32. Use la tabla 3.2.

Para encontrar fácilmente la frecuencia de un ______, se pueden *organizar* los datos como se ha hecho en la tabla 3.2. Vea los encabezados de las columnas: en la primera se indican los eventos y en la segunda se anotan los elementos que ocurrieron de cada evento.

Observe la tabla 3.1. ¿Qué calificaciones hacen que ocurra el evento A. 51-60?

TABLA 3.2

Evento (intervalo de calificaciones)	Elementos correspondientes a los intervalos
A: 51–60	59, 57
B: 61–70	67, 65, 69, 67, 67
C: 71–80	72, 73, 73, 77, 78, 78
D: 81–90	83, 82, 84, 89, 83, 84, 89, 87, 81, 83, 81
E: 91–100	99, 91, 97, 95, 91, 93

evento, 59, 57.

33 Observe que los números de la respuesta correcta al cuadro anterior se han anotado en el renglón del evento A de la tabla 3.2.

Observe la tabla 3.1. ¿Qué calificaciones corresponden al evento E: 91-100?

TABLA 3.1 REPETIDA

Número del Profesor	[illegible]	[illegible]
1	[illegible]	157
2	[illegible]	158
3	[illegible]	[illegible]
4	[illegible]	[illegible]
5	[illegible]	[illegible]
6	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
8	[illegible]	[illegible]
9	[illegible]	[illegible]
10	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
12	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
19	[illegible]	[illegible]
20	[illegible]	[illegible]
21	[illegible]	[illegible]
22	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
24	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]

36 La tabla 3.2 se puede elaborar fácilmente si previamente se *organizan* los datos en la forma presentada en la tabla 3.3 en la cual aparecen los datos *ordenados* en forma creciente (obsérvela). ¿Qué calificaciones de la tabla 3.3 hacen que ocurra el evento A. 51-60?

TABLA 3.3

Calificación en Pedagogía	Número del profesor
57	13
59	7
65	11
67	8
67	22
67	29
69	17
72	6
73	16
73	18
77	3
78	21
78	28
81	26
81	30
83	1
83	15
83	27
84	9
84	19
87	25
88	4
89	12
89	23
91	5
91	20
93	24
95	14
97	10
99	2

57, 59.

90 El primer paso en el proceso de agrupamiento de datos consiste en calcular el rango, el *segundo* consiste en decidir *cuántos* intervalos de clase se usarán. Es usual, dependiendo del número de observaciones, que el número de intervalos varíe entre 5 y 20. Tome la tabla 3.4 y diga cuántos intervalos se usaron para construirla.

TABLA 3.4 REPETIDA

Evento (intervalo de calificaciones)	Frecuencia
A: 51–60	2
B: 61–70	5
C: 71–80	6
D: 81–90	11
E: 91–100	6

5

Observe la tabla 4 3 ¿Cuál es el ancho de los intervalos de clase, del segundo al octavo?

TABLA 4.3 REPETIDA

Intervalo de clase, en minutos	Frecuencia
41-55	3
50-60	4
61-55	[illegible]
66-70	1[illegible]
71-75	11
76-80	11
81-85	11
86-90	6
91-100	8

5.0.

1 Observe la tabla 3.6. La frecuencia acumulada de clase del segundo intervalo es ________. ¿Cuál es el límite real superior de este intervalo, cuando las observaciones se aproximan al entero más cercano?

TABLA 3-6

Evento (intervalo de calificaciones)	Frecuencia	Frecuencia acumulada
A: 51-60	2	2
B: 61-70	5	7
C: 71-80	6	13
D: 81-90	11	24
E: 91-100	6	30
TOTAL:	30	

7, 70.5

Otra manera de calcular las frecuencias relativas acumuladas, consiste en sumarle a la frecuencia relativa de cada intervalo las correspondientes a todos los intervalos anteriores.

Observe la tabla 3.8 ¿Cuál es la frecuencia relativa acumulada del segundo intervalo?

TABLA 3.8 REPETIDA

Intervalos de clase	Frecuencia	Frecuencia relativa	Frecuencia acumulada	Frecuencia relativa acumulada
51-60	2	0.067	2	0.067
61-70	5	0.[illegible]	7	0.233
71-80	6	0.200	13	0.433
81-90	11	[illegible]	24	0.800
91-100	6	0.200	30	1.000
TOTAL:	30	1.000		

0.233.

198 Use la hoja de trabajo 3.2 y la tabla 3.9. Complete la *tabla de distribuciones de frecuencias* que se presenta en la hoja de trabajo, aprovechando los datos ordenados de la tabla. Verifique que la suma de todas las frecuencias sea 30 (número total de datos), y que la suma de todas las frecuencias relativas sea 1.0

Intervalo de estaturas	Elementos que ocurrieron	Frecuencia	Frecuencia relativa	Frecuencia acumulada	Frecuencia relativa acumulada
15[illegible]-162	160, 161	2	0.067 ([illegible])	2	0.067
163-168	163, 163, 163, 167, 167, 167, 167, 168, 168, 168	10	0.333 (33.3%)	[illegible]	0.40[illegible]
169-174	169, 169, 170, 171, 171, 173, 174	7	0.233 (23.3%)	19	0.633
175-180	175, 175, 175, 178, 179	5	0.167 (16.7%)	24	0.800
181-186	181, 181, 183, 184	4	0.133 (13.3%)	28	0.933
187-192	187, 191	2	0.067 (6.7%)	30	1.000
	TOTAL:	30	1.000 (100%)		

194 Observe la tabla 3.9 que presenta los datos ordenados del problema del cuadro anterior.

a) ¿Cuál es el dato de valor mínimo?
b) ¿Cuál es el dato de valor máximo?
c) ¿Cuál es el rango?

TABLA 3.9

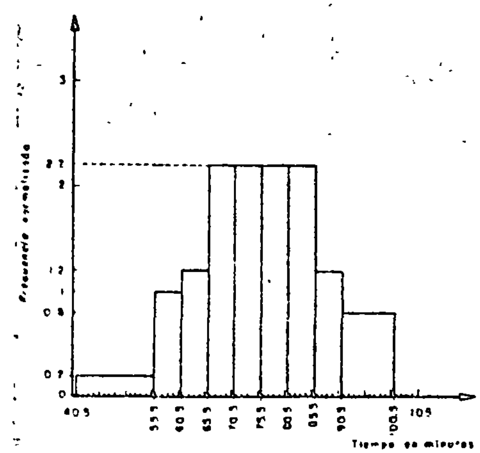
Datos ordenados en forma creciente	
Estatura, en cm	Número del profesor
160	5
161	[illegible]
163	6
163	14
163	22
167	[illegible]
167	13
167	11
167	[illegible]
168	8
168	24
168	30
168	2
169	29
170	25
171	7
171	28
173	23
174	12
175	3
175	9
175	15
178	16
179	10
181	4
181	11
183	19
184	27
187	[illegible]
191	20

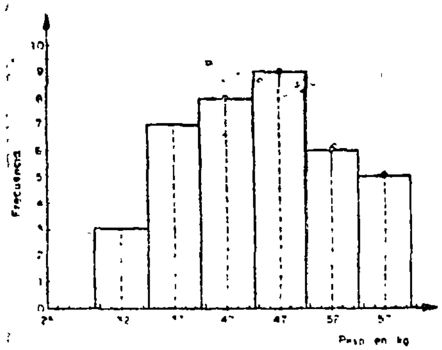
160, 191, 31;(191 - 160 = 31).

41 Use la hoja de trabajo 4.3. ¿Cuánto valen los límites reales superior e inferior, correspondientes a los tres primeros intervalos de clase? Recuerde que los tiempos se aproximaron al minuto más cercano. Después de verificar su respuesta anótela correctamente en la misma hoja de trabajo.

Inferior	Superior
40.5	55.5
55.5	60.5
60.5	65.5

42 Use la hoja de trabajo 4.4, para dibujar el histograma de los datos presentados en la hoja de trabajo 4.3 (recuerde que en el eje vertical tiene que anotar las frecuencias normalizadas). Indique en la figura los límites reales.

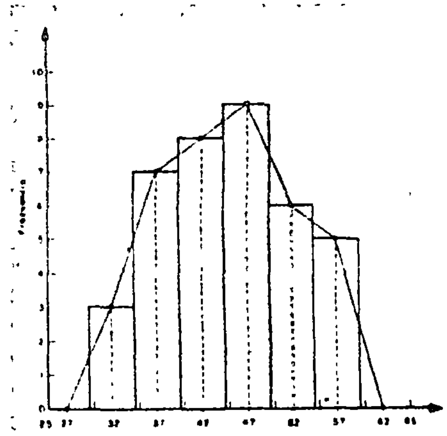




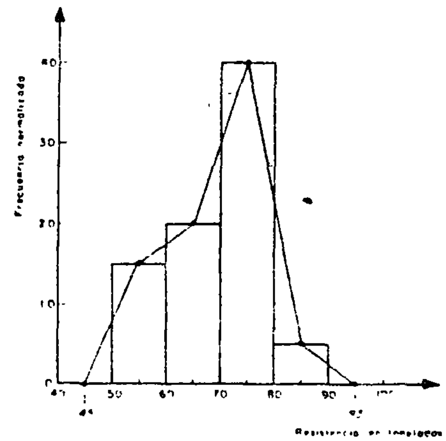
Vea la hoja de trabajo 4 5, en la que se muestra el histograma del cuadro anterior, con todas las marcas de clase señaladas en la parte superior de los rectángulos.

Observe que se han marcado dos puntos sobre el eje horizontal, en 27 y 62, los cuales corresponden a las marcas de clase de dos intervalos con *frecuencia nula*, cada uno de éstos con *igual amplitud* que el intervalo de clase adyacente. Observe, también, que los primeros tres puntos, correspondientes a las marcas de clase 27, 32, y 37, se unieron mediante líneas rectas.

Una los demás puntos directamente sobre el histograma.



Un polígono, como el dibujado en el cuadro anterior, que une todas las marcas de clase señaladas en la parte superior de los rectángulos, se conoce como *polígono de frecuencias*. Para dibujar los extremos del polígono de frecuencias, es necesario indicar dos marcas de clase adicionales, adyacentes una a la _____ (derecha/izquierda) del primer intervalo y la otra a la _________ del último.



68 En el histograma del cuadro anterior, *todos* los intervalos de clase tienen el *mismo* ancho.

Demostraremos a continuación que, cuando esto sucede, la suma de las *áreas* de todos los rectángulos, es *igual al área* comprendida entre el polígono de frecuencias y el eje horizontal. Para esto debemos recordar que las áreas de dos triángulos semejantes __________ (son/no son) proporcionales.

son

69 En la figura 4.1 se reproduce el histograma y el polígono de frecuencias del cuadro 67.

Observe que los triángulos *a* y *b* tienen igual área por ser triángulos semejantes y porque sus lados son iguales; por lo mismo, los triángulos *c* y *d* tienen igual área.

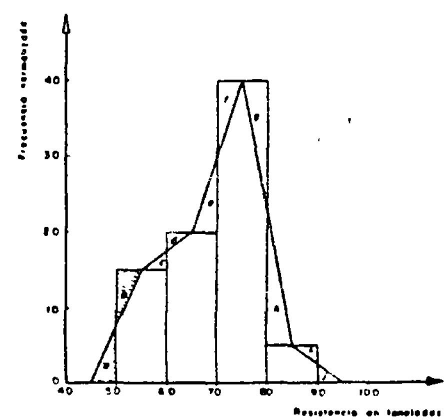


Fig 4.

1. El triángulo _____ (cuál) tiene igual área que el *f*.
2. El triángulo *h* tiene igual área que el ____.
3. El triángulo ____ tiene igual área que el *l*.

5

Intervalo de clase	Frecuencia acumulada	Frecuencia acumulada complementaria
41-55	3	69 (72-3=69)
56-60	8	64 (72-8=64)
61-65	14	
66-70	25	
71-75	36	
76-80	47	
81-85	58	
86-90	64	
91-100	72	

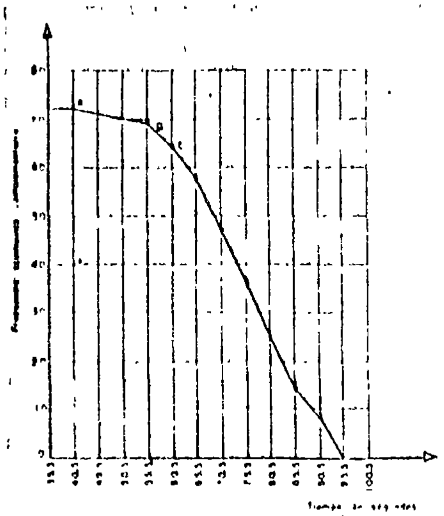
68, 47, 36, 25, 14, 8, 0

---

109 Tome la hoja de trabajo 4.10 y complete la curva de frecuencias acumuladas complementarias correspondiente a los datos presentados en la tabla 4.5. Observe que se anotó una frecuencia acumulada complementaria de 72 para un tiempo de 40.5, puesto que todos los datos fueron mayores que 40.5 (punto *A*). El punto B corresponde al límite real superior de 55.5 el cual tiene una frecuencia acumulada complementaria de 69. Al siguiente límite real superior (60.5), corresponde una frecuencia acumulada complementaria de 64 (punto *C*).

TABLA 4.5 REPETIDA

Intervalo de clase, en minutos	Límites reales		Frecuencia acumulada	Frecuencia acumulada complementaria
	Inferior	Superior		
41-55	40.5	55.5	3	69
56-60	55.5	60.5	8	64
61-65	60.5	65.5	14	58
66-70	65.5	70.5	25	47
71-75	70.5	75.5	36	36
76-80	75.5	80.5	47	25
81-85	80.5	85.5	58	14
86-90	85.5	90.5	64	8
91-100	90.5	100.5	72	0



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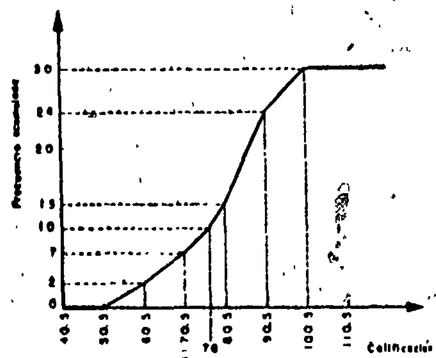
98 Observe que la siguiente curva de distribución de frecuencias acumuladas presenta las líneas horizontales mencionadas en los dos cuadros anteriores. ¿Cuál es la frecuencia de los valores:

menores que 106

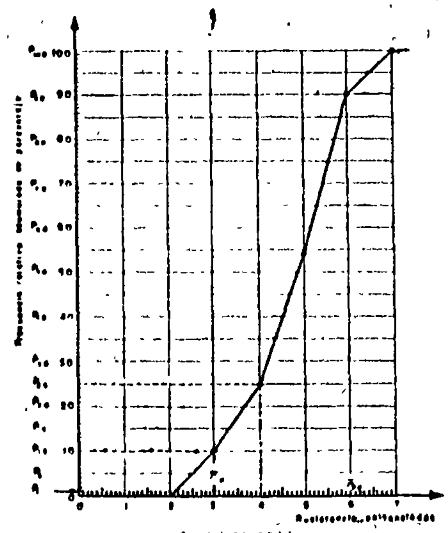
menores que 43

menores que 78?



30, 0, 10.

147 Observe la figura 4.4.
¿Cuánto vale el 49º percentil?



4.8 (aproximadamente).

5 A la clase de mayor frecuencia en una distribución se le conoce con el nombre de *modo, moda o valor modal.* Observe la figura 5.1. El modo corresponde a la clase ______________ (sarampión/tos ferina)

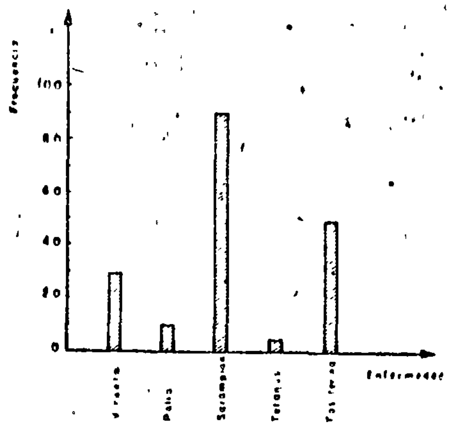


Fig 5.1 REPETIDA

sarampión

---

103 Si los datos están agrupados, se puede utilizar una tabla como la siguiente para calcular la varianza mediante la fórmula simplificada. Complete y calcule la variancia (haga todo el trabajo en esta hoja). Recuerde que $N = \Sigma f_i$.

x	f	xf	x^2	x^2f
3	2	6	9	18
4	1	4	16	16
6	3	10	36	
8	5	40	64	
10	4	40	100	
TOTAL:				

$\bar{X}$ = ______________

$\bar{X}^2$ = ______________

$\overline{X^2}$ = ______________

S^2 = ______________

HOJA DE TRABAJO 6.4

X, en pesos (marca de clase)	f (frecuencia)	Xf	X^2	X^2f
30	149			
100	280			
200	425			
300	82			
400	48			
600	5			
800	3			
TOTAL:				

Table 8.1

Standard Errors for Some Sampling Distributions

Sampling Distribution	Standard Error	Special Remarks
Means	$\sigma_{\bar{X}} = \dfrac{\sigma}{\sqrt{N}}$	This is true for large or small samples. The sampling distribution of means is very nearly normal for $N \geqq 30$ even when the population is non-normal. $\mu_{\bar{X}} = \mu$, the population mean in all cases.
Proportions	$\sigma_P = \sqrt{\dfrac{p(1-p)}{N}} = \sqrt{\dfrac{pq}{N}}$	The remarks made for means apply here as well. $\mu_P = p$ in all cases.
Standard Deviations	(1) $\sigma_s = \dfrac{\sigma}{\sqrt{2N}}$ (2) $\sigma_s = \sqrt{\dfrac{\mu_4 - \mu_2^2}{4N\mu_2}}$	For $N \geqq 100$, the sampling distribution of s is very nearly normal. σ_s is given by (1) only if the population is normal (or approximately normal). If the population is non-normal, (2) can be used. Note that (2) reduces to (1) when $\mu_2 = \sigma^2$ and $\mu_4 = 3\sigma^4$, which is true for normal populations. For $N \geqq 100$, $\mu_s = \sigma$ very nearly.
Medians	$\sigma_{\text{med.}} = \sigma\sqrt{\dfrac{\pi}{2N}} = \dfrac{1.2533\sigma}{\sqrt{N}}$	For $N \geqq 30$, the sampling distribution of the median is very nearly normal. The given result holds only if the population is normal (or approximately normal). $\mu_{\text{med.}} = \mu$.
First and Third Quartiles	$\sigma_{Q_1} = \sigma_{Q_3} = \dfrac{1.3626\sigma}{\sqrt{N}}$	The remarks made for medians apply here as well. μ_{Q_1} and μ_{Q_3} are very nearly equal to the first and third quartiles of the population. Note that $\sigma_{Q_2} = \sigma_{\text{med}}$
Deciles	$\sigma_{D_1} = \sigma_{D_9} = \dfrac{1.7094\sigma}{\sqrt{N}}$ $\sigma_{D_2} = \sigma_{D_8} = \dfrac{1.4288\sigma}{\sqrt{N}}$ $\sigma_{D_3} = \sigma_{D_7} = \dfrac{1.3180\sigma}{\sqrt{N}}$ $\sigma_{D_4} = \sigma_{D_6} = \dfrac{1.2680\sigma}{\sqrt{N}}$	The remarks made for medians apply here as well. $\mu_{D_1}, \mu_{D_2}, \ldots$ are very nearly equal to the first, second, ... deciles of the population. Note that $\sigma_{D_5} = \sigma_{\text{med.}}$
Semi-interquartile Ranges	$\sigma_Q = \dfrac{0.7867\sigma}{\sqrt{N}}$	The remarks made for medians apply here as well. μ_Q is very nearly equal to the population semi-interquartile range.
Variances	(1) $\sigma_{s^2} = \sigma^2\sqrt{\dfrac{2}{N}}$ (2) $\sigma_{s^2} = \sqrt{\dfrac{\mu_4 - \mu_2^2}{N}}$	The remarks made for standard deviation apply here as well. Note that (2) yields (1) in case the population is normal. $\mu_{s^2} = \sigma^2(N-1)/N$, which is very nearly σ^2 for large N.
Coefficients of Variation	$\sigma_V = \dfrac{v}{\sqrt{2N}}\sqrt{1+2v^2}$	Here $v = \sigma/\mu$ is the population coefficient of variation. The given result holds for normal (or nearly normal) populations and $N \geqq 100$.

(a) The graph of β vs. p, shown in Fig. 10-7(a), is called the *operating characteristic curve* or *OC curve* of the decision rule or test of hypothesis.

The distance from the maximum point of the *OC* curve to the line $\beta = 1$ is equal to $\alpha = .0358$, the level of significance of the test.

In general, the sharper the peak of the *OC* curve the better is the decision rule for rejecting hypotheses which are not valid.

(b) The graph of $(1-\beta)$ vs. p, shown in Fig. 10-7(b), is called the *power curve* of the decision rule or test of hypothesis. This curve is obtained simply by inverting the *OC* curve, so that actually both graphs are equivalent.

The quantity $(1-\beta)$ is often called a *power function* since it indicates the ability or *power of a test* to reject hypotheses which are false, i.e. should be rejected. The quantity β is also called the *operating characteristic function* of a test.

13. A company manufactures rope whose breaking strengths have a mean of 300 lb and standard deviation 24 lb. It is believed that by a newly developed process the mean breaking strength can be increased.

(a) Design a decision rule for rejecting the old process at a .01 level of significance if it is agreed to test 64 ropes.

(b) Under the decision rule adopted in (a), what is the probability of accepting the old process when in fact the new process has increased the mean breaking strength to 310 lb? Assume the standard deviation is still 24 lb.

Solution:

(a) If μ is the mean breaking strength, we wish to decide between the hypotheses:

H_0: $\mu = 300$ lb, and the new process is the same as the old one.
H_1: $\mu > 300$ lb, and the new process is better than the old one.

For a one-tailed test at a .01 level of significance, we have the following decision rule [refer to Fig. 10-8(a)]:

(1) Reject H_0 if the z score of the sample mean breaking strength is greater than 2.33.

(2) Accept H_0 otherwise.

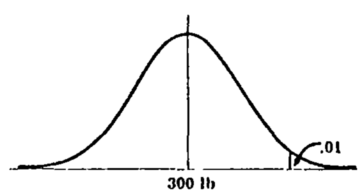


Fig. 10-8(a)

Since $z = \dfrac{\bar{X}-\mu}{\sigma/\sqrt{N}} = \dfrac{\bar{X}-300}{24/\sqrt{64}}$, $\bar{X} = 300 + 3z$. Then if $z > 2.33$, $\bar{X} > 300 + 3(2.33) = 307.0$ lb.

Thus the above decision rule becomes:

(1) Reject H_0 if the mean breaking strength of 64 ropes exceeds 307.0 lb.

(2) Accept H_0 otherwise.

(b) Consider the two hypotheses H_0: $\mu = 300$ lb and H_1: $\mu = 310$ lb. The distributions of mean breaking strengths corresponding to these two hypotheses are represented respectively by the left and right normal distributions of Fig. 10-8(b).

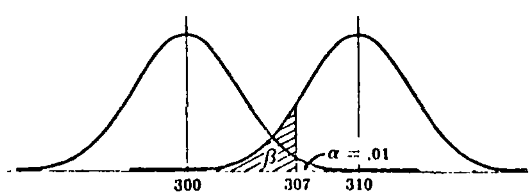


Fig. 10-8(b)

The probability of accepting the old process when the new mean breaking strength is actually 310 lb is represented by the region of area β in Fig. 10-8(b). To find this, note that 307.0 lb in standard units $= (307.0 - 310)/3 = -1.00$; hence

$$\beta = (\text{area under right-hand normal curve to left of } z = -1.00) = .1587$$

This is the probability of accepting H_0: $\mu = 300$ lb when actually H_1: $\mu = 310$ lb is true, i.e. it is the probability of making a Type II error.

14. Construct (a) an *OC* curve and (b) a power curve for Problem 13, assuming that the standard deviation of breaking strengths remains at 24 lb.

Solution:

By reasoning similar to that used in Problem 13(b), we can find β for the cases where the new

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Referencia:
Fabrycky, Ghare y Torgersen, "Industrial operations research", Ed. Prentice-Hall (1972)

11

11.1 THE CONCEPT OF ACCEPTANCE SAMPLING

The quality of a group of items may be verified in one of three ways. Every item in the lot may be inspected, a sample of items may be taken from the lot and inspected, or no inspection may be used. In the third case it is assumed that the quality of the lot certainly exceeds some minimum acceptable standard. In the former cases, the lot may be accepted or rejected, depending upon the outcome of the inspection process.

The level of verification chosen should consider the cost of inspection measured against the cost of accepting and perhaps using defective items. In general, acceptance sampling will be more economical than 100 per cent inspection when the occurrence of a defective in an accepted lot is not prohibitively expensive or when an inspection process requires the destruction of the item. Acceptance sampling will be more economical than no inspection when some expense is incurred in accepting defectives and the number of defectives differ from one lot to the next. The concept of acceptance sampling is presented in this section.

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making a Type I error. If three sigma limits are used on the $\bar{X}$ chart, this probability is approximately 0.0027, an unlikely event.

Of at least equal concern to the decision maker is the probability of making or not making a Type II error. When changes in the pattern of variation do occur, the decision maker is concerned with the model's ability to detect these changes. The probability of making a Type II error can be demonstrated with an operating characteristic or OC curve. An OC curve for an $\bar{X}$ chart of three sigma limits is illustrated in Figure 10.7. The ordinate is the probability of not detecting a shift in the mean of a pattern of variation, assuming that only the mean and not the dispersion has shifted. The magnitude of the shift in the mean is defined in terms of k as $\mu + k\sigma_{\bar{x}}$. This permits one such OC curve to describe all $\bar{X}$ charts with three sigma limits. Superimposed upon the OC curve of Figure 10.7 are a series of distributions

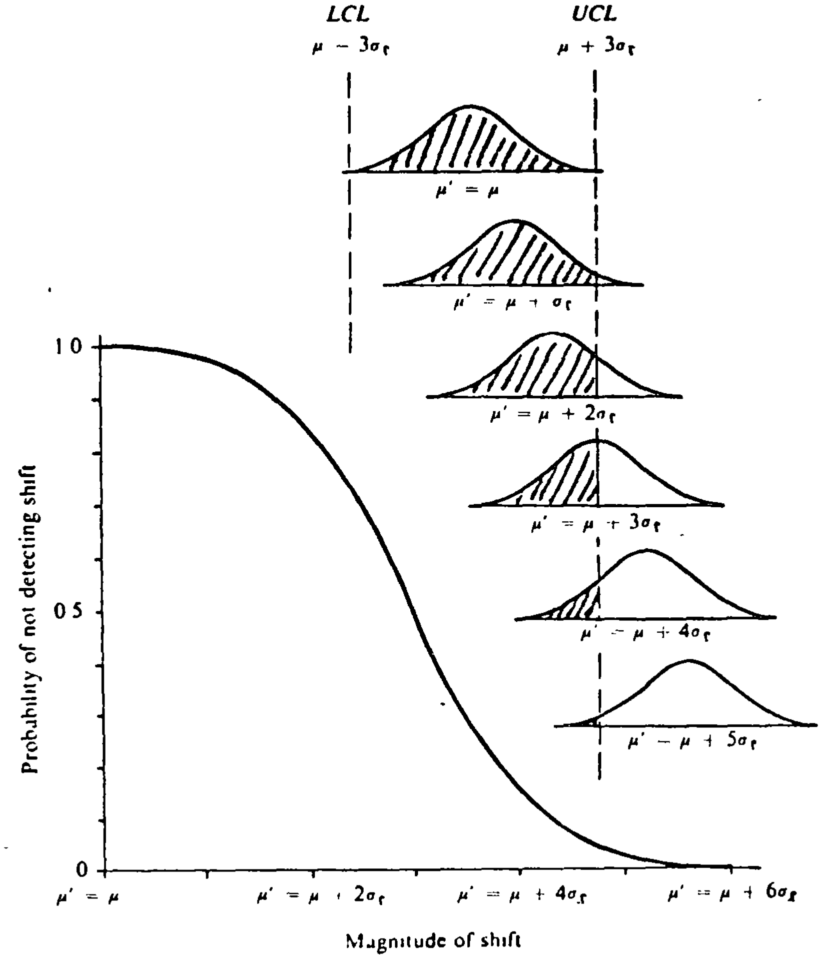

Figure 10.7. An operating characteristic curve for an $\bar{X}$ chart.

Acceptance sampling plans. The most elementary acceptance sampling plan calls for the random selection of a sample of size n, from a lot containing N items. The entire lot is then accepted if the number of defectives found in the sample is equal to or less than c, the acceptance number. For example, a sampling inspection plan might be defined as $N = 1{,}000$, $n = 50$, and $c = 1$. This designation means that a sample of 50 items is to be taken from the lot of 1,000. If zero or one defective is found in the sample, the whole lot is accepted. If more than one defective is found, the lot is rejected. A rejected lot can either be returned to the producer or it can be retained and subjected to a 100 per cent screening process. The former action is called a *nonrectifying inspection program*, the latter, a *rectifying* inspection program.

The type of inspection sampling described by N, n, and c uses inspection by attributes and a single sample of size n. Other attribute inspection plans might use two samples before requiring the acceptance or rejection of a lot. A third procedure might use multiple samples or a sequential sampling process in evaluating a lot. Each of these methods—single, double, and multiple sampling—rests upon a system of inspection by attributes of items logically grouped into lots. When it is not feasible to divide a continuous production process into discrete lots, a special class of attribute sampling methods must be used. These continuous sampling models verify the quality of the process output through inspection of a proportion of the items produced.

Inspection may also be by variables. Here a measurement is obtained and recorded as a continuous dimension, subject only to the limitations of the measuring instrument or the convenience of measurement rather than as a simple classification of acceptable versus defective units. Acceptance sampling by variables represents a whole class of acceptance sampling models, each member of which still retains the element of a sample selected from a discrete lot, but with quality verified through the measuring of a continuous dimension.

The operating characteristics curve. Acceptance sampling plans attempt to discriminate between lots of acceptable and lots of unacceptable items. The relative ability of a sampling plan to meet this objective can be demonstrated with an operating characteristic curve. An OC curve defines the probability of a lot being accepted (or finding c or fewer defectives in a sample) for different levels of proportion defective.

An operating characteristic curve for the sampling plan $N = 1{,}000$, $n = 50$, $c = 1$ is illustrated in Figure 11.1. The abscissa refers to the proportion defective in the lot. The ordinate refers to the probability of accepting a lot at a specified level of proportion defective. Note that if N contains no defectives and if $p = 0$, then the lot is certain to be accepted. If the lot contains 10 defectives and if $p = 0.01$, then the probability of accepting the

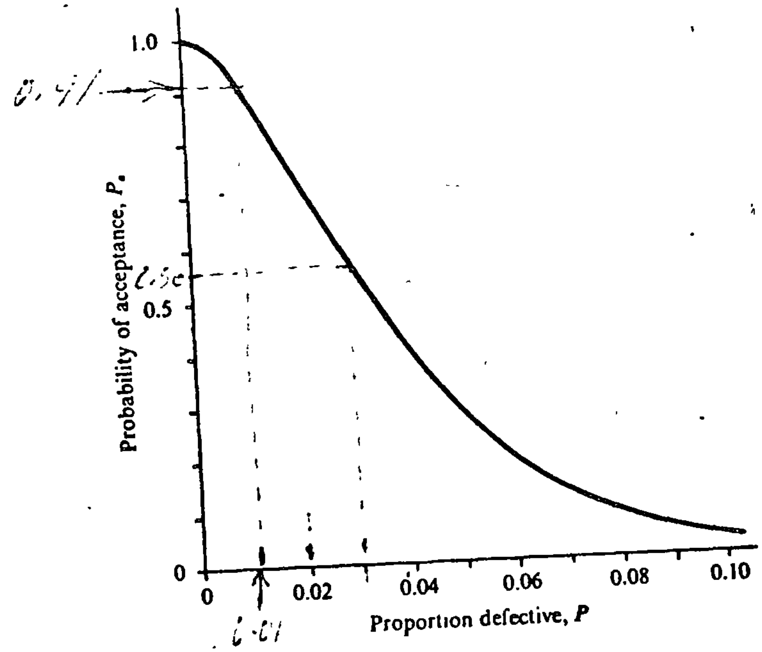

Figure 11.1. An operating characteristic curve for the sampling plan $n = 50$, $c = 1$.

lot is 0.91. The probability distribution appropriate to these calculations is the hypergeometric. For ease of calculation, however, the Poisson distribution is used as an approximation, as was illustrated in Figure 3.8. Thus, the Thorndike chart of Figure 3.2 may be used to develop an OC curve quickly. As an example, at $p = 0.03$ in Figure 11.1, $np = (50)(0.03) = 1.50$ and the probability of the occurrence of one or fewer defects would be 0.56. Note that because the Poisson distribution is used as an approximation, the OC curve is independent of the lot size.

A good sampling plan will have a high probability of accepting those lots which contain few defectives and a low probability of accepting lots having an excessive number of defectives. The OC curve illustrates how well a given sampling plan discriminates between good and bad lots. *Good* and *bad* are relative terms, and a lot containing 1 per cent defective might be considered quite good in one instance and very poor in another. Consider Figure 11.2 which illustrates a number of OC curves with only the acceptance number, c, differing in each case. The relative shapes of these curves are quite similar in that they are nearly parallel through their middle sections. In effect, increasing the acceptance number slides the OC curve to the right. This is indicative of a definition of good lots which contain more defectives. As an example, the sampling plan $c = 3$, $n = 50$ might be used to accept

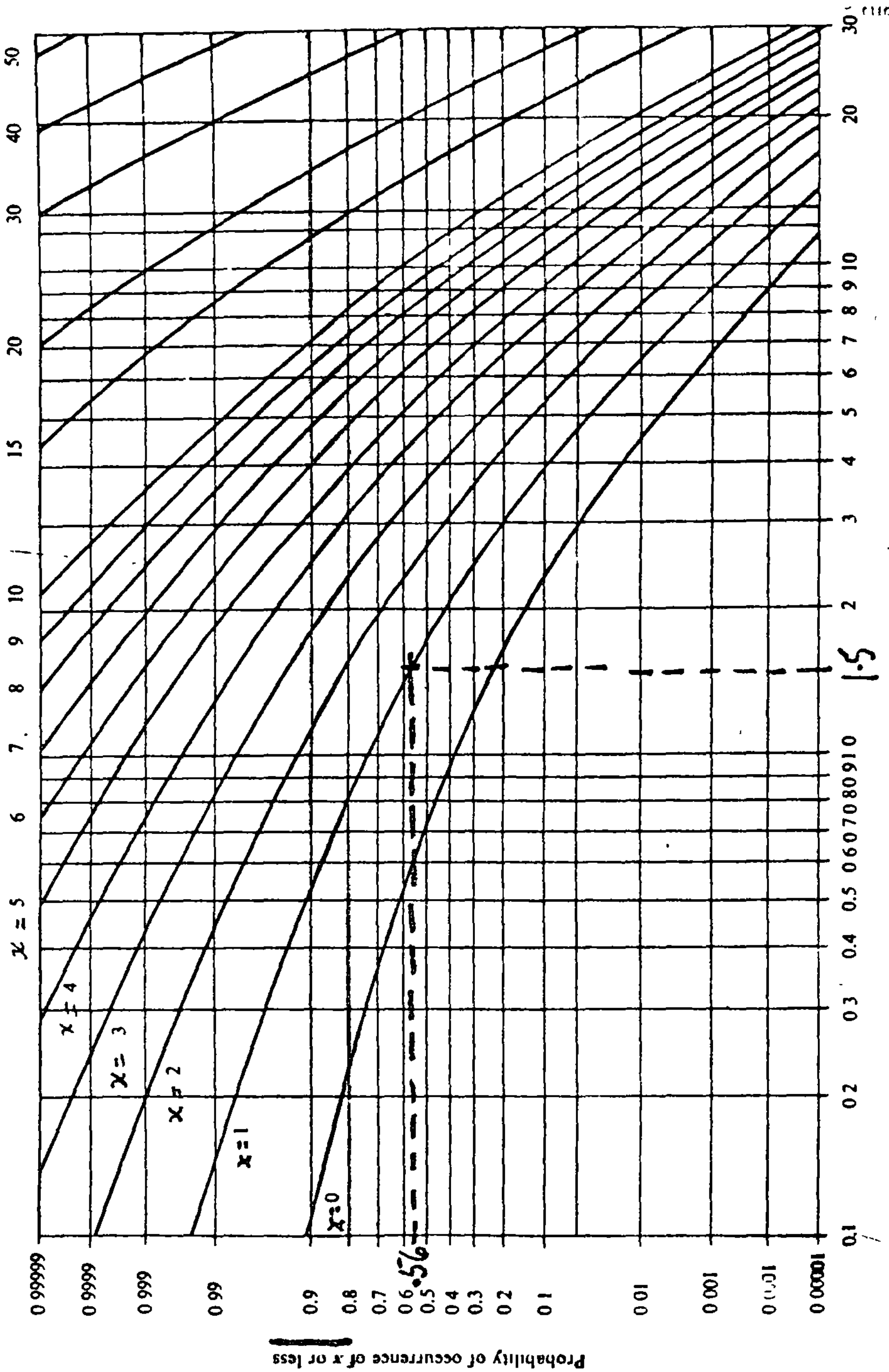

Figure 3.2. Cumulative probability curves for the Poisson distribution (modified Thorndike Chart). Reproduced, with permission, from H. F. Dodge and H. G. Romig, *Sampling Inspection Tables*, John Wiley & Sons, Inc., 1959.

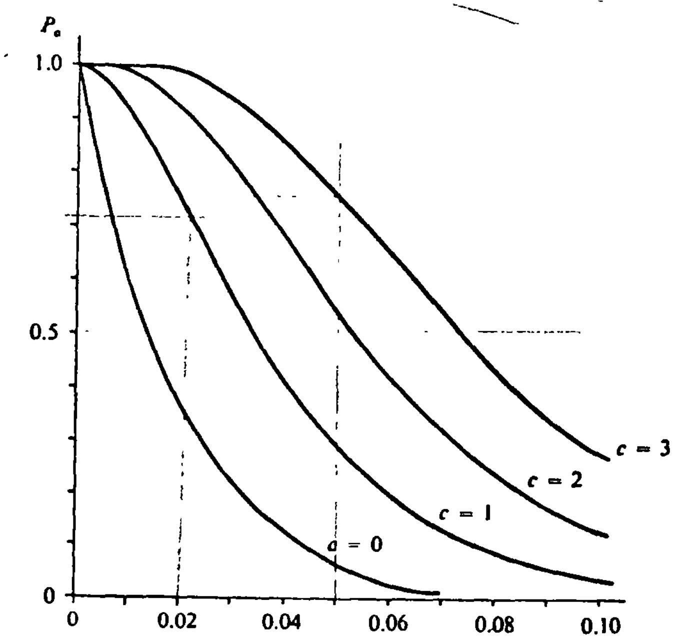

Figure 11.2. *OC* curves for different acceptance numbers with a constant sample size, $n = 50$.

lots where material up to 5 per cent defective is considered to represent a good lot. If no more than 2 per cent defective is considered acceptable, then the sampling plan $c = 1$, $n = 50$ might be employed. This is true only in approximate terms because the shape of the OC curve is not solely dependent upon the sample size. Actually, both sample size and acceptance number are parameters upon which the form of the OC curve depends.

Once an acceptable proportion defective is defined, the relative ability of a sampling model to discriminate between lots containing more or fewer defectives will, in large measure, be dependent upon the sample size. As an example, consider the OC curves of Figure 11.3, each of which contains the same ratio of acceptance number to sample size. Note that as the sample size increases, the OC curve becomes steeper. In general, this is desirable in a sampling plan, although the expense involved in this greater discriminating ability is the cost of a larger sample size. The ideal discrimination of a vertical line is indicated in Figure 11.3 with a dashed line. This, however, can be achieved only with 100 per cent inspection.

Consumer and producer risks. Two parties are involved in an inspection sampling procedure, the party submitting the lot and the party to whom the lot is consigned if accepted. These two parties are referred to as the *producer* and the *consumer*, respectively. The parties may represent a seller and a

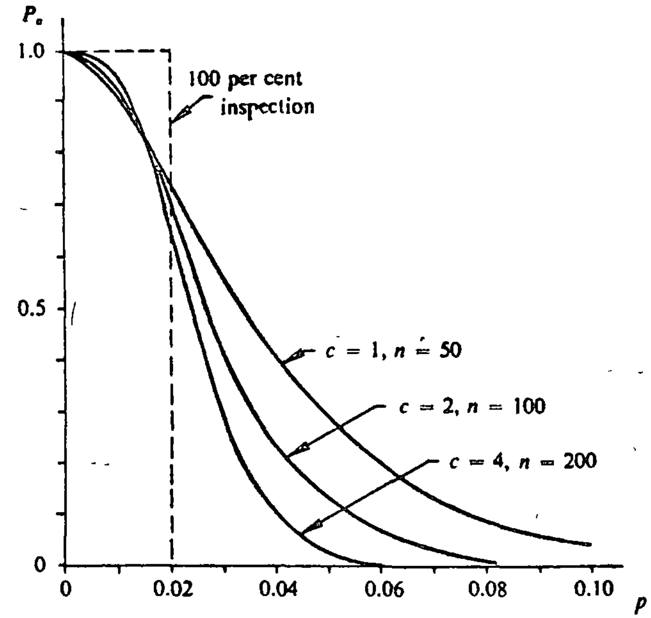

Figure 11.3. *OC* curves for three sampling plans, each with the same ratio of acceptance number to sample size.

buyer of a product, or they may represent two departments within the same organization. As an example, castings from a foundry department may be delivered for acceptance to the machining department In another situation, the producer may be an accounting department and the consumer may be represented by an auditor who either accepts or rejects a number of accounting records against some criterion of accounting quality. In each case, the producer usually desires that material relatively free from defectives have a high probability of being accepted. The consumer desires that it will be unlikely for the lot to be accepted if it contains a high proportion of defectives.

The concept of producer and consumer risks can be defined in terms of two points on an operating characteristic curve. The producer risk point occurs at a fraction defective, p_1, the consumer risk point occurs at p_2. Four values are used to specify these two points which, in turn, may be used to construct the OC curve for a specific acceptance sampling plan.

(1) Acceptable quality level, AQL. This indicates a good level of quality and low proportion of fraction defective, referred to as p_1, for which it is desired to have a high probability of acceptance.
(2) Producer's risk, α: The probability that lots of the quality level given as the AQL will not be accepted where $\alpha = 1 - P_a$ In effect, this is the probability of making a Type I error, that is, of rejecting a lot when it should be accepted.

(3) Lot tolerance per cent defective, LTPD: This level of quality, given as p_2, is deemed to be quite poor and it is desired to reject lots of this quality or at least have a low probability of acceptance.
(4) Consumer's risk, β: The probability, P_a, that lots of a quality level at the LTPD will be accepted. A value of $P_a = 0.10$ at p_2 is often used in acceptance sampling. This probability represents the likelihood of making a Type II or β error, that is, of accepting a lot when it should be rejected.

Each of these values is illustrated on the OC curve of Figure 11.4. The development of a sampling plan from the producer risk point and the consumer risk point is presented in the next section.

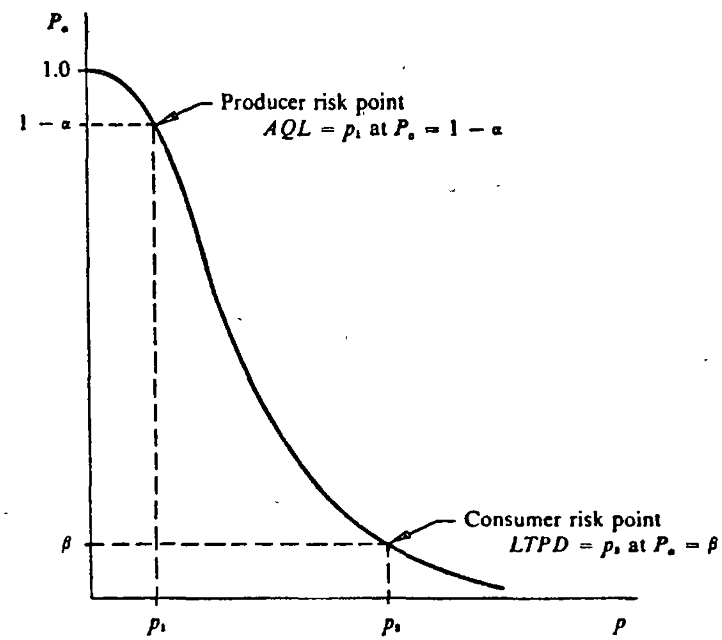

Figure 11.4. An *OC* curve passing through a Consumer Risk Point and a Producer Risk Point and possessing a unique value of *n* and *c*.

11.2 ACCEPTANCE SAMPLING BY ATTRIBUTES

Most acceptance sampling plans involve inspection by attributes. Often a unit can be assessed only in the two-valued classification of acceptable or defective. In other situations, it may be advantageous to take a continuous dimension and reduce it to a dichotomous assessment of within specifications and acceptable, or defective and outside specification limits. In either case, the function of the acceptance sampling model is to acc--- those lots containing few defectives and reject those lots containing many defectives. This objective is often defined in terms of a producer and a consumer risk.

Developing a single sampling plan. When two required points, such as a consumer and a producer risk point, are given as the basis for a sampling plan, the effect is to require the solution to two equations for two unknowns. An iterative, trial-and-error solution, using the Poisson distribution as an approximation, can be easily effected with the Thorndike chart of Figure 3.2, or the cumulative Poisson tables in Appendix A, Table A.1.

As an example, assume that a single sampling plan is desired which will yield an OC curve passing through a producer risk of $\alpha = 0.05$ at an AQL of 0.01, and a consumer risk of $\beta = 0.10$ at an LTPD of 0.04. The solution is facilitated if a table is constructed as illustrated in Table 11.1. This table permits the solution for n and c in the following equations:

$$(1 - 0.05) = \sum_{0}^{c} \frac{e^{-0.01n}(0.01n)^c}{c!}$$

$$0.10 = \sum_{0}^{c} \frac{e^{-0.04n}(0.04n)^c}{c!}.$$

These equations represent the producer and consumer risk points, respectively.

Table 11.1. A TABLE USED TO DETERMINE A SINGLE SAMPLING PLAN APPROXIMATING α = 0.05 AT AQL = 0.01 AND β = 0.10 AT LTPD = 0.04

c	p_1n ($P_a = 0.95$)	p_2n ($P_a = 0.10$)	$\frac{p_2}{p_1}$
0	0.05	2.31	46.2
1	0.35	3.89	11.1
2	0.82	5.33	6.50
3	1.36	6.68	4.91
4	1.97	8.00	4.06
5	2.61	9.30	3.56

$\frac{\text{LTPD}}{\text{AQL}}$

In Table 11.1, if an acceptance number of $c = 0$ is required and if $P_a = 0.95$, then p_1n must be 0.05. This value was obtained by interpolation in the cumulative Poisson tables. Less precise values may be obtained more quickly from the Thorndike chart, although this specific value lies outside the limits of the chart. The second value, for $c = 1$ at $P_a = 0.95$, yields $p_1n = 0.35$. If $c = 2$ and $P_a = 0.95$, then $p_1n = 0.82$, and so forth. This process is

repeated for $P_a = 0.10$ to find the values given in the third column. The fourth column in Table 11.1 is completed by recording the ratio of p_2/p_1. In this example, the ratio of fraction defective of consumer to producer risk points was given as 0.04/0.01 = 4.0. Therefore, this process is continued until the desired ratio of $p_2/p_1 = 4.0$ is bracketed.

The desired sampling plan calls for an acceptance number somewhere between $c = 4$ and $c = 5$. Because both the acceptance number and the sample size must be integers, it is not possible to achieve the precise requirement that was given. One of the four plans listed in Table 11.2 must be selected with the associated degree of protection. The data from Table 11.2 are developed directly from Table 11.1 as follows: With $c = 4$ and $P_a = 0.95$,

Table 11.2. FOUR SAMPLING PLANS WHICH BRACKET α = 0.05 AT AQL = 0.01 AND β = 0.10 AT LTPD = 0.04

Plan	α *at* p_1	β *at* p_2
$c = 4, n = 197$	0.050	0.107
$c = 4, n = 200$	0.053	0.100
$c = 5, n = 261$	0.050	0.052
$c = 5, n = 233$	0.032	0.100

p_1n was taken to be 1.97, and if $p = 0.01$ at the producer risk point, then $n = 197$. This sampling plan of $c = 4$, $n = 197$ will yield an $\alpha = 0.05$ as required but will yield a $\beta = 0.107$ which is slightly higher than desired. If $c = 4$ and it is desired that the OC curve go through the consumer risk point, then $n = p_2n/p_2 = 8.00/0.04 = 200$. With $c = 4$ and $n = 200$, β is established at 0.10 and α will be $1 - P_a$ or $1 - 0.947 = 0.053$. Similarly, α and β can be found for $c = 5$.

The four plans of Table 11.2 were obtained by alternating in holding α as required and solving for β and then maintaining β while solving for α. These four plans are sketched in Figure 11.5 with the risk point bracketing effect magnified so that it will be more evident. Once these four plans have been defined, it is likely that a plan will be selected and used which results in a compromise in regard to α and β. In this example $c = 4$ yields two plans fairly close to the consumer and producer risk points. One or the other might be selected. The plan $c = 4$, $n = 200$ would have the advantage of a convenient sample size which would facilitate subsequent computations. In other cases, the average sample size for one or the other acceptance number could be used.

Average outgoing quality. When rejected lots are returned to the supplier, the acceptance sampling plan does not significantly improve the quality level of lots submitted to the plan. A few defectives may be detected and discarded

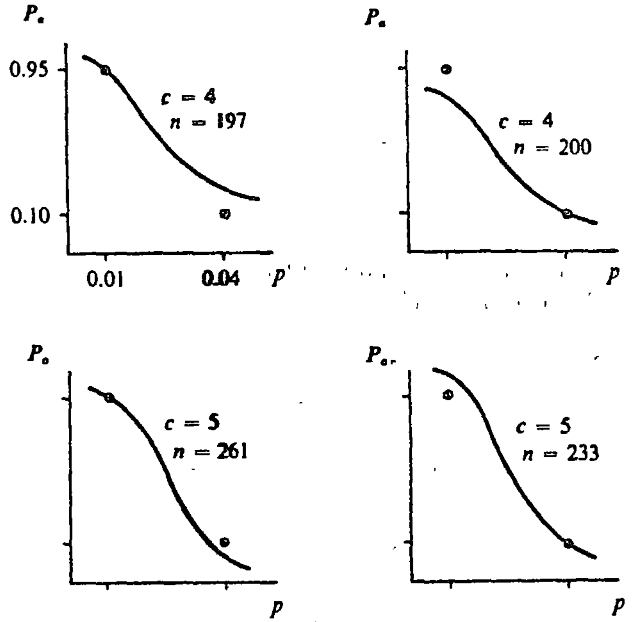

Figure 11.5. The four sampling plans of Table 11.2 (not to scale).

from samples of accepted lots, but no profound improvements can be realized here without resulting in the rejection of the lot. The sampling plan can and should function as a screening process and permit the acceptance of good lots and the rejection and return of poor quality lots. This will result in some improvement if there is a large variation in the level of quality from one lot to another.

When a rectifying inspection program is employed and rejected lots are subjected to a 100 per cent inspection, significant and predictable quality improvements can be realized. Under a rectifying inspection program, an average outgoing level, AOQ, and an average inspection load, I, can be predicted for varying levels of incoming fraction defective. In addition, an average outgoing quality limit, AOQL, the worst possible average outgoing quality level, can be forecast and related to a specific incoming level of fraction defective. This latter value gives assurance regarding the poorest average quality level that might leave the inspection station.

If it is assumed that all lots arriving at an inspection station contain the same proportion of defectives, p, and if rejected lots are subjected to 100 per cent inspection,

$$\text{AOQ} = \frac{P_a(p)(N-n)}{N - pn - (1-P_a)p(N-n)}. \tag{11.1}$$

The numerator in Equation (11.1) represents the average number of defectives

in each lot beyond the point of inspection. Defectives will be found only in the proportion of lots which have been accepted, P_a, and will constitute $p(N-n)$ in number. The denominator represents the average lot size, where N is the original lot size, pn is the reduction in size due to defectives found and discarded in the sample, and $(1-P_a)p(N-n)$ is the reduction in lot size due to defectives found and discarded during the 100 per cent screening process. By similar reasoning, the average inspection can be shown to be

$$I = n + (1-P_a)(N-n). \tag{11.2}$$

As an example, if a lot size of $N = 10{,}000$ is assumed and each lot contains 200 defectives, then for the sampling plan previously developed of $n = 200$ and $c = 5$, P_a can be found to be 0.785, and from Equation (11.1),

$$\text{AOQ} = \frac{(0.785)(0.02)(9{,}800)}{10{,}000 - (0.02)(200) - (0.215)(0.02)(9{,}800)}$$
$$= \frac{153.86}{10{,}000 - 4 - 42.14} = 0.01546.$$

The average inspection can be found from Equation (11.2) to be

$$I = 200 + (0.215)(9{,}800) = 2{,}307.00.$$

It should be recognized that these values of AOQ and of I are expected or average values that will be approached in the long run over many lots. In regard to the proportion defective, one specific lot will either contain somewhere between 195 to 200 defectives if the lot is accepted and it is assumed to contain no defectives if the lot is rejected. By the same token, either 200 items will be inspected if the lot is accepted or the total of 10,000 units will be verified if it is rejected. In the long run, however, the foregoing results for AOQ and I will represent the average for all lots submitted at a value of $p = 0.02$.

Under some conditions, it might be desired to retain a constant lot size whether a lot is accepted or rejected and regardless of the number of defectives discarded during the sampling and/or screening process. A constant lot size can be maintained if defectives are replaced by units which are assumed to be selected, inspected, and inserted in the place of the defectives if they are acceptable. Under these conditions of replacement,

$$\text{AOQ} = \frac{P_a(p)(N-n)}{N}. \tag{11.3}$$

The average inspection increases slightly to

$$I = \frac{n + (1-P_a)(N-n)}{1-p}. \tag{11.4}$$

The average outgoing quality and the average inspection will vary as a function of the level of incoming proportion defective. With the sampling plan $n = 200$, $c = 5$, under the condition of nonreplacement of defectives, AOQ and I are given in Table 11.3 and sketched in Figure 11.6 and Figure 11.7. Note that the average outgoing quality increases as the proportion defective in incoming lots increases until it reaches a maximum value. This value is referred to as the *average outgoing quality limit*, AOQL. From this point on, a pronounced number of lots are being rejected and screened under 100 per cent inspection. This latter effect is resulting in a continuing reduction in the average outgoing quality, as can be seen in Figure 11.7. The concept of an average outgoing quality limit is often employed in specifying a sampling

Table 11.3. AVERAGE OUTGOING QUALITY AND AVERAGE INSPECTION FOR THE SINGLE SAMPLING PLAN OF $N = 10{,}000$, $n = 200$, $c = 5$ UNDER RECTIFYING INSPECTION WITHOUT REPLACEMENT

Proportion Defective in Submitted Lots p	*Probability of Acceptance* P_a	*Average Outgoing Quality* AOQ	*Average Inspection* I
0	1.000	0	200.0
0.005	0.999	0.00490	209.8
0.010	0.983	0.00964	366.6
0.015	0.916	0.01349	1,023.2
0.020	0.785	0.01546	2,307.0
0.025	0.616	0.01524	3,924.0
0.030	0.446	0.01334	5,629.2
0.035	0.301	0.01059	7,050.2
0.040	0.191	0.00774	8,128.2
0.045	0.116	0.00533	8,863.2
0.050	0.067	0.00344	9,343.4

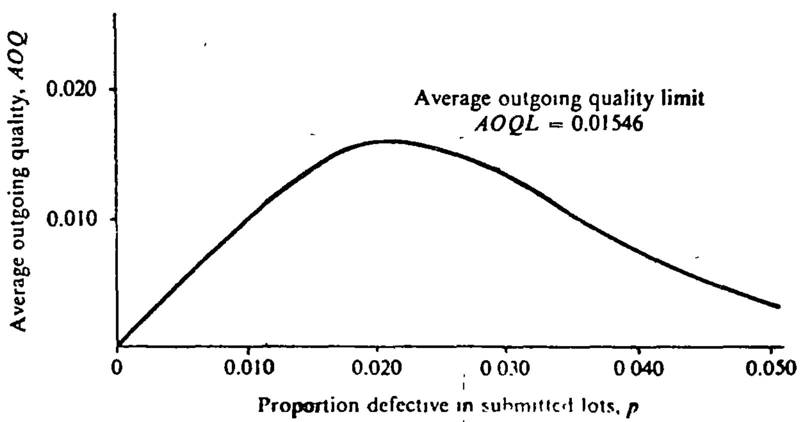

Figure 11.6. Average outgoing quality for the data of Table 11.3.

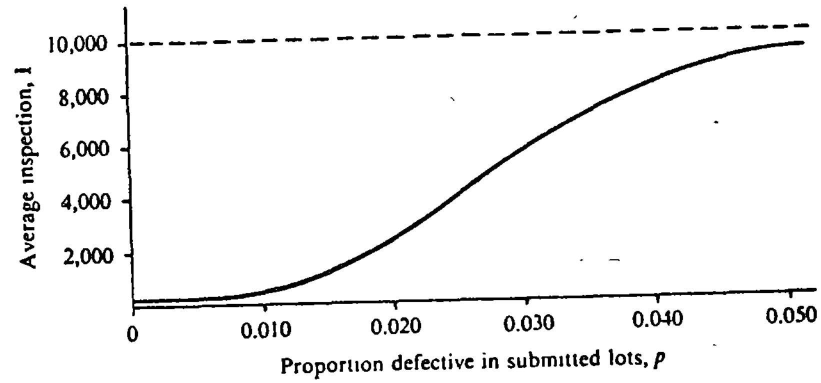

Figure 11.7. Average inspection for the data of Table 11.3.

plan. Sampling plans have been devised for varying values of AOQL and presented in tabular form. With this limit as the worst average quality level that can be expected to occur, a plan with a known AOQL can be selected along with other desired criteria.

The concept of a level of average outgoing quality and a level of average inspection rests upon the assumption of the detection and removal of all defectives from screened lots. Further, the values obtained for AOQ and I are expected values that will occur in the long run. Over a short time period some variation from these values can be expected. The concept of an average outgoing quality level and average inspection has found wide application in the field of product acceptance. In recent years it has also been found applicable in auditing accounting records and verifying clerical activities.

Double sampling plans. A single sampling plan requires a decision to accept or reject a lot on the basis of the evidence offered from a single sample. A double sampling plan permits the acceptance or rejection of a lot after a single sample, but also permits the alternative of taking a second sample before making the decision. A double sampling plan is defined with a lot size and two sample sizes and two acceptance numbers, designated respectively as N, n_1, n_2, c_1, and c_2, with c_2 always larger than c_1.

Under a double sampling program, a sample n_1 is taken from the lot N. If c_1 or fewer defectives are detected, the lot is accepted. If more than c_2 defectives are found, the lot is rejected. If c_2 or less, but more than c_1 defectives are found, then a second sample n_2 is taken. The lot is finally accepted if c_2 or fewer defectives are found in the combined sample of $n_1 + n_2$. The lot is rejected if more than c_2 defectives are found in $n_1 + n_2$.

The operation of a double sampling plan and the OC curve which defines this plan can be illustrated with an example. Assume that $N = 10{,}000$, $n_1 = 50$, $n_2 = 80$, $c_1 = 0$, and $c_2 = 3$. This plan states that if no defectives, $c_1 = 0$, are found in the first sample of $n_1 = 50$, the lot is immediately accepted. If more than $c_2 = 3$ defectives are found, the lot is immediately rejected. If 1, 2, or 3 defectives are found, then a second sample, $n_2 = 80$, is taken. If $c_2 = 3$ or fewer total defectives are found in the combined sample of $n_1 + n_2 = 130$, the lot is accepted. If more than $c_2 = 3$ are found, it is rejected.

The probabilities associated with each of these alternatives can be calculated for any value of proportion defective. In the preceding example, assume

Table 11.4. A COMPUTATIONAL SCHEME FOR FINDING P_{a2} AT $p = 0.02$ IF $N = 10{,}000$, $n_1 = 50$, $n_2 = 80$, $c_1 = 0$, AND $c_2 = 3$

Defects in n_1	*Probability*	*Defects in n_2 to Accept*	*Probability*
1	0.368	2 or less	0.783
2	0.184	1 or less	0.525
3	0.061	0	0.202

that the lot of 10,000 units contains 200 defectives, $p = 0.02$. The probability of accepting this lot on the first sample, P_{a1}, is the probability of finding no defectives in a sample of 50 taken from a lot with $p = 0.02$. Using the Poisson approximation, for $c = 0$ at $np = 1.00$, gives $P_{a1} = 0.368$. The probability of rejecting on the first sample is the probability of finding more than three defectives in the sample. This is $1 - P$(3 or fewer) and $P_{r1} = 0.019$.

The probability of accepting the lot after inspecting the second sample requires the use of conditional probabilities as demonstrated in Table 11.4. This probability is

$$P_{a2} = (0.368)(0.783) + (0.184)(0.525) + (0.061)(0.202) = 0.397.$$

The probability of rejection on the second sample must therefore be

$$\begin{aligned} P_{r2} &= 1 - P_{a1} - P_{r1} - P_{a2} \\ &= 1 - 0.368 - 0.019 - 0.397 = 0.216. \end{aligned}$$

The total probability of acceptance is

$$P_{a1} + P_{a2} = 0.368 + 0.397 = 0.765.$$

And the total probability of rejection is

$$P_{r1} + P_{r2} = 0.019 + 0.216 = 0.235.$$

', the probability of making a decision to accept or reject the lot afte e

first sample is

$$P_{a1} + P_{r1} = 0.368 + 0.019 = 0.387.$$

And the probability of requiring a second sample is

$$P_{a2} + P_{r2} = 0.397 + 0.216 = 0.613.$$

Similar probabilities can be calculated for varying levels of fraction defective. These may be used to develop the OC curves illustrated in Figure 11.8.

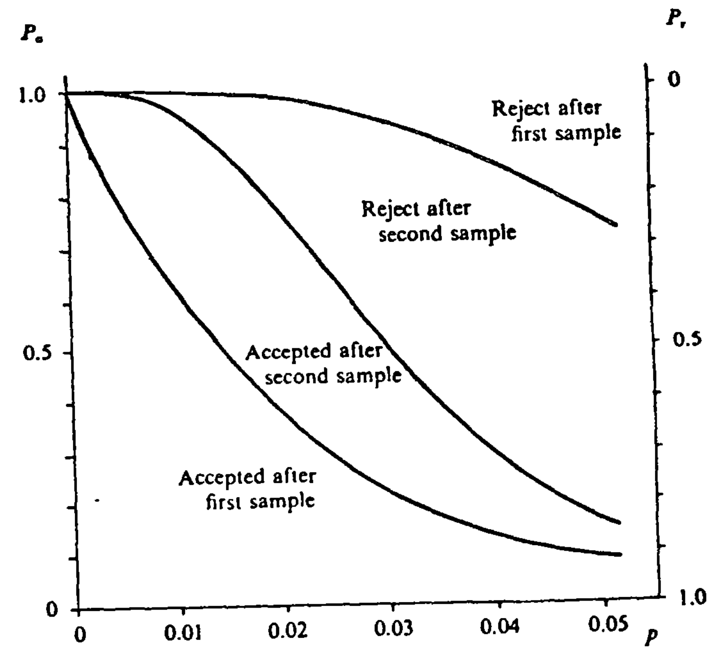

Figure 11.8. *OC* curves for the double sampling plan $n_1 = 50$, $n_2 = 80$, $c_1 = 0$, $c_2 = 3$.

The average number of units inspected as a sample of each lot, ASN, will vary with the proportion defective, and thus, with the probability of making a decision on the first sample. This probability is

$$\text{ASN} = n_1 + n_2(1 - P_{a1} - P_{r1}). \quad (11.5)$$

Under a rectifying inspection program, the average inspection and average outgoing quality can be developed both for the case of nonreplacement and for the case of replacement of defectives. The average number of items inspected with the nonreplacement of defectives will be

$$I = n_1 + n_2(1 - P_{a1}) + (N - n_1 - n_2)(1 - P_{a1} - P_{a2}). \qquad (11.6)$$

As with single sampling, the AOQ is the ratio of the number of accepted defectives to the lot size. Defectives will be retained when they are not detected after a lot is accepted on the first or the second sample. If defectives which are found are not replaced to maintain a constant lot size, this lot size will be reduced by the elimination of defectives during the sampling and screening process. Thus, this proportion can be expressed as

$$\text{AOQ} = \frac{P_{a1}p(N - n_1) + P_{a2}p(N - n_1 - n_2)}{N - pI}. \qquad (11.7)$$

The equations for average inspection and average outgoing quality can be developed in a similar manner for the case of replacement of defectives.

Double sampling plans have the advantage of permitting the acceptance of good lots and the rejection of very poor lots with less inspection than a single sampling plan with a comparable OC curve. The double sampling plan also has the psychological advantage of giving a marginal lot a second chance, by permitting the taking of a second sample. The obvious disadvantage is the fluctuation in inspection workload that occurs as the quality level of incoming material varies.

Multiple and sequential sampling plans. A double sampling plan may defer a decision to accept or reject a lot until a second sample has been taken. A further extension of this is possible under multiple and sequential sampling. A multiple sampling plan is a simple extension of double sampling and may call for three or more samples before a decision is made. Sequential sampling is different only in that it does not call for specific sample sizes of n_1, n_2, n_3, etc., but calls for a continuous sequential sampling of units until the decision is made to accept or reject the lot. Sequential sampling is the limiting case of multiple sampling where $n_1 = n_2 = n_3 = \cdots = n_n = 1$.

The number of items inspected in sequential sampling is determined by the cumulative results of the inspection process. The sampling plan is defined by h_1, h_2, and s. This results in two parallel limit lines

$$c = h_2 + sn. \qquad (11.8)$$

$$c = -h_1 + sn. \qquad (11.9)$$

These limit lines are illustrated in Figure 11.9. They divide the area into regions of rejection, continued sampling, and acceptance. As soon as one of these two limit lines is reached or crossed, the lot is accepted or rejected.

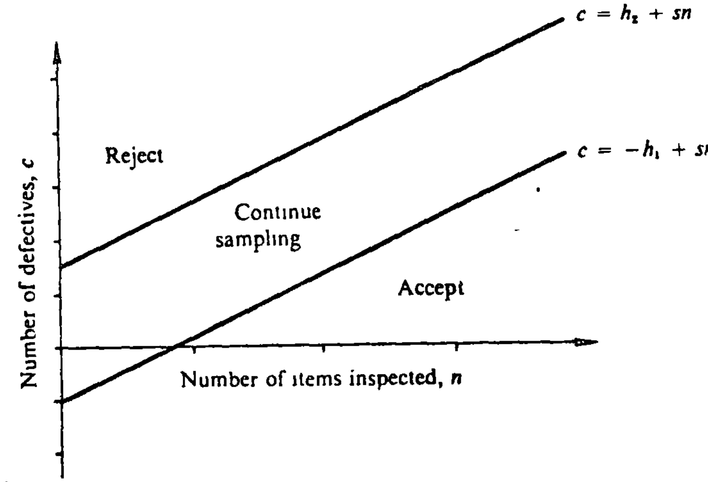

Figure 11.9. A graphical representation of a sequential sampling plan.

A sequential sampling plan can be developed that will meet specified producer and consumer risk points. With α, β, p_1, and p_2 as illustrated in Figure 11.4, it can be shown[1] that

$$h_1 = \frac{\log [(1 - \alpha)/\beta]}{\log \{[p_2(1 - p_1)]/[p_1(1 - p_2)]\}} \qquad (11.10)$$

$$h_2 = \frac{\log [(1 - \beta)/\alpha]}{\log \{[p_2(1 - p_1)]/[p_1(1 - p_2)]\}} \qquad (11.11)$$

$$s = \frac{\log [(1 - p_1)/(1 - p_2)]}{\log \{[p_2(1 - p_1)]/[p_1(1 - p_2)]\}}. \qquad (11.12)$$

For example, consider a sequential sampling plan defined as $h_1 = 1.00$, $h_2 = 1.50$, and $s = 0.12$. Assume a lot containing no defectives is submitted to this plan. How large a sample will be necessary to accept the lot? Acceptance will be possible when the line of $c = -1.00 + 0.120n$ is reached or crossed at $c = 0$ into the region of acceptance. With a sample of $n = 9$, this will be possible. As a second example, assume a lot is rejected after the twentieth unit was found to be a defective. How many total defectives would have to be found in the sample of 20? If rejection occurred on the twentieth unit, then at $n = 20$ the total number of defectives found must have just reached or exceeded $1.500 + 0.120(20)$ or $c = 4$.

Multiple and sequential sampling plans may be expressed as OC curves, or they may be developed from consumer and producer risk points on an OC curve. The advantages in their use are extensions of the advantages of double

[1] A. Wald, *Sequential Analysis* (New York. John Wiley & Sons, Inc., 1947).

sampling plans over single sampling plans of comparable protection. Very good lots and rather poor lots can be accepted or rejected with even smaller sample sizes, but the inspection workload problem becomes still more pronounced and dependent upon the quality level of the incoming material.

Sampling plans for continuous production. Under some conditions, the formation of inspection lots may be artificial. Where production is continuous or flows on a conveyor line, the use of inspection lots may be impractical and expensive. To meet the need for a sampling plan to verify the quality of a continuous production process, Dodge developed his CSP-1 plan[2] which can be described as follows: Inspect every unit, until "i" consecutive units have been found without detecting a defective. At this point, continue inspection by only verifying the fraction "f" of the units in such a fashion as to ensure an unbiased sample. As soon as a defective is found, return to 100 per cent inspection. A continuous sampling plan under this type of a rectifying inspection program is defined by i and f. The relationship between i and f and AOQL is illustrated in Figure 11.10. Note that with $i = 50$ and $f = 0.20$, an AOQL of approximately 0.015 can be expected.

The functioning of a sampling plan for continuous production will depend upon the level of fraction defective encountered in the production flow. The average number of pieces that will be inspected under the 100 per cent inspection portion of the cycle will be

$$u = \frac{1 - (1 - p)^i}{(p)(1 - p)^i}. \qquad (11.13)$$

And the average number of units passed under the sampling portion will be

$$v = \frac{1}{fp}. \qquad (11.14)$$

Thus, the average cycle will consist of $u + v$ total units. The average proportion of the produced units which must be inspected will than be

$$F = \frac{u + fv}{u + v}. \qquad (11.15)$$

And the average proportion of produced units which will be accepted without inspection will be

$$P_a = 1 - F = \frac{v(1 - f)}{u + v}. \qquad (11.16)$$

[2] H. F. Dodge, "A Sampling Plan for Continuous Production," *Annals of Mathematical Statistics*, XIV, 1943, pp. 279.

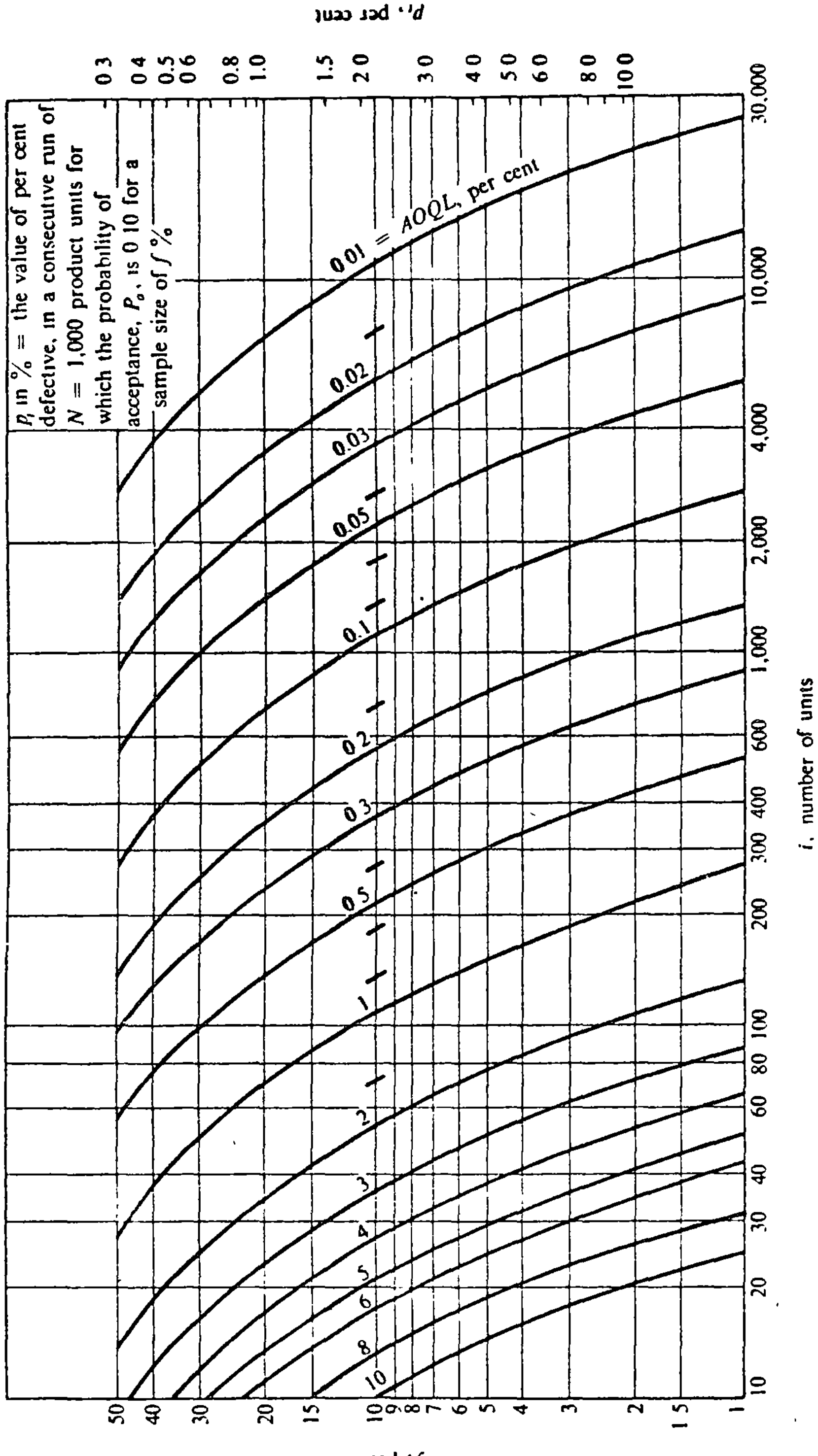

Figure 11.10. Curves for determining values of f and i for a given value of AOQL in Dodge's plan for continuous production. (*Reproduced by permission from "A Sampling Inspection Plan for Continuous Production" by H. F. Dodge.*)

20

The last equation is the equivalent of the probability of acceptance under lot-by-lot inspection and can be used to construct an OC curve for a given sampling plan.

As an example, with $p = 0.05$ and a sampling plan of $f = 0.20$, $i = 50$, the average number of pieces inspected following the finding of a defective is

$$u = \frac{1 - (0.95)^{50}}{(0.05)(0.95)^{50}} = \frac{1 - 0.077}{(0.05)(0.077)} = 240.$$

The average number of pieces passed under the sampling procedure is

$$v = \frac{1}{(0.20)(0.05)} = 100.$$

The average proportion of total units inspected is

$$F = \frac{240 + 20}{340} = 0.765.$$

And the probability of acceptance at $p = 0.05$ is

$$P_a = 1 - 0.765 = 0.235.$$

Because this is a rectifying inspection program, it is possible to develop the AOQ at varying levels of fraction defective. The AOQ expresses the proportion of defectives which are accepted. If it is assumed that detected defectives are replaced to maintain a constant rate of production flow, then the

$$\text{AOQ} = P_a(p) = \frac{v(1 - f)(p)}{u + v}. \tag{11.17}$$

This equation can be used to develop an average outgoing quality function similar to the one in Figure 11.6. It will then verify the AOQL expressed in Figure 11.10.

11.3 ACCEPTANCE SAMPLING BY VARIABLES

If a quality characteristic can be measu… it is possible to devise an acceptance sampling plan that will verify the quality of a lot under inspection by variables. In some cases, however, the quality characteristic is observable only as an attribute. In other cases, the cost of attribute assessment under a go, no-go arrangement is much more economical than variable inspection. And finally, in still other situations, acceptance criteria may have to be applied to many quality characteristics. Although this is feasible with one plan under

21

attribute verification, the use of variables inspection will require as many inspection plans as there are significant quality characteristics.

If these limitations are not serious, some advantages can be gained by employing an acceptance sampling plan using variables inspection. Superior protection in the form of a steeper OC curve can be achieved under variables inspection with the same sample size. As a corollary of this advantage, comparable protection can be obtained with a smaller sample size. This might be a very desirable advantage when a unit must be destroyed to be tested. The second major advantage of variables inspection lies within the records of the data which are collected. Variables data will be more useful when marginal product performance must be assessed and will provide a better basis for a quality improvement program. In addition, errors of measurement will be more noticeable under variables inspection.

Two classes of acceptance sampling by variables are considered in this section. The first assumes that the population variance is known and constant. An example problem will be presented for this assumption. The second case will deal with the situation where the variance is unknown or assumed to vary from lot to lot.

Known and constant sigma plans. A variables sampling plan can be defined with a sample size of n, and an acceptance average of the sample referred to as $\bar{X}_a$. As an illustration, a variables sampling plan used to test the breaking strength of concrete might be defined as $n = 10$, $\bar{X}_a = 4{,}900$ psi. This plan calls for testing 10 specimens and accepting the lot if the mean breaking strength of the 10 specimens equals or exceeds 4,900 psi. If it can be assumed that the population is normally distributed with a variance that is known and constant, a variables sampling plan can be developed that will yield an OC curve meeting specified producer and consumer risk points.

As an example, assume that steel castings are produced in a batch process and records indicate that the distribution of yield points can be assumed to be normal with $\sigma = 3{,}000$ psi. Castings with a yield strength of 62,000 psi are considered good and should be accepted 95 per cent of the time. A yield strength of only 59,000 psi is not considered good, and castings from this batch should be rejected 90 per cent of the time. The consumer and producer risk points are thus specified as $\alpha = 0.05$ at 62,000 psi, and $\beta = 0.10$ at 59,000 psi.

If lots at the producer risk point are to be accepted as indicated, the following relationship is applicable

$$\frac{\bar{X}_a - 62{,}000}{3{,}000/\sqrt{n}} = -1.645$$

whe[re] −1.645 refers to the standard normal deviate which defines the are[a]

and thus the probability of acceptance of 0.95. Similarly, at the consumer risk point, the relationship is

$$\frac{\bar{X}_a - 59{,}000}{3{,}000/\sqrt{n}} = +1.282$$

where +1.282 defines the probability of 0.10. These two equations are solved for the two unknowns, $\bar{X}_a$ and n. This solution yields the variables sampling plan, $n = 9$, $\bar{X}_a = 60{,}318$. The sampling plan does not yield an OC curve that passes precisely through the two desired points, since n must be an integer. The indicated value of $\bar{X}_a$ is a compromise between the two values which would yield OC curves passing exactly through the one or the other point

One can construct a complete OC curve for the preceding variables sampling plan by calculating the probability of acceptance at varying levels of batch yield strength. For example, with the yield strength assumed to be 60,000 psi, this represents a standard normal deviate of

$$\frac{60{,}318 - 60{,}000}{3{,}000/\sqrt{9}} = +0.318.$$

This deviation coresponds to $P_a = 0.375$. The complete OC curve for the example problem is shown in Figure 11.11.

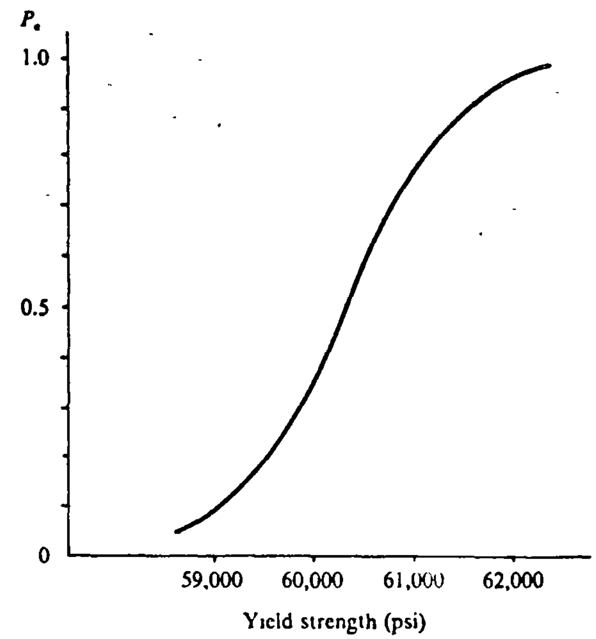

Figure 11.11. OC curve for the variables sampling plan $n = 9$, $\bar{X} = 60{,}318$ psi with $\sigma = 3{,}000$ psi.

The example solution considered only one tolerance: a lower limit of 59,000 psi at which it was desired to accept no more than 1 lot in 10. If an upper tolerance limit also exists, then an upper and a lower acceptance limit, designated $\bar{X}_{U_a}$ and $\bar{X}_{L_a}$, respectively, must be specified along with the sample size. Under these conditions, the acceptance criteria can be developed as if two separate single limit plans were to be used. Two equations would be developed for each limit and a solution obtained for $\bar{X}_{U_a}$, $\bar{X}_{L_a}$, and n. A modified value of OC would have to be considered in order to provide for the possibility of making a Type I error by rejecting a lot at either acceptance limit. If the quality level corresponding to α lies midway between the tolerance limits, then $\alpha/2$ can be used in obtaining the standard normal deviates

When a specification rather than a tolerance is provided, another approach might be used in developing variables sampling plans. In this case the proportion defective can be calculated as the area within the distribution which lies outside the specification limit. Values of α and β will correspond to proportion defectives and an acceptance limit can be obtained from the same parameters used to obtain attribute plans.

Unknown and variable sigma plans. When the variance of the population being sampled is unknown or is assumed to vary from one lot to the next, it must be estimated from the sample. The student's "t" distribution (a distribution not described or tabulated herein) should be used as the test statistic. In effect, a sample of size n is drawn from the lot, and the population mean and standard deviation are estimated from this sample, where

$$s = \sigma\sqrt{\frac{n}{n-1}}.$$

The decision statistic is

$$\frac{\bar{X} - \bar{X}_a}{s/\sqrt{n}}$$

where $\bar{X}_a$ is the AQL. If the decision statistic is numerically equal to or less than the "t" deviate at a probability α, the lot would be accepted.

The difficulty in working with unknown sigma plans is that the OC curve is dependent upon the population variance. If this variance changes from one lot to another, no meaningful OC curve can be developed.

11.4 SYSTEMS OF ACCEPTANCE SAMPLING PLANS

Recently, a number of systems of acceptance sampling plans have been developed which have facilitated the widespread use of acceptance sampling in industry. These systems generally serve to bridge the gap between

academic interest in developing such plans and the industrial need for acceptance sampling. Although many systems of sampling plans are available, only three of the more widely used will be introduced in this section. The first two were developed for the U.S. Department of Defense and consist of sets of acceptance sampling plans. The first is for inspection by attributes (MIL-STD-105D),[3] and the second is for inspection by variables (MIL-STD-414).[4] Both are applicable under nonrectifying inspection. The third system is referred to as the *Dodge–Romig tables*[5] and consists of four sets of tables for inspection by attributes under a rectifying inspection procedure.

U.S. Department of Defense sampling plans. MIL-STD-105D has evolved from its inception in 1942 through four revisions to its present form. The latest revision was an international undertaking by a committee made up of personnel from military agencies of Great Britain, Canada, and the United States. It is not only the military use in accepting products under government procurement contracts which has made the system so widely known. Industry has also been quick to adopt this standard to meet its own acceptance sampling needs.

This system rests upon and first introduced the concept of an Acceptable Quality Level, AQL, which was defined and illustrated in Figure 11.4. The acceptance criteria are selected to protect the producer against the rejection and the return of submitted lots of this quality level or better. In conjunction with the concept of an AQL, the system includes the use of "tightened inspection" and "reduced inspection" as alternatives which are available to protect the consumer if it is necessary and justified in light of the previous quality history of the producer. A plan under tightened inspection will yield a steeper OC curve and one under reduced inspection will yield a flatter OC curve. One or the other may be called upon in lieu of "normal inspection" if certain criteria are met.

Another interesting aspect of MIL-STD-105D is the provision for classifying defects on the basis of their severity. Definitions are given for varying levels of the seriousness of defects, and the acceptable quantity of each level is built into the acceptance sampling plan. Thus, more minor defects would be permitted, and only one or a few critical defects may be grounds for the rejection of a lot. This standard includes sets of single, double, and multiple sampling plans and specifies a sample size that is dependent upon and increases with lot size in an absolute sense but decreases in a relative sense.

[3] *Military Standard 105D, Sampling Procedures and Tables for Inspection by Attributes* (Washington, D.C.: Government Printing Office, 1963.)

[4] *Military Standard 414, Sampling Procedures and Tables for Inspection by Variables for Percent Defective* (Washington, D.C.: Government Printing Office, 1957.)

[5] H. F. Dodge and H. G. Romig *Sampling Inspection Tables—Single and Double Sampling*, 2nd ed. (New York: John Wiley & Sons, Inc., 1959.)

MIL-STD-414 is quite similar to MIL-STD-105D in that it is based on the concept of an AQL; uses lot-by-lot acceptance; provides for normal, tightened, or reduced inspection depending on the previous quality history of the supplier; and relates sample size to the size of the lot. MIL-STD-144 uses variables rather than attributes inspection. It can be used with either single or double specification limits and provides two sets of tables: one for the case of "variability known" and the other for "variability unknown." With the latter set, the variability of the lot may be estimated through a "standard deviation method" as previously described or a "range method."

In the simple case of a single specification limit with known lot variability, a sampling plan of $n = 8$ and $k = 1.68$ would be obtained under inspection level II, with a lot size of 1,500, at normal inspection and an AQL = 0.015 (1.50 per cent). If it were further assumed that the process had a variability of $\sigma = 0.010$ inches and only a lower specification limit of 1.000 inches, then the acceptance criteria would be met if

$$\frac{\bar{X} - 1.000}{0.010} \geq 1.68$$

where $\bar{X}$ is the mean of the sample of eight units.

The Dodge–Romig sampling plans. The Dodge–Romig volume is based on a rectifying inspection program and contains the following four sets of tables:

(1) Single-sampling lot tolerance tables.
(2) Double-sampling lot tolerance tables.
(3) Single-sampling AOQL tables.
(4) Double-sampling AOQL tables.

The first two sets of tables contain sampling plans assuming $\beta = 0.10$ for lot tolerance per cent defectives, LTPD, of 0.5 per cent, 1.0 per cent, 2.0 per cent, 3.0 per cent, 4.0 per cent, 5.0 per cent, 7.0 per cent, and 10.0 per cent. Table 11.5 is a single-sampling lot tolerance table with LTPD = 0.05 or 5 per cent. In effect, all the plans on this one table have OC curves which pass through the consumer risk point of LTPD = 0.05 at $\beta = 0.10$. The six columns in this table are for different values of process average per cent defective. The plan selected at a given column of process average and row of lot size will minimize total inspection under a rectifying inspection program while providing the desired consumer protection. Although these tables are designed to minimize total inspection under a rectifying inspection program and yield the indicated value of AOQL, they can be used under nonrectifying insp[illegible]on and still yield the indicated consumer risk point protection [illegible]he second two sets of tables contain sampling plans with AOQL valu

26

Table 11.5. EXAMPLE OF DODGE–ROMIG SINGLE-SAMPLING LOT TOLERANCE TABLES
(Lot tolerance per cent defective = 5.0; Consumer's risk = 0.10)

Process Average (%)	0–0.05			0.06–0.50			0.51–1.00			1.01–1.50			1.51–2.00			2.01–2.50		
Lot Size	n	c	AOQL (%)	n	c	AOQL (%)	n	c	AOQL (%)	n	c	AOQL (%)	n	c	AOQL (%)	n	c	AOQL (%)
1–30	All	0	0	All	0	0	All	0	0	All	0	0	All	0	0	All	0	0
31–50	30	0	0.49	30	0	0.49	30	0	0.49	30	0	0.49	30	0	0.49	30	0	0.49
51–100	37	0	0.63	37	0	0.63	37	0	0.63	37	0	0.63	37	0	0.63	37	0	0.63
101–200	40	0	0.74	40	0	0.74	40	0	0.74	40	0	0.74	40	0	0.74	40	0	0.74
201–300	43	0	0.74	43	0	0.74	70	1	0.92	70	1	0.92	95	2	0.99	95	2	0.99
301–400	44	0	0.74	44	0	0.74	70	1	0.99	100	2	1.0	120	3	1.1	145	4	1.1
401–500	45	0	0.75	75	1	0.95	100	2	1.1	100	2	1.1	125	3	1.2	150	4	1.2
501–600	45	0	0.76	75	1	0.98	100	2	1.1	125	3	1.2	150	4	1.3	175	5	1.3
601–800	45	0	0.77	75	1	1.0	100	2	1.2	130	3	1.2	175	5	1.4	200	6	1.4
801–1,000	45	0	0.78	75	1	1.0	105	2	1.2	155	4	1.4	180	5	1.4	225	7	1.5
1,001–2,000	45	0	0.80	75	1	1.0	130	3	1.4	180	5	1.6	230	7	1.7	280	9	1.8
2,001–3,000	75	1	1.1	105	2	1.3	135	3	1.4	210	6	1.7	280	9	1.9	370	13	2.1
3,001–4,000	75	1	1.1	105	2	1.3	160	4	1.5	210	6	1.7	305	10	2.0	440	15	2.2
4,001–5,000	75	1	1.1	105	2	1.3	160	4	1.5	235	7	1.8	330	11	2.0	440	16	2.2
5,001–7,000	75	1	1.1	105	2	1.3	185	5	1.7	260	8	1.9	350	12	2.2	490	18	2.4
7,001–10,000	75	1	1.1	105	2	1.3	185	5	1.7	260	8	1.9	380	13	2.2	535	20	2.5
10,001–20,000	75	1	1.1	135	3	1.4	210	6	1.8	285	9	2.0	425	15	2.3	610	23	2.6
20,001–50,000	75	1	1.1	135	3	1.4	235	7	1.9	305	10	2.1	470	17	2.4	700	27	2.7
50,001–100,000	75	1	1.1	160	4	1.6	235	7	1.9	355	12	2.2	515	19	2.5	770	30	2.8

From H. F. Dodge and H. G. Romig, *Sampling Inspection Tables* (New York: John Wiley & Sons, Inc., 1959), *reprinted by permission.*

27

of 0.10 per cent, 0.25 per cent, 0.50 per cent, 0.75 per cent, 1.0 per cent, 1.5 per cent, 2.0 per cent, 2.5 per cent, 3.0 per cent, 4.0 per cent, 5.0 per cent, 7.0 per cent, and 10.0 per cent. Table 11.6 is a double-sampling AOQL table with the AOQL = 0.02 or 2 per cent. All the plans listed on this table will yield this value of AOQL. The selection criterion is again one of minimizing total inspection if rejected lots are subject to 100 per cent screening.

11.5 THE ECONOMY OF ACCEPTANCE SAMPLING

An acceptance sampling plan may be described as a formalized procedure designed to assess the quality of a product group with some predetermined probability of error. This assessment is an operation that should be performed with economy. Economy requires that one develop an effectiveness function that relates the variables under direct control of the decision maker with those not under his direct control. In acceptance sampling, the decision maker can specify the sampling plan to be used. The quality characteristics of the product group, the costs of assessment, and the costs of accepting defectives are not directly under his control. Therefore, in selecting a sampling plan that will result in a minimum total system cost, he must consider these parameters.

In practice, it is difficult to ascertain the precise costs of inspection and costs of accepting defectives. Often, these costs are assumed to be linear with little empirical justification. The quality characteristics of a product group may also be difficult to estimate, although the quality history of a producer could serve as a guide. In spite of these difficulties, an economic evaluation is useful in the selection of an acceptance sampling plan.

Total system cost under rectifying inspection. If it is assumed that defectives are replaced to maintain a constant lot size, the expected total cost per lot will be

$$TC = (\text{AOQ})(N)(C_d) + (I)(C_i). \qquad (11.18)$$

The cost of accepting a defective is designated C_d and the cost of inspecting one item is C_i. Substituting Equation (11.3) for AOQ and Equation (11.4) for I reduces Equation (11.18) to

$$TC = (P_a)(p)(N - n)C_d + \left[\frac{n + (1 - P_a)(N - n)}{(1 - p)}\right]C_i. \qquad (11.19)$$

If the expected level of defectives varies from one lot to another, it will be necessary to apply Equation (11.19) to each group. The total system cost will then be a weighted average based on the fraction of lots having each level of defectives.

Table 11.6. EXAMPLE OF DODGE–ROMIG DOUBLE-SAMPLING AOQL TABLES
(Average outgoing quality limit = 2.0 per cent)

Process Average (%)	0–0.04						0.05–0.40						0.41–0.80					
Lot Size	Trial 1		Trial 2			p_t (%)	Trial 1		Trial 2			p_t (%)	Trial 1		Trial 2			p_t (%)
	n_1	c_1	n_2	n_1+n_2	c_2		n_1	c_1	n_2	n_1+n_2	c_2		n_1	c_1	n_2	n_1+n_2	c_2	
1–15	All	0	—	—	—	—	All	0	—	—	—	—	All	0	—	—	—	—
16–50	14	0	—	—	—	13.6	14	0	—	—	—	13.6	14	0	—	—	—	13.6
51–100	21	0	12	33	1	11.7	21	0	12	33	1	11.7	21	0	12	33	1	11.7
101–200	24	0	13	37	1	11.0	24	0	13	37	1	11.0	24	0	13	37	1	11.0
201–300	26	0	15	41	1	10.4	26	0	15	41	1	10.4	29	0	31	60	2	9.1
301–400	26	0	16	42	1	10.3	26	0	16	42	1	10.3	30	0	35	65	2	9.0
401–500	27	0	16	43	1	10.3	30	0	35	65	2	9.0	30	0	35	65	2	9.0
501–600	27	0	16	43	1	10.3	31	0	34	65	2	8.9	35	0	55	90	3	7.9
601–800	27	0	17	44	1	10.2	31	0	39	70	2	8.8	35	0	60	95	3	7.7
801–1,000	27	0	17	44	1	10.2	32	0	38	70	2	8.7	36	0	69	95	3	7.6
1,001–2,000	33	0	37	70	2	8.5	33	0	37	70	2	8.5	37	0	73	100	3	7.5
2,001–3,000	34	0	41	75	2	8.2	34	0	41	75	2	8.2	41	0	84	125	4	7.0
3,001–4,000	34	0	41	75	2	8.2	38	0	62	100	3	7.3	41	0	89	130	4	6.9
4,001–5,000	34	0	41	75	2	8.2	38	0	62	100	3	7.3	42	0	88	130	4	6.9
5,001–7,000	35	0	40	75	2	8.1	38	0	62	100	3	7.3	44	0	116	160	5	6.4
7,001–10,000	35	0	40	75	2	8.1	38	0	62	100	3	7.3	45	0	115	160	5	6.2
10,001–20,000	35	0	40	75	2	8.1	39	0	66	105	3	7.2	45	0	115	160	5	6.3
20,001–50,000	35	0	40	75	2	8.1	43	0	92	135	4	6.6	47	0	148	195	6	6.0
50,001–100,000	35	0	45	80	2	8.0	43	0	92	135	4	6.6	85	1	185	270	8	5.2

Process Average (%)	0.81–1.20						1.21–1.60						1.61–2.00					
Lot Size	Trial 1		Trial 2			p_t (%)	Trial 1		Trial 2			p_t (%)	Trial 1		Trial 2			p_t (%)
	n_1	c_1	n_2	n_1+n_2	c_2		n_1	c_1	n_2	n_1+n_2	c_2		n_1	c_1	n_2	n_1+n_2	c_2	
1–15	All	0	—	—	—	—	All	0	—	—	—	—	All	0	—	—	—	—
16–50	14	0	—	—	—	13.6	14	0	—	—	—	13.6	14	0	—	—	—	13.6
51–100	21	0	12	33	1	11.7	21	0	12	33	1	11.7	23	0	23	46	2	10.9
101–200	27	0	28	55	2	9.5	27	0	28	55	2	9.6	27	0	28	55	2	9.6
201–300	29	0	31	60	2	9.1	32	0	48	80	3	8.4	32	0	48	80	3	8.4
301–400	33	0	52	85	3	8.2	33	0	52	85	3	8.2	36	0	69	105	4	7.6
401–500	34	0	58	90	3	7.9	36	0	74	110	4	7.5	60	1	90	150	6	7.0
501–600	35	0	55	90	3	7.9	37	0	78	115	4	7.4	65	1	95	160	6	6.8
601–800	38	0	82	120	4	7.3	38	0	82	120	4	7.3	70	1	120	190	7	6.4
801–1,000	38	0	87	125	4	7.2	70	1	100	170	6	6.5	70	1	145	215	8	6.2
1,001–2,000	43	0	112	155	5	6.5	80	1	160	240	8	5.8	110	2	205	315	11	5.5
2,001–3,000	75	1	115	190	6	6.1	115	2	195	310	10	5.3	160	3	310	470	15	4.7
3,001–4,000	80	1	140	220	7	5.8	120	2	255	373	12	5.0	235	5	415	650	20	4.3
4,001–5,000	80	1	175	255	8	5.5	125	2	285	410	13	4.9	275	6	475	750	23	4.2
5,001–7,000	85	1	205	290	9	5.3	125	2	320	445	14	4.8	280	6	575	855	26	4.1
7,001–10,000	85	1	210	295	9	5.2	165	3	335	500	15	4.5	320	7	645	965	29	4.0
10,001–20,000	90	1	260	350	11	5.1	170	3	425	595	18	4.4	395	9	835	1,230	37	3.9
20,001–50,000	130	2	300	430	13	4.7	205	4	515	720	22	4.3	480	11	1,090	1,570	46	3.[illegible]
50,001–100,000	135	2	345	480	14	4.5	250	5	615	865	26	4.1	580	13	1,460	2,040	58	3.[illegible]

From H. F. Dodge and H. G. Romig, *Sampling Inspection Tables* (New York: John Wiley & Sons, Inc., 1959), *reprinted by permission.*

As an example of the application of the foregoing model, consider the data of Table 11.3. The sampling plan was $N = 10{,}000$, $n = 200$, and $c = 5$. Suppose that the cost of accepting a defective is \$5.00 and the cost of inspecting one item is \$0.10. It is estimated that half of the lots submitted will contain no defectives, one-fourth of the lots will contain 2 per cent defective, and the remaining one-fourth of the lots will contain 4 per cent defective. Under these conditions, the total cost for each level of defectives will be

$$TC_{p=0} = (1)(0)(9{,}800)(\$5.00) + \left[\frac{200 + (1-1)(10{,}000-200)}{(1-0)}\right]\$0.10$$
$$= \$20.00$$
$$TC_{p=0.02} = (0.785)(9{,}800)(0.02)(\$5.00)$$
$$+ \left[\frac{200 + (1-0.785)(10{,}000-200)}{(1-0.02)}\right]\$0.10$$
$$= \$1{,}003.70$$
$$TC_{p=0.04} = (0.191)(9{,}800)(0.04)(\$5.00)$$
$$+ \left[\frac{200 + (1-0.191)(10{,}000-200)}{(1-0.04)}\right]\$0.10$$
$$= \$1{,}201.52.$$

The weighted total cost will be

$$TC = \tfrac{1}{2}TC_{p=0} + \tfrac{1}{4}TC_{p=0.02} + \tfrac{1}{4}TC_{p=0.04}$$
$$= \tfrac{1}{2}(\$20.00) + \tfrac{1}{4}(\$1{,}003.70) + \tfrac{1}{4}(\$1{,}201.52) = \$561.31.$$

The total system cost under this sampling plan may now be compared with no inspection and with 100 per cent inspection. With no inspection, the only cost would be that of accepting defectives. This is computed as

$$TC = [\tfrac{1}{2}(0) + \tfrac{1}{4}(0.02)(10{,}000) + \tfrac{1}{4}(0.04)(10{,}000)]\$5.00 = \$750.00.$$

Under 100 per cent screening, the cost is that of inspection. This is computed as

$$TC = (10{,}000)\$0.10 = \$1{,}000.00.$$

In this example, the sampling plan is more economical than either no inspection or the complete 100 per cent screening of every item. This does not mean, however, that this is the most economical sampling plan available. The minimum cost sampling plan would have to be found by trial-and-error methods.

Total system cost under nonrectifying inspection. Under a nonrectifying inspection program, only accepted lots are retained, and the total cost must

be adjusted to reflect the inspection costs of lots which are returned. The solution to the problem of the previous example under nonrectifying inspection can be obtained from

$$TC = (\text{ALQ})(N)(C_d) + \frac{(n)(C_i)}{p_{\text{AL}}}. \qquad (11.20)$$

The proportion defective in accepted lots is designated ALQ, and the proportion of accepted lots is p_{AL}. Table 11.7 gives the computations necessary for the application of Equation (11.20). The total cost is

$$TC = (0.00784)(10{,}000)(\$5.00) + \frac{(200)(\$0.10)}{0.7440} = \$418.88.$$

Table 11.7. A COMPUTATIONAL SCHEME FOR FINDING THE PROPORTION OF ACCEPTED LOTS AND THE PROPORTION DEFECTIVE IN ACCEPTED LOTS UNDER NONRECTIFYING INSPECTION

Proportion Defective (A)	*Proportion of Lots* (B)	*Probability of Acceptance* (C)	*Proportion of Accepted Lots* $p_{\text{AL}} = \sum BC$	*Proportion Defective in Accepted Lots* $\text{ALQ} = \frac{\sum ABC}{\sum BC}$
0	0.50	1.000	0.5000	0
0.02	0.25	0.785	0.1962	0.00393
0.04	0.25	0.191	0.0478	0.00191
			0.7440	$\frac{0.00584}{0.7440} =$ 0.00784

The foregoing solution is fairly simple, although it is only an approximation in that a few defectives can be expected to be found and discarded in the sample of accepted lots. A correction for this omission should not, however, appreciably alter the preceding answer.

The economy of acceptance sampling reduces to selecting a sampling plan which minimizes the costs of inspection and the costs of accepting defectives. The decision maker is often faced with making this decision on the basis of incomplete and sometimes erroneous data. He must make his decision with the available data and rely upon intuition and some subjective judgment to carry the evaluation through to a final sampling procedure. In a practical case, a sampling plan would probably be selected from an established system or table of acceptance sampling plans. These plans might specify error criteria, and an intuitive reconciliation would have to be made between these criteria and some estimate of sampling and other costs. Attempts would b made to facilitate the identification of defectives by removing as much of the subjective human element as possible. Nevertheless, it might well be noted that the identification of defectives might be superior under a sampling plan than under 100 per cent inspection.

QUESTIONS

11.1. How does the objective of an inspection sampling plan differ from that of a control model?

11.2. What is the difference between a rectifying and a nonrectifying inspection program?

11.3. What does an operating characteristic curve illustrate?

11.4. Discuss the general relationship between the sample size and the form of an OC curve.

11.5. Define and illustrate the consumer and the producer risk points on an OC curve.

11.6. Why is it usually necessary to bracket the two desired points on an OC curve?

11.7. Discuss the relationship between the average outgoing quality limit and the average outgoing quality.

11.8. List the relative advantages and disadvantages of single, double, and multiple or sequential sampling.

11.9. Under what conditions is it desirable to use a sampling plan for continuous production rather than a lot-by-lot plan?

11.10. What are the limitations in the use of acceptance sampling by variables? What are the advantages?

11.11. What are the unique features of MIL-STD-105D and MIL-STD-414?

11.12. What sets of tables are included in the Dodge–Romig system?

11.13. What costs are associated with the operation of an acceptance sampling plan?

11.14. Discuss acceptance sampling in terms of variables directly under control of a decision maker and variables not directly under his control.

PROBLEMS

11.1. Sketch the OC curves for the sampling plans $c = 0$, $n = 100$; $c = 1$, $n = 100$; $c = 2$, $n = 100$.

11.2. Sketch the OC curves for the sampling plans $c = 1$, $n = 100$; $c = 2$, $n = 200$; $c = 5$, $n = 500$.

11.3. Develop the four single sampling plans which bracket the producer and consumer risk points of $\alpha = 0.05$ at AQL $= 0.02$ and $\beta = 0.10$ at LTPD $= 0.05$.

11.4. Develop the plans which bracket $\alpha = 0.05$ at AQL $= 0.02$ and $\beta = 0.10$ at LTPD $= 0.10$.

11.5. Sketch I and AOQ, and specify AOQL for the plan $c = 1$, $n = 100$, and $N = 1{,}000$ under rectifying inspection with nonreplacement of defectives. Do the same for $c = 0$, $n = 50$, and $N = 1{,}000$.

11.6. Sketch the OC curves for the double sampling plan $N = 1{,}000$, $n_1 = 50$, $n_2 = 80$, $c_1 = 0$, and $c_2 = 3$. Do the same for the double sampling plan $N = 1{,}000$, $n_1 = 85$, $n_2 = 120$, $c_1 = 1$, and $c_2 = 6$.

11.7. What is the probability of making a decision on the first sample for each of the plans in the previous problem, at $p = 0.02$, $p = 0.03$, and $p = 0.05$?

11.8. Calculate ASN, I, and AOQ under rectifying inspection with replacement of defectives for the double sampling plans given below:
(a) $N = 2{,}000$, $n_1 = 60$, $n_2 = 80$, $c_1 = 0$, and $c_2 = 4$.
(b) $N = 2{,}000$, $n_1 = 80$, $n_2 = 110$, $c_1 = 2$, and $c_2 = 5$.

11.9. Define the sequential sampling plan that meets the producer and consumer risk points of $\alpha = 0.05$ at AQL $= 0.010$ and $\beta = 0.10$ at LTPD $= 0.050$.

11.10. With the plan of Problem 11.9, what are the minimum number of units which would have to be inspected to accept a lot?

11.11. With the continuous sampling plan $i = 200$ and $f = 0.10$, develop an OC curve, calculate AOQL, and compare this latter value with that obtained from Figure 11.10.

11.12. Specify the variables sampling plan with a known and constant $\sigma = 2.00$ inches and a consumer and producer risk point of $\alpha = 0.05$ at 2.00 and $\beta = 0.10$ at 3.50 inches.

11.13. In 1215, the Chinese city of Yen-King, the modern Peking, was besieged by the Mongols under Genghis Khan. When all the metal inside the city had been used up for cannon balls, the defenders began melting down silver and eventually gold, and their ancient muzzleloaders finally poured golden shot into the Mongols' camp. In the end, however, the city was taken and destroyed, later to be rebuilt by Kublai Khan. The resistance to the siege was reported to have been influenced by the quality control procedures used in conjunction with the casting of the cannon balls.

Two hundred and fifty lots of 50 silver cannon balls each were cast within the city and delivered to the guns on the walls. There they were subjected to the nonrectifying inspection program by attributes of $N = 50$, $n = 10$, and $c = 0$. Rejected lots were returned where they were melted down into coins to be used as bribe money. After the silver balls in accepted lots were exhausted, gold balls were cast, verified under a Dodge CSP-1 plan, obtained from the still undiscovered Western Hemisphere, and fired. The specific plan called for $i = 50$, $f = 0.10$.

Archaeologists have since discovered that half of the lots of silver balls contained no defectives, whereas the other half were probably 10 per cent defective. The gold balls were probably all 5 per cent defective. They also discovered that when a cannon had fired 10 defective balls it became inoperative, and that the city began its resistance with 150 cannon and it fell when all the cannon became inoperative How many gold cannon balls were fired?

11.14. It costs $2 to accept a defective item and $0.08 to inspect each item. Determine the cost of a sampling plan with $N = 1{,}500$, $n = 200$, and $c = 2$. The distribution given below shows the defectives:

Proportion Defective	*Proportion of Lots*
0.00	0.10
0.01	0.10
0.02	0.20
0.03	0.30
0.04	0.20
0.05	0.10

Compare the cost to that of 100 per cent inspection and no inspection.

11.15. Solve Problem 11.14 under a nonrectifying inspection program.

Para un producto se exige una calidad mínima x_0

El consumidor rechaza con la regla siguiente

Toma una muestra de 3 y si su promedio resulta menor que x_0, rechaza

$$x = \frac{f_1 + f_2 + f_3}{3}$$

x es normal m_x, σ_x

$$m_x = m_f$$

$$P_R = P[x \leq x_0]$$

$$\sigma_x = \sigma_f / \sqrt{3}$$

El productor se cubre buscando para el producto una calidad media $m_f > x_0$

sea

$$m_f = x_0 + k\,\sigma_f$$

y σ_f conocido

CUANTO DEBE VALER k

PROBABILIDAD DE RECHAZO

$$P_R = P[x \leq x_0] = P[x \leq m_f - k\sigma_f] = P\left[\frac{x - m_x}{\sigma_x} \leq -\frac{k\,\sigma_f}{\sigma_f/\sqrt{3}}\right]$$

$$P_R = P[z \leq z_0] = P\left[z \leq -\frac{k}{\sqrt{3}}\right]$$

z es normal estandarizada

$$z_0 = -k\sqrt{3} \qquad k = -\frac{z_0}{\sqrt{3}} = -\frac{z_0}{1{,}73}$$

P_R	Z_α	k
0.1	-1.28	0.68
0.05	-1.64	0.95
0.01	-2.33	1.35
0.001	-3.08	1.73

Si se quiere disminuir la probabilidad de rechazo hay que aumentar la media pero eso cuesta.

Hay que balancear

Solución óptima

Se quiere la solución que involucre un costo mínimo

El costo está compuesto por

C_p = Costo de producción, aumenta al crecer m_f

Supongamos

$$C_p = C_0[1 + c_1(m_f - x_0)]$$

C_R = Costo del rechazo. Cada unidad (o lote) rechazado implica un costo supongamos

$$C_R = c_2 C_0$$

Costo total $\quad C_T = C_p + C_R P_R$

$$C_T = C_0 + C_0 c_1(m_f - x_0) + c_2 C_0 P_R$$

$$C_T = C_0[1 + c_1 k \sigma_f + c_2 P_R]$$

Consideremos los siguientes valores

$c_1 = 0.01$ y $\sigma_f = 50$ $\quad c_2 = 10$

$$C_T = C_0 [1 + 0.5k + 10 P_R]$$

El valor mínimo puede encontrarse analizando la Tabla.

P_R	k	C_T/C_0		
		$\sigma_f = 50$	$\sigma_f = 35$	$\sigma_f = 20$
0.1	0.68	2.34	2.24	2.14
0.05	0.95	1.97	1.85	1.69
0.01	1.35	1.78	1.57	1.37
0.001	1.73	1.87	1.62	1.35

σ_f	k_{opt}
50	1.4
35	1.7
20	1.9

Para el valor de σ_f supuesto resulta que el k optimo es el que corresponde a una probabilidad de rechazo de aproximadamente 1%. Puede afinarse interpolando. Se obtendria $k_{opt} = 1.4$

Si $x_0 = 200$ kg/cm²

Hay que proporcionar el producto para una calidad media $m_f = x_0 + k\sigma_f$

$$m_f = 200 + 1.4 \times 50 = 270$$

VARIAS ALTERNATIVAS EN σ_f

El productor va a empezar sus operaciones y puede elegir entre tres sistemas de producción ya existentes en el mercado y de distinto grado de automatización y precisión

El sistema A da lugar a $\sigma_f = 50$ y a un $C_0 = 200$
" " B " " " $\sigma_f = 35$ " " $C_0 = 210$
" " C " " " $\sigma_f = 20$ $C_0 = 240$

¿Cuál de los tres sistemas le conviene utilizar?

El que de lugar al costo total mínimo.

La expresión obtenida

$$C_T = C_0 \left[1 + c\,k\,\sigma_f + c_2 P_R \right]$$

o sea

$$C_T = C_0 \left[1 + 0{,}01\,k\,\sigma_f + 10\,P_R \right]$$

en cada caso el productor daría la calidad optima de acuerdo con el criterio anterior o sea usaría un k_{optimo}

Sistema	k_{opt}	m_f	C_T/C_0	C_T
A	1,4	270	1.75	350
B	1,7	259	1.55	326
C	1,9	238	1.35	324

según lo anterior la solución C es la más adecuada

DIRECTORIO DE ASISTENTES AL CURSO DE CONTROL DE CALIDAD (DEL 29 DE ENERO AL 16 DE MARZO DE 1973)

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