestabilidad.

De la ecuación 9-4 se ve que cuando  $P < P_{cr}$  la deflexión  $y$  desaparece al anularse la carga  $W$ . Solamente cuando  $P = P_{cr}$  la deflexión  $y$  no desaparece aunque  $W$  se haga cero, sino se vuelve indeterminada. Éste es un fenómeno característico de la inestabilidad.

Además, vemos de la ecuación 9-4 que la relación entre  $y$  y  $W$  no es lineal, en virtud de que  $y$  no sólo depende de  $W$ , sino también de  $P$ . Para un valor dado de  $P \neq P_{cr}$ ,  $y$  varía linealmente con  $W$ , pero por ser  $y$  una función trigonométrica de  $P$ , rápidamente tiende a infinito cuando  $P$  tiende a  $P_{cr}$ . Esta no linealidad es otra característica de la inestabilidad, y es contraria al comportamiento, normalmente supuesto lineal, de los sistemas estructurales, en los cuales comúnmente se supone que un pequeño cambio en las condiciones de carga produce un pequeño cambio proporcional en los esfuerzos o desplazamientos.

El fenómeno de inestabilidad resulta del hecho de que el cambio en la geometría de la estructura (deformación) influye en las condiciones de equilibrio (Ec. 9-1). En el análisis convencional de estructuras se desprecia la deformación al considerar las condiciones de equilibrio. Por otra parte, el tomar en cuenta el cambio en la geometría de una estructura no siempre conduce a la inestabilidad. Algunos sistemas son estables en sí, como por ejemplo en el caso de tensión axial, la cual tiende siempre a

enderezar un miembro sujeto a flexión  $y$ , por lo tanto, la tensión nunca conduce a la inestabilidad. Por otra parte, la compresión tiende a incrementar la curvatura de un miembro sujeto a flexión y consiguientemente puede conducir a la inestabilidad.

En los sistemas estructurales pueden presentarse varias formas de pandeo (inestabilidad). Una columna esbelta, tal como la examinada anteriormente, puede fallar por pandeo translacional, o sea por una translación de su sección transversal sin cambio de forma. Algunas formas de sección transversal pueden fallar por pandeo torsional, cuando la sección gira al mismo tiempo que se traslada respecto a su posición original, o por pandeo local, cuando parte de la sección transversal (generalmente una placa delgada) falla por inestabilidad local antes de que la sección completa de la columna se pandee por translación o rotación. Los sistemas reticulados pueden fallar también por inestabilidad total, y a este modo de falla se le llama "inestabilidad general de la estructura". En las siguientes secciones del capítulo se estudian las cargas de pandeo correspondientes a algunos modos de inestabilidad. Un tratamiento más detallado del pandeo de los miembros y de las estructuras de acero se puede encontrar en otras fuentes.<sup>1, 2, 3</sup>

## 9-2 PANDEO PLÁSTICO DE MIEMBROS PRISMÁTICOS CARGADOS AXIALMENTE

La carga crítica de pandeo definida por la ecuación 9-5 es válida para miembros prismáticos rectos, idealizados mediante las siguientes hipótesis:

- El material es linealmente elástico y no se excede en ningún caso el esfuerzo correspondiente a su límite de proporcionalidad.
- El módulo elástico del material es el mismo en tensión que en compresión.
- El material es perfectamente homogéneo e isotrópico.
- El miembro es perfectamente recto inicialmente, y la aplicación de la carga axial es perfectamente concéntrica con el centroide de su sección transversal.
- Los extremos del miembro son articulaciones perfectas sin fricción, soportados en forma tal que su acortamiento no está restringido.
- La sección del miembro no se tuerce y sus elementos no sufren pandeo local.
- El miembro se encuentra totalmente libre de esfuerzos residuales.
- Se puede utilizar la aproximación de deformaciones pequeñas para definir la curvatura del eje deformado de la columna.

Para un miembro ideal como éste la inestabilidad se caracteriza por

una deflexión y igual a cero con cargas  $P$  que aumentan hasta el valor crítico  $P_{cr}$  y por una bifurcación en el punto de carga crítica, ya sea con  $y$  igual a cero o con una  $y$  indeterminada, que satisfaga la solución matemática. En la Fig. 9-2 se muestra esta relación mediante la línea llena  $OAB$ . En realidad, como las condiciones (c) y (d) no pueden satisfacerse por completo ni aun con cuidado y precisión extremos, la deflexión lateral crece en forma progresiva, pero tiene un incremento súbito con cargas cercanas a  $P_{cr}$ , lo cual se muestra por medio de la línea punteada  $OA'B'$  (Fig. 9-2).

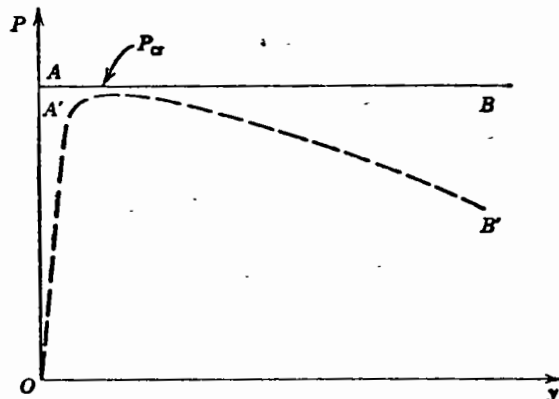


Fig. 9-2

Para una columna ideal, que satisfaga las hipótesis (a) hasta (h), el esfuerzo axial está distribuido uniformemente sobre la sección transversal para todos los valores de carga hasta llegar a la carga crítica, y por lo tanto el esfuerzo crítico  $f_{cr}$  puede definirse como sigue:

$$f_{cr} = \frac{P_{cr}}{A} = \frac{\pi^2 EI}{L^2 A} = \frac{\pi^2 E}{(L/r)^2} \quad (9-6a)$$

donde  $r = \sqrt{I/A}$  es el radio de giro mínimo de la sección transversal.

Resulta conveniente algunas veces expresar esta ecuación en una forma adimensional, dividiendo ambos términos entre el esfuerzo de fluencia  $F_y$  del material, en cuyo caso la ecuación queda:

$$\frac{f_{cr}}{F_y} = \frac{1}{\lambda^2} \quad (9-6b)$$

donde  $\lambda = (L/r)(F_y/\pi^2 E)^{1/2}$ .

La ecuación 9-6a, conocida usualmente como fórmula de Euler, en honor del matemático suizo Euler, quien obtuvo la expresión para la carga crítica  $P_{cr}$  en 1757, define el esfuerzo crítico como una función del módulo elástico  $E$  de la relación de esbeltez  $L/r$ , una característica geométrica

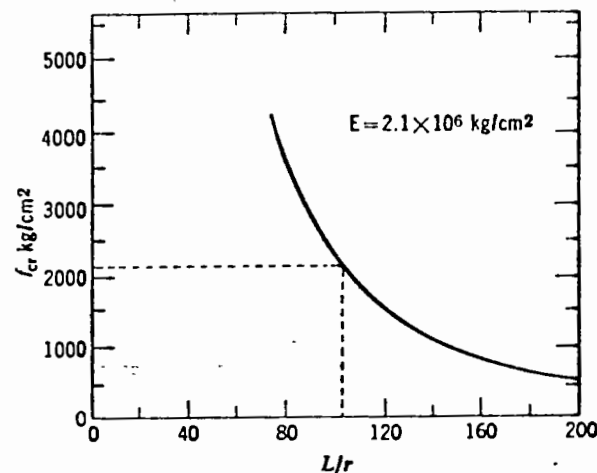


Fig. 9-3

adimensional de una columna ideal. Para el caso de las columnas de acero, la variación de  $f_{cr}$  con  $L/r$ , definida por la ecuación 9-6, se muestra en la figura 9-3.

Los experimentos indican que para columnas de acero esbeltas, con valores de  $L/r$  mayores de 100 ó 120, los resultados de los ensayos se apegan notablemente a los valores obtenidos mediante la ecuación 9-6, mientras que para columnas más cortas, o menos esbeltas, los resultados experimentales se desvían de los valores ideales de pandeo elástico, principalmente porque los esfuerzos locales exceden el límite de proporcionalidad del material.

### 9.3 PANDEO INELÁSTICO DE MIEMBROS PRISMÁTICOS CARGADOS AXIALMENTE

La suposición de un comportamiento linealmente elástico del material de una columna ideal sólo es válida mientras el esfuerzo crítico  $f_{cr}$  no excede el límite de proporcionalidad  $f_p$ . En las columnas reales, el material posee un diagrama esfuerzo deformación con una porción curva por arriba del límite de proporcionalidad, como lo indica la figura 9-4. En un cierto esfuerzo  $f > f_p$ , la pendiente de la curva esfuerzo deformación se define por medio del módulo tangente  $E_t$ , el cual es más pequeño que el módulo inicial  $E$ , basado en la relación lineal  $f-\epsilon$ . Por tanto, la carga crítica  $P_{cr}$  que se basa en la suposición de un comportamiento linealmente elástico del material ya no es válida cuando el esfuerzo crítico excede el límite de proporcionalidad.



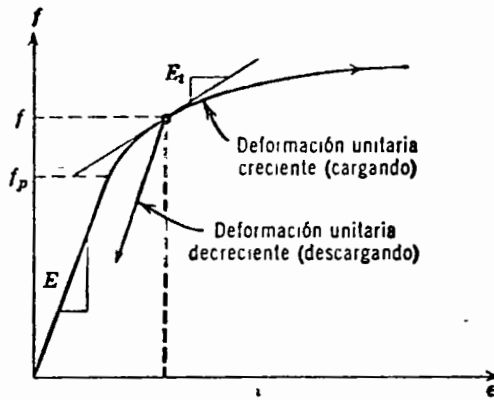


Fig. 9-4

Engesser, en 1889, sugirió que la carga crítica de pandeo de una columna cargada axialmente dentro del rango inelástico se puede definir por medio de la ecuación de Euler siempre y cuando se reemplace el módulo de elasticidad  $E$  por el módulo tangente  $E_t$  que corresponda al esfuerzo crítico; es decir:

$$f_{cr} = \frac{\pi^2 E_t}{(L/r)^2} \quad (9-7)$$

Esta relación se basa en la hipótesis de que la deformación de todas las fibras de la sección transversal está controlada por la ley  $(df/d\epsilon) = E_t$ , o sea que no tiene lugar ninguna descarga de las fibras. Sin embargo, si la columna está ligeramente curvada, cualquier incremento en la curvatura origina un aumento del esfuerzo de compresión en el lado cóncavo y una disminución del mismo en el lado convexo. A la disminución del esfuerzo de compresión en el lado convexo seguirá una relación lineal esfuerzo-deformación, mientras que al aumento del esfuerzo de compresión en el lado cóncavo seguirá la relación no lineal  $(df/d\epsilon) = E_t$ . Este concepto, propuesto por Considere y desarrollado por Von Karman, conduce a la llamada teoría del módulo reducido, donde  $E_t < E_r < E$  y el esfuerzo crítico  $f_{cr}$  se da por

$$f_{cr} = \frac{\pi^2 E_r}{(L/r)^2} \quad (9-8)$$

Shanley ha demostrado que el esfuerzo crítico depende de las condiciones que preceden al pandeo, y que la teoría del módulo tangente fija un límite inferior del valor real del esfuerzo crítico. Por consiguiente, la ecuación 9-7 se acepta ahora generalmente como la solución apropiada para el esfuerzo crítico dentro del rango inelástico.

La ecuación de Engesser (véase la ecuación 9-7) no puede resolverse directamente porque  $E_t$  y  $f_t = f_{cr}$  son interdependientes, y  $f_{cr}$  debe conocerse antes de que se encuentre  $E_t$ . Se pueden hacer, sin embargo, varios tanteos dando valores a  $f_{cr}$  y obteniendo los correspondientes de  $E_t$ , hasta que ambos sean consistentes y correspondan el uno al otro, basándose en la curva esfuerzo-deformación específica para cada caso. Para encontrar la solución resulta conveniente trazar los valores de  $E_t$  y de  $f$  en un sistema coordenado, como se muestra en la figura 9-5.

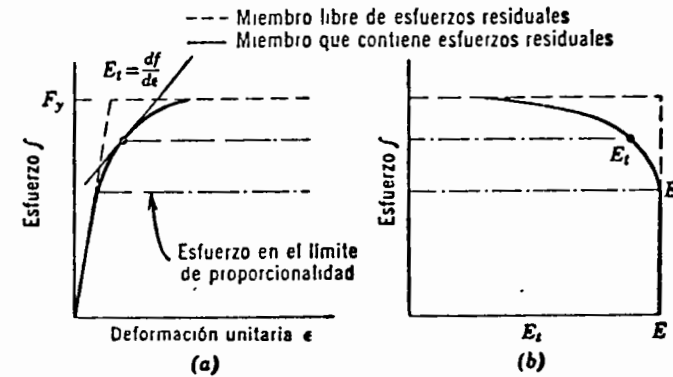


Fig. 9-5 Módulo tangente para el acero dulce.

Substituyendo los valores correspondientes de  $E_t$  y de  $f_t$  en la ecuación 9-7 pueden determinarse los valores de la relación de esbeltez  $L/r$  para cada caso, como

$$\frac{L}{r} = \left( \frac{\pi^2 E_t}{f_t} \right)^{1/4} \quad (9-9)$$

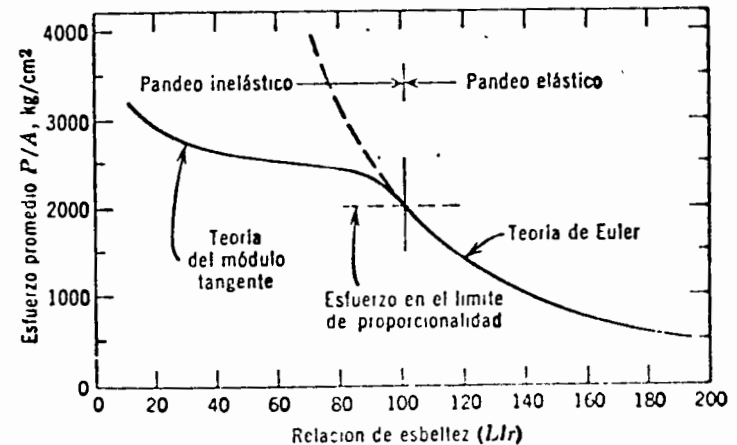


Fig. 9-6 Curva esfuerzo/relación de esbeltez para columnas de acero estructural al carbono.

y la curva del módulo tangente para el esfuerzo crítico en el rango inelástico puede trazarse como una extensión de la curva de Euler para el rango elástico, tal y como se aprecia en la figura 9-6.

#### 9.4 PANDEO DE MIEMBROS PRISMÁTICOS CARGADOS EXCÉNTRICAMENTE

La naturaleza de la carga en un miembro a compresión es siempre un factor determinante en su resistencia. Rara vez sucede que las cargas sean concéntricas con los centroides de las secciones transversales y, además de las excentricidades introducidas por la configuración geométrica de la estructura, existen otras, llamadas excentricidades accidentales, que introducen flexión en el miembro. En esta parte del libro se considera el caso de un miembro cargado excéntricamente, en el cual el plano de la flexión producida por la excentricidad coincide con el plano de pandeo, como se muestra en la Fig. 9-7.

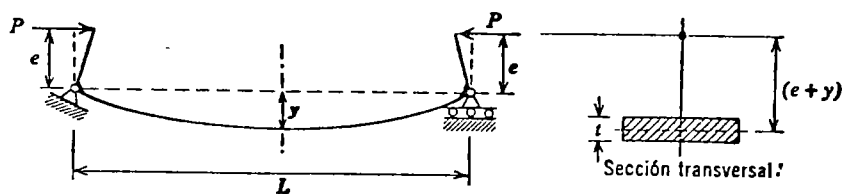


Fig. 9-7. Pandeo bajo carga excéntrica.

No obstante que en el caso de un miembro ideal cargado axialmente el pandeo puede caracterizarse por una deflexión lateral repentina que ocurre cuando la carga alcanza el valor crítico, en el caso de una columna cargada excéntricamente la deflexión aumenta gradualmente conforme aumenta la carga, por lo que debe ampliarse la definición de la carga crítica.

Consideremos un miembro esbelto en compresión, articulado en sus extremos y cargado excéntricamente (Fig. 9-7). Podemos describir el comportamiento de esta viga-columna en el rango elástico como sigue:

$$\text{equilibrio: } M_x = Pe + Py \quad (9-10)$$

$$\text{geometría: } M_x = -EI \frac{d^2y}{dx^2} \quad (9-11)$$

$$\text{por tanto: } EI \frac{d^2y}{dx^2} + Py + Pe = 0 \quad (9-12)$$

$$\text{cuya solución es: } y = e \left[ \frac{\cos(\gamma L/2) - \alpha x}{\cos(\gamma L/2)} - 1 \right] \quad (9-13)$$

donde  $\alpha = (P/EI)^{1/2}$ . La forma de la Ec. 9-13 es semejante a la de la Ec. 9-4, y por tanto puede concluirse que la carga crítica de pandeo, que corresponde a una deformación infinita, puede obtenerse haciendo  $\cos(\alpha L/2) = 0$ , lo que lleva a  $P = P_{cr} = \pi^2 EI/L^2$ , como en la ecuación 9-5.

Esta solución es correcta si el esfuerzo máximo  $f_m$  debido al efecto combinado de la carga  $P$ , la excentricidad  $e$  y la deflexión máxima  $y_m$  (en  $x = L/2$ ), es menor que el esfuerzo del límite de proporcionalidad del material,  $f_p$ . Este esfuerzo  $f_m$  es

$$f_m = \frac{P}{A} + \frac{P(e + y_m)c}{I} \quad (9-14)$$

donde  $c$  es la distancia del eje centroidal de la sección transversal a la fibra extrema de la misma.

Sustituyendo  $y_{m, \max}$  de la ecuación 9-13 en la 9-14, se tiene

$$f_m = \frac{P}{A} \left( 1 + \frac{ec}{r^2} \sec \frac{L}{2r} \sqrt{\frac{P}{AE}} \right) \quad (9-15)$$

Conforme  $P$  se acerca a la carga de Euler  $\pi^2 EI/L^2$ , la contribución del esfuerzo de flexión al valor de  $f_m$  aumenta rápidamente y, por consiguiente,  $f_m$  puede alcanzar el valor del esfuerzo en el límite proporcional con valores relativamente bajos del esfuerzo promedio  $P/A$ . Por tanto, la validez de la ecuación 9-13 para evaluación de la carga crítica (de pandeo) de un miembro cargado excéntricamente es mucho más limitada que para los miembros cargados axialmente.

Con el objeto de evaluar en forma adecuada la carga de pandeo de un miembro cargado excéntricamente, es necesario considerar el comportamiento inelástico a esfuerzos mayores que el límite de proporcionalidad del material. Este problema, formulado por Von Karman, requiere determinar la relación existente entre la carga  $P$  y la deflexión y en el rango inelástico, y depende de la relación esfuerzo-deformación del material, de la esbeltez del miembro y de la geometría de la sección transversal. En general, requiere una solución numérica, en virtud de que no es factible una solución matemática precisa. Estas soluciones numéricas pueden encontrarse descritas en detalle en otras fuentes.<sup>1, 2</sup>

Para que se pueda apreciar lo que significan tales soluciones, consideremos la curva de  $P$  contra la deformación máxima  $y_m$  que se muestra en la figura 9-8.

La figura muestra las relaciones carga-deformación correspondientes a un miembro de longitud, sección transversal y características esfuerzo-deformación conocidas, calculadas suponiendo comportamientos elástico e inelástico.

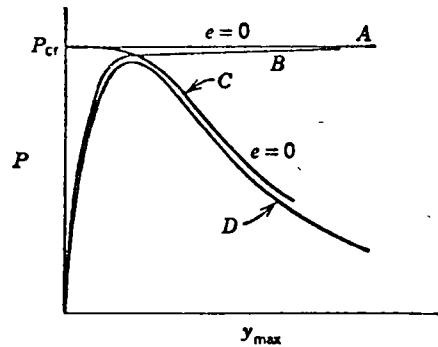


Fig. 9-8. Características de pandeo A, elástico,  $e = 0$ ; B, elástico,  $e > 0$ ; C, inelástico,  $e = 0$ , y D, inelástico,  $e > 0$ .

La curva A corresponde a la respuesta elástica para carga axial y la B a la misma respuesta elástica para carga excéntrica; ambas tienden al mismo valor de la carga crítica de pandeo (de Euler).

La curva C, que corresponde a la respuesta inelástica calculada para carga axial, muestra la naturaleza inestable de esta respuesta, esto es, carga  $P$  decreciente con deflexión y creciente. El punto correspondiente a la carga de pandeo está dado por una carga de Euler modificada, basada en el módulo tangente. La curva D corresponde a la respuesta inelástica calculada para una columna cargada excéntrica. La rama ascendente de esta curva representa una condición de equilibrio estable, mientras que la descendente representa el equilibrio inestable. La carga crítica corresponde a la parte más alta de la curva, entre la respuesta estable y la inestable.

Al definir la carga crítica de columnas cargadas excéntrica se ha sugerido que se tome como valor crítico de la carga el primero que produzca un esfuerzo en la fibra externa igual al esfuerzo de fluencia del material, calculado con base en la teoría lineal elástica. Esa carga puede obtenerse de la Ec. 9-15, haciendo  $f_m$  igual a  $F_v$  y despejando  $P$ .

No es posible obtener una solución inmediata matemáticamente, ya que la carga  $P$  es al mismo tiempo un argumento del término trigonométrico de la ecuación; sin embargo, sí es posible obtener una solución gráfica.

El esfuerzo promedio  $P/A$  correspondiente a este criterio de carga límite se define en la Ec. 9-16 y se muestra en la figura 9-9.

$$\frac{P}{A} = \frac{F_v}{1 + \frac{ec}{r^2} \sec \frac{L}{2r} \sqrt{\frac{P}{AE}}} \quad (9-16)$$

La carga límite  $P$ , definida por la ecuación 9-16, y la carga crítica de pandeo  $P_{cr}$  para una columna cargada excéntrica, obtenida mediante un análisis apropiado basado en un comportamiento inelástico, representan dos criterios de diseño básicamente diferentes. La diferencia entre estos valores varía dentro de un rango muy amplio: puede ser pequeña o grande, dependiendo de la forma de la sección transversal, de la esbeltez de la columna, de la excentricidad relativa de la carga y de las características esfuerzo-deformación del material.

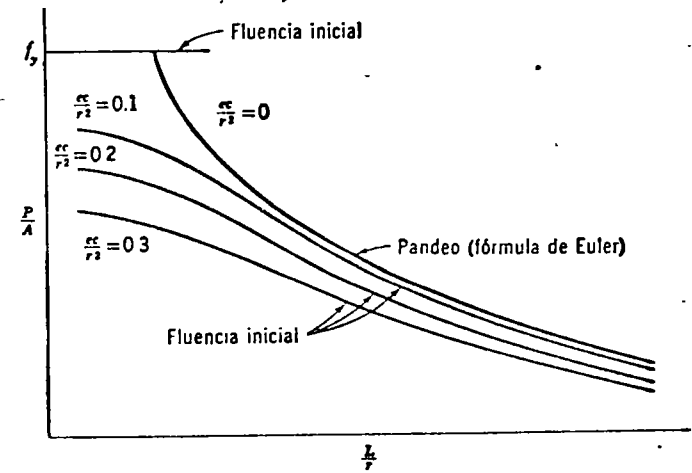


Fig. 9-9 Fluencia bajo compresión excéntrica.

No obstante que la llamada fórmula de la secante (Ec. 9-16) puede ser útil para formular el criterio de diseño para miembros en compresión debe, sin embargo, hacerse notar que las cargas obtenidas con este criterio no están directamente relacionadas con la carga real de pandeo.

### 9-5 INFLUENCIA DE LOS ESFUERZOS RESIDUALES

La etapa final de la manufactura de los perfiles de acero laminados en caliente es la del enfriado de los mismos, a partir de las altas temperaturas requerdas para laminar el lingote y darle la forma deseada, hasta la temperatura ambiente. Como la rapidez de enfriado depende del espesor de la parte en cuestión, en un determinado perfil laminado de espesores no uniformes las porciones delgadas se enfrían primero y aparecen en ellas esfuerzos internos de compresión, mientras que las porciones gruesas del perfil se enfrían al final y desarrollan esfuerzos internos de tensión. A es-

tos esfuerzos internos debidos al enfriado no uniforme de la sección se les conoce como esfuerzos residuales, y varían aproximadamente como se indica en la figura 9-10a. Los esfuerzos residuales se introducen también en los perfiles estructurales de acero por algunas operaciones de fabricación, tales como las de enderezado y las de soldadura.

El efecto neto de los esfuerzos residuales es alterar el diagrama esfuerzo-deformación del perfil estructural, comparado con el de un espécimen ideal del material que lo compone. Si en lugar de probar un espécimen de laboratorio para determinar el diagrama esfuerzo-deformación se utilizara un tramo corto del perfil entero, se obtiene una curva realista, que incluye el efecto de los esfuerzos residuales. Se obtiene así una curva que puede considerarse como una curva esfuerzo-deformación promedio, y los valores del módulo tangente obtenidos con ella indican la presencia de esfuerzos residuales así como también la variación de la resistencia de fluencia a través de la sección.

Este efecto puede ilustrarse por la curva esfuerzo-deformación de la Fig. 9-10b, en la cual se indica con la línea de puntos el comportamiento de un espécimen de prueba ordinario y con la línea continua el de un tramo corto de columna. De esta curva se puede obtener el valor adecuado para el módulo tangente  $E_t$ , para toda la sección transversal. En el caso de que no se pueda disponer de una prueba real de un tramo corto de la columna, se puede suponer una distribución idealizada de los esfuerzos residuales, como se muestra en la Fig. 9-10a.

Un resumen de las investigaciones realizadas sobre los esfuerzos residuales en los perfiles estructurales laminados \* indica que el valor promedio del máximo esfuerzo residual de compresión es aproximadamente  $f_{rc} = 0.3 F_y$ .

La figura 9-11 nos muestra los resultados de una investigación \* en la cual se trazaron curvas de la capacidad de distintas columnas de sección WF, cargadas concéntricamente. Las curvas se trazaron en la forma adimensional ( $f_c/F_y$ ) contra  $\lambda$ , para tres casos. El caso A corresponde al pandeo elástico sin el efecto de los esfuerzos residuales, el B corresponde al pandeo inelástico respecto al eje fuerte (de mayor resistencia), utilizando los esfuerzos residuales mostrados en la determinación del comportamiento inelástico, y el caso C al pandeo inelástico respecto al eje débil (de menor resistencia).

La curva designada D en la Fig. 9-11 representa lo que se ha definido empíricamente como "la resistencia básica de la columna"; es una parábola tangente a la curva de Euler en el punto  $f_{cr} = 1/2 F_y$ , y tiene una tangencia horizontal en el punto  $\lambda = 0$ .

Esta curva básica es conservadora para el caso de flexión respecto al eje fuerte y ligeramente del lado de la inseguridad para la flexión al eje débil.

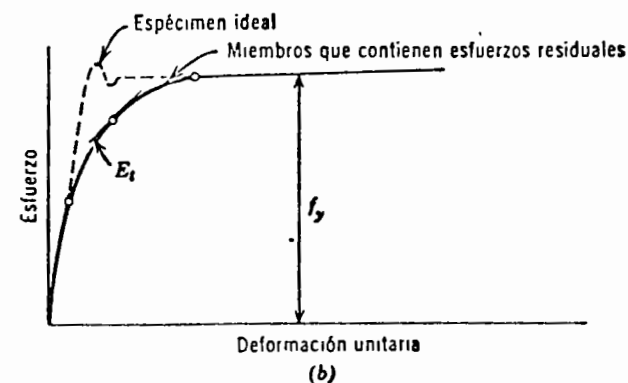
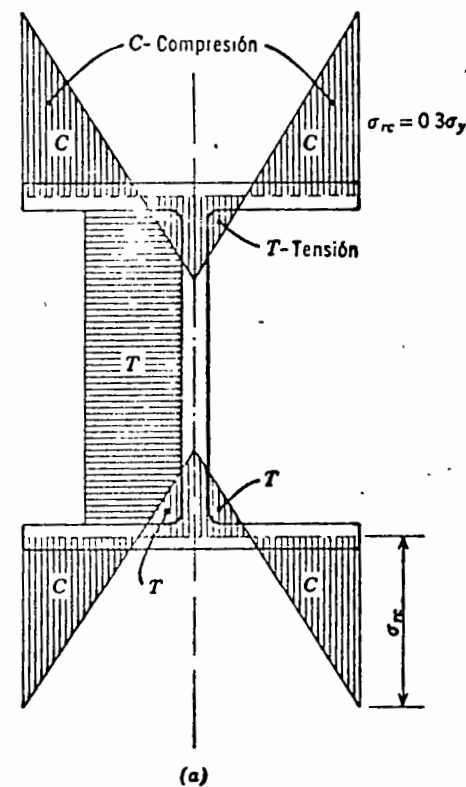


Fig. 9-10 Esfuerzos residuales en perfiles laminados. (a) Distribución de esfuerzos residuales supuesta en un perfil laminado (b) Influencia de los esfuerzos residuales en el diagrama esfuerzo-deformación unitaria.

9.6 PANDEO TORSIONAL

El pandeo primario considerado anteriormente es producido por la flexión sin torcimiento, es decir, las secciones se desplazan de sus posiciones originales exclusivamente por medio de una translación sin rotación. Los miembros de paredes delgadas y secciones transversales de forma abierta son algunas veces débiles a la torsión, contrariamente a los de secciones en cajón o con paredes gruesas, motivo por el cual pueden pandearse por torcimiento más que por flexión. El pandeo torsional ocurre cuando la rigidez torsional del miembro es apreciablemente más pequeña que su rigidez flexionante.<sup>5</sup>

Además de los modos de pandeo por flexión y por torsión, algunas secciones fallan en un modo combinado de pandeo por torsión-flexión. Con-

sidérese la columna de sección T mostrada en la Fig. 9-12, cargada a través del centroide de la sección, la cual se supone que se pandea lateralmente por flexión. A causa de la forma curvada del eje de la columna, la carga vertical deja de ser normal a la sección transversal plana, y tiene una componente normal y una cortante respecto a la sección. Esta componente cortante centroidal no pasa a través del centro de cortante y por consiguiente causa un torcimiento de la columna. Se puede ver así que no es posible el pandeo lateral puro de esta sección T; esto es, la flexión lateral está acompañada de un torcimiento, lo cual da como resultado un modo de pandeo por flexotorsión. La carga crítica  $P_{cr}$  de dicho modo de pandeo es menor que la carga de Euler  $P_{E,cr}$  para el pandeo lateral puro respecto al eje xx. La diferencia entre  $P_{cr}$  y  $P_{E,cr}$  puede ser pequeña para columnas largas con rigidez adecuada, pero para columnas torsionalmente débiles y de longitud intermedia, esta diferencia es apreciable.

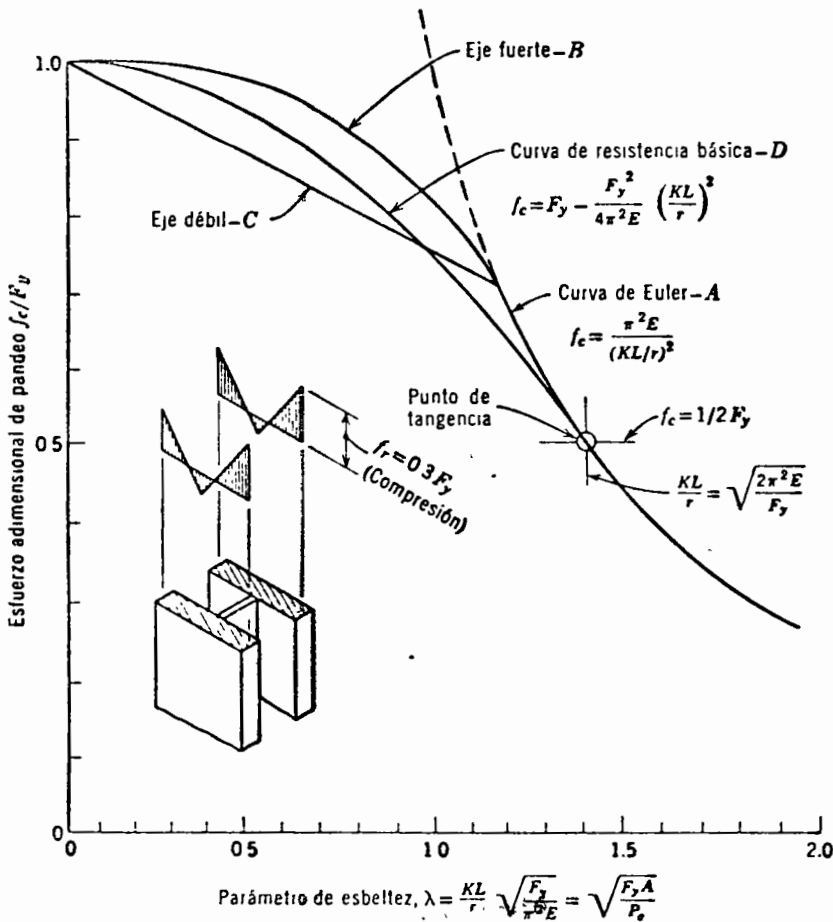


Fig. 9-11 Efecto de los esfuerzos residuales en la resistencia de columnas de sección de alas anchas (WF) (Ref. 4).

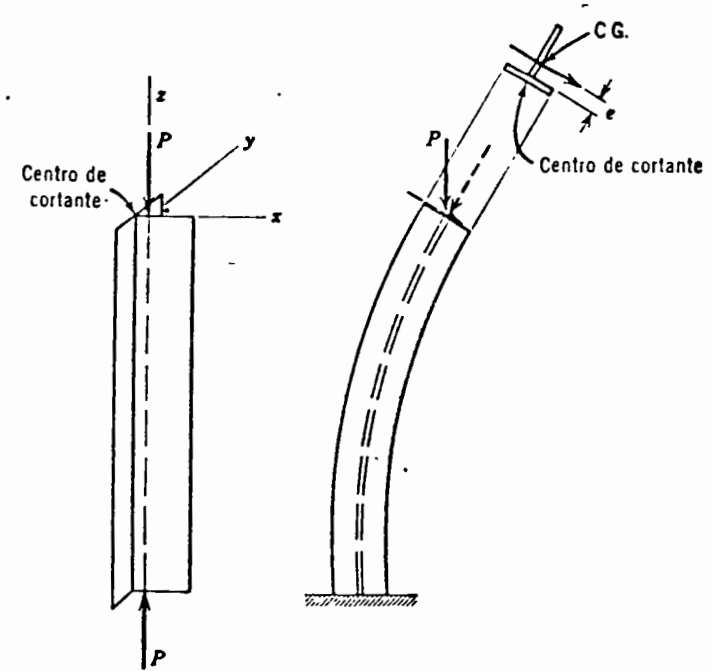


Fig. 9-12 Pandeo por torsion y flexion combinadas.

La magnitud de la carga crítica  $P_{E,cr}$  para pandeo torsional puro puede determinarse considerando la torsión restringida combinada con compresión, tal como la carga de Euler puede determinarse considerando la flexión combinada con compresión. La deducción matemática de  $P_{E,cr}$ , omitida aquí, da como resultado la siguiente expresión.<sup>5</sup>

$$P_{z,cr} = \frac{1}{r_z^2} \left( \frac{\pi^2 EK_b}{L^2} + GK_t \right) = \frac{GK_t}{r_z^2} \left( 1 + \pi^2 \frac{a^2}{L^2} \right) \quad (9-17)$$

donde  $r_z$  = radio de giro polar respecto al eje centroidal  $z$

$E, G$  = módulo de Young y módulo al cortante, respectivamente

$K_b$  = constante de torsión-flexión — propiedad de la sección transversal (ver Sec. 8-7)

$K_t$  = constante de rigidez torsional (ver Sec. 8-7)

$a = (EK_b/GK_t)^{1/2}$  (ver Sec. 8-7)

El modo de pandeo, es decir, por flexión pura, por torsión pura, o por torsión y flexión combinadas, depende de la excentricidad de la carga, de la localización del centro de cortante y de la simetría de la sección transversal. Despreciando los efectos de la fluencia o del pandeo locales, un miembro articulado en sus extremos y sometido a carga excéntrica se pandeará bajo una carga crítica  $P_{cr}$ , ya sea por flexión o por torsión, o bien por una combinación de ambas. La magnitud de esta carga está dada por la menor de las raíces de la Ec. 9-18.

$$(P_{cr} - P_{x,cr})(P_{cr} - P_{y,cr})(\alpha P_{cr} - P_{z,cr}) - P_{cr}^2 [\beta_x(P_{cr} - P_{x,cr}) + \beta_y(P_{cr} - P_{y,cr})] = 0 \quad (9-18)$$

donde  $P_{x,cr}$  y  $P_{y,cr}$  son las cargas críticas de Euler para pandeo respecto a los ejes principales  $x$  y  $y$ , respectivamente;  $P_{z,cr}$  es la carga crítica para pandeo torsional puro (véase Ec. 9-17); y  $\alpha, \beta_x, \beta_y$  son coeficientes que dependen de las propiedades geométricas de la sección transversal, dados por

$$\begin{aligned} \alpha &= 1 - \frac{e_x x_o - e_y y_o}{r_x^2 + x_c^2 + y_c^2} \\ \beta_x &= \frac{(x_c - e_x)^2}{r_x^2 + x_c^2 + y_c^2} \\ \beta_y &= \frac{(y_c - e_y)^2}{r_x^2 + x_c^2 + y_c^2} \end{aligned} \quad (9-19)$$

donde  $e_x$  y  $e_y$  son las excentricidades de la carga con respecto a los ejes principales  $x$  y  $y$ , respectivamente;  $x_c, y_c$ , son las coordenadas del centro de cortante con respecto a los ejes principales  $x$  e  $y$ ; y  $x_o, y_o$  son las coordenadas de un punto característico de la forma de la sección transversal, definido por

$$\begin{aligned} x_o &= 2x_c - \frac{\int_A x(x^2 + y^2) dA}{I_y} \\ y_o &= 2y_c - \frac{\int_A y(x^2 + y^2) dA}{I_x} \end{aligned} \quad (9-20)$$

De la consideración de los siguientes casos especiales puede verse el significado de la excentricidad de la carga, de la localización del centro de cortante y de la simetría de la sección:

**Caso 1. Sección con dos ejes de simetría, centro de cortante coincidiendo con el centroide**

$$x_c = y_c = 0, \quad x_o = y_o = 0, \quad \alpha = 1, \quad \beta_x = \left( \frac{e_x}{r_x} \right)^2, \quad \beta_y = \left( \frac{e_y}{r_y} \right)^2$$

Si la carga  $P$  es concéntrica, es decir, si  $e_x = e_y = 0$ , entonces, de la Ec. 9-18,

$$(P_{cr} - P_{x,cr})(P_{cr} - P_{y,cr})(P_{cr} - P_{z,cr}) = 0 \quad (9-21)$$

y la carga crítica  $P_{cr}$  es el menor de los valores de las tres raíces  $P_{x,cr}, P_{y,cr}, P_{z,cr}$ ; la columna falla por pandeo lateral puro, definido por la carga de Euler, o por torcimiento puro (Ec. 9-17).

**Caso 2. Sección con un eje de simetría, simétrica respecto al eje  $x$**

$$\begin{aligned} y_c = y_o = 0, \quad \alpha &= 1 - \frac{e_x x_o}{r_x^2 + x_c^2}, \quad \beta_x = \frac{(x_c - e_x)^2}{r_x^2 + x_c^2}, \\ \beta_y &= \frac{e_y^2}{r_x^2 + x_c^2} \end{aligned}$$

Si la carga  $P$  pasa a través del centroide,  $e_x = e_y = 0$ ; entonces

$$\alpha = 1, \quad \beta_y = 0, \quad \beta_x = \frac{x_c^2}{r_x^2 + x_c^2}$$

En este caso puede demostrarse que la menor de las raíces de la Ec. 9-18 es, o bien  $P_{cr} = P_{y,cr}$ , o menor que  $P_{x,cr}$  o  $P_{z,cr}$ .

Si la carga pasa a través del centro de cortante,  $e_x = x_c, e_y = y_c$ , y entonces

$$\beta_x = \beta_y = 0, \quad \alpha = 1 - \frac{e_x x_o}{r_x^2 + x_c^2}$$

y la carga crítica  $P_{cr}$  es el menor de los valores de las tres raíces  $P_{x,cr}, P_{y,cr},$  y  $P_{z,cr}/\alpha$ . Debe hacerse notar que la carga  $P_{x,cr}$  ó  $P_{y,cr}$  es una carga crítica de pandeo solamente cuando es menor que  $P_{z,cr}/\alpha$  y el centro de cortante coincide con el centroide o la carga se aplica en el centro de cortante. En el último de estos casos, si  $x$  es eje de simetría, la carga aplicada en el centro de cortante es excéntrica con respecto al  $e$ , y, si  $P_{y,cr}$  es menor que  $P_{x,cr}$ , el pandeo no es del tipo repentino, sino que ocurre con

forme aumentan las deformaciones por flexión, en forma aproximadamente hiperbólica, con el incremento de carga  $P$ .

### 9.7 LONGITUD EFECTIVA DE LOS MIEMBROS EN COMPRESIÓN

En la deducción de las fórmulas de la secante y de Euler se supuso que los extremos del miembro estaban en libertad de girar, tal como lo muestra la Fig. 9-13a. En las estructuras reales esa condición idealizada es la excepción más que la regla, puesto que los extremos por lo general están remachados o soldados a otros miembros y, por lo mismo, restringidos contra la rotación. Aun cuando los miembros están articulados en sus extremos, la libertad de girar existe solamente respecto a uno de los ejes de la sección, y hay fricción entre las articulaciones y el miembro, la cual evita una rotación completamente libre. La importancia de esta restricción al giro varía grandemente en las diferentes estructuras. Si, por ejemplo, el miembro en compresión está ligado rigidamente a elementos relativamente rígidos, la condición de apoyo se acerca a la del empotramiento total (Fig. 9-13b). Si está articulado en uno de sus extremos y rigidamente empotrado en el otro, se deformará como se indica en la Fig. 9-14a. Algunas veces, un miembro en compresión puede tener un extremo completamente libre, es decir, no solamente libre de girar en cualquier sentido, sino también de trasladarse, como se muestra en la Fig. 9-14b. En todos los casos, la resistencia de un miembro en compresión de longitud real  $L$ , con cualquier grado de restricción en sus extremos, puede compararse con la de un miembro biarticulado de longitud  $L_e$ , de modo que el miembro equivalente tenga la misma resistencia que el real.

El significado físico de la longitud equivalente se hace evidente si con-

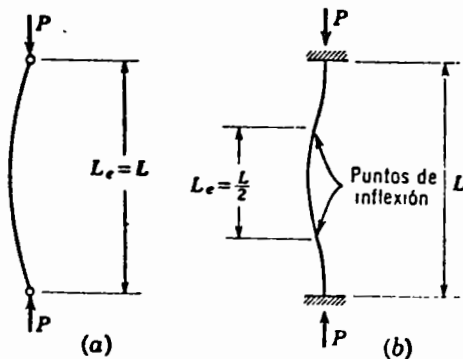


Fig. 9-13 Longitudes efectivas de columnas (a) columna biarticulada y (b) columna doblemente empotrada.

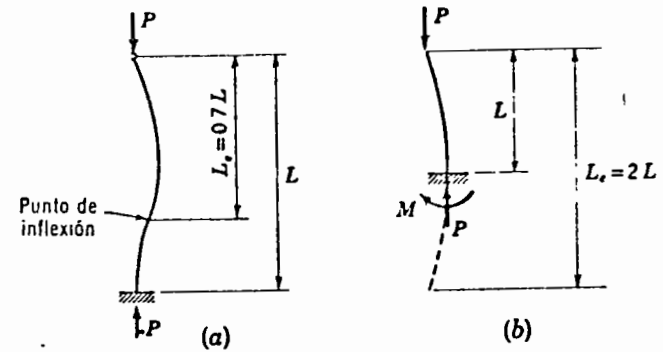


Fig. 9-14 (a) un extremo empotrado y el otro articulado, y (b) un extremo empotrado y el otro libre.

sideramos la forma del miembro pandeado (modo de pandeo). Como ejemplo, consideremos un miembro biarticulado de longitud  $L_e$  y otro de longitud  $L = 1/2L_e$ , el cual está empotrado en un extremo pero libre en el otro (Fig. 9-14b). Resulta obvio que la forma de los dos miembros pandeados es la misma y, por tanto, su carga crítica de pandeo es también la misma. En otras palabras, la longitud "efectiva"  $L_e$  de la columna de la Fig. 9-14b es  $L_e = 2L$  y la carga crítica de pandeo es

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 EI}{4L^2} \quad (9-22)$$

La longitud "efectiva" se llama también en ocasiones longitud "sin soporte" o longitud "sin arriostrar". Esta terminología puede ser confusa en vista de que la longitud efectiva puede ser mayor o menor que la distancia entre apoyos, la cual es la longitud real no soportada. Mediante el uso adecuado de esta longitud "efectiva", la mayor parte de las fórmulas deducidas para las columnas biarticuladas pueden aplicarse a columnas con otras condiciones de extremo.

El concepto de longitud "efectiva" está basado en gran parte en su utilización en la fórmula de Euler, y su uso en otros tipos de fórmulas puede o no ser correcto. Definiendo la longitud efectiva de una columna en términos de su longitud total sin soporte y del coeficiente de fijación  $C$  de sus extremos, la carga crítica de Euler para un miembro con restricciones en sus extremos está dada por la siguiente ecuación:

$$P_{cr} = \frac{C\pi^2 EI}{L^2} = \frac{\pi^2 EI}{L_e^2/C} \quad (9-23)$$

Si la longitud efectiva  $L_e$  se define como  $L_e/\sqrt{C} = KL$ , la Ec. 9-23 se simplifica a:

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} \quad (9-24)$$





## 10-3 COLUMNAS SUJETAS A CARGA AXIAL Y FLEXIÓN

El diseño de un miembro sujeto a compresión axial y a flexión simultáneas (viga-columna) se realiza, por lo general, mediante un procedimiento de aproximaciones sucesivas, en donde se supone una sección y se revisa a continuación su efectividad, mediante el criterio apropiado. Esta revisión de esfuerzos y resistencias es casi siempre un problema sencillo, una vez que se ha establecido un criterio específico, como por ejemplo el del AISC. La selección de una sección óptima, tanto estructural como económicamente, requiere una mayor habilidad. Se presentarán aquí algunos métodos aproximados de diseño, después de que se discuta el desarrollo del criterio de diseño apropiado; por lo común, estos criterios toman la forma de ecuaciones de interacción, las cuales se discutieron en términos generales en la Sec. 8-14.

**Diseño por esfuerzos permisibles.** La forma básica del criterio de diseño es una ecuación lineal de interacción.

$$\frac{P}{P_o} + \frac{M_x}{M_o} \leq 1.0 \quad (10-24)$$

donde  $P_o$  y  $M_o$  son los valores permisibles de la carga axial y del momento, cuando éstos actúan por separado, y  $P$  y  $M_x$  son los valores de diseño de la carga axial y el momento flexionante que actúan simultáneamente. El momento  $M_x$  se toma en una sección crítica y depende tanto del momento flexionante primario, debido a las cargas transversales y los momentos de extremo, como del momento flexionante secundario, debido a la carga axial y a la deformación del miembro.

Para diseño, usualmente es conveniente escribir la Ec. 10-24 en términos de los esfuerzos, como sigue:

$$\frac{f_a}{F_a} + \frac{f_{bx}}{F_b} \leq 1.0 \quad (10-25)$$

donde  $F_a$  es el esfuerzo permisible para carga axial,  $F_b$  es el esfuerzo permisible para flexión, y  $f_a = P/A$ ,  $f_{bx} = M_x/S_x$ .

Se toman en cuenta dos modos de falla: la verificación de la fluencia local en los extremos arriostros de los miembros y la verificación de la fluencia general o inestabilidad.

Para tomar en cuenta la limitación por fluencia en una sección arriostros, donde no existe la contribución de la flexión secundaria y donde la capacidad de carga axial está determinada por la resistencia de fluencia  $F_y$  del material, en las Especificaciones AISC se usan los siguientes esfuerzos permisibles:

$$F_a = 0.6F_y \quad \text{y} \quad F_b = 0.65F_y \quad \text{ó} \quad 0.60F_y.$$

El esfuerzo de flexión  $f_{bx}$  se toma, en los extremos arriostros, como  $f_{bx} = f_b = M_o/S$ , donde  $M_o$  es el momento flexionante primario en dicho extremo. El criterio de diseño se convierte entonces en

$$\frac{f_a}{0.6F_y} + \frac{f_b}{F_b} \leq 1.0 \quad (10-26)$$

Para tomar en cuenta la fluencia general o inestabilidad entre extremos arriostros, el esfuerzo permisible  $F_a$  para carga axial debe tomarse con la debida consideración de la relación de esbeltez ( $KL/r$ ) y de la resistencia de fluencia  $F_y$ , según se describió en la Sec. 10-2. El esfuerzo permisible para flexión  $F_b$  se toma como una fracción adecuada (0.65 o 0.60) de la resistencia de fluencia. La definición del esfuerzo de flexión  $f_{bx} = M_x/S_x$  es algo más compleja porque depende de la evaluación del momento  $M_x$  en la sección crítica; teóricamente, el valor de  $M_x$  puede determinarse mediante el análisis adecuado de "viga-columna" discutido en la Sec. 8-13. Para el desarrollo de un criterio de diseño se han deducido fórmulas para los casos más comunes.

Al desarrollar estas fórmulas aproximadas se representa al esfuerzo  $f_{bx}$  como un múltiplo o una fracción del esfuerzo de compresión por flexión primaria en la sección considerada; por ejemplo:

$$f_{bx} = \alpha f_b = \alpha \frac{M_{ox}}{S} \quad (10-27)$$

donde  $M_{ox}$  es el momento flexionante primario en la sección. En ausencia de cargas transversales entre los puntos de apoyo,  $M_{ox}$  se toma como el mayor de los momentos en dichos apoyos. Cuando existen cargas transversales intermedias, se usa como  $M_{ox}$  el momento máximo entre los puntos de apoyo.

El factor de amplificación  $\alpha$  debe tomar en cuenta el incremento no lineal en el momento flexionante total y en los esfuerzos correspondientes cuando se incrementan proporcionalmente todas las cargas, axiales y de flexión transversal, por medio de un factor de carga  $n$ . Si la carga axial de servicio es  $P$ , el incremento crítico en el momento primario ocurrirá cuando la carga axial sea  $nP$ .

Consideremos una viga-columna bi-articulada, con excentricidades iguales  $e$  en sus extremos y una carga de servicio  $P$ , de modo que el momento primario  $M_o = Pe$ . La condición crítica de diseño estará representada por  $nM_x$  cuando se incrementa la carga axial a  $nP$ ; este momento ocurre en el centro de la longitud y puede calcularse aproximadamente, de una manera similar a la que se utilizó para calcular las deflexiones aproximadas, Ec. 8-61:

$$nM_x = \pi nM_o = nM_o \frac{1}{(1 - nP/P_{cr})} \quad (10-28)$$

donde  $P_{cr}$  es la carga crítica de Euler. El factor  $(nP/P_{cr})$  es

$$\left(\frac{nP}{P_{cr}}\right) = \left(\frac{nP/A}{P_{cr}/A}\right) = \left(\frac{f_a}{F'_e}\right) \quad (10-29)$$

donde  $f_a = P/A$  y  $F'_e = (P_{cr}/n \cdot A)$ , y está dado por la Ec. 10.4. El momento de diseño  $M_x$  y los esfuerzos de flexión correspondientes  $F_{bx} = M_x/S_x$  son

$$M_x = \alpha M_o = M_o \frac{1}{(1 - f_a/F'_e)} \quad (10-30)$$

y

$$f_{bx} = \frac{M_x}{S_x} = \frac{M_o}{(1 - f_a/F'_e)S_x} = f_b \frac{1}{(1 - f_a/F'_e)} \quad (10-31)$$

En este caso particular, cuando los momentos en los extremos son iguales y no existe carga transversal sobre la viga-columna entre los extremos, el factor de amplificación es simplemente

$$\alpha = \frac{1}{(1 - f_a/F'_e)} \quad (10-32)$$

Cuando los momentos varían a lo largo del miembro, el factor de amplificación puede escribirse como

$$\alpha = \frac{C_m}{(1 - f_a/F'_e)} \quad (10-33)$$

donde el coeficiente  $C_m$  varía según la variación de momentos flexionantes a lo largo del miembro; entonces, si la columna está sujeta a momentos  $M_1$  y  $M_2$  en ambos extremos, de modo que  $M_x = \beta M_1$ , y  $1 > \beta > -1$ , y se evita la translación de los extremos, puede diseñarse para un momento primario equivalente  $C_m M_1$ , el cual es constante a lo largo de la columna. Se han propuesto diferentes expresiones empíricas para  $C_m$ . Una de ellas es

$$C_m = \sqrt{0.3 + 0.4\beta + 0.3\beta^2} \quad (10-34)$$

Para simplificar esta expresión, el AISC usa

$$C_m = 0.6 + 0.4\beta \geq 0.4 \quad (10-35)$$

En la Fig. 10-2 se muestran ambas expresiones. Cuando no se evita la translación lateral de los extremos, se hace posible un modo diferente de pandeo, por esta razón, el AISC pone un límite más conservador al valor de  $C_m$  (Fig. 10-2),

$$C_m = 0.6 + 0.4\beta \geq 0.85 \quad (10-36)$$

Los factores de amplificación anteriores corresponden a vigas-columnas sin cargas transversales aplicadas entre sus extremos. A continuación

se mostrará un caso que se presenta en armaduras, cuando el cordón de compresión está sujeto a cargas transversales entre los nudos; se supone que no existe translación lateral en los extremos. En la Fig. 10-3 se muestran las cargas de servicio, la configuración de la cuerda deformada y los diagramas de momentos flexionantes primario y total. Sean  $n$  el factor de carga correspondiente a la condición crítica y  $nM_x$  el momento máximo que ocurrirá en una sección correspondiente al momento máximo primario  $M_x$ . Entonces, cuando se incrementan las cargas proporcionalmente según el factor  $n$ :

$$nM_x = nM_x + nP \cdot ny \quad (10-37)$$

donde  $ny = ny_o \left(\frac{1}{1 - nP/P_{cr}}\right)$  de la Ec. 8-61, donde  $P_{cr} = [\pi^2 EI/(KL)^2]$

Entonces

$$M_x = \alpha M_x = \left[1 + \frac{nPy_o}{M_x(1 - nP/P_{cr})}\right] M_x = \frac{\left(1 - \frac{nP}{P_{cr}} + \frac{nPy_o}{M_x}\right)}{(1 - nP/P_{cr})} \cdot M_x \quad (10-38)$$

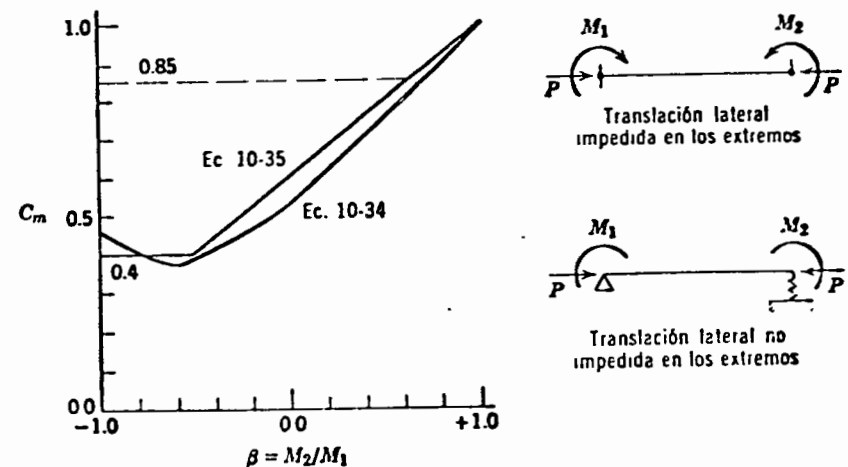


Fig. 10-2 Influencia del gradiente de momento sobre  $C_m$ .

El factor de amplificación  $\alpha$  puede expresarse en términos de  $C_m$ ,  $f_a$  y  $F'_e$ , como antes:

$$\alpha = \left[1 + \frac{nPy_o}{M_x(1 - nP/P_{cr})}\right] = \frac{\left[1 - \frac{nP}{P_{cr}} \left(1 - \frac{P_{cr}y_o}{M_x}\right)\right]}{(1 - nP/P_{cr})} = \frac{C_m}{(1 - f_a/F'_e)} \quad (10-39)$$

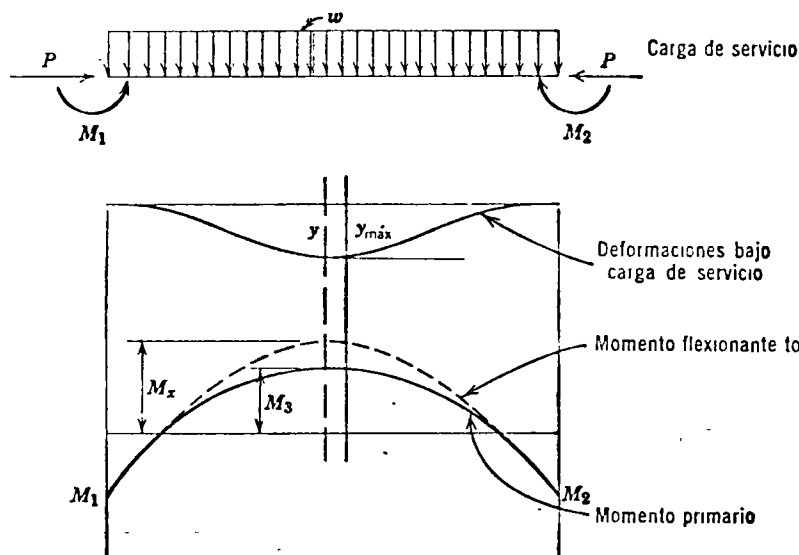


Fig. 10-3 Diagramas de momentos primarios y totales y configuración del miembro deformado.

donde

$$C_m = \left[ 1 - \frac{f_a}{F_c} \left( 1 - \frac{P_{cr} y_o}{M_3} \right) \right] \quad (10-40)$$

Puede resumirse la determinación de las cargas permisibles AISC para miembros sujetos a compresión y flexión combinadas, como sigue:

(a) Los esfuerzos debidos a carga axial  $P$  y momento flexionante  $M$  en una sección considerada se calculan de manera convencional, de modo que  $f_a = P/A$  y  $f_b = M/S$ . Estos esfuerzos deben satisfacer una fórmula de interacción apropiada.

(b) En los puntos arriostrados en el plano de flexión, la fórmula de interacción está dada por la Ec. 10-26.

(c) En general, los esfuerzos deben satisfacer una ecuación de interacción de la forma:

$$\frac{f_a}{F_a} + \alpha \frac{f_b}{F_b} \leq 1.0 \quad (10-41)$$

En esta ecuación  $F_a$  se determina como el esfuerzo permisible en la columna si sólo existiera la fuerza axial  $P$ . De manera similar,  $F_b$  es el esfuerzo de compresión debido a la flexión permisible si existiera únicamente el momento flexionante  $M$ . A continuación, se presenta una discusión de la determinación del factor de amplificación  $\alpha$ .

(d) La forma general de  $\alpha$  está dada por la Ec. 10-33; sin embargo, cuando  $f_a/F_a \leq 0.15$ , el factor de amplificación  $\alpha$  debe tomarse como la

unidad. Cuando  $f_a/F_a > 0.15$ , el factor de amplificación depende de la magnitud del esfuerzo axial  $f_a$ , de la relación de esbeltez efectiva ( $KL/r$ ), de la variación del momento flexionante a lo largo del miembro y de la presencia o ausencia de translación lateral. Para condiciones especiales, los valores del coeficiente de momento equivalente  $C_m$  y los valores correspondientes del factor de amplificación  $\alpha$  pueden obtenerse de las Ecs. 10-33, 10-35 y 10-40.

Es posible hacer una estimación preliminar de los requisitos de la sección, basados en el criterio de esfuerzos permisibles del AISC, mediante el cálculo de una carga equivalente de compresión axial  $P_{eq}$ , correspondiente a la combinación de compresión y flexión. Entonces, el área requerida de la sección transversal se estima a partir de la Ec. 10-1, donde  $P$  se convierte en  $P_{eq}$  y para el diseño preliminar se supone el esfuerzo permisible  $F_a$ . La carga equivalente se puede definir a partir de la ecuación de interacción, como sigue:

$$\frac{f_a}{F_a} + \alpha \frac{f_b}{F_b} \leq 1.0 \quad (10-42)$$

ó

$$\frac{Af_a}{AF_a} + \frac{\alpha f_b S}{F_b S} \leq 1.0 \quad (10-43)$$

ó

$$Af_a + \frac{\alpha f_b \cdot S}{F_b} \times \frac{AF_a}{S} \leq AF_a \quad (10-44)$$

ó

$$P + \frac{F_a}{F_b} \cdot B \cdot \alpha \cdot M \leq AF_a \quad (10-45)$$

donde  $B = A/S$  es una característica geométrica de una sección dada, llamada "factor de flexión", definida previamente en la Sec. 7-4. También

$$\frac{P + \alpha(F_a/F_b) \cdot B \cdot M}{F_a} = A = \frac{P_{eq}}{F_a} \quad (10-46)$$

ó

$$P_{eq} = P + \alpha \frac{F_a}{F_b} \cdot B \cdot M \quad (10-47)$$

Cuando existen momentos flexionantes con respecto a ambos ejes  $x$  e  $y$  de una columna, la carga equivalente se convierte en

$$P_{eq} = P + \alpha_x \frac{F_a}{F_{b,x}} B_x M_x + \alpha_y \frac{F_a}{F_{b,y}} B_y M_y \quad (10-48)$$

Entonces, suponiendo un valor de  $F_a$ , la sección requerida estimada se convierte en:

$$A = \frac{P_{eq}}{F_a} \quad (10-49)$$

Puede seleccionarse la sección económica más próxima, y verificarla posteriormente bajo las cargas  $P$ ,  $M_x$  y  $M_y$  combinadas. Los valores de  $B_x = A/S_x$  y de  $B_y = A/S_y$  varían ampliamente para diferentes secciones; están tabulados en el Manual AISC, págs. 3-13 a 3-56, para secciones típicas de columnas, y en la Fig. 7-10 se muestra una gráfica de valores aproximados.

Para perfiles tipo  $W^F$ , los valores de  $B_x$  varían de 0.18 a 0.25, y los de  $B_y$ , de 0.443 a 2.25, correspondiendo los valores mayores a los perfiles de menor tamaño. Como estos valores deben estimarse para un diseño preliminar, en caso de que el diseñador no pueda hacer una estimación juiciosa basada en su experiencia, puede usarse para  $B_x$  un valor de 0.25 y para  $B_y$  uno de 0.75.

Los valores de  $F_{ax}/F_{bx}$  y de  $F_{ay}/F_{by}$  dependen de las relaciones de esbeltez, que determinan  $F_{ax}$  y  $F_{ay}$ ; para Acero A36 puede utilizarse 0.75 como primera aproximación. Los factores de amplificación  $\alpha_x$  y  $\alpha_y$  dependen de los momentos en los extremos, del esfuerzo axial  $f_a$  y de las relaciones de esbeltez, que determinan  $F_{cx}$  y  $F_{cy}$ ; como una primera aproximación, pueden tomarse los valores de  $\alpha_x$  y  $\alpha_y$  como la unidad. Sustituyendo estos valores aproximados, la Ec. 10-48 se convierte en:

$$P_{eq} = P + (1.0)(0.75)(0.25)M_x + (1.0)(0.75)(0.75)M_y \\ = P + 0.2M_x + 0.6M_y \quad (10-50)$$

La expresión anterior sólo es una burda primera aproximación para perfiles tipo  $W^F$ , que puede usarse únicamente cuando el diseñador está incapacitado para realizar una elección más juiciosa.

**Ejemplo 10-2.** Seleccionar una sección tipo  $W^F$  de acero A36 para soportar las cargas mostradas (Fig. 1). La columna, con una longitud de 12 pies, forma parte de un marco continuo de un edificio de varios pisos, arriostrado contra movimientos laterales en el plano  $x$ - $z$ , pero no en el plano  $y$ - $z$ . Un análisis aproximado indicó que  $(KL)_x = 1.25 \times 12 = 15$  pies y que  $(KL)_y = 12$  pies.

**Solución**

Usando la Ec. 10-50:

$$P_{eq} = P + 0.2 M_x = 400 + 0.2 \times 2000 = 800 \text{ kips}$$

Supongamos  $F_a = 19$  kips/plg<sup>2</sup>

$$A \text{ estimada} = \frac{P_{eq}}{F_a} = \frac{800}{19} = 42 \text{ plg}^2$$

Probemos una sección 12  $W^F$  133 (ver AISC, págs. 1-15)

$$A = 39.11 \text{ plg}^2 \quad r_x = 5.59 \text{ plg} \quad r_y = 3.16 \text{ plg}$$

$$S_x = 182.5 \text{ plg}^3$$

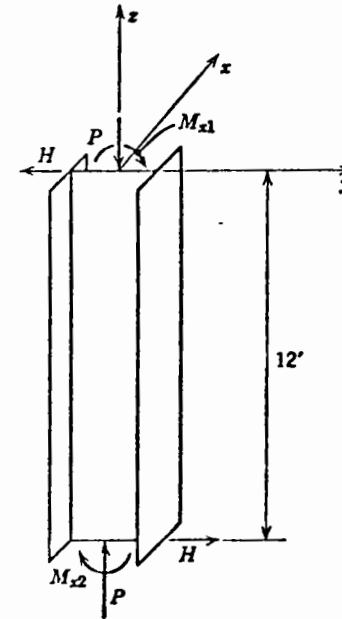


Fig. 1  $P = 400$  kips,  $M_{x1} = 2,000$  kips-plg,  $M_{x2} = 1,800$  kips-plg.

Para una sección compacta, arriostrada adecuadamente,  $F_b = 24$  kips-plg<sup>2</sup>.

Verificaremos la suficiencia de una sección 12  $W^F$  133, usando la Ec. 10-42:

$$f_a = \frac{P}{A} = \frac{400}{39.11} = 10.2 \text{ kip/plg}^2$$

$$f_b = \frac{M}{S} = \frac{2000}{182.5} = 11.0 \text{ kip/plg}^2$$

$$\left(\frac{KL}{r}\right)_x = \left(15 \times \frac{12}{5.59}\right) = 32.2$$

$$\left(\frac{KL}{r}\right)_y = \left(12 \times \frac{12}{3.16}\right) = 45.5$$

$$F_a \left[ \text{para } \left(\frac{KL}{r}\right)_y = 45.5, \text{ ver Tabla 10-1} \right] \quad F_a = 18.7 \text{ kip/plg}^2$$

$$F_e' \left[ \text{para } \left( \frac{KL}{r} \right)_e = 32.2, \text{ ver Tabla 10-1} \right] \quad F_e' = 144 \text{ kip/plg}^2$$

$$C_m (\text{ver Eq. 10-36}, \beta = -\frac{1800}{2000} = -0.9) \quad C_m = 0.85$$

$$\alpha = (C_m)/(1 - f_a/F_e') = 0.85/(1 - 10.2/144) = 0.916$$

Entonces, en la Ec. 10-42:

$$\frac{f_a}{F_a} + \alpha \frac{f_b}{F_b} = \frac{10.2}{18.77} + 0.916 \frac{11.0}{24.0} = 0.544 + 0.42 = 0.964 < 1.0$$

Usar la sección 12 WF 133; es adecuada.

**Diseño plástico.** El diseño plástico de columnas en marcos rígidos puede ser efectivo cuando se evita el desplazamiento lateral mediante un contraventeo diagonal, o bien por medio de la sujeción a un sistema estructural que tenga bastante rigidez lateral, suministrada por muros de cortante o elementos similares. Cuando se permite el desplazamiento lateral, la formación de articulaciones plásticas conduce a una reducción substancial en la estabilidad lateral total. Aún no se ha definido adecuadamente la cantidad de contraventeo lateral requerido para evitar la inestabilidad de los marcos en estructuras altas de edificios con articulaciones plásticas; por esta razón, las Especificaciones AISC de 1963 limitan la aplicación del diseño plástico a vigas continuas o a marcos de uno o dos pisos de altura.

Las columnas de un marco arriostrado pueden diseñarse para satisfacer una ecuación de interacción de la forma general

$$\frac{M}{M_p} = H + K \left( \frac{P}{P_p} \right) + J \left( \frac{P}{P_p} \right)^2 \quad (10-51a)$$

En esta ecuación, los valores de  $H$ ,  $K$  y  $J$  dependen de la configuración de la columna deformada y de las condiciones de carga, así como de su relación de esbeltez. Los valores de estos coeficientes se encuentran tabulados en el Apéndice de las Especificaciones AISC, págs. 5-64 a 5-75; los valores de  $M$  y  $P$  en la Ec. 10-51a son los que se obtienen del análisis plástico de un marco sujeto a carga última, la cual se toma como la carga de servicio multiplicada por un factor de carga de 1.7, u otro valor apropiado. Los valores de  $M_p$  y  $P_p$  son las capacidades plásticas y de carga axial de fluencia de la sección, respectivamente.

Para columnas en marcos rígidos de uno o dos pisos que no estén arriostrados lateralmente, la carga de compresión debe ser tal que

$$\frac{P}{P_p} \leq 0.5 \left( 1 - \frac{L}{70r} \right) \quad (10-51b)$$

Se estima que dentro de estos límites no se presentará la inestabilidad lateral del marco.

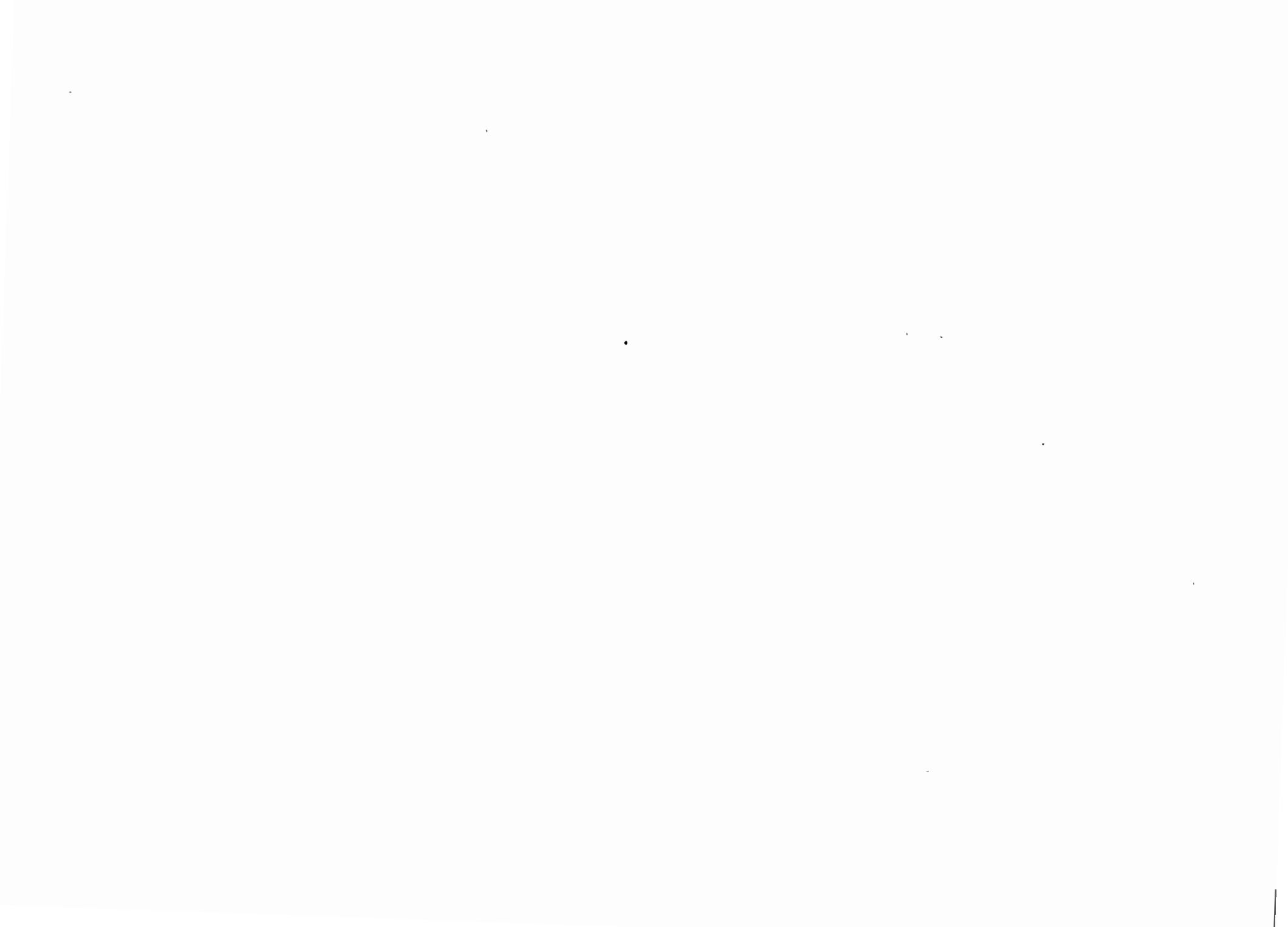
#### 10-4 REQUISITOS DE DISEÑO

En el diseño de un miembro en compresión se conocen por lo general las cargas máximas y la longitud efectiva del miembro, mientras que están por determinarse la forma y las dimensiones de la sección transversal, así como las conexiones. El área de la sección transversal requerida  $A$  está determinada por la carga  $P$  o bien por la carga  $P_{cr}$ , cuando la compresión está combinada con flexión, y el esfuerzo permisible promedio  $F$  de relación  $A = P/F$ ; el esfuerzo permisible  $F$  depende de la relación de esbeltez  $L/r$  y de las condiciones de restricción de los apoyos, cantidades que no se conocen con precisión hasta después de determinar la forma y el tamaño del miembro. Por consiguiente, la solución de este problema únicamente puede realizarse por medio de aproximaciones sucesivas.

**Relación de esbeltez.** Con objeto de reducir el peso del miembro, éste debe desarrollar un esfuerzo permisible promedio elevado. Como el esfuerzo permisible disminuye para valores altos de  $L/r$ , es deseable tener un valor mínimo de  $L/r$  para el área de la sección transversal considerada, siempre y cuando esto no esté en conflicto con otras consideraciones económicas y de estabilidad. Con el valor de  $L$  predeterminado, el problema se reduce a la selección de una sección práctica con el mayor valor posible de  $r$ .

Por lo común, si las cargas de compresión son suficientemente altas, puede seleccionarse un perfil económico en un rango de valores de  $L/r$  no mayores de 80. Si las cargas de compresión son ligeras, las áreas calculadas para una resistencia adecuada son pequeñas y pueden justificarse miembros esbeltos, en el rango de  $L/r$  de 80 a 120. Al usar miembros esbeltos, deben considerarse los efectos de las cargas accidentales y de las cargas de construcción, generalmente despreciados en el análisis. Los trabajadores pueden usar frecuentemente los miembros de arriostramiento como andadores, durante y después de la construcción, pueden ser utilizados, también, como soporte para algún equipo temporal o auxiliar imprevisto; y existe también la posibilidad de tener vibraciones en ellos. Por estas razones, la mayoría de las especificaciones estipulan que la relación de esbeltez  $L/r$  no debe exceder un valor máximo especificado, que se toma por lo general como 200.

**Cargas de construcción.** Las cargas aplicadas durante la construcción son generalmente más pequeñas que las cargas completas de diseño, pero



### 3.- Miembros a Flexocompresión

Por separado se están proporcionando al estudiante copias de parte de los capítulos 8, 9 y 10 del libro "Diseño de Estructuras de Acero", de Bresler, Lin y Scalzi (Editorial Limusa-Wiley), que le permitirán recordar el problema básico de los miembros sometidos a flexocompresión. Se sugiere su lectura para familiarizarlo con el resumen que a continuación se ofrece, y con los problemas que resolveremos después de éste.

La presencia de esfuerzos residuales, la curvatura inicial del eje del miembro y las excentricidades de las cargas aplicadas respecto al eje del mismo, hacen que la resistencia de un miembro cargado axialmente sea menor que la que teóricamente pudiera esperarse de un miembro perfectamente recto, sin esfuerzos residuales y cargado precisamente en forma axial.

Una columna cargada axialmente pero que no es perfectamente recta, o bien que está cargada con ligeras excentricidades se deflexionará lateralmente conforme se incrementa la carga. Timoshenko encontró que si la elástica de la columna deflexionada inicialmente se aproxima a la forma de una media onda de la curva senoide, cuya ordenada inicial al centro del claro (falta de rectitud) es  $Y_0$ , la ordenada final en el mismo centro del claro, después de aplicada la carga axial, se puede valorar con la ecuación

$$Y = Y_0 \left( \frac{1}{1 - \frac{P}{P_e}} \right) \quad \text{----- (8)}$$

expresión semejante a la ecuación (4) (Hoja No. 16) vista anteriormente; el significado de las literales es el mismo que en la ec. (4).

El término entre parentesis tiende a aumentar la deflexión inicial, motivo por el cual se le denomina "factor de magnificación" ó "factor de amplificación".

La carga crítica de Euler  $P_e$  no está limitada en este caso al rango elástico, sino que es un valor "teórico", aunque el esfuerzo crítico de pandeo  $\frac{\pi^2 EI}{L^2}$  exceda  $F_y$ .

Si la carga que actúa en los extremos del miembro inicialmente recto ( $Y_0 = 0$ ) se aplica con una excentricidad  $e$  del centroide de la sección, entonces, la deflexión total del miembro en su centro del claro, después de aplicar la carga  $P$  es:

$$Y = e \sec \frac{\pi}{2} \sqrt{\frac{P}{P_e}} \quad \text{----- (9)}$$

Del examen cuidadoso de esta expresión, se puede observar que conforme  $P$  se aproxima a  $P_e$ ,  $Y$  crece indefinidamente, tendiendo a infinito.

Los esfuerzos máximos en el miembro flexocomprimido  $f_m$ , que es la suma de los esfuerzos debidos a la carga axial más los esfuerzos producidos por el momento flexionante son:

$$f_m = \frac{P}{A} + \frac{PY}{I} c = \frac{P}{A} \left[ 1 + \frac{Y_0 c}{r^2} \left( \frac{1}{1 - \frac{P}{P_e}} \right) \right] \quad \text{----- (10)}$$

y para un miembro inicialmente recto pero con carga excéntrica:

$$f_m = \frac{P}{A} \left[ 1 + \frac{ec}{r^2} \sec \frac{\pi}{2} \sqrt{\frac{P}{P_e}} \right] \quad \text{----- (11)}$$

La carga que origina la cedencia inicial en las fibras exteriores de un determinado miembro dado, puede obtenerse de las ecuaciones (10) y (11) anteriores, substituyendo el valor de  $F_y$  por  $f_m$ .

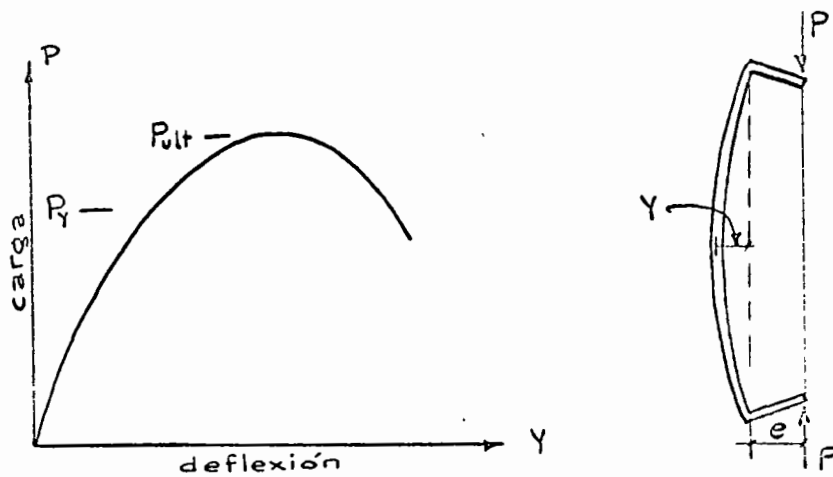
Se puede demostrar que la cedencia inicial en las fibras exteriores se presenta cuando el esfuerzo promedio es menor que el esfuerzo crítico de Euler que el esfuerzo de cedencia del material. Para un valor constante de la relación  $c/r^2$ , los efectos de la curvatura inicial y de la excentricidad accidental de la carga son similares para valores iguales de  $Y_0$  y de  $e$ , haciéndose prácticamente iguales en columnas con relaciones de esbeltez bajas e intermedias.



La resistencia última de una columna no se alcanza sino hasta que una parte de su sección transversal ha fluido plásticamente, siendo ésta por lo tanto, un poco mayor que su resistencia a la cedencia inicial. Esta diferencia de resistencias es pequeña para columnas de sección transversal en forma de perfil de patines anchos (Wide flange), flexionados respecto a su eje de mayor resistencia, pero en cambio, para los perfiles sólidos de sección cuadrada o rectangular, la diferencia es marcada.

Antiguamente, la fórmula de la secante (11) se utilizaba ampliamente en virtud de que al presuponer un valor arbitrario de la "excentricidad equivalente"  $ec/r^2$ , de 0.25, para considerar la falta de rectitud inicial y la excentricidad accidental de la carga, y también que la resistencia máxima de la columna se alcanzaba cuando  $f_m$  adquiría el valor de  $f_y$ , las cargas así calculadas se apegaban bastante a las cargas máximas observadas en las pruebas hechas con columnas en el laboratorio, sin embargo, las investigaciones posteriores demostraron que la influencia de los esfuerzos residuales en la resistencia de la columna era a menudo mayor aún que la de la falta de rectitud inicial del mismo y la excentricidad accidental de la carga.

Conforme la carga aumenta, la suma de los esfuerzos debidos a la flexión y los esfuerzos de compresión debidos a la carga aumentan hasta alcanzar  $F_y$  en las fibras exteriores de la sección al centro del claro. A la carga que produce esta condición se le llama  $P_y$ . La relación entre la carga y la deflexión del miembro en el centro del claro no es una relación lineal aún dentro de este rango de esfuerzos elásticos, en virtud del factor de magnificación  $\frac{1}{1 - \frac{P}{P_c}}$  (ver figura 11).



Si la carga en la columna sigue aumentando más allá de  $P_D$ , la fluencia progresa a través de la sección transversal del miembro y como la parte de éste que ya ha fluido plásticamente, contribuye poco a la rigidez del miembro, ésta rigidez va disminuyendo también. Como cada incremento en la deflexión lateral representa también un incremento en el momento flexionante y éste origina un aumento también en la porción del miembro que ya ha fluido plásticamente y éste a su vez una reducción de la rigidez del miembro, se puede ver que los incrementos de carga requeridos para producir cada incremento en la deflexión lateral de la columna, se hacen más y más pequeños cada vez reduciéndose a cero en la carga última. Al seguir creciendo la deflexión lateral de la pieza, la carga axial que puede resistir disminuye, de modo que termina por fallar por inestabilidad originada por flexión excesiva en el plano de los momentos aplicados.

En los miembros muy cortos, la carga última es la que causa la plastificación total de su sección transversal. También si una viga-columna se somete a flexión respecto a su eje de mayor resistencia y no está soportada lateralmente en forma adecuada, puede darse el caso que falle prematuramente por pandeo lateral torsional.

Los esfuerzos residuales que ocasionan la cedencia prematura y su correspondiente <sup>disminución</sup> de rigidez, pueden también reducir tanto la carga que produce la fluencia inicial como la carga última de la viga-columna. Análogamente, una falta

de rectitud inicial que incrementa el momento flexionante producido por una determinada condición de carga, reduce tanto la carga que produce la fluencia inicial como la carga última.

La carga de diseño para una viga columna puede basarse en la combinación de cargas que ocasione la cedencia inicial o la carga última. La determinación de cada una de estas, se considerará en seguida.

Determinación de la carga que inicia la cedencia  $P_y$

a) Método "directo" para miembros soportados lateralmente

Como se explicó anteriormente, el momento flexionante total en el centro del claro de una columna inicialmente deflexionada una ordenada  $Y_0$  en ese punto, y después de que la carga axial se le ha aplicado, esta dado por la expresión:

$$M = P \cdot Y$$

Si reemplazamos por  $Y$  su valor según la ecuación (8), tenemos:

$$M = P Y_0 \left( \frac{1}{1 - \frac{P}{P_c}} \right) \quad \text{----- } \textcircled{12}$$

Si además, la pieza cuenta con un momento inicial  $M_0$  al centro del claro, entonces el momento flexionante total al centro del claro es

$$M = M_0 + P Y_0 \left( \frac{1}{1 - \frac{P}{P_c}} \right) \quad \text{---- } \textcircled{13}$$

Ecuación semejante a la ecuación (5), excepto por el signo (-) del denominador del término entre paréntesis, lo cual hace que dicho término funcione como un "factor de magnificación" de la deflexión transversal de la viga, y consecuentemente, del momento flexionante total al centro del claro

No obstante de que el valor de la ordenada aproximada  $y$  - calculado por la ecuación No. (8) fué derivado partiendo - de la consideración de que la elástica deformada de la pie - za era una media onda de senoide, se ha visto que el momen - to calculado con la expresión (13) dá valores de una apro - ximación del orden 2% del momento exacto para los casos más comunes de carga, donde el momento máximo ocurre al centro del claro.

La ecuación (13) se puede reescribir como

$$M = M_0 \left[ \frac{1 + \psi \frac{P}{P_c}}{1 - \frac{P}{P_c}} \right] \quad \text{----- (14)}$$

donde  $\psi$  se define como

$$\psi = \frac{P_c Y_0}{M_0} - 1 = \frac{\pi^2 EI}{L^2} \frac{Y_0}{M_0} - 1 \quad \text{----- (15)}$$

El esfuerzo máximo  $f_m$  debido a la compresión axial y a la flexión, será

$$f_m = \frac{P}{A} + \frac{M}{I} c = \frac{P}{A} + \left[ \frac{1 + \psi \frac{P}{P_c}}{1 - \frac{P}{P_c}} \right] \frac{M_0 c}{I} \quad \text{----- (16)}$$

El valor de  $P$  que hace  $f_m$  igual a  $F_y$  es la carga axial que inicia la fluencia del miembro,  $P_y$ .

El término  $\psi$  puede valuarse numericamente para varias con - diciones de carga por ejemplo, para el caso de una carga - uniformemente distribuida  $w$  y extremos apoyados libremente.

$$\psi = \frac{\pi^2 EI}{L^2} \frac{\frac{5}{384} \frac{w L^4}{EI}}{\frac{w L^4}{8}} - 1 = 0.0281$$

La tabla 7, proporciona los valores de  $\psi$  para distintas - condiciones de carga transversal.

b) Ecuaciones de interacción

Las especificaciones AISC 1969 establecen ecuaciones de interacción para el caso de la combinación de esfuerzos de flexión (uniaxial o biaxial) y carga axial, que fueron desarrolladas a partir de los métodos descritos anteriormente. Las ecuaciones se han pensado para tomar en cuenta los posibles efectos del pandeo lateral. Cuando el esfuerzo axial  $f_a$  es la columna es menor del 15% del esfuerzo axial permisible para columnas cargadas axialmente  $F_a$ , es posible simplificar las ecuaciones de interacción despreciando el efecto flexionante originado por la carga axial y la deflexión de la columna. Para mayores valores de  $f_a$ , se hace necesario considerar ese momento secundario, en virtud de que el factor de magnificación  $1/(1- P/P_e)$  no es despreciable.

Las ecuaciones de AISC son:

Si  $\frac{f_a}{F_a} \leq 0.15$

$$\frac{f_a}{F_a} + \frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}} \leq 1.0 \text{ ---- } (17) \quad (\text{AISC 1.6-2})$$

Si  $\frac{f_a}{F_a} > 0.15$

$$\frac{f_a}{F_a} + \frac{C_{mx} f_{bx}}{\left(1 - \frac{f_a}{F_{e_x}}\right) F_{bx}} + \frac{C_{m_y} f_{by}}{\left(1 - \frac{f_a}{F_{e_y}}\right) F_{by}} \leq 1.0 \text{ --- } (18) \quad (\text{AISC 1.6-1})$$

En puntos de arriostamiento lateral,  $F_a = 0.6 F_y$  por lo tanto, la ec. (17) dá

$$\frac{f_a}{0.6 F_y} + \frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}} \leq 1.0 \text{ ---- } (19) \quad (\text{AISC 1.6-1b})$$

En estas fórmulas

$F_a$  = Esfuerzo axial permisible en la columna, como si solo existiera carga axial (Ver tablas 2, 3, 4 y 5)

$F_b$  = Esfuerzo permisible de compresión debida a la flexión como si solamente existiera esta. (sección 1.5.1.4. - AISC)

$$F'_{e_x} = \frac{10\,480\,000}{\left(\frac{K_x l_x}{r_x}\right)^2} ; \quad F'_{e_y} = \frac{10\,480\,000}{\left(\frac{K_y l_y}{r_y}\right)^2} , \quad \text{es el esfuerzo}$$

crítico de pandeo de Euler (teórico) dividido por un factor de seguridad de 2, tomado con la relación de esbeltez  $\frac{Kl}{r}$  correspondiente al plano de flexión que se revisa

fa = esfuerzo existente axial de compresión  $\left(\frac{P}{A}\right)$

fb = esfuerzo existente de flexión  $\left(\frac{M}{S}\right)$

Cm = coeficiente cuyo valor puede tomarse como sigue:

1.- Para miembros en compresión, sujetos a traslación lateral de sus uniones,  $C_m = 0.85$ .

2.- Para miembros en compresión con apoyos totalmente empotrados, en marcos arriostrados contra la translación de sus juntas, sin estar sujetos a cargas transversales entre sus apoyos en el plano de flexión:

$$C_m = 0.6 + 0.4 \frac{M_1}{M_2} , \quad (\text{pero no menor de } 0.4) , \quad \text{donde } \frac{M_1}{M_2}$$

es la relación del menor al mayor de los momentos extremos de la porción del miembro sin arriostrar, en el plano de flexión bajo consideración.  $M_1/M_2$  es positiva cuando el miembro se flexiona con curvatura simple, y negativa cuando adquiere curvatura doble.

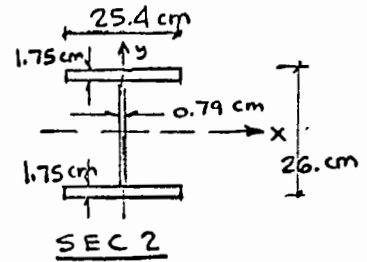
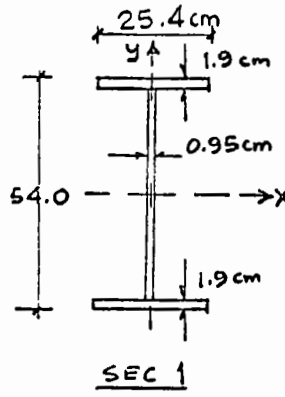
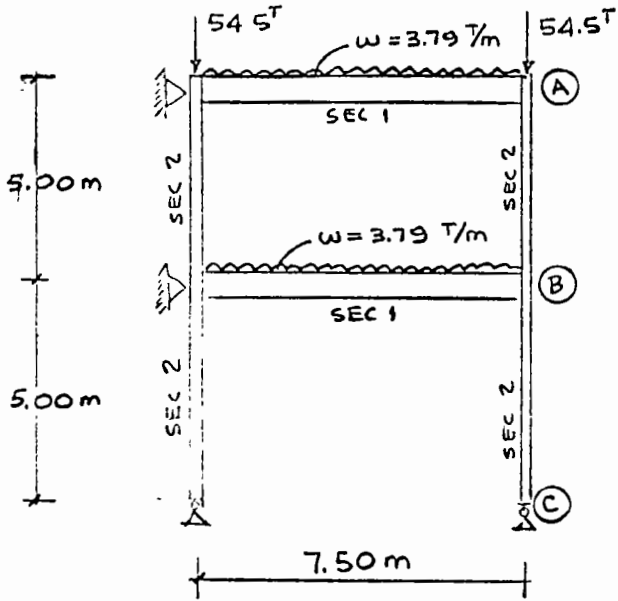
3.- Para miembros en compresión en marcos arriostrados contra la translación de sus juntas en el plano de carga y sujetos a cargas transversales entre sus apoyos, el valor de "Cm" puede determinarse por un análisis racional\*; sin embargo, en lugar de dicho análisis, los siguientes valores pueden aplicarse: Para miembros cuyos extremos están empotrados  $C_m = 0.85$  y  $C_m = 1.0$  en caso contrario.

$$(*) \quad C_{m_x} = 1 + \psi \frac{f_a}{F'_{e_x}} ; \quad C_{m_y} = 1 + \psi \frac{f_a}{F'_{e_y}} \quad \text{--- (20)}$$

Las tablas 7 y 8 contienen los valores de  $C_m$  para distintas condiciones. Se sugiere al estudiante leer el comentario de las especificaciones AISC, sección 1.6 para una mejor comprensión de estas especificaciones.

Ejemplo 1

Revisar de acuerdo a las especificaciones AISC 1969, las columnas AB y BC del marco siguiente. Acero ASTM A-36.- Tómesese  $K_y = 1.0$



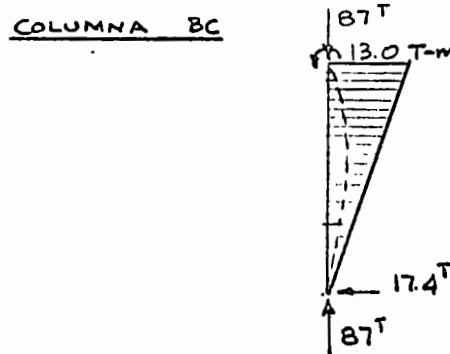
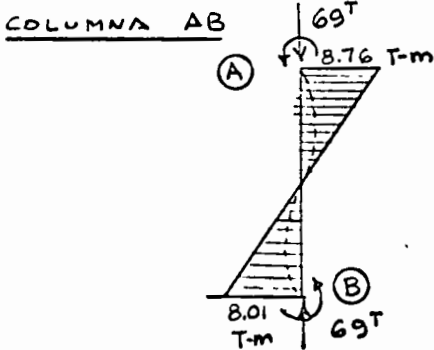
$I_{xx} = 51186 \text{ cm}^4$   
 $S_{xx} = 1896 \text{ cm}^3$   
 $A = 107.8 \text{ cm}^2$   
 $r_x = 11.39 \text{ cm}$

**SEC 2**  
 $I_{xx} = 13822 \text{ cm}^4$   
 $S_{xx} = 1063 \text{ cm}^3$   
 $A = 106.57 \text{ cm}^2$   
 $r_x = 11.39 \text{ cm}$   
 $r_y = 6.69 \text{ cm}$

NOTA: el plano del marco coincide con el plano de las almas de las vigas y columnas

SOLUCION:

Resolviendo el marco, se encuentran los siguientes elementos mecánicos:



$$\left(\frac{I}{L}\right)_{\text{TRABE}} = \frac{51186}{750} = 68.2$$

$$\left(\frac{I}{L}\right)_{\text{COL}} = \frac{13822}{500} = 27.6$$

$$G = \frac{\sum \left(\frac{I}{L}\right)_{\text{COL}}}{\sum \left(\frac{I}{L}\right)_{\text{TRABE}}}$$

$$G_{\text{SUP}} = \frac{27.6}{68.2} = 0.405 \quad \left. \vphantom{G_{\text{SUP}}} \right\} K_x = 0.705 \text{ (A-B)}$$

$$G_{\text{INF}} = \frac{2 \times 27.6}{68.2} = 0.810$$

$$G_{\text{SUP}} = G_{\text{INF col AB}} = 0.810 \quad \left. \vphantom{G_{\text{SUP}}} \right\} K_x = 0.845$$

$$G_{\text{INF}} = 10 \text{ (Valor recomendado)}$$

Revisión

COLUMNA AB

$$f_a = \frac{69000}{106.57} = 647 \text{ Kg/cm}^2$$

$$f_{b_x} = \frac{8.76 \times 10^5}{1063} = 824 \text{ Kg/cm}^2$$

$$\frac{K_x l}{r_x} = \frac{0.705 \times 500}{11.39} = 31 \quad (\text{en el plano de la flexión}) \therefore F'_e = 10910 \text{ Kg/cm}^2$$

$$\frac{K_y l}{r_y} = \frac{1.0 \times 500}{6.69} = 74.7 \quad (\text{rígido para evaluar } F_a) \therefore F_a = 1120 \text{ Kg/cm}^2$$

$$\frac{f_a}{F_a} = \frac{647}{1120} = 0.577 > 0.15 \therefore \text{utilizamos la fórmula (18) (AISC 1.6-1)}$$

Revisando si la sección es compacta:

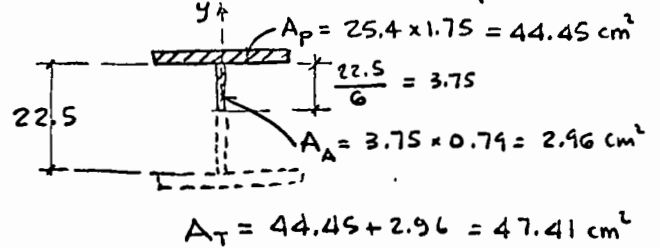
$$\frac{b}{2t} = \frac{25.4}{2 \times 1.75} = 7.26 < 8.7 \quad \therefore \checkmark$$

$$\frac{d}{t} = \frac{(26.0 - 2 \times 1.75)}{0.79} = 28.5 < 42.8 \quad \therefore \checkmark \quad \left( \frac{f_a}{F_y} = \frac{647}{2530} = 0.256 > 0.16 \right)$$

$$\lambda_b = 500 \text{ cm} > 12.7 b = 12.7 \times 25.4 = 323 \text{ cm} \therefore \text{no es compacta.}$$

$$r_t = \sqrt{\frac{\frac{1.75 \times 25.4^3}{12}}{47.41}} = \sqrt{50.4} = 7.1 \text{ cm}$$

$$\frac{l}{r_t} = \frac{500}{7.1} = 70.4$$



$$C_b = 1.75 + 1.05 \left( -\frac{8.01}{8.76} \right) + 0.3 \left( -\frac{8.01}{8.76} \right)^2 = 1.04$$

$$53 - \sqrt{C_b} = 54.06$$

$$119 - \sqrt{C_b} = 121.38$$

$$\text{Como } 54.6 < \frac{l}{r_t} < 121.38 \quad F_{b_x} = 1690 - \frac{\left( \frac{l}{r_t} \right)^2}{16.8 C_b} \quad (\text{AISC 1.5.1.4.6a})$$

$$= 1690 - \frac{70.4^2}{16.8 \times 1.04} = 1662 \text{ Kg/cm}^2$$

$$C_{m_x} = 0.6 + 0.4 \left( -\frac{8.01}{8.76} \right) = 0.234 < 0.4 \quad C_{m_x} = 0.4$$



Aplicando la fórmula de interacción (18).

$$0.577 + \frac{0.4 \times 824}{\left(1 - \frac{647}{10910}\right) 1662} = 0.577 + 0.211 = 0.788 < 1.0 \quad \therefore \checkmark$$

Adicionalmente revisamos en el apoyo más esforzado (A), ecuación (19)

$$\frac{647}{1520} + \frac{824}{1662} = 0.426 + 0.496 = 0.922 < 1.0 \quad \therefore \text{correcta}$$

Conclusión: La sección de la columna AB es satisfactoria.

### COLUMNA BC

$$f_a = \frac{87000}{106.57} = 816 \text{ Kg/cm}^2$$

$$f_{bx} = \frac{13 \cdot 10^5}{1063} = 1223 \text{ Kg/cm}^2$$

$$\frac{K_x l}{r_x} = \frac{0.845 \times 500}{11.39} = 37.2 \quad \rightarrow \therefore F'_{ex} = 7580 \text{ Kg/cm}^2$$

$$\frac{K_y l}{r_y} = \frac{1.10 \times 500}{6.69} = 74.7 \quad (\text{rige para } F_2) \quad \therefore F_2 = 1120 \text{ Kg/cm}^2$$

$$\frac{f_a}{F_2} = \frac{816}{1120} = 0.729 > 0.15 \quad \therefore \text{usaremos la ecuación (18)}$$

$$C_b = 1.75, \quad 53\sqrt{C_b} = 70.2; \quad 119\sqrt{C_b} = 157.6$$

$$\text{como } 70.2 < \frac{l}{r_t} = 70.4 \leq 157.6$$

$$F_{bx} = 1690 - \frac{70.4^2}{16.8 \times 1.75} = 1521 \text{ Kg/cm}^2$$

$$C_m = 0.6$$

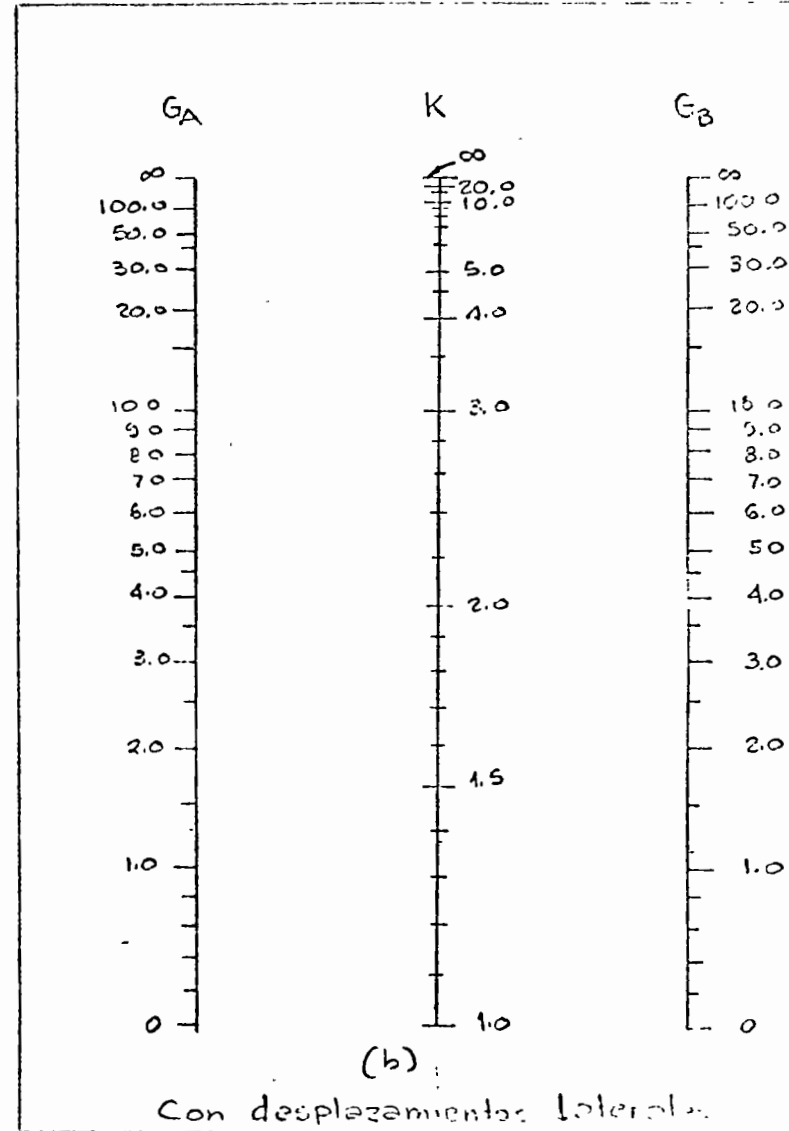
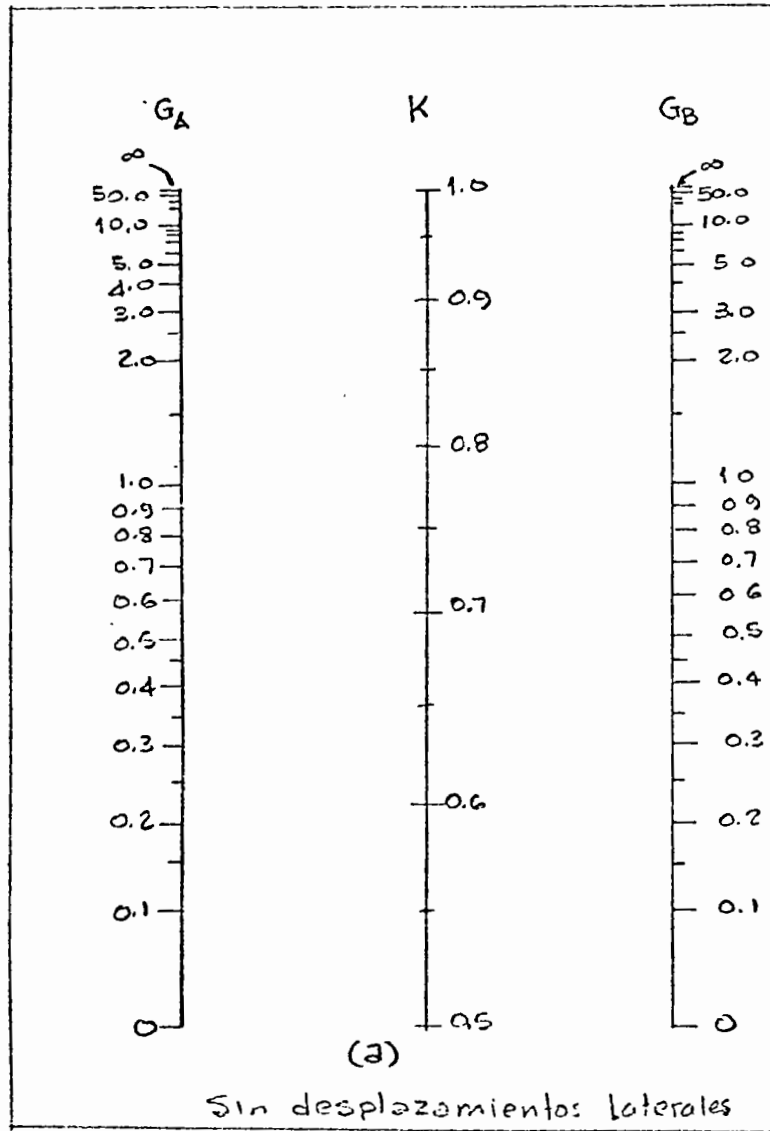
Aplicando interacción, (ecuación (15))

$$0.729 + \frac{0.6 \times 1223}{\left(1 - \frac{816}{7580}\right) 1521} = 0.729 + 0.325 = 1.054 \approx 1.0 \quad \therefore \checkmark$$

Adicionalmente, en el apoyo (B) (ecuación (19))

$$\frac{816}{1520} + \frac{1223}{1521} = 0.537 + 0.804 = 1.341 > 1.0 \quad \therefore \text{La columna BC}$$

no es satisfactoria y habrá que reforzar su sección.



NOTA 1: Extremos articulados o bien apoyados pero sin estar rígidamente unidos a la cimentación tomar  $G=10$

Extremos empotrados tomar  $G=1.0$  (o más si se justifica)

Nomograma para encontrar las longitudes efectivas de columnas en marcos continuos.

$$G = \frac{\sum \frac{I_{COL}}{L_{COL}}}{\sum \frac{I_{TRABE}}{L_{TRABE}}}$$

NOTA 2: Multiplicar  $\frac{1}{G}$  por los  $L_{COL}$  en la condición de apoyo de los extremos lejanos son bien conocidos -

para el caso (a) { Extr. lejanos articulados, etc.  
para el caso (b) { Extr. lejanos empotrados, etc.  
o viceversa en cualquier caso

TABLA 7

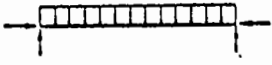
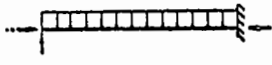
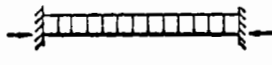
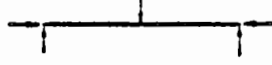
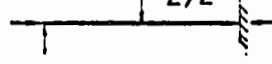

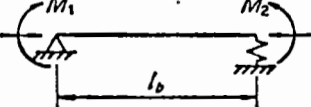
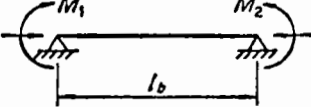
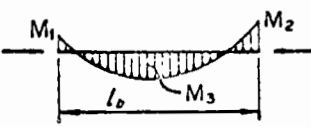
Caso	$\psi$	$C_m$
	0	1.0
	-0.3	$1 - 0.3 \frac{f_o}{F'_o}$
	-0.4	$1 - 0.4 \frac{f_o}{F'_o}$
	-0.2	$1 - 0.2 \frac{f_o}{F'_o}$
	-0.4	$1 - 0.4 \frac{f_o}{F'_o}$
	-0.6	$1 - 0.6 \frac{f_o}{F'_o}$

TABLA 8

Categoría	Condiciones de carga ( $f_o > 0.15 F'_o$ )	$f_b$	$C_m$	Observaciones
A	Momento máximo en el extremo; sin impedir la traslación de la junta.	$\frac{M_2}{S}$	0.85	 $M_1 < M_2$ ; $\frac{M_1}{M_2}$ positivo como se muestra Comprobar fórmulas (7a) y (7b)
B	Momento máximo en el extremo; sin cargas transversales; impidiendo la traslación de la junta	$\frac{M_2}{S}$	$(0.4 \frac{M_1}{M_2} + 0.6)$ pero no menor de 0.4	 Comprobar ambas fórmulas (7a) y (7b)
C	Carga transversal; impidiendo la traslación de la junta	$\frac{M_2}{S}$ usando fórmula (7b) $\frac{M_2}{S}$ usando fórmula (7a)	$1 + \psi \frac{f_o}{F'_o}$	 Comprobar ambas fórmulas (7a) y (7b)



**SPECIFICATION  
FOR THE  
DESIGN,  
FABRICATION  
& ERECTION  
OF  
STRUCTURAL  
STEEL FOR  
BUILDINGS**

**FEBRUARY 12, 1969**

**AMERICAN INSTITUTE  
OF STEEL CONSTRUCTION  
101 PARK AVENUE, NEW YORK, N.Y. 10017**

## Preface

Research completed since the last revision of the *AISC Specification for the Design, Fabrication and Erection of Structural Steel for Buildings* in 1963, together with the publication of new ASTM specifications covering grades of structural steel often affording improved economy, account for the additions and most of the changes in this revision of the AISC Specification.

Among the new provisions attributable to recent research are those covering the use of hybrid flexural members, that is, beams and girders having higher strength steel in the flanges than in the web. Also included are: a more rational set of working stresses for fillet welds that consider the mechanical properties of both weld and base metal and cover a much wider strength range than heretofore; the extension of existing working stress provisions and geometric limitations, expressed in terms of specified minimum yield stress, to steels having a yield stress of 100 ksi; and the extension of plastic design rules to cover braced multi-story structures and steels having a yield stress up to 65 ksi.

As in the past, in order to avoid reference to proprietary steels which may be available from but one source, only steels which can be identified by ASTM specifications are listed. However, steels covered by ASTM specifications but subject to more costly manufacturing and inspection techniques than deemed essential for structures covered by this Specification are not listed, even though they may provide all of the necessary characteristics of less expensive steels which are listed.

Steels covered by the listed ASTM specifications which have been adopted since the 1963 revision of the AISC Specification have been available for some time as proprietary products and considerable experience has already been acquired in their use. Also listed for the first time are several grades of steel having less frequent applications in building construction, which have been covered by ASTM specifications for many years. Their inclusion is for clarification. They have proven entirely satisfactory when used in accordance with the provisions of the AISC Specification.

With the extension of plastic design to steels having a yield stress in excess of the previous 36 ksi limitation, more restrictive width-thickness ratios are imposed on compression elements in Sect. 2.7 than in Sect. 1.5.1.4.1. This is because the required plastic hinge rotations in structures designed according to the provisions of Part 2 may be considerably greater than in designs executed according to the provisions of Sect. 1.5.1.4.1. The term *compact section* will continue to apply to members meeting the geometric limitations of Sect. 1.5.1.4.1 with respect to profile; shapes conforming to the requirements of Sect. 2.7 may be referred to as *plastic design sections*.

In order to simplify design calculations, the forces, stresses, and formulas related to them are now expressed in kips or kips per square inch instead of pounds or pounds per square inch as in the past.

As used throughout the Specification, the term *structural steel* refers exclusively to those items enumerated in Section 2 of the AISC Code of Standard Practice for Steel Buildings and Bridges, and nothing herein contained is intended as a recommended practice for skylights, fire escapes, or other items not specifically enumerated in that Code. For the design of cold-formed steel structural members, whose profiles contain rounded corners and slender flat elements, the provisions of the American Iron and Steel Institute Specification for the Design of Cold-Formed Steel Structural Members are recommended.

Many provisions of the Specification, notably in the sections dealing with fabrication and erection practices, have evolved from years of shop and field experience and need no further elaboration. Others are the outgrowth of recent extensive research. A separate *Commentary*, providing the background for such provisions, published by the American Institute of Steel Construction, is available at no cost to users of the Specification.

By the Committee,

Milton E. Eliot, Chairman	Edwin H. Gaylord	William A. Milek, Jr.
William C. Alsmeyer	John A. Gilligan	William H. Munse
Stephenson B. Barnes	John D. Griffiths	Anthony Nassetta
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Robert R. Gavin	Carl A. Metz	Charles A. Zwissler

February 12, 1969

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# Nomenclature

- A<sub>b</sub>** Nominal body area of a fastener  
**A<sub>c</sub>** Actual area of effective concrete flange in composite design  
**A<sub>bc</sub>** Planar area of web at beam-to-column connection  
**A<sub>f</sub>** Area of compression flange  
**A<sub>s</sub>** Area of steel beam in composite design  
**A<sub>sr</sub>** Area of reinforcing steel providing composite action at point of negative moment  
**A<sub>st</sub>** Cross-sectional area of stiffener or pair of stiffeners  
**A<sub>w</sub>** Area of girder web  
**C** Ratio of bolt tensile strength to tensile strength of connected part  
**C<sub>a</sub>** Coefficient used in Table 1-A  
**C<sub>b</sub>** Bending coefficient dependent upon moment gradient; equal to

$$1.75 + 1.05 \left( \frac{M_1}{M_2} \right) + 0.3 \left( \frac{M_1}{M_2} \right)^2$$

- C<sub>c</sub>** Column slenderness ratio dividing elastic and inelastic buckling; equal to

$$\sqrt{\frac{2\pi^2 E}{F_y}}, \quad \text{except in Appendix C}$$

- C<sub>m</sub>** Coefficient applied to bending term in interaction formula and dependent upon column curvature caused by applied moments  
**C<sub>p</sub>** Stiffness factor for primary member in a flat roof  
**C<sub>s</sub>** Stiffness factor of secondary member in a flat roof  
**C<sub>o</sub>** Ratio of "critical" web stress, according to the linear buckling theory, to the shear yield stress of web material; equal to

$$\frac{\pi^2 E k \sqrt{3}}{12(1 - \nu^2)(h/t)^2 F_y} \quad \text{or} \quad \frac{190}{h/t} \sqrt{\frac{k}{F_y}} \quad (\text{See Sect. 1.10.5.2})$$

- C<sub>1</sub>** Ratio of beam yield stress to column yield stress  
**C<sub>2</sub>** Ratio of column yield stress to stiffener yield stress  
**D** Factor depending upon type of transverse stiffeners  
**E** Modulus of elasticity of steel (29,000 kips per square inch)  
**E<sub>c</sub>** Modulus of elasticity of concrete  
**F** Load factor in plastic design  
**F<sub>a</sub>** Axial stress permitted in the absence of bending moment  
**F<sub>as</sub>** Axial compressive stress, permitted in the absence of bending moment, for bracing and other secondary members  
**F<sub>b</sub>** Bending stress permitted in the absence of axial force  
**F'<sub>b</sub>** Allowable bending stress in compression flange of plate girders as reduced for hybrid girders or because of large web depth-to-thickness ratio  
**F'<sub>e</sub>** Euler stress divided by factor of safety; equal to

$$\frac{12\pi^2 E}{23(Kl_b/r_b)^2}$$

- F<sub>p</sub>** Allowable bearing stress

- F<sub>sr</sub>** Stress range  
**F<sub>t</sub>** Allowable tensile stress  
**F<sub>v</sub>** Allowable shear stress  
**F<sub>y</sub>** Specified minimum yield stress of the type of steel being used (kips per square inch). As used in this Specification, "yield stress" denotes either the specified minimum yield point (for those steels that have a yield point) or specified minimum yield strength (for those steels that do not have a yield point).  
**F<sub>yr</sub>** Yield stress of reinforcing steel providing composite action at point of negative moment  
**I<sub>d</sub>** Moment of inertia of steel deck on a flat roof  
**I<sub>p</sub>** Moment of inertia of primary member in flat roof framing  
**I<sub>s</sub>** Moment of inertia of secondary member in flat roof framing  
**I<sub>tr</sub>** Moment of inertia of transformed composite section  
**K** Effective length factor  
**L** Span length (feet)  
**L<sub>p</sub>** Length of primary member in a flat roof (feet)  
**L<sub>s</sub>** Length of secondary member in a flat roof (feet)  
**M** Moment (kip-feet)  
**M<sub>1</sub>** Smaller moment at end of unbraced length of beam-column  
**M<sub>2</sub>** Larger moment at end of unbraced length of beam-column  
**M<sub>D</sub>** Moment produced by dead load  
**M<sub>L</sub>** Moment produced by live load  
**M<sub>m</sub>** Critical moment that can be resisted by a plastically designed member in absence of axial load  
**M<sub>o</sub>** Reduced plastic moment  
**M<sub>p</sub>** Plastic moment  
**N** Length of bearing of applied load (inches)  
**N<sub>1</sub>** Number of shear connectors equal to  $V_h/q$  or  $V'_h/q$ , as applicable  
**N<sub>2</sub>** Number of shear connectors required where closer spacing is needed adjacent to point of zero moment  
**P** Applied load (kips)  
**P<sub>cr</sub>** = 1.70  $A F_a$   
**P<sub>s</sub>** = 1.92  $A F'_s$   
**P<sub>y</sub>** Plastic axial load; equal to profile area times specified minimum yield stress (kips)  
**Q<sub>a</sub>** Ratio of effective profile area of an axially loaded member to its total profile area  
**Q<sub>s</sub>** Axial stress reduction factor where width-thickness ratio of unstiffened elements exceeds limiting value given in Sect. 1.9.1.2  
**R** Reaction or concentrated transverse load applied to beam or girder (kips)  
**S** Spacing of secondary members in a flat roof (feet)  
**S<sub>eff</sub>** Effective section modulus corresponding to partial composite action  
**S<sub>s</sub>** Section modulus of steel beam used in composite design, referred to the bottom flange  
**S<sub>tr</sub>** Section modulus of transformed composite cross-section, referred to the bottom flange  
**T<sub>b</sub>** Proof load of a high strength bolt (kips)  
**V** Static shear on beam (kips)  
**V<sub>h</sub>** Total horizontal shear to be resisted by connectors and all composite action (kips)



- $V_h$  Total horizontal shear to be resisted by connectors in providing partial composite action (kips)
- $V_u$  Statical shear produced by "ultimate" load in plastic design (kips)
- $Y$  Ratio of yield stress of web steel to yield stress of stiffener steel
- $a$  Clear distance between transverse stiffeners
- $a'$  Distance required at ends of welded partial length cover plate to develop stress
- $b$  Effective width of concrete slab; actual width of stiffened and unstiffened compression elements
- $b_e$  Effective width of stiffened compression element
- $b_f$  Flange width of rolled beam or plate girder
- $c$  Distance from neutral axis to extreme fiber of beam
- $d$  Depth of beam or girder. Also diameter of roller or rocker bearing
- $d_c$  Column web depth clear of fillets
- $e$  Horizontal displacement, in the direction of the span, between top and bottom of simply supported beam at its ends
- $f$  Axial compression load on member divided by effective area (kips per square inch)
- $f_a$  Computed axial stress
- $f_b$  Computed bending stress
- $f'_c$  Specified compression strength of concrete
- $f_t$  Computed tensile stress
- $f_s$  Computed shear stress
- $f_{sv}$  Shear between girder web and transverse stiffeners (kips per linear inch of single stiffener or pair of stiffeners)
- $g$  Transverse spacing between fastener gage lines
- $h$  Clear distance between flanges of a beam or girder
- $k$  Coefficient relating linear buckling strength of a plate to its dimensions and condition of edge support. Also distance from outer face of flange to web toe of fillet of rolled shape or equivalent distance on welded section
- $l$  Actual unbraced length (inches)
- $l_b$  Actual unbraced length in plane of bending (inches)
- $l_{cr}$  Critical unbraced length adjacent to plastic hinge (inches)
- $n$  Modular ratio; equal to  $E/E_c$
- $q$  Allowable horizontal shear to be resisted by a shear connector
- $r$  Governing radius of gyration
- $r_b$  Radius of gyration about axis of concurrent bending
- $r_v$  Lesser radius of gyration
- $s$  Spacing (pitch) between successive holes in line of stress
- $t$  Girder, beam, or column web thickness
- $t_b$  Beam flange thickness at rigid beam-to-column connection
- $t_f$  Flange thickness
- $t_t$  Thickness of thinner part joined by partial penetration groove weld
- $w$  Length of channel shear connectors
- $x$  Subscript relating symbol to strong axis bending
- $y$  Subscript relating symbol to weak axis bending
- $\alpha$  Ratio of hybrid girder web yield stress to flange yield stress
- $\beta$  Ratio  $S_{cr}/S_x$  or  $S_{cyy}/S_y$
- $\nu$  Poisson's ratio, may be taken as 0.3 for steel

## SPECIFICATION FOR THE

# Design, Fabrication and Erection of Structural Steel for Buildings

## PART 1

### SECTION 1.1 PLANS AND DRAWINGS

#### 1.1.1 Plans

The plans (design drawings) shall show a complete design with sizes, sections, and the relative locations of the various members. Floor levels, column centers, and offsets shall be dimensioned. Plans shall be drawn to a scale large enough to convey the information adequately.

Plans shall indicate the type or types of construction (as defined in Sect. 1.2) to be employed, and they shall be supplemented by such data concerning the assumed loads, shears, moments and axial forces to be resisted by all members and their connections, as may be required for the proper preparation of the shop drawings.

Where joints are to be assembled with high strength bolts and are required to resist shear between the connected parts, the plans shall indicate the type of connections to be provided, namely, friction or bearing.

Camber of trusses, beams and girders, if required, shall be called for on the design drawings.

#### 1.1.2 Shop Drawings

Shop drawings, giving complete information necessary for the fabrication of the component parts of the structure, including the location, type and size of all rivets, bolts and welds, shall be prepared in advance of the actual fabrication. They shall clearly distinguish between shop and field rivets, bolts and welds.

Shop drawings shall be made in conformity with the best modern practice and with due regard to speed and economy in fabrication and erection.

#### 1.1.3 Notations for Welding

Note shall be made on the plans and on the shop drawings of those joints or groups of joints in which it is especially important that the welding sequence and technique of welding be carefully controlled to minimize welding under restraint and to avoid undue distortion.

Weld lengths called for on the plans and on the shop drawings shall be the net effective lengths.

### 1.1.4 Standard Symbols and Nomenclature

Welding symbols used on plans and shop drawings shall preferably be the American Welding Society symbols. Other adequate welding symbols may be used, provided a complete explanation thereof is shown on the plans or drawings.

Unless otherwise noted, the standard nomenclature contained in the joint AISC-SJI *Standard Specifications for Open Web Steel Joists and Long-span Steel Joists*, latest edition, shall be used in describing steel joists.

## SECTION 1.2 TYPES OF CONSTRUCTION

Three basic types of construction and associated design assumptions are permissible under the respective conditions stated hereinafter, and each will govern in a specific manner the size of members and the types and strength of their connections.

Type 1, commonly designated as "rigid-frame" (continuous frame), assumes that beam-to-column connections have sufficient rigidity to hold virtually unchanged the original angles between intersecting members.

Type 2, commonly designated as "simple" framing (unrestrained, free-ended), assumes that, in so far as gravity loading is concerned, the ends of beams and girders are connected for shear only, and are free to rotate under gravity load.

Type 3, commonly designated as "semi-rigid framing" (partially restrained), assumes that the connections of beams and girders possess a dependable and known moment capacity intermediate in degree between the rigidity of Type 1 and the flexibility of Type 2.

The design of all connections shall be consistent with the assumptions as to type of construction called for on the design drawings.

Type 1 construction is unconditionally permitted under this Specification. Two different methods of design are recognized. Within the limitations laid down in Sect. 2.1, members of continuous frames, or continuous portions of frames, may be proportioned, on the basis of their maximum predictable strength, to resist the specified design loads multiplied by the prescribed load factors. Otherwise Type 1 construction shall be designed, within the limitations of Sect. 1.5, to resist the stresses produced by the specified design loads, assuming moment distribution in accordance with the elastic theory.

Type 2 construction is permitted under this Specification, subject to the stipulations of the following paragraph wherever applicable.

In tier buildings designed as Type 2 construction (that is, with beam-to-column connections other than wind connections assumed flexible under gravity loading) the wind moments may be distributed among selected joints of the frame provided that

1. The connections and connected members have capacity to resist the wind moments.
2. The girders are adequate to carry the full gravity load as "simple beams."
3. The connections have adequate inelastic rotation capacity to avoid overstress of the fasteners or welds under combined gravity and wind loading.

Type 3 (semi-rigid) construction will be permitted only upon evidence that the connections to be used are capable of furnishing, as a minimum, a predictable proportion of full end restraint. The proportioning of main members joined by such connections shall be predicated upon no greater degree of end restraint than this minimum.

Types 2 and 3 construction may necessitate some non-elastic but self-limiting deformation of a structural steel part.

## SECTION 1.3 LOADS AND FORCES

### 1.3.1 Dead Load

The dead load to be assumed in design shall consist of the weight of steelwork and all material permanently fastened thereto or supported thereby.

### 1.3.2 Live Load

The live load, including snow load if any, shall be that stipulated by the Code under which the structure is being designed or that dictated by the conditions involved. Snow load shall be considered as applied either to the entire roof area or to a portion of the roof area, and any probable arrangement of loads resulting in the highest stresses in the supporting member shall be used in the design.

### 1.3.3 Impact

For structures carrying live loads which induce impact, the assumed live load shall be increased sufficiently to provide for same.

If not otherwise specified, the increase shall be:

For supports of elevators . . . . .	100 percent
For traveling crane support girders and their connections . . . . .	25 percent
For supports of light machinery, shaft or motor driven, not less than . . . . .	20 percent
For supports of reciprocating machinery or power driven units, not less than . . . . .	50 percent
For hangers supporting floors and balconies . . . . .	33 percent

### 1.3.4 Crane Runway Horizontal Forces

The lateral force on crane runways to provide for the effect of moving crane trolleys shall, if not otherwise specified, be 20 percent of the sum of the weights of the lifted load and of the crane trolley (but exclusive of other parts of the crane). The force shall be assumed to be applied at the top of the rail, one-half on each side of the runway, and shall be considered as acting in either direction normal to the runway rail.

The longitudinal force shall, if not otherwise specified, be taken as 10 percent of the maximum wheel loads of the crane applied at the top of rail.

### 1.3.5 Wind

Proper provision shall be made for stresses caused by wind both during erection and after completion of the building.

### 1.3.6 Other Forces

Structures in localities subject to earthquakes, hurricanes and other extraordinary conditions shall be designed with due regard for such conditions.

### 1.3.7 Minimum Loads

In the absence of any applicable building code requirements, the loads referred to in Sect. 1.3.1, 1.3.2, 1.3.5 and 1.3.6 above shall be not less than those recommended in the *USA Standard Building Code Requirements for Minimum Design Loads in Buildings and Other Structures*, USASI A58.1, latest edition.

## SECTION 1.4 MATERIAL

### 1.4.1 Structural Steel

1.4.1.1 Material conforming to one of the following listing (latest date of issue) is approved for use under this Specification:

*Structural Steel*, ASTM A36

*Welded and Seamless Steel Pipe*, ASTM A53, Grade B

*High-Strength Low-Alloy Structural Steel*, ASTM A242

*High-Strength Low-Alloy Hot-Rolled Steel Sheet and Strip*, ASTM A375

*High-Strength Structural Steel*, ASTM A440

*High-Strength Low-Alloy Structural Manganese Vanadium Steel*, ASTM A441

*Cold-Formed Welded and Seamless Carbon Steel Structural Tubing in Rounds and Shapes*, ASTM A500

*Hot-Formed Welded and Seamless Carbon Steel Structural Tubing*, ASTM A501

*Structural Steel with 42,000 psi Minimum Yield Point*, ASTM A529

*Hot-Rolled Carbon Steel Sheets and Strip, Structural Quality*, ASTM A570, Grades D and E

*High-Strength Low-Alloy Columbium-Vanadium Steels of Structural Quality*, ASTM A572

*High-Strength Low-Alloy Structural Steel with 50,000 psi Minimum Yield Point to 4 in. Thick*, ASTM A588

*High-Yield Strength Quenched and Tempered Alloy Steel Plate, Suitable for Welding*, ASTM A514. (Quenched and tempered alloy steel structural shapes and seamless mechanical tubing meeting all of the mechanical and chemical requirements of A514 steel, except that the specified maximum tensile strength may be 140,000 psi for structural shapes and 145,000 psi for seamless mechanical tubing, shall be considered as A514 steel.)

Certified mill test reports or certified reports of tests made by the fabricator or a testing laboratory in accordance with ASTM A6 and the governing specification shall constitute sufficient evidence of conformity with one of the above ASTM specifications. Additionally, the fabricator shall, if requested, provide an affidavit stating that the structural steel furnished meets the requirements of the grade specified.

1.4.1.2 Unidentified steel, if free from surface imperfections, may be used for parts of minor importance, or for unimportant details, where the precise physical properties of the steel and its weldability would not affect the strength of the structure.

### 1.4.2 Other Metals

Cast steel shall conform to one of the following specifications, latest edition:

*Mild-to-Medium-Strength Carbon-Steel Castings for General Application*, ASTM A27, Grade 65-35

*High-Strength Steel Castings for Structural Purposes*, ASTM A148, Grade 80-50

Certified test reports shall constitute sufficient evidence of conformity with the specifications.

Steel forgings shall conform to one of the following specifications, latest edition:

*Carbon Steel Forgings for General Industrial Use*, ASTM A235, Class C1, F and G. (Class C1 Forgings that are to be welded shall be ordered in accordance with Supplemental Requirements S5 of A235.)

*Alloy Steel Forgings for General Industrial Use*, ASTM A237, Class A

Certified test reports shall constitute sufficient evidence of conformity with the specifications.

### 1.4.3 Rivets

Rivets shall conform to the provisions of the *Specification for Structural Rivets*, ASTM A502, Grade 1 or Grade 2, latest edition:

Manufacturer's certification shall constitute sufficient evidence of conformity with the specifications.

### 1.4.4 Bolts

High strength steel bolts shall conform to one of the following specifications, latest edition:

*High Strength Bolts for Structural Steel Joints, Including Suitable Nuts and Plain Hardened Washers*, ASTM A325

*Quenched and Tempered Steel Bolts and Studs*, ASTM A449

*Quenched and Tempered Alloy Steel Bolts for Structural Steel Joints*, ASTM A490

Other bolts shall conform to the *Specification for Low-Carbon Steel Externally and Internally Threaded Standard Fasteners*, ASTM A307, latest edition, hereinafter designated as A307 bolts.

Manufacturer's certification shall constitute sufficient evidence of conformity with the specifications.

### 1.4.5 Filler Metal for Welding

Welding electrodes for manual shielded metal-arc welding shall conform to the *Specification for Mild Steel Covered Arc-Welding Electrodes*, AWS A5.1, latest edition, or the *Specification for Low-Alloy Steel Covered Arc-Welding Electrodes*, AWS A5.5, latest edition.

Bare electrodes and granular flux used in the submerged-arc process shall conform to F60 or F70 AWS-flux classifications of the *Specification for Bare Mild Steel Electrodes and Fluxes for Submerged Arc Welding*, AWS A5.17, latest edition, or the provisions of Sect. 1.17.3.

E60S or E70S electrodes used in the gas metal-arc process shall conform to the *Specification for Mild Steel Electrodes for Gas Metal-Arc Welding*, AWS A5.18, latest edition, or the provisions of Sect. 1.17.3; E60T or E70T electrodes used in the flux cored-arc process shall conform to the *Specification for Mild Steel Electrodes for Flux-Cored-Arc Welding*, AWS A5.20, latest edition, or the provisions of Sect. 1.17.3.

Manufacturer's certification shall constitute sufficient evidence of conformity with the specifications.

## SECTION 1.5 ALLOWABLE STRESSES\*

Except as provided in Sects. 1.6, 1.7, 1.10, 1.11 and in Part 2, all components of the structure shall be so proportioned that the stress, in kips per square inch, shall not exceed the following values, except as they are rounded off in Appendix A.

### 1.5.1 Structural Steel

#### 1.5.1.1 Tension

On the net section, except at pin holes:

$$F_t = 0.60F_y$$

but not more than 0.5 times the minimum tensile strength of the steel.

On the net section at pin holes in eyebars, pin-connected plates or built-up members:

$$F_t = 0.45F_y$$

For tension on threaded parts see Table 1.5.2.1.

#### 1.5.1.2 Shear

On the gross section:  $F_v = 0.40F_y$

(The gross section of rolled and fabricated shapes may be taken as the product of the overall depth and the thickness of the web. See Sect. 1.10 for reduction required for thin webs. For discussion of high shear stress within boundaries of rigid connections of members whose webs lie in a common plane, see Commentary Sect. 1.5.1.2.)

#### 1.5.1.3 Compression

1.5.1.3.1 On the gross section of axially loaded compression members when  $Kl/r$ , the largest effective slenderness ratio of any unbraced segment as defined in Sect. 1.8, is less than  $C_c$ :

$$F_a = \frac{\left[1 - \frac{(Kl/r)^2}{2C_c^2}\right] F_y}{\frac{5}{3} + \frac{3(Kl/r)}{8C_c} - \frac{(Kl/r)^3}{8C_c^3}} \quad (1.5-1)$$

where

$$C_c = \sqrt{\frac{2\pi^2 E}{F_y}}$$

\* See Appendix A for tables of numerical values for various grades of steel corresponding to provisions of this Section.

1.5.1.3.2 On the gross section of axially loaded compression members when  $Kl/r$  exceeds  $C_c$ :

$$F_a = \frac{12\pi^2 E}{23(Kl/r)^2} \quad (1.5-2)$$

1.5.1.3.3 On the gross section of axially loaded bracing and secondary members, when  $l/r$  exceeds 120\*:

$$F_{as} = \frac{F_a \text{ (by Formula (1.5-1) or (1.5-2))}}{1.6 - \frac{l}{200r}} \quad (1.5-3)$$

1.5.1.3.4 On the gross area of plate girder stiffeners:

$$F_a = 0.60F_y$$

1.5.1.3.5 On the web of rolled shapes at the toe of the fillet (crippling, see Sect. 1.10.10):

$$F_a = 0.75F_y$$

#### 1.5.1.4 Bending

1.5.1.4.1 Tension and compression on extreme fibers of compact hot-rolled or built-up members (except hybrid girders and members of A514 steel) symmetrical about, and loaded in, the plane of their minor axis and meeting the requirements of this section:

$$F_b = 0.66F_y$$

In order to qualify under this section a member must meet the following requirements:

- The flanges shall be continuously connected to the web or webs.
- The width-thickness ratio of unstiffened projecting elements of the compression flange, as defined in Sect. 1.9.1.1, shall not exceed  $52.2/\sqrt{F_y}$ .
- The width-thickness ratio of stiffened elements of the compression flange, as defined in Sect. 1.9.2.1, shall not exceed  $190/\sqrt{F_y}$ .
- The depth-thickness ratio of the web or webs shall not exceed the value

$$d/t = 412 \left(1 - 2.33 \frac{f_a}{F_y}\right) / \sqrt{F_y} \quad (1.5-4)$$

- except that it need not be less than  $257/\sqrt{F_y}$ .
- The compression flange shall be supported laterally at intervals not to exceed  $76.0b/\sqrt{F_y}$  nor  $\frac{20,000}{(d/A)F_y}$

Except for hybrid girders and members of A514 steel, beams and girders (including members designed on the basis of composite action) which meet the requirements of sub-paragraphs a, b, c, d and e above and are continuous over supports or are rigidly framed to columns by means of rivets,

\* For this case,  $K$  is taken as unity.

high strength bolts or welds, may be proportioned for  $\frac{9}{10}$  of the negative moment produced by gravity loading which are maximum at points of support, provided that, for such members, the maximum positive moment shall be increased by  $\frac{1}{10}$  of the average negative moments. This reduction shall not apply to moments produced by loading on cantilevers. If the negative moment is resisted by a column rigidly framed to the beam or girder, the  $\frac{1}{10}$  reduction may be used in proportioning the column for the combined axial and bending loading, provided that the stress,  $f_a$ , due to any concurrent axial load on the member, does not exceed  $0.15F_u$ .

**1.5.1.4.2** Members (except hybrid girders and members of A514 steel) which meet the requirements of Sect. 1.5.1.4.1 except that  $b_f/2t_f$ , exceeds  $52.2/\sqrt{F_y}$  but is less than  $95.0/\sqrt{F_y}$ , may be designed on the basis of an allowable bending stress

$$F_b = F_y \left[ 0.733 - 0.0014 \left( \frac{b_f}{2t_f} \right) \sqrt{F_y} \right] \quad (1.5-5)$$

**1.5.1.4.3** Tension and compression on extreme fibers of doubly-symmetrical I- and H-shape members meeting the requirements of Sect. 1.5.1.4.1, subparagraphs a and b, and bent about their minor axis (except members of A514 steel); solid round and square bars; and solid rectangular sections bent about their weaker axis:

$$F_b = 0.75F_y$$

**1.5.1.4.4** Tension and compression on extreme fibers of box-type flexural members whose compression flange or web width-thickness ratio does not meet the requirements of Sect. 1.5.1.4.1 but does conform to the requirements of Sect. 1.9 and whose compression flange is braced laterally at intervals not exceeding  $2,500/F_y$  times the transverse distance out-to-out of the webs:

$$F_b = 0.60F_y$$

**1.5.1.4.5** Tension on extreme fibers of flexural members not covered in Sect. 1.5.1.4.1, 1.5.1.4.2, 1.5.1.4.3 or 1.5.1.4.4:

$$F_b = 0.60F_y$$

**1.5.1.4.6a** Compression on extreme fibers of flexural members included under Sect. 1.5.1.4.5, having an axis of symmetry in, and loaded in, the plane of their web, and compression on extreme fibers of channels\* bent about their major axis: the larger value computed by Formulas (1.5-6a) or (1.5-6b) and (1.5-7) as applicable (unless a higher value can be justified on the basis of a more precise analysis\*\*), but not more than  $0.60F_y$ .

When  $\sqrt{\frac{102 \times 10^3 C_b}{F_y}} \leq \frac{l}{r_T} \leq \sqrt{\frac{510 \times 10^3 C_b}{F_y}}$

$$F_b = \left[ \frac{2}{3} - \frac{F_y(l/r_T)^2}{1,530 \times 10^3 C_b} \right] F_y \quad (1.5-6a)$$

When  $l/r_T \geq \sqrt{\frac{510 \times 10^3 C_b}{F_y}}$

$$F_b = \frac{170 \times 10^3 C_b}{(l/r_T)^2} \quad (1.5-6b)$$

Or, when the compression flange is solid and approximately rectangular in cross-section and its area is not less than that of the tension flange

$$F_b = \frac{12 \times 10^3 C_b}{l_d/A_f} \quad (1.5-7)$$

In the foregoing,

$l$  = distance between cross-sections braced against twist or lateral displacement of the compression flange

$r_T$  = radius of gyration of a section comprising the compression flange plus one-third of the compression web area, taken about an axis in the plane of the web

$A_f$  = area of the compression flange

$C_b$  =  $1.75 + 1.05 (M_1/M_2) + 0.3 (M_1/M_2)^2$ , but not more than 2.3\*, where  $M_1$  is the smaller and  $M_2$  the larger bending moment at the ends of the unbraced length, taken about the strong axis of the member, and where  $M_1/M_2$ , the ratio of end moments, is positive when  $M_1$  and  $M_2$  have the same sign (reverse curvature bending) and negative when they are of opposite signs, (single curvature bending). When the bending moment at any point within an unbraced length is larger than that at both ends of this length, the value of  $C_b$  shall be taken as unity.  $C_b$  shall also be taken as unity in computing the value of  $F_{bx}$  and  $F_{by}$  to be used in Formula (1.6-1a). See Sect. 1.10 for further limitation in plate girder flange stress.

For hybrid plate girders,  $F_y$  for Formulas (1.5-6a) and (1.5-6b) is the yield stress of the compression flange. Formula (1.5-7) shall not apply to hybrid girders.

**1.5.1.4.6b** Compression on extreme fibers of flexural members included under Sect. 1.5.1.4.5, but are not included in Sect. 1.5.1.4.6a:

$$F_b = 0.60F_y$$

provided that sections bent about their major axis are braced laterally in the region of compression stress at intervals not exceeding  $76.0b_f/\sqrt{F_y}$ .

**1.5.1.5 Bearing** (on contact area)

**1.5.1.5.1** Milled surfaces, including bearing stiffeners and pins in reamed, drilled, or bored holes:

$$F_p = 0.90F_y^{**}$$

\* Only Formula (1.5-7) applicable to channels.

\*\* See Commentary Sects. 1.5.1.4.5 and 1.5.1.4.6, last two paragraphs.

\*  $C_b$  can be conservatively taken as unity. For smaller values see Appendix A, Fig. A1, p. 5-104.

\*\* When parts in contact have different yield stresses,  $F_y$  shall be the smaller value.

1.5.1.5.2 Expansion rollers and rockers, kips per linear inch:

$$F_p = \left( \frac{F_v - 13}{20} \right) 0.66d$$

where  $d$  is the diameter of roller or rocker in inches.

1.5.2 Rivets, Bolts, and Threaded Parts

1.5.2.1 Allowable tension and shear stresses on rivets, bolts and threaded parts (kips per square inch of area of rivets before driving or unthreaded-body area of bolts and threaded parts except as noted) shall be as given in Table 1.5.2.1. High strength bolts required to support applied load by means of direct tension shall be so proportioned that their average tensile stress, computed on the basis of nominal bolt area and independent of any initial tightening force, will not exceed the appropriate stress given in Table 1.5.2.1. The applied load shall be the sum of the external load and any tension resulting from prying action produced by deformation of the connected parts.

TABLE 1.5.2.1

Description of Fastener	Tension ( $F_t$ )	Shear ( $F_v$ )	
		Friction-Type Connections	Bearing-Type Connections
A502, Grade 1, hot-driven rivets	20.0		15.0
A502, Grade 2, hot-driven rivets	27.0		20.0
A307 bolts	20.0 <sup>1</sup>		10.0
Threaded parts <sup>2</sup> of steel meeting the requirements of Sect. 1.4.1	0.60 $F_v$ <sup>1</sup>		0.30 $F_v$
A325 and A449 bolts, when threading is <i>not</i> excluded from shear planes	40.0 <sup>2</sup>	15.0	15.0
A325 and A449 bolts, when threading is excluded from shear planes	40.0 <sup>2</sup>	15.0	22.0
A490 bolts, when threading is <i>not</i> excluded from shear planes	54.0 <sup>2,4</sup>	20.0	22.5
A490 bolts, when threading is excluded from shear planes	54.0 <sup>2,4</sup>	20.0	32.0

<sup>1</sup> Applied to tensile stress area equal to  $0.7854 \left( D - \frac{0.9743}{n} \right)^2$  where  $D$  is the major thread diameter and  $n$  is the number of threads per inch.

<sup>2</sup> Applied to the nominal bolt area.

<sup>3</sup> Since the nominal area of an upset rod is less than the stress area, the former area will govern.

<sup>4</sup> Static loading only.

TABLE 1.5.3

Kind of Stress	Permissible Stress	Required Electrode <sup>4</sup>	"Matching Base Metal"
Tension and Compression parallel to axis of any complete penetration groove weld	Same as for base metal <sup>1</sup>		
Tension normal to effective throat of complete-penetration groove weld	Same as allowable tensile stress for base metal <sup>1</sup>		
Compression normal to effective throat of complete or partial-penetration groove weld	Same as allowable compressive stress for base metal <sup>1</sup>		
Shear on effective throat of complete-penetration groove weld	Same as allowable shear stress for base metal <sup>1</sup>		
Shear stress on effective <sup>2</sup> throat of fillet weld and partial-penetration groove weld regardless of direction of application of load; tension normal <sup>3</sup> to the axis on the effective throat of a partial-penetration groove weld; and shear stress on effective area of a plug or slot weld. The given stresses shall also apply to such welds made with the specified electrode on steel having a yield stress greater than that of the "matching" base metal. The permissible stress, regardless of electrode classification used, shall not exceed that given in the table for the weaker "matching" base metal being joined.	18.0 ksi	AWS A5.1, E60XX electrodes AWS A5.17, F6X-EXXX flux-electrode combination AWS A5.20, E60T-X electrodes	A500 Grade A570 Grade
	21.0 ksi	AWS A5.1 or A5.5, E70XX electrodes AWS A5.17, F7X-EXXX flux-electrode combination AWS A5.18, E70S-X or E70U-1 electrodes AWS A5.20, E70T-X electrodes	A36 A53 Grade A242 A375 A441 A500 Grade A501 A529 A570 Grade A572 Grade 42 to 60 A588
	24.0 ksi	AWS A5.5, E80XX electrodes Grade 80 Submerged Arc, Gas Metal-Arc or Flux Cored Arc Weld Metal	A572 Grade 65
	27.0 ksi	AWS A5.5, E90XX electrodes Grade 90 Submerged Arc, Gas Metal-Arc or Flux Cored Arc Weld Metal	A514 over 2 in. thick
	30.0 ksi	AWS A5.5, E100XX electrodes Grade 100 Submerged Arc, Gas Metal-Arc or Flux Cored Arc Weld Metal	A514 over 2 in. thick
	33.0 ksi	AWS A5.5, E110XX electrodes Grade 110 Submerged Arc, Gas Metal-Arc or Flux Cored Arc Weld Metal	A514 2 1/4 and less thickness

<sup>1</sup> The electrode or flux specified in Table 1.17.2 shall be used.

<sup>2</sup> For definition of effective throat of fillet welds and partial penetration groove welds see § 1.14.7.

<sup>3</sup> Fillet welds and partial penetration groove welds joining the component elements of built-up members, such as flange-to-web connections, may be designed without regard to the tension or compressive stress in these elements parallel to the axis of the welds.

<sup>4</sup> Only low-hydrogen electrodes shall be used on A242, A441, A514, A572 and A588.

1.5.2.2 Allowable bearing stress on projected area of bolts in beam connections and on rivets:

$$F_p = 1.35F_y$$

where  $F_y$  is the yield stress of the connected part. (Bearing stress is not restricted in friction-type connections assembled with A325, A449 or A490 bolts.)

### 1.5.3 Welds

Except as modified by the provisions of Sect. 1.7, welds shall be proportioned to meet the stress requirements given in Table 1.5.3.

### 1.5.4 Cast Steel and Steel Forgings

Allowable stresses same as those provided in Sect. 1.5.1, where applicable.

### 1.5.5 Masonry Bearing

In the absence of Code regulations the following stresses apply:

On sandstone and limestone . . . . .	$F_p = 0.40$ ksi
On brick in cement mortar . . . . .	$F_p = 0.25$ ksi
On the full area of a concrete support . . . . .	$F_p = 0.25f'_c$
On one-third of this area . . . . .	$F_p = 0.375f'_c$

where  $f'_c$  is the specified compression strength of the concrete.

### 1.5.6 Wind and Seismic Stresses

Allowable stresses may be increased one-third above the values provided in Sect. 1.5.1, 1.5.2, 1.5.3, 1.5.4 and 1.5.5 when produced by wind or seismic loading, acting alone or in combination with the design dead and live loads, provided the required section computed on this basis is not less than that required for the design dead and live load and impact (if any), computed without the one-third stress increase.

## SECTION 1.6 COMBINED STRESSES

### 1.6.1 Axial Compression and Bending

Members subjected to both axial compression and bending stresses shall be proportioned to satisfy the following requirements:

$$\frac{f_a}{F_a} + \frac{C_{mx}f_{bx}}{\left(1 - \frac{f_a}{F'_{ax}}\right) F_{bx}} + \frac{C_{my}f_{by}}{\left(1 - \frac{f_a}{F'_{ay}}\right) F_{by}} \leq 1.0 \quad (1.6-1a)$$

$$\frac{f_a}{0.60F_y} + \frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}} \leq 1.0 \quad (1.6-1b)$$

When  $\frac{f_a}{F_a} \leq 0.15$ , Formula (1.6-2) may be used in lieu of Formulas (1.6-1a) and (1.6-1b)

$$\frac{f_a}{F_a} + \frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}} \leq 1.0 \quad (1.6-2)$$

In Formulas (1.6-1a), (1.6-1b), and (1.6-2) the subscripts  $x$  and  $y$ , combined with subscripts  $b$ ,  $m$  and  $e$ , indicate the axis of bending about which a particular stress or design property applies, and

$F_a$  = axial stress that would be permitted if axial force alone existed  
 $F_b$  = compressive bending stress that would be permitted if bending moment alone existed

$F'_e = \frac{12\pi^2 E}{23(Kl_b/r_b)^2}$  (In the expression for  $F'_e$ ,  $l_b$  is the actual unbraced length in the plane of bending and  $r_b$  is the corresponding radius of gyration.  $K$  is the effective length factor in the plane of bending. As in the case of  $F_a$ ,  $F_b$  and  $0.6 F_y$ ,  $F'_e$  may be increased one-third in accordance with Sect. 1.5.6.)

$f_a$  = computed axial stress  
 $f_b$  = computed compressive bending stress at the point under consideration

$C_m$  = a coefficient whose value shall be taken as follows:

1. For compression members in frames subject to joint translation (sidesway),  $C_m = 0.85$ .
2. For restrained compression members in frames braced against joint translation and not subject to transverse loading between their supports in the plane of bending,

$$C_m = 0.6 - 0.4 \frac{M_1}{M_2}, \text{ but not less than } 0.4,$$

where  $M_1/M_2$  is the ratio of the smaller to larger moments at the ends of that portion of the member unbraced in the plane of bending under consideration.  $M_1/M_2$  is positive when the member is bent in reverse curvature and negative when it is bent in single curvature.

3. For compression members in frames braced against joint translation in the plane of loading and subjected to transverse loading between their supports, the value of  $C_m$  may be determined by rational analysis. However, in lieu of such analysis, the following values may be used: (a) for members whose ends are restrained,  $C_m = 0.85$ ; (b) for members whose ends are unrestrained,  $C_m = 1.0$ .

### 1.6.2 Axial Tension and Bending

Members subject to both axial tension and bending stresses shall be proportioned at all points along their length to satisfy the requirements of Formula (1.6-1b) where  $f_b$  is the computed bending tensile stress. However, the computed bending compressive stress, taken alone, shall not exceed the applicable value according to Sect. 1.5.1.4.

### 1.6.3 Shear and Tension

Rivets and bolts subject to combined shear and tension shall be so proportioned that the tension stress, in kips per square inch, produced by forces applied to the connected parts, shall not exceed the following:

- For A502 Grade 1 rivets . . . . .  $F_t = 28.0 - 1.6f_s \leq 20.0$
- For A502 Grade 2 rivets. . . . .  $F_t = 38.0 - 1.6f_s \leq 27.0$
- For A307 bolts (applied to stress area)  $F_t = 28.0 - 1.6f_s \leq 20.0$
- For A325 and A449 bolts in bearing-type joints . . . . .  $F_t = 50.0 - 1.6f_s \leq 40.0$
- For A490 bolts in bearing-type joints . . . . .  $F_t = 70.0 - 1.6f_s \leq 54.0$

where  $f_s$ , the shear stress produced by the same forces, shall not exceed the value for shear given in Sect. 1.5.2.

For bolts used in friction-type joints, the shear stress allowed in Sect. 1.5.2 shall be reduced so that:

- For A325 and A449 bolts . . . . .  $F_s \leq 15.0(1 - f_t A_b / T_b)$
- For A490 bolts. . . . .  $F_s \leq 20.0(1 - f_t A_b / T_b)$

where  $f_t$  is the average tensile stress due to a direct load applied to all of the bolts in a connection and  $T_b$  is the specified pretension load of the bolt.

## SECTION 1.7 MEMBERS AND CONNECTIONS SUBJECT TO REPEATED VARIATION OF STRESS (FATIGUE)

### 1.7.1 General

Fatigue, as used in this Specification, is defined as the damage that may result in fracture after a sufficient number of fluctuations of stress. Stress range is defined as the magnitude of these fluctuations. In the case of a stress reversal, stress range shall be computed as the numerical sum of maximum repeated tensile and compressive stresses or the sum of maximum shearing stresses of opposite direction at a given point, resulting from differing arrangements of live load.

Few members or connections in conventional buildings need to be designed for fatigue, since most load changes in such structures occur only a small number of times or produce only minor stress fluctuations. The occurrence of full design wind or earthquake loads is too infrequent to warrant consideration in fatigue design. However, crane runways and supporting structures for machinery and equipment are often subject to fatigue loading conditions.

### 1.7.2 Design for Fatigue

Members and their connections, subject to fatigue loading as defined in Appendix B, shall be proportioned to satisfy the stress range limitations as provided therein.

## SECTION 1.8 STABILITY AND SLENDERNESS RATIOS

### 1.8.1 General

General stability shall be provided for the structure as a whole and for each compression element.

In determining the slenderness ratio of an axially loaded compression member except as provided in Sect. 1.5.1.3.3, the length shall be taken as its effective length  $Kl$  and  $r$  as the corresponding radius of gyration.

### 1.8.2 Sidesway Prevented

In frames where lateral stability is provided by adequate attachment to diagonal bracing, shear walls, an adjacent structure having adequate lateral stability, or to floor slabs or roof decks secured horizontally by walls or bracing systems parallel to the plane of the frame, and in trusses, the effective length factor,  $K$ , for the compression members shall be taken as unity, unless analysis shows that a smaller value may be used.

### 1.8.3 Sidesway Not Prevented

In frames where lateral stability is dependent upon the bending stiffness of rigidly connected beams and columns, the effective length  $Kl$  of compression members, shall be determined by a rational method and shall not be less than the actual unbraced length.

### 1.8.4 Maximum Ratios

The slenderness ratio,  $Kl/r$ , of compression members shall not exceed 200.

The slenderness ratio,  $Kl/r$ , of tension members, other than rods, preferably should not exceed:

- For main members . . . . . 240
- For bracing and other secondary members . . . . . 300

## SECTION 1.9 WIDTH-THICKNESS RATIOS

### 1.9.1 Unstiffened Elements Under Compression

1.9.1.1 Unstiffened (projecting) compression elements are those having one free edge parallel to the direction of compression stress. The width of unstiffened plates shall be taken from the free edge to the first row of fasteners or welds; the width of legs of angles, channel and zee flanges, and stems of tees shall be taken as the full nominal dimension; the width of flanges of I-shape members and tees shall be taken as one-half the full nominal width. The thickness of a sloping flange shall be measured halfway between a free edge and the corresponding face of the web.

1.9.1.2 Unstiffened elements subject to axial compression or compression due to bending shall be considered as fully effective when the ratio of width to thickness is not greater than the following:

- Single-angle struts; double-angle struts with separators. . . . .  $76.0/\sqrt{F_v}$
- Struts comprising double angles in contact; angles or plates projecting from girders, columns or other compression members; compression flanges of beams; stiffeners on plate girders . . . . .  $95.0/\sqrt{F_v}$
- Stems of tees . . . . .  $127/\sqrt{F_v}$

When the actual width-to-thickness ratio exceeds these values, the design stress shall be governed by the provisions of Appendix C.



## 1.9.2 Stiffened Elements Under Compression

1.2.1 Stiffened compression elements are those having lateral support along both edges which are parallel to the direction of the compression stress. The width of such elements shall be taken as the distance between nearest lines of fasteners or welds, or between the roots of the flanges in the case of rolled sections.

1.9.2.2 Stiffened elements subject to axial compression, or to uniform compression due to bending as in the case of the flange of a flexural\* member, shall be considered as fully effective when the ratio of width to thickness is not greater than the following:

Flanges of square and rectangular sections of uniform thickness . . . . .	$238/\sqrt{F_y}$
Unsupported width of cover plates perforated with a succession of access holes** . . . . .	$317/\sqrt{F_y}$
All other uniformly compressed stiffened elements . . . . .	$253/\sqrt{F_y}$

Except in the case of perforated cover plates, when the actual width-to-thickness ratio exceeds these values the design shall be governed by the provisions of Appendix C.

## SECTION 1.10 PLATE GIRDERS AND ROLLED BEAMS

### 1.10.1 Proportions

Riveted and welded plate girders, cover-plated beams and rolled beams shall in general be proportioned by the moment of inertia of the gross section. No deduction shall be made for shop or field rivet or bolt holes in either flange, except that in cases where the reduction of the area of either flange by such holes, calculated in accordance with the provisions of Sect. 1.14.3, exceeds 15 percent of the gross flange area, the excess shall be deducted.

Hybrid girders may be proportioned by the moment of inertia of their gross section,† subject to the applicable provisions in Sect. 1.10, provided that they are not required to resist an axial force greater than  $0.15F_y$  times the area of the gross section, where  $F_y$  is the yield stress of the flange material. To qualify as hybrid girders the flanges at any given section shall have the same cross-sectional area and be made of the same grade of steel.

### 1.10.2 Web

The clear distance between flanges, in inches, shall not exceed

$$\frac{14,000}{\sqrt{F_y(F_y + 16.5)}}$$

times the web thickness, where  $F_y$  is the yield stress of the compression flange, except that it need not be less than  $2,000/\sqrt{F_y}$  when transverse stiffeners are provided, spaced not more than  $1\frac{1}{2}$  times the girder depth

\* Webs of flexural members are covered by the provisions of Sects. 1.10.2 and 1.10.6 and are not subject to the provisions of this section.

\*\* Assumes net area of plate at widest hole as basis for computing compression stress.

† No limit is placed on the web stresses produced by the applied bending moment for which a hybrid girder is designed, except as provided in Sect. 1.7 and Appendix B

## 1.10.3 Flanges

The thickness of outstanding parts of flanges shall conform to the requirements of Sect. 1.9.1.2.

Flanges of welded plate girders may be varied in thickness or width by splicing a series of plates or by the use of cover plates.

The total cross-sectional area of cover plates of riveted girders shall not exceed 70 percent of the total flange area.

### 1.10.4 Flange Development

Rivets, high strength bolts or welds connecting flange to web, or cover plate to flange, shall be proportioned to resist the total horizontal shear resulting from the bending forces on the girder. The longitudinal distribution of these rivets, bolts or intermittent welds shall be in proportion to the intensity of the shear. But the longitudinal spacing shall not exceed the maximum permitted, respectively, for compression or tension members in Sect. 1.18.2.3 or 1.18.3.1. Additionally, rivets or welds connecting flange to web shall be proportioned to transmit to the web any loads applied directly to the flange unless provision is made to transmit such loads by direct bearing.

Partial length cover plates shall be extended beyond the theoretical cut-off point and the extended portion shall be attached to the beam or girder by rivets, high strength bolts (friction-type joint), or fillet welds adequate, at the applicable stresses allowed in Sect. 1.5.2 or 1.5.3 or Sect. 1.7, to develop the cover plate's portion of the flexural stresses in the beam or girder at the theoretical cut-off point. In addition, for welded cover plates, the welds connecting the cover plate termination to the beam or girder in the length  $a'$ , defined below, shall be adequate, at the allowed stresses, to develop the cover plate's portion of the flexural stresses in the beam or girder at the distance  $a'$  from the end of the cover plate.\* The length  $a'$ , measured from the end of the cover plate, shall be:

1. A distance equal to the width of the cover plate when there is a continuous weld equal to or larger than  $\frac{3}{4}$  of the plate thickness across the end of the plate and continued welds along both edges of the cover plate in the length  $a'$ .
2. A distance equal to  $1\frac{1}{2}$  times the width of the cover plate when there is a continuous weld smaller than  $\frac{3}{4}$  of the plate thickness across the end of the plate and continued welds along both edges of the cover plate in the length  $a'$ .
3. A distance equal to 2 times the width of the cover plate when there is no weld across the end of the plate but continuous welds along both edges of the cover plate in the length  $a'$ .

### 1.10.5 Stiffeners

1.10.5.1 Bearing stiffeners shall be placed in pairs at unframed ends on the webs of plate girders and where required\*\* at points of concentrated

\* This may require the cover plate termination to be placed at a point in the beam or girder that has lower bending stress than the stress at the theoretical cut-off point.

\*\* For provisions governing welded plate girders, see Sect. 1.10.10.

loads. Such stiffeners shall have a close bearing against the flange, or flanges, through which they receive their loads or reactions, and shall extend approximately to the edge of the flange plates or flange angles. They shall be designed as columns subject to the provisions of Sect. 1.5.1, assuming the column section to comprise the pair of stiffeners and a centrally located strip of the web whose width is equal to not more than 25 times its thickness at interior stiffeners or a width equal to not more than 12 times its thickness when the stiffeners are located at the end of the web. The effective length shall be taken as not less than  $\frac{3}{4}$  of the length of the stiffeners in computing the ratio  $l/r$ . Only that portion of the stiffener outside of the flange angle fillet or the flange-to-web welds shall be considered effective in bearing.

**1.10.5.2** Except as hereinafter provided, the largest average web shear,  $f_v$ , in kips per square inch, computed for any condition of complete or partial loading, shall not exceed the value given by Formula (1.10-1).

$$F_v = \frac{F_y}{2.89} (C_v) \leq 0.4F_y \quad (1.10-1)$$

where

$$C_v = \frac{45,000k}{F_y(h/t)^2}, \text{ when } C_v \text{ is less than } 0.8$$

$$= \frac{190}{h/t} \sqrt{\frac{k}{F_y}}, \text{ when } C_v \text{ is more than } 0.8$$

$$k = 4.00 + \frac{5.34}{(a/h)^2}, \text{ when } a/h \text{ is less than } 1.0$$

$$= 5.34 + \frac{4.00}{(a/h)^2}, \text{ when } a/h \text{ is more than } 1.0$$

$t$  = thickness of web, in inches

$a$  = clear distance between transverse stiffeners, in inches

$h$  = clear distance between flanges, in inches

Alternatively, for girders other than hybrid girders, if intermediate stiffeners are provided and spaced to satisfy the provisions of Sect. 1.10.5.3 and if  $C_v \leq 1$ , the allowable shear given by Formula (1.10-2) may be used in lieu of the value given by Formula (1.10-1).

$$F_v = \frac{F_y}{2.89} \left[ C_v + \frac{1 - C_v}{1.15\sqrt{1 + (a/h)^2}} \right] \leq 0.4F_y \quad (1.10-2)^*$$

**1.10.5.3** Intermediate stiffeners are not required when the ratio  $h/t$  is less than 260 and the maximum web shear stress  $f_v$  is less than that permitted by Formula (1.10-1).

The spacing of intermediate stiffeners, where stiffeners are required, shall be such that the web shear stress will not exceed the value for  $F_v$  given by Formulas (1.10-1) or (1.10-2), as applicable, and the ratio  $a/h$  shall not exceed  $\left(\frac{260}{h/t}\right)^2$ , nor 3.0.

\* Formula (1.10-2) recognizes the contribution of tension field action. For values of  $F_v$  provided by this formula, see Tables 3-36 through 3-100 in Appendix A.

In girders designed on the basis of tension field action, the spacing between stiffeners at end panels and panels containing large holes shall be such that the smaller panel dimension,  $a$  or  $h$ , shall not exceed  $348t/\sqrt{f_v}$ .

**1.10.5.4** The moment of inertia of a pair of intermediate stiffeners, or a single intermediate stiffener, with reference to an axis in the plane of the web, shall not be less than  $(h/50)^4$ .

The gross area, in square inches, of intermediate stiffeners spaced as required for Formula (1.10-2) (total area, when stiffeners are furnished in pairs) shall be not less than that computed by Formula (1.10-3).

$$A_{st} = \frac{1 - C_v}{2} \left[ \frac{a}{h} - \frac{(a/h)^2}{\sqrt{1 + (a/h)^2}} \right] Y D h t \quad (1.10-3)$$

where

$C_v$ ,  $a$ ,  $h$  and  $t$  are defined in Sect. 1.10.5.2

$Y = \frac{\text{yield stress of web steel}}{\text{yield stress of stiffener steel}}$

$D = 1.0$  for stiffeners furnished in pairs  
 $= 1.8$  for single angle stiffeners  
 $= 2.4$  for single plate stiffeners

When the greatest shear stress  $f_v$  in a panel is less than that permitted by Formula (1.10-2) this gross area requirement may be reduced in like proportion.

Intermediate stiffeners required by Formula (1.10-2) shall be connected for a total shear transfer, in kips per linear inch of single stiffener or pair of stiffeners, not less than that computed by the formula

$$f_{vs} = h \sqrt[3]{\left(\frac{F_y}{340}\right)^3} \quad (1.10-4)$$

where  $F_y$  = yield stress of web steel.

This shear transfer may be reduced in the same proportion that the largest computed shear stress  $f_v$  in the adjacent panels is less than that permitted by Formula (1.10-2). However, rivets and welds in intermediate stiffeners which are required to transmit to the web an applied concentrated load or reaction shall be proportioned for not less than the applied load or reaction.

Intermediate stiffeners may be stopped short of the tension flange a distance not to exceed 4 times the web thickness, provided bearing is not needed to transmit a concentrated load or reaction. When single stiffeners are used they shall be attached to the compression flange, if it consists of a rectangular plate, to resist any uplift tendency due to torsion in the plate. When lateral bracing is attached to a stiffener, or a pair of stiffeners, these, in turn, shall be connected to the compression flange to transmit 1 percent of the total flange stress, unless the flange is composed only of angles.

Rivets connecting stiffeners to the girder web shall be spaced not more than 12 inches on center. If intermittent fillet welds are used, the clear distance between welds shall not be more than 16 times the web thickness nor more than 10 inches.

### 1.10.6 Reduction in Flange Stress

When the web depth-to-thickness ratio exceeds  $760/\sqrt{F_b}$ , the maximum stress in the compression flange shall not exceed

$$F'_b \leq F_b \left[ 1.0 - 0.0005 \frac{A_w}{A_f} \left( \frac{h}{t} - \frac{760}{\sqrt{F_b}} \right) \right] \quad (1.10-5)$$

where

$F_b$  = applicable bending stress given in Sect. 1.5.1

$A_w$  = area of the web

$A_f$  = area of compression flange

The maximum stress in either flange of a hybrid girder shall not exceed the value given by Formula (1.10-5) nor

$$F'_b \leq F_b \left[ \frac{12 + \left( \frac{A_w}{A_f} \right) (3\alpha - \alpha^3)}{12 + 2 \left( \frac{A_w}{A_f} \right)} \right] \quad (1.10-6)$$

where  $\alpha$  = ratio of web yield stress to flange yield stress.

### 1.10.7 Combined Shear and Tension Stress

Plate girder webs, which depend upon tension field action as provided in Formula (1.10-2) shall be so proportioned that bending tensile stress, due to moment in the plane of the girder web, shall not exceed  $0.6F_v$  nor

$$\left( 0.825 - 0.375 \frac{f_v}{F_v} \right) F_v \quad (1.10-7)$$

where

$f_v$  = computed average web shear stress (total shear divided by web area)

$F_v$  = allowable web shear stress according to Formula (1.10-2)

The allowable shear stress in the webs of girders having A514 flanges and webs shall not exceed the values given by Formula (1.10-1) if the flexural stress in the flange,  $f_b$ , exceeds  $0.75F_b$ .

### 1.10.8 Splices

Groove welded splices in plate girders and beams shall be complete penetration groove welds and shall develop the full strength of the smaller spliced section. Other types of splices in cross-sections of plate girders and in beams shall develop the strength required by the stresses, at the point of splice.

### 1.10.9 Horizontal Forces

The flanges of plate girders supporting cranes or other moving loads shall be proportioned to resist the horizontal forces produced by such loads. (See Sect. 1.3.4.)

### 1.10.10 Web Crippling

1.10.10.1 Webs of beams and welded plate girders shall be so proportioned that the compressive stress at the web toe of the fillets, resulting from concentrated loads not supported by bearing stiffeners, shall not exceed the value of  $0.75F_v$ ; otherwise, bearing stiffeners shall be provided. The governing formulas shall be:

For interior loads,

$$\frac{R}{t(N + 2k)} \leq 0.75F_v \quad (1.10-8)$$

For end-reactions,

$$\frac{R}{t(N + k)} \leq 0.75F_v \quad (1.10-9)$$

where

$R$  = concentrated load or reaction, in kips

$t$  = thickness of web, in inches

$N$  = length of bearing in inches (not less than  $k$  for end reactions)

$k$  = distance from outer face of flange to web toe of fillet, in inches

1.10.10.2 Webs of plate girders shall also be so proportioned or stiffened that the sum of the compression stresses resulting from concentrated and distributed loads, bearing directly on or through a flange plate, upon the compression edge of the web plate, and not supported directly by bearing stiffeners, shall not exceed

$$\left[ 5.5 + \frac{4}{(a/h)^2} \right] \frac{10,000}{(h/t)^2} \text{ kips per square inch} \quad (1.10-10)$$

when the flange is restrained against rotation, nor

$$\left[ 2 + \frac{4}{(a/h)^2} \right] \frac{10,000}{(h/t)^2} \text{ kips per square inch} \quad (1.10-11)$$

when the flange is not so restrained.

These stresses shall be computed as follows:

Concentrated loads and loads distributed over partial length of a panel shall be divided by the product of the web thickness and the girder depth or the length of panel in which the load is placed, whichever is the lesser panel dimension.

Any other distributed loading, in kips per linear inch of length, shall be divided by the web thickness.

### 1.10.11 Rotational Restraint at Points of Support

Beams, girders and trusses shall be restrained against rotation, about their longitudinal axis, at points of support.

## SECTION 1.11 COMPOSITE CONSTRUCTION

### 1.11.1 Definition

Composite construction shall consist of steel beams or girders supporting a reinforced concrete slab, so inter-connected that the beam and slab act together to resist bending. When the slab extends on both sides of the beam, the effective width of the concrete flange shall be taken as not more than one-fourth of the span of the beam, and its effective projection beyond the edge of the beam shall not be taken as more than one-half the clear distance to the adjacent beam, nor more than eight times the slab thickness. When the slab is present on only one side of the beam, the effective width of the concrete flange (projection beyond the beam) shall be taken as not more than one-twelfth of the beam span, nor six times its thickness, nor one-half the clear distance to the adjacent beam.

Beams totally encased 2 inches or more on their sides and soffit in concrete cast integrally with the slab may be assumed to be inter-connected to the concrete by natural bond, without additional anchorage, provided the top of the beam is at least  $1\frac{1}{2}$  inches below the top and 2 inches above the bottom of the slab, and provided that the encasement has adequate mesh or other reinforcing steel throughout the whole depth and across the soffit of the beam to prevent spalling of the concrete. When shear connectors are provided in accordance with Sect. 1.11.4, encasement of the beam to achieve composite action is not required.

### 1.11.2 Design Assumptions

**1.11.2.1** Encased beams shall be proportioned to support unassisted all dead loads applied prior to the hardening of the concrete (unless these loads are supported temporarily on shoring) and, acting in conjunction with the slab, to support all dead and live loads applied after hardening of the concrete, without exceeding a computed bending stress of  $0.66F_y$ , where  $F_y$  is the yield stress of the steel beam. The bending stress produced by loads after the concrete has hardened shall be computed on the basis of the section properties of the composite section. Concrete tension stresses shall be neglected. Alternatively, the steel beam alone may be proportioned to resist unassisted the positive moment produced by all loads, live and dead using a bending stress equal to  $0.76F_y$ , in which case temporary shoring is not required.

**1.11.2.2** When shear connectors are used in accordance with Sect. 1.11.4 the composite section shall be proportioned to support all of the loads without exceeding the allowable stress prescribed in Sect. 1.5.1.4, even when the steel section is not shored during construction.

Reinforcement parallel to the beam within the effective width of the slab, when anchored in accordance with the provisions of the applicable code, may be included in computing the properties of composite section subject to negative bending moment, provided shear connectors are furnished in accordance with the requirements of Sect. 1.11.4. The section properties of the composite section shall be computed in accordance with the elastic theory. Concrete tension stresses shall be neglected. The compression area of the concrete on the compression side of the neutral axis shall be treated as an equivalent area of steel by dividing it by the modular ratio  $n$ .

In cases where it is not feasible or necessary to provide adequate connectors to satisfy the horizontal shear requirements for full composite action, the effective section modulus shall be determined as

$$S_{eff} = S_s + \frac{V'_h}{V_h} (S_{tr} - S_s) \quad (1.11-1)$$

where

$V_h$  and  $V'_h$  are as defined in Sect. 1.11.4

$S_s$  = section modulus of the steel beam referred to its bottom flange

$S_{tr}$  = section modulus of the transformed composite section referred to its bottom flange

For construction without temporary shoring, the value of the section modulus of the transformed composite section used in stress calculations (referred to the bottom flange of the steel beam) shall not exceed

$$S_{tr} = \left( 1.35 + 0.35 \frac{M_L}{M_D} \right) S_s \quad (1.11-2)$$

where  $M_L$  is the moment caused by loads applied subsequent to the time when the concrete has reached 75 percent of its required strength,  $M_D$  is the moment caused by loads applied prior to this time, and  $S_s$  is the section modulus of the steel beam (referred to its bottom flange). The steel beam alone, supporting the loads before the concrete has hardened, shall not be stressed to more than the applicable bending stress given in Sect. 1.5.1.

The actual section modulus of the transformed composite section shall be used in calculating the concrete flexural compression stress and, for construction without temporary shores, this stress shall be based upon loading applied after the concrete has reached 75 percent of its required strength. The stress in the concrete shall not exceed  $0.45f'_c$ .

### 1.11.3 End Shear

The web and the end connections of the steel beam shall be designed to carry the total dead and live load.

### 1.11.4 Shear Connectors

Except in the case of encased beams as defined in Sect. 1.11.1, the entire horizontal shear at the junction of the steel beam and the concrete slab shall be assumed to be transferred by shear connectors welded to the top flange of the beam and embedded in the concrete. For full composite action with concrete subject to flexural compression, the total horizontal shear to be resisted between the point of maximum positive moment and points of zero moment shall be taken as the smaller value using Formulas (1.11-3) and (1.11-4).

$$V_h = \frac{0.85f'_c A_c}{2} \quad (1.11-3)$$

and

$$V_h = \frac{A_s F_y}{2} \quad (1.11-4)$$

where

- $f_c$  = specified compression strength of concrete
- $A_c$  = actual area of effective concrete flange defined in Sect. 1.11.1
- $A_s$  = area of steel beam

In continuous composite beams where longitudinal reinforcing steel is considered to act compositely with the steel beam in the negative moment regions, the total horizontal shear to be resisted by shear connectors between an interior support and each adjacent point of contraflexure shall be taken as

$$V_h = \frac{A_{sr} F_{yr}}{2} \quad (1.11-5)$$

where

- $A_{sr}$  = total area of longitudinal reinforcing steel at the interior support located within the effective flange width specified in Sect. 1.11.1
- $F_{yr}$  = specified minimum yield stress of the longitudinal reinforcing steel

For full composite action, the number of connectors resisting the horizontal shear,  $V_h$ , each side of the point of maximum moment, shall not be less than that determined by the relationship  $V_h/q$ , where  $q$ , the allowable shear load for one connector, is given in Table 1.11.4. Working values for use with concrete having aggregate not conforming to ASTM C33 and for connector types other than those shown in Table 1.11.4 must be established by a suitable test program.

TABLE 1.11.4

Connector	Allowable Horizontal Shear Load ( $q$ ) (kips)		
	(Applicable only to concrete made with ASTM C33 aggregates)		
	$f'_c$ (kips per square inch)		
	3.0	3.5	4.0
½" diam. × 2" hooked or headed stud	5.1	5.5	5.9
⅝" diam. × 2½" hooked or headed stud	8.0	8.6	9.2
¾" diam. × 3" hooked or headed stud	11.5	12.5	13.3
⅞" diam. × 3½" hooked or headed stud	15.6	16.8	18.0
3" channel, 4.1 lb.	4.3w	4.7w	5.0w
4" channel, 5.4 lb.	4.6w	5.0w	5.3w
5" channel, 6.7 lb.	4.9w	5.3w	5.6w

$w$  = length of channel in inches.

For incomplete composite action with concrete subject to flexural compression, the horizontal shear,  $V_h$ , to be used in computing  $S_{eff}$  shall be taken as the product of  $q$  times the number of connectors furnished between the point of maximum moment and the nearest point of zero moment.

The connectors required each side of the point of maximum moment in an area of positive bending may be uniformly distributed between that point and adjacent points of zero moment, except that  $N_2$ , the number of

shear connectors required between any concentrated load in that area and the nearest point of zero moment, shall be not less than that determined by Formula (1.11-6).

$$N_2 = \frac{N_1 \left[ \frac{M\beta}{M_{max}} - 1 \right]}{\beta - 1} \quad (1.11-6)$$

where

- $M$  = moment (less than the maximum moment) at a concentrated load point
- $N_1$  = number of connectors required between point of maximum moment and point of zero moment, determined by the relationship  $V_h/q$  or  $V'_h/q$ , as applicable
- $\beta$  =  $\frac{S_{tr}}{S_s}$  or  $\frac{S_{eff}}{S_s}$ , as applicable

Connectors required in the region of negative bending on a continuous beam may be uniformly distributed between the point of maximum moment and each point of zero moment.

Shear connectors shall have at least 1 inch of concrete cover in all directions. Unless located directly over the web, the diameter of studs shall not be greater than 2.5 times the thickness of the flange to which they are welded.

## SECTION 1.12 SIMPLE AND CONTINUOUS SPANS

### 1.12.1 Simple Spans

Beams, girders and trusses shall ordinarily be designed on the basis of simple spans whose effective length is equal to the distance between centers of gravity of the members to which they deliver their end reactions.

### 1.12.2 End Restraint

When designed on the assumption of full or partial end restraint, due to continuous, semi-continuous or cantilever action, the beams, girders and trusses, as well as the sections of the members to which they connect, shall be designed to carry the shears and moments so introduced, as well as all other forces, without exceeding at any point the unit stresses prescribed in Sect. 1.5.1; except that some non-elastic but self-limiting deformation of a part of the connection may be permitted when this is essential to the avoidance of overstressing of fasteners.

## SECTION 1.13 DEFLECTIONS, VIBRATION, AND PONDING

### 1.13.1 Deflections

Beams and girders supporting floors and roofs shall be proportioned with due regard to the deflection produced by the design loads. Beams and girders supporting plastered ceilings shall be so proportioned that the maximum live load deflection does not exceed  $\frac{1}{360}$  of the span.

### 1.13.2 Vibration

Beams and girders supporting large open floor areas free of partitions or other sources of damping, where transient vibration due to pedestrian traffic might not be acceptable, shall be designed with due regard for vibration.

### 1.13.3 Ponding

Unless a roof surface is provided with sufficient slope toward points of free drainage or adequate individual drains to prevent the accumulation of rain water, the roof system shall be investigated by rational analysis to assure stability under ponding conditions, except as follows:

The roof system shall be considered stable and no further investigation will be needed if

$$C_p + 0.9C_s \leq 0.25 \text{ and } I_d \geq 25S^4/10^6$$

where

$$C_p = \frac{32L_p L_p^4}{10^7 I_p} \text{ and } C_s = \frac{32S L_s^4}{10^7 I_s}$$

$L_p$  = Column spacing in direction of girder, feet (length of primary members)

$L_s$  = Column spacing perpendicular to direction of girder, feet (length of secondary member)

$S$  = Spacing of secondary members, feet

$I_p$  = Moment of inertia for primary members, inches<sup>4</sup>

$I_s$  = Moment of inertia for secondary member, inches<sup>4</sup>

$I_d$  = Moment of inertia of the steel deck supported on secondary members, inches<sup>4</sup> per foot

For trusses and steel joists, the moment of inertia,  $I_s$ , shall be decreased 15 percent when used in the above formulas. A steel deck shall be considered a secondary member when it is directly supported by the primary members.

Total bending stress due to dead loads, gravity live loads (if any) and ponding shall not exceed  $0.80F_y$  for primary and secondary members. Stresses due to wind or seismic forces need not be included in a ponding analysis.

## SECTION 1.14 GROSS AND NET SECTIONS

### 1.14.1 Definitions

The gross section of a member at any point shall be determined by summing the products of the thickness and the gross width of each element as measured normal to the axis of the member. The net section shall be determined by substituting for the gross width the net width computed in accordance with Sects. 1.14.3 to 1.14.6, inclusive.

### 1.14.2 Application

Unless otherwise specified, tension members shall be designed on the basis of net section. Compression members shall be designed on the basis of gross section. Beams and girders shall be designed in accordance with Sect. 1.10.1.

### 1.14.3 Net Section

In the case of a chain of holes extending across a part in any diagonal or zigzag line, the net width of the part shall be obtained by deducting from the gross width the sum of the diameters of all the holes in the chain, and adding, for each gage space in the chain, the quantity

$$\frac{s^2}{4g}$$

where

$s$  = longitudinal spacing (pitch, in inches) of any two consecutive holes

$g$  = transverse spacing (gage, in inches) of the same two holes

The critical net section of the part is obtained from that chain which gives the least net width; however, the net section taken through a hole shall in no case be considered as more than 85 percent of the corresponding gross section.

In determining the net section across plug or slot welds, the weld metal shall not be considered as adding to the net area.

### 1.14.4 Angles

For angles, the gross width shall be the sum of the widths of the legs less the thickness. The gage for holes in opposite legs shall be the sum of the gages from back of angles less the thickness.

### 1.14.5 Size of Holes

In computing net area the diameter of a rivet or bolt hole shall be taken as  $\frac{1}{8}$ -inch greater than the nominal diameter of the rivet or bolt.

### 1.14.6 Pin-Connected Members

Eyebars shall be of uniform thickness without reinforcement at the pin holes.\* They shall have "circular" heads in which the periphery of the head beyond the pin hole is concentric with the pin hole. The radius of transition between the circular head and the body of the eyebar shall be equal to or greater than the diameter of the head.

The width of the body of the eyebar shall not exceed 8 times its thickness, and the thickness shall not be less than  $\frac{1}{2}$ -inch. The net section of the head through the pin hole, transverse to the axis of the eyebar, shall not be less than 1.33 nor more than 1.50 times the cross-sectional area of the body of the eyebar. The diameter of the pin shall not be less than  $\frac{7}{8}$  the width of the body of the eyebar. The diameter of the pin hole shall not be more than  $\frac{1}{32}$ -inch greater than the diameter of the pin. For steels having a yield stress greater than 70 ksi, the diameter of the pin hole shall not exceed 5 times the plate thickness.

The minimum net section across the pin hole, transverse to the axis of the member, in pin-connected plates and built-up members shall be determined at the stress allowed for such sections in Sect. 1.5.1.1. The net section beyond the pin hole, parallel to the axis of the member, shall not be less than  $\frac{2}{3}$  of the net section across the pin hole. The corners beyond the pin hole may be cut

\* Members having a different thickness at the pin hole location termed "built-up."

at 45° to the axis of the member provided the net section beyond the pin hole on a line perpendicular to the cut is not less than that required beyond the pin hole parallel to the axis of the member. The parts of members built up at the pin hole shall be attached to each other by sufficient fasteners to support the stress delivered to them by the pin.

The distance transverse to the axis of a pin-connected plate or any separated element of a built-up member, from the edge of the pin hole to the edge of the member or element, shall not exceed 4 times the thickness at the pin hole. The diameter of the pin hole shall not be less than 1.25 times the smaller of the distances from the edge of the pin hole to the edge of a pin-connected plate or separated element of a built-up member at the pin hole. The diameter of the pin hole shall not be more than 1/32-inch greater than the diameter of the pin. In the case of pin-connected plates of uniform thickness, for steels having a yield stress greater than 70 ksi, the diameter of the pin hole shall not exceed 5 times the plate thickness.

Thickness limitations on both eyebars and pin-connected plates may be waived whenever external nuts are provided so as to tighten pin plates and filler plates into snug contact. When the plates are thus contained, the allowable stress in bearing shall be no greater than as specified in Sect. 1.5.1.5.1.

#### 1.14.7 Effective Areas of Weld Metal

The effective area of groove and fillet welds shall be considered as the effective length of the weld times the effective throat thickness.

The effective shearing area of plug and slot welds shall be considered as the nominal cross-sectional area of the hole or slot, in the plane of the faying surface.

The effective area of fillet welds in holes and slots shall be computed as above specified for fillet welds, using  $t$  or effective length, the length of centerline of the weld through the center of the plane through the throat. However, in the case of overlapping fillets, the effective area shall not exceed the nominal cross-sectional area of the hole or slot, in the plane of the faying surface.

The effective length of a fillet weld shall be the overall length of full-size fillet including returns.

The effective length of a groove weld shall be the width of the part joined.

The effective throat thickness of a fillet weld shall be the shortest distance from the root to the face of the diagrammatic weld, except that, for fillet welds made by the submerged arc process, the effective throat thickness shall be taken equal to the leg size for 3/8-inch and smaller fillet welds, and equal to the theoretical throat plus 0.11-inch for fillet welds over 3/8-inch.

The effective throat thickness of a complete penetration groove weld (i.e., a groove weld conforming to the requirements of Sect. 1.23.6) shall be the thickness of the thinner part joined.

The effective throat thickness of single and double partial penetration groove welds shall be the depth of the groove, except that the effective throat thickness of a bevel joint made by manual shielded metal-arc welding shall be 1/8-inch less than the depth of the groove, and the effective throat thickness of each weld shall be not less than  $\sqrt{t_i/6}$ , where  $t_i$  is the thickness of the thinner part connected by the weld.

## SECTION 1.15 CONNECTIONS

### 1.15.1 Minimum Connections

Connections carrying calculated stresses, except for lacing, sag bars, and girts, shall be designed to support not less than 6 kips.

### 1.15.2 Eccentric Connections

Axially stressed members meeting at a point shall have their gravity axes intersect at a point if practicable; if not, provision shall be made for bending stresses due to the eccentricity.

### 1.15.3 Placement of Rivets, Bolts, and Welds

Except as hereinafter provided, groups of rivets, bolts or welds at the ends of any member transmitting axial stress into that member shall have their centers of gravity on the gravity axis of the member unless provision is made for the effect of the resulting eccentricity. Except in members subject to repeated variation in stress, as defined in Sect. 1.7, disposition of fillet welds to balance the forces about the neutral axis or axes for end connections of single angle, double angle, and similar type members is not required. Eccentricity between the gravity axes of such members and the gage lines for their riveted or bolted end connections may be neglected.

### 1.15.4 Unrestrained Members

Except as otherwise indicated by the designer, connections of beams, girders or trusses shall be designed as flexible, and may ordinarily be proportioned for the reaction shears only.

Flexible beam connections shall permit the ends of the beam to rotate sufficiently to accommodate its deflection by providing for a horizontal displacement of the top flange determined as follows:

$$e = 0.007d, \text{ when the beam is designed for full uniform load and for live load deflection not exceeding } \frac{1}{360} \text{ of the span}$$

$$= \frac{f_b L}{3,600}, \text{ when the beam is designed for full uniform load producing the stress } f_b \text{ at mid-span}$$

where

$$e = \text{the horizontal displacement of the end of the top flange, in the direction of the span, in inches}$$

$$f_b = \text{the flexural stress in the beam at mid-span, in kips per square inch}$$

$$d = \text{the depth of the beam, in inches}$$

$$L = \text{the span of the beam, in feet}$$

### 1.15.5 Restrained Members

Fasteners or welds for end connections of beams, girders and trusses not conforming to the requirements of Sect. 1.15.4 shall be designed for the combined effect of end reaction shear and tensile or compressive stresses resulting from moment induced by the rigidity of the connection when the member is fully loaded.\*

\* For a discussion of high column web shear stress opposite rigid beam connections, see Commentary Sect. 1.5.1.2.

When fully restrained beams are framed to the flange of an I- or H-shape column, stiffeners shall be provided on the column web as follows:

$$\text{Opposite the compression flange when } t < \frac{C_1 A_f}{t_b + 5k} \quad (1.15-1)$$

$$\text{or when } t \leq \frac{d_c}{5\sqrt{F_y}} \quad (1.15-2)$$

$$\text{Opposite the tension flange when } t_f < 0.4\sqrt{C_1 A_f} \quad (1.15-3)$$

where

- $t$  = thickness of web to be stiffened
- $k$  = distance from outer face of flange to web toe of fillet of member to be stiffened, if a member is a rolled shape  
= flange thickness plus the distance to the farthest toe of the connecting weld, if a member is a welded section
- $t_b$  = thickness of flange delivering concentrated load
- $t_f$  = thickness of flange of member to be stiffened
- $A_f$  = area of flange delivering concentrated load
- $d_c$  = column web depth clear of fillets
- $C_1$  = ratio of beam flange yield stress to column yield stress
- $C_2$  = ratio of column yield stress to stiffener yield stress

The area of such stiffeners,  $A_{st}$ , shall be such that

$$A_{st} \geq [C_1 A_f - t(t_b + 5k)]C_2 \quad (1.15-4)$$

Their ends shall be welded to the inside face of the flange opposite the concentrated tensile load, so as to transfer the load from the beam flange to the column web. The stiffeners may be fitted against the inside face of the flange opposite the concentrated compression load. When the concentrated load delivered by a beam occurs on one side only, the web stiffener need not exceed one-half the depth of the member, but the welding connecting it to the web shall be sufficient to develop  $F_y A_{st}$ .

### 1.15.6 Fillers

When rivets or bolts carrying computed stress pass through fillers thicker than  $\frac{1}{4}$ -inch, except in friction-type connections assembled with high strength bolts, the fillers shall be extended beyond the splice material and the filler extension shall be secured by enough rivets or bolts to distribute the total stress in the member uniformly over the combined section of the member and the filler, or an equivalent number of fasteners shall be included in the connection.

In welded construction, any filler  $\frac{1}{4}$ -inch or more in thickness shall extend beyond the edges of the splice plate and shall be welded to the part on which it is fitted with sufficient weld to transmit the splice plate stress, applied at the surface of the filler as an eccentric load. The welds joining the splice plate to the filler shall be sufficient to transmit the splice plate stress and shall be long enough to avoid overstressing the filler along the toe of the weld. Any filler less than  $\frac{1}{4}$ -inch thick shall have its edges made flush with the edges of the splice plate and the weld size shall be the sum of the size necessary to carry the splice plate stress plus the thickness of the filler plate.

### 1.15.7 Connections of Tension and Compression Members in Trusses

The connections at ends of tension or compression members in trusses shall develop the force due to the design load, but not less than 50 percent of the effective strength of the member.

### 1.15.8 Compression Members with Bearing Joints

Where columns bear on bearing plates, or are finished to bear at splices, there shall be sufficient rivets, bolts, or welds to hold all parts securely in place.

Where other compression members are finished to bear, the splice material and its riveting, bolting or welding shall be arranged to hold all parts in line and shall be proportioned for 50 percent of the computed stress.

All of the foregoing joints shall be proportioned to resist any tension that would be developed by specified lateral forces acting in conjunction with 75 percent of the calculated dead load stress and no live load.

### 1.15.9 Combination of Welds

If two or more of the general types of weld (groove, fillet, plug, slot) are combined in a single joint, the effective capacity of each shall be separately computed with reference to the axis of the group, in order to determine the allowable capacity of the combination.

### 1.15.10 Rivets and Bolts in Combination with Welds

In new work, rivets, A307 bolts, or high strength bolts used in bearing-type connections, shall not be considered as sharing the stress in combination with welds. Welds, if used, shall be provided to carry the entire stress in the connection. High strength bolts installed in accordance with the provisions of Sect. 1.16.1 as a friction-type connection prior to welding may be considered as sharing the stress with the welds.

In making welded alterations to structures, existing rivets and properly tightened high strength bolts may be utilized for carrying stresses resulting from existing dead loads, and the welding need be adequate only to carry all additional stress.

### 1.15.11 High Strength Bolts (in Friction-Type Joints) in Combination with Rivets

In new work and in making alterations, rivets and high strength bolts, installed in accordance with the provisions of Sect. 1.16.1 as friction-type connections, may be considered as sharing the stresses resulting from dead and live loads.

### 1.15.12 Field Connections

Rivets, high strength bolts or welds shall be used for the following connections:

- Column splices in all tier structures 200 feet or more in height.
- Column splices in tier structures 100 to 200 feet in height, if the least horizontal dimension is less than 40 percent of the height.
- Column splices in tier structures less than 100 feet in height, if the least horizontal dimension is less than 25 percent of the height.



TABLE 1.16.5

Rivet or Bolt Diameter (Inches)	Minimum Edge Distance for Punched, Reamed or Drilled Holes (Inches)	
	At Sheared Edges	At Rolled Edges of Plates, Shapes or Bars or Gas Cut Edges**
$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{4}$
$\frac{3}{8}$	$1\frac{1}{8}$	$\frac{3}{4}$
$\frac{1}{2}$	$1\frac{1}{4}$	1
$\frac{5}{8}$	$1\frac{3}{8}$ *	$1\frac{1}{8}$
1	$1\frac{3}{8}$ *	$1\frac{1}{4}$
$1\frac{1}{8}$	2	$1\frac{1}{2}$
$1\frac{1}{4}$	$2\frac{1}{4}$	$1\frac{3}{4}$
Over $1\frac{1}{4}$	$1\frac{1}{4} \times \text{Diameter}$	$1\frac{1}{4} \times \text{Diameter}$

\* These may be  $1\frac{1}{4}$ -in. at the ends of beam connection angles.

\*\* All edge distances in this column may be reduced  $\frac{1}{8}$ -in. when the hole is at a point where stress does not exceed 25% of the maximum allowed stress in the element.

Connections of all beams and girders to columns and of any other beams and girders on which the bracing of columns is dependent, in structures over 125 feet in height.

Roof-truss splices and connections of trusses to columns, column splices, column bracing, knee braces and crane supports, in all structures carrying cranes of over 5-ton capacity.

Connections for supports of running machinery, or of other live loads which produce impact or reversal of stress.

Any other connections stipulated on the design plans.

In all other cases field connections may be made with A307 bolts.

For the purpose of this Section, the height of a tier structure shall be taken as the vertical distance from the curb level to the highest point of the roof beams, in the case of flat roofs, or to the mean height of the gable, in the case of roofs having a rise of more than  $2\frac{2}{3}$  in 12. Where the curb level has not been established, or where the structure does not adjoin a street, the mean level of the adjoining land shall be used instead of curb level. Penthouses may be excluded in computing the height of structure.

## SECTION 1.16 RIVETS AND BOLTS

### 1.16.1 High Strength Bolts

Use of high strength bolts shall conform to the provisions of the *Specifications for Structural Joints Using ASTM A325 or A490 Bolts* as approved by the Research Council on Riveted and Bolted Structural Joints. ASTM A449 bolts no greater than  $1\frac{1}{2}$  inches in diameter may be used in lieu of ASTM A325 bolts, provided that a hardened washer is installed under the bolt head. However, nuts used with A449 bolts shall meet the requirements of ASTM A325.

### 1.16.2 Effective Bearing Area

The effective bearing area of rivets and bolts shall be the diameter multiplied by the length in bearing, except that for countersunk rivets and bolts half the depth of the countersink shall be deducted.

### 1.16.3 Long Grips

Rivets and A307 bolts which carry calculated stress, and the grip of which exceeds 5 diameters, shall have their number increased 1 percent for each additional  $\frac{1}{16}$ -inch in the grip.

### 1.16.4 Minimum Pitch

The minimum distance between centers of rivet and bolt holes shall be not less than  $2\frac{2}{3}$  times the nominal diameter of the rivet or bolt but preferably not less than 3 diameters.

### 1.16.5 Minimum Edge Distance

The minimum distance from the center of a rivet or bolt hole to any edge, used in design or in preparation of shop drawings, shall be that given in Table 1.16.5.

### 1.16.6 Minimum Edge Distance in Line of Stress

**1.16.6.1** In connections of tension members, where there are not more than two rivets in a line parallel to the direction of stress, the distance from the center of the end rivet to that end of the connected part toward which the stress is directed shall be not less than the area of the rivet divided by the thickness of the connected part for rivets in single shear or twice this distance for rivets in double shear.

**1.16.6.2** In bearing-type connections of tension members, where there are not more than two high strength bolts in a line parallel to the direction of stress, the distance from the center of the end bolt to that end of the connected part toward which the stress is directed shall be not less than  $A_b C/t$  for single shear or  $2A_b C/t$  for double shear, where  $A_b$  is the nominal cross-sectional area of the bolt,  $t$  is the thickness of the connected part, and  $C$  is the ratio of specified minimum tensile strength of the bolt to the specified minimum tensile strength of the connected part.

**1.16.6.3** However, the end distance prescribed in Sects. 1.16.6.1 and 1.16.6.2 may be decreased in such proportion as the fastener stress is less than that permitted in Sect. 1.5.2, but it shall not be less than the distance specified in Sect. 1.16.5 and need not exceed  $1\frac{1}{2}$  times the transverse spacing of fasteners.

**1.16.6.4** When more than two fasteners are provided in the line of stress, the provisions of Sect. 1.16.5 shall govern.

### 1.16.7 Maximum Edge Distance

The maximum distance from the center of any rivet or bolt to the nearest edge of parts in contact with one another shall be 12 times the thickness of the plate, but shall not exceed 6 inches.

**1.17.1 Welder, Tacker, and Welding Operator Qualifications**

Welds shall be made only by welders, tackers, and welding operators who have been previously qualified by tests as prescribed in the *Code for Welding in Building Construction*, AWS D1.0-69, of the American Welding Society to perform the type of work required.

**1.17.2 Qualification of Weld and Joint Details**

Weld grooves for complete and partial penetration welds which are accepted without welding procedure qualification under the provisions of AWS D1.0-69, may be used under this specification without welding procedure qualification.

Joint forms, details, welding processes, or welding procedures other than those included in the foregoing may be employed provided they shall have been qualified in accordance with the requirements of AWS D1.0-69.

The electrodes or flux specified in Table 1.17.2 shall be used in making complete penetration groove welds designed on the basis of the allowable stresses for the base metal, as provided in Table 1.5.3. The electrodes and fluxes as listed in Table 1.5.3 may be used in making fillet welds and partial penetration groove welds.

Welding of A440 steel is not recommended.

**1.17.3 Submerged-Arc, Gas Metal-Arc, and Flux Cored-Arc Welding of High Strength Steel**

Electrodes for use in submerged-arc, gas metal-arc, and flux cored-arc welding listed in Tables 1.5.3 and 1.17.2 by grade designation and not covered in AWS A5.17, A5.18 or A5.20, shall meet the provisions of Sections 412, 417 or 418 of AWS D1.0-69, as applicable.

**1.17.4 Electroslag and Electrode Gas Welding**

Weld metal deposited by the electroslag or electrode gas welding process shall conform to the requirements of Article 422 of AWS D1.0-69. Weldments of A514 steel, made by either process, shall be quenched and tempered after welding.

**1.17.5 Minimum Size of Fillet Welds**

In joints connected only by fillet welds, the minimum size of fillet weld to be used shall be as shown in Table 1.17.5. Weld size is determined by the thicker of the two parts joined, except that the weld size need not exceed the thickness of the thinner part joined unless a larger size is required by calculated stress:

TABLE 1.17.5

Material Thickness of Thicker Part Joined (Inches)	Minimum Size of Fillet Weld (Inches)	Material Thickness of Thicker Part Joined (Inches)	Minimum Size of Fillet Weld (Inches)
To 1/4 inclusive	1/8	Over 1 1/2 to 2 1/4 Over 2 1/4 to 6 Over 6	3/8
Over 1/4 to 1/2	3/16		1/2
Over 1/2 to 3/4	1/4		5/8
Over 3/4 to 1 1/2	5/16		

TABLE 1.17.2

Base Metal <sup>2</sup>	Welding Process <sup>1</sup> :			
	Shielded Metal-Arc	Submerged-Arc	Gas Metal-Arc	Flux Cored-Arc
ASTM A36, A53 Gr. B, A375, A500, A501, A529, and A570 Gr. D and E	AWS A5.1 or A5.5, E60XX or E70XX <sup>3</sup>	AWS A5.17 F6X or F7X-EXXX	AWS A5.18 E70S-X or E70U-1	AWS 5.20 E60T-X or E70T-X (except EXXT-2 and EXX-3)
ASTM A242, A441, A572 Grades 42 thru 60 and A588 <sup>4</sup>	AWS A5.1 or A5.5, E70XX <sup>3</sup>	AWS A5.17 F7X-EXXX	AWS A5.18 E70S-X or E70U-1	AWS 5.20 E70T-X (except E70T-2 and E70T-3)
ASTM A572 Grade 65	AWS A5.5 E80XX <sup>3</sup>	Grade F80	Grade E80S	Grade E80T
ASTM A514 over 2 1/2" thick	AWS A5.5 E100XX <sup>3</sup>	Grade F100	Grade E100S	Grade E100T
ASTM A514 2 1/2" thick and under	AWS A5.5 E110XX <sup>3</sup>	Grade F110	Grade E110S	Grade E110T

Use of the same type filler metal having next higher mechanical properties is permitted.

<sup>1</sup> When welds are to be stress relieved the deposited weld metal shall not exceed 0.05 percent vanadium.

See Article 422 of AWS D1.0-69 for electroslag and electrode gas weld metal requirements.

<sup>2</sup> In joints involving base metals of different yield strengths, filler metals applicable to the lower yield strength may be used.

<sup>3</sup> For architectural exposed bare unpainted applications, the deposited weld metal shall have similar atmospheric corrosion resistance and coloring characteristics as the base metal used. The steel manufacturer's recommendation shall be followed.

<sup>4</sup> Low hydrogen classifications.

### 1.17.11 Maximum Effective Size of Fillet Welds

The maximum size of a fillet weld that may be assumed in the design of a connection shall be such that the stresses in the adjacent base material do not exceed the values allowed in Sect. 1.5.1. The maximum size that may be used along edges of connected parts shall be:

1. Along edges of material less than  $\frac{1}{4}$ -inch thick, the maximum size may be equal to the thickness of the material.
2. Along edges of material  $\frac{1}{4}$ -inch or more in thickness, the maximum size shall be  $\frac{1}{16}$ -inch less than the thickness of the material, unless the weld is especially designated on the drawings to be built out to obtain full throat thickness.

### 1.17.7 Length of Fillet Welds

The minimum effective length of a strength fillet weld shall be not less than 4 times the nominal size, or else the size of the weld shall be considered not to exceed one-fourth of its effective length.

If longitudinal fillet welds are used alone in end connections of flat bar tension members, the length of each fillet weld shall be not less than the perpendicular distance between them. The transverse spacing of longitudinal fillet welds used in end connections shall not exceed 8 inches, unless the design otherwise prevents excessive transverse bending in the connection.

### 1.17.8 Intermittent Fillet Welds

Intermittent fillet welds may be used to transfer calculated stress across a joint or faying surfaces when the strength required is less than that developed by a continuous fillet weld of the smallest permitted size, and to join components of built-up members. The effective length of any segment of intermittent fillet welding shall be not less than 4 times the weld size with a minimum of  $1\frac{1}{2}$  inches.

### 1.17.9 Lap Joints

The minimum amount of lap on lap joints shall be 5 times the thickness of the thinner part joined and not less than 1 inch. Lap joints joining plates or bars subjected to axial stress shall be fillet welded along the end of both lapped parts except where the deflection of the lapped parts is sufficiently restrained to prevent opening of the joint under maximum loading.

### 1.17.10 End Returns of Fillet Welds

Side or end fillet welds terminating at ends or sides, respectively, of parts or members shall, wherever practicable, be returned continuously around the corners for a distance not less than twice the nominal size of the weld. This provision shall apply to side and top fillet welds connecting brackets, beam seats and similar connections, on the plane about which bending moments are computed. End returns shall be indicated on the design and detail drawings.

### 1.17.11 Fillet Welds in Holes and Slots

Fillet welds in holes or slots may be used to transmit shear in lap joints or to prevent the buckling or separation of lapped parts, and to join components of built-up members. Such fillet welds may overlap, subject to the provisions of Sect. 1.14.7. Fillet welds in holes or slots are not to be considered plug or slot welds.

### 1.17.12 Plug and Slot Welds

Plug or slot welds may be used to transmit shear in a lap joint or to prevent buckling of lapped parts and to join component parts of built-up members.

The diameter of the holes for a plug weld shall be not less than the thickness of the part containing it plus  $\frac{5}{16}$ -inch, rounded to the next greater odd  $\frac{1}{16}$ -inch, nor greater than  $2\frac{1}{4}$  times the thickness of the weld metal.

The minimum center-to-center spacing of plug welds shall be 4 times the diameter of the hole.

The length of slot for a slot weld shall not exceed 10 times the thickness of the weld. The width of the slot shall be not less than the thickness of the part containing it, plus  $\frac{5}{16}$ -inch, rounded to the next greater odd  $\frac{1}{16}$ -inch, nor shall it be greater than  $2\frac{1}{4}$  times the thickness of the weld. The ends of the slot shall be semicircular or shall have the corners rounded to a radius not less than the thickness of the part containing it, except those ends which extend to the edge of the part.

The minimum spacing of lines of slot welds in a direction transverse to their length shall be 4 times the width of the slot. The minimum center-to-center spacing in a longitudinal direction on any line shall be 2 times the length of the slot.

The thickness of plug or slot welds in material  $\frac{5}{8}$ -inch or less in thickness shall be equal to the thickness of the material. In material over  $\frac{5}{8}$ -inch in thickness, it shall be at least one-half the thickness of the material but not less than  $\frac{5}{8}$ -inch.

## SECTION 1.18 BUILT-UP MEMBERS

### 1.18.1 Open Box-Type Beams and Grillages

Where two or more rolled beams or channels are used side-by-side to form a flexural member, they shall be connected together at intervals of not more than 5 feet. Through-bolts and separators may be used, provided that in beams having a depth of 12 inches or more, no fewer than 2 bolts shall be used at each separator location. When concentrated loads are carried from one beam to the other, or distributed between the beams, diaphragms having sufficient stiffness to distribute the load shall be riveted, bolted or welded between the beams. Where beams are exposed, they shall be sealed against corrosion of interior surfaces, or spaced sufficiently far apart to permit cleaning and painting.

### 1.18.2 Compression Members

1.18.2.1 All parts of built-up compression members and the transverse spacing of their lines of fasteners shall meet the requirements of Sects. 1.8 and 1.9.

1.18.2.2 At the ends of built-up compression members bearing on base plates or milled surfaces, all components in contact with one another shall be connected by rivets or bolts spaced longitudinally not more than 4 diameters apart for a distance equal to  $1\frac{1}{2}$  times the maximum width of the member, or by continuous welds having a length not less than the maximum width of the member.

1.18.2.3 The longitudinal spacing for intermediate rivets, bolts or intermittent welds in built-up members shall be adequate to provide for the transfer of calculated stress. However, where a component of a built-up compression member consists of an outside plate, the maximum spacing shall not exceed the thickness of the thinner outside plate times  $127/\sqrt{F_v}$  when rivets are provided on all gage lines at each section, or when intermittent welds are provided along the edges of the components, but this spacing shall not exceed 12 inches. When rivets or bolts are staggered, the maximum spacing on each gage line shall not exceed the thickness of the thinner outside plate times  $190/\sqrt{F_v}$  nor 18 inches. The maximum longitudinal spacing of rivets, bolts or intermittent welds connecting two rolled shapes in contact with one another shall not exceed 24 inches.

1.18.2.4 Compression members composed of two or more rolled shapes separated from one another by intermittent fillers shall be connected to one another at these fillers at intervals such that the slenderness ratio  $l/r$  of either shape, between the fasteners, does not exceed the governing slenderness ratio of the built-up member. The least radius of gyration  $r$  shall be used in computing the slenderness ratio of each component part.

1.18.2.5 Open sides of compression members built up from plates or shapes shall be provided with lacing having tie plates at each end, and at intermediate points if the lacing is interrupted. Tie plates shall be as near the ends as practicable. In main members carrying calculated stress the end tie plates shall have a length of not less than the distance between the lines of rivets, bolts or welds connecting them to the components of the member. Intermediate tie plates shall have a length not less than one-half of this distance. The thickness of tie plates shall be not less than  $1/50$  of the distance between the lines of rivets, bolts or welds connecting them to the segments of the members. In riveted and bolted construction the pitch in tie plates shall be not more than 6 diameters and the tie plates shall be connected to each segment by at least three fasteners. In welded construction, the welding on each line connecting a tie plate shall aggregate not less than one-third the length of the plate.

1.18.2.6 Lacing, including flat bars, angles, channels or other shapes employed as lacing, shall be so spaced that the ratio  $l/r$  of the flange included between their connections shall not exceed the governing ratio for the member as a whole. Lacing shall be proportioned to resist a shearing stress normal to the axis of the member equal to 2 percent of the total compressive stress in the member. The ratio  $l/r$  for lacing bars arranged in single systems shall not exceed 140. For double lacing this ratio shall not exceed 200. Double lacing bars shall be joined at their intersections. In determining the required section for lacing bars, Formula (1.5-1) or (1.5-2) shall be used,  $l$  being taken as the unsupported length of the lacing bar between rivets or welds connecting it to the components of the built-up member for single lacing and 70 percent of that distance for double lacing. The inclination of lacing bars to the axis of the member shall preferably be not less than 60 degrees for single lacing and 45 degrees for double lacing. When the distance between the lines of rivets or welds in the flanges is more than 15 inches, the lacing shall preferably be double or be made of angles.

1.18.2.7 The function of tie plates and lacing may be performed by continuous cover plates perforated with a succession of access holes. The width of such plates at access holes, as defined in Sect. 1.9.2, is assumed available to resist axial stress, provided that: the width-to-thickness ratio conforms to the limitations of Sect. 1.9.2; the ratio of length (in direction of stress) to width of hole shall not exceed 2; the clear distance between holes in the direction of stress shall be not less than the transverse distance between nearest lines of connecting rivets, bolts or welds; and the periphery of the holes at all points shall have a minimum radius of  $1\frac{1}{2}$  inches.

### 1.18.3 Tension Members

1.18.3.1 The longitudinal spacing of rivets, bolts and intermittent fillet welds connecting a plate and a rolled shape in a built-up tension member, or two plate components in contact with one another, shall not exceed 24 times the thickness of the thinner plate nor 12 inches. The longitudinal spacing of rivets, bolts and intermittent welds connecting two or more shapes in contact with one another in a tension member shall not exceed 24 inches. Tension members composed of two or more shapes or plates separated from one another by intermittent fillers shall be connected to one another at these fillers at intervals such that the slenderness ratio of either component between the fasteners does not exceed 240.

1.18.3.2 Either perforated cover plates or tie plates without lacing may be used on the open sides of built-up tension members. Tie plates shall have a length not less than two-thirds the distance between the lines of rivets, bolts or welds connecting them to the components of the member. The thickness of such tie plates shall not be less than  $1/50$  of the distance between these lines. The longitudinal spacing of rivets, bolts or intermittent welds at tie plates shall not exceed 6 inches. The spacing of tie plates shall be such that the slenderness ratio of any component in the length between tie plates will not exceed 240.

## SECTION 1.19 CAMBER

### 1.19.1 Trusses and Girders

Trusses of 80 feet or greater span should generally be cambered for approximately the dead load deflection. Crane girders of 75 feet or greater span should generally be cambered for approximately the dead and half live load deflection.

### 1.19.2 Camber for Other Trades

If any special camber requirements are necessary in order to bring a loaded member into proper relation with the work of other trades, as for the attachment of runs of sash, the requirements shall be set forth on the plans and on the detail drawings.

### 1.19.3 Erection

Beams and trusses detailed without specified camber shall be fabricated so that after erection any minor camber due to rolling or shop assembly shall be upward. If camber involves the erection of any member under straining force, this shall be noted on the erection diagram.

## SECTION 1.20 EXPANSION

adequate provision shall be made for expansion and contraction appropriate to the service conditions of the structure.

## SECTION 1.21 COLUMN BASES

### 1.21.1 Loads

Proper provision shall be made to transfer the column loads, and moments if any, to the footings and foundations.

### 1.21.2 Alignment

Column bases shall be set level and to correct elevation with full bearing on the masonry.

### 1.21.3 Finishing

Column bases shall be finished in accordance with the following requirements:

1. Rolled steel bearing plates, 2 inches or less in thickness, may be used without planing, provided a satisfactory contact bearing is obtained; rolled steel bearing plates over 2 inches but not over 4 inches in thickness may be straightened by pressing; or, if presses are not available, by planing for all bearing surfaces (except as noted under requirement 3 of this Section), to obtain a satisfactory contact bearing; rolled steel bearing plates over 4 inches in thickness shall be planed for all bearing surfaces (except as noted under requirement 3 of this Section).
2. Column bases other than rolled steel bearing plates shall be planed for all bearing surfaces (except as noted under requirement 3 of this Section).
3. The bottom surfaces of bearing plates and column bases which are grouted to insure full bearing contact on foundations need not be planed.

## SECTION 1.22 ANCHOR BOLTS

Anchor bolts shall be designed to provide resistance to all conditions of tension and shear at the bases of columns, including the net tensile components of any bending moments which may result from fixation or partial fixation of columns.

## SECTION 1.23 FABRICATION

### 1.23.1 Straightening Material

Rolled material, before being laid off or worked, must be straight within the tolerances allowed by ASTM Specification A6. If straightening is necessary, it may be done by mechanical means or by the application of a limited amount of localized heat. The temperature of heated areas, as measured by approved methods, shall not exceed 1100°F for A514 steel nor 1200°F for other steels.

### 1.23.2 Oxygen Cutting

Oxygen cutting shall preferably be done by machine. Oxygen cut edges which will be subjected to substantial stress or which are to have weld metal deposited on them shall be reasonably free from gouges; occasional notches or gouges not more than  $\frac{3}{16}$ -inch deep will be permitted. Gouges greater than  $\frac{3}{16}$ -inch that remain from cutting shall be removed by grinding. All re-entrant corners shall be shaped notch-free to a radius of at least  $\frac{1}{2}$ -inch.

### 1.23.3 Planing of Edges

Planing or finishing of sheared or gas cut edges of plates or shapes will not be required unless specifically called for on the drawings or included in a stipulated edge preparation for welding.

### 1.23.4 Riveted and Bolted Construction—Holes

Holes for rivets or bolts shall be  $\frac{1}{16}$ -inch larger than the nominal diameter of the rivet or bolt. If the thickness of the material is not greater than the nominal diameter of the rivet or bolt plus  $\frac{1}{8}$ -inch, the holes may be punched. If the thickness of the material is greater than the nominal diameter of the rivet or bolt plus  $\frac{1}{8}$ -inch, the holes shall be either drilled from the solid, or sub-punched and reamed. The die for all sub-punched holes, and the drill for all sub-drilled holes, shall be at least  $\frac{1}{16}$ -inch smaller than the nominal diameter of the rivet or bolt. Holes in A514 steel plates over  $\frac{1}{2}$ -inch thick shall be drilled.

### 1.23.5 Riveted and High Strength Bolted Construction—Assembling

All parts of riveted members shall be well pinned or bolted and rigidly held together while riveting. Drifting done during assembling shall not distort the metal or enlarge the holes. Holes that must be enlarged to admit the rivets or bolts shall be reamed. Poor matching of holes shall be cause for rejection.

Rivets shall be driven by power riveters, of either compression or manually-operated type, employing pneumatic, hydraulic or electric power. After driving they shall be tight and their heads shall be in full contact with the surface.

Rivets shall ordinarily be hot-driven, in which case their finished heads shall be of approximately hemispherical shape and shall be of uniform size throughout the work for the same size rivet, full, neatly finished and concentric with the holes. Hot-driven rivets shall be heated uniformly to a temperature not exceeding 1950° F; they shall not be driven after their temperature has fallen below 1000° F.

Rivets may be driven cold if approved measures are taken to prevent distortion of the riveted material. The requirements for hot-driven rivets shall apply except as modified in the *Tentative Specifications for Cold-Driven Rivets* of the Industrial Fasteners Institute.

Surfaces of high strength bolted parts in contact with the bolt head and nut shall not have a slope of more than 1:20 with respect to a plane normal to the bolt axis. Where the surface of a high strength bolted part has a slope of more than 1:20, a beveled washer shall be used to compensate for the lack of parallelism. High strength bolted parts shall fit solidly together when assembled and shall not be separated by gaskets or any other interposed compressible materials. When assembled, all joint surfaces, including those

adjacent to the washers, shall be free of scale except tight mill scale. They shall be free of dirt, loose scale, burrs, and other defects that would prevent solid seating of the parts. Contact surfaces within friction-type joints shall be free of oil, paint, lacquer or galvanizing.

All A325, A449, and A490 bolts shall be tightened to a bolt tension not less than that given in Table 1.23.5. Tightening shall be done by the

TABLE 1.23.5

Bolt Size, Inches	Minimum Bolt Tension, <sup>1</sup> Kips	
	A325 and A449 Bolts	A490 Bolts
½	12	15
¾	19	24
¾	28	35
⅞	39	49
1	51	64
1⅛	56	80
1¼	71	102
1⅝	85	121
1¾	103	148
Over 1½		0.7 × T.S.

<sup>1</sup> Equal to 70 percent of specified minimum tensile strengths of bolts, rounded off to the nearest kip.

turn-of-nut method\* or with properly calibrated wrenches. Bolts tightened by means of a calibrated wrench shall be installed with a hardened washer under the nut or bolt head, whichever is the element turned in tightening. Hardened washers are not required when bolts are tightened by the turn-of-nut method, except that hardened washers are required under the nut and bolt head when A490 bolts are used to connect material having a specified yield point less than 40 ksi and a hardened washer is required under the head of A449 bolts used in lieu of A325 bolts.

### 1.23.6 Welded Construction

Surfaces to be welded shall be free from loose scale, slag, rust, grease, paint and any other foreign material except that mill scale which withstands vigorous wire brushing may remain. Joint surfaces shall be free from fins and tears. Preparation of edges by gas cutting shall, wherever practicable, be done by a mechanically guided torch.

Parts to be fillet welded shall be brought in as close contact as practicable and in no event shall be separated by more than ⅜-inch. If the separation is ⅜-inch or greater, the size of the fillet welds shall be increased by the amount of the separation. The separation between faying surfaces of lap joints and butt joints on a backing structure shall not exceed ⅜-inch. The fit of joints at contact surfaces which are not completely sealed by welds, shall be close enough to exclude water after painting.

\* See Commentary, Sect. 1.23.5.

Abutting parts to be butt welded shall be carefully aligned. Misalignments greater than ⅛-inch shall be corrected and, in making the correction, the parts shall not be drawn into a sharper slope than 2 degrees (⅜-inch in 12 inches).

The work shall be positioned for flat welding whenever practicable.

In assembling and joining parts of a structure or of built-up members, the procedure and sequence of welding shall be such as will avoid needless distortion and minimize shrinkage stresses. Where it is impossible to avoid high residual stresses in the closing welds of a rigid assembly, such closing welds shall be made in compression elements.

In the fabrication of cover-plated beams and built-up members, all shop splices in each component part shall be made before such component part is welded to other parts of the member. Long girders or girder sections may be made by shop splicing not more than three subsections, each made in accordance with this paragraph.

All complete penetration groove welds made by manual welding, except when produced with the aid of backing material or welded in the flat position from both sides in square-edge material not more than ⅝-inch thick with root opening not less than one-half the thickness of the thinner part joined, shall have the root of the initial layer gouged out on the back side before welding is started from that side, and shall be so welded as to secure sound metal and complete fusion throughout the entire cross-section. Oxygen gouging shall not be permitted on ASTM A514 steel; all carbon deposits shall be removed by grinding after arc gouging A514 steel. Groove welds made with use of a backing of the same material as the base metal shall have the weld metal thoroughly fused with the backing material. Backing strips need not be removed. If required, they may be removed by gouging or gas cutting after welding is completed, provided no injury is done to the base metal and weld metal and the weld metal surface is left flush or slightly convex with full throat thickness.

Groove welds shall be terminated at the ends of a joint in a manner that will ensure their soundness. Where possible, this should be done by use of extension bars or run-off plates. Extension bars or run-off plates, if used, shall be removed upon completion of the weld and the ends of the weld made smooth and flush with the abutting parts.

Base metal shall be preheated as required to the temperature called for in Table 1.23.6 prior to welding, except tack welding which is to be remelted and incorporated into continuous submerged-arc welds. When base metal not otherwise required to be preheated is at a temperature below 32° F, it shall be preheated to at least 70° F prior to tack welding or welding. Preheating shall bring the surface of the base metal within 3 inches of the point of welding to the specified preheat temperature, and this temperature shall be maintained as a minimum interpass temperature while welding is in progress. Minimum preheat and interpass temperatures shall be as specified in Table 1.23.6. Heat input for the welding of ASTM A514 steel should not exceed the steel producer's recommendations or suggestions.

Where required, intermediate layers of multiple-layer welds may be peened with light blows from a power hammer, using a round-nose tool. Peening shall be done after the weld has cooled to a temperature warm to the hand. Care shall be exercised to prevent scaling, or flaking of weld and base metal from over-peening.

TABLE 1.23.6  
Minimum Preheat and Interpass Temperature, °F<sup>1</sup>

Thickest Part at Point of Welding (inches)	Welding Process			
	Shielded Metal-Arc Welding with other than Low Hydrogen Electrodes	Shielded Metal-Arc Welding with Low Hydrogen Electrodes; Submerged Arc Welding; Gas Metal-Arc Welding; or Flux Cored Arc Welding	Shielded Metal-Arc Welding with Low Hydrogen Electrodes; Submerged Arc Welding with Carbon or Alloy Steel Wire, Neutral Flux; Gas Metal-Arc Welding; or Flux Cored Arc Welding	Submerged Arc Welding with Carbon Steel Wire, Alloy Flux
To ¼, incl.	ASTM A36; A53 Grade B; A375; A500; A501; A529; A570 Grades D and E	ASTM A36; A242 Weldable Grade; A375; A441; A529; A570 Grades D & E; A572 Grades 42, 45, and 50; A588	ASTM A572 Grades 55, 60, and 65	ASTM A514
Over ¼ to 1½, incl.	None <sup>1</sup>	None <sup>1</sup>	70	50
Over 1½ to 2½, incl.	150	70 <sup>1</sup>	150	125
Over 2½	225	150 <sup>1</sup>	225	175
	300	225	300	225
				400

<sup>1</sup> Welding shall not be done when the ambient temperature is lower than 0° F. When the base metal is below the temperature listed for the welding process being used and the thickness of material being welded, it shall be preheated (except as otherwise provided) in such manner that the surface of the parts on which weld metal is being deposited are at or above the specified minimum temperature for a distance equal to the thickness of the part being welded, but not less than 3 in., both laterally and in advance of the welding. Preheat and interpass temperatures must be sufficient to prevent crack formation. Temperature above the minimum shown may be required for highly restrained weld. For A514 steel the maximum preheat and interpass temperature shall not exceed 400° F for thicknesses up to 1½ in., inclusive, and 450° F for greater thicknesses.

<sup>2</sup> When base metal temperature is below 32° F, preheat base metal to at least 70° F and maintain this minimum temperature during welding.

<sup>3</sup> This provision also applies to A36 steel in thicknesses up to 1 in.

<sup>4</sup> Minimum preheat for A36 steel in thicknesses up to 2 in. shall be 50° F.

When required by the plans or specifications, welded assemblies shall be stress relieved by heat treating in accordance with the provisions of Article 310 of AWS D1.0-69.

The technique of welding employed, the appearance and quality of welds made, and the methods used in correcting defective work shall conform to Section 3—Workmanship and Section 4—Technique of the *Code for Welding in Building Construction*, D1.0-69, of the American Welding Society, except that the tolerance for flatness of girder webs given in Article 305 need not apply for statically loaded girders.

### 1.23.7 Finishing

Compression joints depending upon contact bearing shall have the bearing surfaces prepared to a common plane by milling, sawing or other suitable means.

### 1.23.8 Tolerances

#### 1.23.8.1 Straightness

Structural members consisting primarily of a single rolled shape shall, unless otherwise specified, be straight within the appropriate tolerances allowed by ASTM Specification A6 or as prescribed in the following paragraph. Built-up structural members fabricated by riveting or welding, unless otherwise specified, shall be straight within the tolerances allowed for wide flange shapes by ASTM Specification A6 or by the requirements of the following paragraph.

Compression members shall not deviate from straightness by more than 1/1000 of the axial length between points which are to be laterally supported.

Completed members shall be free from twists, bends, and open joints. Sharp kinks or bends shall be cause for rejection of material.

#### 1.23.8.2 Length

A variation of 1/32-inch is permissible in the overall length of members with both ends finished for contact bearing as in Sect. 1.23.7.

Members without ends finished for contact bearing, which are to be framed to other steel parts of the structure, may have a variation from the detailed length not greater than 1/16-inch for members 30 feet or less in length, and not greater than 1/8-inch for members over 30 feet in length.

## SECTION 1.24 SHOP PAINTING

### 1.24.1 General Requirements

Unless otherwise specified, steelwork which will be concealed by interior building finish need not be painted; steelwork to be encased in concrete shall not be painted. Unless specifically exempted, all other steelwork shall be given one coat of shop paint, applied thoroughly and evenly to dry surfaces which have been cleaned, in accordance with the following paragraph, by brush, spray, roller coating, flow coating, or dipping, at the election of the fabricator.

After inspection and approval and before leaving the shop, all steelwork specified to be painted shall be cleaned by hand-wire brushing, or by other methods elected by the fabricator, of loose mill scale, loose rust, weld slag or flux deposit, dirt and other foreign matter. Oil and grease deposits shall be

removed by solvent. Steelwork specified to have no shop paint shall, after fabrication, be cleaned of oil or grease by solvent cleaners and be cleaned of dirt and other foreign material by thorough sweeping with a fiber brush.

The shop coat of paint is intended to protect the steel for only a short period of exposure, even if it is a primer for subsequent painting to be performed in the field by others.

#### 1.24.2 Inaccessible Surfaces

Surfaces inaccessible after assembly shall be treated in accordance with Sect. 1.24.1 before assembly.

#### 1.24.3 Contact Surfaces

Contact surfaces shall be cleaned in accordance with Sect. 1.24.1 before assembly but shall not be painted.

#### 1.24.4 Finished Surfaces

Machine finished surfaces shall be protected against corrosion by a rust-inhibiting coating that can be easily removed prior to erection or which has characteristics that make removal unnecessary prior to erection.

#### 1.24.5 Surfaces Adjacent to Field Welds

Unless otherwise provided, surfaces within two inches of any field weld location shall be free of materials that would prevent proper welding or produce objectionable fumes while welding is being done.

### SECTION 1.25 ERECTION

#### 1.25.1 Bracing

The frame of steel skeleton buildings shall be carried up true and plumb, within the limits defined in Section 7(h) of the *AISC Code of Standard Practice*, and temporary bracing shall be introduced wherever necessary to take care of all loads to which the structure may be subjected, including equipment and the operation of same. Such bracing shall be left in place as long as may be required for safety.

Wherever piles of material, erection equipment or other loads are carried during erection, proper provision shall be made to take care of stresses resulting from such loads.

#### 1.25.2 Adequacy of Temporary Connections

As erection progresses, the work shall be securely bolted, or welded, to take care of all dead load, wind and erection stresses.

#### 1.25.3 Alignment

No riveting, permanent bolting or welding shall be done until as much of the structure as will be stiffened thereby has been properly aligned.

#### 1.25.4 Field Welding

Any shop paint on surfaces adjacent to joints to be field welded shall be wire brushed to reduce the paint film to a minimum.

#### 1.25.5 Field Painting

Responsibility for touch-up painting and cleaning, as well as for general painting shall be allocated in accordance with accepted local practices and this allocation shall be set forth explicitly in the contract.

### SECTION 1.26 QUALITY CONTROL

#### 1.26.1 General

The fabricator shall provide quality control procedures to the extent that he deems necessary to assure that all work is performed in accordance with this Specification. In addition to the fabricator's quality control procedures, material and workmanship at all times may be subject to inspection by qualified inspectors representing the purchaser. If such inspection by representatives of the purchaser will be required, it shall be so stated in the information furnished to the bidders.

#### 1.26.2 Cooperation

As far as possible all inspection by representatives of the purchaser shall be made at the fabricator's plant. The fabricator shall cooperate with the inspector, permitting access for inspection to all places where work is being done. The purchaser's inspector shall so schedule his work as to provide the minimum interruption to the work of the fabricator.

#### 1.26.3 Rejections

Material or workmanship not in reasonable conformance with the provisions of this Specification may be rejected at any time during the progress of the work. The fabricator shall receive copies of all reports furnished to the purchaser by the inspection agency.

#### 1.26.4 Inspection of Welding

The inspection of welding shall be performed in accordance with the provisions of Section 6 of the *Code for Welding in Building Construction, D1.0-69*, of the American Welding Society.

When non-destructive testing is required, the process, extent, technique and standards of acceptance shall be clearly defined in information furnished to the bidders.

#### 1.26.5 Identification of High Strength Steel

Steel which is used for main components and which is required to have a yield stress greater than 36 kips per square inch shall, at all times in the fabricator's plant, be marked to identify its ASTM Specification. Identification of such steel in completed members or assemblies shall be marked by painting the ASTM Specification designation on the piece, over any shop coat of paint, prior to shipment from the fabricator's plant.



## PART 2

### SECTION 2.1 SCOPE

Subject to the limitations contained herein, simple or continuous beams, one and two-story rigid frames, braced multi-story rigid frames, and similar portions of structures rigidly constructed so as to be continuous over at least one interior support,\* may be proportioned on the basis of plastic design, i.e., on the basis of their maximum strength. This strength, as determined by rational analysis, shall not be less than that required to support a factored load equal to 1.7 times the given live load and dead load or 1.3 times these loads acting in conjunction with 1.3 times any specified wind or earthquake forces.

Rigid frames shall satisfy the requirements for Type 1 construction in the plane of the frame as provided in Sect. 1.2. Type 2 construction is permitted for members between rigid frames. Connections joining a portion of a structure designed on the basis of plastic behavior with a portion not so designed need be no more rigid than ordinary seat-and-cap angle or standard web connections.

Where plastic design is used as the basis for proportioning continuous beams and structural frames, the provisions relating to allowable working stress, contained in Part 1, are waived. Except as modified by these rules, however, all other pertinent provisions of Part 1 shall govern.

It is not recommended that crane runways be designed continuous over interior vertical supports on the basis of maximum strength. However, rigid frame bents supporting crane runways may be considered as coming within the scope of the rules.

### SECTION 2.2 STRUCTURAL STEEL

Structural steel shall conform to one of the following specifications, latest edition:

*Structural Steel, ASTM A36*

*High-Strength Low-Alloy Structural Steel, ASTM A242*

*High-Strength Low-Alloy Structural Manganese Vanadium Steel, ASTM A441*

*Structural Steel with 42,000 psi Minimum Yield Point, ASTM, A529*

*High-Strength Low-Alloy Columbium-Vanadium Steels of Structural Quality, ASTM A572*

*High-Strength Low-Alloy Structural Steel with 50,000 psi Minimum Yield Point to 4 in. Thick, ASTM A588*

\* As used here, "interior support" may be taken to include a rigid frame knee formed by the junction of a column and a sloping or horizontal beam or girder.

### SECTION 2.3 VERTICAL BRACING SYSTEM

The vertical bracing system for a plastically designed braced multi-story frame shall be adequate, as determined by a rational analysis, to:

1. Prevent buckling of the structure under factored gravity loads
2. Maintain the lateral stability of the structure, including the overturning effects of drift, under factored gravity plus factored horizontal loads.

The vertical bracing system may be considered to function together with in-plane shear-resisting exterior and interior walls, floor slabs, and roof decks, if these walls, slabs, and decks are secured to the structural frames. The columns, girders, beams, and diagonal members, when used as the vertical bracing system, may be considered to comprise a vertical-cantilever, simply-connected truss in the analyses for frame buckling and lateral stability. Axial deformation of all members in the vertical bracing system shall be included in the lateral stability analysis. The axial force in these members, caused by factored gravity plus factored horizontal loads, shall not exceed  $0.85P_y$ , where  $P_y$  is the product of yield stress times area of the member.

Girders and beams included in the vertical bracing system of a braced multi-story frame shall be proportioned for axial force and moment caused by the concurrent factored horizontal and gravity loads, in accordance with Formula (2.4-2), with  $P_{cr}$  taken as the maximum axial strength of the beam, based on the actual slenderness ratio between braced points in the plane of bending.

### SECTION 2.4 COLUMNS

In the plane of bending of columns which would develop a plastic hinge at ultimate loading, the slenderness ratio  $l/r$  shall not exceed  $C_c$ , defined in Sect. 1.5.1.3.

The maximum strength of an axially loaded compression member shall be taken as

$$P_{cr} = 1.7AF_a \quad (2.4-1)$$

where  $A$  is the gross area of the member and  $F_a$ , as defined by Formula (1.5-1), is based upon the applicable slenderness ratio.\*

Members subject to combined axial load and bending moment shall be proportioned so as to satisfy the following interaction formulas:

$$\frac{P}{P_{cr}} + \frac{C_m M}{\left(1 - \frac{P}{P_c}\right) M_m} \leq 1.0 \quad (2.4-2)$$

$$\frac{P}{P_y} + \frac{M}{1.18M_p} \leq 1.0; \quad M \leq M_p \quad (2.4-3)$$

\* See Commentary p. 5-162.

in which

- $M$  = maximum applied moment  
 $P$  = applied axial load  
 $P_e = (23/12) AF'_e$ , where  $F'_e$  is as defined in Sect. 1.6.1  
 $C_m$  = coefficient defined in Sect. 1.6.1  
 $M_m$  = maximum moment that can be resisted by the member in the absence of axial load

For columns braced in the weak direction:

$$M_m = M_p$$

For columns unbraced in the weak direction:

$$M_m = \left[ 1.07 - \frac{(l/r_y)\sqrt{F_y}}{3,160} \right] M_p \leq M_p \quad (2.4-4)$$

## SECTION 2.5 SHEAR

Unless reinforced by diagonal stiffeners or a doubler plate, the webs of columns, beams, and girders, including areas within the boundaries of the connections, shall be so proportioned that

$$V_u \leq 0.55F_y t d \quad (2.5-1)$$

where  $V_u$  is the shear, in kips, that would be produced by the required factored loading,  $d$  is the depth of the member, and  $t$  is its web thickness.

## SECTION 2.6 WEB CRIPPLING

Web stiffeners are required on a member at a point of load application where a plastic hinge would form.

At points on a member where the concentrated load delivered by the flanges of a member framing into it would produce web crippling opposite the compression flange or high tensile stress in the connection of the tension flange, web stiffeners are required in accordance with the provisions of Sect. 1.15.5.

## SECTION 2.7 MINIMUM THICKNESS (WIDTH-THICKNESS RATIOS)

The width-thickness ratio for flanges of rolled I or W shapes and similar built-up single-web shapes that would be subjected to compression involving hinge rotation under ultimate loading shall not exceed the following values:

$F_y$	$b_f/2t_f$
36	8.5
42	8.0
45	7.4
50	7.0
55	6.6
60	6.3
65	6.0

The thickness of sloping flanges may be taken as their average thickness.

The width-thickness ratio of similarly compressed flange plates in box sections and cover-plates shall not exceed  $190/\sqrt{F_y}$ . For this purpose the width of a cover-plate shall be taken as the distance between longitudinal lines of connecting rivets, high strength bolts or welds.

The depth-thickness ratio of webs of members subjected to plastic bending shall not exceed the value given by Formula (2.7-1a) or (2.7-1b), as applicable.

$$\frac{d}{t} = \frac{412}{\sqrt{F_y}} \left( 1 - 1.4 \frac{P}{P_y} \right) \quad \text{when } \frac{P}{P_y} \leq 0.27 \quad (2.7-1a)$$

$$\frac{d}{t} = \frac{257}{\sqrt{F_y}} \quad \text{when } \frac{P}{P_y} > 0.27 \quad (2.7-1b)$$

## SECTION 2.8 CONNECTIONS

All connections, the rigidity of which is essential to the continuity assumed as the basis of the analysis, shall be capable of resisting the moments, shears and axial loads to which they would be subjected by the full factored loading, or any probable partial distribution thereof.

Corner connections (haunches), tapered or curved for architectural reasons, shall be so proportioned that the full plastic bending strength of the section adjacent to the connection can be developed, if required.

Stiffeners shall be used, as required, to preserve the flange continuity of interrupted members at their junction with other members in a continuous frame. Such stiffeners shall be placed in pairs on opposite sides of the web of the member which extends continuously through the joint.

High strength bolts, A307 bolts, rivets, and welds shall be proportioned to resist the forces produced at factored load, using stresses equal to 1.7 times those given in Part 1. In general, groove welds are preferable to fillet welds, but their use is not mandatory.

High strength bolts may be used in joints having painted contact surfaces when these joints are of such size that the slip required to produce bearing would not interfere with the formation, at factored loading, of the plastic hinges assumed in the design.

## SECTION 2.9 LATERAL BRACING

Members shall be adequately braced to resist lateral and torsional displacements at the plastic hinge locations associated with the failure mechanism. The laterally unsupported distance,  $l_{cr}$ , from such braced hinge locations to similarly braced adjacent points on the member or frame shall not exceed the value determined from Formula (2.9-1a) or (2.9-1b), as applicable.

$$\frac{l_{cr}}{r_y} = \frac{1,375}{F_y} + 25 \quad \text{when } +1.0 > \frac{M}{M_p} > -0.5 \quad (2.9-1a)$$

$$\frac{l_{cr}}{r_y} = \frac{1,375}{F_y} \quad \text{when } -0.5 > \frac{M}{M_p} > -1.0 \quad (2.9-1b)$$

where

- $r_y$  = the radius of gyration of the member about its weak axis
- $M$  = the lesser of the moments at the ends of the unbraced segment
- $M/M_p$  = the end moment ratio, is positive when the segment is bent in reverse curvature and negative when bent in single curvature.

The foregoing provisions need not apply in the region of the last hinge to form in the failure mechanism assumed as the basis for proportioning a given member, nor in members oriented with their weak axis normal to the plane of bending. However, in the region of the last hinge to form, and in regions not adjacent to a plastic hinge, the maximum distance between points of lateral support shall be such as to satisfy the requirements of Formulas (1.5-6a), (1.5-6b) or (1.5-7) as well as Formulas (1.6-1a) and (1.6-1b) in Part 1 of this Specification. For this case the value of  $f_a$  and  $f_b$  shall be computed from the moment and axial force at factored loading, divided by the applicable load factor.

Members built into a masonry wall and having their web perpendicular to this wall can be assumed to be laterally supported with respect to their weak axis of bending.

## SECTION 2.10 FABRICATION

The provisions of Part 1 with respect to workmanship shall govern the fabrication of structures, or portions of structures, designed on the basis of maximum strength, subject to the following limitations:

The use of sheared edges shall be avoided in locations subject to plastic hinge rotation at factored loading. If used they shall be finished smooth by grinding, chipping or planing.

In locations subject to plastic hinge rotation at factored loading, holes for rivets or bolts in the tension area shall be sub-punched and reamed or drilled full size.

## APPENDIX A

	Yield Stress — $F_y$ (ksi)		
	36.0	42.0	45.0
<b>SECTION 1.5 ALLOWABLE STRESSES</b>			
<b>1.5.1.1 Tension</b>			
Tension on the net section, except at pin holes: $F_t = 0.60F_y \leq 0.50F_{TS}$ where $F_{TS}$ = minimum tensile strength	22.0	25.2	27.0
Tension on the net section at pin holes in eyebars, pin-connected plates or built-up members: $F_t = 0.45F_y$	16.2	19.0	20.3
<b>1.5.1.2 Shear</b>			
Shear on the gross section (see Table 3 for reduced values for girder webs): $F_v = 0.40F_y$	14.5	17.0	18.0
<b>1.5.1.3 Compression</b>			
1.5.1.3.1 Compression on the gross section of axially loaded compression members when $Kl/r$ is less than $C_c$ : Formula (1.5-1) $F_a = \frac{\left[1 - \frac{(Kl/r)^2}{2C_c^2}\right] F_y}{\frac{5}{3} + \frac{3(Kl/r)}{8C_c} - \frac{(Kl/r)^2}{8C_c^2}}$	Table 1-36	Table 1-42	Table 1-45
1.5.1.3.2 Compression on the gross section of axially loaded compression members when $Kl/r$ exceeds $C_c$ : Formula (1.5-2) $F_a = \frac{12\pi^2 E}{23(Kl/r)^2}$	Table 1-36	Table 1-42	Table 1-45
1.5.1.3.3 Compression on the gross section of axially loaded bracing and secondary members when $l/r$ exceeds 120: Formula (1.5-3) $F_{as} = \frac{F_a \text{ [by Formula (1.5-1) or (1.5-2)]}}{1.6 - \frac{l}{200r}}$	Table 1-36	Table 1-42	Table 1-45

Yield Stress — $F_y$ (ksi)					
50.0	55.0	60.0	65.0	90.0	100.0
30.0	33.0	36.0	39.0	52.5*	57.5*
22.5	24.8	27.0	29.3	40.5	45.0
20.0	22.0	24.0	26.0	36.0	40.0
Table 1-50	Table 1-55	Table 1-60	Table 1-65	Table 1-90	Table 1-100
Table 1-50	Table 1-55	Table 1-60	Table 1-65	Table 1-90	Table 1-100
Table 1-50	Table 1-55	Table 1-60	Table 1-65	Table 1-90	Table 1-100

\* Value equal to  $\frac{1}{50}$  times minimum tensile strength ( $= 0.50F_{TS}$ )

	Yield Stress — $F_y$ (ksi)		
	36.0	42.0	45.0
<b>1.5.1.3 Compression (cont'd)</b>			
<b>1.5.1.3.4</b> Compression on the gross area of plate girder stiffeners:  $F_a = 0.60F_y$	22.0	25.2	27.0
<b>1.5.1.3.5</b> Compression on the web of rolled shapes at the toe of fillet:  $F_a = 0.75F_y$	27.0	31.5	33.8
<b>1.5.1.4 Bending</b>			
<b>1.5.1.4.1</b> Tension and compression for compact, adequately braced members symmetrical about, and loaded in, the plane of their minor axis:  $F_b = 0.66F_y$	24.0	28.0	29.7
when			
a. Flanges are continuously connected to web			
b. $b_f/2t_f \leq 52.2/\sqrt{F_y}$	8.7	8.1	7.8
c. $b_e/t_f \leq 190/\sqrt{F_y}$	31.7	29.3	28.3
d. Use Formula (1.5-4):  $d/t \leq 412 \left( 1 - 2.33 \frac{f_a}{F_y} \right) / \sqrt{F_y}$	68.7 - 4.4 $f_a$	63.6 - 3.5 $f_a$	61.4 - 3.2 $f_a$
except that $d/t$ need not be less than $257/\sqrt{F_y}$	42.8	39.7	38.3
e. $l_b \leq 76.0b_f/\sqrt{F_y}$	12.7 $b_f$	11.7 $b_f$	11.3 $b_f$
and			
$l_b \leq \frac{20,000}{(d/A_f)F_y}$	$\frac{556}{d/A_f}$	$\frac{476}{d/A_f}$	$\frac{444}{d/A_f}$

	Yield Stress — $F_y$ (ksi)					
	50.0	55.0	60.0	65.0	90.0	100.0
	30.0	33.0	36.0	39.0	54.0	60.0
	37.5	41.3	45.0	48.8	67.5	75.0
	33.0	36.3	39.6	42.9	—	—
	7.4	7.0	6.7	6.5	—	—
	26.9	25.6	24.5	23.6	—	—
	58.3 - 2.7 $f_a$	55.6 - 2.4 $f_a$	53.2 - 2.1 $f_a$	51.1 - 1.8 $f_a$	—	—
	36.3	34.7	33.2	31.9	—	—
	10.7 $b_f$	10.2 $b_f$	9.8 $b_f$	9.4 $b_f$	—	—
	$\frac{400}{d/A_f}$	$\frac{364}{d/A_f}$	$\frac{333}{d/A_f}$	$\frac{308}{d/A_f}$	—	—

		Yield Stress — $F_y$ (ksi)																																									
		36.0	42.0	45.0																																							
<b>1.5.1.4 Bending (cont'd)</b>																																											
<b>1.5.1.4.2</b>																																											
Tension and compression for members which meet the requirements of Sect. 1.5.1.4.1 except subparagraph b:																																											
when																																											
$\frac{52.2}{\sqrt{F_y}} < \frac{b_f}{2t_f}$		8.7	8.1	7.8																																							
and																																											
$\frac{b_f}{2t_f} < \frac{95.0}{\sqrt{F_y}}$		15.8	14.7	14.2																																							
use Formula (1.5-5):																																											
$F_b = F_y \left[ 0.733 - 0.0014 \left( \frac{b_f}{2t_f} \right) \sqrt{F_y} \right]$																																											
		<table border="1"> <thead> <tr> <th></th> <th><math>\frac{b_f}{2t_f}</math></th> </tr> </thead> <tbody> <tr><td rowspan="10">Values of <math>F_b</math></td><td>7.0</td><td>—</td><td>—</td><td>—</td></tr> <tr><td>8.0</td><td>—</td><td>—</td><td>29.6</td></tr> <tr><td>9.0</td><td>23.7</td><td>27.3</td><td>29.2</td></tr> <tr><td>10.0</td><td>23.4</td><td>27.0</td><td>28.8</td></tr> <tr><td>11.0</td><td>23.1</td><td>26.6</td><td>28.4</td></tr> <tr><td>12.0</td><td>22.8</td><td>26.7</td><td>27.9</td></tr> <tr><td>13.0</td><td>22.5</td><td>25.8</td><td>27.5</td></tr> <tr><td>14.0</td><td>22.1</td><td>25.5</td><td>27.1</td></tr> <tr><td>15.0</td><td>22.0</td><td>—</td><td>—</td></tr> </tbody> </table>				$\frac{b_f}{2t_f}$	Values of $F_b$	7.0	—	—	—	8.0	—	—	29.6	9.0	23.7	27.3	29.2	10.0	23.4	27.0	28.8	11.0	23.1	26.6	28.4	12.0	22.8	26.7	27.9	13.0	22.5	25.8	27.5	14.0	22.1	25.5	27.1	15.0	22.0	—	—
	$\frac{b_f}{2t_f}$																																										
Values of $F_b$	7.0	—	—	—																																							
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	13.0	22.5	25.8	27.5																																							
	14.0	22.1	25.5	27.1																																							
	15.0	22.0	—	—																																							
	<b>1.5.1.4.3</b>																																										
Tension and compression for: doubly-symmetrical I and H shape members meeting the requirements of Sect. 1.5.1.4.1, except subparagraphs c, d and e, and bent about their minor axis (except members of A514 steel); solid round and square bars; and solid rectangular bars bent about their weaker axis:																																											
$F_b = 0.75F_y$		27.0	31.5	33.8																																							
<b>1.5.1.4.4</b>																																											
Tension and compression for box-type flexural members not included in Sect. 1.5.1.4.1, but which meet the requirements of Sect. 1.9:																																											
$F_b = 0.60F_y$		22.0	25.2	27.0																																							
when																																											
$\leq 2500b_e/F_y$		69.4 $b_e$	59.5 $b_e$	55.6 $b_e$																																							

Yield Stress — $F_y$ (ksi)					
50.0	55.0	60.0	65.0	90.0	100.0
7.4	7.0	6.7	6.5	—	—
13.4	12.8	12.3	11.8	—	—
—	—	39.4	42.5	—	—
32.7	35.7	38.8	41.8	—	—
32.2	35.2	38.1	41.0	—	—
31.7	34.6	37.5	40.3	—	—
31.2	34.0	36.8	39.6	—	—
30.7	33.5	36.2	—	—	—
30.2	—	—	—	—	—
—	—	—	—	—	—
—	—	—	—	—	—
37.5	41.3	45.0	48.8	—	—
30.0	33.0	36.0	39.0	54.0	60.0
50.0 $b_e$	45.5 $b_e$	41.7 $b_e$	38.5 $b_e$	27.6	25.0 $b_e$

	Yield Stress — $F_y$ (ksi)		
	36.0	42.0	45.0
<b>1.5.1.4 Bending (cont'd)</b>			
1.5.1.4.5 Tension for flexural members not covered in Sect. 1.5.1.4.1, 1.5.1.4.2, 1.5.1.4.3 or 1.5.1.4.4:  $F_b = 0.60F_y$	22.0	25.2	27.0
1.5.1.4.6a Compression for flexural members included under Sect. 1.5.1.4.5, having an axis of symmetry in, and loaded in, the plane of their web; compression for channels bent about their major axis: The larger value computed by Formula (1.5-6a) or (1.5-6b) and Formula (1.5-7), but not more than  $F_b = 0.60F_y$  when  $l/r_T \leq \sqrt{\frac{102 \times 10^3 \times C_b}{F_y}}$  When this limit is exceeded, use Formula (1.5-6a):  $F_b = \left[ \frac{2}{3} - \frac{F_y(l/r_T)^2}{1,530 \times 10^3 \times C_b} \right] F_y^*$  unless  $l/r_T \geq \sqrt{\frac{510 \times 10^3 \times C_b}{F_y}}$  in which case, use Formula (1.5-6b):  $F_b = \frac{170 \times 10^3 \times C_b}{(l/r_T)^2}$  When the compression flange is solid and approximately rectangular in cross-section and its area is not less than that of the tension flange, use Formula (1.5-7):  $F_b = \frac{12 \times 10^3 \times C_b}{ld/A_f}$	22.0	25.2	27.0
	$53\sqrt{C_b}$	$49\sqrt{C_b}$	$48\sqrt{C_b}$
	$24.0 - \frac{(l/r)^2}{1181C_b}$	$28.0 - \frac{(l/r)^2}{867C_b}$	$30.0 - \frac{(l/r)^2}{756C_b}$
	$119\sqrt{C_b}$	$110\sqrt{C_b}$	$106\sqrt{C_b}$

	Yield Stress — $F_y$ (ksi)					
	50.0	55.0	60.0	65.0	90.0	100.0
	30.0	33.0	36.0	39.0	54.0	60.0
	$45\sqrt{C_b}$	$43\sqrt{C_b}$	$41\sqrt{C_b}$	$40\sqrt{C_b}$	$34\sqrt{C_b}$	$32\sqrt{C_b}$
	$33.3 - \frac{(l/r)^2}{612C_b}$	$36.7 - \frac{(l/r)^2}{506C_b}$	$40.0 - \frac{(l/r)^2}{425C_b}$	$43.3 - \frac{(l/r)^2}{362C_b}$	$60.0 - \frac{(l/r)^2}{189C_b}$	$66.7 - \frac{(l/r)^2}{153C_b}$
	$101\sqrt{C_b}$	$96\sqrt{C_b}$	$92\sqrt{C_b}$	$89\sqrt{C_b}$	$75\sqrt{C_b}$	$71\sqrt{C_b}$

\*For values of  $C_b$  see Fig. A1, p. 5-104.

	Yield Stress — $F_y$ (ksi)		
	36.0	42.0	45.0
<b>1.5.1.4. Bending (cont'd)</b>			
1.5.1.4.6b Compression for flexural members included under Sect. 1.5.1.4.5, which do not satisfy the requirements of Sect. 1.5.1.4.6a, and which if bent about their major axis are braced so that  $l_b \leq (76.0b_f/\sqrt{F_y})$ $F_b = 0.60F_y$	12.7b <sub>f</sub> 22.0	11.7b <sub>f</sub> 25.2	11.3b <sub>f</sub> 27.0
<b>1.5.1.5 Bearing (on contact area)</b>			
1.5.1.5.1 Bearing on milled surfaces, including bearing stiffeners and pins in reamed, drilled, or bored holes:  $F_p = 0.90F_y$	33.0	38.0	40.5
1.5.1.5.2 Bearing on expansion rollers and rockers:  $F_p = \left(\frac{F_y - 13}{20}\right) 0.66d$	0.76d	0.96d	1.06d
<b>1.5.2 Rivets, Bolts, and Threaded Parts</b>			
1.5.2.2 Bearing on projected area of bolts in bearing-type connections and on rivets:  $F_p = 1.35F_y$	48.6	56.7	60.8
<b>SECTION 1.9 WIDTH-THICKNESS RATIOS</b>			
<b>1.9.1 Unstiffened Elements Under Compression</b>			
1.9.1.2 Maximum width-to-thickness ratios for unstiffened elements of:  Single-angle struts; double-angle struts with separators: $76.0/\sqrt{F_y}$  Double-angle struts in contact; angles or plates projecting from girders, columns or other compression members, compression flanges of beams; stiffeners on plate girders: $95.0/\sqrt{F_y}$ Stems of tees: $127/\sqrt{F_y}$	12.7     15.8 21.2	11.7     14.7 19.6	11.3     14.2 18.9

Yield Stress — $F_y$ (ksi)					
50.0	55.0	60.0	65.0	80.0	100.0
10.7b <sub>f</sub> 30.0	10.2b <sub>f</sub> 33.0	9.8b <sub>f</sub> 36.0	9.4b <sub>f</sub> 39.0	8.0b <sub>f</sub> 54.0	7.6b <sub>f</sub> 60.0
45.0	49.5	54.0	58.5	81.0	90.0
1.22d	1.39d	1.55d	1.72d	2.54d	2.87d
67.5	74.3	81.0	87.8	121.5	135.0
10.7	10.2	9.8	9.4	8.0	7.6
13.4	12.8	12.3	11.8	10.0	9.5
18.0	17.1	16.4	15.8	13.4	12.7



	Yield Stress — $F_y$ (ksi)		
	36.0	42.0	45.0
<b>1.9.2 Stiffened Elements Under Compression</b>			
<b>1.9.2.2</b>			
Maximum width-to-thickness ratios for stiffened elements of:			
Flanges of square and rectangular sections of uniform thickness:			
$\frac{238}{\sqrt{F_y}}$	39.7	36.7	35.5
Unsupported width of perforated cover plates:			
$\frac{317}{\sqrt{F_y}}$	52.8	48.9	47.3
All other uniformly compressed elements:			
$\frac{253}{\sqrt{F_y}}$	42.2	39.0	37.7
<b>SECTION 1.10 PLATE GIRDERS AND ROLLED BEAMS</b>			
<b>1.10.1 Proportions</b>			
Maximum axial force resisted by hybrid girders:			
$P = 0.15F_y A$	5.4A	6.3A	6.8A
where $A$ = gross sectional area			
<b>1.10.2 Web</b>			
Maximum clear distance between flanges:			
$\frac{14,000}{\sqrt{F_y(F_y + 16.5)}} t$	322t	282t	266t
When transverse stiffeners are spaced $1.5 \times d$ or less, the clear distance between flanges need not be less than			
$\frac{2,000}{\sqrt{F_y}} t$	333t	309t	298t
where $t$ = thickness of web $F_y$ = yield stress of compression flange			

Yield Stress — $F_y$ (ksi)					
50.0	55.0	60.0	65.0	90.0	100.0
33.7	32.1	30.7	29.5	25.1	23.8
44.8	42.7	40.9	39.3	33.4	31.7
35.8	34.1	32.7	31.4	26.7	25.3
7.5A	8.3A	9.0A	9.8A	13.5A	15.0A
243t	223t	207t	192t	143t	130t
283t	270t	258t	248t	211t	200t

	Yield Stress — $F_y$ (ksi)																																														
	36.0	42.0	45.0																																												
<b>1.10.5 Stiffeners</b>																																															
1.10.5.2 Largest average web shear, $F_v$ , by Formula (1.10-1) or (1.10-2), as applicable	Table 3-36	Table 3-42	Table 3-45																																												
1.10.5.4 Required gross area of intermediate stiffeners, by Formula (1.10-3)	Table 3-36	Table 3-42	Table 3-45																																												
Intermediate stiffeners required by Formula (1.10-2) shall be connected for a total shear transfer not less than  $f_v = h \sqrt{\left(\frac{F_v}{340}\right)^2}$ where $F_v =$ yield stress of web steel	0.034h	0.043h	0.048h																																												
<b>1.10.7 Combined Shear and Tension Stress</b>																																															
Bending tensile stress due to moment in the plane of the web shall not exceed $0.6F_y$ , nor Formula (1.10-7):  $F_b = \left(0.825 - 0.375 \frac{f_v}{F_v}\right) F_y$																																															
	Values of $F_b$																																														
	<table border="1" style="margin: auto;"> <tr> <td style="padding: 0 5px;"><math>f_v/F_v</math></td> <td></td> <td></td> <td></td> </tr> <tr> <td>0.1</td> <td>22.0</td> <td>25.2</td> <td>27.0</td> </tr> <tr> <td>0.2</td> <td>22.0</td> <td>25.2</td> <td>27.0</td> </tr> <tr> <td>0.3</td> <td>22.0</td> <td>25.2</td> <td>27.0</td> </tr> <tr> <td>0.4</td> <td>22.0</td> <td>25.2</td> <td>27.0</td> </tr> <tr> <td>0.5</td> <td>22.0</td> <td>25.2</td> <td>27.0</td> </tr> <tr> <td>0.6</td> <td>22.0</td> <td>25.2</td> <td>27.0</td> </tr> <tr> <td>0.7</td> <td>20.3</td> <td>23.6</td> <td>25.3</td> </tr> <tr> <td>0.8</td> <td>18.9</td> <td>22.1</td> <td>23.6</td> </tr> <tr> <td>0.9</td> <td>17.6</td> <td>20.5</td> <td>21.9</td> </tr> <tr> <td>1.0</td> <td>16.2</td> <td>18.9</td> <td>20.3</td> </tr> </table>	$f_v/F_v$				0.1	22.0	25.2	27.0	0.2	22.0	25.2	27.0	0.3	22.0	25.2	27.0	0.4	22.0	25.2	27.0	0.5	22.0	25.2	27.0	0.6	22.0	25.2	27.0	0.7	20.3	23.6	25.3	0.8	18.9	22.1	23.6	0.9	17.6	20.5	21.9	1.0	16.2	18.9	20.3		
$f_v/F_v$																																															
0.1	22.0	25.2	27.0																																												
0.2	22.0	25.2	27.0																																												
0.3	22.0	25.2	27.0																																												
0.4	22.0	25.2	27.0																																												
0.5	22.0	25.2	27.0																																												
0.6	22.0	25.2	27.0																																												
0.7	20.3	23.6	25.3																																												
0.8	18.9	22.1	23.6																																												
0.9	17.6	20.5	21.9																																												
1.0	16.2	18.9	20.3																																												
<b>1.10.10 Web Crippling</b>																																															
1.10.10.1 Bearing stiffeners are not required under interior concentrated loads when, by Formula (1.10-8)																																															
$\frac{R}{t(N + 2k)} \leq 0.75F_y$	27.0	31.5	33.8																																												
or under end reactions when, by Formula (1.10-9)																																															
$\frac{R}{t(N + k)} \leq 0.75F_y$	27.0	31.5	33.8																																												

	Yield Stress — $F_y$ (ksi)					
	50.0	55.0	60.0	65.0	90.0	100.0
Table 3-50	Table 3-55	Table 3-60	Table 3-65	Table 3-90	Table 3-100	
Table 3-50	Table 3-55	Table 3-60	Table 3-65	Table 3-90	Table 3-100	
0.056h	0.065h	0.074h	0.084h	0.136h	0.160h	
30.0	33.0	36.0	39.0	54.0	60.0	
30.0	33.0	36.0	39.0	54.0	60.0	
30.0	33.0	36.0	39.0	54.0	60.0	
30.0	33.0	36.0	39.0	54.0	60.0	
30.0	33.0	36.0	39.0	54.0	60.0	
28.1	30.9	33.8	36.6	50.6	56.3	
26.3	28.9	31.5	34.1	47.3	52.5	
24.4	26.8	29.3	31.7	43.9	48.8	
22.5	24.8	27.0	29.3	40.5	45.0	
37.5	41.3	45.0	48.8	67.5	75.0	
37.5	41.3	45.0	48.8	67.5	75.0	

	Yield Stress — $F_y$ (ksi)		
	36.0	42.0	45.0
<b>SECTION 1.11 COMPOSITE CONSTRUCTION</b>			
<b>1.11.2 Design Assumptions</b>			
<b>1.11.2.1</b> Tension and compression for encased composite beams based upon the section properties of the composite section: $F_b = 0.66F_y$	24.0	28.0	29.7
Tension and compression for encased composite beams based upon the section properties of the steel beam alone: $F_b = 0.76F_y$	27.4	31.9	34.2
<b>SECTION 1.13 DEFLECTIONS, VIBRATION AND PONDING</b>			
<b>1.13.3 Ponding</b>			
Total bending stress due to dead loads, gravity live loads (if any) and ponding, for primary and secondary members: $F_b = 0.80F_y$	28.8	33.6	36.0
<b>SECTION 1.18 BUILT-UP MEMBERS</b>			
<b>1.18.2 Compression Members</b>			
<b>1.18.2.3</b> Maximum longitudinal spacing for intermediate rivets, bolts or intermittent welds in built-up members having a component consisting of an outside plate shall not exceed 12 in. nor $\frac{127}{\sqrt{F_y}} t$ where $t$ = thickness of thinnest outside plate	21.2t	19.6t	18.9t
Maximum longitudinal spacing when rivets or bolts are staggered shall not exceed 18 in. nor $\frac{190}{\sqrt{F_y}} t$ where $t$ = thickness of thinnest outside plate	31.7t	29.3t	28.3t

Yield Stress — $F_y$ (ksi)					
50.0	55.0	60.0	65.0	90.0	100.0
33.0	36.3	39.6	42.9	59.4	66.0
38.0	41.8	45.6	49.4	68.4	76.0
40.0	44.0	48.0	52.0	72.0	80.0
18.0t	17.1t	16.4t	15.8t	13.4t	12.7t
26.9t	25.6t	24.5t	23.6t	20.0t	19.0t

	Yield Stress — $F_y$ (ksi)		
	36.0	42.0	45.0
<b>SECTION 2.3 VERTICAL BRACING SYSTEM</b>			
Maximum axial force due to factored gravity plus factored horizontal loads in members comprising the vertical bracing system:  $P = 0.85F_y A$ where $A$ = gross area of the member	30.6A	35.7A	38.3A
<b>SECTION 2.4 COLUMNS</b>			
The ratio of critical moment of a column without axial load and unbraced in the weak direction to the plastic moment of the column section shall not exceed 1 nor Formula (2.4-4):  $\frac{M_n}{M_p} \leq \left[ 1.07 - \frac{(l/r_v)\sqrt{F_y}}{3,160} \right]$	$1.07 - \frac{(l/r_v)}{527}$	$1.07 - \frac{(l/r_v)}{488}$	$1.07 - \frac{(l/r_v)}{471}$
<b>SECTION 2.5 SHEAR</b>			
Shear in unreinforced webs of columns, beams and girders due to factored loading: Formula (2.5-1) $V_n = 0.55F_y t d$ where $t$ = thickness of web $d$ = depth of member	19.8td	23.1td	24.8td
<b>SECTION 2.7 MINIMUM THICKNESS (WIDTH-THICKNESS RATIOS)</b>			
Maximum width-thickness ratios of similarly compressed flange plates in box sections and cover plates:  $\frac{b}{t} \leq \frac{190}{\sqrt{F_y}}$	31.7	29.3	28.3
Maximum depth-thickness ratios of webs of members subjected to plastic bending without axial load: When $P/P_y \leq 0.27$ , use Formula (2.7-1a):  $\frac{d}{t} \leq \frac{412}{\sqrt{F_y}} \left( 1 - 1.4 \frac{P}{P_y} \right)$	$68.7 - 96.1 \frac{P}{P_y}$	$63.6 - 89.0 \frac{P}{P_y}$	$61.4 - 86.0 \frac{P}{P_y}$
When $P/P_y > 0.27$ , use Formula (2.7-1b):  $\frac{d}{t} \leq \frac{257}{\sqrt{F_y}}$	42.8	39.6	38.3

Yield Stress — $F_y$ (ksi) <					
50.0	55.0	60.0	65.0	90.0	100.0
42.5A	46.8A	51.0A	55.3A	—	—
$1.07 - \frac{(l/r_v)}{447}$	$1.07 - \frac{(l/r_v)}{426}$	$1.07 - \frac{(l/r_v)}{408}$	$1.07 - \frac{(l/r_v)}{392}$	—	—
27.5td	30.3td	33.0td	35.8td	—	—
26.9	25.6	24.5	23.6	—	—
$58.3 - 81.6 \frac{P}{P_y}$	$55.6 - 77.8 \frac{P}{P_y}$	$53.2 - 74.5 \frac{P}{P_y}$	$51.1 - 71.5 \frac{P}{P_y}$	—	—
36.3	34.7	33.2	31.9	—	—

	Yield Stress — $F_y$ (ksi)		
	36.0	42.0	45.0
<b>SECTION 2.9 LATERAL BRACING</b>			
Maximum critical slenderness ratio, $l_{cr}/r_y$ , from braced hinge locations to similarly braced adjacent points on a beam or frame:			
When $1.0 > \frac{M}{M_p} > -0.5$ , use Formula (2.9-1a): $\frac{l_{cr}}{r_y} \leq \frac{1,375}{F_y} + 25$	63.2	57.7	55.6
When $-0.5 > \frac{M}{M_p} > -1.0$ , use Formula (2.9-1b): $\frac{l_{cr}}{r_y} \leq \frac{1,375}{F_y}$	38.2	32.7	30.6

Yield Stress — $F_y$ (ksi)					
50.0	55.0	60.0	65.0	90.0	100.0
52.5	50.0	47.9	46.2	—	—
27.5	25.0	22.9	21.2	—	—

TABLE 1-36

ALLOWABLE STRESS (KSI)  
FOR COMPRESSION MEMBERS OF 36 KSI SPECIFIED YIELD STRESS STEEL

TABLE 1-42

ALLOWABLE STRESS (KSI)  
FOR COMPRESSION MEMBERS OF 42 KSI SPECIFIED YIELD STRESS STEEL

F<sub>y</sub> = 36 ksi

Main and Secondary Members Kl/r not over 120			Main Members Kl/r 121 to 200		Secondary Members* l/r 121 to 200	
Kl/r	F <sub>a</sub> (ksi)	F <sub>a</sub> (ksi)	Kl/r	F <sub>a</sub> (ksi)	l/r	F <sub>as</sub> (ksi)
1	21.56	41.19	11	81.15	24	121.10
2	21.52	42.19	03	82.15	13	122.09
3	21.48	43.18	95	83.15	02	123.00
4	21.44	44.18	86	84.14	90	124.90
5	21.39	45.18	78	85.14	79	125.80
6	21.35	46.18	70	86.14	67	126.70
7	21.30	47.18	61	87.14	56	127.59
8	21.25	48.18	53	88.14	44	128.49
9	21.21	49.18	44	89.14	32	129.40
10	21.16	50.18	35	90.14	20	130.30
11	21.10	51.18	26	91.14	09	131.21
12	21.05	52.18	17	92.13	97	132.12
13	21.00	53.18	08	93.13	84	133.03
14	20.95	54.17	99	94.13	72	134.94
15	20.89	55.17	90	95.13	60	135.86
16	20.83	56.17	81	96.13	48	136.78
17	20.78	57.17	71	97.13	36	137.70
18	20.72	58.17	62	98.13	24	138.62
19	20.66	59.17	53	99.13	12	139.54
20	20.60	60.17	43	100.12	01	140.47
21	20.54	61.17	33	101.12	89	141.39
22	20.48	62.17	24	102.12	77	142.32
23	20.41	63.17	14	103.12	65	143.25
24	20.35	64.17	04	104.12	53	144.18
25	20.28	65.16	94	105.12	41	145.12
26	20.22	66.16	84	106.12	29	146.05
27	20.15	67.16	74	107.12	17	147.99
28	20.08	68.16	64	108.11	05	148.93
29	20.01	69.16	54	109.11	93	149.87
30	19.94	70.16	44	110.11	81	150.81
31	19.87	71.16	33	111.11	69	151.75
32	19.80	72.16	23	112.11	57	152.69
33	19.73	73.16	12	113.11	45	153.63
34	19.65	74.16	01	114.11	33	154.57
35	19.58	75.15	90	115.10	21	155.51
36	19.50	76.15	80	116.10	09	156.45
37	19.42	77.15	69	117.10	97	157.39
38	19.35	78.15	58	118.10	85	158.33
39	19.27	79.15	47	119.10	73	159.27
40	19.19	80.15	36	120.10	61	160.21

\* K taken as 1.0 for secondary members.

Note: C<sub>c</sub> = 126.1

F<sub>y</sub> = 42 ksi

Main and Secondary Members Kl/r not over 120			Main Members Kl/r 121 to 200		Secondary Members* l/r 121 to 200	
Kl/r	F <sub>a</sub> (ksi)	F <sub>a</sub> (ksi)	Kl/r	F <sub>a</sub> (ksi)	l/r	F <sub>as</sub> (ksi)
1	25.15	41.21	98	81.16	92	121.10
2	25.10	42.21	87	82.16	80	122.03
3	25.05	43.21	77	83.16	68	123.97
4	24.99	44.21	66	84.16	56	124.90
5	24.94	45.21	55	85.16	44	125.80
6	24.88	46.21	44	86.16	32	126.70
7	24.82	47.21	33	87.16	20	127.59
8	24.76	48.21	22	88.15	08	128.49
9	24.70	49.21	10	89.15	96	129.40
10	24.63	50.20	99	90.15	84	130.30
11	24.57	51.20	87	91.15	72	131.21
12	24.50	52.20	76	92.15	60	132.12
13	24.43	53.20	64	93.15	48	133.03
14	24.36	54.20	52	94.14	36	134.94
15	24.29	55.20	40	95.14	24	135.86
16	24.2	56.20	28	96.14	12	136.78
17	24.15	57.20	16	97.14	01	137.70
18	24.07	58.20	03	98.14	89	138.62
19	24.00	59.19	91	99.14	77	139.54
20	23.92	60.19	79	100.13	65	140.47
21	23.84	61.19	66	101.13	53	141.39
22	23.76	62.19	53	102.13	41	142.32
23	23.68	63.19	40	103.13	29	143.25
24	23.59	64.19	27	104.13	17	144.18
25	23.51	65.19	14	105.13	05	145.12
26	23.42	66.19	01	106.12	93	146.05
27	23.33	67.18	88	107.12	81	147.99
28	23.24	68.18	75	108.12	69	148.93
29	23.15	69.18	61	109.12	57	149.87
30	23.06	70.18	48	110.12	45	150.81
31	22.97	71.18	34	111.12	33	151.75
32	22.88	72.18	20	112.11	21	152.69
33	22.78	73.18	06	113.11	09	153.63
34	22.69	74.17	92	114.11	97	154.57
35	22.59	75.17	78	115.11	85	155.51
36	22.49	76.17	64	116.11	73	156.45
37	22.39	77.17	50	117.10	61	157.39
38	22.29	78.17	35	118.10	49	158.33
39	22.19	79.17	21	119.10	37	159.27
40	22.08	80.17	06	120.10	25	160.21

\* K taken as 1.0 for secondary members.

Note: C<sub>c</sub> = 116.7

TABLE 1-45

ALLOWABLE STRESS (KSI)  
FOR COMPRESSION MEMBERS OF 45 KSI SPECIFIED YIELD STRESS STEEL

TABLE 1-50

ALLOWABLE STRESS (KSI)  
FOR COMPRESSION MEMBERS OF 50 KSI SPECIFIED YIELD STRESS STEEL

F<sub>y</sub> = 45 ksi

Main and Secondary Members Kl/r not over 120			Main Members Kl/r 121 to 200			Secondary Members* l/r 121 to 200		
Kl/r	F <sub>a</sub> (ksi)	F <sub>a</sub>	Kl/r	F <sub>a</sub> (ksi)	F <sub>a</sub>	l/r	F <sub>a</sub> (ksi)	F <sub>a</sub>
1	26.95	41.23.39	81	17.67	121	10.20	161	5.76
2	26.89	42.23.27	82	17.51	122	10.03	162	5.69
3	26.83	43.23.15	83	17.34	123	9.87	163	5.62
4	26.77	44.23.03	84	17.17	124	9.71	164	5.55
5	26.71	45.22.90	85	17.00	125	9.56	165	5.49
6	26.64	46.22.78	86	16.82	126	9.41	166	5.42
7	26.58	47.22.65	87	16.65	127	9.26	167	5.35
8	26.51	48.22.53	88	16.48	128	9.11	168	5.29
9	26.44	49.22.40	89	16.30	129	8.97	169	5.23
10	26.37	50.22.27	90	16.12	130	8.84	170	5.17
11	26.30	51.22.14	91	15.95	131	8.70	171	5.11
12	26.22	52.22.01	92	15.77	132	8.57	172	5.05
13	26.15	53.21.88	93	15.59	133	8.44	173	4.99
14	26.07	54.21.74	94	15.40	134	8.32	174	4.93
15	25.99	55.21.61	95	15.22	135	8.19	175	4.88
16	25.91	56.21.47	96	15.04	136	8.07	176	4.82
17	25.82	57.21.33	97	14.85	137	7.96	177	4.77
18	25.74	58.21.19	98	14.66	138	7.84	178	4.71
19	25.65	59.21.05	99	14.47	139	7.73	179	4.66
20	25.57	60.20.91	100	14.28	140	7.62	180	4.61
21	25.48	61.20.77	101	14.09	141	7.51	181	4.56
22	25.39	62.20.63	102	13.90	142	7.41	182	4.51
23	25.29	63.20.48	103	13.71	143	7.30	183	4.46
24	25.20	64.20.34	104	13.51	144	7.20	184	4.41
25	25.11	65.20.19	105	13.32	145	7.10	185	4.36
26	25.01	66.20.04	106	13.12	146	7.01	186	4.32
27	24.91	67.19.89	107	12.92	147	6.91	187	4.27
28	24.81	68.19.74	108	12.72	148	6.82	188	4.23
29	24.71	69.19.59	109	12.52	149	6.73	189	4.18
30	24.61	70.19.43	110	12.31	150	6.64	190	4.14
31	24.50	71.19.28	111	12.11	151	6.55	191	4.09
32	24.40	72.19.12	112	11.90	152	6.46	192	4.05
33	24.29	73.18.97	113	11.69	153	6.38	193	4.01
34	24.18	74.18.81	114	11.49	154	6.30	194	3.97
35	24.07	75.18.65	115	11.29	155	6.22	195	3.93
36	23.96	76.18.49	116	11.10	156	6.14	196	3.89
37	23.85	77.18.33	117	10.91	157	6.06	197	3.85
38	23.74	78.18.17	118	10.72	158	5.98	198	3.81
39	23.62	79.18.00	119	10.55	159	5.91	199	3.77
40	23.51	80.17.84	120	10.37	160	5.83	200	3.73

\* K taken as 1.0 for secondary members.

Note: C<sub>c</sub> = 112.8

F<sub>y</sub> = 50 ksi

Main and Secondary Members Kl/r not over 120			Main Members Kl/r 121 to 200			Secondary Members* l/r 121 to 200		
Kl/r	F <sub>a</sub> (ksi)	F <sub>a</sub>	Kl/r	F <sub>a</sub> (ksi)	F <sub>a</sub>	l/r	F <sub>a</sub> (ksi)	F <sub>a</sub>
1	29.94	41.25.69	81	18.81	121	10.20	161	5.76
2	29.87	42.25.55	82	18.61	122	10.03	162	5.69
3	29.80	43.25.40	83	18.41	123	9.87	163	5.62
4	29.73	44.25.26	84	18.20	124	9.71	164	5.55
5	29.66	45.25.11	85	17.99	125	9.56	165	5.49
6	29.58	46.24.96	86	17.79	126	9.41	166	5.42
7	29.50	47.24.81	87	17.58	127	9.26	167	5.35
8	29.42	48.24.66	88	17.37	128	9.11	168	5.29
9	29.34	49.24.51	89	17.15	129	8.97	169	5.23
10	29.26	50.24.35	90	16.94	130	8.84	170	5.17
11	29.17	51.24.19	91	16.72	131	8.70	171	5.11
12	29.08	52.24.04	92	16.50	132	8.57	172	5.05
13	28.99	53.23.88	93	16.29	133	8.44	173	4.99
14	28.90	54.23.72	94	16.06	134	8.32	174	4.93
15	28.80	55.23.55	95	15.84	135	8.19	175	4.88
16	28.71	56.23.39	96	15.62	136	8.07	176	4.82
17	28.61	57.23.22	97	15.39	137	7.96	177	4.77
18	28.51	58.23.06	98	15.17	138	7.84	178	4.71
19	28.40	59.22.89	99	14.94	139	7.73	179	4.66
20	28.30	60.22.72	100	14.71	140	7.62	180	4.61
21	28.19	61.22.55	101	14.47	141	7.51	181	4.56
22	28.08	62.22.37	102	14.24	142	7.41	182	4.51
23	27.97	63.22.20	103	14.00	143	7.30	183	4.46
24	27.86	64.22.02	104	13.77	144	7.20	184	4.41
25	27.75	65.21.85	105	13.53	145	7.10	185	4.36
26	27.63	66.21.67	106	13.29	146	7.01	186	4.32
27	27.52	67.21.49	107	13.04	147	6.91	187	4.27
28	27.40	68.21.31	108	12.80	148	6.82	188	4.23
29	27.28	69.21.12	109	12.57	149	6.73	189	4.18
30	27.15	70.20.94	110	12.34	150	6.64	190	4.14
31	27.03	71.20.75	111	12.12	151	6.55	191	4.09
32	26.90	72.20.56	112	11.90	152	6.46	192	4.05
33	26.77	73.20.38	113	11.69	153	6.38	193	4.01
34	26.64	74.20.19	114	11.49	154	6.30	194	3.97
35	26.51	75.19.99	115	11.29	155	6.22	195	3.93
36	26.38	76.19.80	116	11.10	156	6.14	196	3.89
37	26.25	77.19.61	117	10.91	157	6.06	197	3.85
38	26.11	78.19.41	118	10.72	158	5.98	198	3.81
39	25.97	79.19.21	119	10.55	159	5.91	199	3.77
40	25.83	80.19.01	120	10.37	160	5.83	200	3.73

\* K taken as 1.0 for secondary members.

Note: C<sub>c</sub> = 107.0







TABLE 1-100

ALLOWABLE STRESS (KSI)  
FOR COMPRESSION MEMBERS OF 100 KSI SPECIFIED YIELD STRESS STEEL

F<sub>y</sub> = 100 ksi

Main and Secondary Members Kl/r not over 120			Main Members Kl/r 121 to 200		Secondary Members l/r 121 to 200	
Kl/r	F <sub>a</sub> (ksi)	F <sub>a</sub> (ksi)	Kl/r	F <sub>a</sub> (ksi)	l/r	F <sub>a</sub> (ksi)
1	59.82	41.46	121	10.20	161	5.76
2	59.62	42.45	122	10.03	162	5.69
3	59.42	43.45	123	9.87	163	5.62
4	59.21	44.44	124	9.71	164	5.55
5	58.99	45.44	125	9.56	165	5.49
6	58.76	46.43	126	9.41	166	5.42
7	58.53	47.43	127	9.26	167	5.35
8	58.28	48.42	128	9.11	168	5.29
9	58.03	49.42	129	8.97	169	5.23
10	57.77	50.41	130	8.84	170	5.17
11	57.50	51.41	131	8.70	171	5.11
12	57.22	52.40	132	8.57	172	5.05
13	56.93	53.40	133	8.44	173	4.99
14	56.64	54.39	134	8.32	174	4.93
15	56.34	55.38	135	8.19	175	4.88
16	56.03	56.38	136	8.07	176	4.82
17	55.72	57.37	137	7.96	177	4.77
18	55.39	58.37	138	7.84	178	4.71
19	55.06	59.36	139	7.73	179	4.66
20	54.72	60.36	140	7.62	180	4.61
21	54.38	61.35	141	7.51	181	4.56
22	54.03	62.34	142	7.41	182	4.51
23	53.67	63.34	143	7.30	183	4.46
24	53.30	64.33	144	7.20	184	4.41
25	52.93	65.33	145	7.10	185	4.36
26	52.55	66.32	146	7.01	186	4.32
27	52.17	67.31	147	6.91	187	4.27
28	51.78	68.31	148	6.82	188	4.23
29	51.38	69.30	149	6.73	189	4.18
30	50.97	70.29	150	6.64	190	4.14
31	50.56	71.29	151	6.55	191	4.09
32	50.15	72.28	152	6.46	192	4.05
33	49.72	73.27	153	6.38	193	4.01
34	49.29	74.27	154	6.30	194	3.97
35	48.86	75.26	155	6.22	195	3.93
36	48.42	76.25	156	6.14	196	3.89
37	47.97	77.25	157	6.06	197	3.85
38	47.51	78.24	158	5.98	198	3.81
39	47.05	79.23	159	5.91	199	3.77
40	46.59	80.23	160	5.83	200	3.73

TABLE 1-A

VALUES OF C<sub>c</sub>  
For determining F<sub>a</sub> from equation F<sub>a</sub> = C<sub>a</sub>F<sub>y</sub>, for all grades of steel

Kl/r	C <sub>c</sub>	Kl/r	C <sub>c</sub>	Kl/r	C <sub>c</sub>	Kl/r	C <sub>c</sub>
.01	.599	.26	.548	.51	.472	.76	.375
.02	.597	.27	.546	.52	.469	.77	.371
.03	.596	.28	.543	.53	.465	.78	.366
.04	.594	.29	.540	.54	.462	.79	.362
.05	.593	.30	.538	.55	.458	.80	.357
.06	.591	.31	.535	.56	.455	.81	.353
.07	.589	.32	.532	.57	.451	.82	.348
.08	.588	.33	.529	.58	.447	.83	.344
.09	.586	.34	.527	.59	.444	.84	.339
.10	.584	.35	.524	.60	.440	.85	.335
.11	.582	.36	.521	.61	.436	.86	.330
.12	.580	.37	.518	.62	.432	.87	.325
.13	.578	.38	.515	.63	.428	.88	.321
.14	.576	.39	.512	.64	.424	.89	.316
.15	.574	.40	.509	.65	.420	.90	.311
.16	.572	.41	.506	.66	.416	.91	.306
.17	.570	.42	.502	.67	.412	.92	.301
.18	.568	.43	.499	.68	.408	.93	.296
.19	.565	.44	.496	.69	.404	.94	.291
.20	.563	.45	.493	.70	.400	.95	.286
.21	.561	.46	.489	.71	.396	.96	.281
.22	.558	.47	.486	.72	.392	.97	.276
.23	.556	.48	.483	.73	.388	.98	.271
.24	.553	.49	.479	.74	.384	.99	.266
.25	.551	.50	.476	.75	.379	1.00	.261

TABLE 1-B

VALUES OF C<sub>c</sub>  
For use in Formulas (1.5-1), (1.5-2), and (1.5-3),  
Sect. 1.5.1.3, and in Table 1-A

F <sub>y</sub> (ksi)	C <sub>c</sub>	F <sub>y</sub> (ksi)	C <sub>c</sub>
33	131.7	46	111.6
35	127.9	50	107.0
36	126.1	55	102.0
39	121.2	60	97.1
40	119.6	65	93.8
42	116.7	90	79.8
45	112.8	100	75.7

\* K taken as 1.0 for secondary members.

Note: = 75.7

TABLE 2

VALUES OF  $F'_c$  (ksi)

For use in Formula (1 6-1a), Sect. 1.6.1, for all grades of steel

$\frac{Kl_b}{r_b}$	$F'_c$ (ksi)	$\frac{Kl_b}{r_b}$	$F'_c$ (ksi)	$\frac{Kl_b}{r_b}$	$F'_c$ (ksi)	$\frac{Kl_b}{r_b}$	$F'_c$ (ksi)	$\frac{Kl_b}{r_b}$	$F'_c$ (ksi)
21	338.62	51	57.41	81	22.76	111	12.12	141	7.51
22	308.54	52	55.23	82	22.21	112	11.90	142	7.41
23	282.29	53	53.16	83	21.68	113	11.69	143	7.30
24	259.26	54	51.21	84	21.16	114	11.49	144	7.20
25	238.93	55	49.37	85	20.67	115	11.29	145	7.10
26	220.90	56	47.62	86	20.19	116	11.10	146	7.01
27	204.84	57	45.96	87	19.73	117	10.91	147	6.91
28	190.47	58	44.39	88	19.28	118	10.72	148	5.82
29	177.56	59	42.90	89	18.85	119	10.55	149	6.73
30	165.92	60	41.48	90	18.44	120	10.37	150	6.64
31	155.39	61	40.13	91	18.03	121	10.20	151	6.55
32	145.83	62	38.85	92	17.64	122	10.03	152	6.46
33	137.13	63	37.62	93	17.27	123	9.87	153	6.38
34	129.18	64	36.46	94	16.90	124	9.71	154	6.30
35	121.90	65	35.34	95	16.55	125	9.56	155	6.22
36	115.22	66	34.28	96	16.20	126	9.41	156	6.14
37	109.08	67	33.27	97	15.87	127	9.26	157	6.06
38	103.42	68	32.29	98	15.55	128	9.11	158	5.98
39	98.18	69	31.37	99	15.24	129	8.97	159	5.91
40	93.33	70	30.48	100	14.93	130	8.84	160	5.83
41	88.83	71	29.62	101	14.64	131	8.70	161	5.76
42	84.65	72	28.81	102	14.35	132	8.57	162	5.69
43	80.76	73	28.02	103	14.08	133	8.44	163	5.62
44	77.13	74	27.27	104	13.81	134	8.32	164	5.55
45	73.74	75	26.55	105	13.54	135	8.19	165	5.49
46	70.57	76	25.85	106	13.29	136	8.07	166	5.42
47	67.60	77	25.19	107	13.04	137	7.96	167	5.35
48	64.81	78	24.54	108	12.80	138	7.84	168	5.29
49	62.20	79	23.93	109	12.57	139	7.73	169	5.23
50	59.73	80	23.33	110	12.34	140	7.62	170	5.17

$$F'_c = \frac{12\pi^2 E}{23(Kl_b/r_b)^2}$$

All grades of steel

TABLE 3-36

ALLOWABLE SHEAR STRESSES ( $F_v$ ) IN PLATE GIRDERS (ksi)  
FOR 36 KSI SPECIFIED YIELD STRESS STEEL

(*Italic values indicate gross area, as percent of web area, required for pairs of intermediate stiffeners of 36 ksi yield stress steel.*)\*

Slenderness ratios $h/t$ : web depth to web thickness	Aspect ratios $a/h$ : stiffener spacing to web depth														over		
	0.5	0.6	0.7	0.8	0.9	1.0	1.2	1.4	1.6	1.8	2.0	2.5	3.0				
	60											14.5	14.5	14.5		14.5	14
70										14.5	14.5	14.5	14.4	14.2	13.8	13.6	13
80						14.5	14.5	14.0	13.4	13.0	12.6	12.4	12.2	12.0	11.8	11.6	11
90					14.5	14.3	13.4	12.5	12.2	11.9	11.8	11.6	11.3	11.1	10.9	10.7	10
100			14.5	13.9	12.8	12.3	11.9	11.6	11.3	11.1	10.9	10.7	10.5	10.3	10.1	9.9	8
110		14.5	13.8	12.6	12.2	11.9	11.5	11.0	10.5	10.1	9.8	9.5	9.2	8.9	8.6	8.3	6
120		14.3	12.7	12.2	11.8	11.5	11.0	10.5	10.2	9.8	9.4	9.0	8.6	8.2	7.8	7.4	5
130	14.5	13.2	12.2	11.9	11.5	11.0	10.3	9.7	9.2	8.8	8.3	7.9	7.5	7.1	6.7	6.3	4
140	14.2	12.4	12.0	11.6	11.0	10.5	9.8	9.2	8.7	8.3	7.9	7.5	7.1	6.7	6.3	5.9	4
150	13.2	12.2	11.8	11.2	10.6	10.1	9.4	8.8	8.3	7.9	7.5	7.1	6.7	6.3	5.9	5.5	3
160	12.4	12.0	11.5	10.9	10.3	9.8	9.1	8.5	8.0	7.6	7.2	6.8	6.4	6.0	5.6	5.2	3
170	12.3	11.8	11.2	10.6	10.1	9.6	8.9	8.3	7.7	7.3	6.9	6.5	6.1	5.7	5.3	4.9	2
180	12.1	11.6	10.9	10.4	9.9	9.4	8.7	8.1	7.5	7.1	6.7	6.3	5.9	5.5	5.1	4.7	2
200	11.9	11.2	10.5	10.0	9.5	9.1	8.3	7.7	7.2	6.8	6.4	6.0	5.6	5.2	4.8	4.4	2
220	11.5	10.8	10.3	9.7	9.3	8.8	8.1	7.5	7.0	6.6	6.2	5.8	5.4	5.0	4.6	4.2	1
240	11.2	10.6	10.0	9.5	9.1	8.6	8.0	7.4	6.9	6.5	6.1	5.7	5.3	4.9	4.5	4.1	1
260	11.0	10.4	9.9	9.4	8.9	8.5	7.9	7.3	6.8	6.4	6.0	5.6	5.2	4.8	4.4	4.0	1
280	10.8	10.2	9.7	9.2	8.7	8.3	7.7	7.1	6.6	6.2	5.8	5.4	5.0	4.6	4.2	3.8	
300	10.7	10.1	9.6	9.1	8.6	8.2	7.6	7.0	6.5	6.1	5.7	5.3	4.9	4.5	4.1	3.7	
320	10.5	10.0	9.5	9.0	8.5	8.1	7.5	6.9	6.4	6.0	5.6	5.2	4.8	4.4	4.0	3.6	

Girders so proportioned that the computed shear is less than that given in right-hand column do not require intermediate stiffeners.

\* For single angle stiffeners, multiply by 1.8; for single plate stiffeners, multiply by 2.4.

ALLOWABLE SHEAR STRESSES ( $F_v$ ) IN PLATE GIRDERS (KSI)  
FOR 42 KSI SPECIFIED YIELD STRESS STEEL

(*Italic values indicate gross area, as percent of web area, required for pairs of intermediate stiffeners of 42 ksi yield stress steel.*) \*

Slenderness ratios $h/t$ : web depth to web thickness	Aspect ratios $a/h$ : stiffener spacing to web depth													
	0.5	0.6	0.7	0.8	0.9	1.0	1.2	1.4	1.6	1.8	2.0	2.5	3.0	over 3
50														17.0
60								17.0	17.0	17.0	17.0	17.0	17.0	16.4
70						17.0	17.0	16.5	16.0	15.6	15.3	14.9	14.6	14.1
80				17.0	17.0	16.3	15.2	14.5	14.2	14.0	13.8	13.5	13.3	12.3
90			17.0	16.6	15.4	14.5	14.1	13.7	13.4	13.1	12.9	12.5	12.1	10.3
100		17.0	16.4	15.0	14.3	14.0	13.5	13.0	12.5	12.1	11.7	11.0	10.5	8.3
110		16.8	15.0	14.2	13.9	13.5	12.7	12.0	11.5	11.0	10.6	9.9	9.4	6.9
120	17.0	15.4	14.3	13.9	13.4	12.8	12.0	11.3	10.7	10.2	9.8	9.1	8.5	5.8
130	16.5	14.5	14.0	13.5	12.8	12.3	11.4	10.7	10.1	9.6	9.2	8.4	7.9	4.9
140	15.3	14.2	13.7	13.0	12.4	11.8	10.9	10.2	9.7	9.2	8.7	7.9	7.3	4.2
150	14.5	14.0	13.3	12.6	12.0	11.4	10.6	9.9	9.3	8.8	8.3	7.5	6.9	3.7
160	14.3	13.8	13.0	12.3	11.7	11.1	10.3	9.6	9.0	8.4	8.0	7.2		3.2
170	14.1	13.4	12.7	12.0	11.4	10.9	10.0	9.3	8.7	8.2	7.7			2.9
180	14.0	13.1	12.4	11.8	11.2	10.7	9.8	9.1	8.5	8.0	7.5			2.6
200	13.5	12.7	12.0	11.4	10.9	10.3	9.5	8.8	8.1					2.1
220	13.1	12.4	11.7	11.1	10.6	10.1	9.2	8.5						1.7
240	12.8	12.1	11.5	10.9	10.4	9.9								1.4
260	12.6	11.9	11.3	10.8	10.3	9.8								1.2
280	12.4	11.8	11.2	10.7										
300	12.3	11.7	11.1											

Girders so proportioned that the computed shear is less than that given in right-hand column do not require intermediate stiffeners.

\* For areas of other intermediate stiffeners, multiply *italic values* by appropriate factor below:

Stiffener Steel Grade	Pairs of Stiffeners	Single Angle Stiffeners	Single Plate Stiffeners
$F_v = 42$ ksi	1.0	1.8	2.4
$F_v = 36$ ksi	1.2	2.1	2.8

TABLE 3-45 ( )  
ALLOWABLE SHEAR STRESSES ( $F_v$ ) IN PLATE GIRDERS (KSI)  
FOR 45 KSI SPECIFIED YIELD STRESS STEEL

(*Italic values indicate gross area, as percent of web area, required for pairs of intermediate stiffeners of 45 ksi yield stress steel.*) \*

Slenderness ratios $h/t$ : web depth to web thickness	Aspect ratios $a/h$ : stiffener spacing to web depth													
	0.5	0.6	0.7	0.8	0.9	1.0	1.2	1.4	1.6	1.8	2.0	2.5	3.0	
50														18.0
60								18.0	18.0	18.0	18.0	18.0	18.0	17.7
70						18.0	18.0	18.0	17.1	16.6	16.2	15.9	15.5	15.3
80						18.0	17.9	16.8	15.7	15.3	15.0	14.7	14.5	14.2
90						18.0	17.2	15.9	15.3	14.9	14.5	14.1	13.8	13.5
100		18.0	17.0	15.5	15.1	14.8	14.2	13.6	13.0	12.5	12.1	11.3	10.8	
110		17.4	15.5	15.1	14.7	14.2	13.3	12.6	12.0	11.5	11.0	10.2	9.7	
120	18.0	15.9	15.2	14.7	14.1	13.5	12.6	11.8	11.2	10.7	10.2	9.4	8.8	
130	17.1	15.4	14.9	14.2	13.5	12.9	12.0	11.2	10.6	10.1	9.6	8.8	8.1	
140	15.9	15.1	14.5	13.7	13.0	12.4	11.5	10.8	10.1	9.6	9.1	8.2	7.6	
150	15.4	14.9	14.1	13.3	12.7	12.1	11.2	10.4	9.8	9.2	8.7	7.8	7.2	
160	15.2	14.5	13.7	13.0	12.3	11.8	10.9	10.1	9.4	8.9	8.4	7.5		3
170	15.0	14.2	13.4	12.7	12.1	11.5	10.6	9.8	9.2	8.6	8.1			2
180	14.8	13.9	13.2	12.5	11.9	11.3	10.4	9.6	9.0	8.4	7.9			2
200	14.3	13.5	12.8	12.1	11.5	11.0	10.1	9.3	8.6					2
220	13.9	13.2	12.5	11.8	11.3	10.7	9.8	9.0						1
240	13.6	12.9	12.3	11.6	11.1	10.5								1
260	13.4	12.7	12.1	11.5	10.9	10.4								1
280	13.2	12.6	11.9	11.4										
300	13.1	12.4	11.8											

Girders so proportioned that the computed shear is less than that given in right-hand color do not require intermediate stiffeners.

\* For areas of other intermediate stiffeners, multiply *italic values* by appropriate factor below:

Stiffener Steel Grade	Pairs of Stiffeners	Single Angle Stiffeners	Single Plate Stiffeners
$F_v = 45$ ksi	1.0	1.8	1
$F_v = 36$ ksi	1.3	2.3	3.0

TABLE 3-50

ALLOWABLE SHEAR STRESSES ( $F_v$ ) IN PLATE GIRDERS (KSI)  
FOR 50 KSI SPECIFIED YIELD STRESS STEEL

(*Italic values indicate gross area, as percent of web area, required for pairs of intermediate stiffeners of 50 ksi yield stress steel.*)\*

Slenderness ratios $h/t$ : web depth to web thickness	Aspect ratios $a/h$ : stiffener spacing to web depth													
	0.5	0.6	0.7	0.8	0.9	1.0	1.2	1.4	1.6	1.8	2.0	2.5	3.0	over 3
50										20.0	20.0	20.0	20.0	20.0
60							20.0	20.0	20.0	19.9	19.5	18.9	18.6	17.9
70					20.0	20.0	18.9	18.0	17.4	17.1	16.9	16.6	16.3	15.3
80			20.0	20.0	18.9	17.8	17.0	16.6	16.2	15.9	15.7	15.2	14.9	13.0
90			19.9	18.1	17.1	16.7	16.2	15.7	15.1	14.6	14.2	13.4	12.8	10.3
100		20.0	17.9	17.0	16.5	16.1	15.2	14.4	13.8	13.2	12.8	11.9	11.3	8.3
110	20.0	18.3	17.0	16.5	16.0	15.3	14.3	13.4	12.8	12.2	11.7	10.8	10.2	6.9
120	19.5	17.2	16.6	16.0	15.2	14.5	13.5	12.7	12.0	11.4	10.9	10.0	9.3	5.8
130	18.0	16.8	16.3	15.4	14.6	14.0	12.9	12.1	11.4	10.8	10.3	9.3	8.6	4.9
140	17.2	16.6	15.7	14.9	14.2	13.5	12.5	11.6	10.9	10.3	9.8	8.8	8.1	4.2
150	16.9	16.2	15.3	14.5	13.8	13.1	12.1	11.3	10.5	9.9	9.4	8.4	7.7	3.7
160	16.7	15.8	14.9	14.2	13.5	12.8	11.8	11.0	10.2	9.6	9.1	8.0		3.2
170	16.5	15.5	14.6	13.9	13.2	12.6	11.6	10.7	10.0	9.4	8.8			2.9
180	16.2	15.2	14.4	13.7	13.0	12.4	11.4	10.5	9.8	9.1	8.6			2.6
200	15.7	14.8	14.0	13.3	12.6	12.0	11.0	10.2	9.4					2.1
220	15.3	14.4	13.7	13.0	12.4	11.8	10.8	9.9						1.7
240	15.0	14.2	13.5	12.8	12.2	11.6								1.4
260	14.8	14.0	13.3	12.7	12.0	11.5								1.2
280	14.6	13.9	13.2	12.5										

Girders so proportioned that the computed shear is less than that given in right-hand column do not require intermediate stiffeners.

\* For areas of other intermediate stiffeners, multiply *italic values* by appropriate factor below:

Stiffener Steel Grade	Pairs of Stiffeners	Single Angle Stiffeners	Single Plate Stiffeners
$F_v = 50$ ksi	1.0	1.8	2.4
$F_v = 36$ ksi	1.4	2.5	3.3

TABLE 3-55

ALLOWABLE SHEAR STRESSES ( $F_v$ ) IN PLATE GIRDERS (KSI)  
FOR 55 KSI SPECIFIED YIELD STRESS STEEL

(*Italic values indicate gross area, as percent of web area, required for pairs of intermediate stiffeners of 55 ksi yield stress steel.*)\*

Slenderness ratios $h/t$ : web depth to web thickness	Aspect ratios $a/h$ : stiffener spacing to web depth													
	0.5	0.6	0.7	0.8	0.9	1.0	1.2	1.4	1.6	1.8	2.0	2.5	3.0	over 3
50														
60						22.0	22.0	22.0	21.3	20.8	20.5	19.9	19.5	18.6
70					22.0	22.0	21.3	19.8	19.0	18.6	18.4	18.1	17.7	17.4
80			22.0	21.4	19.8	18.9	18.3	17.8	17.4	17.1	16.8	16.0	15.4	14.8
90		22.0	20.9	19.0	18.5	18.1	17.4	16.6	15.9	15.3	14.8	13.9	13.3	12.8
100		21.2	19.0	18.4	17.9	17.3	16.2	15.3	14.6	14.0	13.4	12.5	11.8	11.3
110	22.0	19.2	18.5	17.9	17.1	16.3	15.2	14.3	13.6	12.9	12.4	11.4	10.6	10.1
120	20.5	18.7	18.1	17.2	16.4	15.6	14.5	13.6	12.8	12.1	11.6	10.5	9.8	9.3
130	19.0	18.3	17.5	16.6	15.8	15.0	13.9	13.0	12.2	11.5	11.0	10.0	9.1	8.6
140	18.7	18.0	17.0	16.1	15.3	14.6	13.5	12.5	11.7	11.1	10.5	9.4	8.6	8.1
150	18.4	17.5	16.5	15.7	14.9	14.2	13.1	12.1	11.3	10.7	10.1	8.9	8.1	7.6
160	18.2	17.1	16.2	15.3	14.6	13.9	12.8	11.8	11.0	10.3	9.8	8.6	7.8	7.3
170	17.8	16.8	15.9	15.1	14.3	13.6	12.5	11.6	10.8	10.1	9.5	8.4	7.6	7.1
180	17.5	16.5	15.6	14.8	14.1	13.4	12.3	11.4	10.6	9.9	9.3	8.2	7.4	6.9
200	17.0	16.1	15.2	14.5	13.8	13.1	12.0	11.0	10.2					
220	16.6	15.7	14.9	14.2	13.5	12.9	11.8	10.8						
240	16.3	15.5	14.7	14.0	13.3	12.7								
260	16.1	15.3	14.5	13.8	13.2	12.5								
280	9.6	11.2	12.2	12.8	13.0	13.0								

Girders so proportioned that the computed shear is less than that given in right-hand column do not require intermediate stiffeners.

\* For areas of other intermediate stiffeners, multiply *italic values* by appropriate factor below:

Stiffener Steel Grade	Pairs of Stiffeners	Single Angle Stiffeners	Single Plate Stiffeners
$F_v = 55$ ksi	1.0	1.8	2.4
$F_v = 36$ ksi	1.5	2.8	3.7

TABLE 3-60

ALLOWABLE SHEAR STRESSES ( $F_v$ ) IN PLATE GIRDERS (KSI)  
FOR 60 KSI SPECIFIED YIELD STRESS STEEL

(*Italic values indicate gross area, as percent of web area, required for pairs of intermediate stiffeners of 60 ksi yield stress steel.*)\*

		Aspect ratios $a/h$ : stiffener spacing to web depth													
		0.5	0.6	0.7	0.8	0.9	1.0	1.2	1.4	1.6	1.8	2.0	2.5	3.0	over 3
Slenderness ratios $h/t$ : web depth to web thickness	40														24.0
	50														24.0
	60					24.0	24.0	24.0	23.1	22.3	21.8	21.4	20.8	20.5	23.5
	70				24.0	23.7	22.2	20.7	20.3	19.9	19.5	19.3	18.7	18.4	19.6
	80			24.0	22.4	20.7	20.3	19.6	19.0	18.6	18.0	17.5	16.6	15.9	13.0
	90		24.0	21.8	20.5	19.9	19.4	18.5	17.5	16.7	16.1	15.5	14.5	13.8	10.3
	100	24.0	22.1	20.4	19.8	19.3	18.4	17.2	16.2	15.4	14.7	14.1	13.0	12.2	8.3
	110	23.3	20.6	19.9	19.2	18.2	17.4	16.2	15.2	14.4	13.7	13.1	11.9	11.1	6.9
	120	21.4	20.2	19.4	18.4	17.5	16.7	15.4	14.4	13.6	12.9	12.3	11.1	10.2	5.8
	130	20.5	19.8	18.7	17.7	16.9	16.1	14.9	13.9	13.0	12.3	11.6	10.4	9.6	4.9
	140	20.2	19.3	18.2	17.2	16.4	15.6	14.4	13.4	12.5	11.8	11.2	9.9	9.0	4.2
	150	19.9	18.8	17.8	16.8	16.0	15.3	14.0	13.0	12.1	11.4	10.8	9.5	8.6	3.7
	160	19.6	18.4	17.4	16.5	15.7	15.0	13.7	12.7	11.8	11.1	10.4	9.2	8.4	3.2
	170	19.2	18.1	17.1	16.2	15.4	14.7	13.5	12.5	11.6	10.8	10.2			2.9
180	18.9	17.8	16.8	16.0	15.2	14.5	13.3	12.2	11.4	10.6	9.9			2.6	
200	18.3	17.3	16.5	15.6	14.9	14.2	13.0	11.9	11.0					2.1	
220	18.0	17.0	16.2	15.4	14.6	13.9	12.7	11.7						1.7	
240	17.7	16.8	15.9	15.2	14.4	13.7								1.4	
260	17.4	16.6	15.8	15.0	14.3	13.6								1.2	

TABLE 3-65

ALLOWABLE SHEAR STRESSES ( $F_v$ ) IN PLATE GIRDERS (KSI)  
FOR 65 KSI SPECIFIED YIELD STRESS STEEL

(*Italic values indicate gross area, as percent of web area, required for pairs of intermediate stiffeners of 65 ksi yield stress steel.*)\*

		Aspect ratios $a/h$ : stiffener spacing to web depth													
		0.5	0.6	0.7	0.8	0.9	1.0	1.2	1.4	1.6	1.8	2.0	2.5	3.0	0
Slenderness ratios $h/t$ : web depth to web thickness	40														26.0
	50														26.0
	60					26.0	26.0	25.2	24.0	23.2	22.7	22.3	21.9	21.6	22.2
	70			26.0	26.0	24.6	23.1	22.1	21.5	21.1	20.7	20.4	19.8	19.4	19.1
	80		26.0	25.6	23.3	22.2	21.6	20.9	20.3	19.4	18.7	18.2	17.1	16.4	16.1
	90		25.6	22.7	21.9	21.3	20.8	19.4	18.4	17.5	16.8	16.2	15.0	14.2	13.9
	100	26.0	23.0	21.9	21.2	20.4	19.4	18.1	17.1	16.2	15.4	14.8	13.6	12.7	12.4
	110	24.3	22.1	21.4	20.4	19.4	18.5	17.2	16.1	15.2	14.4	13.7	12.5	11.6	11.3
	120	22.4	21.6	20.7	19.6	18.6	17.7	16.4	15.3	14.4	13.6	12.9	11.6	10.7	10.4
	130	22.1	21.2	20.0	18.9	18.0	17.1	15.8	14.7	13.8	13.0	12.3	11.0	10.0	9.7
	140	21.7	20.6	19.4	18.4	17.5	16.7	15.4	14.3	13.3	12.5	11.8	10.5	9.5	9.2
	150	21.4	20.1	19.0	18.0	17.1	16.3	15.0	13.9	12.9	12.1	11.4	10.1	9.1	8.8
	160	20.9	19.7	18.6	17.7	16.8	16.0	14.7	13.6	12.6	11.8	11.1	9.7	8.7	8.4
	170	20.5	19.4	18.3	17.4	16.6	15.8	14.5	13.3	12.4	11.5	10.8			10.8
180	20.2	19.1	18.1	17.2	16.3	15.6	14.3	13.1	12.2	11.3	10.6			10.6	
200	19.7	18.6	17.7	16.8	16.0	15.2	13.9	12.8						12.8	
220	19.3	18.3	17.4	16.5	15.7	15.0	13.7	12.5						12.5	
240	19.0	18.1	17.2	16.3	15.5	14.8								14.8	

Girders so proportioned that the computed shear is less than that given in right-hand column do not require intermediate stiffeners.

\* For areas of other intermediate stiffeners, multiply *italic values* by appropriate factor below:

Stiffener Steel Grade	Pairs of Stiffeners	Single Angle Stiffeners	Single Plate Stiffeners
$F_v = 65$ ksi	1.0	1.8	2.4
$F_v = 36$ ksi	1.8	3.3	4.3

Girders so proportioned that the computed shear is less than that given in right-hand column do not require intermediate stiffeners.

\* For areas of other intermediate stiffeners, multiply *italic values* by appropriate factor below:

Stiffener Steel Grade	Pairs of Stiffeners	Single Angle Stiffeners	Single Plate Stiffeners
$F_v = 60$ ksi	1.0	1.8	2.4
$F_v = 50$ ksi	1.7	3.0	4.0

TABLE 3-90

ALLOWABLE SHEAR STRESSES ( $F_v$ ) IN PLATE GIRDERS (KSI)  
FOR 90 KSI SPECIFIED YIELD STRESS STEEL

(*Italic values indicate gross area, as percent of web area, required for pairs of intermediate stiffeners of 90 ksi yield stress steel.*)\*

		Aspect ratios $a/h$ : stiffener spacing to web depth													
		0.5	0.6	0.7	0.8	0.9	1.0	1.2	1.4	1.6	1.8	2.0	2.5	3.0	over 3
Slenderness ratios $h/t$ : web depth to web thickness	40														
	50					36.0	36.0	35.5	33.9	32.8	32.0	31.4	30.7	30.3	28.8
	60			36.0	36.0	33.8	31.8	30.5	29.7	29.1	28.5	28.1	27.3	26.7	23.1
	70		36.0	34.4	31.3	30.4	29.6	28.6	27.3	26.2	25.2	24.4	22.9	21.9	17.0
	80	36.0	33.8	30.8	29.9	29.1	27.9	26.1	24.6	23.4	22.4	21.5	19.9	18.8	13.0
	90	34.9	30.9	29.9	28.8	27.3	26.1	24.3	22.8	21.5	20.5	19.6	17.8	16.6	10.3
	100	31.4	30.1	28.9	27.3	26.0	24.7	22.9	21.4	20.1	19.1	18.1	16.4	15.1	8.3
	110	30.6	29.4	27.7	26.2	24.9	23.8	22.0	20.4	19.1	18.0	17.1	15.3	14.0	6.9
	120	30.0	28.4	26.8	25.4	24.2	23.0	21.2	19.7	18.4	17.3	16.3	14.4	13.1	5.8
	130	29.4	27.6	26.1	24.8	23.6	22.5	20.7	19.1	17.8	16.7	15.7	13.8	12.4	4.9
	140	28.7	27.0	25.6	24.3	23.1	22.0	20.2	18.6	17.3	16.2	15.2	13.3	11.9	4.2
	150	28.1	26.5	25.2	23.9	22.7	21.6	19.8	18.3						
	160	27.6	26.1	24.8	23.6	22.4	21.3	19.5	18.0						
	170	27.3	25.8	24.5	23.3	22.1	21.1	19.3	17.7						
	180	26.9	25.5	24.2	23.0	21.9	20.9	19.1	17.5						
	200	26.4	25.1	23.9	22.7	21.6	20.5	18.7	17.2						

Girders so proportioned that the computed shear is less than that given in right-hand column do not require intermediate stiffeners.

\* For areas of other intermediate stiffeners, multiply *italic values* by appropriate factor below:

Stiffener Steel Grade	Pairs of Stiffeners	Single Angle Stiffeners	Single Plate Stiffeners
$F_v = 90$ ksi	1.0	1.8	2.4
$F_v = 36$ ksi	2.5	4.5	6.0

TABLE 3-100

ALLOWABLE SHEAR STRESSES ( $F_v$ ) IN PLATE GIRDERS (KSI)  
FOR 100 KSI SPECIFIED YIELD STRESS STEEL

(*Italic values indicate gross area, as percent of web area, required for pairs of intermediate stiffeners of 100 ksi yield stress steel.*)\*

		Aspect ratios $a/h$ : stiffener spacing to web depth													
		0.5	0.6	0.7	0.8	0.9	1.0	1.2	1.4	1.6	1.8	2.0	2.5	3.0	over 3
Slenderness ratios $h/t$ : web depth to web thickness	30														
	40												40.0	40.0	40.0
	50												40.0	40.0	40.0
	60				40.0	40.0	40.0	37.5	35.7	34.6	34.1	33.7	32.9	32.4	30.0
	70		40.0	36.3	34.1	33.2	32.3	30.7	29.1	27.8	26.7	25.8	24.0	22.8	17.0
	80	40.0	35.7	33.8	32.8	31.5	30.0	28.0	26.4	25.0	23.9	22.9	21.0	19.7	13.0
	90	36.8	33.9	32.8	31.1	29.6	28.2	26.2	24.5	23.1	21.9	20.9	19.0	17.6	10.0
	100	34.3	33.1	31.3	29.7	28.2	26.9	24.9	23.2	21.7	20.5	19.5	17.5	16.0	8.0
	110	33.6	32.0	30.2	28.6	27.2	25.9	23.9	22.2	20.7	19.5	18.4	16.4	14.9	6.0
	120	33.0	31.0	29.3	27.8	26.4	25.2	23.2	21.4	20.0	18.7	17.6	15.6	14.0	5.0
	130	32.1	30.2	28.6	27.1	25.8	24.6	22.6	20.9	19.4	18.1	17.0	14.9	13.4	4.0
	140	31.4	29.6	28.1	26.6	25.3	24.1	22.1	20.4						
	150	30.8	29.1	27.6	26.2	25.0	23.8	21.8	20.0						
	160	30.3	28.7	27.3	25.9	24.6	23.5	21.5	19.7						
	170	29.9	28.4	27.0	25.6	24.4	23.2	21.2	19.5						
	180	29.6	28.1	26.7	25.4	24.2	23.0	21.0	19.2						
	200	29.1	27.7	26.3	25.0	23.8	22.7	20.7	18.9						

Girders so proportioned that the computed shear is less than that given in right-hand column do not require intermediate stiffeners.

\* For areas of other intermediate stiffeners, multiply *italic values* by appropriate factor below:

Stiffener Steel Grade	Pairs of Stiffeners	Single Angle Stiffeners	Single Plate Stiffeners
$F_v = 100$ ksi	1.0	1.8	2.4
$F_v = 36$ ksi	2.8	5.0	6.7

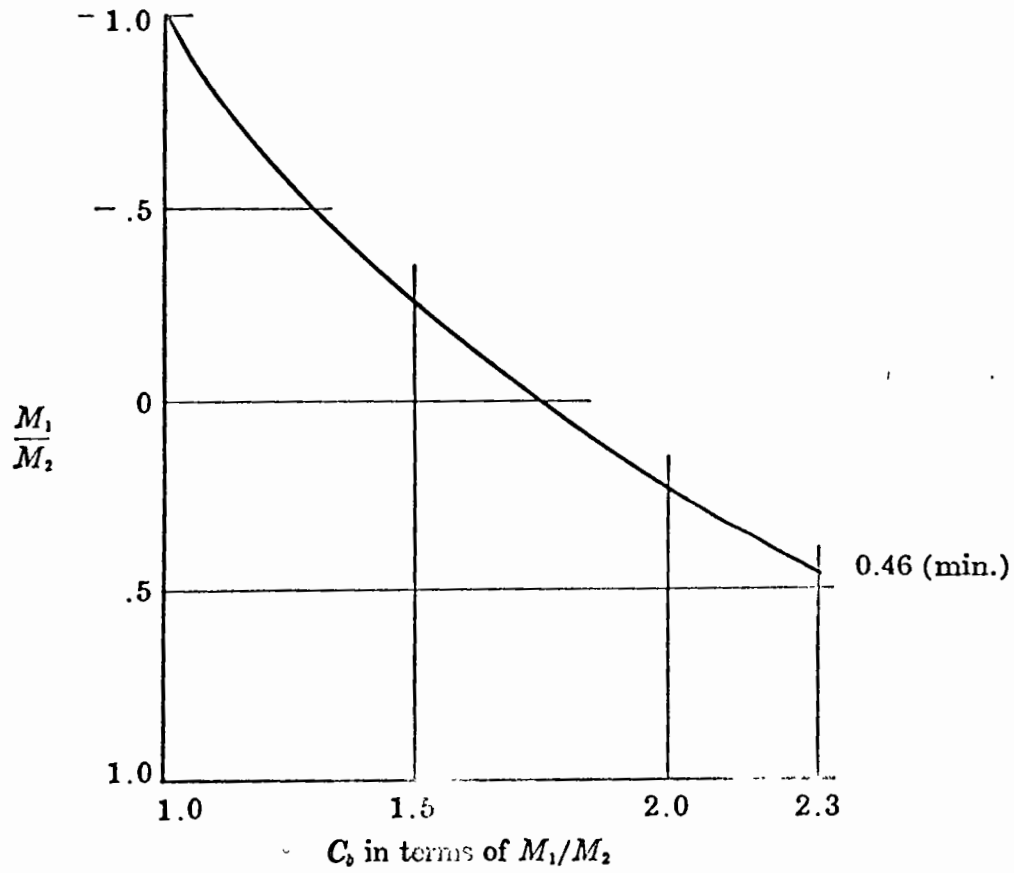


Fig. A1

## APPENDIX B

### Fatigue



## SECTION B1 LOADING CONDITIONS AND TYPE AND LOCATION OF MATERIAL

In the design of members and connections subject to repeated variation of live load stress, consideration shall be given to the number of stress cycles, the expected range of stress, and type and location of member or detail.

Loading conditions shall be classified as shown in Table B1.

TABLE B1

Loading Condition	Number of Loading Cycles	
	From	To
1	20,000 <sup>1</sup>	100,000 <sup>1</sup>
2	100,000	500,000 <sup>2</sup>
3	500,000	2,000,000 <sup>3</sup>
4	Over 2,000,000	

<sup>1</sup> Approximately equivalent to two applications every day for 25 years.

<sup>2</sup> Approximately equivalent to ten applications every day for 25 years.

<sup>3</sup> Approximately equivalent to fifty applications every day for 25 years.

<sup>4</sup> Approximately equivalent to two hundred applications every day for 25 years.

The type and location of material shall be categorized as shown in Table B2.

## SECTION B2 ALLOWABLE STRESSES

The maximum stress shall not exceed the basic allowable stress provided in Sects. 1.5 and 1.6 of this Specification, and the maximum range of stress shall not exceed that given in Table B3 except that, in the case of stress reversal only, the value  $F'_{sr}$  given by Formula (B1) may be used as the stress range for those categories marked with an asterisk in Table B2.

$$F'_{sr} = \left( \frac{f_t + f_c}{f_t + 0.6f_c} \right) F_{sr} \quad (\text{B1})$$

where  $f_t$  and  $f_c$  are, respectively, calculated tensile and compressive stresses considered as positive quantities, and  $F_{sr}$  is the allowable stress range given in Table B3.

TABLE B2

General Condition	Situation	Kind of Stress <sup>1</sup>	Stress Category (See Table B3)	Illustrative Example Nos. (See Fig. B1) <sup>2</sup>
Plain material	Base metal with rolled or cleaned surfaces.	T or Rev.	A	1, 2
Built-up members	Base metal and weld metal in members, without attachments, built up of plates or shapes connected by continuous full penetration groove welds parallel to the direction of applied stress.	Rev. Rev. T or C	B* <sup>3</sup> B B	3 4 3, 4
	Base metal and weld metal in members, without attachments, built up of plates or shapes connected by continuous fillet welds parallel to the direction of applied stress.	T, C or Rev.	B	4, 5, 6
	Calculated flexural stress, $f_t$ , at toe of welds on girder webs or flanges adjacent to welded transverse stiffeners: When $f_v \leq F_v/2$ When $f_v > F_v/2$ where $F_v$ = allowable shear stress.	T or Rev. T or Rev.	C D	7 7
	Base metal at end of partial length welded cover plates having square or tapered ends, with or without welds across the ends.	T, C or Rev.	E	5

<sup>1</sup> "T" signifies range in tensile stress only; "C" signifies range in compressive stress only; "Rev." signifies a range involving reversal of tensile or compressive stress; "S" signifies range in shear including shear stress reversal.

<sup>2</sup> These examples are provided as guide lines and are not intended to exclude other reasonably similar situations.

<sup>3</sup> Form B1) applicable in situations identified by asterisk (\*).

<sup>4</sup> Where stress reversal is involved, use of A307 bolts is not recommended.

TABLE B2 (continued)

General Condition	Situation	Kind of Stress <sup>1</sup>	Stress Category. (See Table B3)	Illustrative Example Nos. (See Fig. B1) <sup>2</sup>
Mechanically fastened connections	Base metal at net section of high-strength bolted connections, except bearing-type connections subject to stress reversal and axially loaded joints which induce out-of-plane bending in connected material.	T or Rev.	A	8
	Base metal at net section of other mechanically fastened joints. <sup>4</sup>	T or Rev.	B	8, 9
Groove welds	Base metal and weld metal at full penetration groove welded splices of parts of similar cross section ground flush, with grinding in the direction of applied stress and with weld soundness established by radiographic or ultrasonic inspection.	T or Rev.	A	10
	Base metal and weld metal at full penetration groove welded splices of rolled and welded sections having similar profiles, when welds are ground flush.	T or Rev.	B	11
	Base metal and weld metal in or adjacent to full penetration groove welded splices at transitions in width or thickness, with welds ground to provide slopes no steeper than 1 to 2½, with grinding in the direction of applied stress, and with weld soundness established by radiographic or ultrasonic inspection.	T or Rev.	B	12, 13

TABLE B2 (continued)

General Condition	Situation	Kind of Stress <sup>1</sup>	Stress Category. (See Table B3)	Illustrative Example Nos. (See Fig. B1) <sup>2</sup>
Groove welds (cont'd)	Base metal and weld metal in or adjacent to full penetration groove welded splices, with or without transitions having slopes no greater than 1 to 2½, when reinforcement is not removed and/or weld soundness is not established by radiographic or ultrasonic inspection.	T Rev. T or Rev.	C C* C	10 10 11, 12, 13
	Base metal or weld metal in or adjacent to full penetration groove welds in tee or cruciform joints.	T Rev.	D D*	14 14
	Base metal at details attached by groove welds subject to transverse and/or longitudinal loading.	T, C or Rev.	E	15
	Weld metal of partial penetration transverse groove welds, based on effective throat area of the weld or welds.	T or Rev.	G	16
	Base metal at intermittent fillet welds.	T, C or Rev.	E	
Fillet welded connections	Base metal at junction of axially loaded members with fillet welded end connections. Welds shall be disposed about the axis of the member so as to balance weld stresses.	T, C or Rev.	E	17, 18, 19, 20
	Continuous or intermittent longitudinal or transverse fillet welds (except transverse fillet welds in tee joints) and continuous fillet welds	S	F	5, 17, 18, 19, 21

TABLE B2 (continued)

General Condition	Situation	Kind of Stress <sup>1</sup>	Stress Category. (See Table B3)	Illustrative Example Nos. (See Fig. B1) <sup>2</sup>
Fillet welded connections (cont'd)	subject to shear parallel to the weld axis in combination with shear due to flexure.			
	Transverse fillet welds in tee joints.	S	G	20
Miscellaneous details	Base metal adjacent to short (2 in. maximum length in direction of stress) welded attachments.	C T or Rev.	C D	22, 23, 24 22, 23, 24, 25
	Base metal adjacent to longer fillet welded attachments.	T, C or Rev.	E	26
	Base metal at plug or slot welds.	T, C or Rev.	E	27
	Shear stress on nominal area of stud-type shear connectors.	S	G	22
	Shear on plug or slot welds.	S	G	27

TABLE B3

Category (From Table B2)	Allowable Range of Stress, $F_{rr}$ (ksi)			
	Loading Condition 1 $F_{rr1}$	Loading Condition 2 $F_{rr2}$	Loading Condition 3 $F_{rr3}$	Loading Condition 4 $F_{rr4}$
A <sup>1</sup>	40	32	24	24
B	33	25	17	15
C	28	21	14	12
D	24	17	10	9
E	17	12	7	6
F	17	14	11	9
G	15	12	9	8

<sup>1</sup> For A514 steels in Category A, substitute the following values:  $F_{rr1} = 45$ ,  $F_{rr2} = 35$ ,  $F_{rr3} = 25$  and  $F_{rr4} = 25$ .

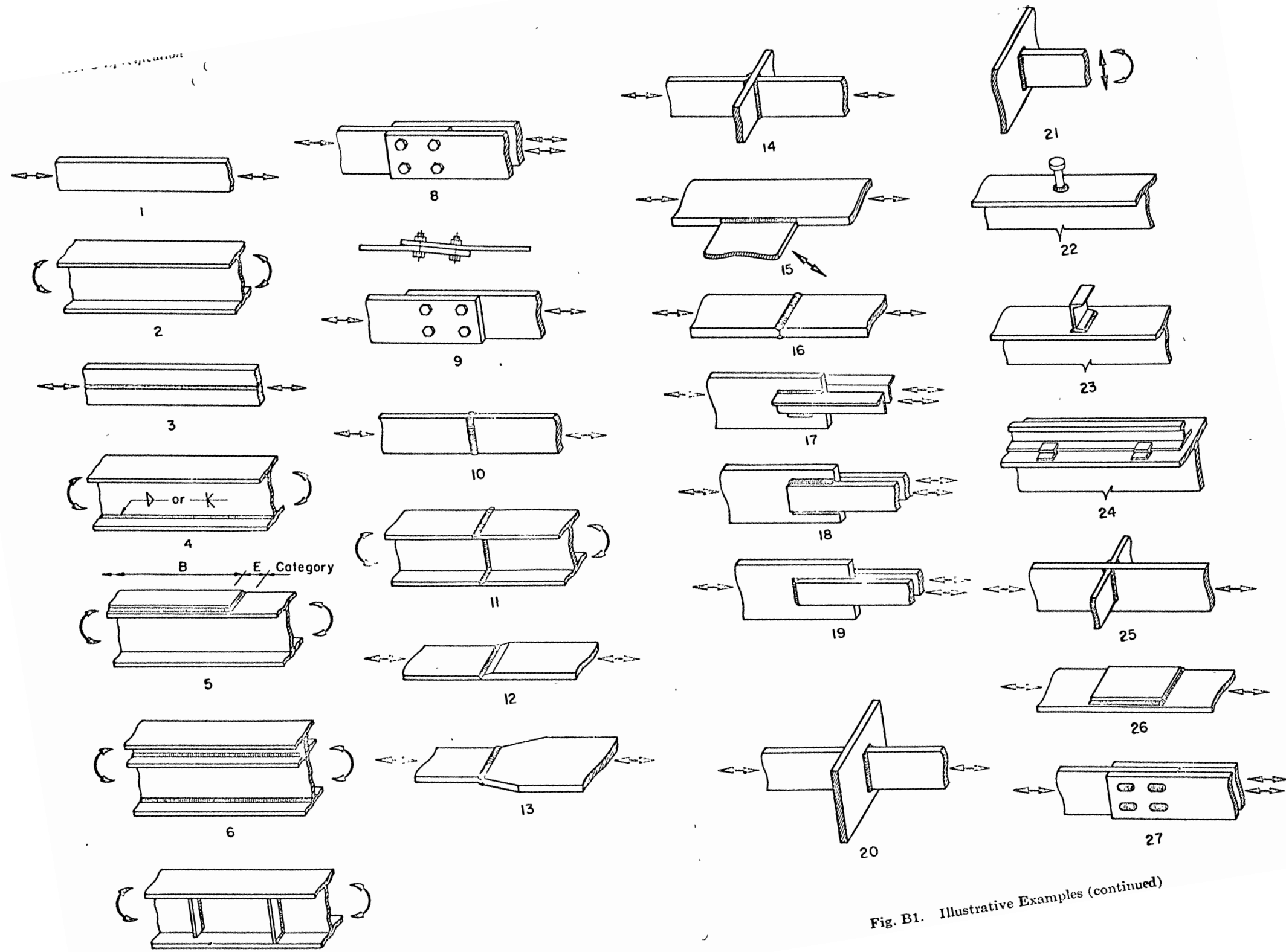


Fig. B1. Illustrative Examples (continued)

## APPENDIX C

### Slender Compression Elements

#### SECTION C1 GENERAL

Axially loaded members and flexural members containing elements subject to compression and having a width-thickness ratio in excess of the applicable limit given in Sect. 1.9 shall be proportioned to meet the requirements of this Appendix.

#### SECTION C2 STRESS REDUCTION FACTOR—UNSTIFFENED COMPRESSION ELEMENTS

Except as hereinafter provided, unstiffened compression elements whose width-thickness ratio exceeds the applicable limit given in Sect. 1.9.1.2 shall be subject to a reduction factor  $Q_s$ . The value of  $Q_s$  shall be determined by Formulas (C2-1) to (C2-6), as applicable, where  $b$  is the width of the unstiffened element as defined in Sect. 1.9.1.1. When such elements comprise the compression flange of a flexural member the maximum allowable bending stress shall not exceed  $0.6F_y Q_s$ , nor the applicable value as provided in Sect. 1.5.1.4.6. The allowable stress of axially loaded compression members shall be modified by the appropriate reduction factor  $Q_s$ , as provided in Sect. C5.

For single angles:

When  $76.0/\sqrt{F_y} < b/t < 155/\sqrt{F_y}$ :

$$Q_s = 1.340 - 0.00447(b/t)\sqrt{F_y} \quad (C2-1)$$

When  $b/t \geq 155/\sqrt{F_y}$ :

$$Q_s = 15,500/[F_y(b/t)^2] \quad (C2-2)$$

For angles or plates projecting from columns or other compression members, and for compression flanges of girders:

When  $95.0/\sqrt{F_y} < (b/t) < 176/\sqrt{F_y}$ :

$$Q_s = 1.415 - 0.00437(b/t)\sqrt{F_y} \quad (C2-3)$$

When  $b/t \geq 176/\sqrt{F_y}$ :

$$Q_s = 20,000/[F_y(b/t)^2] \quad (C2-4)$$

For stems of tees:

When  $127/\sqrt{F_y} < (b/t) < 176/\sqrt{F_y}$ :

$$Q_s = 1.908 - 0.00715(b/t)\sqrt{F_y} \quad (C2-5)$$

When  $b/t \geq 176/\sqrt{F_y}$ :

$$Q_s = 20,000/[F_y(b/t)^2] \quad (C2-6)$$

However, the proportions of channels and tees shall in any case conform to the limits given in Table C1.

TABLE C1  
Limiting Proportions for Channels and Tees

Shape	Ratio of flange width to profile depth	Ratio of flange thickness to web or stem thickness
Built-up or Rolled Channels	$\leq 0.25$	$\leq 3.0$
	$\leq 0.50$	$\leq 2.0$
Built-up Tees	$\geq 0.50$	$\geq 1.25$
Rolled Tees	$\geq 0.50$	$\geq 1.10$

### SECTION C3 EFFECTIVE WIDTH—STIFFENED COMPRESSION ELEMENTS

When the width-thickness ratio of a uniformly compressed stiffened element (except perforated cover plates) exceeds the applicable limit given in Sect. 1.9.2.2, a reduced effective width,  $b_e$ , shall be used in computing the flexural design properties of the section containing the element and the permissible axial stress, except that the ratio  $b_e/t$  need not be taken as less than the applicable value permitted in Sect. 1.9.2.2.

For the flanges of square and rectangular sections of uniform thickness:

$$b_e = \frac{253t}{\sqrt{f}} \left( 1 - \frac{50.3}{(b/t)\sqrt{f}} \right) \leq b \quad (C3-1)$$

For other uniformly compressed elements:

$$b_e = \frac{253t}{\sqrt{f}} \left( 1 - \frac{44.3}{(b/t)\sqrt{f}} \right) \leq b \quad (C3-2)$$

where

$b$  = actual width of a stiffened compression element as defined in Sect. 1.9.2.1

$t$  = its thickness

$f$  = compressive stress in the element computed on the basis of its section properties as provided hereinafter. In the case of axial loading and flexure on extreme fibers,  $f = 0.6F_y Q_s$ , except as otherwise provided for wind and seismic loading

When the allowable stresses are increased due to wind or seismic loading, in accordance with the provisions of Sect. 1.5.6, the effective width  $b_e$  shall be determined on the basis of 0.75 times the stress caused by wind or seismic loading acting alone or in combination with the design dead and live loading.

### SECTION C4 DESIGN PROPERTIES

Properties of sections shall be determined in accordance with conventional methods, using the full cross-section of the member except as follows:

In computing the moment of inertia and section modulus of flexural members, with respect to the axis of bending under consideration, the

effective width of stiffened compression elements parallel to the axis of bending and having a width-thickness ratio in excess of the applicable limit given in Sect. 1.9.2.2, rather than the actual width, shall be used and the axis of bending shall be located accordingly, except that, for sections otherwise symmetrical, the properties may conservatively and more easily be computed using a corresponding effective area on the tension side of the neutral axis as well. That portion of the area which is neglected in arriving at the effective area shall be located at and symmetrically about the center line of the stiffened element to which it applies.

The stress  $f_a$  due to axial loading and the radius of gyration  $r$  shall be computed on the basis of actual cross-sectional area. However, the allowable axial stress  $F_a$ , as provided in Sect. C5, shall be subject to the form factor

$$Q_a = \frac{\text{effective area}}{\text{actual area}}$$

where the effective area is equal to the actual area less  $\Sigma(b - b_e)t$ .

### SECTION C5 AXIALLY LOADED COMPRESSION MEMBERS

The allowable stress for axially loaded compression members containing unstiffened or stiffened elements shall not exceed:

$$F_a = \frac{Q_s Q_a \left[ 1 - \frac{(Kl/r)^2}{2C_c^2} \right] F_y}{\frac{5}{3} + \frac{3(Kl/r)}{8C_c} - \frac{(Kl/r)^3}{8C_c^3}} \quad (C5-1)$$

where

$$C_c = \sqrt{\frac{2\pi^2 E}{Q_s Q_a F_y}}$$

when the largest effective slenderness ratio of any unbraced segment of the member is less than  $C_c$ , nor the value given by Formula (1.5-2) or (1.5-3) when  $Kl/r$  exceeds  $C_c$  or  $l/r$  exceeds 120, as applicable.

### SECTION C6 COMBINED AXIAL AND FLEXURAL STRESS

In applying the provisions of Sect. 1.6 to members subject to combined axial and flexural stress and containing stiffened elements whose width-thickness ratio exceeds the applicable limit given in Sect. 1.9, the stresses  $F_a$ ,  $f_{bx}$  and  $f_{by}$  shall be calculated on the basis of the section properties as provided in Sects. C4 and C5, as applicable. The allowable bending stress,  $F_b$ , for members containing unstiffened elements whose width-thickness ratio exceeds the applicable limit given in Sect. 1.9 shall be the smaller value,  $0.6F_y Q_s$  or that provided in Sect. 1.5.1.4.6.

# Commentary

## ON THE SPECIFICATION FOR THE DESIGN, FABRICATION AND ERECTION OF STRUCTURAL STEEL FOR BUILDINGS

### INTRODUCTION

In the belief that the designer can make more efficient use of the Specification if he knows the basis for its various provisions, this Commentary has been prepared.

Many provisions, notably in the sections dealing with fabrication and erection practices, have evolved from years of shop and field experience and need no further elaboration. Attention is directed primarily to less widely understood measures and particularly to modifications appearing for the first time. Many of these are the outgrowth of extensive research which has been carried out in recent years.

Part 1 of the Specification includes all of the provisions necessary for a working-stress design covering all three types of construction. Part 2 covers provisions applicable to plastic design.

### SECTION 1.2 TYPES OF CONSTRUCTION

In order that adequate instructions can be issued to the shop and erection forces, the basic assumptions underlying the design must be thoroughly understood by all concerned. As in the earlier AISC Specification, these assumptions are classified under three separate but generally recognized types of construction.

For better clarity, the provisions covering tier buildings of Type 2 construction designed for wind loading have been reworded in the current Specification, but without change in intent. Justification for these provisions has been discussed by Surochnikoff,\* Disque\*\* and others.

### SECTION 1.3 LOADS AND FORCES

The Specification does not presume to establish the loading requirements for which structures should be designed. In most cases these are adequately covered in the applicable local building codes. Where such is not the case, the generally recognized standards of the USA Standards Institute are recommended as the basis for design.

\* Surochnikoff, B. Wind Stresses in Semi-Rigid Connections of Steel Framework, 1950 ASCE Transactions.

\*\* Disque, R. O. Wind Connections with Simple Framing, AISC Engineering Journal, Vol. 1, No. 3.

## SECTION 1.4 MATERIAL

The 1961 edition of the Specification provided for the use of structural steel having a specified minimum yield point up to, but not exceeding, 50 kips per square inch. The grades of structural steel now approved for use under the Specification, covered by ASTM standards adopted since that time, extend the yield stress to 100 kips per square inch.

A number of other ASTM specifications are also now listed, covering types of material having infrequent application but suitable for use under the Specification.

Some of these ASTM standards specify a minimum yield point, while others specify a minimum yield strength. The term "yield stress" is used in the Specification as a generic term to denote either the yield point or the yield strength. However, the specified terms "yield point" and "yield stress" are used where they are uniquely applicable.

In keeping with the inclusion of steels of several strength grades, a number of corresponding specifications for cast steel forgings and other appurtenant materials such as rivets, bolts, and welding electrodes are also included.

When requested to do so, the fabricator must make affidavit that all steel specified to a yield stress in excess of 36 kips per square inch has been provided in accordance with the plans and Specification.

## SECTION 1.5 ALLOWABLE STRESSES

### 1.5.1. Structural Steel

Because of the introduction of steels having various specified minimum yield stresses, it is convenient to express permissible working stresses in terms of yield stress,  $F_y$ .

Where provisions are given in terms of  $F_y$  together with numerical values, it should be noted that, throughout the Specification, all stresses including the applicable value of  $F_y$  are expressed in kips per square inch.

For ready reference, numerical values are presented in Appendix A for several of the yield stress levels represented in Sect. 1.4.1.

#### 1.5.1.1 Tension

The 5/3 factor of safety with respect to yield stress used in determining the basic working stress for the newer and stronger steels is the same as that provided since the Specification was first adopted.

However, a further precaution has been added, applicable only at the net section of axially loaded members. Here a factor of safety of 2 with respect to specified minimum tensile strength must also be provided. This latter provision, of course, would apply only to steel having a yield stress-to-tensile strength ratio 5/6 or greater.

The working stress at the net section at pin holes is based upon research\* and experience with eye-bars.

\* Johnston, B. G. Pin-Connected Plate Links, 1939 ASCE Transactions.

### 1.5.1.2 Shear

While the shear yield stress of structural steel has been variously estimated as between one-half and five-eighths of the tension and compression yield stress and is frequently taken as  $F_y/\sqrt{3}$ , it will be noted that the permissible working value is given as two-thirds the recommended basic allowable tensile stress, substantially as it has been since the first edition of the AISC Specification, published in 1923. This apparent reduction in factor of safety is justified by the minor consequences of shear yielding, as compared with those associated with tension and compression yielding, and by the effect of strain hardening.

The webs of rolled shapes are all of such thickness that shear is seldom the criterion for design. However, the web shear stresses are generally high within the boundaries of the rigid connection of two or more members whose webs lie in a common plane. Such webs should be reinforced when the web thickness is less than

$$\frac{32M}{A_{bc}F_y}$$

where  $M$  is the algebraic sum of clockwise and counter-clockwise moments (in kip-feet) applied on opposite sides of the connection boundary and  $A_{bc}$  is the planar area of the connection web, expressed in square inches. This expression is based upon the assumption that the moment  $M$  is resisted by a couple having an arm equal to  $0.95d_b$ , where  $d_b$  is the depth of the member introducing the moment. Designating as  $d_c$  the depth of the member entering the joint more or less at right angles to it, and noting that  $A_{bc}$  is approximately equal to  $d_b \times d_c$ , the minimum thickness of the web not requiring reinforcement can be computed from the equation

$$\text{allowable shear stress} = 0.40F_y = \frac{12M}{0.95A_{bc}t_{\min}}$$

### 1.5.1.3 Compression

1.5.1.3.1 Formulas (1.5-1) and (1.5-2) are founded upon the basic column strength estimate suggested by the Column Research Council.\* This estimate assumes that the upper limit of elastic buckling failure is defined by an average column stress equal to one-half of yield stress. The slenderness ratio  $C_c$ , corresponding to this limit, can be expressed in terms of the yield stress of a given grade of structural steel as

$$C_c = \sqrt{\frac{2\pi^2E}{F_y}}$$

A variable factor of safety has been applied to the column strength estimate to obtain allowable working stresses. For very short columns this factor has been taken as equal to, or only slightly greater than, that required for members axially loaded in tension, and can be justified by the insensitivity of such members to accidental eccentricities. For longer columns, approaching the Euler slenderness range, the factor is increased 15 percent, to approximately the value provided in the AISC Specification since it was first published 46 years ago.

\* Column Research Council Guide to Design Criteria for Metal Compression Members, Second Edition, Eqs. (2.11) and (2.12).



In order to provide a smooth transition between these limits, the factor of safety has been arbitrarily defined by the algebraic equivalent of a quarter sine curve whose abscissas are the ratio of given  $Kl/r$  values to the limiting value  $C_c$ , and whose ordinates vary from 5/3 when  $Kl/r$  equals 0 to 23/12 when  $Kl/r$  equals  $C_c$ . Substituting  $12\pi^2E/23$  for the previous rounded-off value, 149,000,000, in Formula (1.5-2) affords an exact convergence with Formula (1.5-1).

Tables giving the permissible stress for columns and other compression members for a number of the approved structural steels are included in Appendix A of the Specification for the convenience of the designer.

**1.5.1.3.2** Formula (1.5-2), covering columns slender enough to fail by elastic buckling, is based upon a constant factor of safety of 23/12 with respect to the elastic (Euler) column strength.

**1.5.1.3.3** By dividing the values obtained from Formulas (1.5-1) and (1.5-2) by the factor  $\left(1.6 - \frac{l}{200r}\right)$  when  $l/r$  exceeds 120, to obtain Formula (1.5-3), substantially the same allowable stresses are still recommended for bracing and secondary members as those formerly given by the Rankine-Gordon formula which, until 1961, had been included in the AISC Specification since its first adoption.

The more liberal working stress for this type of member was justified in part by the relative unimportance of such members and in part by the greater effectiveness of end restraint likely to be present at their ends.

Since Formula (1.5-3) does not take advantage of end restraint, the full unbraced length of the member (rather than a reduced effective length, assuming  $K < 1.0$ ) should always be used, and the formula should be restricted to members which are more or less fixed against rotation and translation at braced points.

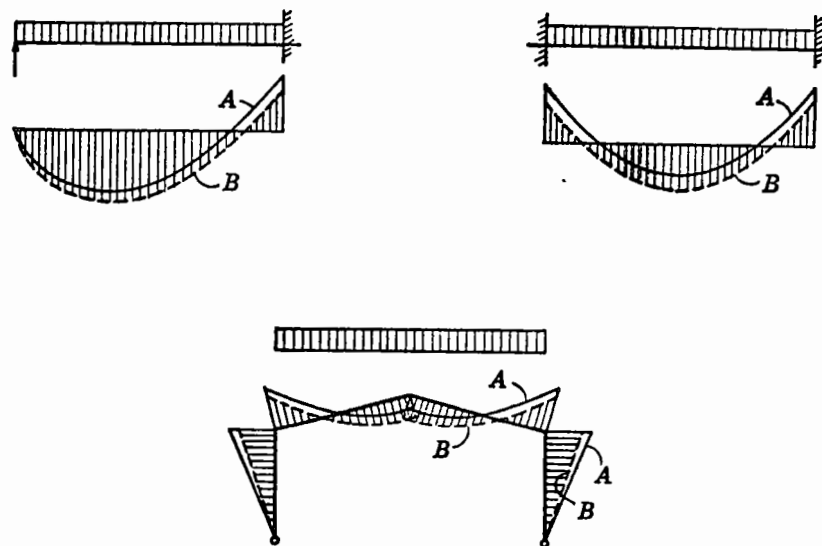
### 1.5.1.4 Bending

**1.5.1.4.1** When flexural members are proportioned in accordance with the provisions of Sects. 1.9.1.2 and 1.9.2.2 and are adequately braced to prevent the lateral displacement of the compression flange, they provide bending resistance equal at least to the product of their section modulus and yield stress, even when the width-thickness ratio of compressed elements of their profile is such that local buckling may be imminent.

Research in plastic design has demonstrated that local buckling will not occur in homogeneous sections meeting the requirements of subparagraphs a to e, inclusive, of Sect. 1.5.1.4.1 before the full plastic moment is reached. Practically all W- and I-shapes of A36 steel and a large proportion of these shapes having a yield stress of 50 ksi meet these provisions and are termed "compact" sections. It is obvious that the possibility of overload failure in bending of such rolled shapes must involve a higher level of stress (computed on the basis of  $M/S$ ) than members having more slender compression elements. Since the shape factor of W- and I-beams is generally in excess of 1.12, the allowable bending stress for such members has been raised 10 percent from  $0.60F_y$  to  $0.66F_y$ .

The further provision, permitting the arbitrary redistribution of 10 percent of the moment at points of support, due to gravity loading, gives partial recognition to the philosophy of plastic design. Subject to the re-

strictions provided in Sect. 1.5.1.4.1, continuous framing consisting of compact members may safely be proportioned on the basis of the working stress provisions of Part 1 of the Specification when the moments, before redistribution, are determined on the basis of an elastic analysis. Fig. C 1.5.1 illustrates the application of this provision by comparing calculated moment diagrams with the diagrams as altered by this provision.



A = Actual moment diagram  
B = Modified diagram corresponding to 10 percent moment reduction allowance at interior supports

Fig. C 1.5.1

In order to assure maximum advantage of moment redistribution, designs should be executed in accordance with the rules for plastic design given in Part 2. However, for many cases commonly encountered, the provisions of Sect. 1.5.1.4.1 afford approximately the same overall economy.

**1.5.1.4.2** Formula (1.5-5) avoids an abrupt transition between an allowable bending stress of  $0.66F_y$  when the half-flange width-to-thickness ratio of laterally supported compression flanges exceeds  $52.2/\sqrt{F_y}$ , and  $0.60F_y$  when this ratio is no more than  $95.0/\sqrt{F_y}$ . The assured hinge rotation capacity in this range is too small to permit redistribution of computed moment.

**1.5.1.4.3** The 25 percent increase in allowable bending stress for compact sections and solid rectangular bars bent about their weak axis, as well as for square and rectangular bars, is based upon the favorable shape factor present when these sections are bent about their weakest axis, and the fact that, in this position, they are not subject to lateral-torsional buckling. While the plastic bending strength of these shapes, bent in this direction, is considerably more than 25 percent in excess of their elastic bending strength, full advantage is not taken of this fact in order to provide elastic behavior at service loading.

1.5.1.4.4. Box-type members are torsionally very stiff.\* The critical flexural stress due to lateral-torsional buckling, for the compression flange of a box-type beam loaded in the plane of its minor axis so as to bend about its major axis, can be obtained using Formula (1.5-1) with an equivalent slenderness ratio, by the expression

$$\left(\frac{l}{r}\right)_{\text{equivalent}} = \sqrt{\frac{5.1S_x}{\sqrt{J}I_y}}$$

where  $l$  is the distance between points of lateral support and  $S_x$ ,  $I_y$  and  $J$  are, respectively, the major axis section modulus, minor axis moment of inertia and the torsional constant of the beam cross-section. It can be shown that, when  $d < 10b$  and  $l/b < 2,500/F_y$ , the allowable compression flange stress indicated by the above equation will approximate  $0.60F_y$ . Beyond this limit deflection rather than stress is likely to be the design criterion.

1.5.1.4.5 and 1.5.1.4.6 The allowable bending stress for all other flexural members is given as  $0.60F_y$ , provided the compression flange is braced laterally at relatively close intervals ( $U/b_f \leq 76.0/\sqrt{F_y}$ ).

Members bent about their major axis and having an axis of symmetry in the plane of loading may be adequately braced laterally at greater intervals if the maximum bending stress is reduced sufficiently to prevent premature buckling of the compression flange. Mathematical expressions affording an exact estimate of the buckling strength of such members, which take into account their torsional rigidity about their longitudinal axis (St. Venant torsion) as well as the bending stiffness of their compression flange between points of lateral support (warping torsion), are too complex for general design office use. Furthermore, their accuracy is dependent upon the validity of assumptions regarding restraint at points of lateral support and conditions of loading which, at best, can be no more than engineering judgments.

The combination of Formulas (1.5-6a) or (1.5-6b) and (1.5-7) provides a reasonable design criterion in more convenient form.

As in Formula (4) of the 1963 edition of the Specification, Formulas (1.5-6a) and (1.5-6b) are based on the assumption that only the bending stiffness of the compression flange will prevent the lateral displacement of that element between bracing points. The new Formulas (1.5-6a) and (1.5-6b) differ from the earlier Formula (4) in two ways:

1. Whereas the earlier provisions required no stress reduction when  $l/r$  was less than 40 (regardless of yield stress value) and then a reduction to the value obtained from the parabolic expression, the new formulas, by increasing  $F_b$  at  $l = 0$  from  $0.60F_y$  to  $2F_y/3$ , provides a continuous stress relationship with the unbraced length when  $F_b$  is reduced from the maximum permissible value of  $0.60F_y$ .
2. Whereas the earlier single Formula (4) applied even in the range of elastic buckling stress (on the assumption that Formula (5) would govern), the replacement of Formula (4) is liberalized in this range by the addition of an Euler-type expression, since this assumption is not always correct.

Formula (1.5-7) is a convenient approximation which assumes the presence of both lateral bending resistance and St. Venant torsional resistance. Due to the difference between flange and web yield strength, it is desirable to base the lateral buckling resistance solely on warping torsion of the flange. Hence, use of Formula (1.5-7) is not permitted for such members. Its agreement with more exact expressions for the buckling strength of intermittently braced flexural members\* is closest for homogeneous sections having substantial resistance to St. Venant torsion, identifiable in the case of doubly-symmetrical sections by a relatively low  $d/A_f$  ratio.

For some sections having a compression flange area distinctly smaller than the tension flange area, Formula (1.5-7) may be unconservative; hence, its use is limited to sections whose compression flange area is at least as great as the tension flange. In plate girders, which usually have a much higher  $d/A_f$  ratio than rolled I- and W-shapes, Formula (1.5-7) may err grossly on the conservative side. For such members the larger stress permitted by Formula (1.5-6a) and, at times by Formula (1.5-6b), affords the better estimate of buckling strength. While these latter formulas underestimate this strength somewhat because they ignore the St. Venant torsional rigidity of the profile, this rigidity for such sections is relatively small and the margin of overconservatism, therefore, is likewise small.

It should be noted that Formula (1.5-7), like the more precise, complex expressions it replaces, is written for the case of elastic buckling. A transition is not provided for this formula in the inelastic stress range because, when actual conditions of load application and variation in bending moment are considered, any unconservative error without it must be small.

Singly-symmetrical, built-up, I-shape members, such as some crane girders, often have an increased compression flange area in order to resist bending due to lateral loading action in conjunction with the vertical loads. Such members usually can be proportioned for the full permissible bending stress when that stress is produced by the combined vertical and horizontal loading. Where the failure mode of a singly-symmetrical I-shape member having a larger compression than tension flange would be by lateral buckling, the permissible bending stress can be obtained by using Formula (1.5-6a) or (1.5-7).

Through the introduction of the modifier\*\*  $C_b$ , some liberalization in stress is permissible when there is moment gradient over the unbraced length except where, in the case of combined bending and axial compression, this adjustment is provided by the factor  $C_m$  in Formula (1.6-1a).

Formulas (1.5-6a) and (1.5-6b) may be refined to include both St. Venant and warping torsion (thereby eliminating the need for Formula (1.5-7) by substituting a derived value for  $r$ . This equivalent radius of gyration,  $r_{\text{equivalent}}$ , can be obtained by equating the appropriate expression giving the critical elastic bending stress for the compression flange of a beam† with that of an axially loaded column.‡

\* Column Research Council Guide to Design Criteria for Metal Compression Members, Second Edition, Eq. 4.8.

\*\* Ibid., Eq. 4.13.

† Ibid., Eqs. (4.9c), (4.30), (4.31) or (4.32).

‡ Ibid., Eq. (2.2).

\* Column Research Council Guide to Design Criteria for Metal Compression Members, Second Edition, Sect. 4.2.

For the case of a doubly-symmetrical I-shape beam,

$$r_{equiv}^2 = \frac{C_b I_y}{2S_x} \sqrt{d^2 + \frac{0.156l^2 J}{I_y}}$$

where  $C_b$  is as defined in Sect. 1.5.1.4.6a,  $I_y$  is the minor axis moment of inertia of the member,  $S_x$  is its major axis section modulus, and

$$J = \frac{2bt_f^3}{3} + \frac{dt^3}{3}$$

### 1.5.1.5 Bearing

**1.5.1.5.1** As used throughout the Specification the terms “milled surface,” “milled” or “milling” are intended to include surfaces which have been accurately sawed or finished to a true plane by any suitable means. The recommended bearing stress on pins is not the same as for rivets. The lower value, nine-tenths of the yield stress of the part containing the pin hole, provides a safeguard against instability of the plate beyond the hole,\* which is considerably larger than a rivet hole.

## 1.5.2 Rivets, Bolts, and Threaded Parts

### 1.5.2.1 Tension

As in earlier editions, permissible stresses for rivets are given in terms applicable to the nominal cross-sectional area of the rivet before driving. For greater convenience in the proportioning of high strength bolted connections, permissible stresses for the bolts are given in terms applicable to their nominal body area, i.e., the area of the unthreaded shank. However, for A307 bolts (which are available in sizes up to 4 in. in diameter) and threaded parts other than high strength bolts, the allowable tensile stress is applicable to a stress area equal to  $0.7854 [D - (0.9743/n)]^2$ . This area (intermediate between gross area and area at the root of the thread) when multiplied by the mechanical properties of the unthreaded material, has been found to more closely predict the tensile strength of larger diameter threaded parts, such as might be used for anchor bolts or upset rods.

In recognition of the protection against notch effect in the threading, assured by the required initial tightening of high strength bolts, the Research Council on Riveted and Bolted Structural Joints has recommended a relatively higher working stress in tension for high strength bolts.

Any additional fastener tension resulting from prying action due to distortion of the connection details should be added to the stress calculated directly from the applied tension in proportioning fasteners for an applied tensile force, using the specified working stresses. Depending upon the relative stiffness of the fasteners and the connection material, this prying action may be negligible or it may be a substantial part of the total tension in the fasteners.\*\*

\* Johnston, B. G. Pin-Connected Plate Links, 1939 ASCE Transactions.

\*\* Munse, W. H. Research on Bolted Connections, 1956 ASCE Transactions, p. 1265.

### 1.5.2.1 Shear

Connections which transmit load by means of shear in their fasteners are categorized as “friction-type” or “bearing-type”. The former depend upon sufficiently high clamping force to prevent slip of the connected parts. The latter depend upon contact of the fasteners against the sides of their holes to transfer the load from one connected part to another.

The amount of clamping force developed by shrinkage of a rivet after cooling and by A307 bolts is unpredictable and generally insufficient to prevent complete slippage at the permissible working stress. Hence riveted connections and connections made with A307 bolts for shear are treated as bearing-type. The high clamping force produced by properly tightened high strength bolts is sufficient to prevent slip of the connected parts when an equal number of these bolts are substituted for the rivets of equal size that would be required to transmit a given load — A325 bolts for A502 Grade 1 rivets and A490 bolts for A502 Grade 2 rivets.

The efficiency of threaded fasteners in resisting shear in bearing-type connections is reduced when the threading extends into the shear plane between the connected parts. In the case of high strength bolts, two allowable shear stress values are given: one where threading is excluded from the shear plane and one where it is not. Since it is not customary to control this feature in the case of A307 bolts, it is assumed that threading may extend into the shear plane and the allowable shear value, applicable to the gross area, is reduced accordingly.

### 1.5.2.2 Bearing

Bearing values are provided, not as a protection to the fastener, because it needs no such protection, but as an index of the efficiency of net sections computed in accordance with Sect. 1.14.3. The same index is valid for joints assembled with rivets or with bolts, regardless of fastener shear strength or the presence or absence of threads in the bearing area. Tests of riveted joints\* have shown that the tensile strength of the connected part is not impaired when the bearing pressure on the computed contact area of the fastener is as much as  $2\frac{1}{4}$  times the tensile stress permitted on the net area of the part. In this investigation the contact (bearing) area was computed, according to the usual convention, as the product of nominal fastener diameter and thickness of the connected part. No difference was observed between single-shear bearing and enclosed bearing. Based on these findings, the recommended working stress is the same for single-shear and double-shear bearing, and approximately equal to  $2\frac{1}{4}$  times the tensile working stress recommended for determining required net area.

## 1.5.3 Welds

As in the past, the allowable working stresses for statically loaded full-penetration welds are the same as those permitted for the base metal, provided the mechanical properties of the electrodes used are such as to match or exceed those of the weakest grade of base metal being joined.

\* Jones, Jonathan Effect of Bearing Ratio on Static Strength of Riveted Joints, 1958 ASCE Transactions.

In earlier editions of the AISC Specification, working stresses were not given for fillet welds made with electrodes stronger than the E70 classification. The stresses that were given were known to be overly conservative for their recommended use with E60 and E70 classifications. Based upon recent tests,\* the allowable stress on fillet welds, deposited on "matching" base metal or steel having mechanical properties higher than those specified for such base metal, is now given in terms of the specified tensile strength of the weld metal.

As in the past, the same working value is given to a transverse as to a longitudinal weld, even though the force that the former can resist is substantially greater than that of the latter. In the case of tension on the throat of partial penetration groove welds normal to their axis (more nearly analagous to that of transverse than longitudinal fillets), the working stress is conservatively taken the same as for fillet welds.

When partial penetration groove welds are so disposed that they are stressed primarily in compression, bearing, or in tension parallel to the longitudinal axis of the groove, they may be proportioned to resist such stress at the same unit value permitted in the base metal.

#### 1.5.4. Cast Steel

In keeping with the inclusion of high strength low-alloy steels, the Specification recognizes high strength steel castings. Allowable stresses are expressed in terms of the specified minimum yield stress for castings.

### SECTION 1.6 COMBINED STRESSES

#### 1.6.1 Axial Compression and Bending

The application of moment along the unbraced length of axially loaded members, with its attendant axial displacement in the plane of bending, generates a secondary moment equal to the product of resulting eccentricity and the applied axial load, which is not reflected in the computed stress  $f_b$ . To provide for this added moment in the design of members subject to combined axial and bending stress, Formula (1.6-1a) requires that  $f_b$  be amplified by the factor

$$\frac{1}{\left(1 - \frac{f_a}{F'_e}\right)}$$

Depending upon the shape of the applied moment diagram (and, hence, the critical location and magnitude of the induced eccentricity), this factor may overestimate the extent of the secondary moment. To take care of this condition the amplification factor is modified, as required, by a reduction factor  $C_m$ .

When bending occurs about both the  $x$ - and  $y$ -axes, the bending stress calculated about each axis is adjusted by the value of  $C_m$  and  $F'_e$  corresponding to the distribution of moment and the slenderness ratio in its plane of bending, and is then taken as a fraction of the stress permitted for bending about that axis, with due regard to the unbraced length of compression flange where this is a factor.

\* Higgin, L. R. and Preece, F. R. Proposed Working Stresses for Fillet Welds in Building Construction, *Welding Journal Research Supplement*, Oct., 1968.

When the computed axial stress is no greater than 15 percent of the permissible axial stress, the influence of

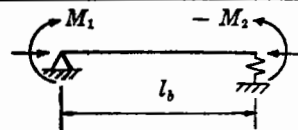
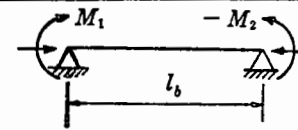
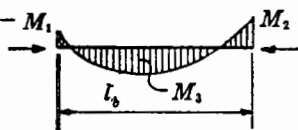
$$\frac{C_m}{\left(1 - \frac{f_a}{F'_e}\right)}$$

is generally small and may be neglected, as provided in Formula (1.6-2). However, its use in Formula (1.6-1a) is not intended to permit a value of  $f_b$  greater than  $F_b$  when the value of  $C_m$  and  $f_a$  are both small.

Depending upon the slenderness ratio of the given unbraced length of a member in the plane of bending, the combined stress computed at one or both ends of this length may exceed the combined stress at all intermediate points where lateral displacement is created by the applied moments. The limiting value of the combined stress in this case is established by Formula (1.6-1b).

The classification of members subject to combined axial compression and bending stresses is dependent upon two conditions: the stability against sidesway of the frame of which they are an integral part, and the presence or absence of transverse loading between points of support in the plane of bending. Three categories and the appropriate provisions of Sect. 1.6.1 are listed in Table C 1.6.1.1.

TABLE C 1.6.1.1

Category	Loading conditions ( $f_a > 0.15F_a$ )	$f_b$	$C_m$	Remarks
A	Computed moments maximum at end; joint translation not prevented	$\frac{M_2}{S}$	0.85	 $M_1 < M_2$ ; $\frac{M_1}{M_2}$ negative as shown. Check both Formulas (1.6-1a) & (1.6-1b)
B	Computed moments maximum at end; no transverse loading; joint translation prevented	$\frac{M_2}{S}$	$\left(0.6 \pm 0.4 \frac{M_1}{M_2}\right)$ but not less than 0.4	 Check both Formulas (1.6-1a) & (1.6-1b)
C	Transverse loading; joint translation prevented	$\frac{M_2}{S}$ Using Formula (1.6-1b) $\frac{M_1}{S}$ Using Formula (1.6-1a)	$1 + \psi \frac{f_a}{F'_e}$	 Check both Formulas (1.6-1a) & (1.6-1b)

**Note**  $f_b$  is defined as the computed bending stress at the point under consideration. In the absence of transverse loading between points of support,  $f_b$  is computed from the larger of the moments at these points of support. When intermediate transverse loading is present, the larger moment at one of the two supported points is used to compute  $f_b$  for use in Formula (1.6-1b). The maximum moment between points of support, however, is used to compute the bending stress for use in Formula (1.6-1a).

Category A covers columns in frames subject to sidesway, i.e., frames which depend upon the bending stiffness of their several members for overall lateral stability. For determining the value of  $F_a$  and  $F'_e$ , the effective length of such members, as discussed hereinafter under Sect. 1.8, is never less than the actual length, unbraced in the plane of bending, and may be greater than this length. The actual length is used in computing moments. For this case the value of  $C_m$  can be taken as

$$C_m = 1 - 0.18f_a/F'_e.$$

However, under the combination of compression stress and bending stress most affected by the amplification factor, a value of 0.15 can be substituted for  $0.18f_a/F'_e$ . Hence, a constant value of 0.85 is recommended for  $C_m$  here.

Category B applies to columns not subject to transverse loading in frames where sidesway is prevented. For determining the value of  $F_a$  and  $F'_e$ , the effective length of such members is never greater than the actual unbraced length and may be somewhat less. The actual length is used in computing moments.

For this category, the greatest eccentricity, and hence the greatest amplification, occurs when  $M_1$  and  $-M_2$  are numerically equal and cause single curvature. It is least when they are numerically equal and of a direction to cause reverse curvature.

To evaluate properly the relationship between end moment and amplified moment, the concept of an equivalent moment,  $M_e$ , to be used in lieu of the numerically smaller end moment, has been suggested.  $M_e$  can be defined as the value of equal end moments of opposite signs which would cause failure at the same concurrent axial load as would the given unequal end moments.

Then  $M_e/M_2$  can be written\*\* in terms of  $\pm M_1/M_2$  as

$$\frac{M_e}{M_2} = C_m = \sqrt{0.3 \left(\frac{M_1}{M_2}\right)^2 - 0.4 \left(\pm \frac{M_1}{M_2}\right) + 0.3}$$

It has been noted† that the simpler formulation

$$C_m = 0.6 - 0.4 \left(\pm \frac{M_1}{M_2}\right) \geq 0.4$$

affords a good approximation to this expression. When  $M_1/M_2$  is greater

\* The sign convention for moments here and in Sect. 1.6 is that generally used in frame analysis: It should not be confused with the beam sign convention used in many textbooks. Moments are considered positive when acting clockwise about a fixed point, negative when acting counter-clockwise.

\*\* Column Research Council Guide to Design Criteria for Metal Compression Members, p. 163. (Discussion in the Guide uses beam sign convention.)

† Austin, W. J. Strength and Design of Metal Beam-Columns, ASCE Journal of the Structural Division, April, 1961.

than 0.5 the combined axial and bending stress is usually limited by general yielding rather than by stability, in which case Formula (1.6-1b) would govern. Therefore, a tentatively selected column section should be tested by both Formulas (1.6-1a) and (1.6-1b).

Category C is exemplified by the compression chord of a truss, subject to transverse loading between panel points. For this case the value for  $C_m$  can be computed using the expression

$$C_m = 1 + \psi \frac{f_a}{F'_e}$$

where

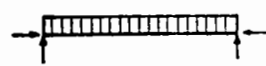
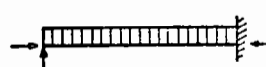

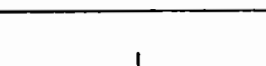
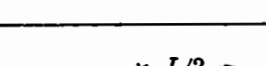

$$\psi = \frac{\pi^2 \delta_0 EI}{M_0 L^2} - 1$$

$\delta_0$  = maximum deflection due to transverse loading

$M_0$  = maximum moment between supports due to transverse loading

Values for  $\psi$  for several conditions of loading and end restraint are given in Table C 1.6.1.2.

TABLE C 1.6.1.2

Case	$\psi$	$C_m$
	0	1.0
	-0.3	$1 - 0.3 \frac{f_a}{F'_e}$
	-0.4	$1 - 0.4 \frac{f_a}{F'_e}$
	-0.2	$1 - 0.2 \frac{f_a}{F'_e}$
	-0.4	$1 - 0.4 \frac{f_a}{F'_e}$
	-0.6	$1 - 0.6 \frac{f_a}{F'_e}$

Note that  $F_u$  is governed by the maximum slenderness ratio, regardless of the plane of bending.  $F'_u$ , on the other hand, is always governed by the slenderness ratio in the plane of bending. Thus, when flexure is about the strong axis only, two different values of slenderness ratio may be required in solving a given problem.

### 1.6.2 Axial Tension and Bending

Contrary to the behavior in compression members, axial tension tends to reduce the bending stress between points of lateral support because the secondary moment, which is the product of the deflection and the axial tension, is opposite in sense to the applied moment, instead of being of the same sense and additive, as in columns.

### 1.6.3 Shear and Tension

Tests have shown\* that the strength of rivets subject to combined tension and shear resulting from externally applied forces (in addition to existing internal shrinkage stresses) can be closely defined by either (1) an ellipse, or (2) three straight lines, as shown in Fig. C 1.6.3.1.

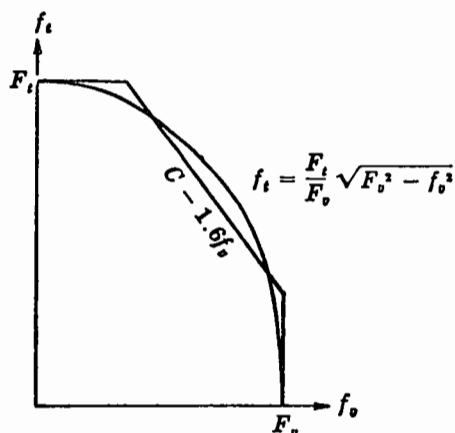


Fig. C 1.6.3.1

In most cases the latter representation is the more simple of application, since it requires no modification of the stress recommended for either shear or tension when these stresses act in conjunction, respectively, with relatively large concurrent tension or shear stresses. Therefore, it is the only one given in Sect. 1.6.3, since the inclusion of more than one method is hardly warranted. However, solutions based upon use of the ellipse are equally valid and should be allowed. Any differences in the number of fasteners required by the two prescriptions would be small.

Similar interaction formulas have been derived for the other approved types of fasteners from ellipses constructed with major and minor axis half-lengths equal, respectively, to the tension and shear stress given in Sect. 1.5.2.

\* Higgins R. and Munse, W. H. How Much Combined Stress Can A Rivet Take? *Engineering News-Record*, Dec. 4, 1962.

## SECTION 1.7 MEMBERS AND CONNECTIONS SUBJECT TO REPEATED VARIATION OF STRESS (FATIGUE)

Because most members in building frames need not be designed for fatigue, the provisions covering such designs have been placed in Appendix B.

Where fatigue is a design consideration, its severity is most significantly affected by the number of load applications and the magnitude of the stress range. It is aggravated by the presence of stress raisers to a varying degree, depending on the particular detail. Consequently, when fatigue is of concern, all the applicable provisions of Appendix B must be satisfied.

Members or connections subject to less than 20,000 cycles of loading will not involve a fatigue condition except in the case of repeated loading involving large ranges of stress. For such conditions the admissible range of stress can conservatively be taken as  $1\frac{1}{2}$  times the applicable value given in Table B3 for Loading Condition 1.

Except where indicated by "C" under "Kind of Stress" in Table B2, fluctuation in stress which does not involve tensile stress is not considered a fatigue situation.

When fabrication details involving more than one category occur at the same location in a member, the stress range at that location must be limited to that of the most restrictive category. By locating notch-producing fabrication details in regions subject to a small range of stress, the need for a member larger than required by static loading will often be eliminated.

The use of a constant stress range, which can be read directly from a table for a particular category and loading condition, greatly simplifies designs involving fatigue when compared with designs based on maximum or minimum allowable stress obtained from fatigue strength formulas on the basis of a stress ratio.

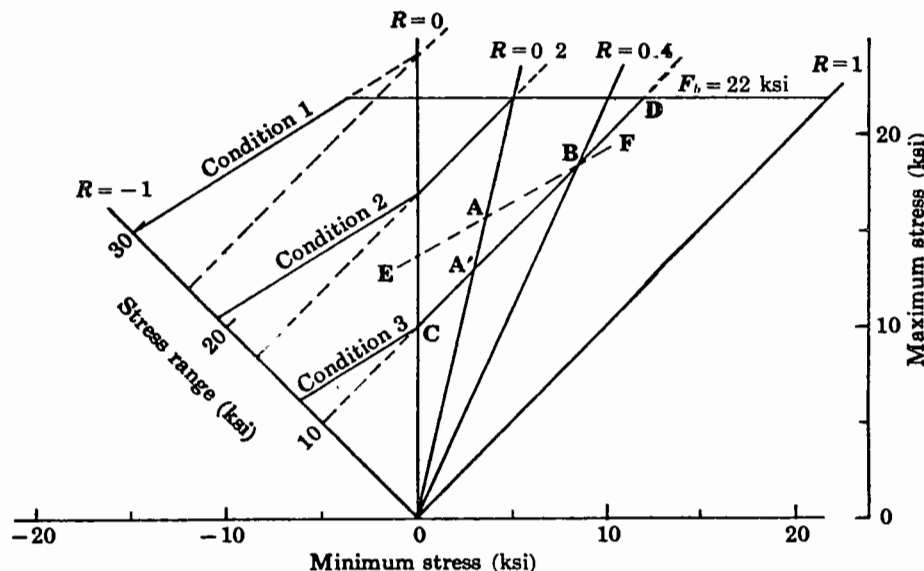


Fig. C 1.7.1

The reason for this shift in design criteria is apparent when the provisions of Appendix B are presented in the form of the familiar modified Goodman diagram often used as a design aid in lieu of such formulas. In Fig. C 1.7.1 the provisions of a category "D\*" detail of A36 Steel are plotted diagrammatically in this form. With maximum stress and stress ratio as the governing parameters, note that points A and B define substantially different critical maximum stress, with only slightly different stress ratios. However, with the line CD drawn parallel to the 45° boundary line representing static loading (min = max;  $R = 1$ ) the permissible range in stress for points A' and B (or any point between C and D) is the same. Only minor change in stress range would result had the slope of line CD been varied somewhat from 1 on 1, as often indicated in earlier evaluations of fatigue test results and as indicated by the line EF.

The allowable range of fluctuating tensile stress for Loading Condition 3, regardless of maximum stress value, can be read on the maximum stress scale and is represented by the distance OC.

This is the value  $F_{r,3}$  given in Table B3. It might also be read from a scale plotted on the  $R = -1$  boundary line, so laid off that

$$\text{stress range scale : max stress scale} = 1:\sqrt{2}$$

In developing the stress range values given in Table B3, published fatigue data and data obtained in continuing research were reviewed. In adopting a constant stress range basis for designs involving fatigue (in the interest of a simpler design procedure), it was realized that a number of known characteristics of fatigue strength data would not be taken into consideration. For example, except for A514 steel in category "A", the provisions do not recognize any increase in fatigue strength for the higher strength steels, as compared with that of A36 steel. For a particular category, this increased strength varies for the different steels depending upon the number of cycles of repeated loading.

As a consequence, the provisions may not provide a uniform factor of safety for the different strength steels. However, deviations from a uniform factor of safety are on the conservative side. Comparison of the fatigue provisions of this Specification with available test data indicate that the safety factors inherent in the recommended fatigue provisions are commensurate with static stress provisions.

In a few instances, identified by asterisks in Table B2, the extent of this conservatism warranted the liberalization provided by Formula (B1), which was derived from the expression for maximum permissible fatigue stress:

$$F_r = \frac{f_{r0}}{1 - mR}$$

where

- $R$  = Stress ratio, having a negative value with reversal of stress
- $f_{r0}$  = Maximum permissible stress when  $R = 0$
- $m$  = Slope of a fatigue strength line as presented in a modified Goodman diagram ( $m \sim 0.6$ )

Substituting  $f_t$  for  $F_r$ ,  $F_{sr}$  for  $f_{r0}$ , 0.6 for  $m$ , and  $-(f_c/f_t)$  for  $R$ , and noting that  $F'_{sr} = f_t + f_c$ ,

$$F'_{sr} = \frac{f_t + f_c}{f_t + 0.6f_c} F_{sr} \quad (B1)$$

Since Fig. C 1.7.1 was drawn for category "D\*", where Formula (B1) applies when a reversal of stress is involved, the fatigue strength lines (shown solid) represent the liberalization in stress range provided by Formula (B1) as compared with the dashed lines which would govern for category "D".

While greater fatigue strength than indicated by the provisions of Appendix B is attainable using special treatment, and is often provided in the case of manufactured products, the application of such treatment to as-fabricated structural steel is seldom economical. An exception is the grinding flush of full penetration groove welded splices which must be located where the alternate to the higher stress range permitted would be a substantial increase in required member size.

## SECTION 1.8 STABILITY AND SLENDERNESS RATIOS

Considerable attention has been given in the technical literature to the subject of "effective" column length (as contrasted with actual unbraced length) as a factor in estimating column strength. The topic is reviewed at some length in Sect. 2.8 of the *Guide to Design Criteria for Metal Compression Members*.

Two conditions, opposite in their effect upon column strength under axial loading, must be considered. If enough axial load is applied to the columns in a frame dependent entirely upon its own bending stiffness for stability against sidesway, i.e., uninhibited lateral movement, as shown in

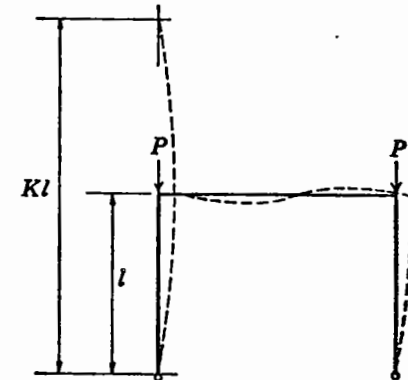


Fig. C 1.8.1

Fig. C 1.8.1, the "effective" length of these columns will exceed their actual length. On the other hand, if the same frame were braced in such a way that lateral movement of the tops of the columns with respect to their bases (translation or sidesway) were prevented, the effective length would be less than the actual length, due to the restraint (resistance to joint rotation) provided by the horizontal member. The ratio  $K$ , effective column length to actual unbraced length, may be greater or less than 1.0.

The theoretical  $K$ -values for six idealized conditions in which joint rotation and translation are either fully realized or non-existent are tabulated in Table C 1.8.1. Also shown are suggested design values recommended by the Column Research Council for use when these conditions are approximated in actual design. In general, these suggested values are slightly higher than their theoretical equivalents, since joint fixity is seldom fully realized.

TABLE C 1.8.1

	(a)	(b)	(c)	(d)	(e)	(f)
Buckled shape of column is shown by dashed line						
Theoretical $K$ value	0.5	0.7	1.0	1.0	2.0	2.0
Recommended design value when ideal conditions are approximated	0.65	0.80	1.2	1.0	2.10	2.0
End condition code			Rotation fixed and translation fixed			
			Rotation free and translation fixed			
			Rotation fixed and translation free			
			Rotation free and translation free			

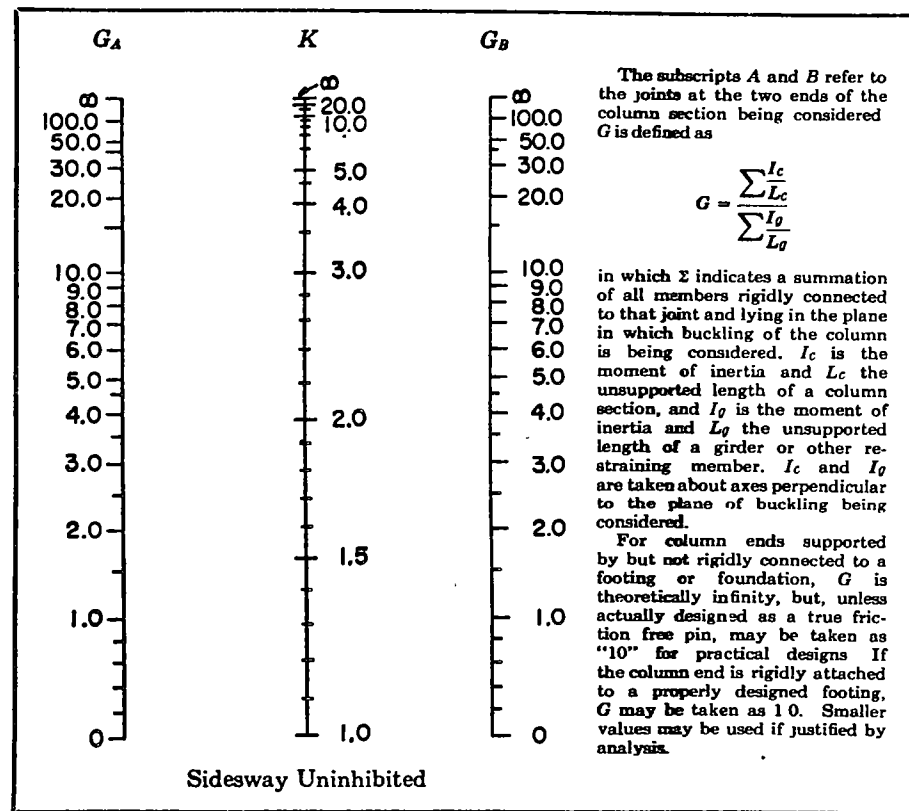
If the column base in case (f) of Table C 1.8.1 were truly pinned,  $K$  would actually exceed 2.0 for a frame such as that pictured in Fig. C 1.8.1, because the flexibility of the horizontal member would prevent realization of full fixity at the top of the column. On the other hand, it has been shown\* that the restraining influence of foundations, even where these footings are designed only for vertical load, can be very substantial in the case of flat-ended column base details with ordinary anchorage. For this condition, a design  $K$ -value of 1.5 would generally be conservative in case (f).

While ordinarily the existence of masonry walls provides enough lateral support for tier building frames to prevent sidesway, the increasing use of light curtain wall construction and wide column spacing, for high-rise structures not provided with a positive system of diagonal bracing, can create a situation where only the bending stiffness of the frame itself provides this support.

In this case the effective length factor,  $K$ , for an unbraced length of column,  $l$ , is dependent upon the amount of bending stiffness provided by the other in-plane members entering the joint at each end of the unbraced

segment. If the combined stiffness provided by the beams is sufficiently small, relative to that of the unbraced column segments,  $Kl$  could exceed two or more story heights.\*

Several rational methods are available by means of which the effective length of the columns in a laterally unbraced frame can be estimated with sufficient accuracy. These range from simple interpolation between the idealized cases shown in Table C 1.8.1 to very complex analytical procedures.



Alignment Chart for Effective Length of Columns in Continuous Frames

Fig. C 1.8.2

Once a trial selection of framing members has been made, the use of the alignment chart in Fig. C 1.8.2 affords a fairly rapid method for determining suitable  $K$ -values.

If roof decks or floor slabs, anchored to shear walls or vertical plane bracing systems, are counted upon to provide lateral support for individual columns in a building frame, due consideration must be given to their stiffness when functioning as a horizontal diaphragm.\*\*

\* Bleich, F. Buckling Strength of Metal Structures, pp. 260-265.

\*\* Winter, G. Lateral Bracing of Columns and Beams, *ASCE Journal of the Structural Division*, March, 1958.

\* Galar, T. V. Influence of Partial Base Fixity on Frame Stability, *ASCE Journal of the Structural Division*, May, 1960.



While translation of the joints in the plane of a truss is inhibited and, due to end restraint, the effective length of compression members might therefore be assumed as less than the distance between panel points, it is usual practice to take  $K$  as equal to 1.0, since, if all members of the truss reached their ultimate load capacity simultaneously the restraints at the ends of the compression members would disappear or, at least, be greatly reduced.

The slenderness limitations recommended for tension members are not essential to the structural integrity of such members; they merely afford a degree of stiffness such that undesirable lateral movement ("slapping" or vibration) will be avoided. These limitations are not mandatory.

## SECTION 1.9 WIDTH-THICKNESS RATIOS

When the width-thickness ratio of the compressed elements in a member does exceed the applicable limit specified in Sects. 1.9.1.2 or 1.9.2.2, no reduction in allowable stress is necessary in order to prevent local buckling. The design of members containing compression elements having a width-thickness ratio somewhat in excess of these limits is generally conservative if the area provided by the excessive width is ignored, as has been permitted in earlier editions of the Specification.

This expediency, in the case of unstiffened elements, raises a question as to eccentricity between actual and admissible cross-sectional area axes, makes no provision for computing an "effective" section modulus, and may even result in unconservative design. For the infrequent situation where width-thickness ratios substantially in excess of the limits given in Sect. 1.9 are involved, the provisions of Appendix C afford a better design procedure.

Formulas (C2-1) to (C2-6) are based upon the expression\* for critical buckling stress for a plate having one or both edges parallel to an in-plane compressive force supported against lateral deflection, with or without torsional restraint along these edges. For this case

$$\sigma_c = k \left[ \frac{\pi^2 E \sqrt{\eta}}{12(1 - \nu^2)(b/t)^2} \right] \quad (C1)$$

where  $\eta$  is the ratio of the tangent modulus to the elastic modulus,  $E_t/E$ , and  $\nu$  is Poisson's ratio. The idealized value  $k = 0.425$ , assumes nothing more than knife-edge lateral support, applied along one edge of the unstiffened element, at the mid-plane of the element providing it. Some increase in this value is warranted because of the torsional restraint provided by the supporting element and because of the difference between  $b$ , as defined in Sect. 1.9.1.2, and the theoretical width  $b$ .

In the interest of simplification, when  $\sqrt{\eta} < 1.0$  a linear formula is substituted for the theoretical expression. Its agreement with the latter may be judged by the comparison shown in Fig. C 1.9.1.

Formula (C2-5) assumes a decrease in the torsional restraint characteristic of tees cut from rolled shapes, which might be expected of tees of quite different proportions formed by welding two plates together.

\* Column Research Council Guide to Design Criteria for Metal Compression Members, Sect. 3.3.

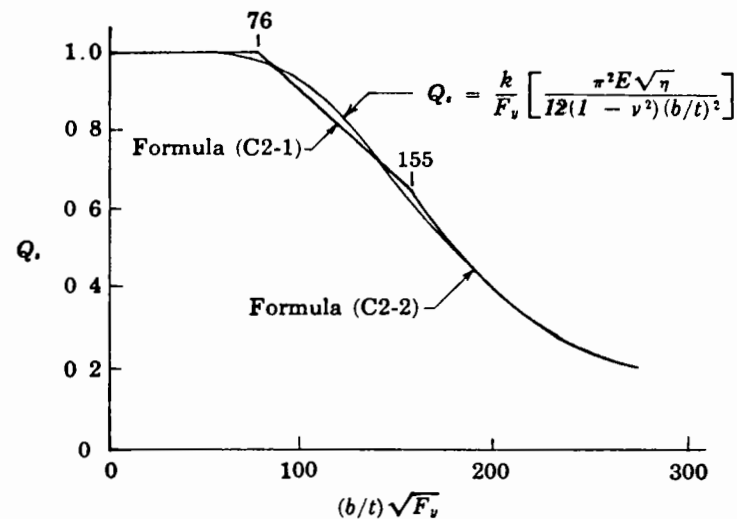


Fig. C 1.9.1

It has been shown\* that singly-symmetrical members whose cross-section consists of elements having large width-thickness ratios may fail by twisting under a smaller axial load than that associated with general column failure. Such is not generally the case with hot-rolled shapes. To guard against this type of failure, particularly when relatively thin-walled members are fabricated from plates, Table C1 places an upper limit on the proportions permissible for channels and tees.

With both edges parallel to the applied load supported against local buckling, stiffened compression elements can support a load producing an average stress,  $\sigma_c$ , greater than that given in the above expression for critical plate buckling stress. This is true even when  $k$  is taken as 4.0, applicable to the case where both edges are simply supported, or a value between 4.0 and 6.97, applicable when some torsional restraint is also provided along these edges.

A better estimate of the compressive strength of stiffened elements, based upon an "effective width" concept, was first proposed by von Karman.\*\* This was later modified by Winter† to provide a transition between very slender elements and stockier elements shown by tests to be fully effective.

As modified, the ratio of effective width to actual width increases as the level of compressive stress applied to a stiffened element in a member is decreased, and takes the form

$$\frac{b_e}{t} = 1.9 \sqrt{\frac{E}{f}} \left[ 1 - \frac{C}{(b/t)} \sqrt{\frac{E}{f}} \right]$$

\* Chajes, A. and Winter, G. Torsional Flexural Buckling of Thin-Walled Members, ASCE Journal of the Structural Division, August, 1965.

\*\* v. Karman, T., Sechler, E. E. and Donnell, L. H. The Strength of Thin Plates in Compression, 1932 ASME Transactions, Vol. 54, APM-54-5, p. 53.

† Winter, G. Strength of Steel Compression Flanges, 1947 ASCE Transactions.

where  $f$  is the level of uniformly distributed stress to which the element would be subjected based upon the design of the member, and  $C$  is an arbitrary constant based on engineering judgment supported by observed test results.

Obviously, holding the effective width of a stiffened element to no greater value than given by the limits provided in Sect. 1.9.2.2 is unnecessarily conservative when the maximum uniformly distributed design stress is substantially less than  $0.6F_y$ , or when  $b/t$  is considerably in excess of the limit given in Sect. 1.9.2.2.

For the case of square and rectangular box sections, the sides of which, in their buckled condition, afford negligible torsional restraint for one another along their corner edges, the value of  $C$  reflected in Formula (C3-1) is higher than for the other case, thereby providing a slightly more conservative evaluation of effective width. For cases where appreciable torsional restraint is provided, as for example the web of an I-shape column, the value of  $C$  implicit in Formula (C3-2) is decreased slightly. As in earlier editions of the AISC Specification, for such cases no reduction from actual width is required when the width-thickness ratio does not exceed  $253/\sqrt{F_y}$  and, for greater widths, the effective width may be taken as equal to  $253t/\sqrt{F_y}$ . If the actual width-thickness is substantially greater than  $253/\sqrt{F_y}$ , however, a larger effective width can be obtained using Formula (C3-2) rather than the earlier provisions.

In computing the section modulus of a member subject to bending, the area of stiffened elements parallel to the axis of bending and subject to compressive stress must be based upon their effective rather than actual width. In computing the effective area of a member subject to axial loading, the effective rather than actual area of all stiffened elements must be used. However, the radius of gyration of the actual cross-section together with the form factor  $Q_a$  may be used in determining the allowable axial stress. If the cross-section contains an unstiffened element, this allowable stress must be modified by the reduction factor  $Q_s$ .

## SECTION 1.10 PLATE GIRDERS AND ROLLED BEAMS

### 1.10.1 Proportions

As in earlier editions, it is provided\* that flexural members be proportioned to resist bending on the basis of the moment of inertia of their gross cross-section, with the stipulation that holes in the flanges having an area in excess of 15 percent of the gross flange area must be deducted. This provision is now extended to include the design of hybrid flexural members whose flanges are fabricated from a stronger grade of steel than that in their web. As in the case of flexural members having the same grade of steel throughout their cross-section, their bending strength is defined by the product of the section modulus of the gross cross-section multiplied by the allowable bending stress. On this basis the stress in the web, at its junction with the flanges, may even exceed the yield stress of the web material, but under strains controlled by the elastic state of stress in the stronger

\* Lilly, and Carpenter, S. T. Effective Moment of Inertia of a Riveted Plate Girder, 1940 ASCE Transactions.

flanges. Numerous tests, summarized in a recent report,\* have shown that, with only minor adjustment in the basic allowable bending stress as provided in Formula (1.10-5), the bending strength of a hybrid member is predictable within the same degree of accuracy as that of a homogeneous one.

### 1.10.2 Web

The limiting web depth-thickness ratio, included in the 1961 edition of the AISC Specification to prevent vertical buckling of the compression flange into the web before attainment of yield stress in the flange due to flexure, may now be increased when transverse stiffeners are provided, spaced not more than  $1\frac{1}{2}$  times the girder depth on centers.

The earlier provision, which was based on an analysis\*\* that placed no limitation on the spacing of transverse stiffeners, correlated reasonably well with tests performed on girders made of A7 steel having a specified yield stress of 33 ksi. The more liberal provision ( $h/t \leq 2000/\sqrt{F_y}$ ) is based upon more recent tests† on both homogeneous and hybrid girders with flanges having a specified yield stress of 100 ksi and a web of similar or weaker steel.

### 1.10.4 Flange Development

If a partial length cover plate is to function as an integral part of a beam or girder at the theoretical cut-off point beyond which it is not needed, it must be developed in an extension beyond this point by enough rivets, high strength bolts, or welding to support its portion of the flexural stresses (i.e., the stresses which the plate would have received had it been extended the full length of the member). The cover plate force to be developed by the fasteners in the extension is equal to

$$\frac{MQ}{I}$$

where

$M$  = Moment at beginning of extension

$Q$  = Statical moment of cover plate area about neutral axis of cover-plated section

$I$  = Moment of inertia of cover-plated section

When the nature of the loading is such as to produce repeated variations of stress, the fasteners must be proportioned in accordance with the provisions of Sect. 1.7.

In the case of welded cover plates it is further provided that the amount of stress that may be carried by a partial length cover plate, at a distance  $a'$  in from its actual end, may not exceed the capacity of the terminal welds

\* Design of Hybrid Steel Beams, Report of Subcommittee 1 of the Joint ASCE-AASHTO Committee on Flexural Members, ASCE Journal of the Structural Division, June, 1968.

\*\* Basler, K. and Thürlimann, B. Strength of Plate Girders in Bending, ASCE Journal of the Structural Division, August, 1963.

† Design of Hybrid Steel Beams, Report of Subcommittee 1 of the Joint ASCE-AASHTO Committee on Flexural Members, p. 1412, ASCE Journal of the Structural Division, June, 1968.

deposited along its edges and optionally across its end within this distance  $a'$ . If the moment, computed by equating  $MQ/I$  to the capacity of the welds in this distance, is less than the value at the theoretical cut-off point, either the size of the welds must be increased or the end of the cover plate must be extended to a point such that the moment on the member at the distance  $a'$  from the end of the cover plate is equal to that which the terminal welds will support.

### 1.10.5 Stiffeners

To provide better clarity, the provisions of Sect. 1.10.5 have been rearranged in the current edition of the Specification, but without substantive change of the provisions in Sect. 1.10.5 of the 1963 adoption.

Provisions governing the design of plate girders prior to the 1961 revision were based upon the assumption that the limit of structural usefulness of a girder web is attained when the level of stress in the web reaches the so-called "buckling" stage. Unlike columns, however, which actually are on the verge of collapse as their buckling stage is approached, the panels of a plate girder web, bounded on all sides by the girder flanges or transverse stiffeners, are capable of carrying loads far in excess of their "web buckling" load. Upon reaching the theoretical buckling limit, very slight lateral displacements will have developed in the web. Nevertheless, they are of no structural significance because other means are still present to assist in resisting further loading.

When transverse stiffeners are properly spaced and strong enough to act as compression struts, membrane stresses, due to shear forces greater than those associated with the theoretical buckling load, form diagonal tension fields. The resulting combination in effect provides a Pratt truss which, without producing yield stress in the steel, furnishes the capacity to resist applied shear forces unaccounted for by the linear buckling theory.

Analytical methods based upon this action have been developed\* and corroborated in an extensive program of tests.\*\* These methods form the basis for Formula (1.10-2). Use of tension field action is not counted upon when

$$\frac{0.6F_y}{\sqrt{3}} \leq F_v \leq 0.4F_y$$

or where

$$a/h > 3.0$$

Pending further investigation, it is not recommended for hybrid girders.

When the computed average shear stress in the web is less than that permitted by Formula (1.10-1), intermediate stiffeners are not required provided the depth of girders is limited to not more than 260 times the web thickness. Such girders do not depend upon tension field action.

\* Basler, K. Strength of Plate Girders in Shear, *ASCE Journal of the Structural Division*, October, 1961.

\*\* Basler, K., Yen, B. T., Mueller, J. A. and Thürlimann, B. Web Buckling Tests on Welded Plate Girders, *Welding Research Council Bulletin No. 64*.

In order to facilitate handling during fabrication and erection, when intermediate stiffeners are required, the panel aspect ratio  $a/h$  is arbitrarily limited to not more than

$$\left( \frac{260}{h/t} \right)^2$$

with a maximum spacing of 3 times the girder depth.

When required, their maximum permissible longitudinal spacing is dependent upon three parameters:  $a/h$ ,  $h/t$  and  $f_v$ . For the convenience of the designer, their relationship with one another is presented in Tables 3-36 through 3-100 of Appendix A for several specified yield stresses covered by the Specification. Given the shear diagram produced by the design loads and a desired depth of girder, it is only necessary to select a web thickness (with due regard for limitations placed on  $h/t$  ratios) such that the web shear stress will be equal to or less than the maximum permitted value. With the resulting value for  $h/t$  and the computed shear stress, the required aspect ratio  $a/h$  can be taken directly from the table. Comparison of the web and stiffener material required with two or three trial web thicknesses will quickly indicate the most economical combination.

The corresponding gross area of intermediate stiffeners, given as a percent of the web area, is shown in italics in the column headed by the required aspect ratio and the line nearest to the selected  $h/t$  ratio. Stiffeners which will provide this area usually will be little, if any, larger than those generally called for. No stiffener areas are shown when the  $a/h$  and  $h/t$  ratios are small enough to permit a shear stress larger than  $0.35F_y$ , which is covered by Formula (1.10-1). For such cases tension field action is not counted upon.

At the ends of the girder, the spacing between adjacent stiffeners is limited to  $11,000t/\sqrt{f_v}$ , to provide an "anchor" for the tension fields developed in interior panels. The stiffeners bounding panels containing large holes likewise are required to be spaced close enough together so that the shear in these panels can be supported without tension field action.

To provide adequate lateral support for the web, all stiffeners are required to have a moment of inertia at least equal to  $(h/50)^4$ . In many cases, however, this provision will be overshadowed by the new gross area requirement. The amount of stiffener area necessary to develop the tension field, which is dependent upon the ratios  $a/h$  and  $h/t$ , is given by Formula (1.10-3). Larger gross areas are required for one-sided stiffeners than for pairs of stiffeners, because of the eccentric nature of their loading.

The amount of shear to be transferred between web and stiffeners is not affected by the eccentricity of loading, and generally is so small that it can be taken care of by the minimum amount of welding or riveting that might be desired. The specified formula

$$f_{vs} = h \sqrt{\left( \frac{F_v}{340} \right)^2}$$

affords a conservative estimate of required shear transfer under any condition of stress permitted by Formula (1.10-2). The shear transfer between web and stiffener due to tension field action and that due to a concentrated load or reaction in line with the stiffener are not additive. The stiffener need only be connected for the larger of the two shears.

### 1.10.6 Reduction in Flange Stress

In regions of maximum bending moment, a portion of a thin web may deflect enough laterally on the compression side of the neutral axis that it does not provide the full bending resistance assumed in proportioning the girder on the basis of its moment of inertia. The compression stress which the web would have resisted is, therefore, shifted to the compression flange. But the relative bending strength of this flange being so much greater than that of the laterally displaced portion of the web, the resulting increase in flange stress is at most only a few percent. By reducing the allowable design stress in the compression flange from  $F_b$  to  $F'_b$ , as provided in Formula (1.10-5), sufficient bending capacity is provided in the flange to compensate for any loss of bending strength in the web due to its lateral displacement.

To compensate for the slight loss of bending resistance when portions of the web of a hybrid flexural member are strained beyond their yield stress limit, Formula (1.10-6)\* provides for a reduced allowable flange bending stress applicable to both flanges. The extent of the reduction is dependent upon the ratio of web area to a flange area and the ratio of web yield stress to flange yield stress.

In order to avoid a more complicated formula, the area and grade of steel in both flanges are required to be the same. Since any reductions in bending strength due to buckling of the web on the compression side of the neutral axis is considerably less in the case of a hybrid girder than for a homogeneous member having the same cross-section, it is not required that Formula (1.10-5) apply when the stress permitted by Formula (1.10-6) is less than that given for the former.

### 1.10.7 Combined Shear and Tension Stress

Unless a flexural member is designed on the basis of tension field action, no stress reduction is required due to the interaction of concurrent bending and shear stress.

It has been shown\*\* that plate girder webs subject to tension field action can be proportioned on the basis of:

1. Maximum permissible bending stress when the concurrent shear is not greater than 0.6 the full permissible value, or
2. Full permissible shear stress when the bending stress is not more than  $\frac{3}{4}$  of the maximum allowable.

Beyond these limits a linear interaction formula is provided in the Specification by Formula (1.10-7).

However, because the webs of homogeneous girders of A514 steel loaded to their full capacity in bending develop more waviness than less heavily stressed girder webs of weaker grades of steel, use of tension field action is limited in the case of A514 steel webs to regions where the concurrent bending stress is no more than  $0.75F_b$ .

\* Design of Hybrid Steel Beams, Report of Subcommittee 1 of the Joint ASCE-AASHO Committee on Flexural Members, *ASCE Journal of the Structural Division*, June, 1968.

\*\* Basler, Strength of Plate Girders Under Combined Bending and Shear, *ASCE Journal of the Structural Division*, October, 1961.

### 1.10.10 Web Crippling

**1.10.10.1** Webs of beams and girders not protected by bearing stiffeners could fail by crippling at points of high stress concentration resulting from the application of concentrated loads or reactions. To guard against this, the stress at the toe of the flange fillet, assumed to be distributed longitudinally a distance no greater than the length of the bearing, plus 1 or 2 times the  $k$ -distance of the flange, depending upon the location of the load, is limited by Formula (1.10-8) or (1.10-9) to  $0.75F_y$ .

**1.10.10.2** As a safeguard against instability of relatively thin plate girder webs, a further limitation has been placed on the amount of load which can be applied directly to the girder flange between stiffeners. Concentrated loads light enough to meet the provisions of Sect. 1.10.10.1 and loading applied longitudinally over partial panel length are treated as if distributed by means of shear over the full panel length within which they occur (or the depth of girder if this is less than the panel length). Taken together with such other distributed loading as may be applied directly to the flange, the total load divided by the web thickness should not exceed the stress permitted by Formula (1.10-10) or (1.10-11). If the flange is prevented from rotation about its longitudinal axis by its contact with a rigid slab, Formula (1.10-10) will govern; otherwise, the more conservative Formula (1.10-11) is applicable.

These formulas are derived\* from a consideration of the elastic buckling strength of the web plate subject to edge loading. The loading is resisted in part by column action and in part by a plate intermittently stiffened in the direction of applied loading.

The formulas are likely to be over-conservative in the case of riveted girders, since they ignore any bending capacity the flange angles may have in spanning between adjacent stiffeners to support the loads.

### 1.10.11 Rotational Restraint at Points of Support

Slender beams and girders resting on top of columns and stayed laterally only in the plane of their top flanges may become unstable due to the flexibility of the column. Unless lateral support is provided for the bottom flange, either by bracing or continuity at the beam-to-column connection, lateral displacement at the top of the column, accompanied by rotation of the beam about its longitudinal axis, may lead to collapse of the framing.

## SECTION 1.11 COMPOSITE CONSTRUCTION

### 1.11.1 Definition

When the dimensions of a concrete slab supported on steel beams are such that the slab can effectively serve as the flange of a composite T-beam, and the concrete and steel are adequately tied together so as to act as a unit, the beam can be proportioned on the assumption of composite action.

Two cases are recognized: fully encased steel beams which depend upon natural bond for interaction with the concrete and those with mechanical anchorage to the slab (shear connectors), which do not have to be encased.

\* Basler, K. New Provisions for Plate Girder Design, *ASCE Proceedings AISC National Engineering Conference*.

### 1.11.2 Design Assumptions

Unless temporary shores are used, beams encased in concrete and interconnected only by means of natural bond must be proportioned to support all of the dead load, unassisted by the concrete, plus the superimposed live load in composite action, without exceeding the allowable bending stress for steel provided in Sect. 1.5.1.

Because the completely encased steel section is restrained from both local and lateral buckling, an allowable stress of  $0.66F_y$ , rather than  $0.60F_y$ , can be applied here. The alternate provision, permitting a stress of  $0.76F_y$ , to be used in designs where a fully encased beam is proportioned to resist all loads unassisted, reflects a common engineering practice where it is desired to eliminate the calculation of composite section properties.

In keeping with the *Tentative Recommendations for the Design and Construction of Composite Beams and Girders for Buildings\**, when shear connectors are used to obtain composite action, this action may be assumed, within certain limits, in proportioning the beam for the moments created by both live and dead loads, even for unshored construction. This liberalization is based upon an ultimate strength concept, although the proportioning of the member is based upon the elastic section modulus of the transformed cross-section.

In order that the maximum bending stress in the steel beam, under service loading, will be well below the level of initial yielding, regardless of the ratio of live-load moment to dead-load moment, the section modulus of the composite cross-section, in tension at the bottom of the beam, for unshored construction, is limited to  $(1.35 + 0.35 M_L/M_D)$  times the section modulus of the bare beam.\*\*

On the other hand, the requirement that flexural stress in the concrete slab, due to actual composite action, be computed on the basis of actual transformed section modulus and limited to the generally accepted working stress limit, is necessary in order to avoid excessively conservative slab-to-beam proportions.

Research at Lehigh University† has shown that, for a given beam and concrete slab, the increase in bending strength intermediate between no composite action and full composite action is directly proportional to the shear resistance developed between the steel and concrete, i.e., the number of shear connectors provided between these limits. At times it may not be feasible, nor even necessary, to provide full composite action. Therefore the Specification recognizes two conditions: full and incomplete composite action.

For the case where the total shear ( $V'_h$ ) developed between steel and concrete each side of the point of maximum moment is less than  $V_h$ , Formula (1.11-1) can be used to derive an effective section modulus,  $S_{eff}$ , having a value less than the section modulus for fully effective composite action,  $S_{tr}$ , but more than that of the steel beam alone.

\* Progress Report of the Joint ASCE-ACI Committee on Composite Construction, *ASCE Journal of the Structural Division*, December, 1960.

\*\* Ibid., Eq. (3).

† Slutter, R. G. and Driscoll, G. C. Flexural Strength of Steel-Concrete Composite Beams, p. 91, *ASCE Journal of the Structural Division*, April, 1965.

### 1.11.4 Shear Connectors

Based upon tests at Lehigh University,\* and a re-examination of previously published test data reported by a number of investigators, more liberal working values are recommended for various types and sizes of shear connectors than in use prior to 1961.

Composite beams in which the longitudinal spacing of shear connectors has been varied according to the intensity of statical shear, and duplicate beams where the required number of connectors were uniformly spaced, have exhibited the same ultimate strength, and the same amount of deflection at normal working loads. Only a slight deformation in the concrete and the more heavily stressed shear connectors is needed to redistribute the horizontal shear to other less heavily stressed connectors. The important consideration is that the total number of connectors, either side of the point of maximum moment, be sufficient to develop the composite action counted upon at that point. The provisions of the Specification are based upon this concept of composite action.

The required shear connectors can generally be spaced uniformly between the points of maximum and zero moment.\* However, certain loading patterns can produce a condition where closer spacing is required over a part of this distance.

Consider, for example, the case of a uniformly loaded simple beam also required to support two equal concentrated loads, symmetrically disposed about midspan, of such magnitude that the moment at the concentrated loads is only slightly less than the maximum moment at midspan. The number of shear connectors ( $N_2$ ) required between each end of the beam and the adjacent concentrated load would be only slightly less than the number ( $N_1$ ) required between each end and midspan.

Formula (1.11-6) is provided as a check to determine whether the number of connectors,  $N_1$ , required to develop  $M_{max}$  would, if uniformly distributed, provide  $N_2$  connectors between one of the concentrated loads and the nearest point of zero moment. It is based upon the requirement that

$$S_{eff}:S_{tr} = M:M_{max}$$

where

$$0 < M < M_{max}$$

$S_{eff}$  = section modulus corresponding to the minimum amount of incomplete composite action required at the section subject to the moment  $M$

$$V'_h:V_h = N_2:N_1$$

In computing the section modulus at points of maximum negative bending, reinforcement parallel to the steel beam and lying within the effective width of slab may be included, provided such reinforcement is properly anchored beyond the region of negative moment. However, enough shear connectors are required to transfer, from the slab to the steel beam, one-half of the ultimate tensile strength of the reinforcement.

\* Slutter, R. G. and Driscoll, G. C. Flexural Strength of Steel-Concrete Composite Beams, p. 91, *ASCE Journal of the Structural Division*, April 1965.

The working values for various types of shear connectors are based upon a factor of safety of approximately 2.50 against their demonstrated ultimate strength.

Working values for use with concrete having aggregate not conforming to ASTM C33 and for connector types other than those shown in Table 1.11.4 must be established by a suitable testing program.

The values of  $q$  in Table 1.11.4 must not be confused with shear connection values suitable for use when the required number is measured by the parameter  $VQ/I$ , where  $V$  is the total shear at any given cross-section. Such a misuse could result in providing less than half the number required by Formula (1.11-3), (1.11-4) or (1.11-5).

Stud welds not located directly over the web of a beam tend to tear out of a thin flange before attaining their full shear-resisting capacity. To guard against this contingency, the size of a stud not located over the beam web is limited to  $2\frac{1}{2}$  times the flange thickness.

## SECTION 1.13 DEFLECTIONS, VIBRATION AND PONDING

### 1.13.1 Deflections

Although deformation, rather than stress, is sometimes the criterion of satisfactory design, there is no single scale by which the limit of tolerable deflection can be defined. Where limitations on flexibility are desirable, they are often dictated by the nature of collateral building components, such as plastered walls and ceilings, rather than by considerations of human comfort and safety. The admissible amount of movement varies with the type of component.

Obviously, the most satisfactory solution must rest upon the sound judgment of qualified engineers. As a guide, but only a guide, the following rules are suggested:

The depth of fully stressed beams and girders in floors should, if practicable, be not less than  $F_y/800$  times the span. If members of less depth are used, the unit stress in bending should be decreased in the same ratio as the depth is decreased from that recommended above.

The depth of fully stressed roof purlins should, if practicable, be not less than  $F_y/1,000$  times the span, except in the case of flat roofs.

### 1.13.2 Vibration

Where human comfort is the criterion for limiting motion, as in the case of perceptible vibrations, the limit of tolerable amplitude is dependent, on the one hand, upon the frequency of the vibration and, on the other, the damping effect provided by components of the construction. When such vibrations are caused by running machinery, they should be isolated by effective damping devices or by the use of independent foundations.

The depth of a steel beam supporting large open floor areas free of partitions or other sources of damping should not be less than  $\frac{1}{20}$  of the span, in order to minimize perceptible transient vibration due to pedestrian traffic.

### 1.13.3 Ponding

As used in the Specification, ponding refers to the retention of water due solely to the deflection of flat roof framing. The amount of this water is dependent upon the flexibility of the framing. Lacking sufficient framing stiffness, its accumulated weight can result in collapse of the roof.

Representing the deflected shape of the primary and critical secondary member as a half-sine wave, the weight and distribution of the ponded water can be estimated and, from this, the contribution that the deflection each of these members make to the total ponding deflection can be expressed\* as

$$\Delta_w = \frac{\alpha_p \Delta_o \left[ 1 + \frac{\pi}{4} \alpha_s + \frac{\pi}{4} \rho (1 + \alpha_s) \right]}{1 - \frac{\pi}{4} \alpha_p \alpha_s}$$

for the primary member, and

$$\delta_w = \frac{\alpha_s \delta_o \left[ 1 + \frac{\pi^2}{32} \alpha_p + \frac{\pi^2}{8\rho} (1 + \alpha_p) + 0.185 \alpha_s \alpha_p \right]}{1 - \frac{\pi}{4} \alpha_p \alpha_s}$$

for the secondary member. In these expressions  $\Delta_o$  and  $\delta_o$  are, respectively, the primary and secondary beam deflections due to loading present at the initiation of ponding,  $\alpha_p = C_p/(1 - C_p)$ ,  $\alpha_s = C_s/(1 - C_p)$ , and  $\rho = \delta_o/\Delta_o = C_s/C_p$ .

Using the above expressions for  $\Delta_w$  and  $\delta_w$ , the ratios  $\Delta_w/\Delta_o$  and  $\delta_w/\delta_o$  can be computed for any given combination of primary and secondary beam framing using, respectively, the computed value of parameters  $C_p$  and  $C_s$  defined in the Specification.

Even on the basis of unlimited elastic behavior, it is seen that the ponding deflections would become infinitely large unless

$$\left( \frac{C_p}{1 - C_p} \right) \left( \frac{C_s}{1 - C_s} \right) < \frac{4}{\pi}$$

Since elastic behavior is not unlimited, the effective bending strength available in each member to resist the stress caused by ponding action is restricted to the difference between the yield stress of the member and the stress,  $f_o$ , produced by the total load supported by it before consideration of ponding is included.

Noting that elastic deflection is directly proportional to stress, and providing a factor of safety of 1.25 with respect to stress due to ponding, the admissible amount of ponding deflection in either the primary or critical (midspan) secondary member, in terms of the applicable ratio  $\Delta_w/\Delta_o$  or  $\delta_w/\delta_o$ , can be represented as  $(0.8F_y - f_o)/f_o$ . Substituting this expression for  $\Delta_w/\Delta_o$  and  $\delta_w/\delta_o$  and combining with the foregoing expressions for  $\Delta_w$  and  $\delta_w$ , the relationship between critical values for  $C_p$  and  $C_s$  and the available elastic bending strength to resist ponding is obtained. The curves presented in Figs. C1.13.3.1 and C1.13.3.2 are based upon this relationship. They constitute a design aid for use when a more exact determination of required flat roof framing stiffness is needed than given by the Specification provision that  $C_p + 0.9C_s \leq 0.25$ .

\* Marino, F. J. Ponding of Two-Way Roof Systems, *AISC Engineering Journal*, July, 1966.

Given any combination of primary and secondary framing, the stress index is computed as

$$U_p = \left( \frac{0.8F_v - f_o}{f_o} \right)_p, \text{ for the primary member}$$

$$U_s = \left( \frac{0.8F_v - f_o}{f_o} \right)_s, \text{ for the secondary member}$$

where  $f_o$ , in each case, is the computed bending stress in the member due to the supported loading, neglecting ponding effect. Depending upon geographic location, this loading should include such amount of snow as might also be present, although ponding failures have occurred more frequently

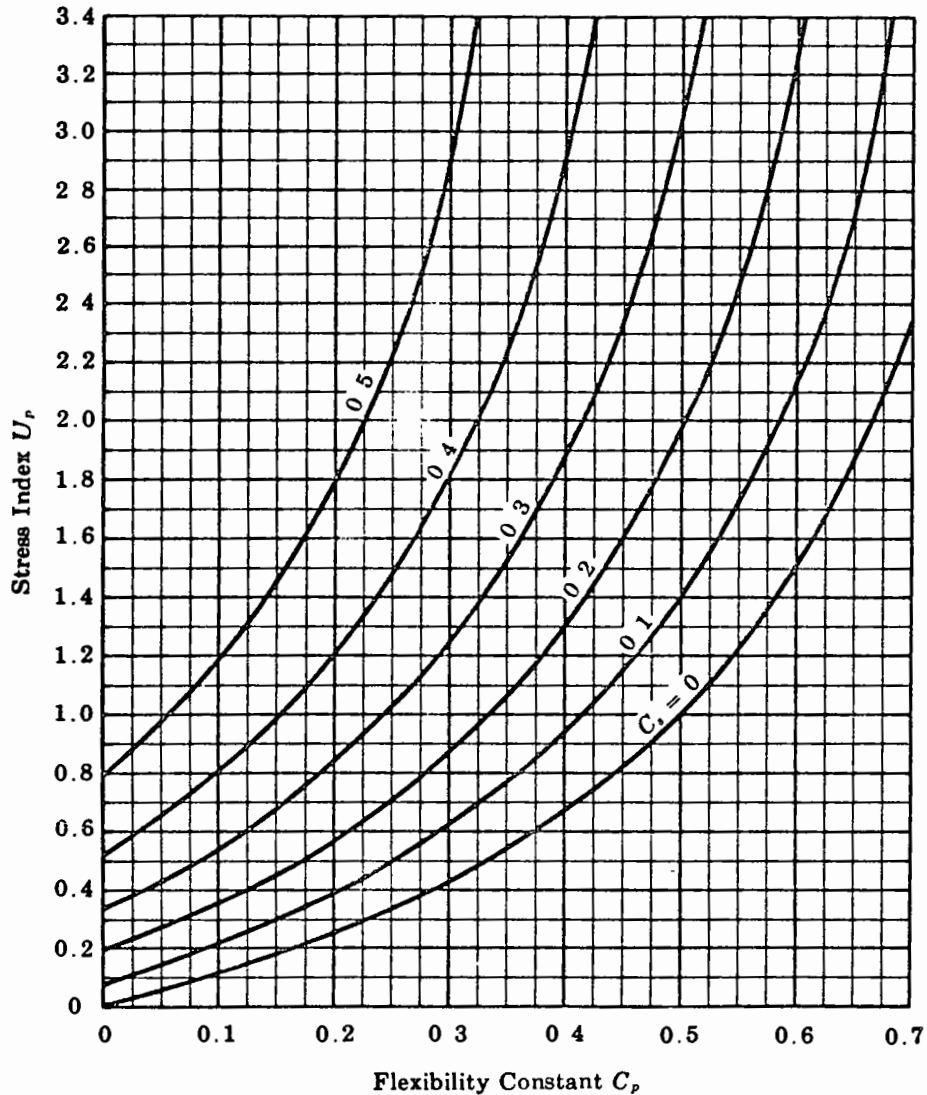


Fig. C1.13.3.1

during torrential summer rains, when the rate of precipitation exceeded the rate of drainage run-off and the resulting hydraulic gradient over large roof areas caused substantial accumulation of water some distance from the eaves.

Given the size, spacing and span of a tentatively selected combination of primary and secondary beams, for example, one may enter Fig. C1.13.3.1 at the level of the computed stress index,  $U_p$ , determined for the primary beam; move horizontally to the computed  $C_s$ -value of the secondary beams; and, thence, downward to the abscissa scale. The combined stiffness of the primary and secondary framing is sufficient to prevent ponding if the flexibility constant read from this latter scale is more than the value of  $C_p$  computed for the given primary member; if not, a stiffer primary or secondary beam, or combination of both, is required.

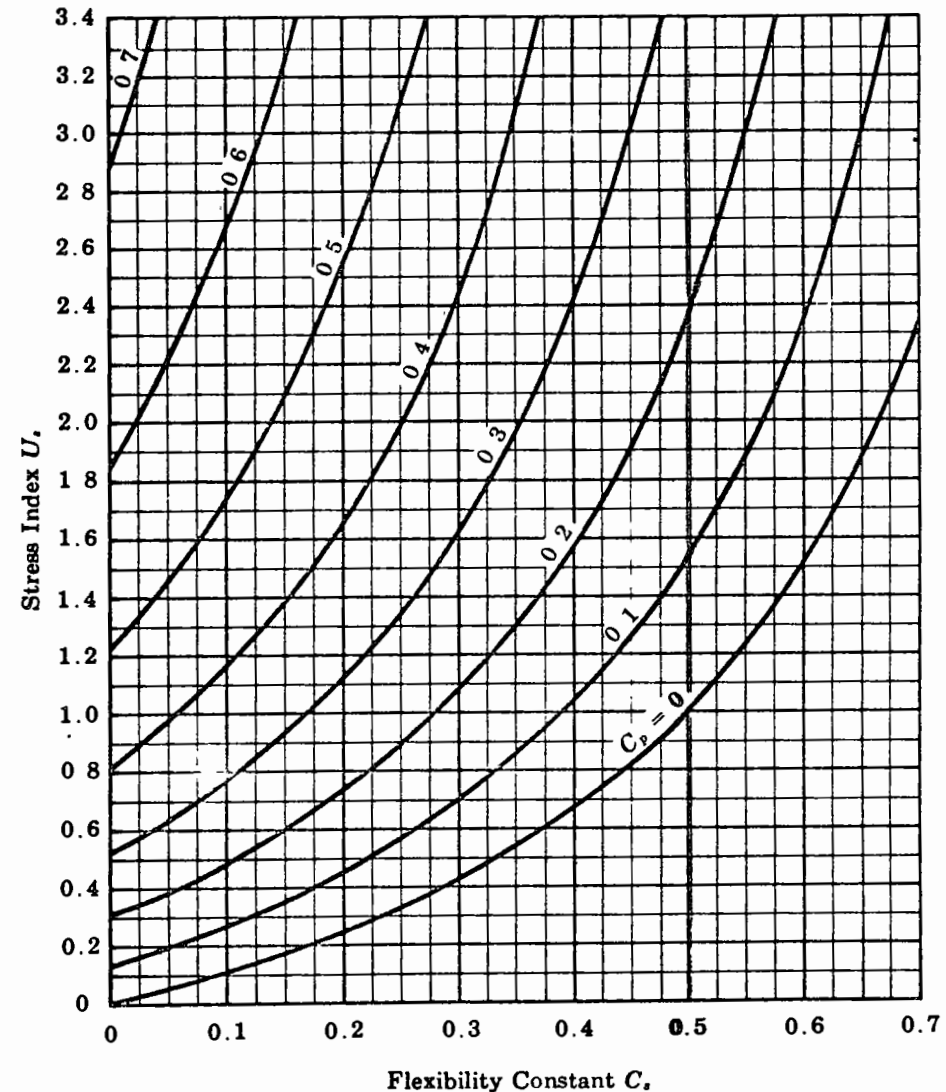


Fig. C1.13.3.2

If the roof framing consists of a series of equally-spaced wall-bearing beams, they would be considered as secondary members, supported on an infinitely stiff primary member. For this case, one would enter Fig. C1.13.3.2. The limiting value of  $C$ , would be determined by the intercept of a horizontal line representing the  $U_s$ -value and the curve for  $C_p = 0$ .

The ponding deflection contributed by a metal deck is usually such a small part of the total ponding deflection of a roof panel, that it is sufficient merely to limit its moment of inertia (per foot of width normal to its span) to 0.000025 times the fourth power of its span length, as provided in the Specification. However, the stability against ponding of a roof consisting of a metal roof deck of relatively slender depth-span ratio, spanning between beams supported directly on columns, may need to be checked. This can be done using Fig. C1.13.3.1 or C1.13.3.2 with the following computed values:

- $U_p$ , the stress index for the supporting beam
- $U_s$ , the stress index for the roof deck
- $C_p$ , the flexibility constant for the supporting beams
- $C_s$ , the flexibility constant for one foot width of the roof deck ( $S = 1.0$ )

Since the shear rigidity of their web system is less than that of a solid plate, the moment of inertia of steel joists and trusses should be taken as somewhat less than that of their chords.

## SECTION 1.14 GROSS AND NET SECTIONS

### 1.14.3 Net Section

Tests\* have indicated that, as the ratio of net to gross section approaches unity, the ultimate tensile strength of a member may be less than the product of the net section multiplied by the tensile strength of the steel determined by standard coupon tests. A precise evaluation of this relationship would depend upon such parameters as hole spacing normal to the applied tension force versus thickness of section, and the ductility of the steel. Pending further investigation, the Specification places the upper limit of the fully effective net section at 85 percent of the gross section.

### 1.14.6 Pin-Connected Members

Forged eyebars have been replaced by pin-connected plates or eyebars flame-cut from plates. Provisions for the proportioning of eyebars contained in the Specification are based upon standards evolved from long experience with forged eyebars. Through extensive destructive testing they have been found to provide balanced designs when these members are flame-cut instead of forged. The somewhat more conservative rules for pin-connected members of non-uniform cross-section and those not having enlarged "circular" heads is likewise based on the results of experimental research.\*\*

\* Schutz, F. W. and Newmark, N. M. The Efficiency of Riveted Structural Joints, *Structural Research Series No. 30, University of Illinois.*

Fisher, J. W. Behavior of Fasteners and Plates With Holes, *ASCE Journal of the Structural Division, December, 1965.*

\*\* Johnson, B. G. Pin Connected Plate Links, *1939 ASCE Transactions.*

Somewhat stockier proportions are provided for eyebars and pin-connected members fabricated from steel having a yield stress greater than 70 ksi, in order to eliminate any possibility of their "dishing" under the higher working stress for which they may be designed.

### 1.14.7 Effective Areas of Weld Metal

In recognition of the deeper penetration obtained by the submerged arc process, fillet welds made by this process may be proportioned on the basis of an effective throat thickness somewhat greater than the perpendicular distance from the root to the diagrammatic weld face. For fillet welds of such size as to require more than a single pass, the recognized increase in throat thickness is held constant.

Provision for the use of partial penetration groove welds, which first appeared in the 1961 AISC Specification, has been extended to cover their use on both sides of a joint, in keeping with similar provisions now included in the AWS Building Code.

## SECTION 1.15 CONNECTIONS

### 1.15.3 Placement of Rivets, Bolts and Welds

Slight eccentricities between the gravity axis of single- and double-angle members and the center of gravity of their connecting rivets or bolts have long been ignored as having negligible effect upon the strength of such members. Tests\* have shown that similar practice is warranted in the case of welded members in statically loaded structures.

### 1.15.5 Restrained Members

Whether or not transverse stiffeners are required on the web of a member opposite the flanges of members rigidly connected to its flanges, as in Fig. C1.15.5.1, depends upon the proportions of these members. Formulas

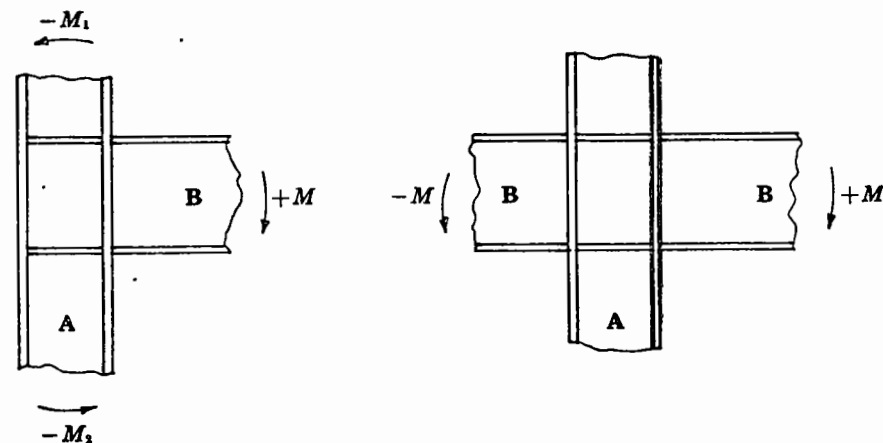


Fig. C1.15.5.1

\* Gibson, G. T. and Wake, B. T. An Investigation of Welded Connections for Angle Tension Members, *The Welding Journal, January 1942, American Welding Society.*



(1.15-1) and (1.15-3) are based on tests\* supporting the concept that, in the absence of transverse stiffeners, the web and flange thickness of member A should be such that these elements will not yield inelastically under concentrated forces delivered by member B and equal to the area of the rigidly connected flange times its yield stress.

Formula (1.15-4), giving the required area of stiffeners when stiffeners are needed, is based upon the same concept.

Formula (1.15-2) limits the slenderness ratio of an unstiffened web of the supporting member, in order to avoid possibility of its buckling.

Since these provisions are based upon the maximum force that can be delivered by the supported member flanges, they obviously would be conservative in the case of less rigidly connected members.

### 1.15.6 Fillers

The practice of securing fillers by means of additional fasteners, so that they are in effect an integral part of a shear-connected component, is not required where a connection is designed as a friction-type joint using high strength bolts. In such connections the resistance to slip between filler and either connected part is comparable to that which would exist between these parts if no fill were required.

### 1.15.10 Rivets and Bolts in Combination with Welds

The sharing of stress between rivets and A307 bolts in a single group of fasteners is not recommended in new work. High strength bolts used in bearing-type connections should not be required to share shear stress with welds. High strength bolts used in friction-type connections, however, because of the rigidity of the connection, may be proportioned to function in conjunction with welds in resisting the transfer of stress across faying surfaces, provided the welds are made after the bolts have been tightened.

In making alterations to existing structures it is assumed that whatever slip is likely to occur in riveted joints or high strength bolted, bearing-type joints will have already taken place. Hence, in such cases the use of welding to resist all contemplated stresses in addition to those produced by existing dead load, present at the time of making the alteration, is permitted.

## SECTION 1.16 RIVETS AND BOLTS

### 1.16.1 High Strength Bolts

Earlier reference to A354 Grade BC bolts has been deleted since the Specification of the Research Council on Riveted and Bolted Structural Joints has been revised to include A490 bolts, which are better suited and more readily available. At the same time, provision for the use of A449 bolts, in lieu of A325 bolts, has been added. These bolts differ from A325 bolts only as to the size of head and conform to the high strength bolts originally called for in the Council's Specification when the use of hardened washers under head and nut was mandatory.

\* Graham, J. D., Sherbourne, A. N., Knabbaz, R. N. and Jensen, C. D. Welded Interior Beam-to-Column Connections, American Institute of Steel Construction.

### 1.16.3 Long Grips

Provisions requiring a decrease in calculated stress for rivets having long grips (by arbitrarily increasing the required number an amount in proportion to the grip length) are not required for high strength bolts. Tests\* have demonstrated that the ultimate shearing strength of high strength bolts having a grip of 8 or 9 diameters is no less than that of similar bolts with much shorter grips.

### 1.16.4 Minimum Pitch

The recommendations for minimum pitch in the spacing of rivets and bolts is dictated solely by the need for driving or wrenching clearance during the installation of these fasteners.

### 1.16.6 Minimum Edge Distance in Line of Stress

The requirements of this section have been revised to provide greater flexibility in their application to various combinations of fastener hardness and yield stress in the connected parts. The earlier provisions, covering the use of A502 Grade 1 rivets in mild carbon steel, have been retained as the basic concept.

## SECTION 1.17 WELDS

### 1.17.2 Qualification of Weld and Joint Details

As in earlier editions, the Specification accepts without further procedure qualification numerous weld and joint details executed in accordance with the provisions of the *AWS Code for Welding in Building Construction*, D1.0-69. Other welding procedures may be used, provided they are qualified to the satisfaction of the designer and the building code authority and are executed in accordance with the provisions of AWS D1.0-69.

## SECTION 1.18 BUILT-UP MEMBERS

Requirements dealing with the detailing of built-up members, which cannot be stated in terms of calculated stress, are based upon judgment, tempered by experience.

The longitudinal spacing of fasteners connecting components of built-up compression members must be so limited that buckling of segments between adjacent fasteners would not occur at less load than that required to develop the ultimate strength of the member as a whole. However, maximum fastener spacing less than that necessary to prevent local buckling may be needed to ensure a close fit-up over the entire faying surface of components designed to be in contact with one another.

Provisions based on this latter consideration, like those giving maximum spacing of stitch fasteners for separated components of built-up tension mem-

\* Bendigo, R. A., Hansen, R. M. and Rumpf, J. L. Long Bolted Joints, *ASCE Journal of the Structural Division*, December, 1963.

bers, are of little structural significance. Hence, some latitude is warranted in relating them to the given dimensions of a particular member.

The provisions governing the proportioning of perforated cover plates are based upon extensive experimental research.\*

## SECTION 1.19 CAMBER

The cambering of flexural members, to eliminate the appearance of sagging or to match the elevation of adjacent building components when the member is loaded, is accomplished in various ways. In the case of trusses and girders the desired curvature can be built in during assembly of the component parts. Within limits, rolled beams can be cold-cambered at the producing mill.

The local application of heat has come into common use as a means of straightening or cambering beams and girders. The method depends upon an ultimate shortening of the heat-affected zones. A number of such zones, on the side of the member that would be subject to compression during cold-cambering or "gagging", are heated enough to be "upset" by the restraint provided by surrounding unheated areas. Shortening takes place upon cooling.

While the final curvature of camber produced by any of these methods can be controlled to a remarkable degree, it must be realized that some tolerance, to cover workmanship error and permanent change due to handling, is inevitable.

## SECTION 1.20 EXPANSION

As in the case of deflections, the satisfactory control of expansion cannot be reduced to a few simple rules, but must depend largely upon the good judgment of qualified engineers.

The problem is more serious in buildings having masonry wall enclosures than where the walls consist of prefabricated units. Complete divorcement of the framing, at widely spaced expansion joints, is generally more satisfactory than more frequently located devices dependent upon the sliding of parts in bearing, and usually less expensive than rocker or roller expansion bearings.

## SECTION 1.23 FABRICATION

### 1.23.1 Straightening Material

The use of heat for straightening or cambering members is permitted for A514 steel, as it is for other steels. However, the maximum temperature permitted for such straightening is 1100°F for A514 steel, as contrasted with 1200°F for other steels.

### 1.23.5 Riveted and High Strength Bolted Construction Assembling

Even when used in bearing-type shear connections, high strength bolts are required to be tightened to their proof load in the case of A325 and A449 bolts, and to 0.7 of their tensile strength in the case of A490 bolts.

\* Stang, A. H. and Jaffe, B. S. Perforated Cover Plates for Steel Columns, Research Paper RP1861, National Bureau of Standards.

This may be done either by the turn-of-nut method\* or by a calibrated wrench. Since fewer fasteners and stiffer connected parts are involved than is generally the case with A307 bolts, the greater clamping force is recommended in order to ensure solid seating of the connected parts.

### 1.23.6 Welded Construction

Inclusion of a number of grades of steel in the Specification has created the need for a greater control of preheat and interpass temperature in welding. The rules given reflect present practices as indicated by the standards of the American Welding Society.

## SECTION 1.24 SHOP PAINTING

The shop painting of structural steel not to be encased in concrete is not mandatory. Steelwork to be covered up by the building finish will be shop painted only if required by the plans and job specification. The surface condition of steel framing disclosed by the demolition of long-standing buildings has been found to be unchanged from the time of its erection, except at isolated spots where leakage may have occurred. Where such leakage is not eliminated the presence or absence of a shop coat is of minor influence.\*\*

The Specification does not define the type of paint to be used when a shop coat is required. Conditions of exposure and individual preferences with regard to finish paint are factors which have a bearing on the selection of the proper primer. Hence, a single formulation would not suffice.†

## SECTION 1.26 QUALITY CONTROL

Starting at the producing mill, and continuing in the fabricator's plant, steel required to have a yield stress in excess of 36 kips per square inch must at all times be so marked as to identify the ASTM specification and grade to which it conforms.

\* See Specification for Structural Joints Using ASTM A325 Or A490 Bolts Research Council on Riveted and Bolted Structural Joints.

\*\* Bigos, J., Smith, G. W., Ball, E. F. and Foehl, P. J. Shop Paint and Painting Practice, 1954 Proceedings AISC National Engineering Conference

† For a comprehensive treatment of the subject, see Systems Specifications, Steel Structures Painting Manual, Volume 2, published by the Steel Structures Painting Council, Pittsburgh, Pa.

## SECTION 2.1 SCOPE

When provisions for plastic design were first introduced into the AISC Specification in 1961, their use was limited to one- and two-story rigid frames. However, as noted in the *Commentary* at that time, they were not ruled out in the case of beam design for multi-story buildings if resistance to lateral forces applied to the building was provided by means other than the bending stiffness of these beams.

The bending strength of a compact flexural member is greater than its strength at initial yielding, in an amount measured by the shape factor  $f$  of its profile; a non-compact member (meeting the provisions of Sect. 1.9, but not those of Sect. 2.7), usually has little reserve strength beyond the elastic limit, because of buckling. Hence, for such members it may be said that the effective shape factor is 1.0.

The superior bending strength of compact sections is recognized in Part 1 of the Specification by increasing the allowable bending stress to  $0.66F_v$ . By the same token, the logical load factor for plastically designed beams is given by the equation  $F = \frac{F_v}{0.66F_v} \cdot (f)$ . For such shapes listed in the *AISC Steel Construction Manual*, the variation of  $(f)$  is from 1.10 to 1.23 with a mode of 1.12. Then the corresponding load factor must vary from 1.67 to 1.86 with a mode of 1.70.

Such a load factor is consistent and in better balance with that inherent in the allowable working stresses for tension members and deep plate girders. While a load factor of 1.7, comparable to the basic 5/3 factor of safety inherent in working stress design, was specified for beams, the recommended load factor for frames as a whole was 1.85, pending further investigation of columns and frame stability problems.

Research which has been completed since 1961\* has provided a better understanding of the ultimate strength of heavily loaded columns subjected to concurrent bending moments. Based upon this information, the load factor of frames has been made the same as that provided for members subject only to bending. Consistent with this change, the load factor to be used in designing for gravity loading combined with wind or seismic loading has been reduced from 1.4 to 1.3.

Based on continuing research at Lehigh University on multi-story framing,\*\* application of the Specification provisions has been extended to include the complete design of planar frames in high-rise buildings, provided they are braced to take care of any lateral loading. Systematic procedures for application of plastic design in proportioning the members of such frames have been developed† and are available in the current literature.

\* Van Kuren, R. C. and Galambos, T. V. Beam Column Experiments, *ASCE Journal of the Structural Division*, April, 1964.

\*\* Driscoll, G. C. et al. Plastic Design of Multi-Story Frames—Lecture Notes, *Fritz Engineering Laboratory Report No. 273.20, Lehigh University, August, 1965.*

Driscoll, G. C. Lehigh Conference on Plastic Design of Multi-Story Frames—A Summary, *AISC Engineering Journal*, April, 1966.

† Plastic Design of Braced Multi-Story Steel Frames, *American Iron and Steel Institute*, 1968.

Lu, Le-Wu Design of Braced Multi-Story Frames by the Plastic Method, *AISC Engineering Journal*, January, 1967.

## SECTION 2.2 STRUCTURAL STEEL

The 1961 AISC Specification limited the use of plastic design to steels having a specified minimum yield point no higher than 36 ksi. Most of the experimental verification of provisions for plastic design contained in the Specification at that time had used steel of about this strength.

By 1965 the applicability of such provisions, with only minor modifications, to high-strength low-alloy steel furnished to a specified yield point of 50 ksi, had been established.\* With the advent of ASTM Specification A572 in 1966, further investigation was undertaken which indicated their applicability for all grades covered by that standard.\*\*

On the basis of these investigations, the list of steels covered by ASTM standard specifications has been increased accordingly.

## SECTION 2.3 VERTICAL BRACING SYSTEM

While resistance to wind and seismic loading can be provided in moderate height buildings by means of concrete or masonry shear walls, which also provide for overall frame stability at factored gravity loading, taller building frames must provide this resistance acting alone. This can be achieved in one of two ways: either by a system of bracing or by a moment-resisting frame.

In moment resisting frames, designed in accordance with the provisions of Part 1 of the Specification, the necessary resistance to lateral loading is provided by the bending strength of the beams and columns rigidly connected to one another. Distribution of bending moments is based upon an assumption of completely elastic frame behavior; column strength is based upon an effective unbraced length generally greater than the actual unbraced length.

Neither of these assumptions apply in the analysis of unbraced, plastically designed high-rise frames, although appropriate analytical procedures have been proposed.† Pending further study, design of such framing more than two stories in height, in accordance with the provisions of Part 2 of the Specification, is restricted to fully-braced systems. The role and requirements of such systems‡ are defined by the provisions of Sect. 2.3.

The limitation on axial force of  $0.85P_y$  is inserted as a simple means of compensating for three possible effects:¶

- a) Loss of stiffness due to residual stress
- b) Effect of secondary moments from the vertical bracing system
- c) Lateral torsional buckling effect

\* Adams, P. F., Lay, M. G. and Galambos, T. V. Experiments on High Strength Steel Members, *Welding Research Council Bulletin No. 110.*

\*\* Plastic Design in Steel, *ASCE Manual of Engineering Practice No. 41, Second Edition, Section 5.1.*

† Driscoll, G. C. et al. Plastic Design of Multi-Story Frames—Lecture Notes, Chapter 14, *Fritz Engineering Laboratory Report No. 273.20, Lehigh University, August, 1965.*

‡ Lu, Le-Wu Design of Braced Multi-Story Frames by the Plastic Method, *AISC Engineering Journal*, January, 1967.

¶ Plastic Design in Steel, *ASCE Manual of Engineering Practice No. 41, Second Edition, Chapter 10.*

## SECTION 2.4 COLUMNS

Based on research completed since the previous edition of the Specification, provisions for design of beam-columns have been extensively revised.

Formulas (2.4-2) and (2.4-3)\* will be recognized as similar in type to Formulas (1.6-1a) and (1.6-1b) in Part 1, except that they are written in terms of factored loads and moments, instead of allowable stresses at service loading. As in the case of Formulas (1.6-1a) and (1.6-1b),  $P_{cr}$  is computed on the basis of the larger slenderness ratio for any given unbraced length.\*\*

A column is considered to be fully braced if the slenderness ratio  $l/r_y$  between the braced points is less than or equal to that specified in Sect. 2.9. For limiting values of  $l/r_y$  applicable to various yield stress steels and end moment ratios, see Sect. 2.9 in Appendix A.

When the unbraced length ratio of a member bent about its strong axis exceeds the limit specified in Sect. 2.9, the rotation capacity of the member may be impaired, due to the combined influence of lateral and torsional deformation, to such an extent that plastic hinge action within the member cannot be counted upon. However, if the computed value of  $M$  is small enough so that the limitations of Formulas (2.4-2) and (2.4-3) are met, the member will be strong enough to function at a joint where the required hinge action is provided in another member entering the joint. An assumed reduction in moment-resisting capacity is provided by using the value  $M_m$ , computed from Formula (2.4-4), in Formula (2.4-2).

Formula (2.4-4) was developed empirically on the basis of test observations and provides an estimate of the critical lateral buckling moment, in the absence of axial load, for the case where  $M_1/M_2 = -1.0$ . For other values of  $M_1/M_2$ , adjustment is provided by using the appropriate  $C_m$  value as defined in Sect. 1.6.1.

Formula (2.4-4) is to be used only in connection with Formula (2.4-2).

Space frames containing plastically designed planar rigid frames are assumed to be supported against sidesway normal to these frames. Depending upon other conditions of restraint, the basis for determination of proper values for  $P_{cr}$  and  $P_e$  and  $M_m$ , for a plastically designed column oriented to resist bending about its strong axis, is outlined in Table C 2.4.1. In each case  $l$  is the distance between points of lateral support corresponding to  $r_x$  or  $r_y$ , as

TABLE C 2.4.1

	Braced Planar Frames	One- and Two-Story Unbraced Planar Frames
$P_{cr}$	Use larger ratio, $\frac{l}{r_y}$ or $\frac{l}{r_x}$	<sup>1</sup> Use larger ratio, $\frac{l}{r_y}$ or $\frac{Kl}{r_x}$
$P_e$	Use $l/r_x$	<sup>1</sup> Use $Kl/r_x$
$M_m$	Use $l/r_y$	Use $l/r_y$

<sup>1</sup> Webs of columns assumed to be in plane of frame.

\* Driscoll, G. C. et al. Plastic Design of Multi-Story Frames—Lecture Notes, Eq. (4.6) and Eq. (4.7), Fritz Engineering Laboratory Report No. 273.20, Lehigh University, August, 1965.

\*\* Ibid., 24.

† Ibid., p. 4.26

applicable. When  $K$  is indicated, its value is governed by the provisions of Sect. 1.8.3 of the Specification. Elsewhere,  $Kl/r = l/r$ .

## SECTION 2.5 SHEAR

Using the von Mises criterion, the average stress at which an unreinforced web would be fully yielded in pure shear can be expressed as  $F_y/\sqrt{3}$ . It has been observed\* that the plastic bending strength of an I-shape beam is not appreciably reduced until shear yielding occurs over the full effective depth, which may be taken as the distance between the centroids of its flanges (approx. 0.95 times its actual depth). Thus

$$V_u = \frac{0.95F_y}{\sqrt{3}} dt = 0.55F_y dt$$

Shear stresses are generally high within the boundaries of a rigid connection of two or more members whose webs lie in a common plane. Assuming the moment  $+M$ , in Fig. C 2.5.1, expressed in kip-feet, to be resisted by a couple of forces at the centroid of the beam flanges, the shear, in kips, produced in beam-to-column connection web  $abcd$  can be computed as

$$V = \frac{+12M}{0.95d_b}$$

when  $V = V_u = 0.55F_y d_c t$

$$\text{Req'd } t = \frac{12M}{0.95d_b \times 0.55F_y d_c} = \frac{23M}{A_{bc} F_y}$$

where  $A_{bc}$  is the planar area  $abcd$  and  $F_y$  is expressed in kips per square inch.

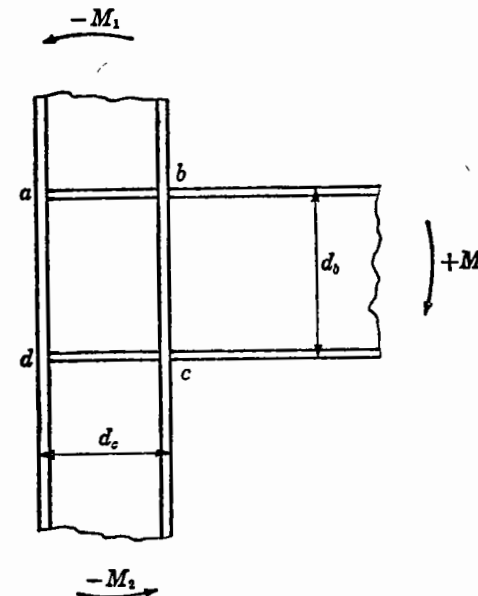


Fig. C 2.5.1

\* Plastic Design in Steel, ASCE Manual of Engineering Practice No. 41, Second Edition, Section 6.1.

If the thickness of the web panel is less than that given by this formula, the deficiency may be compensated by a pair of diagonal stiffeners or by a reinforcing plate in contact with the web panel and welded around its boundary to the column flanges and horizontal stiffeners.

## SECTION 2.6 WEB CRIPPLING

Usually stiffeners are needed, as at *ab* and *dc* in Fig. C 2.5.1, in line with the flanges of a beam rigidly connected to the flange of a second member so located that their webs lie in the same plane, in order to prevent crippling of the web of the latter opposite the compression flange of the former. A stiffener may also be required opposite the tension flange, in order to protect the weld joining the two flanges; otherwise the stress in the weld might be too great in the region of the beam web, due to lack of bending stiffness in the flange to which the beam is connected. Since their design is based upon equating the plastic resisting capacity of the supporting member to the plastic moment delivered by the supported member, Formulas (1.15-1), (1.15-2), (1.15-3) and (1.15-4) are equally applicable to allowable stress design and plastic design.

When stiffeners are required, as an alternative to the usual pair of horizontal plates, vertical plates parallel to but separated from the web as shown in Fig. C 2.6.1 may prove advantageous.

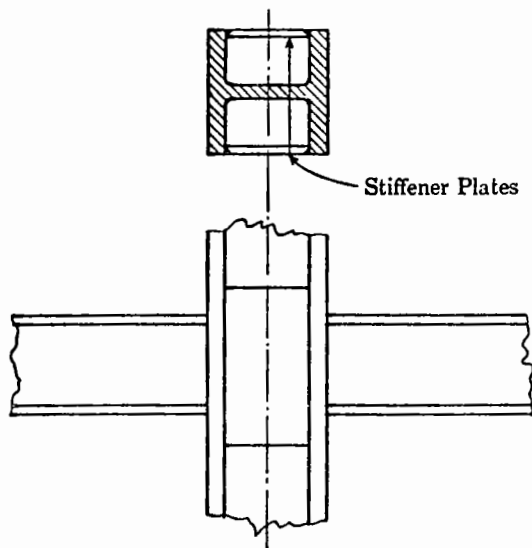


Fig. C 2.6.1

## SECTION 2.7 MINIMUM THICKNESS (WIDTH-THICKNESS RATIOS)

In extending the provisions for plastic design to steels having a yield point higher than 36 ksi, considerable research\* has been required in order to

\* Plastic Design in Steel, ASCE Manual of Engineering Practice No. 41, Second Edition, Section 6.2.

define limiting flange and web width-thickness ratios below which ample plastic hinge rotations could be relied upon without reduction in the  $M_p$ -value due to local buckling.

These studies have shown that the limiting width-thickness ratio is not exactly proportional to  $1/\sqrt{F_y}$ , although the discrepancy using such a relationship, within the range of yield stress presently permitted by the Specification, is not large. Expressions including other pertinent factors are complex and involve use of mechanical properties that have not been clearly defined. Tabular values for limiting flange width-thickness ratios are given in the Specification for the approved grades of steel.

No change in basic philosophy is involved in extending the earlier expression for limiting web depth-thickness ratio to stronger steels. Formulas (2.7-1a) and (2.7-1b) are derived, with minor adjustments for better correlation with observed test results, by multiplying Formula (25) of the 1963 Specification by the factor  $\sqrt{36/F_y}$ , in order to cover the accepted range in yield point stress. Formula (2.7-1a) is identical to Formula (1.5-4) in Part 1, except that it is written in terms of factored loads instead of allowable stresses at service loading.

## SECTION 2.8 CONNECTIONS

Connections located outside of regions where hinges would have formed at ultimate load can be treated in the same manner that similar connections in frames designed in accordance with the provisions of Part 1 would be treated. Since the moments and forces to be resisted will be those corresponding to the factored loading, the permissible stresses to be used in proportioning parts of the connections can be taken as 1.7 times those given in Sects. 1.5 and 1.6 of the Specification.

The same procedure is valid in proportioning connections located in the region of a plastic hinge. Connections required to resist moments and forces due to wind and earthquake loads combined with gravity loading factored to 1.3, and proportioned on the basis of limiting stresses equal to 1.7 times those given in Sects. 1.5 and 1.6, provide a balance between frame strength and connection strength, provided they are adequate to resist gravity loading alone, factored to 1.7.

The width-thickness ratio and unbraced length of all parts of the connection that would be subject to compression stresses in the region of a hinge must meet the requirements given in Part 2, and sheared edges and punched holes must not be used in portions of the connection subject to tension.

When a haunched connection is proportioned elastically for the moments that would exist within its length, the continuous frame can be analyzed as a mechanism having a hinge at the small end of the haunch, rather than at the intersection point between connected members,\* with some attendant economy.

Tests\*\* have shown that splices assembled with high strength bolts are capable of developing the  $M_p$ -value of the gross cross-section of the connected

\* Plastic Design in Steel, ASCE Manual of Engineering Practice No. 41, Second Edition, Chapter 8.

\*\* Douty, R. T. and McGuire, W. High Strength Bolted Moment Connections, ASCE Journal of the Structural Division, April, 1965.

part. It has also been demonstrated\* that beam-to-column connections involving use of welded or mechanically fastened fittings, instead of full penetration groove welds matching the full member cross-section, not only are capable of developing the  $M_p$ -value of the member, but that the resulting hinge rotation can be reversed several times without failure.

## SECTION 2.9 LATERAL BRACING

Portions of members that would be required to rotate inelastically as a plastic hinge, in reducing a continuous frame to a mechanism at ultimate load, need more bracing than similar parts of a continuous frame designed in accordance with the elastic theory. Not only must they reach yield point at a load factor of 1.7, they must also strain inelastically to provide the necessary hinge rotation. This is not true at the last hinge to form, since the factored load is assumed to have been reached when this hinge starts to rotate. When bending takes place about the strong axis, any I-shape member tends to buckle out of the plane of bending. It is for this reason that lateral bracing is needed. The same tendency exists with highly stressed members in elastically designed frames, and in portions of plastically designed frames outside of the hinge areas, but here the problem is less severe since hinge rotation is not involved.

For the limited range of steels recognized as suitable for plastic design in earlier editions of the Specification,  $l_{cr}$ , the allowable unbraced length of compression flanges subject to plastic bending, was given as

$$\left(60 - 40 \frac{M}{M_p}\right) r_y > l_{cr} \geq 35r_y$$

where  $M/M_p$ , the ratio of end moments, was considered positive only when the unbraced length was bent in single curvature.

Based on research seeking to extend the application of plastic design to stronger steels, it was noted\*\* that this expression could be unduly conservative in the region where  $-0.5 < M/M_p < 0$ .† The new provision reflects this and also includes a more conservative approach in the region where  $0 < M/M_p < +1.0$ .†

Both Formulas (2.9-1a) and (2.9-1b) are empirical expressions which closely approximate the suggested revisions.‡

\* Popov, E. P. and Pinkney, R. B. Behavior of Steel Building Connections Subjected to Inelastic Strain Reversals, *Bulletin Nos. 13 and 14, American Iron and Steel Institute, November, 1968.*

\*\* Lay, M. G. and Galambos, T. V. Inelastic Beams Under Moment Gradient, *ASCE Journal of the Structural Division, February, 1967, p. 390.*

† In keeping with similar usage of the parameter  $M/M_p$  in Sect. 1.6 of the Specification, the sign convention adopted in Formulas (2.9-1a) and (2.9-1b) and used here is that generally found to be more convenient in frame analysis, namely that clockwise moments about a fixed point are positive and counterclockwise moments are negative.

‡ Plastic Design in Steel, *ASCE Manual of Engineering Practice No. 41, Second Edition, Section 6.3.*

**SUPPLEMENT NO. 1**

**TO THE  
SPECIFICATION  
FOR THE  
DESIGN,  
FABRICATION  
& ERECTION  
OF  
STRUCTURAL  
STEEL FOR  
BUILDINGS**

**(ADOPTED FEBRUARY 12, 1969)**

**Effective November 1, 1970**

**AMERICAN INSTITUTE  
OF STEEL CONSTRUCTION  
101 PARK AVENUE, NEW YORK, N.Y. 10017**

# Supplement No. 1

## TO THE SPECIFICATION FOR THE DESIGN, FABRICATION AND ERECTION OF STRUCTURAL STEEL FOR BUILDINGS (*Adopted February 12, 1969*)

Effective November 1, 1970

### SECTION 1.4 MATERIAL

#### 1.4.1 Structural Steel

1.4.1.1 After the words "ASTM A514", delete the following:

"(Quenched and tempered alloy steel structural shapes and seamless mechanical tubing meeting all of the mechanical and chemical requirements of A514 steel, except that the specified maximum tensile strength may be 140,000 psi for structural shapes and 145,000 psi for seamless mechanical tubing, shall be considered as A514 steel.)"

Add to the list of approved materials:

*"Hot-Formed Welded and Seamless High-Strength Low-Alloy Structural Tubing, ASTM A618"*

Add a new Section as follows:

#### 1.4.6 Stud Shear Connectors

Steel stud shear connectors shall conform to the requirements of Articles 429 and 430, *Code for Welding in Building Construction*, AWS D1.0-69, of the American Welding Society.

Manufacturer's certification shall constitute sufficient evidence of conformity with specifications."

### SECTION 1.5 ALLOWABLE STRESSES

#### 1.5.1 Structural Steel

##### 1.5.1.3 Compression

1.5.1.3.1 Immediately following the words "compression members", add: "whose cross-sections meet the provisions of Sect. 1.9"



**1.5.1.4 Bending**

1.5.1.4.1 Delete subparagraph d in its entirety and substitute the following:

"d. The depth-thickness ratio of the web or webs shall not exceed the value given by Formulas (1.5-4a) or (1.5-4b) as applicable.

$$\frac{d}{t} = \frac{412}{\sqrt{F_y}} \left( 1 - 2.33 \frac{f_a}{F_y} \right) \text{ when } \frac{f_a}{F_y} \leq 0.16 \quad (1.5-4a)$$

$$\frac{d}{t} = \frac{257}{\sqrt{F_y}} \quad \text{when } \frac{f_a}{F_y} > 0.16 \quad (1.5-4b)"$$

1.5.1.4.2 Immediately following the words "of Sect. 1.5.1.4.1" add a comma, and immediately following the words "except that  $b_f/2t_f$ " delete the comma.

Change formula number "(1.5-5)" to "(1.5-5a)".

1.5.1.4.3 Add a second paragraph as follows:

"Doubly-symmetrical I- and H-shape members bent about their minor axis (except hybrid girders and members of A514 steel) meeting the requirements of Sect. 1.5.1.4.1, subparagraph a, except where  $b_f/2t_f$  exceeds  $52.2/\sqrt{F_y}$  but is less than  $95.0/\sqrt{F_y}$ , may be designed on the basis of an allowable bending stress

$$F_b = F_y \left[ 0.933 - 0.0035 \left( \frac{b_f}{2t_f} \right) \sqrt{F_y} \right] \quad (1.5-5b)"$$

1.5.1.4.6a Immediately following the words "under Sect. 1.5.1.4.5," add: "and meeting the requirements of Sect. 1.9.1.2,".

1.5.1.4.6b Immediately following the words "under Sect. 1.5.1.4.5," add: "and meeting the requirements of Sect. 1.9.1.2,".

**SECTION 1.10 PLATE GIRDERS AND ROLLED BEAMS****1.10.5 Stiffeners**

1.10.5.3 In the third paragraph, immediately following the words "holes shall be such that", delete: "the smaller panel dimension,  $a$  or  $h$ , shall not exceed  $348t/\sqrt{f_s}$ " and substitute the words " $f_s$  does not exceed the value given by Formula (1.10-1)".

**SECTION 1.11 COMPOSITE CONSTRUCTION****1.11.2 Design Assumptions**

1.11.2.2 At the beginning of the fourth paragraph, delete the words "For construction without temporary shoring, the value of the section modulus of the transformed composite section used in stress calculations (referred to the bottom flange of the steel beam) shall not exceed", and substitute the following:

"For construction without temporary shoring, the bottom flange steel stress may be computed from the total load moment and the actual transformed section modulus  $S_{tr}$ , except that the numerical value of  $S_{tr}$  so used shall not exceed that of Formula (1.11-2). This stress shall not exceed the appropriate value of Sect. 1.5.1."

**SECTION 1.15 CONNECTIONS****1.15.5 Restrained Members**

In the first line of the second paragraph delete the words "fully restrained".

**SECTION 1.23 FABRICATION****1.23.1 Straightening Material**

Delete this subhead and the entire text of the paragraph beginning with the words "Rolled material", and substitute a new subheading and paragraph reading as follows:

**"1.23.1 Cambering, Curving, and Straightening**

The local application of heat or mechanical means may be used to introduce or correct camber, curvature, and straightness. The temperature of heated areas, as measured by approved methods, shall not exceed 1100°F for A514 steel nor 1200°F for other steels."

**1.23.6 Welded Construction**

In Table 1.23.6, for thickness "To  $\frac{3}{4}$ , incl.", in the second column under the heading "Welding Process", change "None" to "None\*."

# Errata to the AISC Specification

The following corrections should be made to all copies of the 7/69 printing of the Specification. They have already been incorporated into subsequent printings and all copies of the 7th Edition Manual of Steel Construction.

## NOMENCLATURE

In the definition of  $C_e$ , delete: “, except in Appendix C”.

Delete the definition of  $T_b$  and substitute the following:

“Specified pretension of a high strength bolt (kips)”

## SECTION 1.5 ALLOWABLE STRESSES

### 1.5.3 Welds

In Table 1.5.3, under the column headed “Kind of Stress”, make the following corrections:

In the fourth descriptive block, immediately following the words “groove weld”, add: “and partial-penetration groove weld”.

In the fifth descriptive block, immediately following the words “fillet weld” in the second line, delete: “and partial-penetration groove weld”.

## SECTION 1.10 PLATE GIRDERS AND ROLLED BEAMS

### 1.10.1 Proportions

In the first sentence, immediately following the words “and rolled”, add: “or welded”.

## SECTION 1.15 CONNECTIONS

### 1.15.5 Restrained Members

In the second paragraph, delete the formula identified by the designation (1.15-2) and substitute the formula

$$t \leq \frac{d_c \sqrt{F_y}}{180}$$

## SECTION 2.9 LATERAL BRACING

In Formula (2.9-1b), immediately after “-0.5”, change the symbol “>” to “≥”.

## APPENDIX A

In Table 1-65, in the bottom footnote, change the number “93.3” to “93.8”.

## APPENDIX B

### SECTION B2 ALLOWABLE STRESSES

In Table B2, for the General Condition “Groove welds”, second Situation, add the number “10” in the column headed “Illustrative Example Nos.”.

## APPENDIX C

### SECTION C2 STRESS REDUCTION FACTOR—UNSTIFFENED COMPRESSION ELEMENTS

Delete the last sentence, and substitute the following:

“However, unstiffened elements of channels and tees whose proportions exceed the limits of Sect. 1.9.1.2 shall conform to the limits given in Table C1.”

### SECTION C5 AXIALLY LOADED COMPRESSION MEMBERS

Change the term “ $C_c$ ” to “ $C'_c$ ” in both formulas and in the text.

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**SPECIFICATIONS FOR**

# **Structural Joints Using ASTM A325 or A490 Bolts**

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**Approved by the Research Council on Riveted and Bolted Structural Joints  
of the Engineering Foundation, September 1, 1966**

**Endorsed by American Institute of Steel Construction**

**Endorsed by Industrial Fasteners Institute**



**AMERICAN INSTITUTE OF STEEL CONSTRUCTION**

**101 Park Avenue, New York, N. Y. 10017**

## SPECIFICATION FOR

# Structural Joints Using ASTM A325 or A490 Bolts

Approved by Research Council on Riveted and Bolted Structural Joints of the  
Engineering Foundation, September 1, 1966  
Endorsed by American Institute of Steel Construction, Inc.  
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## 1 Scope

- (a) This specification covers the design and assembly of structural joints using ASTM A325 high-strength carbon steel bolts, ASTM A490 high-strength alloy steel bolts, or equivalent fasteners, tightened to a specified tension. The bolts are used in holes having a nominal diameter slightly larger than the nominal bolt size.
- (b) Construction shall conform to an applicable existing code or specification for structures of wrought iron, carbon structural steel or high-strength steel, except as otherwise provided herein.
- (c) Joints required to resist shear between their connected parts are designated as either *friction-type* or *bearing-type* connections. Shear connections subjected to stress reversal, severe stress fluctuation, or where slippage would be undesirable, shall be *friction-type*.
- (d) The attached Commentary provides guidance in the application of the specification.

## 2 Bolts, Nuts and Washers

- (a) Except as provided in paragraph (d) of this section, bolts, nuts and circular washers if required, shall conform to requirements of the current edition of the specifications of the American Society for Testing and Materials for High-Strength Carbon Steel Bolts for Structural Steel Joints, ASTM A325, or for Quenched and Tempered Alloy Steel Bolts for Structural Steel Joints, ASTM A490. The designer shall specify the grade of bolts to be used.
- (b) Except as provided in paragraph (d) of this section, bolt dimensions shall conform to the current requirements of the American Standards Association for heavy hex structural bolts, ASA Standard B18.2.1.
- (c) Except as provided in paragraph (d) of this section, nut dimensions shall conform to current requirements of the American Standards Association for heavy hex nuts, ASA Standard B18.2.2.
- (d) Other fasteners which meet the chemical composition requirements of ASTM specification A325 or A490 and which meet the mechanical property requirements of the same specification in full-size tests and which have body diameter and bearing areas under the head and nut, or their equivalent, not less than those provided by a bolt and nut of the same nominal dimensions prescribed by paragraphs 2(b) and

Table 1 Washer Dimensions<sup>a</sup>

Bolt Size D	Circular Washers				Square or Rectangular Beveled Washers for American Standard Beams and Channels		
	Nominal Outside Diameter	Nominal Diameter of Hole	Thickness		Minimum Side Dimension	Mean Thickness	Slope or Taper in Thickness
			Min.	Max.			
1/2	1 1/16	17/32	0.097	0.177	1 3/4	5/16	1:6
5/8	1 1/16	21/32	0.122	0.177	1 3/4	5/16	1:6
3/4	1 1/32	13/16	0.122	0.177	1 3/4	5/16	1:6
7/8	1 3/4	1 1/16	0.136	0.177	1 3/4	5/16	1:6
1	2	1 1/16	0.136	0.177	1 3/4	5/16	1:6
1 1/8	2 1/4	1 1/4	0.136	0.177	2 1/4	5/16	1:6
1 1/4	2 1/2	1 3/8	0.136	0.177	2 1/4	5/16	1:6
1 1/2	2 3/4	1 1/2	0.136	0.177	2 1/4	5/16	1:6
1 3/4	3	1 5/8	0.136	0.177	2 1/4	5/16	1:6
2	3 3/8	1 7/8	0.178 <sup>b</sup>	0.28 <sup>b</sup>			
	3 3/4	2 1/8	0.178 <sup>b</sup>	0.28 <sup>b</sup>			
Over 2 to 4 incl.	2D - 1/2	D + 1/8	0.24 <sup>c</sup>	0.34 <sup>c</sup>			

<sup>a</sup> Dimensions in inches. (Tolerances as noted in Table 1-A.)  
<sup>b</sup> 1/16 in. nominal.  
<sup>c</sup> 1/4 in. nominal.

3 Bolted Parts

- (a) The slope of surfaces of bolted parts in contact with the bolt head and nut shall not exceed 1:20 with respect to a plane normal to the bolt axis. Bolted parts shall fit solidly together when assembled and shall not be separated by gaskets or any other interposed compressible material. Holes may be punched, subpunched and reamed, or drilled, as required by the applicable code or specification and shall be a nominal diameter not more than 1/16 in. in excess of the nominal bolt diameter.
- (b) When assembled, all joint surfaces, including those adjacent to the bolt heads, nuts or washers, shall be free of scale, except tight mill scale, and shall also be free of burrs, dirt and other foreign material that would prevent solid seating of the parts.
- (c) Contact surfaces within *friction-type* joints shall be free of oil, paint, lacquer or galvanizing.

4 Allowable Working Stresses

- (a) *Design Stresses.* The allowable working stresses for A325 and A490 bolts specified in the following paragraphs are given, respectively, for bridges and buildings in Table 2. As used in paragraphs (b) and (c), nominal bolt area is defined as the area corresponding to the nominal diameter of the bolt.

2(c), may be used. Such alternate fasteners may differ in other dimensions from those of the specified bolts and nuts. Their installation procedure may differ from those specified in paragraphs 5(c) and 5(d) and their inspection may differ from that specified in Section 6. When a different installation procedure or inspection is used, it shall be detailed in a supplemental specification applying to the alternate fastener and this specification must be approved by the engineer responsible for the design of the structure.

- (e) Circular washers and square or rectangular beveled washers shall conform to the dimensions in Table 1 within tolerances given in Table 1-A. Beveled washers shall taper in thickness. Washers shall have no raised markings on their bearing surfaces.

Where necessary, washers may be clipped on one side to a point not closer than 1/8 of the bolt diameter from the center of the washer.

Table 1-A Washer Dimension Tolerances (inches)

Dimension	Washer Size	
	To 1 1/4 in. Nominal Bolt Size, incl.	Over 1 1/4 in. Nominal Bolt Size
Nominal diameter of hole	-0; +1/32	-0; +1/16
Nominal outside dimensions	-1/32; +1/4	-1/32; +1/4
Flatness; max. deviation from straight edge placed on "cut" side shall not exceed	0.01	0.015
Burr shall not project above immediately adjacent washer surface more than	0.01	0.015

Table 2 Allowable Working Stresses for Fasteners<sup>a</sup>

Specification Paragraph	Loading Conditions	ASTM A325 Bolts		ASTM A490 Bolts	
		Bridges	Buildings	Bridges	Buildings
4(b)	Applied tension, psi	36,000	40,000	48,000 <sup>b</sup>	54,000 <sup>b</sup>
4(c)	Shear, psi				
	1. Friction-type connection	13,500	15,000	18,000	20,000
	2. Bearing-type connection, shear plane through threads	13,500	15,000	20,000	22,500
	3. Bearing-type connection, threads excluded	20,000	22,000	29,000	32,000
4(d)	Bearing, psi <sup>c</sup>	1.22 F <sub>y</sub>	1.35 F <sub>y</sub>	1.22 F <sub>y</sub>	1.35 F <sub>y</sub>

<sup>a</sup> The tabulated stresses, except for bearing stress, apply to bolts used in any grade of steel.  
<sup>b</sup> Static loading only.  
<sup>c</sup> F<sub>y</sub> = Specified minimum yield point of the lowest strength connected part. The bearing stress shall not be more than the specified minimum tensile strength of the lowest strength connected material.

- (b) *Applied Tension.* Bolts required to support applied load by means of direct tension shall be so proportioned that their average tensile stress, computed on the basis of nominal bolt area and independent of any initial tightening force, will not exceed the appropriate stress given in Table 2. The applied load shall be the sum of the external load and any tension resulting from prying action produced by deformation of the connected parts.

(c) **Shear**

1. Bolts in *friction-type* connections assembled in accordance with the requirements of paragraph 3(c) shall be proportioned on the basis of the appropriate stress given in Table 2. These shear stresses may be used to proportion high-strength bolts used in combination with rivets or welds designed in accordance with the provisions of the applicable code or specification. In *friction-type* connections there need be no consideration of bearing, and fillers need not be "developed." However, eccentricity of forces at short thick fillers must be considered.

2. Bolts in *bearing-type* connections having thread in a plane of contact surfaces of the connected parts shall be proportioned on the basis of the appropriate stress given in Table 2.

3. Bolts in *bearing-type* connections, where bolt threads are excluded from the shear planes of the contact surfaces between the connected parts, shall be proportioned on the basis of the appropriate stress given in Table 2.

(d) **Bearing.** In *bearing-type* connections the computed bearing pressure, assumed to be distributed over an area equal to the nominal bolt diameter times the thickness of the connected part, shall not exceed the appropriate stress given in Table 2.

In *bearing-type* connections having no more than two bolts in a line parallel to the direction of stress, the distance between the center of the nearest bolt and that end of the connected member towards which the pressure from the bolt is directed shall be not less than  $AC/t$  for single shear or  $2AC/t$  for double shear, where  $A$  is the nominal cross-sectional area of the fastener,  $t$  is the thickness of the connected part and  $C$  is the ratio of specified minimum tensile strength of the fastener to the specified minimum tensile strength of the connected part. This end distance may be proportionately less where the shear stress per bolt is less than that permitted in this section, but not less than  $1\frac{1}{2}$  times the bolt diameter. It need not exceed  $1\frac{1}{2}$  times the transverse spacing of the fasteners.

(e) **Increase in Working Stress.** Increase in working stress allowed in the applicable code or specification may be applied to the stresses given in this section (see Commentary for *Shear; Friction-Type Connections*).

**5 Installation**

(a) **Fastener Tension.** Each fastener shall be tightened to provide, when all fasteners in the joint are tight, at least the minimum tension shown in Table 3 for the size and grade of fastener used.

Threaded bolts shall be tightened with properly calibrated wrenches or by the turn-of-nut method. If required because of bolt entering and wrench operation clearances, tightening by either procedure may be done by turning the bolt while the nut is prevented from rotating.

Impact wrenches, if used, shall be of adequate capacity and sufficiently supplied with air to perform the required tightening of each bolt in approximately ten seconds.

Table 3 Fastener Tension

Bolt Size, In Inches	Minimum Fastener Tension <sup>a</sup> in Thousands of Pounds (kips)	
	A325 Bolts	A490 Bolts
$\frac{1}{2}$	12	15
$\frac{5}{8}$	19	24
$\frac{3}{4}$	28	35
$\frac{7}{8}$	39	49
1	51	64
$1\frac{1}{8}$	56	80
$1\frac{1}{4}$	71	102
$1\frac{3}{8}$	85	121
$1\frac{1}{2}$	103	148
Over $1\frac{1}{2}$		$0.7 \times T.S.$

<sup>a</sup> Equal to 70 percent of specified minimum tensile strengths of bolts, rounded off to the nearest kip.

(b) **Washers.** A325 fasteners meeting the provisions of Section 2 may be installed without hardened washers when tightening is by the turn-of-nut method. A490 bolts installed by the turn-of-nut method and A325 or A490 bolts tightened by the calibrated wrench method (i.e., by torque control) shall have a hardened washer under the element (nut or bolt head) turned in tightening. Two hardened washers shall be used with all A490 bolts used to connect material having a specified minimum yield point less than 40 ksi.

Where an outer face of the bolted parts has a slope greater than 1:20 with respect to a plane normal to the bolt axis, a beveled washer shall be used to compensate for the lack of parallelism.

(c) **Calibrated Wrench Tightening.** When calibrated wrenches are used to provide the bolt tension specified in paragraph 5(a) their setting shall be such as to induce a bolt tension 5% to 10% in excess of this value. These wrenches shall be calibrated at least once each working day by tightening, in a device capable of indicating actual bolt tension, not less than three typical bolts of each diameter from the bolts being installed. Power wrenches shall be adjusted to stall or cut-out at the selected tension. If manual torque wrenches are used the torque indication corresponding to the calibrating tension shall be noted and used in the installation of all bolts of the tested lot. Nuts shall be in tightening motion when torque is measured. When using calibrated wrenches to install several bolts in a single joint, the wrench shall be returned to "touch up" bolts previously tightened, which may have been loosened by the tightening of subsequent bolts, until all are tightened to the prescribed amount.

(d) **Turn-of-Nut Tightening.** When the turn-of-nut method is used to provide the bolt tension specified in paragraph 5(a), there shall first be enough bolts brought to a "snug tight" condition to insure that the parts of the joint are brought into good contact with each other. Snug tight is defined as the tightness attained by a few impacts of an impact wrench or the full effort of a man using an ordinary spud wrench. Following this initial operation, bolts shall be placed in any remaining holes in the connection and brought to snug tightness. All bolts in the joint shall then be maintained additional

Table 4 Nut Rotation<sup>a</sup> from Snug Tight Condition

Disposition of Outer Faces of Bolted Parts	
Both faces normal to bolt axis, or one face normal to axis and other face sloped not more than 1:20 (bevel washer not used)	
Both faces sloped not more than 1:20 from normal to bolt axis (bevel washers not used)	
Bolt length <sup>b</sup> not exceeding 8 diameters or 8 inches	Bolt length <sup>b</sup> exceeding 8 diameters or 8 inches
For all length of bolts	
½ turn	¾ turn
¾ turn	

<sup>a</sup> Nut rotation is rotation relative to bolt regardless of the element (nut or bolt) being turned. Tolerance on rotation: 30° over or under. For coarse thread heavy hex structural bolts of all sizes and length and heavy hex semi-finished nuts.

<sup>b</sup> Bolt length is measured from underside of head to extreme end of point.

by the applicable amount of nut rotation specified in Table 4, with tightening progressing systematically from the most rigid part of the joint to its free edges. During this operation there shall be no rotation of the part not turned by the wrench.

6 Inspection

- (a) The Inspector shall determine that the requirements of Sections 2, 3 and 5 of this specification are met in the work. When the calibrated wrench method of tightening is used, the Inspector shall have full opportunity to witness the calibration tests prescribed in paragraph 5(c).
- (b) The Inspector shall observe the installation and tightening of bolts to determine that the selected tightening procedure is properly used and shall determine that all bolts are tightened. This inspection will ordinarily assure that the specified bolt tightness is attained.
- (c) When there is need for more inspection of bolt tightness than that provided in paragraph 6(b), the following arbitration inspection shall be used unless a more extensive or different procedure is specified in the inquiry and order for the work:

1. The Inspector shall use an *inspecting wrench* which may be either a torque wrench or a power wrench that can be adjusted in accordance with the requirements of paragraph 5(c).

2. Three bolts of the same grade, size\* and condition as those under inspection shall be placed individually in a calibration device capable of indicating bolt tension. The surface under the part to be turned in tightening each bolt shall be like that under the corresponding part in the structure; i.e., there shall be a washer under the part turned if washers are so used in the structure or, if no washer is used, the material abutting the part turned shall be of the same specification as that in the structure.

3. When the *inspecting wrench* is a torque wrench, each bolt specified in paragraph 6(c)2 shall be tightened in the calibration device by any convenient means to the minimum tension specified for its size in paragraph 5(a). The *inspecting wrench* then shall be

\* Length may be any length representative of bolts used in the structure.

applied to the tightened bolt and the torque necessary to turn nut or head 5 degrees (approximately 1 inch or 12 inch radius) in the tightening direction shall be determined. The average torque measured in the tests of three bolts shall be taken as the *job inspecting torque* to be used in the manner specified in paragraph 6(c)5.

4. When the *inspecting wrench* is a power wrench it shall be adjusted so that it will tighten each bolt specified in paragraph 6(c)2 to a tension at least 5 but not more than 10% greater than the minimum tension specified for its size in paragraph 5(a). This setting of wrench shall be taken as the *job inspecting torque* to be used in the manner specified in paragraph 6(c)5.

5. Bolts represented by the sample prescribed in paragraph 6(c)2 which have been tightened in the structure shall be inspected by applying, in the tightening direction, the *inspecting wrench* and its *job inspecting torque* to 10% of the bolts, but not less than two bolts, selected at random in each connection. If no nut or bolt head is turned by this application of the *job inspecting torque*, the connection shall be accepted as properly tightened. If any nut or bolt head is turned by the application of the *job inspecting torque*, this torque shall be applied to all bolts in the connection, and all bolts whose nut or head is turned by the *job inspecting torque* shall be tightened and re-inspected, or alternatively, the fabricator or erector, at his option may re-tighten all of the bolts in the connection and then re-submit the connection for the specified inspection.

COMMENTARY

C1 Scope

When first approved by the Research Council on Riveted and Bolted Structural Joints of the Engineering Foundation, January, 1951, the Specification for Assembly of Structural Joints Using High-Strength Bolts merely permitted the substitution of a like number of A325 high-strength bolts for hot-driven ASTM A141 steel rivets of the same nominal diameter. It was required that all contact surfaces be free of paint. As revised in 1954, the omission was required to apply only to "joints subjected to stress reversal, impact or vibration, or to cases where stress redistribution due to joint slippage would be undesirable." This relaxation of the earlier provision recognized the fact that, in a great many cases, movement of the connected parts that brings the bolts into bearing against the sides of their holes is in no way detrimental. When the nature of the loading—whether static or cyclic—is such that fatigue-type failure or reversal of movement will not occur, the high clamping force in the bolts provides a rigid assembly in the "slipped" position, and the shear strength of the high-strength bolts, when threads are excluded from contact surface shear planes, is substantially greater than that of hot-driven rivets required to function under similar circumstances. Since allowable stresses as well as the requirements for treatment of contact surfaces appropriate to these service conditions are different, the present specification recognizes two kinds of shear connections, designated as *friction-type* and *clamping-type*, respectively.

Just how much stronger the high-strength bolts are in resisting actual shearing forces and what effect the higher stresses in the bolts have upon the strength of the connected parts have been the subjects of extensive study in the bolt sizes generally used in construction sponsored by the Research Council since 1954. The results of these studies, together with improvements in installation practices which are the outgrowth of extensive experience in the use of high-strength bolts, formed the background for the 1960 edition. The 1962 revision reflected the results of additional research which had shown that washers may be omitted from A325 bolt assemblies. This revision incorporates the results of research conducted since that time, especially on A490 bolts.

The increasing use of high-strength steels has created the need for bolts substantially stronger than A325, in order to resist, with well-proportioned joints, the much greater forces that they support. To meet this need, a new ASTM standard, A490, has been developed.

When provisions for the use of these bolts were included in the Specification in 1964 it was required that they be tightened to their specified proof load, as was required for the installation of A325 bolts. However, the ratio of proof load to specified minimum tensile strength is approximately 0.7 for A325 bolts, whereas it is 0.8 for A490 bolts. Calibration studies have shown that high strength bolts have ultimate load capacities in torqued tension which vary from about 80% to 90% of the direct tensile strength.<sup>1</sup> Hence, if minimum strength bolts were supplied and they experienced the maximum reduction due to torquing, there is a possibility that these bolts could not be tightened to proof load by any method of installation. Also, statistical studies have shown that, tightening to the 0.8 ratio under calibrated wrench control may result in some "twist-off" bolt failures during installation or in some cases a slight amount of undertightening.<sup>2</sup> Therefore the required installed tension for A490 bolts has been reduced to 70 percent of the specified minimum tensile strength. For consistency, but with only minor change, the initial tension required for A325 bolts has also been set at 0.7 of their specified minimum tensile strength and at the same time the values in Table 3 have been rounded off to the nearest kip.

Because greater clamping force is used with A490 bolts it is required that hardened washers, conforming to the requirements of ASTM Specification A325, be installed under both the nut and bolt head when A490 bolts are used in steels having a yield point less than 40 ksi and under the turned element when they are used in higher-strength steels.

## C2 Bolts, Nuts and Washers

In this edition of the specification a single type of fastener, available in two strength grades (A325 and A490) is described as a principal type but conditions for acceptance of other types of fasteners are provided.

<sup>1</sup> "Calibration of Alloy Steel Bolts," by Christopher, R. J., Kulak, G. L., and Fisher, J. W., *Journal of the Structural Division*, ASCE, Vol. 92, No. ST2, Proc. Paper 4768, April, 1966, pp. 19-40.

<sup>2</sup> "The Specification of Minimum Preloads for Structural Bolts," by Gill, P. J., Memorandum, G. K. N. Group Research Laboratory, England, 1966 (Unpublished Report).

Heavy hex structural bolts manufactured to ASTM Specification A325, the dimensions for which are shown in Table 5 and Figure 1, are identified on the top of the head by three radial lines, the legend "A325", and the manufacturer's symbol. Bolts manufactured to ASTM Specification A490 are marked with the legend "A490" and the manufacturer's symbol. Heavy hex nuts manufactured to ASTM Specification A325 are identified on at least one face by three circumferential marks, or by the number "2", "2H", "D" or "DH" and the manufacturer's mark. Heavy hex nuts for use on A490 bolts are identified with the legend "2H" or "DH" and the manufacturer's mark. A490 bolts may be used in lieu of A325 bolts if expedient on the basis of availability, in which case their initial tension need not exceed that required for A325 bolts.

Heavy hex structural bolts have shorter thread lengths than other standard bolts. By making the body length of the bolt the control dimension it has been possible to exclude the thread from all shear planes, except in the case of thin outside parts adjacent to the nut. Depending on the amount of

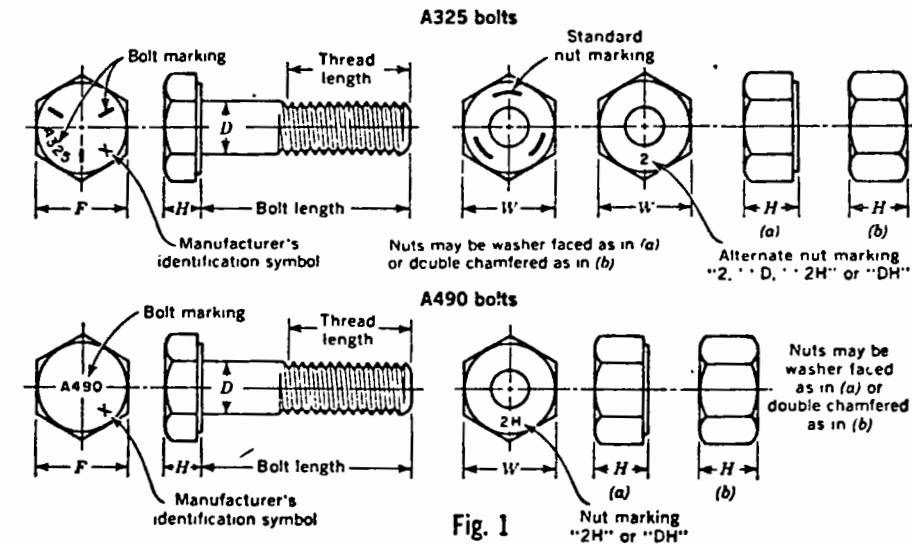


Table 5

Nominal bolt size, D	Bolt Dimensions, in Inches			Nut Dimensions, in Inches	
	Heavy Hex Structural Bolts			Heavy Hex Nuts	
	Width across flats F	Height, H	Thread length	Width across flats W	Height, H
1/2	7/8	5/16	1	7/8	31/64
5/8	1 1/16	23/64	1 1/4	1 1/16	39/64
3/4	1 1/4	15/32	1 3/8	1 1/4	47/64
7/8	1 7/16	39/64	1 1/2	1 7/16	51/64
1	1 3/8	39/64	1 3/4	1 3/8	61/64
1 1/8	1 13/16	1 1/16	2	1 13/16	1 1/64
1 1/4	2	2 1/32	2	2	1 7/32
1 3/8	2 3/16	2 1/32	2 1/4	1 1/2	1 11/32
1 1/2	2 3/8	1 1/16	2 1/4	1 3/4	1 11/32



Bolt length added to adjust for incremental stock lengths, the full thread may extend into the grip as much as  $\frac{3}{8}$  inch for  $\frac{1}{2}$  inch,  $\frac{5}{8}$  inch,  $\frac{3}{4}$  inch,  $\frac{7}{8}$  inch,  $\frac{1}{4}$  inch and  $1\frac{1}{2}$  inch diameter bolts and as much as  $\frac{1}{2}$  inch for 1 inch,  $1\frac{1}{8}$  inch, and  $1\frac{3}{8}$  inch diameter bolts. Inclusion of some of the thread run-out into the plane of shear is permissible. At the other extreme, care should be taken to provide sufficient thread for nut tightening to keep the nut from jamming into the thread run-out. When the thickness of an outside part adjacent to the nut is less than these values it may be necessary to call for the next increment of bolt length together with a sufficient number of flat circular washers to insure full seating of the nut. Then the higher working value in shear permitted in bearing-type joints can still be the basis for determining the number of bolts in the connection.

In order to determine the required bolt length, the value shown in Table 6 should be added to the grip (that is, the total thickness of all connected material, exclusive of washers).

Table 6

Bolt Size, in Inches	To Determine Required Bolt Length Add to Grip, in Inches
$\frac{1}{2}$	$\frac{11}{16}$
$\frac{3}{8}$	$\frac{7}{8}$
$\frac{3}{4}$	1
$\frac{7}{8}$	$1\frac{1}{8}$
1	$1\frac{1}{4}$
$1\frac{1}{8}$	$1\frac{1}{2}$
$1\frac{1}{4}$	$1\frac{3}{8}$
$1\frac{3}{8}$	$1\frac{3}{4}$
$1\frac{1}{2}$	$1\frac{7}{8}$

The preceding values are generalized, with due allowance for manufacturing tolerances, to provide for the use of a heavy hex nut, with a "full nut" when installed. For each hardened flat washer that is used, add  $\frac{5}{32}$  inch, and for each beveled washer add  $\frac{5}{16}$  inch. The length determined by the use of Table 6 should be adjusted to the next longer  $\frac{1}{4}$  inch.

The circular washer dimensions shown in Table 1 are somewhat reduced from those tabulated in 1962 and earlier editions. They have been developed on the principle that the primary function of the washer is to provide a non-galling surface under the part turned in tightening. As discussed more fully under Section C5 of this Commentary, tests have shown that washers play only a minor role in distributing the pressure due to bolt tension. Hence, no consideration is given to this function and the minimum thickness for commonly used washers has been reduced by one or two gages. The maximum thickness is now alike for all washers up to and including the  $1\frac{1}{2}$  inch size, so that these washers can be produced from a single stock of material.

### C3 Bolted Parts

Joints which must transmit the forces in adjacent parts by means of shear are divided into two categories in the current specification; *friction-type* and *bearing-type*. High initial bolt tension provides worthwhile advantages, therefore the same initial tensioning is recommended for *bearing-type* connections

as for the *friction-type*. Among these benefits are overall joint rigidity, better stress pattern and security against nut loosening.

### C4 Allowable Working Stresses

While the provisions contained in the Council specification to a limited extent affect general design considerations, it is not the intent to present a complete design specification. Only those features influenced by the properties of high-strength bolts, as distinct from other types of fasteners, are included. Working stresses applicable to bridges and to buildings (two values differing by about 10%) reflect the historic difference in basic stress between the AREA and AASHTO Specifications governing bridge design and the AISC Specification governing the design of buildings and similar structures. Except as modified by the provisions of the Council's specification, it is assumed that all of the applicable provisions of the standard specifications under which the structure is designed will be observed.

#### Tension

The working stresses recommended are intended to apply to the calculated bolt load plus any tension resulting from prying action produced by deformation of the connected parts. When subjected in tension to the recommended working value (approximately equal to two-thirds of the initial tightening force) high-strength bolts will experience little if any actual change in stress. Since the tensile strength of the A490 bolt is approximately one-third greater than the corresponding average value for the A325 bolt, this ratio has been used to set the allowable tensile stress for the A490 bolt.

Tests<sup>3</sup> on properly tightened A325 bolts have demonstrated that their fatigue strength is not adversely affected by repeated applied tension of this amount.

Similar studies on A490 bolts are under way. Pending completion of these studies the allowable working stress in tension for A490 bolts, given in Section 4(a), is intended for static loading only and no recommendation covering cyclic applied loading in tension is made.

#### Shear: Friction-Type Connections

No change has been made in the recommended working value for A325 bolts used in *friction-type* joints. They are, as heretofore, given the "shear" value recommended in the applicable design specification for hot-driven ASTM A141 steel rivets of the same nominal diameter. The one-third increase in required tightening tension mentioned under *Tension* is the justification for the one-third increase in working stress for A490 bolts used in *friction-type* connections. Resistance to slip is dependent upon the amount of bolt clamping force and the nature of the contact surfaces in a given connection, and is independent of the working stress for which the connected parts are proportioned.

Connections having contact surfaces of unrusted mill scale offer the least resistance to slip of any unpainted joints; rusted surfaces which have been well cleaned may provide up to two times as much resistance. The recommended "shear" value using A325 bolts to connect parts having a specified

<sup>3</sup> "Research on Bolted Connections," by William H. Munse, *Transactions, ASCE*, Vol. 121, 1956, pp. 1255-1266.

eld point of about 33 ksi, based on numerous tests<sup>4,5,6,7</sup> correlates with a  $\mu$  coefficient of 0.35. Similar observations have been made in tests of joints of higher-strength steels.<sup>8</sup> While lower coefficients have been observed in some laboratory tests of joints having contact surfaces of tight unruled mill scale, or surfaces made smooth by grinding, a slip factor of 0.35 is more representative of values likely to be encountered in actual construction.

Applying this value to the recommended minimum bolt tension, the factor of safety against slip can be computed as

$$N = \frac{(0.35) (\text{minimum bolt tension})}{(\text{allowable shear stress}) (\text{nominal bolt area})} \quad (1)$$

For  $\frac{7}{8}$  inch and 1 inch A325 bolts,  $N$  equals 1.68 for bridges designed in accordance with the AASHTO and AREA Specifications, and 1.52 for structures designed in accordance with the AISC Specification. These factors of safety against slip compare with design factors of safety against yield of the connected parts of 1.83 and 1.67, respectively. For other sizes of A325 bolts, the values of  $N$  are within 10% of those for  $\frac{7}{8}$  inch and 1 inch bolts. For A490 bolts, the  $N$  values are approximately the same as for A325 bolts.

Under repeated loading the factor of safety against slip indicates the margin against the condition where a reduced fatigue strength may develop. Under static load conditions it may represent the margin against a one-time displacement movement, as under lateral shock or maximum wind loading, which is seldom likely to be reversed. A factor of safety against slip, lower than that implicit in the design stress used in proportioning the connected parts, is acceptable except where there must not be movement under such overloads as may occur within the allowable design stress factor of safety.

When the allowable "shear" value is increased one-third for wind the value of  $N$  in the above equation approaches unity. If the satisfactory performance of the structure depends upon joints which must not move, the designer should so proportion these joints as to satisfy himself that the margin against slip is adequate.

Connections of the type shown in Figure 2(a) in which some of the bolts (A) lose a part of their clamping force due to applied tension, suffer no overall loss of frictional shear resistance. The bolt tension produced by the moment is coupled with a compensating compressive force (C) on the other side of the axis of bending. In a connection of the type shown in Figure 2(b), however, all of the fasteners (B) receive applied tension which reduces the initial compression at the contact surface. If bolts are used, and slip under load cannot

be tolerated, the working value of the bolts in shear should be reduced in proportion to the ratio of residual tension to initial tension.

Because bolts in *friction-type* connections do not depend upon bearing against the sides of their holes, those provisions of the general design specifications intended to guard against high bearing stresses, and bending of the bolt due to bearing, are waived.

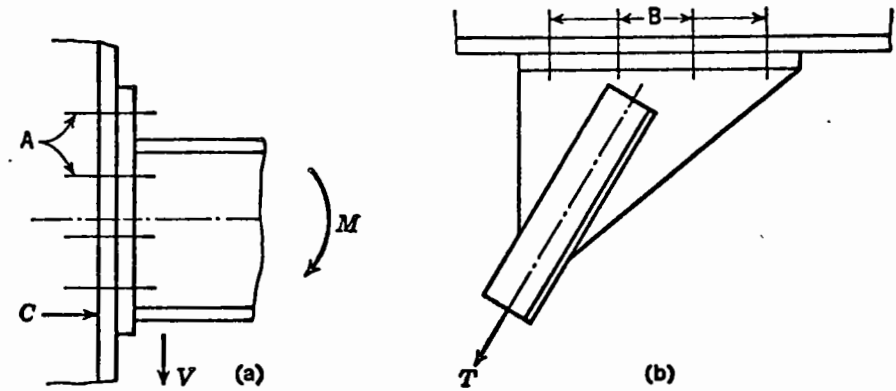


Fig. 2

### Shear: Bearing-Type Connections

In connections where the bolts may bear against the holes in the connected parts, the allowable stress of bolts is dependent upon the presence or absence of bolt threading at the plane of contact surfaces where shearing occurs. If the unthreaded shank of an A325 bolt is available to resist this shear at all planes where it occurs, tests<sup>6,7</sup> have shown that a shear stress equal to 20 ksi for bridges and 22 ksi for buildings (based on nominal fastener area) affords at least as large a factor of safety against high strength bolt shear failure as that provided in the standard design specification for rivets. On the other hand, it was found that failure occurs at 15% less load when threading is present at one of the two shear planes of an enclosed part, and at 30% less load when threads are present in both shear planes. This latter failure load could be expected also for single-shear joints with threads in the shear plane. Similar observations have been made from tests using ordinary bolts. They merely reflect the ratio of area at the root of thread to the nominal bolt area. The allowable shear stresses for A325 bolts with thread in a shear plane (13.5 ksi for bridges and 15 ksi for buildings as shown in Table 2) are developed by applying the above 30% reduction to the stresses allowed in unthreaded shanks.

The shear stresses allowed for A490 bolts in *bearing-type* connections have been similarly determined from tests<sup>9,10,11</sup> to give them at least as much factor of safety against failure as is provided for rivets made of A141 steel.

<sup>4</sup> "High-Strength Bolts in Structural Joints: A Symposium: Slip of Joints under Static Loads," by R. A. Hechtman, D. R. Young, A. G. Chin, and E. R. Savikko *Transactions, ASCE*, Vol. 120, 1955, pp. 1335-1352.

<sup>5</sup> "Effects of Fabrication Techniques," by Desi D. Vasarhelyi, Said Y. Beano, Donald B. Madison, Zung-An Lu, and Umseh C. Vasishth, *Transactions, ASCE*, Vol. 126, Part II, 1961, pp. 764-796.

<sup>6</sup> "Static Tension Tests of Compact Bolted Joints," by Robert T. Foreman and John L. Rumpf, *Transactions, ASCE*, Vol. 126, Part II, 1961, pp. 228-254.

<sup>7</sup> "Long Bolted Joints," by R. A. Bendigo, R. H. Hansen and J. L. Rumpf, *ASCE Journal*, Vol. 89, ST6, December, 1963.

<sup>8</sup> "Static Tension Tests of A440 Steel Joints Connected with A325 Bolts," by W. Fisher, J. Ramseier and L. S. Beedle, *Publications, IABSE*, Vol. 23, 1963.

<sup>9</sup> "High-Strength Bolts Subjected to Combined Tension and Shear," by E. Chesson, Jr., N. L. Faustino and W. H. Munse, *ASCE Journal*, Vol. 91, ST5, October, 1965.

<sup>10</sup> "Shear Strength of High Strength Bolts," by James J. Wallaert and John W. Fisher, *ASCE Journal*, Vol. 91, ST3, June, 1965.

<sup>11</sup> "A490 Steel Joints Connected by A490 Bolts," by John H. Sterling and John W. Fisher, *ASCE Journal*, Vol. 92, ST3, June, 1966.

For both A325 and A490 bolts, it may be noted that no special allowance is made for the condition where a bolt in double shear has unthreaded shank in one shear plane and threaded section in the other. This does not deny designers the advantage of such an analysis. It recognizes, however, that any use of the advantage requires knowledge of bolt placement that is not ordinarily available to designer or detailer, and that the fully conservative procedure is to use the lower allowable shear stress for all shear planes when the joint detail allows bolt thread in any shear plane of the joint.

### Bearing

Tests<sup>12, 13, 14</sup> have shown that bearing pressure on rivets in double or single shear, computed on the basis of an area equal to the product of the part thickness and nominal rivet diameter has no significant effect on the strength of the connected parts of A7 steel when this pressure is not more than 2.25 times the tensile stress applied to the net area of these parts. It would appear that the ratio of fastener spacing normal to the line of force, to fastener diameter, rather than unit pressure per se, is the critical factor, and that computed bearing stress is simply a convenient index of effective net section. In consequence, no increase in allowable bearing value seems warranted when high-strength bolts are substituted for rivets. In some high-strength steels it is, however, limited by the tensile strength of the steel and provision to recognize this is made in Table 2.

When there are not more than two bolts in the line of stress and the pressure from the bolt is directed toward the end of a connected part, an increase in end distance, above that required for rivets under similar circumstances, is recommended. To insure that the end fastener will not tear out of the connected part before the full tensile strength of the net section is attained it as long been required that the end distance of a connected part having substantially the same mechanical properties as the connecting rivets be not less than the nominal area of the rivet divided by the part thickness and multiplied by the number of shears applied to the part. This rule is retained for use with high-strength bolts but the end distance is increased in proportion to the ratio of bolt tensile strength to the tensile strength of the part. Above a length equal to  $1\frac{1}{2}$  times the transverse bolt spacing, failure by rupture along the net plate section, at full fastener efficiency, is assured.

### 25 Installation

Tests<sup>15</sup> have shown that a hardened washer is not needed to prevent minor bolt relaxation resulting from the high stress concentration under the bolt head or nut in connections assembled with A325 bolts. Such relaxations were less

than 5% of the initial tension; took place within hours of bolt tightening, after which further loss of tension was negligible; and were substantially the same with and without the use of washers. Tests<sup>1</sup> have also shown that any galling which may take place where nuts for A325 bolts are tightened directly against the connected parts is not detrimental to the static or fatigue strength of the joint. However, to minimize irregularity in the torque-tension ratio where bolts are tightened by the calibrated wrench method, a washer is still required under the nut or bolt head which is turned in tightening. Otherwise, the use of flat circular washers is no longer required with A325 bolts. They are required with A490 bolts in A7 and A36 steel parts, to reduce galling and brinelling of these parts. In high-strength steel they are only required to prevent galling of the rotated part.

Bolts installed by torquing can sustain additional direct tension loads without any apparent reduction in their ultimate strength. Because of this reserve strength, it is apparent that if the fastener does not fail while being installed, it will not fail thereafter, provided the loads to which it is subjected do not exceed those for which it has been designed.

Without preference, the Council endorses both the calibrated wrench and the turn-of-nut methods for bolt tightening.

Earlier editions of the Council's specifications have listed torque values described as the approximate equivalent of the minimum bolt tension specified for various size bolts. It was explained that these values were no more than observed experimental averages, and that the value to be used, both in installing bolts and in inspection procedures, should be that determined by the actual condition of the application. This point cannot be emphasized too much. The present specification requires that both torque and impact wrenches be calibrated, by means of a device capable of measuring the actual tension produced by a given wrench effort applied to a representative sample, when the tightening of bolts is controlled on the basis of calibrated wrench operation.

Hydraulic calibrating devices capable of indicating bolt tension undergo a slight deformation under large bolt heads. Hence the nut rotation corresponding to a given tension reading may be somewhat larger than it would be if the same bolt were tightened against a solid steel abutment. Stated differently, the reading of the calibrating device tends to under-estimate the tension which a given rotation of the turned element would induce in a bolt in an actual joint. This should be borne in mind when using such devices to establish a tension-rotation relationship.

Instead of suggesting one full turn of the nut from a finger-tight position,<sup>16</sup> when tightening is controlled by the turn-of-nut prescription, a somewhat smaller rotation, from a *snug-tight* condition, is now specified in Table 4.<sup>1, 17, 18, 19</sup> On an average, the bolt tension provided by either pre-

<sup>12</sup> "Bearing Ratio Effect on Static Strength of Riveted Joints," by Jonathan Jones *Transactions*, ASCE, Vol. 123, 1958, pp. 964-972.

<sup>13</sup> "The Effect of Bearing Pressure on the Static Strength of Riveted Connections," Bulletin No. 454, Univ. of Illinois, Engrg. Experiment Sta., Urbana, Ill., July 1959.

<sup>14</sup> "Effect of Bearing Pressures on Fatigue Strength of Riveted Connections," by E. Chesson, Jr., J. F. Parola, and W. H. Munse, Univ. of Illinois, Engrg. Experiment Sta. Bulletin No. 481, 1965.

<sup>15</sup> "Studies of The Behavior of High-Strength Bolts and Bolted Joints," by E. Chesson, Jr., and W. H. Munse, Univ. of Illinois Engrg. Experiment Sta. Bulletin No. 469, 1964.

<sup>16</sup> "Tightening High-Strength Bolts," by F. P. Drew, Proc. Sep. No. 786, ASCE, Vol. 81, August, 1955.

<sup>17</sup> "Installation and Tightening of High-Strength Bolts," by E. F. Ball, J. J. Higgins, *Transactions*, ASCE, Vol. 126, 1961, pp. 797-820.

<sup>18</sup> "Calibration of A325 Bolts," by J. L. Rumpf and J. W. Fisher, Journal of the Structural Division, ASCE Vol. 89, No. ST6, Proc. Paper 3731, December, 1963, pp. 215-234.

<sup>19</sup> "Calibration Tests of A490 High-Strength Bolts," by G. H. Sterling, E. W. J. Troup, E. Chesson and J. W. Fisher, *ASCE Journal*, Vol. 19, ST5, Oct. 1965.

scription is approximately the same. However, measuring the nut rotation from a snug-tight condition, which necessitates first drawing the several parts of the connection tightly together, has been found to produce more uniform bolt tension.

Tests<sup>19</sup> have shown that long A490 bolts require a somewhat greater nut rotation in order to achieve the bolt tension shown in Table 3. Although the need does not exist with A325 bolts, the  $\frac{2}{3}$  turn provision has been applied to the A325 bolts as well, in the interest of uniformity in field practice.

The percentage of bolts in a given connection which must be made snug-tight in order to compact the joint will depend upon the stiffness of the several connected parts and their initial straightness. In extreme cases it may be necessary to snug-up bolts in all of the holes not used for pinning, in order to seat the parts.

After the parts are suitably drawn together, bolts are installed in any remaining open holes, tightened to a snug-tight condition, and all nuts are then rotated by the prescribed amount, after which bolts are installed in the holes originally pinned, and tightened using the same procedure.

Tightening of the bolts in a joint should commence at the most rigidly fixed or stiffest point, and progress toward the free edges, both in the initial snugging up and in the final tightening. During tightening the bolt head or the nut should be held by a hand wrench to prevent turning.

### C6 Inspection

Bolts, nuts and washers are normally received with a light residual coating of oil. This coating is not detrimental even to friction-type connections and need not be removed.

Bolts tightened by the turn-of-nut method may have the outer face of the nut match-marked with the protruding bolt point before final tightening, thus affording the inspector visual means of noting the actual nut rotation. Such marks can be made by the wrench operator with a crayon or dab of paint, after the bolts have been brought up snug tight.

The sides of bolt heads and nuts tightened with an impact wrench will appear slightly peened and thus indicate that the wrench has been applied to the fastener.

If a torque wrench is used to inspect bolts the procedure to be followed is described in detail in Section 6(c) of the Specification.

Where no washers are used, torque readings will be relatively high and may vary considerably. For this case the use of a torque multiplier device may be necessary.

## 15. FORMULÆ FOR RIGID FRAMES

The formulæ given in this section are based on Professor Kleinogel's *Rahmenformeln* and *Mehrstielige Rahmen*, published by Wilhelm Ernst & Sohn of Berlin, to whom grateful acknowledgment is made. The formulæ are applicable to frames which are symmetrical about a central vertical axis, with the single exception of the triangular frame, and in which each member is of constant moment of inertia.

Formulæ are given for the following types of frame:

### *Single-storey Frame*

- Frame I. Hingeless rectangular portal frame.
- II. Two-hinged rectangular portal frame.
- III. Hingeless gable frame with vertical legs.
- IV. Two-hinged gable frame with vertical legs.
- V. Hingeless frame with skew corners.
- VI. Two-hinged frame with skew corners.
- VII. Two-hinged triangular frame.

### *Multi-bay Frame*

- VIII. Twin Gable Frame with hinged feet.

The loadings are so arranged that dead, snow and wind loads may be reproduced on all the frames. For example, wind suction acting normal to the sloping rafters of a building may be divided into horizontal and vertical components, for which appropriate formulæ are given, although all the signs must be reversed because the loadings shown in the tables act inwards, not outwards as in the case of suction. Crane loads, including surge, are also shown in a number of the single-storey frames.

It should be noted that, with few exceptions, the loads between node or panel points are uniformly distributed over the *whole* member. It is appreciated that it is normal practice to impose loads on frames through purlins, side rails or beams. By using the coefficients in Fig. 1, however, allowance can be made for many other symmetrically placed loads on the cross-beams of Frames I and II shown above, where the difference in effect is sufficient to warrant the corrections being made. The indeterminate B.M.s in the whole frame are calculated as though the loads were uniformly distributed over the beam being considered, and then all are adjusted by multiplying by the appropriate coefficient in Fig. 1. It may be of interest to state why these adjustments can be made. In any statically indeterminate structure the indeterminate moments vary directly with the value of the following quantity:

CONVERSION COEFFICIENTS FOR SYMMETRICAL LOADS

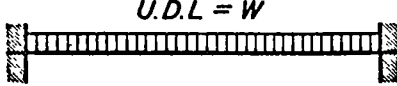
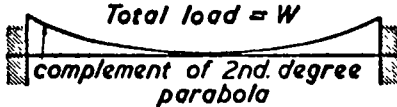
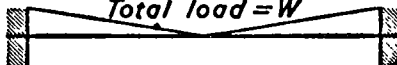
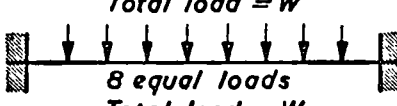
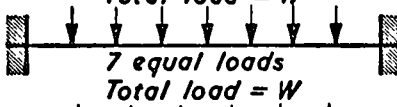
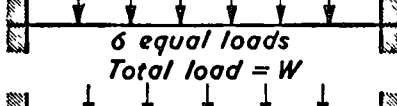
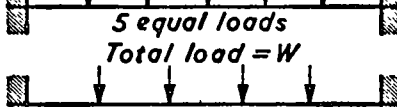
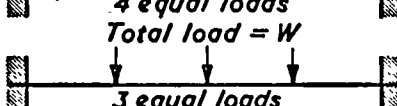
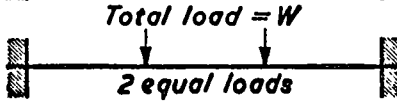
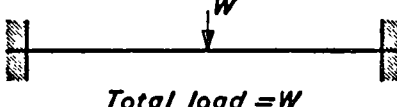

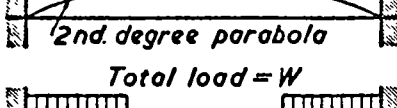
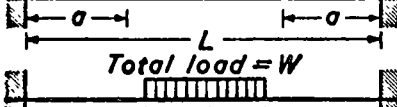
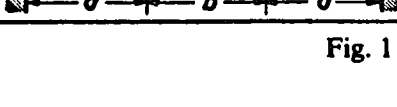

 <p>U.D.L. = W</p>	1.00
 <p>Total load = W complement of 2nd. degree parabola</p>	0.60
 <p>Total load = W</p>	0.75
 <p>Total load = W 8 equal loads</p>	1.111
 <p>Total load = W 7 equal loads</p>	1.125
 <p>Total load = W 6 equal loads</p>	1.143
 <p>Total load = W 5 equal loads</p>	1.167
 <p>Total load = W 4 equal loads</p>	1.20
 <p>Total load = W 3 equal loads</p>	1.25
 <p>Total load = W 2 equal loads</p>	1.333
 <p>Total load = W</p>	1.50
 <p>Total load = W</p>	1.25
 <p>Total load = W 2nd. degree parabola</p>	1.20
 <p>Total load = W</p>	$\frac{a(3L-2a)}{L^2}$
 <p>Total load = W</p>	$\frac{(3L^2-b^2)}{2L^2}$

Fig. 1

$$\frac{\text{Area of the free B.M. diagram}}{EI}$$

Where the loaded member is of constant cross-section,  $EI$  may be ignored.

Consider, as an example, the case of an encastred beam of constant cross-section and of length  $L$  carrying a U.D.L. of  $W$ . Then the area of the free B.M. diagram

$$= \frac{WL}{8} \times \frac{2L}{3} = \frac{WL^2}{12}.$$

If, however,  $W$  were a central point load, the area of the free B.M. diagram would be

$$\frac{WL}{4} \times \frac{L}{2} = \frac{WL^2}{8}.$$

The F.E.M.s due to the two types of loadings are  $WL/12$  and  $WL/8$  respectively, thus demonstrating that the indeterminate moments vary with the area of the free B.M. diagram and proving that the indeterminate moments are in the proportion of 1 : 1.5.

No rules can be laid down for the effect on the reactions of a change in the mode of application of the load, although sometimes they will vary with the indeterminate moments. Consider a simple rectangular portal with hinged feet. If a U.D.L. placed over the whole of the beam is replaced by a central point load of the same magnitude, then the knee moments will increase by 50 per cent with a corresponding increase in the horizontal thrusts  $H$ , while the vertical reactions  $V$  will remain the same.

Although the foregoing remarks relating to the indeterminate moments resulting from symmetrical loads apply to all rectangular portals, the rule applies for asymmetrical loads imposed upon the cross-beam of a rectangular portal frame with hinged feet. If a vertical U.D.L. on the cross-beam is replaced by any vertical load of the same magnitude, then the indeterminate moments vary with the areas of the respective free B.M. diagrams.

No doubt readers who use the tables frequently will learn short cuts, but it is not inappropriate to mention some. For example, if a U.D.L. of  $W$  over the whole of a single-bay symmetrical frame is replaced by a U.D.L. of the same magnitude of  $W$  over either the left-hand or right-hand half of the frame, the horizontal thrust at the feet is unaltered. If the frame has a pitched roof then the ridge moments will also be unaltered.

It will be noted that the formulæ for the load  $P$  on a single crane bracket are related to those for loads  $P$  on both crane brackets. Consider Fig. 2.

Then  $M_A (=M_E)$  and  $M_F (=M_D)$  in Fig. 2 (b) are equal to  $(M_A + M_E)$  and  $(M_B + M_D)$  respectively in Fig. 2 (a), while  $M_C$  in Fig. 2 (b) is double the value of  $M_C$  in Fig. 2 (a).

When frames have hinged feet, the moments resulting from surge loads  $P$  can be written down without calculation, although the frames are nominally statically indeterminate. The moments at both the loads and at the knees

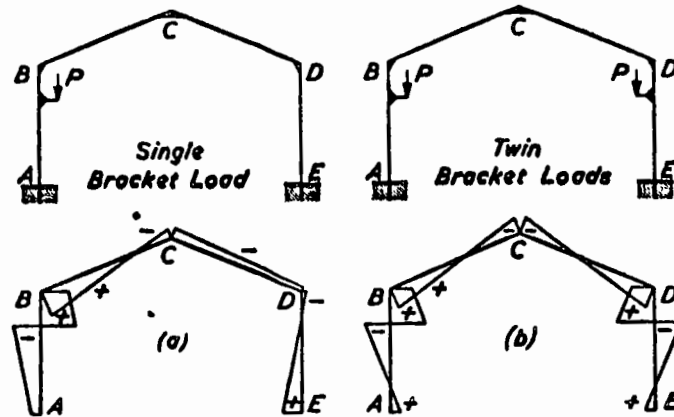


Fig. 2

are equal to  $Pa$ , where  $a$  is the height of the point of application of the loads above the feet of the frame.

The charts on pages 353 to 405 have been prepared to assist in the design of rectangular frames or frames with roof pitches of 1 in 5 or 1 in 2.5. Results for intermediate pitches may be interpolated with reasonable accuracy.

The charts on pages 406 to 419 are for two-bay portal frames with a roof pitch of 1 in 2.5 only.

#### Arrangement of Formulæ

Each set of formulæ is treated as a separate chapter. The data required for each frame, together with the constants to be used in the various formulæ, are given on the first page of each chapter. This general information is followed by the detailed formulæ for the various loading conditions, each of which is illustrated by two diagrams placed side by side, the left-hand diagram giving a loading condition and the right-hand one giving the appropriate B.M. and reaction diagram. It should be noted, however, that some B.M.s change their signs as the frames change their proportions. This will be appreciated by examining the charts.

For simple frames, i.e. for single-bay, single-storey frames, the formulæ for reactions immediately follow the formulæ for B.M.s for each load. For multi-storey or multi-bay frames the formulæ for B.M.s are given first in a group and are followed by formulæ for reactions, shears and thrusts, also in a group.

Considering the simple frames only, the kind of formula depends on the degree of indeterminacy and the shape of the frame. Auxiliary Coefficients  $X$  are introduced whenever the direct expressions become complicated or for other reasons of expediency.

No hard and fast rules can be laid down for the nomenclature and it must be noted that each set of symbols and constants applies only to the particular frame under consideration, although, of course, an attempt has been made to produce similarity in the types of symbols.



5

The formulæ for multi-storey or multi-bay frames may seem less complicated than for simple frames, but they are based on numerous constants and composite coefficients which must be accurately computed.

*Sign Conventions*

All computations must be carried out algebraically, hence every quantity must be given its correct sign. The results will then be automatically correct in sign and magnitude.

The direction of the load or applied moment shown in the left-hand diagram for each load condition is considered to be positive. If the direction of the load or moment is reversed, the signs of all the results obtained from the formulæ as printed must be reversed.

For simple frames, the moments causing tension on the inside faces of the frame are considered to be positive. Upward vertical reactions and inward horizontal reactions are also positive.

For multi-storey or multi-bay frames the same general rules apply to moments and vertical reactions. However, in the case of a two-span portal frame, for example, a problem arises with the central column. It is assumed in the formulæ that the central column belongs to the left-hand bay, so that if the column bends inwards towards this bay, the moment is positive. Similarly, in a multi-storey frame, a cross-beam is associated with the storey below it, tension on the lower face providing a positive B.M.

Horizontal reactions at the feet of multi-storey or multi-bay frames have been given signs in the diagrams but the appropriate rules are given on the first page dealing with reactions in each chapter. In general it may be said that these thrusts bear the same sign as the moment which they create in the joint at the top of the column upon which they act. It should be noted that this system is opposite to that which operates for simple frames.

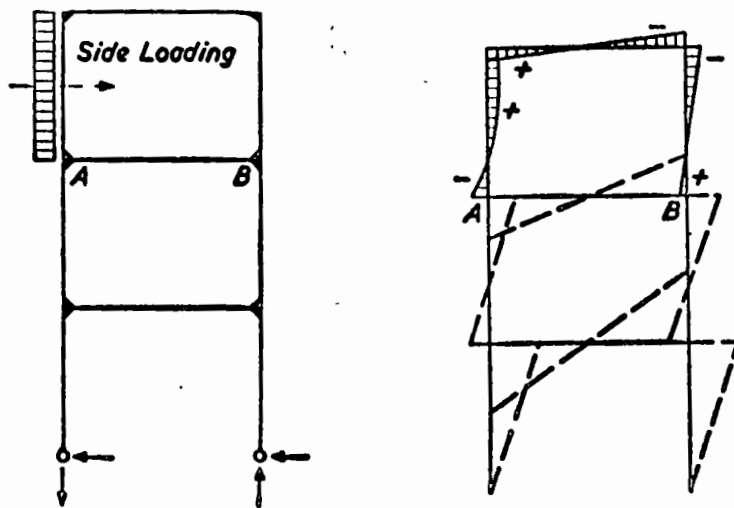


Fig. 3

*Checking Calculations for Indeterminate Frames*

Calculations for indeterminate frames may be checked by using some other method of analysis, but it is also possible to check any frame or portion of a frame, such as that above the line  $AB$  in Fig. 3 by ensuring that

1. The three fundamental statical equations, i.e.  $\Sigma H=0$ ,  $\Sigma V=0$  and  $\Sigma M=0$ , have been satisfied and, in addition, either that

2. The sum of the areas of the  $M/EI$  diagram above any line, such as  $AB$ , is zero if  $A$  and  $B$  are fully fixed; or

3. The sum of the moments, with respect to the base  $AB$ , of the areas of the  $M/EI$  diagram above the line  $AB$  is zero if  $A$  and  $B$  are partially restrained (as shown in the Figure) or are hinged.

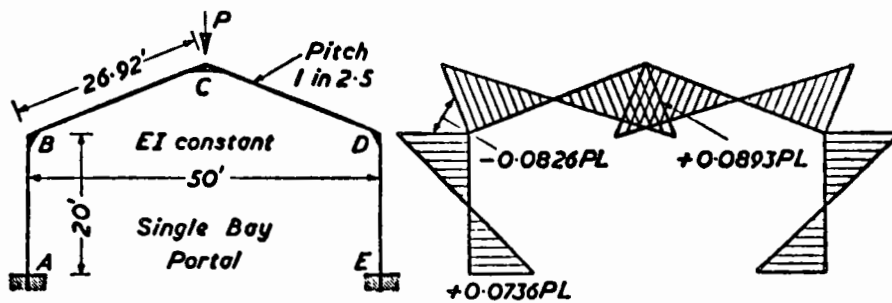


Fig. 4

It is of interest to note that the underlying principles in Rules 2 and 3 above are those used in the application of the Column Analogy method of analysis.

As an example of Rule 2, consider the frame in Fig. 4, where  $EI$  is constant.

Then the sum of the areas of the  $M/EI$  diagram, considering the legs first, is

$$\frac{2}{EI} \left[ \frac{(+0.0736 - 0.0826) \times 20}{2} \right] + \frac{2}{EI} \left[ \frac{(-0.0826 + 0.0893) \times 26.92}{2} \right]$$

$$= \frac{-0.180 + 0.180}{EI} = 0.$$

Thus demonstrating that the moments calculated are correct.

Now consider the frame in Fig. 5, an example for Rule 3.

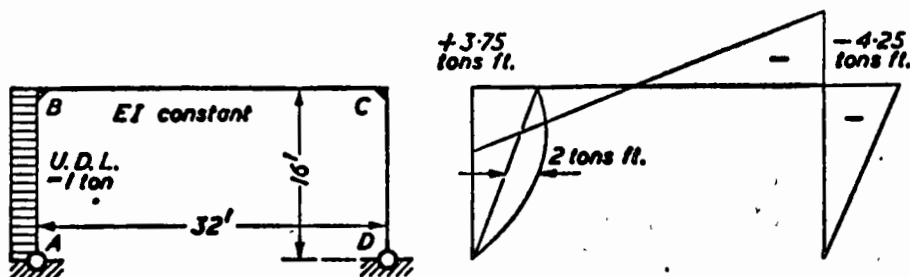


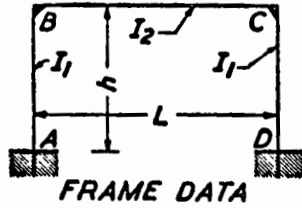
Fig. 5

Then the sum of the moments of the areas of the  $M/EI$  diagram, working from  $A$  round to  $D$ , is

$$\frac{1}{EI} \left\{ \left[ \frac{3.75 \times 16}{2} \times \frac{2 \times 16}{3} \right] + \left[ \frac{2 \times 2 \times 16}{3} \times \frac{16}{2} \right] \right. \\ \left. + \left[ \frac{(3.75 - 4.25) \times 32}{2} \times 16 \right] - \left[ \frac{4.25 \times 16}{2} \times \frac{2 \times 16}{3} \right] \right\} \\ = \frac{1}{EI} \left[ \frac{32}{3} (30 + 16 - 12 - 34) \right] = 0$$

Demonstrating again that the calculations are correct.

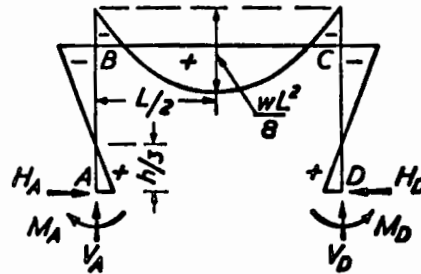
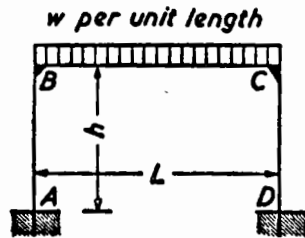
Frame I



Coefficients:

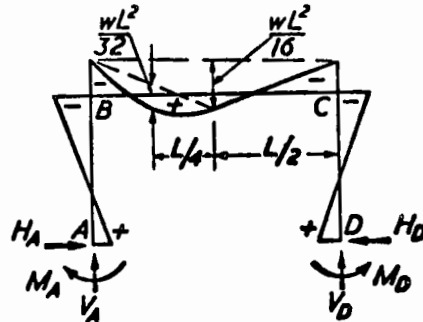
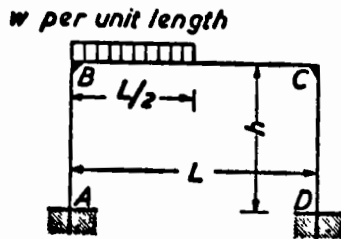
$$k = \frac{I_2}{I_1} \cdot \frac{h}{L}$$

$$N_1 = k + 2 \quad N_2 = 6k + 1$$



$$M_A = M_D = \frac{wL^2}{12N_1} \quad M_B = M_C = -\frac{wL^2}{6N_1} = -2M_A$$

$$M_{max} = \frac{wL^2}{8} + M_B \quad V_A = V_D = \frac{wL}{2} \quad H_A = H_D = \frac{3M_A}{h}$$

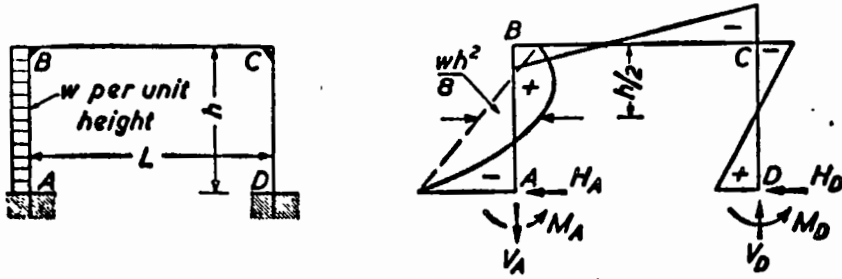


$$M_A = \frac{wL^2}{8} \left[ \frac{1}{3N_1} - \frac{1}{8N_2} \right] \quad M_B = -\frac{wL^2}{8} \left[ \frac{2}{3N_1} + \frac{1}{8N_2} \right]$$

$$M_D = \frac{wL^2}{8} \left[ \frac{1}{3N_1} + \frac{1}{8N_2} \right] \quad M_C = -\frac{wL^2}{8} \left[ \frac{2}{3N_1} - \frac{1}{8N_2} \right]$$

$$V_D = \frac{wL}{8} \left[ 1 - \frac{1}{4N_2} \right] \quad V_A = \frac{wL}{2} - V_D \quad H_A = H_D = \frac{wL^2}{8hN_1}$$

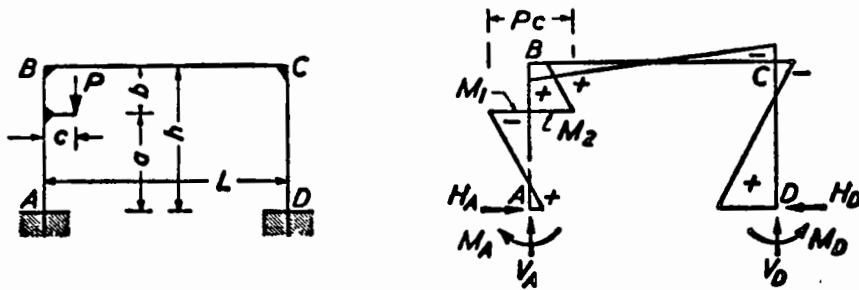
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$$M_A = \frac{wh^2}{4} \left[ -\frac{k+3}{6N_1} - \frac{4k+1}{N_2} \right] \quad M_B = \frac{wh^2}{4} \left[ -\frac{k}{6N_1} + \frac{2k}{N_2} \right]$$

$$M_D = \frac{wh^2}{4} \left[ -\frac{k+3}{6N_1} + \frac{4k+1}{N_2} \right] \quad M_C = \frac{wh^2}{4} \left[ -\frac{k}{6N_1} - \frac{2k}{N_2} \right]$$

$$H_D = \frac{wh(2k+3)}{8N_1} \quad H_A = -(wh - H_D) \quad V_A = -V_D = -\frac{wh^2k}{LN_2}$$



Constants:  $a_1 = \frac{a}{h}$      $b_1 = \frac{b}{h}$

$$X_1 = \frac{Pc}{2N_1} [1 + 2b_1k - 3b_1^2(k+1)] \quad X_2 = \frac{Pcka_1(3a_1 - 2)}{2N_1}$$

$$X_3 = \frac{3Pcka_1}{N_2}$$

$$M_A = +X_1 - \left( \frac{Pc}{2} - X_3 \right) \quad M_B = +X_2 + X_3$$

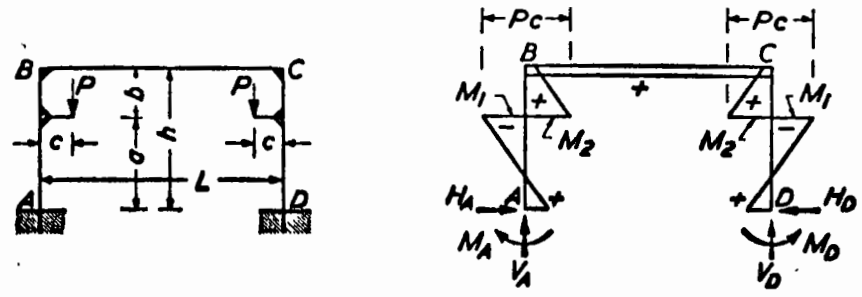
$$M_D = +X_1 + \left( \frac{Pc}{2} - X_3 \right) \quad M_C = +X_2 - X_3$$

$$H_A = H_D = \frac{Pc}{2h} + \frac{X_1 - X_2}{h} \quad V_D = \frac{2X_3}{L} \quad V_A = P - V_D$$

$$M_1 = M_A - H_A a \quad M_2 = M_B + H_D b$$

Extract: 'Kleinlogel, Rahmenformeln' 11. Auflage Berlin—Verlag von Wilhelm Ernst & Sohn.

FRAME I



Constants:  $a_1 = \frac{a}{h}$      $b_1 = \frac{b}{h}$

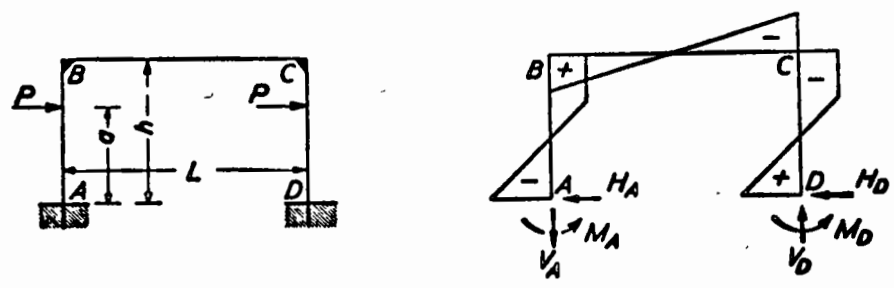
$$X_1 = \frac{Pc}{2N_1} [1 + 2b_1k - 3b_1^2(k+1)] \quad X_2 = \frac{Pcka_1(3a_1 - 2)}{2N_1}$$

$$M_A = M_D = \frac{Pc}{N_1} [1 + 2b_1k - 3b_1^2(k+1)] = 2X_1$$

$$M_B = M_C = \frac{Pcka_1(3a_1 - 2)}{N_1} = 2X_2$$

$$V_A = V_D = P \quad H_A = H_D = \frac{Pc + M_A - M_B}{h}$$

$$M_1 = M_A - H_A a \quad M_2 = M_B + H_D b$$



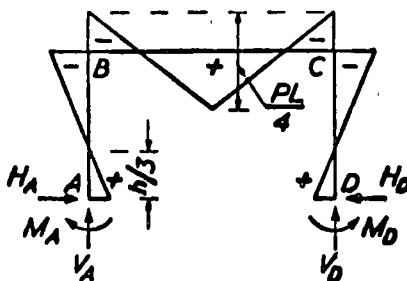
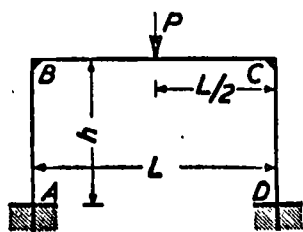
Constants:  $a_1 = \frac{a}{h}$      $X_1 = \frac{3Paa_1k}{N_2}$

$$M_A = -Pa + X_1 \quad M_B = X_1$$

$$M_D = +Pa - X_1 \quad M_C = -X_1$$

$$V_A = -V_D = -\frac{2X_1}{L} \quad H_A = -H_D = -P$$

Extract: 'Kleinlogel, Rahmenformeln' 11. Auflage Berlin—Verlag von Wilhelm Ernst & Sohn.

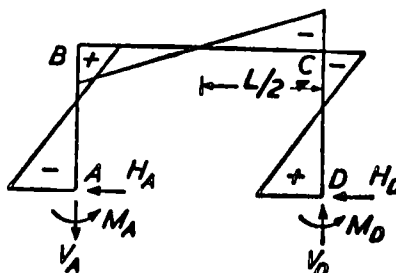
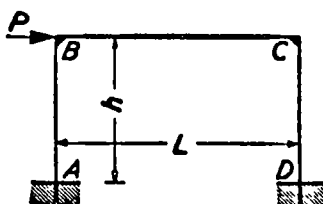


$$M_A = M_D = +\frac{PL}{8N_1}$$

$$M_B = M_C = -2M_A$$

$$V_A = V_D = \frac{P}{2}$$

$$H_A = H_D = \frac{3M_A}{h}$$



$$M_A = -\frac{Ph}{2} \cdot \frac{3k+1}{N_2}$$

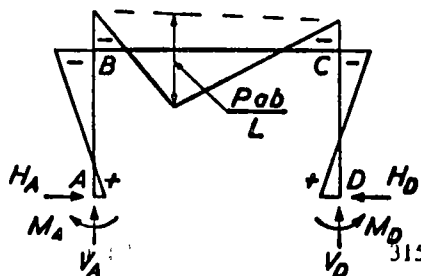
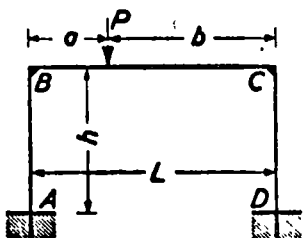
$$M_B = +\frac{Ph}{2} \cdot \frac{3k}{N_2}$$

$$M_D = +\frac{Ph}{2} \cdot \frac{3k+1}{N_2}$$

$$M_C = -\frac{Ph}{2} \cdot \frac{3k}{N_2}$$

$$H_A = -H_D = -\frac{P}{2}$$

$$V_A = -V_D = -\frac{2M_B}{L}$$



Constants:  $a_1 = a/L$        $b_1 = b/L$

$$M_A = +\frac{Pab}{L} \left[ \frac{1}{2N_1} - \frac{b_1 - a_1}{2N_2} \right]$$

$$M_B = -\frac{Pab}{L} \left[ \frac{1}{N_1} + \frac{b_1 - a_1}{2N_2} \right]$$

$$M_D = +\frac{Pab}{L} \left[ \frac{1}{2N_1} + \frac{b_1 - a_1}{2N_2} \right]$$

$$M_C = -\frac{Pab}{L} \left[ \frac{1}{N_1} - \frac{b_1 - a_1}{2N_2} \right]$$

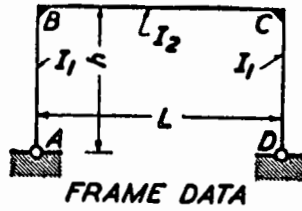
$$V_A = Pb_1 \left[ 1 + \frac{a_1(b_1 - a_1)}{N_2} \right]$$

$$V_D = P - V_A$$

$$H_A = H_D = \frac{3Pab}{2LhN_1}$$

Extract: 'Kleinlogel, Rahmenformeln' 11. Auflage Berlin—Verlag von Wilhelm Ernst & Sohn.

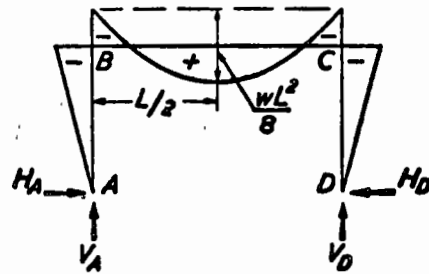
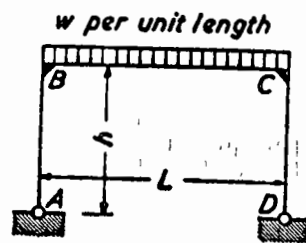
Frame II



Coefficients:

$$k = \frac{I_2}{I_1} \cdot \frac{h}{L}$$

$$N = 2k + 3$$

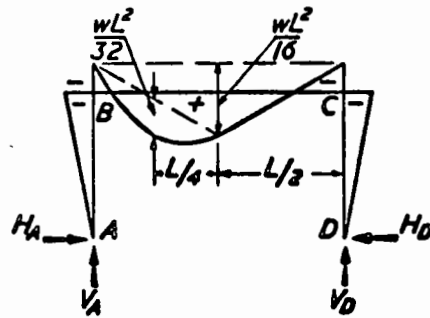
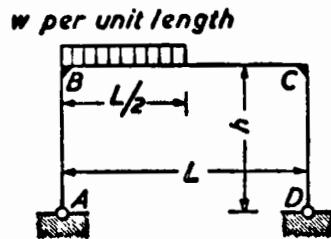


$$M_B = M_C = -\frac{wL^2}{4N}$$

$$M_{\max} = \frac{wL^2}{8} + M_B$$

$$V_A = V_D = \frac{wL}{2}$$

$$H_A = H_D = -\frac{M_B}{h}$$



$$M_B = M_C = -\frac{wL^2}{8N}$$

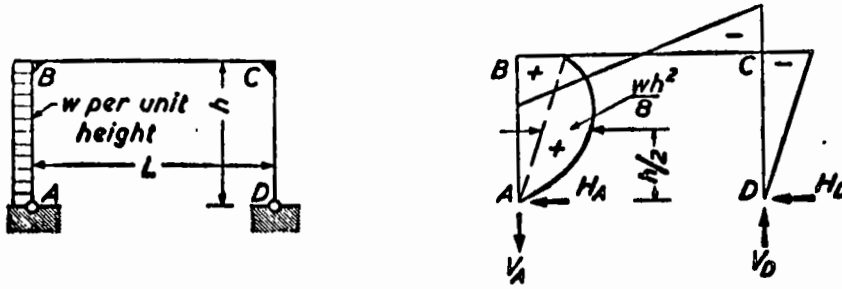
$$V_A = \frac{3wL}{8}$$

$$V_D = \frac{wL}{8}$$

$$H_A = H_D = -\frac{M_B}{h}$$

Extract: 'Kleinlogel, Rahmenformeln' 11. Auflage Berlin—Verlag von Wilhelm Ernst & Sohn.

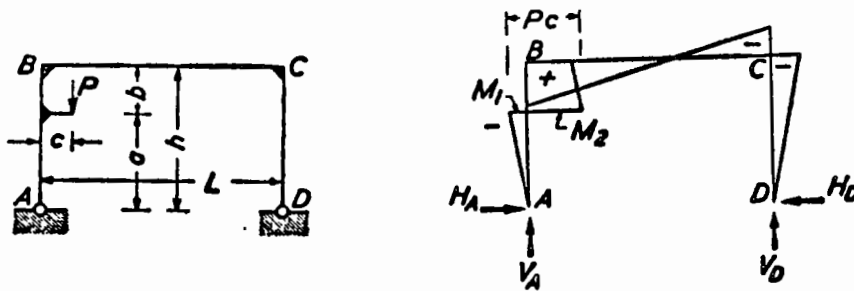




$$M_B = \frac{wh^2}{4} \left[ -\frac{k}{2N} + 1 \right] \quad H_D = -\frac{M_C}{h}$$

$$M_C = \frac{wh^2}{4} \left[ -\frac{k}{2N} - 1 \right] \quad H_A = -(wh - H_D)$$

$$V_A = -V_D = -\frac{wh^2}{2L}$$



Constant:  $a_1 = \frac{a}{h}$

$$M_B = \frac{Pc}{2} \left[ \frac{(3a_1^2 - 1)k}{N} + 1 \right] \quad H_A = H_D = -\frac{M_C}{h}$$

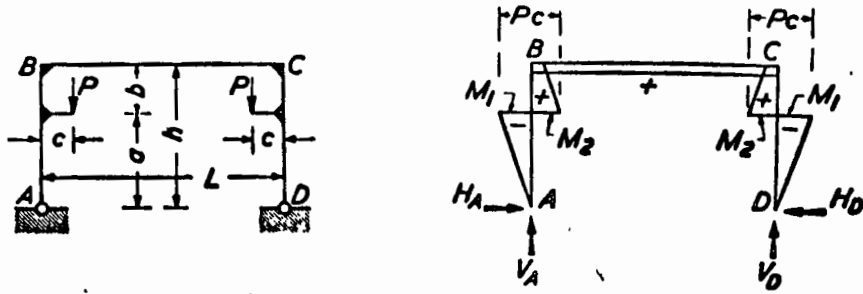
$$M_C = \frac{Pc}{2} \left[ \frac{(3a_1^2 - 1)k}{N} - 1 \right]$$

$$V_D = \frac{Pc}{L} \quad V_A = P - V_D$$

$$M_1 = -H_A a \quad M_2 = Pc - H_A a$$

Extract: 'Kleinlogel, Rahmenformeln' 11. Auflage Berlin—Verlag von Wilhelm Ernst & Sohn.

FRAME II

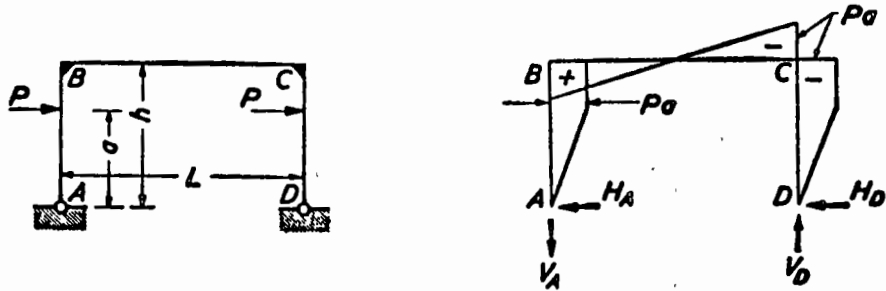


Constant:  $a_1 = \frac{a}{h}$

$$M_B = M_C = \frac{Pc(3a_1^2 - 1)k}{N}$$

$$H_A = H_D = \frac{Pc - M_B}{h} \quad V_A = V_D = P$$

$$M_1 = -H_A a \quad M_2 = Pc - H_A a$$

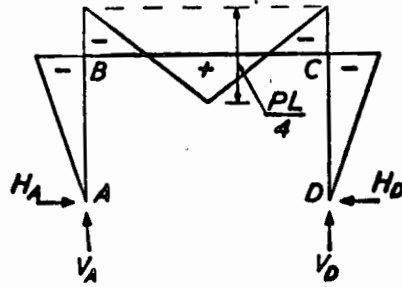
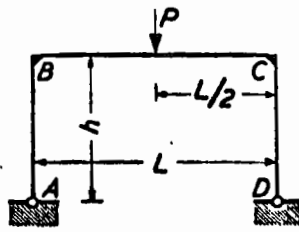


$$M_B = -M_C = Pa \quad H_A = H_D = P$$

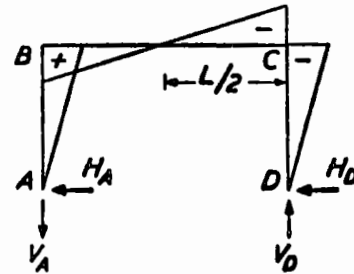
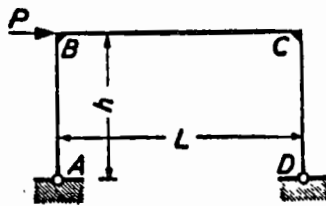
$$V_A = -V_D = -\frac{2Pa}{L}$$

Moment at loads =  $\pm Pa$

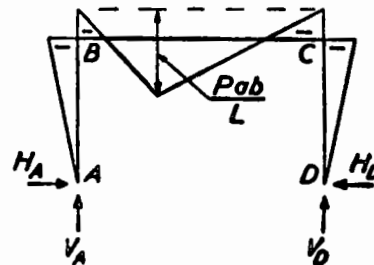
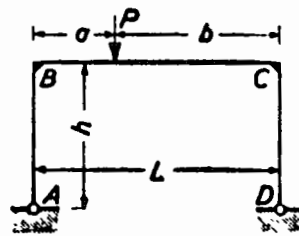
Extract: 'Kleinlogel, Rahmenformeln' 11. Auflage Berlin—Verlag von Wilhelm Ernst & Sohn.



$$M_B = M_C = -\frac{3PL}{8N} \quad V_A = V_D = \frac{P}{2} \quad H_A = H_D = -\frac{1M_B}{h}$$



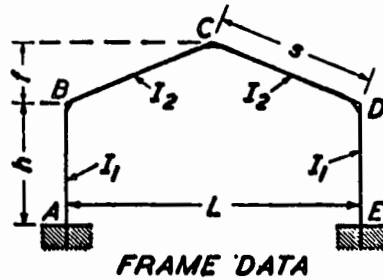
$$M_B = -M_C = +\frac{Ph}{2} \\ V_A = -V_D = -\frac{Ph}{L} \quad H_A = -H_D = -\frac{P}{2}$$



$$M_B = M_C = -\frac{Pab}{L} \cdot \frac{3}{2N} \\ V_A = \frac{Pb}{L} \quad V_D = \frac{Pa}{L} \quad H_A = H_D = -\frac{M_B}{h}$$

Extract: 'Kleinogel, Rahmenformeln' 11. Auflage Berlin—Verlag von Wilhelm Ernst & Sohn.

## Frame III



Coefficients:

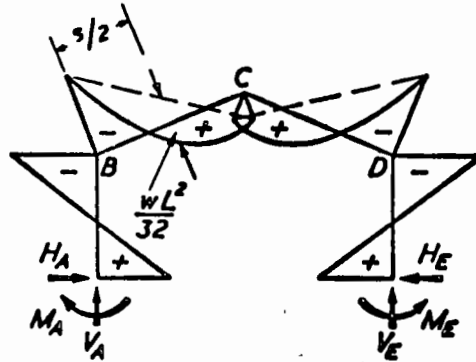
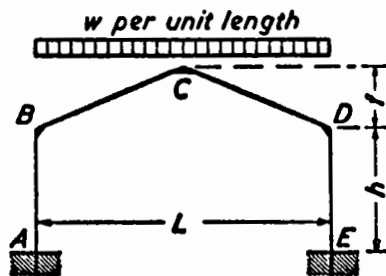
$$k = \frac{I_2}{I_1} \cdot \frac{h}{s} \quad \phi = \frac{f}{h}$$

$$m = 1 + \phi$$

$$B = 3k + 2 \quad C = 1 + 2m$$

$$K_1 = 2(k + 1 + m + m^2) \quad K_2 = 2(k + \phi^2)$$

$$R = \phi C - k \quad N_1 = K_1 K_2 - R^2 \quad N_2 = 3k + B$$



$$M_A = M_E = \frac{wL^2}{16} \cdot \frac{k(8 + 15\phi) + \phi(6 - \phi)}{N_1}$$

$$M_B = M_D = -\frac{wL^2}{16} \cdot \frac{k(16 + 15\phi) + \phi^2}{N_1}$$

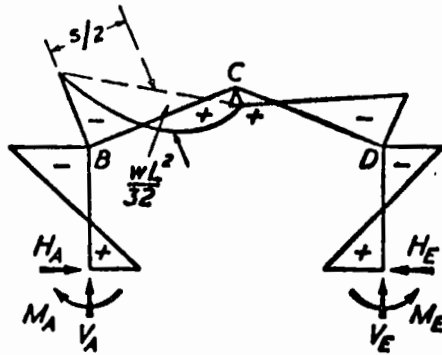
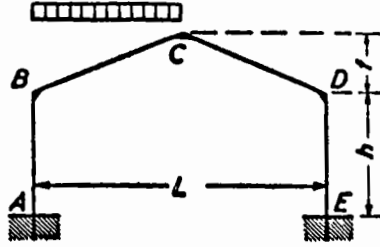
$$M_C = \frac{wL^2}{8} - \phi M_A + m M_B$$

$$V_A = V_E = \frac{wL}{2} \quad H_A = H_E = \frac{M_A - M_B}{h}$$

Extract: 'Kleinlogel, Rahmenformeln' 11. Auflage Berlin—Verlag von Wilhelm Ernst & Sohn.

FORMULÆ FOR RIGID FRAMES

w per unit length



$$\text{Constants: } *X_1 = \frac{wL^2}{32} \cdot \frac{k(8+15\phi) + \phi(6-\phi)}{N_1}$$

$$*X_2 = \frac{wL^2}{32} \cdot \frac{k(16+15\phi) + \phi^2}{N_1} \quad X_3 = \frac{wL^2}{32N_2}$$

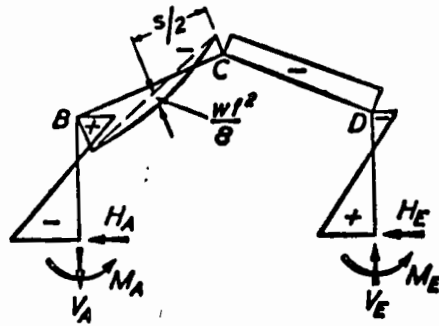
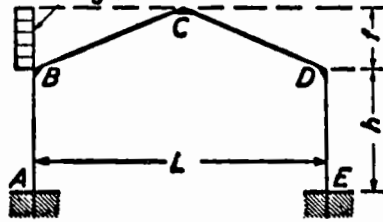
$$M_A = +X_1 - X_3 \quad M_B = -X_2 - X_3 \quad M_C = +X_1 + X_3 \quad M_D = -X_2 + X_3$$

$$*M_C = \frac{wL^2}{16} - \phi X_1 - mX_2$$

$$V_E = \frac{wL}{8} - \frac{2X_3}{L} \quad V_A = \frac{wL}{2} - V_E \quad H_A = H_E = \frac{X_1 + X_2}{h}$$

\* Note that  $X_1$ ,  $-X_1$  and  $M_C$  are respectively half the values of  $M_A (=M_E)$ ,  $M_B (=M_D)$  and  $M_C$  from the previous set of formulæ where the whole span was loaded.

w per unit height



$$\text{Constants: } X_1 = \frac{wf^2}{8} \cdot \frac{k(9\phi+4) + \phi(6+\phi)}{N_1}$$

$$X_2 = \frac{wf^2}{8} \cdot \frac{k(8+9\phi) - \phi^2}{N_1} \quad X_3 = \frac{wf^2}{8} \cdot \frac{4B+\phi}{N_2}$$

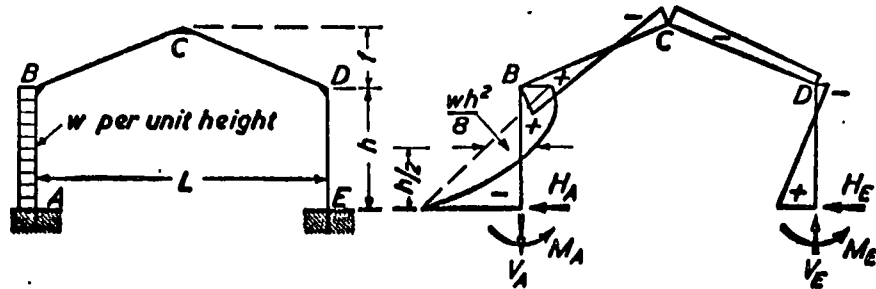
$$M_A = -X_1 - X_3 \quad M_B = +X_2 + \left(\frac{wf^2}{2} - X_3\right)$$

$$M_E = -X_1 + X_3 \quad M_D = +X_2 - \left(\frac{wf^2}{2} - X_3\right)$$

$$M_C = -\frac{wf^2}{4} + \phi X_1 + mX_2$$

$$V_A = -V_E = -\frac{wf^2(2+\phi)}{2L} + \frac{2X_3}{L} \quad H_E = \frac{wf^2}{2} - \frac{X_1 + X_2}{h} \quad H_A = -\left(\frac{wf^2}{2} - H_E\right)$$

FRAME III



$$\text{Constants: } X_1 = \frac{wh^2}{8} \cdot \frac{k(k+6) + k\phi(15+16\phi) + 6\phi^2}{N_1}$$

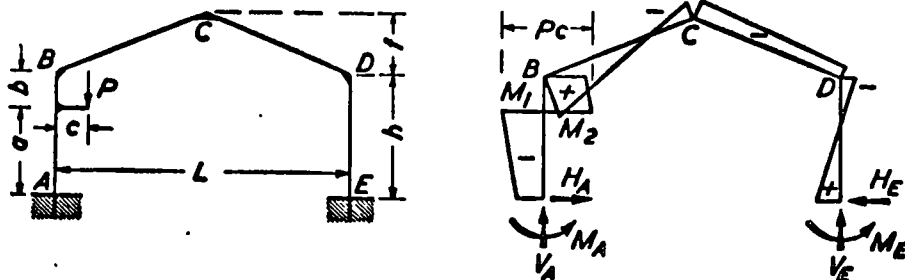
$$X_2 = \frac{wh^2k(9\phi + 8\phi^2 - k)}{8N_1} \quad X_3 = \frac{wh^2(2k+1)}{2N_2}$$

$$M_A = -X_1 - X_3 \quad M_B = +X_2 + \left(\frac{wh^2}{4} - X_3\right)$$

$$M_E = -X_1 + X_3 \quad M_D = +X_2 - \left(\frac{wh^2}{4} - X_3\right)$$

$$M_C = -\frac{whf}{4} + \phi X_1 + mX_2$$

$$V_A = -V_E = -\frac{wh^2}{2L} + \frac{2X_3}{L} \quad H_E = \frac{wh}{4} - \frac{X_1 + X_2}{h} \quad H_A = -(wh - H_E)$$



$$\text{Constants: } a_1 = \frac{a}{h} \quad b_1 = \frac{b}{h}$$

$$Y_1 = Pc[2\phi^2 - (1 - 3b_1^2)k] \quad Y_2 = Pc[\phi C + (3a_1^2 - 1)k]$$

$$X_1 = \frac{Y_1 K_1 - Y_2 R}{2N_1} \quad X_2 = \frac{Y_2 K_2 - Y_1 R}{2N_1} \quad X_3 = \frac{Pc}{2} \cdot \frac{B - 3(a_1 - b_1)k}{N_2}$$

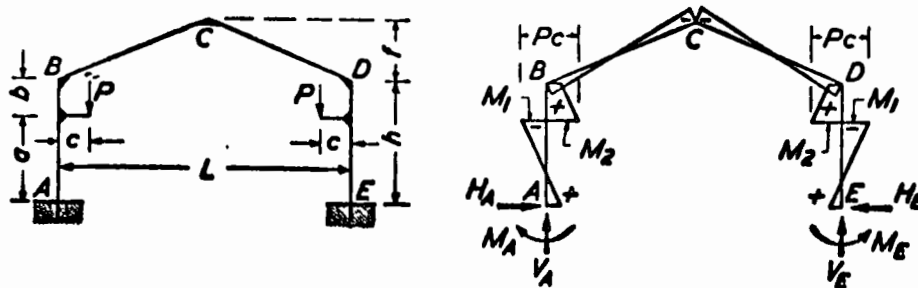
$$M_A = -X_1 - X_3 \quad M_B = +X_2 + \left(\frac{Pc}{2} - X_3\right)$$

$$M_E = -X_1 + X_3 \quad M_D = +X_2 - \left(\frac{Pc}{2} - X_3\right) \quad M_C = -\frac{\phi Pc}{2} + \phi X_1 + mX_2$$

$$M_1 = M_A - H_A a \quad M_2 = M_B + H_E b$$

$$V_E = \frac{Pc - 2X_3}{L} \quad V_A = P - V_E \quad H_A = H_E = \frac{Pc}{2h} - \frac{X_1 + X_2}{h}$$

Extract: 'Kleinogel, Rahmenformeln' 11. Auflage Berlin—Verlag von Wilhelm Ernst & Sohn.



Constants:  $a_1 = \frac{a}{h}$      $b_1 = \frac{b}{h}$

$$Y_1 = Pc[2\phi^2 - (1 - 3b_1^2)k]$$

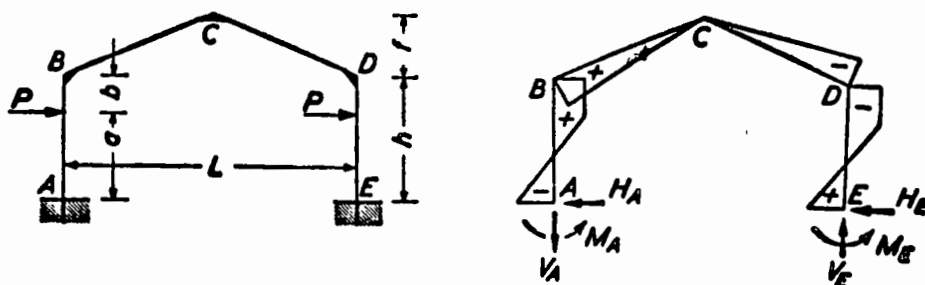
$$Y_2 = Pc[\phi C + (3a_1^2 - 1)k]$$

$$M_A = M_E = \frac{Y_2 R - Y_1 K_1}{N_1} \quad M_B = M_D = \frac{Y_2 K_2 - Y_1 R}{N_1}$$

$$M_C = -\phi(Pc + M_A) + mM_B$$

$$V_A = V_D = P \quad H_A = H_E = \frac{Pc + M_A - M_B}{h}$$

$$M_1 = M_A - H_A a \quad M_2 = M_B + H_E b$$



Constant:  $X_1 = \frac{Pa(B + 3b_1 k)}{N_2}$

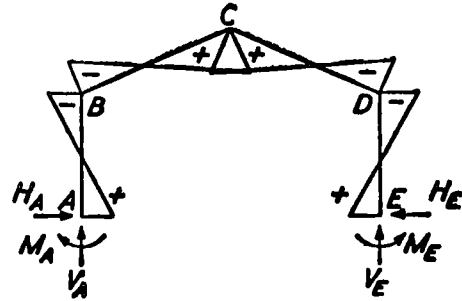
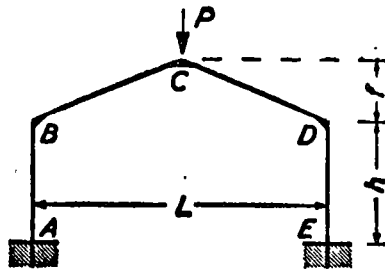
$$M_A = -M_E = -X_1 \quad M_B = -M_D = Pa - X_1 \quad M_C = 0$$

$$V_A = -V_E = -2\left[\frac{Pc}{L} \frac{X_1}{L}\right] \quad H_A = -H_E = -P$$

Extract: 'Kleinlogel, Rahmenformeln' 11. Auflage Berlin—Verlag von Wilhelm Ernst & Sohn.

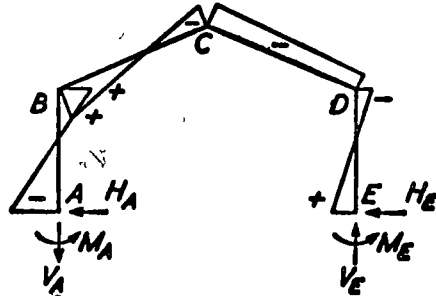
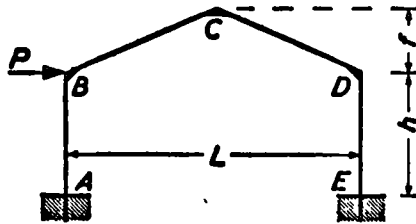
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FRAME III



$$M_A = M_E = \frac{3PL(k + 2k\phi + \phi)}{4N_1} \quad M_B = M_D = -\frac{3PLkm}{2N_1}$$

$$M_C = \frac{PL}{4} - \phi M_A + mM_B \quad V_A = V_E = P/2 \quad H_A = H_E = \frac{M_A - M_B}{h}$$

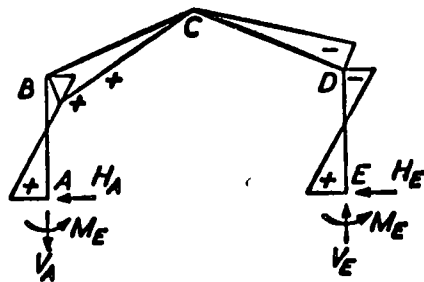
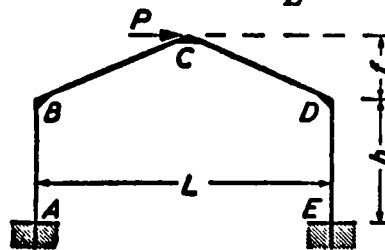


Constants:  $X_1 = \frac{3Pf(k + 2\phi k + \phi)}{2N_1}$      $X_2 = \frac{3Pfmk}{N_1}$      $X_3 = \frac{PhB}{2N_2}$

$$M_A = -X_1 - X_3 \quad M_B = +X_2 + \left(\frac{Ph}{2} - X_3\right)$$

$$M_E = -X_1 + X_3 \quad M_D = +X_2 - \left(\frac{Ph}{2} - X_3\right) \quad M_C = -\frac{Pf}{2} + \phi X_1 + mX_2$$

$$V_A = -V_E = -\frac{Ph - 2X_3}{L} \quad H_B = \frac{P}{2} - \frac{X_1 + X_2}{h} \quad H_A = -(P - H_E)$$



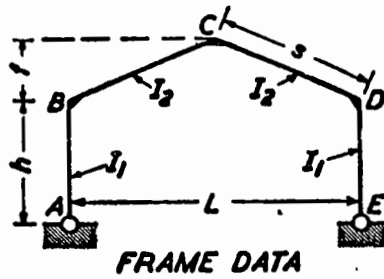
$$M_A = -M = -\frac{PhB}{2N_2} \quad M_B = -M_D = +\frac{3Phk}{2N_2} \quad M_C = 0$$

$$V_A = -V_E = -\frac{P(h+f) + 2M_A}{L} \quad H_A = -H_E = -\frac{P}{2}$$

Extract: 'Kleinlogel, Rahmenformeln' 11. Auflage Berlin—Verlag von Wilhelm Ernst & Sohn.



Frame IV



FRAME DATA

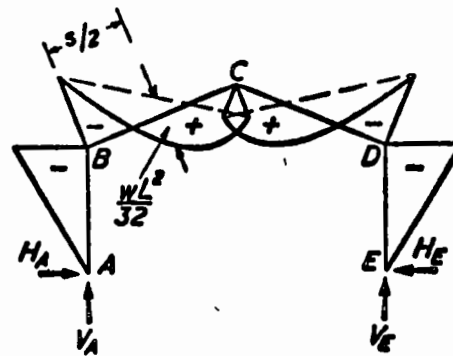
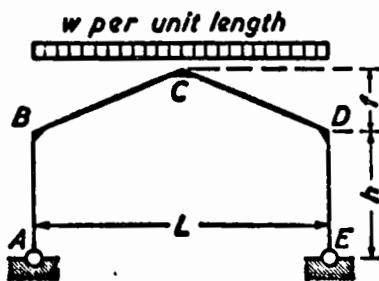
Coefficients:

$$k = \frac{I_2}{I_1} \cdot \frac{h}{s}$$

$$\phi = \frac{f}{h}$$

$$m = 1 + \phi$$

$$B = 2(k+1) + m \quad C = 1 + 2m \quad N = B + mC$$

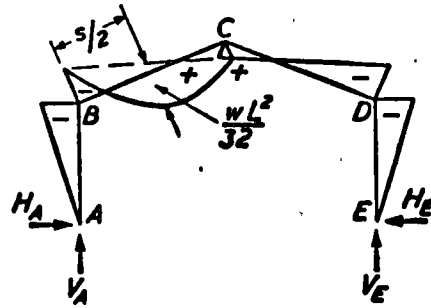
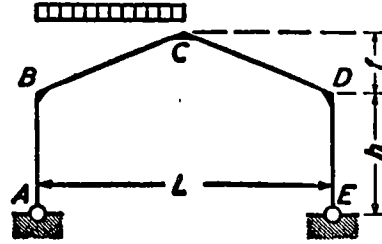


$$M_B = M_D = -\frac{wL^2(3+5m)}{16N} \quad M_C = \frac{wL^2}{8} + mM_B$$

$$H_A = H_E = -\frac{M_B}{h} \quad V_A = V_E = \frac{wL}{2}$$

Extract: 'Kleinlogel, Rahmenformeln' 11. Auflage Berlin—Verlag von Wilhelm Ernst & Sohn.

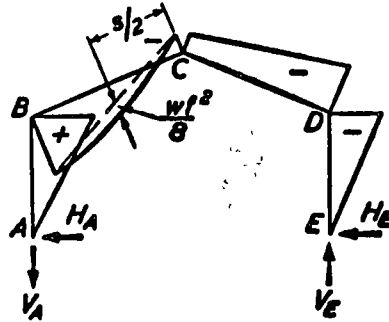
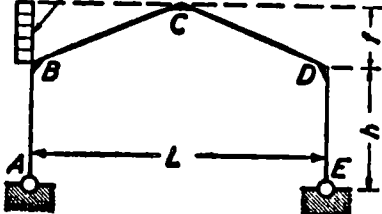
w per unit length



$$M_B = M_D = -\frac{wL^2(3+5m)}{32N} \quad M_C = \frac{wL^2}{16} + mM_B$$

$$H_A = H_E = -\frac{M_B}{h} \quad V_A = \frac{3wL}{8} \quad V_E = \frac{wL}{8}$$

w per unit height



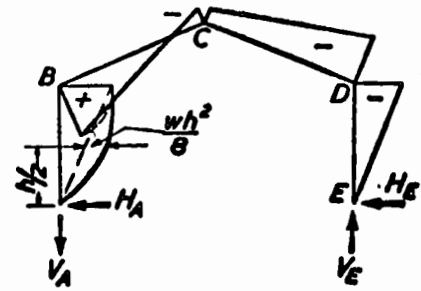
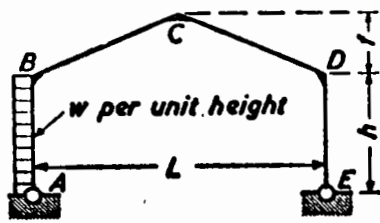
$$\text{Constant: } X = \frac{wf^2(C+m)}{8N}$$

$$M_B = +X + \frac{wfh}{2} \quad M_C = -\frac{wf^2}{4} + mX$$

$$M_D = +X - \frac{wfh}{2} \quad V_A = -V_E = -\frac{wfh(1+m)}{2L}$$

$$H_A = -\frac{X}{h} - \frac{wf}{2} \quad H_E = -\frac{X}{h} + \frac{wf}{2}$$

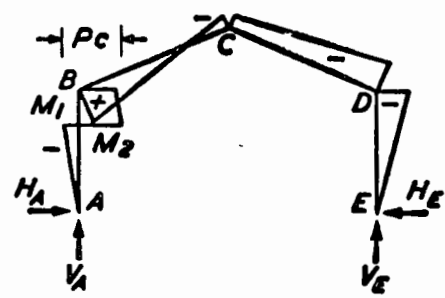
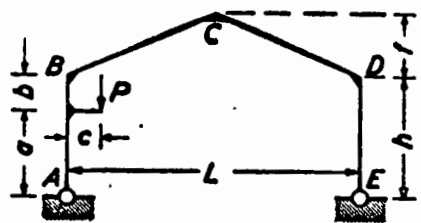
Extract: 'Kleinogel, Rahmenformeln' 11. Auflage Berlin—Verlag von Wilhelm Ernst & Sohn.



$$M_D = -\frac{wh^2}{8} \cdot \frac{2(B+C)+k}{N} \quad M_B = \frac{wh^2}{2} + M_D$$

$$M_C = \frac{wh^2}{4} + mM_D$$

$$V_A = -V_E = -\frac{wh^2}{2L} \quad H_E = -\frac{M_D}{h} \quad H_A = -(wh - H_E)$$



$$\text{Constants: } a_1 = \frac{a}{h} \quad X = \frac{Pc}{2} \cdot \frac{B+C-k(3a_1^2-1)}{N}$$

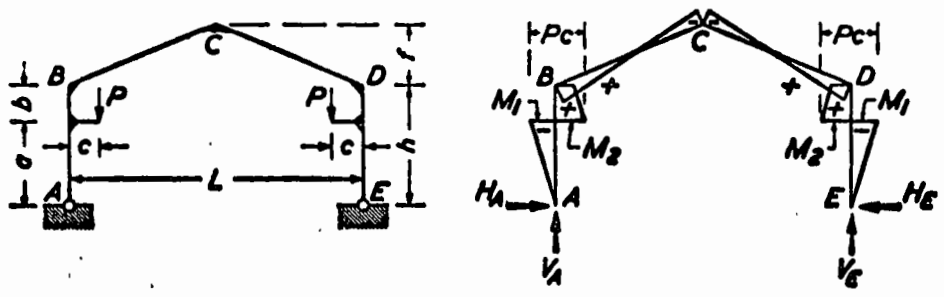
$$M_B = Pc - X \quad M_D = -X \quad M_C = \frac{Pc}{2} - mX$$

$$M_1 = -a_1X \quad M_2 = Pc - a_1X$$

$$V_E = \frac{Pc}{L} \quad V_A = P - V_E \quad H_A = H_E = \frac{X}{h}$$

Extract: 'Kleinlogel, Rahmenformeln' 11. Auflage Berlin—Verlag von Wilhelm Ernst & Sohn.

FRAME IV

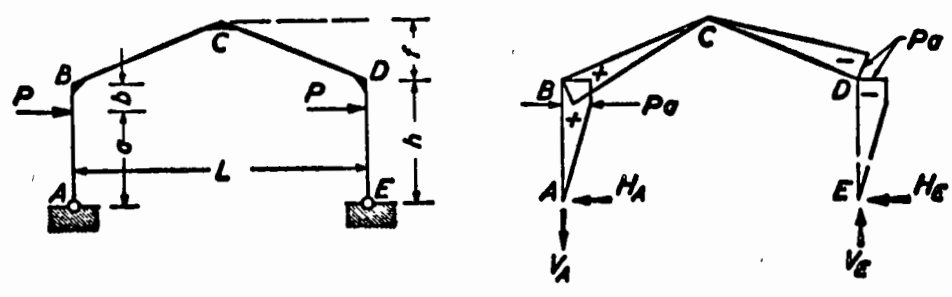


Constant:  $a_1 = \frac{a}{h}$

$$M_B = M_D = Pc \cdot \frac{\phi C + k(3a_1^2 - 1)}{N} \quad M_C = -\phi Pc + mM_B$$

$$H_A = H_E = \frac{Pc - M_B}{h} \quad V_A = V_E = P$$

$$M_1 = -a_1(Pc - M_B) \quad M_2 = (1 - a_1)Pc + a_1M_B$$



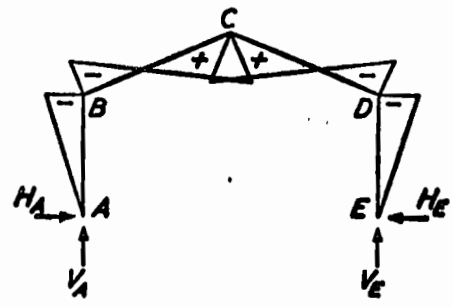
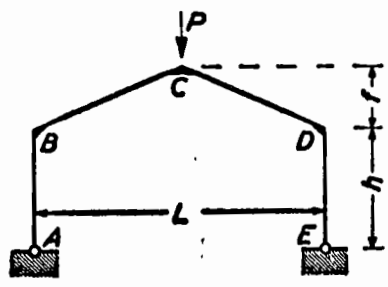
$$M_B = -M_D = Pa \quad M_C = 0$$

$$H_A = -H_E = -P \quad V_A = -V_E = -\frac{2Pa}{L}$$

Moment at loads =  $\pm Pa$

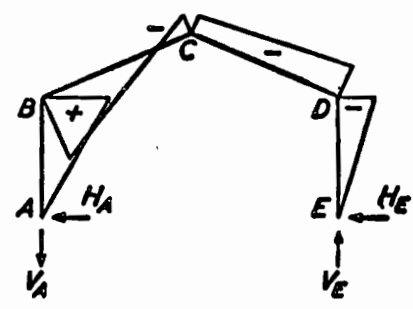
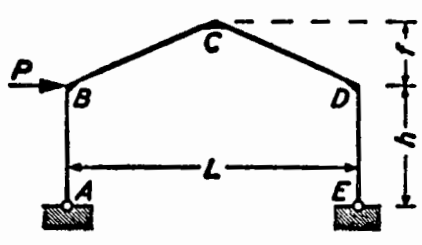
Extract: 'Kleinlogel, Rahmenformeln' 11, Auflage Berlin—Verlag von Wilhelm Ernst & Sohn.

FORMULÆ FOR RIGID FRAMES



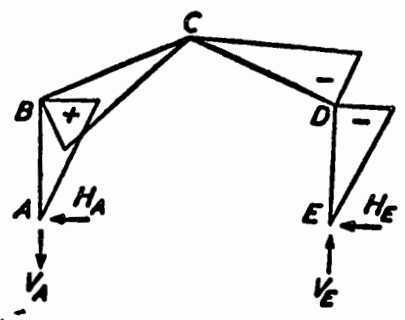
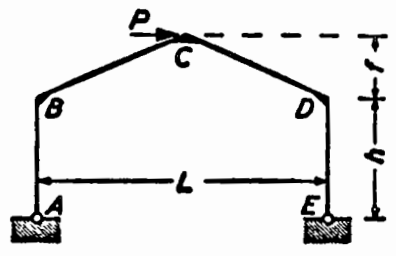
$$M_B = M_D = -\frac{PL}{4} \cdot \frac{C}{N} \quad M_C = +\frac{PL}{4} \cdot \frac{B}{N}$$

$$V_A = V_E = \frac{P}{2} \quad H_A = H_E = -\frac{M_B}{h}$$



$$M_D = -\frac{Ph(B+C)}{2N} \quad M_B = Ph + M_D \quad M_C = \frac{Ph}{2} + mM_D$$

$$V_A = -V_E = -\frac{Ph}{L} \quad H_E = -\frac{M_D}{h} \quad H_A = -(P - H_E)$$

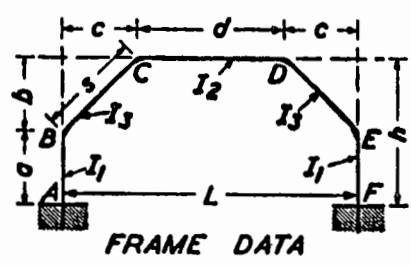


$$M_B = -M_D = +\frac{Ph}{2} \quad M_C = 0 \quad V_A = -V_E = -\frac{Phm}{L} \quad H_A = -H_E = -\frac{P}{2}$$

Extract: 'Kleinlogel, Rahmenformeln' 11. Auflage Berlin—Verlag von Wilhelm Ernst & Sohn.

FRAME V

Frame V



FRAME DATA

Coefficients:

$$k_1 = \frac{I_3}{I_1} \cdot \frac{a}{s} \quad k_2 = \frac{I_3}{I_2} \cdot \frac{d}{s}$$

$$c_1 = \frac{c}{L} \quad d_1 = \frac{d}{L}$$

$$\phi = \frac{b}{a} \quad m = \frac{h}{a} = 1 + \phi$$

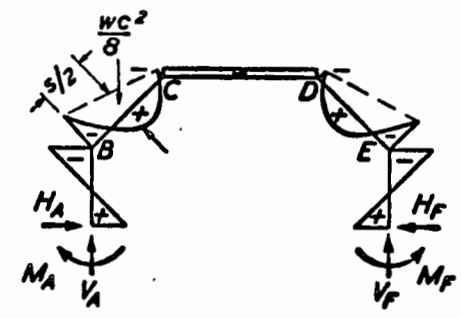
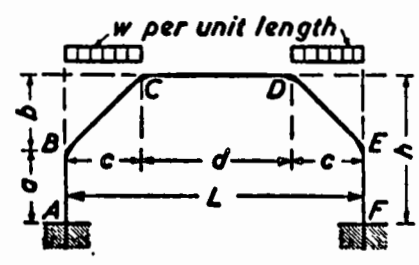
$$(2c_1 + d_1 = 1)$$

$$C_1 = \phi(2 + 3k_2) \quad K_1 = 2(k_1 + 1) + m(1 + C_2)$$

$$C_2 = 1 + m(2 + 3k_2) \quad K_2 = 2k_1 + \phi C_1$$

$$R = \phi C_2 - k_1 \quad N_1 = K_1 K_2 - R^2$$

$$B = 3k_1 + 2 + d_1 \quad C_3 = 1 + d_1(2 + k_2) \quad N_2 = 3k_1 + B + d_1 C_3$$



Constants:  $Y_1 = \frac{wc^2}{4}(2C_1 + \phi) \quad Y_2 = \frac{wc^2}{4}(2C_2 + 1 + m)$

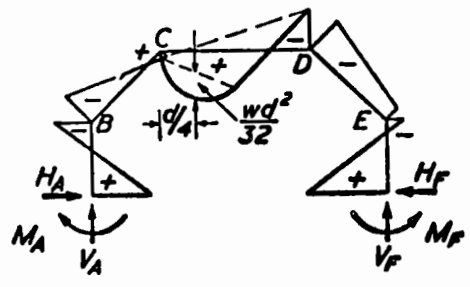
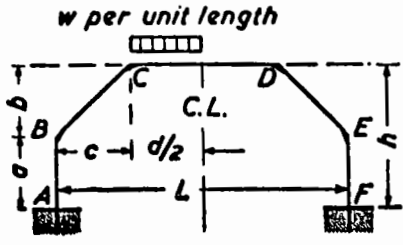
$$M_A = M_F = \frac{Y_1 K_1 - Y_2 R}{N_1} \quad M_B = M_E = -\frac{Y_2 K_2 - Y_1 R}{N_1}$$

$$M_C = M_D = \frac{wc^2}{2} - \phi M_A + m M_B$$

$$V_A = V_F = wc \quad H_A = H_F = \frac{M_A - M_B}{a}$$

Extract: 'Kleinlogel, Rahmenformeln' 11. Auflage Berlin—Verlag von Wilhelm Ernst & Sohn.

FORMULÆ FOR RIGID FRAMES



Constants:  $Y_3 = \frac{wd^2}{32}(8c_1C_3 + d_1k_2)$

$Y_1 = \frac{wd}{4}(2cC_1 + d\phi k_2)$       $Y_2 = \frac{wd}{4}(2cC_2 + dm k_2)$

$X_1 = \frac{Y_1K_1 - Y_2R}{2N_1}$       $X_2 = \frac{Y_2K_2 - Y_1R}{2N_1}$       $X_3 = \frac{Y_3}{2N_2}$

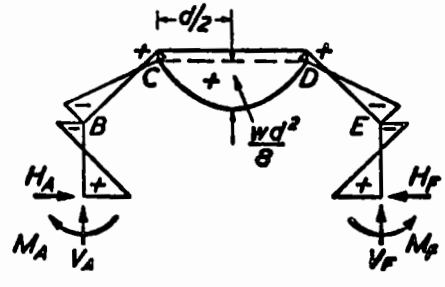
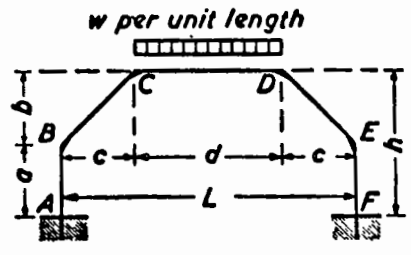
$M_A = +X_1 - X_3$       $M_B = -X_2 - X_3$       $H_A = H_F = \frac{X_1 + X_2}{a}$

$M_F = +X_1 + X_3$       $M_E = -X_2 + X_3$

$M_C = \frac{wdc}{4} - \phi X_1 - mX_2 + \left(\frac{c_1wd^2}{8} - d_1X_3\right)$

$M_D = \frac{wdc}{4} - \phi X_1 - mX_2 - \left(\frac{c_1wd^2}{8} - d_1X_3\right)$

$V_A = \frac{wd(4c + 3d) + 16X_3}{8L}$       $V_F = \frac{wd(4c + d) - 16X_3}{8L}$



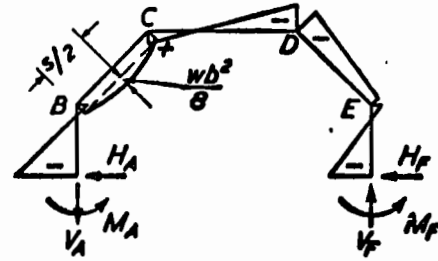
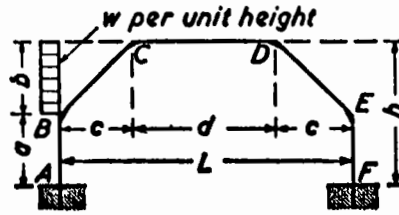
Constants:  $Y_1 = wd\left(cC_1 + \frac{d\phi k_2}{2}\right)$       $Y_2 = wd\left(cC_2 + \frac{dm k_2}{2}\right)$

$X_1 = \frac{Y_1K_1 - Y_2R}{2N_1}$       $X_2 = \frac{Y_2K_2 - Y_1R}{2N_1}$

$M_A = M_F = X_1$       $M_B = M_E = -X_2$       $M_C = M_D = \frac{wdc}{2} - \phi X_1 - mX_2$

$V_A = V_F = \frac{wd}{2}$       $H_A = H_F = \frac{X_1 + X_2}{a}$

Extract: 'Kleinlogel, Rahmenformeln' 11. Auflage Berlin—Verlag von Wilhelm Ernst & Sohn.



$$\text{Constants: } Y_1 = \frac{wb^2}{4}(2C_1 - \phi)$$

$$Y_2 = \frac{wb^2}{4}(2C_2 - 1 - m) \quad Y_3 = wab(B + d_1C_3) + \frac{wb^2}{4}(2d_1C_3 + 1 + d_1)$$

$$X_1 = \frac{Y_1K_1 - Y_2R}{2N_1} \quad X_2 = \frac{Y_2K_2 - Y_1R}{2N_1} \quad X_3 = \frac{Y_3}{2N_2}$$

$$M_A = -X_1 - X_3 \quad M_B = +X_2 + \frac{wab}{2} - X_3$$

$$M_F = -X_1 + X_3 \quad M_E = +X_2 - \frac{wab}{2} + X_3$$

$$M_C = -\frac{wb^2}{4} + \phi X_1 + mX_2 + \frac{d_1}{2} \left( wab + \frac{wb^2}{2} - 2X_3 \right)$$

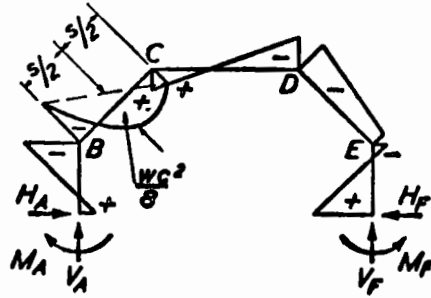
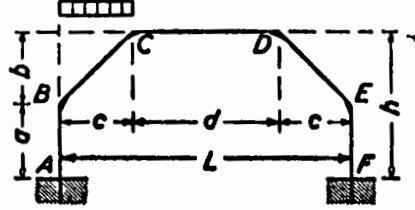
$$M_D = -\frac{wb^2}{4} + \phi X_1 + mX_2 - \frac{d_1}{2} \left( wab + \frac{wb^2}{2} - 2X_3 \right)$$

$$V_A = -V_F = \frac{2wab + wb^2 - 4X_3}{2L}$$

$$H_F = \frac{wb}{2} - \frac{X_1 + X_2}{a} \quad H_A = -(wb - H_F)$$



w per unit length



Constants:  $Y_1 = \frac{wc^2}{4}(2C_1 + \phi)$   $Y_2 = \frac{wc^2}{4}(2C_2 + 1 + m)$   $Y_3 = \frac{wc^2}{4}(2d_1C_3 + 1 + d_1)$

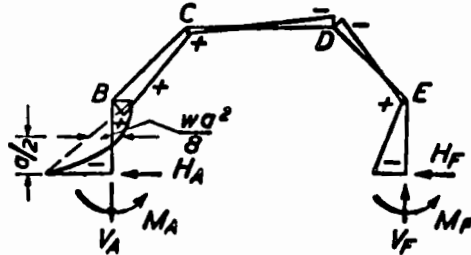
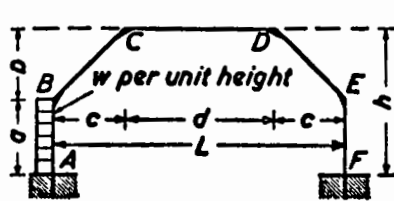
$$X_1 = \frac{Y_1K_1 - Y_2R}{2N_1} \quad X_2 = \frac{Y_2K_2 - Y_1R}{2N_1} \quad X_3 = \frac{Y_3}{2N_2}$$

$M_A = +X_1 - X_3$     $M_B = -X_2 - X_3$     $M_F = +X_1 + X_3$     $M_E = -X_2 + X_3$

$$M_C = \frac{wc^2}{4} - \phi X_1 - mX_2 + \frac{d_1}{2} \left( \frac{wc^2}{2} - 2X_3 \right)$$

$$M_D = \frac{wc^2}{4} - \phi X_1 - mX_2 - \frac{d_1}{2} \left( \frac{wc^2}{2} - 2X_3 \right)$$

$$V_F = \frac{wc^2 - 4X_3}{2L} \quad V_A = wc - V_F \quad H_A = H_F = \frac{X_1 + X_2}{a}$$



Constants:  $Y_1 = \frac{wa^2}{4}(2\phi C_1 + k_1)$   $Y_2 = \frac{wa^2}{4}(2\phi C_2 - k_1)$   $Y_3 = \frac{wa^2}{2}(B + d_1C_3 + k_1)$

The formulæ for  $X_1$ ,  $X_2$  and  $X_3$  are the same as above.

$$M_A = -X_1 - X_3 \quad M_B = +X_2 + \left( \frac{wa^2}{4} - X_3 \right)$$

$$M_F = -X_1 + X_3 \quad M_E = +X_2 - \left( \frac{wa^2}{4} - X_3 \right)$$

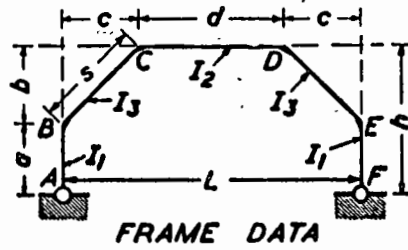
$$M_C = -\frac{wa^2\phi}{4} + \phi X_1 + mX_2 + d_1 \left( \frac{wa^2}{4} - X_3 \right)$$

$$M_D = -\frac{wa^2\phi}{4} + \phi X_1 + mX_2 - d_1 \left( \frac{wa^2}{4} - X_3 \right)$$

$$V_A = -V_F = -\frac{2}{L} \left( \frac{wa^2}{4} - X_3 \right) \quad H_F = \frac{wa}{4} - \frac{X_1 + X_2}{a} \quad H_A = - (wa - H_F)$$

Extract: 'Kleinlogel, Rahmenformeln' 11. Auflage Berlin—Verlag von Wilhelm Ernst & Sohn.

Frame VI



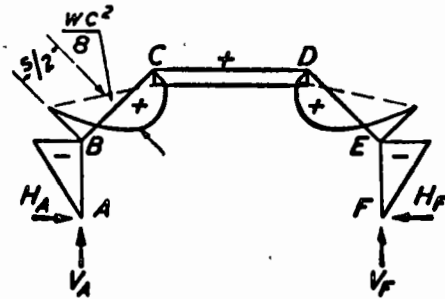
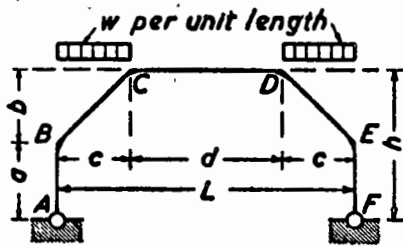
Coefficients:

$$k_1 = \frac{I_3}{I_1} \cdot \frac{a}{s} \quad k_2 = \frac{I_3}{I_2} \cdot \frac{d}{s}$$

$$a_1 = \frac{a}{h} \quad c_1 = \frac{c}{L}$$

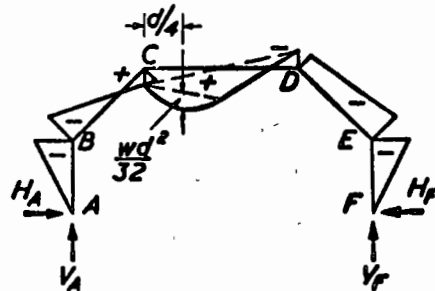
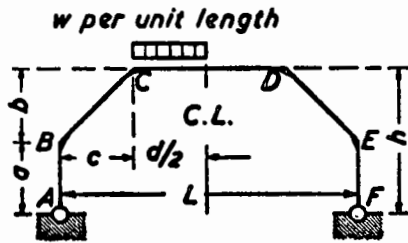
$$B = 2a_1(k_1 + 1) + 1 \quad C = a_1 + 2 + 3k_2$$

$$N = a_1B + C$$



Constant:  $X = \frac{wc^2}{4} \cdot \frac{3a_1 + 5 + 6k_2}{N}$

$$M_B = M_E = -a_1X \quad M_C = M_D = \frac{wc^2}{2} - X \quad V_A = V_F = wc \quad H_A = H_F = \frac{X}{h}$$

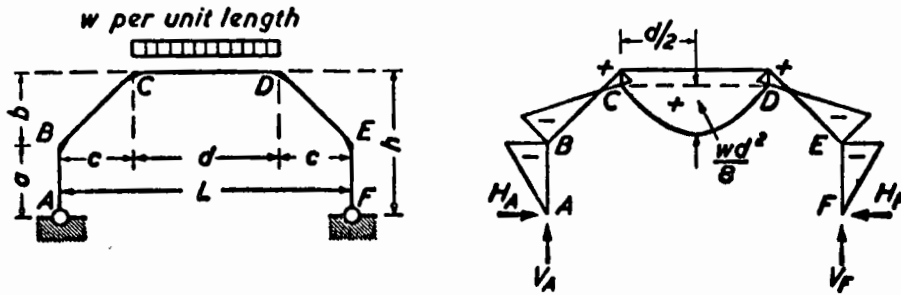


Constant:  $X = \frac{wd(2cC + dk_2)}{8N}$

$$M_C = \frac{c_1 wd}{8}(3d + 4c) - X \quad M_D = \frac{c_1 wd}{8}(4c + d) - X \quad M_B = M_E = -a_1X$$

$$V_A = \frac{wd(3d + 4c)}{8L} \quad V_F = \frac{wd(4c + d)}{8L} \quad H_A = H_F = \frac{X}{h}$$

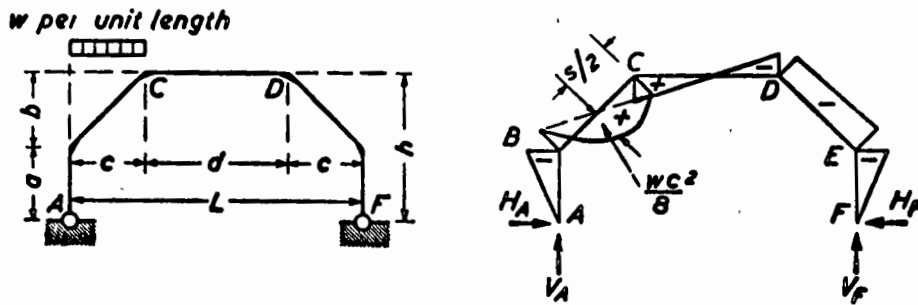
Extract: 'Kleinlogel, Rahmenformeln' 11. Auflage Berlin—Verlag von Wilhelm Ernst & Sohn.



$$\text{Constant: } X = \frac{wd}{4} \cdot \frac{2cC + dk_2}{N}$$

$$M_B = M_E = -a_1 X \quad M_C = M_D = \frac{wdc}{2} - X$$

$$V_A = V_F = \frac{wd}{2} \quad H_A = H_F = \frac{X}{h}$$

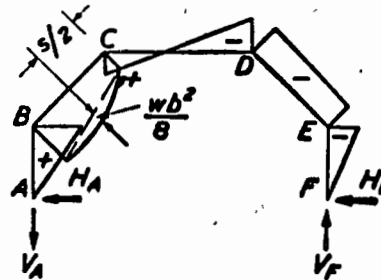
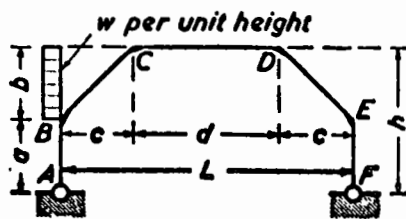


$$\text{Constant: } X = \frac{wc^2(2C + a_1 + 1)}{8N}$$

$$M_B = M_E = -a_1 X \quad M_C = (1 - c_1) \frac{wc^2}{2} - X$$

$$V_F = \frac{wc^2}{2L} \quad V_A = wc - V_F \quad H_A = H_F = \frac{X}{h}$$

Extract: 'Kleinlogel, Rahmenformeln' 11. Auflage Berlin—Verlag von Wilhelm Ernst & Sohn.

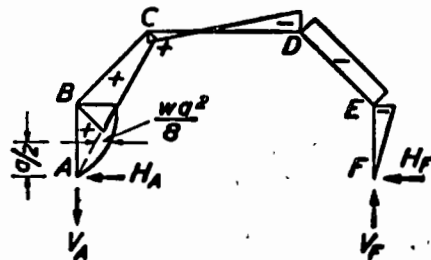
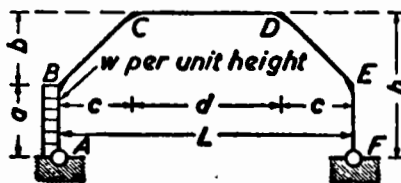


$$\text{Constant: } X = \frac{wb}{8} \cdot \frac{4a(B+C) + b(2C+a_1+1)}{N}$$

$$M_B = wba - a_1X \quad M_E = -a_1X$$

$$M_C = V_F(L-c) - X \quad M_D = -X + V_F \cdot c$$

$$V_A = -V_F = -\frac{wb(a+h)}{2L} \quad H_F = \frac{X}{h} \quad H_A = -(wb - H_F)$$



$$\text{Constant: } X = \frac{wa^2}{8} \cdot \frac{2(B+C) + a_1k_1}{N}$$

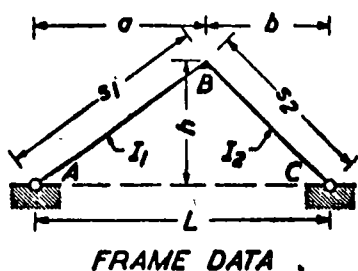
$$M_B = \frac{wa^2}{2} - a_1X \quad M_E = -a_1X$$

$$M_C = V_F(L-c) - X \quad M_D = -X + V_F \cdot c$$

$$V_A = -V_F = -\frac{wa^2}{2L} \quad H_F = \frac{X}{h} \quad H_A = -(wa - H_F)$$

Extract: 'Kleinlogel, Rahmenformeln' 11. Auflage Berlin—Verlag von Wilhelm Ernst & Sohn.

Frame VII

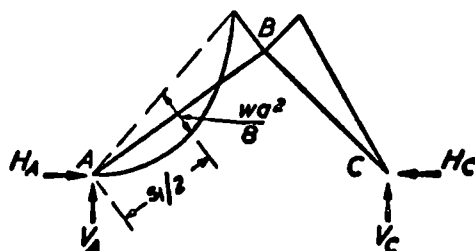
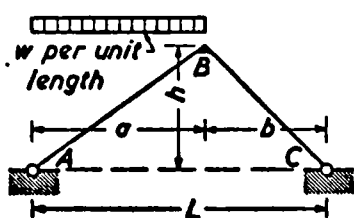


Coefficients:

$$k = \frac{I_1}{I_2} \cdot \frac{s_2}{s_1} \quad N = 1 + k$$

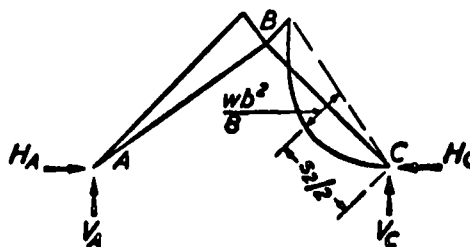
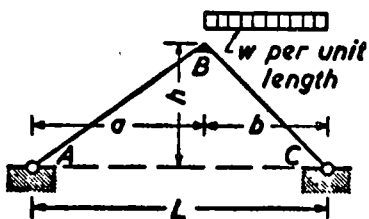
$$a_1 = \frac{a}{L} \quad b_1 = \frac{b}{L}$$

$$(a_1 + b_1 = 1)$$



$$M_B = -\frac{wa^2}{8N} \quad V_C = \frac{wa^2}{2L} \quad V_A = wa - V_C$$

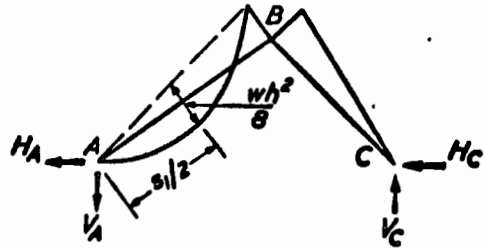
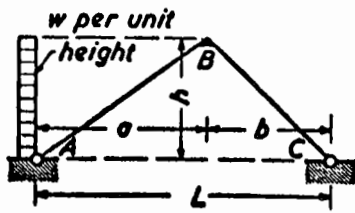
$$H_A = H_C = \frac{wa^2 b_1}{2h} - \frac{M_B}{h}$$



$$M_B = -\frac{wb^2 k}{8N} \quad V_A = \frac{wb^2}{2L} \quad V_C = wb - V_A$$

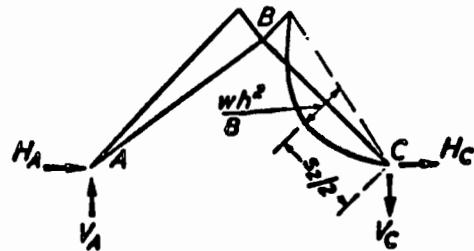
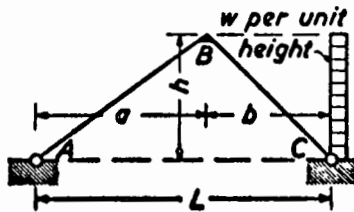
$$H_A = H_C = \frac{wb^2 a_1}{2h} - \frac{M_B}{h}$$

Extract: 'Kleinogel, Rahmenformeln' 11. Auflage Berlin—Verlag von Wilhelm Ernst & Sohn.



$$M_B = -\frac{wh^2}{8N} \quad V_A = -V_C = -\frac{wh^2}{2L}$$

$$H_C = \frac{whb_1}{2} - \frac{M_B}{h} \quad H_A = -(wh - H_C)$$



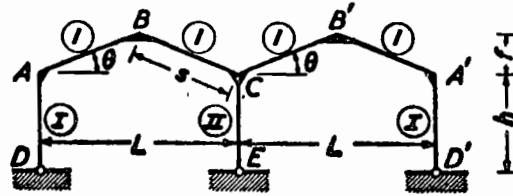
$$M_B = -\frac{wh^2k}{8N} \quad V_A = -V_C = \frac{wh^2}{2L}$$

$$H_A = \frac{wha_1}{2} - \frac{M_B}{h} \quad H_C = -(wh - H_A)$$

Extract: 'Kleinlogel, Rahmenformeln' 11. Auflage Berlin—Verlag von Wilhelm Ernst & Sohn.

FRAME VIII

Frame VIII



FRAME DATA

Constants:

$$\phi = \frac{f}{h} \quad x_I = \frac{I_1 h}{I_1 s} \quad x_{II} = \frac{I_1 h}{I_{II} s} \quad \cos \theta = \frac{L}{2s} \quad \sin \theta = \frac{f}{s}$$

$$N_1 = 8x_I + 12(1 + \phi) + 7\phi^2 \quad N_2 = 2x_I + 12(1 + \phi) + 4\phi^2 + 4x_{II}$$

Influence Coefficients:

$$n_{11} = \frac{2}{N_1} \quad n_{12} = n_{21} = \frac{2 + 3\phi}{2N_1} \quad n_{22} = \frac{x_I + 2 + 3\phi + 2\phi^2}{N_1}$$

$$L_1 = \frac{2 + 5\phi}{4N_1} \quad c_1 = \frac{6 + 7\phi}{4N_1} \quad y_{11} = 2\phi c_1$$

$$L_2 = \frac{2x_I + 2 - \phi - 2\phi^2}{4N_1} \quad c_2 = \frac{6x_I + 6 + \phi}{4N_1} \quad y_{12} = 2\phi c_2$$

$$n = \frac{1}{2N_2} \quad L' = \frac{1 + \phi}{2N_2} \quad c = \frac{3 + 2\phi}{2N_2} \quad r = \frac{1}{N_2}$$

$$y_{13} = \frac{6 + 9\phi + 4\phi^2 + 4x_{II}}{2N_2} \quad y_{23} = \frac{6 + 3\phi + 4x_{II}}{2N_2}$$

$$y_{14} = \frac{2x_I + 6 + 3\phi}{2N_2} \quad y_{24} = \frac{2x_I + 6 + 9\phi + 4\phi^2}{2N_2}$$

$$(y_{13} + y_{14} = 0.5) \quad (y_{23} + y_{24} = 0.5)$$

Composite Influence Coefficients:

$$s_1 = +n_{11} + L_1 \quad s'_1 = -L_1 + n_{21} \quad r_1 = -n_{11} + n_{21}$$

$$s_2 = +n_{12} - L_2 \quad s'_2 = +L_2 + n_{22} \quad r_2 = -n_{12} + n_{22}$$

$$s' = L' + n \quad r = 2n$$

$$m_1 = \frac{+s_1 - s'_1}{2} = L_1 - \frac{r_1}{2} \quad m_2 = \frac{-s_2 + s'_2}{2} = L_2 + \frac{r_2}{2}$$

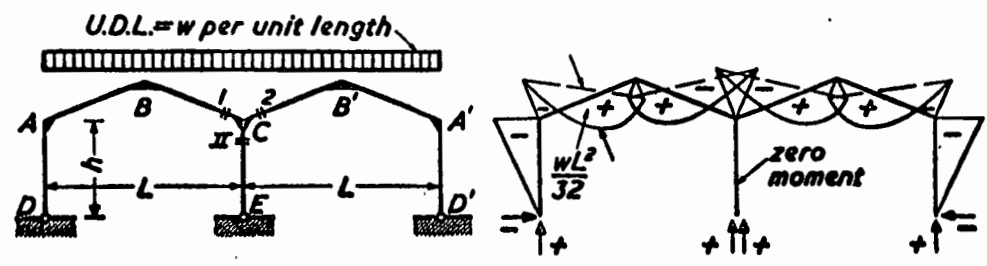
$$y_3 = y_{23} + c\phi \quad y_{13} - y_{23} - 2c\phi = 0$$

$$y_4 = y_{24} - c\phi \quad y_{14} - y_{24} + 2c\phi = 0$$

Note.—The four rafters of equal length AB, BC, CB' and B'A' with the member number 1 are allocated another series of member numbers according to the figure on page 341 as a distinguishing mark for the static values referring to these rafters.

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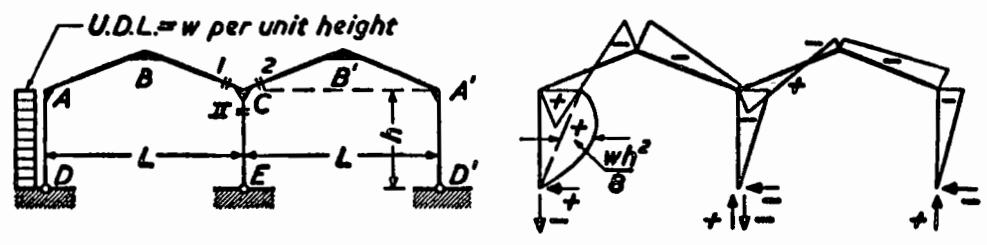


$$M_A = M_{A'} = -\frac{wL^2}{4}(m_1 + 2c_1)$$

$$M_{C1} = M_{C2} = -\frac{wL^2}{4}(m_2 + 2c_2)$$

$$M_{CII} = 0$$

$$M_B = M_{B'} = +\frac{wL^2}{8} + \frac{M_A(1 + 2\phi) + M_{C1}}{2}$$



$$M_A = \frac{wh^2}{4} \cdot x_f(-n_{11} - n) + \frac{wh^2}{2} (+y_{11} + y_{13})$$

$$M_{A'} = \frac{wh^2}{4} \cdot x_f(-n_{11} + n) + \frac{wh^2}{2} (+y_{11} - y_{13})$$

$$M_{C1} = \frac{wh^2}{4} \cdot x_f(+n_{12} - n) + \frac{wh^2}{2} (+y_{12} - y_{14})$$

$$M_{C2} = \frac{wh^2}{4} \cdot x_f(+n_{12} + n) + \frac{wh^2}{2} (+y_{12} + y_{14})$$

$$M_{CII} = -2 \left[ \frac{wh^2}{4} \cdot x_f n + \frac{wh^2}{2} \cdot y_{14} \right]$$

$$M_B = -\frac{wh^2\phi}{2} + \frac{M_A(1 + 2\phi) + M_{C1}}{2}$$

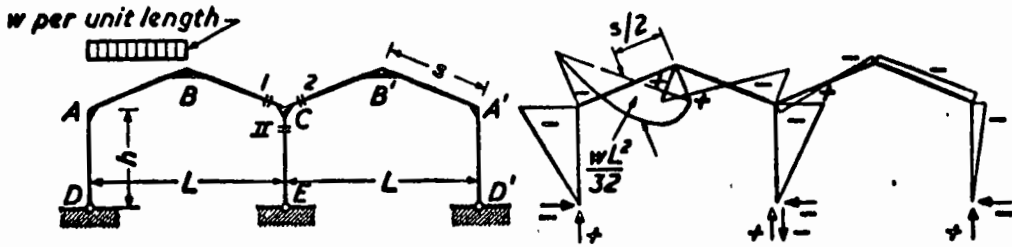
$$M_{B'} = \frac{M_{C2} + M_{A'}(1 + 2\phi)}{2}$$

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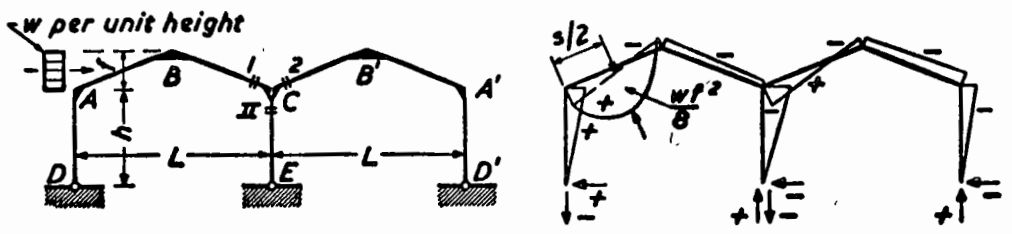


$$M_A = \frac{wL^2}{16}(-s_1 - s' - 2c_1 - 2c) \quad M'_A = \frac{wL^2}{16}(-s_1 + s' - 2c_1 + 2c)$$

$$M_{C1} = \frac{wL^2}{16}(+s_2 - s' - 2c_2 - 2c) \quad M_{C2} = \frac{wL^2}{16}(+s_2 + s' - 2c_2 + 2c)$$

$$M_{CII} = -\frac{wL^2}{8}(s' + 2c)$$

$$M_B = \frac{wL^2}{16} + \frac{M_A(1 + 2\phi) + M_{C1}}{2} \quad M'_B = \frac{M_{C2} + M'_A(1 + 2\phi)}{2}$$



Constant:  $W_1 = wfh$

$$M_A = \frac{wf^2}{4}(-s_1 - s') + \frac{3wf^2}{2}(+c_1 + c) + W_1 y_{23}$$

$$M'_A = \frac{wf^2}{4}(-s_1 + s') + \frac{3wf^2}{2}(+c_1 - c) - W_1 y_{23}$$

$$M_{C1} = \frac{wf^2}{4}(+s_2 - s') + \frac{3wf^2}{2}(+c_2 + c) - W_1 y_{24}$$

$$M_{C2} = \frac{wf^2}{4}(+s_2 + s') + \frac{3wf^2}{2}(+c_2 - c) + W_1 y_{24}$$

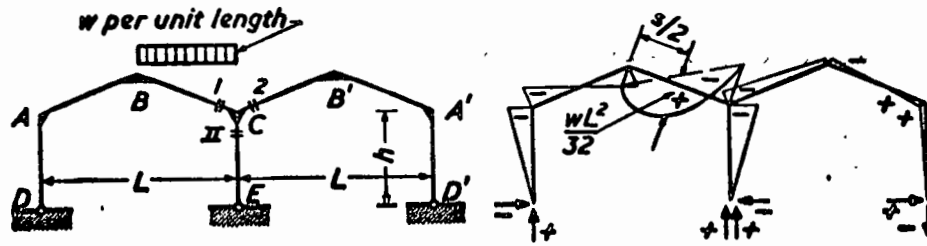
$$M_{CII} = -\frac{wf^2 s'}{2} + 3wf^2 c - 2W_1 y_{24}$$

$$M_B = -\frac{3wf^2}{4} + \frac{M_A(1 + 2\phi) + M_{C1}}{2}$$

$$M'_B = \frac{M_{C2} + M'_A(1 + 2\phi)}{2}$$

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FORMULÆ FOR RIGID FRAMES

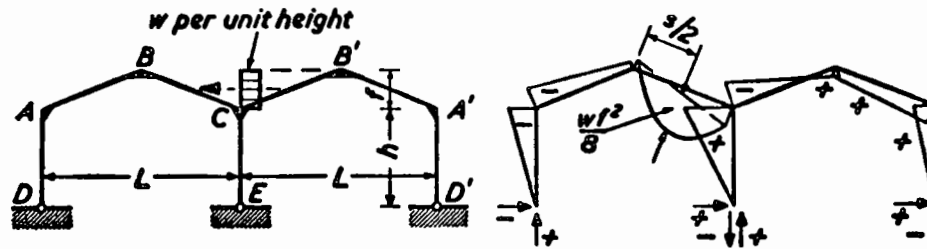


$$M_A = \frac{wL^2}{16} (+s'_1 - s' - 2c_1 - 2c) \quad M'_A = \frac{wL^2}{16} (+s'_1 + s' - 2c_1 + 2c)$$

$$M_{C1} = \frac{wL^2}{16} (-s'_2 - s' - 2c_2 - 2c) \quad M_{C2} = \frac{wL^2}{16} (-s'_2 + s' - 2c_2 + 2c)$$

$$M_{CII} = -\frac{wL^2}{8} (s' + 2c)$$

$$M_B = \frac{wL^2}{16} + \frac{M_A(1+2\phi) + M_{C1}}{2} \quad M'_B = \frac{M_{C2} + M'_A(1+2\phi)}{2}$$



Constant:  $W_1 = wfh$

$$M_A = \frac{wf^2}{4} (+s'_1 - s' - 2c_1 - 2c) - W_1 y_{23}$$

$$M'_A = \frac{wf^2}{4} (+s'_1 + s' - 2c_1 + 2c) + W_1 y_{23}$$

$$M_{C1} = \frac{wf^2}{4} (-s'_2 - s' - 2c_2 - 2c) + W_1 y_{24}$$

$$M_{C2} = \frac{wf^2}{4} (-s'_2 + s' - 2c_2 + 2c) - W_1 y_{24}$$

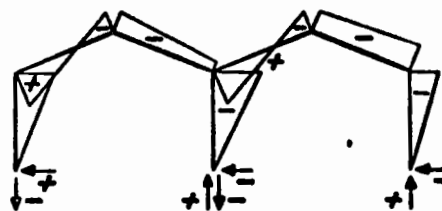
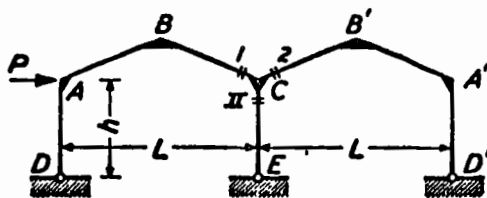
$$M_{CII} = -\frac{wf^2}{2} (s' + 2c) + 2W_1 y_{24}$$

$$M_B = \frac{wf^2}{4} + \frac{M_A(1+2\phi) + M_{C1}}{2} \quad M'_B = \frac{M_{C2} + M'_A(1+2\phi)}{2}$$

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$$M_A = M(+y_{11} + y_{13})$$

where  $M = Ph$

$$M'_A = M(+y_{11} - y_{13})$$

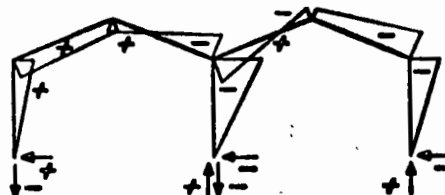
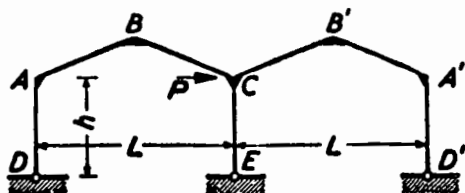
$$M_{C1} = M(+y_{12} - y_{14})$$

$$M_{C2} = M(+y_{12} + y_{14})$$

$$M_{C11} = -2My_{14}$$

$$M_B = \frac{M_A(1 + 2\phi) + M_{C1} - Pf}{2}$$

$$M'_B = \frac{M_{C2} + M'_A(1 + 2\phi)}{2}$$



$$M_A = +My_{23}$$

where  $M = Ph$

$$M'_A = -My_{23}$$

$$M_{C1} = -My_{24}$$

$$M_{C2} = +My_{24}$$

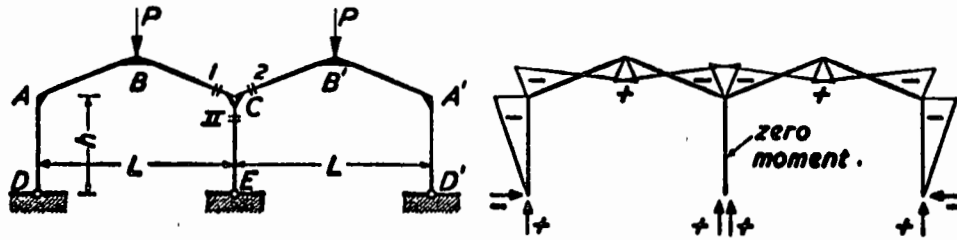
$$M_{C11} = -2My_{24}$$

$$M_B = \frac{M_A(1 + 2\phi) + M_{C1}}{2}$$

$$M'_B = \frac{M_{C2} + M'_A(1 + 2\phi)}{2}$$

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FORMULÆ FOR RIGID FRAMES

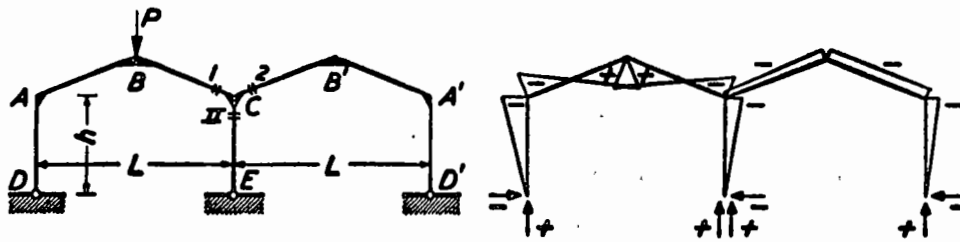


$$M_A = M'_A = -M_{C1}$$

$$M_{C1} = M_{C2} = -M_{C2} \quad \text{where } M = PL$$

$$M_{CII} = 0$$

$$M_B = M'_B = \frac{PL}{4} + \frac{M_A(1+2\phi) + M_{C1}}{2}$$



$$M_A = \frac{M}{2}(-c_1 - c)$$

$$M'_A = \frac{M}{2}(-c_1 + c) \quad \text{where } M = PL$$

$$M_{C1} = \frac{M}{2}(-c_2 - c)$$

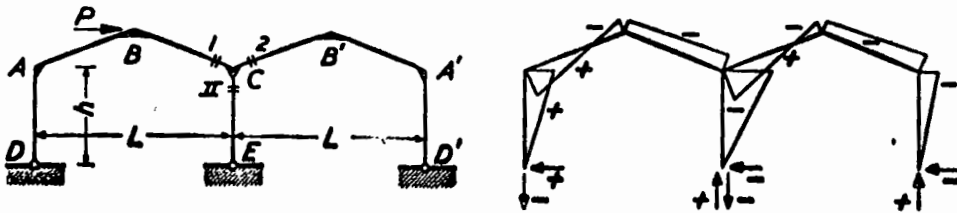
$$M_{C2} = \frac{M}{2}(-c_2 + c) \quad M_{CII} = -Mc$$

$$M_B = \frac{PL}{4} + \frac{M_A(1+2\phi) + M_{C1}}{2}$$

$$M'_B = \frac{M_{C2} + M'_A(1+2\phi)}{2}$$

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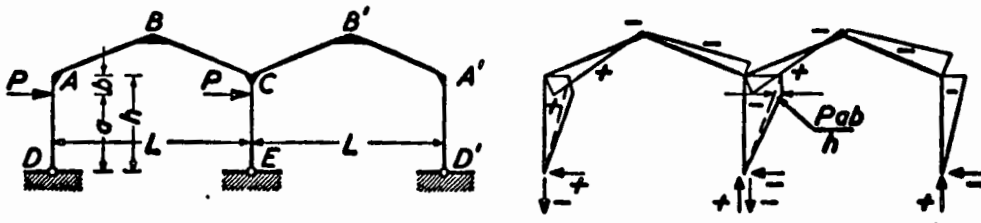
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$$\begin{aligned}
 M_A &= M(+c_1\phi + y_3) \\
 M'_A &= M(+c_1\phi - y_3) \\
 M_{C1} &= M(+c_2\phi - y_4) \\
 M_{C2} &= M(+c_2\phi + y_4) \\
 M_B &= \frac{M_A(1+2\phi) + M_{C1} - Pf}{2} \\
 M'_B &= \frac{M_{C2} + M'_A(1+2\phi)}{2}
 \end{aligned}$$

where  $M = Ph$

$$M_{CII} = -2My_4$$

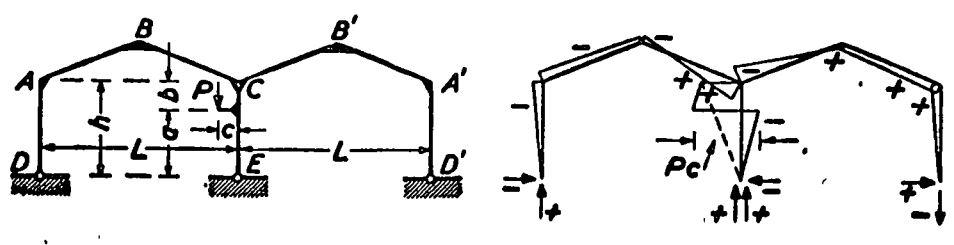


Constants:  $a_1 = a/h$      $b_1 = b/h$

$$\begin{aligned}
 R_I &= Pba_1(1+a_1)x_I & L_{II} &= Pab_1(1+b_1)x_{II} \\
 M_A &= R_I(-n_{11}-n) + Pa(+y_{11}+y_{13}) + L_{II}r + Pay_{23} \\
 M'_A &= R_I(-n_{11}+n) + Pa(+y_{11}-y_{13}) - L_{II}r - Pay_{23} \\
 M_{C1} &= R_I(+n_{12}-n) + Pa(+y_{12}-y_{14}) + L_{II}r - Pay_{24} \\
 M_{C2} &= R_I(+n_{12}+n) + Pa(+y_{12}+y_{14}) - L_{II}r + Pay_{24} \\
 M_{CII} &= -2(R_I n + Pay_{14} - L_{II}r + Pay_{24}) \\
 M_B &= \frac{M_A(1+2\phi) + M_{C1} - Pa\phi}{2} \\
 M'_B &= \frac{M_{C2} + M'_A(1+2\phi)}{2}
 \end{aligned}$$

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FORMULÆ FOR RIGID FRAMES



Constants:  $a_1 = a/h$      $b_1 = b/h$

$$L_{II} = Pc(1 - 3a_1^2)x_{II}$$

$$M_A = -L_{II}r - Pcy_{23}$$

$$M'_A = +L_{II}r + Pcy_{23}$$

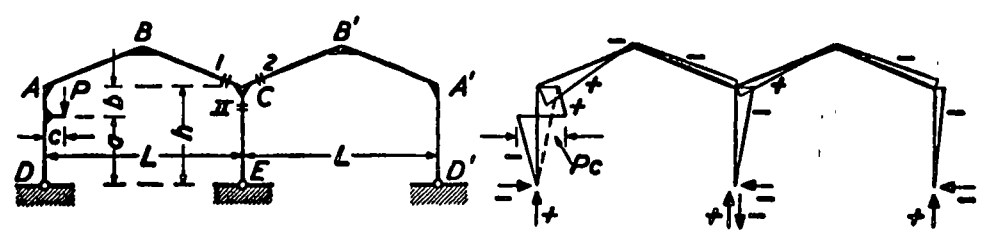
$$M_{C1} = -L_{II}r + Pcy_{24}$$

$$M_{C2} = +L_{II}r - Pcy_{24}$$

$$M_{CII} = 2(-L_{II}r + Pcy_{24})$$

$$M_B = \frac{M_A(1 + 2\phi) + M_{C1}}{2}$$

$$M'_B = \frac{M_{C2} + M'_A(1 + 2\phi)}{2}$$



Constants:  $a_1 = a/h$      $b_1 = b/h$

$$R_I = Pc(1 - 3a_1^2)x_I$$

$$M_A = R_I(-n_{11} - n) + Pc(+y_{11} + y_{13})$$

$$M'_A = R_I(-n_{11} + n) + Pc(+y_{11} - y_{13})$$

$$M_{C1} = R_I(+n_{12} - n) + Pc(+y_{12} - y_{14})$$

$$M_{C2} = R_I(+n_{12} + n) + Pc(+y_{12} + y_{14})$$

$$M_{CII} = -2(R_I n + Pcy_{14})$$

$$M_B = \frac{M_A(1 + 2\phi) + M_{C1} - Pc\phi}{2}$$

$$M'_B = \frac{M_{C2} + M'_A(1 + 2\phi)}{2}$$

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FRAME VIII

Formulae for Support Reactions, Shear and Axial Forces.

Nomenclature:

$V$  = vertical reaction or axial force in columns.

$N$  = axial force in rafters or columns.

$T$  = shear force at ends of rafters or columns.

$H$  = horizontal reactions in columns.

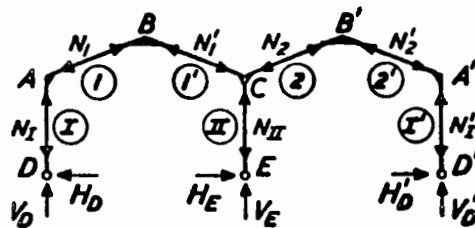
(a) For all loads:

$$M_{CII} = M_{C1} - M_{C2}$$

$$V_E = V_{C1} + V_{C2}$$

$$V_1 = \frac{-M_A + M_{C1}}{L}$$

$$V_2 = \frac{-M_{C2} + M'_A}{L}$$



All the values of  $H$  and  $V$  shown are positive

$$H_I = \frac{M_A}{h} \quad H_{II} = \frac{M_{CII}}{h} \quad H'_I = \frac{M'_A}{h}$$

The axial thrusts in the columns are:

$$N_I = V_D \quad N_{II} = V_E \quad N'_I = V'_D$$

The horizontal thrusts in unloaded columns are:

$$H_D = H_I \quad H_E = H_{II} \quad H'_D = H'_I$$

The shear forces in unloaded columns are:

$$T_I = +H_I \quad T_{II} = -H_{II} \quad T'_I = -H'_I$$

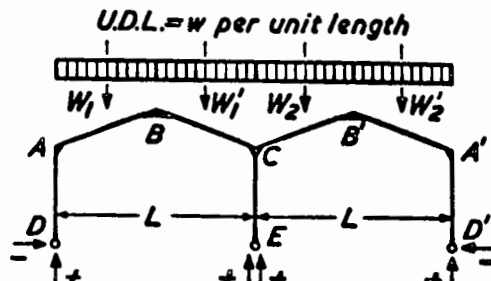
(b) For vertical U.D.L. over the whole frame:

$$V_D = +V_1 + \frac{wL}{2}$$

$$V_{C2} = +V_2 + \frac{wL}{2}$$

$$V_{C1} = -V_1 + \frac{wL}{2}$$

$$V'_D = -V_2 + \frac{wL}{2}$$



Reactions shown are those for total U.D.L.

For vertical U.D.L. over the extreme left rafter:

$$V_D = +V_1 + \frac{3wL}{8} \quad V_{C2} = +V_2$$

$$V_{C1} = -V_1 + \frac{wL}{8} \quad V'_D = -V_2$$

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FORMULÆ FOR RIGID FRAMES

For vertical U.D.L. over the second rafter from the left:

$$V_D = +V_1 + \frac{wL}{8} \quad V_{C2} = +V_2$$

$$V_{C1} = -V_1 + \frac{3wL}{8} \quad V'_D = -V_2$$

The shear forces in the rafters for all vertical U.D.L.s are:

$$T_{L1} = +V_D \cos \theta + H_1 \sin \theta \quad T_{R1} = T_{L1} - W_1 \cos \theta$$

$$T'_{R1} = -V_{C1} \cos \theta - H_1 \sin \theta \quad T'_{L1} = T'_{R1} + W'_1 \cos \theta$$

$$T_{L2} = +V_{C2} \cos \theta + H'_1 \sin \theta \quad T_{R2} = T_{L2} - W_2 \cos \theta$$

$$T'_{R2} = -V'_D \cos \theta - H'_1 \sin \theta \quad T'_{L2} = T'_{R2} + W'_2 \cos \theta$$

where the suffix *L1* refers to the left-hand end of the first rafter from the left, *R1* refers to the right-hand end of the first rafter, and so on, and where  $W_1, W'_1, W_2$  and  $W'_2$  are the U.D.L.s on the first, second, third and fourth rafters from the left.

The axial thrusts for all vertical U.D.L.s are:

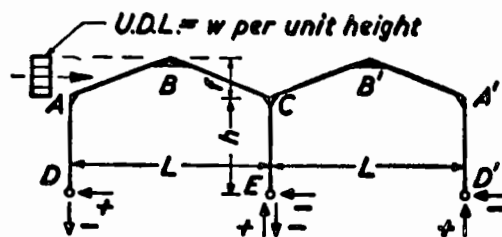
$$N_{1A} = +V_D \sin \theta - H_1 \cos \theta \quad N_{1B} = N_{1A} - W_1 \sin \theta$$

$$N'_{1C} = +V_{C1} \sin \theta - H_1 \cos \theta \quad N'_{1B} = N'_{1C} - W'_1 \sin \theta$$

$$N_{2C} = +V_{C2} \sin \theta - H'_1 \cos \theta \quad N_{2B} = N_{2C} - W_2 \sin \theta$$

$$N_{2A} = +V'_D \sin \theta - H'_1 \cos \theta \quad N'_{2B} = N_{2A} - W'_2 \sin \theta$$

where  $W_1, W'_1, W_2$  and  $W'_2$  are the U.D.L.s on the first, second, third and fourth rafters respectively from the left.



(c) For horizontal U.D.L. applied to the extreme left rafter:

$$V_D = +V_1 - \frac{wf^2}{2L} = -V_{C1}$$

$$V_{C2} = +V_2 = -V'_D$$

$$H_{C1} = H_1 - wf \quad H_{C2} = H'_1$$

The shear forces in the rafters are:

$$T_{L1} = +V_D \cos \theta + H_D \sin \theta \quad T_{L2} = -V'_D \cos \theta + H_{C2} \sin \theta$$

$$T_{R1} = T_{L1} - wf \sin \theta \quad T_{R2} = T_{L2}$$

$$T'_1 = +V_D \cos \theta - H_{C1} \sin \theta \quad T'_2 = -V'_D \cos \theta - H'_D \sin \theta$$

The axial thrusts are:

$$N_{1A} = +V_D \sin \theta - H_D \cos \theta \quad N_{2C} = -V'_D \sin \theta - H_{C2} \cos \theta$$

$$N_{1B} = N_{1A} + wf \cos \theta \quad N_{2B} = N_{2C}$$

$$N'_1 = -V_D \sin \theta - H_{C1} \cos \theta \quad N'_2 = +V'_D \sin \theta - H'_D \cos \theta$$

Extract: 'Kleinlogel, Mehrstielige Rahmen', Band I+II. Berlin—Verlag von Wilhelm Ernst & Sohn.



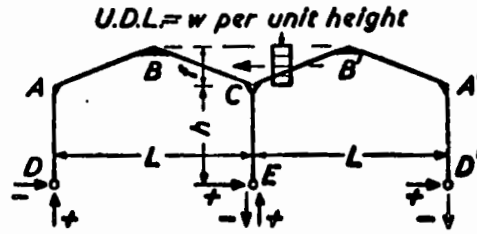
(d) For horizontal U.D.L. acting to the left on the second rafter from the left:

$$V_D = +V_1 + \frac{wf^2}{2L} = -V_{C1}$$

$$V_{C2} = +V_2 = -V'_D$$

$$H_{C1} = H_I + wf$$

$$H_{C2} = H'_I$$



The shear forces in the rafters are:

$$T'_{R1} = +V_D \cos \theta - H_{C1} \sin \theta \quad T'_{R2} = -V'_D \cos \theta - H'_D \sin \theta$$

$$T'_{L1} = T'_{R1} + wf \sin \theta \quad T'_{L2} = T'_{R2}$$

$$T_1 = +V_D \cos \theta + H_D \sin \theta \quad T_2 = -V'_D \cos \theta + H_{C2} \sin \theta$$

The axial thrusts are:

$$N'_{1C} = -V_D \sin \theta - H_{C1} \cos \theta \quad N'_{2A} = +V'_D \sin \theta - H'_D \cos \theta$$

$$N'_{1B} = N'_{1C} + wf \cos \theta \quad N'_{2B} = N_{2A}$$

$$N_1 = +V_D \sin \theta - H_D \cos \theta \quad N_2 = -V'_D \sin \theta - H_{C2} \cos \theta$$

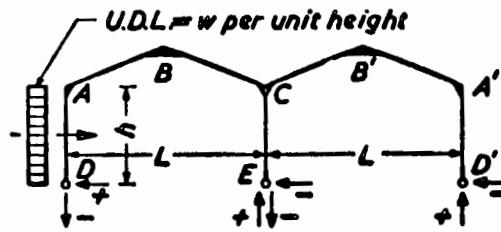
(e) For horizontal U.D.L. applied to extreme left column:

$$V_D = +V_1 \quad V_{C2} = +V_2$$

$$V_{C1} = -V_1 \quad V'_D = -V_2$$

$$H_D = H_I + \frac{wh}{2} \quad H_E = H_{II}$$

$$H'_D = H'_I$$



The shear forces in the columns are:

$$T_D = +H_D \quad T_E = -H_E \quad T'_D = -H'_D$$

$$T_A = +H_D - wh \quad T_C = -H_E \quad T'_A = -H'_D$$

The shear forces in the rafters are:

$$T_1 = +V_1 \cos \theta + T_A \sin \theta \quad T_2 = +V_2 \cos \theta - T'_A \sin \theta$$

$$T'_1 = +V_1 \cos \theta - T_A \sin \theta \quad T'_2 = +V_2 \cos \theta + T'_A \sin \theta$$

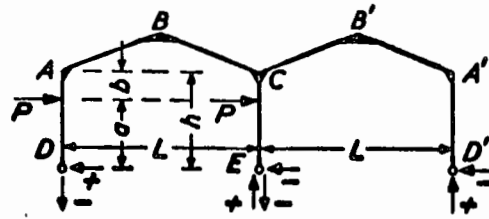
The axial thrusts in the rafters are:

$$N_1 = +V_1 \sin \theta - T_A \cos \theta \quad N_2 = +V_2 \sin \theta + T'_A \cos \theta$$

$$N'_1 = -V_1 \sin \theta - T_A \cos \theta \quad N'_2 = -V_2 \sin \theta + T'_A \cos \theta$$

Extract: 'Kleinlogel, Mehrstellige Rahmen', Band I+II. Berlin—Verlag von Wilhelm Ernst & Sohn.

(f) Gantry crane loads:



(1) Surge loads:

$$V_D = +V_1 \quad V_{C2} = +V_2$$

$$V_{C1} = -V_1 \quad V'_D = -V_2$$

$$H_D = H_I + \frac{Pb}{h} \quad H_E = H_{II} - \frac{Pb}{h}$$

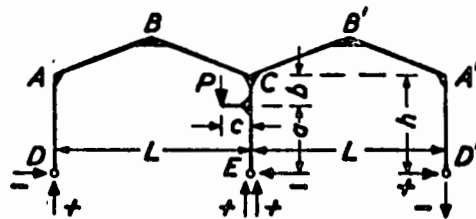
$$H'_D = H'_I$$

The shear forces in the column are:

$$T_D = +H_D \quad T_E = -H_E \quad T'_D = -H'_D$$

$$T_A = +H_D - P \quad T_C = -H_E - P \quad T'_A = -H'_D$$

(2) Bracket load on central column CE:



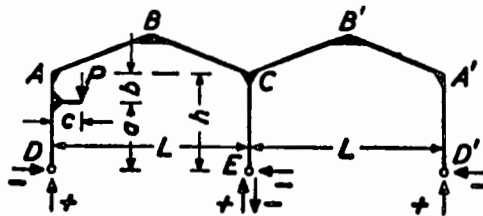
$$V_D = +V_1 \quad V_{C2} = +V_2$$

$$V_{C1} = P - V_1 \quad V'_D = -V_2$$

$$H_D = H_I \quad H_E = H_{II} - \frac{Pc}{h}$$

$$H'_D = H'_I$$

(3) Bracket load on column AD:



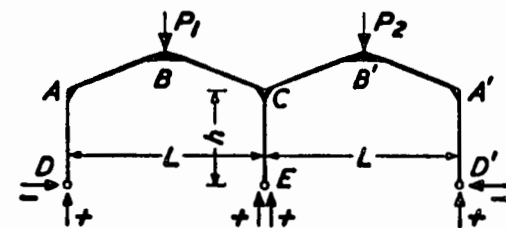
$$V_D = P + V_1 \quad V_{C2} = +V_2$$

$$V_{C1} = -V_1 \quad V'_D = -V_2$$

$$H_D = H_I - \frac{Pc}{h} \quad H_E = H_{II}$$

$$H'_D = H'_I$$

(g) For vertical point loads on the ridges:



$$V_D = +V_1 + \frac{P_1}{2}$$

$$V_{C1} = -V_1 + \frac{P_1}{2}$$

$$V_{C2} = +V_2 + \frac{P_2}{2}$$

$$V'_D = -V_2 + \frac{P_2}{2}$$

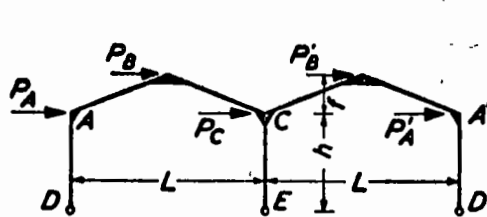
Reactions shown are applicable when  $P_1 = P_2$

## FRAME VIII.

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$$\begin{aligned}
 T_1 &= +V_D \cos \theta + H_1 \sin \theta & N_1 &= +V_D \sin \theta - H_1 \cos \theta \\
 T_1' &= -V_{C1} \cos \theta - H_1 \sin \theta & N_1' &= +V_{C1} \sin \theta - H_1 \cos \theta \\
 T_2 &= +V_{C2} \cos \theta + H_1' \sin \theta & N_2 &= +V_{C2} \sin \theta - H_1' \cos \theta \\
 T_2' &= -V_D' \cos \theta - H_1' \sin \theta & N_2' &= +V_D' \sin \theta - H_1' \cos \theta
 \end{aligned}$$

(h) For horizontal point loads applied at joints:



Reactions vary with the number and value of the loads

$$V_D = +V_1 - \frac{P_B f}{L}$$

$$V_{C2} = +V_2 - \frac{P_B' f}{L}$$

$$V_{C1} = -V_D$$

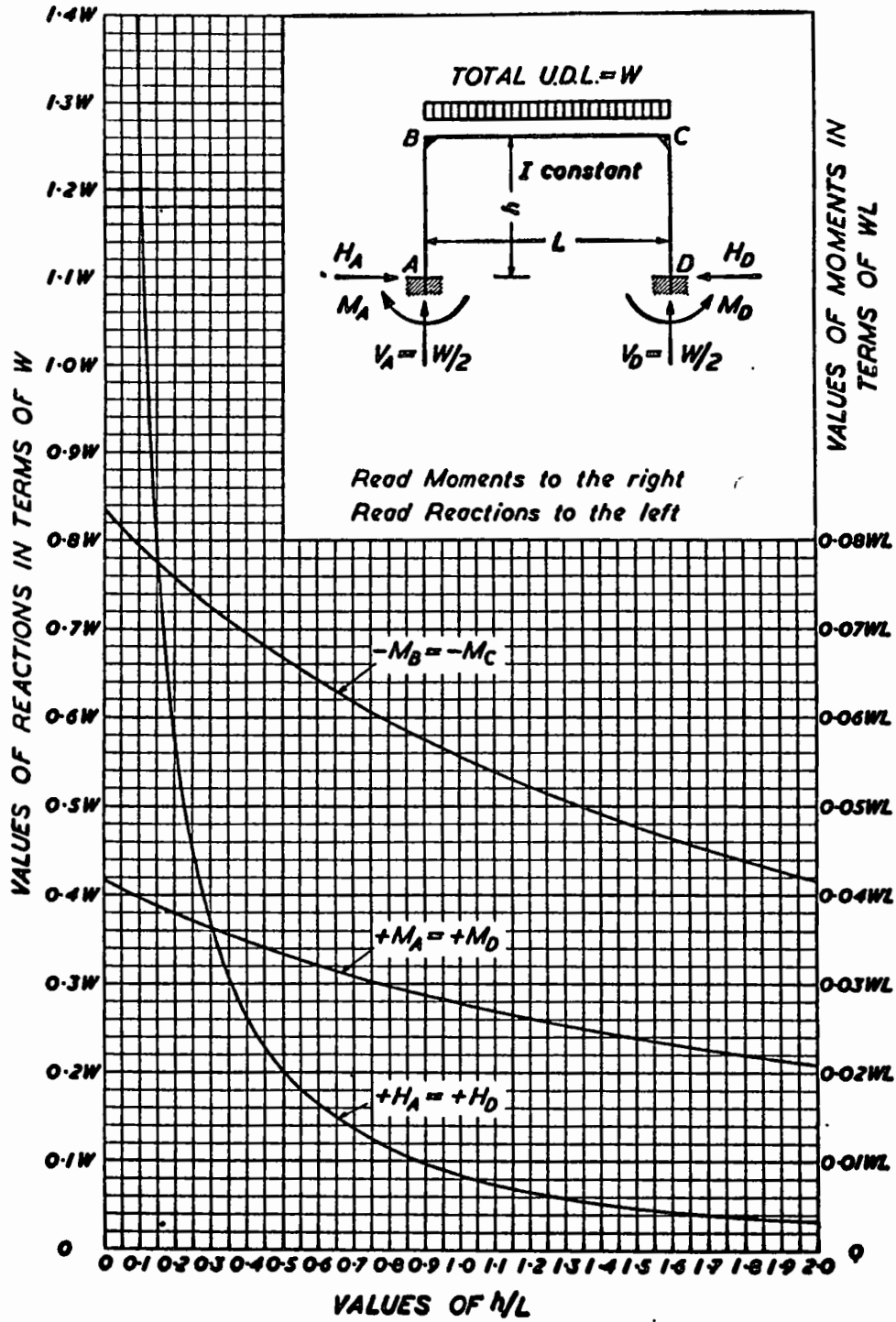
$$V_D' = -V_{C2}$$

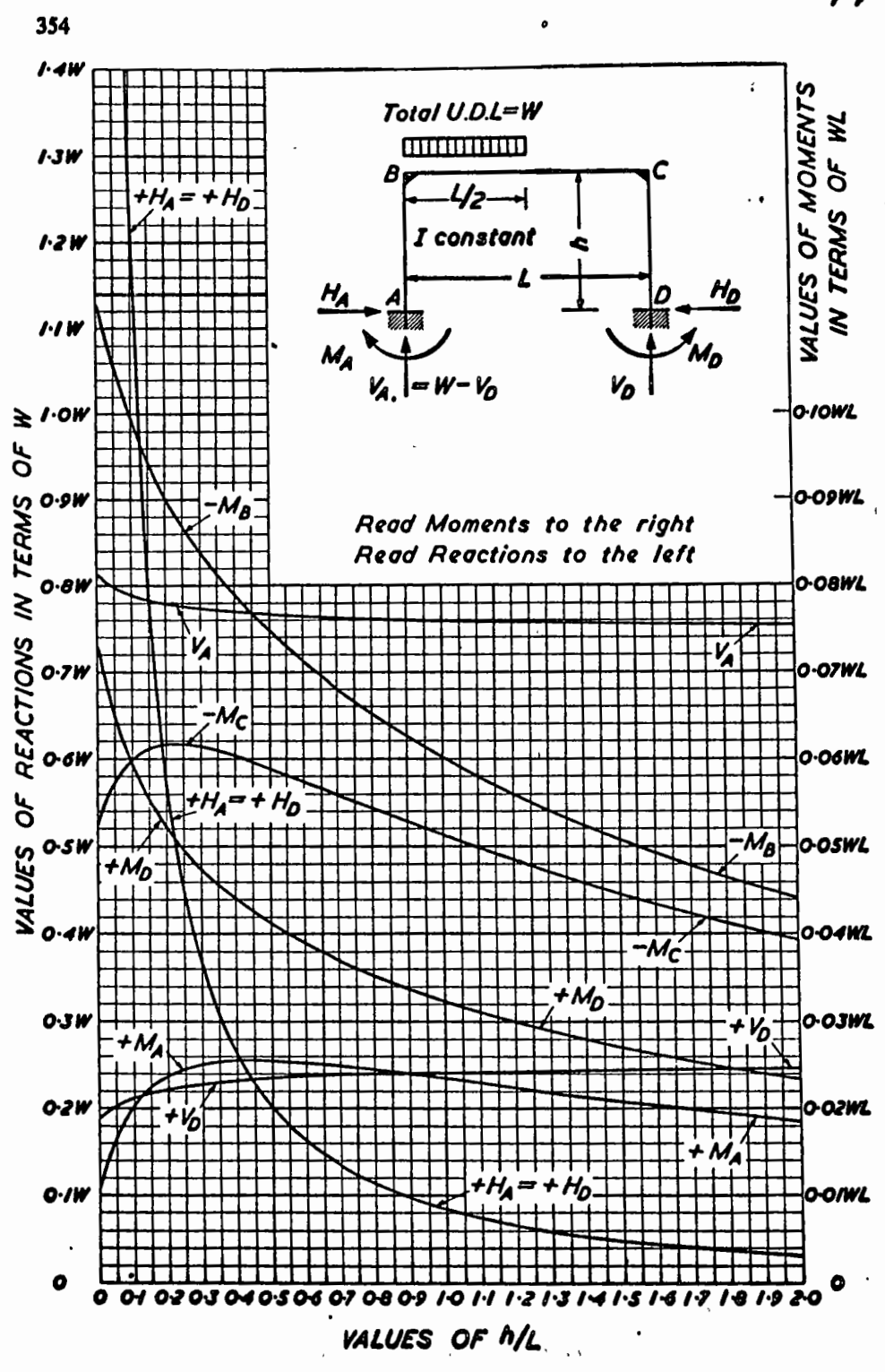
$$H_{C1} = H_1 - P_A - P_B$$

$$H_{C2} = H_1' + P_B' + P_A'$$

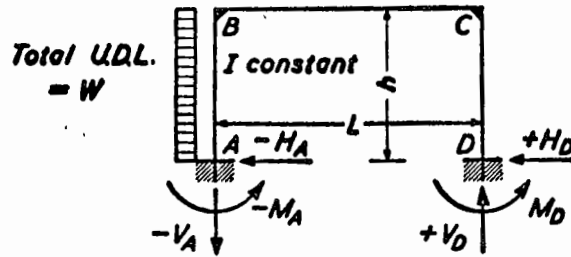
$$\begin{aligned}
 T_1 &= +V_D \cos \theta + (H_1 - P_A) \sin \theta & N_1 &= +V_D \sin \theta - (H_1 - P_A) \cos \theta \\
 T_1' &= +V_D \cos \theta - H_{C1} \sin \theta & N_1' &= -V_D \sin \theta - H_{C1} \cos \theta \\
 T_2 &= -V_D' \cos \theta + H_{C2} \sin \theta & N_2 &= -V_D' \sin \theta - H_{C2} \cos \theta \\
 T_2' &= -V_D' \cos \theta - (H_1' + P_A') \sin \theta & N_2' &= +V_D' \sin \theta - (H_1' + P_A') \cos \theta
 \end{aligned}$$

16. RIGID FRAME CHARTS

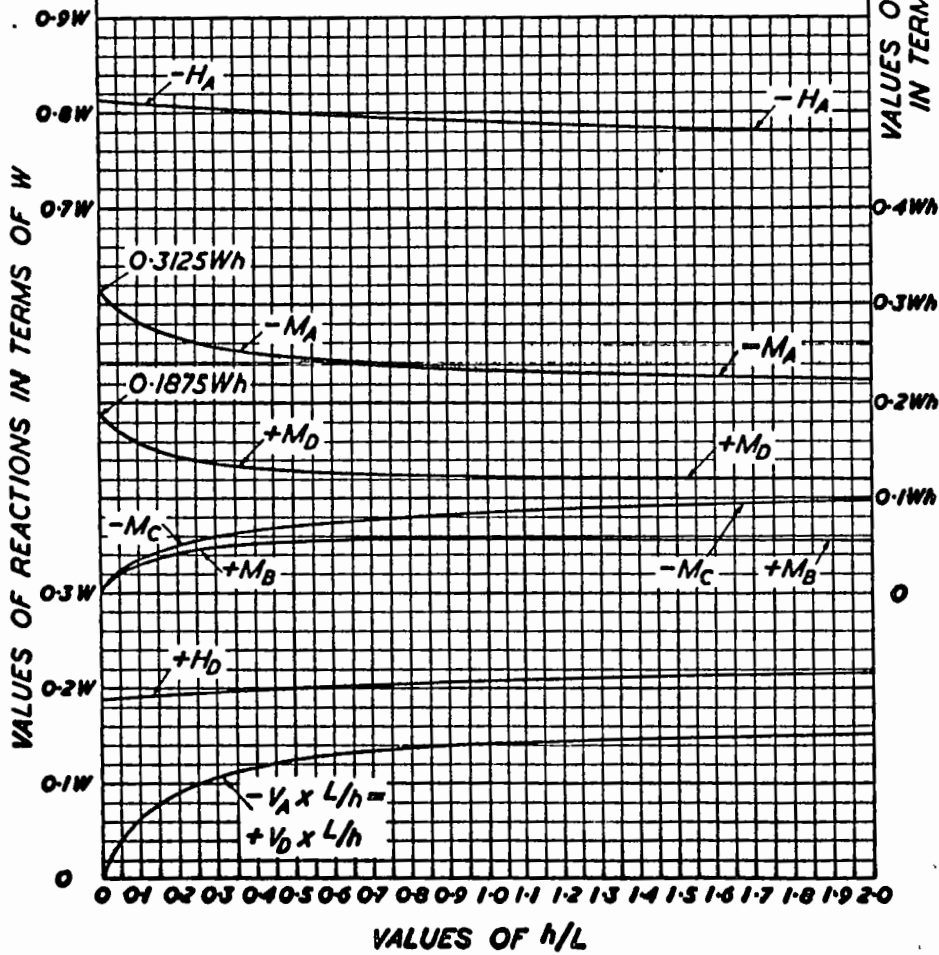


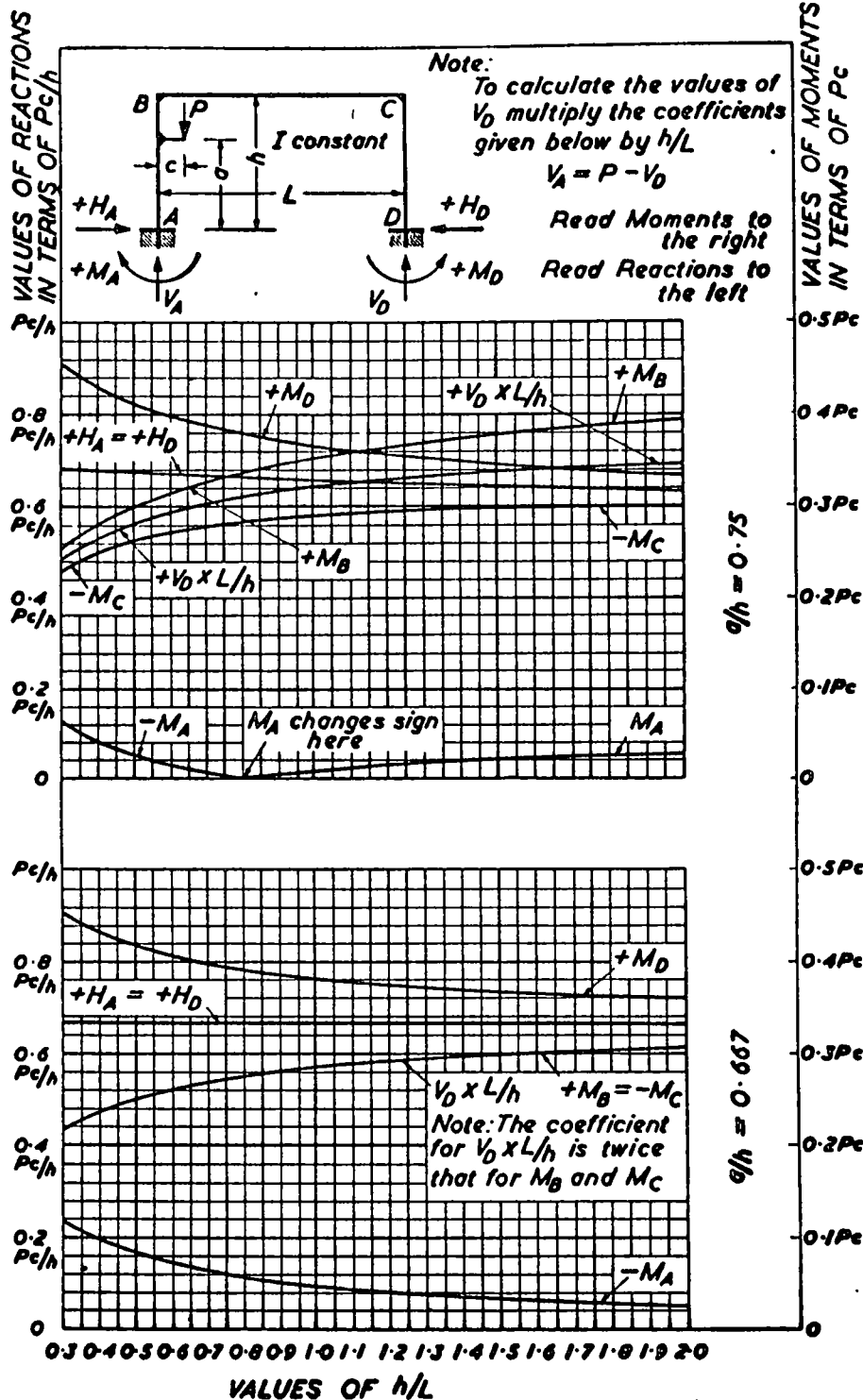


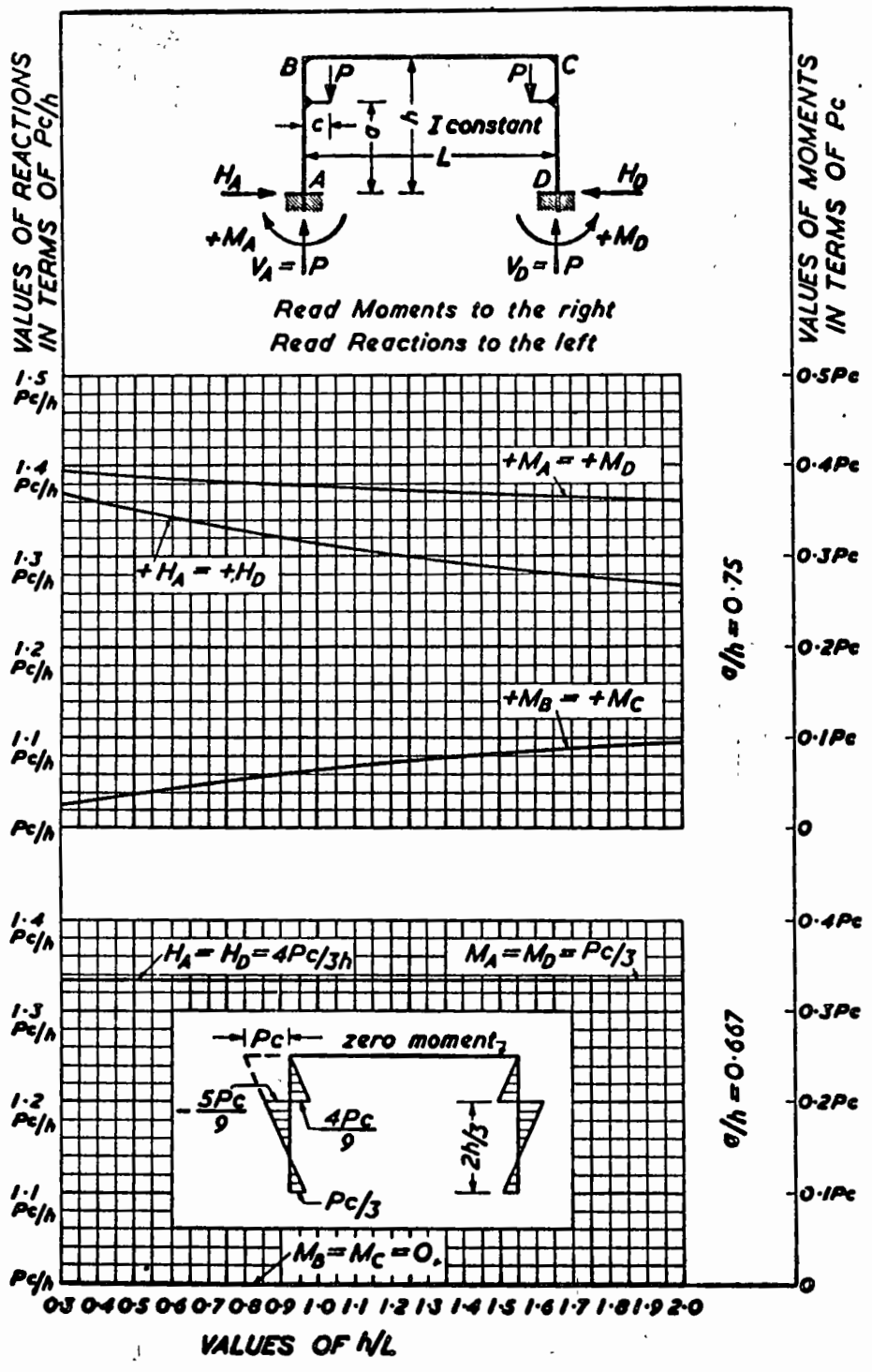
Note. To calculate  $-V_A$  or  $+V_D$  multiply the coefficient given below by  $h/L$



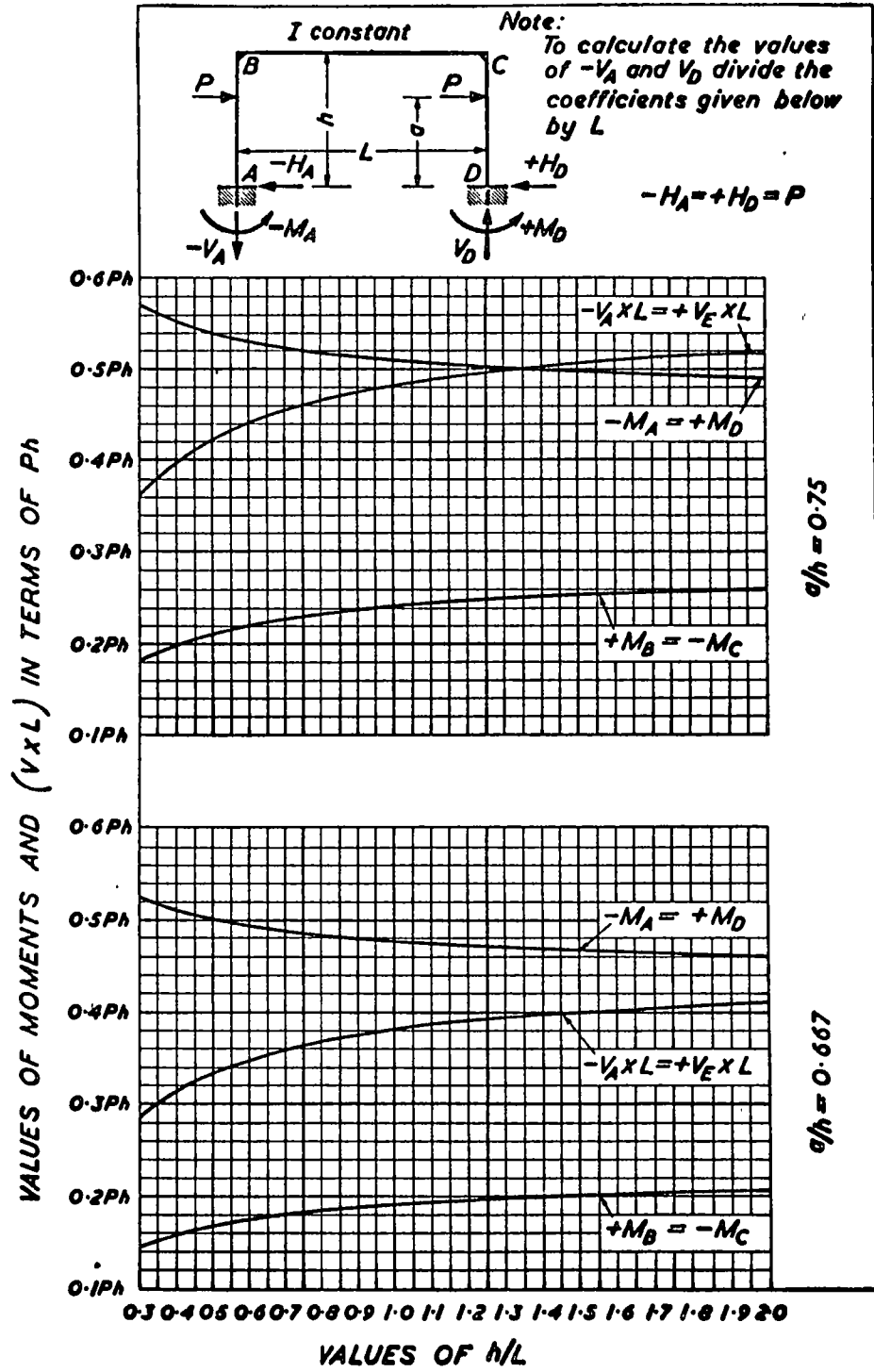
Read Moments to the right  
Read Reactions to the left

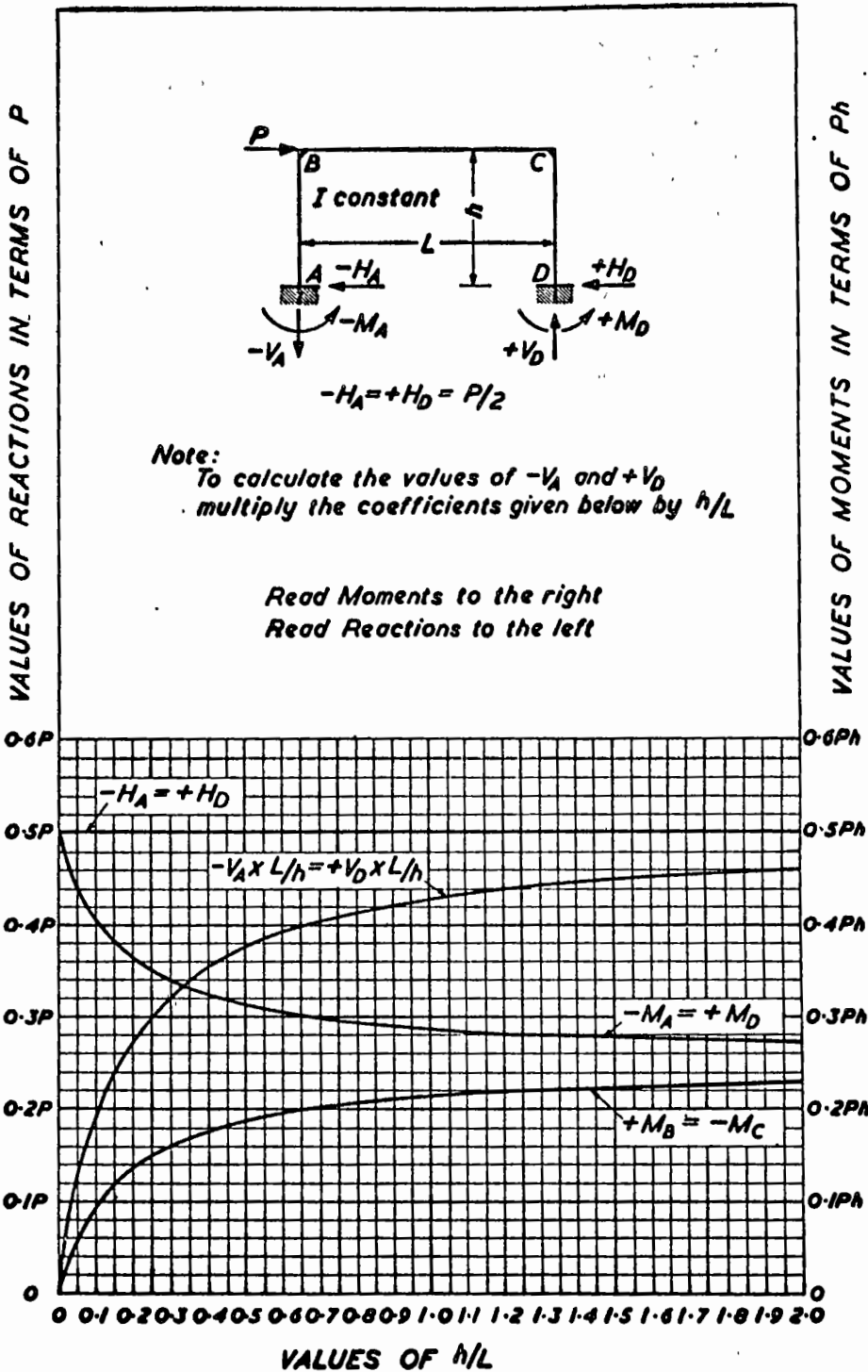


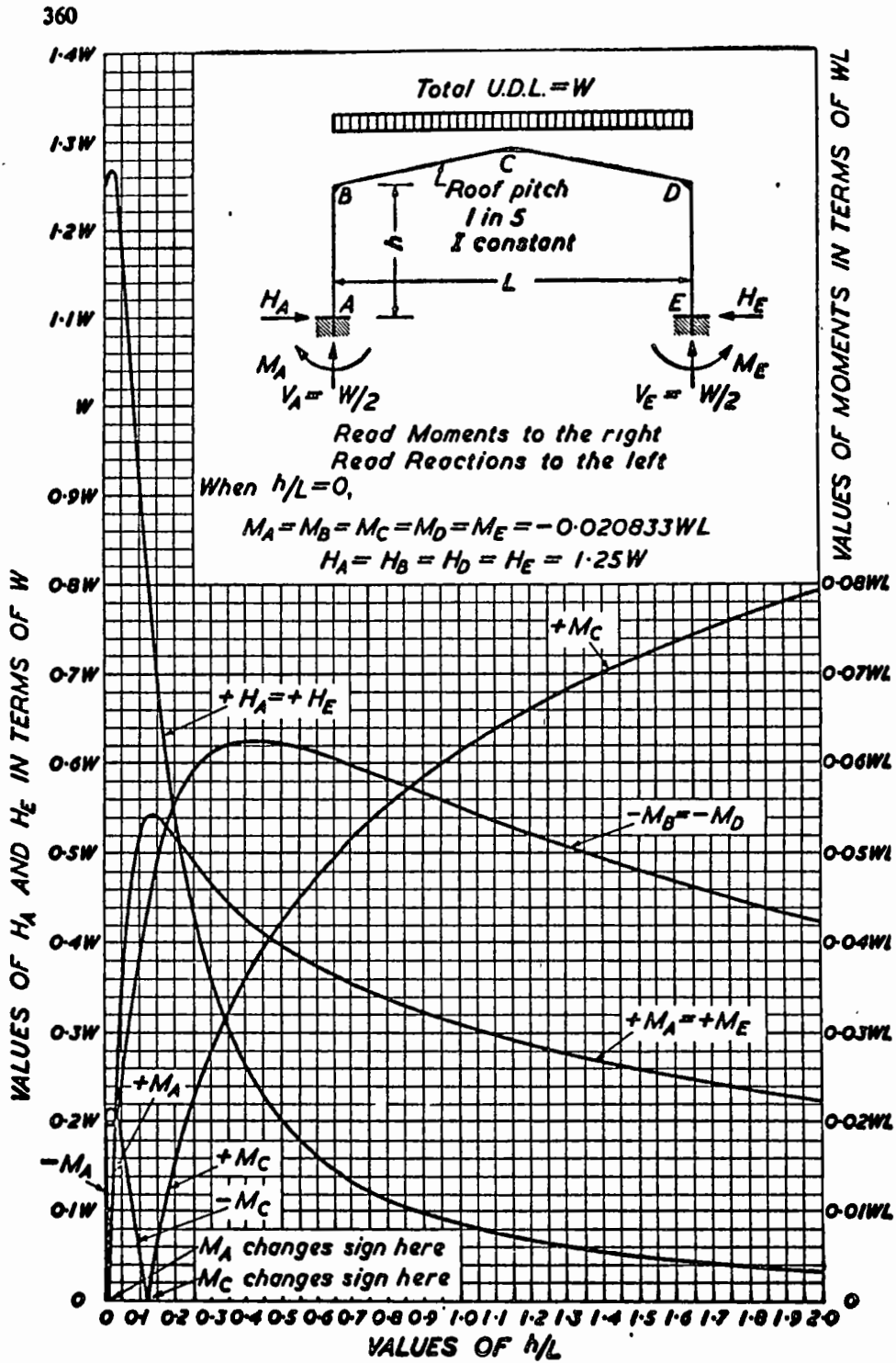


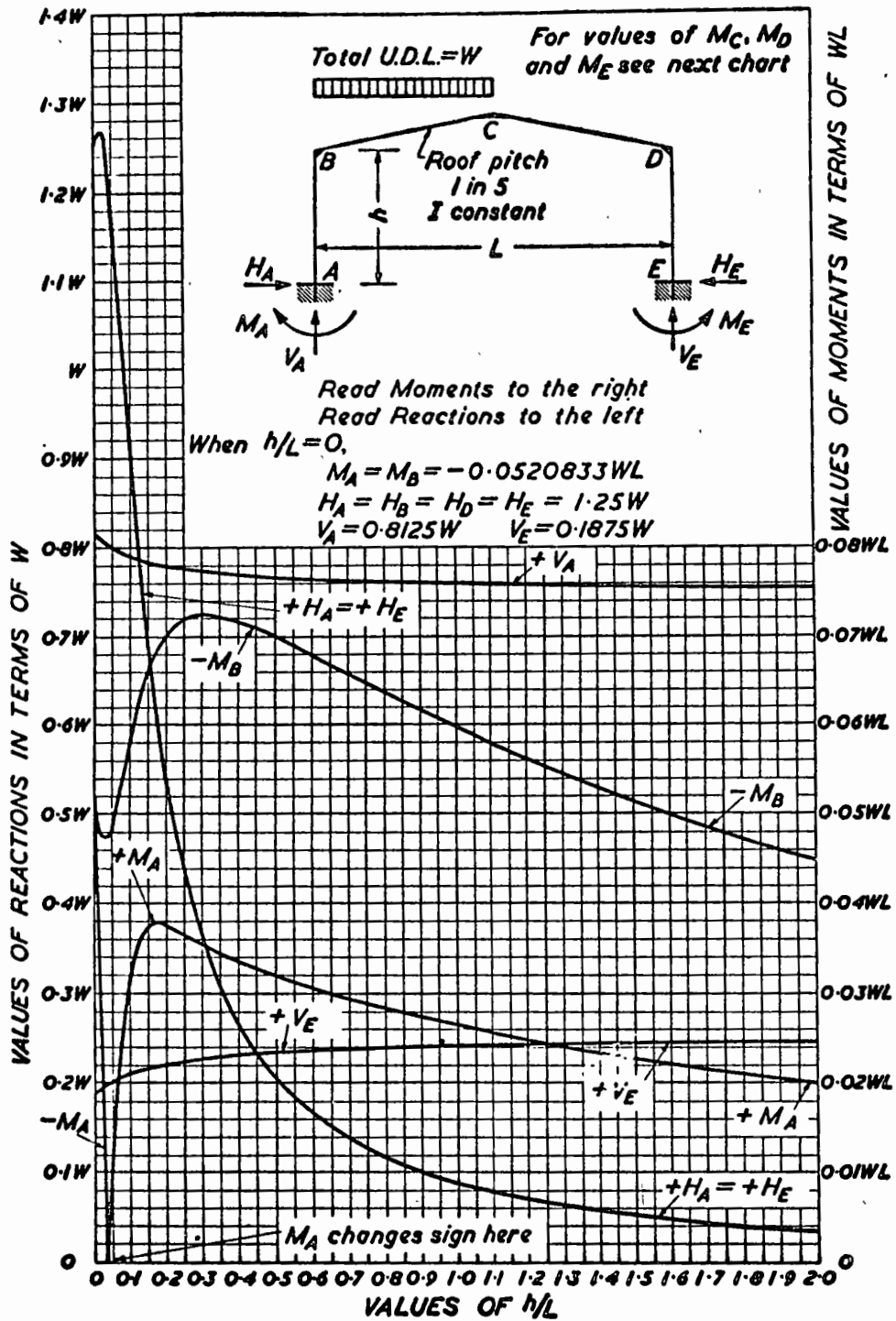


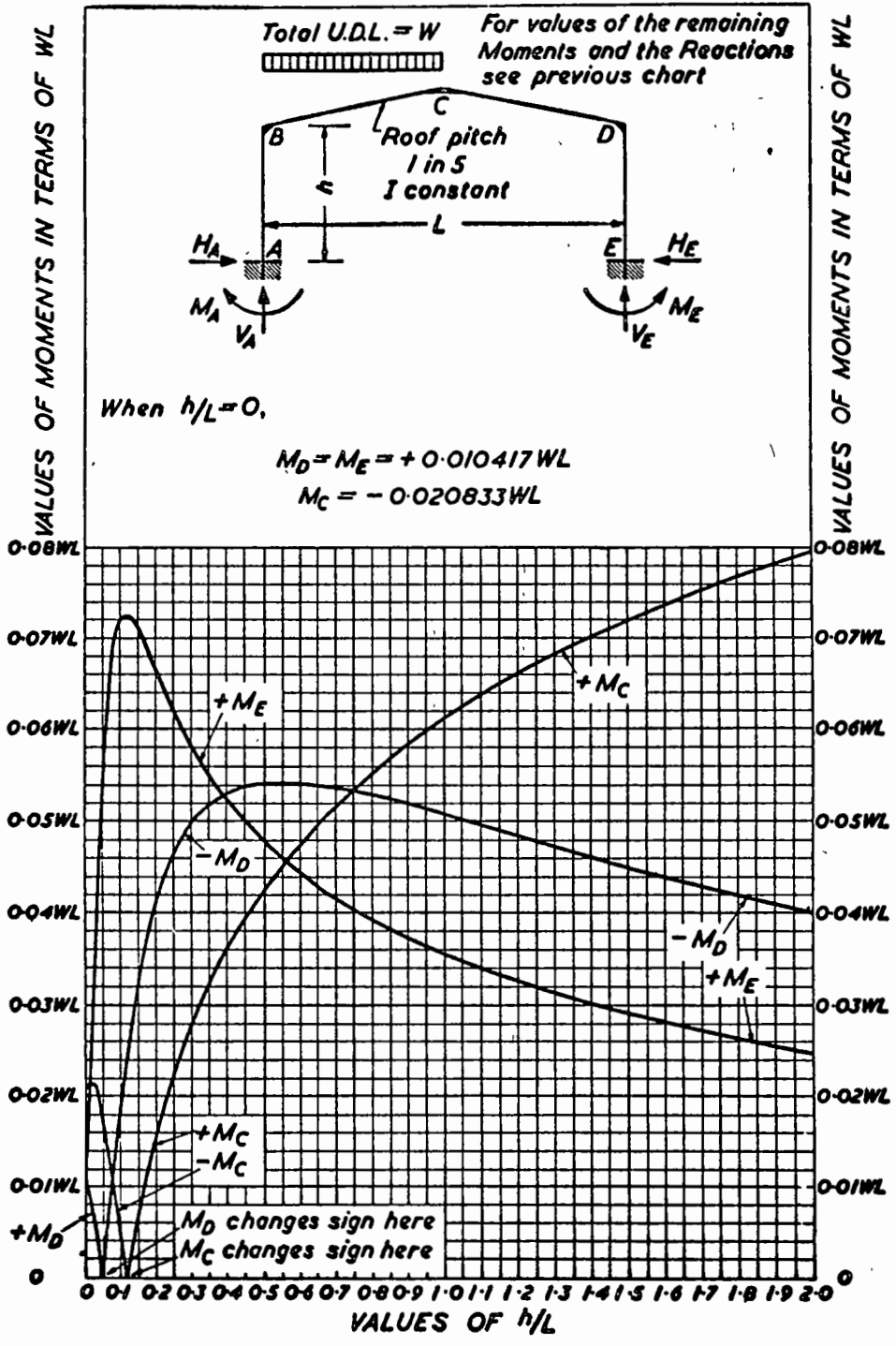


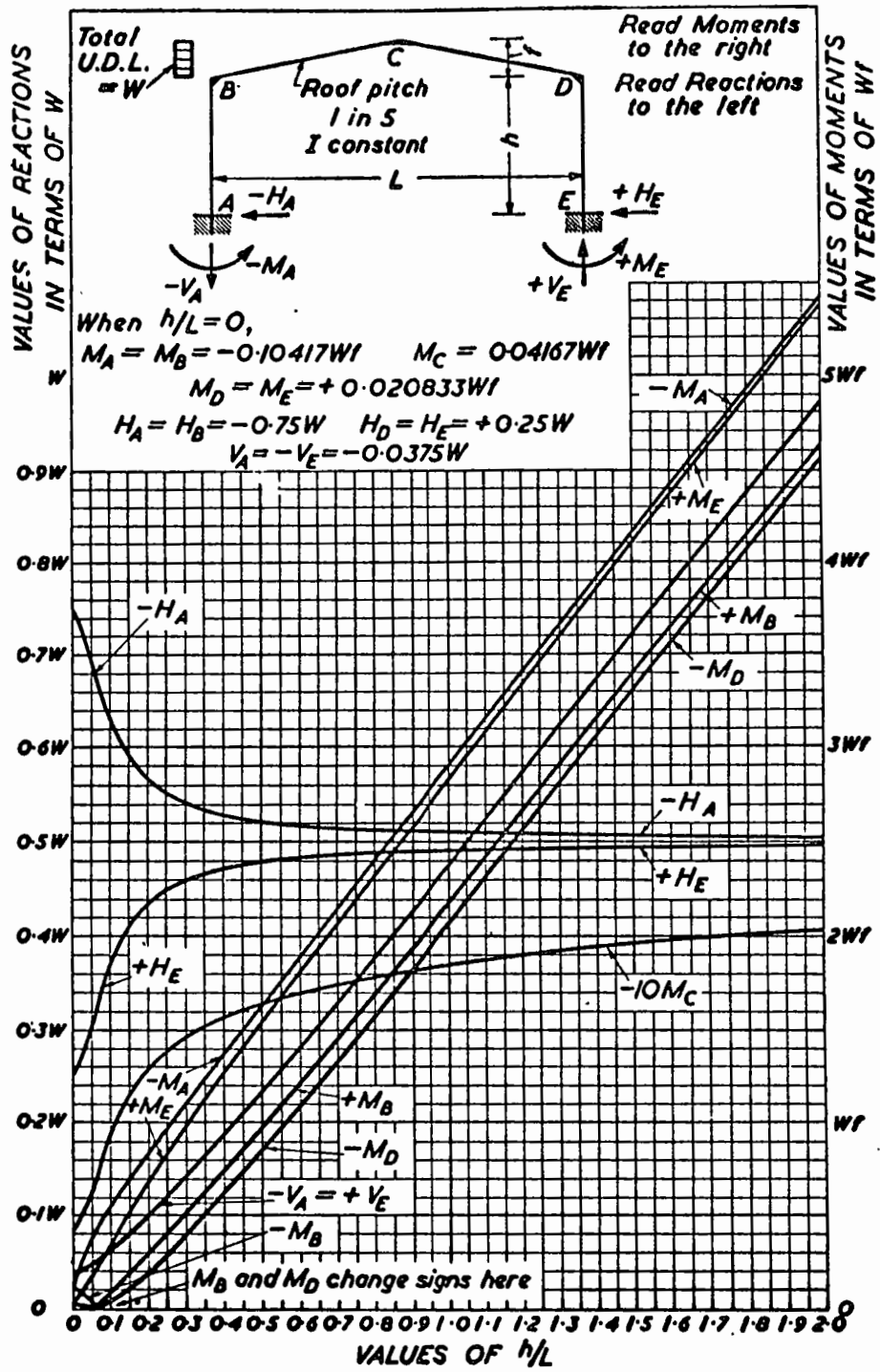


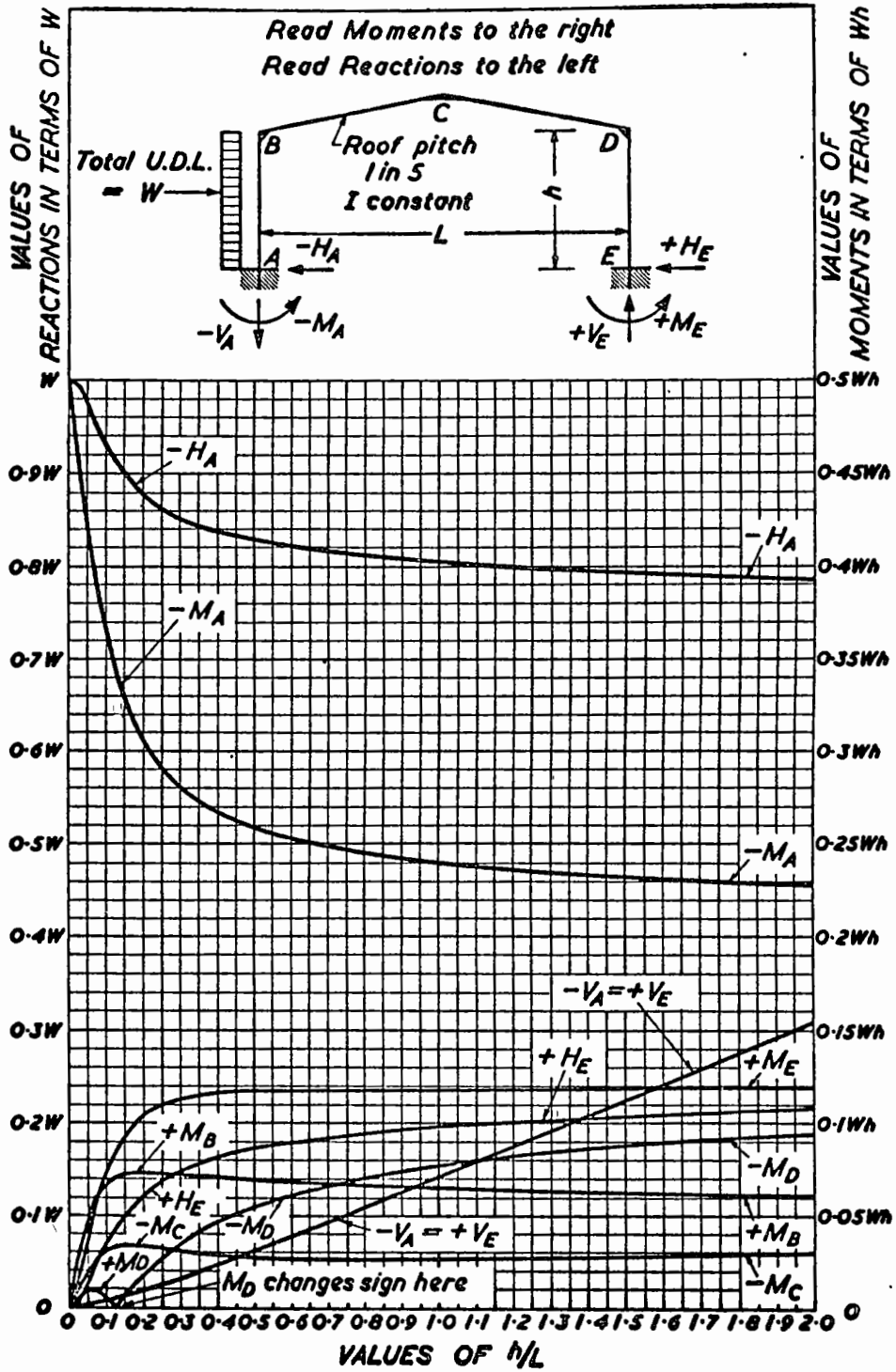


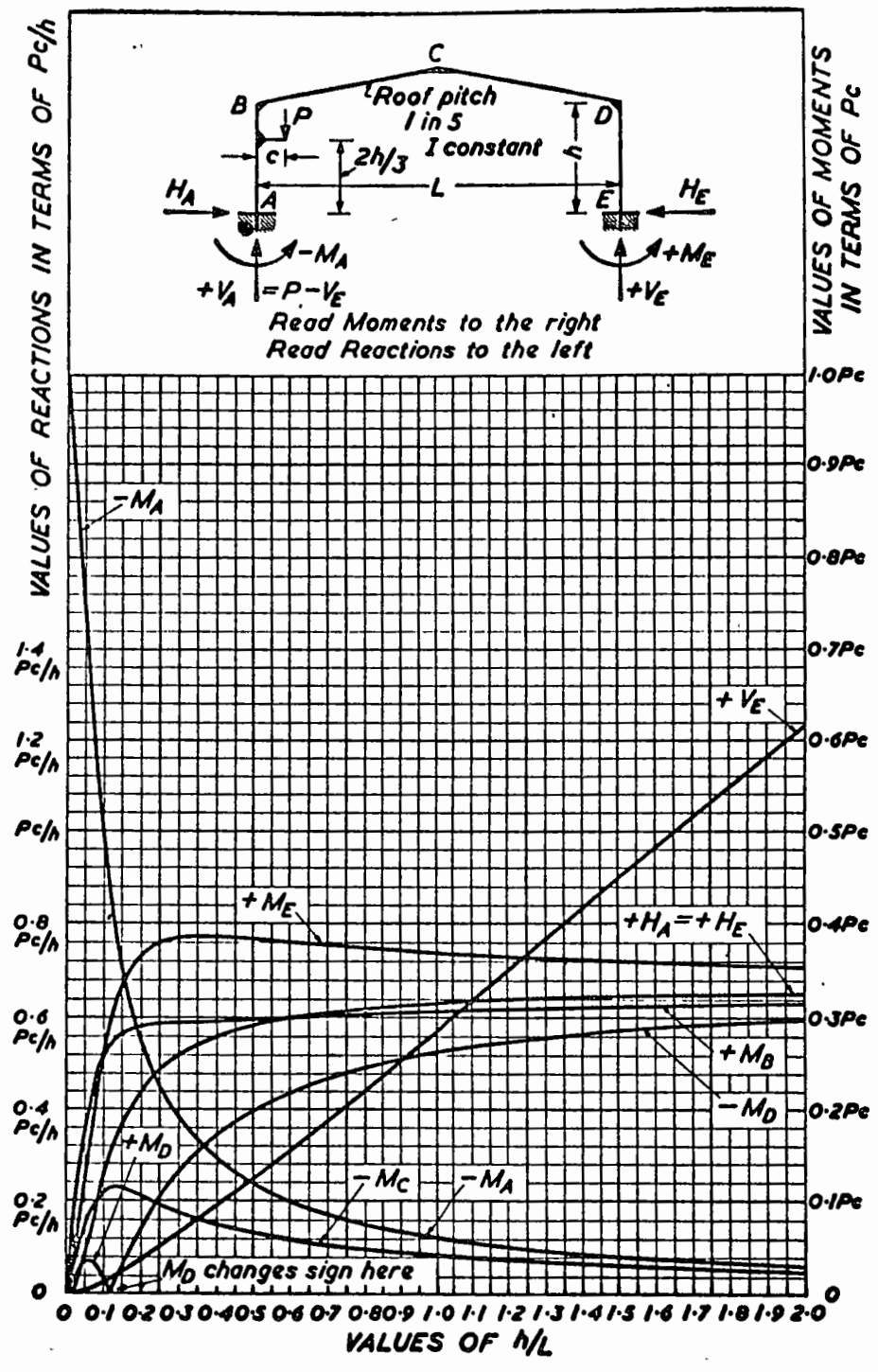




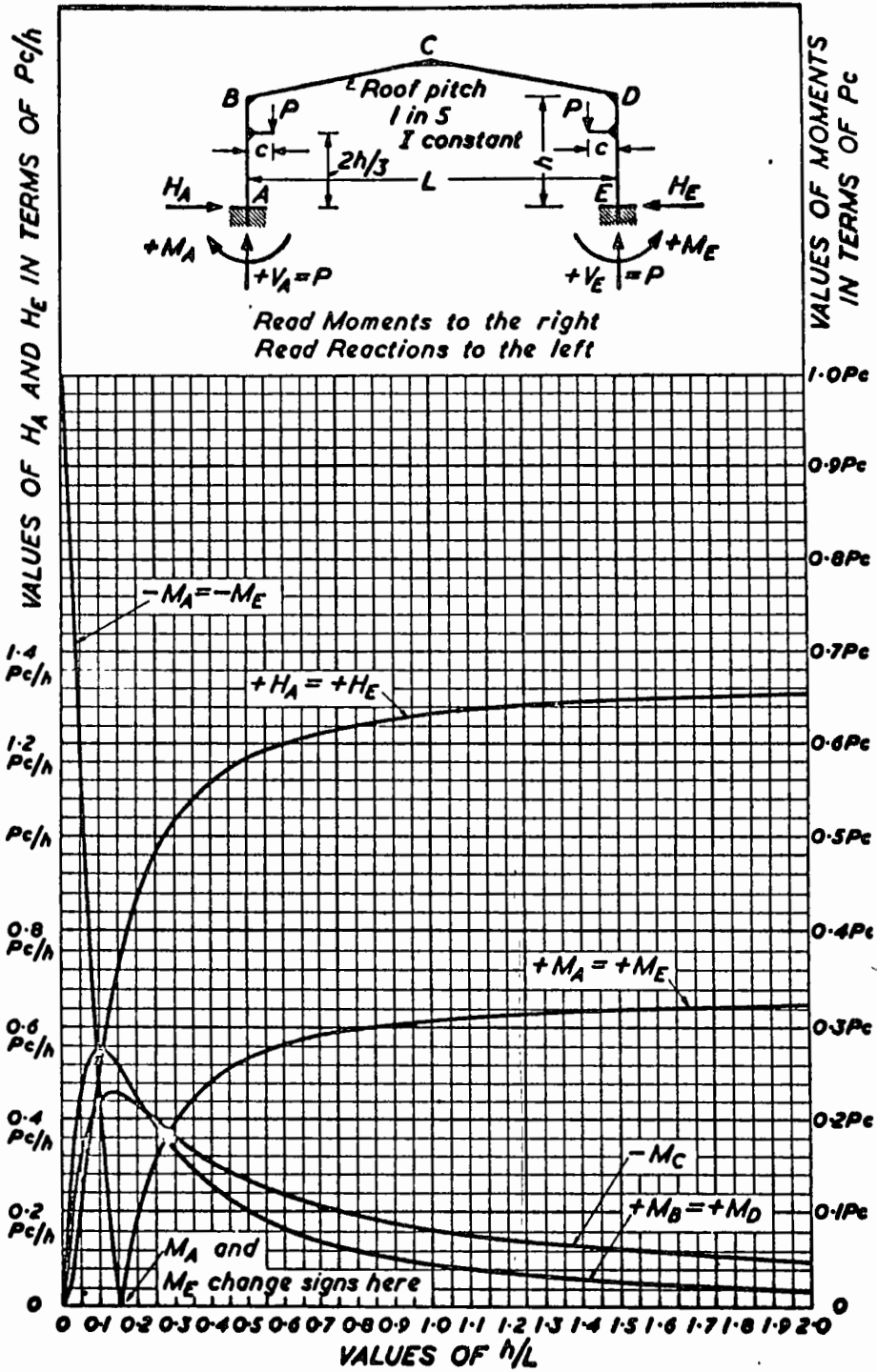


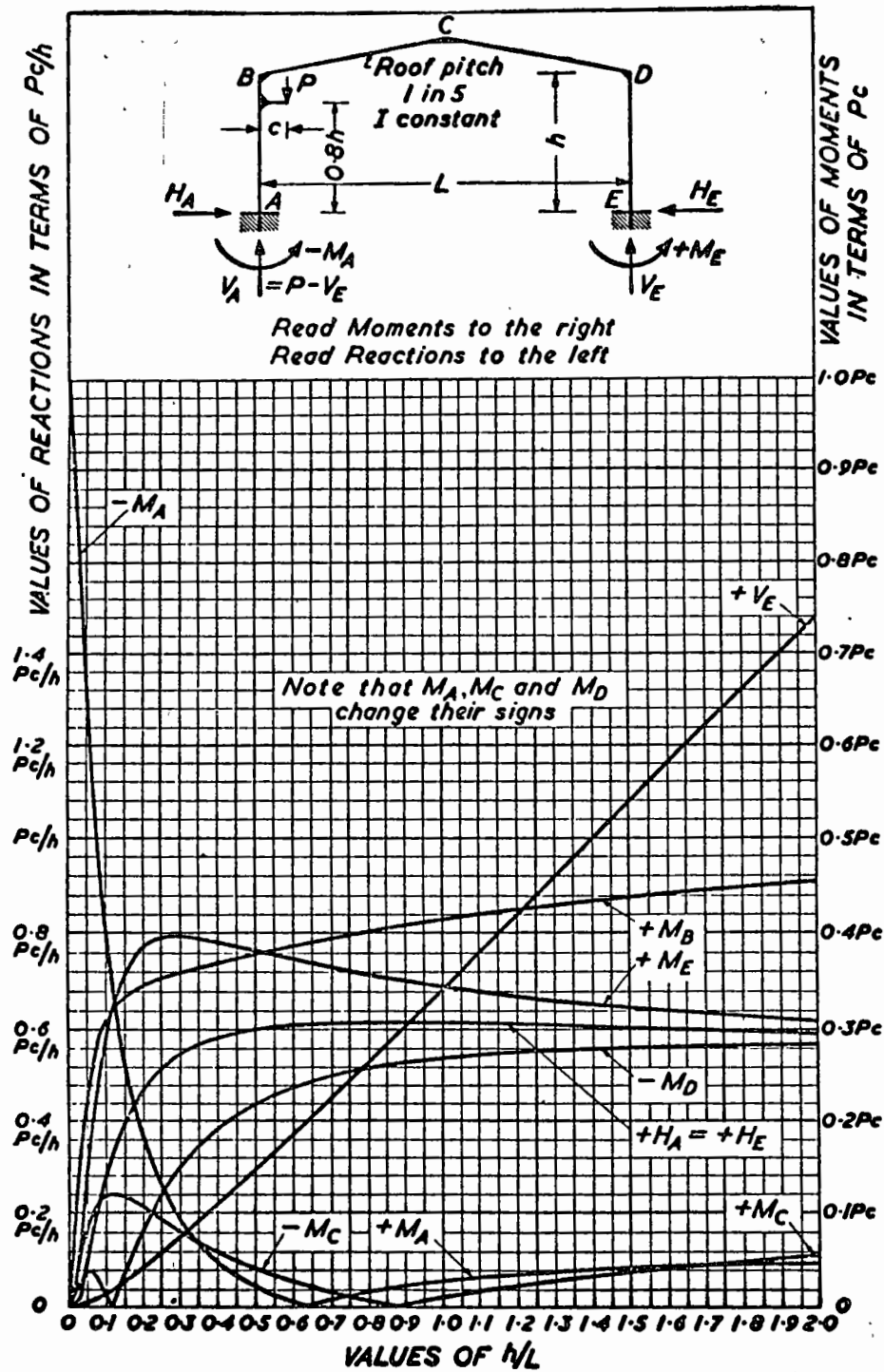


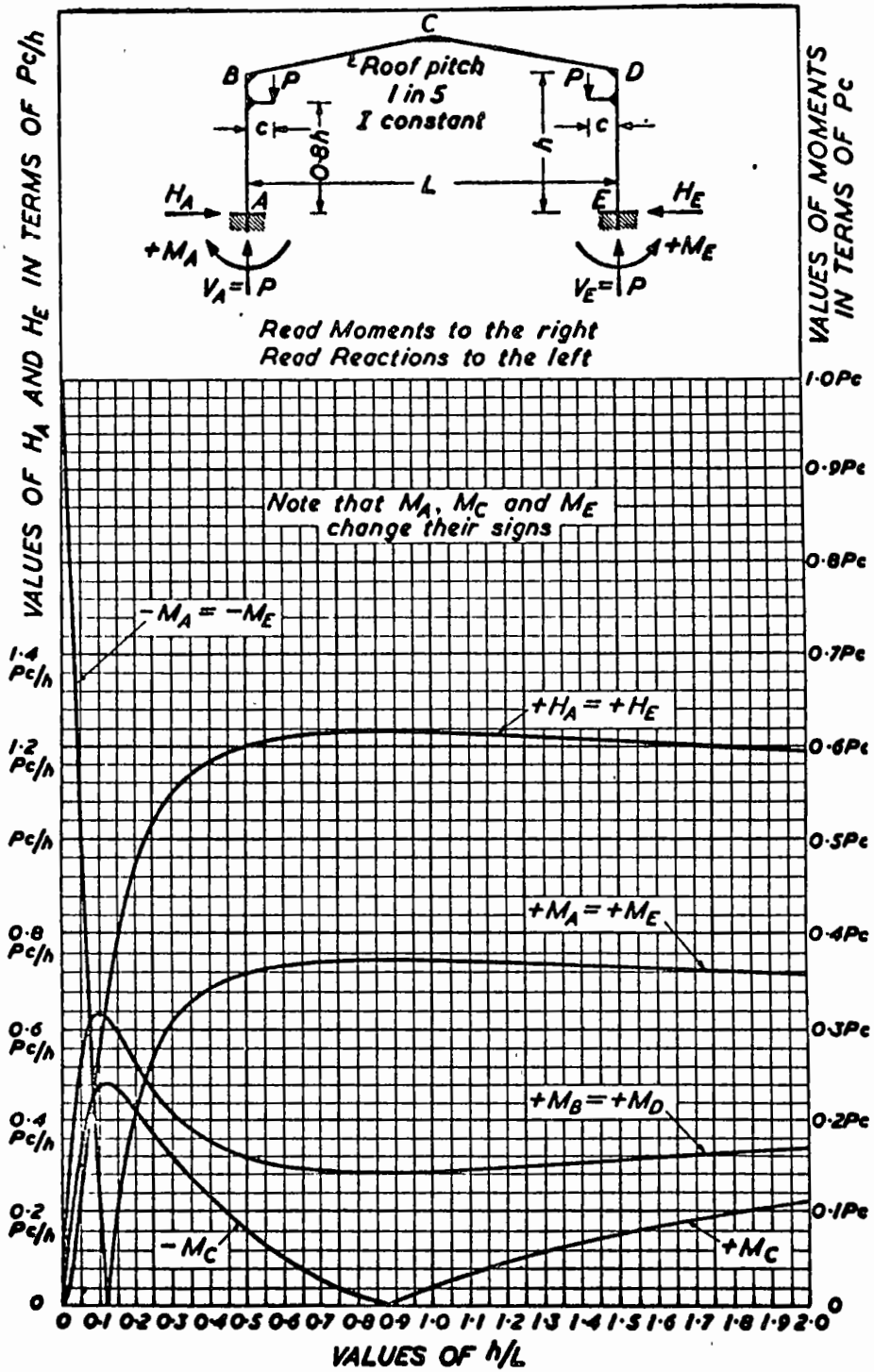




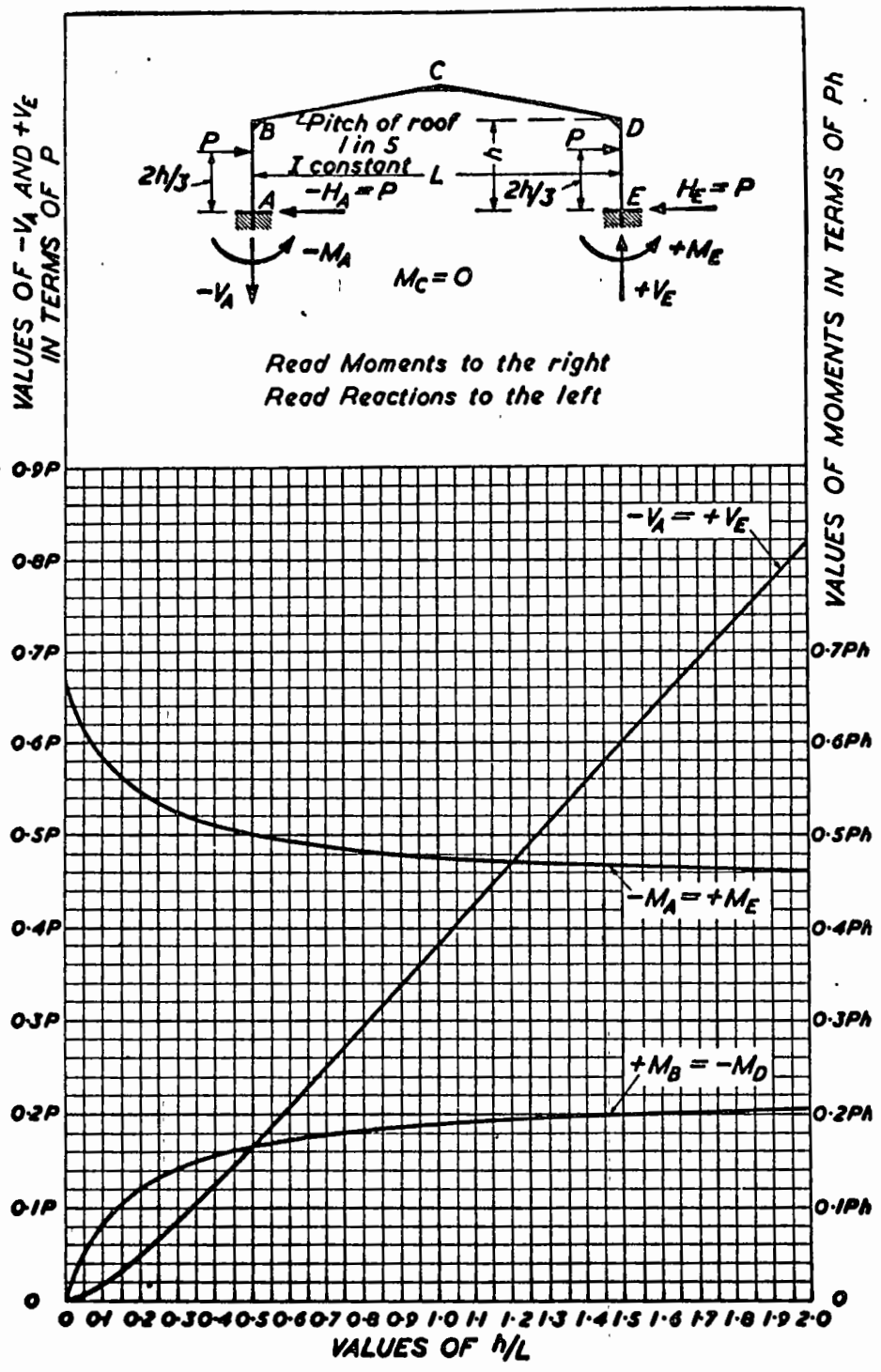


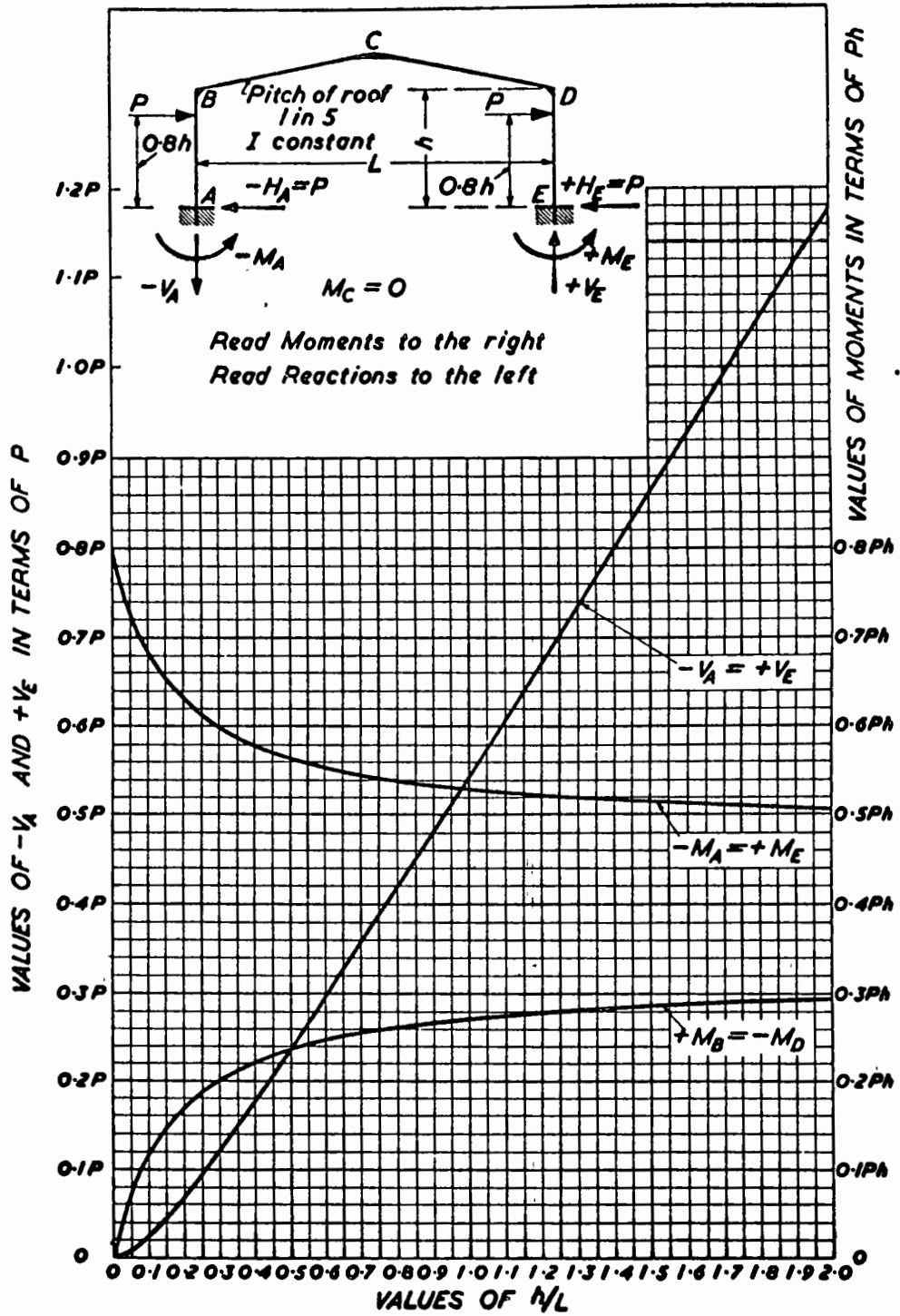


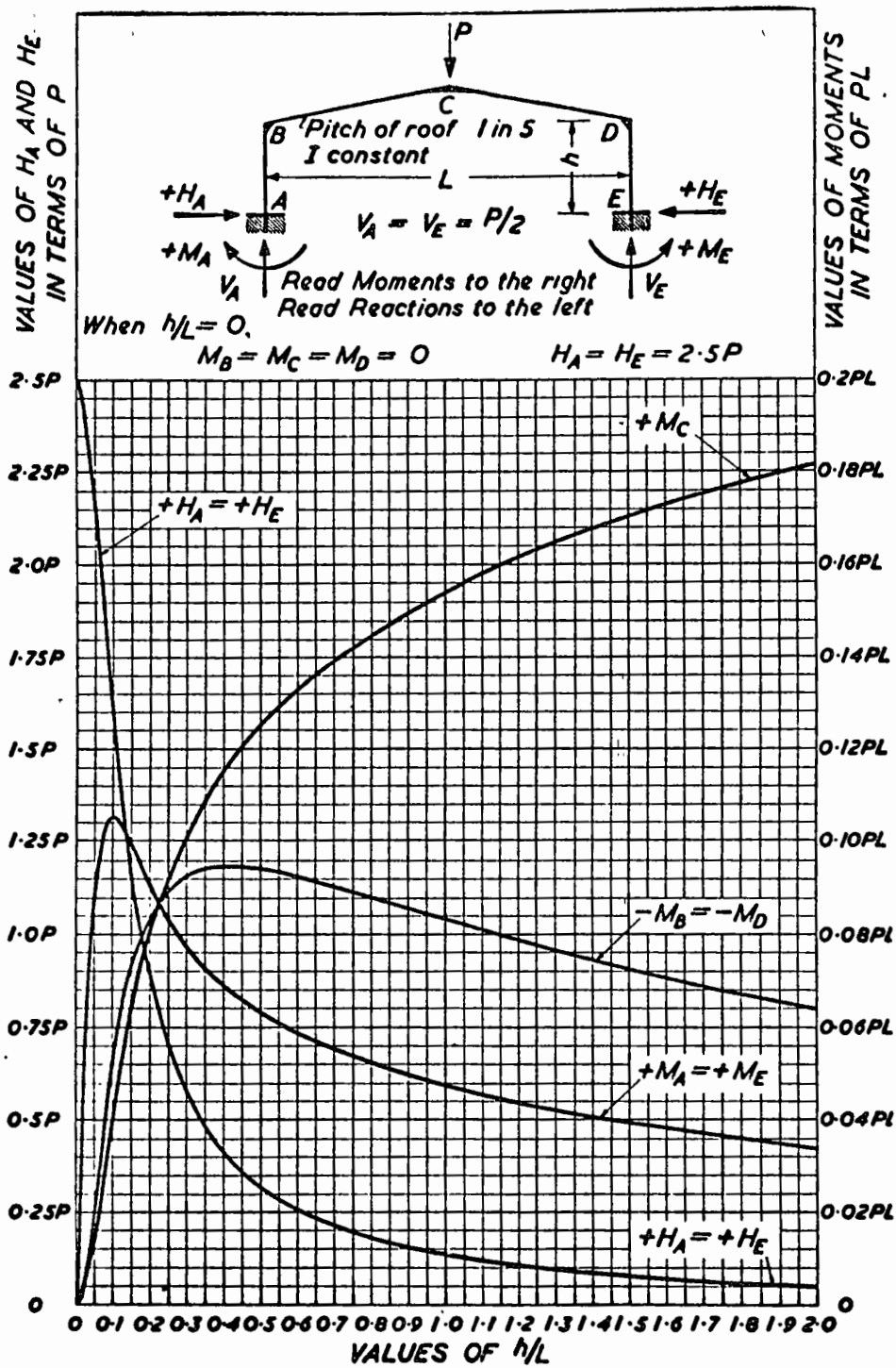


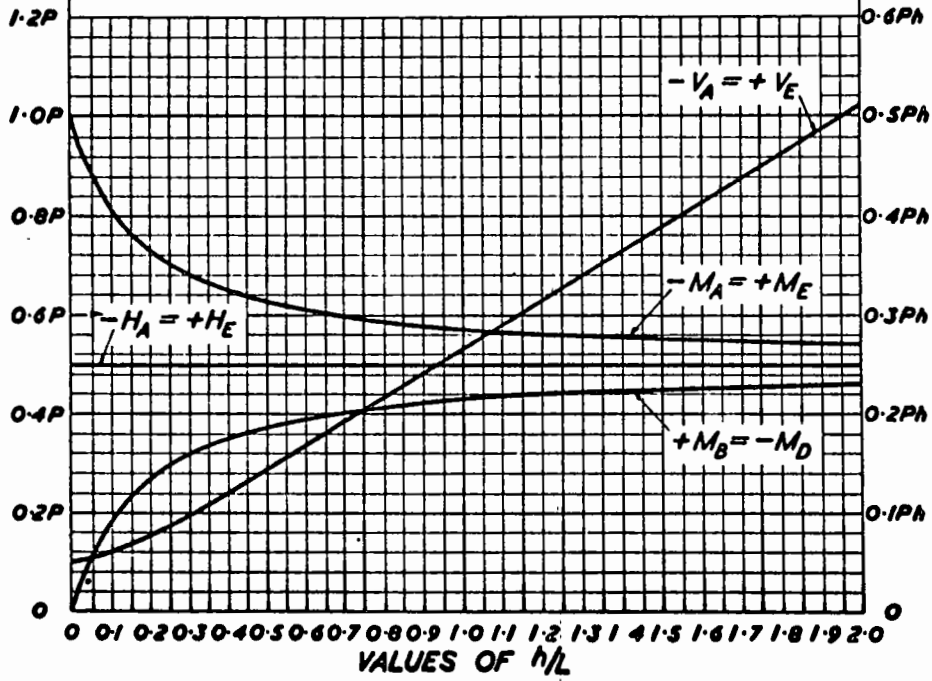
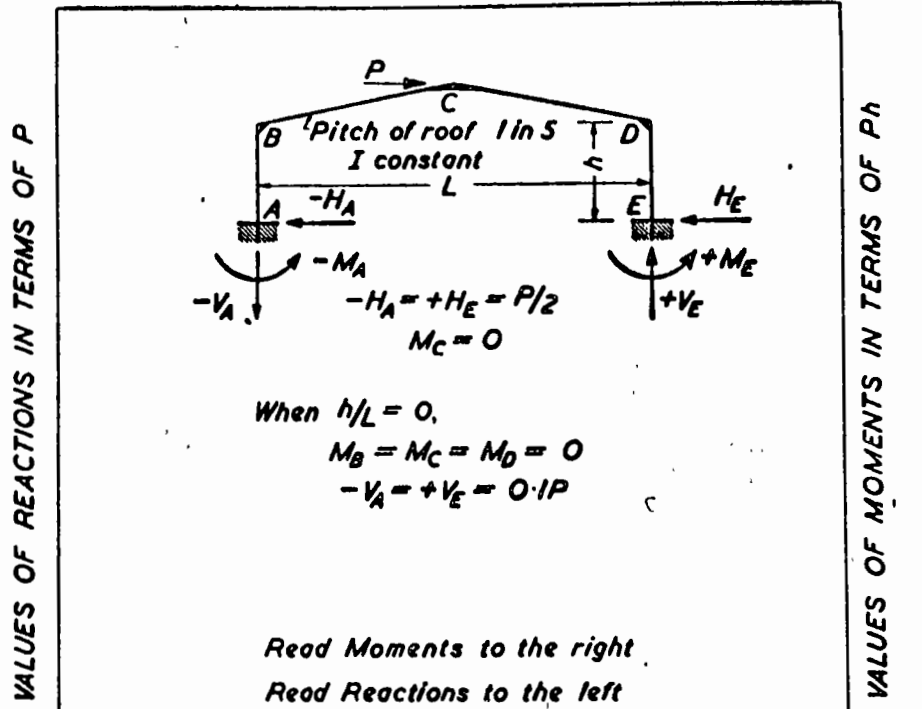


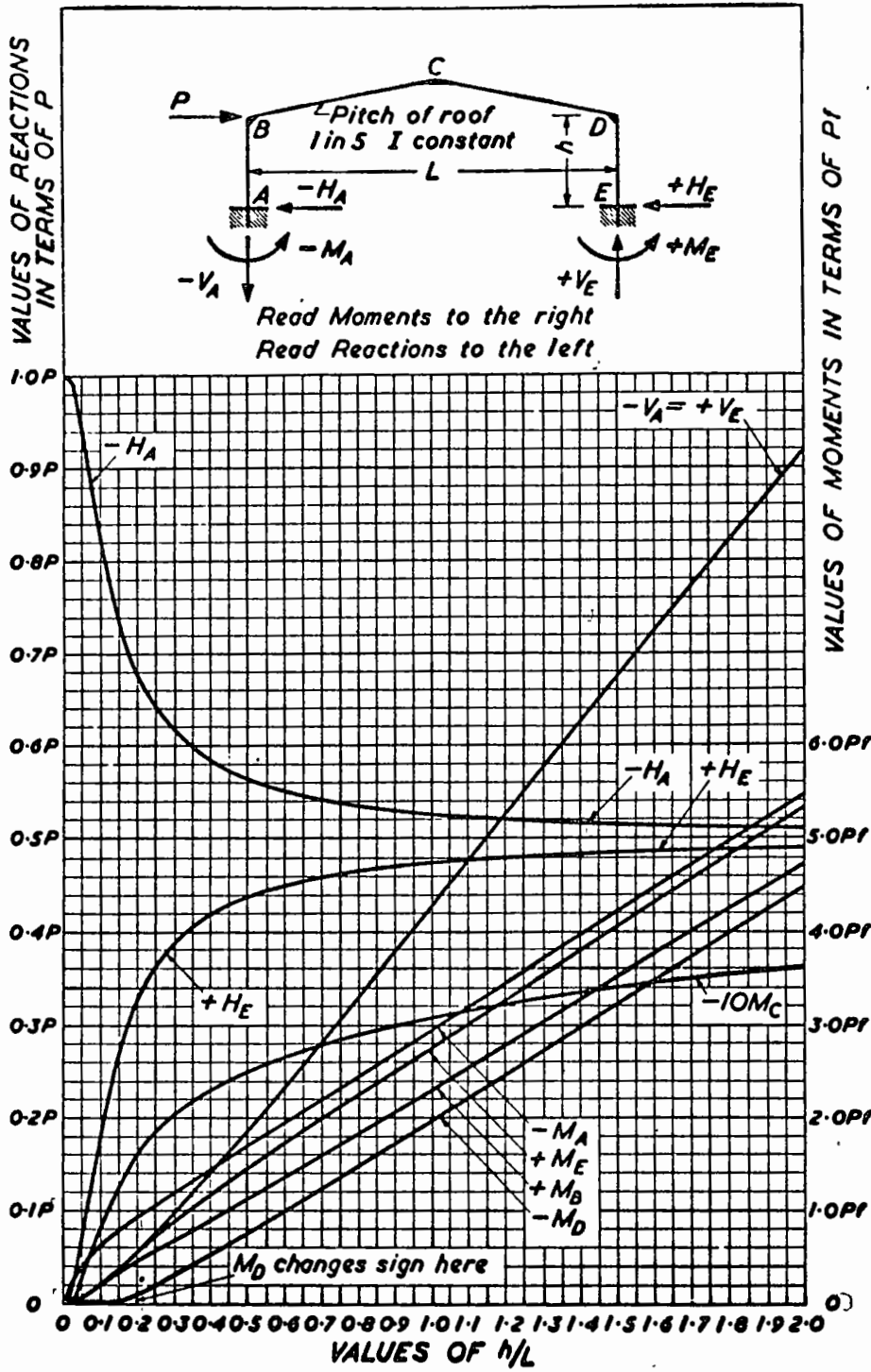
64



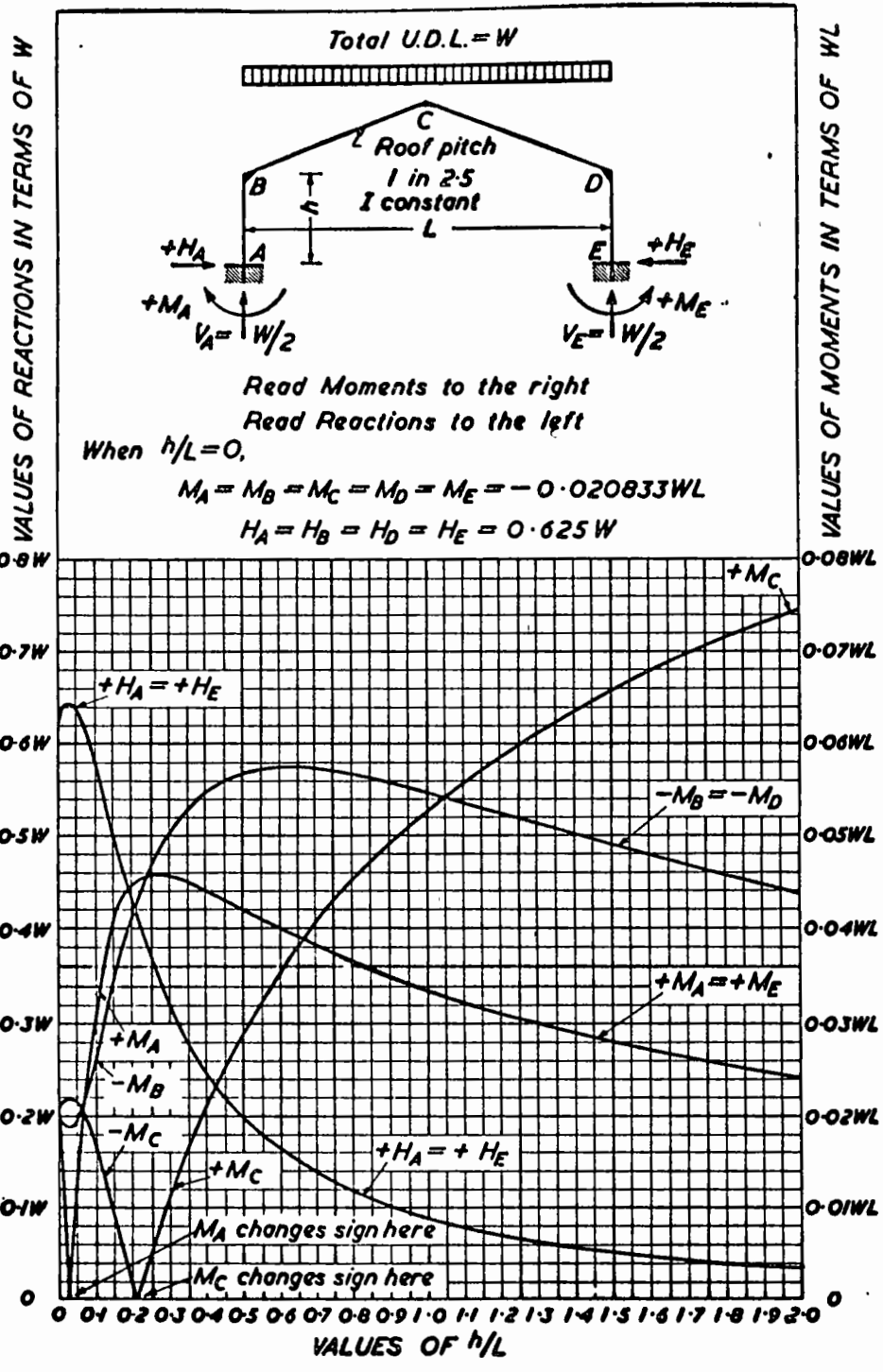


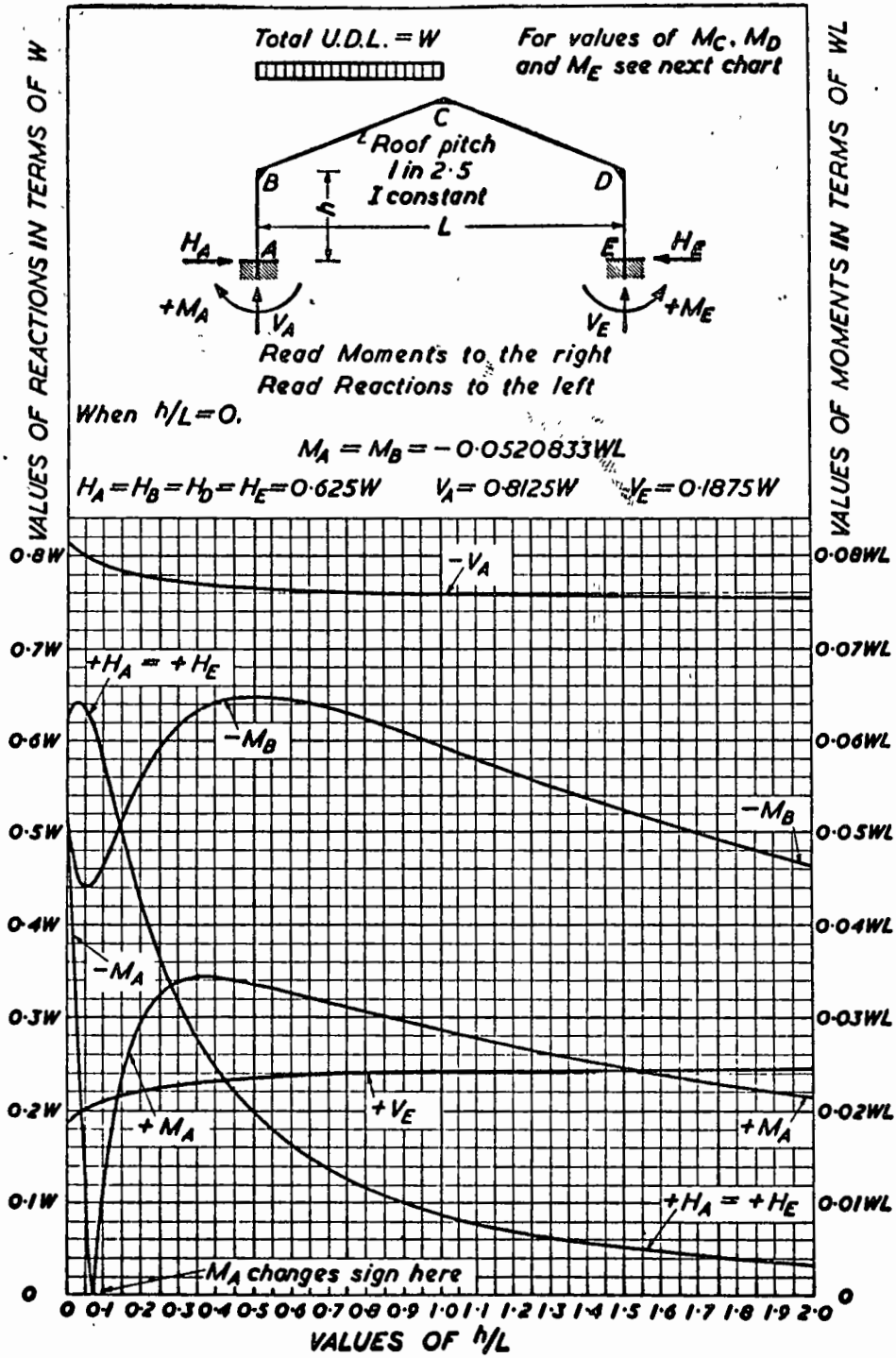


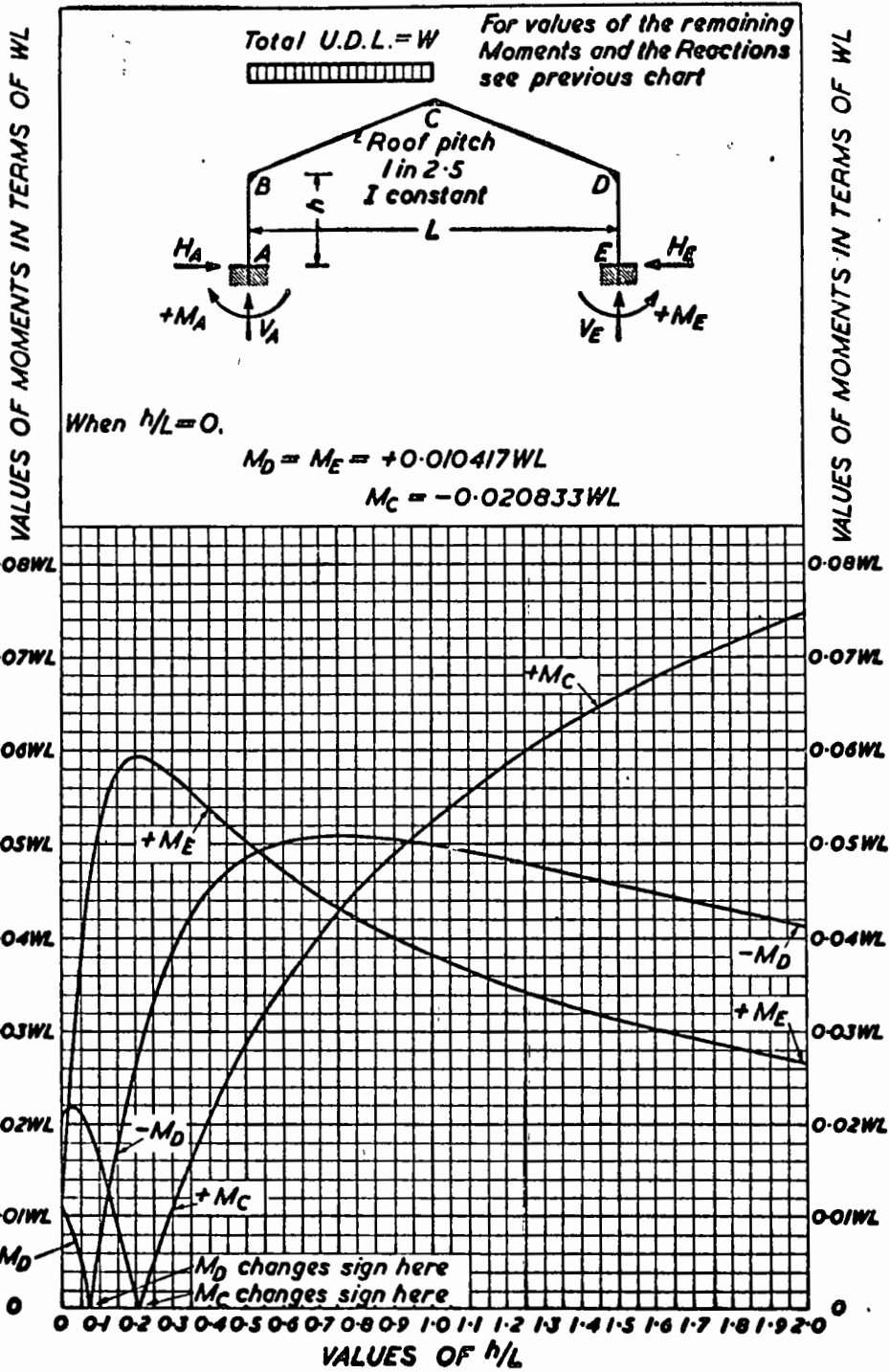


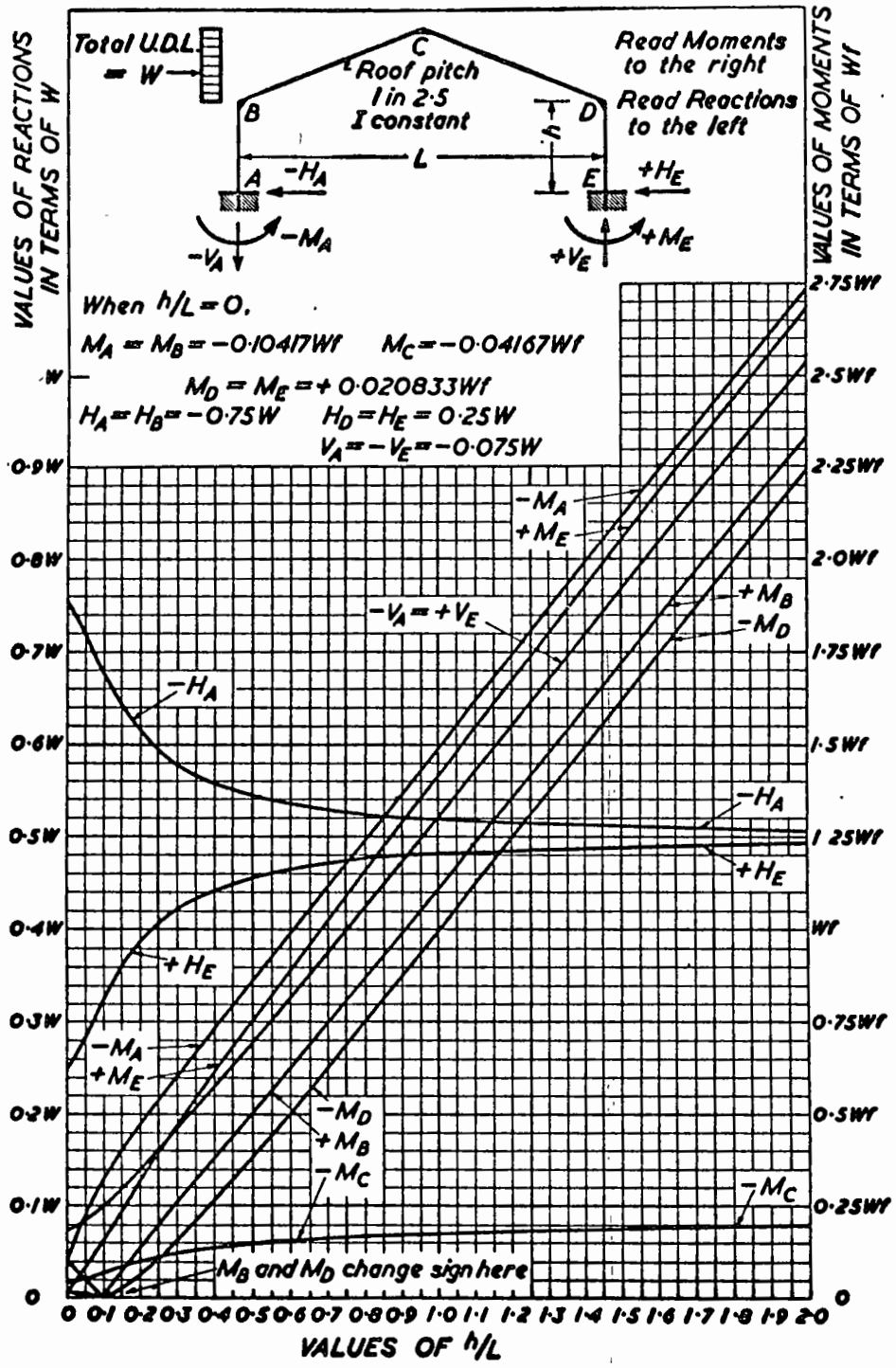


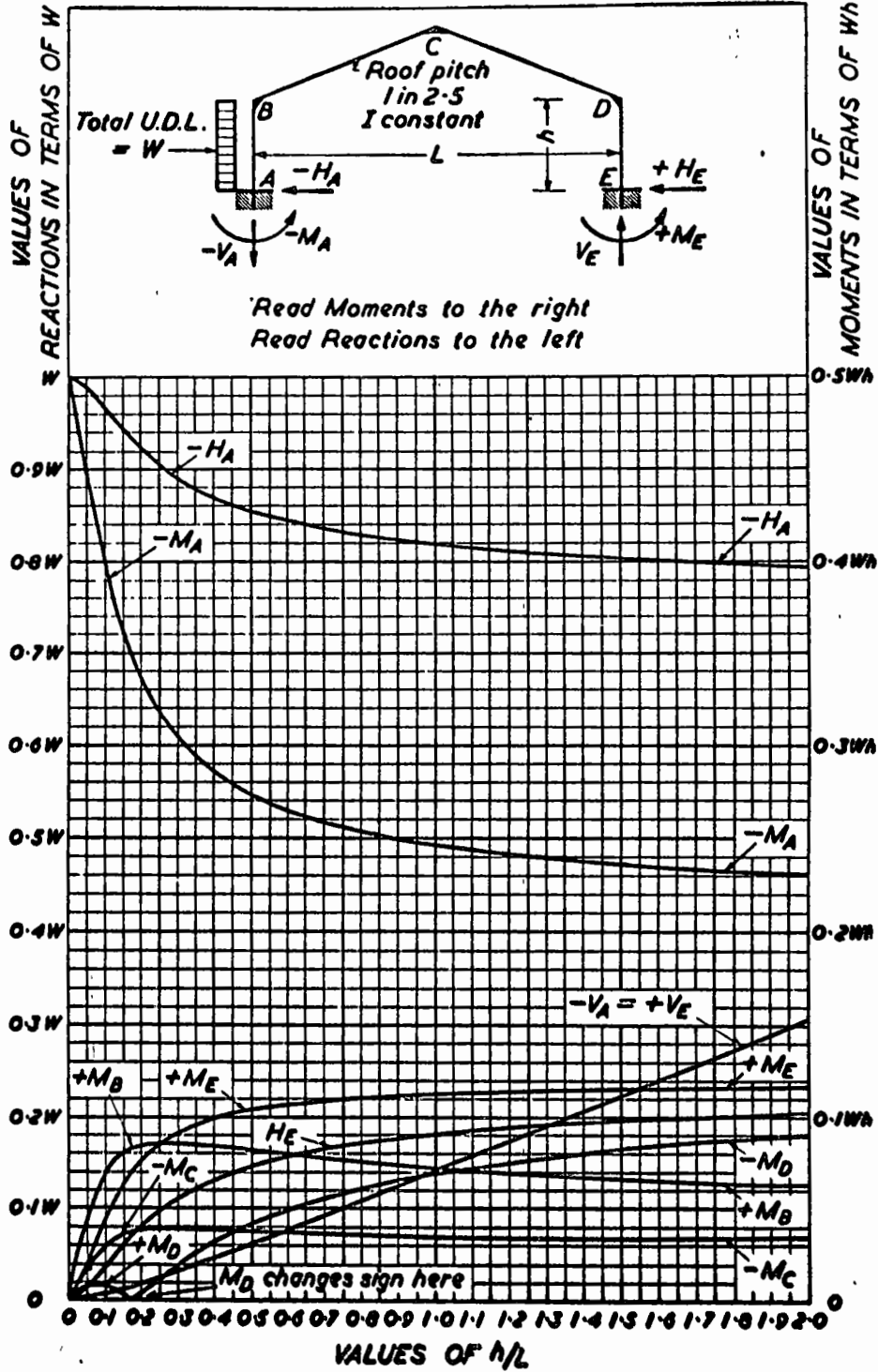


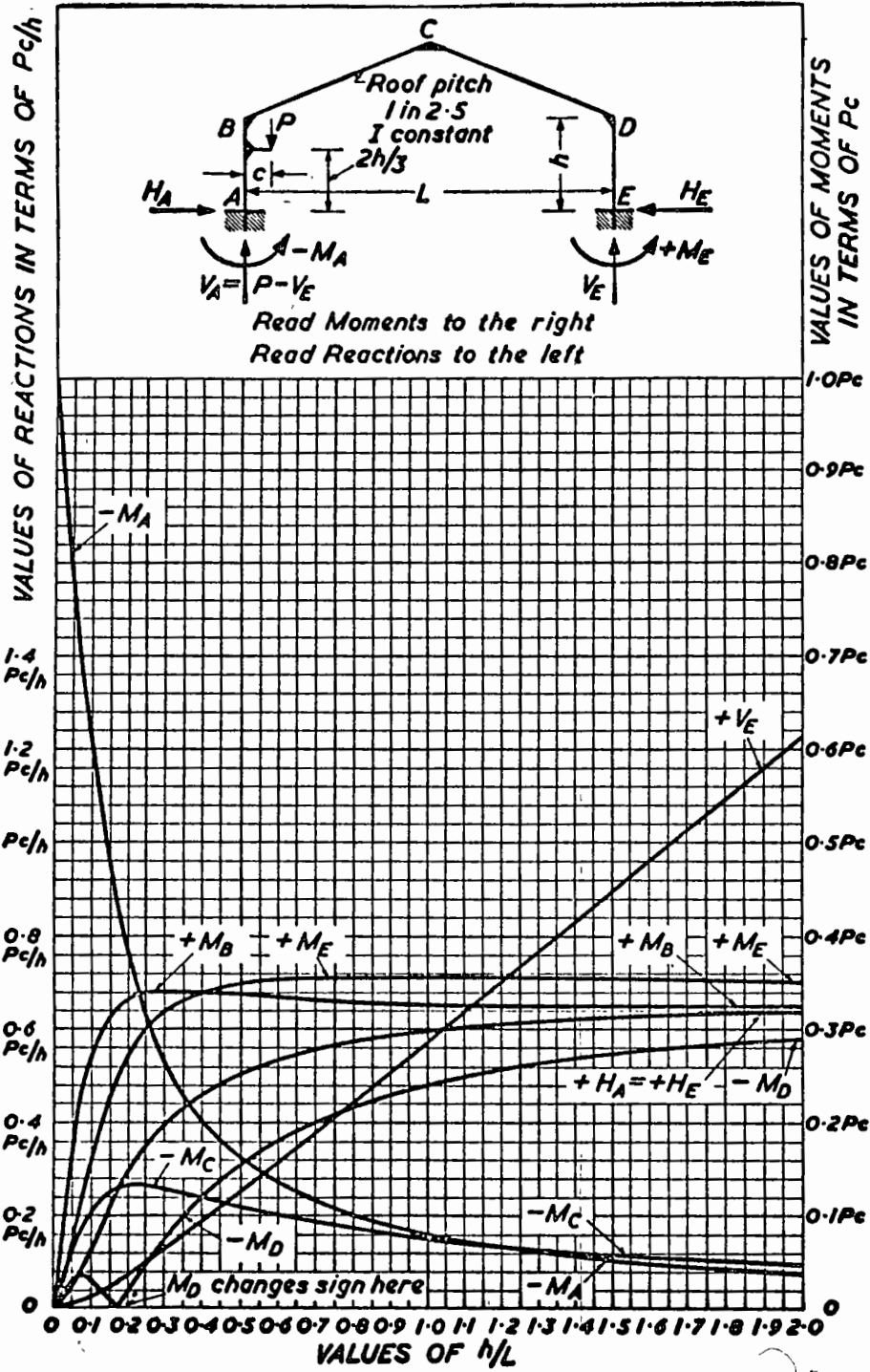


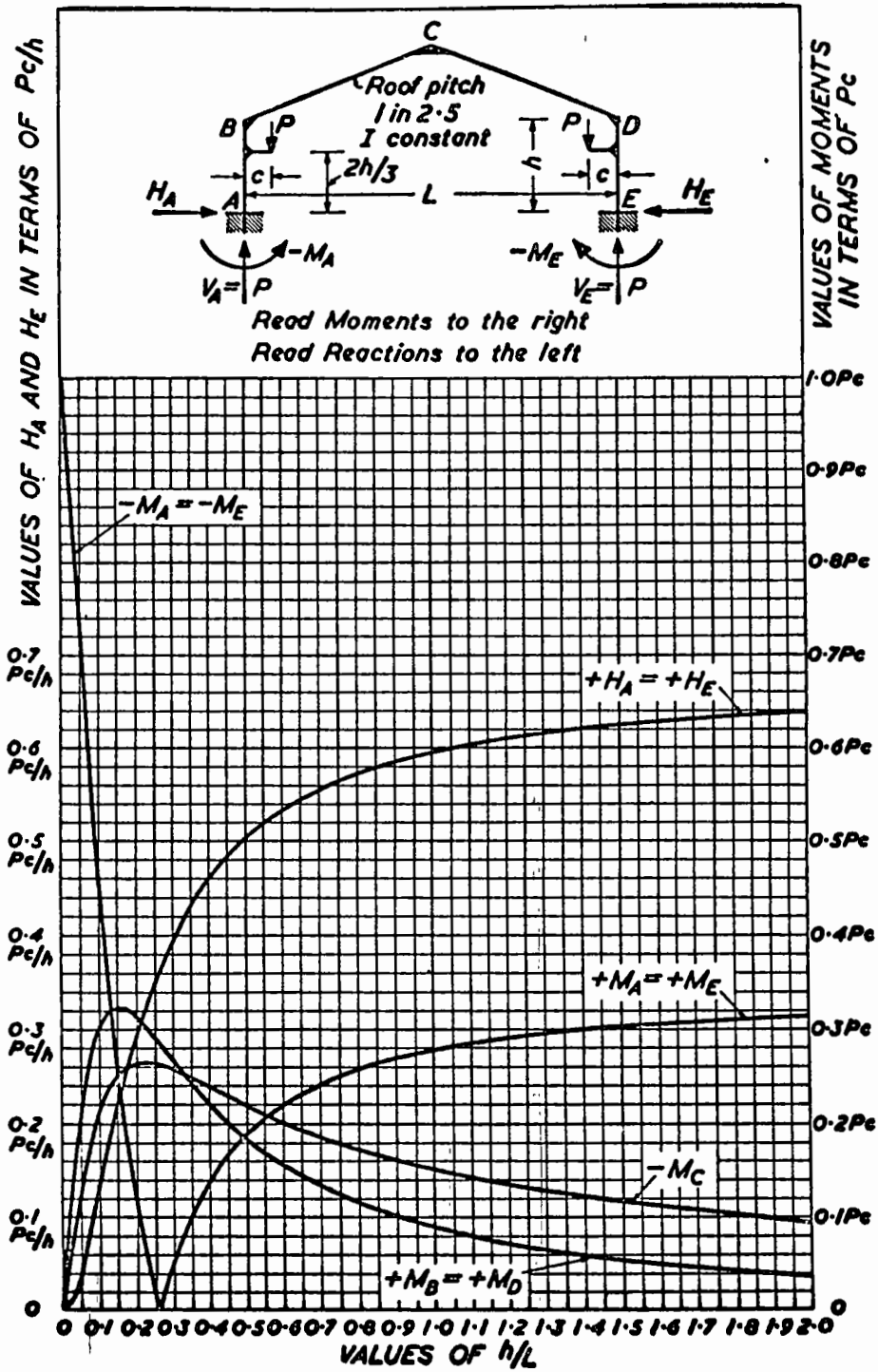


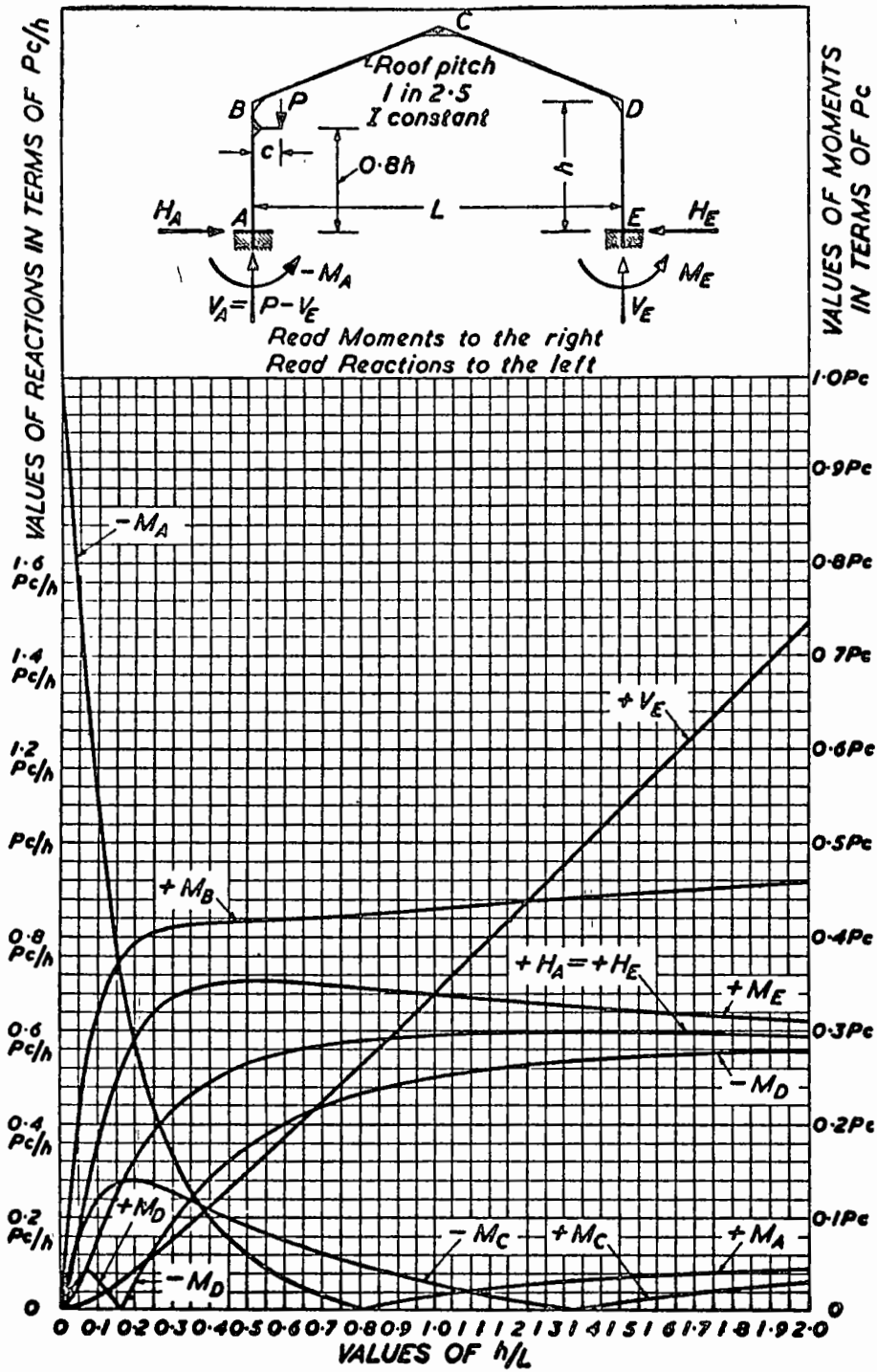




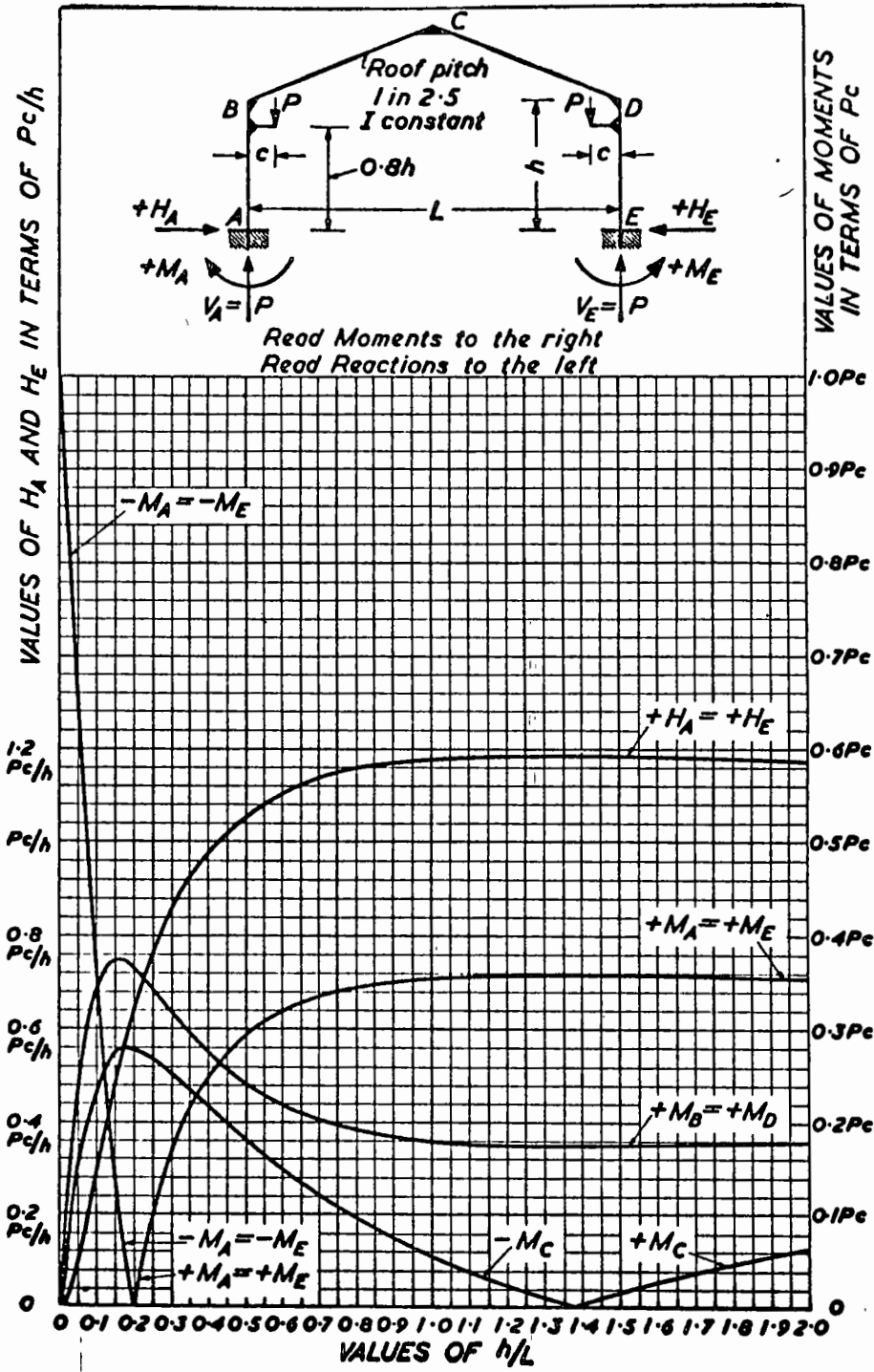


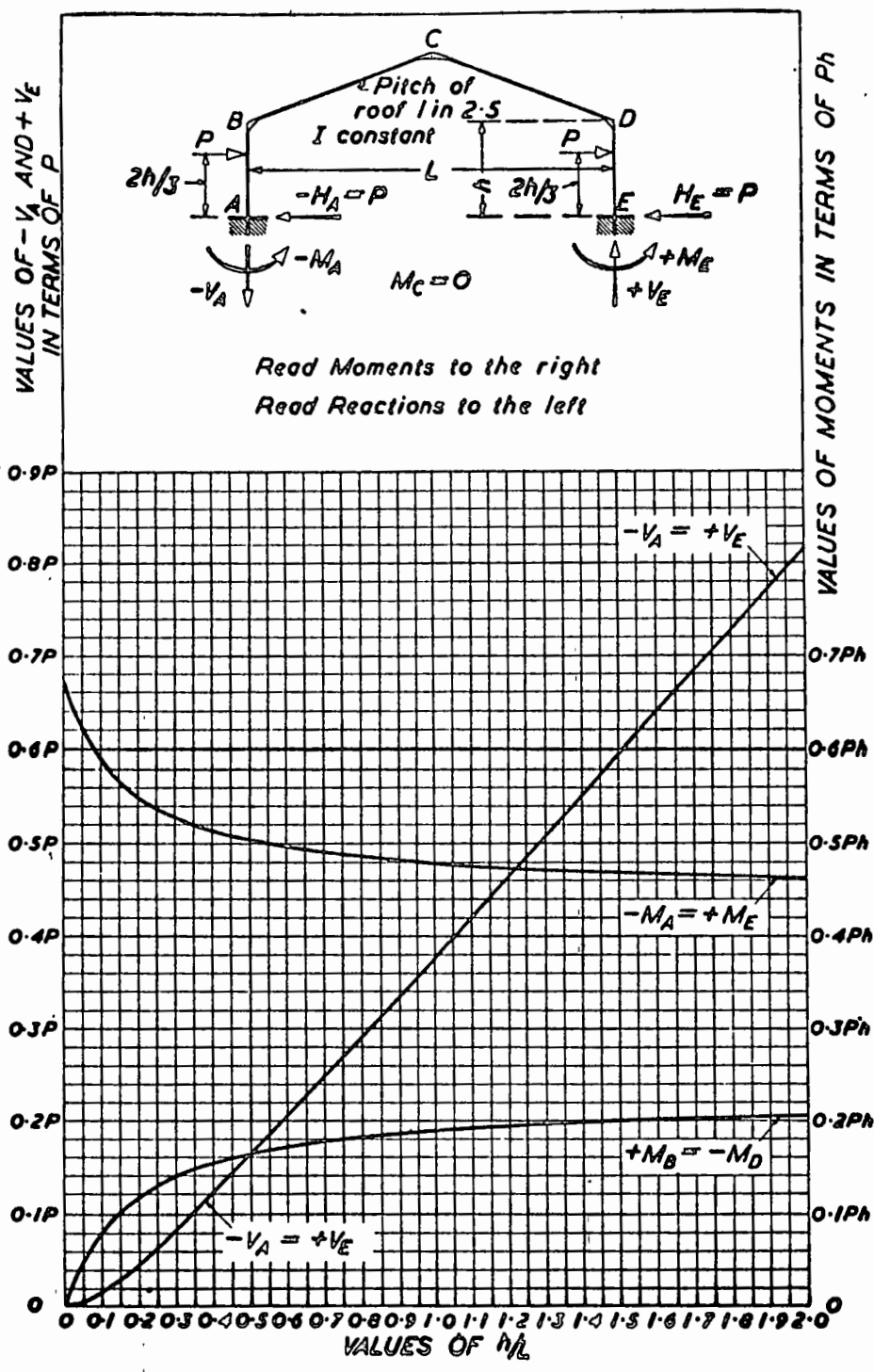


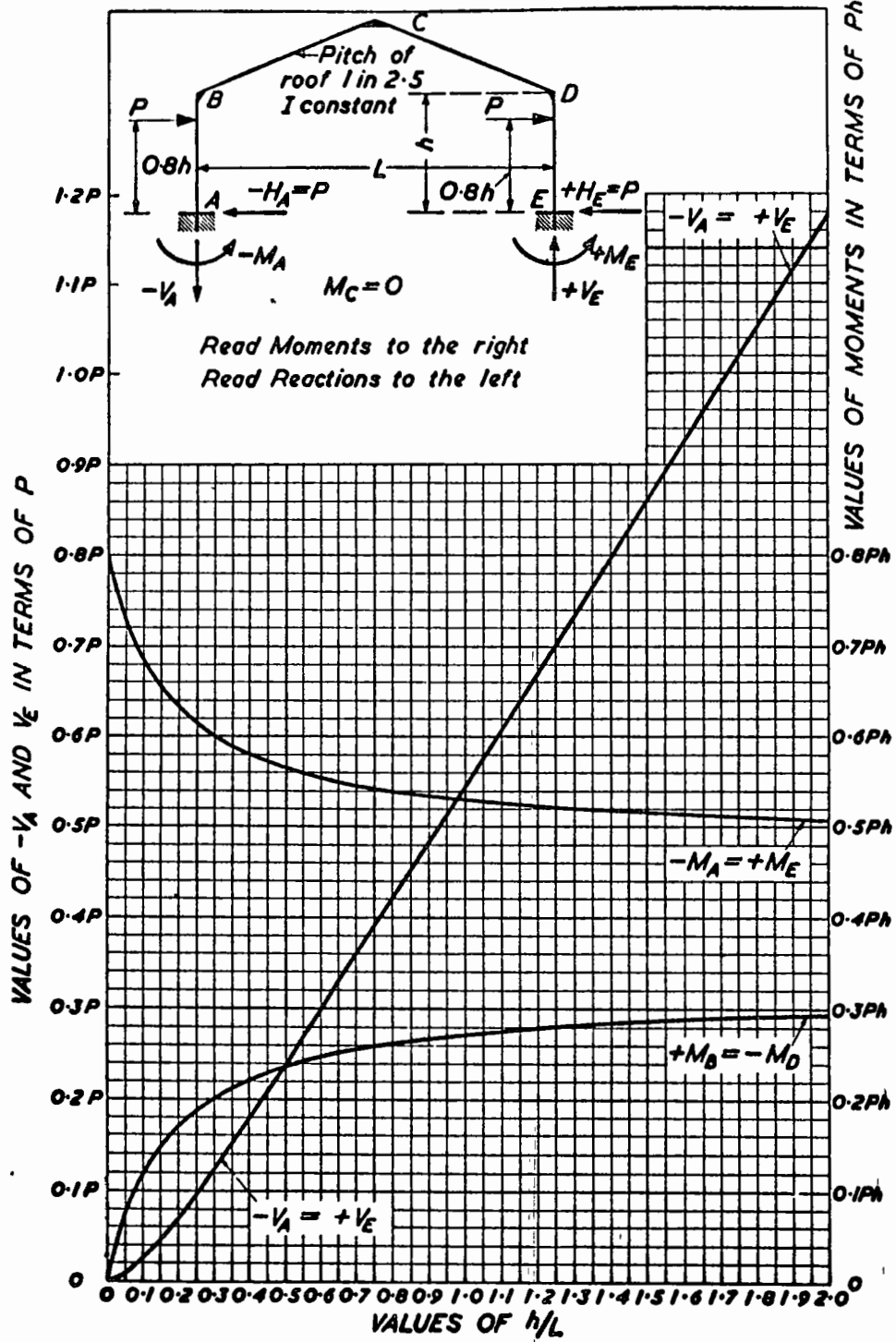


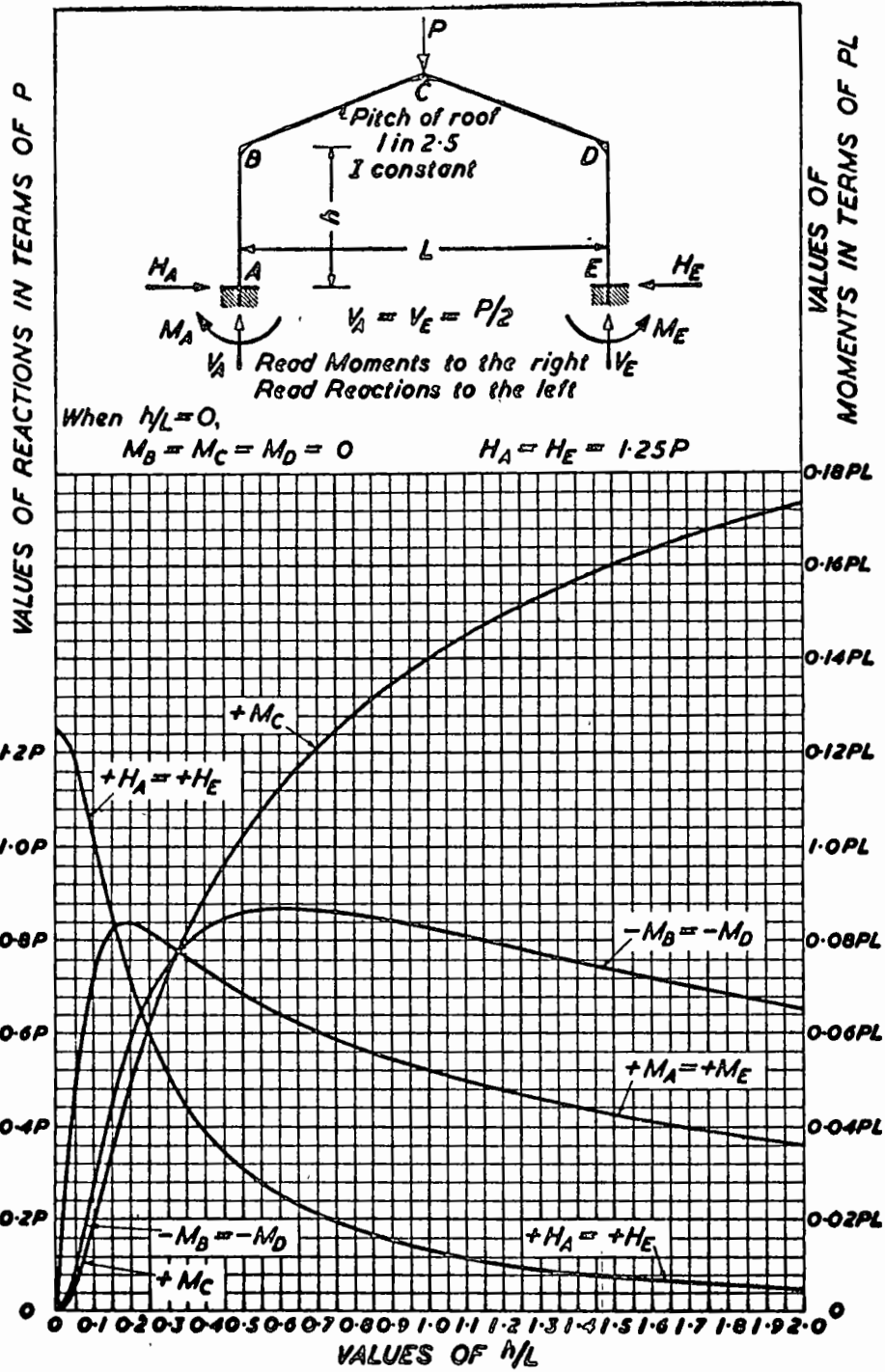


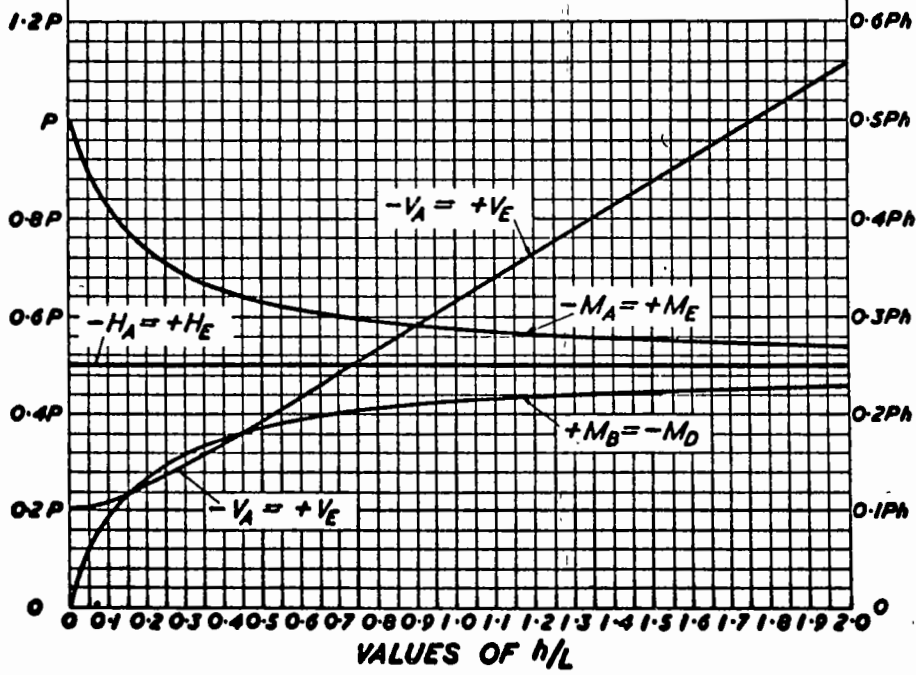
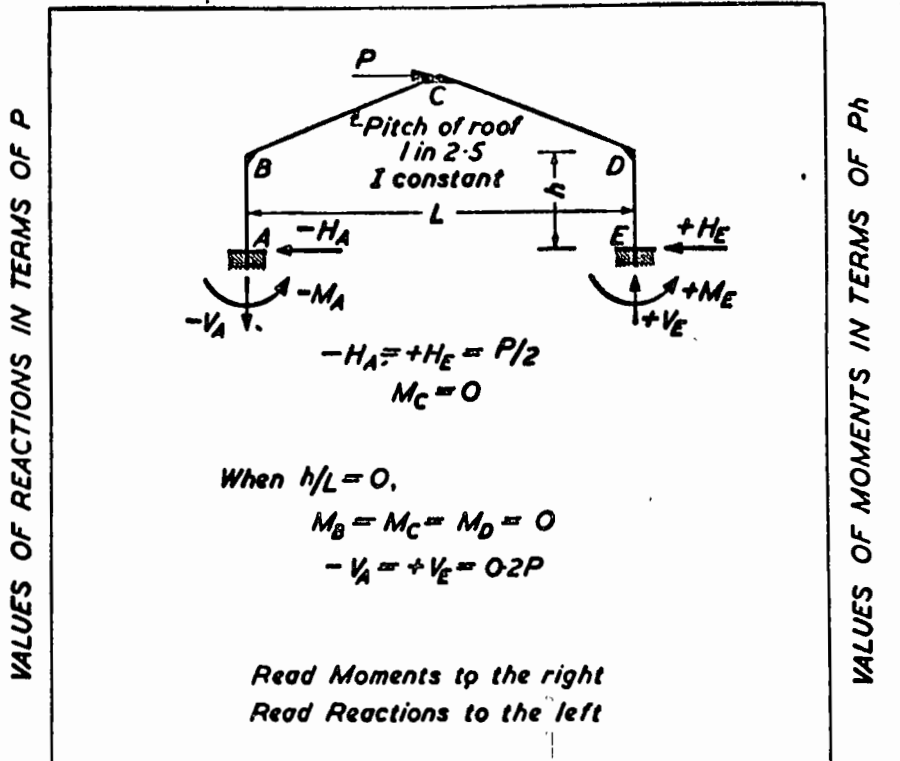


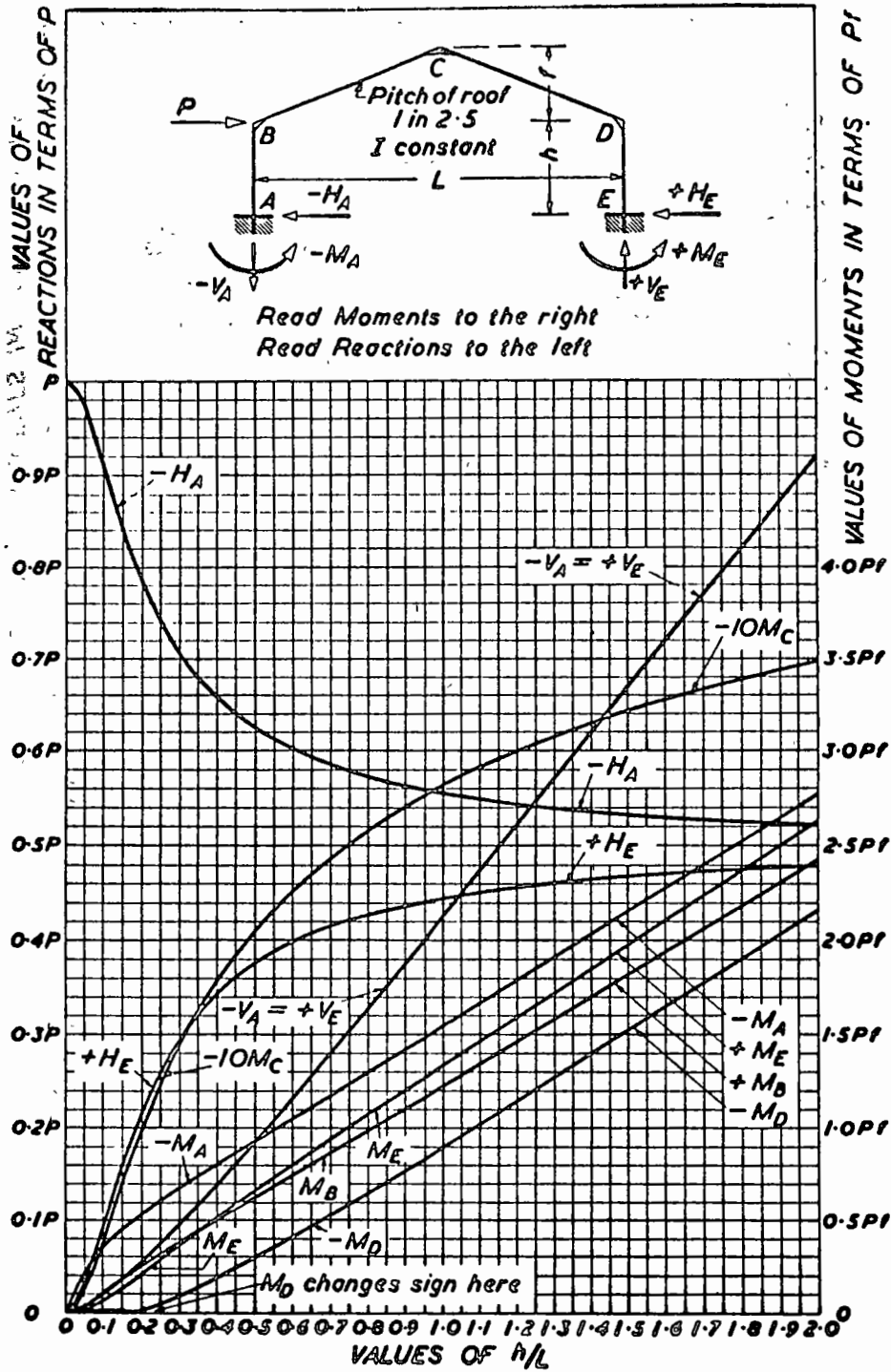












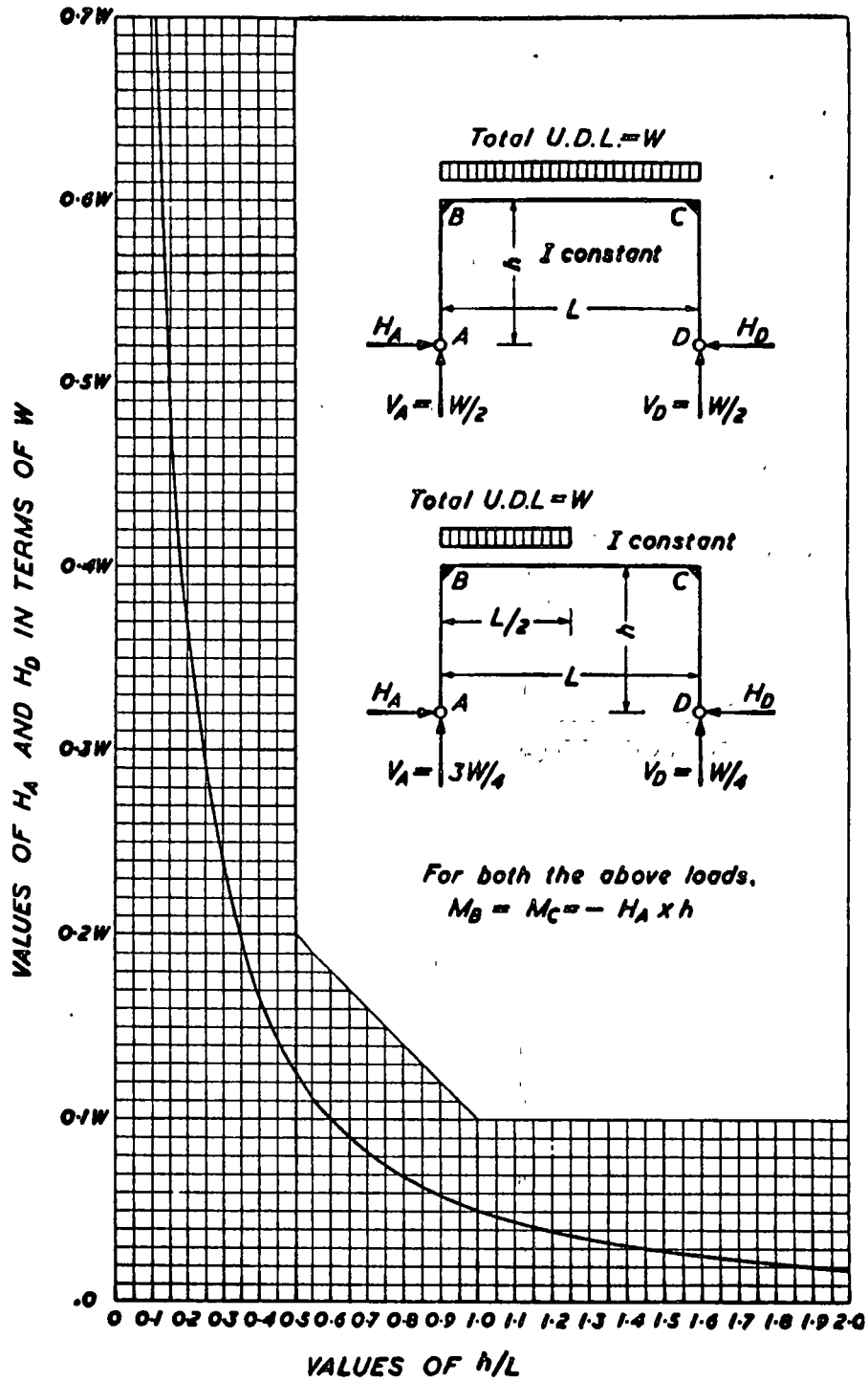
VALUES OF REACTIONS IN TERMS OF P

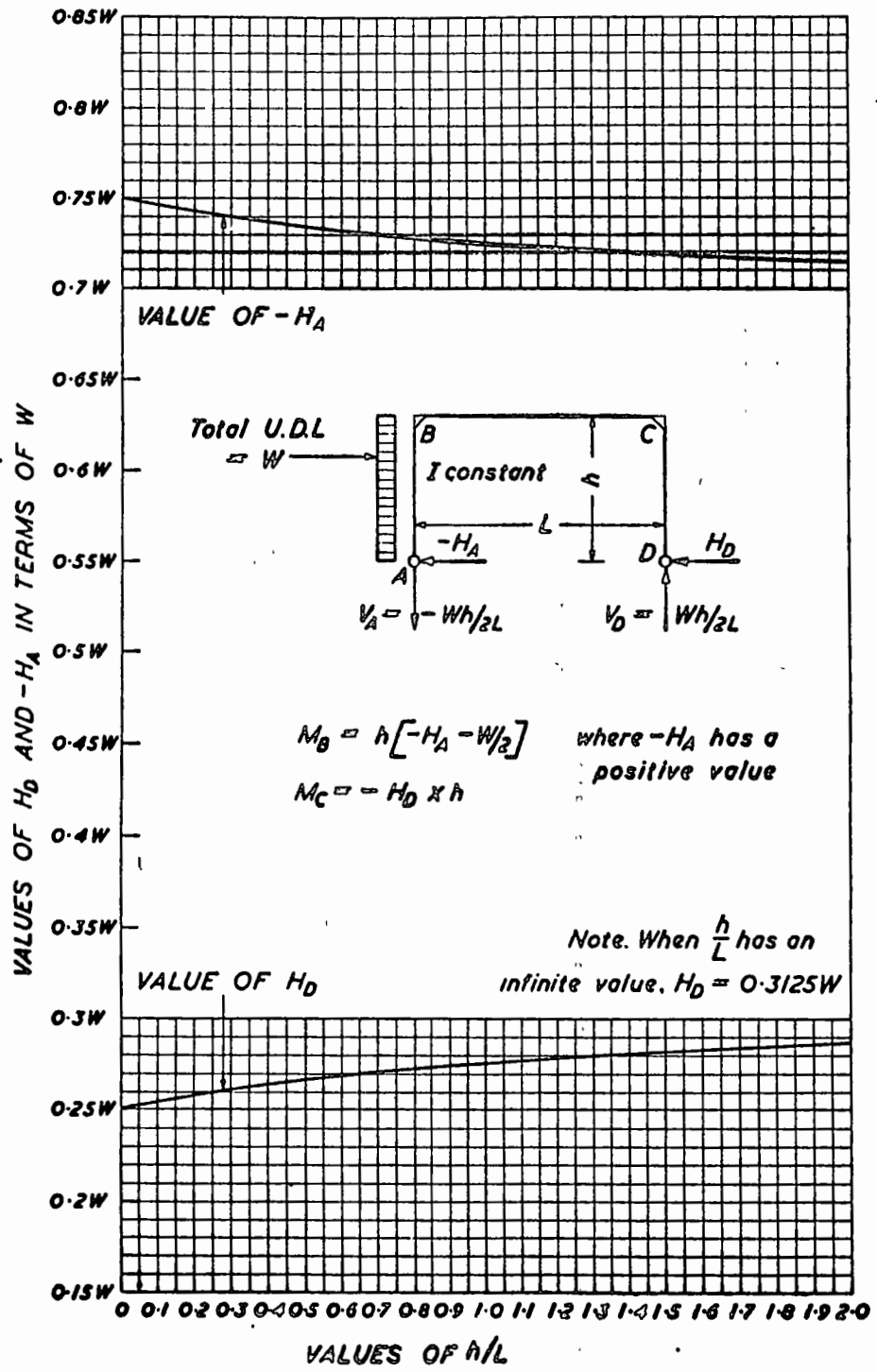
VALUES OF MOMENTS IN TERMS OF P

0.9P  
0.8P  
0.7P  
0.6P  
0.5P  
0.4P  
0.3P  
0.2P  
0.1P  
0

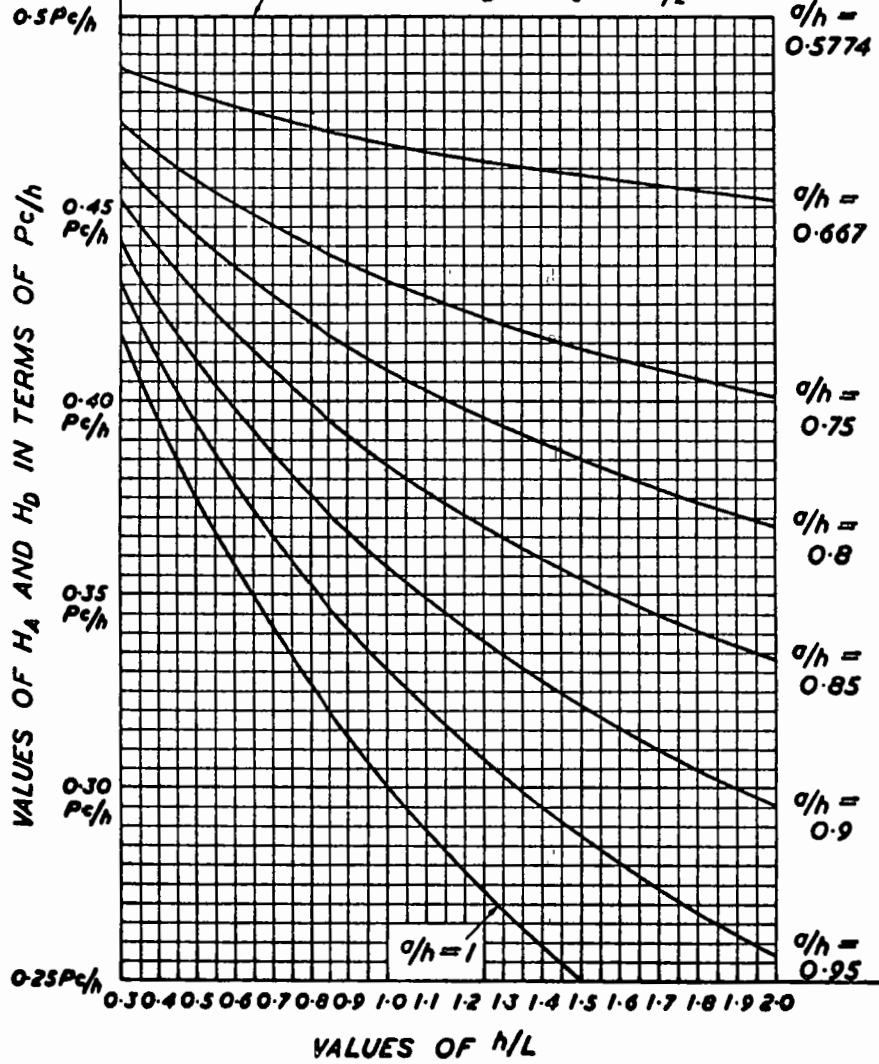
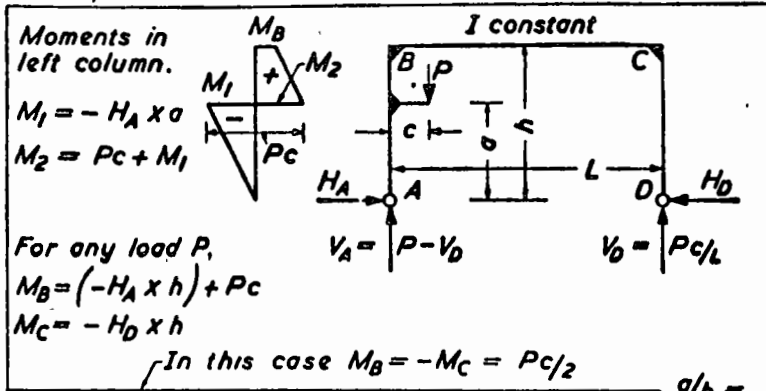
4.0P  
3.5P  
3.0P  
2.5P  
2.0P  
1.5P  
1.0P  
0.5P  
0

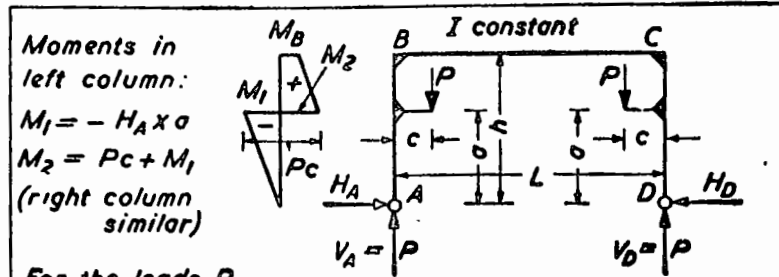
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 1.9 2.0  
VALUES OF h/L











Moments in left column:

$$M_1 = -H_A \times a$$

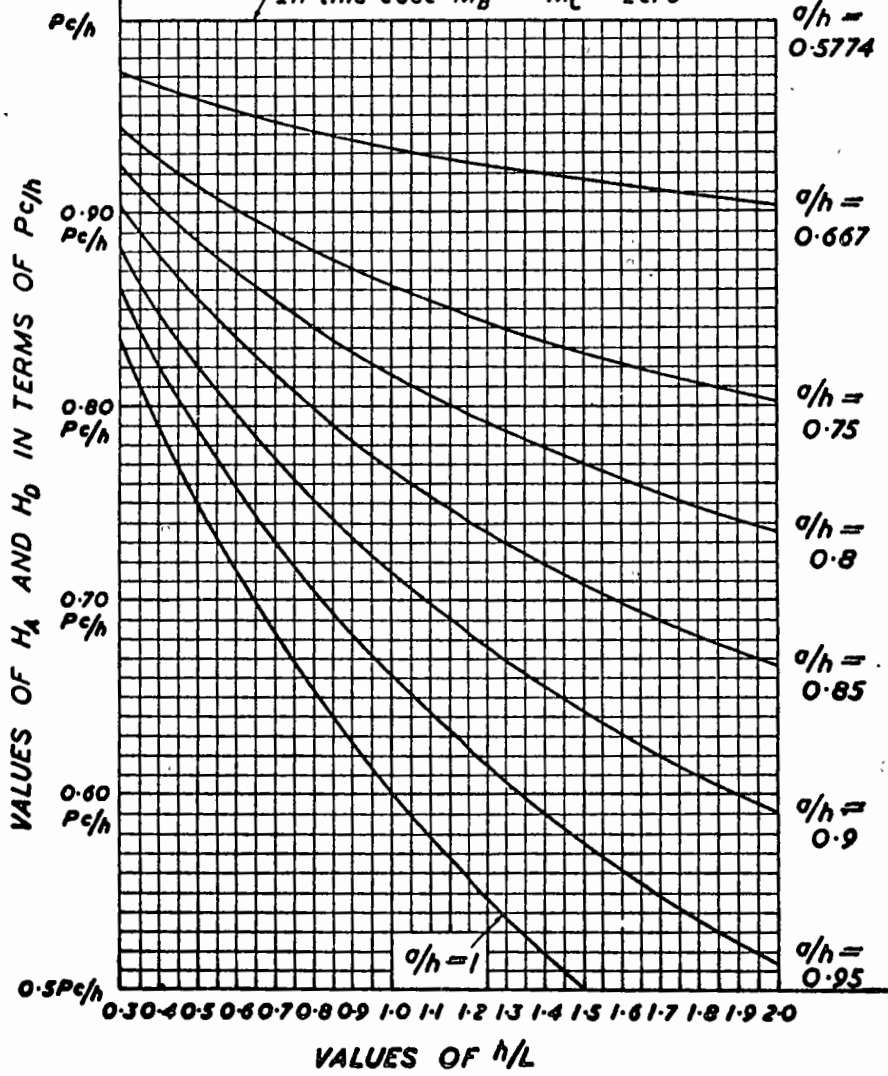
$$M_2 = Pc + M_1$$

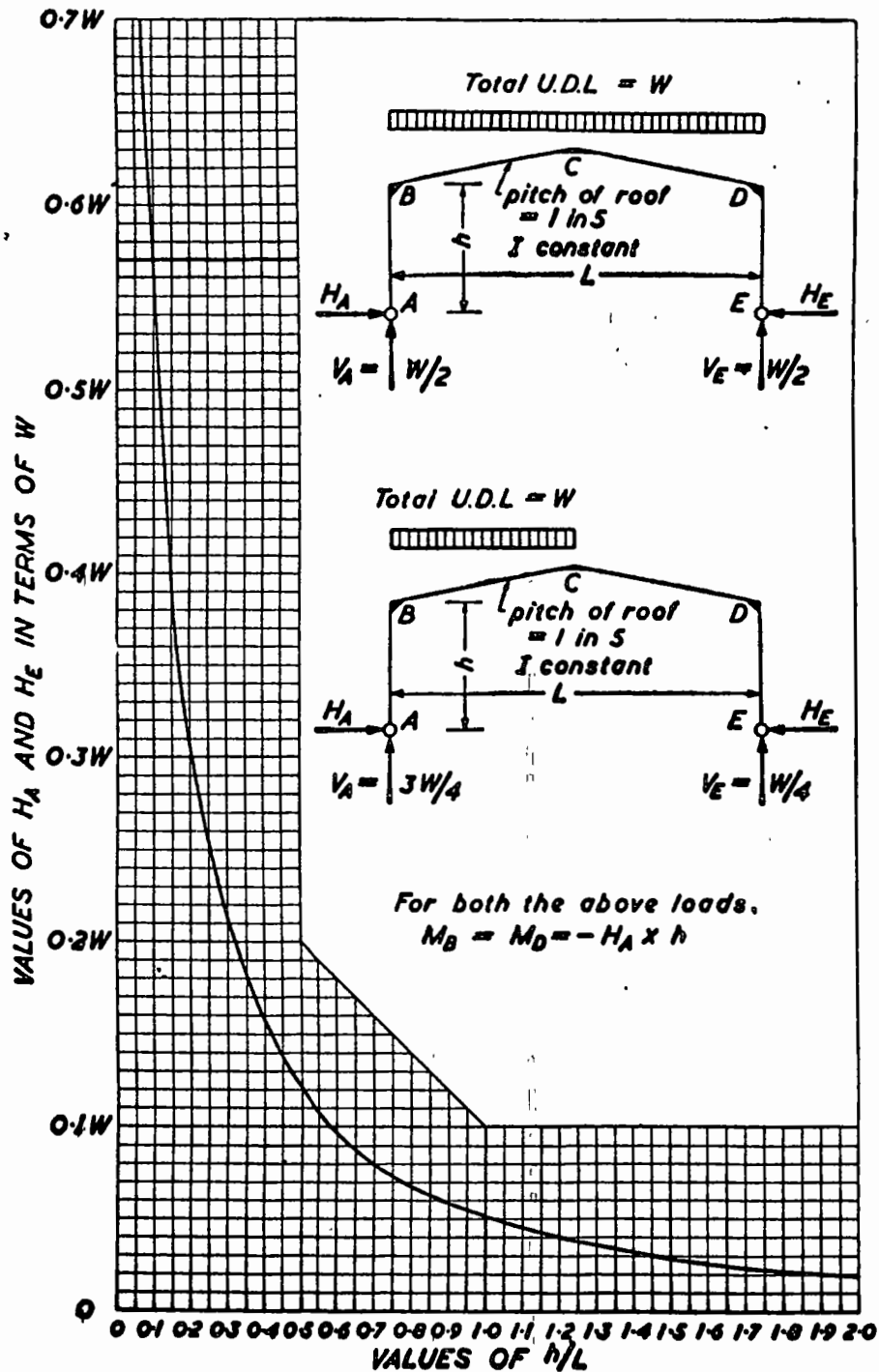
(right column similar)

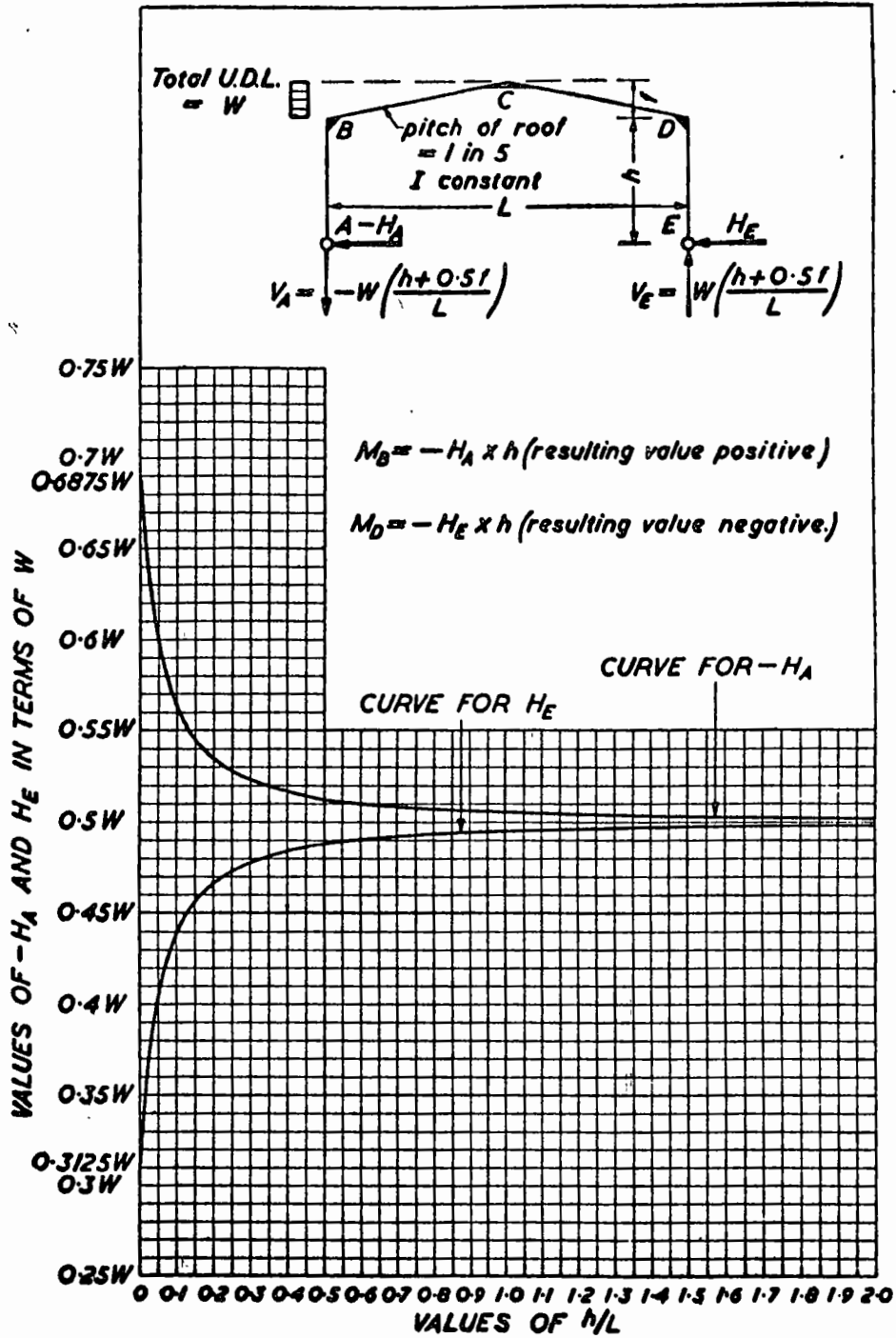
For the loads P,

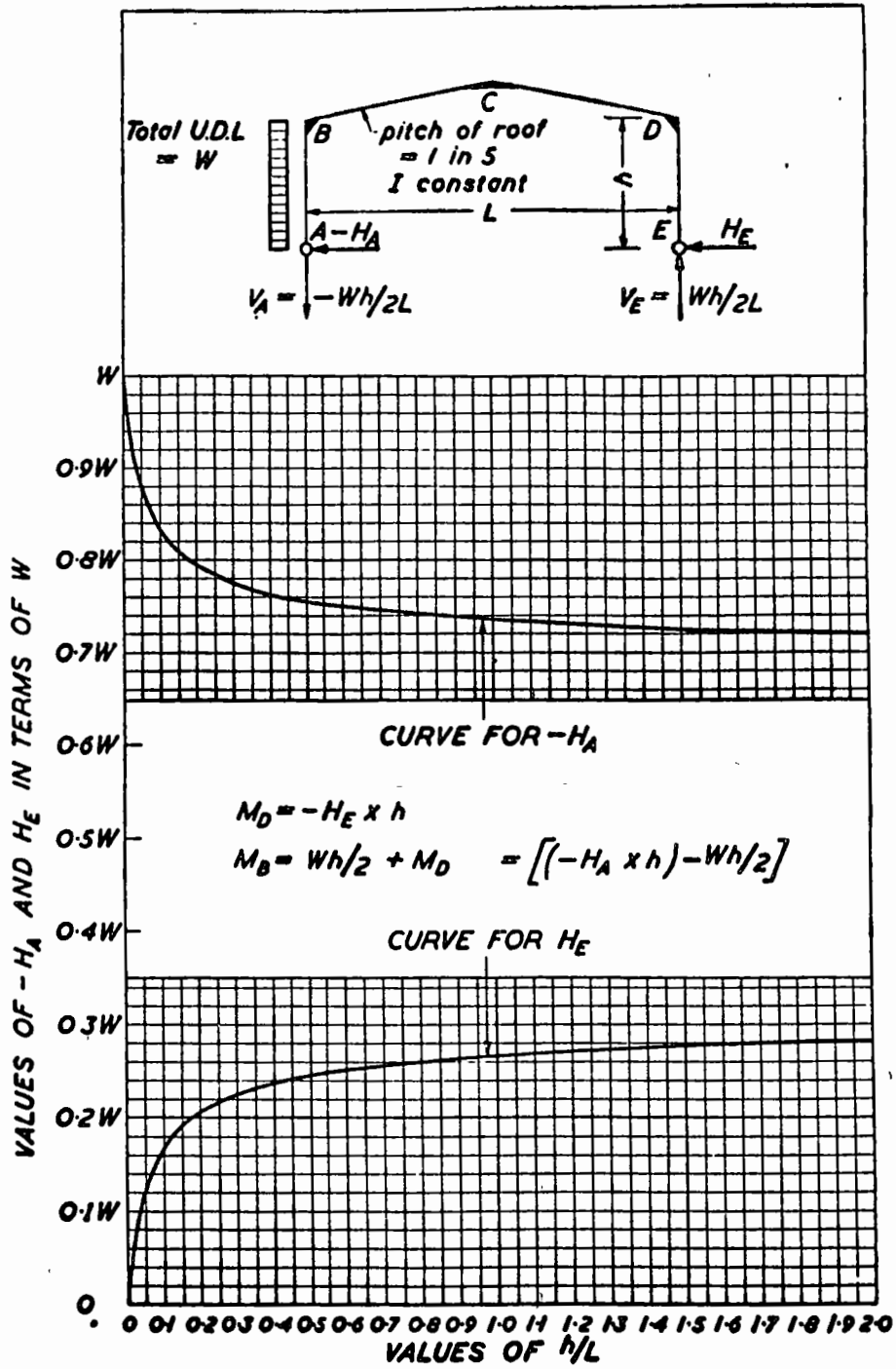
$$M_B = M_C = (-H_A \times h) + Pc$$

In this case  $M_B = M_C = \text{zero}$

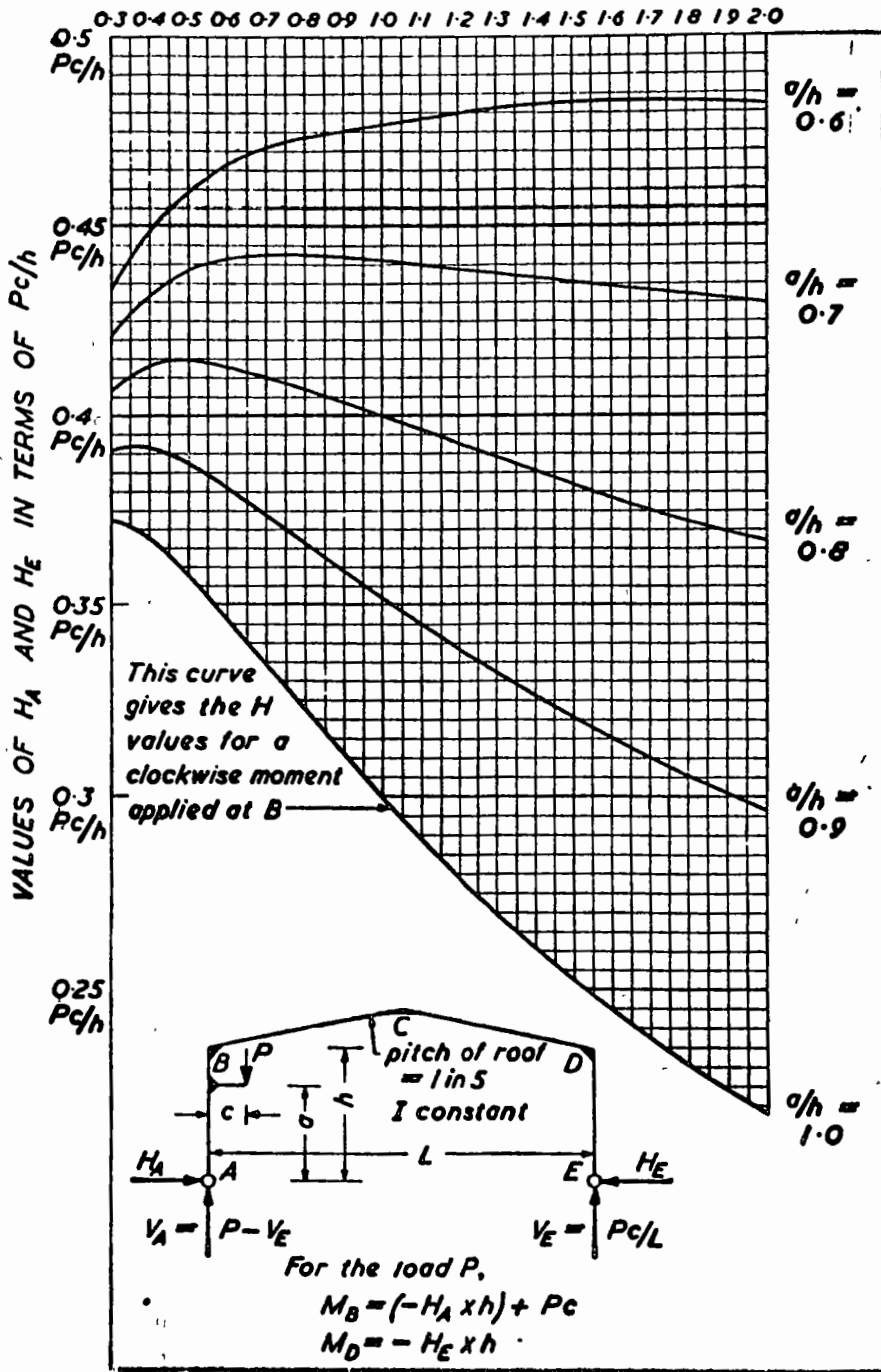


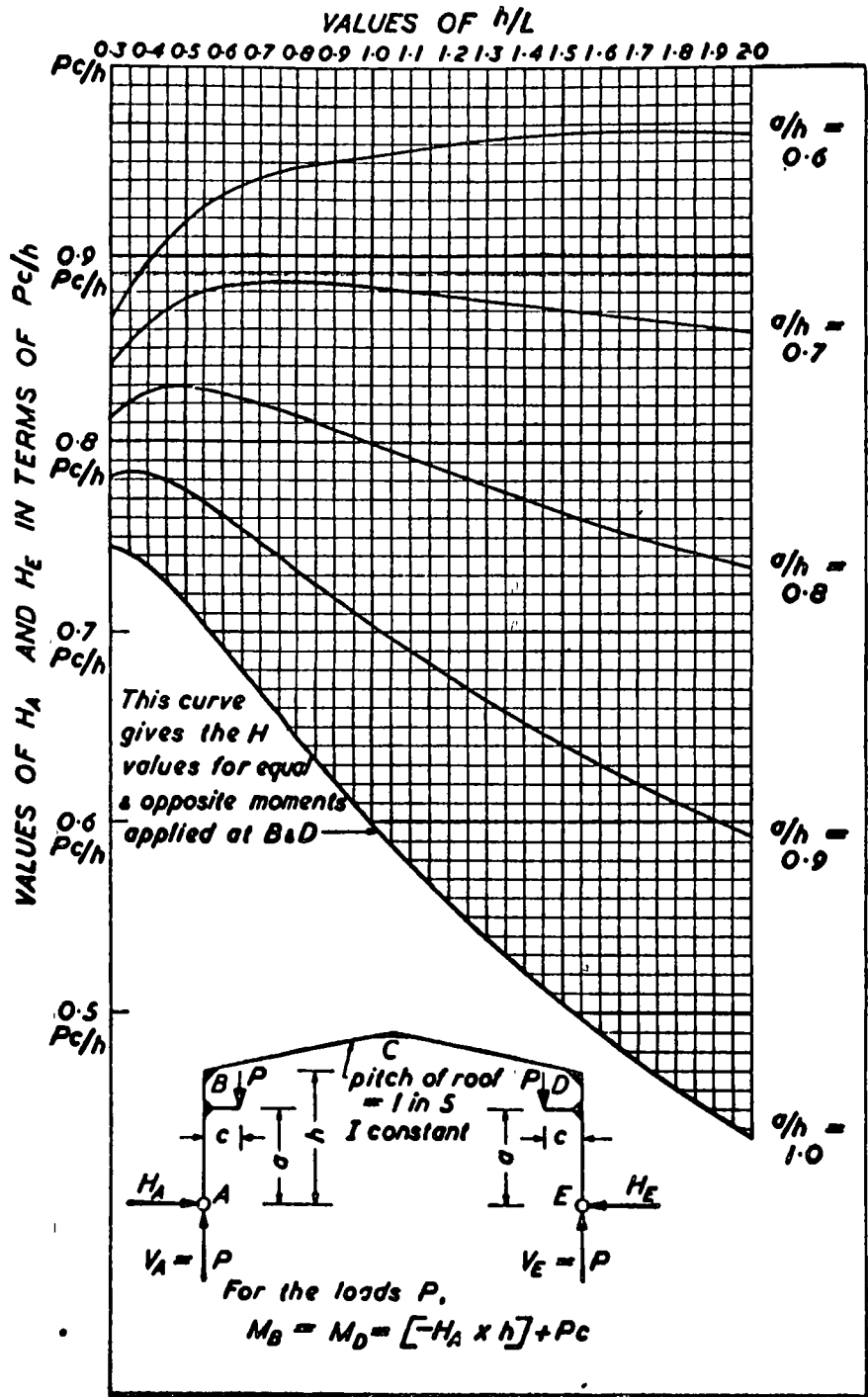


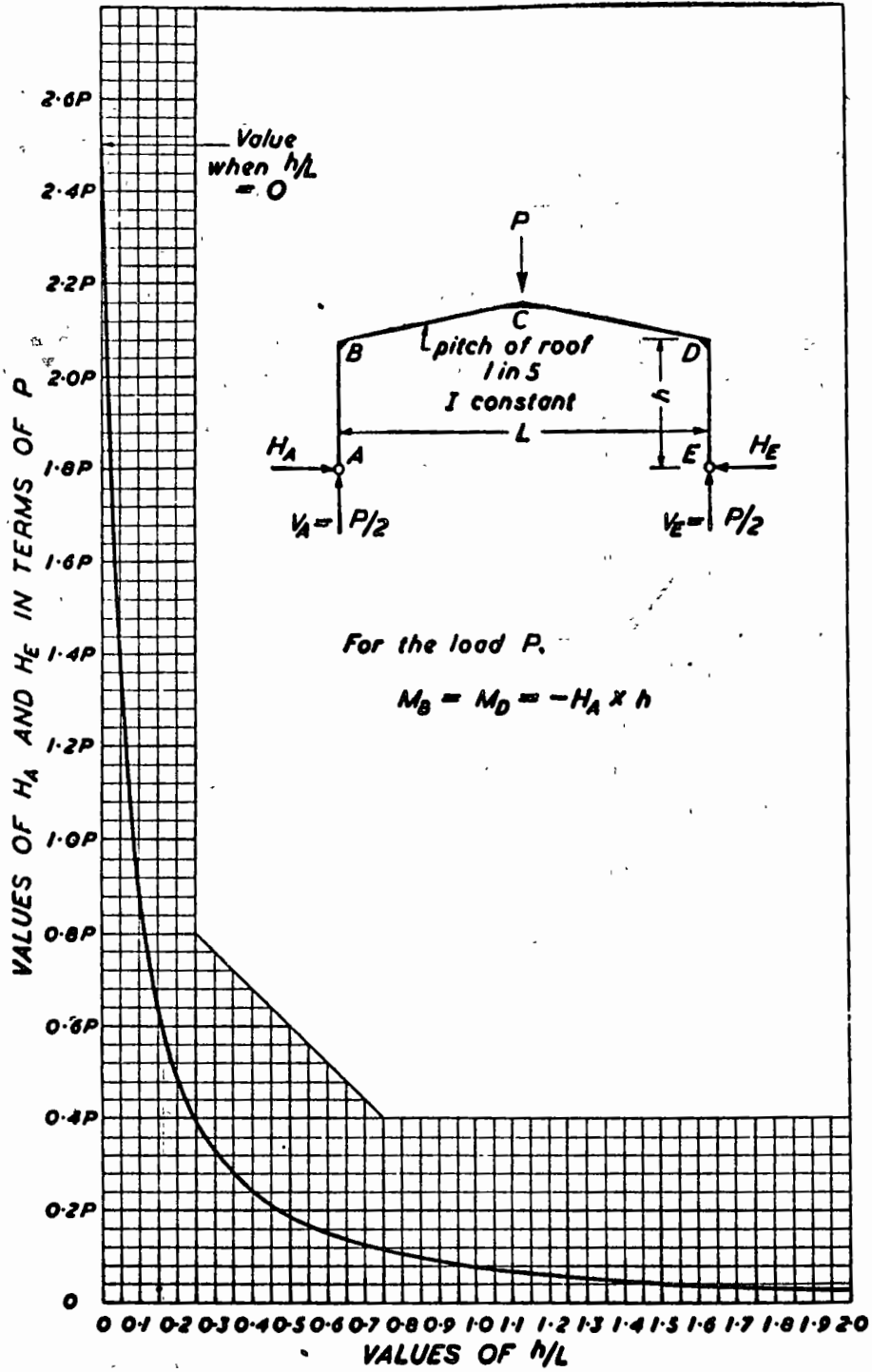




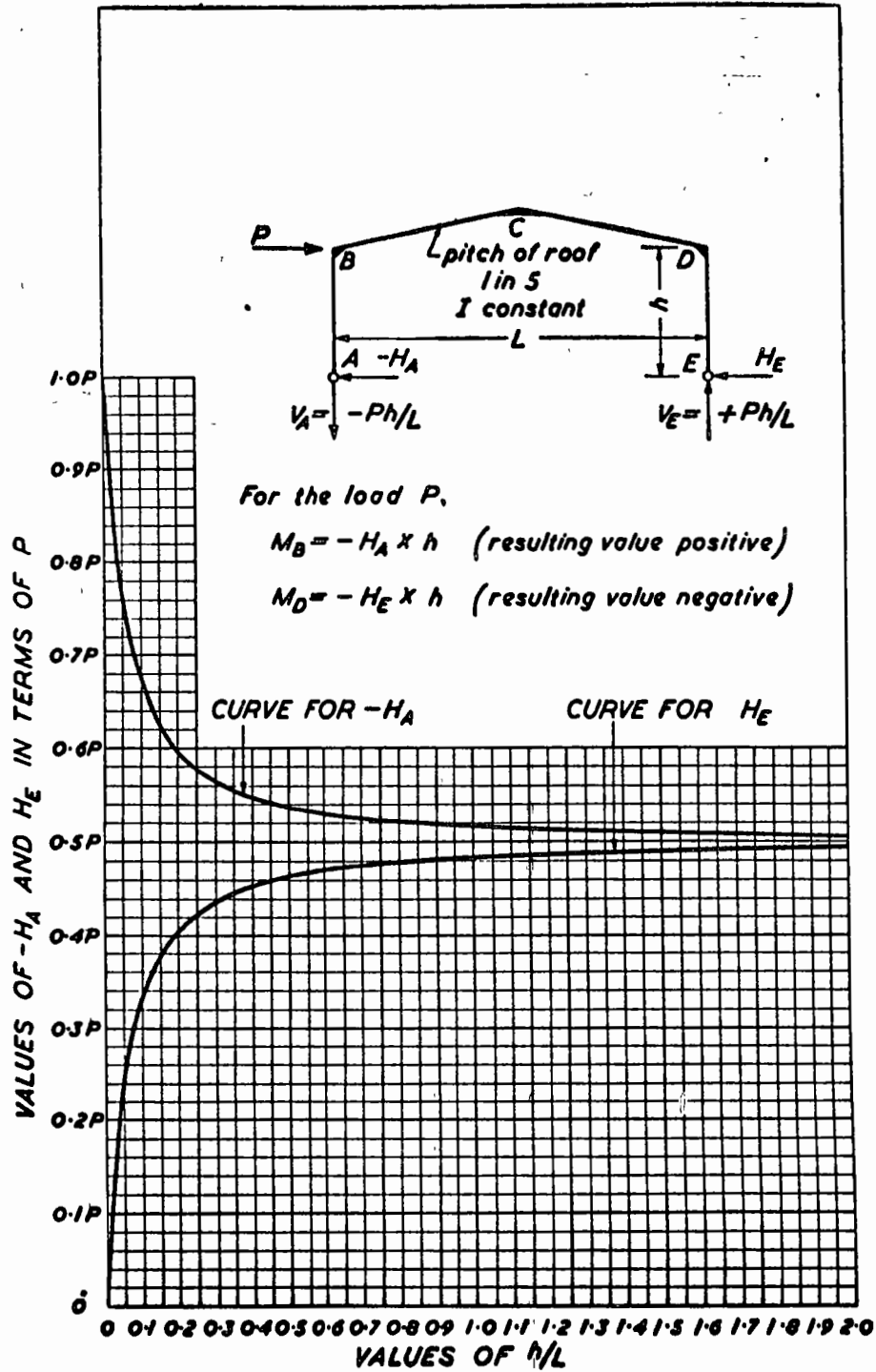
VALUES OF  $h/L$

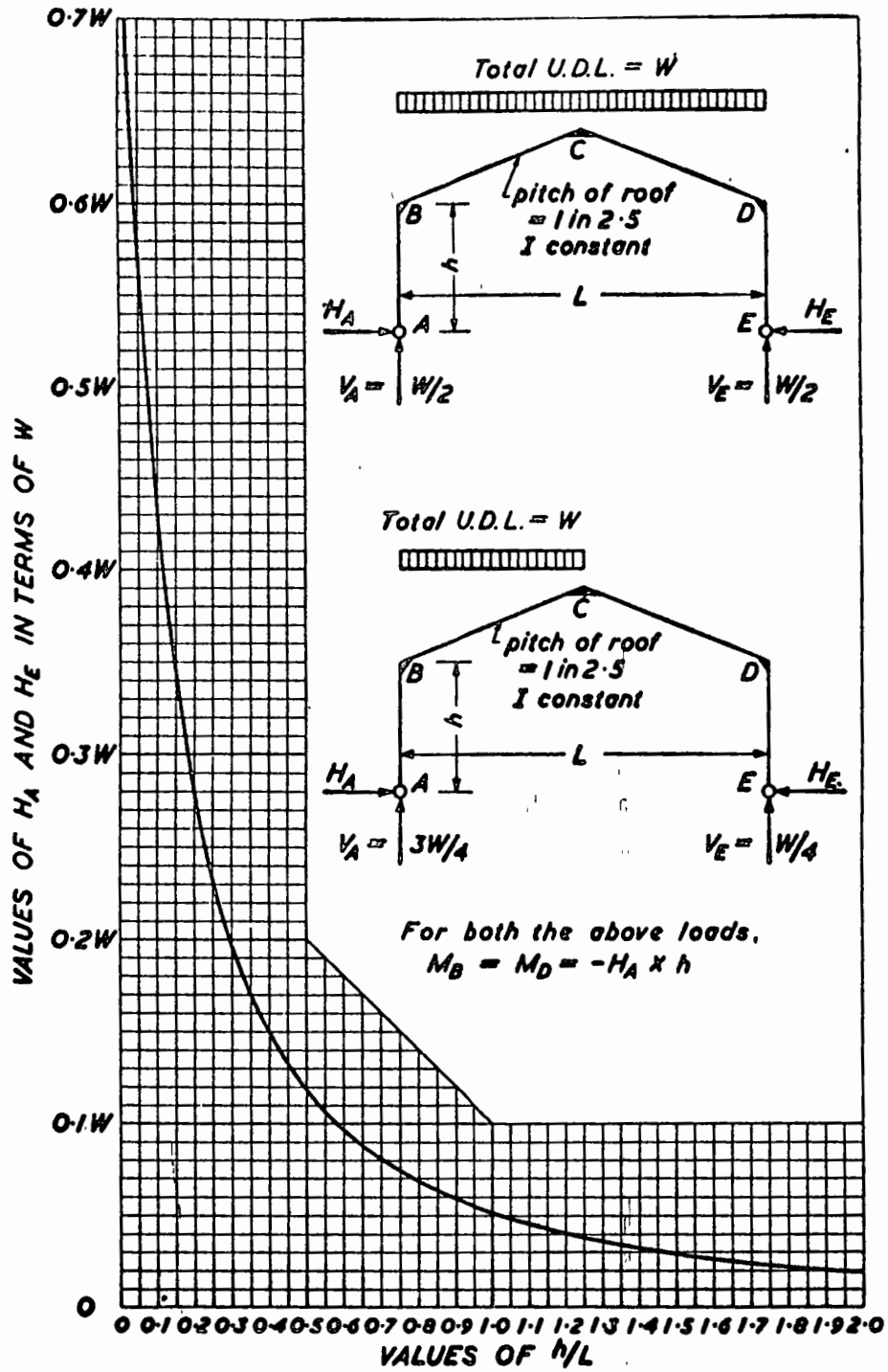


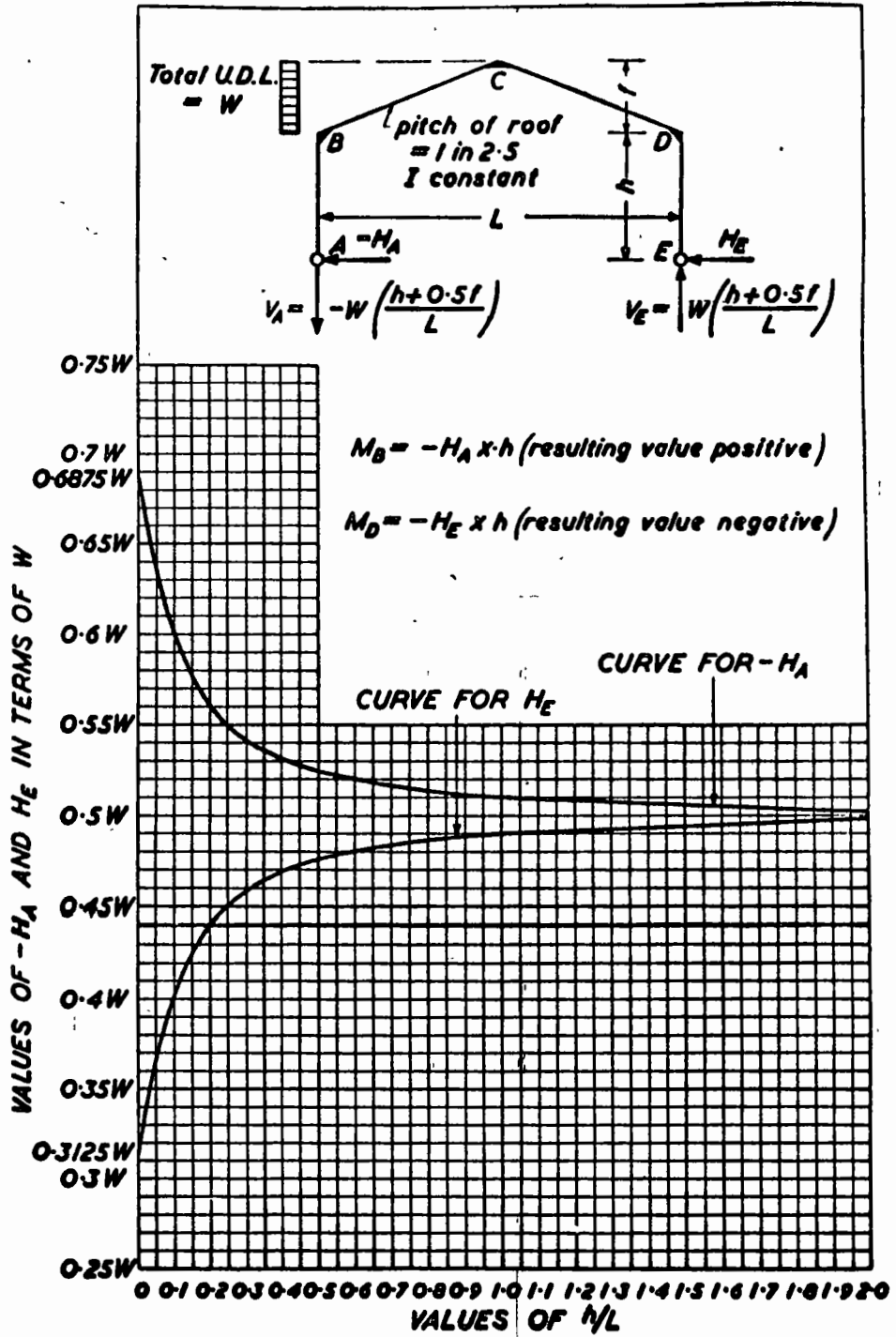


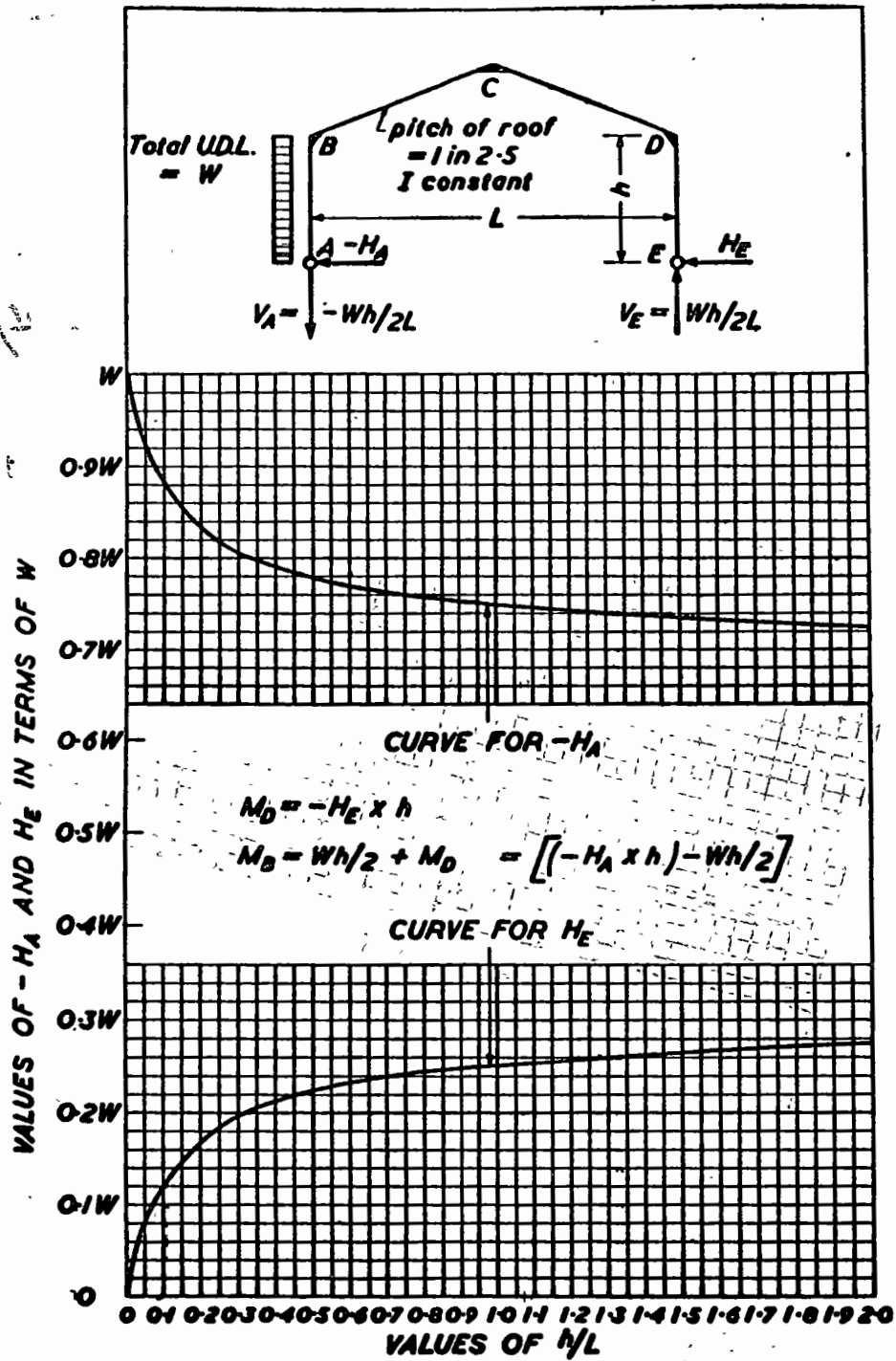




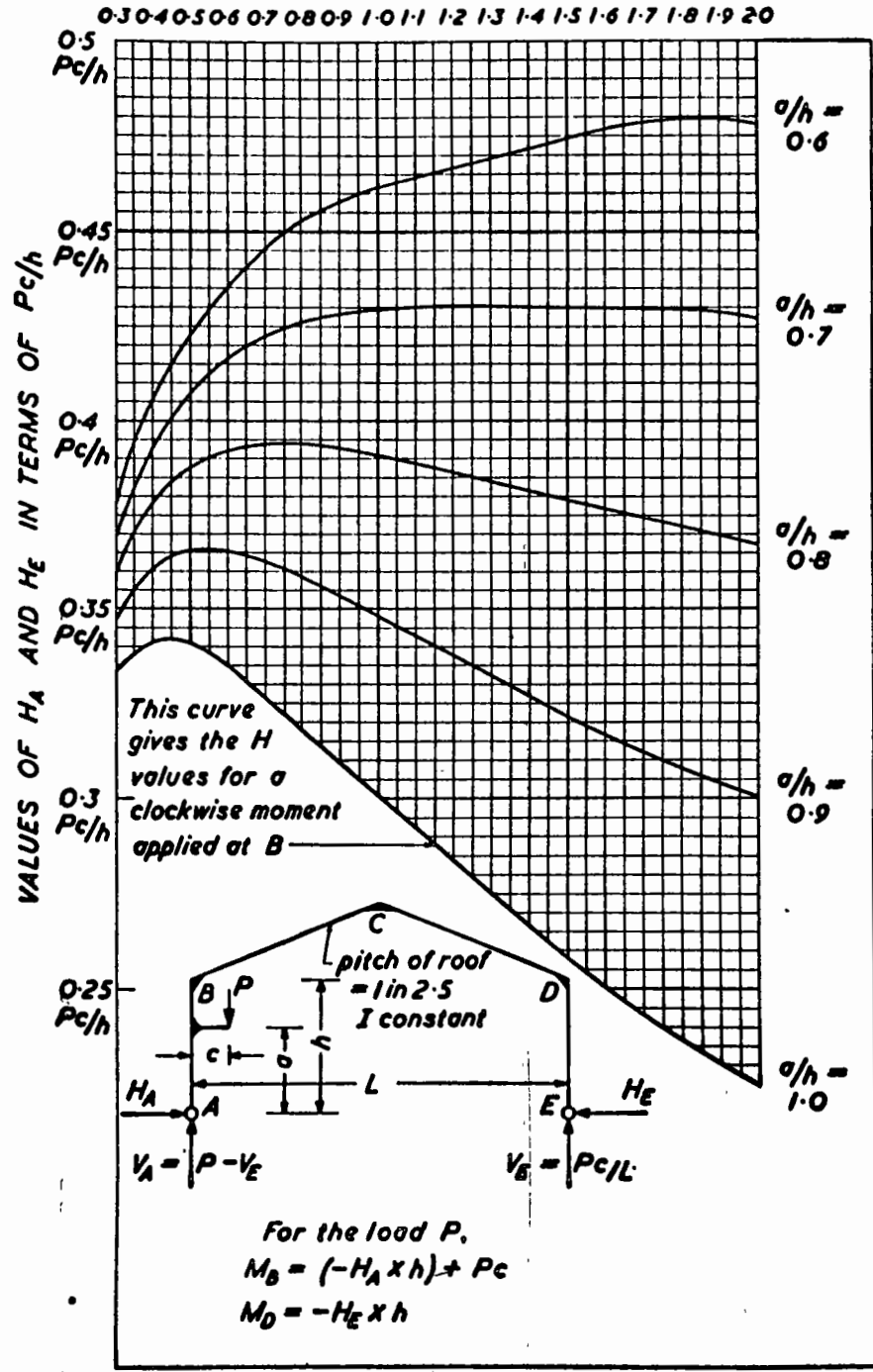




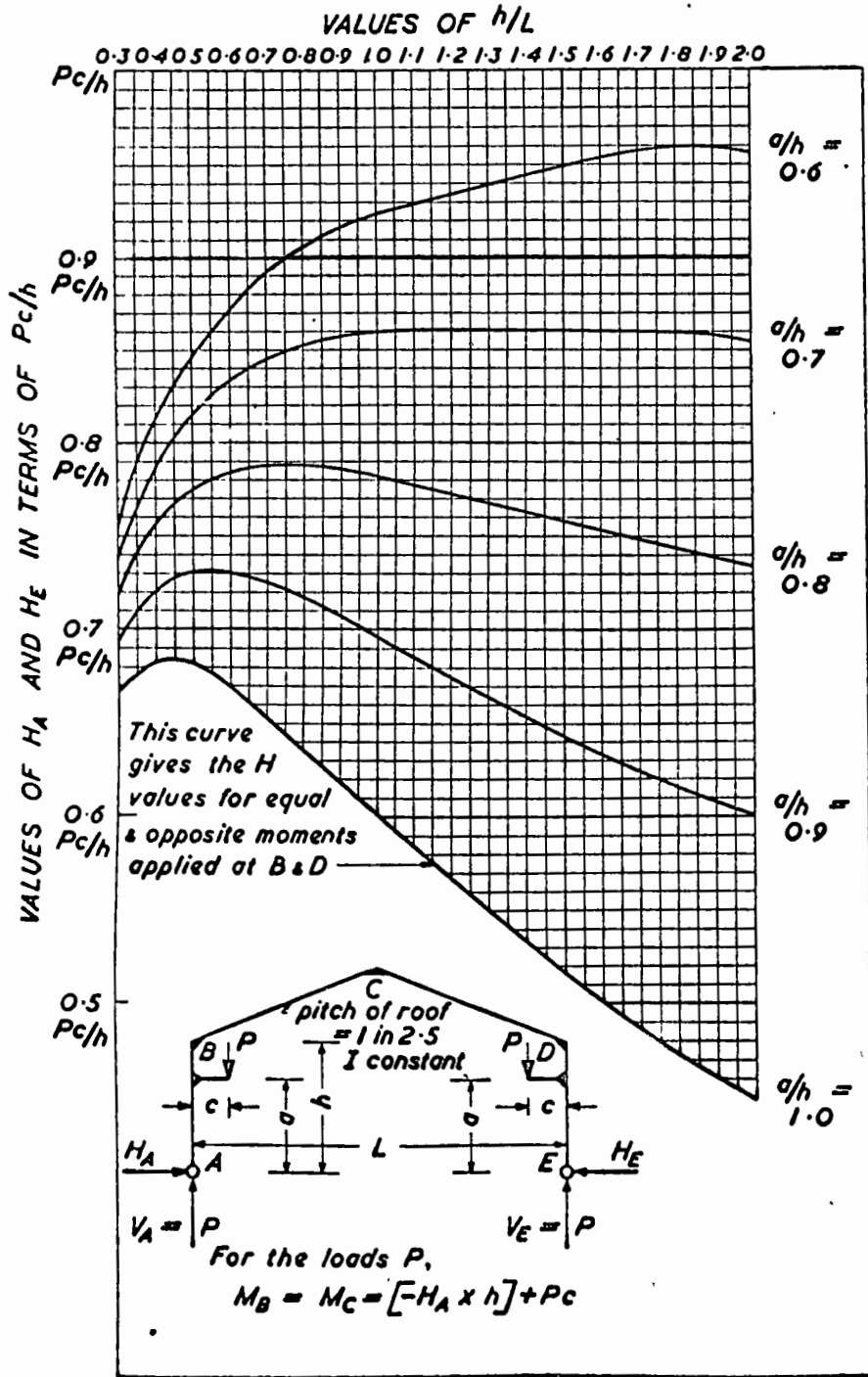


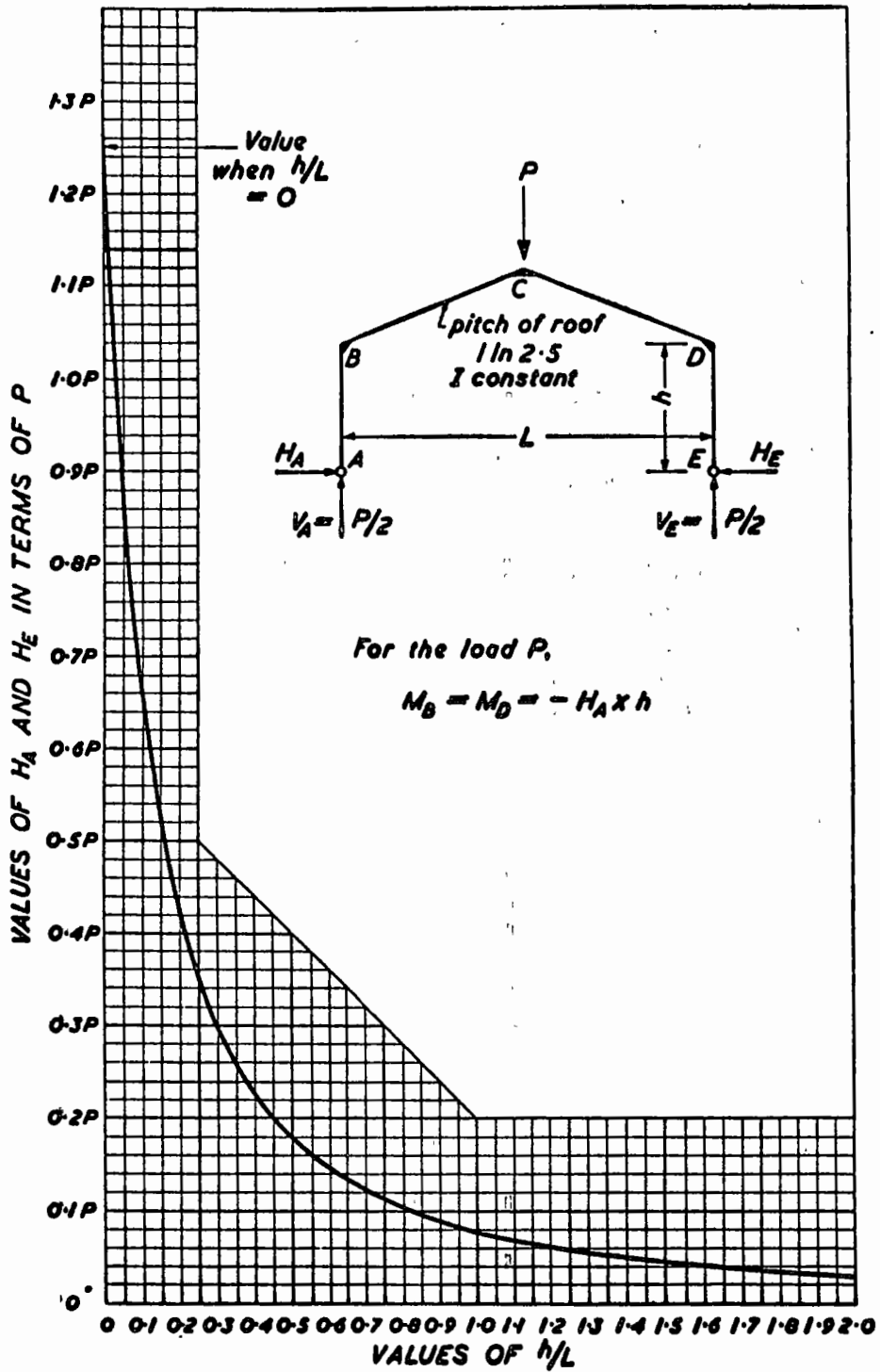


VALUES OF  $h/L$

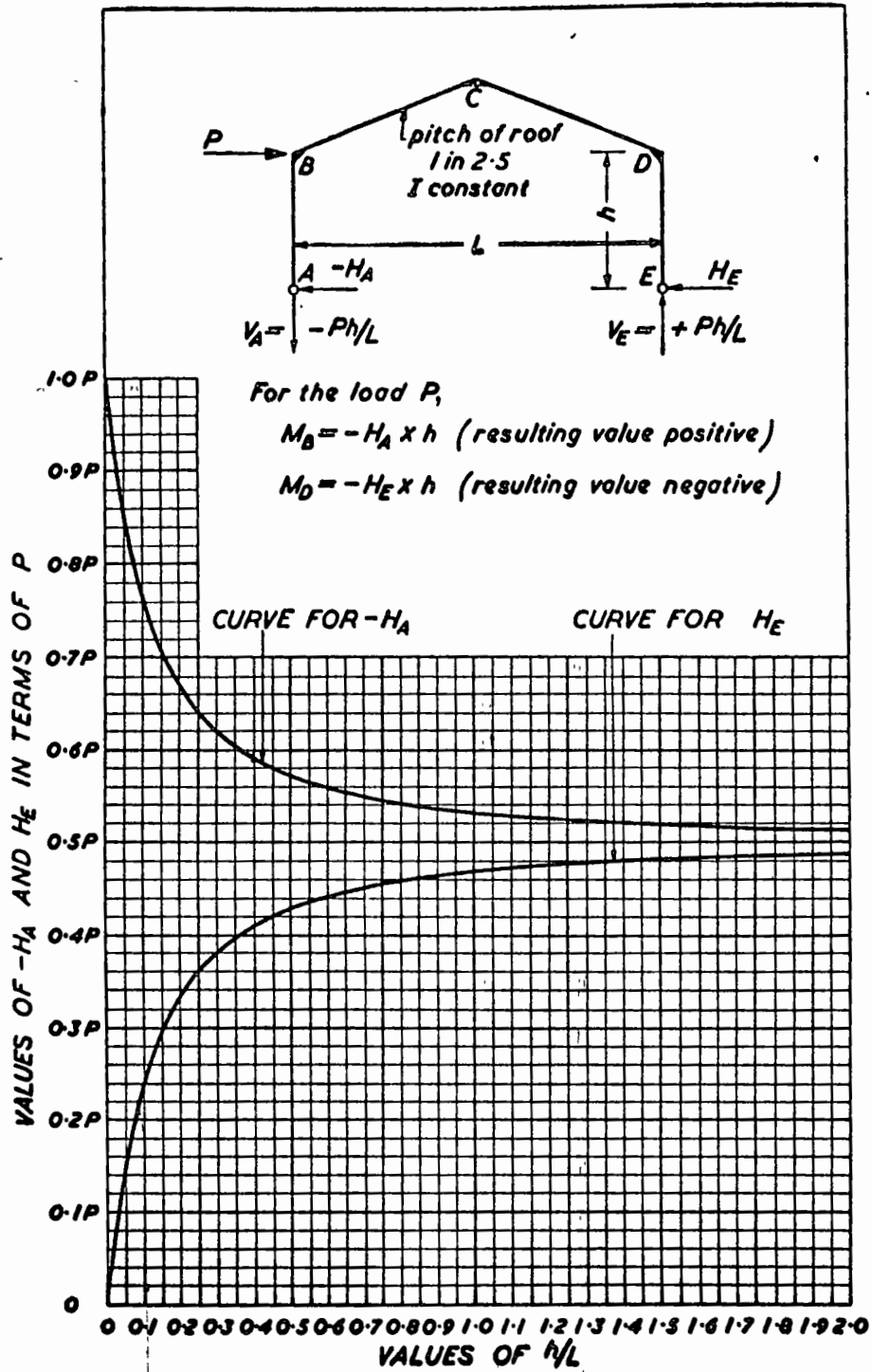


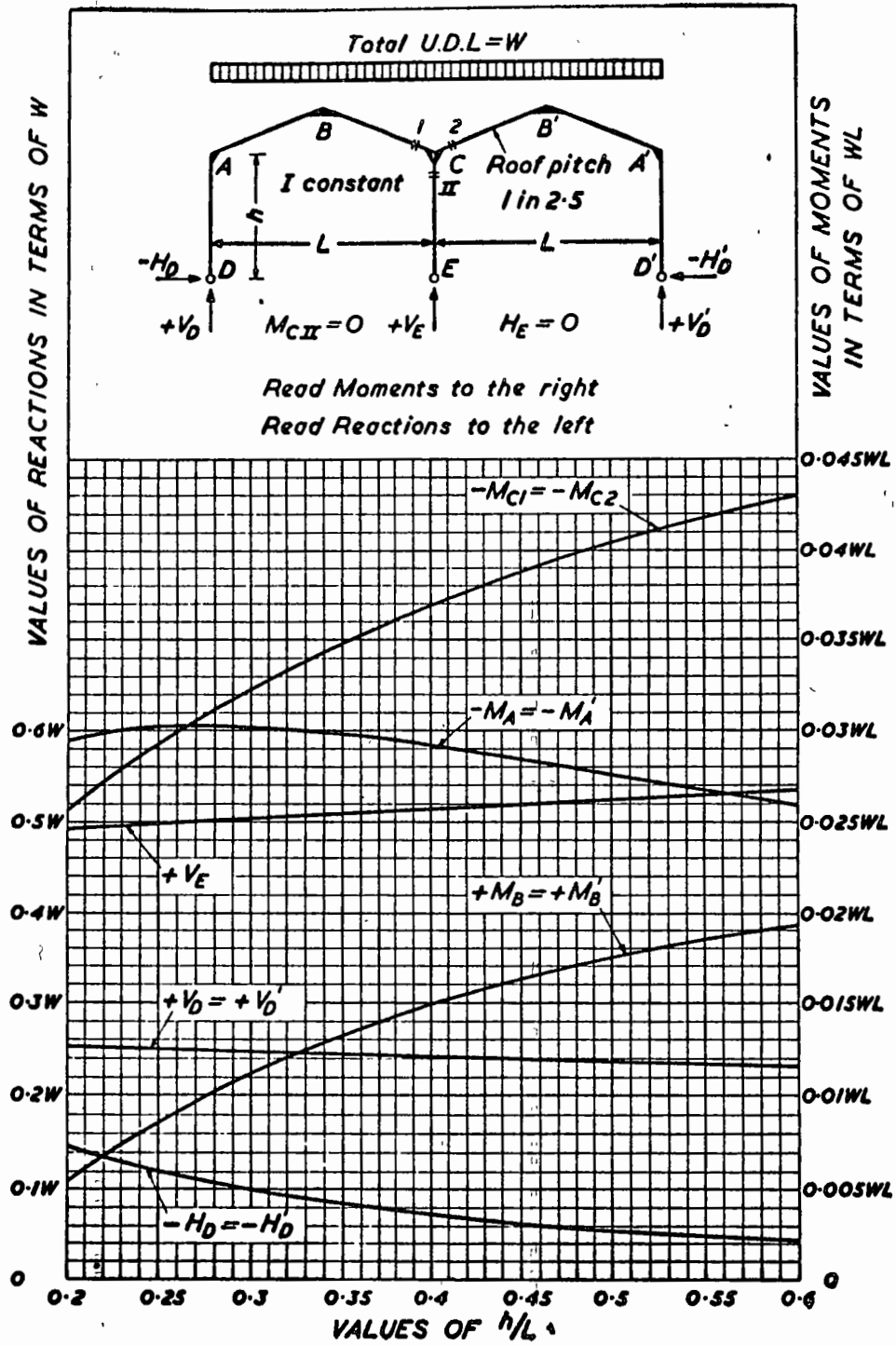


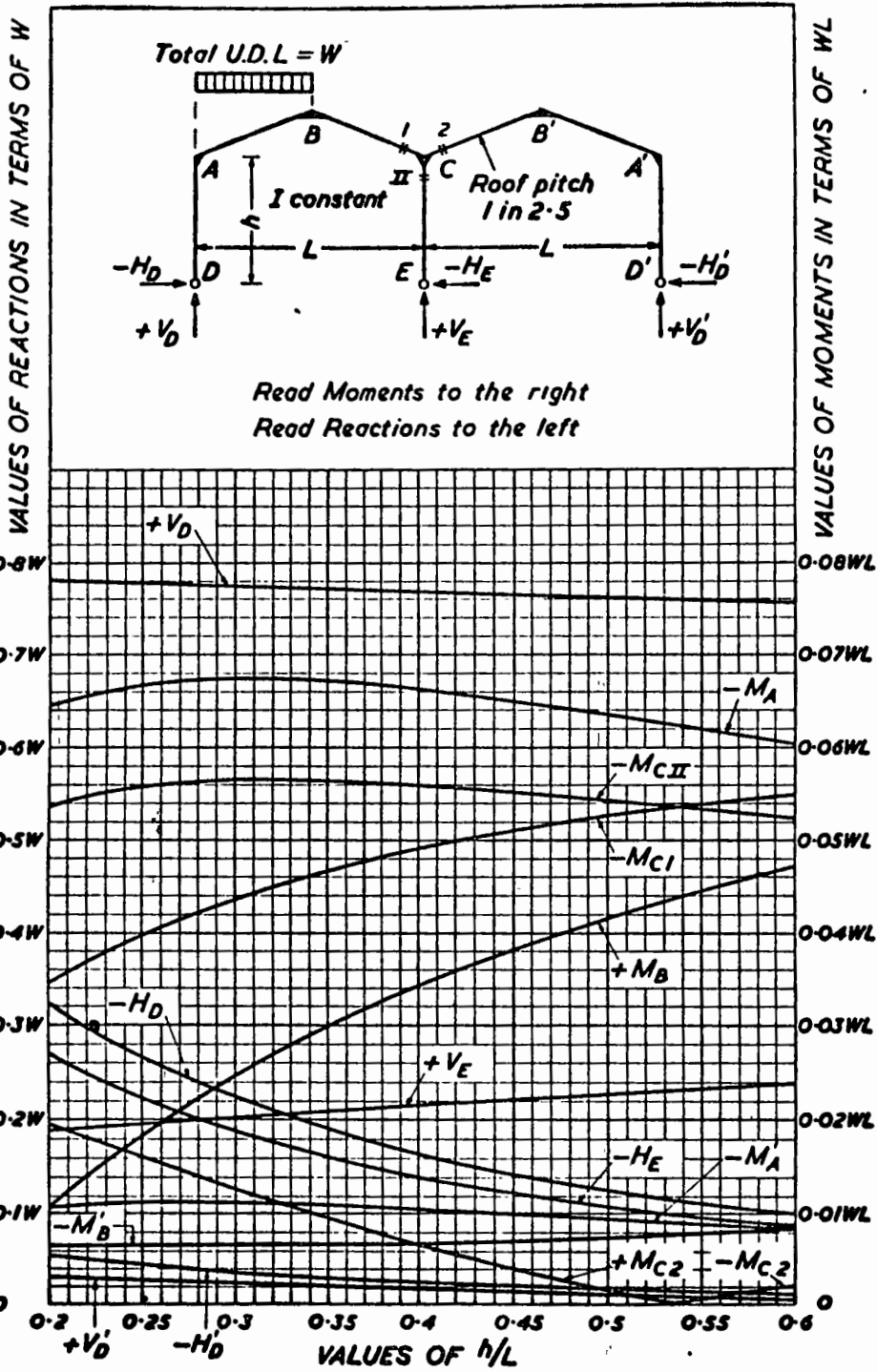


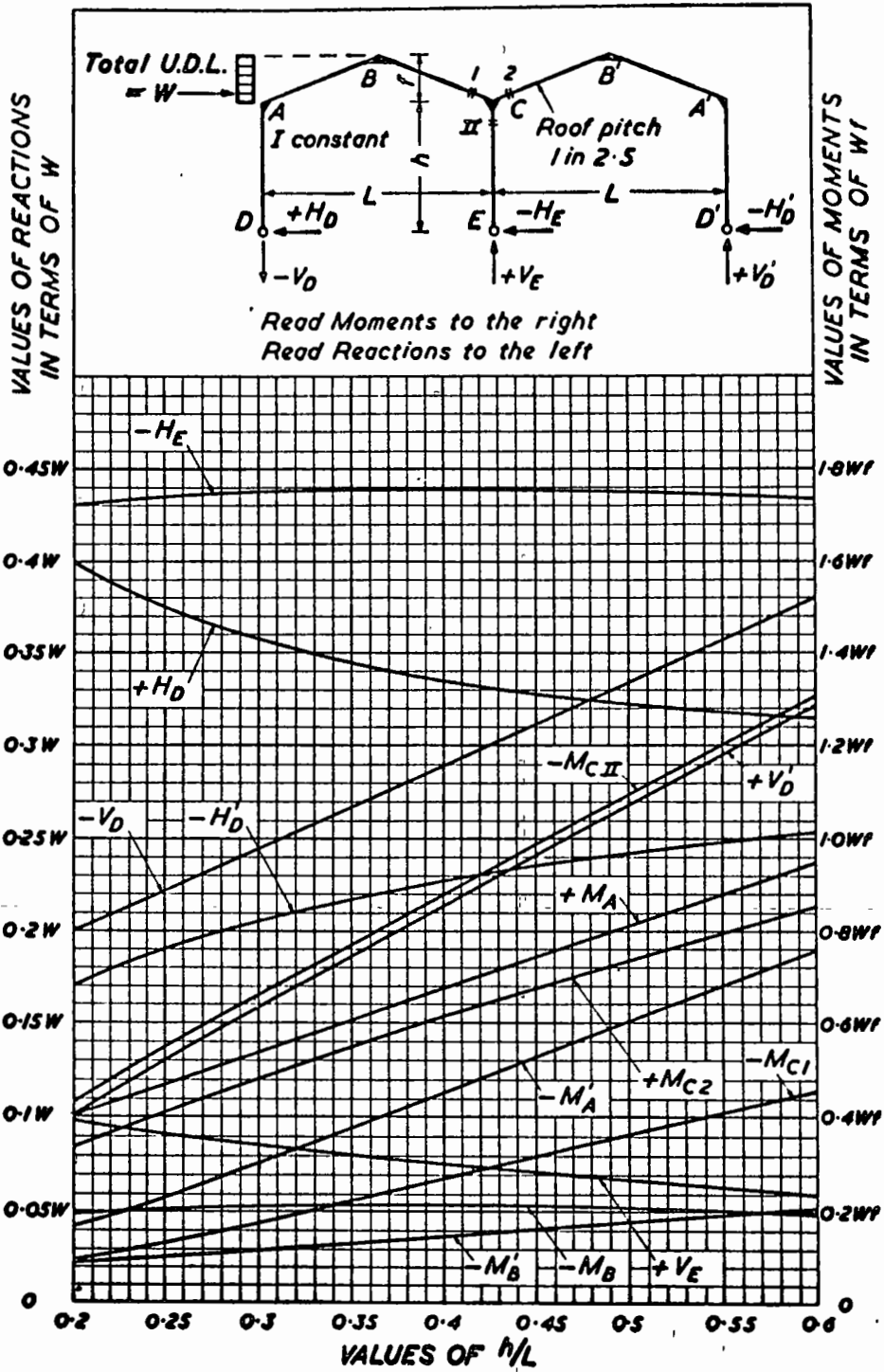


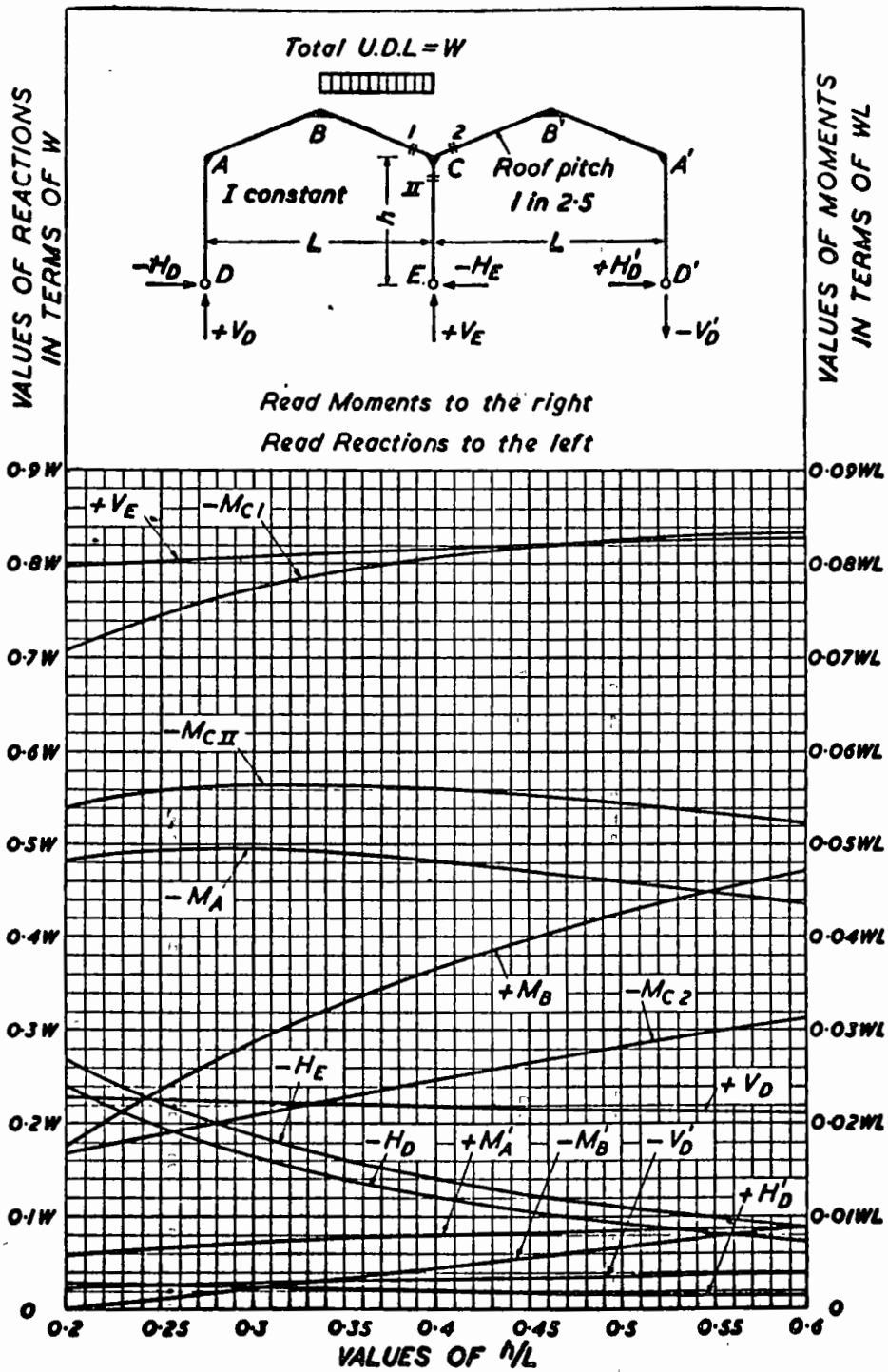


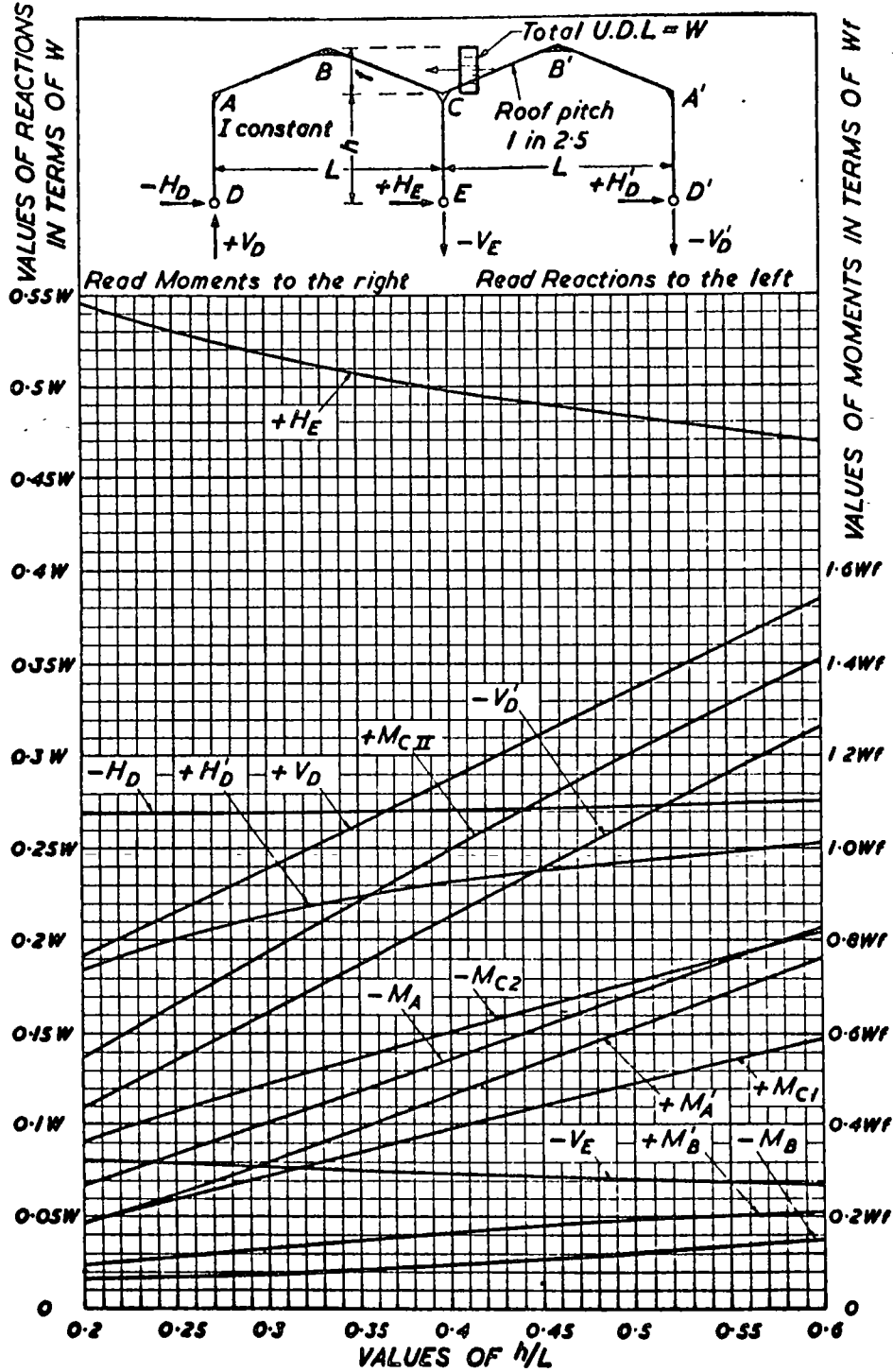


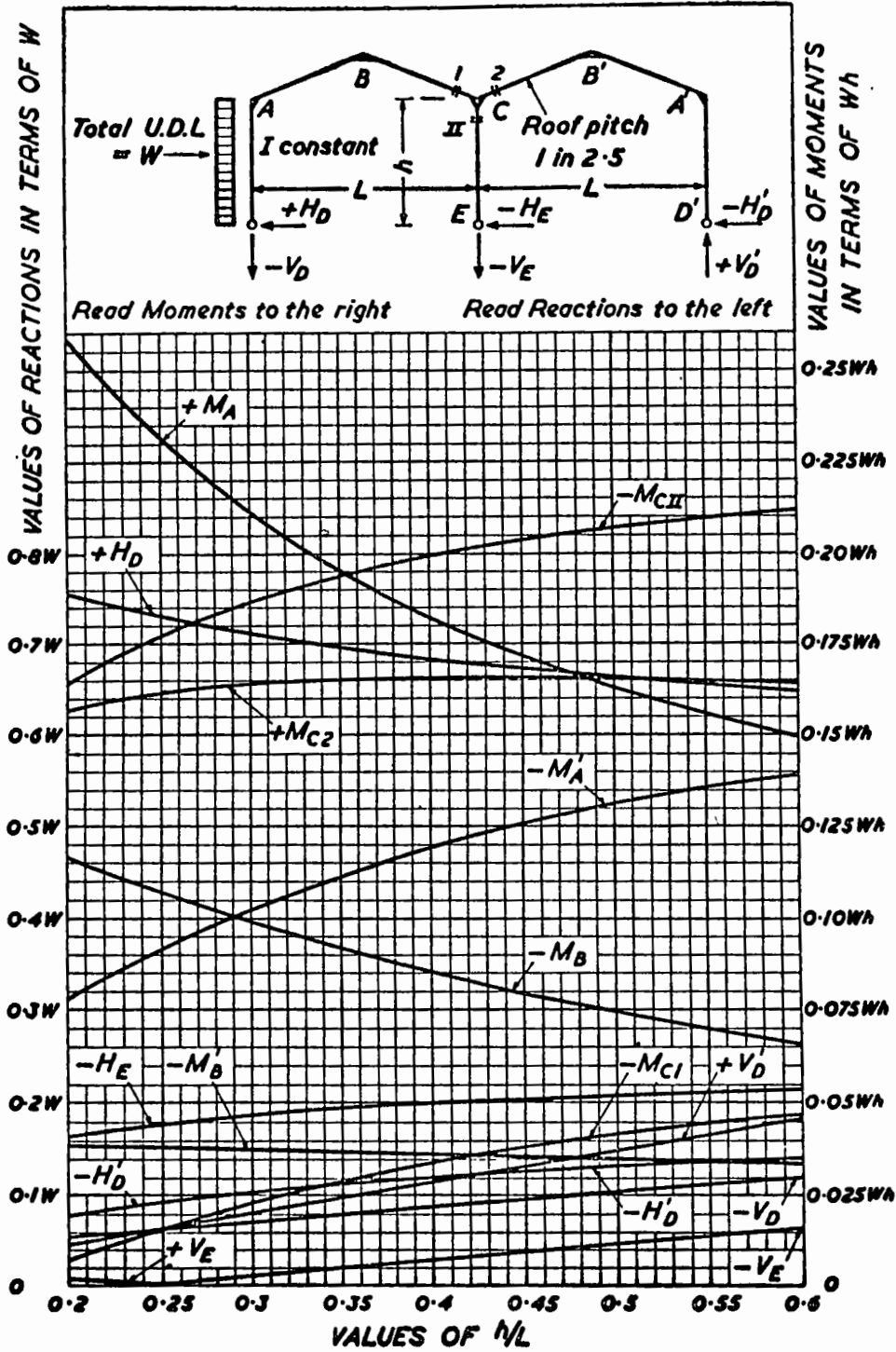


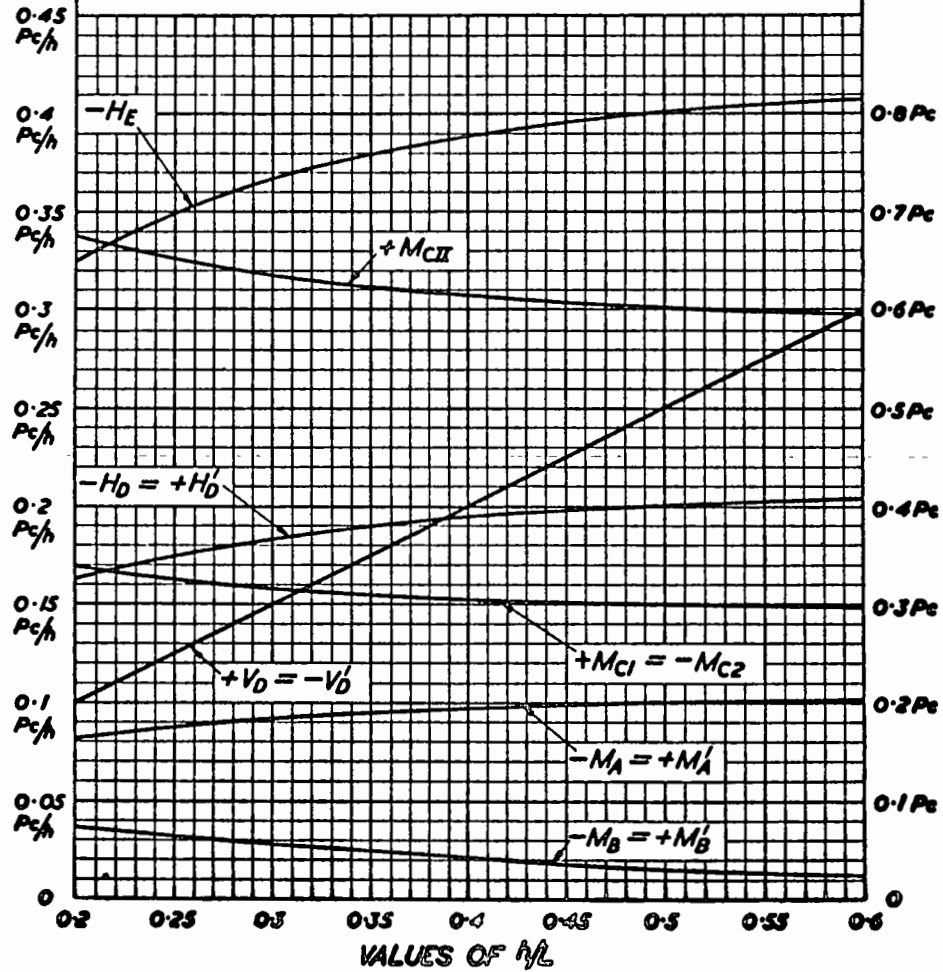
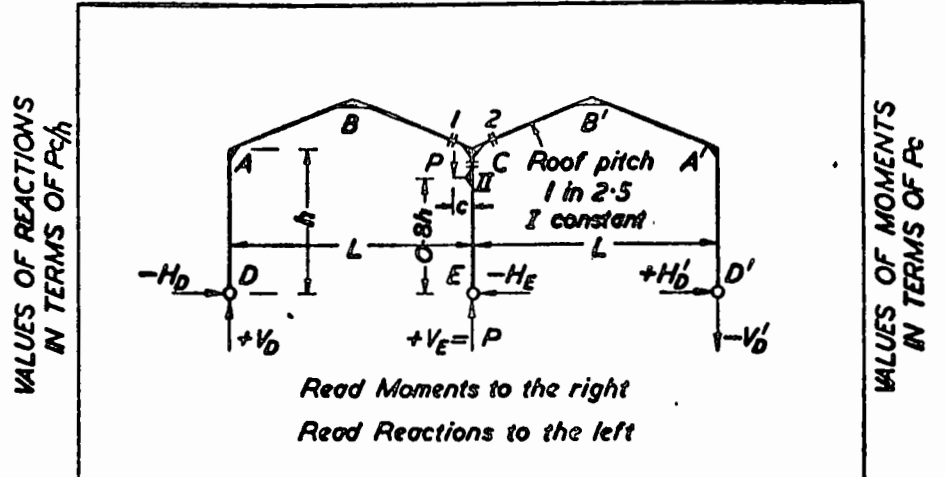




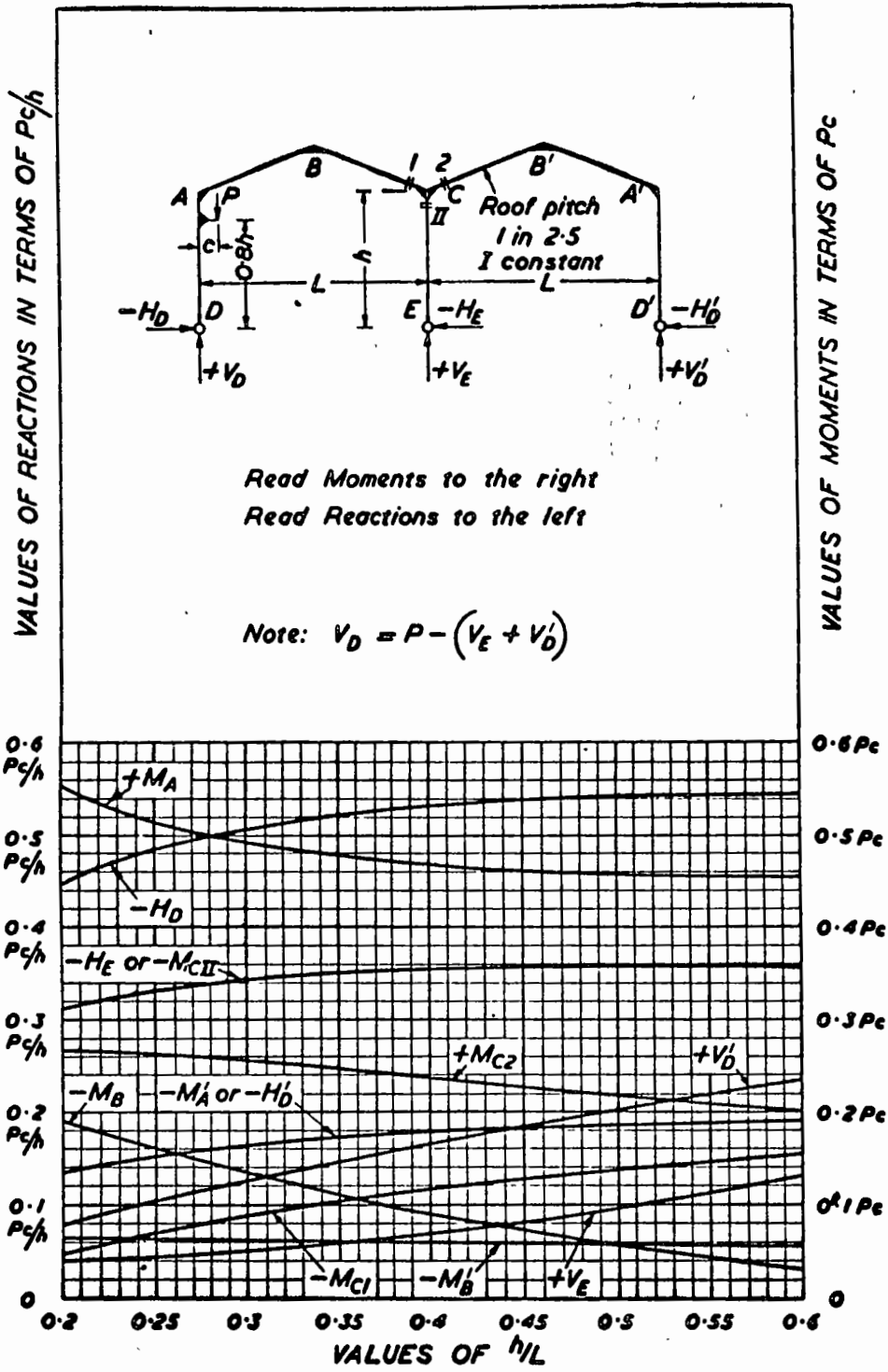


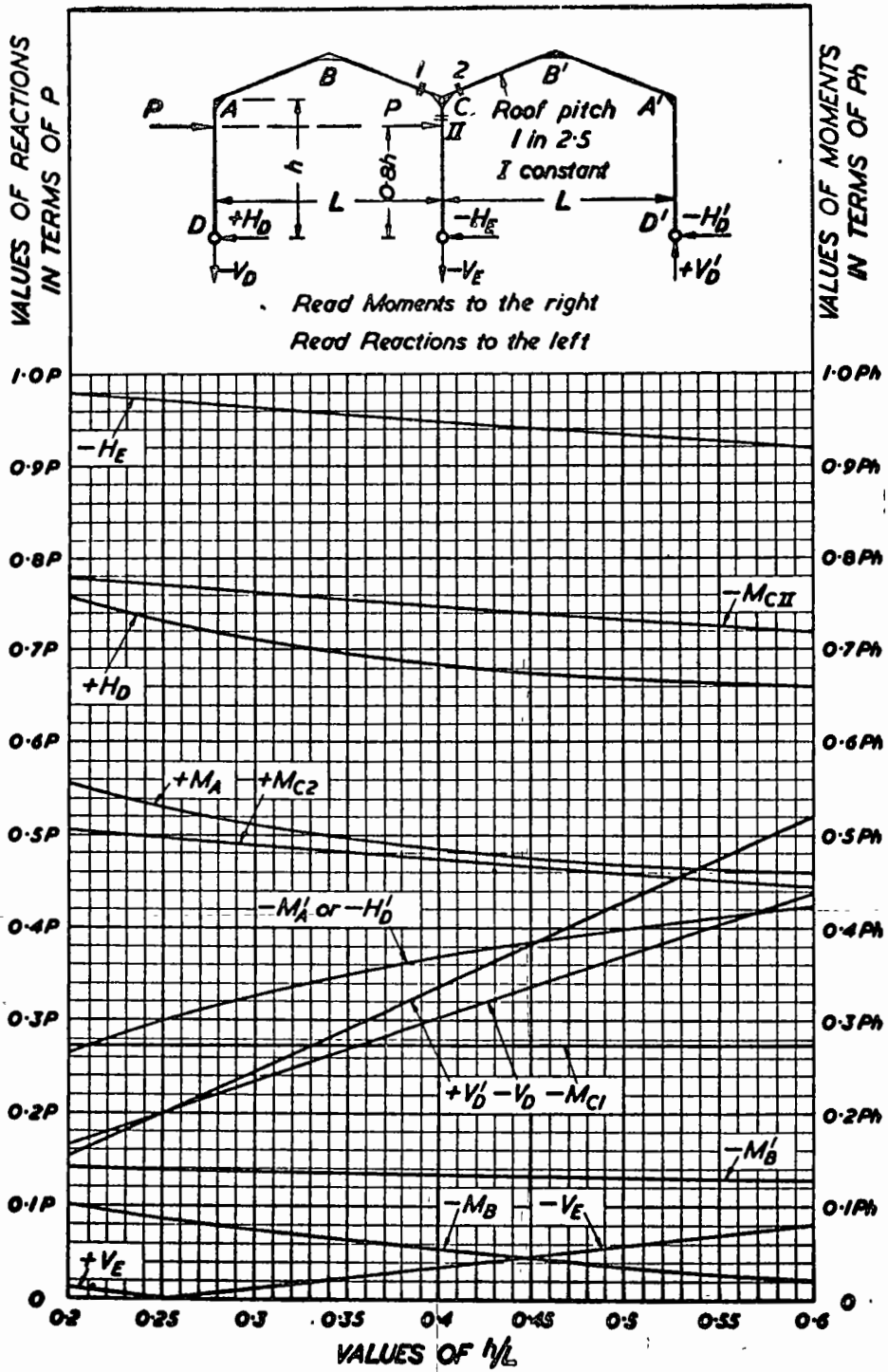


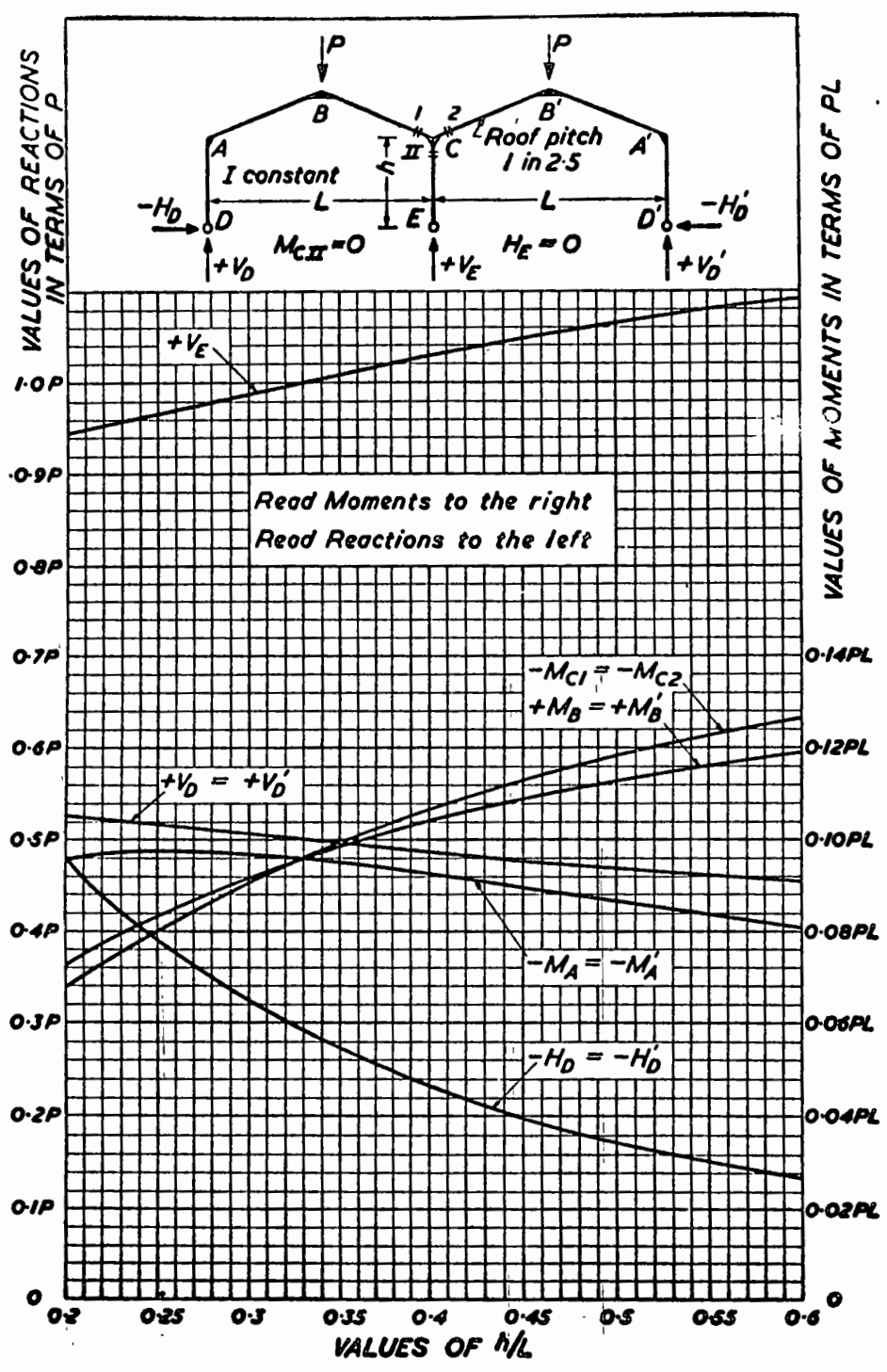


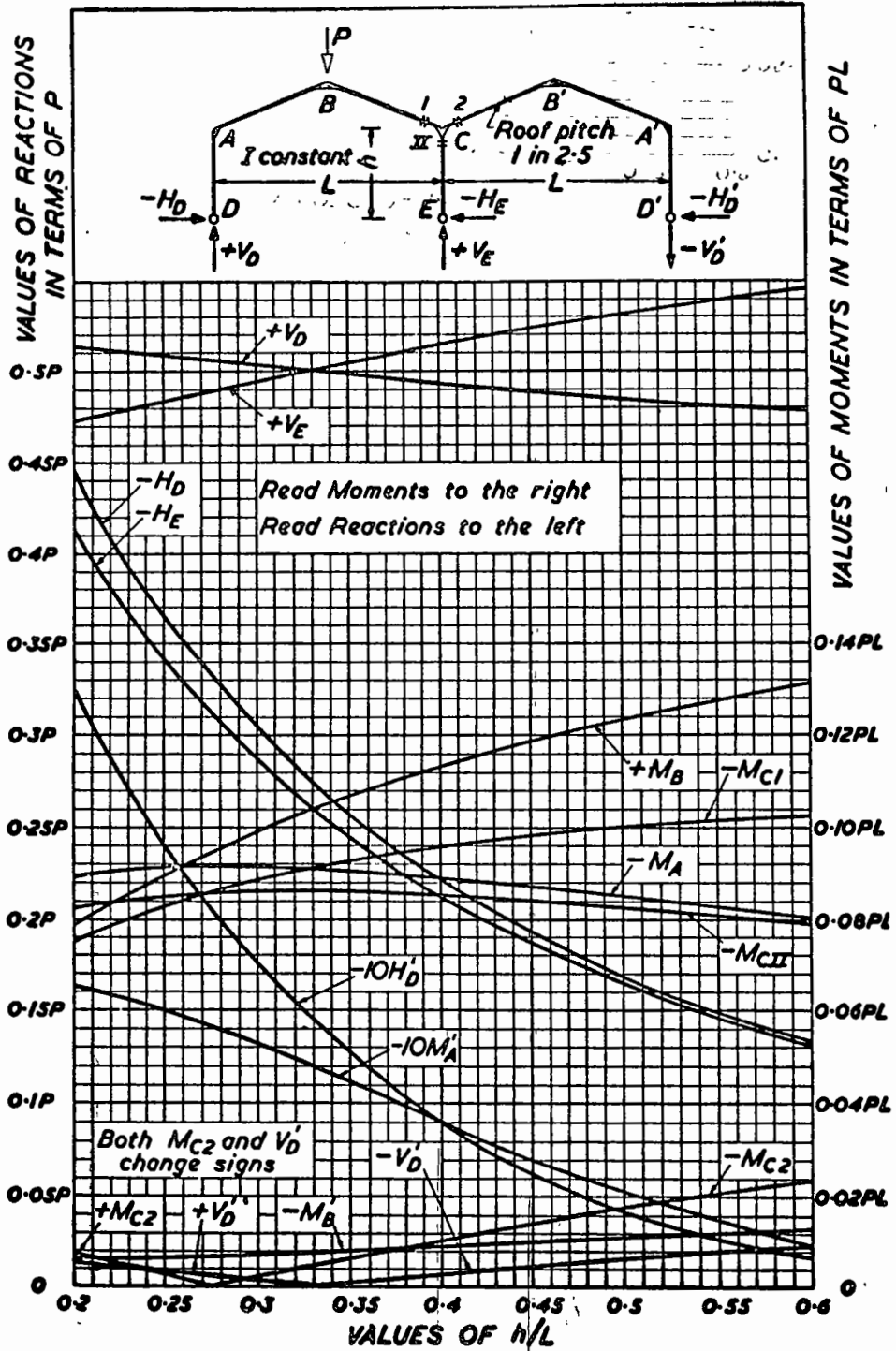


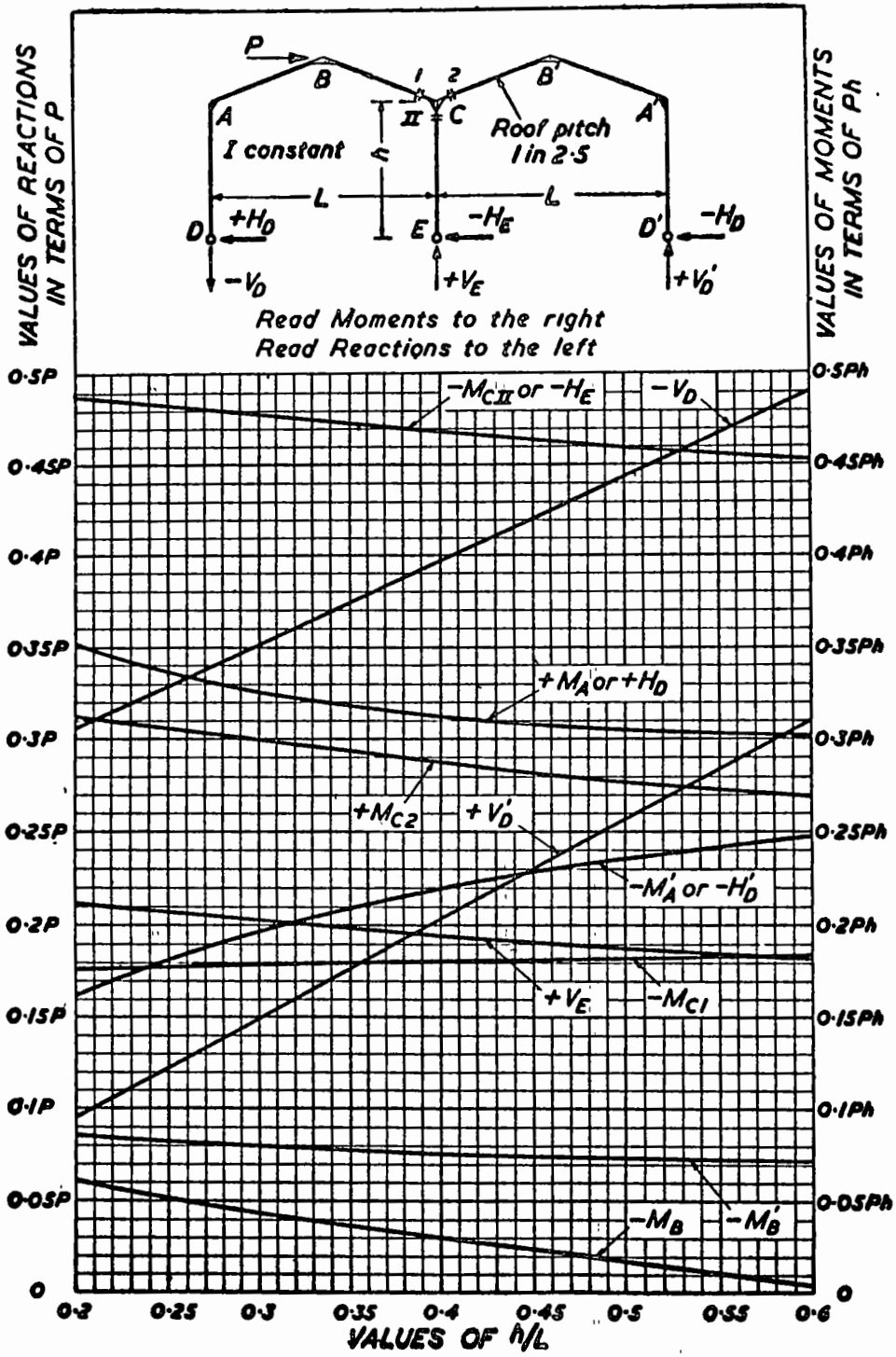


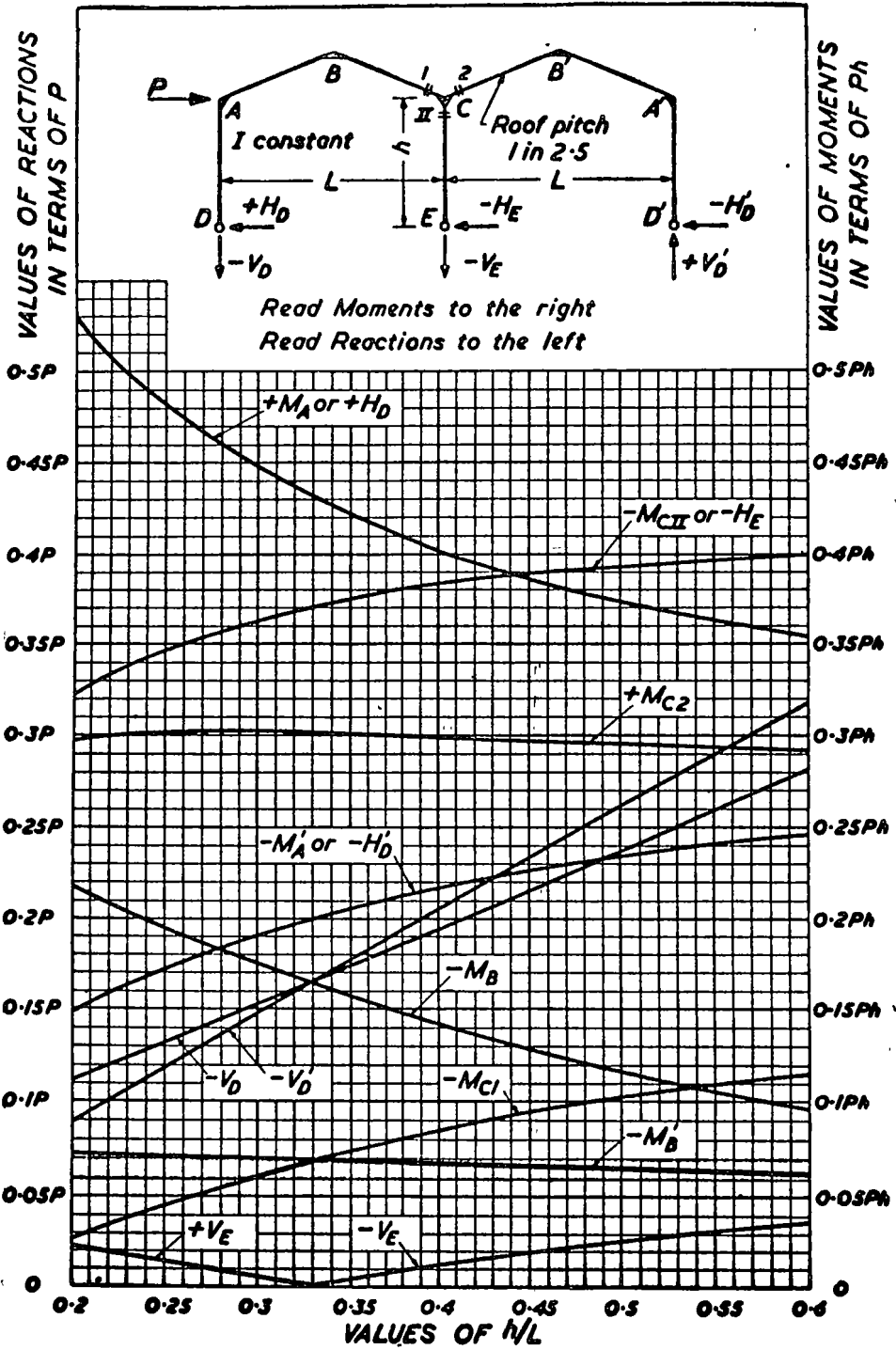


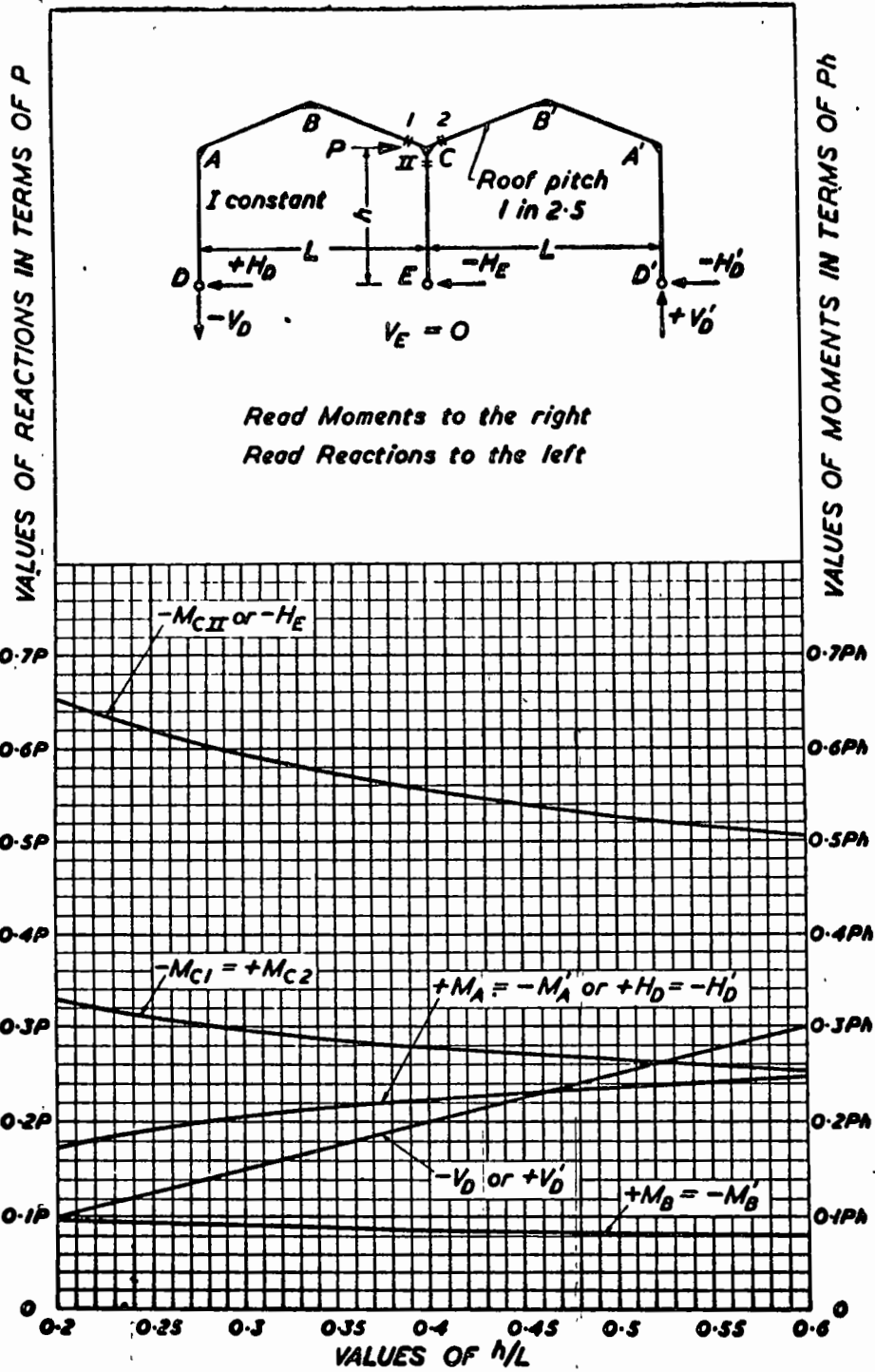












## RIGID-FRAME JOINTS

ALTHOUGH the general principles on which the design of rigid joints is based are fairly well established, the details offer considerable scope for ingenuity. While these joints may be riveted, bolted or welded, the theory to be developed in the following pages will, for simplicity, be largely associated with welding.

A variety of sections may be used in rigid frames, but the most common are Universal Beams and Columns or built-up plate sections.

### Knees for Rectangular Frames

Consider a simple knee for a rectangular frame, as shown in Fig. 42. It does not matter, in principle, whether the cross-beam butts against the column or rests on top of it, but it is assumed that there are two sets of stiffeners, so that the web of the joint has a rectangular frame around it.

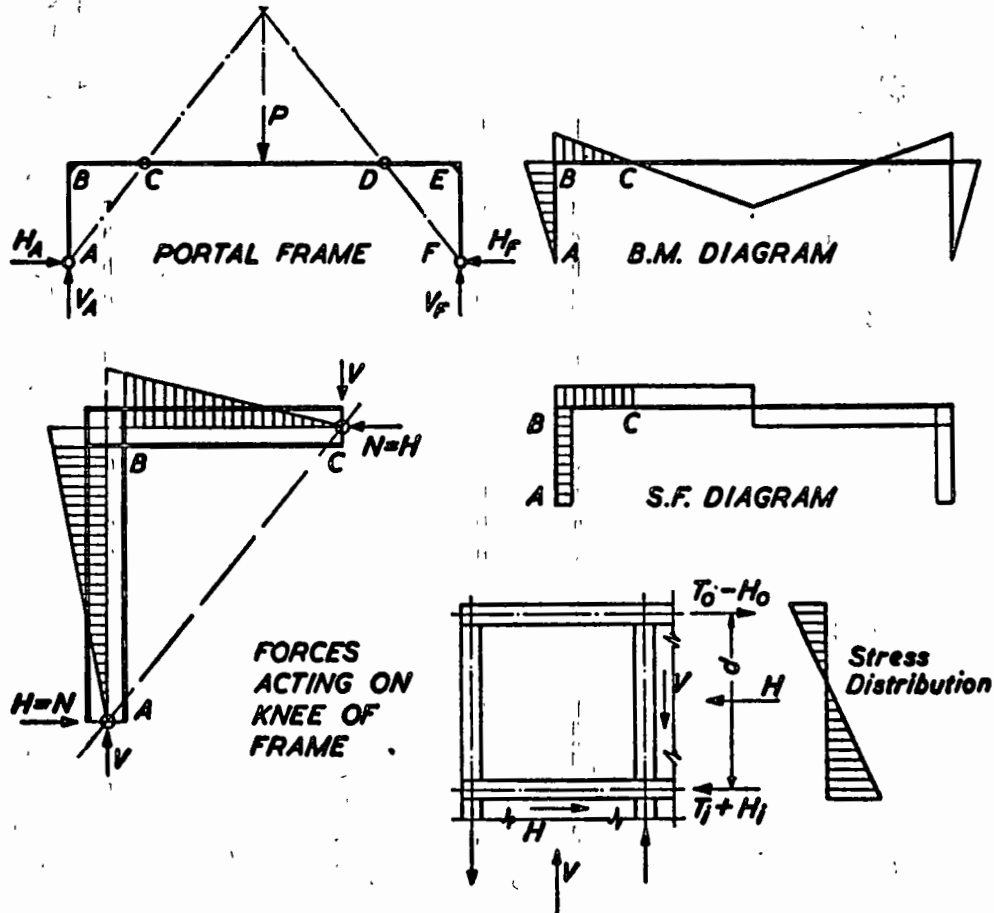


Fig. 42



At any section of a member, the stresses may be found from the normal expression of stress derived in the chapter on Bending and Axial Stresses:

$$f = \frac{P}{A} \pm \frac{M_{xx}y}{I_{xx}}$$

where  $f$  = the stress in any fibre,

$P$  = the longitudinal thrust (i.e.  $N$  or  $H$  in this case),

$A$  = the cross-sectional area of the member,

$M_{xx}$  = the B.M. at the section,

$y$  = the distance from the neutral axis to the fibre being considered

and  $I_{xx}$  = the moment of inertia of the member.

Whilst the maximum B.M. in the frame occurs at the intersection of the neutral axes of the girder and column, the B.M. taken for design purposes can be that at the limits of the knee, i.e. in line with the inside flange of the girder or column.

The shear stress may be found in the strictly accurate manner, giving the distribution shown in Fig. 43 or, in accordance with B.S. 449, by dividing the shear force by the gross web area. Considering the forces applied to the joint, if

$f_o$  and  $f_i$  = the average bending stresses in the outside and inside flanges respectively;

$A_o$  and  $A_i$  = the cross-sectional areas of the outside and inside flanges respectively;

$T_o$  and  $T_i$  = the forces in the outside and inside flanges respectively; while  $H_o$  and  $H_i$  = the components of the horizontal or normal thrust  $H$  (or  $N$ ) in the outside and inside flanges respectively; then

$$T_o = A_o f_o \quad \text{and} \quad T_i = A_i f_i$$

$$H_o = \frac{A_o}{A} H \quad \text{and} \quad H_i = \frac{A_i}{A} H.$$

Consequently, the girder will impose a tensile force of  $T_o - H_o$  in the outside flange and a compressive force of  $T_i + H_i$  in the inside flange at the boundaries of the knee.

The foregoing analysis is theoretical and can safely be simplified by assuming that the flanges of the girder take the whole of the B.M. and transmit the whole of the thrusts, while the web transmits only the shear.

Then,  $T_o = T_i = \frac{M}{d}$  (where  $d$  is the depth of the girder)

$$H_o = \frac{A_o}{A_o + A_i} \cdot H$$

$$H_i = \frac{A_i}{A_o + A_i} \cdot H.$$

In rolled sections, where the flanges are equal in size,  $H_o = H_i$ .

As for the knee itself, experimental evidence shows that there is no tensile stress at the extreme corner as the load takes a direct path across the web.

It is possible to assume, therefore, that the tensile forces in the outer flanges vary uniformly from a maximum at points in line with the inside flanges of the frame, to zero at the outside corner, as shown in Fig. 43.

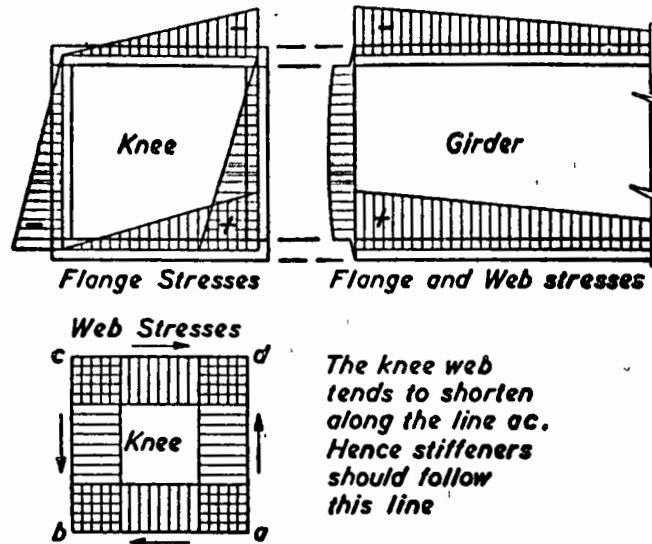


Fig. 43

Each of the flange loads is transmitted into the knee web plate within the lengths of its sides, and this plate is the only means by which the B.M. is transferred from the girder to the column. Consequently, there are heavy shear forces in the knee.

Considering the shear, if

$L$  = the length of the side of the web plate being considered,

$t$  = the web thickness

and  $T$  = the total thrust in the flange,

then, the shear per unit length of plate =  $\frac{T}{L}$ ,

while the shear stress =  $\frac{T}{L \times t}$ .

In welded knees, the load per unit length of fillet weld (one each side of the web plate) is  $T/2L$ .

In the top edge of the web plate the thrust  $T$  is equal to  $T_o - H_o$ , while in the bottom edge it is equal to  $(T_i + H_i) - H$ , as shown in Fig. 42. But  $(T_i + H_i) - H = (T_i + H_i) - (H_i + H_o) = T_i - H_o = T_o - H_o$ . Therefore the shear forces in the top and bottom edges of the web plate are equal. Similarly, those in the outside and inside vertical edges are equal.

If the forces in the web tend to cause overstressing, the web may be increased in thickness or provided with suitable stiffeners. The normal procedure for simple knees is to use diagonal stiffeners.

## American Research on Rectangular Knees

Much of the experimental work carried out on the knees of portal frames has been done in America, particularly by the American Bureau of Standards and at Lehigh University. One of the more easily adaptable groups of formulae for rectangular web plates, attributed to Osgood, was published in Research Paper, R.P. 1130 (reference 4). The theory and example which follow are based on extracts from this paper.

Consider a flat rectangular plate of uniform thickness  $t$ , loaded by forces and couples, as shown in Fig. 44. For equilibrium,

$$M_x - a(2F_{xy} + F_y) = M_y - b(2F_{yx} + F_x).$$

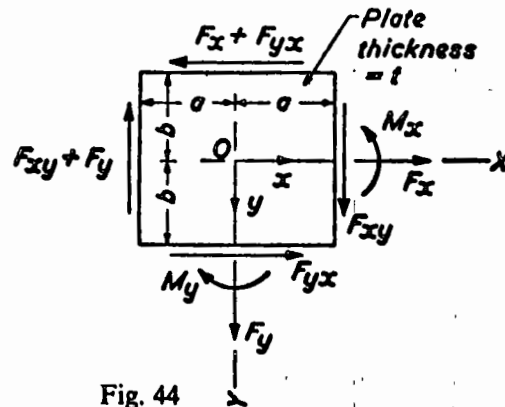


Fig. 44

It is required to determine the stress conditions in the plate, assuming that the normal stresses  $f_x$  and  $f_y$  along the boundaries  $x=a$  and  $y=b$  respectively are uniformly varying and along the boundaries  $x=-a$  and  $y=-b$  are everywhere zero.

Such a condition may be derived from the Airy Stress Function:

$$\phi = b_2xy + \frac{1}{6}(3c_3 + d_4y)(a+x)y^2 + \frac{1}{6}(3b_3 + b_4x)(b+y)x^2.$$

It can be shown that:

$$b_2 = \frac{1}{4abt} \left[ M_x - a(2F_{xy} + F_y) - \frac{3}{2}(M_x + M_y) \right]$$

$$b_3 = \frac{F_y}{4abt}, \quad c_3 = \frac{F_x}{4abt}$$

$$b_4 = \frac{3M_y}{4a^3bt}, \quad d_4 = \frac{3M_x}{4ab^3t}, \text{ from which it may be derived that:}$$

$$f_x = \frac{1}{4abt} \left( F_x + \frac{3M_x}{b^2}y \right) (a+x)$$

$$f_y = \frac{1}{4abt} \left( F_y + \frac{3M_y}{a^2}x \right) (b+y)$$

$$v_{xy} = -\frac{1}{4abt} \left[ M_x - a(2F_{xy} + F_y) + F_y \cdot x + F_x \cdot y - \frac{3M_y}{2} \left( 1 - \frac{x^2}{a^2} \right) - \frac{3M_x}{2} \left( 1 - \frac{y^2}{b^2} \right) \right]$$

where

$v_{xy}$  = the shear stress

$$F_x = -(1-k-j)H - \left( \frac{m}{1-k+j} - \frac{n}{1+k-j} \right) \frac{M}{b}$$

$$F_y = -(1-2p)V$$

$$F_{xy} = V - pV - \frac{r}{p}(M_o - Hb)$$

$$F_{yx} = H - jH - \frac{nM}{(1+k-j)b}$$

$$M_x = - \left( 1 - \frac{m}{1-k+j} - \frac{n}{1+k-j} \right) M$$

$$M_y = -(1-2r)(M_o - Hb).$$

These formulæ have been quoted in their original form, but the stresses  $f_x$  and  $f_y$  will be negative if compressive and positive if tensile, the signs being opposite to those given elsewhere in this section.

In the foregoing formulæ  $p$  and  $r$  are the proportions of  $V$  and  $M$  respectively, which are taken by each flange of the column at the edge  $y=b$ . If a flange is not wholly continuous at the knee, as in riveted construction, it will transmit stress only partially across the discontinuous section. Consequently,  $k$  and  $j$  are the proportions of  $H$ , and  $m$  and  $n$  are the proportions of  $M$ , which are taken by the top and bottom flanges of the beam portion of the knee at the edge  $x=a$ . Had there been no discontinuity,  $k$  would equal  $j$  and  $m$  would equal  $n$ . It is further assumed that the flanges carry no transverse shear.

For a welded frame, as in Fig. 45,

$$M_o = V(a+A) = H(b+B),$$

$$M = M_o - Va$$

$$M_x = -M(1-2n),$$

$$M_y = -(M_o - Hb)(1-2p)$$

$$F_x = -(1-2j)H,$$

$$F_y = -(1-2p)V$$

$$F_{xy} = V - pV - \frac{r}{a}(M_o - Hb)$$

$$F_{yx} = H - jH - \frac{nM}{b}$$

$$j = \frac{\text{Area of one flange of beam}}{\text{Total sectional area of beam}}$$

$$n = \frac{\text{Moment of inertia } I \text{ of one flange of beam}}{\text{Total } I \text{ for the beam}}, \text{ and}$$

$p$  and  $r$  = the corresponding quantities for the column.

#### Principal Stresses and Greatest Shear Stresses

Although not required by B.S. 449:1959, the principal stresses in the knee web of a frame can be computed from the usual formula:

$$f = \frac{f_x + f_y}{2} \pm \sqrt{\left[ \left( \frac{f_x - f_y}{2} \right)^2 + v_{xy}^2 \right]}.$$

Normally the greatest stress occurs at the inside corner of the knee where  $x = +a$ ,  $y = +b$ .

120

The greatest shear stress in the web occurs at the point:

$$x = -\frac{b_3}{b_4}, \quad y = \frac{c_3}{d_4}, \quad \text{i.e. } x = -\frac{F_y \cdot a^2}{3M_y}, \quad y = -\frac{F_x \cdot b^2}{3M_x},$$

the maximum stress being computed from the formula:

$$v_{\max} = \sqrt{\left[\left(\frac{f_x - f_y}{2}\right)^2 + v_{xy}^2\right]}.$$

It will be seen that it is necessary to compute two sets of coefficients, first to obtain the principal stresses, and then to obtain the greatest shear stress. Now the point where the greatest shear stress occurs is very near the centre of the web. Noting this fact, a Canadian engineer, Prof. D. T. Wright, has produced a formula for the maximum shear stress which gives results within about 2 per cent of the exact figures derived from Osgood's formula, viz.:

$$v = \frac{M_c}{4abt} \left(1 + \frac{a^2 t}{3Z_a} + \frac{b^2 t}{3Z_b}\right),$$

where  $M_c = M_O - H \cdot b - V \cdot a$  = the moment of the inside corner of the frame.

$Z_a$  = the section modulus of the knee along a horizontal axis, the section including the vertical flanges as well as the web plate, and  $Z_b$  = the corresponding section modulus along a vertical axis.

It is often quicker to use this formula than that evolved by Osgood.

The various formulæ will be demonstrated by an example.

*Example.* Fig. 45 shows a square knee for a rectangular portal frame in which the flanges are 12 in. x 1 in. in cross-section, and the web plate is 40 in. x 40 in. x 1/2 in. thick.

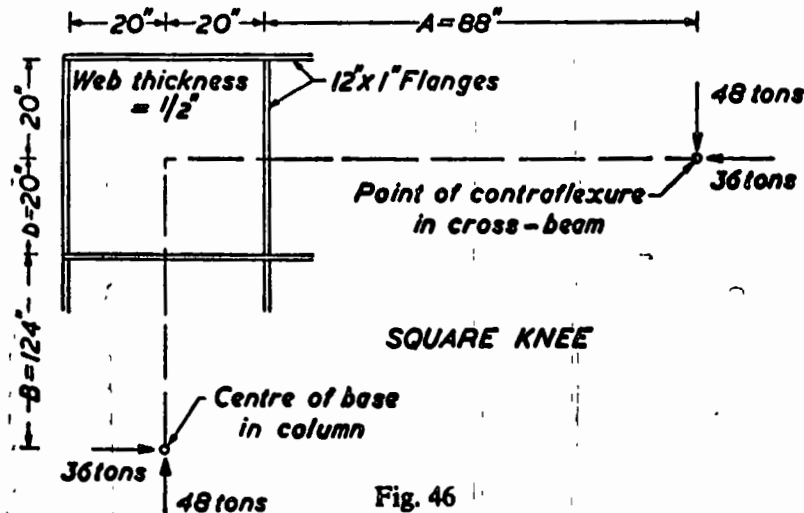


Fig. 46

Using Osgood's formulæ,

$a = b = 20$ in.	$t = 0.5$ in.
$A = 88$ in.	$B = 124$ in.
$V = 48$ tons.	$H = 36$ tons.

The proportion of the reaction  $V$  taken by each flange of the column,

$$p = \frac{\text{Area of one flange}}{\text{Area of whole section}} = \frac{A_f}{2A_f + A_w}$$

(where  $f$  and  $w$  refer to flange and web respectively)

$$= \frac{(12 \times 1)}{(2 \times 12 \times 1) + (40 \times 0.5)} = 0.2727.$$

The proportion of the moment  $M$  taken by each flange

$$r = \frac{I_f}{2I_f + I_w} = \frac{12 \times 1 \times 20 \cdot 5^2}{2(12 \times 1 \times 20 \cdot 5^2) + (0.5 \times 40^3)/12}$$

$$= \frac{5043}{12,753} = 0.3955.$$

The proportion of the thrust  $H$  taken by each flange of the beam,

$$k = j = 0.2727 \quad (\text{as above for } p).$$

The proportion of the moment  $M$  taken by each flange,

$$m = n = 0.3955 \quad (\text{as above for } r).$$

The moment about the centre of gravity of the knee web,

$$M_o = V(a + A) = H(b + B)$$

$$= 48(1.67 + 7.33)$$

$$= 432 \text{ tons ft.}$$

The moment at the junction with the beam (where  $x = a$ ),

$$M = M_o - Va = 432 - (48 \times 1.67)$$

$$= 352 \text{ tons ft.}$$

$$F_x = -(1 - 2j)H = -16.36 \text{ tons.}$$

$$F_y = -(1 - 2p)V = -21.82 \text{ tons.}$$

$$F_{xy} = V - pV - \frac{r}{a}(M_o - Hb)$$

$$= 48 - (0.2727 \times 48) - \frac{0.3955(432 \times 12 - 36 \times 20)}{20}$$

$$= -53.37 \text{ tons.}$$

$$F_{yz} = H - jH - \frac{nM}{b}$$

$$= 36 - (0.2727 \times 36) - \frac{0.3955 \times 352 \times 12}{20}$$

$$= -57.35 \text{ tons.}$$

$$M_x = -M(1 - 2m)$$

$$= -(352 \times 12)(1 - 0.7910)$$

$$= -882.82 \text{ tons in.}$$

$$M_y = -(M_o - Hb)(1 - 2r)$$

$$= -(432 \times 12 - 36 \times 20)(1 - 0.7910)$$

$$= -932.98 \text{ tons in.}$$

Checking for equilibrium,

$$M_x - a(2F_{xy} + F_y) = M_y - b(2F_{yx} + F_x)$$

$$-882.82 - 20(-106.73 - 21.82) = -932.98 - 20(-114.69 - 16.36)$$

or  $1688.18 = 1688.10$ ,

which satisfies the expression  $\Sigma M = 0$  with reasonable accuracy.

### Stresses

The greatest compressive stresses occur at the inside corner where  $x = a$ ,  $y = b$ .

Considering the stresses at this point,

$$f_x = \frac{1}{4abt} \left( F_x + \frac{3M_x \cdot y}{b^2} \right) (a + x)$$

$$= \frac{1}{4 \times 20 \times 20 \times 0.5} \left( -16.36 + \frac{-3 \times 882.82 \times 20}{20^2} \right) (20 + 20)$$

$$= -7.44 \text{ tons per sq. in.}$$

$$f_y = \frac{1}{4abt} \left( F_y + \frac{3M_y \cdot x}{a^2} \right) (b + y)$$

$$= \frac{1}{4 \times 20 \times 20 \times 0.5} \left( -21.82 + \frac{-3 \times 932.98 \times 20}{20^2} \right) (20 + 20)$$

$$= -8.088 \text{ tons per sq. in.}$$

$$v_{xy} = -\frac{1}{4abt} [M_x - a(2F_{xy} + F_y) + F_y \cdot x + F_x \cdot y + 0 + 0]$$

$$= -\frac{1}{4 \times 20 \times 20 \times 0.5} [-882.82 - 20(-106.7 - 21.82) - (21.82 \times 20) - (16.36 \times 20)]$$

$$= -1.156 \text{ tons per sq. in.}$$

The shear stress has been computed as it is needed for the calculation of the principal stresses, but this stress is not the greatest shear stress in the knee. This will be computed later. The maximum principal stress, in compression, is:

$$f_{\max} = \frac{f_x + f_y}{2} - \sqrt{\left[ \left( \frac{f_x - f_y}{2} \right)^2 + v_{xy}^2 \right]}$$

$$= \frac{-7.44 - 8.088}{2} - \sqrt{\left[ \left( \frac{-7.44 + 8.088}{2} \right)^2 + 1.156^2 \right]} = -7.764 - 1.200$$

$$= -8.964 \text{ tons per sq. in.}$$

The formula evolved by Prof. Wright will be used to find the greatest shear stress:

$$v = \frac{M_c}{4abt} \left( 1 + \frac{a^2 t}{3Z_a} + \frac{b^2 t}{3Z_b} \right)$$

Now

$$M_c = M_O - H \cdot b - V \cdot a$$

$$= (432 \times 12) - (36 \times 20) - (48 \times 20)$$

$$= 3504 \text{ tons in.}$$

$$Z_a = Z_b = \text{Total } I/a = 12,753/20 \text{ in.}^3$$

Hence,

$$v = \frac{3504}{4 \times 20 \times 20 \times 0.5} \left( 1 + \frac{2 \times 20^2 \times 0.5 \times 20}{3 \times 12,753} \right)$$

$$= 5.59 \text{ tons per sq. in.}$$

### Knees for Rigid Frames with Pitched Roofs

It is quite common to haunch the knees of frames with pitched roofs, as shown in Fig. 47, or by curving the inner flange, a method of treatment to be described later.

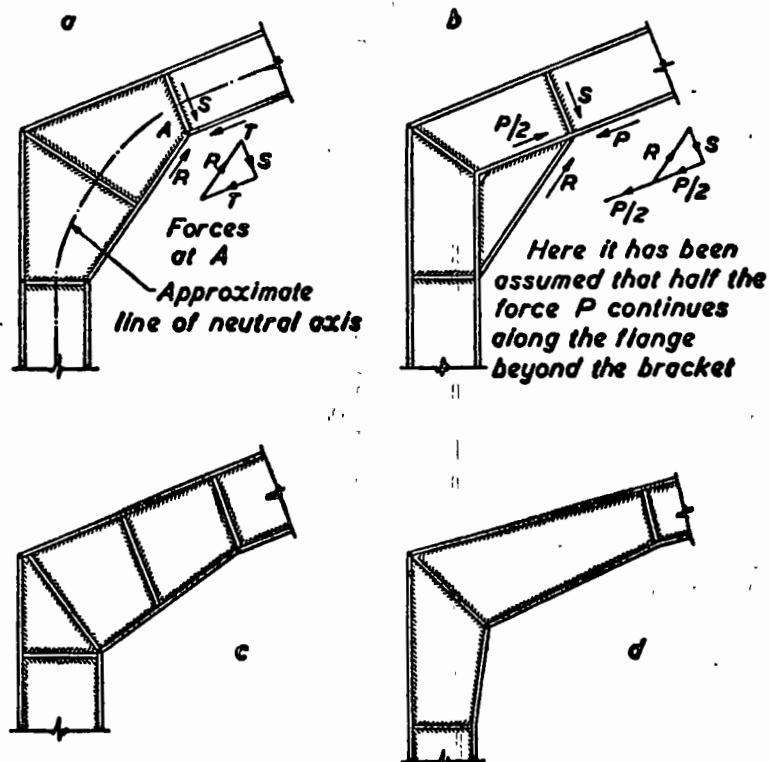


Fig. 47

There is ample experimental evidence to show that the neutral axis of stress in haunched knees moves towards the inside of the frame, as shown in Fig. 47 (a). Consequently the approximate force in the inside flange may be found by assuming that the neutral axis occurs at the third point along the diagonal from the inside flange to the outside corner of the frame. Alternatively the force in the flange may be resolved from the triangle of forces shown in Fig. 47 (a), the force being known that in the inside flange



of the rafter. In the type of joint shown in Fig. 47 (b) it is often assumed that the whole of the inside flange forces are resolved into the bracket flange, but, as an alternative, it may be assumed that, say, one-half of the force continues along the flange of the main member, as shown in the diagram.

The stresses in knees of the type shown in Fig. 47 (a), (c) and (d) can be found by using Vierendeel's Tapered Beam formulæ or Olander's formulæ which are described later. See Figs. 60 and 62. Considering the knee in Fig. 48, all the sections between *AA* and *BB*, except in the hatched areas can be analysed by the above formulæ. However, if the knee is not overstressed elsewhere, it is very unlikely to be so in the hatched zones.

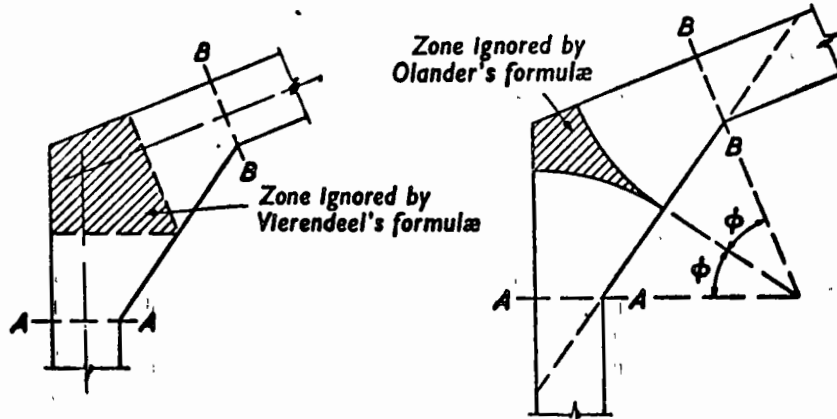
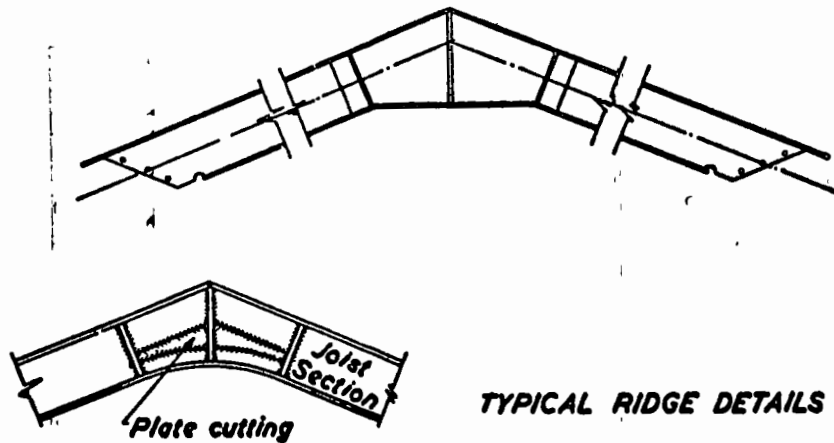


Fig. 48

The forces in the stiffeners at the limits of the knee are found by a resolution of forces, as at point *A* in Fig. 47 (a) where *S* is the appropriate stiffener force, and the stiffeners and welds are designed accordingly. The remaining stiffeners inside the knee can be of nominal size, their primary function being to prevent local buckling of the web and lateral failure of the inner flange.



TYPICAL RIDGE DETAILS

Fig. 49

### Ridges in Pitched Roofs

Ridges in pitched roofs are designed in precisely the same manner as obtuse-angled knees. Normally they present less difficulty than knee joints as the angle between the rafters is very obtuse and the forces are much less than in knees. If joist sections are employed it may be unnecessary to add brackets, while in lightly loaded structures it is sufficient to butt weld the ends of the rafters. Some typical joints are shown in Fig. 49.

### Knees with Curved Flanges

It is probable that joints with curved flanges will always be the subject of some controversy but here it is proposed to describe and illustrate the articles published or research work carried out both in this country and abroad.

It is of some interest to consider a formula, known as the Winkler-Résal formula, which can be used for an *initially* curved bar with parallel flanges, such as that shown in Fig. 50.

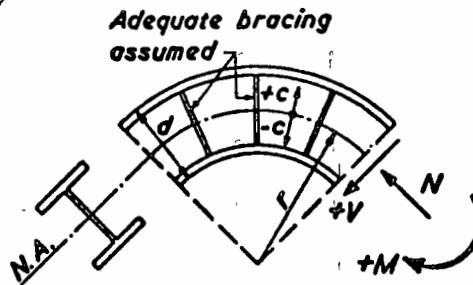


Fig. 50

The formula for the stress in any fibre of the bar is

$$f = \frac{N}{A} - \frac{M}{r \cdot A} - \frac{M \cdot c}{U} \times \frac{r}{r + c}$$

where  $N$  = the normal thrust,

$A$  = the cross-sectional area of the bar,

$M$  = the applied B.M.

$r$  = the initial radius of curvature of the bar taken to the neutral axis N.A. of the section,

$c$  = the distance from the N.A. to the fibre being considered, being positive when measured away from the centre of curvature and negative when measured towards it

and  $U$  = a figure analogous to the moment of inertia  $I$  and which may be replaced by  $I$  when the value of  $r$  is greater than twice the depth  $d$  of the bar.

When  $r$  is less than twice  $d$  and the section is composed of rectangles,

$$U = r^2(2.30258r \sum b \cdot \log w_1/w_2 - A),$$

where the symbols have the significance shown in Fig. 51.

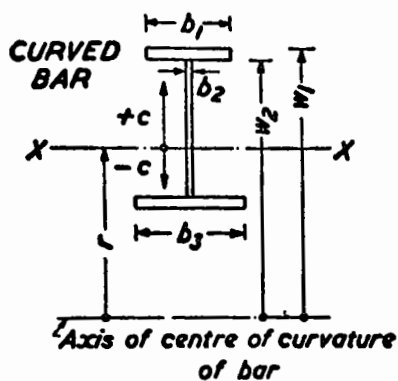


Fig. 51

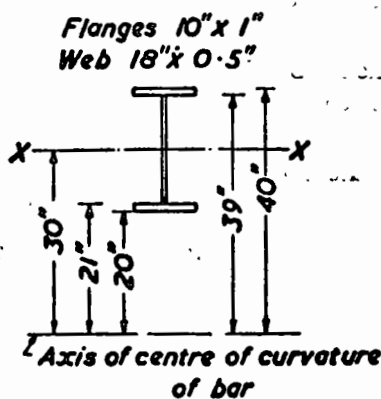


Fig. 52

Suppose the member is of the section shown in Fig. 52, the properties being as follows:

Flanges = 10 in. x 1 in.    Web = 18 in. x 0.5 in.  
 Area  $A = 29$  sq. in.     $r = 30$  in. ( $r/d = 1.5$ ).

Then,  $U = r^2(2.30258r \sum b \cdot \log w_1/w_2 - A)$   
 Now,  $10 \log 40/39 = 0.10995$   
 $0.5 \log 39/21 = 0.13442$   
 $10 \log 21/20 = 0.21189$   
 $\sum b \cdot \log w_1/w_2 = 0.45626$

Therefore,  $U = 30^2(2.30258 \times 30 \times 0.45626 - 29)$   
 $= 2265.53 \text{ in.}^4$

It is interesting to note that the moment of inertia  $I = 10 \times 20^3/12 + 9.5 \times 18^3/12 = 2049 \text{ in.}^4$ .

It will be observed that the general form of the stress equation resembles the normal stress formula, i.e.  $N/A \pm M/Z$ .

The Winkler-Résal formula should be used for curved members when  $r/d$  is less than 2.5. When this ratio exceeds 2.5, the normal formula can be used with safety.

**Example.** Considering the member shown in Fig. 52, and assuming that it is subjected to a B.M. of 1900 tons in.,

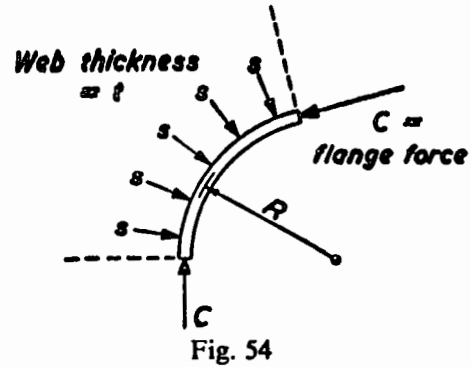
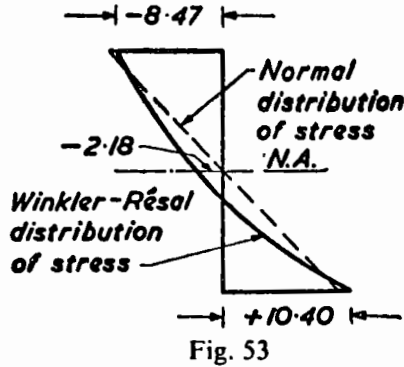
$$f = \frac{N}{A} - \frac{M}{r \cdot A} - \frac{M \cdot c}{U} \times \frac{r}{r+c} = \frac{N}{A} - \frac{M}{r \cdot A} - \frac{M \cdot r}{U} \times \frac{c}{r+c}$$

$$= 0 - \frac{1900}{30 \times 29} - \frac{1900 \times 30}{2266} \times \frac{c}{30+c}$$

$$= -2.184 - 25.160 \times \frac{c}{30+c}$$

If values are plotted for various depths  $c$  from the neutral axis, the stress diagram shown in Fig. 53 is obtained.

It should be noted that the Winkler-Résal formula makes suitable allowance for the shift of the neutral plane of bending from the N.A. of the section towards the inner flange.



Now the change in direction of the force in the flange of a curved member induces radial stresses in the web which can be calculated from the following formula, due to Professor Campus of Liège:

$$s = C/Rt, \text{ (see Fig. 54 )}$$

where  $s$  = the unit radial stress,

$C$  = the total flange force,

$R$  = the radius of curvature of the flange being considered

and  $t$  = web thickness.

Professor Magnel (reference 6) states that the radial stress should be added to that due to the shear across the web.

The radial force is applied at the junction of web and flange and causes cross-bending, the edges of the flanges moving away from the centre of curvature when the flange is compressed and towards it when it is in tension, as shown in Fig. 54.

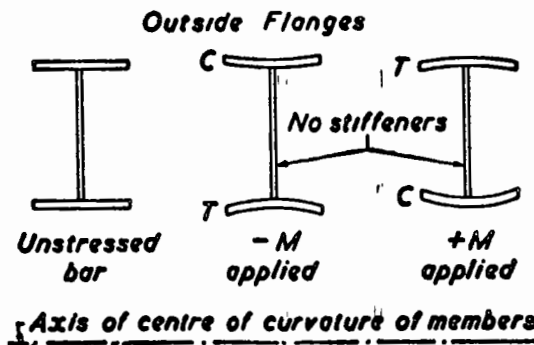


Fig. 55

Dr. Hans H. Bleich (reference 7) investigated the effects of this phenomenon and produced two coefficients  $\nu$  and  $\mu$  (nu and mu), the first being associated with the longitudinal stresses in the flanges and the second the transverse stresses.

If  $f$  is the mean stress derived from the Winkler-Résal formula, then  $\text{max. } f = f/\nu$  and  $f' = \mu \text{ max. } f$ , where  $\nu$  and  $\mu$  have the following values with respect to the expression  $b^2/Rt$ , the symbols for which are shown in Fig. 56 :

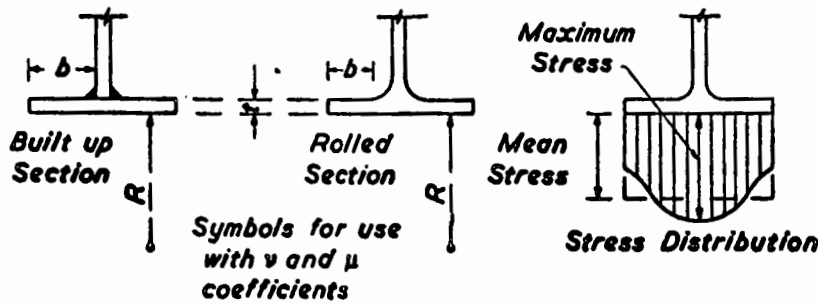


Fig. 56

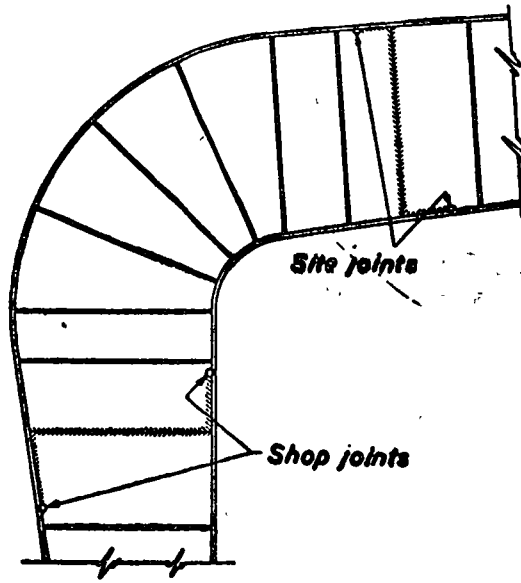
Bleich's Coefficients

$\frac{b^2}{Rt}$	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$\nu$	1.000	0.994	0.977	0.950	0.915	0.878	0.838	0.800	0.762	0.726
$\mu$	0	0.297	0.580	0.836	1.056	1.238	1.382	1.495	1.577	1.636
$\frac{b^2}{Rt}$	1.0	1.1	1.2	1.3	1.4	1.5	2.0	3.0	4.0	5.0
$\nu$	0.693	0.663	0.636	0.611	0.589	0.569	0.495	0.414	0.367	0.334
$\mu$	1.677	1.703	1.721	1.728	1.732	1.732	1.707	1.671	1.680	1.700

If the radial forces tend to lead to overstressing, the flanges must be braced either by entire web stiffeners or by small gussets, the spacing of which is a matter for engineering judgment as no rules, mathematical or empirical, have been derived.

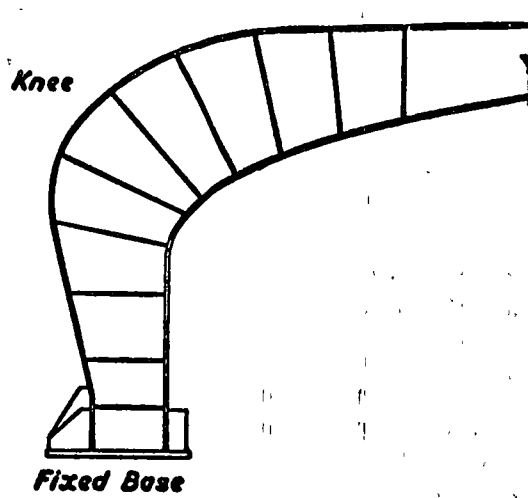
Some practical examples of curved knees closely resembling curved bars are shown in Figs. 57 and 58, the latter being described in reference 8, as well as in other papers.

Generally knees are not shaped like curved bars, the majority being of the type shown in Figs. 59 (a) and (b) where the outside flange is straight. The rapid change of section at the knee and the curvature of the centre-line affect both the magnitude and the distribution of the fibre stresses.



*Portal Knee  
for Bus Garage*

Fig. 57



*Portal Knee  
for Bridge*

Fig. 58

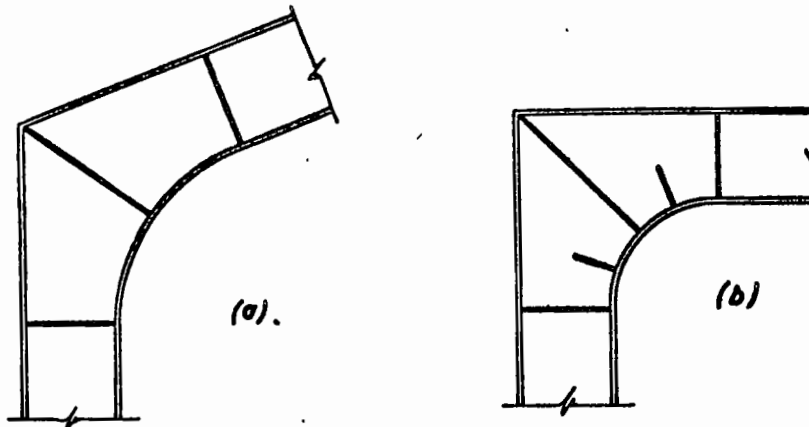


Fig. 59

Professor Vierendeel devised Tapered Beam formulæ for the knee shown in Fig. 60, the formulæ being as follows:

On any section  $AA$ ,

$$f_o = \frac{P}{A} - \frac{Ma_o}{I}$$

$$f_i = \frac{P}{A} + \frac{Ma_i}{I}$$

$$v = \frac{1}{bd}(V + f_i F_i \sin \phi),$$

where  $f_o$  and  $f_i$  = the mean stresses in the outside and inside flanges, respectively,  
 $a_o$  and  $a_i$  = the distances of the centroids of the outside and inside flanges, respectively, from the axis shown,  
 $v$  = the shear stress

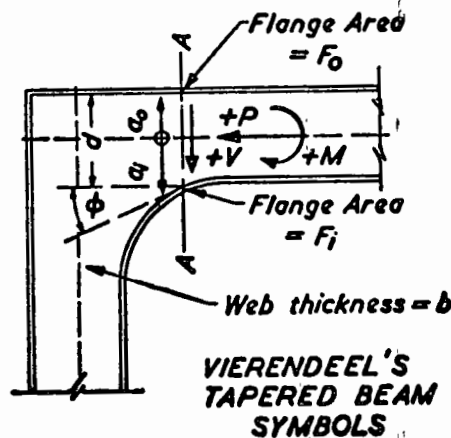


Fig. 60

$M$ ,  $V$  and  $P$  = the bending moment, shear force and thrust at the section  $AA$ ,

$b$  = the web thickness,

$d$  = the web depth,

$$A = bd + F_o + F_1 \cos \phi$$

and 
$$I = bd^3/12 + F_o a_o^2 + F_1 \cos \phi a_1^2.$$

These formulæ are logical and give reasonable results.

When the mean flange stresses have been calculated, the maximum stresses may be computed using Dr. H. H. Bleich's coefficients and suitable stiffeners added where necessary.

It should be noted that the Tapered Beam formulæ can be applied to knees of the type shown in Fig. 47.

In the U.S.A. a number of investigators have produced formulæ for curved knees, using circular sections as shown in Fig. 61.

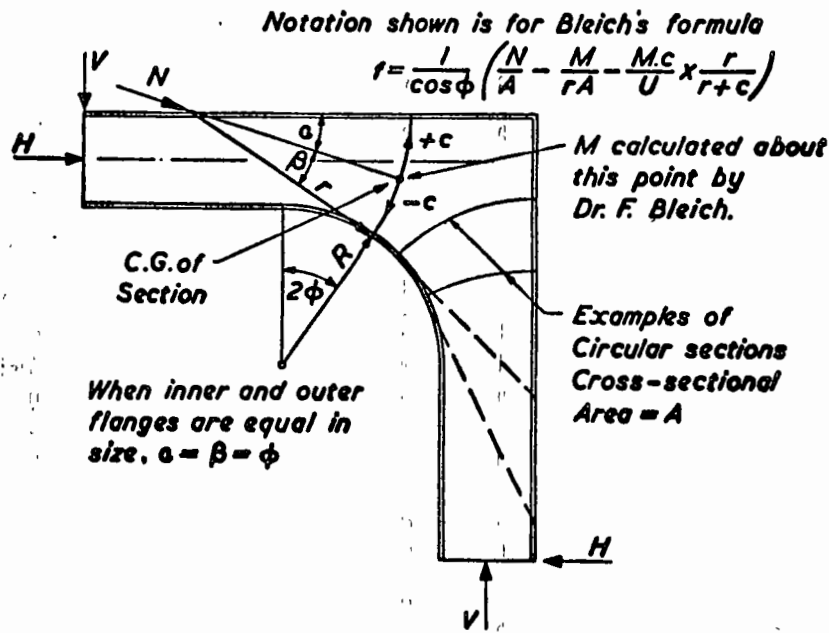


Fig. 61

Osgood's formulæ (reference 5) are mathematically exact but are lengthy and cumbersome for use in design and are not quoted here.

Dr. Friedrich Bleich (reference 9) adapted the Winkler-Résal formula, adjusting it by multiplying the whole expression by  $1/\cos \phi$ . It will be noted from Fig. 61 that  $r$  and  $c$  are measured in a different way, but the principles of calculation are identical with those for Example 1.

The amount of work involved in using Bleich's method is less than with Osgood's method but greater than that with a recent method (reference 10) due to Harvey C. Olander, which gives reasonable results and can be recommended for use in design.



132

Olander states that the method is simply to take circular sections that cut the extreme fibres at right angles, such as section *AB*, Fig. 62, develop the section as shown and obtain the cross-sectional area *A* and moment of inertia *I* of the developed section. Next, resolve all forces to the right of section *AB* into the values  $P_O$  and  $M_O$  about the point *O*, the centre of the arc.  $P_O$  passes through the centre of gravity of section *AB*, and  $M_O$  is the moment of the forces about *O*. Then, with these values, the stresses are

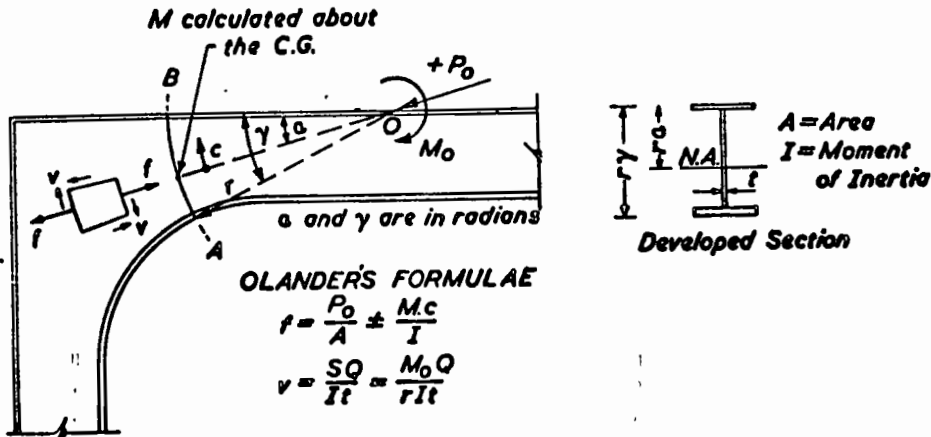


Fig. 62

calculated as for an ordinary beam, except that the shear is determined from  $M_O$ . The total shear on section  $AB = S = M_O/r$ . Then the unit shear along the section,

$$v = \frac{SQ}{It} = \frac{M_O Q}{rIt}$$

where *Q* is the statical moment of the area of the section about the point being considered. (Cf. British notation,  $v = S \cdot a \cdot y/It$ .)

The stresses normal to the section,

$$f = \frac{P_O}{A} \pm \frac{M \cdot c}{I}$$

where *M* is the B.M. at the C.G. of the section *AB*.

An example will help to explain the method.

*Example.* Fig. 62 shows a curved knee joining a 16-in. × 12-in. × 110-lb. B.F.B. to a 24-in. × 12-in. × 165-lb. B.F.B. The properties of the two B.F.B.s are as follows:

B.F.B. (in.)	Flange Thickness (in.)	Web Thickness (in.)	Area <i>A</i> (sq. in.)	$Z_{xx}$ (in. <sup>3</sup> )
16 × 12	1.00	0.55	32.35	188.53
24 × 12	1.36	0.70	48.53	404.79

The cross-beam is chosen to resist the B.M. in the centre of the beam and will be understressed at its junction with the knee. If the radius of the inner flange is 48 in., then the radius will equal twice the depth of the larger section or three times that of the smaller section.

It is convenient to continue the flange thicknesses of the smaller B.F.B. around the knee, i.e. to use a 12-in. x 1-in. plate. The web of the knee will be 0.625 in. thick, i.e. some convenient thickness between those for the two B.F.B.s.

The knee can be divided into any number of circular sections. As, however, the greatest stress is situated just inside the knees, these areas should be investigated. Consequently, sections are usually chosen at 15° or 18° intervals along the inside of the knee.

If 18° intervals are used in this case, there will be six cross-sections to consider, as shown in Fig. 63.

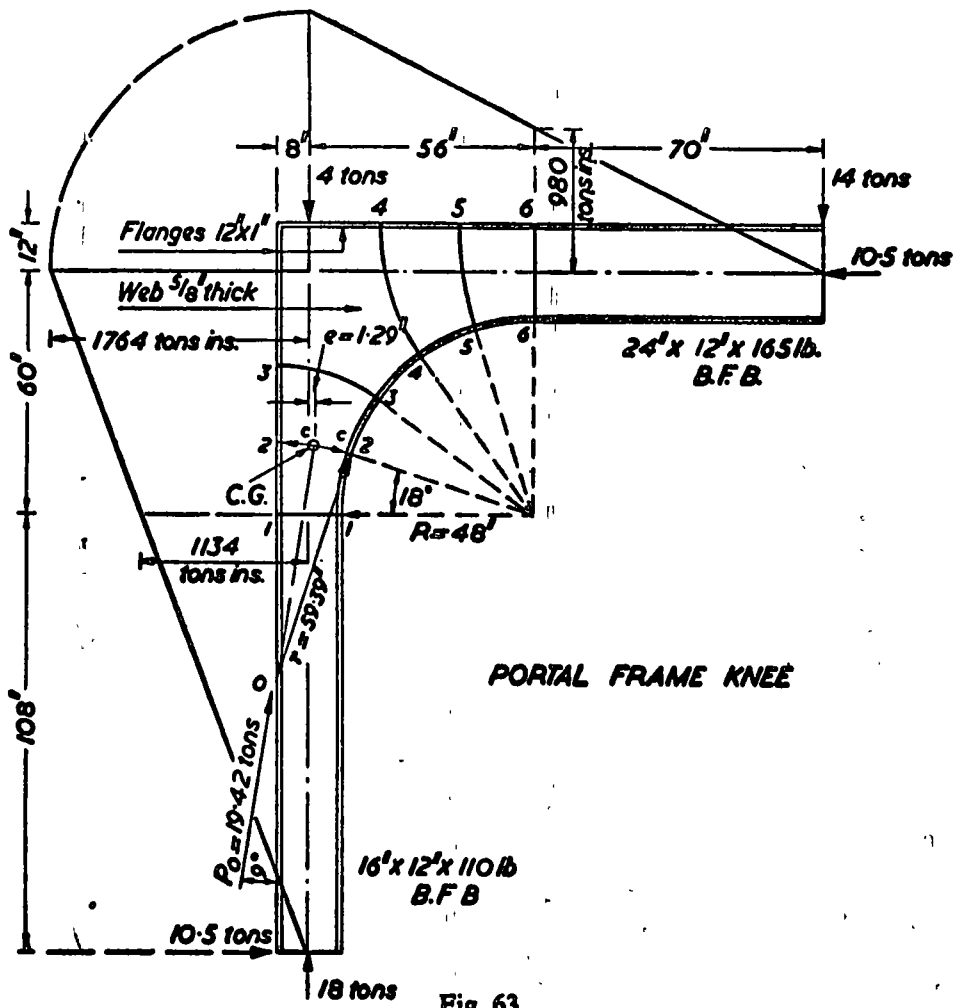


Fig. 63

It is advisable to check that sections 1-1 and 6-6 are in order first, before embarking upon the knee proper.

Considering section 1-1,

$$P = 18 \text{ tons.} \quad A = 31.19 \text{ sq. in.}$$

$$M_{xx} = 1134 \text{ tons in.} \quad Z_{xx} = 144.5 \text{ in.}^3.$$

Then

$$f = \frac{P}{A} \pm \frac{M_{xx}}{Z_{xx}}$$

$$= \frac{18}{31.19} \pm \frac{1134}{144.5}$$

$$= 6.56 \text{ or } -5.46 \text{ tons per sq. in.}$$

Using the normal B.S. 449 procedure for joists, the shear stress

$$v = \frac{\text{load}}{\text{gross web area}} = \frac{10.5}{16 \times 0.55}$$

$$= 1.19 \text{ tons per sq. in.}$$

Considering section 6-6, the knee section should be taken, as the properties of the adjoining B.F.B. are greater in magnitude than those for the knee.

Then

$$I_{xx} = \frac{12 \times 24^3}{12} - \frac{11.375 \times 22^3}{12}$$

$$= 3731 \text{ in.}^4.$$

and

$$Z_{xx} = \frac{3731}{12} = 311 \text{ in.}^3.$$

$$A = 2(12 \times 1) + (22 \times 0.625)$$

$$= 37.75 \text{ sq. in.}$$

Therefore

$$f = \frac{P}{A} \pm \frac{M_{xx}}{Z_{xx}}$$

$$= \frac{10.5}{37.75} \pm \frac{980}{311}$$

$$= 3.43 \text{ or } -2.87 \text{ tons per sq. in.}$$

and

$$v = \frac{14}{24 \times 0.625}$$

$$= 0.93 \text{ ton per sq. in.}$$

Consequently, the stresses at sections 1-1 and 6-6 are acceptable.

Although Oländer's method is quicker than some other methods of analysis, a certain amount of time is taken calculating the geometrical properties of the circular sections. It is desirable to devise some system of tabulation for the calculations.

Fig. 64 shows the symbols which have to be calculated for each section.

$$r = \frac{h + R(1 - \cos 2\phi)}{\sin 2\phi}$$

$$a = R \cdot \sin 2\phi$$

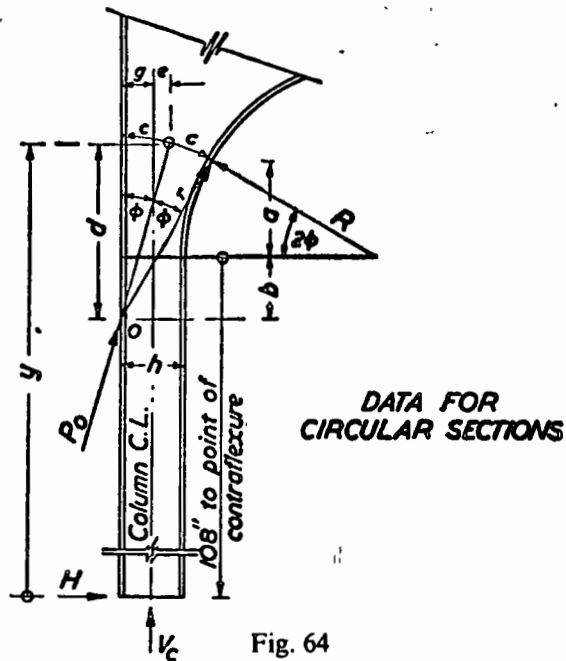
$$(a + b) = r \cdot \cos 2\phi$$

$$d = r \cdot \cos \phi$$

$$c = \phi \cdot r \quad (\phi \text{ in radians})$$

$$(g + e) = r \cdot \sin \phi,$$

where  $h$ ,  $R$ ,  $\phi$  and  $g$  are known quantities.



For values of  $2\phi$  not exceeding  $45^\circ$  the radius  $r$  is measured along the column, but between  $45^\circ$  and  $90^\circ$  it is measured along the cross-beam. (The angle subtended by the inner flange should always be bisected in this way, even if the angle is acute as in the case of a frame with a pitched roof.)

The area and moment of inertia of the 'I'-sections traced by each circular section are found in the normal way, the total depth being the arc length.

For sections 2-2 and 3-3,

$$P_0 = V_c \cos \phi + H \sin \phi$$

$$M = H \cdot y - V_c \cdot e.$$

For sections 5-5 and 4-4,

$$P_0 = H \cos \phi + V_b \sin \phi$$

$$M = V_b \cdot x - H \cdot e.$$

where  $y$  is the vertical distance between the bottom of the column and the C.G. of the section,  $x$  is the horizontal distance from the end of the beam to the C.G. of the section and  $V_c$  and  $V_b$  refer to the column and beam respectively.

Proceeding, the relevant properties of, and the stresses in, the four sections inside the knee are as follows, the units being tons and inches:

Section	$P_0$	$A$	$M$	$e$	$I_{xx}$	$\frac{P_0}{A} \pm \frac{M \cdot c}{I_{xx}}$
2-2	19.42	34.41	1289	9.33	2112	+6.26 or -5.13
3-3	20.38	39.57	1400	13.46	4833	+4.41 or -3.38
4-4	14.31	44.88	1430	17.70	9041	+3.18 or -2.48
5-5	12.56	39.49	1217	13.40	4481	+3.92 or -3.36

The complete flange stresses have been plotted in Fig. 65.

RIGID-FRAME JOINTS

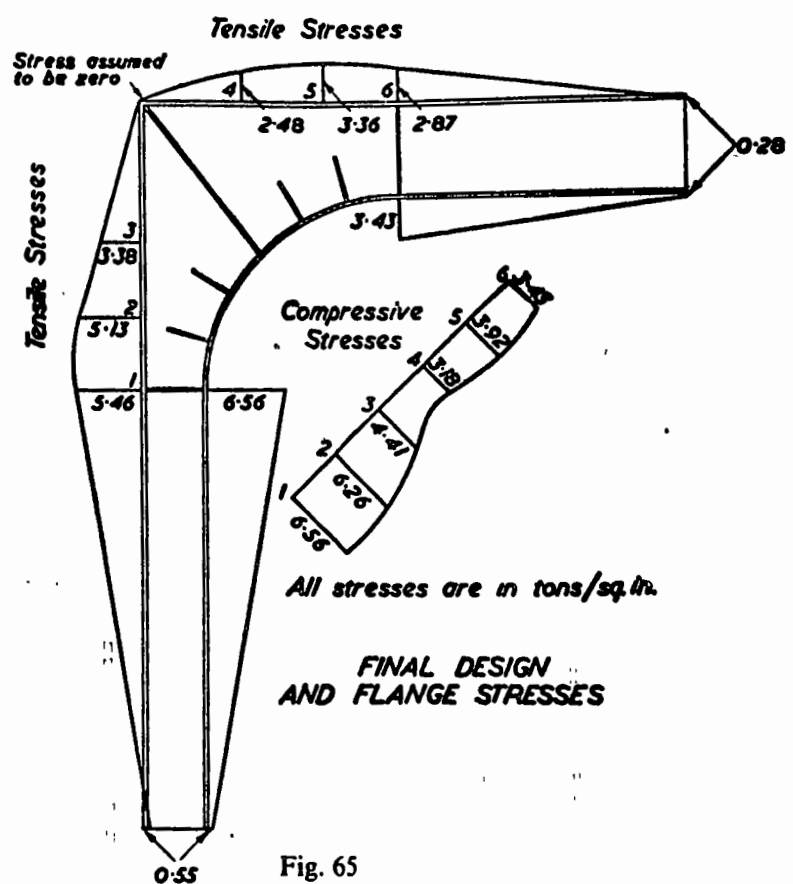


Fig. 65

The stresses given for the inner flange presuppose that it is properly stiffened to resist cross-bending. If unstiffened, the stresses can be calculated by Dr. H. H. Bleich's formulæ. (See p. 783.)

Considering section 2-2,

$$\text{max. } f = \frac{f}{v}$$

The value of  $b^2/Rt$  for the flange

$$= \left( \frac{12 - 0.625}{2} \right)^2 \times \frac{1}{48 \times 1} = 0.67.$$

Hence  $v = 0.81$  (by interpolation) and  $\text{max. } f = \frac{f}{v}$

$$= \frac{6.26}{0.81} = 7.73 \text{ tons per sq. in.}$$

Now the cross-bending stress

$$f' = \mu \text{ max. } f$$

$$\mu = 1.46 \text{ (by interpolation).}$$

Therefore  $f' = 1.46 \times 7.73$   
 $= 11.29 \text{ tons per sq. in.}$

As this exceeds the permissible stress, stiffeners are required.

The radial stress in the web is greatest where the flange stress is greatest, i.e. somewhere between sections 1-1 and 2-2. Taking the flange stress at 1-1 and using Campus's formula,

$$s = \frac{C}{Rt} = \frac{\text{Area} \times \text{unit stress}}{Rt}$$

$$= \frac{12 \times 1 \times 6.56}{48 \times 0.625}$$

$$= 2.62 \text{ tons per sq. in.}$$

Even if the shear stress at 1-1, 1.19 tons per sq. in., is added to this stress, the total stress is still low.

Olander's method of obtaining the shear stresses in the knee can be demonstrated for section 2-2.

Now 
$$v = \frac{M_o Q}{rIt}$$

Considering Fig. 64, the value of the distance  $b$  for section 2-2 = 41.65 in. and the vertical distance from  $O$  to the bottom of the column is 66.35 in.

$$M_o = (10.5 \times 66.35) + (18 \times 8)$$

$$= 841 \text{ tons in.}$$

Considering Fig. 66,

$$Q = (0.625 \times 8.33) \frac{8.33}{2} + (1 \times 12 \times 8.83)$$

$$= 127.7 \text{ in.}^3$$

Hence, 
$$v = \frac{841 \times 127.7}{59.39 \times 2112 \times 0.625}$$

$$= 1.37 \text{ tons per sq. in.}$$

This stress is low and those at other sections

are lower still.

It follows that the stiffeners required to prevent cross-bending of the flange can be nominal in size. A thickness of  $\frac{3}{8}$  in. would be suitable.

### Valley Joints

The principles involved in the design of the Y-shaped valley joints in multi-bay construction are the same as those for knee joints. It is quite reasonable to design the inner flange of each rafter section as though the joint were a knee, the other rafter being ignored, as shown in Fig. 67. The detailing should be as simple as possible. Two examples showing haunched and curved inner flanges are shown in Figs. 68 and 69 (references 11 and 12 respectively).

To reduce the cost of heating the building or for æsthetic reasons roofs are often

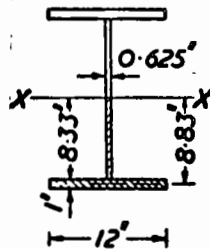


Fig. 66

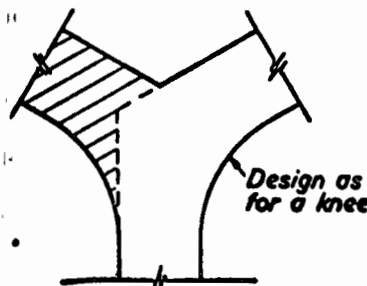
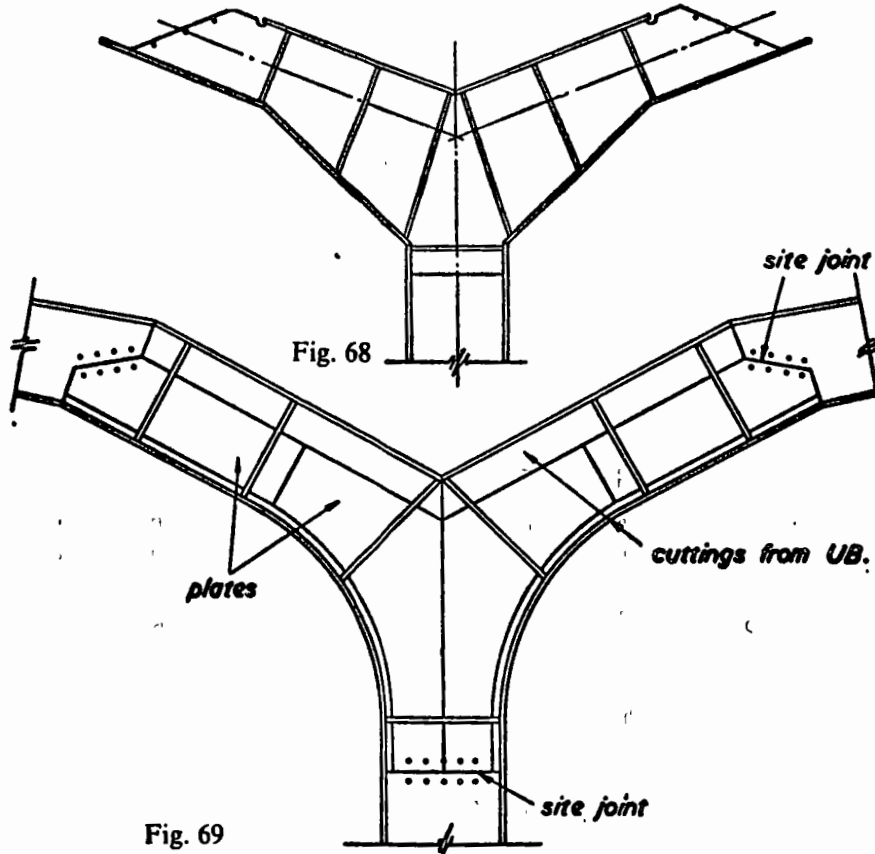


Fig. 67

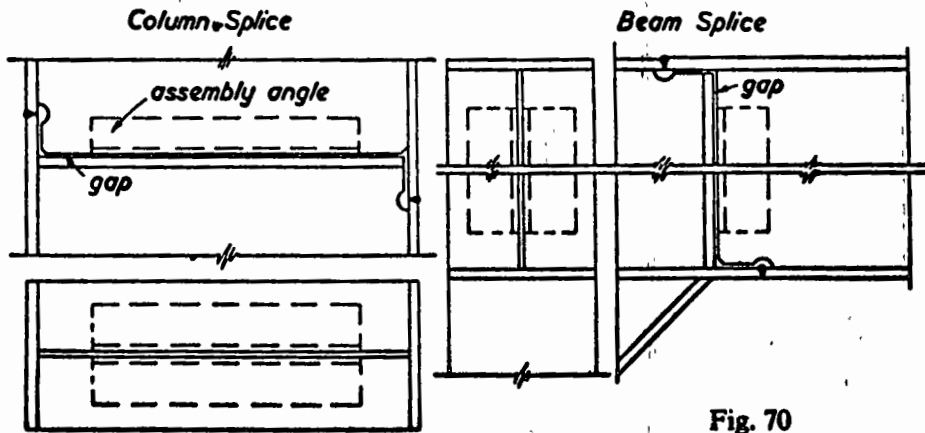
made of low pitch, and practical difficulties can arise in accommodating the valley gutter. The problem may be overcome as shown in Fig. 69, the



pitch being fairly steep at the feet of the rafters and low over the central portion of the roof.

**Splice Connections**

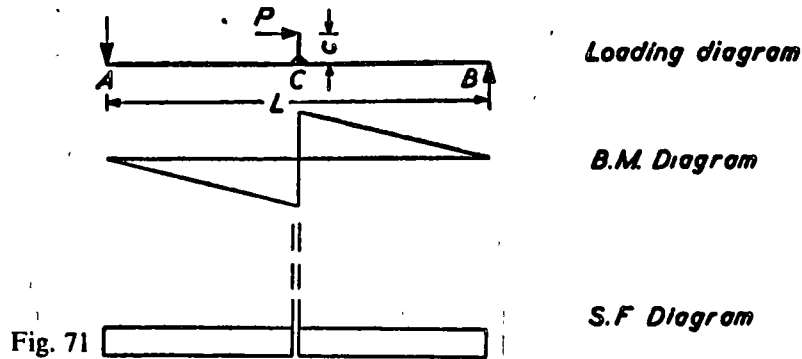
Splice connections in rigid frames should be arranged at or near the



dead-load points of contraflexure. Figs. 49, 57, 68 and 69 all incorporate splice connections. Some typical joints for a column and cross-beam are shown in Fig. 70, but it should be noted that it is not essential that joints should occur at stiffeners. Further details may be found in reference 13.

### Rigid Joints in Multi-storey Buildings

Although some of the calculations involved in designing a joint in a multi-storey building may be lengthy, the underlying principles on which the work is based are comparatively elementary.

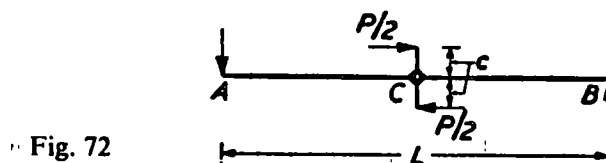


Consider the simply supported prismatic beam shown in Fig. 71. When the load  $P$  is applied, a moment  $M$  is induced at  $C$  which is distributed between the portions of the beam  $AC$  and  $CB$  as shown. The reactions at the supports are:

$$R_A = -\frac{M}{L} \quad \text{and} \quad R_B = +\frac{M}{L}.$$

Now the shear in  $AC$  and  $CB$  may be obtained by calculating the slope of the B.M. diagram. Hence, the value of the shear  $= M/L$ , which is, of course, equal to the reactions at the supports  $A$  or  $B$ .

At  $C$  the moment diagram undergoes an abrupt vertical change of moment, equal to  $M$ , and the corresponding shear is infinite. This results from the assumption that the moment is applied at a point, as drawn in Fig. 71.



Much the same state would result if the moment  $M$  were applied through a couple as shown in Fig. 72.

If, however, the moment  $M$  were to be applied through a couple formed by two equal and opposite forces of value  $P$  as shown in Fig. 73, then the change of moment at  $C$  would be less abrupt and the S.F. between the two loads could be calculated.



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Thus,

$$V = \frac{M_{CA} + M_{CB}}{c} = \frac{P(L-c)}{L}$$

As before, the shear on either side of the applied moment would be of constant value, being equal to  $cP/L$ .

Now it is proposed to give a practical example to demonstrate how such joints can be treated.

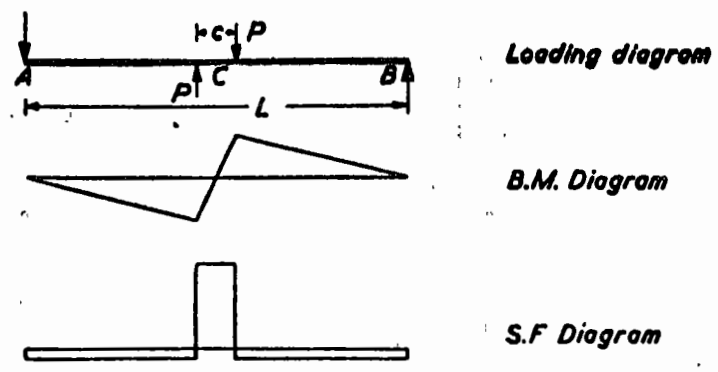


Fig. 73

*Example.* Fig. 74 shows a portion of a rigid-frame building, consisting of a section of girder *BD* and sections of two columns *AB* and *BC*; *A*, *C* and *D* being the points of contraflexure in the B.M. diagram for the worst conditions of loading on this portion of the frame.

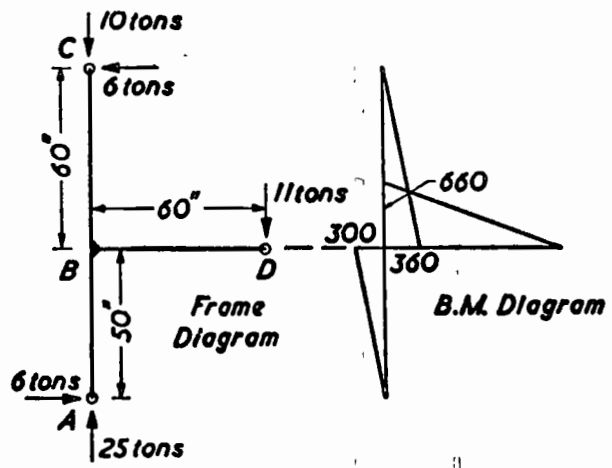


Fig. 74

The maximum B.M.s at the junction of the neutral axes of the girder and columns are as follows:

- $M_{BC} = +360$  tons in.
- $M_{BA} = -300$  tons in.
- $M_{BD} = -660$  tons in.

It is not proposed to design the columns or cross-beam. It will be assumed that the sections shown in Fig. 75 are adequate.

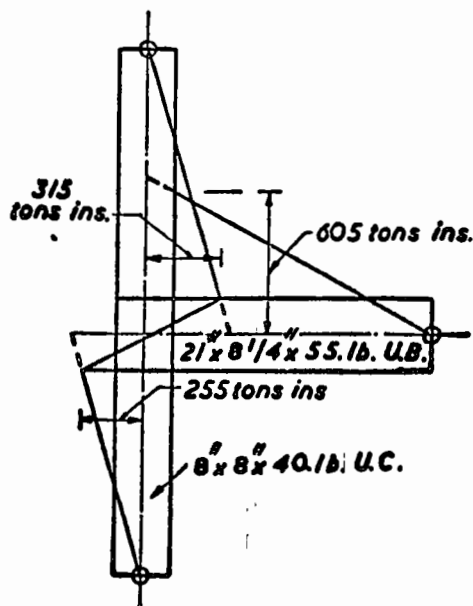


Fig. 75

Considering the joint at *B*, the shear across *B* is calculated from the slope of the B.M. diagram.

$$\text{Hence } V = \frac{315 \cdot 0 + 255 \cdot 0}{15} = 38 \text{ tons.}$$

Now the web thickness of a 8-in. x 8-in. x 40-lb. u. c. is 0.365 in. and the gross area of the web = 8-in. x 0.365-in. = 2.92 sq. in.

Therefore the shear stress in the web,

$$v = \frac{38}{2.92} = 13.01 \text{ tons per sq. in.}$$

This stress exceeds the maximum permissible shear stress of 7.0 tons per sq. in. A number of expedients may be adopted to increase the resistance to shear.

One of the most common methods is to weld two rectangular plates to the web of the column. The clear web depth of a 8-in. x 8-in. x 40-lb. column is 6.33 in. If two plates, 6 1/2 in. wide and 1/4 in. thick, were welded, one each side of the web, as shown in Fig. 76, then the shear stress would be,

$$v = \frac{38}{8 \times 0.865} = 5.49 \text{ tons per sq. in.}$$

which would be acceptable.

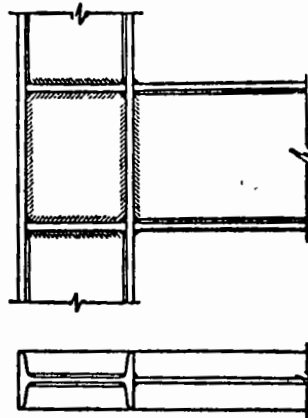


Fig. 76

It will be noticed that two sets of stiffeners have been introduced at the level of the flanges of the girder. These should be of approximately the same section as the girder flanges and, in this case, are made  $\frac{1}{2}$  in. thick.

Instead of thickening up the web some kind of triangulated system of stiffeners may be used, such as those shown in Fig. 77. In such cases the diagonal or oblique stiffeners may supplement the strength of the web, or the web may be entirely ignored, when the stiffeners form a truss framework and are designed accordingly.

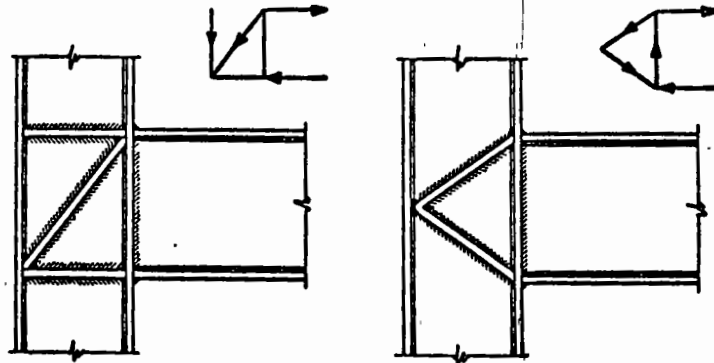


Fig. 77

Another method to avoid overstressing the web of the column is to deepen the end of the girder or to provide brackets at the junction of the girder and column, the object being to reduce the steep slope of the B.M. diagram, i.e. to reduce the shear force in the joint. Various expedients are shown in Fig. 78.

#### Open-frame or Vierendeel Girders

The most obvious characteristic of open-frame girders, some examples of which are shown in Fig. 79, is the complete absence of diagonal members in

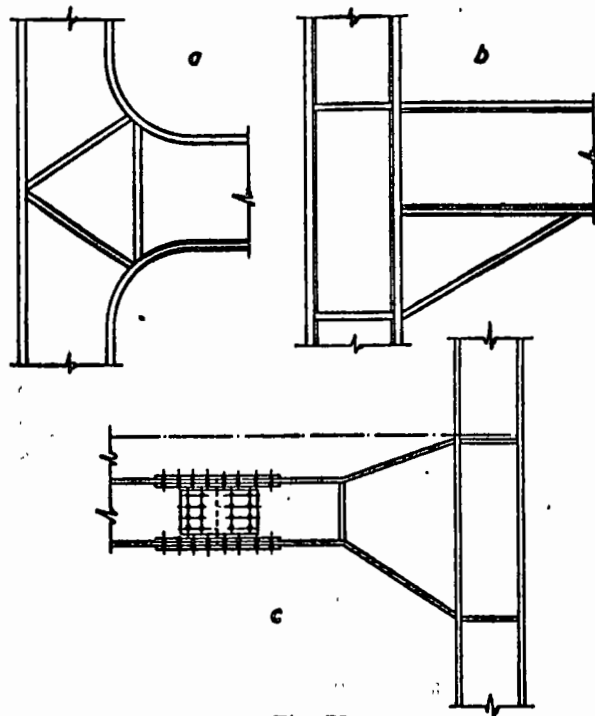


Fig. 78

the panels, the girders depending on the rigidity of the joints for their stability.

The open-frame girder is not commonly used in Great Britain, but many examples exist in Europe, particularly in Belgium, where they are associated with the pioneer work and development of the late Professor *Vierendeel* after whom they are usually named.

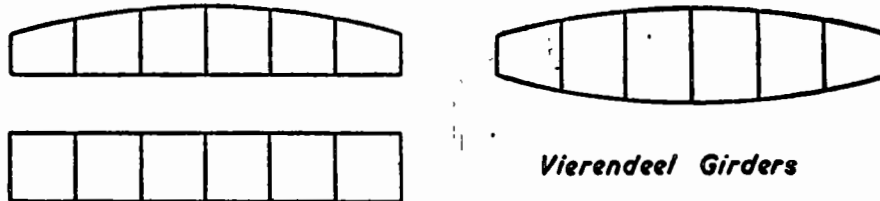


Fig. 79

Although there are many *Vierendeel* bridges in Belgium, only one such bridge, for pedestrians, has been built in Great Britain. Examples in structures taking static loads have usually resulted from a demand for a free unobstructed space where the use of diagonals has been precluded. Consequently, *Vierendeel* girders can be used for clerestory lighting in churches and other structures or for spanning any gap where a plate girder or truss would be used but for the fact that the web would provide an obstruction.

Many foreign authors have devoted whole books to the analysis of Vierendeel girders, but, unfortunately, the literature relating to the design of joints is comparatively scanty. Here, it is proposed to deal with girders with parallel top and bottom booms. Provided that the booms are parallel, analysis by Slope Deflection is possible but lengthy, but if the top and bottom booms are of identical section in each panel the girders may be analysed quickly and accurately by Naylor's application of Moment Distribution, as demonstrated in Example 15 in the section on that method of analysis. See page 273

The analysis and design of a joint for a multi-storey building frame has just been given. Now a single-bay, multi-storey building of the type described in Examples 9 and 11 in the section on Moment Distribution is really a Vierendeel girder erected vertically, so that the principles underlying the design have already been described. However, it is usual for the axial forces in the members of Vierendeel girders to be very great compared with those in the analogous members in building frames.

Most of the loads applied to Vierendeel girders are applied at the panel points but sometimes the booms take comparatively light loads, usually uniformly distributed, when a design by Moment Distribution will incorporate the devices associated with inter-panel loading (c.f. Example 16 in Moment Distribution). Inter-panel loading can also be treated by Slope Deflection.

#### *Formulae for Joints*

Vierendeel evolved a number of formulae for different types of joint. For the 'T'-joint shown in Fig. 80, the formulae resemble the Tapered Beam formulae given earlier in this section, viz.:

$$f_1 = \frac{P}{A} - \frac{Ma_1}{I}$$

$$f_2 = \frac{P}{A} + \frac{Ma_2}{I}$$

$$v = \frac{1}{bd} [V - f_1 F_1 \sin \phi_1 + f_2 F_2 \sin \phi_2] \text{ (using appropriate signs for } f_1 \text{ and } f_2)$$

$$A = bd + F_1 \cos \phi_1 + F_2 \cos \phi_2$$

$$I = \frac{bd^3}{12} + F_1 \cos \phi_1 a_1^2 + F_2 \cos \phi_2 a_2^2$$

These formulae may be used for a joint in a multi-storey building where the radius of the top flange is small so that the flange does not project above floor level. Normally in Vierendeel girders  $\phi_1 = \phi_2$ .

This type of joint is not used in Belgium for bridges or other structures taking dynamic loading. It is the invariable practice to employ the type shown in Fig. 81, where the posts are planted on the booms and the radii of curvature may be as much as one-third of the panel length.

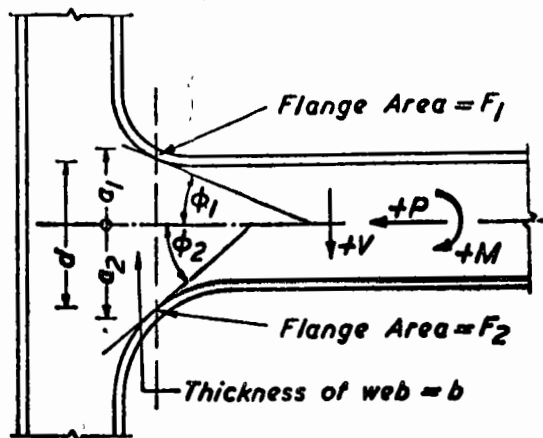


Fig. 80

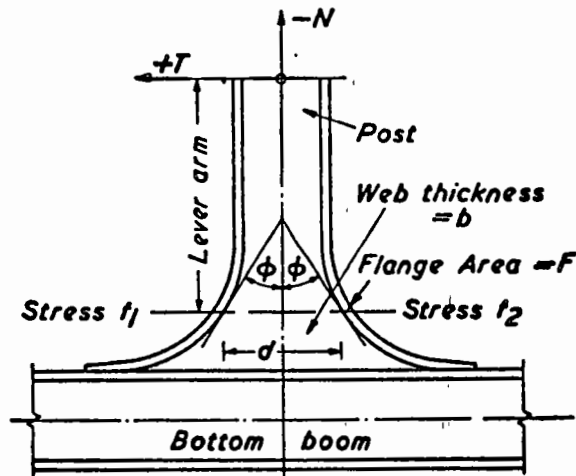


Fig. 81

In this case the post is symmetrical about the vertical axis, and the flange stresses due to the moment only are:

$$f_1 = -f_2 = \frac{M}{d(F \cos \phi + bd/6)}$$

$M$  equals  $T$  times the lever arm about the section considered.

At the junction of post and boom, as  $\cos \phi = 0$ , the moment is taken by the web only.

The stresses due to  $-N = -N/A$ . The values of  $v$  and  $A$  are found as before.

The B.M. diagram for a boom is drawn as shown in Fig. 82. The lines between the tangent points of the post are parabolas.

Professor Magnel of Ghent (reference 6) quoted the following formulæ for the section shown in Fig. 83:

$$\begin{aligned} T &= P_3 + P_1 \sin \phi - P_2 \sin \theta \\ T \cdot OX - M &= P_3 \cdot OX \\ N &= P_1 \cos \phi + P_2 \cos \theta, \end{aligned}$$

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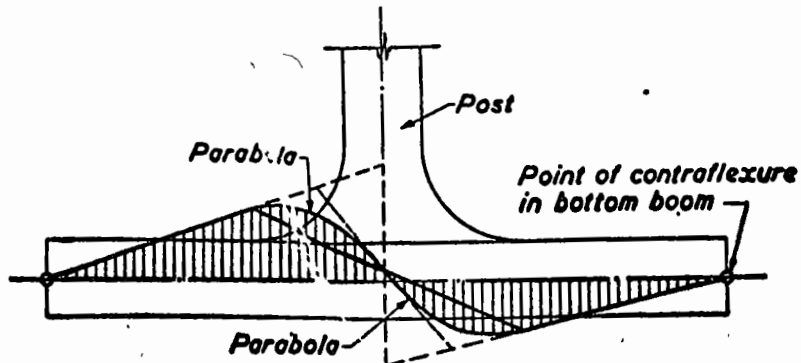


Fig. 82

from which

$$P_1 = \frac{N \cdot \sin \theta + \frac{M}{OX} \cos \theta}{\sin(\phi + \theta)}$$

$$P_2 = \frac{N \cdot \sin \phi - \frac{M}{OX} \cos \phi}{\sin(\phi + \theta)}$$

$$P_3 = T - \frac{M}{OX}$$

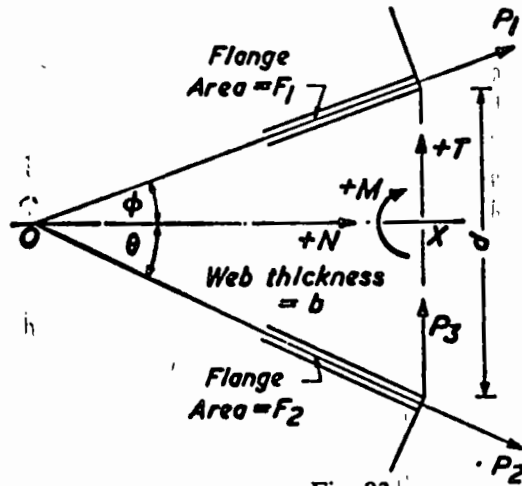


Fig. 83

The member shown has straight sloping sides, but the same formulæ could be used for curved flanges although Magnel stated that the formulæ should not be used when  $(\phi + \theta)$  exceeds  $90^\circ$ . However, he quoted experiments carried out by Professor Campus in which he proved that in joints such as that shown in Fig. 81 the critical sections were those in which  $(\phi + \theta)$  varied from  $0$  to  $90^\circ$ .

The stresses derived from Magnel's formulæ are as follows:

$$f_1 = \frac{P_1}{F_1}; \quad f_2 = \frac{P_2}{F_2}; \quad \nu = \frac{P_3}{bd}$$

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"TEORIA DEL PUNZONADO Y DEL CORTE  
CON CIZALLA"

¿Quién decide en su fábrica si se debe punzonar, taladrar, cortar con cizalla o con sierra? El punzonado y el corte con cizalla son en grado considerable los procesos menos costosos, sin embargo, en algunas fábricas se necesita algo muy cercano a un decreto presidencial para aceptar estos métodos. Los cursos de manejo de máquinas que se imparte en escuelas secundarias, escuelas técnicas, colegios y universidades, dedican muchas horas a enseñar a nuestros futuros ingenieros, superintendentes y ejecutivos, como taladrar, fresar y cortar con sierra. Tales entrenamientos los hace pensar siempre en como maquinar el metal. Pero si queremos obtener utilidades, necesitamos empezar a pensar en punzonar y cortar el metal.

Un fabricante de puertas para cajas de seguridad, por verdadero accidente probó el sistema de punzonado para hacer las perforaciones en las cerraduras (perforaciones con diámetro de 11/16" a 1") a través de acero inoxidable de 1/2" de espesor y en acero rolado en frío C-1018 y ahorró miles de pesos al año, en comparación con lo que estaba gastando con los métodos de taladrado que había usado hasta entonces. Su empresa había estado taladrando estas partes por años y se había mostrado complacida con el sistema, hasta que se recibió un tipo equivocado de acero inoxidable que se endurecía al ser taladrado. Era casi imposible taladrarlo con rapidez suficiente como para hacer costeable el proceso, además de que los taladros se quemaban continuamente. En medio del problema

apareció en escena un técnico con una prensa hidráulica que quería demostrar. Probó como se realizaba el trabajo de punzonado a través de aquel duro material. Los resultados fueron excelentes y la prensa se compró.

Para la mayoría de nosotros es fácil decir "con una mirada que el proceso usar, si el éxito de punzonar o cortar con cizalla es marginal merece experimentación. Este informe tiene la intención de familiarizar al escéptico con los procesos de punzonado y de corte con cizalla y ayudar a todos a comprender estos procesos y sus limitaciones.

Hay muchas operaciones - en las que se usan prensas - que se realizan aplicando los mismos principios del corte, por lo tanto, el comprender la teoría de este tipo de corte le permite a uno comprender las siguientes operaciones:

A. Corte con cizalla.

- a. Acción de cortar con dados o cuchillas.
- b. Inclinación entre dos filos cortantes.
- c. Corte logrado por medio de una herramienta para cortar metal y otros materiales, debido a un movimiento de acercamiento de dos orillas afiladas, que llegan a juntarse.

B. Punzonar

- a. Un término general que describe el proceso de cortar con troquel o dado, una perforación en una lámina o placa de alguna pieza.

Perforado es un término que se usa también para describir el proceso.

- b. Punzón es la parte del troquel que podría considerarse el "Macho" y generalmente es el miembro superior.
- C. Estampar: El proceso de cortar o sacar una pieza de un material con un contorno predeterminado.
- D. Recortar: Acción que consiste en realizar cortes de varias formas en la orilla de una tira o de una pieza.
- E. Niblar: Acción de realizar un corte progresivo a gran velocidad, ya sea para lograr una orilla de acabado recto o una orilla ondulada.
- F. Semi-Corte: Acción de cortar a lo largo de una línea en la pieza de trabajo, sin producir desperdicio o pedazo de material que se separe de la pieza de trabajo.

#### La acción del corte cuando se trabaja en metal

El material se corta como resultado de dos filos agudos con acción de cerrar, forzando al cortante hasta su punto de fractura o más allá de su resistencia a la rotura. Hay esfuerzo tanto de tensión como de compresión, la parte superior está momentáneamente bajo tensión, mientras que el fondo bajo el dado o la cuchilla está bajo compresión. Cuando se pasa del límite elástico del material, se le somete a una presión que produce la fractura y lo hace romperse completamente del metal. La Figura 1 ilustra el esfuerzo en una parte que se corta o que como sucede en este caso,

do que s  
de A

Hay tres pasos fundamentales en el corte que se muestran en la Figura 2. La presión aplicada por el punzón hace que el material se deforme hacia la apertura del dado. Cuando se excede el límite elástico del material, puede retirarse el punzón, pero el material quedará retallado. En la segunda etapa, el punzón penetra en el material y corta hasta el punto de la fractura. Cuando el punzón avanza (diagrama inferior) las fracturas se extienden rápidamente de cada orilla y la carga se reduce hasta la cantidad que se necesita para desprender el material del dado.

La acción de cortar con cizalla produce cuatro características inherentes que se encuentran tanto en el material paterno como en la pieza cortada (o punzonada). (Figura 3). Estas son:

- 1.- Deformación plástica.
- 2.- Banda vertical de corte bruñido.
- 3.- Fractura angular.
- 4.- Rebaba causada por la fractura que se inicia encima de la orilla del corte.

El grado de cada una de las características arriba mencionadas, depende de lo siguiente:

- 1.- Espesor del material.
- 2.- Tipo y dureza del material.
- 3.- Espacio libre entre las cuchillas o punzón y dado.
- 4.- Condición de las cuchillas o punzones y dados.
- 5.- Apoyo o firmeza del material a ambos lados del corte.

Si todas las condiciones dadas arriba se satisfacen, y la condición de la orilla de corte no es aceptable, hay otros métodos de troquelado que pueden emplear. Uno de los métodos de acabado más comunes

es usar una rima que quita una pequeña cantidad de material, para -  
eliminar el ángulo de fractura.

### DEFORMACION

Tanto el tipo como la dureza y el espesor del material tiene efecto en la cantidad de deformación. Entre más suave y grueso es el material, más se deforma. El espacio libre entre las cuchillas y el apoyo del material, en ambos lados del corte también tiene un gran efecto. La Figura 3 muestra una perforación que se punzona con exceso de espacio libre y puede notarse la deformación que aparece alrededor de la superficie superior. Sostener el material casi nunca es problema cuando se trata de hacer una perforación, ya que el punzón sostiene firmemente el material contra el dado y la fuerza cortante sostiene el desperdicio o la pieza con firmeza contra la cara del punzón. Sin embargo, al hacer un corte recto en una solera o barra larga es necesario usar un sujetador para evitar que la solera se retuerza ante la cuchilla inferior (Figura 4). La combinación de los esfuerzos de tensión y compresión tal como son mostrados en la Figura 1, causan esta deformación. Un apoyo opuesto al sujetador produce los mejores resultados. Nótese que sin el apoyo, el material se dobla antes de fracturarse y queda una impresión angular de la cuchilla en el extremo de la solera.

### PENETRACION

La penetración es la suma de la distancia de la deformación y de la altura del bruñido vertical, tal como se ve en la Figura 5. Se expresa como porcentaje del espesor del material y es definida como la distancia que el punzón debe recorrer antes de que se fracture -

el metal. El porcentaje de penetración varía según el tipo y dureza del material. A medida que el material es más duro, el porcentaje de penetración disminuye.

### ESPACIO LIBRE

La fractura angular y la calidad del corte de la perforación o del estampado, depende mucho de la cantidad de espacio libre que hay entre el punzón y el dado. La Figura 3 muestra el espacio libre así como la distancia entre los dos filos de corte. Sin un espacio libre adecuado, el material no se fracturará limpiamente. La Figura 2 muestra como hay fracturas superiores e inferiores que deben encontrarse y cuando esto sucede se produce una perforación limpia, con un consumo mínimo de esfuerzo. Cuando falta espacio libre se produce un defecto que se conoce como "rompimiento secundario", según puede verse en la Figura 6. Cuando no se logró que se encontraran las dos líneas de fractura, eso deja un anillo de material que debe ser forzado hasta lograr romperlo, con un gasto adicional de energía. El grado de rompimiento secundario disminuye a medida que el espacio libre aumenta hasta que se llega al espacio adecuado. Las herramientas desafiladas crean el efecto de un espacio libre demasiado pequeño, además de producir rebaba. Un exceso de espacio libre entre los filos de corte produce un exceso de deformación plástica, una rebaba grande, y un ángulo muy alto de fractura como se muestra en la Figura 6. Por lo tanto el espacio libre adecuado puede definirse como aquel que no causa rompimiento secundario y que produce un mínimo de deformación plástica y de rebaba.

El espacio libre correcto varía de acuerdo con el espesor y el tipo de material. Como se vió antes en la Figura 5, el grado de penetración o fractura depende del tipo y dureza del material. El espacio libre es expresado como porcentaje de espesor del material y debe ser calificado para indicar si significa espacio libre por lado, o espacio libre de diámetro total. Hay muchas opiniones sobre como representar el mejor porcentaje de espacio libre y la Figura 7 es una tabla que ha demostrado funcionar bastante bien. El espacio libre adecuado se encuentra mejor por medio de pruebas prácticas. No hay fórmulas ni tablas que nos den los espacios libres exactos.

#### DONDE APLICAR ESPACIOS LIBRES

Como es obvio cuando se estampa o se punzona exactamente a la medida, debe tomarse en cuenta el espacio libre. La Figura 3 es un buen ejemplo de esto. El dado es más grande que el punzón en la extensión de espacio libre que se necesita para producir una fractura limpia. Si la perforación se va a hacer exactamente de una medida, el punzón debe ser de esa medida y el dado tiene que ser de un tamaño mayor, con una diferencia que permita un espacio libre adecuado. A la inversa, si la operación consiste en estampar un disco que tiene que ser de tamaño exacto, el dado debe ser de ese tamaño y el punzón debe ser menor para permitir el espacio libre adecuado.

Una perforación punzonada se encoge muy ligeramente y debe tomarse en cuenta esto si se requiere alta precisión. La cantidad - - varía de acuerdo con el tipo y dureza del material, pero normalmente no excede de .002". El punzonado o estampado que se hace - de modo consistente produce piezas muy exactas.

#### LA VIDA DE LA HERRAMIENTA

El espacio libre tiene un efecto considerable sobre la vida de la herramienta. Se logra un máximo de vida para ella cuando el espacio libre es correcto. El espacio libre insuficiente reduce considerablemente la vida de la herramienta, debido al resorteo y a la fusión en frío del material ante el punzón y el dado. Cuando se punzonan materiales más duros, un espacio libre insuficiente - crea un esfuerzo mucho mayor en el filo y hace que el material se despostille y rompa. El señor L. R. Allingham en su Informe Técnico ASTME # 622, citó el caso de un troquel de estampado con - - 2-1/2% de espacio libre por lado que se rompió después de hacer solo 1,000 piezas. Una vez rectificado el troquel, el espacio libre se aumentó al 13% y se produjeron 240,000 piezas sin desgaste o deformación evidente en el troquel.

Un espacio libre excesivo tiene también un efecto adverso en la vida de la herramienta, debido a que el material se extiende en las orillas de corte, y hace que los filos se rompan prematuramente.

Un lubricante para troqueles y herramientas de buena calidad alarga considerablemente la vida del punzón y el dado.



## EL SEPARADOR O BOTADOR

El material que se desprende o desperdicia es normalmente empujado a través de la abertura que hay en el dado por el punzón que se introduce ligeramente en el dado. Esto resuelve la mitad del problema, pero algunas veces la parte más difícil es separar o botar el punzón de la pieza que se perforó.

El material se adhiere al punzón debido a las tendencias de contracción y de fusión del material y varía en magnitud, según el espacio libre entre el punzón y el dado, además el tipo del material. Los materiales suaves como el aluminio y el cobre son generalmente los más difíciles de separar.

La fuerza para botar o separar puede ser tan elevada como el 10% de la fuerza que se necesita para punzonar o estampar, lo cual puede ser el caso cuando hay insuficiente espacio libre entre el punzón y el dado. Con espacio libre excesivo, la fuerza para botar o separar se reduce hasta un 2% de la fuerza para punzonar y con espacio libre adecuado normalmente requiere de un 3 a un 4% de la fuerza de punzonado, siempre y cuando las orillas del punzón y dado estén bien afiladas. Si la acción de botar tiene lugar cerca de la orilla de la pieza a punzonar, la fuerza necesaria será menor, porque el metal puede ceder o doblarse ligeramente al liberarse del punzón. Los botadores deben ser diseñados para ejercer de un 5 a un 6% de la fuerza que se necesita para punzonar o estampar en trabajo normal.

La separación del material (Figura 8) puede realizarse con una placa de uretano, que sostiene el material firmemente contra el dado o una placa botadora positiva, que es colocada a corta distancia por encima del material. Con un botador positivo el material es levantado por el punzón al subir éste, hasta que el material toca el botador y se desprende del punzón.

Un lubricante de buena calidad para troqueles, reducirá considerablemente la fuerza para botar o separar especialmente cuando se trata de material suave, como el cobre y el aluminio.

#### EFECTO DEL ESPACIO LIBRE SOBRE LAS NECESIDADES DE FUERZA Y ENERGIA

El espacio libre tiene efecto, o ninguno sobre la cantidad de fuerza que se necesita para cortar o punzonar. Se han realizado muchos estudios que apoyan esta teoría y el resultado obtenido en uno de ellos se ve en la Figura 9. Las curvas de la izquierda y del centro representan acero suave de igual espesor cortado uno con espacio libre insuficiente y el otro con espacio libre adecuado.

La altura de estas dos curvas es aproximadamente la misma y la curva con espacio libre insuficiente es la más elevada.

Nótese el área que hay bajo las curvas de la izquierda y del centro que representa las exigencias de energía. El espacio libre insuficiente aumenta considerablemente las exigencias de trabajo (7500 pulgadas-libras comparadas con 4300 pulgadas-libras) como resultado del rompimiento secundario. La fuerza del corte está-

presente a través de todo el espesor del material comparada con la curva del centro, donde la fuerza desciende hasta aproximadamente la mitad del material. Es perfectamente obvio en este punto, ver porque la vida de las herramientas se reduce, la herramienta realiza un 75% más de trabajo.

La curva de la derecha en la Figura 9, muestra un corte en acero duro (0.90 de carbón). La fuerza que se necesita para lograr la fractura es mayor para este tipo de acero que para el acero suave, pero la fractura empieza más pronto y una vez que se inicia, termina más rápidamente. Se trabaja menos (ver el área que está abajo de las curvas) en el material duro que en el material suave.

#### FUERZA DE CORTE

La fórmula para obtener la fuerza "F" que se necesita para cortar, estampar o punzonar un material dado, suponiendo que no haya viaje en el punzón o en el troquel es:

Fórmula básica de corte: Fuerza = Area x Esfuerzo cortante

$F = LTS$  --- Para cualquier forma de corte

$F = DTS$  --- Para perforaciones redondas

O sea  $L$  = Largo de corte, en pulgadas

$T$  = Espesor del material en pulgadas

$S$  = Resistencia al corte del material, en libras por pulgada cuadrada

$D$  = Diámetro en pulgadas

La resistencia al corte del material, es calculada en libras por pulgada cuadrada y para algunos materiales puede encontrarse en la Figura 10. Aquí se representa la fuerza que se necesita para cortar

en dos una barra cuadrada de 1" (Figura 11). La misma regla se aplica a barras de diversas formas en operaciones de punzonado y estampado, tal como se muestra. Para punzonar y estampar, el perímetro de corte se multiplica por el espesor del material, para encontrar el área. Nótese en la Figura 10 que el esfuerzo cortante y de resistencia a la tensión no son la misma. Además la resistencia a punto cedente no puede usarse como el esfuerzo cortante del mismo, puesto que no son iguales. Si el esfuerzo cortante de un material no se dá ni se conoce, puede calcularse una simple prueba en una prensa hidráulica haciendo una perforación.

Se calculó el tonelaje que se requiere para punzonar o estampar perforaciones redondas para varios espesores y para comodidad de nuestros lectores, se muestra en la Figura 12. Por ejemplo, una perforación de 1/2" de diámetro punzonado a través de acero suave de 1/4" de espesor, requiere 9.8 toneladas. La gráfica puede usarse también para otros materiales multiplicando el resultado dado en ella, por el factor que se ofrece en la Figura 10, para cada material. Por ejemplo, el factor para aluminio 6061-T6 es .58 lo que significa que su esfuerzo cortante es equivalente al 58% de la que se necesita para el acero suave. Una perforación de 1/2" de diámetro hecho en aluminio 6061-T6 de 1/4" de espesor requiere  $9.8 \times .58$ , o sea 5.68 toneladas. Las exigencias de tonelaje para perforaciones de más de 1" de diámetro pueden obtenerse sumando los tonelajes para dos o más diámetros, que dan como total el diámetro deseado. Por ejemplo, una perforación de 1-1/2"

de diámetro a través de acero suave de 1/4" de espesor, sería el de 1/2" a través de 1/4", más 1" a través de 1/4", o sea - - - - 9.8 + 19.7 = 29.5 toneladas.

#### FUERZA DE CORTE CON VIAJE

Hay muchos casos en que se realiza el corte en forma progresiva, lo cual reduce la fuerza que se necesita. Esto se logra escalonando los punzones cuando se usa más de uno, de tal manera que los dos no corten al mismo tiempo, o afilando un ángulo al punzón y dado. En ambos métodos el trabajo se realiza a través de una mayor distancia, usando menor fuerza, pero el trabajo total realizado es el mismo que si no se hubiera escalonado el punzón o si no se hubiera usado el viaje. La Figura 13 ilustra tres punzones, cada uno escalonado igual a la mitad del espesor del material. Con espacio libre adecuado al punzonar acero suave, la fuerza máxima requerida sería igual a la que se necesitaría para un solo punzonado. La curva de trabajo en lo alto de la figura muestra la fuerza máxima que incluye además, la necesidad de que el volante de una prensa mecánica tiene que proporcionar tres veces más energía, lo cual puede ser un problema en algunas prensas. La mayor parte de las prensas hidráulicas tiene su capacidad total a lo largo de toda la carrera del pistón con suficiente energía.

Si el espacio libre del dado para hacer las tres perforaciones fuera insuficiente, la fuerza máxima requerida sería afectada de manera considerable, como muestra la curva punteada. La necesidad de fuerza en un punzón debe disminuir antes de que el - -

siguiente se ponga en contacto con el material. La Figura 9 - ilustra el problema del espacio libre insuficiente.

Muchos cortes se realizan con una superficie plana de corte y la otra superficie de corte puesta en ángulo. Para evitar la deformación de la pieza cortada que sea (el disco o la pieza que se saca) si posteriormente la pieza se va a usar, el viaje o ángulo de corte se hace en el dado, de otra manera en el punzón. La Figura 14 muestra un punzón y un dado paralelos, con el área correspondiente que debe ser cortada, comparada con un punzón que tiene viaje igual al espesor del material y su área de corte correspondiente. El área de corte varía según la cantidad de espacio libre que haya entre el punzón y el dado y esta figura muestra la importancia de un espacio libre adecuado.

Un ejemplo de la necesidad de un espacio libre adecuado se encuentra en el caso de una prensa hidráulica de 30 toneladas que perfora 2" de diámetro a través de acero suave de 1/4" con un punzón que tiene viaje de 1/8". Sin embargo con espacio libre suficiente, el tonelaje requerido se acerca al tonelaje completo del punzón y del dado sin viaje (40 toneladas) y la prensa de 30 toneladas no podrá hacer el trabajo. En este caso debe usarse un dado 1/32" más grande que el punzón, para proporcionar el espacio libre adecuado.

Hay muchas formas de aplicar el viaje a una cuchilla o a un punzón y dado. El sistema usado comúnmente es un viaje inclinado de un lado hacia el otro, normalmente a razón de 3/8" por pie. La Figura 15 muestra varios métodos de aplicar el viaje al punzón o dado. Es importante mantener equilibrada la carga del

punzón y del dado para evitar presiones laterales sobre el punzón, que harían que el punzón se apretara al golpear o entrar - el dado. Este mismo problema también se encuentra cuando se ha ce una parte de una perforación en la orilla de una lámina.

Un punzón o dado con viaje es una forma económica de extender - la capacidad de la prensa siempre y cuando ésta posea suficiente energía para aceptar el esfuerzo adicional. Muchas prensas me cánicas (con volante) no tienen suficiente energía para aprovechar el viaje del punzón y dado. De hecho el viaje puede reducir su capacidad, si no es adecuadamente aplicado.

#### RELACION DE DIAMETRO A ESPESOR

Toda cosa buena tiene sus limitaciones. El punzonado no es dife rente. Una limitación es que no se puede hacer perforaciones - muy pequeñas a través de material muy grueso. Por ejemplo, una perforación de 1/4" de diámetro a través de acero suave de 1" - de espesor. En este caso la perforación tendría que ser taladra da, pero ¿cual es el punto límite?

La vieja regla que normalmente se aplica de que el diámetro del punzonado debe ser cuando menos "igual" al espesor del material - ha costado a la industria injustificadamente miles de pesos. - Un fabricante de estructuras de acero hizo que el arquitecto au mentara las especificaciones en el tamaño de unas perforaciones de 13/16" de diámetro a 15/16", para poder hacer varios miles - de perforaciones en viguetas de 1" de espesor. Consintió en pa gar la cantidad extra que significó comprar tornillos más grandes, ya que eso era mucho mejor que tener que taladrar. - - -

Desafortunadamente no sabía que con su prensa hidráulica portátil podía hacer perforaciones de 13/16" de diámetro a través de acero suave de 1" de espesor sin ningún problema.

Cuando se consideran todos los factores que intervienen en la limitación de la relación entre espesor y diámetro, es posible llegar a una nueva tabla de relaciones más realista, para aplicaciones sin choque.

El diámetro del punzón debe ser tal que la fuerza compresiva del punzón sea mayor que la fuerza requerida para perforar.

La fuerza para perforar puede encontrarse multiplicando el espesor del material por su esfuerzo cortante (en libras por pulgada cuadrada) y multiplicando después esto por el largo del corte.

Ahora veamos cómo puede usarse esto para determinar si un punzón resistirá al ser usado en una prensa hidráulica. Los siguientes factores deben ser considerados:

A = Area de la sección transversal del punzón, tal como es determinada por la medida y forma (Figura 16)

T = Espesor del material que es perforado

S<sub>S</sub> = esfuerzo cortante del material que es perforado

S<sub>C</sub> = Resistencia a la compresión en el punzón

L = Largo del corte

La resistencia a la compresión en el punzón puede calcularse mediante la siguiente fórmula:

$$S_c = \frac{T \times S_s \times L}{A}$$

A



La máxima resistencia a la compresión permisible ( $S_c$ ) depende desde luego del tipo de acero para herramientas del cual está hecho el punzón y su dureza. Un buen acero para herramientas endurecido al aceite y resistente al choque, resistirá una fuerza compresiva de 300,000 libras por pulgada cuadrada antes de romperse y puede usarse con margen de seguridad hasta 250,000 libras por pulgada cuadrada (PSI) con buena vida para la herramienta.

Las curvas de la Figura 17 están basadas en estos valores de fuerza de punzonado y para un esfuerzo constante conocido (del material que se está perforando), las curvas proporcionan las relaciones recomendadas de espesor a diámetro.

La curva mostrada como una línea sólida representa la resistencia a la rotura de (300,000 libras por pulgada cuadrada), la curva punteada por su parte representa el esfuerzo de trabajo que se recomienda usar (250,000 libras por pulgada cuadrada). Por ejemplo, para perforar acero suave con esfuerzo cortante de 50,000 libras por pulgada cuadrada, la relación de espesor a diámetro que se recomienda es  $1-1/4"$  a  $1"$ , y la relación máxima es  $1-1/2"$  a  $1"$ . Por lo tanto, no hay peligro en hacer una perforación de  $1"$  de diámetro a través de acero suave de  $1-1/4"$  de espesor.

Con frecuencia se usa una relación entre  $1-1/4"$  y  $1-1/2"$  a  $1"$  para acero suave, pero la vida del punzón es menor. Una empresa dedicada a estructuras, por ejemplo, hace perforaciones de

de  $13/16$ " a través de vigas de ala ancha de  $1-1/8$ " de espesor, usando una prensa hidráulica portátil de 96 toneladas. En esta relación de 1.38 a 1, el punzón hace entre 200 y 500 perforaciones antes de fallar por fatiga.

#### Tamaño mínimo de perforación

La segunda gráfica (Figura 18) muestra el diámetro mínimo de perforación que se puede hacer a través de un espesor dado de material. Se ilustran tres materiales diferentes.

Para usar la gráfica localice el espesor del material en la escala vertical y siga horizontalmente hasta el extremo inferior del área sombreada, para llegar al material que está siendo perforado. De este punto de intersección, descienda a la escala horizontal y lea directamente el diámetro mínimo de perforación que se recomienda.

La orilla superior de las áreas sombreadas representa el punto de rotura del punzón. Si se trabaja dentro de las áreas sombreadas eso significa una vida más corta para el punzón. Por ejemplo, al perforar acero suave de  $3/4$ " de espesor,  $19/32$ " es el diámetro mínimo de perforación recomendado, si se usa un punzón de  $1/2$ " de diámetro, fallará.

Un punzón falla en cualquiera de dos formas, cuando se le sobrecarga. Si su límite de elasticidad es ligeramente excedido, el punzón se expandirá al avanzar en el material. Se necesitará entonces una fuerza muy grande para botar o separar el material y esto hará que el punzón se rompa en la punta o debajo de la cabeza.

El segundo tipo de falla ocurre cuando la resistencia a la compresión es considerablemente excedida y el punzón simplemente se pandea antes de penetrar en el material.

Recuérdese que hemos estado hablando de punzones para perforación normal, tal como se usan en la mayor parte de los trabajos que se realizan en lámina, acero estructural o en placa.

Hay punzones con guías especialmente (Figura 19) para perforar material hasta de 1/2" de espesor; estos ofrecen relaciones de espesor a diámetro tan elevados, como de 2 á 1 en el acero suave.

#### Limitaciones del punzonado

El punzonado puede causar distorsión. En algunos casos no hay nada que pueda hacerse para evitarlo; pero en otros casos se pueden tomar medidas para reducirlo al mínimo.

Un problema común es la cercanía de la perforación a la orilla de una pieza. Si la perforación está demasiado cerca, hace que la orilla sufra una combadura. De preferencia debe permitirse -- dos veces el espesor del material, desde la orilla de la perforación a la orilla de la pieza.

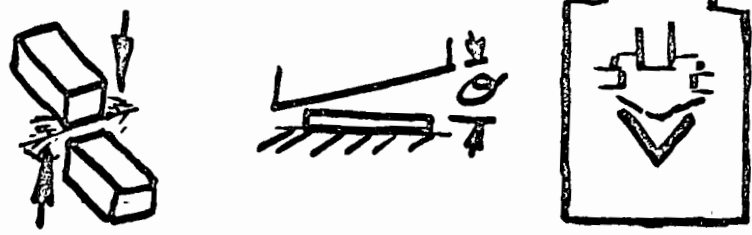
Otro frecuente problema que se presenta en el trabajo sobre solera es la curvatura producida por el punzonado fuera del centro. Por ejemplo, esto sucede cuando se hace una fila de perforaciones de 13/16" de diámetro, en centros de 3", en una solera de acero suave de 6" de ancho y de 3/4" de espesor, donde la línea central de punzonado es de 2" de cada lado. La figura 20 muestra el tipo de distorsión que se produce y las perforaciones deben estar en el centro, siempre que sea posible, y la solera no debe ser demasiado angosta. Un espacio libre insuficiente causará una mayor distorsión, debido a las fuerzas externas que se producen.

Esta breve introducción debe estimular su mente y su imaginación y hacer que empiece a "pensar en punzonar en lugar de hacer virutas".

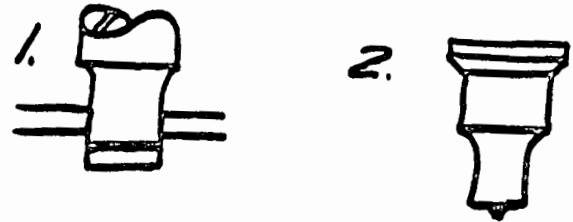
Las aplicaciones del trabajo de punzonar, estampar, recortar y -niblar si se comprenden y se usan adecuadamente, pueden aumentar considerablemente sus ganancias y mejorar su posición con respecto a sus competidores.

NOTA: AGRADECÉMOS A W. A. WHITNEY CORP. Y A METAL FABRICATING INSTITUTE INC., AMBOS DE ROCKFORD, ILLINOIS, EE. UU. - POR SU PERMISO Y COOPERACION PARA PRESENTAR ESTE ARTICULO EN ESPAÑOL.

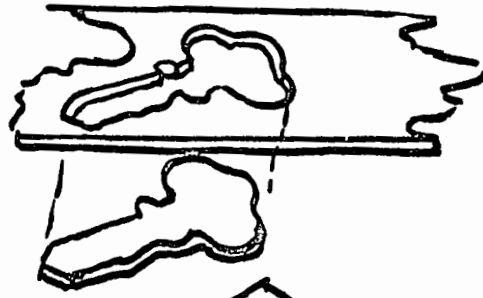
**A. CORTE CON CIZALLA**



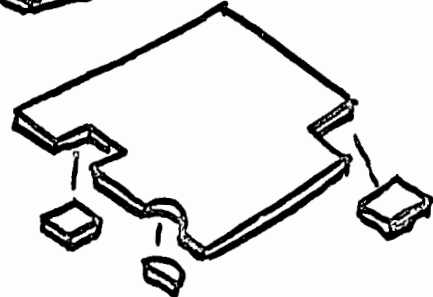
**B. PUNZONAR**



**C. ESTAMPAR**



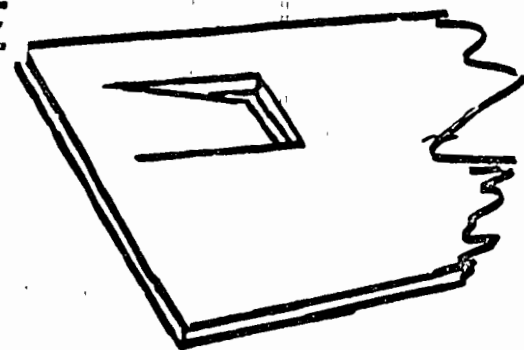
**D. RECORTAR**



**E. NIBLAR**



**F. SEMI-CORTE**



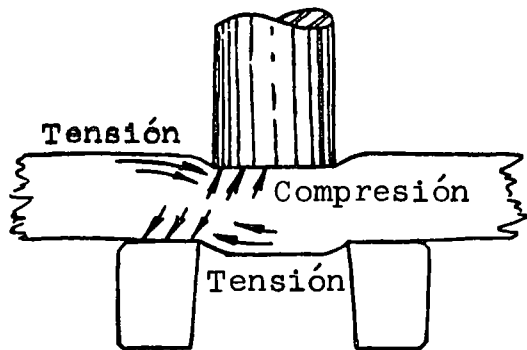


Figura 1.- Dirección de esfuerzos en el corte de metal.

- A. Deformación
- B. Bruñido Vertical
- C. Fractura
- D. Rebaba

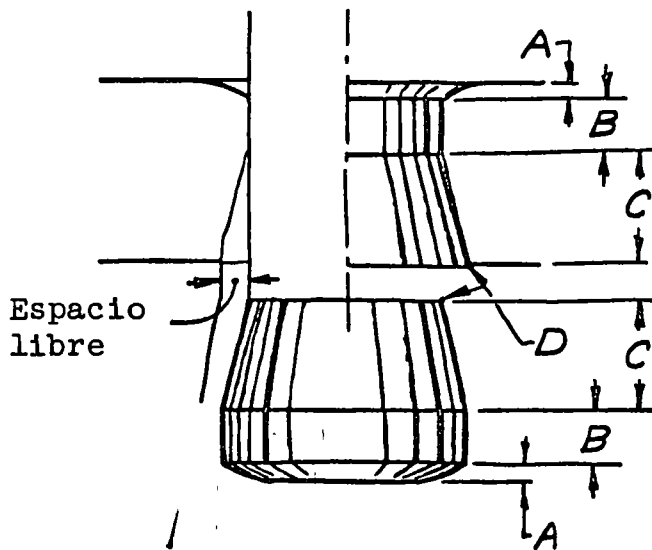


Figura 3.- Características de la orilla de corte

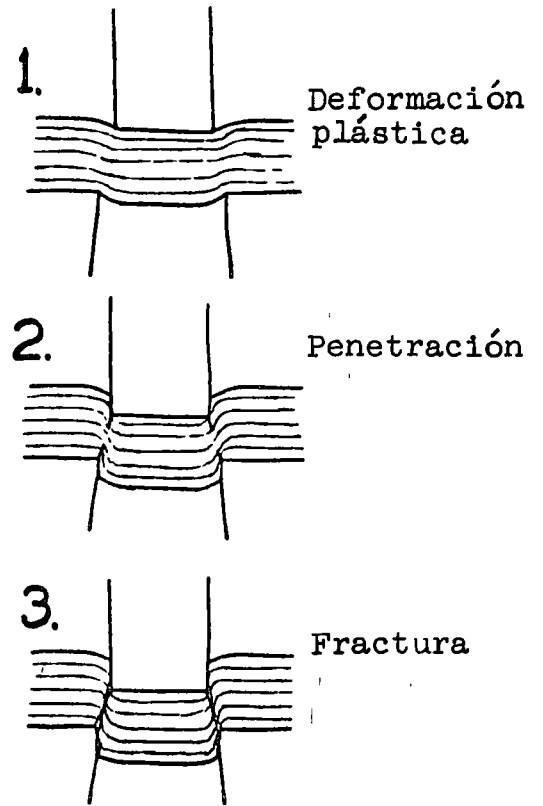


Figura 2.- Pasos en el corte del metal

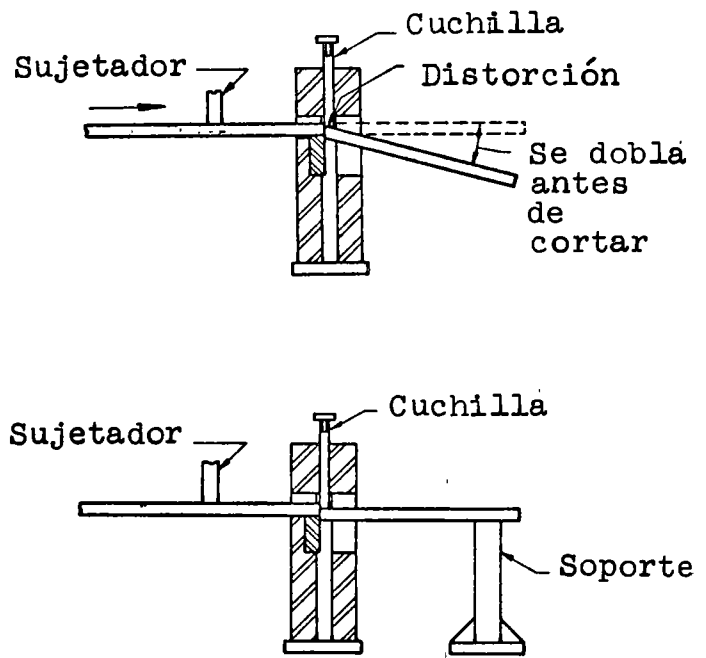
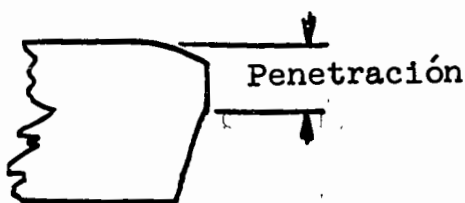
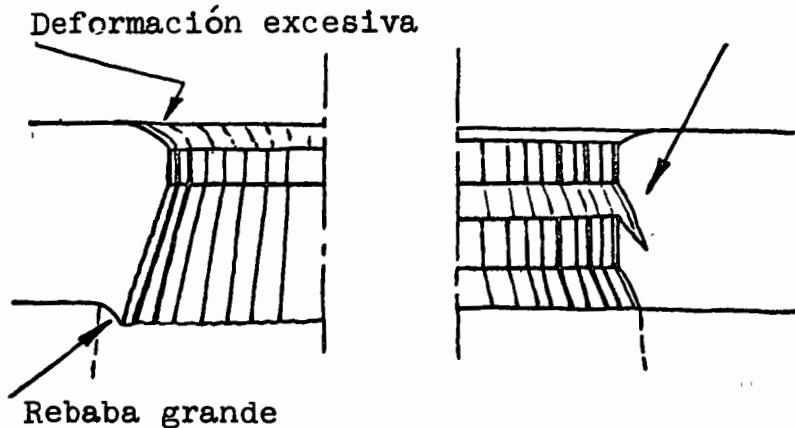


Figura 4.- Corte sencillo de una solera plana, con sujetador sin apoyo posterior y con apoyo posterior

Material	Porcentaje de Penetración "p"	% Penetración
Aluminio		60
Cobre		55
Latón		50
Bronce		25
Acero 0.10C-Recocido		50
-Rolado en frío		38
Acero 0.30C - Recocido		33
-Rolado en frío		22
Acero al Silicio		30
Niquel		55

Rompimiento Secundario



Excesivo

Espacio libre

Insuficiente

Figura 6.- Efecto del espacio libre

Figura 5.- La penetración varía, según el tipo de material

Porcentaje del espacio libre total.

Cobre	8%
Latón	8%
Acero Suave	10%
Acero 0.50C	12%
Aluminio	10%
Acero inoxidable	12%

Figura 7.- Espacio libre total, expresado como un porcentaje del espesor del material.

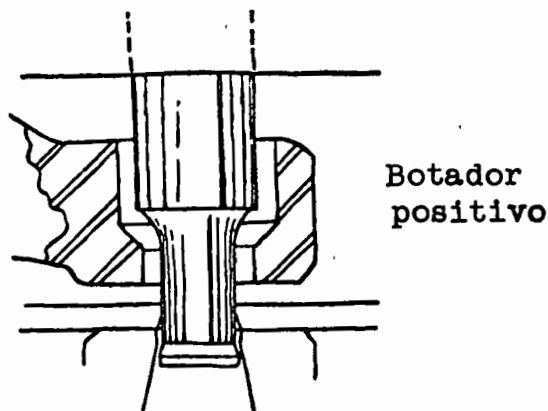
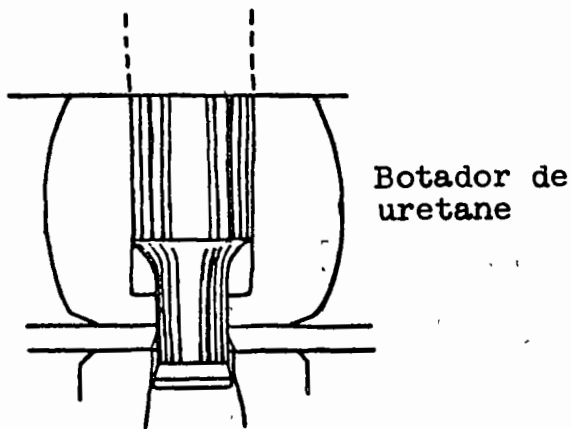
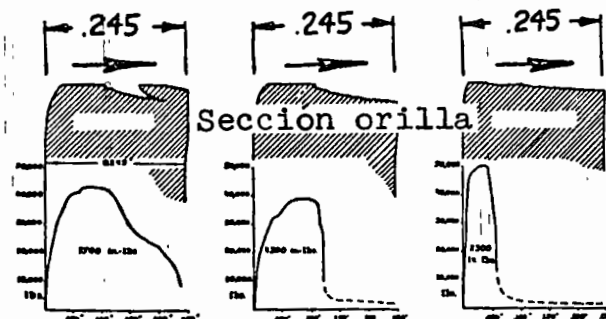


Figura 8.- El botador de resorte, uretano ó el positivo separa el material del punzón.



Acero suave Espacio libre insuficiente      Acero suave Espaciolibre adecuado      Acero duro Esp libre adecuado.

Figura 9.- Curva de esfuerzo que muestra el efecto del espacio libre y la dureza del material

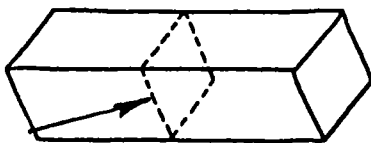
Figura 10.- Esfuerzo cortante de algunos materiales, en libras - por pulgada cuadrada  
El promedio de la resistencia a la rotura de los mate riales.

Material	Libras por pulgada cuadrada		
	Factor	Esfuerzo	Tensión
<b>Aluminio</b>			
1100-0	.19	9,500	13,000
1100-H14	.22	11,000	18,000
3003-H14	.28	14,000	22,000
2024-T4	.82	41,000	68,000
5005-H18	.32	16,000	29,000
6063-T5	.36	18,000	30,000
6061-T4	.48	24,000	35,000
6061-T6	.58	29,000	41,000
7075-T6	.98	49,000	82,000
<b>Latón, laminado hoja-suave</b>			
1/2 Duro	.88	44,000	65,000
Duro	1.20	50,000	78,000
<b>Cobre, 1/4 Duro</b>			
Duro	.70	35,000	50,000
<b>Acero, Suave A-7 Estructural</b>			
Placa para caldera	1.10	55,000	70,000
Estructural A-36	1.20	60,000	85,000
Estructural Cor-ten (ASTM A242)	1.28	64,000	90,000
Rolado en frío C-1018	1.20	60,000	85,000
Rolado en Caliente C-1050	1.40	70,000	100,000
Rolado en caliente C-1095	2.20	110,000	150,000
<b>Rolado en caliente C-1095</b>			
recocido	1.64	82,000	110,000
Inoxidable 302 recocido	1.40	70,000	90,000
<b>Inoxidable 304 Rolado en frío</b>			
	1.40	70,000	90,000
<b>Inoxidable 316 Rolado en frío</b>			
	1.40	70,000	90,000

$$F = \pi DTS = 3.14 \times 2 \times 1/4 \times 50,000 = 78,500 \text{ Libras}$$

$$\text{ó}$$

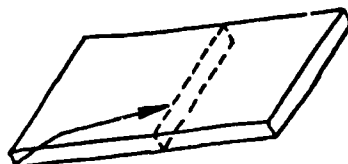
$$39.25 \text{ Toneladas}$$



$$F = LTS = 1 \times 1 \times 50,000 = 50,000 \text{ Libras}$$

$$\text{ó}$$

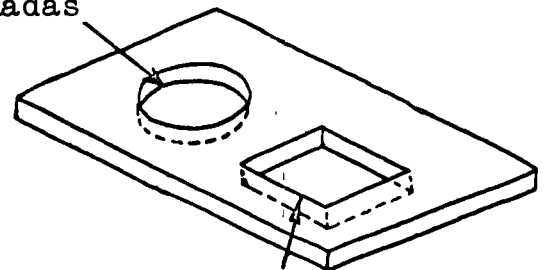
$$25 \text{ Toneladas}$$



$$F = LTS = 4 \times 1/4 \times 50,000 = 50,000 \text{ Libras}$$

$$\text{ó}$$

$$25 \text{ Toneladas}$$



$$F = 2 (A+B) TS = 2 (2+2) \times 1/4 \times 50,000 = 100,000 \text{ Libras}$$

$$\text{ó}$$

$$50 \text{ Toneladas}$$

Figura 11.- Planos de corte para varias secciones y perforaciones



# DATOS DE INGENIERIA

## PERFORACIONES SENCILLAS

Cuando se usa una prensa para hacer perforaciones sencillas se puede usar la tabla para determinar la presión (tonelaje) requerido para hacer dicha perforación. Esta tabla está basada sobre acero dulce de esfuerzo cortante de 50,000 P.S.I.

*EJEMPLO:* Para hacer una perforación de 15/16" de diámetro en acero dulce calibre 10 la presión requerida será de 10.0 toneladas.

## EFECTO DE VIAJE EN EL PUNZON

El viaje en un punzón o dado reduce la presión requerida para hacer una perforación. Consúltenos para mayor información.

Todos los punzones del estilo 28XX desde 1.453" de diámetro llevan un viaje de 1/8" como norma.

Diámetro de Perforación en Pulgadas	TONELAJE REQUERIDO (Calibre de acero con su equivalente en pulgadas)											
	20 .036	18 .048	16 .060	14 .075	12 .105	11 .120	10 .135	3/16 .187	1/4 .250	5/16 .312	3/8 .375	1/2 .500
1/8	.35	.47	.59	.74	1.0	1.2						
3/16	.53	.71	.89	1.1	1.6	1.8	2.0	2.8				
1/4	.71	.94	1.2	1.5	2.1	2.4	2.7	3.7	4.9			
5/16	.88	1.2	1.5	1.9	2.6	3.0	3.3	4.6	6.2	7.8		
3/8	1.1	1.4	1.8	2.2	3.1	3.5	4.0	5.5	7.4	9.2	11.1	
7/16	1.2	1.7	2.1	2.6	3.6	4.1	4.6	6.5	8.6	10.8	13.0	17.2
1/2	1.4	1.9	2.4	2.9	4.1	4.7	5.3	7.4	9.8	12.3	14.8	19.7
9/16	1.6	2.1	2.7	3.3	4.7	5.3	6.0	8.3	11.0	13.8	16.6	22.1
5/8	1.8	2.4	2.9	3.7	5.2	5.9	6.6	9.2	12.3	15.4	18.5	24.6
11/16	1.9	2.6	3.2	4.1	5.7	6.5	7.3	10.2	13.5	16.9	20.3	27.1
3/4	2.1	2.8	3.5	4.4	6.2	7.1	8.0	11.1	14.8	18.4	22.1	29.5
13/16	2.3	3.1	3.8	4.8	6.7	7.7	8.6	12.0	16.0	20.0	24.0	32.0
7/8	2.5	3.3	4.1	5.2	7.2	8.3	9.3	12.9	17.2	21.5	25.8	34.4
15/16	2.7	3.5	4.4	5.5	7.7	8.8	10.0	13.8	18.5	23.0	27.7	36.9
1	2.8	3.8	4.7	5.9	8.3	9.4	10.6	14.8	19.7	24.6	29.5	39.4
1-1/2	4.2	5.6	7.0	8.8	12.3	14.1	15.8	22.1	34.4	42.9	51.5	68.7
2	5.6	7.5	9.4	11.7	16.4	18.8	21.1	29.5	39.3	49.1	58.9	78.5
2-1/2	7.1	9.4	11.7	14.7	20.5	23.6	26.4	36.8	49.1	61.4	73.6	98.2
3	8.5	11.3	14.1	17.6	24.6	28.2	31.7	44.2	58.9	73.6	88.4	118
3-1/2	9.9	13.1	16.4	20.5	28.8	32.7	37.0	51.5	68.7	85.9	103	137
4	11.3	15.0	18.8	23.5	32.8	37.6	42.2	58.9	78.5	98.2	118	157
4-1/2	12.7	16.9	21.2	26.4	37.0	42.4	47.5	66.3	88.4	110	133	177
5	14.1	18.7	23.5	29.3	41.1	47.1	52.8	73.6	98.2	123	147	196

Para material que no es de 50,000 P.S.I. de esfuerzo cortante se usa un factor para calcular la presión requerida para perforar según la tabla a la derecha.

*EJEMPLO:* Para hacer una perforación de 15/16" de diámetro en acero inoxidable 18-8 calibre 10 (70,000 P.S.I. de esfuerzo cortante) la presión requerida (según la tabla) es de 10.0 toneladas. El factor es de 1.4 ó 10.0 toneladas x 1.4 = 14.0 toneladas de presión requerida.

TABLA DE FACTORES

TIPO DE MATERIAL	Ton. P.S.I.	Esfuerzo Cortante P.S.I.	Factor
Aluminio (Hoja 1/2 duro)	9.5	19,000 P.S.I.	.38
Bronce (Hoja 1/2 duro)	17.5	35,000 P.S.I.	.70
Cobre (Rolado)	14.0	28,000 P.S.I.	.56
Acero Dulce	25.0	50,000 P.S.I.	1.00
Acero, ASTM-A36	30.0	60,000 P.S.I.	1.20
Acero, Carbón 50	35.0	70,000 P.S.I.	1.40
Acero, Estirado en Frío	30.0	60,000 P.S.I.	1.20
Acero Inoxidable (18-8)	35.0	70,000 P.S.I.	1.40

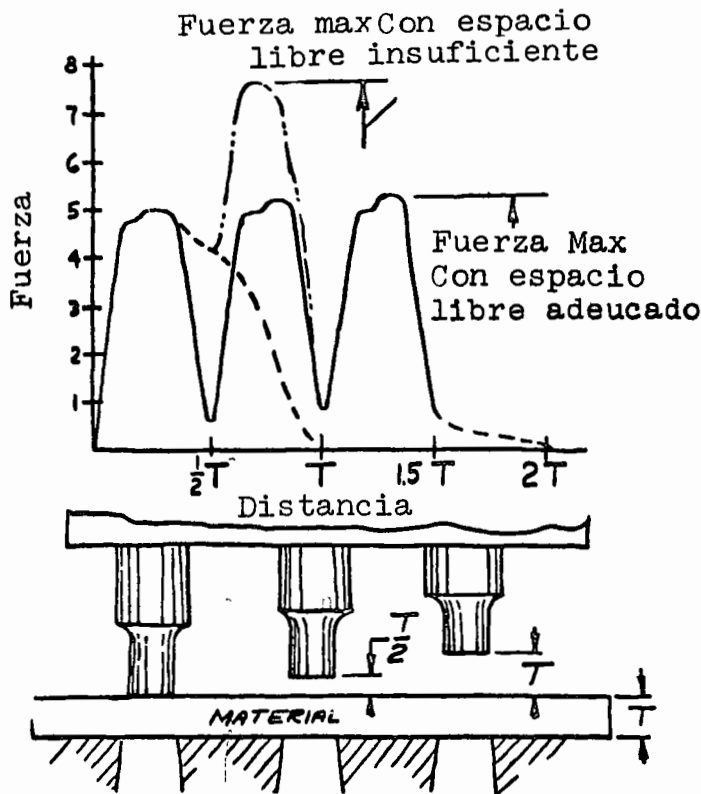


Figura 13.- Escalonado un grupo de punzones se reduce el tonelaje que se requiere para perforar.

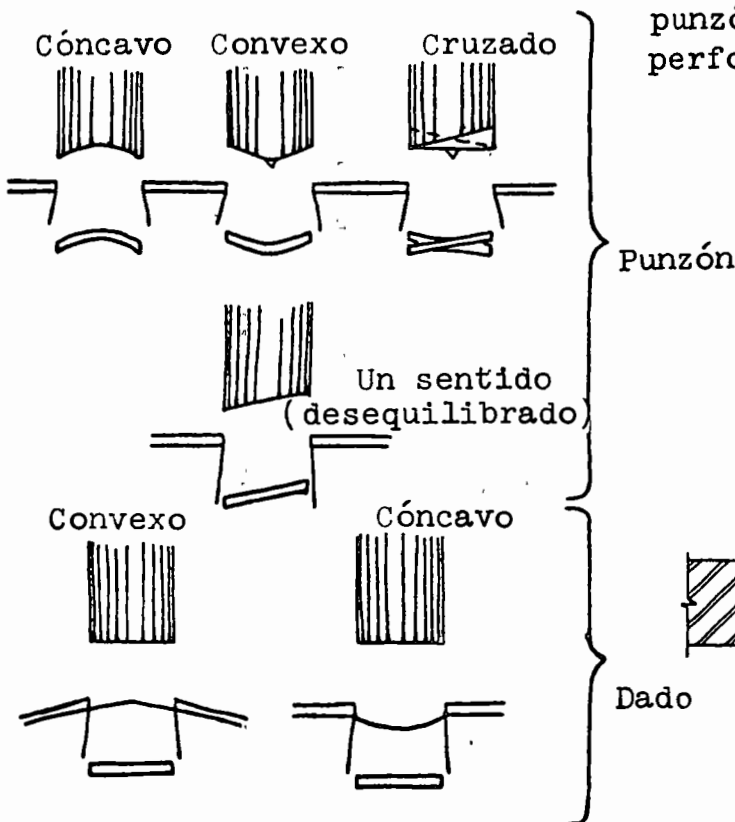
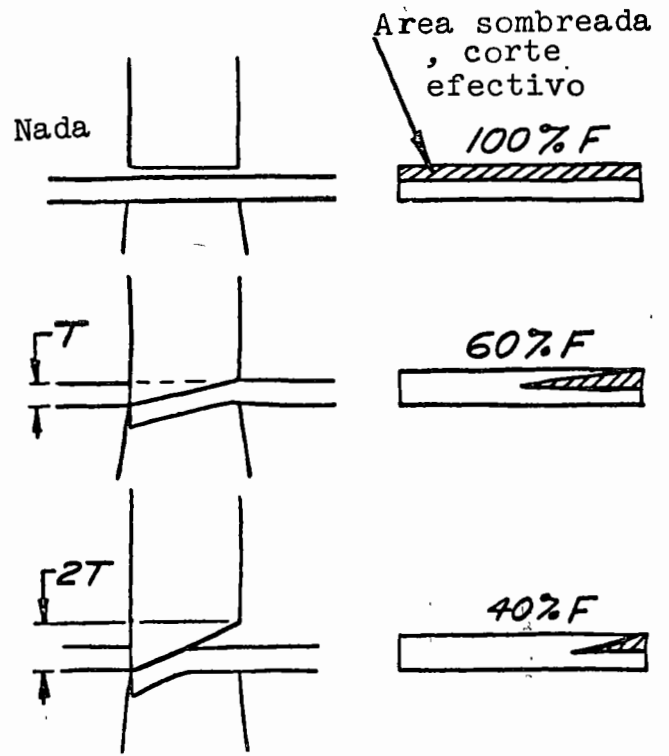


Figura 15.- Varios métodos de aplicar el viaje a un punzón y dado.



Corte -  $1/2 T = 70\% F$

Todo basado en un espacio libre adecuado

Figura 14.- Efecto del vaje en un punzón, dado o cuchilla que se aplica a perforaciones redondas y cortes rectos.

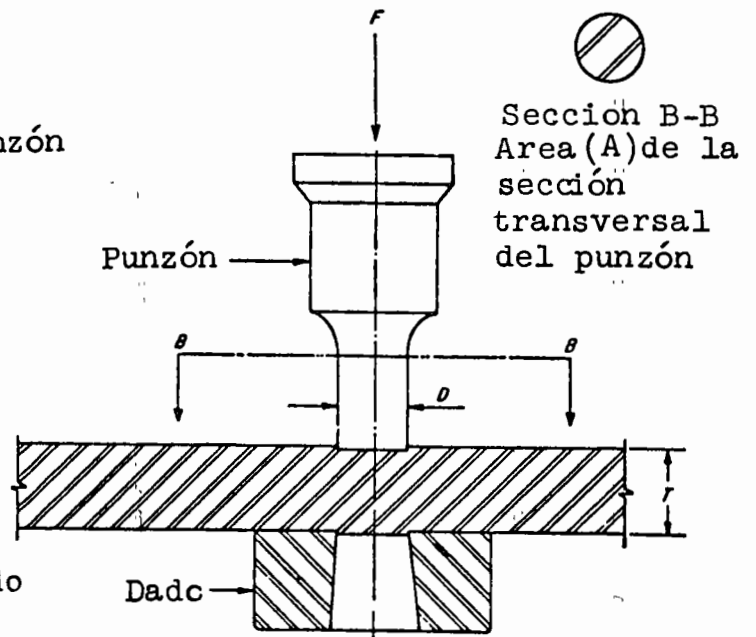
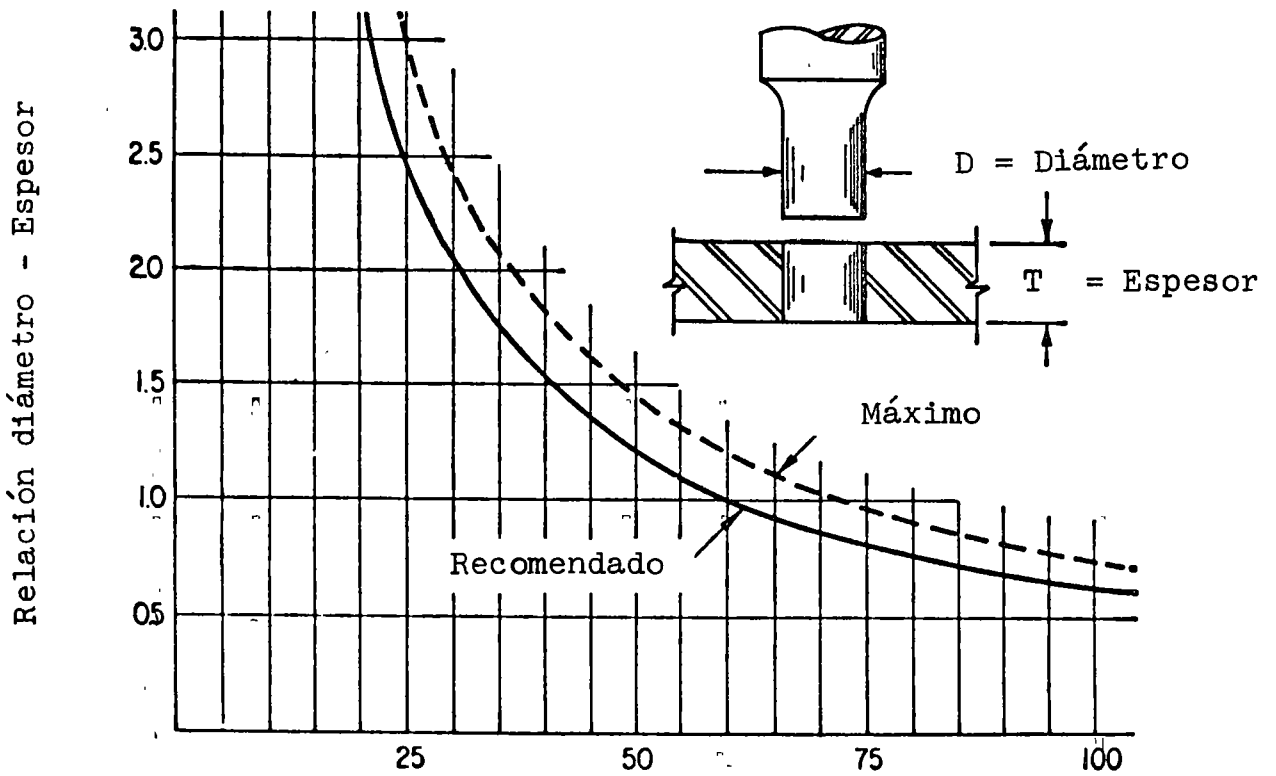


Figura 16.- Relación del punzón respecto al material que es perforado.



Esfuerzo cortante en libras por pulgada cuadrada x 100

Figura 17.- Relación del espesor del material respecto al diámetro de punzonado para valores dados de esfuerzo cortante.

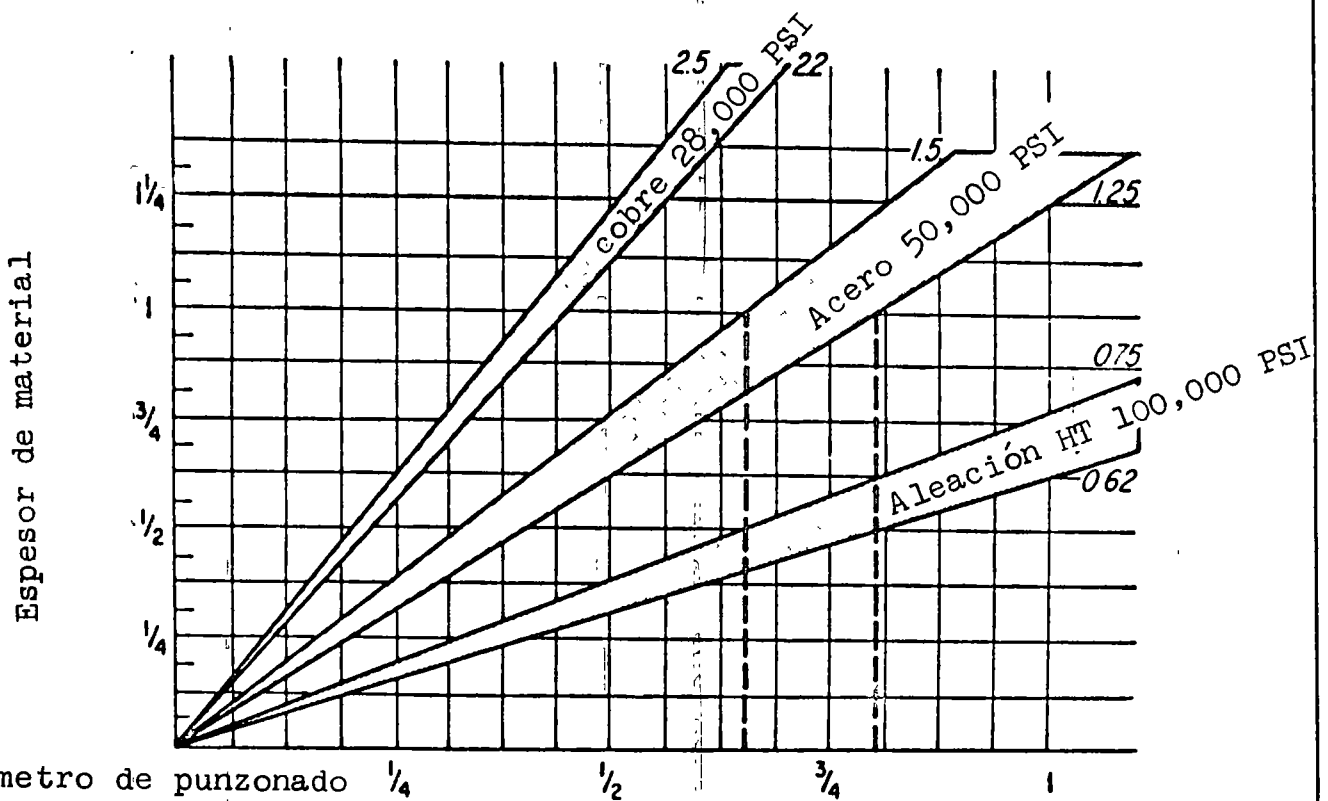


Figura 18.- Mínimo de diámetro de perforación que puede hacerse a través de cierto espesor dado de material.

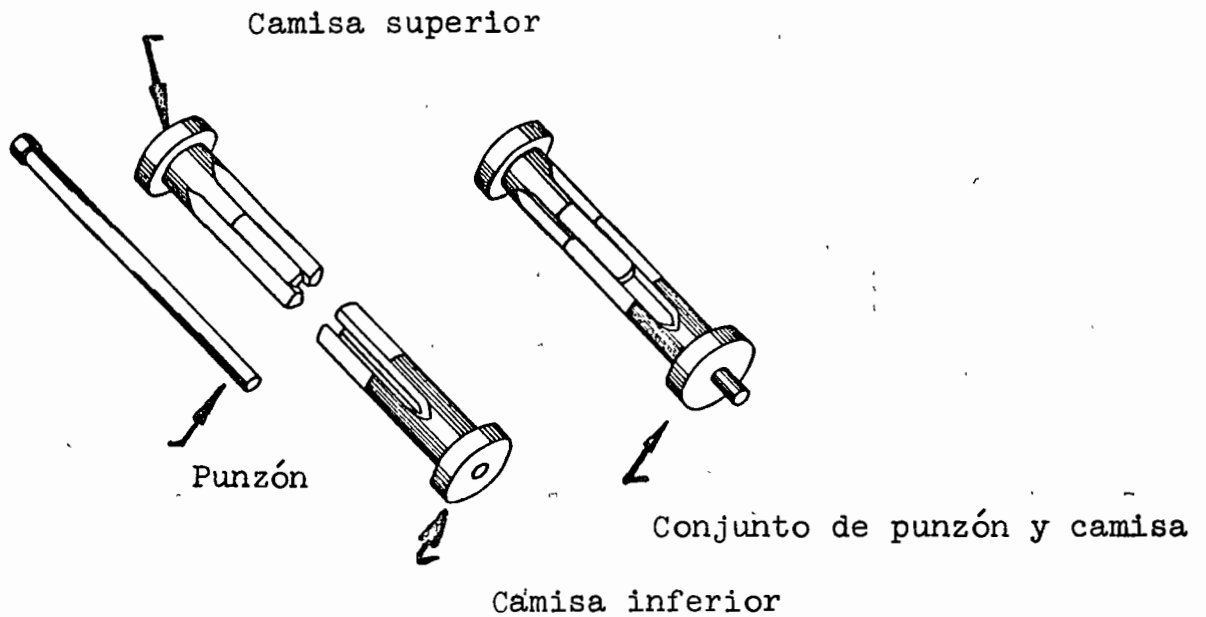


Figura 19.- Un punzón con guía para mayores relaciones de espesor a diámetro, apoya el punzón de principio a fin.

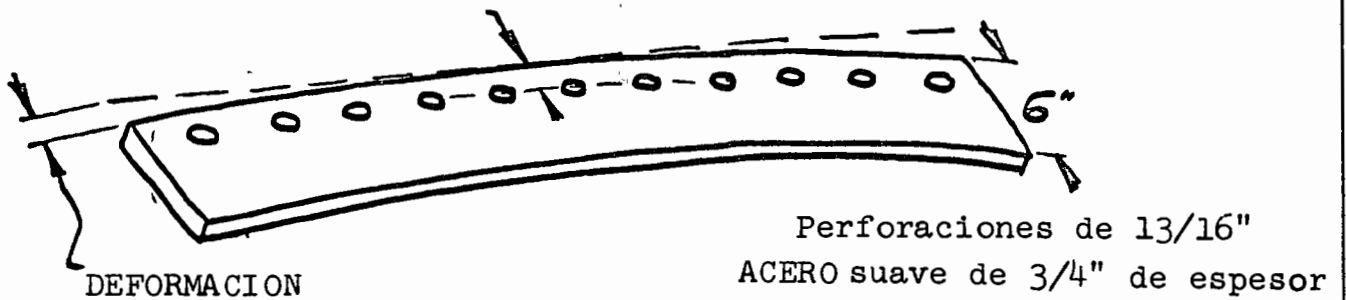
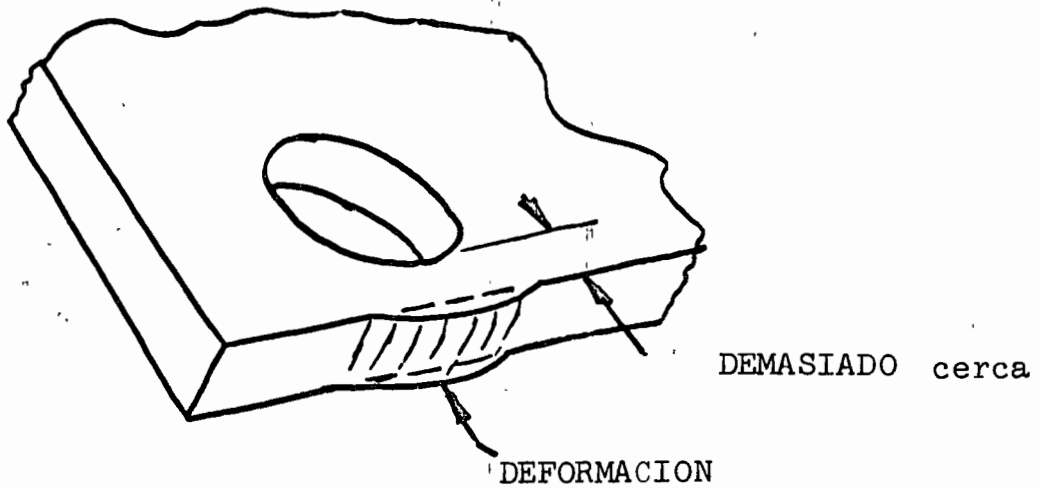


Figura 20.- Deformación debida a muchas perforaciones fuera del centro en solera angosta.

# Esfuerzos permisibles para soldaduras (AISC 1969).

TABLA 1.5.3 (AISC)

TIPO DE ESFUERZO	ESF. PERM. (Kg/cm <sup>2</sup> )	ELECTRODO REQUERIDO <sup>4</sup>	METAL BASE "AFIN" <sup>4</sup>
Tensión y compresión axial paralela al eje de cualquier soldadura de ranura (a tope) de penetración completa.	El mismo que el del metal base <sup>1</sup>		
Tensión perpendicularmente a la garganta efectiva de soldaduras de ranura (a tope) de penetración completa.	El esfuerzo permisible a la tensión del metal base <sup>1</sup>		
Compresión perpendicularmente a la garganta efectiva de soldaduras de ranura (a tope) de penetración completa.	El esfuerzo permisible a la compresión del metal base <sup>1</sup>		
Cortante en la garganta efectiva de soldaduras de ranura (a tope) de penetración completa o parcial.	El esfuerzo permisible al corte en el metal base <sup>1</sup>		
Cortante en la garganta efectiva <sup>2</sup> de soldaduras de filete, independientemente de la dirección de aplicación de la carga.	1265 Kg/cm <sup>2</sup>	Electrodos AWS A5.1, E-60XX Electrodos combinados con fundente AWS A5.17, FGX-EXXX Electrodos A5.20, E60T-X	A500 Grado A A570 Grado D
Tensión perpendicularmente <sup>3</sup> al eje de la garganta efectiva de una soldadura de ranura (a tope) de penetración parcial. Cortante en el área efectiva de una soldadura de tapón o de ranura. Los esfuerzos permisibles que se especifican se aplicarán también a soldaduras hechas con los electrodos indicados sobre un acero con un esfuerzo de cedencia mayor que el del metal base "afin". El esfuerzo permisible, independientemente de la clasificación del electrodo empleado, no excederá del que se indica en la tabla para el metal base "afin" más débil que se emplee en la junta.	1480 Kg/cm <sup>2</sup>	Electrodos AWS A5.1 ó A5.5, E-70XX Electrodos combinados con fundente AWS A5.17, F7X-EXXX Electrodos AWS A5.18, E70S-X ó E70U-1 Electrodos AWS A5.20, E70T-X	A-36 A-53 Grado B A-242 A-375 A-441 A-500 Grado B A-501 A-529 A-570 Grado E A-572 Grados 42 a 60 A-588
	1690 Kg/cm <sup>2</sup>	Electrodos AWS A5.5, E-80XX Arco sumergido Grado 80, Arco Metálico con gas, o Arco metálico de aportación con relleno de fundente.	A-572 Grado 65

Notas:

1. Se utilizará el electrodo o el fundente especificado en la tabla 1.17.2
2. Para definición de la garganta efectiva de una soldadura de filete y de las soldaduras de ranura de penetración parcial, véase la sección 1.14.7.
3. Las soldaduras de filete y las de ranura de penetración parcial que unen los elementos que forman un miembro compuesto, tales como las conexiones de patines a alma, se podrán diseñar sin importar la dirección del esfuerzo de tensión o compresión en estos elementos, paralelos a los ejes de las soldaduras.
4. Se utilizarán solamente electrodos de bajo hidrogeno sobre aceros A-242, A-441, A-514, A-572 y A-588.

TABLA 1.17.2 (AISC)

Metal base <sup>3</sup>	Proceso de soldadura 1/2			
	Arco metálico recub.	Arco sumergido	Arco metálico c/gas	Arco c/relleno de fundente
ASTM A36, A53 Gr. B, A375, A500, A501, A529 y A570 Grados D y E.	AWS A5.1 ó A5.5, E-60XX ó E-70XX	AWS A5.17 F6X ó F7X-EXXX	AWS A5.18 E70S-X ó E70U-1	AWS 5.20 E60T-X ó E70T-X (excepto EXXT-2 y EXX-3)
ASTM A-242, A-441, A-572 Grados 42 a 60 y A588 <sup>4</sup>	AWS A5.1 ó A5.5, E-70XX <sup>5</sup>	AWS A5.17 F7X-EXXX	AWS A5.18 E70S-X ó E70U-1	AWS 5.20 E70T-X (excepto E-70T-2 y E-70T-3)
ASTM A572 Grado 65	AWS A5.5 E-80XX <sup>5</sup>	Grado F80	Grado E80S	Grado E80T

Se permite el uso de metal de relleno cuyas propiedades mecánicas sean las más próximas pero superiores.

1. Cuando las soldaduras se vayan a relevar de esfuerzos, el metal de aportación en la soldadura no deberá contener más del 0.05 por ciento de vanadio.
2. Véanse las especificaciones AWS D1.0-69, artículo 422 para los requisitos del metal de aportación en soldaduras de electrogas y electroescoria.
3. En aquellas juntas que contengan metales de distinto esfuerzo de cedencia, podrán utilizarse material de relleno de aquellos aceros cuyo esfuerzo de cedencia sea el menor de los que entran en la junta.
4. Para el caso de soldaduras de metales que quedarán expuestos a la intemperie por fines arquitectónicos, el metal de aportación deberá además tener características de resistencia a la corrosión atmosférica y coloración de su oxidación, semejantes a las del metal base utilizado. Sígase las recomendaciones del fabricante del acero.
5. Clasificaciones correspondientes a electrodos de bajo hidrógeno.

Recomendaciones AISC para juntas soldadas.

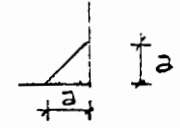
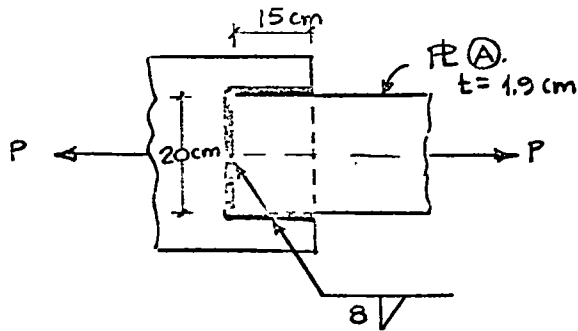
- 1.- Long. mín. de un filete =  $4a$  ( $a$  es el espesor del filete 
- 2.-  $a_{max} = 1/4"$  (0.63cm) para material de  $t = 1/4"$  de espesor. Para materiales más gruesos  $a_{max} = t - 1/16"$
- 3.- Las dimensiones mínimas de los filetes de soldadura estarán dadas por la tabla siguiente:

TABLA 1.17.5 (AISC)

Espesor de la placa más gruesa en la unión		Tamaño mínimo de la soldadura	
cm	pulg.	cm.	pulg.
hasta 0.63 incl.	hasta 1/4" inclusive	0.32	1/8"
de 0.63 a 1.27	de 1/4" a 1/2"	0.48	3/16"
de 1.27 a 1.90	de 1/2" a 3/4"	0.64	1/4"
de 1.90 a 3.81	de 3/4" a 1 1/2"	0.79	5/16"
de 3.81 a 6.35	de 1 1/2" a 2 1/4"	0.95	3/8"
de 6.35 a 15.24	de 2 1/4" a 6"	1.27	1/2"
mayor de 15.24	mayor de 6"	1.59	5/8"

### EJEMPLO 1

Cual es la capacidad de carga  $P$  permisible para la conexión de la figura si se utiliza acero A-36, y electrodos E-70XX.



Solución:

Capacidad de la soldadura de filete de 8mm.

$$1480 \times 0.707 \times 0.8 = 837 \text{ Kg/cm.}$$

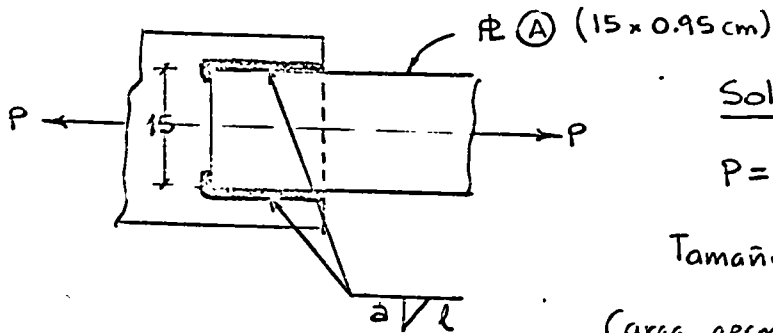
$$\text{Capacidad de la soldadura} = 837(20 + 2 \times 15) = 41850 \text{ Kg}$$

$$\text{Capacidad de la placa } \textcircled{A} \text{ a la tensión} = 1520 \times 20 \times 1.9 = 57760 \text{ Kg.}$$

$$\text{Capacidad de la conexión } \underline{P = 41850 \text{ Kg}}$$

### EJEMPLO 2

Calcule la soldadura requerida para que la junta mostrada desarrolle la capacidad permisible de tensión en la placa  $\textcircled{A}$ . Acero A-572 Grado 65 ( $f_y = 4570 \text{ Kg/cm}^2$ ) y electrodos E-80XX



Solución

$$P = 0.6 \times 4570 \times 15 \times 0.95 = 39074 \text{ Kg}$$

$$\text{Tamaño max. de sold.} = \frac{3}{8} - \frac{1}{16} = \frac{5}{16} = 0.79 \text{ cm}$$

$$\text{Carga permisible de la sold.} = 1690 \times 0.707 \times 0.79 = 944 \text{ Kg/cm}$$

$$\text{longitud requerida de la soldadura por cordón } l = \frac{39074}{2 \times 944} = 20.7 \text{ cm}$$

$$\text{utilizando vueltas de long. mínima de } 2 \times 0.79 = 1.58 \text{ cm (sean de 1.7 cm)}$$

$$l = 20.7 - 1.7 = 19 \text{ cm. por cordón.}$$

### EJEMPLO 3

Diseñe la soldadura para unir un tensor formado por una canal de 305mm pesada, a una placa de 1.27cm, como se muestra en la figura. El acero de la canal es A-36; el de la placa es A-242 y la soldadura si hace con electrodos E-70XX. La carga que se transmitirá es de 115 ton., y debido a requisitos arquitectónicos, la canal no puede traspasar más de 12 cm dentro de la placa de 1.27cm..

Solución.

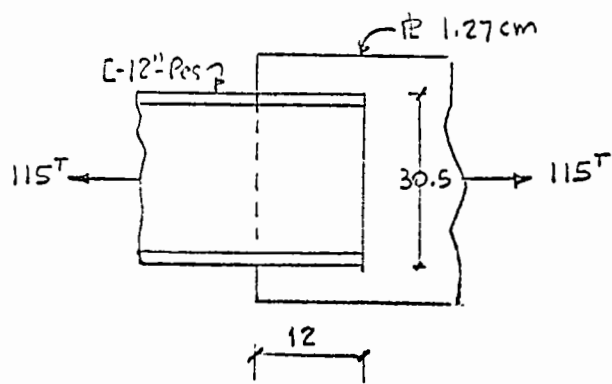
El espesor del alma de la [ es 1.92 cm, > 1.27

Tamaño max. de la soldadura =  $\frac{1}{2} - \frac{1}{16} = \frac{7}{16} = 1.11 \text{ cm}$

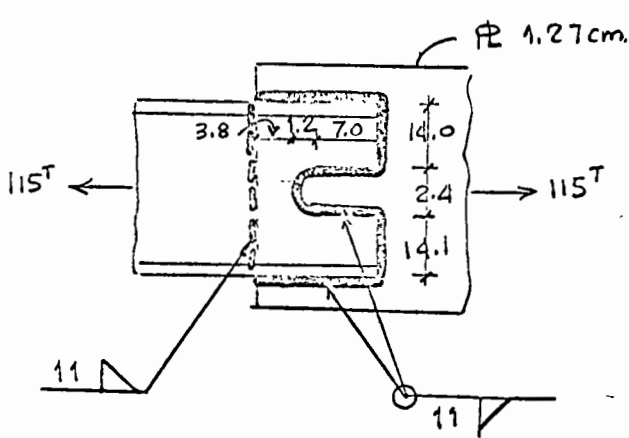
Cap. sold =  $1480 \times 0.707 \times 1.11 = 1161 \text{ Kg/cm}$ .

$l_{req} = \frac{115000}{1161} = 99.0 \text{ cm}$

long. disponible =  $2 \times 12 + 2 \times 30.5 = 85 \text{ cm} < 99.0 \text{ cm}$ .  $\therefore$  hay que hacer una ranura para alojar la longitud faltante  $99.0 - 85 = 14 \text{ cm}$ .



Dimensiones de la ranura (Ver AISC 1.17.12)



- a)  $1.27 + 0.8 = 2.07 \text{ cm}$
- b)  $2.25 \times 1.27 = 2.86 \text{ cm}$ .

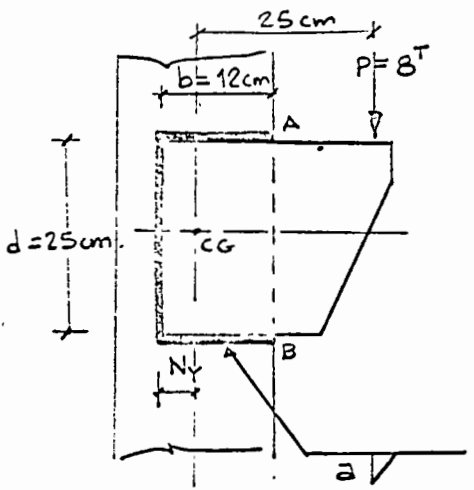
Usamos 2.38 cm ( $\frac{15}{16}$ ), (en virtud de que los punzones estructurales se fabrican en un número non de 16 avos de pulgada)  
La longitud de la ranura

$l_{max} = 10 \times 1.27 = 12.7 \text{ cm} > 7.0 \text{ cm, req.}$

$\therefore$  Usar soldadura de ranura de  $7.0 \times 2.4 \text{ cm}$

EJEMPLO 4

Diseñar el tamaño de la soldadura de filete de la figura siguiente; el acero es A-36, y los electrodos E-70XX.



Solución

de la tabla II  $N_y = \frac{b^2}{2b+d} = \frac{12^2}{2 \times 12 + 25} = 2.94 \text{ cm}$ .

$J_s = \frac{(2b+d)^3}{12} - \frac{b^2(b+d)^2}{2b+d} = \frac{(2 \times 12 + 25)^3}{12} - \frac{12^2(12+25)^2}{2 \times 12 + 25} = 5780 \text{ cm}^3$

$A_s = 25 + 2 \times 12 = 49 \text{ cm}$

$f_1 = \frac{P}{A_s} = \frac{8000}{49} = 163 \text{ Kg/cm}$

$f_{2H} = \frac{T}{J} Y = \frac{8000 \times 25}{5780} \times 12.5 = 433 \text{ Kg/cm}$

$f_{2V} = \frac{T}{J} X = \frac{8000 \times 25}{5780} (12 - 2.94) = 313 \text{ Kg/cm}$



el esfuerzo cortante total en A vale:

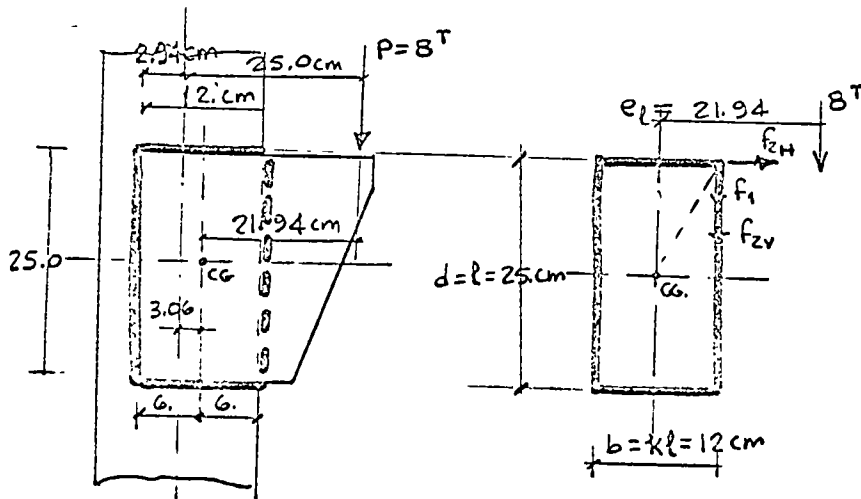
$$f_{TOT} = \sqrt{(f_1 + f_{2V})^2 + \overline{f_{2H}}^2} = \sqrt{(163 + 313)^2 + 433^2} = 644 \text{ Kg/cm}$$

el tamaño requerido del filete es:  $644 \text{ Kg/cm} = 1480 \times 0.707 \cdot a$

$$\therefore a = \frac{644}{1480} = 0.62 \text{ cm} ; \text{ sea } a = 6.3 \text{ mm } \left(\frac{1}{4}''\right)$$

### EJEMPLO 5

Sea el mismo caso del problema anterior, pero colocando además un cordón de soldadura por detrás de la placa-ménsula.



Solución: (Ver tabla II)

$$A_s = 2(25+12) = 74 \text{ cm} \quad J_s = \frac{(b+d)^3}{6} = \frac{(25+12)^3}{6} = 8420 \text{ cm}^3$$

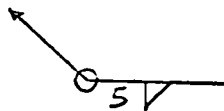
$$f_1 = \frac{8000}{74} = 108 \text{ Kg/cm}$$

$$f_{2V} = \frac{8000 \cdot 21.24}{8420} \times 6 = 125 \text{ Kg/cm}$$

$$f_{2H} = \frac{8000 \cdot 21.94}{8420} \times 12.5 = 260.6 \text{ Kg/cm}$$

$$f_{tot} = \sqrt{(108 + 125)^2 + 260.6^2} = 350 \text{ Kg/cm} = 1480 \times 0.707 \cdot a$$

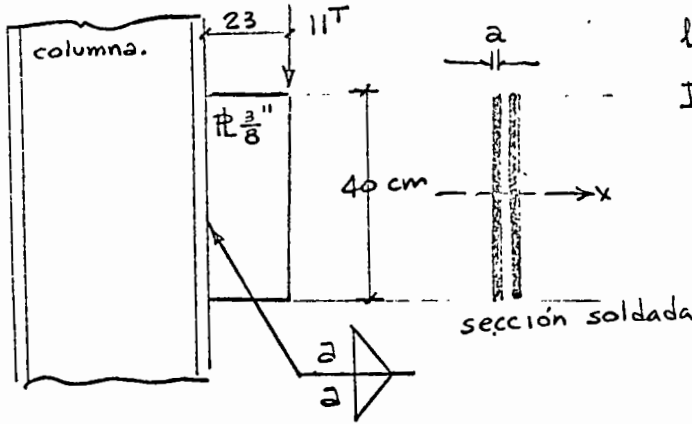
$$\therefore a = \frac{350}{1480 \cdot 0.707} = 0.33 \text{ cm} \quad \therefore \text{ sea } a = 0.48 \text{ cm} = \frac{3}{16}''$$



### EJEMPLO 6

Determine el tamaño del filete de sold.  $\exists$  requerido para resistir la carga  $P = 11 \text{ ton}$ , aplicada como se muestra en la figura. Acero A-36 y Electrodo E-70XX

#### Solución



$$l = 2 \times 40 = 80 \text{ cm}$$

$$I_{xx} = 2 \times \frac{40^3}{12} = 10667 \text{ cm}^3$$

$$f_1 = \frac{11000}{80} = 137.5 \text{ Kg/cm}$$

$$f_2 = \frac{11000 \times 23}{10667} \times 20 = 474.4 \text{ Kg/cm}$$

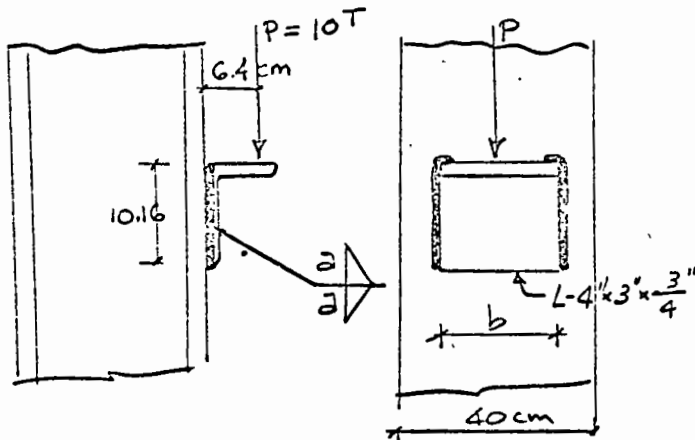
$$f_{\text{tot}} = \sqrt{137.5^2 + 474.4^2} = 494 \text{ Kg/cm}$$

$$a = \frac{494}{1480 \times 0.707} = 0.472 \text{ cm}, \text{ sea } a = \frac{3}{16}'' = 0.48 \text{ cm}$$

$$a = \frac{3}{16}'' < \frac{3}{8}'' - \frac{1}{16}'' = \frac{5}{16}'' \therefore \checkmark$$

### EJEMPLO 7

Utilizando electrodos E-80XX y acero A572-65 determine el tamaño de la soldadura requerido para la conexión de asiento de la figura.



#### Solución

$$M = 10000 \times 6.4 = 64000 \text{ Kg-cm}$$

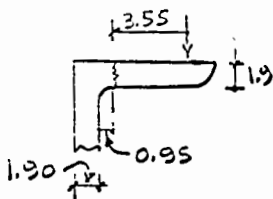
$$f_1 = \frac{10000}{2 \times 10.16} = 492 \text{ Kg/cm}$$

$$f_2 = \frac{64000}{\frac{2 \times 10.16^2}{6}} = 1860 \text{ Kg/cm}$$

$$f_r = \sqrt{492^2 + 1860^2} = 1924 \text{ Kg/cm}^2$$

$$a = \frac{1924}{1690 \times 0.707} = 1.61 \text{ cm}$$

$$\text{sea } a = \frac{11}{16}'' = 1.75 \text{ cm.} \quad \left( a_{\text{max}} = \frac{3}{4}'' - \frac{1}{16}'' = \frac{11}{16}'' \therefore \checkmark \right)$$



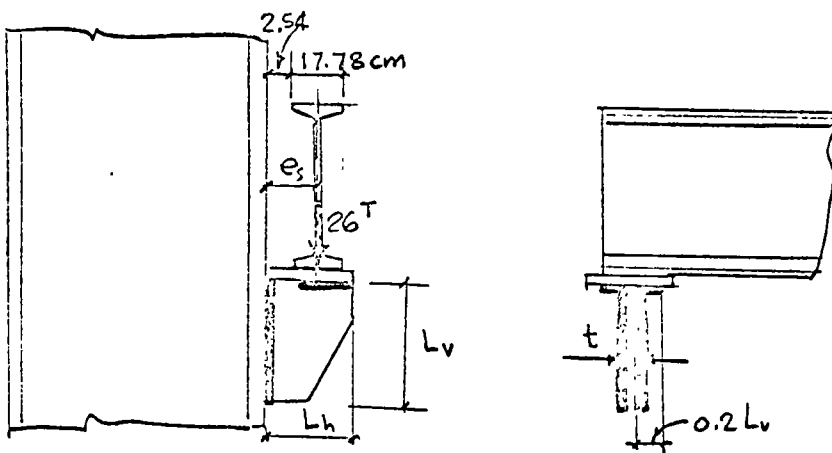
$$M = 10000 \times 3.55 = 35500 \text{ Kg-cm}$$

$$S = \frac{1.9^2 \times b}{6}; \quad F_b = 0.75 F_y = 3428 \text{ Kg/cm}^2; \quad M_r = 3428 \times \frac{1.9^2 b}{6}$$

$$\therefore b = \frac{35500 \times 6}{3428 \times 1.9} = 32.7 \text{ cm} \text{ sea } 33 \text{ cm.}$$

### EJEMPLO 3

Diseñe el asiento atiesado de una viga cuya reacción es de 26T. Use Acero A-36 y electrodos E-70XX



$$e_s = 2.54 + \frac{17.78}{2} = 11.43 \text{ cm}$$

$$L_h = 2.54 + 17.78 = 20.32 \text{ cm}$$

#### Solución

Determinación del espesor de la placa:

$$A = t L_h$$

$$S = \frac{t L_h^2}{6}$$

$$M = R(e_s - \frac{L_h}{2})$$

$$f = \frac{R}{A} + \frac{M}{S} = \frac{R}{t L_h} + \frac{R(e_s - \frac{L_h}{2})}{\frac{t L_h^2}{6}} = \frac{R(6e_s - 2L_h)}{t L_h^2}$$

$$\therefore t = \frac{R(6e_s - 2L_h)}{f L_h^2}$$

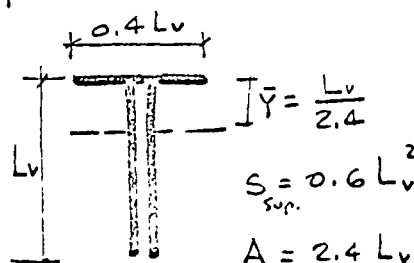
si  $f = 0.6 F_T = 1520 \text{ kg/cm}^2$ ,  $R = 26,000 \text{ kg}$ ,  $e_s = 11.43 \text{ cm}$  y  $L_h = 20.32 \text{ cm}$

$$t = \frac{26000(6 \times 11.43 - 2 \times 20.32)}{1520 \times 20.32^2} = 1.16 \text{ cm.} \quad \text{sea } t = \frac{7}{16} = 1.11 \text{ cm}$$

$$\text{y } a = \frac{5}{16} = 0.79 \text{ cm}$$

Determinación de la longitud del atiesador  $L_v$ :

La sección soldada es:



$$f_1 = \frac{R}{A} = \frac{R}{2.4 L_v}$$

$$f_2 = \frac{M}{S} = \frac{R e_s}{0.6 L_v^2}$$

$$f_{\text{tot}} = \sqrt{f_1^2 + f_2^2} = \sqrt{\left(\frac{R}{2.4 L_v}\right)^2 + \left(\frac{R e_s}{0.6 L_v^2}\right)^2} = \frac{R}{2.4 L_v^2} \sqrt{L_v^2 + 16 e_s^2}$$

$$a = \frac{f_{\text{tot}}}{F_{\text{perm}}} \quad \gamma \quad F_{\text{perm}} = 1480 \text{ Kg/cm}^2 \times 0.707 = 1046 \text{ Kg/cm}^2$$

$$\frac{R}{a} = \frac{1046 \times 2.4 L_v^2}{\sqrt{L_v^2 + 16 e_s^2}} = \frac{2511 L_v^2}{\sqrt{L_v^2 + 16 e_s^2}}$$

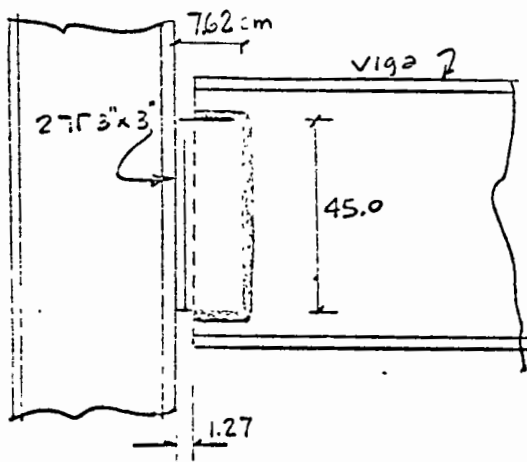
$$\therefore L_v = \sqrt{\frac{B}{2} \left[ B + \sqrt{B^2 + 64 e_s^2} \right]} \quad \text{donde } B = \frac{R}{2511 a}$$

en nuestro caso  $B = \frac{26000}{2511 \times 0.79} = 13.11$

$$L_v = \sqrt{6.55 \left[ 13.11 + \sqrt{171.8 + 64 \times 11.43^2} \right]} = 26.28 \text{ cm} \quad \text{seam } 10\frac{1}{2}'' = 26.67 \text{ cm.}$$

### EJEMPLO 9.

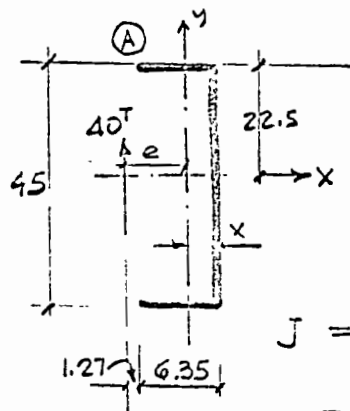
Determine la dimensión  $a$  del filete de soldadura requerido para la conexión de dos ángulos al alma de la viga mostrado en la figura.



Reacción =  $40^T$   
Electrodos E-70XX  
Acero A-36

### Solución

La sección soldada tiene la forma:



$$\bar{x} = \frac{2 \times 6.35 \times \frac{6.35}{2}}{45 + 2 \times 6.35} = 0.7 \text{ cm}$$

$$e = 7.62 - 0.70 = 6.92 \text{ cm}$$

$$J = \frac{(2b+d)^3}{12} - \frac{b^2(b+d)^2}{2b+d}$$

$$J = \frac{(12.7+45)^3}{12} - \frac{6.35^2(6.35+45.0)^2}{12.7+45} = 16008 - 1843 = 14165 \text{ cm}^3$$

$$A = 2 \times 6.35 + 45 = 57.7 \text{ cm}$$

Esfuerzos: (en el punto A)

$$f_1 = \frac{40000/2}{57.7} = 347 \text{ Kg/cm}$$

$$f_{2V} = \frac{(40000/2) 6.92}{14165} (6.35 - 0.7) = 55. \text{ Kg/cm}$$

$$f_{2H} = \frac{(40000/2) 6.92}{14165} \cdot 22.5 = 220 \text{ Kg/cm}$$

$$f_{tot} = \sqrt{(347+55)^2 + 220^2} = 459 \text{ Kg/cm}$$

$$\bar{\sigma} = \frac{459}{1406} = 0.32 \text{ cm} \quad \text{sea } \underline{\text{soldadura de } 1/8''}$$



# CHAPTER 5

## Connections—Bolted or Riveted

THE BEAMS, GIRDERS, TRUSSES, COLUMNS and other members which form a completed structure are designed to support certain loads. Each of these members must transmit its load through structural joints to supporting members. For example, beams transmit loads to girders; girders transmit tributary beam loads to trusses or columns; and finally, columns transmit their loads to footings or piers in the ground. Each member in this system must be provided with suitable connections to transfer its assigned load to other members safely, economically, and in conformance with applicable specifications.

All joints require a means of fastening—either bolts, rivets, or welds. In addition, most structural joints require detail connection material, made of angles, plates, or pieces of rolled beams. This chapter is primarily concerned with bolts and rivets, generally identified as fasteners. Chapter 6 will cover connections made by welds.

### TYPES OF FASTENERS

#### Rivets

Rivets are manufactured to ASTM Designation A502, which includes carbon steel rivets, Grade 1, for general purposes, and carbon manganese steel rivets, Grade 2,

suitable for use with high-strength carbon and high-strength low alloy structural steels. Grade 2 rivets are identified by the numeral 2 on the manufactured head; Grade 1 rivets may be identified by the numeral 1, or may be left unmarked. The characteristic button and countersunk heads are shown dimensioned in Manual Part 4. Figure 5-1 shows the appearance of the manufactured and driven rivet heads used in structural steel fabrication.

For the sake of brevity, these grades will be referred to as A502-1 and A502-2 throughout this book.

#### A307 Bolts

These bolts, frequently termed unfinished, machine, plain, common, or rough bolts, are employed for shop and field connections wherever their use is permitted. Although A307 bolts are prohibited in certain major connections by AISC Specification Sect. 1.15.12, they have wide application on filler beams and in other non-critical areas. Their manufacture and testing is covered by ASTM A307. The dimensions of square and hexagon heads and nuts, and countersunk heads, as well as thread proportions and lengths, are listed in tables on "Bolt Data—Threaded Fasteners", Manual Part 4. Figure 5-2 gives bolt and nut nomenclature and shows the various heads and nuts of A307 bolts.

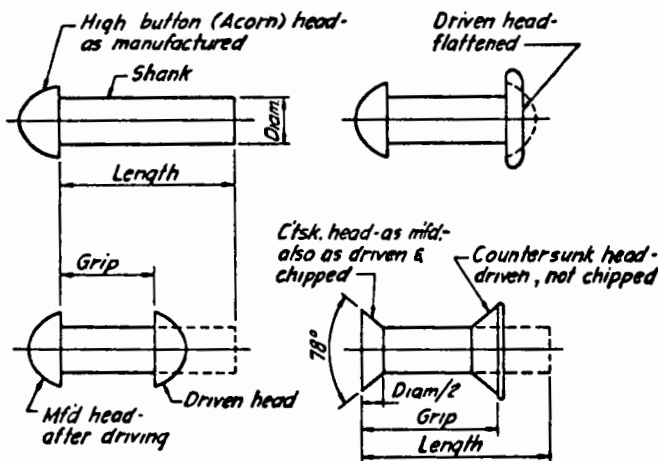


Fig. 5-1. Rivet nomenclature

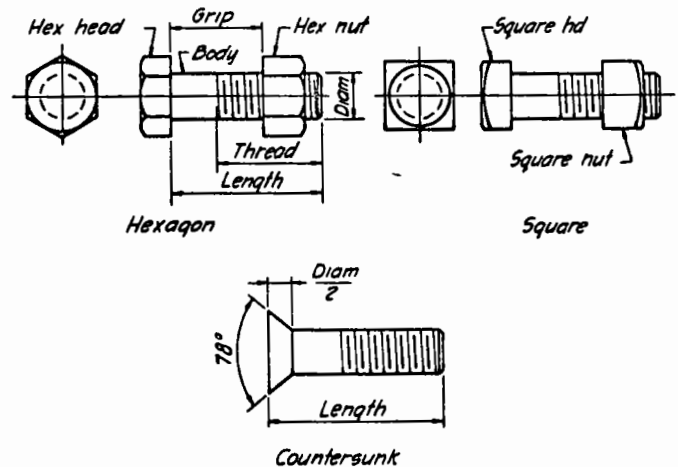


Fig. 5-2. Bolt and nut nomenclature

**A325 and A490 Bolts**

High-strength structural bolts are used almost exclusively where fasteners are required in field connections, and to a considerable extent in shop connections, for all types of structures. They are furnished in two grades: ASTM A325 high-strength carbon steel bolts and ASTM A490 high-strength alloy steel bolts. These specifications cover the manufacture and testing of the bolts, nuts, and hardened washers required for high-strength bolting.

Whereas the A490 specification is limited to a single description of bolt, the A325 specification covers three bolt types: Type 1 bolts are made of medium carbon steel; Type 2 bolts are generally described as being made of low carbon martensite steel (a material furnished preferentially by some manufacturers), and Type 3 bolts are made of corrosion-resistant, weathering steel that is compatible in this respect with ASTM A242 or A588 steels. It should be noted that Type 1 bolts are the same as bolts furnished to A325 specifications prior to 1970, and that their use continues to be indicated for all appli-

cations not specifically calling for Type 2 or 3 bolts. In the event an order does not stipulate a bolt type, Type 1 bolts will be furnished, although Type 2 or Type 3 bolts may be substituted at the manufacturer's option, subject to the approval of the purchaser. Nuts and washers for Type 1 and 2 bolts are identical, but Type 3 bolts must be provided with Type 3 nuts and washers to furnish the necessary corrosion-resistant and weathering properties. Since the three types of A325 bolts have the same mechanical properties, references to A325 bolts in this book will omit the type designation.

The *Specification for Structural Joints Using ASTM A325 or A490 Bolts*, approved by the Research Council on Riveted and Bolted Structural Joints of the Engineering Foundation, establishes bolt head, nut, and washer dimensions, allowable stresses, and rules for installation. Hereafter, in this chapter, this will be referred to as the "Council Specification".

A325 and A490 bolts are distinguished from A307 bolts by various codings, the ASTM designation, and the manufacturer's mark. Figure 5-3 shows the appearance and code markings of high-strength bolts.

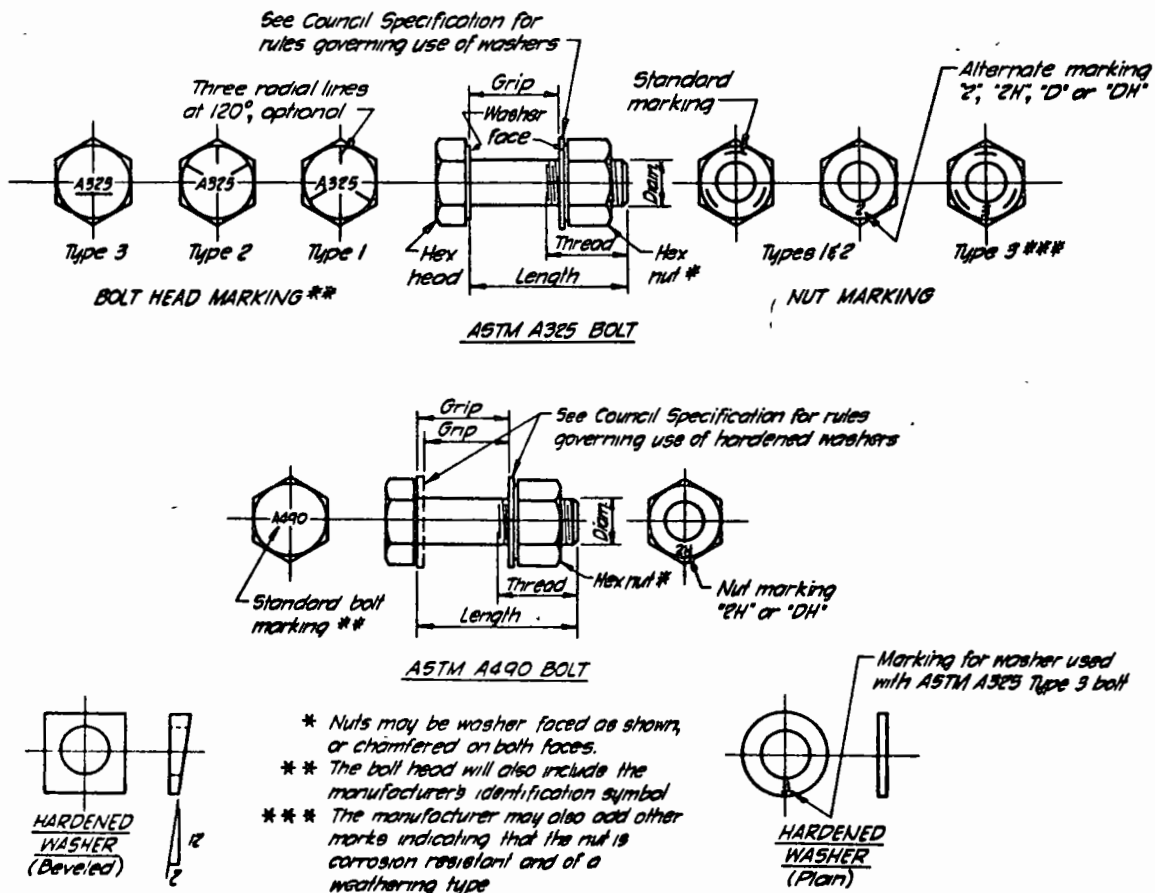


Fig. 5-3. High strength bolt nomenclature



**Installation of A325 and A490 Bolts**—High-strength bolts are tightened either by the calibrated wrench method or by the turn-of-nut method.

In the calibrated wrench method, power wrenches are preset to cease turning when the selected tension range is reached. If manual torque wrenches are employed, the torque indication corresponding to the calibrating tension is noted and used. Accurate tightening depends on a uniform resistance to nut (or bolt) rotation when tensioning is done by a calibrated wrench. Since galling or excessive friction between the turned part and the material connected may render calibration ineffective, the Council Specification calls for a plain smooth hardened washer under the element turned (head or nut) for bolts tightened by this method. To prevent bending of high-strength bolts, as well as galling during tightening, the Council Specification further stipulates the use of beveled washers when an outer face of joint material has a slope greater than 1 in 20. The type of beveled washer used to compensate for the slope of S-beam flanges and American Standard channel flanges, which have a slope in excess of 1 in 20, is shown in Fig. 5-3. Since both bolt and washer are hardened, uniform resistance to rotation is achieved.

In the turn-of-nut method, tensioning is obtained by tightening the bolt to a snug tight fit, and then rotating the nut (or bolt) a specified additional fraction of a turn. Since galling or friction will not affect the amount of tension induced, plain hardened washers are not required with A325 bolts tightened by this method. However, beveled washers will be required if the slope exceeds 1 in 20.

The extremely high preloads used with the A490 type of bolt necessitate plain hardened washers, as well as beveled washers when required, under the turned element for all methods of assembling. A second hardened washer must be furnished if either outer ply of connected material has a yield stress less than 40.0 ksi.

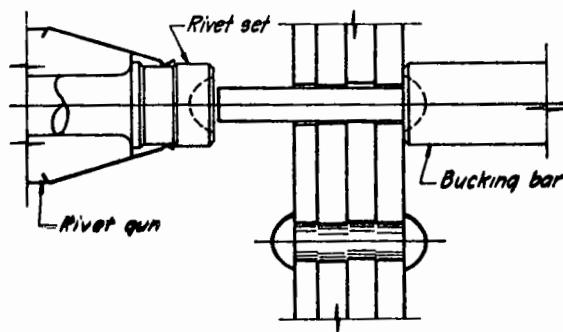


Fig. 5-4. Rivet before and after driving

**HOLES FOR FASTENERS**

Holes for rivets and A307 bolts are made  $\frac{1}{16}$ -in. larger in diameter than the nominal size of the fastener body. This provides a certain amount of play in the holes, which compensates for small misalignments in hole location or assembly, and aids in the shop and field entry of fasteners. Hot-driven rivets will upset and substantially fill the holes (Fig. 5-4). A307 bolts may also be used with holes slotted for adjustment, provided the slot is perpendicular to the direction of loading. In addition, these bolts are customarily assembled with regular structural washers placed over slots which occur in the outer plies of material.

Although most holes for high-strength bolts are also made  $\frac{1}{16}$ -in. larger in diameter than the bolt body, certain conditions encountered in field erection require greater adjustment than this clearance can provide. Consequently, on the basis of extensive laboratory testing, the Council Specification sanctions the limited use of oversize holes and slots, subject to the approval of the designer.\*

\* See the Council Specification for permissible sizes of slots and oversize holes, and rules governing the assembly of high-strength bolts in friction and bearing-type connections.

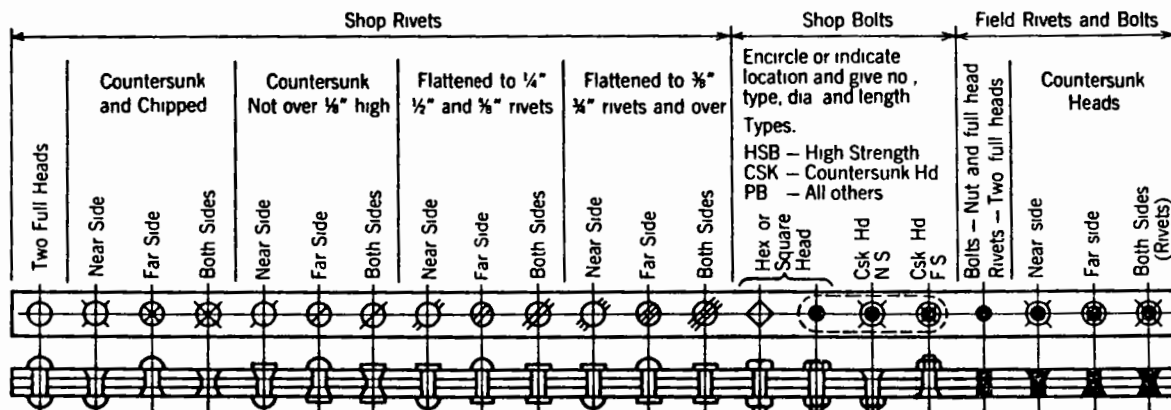


Figure 5-5

**RIVET AND BOLT SYMBOLS**

Shop and field rivets are identified on drawings by conventional signs or symbols which describe the type of head and, should there be differences, distinguish between near and far side heads. Shop bolts are likewise identified, except that additional notes are required to specify the type, diameter, and length of bolts. On detail drawings, the symbols for field rivets and bolts are the same. General or special notes on erection plans specify the type of field fasteners. Figure 5-5 shows conventional signs or symbols for rivets and bolts.

**FASTENER LOADS AND STRESSES**

An external force applied to a member is called a load. The internal force in a member resisting a load is called a stress.

When a fastener is required to resist a load which tends to stretch it in the direction of its length, as in Fig. 5-6a, it is said to be loaded in *tension*; the load creates a *tensile stress* in the fastener.

In most structural connections, the fastener is required to prevent the movement of connected material in a direction transverse to the length of the fastener, as

in Fig. 5-6b. In such cases the fastener is said to be loaded in *shear*. A *shearing stress* is created in the fastener.

Two basic types of connections are used to transmit shear loads. A *bearing-type connection* is one in which the fasteners bear against the sides of the holes in the connected material. All shear connections using rivets or A307 bolts are bearing-type connections. Under certain conditions explained later in this chapter, high strength bolts are also used in bearing type connections. A *friction-type connection* is one in which the fasteners clamp the connected parts together with an extremely high pressure, so that the shearing force is resisted by the friction between the connected parts, and not by a shear stress on the fasteners. *Only high-strength bolts are used in friction-type connections.*

When a fastener transmits a shear load in a bearing-type connection, as in Fig. 5-6c, a *bearing stress* is also set up in the connected material. In friction-type shear connections, the fasteners set up *compressive stresses* in areas surrounding the bolts (Fig. 5-6d) which in turn induce friction between the plies of connected material. Bearing stresses, therefore, are not present.

Figure 5-7 illustrates the basic functions of fasteners in a connection:

- (1) An S beam suspended from a bracket supports a load  $P$ , which is transmitted to the bracket angles by the bolts marked A. Bolts A resist the downward pull of  $P$ ; each bolt supports a share of the load, and is stretched in the direction of its length. These bolts are loaded in *tension*.

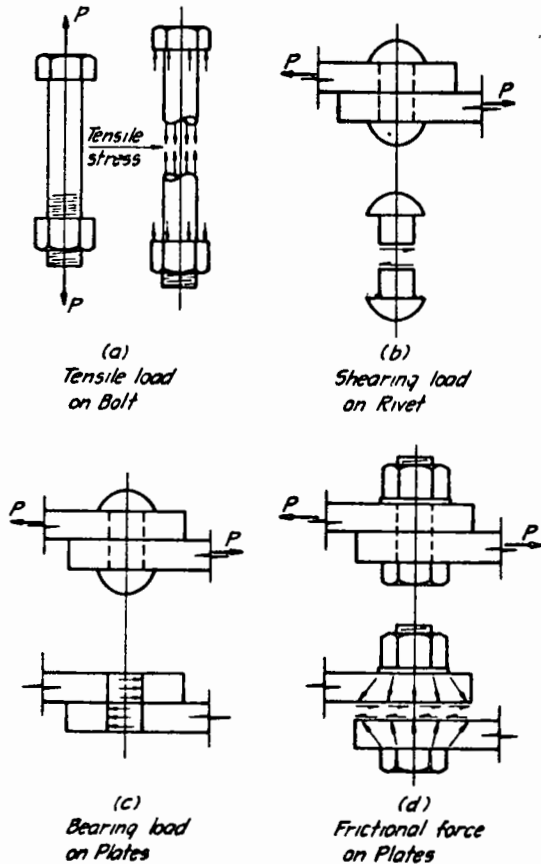


Figure 5-6

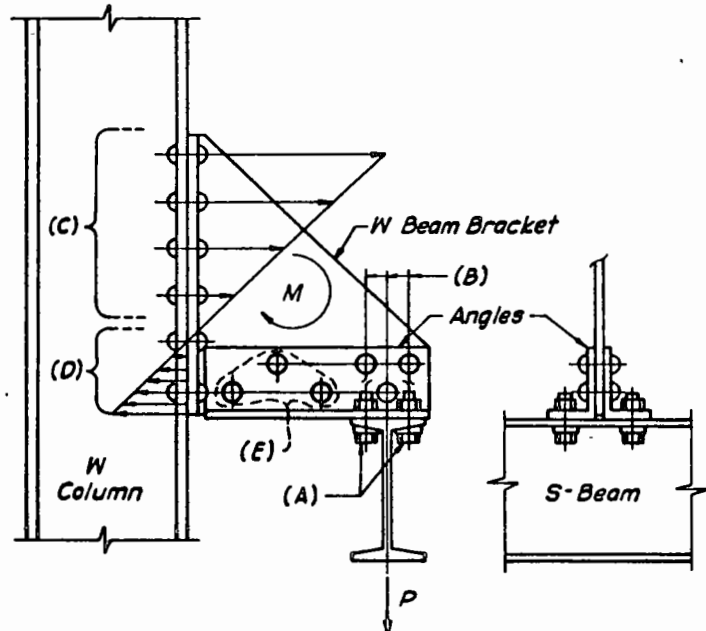


Figure 5-7

- (2) The load from bolts **A** passes through the two bracket angles and is transferred by rivets **B** into the bracket web. These rivets prevent the angles from moving downward and in doing so resist a shearing force between the contact surfaces of the angles and the bracket web. Rivets **B** are loaded in *shear*.
- (3) The rivets attaching the bracket to the flange of the W column are divided into groups **C** and **D**, in accordance with the loads they support. The entire group, **C + D**, is affected by the downward force  $P$ , and each rivet is loaded in shear. But, due to the position and direction of  $P$ , there is set up also a rotating force or moment  $M$ , which tends to rotate the bracket in a clockwise direction, pulling the top away from the column and pushing the bottom toward it. The pull at the top of the bracket is resisted by the rivets in group **C**. Rivets **C** are therefore loaded in tension (in varying degrees) as well as in shear, and are said to be stressed in *combined shear and tension*. Rivets **D**, in the lower part of the connection, where the bracket presses against the column, are stressed in shear alone. The compressive load is transmitted through metal-to-metal bearing between bracket and column flanges, and is not carried by the rivets. The diagonal line represents the assumed distribution of horizontal load intensity from top to bottom of the bracket.
- (4) Rivets **E** clamp the angles to the bracket web and thereby stiffen its bottom edge against buckling. When used in this way they are called *stitch rivets*. Stitch rivets carry no calculable stresses; their primary function is to stitch together component parts to insure action of the member as a unit, rather than an assemblage of loose pieces. Stitch rivets are also employed to seal the edges of contact surfaces against moisture and rusting. The maximum spacing of fasteners for stitching or sealing purposes is limited by specification provisions.

The use of fasteners is not limited to transferring loads to and from structural connections. Fasteners frequently serve to transfer calculated stresses between the main material elements comprising built-up members.

### Stresses

As explained in Chapter 4, the term *stress* refers to the stress (internal force) per unit area. Stresses are expressed numerically in kips per square inch (ksi).

Computed stresses are the stresses created within a member by an applied load. **Allowable stresses** are the *maximum* stresses permitted in a structural member by a design specification.

Allowable tension and shear stresses for rivets and bolts and allowable bearing stresses in material connected by rivets and bolts are listed in the AISC Specification, Sect. 1.5.2. Allowable stresses for fasteners used in bridge work can be found in the AASHTO\* *Standard Specifications for Highway Bridges*, and in the AREA\*\* *Specifications for Steel Railway Bridges*. The following discussion of stresses in fasteners is based on and limited to provisions in the AISC Specification, Manual Part 5. In drafting practice, the design specifications for a project (AISC, AASHTO, AREA, or other applicable specification) must be used.

The nomenclature most commonly used in computations involving fasteners employs the following symbols:

- $A_b$  = Nominal body area of a fastener (cross-sectional area based on nominal diameter), sq in.
- $F_t$  = Allowable tensile stress, ksi.
- $F_v$  = Allowable shear stress, ksi.
- $F_p$  = Allowable bearing stress, ksi.
- $F_y$  = Specified minimum yield stress of the type of steel being used, ksi.
- $f_t$  = Computed tensile stress, ksi.
- $f_v$  = Computed shear stress, ksi.
- $f_p$  = Computed bearing stress, ksi.
- $f_R$  = Computed shear or bearing value of one fastener, kips.
- $r_v$  = Allowable shear or bearing value of one fastener, kips.
- $r_t$  = Allowable tension value of one fastener, kips.

### Shear in Rivets and A307 Bolts

If two plates are connected by a bolt as in Fig. 5-8a, and equal opposing forces  $P$  act on them, the tendency of these plates to slide past one another along their contact or *faying* surfaces is resisted by the bolt. The bolt is stressed on one transverse section, X-X, identified as a *shear plane*. It is said to be loaded in *single shear*.

In Fig. 5-8c, the rivet shown is required to transmit the forces  $P$  from each of the two outside plates into the middle plate. In this case the rivet is stressed in two shear planes, Y-Y and Z-Z. Each transverse section transmits a load  $P$  to equalize the force  $2P$  in the middle plate. This condition of loading is called *double shear*.

Although single and double shear connections account for most riveted and bolted joints, multiple plate arrangements can also involve *triple* and *quadruple* shear. For example, in Figs. 5-8e and 5-8f, the plates are alternately placed to utilize three and four shear planes, with proportionate increases in the total force one bolt is capable of transmitting.

\* American Association of State Highway Officials

\*\* American Railway Engineering Association

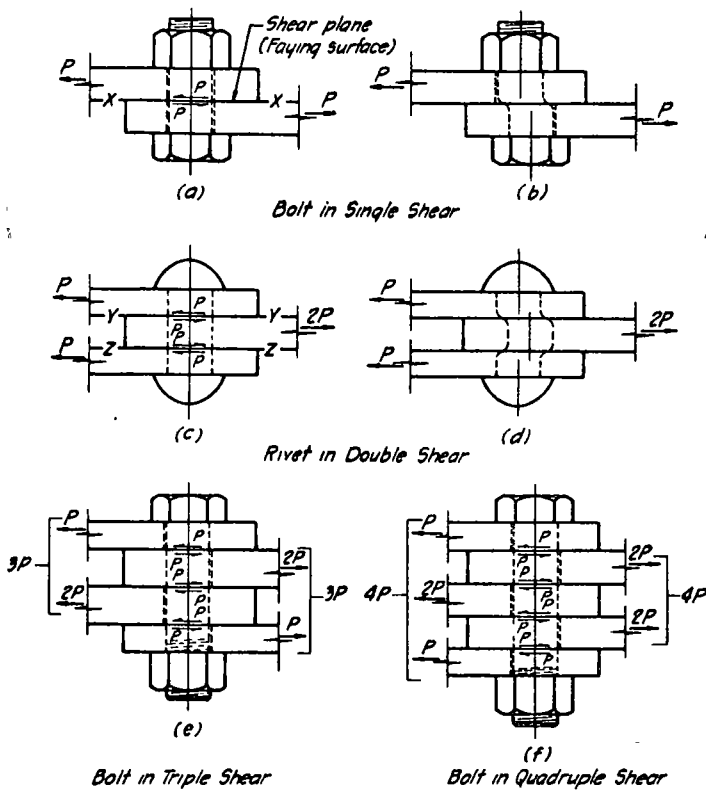


Figure 5-8

Every fastener has a specific load capacity. If this capacity is exceeded by the applied load, the fastener will “fail”. For example, if the fasteners in Figs. 5-8a and 5-8c did not have sufficient strength to resist the loads  $P$ , the plates would shear the fastener shanks as illustrated in Figs. 5-8b and 5-8d.

The ability of a fastener to resist shear at any transverse plane is dependent on its nominal body area  $A_b$  and its allowable stress  $F_v$ . The product of these two quantities,  $F_v \times A_b$ , is the **allowable single shear value** of the fastener. Twice this product,  $2(F_v \times A_b)$  is the **allowable double shear value**; three times the product, the **allowable triple shear value**, etc. For convenience, these allowable shear values are labeled  $r_v$ , which is the total shear load that one fastener can support in a specific condition of shear. These shear relationships can be expressed by the equations:

$$r_v \text{ (single shear)} = (F_v \times A_b)$$

$$r_v \text{ (double shear)} = 2(F_v \times A_b)$$

Numerical values of  $F_v$  are established by specification to provide an adequate margin of safety for each fastener type. The table in Section 1.5.2 of the AISC Specification lists, in part, the following allowable shear stresses for bearing-type connections:

Fastener	$F_v$ , ksi
A502-1 rivets	15.0
A502-2 rivets	20.0
A307 bolts	10.0

Thus, a  $\frac{7}{8}$ -in. A502-1 rivet ( $A_b = 0.6013$  sq in.) has an allowable single shear value of:

$$r_v = 15.0 \times 0.6013 = 9.02 \text{ kips}$$

The same rivet has a double shear value of:

$$r_v = 2(15.0 \times 0.6013) = 18.04 \text{ kips}$$

Triple and quadruple shear values of this rivet would be, respectively, three and four times the single shear value. In a similar manner, the single and double shear values of a  $\frac{7}{8}$ -in. A502-2 rivet ( $F_v = 20.0$  ksi) would be calculated as 12.03 and 24.05 kips, respectively.

In actual structures, more than one fastener is required for a connection. The draftsman must (1) determine how many fasteners of a certain type and size are needed to support an applied load, or (2) analyze a connection to determine the stress in the fasteners under a given load. These computations are illustrated in the following problem:

**Example 1**—Figure 5-9 represents a hanger connection in which a gusset plate transmits a 60.0 kip load from a pair of angles into a structural channel. The gusset is connected to the channel by four 1-in. A502-2 rivets ( $A_b = 0.7854$  sq in.), and  $\frac{7}{8}$ -in. A307 bolts are used to attach the angles to the gusset.

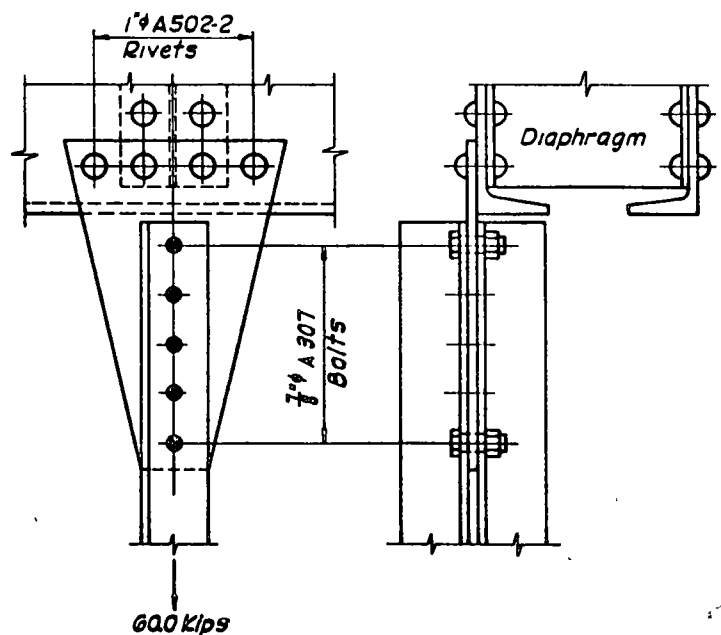


Figure 5-9

**Required:** (a) Calculate the shear  $f_v$  in the 1-in. rivets and compare with the allowable  $F_v$ , and (b) determine the number of  $\frac{7}{8}$ -in. bolts needed to fasten the angles to the gusset.

**Solution:** Examination of the sketch reveals that the 1-in. rivets are in single shear, and that the  $\frac{7}{8}$ -in. bolts are in double shear.

(a) Each 1-in. rivet carries one-quarter of the 60.0 kip load, or  $60.0/4 = 15.0$  kips. This equal distribution of load to a group of fasteners is a basic assumption in the design of connections. It is applicable when the center of the load, producing shear on the fastener group, passes through the center of gravity of the group. Each rivet is stressed in *one* shear area (single shear) of 0.7854 sq in. The computed stress is equal to the load divided by the cross-sectional area of the rivet, thus:

$$f_v = 15.0/0.7854 = 19.1 \text{ ksi}$$

This is less than the allowable stress,  $F_v = 20.0$  ksi; therefore the connection is satisfactory. If  $f_v$  had exceeded  $F_v$ , a larger number of rivets would have been required.

(b) Each  $\frac{7}{8}$ -in. A307 bolt has a double shear value of  $r_v = 2(F_v \times A_b) = 2(10.0 \times 0.6013) = 12.03$  kips. The number of bolts required is the total load divided by the double shear value of one bolt, or

$$60.0/12.03 = 4.99; \text{ use 5 bolts.}$$

The Manual provides tables of single and double shear values for each type and specification of structural fastener in sizes from  $\frac{5}{8}$ -in. to  $1\frac{1}{2}$ -in. These tables (at the beginning of Part 4) eliminate the step of computing  $r_v$  using  $F_v$  and  $A_b$ .

#### Bearing in Riveted or Bolted Shear Connections

Any structural unit is only as strong as its weakest element. Although the fasteners in a structural joint may be strong enough internally to transmit the applied forces, the joint will fail unless the material joined is capable of transmitting these forces into the fasteners. Figure 5-10 illustrates how a thin connecting plate in a bearing-type shear connection might fail by a bearing or a shearing failure before the full shear value of the fastener can become effective.

The ability of a joint to resist this type of failure depends on the bearing value of the connected material. Figure 5-11a illustrates *single shear bearing*. The opposing forces  $P$  act on the body of the fastener through the sides of the hole. The surfaces over which these forces are assumed to bear uniformly are the projected areas,  $bcd$  and  $efgh$ , shown in the cutaway view, Fig. 5-11b. These projected areas are calculated by multiplying the plate thickness by the nominal fastener diameter.

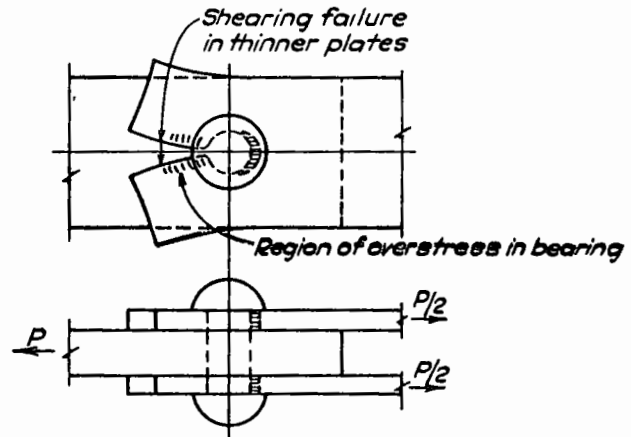


Figure 5-10

Figure 5-11c illustrates *double shear bearing*. Principles and assumptions here are the same as for single shear bearing. Note, however, that the amount of bearing pressure in the outer plates is only half of that in the center plate. Bearing in triple and quadruple shear joints is similar, the total bearing stresses being proportional to the forces applied to the individual plies of material.

The pressure which may be applied safely in bearing is not dependent on the fastener material, but rather on the material being fastened. Allowable bearing values are established by specification and are based on the yield stresses of the types of material connected. The allowable bearing stress\* in the AISC Specification is:

$$F_p = 1.35 F_v$$

where  $F_p$  is the allowable bearing stress and  $F_v$  is the specified minimum yield stress of the connected material. Thus, for connected material of A36 steel ( $F_v = 36.0$  ksi),  $F_p = 1.35 \times 36.0 = 48.6$  ksi.

The allowable bearing value,  $r_v$ , is equal to  $F_p$  multiplied by the projected bearing area of the fastener. Thus, for a 1-in. diameter fastener bearing on a  $\frac{1}{2}$ -in. thick A36 steel plate, the allowable bearing value would be  $r_v = F_p \times \text{fastener diameter} \times \text{plate thickness} = 48.6 \times 1 \times \frac{1}{2} = 24.3$  kips.

If a joint is made up of material having different  $F_v$  values (for example, A36 steel connected to A572 Gr. 50 steel), the bearing value for each type of material must be considered.

In the case of countersunk head rivets and bolts, a special rule governs the calculation of the allowable bearing value. AISC Specification Sect. 1.16.2 stipulates that the bearing area be computed as the product of the nominal fastener diameter and the length in bearing *less one-half of the depth of countersink*. For the standard

\* See Manual Part 5, Commentary on AISC Specification, Sect. 1.5.2.2, for background information on this formula.

countersunk head dimensions given in Manual Part 4, this means that the effective thickness of the ply of material in which the countersink is made is less than the actual thickness by an amount equal to one-quarter of the nominal fastener diameter as shown in Fig. 5-11d. Because this relationship substantially reduces the effective bearing area, countersinking is used only when the outer surface of the connected material must be free of all obstructions.

There are two possible limitations to the capacity of a shear connection using bearing-type fasteners. One is the *shear capacity of the fastener*; the other is the *bearing capacity of the connected material*. Therefore, it is necessary to check riveted and bolted bearing-type connections for both shear and bearing. The lesser value so obtained will determine the joint capacity.

**Example 2**—In Fig. 5-9, assume that the angle and gusset material is  $\frac{3}{8}$ -in. thick, and that the web of the supporting channel is  $\frac{1}{2}$ -in. thick. All material is ASTM A36, for which  $F_p = 48.6$  ksi.

**Required:** Compute the allowable bearing values associated with the fasteners in each group. Compare these with the corresponding allowable shear values of the fasteners.

**Solution:**

(a) The allowable bearing value of the  $\frac{3}{8}$ -in. gusset with respect to the  $\frac{1}{8}$ -in. bolts is

$$r_v = \frac{3}{8} \times \frac{1}{8} \times 48.6 = 15.95 \text{ kips}$$

The allowable double shear value of one  $\frac{1}{8}$ -in. A307 bolt is

$$r_v = 2 (10.0 \times 0.6013) = 12.03 \text{ kips}$$

The shear capacity of the bolts is lower; therefore shear determines the joint capacity.

(b) The allowable bearing value of the  $\frac{3}{8}$ -in. gusset with respect to the 1-in. rivets is

$$r_v = \frac{3}{8} \times 1 \times 48.6 = 18.23 \text{ kips}$$

The allowable single shear value of one 1-in. A502-2 rivet is

$$r_v = 20.0 \times 0.7854 = 15.71 \text{ kips}$$

The shear capacity of the rivets is lower; therefore shear governs.

No computation is made for bearing values of the  $\frac{3}{8}$ -in. angles or the  $\frac{1}{2}$ -in. channel web because in each case the weaker part of the joint was investigated first and proved satisfactory. Bearing on each of the two

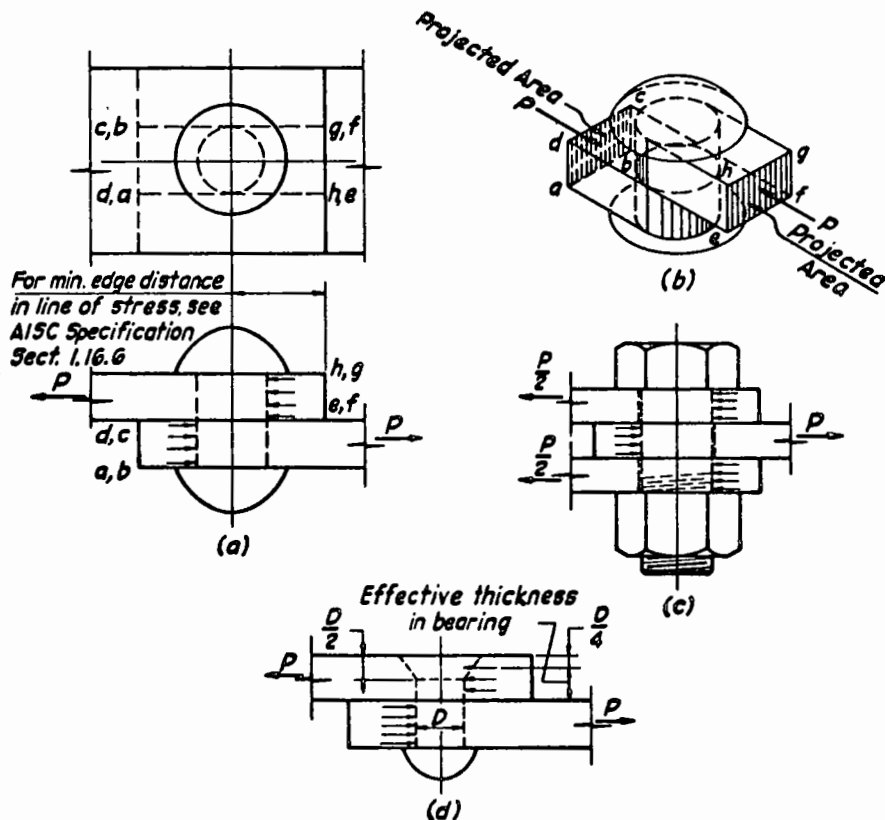


Figure 5-11

$\frac{3}{8}$ -in. angles can be only half of that on the enclosed  $\frac{3}{8}$ -in. gusset; likewise, if bearing on the  $\frac{3}{8}$ -in. gusset with respect to the 1-in. fasteners is safe, the  $\frac{1}{2}$ -in. channel web will provide even greater safety. Much computation to investigate bearing can be eliminated by similar reasoning.

The shear values governed in both parts of this example. Although this is frequently the case when fastener sizes and specifications are suited to material strength and thickness, *bearing capacity must be verified in all connections.*

The AISC Manual can be of considerable assistance in solving problems involving shear and bearing relationships. Tables of fastener bearing values for steels of various  $F_v$  values, tabulated by fastener diameter and thickness of connected material, are given on pages facing the tables of single and double shear values at the beginning of Manual Part 4. Thus shear and bearing values may be compared quickly, and the limiting value can be determined without computation.

**Shear in High-Strength Bolted Connections**

**Friction-Type Connections**—In friction-type joints the shear load is resisted by friction between the contact surfaces of the connected members, rather than by shear and bearing on the bolts. Friction-type joints are employed in connections subject to reversal of stress, severe stress fluctuations, or where joint slippage would be undesirable.

The frictional resistance required to transfer the shear loads depends jointly on the high clamping forces exerted by properly tightened bolts and the condition of the surfaces in contact. Consequently, specifications require that all contact surfaces, including those under bolt heads, nuts or washers, shall be in solid contact, free of loose mill scale, burrs, dirt, and other foreign matter. It is further stipulated that contact surfaces *within* the joint shall be free of any applied coating that would tend to lubricate the surfaces and so reduce their frictional resistance. Coatings not considered to be detrimental to frictional resistance include hot dip galvanizing inorganic zinc rich paints, and metallized zinc or aluminum.\*

Since friction alone transmits the load, no shear stress is set up in the bolt shank nor does the connected material bear against the bolt. As a result, neither the material specification nor the relative thicknesses of joint components affect the joint's resistance to shear and it is unnecessary to investigate the strength of such connections in bearing.

The amount of frictional resistance caused by the clamping action of a high-strength bolt is directly related to the cross-sectional (nominal body) area of the bolt. For this reason, connection capacities can be computed using *equivalent values of  $F_v$* , similar to the method used with rivets and A307 bolts. The AISC Specification lists equivalent allowable "shear" stresses for A325 and A490 bolts for *friction-type* connections shown in the following table:

Fastener	$F_v$ , ksi
A325 Bolts	15.0
A490 Bolts	20.0

As with rivets and A307 bolts,  $r_v$  shear values are determined by the product of  $F_v$  and  $A_b$  for each plane in which shear is to be resisted.

Computations for the design of friction-type high-strength bolted connections follow the procedures previously outlined for rivets and A307 bolts when shear is the sole consideration. Single and double "shear" values of  $r_v$  for high-strength bolts, in sizes ranging from  $\frac{5}{8}$ -in. to  $1\frac{1}{2}$ -in., are listed at the beginning of Manual Part 4.

Referring to the table of  $F_v$  values for "Rivets and Threaded Fasteners," Manual Part 4, note that A502-1 rivets and A325 bolts have the same allowable stresses; accordingly, *these two fasteners can be used interchangeably on a one-for-one basis.* A490 bolts, with a higher "shear" value, are more commonly used for connecting members fabricated of high strength steels. They are also used in other cases where the more compact connections they can provide justifies their greater cost.

**Bearing-Type Connections**—In bearing-type joints, the shear load may be resisted by the body of the high-strength bolt, bearing against the sides of the holes in the connected material. The high clamping force of the high-strength bolts contributes to the connection rigidity, but the shear load is not considered to be resisted by the friction between the connected parts.

High-strength bolted bearing-type connections are used when the slip between joint surfaces, necessary to bring the bolts into bearing in holes in the connected material, can be tolerated. Such connections are suitable for static loads; or for repeated loadings which do not involve reversal of stress or severe stress fluctuations.

As with friction-type joints, the faying surfaces must be brought into solid contact, and no loose mill scale, burrs, dirt or other foreign material is permitted. However, the presence of paint, oil, lacquer, or galvanizing is not prohibited.

\* See Council Specification for details of application and surface treatment requirements for these permissible coatings.

Two strength levels are allowed for each type of high-strength bolt in a bearing type connection. The AISC Specification lists  $F_v$  values for A325 and A490 bolts in bearing-type connections as follows:

Fastener	$F_v$ , ksi	
	A Threads in shear plane	B Threads not in shear plane
A325 Bolts	15.0*	22.0
A490 Bolts	22.5	32.0

Note that full advantage of bearing-type connections can only be realized when bolt threads are not allowed to cross a shear plane (column B). If this condition is not complied with, the lower  $F_v$  values (column A) must be used.

Thread lengths on high-strength bolts are made shorter than those on other types of bolts for the express purpose of assuring a minimum encroachment of threads into the grip (total thickness of all plies of connected material). For the convenience of the draftsman, fabricators usually provide lists of bolt lengths for various diameters and grips, which take into account washer and nut thicknesses and allow for an adequate extension or "stick-through" of the bolt end beyond the nut. Lengths so determined are entirely satisfactory for bolts in friction-type connections. However, if it is desired to use the higher  $F_v$  values permissible in bearing-type connections, lengths called for must be such that the amount of thread within the grip is less than the thickness of the thinnest outside joint component.

Figure 5-12a shows a 4-ply joint in which the thread length extends across shear plane X-X and the entire bearing thickness of plate P is on the threads. The fact that plane Y-Y is not crossed by the threads and plates M, N and O are not bearing solely on threads is generally ignored. The entire joint is rated by the lower  $F_v$  values (column A).

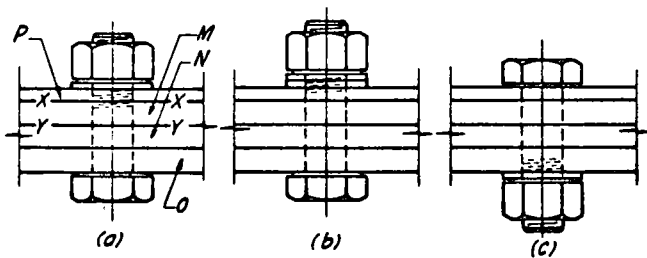


Figure 5-12

\* Note that this value is the same as for high-strength bolts in friction-type connections.

In the above situation it is possible to use the higher  $F_v$  values (column B) provided that (1) a longer bolt is used, furnished with additional hardened washers if necessary to permit full tightening of the nut (Fig. 5-12b), or (2) the bolt is assembled as shown in Fig. 5-12c, with the nut adjacent to the ply of material thick enough to keep the threads from the shear plane. If the latter method is to be used, the draftsman must give instructions to control the field assembly of bolts.

Analysis of the strength of bearing-type high-strength bolted connections follows the same procedures outlined for rivets and A307 bolts, except that the allowable  $F_v$  values are different. Single and double shear values of  $r_v$  for high strength bolts, in sizes ranging from 5/8-in. to 1 1/2-in., are listed at the beginning of Manual Part 4.

Tension in Fasteners

Permissible tension in rivets and high-strength bolts is measured by the product of the nominal body area,  $A_b$ , and the allowable tensile stress  $F_t$ . Thus, the allowable tension value  $r_t$  of a fastener equals the product of  $F_t \times A_b$ . Allowable  $F_t$  values for these fasteners are listed in the AISC Specification as follows:

Fastener	$F_t$ , ksi
A502-1 Rivets	20.0
A502-2 Rivets	27.0
A325 Bolts	40.0
*A490 Bolts	54.0

Body areas, in square inches, are based on the nominal body diameters of high-strength bolts, with no deduction for thread depth, and on the nominal diameters of rivets before driving.

Permissible tension in A307 bolts and threaded parts made of the structural steels covered by AISC Specification, Sect. 1.4.1 is based on the product of the allowable unit tensile stress  $F_t$  and a "tensile stress area" expressed as

$$0.7854 \left[ D - \frac{0.9743}{n} \right]^2$$

where  $D$  = the major thread diameter and  $n$  = the number of threads per inch. The allowable  $F_t$  values for A307 bolts and threaded parts of the steels mentioned above are listed in the AISC Specification as follows:

Fastener	$F_t$ , ksi
A307 Bolts	20.0
Other threaded parts	0.60 $F_v$

\* Static loading only.



Tensile stress areas for bolts or other threaded parts from  $\frac{1}{4}$ -in. diameter to 6-in. diameter are given in "Bolt Data—Threaded Fasteners" in Manual Part 4. Tension values for bolts or other threaded parts, in the most commonly used diameters, based on either nominal body areas or tensile stress areas, may be read directly from tables provided at the beginning of Manual Part 4.

Flattened or countersunk head fasteners should be avoided in joints where they will be stressed in tension. Allowable  $F_t$  values are based on "full head" rivets, and on bolts with hexagon heads and nuts.

**Example 3**—To illustrate the basic tension relationships in fasteners, assume that the four fasteners shown in Fig. 5-13 are  $\frac{7}{8}$ -in. diameter ( $A_b = 0.6013$  sq in.) A502-1 rivets. Based on AISC values of  $F_t$ , find the maximum permissible value of  $P$ .

**Solution:** The allowable value of one fastener is

$$\begin{aligned} r_t &= 20.0 \times 0.6013 \\ &= 12.03 \text{ kips} \end{aligned}$$

Since the four bolts are symmetrically placed with respect to the load, each is assumed to take an equal share. Therefore each fastener supports  $\frac{1}{4}$  of the load

$$\begin{aligned} *P &= 4r_t = 4 \times 12.03 \\ &= 48.12 \text{ kips} \end{aligned}$$

These computations illustrate principles only. In practice it is recommended that allowable fastener tension values be read direct from tables in Manual Part 4.

**Example 4**—In Fig. 5-13, assume that force  $P$  equals 120 kips. What is the computed tensile stress  $f_t$  if the fasteners are four 1-in. diameter A325 bolts, each with an area of 0.7854 sq in.? Would such a connection be safe?

**Solution:** Assume each fastener supports  $\frac{1}{4}$  of the load.

$$\begin{aligned} P/4 &= 120.0/4 \\ &= 30.0 \text{ kips} \end{aligned}$$

The stress per unit area is

$$\begin{aligned} f_t &= 30.0/0.7854 \\ &= 38.2 \text{ ksi} < F_t \text{ of } 40.0 \text{ ksi} \end{aligned}$$

This shows that the computed tensile stress is less than the allowable tensile stress. Assuming no other loading, the connection is safe.\*

\* The effect of prying force is disregarded in these examples, but should be investigated in critical applications. See section on "Prying Forces", later in this chapter.

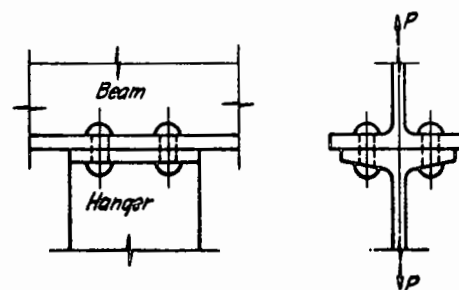


Figure 5-13

### Fasteners in Combined Shear and Tension

Fasteners subjected simultaneously to shear and tension loads require special analysis to assure conformance to specification provisions for  $F_v$  and  $F_t$ . Joints with this type of combined loading occur frequently in end connections of diagonal bracing members (Fig. 5-14).

AISC Specification Sect. 1.6.3, provides interaction formulas establishing allowable stresses for fasteners subject to combined shear and tension. Therefore, in analyzing this type of connection, stresses for shear and tension are considered separately and are not combined as a single resultant force on the fastener. Since the formulas are based on stresses, and not on loads per fastener, it is necessary to express shear, tension, moments of inertia, and section moduli in terms of areas rather than in terms of points (see Appendix page 5-A4, "Fastener Areas Considered").

**Bearing-Type Connections**—Reference to the Specification shows that allowable stresses for fasteners in bearing-type connections are limited by expressions such as:

$$\text{A502-1 Rivets: } F_t = 28.0 - 1.6 f_v \leq 20.0$$

This means that the allowable tensile stress,  $F_t$  is equal to 28.0 ksi less 1.6  $\times$  the computed shear stress  $f_v$  in ksi, but that the result cannot be greater than 20.0 ksi. It should be understood that the computed  $f_v$  shall not exceed the allowable  $F_v$  of AISC Specification Sect. 1.5.2.1.

**Example 5**—In Fig. 5-14a a diagonal force  $P$  of 90 kips is applied to the bracing connection, subjecting the  $\frac{7}{8}$ -in. A502-1 rivets to both shear and tension loads.

**Required:** Compute the shear and tensile stresses in the rivets and compare them with the allowable stresses permitted by the AISC Specification.

**Solution:** The diagonal force  $P$  of 90 kips is shown resolved graphically into two equivalent vectors, a shear force  $P_v$  of 45.0 kips and a tension force  $P_t$  of 77.9 kips. Since the load  $P$  is acting at the center of gravity of the rivet group, components  $P_v$  and  $P_t$  load the rivets uniformly in both shear and tension.

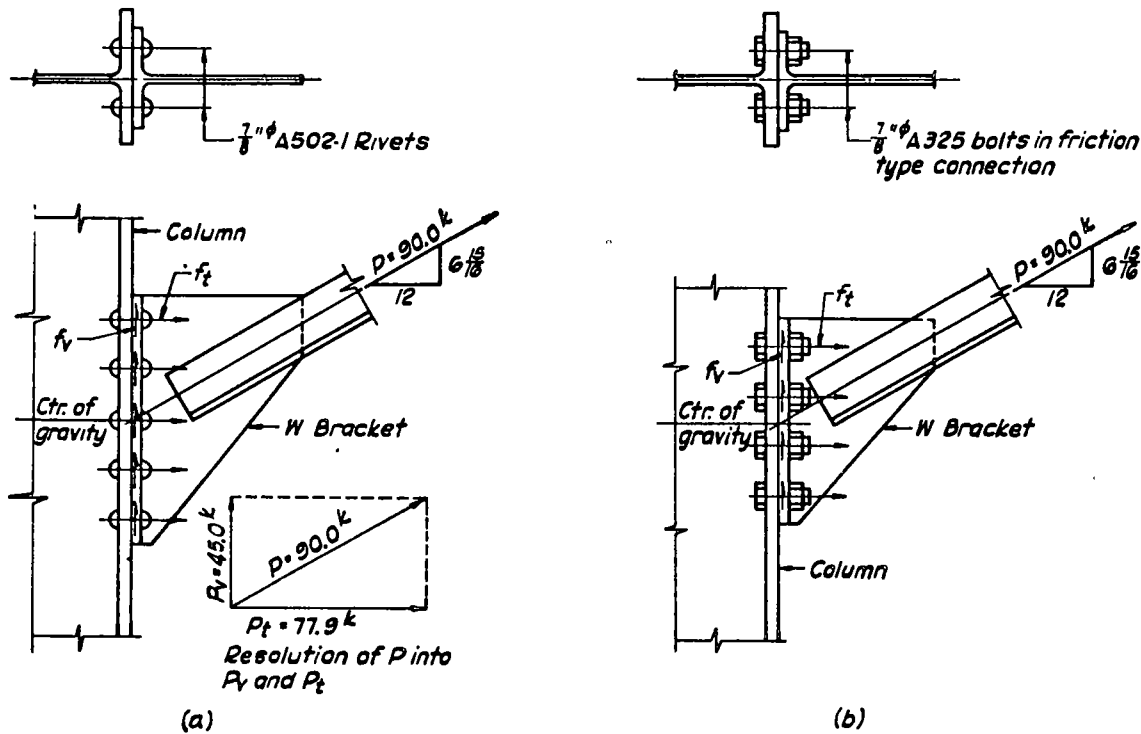


Figure 5-14

The area  $A_b$  of one rivet = 0.6013 sq in. The area of the group,  $\Sigma A_b = 10 \times 0.6013 = 6.01$  sq in. Then,

$$f_v = \frac{P_v}{\Sigma A_b} = \frac{45.0}{6.01} = 7.49 \text{ ksi} < F_v = 15.0 \text{ ksi}$$

$$f_t = \frac{P_t}{\Sigma A_b} = \frac{77.9}{6.01} = 12.96 \text{ ksi.}$$

By the interaction formula for A502-1 rivets,

$$F_t = 28.0 - 1.6 \times 7.49 = 16.02 \text{ ksi}$$

Since this allowable  $F_t = 16.02$  ksi is less than the upper limit of 20.0 ksi and is greater than the computed  $f_t = 12.96$  ksi, the shear-tension relationship is compatible.

**Friction-Type Connections**—Interaction formulas for high-strength bolts in friction-type connections subject to combined loading differ from bearing-type connections in that the allowable “shear” stress  $F_v$  is reduced in value by an amount based on the computed tensile stress and the ratio of the fastener area to the specified pretension load of the bolt.\* The applicable formula for A325 bolts, also given in AISC Specification Sect. 1.6.3, is expressed as

$$F_v \leq 15.0 (1 - f_t \times A_b / T_b)$$

in which  $A_b$  is the area of the bolt body in square inches,  $T_b$  is its specified pretension load, and it is understood that the computed  $f_t$  shall not exceed the allowable  $F_t$  of AISC Specification Sect. 1.5.2.1.

**Example 6**—In Fig. 5-14b, a diagonal force  $P$  of 90 kips is applied to the bracing connection, subjecting the  $\frac{1}{8}$ -in. A325 bolts to both shear and tension loads.

**Required:** Compute the shear and tensile stresses in the bolts and compare them to the allowable stresses permitted by the AISC Specification.

**Solution:** As in Example 5, the diagonal force produces  $P_v = 45.0$  kips and  $P_t = 77.9$  kips.

$$\Sigma A_b = 8 \times 0.6013 = 4.81 \text{ sq in.}$$

$$f_v = \frac{P_v}{\Sigma A_b} = \frac{45.0}{4.81} = 9.36 \text{ ksi}$$

$$f_t = \frac{P_t}{\Sigma A_b} = \frac{77.9}{4.81} = 16.20 \text{ ksi}$$

The specified pretension load of one  $\frac{1}{8}$ -in. A325 bolt is 39\* kips. By the interaction formula,

$$F_v \leq 15.0 \left( 1 - 16.20 \times \frac{0.6013}{39} \right) = 11.25 \text{ ksi}$$

Since  $f_v = 9.36$  ksi is less than the allowable shear stress  $F_v = 11.25$  ksi, and  $f_t = 16.20$  ksi is less than the upper limit  $F_t = 40.0$  ksi, the computed values of both  $f_v$  and  $f_t$  are within Specification limits.

\* Values for specified pretension loads are listed by bolt specification and diameter under the heading “Minimum Fastener Tension” in Table 3 of the Council Specification, Manual Part 5.

**Stresses in Eccentrically Loaded Fastener Groups**

Thus far in this chapter, only concentric loads on fastener groups have been discussed.

Stresses in the fasteners have been treated as if the applied loads passed through the centers of gravity of the fastener groups. Each fastener was assumed to provide the *same amount of resistance* in supporting the load. The load per fastener was calculated by dividing the total load by the total number of fasteners. The direction of the resistance or reaction was assumed to be parallel and opposite to that of the applied load.

However, when a load is applied *eccentrically* to a group of fasteners it does not pass through the center of gravity of the group. Consequently the fasteners must resist a *twisting or rotating effect* in addition to resisting the equal load each must take if loaded concentrically.

**Eccentric Loading Producing Shear Only**—In this case it is convenient and conservative to *imagine the eccentric load replaced by a concentric load of the same amount and direction passed through the group's center of gravity and by a moment which tends to rotate the fasteners about the group's center of gravity.*\*

Figure 5-15a shows three fasteners, A, B and C, with load  $p$  applied at distance  $l$  from the center of gravity (c.g.) of the group.

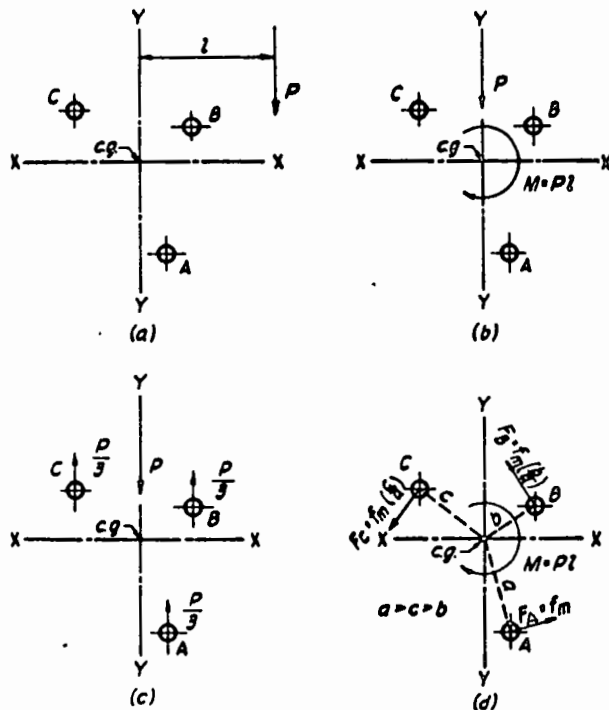


Figure 5-15

\* For a more precise method of analysis, see "Treatment of Eccentrically-loaded Connections in the AISC Manual," AISC Engineering Journal, Vol. 8, No. 2.

Figure 5-15b shows the same group with  $P$  applied at the center of gravity and moment  $M = Pl$ , tending to rotate the group about its center of gravity. The effect on the three fasteners is identical for each case. For determination of center of gravity of unsymmetrical fastener groups, refer to Appendix page 5-A2.

If equilibrium is to be maintained, each of the three fasteners supports one-third of the load ( $P/3$ ) as shown in Fig. 5-15c. Note that each reacting force is directed upward, opposing load  $P$ . Letting  $f_1$  equal the reaction of each fastener due to the imagined concentric load,  $f_1 = P/3$  or, in more general terms,

$$f_1 = P/n \tag{5.1}$$

where  $n$  = the number of fasteners in the group.

Equilibrium of this system also requires that each fastener resist its share of the moment  $M$ . The forces  $F_A$ ,  $F_B$  and  $F_C$  shown in Fig. 5-15d oppose this moment by reacting along lines at right angles to their respective lever arms,  $a$ ,  $b$  and  $c$ . The moment tending to rotate the group about its center of gravity is balanced by the individual reactions applied in an opposite direction about the same center of gravity.

In studying the stresses in a beam resisting bending moment (Chapter 4) it was noted that the bending stress at the neutral axis of the beam is zero, and that the intensity of stress varies directly as the distance from the neutral axis, reaching a maximum value at the outermost fibers.

A similar condition is assumed to exist in the case of eccentrically loaded fastener groups. If there had been a fastener at point c.g. in Fig. 5-15d, it would have contributed nothing in resisting the moment force. On the other hand, fastener A, which is farthest from point c.g., will provide the greatest reaction to rotative force. If  $f_m$  is this maximum reaction due to moment,

$$F_A = f_m$$

The reactions  $F_B$  and  $F_C$  will be less than  $F_A$  and the forces they contribute will be directly proportional to the ratios  $b/a$  and  $c/a$ , respectively:

$$F_B = F_A(b/a) = f_m(b/a)$$

$$F_C = F_A(c/a) = f_m(c/a)$$

The resisting moments (force times lever arm) of these three fasteners with respect to the center of gravity are:

$$M_A = f_m a$$

$$M_B = f_m(b/a) \times b = f_m b^2/a$$

$$M_C = f_m(c/a) \times c = f_m c^2/a$$

If the term  $M_r$  is assigned to the total resisting moment of the fastener group, then,

$$M_r = f_m a + \frac{f_m b^2}{a} + \frac{f_m c^2}{a}$$

Multiplying  $f_m a$  by  $a/a (=1)$ , to provide the same denominator for all expressions for moment, the equation becomes:

$$M_r = \frac{f_m a^2}{a} + \frac{f_m b^2}{a} + \frac{f_m c^2}{a}$$

or

$$M_r = f_m \left( \frac{a^2 + b^2 + c^2}{a} \right)$$

In the above expression the sum of the squares of the distances from the center of gravity to each fastener in the group ( $a^2 + b^2 + c^2$ ) is defined as the **polar moment of inertia,  $I_p$** . The polar moment of inertia is the same in any direction about the pole (located at the center of gravity). Then,

$$M_r = f_m I_p / a$$

To achieve equilibrium, the internal or resisting moment  $M_r$  must equal the external moment  $Pl = M$ . Then,

$$M_r = M = f_m I_p / a$$

and the moment force on the most highly stressed fastener is,

$$f_m = Ma / I_p \tag{5.2}$$

Combining  $f_1$  from Equation (5.1) and  $f_m$  from Equation (5.2) graphically, by scaling force magnitudes and completing the force parallelogram as shown in Fig. 5-16, produces the total resultant force  $f_R$  which fastener A must provide. Similar analyses for fasteners B and C would show that, for the forces represented vectorially in Fig. 5-16, fastener A has the highest combined stress in the group.

However, it should be noted that, depending on the direction of moment and the relative magnitudes of  $f_1$  and  $f_m$ , the fastener most remote from the c.g. of an unsymmetrical group may not be the most highly stressed. A positive check can be made by the construction shown in Fig. 5-16. Point *i* is established on the X-X axis (i.e., the centroidal axis perpendicular to the load) by drawing a line through fastener A perpendicular to vector  $f_R$ . The fastener most remote from point *i* is the most highly stressed. For example, if distance  $Bi$  should prove to be greater than  $Ai$ , Equation (5.2) would become

$$f_m = Mb / I_p$$

and the force parallelogram at fastener B would give the maximum  $f_R$ .

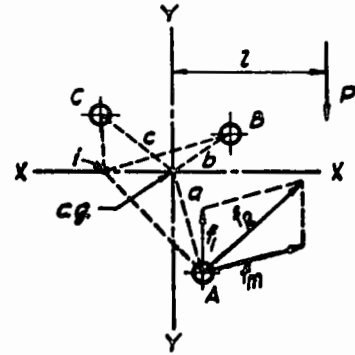


Figure 5-16

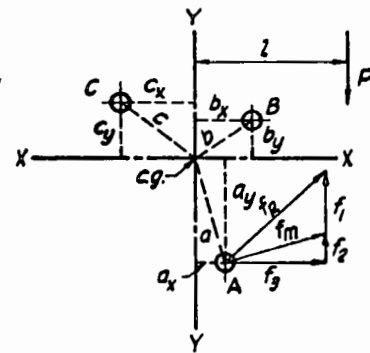


Figure 5-17

Although determining  $f_R$  by the above method is direct, it is usually quicker to calculate  $I_p$  by tabulating values of  $I_x$  and  $I_y$  as outlined in the Appendix to this Chapter. Furthermore, it is convenient to resolve  $f_m$  into its vertical and horizontal components for mathematical combination with  $f_1$  to obtain  $f_R$ .

To illustrate this, Fig. 5-17 shows fasteners A, B and C located by their coordinates:  $a_x, a_y; b_x, b_y; c_x, c_y$ . Using these distances to compute  $I_x$  and  $I_y$ ,

$$\Sigma I_x = (a_y)^2 + (b_y)^2 + (c_y)^2$$

and

$$\Sigma I_y = (a_x)^2 + (b_x)^2 + (c_x)^2$$

$I_p$  can then be calculated by adding  $\Sigma I_x$  and  $\Sigma I_y$ .

Figure 5-17 also shows force  $f_m$  resolved into a vertical component  $f_2$  and a horizontal component  $f_3$ . These forces are calculated by using Equation (5.2) thus:

$$f_2 = Ma_x / I_p \text{ and } f_3 = Ma_y / I_p$$

The right triangle with a base of  $f_3$  and a rise of  $(f_1 + f_2)$  can then be solved to get the single resultant,

$$f_R = \sqrt{(f_3)^2 + (f_1 + f_2)^2}$$

This resultant  $f_R$  is the same as shown developed graphically in Fig. 5-16, and is used to compare with the allowable fastener shear or bearing value  $r_p$ .

The random fastener group employed in the foregoing discussion will seldom be encountered in actual practice. It is a general case chosen to clarify the principles involved in eccentrically loaded connections. Although the usual fastener pattern is symmetrical about one or both axes, the calculations required are nevertheless time consuming.

To expedite the analysis of eccentric loads on fastener groups, numerical coefficients have been developed which reduce the amount of computation required. Tables X through XIII, Manual Part 4, list coefficients for several commonly used fastener groups.

It has been observed that where a force produces a moment in the plane of the fastener group, computations using the actual moment arm ( $l_{act}$ ) result in higher moment stresses than those obtained in laboratory tests. To compensate for this, empirical formulas have been developed which reduce the actual moment arm to an effective moment arm ( $l_{eff}$ ) and thereby reduce the moment. The amount of this reduction depends on (1) the fastener arrangement, i.e., in a single row or in two or more rows, and (2) the number of fasteners in any one row. Where a single row of fasteners occurs,

$$l_{eff} = l_{act} - \left( \frac{1 + 2n}{4} \right)$$

and where two or more rows occur,

$$l_{eff} = l_{act} - \left( \frac{1 + n}{2} \right)$$

where  $n$  = the number of fasteners in one row.

Use of these formulas requires symmetry in the fastener groups. The formulas are not applicable where moments induce combinations of shear and tension or tension only in fasteners.

In the examples that follow, calculations for stresses in bracket plates and tee stubs are not included, since such stresses were discussed in Chapter 4.

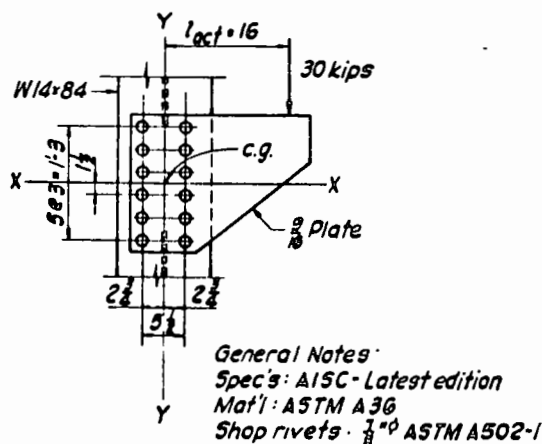


Figure 5-18

**Example 7**—Using the data of Fig. 5-18, convert moment arm  $l_{act}$  to  $l_{eff}$  and find the maximum resultant force  $f_R$  at the extreme rivet. Compare this value with the allowable  $r_v$  value. Solve by (a) using polar moment of inertia and (b) using coefficients from the Manual.

**Solution:**

(a)  $l_{act} = 16$  in.

$$l_{eff} = l_{act} - \left( \frac{1 + n}{2} \right) = 16 - \left( \frac{1 + 6}{2} \right) = 12.5$$
 in.

$$I_x = 4 \times (1.5^2 + 4.5^2 + 7.5^2) = 315.00$$
 in.<sup>2</sup>

$$I_y = 12 \times (2.75^2) = 90.75$$
 in.<sup>2</sup>

$$I_p = 405.75$$
 in.<sup>2</sup>

$$f_1 = 30/12 = 2.50$$
 kips

$$f_2 = \frac{Pl_{eff}(a_x)}{I_p} = \frac{30 \times 12.5 \times 2.75}{405.75} = 2.54$$
 kips

$$f_3 = \frac{Pl_{eff}(a_y)}{I_p} = \frac{30 \times 12.5 \times 7.5}{405.75} = 6.93$$
 kips

$$f_R = \sqrt{6.93^2 + (2.50 + 2.54)^2} = 8.57$$
 kips

Allowable  $r_v$  for 7/8-in. A502-1 rivet = 9.02 kips > 8.57 kips. Therefore the connection is adequate.

(b) Coefficients for solving the given pattern of fasteners (two-row with 5 1/2-in. gage) are found in Table XII of Manual Part 4.

In the column headed  $n = 6$ , read directly,

$$C = 3.62 \text{ for } l_{eff} = 12$$

and

$$C = 3.18 \text{ for } l_{eff} = 14$$

The difference in  $l_{eff}$  is 2 and in  $C$  is 0.44. Interpolating for  $l_{eff} = 12.5$ ,  $C = 0.5/2 \times 0.44 = 0.11$  less than  $C$  for  $l_{eff} = 12$ . Then  $C = 3.62 - 0.11 = 3.51$ .

Rearranging equation  $C = P/r_v$  and substituting  $f_R$  for  $r_v$ ,

$$f_R = P/C = 30/3.51 = 8.55$$
 kips

which compares closely with the 8.57 kip value resulting from solution (a).

In this example there are four outermost fasteners, each the same distance from the center of gravity (Fig. 5-18). Although it might seem that each of these fasteners will be equally stressed, such is not the case.

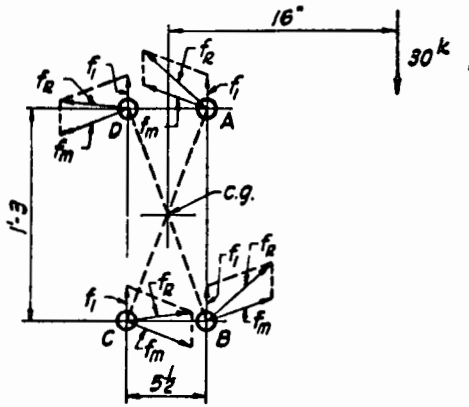
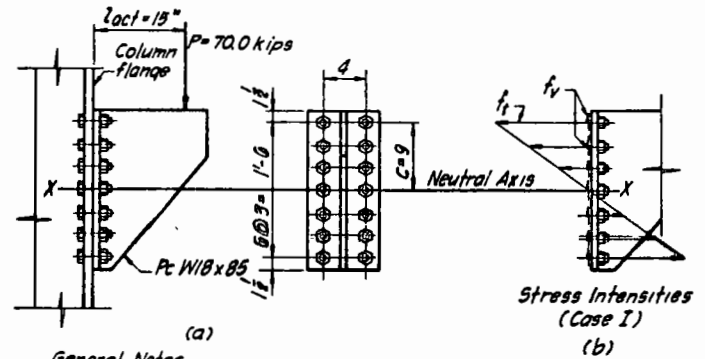


Figure 5-19

Figure 5-19 shows these fasteners, labeled A, B, C and D, with completed force parallelograms. It should be noted that although  $f_m$  and  $f_t$  are of the same magnitude at each point, their relative lines of action result in greater values of  $f_R$  for fasteners A and B than for C and D. Although this difference in magnitude does not affect the result in solving for maximum  $f_R$  in a symmetrical situation, it must be considered if the pattern is unsymmetrical (see Appendix, Fig. 5A-5).

**Eccentric Loading Producing Combined Shear and Tension**—Figure 5-20a shows the details of a W bracket connected to a column flange. The eccentric load  $P$  produces a moment normal (at right angles) to the plane of the fastener group. For this type of loading no reduction is permitted in the moment arm and the full  $l_{act}$  is used. In the analysis, it is assumed that vertical shear is distributed equally among all the fasteners, and the moment is resisted by the upper fasteners loaded in tension and the lower part of the bracket pressed horizontally against the column shaft. Since both shear and tension are present, maximum allowable fastener stresses are governed by the interaction formulas of AISC Specification Sect. 1.6.3. Their application here follows the same rules stated previously under the heading “Fasteners in Combined Shear and Tension”. The vertical shear per fastener is determined by dividing the load by the fastener area times the number of fasteners in the group. Cases I and II describe the two principal methods generally accepted for computing the *moment capacity and maximum tensile stress on the outermost fasteners in a fastener group that is loaded in this manner.*\*

\* The effect of prying force is disregarded in these cases, but should be investigated in critical applications. See section on “Prying Forces” later in this chapter.



General Notes  
 Specs: AISC - Latest edition  
 Mat'l: ASTM A30  
 Shop bolts: 7/8" ASTM A925  
 in bearing type connection  
 threads excluded from  
 shear planes.

Figure 5-20

**Case I**—In this method, it is assumed that the neutral axis is at the center of gravity of the group, and that fasteners above the axis are loaded in tension and those below are “loaded in compression” as shown in Fig. 5-20b. It is further assumed that all fasteners contribute to the section modulus of the group. The tension stress  $f_t$  in any fastener varies directly as its vertical distance from the center of gravity of the group. The shear stress  $f_v$  is assumed to be equal in each fastener. The maximum fastener stress occurs at the top row where the tensile stress is maximum.

*Analysis by Case I is recommended for high-strength bolts in friction- or bearing-type connections. Its use with rivets or A307 bolts will give conservative results.*

It should be noted that *high-strength bolts in friction-type connections, loaded as described for Case I, are exempt from the provisions of the interaction formulas.* Although the friction which produces bolt “shear” is relaxed at the top of the bracket, it is increased at the bottom, and the total shear resistance of the connection remains virtually unchanged. For this case, the reduction in allowable shear stress  $F_v$  required by AISC Specification Sect. 1.6.3 is not applicable. Therefore, the allowable shear and tension, considered separately, must conform to the provisions of AISC Specification Sect. 1.5.2.1.

**Case II**—In this method, it is assumed that the neutral axis is located somewhat below the center of gravity of the fastener group. The tension stress  $f_t$  in the upper fasteners varies in direct proportion to their distances from the neutral axis; fasteners below this axis are assumed to resist shear only. The compression force due to the moment is transmitted horizontally through a small area of the bracket flange between the neutral axis and the bottom of the bracket as shown in Fig. 5-22d.

Calculations for the section modulus are based on the fastener areas above the neutral axis and the small compression area below. The shear stress  $f_v$  is assumed to be equal in each fastener. The maximum fastener stress occurs at the top row where the tensile stress is maximum. The bearing stress at the bottom of the bracket must be checked.

Since the strong clamping effect of high-strength bolts largely precludes the assumed downward shifting of the neutral axis, use of Case II should be restricted to brackets fastened by rivets or A307 bolts.

Application of Case II is more time-consuming than Case I, but results in greater load capacities for the bracket fasteners. The extra work involved may be warranted where critical loadings or many identical connections are present.

**Location of Neutral Axis for Case II**—In applying the method of Case II to bracket fasteners, it is first necessary to locate the neutral axis. The theoretical approach to this is extremely complicated, and it is customary to assume a trial position at  $\frac{1}{6}$  of the total bracket depth, measured upward from the bottom. In Fig. 5-21 this axis is indicated by line X-X.

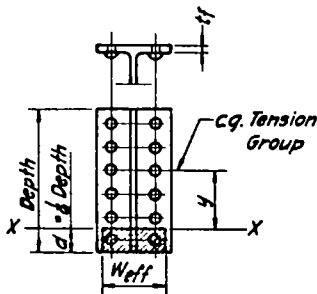


Figure 5-21

Although some authorities sanction use of the entire flange width for bearing in the compression area, it is recommended that an effective flange width  $W_{eff}$  limited to  $8t_f$  be used. In this expression,  $t_f$  is the thickness of bracket flange and 8 is an arbitrary constant intended to guard against the use of flanges too thin to evenly distribute the bearing forces. The formula  $W_{eff} \leq 8t_f$  is applicable to bracket flanges made from W or S-beams, welded plates, or angles. Where sloping flanges are encountered, the value of  $t_f$  should be the average flange thickness.

Having assumed the location of the neutral axis and established a width of compression zone, it is possible to check the equilibrium of fastener tension areas against the compression area by equating moments of areas about the trial axis. In Fig. 5-21 this equation would be

$$W_{eff} \times d \times d/2 \approx \Sigma A_b \times y$$

in which  $\Sigma A_b$  is the area of all fasteners above line X-X and  $y$  is the distance from line X-X to the center of gravity of this same group. Marked differences between tension and compression area moments may be corrected by adjustment of  $d$  until a reasonable equality is achieved.

**Example 8 (Case I)**—Using the data of Fig. 5-20a, compute the maximum shear and tension stresses in the fasteners and compare these values with allowable values. Check the results by the applicable interaction formula of AISC Specification Sect. 1.6.3. (Such a check must always be made when A325 bolts are used in a bearing-type connection.) Assume that the moment tends to stress the fastener group about a neutral axis through its center of gravity, as shown in Fig. 5-20b.

**Solution:**

(a) Shear stress:

$$f_v = P/A_b N,$$

where

$P$  = Load in kips

$A_b$  = Area of one fastener, sq in.

$N$  = Total number of fasteners

$$\text{Substituting, } f_v = \frac{70.0}{0.6013 \times 14} = 8.32 \text{ ksi}$$

Allowable  $F_v = 22.0 \text{ ksi} > 8.32 \text{ ksi}$ ; shear stress is satisfactory.

(b) Moment of inertia and section modulus:

$$I_x = A_b \Sigma (d_v)^2 \text{ (see Appendix, page 5-A4)}$$

$$4 \times 3^2 = 36$$

$$4 \times 6^2 = 144$$

$$4 \times 9^2 = 324$$

$$\Sigma (d_v)^2 = 504 \text{ in.}^2$$

$$I_x = 0.6013 \times 504 = 303.1 \text{ in.}^4$$

$$S = I_x/c = 303.1/9 = 33.68 \text{ in.}^3$$

(c) Tension stress:

$$f_t = M/S = \frac{70.0 \times 15}{33.68} = 31.18 \text{ ksi}$$

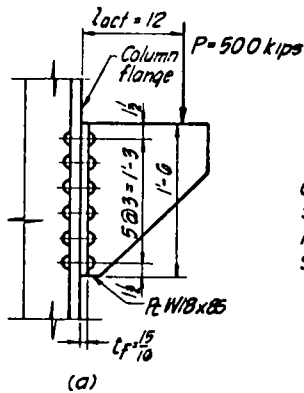
(d) Allowable  $F_t$  by interaction formula:

$$F_t = 50.0 - 1.6 f_v \leq 40.0$$

$$= 50.0 - 1.6 \times 8.32 = 36.69 < 40.0 \text{ ksi}$$

Since  $f_t = 31.18 \text{ ksi} < F_t = 36.69 \text{ ksi}$ , the shear-tension relationship is compatible.

Had the connection in this example been of the *friction-type*, the allowable values  $F_v = 15.0$  ksi and  $F_t = 40.0$  ksi, AISC Specification Sect. 1.5.2.1, are also well above the  $f_v = 8.32$  and  $f_t = 31.18$  computed in steps (a) and (c). As noted in the discussion of Case I, the interaction formula would not have applied.



General Notes  
 Spec's : AISC - Latest edition  
 Mat'l : ASTM A36  
 Shop rivets : 7/8" φ ASTM A502-1

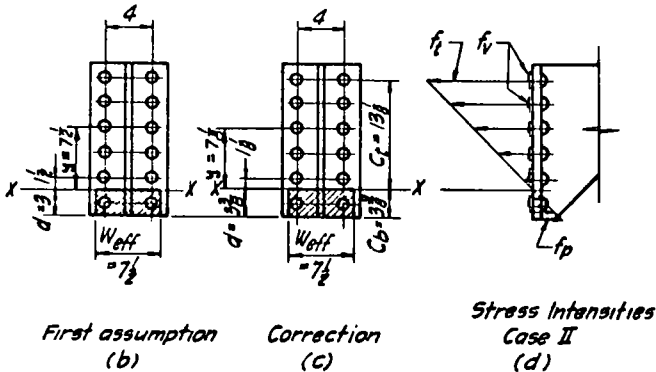


Figure 5-22

**Example 9 (Case II)**—Using the data of Fig. 5-22a, compute the maximum shear and tension stresses in the fasteners and compare these with the allowable values. Check results by the applicable interaction formula of AISC Specification Sect. 1.6.3. Assume a trial neutral axis at  $1/6$  of the bracket depth, measured upward from the bottom. Compute bearing stress at the bottom of the bracket and compare with the allowable bearing stress  $F_p$  from AISC Specification Sect. 1.5.1.5.1.

**Solution:**

(a) Shear stress:

$$f_v = \frac{P}{A_b N}$$

$$= \frac{50.0}{0.6013 \times 12} = 6.93 < F_v = 15.0 \text{ ksi o.k.}$$

(b) Locate neutral axis:

First assumption (see Fig. 5-22b):

$$d = \frac{1}{6} \times 18 = 3 \text{ in.}$$

$$W_{eff} = 8 \times \frac{15}{16} = 7.5 \text{ in.}$$

$$W_{eff} \times d \times \frac{d}{2} \approx \Sigma A_b \times y$$

$$7.5 \times 3 \times 1.5 = 10 \times 0.6013 \times 7.5$$

$$33.75 \neq 45.10$$

This result indicates need of an upward adjustment of the neutral axis.

Correction (see Fig. 5-22c):

Assume the neutral axis to be located at a distance 3.375 in. from the bottom edge of the bracket.

$$7.5 \times 3.375 \times 1.688 = 10 \times 0.6013 \times 7.125$$

$$42.73 \approx 42.84$$

This result is reasonably close to balancing and the location of the neutral axis at 3.375 in. from the bottom will be used for subsequent computations.

(c) Moment of inertia and section modulus:

$I_x$  (Fastener area) =  $A_b \Sigma (d_v)^2$  (see Appendix to this Chapter)

$$2 \times 1.125^2 = 2.5$$

$$2 \times 4.125^2 = 34.0$$

$$2 \times 7.125^2 = 101.5$$

$$2 \times 10.125^2 = 205.0$$

$$2 \times 13.125^2 = 344.5$$

$$\Sigma (d_v)^2 = 687.5 \text{ in.}^2$$

and

$$A_b \Sigma (d_v)^2 = 0.6013 \times 687.5$$

$$= 413.4 \text{ in.}^4$$

$$I_x \text{ (compression area)*} = \frac{W_{eff} \times d^3}{3}$$

$$= \frac{7.5 \times (3.375)^3}{3} = 96.1 \text{ in.}^4$$

$$I_x \text{ (total)} = 413.4 + 96.1 = 509.5 \text{ in.}^4$$

$$S_{top} = I_x / c_t = 509.5 / 13.125 = 38.8 \text{ in.}^3$$

$$S_{bottom} = I_x / c_b = 509.5 / 3.375 = 151.0 \text{ in.}^3$$

\* See "Properties of Geometric Sections", Manual Part 6.



(d) Tension stress:

$$f_t = M/S_{top} = \frac{50.0 \times 12}{38.8} = 15.46 \text{ ksi}$$

(e) Allowable  $F_t$  by interaction formula:

$$\begin{aligned} F_t &= 28.0 - 1.6 f_s \leq 20.0 \\ &= 28.0 - 1.6 \times 6.93 \\ &= 16.91 < 20.0 \text{ ksi} \end{aligned}$$

Since  $f_t = 15.46 < F_t = 16.91$  ksi, the shear-tension relationship is compatible.

(f) Check bearing stress at bottom of bracket:

$$\begin{aligned} f_p &= M/S_{bott} = \frac{50.0 \times 12}{151.0} \\ &= 3.97 \text{ ksi} < F_p = 0.90 F_y = 33.0 \text{ ksi o.k.} \end{aligned}$$

**Secondary Loading of Fasteners in Tension**—In Examples 3, 4, 5, 6, 8, and 9, which illustrated fasteners loaded in tension alone or in tension and shear combined, calculations were limited to proportioning fasteners to resist primary loading only. Actually, tensile stress can be created by conditions other than externally applied loads. These secondary conditions include (1) prestress in the fastener and (2) prying force on the fastener.

**Prestress** in a fastener may be caused by shrinkage, as when hot driven rivets cool, or by tightening of high strength bolts to specification requirements. Since tensile stresses exist in such prestressed fasteners before any external tensile load is applied, it might seem that the stresses resulting from the external (primary) load should be added to the initial prestress. However, it can be demonstrated that external tensile forces applied to such fasteners do not substantially affect the stresses in the fasteners until the externally applied load exceeds the initial prestress.

Figure 5-23a shows two plates gripped by a prestressed bolt, with applied loads  $P = (P/2 + P/2)$  tending to pull them apart. Figure 5-23b represents one-half of this joint, with loads  $P/2$  removed to show equilibrium of the internal forces due to prestress alone.  $T$  is the pre-load force, applied by the bolt head (or nut) at the upper surface of the plate.  $C$  (equal to  $T$ ) represents the compressive force at the faying surface in plane X-X.

If an external load  $P$ , with  $P < T$ , were applied at plane X-X as shown in Fig. 5-23c, the compressive force  $C$  would be reduced by the value of  $P$ , and  $C = T - P$ . If  $P$  were then increased to equal  $T$ ,  $C = 0$ , and the plates would be on the verge of separating. Since any further increase in  $P$  would cause the plates to separate and the bolts to stretch, the bolt load  $T$  would thereafter be equal to  $P$ .

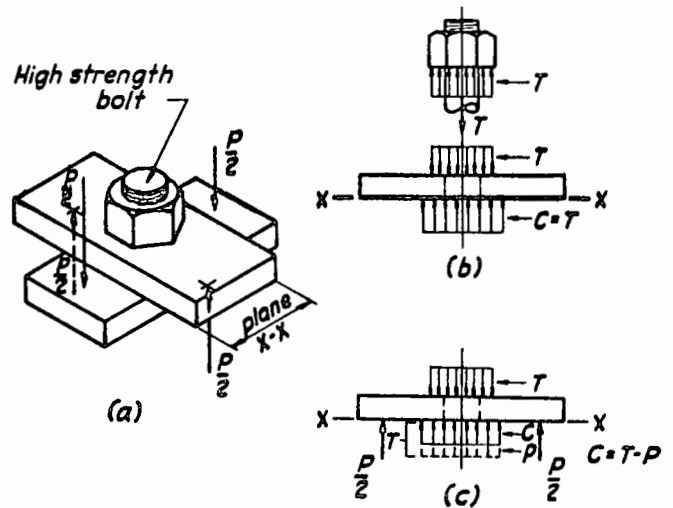


Figure 5-23

Stated briefly, this means that the bolt tension would not increase unless the plates separate, and this would not happen until the external load exceeded the compressive force between the plates. The action described would also apply to hot-driven rivets, since such rivets are prestressed due to shrinking.

In practice, load  $P$  is usually applied at some intermediate point between plane X-X and the top surface of the plate. Consequently, the relative resistance to distortion between plate and bolt becomes a factor which is dependent upon the relative stiffness of the fastener and the connected parts. For a normally proportioned joint, where  $P < T$ , any increase in bolt tension can be ignored since the compressed area, and hence the stiffness of the connected parts, far exceeds the area of the fastener.

Fasteners in a connection subject to many applications of a fluctuating tensile load may fail if the tension so induced approaches the magnitude of the pretension in the fasteners resulting from shrinkage (in the case of rivets) or nut tightening (in the case of bolts). Since the amount of pretension in a rivet due to shrinkage is uncertain, high-strength bolts should be used in connections subject to many repetitions of large tensile forces.

**Prying forces** on fasteners in tension may be even more critical than prestress forces, because the stresses created are *added* to those resulting from externally applied loads. The prying effect can occur in any connection in which fasteners in tension transmit loads through tee flanges, the legs of angles, or by any other means whereby external edges of joint components act as fulcrums to pry on fasteners. The only generalization that can be made is that prying effects are the least (or even nonexistent) for members with thick flanges and minimum gages for the attaching fasteners.

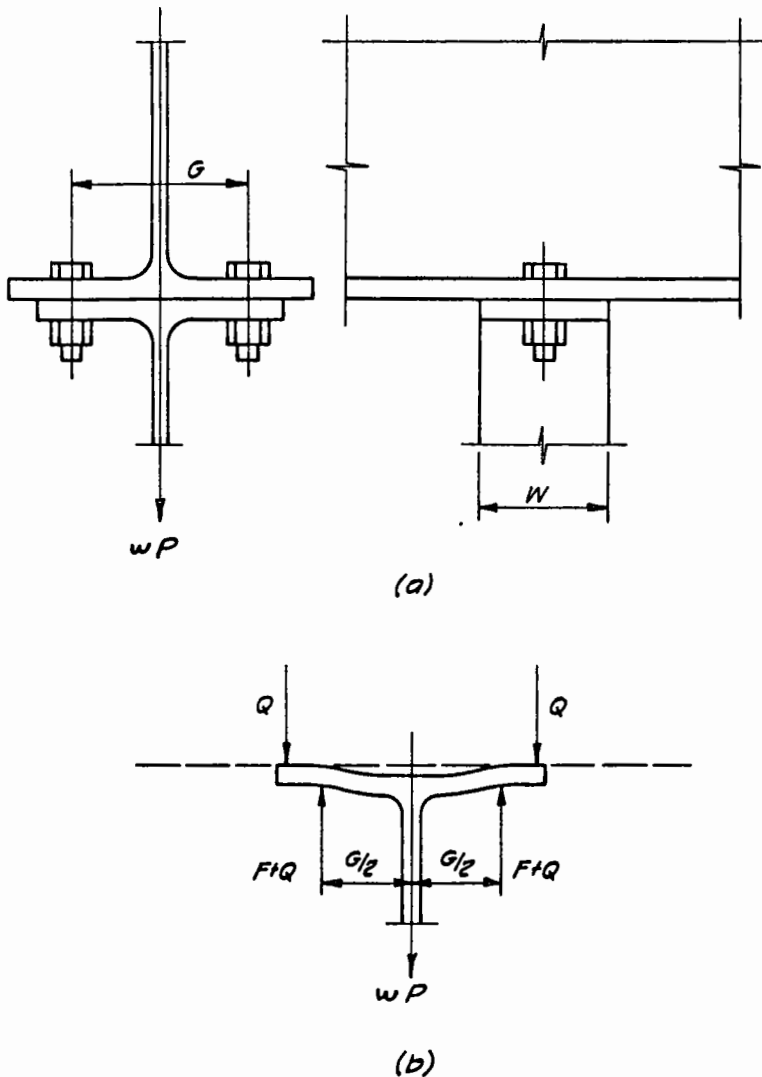


Figure 5-24

Figure 5-24a represents a hanger connection of length  $w$ , which is fastened to the bottom flange of a supporting beam. Letting  $P$  denote the load per inch of connection length, it will be seen that the fitting resists a downward force,  $wP$ , by means of two bolts loaded in tension. From the sketch, it might appear that each bolt will be required to support not more than a load of  $wP/2$ . However, as is shown in Fig. 5-24b, load  $wP$  will bend the tee flange so that the outer portions, between the bolt lines and the edges of the tee flange, will press upward, an action resisted by prying forces  $Q$ . Each force  $Q$  is assumed to be a line load, applied near the edge of the tee flange, and effective uniformly over  $w$ , the length tributary to one bolt. Forces  $F$  are the upward reactions applied by the bolts in resisting external

load  $wP$ , from which it will be seen that  $F = wP/2$ . In order to balance the system of vertical forces,  $F$  must be increased by  $Q$ , so that the total upward reaction applied by each bolt to the tee flange is  $F + Q$ . It follows that  $F + Q$  must not exceed  $r_t$ , the value of one bolt in tension, or  $r_t \geq F + Q$ .

With  $r_t$  and  $F$  given, the problem is to find the value of  $Q$ . Since  $Q$  depends on a number of factors, including material and fastener specifications, fastener size, and the geometry of the connection, it may differ appreciably from one connection to another. Although the theoretical determination of  $Q$  is highly complex, research\* has provided empirical formulas which furnish values closely approximating test results.

For connections using A325 bolts and A36 material:

$$Q = F \left[ \frac{100b(d_b)^2 - 18w(t_f)^2}{70a(d_b)^2 + 21w(t_f)^2} \right] \quad (5.3)$$

For connections using A490 bolts and A36 material:

$$Q = F \left[ \frac{100b(d_b)^2 - 14w(t_f)^2}{62a(d_b)^2 + 21w(t_f)^2} \right] \quad (5.4)$$

where

- $Q$  = Prying force per fastener, kips
- $F$  = Externally applied load per fastener, kips
- $w$  = Length of flange tributary to each bolt, in.
- $d_b$  = Nominal bolt diameter, in.
- $a$  = Distance from fastener line to edge of flange, but not more than  $2t_f$ , in.
- $t_f$  = Thickness of angle or flange of tee, in.\*\*
- $b$  = Distance from fastener line to near face of outstanding leg of angle or web of tee, less  $1/16$ -in.

Figure 5-25 represents the geometry of typical hanger connections, employing either tees or angles, showing the dimensions used in Equations (5.3) and (5.4). Note that, regardless of the actual thickness of the tee flange or angles selected, the lesser value, whether it be  $t_f$  or  $t_f'$ , must be used in Equations (5.3) and (5.4) and in the calculations for bending stress. All other variables will remain those of the hanger tee or angles. It will be observed in Fig. 5-25 that  $w$  is the length of hanger tributary to one transverse row of fasteners, regardless of the total number required for the hanger.

In the design of a hanger connection, the given data will include the load,  $P$ , the size or shape description of the supporting member and, usually, the size and specification of the fasteners. The gage,  $G$ , should be the minimum dimension permitted by clearance requirements for

\* Behavior of Bolts in Tee-Connections Subject to Prying Action, Structural Research Series No. 353, University of Illinois, Sept. 1969.

\*\* Use average thickness for tees with sloping flanges.

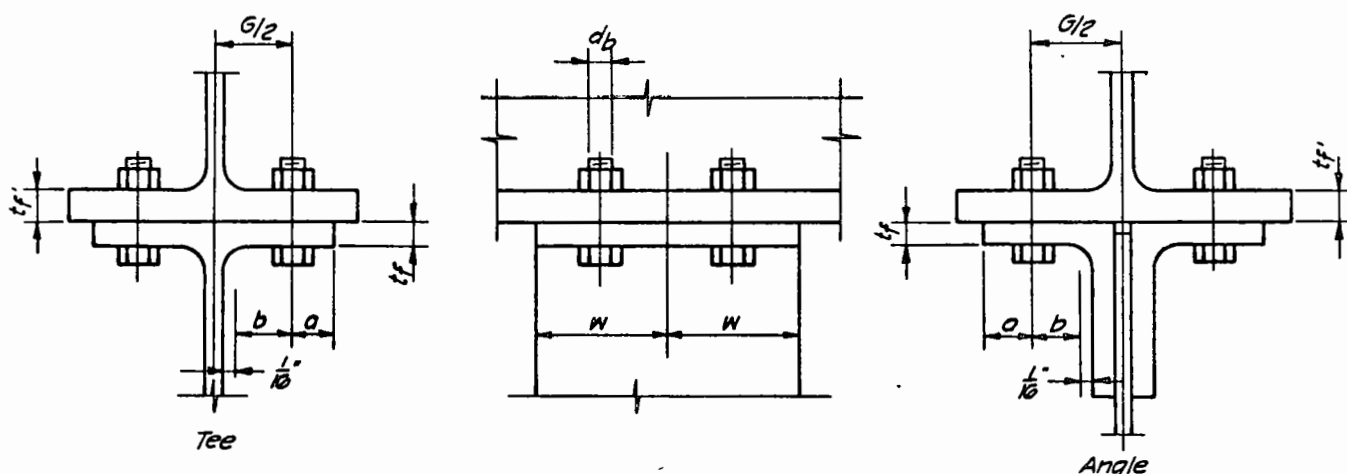


Figure 5-25

entering and tightening the bolts. Data which must be assumed, or otherwise determined, includes a trial section for the fitting (A36 steel) and concurrently, the number of fasteners and dimension  $w$ . From the given data and the geometry of the trial section, all variables are given at least tentative values and can be substituted in Equation (5.3) or (5.4), as applicable.

If the resulting value of  $Q$  is such that  $F + Q$  is equal to or less than  $r_t$ , and the allowable bending stress in the attached flange is not exceeded, the design is adequate. If  $F + Q$  is greater than  $r_t$ , or if the flange bending stress is greater than  $F_b$ , adjustment of the trial section must be made.

**Example 10**—Design a tee-section hanger connection, using A36 steel, to support a load of 60 kips, suspended from the bottom of a W36×230 beam. Fasteners are to be 1-in. diam. A325 high strength bolts on a 5-in. gage. For the W36×230 beam,  $t_f = 1.260$  in. and the tension value of the 1-in. bolt =  $r_t = 31.42$  kips.

**Solution:**

1. Trial bolts: Considering the load of 60 kips and  $r_t = 31.42$  kips, it is evident that one row of two bolts would be sufficient *only* if prying action were negligible.

Try two rows with four bolts, total. Then,  
 $F = 60/4 = 15.0$  kips.

2. Trial section: In determining a trial section it is convenient to use the table of hanger load capacities found in Manual Part 4 under the heading "Hanger Type Connections". Loads  $P$ , in kips per linear inch, are tabulated for corresponding values of  $b$  and  $t_f$ . Although the relationship,  $a = b/2$ , upon which the stress calculations are based, will seldom be true for available rolled sections, the  $P$  values are adequate for the present purpose.

The procedure will be to estimate dimension  $b$  and assume a value of  $P$  from which a trial thickness  $t_f$  will be established. Based on gage  $G = 5$  in.,  $b = 5/2 - 1/4 = 2 1/4$  in. To arrive at a value for  $P$ , the tributary length  $w$  must be assumed. This was arbitrarily set at 5 in. which gives a total fitting length of 10 in. Then, the load  $P$ , per linear inch =  $60/10 = 6.0$  kips. In the table, on the line for  $b = 2 1/4$ , the value of  $P$  next higher than 6.0 kips is 6.13 kips which calls for  $t_f = 7/8$ -in.

Try a tee cut from a W18×85 beam. This section has a flange width of  $8 7/8$  in., acceptable for the 5-in. gage, and a flange thickness  $t_f$  of 0.911 in., which is a little on the conservative side. Note also that the thickness of the supporting beam flange, 1.260 in., is greater than the thickness of the tee, 0.911 in.

Figure 5-26 shows the geometry of the fitting with variables  $w$ ,  $d_b$ ,  $a$ ,  $t_f$ , and  $b$  converted to decimals for substitution in applicable Equation (5.3).

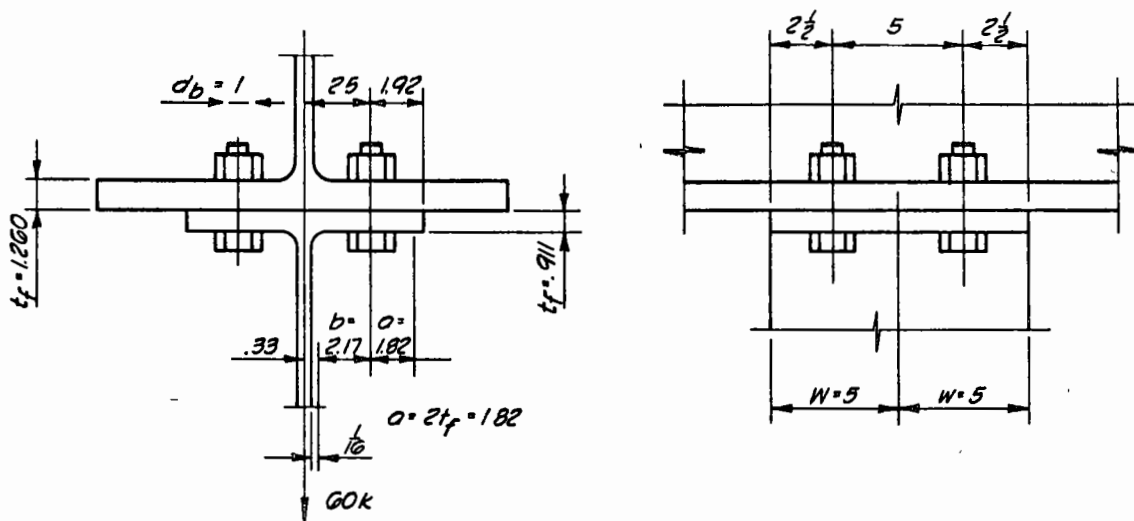


Figure 5-26

3. Solve for  $Q$  by Equation (5.3):

$$Q = 15 \left[ \frac{100 \times 2.17 \times 1^2 - 18 \times 5 \times 0.911^2}{70 \times 1.82 \times 1^2 + 21 \times 5 \times 0.911^2} \right]$$

$$= 15 \times 0.663 = 9.95 \text{ kips}$$

$$F + Q = 15 + 9.95 = 24.95 < 31.42 \text{ kips o.k.}$$

4. Check bending of tee flange:

Allowable  $F_b$  in tee flange =  $0.75F_u$  ksi (see Specification Sect. 1.5.1.4.3). For A36 steel,  
 $F_b = 27.0$  ksi

$$M_2 = Q \times a = 9.95 \times 1.82$$

$$= 18.11 \text{ kip in. Governs}$$

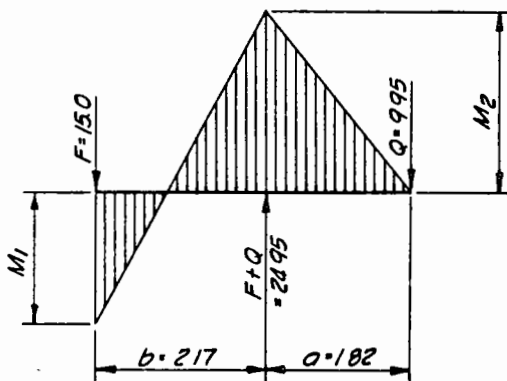
Substituting in the flexure formula,

$$f_b = \frac{Mc}{I}$$

$$= \frac{18.11 \times (0.911/2)}{\frac{1}{12} \times 5 \times 0.911^3}$$

$$= 26.18 < 27.0 \text{ ksi o.k.}$$

5. Use tee cut from W18×85, 10 in. long with four 1-in. diam A325 high-strength bolts on 5-in. centers and a 5-in. gage.



In the moment diagram above,

$$M_1 = (F + Q)b - Q(a + b)$$

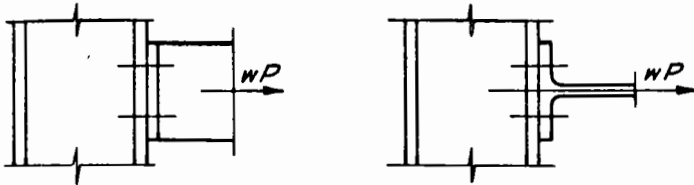
$$= (24.95 \times 2.17) - (9.95 \times 3.99)$$

$$= 14.44 \text{ kip-in.}$$

### Prying Forces—General Considerations

**Rivets and A307 bolts**—Structural rivets and A307 bolts have long been used in hanger and bracket type connections with no evident distress from prying action. This is due in part to the ductile nature of the fasteners and in part to the fact that their tension capacities are so limited that connection material of moderate thickness is rigid enough to reduce prying action to a minimum. Equations similar to those for high-strength bolts are not available for rivets and A307 bolts. However, an assumed value of  $Q$  equal to  $F/2$  will generally provide a conservative design. This means simply that the  $r_t$  value of the fastener should be equal to or greater than 1.5 times the calculated tensile requirement for the fastener.

*Orientation of hangers or brackets*—Whether or not the web of a hanger or tension bracket is parallel with that of the supporting member does not alter the procedure in calculating prying effect.



*Moment brackets*—In the case of a moment bracket attached with fasteners, prying action should be considered on the pair of fasteners carrying the maximum tensile stress. For high-strength bolts, assume  $w$  equal to the fastener pitch and proceed as for a hanger connection. For rivets and A307 bolts use  $Q = F/2$  and proportion fasteners accordingly.

*Fasteners on four gage lines*—On hangers or brackets with four gage lines, fasteners on the two outer lines are not considered to contribute to the tensile capacity of the fitting. Use only the inner lines, with  $a = 2t_f$ .

*Fasteners used at full tensile strength*—When the full tensile strength of fasteners is needed to resist the applied tension force,  $F$ , the hanger or bracket flange should be designed for twice the bending moment used in computing the values for  $P$  which appear in the “Hanger Type Connection” table in Manual Part 4. This may be done by entering the table at twice the  $b$  dimension and following across to where the  $P$  value first exceeds  $2r_t/w$ . The heading of this column will be the required  $t_f$ . When this doubled  $b$  value is beyond the limits of the table, an alternate approach is suggested, wherein the table is entered at the actual value of  $b$ . Proceed as before to the  $P$  value first exceeding  $2r_t/w$  and increase the indicated  $t_f$  by 40%. Selection of a rolled or built-up tee with  $t_f$  values so calculated will provide a fitting for which prying action need not be considered.



## PROBLEMS/CHAPTER 5

### Connections—Bolted or Riveted

#### SHEAR IN RIVETS AND A307 BOLTS

**Problem 1**—Calculate the allowable single shear values of the following fasteners. Check answers by referring to tables “Rivets and Threaded Fasteners—Shear” in Manual Part 4.

- (a)  $\frac{3}{4}$ -in. A502-1 rivet      (c)  $\frac{3}{4}$ -in. A307 bolt  
(b) 1-in. A502-1 rivet      (d) 1-in. A307 bolt

**Problem 2**—Calculate the allowable double shear values of the following fasteners. Check answers as for Problem 1.

- (a)  $\frac{7}{8}$ -in. A502-1 rivet      (c)  $\frac{7}{8}$ -in. A307 bolt  
(b)  $\frac{7}{8}$ -in. A502-2 rivet      (d)  $\frac{3}{4}$ -in. A502-1 rivet

**Problem 3**—Referring to tables “Framed Beam Connections—Bolted or Riveted, Table I” in Manual Part 4, check the total shear values shown in Tables I-A1 through I-A10 for the following connections. Use A502-1 rivets in the web legs and A307 bolts in the outstanding legs.

- (a) 10 rows,  $\frac{3}{4}$ -in.      (d) 7 rows,  $\frac{3}{4}$ -in.  
(b) 9 rows,  $\frac{7}{8}$ -in.      (e) 6 rows, 1-in.  
(c) 8 rows, 1-in.      (f) 5 rows,  $\frac{3}{4}$ -in.

**Note:** Use single and double shear values given in tables “Rivets and Threaded Fasteners—Shear” in Manual Part 4. Give values to three significant digits and indicate which group of fasteners governs the connection value.

**Problem 4**—Refer to the hanger connection shown in Fig. 5-9 of the Text, with a 60 kip load. Using tables of “Rivets and Threaded Fasteners—Shear” in Manual Part 4:

- (a) Calculate the number of 1-in. A502-1 rivets required to connect the gusset plate to the back of the channel.  
(b) Calculate the number of 1-in. A307-bolts required to connect the angles to the gusset.  
(c) Draw a sketch similar to the one appearing in Fig. 5-9 of the Text, using the fasteners as calculated in (a) and (b) above.

#### BEARING IN RIVETED OR BOLTED SHEAR CONNECTIONS

**Problem 5**—Calculate the allowable bearing value for the following fasteners. Indicate with an “X” any value exceeding the allowable  $r$ , for double shear. Check answers by referring to tables “Rivets and Threaded Fasteners—Bearing” in Manual Part 4.

- (a)  $\frac{3}{4}$ -in. A502-1 rivet bearing on  $\frac{1}{4}$ -in. thick material with  $F_v = 36.0$  ksi  
(b)  $\frac{7}{8}$ -in. A502-1 rivet bearing on  $\frac{3}{8}$ -in. thick material with  $F_v = 42.0$  ksi  
(c) 1-in. A307 bolt bearing on  $\frac{1}{2}$ -in. thick material with  $F_v = 36.0$  ksi  
(d)  $\frac{7}{8}$ -in. A502-2 rivet bearing on 0.437 in. thick material with  $F_v = 50.0$  ksi

**Note:** Refer to Appendix A of AISC Specification Sect. 1.5.2, “Rivets, Bolts and Threaded Parts,” for  $F_v$  values corresponding to the given  $F_v$  values.

**Problem 6**—Refer to tables “Framed Beam Connections—Bolted or Riveted, Table I” in Manual Part 4 and verify the total bearing values shown in Tables I-B1 through I-B10 for the following connections. Note that bearing values given in these tables are based on 1-in. thick material.

- (a) 2 rows of  $\frac{3}{4}$ -in. fasteners in material with  $F_v = 36.0$  ksi  
(b) 2 rows of 1-in. fasteners in material with  $F_v = 36.0$  ksi  
(c) 8 rows of  $\frac{7}{8}$ -in. fasteners in material with  $F_v = 45.0$  ksi  
(d) 10 rows of 1-in. fasteners in material with  $F_v = 36.0$  ksi  
(e) 4 rows of  $\frac{3}{4}$ -in. fasteners in material with  $F_v = 42.0$  ksi  
(f) 6 rows of  $\frac{7}{8}$ -in. fasteners in material with  $F_v = 50.0$  ksi

**Note:** Solve these problems by use of the basic formula  $F_v = 1.35F_p$ , where  $F_p$  is the allowable bearing stress in ksi. Give answers to three significant digits.

**Problem 7**—What thickness of A36 material ( $F_y = 36.0$  ksi) is required to develop the single and double shear values of the following rivets? Give answers to three decimal places, with equivalent fractional dimensions in sixteenths of an inch.

- (a)  $\frac{3}{4}$ -in. A502-1 rivet
- (b)  $\frac{1}{8}$ -in. A502-1 rivet
- (c)  $\frac{3}{4}$ -in. A502-2 rivet
- (d)  $\frac{1}{8}$ -in. A502-2 rivet

**Problem 8**—In Problem 4, referring to Fig. 5-9 in the Text, a 60.0 kip load was supported by 1-in. A502-1 rivets connecting the gusset plate to the back of the channel and 1-in. A307 bolts connecting the angles to the gusset plate. Assume that the angle and the web of the channel are both  $\frac{3}{8}$ -in. thick and that all material is A36 steel. Calculate the number of rivets and bolts required in these connections if:

- (a) The gusset plate is  $\frac{1}{16}$ -in. thick
- (b) The gusset plate is  $\frac{1}{4}$ -in. thick.

Note: Use shear and bearing values from tables of "Rivets and Threaded Fasteners" in Manual Part 4.

**HIGH-STRENGTH BOLTED CONNECTIONS**

**Problem 9**—Calculate the allowable single "shear" value of the following high-strength bolts. Give answers to two decimal places. Refer to tables of "Rivets and Threaded Fasteners—Shear" for allowable single shear values corresponding to suffix letters F, N and X.

- (a)  $\frac{3}{4}$ -in. A325-F
- (b)  $\frac{1}{8}$ -in. A325-F
- (c) 1-in. A490-F
- (d)  $\frac{3}{4}$ -in. A325-N
- (e)  $\frac{1}{8}$ -in. A325-X
- (f) 1-in. A490-N

**Problem 10**—Refer to tables "Framed Beam Connections—Bolted or Riveted, Table I" in Manual Part 4 and perform the calculations necessary to verify the shear or bearing values listed for the following high-strength bolted connections:

- (a) Shear value: 8 rows  $\frac{1}{8}$ -in. A325-F
- (b) Shear value: 8 rows  $\frac{1}{8}$ -in. A325-X
- (c) Shear value: 6 rows 1-in. A490-X
- (d) Shear value: 6 rows 1-in. A490-N
- (e) Bearing value: 8 rows  $\frac{1}{8}$ -in. A325 in 1-in. thick material,  $F_y = 36.0$  ksi
- (f) Bearing value: 6 rows 1-in. A325 in 1-in. thick material,  $F_y = 45.0$  ksi
- (g) Shear value: 4 rows  $\frac{3}{4}$ -in. A490-F
- (h) Shear value: 4 rows  $\frac{3}{4}$ -in. A325-X

**Problem 11**—Using data in Fig. P5.1, calculate the number of fasteners required to connect the two  $\frac{5}{16}$ -in. angles to the gusset plate. Use the following high-strength bolts:

- (a)  $\frac{3}{4}$ -in. A325-F
- (b)  $\frac{3}{4}$ -in. A325-X
- (c)  $\frac{3}{4}$ -in. A325-N
- (d)  $\frac{3}{4}$ -in. A490-F

**Problem 12**—Refer to Fig. P5.1 and calculate the number of fasteners required to connect the gusset plate to the web of the channel. Use the following high-strength bolts:

- (a)  $\frac{1}{8}$ -in. A325-F
- (b)  $\frac{1}{8}$ -in. A325-X
- (c)  $\frac{1}{8}$ -in. A325-N
- (d)  $\frac{1}{8}$ -in. A490-F

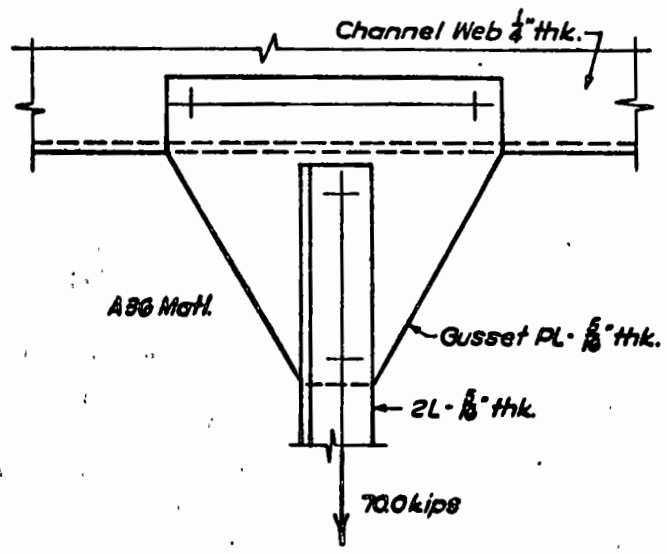


Figure P5.1

**TENSION IN FASTENERS**

**Problem 13**—Calculate the allowable tension value  $r_t$  of the following fasteners. Check answers by referring to tables "Rivets and Threaded Fasteners—Tension" in Manual Part 4.

- (a) 1-in. A502-1 rivet
- (b) 1-in. A325 bolt
- (c) 1-in. A307 bolt\*
- (d) 1-in. A490 bolt
- (e) 1-in. A502-2 rivet
- (f) 1-in. threaded rod, A36 material\*

\* In calculating tension values of A307 bolts and other threaded parts of various  $F_y$  values, use tensile stress area as defined in AISC Specification Sect. 1.5.2.1 and listed in the above Manual reference.



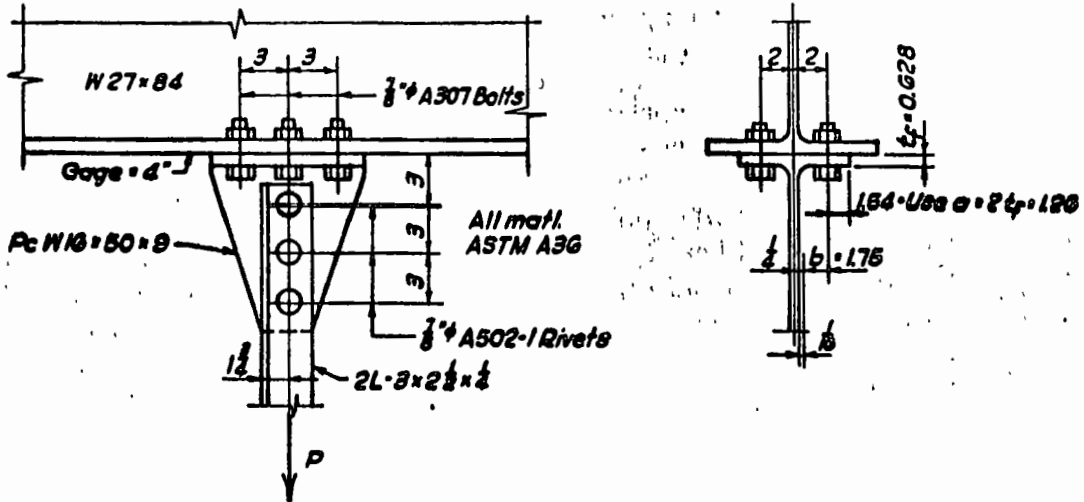


Figure P5.2

**Problem 14**—Refer to Fig. 5-13 in the Text and change the number of fasteners from 4 to 6. Calculate the maximum permissible value of load  $P$ , using the following fasteners:

- (a)  $\frac{1}{8}$ -in. A502-1 rivets
- (b)  $\frac{1}{8}$ -in. A325 bolts
- (c)  $\frac{1}{8}$ -in. A307 bolts
- (d)  $\frac{1}{8}$ -in. A490 bolts

Note: Use allowable loads from tables "Riveted and Threaded Fasteners—Tension" in Manual Part 4. For the purpose of this problem, disregard any effect prying might have on the fasteners.

**Problem 15**—Using the data given in Fig. P5.2, calculate the maximum load  $P$ , in kips, that can be supported by the hanger connection. Perform the following:

- (a) Calculate the capacity of the rivets attaching the tee stem.
- (b) Calculate the capacity of the bolts attaching the tee flange to the supporting beam. Assume that prying action will increase the tensile load on the bolts by 50%.
- (c) Calculate the bending capacity of the tee flange.
- (d) Summarize the values from (a) through (c) and indicate the limiting value of  $P$ .

**Problem 16**—Using the data given in Fig. P5.3, design a welded built-up hanger connection which will permit use of the fasteners at their calculated tension values, excluding the prying effect. Use the method outlined in Chapter 5 of the Text, in the paragraph headed "Fasteners used at full tensile strength".

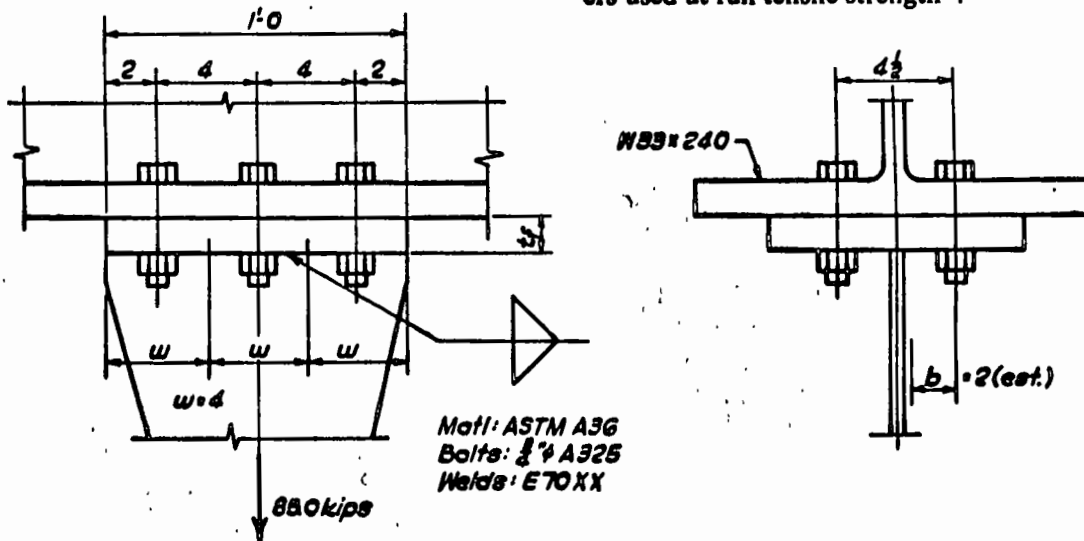


Figure P5.3

**FASTENERS IN COMBINED SHEAR AND TENSION**

**Problem 17**—Referring to Fig. P5.4, perform the following operations:

- Calculate the number of bolts required to attach the bracing angle to the gusset plate.
- Calculate the number of rivets required to attach the gusset plate to the connection angles.
- Calculate the number of rivets required to attach the connection angles to the column flange. Assume that prying action will increase the tensile load on the rivets by 50%.
- Calculate the thickness of the connection angles.
- Draw a dimensioned layout of the connection, using a scale of 1½ in. = 1 ft. Bill connection material and show cutting sketch for nested plates.
- Calculate the tensile stress in the gusset plate at the first hole in the bracing angle. Scale gross width from layout.

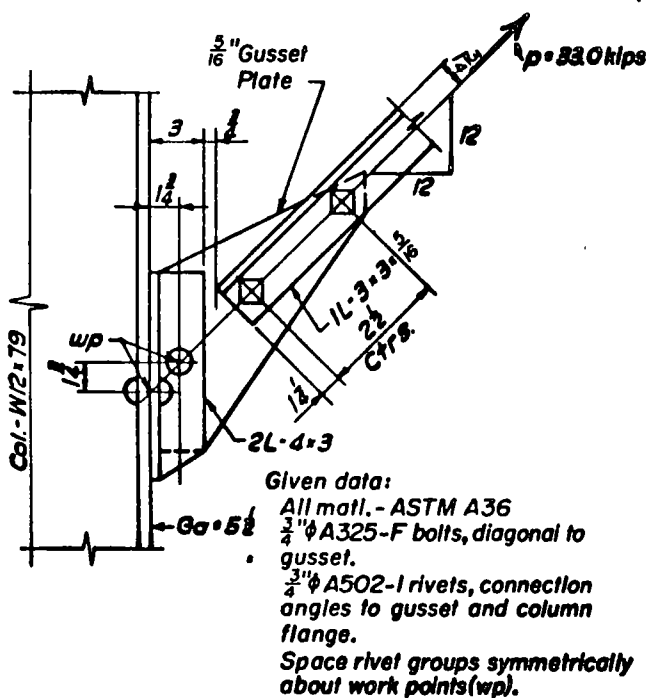


Figure P5.4

**ECCENTRIC CONNECTIONS—FASTENERS IN SHEAR ONLY**

**Problem 18**—Using the data in Fig. P5.5, find the resultant values of  $f_R$  at the extreme fasteners in the following ways:

- Solve for maximum  $f_R$  by the method of Example 7 in Chapter 5 of the Text.
- Check the answer by using the tables of eccentric loads on fasteners in Manual Part 4.
- Make a graphical construction showing the reactions  $f_1, f_2, f_3,$  and  $f_R$  at the four extreme fasteners, A, B, C and D. Use scale of 1-in. equals 4 kips. Compare scaled values of  $f_R$  for fasteners A and B with calculated values from (a). Check scaled values of  $f_R$  for fasteners C and D by a mathematical solution.

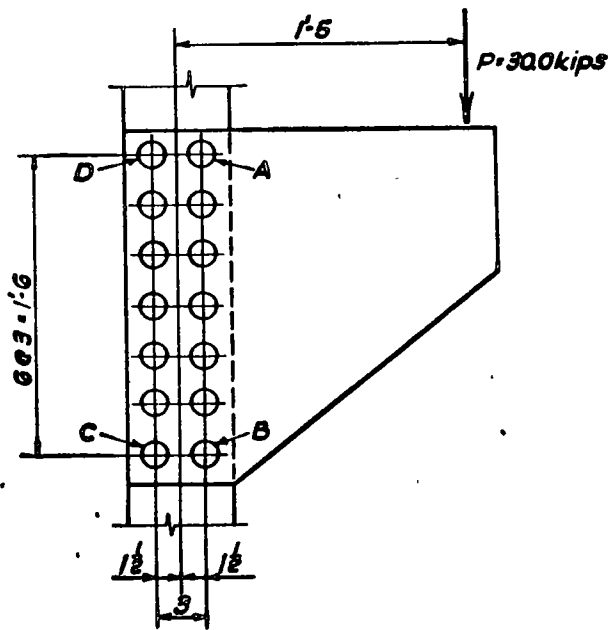


Figure P5.5

**Problem 19**—Using the data in Fig. P5.6, find the resultant force  $f_R$  at the extreme fasteners in the following ways:

- (a) Solve for  $f_R$  by the method of Example 7 in Chapter 5 of the Text.
- (b) Check the answer by using the tables of eccentric loads on fasteners in Manual Part 4.

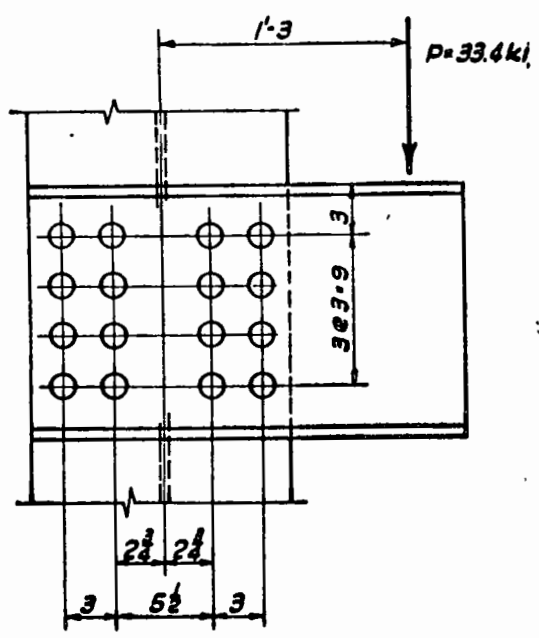


Figure P5.6

**ECCENTRIC CONNECTIONS—FASTENERS IN COMBINED SHEAR AND TENSION**

**Problem 20**—Using the data in Fig. P5.7 and the several fastener specifications listed below, calculate the shear and tension stresses in the top row of fasteners and compare these values with the allowable values. Use the interaction formulas of AISC Specification Sect. 1.6.3 for allowable tension values where applicable. Consider prying action on the fasteners and check the bending stress in the tee flange.

- (a) A325-X bolts (use Case I from Text Chapter 5)
- (b) A325-F bolts (use Case I from Text Chapter 5)
- (c) A502-1 rivets (use Case I from Text Chapter 5)
- (d) A502-1 rivets (use Case II from Text Chapter 5)

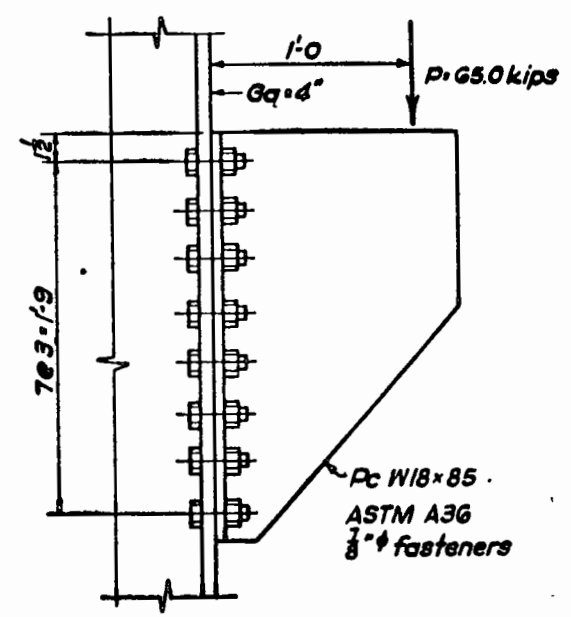


Figure P5.7

## PROBLEMS/CHAPTER 6

### Connections—Welded

#### STRESSES IN CONCENTRICALLY LOADED FILLET WELDS

**Problem 1**—Two  $6 \times \frac{5}{16}$ -in. bars of ASTM A36 steel are to be fillet welded as shown in Fig. P6.1.

- (a) Compute the size of fillet weld required, using E70XX electrodes. Solve by calculating the number of  $\frac{1}{16}$  inches of weld size required. Draw a sketch and show the weld symbol.
- (b) Investigate the capacity of the bars to resist the stress concentration due to weld. Check the answer by use of Table 6A-II in the Appendix to Text Chapter 6.

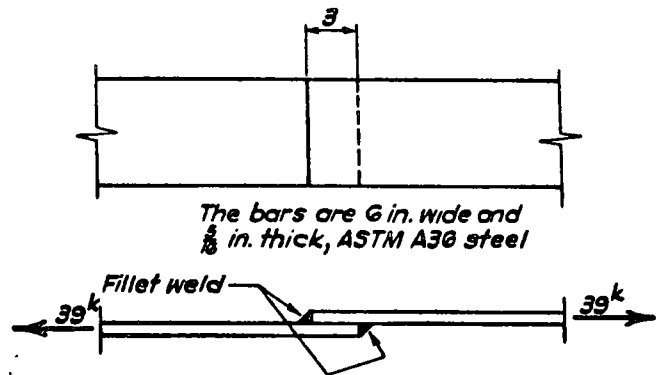


Figure P6.1

**Problem 2**—Two  $\frac{3}{8}$ -in. thick angles are to be fillet welded at heel and toe to the  $\frac{1}{2}$ -in. thick stem of a tee section, as shown in Fig. P6.2. All material is ASTM A36 steel.

- (a) Compute the size of fillet weld required, using E70XX electrodes. Solve by calculating the number of  $\frac{1}{16}$  inches of weld size required. Draw a sketch and show the weld symbol.
- (b) Investigate the capacity of the tee web to resist the stress concentration due to weld. Check the answer by use of Table 6A-II in the Appendix to Text Chapter 6.

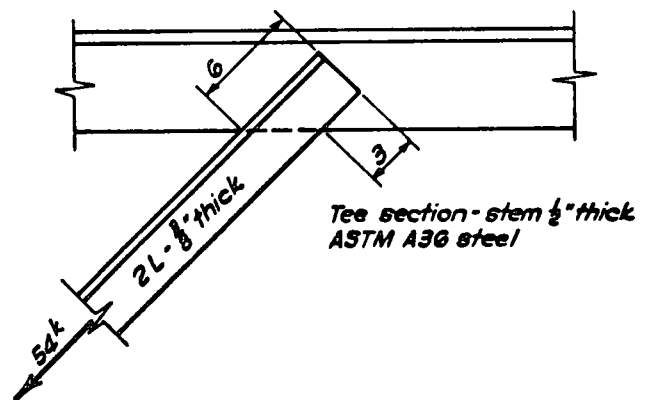


Figure P6.2

**Problem 3**—Determine the length and size of fillet weld required to connect the  $7 \times \frac{3}{8}$ -in. bar to flange of column as shown in Fig. P6.3. Do not weld full width of bar unless required by strength. Draw sketch and show welding symbol.

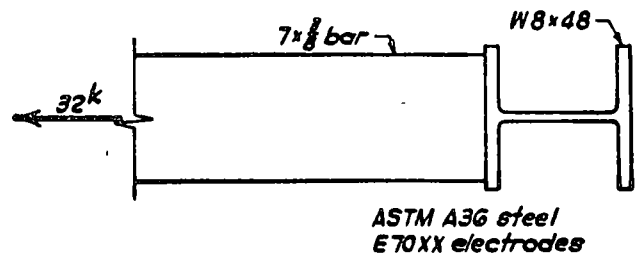


Figure P6.3

**Problem 4**—Determine the length and size of fillet weld required to connect the  $7 \times \frac{3}{8}$ -in. bar to flanges of column as shown in Fig. P6.4. Draw sketch and show welding symbol. (Note: The theoretical distance of  $7\frac{1}{8}$  in. between flanges of the  $W8 \times 48$  may vary; see Rolling Tolerances for W shapes under "Standard Mill Practice" in Manual Part 1. For the purpose of this problem it is assumed that the distance is  $7\frac{1}{8}$  in.)

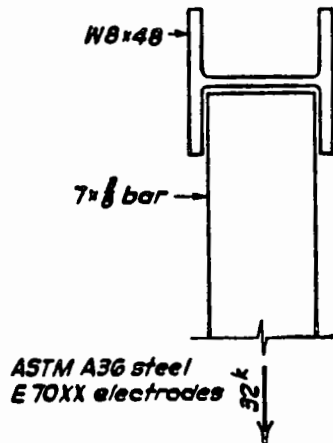


Figure P6.4

### STRESSES IN ECCENTRICALLY LOADED FILLET WELDS

**Problem 5**—Find size of fillet weld required to connect the plate to the face of column in Fig. P6.5. Construct a force diagram at the point of maximum stress. Draw sketch of connection and show welding symbol.

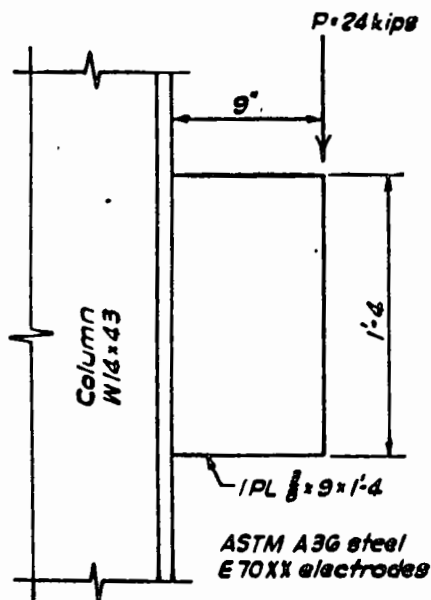


Figure P6.5

**Problem 6**—Find required size of fillet weld to fasten the end connection angles to the beam web, based on data shown in Fig. P6.6. Check answer by use of tables "Eccentric Loads on Weld Groups" in Manual Part 4. Construct force diagram at point of maximum stress.

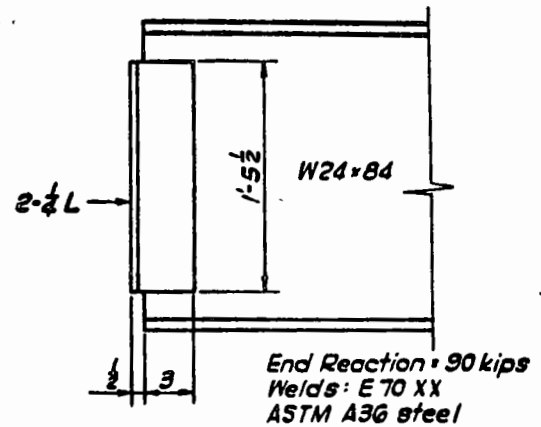


Figure P6.6

**Problem 7**—For the connection shown in Fig. P6.7:

- Determine size of fillet welds required to connect the plate to the face of column, welded as shown. Solve by means of the eccentric load tables in Manual Part 4.
- Calculate the required thickness of plate.
- Draw a sketch of the connection.

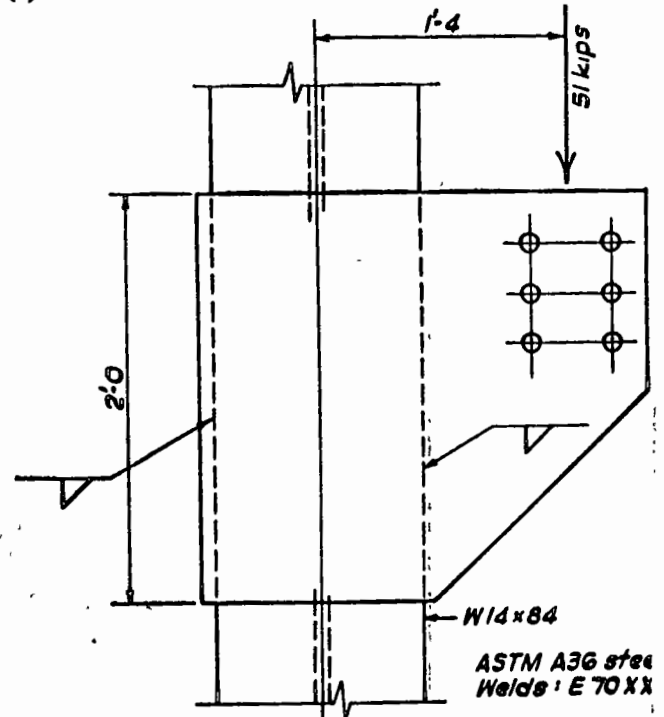


Figure P6.7

**Problem 8**—Referring to Fig. P6.7, by welding along the top and bottom edges of plate in addition to the vertical edges, the plate size can be decreased.

- (a) Determine the length of plate required, using the minimum size fillet weld permitted by Sect. 1.17.5 of the Specification. (Suggestion: Try a plate 13 in. long.)

- (b) Calculate the required thickness of plate.
- (c) Draw a sketch of the connection.

**WELDING SYMBOLS**

**Problem 9**—Make a sketch of each joint shown in Fig. P6.8, indicating what each welding symbol means by adding required welds, dimensions, and notes.

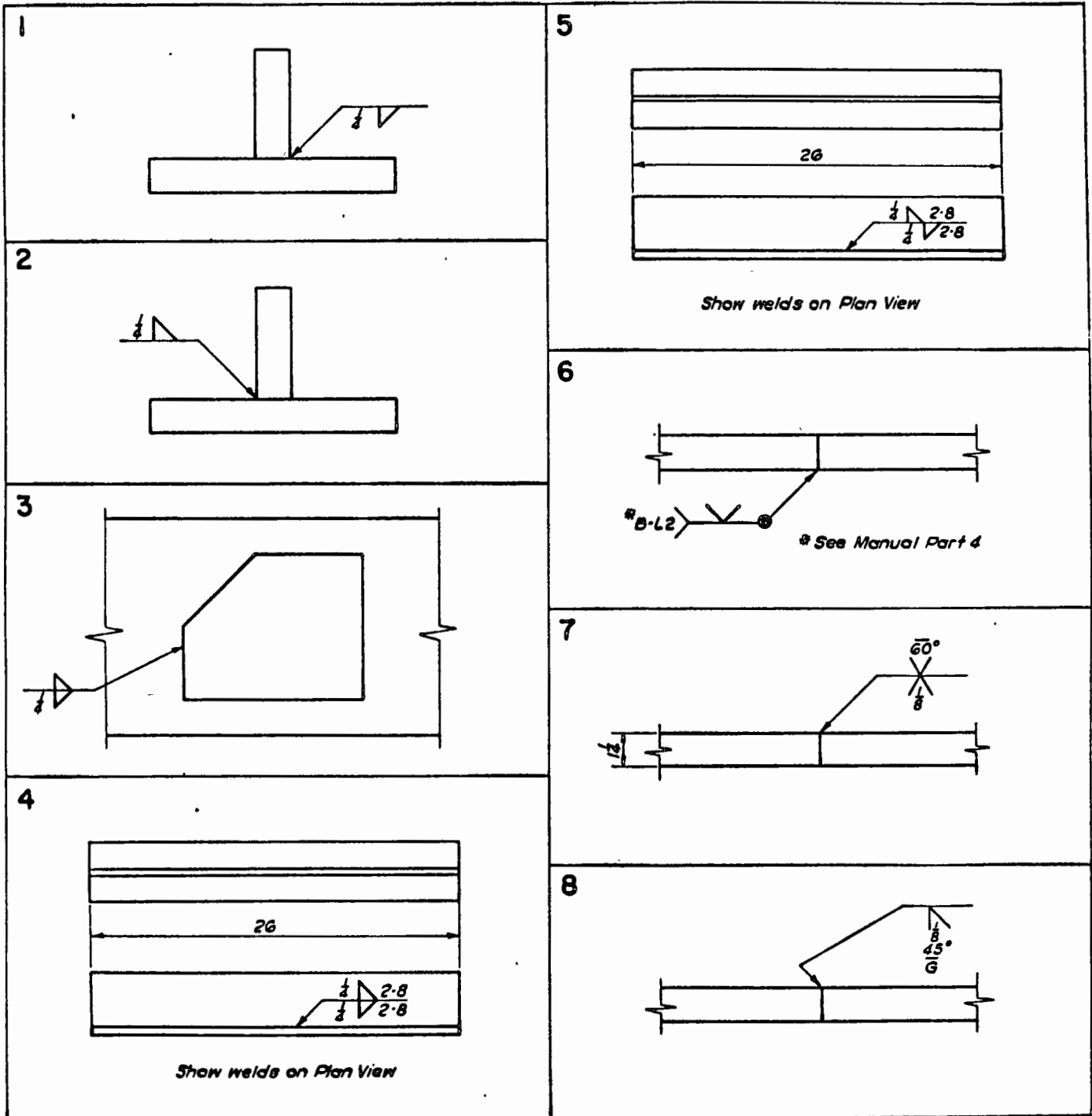


Figure P6.8

## PROBLEMS/CHAPTER 7

# Framed and Seated Beam Connections—Bolted or Riveted

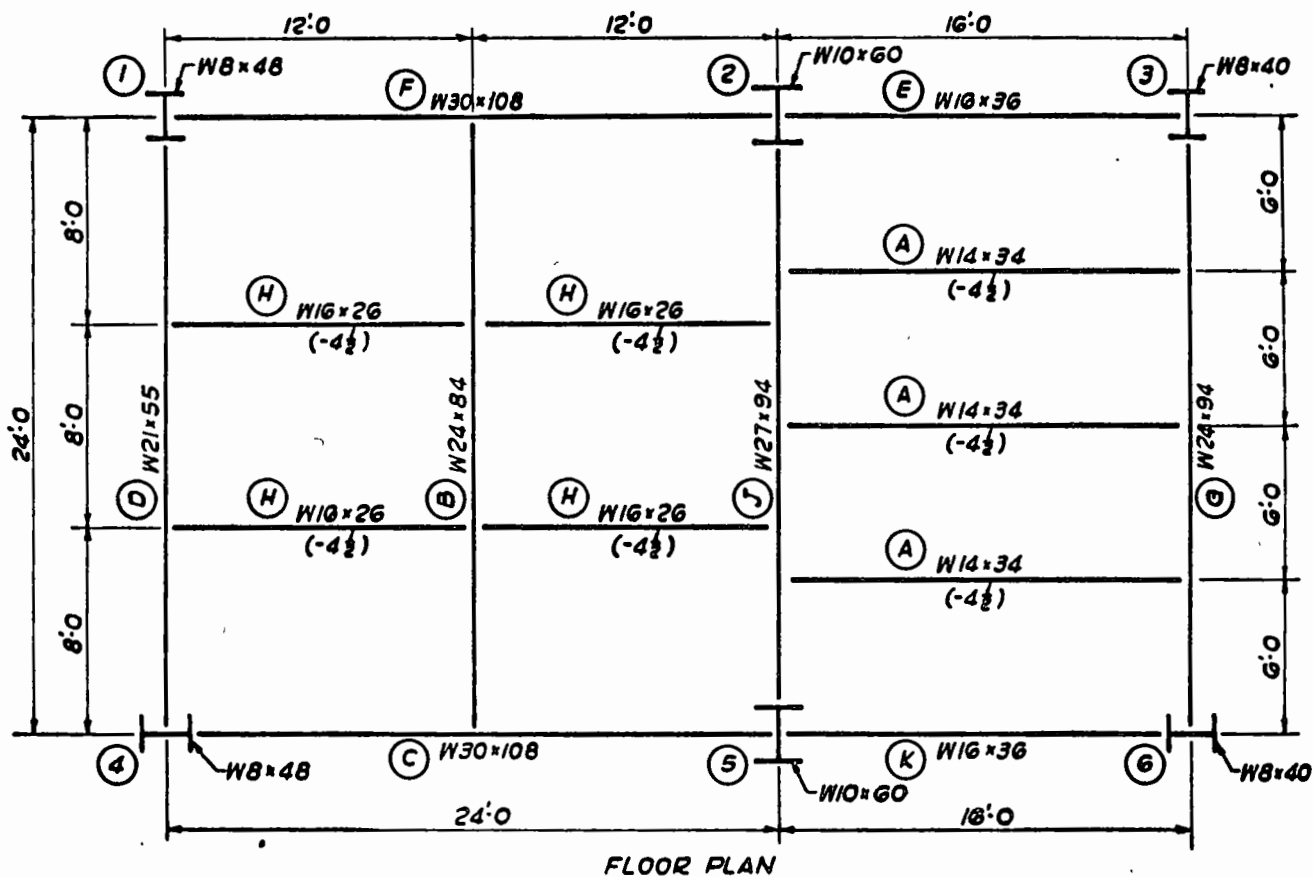
### FRAMED BEAM CONNECTIONS

In this chapter, all beams refer to the floor plan in Fig. P7.1, unless otherwise noted.

**Problem 1**—Select the end connections for beam A

(W14×34), using framing angles shop bolted to the ends of the beam.

**Problem 2**—If  $\frac{3}{4}$ -in. A307 bolts are used in place of the  $\frac{7}{8}$ -in. A307 bolts in Problem 1, will the same 3-row connection from Table I be adequate?



*Tops of all beams 3" below finished floor except those indicated otherwise, thus (-4½).*

**General Notes:**

Specifications: AISC, latest edition.

Material: ASTM A36

**Shop and Field Fasteners:**

Beams to Columns:  $\frac{3}{8}$ " A325 bolts, friction type connections.

Beams to Beams:  $\frac{7}{8}$ " A307 bolts,

Connection of all beams to be designed to support one-half the total uniform load capacity shown in tables for Allowable Loads on Beams in AISC Manual of Steel Construction.

Figure P7.1

**Problem 3**—Select the end connections for beam B (W24×84), using framing angles shop bolted to the ends of the beam.

**Problem 4**—Same as Problem 3 except the reaction is 55 kips.

**Problem 5**—Same as Problem 3 except with another W24×84 beam framing opposite, as shown in Fig. P7.2.

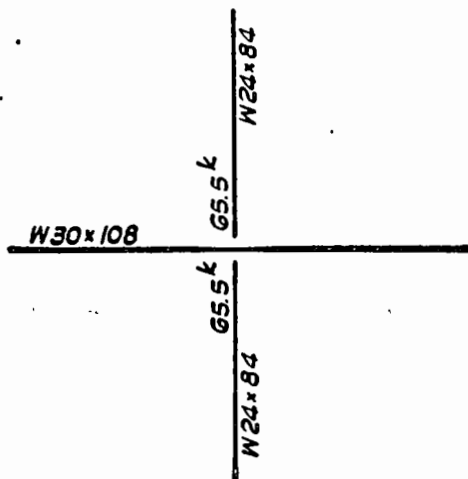


Figure P7.2

**Problem 6**—Same as Problem 5 except the supporting beam is W27×84 instead of W30×108.

**Problem 7**—Same as Problem 5 except the reactions are 55 kips.

**Problem 8**—Same as Problem 6 except the reactions are 55 kips.

**Problem 9**—Select the connection for beam C (W30×108) to column 4, using framing angles connected as shown in Fig. P7.3. Determine the gage in the web leg of the connection angles.

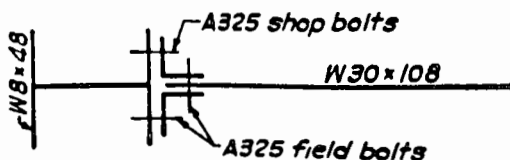


Figure P7.3

**Problem 10**—Same as Problem 9 except use  $\frac{3}{4}$ -in. A325 bolts in a friction-type connection, for both shop and field fasteners.

**Problem 11** Same as Problem 9 except use  $\frac{7}{8}$ -in. A325 bolts in a bearing-type connection with threads excluded from shear planes, for both shop and field fasteners.

**Problem 12**—Same as Problem 9, except use  $\frac{3}{4}$ -in. A325 bolts in a bearing-type connection with threads excluded from shear planes, for both shop and field fasteners.

**Problem 13**—A W24×120 beam with an end reaction of 190 kips connects to a W30×172 girder. Select the required connection, using framing angles shop riveted to the W24 with  $\frac{7}{8}$ -in. A502-1 rivets and field bolted to the W30 with  $\frac{7}{8}$ -in. A325 bolts in a friction-type connection. Tops of beams are flush and all steel is ASTM A36. Determine the gages and size of angles.

**Problem 14**—Same as Problem 13 except change the reaction to 140 kips.

**Problem 15**—Design the connection for a W27×94 beam to a W14×142 column, using framing angles, as shown in Fig. P7.4. The W27×94 is uniformly loaded on a 16-ft span. Main material is ASTM A572 grade 50 steel and detail material is ASTM A36 steel. Fasteners are A490 bolts in bearing-type connections with threads excluded from shear planes, of the diameters called for in Fig. P7.4. Determine the gage in the web

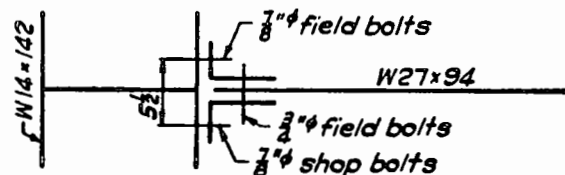


Figure P7.4

legs of the connection angles and the size of angles, based on fasteners lining up on the same gage lines (not staggered).

**Problem 16**—Same as Problem 15 except span of W27×94 beam is 20 ft.

#### UNSTIFFENED SEATED CONNECTIONS

**Problem 17**—Design the end connection for beam A (W14×34), using a seat angle shop bolted to the web of the W24×94. Draw a sketch of the connection, similar to Fig. 7-16 in the Text.

**Problem 18**—If  $\frac{3}{4}$ -in. instead of  $\frac{7}{8}$ -in. A307 bolts are used in the seat angle in Problem 17, will the Type E connection with 6 bolts be adequate?



**Problem 19**—Design the connection of beam D (W21 × 55) to column 4, using a seat angle shop bolted to the web of the column. Draw a sketch of the connection, similar to Fig. 7-17(b) in the Text.

**Problem 20**—If 3/4-in. instead of 7/8-in. bolts are used in the seat angle in Problem 19, will the Type C connection with 6 bolts be adequate?

**Problem 21**—Design the connection of beam E (W16 × 36) to column 3, except add a W16 × 36 beam connecting at the same elevation and with the same reaction to the other side of the column, as shown in Fig. P7.5. Use

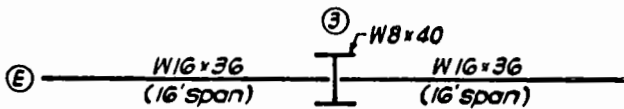


Figure P7.5

seat angles shop fastened to the web of the column with 7/8-in. A502-1 rivets. Draw a sketch of the connection, similar to Fig. 7-17(b) in the Text.

**Problem 22**—Same as Problem 21, except change the reaction to 23 kips and use 3/4-in. A502-1 rivets in the seat angle.

**Problem 23**—Design the connection of a W16 × 45 beam to the web of a W12 × 65 column as shown in Fig. P7.6,

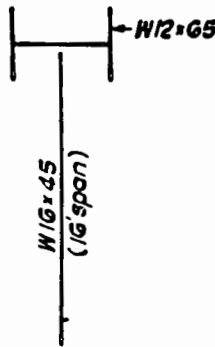


Figure P7.6

using unstiffened seat fastened to the column with 3/4-in. A490 bolts in a bearing-type connection with threads excluded from the shear planes. The reaction is one-half the allowable uniform load for the 16-ft span. Main material is ASTM A572 grade 50 steel and detail material is ASTM A36 steel. Draw a sketch of the connection similar to Fig. 7-17(b) in the Text.

**Problem 24**—Same as Problem 23 except the W16 × 45 is on an 18-ft span and fasteners are 7/8-in. A490 bolts in a friction-type connection.

**STIFFENED SEATED CONNECTIONS**

**Problem 25**—Design the connection of beam F (W30 × 108) to column 1, using a stiffened seated connection. Draw a sketch of the connection, similar to Fig. 7-22 in the Text.

**Problem 26**—Same as Problem 25 except change the reaction to 85 kips and use 3/4-in. A325 bolts in a friction-type connection. Draw a sketch of the connection.

**Problem 27**—Design the connection of beam G (W24 × 94) to column 6, using a stiffened seated connection. Draw a sketch of the connection.

**Problem 28**—Same as Problem 27 except change the reaction to 90 kips and use 1-in. A325 bolts in a friction-type connection. Draw a sketch of the connection.

**Problem 29**—Design the connection of two W24 × 68 beams to the web of a W8 × 35 column as shown in Fig. P7.7, using a Type B stiffened seated connection. Draw a sketch of the connection.

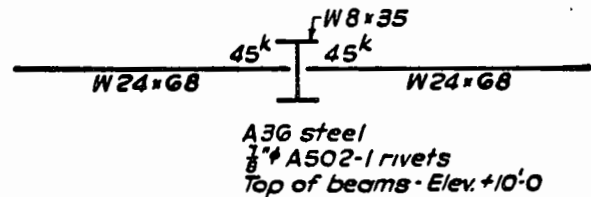


Figure P7.7

**Problem 30**—Same as Problem 29 except use 1-in. A502-1 rivets. Draw a sketch of the connection.

**Problem 31**—Design the connection of two W36 × 135 beams to the web of a W14 × 87 column as shown in Fig. P7.8. Draw a sketch of the connection.

**Problem 32**—Same as Problem 31, except the beams are W36 × 150 and the span is 36 ft. Stiffener angles are A572 grade 50 steel.

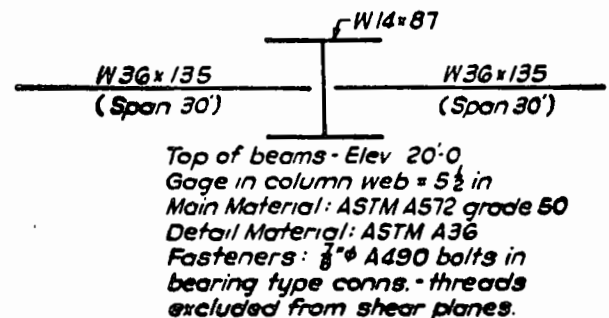


Figure P7.8

# PROBLEMS/CHAPTER 8

## Framed and Seated Beam Connections—Welded

### FRAMED BEAM CONNECTIONS

In this chapter, all beams, unless otherwise noted, refer to the floor plan in Fig. P7.1, Problems/Chapter 7.

**Problem 1**—Select the end connections for beam A (W14×34), using framing angles shop welded to the W14 with E70XX electrodes and field bolted to the supporting beam with 3/4-in. A307 bolts.

**Problem 2**—Same as Problem 1, except change reaction to 35 kips and use 7/8-in. A307 bolts.

**Problem 3**—Select the connection of beam H (W16×26) to beam B (W24×84), using framing angles shop welded to the W16 with E70XX electrodes and field bolted to the W24 with 7/8-in. A307 bolts.

**Problem 4**—Same as Problem 3 except change reaction to 29 kips and use 3/4-in. A307 bolts.

**Problem 5**—Select the connection of beam B (W24×84) to beam F (W30×108), using framing angles shop welded to the W24 with E70XX electrodes and field bolted to the W30 with 3/4-in. A325 bolts in a friction-type connection.

**Problem 6**—Same as Problem 5 except use 7/8-in. A307 bolts.

**Problem 7**—Select the connection for the W24×100 beams shown in Fig. P8.1, using framing angles shop welded to the W24 with E70XX electrodes and field bolted to the W36 with 3/4-in. diameter A490 bolts in a bearing-type connection with threads excluded from shear planes.

**Problem 8**—Same as Problem 7 except the span of the W24×100 beams is 22 ft.

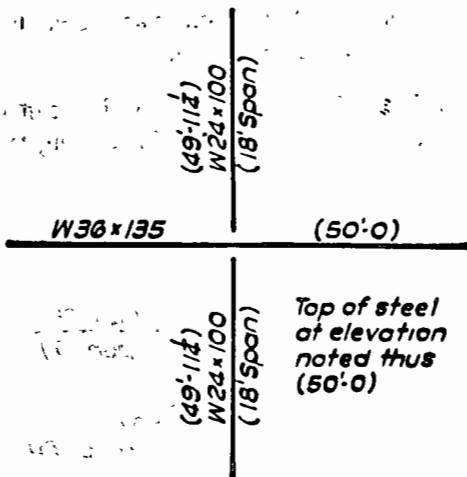
**Problem 9**—Select the connection of beam C (W30×108) to column 4, using framing angles shop welded to the column with E70XX electrodes and field bolted to the W30 with 7/8-in. A325 bolts in a bearing-type connection with threads in shear planes.

**Problem 10**—Same as Problem 9 except use 3/4-in. A325 bolts in a bearing-type connection with threads in shear planes, and change the size of column to W12×65.

**Problem 11**—Select the connection of beam G (W24×94) to column 3, using framing angles shop welded to the column with E70XX electrodes and field bolted to the W24 with 7/8-in. A325 bolts in a friction-type connection.

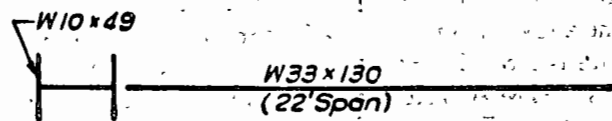
**Problem 12**—Same as Problem 11, except use 1-in. A325 bolts in a friction-type connection, with a reaction of 110 kips.

**Problem 13**—Select the connection for the W33×130 beam shown in Fig. P8.2, using framing angles shop welded to the column with E70XX electrodes and field bolted to the W33 with 3/4-in. diam. A490 bolts in a bearing-type connection with threads excluded from shear planes.



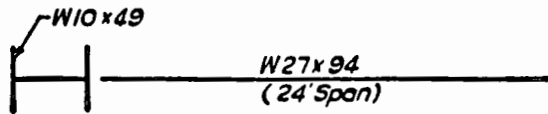
Main Material: ASTM A572 Grade 50  
Detail Material: ASTM A36

Figure P8.1



Main Material: ASTM A572 Grade 50  
Detail Material: ASTM A36

Figure P8.2

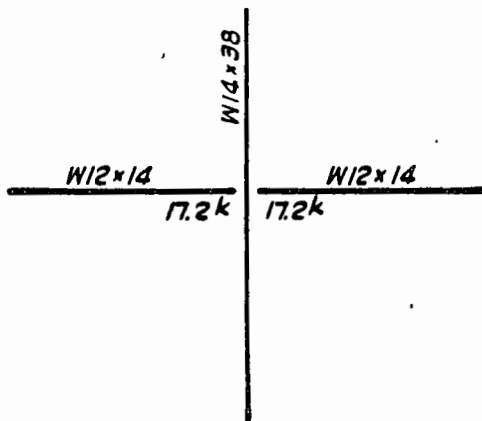


Main Material: ASTM 572 Grade 50  
Detail Material: ASTM A36

Figure P8.3

**Problem 14**—Select the connection for the W27×94 shown in Fig. P8.3, using framing angles shop welded to the column with E70XX electrodes and field bolted to the W27 with 3/4-in. diam. A490 bolts in a bearing-type connection with threads included in shear planes.

**Problem 15**—Select the connection of the W12×14 beam to the W14×38 girder shown in Fig. P8.4. Use framing angles shop welded to the W12×14 and field welded to the W14×38 with E70XX electrodes.



All beams flush top  
A36 steel

Figure P8.4

**Problem 16**—Select the connection of the W12×27 beam to the W16×50 girder shown in Fig. P8.5. Use framing angles shop welded to the W12×27 and field welded to the W16×50 with E70XX electrodes.

**Problem 17**—Select the connection of beam H (W16×26) to beam B (W24×84), using framing angles shop welded to the W16 and field welded to the W24×84 with E70XX electrodes.

**Problem 18**—Same as Problem 17 except the reaction is half the allowable uniform load for the 10-ft span.

**Problem 19**—Select the connection of beam D (W21×55) to column 1, using framing angles shop welded to the beam and field welded to the column with E70XX electrodes. Main material is ASTM A572 grade 50 and detail material is ASTM A36.

**Problem 20**—Select the connection of beam B (W24×84) to beam F (W30×108), using framing angles shop welded to the W24 and field welded to the W30 with E70XX electrodes. Main material is ASTM A572 grade 50 and detail material is ASTM A36.

**UNSTIFFENED SEATED CONNECTIONS**

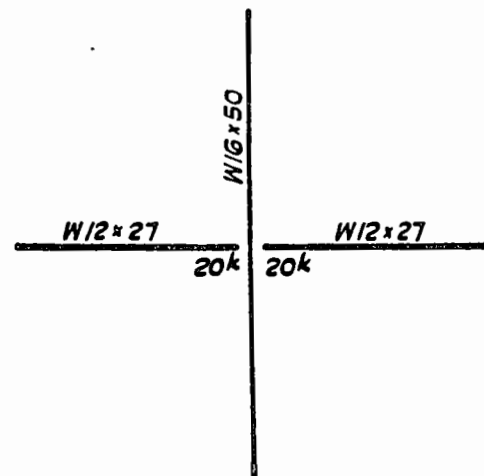
**Problem 21**—Design the end connection for beam A (W14×34) to beam G (W24×94), using a seat angle shop welded to the W24 with E70XX electrodes. The beam is to be field bolted to the side angle with 3/4-in. A307 bolts. Before making calculations, draw a sketch to determine the space available for the seat angle.

**Problem 22**—Same as Problem 21 except the reaction is 30.0 kips.

**Problem 23**—Design the connection of beam H (W16×26) to beam J (W27×94), using a seat angle shop welded to the W27 with E70XX electrodes. The beam is to be field bolted to the side angle with 3/4-in. A307 bolts. Before making calculations, draw a sketch to determine the space available for the seat angle.

**Problem 24**—Same as Problem 23 except reaction is 19.0 kips.

**Problem 25**—Design the connection of beam E (W16×36) to column 2, using a seat angle shop welded to the column with E70 electrodes. The beam is to be field welded to the seat and top angles. Determine the width to which the bottom flange of the W16×36 must be cut and draw a sketch showing the bottom flange cuts.



All beams flush top  
A36 steel

Figure P8.5

**Problem 26**—Same as Problem 25 except the reaction is 19.5 kips and the column is a  $W12 \times 65$ .

**Problem 27**—Design the connection of a  $W21 \times 55$  beam to the flange of a  $W14 \times 87$  column, using a seat angle shop welded to the column with E70XX electrodes. The beam is uniformly loaded on a span of 24 ft. Main material is ASTM A572 grade 50 steel and detail material is ASTM A36 steel. The beam is to be field bolted to the seat and top angles with  $\frac{3}{4}$ -in. A307 bolts. The gage in the beam flanges is  $5\frac{1}{2}$  in.

**Problem 28**—Same as Problem 27 except the beam is  $W16 \times 40$ . Use a  $3\frac{1}{2}$ -in. gage in the beam flanges.

### STIFFENED SEATED CONNECTIONS

**Problem 29**—Design a two-plate welded stiffened seat to connect a  $W24 \times 84$  beam to a  $W14 \times 158$  column as shown in Fig. P8.6. The beam is to be field welded to the seat and top angles. Use E70XX electrodes. Draw a sketch of the connection similar to sketches accompanying examples preceding Table VIII in Manual Part 4.

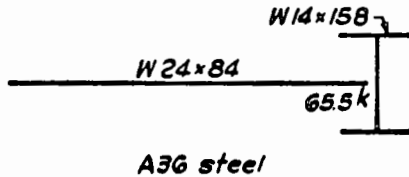


Figure P8.6

**Problem 30**—Same as Problem 29 except the reaction is 85 kips.

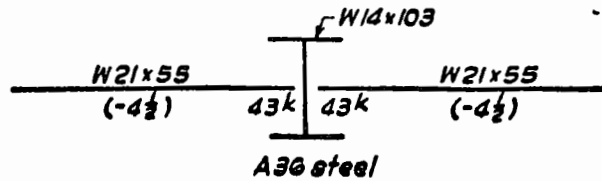


Figure P8.7

**Problem 31**—Design a two-plate welded stiffened seat to connect a  $W21 \times 55$  beam to a  $W14 \times 103$  column as shown in Fig. P8.7. Use E70XX electrodes. The beam is to be field bolted to the seat and top angles with  $\frac{7}{8}$ -in. A325 bolts. Draw a sketch of the connection similar to sketches accompanying examples preceding Table VIII in Manual Part 4. Use a  $5\frac{1}{2}$ -in. gage in the beam flanges.

**Problem 32**—Same as Problem 31 except reaction is 57 kips.

**Problem 33**—Design a two-plate welded stiffened seat to connect beam F ( $W30 \times 108$ ) to column 2 (Fig. P7.1), except use ASTM A572 grade 50 steel for the main material. Detail material is ASTM A36. The beam is to be bolted to the seat and top angles. Draw sketch of connection similar to sketches accompanying examples preceding Table VIII in Manual Part 4.

**Problem 34**—Same as Problem 33 except the reaction is 150 kips.

## SOLUTIONS/CHAPTER 5

### Connections—Bolted or Riveted

#### SHEAR IN RIVETS AND A307 BOLTS

**Problem 1**—The allowable single shear value of a fastener is expressed by the formula  $r_s = F_s \times A_s$ .

- (a)  $\frac{3}{4}$ -in. A502-1 rivet:  $r_s = 15.0 \times 0.4418$   
 $= 6.63$  kips
- (b) 1-in. A502-1 rivet:  $r_s = 15.0 \times 0.7854$   
 $= 11.78$  kips
- (c)  $\frac{3}{4}$ -in. A307 bolt:  $r_s = 10.0 \times 0.4418$   
 $= 4.42$  kips
- (d) 1-in. A307 bolt:  $r_s = 10.0 \times 0.7854$   
 $= 7.85$  kips

**Problem 2**—The allowable double shear value of a fastener is expressed by the formula  $r_s = 2(F_s \times A_s)$ .

- (a)  $\frac{7}{8}$ -in. A502-1 rivet:  $r_s = 2(15.0 \times 0.6013)$   
 $= 18.04$  kips
- (b)  $\frac{7}{8}$ -in. A502-2 rivet:  $r_s = 2(20.0 \times 0.6013)$   
 $= 24.05$  kips
- (c)  $\frac{7}{8}$ -in. A307 bolt:  $r_s = 2(10.0 \times 0.6013)$   
 $= 12.03$  kips
- (d)  $\frac{3}{4}$ -in. A502-1 rivet:  $r_s = 2(15.0 \times 0.4418)$   
 $= 13.25$  kips

**Problem 3**—Shear values of "Framed Beam Connections—Bolted or Riveted, Table I" in Manual Part 4 are verified as follows:

- (a) 10 rows of  $\frac{3}{4}$ -in. fasteners (Table I-A10):  
 Web legs (A502-1 in D.S.):  $10 \times 13.25 = 133$  kips  
 Outstanding legs (A307 in S.S.):  
 $20 \times 4.42 = 88.4$  kips (governs)
- (b) 9 rows of  $\frac{7}{8}$ -in. fasteners (Table I-A9):  
 Web legs (A502-1 in D.S.):  $9 \times 18.04 = 162$  kips  
 Outstanding legs (A307 in S.S.):  
 $18 \times 6.01 = 108$  kips (governs)
- (c) 8 rows of 1-in. fasteners (Table I-A8):  
 Web legs (A502-1 in D.S.):  $8 \times 23.56 = 188$  kips  
 Outstanding legs (A307 in S.S.):  
 $16 \times 7.85 = 126$  kips (governs)
- (d) 7 rows of  $\frac{3}{4}$ -in. fasteners (Table I-A7):  
 Web legs (A502-1 in D.S.):  $7 \times 13.25 = 92.8$  kips  
 Outstanding legs (A307 in S.S.):  
 $14 \times 4.42 = 61.9$  kips (governs)

- (e) 6 rows of 1-in. fasteners (Table I-A6):  
 Web legs (A502-1 in D.S.):  $6 \times 23.56 = 141$  kips  
 Outstanding legs (A307 in S.S.):  
 $12 \times 7.85 = 94.2$  kips (governs)
- (f) 5 rows of  $\frac{3}{4}$ -in. fasteners (Table I-A5):  
 Web legs (A502-1 in D.S.):  $5 \times 13.25 = 66.3$  kips  
 Outstanding legs (A307 in S.S.):  
 $10 \times 4.42 = 44.2$  kips (governs)

**Problem 4**—Refer to the hanger connection shown in Fig. 5-9 in the Text. Use the 60 kip load.

- (a) The required number of 1-in. A502-1 rivets, based on the single shear value of 11.78 kips, is  $60/11.78 = 5.1$ ; use 6 rivets.
- (b) The required number of 1-in. A307 bolts, based on the double shear value of 15.71 kips, is  $60/15.71 = 3.8$ ; use 4 bolts.
- (c) The sketch of this connection (required, but not shown) is the same as shown in Fig. 5-9 of the Text, except that 6 instead of 4 rivets will be used to connect the gusset plate to the channel web and 4 instead of 5 bolts will be used to connect the angles to the gusset plate.

#### BEARING IN RIVETED OR BOLTED SHEAR CONNECTIONS

**Problem 5**—The allowable bearing value of a fastener is expressed by the formula  $r_b = t \times \text{fastener diameter} \times F_p$ , where  $t$  is the thickness of material and  $F_p$  is the allowable bearing stress corresponding to the  $F_s$  value of the material.

- (a)  $\frac{3}{4}$ -in. A502-1 rivet in  $\frac{1}{4}$ -in. material,  $F_p = 36.0$  ksi:  
 $F_p = 48.6$  ksi  
 $r_b = 0.25 \times 0.75 \times 48.6 = 9.11$  kips
- (b)  $\frac{7}{8}$ -in. A502-1 rivet in  $\frac{3}{8}$ -in. material,  $F_p = 42.0$  ksi:  
 $F_p = 56.7$  ksi  
 $r_b = 0.375 \times 0.875 \times 56.7 = 18.6$  kips (X)
- (c) 1-in. A307 bolt in  $\frac{1}{2}$ -in. material,  $F_p = 36.0$  ksi:  
 $F_p = 48.6$  ksi  
 $r_b = 0.5 \times 1.0 \times 48.6 = 24.3$  kips (X)

- (d)  $\frac{7}{8}$ -in. A502-2 rivet in 0.437 in. matl.,  $F_v = 50$  ksi:  
 $F_p = 67.5$  ksi  
 $r_s = 0.437 \times 0.875 \times 67.5 = 25.8$  kips (X)

Since the table does not permit direct reading of the bearing on a 0.437 in. thickness, use the value for 1-in. thick material and multiply by the given thickness to verify this last  $r_s$  value:  
 $59.1 \times 0.437 = 25.8$  kips o.k.

**Problem 6**—The bearing value of a connection is the number of fasteners  $\times$  fastener diameter  $\times$  material thickness  $\times F_p$ , where  $F_p = 1.35F_v$ . In this problem the material thickness is 1-in. for all cases.

- (a) 2 rows of  $\frac{3}{4}$ -in. fasteners,  $F_v = 36.0$  ksi (Table I-B2):  
 $2 \times \frac{3}{4} \times 1 \times 1.35 \times 36.0 = 72.9$  kips
- (b) 2 rows of 1-in. fasteners,  $F_v = 36.0$  ksi (Table I-B2):  
 $2 \times 1 \times 1 \times 1.35 \times 36.0 = 97.2$  kips
- (c) 8 rows of  $\frac{7}{8}$ -in. fasteners,  $F_v = 45.0$  ksi (Table I-B8):  
 $8 \times \frac{7}{8} \times 1 \times 1.35 \times 45.0 = 425$  kips
- (d) 10 rows of 1-in. fasteners,  $F_v = 36.0$  ksi (Table I-B10):  
 $10 \times 1 \times 1 \times 1.35 \times 36.0 = 486$  kips
- (e) 4 rows of  $\frac{3}{4}$ -in. fasteners,  $F_v = 42.0$  ksi (Table I-B4):  
 $4 \times \frac{3}{4} \times 1 \times 1.35 \times 42.0 = 170$  kips
- (f) 6 rows of  $\frac{7}{8}$ -in. fasteners,  $F_v = 50.0$  ksi (Table I-B6):  
 $6 \times \frac{7}{8} \times 1 \times 1.35 \times 50.0 = 354$  kips

**Problem 7**—The bearing value of a fastener in material of a given  $F_v$  value can be expressed as  $t \times$  the bearing value of the fastener in a 1-in. thickness of that material. Equating this to the single shear value of the fastener,  $r_s = t \times$  bearing in 1-in. material, and rearranging,

$$\text{Required } t = \frac{r_s}{\text{Bearing in 1-in. material}}$$

results in a value of  $t$  for which the bearing value of the given material equals the shear value of the fastener. Likewise, in the equation

$$\text{Required } t = \frac{2r_s}{\text{Bearing in 1-in. material}}$$

$t$  is the thickness of the given material whose bearing value will equal the double shear value of the fastener. Refer in Manual Part 4 to tables "Rivets and Threaded Fasteners—Shear" for single and double shear fastener values, and to tables "Rivets and Threaded Fasteners—

Bearing" for bearing values in 1-in. thick material of the specified  $F_v$  value.

- (a)  $\frac{3}{4}$ -in. A502-1 rivet:  
 For single shear,  $t = 6.63/36.5 = 0.182$  in.;  $\frac{3}{16}$ -in.  
 For double shear,  $t = 13.25/36.5 = 0.363$  in.;  $\frac{3}{8}$ -in.
- (b)  $\frac{7}{8}$ -in. A502-1 rivet:  
 For single shear,  $t = 9.02/42.5 = 0.212$  in.;  $\frac{1}{4}$ -in.  
 For double shear,  $t = 18.04/42.5 = 0.424$  in.;  $\frac{7}{16}$ -in.
- (c)  $\frac{3}{4}$ -in. A502-2 rivet:  
 For single shear,  $t = 8.84/36.5 = 0.242$  in.;  $\frac{1}{4}$ -in.  
 For double shear,  $t = 17.67/36.5 = 0.484$  in.;  $\frac{1}{2}$ -in.
- (d)  $\frac{7}{8}$ -in. A502-2 rivet:  
 For single shear,  $t = 12.03/42.5 = 0.283$  in.;  $\frac{5}{16}$ -in.  
 For double shear,  $t = 24.05/42.5 = 0.566$  in.;  $\frac{5}{8}$ -in.

**Problem 8**—Refer to the hanger connection in Text Fig. 5-9. Load is 60.0 kips. Angle and channel web thickness is  $\frac{3}{8}$ -in. All material is A36 steel. 1-in. A502-1 rivets connect the gusset plate to the channel and 1-in. A307 bolts connect the angles to the gusset plate.

- (a) Gusset plate is  $\frac{7}{16}$ -in. thick:  
 Connection of gusset plate to web of channel:  
 Single shear value of 1-in. rivet = 11.78 kips (governs)  
 Bearing of 1-in. rivet on  $\frac{3}{8}$ -in. material = 18.2 kips  
 Number of rivets required =  $60/11.78 = 5.1$ ;  
 use 6 rivets
- Connection of angles to gusset plate:  
 Double shear value of 1-in. A307 bolt = 15.71 kips (governs)  
 Bearing of 1-in. bolt on  $\frac{7}{16}$ -in. material = 21.3 kips  
 Number of bolts required =  $60/15.71 = 3.8$ ; use 4 bolts
- (b) Gusset plate is  $\frac{1}{4}$ -in. thick:  
 Connection of gusset plate to web of channel:  
 Single shear value of 1-in. rivet = 11.78 kips (governs)  
 Bearing of 1-in. rivet in  $\frac{1}{4}$ -in. material = 12.2 kips  
 Number of rivets required =  $60/11.78 = 5.1$ ; use 6 rivets
- Connection of angles to gusset plate:  
 Double shear value of 1-in. A307 bolt = 15.71 kips  
 Bearing of 1-in. bolt on  $\frac{1}{4}$ -in. material = 12.2 kips (governs)  
 Number of bolts required =  $60/12.2 = 4.92$ ; use 5 bolts

## HIGH STRENGTH BOLTED CONNECTIONS

**Problem 9**—The allowable single “shear” value of high strength bolts may be calculated by the formula  $r_s = F_s \times A_b$ .

- (a)  $\frac{3}{4}$ -in. A325-F:  
 $r_s = 15.0 \times 0.4418 = 6.63$  kips
- (b)  $\frac{7}{8}$ -in. A325-F:  
 $r_s = 15.0 \times 0.6013 = 9.02$  kips
- (c) 1-in. A490-F:  
 $r_s = 20.0 \times 0.7854 = 15.71$  kips
- (d)  $\frac{3}{4}$ -in. A325-N:  
 $r_s = 15.0 \times 0.4418 = 6.63$  kips
- (e)  $\frac{7}{8}$ -in. A325-X:  
 $r_s = 22.0 \times 0.6013 = 13.23$  kips
- (f) 1-in. A490-N:  
 $r_s = 22.5 \times 0.7854 = 17.67$  kips

**Problem 10**—The values shown in the tables “Framed Beam Connections—Bolted or Riveted, Table I” in Manual Part 4 may be verified as follows:

- (a) Shear value, 8 rows  $\frac{7}{8}$ -in. A325-F:  
 Outstanding legs (S.S.):  $16 \times 9.02 = 144$  kips  
 Web legs (D.S.):  $8 \times 18.04 = 144$  kips
- (b) Shear value, 8 rows  $\frac{7}{8}$ -in. A325-X:  
 Outstanding legs (S.S.):  $16 \times 13.23 = 212$  kips  
 Web legs (D.S.):  $8 \times 26.46 = 212$  kips
- (c) Shear value, 6 rows 1-in. A490-X:  
 Outstanding legs (S.S.):  $12 \times 25.13 = 302$  kips  
 Web legs (D.S.):  $6 \times 50.27 = 302$  kips
- (d) Shear value, 6 rows 1-in. A490-N:  
 Outstanding legs (S.S.):  $12 \times 17.67 = 212$  kips  
 Web legs (D.S.):  $6 \times 35.34 = 212$  kips
- (e) Bearing value, 8 rows  $\frac{7}{8}$ -in. A325 in 1-in. thick material,  $F_v = 36.0$  ksi:  
 For  $F_v = 36.0$  ksi,  $F_p = 48.6$  ksi  
 Required value =  $8 \times \frac{7}{8} \times 48.6 = 340$  kips
- (f) Bearing value, 6 rows 1-in. A325 in 1-in. thick material,  $F_v = 45.0$  ksi:  
 For  $F_v = 45.0$  ksi,  $F_p = 60.8$  ksi  
 Required value =  $6 \times 1 \times 60.8 = 365$  kips
- (g) Shear value, 4 rows  $\frac{3}{4}$ -in. A490-F:  
 Outstanding legs (S.S.):  $8 \times 8.84 = 70.7$  kips  
 Web legs (D.S.):  $4 \times 17.67 = 70.7$  kips
- (h) Shear value, 4 rows  $\frac{3}{4}$ -in. A325-X:  
 Outstanding legs (S.S.):  $8 \times 9.72 = 77.8$  kips  
 Web legs (D.S.):  $4 \times 19.44 = 77.8$  kips

**Problem 11**—The number of fasteners required to connect the angles to the gusset plate may be calculated as follows:

- (a)  $\frac{3}{4}$ -in. A325-F:  
 Double shear value of one fastener = 13.25 kips  
 Number of bolts required =  $70/13.25 = 5.3$ ;  
 use 6 bolts
- (b)  $\frac{3}{4}$ -in. A325-X:  
 Double shear value of one fastener = 19.44 kips  
 Bearing value of one fastener on the  $\frac{5}{16}$ -in. plate = 11.4 kips (governs)  
 Number of bolts required =  $70/11.4 = 6.1$ ;  
 use 7 bolts
- (c)  $\frac{3}{4}$ -in. A325-N:  
 Double shear value of one fastener = 13.25 kips  
 Bearing value of one fastener on the  $\frac{5}{16}$ -in. plate = 11.4 kips (governs)  
 Number of bolts required =  $70/11.4 = 6.1$ ;  
 use 7 bolts
- (d)  $\frac{3}{4}$ -in. A490-F:  
 Double shear value of one bolt = 17.67 kips  
 Number of bolts required =  $70/17.67 = 4.0$ ;  
 use 4 bolts

**Problem 12**—The number of fasteners required to connect the gusset plate to the channel web may be calculated as follows:

- (a)  $\frac{7}{8}$ -in. A325-F:  
 Single shear value of one bolt = 9.02 kips  
 Number of bolts required =  $70/9.02 = 7.8$ ;  
 use 8 bolts
- (b)  $\frac{7}{8}$ -in. A325-X:  
 Single shear value of one bolt = 13.23 kips  
 Bearing value of one bolt on the  $\frac{1}{4}$ -in. channel web = 10.6 kips (governs)  
 Number of bolts required =  $70/10.6 = 6.6$ ;  
 use 7 bolts
- (c)  $\frac{7}{8}$ -in. A325-N:  
 Single shear value of one bolt = 9.02 kips (governs)  
 Bearing value of one bolt on the  $\frac{1}{4}$ -in. channel web = 10.6 kips  
 Number of bolts required =  $70/9.02 = 7.8$ ;  
 use 8 bolts
- (d)  $\frac{7}{8}$ -in. A490-F:  
 Single shear value of one bolt = 12.03 kips  
 Number of bolts required =  $70/12.03 = 5.8$ ;  
 use 6 bolts

## TENSION IN FASTENERS

**Problem 13**—The allowable tension value of fasteners may be calculated by the formula  $r_t = F_t \times A_b$ .

- (a) 1-in. A502-1 rivet:  
 $r_t = 20.0 \times 0.7854 = 15.71$  kips
- (b) 1-in. A325 bolt:  
 $r_t = 40.0 \times 0.7854 = 31.42$  kips
- (c) 1-in. A307 bolt:  
 $r_t = 20.0 \times 0.6057 = 12.11$  kips
- (d) 1-in. A490 bolt:  
 $r_t = 54.0 \times 0.7854 = 42.41$  kips
- (e) 1-in. A502-2 rivet:  
 $r_t = 27.0 \times 0.7854 = 21.21$  kips
- (f) 1-in. threaded rod, A36 material:  
 $r_t = 22.0 \times 0.6057 = 13.33$  kips

**Problem 14**—The maximum permissible load,  $P$ , for the hanger connection of Fig. 5-13 in the Text, using 6 fasteners of the types specified, may be calculated as follows (the effect of prying action is not considered):

- (a)  $\frac{7}{8}$ -in. A502-1 rivets:  
 $P = 6 \times 12.03 = 72.2$  kips
- (b)  $\frac{7}{8}$ -in. A325 bolts:  
 $P = 6 \times 24.05 = 144$  kips
- (c)  $\frac{7}{8}$ -in. A307 bolts:  
 $P = 6 \times 9.23 = 55.4$  kips
- (d)  $\frac{7}{8}$ -in. A490 bolts:  
 $P = 6 \times 32.47 = 195$  kips

**Problem 15**—Calculations used to determine the maximum capacity,  $P$ , in kips, of the given hanger connection are as follows:

- (a) Value of three  $\frac{7}{8}$ -in. A502-1 rivets attaching the hanger angles to the tee stem:  
 $r_t$  (D.S.) = 18.04 kips;  $P = 3 \times 18.04 = 54.1$  kips  
 $r_t$  (Bearing) = 15.9 kips;  $P = 3 \times 15.9 = 47.7$  kips
- (b) Value of six  $\frac{7}{8}$ -in. A307 bolts attaching the tee to the supporting beam:

From the table "Rivets and Threaded Fasteners—Tension" in Manual Part 4, the tensile capacity,  $r_t$ , of one such fastener is found to be 9.23 kips. Using the nomenclature for problems in prying action (from "Hanger Type Connections" in Manual Part 4) where  $F$  and  $Q$  are, respectively, the externally applied load and prying force on one bolt:

$$F + Q = 9.23 \text{ kips}$$

But, since it is assumed that  $Q = 0.5F$ :

$$1.5F = 9.23 \text{ kips}$$

and the maximum allowable externally applied force  $F = 6.15$  kips per bolt. The capacity of 6 bolts in resisting the applied load in tension is

$$P = 6 \times 6.15 = 36.9 \text{ kips}$$

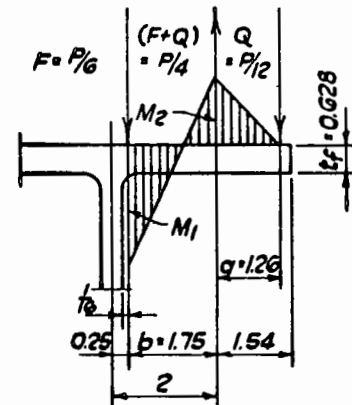


Figure S5.1

- (c) In calculating the bending capacity of the tee flange, the same relations are assumed for  $F$  and  $Q$  (see Fig. S5.1) as for the fasteners in (b) above. Expressing  $F$  and  $Q$  in terms of  $P$  and using the 3-in. length of flange tributary to one bolt:

$$F = P/6 \text{ (6 fasteners)}$$

$$Q = 0.5 \times P/6 = P/12$$

$$F = Q = P/6 + P/12 = P/4$$

$$M_1 = (1.75 \times P/4) - (3.01 \times P/12) = 0.1867P \text{ kip-in.}$$

$$M_2 = 1.26 \times P/12 = 0.105P \text{ kip-in.}$$

$M_1$  governs.

The section modulus of a 3-in. length of tee flange is

$$S = \frac{3 \times 0.628^3}{6} = 0.197 \text{ in.}^3$$

From the formula  $F_b = M/S$ , when  $F_b = 27.0$  ksi (see AISC Specification, Sect. 1.5.1.4.3), the allowable bending moment  $M = 0.197 \times 27.0 = 5.32$  kip-in. Equating  $M_1$  to the allowable bending moment,  $0.1867P = 5.32$  and  $P = 28.5$  kips, the connection capacity produced by resistance to bending in the tee flange.



- (d) Summarizing the above results:  
 From (a), the capacity of the rivets in D.S. is 54.1 kips.  
 From (a), the capacity of the rivets in bearing is 47.7 kips.  
 From (b), the capacity of the bolts in tension is 36.9 kips.  
 From (c), the capacity of the tee flange in bending is 28.5 kips (governs).  
 The maximum capacity of the connection is  $P = 28.5$  kips

**Problem 16**—Calculations for the design of the hanger connection are as follows:

- (a) Flange plate thickness:  
 $b$  (estimated) = 2 in.;  $w = 4$  in.  
 Calculated tensile stress in one fastener  
 $= 85/6 = 14.17$  kips  
 $r_t$  for one  $\frac{3}{4}$ -in. diam. A325 bolt  
 $= 17.67 > 14.17$  kips o.k.  
 For the purpose of this example, let  $r_t = 14.17$  kips. Then,

$$2r_t/w = (2 \times 14.17)/4 = 7.09 \text{ kips}$$

Using the "alternate approach" (Text, Chapter 5), enter the table under "Hanger Type Connections" (Manual, Part 4) with  $b = 2$ , and follow across to the value next larger than 7.09 which is 7.91. The column heading above 7.91 calls for  $t_f = 1\frac{5}{16}$ -in. Increasing this value by 40%,  $1\frac{5}{16} \times 1.40 = 1.31$  in., or  $1\frac{5}{16}$  in. Use a flange plate  $1\frac{5}{16} \times 10 \times 1'-0$ .

- (b) Welds:  
 Minimum weld size for  $1\frac{5}{16}$ -in. plate =  $\frac{5}{16}$ -in. (AISC Specification Sect. 1.17.5).  
 Total weld length available =  $2 \times 12 = 24$  in.  
 Total weld capacity =  $0.928 \times 5 \times 24 = 111 > 85$  kips o.k.  
 (c) Tee stem thickness:  
 Plate thickness necessary to develop shear from the paired  $\frac{5}{16}$ -in. welds is 0.42 in. (Table 6A-II in Appendix to Chapter 6 of the Text). Use a  $\frac{7}{16}$ -in. plate. A recalculation of  $b$  gives  $1\frac{3}{32}$ ; use 2 in. as estimated.

Note that although the tension stress in the plate would have required a thickness of  $t = 85/(12 \times 22) = 0.322$  in., say  $\frac{3}{8}$ -in., it is generally considered good practice to provide a plate that will develop the full strength of the required welds.

**FASTENERS IN COMBINED SHEAR AND TENSION**

**Problem 17**—Using the given sketch and data, the design of the bracing connection is as follows:

- (a) The number of bolts required to attach the bracing angle to the gusset plate is  $P/r_s = 33.0/6.63 = 5.0$ ; use five  $\frac{3}{4}$ -in. A325-F bolts.

- (b) The number of rivets required to attach the gusset plate to the connection angles is  $P/r_s$ .

$$r_s \text{ in D.S.} = 13.25 \text{ kips}$$

$$\text{No. req'd for shear} = 33/13.25 = 2.5$$

$$r_s \text{ in bearing in the } \frac{5}{16}\text{-in. plate} = 11.4 \text{ kips}$$

$$\text{No. req'd for bearing} = 33/11.4 = 2.9 \text{ (governs)}$$

Use three  $\frac{3}{4}$ -in. A502-1 rivets.

- (c) For the attachment of the connection angles to the column flange, assume six  $\frac{3}{4}$ -in. A502-1 rivets in two lines of three rivets each. Because of the  $1\frac{3}{4}$ -in. offset between work points, it will be convenient to use  $3\frac{1}{2}$ -in. spacing in both legs of the connection angles. As shown in Fig. S5.2, the diagonal force,  $P$ , of 33.0 kips is resolved graphically into a vertical shear force of  $P_v = 23.3$  kips and a horizontal tension force of  $P_t = 23.3$  kips.

Fastener stresses are:

$$f_s = \frac{P_v}{\Sigma A_b} = \frac{23.3}{6 \times 0.4418} = 8.79 \text{ ksi}$$

$$f_t = \frac{P_t}{\Sigma A_b} = \frac{23.3}{6 \times 0.4418} = 8.79 \text{ ksi}$$

Allowable tensile stress from the interaction formula (AISC Specification Sect. 1.6.3) is:

$$\begin{aligned} F_t &= 28.0 - 1.6f_s \leq 20.0 \\ &= 28.0 - (1.6 \times 8.79) \leq 20.0 \\ &= 13.9 < 20.0 \text{ ksi} \end{aligned}$$

Since  $f_s = 8.79 \text{ ksi} < F_s = 15.0 \text{ ksi}$  and  $f_t = 8.79 \text{ ksi} < F_t = 13.9 \text{ ksi}$

the fasteners are compatible in shear and tension.

Increasing the tension stress on these rivets by 50% to provide for the assumed prying force results in

$$\begin{aligned} f_t &= 8.79 \times 1.5 \\ &= 13.2 \text{ ksi} < \text{allowable } F_t = 13.9 \text{ ksi o.k.} \end{aligned}$$

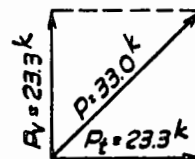


Figure S5.2

- (d) Assume a trial angle thickness of  $\frac{1}{2}$ -in. (See Fig. S5.3). Using the nomenclature for problems in prying action (from "Prying Forces" in Chapter 5 of the Text), where  $F$  and  $Q$  are, respectively, the applied forces and prying forces on one fastener,  $F = 23.3/6 = 3.88$  kips and  $Q$ , assumed to be  $0.5F = 3.88 \times 0.5 = 1.94$  kips. The total bolt load is  $F + Q = 5.82$  kips. Substituting these values in the moment equations for  $M_1$  and  $M_2$ ,

$$M_1 = (5.82 \times 2.03) - (1.94 \times 3.03) = 5.93 \text{ kip-in.}$$

$$M_2 = 1.94 \times 1.0 = 1.94 \text{ kip-in.}$$

$$M_1 = 5.93 \text{ kip-in. governs.}$$

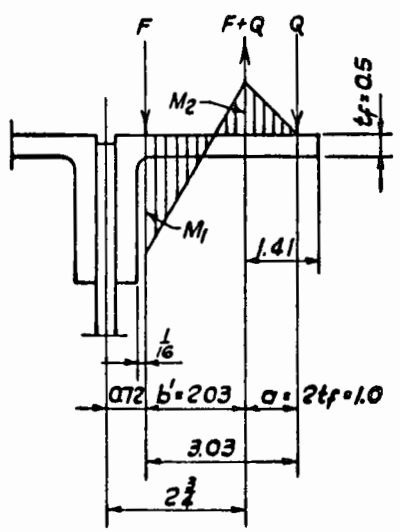


Figure S5.3

Since these moment values are based on the forces applicable to one fastener, the section modulus of the angle leg will be calculated for  $3\frac{1}{2}$  in., the length of angle tributary to one fastener.

$$S = \frac{3.50 \times 0.5^3}{6} = 0.146 \text{ in.}^3$$

The stress in bending is calculated by the formula  $f_b = M/S = 5.93/0.146 = 40.6 \text{ ksi} > \text{allowable } F_b = 27.0 \text{ ksi n.g.}$

Since the allowable bending stress is exceeded, it will be necessary to redesign the connection to provide a greater section modulus. This may be done by increasing either the angle thickness or the fastener spacing, or both. For this problem it was decided to increase the angle thickness to  $\frac{5}{8}$ -in. The connection geometry for this thickness results in  $t_f = 0.625$  in.,  $a = 1.25$  in., and  $b = 1.91$  in.

Recalculating moments, section modulus and bending stress as above,

$$M_1 = 4.99 \text{ kip-in. (governs)}$$

$$M_2 = 2.43 \text{ kip-in.}$$

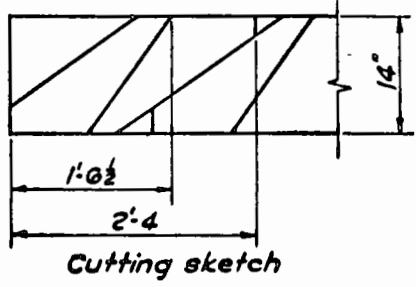
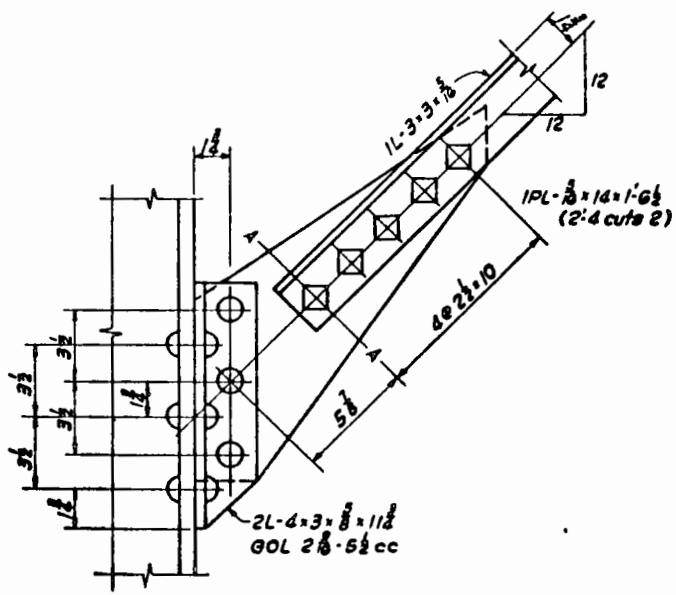
$$S = 0.228 \text{ in.}^3$$

$$f_b = 4.99/0.228 = 21.9 < 27.0 \text{ ksi o.k.}$$

Use  $4 \times 3 \times \frac{5}{8}$  angles.

Since the calculations are based on a tributary length of angle of  $3\frac{1}{2}$  in. on all fasteners attaching the angles to the column, a minimum end edge distance of  $1\frac{3}{4}$  in. will be required.

- (e) For layout of connection, see Fig. S5.4.
- (f) Tensile stress in plate at the first hole in the bracing angle (on line A-A, Fig. S5.4):  
 Scaled width of plate at A-A = 6 in.  
 Net width, with one hole out, =  $6 - \frac{7}{8} = 5\frac{1}{8}$  in.  
 Net section =  $5.125 \times 0.313 = 1.60 \text{ in.}^2$   
 Tensile stress  $f_t = P/A = 33.0/1.60 = 20.6 \text{ ksi} < F_t = 22.0 \text{ ksi o.k.}$



Cutting sketch

Figure S5.4

### ECCENTRIC CONNECTIONS—FASTENERS IN SHEAR ONLY

**Problem 18**—The resultant forces,  $f_R$ , at the extreme fasteners are calculated as follows (see Fig. S5.5):

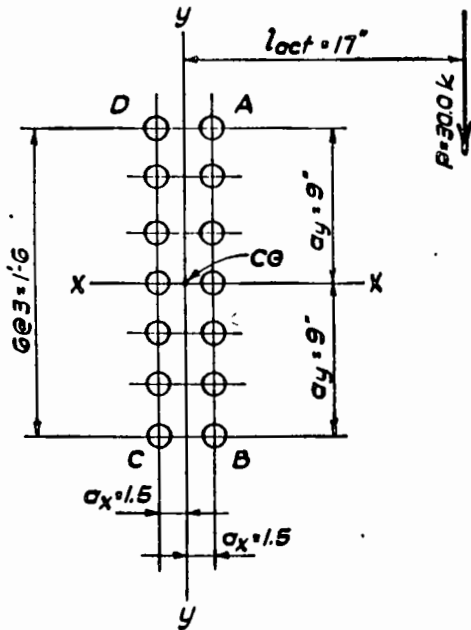


Figure S5.5

- (a) Find maximum  $f_R$ :

$$l_{eff} = l_{oct} - \left(\frac{1+n}{2}\right) = 17 - \left(\frac{1+7}{2}\right) = 13 \text{ in.}$$

$$I_x = 4(3^2 + 6^2 + 9^2) = 504.0 \text{ in.}^2$$

$$I_y = 14 \times 1.5^2 = 31.5 \text{ in.}^2$$

$$I_p = 535.5 \text{ in.}^2$$

$$f_1 = 30/14 = 2.14 \text{ kips}$$

$$f_2 = \frac{Pl_{eff}(a_x)}{I_p} = \frac{30 \times 13 \times 1.5}{535.5} = 1.09 \text{ kips}$$

$$f_3 = \frac{Pl_{eff}(a_y)}{I_p} = \frac{30 \times 13 \times 9}{535.5} = 6.55 \text{ kips}$$

$$f_R = \sqrt{f_3^2 + (f_1 + f_2)^2} = \sqrt{6.55^2 + (2.14 + 1.09)^2} = 7.30 \text{ kips}$$

- (b) Check maximum  $f_R$  by use of tables of eccentric loads on fasteners in Manual Part 4.

From Table XI:

For  $l_{eff} = 12$  and  $n = 7$ :  $C = 4.40$

For  $l_{eff} = 14$  and  $n = 7$ :  $C = 3.85$

Interpolating for  $l_{eff} = 13$ :  $C = (4.40 + 3.85)/2 = 4.125$

$$r_s = f_R = P/C = 30/4.125 = 7.27 \approx 7.30 \text{ kips o.k.}$$

- (c) For graphical construction, see Fig. S5.6. Scaled values of  $f_R$  for fasteners A and B are identical with calculated maximum value of  $f_R = 7.3$  kips.

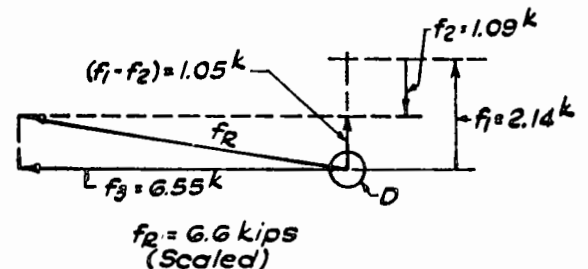
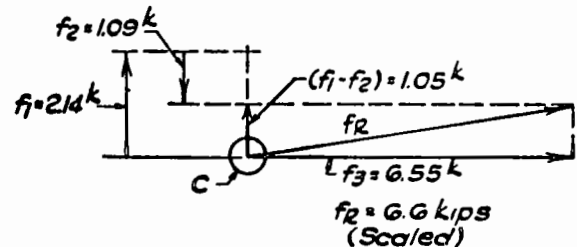
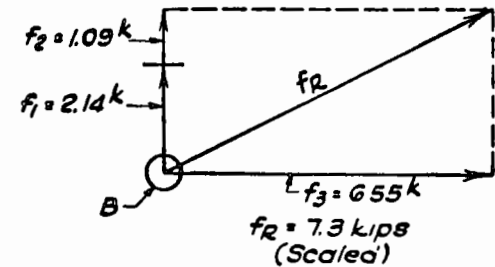
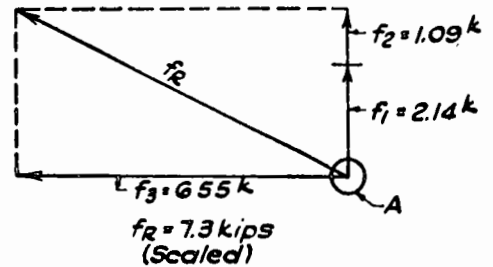


Figure S5.6

Calculations for fasteners C and D:

$$f_R = \sqrt{f_3^2 + (f_1 - f_2)^2} = \sqrt{6.55^2 + (2.14 - 1.09)^2} = 6.63 \approx 6.6 \text{ kips (as scaled) o.k.}$$

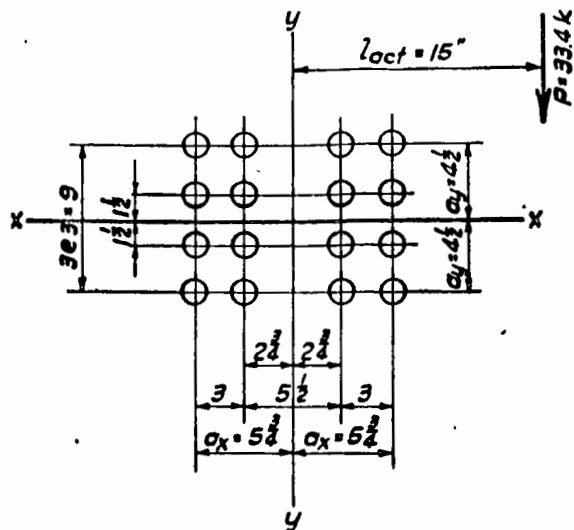


Figure S5.7

**Problem 19**—The resultant force at the extreme fasteners is calculated from Fig. S5.7 as follows.

(a) Find maximum  $f_R$ :

$$l_{eff} = l_{act} - \left(\frac{1+n}{2}\right) = 15 - \left(\frac{1+4}{2}\right) = 12.5 \text{ in.}$$

$$I_x = 8(1.5^2 + 4.5^2) = 180.0 \text{ in.}^2$$

$$I_y = 8(2.75^2 + 5.75^2) = 325.0 \text{ in.}^2$$

$$I_p = 505.0 \text{ in.}^2$$

$$f_1 = 33.4/16 = 2.09 \text{ kips}$$

$$f_2 = \frac{Pl_{eff}(a_x)}{I_p} = \frac{33.4 \times 12.5 \times 5.75}{505.0} = 4.75 \text{ kips}$$

$$f_3 = \frac{Pl_{eff}(a_y)}{I_p} = \frac{33.4 \times 12.5 \times 4.5}{505.0} = 3.72 \text{ kips}$$

$$f_R = \sqrt{f_3^2 + (f_1 + f_2)^2} = \sqrt{3.72^2 + (2.09 + 4.75)^2} = 7.79 \text{ kips}$$

(b) Check maximum  $f_R$  by use of tables of eccentric loads on fasteners in Manual Part 4.

From Table XIII:

$$\text{For } l_{eff} = 12 \text{ and } n = 4: \quad C = 4.42$$

$$\text{For } l_{eff} = 14 \text{ and } n = 4: \quad C = 3.93$$

Interpolating for  $l_{eff} = 12.5$ :

$$C = 4.42 - \frac{1}{4}(4.42 - 3.93) = 4.42 - 0.12 = 4.30$$

$$r_o = f_R = P/C = 33.4/4.30 = 7.77 \approx 7.79 \text{ kips o.k.}$$

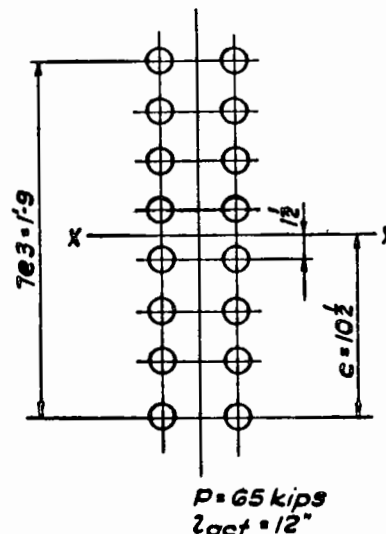


Figure S5.8

### ECCENTRIC CONNECTIONS—FASTENERS IN COMBINED SHEAR AND TENSION

**Problem 20**—Analysis of the bracket connection (see Fig. S5.8) utilizing the several given fastener specifications is as follows:

(a)  $\frac{7}{8}$ -in. A325-X bolts. Solve steps (1) through (4) as in Example 8 (Case I) in Chapter 5 of the Text:

(1) Shear stress:

$$f_v = \frac{P}{A_b N} = \frac{65.0}{0.6013 \times 16} = 6.76 \text{ ksi} < f_v = 22.0 \text{ ksi o.k.}$$

(2) Moment of inertia and section modulus:

$$I_x = A_b \Sigma(d_y)^2 = 4 \times 1.5^2 = 9$$

$$4 \times 4.5^2 = 81$$

$$4 \times 7.5^2 = 225$$

$$4 \times 10.5^2 = 441$$

$$\Sigma(d_y)^2 = 756 \text{ in.}^2$$

$$I_x = 0.6013 \times 756 = 455 \text{ in.}^4$$

$$S = I_x/c = 455/10.5 = 43.3 \text{ in.}^3$$

(3) Tension stress:

$$f_t = M/S = \frac{65.0 \times 12}{43.3} = 18.01 \text{ ksi}$$

(Note that  $l_{eff} = l_{act}$  for problems involving combined shear and tension in fasteners.)

(4) Allowable  $F_t$  by the interaction formula:

$$F_t = 50.0 - 1.6f_v \leq 40.0 = 50.0 - (1.6 \times 6.76) = 39.2 < 40.0 \text{ ksi o.k.}$$

Since  $f_t = 18.01 \text{ ksi} < F_t = 39.2 \text{ ksi}$ , the shear-tension relationship is compatible.

## (5) Prying action on fasteners:

From the nomenclature and methods for solving prying action problems (see "Prying Forces" in Chapter 5 of the Text),  $F$  is defined as the external load on one fastener. In this problem,  $F$  is the tensile force on the most highly stressed fastener  $= f_t \times A_b = 18.01 \times 0.6013 = 10.83$  kips.

With  $F = 10.83$  kips,  $w = 3$  in. (fastener pitch),  $d_b = 7/8$ -in. and  $t_f = 0.911$  in. given, and, from Fig. S5.9,  $a = 1.82$  in. and  $b = 1.67$  in., solve

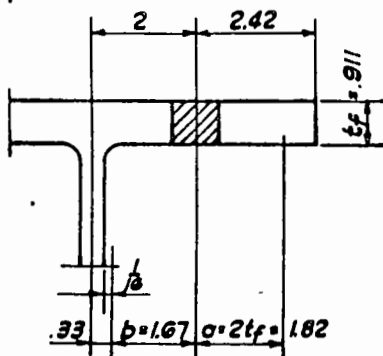


Figure S5.9

for  $Q$  (the prying force on one bolt) by substituting the above values in Equation (5.3) of Chapter 5 of the Text:

$$Q = 10.83 \times \left[ \frac{100 \times 1.67 \times 0.875^2 - 18 \times 3 \times 0.911^2}{70 \times 1.82 \times 0.875^2 + 21 \times 3 \times 0.911^2} \right]$$

$$= 10.83 \times 0.555 = 6.01 \text{ kips}$$

The total tension force on one bolt is:

$$F + Q = 10.83 + 6.01 = 16.84 \text{ kips}$$

Allowable  $F_t = 39.2$  ksi (from the interaction formula)

$$\text{Allowable } r_t = 39.2 \times 0.6013 = 23.57 \text{ kips}$$

Since  $F + Q = 16.84$  kips  $<$  allowable  $r_t = 23.57$  kips, the fasteners are not overstressed by prying action.

## (6) Bending stress in the tee flange:

Substituting the values of  $Q$  and  $F + Q$  in the moment equations for  $M_1$  and  $M_2$  (see Fig. S5.10):

$$M_1 = (1.67 \times 16.84) - (3.49 \times 6.01)$$

$$= 7.15 \text{ kip-in.}$$

$$M_2 = 1.82 \times 6.01$$

$$= 10.93 \text{ kip-in. (governs)}$$

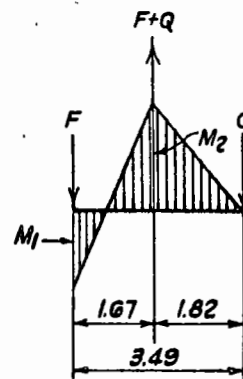


Figure S5.10

The section modulus of a 3-in. length of flange is:

$$S = \frac{3.0 \times 0.911^2}{6} = 0.415 \text{ in.}^3$$

The stress in bending is:

$$f_b = M/S = 10.93/0.415$$

$$= 26.3 \text{ ksi} < \text{allowable stress} = 27.0 \text{ ksi o.k.}$$

(b)  $7/8$ -in. A325-F bolts: The solution of this part of the problem is the same as for (a) above, except for the omission of step 4:

## (1) Shear stress (see step 1 in part (a) above):

$$f_v = 6.76 \text{ ksi} < F_v = 15.0 \text{ ksi o.k.}$$

## (2) Moment of inertia and section modulus (see step 2 in part (a) above):

$$I_x = 455 \text{ in.}^4$$

$$S = 43.3 \text{ in.}^3$$

## (3) Tension stress (see step 3 in part (a) above):

$$f_t = 18.01 \text{ ksi} < F_t = 40.0 \text{ ksi o.k.}$$

(4) High strength bolts in friction-type connections are exempt from the limitations on allowable  $F_t$  imposed by the interaction formulas.  $F_t$  remains 40.0 ksi

## (5) Prying action on fasteners (see step 5 in part (a) above):

$$F = 10.83 \text{ kips}$$

$$Q = 6.01 \text{ kips}$$

$$F + Q = 16.84 \text{ kips}$$

$$\text{Allowable } r_t = 40.0 \times 0.6013 = 24.05 \text{ kips}$$

Since  $F + Q = 16.84$  kips  $<$   $r_t = 24.05$  kips, the fasteners are not overstressed by prying action.

## (6) Bending in the tee flange (see step 6 in part (a) above):

$$f_b = 26.3 \text{ ksi} < \text{allowable } F_b = 27.0 \text{ ksi o.k.}$$

(c)  $\frac{1}{8}$ -in. A502-1 rivets: The investigation of the bracket connection when Case I is used with A502-1 rivets is as follows:

- (1) Shear stress (see step 1 in part (a) above):  
 $f_v = 6.76 \text{ ksi} < F_v = 15.0 \text{ ksi o.k.}$
- (2) Moment of inertia and section modulus (see step a-2 above):  
 $I_x = 455 \text{ in.}^4$   
 $S = 43.3 \text{ in.}^3$
- (3) Tension stress (see step 3 of part (a) above):  
 $f_t = 18.01 \text{ ksi}$
- (4) Allowable  $F_t$  by interaction formula:  
 $F_t = 28.0 - 1.6f_v \leq 20.0$   
 $= 28.0 - (1.6 \times 6.76) = 17.2 < 20.0 \text{ ksi}$   
 However,  $f_t = 18.01 \text{ ksi} > \text{allowable } F_t = 17.2 \text{ ksi}$ , and the shear-tension relationship is not compatible. If the assumptions of Case I are to be used, the connection must be redesigned.

(d)  $\frac{1}{8}$ -in. A502-1 rivets: The investigation of the bracket connection when Case II is used with A502-1 rivets is as follows:

- (1) Shear stress (see step 1 in part (a) above):  
 $f_v = 6.76 \text{ ksi} < F_v = 15.0 \text{ ksi o.k.}$

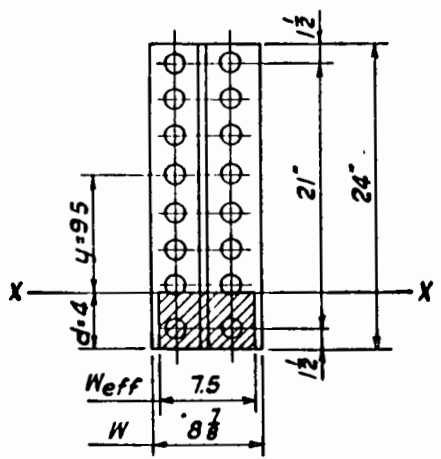


Figure S5.11

(2) Locate neutral axis (see Example 9 in Chapter 5 of the Text):

First assumption: Try location  $\frac{1}{6}$  of the depth of the bracket from the bottom (see Fig. S5.11).  
 $\frac{1}{6} \times 24 = 4 \text{ in.}$   
 $W_{eff} = 8 \times \frac{15}{16} = 7.5 \text{ in.}$   
 $W_{eff} \times d \times d/2 = \Sigma A_b y$   
 $7.5 \times 4 \times 2 = 14 \times 0.6013 \times 9.5$   
 $60 \neq 80$

This marked inequality indicates the need of an upward adjustment of the neutral axis.

Correction: Move the neutral axis up  $\frac{1}{2}$ -in. This will place it on the second row of rivets from the bottom of the bracket (see Fig. S5.12):

$$W_{eff} \times d \times d/2 = \Sigma A_b y$$

$$7.5 \times 4.5 \times 2.25 = 12 \times 0.6013 \times 10.5$$

$$75.9 \approx 75.8$$

This result is close to balancing the moments of area, so the corrected location of the neutral axis will be used in subsequent calculations.

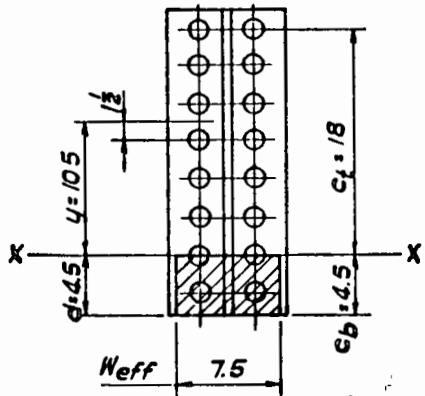


Figure S5.12

(3) Moment of inertia and section modulus:

$$I_x = A_b \Sigma (d_r)^2 \text{ (fasteners)}$$

$$\Sigma (d_r)^2 = 2(3^2 + 6^2 + 9^2 + 12^2 + 15^2 + 18^2)$$

$$= 1638 \text{ in.}^2$$

$$I_x \text{ (fasteners)} = 0.6013 \times 1638 = 985 \text{ in.}^4$$

$$I_x \text{ (compression area)} = \frac{W_{eff} \times d^3}{3}$$

$$= \frac{7.5 \times (4.5)^3}{3} = 228 \text{ in.}^4$$

$$I_x \text{ (total)} = 985 + 228 = 1213 \text{ in.}^4$$

$$S_{top} = I_x / c_1 = 1213 / 18 = 67.4 \text{ in.}^3$$

$$S_{bot} = I_x / c_2 = 1213 / 4.5 = 270 \text{ in.}^3$$

(4) Tension stress (fasteners):

$$f_t = M / S_{top} = \frac{65 \times 12}{67.4} = 11.57 \text{ ksi}$$

(5) Allowable  $F_t$  by the interaction formula:

$$F_t = 28.0 - 1.6f_v \leq 20.0$$

$$= 28.0 - 1.6 \times 6.76 = 17.2 \text{ ksi} < 20.0 \text{ ksi}$$

Since  $f_t = 11.57 \text{ ksi} < F_t = 17.2 \text{ ksi}$ , the shear-tension relationship is compatible.

(6) Bearing stress at the bottom of the bracket:

$$f_p = M/S_{bot} = \frac{65 \times 12}{270} = 2.89 \text{ ksi}$$

$$\text{Allowable } F_p = 0.90F_y = 33.0 \text{ ksi}$$

Since 2.89 ksi < 33.0 ksi, bearing stress is o.k.

(7) Prying action on fasteners:

The external load,  $F$ , on the most highly stressed rivet is:

$$F = f_t \times A_b = 11.57 \times 0.6013 = 6.96 \text{ kips}$$

For rivets,  $Q = 0.5F$

$$= 0.5 \times 6.96 = 3.48 \text{ kips}$$

$$F + Q = 6.96 + 3.48 = 10.44 \text{ kips}$$

Since the allowable  $F_t = 17.2$  (see step 5 in part (d) above), the allowable load per rivet,  $r_t = 17.2 \times 0.6013 = 10.34$  kips.

Although the allowable  $r_t = 10.34$  kips is slightly less than the total load,  $F + Q = 10.44$  kips due to external and prying forces, the results will be acceptable.

(8) Bending stress in the tee flange:

Calculating the moments  $M_1$  and  $M_2$ , and the section modulus as in step 6 of part (a) above:

$$M_1 = (1.67 \times 10.44) - (3.49 \times 3.48) \\ = 5.28 \text{ kip-in.}$$

$$M_2 = 1.82 \times 3.48 = 6.33 \text{ kip-in. (governs)}$$

$$S = \frac{3.0 \times 0.911^2}{6} = 0.415 \text{ in.}^3$$

$$f_b = M/S = 6.33/0.415 \\ = 15.3 \text{ ksi} < \text{allowable stress} = 27.0 \text{ ksi} \\ \text{o.k.}$$

## SOLUTIONS/CHAPTER 6

### Connections—Welded

#### STRESSES IN CONCENTRICALLY LOADED FILLET WELDS

Problem 1—See Fig. S6.1.

(a) Length available for welding:  $2 \times 6 = 12$  in.

Required shear value of weld per lin. in.:

$$f_R = 39/12 = 3.25 \text{ kips/lin. in.}$$

$$f_R = 0.928D$$

$$D = 3.25/0.928 = 3.49 \text{ sixteenths}$$

Use  $\frac{1}{4}$ -in. fillet weld.

(b) The capacity of the bars to resist stress concentration due to weld may be determined from the formula  $f_R \leq F_t \times t$ :

$$F_t \times t = 22.0 \times \frac{5}{16} = 6.88 \text{ kips/lin. in.}$$

$$f_R = 3.25 < 6.88 \text{ kips/lin. in. o.k.}$$

By referring to Table 6A-II in Appendix to Chapter 6, it is found that a  $\frac{1}{4}$ -in. fillet weld, made with E70XX electrodes on one side of the bar, will not cause overstress in tension unless the thickness of the bar is less than  $\frac{1}{2} \times 0.337 = 0.168$  in. Hence,  $\frac{5}{16}$ -in. thickness o.k.

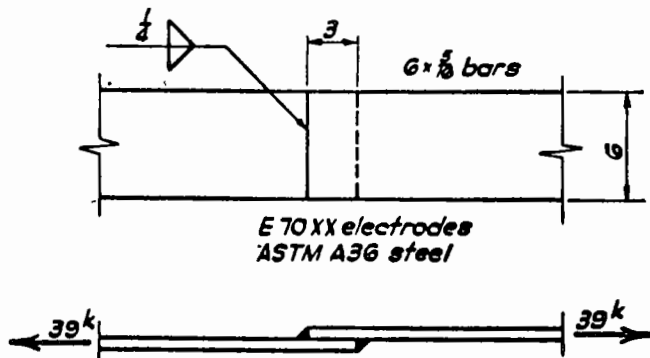


Figure S6.1

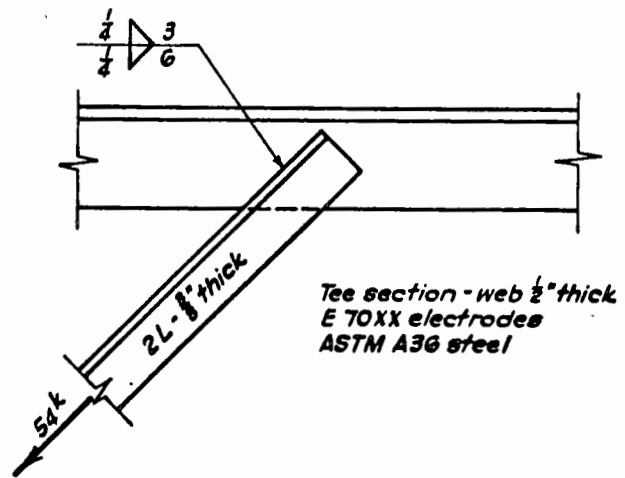


Figure S6.2

Problem 2—See Fig. S6.2.

(a) Length available for welding:  $2(6 + 3) = 18$  in.

Required shear value of weld per lin. in.:

$$f_R = 54/18 = 3.0 \text{ kips/lin. in.}$$

$$f_R = 0.928D$$

$$D = 3.0/0.928 = 3.2 \text{ sixteenths}$$

Use  $\frac{1}{4}$ -in. fillet weld.

(b) The capacity of the tee stem to resist stress concentration due to weld may be determined from the formula  $f_R \leq F_v \times t$ :

$$F_v \times t = 14.5 \times \frac{1}{2} = 7.25 \text{ kips/lin. in.}$$

$$f_R = 2 \times 3.0 = 6.0 \text{ kips/lin. in. (welds on both sides of tee stem)} < 7.25 \text{ kips/lin. in. o.k.}$$

It will be noted by reference to Table 6A-II in the Appendix to Chapter 6 that the minimum thickness of A36 steel stressed in shear by the two  $\frac{1}{4}$ -in. welds (E70XX electrode) should be 0.512 in., which slightly exceeds the  $\frac{1}{2}$ -in. tee web thickness. However, the indicated overstress does not occur, since the  $\frac{1}{4}$ -in. welds are not fully loaded. This is evident from the above solution, in which  $D = 3.2$ , showing that a weld size only slightly larger than  $\frac{3}{16}$ -in. is required.



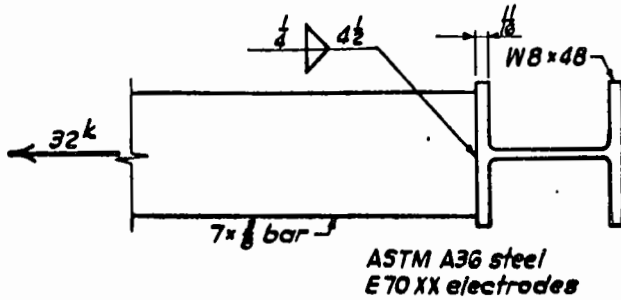


Figure S6.3

Problem 3—See Fig. S6.3.

(a) Select size of fillet:

Minimum size of fillet, according to Specification Sect. 1.17.5, for  $1\frac{1}{16}$ -in. material (thickness of column flange) is  $\frac{1}{4}$ -in.

Maximum size of fillet so as not to overstress  $\frac{3}{8}$ -in. plate in tension is  $\frac{1}{4}$ -in., from Table 6A-II in Appendix to Chapter 6.

Hence, a  $\frac{1}{4}$ -in. fillet weld will be used.

(b) Strength of weld:

For  $\frac{1}{4}$ -in. fillet weld, E70XX electrodes:

$$f_R = 0.928D = 0.928 \times 4 = 3.71 \text{ kips/lin. in.}$$

(c) Length of weld required:

$$l = 32/3.71 = 8.6$$

Round off to 9 in. ( $4\frac{1}{2}$ -in. on top and bottom of plate.)

Problem 4—See Fig. S6.4.

(a) Select size of fillet weld:

Minimum size of fillet according to Specification Sect. 1.17.5 for  $1\frac{1}{16}$ -in. material (thickness of column flange) is  $\frac{1}{4}$ -in.

Maximum size of fillet so as not to overstress  $\frac{3}{8}$ -in. bar in shear is about  $\frac{3}{16}$ -in., from Table 6A-II in Appendix to Chapter 6.

Hence, a  $\frac{1}{4}$ -in. fillet will be used, reduced in strength by the proportion which the actual thickness of plate bears to the required thickness.

(b) Strength of weld:

For  $\frac{1}{4}$ -in. fillet weld:

$$f_R = 0.928D = 0.928 \times 4 = 3.71 \text{ kips/lin. in.}$$

This value of  $f_R$  must be reduced in order not to overstress the  $\frac{3}{8}$ -in. plate in shear. According to Table 6A-II in Appendix to Chapter 6, required plate thickness is 0.512 in. Hence,

$$\text{corrected } f_R = \frac{0.375}{0.512} \times 3.71 = 2.72 \text{ kips/lin. in.}$$

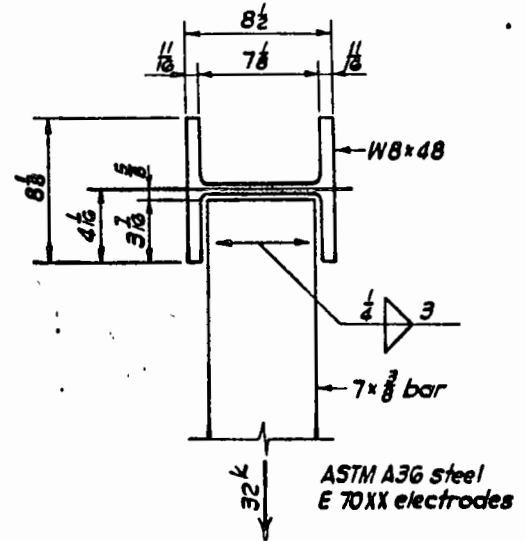


Figure S6.4

(c) Length of weld required:

$$l = 32/2.72 = 11.8 \text{ in.}$$

Round off to 12 in. and use 3 in. of weld near and far side at both edges of bar.

Note: Comparing the solutions of Problems 3 and 4, it will be noted that 9 in. of  $\frac{1}{4}$ -in. fillet weld is required in Problem 3, and 12 in. is required in Problem 4, for the same force of 32 kips. This is because of the reduced value of the  $\frac{1}{4}$ -in. weld in Problem 4 in order not to overstress the  $7 \times \frac{3}{8}$  bar in shear.

### STRESSES IN ECCENTRICALLY LOADED FILLET WELDS

Problem 5—See Fig. 6.5.

$$f_i = \frac{P}{2l} = \frac{24}{2 \times 16} = 0.75 \text{ kips/lin. in.}$$

$$f_m = \frac{Pe}{2(l^2/6)} = \frac{24 \times 9}{2(16^2/6)} = \frac{216}{2 \times 42.7} = 2.53 \text{ kips/lin. in.}$$

$$f_R = \sqrt{(f_i)^2 + (f_m)^2} = \sqrt{0.75^2 + 2.53^2} \\ = \sqrt{0.56 + 6.40} = 2.64 \text{ kips/lin. in.}$$

$$D = \frac{f_R}{0.928} = \frac{2.64}{0.928} = 2.84 \text{ sixteenths}$$

Use  $\frac{3}{16}$ -in. fillet welds.

The  $\frac{3}{16}$ -in. fillet weld is compatible with the  $\frac{1}{2}$ -in. column flange thickness (refer to Specification Sect. 1.17.5).

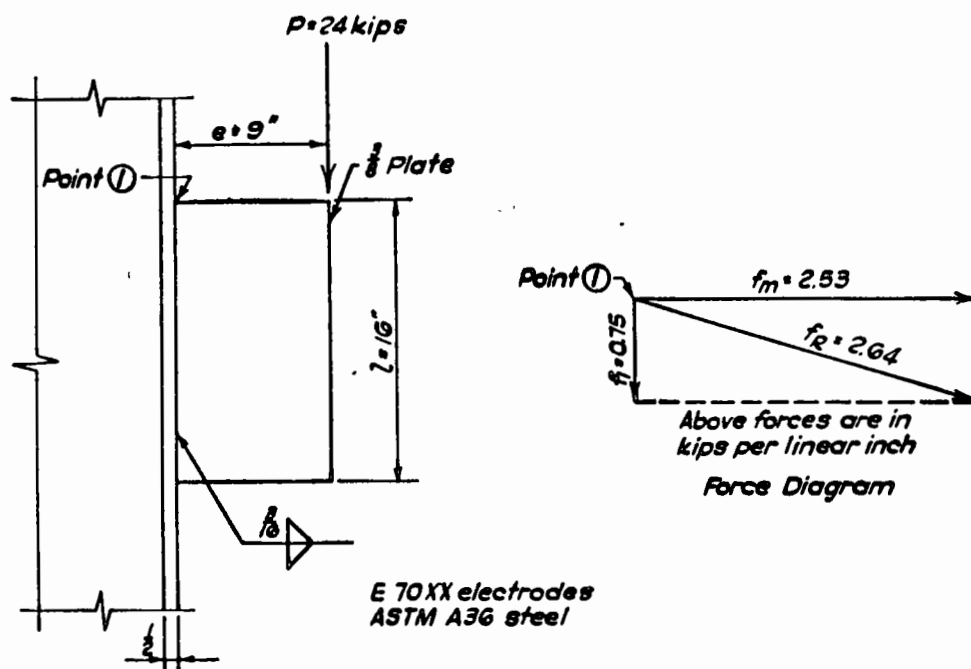


Figure S6.5

Investigate shear stress concentration in the  $\frac{3}{8}$ -in. plate:

$$f_R \leq F_v \times t$$

$$F_v \times t = 14.5 \times 0.375 = 5.43 \text{ kips/lin. in.}$$

$$\begin{aligned} f_R &= 2 \times 2.64 \text{ (welds both sides)} \\ &= 5.28 < 5.43 \text{ kips/lin. in. o.k.} \end{aligned}$$

Note: This is a conservative analysis, as only the vertical force produces shear in the plate. As will be observed from the force diagram above, the vertical force  $f_i$  is relatively small compared with the horizontal force  $f_m$  which produces tension in the plate. As the allowable tensile stress  $F_t = 22.0$  ksi and the allowable shear stress  $F_v = 14.5$  ksi, it is unnecessary to investigate the tensile stress concentration when the shear stress investigation discloses no overstress.

Problem 6—See Fig. S6.6.

Center of gravity of weld group:

$$\bar{X} = \frac{2 \times 3 \times 1.5}{3 + 3 + 17.5} = \frac{9.0}{23.5} = 0.38 \text{ in.}$$

$$d_{xv} = 0.38$$

$$d_{xh} = 1.50 - 0.38 = 1.12$$

$$e = 3.50 - 0.38 = 3.12$$

Polar Moment of Inertia  $I_p$  of weld group:

$$I_x \text{ (vert. weld)} = \frac{I_v^3}{12} = \frac{17.5^3}{12} = 447$$

$$\begin{aligned} I_x \text{ (horiz. weld)} &= 2I_h(d_v)^2 \\ &= 2 \times 3.0 \times 8.75 = 459 \\ \Sigma I_x &= 906 \end{aligned}$$

$$I_y \text{ (vert. weld)} = I_v(d_x)^2 = 17.5 \times 0.38^2 = 3$$

$$\begin{aligned} I_y \text{ (horiz. weld)} &= 2 \left[ \frac{I_h^3}{12} + I_h(d_x)^2 \right] \\ &= 2 \left[ \frac{3^3}{12} + 3(1.12)^2 \right] \\ &= 2(2.25 + 3 \times 1.25) = 12 \\ \Sigma I_y &= 15 \end{aligned}$$

$$I_p = I_x + I_y = 906 + 15 = 921 \text{ in.}^3$$

Since this  $I_p$  is for one weld group only, and there are two weld groups (one for each angle), the load  $P$  will be divided by two in subsequent calculations.

Components:

$$f_1 = \frac{P/2}{l} = \frac{90/2}{17.5 + 3 + 3} = \frac{45}{23.5} = 1.91 \text{ kips/lin. in.}$$

$$f_2 = \frac{MC_x}{I_p} = \frac{90/2 \times 3.12 \times 2.62}{921} = 0.40 \text{ kips/lin. in.}$$

$$f_3 = \frac{MC_y}{I_p} = \frac{90/2 \times 3.12 \times 8.75}{921} = 1.33 \text{ kips/lin. in.}$$

$$\begin{aligned} f_R &= \sqrt{(f_3)^2 + (f_1 + f_2)^2} \\ &= \sqrt{1.33^2 + (1.91 + 0.40)^2} = \sqrt{1.77 + 5.34} \\ &= 2.67 \text{ kips/lin. in.} \end{aligned}$$

$$\text{Required } D = \frac{2.67}{0.928} = 2.88 \text{ sixteenths}$$

Use  $\frac{3}{16}$ -in. fillet weld.

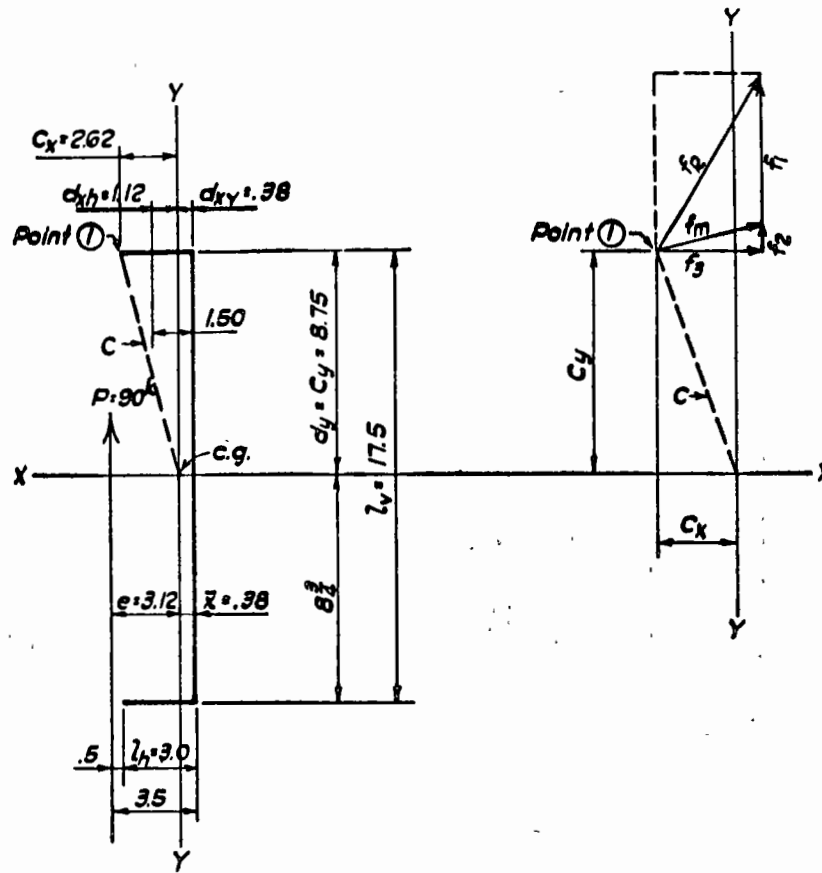


Figure S6.6

From Table 6A-II in Appendix to Chapter 6, a  $\frac{3}{16}$ -in. fillet weld made with E70XX electrodes on both sides of an A36 beam web requires a web thickness of 0.38 in. to avoid overstressing the base material in shear.\* The web thickness of a W24×E4 is 0.47 in., and is therefore adequate.

The above answer will be checked, using Table XVI in Manual Part 4:

$$\begin{aligned}
 al &= 3.12 \\
 kl &= 3.0 \\
 l &= 17.5 \\
 a &= 3.12/17.5 = 0.18 \\
 k &= 3.0/17.5 = 0.17
 \end{aligned}$$

By interpolation,  $C = 0.89$ .

$$D = \frac{P}{Cl} = \frac{90/2}{0.89 \times 17.5} = 2.89 \text{ sixteenths}$$

\* The use of Table 6A-II for determination of required minimum web thickness in connections of this type welded in a conservative answer. As explained in Chapter 8 under Welded Connections, Table III in Manual Part 4 may be used for such connections. This table shows that a minimum web thickness of 0.31 in. is required for the connection in this problem.

This is a good check of the 2.88 sixteenths computed without the use of the eccentric load tables.

Problem 7—See Fig. S6.7.

(a) Determine size of fillet weld:  
Using Table XIV in Manual Part 4:

$$\begin{aligned}
 P &= 51 \text{ kips} \\
 al &= 16 \\
 kl &= 12 \\
 l &= 24 \\
 a &= 16/24 = 0.67 \\
 k &= 12/24 = 0.50
 \end{aligned}$$

From table, by interpolation:

$$\begin{aligned}
 C &= 0.592 \\
 D &= \frac{P}{CC_1l} = \frac{51}{0.592 \times 1 \times 24} \\
 &= 3.6 \text{ sixteenths}
 \end{aligned}$$

Use  $\frac{1}{4}$  in. fillet welds.

Flange thickness of W14×84 is  $\frac{3}{4}$ -in. Specification Sect. 1.17.5 requires minimum fillet weld of  $\frac{1}{4}$ -in. for  $\frac{3}{4}$ -in. thickness of material. Hence,  $\frac{1}{4}$ -in. weld is o.k.

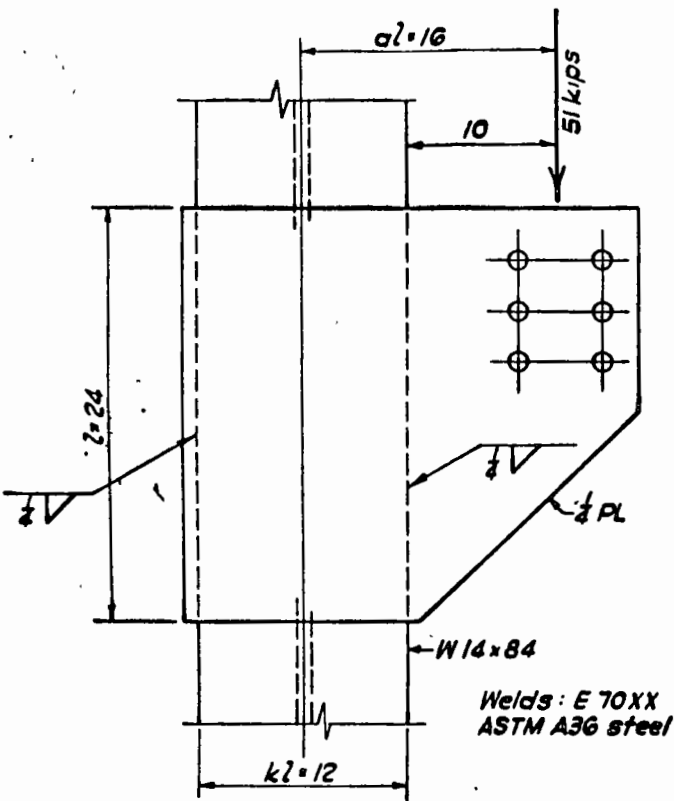


Figure S6.7

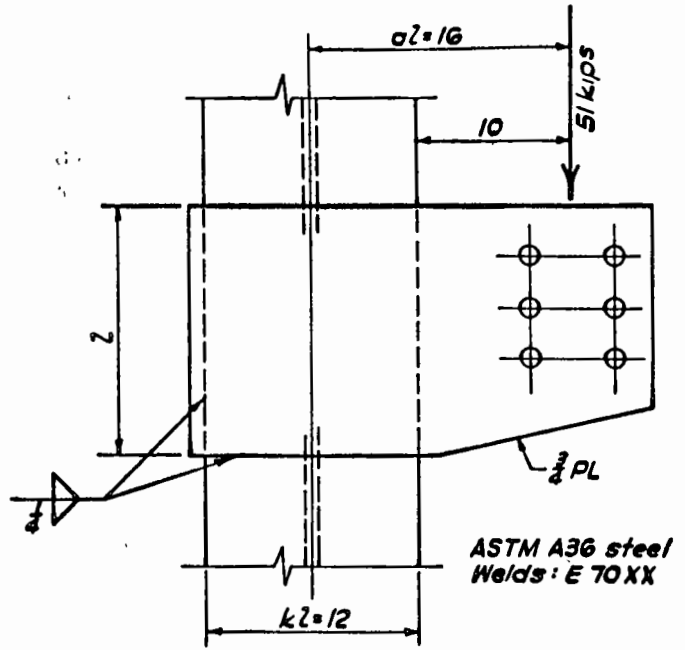


Figure S6.8

Problem 8—See Fig. S6.8.

- (a) Determine length of plate required: Specification Sect. 1.17.5 requires a 1/4-in. minimum fillet weld for 3/4-in. thickness of column flange. Try a plate length of 13 in. and determine whether a 1/4-in. weld is adequate.

Using Table XVIII in Manual Part 4:

$$\begin{aligned}
 P &= 51 \text{ kips} \\
 al &= 16 \quad kl = 12 \\
 l &= 13 \\
 a &= 16/13 = 1.23 \\
 k &= 12/13 = 0.92 \text{ (use 0.9)}
 \end{aligned}$$

Note: In determining the size of fillet weld from the tables, it is generally satisfactory to use values of either  $a$  or  $k$  to the nearest tenth (one decimal place). This usually eliminates the time-consuming operation of interpolation for both  $a$  and  $k$ . If the resulting answer for size of fillet weld is very close to a whole number, such as 3.9 or 4.1, the values of  $C$  and  $D$  can be recalculated by using values of  $a$  and  $k$  carried out to two decimal places, in order to obtain a more precise answer.

From the table, by interpolation:

$$\begin{aligned}
 C &= 1.01 \\
 D &= \frac{P}{CC'l} = \frac{51}{1.01 \times 1 \times 13} \\
 &= 3.9 \text{ sixteenths or } \frac{1}{4}\text{-in.}
 \end{aligned}$$

- (b) Required thickness of plate:

Review "Bending Stresses in Bracket Plates" in Chapter 4. Critical section is at edge of flange, which is 10 in. from center of application of load.

$$S = bd^2/6, \text{ where } b = \text{plate thickness and } d = \text{plate length}$$

$$F_b = \frac{M}{S} = \frac{M}{(bd^2/6)} \text{ or } b = \frac{6M}{F_b d^2}$$

Substituting  $F_b = 22 \text{ ksi}$ ,  $M = 51 \times 10 = 510 \text{ kip-in.}$ , and  $d = 24 \text{ in.}$ ,

$$b = \frac{6 \times 510}{22 \times 24^2} = 0.241 \text{ in. or } \frac{1}{4}\text{-in.}$$

In order that the 1/4-in. fillet weld does not cause shear overstress in the plate, the plate should be  $0.51/2 = 0.255 \text{ in.}$  thick, according to Table 6A-II in Appendix to Chapter 6. This is slightly more than 1/4-in. but not enough to cause significant overstress, especially as part of the force from the weld causes tensile stress in the plate instead of shear stress, as shown in previous problems.

Use 1/4-in. plate.

Although the value of  $D$  is close to a whole number, it is unnecessary to recalculate using  $k = 0.92$  (instead of 0.9), because it will be noticed that a larger value of  $k$  will result in a larger value of  $C$  and consequently a smaller value of  $D$ .

Hence, the  $\frac{1}{4}$ -in. weld is adequate.

(b) Required thickness of plate:

The critical section is at the edge of flange, which is 10 in. from the center of application of the load; therefore,  $M = 51 \times 10 = 510$  kip-in.

Continue to assume a plate length of 13 in.

Using the formula developed in the solution to Problem 7, part b:

$$b = \frac{6M}{F_b d^2} = \frac{6 \times 510}{22 \times 13^2} = 0.82 \text{ in}$$

It is undesirable to use a plate over  $\frac{3}{4}$ -in. thick, because a  $\frac{5}{16}$ -in. weld would be required in accordance with Specification Sect. 1.17.5. Therefore, try a plate length of 14 in. Then,

$$b = \frac{6 \times 510}{22 \times 14^2} = 0.71 \text{ in.}$$

A  $\frac{3}{4}$ -in. plate 14 in. long will be used.

WELDING SYMBOLS

Problem 9—See Fig. S6.9.

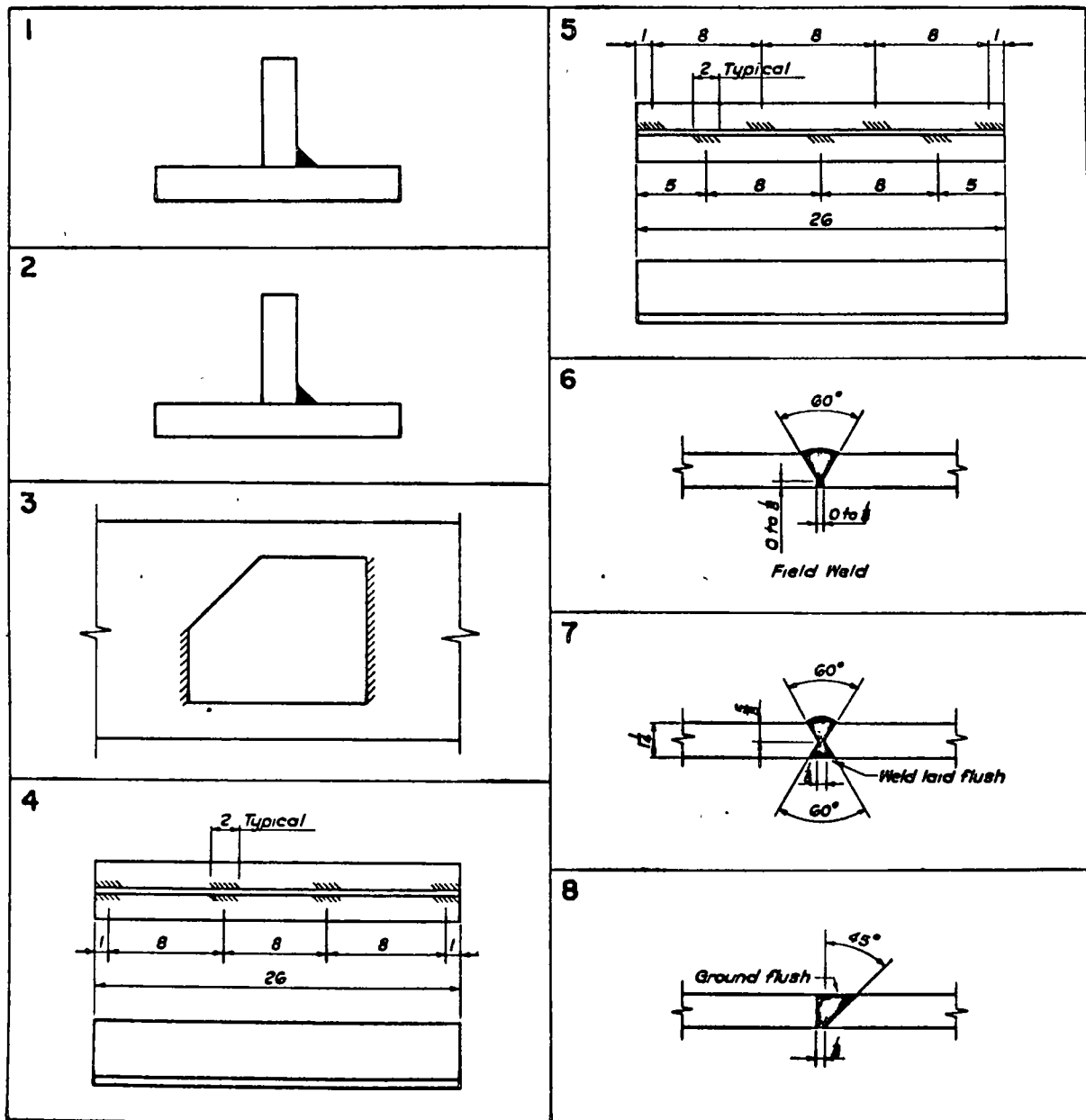


Figure S6.9

## SOLUTIONS/CHAPTER 7

# Framed and Seated Beam Connections—Bolted or Riveted

### FRAMED BEAM CONNECTIONS

**Problem 1—End connections for beam A:**

*Given:*

- W14×34: Web  $t_w = 0.287$  in.
- W27×94: Web  $t_w = 0.490$  in.
- W24×94: Web  $t_w = 0.516$  in.
- Steel: ASTM A36
- Shop and field fasteners:  $\frac{7}{8}$ -in. A307 bolts
- Reaction:  $\frac{1}{2}$  allowable uniform load for 16-ft span  
= 24.5 kips.

*Solution:*

Try 3 rows, Table I, which is the minimum connection for a W14:

- (1) Web connection (three  $\frac{7}{8}$ -in. A307 bolts):
  - (a) From Table I-A3:  
Double shear value = 36.1 > 24.5 kips o.k.
  - (b) Angle thickness =  $\frac{1}{4}$ -in.
  - (c) From Table I-B3:  
Bearing value =  $0.287 \times 128$   
= 36.7 > 24.5 kips o.k.
- (2) Outstanding legs (six  $\frac{7}{8}$ -in. A307 bolts):
  - (a) From Table I-A3:  
Single shear value = 36.1 > 24.5 kips o.k.
  - (b) Angle thickness =  $\frac{1}{4}$ -in.
  - (c) The thickness of connection angles is less than the thickness of web of the W24 or W27 and therefore governs bearing value. It is not necessary to calculate the bearing capacity of fasteners, because angle thicknesses listed in the tables have been proportioned to be adequate in bearing for the given loads.

The 3-row connection from Table I with 2L4 x  $3\frac{1}{2}$  x  $\frac{1}{4}$  x  $8\frac{1}{2}$  is satisfactory.

**Problem 2—End connection for beam A with  $\frac{3}{4}$ -in. A307 bolts:**

*Solution* (student to show all steps):

The 3-row connection from Table I with 2L4 x  $3\frac{1}{2}$  x  $\frac{1}{4}$  x  $8\frac{1}{2}$  is adequate.

**Problem 3—End connection for beam B (W24×84).**

*Given:*

- W24×84: Web  $t_w = 0.470$  in.
- W30×108: Web  $t_w = 0.548$  in.
- Steel: ASTM A36.
- Shop and field fasteners:  $\frac{7}{8}$ -in. A307 bolts.
- Reaction:  $\frac{1}{2}$  allowable uniform load for 24-ft span  
= 65.5 kips.

*Solution:*

Examination of Table I shows that there are four connections available for a W24 beam. These are 4 rows, 5 rows, 6 rows, and 7 rows. Based on  $\frac{7}{8}$ -in. A307 bolts in the outstanding legs, it can be readily seen that the shear values of 48.1 kips and 60.2 kips for 4 rows and 5 rows, respectively, are inadequate. Therefore, try 6 rows:

- (1) Web connection (six  $\frac{7}{8}$ -in. A307 bolts):
  - (a) From Table I-A6:  
Double shear value = 72.2 > 65.5 kips o.k.
  - (b) Angle thickness =  $\frac{1}{4}$ -in.
  - (c) From Table I-B6:  
Bearing value =  $0.470 \times 255$   
= 120 > 65.5 kips o.k.
- (2) Outstanding legs (twelve  $\frac{7}{8}$ -in. A307 bolts):
  - (a) From Table I-A6:  
Single shear value = 72.2 > 65.5 kips o.k.
  - (b) Angle thickness =  $\frac{1}{4}$ -in.
  - (c) Bearing value is o.k., since web thickness of 0.548 in. is more than the  $\frac{1}{4}$ -in. thickness of connection angles.

The 6-row connection from Table I with 2L4 x  $3\frac{1}{2}$  x  $\frac{1}{4}$  x  $1'-5\frac{1}{2}$  is satisfactory.

**Problem 4**—End connection for beam B, using reaction of 55 kips.

**Solution** (student to show all steps):

The 5-row connection from Table I with  $2L4 \times 3\frac{1}{2} \times \frac{1}{4}$  x  $1'-2\frac{1}{2}$  is satisfactory.

**Problem 5**—End connection for  $W24 \times 84$  (see Fig. S7.1).

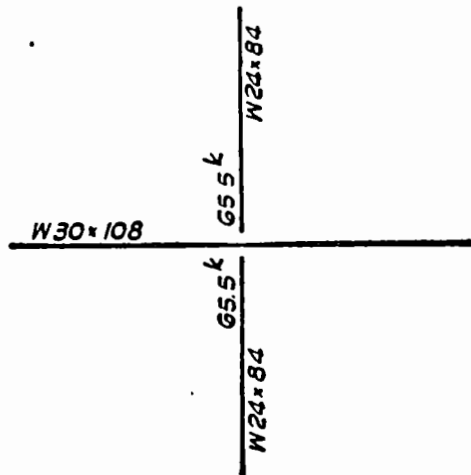


Figure S7.1

**Solution:**

Same as Problem 3 up to the following step:

(2) Outstanding legs (twelve  $\frac{7}{8}$ -in. A307 bolts):

The bolts through the outstanding legs must support the reactions from both beams ( $65.5 + 65.5 = 131$  kips) in double shear and in bearing on the  $W30 \times 108$  web.

(a) From Table I-A6:

$$\begin{aligned} \text{Double shear value} &= 72.2 + 72.2 \\ &= 144.4 > 131 \text{ kips o.k.} \end{aligned}$$

(b) Angle thickness =  $\frac{1}{4}$ -in.

(c) Bearing value:

The thickness of the  $W30 \times 108$  beam web is 0.548 in. This is more than the combined thickness of the  $\frac{1}{4}$ -in. angles connecting on opposite sides of the web. Therefore, the angle thickness governs and, as explained in the solution to Problem 1 and in the text, no calculations are necessary, as the thicknesses of angles listed in Tables I and II have been proportioned to be adequate in bearing for the given loads.

(3) Transverse spacing between holes in outstanding legs:

The minimum gage of holes in outstanding legs can be calculated as follows:

$$\text{Thickness of angle} = \frac{1}{4}\text{-in.}$$

$$\text{Height of hex nut} = \frac{3}{4}\text{-in. (see "Threaded Fasteners—Nuts", Manual Part 4)}$$

$$\text{Projection} = \frac{1}{4}\text{-in.}$$

$$\text{Wrench clearance } F = 1\frac{3}{8}\text{-in. (from "Rivets and Threaded Fasteners—Erection Clearances", Manual Part 4)}$$

$$\text{Total} = 2\frac{5}{8}\text{ in.}$$

This requires a transverse spacing of  $2\frac{5}{8} + \frac{1}{2} + 2\frac{5}{8} = 5\frac{3}{4}$  in. when used with the  $\frac{1}{2}$ -in. web thickness of the  $W24 \times 84$ . A 6-in. transverse spacing would be a good choice with a  $2\frac{3}{4}$ -in. gage in 4-in. outstanding legs of connection angles.

The 6-row connection from Table I with  $2L4 \times 3\frac{1}{2} \times \frac{1}{4}$  x  $1'-5$  will be used.

**Problem 6**—Same as Problem 5, except supporting beam is  $W27 \times 84$ .

**Solution:**

Same as Problem 5, except step 2(c) is as follows:

2(c) Bearing value:

The thickness of the  $W27 \times 84$  beam web is 0.463 in., which is less than twice the thickness of the  $\frac{1}{4}$ -in. angles, and therefore governs.

$$\begin{aligned} \text{Bearing value} &= 2 \times 0.463 \times 255 \\ &= 236 > 131 \text{ kips o.k.} \end{aligned}$$

**Problem 7**—Same as Problem 5, except reaction is 55 kips.

**Solution** (student to show all steps):

The 5-row connection from Table I with  $2L4 \times 3\frac{1}{2} \times \frac{1}{4}$  x  $1'-2\frac{1}{2}$  is satisfactory.

**Problem 8**—Same as Problem 6, except reaction is 55 kips.

**Solution** (student to show all steps):

The 5-row connection from Table I with  $2L4 \times 3\frac{1}{2} \times \frac{1}{4}$  x  $1'-2\frac{1}{2}$  is satisfactory.

**Problem 9—End connection of beam C to column 4.**

**Given:**

W30×108: Web  $t_w = 0.548$  in.  
 Steel: ASTM A36  
 Shop and field fasteners:  $\frac{7}{8}$ -in. A325-F bolts  
 Reaction:  $\frac{1}{2}$  allowable uniform load for 24-ft span  
 = 100 kips

**Solution:**

Try 6 rows, Table I:

- (1) Web connection (six  $\frac{7}{8}$ -in. A325-F bolts):
- From Table I-A6:  
Double shear value = 108 > 100 kips o.k.
  - Angle thickness =  $\frac{1}{4}$ -in.
  - Bearing not a factor in friction-type connections.
- (2) Outstanding legs (twelve  $\frac{7}{8}$ -in. A325-F bolts):
- From Table I-B6:  
Single shear value = 108 > 100 kips o.k.
  - Angle thickness =  $\frac{1}{4}$ -in.
  - Bearing not a factor in friction-type connections.

- (3) Gage in web leg of connection angles:
- In this case it is necessary for the impact wrench to clear the nut and projecting shank of the A325 bolt through the column flange.

Thickness of angle =  $\frac{1}{4}$ -in.  
 Height of nut =  $\frac{7}{8}$ -in. (see A325 bolt specification, Manual Part 5)  
 Projection of shank =  $\frac{3}{8}$ -in.  
 Wrench clearance  $F = 1\frac{3}{8}$  in. (from "Rivets and Threaded Fasteners—Erection Clearances", Manual Part 4)

Minimum gage =  $2\frac{7}{8}$  in.

If the head of the bolts through the column flange were placed on the outside of the column flange, a smaller gage could be used, but this would require special instructions regarding placing of bolts.

The 6-row connection from Table I with 2L4 x 4 x  $\frac{1}{4}$  x 1'-5 $\frac{1}{2}$  is satisfactory.

**Problem 10—Connection of beam C (W30×108) to column 4 with  $\frac{3}{4}$ -in. A325 bolts in friction-type connections.**

**Solution (student to show all steps):**

The 8-row connection from Table I with 2L4 x 4 x  $\frac{1}{4}$  x 1'-11 $\frac{1}{2}$  is satisfactory.

**Problem 11—Connection of beam C (W30×108) to column 4 using  $\frac{7}{8}$ -in. A325 bolts, bearing-type connections with threads excluded from shear planes.**

**Given:**

W30×108: Web  $t_w = 0.548$  in.  
 W8×48: Flange  $t_f = 0.683$  in.  
 Steel: ASTM A36  
 Shop and field fasteners:  $\frac{7}{8}$ -in. A325-X bolts  
 Reaction: 100 kips (from Problem 9)

**Solution:**

Try 5 rows, Table I, which is the minimum for a W30 member:

- (1) Web connection (five  $\frac{7}{8}$ -in. A325-X bolts):
- From Table I-A5:  
Double shear value = 132 > 100 kips o.k.
  - Angle thickness =  $\frac{3}{8}$ -in.
  - From Table I-B5:  
Bearing value =  $0.548 \times 213$   
= 117 > 100 kips o.k.
- (2) Outstanding legs (ten  $\frac{7}{8}$ -in. A325-X bolts):
- From Table I-A5:  
Single shear value = 132 > 100 kips o.k.
  - Angle thickness =  $\frac{3}{8}$ -in.
  - Bearing is adequate as flange thickness of W8×48 column is 0.683 in. >  $\frac{3}{8}$ -in. angle thickness.

The 5-row connection from Table I, using 2L4 x 4 x  $\frac{3}{8}$  x 1'-2 $\frac{1}{2}$  is satisfactory.

**Problem 12—Connection of beam C (W30×108) to column 4 with  $\frac{3}{4}$ -in. A325 bolts in bearing-type connections with threads excluded from shear planes.**

**Solution (student to show all steps):**

The 6-row connection from Table I using 2L4 x 4 x  $\frac{5}{16}$  x 1'-5 $\frac{1}{2}$  is satisfactory.



**Problem 13**—Connection of W24×120 beam to W30×172 girder.

**Given:**

W24×120: Web  $t_w = 0.556$  in.

Steel: ASTM A36

Shop fasteners:  $\frac{7}{8}$ -in. A502-1 rivets

Field fasteners:  $\frac{7}{8}$ -in. A325-F bolts.

Reaction: 190 kips

**Solution:**

Examination of Table I shows that the largest connection for W24 beams has 7 rows and a shear capacity of 126 kips, using  $\frac{7}{8}$ -in. A502-1 rivets. Therefore, proceed to Table II and note that the connection with 7 rows of A502-1 rivets has a shear capacity of 198 kips, which appears to be satisfactory and will be investigated.

(1) Web connection (eleven  $\frac{7}{8}$ -in. A502-1 rivets):

(a) From Table II-A7:

Double shear value = 198 > 190 kips o.k.

(b) Angle thickness =  $\frac{3}{8}$ -in.

(c) From Table II-B7:

Bearing value =  $0.556 \times 468$   
= 260 > 190 kips o.k.

(2) Outstanding legs (twenty-two  $\frac{7}{8}$ -in. A325-F bolts):

(a) From Table II-A7:

Single shear value = 198 > 190 kips o.k.

(b) Angle thickness =  $\frac{3}{8}$ -in.

(c) Bearing not a factor in friction-type connections.

(3) Gages and size of angle:

For web legs, use gages of  $2\frac{1}{4}$  and  $2\frac{1}{2}$  in a 6-in. leg.

For outstanding legs, minimum gage is determined as follows:

With connection on only one side of the W30×172, the bolts can be tightened from the side opposite the W24×120 and the minimum gage would be:

Thickness of angle	$\frac{3}{8}$ -in.
Height of rivet head	$\frac{5}{8}$ -in.
$\frac{1}{2}$ width of bolt head	$\frac{7}{8}$ -in.
Minimum gage	$1\frac{7}{8}$ in.

Use gages of  $2\frac{1}{4}$  and  $2\frac{1}{2}$  in a 6 in. leg.

The 7-row connection from Table II with 2L6 x 6 x  $\frac{3}{8}$  x  $1'-8\frac{1}{2}$  is satisfactory.

**Problem 14**—Connection of W24×120 to W30×172 with reaction of 140 kips.

**Solution** (student to show all steps):

The 5-row connection from Table II with 2L6 x 6 x  $\frac{3}{8}$  x  $1'-2\frac{1}{2}$  is satisfactory.

**Problem 15**—Connection of W27×94 (16-ft span) to flange of W14×142 column.

**Given:**

W27×102: Web  $t_w = 0.490$  in.

W14×142: Flange  $t_f = 1.063$  in.

Steel: Main material A572 grade 50 ( $F_y = 50$  ksi)  
Detail material A36

Fasteners:  $\frac{3}{4}$ -in. A490-X bolts in beam web

$\frac{7}{8}$ -in. A490-X bolts in column flange

Reaction:  $\frac{1}{2}$  allowable uniform load for 16-ft span  
= 167 kips

**Solution:**

Try 6 rows, Table I:

(1) Web connection (six  $\frac{3}{4}$ -in. A490-X bolts):

(a) From Table I-A6:

Double shear value = 170 > 167 kips o.k.

(b) Angle thickness =  $\frac{7}{16}$ -in.

(c) From Table I-B6:

Bearing value =  $0.490 \times 304$   
= 149 < 167 kips n.g.

Therefore, try 7 rows.

(d) From Table I-B7:

Bearing value =  $0.490 \times 354$   
= 173 > 167 kips o.k.

(2) Outstanding legs (fourteen  $\frac{7}{8}$ -in. A490-X bolts):

(a) From Table I-A7:

Single shear value = 269 > 167 kips o.k.

(b) Angle thickness =  $\frac{1}{2}$ -in.

(c) Note that required thickness of web legs is  $\frac{7}{16}$ -in. Hence, investigate bearing capacity of outstanding leg based on  $\frac{7}{16}$ -in. thickness:

$14 \times \frac{7}{8} \times \frac{7}{16} \times 48.6 = 260 > 167$  kips o.k.

(3) Gage in web legs and size of angles:

The *Specification for Structural Joints Using ASTM A325 or A490 Bolts* requires a hardened washer to be used under the turned element (head or nut) for A490 bolts. Moreover, a hardened washer is required under head or nut if in contact with steel having an  $F_y$  less than 40 ksi.

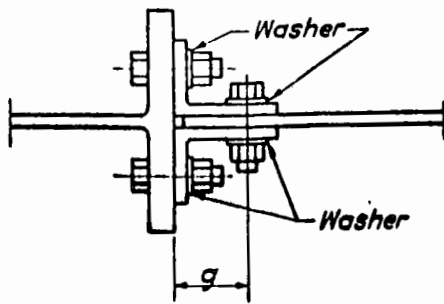


Figure S7.2

Minimum gage  $g$  (see Fig. S7.2) is calculated as follows:

Thickness of angle	=	$\frac{7}{16}$ -in.
"    " washer	=	$\frac{3}{16}$ -in.
Height of $\frac{7}{8}$ in. diam. nut	=	$\frac{7}{8}$ -in.
Projection	=	$\frac{7}{16}$ -in.
Clearance for tightening $\frac{3}{4}$ -in. diam. nut	=	$\frac{1}{4}$ in.
Minimum gage	=	$3\frac{3}{16}$ in.

(Note: A smaller gage could be used if the shop were instructed to place the bolts through the column flange with the heads on the outside of the flange.)

Referring to Table 1.16.5 in the AISC Specification, the minimum edge distance with a  $\frac{3}{4}$ -in. diameter fastener is 1 in. (to a rolled edge). Therefore, a 4-in. leg may be used.

A 7-row connection from Table I using  $2L6 \times 4 \times \frac{7}{16}$  x  $1'-8\frac{1}{2}$  will be used.

**Problem 16**—Connection of  $W27 \times 94$  (20-ft span) to flange of  $W14 \times 142$  column.

*Solution* (student to show all steps):

A 6-row connection from Table I using  $2L4 \times 4 \times \frac{7}{16}$  x  $1'-5\frac{1}{2}$  will be used.

### UNSTIFFENED SEATED CONNECTIONS

**Problem 17**—Connection of beam A ( $W14 \times 34$ ) to  $W24 \times 94$  with  $\frac{7}{8}$ -in. A307 bolts.

*Given:*

$W14 \times 34$ : Web  $t_w = \frac{5}{16}$ -in.

$W24 \times 94$ : Web  $t_w = \frac{1}{2}$ -in.

Steel: ASTM A36

Fasteners:  $\frac{7}{8}$ -in. A307 bolts

Reaction:  $\frac{1}{2}$  allowable uniform load for 16-ft span = 24.5 kips

*Solution:*

(1) Enter Table V-C:

(a) Shear capacity of a Type E connection with six  $\frac{7}{8}$ -in. A307 bolts = 36.1 > 24.5 kips o.k. It will be noted that a Type C connection has the same shear capacity, but there is not enough space available on the web of the  $W24 \times 94$  for this type of connection.

(b) Bearing capacity is obviously o.k., as the web thickness of a  $W24 \times 94$  is  $\frac{1}{2}$ -in. and it will be observed from the Table of Allowable Loads in Bearing for Rivets and Threaded Fasteners, Manual Part 4, that any thickness over  $\frac{1}{8}$ -in. has a bearing capacity greater than the single shear value of 6.01 kips for a  $\frac{7}{8}$ -in. A307 bolt.

(2) Enter Table V-A:

Angle length = 8 in.

Beam web thickness =  $\frac{5}{16}$ -in.

Outstanding leg capacity for  $\frac{3}{4}$ -in. angle thickness = 26.5 > 24.5 kips o.k.

(3) Enter Table V-D:

Opposite the Type E connection, note that the smallest  $\frac{3}{4}$ -in. angle available is the 6 x 4 size.

*Detail Data* (sketch required but not shown):

Seat angle:  $1L6 \times 4 \times \frac{3}{4} \times 8$

Side angle:  $1L3\frac{1}{2} \times 3\frac{1}{2} \times \frac{1}{4} \times 5\frac{1}{2}$

**Problem 18**—Connection of beam A ( $W14 \times 34$ ) to  $W24 \times 94$  with  $\frac{3}{4}$ -in. A307 bolts.

*Solution* (student to show all steps and draw sketch):

Use a Type E connection with six  $\frac{3}{4}$ -in. A307 bolts. (Detail data same as for Problem 17.)

**Problem 19**—Connection of beam D ( $W21 \times 55$ ) to column 4.

*Given:*

$W21 \times 55$ : Web  $t_w = \frac{3}{8}$ -in.

$W8 \times 48$ : Web  $t_w = \frac{3}{8}$ -in.

Steel: ASTM A36

Shop fasteners:  $\frac{7}{8}$ -in. A325-F bolts

Reaction:  $\frac{1}{2}$  allowable uniform load for 24-ft span = 36.5 kips

*Solution:*

(1) Enter Table V-C:

(a) Shear capacity of Type C connection with six  $\frac{7}{8}$ -in. A325-F bolts = 54.1 > 36.5 kips o.k.

(b) Bearing not a factor in friction-type connections.

(2) Enter Table V-A:

Angle length = 6 in.

Beam web thickness =  $\frac{3}{8}$ -in.

Outstanding leg capacity for 1-in. angle thickness = 37.6 > 36.5 kips o.k.

(3) Enter Table V-D:

Opposite the Type C connection, note that the smallest 1-in. thick angle is the 8 x 4 size. However, with a 1-in. seat angle it is advisable to use the 9 x 4 size. The minimum first gage for a 1-in. seat angle with  $\frac{7}{8}$ -in. A325 bolts is  $2\frac{1}{4}$  in. (see "Minimum Gages in Connections Using A325 Bolts"). The minimum pitch for  $\frac{7}{8}$ -in. fasteners allowed by Section 1.16.4 of the AISC Specification is  $2\frac{3}{8}$  in. Gages of  $2\frac{1}{4}$ ,  $2\frac{3}{8}$ ,  $2\frac{3}{8}$  would result in only a 1-in. edge distance, whereas the minimum allowed by Sect. 1.16.5 is  $1\frac{1}{8}$  in. Therefore, use a 9-in. leg with gages of  $2\frac{1}{4}$ ,  $2\frac{3}{4}$ ,  $2\frac{3}{4}$ .

Use a Type C connection with six  $\frac{7}{8}$ -in. A325-F bolts and the following detail data.

*Detail Data* (sketch required but not shown):

Seat angle: 1L9 x 4 x 1 x 6

Top angle: 1L4 x 3 x  $\frac{1}{4}$  x 6 (use gage of  $1\frac{3}{4}$  in 3-in. vertical leg)

**Problem 20**—Connection of beam D (W21X55) to column 4 with  $\frac{3}{4}$ -in. A325-F bolts.

*Solution* (student to show all steps and draw sketch):

Use a Type C connection with six  $\frac{3}{4}$ -in. A325-F bolts, and the same detail data as for Problem 19.

(Note: An 8 x 4 x 1 seat angle can be used with gages of  $2\frac{1}{4}$ ,  $2\frac{1}{4}$ ,  $2\frac{1}{4}$ .)

**Problem 21**—Connection of beam E (W16X36) to column 3, with W16X36 on opposite side of column web, using  $\frac{7}{8}$ -in. shop rivets.

*Given:*

W16X36: Web  $t_w = \frac{5}{16}$ -in.

W8X40: Web  $t_w = \frac{3}{8}$ -in.

Steel: ASTM A36

Shop fasteners:  $\frac{7}{8}$ -in. A502-1 rivets

Reaction:  $\frac{1}{2}$  allowable uniform load for 16-ft span = 28.5 kips

*Solution:*

(1) Enter Table V-C:

(a) Shear capacity of Type B connection with four  $\frac{7}{8}$ -in. rivets = 36.1 > 28.5 kips o.k.

(b) The rivets support the reaction from both beams, which is  $28.5 + 28.5 = 57.0$  kips.

Bearing value of four  $\frac{7}{8}$ -in. rivets on the  $\frac{3}{8}$ -in. column web is

$$4 \times 15.9 = 63.6 > 57.0 \text{ kips o.k.}$$

(2) Enter Table V-A:

Angle length = 6 in.

Beam web thickness =  $\frac{5}{16}$ -in.

Outstanding leg capacity for 1-in. angle thickness = 32.0 > 28.5 kips o.k.

(3) Enter Table V-D:

Opposite the Type B connection, note that the smallest 1-in. thick angle is the 8 x 4 size.

Use a Type B connection with four  $\frac{7}{8}$ -in. rivets and the following detail data.

*Detail Data* (sketch required but not shown):

Seat angle: 1L8 x 4 x 1 x 6 (use gages of  $2\frac{1}{2}$  and 3 in the 8-in. leg)

Top angle: 1L4 x  $3\frac{1}{2}$  x  $\frac{1}{4}$  x 6 (use  $2\frac{1}{4}$  gage in the  $3\frac{1}{2}$ -in. vertical leg)

**Problem 22**—Connection of W16X36 to column 3, with W16X36 on opposite side of column web, using  $\frac{3}{4}$ -in. rivets. Reaction of each beam 23 kips.

*Solution* (student to show all steps and draw sketch):

Use a Type B connection with four  $\frac{3}{4}$ -in. A502-1 rivets and the following detail data.

*Detail Data:*

Seat angle: 1L6 x 4 x  $\frac{3}{4}$  x 6 (use gages of  $1\frac{3}{4}$  and 3 in the 6-in. leg)

Top angle: 1L4 x  $3\frac{1}{2}$  x  $\frac{1}{4}$  x 6 (use  $2\frac{1}{4}$  gage in the  $3\frac{1}{2}$ -in. leg)

**Problem 23**—Connection of W16X45 beam to web of W12X65 column.

*Given:*

W16X45: Web  $t_w = \frac{3}{8}$ -in.

W12X65: Web  $t_w = \frac{3}{8}$ -in.

Steel: Main material ASTM A572 Grade 50;

Detail material ASTM A36

Shop fasteners:  $\frac{3}{4}$ -in. A490-X bolts

Reaction:  $\frac{1}{2}$  allowable uniform load on 16-ft span = 50 kips

**Solution:**

(1) Enter Table V-C:

(a) Shear capacity of Type B connection with four  $\frac{3}{4}$ -in. A490-X bolts =  $56.6 > 50$  kips o.k.(b) Bearing capacity of four  $\frac{3}{4}$  in. bolts on  $\frac{3}{8}$ -in. column web is  $4 \times 19.0 = 76.0 > 50$  kips o.k.

(2) Enter Table V-B:

On web of W12×65, a gage of  $5\frac{1}{2}$  in. and a seat angle 8 in. long will be used; beam web thickness =  $\frac{3}{8}$ -in.Outstanding leg capacity for 1-in. angle thickness =  $51.7 > 50$  kips o.k.

(3) Enter Table V-D:

Opposite the Type B connection, note that the smallest 1-in. thick angle available is the 8 x 4 size.

Use a Type B connection with four  $\frac{3}{4}$ -in. A490-X bolts and the following detail data. Use hardened washers under bolt heads or nuts in contact with A36 steel.**Detail Data** (sketch required but not shown):Seat angle: 1 L8 x 4 x 1 x 8 (use gages of  $2\frac{1}{2}$  and 3 in the 8-in. leg)Top angle: 1 L4 x 3 x  $\frac{1}{4}$  x 8 (use  $1\frac{3}{4}$  gage in the 3-in. vertical leg)**Problem 24**—Connection of W16×45 beam to web of W12×65 column.**Solution** (student to show all steps and draw sketch):Use a Type B connection with four  $\frac{7}{8}$ -in. A490-F bolts and the following detail data.**Detail Data:**Seat angle: 1 L6 x 4 x  $\frac{7}{8}$  x 8 (gages of  $2\frac{1}{4}$  and  $2\frac{1}{2}$  in the 6-in. leg)Top angle: 1 L4 x 3 x  $\frac{1}{4}$  x 8 (gage of  $1\frac{3}{4}$  in the 3-in. vertical leg)Note: Gage of  $2\frac{1}{4}$  in seat angle is minimum for  $\frac{7}{8}$ -in. thickness and  $\frac{7}{8}$ -in. A490 bolts. The gage is calculated as follows:

Thickness of angle	$\frac{7}{8}$ -in.
Fillet of 6 x 4 x $\frac{7}{8}$ angle	$\frac{1}{2}$ -in.
$\frac{1}{2}$ diam. of washer	$\frac{7}{8}$ -in.*
Minimum gage	$2\frac{1}{4}$ in.

\* Hardened washer is required under head of an A490 bolt in contact with material having  $F_u$  less than 40 ksi.**STIFFENED SEATED CONNECTIONS****Problem 25**—Connection of beam F to column 1.**Given:**W30×108: Web  $t_w = \frac{9}{16}$ -in.W8×48: Web  $t_w = \frac{3}{8}$ -in.

Steel: ASTM A36

Shop fasteners:  $\frac{7}{8}$ -in. A325-F boltsReaction:  $\frac{1}{2}$  allowable uniform load for 24-ft span = 100 kips**Solution:**

(1) Required length of bearing:

From beam load tables:

$$R = 75 \text{ kips}$$

$$R_t = 14.8 \text{ kips/lin. in.}$$

Req'd length of bearing =

$$3.5 + \frac{100 - 75}{14.8} = 5.2 \text{ in.}$$

(2) Number of fasteners required:

(a) From Table VII-B:

Six  $\frac{7}{8}$ -in. A325-F bolts in one vertical row (in each of two stiffener angles) have a shear capacity of  $108 > 100$  kips o.k.

(b) Bearing not a factor (friction-type connection).

(3) Size and thickness of stiffener angles:

From Table VII-A:

Use stiffeners with 5-in. outstanding legs, which furnish  $5\frac{1}{4}$ -in. length of bearing (5.2 in. required).Bearing capacity of two angles  $\frac{3}{8}$ -in. thick is  $111 > 100$  kips o.k.Use a Type A connection with six  $\frac{7}{8}$ -in. A325-F bolts in each stiffener, and the following detail data.**Detail Data** (sketch required but not shown):Seat angle: 1 L6 x 6 x  $\frac{3}{8}$  x 6Stiffener angles: 2 L5 x 3 x  $\frac{3}{8}$  x  $1'-5\frac{5}{8}$  (5-in. OSL)Filler: 1 Bar 6 x  $\frac{3}{8}$  x  $11\frac{1}{2}$ Top angle: 1 L4 x 3 x  $\frac{1}{4}$  x 6**Problem 26**—Connection of beam F to column 1 with reaction of 85 kips and  $\frac{3}{4}$ -in. A325 bolts in friction-type connection.**Solution** (student to show all steps and draw sketch):Use a Type A connection with seven  $\frac{3}{4}$ -in. A325-F bolts in each stiffener, and the following detail data.**Detail Data:**Seat angle: 1 L6 x 6 x  $\frac{3}{8}$  x 6Stiffener angles: 2 L4 x 3 x  $\frac{3}{8}$  x  $1'-8\frac{5}{8}$  (4-in. OSL)Filler: 1 Bar 6 x  $\frac{3}{8}$  x  $1'-2\frac{1}{2}$ Top angle: 1 L4 x 3 x  $\frac{1}{4}$  x 6

**Problem 27—Connection of beam G to column 6.****Given:**W24×94: Web  $t_w = \frac{1}{2}$ -in.W8×40: Web  $t_w = \frac{3}{8}$ -in.

Steel: ASTM A36

Shop fasteners:  $\frac{7}{8}$ -in. A325-F boltsReaction:  $\frac{1}{2}$  allowable uniform load for 24-ft span  
= 73.5 kips**Solution:****(1) Required length of bearing:**

From beam load tables:

$$R = 71 \text{ kips}$$

$$R_t = 13.9 \text{ kips}$$

Req'd length of bearing =

$$3.5 + \frac{73.5 - 71}{13.9} = 3.7 \text{ in.}$$

**(2) Number of fasteners required:****(a) From Table VII-B:**Five  $\frac{7}{8}$ -in. A325-F bolts in one vertical row  
(in each of two stiffener angles) have a shear capacity of 90.2 > 73.5 kips o.k.**(b) Bearing not a factor (friction-type connection).****(3) Size and thickness of stiffener angles:**

From Table VII-A:

Use stiffeners with 4-in. outstanding leg which furnishes  $4\frac{1}{4}$ -in. length of bearing > 3.7 in. required.Bearing capacity of two angles  $\frac{3}{8}$ -in. thick is 86.6 > 73.5 kips o.k.Use a Type A connection with five  $\frac{7}{8}$ -in. A325-F bolts in each stiffener, and the following detail data.**Detail Data (sketch required but not shown):**Seat angle: 1 L6 x 6 x  $\frac{3}{8}$  x 6Stiffener angles: 2 L4 x 3 x  $\frac{3}{8}$  x 1'-2 $\frac{5}{8}$  (4-in. OSL)Filler: 1 Bar 6 x  $\frac{3}{8}$  x 8 $\frac{1}{2}$ Top angle: 1 L4 x 3 x  $\frac{1}{4}$  x 6**Problem 28—Connection of beam G to column 6 with reaction of 90 kips and 1-in. A325 bolts in friction-type connection.****Solution (student to show all steps and draw sketch):**

Use a Type A connection with four 1-in. A325-F bolts in each stiffener, and the following detail data.

**Detail Data:**Seat angle: 1 L6 x 6 x  $\frac{3}{8}$  x 6Stiffener angles: 2 L5 x 3 x  $\frac{5}{16}$  x 11 $\frac{5}{8}$  (5-in. OSL)Filler: 1 Bar 6 x  $\frac{3}{8}$  x 5 $\frac{1}{2}$ Top angle: 1 L4 x 3 x  $\frac{1}{4}$  x 6**Problem 29—Connection of W24×68 beams to both sides of the web of the W8×35 column (use a Type B connection).****Given:**W24×68: Web  $t_w = \frac{7}{16}$ -in.W8×35: Web  $t_w = \frac{5}{16}$ -in.

Steel: ASTM A36

Shop fasteners:  $\frac{7}{8}$ -in. A502-1 rivets

Reaction: 45 kips each beam

**Solution:****(1) Required length of bearing:**From beam load tables:  $R = 55$  kipsHence,  $3\frac{1}{2}$ -in. length of bearing is adequate.**(2) Number of fasteners required:****(a) From Table VII-B:**Five  $\frac{7}{8}$ -in. rivets in one row (in a type B connection) have a shear capacity of  $90.2/2 = 45.1 > 45$  kips o.k.**(b) Bearing capacity:**The rivets support the reactions from both beams, which is  $45 + 45 = 90$  kips.Bearing value of five  $\frac{7}{8}$ -in. rivets on  $\frac{5}{16}$ -in. column web is  $5 \times 13.3 = 66.5 < 90$  kips n.g.Number of  $\frac{7}{8}$ -in. rivets req'd =  $90/13.3 = 6.8$ 

Use 7 rivets.

**(3) Size and thickness of stiffener angles:**

From Table VII-A:

Use stiffener with  $3\frac{1}{2}$ -in. outstanding leg (in accordance with  $3\frac{1}{2}$ -in. length of bearing called for in step 1.Bearing capacity of one angle  $\frac{1}{2}$ -in. thick is  $99/2 = 49.5 > 45$  kips o.k.Use a Type B connection with seven  $\frac{7}{8}$ -in. A502-1 rivets and the following detail data.**Detail Data (for each side of column web; sketch required but not shown):**Seat angle: 1 L6 x 4 x  $\frac{3}{8}$  x 6Stiffener angle: 1 L3 $\frac{1}{2}$  x 3 x  $\frac{1}{2}$  x 1'-8 $\frac{5}{8}$  (3 $\frac{1}{2}$ -in. OSL)Filler: 1 Bar 3 x  $\frac{3}{8}$  x 1'-2 $\frac{1}{2}$ Top angle: 1 L4 x 3 $\frac{1}{2}$  x  $\frac{1}{4}$  (use a 2 $\frac{1}{4}$ -in. gage in the  $3\frac{1}{2}$ -in. leg)

Note: An optional solution would be to use two stiffener angles with 4 rivets in each stiffener. This would require one less rivet (8 instead of 9) in the connection, but would require one more piece (each side of column web) to be fabricated and handled. Hence, the solution with 7 rivets in a single stiffener would generally be considered more economical.

**Problem 30**—Connection of W24×68 beams to both sides of web of a W8×35 column, using 1-in. A502-1 rivets in a Type B connection.

**Solution** (student to show all steps):

Use a Type B connection with six 1-in. A502-1 rivets and the following detail data.

**Detail Data** (student to draw sketch):

Seat angle: 1L6 x 4 x  $\frac{3}{8}$  x 6

Stiffener angle: 1L3 $\frac{1}{2}$  x 3 x  $\frac{1}{2}$  x 1'-5 $\frac{5}{8}$  (3 $\frac{1}{2}$ -in. OSL)

Filler: 1 Bar 3 x  $\frac{3}{8}$  x 11 $\frac{1}{2}$

Top angle: 1L4 x 3 $\frac{1}{2}$  x  $\frac{1}{4}$  x 6

**Problem 31**—Connection of W36×135 beams to both sides of web of a W14×87 column.

**Given:**

W36×135: Web  $t_w = \frac{5}{8}$ -in.

W14×87: Web  $t_w = \frac{7}{16}$ -in.

Steel: Main Material: ASTM A572 Grade 50

Detail Material: ASTM A36

Fasteners:  $\frac{7}{8}$ -in. A490-X bolts

Reaction:  $\frac{1}{2}$  allowable uniform load for 30-ft span

$$= \frac{293}{2} = 146.5 \text{ kips}$$

**Solution:**

(1) Required length of bearing:

From beam load tables:

$$R = 116 \text{ kips}$$

$$R_t = 22.4 \text{ kips}$$

Req'd length of bearing

$$= 3.5 + \frac{146.5 - 116}{22.4}$$

$$= 4.9 \text{ in.}$$

(2) Number of fasteners required:

(a) From Table VII-B:

Four  $\frac{7}{8}$ -in. A490-X bolts in one vertical row (in each of two angles) have a shear capacity of 154 > 146.5 kips o.k.

(b) Bearing Capacity:

The bolts support the reaction from both beams, which is  $146.5 + 146.5 = 293$  kips.

Bearing value of eight  $\frac{7}{8}$ -in. bolts on the  $\frac{7}{16}$ -in. column web is  $8 \times 25.8 \text{ kips} = 206.4 < 293$  kips n.g.

$$\text{Number of bolts req'd} = 293/25.8 = 11$$

Use 12 bolts

(3) Size and thickness of stiffener angles:

From Table VII-A:

Use stiffeners with 5-in. outstanding legs, which furnish 5 $\frac{1}{4}$ -in. length of bearing > 4.9 in. o.k.

Bearing capacity of two angles  $\frac{1}{2}$ -in. thick is 149 > 146.5 kips o.k.

With a column gage of 5 $\frac{1}{2}$  in., it would not be economical to place the stiffener angles back to back. The maximum permissible separation between outstanding legs is  $2 \times (k - \text{stiffener thickness}) = 2\frac{3}{8}$  in.

Use stiffener with 1 $\frac{3}{4}$ -in. gage in 3-in. leg, resulting in a separation of 2 in.

Use a Type A connection with six  $\frac{7}{8}$ -in. A490-X bolts in each stiffener, and the following detail data.

**Detail Data** (sketch required but not shown):

Seat angle: 1L6 x 6 x  $\frac{3}{8}$  x 8

Stiffener angles: 2L5 x 3 x  $\frac{1}{2}$  x 1'-5 $\frac{5}{8}$  (5-in. OSL)

Filler: 1 Bar 8 x  $\frac{3}{8}$  x 11 $\frac{1}{2}$

Top angle: 1L4 x 3 $\frac{1}{2}$  x  $\frac{1}{4}$  x 8

**Problem 32**—Connection of W36×150 beams to both sides of the web of a W14×87 column.

**Solution** (student to show all steps):

Use a Type A connection with six  $\frac{7}{8}$ -in. A490-X bolts in each stiffener, and the following detail data.

**Detail Data** (student to draw sketch):

Seat angle: 1L6 x 6 x  $\frac{3}{8}$  x 8

Stiffener angles: 2L5 x 3 x  $\frac{7}{16}$  x 1'-5 $\frac{5}{8}$  (5-in. OSL)

Filler: 1 Bar 8 x  $\frac{3}{8}$  x 11 $\frac{1}{2}$

Top angle: 1L4 x 3 $\frac{1}{2}$  x  $\frac{1}{4}$  x 8

## SOLUTIONS/CHAPTER 8

# Framed and Seated Beam Connections—Welded

### FRAMED BEAM CONNECTIONS

**Problem 1**—End connections for beam A (W14×34) with E70XX electrodes and 3/4-in. A307 bolts.

**Given:**

- W14×34: Web  $t_w = 0.287$  in.
- W27×94: Web  $t_w = 0.490$  in.
- W24×94: Web  $t_w = 0.516$  in.
- Steel: ASTM A36
- Shop welds: E70XX
- Field bolts: 3/4-in. A307
- Reaction: 1/2 allowable uniform load for 16-ft span = 24.5 kips

**Solution:**

- (1) Outstanding legs:
    - Refer to Table I; try 3 rows, angles 8 1/2-in. long.
    - (a) From Table I-A3:
      - Single shear value (6 bolts) = 26.5 > 24.5 kips o.k.
    - (b) Thickness of angles = 1/4-in.
    - (c) Bearing value:
      - Minimum thickness of connection angles is 1/4-in., < the web thickness of either the W24×94 or the W27×94. Bearing value is therefore o.k., as angle thicknesses in Table I have been proportioned to be adequate in bearing.
  - (2) Web legs:
    - From Table III, under Weld A,  $L = 8 1/2$  in.
    - (a) 3/16-in. weld capacity = 42.8 > 24.5 kips o.k.
    - (b) Angle thickness of 1/4-in. o.k. for 3/16-in. fillet weld.
    - (c) Min web thickness reqd = 0.28 < 0.287 o.k.
- Use 2L4 x 3 x 1/4 x 8 1/2 (3-in. legs against W14 web); 3/16-in. welds, E70XX; six 3/4-in. A307 field bolts.

**Problem 2**—End connections for W14×34, reaction 35 kips, with E70XX electrodes and 7/8-in. A307 bolts.

**Solution:**

- Similar to Problem 1 (student to show all steps).
- Use: 2L4 x 3 x 1/4 x 8 1/2 (3-in. leg against W14 web); 3/16-in. shop welds, E70XX; six 7/8-in. A307 field bolts.

**Problem 3**—Connection of beam H (W16×26) to beam B (W24×84) with E70XX electrodes and 7/8-in. A307 bolts.

**Given:**

- W16×26: Web  $t_w = 0.250$  in.
- W24×84: Web  $t_w = 0.470$  in.
- Steel: ASTM A36
- Shop Welds: E70XX
- Field Bolts: 7/8-in. A307
- Reaction: 1/2 allowable uniform load for 12-ft span = 25.5 kips

**Solution:**

- (1) Outstanding legs:
  - Refer to Table I; try 3 rows, angles 8 1/2-in. long. The connection bolts through the outstanding legs must support the reactions from both W16 beams framing on opposite sides of the W24 (25.5 + 25.5 = 51.0 kips).
  - (a) From Table I-A3:
    - Double shear value = 36.1 + 36.1 = 72.2 > 51.0 kips o.k.
  - (b) Angle thickness = 1/4-in.
  - (c) From Table I-B3:
    - Web thickness of the W24 is less than twice the thickness of the connection angles and therefore governs.
    - Bearing value = 2 × 0.470 × 128 = 120 > 51.0 kips o.k.
- (2) Web legs:
  - From Table III, under Weld A,  $L = 8 1/2$  in.
  - (a) Capacity of 3/16-in. weld = 42.8 > 25.5 kips o.k.
  - (b) Angle thickness of 1/4-in. o.k. for 3/16-in. fillet weld.
  - (c) Minimum web thickness required = 0.28 > 0.25 in. n.g.
    - Therefore, reduce capacity of weld: 0.25/0.28 × 42.8 = 38.2 > 25.5 kips o.k.

Use 2L4 x 3 x 1/4 x 8 1/2 (3-in. legs against W16 beam web); 3/16-in. shop welds, E70XX; six 7/8-in. A307 field bolts

**Problem 4**—Connection of beam H (W16×26), reaction 29 kips, to beam B (W24×84) with E70XX electrodes and  $\frac{3}{4}$ -in. A307 bolts.

**Solution:**

Similar to Problem 3 (student to show all steps).

Use: 2L4 x 3 x  $\frac{1}{4}$  x 11 $\frac{1}{2}$ ;  $\frac{3}{16}$ -in. shop welds, E70XX; eight  $\frac{3}{4}$ -in. A307 field bolts.

**Problem 5**—Connection of beam B (W24×84) to beam F (W30×108) with E70XX electrodes and  $\frac{3}{4}$ -in. A325 bolts, friction-type connection.

**Given:**

W24×84: Web  $t_w = 0.470$  in.

W30×108: Web  $t_w = 0.548$  in.

Steel: ASTM A36

Shop Welds: E70XX

Field Bolts:  $\frac{3}{4}$ -in. A325-F

Reaction:  $\frac{1}{2}$  allowable uniform load for 24-ft span  
= 65.5 kips

**Solution:**

(1) Outstanding legs:

Refer to Table I; try 5 rows, angles 1'-2 $\frac{1}{2}$  long.

(a) From Table I-A5:

Single shear value = 66.3 > 65.5 kips o.k.

(b) Angle thickness =  $\frac{1}{4}$ -in.

(c) Bearing o.k. as web thickness of W30 is greater than angle thickness (see Problem 1 above).

(2) Web legs:

From Table III, under Weld A,  $L = 1'-2\frac{1}{2}$ .

(a) Capacity of  $\frac{3}{16}$ -in. weld = 76.6 > 65.5 kips o.k.

(b) Angle thickness of  $\frac{1}{4}$ -in. o.k. for  $\frac{3}{16}$ -in. weld.

(c) Minimum web thickness required = 0.30 < 0.47 in. o.k.

Use 2L4 x 3 x  $\frac{1}{4}$  x 1'-2 $\frac{1}{2}$  (3-in. leg against W24 beam web);  $\frac{3}{16}$ -in. shop welds, E70XX; ten  $\frac{3}{4}$ -in. A325-F field bolts.

**Problem 6**—Connection of beam B (W24×84) to beam F (W30×108) with E70XX electrodes and  $\frac{7}{8}$ -in. A307 bolts.

**Solution:**

Similar to Problem 5 (student to show all steps).

Use 2L4 x 3 x  $\frac{1}{4}$  x 1'-5 $\frac{1}{2}$ ;  $\frac{3}{16}$ -in. shop welds, E70XX; twelve  $\frac{7}{8}$ -in. A307 field bolts.

**Problem 7**—Connection of two W24×100 beams on opposite sides of the W36×135, using E70XX electrodes and A490 bolts, bearing-type connection with threads excluded from the shear planes. Span of the W24 beams is 18 ft.

**Given:**

W24×100: Web  $t_w = 0.468$  in.

W36×135: Web  $t_w = 0.598$  in.

Steel: Main material: ASTM A572 grade 50

Detail material: ASTM A36

Shop welds: E70XX

Field bolts:  $\frac{3}{4}$ -in. A490-X

Reaction:  $\frac{1}{2}$  allowable uniform load for 18 ft span  
= 152 kips

**Solution:**

(1) Outstanding legs:

Refer to Table I; try 6 rows, angles 1'-5 $\frac{1}{2}$  long. The connection bolts through the outstanding legs must support the reactions from both W24 framing on opposite sides of the W36 (152 + 152 = 304 kips).

(a) From Table I-A6:

Double shear value =  $2 \times 170$   
= 340 > 304 kips o.k.

(b) Angle thickness =  $\frac{7}{16}$ -in.

(c) From Table I-B6:

As the W36 beam and the connection angles are different kinds of steel, the bearing value of each will be investigated.

Bearing value W36 web =  $2 \times 0.598 \times 304$   
= 364 > 304 kips o.k.

Bearing value of angles =  $2 \times \frac{7}{16} \times 219$   
= 192 > 152 kips o.k.

(2) Web legs:

From Table III, under Weld A,  $L = 1'-5\frac{1}{2}$

(a) Capacity of  $\frac{5}{16}$ -in. weld = 157 > 152 kips o.k.

(b) Angle thickness of  $\frac{7}{16}$ -in. o.k. for  $\frac{5}{16}$ -in. weld.

(c) Minimum web thickness required = 0.38 < 0.468 in. o.k.

Use 2L4 x 3 x  $\frac{7}{16}$  x 1'-5 $\frac{1}{2}$  (3-in. leg against W24 beam web);  $\frac{5}{16}$ -in. shop welds, E70XX; twelve  $\frac{3}{4}$ -in. A490-X field bolts, with 24 hardened washers\*.

\* The Specification for Structural Joints Using A325 or A490 Bolts requires a hardened washer to be used under the turned element (head or nut) for A490 bolts. Moreover, a hardened washer is required under the head or nut if in contact with steel having  $F_u$  less than 40 ksi.



**Problem 8**—Connection of W24×100 beams on opposite sides of W36×135, using E70XX electrodes and A490 bolts, bearing type connection with threads excluded from shear planes. Span of W24 beams is 22 ft.

**Solution:**

Similar to Problem 7 (student to show all steps).

Use 2L4 x 3 x  $\frac{7}{16}$  x 1'-2 $\frac{1}{2}$  (3-in. leg against W24 web);  $\frac{5}{16}$ -in. welds, E70XX; ten  $\frac{3}{4}$ -in. A490-X field bolts with 20 hardened washers (see footnote to Problem 7).

**Problem 9**—Connection of beam C (W30×108) to Column 4 (W8×48), using E70XX electrodes and  $\frac{7}{8}$ -in. A325 field bolts, bearing-type connection with threads in shear planes.

**Given:**

W30×108: Web  $t_w = 0.548$  in.

W8×48: Flange:  $8\frac{1}{8}$  x  $1\frac{1}{16}$ -in.

Steel: ASTM A36

Shop welds: E70XX

Field bolts:  $\frac{7}{8}$ -in. A325-N

Reaction:  $\frac{1}{2}$  allowable uniform load for 24-ft span  
= 100 kips

**Solution:**

(1) Web legs:

Refer to Table I; try 6 rows, angles 1'-5 $\frac{1}{2}$  long:

(a) From Table I-A6:

Double shear value = 108 > 100 kips o.k.

(b) Angle thickness =  $\frac{1}{4}$ -in.

(c) As the web thickness of 0.548 in. is greater than twice the thickness of the angles, angle thickness governs and bearing is o.k. (see step 1(c) of Problem 1).

(2) Outstanding legs:

From Table III, under weld B,  $L = 1'-5\frac{1}{2}$

(a) Capacity of  $\frac{1}{4}$ -in. weld = 101 > 100 kips o.k.

(b) Angle thickness of  $\frac{1}{4}$ -in. from Table I-A6 must be increased to  $\frac{5}{16}$ -in. ( $\frac{1}{16}$ -in. larger than size of fillet weld, AISC Specification Sect. 1.17.6)

(c) Shear capacity of connected material:

Thickness of angles is less than column flange thickness and therefore governs. As stated in text, with E70XX welds and angles of steel having  $F_y = 36$  ksi., if the angles are  $\frac{1}{16}$ -in. thicker than the fillet weld size, the weld shear per linear inch will not exceed the shear value of the angles per linear inch.

Use 2L3 $\frac{1}{2}$  x 3 x  $\frac{5}{16}$  x 1'-5 $\frac{1}{2}$  (3-in. legs against column flange);  $\frac{1}{4}$ -in. shop welds, E70XX; six  $\frac{7}{8}$ -in. A325-N field bolts.

**Problem 10**—Connection of beam C (W30×108) to W12×65 column, using E70XX electrodes and  $\frac{3}{4}$ -in. A325 field bolts, bearing-type connection with threads in shear planes.

**Solution:**

Similar to the solution Problem 9 (student to show all steps).

Use 2L4 x 3 $\frac{1}{2}$  x  $\frac{5}{16}$  x 1'-11 $\frac{1}{2}$  (4-in. legs against column flange);  $\frac{1}{4}$ -in. shop welds, E70XX; eight  $\frac{3}{4}$ -in. A325-N field bolts.

**Problem 11**—Connection of beam G (W24×94) to column 3 (W8×40), using E70XX electrodes and  $\frac{7}{8}$ -in. A325 bolts, friction-type connection.

**Given:**

W24×94: Web  $t_w = 0.516$  in.

W8×40: Flange:  $8\frac{1}{8}$  x  $\frac{9}{16}$ -in.

Steel: ASTM A36

Shop welds: E70XX

Field bolts:  $\frac{7}{8}$ -in. A325-F

Reaction:  $\frac{1}{2}$  allowable uniform load for 24-ft span  
= 73.5 kips

**Solution:**

(1) Web legs:

Refer to Table I; try 5 rows, angles 1'-2 $\frac{1}{2}$  long:

(a) From Table I-A5:

Double shear value = 90.2 > 73.5 kips o.k.

(b) Angle thickness =  $\frac{1}{4}$ -in.

(c) Bearing not a factor in friction-type connections.

(2) Outstanding legs:

From Table III, under Weld B,  $L = 1'-2\frac{1}{2}$ .

(a) Capacity of  $\frac{1}{4}$ -in. weld = 76.6 > 73.5 kips o.k.

(b) Angle thickness of  $\frac{1}{4}$ -in. from Table I-A5 must be increased to  $\frac{5}{16}$ -in. ( $\frac{1}{16}$ -in. larger than fillet weld size, as per AISC Specification Sect. 1.17.6).

(c) Investigate shear capacity of connected material:

Thickness of angles is less than column flange thickness and therefore governs. Shear capacity of angles o.k. as explained in step 2-c of Problem 9.

Use 2L3 $\frac{1}{2}$  x 3 x  $\frac{5}{16}$  x 1'-2 $\frac{1}{2}$  (3-in. leg against column flange);  $\frac{1}{4}$ -in. shop welds, E70XX; five  $\frac{7}{8}$ -in. A325-F field bolts.

**Problem 12**—Connection of beam G (W24×94) to column 3 (W8×40), using E70XX electrodes and 1-in. A325 bolts, friction-type connection, with reaction of 110 kips.

**Solution:**

Similar to Problem 11 (student to show all steps).  
Use 2L3½ x 3 x 7/16 x 1'-2½ (3-in. leg against column flange); 3/8-in. shop welds, E70XX; five 1-in. A325-F field bolts.

**Alternate Solution:**

2L3½ x 3 x 3/8 x 1'-5½ (3-in. leg against column flange); 5/16-in. shop welds, E70XX; six 1-in. A325-F bolts.

This alternate solution may be more economical than the previous solution, as it avoids the use of 3/8-in. fillet welds, which require two passes by the welder.\*

Note that it requires longer but thinner angles and an extra bolt.

**Problem 13**—Connection of W33×130 to W10×49 column, using E70XX electrodes and 3/4-in. A490 bolts, bearing-type connection with threads excluded from shear planes.

**Given:**

W33×130: Web  $t_w = 0.580$  in.  
W10×49: Flange:  $10 \times 9/16$  in.  
Steel: Main material: ASTM A572 grade 50  
Detail material: ASTM A36  
Shop welds: E70XX  
Field bolts: 3/4-in. A490-X  
Reaction: 1/2 allowable uniform load for 22-ft span = 203 kips

**Solution:**

(1) Web legs:

Refer to Table I; try 8 rows, angles 1'-11½ long:

(a) From Table I-A8:

Double shear value = 226 > 203 kips o.k.

\* Refer to "Economy in Selection of Welds" in Chapter 6 of textbook.

(b) Angle thickness = 7/16-in.

(c) As the W33 beam and the connection angles are different kinds of steel, the bearing value of each will be investigated.

From Table I-B8:

Bearing value of W33 web =  $0.580 \times 405 = 235 > 203$  kips o.k.

Bearing value of 2 angles =  $(2 \times 7/16) \times 292 = 255 > 203$  kips o.k.

(2) Outstanding legs:

From Table III, under Weld B,  $L = 1'-11½$ .

(a) Capacity of 3/8-in. weld = 223 > 203 kips o.k.

(b) Angle thickness of 7/16-in. o.k. for 3/8-in. weld.

(c) Investigate shear capacity of connected material:

Thickness of angles is less than the column flange thickness and therefore governs. Shear capacity of angles o.k. as explained in solution to Problem 9.

Use 2L4 x 3½ x 7/16 x 1'-11½ (4-in. leg against column flange); 3/8-in. shop welds, E70XX; eight 3/4-in. A490-X field bolts with 16 hardened washers.

**Alternate Solution:**

2L4 x 3½ x 7/16 x 2'-2½ (4-in. leg against column flange); 5/16-in. shop welds, E70XX; nine 3/4-in. A490-X field bolts with 18 hardened washers.

Refer to comments under Alternate Solution to Problem 12.

**Problem 14**—Connection of W27×94 to W10×49 column, using E70XX electrodes and 3/4-in. A490 bolts, bearing-type connection with threads in shear planes.

**Solution:**

Similar to Problem 13 (student to show all steps).

Use 2L3½ x 3 x 3/8 x 1'-5½ (3-in. leg against column flange); 5/16-in. shop welds, E70XX; six 3/4-in. A490-N field bolts with 12 hardened washers.

**Problem 15**—Connection of W12×14 to W14×38 with reaction of 17.2 kips, using E70XX electrodes.

**Given:**

W12×14: Web  $t_w = 0.198$  in.

W14×38: Web  $t_w = 0.313$  in.

Steel: ASTM A36

Welds: E70XX

Reaction: 17.2 kips

**Solution:**

(1) Enter Table IV and note that either of the following appears to satisfy the requirements:

(a)  $L = 5$  in.; Weld A = 29.5 kips,  $\frac{1}{4}$ -in. weld; Weld B = 19.5 kips,  $\frac{5}{16}$ -in. weld

(b)  $L = 6$  in.; Weld A = 29.5 kips,  $\frac{3}{16}$ -in. weld; Weld B = 21.7 kips,  $\frac{1}{4}$ -in. weld

The length of 5 in. is slightly less than half the  $T$ -distance of  $10\frac{3}{8}$  in. for the W12×14. It could be considered satisfactory, but in this case the longer length is a better choice because the connections on both sides of the W14×38 tend to overstress the web of the W14. Use 6-in.

(2) Weld A:

(a) Capacity of  $\frac{3}{16}$ -in. weld = 29.5 > 17.2 kips o.k.

(b) Min web  $t_w$  reqd = 0.27 > 0.198 in. n.g.

Therefore reduce capacity of weld:

$$0.198/0.27 \times 29.5 = 21.6 > 17.2 \text{ kips o.k.}$$

(3) Weld B:

(a) Capacity of  $\frac{1}{4}$ -in. weld = 21.7 > 17.2 kips o.k.

(b) Check shear capacity of connected material:

Shear capacity of W14×38 web =  $14.5 \times 1 \times 0.313 = 4.54$  kips/lin. in.

Weld shear (both sides of web) =  $2 \times 0.928 \times 4 = 7.42 > 4.54$  kips/lin. in. n.g.

Therefore, reduce capacity of weld:  $4.54/7.42 \times 21.7 = 13.3 < 17.2$  kips n.g.

Hence, try a connection with length of 7 in.

Capacity of  $\frac{1}{4}$ -in. fillet for Weld B = 28.3 kips

Reduce capacity of weld  $4.54/7.42 \times 28.3 = 17.3 > 17.2$  kips o.k.

Use 2L3 x 3 x  $\frac{5}{16}$  x 7;  $\frac{3}{16}$ -in. shop welds (E70XX) on web legs;  $\frac{1}{4}$ -in. field welds (E70XX) on outstanding legs.

**Problem 16**—Connection of W12×27 to W16×50, with reaction of 20.0 kips, using E70XX electrodes.

**Solution:**

Similar to Problem 15 (student to show all steps).

Use 2L3 x 3 x  $\frac{5}{16}$  x 7;  $\frac{3}{16}$ -in. welds (E70XX) on web legs;  $\frac{1}{4}$ -in. field welds (E70XX) on outstanding legs.

**Problem 17**—Connection of beam II (W16×26) to beam B (W24×84), using E70XX electrodes.

**Given:**

W16×26: Web  $t_w = 0.250$  in.

W24×84: Web  $t_w = 0.470$  in.

Steel: ASTM A36

Welds: E70XX

Reaction:  $\frac{1}{2}$  allowable uniform load for 12-ft span = 25.5 kips

**Solution:**

(1) Enter Table IV and note that either of the following appears to satisfy the requirements:

(a)  $L = 6$  in.; Weld A = 39.3 kips,  $\frac{1}{4}$ -in. weld; Weld B = 27.1 kips,  $\frac{5}{16}$ -in. weld.

(b)  $L = 7$  in.; Weld A = 34.7 kips,  $\frac{3}{16}$ -in. weld; Weld B = 28.3 kips,  $\frac{1}{4}$ -in. weld.

The length of 6 in. is less than half the  $T$ -distance of  $13\frac{3}{4}$ -in. for the W16×26, so should not be used. Hence, try the length of 7 in.

(2) Weld A:

(a) Capacity of  $\frac{3}{16}$ -in. weld = 34.7 > 25.5 kips o.k.

(b) Minimum web thickness required = 0.27 > 0.25 in. n.g.

Hence, reduce capacity of weld:  $0.25/0.27 \times 34.7 = 32.1 > 25.5$  kips o.k.

(3) Weld B:

(a) Capacity of  $\frac{1}{4}$ -in. weld = 28.3 > 25.5 kips o.k.

(b) Investigate shear capacity of connected material:

Shear capacity of W24×84 web =  $14.5 \times 1 \times 0.470 = 6.82$  kips/lin. in.

Weld shear (both sides of web) =  $2 \times 0.928 \times 4 = 7.42 > 6.82$  kips/lin. in. n.g.

Hence, reduce capacity of weld:  $6.82/7.42 \times 28.3 = 26.0 > 25.5$  kips o.k.

Use 2L3 x 3 x  $\frac{5}{16}$  x 7;  $\frac{3}{16}$ -in. shop welds (E70XX) on web legs;  $\frac{1}{4}$ -in. field welds (E70XX) on outstanding legs.

**Problem 18**—Connection of beam H (W16×26) to beam B (W24×84), using E70XX electrodes. Reaction based on 10-ft span.

**Solution:**

Similar to Problem 17 (student to show all steps).

Use 2L3 x 3 x  $\frac{5}{16}$  x 8;  $\frac{3}{16}$ -in. shop welds (E70XX) on web legs;  $\frac{1}{4}$ -in. field welds (E70XX) on outstanding legs.

**Problem 19**—Connection of beam D (W21×55) to column 1 (W8×48), using E70XX electrodes. Main material is ASTM A572 Grade 50 and detail material is ASTM A36.

**Given:**

- W21×55: Web  $t_w = 0.375$  in.
- W8×48: Flange:  $8\frac{1}{8} \times 1\frac{1}{16}$ -in.
- Steel: Main material: ASTM A572 grade 50  
Detail material: ASTM A36
- Welds: E70XX
- Reaction:  $\frac{1}{2}$  allowable uniform load for 24-ft span  
= 50.0 kips

**Solution:**

- (1) Enter Table IV and note that either of the following appears to satisfy the requirements:
  - (a)  $L = 9$  in.; Weld A = 60.6 kips,  $\frac{1}{4}$ -in. weld;  
Weld B = 53.7 kips,  $\frac{5}{16}$ -in. weld.
  - (b)  $L = 10$  in.; Weld A = 51.0 kips,  $\frac{3}{16}$ -in. weld;  
Weld B = 50.5 kips,  $\frac{1}{4}$ -in. weld.

Length of 9 in. will be selected because it requires less length of weld. It is slightly less than half the  $T$ -distance of  $18\frac{1}{2}$  in. for the W21, but can be considered satisfactory.

(2) Weld A:

- (a) Capacity of  $\frac{1}{4}$ -in. weld = 60.6 > 50.0 kips o.k.
- (b) Minimum web thickness required = 0.27 in. < 0.375 in. o.k.

(3) Weld B:

- (a) Capacity of  $\frac{5}{16}$ -in. weld = 53.7 > 50.0 kips o.k.
- (b) Investigate shear capacity of connected material. Thickness of angles is less than column flange thickness and therefore governs. Shear capacity of angles is adequate and no calculations are necessary, as explained in Problem 9.

Use 2L3 x 3 x  $\frac{3}{8}$  x 9;  $\frac{1}{4}$ -in. weld (E70XX) on web legs;  $\frac{5}{16}$ -in. weld (E70XX) on outstanding legs.

**Problem 20**—Connection of beam B (W24×84) to beam F (W30×108), using E70XX electrodes. Main material is ASTM A572 grade 50 and detail material is ASTM A36.

**Solution:**

Similar to previous problems (student to show all steps).

Use 2L3 x 3 x  $\frac{3}{8}$  x 1'-2;  $\frac{1}{4}$ -in. weld (E70XX) on web legs;  $\frac{5}{16}$ -in. weld (E70XX) on outstanding legs.

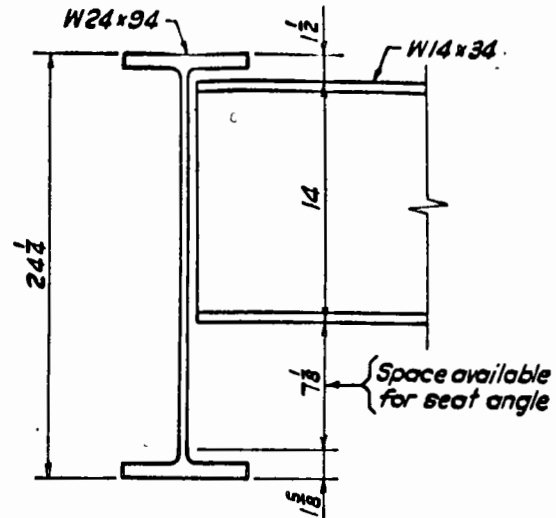


Figure S8.1

**UNSTIFFENED SEATED CONNECTIONS**

**Problem 21**—Connection of beam A (W14×34) to beam G (W24×94). See Fig. S8.1.

**Given:**

- W14×34: Web  $t_w = \frac{5}{16}$ -in.  
Flange:  $6\frac{3}{4} \times \frac{7}{16}$ -in.
- W24×94: Web  $t_w = \frac{1}{2}$ -in.
- Steel: ASTM A36
- Welds: E70XX; Field bolts:  $\frac{3}{4}$ -in. A307
- Reaction:  $\frac{1}{2}$  allowable uniform load for 16-ft span  
= 24.5 kips

**Solution:**

- (1) Select seat angle length:  
Beam flange is  $6\frac{3}{4}$  in. wide. For bottom flange of beam to clear weld returns, use  $8\frac{3}{4}$  in.
- (2) From Table VI-A:  
Capacity of  $\frac{3}{4}$ -in. angle = 26.5 > 24.5 kips o.k.
- (3) From Table VI-C:  
Capacity of 6 x 4 angle with  $\frac{5}{16}$ -in. weld = 27.3 > 24.5 kips o.k.  
Capacity of 7 x 4 angle with  $\frac{1}{4}$ -in. weld = 28.5 > 24.5 kips o.k.  
Use 6 x 4 angle with  $\frac{5}{16}$ -in. welds.\*

Use seat angle 6 x 4 x  $\frac{3}{4}$  x  $8\frac{3}{4}$  with  $\frac{5}{16}$ -in. welds to W24 and  $\frac{1}{4}$ -in. welds to W14; side angle 4 x  $3\frac{1}{2}$  x  $\frac{1}{4}$  x 6 with  $\frac{3}{16}$ -in. welds connecting 4-in. leg to web of W24 and two  $\frac{3}{4}$ -in. A307 bolts to W14; welds E70XX.

\* As a general rule, choose the connection requiring less length of weld and smaller size angle, except avoid welds larger than  $\frac{5}{16}$ -in., as stated in text. In the case of unstiffened seats, the capacity of the supporting beam web, or column web or flange, to receive the weld need not be investigated, as explained in text.

**Problem 22**—Same as Problem 21 except reaction is 30.0 kips. See Fig. S8.1.

**Solution:**

Similar to Problem 21 (student to show all steps).

Use seat angle  $7 \times 4 \times \frac{7}{8} \times 8\frac{3}{4}$  with  $\frac{5}{16}$ -in. welds; side angle  $4 \times 3\frac{1}{2} \times \frac{1}{4} \times 6$  with  $\frac{3}{16}$ -in. welds connecting 4-in. leg to W24 and two  $\frac{3}{4}$ -in. A307 bolts to W14; welds E70XX.

**Problem 23**—Connection of beam H (W16×26) to beam J (W27×94). See Fig. S8.2.

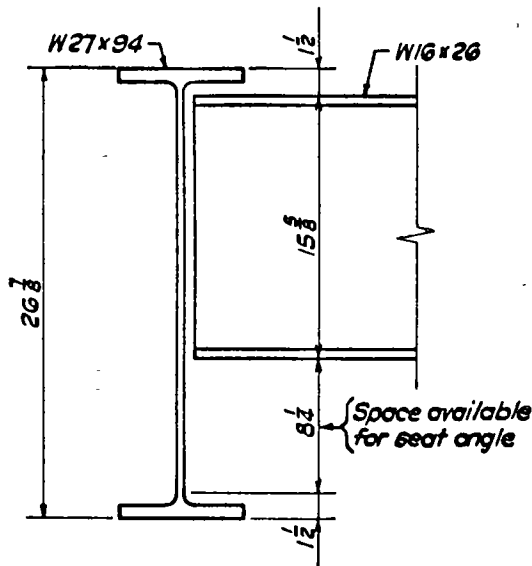


Figure S8.2

**Given:**

W16×26: Web  $t_w = \frac{1}{4}$ -in.  
 Flange:  $5\frac{1}{2} \times \frac{3}{8}$ -in.  
 W27×94: Web  $t_w = \frac{1}{2}$ -in.  
 Steel: ASTM A36  
 Welds: E70XX  
 Field bolts:  $\frac{3}{4}$ -in. A307  
 Reaction:  $\frac{1}{2}$  allowable uniform load for 12-ft span  
 = 25.5 kips

**Solution:**

(1) Select seat angle length:

Beam flange is  $5\frac{1}{2}$  in. wide. In order for bottom flange of beam to clear weld returns, seat angle should be  $7\frac{1}{2}$  in. long.

(2) From Table VI-A:

Based on a length of 8 in., capacity of a  $\frac{7}{8}$ -in. angle is 25.5 kips o.k. Therefore, use length of 8 in., instead of  $7\frac{1}{2}$  in.

(3) From Table VI-C:

The  $\frac{7}{8}$ -in. thickness of seat angle requires a  $\frac{5}{16}$ -in. minimum fillet weld (Specification Sect. 1.17.5). Capacity of  $6 \times 4$  angle with  $\frac{5}{16}$ -in. weld =  $27.3 > 25.5$  kips o.k.

Use seat angle  $6 \times 4 \times \frac{7}{8} \times 8$  with  $\frac{5}{16}$ -in. welds; side angle  $4 \times 3\frac{1}{2} \times \frac{1}{4} \times 6$  with  $\frac{3}{16}$ -in. weld connecting 4-in. leg to web of W27 and two  $\frac{3}{4}$ -in. A307 bolts to W16; welds E70XX.

**Problem 24**—Connection of beam H (W16×26) to beam J (W27×94) if reaction is 19.0 kips. See Fig. S8.2.

**Solution:**

Similar to previous problems (student to show all steps).

Use seat angle  $5 \times 3\frac{1}{2} \times \frac{3}{4} \times 7\frac{1}{2}$  with  $\frac{5}{16}$ -in. welds to W27 and  $\frac{1}{4}$ -in. welds to W16; side angle  $4 \times 3\frac{1}{2} \times \frac{1}{4} \times 6$  with  $\frac{3}{16}$ -in. welds connecting 4-in. leg to web of W27 and two  $\frac{3}{4}$ -in. A307 bolts to W16; welds E70XX.

**Problem 25**—Connection of beam E (W16×36) to column 2 (W10×60).

**Given:**

W16×36: Web  $t_w = \frac{5}{16}$ -in.  
 Flange:  $7 \times \frac{1}{16}$ -in.  
 W10×60: Web  $t_w = \frac{1}{16}$ -in.  
 $T = 7\frac{3}{4}$  in.  
 Steel: ASTM A36  
 Welds: E70XX  
 Reaction:  $\frac{1}{2}$  allowable uniform load for 16-ft span  
 = 28.5 kips

**Solution:**

(1) Select seat angle length:

Since clear distance  $T$  on column web is  $7\frac{3}{4}$  in., use an angle length of 6 in., which will provide space for welding the ends of the seat angle to the column web.

(2) From Table VI-A:

Capacity of 1-in. angle =  $32.0 > 28.5$  kips o.k.

**(3) From Table VI-C:**

The 1-in. angle requires a minimum fillet weld of  $\frac{5}{16}$ -in. (Specification Sect. 1.17.5).

Capacity of 7 x 4 angle with  $\frac{5}{16}$ -in. weld = 35.6 > 28.5 kips o.k.

However, 7 x 4 angle is not available in 1-in. thickness, so 8 x 4 angle must be used.

**(4) Cutting bottom flange:**

The bottom flange of the W16X36 must be cut to provide space for welding the toes of the flange to the seat angle and also to clear the return welds at the heel of the seat angle. The remaining flange must be wide enough to accommodate holes for temporary erection bolts. If these are placed on a gage of 3 in. the flange should be 5 in. wide, based on a 1-in. edge distance (Specification Sect. 1.16.5). This will provide sufficient space for the  $\frac{1}{4}$ -in. fillet weld connecting the beam flange to the seat angle, but the 5-in. flange width may not clear the return welds at the heel of the seat angle. For this reason the end of the bottom flange should be cut at 45° for a distance of 1 in. (see Fig. S8.3).

Use seat angle 8 x 4 x 1 x 6 with  $\frac{5}{16}$ -in. welds to column and W16; top angle 4 x 4 x  $\frac{1}{4}$  x 4 with  $\frac{3}{16}$ -in. welds to column and beam flange; welds E70XX.

Note: It is assumed that erection bolts are not required in the top flange; thus, the 4-in. length is satisfactory for the top angle. This angle must be shipped loose and field welded to the column to permit erection of the beam.

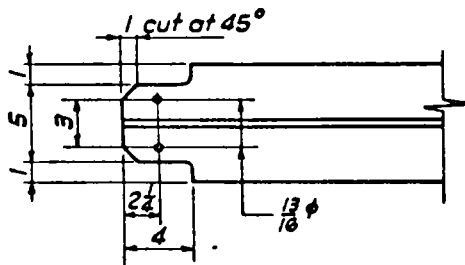


Figure S8.3

**Problem 26**—Same as Problem 25 except column is W12X65 and reaction is 19.5 kips.

**Solution:**

Similar to Problem 25 (student to show all steps). Cut bottom flange of beam to 6 in. to clear return welds and provide space for welding flange to seat angle. (Student to draw sketch showing holes and cuts.)

Use seat angle 5 x 3 $\frac{1}{2}$  x  $\frac{5}{8}$  x 4 with  $\frac{5}{16}$ -in. welds to column and  $\frac{1}{4}$ -in. welds to W16; top angle 4 x 4 x  $\frac{1}{4}$  x 4 with  $\frac{3}{16}$ -in. welds to column and beam flange; welds E70XX.

**Problem 27**—Connection of a W21X55 to the flange of a W14X87 column. Span of beam is 24 ft.

**Given:**

W21X55:  $t_w = \frac{3}{8}$ -in.

Flange: 8 $\frac{1}{4}$  x  $\frac{1}{2}$ -in.

W14X87: Flange 14 $\frac{1}{2}$  x  $1\frac{1}{4}$ -in.

Steel: Main material: ASTM A572 grade 50

Detail material: ASTM A36

Welds: E70XX

Field bolts:  $\frac{3}{4}$ -in. A307

Reaction:  $\frac{1}{2}$  allowable uniform load for 24-ft span = 50 kips.

**Solution:**

**(1) Select seat angle length:**

Column flange is 14 $\frac{1}{2}$  in. wide and width of beam flange is 8 $\frac{1}{4}$ -in.

The seat angle should be 10 $\frac{1}{4}$  in. long in order for the bottom flange of the beam to clear the weld returns at the heel of the seat angle.

**(2) From Table VI-B:**

Capacity of 1-in. angle = 51.7 > 50 kips o.k.

**(3) From Table VI-C:**

The 1-in. angle requires a minimum fillet weld of  $\frac{5}{16}$ -in. (Specification Sect. 1.17.5)

Capacity of 9 x 4 angle with  $\frac{5}{16}$ -in. weld = 53.8 > 50 kips o.k.

**(4) Design top angle connection:**

Because the top angle is field bolted to the beam, its length of top angle depends on the gage in the beam flange. Use 8-in. long angle

A  $\frac{1}{4}$ -in. minimum fillet weld connecting the top angle to the column is required by the  $1\frac{1}{16}$ -in. column flange thickness (Specification Sect. 1.17.5). Thus, a top angle  $\frac{5}{16}$ -in. thick is required (Specification Sect. 1.17.6).

Use seat angle 9 x 4 x 1 x 10 $\frac{1}{4}$  with  $\frac{5}{16}$ -in. welds to column; top angle 4 x 4 x  $\frac{5}{16}$  x 8 with  $\frac{1}{4}$ -in. welds to column; four  $\frac{3}{4}$ -in. A307 bolts, seat and top angles to beam; E70XX welds.

**Problem 28**—Connection of a W16X40 to the flange of a W14X87 column. Material is ASTM A572 grade 50 steel. Span of beam is 24 ft.

**Solution:**

Similar to Problem 27 (student to show all steps).

Use seat angle 7 x 4 x  $\frac{3}{4}$  x 9 with  $\frac{5}{16}$ -in. welds to column; top angle 4 x 4 x  $\frac{5}{16}$  x 6 with  $\frac{1}{4}$ -in. welds to column; four  $\frac{3}{4}$ -in. A307 bolts, seat and top angles to beam; E70XX welds.

**STIFFENED SEATED CONNECTIONS**

**Problem 29**—Connection of a W24×84 to web of W14×158 column. Reaction is 65.5 kips.

**Given:**

- W24×84: Web  $t_w = \frac{1}{2}$ -in.
- Flange:  $9 \times \frac{3}{4}$ -in.
- W14×158: Web  $t_w = \frac{3}{4}$ -in.;  $T = 11\frac{1}{4}$  in.
- Steel: ASTM A36
- Welds: E70XX
- Reaction: 65.5 kips

**Solution:**

(1) Required stiffener width  $W$ :

Required length of bearing

$$N = 3.5 + \frac{\text{Reaction} - R}{R_t}$$

$$= 3.5 + \frac{65.5 - 64}{12.7} = 3.6 \text{ in.}$$

$$W = 3.6 + 0.5 \text{ (setback)} = 4.1 \text{ in. Use 5 in.}$$

(2) Length of stiffener plate  $L$  and weld size:

(a) In order not to overload the column web in shear, the maximum weld size is  $1 \times \frac{3}{4} = \frac{3}{4}$ -in.\*

Try a  $\frac{5}{16}$ -in. weld (minimum permitted for column web).

(b) From Table VIII:

$L$  of 11 in. with  $\frac{5}{16}$ -in. weld has a capacity of 69.4 > 65.5 kips o.k.

(3) Thickness  $t$  of stiffener plate and seat plate:

$$** t = 2 \times \text{weld size} \geq \text{beam web } t_w$$

$$= 2 \times \frac{5}{16} = \frac{5}{8} > \frac{1}{2}\text{-in. o.k.}$$

\* The whole number 1 is obtained by rounding off the more precise figure of 0.98 (see the table in the section on "Stiffened Seated Connections", Chapter 8 of the Textbook). This may be rounded to the whole number 1. Similarly, for connections on both sides of the web, a figure of  $\frac{1}{2}$  may be used instead of the more precise 0.49.

\*\* The whole number 2 is used instead of the more precise figure of 2.05 (see the formula for Min.  $t$ , "Stiffened Seated Connections", Chapter 8 of the Textbook and the examples accompanying Table VIII in the Manual).

(4) Seat plate length and welds:

(a) Nominal  $\frac{1}{4}$ -in. weld 2 in. long is used to connect the beam flange to the seat plate. Note that a  $\frac{1}{4}$ -in. weld satisfies the requirements of Specification Sect. 1.17.5.

(b) Minimum length of seat plate  
= flange width + (4 × weld size)  
=  $9 + (4 \times \frac{1}{4}) = 10$  in.

Use 10 in.

(c) Minimum length of  $\frac{5}{16}$ -in. weld to attach seat plate to column web =  $2 \times 0.2L = 2 \times 0.2 \times 11 = 4.4$  in.

This also establishes a 4.4-in. (2.2 in. on each side of stiffener) minimum length of weld to connect the stiffener to the seat plate. However, to simplify layout and welding, the seat plate will be welded full length to the column web and the stiffener plate will be welded full width to the seat plate.

(5) Top angle:

A nominal  $\frac{1}{4}$ -in. weld will be used to connect the top angle to the beam flange and to the column web, welding only at the toes of the angle. This size weld meets the requirements of Specification Sect. 1.17.5. Angle must be  $\frac{5}{16}$ -in. thick to accept the  $\frac{1}{4}$ -in. weld (Specification Sect. 1.17.6). Top angle will be made 4 in. long.

Use: Seat plate:  $5 \times \frac{5}{8} \times 10$

Stiffener plate:  $5 \times \frac{5}{8} \times 11$ , with  $\frac{5}{16}$ -in. weld

Top angle:  $4 \times 4 \times \frac{5}{16} \times 4$ , with  $\frac{1}{4}$ -in. weld

Welds: E70XX

Sketch required but not shown.

**Problem 30**—Connection of W24×84 to web of W14×158 column with reaction of 85 kips.

**Solution:**

Similar to Problem 29 (student to show all steps and draw sketch).

Use: Seat plate:  $6 \times \frac{5}{8} \times 10$

Stiffener plate:  $6 \times \frac{5}{8} \times 11$ , with  $\frac{5}{16}$ -in. weld

Top angle:  $4 \times 4 \times \frac{5}{16} \times 4$ , with  $\frac{1}{4}$ -in. weld

Welds: E70XX

**Problem 31**—Connection of W21×55 beams framing on opposite sides of a W14×103 column web, with reaction of 43 kips for each beam.

**Given:**

W21×55: Web  $t_w = \frac{3}{8}$ -in.  
 Flange:  $8\frac{1}{4} \times \frac{1}{2}$ -in.  
 W14×103: Web  $t_w = \frac{1}{2}$ -in.;  $T = 11\frac{1}{4}$  in.  
 Steel: ASTM A36  
 Welds: E70XX; Field bolts:  $\frac{7}{8}$ -in. A325  
 Reaction: 43 kips

**Solution:**

(1) Required stiffener width  $W$ :

Required length of bearing

$$N = 3.5 + \frac{\text{Reaction} - R}{R_t} = 3.5 + \frac{43 - 47}{10.1} = 3.5 \text{ in. (disregarding negative quantity)}$$

Width  $W = 3.5 + 0.5$  (setback) = 4.0 in. Use 4 in.

(2) Length of stiffener plate  $L$  and weld size:

(a) Because the stiffened seats frame opposite each other and the vertical welds are in line, the maximum E70XX weld size is limited to  $\frac{1}{2}$  times the web thickness, so as not to overstress column web in shear.

$$\frac{1}{2} \times \frac{1}{2}\text{-in.} = \frac{1}{4}\text{-in. Use } \frac{1}{4}\text{-in. fillet weld.}$$

(b) From Table VIII:

$L$  of 9 in. with  $\frac{1}{4}$ -in. fillet weld has a capacity of 46.1 > 43 kips o.k.

(3) Thickness  $t$  of stiffener plate and seat plate:

$$t = 2 \times \text{weld size} \geq \text{beam web } t_w = 2 \times \frac{1}{4} = \frac{1}{2} > \frac{3}{8}\text{-in. o.k.}$$

(4) Seat plate length and welds:

(a) Gage in beam flange is  $5\frac{1}{2}$  in.;  $T$ -distance of column web is  $11\frac{1}{4}$  in. Seat plate will be made  $8\frac{1}{2}$  in. long.

(b) Minimum length of  $\frac{1}{4}$ -in. weld to attach seat plate to column web =  $2 \times 0.2L = 3.6$  in.

This also establishes a 3.6 in. (1.8 in. on each side of stiffener) minimum length of weld to connect the stiffener to the seat plate. However, to simplify layout and welding, the seat plate will be welded full length to the column web and the stiffener plate will be welded full width to the seat plate.

Use (each side of column web):

Seat plate:  $4 \times \frac{1}{2} \times 8\frac{1}{2}$   
 Stiffener plate:  $4 \times \frac{1}{2} \times 9$ , with  $\frac{1}{4}$ -in. weld  
 Top angle:  $4 \times 3\frac{1}{2} \times \frac{1}{4} \times 8\frac{1}{2}$ , with 4-in. leg bolted for shipment to column web  
 Ten  $\frac{7}{8}$ -in. A325 bolts (total)  
 Welds: E70XX  
 (Sketch required but not shown.)

**Problem 32**—Connection of W21×55 beams framing on opposite sides of a W14×103 column web, with a reaction of 57 kips for each beam.

**Solution:**

Similar to Problem 31 (student to show all steps and draw sketch).

Use (each side of column web):

Seat plate:  $5 \times \frac{1}{2} \times 8\frac{1}{2}$   
 Stiffener plate:  $5 \times \frac{1}{2} \times 1'0$ , with  $\frac{1}{4}$ -in. weld  
 Top angle:  $4 \times 3\frac{1}{2} \times \frac{1}{4} \times 8\frac{1}{2}$ , with 4-in. leg bolted for shipment to column web  
 Ten  $\frac{7}{8}$ -in. A325 bolts (total)  
 Welds: E70XX

**Problem 33**—Connection of beam F (W30×108) to column 2 (W10×60). Main material is ASTM A572 grade 50 steel.

**Given:**

W30×108: Web  $t_w = \frac{9}{16}$ -in.  
 Flange:  $10\frac{1}{2} \times \frac{3}{4}$ -in.  
 W10×60: Web  $t_w = \frac{7}{16}$ -in.,  $T = 7\frac{3}{4}$  in.  
 Steel: Main material: ASTM A572 grade 50  
 Detail material: ASTM A36  
 Welds: E70XX  
 Field bolts:  $\frac{7}{8}$ -in. A325  
 Reaction:  $\frac{1}{2}$  allowable uniform load for 24-ft span = 137.5 kips.

**Solution:**

(1) Required stiffener width  $W$ :

Required length of bearing

$$N = 3.5 + \frac{\text{Reaction} - R}{R_t} = 3.5 + \frac{137.5 - 104}{20.6} = 5.1 \text{ in.}$$

Width  $W = 5.1 + 0.5$  (setback) = 5.6 in. Use 6 in.

(2) Length of stiffener plate  $L$  and weld size:

(a) In order not to overload the column web in shear, the maximum weld size is  $1.34 \times \frac{7}{16} = 0.59$  or  $\frac{5}{8}$ -in. Use  $\frac{5}{16}$ -in. fillet weld.

(b) From Table VIII:

$L$  of 18 in. with  $\frac{5}{16}$ -in. weld has a capacity of 137 kips, which is approximately equal to 137.5 kips and is o.k.

(3) Thickness  $t$  of stiffener plate and seat plate:

$$t = 2 \times \text{weld size} \geq \text{beam web } t_w = 2 \times \frac{5}{16} = \frac{5}{8} > \frac{9}{16}\text{-in. o.k.}$$

(4) Seat plate length and welds:

(a)  $T$ -distance of W10×60 column web is  $7\frac{3}{4}$ -in.



- (b) Minimum length of  $\frac{5}{16}$ -in. weld to attach seat plate to column web is  $2 \times 0.2L = 2 \times 0.2 \times 18 = 7.2$  in.

This requires a seat plate length of  $7.2 + \frac{5}{8} = 7.83$  in. An 8-in. long seat plate will be used. As the  $T$ -distance is  $7\frac{3}{4}$  in., the plate may "ride" the fillet  $\frac{1}{8}$  in. at each end. This is not considered objectionable.

This also establishes the minimum length of weld required to connect the stiffener to the seat plate as explained in Problems 29 and 31.

Note: If the seat plate had not been long enough to accommodate the required length of weld connecting it to the column, a shorter length of stiffener plate  $L$  and a larger size fillet weld would have been used. This would reduce the required length of weld between the seat plate and the column web.

- (c) Top and bottom beam flanges will be cut to 6 in. and a  $3\frac{1}{2}$ -in. transverse gage will be used for the bolt holes.

**Use**

Seat plate:  $6 \times \frac{5}{8} \times 8$

Stiffener plate:  $6 \times \frac{5}{8} \times 1'6$ , with  $\frac{5}{16}$ -in. weld

Top angle:  $3\frac{1}{2} \times 3\frac{1}{2} \times \frac{1}{4} \times 6$  (bolted for shipment to column web)

Ten  $\frac{7}{8}$ -in. A325 bolts (total)

Welds: E70XX

Sketch required but not shown.

**Problem 34**—Connection of beam F (W30 $\times$ 108) to column 2 (W10 $\times$ 60). Reaction is 150 kips. Main material is ASTM A572 grade 50 steel.

**Solution:**

Similar to Problem 33 (student to show all steps and draw sketch).

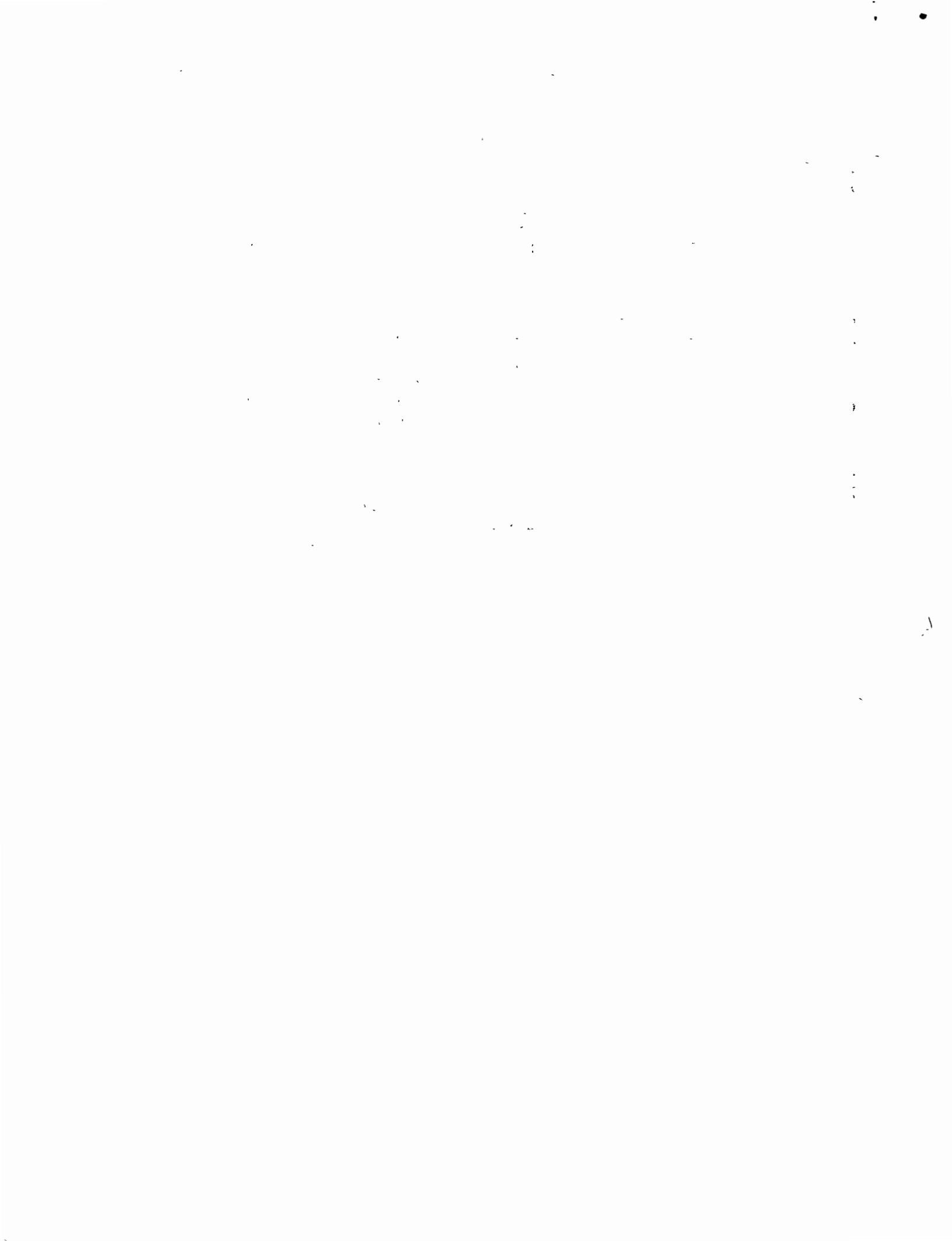
**Use:** Seat plate:  $7 \times \frac{3}{4} \times 8$

Stiffener plate:  $7 \times \frac{3}{4} \times 1'6$ , with  $\frac{3}{8}$ -in. weld

Top angle:  $3\frac{1}{2} \times 3\frac{1}{2} \times \frac{1}{4} \times 6$  (bolted for shipment to column web)

Six  $\frac{7}{8}$ -in. A325 bolts

Welds: E70XX



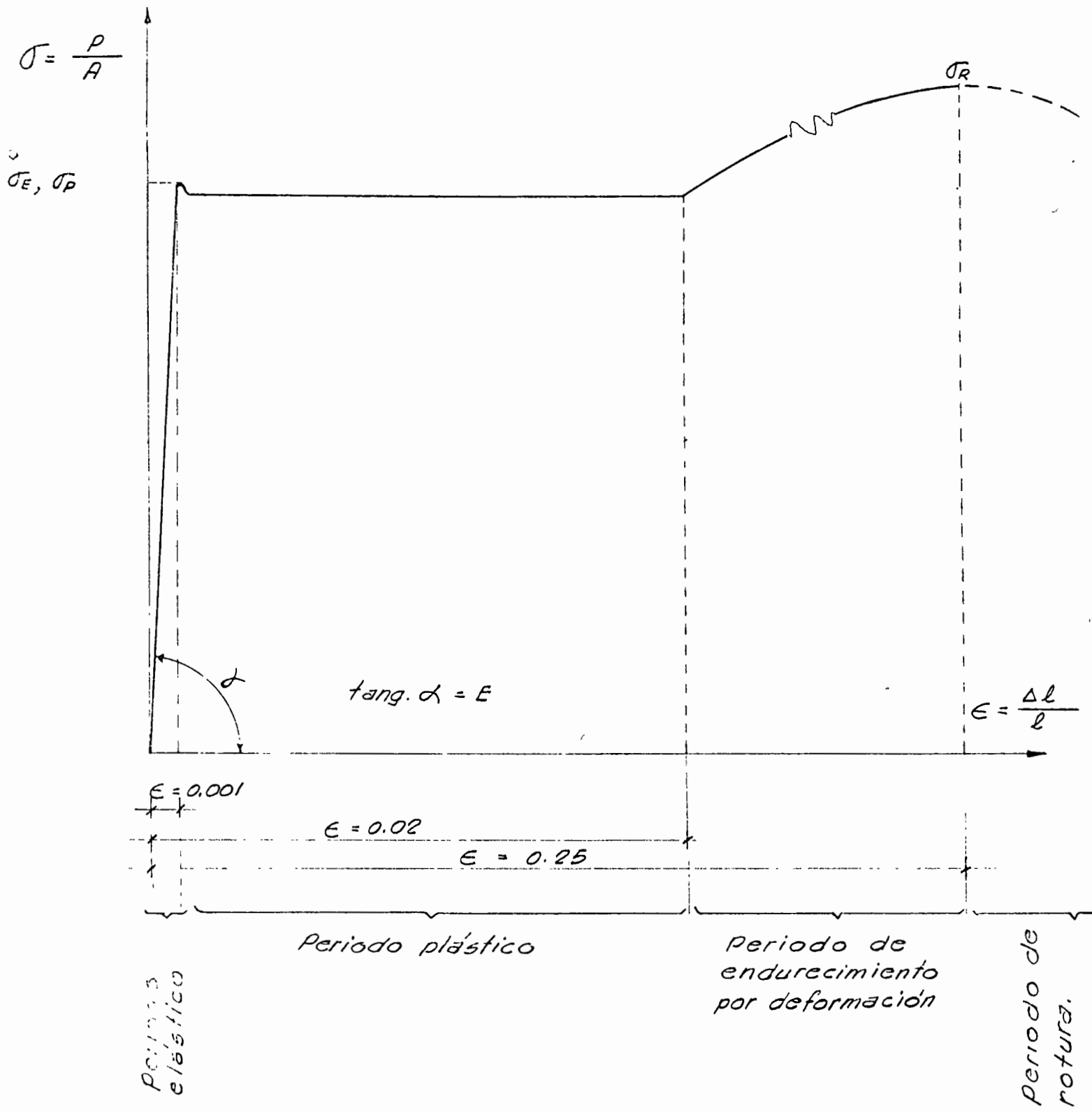
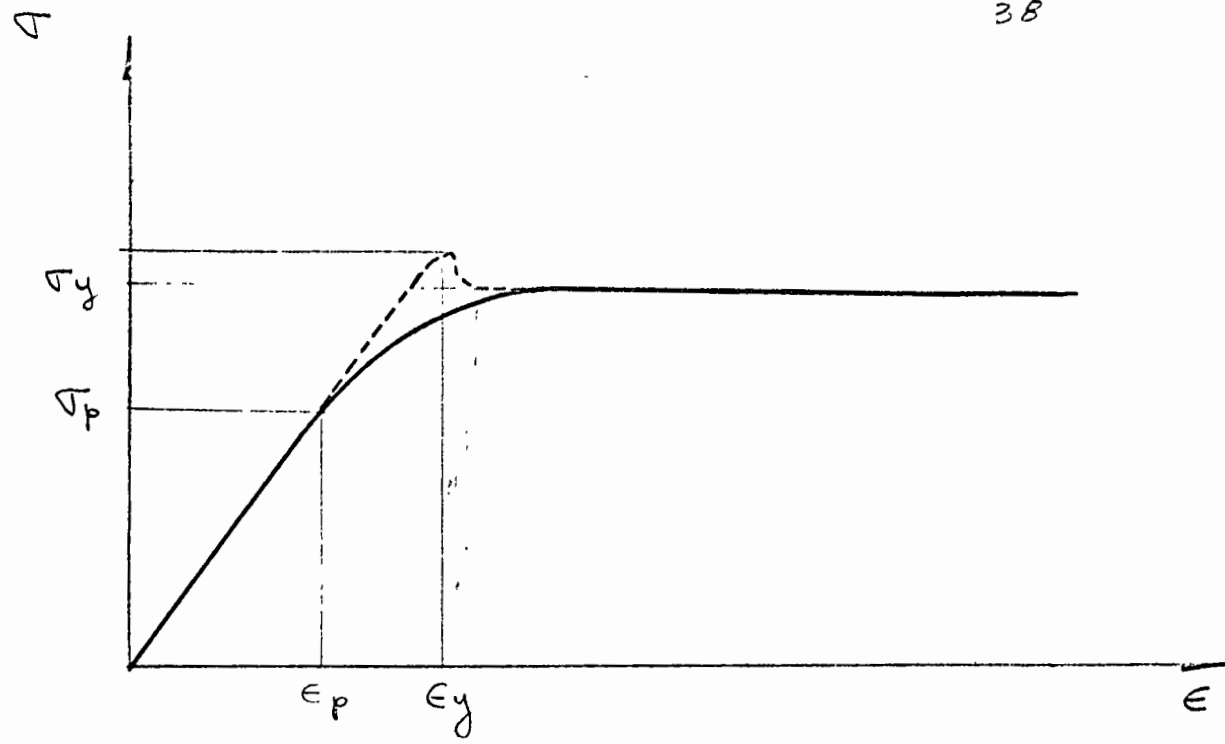
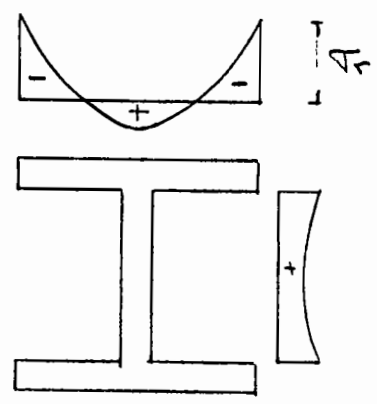


FIGURA 1



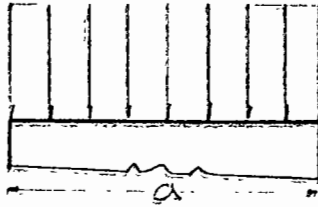
GRAFICA TÍPICA ESFUERZO-DEFORMACIÓN PARA UN PERFIL DE ACERO ESTRUCTURAL



ESFUERZOS RESIDUALES TÍPICOS EN UN PERFIL LAMINADO

FIGURA 2

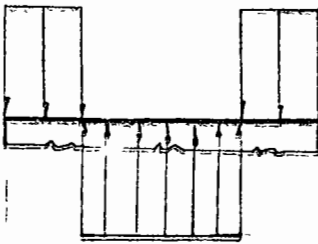
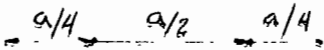
# FIGURA 3



$\sigma = \frac{P}{A}$  Esfuerzos debidos a las cargas.

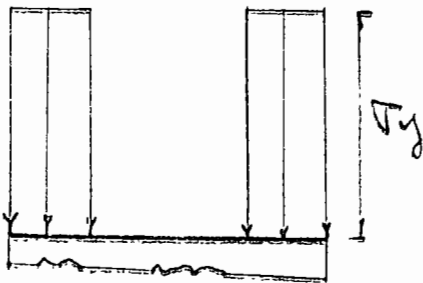
+

+



$\frac{\sigma_y}{2}$   
 $\frac{\sigma_y}{2}$

Distribucion de esfuerzos residuales.



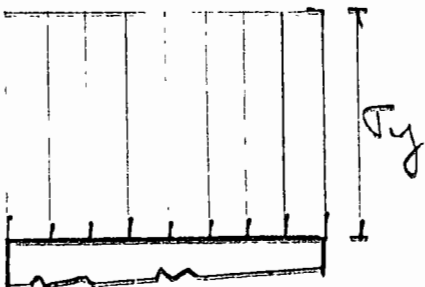
El perfil se comporta elásticamente hasta que:

$$\frac{P}{A} = \frac{\sigma_y}{2}$$

Entonces:

$$\epsilon = \frac{P}{AE} = \frac{\frac{A}{2} \sigma_y}{AE}$$

$$\epsilon = \frac{\sigma_y}{2E} = \frac{\epsilon_y}{2}$$



Si se aumenta la carga se llega finalmente a  $\sigma_y$  en toda la sección:

$$\Delta \epsilon = \frac{\Delta P}{\frac{A}{2} E} = \frac{\frac{A}{2} \sigma_y}{\frac{A}{2} E} = \frac{\sigma_y}{E} = \epsilon_y$$

CONCRETO

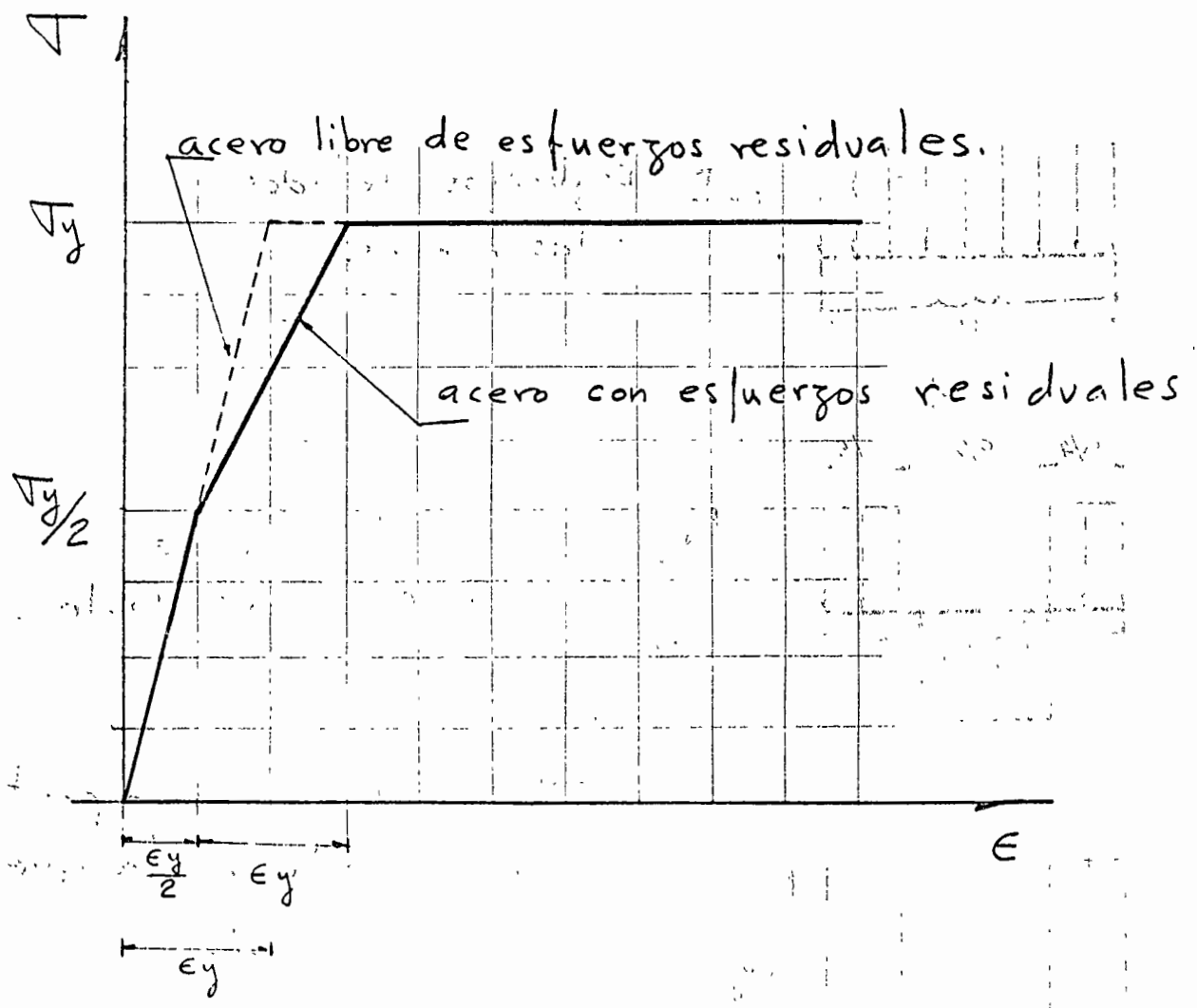
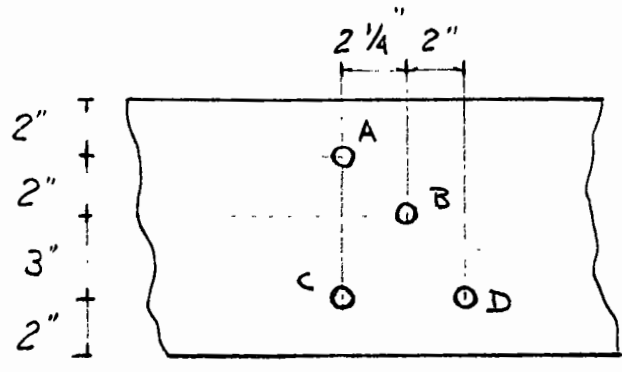


FIGURA 4

# FIGURA 5.

(tomado de Beedle pg 170)



$$B_n = B - \sum \phi + \sum_{n=1}^n \frac{S_n^2}{4q_n}$$

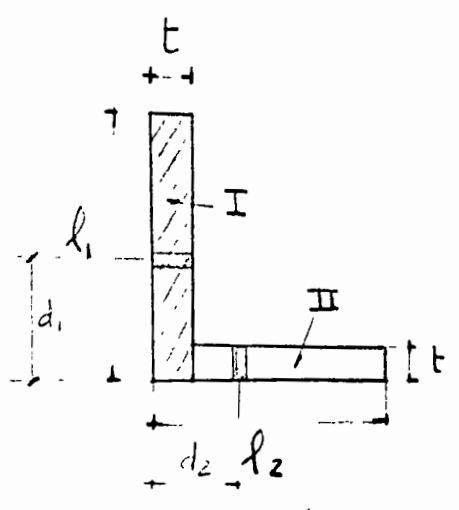
$$A_n = B_n \cdot t$$

A-C;  $B_n = 9 - 2(1) + 0 = 7''$

A-B-C;  $B_n = 9 - 3(1) + \frac{2.25^2}{4(2)} + \frac{2.25^2}{4(3)} = 7.06''$

A-B-D;  $B_n = 9 - 3(1) + \frac{2.25^2}{4(2)} + \frac{2^2}{4(3)} = 6.97''$

La sección crítica es ABD



Area de I =  $l_1 \cdot t$

Area de II =  $(l_2 - t) \cdot t$

$$B = (l_1 + l_2 - t)$$

$$B = \frac{A}{t}$$

$$d_1 = d_1 + d_2 - t$$

# Especificaciones relativas a tensión.

Esfuerzo permisible:

$$\sigma \leq 0.6 \sigma_y$$

$$\sigma \leq 0.5 \sigma_T$$

A.I.S.C.

C.F.E.

1.5.1.1

—

1.5.1.1

—

En zonas de pasadores

$$\sigma \leq 0.45 \sigma_y$$

1.5.1.1

—

Area neta:

$$A_n \leq 0.85 A$$

1.14.3

1.5 a

Diámetro de agujeros para  
di seño

$$\phi_d = \phi + 3 \text{ mm.}$$

1.14.5

1.5 a

Relación de esbeltez  
(recomendación)

$$\text{miembros principales } \frac{L}{r} \leq 240$$

1.8.4

1.4

$$\text{miembros secundarios } \frac{L}{r} \leq 300$$

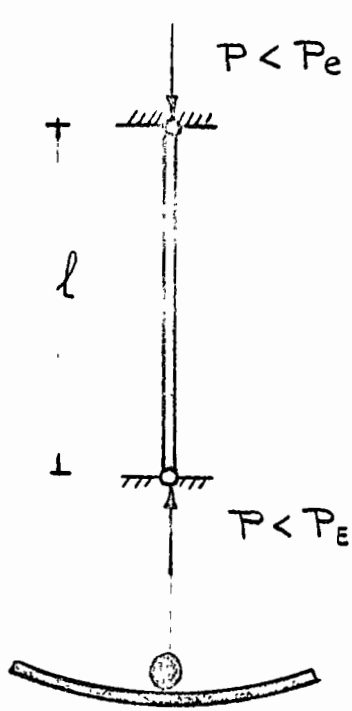
1.8.4

1.4

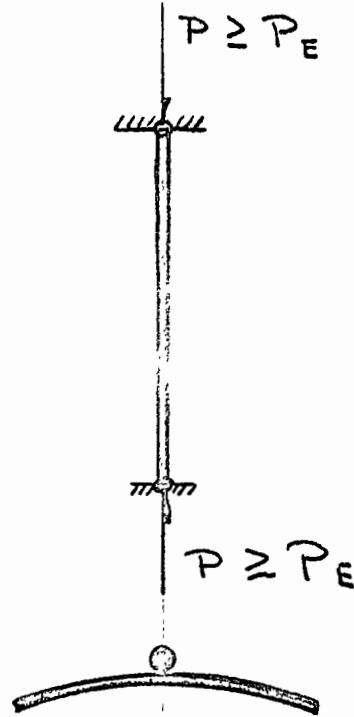
FIGURA 6



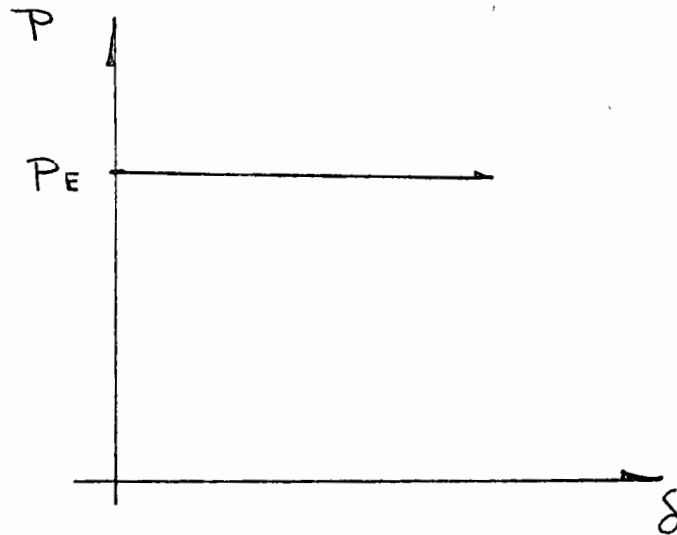
# FIGURA C1



Equilibrio Estable.

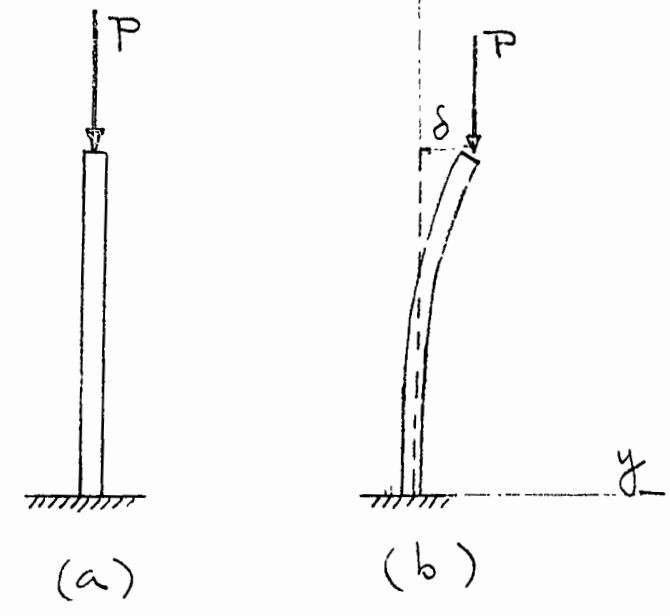


Equilibrio Inestable.



Bifurcación de la posición de equilibrio

FIGURA  
C2



Dos configura-  
ciones de equili-  
brio posibles

Si el material es elástico y las deformaciones son pequeñas:

$$EI \frac{d^2 y}{dx^2} = P(\delta - y)$$

si  $k^2 = \frac{P}{EI}$

$$\frac{d^2 y}{dx^2} + k^2 y = k^2 \delta$$

la solución es:  $y = \delta + A \cos kx + B \sin kx$

como:  $y = 0$  para  $x = 0$   $A = -\delta$

y como  $\frac{dy}{dx} = 0$  para  $x = 0$   $B = 0$

$$y = \delta (1 - \cos kx)$$

para  $x = l$   $\delta = \delta (1 - \cos kl)$

Se cumple para  $\delta = 0$  y para  $\cos kl = 0$   
esto es  $kl = (2n+1) \frac{\pi}{2} = l \sqrt{\frac{P}{EI}}$

$$P_{min} = P_{cr} = \frac{\pi^2 EI}{4l^2}$$

# LONGITUDES DE PANDEO

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

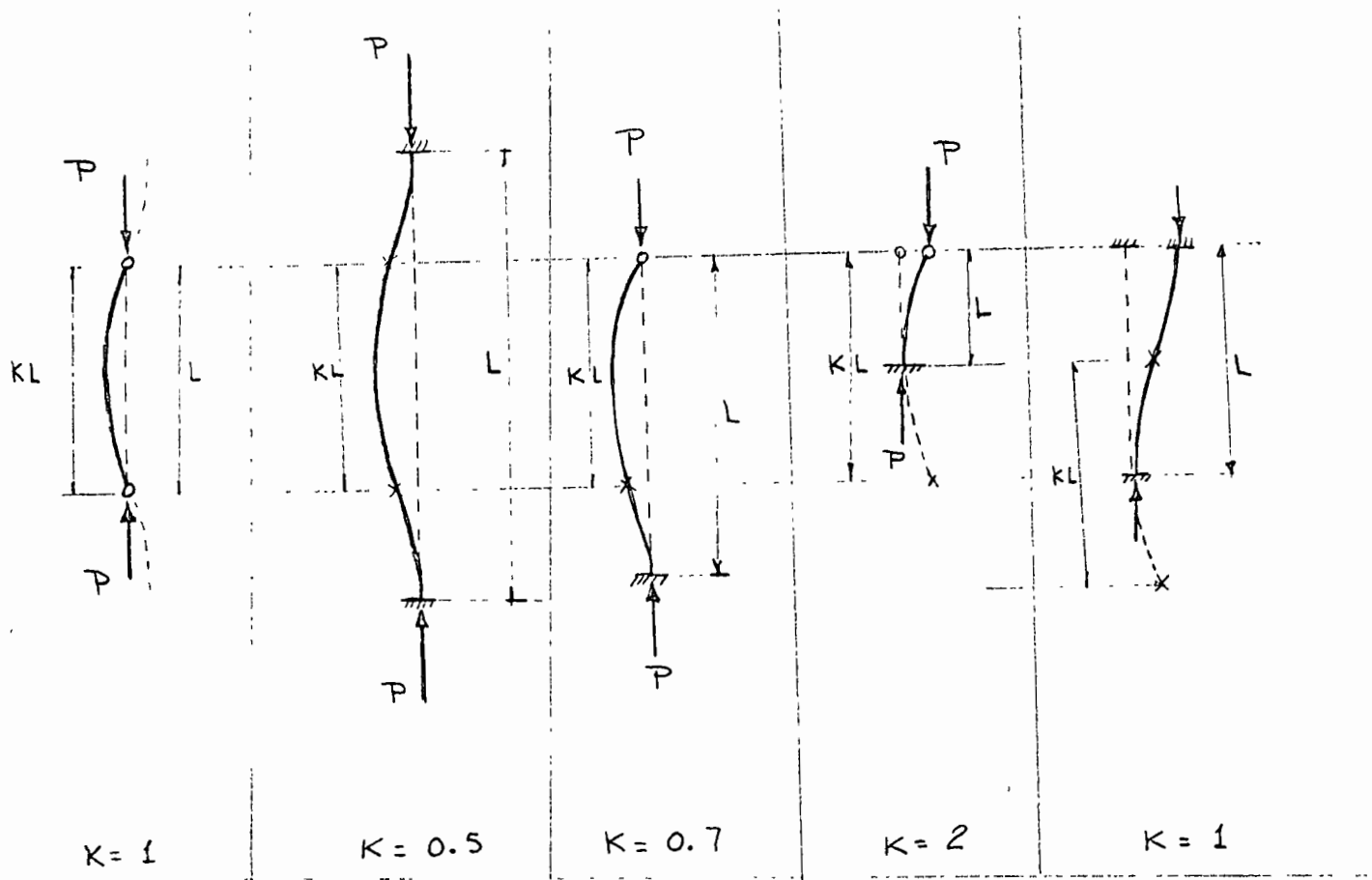
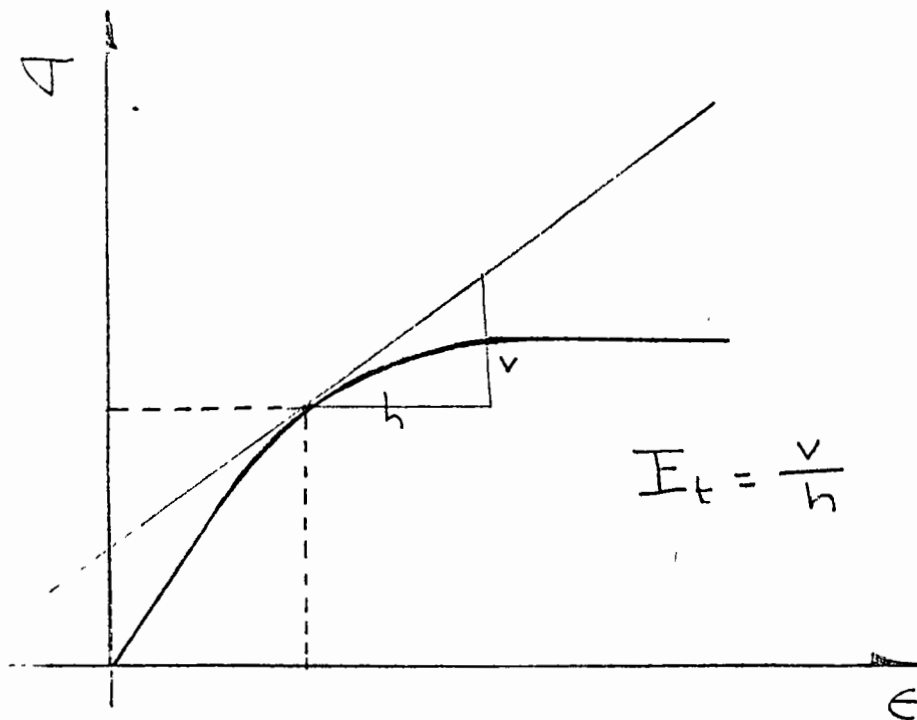


FIGURA C3

Límite de aplicación de la fórmula de Euler

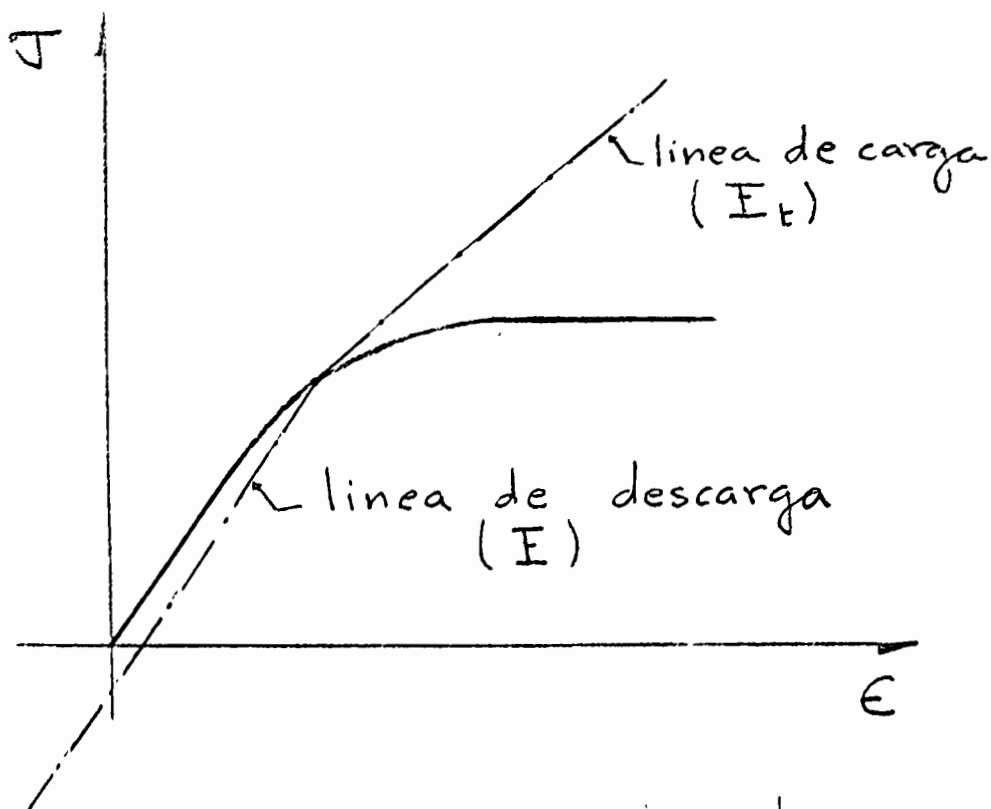
$$\sigma_{cr} = \frac{\pi^2 E}{(kl/r)^2} = \sigma_p$$

$$\therefore \frac{l}{r} = \pi \sqrt{\frac{E}{\sigma_p}}$$



$$\sigma_{cr} = \frac{\pi^2 E_t}{\left(\frac{kl}{r}\right)^2}$$

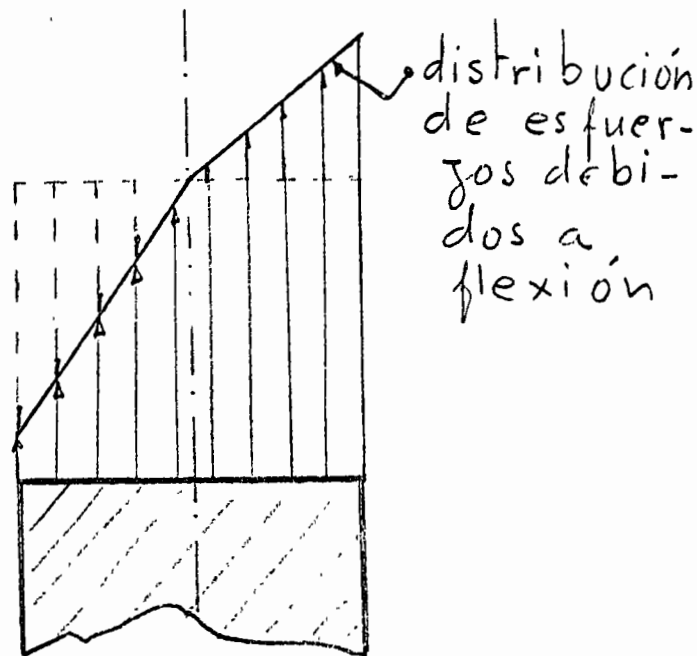
FIGURA C4



$$\frac{d^2 y}{dx^2} \underbrace{(EI_1 + E_T I_2)}_{E_r} = P y$$

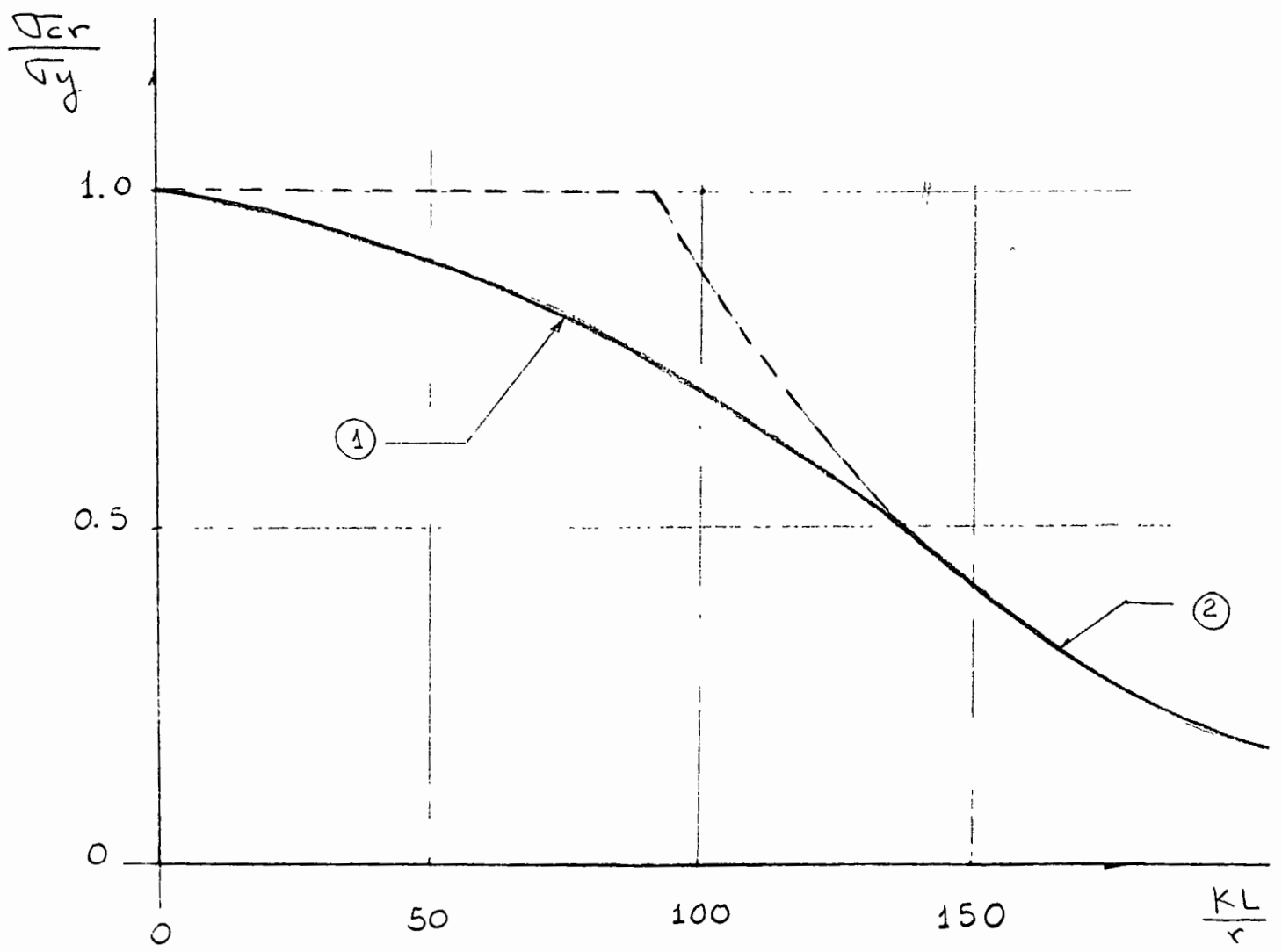
$$P_{cr} = \frac{\pi^2 E_r}{l^2}$$

Teoría del  
módulo doble..



Distribución de esfuerzos al deformarse la columna.-

# FIGURA C-6



CURVA  $F_{cr} - \frac{L}{r}$

A.I.S.C. ①  $\left(1 - \frac{(KL/r)^2}{2C_c^2}\right)$  ----- pandeo inelástico  
 ② -----  $\frac{\pi^2 E}{(KL/r)^2}$  ----- pandeo elástico (fórmula de Euler)

FIGURA C6aPANDEO POR TORSIÓN

Ecuación básica de partida:

$$E K_1 \frac{d^3 \beta}{d\zeta^3} - G K \frac{d\beta}{d\zeta} = -T$$

$K =$  constante de torsión ;  $\sum \frac{bt^3}{3}$  (sección compuesta de rectángulos)

$T =$  momento torsionante

$\beta =$  giro alrededor del eje longitudinal.

$K_1 =$  constante de alabeo

Esfuerzo crítico de pandeo por torsión:

$$\sigma_{ct} = \frac{E}{2(1+\mu)} \left(\frac{t}{b}\right)^2 \quad \text{para un ángulo}$$

$$\sigma_{crt} = \frac{1}{J} \left( GK + \frac{\pi^2 E K_1}{L^2} \right) \quad \text{para una viga}$$

# Formulas del AISC para diseño de Columnas a Compresión Axial.

		Especificación	
		AISC	C F E
$F_a = \frac{\left[1 - \frac{(KL/r)^2}{2C_c^2}\right] \sigma_y}{\frac{5}{5} + \frac{3(KL/r)}{8C_c} - \frac{(KL/r)^3}{8C_c^3}} = \frac{\sigma_{cr}}{C.S.}$ <p>para <math>\frac{KL}{r} &lt; \sqrt{\frac{2\pi^2 E}{\sigma_y}} = C_c</math> (limite de validez de la fórmula de Euler)</p>		1.5.1.3.1 (1.5.1)	R 2.4.2.
$F_a = \frac{12\pi^2 E}{23(KL/r)^2} = \frac{\sigma_{cr}}{C.S.}$ <p>para <math>\frac{KL}{r} &gt; C_c</math></p>		1.5.1.3.2 (1.5.2)	R 2.4.1.
<p>Contraentes y miembros secundarios</p> $F_{as} = \frac{F_a}{1.6 - \frac{L}{200r}} \quad \left(\text{para } \frac{L}{r} > 120\right)$		1.5.1.3.3 (1.5.3)	
<p>Aties. loras de traves armadas</p> $F_a = 0.6 \sigma_y$		1.5.1.3.4	
<p>Almas de viguetas</p> $F_a = 0.75 \sigma_y$		1.5.1.3.5	
$\frac{KL}{r} \leq 200$		1.8.4	R 2.5.



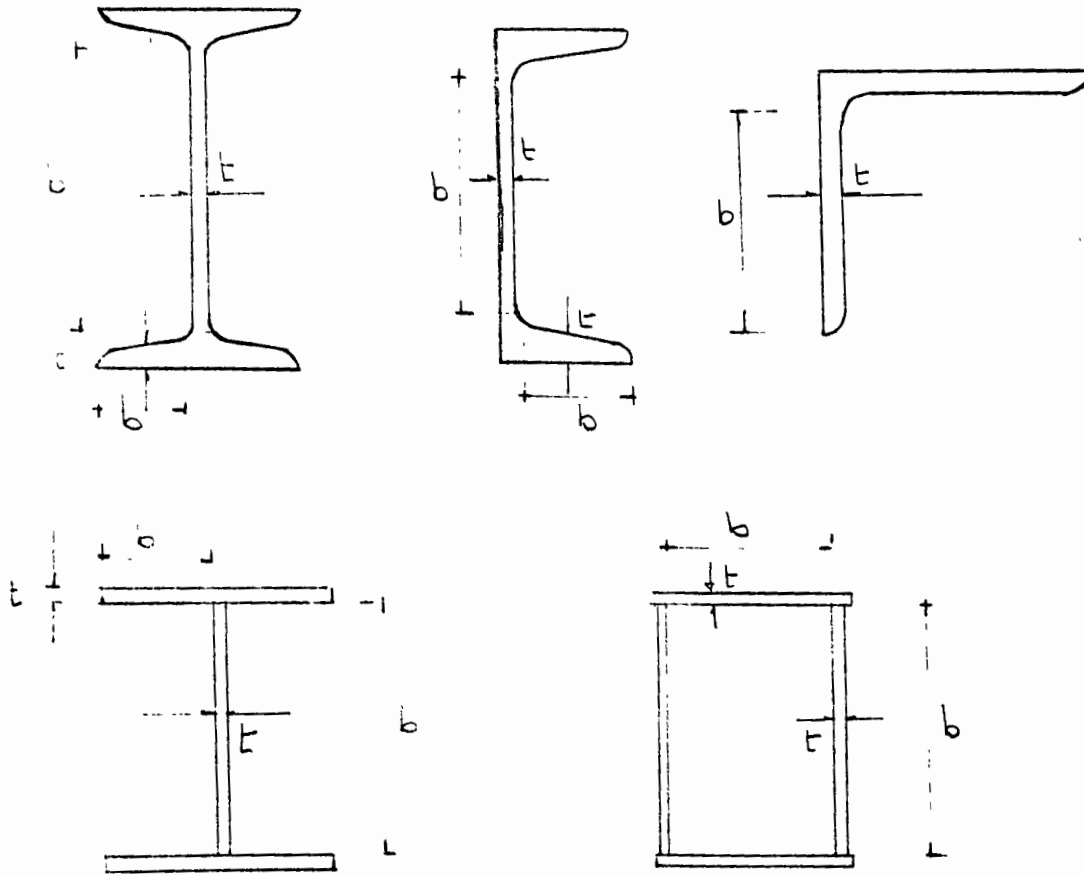
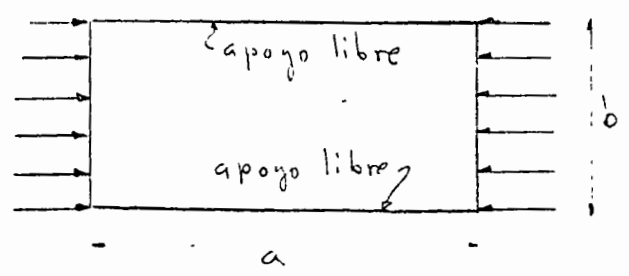


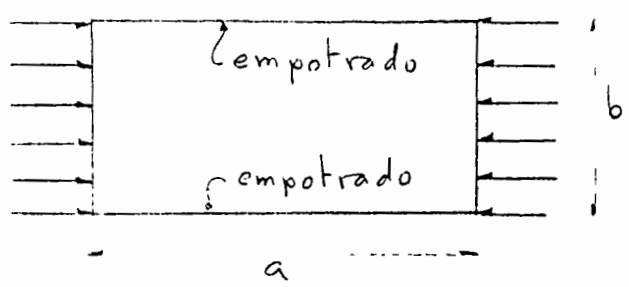
FIGURA C-7

# FIGURA C8

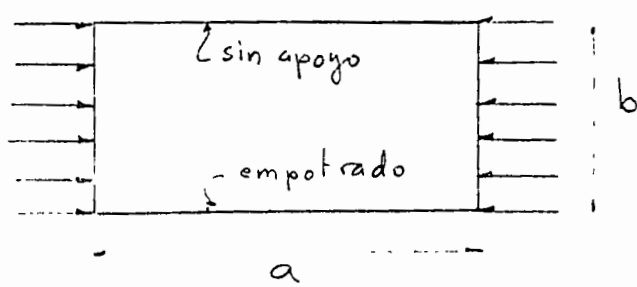
$a \gg b$



$K = 4$



$K = 7$



$K = 1.3$

## COEFICIENTES DE PANELO DE PLACAS (K)

$$\sigma_{cr} = \frac{\pi^2 E \sqrt{G}}{12(1-\mu^2)} \left(\frac{t}{b}\right)^2 K$$

## PANDEO LOCAL

53

$$\sigma_{cr} = \frac{\pi^2 E \sqrt{\zeta}}{12(1-\mu^2)} \left(\frac{t}{b}\right)^2 K \quad [\text{Ec. PL1}]$$

En el rango elástico  $\zeta = 1$   
Para que no se presente pandeo local:

$$\frac{\pi^2 E}{(L/r)^2} \leq \frac{\pi^2 E}{12(1-\mu^2)} \left(\frac{t}{b}\right)^2 K$$

$$y \quad \frac{b}{t} \leq 0.3 \frac{L}{r} \sqrt{K}$$

En el rango inelástico

$$\zeta = \frac{(\sigma_y - \sigma_{cr}) \sigma_{cr}}{(\sigma_y - \sigma_p) \sigma_p}$$

Para que no haya pandeo local::

$$\left[1 - \frac{L/r}{2C_c^2}\right] \sigma_y \leq \frac{\pi^2 E \sqrt{\zeta}}{12(1-\mu^2)} \left(\frac{t}{b}\right)^2 K$$

$$\frac{b}{t} \leq \frac{38.2 \sqrt{L/r}}{\sqrt[4]{31802 - (L/r)^2}} \sqrt{K} \quad (\text{Acero A-36})$$

Y en general

$$\frac{b}{t} \leq c \sqrt{K}$$

FIGURA C9

Criterio del AISC para pandeo local:

$$\sigma_y = K \frac{E \pi^2}{12(1-\mu^2) \left(\frac{b}{t}\right)^2}$$

$$\mu = 0.3$$

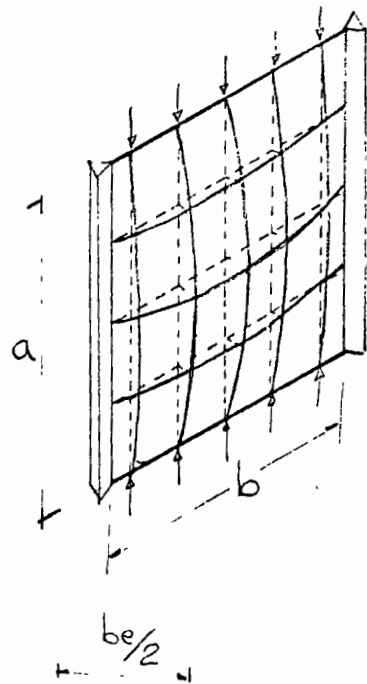
$$\frac{\sigma_y}{K} = \frac{1900000}{\left(\frac{b}{t}\right)^2}$$

$$\frac{b}{t} = \frac{1380}{\sqrt{\sigma_y/K}}$$

Si  $K = 4$

$$\frac{b}{t} = \frac{2760}{\sqrt{\sigma_y}}$$

FIGURACIÓ



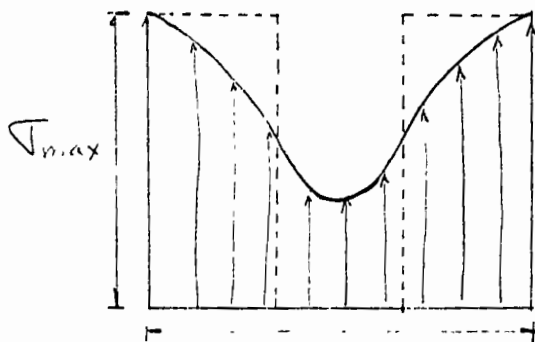
La placa es completamente efectiva hasta:

$$\frac{b}{t} < 0.95 \sqrt{\frac{E}{J_{max}}}$$

para valores mayores de  $\frac{b}{t}$ :

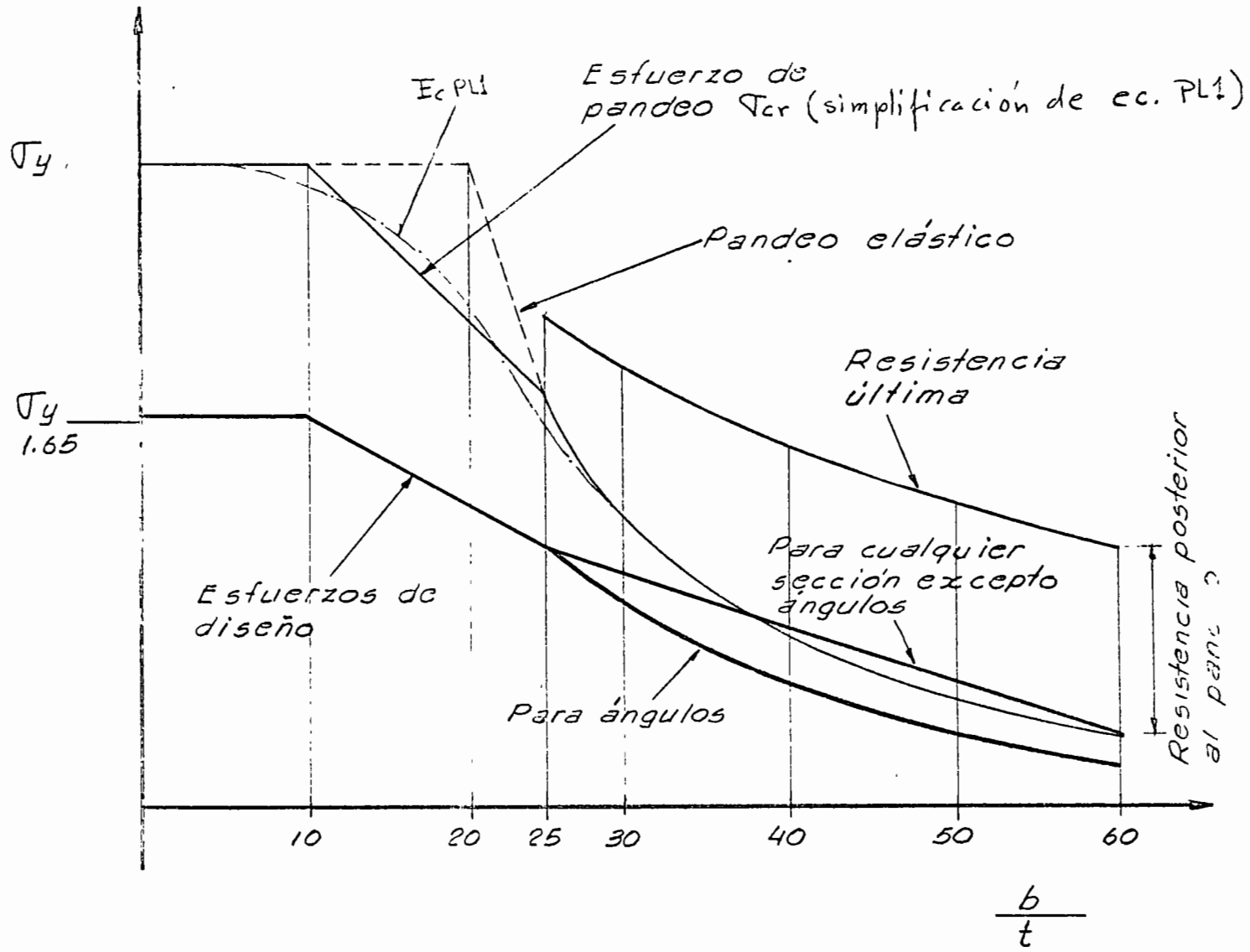
$$\frac{be}{t} = 1.9 \sqrt{\frac{E}{J_{max}}} \left( 1 - 0.475 \frac{t}{b} \sqrt{\frac{E}{J_{max}}} \right)$$

(ec. PL2)



RESISTENCIA POSTERIOR AL PANDEO

# FIGURA C 12



ESFUERZOS DE PANDEO PARA PERFILES DE CALIBRE LIGERO NO ATIESADOS

# TABLA C13

## ESPECIFICACIONES AISI. y AISC, COLUMNAS CON CARGA AXIAL DE PERFILES DELGADOS

Para placas no atiesadas:

$$\frac{b}{t} < 10 \quad \sigma_p = \sigma_y / 1.65 = \sigma_a$$

$$10 < \frac{b}{t} < 25 \quad \sigma_p = (1.67 \sigma_a - 605) - \frac{1}{15} (\sigma_a - 905) \frac{b}{t}$$

$$25 < \frac{b}{t} < 60 \quad \sigma_p = \frac{568000}{(b/t)^2} \quad (\text{para ángulos})$$

$$\sigma_p = 1400 - 282 \frac{b}{t} \quad (\text{otros perfiles})$$

$$Q_s = \frac{\sigma_p}{\sigma_a}$$

Para perfiles atiesados:

$$Q_a = \frac{A_{ef}}{A}$$

### ESFUERZOS PERMISIBLES EN COLUMNAS AISI

$$\text{Para } \frac{KL}{r} < C_c' = \sqrt{\frac{2\pi^2 E}{Q\sigma_y}} ; \quad F_a = \frac{Q\sigma_y}{F.S.} \left[ 1 - \frac{Q\sigma_y}{4\pi^2 E} \left(\frac{L}{r}\right)^2 \right]$$

$$\text{Para } \frac{KL}{r} > C_c' ; \quad F_a = \frac{10450000}{(KL/r)^2}$$

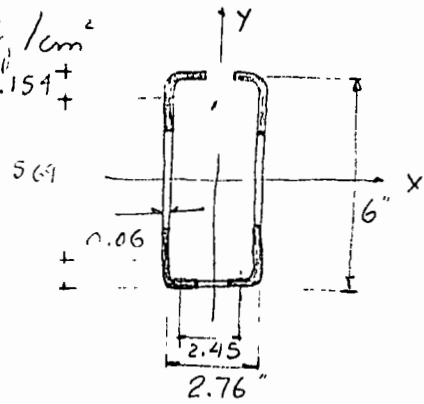
### ESFUERZOS PERMISIBLES EN COLUMNAS AISC

$$\text{Para } KL/r < C_c = \sqrt{\frac{2\pi^2 E}{Q_s Q_a \sigma_y}} \\ F_a = \frac{Q_s Q_a \left[ 1 - (KL/r)^2 / 2C_c^2 \right] \sigma_y}{F.S.}$$

Ejemplo del diseño de un perfil de lámina delgada.

Acero con  $\sigma_y = 2320 \text{ Kg/cm}^2$

$0.154 = R + t$   
 $R = 0.094$



Area =  $6.6 \text{ cm}^2$   
 $r_y = 3.15 \text{ cm}$   
 $L = 2.30 \text{ m}$

$\frac{L}{r_y} = \frac{230}{3.15} = 73$

Area efectiva:

$\frac{b_e}{t} = 1.9 \sqrt{\frac{E}{\sigma_y}} \left( 1 - 0.475 \frac{t}{b} \sqrt{\frac{E}{\sigma_y}} \right)$   
 $\frac{b_e}{t} = 56 \left( 1 - 14 \frac{t}{b} \right)$

Para los lados largos  $\frac{b}{t} = \frac{5.69}{0.06} = 95$ ;  $\frac{b_e}{t} = 48$ ;  $b_e = 2.9$ "

Para los lados cortos  $\frac{b}{t} = \frac{2.45}{0.06} = 40.8$ ;  $\frac{b_e}{t} = 37$ ;  $b_e = 2.2$ "

Area no efectiva =  $2 \left[ (5.69 - 2.90) + (2.45 - 2.20) \right] 0.06 = 0.365 \text{ in}^2 = 2.35 \text{ cm}^2$

Area efectiva =  $6.6 - 2.35 = 4.25 \text{ cm}^2$

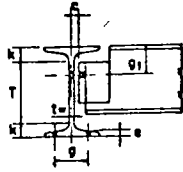
$Q_a = \frac{4.25}{6.6} = 0.645$

$C_c = \sqrt{\frac{2 \pi^2 E}{\sigma_y}} = 165 > \frac{L}{r_y} \therefore$

$\sigma_p = \frac{\sigma_y}{FS} \left[ 1 - \frac{Q_a \sigma_y}{4 \pi^2 E} \left( \frac{L}{r} \right)^2 \right] = \frac{0.645 \times 2320}{1.95} \left[ 1 - \frac{0.645 \times 2320}{4 \pi^2 E} (73)^2 \right] = 690 \text{ Kg/cm}^2$

$\rightarrow = 6.6 \times 690 = 4750 \text{ Kg}$





VIGAS I PERFIL STANDARD

I P S

Dimensiones para detallar

PERFIL	PESO Kg/m.	SERIALTE d mm	PATIN Ancho b mm	ESPESOR DEL ALMA t mm	DIMENSIONES					CANT. DE AGUJERES	G.M.P. en perfil mm.		
					a	T	h	g'	e				
3	8.48	76	60	6	4	28	48	14	38	5	36	6	9.5
76.2	11.16	76	64	6	10	28	48	14	38	7	36	6	9.5
4	11.46	102	68	8	5	32	70	16	51	5	38	8	12.7
101.6	14.14	102	71	8	8	32	70	16	51	6	38	8	12.7
5	14.88	127	76	8	6	35	91	18	51	5	40	8	12.7
127.0	21.95	127	83	8	13	35	91	18	51	8	40	8	12.7
6	18.60	152	85	10	6	39	114	19	51	5	44	10	15.8
152.4	25.67	152	92	10	13	39	114	19	51	8	44	10	15.8
7	22.77	178	92	10	6	44	136	21	57	5	57	10	15.8
177.8	29.76	178	98	10	11	44	136	21	57	8	57	10	15.8
8	27.38	203	102	11	8	48	159	22	57	6	57	11	19.0
203.2	34.23	203	105	11	11	48	159	22	57	8	57	11	19.0
10	37.80	254	118	13	8	55	204	25	64	6	70	13	19.0
254	52.09	254	127	13	16	55	204	25	64	10	70	13	19.0
12	47.32	305	127	14	10	60	247	29	64	7	76	14	19.0
305	52.09	305	129	14	11	60	247	29	64	7	76	14	19.0

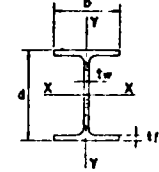


ALTOS HORNOS DE MEXICO, S. A.

VIGAS I PERFIL RECTANGULAR

I P R

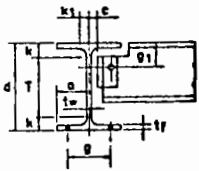
Propiedades para diseño



PERFIL d x b	PESO A	PESOTE d	PATIN		ESPESOR DEL ALMA t	d	EJE X X			EJE Y Y		
			Ancho b	Espesor d			I	S	r	I	S	r
L P R 6 x 4 152.4 x 101.6	16.13 22.77 30.45	148 152 159	100 102 102	4.9 7.1 10.3	4.3 5.8 6.6	3.02 2.09 -1.51	616 903 1319	83 119 165	6.17 6.30 6.58	78.6 120.2 179.8	15 23 35	2.21 2.30 2.43
L P R 8 x 4 203.2 x 101.6	19.03 24.71 28.58	201 203 206	100 102 102	5.2 6.5 8.0	4.3 5.8 6.2	3.87 3.06 2.52	1282 1644 1998	127 162 193	8.20 16.44 8.35	82.8 109.0 137.3	16 21 27	2.08 2.10 2.19
L P R 8 x 5 1/4 203.2 x 133.4	32.26 37.93	203 206	133 133	7.8 9.6	5.8 6.3	1.96 1.61	2348 2880	231 279	8.53 8.71	279.7 353.7	42 52	2.94 3.04
L P R 10 x 4 254.0 x 101.6	21.87 28.38 32.13 36.19	251 254 257 260	100 102 102 100	5.2 6.8 8.4 10.0	4.6 5.8 6.1 6.4	4.83 3.66 2.99 2.55	2160 2864 3405 4004	172 226 265 308	9.96 10.03 10.28 10.52	83.6 116.1 143.6 174.4	17 23 28 34	1.95 2.02 2.11 2.19
L P R 10 x 5 3/4 254.0 x 146.0	39.93 47.42 55.03	251 256 259	146 146 147	8.6 10.9 12.7	6.1 6.4 7.3	1.99 1.61 1.39	4424 5544 6547	352 432 505	10.52 10.82 10.89	403.7 528.5 632.6	55 72 85	3.17 3.32 3.40
L P R 12 x 4 304.8 x 101.6	26.71 31.35 36.25 41.74	302 305 309 313	101 102 102 102	5.7 6.8 8.9 10.8	5.1 5.8 6.1 6.6	5.25 4.53 3.40 2.84	3671 4383 5415 6481	243 256 350 414	11.71 11.81 12.22 12.47	93.6 116.1 152.7 189.3	18 23 30 37	1.87 1.93 2.05 2.13
L P R 12 x 6 1/2 304.8 x 165.1	51.42 58.83 68.32	304 307 311	165 166 167	10.2 11.8 13.7	6.1 6.7 7.7	1.81 1.57 1.36	8495 9923 11687	558 645 752	12.85 12.98 13.08	690.9 824.1 986.4	84 100 118	3.65 3.74 3.80
L P R 12 x 8 304.8 x 203.2	75.94 85.42 94.90	303 306 309	203 204 205	13.1 14.6 16.3	7.5 8.5 9.4	1.14 1.03 0.92	12907 14600 16420	850 953 1060	13.03 13.08 13.15	1835 2081 2347	180 203 229	4.92 4.92 4.97
L P R 14 x 6 3/4 355.6 x 171.4	56.84 64.52 72.06	352 356 359	171 171 172	9.7 11.5 13.0	6.9 7.3 7.9	2.12 1.81 1.61	12053 14117 16036	685 795 895	14.50 14.80 14.90	728 846 1023	85 103 120	3.58 3.70 3.78
L P R 14 x 8 355.6 x 203.2	81.61 91.03 100.58	347 351 354	203 204 205	13.4 15.1 16.7	7.8 8.6 9.4	1.28 1.14 1.03	17856 20183 22562	1027 1150 1275	14.78 14.88 14.98	1877 2135 2393	185 209 234	4.80 4.95 4.87
L P R 16 x 7 406.4 x 177.8	68.32 75.94 85.42 94.84	403 406 409 413	177 178 179 180	10.9 12.7 14.3 15.9	7.6 7.8 8.8 9.6	2.09 1.79 1.60 1.44	18576 21457 24279 27240	923 1055 1186 1322	16.48 16.81 16.87 16.97	920 1103 1270 1448	103 125 143 161	3.65 3.91 3.86 3.91
L P R 18 x 8 3/4 457.2 x 222.2	121.29 132.64 145.99 161.09	454 457 461 465	221 222 223 224	17.4 19.1 21.1 23.1	10.2 11.1 12.1 13.4	1.18 1.08 0.94 0.90	43529 48028 53560 59517	1917 2100 2322 2558	18.95 19.01 19.15 19.23	2926 3267 3688 4137	264 293 331 369	4.90 4.95 5.03 5.01
L P R 18 x 11 3/4 457.2 x 298.4	182.06 199.09 216.19	461 465 469	298 299 301	21.1 23.1 25.2	13.0 14.1 15.1	0.73 0.67 0.62	69706 77106 84653	3021 3313 3606	19.55 19.68 19.78	8407 9615 10638	577 642 708	6.88 6.93 7.01

ALTOS HORNOS DE MEXICO, S. A.





VIGAS I PERFIL RECTANGULAR

I P R

Dimensiones para detallar

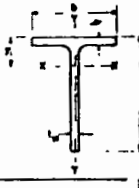
PERFIL	PESO	PERALTE	PATIN		ESPAZOR		DISTANCIAS					GRAMIL
d x b	Kg/m	d	Ancho b	Espesor d	ALMA tw	a	T	k	k1	g1	c	s
		mm	mm.	mm.	mm	mm	mm.	mm.	mm.	mm.	mm.	mm.
L.P.R.	127	149	102	5	5	48	127	11	10	51	5	57
6 x 4	17.9	152	102	6	6	48	124	14	11	51	5	57
152 x 102	23.8	159	102	10	6	48	125	17	11	57	5	57
L.P.R.	14.9	200	102	5	5	48	174	13	10	51	5	57
8 x 4	19.4	203	102	6	6	48	175	14	11	51	5	57
203 x 102	22.4	206	102	8	6	48	174	16	11	51	5	57
L.P.R.	25.3	203	133	8	6	63	171	16	11	51	5	70
8 x 5 1/4	29.8	206	133	10	6	63	172	17	11	57	5	70
203 x 133												
L.P.R.	17.1	251	102	5	5	48	225	13	10	51	5	57
10 x 4	22.4	254	102	6	6	48	226	14	11	51	5	57
254 x 102	25.3	257	102	8	6	48	225	16	11	51	5	57
	28.3	260	102	10	6	48	226	17	11	57	5	57
L.P.R.	31.3	251	146	8	6	70	219	16	11	57	5	70
10 x 5 3/4	37.3	256	146	11	6	70	218	19	11	57	5	70
254 x 146	43.2	259	146	13	8	69	219	20	11	57	6	70
L.P.R.	20.9	302	102	6	6	48	274	14	10	51	5	57
12 x 4	24.6	305	102	6	6	48	273	16	11	51	5	57
305 x 102	28.3	308	102	10	10	46	274	17	11	57	5	57
	32.8	311	102	11	11	45	273	19	11	57	5	57
L.P.R.	40.3	304	165	10	10	77	264	21	13	57	5	89
12 x 6 1/2	46.2	308	165	11	11	77	264	22	13	57	5	89
305 x 165	53.7	311	168	14	14	77	264	24	13	57	5	89
L.P.R.	59.6	305	203	13	8	97	247	29	19	63	6	140
12 x 8	67.1	305	203	14	8	97	245	30	19	63	6	140
305 x 203	74.5	311	206	16	10	98	247	32	21	63	6	140
L.P.R.	44.7	352	171	10	6	82	310	21	14	57	5	89
14 x 6 3/4	50.7	356	171	11	8	81	310	23	14	63	6	89
356 x 171	56.6	359	171	13	8	81	311	24	14	63	6	89
L.P.R.	64.1	346	203	13	8	97	288	29	19	63	6	140
14 x 8	71.5	349	203	14	8	97	289	30	19	63	6	140
356 x 203	79.0	356	203	17	10	96	290	33	21	63	6	140
L.P.R.	53.6	403	178	11	8	85	359	22	14	63	6	89
16 x 7	59.6	406	178	13	8	85	356	25	14	63	6	89
406 x 178	67.1	409	178	14	10	84	359	25	14	63	6	89
	74.5	413	181	16	10	85	359	27	14	63	6	89
L.P.R.	95.4	454	222	17	10	106	390	32	19	70	6	140
18 x 8 3/4	104.3	457	222	19	11	105	391	33	19	70	8	140
457 x 222	114.7	460	222	21	13	104	390	35	21	70	8	140
	126.7	467	225	24	13	106	391	38	21	70	8	140
L.P.R.	143.0	460	298	21	13	142	390	35	21	76	8	140
18 x 11 3/4	156.5	467	298	24	14	142	391	38	21	76	10	140
457 x 298	170.0	470	302	25	16	143	390	40	22	76	10	140

VIGAS T PERFIL RECTANGULAR

(Semi vigas I perfil rectangular)

T P R

Propiedades para diseño



PERFIL	PESO	AREA	PERALTE	PATIN	ESPAZOR		d	EJE X X				EJE Y Y		
d x b	Kg/m	cm²	d	Ancho b	Espesor d	ALMA tw	t	I	S	r	y	I	S	r
			mm.	mm	mm.	mm	mm	cm⁴	cm³	cm	cm	cm⁴	cm³	cm
T.P.R.	6.35	8.06	74	100	4.9	4.3	17.2	37.4	6.5	2.15	1.62	39.3	7.8	2.21
3 x 4	8.95	11.38	76	102	7.1	5.8	13.0	54.1	9.1	2.18	1.70	60.1	11	2.30
76.2 x 101.6	16.90	15.22	79	102	10.3	6.6	12.0	69.0	11	2.13	1.70	89.9	17	2.43
T.P.R.	7.45	9.51	100	100	5.2	4.3	23.2	89.4	11	3.07	2.43	41.4	8.3	2.08
4 x 4	9.70	12.35	101	102	6.5	5.8	17.4	120.7	16	3.12	2.61	54.5	10	2.10
101.6 x 101.6	11.20	14.29	103	102	8.0	6.2	16.6	136.9	17	3.09	2.54	68.6	13	2.19
T.P.R.	12.65	16.13	101	133	7.8	5.8	17.4	133.6	16	2.87	2.13	139.8	21	2.94
4 x 5 1/4	14.90	18.96	103	133	9.6	6.3	16.4	152.3	18	2.84	2.10	176.8	26	3.04
101.6 x 133.3														
T.P.R.	8.55	10.93	125	100	5.2	4.6	27.4	172.7	19	3.98	3.42	41.6	8.3	1.95
5 x 4	11.20	14.19	127	102	6.8	5.8	21.7	227.2	24	3.98	3.47	58.0	11	2.02
127.0 x 101.6	12.65	16.06	128	102	8.4	6.1	21.1	252.6	26	3.96	3.35	71.8	14	2.11
	14.15	18.09	130	102	10.0	6.4	20.5	278.8	28	3.93	3.25	87.2	17	2.19
T.P.R.	15.65	19.96	125	146	8.6	6.1	20.6	262.6	26	3.63	2.69	201.8	27	3.17
5 x 5 3/4	18.65	23.71	128	146	10.9	6.4	20.0	296.3	29	3.53	2.59	264.2	36	3.32
127.0 x 146.0	21.60	27.51	129	147	12.7	7.3	17.7	348.8	33	3.55	2.66	316.3	42	3.40
T.P.R.	10.45	13.35	151	101	5.7	5.1	29.8	320.4	29	4.87	4.47	46.8	9.3	1.87
6 x 4	12.30	15.67	152	102	6.8	5.8	26.1	375.4	34	4.90	4.47	58.0	11	1.93
152.4 x 101.6	14.15	18.12	154	102	8.9	6.1	25.3	424.5	38	4.85	4.24	76.3	15	2.05
	16.40	20.87	156	102	10.8	6.6	23.7	486.9	42	4.82	4.14	94.6	18	2.13
T.P.R.	20.15	25.71	152	165	10.2	6.1	24.9	474.5	39	4.29	3.07	345.4	41	3.65
6 x 6 1/2	23.10	29.41	153	166	11.8	6.7	22.8	541.0	44	4.29	3.09	412.0	49	3.74
152.4 x 165.1	26.85	34.16	155	167	13.7	7.7	20.1	636.8	51	4.31	3.20	495.3	59	3.80
T.P.R.	29.80	37.97	151	203	13.1	7.5	20.3	599.3	48	3.96	2.74	915.7	90	4.92
6 x 8	33.55	42.71	153	204	14.6	8.5	17.9	690.9	55	4.03	2.87	1040	101	4.92
152.4 x 203.2	37.25	47.45	154	205	16.3	9.4	16.4	778.3	62	4.06	2.97	1173	114	4.97
T.P.R.	22.35	28.42	176	171	9.7	6.9	25.7	790.8	58	5.28	4.03	364.0	42	3.58
7 x 6 3/4	25.35	32.26	178	171	11.5	7.3	24.4	878.2	63	5.20	3.93	443.0	51	3.70
177.8 x 171.4	28.30	36.03	179	172	13.0	7.9	22.6	978.1	69	5.20	3.96	511.5	60	3.78
T.P.R.	32.05	40.80	173	203	13.4	7.8	22.2	924.0	65	4.74	3.37	938.5	92	4.80
7 x 8	35.75	45.51	175	204	15.1	8.6	20.4	1036	73	4.77	3.42	1065	104	4.85
177.8 x 203.2	39.50	50.29	177	205	16.7	9.4	18.8	1152	81	4.77	3.50	1198	117	4.87
T.P.R.	26.80	34.16	201	177	10.9	7.6	26.5	1277	83	6.12	4.82	460.0	51	3.68
8 x 7	29.80	37.97	203	178	12.7	7.8	26.1	1381	87	6.01	4.62	551.5	62	3.81
203.2 x 177.8	33.55	42.71	204	179	14.3	8.8	23.3	1573	99	6.07	4.74	635.0	71	3.86
	37.25	47.42	206	180	15.9	9.6	21.4	1756	110	6.09	4.80	724.0	80	3.91
T.P.R.	47.70	60.64	227	221	17.4	10.2	22.2	2572	144	6.50	4.90	1464	132	4.90
9 x 8 3/4	52.15	66.32	228	222	19.1	11.1	20.5	2834	158	6.52	4.97	1633	146	4.95
228.6 x 222.2	57.35	72.99	230	223	21.1	12.1	19.1	3134	173	6.55	5.05	1844	165	5.03
	63.35	80.54	232	224	23.1	13.4	17.4	3512	195	6.60	5.20	2088	185	5.01
T.P.R.	71.50	91.03	230	298	21.1	13.0	17.7	3550</						

SECCION COMPUESTA  
VIGA I PERFIL RECTANGULAR CON CANAL PERFIL STANDARD

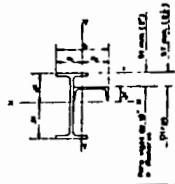
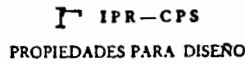
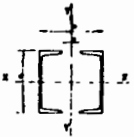


Table with columns: Perfil y peso, Canal, Pese, Area, EJE XX, EJE YY. Rows list various sizes from 16x7 to 18x11 3/4.

SECCION COMPUESTA  
DOS CANALES PERFIL STANDARD



PROPIEDADES PARA DISEÑO

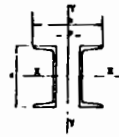
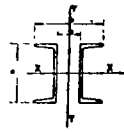


Table with columns: Seccion, Propiedades de la Sección, EJE X-X, EJE Y-Y. Rows list various sizes from 76.2x71.6 to 101.6x152.4.

3

4

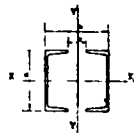




SECCION COMPUESTA  
DOS CANALES PERFIL STANDARD

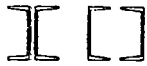
I 2 CPS

□ 2 CPS



PROPIEDADES PARA DISEÑO

SECCION

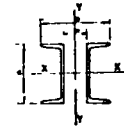


Propiedades de la Sección				EJE X-X				EJE Y-Y				EJE Y-Y			
d x b mm	Peso Kg/m	Area cm <sup>2</sup>		I cm <sup>4</sup>	S cm <sup>3</sup>	r cm	e mm	I cm <sup>4</sup>	S cm <sup>3</sup>	r cm	e mm	I cm <sup>4</sup>	S cm <sup>3</sup>	r cm	e mm
127 0x 88.9	19.94	25.16		616.0	97.0	4.95		80.5	18.1	1.79	0.0	298.4	67.1	3.44	0.0
127 0x 95.8	26.78	33.94		732.6	115.4	4.65		106.3	22.2	1.77	0.0	485.4	101.4	3.78	0.0
127 0x 95.2	19.94	25.16		616.0	97.0	4.95		103.2	21.7	2.02	6.3				
127 0x102.1	26.78	33.94		732.6	115.4	4.65		136.4	26.7	2.00	6.3				
127 0x 98.4	19.94	25.16		616.0	97.0	4.95		116.6	23.7	2.15	9.5				
127 0x105.3	26.78	33.94		732.6	115.4	4.65		154.3	29.3	2.13	9.5				
127 0x127.0	19.94	25.16		616.0	97.0	4.95						696.9	109.7	5.26	38.1
127 0x127.0	26.78	33.94		732.6	115.4	4.65						945.8	149.0	5.28	31.2
127 0x152.4	19.94	25.16		616.0	97.0	4.95						1064.1	139.6	6.50	63.5
127 0x152.4	26.78	33.94		732.6	115.4	4.65						1442.6	189.4	6.52	56.6
127 0x159.7	19.94	25.16		616.0	97.0	4.95		616.0	77.1	4.94	70.9				
127 0x161.7	26.78	33.94		732.6	115.4	4.65		732.6	91.1	4.64	65.1				
127 0x177.8	19.94	25.16		616.0	97.0	4.95						1514.3	170.3	7.76	88.9
127 0x177.8	26.78	33.94		732.6	115.4	4.65						2048.9	230.5	7.77	82.0
152.4x 97.5	24.40	30.84	1082.2	142.0	5.94			112.0	23.0	1.91	0.0	448.5	92.0	3.81	0.0
152.4x103.3	31.26	39.62	1257.0	165.0	5.64			136.3	26.4	1.85	0.0	673.8	130.4	4.13	0.0
152.4x109.6	38.70	49.16	1440.2	189.0	5.41			177.2	32.4	1.90	0.0	941.9	171.9	4.38	0.0
152.4x103.8	24.40	30.84	1082.2	142.0	5.94			140.7	27.1	2.14	6.3				
152.4x109.6	31.26	39.62	1257.0	165.0	5.64			172.0	31.4	2.08	6.3				
152.4x115.9	38.70	49.16	1440.2	189.0	5.41			223.0	38.5	2.13	6.3				
152.4x107.0	24.40	30.84	1082.2	142.0	5.94			157.7	29.5	2.26	9.5				
152.4x112.8	31.26	39.62	1257.0	165.0	5.64			193.1	34.2	2.21	9.5				
152.4x119.1	38.70	49.16	1440.2	189.0	5.41			250.0	42.0	2.25	9.5				
152.4x152.4	24.40	30.84	1082.2	142.0	5.94							1280.4	168.0	6.44	54.9
152.4x152.4	31.26	39.62	1257.0	165.0	5.64			1670.5	219.2	6.49	49.1				
152.4x152.4	38.70	49.16	1440.2	189.0	5.41			2042.1	268.0	6.45	42.8				
152.4x177.8	24.40	30.84	1082.2	142.0	5.94			1823.2	205.0	7.69	80.3				
152.4x177.8	31.26	39.62	1257.0	165.0	5.64			2373.6	267.0	7.74	74.5				
152.4x177.8	38.70	49.16	1440.2	189.0	5.41			2907.9	327.1	7.69	68.2				
152.4x186.4	24.40	30.84	1082.2	142.0	5.94			1082.2	116.2	5.92	88.8				
152.4x187.3	31.26	39.62	1257.0	165.0	5.64			1257.0	134.2	5.63	84.0				
152.4x189.0	38.70	49.16	1440.2	189.0	5.41			1440.2	153.2	5.41	78.4				
152.4x203.2	24.40	30.84	1082.2	142.0	5.94							2465.6	242.6	8.94	105.7
152.4x203.2	31.26	39.62	1257.0	165.0	5.64							3204.4	315.4	8.99	94.9
152.4x203.2	38.70	49.16	1440.2	189.0	5.41							3932.4	387.1	8.96	93.6



ALTOS HORNOS DE MEXICO, S. A.

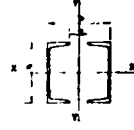
4



SECCION COMPUESTA  
DOS CANALES PERFIL STANDARD

I 2 CPS

□ 2 CPS



PROPIEDADES PARA DISEÑO

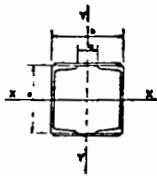
SECCION



Propiedades de la Sección				EJE X-X				EJE Y-Y				EJE Y-Y			
d x b mm	Peso Kg/m	Area cm <sup>2</sup>		I cm <sup>4</sup>	S cm <sup>3</sup>	r cm	e mm	I cm <sup>4</sup>	S cm <sup>3</sup>	r cm	e mm	I cm <sup>4</sup>	S cm <sup>3</sup>	r cm	e mm
177 8x106.2	29.16	36.78		1756.4	197.6	6.91		153.7	28.9	2.04	0.0	643.6	121.2	4.18	0.0
177 8x111.5	36.46	46.20		2006.2	225.6	6.58		186.6	33.5	2.01	0.0	916.0	164.4	4.45	0.0
177 8x116.8	43.90	55.74		2256.0	253.8	6.38		221.2	37.9	1.99	0.0	1229.8	210.6	4.70	0.0
177 8x115.7	29.16	36.78		1756.4	197.6	6.91		210.9	36.5	2.39	9.5				
177 8x121.0	36.46	46.20		2006.2	225.6	6.58		257.2	42.5	2.36	9.5				
177 8x126.3	43.90	55.74		2256.0	253.8	6.38		306.3	48.5	2.34	9.5				
177 8x118.9	29.16	36.78		1756.4	197.6	6.91		233.9	39.3	2.52	12.7				
177 8x124.2	36.46	46.20		2006.2	225.6	6.58		285.6	46.0	2.48	12.7				
177 8x129.5	43.90	55.74		2256.0	253.8	6.38		340.6	52.6	2.47	12.7				
177 8x177.8	29.16	36.78		1756.4	197.6	6.91						2144.4	241.2	7.64	71.6
177 8x177.8	36.46	46.20		2006.2	225.6	6.58						2711.1	305.0	7.66	66.3
177 8x177.8	43.90	55.74		2256.0	253.8	6.38						3267.8	367.6	7.65	61.0
177 8x203.2	29.16	36.78		1756.4	197.6	6.91						2903.3	285.8	8.89	97.0
177 8x203.2	36.46	46.20		2006.2	225.6	6.58						3667.9	361.1	8.91	91.7
177 8x203.2	43.90	55.74		2256.0	253.8	6.38						4277.5	421.1	8.76	86.4
177 8x213.2	29.16	36.78		1756.4	197.6	6.91		1756.4	164.8	6.91	107.0				
177 8x212.7	36.46	46.20		2006.2	225.6	6.58		2006.2	188.8	6.59	108.0				
177 8x213.2	43.90	55.74		2256.0	253.8	6.38		2256.0	211.5	6.36	96.6				
177 8x228.6	29.16	36.78		1756.4	197.6	6.91						3780.9	330.8	10.14	122.4
177 8x228.6	36.46	46.20		2006.2	225.6	6.58						4773.6	417.7	10.17	117.1
177 8x228.6	43.90	55.74		2256.0	253.8	6.38						5756.5	503.7	10.16	111.8
203.2x114.8	34.22	43.36	2688.8	264.6	7.87			201.9	35.2	2.16	0.0	898.8	156.6	4.55	0.0
203.2x119.0	40.92	51.88	2980.2	293.2	7.59			229.5	38.6	2.10	0.0	1190.0	200.0	4.79	0.0
203.2x128.4	55.80	70.84	3637.8	358.0	7.16			315.5	49.1	2.11	0.0	1915.6	295.4	5.20	0.0
203.2x124.3	34.22	43.36	2688.8	264.6	7.87			272.3	43.8	2.50	9.5				
203.2x128.5	40.92	51.88	2980.2	293.2	7.59			311.2	48.4	2.45	9.5				
203.2x137.9	55.80	70.84	3637.8	358.0	7.16			420.9	61.0	2.44	9.5				
203.2x127.5	34.22	43.36	2688.8	264.6	7.87			300.4	47.1	2.63	12.7				
203.2x131.7	40.92	51.88	2980.2	293.2	7.59			344.0	52.2	2.58	12.7				
203.2x141.1	55.80	70.84	3637.8	358.0	7.16			465.6	66.0	2.56	12.7				
203.2x203.2	34.22	43.36	2688.8	264.6	7.87							3382.6	332.9	8.83	88.4
203.2x203.2	40.92	51.88	2980.2	293.2	7.59							4088.8	402.4	8.88	84.2
203.2x203.2	55.80	70.84	3637.8	358.0	7.16							5519.5	545.3	8.84	74.8
203.2x219.2	34.22	43.36	2688.8	264.6	7.87			2688.8	224.3	7.88	124.2				
203.2x239.0	40.92	51.88	2980.2	293.2	7.59			2980.0	249.4	7.58	120.0				
203.2x239.4	55.80	70.84	3637.8	358.0	7.16			3637.8	303.9	7.16	111.0				
203.2x228.6	34.22	43.36	2688.8	264.6	7.87							4409.6	345.8	10.08	113.8
203.2x228.6	40.92	51.88	2980.2	293.2	7.59							5324.3	465.8	10.13	109.6
203.2x228.6	55.80	70.84	3637.8	358.0	7.16							7220.7	631.8	10.09	100.2
203.2x254.0	34.22	43.36	2688.8	264.6	7.87							5676.5	447.0	11.45	139.2
203.2x254.0	40.92	51.88	2980.2	293.2	7.59							6727.2	529.7	11.39	135.0
203.2x254.0	55.80	70.84	3637.8	358.0	7.16							9130.6	719.0	11.35	125.6



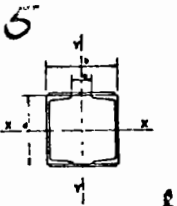
ALTOS HORNOS DE MEXICO, S. A.



SECCION COMPUESTA  
DOS CANALES PERFIL STANDARD Y DOS PLACAS SOLDADAS

2 CPS  
PROPIEDADES PARA DISEÑO

SECCION d x b mm	PLACAS Dimensiones mm	Peso Total Kg/m	Area Total cm <sup>2</sup>	EJE X - X			EJE Y - Y			e mm
				I cm <sup>4</sup>	S cm <sup>3</sup>	r cm	I cm <sup>4</sup>	S cm <sup>3</sup>	r cm	
3 76 x 102	89x 6	21 05	26 66	325.9	73.4	3.50	332	65	3.54	—
	89x 6	23.73	30 14	342.5	77.1	3.37	391	77	3.61	—
	89x 6	26.71	33 88	367.5	82.8	3.29	445	88	3.64	—
	89x10	25.48	32.36	446.1	93.7	3.71	369	73	3.38	—
	89x10	28.10	35.84	462.7	97.2	3.59	428	84	3.45	—
	89x10	31.13	39.58	487.7	102.5	3.52	482	95	3.49	—
	89x13	29.91	37.96	578.7	113.9	3.90	406	80	3.27	—
	89x13	32.59	41.44	595.3	117.2	3.79	465	92	3.35	—
	89x13	35.57	45.18	620.3	122.1	3.71	520	102	3.39	—
4 102 x 102	89x 6	24.93	31 42	645.9	113.1	4.53	408	80	3.60	12.7
	89x 6	30.43	38.66	704.1	123.3	4.27	530	104	3.70	11.6
	89x 8	27.16	34 18	738.6	126.0	4.65	427	84	3.54	13.4
	89x 8	32.66	41.42	796.8	136.0	4.39	548	108	3.63	12.2
	89x10	29.36	37 12	842.4	139.7	4.75	445	88	3.46	14.1
	89x10	34.86	44.36	900.6	149.4	4.51	567	112	3.58	12.8
	89x13	33.78	42.72	1053.2	165.9	4.97	483	95	3.36	15.5
	89x13	39.28	49.96	1111.4	175.0	4.72	604	119	3.48	13.9
	5 127 x 127	114x 6	31.32	39 68	1262.0	180.8	5.64	854	134	4.64
114x 6		38.16	48 46	1378.6	197.5	5.34	1103	174	4.67	14.1
114x 8		34.16	43.24	1437.3	201.3	5.76	893	141	4.54	16.3
114x 8		41.00	52.02	1553.9	217.6	5.47	1142	180	4.69	14.8
114x10		37.02	47.06	1637.6	224.3	5.90	932	147	4.45	17.2
114x10		43.86	55.84	1754.2	240.3	5.61	1181	186	4.60	15.5
114x13		42.71	54.20	2030.8	266.5	6.12	1010	159	4.50	18.8
114x13		49.95	62.98	2147.4	281.8	5.84	1259	198	4.47	16.9
6 152 x 152		140x 6	38.32	48.54	2198.1	266.4	6.72	1569	206	5.68
	140x 6	45.18	57.32	2372.9	287.6	6.44	1959	257	5.84	16.7
	140x 6	52.62	66.86	2556.1	309.8	6.18	2331	306	5.90	15.9
	140x 8	41.80	52.94	2503.7	297.7	6.89	1641	215	5.58	19.0
	140x 8	48.66	61.72	2678.5	318.5	6.59	2031	267	5.72	17.6
	140x 8	56.10	71.26	2861.7	340.3	6.34	2402	315	5.81	16.6
	140x10	45.27	57.60	2837.9	331.1	7.01	1713	225	5.45	20.0
	140x10	52.13	66.38	3012.7	351.5	6.74	2103	276	5.63	18.4
	140x10	59.57	75.92	3195.9	372.9	6.50	2474	325	5.71	17.3
	140x13	52.23	66.34	3498.4	393.5	7.27	1857	244	5.29	21.8
	140x13	59.09	75.12	3673.2	413.2	6.99	2248	295	5.47	20.0
	140x13	66.53	84.66	3856.4	433.8	6.75	2619	344	5.56	18.7
	140x16	59.19	75.24	4215.0	457.6	7.49	2003	266	5.20	23.7
	140x16	66.05	84.02	4398.8	476.6	7.23	2366	311	5.31	21.5
	140x16	73.49	93.56	4573.0	496.5	7.29	2761	363	5.44	20.1

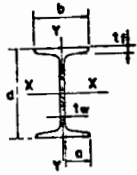


SECCION COMPUESTA  
DOS CANALES PERFIL STANDARD Y DOS PLACAS SOLDADAS

2 CPS  
PROPIEDADES PARA DISEÑO

SECCION d x b mm	PLACAS Dimensiones mm	Peso Total Kg/m	Area Total cm <sup>2</sup>	EJE X - X			EJE Y - Y			e mm
				I cm <sup>4</sup>	S cm <sup>3</sup>	r cm	I cm <sup>4</sup>	S cm <sup>3</sup>	r cm	
7 178 x 178	165x 6	45.60	57 74	3534.3	371.2	7.82	2620	295	6.74	21.9
	165x 6	52.90	67 16	3754.1	397.4	7.51	3187	356	6.89	19.4
	165x 6	60.34	76 70	4033.9	423.7	7.25	3743	421	6.99	18.4
	165x 8	49.72	63 00	4019.3	415.2	7.99	2740	308	6.60	21.9
	165x 8	57.02	72.44	4269.1	441.0	7.68	3306	372	6.76	20.3
	165x 8	64.46	81.96	4518.9	466.8	7.43	3863	436	6.88	19.2
	165x10	53.82	68.24	4518.5	459.1	8.15	2859	322	6.47	22.9
	165x10	61.12	77.66	4768.3	484.5	7.84	3425	385	6.65	21.3
	165x10	68.56	87.20	5018.1	509.9	7.56	3981	448	6.75	20.0
	165x13	62.04	78.72	5557.4	547.0	8.40	3096	348	6.27	24.9
	165x13	69.34	88.14	5807.2	571.6	8.12	3663	412	6.46	23.0
	165x13	76.78	97.68	6057.0	596.2	7.87	4220	475	6.58	21.6
	165x16	70.76	89.20	6668.3	636.3	8.64	3333	375	6.12	27.0
	165x16	77.56	98.62	6918.1	660.1	8.37	3900	438	6.29	24.9
	165x16	85.00	108 16	7167.9	683.9	8.15	4457	501	6.42	23.2
165x19	78.48	99 68	7846 8	727.2	8.86	3573	402	5.98	29.1	
165x19	85.78	109 10	8096.6	750.3	8.60	4139	466	6.16	26.6	
165x19	93.22	118.64	8346.4	773.5	8.40	4696	528	6.29	24.8	
8 203 x 203	190x 6	53.20	67.56	5346.7	495.5	8.89	4114	405	7.80	23.3
	190x 6	59.90	76.08	5637.9	522.5	8.60	4820	474	7.96	22.0
	190x 6	74.78	95.04	6295.7	583.4	8.13	6271	617	8.12	20.4
	190x 8	57.94	73.60	6061.0	553.5	9.08	4297	423	7.64	24.5
	190x 8	64.64	82.12	6352.2	580.1	8.80	5004	492	7.80	23.1
	190x 8	79.52	101.08	7010.0	640.2	8.33	6454	635	8.00	21.2
	190x10	62.68	79.66	6798.4	611.9	9.24	4480	441	7.50	25.7
	190x10	69.38	88.18	7089.6	638.1	8.96	5186	510	7.67	24.1
	190x10	84.26	107.14	7747.4	697.3	8.50	6636	653	7.86	21.9
	190x13	72.16	92.74	8254.9	722.5	9.44	4846	477	7.23	27.8
	190x13	78.86	100.26	8550.1	748.0	9.13	5552	546	7.45	26.0
	190x13	93.74	119.22	9207.9	805.5	8.79	7003	689	7.67	23.6
190x16	81.66	103.94	9940.5	846.0	9.78	5210	513	7.09	29.5	
190x16	88.36	112.46	10231.7	870.7	9.55	5916	582	7.26	28.1	
190x16	103.24	131.32	10889.5	926.7	9.10	7367	725	7.50	25.4	
190x19	91.14	115.94	11640.1	965.2	10.01	5578	549	6.94	32.3	
190x19	97.84	124.46	11931.3	989.3	9.79	6294	618	7.10	30.1	
190x19	112.72	143.42	12589.1	1043.9	9.36	7744	761	7.34	27.0	





VIGAS I PERFIL STANDARD

IPS

Módulo Plástico

Acero A36  $f_y = 2536 \text{ kg/cm}^2$

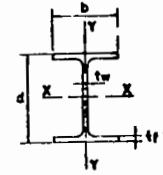
PERFIL	Peso	Módulo Plástico	Momento Plástico	Espeor Pata	Espeor Alma	
$d \times b$ Pulg y mm	Kg/m	Z en $\text{cm}^3$	en Ton cm	tf en mm	tw en mm	
3	76.2 x 59.2	8.48	31.1	79.0	6.6	4.3
	76.2 x 63.7	11.16	37.7	95.6	6.6	8.9
4	101.6 x 67.6	11.46	57.3	145.4	7.4	4.8
	101.6 x 71.0	14.14	65.5	166.2	7.4	8.3
5	127.0 x 76.2	14.88	91.7	232.7	8.3	5.3
	127.0 x 83.4	21.95	121.2	307.5	8.3	12.5
6	152.4 x 84.6	18.60	137.6	349.1	9.1	5.8
	152.4 x 90.5	25.67	172.1	436.3	9.1	11.8
7	177.8 x 93.0	22.77	195.0	494.5	10.0	6.3
	177.8 x 98.0	29.76	236.0	598.4	10.0	11.4
8	203.2 x 101.6	27.38	267.1	677.4	10.8	6.9
	203.2 x 105.6	34.23	314.6	797.9	10.8	11.2
10	254.0 x 118.4	37.80	458.8	1163.6	12.5	7.9
	254.0 x 125.6	52.09	576.8	1462.8	12.5	15.1
12	304.8 x 127.0	47.32	681.7	1728.8	13.8	8.9
	304.8 x 129.0	52.09	727.6	1845.1	13.8	10.9

VIGAS I PERFIL RECTANGULAR

IPR

Módulo Plástico

Acero A36  $f_y = 2536 \text{ kg/cm}^2$



PERFIL	Peso	Módulo Plástico	Momento Plástico	Espeor Pata	Espeor Alma	
$d \times b$ Pulg y mm	Kg/m	Z en $\text{cm}^3$	en Ton cm	tf en mm	tw en mm	
6" x 4"	12.7	92.0	233.0	4.9	4.3	
	152.4 x 101.6	17.9	136.00	345.0	7.1	5.8
	23.8	190.0	482.0	10.3	6.6	
8" x 4"	14.9	136.0	345.0	5.2	4.3	
	203.2 x 101.6	19.4	186.8	474.0	6.5	5.8
	22.4	222.9	565.0	8.0	6.2	
8" x 5 1/4"	25.3	258.9	656.0	7.8	5.8	
	203.2 x 133.4	29.8	312.9	793.0	9.6	6.3
	17.1	186.2	472.0	5.2	4.6	
10" x 4"	22.4	262.2	665.0	6.8	5.8	
	254.0 x 101.6	25.3	304.8	773.0	8.4	6.1
	28.3	353.9	897.0	10.0	6.4	
10" x 5 3/4"	31.3	394.9	1001.0	8.6	6.1	
	254.0 x 146.1	37.3	483.8	1227.0	10.9	6.4
	43.2	568.6	1442.0	12.7	7.3	
12" x 4"	20.9	326.0	826.0	5.7	5.1	
	304.8 x 101.6	24.6	337.5	856.0	6.8	5.8
	28.3	406.4	1031.0	8.9	6.1	
12" x 6 1/2"	32.8	481.7	1221.0	10.8	6.6	
	40.3	622.7	1579.0	10.2	6.1	
	304.8 x 165.1	46.2	721.0	1828.0	11.8	6.7
12" x 8"	53.7	842.3	2136.0	13.7	7.7	
	59.6	943.9	2394.0	13.1	7.5	
	304.8 x 203.2	67.1	1063.5	2697.0	14.6	8.5
14" x 8"	74.5	1189.7	3017.0	16.3	9.4	
	44.7	771.8	1957.0	9.7	6.9	
	355.6 x 171.4	50.7	893.0	2265.0	11.5	7.3
14" x 8"	56.6	1007.8	2556.0	13.0	7.9	
	64.1	1142.1	2896.0	13.4	7.8	
	355.6 x 203.2	71.5	1286.4	3262.0	15.1	8.6
16" x 7"	79.0	1427.3	3620.0	16.7	9.4	
	53.6	1047.1	2655.0	10.9	7.6	
	406.4 x 177.8	59.6	1191.3	3021.0	12.7	7.8
18" x 8 3/4"	67.1	1343.7	3408.0	14.3	8.8	
	74.5	1519.0	3852.0	15.9	9.6	
	95.4	2159.8	5477.0	17.4	10.2	
18" x 11 3/4"	104.3	2371.2	6013.0	19.1	11.1	
	114.7	2630.1	6670.0	21.1	12.1	
	457.2 x 222.2	126.7	2910.3	7380.0	23.1	13.4
18" x 11 3/4"	143.0	3375.7	8561.0	21.1	13.0	
	156.5	3711.6	9412.0	23.1	14.1	
	457.2 x 298.4	170.0	4062.3	10302.0	25.2	15.1

ALTOS HORNOS DE MEXICO, S. A.



ALTOS HORNOS DE MEXICO, S. A.



## CAPACIDAD DE CARGA EN COLUMNAS

### Generalidades:

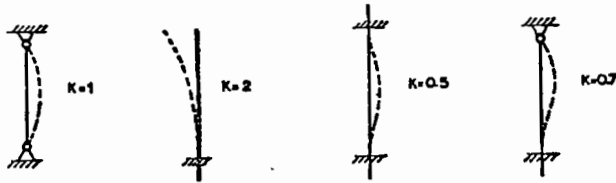
Una pieza estructural esbelta sujeta a compresión se le llama Columna.

La falla principal en una Columna ocurre por flambéo a un esfuerzo unitario inferior al esfuerzo de ruptura del material y depende de la relación de esbeltez que es la razón entre la longitud efectiva de flambéo de la Columna y el radio de giro máximo de la sección.

Las tablas de esfuerzos permisibles para Columnas cargadas axialmente se calcularon variando la relación de esbeltez de 1 a 250 y de acuerdo con las siguientes fórmulas:

$$\text{Relación de esbeltez} = K \frac{L}{r}$$

K=Factor longitud efectiva de flambéo, L=Longitud entre apoyos, r=Radio de giro.



$$f_p = f_y \left[ 1 - \frac{\left( K \frac{L}{r} \right)^2}{2 \left( \frac{L}{r} \right)_c^2} \right] \quad \frac{1}{C.S.} \text{ cuando } K \frac{L}{r} < \left( \frac{L}{r} \right)_c$$

$$f_p = \frac{10.5 \times 10^6}{\left( K \frac{L}{r} \right)^2} \text{ cuando } K \frac{L}{r} > \left( \frac{L}{r} \right)_c < 250$$

En donde

$f_p$  = Esfuerzo permisible en  $K/cm^2$  para carga axial pura

$f_y$  = Esfuerzo al límite elástico

$K \frac{L}{r}$  = Esbeltez efectiva de la columna

$$\left( \frac{L}{r} \right)_c = \sqrt{\frac{2\pi^2 E}{f_y}}$$

$$C.S. = \text{Coeficiente de seguridad} = \frac{5}{3} + \frac{3 \left( K \frac{L}{r} \right)}{8 \left( \frac{L}{r} \right)_c} - \frac{\left( K \frac{L}{r} \right)^3}{8 \left( \frac{L}{r} \right)_c^3}$$

E = Módulo de Elasticidad del acero



Los factores de flexión  $B_x$  y  $B_y$  que aparecen en las Tablas de Capacidad de carga proporcionan una ayuda para el cálculo de columnas con cargas combinadas de momento flexionante y carga axial.

El momento flexionante se multiplica por el factor de flexión  $B_x$  ó  $B_y$  dependiendo si el momento actúa en el eje X-X ó en el eje Y-Y, para transformarlo en una carga axial  $P'$  y sumarse a la carga axial  $P$ . La suma total de  $P+P'$  permite calcular una sección preliminar que puede consultarse en las Tablas.

La selección final se puede hacer usando las siguientes fórmulas:

Para flexión con respecto al eje X-X.

Cuando  $f_p / f_y \leq 0.15$

$$P + P' = P + \left[ B_x M_x \left( \frac{f_p}{f_{bx}} \right) \right] = \text{Carga tabulada requerida.}$$

Cuando  $f_p / f_y \geq 0.15$

$$P + P' = P + \left[ B_x M_x C_m x \left( \frac{f_p}{f_{bx}} \right) \left( 1 - \frac{ax}{P (KL)^2} \right) \right] = \text{Carga tabulada requerida.}$$

$$P + P' = P \left( \frac{f_p}{0.6 f_y} \right) + \left[ B_x M_x \left( \frac{f_p}{f_{bx}} \right) \right] = \text{Carga tabulada requerida.}$$

Para flexión con respecto al eje Y-Y, sustituir los términos correspondientes en las ecuaciones anteriores.

$P$  = Carga axial

$P'$  = Carga axial equivalente =  $M_x B_x$  ó  $M_y B_y$

$$B_x = \frac{A}{S_x} = \frac{\text{Área de la sección}}{\text{Módulo de sección eje X-X}}$$

$$B_y = \frac{A}{S_y} = \frac{\text{Área de la sección}}{\text{Módulo de sección eje Y-Y}}$$

$M_x$  y  $M_y$  = Momento flexionante

$$f_p = \text{Esfuerzo axial calculado} = \frac{P}{A}$$

$f_p$  = Esfuerzo permitido en compresión axial según tablas.

$f_{bx}$  = Esfuerzo de flexión permitido =  $0.60 f_y$

$C_m$  = Coeficiente cuyo valor puede considerarse como sigue:

a) Para miembros en compresión sujetos a traslación lateral de sus uniones  $C_m = 0.85$ .

b) Para miembros en compresión con apoyos totalmente empotrados en marcos arriostrados contra la traslación de sus juntas, sin estar sujetos a cargas transversales entre sus apoyos en el plano de flexión:

$$C_m = 0.6 + 0.4 \frac{M_1}{M_2}, \text{ (Pero no menor que } 0.4 \text{) donde } M_1 / M_2 \text{ es la relación del menor al mayor de los}$$

momentos extremos de la porción del miembro sin arriostrar en el plano de flexión bajo consideración.  $M_1 / M_2$  positiva cuando el miembro se flexiona con curvatura simple y negativa cuando adquiere curvatura doble.

c) Para miembros en compresión en marcos arriostrados contra la traslación de sus juntas en el plano de carga y sujetos a cargas transversales entre sus apoyos, el valor de " $C_m$ " puede determinarse por un análisis racional; sin embargo, en lugar de dicho análisis, los siguientes valores pueden aplicarse:

Para miembros cuyos extremos están empotrados  $C_m = 0.55$  y  $C_m = 1.00$ , en caso contrario.

$$A_x = 10.5 \times 10^3 I_x^2$$

$$A_y = 10.5 \times 10^3 I_y^2$$

$I_x$  e  $I_y$  = Momento de inercia de la sección según Eje X-X ó Eje Y-Y.



ESFUERZOS PERMISIBLES EN COLUMNAS DE ACERO — KG/CM<sup>2</sup>

Miembros principales y secundarios con relación de esbeltez de 1 a 120

$$(K L/r \leq 120)$$

L/r	TIPO DE ACERO			L/r	TIPO DE ACERO			L/r	TIPO DE ACERO		
	A-7	A-36	AH 55		A-7	A-36	AH 55		A-7	A-36	AH 55
10	1392.6	1518.8	2319.8	41.0	1243.1	1346.2	1968.8	81.0	1009.1	1073.8	1395.2
20	1390.1	1516.0	2314.3	42.0	1238.2	1340.5	1956.9	82.0	1002.3	1065.9	1378.2
30	1387.5	1513.1	2308.7	43.0	1233.3	1334.7	1944.9	83.0	995.5	1057.9	1361.2
40	1384.9	1510.1	2302.9	44.0	1228.3	1328.9	1932.8	84.0	988.6	1049.9	1344.0
50	1382.2	1507.0	2296.9	45.0	1223.2	1323.0	1920.5	85.0	981.7	1041.8	1326.7
60	1379.4	1503.8	2290.7	46.0	1218.1	1317.1	1908.1	86.0	974.7	1033.7	1309.3
70	1376.6	1500.5	2284.3	47.0	1212.9	1311.1	1895.6	87.0	967.7	1025.5	1291.7
80	1373.7	1497.2	2277.7	48.0	1207.7	1305.0	1883.0	88.0	960.7	1017.3	1274.0
90	1370.7	1493.8	2271.0	49.0	1202.4	1298.9	1870.2	89.0	953.6	1009.0	1256.1
100	1367.7	1490.3	2264.1	50.0	1197.1	1292.7	1857.3	90.0	946.4	1000.6	1238.1
110	1364.6	1486.7	2256.9	51.0	1191.8	1286.5	1844.3	91.0	939.2	992.2	1220.0
120	1361.4	1483.1	2249.6	52.0	1186.3	1280.2	1831.1	92.0	932.0	983.8	1201.7
130	1358.1	1479.3	2242.2	53.0	1180.9	1273.8	1817.8	93.0	924.7	975.3	1183.3
140	1354.8	1475.5	2234.5	54.0	1175.4	1267.4	1804.4	94.0	917.4	966.7	1164.8
150	1351.5	1471.6	2226.7	55.0	1169.8	1261.0	1790.9	95.0	910.0	958.1	1146.1
160	1348.0	1467.7	2218.7	56.0	1164.2	1254.5	1777.2	96.0	902.6	949.4	1127.2
170	1344.5	1463.6	2210.6	57.0	1158.6	1247.9	1763.4	97.0	895.1	940.7	1108.2
180	1341.0	1459.5	2202.3	58.0	1152.9	1241.3	1749.5	98.0	887.6	931.9	1089.1
190	1337.4	1455.4	2193.8	59.0	1147.1	1234.6	1735.5	99.0	880.1	923.1	1069.8
200	1333.7	1451.1	2185.2	60.0	1141.3	1227.8	1721.3	100.0	872.5	914.2	1050.4
210	1330.0	1446.8	2176.4	61.0	1135.5	1221.0	1707.0	101.0	864.8	905.3	1030.8
220	1326.2	1442.4	2167.4	62.0	1129.6	1214.2	1692.6	102.0	857.1	896.3	1011.1
230	1322.3	1437.9	2158.3	63.0	1123.7	1207.3	1678.1	103.0	849.4	887.2	991.7
240	1318.4	1433.4	2149.0	64.0	1117.7	1200.3	1663.4	104.0	841.6	878.1	970.8
250	1314.4	1428.8	2139.6	65.0	1111.7	1193.3	1648.6	105.0	833.8	868.9	952.4
260	1310.4	1424.1	2130.0	66.0	1105.6	1186.2	1633.7	106.0	825.9	859.7	934.5
270	1306.3	1419.4	2120.2	67.0	1099.5	1179.1	1618.7	107.0	818.0	850.4	917.1
280	1302.1	1414.6	2110.4	68.0	1093.3	1172.0	1603.5	108.0	810.0	841.1	900.2
290	1297.9	1409.7	2100.3	69.0	1087.1	1164.7	1588.3	109.0	802.0	831.7	883.8
300	1293.7	1404.8	2090.1	70.0	1080.9	1157.4	1572.9	110.0	793.9	822.2	867.8
310	1289.3	1399.8	2079.8	71.0	1074.6	1150.1	1557.3	111.0	785.8	812.7	852.2
320	1285.0	1394.7	2069.3	72.0	1068.2	1142.7	1541.7	112.0	777.6	803.1	837.1
330	1280.5	1389.5	2058.7	73.0	1061.8	1135.3	1525.9	113.0	769.4	793.5	822.3
340	1276.0	1384.3	2048.0	74.0	1055.4	1127.8	1510.0	114.0	761.2	783.8	807.9
350	1271.5	1379.1	2037.1	75.0	1048.9	1120.2	1494.0	115.0	752.9	774.0	794.0
360	1266.9	1373.7	2026.0	76.0	1042.4	1112.6	1477.8	116.0	744.5	764.2	780.3
370	1262.3	1368.4	2014.8	77.0	1035.8	1105.0	1461.6	117.0	736.1	754.4	767.0
380	1257.6	1362.9	2003.5	78.0	1029.2	1097.3	1445.2	118.0	727.6	744.4	754.1
390	1252.8	1357.4	1992.1	79.0	1022.6	1089.5	1428.6	119.0	719.1	734.4	741.25
400	1248.0	1351.8	1980.5	80.0	1015.9	1081.7	1412.0	120.0	710.6	724.3	729.2

ESFUERZOS PERMISIBLES EN COLUMNAS DE ACERO — KG/CM<sup>2</sup>

Miembros principales y secundarios con relación de esbeltez de 121 a 250

$$K L/r \begin{matrix} > & 120 \\ < & 250 \end{matrix}$$

L/r	TIPO DE ACERO			L/r	ACERO A7 A36-AH55	L/r	ACERO A7 A-36 AH55
	A-7	A-36	AH 55				
121.0	702.0	714.2	717.2	166.0	381.0	211.0	235.8
122.0	693.3	704.0	705.5	167.0	376.5	212.0	233.6
123.0	684.6	693.8	694.0	168.0	372.0	213.0	231.4
124.0	675.8	683.5	682.9	169.0	367.6	214.0	229.3
125.0	667.0	673.1	672.0	170.0	363.3	215.0	227.1
126.0	658.1	662.6	661.4	171.0	359.1	216.0	225.1
127.0	649.2	651.0	651.0	172.0	354.9	217.0	223.0
128.0	640.2	640.9	640.9	173.0	350.8	218.0	220.9
129.0	631.2	631.0	631.0	174.0	346.8	219.0	218.9
130.0	622.1	621.3	621.3	175.0	342.9	220.0	216.9
131.0	613.0	611.9	611.9	176.0	339.0	221.0	215.0
132.0	602.6	602.6	602.6	177.0	335.2	222.0	213.1
133.0	593.6	593.6	593.6	178.0	331.4	223.0	211.1
134.0	584.8	584.8	584.8	179.0	327.7	224.0	209.3
135.0	576.1	576.1	576.1	180.0	324.1	225.0	207.4
136.0	567.7	567.7	567.7	181.0	320.5	226.0	205.6
137.0	559.4	559.4	559.4	182.0	317.0	227.0	203.8
138.0	551.4	551.4	551.4	183.0	313.5	228.0	202.0
139.0	543.4	543.4	543.4	184.0	310.1	229.0	200.2
140.0	535.7	535.7	535.7	185.0	306.8	230.0	198.5
141.0	528.1	528.1	528.1	186.0	303.5	231.0	196.8
142.0	520.7	520.7	520.7	187.0	300.3	232.0	195.1
143.0	513.5	513.5	513.5	188.0	297.1	233.0	193.4
144.0	506.4	506.4	506.4	189.0	293.9	234.0	191.8
145.0	499.4	499.4	499.4	190.0	290.9	235.0	190.1
146.0	492.6	492.6	492.6	191.0	287.8	236.0	188.5
147.0	485.9	485.9	485.9	192.0	284.8	237.0	186.9
148.0	479.4	479.4	479.4	193.0	281.9	238.0	185.4
149.0	473.0	473.0	473.0	194.0	279.0	239.0	183.8
150.0	466.7	466.7	466.7	195.0	276.1	240.0	182.3
151.0	460.5	460.5	460.5	196.0	273.3	241.0	180.8
152.0	454.5	454.5	454.5	197.0	270.6	242.0	179.3
153.0	448.5	448.5	448.5	198.0	267.8	243.0	177.8
154.0	442.7	442.7	442.7	199.0	265.1	244.0	176.4
155.0	437.0	437.0	437.0	200.0	262.5	245.0	174.9
156.0	431.5	431.5	431.5	201.0	259.9	246.0	173.5
157.0	426.0	426.0	426.0	202.0	257.3	247.0	172.1
158.0	420.6	420.6	420.6	203.0	254.8	248.0	170.7
159.0	415.3	415.3	415.3	204.0	252.3	249.0	169.4
160.0	410.2	410.2	410.2	205.0	249.9	250.0	168.0
161.0	405.1	405.1	405.1	206.0	247.4		
162.0	400.1	400.1	400.1	207.0	245.0		
163.0	395.2	395.2	395.2	208.0	242.7		
164.0	390.4	390.4	390.4	209.0	240.4		
165.0	385.7	385.7	385.7	210.0	238.1		

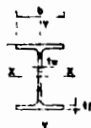


ALTOS HORNOS DE MEXICO, S. A.

ALTOS HORNOS DE MEXICO, S. A.







CAPACIDAD DE CARGA EN TONELADAS

COLUMNAS I PERFIL RECTANGULAR

I P R

DIMENSION	TIPO DE ACERO A 7 A 36 AH 55				TIPO DE ACERO A 7 A 36 AH 55				TIPO DE ACERO A 7 A 36 AH 55				
	Altura	L/r	TIPO DE ACERO		L/r	TIPO DE ACERO		L/r	TIPO DE ACERO		L/r	TIPO DE ACERO	
cm	cm		A	B	AH		A	B	AH		A	B	AH
			A = 16.13 cm <sup>2</sup> B <sub>x</sub> = 0.194 B <sub>y</sub> = 1.080			A = 22.77 cm <sup>2</sup> B <sub>x</sub> = 0.191 B <sub>y</sub> = 0.990			A = 30.45 cm <sup>2</sup> B <sub>x</sub> = 0.185 B <sub>y</sub> = 0.870				
6	50	22.6	21.4	23.2	34.9	21.9	30.2	32.9	49.4	20.6	40.5	44.1	66.4
x	100	45.2	19.7	21.3	30.9	43.9	28.0	30.3	44.0	41.2	37.8	41.0	59.9
	150	67.9	17.6	18.9	25.9	65.8	25.2	27.0	37.3	61.7	34.4	37.0	51.7
4	200	90.5	15.2	16.1	19.8	87.7	21.9	23.2	29.1	82.3	30.5	32.4	41.8
x	250	113.1	12.4	12.8	13.2	109.6	18.1	18.8	19.9	102.9	25.9	27.0	30.2
152.4	300	135.7	9.2	9.2	9.2	131.6	13.8	13.8	13.8	123.5	20.7	21.0	21.0
	350	158.4	6.8	6.8	6.8	153.5	10.1	10.1	10.1	144.0	15.4	15.4	15.4
101.6	400	181.0	5.2	5.2	5.2	175.4	7.8	7.8	7.8	164.6	11.8	11.8	11.8
	450	—	—	—	—	197.4	6.1	6.1	6.1	185.2	9.3	9.3	9.3

P=25.30 A = 32.26 cm<sup>2</sup>  
B<sub>x</sub> = 0.140  
B<sub>y</sub> = 0.770

P=29.80 A = 37.93 cm<sup>2</sup>  
B<sub>x</sub> = 0.136  
B<sub>y</sub> = 0.730

8	50	17.0	43.4	47.2	71.3	16.4	51.1	55.6	84.0
x	100	34.0	41.2	44.7	66.1	32.9	48.6	52.7	78.1
	150	51.0	38.4	41.5	59.5	49.3	45.5	49.2	70.8
51/4	200	68.0	35.3	37.8	51.7	65.8	42.0	45.1	62.1
	250	85.0	31.7	33.6	42.8	82.2	38.0	40.4	52.1
203.2	300	102.0	27.6	28.9	32.5	98.7	33.5	35.1	40.8
x	350	119.0	23.2	23.7	23.9	115.1	28.5	29.3	30.0
133.4	400	136.1	18.3	18.3	18.3	131.6	23.0	23.0	23.0
	450	153.1	14.5	14.5	14.5	148.0	18.2	18.2	18.2
	500	170.1	11.7	11.7	11.7	164.5	14.7	14.7	14.7
	550	187.1	9.7	9.7	9.7	180.9	12.2	12.2	12.2
	600	—	—	—	—	197.4	10.2	10.2	10.2

P=31.30 A = 39.93 cm<sup>2</sup>  
B<sub>x</sub> = 0.113  
B<sub>y</sub> = 0.730

P=37.30 A = 47.42 cm<sup>2</sup>  
B<sub>x</sub> = 0.110  
B<sub>y</sub> = 0.660

P=43.20 A = 55.03 cm<sup>2</sup>  
B<sub>x</sub> = 0.109  
B<sub>y</sub> = 0.650

10	50	15.8	53.9	58.6	88.7	15.1	64.1	69.8	105.6	14.7	74.4	81.0	122.7
x	100	31.5	51.4	55.8	82.8	30.1	61.3	66.6	99.1	29.4	71.3	77.5	115.4
	150	47.3	48.4	52.3	75.5	45.2	58.0	62.7	91.0	44.1	67.6	73.1	106.3
53/4	200	63.1	44.8	48.2	67.0	60.2	54.1	58.1	81.5	58.8	63.2	68.0	95.6
	250	78.9	40.9	43.5	57.1	75.3	49.6	53.0	70.6	73.5	58.2	62.3	83.5
254.0	300	94.6	36.4	38.4	46.0	90.4	44.8	47.3	58.4	88.2	52.8	55.9	69.9
x	350	110.4	31.6	32.7	34.4	105.4	39.4	41.0	44.8	102.9	46.8	48.9	54.5
146.0	400	126.2	26.2	26.3	26.3	120.5	33.5	34.1	34.3	117.6	40.2	41.2	41.7
	450	142.0	20.8	20.8	20.8	135.5	27.1	27.1	27.1	132.4	33.0	33.0	33.0
	500	157.7	16.9	16.9	16.9	150.6	22.0	22.0	22.0	147.1	26.7	26.7	26.7
	550	173.5	13.9	13.9	13.9	165.7	18.1	18.1	18.1	161.8	22.1	22.1	22.1
	600	189.3	11.7	11.7	11.7	180.7	15.2	15.2	15.2	176.5	18.6	18.6	18.6
	650	—	—	—	—	195.8	13.0	13.0	13.0	191.2	15.8	15.8	15.8

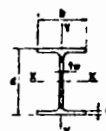


ALTOS HORNOS DE MEXICO, S. A.

CAPACIDAD DE CARGA EN TONELADAS

COLUMNAS I PERFIL RECTANGULAR

I P R



DIMENSION	TIPO DE ACERO A 7 A 36 AH 55				TIPO DE ACERO A 7 A 36 AH 55				TIPO DE ACERO A 7 A 36 AH 55				
	Altura	L/r	TIPO DE ACERO		L/r	TIPO DE ACERO		L/r	TIPO DE ACERO		L/r	TIPO DE ACERO	
cm	cm		A	B	AH		A	B	AH		A	B	AH
			A = 51.42 cm <sup>2</sup> B <sub>x</sub> = 0.092 B <sub>y</sub> = 0.610			A = 58.83 cm <sup>2</sup> B <sub>x</sub> = 0.091 B <sub>y</sub> = 0.590			A = 68.32 cm <sup>2</sup> B <sub>x</sub> = 0.091 B <sub>y</sub> = 0.580				
12	100	27.4	67.1	72.9	108.8	26.8	76.9	83.6	124.8	26.2	89.5	97.2	145.4
x	150	41.1	63.9	69.2	101.2	40.2	73.4	79.5	116.4	39.4	85.5	92.6	135.8
	200	54.8	60.2	64.9	92.2	53.6	69.3	74.7	106.5	52.5	80.9	87.2	124.7
61/2	250	68.5	56.1	60.1	82.1	67.0	64.7	69.4	95.2	65.6	75.7	81.2	112.0
	300	82.2	51.5	54.7	70.7	80.4	59.6	63.4	82.6	78.7	70.0	74.6	97.9
304.8	350	95.9	46.5	48.9	58.1	93.8	54.0	57.0	68.7	91.9	63.7	67.3	82.3
x	400	109.6	41.0	42.5	45.0	107.2	48.0	49.9	53.7	105.0	57.0	59.4	65.1
165.1	450	123.3	35.1	35.5	35.5	120.6	41.5	42.2	42.4	118.1	49.6	50.8	51.4
	500	137.0	28.8	28.8	28.8	134.0	34.4	34.4	34.4	131.2	41.7	41.7	41.7
	550	150.7	23.8	23.8	23.8	147.5	28.4	28.4	28.4	144.4	34.4	34.4	34.4
	600	164.4	20.0	20.0	20.0	160.9	23.9	23.9	23.9	157.5	28.9	28.9	28.9
	650	178.1	17.0	17.0	17.0	174.3	20.3	20.3	20.3	170.6	24.6	24.6	24.6
	700	191.8	14.7	14.7	14.7	187.7	17.5	17.5	17.5	183.7	21.3	21.3	21.3
	750	—	—	—	—	—	—	—	—	196.9	18.5	18.5	18.5

P=59.6 A = 75.94 cm<sup>2</sup>  
B<sub>x</sub> = 0.089  
B<sub>y</sub> = 0.420

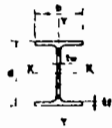
P=67.10 A = 85.42 cm<sup>2</sup>  
B<sub>x</sub> = 0.090  
B<sub>y</sub> = 0.420

P=74.50 A = 94.90 cm<sup>2</sup>  
B<sub>x</sub> = 0.090  
B<sub>y</sub> = 0.410

12	100	20.3	101.2	110.1	145.7	20.3	113.8	123.8	186.4	20.1	126.5	137.7	207.3
x	150	30.5	98.1	106.5	158.3	30.5	110.3	119.4	178.1	30.2	122.7	133.2	194.2
	200	40.7	94.5	102.4	149.8	40.7	106.3	115.2	164.5	40.2	118.3	128.2	187.7
8	250	50.8	90.6	97.8	140.2	50.8	101.9	110.0	157.7	50.3	113.5	122.5	175.9
	300	61.0	86.2	92.7	129.7	61.0	97.0	104.3	145.8	60.4	108.1	116.3	162.9
304.8	350	71.1	81.5	87.3	118.1	71.1	91.7	98.2	132.8	70.4	102.3	109.5	148.6
x	400	81.3	76.5	81.4	105.6	81.3	86.0	91.5	118.7	80.5	96.1	102.3	133.2
203.2	450	91.5	71.1	75.1	92.0	91.5	79.9	84.4	103.5	90.5	89.4	94.5	116.6
	500	101.6	65.3	68.3	77.3	101.6	73.5	76.8	87.0	100.6	82.4	86.2	98.6
	550	111.8	59.2	61.1	63.8	111.8	66.6	68.8	71.8	110.7	74.8	77.4	81.4
	600	122.0	52.7	53.5	53.6	122.0	59.3	60.2	60.3	120.7	66.8	68.0	68.4
	650	132.1	45.7	45.7	45.7	132.1	51.4	51.4	51.4	130.8	58.4	58.3	58.3
	700	142.3	39.4	39.4	39.4	142.3	44.3	44.3	44.3	140.8	50.2	50.2	50.2
	750	152.4	34.3	34.3	34.3	152.4	38.6	38.6	38.6	150.9	43.8	43.8	43.8
	800	162.6	30.2	30.2	30.2	162.6	33.9	33.9	33.9	161.0	38.5	38.5	38.5
	850	172.8	26.7	26.7	26.7	172.8	30.0	30.0	30.0	171.0	34.1	34.1	34.1
	900	182.9	23.8	23.8	23.8	182.9	26.8	26.8	26.8	181.1	30.4	30.4	30.4
	950	193.1	21.4	21.4	21.4	193.1	24.1	24.1	24.1	191.1	27.3	27.3	27.3

ALTOS HORNOS DE MEXICO, S. A.





CAPACIDAD DE CARGA EN TONELADAS

COLUMNAS I PERFIL RECTANGULAR

IPR

DIMENSION	Altura en cm	L/i	TIPO DE ACERO			L/i	TIPO DE ACERO			L/i	TIPO DE ACERO		
			A-7	A-36	AH-55		A-7	A-36	AH-55		A-7	A-36	AH-55
			A = 81.61 cm <sup>2</sup> P=64.10 Bx= 0.079 By= 0.440			A = 91.03 cm <sup>2</sup> P=71.50 Bx= 0.079 By= 0.440			A = 100.58 cm <sup>2</sup> P=79.0 Bx= 0.079 By= 0.430				
14	100	23.8	108.6	118.1	177.7	20.6	121.2	131.9	198.4	20.5	133.9	145.7	219.3
x	200	41.7	101.2	109.6	160.0	41.2	113.1	122.4	179.0	41.1	125.0	135.4	197.9
8	250	52.1	96.8	104.4	149.3	51.5	108.2	116.8	167.2	51.3	119.7	129.2	185.1
	300	62.5	91.9	98.8	137.5	61.9	102.9	110.6	154.3	61.6	113.9	122.4	170.8
	350	72.9	86.7	92.7	124.6	72.2	97.1	103.9	140.1	71.9	107.5	115.0	155.3
	400	83.3	81.1	86.1	110.6	82.5	90.9	96.7	124.7	82.1	100.7	107.1	138.4
355.6	450	93.7	75.0	79.1	95.4	92.8	84.3	88.9	108.1	92.4	93.4	98.6	120.1
x	500	104.2	68.6	71.5	79.0	103.1	77.3	80.7	89.9	102.7	85.7	89.5	100.2
203.2	550	114.6	61.7	63.5	65.3	113.4	69.7	71.9	74.3	112.9	77.4	79.9	82.8
	600	125.0	54.4	54.9	54.8	123.7	61.8	62.5	62.5	123.2	68.7	69.6	69.6
	650	135.4	46.7	46.7	46.7	134.0	53.2	53.2	53.2	134.0	53.2	53.2	53.2
	700	145.8	40.3	40.3	40.3	144.3	45.9	45.9	45.9	143.7	51.1	51.1	51.1
	750	156.2	35.1	35.1	35.1	154.6	40.0	40.0	40.0	154.6	44.5	44.5	44.5
	800	166.7	30.8	30.8	30.8	164.9	35.1	35.1	35.1	164.3	39.1	39.1	39.1
	850	177.1	27.3	27.3	27.3	175.3	31.1	31.1	31.1	174.5	34.7	34.7	34.7
	900	187.5	24.4	24.4	24.4	185.6	27.8	27.8	27.8	184.8	30.9	30.9	30.9
	950	197.9	21.9	21.9	21.9	195.9	24.9	24.9	24.9	195.1	27.8	27.8	27.8

			A = 182.06 cm <sup>2</sup> P=143.00 Bx= 0.060 By= 0.320			A = 199.09 cm <sup>2</sup> P=156.50 Bx= 0.060 By= 0.310			A = 216.19 cm <sup>2</sup> P=170.00 Bx= 0.060 By= 0.310				
18	100	14.5	246.3	268.3	406.1	14.4	269.4	293.4	444.2	14.3	292.7	318.8	482.6
x	200	29.1	236.2	256.6	382.3	28.9	258.5	280.8	418.4	28.5	281.0	305.3	455.1
113/4	250	36.3	230.4	249.8	368.2	36.1	252.2	273.4	403.2	35.7	274.2	297.4	438.8
	300	43.6	224.0	242.4	352.8	43.3	245.2	265.4	386.5	42.8	266.8	288.8	421.0
	350	50.9	217.1	234.4	336.1	50.5	237.8	256.7	368.5	49.9	258.9	279.6	401.7
	400	58.1	209.7	225.8	318.2	57.7	229.8	247.5	349.1	57.1	250.4	269.7	381.0
457.2	450	65.4	201.9	216.7	299.0	64.9	221.4	237.7	328.4	64.2	241.4	259.2	359.0
x	500	72.7	193.7	207.1	278.7	72.2	212.5	227.3	306.5	71.3	231.9	248.1	335.6
298.4	550	79.9	185.0	197.0	257.2	79.4	203.1	216.3	283.2	78.5	221.8	236.5	310.8
	600	87.2	175.9	186.4	234.5	86.6	193.2	204.9	258.6	85.6	211.3	224.2	284.6
	650	94.5	166.4	175.3	210.4	93.8	182.9	192.8	232.6	92.7	200.3	211.4	256.9
	700	101.7	156.4	163.6	185.0	101.0	172.2	180.2	205.2	99.9	188.9	197.9	227.7
	750	109.0	146.0	151.4	160.9	108.2	160.9	167.0	178.5	107.0	176.9	183.9	198.3
	800	116.3	135.1	138.6	141.4	115.4	149.2	153.2	156.9	114.1	164.3	169.2	174.3
	850	123.5	123.8	125.3	125.2	122.7	136.9	138.8	139.0	121.3	151.3	153.8	154.4
	900	130.8	111.9	111.7	111.7	129.9	124.1	123.9	123.9	128.4	137.7	137.7	137.7
	950	138.1	100.3	100.3	100.3	137.1	111.2	111.2	111.2	135.5	123.6	123.6	123.6
	1000	145.3	90.5	90.5	90.5	144.3	100.4	100.4	100.4	142.7	111.5	111.5	111.5
	1050	152.6	82.1	82.1	82.1	151.5	91.1	91.1	91.1	149.8	101.2	101.2	101.2
	1100	159.9	74.8	74.8	74.8	158.7	83.0	83.0	83.0	156.9	92.2	92.2	92.2
	1150	167.2	68.4	68.4	68.4	165.9	75.9	75.9	75.9	164.1	84.3	84.3	84.3
	1200	174.4	62.8	62.8	62.8	173.2	69.7	69.7	69.7	171.2	77.5	77.5	77.5
	1250	181.7	57.9	57.9	57.9	180.4	64.3	64.3	64.3	178.3	71.4	71.4	71.4
	1300	189.0	53.5	53.5	53.5	187.6	59.4	59.4	59.4	185.4	66.0	66.0	66.0
	1350	196.2	49.6	49.6	49.6	194.8	55.1	55.1	55.1	192.6	61.2	61.2	61.2
1400	1400	—	—	—	—	—	—	—	—	199.7	56.9	56.9	56.9

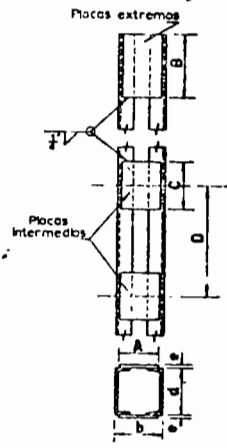
DATOS PARA DETALLE DE COLUMNAS FORMADAS POR 2 CANALES

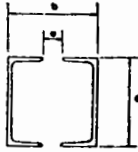
UNIDAS CON PLACAS INTERRUMPIDAS SOLDADAS.

2 C P S

DATOS PARA DETALLAR

Dimensión d x b	Placas Exteriores			Placas Intermedias			Peso Unitario	Espesor e	Dím. Máx. Torn.
	A	B	C	A	C	D			
76x102	89	127	0.57	89	76	675	0.34	6.3	—
76x127	114	127	0.72	114	76	675	0.43	6.3	—
102x102	89	152	0.68	89	102	750	0.45	6.3	13
102x127	114	152	0.87	114	102	750	0.58	6.3	13
102x152	140	152	1.07	140	102	750	0.71	6.3	13
127x127	114	178	1.01	114	127	875	0.72	6.3	13
127x152	140	178	1.25	140	127	875	0.89	6.3	13
127x178	165	178	1.47	165	127	875	1.05	6.3	13
152x152	140	203	1.44	140	152	900	1.07	6.3	16
152x178	165	203	1.68	165	152	900	1.26	6.3	16
152x203	190	203	1.93	190	152	900	1.45	6.3	16
178x178	165	229	2.36	165	178	975	1.84	7.9	16
178x203	190	229	2.72	190	178	975	2.12	7.9	16
178x229	216	229	3.08	216	178	975	2.40	7.9	16
203x203	190	254	3.02	190	203	1050	2.41	7.9	19
203x229	216	254	3.43	216	203	1050	2.74	7.9	19
203x254	241	254	3.83	241	203	1050	3.05	7.9	19





CAPACIDAD DE CARGA EN TONELADAS  
COLUMNAS COMPUESTAS DOS CANALES PERFIL STANDARD

□ 2 C P S

Dimension	Altura en cm	L/r	TIPO DE ACERO			L/r	TIPO DE ACERO			L/r	TIPO DE ACERO		
			A-7	A-36	AH-55		A-7	A-36	AH-55		A-7	A-36	AH-55
2 C P S  3  76.2  c=0	50	18.6	20.6	22.4	33.8	17.7	25.3	27.5	41.5	18.0	30.3	33.0	49.7
	100	37.2	19.4	21.0	30.9	35.5	23.9	25.9	38.3	36.0	28.6	31.0	45.8
	150	55.8	17.9	19.3	27.3	53.2	22.2	24.0	34.2	54.0	26.5	28.6	40.8
	200	74.3	16.2	17.3	23.1	70.9	20.3	21.7	29.4	71.9	24.1	25.8	34.8
	250	92.9	14.2	15.0	18.2	88.7	18.0	19.1	23.8	89.9	21.4	22.6	28.0
	300	111.5	12.0	12.4	13.0	106.4	15.5	16.1	17.5	107.9	18.3	19.0	20.4
	350	130.1	9.5	9.5	9.5	124.1	12.7	12.9	12.8	125.9	14.9	15.0	15.0
	400	148.7	7.3	7.3	7.3	141.8	9.8	9.8	9.8	143.9	11.5	11.5	11.5
	450	167.3	5.8	5.8	5.8	159.6	7.8	7.8	7.8	161.9	9.0	9.0	9.0
	500	185.9	4.7	4.7	4.7	177.3	6.3	6.3	6.3	179.9	7.3	7.3	7.3
550	—	—	—	—	195.0	5.2	5.2	5.2	197.8	6.1	6.1	6.1	

Peso total = 16.08  
Area total = 20.12

Peso total = 21.58  
Area total = 27.36

2 C P S  4  101.6  c=0	50	16.3	27.1	29.5	44.6	14.7	37.0	40.3	61.0
	100	32.6	25.8	28.0	41.5	29.4	35.5	38.5	57.4
	150	48.9	24.2	26.2	37.7	44.1	33.6	36.3	52.8
	200	65.1	22.3	24.0	33.1	58.8	31.4	33.8	47.6
	250	81.4	20.2	21.5	27.9	73.5	29.0	31.0	41.5
	300	97.7	17.9	18.8	22.0	88.2	26.2	27.8	34.7
	350	114.0	15.3	15.8	16.3	102.9	23.3	24.3	27.1
	400	130.3	12.5	12.4	12.4	117.6	20.0	20.5	20.8
	450	146.6	9.8	9.8	9.8	132.4	16.4	16.4	16.4
	500	162.9	8.0	8.0	8.0	147.1	13.3	13.3	13.3
550	179.2	6.6	6.6	6.6	161.8	11.0	11.0	11.0	
600	195.4	5.5	5.5	5.5	176.5	9.2	9.2	9.2	
650	—	—	—	—	191.2	7.9	7.9	7.9	

Peso total = 16.08  
Area total = 20.12

Peso total = 21.58  
Area total = 27.36

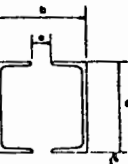
2 C P S  4  101.6  b=101.6	50	12.6	27.4	29.8	45.2	13.5	37.1	40.4	61.2
	100	25.2	26.4	28.7	43.0	27.0	35.7	38.8	58.0
	150	37.8	25.3	27.4	40.4	40.5	34.1	36.9	54.0
	200	50.4	24.0	26.0	37.3	54.1	32.2	34.7	49.3
	250	63.0	22.6	24.3	33.8	67.6	30.0	32.1	44.1
	300	75.6	21.0	22.5	29.9	81.1	27.6	29.4	38.1
	350	88.2	19.3	20.4	25.6	94.6	25.0	26.3	31.6
	400	100.8	17.4	18.3	20.8	108.1	22.1	23.0	24.6
	450	113.4	15.4	15.9	16.4	121.6	19.1	19.4	19.4
	500	125.9	13.3	13.3	13.3	135.1	15.7	15.7	15.7
	550	138.5	11.0	11.0	11.0	148.6	13.0	13.0	13.0
	600	151.1	9.2	9.2	9.2	162.2	10.9	10.9	10.9
	650	163.7	7.9	7.9	7.9	175.7	9.3	9.3	9.3
	700	176.3	6.8	6.8	6.8	189.2	8.0	8.0	8.0
	750	188.9	5.9	5.9	5.9	—	—	—	—



ALTOS HORNOS DE MEXICO, S. A.

CAPACIDAD DE CARGA EN TONELADAS  
COLUMNAS COMPUESTAS DOS CANALES PERFIL STANDARD

□ 2 C P S



Dimension	Altura en cm	L/r	TIPO DE ACERO			L/r	TIPO DE ACERO		
			A-7	A-36	AH-55		A-7	A-36	AH-55
2 C P S  5  127.0  c=0	50	14.5	34.0	37.1	56.1	13.2	46.1	50.2	76.0
	100	29.1	32.6	35.5	52.8	26.5	44.4	48.3	72.1
	150	43.6	31.0	33.5	48.7	39.7	42.4	45.9	67.3
	200	58.1	29.0	31.2	44.0	52.9	40.1	43.3	61.7
	250	72.7	26.8	28.6	38.5	66.1	37.5	40.2	55.4
	300	87.2	24.3	25.8	32.4	79.4	34.6	36.9	48.3
	350	101.7	21.6	22.6	25.6	92.6	31.5	33.2	40.4
	400	116.3	18.7	19.2	19.5	105.8	28.1	29.2	31.8
	450	130.8	15.5	15.4	15.4	119.0	24.4	24.9	25.1
	500	145.3	12.5	12.5	12.5	132.3	20.4	20.4	20.4
550	159.9	10.3	10.3	10.3	145.5	16.8	16.8	16.8	
600	174.4	8.7	8.7	8.7	158.7	14.1	14.1	14.1	
650	189.0	7.4	7.4	7.4	172.0	12.1	12.1	12.1	
700	—	—	—	—	185.2	10.4	10.4	10.4	
750	—	—	—	—	198.4	9.1	9.1	9.1	

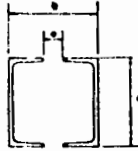
Peso total = 19.94  
Area total = 25.16

Peso total = 26.78  
Area total = 33.94

2 C P S  5  127.0  b=127.0	50	10.1	34.4	37.5	56.9	10.8	46.3	50.5	76.7
	100	20.2	33.5	36.5	54.9	21.5	45.1	49.0	73.7
	150	30.3	32.5	35.3	52.5	32.3	43.6	47.3	70.1
	200	40.4	31.4	34.1	49.7	43.0	41.9	45.3	66.0
	250	50.5	30.1	32.4	46.6	53.8	39.9	43.1	61.3
	300	60.6	28.6	30.8	43.1	64.5	37.8	40.6	56.2
	350	70.7	27.1	29.0	39.3	75.3	35.5	38.0	50.6
	400	80.8	25.4	27.1	35.2	86.0	33.1	35.1	44.4
	450	90.9	23.6	25.0	30.7	96.8	30.4	32.0	37.8
	500	101.0	21.8	22.8	25.9	107.5	27.6	28.7	30.8
	550	111.1	19.7	20.4	21.4	118.3	24.6	25.2	25.5
	600	121.2	17.6	17.9	18.0	129.0	21.4	21.4	21.4
	650	131.3	15.3	15.3	15.3	139.8	18.2	18.2	18.2
	700	141.4	13.2	13.2	13.2	150.5	15.7	15.7	15.7
	750	151.5	11.5	11.5	11.5	161.3	13.7	13.7	13.7
800	161.6	10.1	10.1	10.1	172.0	12.0	12.0	12.0	
850	171.7	9.0	9.0	9.0	182.8	10.7	10.7	10.7	
900	181.8	8.0	8.0	8.0	193.5	9.5	9.5	9.5	
950	191.9	7.2	7.2	7.2	—	—	—	—	



ALTOS HORNOS DE MEXICO, S. A.



**CAPACIDAD DE CARGA EN TONELADAS**  
**COLUMNAS COMPUESTAS DOS CANALES PERFIL STANDARD**

□ 2 C P S

Dimensión	Altura en cm	L/r	TIPO DE ACERO				L/r	TIPO DE ACERO				L/r	TIPO DE ACERO			
			A-7 A-36 AH-55					A-7 A-36 AH-55					A-7 A-36 AH-55			
6	152.4	e=0	Peso total = 24.40 Area total = 30.84				Peso total = 31.26 Area total = 39.62				Peso total = 38.70 Area total = 49.16					
			50	13.1	41.9	45.6	69.1	12.1	53.9	58.7	89.1	11.4	67.0	73.0	110.8	
			100	26.2	40.4	43.9	65.6	24.3	52.2	56.7	85.0	22.8	65.0	70.7	106.2	
			150	39.4	38.6	41.8	61.3	36.4	50.1	54.3	80.1	34.2	62.7	68.0	100.5	
			200	52.5	36.5	39.4	56.3	48.5	47.7	51.6	74.3	45.7	60.0	64.8	94.0	
			250	65.6	34.2	36.7	50.6	60.7	45.1	48.5	67.8	57.1	56.9	61.3	86.6	
			300	78.7	31.6	33.7	44.2	72.8	42.1	45.0	60.6	68.5	53.6	57.4	78.5	
			350	91.9	28.8	30.4	37.1	85.0	38.9	41.3	52.6	79.9	50.0	53.2	69.5	
			400	105.0	25.7	26.8	29.4	97.1	35.4	37.2	43.8	91.3	46.1	48.6	59.7	
			450	118.1	22.4	22.9	23.2	109.2	31.7	32.9	34.9	102.7	41.9	43.7	48.9	
			500	131.2	18.8	18.8	18.8	121.4	27.7	28.2	28.2	114.2	37.4	38.5	39.6	
			550	144.4	15.5	15.5	15.5	133.5	23.3	23.3	23.3	125.6	32.5	32.8	32.7	
600	157.5	13.1	13.1	13.1	145.6	19.6	19.6	19.6	137.0	27.5	27.5	27.5				
650	170.6	11.1	11.1	11.1	157.8	16.7	16.7	16.7	148.4	23.4	23.4	23.4				
700	183.7	9.6	9.6	9.6	169.9	14.4	14.4	14.4	159.8	20.2	20.2	20.2				
750	196.9	8.4	8.4	8.4	182.0	12.6	12.6	12.6	171.2	17.6	17.6	17.6				
800	—	—	—	—	194.2	11.0	11.0	11.0	182.6	15.5	15.5	15.5				
850	—	—	—	—	—	—	—	—	194.1	13.7	13.7	13.7				

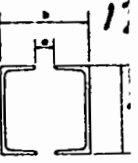
Dimensión	Altura en cm	L/r	TIPO DE ACERO				L/r	TIPO DE ACERO				L/r	TIPO DE ACERO			
			A-7 A-36 AH-55					A-7 A-36 AH-55					A-7 A-36 AH-55			
6	152.4	b=152.4	Peso total = 24.40 Area total = 30.84				Peso total = 31.26 Area total = 39.62				Peso total = 38.70 Area total = 49.16					
			50	8.4	42.3	46.1	70.2	8.9	54.3	59.2	90.0	9.2	67.3	73.4	111.6	
			100	16.8	41.5	45.2	68.2	17.7	53.2	57.9	87.3	18.5	65.8	71.7	108.1	
			150	25.3	40.5	44.0	65.9	26.6	51.8	56.3	84.2	27.7	64.1	69.6	103.9	
			200	33.7	39.4	42.7	63.3	35.5	50.3	54.5	80.5	37.0	62.1	67.3	99.1	
			250	42.1	38.2	41.3	60.3	44.3	48.6	52.6	76.4	46.2	59.8	64.7	93.7	
			300	50.5	36.8	39.8	57.1	53.2	46.7	50.4	71.9	55.5	57.4	61.8	87.7	
			350	58.9	35.4	38.1	53.6	62.1	44.7	48.1	67.0	64.7	54.7	58.8	81.3	
			400	67.3	33.8	36.3	49.8	70.9	42.6	45.6	61.8	73.9	51.9	55.5	74.3	
			450	75.8	32.2	34.4	45.7	79.8	40.3	42.9	56.1	83.2	48.9	51.9	66.8	
			500	84.2	30.5	32.3	41.4	88.7	37.9	40.1	50.0	92.4	45.7	48.2	58.7	
			550	92.6	28.6	30.2	36.7	97.5	35.3	37.1	43.5	101.7	42.3	44.2	50.0	
600	101.0	26.7	27.9	31.8	106.4	32.6	33.9	36.8	110.9	38.7	40.0	42.0				
650	109.4	24.6	25.5	27.0	115.2	29.7	30.6	31.3	120.1	34.9	35.5	35.8				
700	117.8	22.5	23.0	23.3	124.1	26.7	27.0	27.0	129.4	30.9	30.8	30.8				
750	126.3	20.2	20.3	20.3	133.0	23.5	23.5	23.5	138.6	26.9	26.9	26.9				
800	134.7	17.9	17.9	17.9	141.8	20.7	20.7	20.7	147.9	23.6	23.6	23.6				
850	143.1	15.8	15.8	15.8	150.7	18.3	18.3	18.3	157.1	20.9	20.9	20.9				
900	151.5	14.1	14.1	14.1	159.6	16.3	16.3	16.3	166.4	18.7	18.7	18.7				
950	159.9	12.7	12.7	12.7	168.4	14.7	14.7	14.7	175.6	16.7	16.7	16.7				
1000	168.4	11.4	11.4	11.4	177.3	13.2	13.2	13.2	184.8	15.1	15.1	15.1				
1050	176.8	10.4	10.4	10.4	186.2	12.0	12.0	12.0	194.1	13.7	13.7	13.7				
1100	185.2	9.4	9.4	9.4	195.0	10.9	10.9	10.9	—	—	—	—				
1150	193.6	8.6	8.6	8.6	—	—	—	—	—	—	—	—				



ALTOS HORNOS DE MEXICO, S. A.

**CAPACIDAD DE CARGA EN TONELADAS**  
**COLUMNAS COMPUESTAS DOS CANALES PERFIL STANDARD**

□ 2 C P S

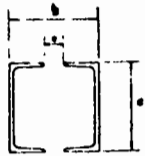


Dimensión	Altura en cm	L/r	TIPO DE ACERO				L/r	TIPO DE ACERO				L/r	TIPO DE ACERO			
			A-7 A-36 AH-55					A-7 A-36 AH-55					A-7 A-36 AH-55			
7	177.8	e=0	Peso total = 29.16 Area total = 36.78				Peso total = 36.46 Area total = 46.20				Peso total = 43.90 Area total = 55.74					
			100	23.9	48.5	52.7	79.1	22.5	61.2	66.5	99.9	21.3	74.1	80.6	121.2	
			150	35.9	46.6	50.5	74.6	33.7	59.0	64.0	94.8	31.9	71.6	77.8	115.4	
			200	47.8	44.4	48.0	69.3	44.9	56.5	61.1	88.8	42.6	68.9	74.5	108.7	
			250	59.8	42.0	45.2	63.4	56.2	53.7	57.9	82.0	53.2	65.8	70.9	101.2	
			300	71.8	39.3	42.1	56.8	67.4	50.7	54.3	74.5	63.8	62.4	67.0	92.9	
			350	83.7	36.4	38.7	49.6	78.7	47.4	50.5	66.3	74.5	58.7	62.7	83.8	
			400	95.7	33.3	35.0	41.7	89.9	43.8	46.3	57.3	85.1	54.7	58.0	73.8	
			450	107.7	29.9	31.1	33.3	101.1	39.9	41.8	47.5	95.7	50.4	53.0	63.1	
			500	119.6	26.3	26.8	27.0	112.4	35.8	36.9	38.4	106.4	45.9	47.7	51.7	
			550	131.6	22.3	22.3	22.3	123.6	31.4	31.8	31.8	117.0	41.0	42.0	42.7	
			600	143.5	18.7	18.7	18.7	134.8	26.7	26.7	26.7	127.7	35.9	35.9	35.9	
650	155.5	16.0	16.0	16.0	146.1	22.7	22.7	22.7	138.3	30.6	30.6	30.6				
700	167.5	13.8	13.8	13.8	157.3	19.6	19.6	19.6	148.9	26.4	26.4	26.4				
750	179.4	12.0	12.0	12.0	168.5	17.1	17.1	17.1	159.6	23.0	23.0	23.0				
800	191.4	10.5	10.5	10.5	179.8	15.0	15.0	15.0	170.2	20.2	20.2	20.2				
850	—	—	—	—	191.0	13.3	13.3	13.3	180.9	17.9	17.9	17.9				
900	—	—	—	—	—	—	—	—	191.5	16.0	16.0	16.0				

Dimensión	Altura en cm	L/r	TIPO DE ACERO				L/r	TIPO DE ACERO				L/r	TIPO DE ACERO			
			A-7 A-36 AH-55					A-7 A-36 AH-55					A-7 A-36 AH-55			
7	177.8	b=177.8	Peso total = 29.16 Area total = 36.78				Peso total = 36.46 Area total = 46.20				Peso total = 43.90 Area total = 55.74					
			100	14.5	49.8	54.2	82.1	15.2	62.4	68.0	102.8	15.7	75.2	81.9	123.8	
			150	21.7	48.8	53.1	79.8	22.8	61.1	66.5	99.8	23.5	73.6	80.0	120.0	
			200	28.9	47.7	51.9	77.3	30.4	59.7	64.8	96.4	31.3	71.8	77.9	115.7	
			250	36.2	46.6	50.5	74.4	38.0	58.1	63.0	92.6	39.2	69.8	75.6	110.9	
			300	43.4	45.3	49.0	71.3	45.6	56.4	61.0	88.4	47.0	67.6	73.1	105.6	
			350	50.7	43.9	47.4	68.0	53.2	54.5	58.8	83.9	54.9	65.2	70.3	99.9	
			400	57.9	42.4	45.7	64.4	60.8	52.5	56.5	79.0	62.7	62.7	67.4	93.8	
			450	65.1	40.9	43.9	60.6	68.4	50.4	54.0	73.8	70.5	60.1	64.3	87.2	
			500	72.4	39.2	41.9	56.5	76.0	48.2	51.4	68.3	78.4	57.2	61.0	80.2	
			550	79.6	37.5	39.9	52.2	83.6	45.8	48.7	62.4	86.2	54.3	57.5	72.8	
			600	86.8	35.6	37.8	47.6	91.2	43.3	45.8	56.2	94.0	51.1	53.9	64.9	
650	94.1	33.7	35.5	42.8	98.8	40.7	42.7	49.6	101.9	47.8	50.0	56.5				
700	101.3	31.7	33.2	37.7	106.4	38.0	39.6	42.9	109.7	44.4	46.0	48.6				
750	108.5	29.6	30.7	32.8	114.0	35.2	36.2	37.3	117.6	40.8	41.7	42.4				
800	115.8	27.5	28.2	28.8	121.6	32.2	32.7	32.8	125.4	37.0	37.3	37.2				
850	123.0	25.2	25.5	25.5	129.2	29.1	29.1	29.1	133.2	33.0	33.0	33.0				
900	130.2	22.8	22.8	22.8	136.8	25.9	25.9	25.9	141.1	29.4	29.4	29.4				
950	137.5	20.4	20.4	20.4	144.4	23.3	23.3	23.3	148.9	26.4	26.4	26.4				
1000	144.7	18.4	18.4	18.4	152.0	21.0	21.0	21.0	156.7	23.8	23.8	23.8				
1050	152.0	16.7	16.7	16.7	159.6	19.1	19.1	19.1	164.6	21.6	21.6	21.6				
1100	159.2	15.2	15.2	15.2	167.2	17.4	17.4	17.4	172.4	19.7	19.7	19.7				
1150	166.4	13.9	13.9	13.9	174.8	15.9	15.9	15.9	180.3	18.0	18.0	18.0				
1200	173.7	12.8	12.8	12.8	182.4	14.6	14.6	14.6	188.1	16.5	16.5	16.5				
1250	180.9	11.8	11.8	11.8	190.0	13.4	13.4	13.4	195.9	15.2	15.2	15.2				
1300	188.1	10.9	10.9	10.9	197.6	12.4	12.4	12.4	—	—	—	—				
1350	195.4	10.1	10.1	10.1	—	—	—	—	—	—	—	—				

ALTOS HORNOS DE MEXICO, S. A.





**CAPACIDAD DE CARGA EN TONELADAS**  
**COLUMNAS COMPUESTAS DOS CANALES PERFIL STANDARD**

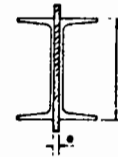
**2 C P S**

Dimensión	Altura en cm	L/e	TIPO DE ACERO			L/e	TIPO DE ACERO			L/e	TIPO DE ACERO			
			A-7	A-36	AH-55		A-7	A-36	AH-55		A-7	A-36	AH-55	
8 203.2 e=0	100	22.0	Peso total = 34.22 Area total = 43.36			104.4	Peso total = 40.92 Area total = 51.88			106.0	Peso total = 55.80 Area total = 70.84			
			50	57.5	62.5		94.0	69.0	75.1		113.0	94.7	103.0	155.2
			150	55.5	60.3		89.3	66.8	72.5		107.7	92.0	99.9	148.9
	200	53.3	57.6	83.8	64.3	69.6	101.7	88.9	96.3	141.5				
	250	51.9	54.7	77.7	61.5	66.4	94.9	85.5	92.4	133.2				
	300	49.0	51.5	70.9	58.4	62.8	87.3	81.7	88.0	124.1				
	350	47.9	47.9	63.4	55.1	58.9	79.1	77.7	83.3	114.2				
	400	41.7	44.1	55.3	51.5	54.7	70.2	73.3	78.2	103.5				
	450	38.2	40.1	46.5	47.6	50.2	60.5	68.7	72.8	91.9				
	500	34.5	35.7	37.7	43.5	45.4	50.0	63.8	67.0	79.4				
	550	30.5	31.0	31.2	39.1	40.2	41.3	56.5	60.9	66.2				
	600	26.2	26.2	26.2	34.5	34.8	34.7	53.0	54.4	55.7				
	650	22.3	22.3	22.3	29.6	29.6	29.6	47.1	47.5	47.4				
	700	19.2	19.2	19.2	25.5	25.5	25.5	40.9	40.9	40.9				
	750	16.8	16.8	16.8	22.2	22.2	22.2	35.6	35.6	35.6				
	800	14.7	14.7	14.7	19.5	19.5	19.5	31.3	31.3	31.3				
850	13.0	13.0	13.0	17.3	17.3	17.3	27.7	27.7	27.7					
900	11.6	11.6	11.6	15.4	15.4	15.4	24.7	24.7	24.7					
950	—	—	—	13.8	13.8	13.8	22.2	22.2	22.2					
1000	—	—	—	—	—	—	20.0	20.0	20.2					

Dimensión	Altura en cm	L/e	TIPO DE ACERO			L/e	TIPO DE ACERO			L/e	TIPO DE ACERO			
			A-7	A-36	AH-55		A-7	A-36	AH-55		A-7	A-36	AH-55	
8 203.2 e=0	100	12.7	Peso total = 34.22 Area total = 43.36			104.4	Peso total = 40.92 Area total = 51.88			106.0	Peso total = 55.80 Area total = 70.84			
			50	54.9	64.2		97.3	70.4	76.7		116.3	96.0	104.5	158.3
			150	58.0	63.1		95.1	69.2	75.3		113.5	94.2	102.5	154.2
	200	56.9	61.9	92.6	67.9	73.8	110.3	92.3	100.2	149.5				
	250	55.8	60.5	89.8	66.4	72.1	106.8	90.1	97.7	144.4				
	300	54.5	59.1	86.8	64.9	70.3	103.0	87.8	95.0	138.7				
	350	53.2	57.5	83.6	63.2	68.3	98.9	85.2	92.1	132.6				
	400	51.7	55.8	80.1	61.3	66.2	94.5	82.5	88.9	126.0				
	450	50.2	54.1	76.4	59.3	63.9	89.8	79.7	85.6	119.0				
	500	48.6	52.2	72.4	57.4	61.6	84.9	76.6	82.1	111.6				
	550	46.9	50.2	68.3	55.3	59.1	79.6	73.5	78.4	103.8				
	600	45.1	48.2	63.9	53.0	56.5	74.1	70.1	74.5	95.5				
	650	43.3	46.0	59.3	50.7	53.8	68.3	66.6	70.4	86.7				
	700	41.4	43.8	54.5	48.3	50.9	62.1	63.0	66.2	77.5				
	750	39.4	41.4	49.4	45.7	48.0	55.7	59.2	61.7	67.8				
	800	37.3	39.0	44.1	43.1	44.9	49.0	55.2	57.1	59.6				
850	35.1	36.5	39.0	40.3	41.7	43.4	51.1	52.2	52.8					
900	32.9	33.8	34.8	37.5	38.3	38.7	46.8	47.2	47.1					
950	30.9	31.1	31.2	34.5	34.8	34.8	42.3	42.3	42.3					
1000	28.1	28.2	28.2	31.4	31.4	31.4	38.1	38.1	38.1					
1050	25.6	25.6	25.6	28.5	28.5	28.5	34.6	34.6	34.6					
1100	23.3	23.3	23.3	25.9	25.9	25.9	31.5	31.5	31.5					
1150	21.3	21.3	21.3	23.7	23.7	23.7	28.8	28.8	28.8					
1200	19.6	19.6	19.6	21.8	21.8	21.8	26.5	26.5	26.5					
1250	18.0	18.0	18.0	20.1	20.1	20.1	24.4	24.4	24.4					
1300	16.7	16.7	16.7	18.6	18.6	18.6	22.6	22.6	22.6					
1350	15.5	15.5	15.5	17.2	17.2	17.2	20.9	20.9	20.9					
1400	14.4	14.4	14.4	16.0	16.0	16.0	19.5	19.5	19.5					
1450	13.4	13.4	13.4	14.9	14.9	14.9	—	—	—					
1500	12.5	12.5	12.5	13.9	13.9	13.9	—	—	—					
1550	11.7	11.7	11.7	—	—	—	—	—	—					

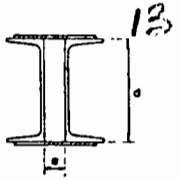


ALTOS HORNOS DE MEXICO, S A



**CAPACIDAD DE CARGA EN TONELADAS**  
**COLUMNAS COMPUESTAS DOS CANALES PERFIL STANDARD**

**2 C P S**

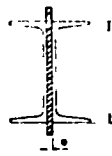


Dimensión	Altura en cm	L/e	TIPO DE ACERO			L/e	TIPO DE ACERO			L/e	TIPO DE ACERO				
			A-7	A-36	AH-55		A-7	A-36	AH-55		A-7	A-36	AH-55		
6 152.4 e=6.3	50	23.4	Peso total = 24.40 Area total = 30.84			106.0	Peso total = 31.26 Area total = 39.62			106.0	Peso total = 38.70 Area total = 49.16				
			40.7	44.3	66.5		23.9	52.2	56.8		85.2	23.5	64.9	70.6	105.9
			100	37.4	40.5		58.6	47.8	47.9		51.7	74.7	46.9	59.6	64.5
	150	33.3	35.7	48.5	42.4	45.3	61.2	40.4	53.0	56.7	77.0				
	200	28.4	30.0	36.2	35.9	37.7	44.9	33.9	45.1	47.6	57.4				
	250	22.7	23.3	23.7	28.3	28.9	29.1	22.7	23.3	23.7	23.7				
300	16.5	16.5	16.5	20.2	20.2	20.2	16.5	16.5	16.5	16.5					
350	12.1	12.1	12.1	14.8	14.8	14.8	12.1	12.1	12.1	12.1					
400	9.3	9.3	9.3	11.4	11.4	11.4	9.3	9.3	9.3	9.3					

Dimensión	Altura en cm	L/e	TIPO DE ACERO			L/e	TIPO DE ACERO			L/e	TIPO DE ACERO				
			A-7	A-36	AH-55		A-7	A-36	AH-55		A-7	A-36	AH-55		
6 152.4 e=9.5	50	22.1	Peso total = 24.40 Area total = 30.84			106.0	Peso total = 31.26 Area total = 39.62			106.0	Peso total = 38.70 Area total = 49.16				
			40.9	44.5	66.8		22.5	52.5	57.1		85.7	22.1	65.2	70.9	106.5
			100	37.8	40.9		59.5	45.0	48.5		52.4	76.1	44.2	60.3	65.3
	150	34.0	36.5	50.2	43.4	46.6	63.8	40.4	54.2	58.2	80.0				
	200	29.5	31.2	39.0	37.5	39.6	49.0	35.9	47.1	49.8	62.2				
	250	24.3	25.2	26.5	30.6	31.6	32.8	24.3	24.3	24.3	24.3				
300	18.4	18.4	18.4	22.8	22.8	22.8	18.4	18.4	18.4	18.4					
350	13.5	13.5	13.5	16.7	16.7	16.7	13.5	13.5	13.5	13.5					
400	10.3	10.3	10.3	12.8	12.8	12.8	10.3	10.3	10.3	10.3					
450	8.2	8.2	8.2	—	—	—	—	—	—	—					

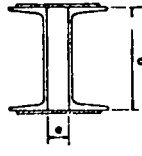


ALTOS HORNOS DE MEXICO,



**CAPACIDAD DE CARGA EN TONELADAS**  
**COLUMNAS COMPLETAS DOS CANALES PERFIL STANDARD**

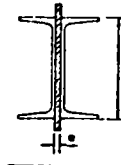
**I 2 C P S**



Dimensión	Altura en cm	L/r TIPO DE ACERO				L/r TIPO DE ACERO				L/r TIPO DE ACERO			
		A-7	A-36	AH-55		A-7	A-36	AH-55		A-7	A-36	AH-55	
2 C P S <b>7</b> 177.8 e=0	50	Peso total = 29.16 Area total = 36.78				Peso total = 36.46 Area total = 46.20				Peso total = 43.90 Area total = 55.74			
	100	24.5	48.4	52.6	78.9	24.9	60.7	66.0	98.9	25.1	73.2	79.6	119.2
	150	49.0	44.2	47.8	68.8	49.8	55.4	59.8	86.0	50.3	66.7	72.0	103.3
	200	73.5	38.9	41.6	55.8	74.6	48.6	51.9	69.3	75.4	58.3	62.3	82.9
	250	98.0	32.6	34.3	40.0	99.5	40.5	42.4	49.0	100.5	48.4	50.7	58.0
	300	122.5	25.3	25.7	25.7	124.4	31.1	31.4	31.4	125.6	36.9	37.2	37.1
	350	147.1	17.9	17.9	17.9	149.3	21.8	21.8	21.8	150.8	25.8	25.8	25.8
	400	171.6	13.1	13.1	13.1	174.1	16.0	16.0	16.0	175.9	18.9	18.9	18.9

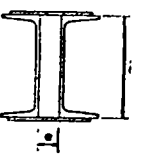
Dimensión	Altura en cm	L/r TIPO DE ACERO				L/r TIPO DE ACERO				L/r TIPO DE ACERO			
		A-7	A-36	AH-55		A-7	A-36	AH-55		A-7	A-36	AH-55	
2 C P S <b>7</b> 177.8 e=9.5	50	Peso total = 29.16 Area total = 36.78				Peso total = 36.46 Area total = 46.20				Peso total = 43.90 Area total = 55.74			
	100	20.8	48.9	53.2	80.1	21.2	61.4	66.8	100.5	21.3	74.1	80.6	121.2
	150	41.7	45.6	49.4	72.1	42.4	57.1	61.8	90.2	42.6	68.9	74.5	108.7
	200	62.5	41.4	44.5	62.0	63.6	51.8	55.6	77.1	63.8	62.4	67.0	92.9
	250	83.3	36.5	38.8	49.9	84.7	45.4	48.2	61.5	85.1	54.7	58.0	73.8
	300	104.2	30.9	32.2	35.6	105.9	38.2	39.7	43.2	106.4	45.9	47.7	51.7
	350	125.0	24.5	24.8	24.7	127.1	29.9	30.0	30.0	127.7	35.9	35.9	35.9
	400	145.8	18.2	18.2	18.2	148.3	22.1	22.1	22.1	148.9	26.4	26.4	26.4

Dimensión	Altura en cm	L/r TIPO DE ACERO				L/r TIPO DE ACERO				L/r TIPO DE ACERO			
		A-7	A-36	AH-55		A-7	A-36	AH-55		A-7	A-36	AH-55	
2 C P S <b>7</b> 177.8 e=12.7	50	Peso total = 29.16 Area total = 36.78				Peso total = 36.46 Area total = 46.20				Peso total = 43.90 Area total = 55.74			
	100	19.8	49.1	53.4	80.4	20.2	61.6	67.0	100.9	20.2	74.3	80.8	121.7
	150	39.7	46.0	49.8	73.0	40.3	57.6	62.4	91.3	40.5	69.4	75.2	110.1
	200	59.5	42.1	45.3	63.6	60.5	52.6	56.6	79.2	60.7	63.4	68.2	95.4
	250	79.4	37.5	40.0	52.3	80.6	46.7	49.7	64.7	81.0	56.3	59.9	77.8
	300	99.2	32.3	33.9	39.2	100.8	40.0	41.9	47.8	101.2	48.1	50.4	57.2
	350	119.0	26.4	27.0	27.2	121.0	32.4	33.0	33.2	121.5	38.9	39.6	39.7
	400	138.9	20.0	20.0	20.0	141.1	24.4	24.4	24.4	141.7	29.1	29.1	29.1



**CAPACIDAD DE CARGA EN TONELADAS**  
**COLUMNAS COMPUSTAS DOS CANALES PERFIL STANDARD**

**I 2 C P S**



Dimensión	Altura en cm	L/r TIPO DE ACERO				L/r TIPO DE ACERO				L/r TIPO DE ACERO			
		A-7	A-36	AH-55		A-7	A-36	AH-55		A-7	A-36	AH-55	
2 C P S <b>7</b> 177.8 e=10.7	100	Peso total = 29.16 Area total = 36.78 e = 107.0				Peso total = 36.46 Area total = 46.20 e = 101.0				Peso total = 43.90 Area total = 55.74 e = 96.6			
	150	14.5	49.8	54.2	82.1	15.2	62.4	68.0	102.8	15.7	75.2	81.9	123.8
	200	28.9	47.7	51.9	77.3	30.4	59.7	64.8	96.4	31.3	71.8	77.9	115.7
	250	36.2	46.6	50.5	74.4	38.0	58.1	63.0	92.6	39.2	69.8	75.6	110.9
	300	43.4	45.3	49.0	71.3	45.6	56.4	61.0	88.4	47.0	67.6	73.1	105.6
	350	50.7	43.9	47.4	68.0	53.2	54.5	58.8	83.9	54.9	65.2	70.3	99.9
	400	57.9	42.4	45.7	64.4	60.8	52.5	56.5	79.0	62.7	62.7	67.4	93.8
	450	65.1	40.9	43.9	60.6	68.4	50.4	54.0	73.8	70.5	60.1	64.3	87.2

Dimensión	Altura en cm	L/r TIPO DE ACERO				L/r TIPO DE ACERO				L/r TIPO DE ACERO			
		A-7	A-36	AH-55		A-7	A-36	AH-55		A-7	A-36	AH-55	
2 C P S <b>7</b> 177.8 e=9.5	500	72.4	39.2	41.9	56.5	76.0	48.2	51.4	68.3	78.4	57.2	61.0	80.2
	550	79.6	37.5	39.9	52.2	83.6	45.8	48.7	62.4	86.2	54.3	57.5	72.8
	600	86.8	35.6	37.8	47.6	91.2	43.3	45.8	56.2	94.0	51.1	53.9	64.9
	650	94.1	33.7	35.5	42.8	98.8	40.7	42.7	49.6	101.9	47.8	50.0	56.5
	700	101.3	31.7	33.2	37.7	106.4	38.0	39.6	42.9	109.7	44.4	46.0	48.6
	750	108.5	29.6	30.7	32.8	114.0	35.2	36.2	37.3	117.6	40.8	41.7	42.4
	800	115.8	27.5	28.2	28.8	121.6	32.2	32.7	32.8	125.4	37.0	37.3	37.2
	850	123.0	25.2	25.5	25.5	129.2	29.1	29.1	29.1	133.2	33.0	33.0	33.0
	900	130.2	22.8	22.8	22.8	136.8	25.9	25.9	25.9	141.1	29.4	29.4	29.4
	950	137.5	20.4	20.4	20.4	144.4	23.3	23.3	23.3	148.9	26.4	26.4	26.4
	1000	144.7	18.4	18.4	18.4	152.0	21.0	21.0	21.0	156.7	23.8	23.8	23.8
	1050	152.0	16.7	16.7	16.7	159.6	19.1	19.1	19.1	164.6	21.6	21.6	21.6
	1100	159.2	15.2	15.2	15.2	167.2	17.4	17.4	17.4	172.4	19.7	19.7	19.7
	1150	166.4	13.9	13.9	13.9	174.8	15.9	15.9	15.9	180.3	18.0	18.0	18.0
	1200	173.7	12.8	12.8	12.8	182.4	14.6	14.6	14.6	188.1	16.5	16.5	16.5
	1250	180.9	11.8	11.8	11.8	190.0	13.4	13.4	13.4	195.9	15.2	15.2	15.2
1300	188.1	10.9	10.9	10.9	197.6	12.4	12.4	12.4	—	—	—	—	
1350	195.4	10.1	10.1	10.1	—	—	—	—	—	—	—	—	

Dimensión	Altura en cm	L/r TIPO DE ACERO				L/r TIPO DE ACERO				L/r TIPO DE ACERO			
		A-7	A-36	AH-55		A-7	A-36	AH-55		A-7	A-36	AH-55	
2 C P S <b>8</b> 203.2 e=0	100	Peso total = 34.22 Area total = 43.36				Peso total = 40.92 Area total = 51.88				Peso total = 55.80 Area total = 70.84			
	150	46.3	52.7	57.0	82.6	47.6	62.8	67.8	97.9	47.4	85.8	92.7	133.9
	200	69.4	47.0	50.4	68.6	71.4	55.6	59.5	80.4	71.1	76.1	81.4	110.2
	250	92.6	40.2	42.4	51.6	95.2	47.1	49.6	59.2	94.8	64.6	68.0	81.5
	300	115.7	32.4	33.2	34.0	119.0	37.3	38.1	38.4	118.5	51.3	52.4	53.0
	350	138.9	23.6	23.6	23.6	142.9	26.7	26.7	26.7	142.2	36.8	36.8	36.8
	400	162.0	17.3	17.3	17.3	166.7	19.6	19.6	19.6	165.9	27.0	27.0	27.0
	450	185.2	13.3	13.3	13.3	190.5	15.0	15.0	15.0	189.6	20.7	20.7	20.7

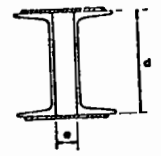
Dimensión	Altura en cm	L/r TIPO DE ACERO				L/r TIPO DE ACERO				L/r TIPO DE ACERO			
		A-7	A-36	AH-55		A-7	A-36	AH-55		A-7	A-36	AH-55	
2 C P S <b>8</b> 203.2 e=9.5	100	Peso total = 34.22 Area total = 43.36				Peso total = 40.92 Area total = 51.88				Peso total = 55.80 Area total = 70.84			
	150	39.8	54.1	58.7	86.0	40.8	64.5	69.9	102.3	40.7	88.2	95.5	139.8
	200	59.8	49.5	53.3	74.8	61.2	58.8	63.3	88.4	61.0	80.4	86.5	120.9
	250	79.7	44.1	47.0	61.5	81.6	52.1	55.5	71.8	81.3	71.3	75.9	98.5
	300	99.6	38.0	39.8	45.9	102.0	44.5	46.5	52.3	101.6	60.9	63.7	72.1
	350	119.5	31.0	31.6	31.9	122.4	35.8	36.3	36.3	122.0	49.1	49.9	50.0
	400	139.4	23.4	23.4	23.4	142.9	26.7	26.7	26.7	142.3	36.7	36.7	36.7
	450	159.4	17.9	17.9	17.9	163.3	20.4	20.4	20.4	162.6	28.1	28.1	28.1





**CAPACIDAD DE CARGA EN TONELADAS**  
**COLUMNAS COMPUESTAS DOS CANALES PERFIL STANDARD**

**I 2 CPS**



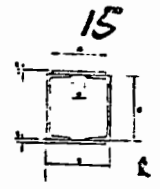
Dimensión	Altura en cm	TIPO DE ACERO				TIPO DE ACERO				TIPO DE ACERO			
		L/r	A-7	A-36	AH-55	L/r	A-7	A-36	AH-55	L/r	A-7	A-36	AH-55
<b>2 CPS</b> <b>8</b> <b>e=127</b>	50	190	58.0	63.1	95.1	19.5	69.3	75.4	113.6	19.4	94.6	103.0	155.2
	100	38.0	54.5	59.1	86.9	38.9	65.0	70.4	103.4	38.8	83.8	96.3	141.3
	150	57.0	50.2	54.1	76.4	58.4	59.7	64.3	90.5	58.1	81.6	87.9	123.8
	200	76.0	45.2	48.2	64.0	77.8	53.5	57.0	75.1	77.5	73.1	78.0	102.9
	250	95.1	39.4	41.5	49.6	97.3	46.3	48.7	57.2	96.9	63.5	66.7	78.6
	300	114.1	33.0	34.0	35.0	116.7	38.3	39.3	40.0	116.3	52.6	53.9	55.0
	350	133.1	25.7	25.7	25.7	136.2	29.4	29.4	29.4	135.7	40.4	40.4	40.4
	400	152.1	19.7	19.7	19.7	155.6	22.5	22.5	22.5	155.0	30.9	30.9	30.9
	450	171.1	15.6	15.6	15.6	175.1	17.8	17.8	17.8	174.4	24.5	24.5	24.5
	500	190.1	12.6	12.6	12.6	194.6	14.4	14.4	14.4	193.8	19.8	19.8	19.8

Dimensión	Altura en cm	TIPO DE ACERO				TIPO DE ACERO				TIPO DE ACERO			
		L/r	A-7	A-36	AH-55	L/r	A-7	A-36	AH-55	L/r	A-7	A-36	AH-55
<b>2 CPS</b> <b>8</b> <b>e=127</b>	100	127	58.9	64.2	97.3	13.2	70.4	76.7	116.3	14.0	96.0	104.5	158.3
	200	25.4	56.9	61.9	92.6	26.4	67.9	73.8	110.3	27.9	92.3	100.2	149.5
	250	31.8	54.8	60.5	89.8	32.9	66.4	72.1	106.8	34.9	90.1	97.7	144.4
	300	38.1	51.5	59.1	86.8	39.5	64.9	70.3	103.0	41.9	87.8	95.0	138.7
	350	44.5	53.2	57.5	83.6	46.1	63.2	68.3	98.9	48.9	85.2	92.1	132.6
	400	50.8	51.7	55.8	80.1	52.7	61.3	66.2	94.5	55.9	82.5	88.9	126.0
	450	57.2	50.2	54.1	76.4	59.3	59.4	63.9	89.8	62.8	79.7	85.6	119.0
	500	63.5	48.6	52.2	72.4	65.9	57.4	61.6	84.9	69.8	76.6	82.1	111.6
	550	69.9	46.9	50.2	68.3	72.5	55.3	59.1	79.6	76.8	73.5	78.4	103.8
	600	76.2	45.1	48.2	63.9	79.1	53.0	56.5	74.1	83.8	70.1	74.5	95.5

**CAPACIDAD DE CARGA**  
**COLUMNAS COMPUESTAS DE 2 CANALES STANDARD**  
**Y 2 PLACAS SOLDADAS**

**O 2 CPS**

**CARGA EN TONELADAS**



Dimensión	Altura en cm	TIPO DE ACERO				TIPO DE ACERO				TIPO DE ACERO			
		L/r	A-7	A-36	AH-55	L/r	A-7	A-36	AH-55	L/r	A-7	A-36	AH-55
<b>3x4</b> <b>76 x 102</b> <b>Pl 6 x 89</b>	50	14.3	36.1	39.3	59.5	14.8	40.8	44.4	67.2	15.2	45.8	49.9	75.4
	100	28.6	34.7	37.7	56.1	29.7	39.0	42.4	63.1	30.4	43.8	47.5	70.7
	150	42.9	32.9	35.6	51.9	44.5	37.0	40.0	58.1	45.6	41.4	44.7	64.8
	200	57.1	30.9	33.3	47.0	59.3	34.5	37.2	52.2	60.8	38.5	41.4	59.0
	250	71.4	28.6	30.6	41.4	74.2	31.8	34.0	45.4	76.0	35.3	37.7	50.1
	300	85.7	26.0	27.6	35.1	89.0	28.7	30.4	37.9	91.2	31.6	33.6	41.2
	350	100.0	23.3	24.4	28.0	103.9	25.4	26.5	29.3	106.4	27.9	29.0	31.4
	400	114.3	20.2	20.8	21.4	118.7	21.8	22.2	22.5	121.6	23.6	24.0	24.1
	450	128.6	16.9	16.9	16.9	133.5	17.7	17.7	17.7	136.8	19.0	19.0	19.0
	500	142.9	13.7	13.7	13.7	148.4	14.4	14.4	14.4	152.0	15.4	15.4	15.4

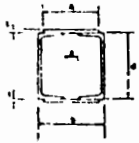
Dimensión	Altura en cm	TIPO DE ACERO				TIPO DE ACERO				TIPO DE ACERO			
		L/r	A-7	A-36	AH-55	L/r	A-7	A-36	AH-55	L/r	A-7	A-36	AH-55
<b>3x4</b> <b>76 x 102</b> <b>Pl 10 x 89</b>	50	14.8	43.8	47.7	72.1	14.5	48.5	52.8	80.0	14.3	53.6	58.4	88.4
	100	29.6	41.9	45.5	67.8	29.0	46.5	50.5	75.3	28.7	51.4	55.9	83.3
	150	44.4	39.7	42.9	62.4	43.5	44.1	47.8	69.5	43.0	48.8	52.9	77.0
	200	59.2	37.1	39.9	56.1	58.0	41.3	44.5	62.7	57.3	45.8	49.3	69.7
	250	74.0	34.2	36.5	48.9	72.5	38.2	40.8	55.0	71.6	42.4	45.4	61.3
	300	88.8	30.9	32.7	40.8	87.0	34.7	36.8	46.3	86.0	38.6	40.9	51.9
	350	103.6	27.4	28.6	31.7	101.4	30.9	32.3	36.6	100.3	34.5	36.1	41.4
	400	118.3	23.5	24.0	24.3	115.9	26.7	27.4	28.0	114.6	29.9	30.8	31.6
	450	133.1	19.2	19.2	19.2	130.4	22.2	22.1	22.1	128.9	25.0	25.0	25.0
	500	147.9	15.5	15.5	15.5	144.9	17.9	17.9	17.9	143.3	20.2	20.2	20.2

Dimensión	Altura en cm	TIPO DE ACERO				TIPO DE ACERO				TIPO DE ACERO			
		L/r	A-7	A-36	AH-55	L/r	A-7	A-36	AH-55	L/r	A-7	A-36	AH-55
<b>3x4</b> <b>76 x 102</b> <b>Pl 13 x 89</b>	50	15.3	51.3	55.8	84.5	14.9	56.0	61.0	92.3	14.7	61.1	66.6	100.7
	100	30.6	49.0	53.2	79.1	29.9	53.7	58.3	86.7	29.5	58.6	63.6	94.7
	150	45.9	46.3	50.0	72.5	44.8	50.8	54.9	79.7	44.2	55.5	60.0	87.2
	200	61.2	43.1	46.3	64.7	59.7	47.4	51.0	71.5	59.0	51.8	55.5	74.4
	250	76.5	39.5	42.1	55.8	74.6	43.6	46.6	62.2	73.7	47.8	51.1	68.4
	300	91.7	35.5	37.4	45.8	89.6	39.4	41.6	51.7	88.5	43.3	45.5	57.2
	350	107.0	31.1	32.3	34.8	104.5	34.7	36.2	39.9	103.2	38.3	40.0	40.5
	400	122.3	26.2	26.6	26.6	119.4	29.7	30.3	30.5	118.0	32.9	33.6	34.1
	450	137.6	21.0	21.0	21.0	134.3	24.1	24.1	24.1	132.7	26.9	26.9	26.9
	500	152.9	17.0	17.0	17.0	149.3	19.5	19.5	19.5	147.5	21.8	21.8	21.8



**CAPACIDAD DE CARGA**  
**COLUMNAS COMPUESTAS DE 2 CANALES STANDARD**  
**Y 2 PLACAS SOLDADAS**

**2 CPS**  
**CARGA EN TONELADAS**



Dimensión	Altura en cm	L/r	TIPO DE ACERO				L/r	TIPO DE ACERO					
			A-7	A-36	AH-55			A-7	A-36	AH-55			
			Peso nominal = 804 Kgs. Peso total = 2493 Kgs. Area total = 3142 cm²					Peso nominal = 1079 Kgs. Peso total = 3043 Kgs. Area total = 3866 cm²					
d x b	50	139	426	464	703	135	525	571	866				
	100	27.8	410	445	664	270	505	549	820				
<b>4x4</b>	150	41.7	390	422	616	405	482	522	763				
	200	55.6	367	395	561	541	454	490	698				
	250	69.4	341	365	497	676	424	454	623				
102 x 102	300	83.3	312	332	426	811	390	415	539				
	350	97.2	281	295	347	946	353	372	446				
Pl 6 x 89	400	111.1	247	255	267	1081	313	325	347				
	450	125.0	210	212	211	1216	269	274	274				
	500	138.9	171	171	171	1351	222	222	222				
	550	152.8	141	141	141	1496	184	184	184				
	600	166.7	119	119	119	1622	154	154	154				
	650	180.6	101	101	101	1757	132	132	132				
	700	194.4	87	87	87	1892	113	113	113				

Dimensión	Altura en cm	L/r	TIPO DE ACERO				L/r	TIPO DE ACERO					
			A-7	A-36	AH-55			A-7	A-36	AH-55			
			Peso nominal = 804 Kgs. Peso total = 2936 Kgs. Area total = 3712 cm²					Peso nominal = 1079 Kgs. Peso total = 3486 Kgs. Area total = 4436 cm²					
d x b	50	145	503	547	828	140	601	655	992				
	100	28.9	482	524	780	279	578	628	937				
<b>4x4</b>	150	43.4	457	495	721	419	550	595	869				
	200	57.8	429	461	651	559	517	557	789				
	250	72.3	396	424	571	698	480	514	699				
102 x 102	300	86.7	360	382	482	838	439	467	598				
	350	101.2	321	336	382	978	395	414	485				
Pl 10 x 89	400	115.6	278	285	292	1117	346	358	373				
	450	130.1	231	230	236	1257	293	295	295				
	500	144.5	187	187	187	1397	239	239	239				
	550	159.0	154	154	154	1536	197	197	197				
	600	173.4	130	130	130	1676	166	166	166				
	650	187.9	110	110	110	1816	141	141	141				
	700	—	—	—	—	1955	122	122	122				

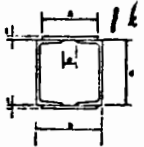
Dimensión	Altura en cm	L/r	TIPO DE ACERO				L/r	TIPO DE ACERO					
			A-7	A-36	AH-55			A-7	A-36	AH-55			
			Peso nominal = 804 Kgs. Peso total = 3378 Kgs. Area total = 4272 cm²					Peso nominal = 1079 Kgs. Peso total = 3928 Kgs. Area total = 4996 cm²					
d x b	50	149	578	629	952	144	677	737	1115				
	100	29.8	553	601	894	287	649	705	1051				
<b>4x4</b>	150	44.6	524	566	823	431	616	667	971				
	200	59.5	489	526	738	575	578	622	878				
	250	74.4	450	481	643	718	534	572	772				
102 x 102	300	89.3	407	430	535	862	486	516	653				
	350	104.2	359	375	413	1006	434	454	519				
Pl 13 x 89	400	119.0	307	314	317	1149	376	387	397				
	450	133.9	250	250	250	1293	314	314	314				
	500	148.8	203	203	203	1437	254	254	254				
	550	163.7	167	167	167	1580	210	210	210				
	600	178.6	141	141	141	1724	176	176	176				
	650	193.5	120	120	120	1868	150	150	150				



ALTOS HORNOS DE MEXICO, S. A.

**CAPACIDAD DE CARGA**  
**COLUMNAS COMPUESTAS DE 2 CANALES STANDARD**  
**Y 2 PLACAS SOLDADAS**

**2 CPS**  
**CARGA EN TONELADAS**



Dimensión	Altura en cm	L/r	TIPO DE ACERO			L/r	TIPO DE ACERO		
			A-7	A-36	AH-55		A-7	A-36	AH-55
			Peso nominal = 997 Kgs. Peso total = 3132 Kgs. Area total = 3968 cm²				Peso nominal = 1339 Kgs. Peso total = 3816 Kgs. Area total = 4846 cm²		
d x b	200	431	489	530	772	428	598	648	944
	250	53.9	467	503	717	535	571	616	878
<b>5x5</b>	300	64.7	442	475	656	642	541	581	805
	350	75.4	415	443	590	749	509	543	725
127 x 127	400	86.2	386	410	518	857	474	502	638
	450	97.0	355	373	440	964	436	459	543
	500	107.8	322	335	359	1071	396	412	444
Pl 6 x 114	550	118.5	287	293	297	1178	354	362	367
	600	129.3	249	249	249	1285	308	308	308
	650	140.1	212	212	212	1392	263	263	263
	700	150.9	183	183	183	1499	226	226	226
	750	161.6	159	159	159	1606	197	197	197
	800	172.4	140	140	140	1713	173	173	173
	850	183.2	124	124	124	1820	154	154	154
	900	194.0	111	111	111	1927	137	137	137

Dimensión	Altura en cm	L/r	TIPO DE ACERO			L/r	TIPO DE ACERO		
			A-7	A-36	AH-55		A-7	A-36	AH-55
			Peso nominal = 997 Kgs. Peso total = 3702 Kgs. Area total = 4706 cm²				Peso nominal = 1339 Kgs. Peso total = 4386 Kgs. Area total = 5584 cm²		
d x b	200	449	576	623	904	435	688	744	1083
	250	56.2	548	590	835	543	655	707	1005
<b>5x5</b>	300	67.4	516	554	759	652	620	666	919
	350	78.7	482	514	675	761	582	621	825
127 x 127	400	89.9	446	472	584	870	541	573	722
	450	101.1	407	426	484	978	497	521	610
	500	112.4	365	376	391	1087	449	466	496
Pl 10 x 114	550	123.6	320	324	323	1196	399	407	410
	600	134.8	272	272	272	1304	345	345	345
	650	146.1	232	232	232	1413	294	294	294
	700	157.3	200	200	200	1522	253	253	253
	750	168.5	174	174	174	1630	221	221	221
	800	179.8	153	153	153	1739	194	194	194
	850	191.0	135	135	135	1848	172	172	172
	900	—	—	—	—	1957	153	153	153

Dimensión	Altura en cm	L/r	TIPO DE ACERO			L/r	TIPO DE ACERO		
			A-7	A-36	AH-55		A-7	A-36	AH-55
			Peso nominal = 997 Kgs. Peso total = 4271 Kgs. Area total = 5420 cm²				Peso nominal = 1339 Kgs. Peso total = 4995 Kgs. Area total = 6298 cm²		
d x b	200	444	665	719	1045	447	771	834	1212
	250	55.6	633	682	967	559	734	791	1120
<b>5x5</b>	300	66.7	597	641	880	671	692	742	1019
	350	77.8	559	596	786	783	647	690	907
127 x 127	400	88.9	517	548	682	895	599	633	786
	450	100.0	473	496	570	1007	546	572	653
	500	111.1	426	440	461	1119	491	507	529
Pl 13 x 114	550	122.2	375	380	381	1230	431	437	437
	600	133.3	320	320	320	1342	367	367	367
	650	144.4	273	273	273	1454	313	313	313
	700	155.6	235	235	235	1566	270	270	270
	750	166.7	205	205	205	1678	235	235	235
	800	177.8	180	180	180	1790	206	206	206
	850	188.9	160	160	160	1902	183	183	183
	900	200.0	142	142	142	—	—	—	—

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ALTOS HORNOS DE MEXICO, S. A.





CAPACIDAD DE CARGA  
COLUMNAS COMPUESTAS DE 2 CANALES STANDARD  
Y 2 PLACAS SOLDADAS

□ 2CPS

CARGA EN TONELADAS

Dimensión	Altura en cm	L/r	TIPO DE ACERO			L/r	TIPO DE ACERO			L/r	TIPO DE ACERO		
			A-7	A-36	AH-55		A-7	A-36	AH-55		A-7	A-36	AH-55
		Peso nominal = 12.20 Kgs Peso total = 38.32 Kgs Area total = 48.54 cm <sup>2</sup>			Peso nominal = 15.63 Kgs Peso total = 45.18 Kgs Area total = 57.32 cm <sup>2</sup>			Peso nominal = 19.35 Kgs Peso total = 52.62 Kgs Area total = 66.86 cm <sup>2</sup>					
d x b	200	35.2	61.7	66.9	98.8	34.2	73.1	79.3	117.3	33.9	85.4	92.6	137.1
	250	44.0	59.6	64.5	93.8	42.8	70.8	76.6	111.7	42.4	82.7	89.5	130.6
	300	52.8	57.4	61.9	88.4	51.4	68.2	73.6	105.5	50.8	79.8	86.1	123.5
	350	61.6	55.0	59.1	82.5	59.9	65.5	70.4	98.8	59.3	76.6	82.4	115.8
	400	70.4	52.4	56.1	76.1	68.5	62.5	67.0	91.5	67.8	73.2	78.5	107.5
	450	79.2	49.6	52.8	69.2	77.1	59.4	63.3	83.8	76.3	69.6	74.3	98.5
	500	88.0	46.6	49.4	61.8	85.6	56.0	59.5	75.5	84.7	65.8	69.8	89.0
	550	96.8	43.5	45.4	51.0	94.2	52.5	55.3	66.6	93.2	61.7	65.1	78.9
	600	105.6	40.2	41.9	45.7	102.7	48.8	51.0	57.0	101.7	57.5	60.1	68.0
	650	114.4	36.8	37.9	38.9	111.3	44.9	46.4	48.6	110.2	53.0	54.9	57.8
	700	123.2	33.1	33.6	33.6	119.9	40.8	41.6	41.9	118.6	48.3	49.4	49.9
	750	132.0	29.2	29.2	29.2	128.4	36.5	36.5	36.5	127.1	43.3	43.4	43.4
	800	140.8	25.7	25.7	25.7	137.0	32.1	32.1	32.1	135.6	38.2	38.2	38.2
	850	149.6	22.8	22.8	22.8	145.5	28.4	28.4	28.4	144.1	33.8	33.8	33.8
	900	158.5	20.3	20.3	20.3	154.1	25.3	25.3	25.3	152.5	30.2	30.2	30.2
	950	167.3	18.2	18.2	18.2	162.7	22.7	22.7	22.7	161.0	27.1	27.1	27.1
	1000	176.1	16.4	16.4	16.4	171.2	20.5	20.5	20.5	169.5	24.4	24.4	24.4
	1050	184.9	14.9	14.9	14.9	179.8	18.6	18.6	18.6	178.0	22.2	22.2	22.2
	1100	193.7	13.6	13.6	13.6	188.4	17.0	17.0	17.0	186.4	20.2	20.2	20.2
	1150	—	—	—	—	196.9	15.5	15.5	15.5	194.9	18.5	18.5	18.5

Dimensión	Altura en cm	L/r	TIPO DE ACERO			L/r	TIPO DE ACERO			L/r	TIPO DE ACERO		
			A-7	A-36	AH-55		A-7	A-36	AH-55		A-7	A-36	AH-55
		Peso nominal = 12.20 Kgs Peso total = 41.0 Kgs Area total = 52.2 cm <sup>2</sup>			Peso nominal = 15.63 Kgs Peso total = 48.66 Kgs Area total = 61.72 cm <sup>2</sup>			Peso nominal = 19.35 Kgs Peso total = 56.10 Kgs Area total = 71.26 cm <sup>2</sup>					
d x b	200	35.8	67.1	72.8	107.4	35.0	78.5	85.2	125.8	34.4	90.8	98.5	145.7
	250	44.8	64.8	70.1	101.8	43.7	75.9	82.2	119.6	43.0	87.9	95.1	138.6
	300	53.8	62.3	67.2	95.7	52.4	73.1	78.9	112.7	51.6	84.7	91.4	130.9
	350	62.7	59.6	64.0	89.1	61.2	70.0	75.3	105.2	60.2	81.3	87.4	122.5
	400	71.7	56.7	60.6	81.9	69.9	66.8	71.5	97.2	68.8	77.6	83.1	113.4
	450	80.6	53.6	57.0	74.2	78.7	63.3	67.4	88.5	77.5	73.6	78.5	103.7
	500	89.6	50.3	53.2	65.9	87.4	59.6	63.1	79.3	86.1	69.5	73.7	93.3
	550	98.6	46.8	49.1	57.1	96.2	55.7	58.5	69.4	94.7	65.0	68.5	82.1
	600	107.5	43.1	44.8	48.1	104.9	51.5	53.7	58.9	103.3	60.4	63.1	70.2
	650	116.5	39.2	40.2	41.0	113.6	47.2	48.6	50.2	111.9	55.5	57.3	59.8
	700	125.4	35.1	35.4	35.3	122.4	42.6	43.2	43.3	120.5	50.4	51.3	51.5
	750	134.4	30.8	30.8	30.8	131.1	37.8	37.7	37.7	129.1	44.9	44.9	44.9
	800	143.4	27.0	27.0	27.0	139.9	33.1	33.1	33.1	137.7	39.5	39.5	39.5
	850	152.3	24.0	24.0	24.0	148.6	29.3	29.3	29.3	146.3	35.0	35.0	35.0
	900	161.3	21.4	21.4	21.4	157.3	26.2	26.2	26.2	154.9	31.2	31.2	31.2
	950	170.3	19.2	19.2	19.2	166.1	23.5	23.5	23.5	163.5	28.0	28.0	28.0
	1000	179.2	17.3	17.3	17.3	174.8	21.2	21.2	21.2	172.1	25.3	25.3	25.3
	1050	188.2	15.7	15.7	15.7	183.6	19.2	19.2	19.2	180.7	22.9	22.9	22.9
	1100	197.1	14.3	14.3	14.3	192.3	17.5	17.5	17.5	189.3	20.9	20.9	20.9
	1150	—	—	—	—	—	—	—	—	197.9	19.1	19.1	19.1

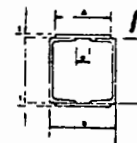


ALTOS HORNOS DE MEXICO, S. A.

CAPACIDAD DE CARGA  
COLUMNAS COMPUESTAS DE 2 CANALES STANDARD  
Y 2 PLACAS SOLDADAS

□ 2CPS

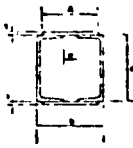
CARGA EN TONELADAS



Dimensión	Altura en cm	L/r	TIPO DE ACERO			L/r	TIPO DE ACERO			L/r	TIPO DE ACERO		
			A-7	A-36	AH-55		A-7	A-36	AH-55		A-7	A-36	AH-55
		Peso nominal = 12.20 Kgs Peso total = 45.27 Kgs Area total = 57.60 cm <sup>2</sup>			Peso nominal = 15.63 Kgs. Peso total = 52.13 Kgs. Area total = 66.38 cm <sup>2</sup>			Peso nominal = 19.35 Kgs. Peso total = 59.57 Kgs. Area total = 75.92 cm <sup>2</sup>					
d x b	200	36.7	72.8	78.9	116.3	35.5	84.3	91.4	134.9	35.0	96.6	104.7	154.7
	250	45.9	70.2	75.9	110.0	44.4	81.4	88.1	124.0	43.8	93.4	101.0	147.0
	300	55.0	67.4	72.6	103.2	53.3	74.3	84.5	120.5	52.5	89.9	97.0	138.5
	350	64.2	64.3	69.1	95.7	62.2	74.9	80.6	112.2	61.3	86.1	92.6	129.3
	400	73.4	61.0	65.2	87.6	71.0	71.3	76.3	103.4	70.1	82.1	87.9	119.4
	450	82.6	57.5	61.2	78.9	79.9	67.5	71.9	93.8	78.8	77.8	82.9	108.7
	500	91.7	53.8	56.8	69.5	88.8	63.4	67.1	83.6	87.6	73.2	77.5	97.3
	550	100.9	45.7	47.3	49.9	97.6	54.5	56.7	61.4	96.5	63.3	65.9	72.2
	600	110.1	36.7	36.7	36.7	106.6	44.7	45.2	45.1	105.1	52.3	53.0	53.0
	650	119.3	28.1	28.1	28.1	115.7	34.5	34.5	34.5	114.1	40.6	40.6	40.6
	700	128.4	22.2	22.2	22.2	124.3	27.3	27.3	27.3	122.6	32.1	32.1	32.1
	750	137.5	18.0	18.0	18.0	133.4	22.1	22.1	22.1	131.7	26.0	26.0	26.0
	800	146.8	16.3	16.3	16.3	142.5	20.0	20.0	20.0	140.8	23.6	23.6	23.6
	850	155.9	—	—	—	151.6	18.3	18.3	18.3	149.9	21.5	21.5	21.5

Dimensión	Altura en cm	L/r	TIPO DE ACERO			L/r	TIPO DE ACERO			L/r	TIPO DE ACERO		
			A-7	A-36	AH-55		A-7	A-36	AH-55		A-7	A-36	AH-55
		Peso nominal = 12.20 Kgs Peso total = 52.23 Kgs Area total = 66.34 cm <sup>2</sup>			Peso nominal = 15.63 Kgs Peso total = 59.09 Kgs Area total = 75.12 cm <sup>2</sup>			Peso nominal = 19.35 Kgs Peso total = 66.53 Kgs Area total = 84.66 cm <sup>2</sup>					
d x b	200	37.8	83.5	90.5	133.1	36.6	95.0	103.0	151.8	36.0	107.3	116.4	171.6
	250	47.3	80.4	86.9	125.6	45.7	91.7	99.1	143.7	45.0	103.6	112.1	162.7
	300	56.7	77.0	82.9	117.3	54.8	88.0	94.8	134.7	54.0	99.6	107.4	152.9
	350	66.2	73.3	78.6	108.3	64.0	84.0	90.2	125.0	62.9	95.2	102.3	142.2
	400	75.6	69.3	74.0	98.5	73.1	79.7	85.2	114.5	71.9	90.5	96.8	130.6
	450	85.1	65.1	69.1	88.0	82.3	75.2	79.9	103.2	80.9	85.5	91.0	118.2
	500	94.5	60.6	63.9	76.7	91.4	70.4	74.3	91.1	89.9	80.2	84.8	105.0
	550	103.9	56.8	59.8	71.9	100.7	59.8	62.0	65.6	98.9	68.7	71.3	76.3
	600	113.4	50.8	52.4	54.1	109.7	48.1	48.2	48.2	107.9	55.8	56.2	56.1
	650	123.2	39.8	39.8	39.8	118.8	36.9	36.9	36.9	116.9	42.9	42.9	42.9
	700	132.3	30.5	30.5	30.5	128.0	29.1	29.1	29.1	125.9	33.9	33.9	33.9
	750	141.4	24.1	24.1	24.1	137.3	23.6	23.6	23.6	135.0	27.5	27.5	27.5
	800	150.5	17.7	17.7	17.7	146.5	21.4	21.4	21.4	144.1	24.9	24.9	24.9
	850	159.6	—	—	—	155.6	19.1	19.1	19.1	153.2	22.7	22.7	22.7

Dimensión	Altura en cm	L/r	TIPO DE ACERO			L/r	TIPO DE ACERO			L/r	TIPO DE ACERO		
			A-7	A-36	AH-55		A-7	A-36	AH-55		A-7	A-36	AH-55
		Peso nominal = 12.20 Kgs Peso total = 59.19 Kgs Area total = 75.24 cm <sup>2</sup>			Peso nominal = 15.63 Kgs. Peso total = 66.05 Kgs. Area total = 84.02 cm <sup>2</sup>			Peso nominal = 19.35 Kgs. Peso total = 73.49 Kgs. Area total = 93.56 cm <sup>2</sup>					
d x b	200	38.5	94.5	102.4									



**CAPACIDAD DE CARGA**  
**COLUMNAS COMPUESTAS DE 2 CANALES STANDARD**  
**Y 2 PLACAS SOLDADAS**

**2 CPS**

**CARGA EN TONELADAS**

Dimensión	Altura en cm	L/r	TIPO DE ACERO			L/r	TIPO DE ACERO			L/r	TIPO DE ACERO		
			A-7	A-36	AH-35		A-7	A-36	AH-35		A-7	A-36	AH-35
d x b	200	29.7	Peso nominal = 14.58 Kgs Peso total = 45.60 Kgs. Area total = 57.74 cm <sup>2</sup>			29.0	Peso nominal = 18.23 Kgs Peso total = 52.90 Kgs. Area total = 67.16 cm <sup>2</sup>			28.6	Peso nominal = 21.95 Kgs. Peso total = 60.34 Kgs. Area total = 76.70 cm <sup>2</sup>		
			74.8	81.2	120.9		87.2	94.7	141.1		99.7	108.3	161.5
			70.8	76.6	111.3		82.7	89.5	130.2		94.7	102.4	149.3
300	44.5	70.8	76.6	111.3	43.5	82.7	89.5	130.2	42.9	94.7	102.4	149.3	
400	59.3	66.1	71.2	100.0	58.1	77.4	83.4	117.5	57.2	88.8	95.6	135.1	
500	74.2	60.9	65.1	87.0	72.6	71.5	76.5	103.0	71.5	82.2	87.9	118.9	
600	89.0	55.1	58.3	72.5	87.1	65.0	68.9	86.7	85.8	74.9	79.4	100.7	
700	103.9	48.7	50.8	56.2	101.6	57.8	60.5	68.5	100.1	66.9	70.0	80.4	
800	118.7	41.7	42.6	43.0	116.1	50.0	51.3	52.3	114.4	58.1	59.8	61.5	
900	133.5	34.0	34.0	34.0	130.6	41.4	41.3	41.3	128.8	48.6	48.6	48.6	
1000	148.4	27.5	27.5	27.5	145.1	33.5	33.5	33.5	143.1	39.3	39.3	39.3	
1100	163.2	22.8	22.8	22.8	159.7	27.7	27.7	27.7	157.4	32.5	32.5	32.5	
1200	178.0	19.1	19.1	19.1	174.2	23.2	23.2	23.2	171.7	27.3	27.3	27.3	
1300	192.9	16.3	16.3	16.3	188.7	19.8	19.8	19.8	186.0	23.3	23.3	23.3	
1400	—	—	—	—	—	—	—	—	200.3	20.1	20.1	20.1	

d x b	200	Peso nominal = 14.58 Kgs Peso total = 49.72 Kgs Area total = 63.00 cm <sup>2</sup>			29.6	Peso nominal = 18.23 Kgs Peso total = 57.02 Kgs Area total = 72.44 cm <sup>2</sup>			29.1	Peso nominal = 21.95 Kgs Peso total = 64.46 Kgs Area total = 81.96 cm <sup>2</sup>		
		81.4	88.1	131.5		93.9	101.9	151.8		106.4	115.6	172.1
		76.9	83.2	120.7		88.9	96.1	139.7		100.9	109.1	158.9
300	45.5	76.9	83.2	120.7	44.4	88.9	96.1	139.7	43.6	100.9	109.1	158.9
400	60.6	71.7	77.1	107.9	59.2	83.1	89.4	125.6	58.1	94.5	101.7	143.3
500	75.8	65.8	70.2	93.4	74.0	76.5	81.7	109.5	72.7	87.2	93.3	125.5
600	90.9	59.2	62.6	77.0	88.8	69.2	73.3	91.3	87.2	79.2	83.9	105.6
700	106.1	52.0	54.1	58.8	103.6	61.2	63.9	70.9	101.7	70.4	73.7	83.3
800	121.2	44.1	44.9	45.0	118.3	52.5	53.7	54.3	116.3	60.8	62.4	63.6
900	136.4	35.6	35.6	35.6	133.1	42.9	42.9	42.9	130.8	50.4	50.3	50.3
1000	151.5	28.8	28.8	28.8	147.9	34.8	34.8	34.8	145.3	40.7	40.7	40.7
1100	166.7	23.8	23.8	23.8	162.7	28.7	28.7	28.7	159.9	33.7	33.7	33.7
1200	181.8	20.0	20.0	20.0	177.5	24.1	24.1	24.1	174.4	28.3	28.3	28.3
1300	197.0	17.1	17.1	17.1	192.5	20.6	20.6	20.6	189.0	24.1	24.1	24.1

d x b	200	Peso nominal = 14.58 Kgs Peso total = 53.82 Kgs. Area total = 68.24 cm <sup>2</sup>			30.1	Peso nominal = 18.23 Kg Peso total = 61.12 Kg Area total = 77.66 cm <sup>2</sup>			29.6	Peso nominal = 21.95 Kg Peso total = 68.56 Kg Area total = 87.20 cm <sup>2</sup>		
		88.0	95.6	142.0		100.5	109.1	162.3		113.0	122.7	182.7
		83.0	89.8	129.9		95.0	102.7	149.1		106.9	115.7	168.1
300	61.8	72.2	83.0	115.7	60.2	88.6	95.3	133.6	59.3	99.9	107.5	151.1
400	77.3	70.6	75.3	99.5	75.2	81.4	86.9	115.8	74.1	92.0	98.3	131.6
500	92.7	63.3	66.7	81.1	90.2	73.4	77.6	95.9	88.9	83.2	88.1	109.7
600	108.2	55.2	57.3	61.2	105.3	64.6	67.3	73.6	103.7	73.6	76.8	85.1
700	123.6	46.3	46.9	46.9	120.3	55.0	56.0	56.3	118.5	63.1	64.5	65.2
800	139.1	37.0	37.0	37.0	135.3	44.5	44.5	44.5	133.3	51.5	51.5	51.5
900	154.6	30.0	30.0	30.0	150.4	36.1	36.1	36.1	148.1	41.7	41.7	41.7
1000	170.0	24.8	24.8	24.8	165.4	29.8	29.8	29.8	163.0	34.5	34.5	34.5
1100	185.5	20.8	20.8	20.8	180.5	25.0	25.0	25.0	177.8	29.0	29.0	29.0
1200	200.9	17.7	17.7	17.7	195.5	21.3	21.3	21.3	192.6	24.7	24.7	24.7

**CAPACIDAD DE CARGA**  
**COLUMNAS COMPUESTAS DE 2 CANALES STANDARD**  
**Y 2 PLACAS SOLDADAS**

**2 CPS**

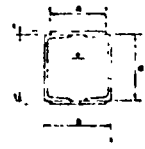
**CARGA EN TONELADAS**



Dimensión	Altura en cm	L/r	TIPO DE ACERO			L/r	TIPO DE ACERO			L/r	TIPO DE ACERO		
			A-7	A-36	AH-35		A-7	A-36	AH-35		A-7	A-36	AH-35
d x b	200	31.9	Peso nominal = 14.58 Kgs Peso total = 62.04 Kgs Area total = 78.72 cm <sup>2</sup>			31.0	Peso nominal = 18.23 Kgs. Peso total = 69.34 Kgs. Area total = 88.14 cm <sup>2</sup>			30.4	Peso nominal = 21.95 Kgs Peso total = 76.78 Kgs Area total = 97.64 cm <sup>2</sup>		
			101.2	109.9	163.0		113.7	123.4	181.4		126.2	137.1	203.8
			101.2	109.9	163.0		113.7	123.4	181.4		126.2	137.1	203.8
300	47.8	95.2	102.8	148.4	46.4	107.2	115.9	167.8	45.6	119.2	128.9	186.9	
400	63.8	88.1	94.6	131.2	61.9	99.6	107.1	149.3	60.8	111.1	119.5	167.1	
500	79.7	80.1	85.3	111.5	77.4	91.1	97.2	125.3	76.0	101.9	108.7	144.4	
600	95.7	71.3	75.0	89.2	92.9	81.6	86.1	104.5	91.2	91.6	96.9	118.9	
700	111.6	61.5	63.5	66.3	108.4	71.2	73.9	78.8	106.4	80.4	83.7	90.6	
800	127.6	50.7	50.8	50.8	123.8	59.7	60.4	60.3	121.6	68.1	69.2	69.4	
900	143.5	40.1	40.1	40.1	139.3	47.7	47.7	47.7	136.8	54.8	54.8	54.8	
1000	159.5	32.5	32.5	32.5	154.8	38.6	38.6	38.6	152.0	44.4	44.4	44.4	
1100	175.4	26.9	26.9	26.9	170.3	31.9	31.9	31.9	167.2	36.7	36.7	36.7	
1200	191.4	22.6	22.6	22.6	185.8	26.8	26.8	26.8	182.4	30.8	30.8	30.8	
1300	—	—	—	—	—	—	—	—	197.6	26.3	26.3	26.3	

d x b	200	Peso nominal = 14.58 Kgs Peso total = 70.76 Kgs Area total = 89.20 cm <sup>2</sup>			31.8	Peso nominal = 18.23 Kgs Peso total = 77.56 Kgs Area total = 98.62 cm <sup>2</sup>			31.2	Peso nominal = 21.95 Kgs Peso total = 85.00 Kgs Area total = 108.16 cm <sup>2</sup>		
		114.4	124.1	184.0		126.9	137.7	204.4		139.4	151.4	224.9
		114.4	124.1	184.0		126.9	137.7	204.4		139.4	151.4	224.9
300	49.0	107.3	115.9	166.9	47.7	119.3	128.9	186.1	46.7	131.4	142.0	205.5
400	65.4	99.0	106.3	146.6	63.6	110.5	118.7	164.7	62.3	122.0	131.1	182.7
500	81.7	89.6	95.3	123.4	79.5	100.6	107.1	141.1	77.9	111.4	118.9	156.6
600	98.0	79.2	83.1	97.1	95.4	89.5	94.2	112.3	93.5	99.7	105.1	127.1
700	114.4	67.6	69.6	71.6	111.3	77.3	79.9	83.6	109.0	67.7	70.0	75.5
800	130.7	54.9	54.8	54.8	127.2	63.9	64.0	64.0	124.6	72.5	73.3	73.1
900	147.1	43.3	43.3	43.3	143.1	50.6	50.6	50.6	140.2	57.8	57.8	57.8
1000	163.4	35.1	35.1	35.1	159.0	41.0	41.0	41.0	155.8	46.8	46.8	46.8
1100	179.7	29.0	29.0	29.0	174.9	33.9	33.9	33.9	171.3	38.7	38.7	38.7
1200	196.1	24.4	24.4	24.4	190.8	28.5	28.5	28.5	186.9	32.5	32.5	32.5

d x b	200	Peso nominal = 14.58 Kgs Peso total = 78.48 Kgs Area total = 99.68 cm <sup>2</sup>			32.5	Peso nominal = 18.23 Kgs. Peso total = 85.78 Kgs. Area total = 109.10 cm <sup>2</sup>			31.8	Peso nominal = 21.95 Kgs Peso total = 93.22 Kgs Area total = 118.64 cm <sup>2</sup>		
		127.5	138.3	204.8		140.0	152.0	225.3		152.6	165.7	245.9
		127.5	138.3	204.8		140.0	152.0	225.3		152.6	165.7	245.9
300	50.2	119.3	128.8	185.0	48.7	131.4	142.0	205.5	47.7	143.5	155.1	223.9
400	66.9	109.7	117.7	161.6	64.9	121.4	130.3	183.0	63.6	132.9	142.8	198.1
500	83.6	98.8	105.0	134.7	81.2	110.0	117.1	152.0	79.5	120.0	128.9	186.6
600	100.3	86.7	90.9	104.1	97.4	97.4	102.3	130.1	95.4	107.7	113.3	135.1
700	117.1	73.4	75.2	76.4	113.6	83.4	85.9	85.7	111.3	93.0	96.1	100.6
800	133.8	58.5	58.5	58.5	129.9	68.0	67.9	67.9	127.2	76.9	77.0	77.0
900	150.5	46.2	46.2	46.2	146.1	53.7	53.7	53.7	143.1	60.8	60.8	60.8
1000	167.2	37.4	37.4	37.4	162.3	43.5	43.5	43.5	159.0	49.3	49.3	49.3
1100	183.9	30.9	30.9	30.9	178.6	35.9	35.9	35.9	174.9	40.7	40.7	40.7
1200	200.											



**CAPACIDAD DE CARGA**  
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**Y 2 PLACAS SOLDADAS**

2 C P S

CARGA EN TONELADAS

Dimensiones	Altura en cm	TIPO DE ACERO			TIPO DE ACERO			TIPO DE ACERO					
		L/r	A-7	A-36	AH-55	L/r	A-7	A-36	AH-55	L/r	A-7	A-36	AH-55
d x b	200	Peso nominal = 17 11 Kgs. Peso total = 53.20 Kgs. Area total = 67.56 cm <sup>2</sup>			Peso nominal = 20 46 Kgs. Peso total = 59 90 Kgs. Area total = 76 08 cm <sup>2</sup>			Peso nominal = 27 90 Kgs. Peso total = 74 78 Kgs. Area total = 95 04 cm <sup>2</sup>					
		25.6	88.7	96.4	144.2	25.1	100.0	108.7	162.7	24.6	125.1	136.0	203.8
8x8	300	35.5	84.8	91.9	135.1	37.7	95.8	103.9	152.8	36.9	120.0	130.1	191.6
	400	51.3	90.4	86.8	124.4	50.3	91.0	98.3	141.1	49.3	114.2	123.3	177.5
203 x 203	500	64.1	75.5	81.1	112.3	62.8	85.6	92.0	127.9	61.6	107.6	115.7	161.5
	600	76.9	70.0	74.7	98.9	75.4	79.6	85.0	113.2	73.9	100.4	107.3	143.7
Pl 6 x 190	700	89.7	64.1	67.8	84.0	87.9	73.1	77.5	97.0	86.2	92.5	98.1	124.1
	800	102.6	57.6	60.2	67.4	100.5	66.1	69.2	79.2	98.5	84.0	88.2	102.6
	900	115.4	50.7	52.1	53.3	113.1	58.5	60.3	62.5	110.8	74.8	77.4	81.2
	1000	128.2	43.1	43.2	43.2	125.6	50.3	50.7	50.6	123.2	65.0	65.8	65.8
	1100	141.0	35.7	35.7	35.7	138.2	41.8	41.8	41.8	135.5	54.4	54.4	54.4
	1200	153.8	30.0	30.0	30.0	150.8	35.1	35.1	35.1	147.8	45.7	45.7	45.7
	1300	166.7	25.5	25.5	25.5	163.3	30.0	30.0	30.0	160.1	38.9	38.9	38.9
	1400	179.5	22.0	22.0	22.0	175.9	25.8	25.8	25.8	172.4	33.6	33.6	33.6
	1500	192.3	19.2	19.2	19.2	188.4	22.5	22.5	22.5	184.7	29.2	29.2	29.2
	1600	—	—	—	—	—	—	—	—	197.0	25.7	25.7	25.7

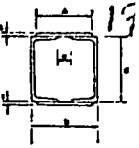
Dimensiones	Altura en cm	TIPO DE ACERO			TIPO DE ACERO			TIPO DE ACERO					
		L/r	A-7	A-36	AH-55	L/r	A-7	A-36	AH-55	L/r	A-7	A-36	AH-55
d x b	200	Peso nominal = 17 11 Kgs. Peso total = 57 94 Kgs. Area total = 73 60 cm <sup>2</sup>			Peso nominal = 20 46 Kgs. Peso total = 64 64 Kgs. Area total = 82 12 cm <sup>2</sup>			Peso nominal = 27 90 Kgs. Peso total = 79 52 Kgs. Area total = 101 08 cm <sup>2</sup>					
		26.2	96.4	104.8	156.7	25.6	107.8	117.1	175.3	25.0	132.9	144.5	216.3
8x8	300	39.3	92.1	99.8	146.4	38.5	103.1	111.8	164.2	37.5	127.4	138.1	203.2
	400	52.4	87.2	94.1	134.5	51.3	97.8	105.5	151.2	50.0	121.0	130.7	187.8
203 x 203	500	65.4	81.7	87.6	120.9	64.1	91.8	98.5	136.5	62.5	113.9	122.4	170.4
	600	78.5	75.5	80.5	105.8	76.9	85.1	90.8	120.2	75.0	106.1	113.3	151.1
Pl 8 x 190	700	91.6	68.8	72.7	89.0	89.7	77.9	82.4	102.1	87.5	97.5	103.3	129.7
	800	104.7	61.6	64.2	70.5	102.6	70.1	73.2	82.0	100.0	88.2	92.4	106.2
	900	117.8	53.7	55.0	55.7	115.4	61.6	63.3	64.8	112.5	78.2	80.7	83.9
	1000	130.9	45.2	45.1	45.1	128.2	52.4	52.5	52.5	125.0	67.4	68.1	67.9
	1100	144.0	37.3	37.3	37.3	141.0	43.4	43.4	43.4	137.5	56.1	56.1	56.1
	1200	157.1	31.3	31.3	31.3	153.8	36.4	36.4	36.4	150.0	47.2	47.2	47.2
	1300	170.2	26.7	26.7	26.7	166.7	31.0	31.0	31.0	162.5	40.2	40.2	40.2
	1400	183.2	23.0	23.0	23.0	179.5	26.8	26.8	26.8	175.0	34.7	34.7	34.7
	1500	196.3	20.0	20.0	20.0	192.3	23.3	23.3	23.3	187.5	30.2	30.2	30.2
	1600	—	—	—	—	—	—	—	—	200.0	26.5	26.5	26.5

Dimensiones	Altura en cm	TIPO DE ACERO			TIPO DE ACERO			TIPO DE ACERO					
		L/r	A-7	A-36	AH-55	L/r	A-7	A-36	AH-55	L/r	A-7	A-36	AH-55
d x b	200	Peso nominal = 17 11 Kgs. Peso total = 62 68 Kgs. Area total = 79 66 cm <sup>2</sup>			Peso nominal = 20 46 Kgs. Peso total = 69 38 Kgs. Area total = 88 18 cm <sup>2</sup>			Peso nominal = 27 90 Kgs. Peso total = 84 26 Kgs. Area total = 107 14 cm <sup>2</sup>					
		26.7	104.2	113.2	169.2	26.1	115.6	125.6	187.8	25.4	140.7	152.9	228.9
8x8	300	40.0	99.5	107.7	157.8	39.1	110.5	119.7	175.6	38.2	134.7	146.0	214.5
	400	53.3	94.0	101.3	144.5	52.2	104.6	112.6	161.3	50.9	127.8	138.0	197.8
203 x 203	500	67.7	87.8	94.2	129.4	65.2	94.0	105.1	145.2	63.6	120.0	128.9	178.9
	600	80.0	91.0	86.2	112.5	78.2	90.7	96.6	127.1	76.1	111.5	119.0	157.4
Pl 10 x 190	700	93.3	73.5	77.5	93.8	91.3	82.7	87.3	107.2	89.1	92.2	108.1	144.5
	800	106.7	65.4	68.0	73.5	104.3	74.0	77.2	85.1	101.8	102.0	96.3	108.8
	900	120.0	56.6	57.7	59.1	117.3	64.7	66.2	67.2	114.5	81.1	83.5	85.8
	1000	133.3	47.0	47.0	47.0	130.4	54.6	54.5	54.5	127.2	69.4	69.5	69.5
	1100	146.7	39.9	39.9	39.9	143.4	45.0	45.0	45.0	139.9	57.4	57.4	57.4
	1200	160.0	32.7	32.7	32.7	156.5	37.8	37.8	37.8	152.7	48.3	48.3	48.3
	1300	173.3	27.8	27.8	27.8	169.5	32.2	32.2	32.2	165.4	41.1	41.1	41.1
	1400	186.7	24.0	24.0	24.0	182.5	27.8	27.8	27.8	178.1	35.5	35.5	35.5
	1500	200.0	20.9	20.9	20.9	195.6	24.2	24.2	24.2	190.8	30.9	30.9	30.9

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2 C P S

CARGA EN TONELADAS



Dimensiones	Altura en cm	TIPO DE ACERO			TIPO DE ACERO			TIPO DE ACERO					
		L/r	A-7	A-36	AH-55	L/r	A-7	A-36	AH-55	L/r	A-7	A-36	AH-55
d x b	200	Peso nominal = 17 11 Kgs. Peso total = 72 16 Kgs. Area total = 92 74 cm <sup>2</sup>			Peso nominal = 20 46 Kgs. Peso total = 78 86 Kgs. Area total = 100 26 cm <sup>2</sup>			Peso nominal = 27 90 Kgs. Peso total = 93 74 Kgs. Area total = 119 22 cm <sup>2</sup>					
		27.7	120.9	131.4	196.1	26.8	131.1	142.4	212.8	26.1	156.2	169.8	253.9
8x8	300	41.5	115.1	124.6	182.1	40.3	125.0	135.4	198.3	39.1	149.4	161.8	237.4
	400	55.3	108.4	116.8	165.7	53.7	118.1	127.3	181.4	52.2	141.4	152.6	218.1
203 x 203	500	69.2	100.8	108.0	147.1	67.1	110.2	118.2	162.2	65.2	132.4	142.2	196.3
	600	83.0	92.4	98.2	126.3	80.5	101.5	108.1	140.7	78.2	122.6	130.7	171.9
Pl 13 x 190	700	96.8	83.2	87.4	103.1	94.0	92.0	97.0	116.9	91.3	111.8	118.1	144.9
	800	110.7	73.2	75.7	79.5	107.4	81.7	84.9	91.3	104.3	100.1	104.4	115.1
	900	124.5	62.3	62.9	62.8	120.8	70.6	71.8	72.1	117.3	87.4	89.6	90.9
	1000	138.3	50.9	50.9	50.9	134.2	58.4	58.4	58.4	130.4	73.8	73.6	73.6
	1100	152.1	42.1	42.1	42.1	147.7	48.3	48.3	48.3	143.4	60.9	60.9	60.9
	1200	166.0	35.3	35.3	35.3	161.1	40.6	40.6	40.6	156.5	51.1	51.1	51.1
	1300	179.8	30.1	30.1	30.1	174.5	34.6	34.6	34.6	169.5	43.6	43.6	43.6
	1400	193.6	26.0	26.0	26.0	187.9	29.8	29.8	29.8	182.5	37.6	37.6	37.6
	1500	—	—	—	—	—	—	—	—	195.6	32.7	32.7	32.7

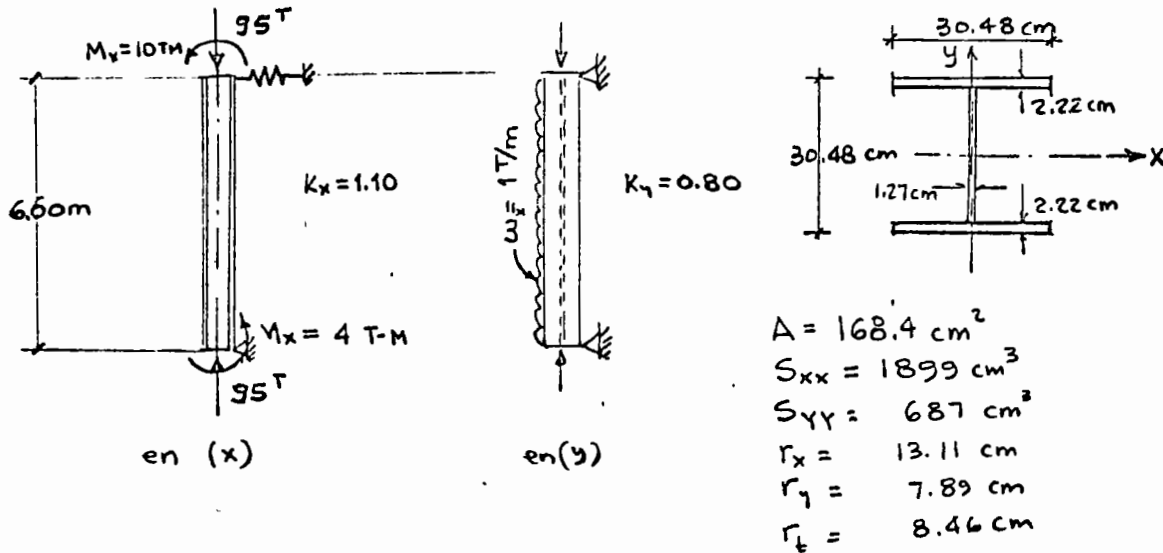
Dimensiones	Altura en cm	TIPO DE ACERO			TIPO DE ACERO			TIPO DE ACERO					
		L/r	A-7	A-36	AH-55	L/r	A-7	A-36	AH-55	L/r	A-7	A-36	AH-55
d x b	200	Peso nominal = 17 11 Kgs. Peso total = 81 66 Kgs. Area total = 103 84 cm <sup>2</sup>			Peso nominal = 20 46 Kgs. Peso total = 88 36 Kgs. Area total = 112 36 cm <sup>2</sup>			Peso nominal = 27 90 Kgs. Peso total = 103 24 Kgs. Area total = 131 32 cm <sup>2</sup>					
		28.2	135.2	146.8	219.0	27.5	146.6	159.2	237.7	26.7	171.8	186.7	279.0
8x8	300	42.3	128.5	139.1	202.9	41.3	139.6	151.1	220.9	40.0	163.9	177.6	260.2
	400	56.4	120.7	130.0	184.0	55.1	131.4	141.7	201.1	53.3	154.9	167.1	238.2
203 x 203	500	70.5	111.9	119.8	162.5	68.9	122.3	131.0	178.7	66.7	144.7	155.2	213.3
	600	84.6	102.2	108.5	138.5	82.6	112.2	119.2	153.7	80.0	133.5	142.1	185.5
Pl 16 x 190	700	98.7	91.6	96.1	111.7	96.4	101.1	106.3	125.8	93.3	121.2	127.7	154.6
	800	112.8	80.1	82.6	85.6	110.2	89.1	92.2	97.2	106.7	107.8	112.1	121.2
	900	126.9	67.5	67.7	67.7	124.0	76.0	76.9	76.8	120.0	93.3	95.2	95.8
	1000	141.0	54.8	54.8	54.8	137.7	62.2	62.2	62.2	133.3	77.6	77.6	77.6
	1100	155.1	45.3	45.3	45.3	151.5	51.4	51.4	51.4	146.7	64.1	64.1	64.1
	1200	169.3	38.1	38.1	38.1	165.3	43.2	43.2	43.2	160.0	53.9	53.9	53.9
	1300	183.4	32.4	32.4	32.4	179.1	36.8	36.8	36.8	173.3	45.9	45.9	45.9
	1400	197.5	28.0	28.0	28.0	192.8	31.7	31.7	31.7	186.7	39.6	39.6	39.6
	1500	—	—	—	—	—	—	—	—	200.0	34.5	34.5	34.5

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## EJEMPLO 2.

Revise la columna de la figura, de acuerdo a las especificaciones AISC, 1969. El acero es ASTM-A-529-62 ( $F_y = 42 \text{ ksi} = 2950 \text{ Kg/cm}^2$ )



Solución:

$$f_a = \frac{95000}{168.4} = 564.1 \text{ Kg/cm}^2$$

$$\frac{K_x l}{r_x} = \frac{1.1 \times 660}{13.11} = 50.3 \quad \rightarrow F'_e = 4145 \text{ Kg/cm}^2 \text{ (tabla 6)}$$

$$\frac{K_y l_y}{r_y} = \frac{0.8 \cdot 600}{7.89} = 60.8 \quad (\text{rige para } F_a) \text{ (tabla 3)} \rightarrow F_2 = 1384 \text{ Kg/cm}^2$$

$$\frac{f_a}{F_2} = \frac{564.1}{1384} = 0.408 > 0.15 \quad \therefore \text{se emplea la ecuación (17)}$$

Revisión para los momentos en el apoyo superior:

$$M_x = 10 \text{ T-M}$$

$$M_y = 0$$

$$P = 95 \text{ T}$$

$$f_{bx} = \frac{10 \times 10^5}{1899} = 527 \text{ Kg/cm}^2$$

Revisando si es compacta

$$\frac{b}{2t} = \frac{30.48}{2 \times 2.22} = 6.86 < 8.1 \quad \therefore \checkmark$$

$$\frac{d}{t} = \frac{30.48 - 2 \times 2.22}{1.27} = 20.5 < 39.7 \quad \therefore \checkmark$$

$$\left( \frac{f_a}{F_y} = \frac{564.1}{2950} = 0.191 > 0.16 \right)$$

$$l_b = 660 > 11.7 \times 30.48 = 357 \text{ cm} \quad \therefore \text{no es compacta}$$

$$C_b = 1.75 + 1.05 \left( -\frac{4}{10} \right) + 0.3 \left( \frac{4}{10} \right)^2 = 1.378$$

$$53 \sqrt{C_b} = 62 ; \quad 119 \sqrt{C_b} = 140$$

$$\frac{l}{r_t} = \frac{660}{8.46} = 78.0$$

Como  $62 < 78 < 140$   $F_{bx} = 1970 - \frac{\left(\frac{l}{r_t}\right)^2}{12.4 C_b}$  (AISC 1.5.1.4.6a)

$$\therefore F_{bx} = 1970 - \frac{78^2}{12.4 \cdot 1.378} = 1614 \text{ Kg/cm}^2$$

$$C_{mx} = 0.85 \quad (\text{por tener desplazamientos})$$

Aplicando (18)

$$0.408 + \frac{0.85 \cdot 527}{\left(1 - \frac{564.1}{4145}\right) 1614} = 0.408 + 0.321 = 0.729 < 1.0 \quad \therefore \checkmark$$

En el apoyo, con ecuación (14)  $(0.6 F_y = 0.6 \cdot 2950 = 1770 \text{ Kg/cm}^2)$

$$\frac{564.1}{1770} + \frac{527}{1614} = 0.319 + 0.326 = 0.645 < 1.0 \quad \therefore \text{correcta}$$

Revisando en el  $\bar{c}$ .

$$M_x = \frac{10 - (-4)}{6.60} \times 3.30 - 4.0 = 3.0 \text{ T-m}$$

$$M_y = \frac{1 \cdot \overline{6.60}^2}{8} = 5.445 \text{ T-m}$$

$$P = 95 \text{ T} \quad ; \quad f_a = 564.1 \text{ Kg/cm}^2 \quad ; \quad F_a = 1384 \text{ Kg/cm}^2 \quad ; \quad \frac{f_a}{F_a} = 0.408$$

$$f_{bx} = \frac{3.0 \cdot 10^5}{1899} = 158 \text{ Kg/cm}^2 \quad ; \quad F'_{ey} = 2836 \text{ Kg/cm}^2 \quad (\text{Para } \frac{K_y L_y}{r_y} = 60.8, \text{Tabla 7})$$

$$f_{by} = \frac{5.445 \cdot 10^5}{687} = 793 \text{ Kg/cm}^2 \quad ; \quad F'_{ex} = 4145 \text{ Kg/cm}^2 \quad (\text{Para } \frac{K_x L_x}{r_x} = 50.3, \text{Tabla 7})$$

$$F_{bx} = 1614 \text{ Kg/cm}^2 \quad ; \quad F_{by} = 0.75 F_y \quad (\text{AISC 1.5.1.4.3}) = 2213 \text{ Kg/cm}^2$$

$$C_{mx} = 0.85 \quad ; \quad C_{my} = 1 + \psi \frac{f_a}{F'_{ey}} \quad ; \quad \psi = 0 \quad \therefore C_m = 1 \quad (\text{Ver tabla 7})$$

Aplicando (18)

$$0.408 + \frac{0.85 \cdot 158}{\left(1 - \frac{564.1}{4145}\right) 1614} + \frac{1 \cdot 793}{\left(1 - \frac{564.1}{2836}\right) 2213} = 0.408 + 0.096 + 0.447 = 0.951 < 1.0 \quad \therefore \checkmark$$

Aplicando (14), para revisar adicionalmente en el apoyo superior

$$\frac{564.1}{1770} + \frac{158}{1614} + \frac{793}{2213} = 0.319 + 0.098 + 0.358 = 0.775 < 1.0 \quad \therefore \checkmark$$

Conclusión: La columna se acepta.

soldaduras están proporcionadas en forma tal que evitan la excentricidad, paralelamente al patín mayor. El acero del ángulo es del tipo ASTM-A-36. Ver Fig. 6.

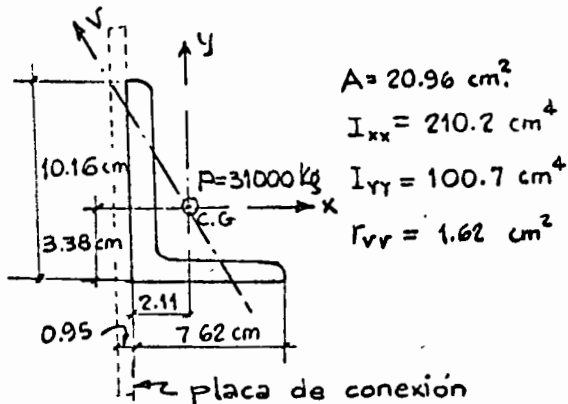


fig. 6

Solución:

- a) esfuerzos de tensión: (suponiendo distribución uniforme de los esfuerzos).

$$f_t = \frac{31000}{20.96} = 1479 \text{ Kg/cm}^2 < 0.6 F_Y = 1520 \text{ Kg/cm}^2, \therefore \checkmark$$

- b) esfuerzos de flexión

$$\text{excentricidad } e_x = 2.11 + \frac{1}{2} \times 0.95 = 2.585 \text{ cm.}$$

$$M_{yy} = 31000 \times 2.585 = 80135 \text{ Kg-cm.}$$

$$f_{bxx} = \frac{80135}{100.7} \times (7.62 - 2.11) = 4385 \text{ Kg/cm}^2 \gg F_Y = 2530 \text{ Kg/cm}^2$$

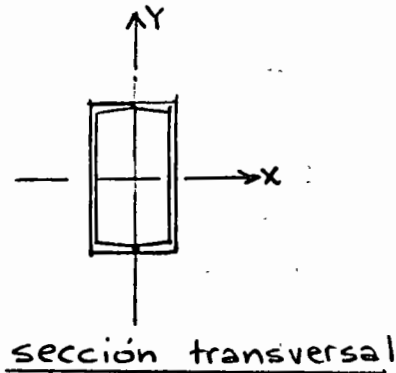
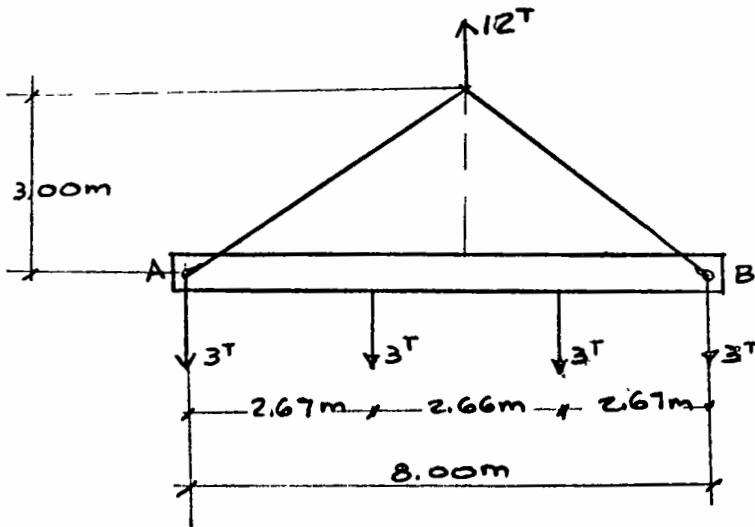
Se vé que los esfuerzos debidos a la flexión (tensión o compresión) son sumamente altos, y que aún sin superponer los a los de tensión axial producidos por la carga de 31000 Kg., exceden con mucho el esfuerzo de cedencia del material. Dichos esfuerzos, por supuesto no son reales, ya que no pueden exceder del esfuerzo de cedencia. Lo anterior implica que existe una plastificación del material cercano a las puntas de los patines no conectados del ángulo, en la zona de la conexión. Aparentemente este tipo de esfuerzos "locales" no son de gran importancia para el A.I.S.C., quien no considera hacer ninguna reducción en cuanto a los esfuerzos permisibles en este tipo de miembros; en cambio la AASHO establece que para considerar el efecto de la excentricidad en las conexiones, el área a considerar el ángulo es la del patín conectado más la mitad de la del patín no conectado. En





### EJEMPLO 3

Diseñar la pieza AB, utilizando dos canales de laminación nacional, colocadas en cajón. Acero A-36. Las cargas  $P$  son estáticas



#### Solución:

$$k_x = k_y = 1$$

$$M_{\max} = 8 \text{ T-m}$$

$$\text{Suponiendo } F_{bx} = 1300 \text{ Kg/cm}^2$$

$$S_{\text{req}} = \frac{8 \times 10^5}{1.3 \times 10^3} = 615 \text{ cm}^3$$

Se proponen 2 [ ] 12<sup>o</sup> Liv.

$$A = 77.80 \text{ cm}^2$$

$$S_{xx} = 699.8 \text{ cm}^3$$

$$r_x = 11.71 \text{ cm}, \quad r_y = 6.33 \text{ cm}$$

$$M_0 = \frac{PL}{3}$$

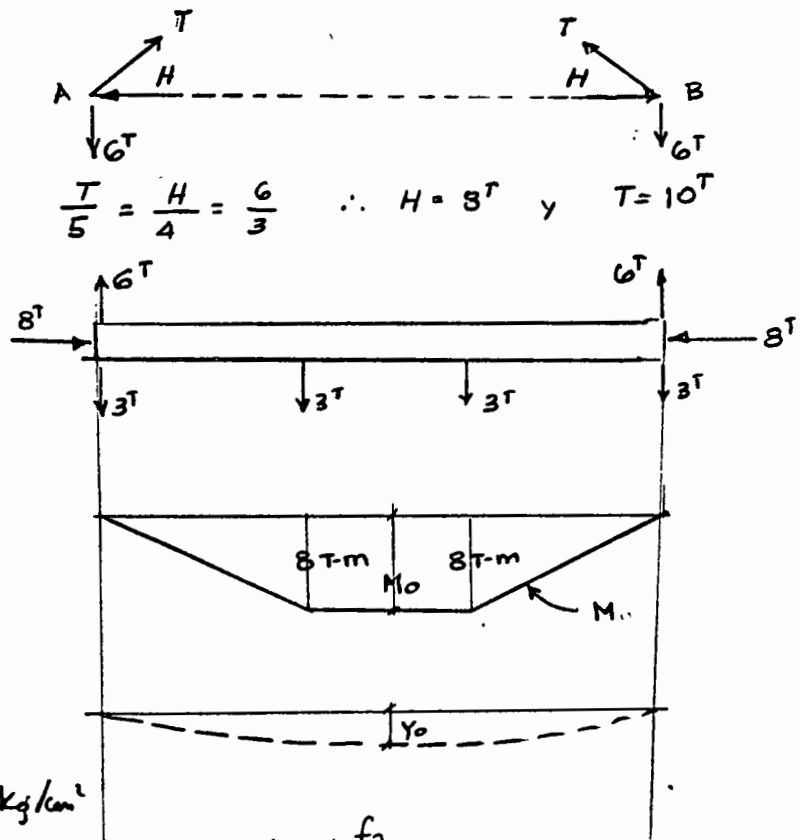
$$Y_0 = \frac{23}{648} \frac{PL^3}{EI}$$

$$\frac{K_y L}{r_x} = \frac{1 \times 800}{11.71} = 68.3 \rightarrow F'_{ex} = 2248 \text{ Kg/cm}^2$$

El esfuerzo axial permisible, lo obtenemos

$$\frac{K_y L}{r_y} = \frac{1 \times 800}{6.33} = 126.4 \rightarrow F_a = 658 \text{ Kg/cm}^2$$

#### Diagrama de cuerpo libre



$$\frac{T}{5} = \frac{H}{4} = \frac{6}{3} \quad \therefore H = 8T \quad \text{y} \quad T = 10T$$

$$C_m = 1 - \psi \frac{f_2}{F_3}$$

$$\psi = \frac{\pi^2 EI}{L^2} \frac{Y_0}{M_0} - 1 = \frac{\pi^2 EI}{L^2} \frac{23}{648} \frac{PL^3}{EI} \frac{3}{PL} - 1$$

$$\psi = 1.05 - 1.0 = 0.05 \quad \therefore C_m \doteq 1.0$$

$$f_a = \frac{8000}{77.80} = 102.8 \text{ Kg/cm}^2$$

$$\frac{f_a}{F_a} = \frac{102.8}{658} = 0.156 > 0.15$$

∴ Aplicando la fórmula (18)

$$\frac{f_a}{F_a} + \frac{C_m \times F_{bx}}{\left(1 - \frac{f_a}{F_{e_x}}\right) F_{bx}} \leq 1.0$$

$$F_{bx} = 0.6 F_y = 1520 \text{ Kg/cm}^2 \quad (\text{sección en cajón})$$

$$f_{bx} = \frac{8 \times 10^5}{699.8} = 1143 \text{ Kg/cm}^2$$

$$0.156 + \frac{1.0 \times 1143}{\left(1 - \frac{102.8}{2248}\right) 1520} = 0.156 + 0.788 = 0.944 < 1.0 \quad \therefore \checkmark$$

0.9543

La revisión por la fórmula (19) no se hace necesaria ya en virtud de que la condición de apoyo no proporciona sujeción lateral.

# Esfuerzos permisibles para soldaduras (AISC 1969).

TABLA 1.5.3 (AISC)

TIPO DE ESFUERZO	ESF. PERM. (Kg/cm <sup>2</sup> )	ELECTRODO REQUERIDO <sup>4</sup>	METAL BASE "AFIN" <sup>4</sup>
Tensión y compresión axial paralela al eje de cualquier soldadura de ranura (a tope) de penetración completa.	El mismo que el del metal base <sup>1</sup>		
Tensión perpendicularmente a la garganta efectiva de soldaduras de ranura (a tope) de penetración completa.	El esfuerzo permisible a la tensión del metal base <sup>1</sup>		
Compresión perpendicularmente a la garganta efectiva de soldaduras de ranura (a tope) de penetración completa.	El esfuerzo permisible a la compresión del metal base <sup>1</sup>		
Cortante en la garganta efectiva de soldaduras de ranura (a tope) de penetración completa o parcial.	El esfuerzo permisible al corte en el metal base <sup>1</sup>		
Cortante en la garganta efectiva <sup>2</sup> de soldaduras de filete, independientemente de la dirección de aplicación de la carga. Tensión perpendicularmente <sup>3</sup> al eje de la garganta efectiva de una soldadura de ranura (a tope) de penetración parcial. Cortante en el área efectiva de una soldadura de tapón o de ranura. Los esfuerzos permisibles que se especifican se aplicarán también a soldaduras hechas con los electrodos indicados sobre un acero con un esfuerzo de cedencia mayor que el del metal base "afin". El esfuerzo permisible, independientemente de la clasificación del electrodo empleado, no excederá del que se indica en la tabla para el metal base "afin" más débil que se emplee en la junta.	1265 Kg/cm <sup>2</sup>	Electrodos AWS A5.1, E-60XX Electrodos combinados con fundente AWS A5.17, F6X-E6XX Electrodos A5.20, E60T-X	A500 Grado A A570 Grado D
	1480 Kg/cm <sup>2</sup>	Electrodos AWS A5.1 ó A5.5, E-70XX Electrodos combinados con fundente AWS A5.17, F7X-E6XX Electrodos AWS A5.18, E70S-X ó E70U-1 Electrodos AWS A5.20, E70T-X	A-36 A-53 Grado B A-242 A-375 A-441 A-500 Grado B A-501 A-529 A-570 Grado E A-572 Grados 42 a 60 A-588
	1690 Kg/cm <sup>2</sup>	Electrodos AWS A5.5, E-80XX Arco sumergido Grado 80, Arco Metálico con gas, o Arco metálico de aportación con relleno de fundente.	A-572 Grado 65

Notas:

1. Se utilizará el electrodo o el fundente especificado en la tabla 1.17.2
2. Para definición de la garganta efectiva de una soldadura de filete y de las soldaduras de ranura de penetración parcial, véase la sección 1.14.7.
3. Las soldaduras de filete y las de ranura de penetración parcial que unen los elementos que forman un miembro compuesto, tales como las conexiones de patines a alma, se podrán diseñar sin importar la dirección del esfuerzo de tensión o compresión en estos elementos, paralelos a los ejes de las soldaduras.
4. Se utilizarán solamente electrodos de bajo hidrogeno sobre aceros A-242, A-441, A-514, A-572 y A-588.

TABLA 1.17.2 (AISC)

Metal base <sup>3</sup>	Proceso de soldadura <sup>1,2</sup>			
	Arco metálico recub.	Arco sumergido	Arco metálico c/gas	Arco c/relleno de fundente
ASTM A36, A53 Gr. B, A375, A500, A501, A529 y A570 Grados D y E.	AWS A5.1 ó A5.5, E-60XX ó E-70XX	AWS A5.17 F6X ó F7X-EXXX	AWS A5.18 E70S-X ó E70U-1	AWS 5.20 E60T-X ó E70T-X (excepto EXXT-2 y EXX-3)
ASTM A-242, A-441, A-572 Grados 42 a 60 y A588 <sup>4</sup>	AWS A5.1 ó A5.5, E-70XX <sup>5</sup>	AWS A5.17 F7X-EXXX	AWS A5.18 E70S-X ó E70U-1	AWS 5.20 E70T-X (excepto E-70T-2 y E-70T-3)
ASTM A572 Grado 65	AWS A5.5 E-80XX <sup>5</sup>	Grado F80	Grado E80S	Grado E80T

Se permite el uso de metal de relleno cuyas propiedades mecánicas sean las más próximas pero superiores.

1. Cuando las soldaduras se vayan a relevar de esfuerzos, el metal de aportación en la soldadura no deberá contener más del 0.05 por ciento de vanadio.
2. Véanse las especificaciones AWS D1.0-69, artículo 422 para los requisitos del metal de aportación en soldaduras de electrogas y electroescoria.
3. En aquellas juntas que contengan metales de distinto esfuerzo de cedencia, podrán utilizarse material de relleno de aquellos aceros cuyo esfuerzo de cedencia sea el menor de los que entran en la junta.
4. Para el caso de soldaduras de metales que quedarán expuestos a la intemperie por fines arquitectónicos, el metal de aportación deberá además tener características de resistencia a la corrosión atmosférica y coloración de su oxidación, semejantes a las del metal base utilizado. Sígase las recomendaciones del fabricante del acero.
5. Clasificaciones correspondientes a electrodos de bajo hidrógeno.

Recomendaciones AISC para juntas soldadas:

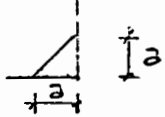
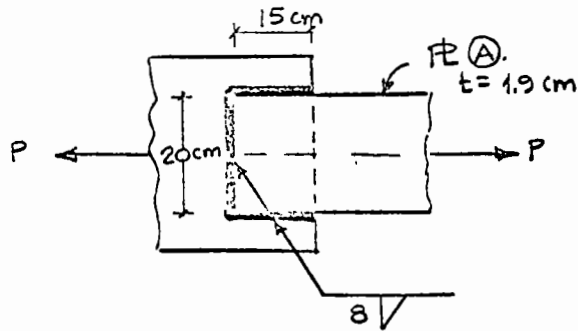
- 1.- Long. mín. de un filete =  $4\bar{a}$  ( $\bar{a}$  es el espesor del filete 
- 2.-  $\bar{a}_{max} = 1/4"$  (0.63cm) para material de  $t = 1/4"$  de espesor. Para materiales más gruesos  $\bar{a}_{max} = t - 1/16"$
- 3.- Las dimensiones mínimas de los filetes de soldadura estarán dadas por la tabla siguiente:

TABLA 1.17.5 (AISC)

Espesor de la placa más gruesa en la unión		Tamaño mínimo de la soldadura	
cm	pulg.	cm.	pulg.
hasta 0.63 incl.	hasta 1/2" inclusive	0.32	1/8"
de 0.63 a 1.27	de 1/4" a 1/2"	0.48	3/16"
de 1.27 a 1.90	de 1/2" a 3/4"	0.64	1/4"
de 1.90 a 3.21	de 3/4" a 1 1/2"	0.79	5/16"
de 3.21 a 6.35	de 1 1/2" a 2 1/4"	0.95	3/8"
de 6.35 a 15.24	de 2 1/4" a 6"	1.27	1/2"
mayor de 15.24	mayor de 6"	1.59	5/8"

### EJEMPLO 1

Cual es la capacidad de carga  $P$  permisible para la conexión de la figura si se utiliza acero A-36, y electrodos E-70XX.



Solución:

Capacidad de la soldadura de filete de 8mm.

$$1480 \times 0.707 \times 0.8 = 837 \text{ Kg/cm.}$$

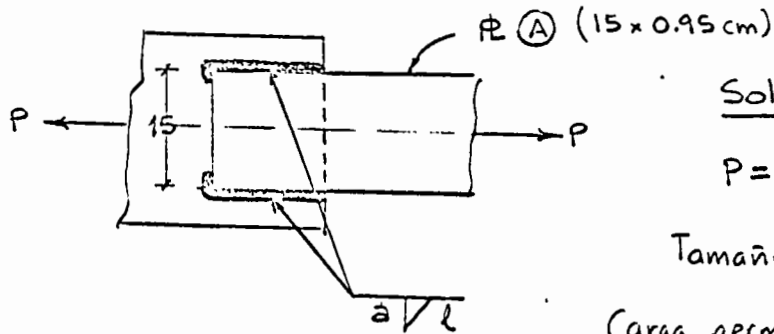
$$\text{Capacidad de la soldadura} = 837(20 + 2 \times 15) = 41850 \text{ Kg}$$

Capacidad de la placa  $\textcircled{A}$  a la tensión =  $1520 \times 20 \times 1.9 = 57760 \text{ Kg.}$

Capacidad de la conexión  $P = \underline{41850 \text{ Kg}}$

### EJEMPLO 2

Calcule la soldadura requerida para que la junta mostrada desarrolle la capacidad permisible de tensión en la placa  $\textcircled{A}$ . Acero A-572 Grado 65 ( $F_y = 4570 \text{ Kg/cm}^2$ ) y electrodos E-80XX



Solución

$$P = 0.6 \times 4570 \times 15 \times 0.95 = 39074 \text{ Kg}$$

$$\text{Tamaño max. de sold.} = 3/8" - 1/16" = 5/16" = 0.79 \text{ cm}$$

$$\text{Carga permisible de la sold.} = 1690 \times 0.707 \times 0.79 = 944 \text{ Kg/cm}$$

$$\text{longitud requerida de la soldadura por cordón } l = \frac{39074}{2 \times 944} = 20.7 \text{ cm}$$

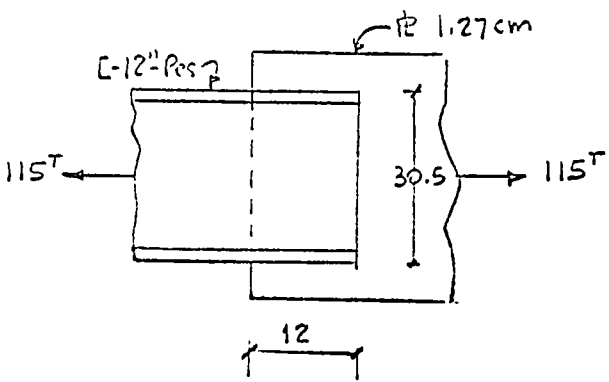
utilizando vueltas de long. mínima de  $2 \times 0.79 = 1.58 \text{ cm}$  (sean de 1.7 cm)

$$l = 20.7 - 1.7 = 19 \text{ cm. por cordón.}$$

### EJEMPLO 3

Diseñe la soldadura para unir un tensor formado por una canal de 305mm pesada, a una placa de 1.27cm, como se muestra en la figura. El acero de la canal es A-36; el de la placa es A-242 y la soldadura se hace con electrodos E-70XX. La carga que se transmitirá es de 115 ton., y debido a requisitos arquitectónicos, la canal no puede traspasar más de 12 cm dentro de la placa de 1.27cm..

Solución.

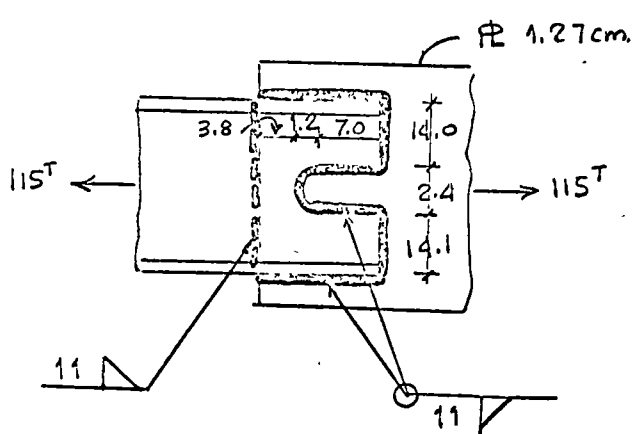


El espesor del alma de la [ es 1.92 cm. > 1.27  
 Tamaño max. de la soldadura =  $\frac{1}{2} - \frac{1}{16} = \frac{7}{16} = 1.11$  cm

Cap. sold =  $1480 \times 0.707 \times 1.11 = 1161$  Kg/cm.

$l_{req} = \frac{115000}{1161} = 99.0$  cm

long. disponible =  $2 \times 12 + 2 \times 30.5 = 85$  cm < 99.0 cm.  $\therefore$  hay que hacer una ranura para alojar la longitud faltante  $99.0 - 85 = 14$  cm.



Dimensiones de la ranura (Ver AISC 1.17.12)

a)  $1.27 + 0.8 = 2.07$  cm

b)  $2.25 \times 1.27 = 2.86$  cm.

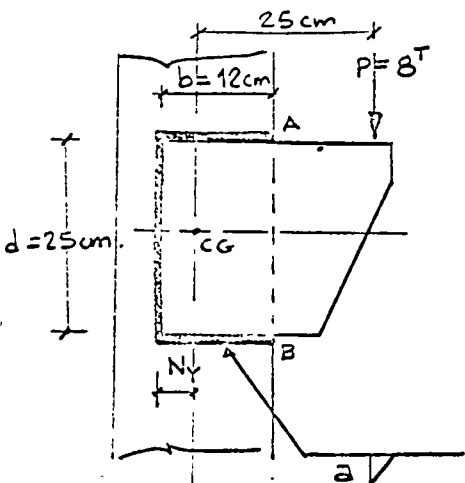
Usamos 2.38 cm ( $\frac{15}{16}$ "), (en virtud de que los punzones estructurales se fabrican en un número non de 16 avos de pulgada)  
 La longitud de la ranura

$l_{max} = 10 \times 1.27 = 12.7$  cm > 7.0 cm, req.

$\therefore$  Usar soldadura de ranura de 7.0 x 2.4 cm

EJEMPLO 4

Diseñar el tamaño de la soldadura de filete de la figura siguiente; el acero es A-36, y los electrodos E-70XX.



Solución

de la tabla II  $N_y = \frac{b^2}{2b+d} = \frac{12^2}{2 \times 12 + 25} = 2.94$  cm.

$J_s = \frac{(2b+d)^3}{12} - \frac{b^2(b+d)^2}{2b+d} = \frac{(2 \times 12 + 25)^3}{12} - \frac{12^2(12+25)^2}{2 \times 12 + 25} = 5780$  cm<sup>3</sup>

$A_s = 25 + 2 \times 12 = 49$  cm

$f_1 = \frac{P}{A_s} = \frac{8000}{49} = 163$  Kg/cm

$f_{2H} = \frac{T}{J} Y = \frac{8000 \times 25}{5780} \times 12.5 = 433$  Kg/cm

$f_{2V} = \frac{T}{J} X = \frac{8000 \times 25}{5780} (12 - 2.94) = 313$  Kg/cm

el esfuerzo cortante total en A vale:

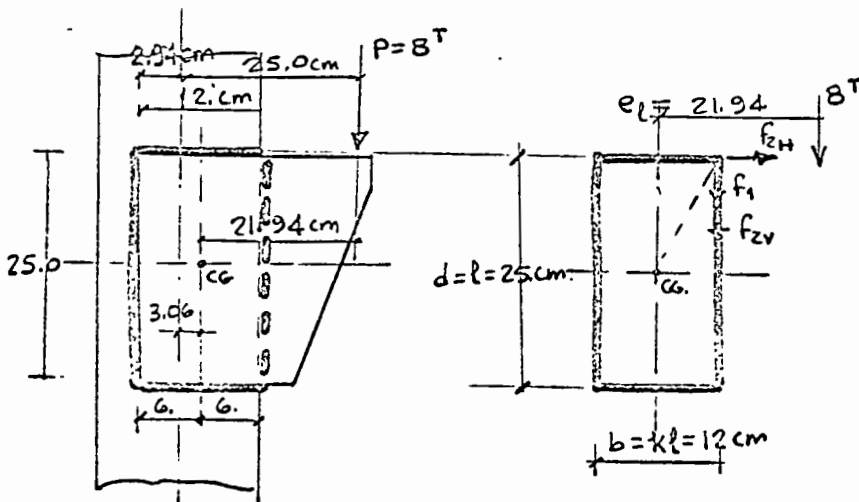
$$f_{TOT} = \sqrt{(f_1 + f_{2V})^2 + f_{2H}^2} = \sqrt{(163 + 313)^2 + 433^2} = 644 \text{ Kg/cm}$$

el tamaño requerido del filete es:  $644 \text{ Kg/cm} = 1480 \times 0.707 \cdot a$

$$\therefore a = \frac{644}{1480} = 0.62 \text{ cm} ; \text{ sea } a = 6.3 \text{ mm } \left(\frac{1}{4}''\right)$$

### EJEMPLO 5

Sea el mismo caso del problema anterior, pero colocando además un cordón de soldadura por detrás de la placa-ménsula.



Solución: (Ver tabla II)

$$A_s = 2(25+12) = 74 \text{ cm} \quad J_s = \frac{(b+d)^3}{6} = \frac{(25+12)^3}{6} = 8420 \text{ cm}^3$$

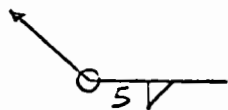
$$f_1 = \frac{8000}{74} = 108 \text{ Kg/cm}$$

$$f_{2V} = \frac{8000 \cdot 21.24}{8420} \times 6 = 125 \text{ Kg/cm}$$

$$f_{2H} = \frac{8000 \cdot 21.94}{8420} \times 12.5 = 260.6 \text{ Kg/cm}$$

$$f_{TOT} = \sqrt{(108+125)^2 + 260.6^2} = 350 \text{ Kg/cm} = 1480 \times 0.707 \cdot a$$

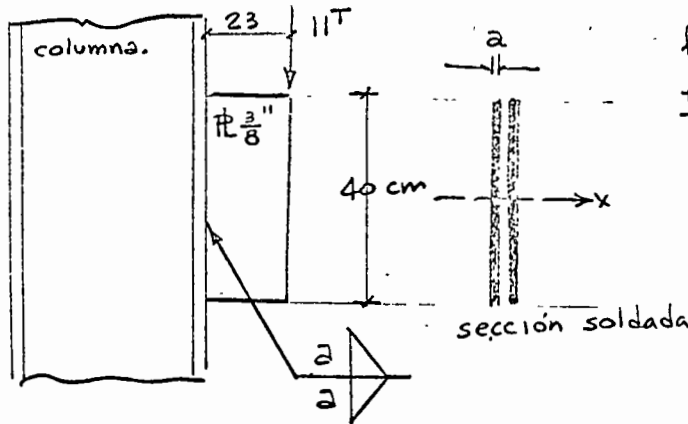
$$\therefore a = \frac{350}{1480 \cdot 0.707} = 0.33 \text{ cm} \quad \therefore \text{ sea } a = 0.48 \text{ cm} = \frac{3}{16}''$$



## EJEMPLO 6

Determine el tamaño del filete de sold.  $\exists$  requerido para resistir la carga  $P = 11 \text{ TON}$ , aplicada como se muestra en la figura. Acero A-36 y Electrodo E-70XX

### Solución



$$l = 2 \times 40 = 80 \text{ cm}$$

$$I_{xx} = 2 \times \frac{40^3}{12} = 10667 \text{ cm}^3$$

$$f_1 = \frac{11000}{80} = 137.5 \text{ Kg/cm}$$

$$f_2 = \frac{11000 \times 23}{10667} \times 20 = 474.4 \text{ Kg/cm}$$

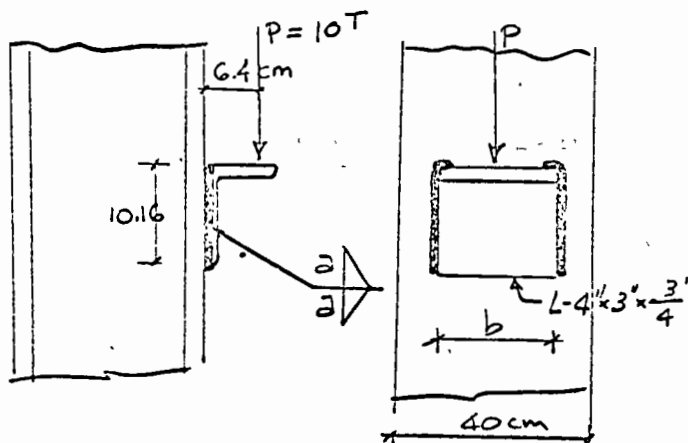
$$f_{\text{TOT}} = \sqrt{137.5^2 + 474.4^2} = 494 \text{ Kg/cm}$$

$$a = \frac{494}{1480 \times 0.707} = 0.472 \text{ cm, sea } a = \frac{3}{16}'' = 0.48 \text{ cm}$$

$$a = \frac{3}{16}'' < \frac{3}{8}'' - \frac{1}{16}'' = \frac{5}{16}'' \therefore \checkmark$$

## EJEMPLO 7

Utilizando electrodos E-80XX y acero A572-65 determine el tamaño de la soldadura requerido para la conexión de asiento de la figura.



### Solución

$$M = 10000 \times 6.4 = 64000 \text{ Kg-cm}$$

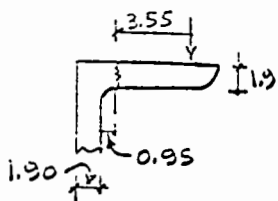
$$f_1 = \frac{10000}{2 \times 10.16} = 492 \text{ Kg/cm}$$

$$f_2 = \frac{64000}{\frac{2 \times 10.16^2}{6}} = 1860 \text{ Kg/cm}$$

$$f_r = \sqrt{492^2 + 1860^2} = 1924 \text{ Kg/cm}^2$$

$$a = \frac{1924}{1690 \times 0.707} = 1.61 \text{ cm}$$

$$\text{sea } a = \frac{11}{16}'' = 1.75 \text{ cm. } \dots \left( a_{\text{max}} = \frac{3}{4}'' - \frac{1}{16}'' = \frac{11}{16}'' \therefore \checkmark \right)$$



$$M = 10000 \times 3.55 = 35500 \text{ Kg-cm}$$

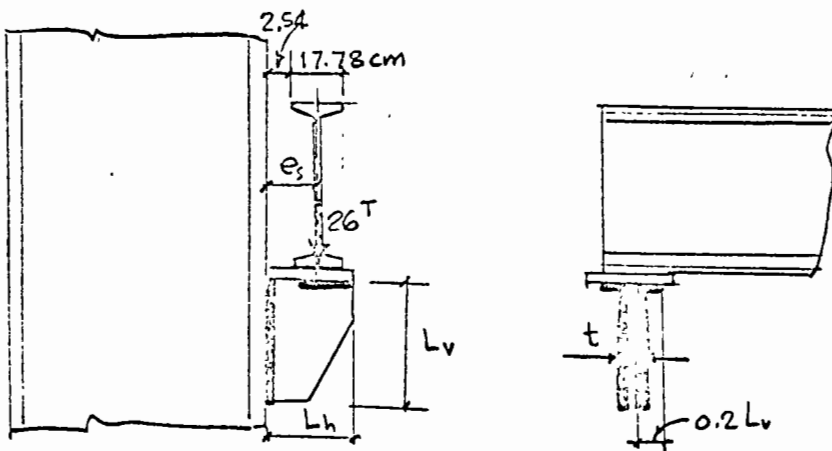
$$S = \frac{1.9^2 \times b}{6}; F_b = 0.75 F_y = 3428 \text{ Kg/cm}^2; M_r = 3428 \times \frac{1.9^2 \times b}{6}$$

$$\therefore b = \frac{35500 \times 6}{3428 \times 1.9} = 32.7 \text{ cm sea } 33 \text{ cm.}$$



### EJEMPLO 8

Diseñe el asiento atiesado de una viga cuya reacción es de  $26^T$ , Use Acero A-36 y electrodos E-70XX



$$e_s = 2.54 + \frac{17.78}{2} = 11.43 \text{ cm}$$

$$L_h = 2.54 + 17.78 = 20.32 \text{ cm}$$

#### Solución

Determinación del espesor de la placa:

$$A = t L_h$$

$$S = \frac{t L_h^2}{6}$$

$$M = R \left( e_s - \frac{L_h}{2} \right)$$

$$f = \frac{R}{A} + \frac{M}{S} = \frac{R}{t L_h} + \frac{R \left( e_s - \frac{L_h}{2} \right)}{\frac{t L_h^2}{6}} = \frac{R (6 e_s - 2 L_h)}{t L_h^2}$$

$$\therefore t = \frac{2 (6 e_s - 2 L_h)}{f L_h^2}$$

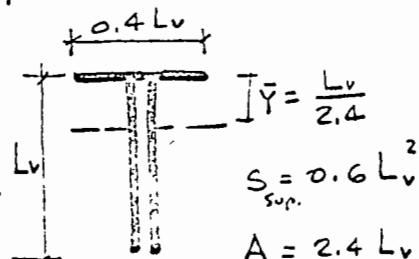
si  $f = 0.6 F_y = 1520 \text{ kg/cm}^2$ ,  $R = 26,000 \text{ kg}$ ,  $e_s = 11.43 \text{ cm}$  y  $L_h = 20.32 \text{ cm}$

$$t = \frac{26000 (6 \times 11.43 - 2 \times 20.32)}{1520 \times 20.32^2} = 1.16 \text{ cm.} \quad \text{sea } t = \frac{7}{16} = 1.11 \text{ cm.}$$

$$\text{y } a = \frac{5}{16} = 0.79 \text{ cm}$$

Determinación de la longitud del atiesador  $L_v$ :

La sección soldada es:



$$f_1 = \frac{R}{A} = \frac{R}{2.4 L_v}$$

$$f_2 = \frac{M}{S} = \frac{R e_s}{0.6 L_v^2}$$

$$f_{tot} = \sqrt{f_1^2 + f_2^2} = \sqrt{\left(\frac{R}{2.4 L_v}\right)^2 + \left(\frac{R e_s}{0.6 L_v^2}\right)^2} = \frac{R}{2.4 L_v^2} \sqrt{L_v^2 + 16 e_s^2}$$

$$a = \frac{f_{tot}}{F_{perm}} \quad \gamma \quad F_{perm} = 1480 \text{ Kg/cm}^2 \times 0.707 = 1046 \text{ Kg/cm}^2$$

$$\frac{R}{a} = \frac{1046 \times 2.4 L_v^2}{\sqrt{L_v^2 + 16 e_s^2}} = \frac{2511 L_v^2}{\sqrt{L_v^2 + 16 e_s^2}}$$

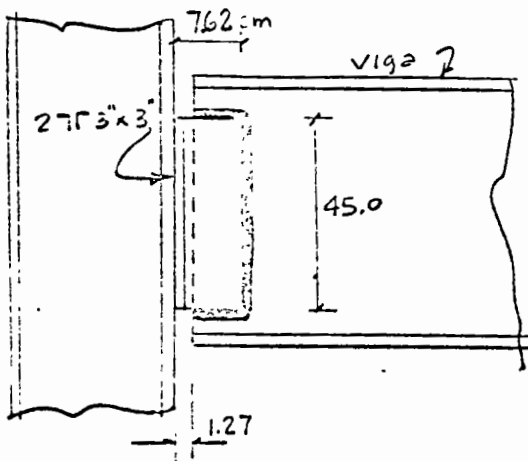
$$\therefore L_v = \sqrt{\frac{B}{2} \left[ B + \sqrt{B^2 + 64 e_s^2} \right]} \quad \text{donde } B = \frac{R}{2511 a}$$

en nuestro caso  $B = \frac{26000}{2511 \times 0.79} = 13.11$

$$L_v = \sqrt{6.55 \left[ 13.11 + \sqrt{171.8 + 64 \times 11.43^2} \right]} = 26.28 \text{ cm} \quad \text{sean } 10\frac{1}{2}'' = 26.67 \text{ cm.}$$

### EJEMPLO 9.

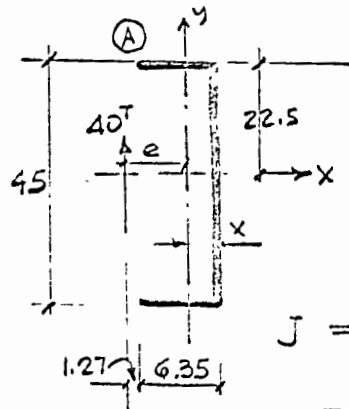
Determine la dimensión  $a$  del filete de soldadura requerido para la conexión de dos ángular al alma de la viga mostrada en la figura.



Reacción =  $40^T$   
Electrodos E-70XX  
Acero A-36

### Solución

La sección soldada tiene la forma:



$$\bar{x} = \frac{2 \times 6.35 \times \frac{6.35}{2}}{45 + 2 \times 6.35} = 0.7 \text{ cm}$$

$$e = 7.62 - 0.70 = 6.92 \text{ cm}$$

$$J = \frac{(2b+d)^3}{12} - \frac{b^2(b+d)^2}{2b+d}$$

$$J = \frac{(12.7 + 45)^3}{12} - \frac{6.35^2 (6.35 + 45.0)^2}{12.7 + 45} = 16008 - 1843 = 14165 \text{ cm}^3$$

$$A = 2 \times 6.35 + 45 = 57.7 \text{ cm}$$

Esfuerzos (en el punto A)

$$f_1 = \frac{40000/2}{57.7} = 347 \text{ Kg/cm}$$

$$f_{2v} = \frac{(40000/2) 6.92}{14165} (6.35 - 0.7) = 55. \text{ Kg/cm}$$

$$f_{2h} = \frac{(40000/2) 6.92}{14165} \cdot 22.5 = 220 \text{ Kg/cm}$$

$$f_{tot} = \sqrt{(347+55)^2 + 220^2} = 459 \text{ Kg/cm}$$

$$\bar{\sigma} = \frac{459}{1406} = 0.32 \text{ cm} \quad \text{sea } \underline{\text{soldadura de } 1/8''}$$



in order to limit damage to the superstructure. Some building codes specify the design axial force in these members as 10 percent of the greater of the vertical forces acting on each of the two footings connected thereby. Since it is the lighter of the two footings that should be forced to move essentially as the heavier one, and not vice versa, the axial force should probably be, in any case, a fraction of the smaller load.

On cohesive ground the danger of the phenomenon we have referred to is unlikely. Often no tie girders are required in the foundation. In some conditions, though, there is danger that cracks may open in the soil. To minimize the probability of crack formation under the building, it is advisable in these cases to use tie girders or other means of tension reinforcement. The amount of reinforcement required is essentially independent of the vertical loads transmitted to the ground even when the soil has an important angle of internal friction. This paradoxical situation results from the function of the reinforcement. The function is not to maintain an integral foundation while the subjacent ground is displaced laterally by cracking but to change the direction of cracks in the surrounding ground so that they circumvent the structure. For the same reason the amount of reinforcement required may be a small fraction of the tension that the soil would resist before cracking. It is also an increasing function of the variability of this strength in the direction of the reinforcement. The authors are unaware of satisfactory criteria for deciding on the amount of reinforcement that should be provided in order to divert the ground's cracks.

### 15.12 The Choice of a Structural Solution

The optimum structural solution is dictated by economic and architectural considerations and depends markedly on the seismicity of the site.

Some materials behave in a distinctly more favorable way than others under the action of repeated alternating loads. Yet by designing the latter materials according to more conservative criteria it is possible, sometimes, to arrive at a solution involving lower capital investment without a comparable increase in the expected actualized cost of failure. On the other hand, architectural advantages may favor choice of the first material. Commercially oriented arguments favoring one material over another, even when supported by test results, should not always be taken at face value.

A structural solution that is optimum when designing without regard for earthquakes does not necessarily remain even acceptable when one designs to withstand intensive ground motions. For example, architectural demands may lead the structural engineer to favor shallow floor systems, perhaps flat plates, when he designs a tall building for modest earthquakes or decides to ignore these phenomena. If he designs to resist strong ground motions, he will try to convince the architect that concessions are in order; otherwise he will produce a very flexible structure requiring wide gaps with nonstructural elements

and hence a special treatment. And if he does not provide the gaps, frequent cracking of walls, as in Fig. 15.23, is sure to occur, and wide separations will be required with respect to property lines. The columns will have huge sections in the first several stories, since their lowest points of contraflexure may be two or three stories above ground level and the effects of vertical loads will greatly magnify those of lateral forces, and special provisions will also be needed for windows, piping fixtures, and so on.

A less obvious dependence of the optimum solution on the design intensities concerns the matter of concentration of rigidity. Take a moment-resisting frame as shown in Fig. 15.24a. If we do not design to resist earthquakes, there will rarely be a reason for supplying it with cross bracing, other than to resist wind pressures and to reduce the probability of overall buckling.

If we design for mild shocks, an arrangement of braces as in Fig. 15.24b may be adequate and architectural requirements may make it difficult to choose a different arrangement. It is true that overturning moment will induce axial compression in one of the columns at the sides of the panels that contain the braces, and it will induce tension in the other column. But the compression will be sufficiently small compared with the compression induced by gravity loads alone that its presence may not even alter the column's design, and the tension will surely not surpass the gravity-load compression.

If we design for higher earthquake intensities these considerations will not apply, and there will be a distinct advantage to distributing the cross braces

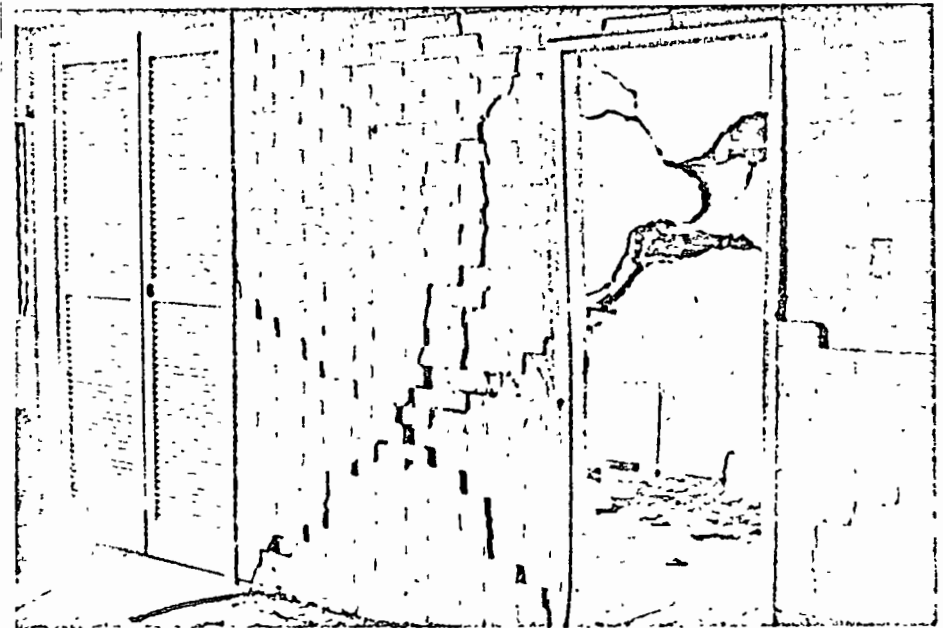


Figure 15.23. Typical cracks in partitions. After Esteva, Diaz de Cossio, and Elorduy (1968).

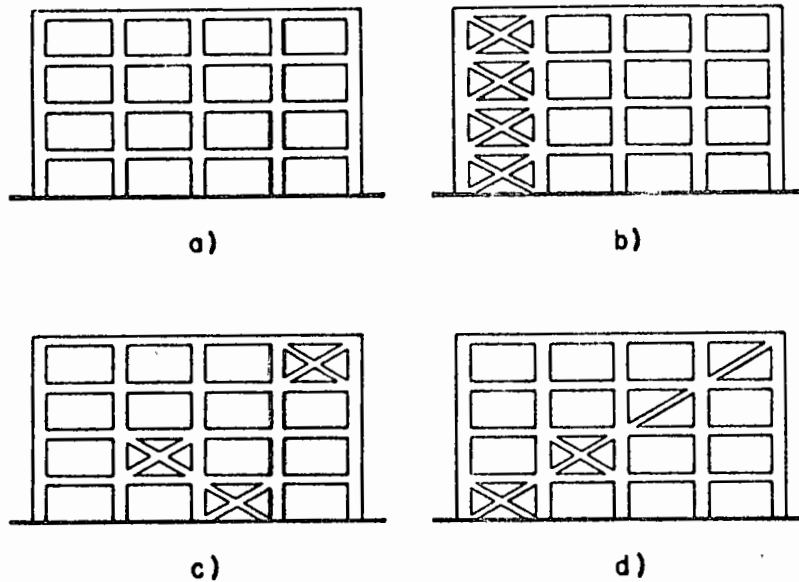


Figure 15.24. Different structural solutions for frame with and without diagonal braces:  
 (a) For no earthquakes or very strong earthquakes  
 (b) For mild earthquakes  
 (c) and (d) For moderate and strong earthquakes.

as in Fig. 15.24c, or as in d, so as to make a large number of columns participate in resisting the overturning moments, simultaneously lengthening the effective moment arm. Both measures will alleviate conditions in the foundation and also increase the structure's rigidity, perhaps making it unnecessary to take special provisions to protect nonstructural elements.

Design for even stronger earthquakes may lead us to dispense with braces altogether, reverting to Fig. 15.24a. The choice might be due to architectural limitations that prevent us from adopting a solution such as those in Fig. 15.24c and d, forcing us to a solution similar to the now very objectionable one in Fig. 15.24b. It might also follow from the importance of ductility under the new conditions of design and the awkwardness of large-section braces and heavy details.

As a second example, consider the building represented in the plan view in Fig. 15.25a. If the ratio of height to least dimension of the base is small or the design intensity is low, this arrangement of shear walls may be good. We must, in this case, take into account the deformations of the floor systems in their own planes and design these systems and the central transverse frames accordingly. For slender buildings designed to resist strong earthquakes the arrangement will no longer be desirable. Concentration of overturning moments in the two shear walls will cause considerable difficulty in the design of the foundation—event uplift and excessive contact pressures—and of the corner columns—

to take important vertical tension and compression. A preferable solution is illustrated in Fig. 15.25b, in which shear walls have been distributed along the entire plan. Should there be architectural objections to this alternative, or should the building be very slender and we wished to design for exceedingly high intensities, it is likely that the total omission of shear walls would prove advantageous.

The desirability to limit the vertical forces induced by overturning moments, coupled with architectural restrictions on shear walls and cross bracing, will often suggest an arrangement of either these walls or these braces distributed in different bays and different vertical planes from one story to the next. The advantages of increased lateral stiffness and of reduced vertical forces more than compensate for the additional stresses that appear in the floor systems in their capacity as horizontal diaphragms as well as in their work as parts of the wall or bracing systems.

Figure 15.26 illustrates another situation in which the earthquake intensities that one wishes to resist influence the choice of a structural solution. Suppose that architectural limitations permitted the distribution of columns shown in Fig. 15.26a. If we do not design for sizeable lateral forces or if the building is relatively short, the most economical arrangement for a reinforced concrete frame is probably as schematized in this figure. Here we take advantage of the intermediate column and make the beam of uniform depth throughout. For intermediate building heights and earthquake intensities, whether we use a steel or a concrete frame, it is preferable to adopt a variable beam depth (Fig. 15.26b). This obviates high concentrations of bending moments and shears in the shorter spans and in the contiguous members. And in the case of tall buildings designed for high seismic intensities, it is even better to omit altogether the column that causes the disparity in spans (Fig. 15.26c).

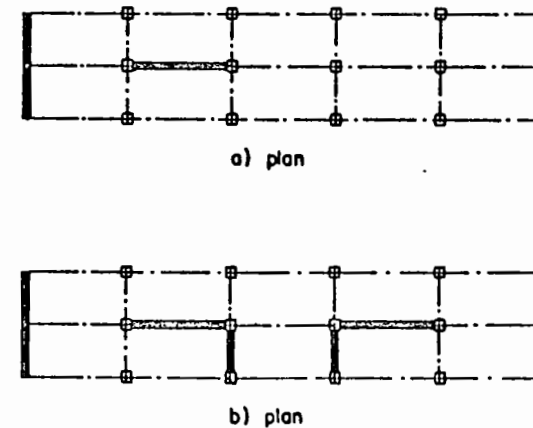


Figure 15.25. Two different arrangements of shear walls:  
 (a) For mild earthquakes and short buildings  
 (b) For strong earthquakes or moderate tall buildings.

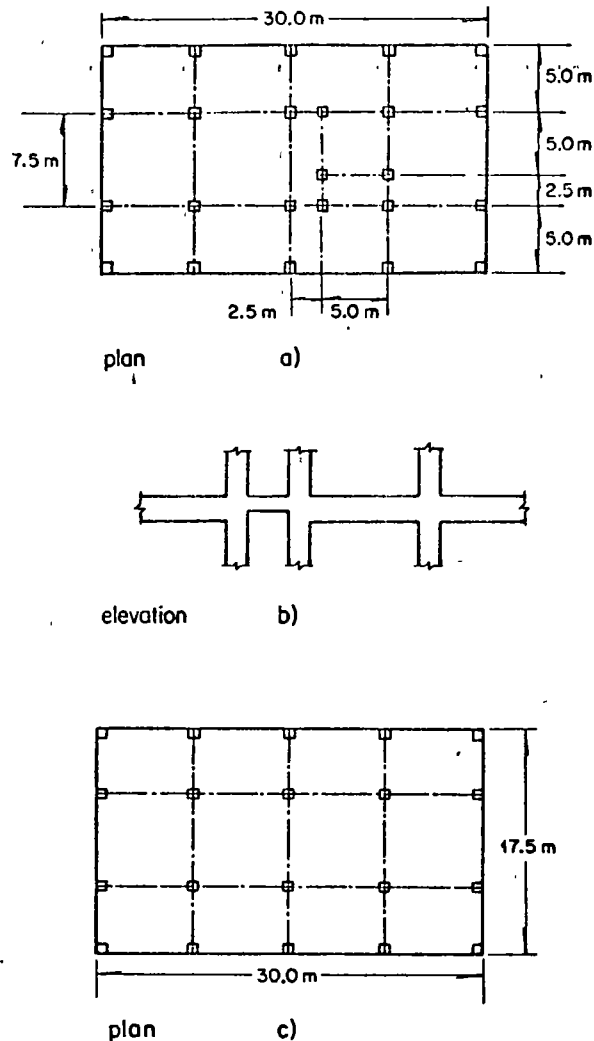


Figure 15.26. Different structural solutions for building with uneven spans.

Contemporary trends in architecture employ a number of structural details of design for which satisfactory criteria are still lacking. Typical among them is the use of eccentric beam-column connections (Fig. 15.27), whether in concrete or in steel. There are numerous examples of local failures of these connections during earthquakes (Rosenblueth, Marsal, and Hiriart, 1958), which show that ignoring the eccentricity produces seriously objectionable designs.

Some structural solutions in reinforced concrete seem ideally suited for certain buildings, except that they call for exceedingly high ductility factors in a few structural members. Sometimes this happens with the combination of shear

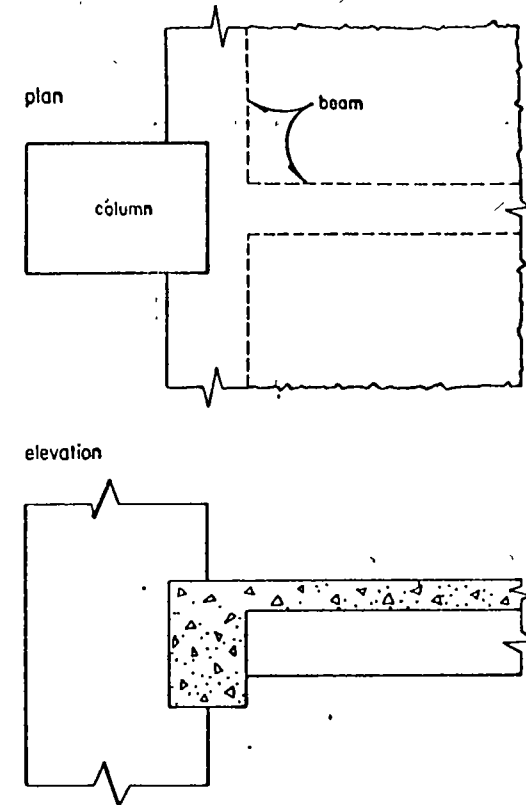


Figure 15.27. Eccentric beam-column connection.

walls and frames that may be entirely satisfactory except for the ductility demand at the beams that are directly connected to the walls in their own plane. The situation may be taken care of by designing these beams in reinforced concrete with sufficient confinement through the use of closely spaced lateral reinforcement and very careful detailing, or by replacing these beams with steel members. If confined concrete is chosen, it may be justified to hide the beams so that spalling at regions of large strains will not be visible following a strong earthquake. If steel is preferred and earthquakes of long duration are expected, the regions near the supports may require special stiffeners to prevent likelihood of the type of failure under repeated loads that we described in Section 13.6.2.

Another example is found in staircase ramps. Very often there is an advantage in replacing them with steel members that carry precast steps.

In many building codes we still find a relic whose origin and meaning are difficult to understand—the demand that the structure be designed so that it “move as a unit.” Presumably this requires design of horizontal diaphragms satisfying compatibility. Codes frequently specify also that buildings of irregular plants (say, *E*, *I*, *L*, and *U* shapes) be provided with construction joints—essentially expansion joints—so as to divide them into rectangular units.

Obviously, horizontal diaphragms should be designed to withstand the forces that, according to rational analysis, will act on them, consistently with those acting in the vertical resisting elements. And if this is done, it will often be found uneconomical to divide the plan into regularly shaped units. The decision should proceed from a comparative study of alternate solutions. Again the optimum will be found to depend on the intensity of earthquakes that the building is expected to resist without serious damage.

For example, the narrow band between axes *A* and *B* and between 1 and 2 in Fig. 15.28 will be called upon to resist small stresses (axial, shearing, and bending) if this structure is designed for small lateral forces. Horizontal bending may become quite high if we design for moderate intensities, making it desirable to add a beam at every floor along axis 4 between *A* and *B*. For higher design intensities we shall probably find it convenient to do without the added beam, introduce a wide expansion joint between *A* and *B*, as shown by the dashed lines in the figure, and cantilever the floor from both *A* and *B* toward this joint.

Figure 15.29 represents a solution commonly used for one- and two-story school buildings in tropical and semitropical countries. When the possibility of strong earthquakes is remote, this arrangement may be desirable because of its functional assets. However, when the structure is called upon to resist strong ground motions in the longitudinal direction, the solution has severe drawbacks. The least of these lies in the torsion induced by the difference in stiffnesses between axes *A* and *B*, because transverse walls and partitions usually have ample strength and rigidity to resist it.

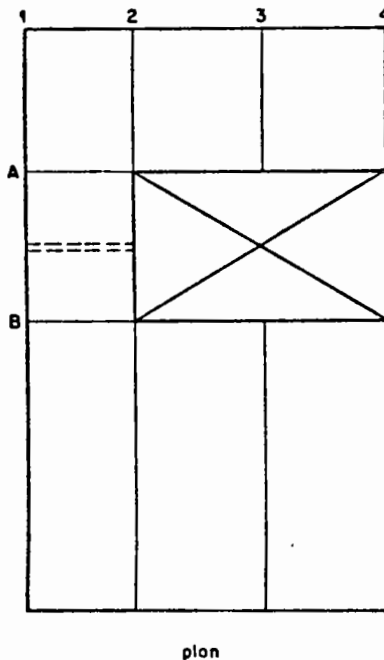


Figure 15.28. Strangled slab.

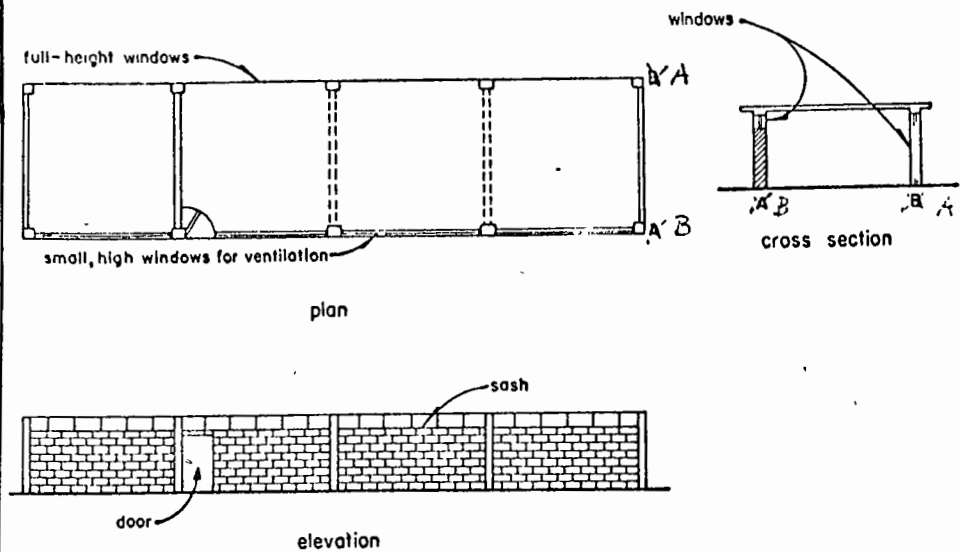


Figure 15.29. Typical school building in tropical country.

The chief disadvantages of this type of structure are: (1) the stiffness of axis *B* is of a higher order of magnitude than that of axis *A* and hence is subjected practically to the entire longitudinal force; (2) unless the columns in axis *B* are designed to resist extremely high shears, their strength in shear will be much smaller than that in flexure, so that they are likely to fail in shear, which involves an undesirably low ductility; (3) the increased stiffness in the longitudinal direction, relative to a solution in which the curtain walls would not participate in resisting horizontal motion, subjects structures of this type, resting on firm ground, to considerably greater spectral accelerations. Failure of columns of axis *B* in diagonal tension is practically inevitable during a strong earthquake. The literature abounds in examples of this sort (Rosenblueth and Prince, 1965). This is tragic because schools normally belong to this type of structure, and the cost of preventing this sort of failure, by making the curtain walls independent of the frames for horizontal motion, is quite low.

A similar situation arises in hot climates when the central longitudinal partitions are interrupted near the top slab to allow cross ventilation (Fig. 15.30). Countless cases of failures of the row of columns having interrupted partitions in structures of this type are found in the literature (Esteva and Nieto, 1967; Esteva, Rascón, and Gutiérrez, 1969).

These situations usually result from wishful thinking, which leads to assume that structural elements—in these examples the curtain walls and central partitions—regarded as nonstructural during the design stage, will not partake of structural action during an earthquake. The same fallacy is responsible for the appearance of large diagonal cracks at the corners of the frames that enclose unreinforced masonry walls (Fig. 15.31).



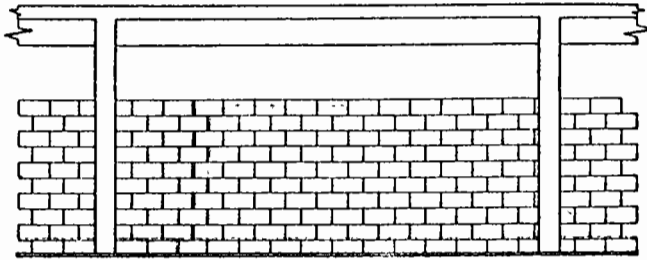


Figure 15.30. Interruption of partition for ventilation in hot climates.

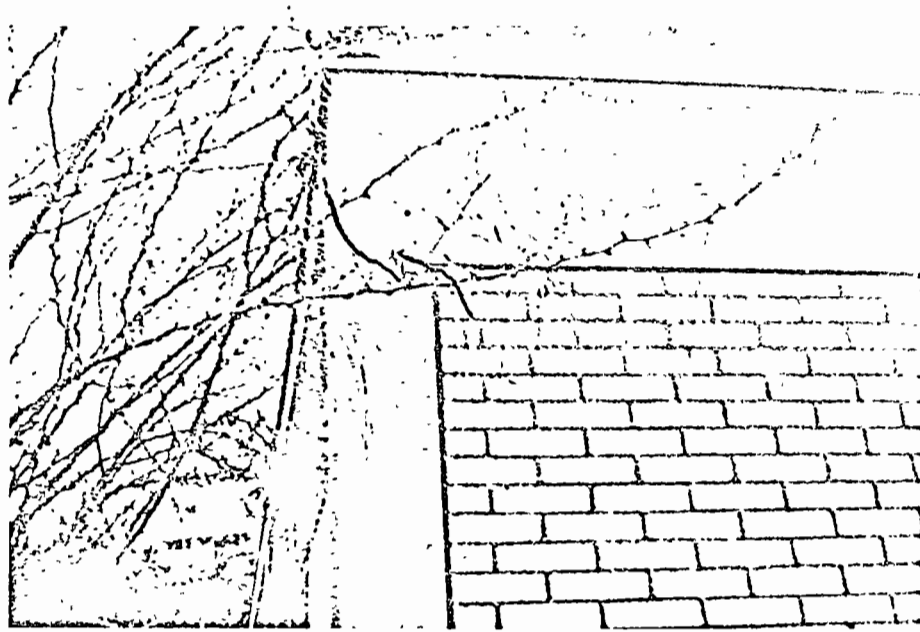


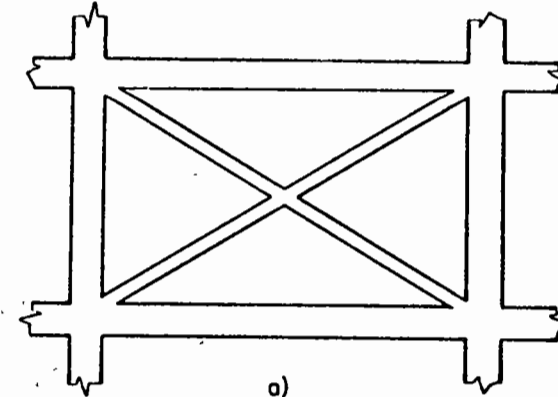
Figure 15.31. Cracking of reinforced concrete frame due to its interaction with infilling wall panel. After Esteva and Nieto (1967).

The number of examples could extend indefinitely to show that dogmatic postures are untenable in the election of optimum structural solutions. Such matters as regional seismicity, local soil conditions, local economic situation, and architectural requirements determine the choice of solutions that are apparently objectionable when judged in the frame of a different set of circumstances.

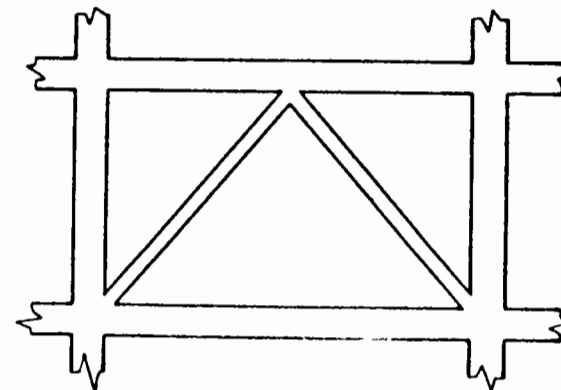
Unprejudiced experience is valuable in lending orders of magnitude to such terms as "tall," "slender," and "intense," which we have used with vagueness in the foregoing paragraphs, as these orders change from site to site. Such experience is also of use in pointing out particularly vulnerable spots in some structural solutions that would otherwise be judged optimistically. This applies to connections between beams and shear walls, for example. Based on experience

with the construction difficulties that *X*-bracing presents at joints rather than with regard to the increase in ductility (at the expense of rigidity) it seems that an engineer will lean toward the use of *A*-braces instead (Fig. 15.32b vs. a).

There has been sustained interest in solutions that may drastically reduce earthquake stresses throughout the structure (see Matsushita and Izumi, 1969). The first analytical attempts in this direction advocated what became known as the "flexible first story" (Green, 1935). It was contended that a sufficiently flexible first story would so lengthen a building's natural periods of vibration that it would reduce the base shear, and hence all stresses in the superstructure, to significantly lower values than is possible with more conventional structural solutions. Important savings would ensue. In order to prevent excessive sway under wind and mild earthquakes, a "fuse" would be provided, consisting for example of brittle and weak, hollow tile partitions. These would fail under a strong shock.



a)



b)

Figure 15.32. Two types of braces:

- (a) *X*-braces
- (b) *A*-braces.

Biot (1943) showed that the first story would have to be impracticably flexible to achieve a significant economy in higher stories. Typically, in a 20-story building a 10-fold increase in the flexibility of the first story will reduce stresses at all higher elevations by no more than about 30 percent. The solution is even less effective than might seem at first, since the large deflections of the first

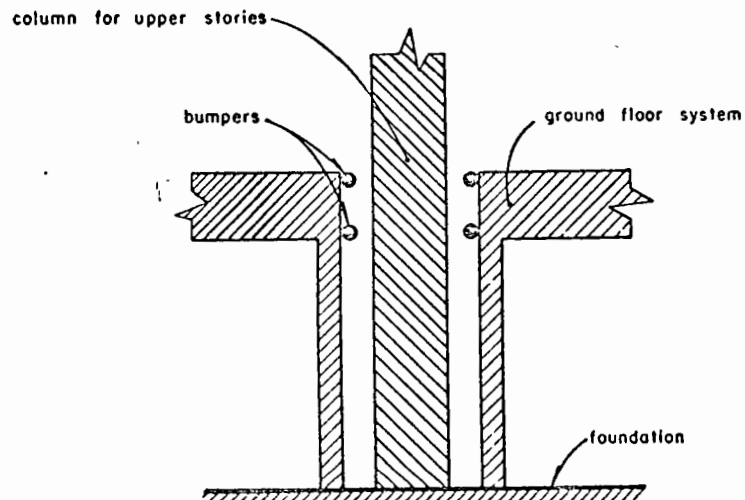


Figure 15.33. Use of hollow basement-columns. After Matsushita and Izumi (1965).

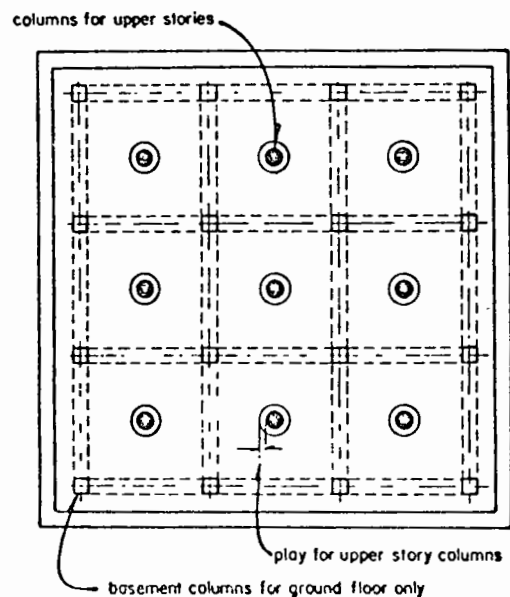


Figure 15.34. Use of double system of columns. After Matsushita and Izumi (1965).

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story bring about large story moments due to the action of gravity forces.

A more practical version of the same solution was proposed at a later date (Matsushita and Izumi, 1965). In it the basement columns are hollow and quite rigid; they enclose very flexible columns that carry the whole superstructure. In this way it is simple to limit the ground-slab displacements, giving rise to bilinear behavior of the system (Fig. 15.33). A variation of this solution is shown in Fig. 15.34. Despite the improvement there is still the matter of increased story moments in the flexible columns.

Other proposals similarly oriented employ the use of soft pads under the basement or ground-story columns [(Fig. 15.35) Joshi (1960)], the use of rollers [(Fig. 15.36) González-Flores (1964)], and the adoption of suspended supports [(Fig. 15.37) Garza-Tamez (1968)]. Some of these solutions are patented. The last two mentioned have been proposed in conjunction with dashpots tending to reduce deflections. To the authors' knowledge no major applications of any of these alternatives have been attempted.

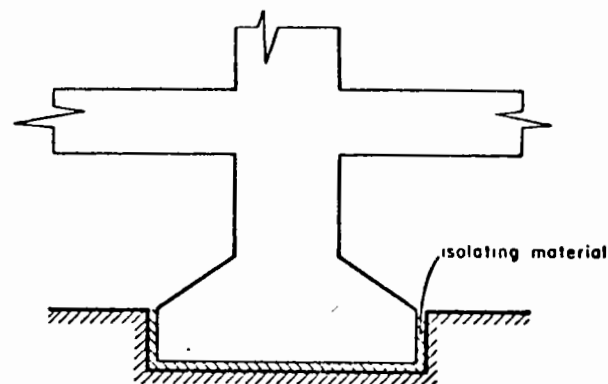
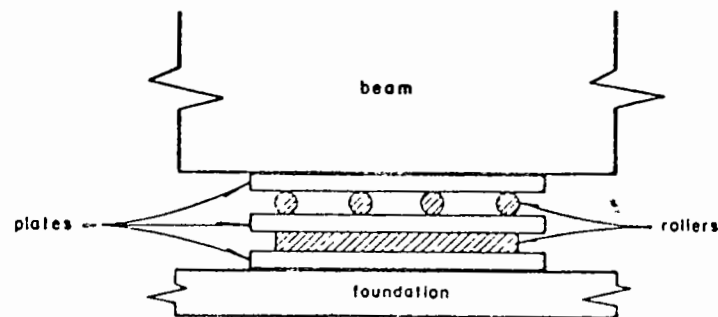


Figure 15.35. Use of rubber pads for partial isolation from earthquake motions. After Joshi (1960).



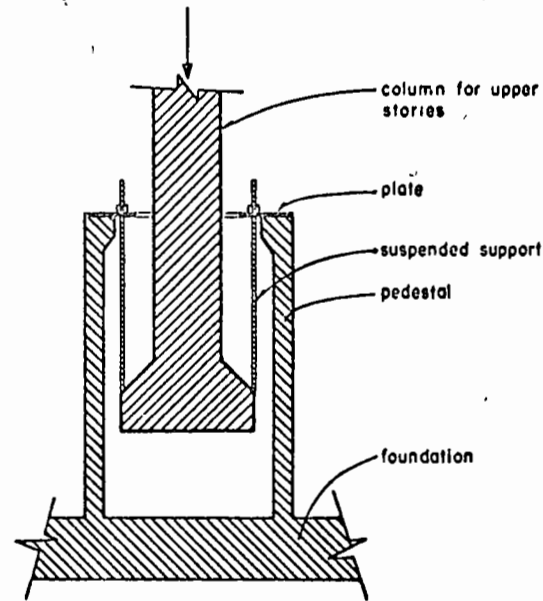


Figure 15.37. Suspended supports. *After Garza-Tamez (1968).*

### 15.13 Structural Synthesis

A very promising approach to earthquake-resistant design of buildings lies in structural synthesis ("direct design"). At least two important steps have been made in this direction. One consists of fixing allowable story drifts in a multi-story building and using a computer program that, iteratively, selects story shear stiffnesses so that the envelope of the drifts produced by a given family of earthquake records is no more and no less than the allowable values (Matsushita and Izumi, 1965). Thus far the method has only been applied in the range of linear behavior.

The second contribution is applicable to single-story buildings of reinforced concrete. Through the use of graphs, it directly furnishes the required column sections and reinforcement given the design spectrum, the mass of the building, its height, the allowable drift, the allowable ductility factor, the concrete strength, and the yield-point stress of the reinforcement (Borges, 1965).

