UNIVERSIDAD NACIONAL AUTÓNOMA DE MÉXICO

FACULTAD DE INGENIERÍA

PROBLEMAS DE ANÁLISIS ESTRUCTURAL

METODO DE CROSS.

EL METODO DE CROSS SE UTILIZA PARA ANALIZAR ESTRUCTURAS FORMADAS POR BARRAS. ES UN METODO DE APROXIMACIONES SUCESIVAS.

PARA EFECTUAR UN ANALISIS POR EL METODO DE CROSS SE LLEVAN A CABO LAS SIGUIENTES ETAPAS.

1.- SE OBTIENEN LAS RIGIDECES ANGULARES DE CADA UNA DE LAS BARRAS QUE FORMAN LA ESTRUCTURA.

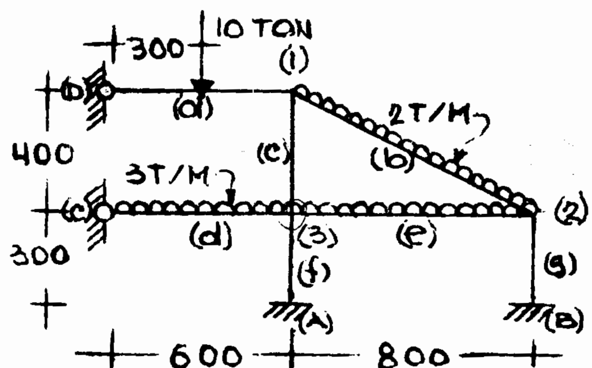
2.- EN CADA NUDO SE EFECTUA LA SUMA DE LAS RIGIDECES ANGULARES DE LAS BARRAS QUE CONCURREN A EL, CON LO CUAL SE OBTIENE LA RIGIDEZ DEL NUDO.

3.- SE EFECTUA EL COCIENTE DE LA RIGIDEZ ANGULAR DE CADA BARRA ENTRE LA RIGIDEZ DEL NUDO A QUE CONCURRE. ESTE COCIENTE RECIBE EL NOMBRE DE FACTOR DE DISTRIBUCION. LA SUMA DE LOS FACTORES DE DISTRIBUCION EN CADA NUDO DEBE SER IGUAL A LA UNIDAD.

4.- SE SUPONEN LAS BARRAS AISLADAS CONSIDERANDOLAS EMPOTRADAS EN LOS EXTREMOS QUE CONCURRAN A UN NUDO. DE ACUERDO A LAS SOLICITACIONES DE CADA BARRA EN PARTICULAR SE OBTIENEN LOS MOMENTOS EN LOS APOYOS DE LAS MISMAS LLAMANDOSE A ESTOS: MOMENTOS DE EMPOTRAMIENTO.

5.- TODO LO ANTERIOR SE VACIA EN UN CROQUIS.

APLICANDOLO AL SIGUIENTE EJEMPLO:



EI=CTE.

CARGAS UNIFORMEMENTE REPARTIDAS PERPENDICULARES AL EJE DE LA BARRA. NO HAY GRADO DE LIBERTAD LINEAL.

ETAPAS 1, 2 y 3

r_{aa} = RIGIDEZ ANGULAR DE LA BARRA

F. D = FACTOR DE DISTRIBUCION

NUDO " 1 "

$r_{aa} = \frac{3EI}{l} = \frac{3EI}{6} = 0.5\ EI$ F.D = $\frac{0.5EI}{1.95EI} = 0.26$

$r_{ab} = \frac{4EI}{l} = \frac{4EI}{8.9} = 0.45EI$ F.D = $\frac{0.45EI}{1.95EI} = 0.23$

$r_{ac} = \frac{4EI}{l} = \frac{4EI}{4} = 1.00EI$ F.D = $\frac{1.00EI}{1.95EI} = 0.51$

$\Sigma = 1.95EI$ $\Sigma = 1.00$

NUDO " 2 "

$r_{ab} = 0.45EI = 0.45\ EI$ F.D = $\frac{0.45EI}{2.28EI} = 0.20$

$r_{ag} = \frac{4EI}{l} = \frac{4EI}{3} = 1.33\ EI$ F.D = $\frac{1.33EI}{2.28EI} = 0.58$

$r_{ae} = \frac{4EI}{l} = \frac{4EI}{8} = 0.50\ EI$ F.D = $\frac{0.50EI}{2.28EI} = 0.22$

$= 2.28\ EI$ $\Sigma = 1.00$

NUDO " 3 "

$r_{ac} = \frac{4EI}{l} = \frac{4EI}{4} = 1.00\ EI$ F.D = $\frac{1.00}{3.33} = 0.30$

$r_{ae} = \qquad = 0.50\ EI$ F.D = $\frac{0.50}{3.33} = 0.15$

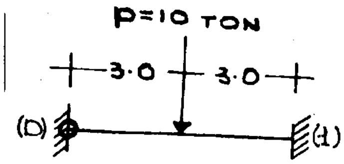
$r_{ad} = \frac{3EI}{l} = \frac{3EI}{6} = 0.50\ EI$ F.D = $\frac{0.50}{3.33} = 0.15$

$r_{af} = \frac{4EI}{l} = \frac{4EI}{3} = 1.33\ EI$ F.D = $\frac{1.33}{3.33} = 0.40$

$\Sigma = 3.33\ EI$ $\Sigma = 1.00$

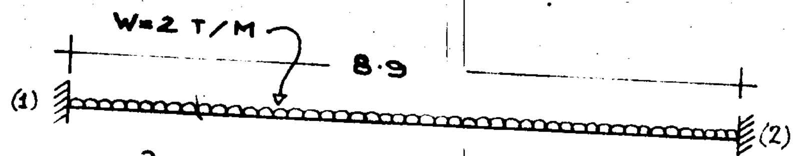
ETAPA 4 (MOMENTOS DE EMPOTRAMIENTO)

BARRA (a)



$$M_1 = \frac{3 \times p \times l}{16} = \frac{3 \times 10 \times 6}{16} = \frac{180}{16} = 11.25$$ (T-M)

BARRA (b)

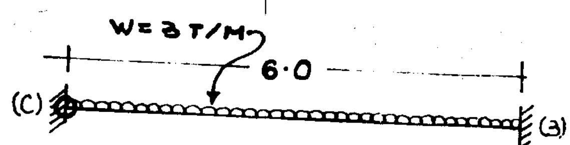


$$M_1 = M_2 = \frac{wl^2}{12} = \frac{2 \times 8.9^2}{12} = 13.2$$ (T-M)

BARRA (c)

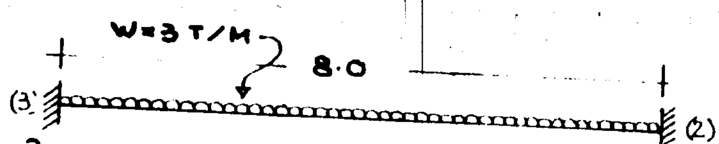
NO TIENE SOLICITACIONES

BARRA (d)



$$M_3 = \frac{wl^2}{8} = \frac{3 \times 6^2}{8} = 13.5$$ (T/M)

BARRA (e)

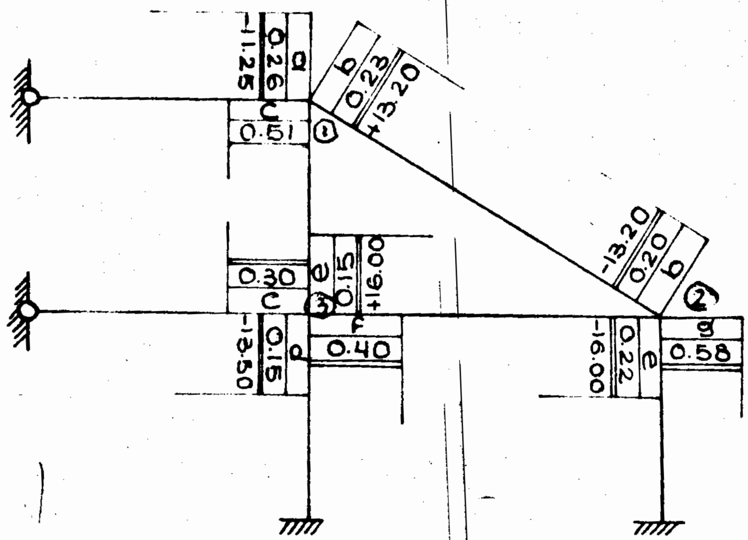


$$M_3 = M_2 = \frac{wl^2}{12} = \frac{3 \times 8^2}{12} = 16$$ (T/M)

BARRAS (f) y (g)

NO TIENEN SOLICITACIONES

ETAPA 5



EN LAS BARRAS SE HA ANOTADO: 1º NOMBRE DE LA BARRA. 2º FACTOR DE DISTRIBUCIÓN DE LA BARRA. 3º MOMENTO DE EMPOTRAMIENTO CON SU SIGNO.

PARA PROSEGUIR CON EL ANALISIS:

1.- SE ESCOGE UN NUDO (DE PREFERENCIA EL QUE TENGA MAYOR MOMENTO DE DESEQUILIBRIO).

2.- SE MULTIPLICA EL MOMENTO DE DESEQUILIBRIO POR CADA FACTOR DE DISTRIBUCION Y SE ANOTA EN LA BARRA CORRESPONDIENTE CON EL SIGNO CAMBIADO (CON ESTO SE TIENE EL NUDO ESCOGIDO EN EQUILIBRIO).

3.- EN CADA BARRA SE MULTIPLICA EL MOMENTO QUE ABSORBE POR EL FACTOR DE TRANSPORTE , Y EL PRODUCTO SE ANOTA EN EL OTRO EXTREMO DE LA BARRA.

ESCOGIENDO EL NUDO (2) :

MOMENTO DE DESEQUILIBRIO = -13.2 -16.0 + 0 = -29.2

PARA LA BARRA (b) :

MOMENTO QUE ABSORBE = -29.2 x 0.2 x (-1) = + 5.84

MOMENTO QUE TRANSPORTA AL NUDO (1) = + 5.84 x 0.5 = +2.92

PARA LA BARRA (g) :

MOMENTO QUE ABSORBE = -29.2 x 0.58 x (-1) = +16.9

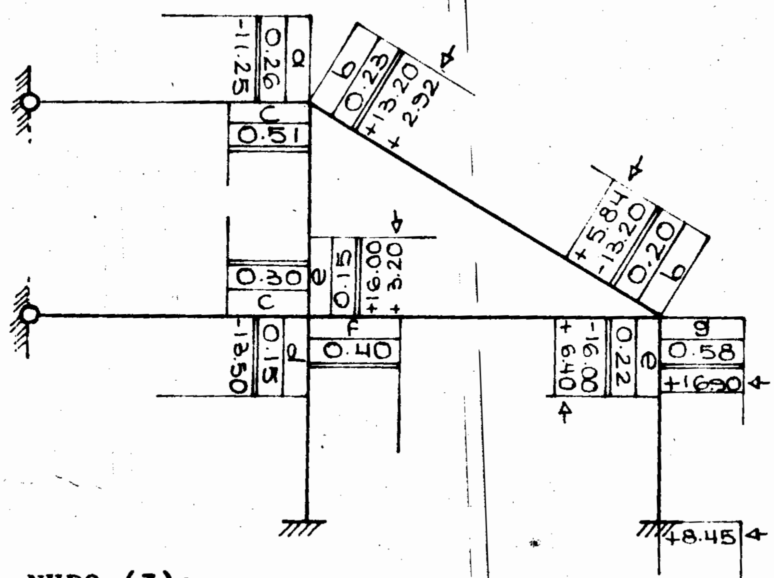
MOMENTO QUE TRANSPORTA AL APOYO = 16.9 x 0.5 = 8.45

PARA LA BARRA (e) :

MOMENTO QUE ABSORBE = -29.2 x 0.22 x (-1) = 6.4

MOMENTO QUE TRANSPORTA AL NUDO (3) = 6.3 x 0.5 = 3.2

VACIANDO LO ANTERIOR EN EL CROQUIS:



ESCOGIENDO EL NUDO (3):

MOMENTO DE DESEQUILIBRIO = 16.0 + 3.2 - 13.5 = 5.7

BARRA (c) :

MOMENTO QUE ABSORBE = 5.7 x 0.3 x (-1) = -.1.71

MOMENTO QUE TRANSPORTA AL NUDO (1) = -1.71 x 0.5 = -0.85

BARRA (e) :

MOMENTO QUE ABSORBE = 5.7 x 0.15 x (-1) = -0.85

MOMENTO QUE TRANSPORTA AL NUDO (2) = -0.85 x 0.5 = -0.43

BARRA (f):

MOMENTO QUE ABSORBE = 5.7 x 0.40 x (-1) = -2.28

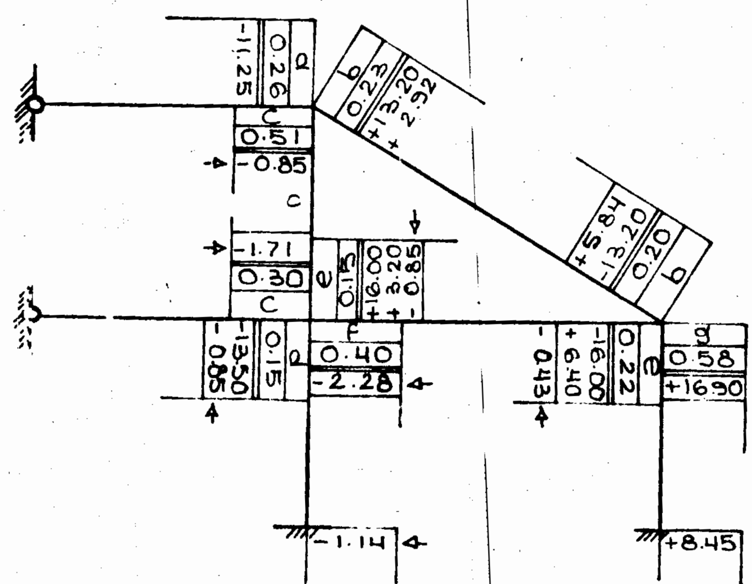
MOMENTO QUE TRANSPORTA AL APOYO = -2.28 x 0.5 =-1.14

BARRA (d):

MOMENTO QUE ABSORBE = 5.7 x 0.15 x (-1) = - 0.85

MOMENTO QUE TRANSPORTA AL APOYO = -0.85 x 0 = 0

VACIANDO LO ANTERIOR EN EL CROQUIS



ESCOGIENDO EL NUDO (1) :

MOMENTO DE DESEQUILIBRIO = -11.25 -0.85 + 13.2 + 2.92 = 4.02

BARRA (b) :

MOMENTO QUE ABSORBE + 4.05 x 0.23 x (-1) = -0.93

MOMENTO QUE TRANSPORTA AL NUDO (2) = -0.93 x 0.5 = -0.46

BARRA (c):

MOMENTO QUE ABSORBE = 4.02 x 0.51 x (-1) = -2.05

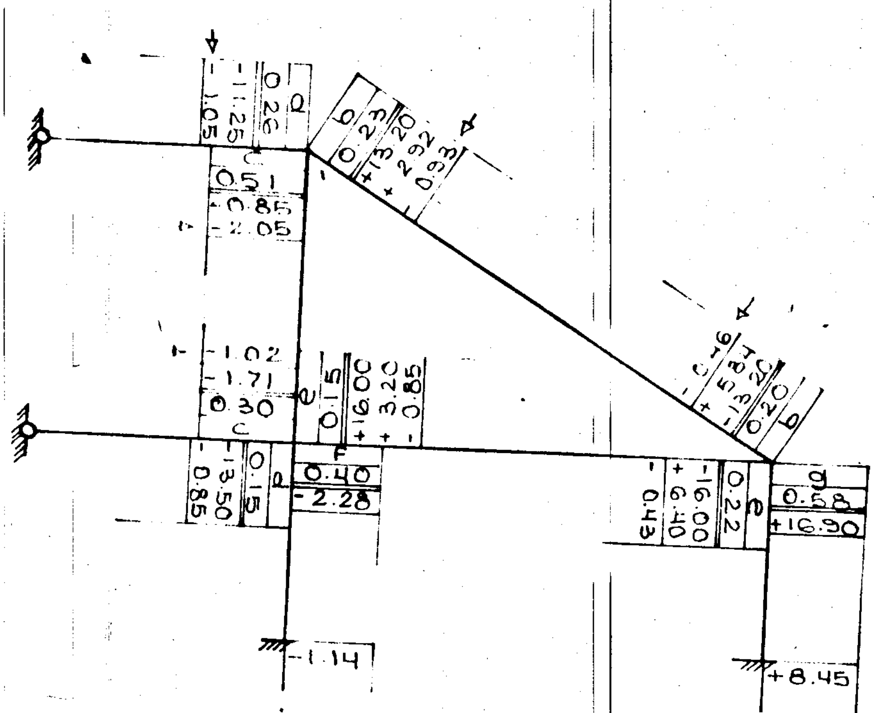
MOMENTO QUE TRANSPORTA AL NUDO (3) = -2.05 x 0.5 = -1.02

BARRA (a):

MOMENTO QUE ABSORBE = 4.02 x 0.26 x (-1) = -1.05

MOMENTO QUE TRANSPORTA AL APOYO = -1.05 x 0 = 0

VACIANDO LO ANTERIOR EN EL CROQUIS:



ESCOGIENDO EL NUDO (3):

MOMENTO DE DESEQUILIBRIO = - 1.02

BARRA (d):

MOMENTO QUE ABSORBE = -1.02 x 0.15 x (-1) = 0.31

MOMENTO QUE TRANSPORTA AL NUDO (1) = 0.31 x 0.5 = 0.15

BARRA (e):

MOMENTOQUE ABSORBE = -1.02 x 0.15 x (-1) = 0.15

MOMENTO QUE TRANSPORTA AL NUDO (2) = 0.15 x 0.5 = 0.08 = 0.75

BARRA (f):

MOMENTO QUE ABSORBE = -1.02 x 0.40 x (-1) = 0.41

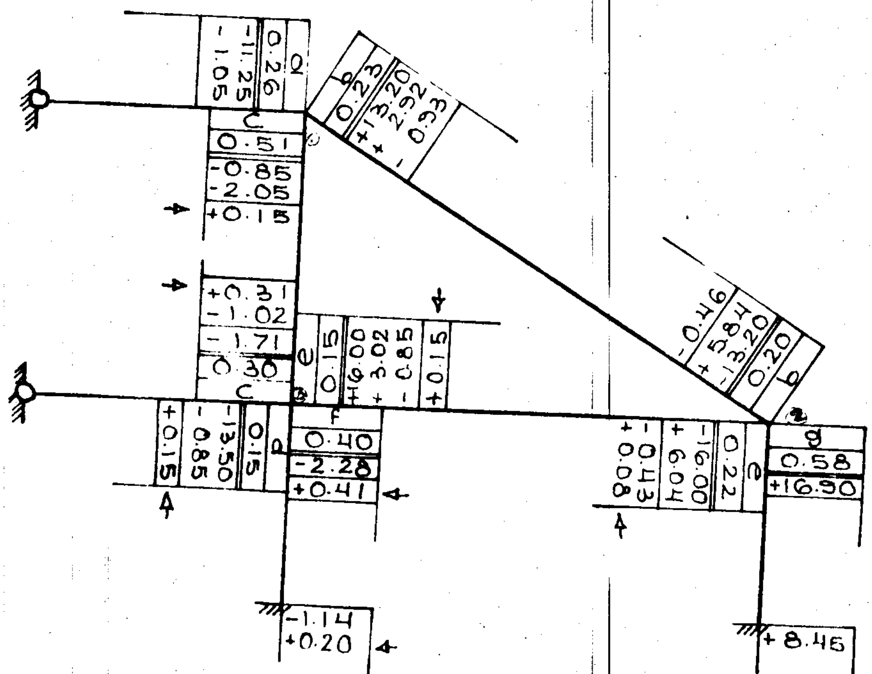
MOMENTO QUE TRANSPORTA AL APOYO = 0.41 x 0.5 = 0.20

BARRA (d) :

MOMENTO QUE ABSORBE = -1.02 x 0.15 x (-1)=+ 0.15

MOMENTO QUE TRANSPORTA AL APOYO = 0.15 x 0 = 0

VACIANDO LO ANTERIOR EN EL CROQUIS :



ESCOGIENDO EL NUDO (2)

MOMENTO DE DESEQUILIBRIO = -0.46 -0.43 + 0.08 = -0.81

BARRA (b):

MOMENTO QUE ABSORBE = -0.81 x 0.20 x (-1) = 0.16

MOMENTO QUE TRANSPORTA AL NUDO (1)= 0.16 x 0.5 = 0.08

BARRA (g):

MOMENTO QUE ABSORBE = -0.81 x 0.58 x (-1) = 0.47

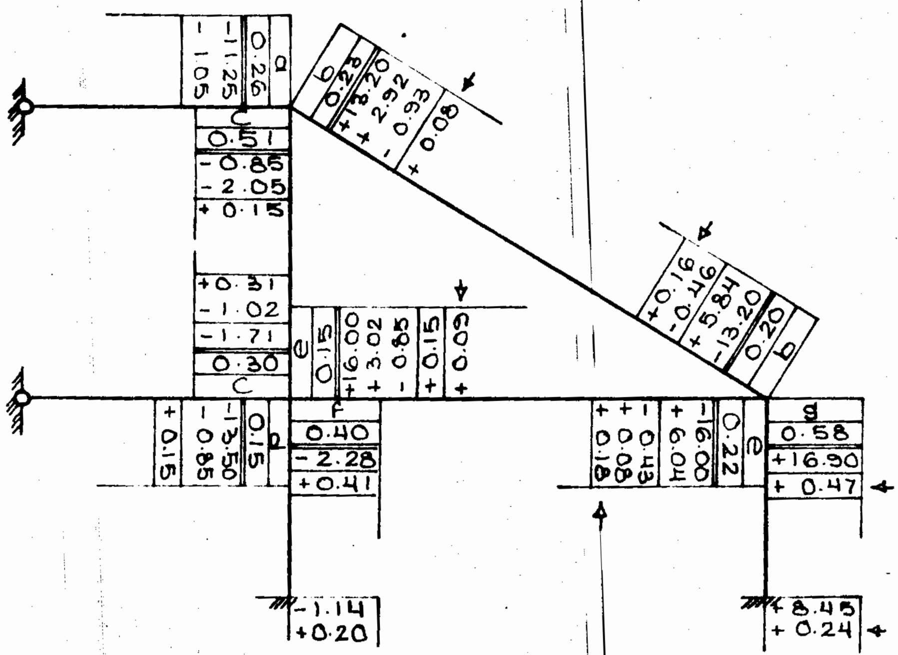
MOMENTO QUE TRANSPORTA AL APOYO = 0.47 x 0.5 = 0.24

BARRA (e):

MOMENTO QUE ABSORBE = -0.81 x 0.22 x (-1) = 0.18

MOMENTO QUE TRANSPORTA AL NUDO (3) = 0.18 x 0.5 = 0.09

VACIANDO LO ANTERIOR EN EL CROQUIS:



ESCOGIENDO EL NUDO (1):

MOMENTO DE DESEQUILIBRIO = 0.15 + 0.08 = 0.23

BARRA (b):

MOMENTO QUE ABSORBE = 0.23 x 0.23 x (-1) = -0.05

MOMENTO QUE TRANSPORTA AL NUDO (2) = -0.05 x 0.5 = -0.03

BARRA (c):

MOMENTO QUE ABSORBE = 0.23 x 0.51 x (-1) = -0.12

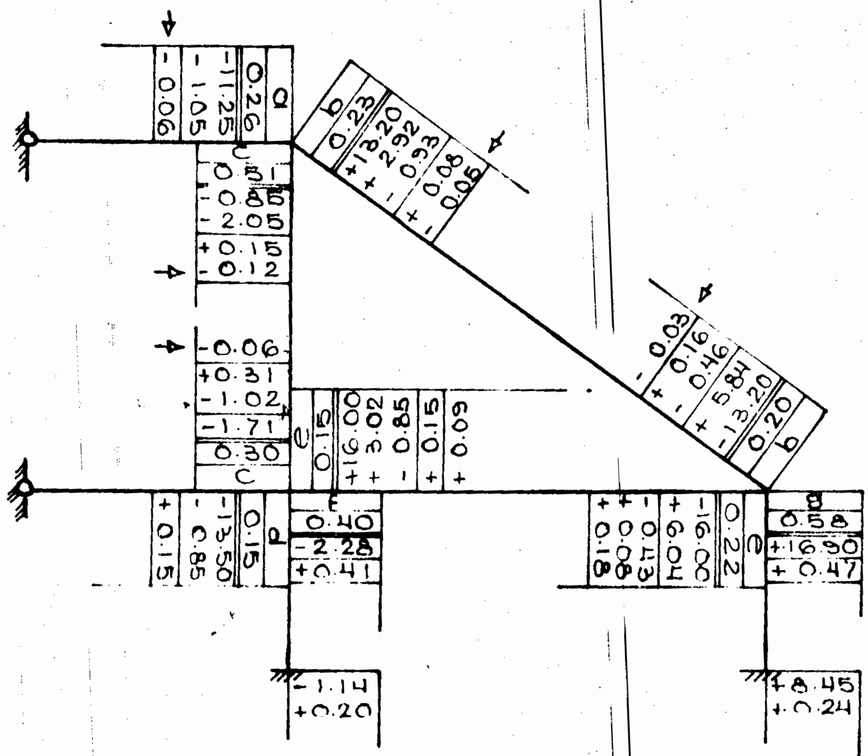
MOMENTO QUE TRANSPORTA AL NUDO (3) = -0.12 x 0.5 = -0.06

BARRA (a):

MOMENTO QUE ABSORBE = 0.23 x 0.26 x (-1) = -0.06

MOMENTO QUE TRANSPORTA AL APOYO = -0.06 x 0 = 0

VACIANDO LO ANTERIOR EN EL CROQUIS:



SUPONGAMOS QUE QUEREMOS LLEGAR A UNA APROXIMACION DE 0.05 EN -- LOS MOMENTOS, NO HAY NINGUN NUDO QUE TENGA UN MOMENTO DE DESEQUILIBRIO MAYOR DE 0.05, POR LO TANDO NO ES NECESARIO HACER MAS ITERACIONES.

PARA CONCLUIR CON EL PROBLEMA SOLO FALTA CONOCER LOS MOMENTOS - FINALES EN LOS EXTREMOS DE LAS BARRAS. PARA OBTENERLOS SE EFECTUA LA SUMA ALGEBRAICA DE LOS MOMENTOS ANOTADOS EN EL EXTREMO - DE CADA BARRA.

NUDO (1):

BARRA (a): -11.25 - 1.05 -0.06 = -12.36

BARRA (b): 13.20 + 2.92 + 0.08 - 0.93 -0.05 = 15.22

BARRA (c): - 0.85 - 2.12 + 0.15 = -2.93

NUDO (2):

BARRA (b): -13.20 - 0.46 - 0.03 + 5,84 + 0.16 = -7.69

BARRA (g): 0 + 16.9 + 0.47 = 17.37

BARRA (e): -16.0 - 0. 43 + 6.04 + 0.08 + 0.18 = - 10.13

NUDO (3):

BARRA (c): 0 - 1.71 - 1.02 - 0.06 + 0.31 = - 2.48

BARRA (e): 16.00 + 3.02 + 0.15 + 0.09 - 0.85 = 18.41

BARRA (f): 0 + 0.41 - 2.28 = - 1.87

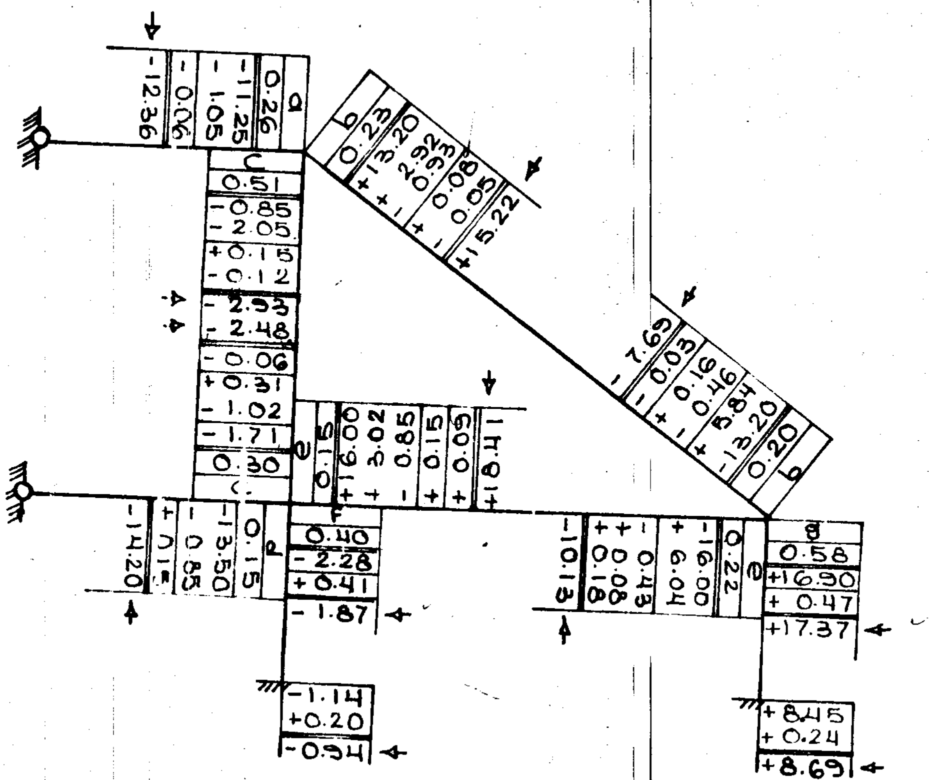
BARRA (d): - 13.50 - 0.85 + 0.15 = - 14. 20

APOYOS

A.- - 1.14 + 0.20 = - 0.94

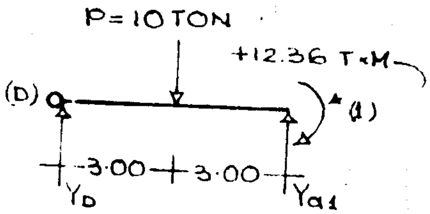
B.- 8.45 + 0.24 = 8.69

VACIANDO LO ANTERIOR EN EL CROQUIS:



CALCULO DE LAS REACCIONES

BARRA (a)



$$\Sigma M_1 = 0$$

$$12.36 + Y_D \times 6 - 10 \times 3 = 0$$

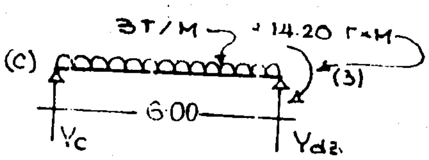
$$Y_D = \frac{30 - 12.36}{6} = \frac{17.64}{6} = 2.9 \text{ TON}$$

$$\Sigma F_y = 0$$

$$Y_D + Y_{a1} - 10 = 0$$

$$Y_{a1} = 10 - Y_D = 10 - 2.9 = 7.1 \text{ TON}$$

BARRA (d)



$$\Sigma M_3 = 0$$

$$14.20 + 6\, Y_C - 3 \times \frac{6^2}{2} = 0$$

$$Y_C = \frac{-14.20 + 1.5 \times 36}{6} = \frac{-14.20 + 54}{6} = \frac{39.8}{6}$$

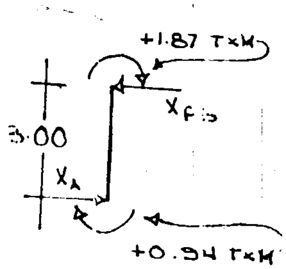
$$Y_C = 6.64 \text{ TON}$$

$$\Sigma F_y = 0$$

$$6.64 - 18 + Y_{d3} = 0$$

$$Y_{d3} = 18 - 6.64 = 11.36 \text{ TON}$$

BARRA (f)



$$\Sigma M_3 = 0$$

$$1.87 + 0.94 - X_A \times 3 = 0$$

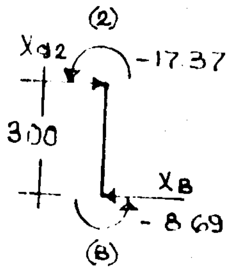
$$X_A = \frac{1.87 + 0.94}{3} = \frac{2.81}{3} = 0.94$$

$$\Sigma M_x = 0$$

$$0.94 - X_{f3} = 0$$

$$X_{f3} = 0.94$$

BARRA (g)



$M_2 = 0$

$-17.37 - 8.69 + 3X_B = 0$

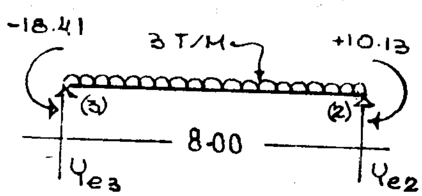
$X_B = \frac{17.37+8.69}{3} = \frac{26.06}{3} = 8.7$ TON

$\Sigma F_X = 0$

$-8.7 + X_{g_2} = 0$

$X_{g_2} = 8.7$ TON

BARRA (e)



$M_2 = 0$

$-18.41 + 10.3 + Y_{w_3} \times 8 - 3\,\frac{8^2}{2} = 0$

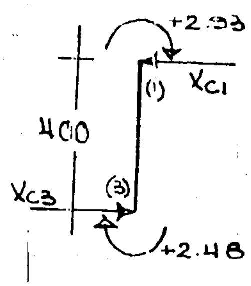
$Y_{e_3} = \frac{18.41-10.3+96.00}{8} = \frac{104.11}{8} = 13$ TON

$\Sigma F_y = 0$

$Y_{e_3} + Y_{e_2} - 24 = 0$

$Y_{e_2} = 24 - 13 = 11$ TON

BARRA (c)



$\Sigma M_3 = 0$

$2.48 + 2.93 - X_{c_1} \times 4 = 0$

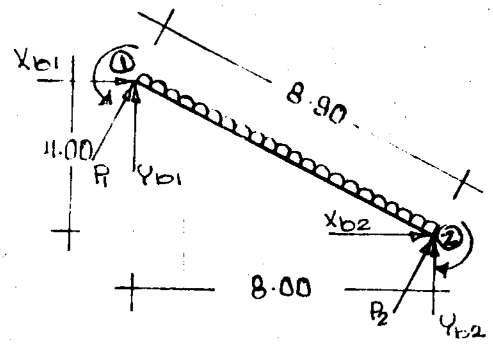
$X_{c_1} = \frac{2.48+2.93}{4} = \frac{5.41}{4} = 1.35$ TON

$\Sigma M_x = 0$

$X_{c_3} - 1.35 = 0$

$X_{c_3} = 1.35$ TON

BARRA (b)



$$\Sigma M_2 = 0$$

$$-15.22+7.69+8.9P_1-2\frac{8.9^2}{2} = 0$$

$$P_1 = \frac{15.22-7.69+80}{8.9} = \frac{87.53}{8.9} = 9.85$$

$$\Sigma F_{y'} = 0$$

$$P_1 + P_2 - 2 \times 8.9 = 0$$

$$P_2 = 17.8 - 9.85 = 7.95 \text{ TON}$$

$$X_{b_1} = 9.85 \frac{4}{8.9} = 4.42 \text{ TON}$$

$$Y_{b_1} = 9.85 \frac{8}{8.9} = 8.85 \text{ TON}$$

$$X_{b_2} = 7.95 \frac{4}{8.9} = 3.57 \text{ TON}$$

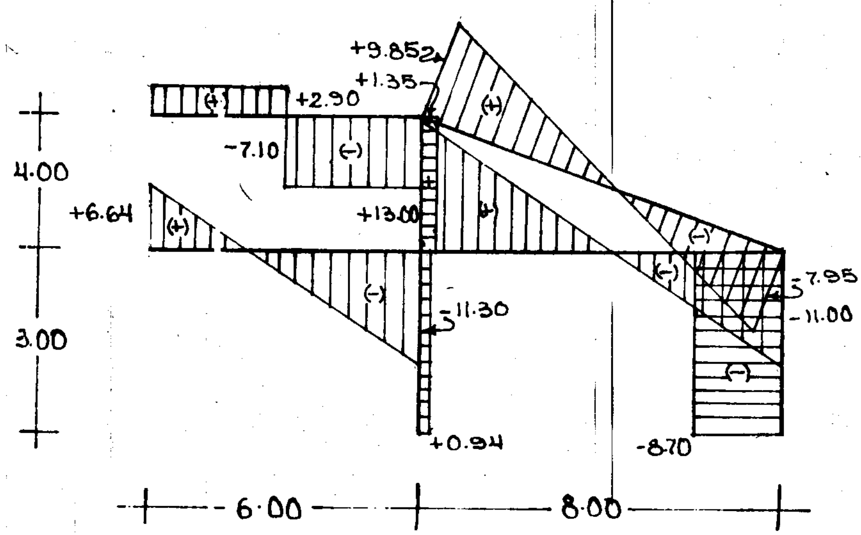
$$X_D = -1.35+4.42 = 2.07 \text{ TON}$$

$$X_C = 1.35-1.87+3.37+8.7 = 11.55 \text{ TON}$$

$$Y_A = 11.36+13.00+7.1+8.85 = 40.31 \text{ TON}$$

$$Y_B = 11.00+7.12 = 18.12 \text{ TON}$$

DIAGRAMA DE FUERZAS CORTANTES



PARA TENER COMPLETO EL DIAGRAMA DE MOMENTOS FLEXIONANTES SOLO - FALTA CALCULAR LOS MOMENTOS MAXIMOS ENTRE LOS APOYOS DE LAS BARRAS. PARA ELLO SE CALCULA EL MOMENTO MAXIMO DE CADA BARRA COMO SIMPLEMENTE APOYADA ES DECIR COMO SI FUESE ISOSTATICA.

M_{isos} = MOMENTO ISOSTATICO

BARRA (a)

$$M_{isos} = \frac{PL}{4} = \frac{10 \times 6}{4} = 15 \text{ T x M}$$

BARRA (b)

$$M_{isos} = \frac{WL^2}{8} = \frac{2 \times 8.9^2}{8} = 19.8 \text{ T x M}$$

BARRA (d)

$$M_{isos} = \frac{WL^2}{8} = \frac{3 \times 6^2}{8} = 13.5 \text{ T x M}$$

BARRA (e)

$$M_{isos} = \frac{WL^2}{8} = \frac{3 \times 8^2}{8} = 24.0 \text{ T x M}$$

BARRAS (c),(f) y (g) NO TIENEN SOLICITACIONES

CALCULO DE MOMENTOS FINALES EN EL CENTRO DEL CLARO

BARRA (a)

$$M_a = \frac{-12.36}{2} + 15 = -6.18+15 = +8.82 \text{ T x M}$$

BARRA (b)

$$M_b = \frac{-15.22-7.69}{2} + 19.80 = -11.46+19.80$$

$$M_b = +8.34 \text{ T x M}$$

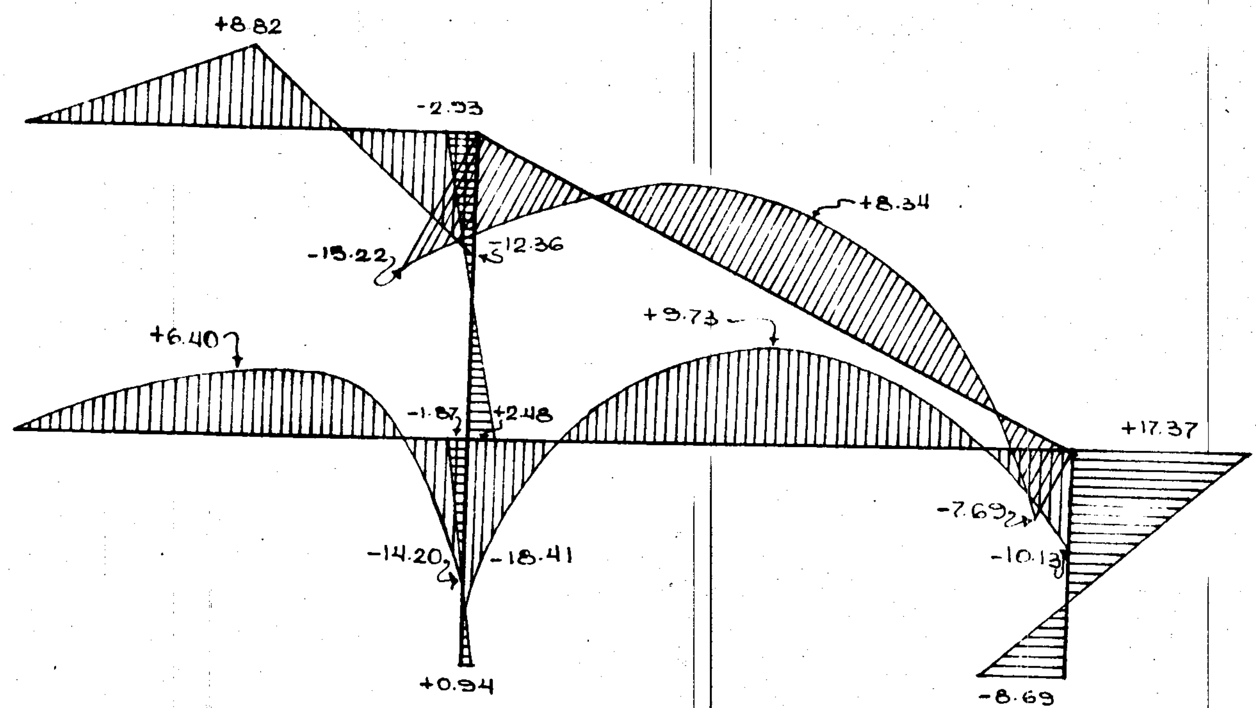
BARRA (d)

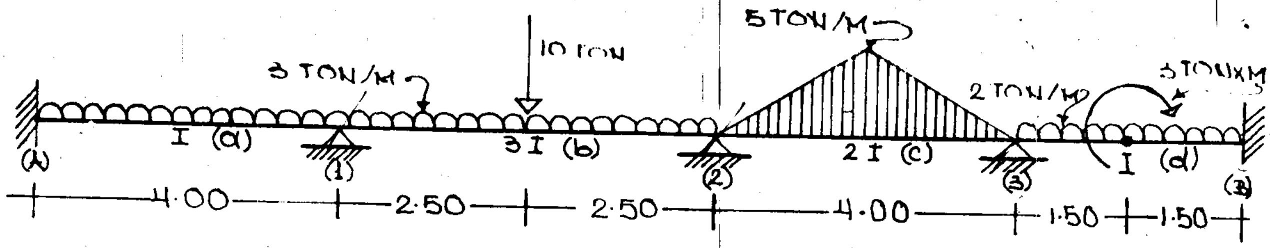
$$M_d = \frac{-14.20}{2} + 13.5 = -7.10+13.5 = +6.40 \text{ T x M}$$

BARRA (e)

$$M_e = \frac{-18.41-10.13}{2} + 24 = -14.27+24 = +9.73 \text{ T x M}$$

DIAGRAMA DE MOMENTOS FINALES





RIGIDECES Y FACTORES DE TRANSPORTE

NUDO (1)

$r_{aa} = \frac{4E(I)}{l} = \frac{4EI}{4} = 1.0\ EI$

$r_{ab} = \frac{4E(I)}{l} = \frac{4E(3I)}{5} = 2.4\ EI$

$\Sigma = 3.4\ EI$

$F.D_a = \frac{1.0\ EI}{3.4\ EI} = 0.29$

$F.D_b = \frac{2.4\ EI}{3.4\ EI} = 0.71$

$\Sigma = 1.00$

NUDO (2)

$r_{ab} = \frac{4E(I)}{l} = \frac{4E(3I)}{5} = 2.4\ EI$

$r_{ac} = \frac{4EI}{l} = \frac{4E(2I)}{4} = 2.0\ EI$

$\Sigma = 4.4\ EI$

$F.D_b = \frac{2.4\ EI}{4.4\ EI} = 0.55$

$F.D_c = \frac{2.0\ EI}{4.4\ EI} = 0.45$

$\Sigma = 1.00$

NUDO (3)

$r_{ac} = \frac{4E(I)}{l} = \frac{4E(2I)}{4} = 2.0\ EI$

$r_{ad} = \frac{3E(I)}{l} = \frac{3EI}{3} = 1.0\ EI$

$\Sigma = 3.0\ EI$

$F.D_c = \frac{2\ EI}{3\ EI} = 0.67$

$F.D_d = \frac{1\ EI}{3\ EI} = 0.33$

$\Sigma = 1.00$

MOMENTOS DE EMPOTRAMIENTO

BARRA (a)

$$M_a = -M_1 = \frac{wl^2}{12} = \frac{3 \times 4^2}{12} = 4 \text{ T x M}$$

$$M_{isos} = \frac{wl^2}{8} = \frac{3 \times 4^2}{8} = 6 \text{ T x M}$$

BARRA (b)

$$M_1 = -M_2 = \frac{wl^2}{12} + \frac{pl}{8} = \frac{3 \times 5^2}{12} + \frac{10 \times 5}{8}$$

$$M_1 = -M_2 = 6.25 + 6.25 = 12.50 \text{ T x M}$$

$$M_{isos} = \frac{wl^2}{8} + \frac{pl}{4} = \frac{3 \times 5^2}{8} = \frac{10 \times 5}{5}$$

$$M_{isos} = 9.38 = 10 = 19.38 \text{ TxM}$$

BARRA (c)

$$M_2 = -M_3 = \frac{5wl^2}{96} = \frac{5 \times 5 \times 4^2}{96} = 4.17 \text{ T x M}$$

$$M_{isos} = \frac{wl^2}{12} = \frac{5 \times 4^2}{12} = 6.67 \text{ T x M}$$

BARRA (d)

$$M_3 = -\frac{M}{8} + \frac{wl^2}{12} = -\frac{3}{8} + \frac{2 \times 3^2}{12} = -0.375 + 1.5$$

$$M_3 = 1.125 \text{ T x M}$$

$$M_B = 0$$

$$M_{isos} = \frac{wl^2}{8} + \frac{M}{2} = \frac{2 \times 3^2}{8} + \frac{3}{2}$$

$$M_{isos} = 2.25 + 1.5 = 3.75 \text{ T x M}$$

	(A)	(a)	(1)		(b)	(2)		(c)	(3)		(d)	(B)
BARRA	a		a	b		b	c		c	d		d
F.D	0		0.29	0.71		0.55	0.45		0.67	0.33		1.00
M.E	+4.00		-4.00	+12.50		-12.50	+4.17		-4.17	+1.20		0.00
1	-1.23	←	-2.46	-6.04	→	-3.02						
2				+3.13	←	+6.25	+5.10	→	+2.55			
3	-0.46	←	-0.91	-2.22	→	-1.11						
4				+0.31	←	+0.61	+0.50	→	+0.25			
5							+0.09	←	+0.17	+0.08		
6	-0.05	←	-0.09	-0.22	→	-0.11						
7						+0.01	+0.01					
8												
M.F	+2.26		-7.46	+7.46		-9.87	+9.87		-1.20	+1.20		0.00

CALCULO DE REACCIONES Y DIAGRAMAS

BARRA (a)

TOMANDO MOMENTOS FLEXIONANTES EN (1)

$$-7.46 = -2.26 + 4Y_a - \frac{3 \times 4^2}{2}$$

$$4Y_a = -7.46 + 2.26 + 24 = 18.80$$

$$Y_a = \frac{18.80}{4} = 4.70 \text{ TON}$$

$$Y_{a_1} = -4.70 + 3 \times 4 = -4.70 + 12.00 = +7.30$$

BARRA (b)

$$-9.87 = -7.46 + Y_{b_1} \times 5 - \frac{3 \times 5^2}{2} - 10 \times 2.5$$

$$-9.87 = -7.46 + 5Y_{b_1} - 37.50 - 25.00$$

$$Y_{b_1} = \frac{-9.87 + 7.46 + 37.50 + 25.00}{5} = \frac{60.09}{5} = 12.02$$

$$Y_{b_2} = -12.02 + 3 \times 5 + 10 = -12.02 + 15 + 10 = 12.98 \text{ TON}$$

BARRA (c)

$$-1.20 = -9.87 + 4Y_{c_2} - \frac{5 \times 4 \times 4}{2 \times 2}$$

$$-1.20 = -9.87 + 4Y_{c_2} - 20$$

$$Y_{c_2} = \frac{-1.20 + 9.87 + 20}{4} = \frac{28.67}{4} = 7.1 \text{ TON}$$

$$Y_{c_3} = -7.17 + \frac{5 \times 4}{2} = -7.17 + 10 = 2.83$$

BARRA (d)

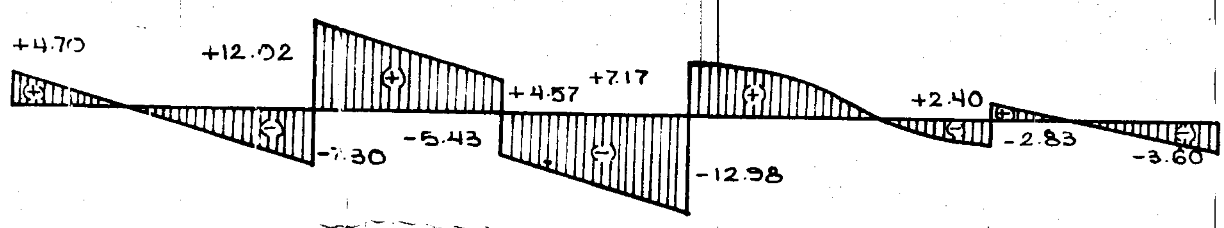
$$0 = -1.20 + 3Y_{d_3} - \frac{2 \times 3^2}{2} + 3$$

$$0 = -1.20 + 3Y_{d_3} - 9 + 3$$

$$Y_{d_3} = \frac{1.20 + 9 - 3}{3} = \frac{7.20}{3} = 2.40 \text{ TON}$$

$$Y_B = -2.40 + 2 \times 3 = -2.40 + 6.00 = 2.60 \text{ TON}$$

DIAGRAMA DE FUERZAS CORTANTES



CALCULO DE MOMENTOS FINALES EN EL CENTRO DEL CLARO

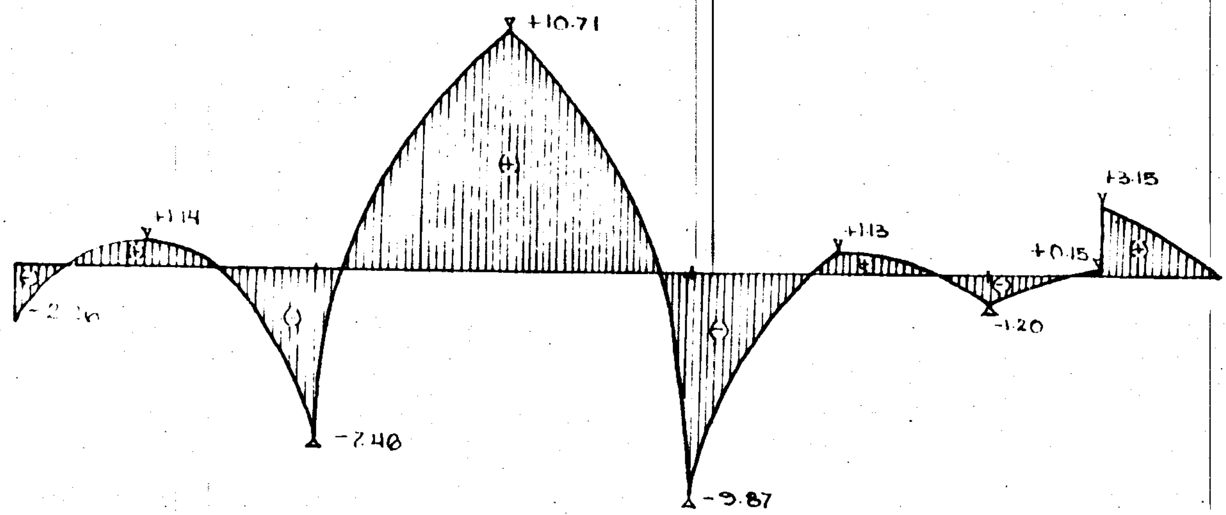
$$M_a = \frac{-2.26 - 7.46}{2} + 6.00 = \frac{-9.72}{2} + 6.00 = -4.86 + 6.00 = +1.14 \text{ T x M}$$

$$M_b = \frac{-7.46 - 9.37}{2} + 19.38 = \frac{-17.33}{2} + 19.38 = -8.67 + 19.38 = +10.71 \text{ T x M}$$

$$M_c = \frac{-9.87-1.20}{2} + 6.67 = \frac{-11.07}{2} + 6.67 = -5.54+6.67 = +1.13 \text{ T x M}$$

$$M_d = -\frac{1.20}{2} + 3.75 = -0.60 + 3.75 = +3.15 \text{ T x M}$$

DIAGRAMA DE MOMENTOS FINALES



CALCULO DE LAS REACCIONES

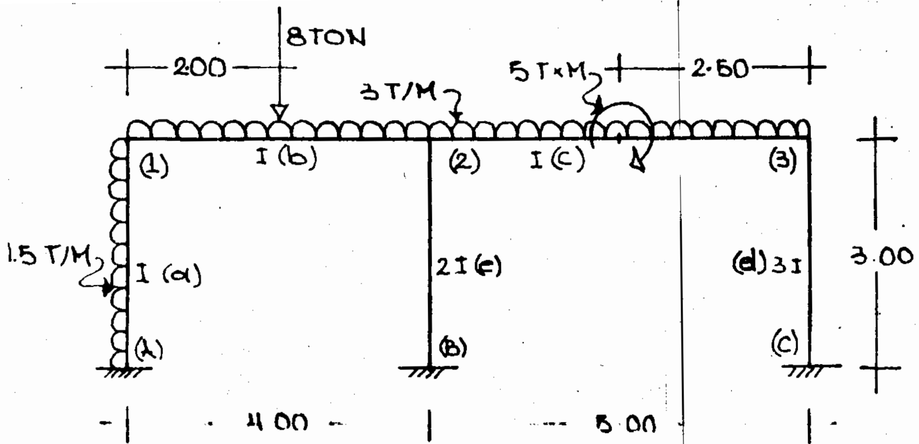
$R_A = Y_A = 4.70$ TON

$R_{(1)} = Y_{a_1} + Y_{b_1} = 7.30 + 12.02 = 19.32$ TON

$R_{(2)} = Y_{b_2} + Y_{c_2} = 12.98 + 7.17 = 20.15$ TON

$R_{(3)} = Y_{c_3} + Y_{d_3} = 2.83 + 2.40 = 5.23$ TON

$R_B = Y_B = 3.60$ TON



RIGIDECES Y FACTORES DE TRANSPORTE

NUDO (1)

$r_{aa} = \frac{4E(I)}{l} = \frac{4EI}{3} = 1.33\ EI$ $F.D_a = \frac{1.33EI}{2.33EI} = 0.57$

$r_{ab} = \frac{4E(I)}{l} = \frac{4EI}{4} = 1.00\ EI$ $F.D_b = \frac{1.00EI}{2.33EI} = 0.43$

$\Sigma = 2.33\ EI$ $\Sigma = 1.00$

NUDO (2)

$r_{ab} = \frac{4E(I)}{l} = \frac{4EI}{4} = 1.00\ EI$ $F.D_b = \frac{1.00EI}{4.46EI} = 0.224$

$r_{ac} = \frac{4E(I)}{l} = \frac{4EI}{5} = 0.80\ EI$ $F.D_c = \frac{0.80EI}{4.46EI} = 0.179$

$r_{ae} = \frac{4E(I)}{l} = \frac{4E(2I)}{3} = 2.66\ EI$ $F.D_e = \frac{2.66EI}{4.46EI} = 0.597$

$\Sigma = 4.46\ EI$ $\Sigma = 1.000$

NUDO (3)

$r_{ac} = \frac{4E(I)}{l} = \frac{4EI}{5} = 0.80\ EI$ $F.D_c = \frac{0.80EI}{4.80EI} = 0.167$

$r_{ad} = \frac{4E(I)}{l} = \frac{4E(3I)}{3} = 4.00\ EI$ $F.D_d = \frac{4.00EI}{4.80EI} = 0.833$

$\Sigma = 4.80\ EI$ $\Sigma = 1.000$

MOMENTOS DE EMPOTRAMIENTO

BARRA (a)

$$M_A = -M_1 = \frac{wl^2}{12} = \frac{1.5x3^2}{12} = 1.13$$

BARRA (b)

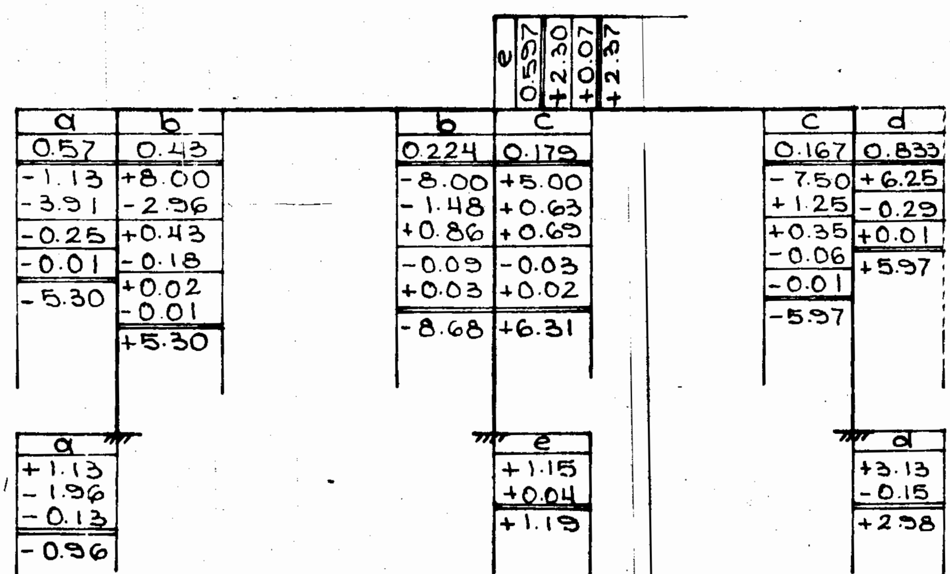
$$M_1 = -M_2 = \frac{wl^2}{12} + \frac{pl}{8} = \frac{3x4^2}{12} + \frac{8x4}{8} = 4+4 = 8 \text{ T x M}$$

BARRA (c)

$$M_2 = \frac{wl^2}{12} - \frac{M}{4} = \frac{3 \text{ x } 5^2}{12} - \frac{5}{4}$$

$$M_2 = 6.35 - 1.25 = 5 \text{ T x M}$$

$$M_3 = \frac{wl^2}{12} - \frac{M}{4} = -6.25 - 1.25 = -7.50 \text{ T x M}$$

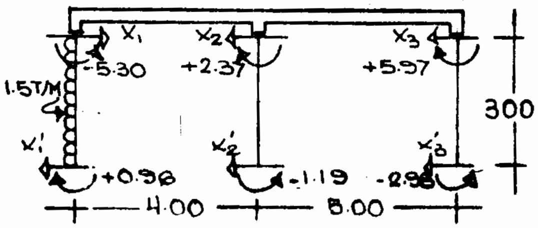


ORDEN DE EQUILIBRIO DE LOS NUDOS : 3,1,2,3,1,2,1,3

EN EL ANALISIS ANTERIOR SE HA CONSIDERADO QUE EL MARCO NO SUFRIO DESPLAZAMIENTO EN EL CABEZAL ES DECIR SE HA CONSIDERADO QUE SE TIENE UN APOYO A LA ALTURA DEL CABEZAL.

A CONTINUACION SE PROCEDE A ENCONTRAR LA FUERZA DE SUJECION ES DECIR LA REACCION DEL APOYO FICTICIO DEL CABEZAL.

ANALISIS DEL CABEZAL



MOMENTOS FLEXIONANTES EN EL NUDO (1)

$$-5.30 = +0.96 - \frac{1.5 \times 3^2}{2} + 3X_1'$$

$$X_1' = \frac{-5.30 - 0.96 + 6.75}{3} = +\frac{0.49}{3} = 0.16 \text{ TON}$$

$$X_1 = 1.5 \times 3 - 0.16 = 4.34 \text{ TON}$$

MOMENTO FLEXIONANTE EN EL NUDO (2)

$$+2.37 = -1.19 + 3X_2'$$

$$X_2' = \frac{2.37 + 1.19}{3} = \frac{3.56}{3} = 1.19$$

$$X_2 = X_2' = 1.19 \text{ TON}$$

MOMENTO FLEXIONANTE EN EL NUDO (3)

$$+5.97 = -2.98 + 3X_3'$$

$$X_3' = \frac{5.97 + 2.98}{3} = \frac{8.95}{3} = 2.98$$

$$X_3 = X_3' = 2.98 \text{ TON}$$

FUERZA DE SUJECION: $F_1 = X_1 + X_2 + X_3$

$$F_1 = 4.34 - 1.19 - 2.98 = 0.17$$

PARA IMPEDIR EL DESPLAZAMIENTO DEL CABEZAL SE NECESITA APLICAR UNA FUERZA $F_1 = 0.17$ TON DE DERECHA A IZQUIERDA EN EL CABEZAL.

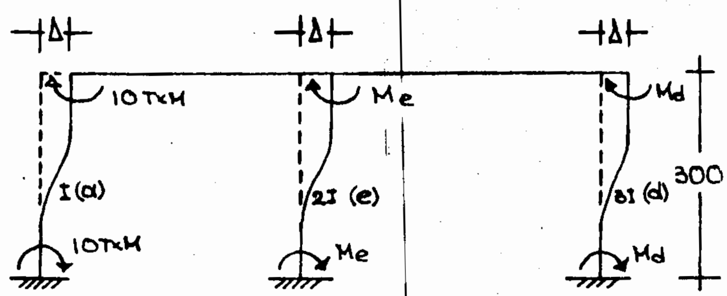
PARA ANALIZAR EL GRADO DE LIBERTAD LINEAL

1º SE SUPONE UN DESPLAZAMIENTO DEL CABEZAL (POR EJEMPLO UN -- DESPLAZAMIENTO TAL QUE PRODUZCA UN MOMENTO DE 10 T x M A UNA DE LAS BARRAS Y SE CALCULA EL VALOR DE DICHO DESPLAZAMIENTO).

2º SE CALCULAN LOS MOMENTOS QUE SE PRODUCEN EN LAS BARRAS DEBIDOS AL DESPLAZAMIENTO OBTENIDO.

3º SE EFECTUA UN ANALISIS DE CROSS UTILIZANDO LOS MOMENTOS CALCULADOS COMO MOMENTOS DE EMPOTRAMIENTO.

1er. PASO



$$r_{1a} = \frac{6E(I)}{l^2} = \frac{6EI}{3^2} = 0.667\ EI$$

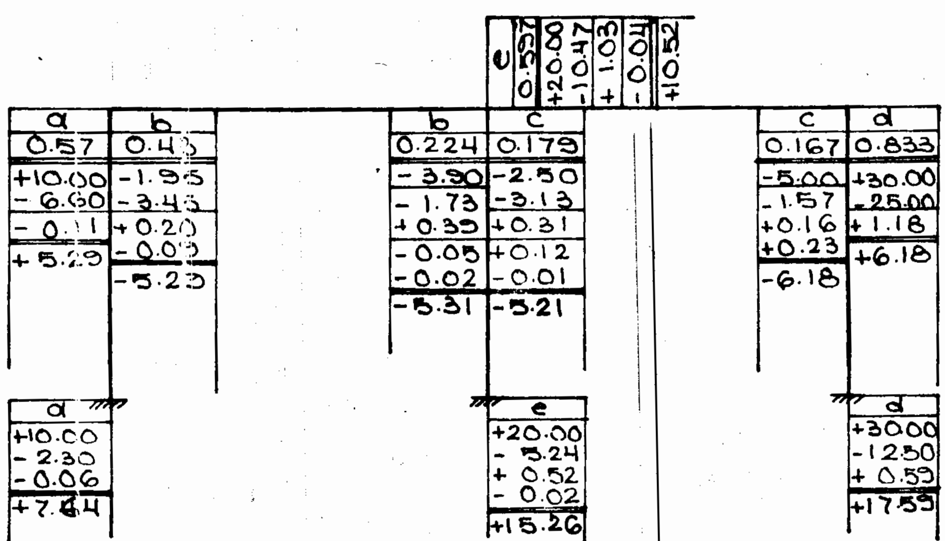
$$\Delta = \frac{M}{r_{1a}} = \frac{10}{0.667EI} = \frac{15}{EI}$$

2º PASO

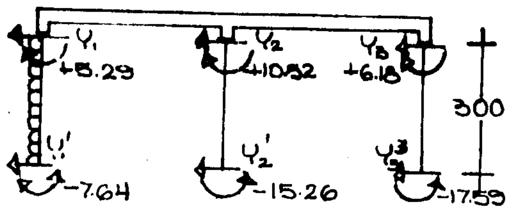
$$r_{1e} = \frac{6E(I)}{l^2} = \frac{6E\times 2I}{3^2} = 1.33\ EI \qquad M_E = \Delta \times r_{1e} = \frac{15}{EI} \times 1.33EI = 20$$

$$r_{1d} = \frac{6E(I)}{l^2} = \frac{6E\times 3I}{3^2} = 2.00\ EI \qquad M_d = \Delta \times r_{1d} = \frac{15}{EI} \times 2\ EI = 30$$

3 er. PASO ORDEN DE EQUILIBRIO DE LOS NUDOS: 3,2,1,2,3,1,2



A CONTINUACION SE EFECTUA EL ANALISIS DEL CABEZAL PARA OBTENER LA FUERZA QUE PRODUJO EL DESPLAZAMIENTO SUPUESTO ($\frac{15}{EI}$).



MOMENTO FLEXIONANTE EN EL NUDO (1)

$+5.29 = -7.64 + 3\,Y'_1$

$Y'_1 = \frac{5.29+7.64}{3} = \frac{12.93}{3} = 4.31$ TON

$Y_1 = Y'_1 = 4.31$ TON

MOMENTO FLEXIONANTE EN EL NUDO (2)

$+10.52 = -15.26 + 3Y'_2$

$Y'_2 = \frac{10.52 + 15.26}{3} = \frac{25.78}{3} = 8.10$

$Y_2 = Y'_2 = 8.10$ TON

MOMENTO FLEXIONANTE EN EL NUDO (3)

$+6.18 = -17.59 + 3Y'_3$

$Y'_3 = \frac{6.18 + 17.59}{3} = \frac{23.77}{3} = 7.93$

$Y_3 = Y'_3 = 7.93$ TON

LA FUERZA QUE PRODUCE EL DESPLAZAMIENTO $= \frac{15}{EI}$ SERA: $F_2 = Y'_1 + Y'_2 + Y'_3$

$F_2 = 4.31 + 8.10 + 7.93 = 20.34$ TON

PARA PRODUCIR LOS ANTERIORES MOMENTOS SE NECESITA APLICAR UNA FUERZA $F_2 = 20.34$ TON DE IZQUIERDA A DERECHA EN EL CABEZAL.

LOS DESPLAZAMIENTOS DEL CABEZAL Y POR LO TANTO LOS MOMENTOS OBTENIDOS EN LA ESTRUCTURA SON DIRECTAMENTE PROPORCIONALES A LA FUERZA QUE LOS PRODUCE , O SEA QUE EXISTE UNA RELACION ENTRE LOS MOMENTOS DEBIDOS AL DESPLAZAMIENTO REAL Y LOS DEBIDOS AL DESPLAZAMIENTO SUPUESTO.

CONSECUENTEMENTE LOS MOMENTOS DEBIDOS AL DESPLAZAMIENTO REAL DEL CABEZAL SERAN IGUALES A LOS OBTENIDOS DEL 2º ANALISIS DE CROSS MULTIPLICADO POR LA CONSTANTE : $C = -\frac{F_1}{F_2}$

LOS MOMENTOS FINALES EN EL EXTREMO DE CADA BARRA SERAN IGUALES AL OBTENIDO EN EL PRIMER CROSS MAS LA CONSTANTE POR EL MOMENTO OBTENIDO EN EL 2º CROSS.

$$CTTE = -\frac{-0.17}{20.34} = 0.00838$$

MOMENTOS FINALES

$$M_{a_1} = -5.30+0.00838x5.29 = -5.30+0.04 = -5.26$$

$$M_{a_A} = -0.96+0.00838x\ 7.64 = -0.96+0.06 = -0.90$$

$$M_{b_1} = +5.30-0.00838x\ 5.29 = +5.30-0.04 = +5.26$$

$$M_{b_2} = -8.68-0.00838x\ 5.31 = -8.68-0.04 = -8.72$$

$$M_{e_2} = +2.37+0.00838x10.52 = +2.37+0.10 = +2.47$$

$$M_{e_b} = +1.19+0.00838x15.26 = +1.19+0.13 = +1.32$$

$$M_{c_2} = +6.31-0.00838x\ 5.21 = +6.31-0.04 = +6.27$$

$$M_{c_3} = -5.97-0.00838x\ 6.18 = -5.97+0.05 = -6.02$$

$M_{d_3} = + 5.97 + 0.00838 \times 6.18 = + 5.97 + 0.05 = + 6.02$

$M_{d_6} = + 2.98 + 0.00838 \times 17.59 = + 2.98 + 0.15 = + 3.13$

CALCULO DE REACCIONES Y DIAGRAMAS

BARRA (a)

$$- 5.26 = MF_1$$

$$- 5.26 = + 0.90 + 3 X_{d_A} - \frac{1.5 \times 3^2}{2}$$

$$X_{d_A} = \frac{-5.26-0.90+6.73}{3} = \frac{0.47}{3} = 0.16$$

$$X_{d_1} = - 0.16 + 1.5 \times 3 = 4.34 \text{ TON}$$

$$M_{isos} = \frac{wl^2}{8} = \frac{1.5 \times 3^2}{8} = 1.69 \text{ T x M}$$

$$M_{℄} = \frac{+0.90-5.26}{2} + 1.69 = \frac{-4.36}{2} + 1.69$$

$$M_{℄} = -2.18 + 1.69 = - 0.49 \text{ T x M}$$

BARRA (b)

$$- 8.72 = MF_2$$

$$- 8.72 = - 5.26 + 4 Y_{b_1} - \frac{3 \times 4^2}{2} - 8 \times 2$$

$$- 8.72 = - 5.26 + 4 Y_{b_1} - 24 - 16$$

$$Y_{b_1} = \frac{-8.72+5.26+24.00+16.00}{4} = \frac{36.54}{4} = 9.15 \text{ TON}$$

$$Y_{b_2} = -9.15 + 3 \times 4 + 8 = 10.85 \text{ TON}$$

$$M_{isos} = \frac{wl^2}{8} + \frac{pl}{4} = \frac{3 \times 4^2}{8} + \frac{8 \times 4}{4} = 6 + 8 = 14.00 \text{ T x M}$$

$$M_{℄} = \frac{-5.26 - 8.72}{2} + 15.00 = \frac{-13.98}{2} + 15.00 = -6.99 + 15.00 = +8.01 \text{ T x M}$$

BARRA (c)

$$-6.02 = MF_3$$

$$-6.02 = -6.22 + 5\,Y_{c_2} - \frac{3x5^2}{2} + 5.00$$

$$-6.02 = -6.22 + 5\,Y_{c_2} - 37.5 + 5.00$$

$$Y_{c_2} = \frac{-6.02+6.22+37.50-5.00}{5} = \frac{32.70}{5} = 6.55 \text{ TON}$$

$$Y_{c_3} = -6.55+3x5.00 = 8.45 \text{ TON}$$

$$M_{isos} = \frac{wl^2}{8} \pm \frac{M}{2} = \frac{3 \times 5^2}{8} \pm \frac{5}{2} = +9.39 \pm 2.5 = \quad M_1 = +6.89 \text{ T x M} \quad M_2 = +11.89 \text{ T x M}$$

$$M_{⊈} = \frac{-6.22-6.02}{2} + 6.89 = -\frac{12.24}{2} + 6.89 = -6.12+6.89 = +0.77 \text{ T x M}$$

BARRA (d)

$$6.02 = MF_3$$

$$6.02 = -3.13 + 3\,X_{c_C}$$

$$X_{c_C} = \frac{6.02+3.13}{3} = \frac{9.15}{3} = 3.05 \text{ TON}$$

$$X_{c_3} = X_{c_C} = 3.05 \text{ TON}$$

$$M_{isos} = 0$$

BARRA (e)

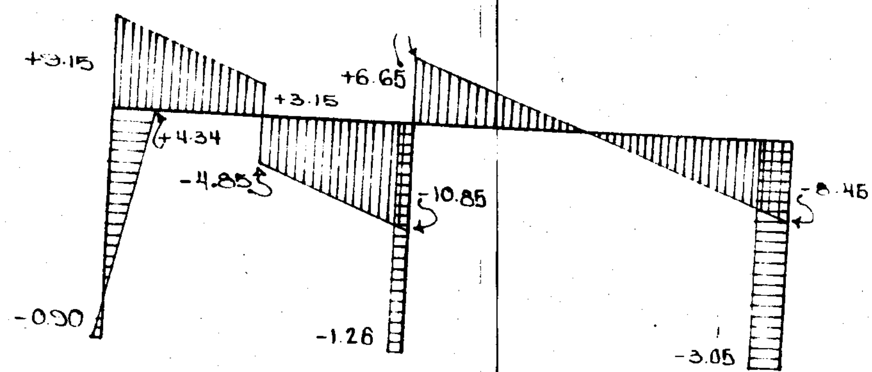
$$2.47 = MF_2$$

$$2.47 = -1.32 + 3\,X_{e_B}$$

$$X_{e_2} = X_{e_B} = 1.26 \text{ TON}$$

$$M_{isos} = 0$$

DIAGRAMA DE FUERZAS CORTANTES



REACCIONES

$X_A = -\ 0.90$ TON

$Y_A = +\ 9.15$ TON

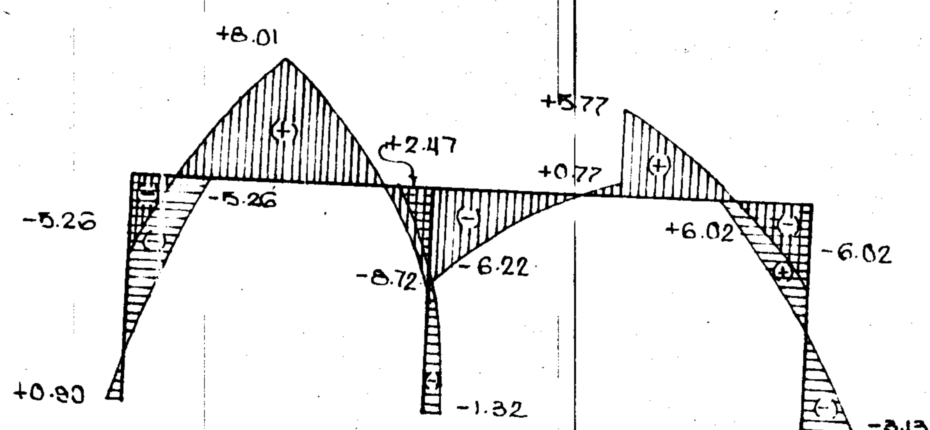
$X_B = -\ 1.26$ TON

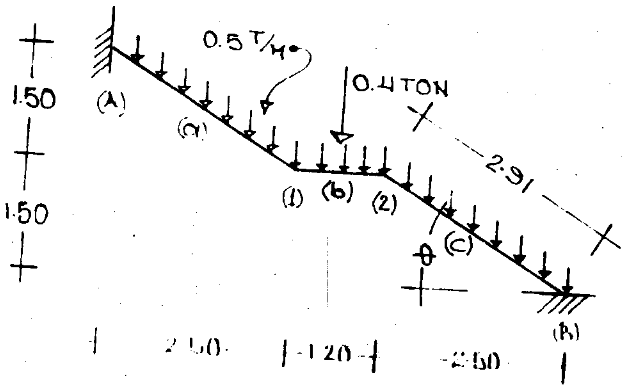
$Y_B = +\ 10.85 + 6.65 = +\ 17.50$ TON

$X_C = -\ 3.05$ TON

$Y_C = +\ 8.45$ TON

DIAGRAMA DE MOMENTOS FINALES





EI = CTE

UN GRADO DE LIBERTAD LINEAL

$l_a = l_c = \sqrt{1.5^2 + 2.5^2} = \sqrt{8.5} = 2.91$

RIGIDECES Y FACTORES DE TRANSPORTE

NUDO (1)

$r_{aa} = \frac{4EI}{l_a} = \frac{4EI}{2.91} = 1.37\ EI$

$r_{ab} = \frac{4EI}{l_b} = \frac{4EI}{1.2} = 3.34\ EI$

$\Sigma = 4.71\ EI$

$F.D_a = \frac{1.37}{4.71} = 0.29$

$F.D_b = \frac{3.34}{4.71} = 0.71$

$\Sigma = 1.00$

NUDO (2)

$r_{ab} = \frac{4EI}{l_b} = \frac{4EI}{1.2} = 3.34\ EI$

$r_{ac} = \frac{4EI}{l_c} = \frac{4EI}{2.91} = 1.37\ EI$

$\Sigma = 4.71\ EI$

$F.D_b = \frac{3.34}{4.71} = 0.71$

$F.D_c = \frac{1.37}{4.71} = 0.29$

$\Sigma = 1.00$

MOMENTOS DE EMPOTRAMIENTO

BARRAS (a) y (c)

$\cos\theta = \frac{2.5}{2.91}$

w PERPENDICULAR AL EJE DE LA BARRA

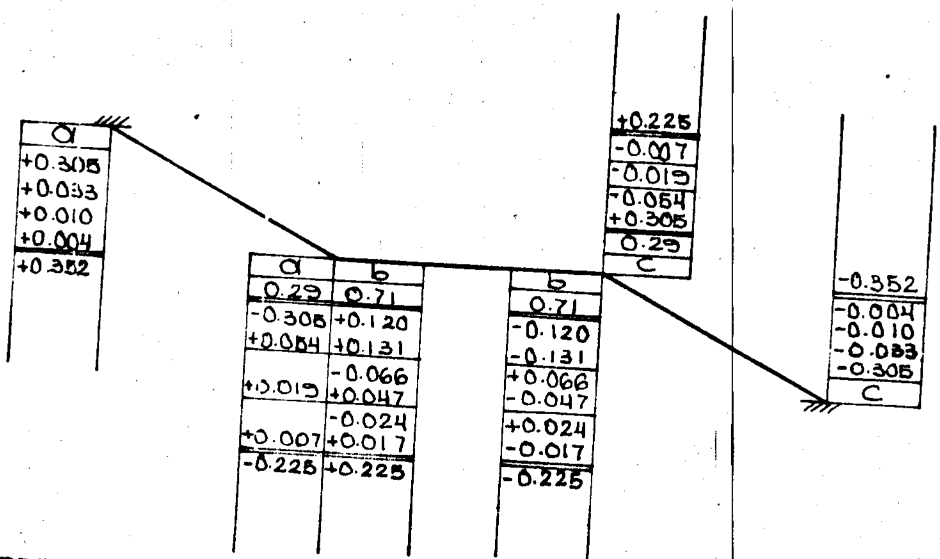
$$w_p = w \cos = 0.5\,\frac{2.5}{2.91} = 0.428 \text{ T x M}$$

$$M_A = M_1 = \frac{w_p l^2}{12} = \frac{0.43 x 2.91^2}{12} = 0.305 \text{ T x M}$$

BARRA (b)

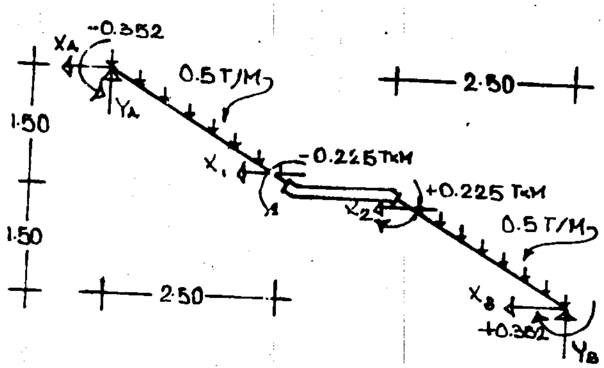
$$M_1 = -M_2 = \frac{wl^2}{12} + \frac{pl}{8} = \frac{0.5 x 1.2^2}{12} + \frac{0.4 x 1.2}{8}$$

$$M_1 = .06 + .06 = 0.12 \text{ T x M}$$



ORDEN DE EQUILIBRIO DE LOS NUDOS: 1 y 2, 1 y 2, 1 y 2

ANALISIS DEL CABEZAL



POR SIMETRIA

$$Y_A = Y_B = \frac{W \text{ TOTAL}}{2}$$

$$Y_A = 0.5x2.5 + 0.5x0.6 + \frac{0.4}{2}$$

$$Y_a = 1.25 + 0.30 + 0.2$$

$$Y_a = 1.75 \text{ TON}$$

$- 0.225 = MF_1$

$- 0.225 = -0.352 + 2.5Y_A - 1.5\ X_A - \frac{0.5 \times 2.5^2}{2}$

$- 0.225 = -0.352 + 2.5 \times 1.5\ X_A - 1.56$

$X_A = \frac{+0.225 - 0.352 + 4.37 - 1.56}{1.5} = \frac{2.683}{1.5} = 1.79$

$X_1 = X_A = 1.79$ TON

$X_2 = X_1 = 1.79$ TON

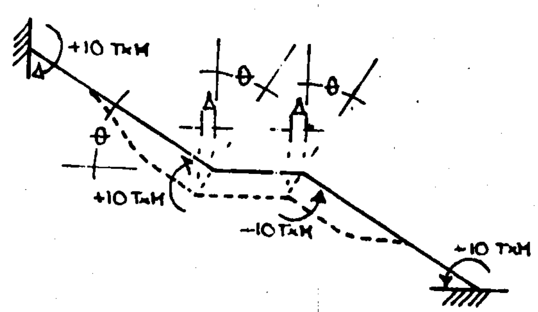
FUERZA DE SUJECION $F_1 = X_1 + X_2 = 1.79 + 1.79$

$F_1 = 3.58$ TON

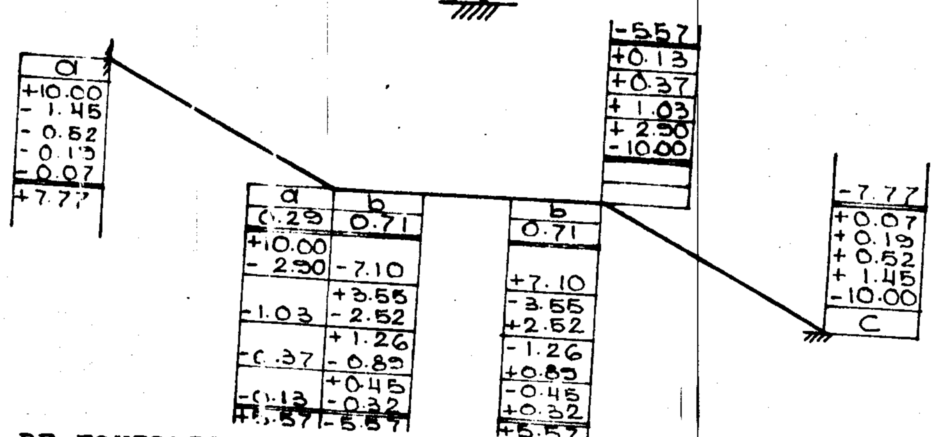
PARA IMPEDIR EL DESPLAZAMIENTO DEL CABEZAL SE DEBE APLICAR UNA FUERZA $F_1 = 3.58$ TON DE IZQUIERDA A DERECHA EN EL CABEZAL.

ANALISIS DEL GRADO DE LIBERTAD LINEAL

AL DESPLAZARSE EL CABEZAL LA ESTRUCTURA SE DEFORMARA DE LA SIGUIENTE MANERA:

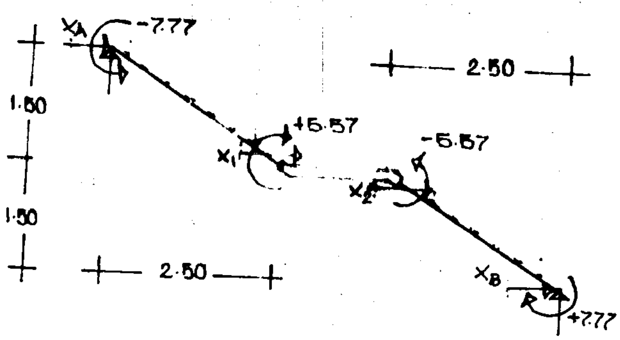


POR SIMETRIA SE CONCLUYE -- QUE LOS MOMENTOS EN (1) y - (2) SON IGUALES Y DE SENTIDO CONTRARIO.



ORDEN DE EQUILIBRIO DE LOS NUDOS: 1 y 2, 1 y 2, 1 y 2, 1 y 2

ANALISIS DEL CABEZAL



$5.57 = MF_1$

$5.57 = -7.77 + 1.5\,X_A$

$X_A = \frac{5.57 + 7.77}{1.5} = \frac{13.34}{1.5}$

$X_A = 8.89$ TON

$X_1 = X_A = 8.89$ TON

$X_2 = X_1 = 8.89$ TON

FUERZA QUE PRODUJO EL DESPLAZAMIENTO $F_2 = X_1 + X_2$

$F_2 = 8.89 + 8.89 = 17.78$ TON

PARA OBTENER LOS ANTERIORES MOMENTOS SE DEBE APLICAR UNA FUERZA $F_2 = 17.78$ TON DE DERECHA A IZQUIERDA EN EL CABEZAL.

CONSTANTE $C = -\frac{F_1}{F_2} = -\frac{3.58}{-17.72} = 0.203$

MOMENTOS FINALES

$M_{a_A} = 0.352+0.203 \times 7.77 = 0.352+1.58 = +1.93$ T x M

$M_{a_1} = -0.225+0.203 \times 5.57 = -0.225 + 1.13 = +0.91$ T x M

$M_{b_1} = +0.225-0.203 \times 5.57 = +0.225 - 1.13 = -0.91$ T x M

$M_{b_2} = +0.225-0.203 \times 5.57 = +0.225 - 1.13 = -0.91$ T x M

$M_{c_2} = -0.225+0.203 \times 5.57 = -0.225 + 1.13 = +0.91$ T x M

$M_{c_B} = -0.352-0.203 \times 5.57 = -0.352 - 1.58 = -1.93$ T x M

CALCULO DE LAS REACCIONES Y DIAGRAMAS.

BARRAS (a) y (c)

$$0.91 = MF_1$$

$$0.91 = -1.93+2.5Y_A - 0.5x\frac{2.5^2}{2}$$

$$0.91 = -1.93+2.5Y_A - 1.56$$

$$Y_A = \frac{0.91+1.93+1.56}{2.5} = \frac{4.40}{2.5} = 1.75$$

$$Y_{a_1} = -1.75+0.5x2.5 = -1.75+1.25 = -0.50 \text{ TON}$$

$$M_{isos} = \frac{wl^2}{8} = \frac{0.5 \times 2.5^2}{8} = 0.39 \text{ T x M}$$

$$M ⊄ = \frac{-1.93+0.91}{2}+0.39 = -\frac{1.02}{2}+0.39 = -0.51+0.39 = -0.12 \text{ T x M}$$

BARRA (b)

$$0.91 = MF_2$$

$$0.91 = 0.91+1.2Y_{b_1} - \frac{0.5x1.2^2}{2} - 0.4x0.6$$

$$0.91 = 0.91+1.2Y_{b_1} - 0.36 - 0.24$$

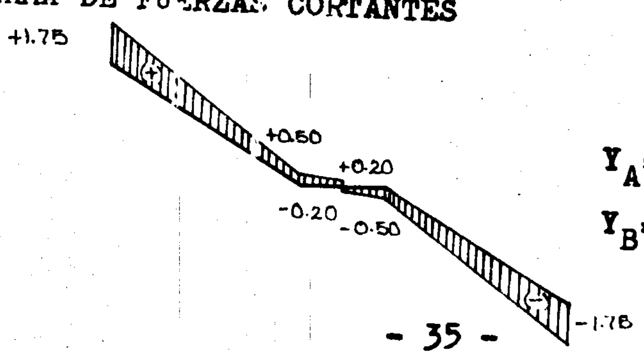
$$Y_{b_1} = \frac{0.91-0.91+0.36+0.24}{1.2} = \frac{0.60}{1.2} = 0.5 \text{ TON}$$

$$Y_{b_2} = -0.5+0.5x1.2+0.4 = 0.5$$

$$M_{isos} = \frac{wl^2}{8} + \frac{pl}{4} = \frac{0.5x1.2^2}{8} + \frac{0.4 \times 1.2}{4} = 0.09 + 0.12 = 0.21 \text{ T x M}$$

$$M ⊄ = \frac{+0.91+0.91}{2} + 0.21 = 0.91 + 0.21 = 1.12 \text{ T x M}$$

DIAGRAMA DE FUERZAS CORTANTES

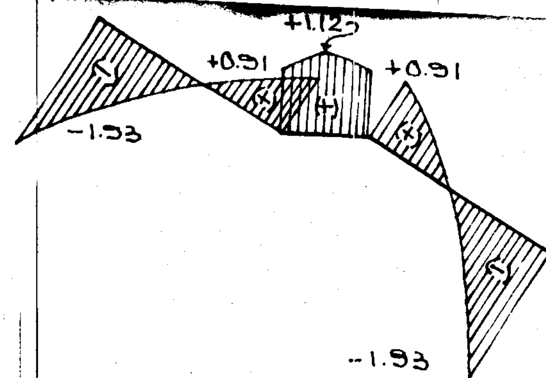


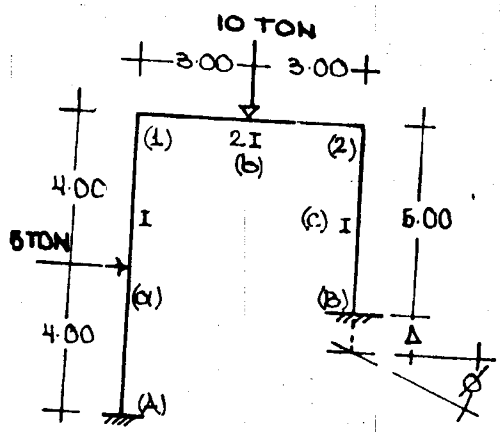
REACCIONES

Y_A = 1.75 TON

Y_B = 1.75 TON

DIAGRAMA DE MOMENTOS FLEXIONANTES





DESPLAZAMIENTOS EN EL APOYO "B"

$$\Delta = \frac{60}{EI}$$

$$\phi = \frac{100}{EI}$$

1 GRADO DE LIBERTAD LINEAL

NUDO (1)

$$r_{aa} = \frac{4E(I)}{l} = \frac{4ExI}{8} = 0.50\ EI$$

$$r_{ab} = \frac{4E(I)}{l} = \frac{4Ex2I}{6} = 1.33\ EI$$

$$\Sigma = 1.83\ EI$$

$$F.D_a = \frac{0.50EI}{1.83EI} = 0.272$$

$$F.D_b = \frac{1.33EI}{1.83EI} = 0.728$$

$$\Sigma = 1.000$$

NUDO (2)

$$r_{ab} = \frac{4E(I)}{l} = \frac{4Ex2I}{6} = 1.33\ EI$$

$$r_{ac} = \frac{4E(I)}{l} = \frac{4ExI}{5} = 0.80\ EI$$

$$\Sigma = 2.13\ EI$$

$$F.D_b = \frac{1.33EI}{2.13EI} = 0.624$$

$$F.D_c = \frac{0.80EI}{2.13EI} = 0.376$$

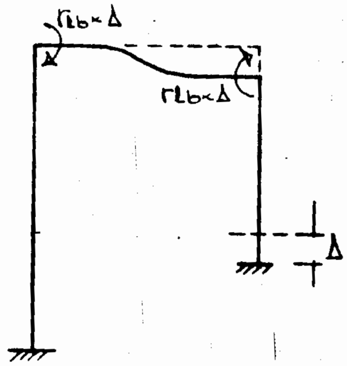
$$\Sigma = 1.000$$

MOMENTOS DE EMPOTRAMIENTO

$$M_A = -M_1 = \frac{pl}{8} = \frac{5 \times 8}{8} = 5\ T \times M$$

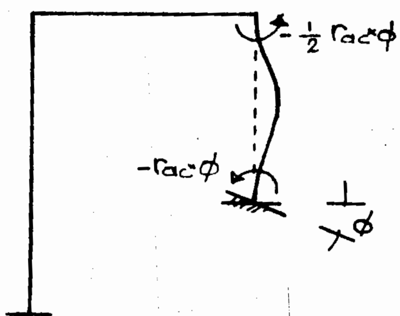
$$M_1 = -M_2 = \frac{pl}{8} = \frac{10x6}{8} = 7.50 \text{ T x M}$$

EL DESPLAZAMIENTO Y EL GIRO DEL APOYO (3) PRODUCIRAN LOS SI GUIENTES MOMENTOS:



$$r_{1b} = \frac{6E(I)}{l^2} = \frac{6Ex2I}{6^2} = \frac{EI}{3}$$

$$M_2 = M_1 = r_{1b} x \Delta = +\frac{EI}{3} x \frac{60}{EI} = 20 \text{ T x M}$$



$$r_{ac} = 0.80 \text{ EI}$$

$$M_B = r_{ac} x \phi = 0.80EI x \frac{100}{EI} = 80 \text{ T x M}$$

$$t_{a_B 2} = \frac{1}{2}$$

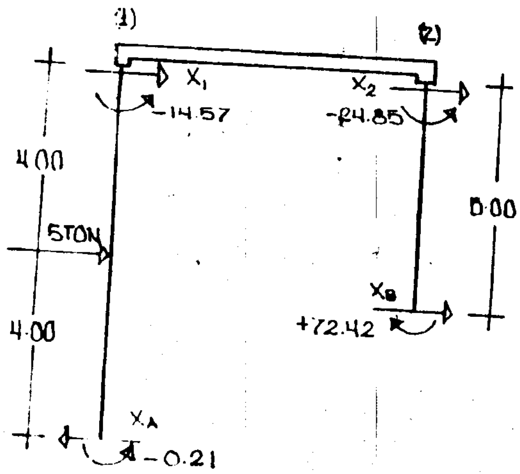
$$M_2 = t_{a_B 2}\ M_B = \frac{1}{2} x 80 = 40 \text{ T x M}$$

a	b	b	c
0.272	0.728	0.624	0.376
-5.00	+7.50	-7.50	-40.00
-8.48	+20.00	+20.00	+10.35
-0.97	+8.58	+17.15	+4.25
-0.11	-22.60	-11.30	+0.48
-0.01	+3.53	+7.05	+0.06
-14.57	-2.56	-1.28	+0.01
	+0.40	+0.80	-24.85
	-0.29	-0.15	
	+0.05	+0.09	
	-0.04	-0.02	
	+14.57	+0.01	
		+24.85	

Apoyo a	Apoyo c
+5.00	-80.00
-4.24	+5.18
-0.49	+2.13
-0.06	+0.24
+0.21	+0.03
	-72.42

ORDEN DE EQUILIBRIO DE LOS NUDOS : 2,1,2,1,2,1,2,1,2

ANALISIS DEL CABEZAL



$- 14.57 = MF_1$

$- 14.57 = -0.21 + 8X_A - 5 \times 4$

$X_A = \frac{-14.57 + 0.21 + 20.00}{8} = \frac{5.64}{8}$

$X_A = 0.71$ TON

$X_1 = -0.71 + 5.00 = 4.29$ TON

$-24.85 = MF_2$

$-24.85 = + 72.42 - 5\ X_B$

$X_B = \frac{24.85 + 72.42}{5} = \frac{97.27}{5}$

$X_B = 19.50$ TON

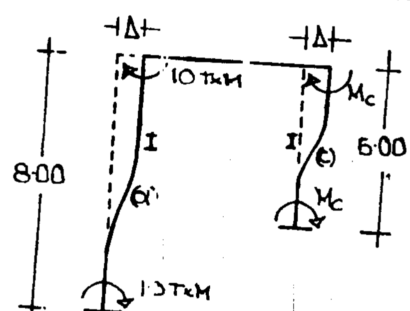
FUERZA DE SUJECION $= X_1 + X_2$

$F_1 = 4.29 + 19.50$

$F_1 = 23.79$ TON

PARA IMPEDIR EL DESPLAZAMIENTO DEL CABEZAL SE DEBE APLICAR UNA FUERZA $F_1 = 23.79$ DE DERECHA A IZQUIERDA EN EL CABEZAL.

ANALISIS DEL GRADO DE LIBERTAD



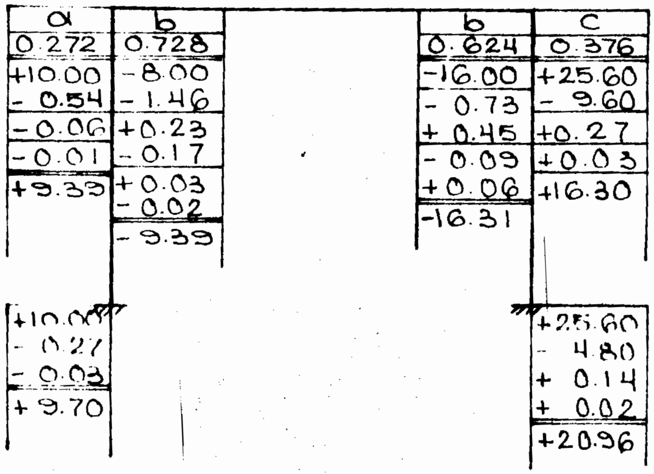
$r_{1a} = \frac{6E(I)}{l^2} = \frac{6EI}{8^2} = 0.0938$ EI

$= \frac{M}{r_e} = \frac{10}{0.0938EI}$

$= \frac{107}{EI}$

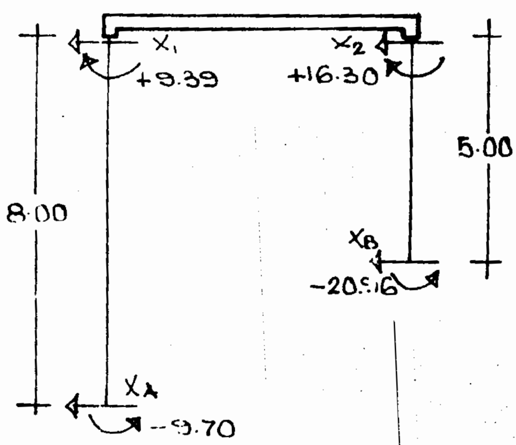
$r_{1_c} = \frac{6E(I)}{l^2} = \frac{6\ EI}{5^2} = 0.24$ EI

$M_c = r_{1c}\ \Delta = 0.34\ EI \times \frac{107}{EI} = 25.6$ T x M



ORDEN DE EQUILIBRIO DE LOS NUDOS: 2,1,2,1,2,1

ANALISIS DEL CABEZAL



$+ 9.39 = MF_1$

$+ 9.39 = - 9.70 + 8.00\, X_A$

$X_A = \frac{9.39+9.70}{8} = \frac{19.09}{8} = 2.38$

$X_1 = X_A = 2.38$ TON

$+ 16.30 = MF_2$

$+ 16.30 = - 20.96 + 5\, X_B$

$X_B = \frac{16.30 + 20.96}{5} = \frac{37.26}{5} = 7.45$

$X_2 = X_B = 7.45$ TON

FUERZA QUE PRODUJO EL DESPLAZAMIENTO $F_2 = X_1 + X_2$

$F_2 = 2.38 + 7.45 = 9.83$ TON

PARA PRODUCIR LOS ANTERIORES MOMENTOS SE DEBE APLICAR UNA FUERZA $F_2 = 9.83$ TON DE IZQUIERDA A DERECHA EN EL CABEZAL.

$$CTTE = -\frac{F_1}{F_2} = -\frac{-23.79}{9.83} = 2.42$$

MOMENTOS FINALES

$M_{a_A} = +0.21 + 2.42 \times 9.70 = 0.21 + 23.50 = +23.71 \; T \times M$

$M_{a_1} = -14.57 + 2.42 \times 9.39 = -14.57 + 22.70 = +8.13 \; T \times M$

$M_{b_1} = +14.57 - 2.42 \times 9.39 = +14.57 - 22.70 = -8.13 \; T \times M$

$M_{b_2} = +24.85 - 2.42 \times 16.31 = +24.85 - 39.50 = -14.65 \; T \times M$

$M_{c_2} = -24.85 + 2.42 \times 16.31 = -24.85 + 39.50 = +14.65 \; T \times M$

$M_{c_B} = -72.42 + 2.42 \times 20.96 = -72.42 + 50.70 = -21.72 \; T \times M$

CALCULO DE REACCIONES Y DIAGRAMAS

BARRA (a)

$$+8.13 = MF_1$$

$$+8.13 = -23.71 + 8 X_A - 5 \times 4.00$$

$$X_A = \frac{8.13+23.71+20.00}{8} = \frac{51.84}{8} = 6.48 \; TON$$

$$X_1 = 6.48 - 5 = 1.48 \; TON$$

$$M_{isos} = \frac{pl}{4} = \frac{5 \times 8}{4} = 10.00 \; T \times M$$

$$M_{\not{c}} = \frac{-23.00+8.13}{2} + 10.00 = -7.44 + 10.00 = +2.56 \; T \times M$$

BARRA (b)

$$-14.65 = MF_2$$

$$-14.65 = +8.13 + 6 Y_1 - 10 \times 3.00$$

$$Y_1 = \frac{-14.65-8.13+30.00}{6} = \frac{7.22}{6} = 1.20$$

$$Y_2 = 1.20 - 10 = -8.80 \text{ TON}$$

$$M_{isos} = \frac{pl}{4} = \frac{10 \times 6.00}{4} = 15.00 \text{ T x M}$$

$$M_{\mathbb{C}} = \frac{+8.13 - 14.65}{2} + 15.00 - 3.26 + 15.00 = +11.74 \text{ T x M}$$

BARRA (c)

$$14.65 = MF_2$$

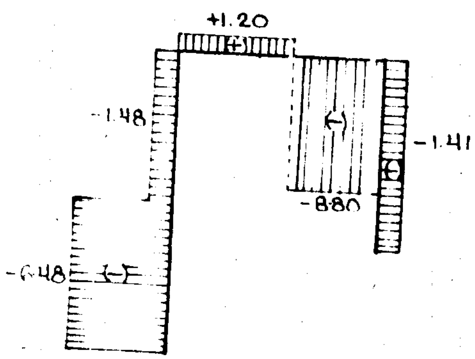
$$14.65 = +21.72 - 5X_B$$

$$X_B = \frac{-14.65 + 21.72}{5} = \frac{7.07}{5} = 1.41$$

$$X_2 = X_B = 1.41 \text{ TON}$$

$$M_{isos} = 0$$

DIAGRAMA DE FUERZAS CORTANTES



REACCIONES

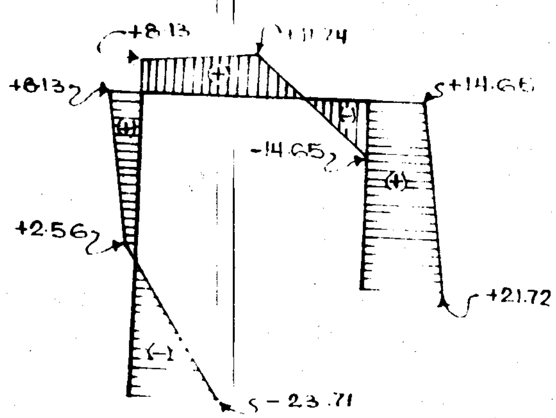
$X_A = -6.48$ TON

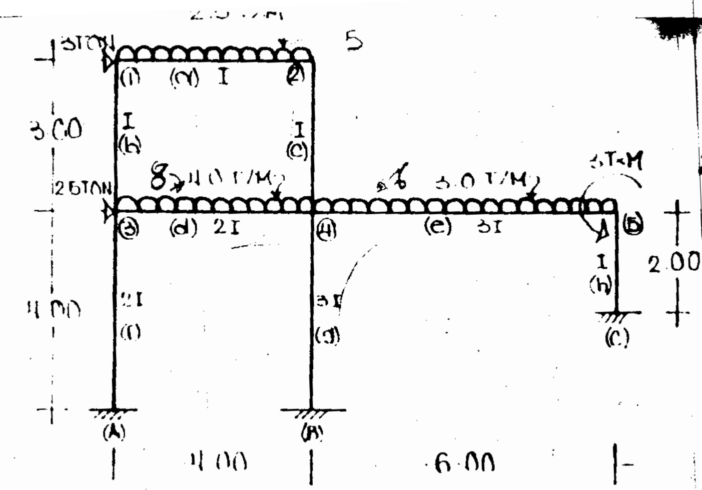
$Y_A = +1.20$ TON

$X_B = -1.41$ TON

$Y_B = +8.80$ TON

DIAGRAMA DE MOMENTOS FINALES





2 GRADOS DE LIBERTAD

LINEAL

NUDO (1)

$$r_{a_a} = \frac{4EI}{1} = \frac{4EI}{4} = 1.00\ EI \qquad F.D_a = \frac{1.00EI}{2.33EI} = 0.43$$

$$r_{a_b} = \frac{4EI}{1} = \frac{4EI}{3} = 1.33\ EI \qquad F.D_b = \frac{1.33EI}{2.33EI} = 0.57$$

$$\Sigma = 2.33\ EI \qquad \Sigma = 1.00$$

NUDO (2)

$$r_{a_a} = \frac{4EI}{1} = \frac{4EI}{4} = 1.00\ EI \qquad F.D_a = \frac{1.00EI}{2.33EI} = 0.43$$

$$r_{a_c} = \frac{4EI}{1} = \frac{4EI}{3} = 1.33\ EI \qquad F.D_c = \frac{1.33EI}{2.33EI} = 0.57$$

$$\Sigma = 2.33\ EI \qquad \Sigma = 1.00$$

NUDO (3)

$$r_{a_b} = \frac{4EI}{1} = \frac{4EI}{3} = 1.33\ EI \qquad F.D_b = \frac{1.33EI}{5.33EI} = 0.250$$

$$r_{a_d} = \frac{4E(I)}{1} = \frac{4Ex2I}{4} = 2.00\ EI \qquad F.D_d = \frac{2.00EI}{5.33EI} = 0.375$$

$$r_{a_f} = \frac{4E(I)}{1} = \frac{4Ex2I}{4} = 2.00\ EI \qquad F.D_f = \frac{2.00EI}{5.33EI} = 0.375$$

$$\Sigma = 5.33\ EI \qquad \Sigma = 1.000$$

NUDO (4)

$$r_{a_c} = \frac{4E(I)}{l} = \frac{4EI}{3} = 1.33\ EI$$

$$r_{a_d} = \frac{4E(I)}{l} = \frac{4Ex2I}{4} = 2.00\ EI$$

$$r_{a_e} = \frac{4E(I)}{l} = \frac{4Ex3I}{6} = 2.00\ EI$$

$$r_{a_g} = \frac{4E(I)}{l} = \frac{4Ex3I}{4} = 3.00\ EI$$

$$\Sigma = 8.33\ EI$$

$$F.D_c = \frac{1.33EI}{8.33EI} = 0.160$$

$$F.D_d = \frac{2\ EI}{8.33EI} = 0.240$$

$$F.D_e = \frac{2\ EI}{8.33EI} = 0.240$$

$$F.D_g = \frac{3\ EI}{8.33EI} = 0.360$$

$$\Sigma = 1.000$$

NUDO (5)

$$r_{a_e} = \frac{4E(I)}{l} = \frac{4Ex3I}{6} = 2.00\ EI$$

$$r_{a_h} = \frac{4E(I)}{l} = \frac{4ExI}{2} = 2.00\ EI$$

$$\Sigma = 4.00\ EI$$

$$F.D_e = \frac{2\ EI}{4} = 0.500$$

$$F.D_h = \frac{2\ EI}{4} = 0.500$$

$$\Sigma = 1.000$$

MOMENTOS DE EMPOTRAMIENTO

BARRA (a)

$$M_1 = -M_2 = \frac{wl^2}{12} = \frac{2.5 \times 4^2}{12} = 3.34\ T \times M$$

BARRA (b), BARRA (c) NO TIENEN CARGAS ENTRE LOS APOYOS

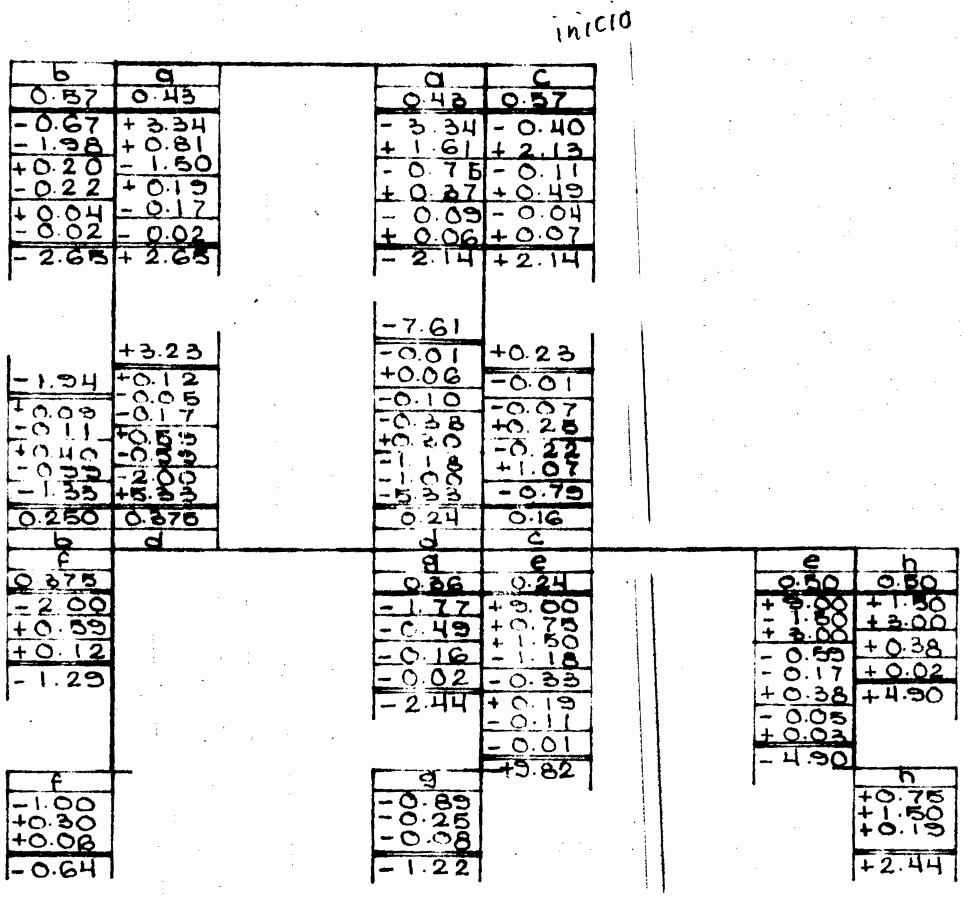
BARRA (d)

$$M_3 = -M_4 = \frac{wl^2}{12} = \frac{4x4^2}{12} = 5.33\ T \times M$$

BARRA (e)

$$M_4 = \frac{wl^2}{12} = \frac{3x6^2}{12} = 9.00\ T \times M$$

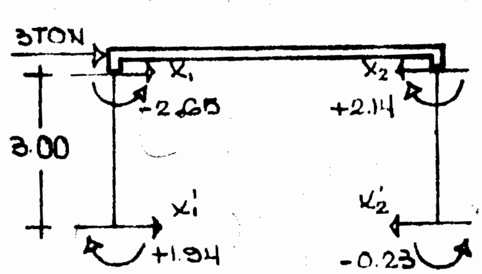
EL MOMENTO EXTERIOR DE -3.00 T x M APLICADO EN EL NUDO (5) PRODUCIRA MOMENTOS DE EMPOTRAMIENTO DE +3 x F.D = 4.5 A LAS BARRAS "e" y "h" TRANSPORTANDO 1.5 x 0.5 = 0.75 T x M A LOS EXTREMOS OPUESTOS.



ORDEN DE QUILIBRIO DE LOS NUDOS: 5,3,4,2,1,3,4,2,5,4,1,3

ANALISIS DE CABEZALES

CABEZAL SUPERIOR



$$-2.65 = MF_1$$

$$-2.65 = 1.94 - 3X_1'$$

$$X_1' = \frac{2.65+1.94}{3} = \frac{4.59}{3} = 1.53 \text{ TON}$$

$$X_1 = X_1' = 1.53 \text{ TON}$$

$$+\,2.14 = MF_2$$

$$+\,2.14 = -0.23 + 3X_2'$$

$$X_2' = \frac{2.14 + 0.23}{3} = \frac{2.37}{3} = 0.79$$

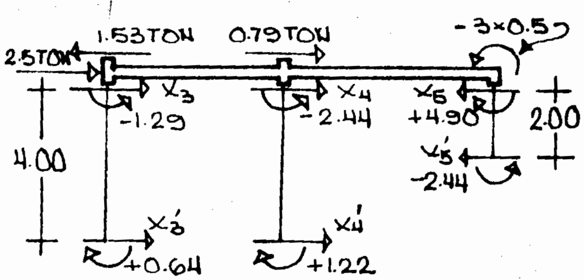
$$X_2 = X_2' = 0.79$$

FUERZA DE SUJECION $F_1 = 3.00 + X_1 - X_2$

$F_1 = 3.00 + 1.53 - 0.79 = 3.74$ TON

PARA IMPEDIR EL DESPLAZAMIENTO DEL CABEZAL ES NECESARIO APLICAR UNA FUERZA $F_1 = 3.74$ TON DE DERECHA A IZQUIERDA EN EL CABEZAL.

CABEZAL INFERIOR



$$-\,1.29 = MF_3$$

$$-\,1.29 = +0.64 - 4\,X_3'$$

$$X_3' = \frac{1.29 + 0.64}{4} = \frac{1.93}{4} = 0.48 \text{ TON}$$

$$X_3 = X_3' = 0.48 \text{ TON}$$

$$-\,2.24 = MF_4$$

$$-\,2.24 = +\,1.22 - 4X_4'$$

$$X_4' = \frac{2.24 + 1.22}{4} = \frac{3.46}{4} = 0.87 \text{ TON}$$

$$X_4 = X_4' = 0.87 \text{ TON}$$

$$-\,1.5 + 4.94 = MF_5$$

$$-\,1.5 + 4.94 = -2.44 + 2X_5'$$

$$X_5' = \frac{-\,1.5 + 4.94 + 2.44}{2} = \frac{5.88}{2} = 2.94$$

$$X_5 = X_5' = 2.94 \text{ TON}$$

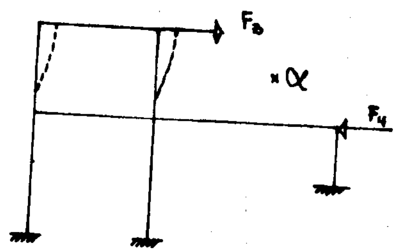
FUERZA DE SUJECION $F_2 = 2.50 - 1.53 + 0.79 + X_3 + X_4 - X_5$

$F_2 = 2.50 - 1.53 + 0.79 + 0.48 + 0.87 - 2.94 = + 0.17$ TON

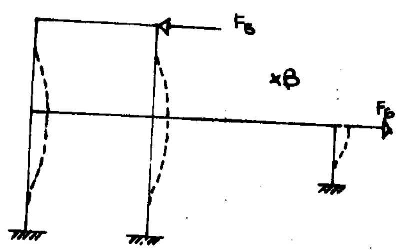
PARA IMPEDIR EL DESPLAZAMIENTO DEL CABEZAL ES NECESARIO APLICAR LE UNA FUERZA $F_2 = + 0.17$ TON DE DERECHA A IZAQUIERDA.

LOS GRADOS DE LIBERTAD SE RESUELVEN UNO A UNO.

EN EL CASO DEL PROBLEMA



F_4 IMPIDE EL MOVIMIENTO DEL NIVEL INFERIOR



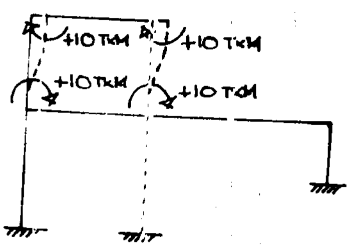
F_5 IMPIDE EL MOVIMIENTO DEL NIVEL SUPERIOR

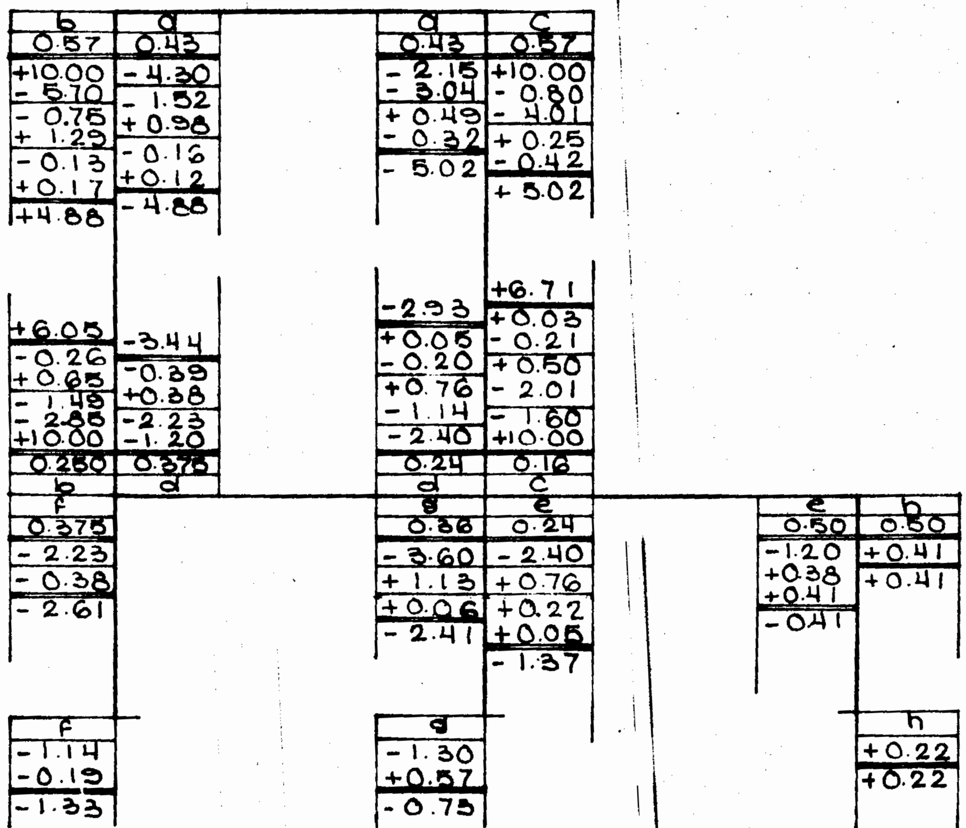
SE FORMARA UN SISTEMA DE 2 ECUACIONES SIMULTANEAS.

$$\alpha F_3 - \beta F_5 = F_1$$

$$-\alpha F_4 + \beta F_6 = F_2$$

ANALISIS DEL 1er. GRADO DE LIBERTAD LINEAL.

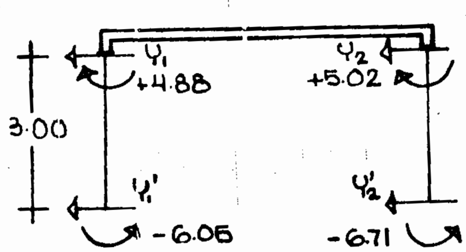




ORDEN DE EQUILIBRIO DE LOS NUDOS: 1,4,2,3,4,1,3,5,1,4

ANALISIS DE CABEZALES

CABEZAL SUPERIOR



$$4.88 = -6.05 + 3Y_1'$$

$$Y_1' = \frac{4.88+6.05}{3} = \frac{10.93}{3} = 3.64 \text{ TON}$$

$$Y_1 = Y_1' = 3.64 \text{ TON}$$

$$5.02 = -6.71 + 3Y_2'$$

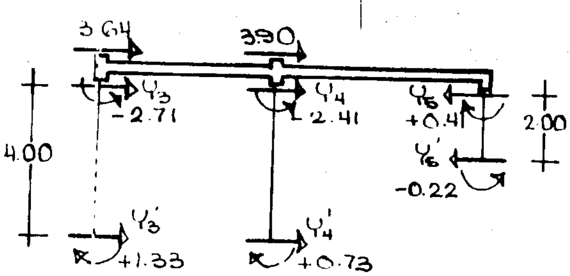
$$Y_2' = \frac{5.02 + 6.71}{3} = \frac{11.73}{3} = 3.90$$

$$Y_2 = Y_2' = 3.90 \text{ TON}$$

$F_3 = Y_1 + Y_2 = 3.64 + 3.90 = 7.54$

$F_3 = 7.54$ TON

CABEZAL INFERIOR



$- 2.71 = 1.33 - 4Y'_3$

$Y'_3 = \frac{2.71+1.33}{4} = \frac{4.04}{4} = 1.01$

$Y_3 = Y'_3 = 1.01$ TON

$- 2.41 = 0.73 - 4Y'_4$

$Y'_4 = \frac{2.41+0.73}{4} = \frac{3.14}{4} = 7.9 \quad 0.79$

$Y_4 = Y'_4 = 0.79$ TON

$0.41 = - 0.22 + 2\ Y'_5$

$Y'_5 = \frac{0.41 + 0.22}{2} = \frac{0.63}{2} = 0.32$

$Y_5 = Y'_5 = 0.32$ TON

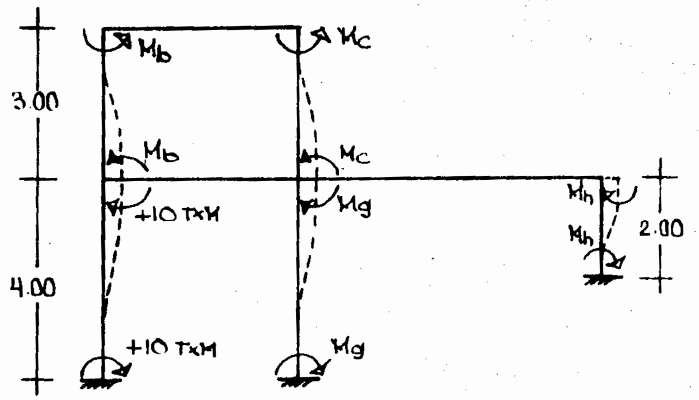
$F_4 = 3.64 + 3.90 + Y_3 + Y_4 - Y_5$

$F_4 = 3.64 + 3.90 + 1.01 + 0.79 - 0.32 = 9.02$

$F_4 = 9.02$ TON

PARA OBTENER LOS MOMENTOS ANTERIORES DEBEMOS APLICAR UNA FUERZA $F_3 = 7.54$ TON DE IZQUIERDA A DERECHA EN EL CABEZAL SUPERIOR Y UNA FUERZA $F_4 = 9.02$ TON DE DERECHA A IZQUIERDA EN EL CABEZAL INFERIOR.

ANALISIS DEL 2o. GRADO DE LIBERTAD LINEAL



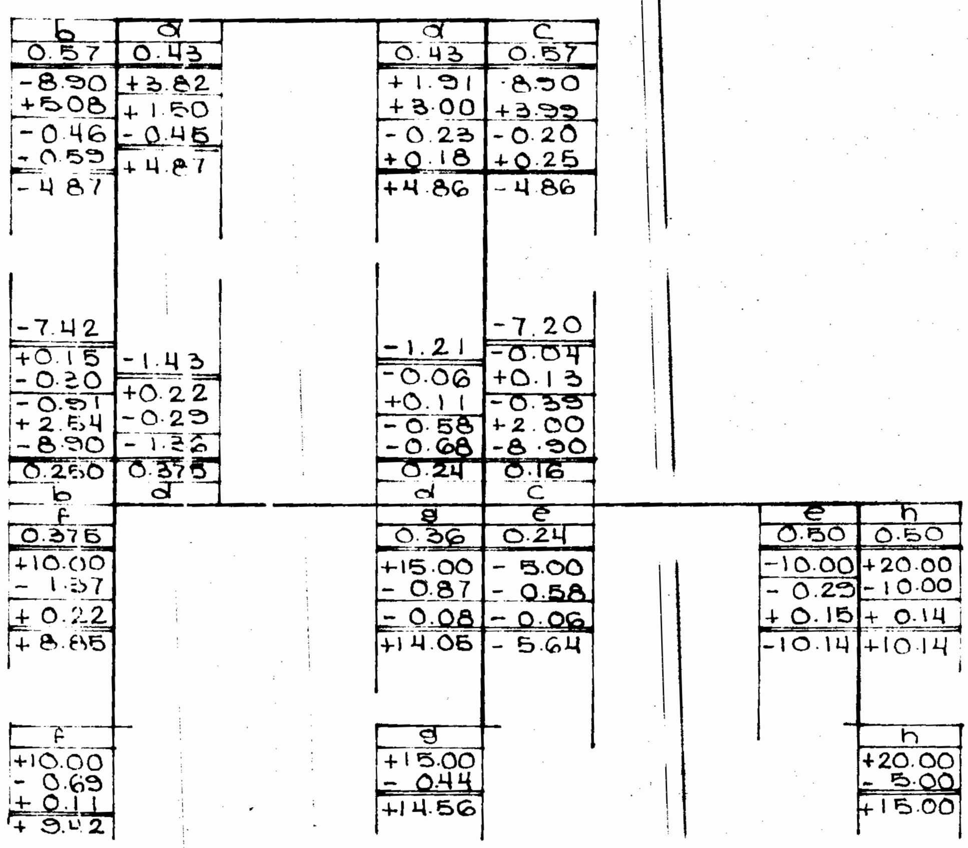
$$r_{1_f} = \frac{6E(I)}{l^2} = \frac{6E\text{x}2I}{4^2} = 0.75\ EI$$

$$\Delta = \frac{M}{r_1} = \frac{10}{0.75EI} = \frac{13.33}{EI}$$

$$r_{1_b} = r_{1_c} = \frac{6E(I)}{l^2} = \frac{6EI}{3^2} = 0.666EI \qquad M_b = M_c = \Delta r_{1_b} = \frac{13.33}{EI}\text{x}0.666EI = 8.90\ T \times M$$

$$r_{1_g} = \frac{6E(I)}{l^2} = \frac{6E3I}{4^2} = 1.123EI \qquad M_g = \Delta r_{1_g} = \frac{13.33}{EI}\text{x}\ 1.123EI = 15.00\ T \times M$$

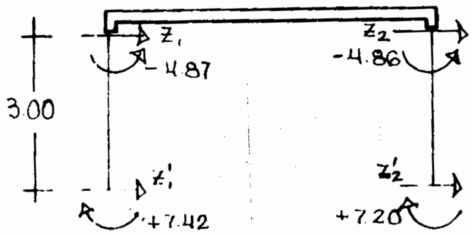
$$r_{1_h} = \frac{6E(I)}{l^2} = \frac{6EI}{2^2} = 1.5\ EI \qquad M_h = \Delta r_{1_h} = \frac{13.33}{EI}\text{x}\ 1.5\ EI = 20.00\ T \times M$$



ORDEN DE EQUILIBRIO DE LOS NUDOS: 5,1,2,3,4,1,3,2,5,4

ANALISIS DE CABEZALES

CABEZAL SUPERIOR



$$-4.87 = 7.42 - 3\,Z_1'$$

$$Z_1' = \frac{7.42+4.87}{3} = \frac{12.29}{3} = 4.10$$

$$Z_1 = Z_1' = 4.10 \text{ TON}$$

$$-4.86 = 7.20 = 3\,Z_2'$$

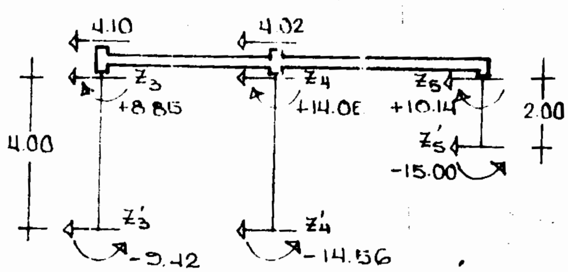
$$Z_2' = \frac{7.20+4.86}{3} = \frac{12.06}{3} = 4.02$$

$$Z_2 = Z_2' = 4.02 \text{ TON}$$

$$F_5 = Z_1 + Z_2 = 4.10 + 4.02 = 8.12$$

$$F_5 = 8.12 \text{ TON}$$

CABEZAL INFERIOR



$$8.85 = -9.42 + 4\,Z_3'$$

$$Z_3' = \frac{8.85+9.42}{4} = \frac{18.27}{4} = 4.57$$

$$Z_3 = Z_3' = 4.57 \text{ TON}$$

$$14.05 = -14.56 + 4Z_4'$$

$$Z_4' = \frac{14.05+14.56}{4} = \frac{28.61}{4} = 7.15$$

$$Z_4 = Z_4' = 7.15 \text{ TON}$$

$$10.14 = -15 + 2Z_5'$$

$$Z_5' = \frac{10.14+15}{2} = \frac{25.14}{2} = 12.57$$

$$Z_5 = Z_5' = 12.57 \text{ TON}$$

$F_6 = 4.10+4.02+Z_3+Z_4+Z_5 = 4.10+4.02+4.57+7.15+12.57 = 32.41$ TON

PARA OBTENER LOS ANTERIORES MOMENTOS DEBEMOS APLICAR UNA FUERZA $F_5 = 8.12$ TON DE DERECHA A IZQUIERDA EN EL CABEZAL SUPERIOR Y UNA FUERZA $F_6 = 32.41$ TON DE IZQUIERDA A DERECHA EN EL CABEZAL INFERIOR.

EL SISTEMA DE DOS ECUACIONES CON DOS INCOGNITAS QUE SE OBTIENE ES EL SIGUIENTE:

$$+\alpha\,7.54 - \beta\,8.12 = +3.74 \qquad (1)$$

$$-\alpha\,9.02 + \beta\,32.41 = +0.17 \qquad (2)$$

DE (1) $\alpha = \frac{3.74+8.12\beta}{7.54}$ (3)

EN (2)

$$-\frac{3.74+8.12\beta}{7.54} \times 9.02 + 32.41\beta = +0.17$$

$$-4.47 - 4.70\beta + 32.41\beta = +0.17$$

$$22.71\beta = 4.64$$

$$\beta = \frac{4.64}{22.71}$$

$$\beta = 0.204$$

EN (3)

$$\alpha = \frac{3.74+8.12\text{x}0.204}{7.54} = \frac{3.74+1.66}{7.54} = \frac{5.40}{7.54}$$

$$\alpha = 0.717$$

LOS MOMENTOS FINALES SERAN IGUALES A LOS OBTENIDOS EN EL 1er. ANALISIS MAS LOS DEL 2o. ANALISIS MULTIPLICADOS POR α MAS LOS DEL 3er. ANALISIS MULTIPLICADOS POR β .

MOMENTOS FINALES

$\alpha = 0.715 \qquad \beta = 0.203$

$M_{a_1} = +2.65-0.717x4.88+0.204x\ 4.87 = +2.65-3.50+0.99 = +\ 0.14$ T x M

$M_{a_2} = -2.14-0.717x5.02+0.204x\ 4.86 = -2.14-3.60+0.99 = -\ 4.75$ T x M

$M_{b_1} = -2.65+0.717x4.88-0.204x\ 4.87 = -2.65+3.50-0.99 = -0.14$ T x M

$M_{b_3} = -1.94+0.717x6.05-0.204x\ 7.42 = -1.94+4.34-1.51 = +\ 0.89$ T x M

$M_{c_2} = +2.14+0.717x5.02-0.204x\ 4.86 = +2.14+3.60-0.99 = +\ 4.75$ T x M

$M_{c_4} = +0.23+0.717x\ 6.71-0.204x\ 7.20 = +0.23+4.81-1.47 = +\ 3.57$ T x M

$M_{d_3} = +3.23-0.717x3.44-0.204x\ 1.43 = +3.23-2.47-0.29 = +\ 0.47$ T x M

$M_{d_4} = -7.61-0.717x2.93-0.204x\ 1.21 = -7.61-2.10-0.25 = -\ 9.96$ T x M

$M_{e_4} = +9.82-0.717x1.37-0.204x\ 5.64 = +9.82-0.98-1.15 = +\ 7.69$ T x M

$M_{e_5} = -4.90-0.717x0.41-0.204x10.14 = -4.90-0.29-2.07 = -\ 7.26$ T x M

$M_{f_3} = -1.29-0.717x2.61+0.204x\ 8.85 = -1.29-1.87+1.81 = -\ 1.35$ T x M

$M_{f_A} = -0.64-0.717x1.33+0.204x\ 9.42 = -0.64-0.95+1.92 = +\ 0.33$ T x M

$M_{g_4} = -2.44-0.717x2.41+0.204x14.05 = -2.44+1.73+2.87 = -\ 1.30$ T x M

$M_{g_B} = -1.22-0.717x0.73+0.204x14.56 = -1.22-0.52+2.97 = +\ 1.23$ T x M

$M_{h_5} = +4.90+0.717x0.41+0.204x10.14 = +4.90+0.29+2.07 = +\ 7.26$ T x M

$M_{h_C} = +2.44+0.717x0.22+0.204x15.00 = +2.44+0.16+3.06 = +\ 5.66$ T x M

CALCULO DE REACCIONES Y DIAGRAMAS.

BARRA (a)

$$-4.75 = MF_2$$

$$-4.75 = -0.14 + 4Y_{a_1} - \frac{2.5x4^2}{2}$$

$$Y_{a_1} = \frac{-4.75+0.14+20.00}{4} = \frac{15.39}{4} = 3.85 \text{ TON}$$

$$Y_{a_2} = 3.85 - 2.5x4 = -6.15 \text{ TON}$$

$$M_{isos} = \frac{wl^2}{8} = \frac{2.5x4^2}{8} = 5.00 \text{ T x M}$$

$$M_{℄} = \frac{-0.14-4.75}{2} + 5.00 = \frac{4.89}{2} + 5.00 = +2.55 \text{ T x M}$$

BARRA (b)

$$-0.14 = MF_1$$

$$-0.14 = -0.89 + 3X_{b_3}$$

$$X_{b_3} = \frac{-0.14+0.89}{3} = \frac{0.75}{3} = 0.25 \text{ TON}$$

$$X_{b_1} = X_{b_3} = 0.25 \text{ TON}$$

$$M_{isos} = 0$$

BARRA (c)

$$4.75 = MF_2$$

$$4.75 = -3.57 + 3X_{c_4}$$

$$X_{c_4} = \frac{4.75+3.57}{3} = \frac{8.32}{3} = 2.77 \text{ TON}$$

$$X_{c_2} = X_{c_4} = 2.77 \text{ TON}$$

$$M_{isos} = 0$$

BARRA (d)

$$- 9.96 = MF_4$$

$$-9.96 = -0.47+4Y_{d_3} - \frac{4 \times 4^2}{2}$$

$$Y_{d_3} = \frac{-9.96+0.47+32.00}{4} = \frac{22.51}{4} = 5.63 \text{ TON}$$

$$Y_{d_4} = 5.63 - 4 \times 4 = -10.37 \text{ TON}$$

$$M_{isos} = \frac{wl^2}{8} = \frac{4x4^2}{8} = 8.00 \text{ T x M}$$

$$M_{℄} = \frac{-0.47-9.96}{2} + 8.00 = - \frac{10.43}{2} + 8.00 = + 2.78 \text{ T x M}$$

BARRA (e)

$$-7.26-1.5 = MF_5$$

$$-7.26-1.5 = -7.69+6Y_{e_4} - \frac{3x6^2}{2}$$

$$Y_{e_4} = \frac{-7.26-1.5+7.69+54.00}{6} = \frac{52.93}{6} = 8.81 \text{ TON}$$

$$Y_{e_5} = 8.80 - 3 \times 6 = -9.20 \text{ TON}$$

$$M_{isos} = \frac{wl^2}{8} = \frac{3 \times 6^2}{8} = 13.50 \text{ T x M}$$

$$M_{℄} = \frac{-7.69-7.26}{2} + 13.50 = + 6.02 \text{ T x M}$$

BARRA (f)

$$- 1.35 = MF_3$$

$$- 1.35 = -0.32 -4X_{f_A}$$

$$X_{f_A} = \frac{1.35-0.33}{4} = \frac{1.02}{4} = 0.25 \text{ TON}$$

$$X_{f_3} = X_{f_A} = 0.26 \text{ TON}$$

$$M_{isos} = 0$$

BARRA (g)

$$-1.30 = MF_4$$

$$-1.30 = -1.23 - 4X_{g_B}$$

$$X_{g_B} = \frac{1.30-1.23}{4} = \frac{0.07}{4} = 0.02 \text{ TON}$$

$$X_{g_4} = X_{g_B} = 0.02 \text{ TON}$$

$$M_{isos} = 0$$

BARRA (h)

$$7.26 = MF_5$$

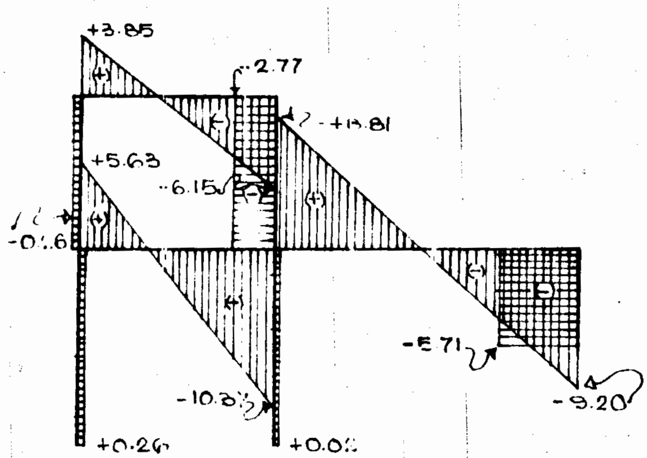
$$-1.5+7.26 = -5.66+2X_{h_C}$$

$$X_{h_C} = \frac{-1.5+7.26+5.65}{2} = \frac{11.41}{2} = 5.71$$

$$X_{h_5} = X_{h_C} = 5.71 \text{ TON}$$

$$M_{isos} = 0$$

DIAGRAMA DE FUERZAS CORTANTES



REACCIONES

$X_A = +0.26$ TON

$Y_A = 3.85+5.63 = 9.48$ TON

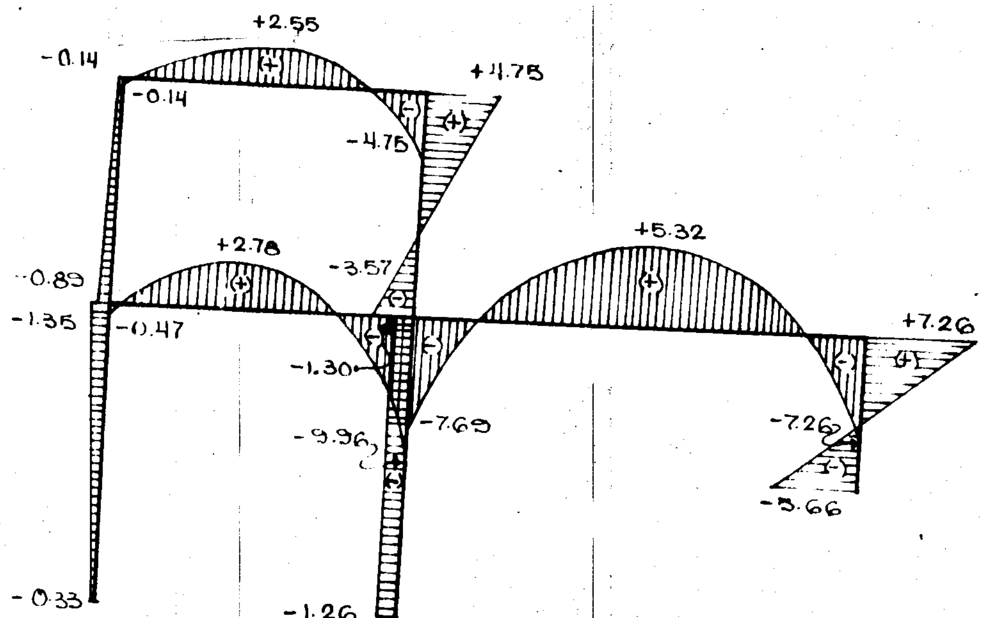
$X_B = +0.02$ TON

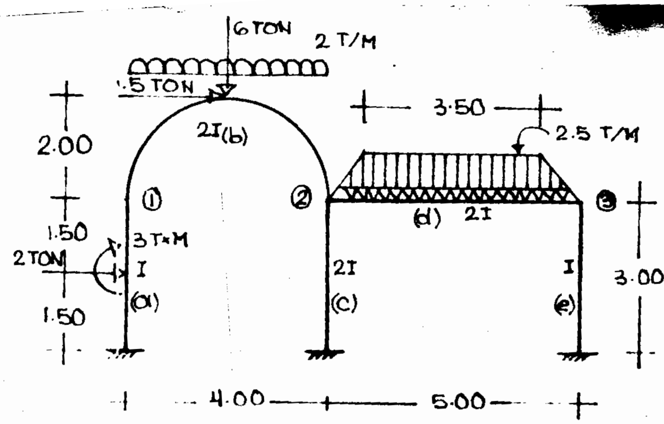
$Y_B = 6.15+10.37+8.81 = +25.33$ TON

$X_C = -5.71$ TON

$Y_C = +9.20$ TON

DIAGRAMA DE MOMENTOS FLEXIONANTES





2 GRADOS DE LIBERTAD LINEAL

NUDO (1)

$r_{aa} = \frac{4E(I)}{l} = \frac{4EI}{3} = 1.33\ EI$ $\qquad F.D_a = \frac{1.33EI}{3.63EI} = 0.367$

$r_{ab} = \frac{2.3E(I)}{r} = \frac{2.3Ex2I}{2} = 2.30\ EI$ $\qquad F.D_b = \frac{2.30EI}{3.63EI} = 0.633$

$= 3.63\ EI$ $\qquad = 1.000$

NUDO (2)

$r_{ab} = \frac{2.3E(I)}{r} = \frac{2.3Ex2I}{2} = 2.30\ EI$ $\qquad F.D_b = \frac{2.30EI}{6.57EI} = 0.350$

$r_{ac} = \frac{4E(I)}{l} = \frac{4Ex2I}{3} = 2.67\ EI$ $\qquad F.D_c = \frac{2.67EI}{6.57EI} = 0.406$

$r_{ad} = \frac{4E(I)}{l} = \frac{4Ex2I}{5} = 1.60\ EI$ $\qquad F.D_d = \frac{1.60EI}{6.57EI} = 0.244$

$= 6.57\ EI$ $\qquad = 1.000$

NUDO (3)

$r_{ad} = \frac{4E(I)}{l} = \frac{4Ex2I}{5} = 1.60\ EI$ $\qquad F.D_d = \frac{1.60EI}{2.93EI} = 0.546$

$r_{ae} = \frac{4E(I)}{l} = \frac{4EI}{3} = 1.33\ EI$ $\qquad F.D_e = \frac{1.33EI}{2.93EI} = 0.454$

$= 2.93\ EI$ $\qquad = 1.000$

MOMENTOS DE EMPOTRAMIENTO

BARRA (a)

$$M_A = -\frac{M}{4} + \frac{pl}{8} = -\frac{3}{4} + \frac{2x3}{8} = -\frac{3}{4} + \frac{3}{4} = 0$$

$$M_1 = -\frac{M}{4} + \frac{pl}{8} = -\frac{3}{4}\ \frac{2x3}{8} = -\frac{3}{4} - \frac{3}{4} = -\frac{6}{4} = -1.5\ T\ x$$

BARRA (b)

$$M_1 = -\frac{WR}{3\pi} - \frac{PxR}{8} + 0.131P_hR = -\frac{2x2}{3\pi} - \frac{6x2}{8} + 0.131x1.5x2$$

$$M_1 = -0.424 - 1.5 + .394 = -1.53 \text{ T x M}$$

$$M_2 = +\frac{WR}{3\pi} + \frac{PxR}{8} + 0.817P_hR = \frac{2x2}{3\pi} + \frac{6x2}{8} + 0.817x1.5 \text{ x } 2$$

$$M_2 = 0.424 + 1.5 + 2.46 = +4.38 \text{ T x M}$$

BARRA (d)

$$M_2 = -M_3 = \frac{1}{96} \text{ wl}(1+b)(5 - \frac{b^2}{l^2})$$

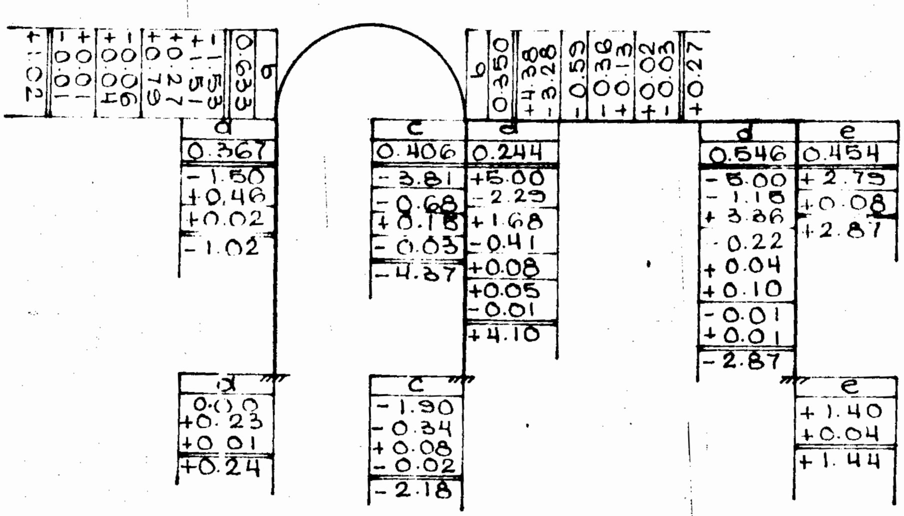
$$M_2 = \frac{1}{96} \text{ x } 2.5x5x(5+3.5)x(5 - \frac{3.5^2}{5^2})$$

$$M_2 = -M_3 = 0.131x8.5x(5-0.49) = 1.11x4.51 =$$

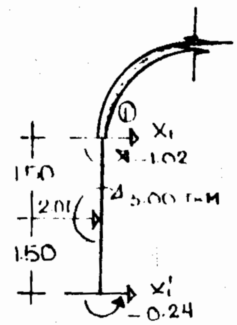
$$M_2 = -M_3 = 5 \text{ T x M}$$

BARRAS (c) y (e) NO TIENEN SOLICITACIONES

t_a(ARCO) = -0.46 $\quad$ t_1(ARCO) = -1.00



ORDEN DE EQUILIBRIO DE LOS NUDOS: 2,3,2,1,3,2,3,1



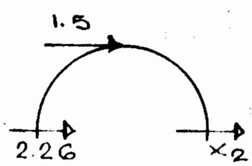
$- 1.02 = MF_1$

$- 1.02 = -0.24 - 3X'_1 + 3 - 2 \times 1.5$

$X'_1 = \frac{1.02-0.24}{3} = \frac{0.78}{3} = +0.26$ TON

$X_1 = 0.26+2 = 2.26$ TON

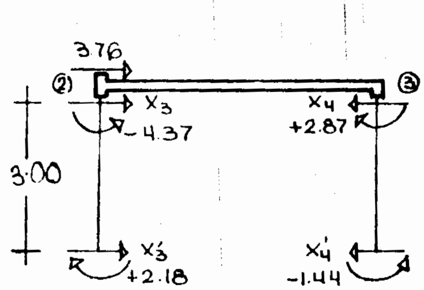
PARA IMPEDIR EL DESPLAZAMIENTO DEL NUDO (1) ES NECESARIO APLICAR UNA FUERZA $F_1 = 2.26$ DE DERECHA A IZQUIERDA EN EL NUDO -- (1)



$\Sigma F_x = 0$

$X_2 = 2.26 + 1.50$

$X_2 = 3.76$ TON



$- 4.37 = \Sigma MF_2$

$- 4.37 = 2.18 - 3X'_3$

$X'_3 = \frac{4.37+2.18}{3} = \frac{6.55}{3} = 2.18$ TON

$X_3 = X'_3 = 2.18$ TON

$+ 2.87 = \Sigma MF_3$

$+ 2.37 = - 1.44 + 3X'_4$

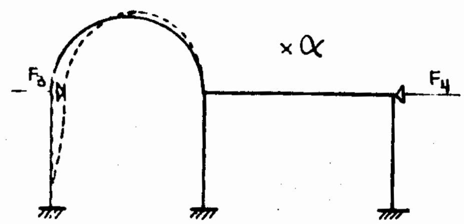
$X'_4 = \frac{2.87+1.44}{3} = \frac{4.31}{3} = 1.44$ TON

$X_4 = X'_4 = 1.44$ TON

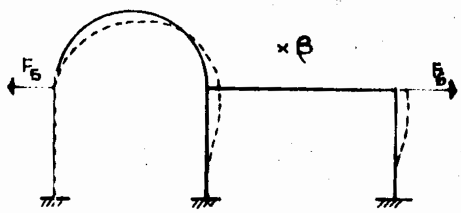
$F_2 = 3.76 + X_3 - X_4 = 3.76 + 2.18 - 1.44 = + 4.50$ TON

$F_2 = 4.50$ TON

PARA IMPEDIR EL DESPLAZAMIENTO DEL CABEZAL ES NECESARIO APLICAR UNA FUERZA F_2 = 4.50 TON DE DERECHA A IZQUIERDA EN EL CABEZAL.



F_2 IMPIDE EL MOVIMIENTO DEL CABEZAL HORIZONTAL

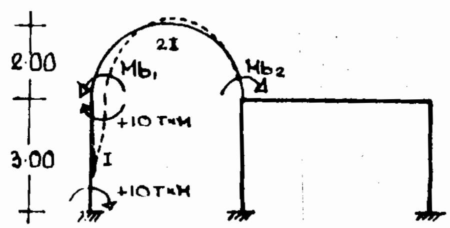


F_5 IMPIDE EL DESPLAZAMIENTO DEL NUDO (1)

$$\alpha F_3 - \beta F_5 = F_1$$

$$\alpha F_4 + \beta F_6 = F_2$$

ANALISIS DEL 1er. GRADO DE LIBERTAD



$$r_{1_a} = \frac{6E(I)}{1^2} = \frac{6EI}{3^2} = 0.667\ EI$$

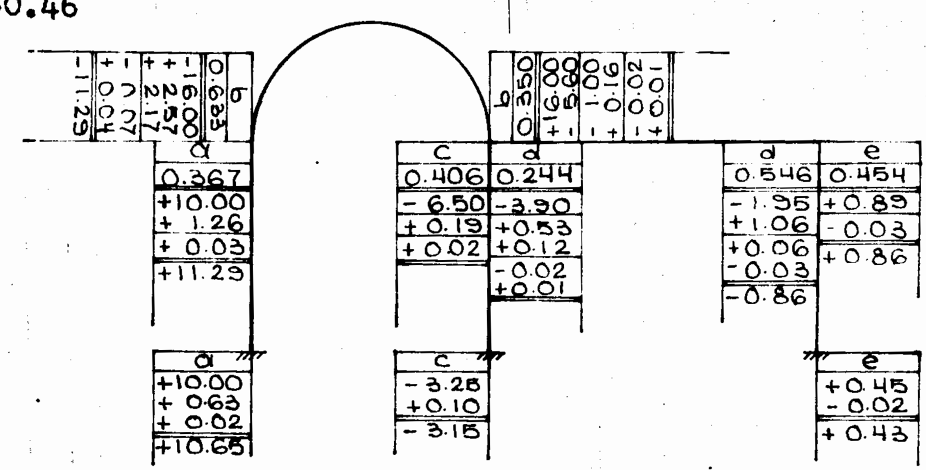
$$= \frac{M_a}{r_{1_a}} = \frac{10}{0.667EI} = \frac{15}{EI}$$

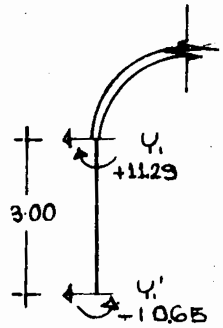
$$r_{1_b} = \frac{2.13E(I)}{r^2} = \frac{2.13E\times 2I}{2^2} = 1.065\ EI$$

$$M_{b_1} = -\Delta r_{1_b} = -\frac{15}{EI} \times 1.065\ EI = -16\ T \times M$$

$$M_{b_2} = M_{b_1} \times t_{1_b} = -15 \times (-1) = +16\ T \times M$$

$$t_a(\text{ARCO}) = -0.46$$





$11.29 = MF_1$

$11.29 = -10.65 + 3Y_1'$

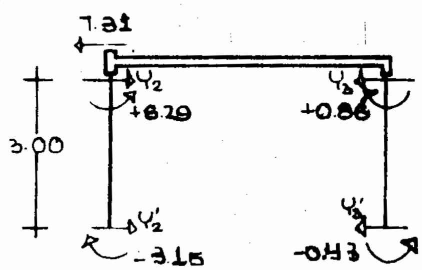
$Y_1' = \frac{11.29+10.65}{3} = \frac{21.94}{3} = 7.31$

$Y_1 = Y_1' = 7.31$ TON

$F_3 = Y_1 = 7.31$ TON

$-6.29 = MF_2$

$-6.29 = 3.15 - 3Y_2'$



$Y_2' = \frac{6.29+3.15}{3} = \frac{9.44}{3} = 3.15$

$Y_2 = Y_2' = 3.15$ TON

$-0.86 = -0.43 + 3Y_3'$

$Y_3' = \frac{0.86+0.43}{3} = \frac{1.29}{3} = 0.43$

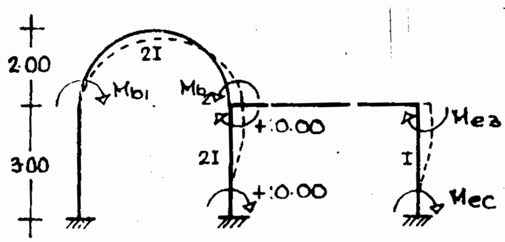
$Y_3 = Y_3' = 0.43$ TON

$F_4 = 7.31-3.15+0.43$

$F_4 = 7.74 - 3.15 = 4.59$ TON

PARA OBTENER LOS ANTERIORES MOMENTOS DEBEMOS APLICAR UNA FUERZA F_3 = 7.31 TON DE IZQUIERDA A DERECHA EN EL NUDO (1) Y UNA FUERZA F_4 = 4.59 TON DE IZQUIERDA A DERECHA EN EL CABEZAL.

ANALISIS DEL 2º GRADO DE LIBERTAD



$r_{1_c} = \frac{6E(I)}{l^2} = \frac{6E\times 2I}{3^2} = 1.333$ EI

$r_{1_b} = \frac{2.13E(I)}{l^2} = \frac{2.13E\times 2I}{2^2} = 1.065$ EI

$$r_{1_e} = \frac{6E(I)}{l^2} = \frac{6EI}{3^2} = 0.667\ EI$$

$$\Delta = \frac{Mc}{r_{1_c}} = \frac{10}{1.333EI} = \frac{7.5}{EI}$$

$$M_{b_2} = -\Delta r_{1_b} = \frac{7.5}{EI} \times 1.065\ EI = 8.00\ T \times M$$

$$M_{b_1} = -M_{b_2}(-1) = 8.00\ T \times M$$

$$M_{e_3} = \Delta r_{1_e} = \frac{7.5}{EI} \times 0.667EI = +5.00\ T \times M$$

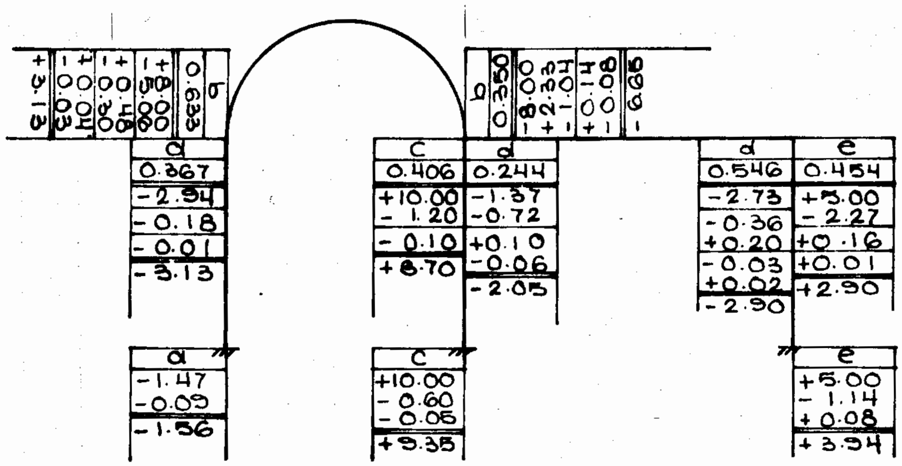
$$M_{c_B} = M_{c_2} = +10\ T \times M$$

$$M_{e_c} = M_{e_3} = +\ 5\ T \times M$$

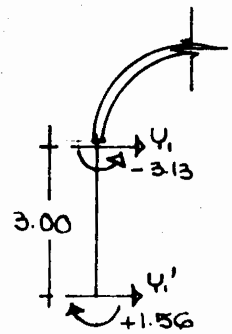
$$M_{b_2} = -\ 8\ T \times M$$

$$M_{b_1} = +\ 8\ T \times M$$

t_a (ARCO) = - 0.46



ORDEN DE EQUILIBRIO DE LOS NUDOS: 1,3,2,1,3,2,1.



$$-3.13 = 1.56 - 3Y'_1$$

$$Y'_1 = \frac{3.13+1.56}{3} = \frac{4.69}{3} = 1.60$$

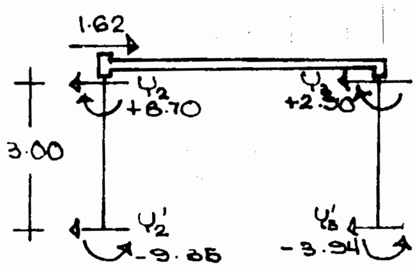
$$Y_1 = Y'_1 = 1.60 \text{ TON}$$

$$F_5 = Y_1 = 1.60 \text{ TON}$$

$$8.70 = -9.35 + 3Y'_2$$

$$Y'_2 = \frac{8.70+9.35}{3} = \frac{18.05}{3} = 6.02$$

$$Y_2 = Y'_2 = 6.02 \text{ TON}$$



$$2.90 = -3.94 + 3\,Y'_3$$

$$Y'_3 = \frac{2.90+3.94}{3} = \frac{6.84}{3} = 2.28$$

$$Y_3 = Y'_3 = 2.28 \text{ TON}$$

$$F_6 = -1.60 + 6.02 + 2.28 = 6.70 \text{ TON}$$

PARA OBTENER LOS ANTERIORES MOMENTOS NECESITAMOS APLICAR UNA -- FUERZA $F_5 = 1.60$ TON DE DERECHA A IZQUIERDA EN EL NUDO (1) Y UNA FUERZA $F_6 = 6.70$ TON DE IZQUIERDA A DERECHA EN EL CABEZAL.

SISTEMA DE ECUACIONES SIMULTANEAS RESULTANTE

$$\alpha\,7.31 - \beta\,1.60 = 2.26 \qquad (1)$$

$$+\alpha\,4.59 + \beta\,6.68 = 4.50 \qquad (2)$$

DE (1) $\alpha = \dfrac{2.26 + 1.60\beta}{7.31}$ (3)

EN (2) $= \dfrac{2.26 + 1.60\beta}{7.31}\;4.59 + 6.70\,\beta = 4.50$

$$1.42 + 1.02\beta + 6.70\beta = 4.50$$

$$7.72\,\beta = 3.08$$

$$\beta = \frac{3.08}{7.72} = 0.40$$

EN (3)

$$\alpha = \frac{2.26+1.62 \times 0.40}{7.31} = \frac{2.26+0.65}{7.31} = \frac{2.91}{7.31} = 0.397$$

$$\alpha = 0.397$$

$$\beta = 0.40$$

MOMENTOS FINALES

$M_{a_A} = +0.24 +0.397 \times 10.65 -0.40 \times 1.56 = +0.24 +4.23 -0.62 = + 3.85$

$M_{a_1} = -1.02 +0.397 \times 11.29 -0.40 \times 3.13 = -1.02 +4.48 -1.25 = + 2.21$

$M_{b_1} = +1.02 -0.397 \times 11.29 +0.40 \times 3.13 = +1.02 -4.48 +1.25 = - 2.21$

$M_{b_2} = +0.27 +0.397 \times 9.55 -0.40 \times 6.65 = +0.27 +3.79 -2.66 = + 1.40$

$M_{c_2} = -4.37 -0.397 \times 6.29 + 0.40 \times 8.70 = -4.37 -2.50 +3.48 = - 3.39$

$M_{c_B} = -2.18 -0.397 \times 3.15 +0.40 \times 9.35 = -2.18 -1.25 +3.74 = + 0.31$

$M_{d_2} = +4.10 -0.397 \times 3.26 -0.40 \times 2.05 = +4.10 -1.29 -0.82 = + 1.99$

$M_{d_3} = -2.87 -0.397 \times 0.86 -0.40 \times 2.90 = -2.87 -0.34 -1.16 = - 4.37$

$M_{e_3} = +2.87 +0.397 \times 0.86 +0.40 \times 2.90 = +2.87 +0.34 +1.16 = + 4.37$

$M_{e_C} = +1.44 +0.397 \times 0.43 +0.40 \times 3.94 = +1.44 +0.17 +1.58 = + 3.19$

CALCULO DE REACCIONES Y DIAGRAMAS.

BARRA (a)

$$+2.21 = MF_1$$

$$+2.21 = -3.85 +3.00 -2 \times 1.50 + 3X_{a_A}$$

$$X_{a_A} = \frac{2.21+3.85}{3} = \frac{6.06}{3} = 2.02$$

$$X_{a_A} = 2.02$$

$$X_{a_1} = 2.02 - 2.00 = 0.02 \text{ TON}$$

$$M_{isos} = \frac{PL}{4} - \frac{M}{2} = \frac{2 \times 3}{4} - \frac{3}{2} = 1.5 - 1.5 = 0$$

$$M_{\text{₵}} = \frac{-3.85+2.21}{2} = -\frac{1.64}{2} = -0.82 \text{ TxM}$$

$$M_{\text{₵}} = -0.82 + 3.00 = +2.18 \text{ T x M}$$

BARRA (b)

$$X_{b_2} = -0.02 + 1.50 = 1.48 \text{ TON}$$

$$X_{b_2} = 1.48 \text{ TON}$$

$$+ 1.40 = MF_2$$

$$+ 1.40 = +2.21 + 1.5 \times 2 - \frac{2 \times 4^2}{2} + Y_{b_1}$$

$$1.40 = +2.21 + 3.00 - 12 - 16 + 4Y_{b_1}$$

$$Y_{b_1} = \frac{1.40-2.21-3.00+12.00+16.00}{4} = \frac{24.19}{4}$$

$$Y_{b_1} = + 6.05 \text{ TON}$$

$$Y_{b2} = +6.05 - 6.00 - 2 \times 4 = 6.05 - 14.00$$

$$Y_{b_2} = - 7.95 \text{ TON}$$

$$M_{isos} = \frac{wl^2}{8} + \frac{P_V l}{4} + P_h \times h = \frac{2 \times 4^2}{8} + \frac{6 \times 4}{4} + 1.5 \times 2$$

$$M_{isos} = +4 + 6 + 3 = 13.00 \text{ T x M}$$

$$M_{\text{₵}} = \frac{2.21+1.40}{2} + 13.00 = \frac{3.61}{2} + 13.00$$

$$M_{\text{₵}} = 1.81 + 13.00 = +14.81 \text{ T x M}$$

BARRA (c)

$$- 3.39 = MF_2$$

$$- 3.39 = 0.31 - 3X_{c_B}$$

$$X_{c_B} = \frac{3.39-0.31}{3} = \frac{3.08}{3} = 1.03 \text{ TON}$$

$$X_{c_2} = X_{c_3} = 1.03 \text{ TON}$$

$$M_{isos} = 0$$

BARRA (d)

$$-4.37 = MF_3$$

$$-4.37 = -1.99 + 5Y_{b_2} - \frac{5+3.5}{2} x\ 2.5\ x\ 2.5$$

$$-4.37 = -1.99 + 5Y_{b_2} - 26.56$$

$$Y_{d_2} = \frac{-4.37+1.96+26.56}{5} = \frac{24.15}{5}$$

$$Y_{d_2} = 4.83 \text{ TON}$$

$$Y_{d_3} = -4.83 + \frac{5+3.5}{2} x\ 2.5 = -4.83+4.25\ x\ 2.5$$

$$Y_{d_3} = -4.83 + 10.63 = 5.40 \text{ TON}$$

$$M_{isos} = \frac{w}{4} \left(\frac{l^2}{2} - \frac{2a^2}{3}\right) = \frac{2.5}{4}\left(\frac{5^2}{2} + \frac{2x3.5^2}{3}\right) = \frac{2.5}{4}(12.5+8.18)$$

$$M_{isos} = \frac{2.5}{4}\ x\ 20.68 = \frac{51.70}{4} = 12.90 \text{ T x M}$$

$$M = \frac{-1.99 - 4.37}{2} + 12.90 = -3.18 + 12.90 = +9.72 \text{ T x M}$$

BARRA (e)

$$+4.37 = MF_3$$

$$+4.37 = -3.19 + 3\ X_{e_C}$$

$$X_{e_C} = \frac{4.37 + 3.19}{3} = \frac{7.56}{3} = 2.52 \text{ TON}$$

$$X_{e_3} = X_{e_C} = 2.52 \text{ TON}$$

$$M_{isos} = 0$$

REACCIONES

$X_A = X_{a_A} = -2.02$ TON

$Y_A = Y_{b_1} = + 6.05$ TON

$X_B = X_{c_B} = + 1.03$ TON

$Y_B = Y_{b_2} + Y_{d_2} = 7.95 + 4.83 = 12.78$ TON

$X_C = X_{e_C} = -2.52$ TON

$Y_C = Y_{d_3} = + 5.40$ TON

DIAGRAMA DE FUERZAS CORTANTES

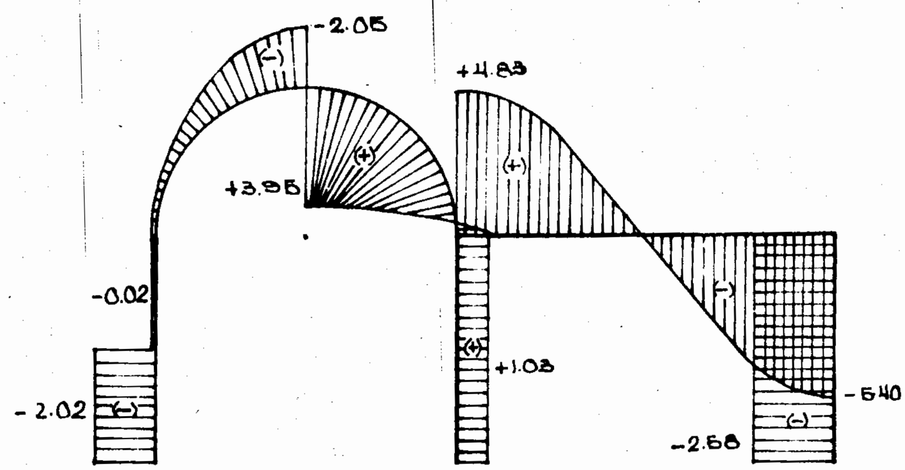
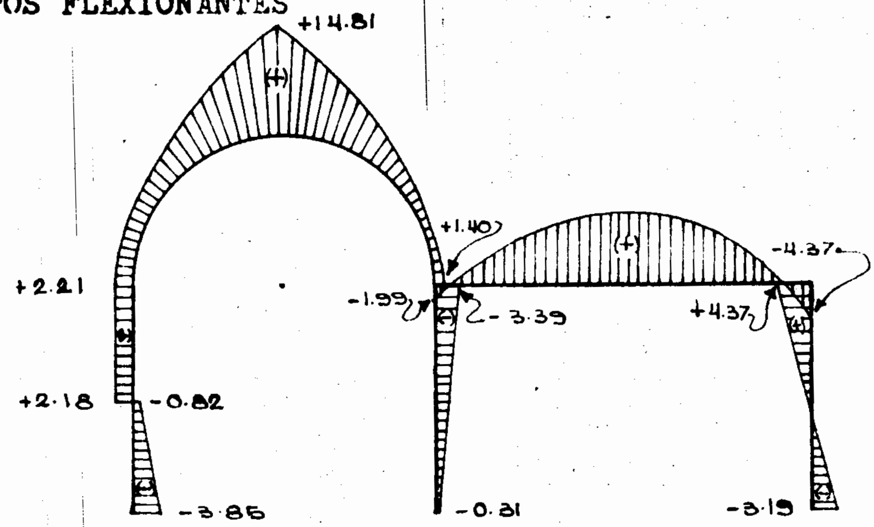
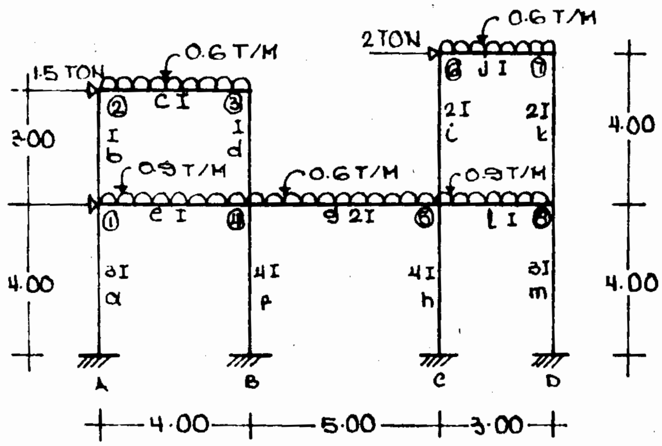


DIAGRAMA DE MOMENTOS FLEXIONANTES





3 GRADOS DE LIBERTAD LINEAL.

NUDO (1)

$r_{a_a} = \frac{4E(I)}{l} = \frac{4Ex3I}{4} = 3.00\ EI$ $\qquad$ $F.D._a = \frac{3.00EI}{5.33EI} = 0.562$

$r_{a_b} = \frac{4E(I)}{l} = \frac{4Ex\ I}{3} = 1.33\ EI$ $\qquad$ $F.D._b = \frac{1.33EI}{5.33EI} = 0.250$

$r_{a_e} = \frac{4E(I)}{l} = \frac{4Ex\ I}{4} = 1.00\ EI$ $\qquad$ $F.D._e = \frac{1.00EI}{5.33EI} = 0.188$

$\Sigma = 5.33\ EI$ $\qquad$ $\Sigma = 1.000$

NUDO (2)

$r_{a_b} = \frac{4E(I)}{l} = \frac{4ExI}{3} = 1.33\ EI$ $\qquad$ $F.D._b = \frac{1.33EI}{2.33EI} = 0.57$

$r_{a_c} = \frac{4E(I)}{l} = \frac{4ExI}{4} = 1.00\ EI$ $\qquad$ $F.D._c = \frac{1.00EI}{2.33EI} = 0.43$

$\Sigma = 2.33\ EI$ $\qquad$ $\Sigma = 1.00$

NUDO (3)

$r_{a_c} = \frac{4E(I)}{l} = \frac{4ExI}{4} = 1.00\ EI$ $\qquad$ $F.D._c = \frac{1.00EI}{2.33EI} = 0.43$

$r_{a_d} = \frac{4E(I)}{l} = \frac{4ExI}{3} = 1.33\ EI$ $\qquad$ $F.D._d = \frac{1.33EI}{2.33EI} = 0.57$

$\Sigma = 2.33\ EI$ $\qquad$ $\Sigma = 1.00$

NUDO (4)

$$r_{a_d} = \frac{4E(I)}{1} = \frac{4ExI}{3} = 1.33 \text{ EI} \qquad F.D._d = \frac{1.33EI}{7.93EI} = 0.167$$

$$r_{a_e} = \frac{4E(I)}{1} = \frac{4ExI}{4} = 1.00 \text{ EI} \qquad F.D._e = \frac{1.00EI}{7.93EI} = 0.126$$

$$r_{a_f} = \frac{4E(I)}{1} = \frac{4Ex4I}{4} = 4.00 \text{ EI} \qquad F.D._f = \frac{4.00EI}{7.93EI} = 0.505$$

$$r_{a_g} = \frac{4E(I)}{1} = \frac{4Ex2I}{5} = 1.60 \text{ EI} \qquad F.D._g = \frac{1.60EI}{7.93EI} = 0.202$$

$$\Sigma = 7.93 \text{ EI} \qquad \Sigma = 1.000$$

NUDO (5)

$$r_{a_g} = \frac{4E(I)}{1} = \frac{4Ex2I}{5} = 1.60 \text{ EI} \qquad F.D._g = \frac{1.60EI}{8.93EI} = 0.179$$

$$r_{a_i} = \frac{4E(I)}{1} = \frac{4Ex2I}{4} = 2.00 \text{ EI} \qquad F.D._i = \frac{2.00EI}{8.93EI} = 0.224$$

$$r_{a_l} = \frac{4E(I)}{1} = \frac{4Ex\ I}{3} = 1.33 \text{ EI} \qquad F.D._l = \frac{1.33EI}{8.93EI} = 0.149$$

$$r_{a_h} = \frac{4E(I)}{1} = \frac{4Ex4I}{4} = 4.00 \text{ EI} \qquad F.D._h = \frac{4.00EI}{8.93EI} = 0.448$$

$$\Sigma = 8.93 \text{ EI} \qquad \Sigma = 1.000$$

NUDO (6)

$$r_{a_i} = \frac{4E(I)}{1} = \frac{4Ex2I}{4} = 2.00 \text{ EI} \qquad F.D._i = \frac{2.00EI}{3.33EI} = 0.60$$

$$r_{a_j} = \frac{4E(I)}{1} = \frac{4Ex\ I}{3} = 1.33 \text{ EI} \qquad F.D._j = \frac{1.33EI}{3.33EI} = 0.40$$

$$\Sigma = 3.33 \text{ EI} \qquad \Sigma = 1.00$$

NUDO (7)

$$r_{a_j} = \frac{4E(I)}{1} = \frac{4Ex\ I}{3} = 1.33 \text{ EI} \qquad F.D._j = \frac{1.33EI}{3.33EI} = 0.40$$

$$r_{a_k} = \frac{4E(I)}{l} = \frac{4E\times 2I}{4} = 2.00\ EI \qquad F.D._k = \frac{2.00\ EI}{3.33\ EI} = 0.60$$

$$\Sigma = 3.33\ EI \qquad \Sigma = 1.00$$

NUDO (8)

$$r_{a_k} = \frac{4E(I)}{l} = \frac{4E\times 2I}{4} = 2.00\ EI \qquad F.D._k = \frac{2.00\ EI}{6.33\ EI} = 0.316$$

$$r_{a_l} = \frac{4E(I)}{l} = \frac{4E\times I}{3} = 1.33\ EI \qquad F.D._l = \frac{1.33\ EI}{6.33\ EI} = 0.210$$

$$r_{a_m} = \frac{4E(I)}{l} = \frac{4E\times 3I}{4} = 3.00\ EI \qquad F.D._m = \frac{3.00\ EI}{6.33\ EI} = 0.474$$

$$\Sigma = 6.33\ EI \qquad \Sigma = 1.000$$

MOMENTOS DE EMPOTRAMIENTO.

BARRA (c)

$$M_2 = -M_3 = \frac{wl^2}{12} = \frac{0.6\times 4^2}{12} = 0.80\ T \times M$$

BARRA (e)

$$M_1 = -M_4 = \frac{wl^2}{12} = \frac{0.9\times 4^2}{12} = 1.20\ T \times M$$

BARRA (g)

$$M_4 = -M_5 = \frac{wl^2}{12} = \frac{0.6\times 5^2}{12} = 1.25\ T \times M$$

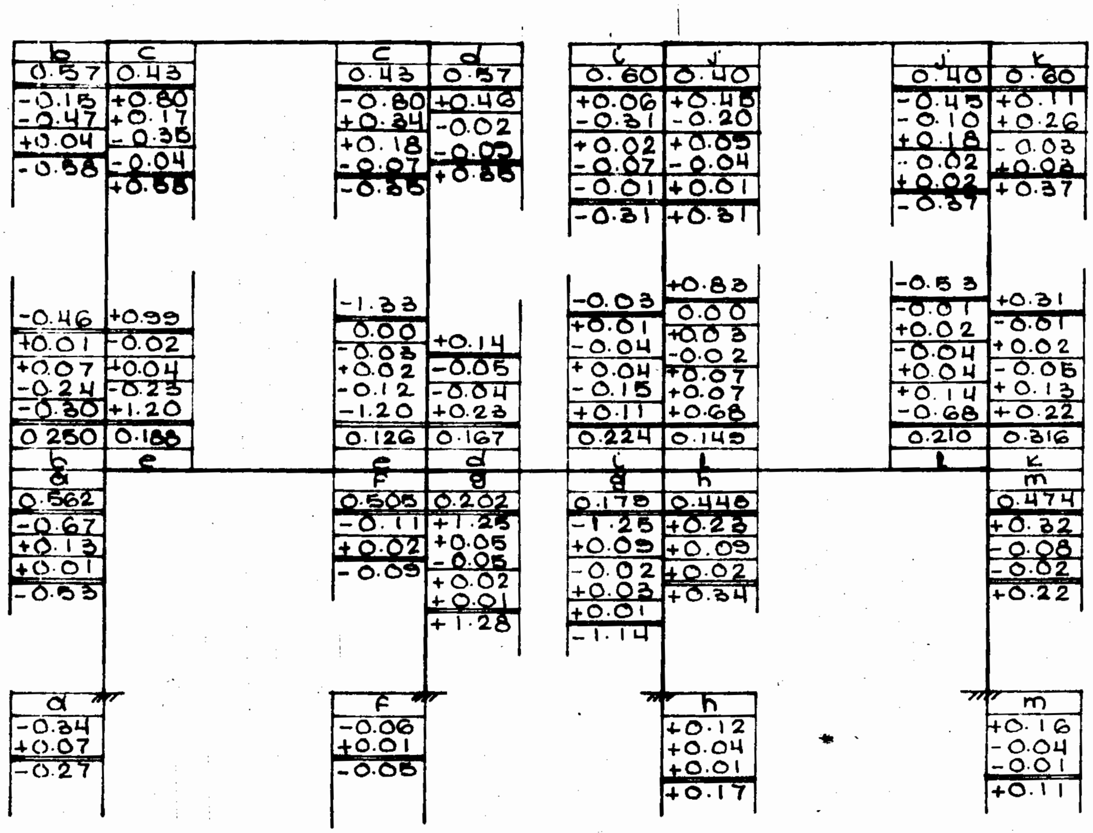
BARRA (j)

$$M_6 = -M_7 = \frac{wl^2}{12} = \frac{0.6\times 3^2}{12} = 0.45\ T \times M$$

BARRA (l)

$$M_5 = -M_8 = \frac{wl^2}{12} = \frac{0.9\times 3^2}{12} = 0.68\ T \times M$$

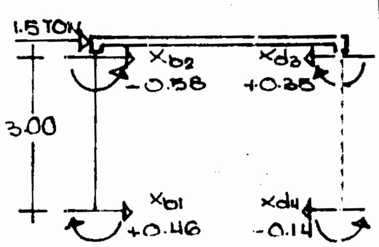
BARRAS (a),(b),(d),(f),(h),(i),(k) y (m) NO TIENEN SOLICITACIONES.



ORDEN DE EQUILIBRIO DE LOS NUDOS: 1,3,2,8,5,6,1,4,5,3,6,7,8,5, 4,1,6.

ANALISIS DE CABEZALES

CABEZAL SUPERIOR IZQUIERDO.



$$-0.58 = MF_2$$

$$-0.58 = +0.46 - 3X_{b_1}$$

$$X_{b_1} = \frac{0.58+0.46}{3} = \frac{1.04}{3} = 0.35 \text{ TON}$$

$$X_{b_2} = X_{b_1} = 0.35 \text{ TON.}$$

$$+0.35 = \Sigma MF_3$$

$$+0.35 = -0.14 + 3X_{d_4}$$

$$X_{d_4} = \frac{0.35+0.14}{3} = \frac{0.49}{3} = 0.16 \text{ TON}$$

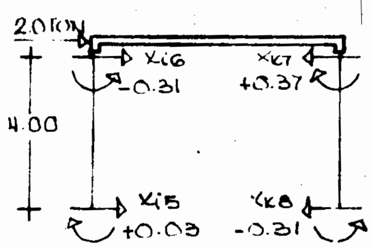
$$X_{d_3} = X_{d_4} = 0.16 \text{ TON}$$

$$F_1 = 1.50 + X_{b_2} - X_{d_3} = 1.50 + 0.35 - 0.16 = 1.69 \text{ TON}$$

$$F_1 = 1.69 \text{ TON}$$

PARA IMPEDIR EL DESPLAZAMIENTO DEL CABEZAL SUPERIOR IZQUIERDO - ES NECESARIO APLICAR UNA FUERZA F_1 = 1.69 TON DE DERECHA A IZQUIERDA EN EL CABEZAL.

CABEZAL SUPERIOR DERECHO.



$$-0.31 = \Sigma MF_6$$

$$-0.31 = +0.03 - 4X_{i_5}$$

$$X_{i_5} = \frac{0.31+0.03}{4} = \frac{0.34}{4} = 0.09 \text{ TON}$$

$$X_{i_6} = X_{i_5} = 0.09 \text{ TON}$$

$$+0.37 = \Sigma MF_7$$

$$+0.37 = -0.31 + 4X_{k_8}$$

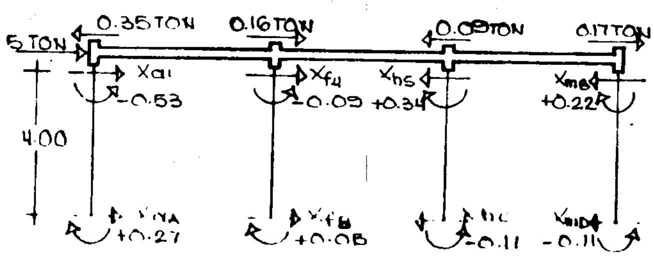
$$X_{k_8} = \frac{0.37+0.31}{4} = \frac{0.68}{4} = 0.17 \text{ TON}$$

$$X_{k_7} = X_{k_8} = 0.17 \text{ TON}$$

$$F_2 = 2.00 + X_{i_6} - X_{k_7} = 2.00 + 0.09 - 0.17 = 1.92 \text{ TON}$$

PARA IMPEDIR EL DESPLAZAMIENTO DEL CABEZAL SUPERIOR DERECHO ES NECESARIO APLICAR UNA FUERZA F_2= 1.92 TON DE DERECHA A IZQUIERDA EN EL CABEZAL.

CABEZAL INFERIOR



$$-0.53 = \Sigma MF_1$$

$$-0.53 = +0.27 - 4X_{a_A}$$

$$X_{a_A} = \frac{0.53+0.27}{4} = \frac{0.80}{4} = 0.20 \text{ TON}$$

$$X_{a_1} = X_{a_A} = 0.20 \text{ TON}$$

$$-0.09 = \Sigma MF_4$$

$$-0.09 = +0.05 - 4X_{f_B}$$

$$X_{f_B} = \frac{-0.09+0.05}{4} = \frac{0.14}{4} = 0.04 \text{ TON}$$

$$X_{f_4} = X_{f_B} = 0.04 \text{ TON}$$

$$+0.34 = \Sigma MF_5$$

$$+0.34 = -0.17 + 4X_{h_c}$$

$$X_{h_C} = \frac{0.37+0.17}{4} = \frac{0.54}{4} = 0.14 \text{ TON}$$

$$X_{h_5} = X_{h_C} = 0.14 \text{ TON}$$

$$+0.22 = \Sigma MF_8$$

$$+0.22 = -0.11 + 4X_{m_B}$$

$$X_{m_D} = \frac{0.22+0.11}{4} = \frac{0.33}{4} = 0.08 \text{ TON}$$

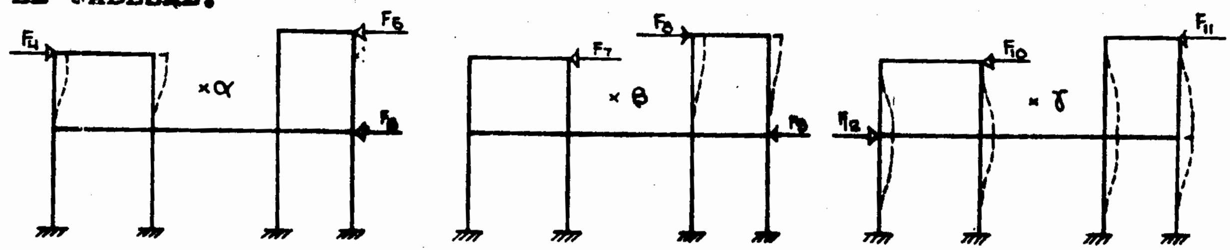
$$X_{m_8} = X_{m_D} = 0.08 \text{ TON}$$

$$F_3 = 5.00 - 0.35 + 0.16 - 0.09 + 0.17 + X_{a_1} + X_{f_4} - X_{h_5} - X_{m_8}$$

$$F_3 = 5.00 - 0.35 + 0.16 - 0.09 + 0.17 + 0.20 + 0.04 - 0.14 - 0.08 = +4.91 \text{ TON}$$

$$F_3 = 4.91 \text{ TON}$$

PARA IMPEDIR EL DESPLAZAMIENTO DEL CABEZAL INFERIOR ES NECESA-- RIO APLICAR UNA FUERZA F_3= 4.91 TON DE DERECHA A IZQUIERDA EN - EL CABEZAL.



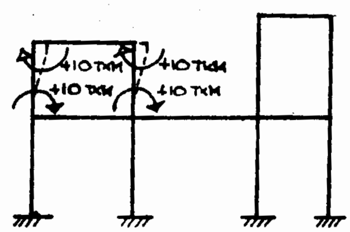
SE FORMARA EL SIGUIENTE SISTEMA DE 3 ECUACIONES SIMULTANEAS.

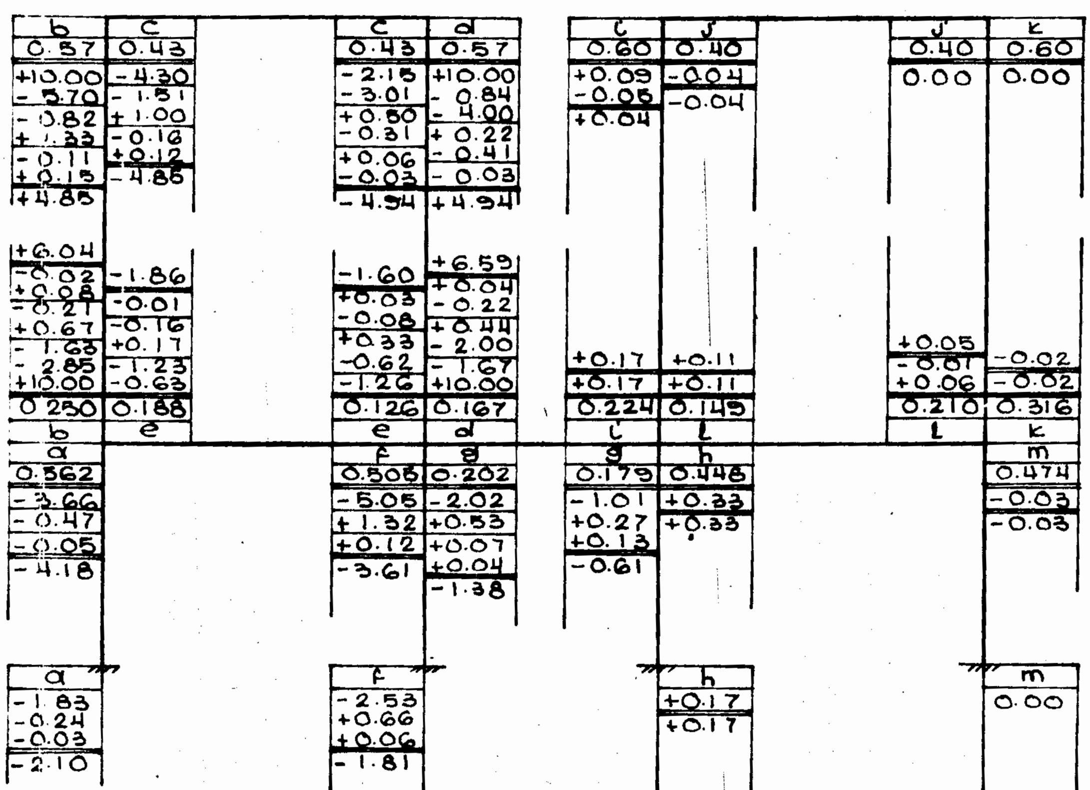
$$\alpha F_4 - \beta F_7 - \gamma F_{10} = - F_1 \qquad (1)$$

$$-\alpha F_5 + \beta F_8 - \gamma F_{11} = - F_2 \qquad (2)$$

$$-\alpha F_6 - \beta F_9 - \gamma F_{12} = - F_3 \qquad (3)$$

ANALISIS DEL 1er. GRADO DE LIBERTAD LINEAL.

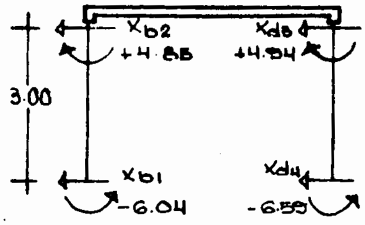




ORDEN DE EQUILIBRIO DE LOS NUDOS: 2,4,3,1,4,2,1,5,3,2,4,6,1,3,8.

ANALISIS DE CABEZALES

CABEZAL SUPERIOR IZQUIERDO



$$4.85 = \Sigma MF_2$$

$$4.85 = -6.04 + 3X_{b_1}$$

$$X_{b_1} = \frac{4.85+6.04}{3} = \frac{10.89}{3} = 3.63 \text{ TON}$$

$$X_{b_2} = X_{b_1} = 3.63 \text{ TON}$$

$$4.94 = \Sigma MF_3$$

$$4.94 = -6.59 + 3X_{d_4}$$

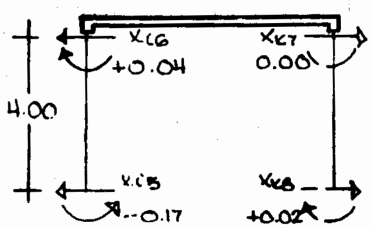
$$X_{d_4} = \frac{4.94 + 6.59}{3} = \frac{11.53}{3} = 3.84 \text{ TON}$$

$$X_{d_3} = X_{d_4} = 3.84 \text{ TON}$$

$$F_4 = X_{b_2} + X_{d_3} = 3.63+3.84 = 7.47 \text{ TON}$$

$$F_4 = 7.47 \text{ TON}$$

CABEZAL SUPERIOR DERECHO



$$0.04 = \Sigma MF_6$$

$$0.04 = -0.17 + 4X_{i_5}$$

$$X_{i_5} = \frac{0.04+0.17}{4} = \frac{0.21}{4} = 0.05 \text{ TON}$$

$$X_{i_6} = X_{i_5} = 0.05 \text{ TON}$$

$$0 = \Sigma MF_7$$

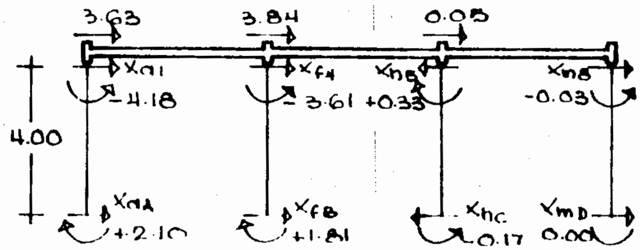
$$0 = 0.02 - 4X_{k_8}$$

$$X_{k_8} = \frac{0.02}{4} = 0.00 \text{ TON}$$

$$X_{k_7} = X_{k_8} = 0.00 \text{ TON}$$

$$F_5 = X_{i_6} = 0.05 \text{ TON}$$

CABEZAL INFERIOR



$$-4.18 = \Sigma MF_1$$

$$-4.18 = 2.10 - 4X_{a_A}$$

$$X_{a_A} = \frac{4.18+2.10}{4} = \frac{6.28}{4} = 1.57 \text{ TON}$$

$$X_{a_1} = X_{a_A} = 1.57 \text{ TON}$$

$$-3.61 = \Sigma MF_4$$

$$-3.61 = 1.81 - 4X_{f_B}$$

$$X_{f_B} = \frac{3.61+1.81}{4} = \frac{5.42}{4} = 1.36 \text{ TON}$$

$$X_{f_4} = X_{f_B} = 1.36 \text{ TON}$$

$$0.33 = \Sigma MF_5$$

$$0.33 = -0.17 + 4X_{h_C}$$

$$X_{h_C} = \frac{0.33+0.17}{4} = \frac{0.50}{4} = 0.13 \text{ TON}$$

$$X_{h_5} = X_{h_C} = 0.13 \text{ TON}$$

$$-0.03 = \Sigma MF_8$$

$$-0.03 = -4X_{m_D}$$

$$X_{m_D} = \frac{0.03}{4} = 0.01 \text{ TON}$$

$$X_{m_8} = X_{m_D} = 0.01 \text{ TON}$$

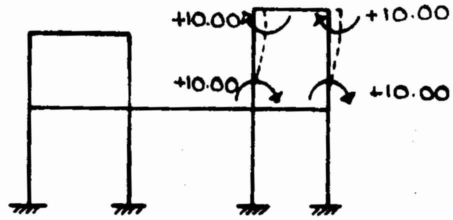
$$F_6 = 3.63 + 3.84 + 0.05 + X_{a_1} + X_{f_4} - X_{h_5} + X_{m_8}$$

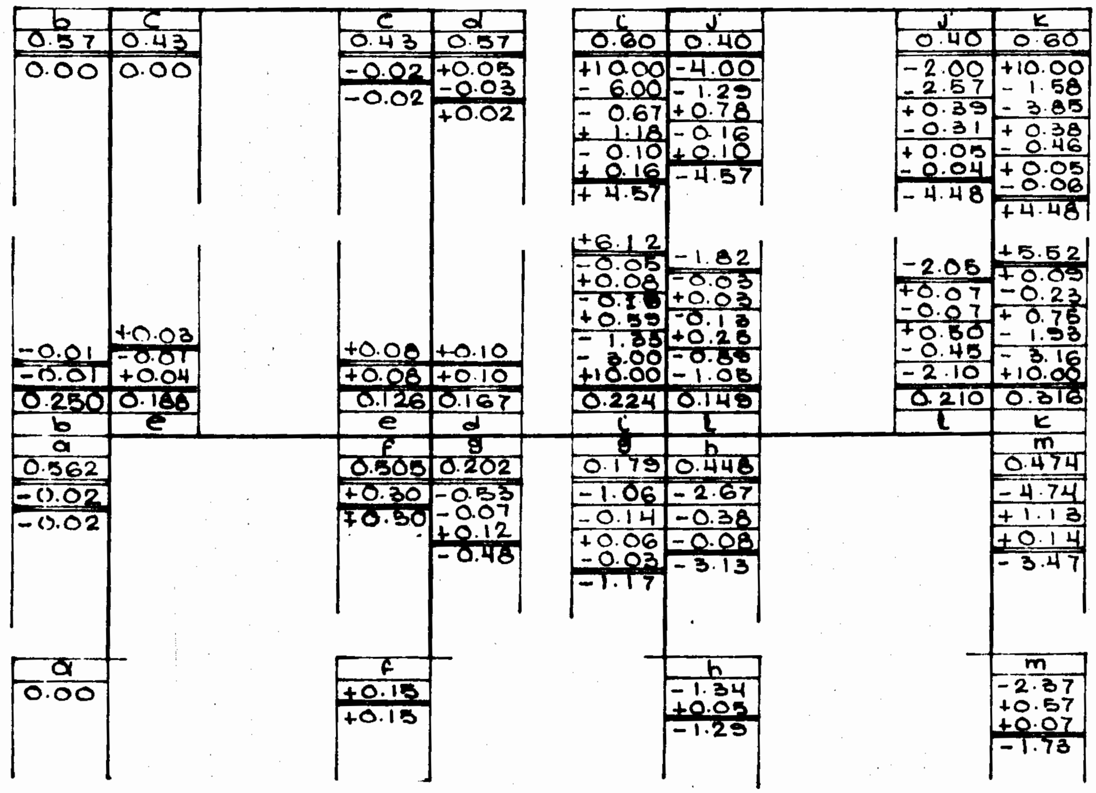
$$F_6 = 3.63 + 3.84 + 0.05 + 1.57 + 1.36 - 0.13 + 0.01 = 10.33 \text{ TON}$$

$$F_6 = 10.33 \text{ TON}$$

PARA PRODUCIR LOS ANTERIORES MOMENTOS ES NECESARIO APLICAR UNA FUERZA $F_4 = 7.47$ TON DE IZQUIERDA A DERECHA EN EL CABEZAL SUPERIOR IZQUIERDO, UNA FUERZA $F_5 = 0.05$ TON DE IZQUIERDA A DERECHA EN EL CABEZAL SUPERIOR DERECHO Y UNA FUERZA $F_6 = 10.33$TON DE DERECHA A IZQUIERDA EN EL CABEZAL INFERIOR.

ANALISIS DEL SEGUNDO GRADO DE LIBERTAD LINEAL.

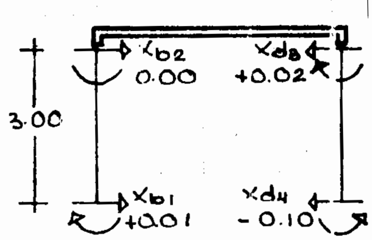




ORDEN DE EQUILIBRIO DE LOS NUDOS: 6,8,7,5,8,6,7,5,4,8,6,5,1,3.

ANALISIS DE CABEZALES.

CABEZAL SUPERIOR IZQUIERDO



$$0 = \Sigma MF_2$$

$$0 = 0.01 - 3X_{b_1}$$

$$X_{b_1} = \frac{0.01}{3} = 0$$

$$X_{b_2} = X_{b_1} = 0$$

$+ 0.02 = \Sigma MF_3$

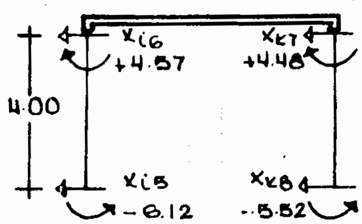
$+ 0.02 = - 0.10 + 3X_{d_4}$

$X_{d_4} = \frac{0.02 + 0.10}{3} = \frac{0.12}{3} = 0.04$ TON

$X_{d_3} = X_{d_4} = 0.04$ TON

$F_7 = X_{d_3} = 0.04$ TON

CABEZAL SUPERIOR DERECHO



$4.57 = \Sigma MF_6$

$4.57 = - 6.12 + 4X_{i_5}$

$X_{i_5} = \frac{4.57+6.12}{4} = \frac{10.69}{4} = 2.67$ TON

$X_{i_6} = X_{i_5} = 2.77$ TON

$4.48 = \Sigma MF_7$

$4.48 = - 5.52 + 4X_{k_8}$

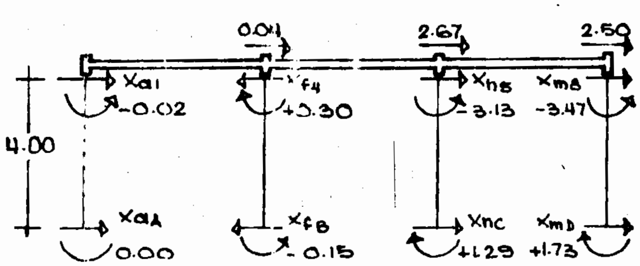
$X_{k_8} = \frac{4.48+5.52}{4} = \frac{10.00}{4} = 2.50$ TON

$X_{k_7} = X_{k_8} = 2.50$ TON

$F_8 = X_{i_6} + X_{k_7} = 2.67 + 2.50 = 5.17$ TON

$F_8 = 5.17$ TON

CABEZAL INFERIOR



$- 0.02 = \Sigma MF_1$

$- 0.02 = - 4X_{a_A}$

$X_{a_A} = \frac{0.02}{4} = 0.01$ TON

$X_{a_1} = X_{a_A} = 0.01$ TON

$0.30 = MF_4$

$0.30 = -0.15 + 4X_{f_B}$

$X_{f_B} = \frac{0.30+0.15}{4} = \frac{0.45}{4} = 0.11$ TON

$X_{f_4} = X_{f_B} = 0.11$ TON

$-3.13 = MF_5$

$-3.13 = 1.29 - 4X_{h_C}$

$X_{h_C} = \frac{3.13+1.29}{4} = \frac{4.42}{4} = 1.10$ TON

$X_{h_5} = X_{h_C} = 1.10$ TON

$-3.47 = MF_8$

$-3.47 = 1.73 - 4X_{m_D}$

$X_{m_D} = \frac{3.47+1.73}{4} = \frac{5.20}{4} = 1.30$ TON

$X_{m_B} = X_{m_D} = 1.30$ TON.

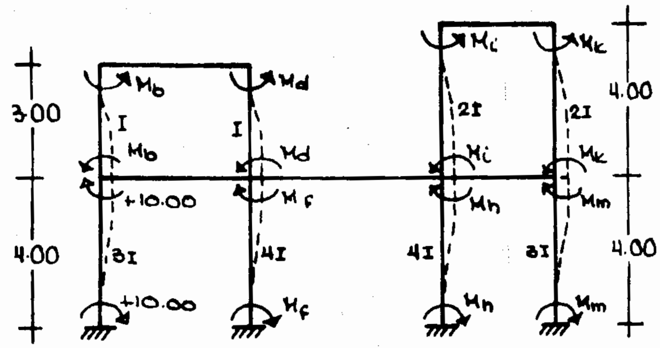
$F_9 = 0.04 + 2.67 + 2.50 + X_{a_7} - X_{f_4} + X_{h_5} + X_{m_8}$

$F_9 = 0.04 + 2.67 + 2.50 + 0.01 - 0.11 + 1.10 + 1.30 = 7.51$ TON

$F_9 = 7.51$ TON

PARA OBTENER LOS ANTERIORES MOMENTOS ES NECESARIO APLICAR UNA FUERZA $F_7 = 0.04$ TON DE IZQUIERDA A DERECHA EN EL CABEZAL SUPERIOR IZQUIERDO, UNA FUERZA $F_8 = 5.17$ TON DE IZQUIERDA A DERECHA EN EL CABEZAL SUPERIOR DERECHO Y UNA FUERZA $F_9 = 7.50$ TON DE DERECHA A IZQUIERDA EN EL CABEZAL INFERIOR.

ANALISIS DEL TERCER GRADO DE LIBERTAD LINEAL.



$$r_{1_a} = \frac{6E(I)}{l^2} = \frac{6Ex3I}{4^2} = \frac{18EI}{16} = 1.125\ EI$$

$$\Delta = \frac{M_a}{r_{1_a}} = \frac{10}{1.125EI} = \frac{8.89}{EI}$$

$$r_{1_b} = r_{1_d} = \frac{6E(I)}{l^2} = \frac{6ExI}{3^2} = 0.667\ EI$$

$$M_b = M_d = \Delta \times r_{1_b} = \frac{8.89}{EI} \times 0.667\ EI = 5.93\ T \times M$$

$$r_{1_f} = r_{1_h} = \frac{6E(I)}{l^2} = \frac{6Ex4I}{4^2} = 1.5\ EI$$

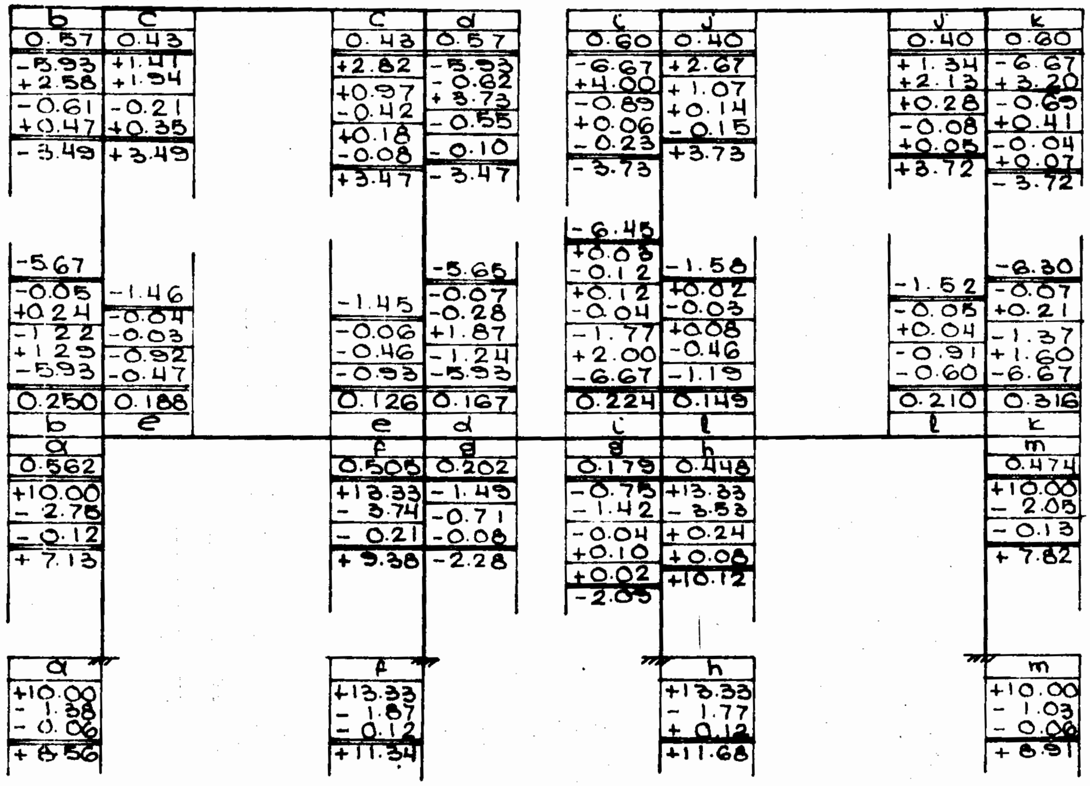
$$M_f = M_h = \Delta \times r_{1_f} = \frac{8.89}{EI} \times 1.5\ EI = 13.33\ T \times M$$

$$r_{1_m} = \frac{6E(I)}{l^2} = \frac{6Ex3I}{4^2} = \frac{18EI}{16} = 1.125\ EI$$

$$M_m = \Delta \times r_{1_m} = \frac{8.89}{EI} \times 1.125\ EI = 10.00\ T \times M$$

$$r_{1_i} = r_{1_k} = \frac{6E(I)}{l^2} = \frac{6Ex2I}{4^2} = 0.75\ EI$$

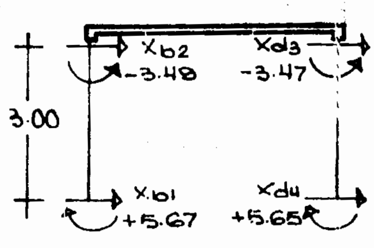
$$M_i = M_k = \Delta \times r_{1_i} = \frac{8.89}{EI} \times 0.75\ EI = 6.67\ T \times M$$



ORDEN DE EQUILIBRIO DE LOS NUDOS: 4,6,5,3,7,2,1,8,3,2,7,4,5,6,
8,1,3,5,7.

ANALISIS DE CABEZALES.

CABEZAL SUPERIOR IZQUIERDO



$$-3.49 = MF_2$$

$$-3.49 = 5.67 - 3X_{b_1}$$

$$X_{b_1} = \frac{3.49+5.67}{3} = \frac{9.16}{3} = 3.05 \text{ TON}$$

$$X_{b_2} = X_{b_1} = 3.05 \text{ TON}$$

$$-3.47 = \Sigma MF_3$$

$$-3.47 = 5.65 - 3X_{d_4}$$

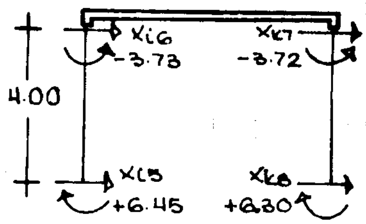
$$X_{d_4} = \frac{3.47 + 5.65}{3} = \frac{9.12}{3} = 3.04 \text{ TON}$$

$X_{d_3} = X_{d_4} = 3.04$ TON

$F_{10} = X_{b_2} + X_{d_3} = 3.05 + 3.04 = 6.09$

$F_{10} = 6.09$ TON

CABEZAL SUPERIOR DERECHO



$- 3.73 = \Sigma MF_6$

$- 3.73 = 6.45 - 3X_{i_5}$

$X_{i_5} = \frac{3.73+6.45}{4} = \frac{10.18}{4} = 2.55$ TON

$X_{i_6} = X_{i_5} = 2.55$ TON

$- 3.72 = \Sigma MF_7$

$- 3.72 = +6.30 - 4X_{k_8}$

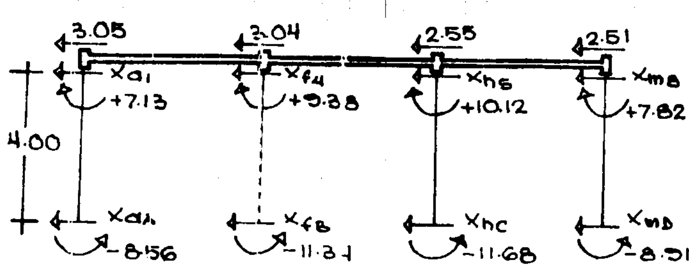
$X_{k_8} = \frac{3.72+6.30}{4} = \frac{10.02}{4} = 2.51$ TON

$X_{k_7} = X_{k_8} = 2.51$ TON

$F_{11} = X_{i_6} + X_{k_7} = 2.55 + 2.51 = 5.06$ TON

$F_{11} = 5.06$ TON

CABEZAL INFERIOR



$7.13 = \Sigma MF_1$

$7.13 = - 8.56 + 4X_{a_A}$

$X_{a_A} = \frac{7.13+8.56}{4} = \frac{15.69}{4} = 3.92$ TON

$X_{a_1} = X_{a_A} = 3.92$ TON

$9.38 = \Sigma MF_4$

$9.38 = - 11.34 + 4X_{f_B}$

$X_{f_B} = \frac{9.38+11.34}{4} = \frac{20.72}{4} = 5.18$ TON

$X_{f_4} = X_{f_B} = 5.18$ TON

$10.12 = \Sigma MF_5$

$10.12 = -11.68 + 4X_{h_C}$

$X_{h_C} = \frac{10.12 + 11.68}{4} = \frac{21.80}{4} = 5.45$ TON

$X_{h_5} = X_{h_C} = 5.45$ TON

$7.82 = \Sigma MF_8$

$7.82 = -8.91 + 4X_{m_D}$

$X_{m_D} = \frac{7.82+8.91}{4} = \frac{16.73}{4} = 4.18$ TON

$X_{m_8} = X_{m_D} = 4.18$ TON

$F_{12} = 3.05 + 3.04 + 2.55 + 2.51 + X_{a_1} + X_{f_4} + X_{h_5} + X_{m_8}$

$F_{12} = 3.05 + 3.04 + 2.55 + 2.51 + 3.92 + 5.18 + 5.45 + 4.18 = 29.88$ TON

PARA PRODUCIR LOS ANTERIORES MOMENTOS ES NECESARIO APLICAR UNA FUERZA $F_{10} = 6.09$ TON DE DERECHA A IZQUIERDA EN EL CABEZAL - SUPERIOR IZQUIERDO, UNA FUERZA $F_{11} = 5.06$ TON DE DERECHA A IZQUIERDA EN EL CABEZAL SUPERIOR DERECHO Y UNA FUERZA $F_{12} = 29.88$ TON DE IZQUIERDA A DERECHA EN EL CABEZAL INFERIOR.

SISTEMA DE ECUACIONES SIMULTANEAS

$\alpha \times 7.47 + \beta \times 0.04 - \gamma \times 6.09 = 1.69$ (1)

$\alpha \times 0.05 + \beta \times 5.17 - \gamma \times 5.06 = 1.92$ (2)

$-\alpha \times 10.33 - \beta \times 7.51 + \gamma \times 29.88 = 4.91$ (3)

DE (1) $\alpha = \dfrac{1.69 - 0.04\,\beta + 6.09\,\gamma}{7.47}$ (4)

(4) EN (2) $\dfrac{1.69 - 0.04\,\beta + 6.09\,\gamma}{7.47} \times 0.05 + 5.17\,\beta - 5.06\,\gamma = 1.92$

$0.0113 - 0.0002\,\beta + 0.0408\,\gamma + 5.17\,\beta - 5.06\,\gamma = 1.92$

$\beta = \dfrac{1.91 + 5.02\,\gamma}{5.17}$ (5)

(5) EN (4) $\alpha = \dfrac{1.69 - 0.04\,\dfrac{1.91 + 5.02\gamma}{5.27} + 6.09\gamma}{7.47} = \dfrac{1.69 - 0.015 - 0.04\,\gamma + 6.09\,\gamma}{7.47}$

$\alpha = \dfrac{1.675 + 6.05\,\gamma}{7.47}$ (6)

(5) y (6) EN (3) $-\dfrac{1.675 + 6.05\,\gamma}{7.47} \times 10.33 - \dfrac{1.91 + 5.02\,\gamma}{5.27} \times 7.51 + 29.88\gamma = 4.9$

$-2.31 - 8.35\,\gamma - 2.72 - 7.15\,\gamma + 29.88\,\gamma = 4.91$

$\gamma = \dfrac{9.94}{14.38} = 0.692$ (7)

(7) EN (5) $\beta = \dfrac{1.91 + 5.02 \times 0.692}{5.17} = \dfrac{5.38}{5.17} = 1.04$ (8)

(7) EN (6) $\alpha = \dfrac{1.675 + 6.05 \times 0.692}{7.47} = \dfrac{5.86}{7.47} = 0.785$

$\alpha = 0.785$; $\beta = 1.04$; $\gamma = 0.692$

MOMENTOS FINALES:

$M_{a_A} = -0.27 - 2.10 \times 0.785 + 0.00 \times 1.04 + 8.56 \times 0.692 = -0.27 - 1.65 + 0.00 + 5.92 = +4.00$

$M_{a_1} = -0.53 - 4.18 \times 0.785 - 0.02 \times 1.04 + 7.13 \times 0.692 = -0.53 - 3.28 - 0.02 - 4.93 = +1.10$

$M_{b_1} = -0.46 + 6.04 \times 0.785 - 0.01 \times 1.04 - 5.67 \times 0.692 = -0.46 + 4.74 - 0.01 - 3.92 = +0.35$

$M_{b_2} = -0.58 + 4.85 \times 0.785 + 0.00 \times 1.04 - 3.49 \times 0.692 = -0.58 + 3.81 + 0.00 - 2.41 = +0.82$

M_{c_2} =+0.58-4.85x0.785+0.00x1.04+ 3.49x0.692=+0.58-3.81+0.00+2.41=-0.82

M_{c_3} =-0.35-4.94x0.785-0.02x1.04+ 3.47x0.692=-0.35-3.88-0.02+2.40=-1.85

M_{d_3} =+0.35+4.94x0.785+0.02x1.04- 3.47x0.692=+0.35+3.88+0.02-2.40=+1.85

M_{d_4} =+0.14+6.59x0.785+0.10x1.04- 5.65x0.692=+0.14+5.17+0.10-3.91=+1.50

M_{e_1} =+0.99-1.86x0.785+0.03x1.04- 1.46x0.692=+0.99-1.46+0.03-1.01=-1.45

M_{e_4} =-1.33-1.60x0.785+0.08x1.04- 1.45x0.692=-1.33-1.25+0.08-1.00=-3.50

M_{f_4} =-0.99-1.86x0.785+0.03x1.04- 1.46x0.692=-0.09-2.83+0.31+6.49=+3.88

M_{f_B} =-0.05-1.81x0.785+0.15x1.04+11.34x0.692=-0.05-1.42+0.16+7.85=+6.94

M_{g_4} =+1.28-1.38x0.785-0.48x1.04- 2.28x0.692=+1.28-1.08-0.50-1.58=-1.88

M_{g_5} =-1.14-0.61x0.785-1.17x1.04- 2.09x0.692=-1.14-0.48-1.22-1.45=-4.29

M_{h_5} =+0.34%0.33x0.785-3.13x1.04+10.12x0.692=+0.34+0.26-3.25+7.00=+4.35

M_{h_C} =+0.17+0.17x0.785-1.29x1.04+11.68x0.692=+0.17+0.13-1.34+8.08=+7.04

M_{i_5} =-0.03+0.17x0.785+6.12x1.04- 6.45x0.692=-0.03+0.13+6.36-4.46=+2.00

M_{i_6} =-0.31+0.04x0.785+4.57x1.04- 3.73x0.692=-0.31+0.03+4.75-2.58=+1.89

M_{j_6} =+0.31-0.04x0.785-4.57x1.04+ 3.73x0.692=+0.31-0.03-4.75+2.58=-1.89

M_{j_7} =-0.37+0.00x0.785-4.48x1.04+ 3.72x0.692=-0.37+0.00-4.66+2.57=-2.46

M_{k_7} =+0.37+0.00x0.785+4.48x1.04- 3.72x0.692=+0.37+0.00+4.66-2.57=+2.46

M_{k_8} =+0.31-0.02x0.785+5.52x1.04- 6.38x0.692=+0.31-0.01+5.74-4.36=+1.68

M_{1_8}=-0.53+0.05x0.785-2.05x1.04- 1.52x0.692=-0.53+0.04-2.13-1.09=-3.67

M_{1_5}=+0.83+0.11x0.785-1.82x1.04- 1.58x0.692=+0.83+0.09-1.89-1.09=-2.06

M_{m_8}=+0.22-0.03x0.785-3.47x1.04+ 7.82x0.692=+0.22-0.02-3.61+5.40=+1.99

M_{m_D}=+0.11+0.00x0.785-1.73x1.04+ 8.91x0.692=+0.11+0.00-1.80+6.16=+4.47

CALCULO DE REACCIONES Y DIAGRAMAS.

BARRA (a)

$$1.10 = \Sigma MF_1$$

$$1.10 = -4.00 + 4X_{a_A}$$

$$X_{a_A} = \frac{4.00+1.10}{4} = \frac{5.10}{4} = 1.28 \text{ TON}$$

$$X_{a_1} = X_{a_A} = 1.28 \text{ TON}$$

$$M_{isos} = 0$$

BARRA (b)

$$+0.82 = \Sigma MF_2$$

$$+0.82 = -0.35 + 3X_{b_1}$$

$$X_{b_1} = \frac{0.82+0.35}{3} = \frac{1.17}{3} = 0.39 \text{ TON}$$

$$X_{b_2} = X_{b_1} = 0.39 \text{ TON}$$

$$M_{isos} = 0$$

BARRA (c)

$$-1.85 = \Sigma MF_3$$

$$-1.85 = +0.82+4Y_{c_2} - \frac{0.6 \times 4^2}{2}$$

$$- 1.85 = + 0.82 + 4Y_{c_2} - 4.80$$

$$Y_{c_2} = \frac{4.80-1.85-0.82}{4} = \frac{2.13}{4} = 0.53 \text{ TON}$$

$$Y_{c_3} = -0.53+0.6x4 = -0.53+2.40 = 1.87 \text{ TON}$$

$$M_{isos} = \frac{wl^2}{8} = \frac{0.6 \times 4^2}{8} = 1.20 \text{ T x M}$$

$$M_{\mathfrak{C}} = \frac{0.82 - 1.85}{2} + 1.20 = - 0.51 + 1.20 = + 0.69$$

BARRA (d)

$$+ 1.85 = \Sigma MF_3$$

$$+ 1.85 = - 1.50 + 3X_{d_4}$$

$$X_{d_4} = \frac{1.85+1.50}{3} = \frac{3.35}{3} = 1.11 \text{ TON}$$

$$X_{d_3} = X_{d_4} = 1.11 \text{ TON}$$

$$M_{isos} = 0$$

BARRA (e)

$$- 3.41 = \Sigma MF_4$$

$$- 3.41 = +1.45 + 4Y_{e_1} - \frac{0.9 \times 4^2}{2}$$

$$- 3.41 = - 1.45 + 4 Y_{e_1} - 7.20$$

$$Y_{e_1} = \frac{7.20+3.41-1.45}{4} = \frac{2.34}{4} = 0.59 \text{ TON}$$

$$Y_{e_4} = -0.59+0.9 \times 4 = 3.01 \text{ TON}$$

$$M_{isos} = \frac{wl^2}{8} = \frac{0.9 \times 4^2}{8} = 1.80$$

$$M_{\mathfrak{C}} = \frac{1.45 - 3.50}{2} + 1.80 = - 1.03 + 1.80 = 0.77 \text{ T x M}$$

BARRA (f)

$$3.88 = \Sigma MF_4$$

$$3.88 = -6.54 + 4X_{f_B}$$

$$X_{f_B} = \frac{6.54+3.88}{4} = \frac{10.42}{4} = 2.61 \text{ TON}$$

$$X_{f_4} = X_{f_B} = 2.61 \text{ TON}$$

$$M_{isos} = 0$$

BARRA (g)

$$-4.29 = MF_5$$

$$-4.29 = 1.88 + 5Y_{g_4} - \frac{0.6 \times 5^2}{2}$$

$$-4.29 = 1.88 + 5Y_{g_4} - 7.50$$

$$Y_{g_4} = \frac{7.50 - 4.29 - 188}{5} = \frac{1.33}{5} = 0.27 \text{ TON}$$

$$Y_{g_5} = -0.27 + 0.6 \times 5 = 2.73 \text{ TON}$$

$$M_{isos} = \frac{wl^2}{8} = \frac{0.6 \times 5^2}{8} = 1.88 \text{ T x M}$$

$$M_{℄} = \frac{1.88 - 4.29}{2} + 1.88 = -1.21 + 1.88 = 0.67 \text{ T x M}$$

BARRA (h)

$$4.35 = \Sigma MF_5$$

$$4.35 = -7.04 + 4X_{h_C}$$

$$X_{h_C} = \frac{7.04+4.35}{4} = \frac{11.39}{4} = 2.85 \text{ TON}$$

$$X_{h_5} = X_{h_C} = 2.85 \text{ TON}$$

$$M_{isos} = 0$$

BARRA (i)

$$1.89 = \Sigma MF_6$$

$$1.89 = -2.00 + 4X_{i_5}$$

$$X_{i_5} = \frac{2.00+1.89}{4} = \frac{3.89}{4} = 0.97 \text{ TON}$$

$$X_{i_6} = X_{i_5} = 0.97 \text{ TON}$$

$$M_{isos} = 0$$

BARRA (j)

$$-2.46 = \Sigma MF_7$$

$$-2.46 = +1.89 - 3Y_{i_6} - \frac{0.6 \times 3^2}{2}$$

$$-2.46 = +1.89 - 3Y_{i_6} - 2.70$$

$$Y_{i_6} = \frac{-2.70+2.46+1.89}{3} = \frac{1.65}{3} = 0.55 \text{ TON}$$

$$Y_{i_7} = 0.55 + 0.6 \times 3 = 2.35 \text{ TON}$$

$$M_{isos} = \frac{wl^2}{8} = \frac{0.6 \times 3^2}{8} = 0.68 \text{ T x M}$$

$$M_{℄} = \frac{1.89 - 2.46}{2} + 0.68 = -0.29 + 0.68 = 0.39 \text{ T x M}$$

BARRA (k)

$$2.46 = \Sigma MF_7$$

$$2.46 = -1.68 + 4X_{k_B}$$

$$X_{k_B} = \frac{2.46+1.68}{4} = \frac{4.14}{4} = 1.03 \text{ TON}$$

$$X_{k_7} = X_{k_B} = 1.03 \text{ TON}$$

$$M_{isos} = 0$$

BARRA (l)

$$-3.67 = \Sigma MF_8$$

$$-3.67 = +2.06 - 3Y_{1_5} - \frac{0.9 \times 3^2}{2}$$

$$-3.67 = +2.06 - 3Y_{1_5} - 4.05$$

$$Y_{1_5} = \frac{3.67 + 2.06 - 4.05}{3} = \frac{1.68}{3} = 0.56 \text{ TON}$$

$$Y_{1_B} = 0.56 + 0.9 \times 3 = 3.26 \text{ TON}$$

$$M_{isos} = \frac{wl^2}{8} = \frac{0.9 \times 3^2}{8} = 1.01 \text{ T x M}$$

$$M_{\mathbb{C}} = \frac{2.06 - 3.67}{2} + 1.01 = -0.81 + 1.01 = 0.20 \text{ T x M}$$

BARRA (m)

$$1.99 = \Sigma MF_8$$

$$1.99 = -4.47 + 4X_{m_D}$$

$$X_{m_D} = \frac{4.47 + 1.99}{4} = \frac{6.46}{4} = 1.62 \text{ TON}$$

$$X_{m_8} = X_{m_D} = 1.62 \text{ TON}$$

CALCULO DE REACCIONES Y DIAGRAMAS

$X_A = X_{a_A} = -1.28$ TON

$Y_A = Y_{c_2} + Y_{e_1} = 0.53 + 0.59 = 1.12$ TON

$X_B = X_{f_B} = -2.61$ TON

$Y_B = Y_{c_3} + Y_{e_4} + Y_{f_4} = 1.87 + 3.01 + 0.27 = 5.15$ TON

$X_C = X_{h_C} = -2.85$ TON

$Y_C = Y_{g_5} + Y_{e_5} + Y_{i_6} = 2.73 + 0.56 + 0.55 = 3.84$ TON

$X_D = X_{m_D} = -1.62$ TON

$Y_D = Y_{1_7} + Y_{1_8} = 2.35 + 3.26 = 5.61$ TON

DIAGRAMA DE FUERZAS CORTANTES

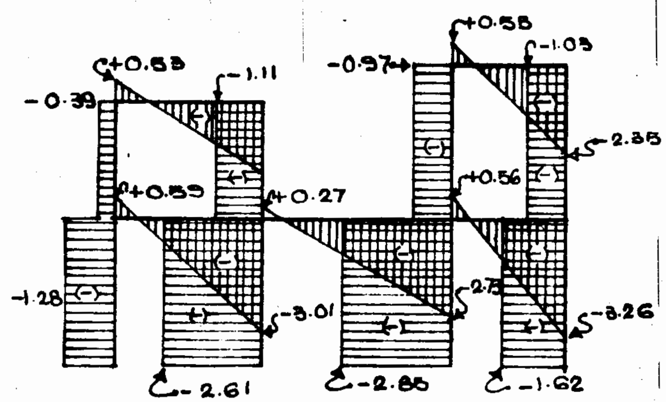
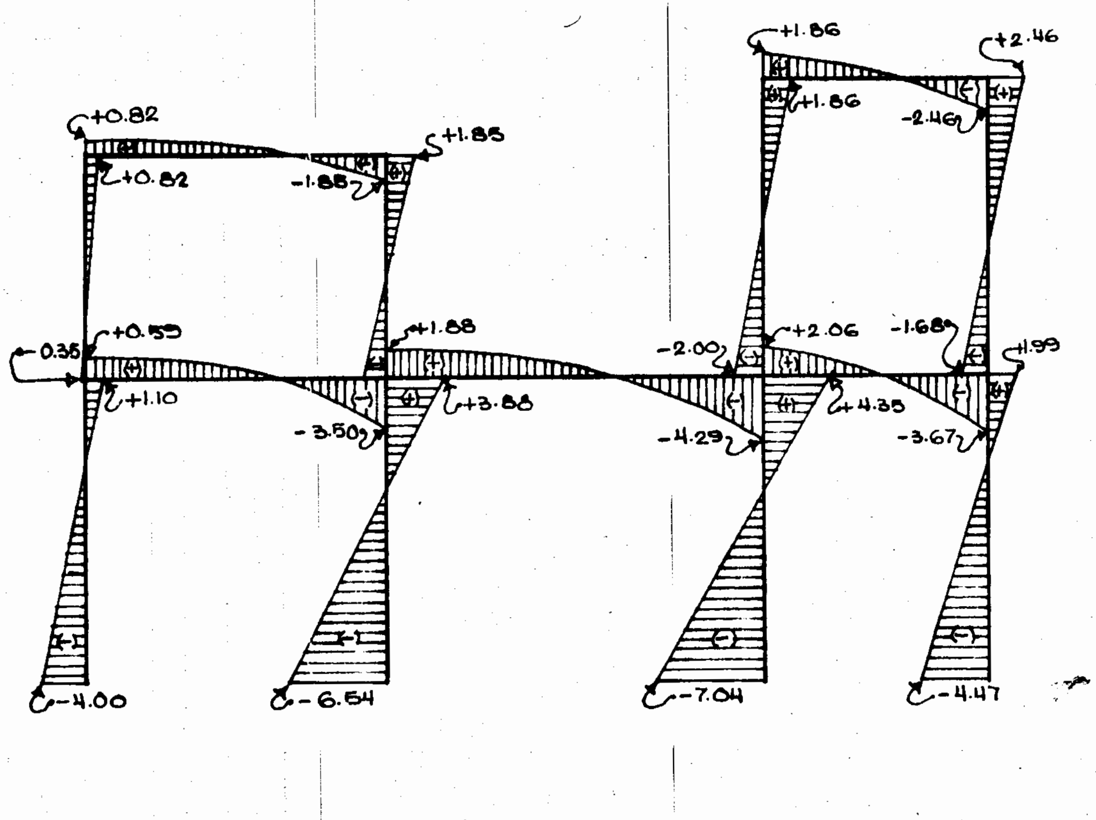
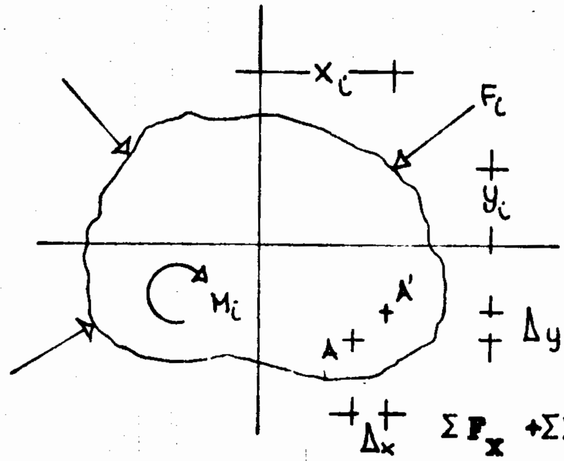


DIAGRAMA DE MOMENTOS FLEXIONANTES



PRINCIPIO DE LOS DESPLAZAMIENTOS VIRTUALES



SI EL CUERPO ESTA EN EQUILIBRIO.

$\Sigma F_x = 0$

$\Sigma F_y = 0$

$\Sigma M = 0$

$$\Sigma F_x + \Sigma F_y + \Sigma M = 0 \qquad (1)$$

SI EXISTE UN PEQUEÑO DESPLAZAMIENTO (SUPUESTO) $\overline{AA'}$ DESCOMPUESTO EN Δ_x y Δ_y , EL TRABAJO EFECTUADO POR LAS FUERZAS "F" SERA:

$$\Delta_x \Sigma F_x + \Delta_y \Sigma F_y$$

COMO $\Sigma F_x = 0$ Y $\Sigma F_y = 0$

$$\Delta_x \Sigma F_x + \Delta_y \Sigma F_y = 0$$

SI EL CUERPO GIRA UN PEQUEÑO ANGULO (SUPUESTO) " α " EL TRABAJO EFECTUADO POR LAS FUERZAS Y PARES SERA:

$$\Sigma M \alpha + \Sigma F_x Y\alpha + \Sigma F_y X\alpha = \alpha(\Sigma M + \Sigma F_x Y + \Sigma F_y X)$$

SUSTITUYENDO (1)

$\Sigma M\alpha + \Sigma F_x Y \alpha + \Sigma F_y X\alpha = 0$ POR LO TANTO:

EN UN CUERPO RIGIDO MANTENIDO EN EQUILIBRIO POR UN SISTEMA DE FUERZAS Y/O PARES, EL TRABAJO VIRTUAL EFECTUADO POR EL SISTEMA DURANTE UN DESPLAZAMIENTO VIRTUAL ES IGUAL A CERO.

PRINCIPIO DEL TRABAJO VIRTUAL:

EL TRABAJO VIRTUAL EXTERNO dwe DE LAS FUERZAS APLICADAS A UNA SECCION DE UN CUERPO SE DISIPARA EN TRABAJO DE ROTACION VIRTUAL Y TRABAJO DE TRASLACION VIRTUAL dw_{rt} Y ENERGIA DE DEFORMACION INTERNA dw_i

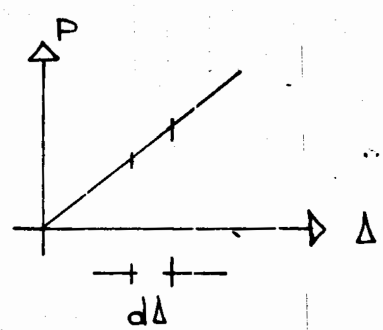
$$dw_e = dw_{rt} + dw_i$$

PERO POR EL PRINCIPIO DE LOS DESPLAZAMIENTOS VIRTUALES $dw_{rt} = 0$ POR LO TANTO:

$$dw_e = dw_i$$

INTEGRANDO A TODO EL CUERPO $W_e = W_i$

SI UN CUERPO MANTENIDO EN EQUILIBRIO POR UN SISTEMA DE FUERZAS SE SUJETA A UNA DEFORMACION VIRTUAL, EL TRABAJO VIRTUAL DEL SISTEMA DE FUERZAS EXTERNO ES IGUAL AL TRABAJO VIRTUAL DEL SISTEMA DE FUERZAS INTERNO.



$$d\zeta = P\,d\Delta$$

$$\zeta = \int_0^{\Delta} P\,d\Delta$$

EN EL RANGO ELASTICO "P" ES PROPORCIONAL A Δ

$$P = K\Delta \qquad (2)$$

$$\zeta = \int_0^{\Delta} K\Delta\,d\Delta$$

$$\zeta = \frac{1}{2} K\Delta^2$$

DE (2)

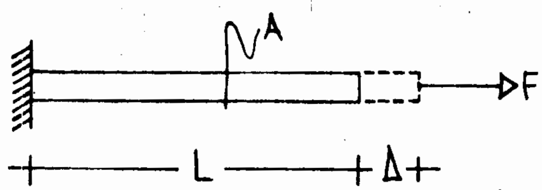
$$\Delta = \frac{P}{K}$$

$$d\Delta = d\,\frac{P}{K}$$

$$\zeta = \int_0^P P\, d\,\frac{P}{K}$$

$$\zeta = \frac{1}{2}\,\frac{P^2}{K}$$

ENERGIA DE DEFORMACION " U " PARA CARGA AXIAL (TRABAJO INTERNO).



$$\Delta = \frac{FL}{AE}$$

$$d\Delta = \frac{F}{AE}\, dx$$

$$U = \int_0^L F\, d\Delta$$

$$U = \int_0^L F\,\frac{F}{AE}\, dx$$

$$U = \frac{1}{2}\,\frac{F^2}{AE}\, L \quad \text{(SI F Y A CTES.)}$$

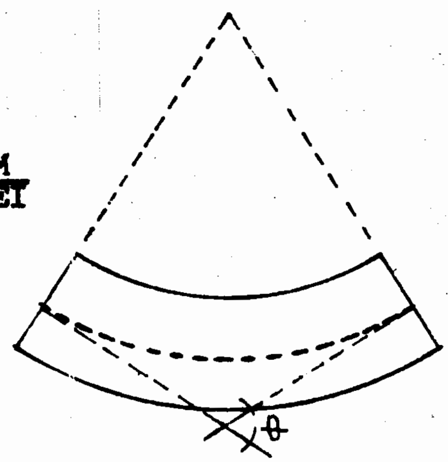
ENERGIA DE DEFORMACION "U" PARA FLEXION

$$\frac{d^2Y}{dx^2} = \frac{M}{EI}$$

$$\frac{d}{dx}\left(\frac{dY}{dx}\right) = \frac{M}{EI}$$

$$\frac{d\theta}{dx} = \frac{M}{EI}$$

$$d\theta = \frac{M}{EI}\, dx$$



$$dU = M\,d\theta$$

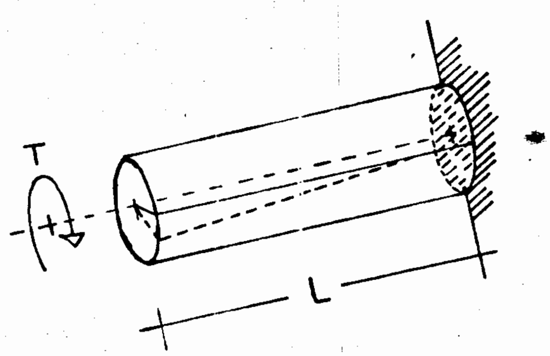
$$dU = M\,\frac{M}{EI}\, dx$$

$$U = \int_0^L M\,\frac{M}{EI}\, dx$$

$$U = \frac{1}{2}\,\frac{M^2}{EI}\, L \quad \text{(SI M Y A SON CTES.)}$$

EI = RIGIDEZ DE MOMENTO FLEXIONANTE.

ENERGIA DE DEFORMACION "U" PARA TORSION.



$$d\phi = \frac{T}{GJ}\, dx$$

$$dU = T\, d\phi$$

$$dU = T\,\frac{T}{GJ}\, dx$$

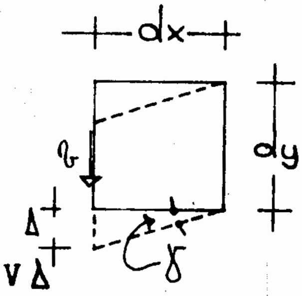
$$U = \int_0^L T\,\frac{T}{GJ}\, dx$$

$$U = \frac{1}{2}\,\frac{T^2}{GJ}\, L \quad \text{(SI T Y A SON CONSTANTES)}$$

GJ = RIGIDEZ DE MOMENTO TORSIONANTE

$$G = \frac{E}{2(1+u)}$$

ENERGIA DE DEFORMACION PARA ESFUERZO CORTANTE



$$\Delta = \gamma \; dx \qquad v = \frac{VQ}{Ib}$$

$$\gamma = \frac{v}{G} \qquad K_s = \frac{AQ}{Ib}$$

$$\Delta = \frac{v}{G} dx \qquad v = K_s \frac{V}{A}$$

$$dU = V\Delta$$

$$dU = V \frac{v}{G} dx$$

$$dU = V \frac{K_s V}{AG} dx$$

$$U = \int_0^L K_s \frac{V^2}{AG} dx$$

$$U = \frac{K_s}{2} \frac{V^2}{AG} L \quad \text{(SI V Y A SON CONSTANTES)}$$

	CARGA AXIAL	FLEXION	TORSION	CORTANTE
U	$\frac{1}{2} P\Delta$	$\frac{1}{2} M\theta$	$\frac{1}{2} T\phi$	$\frac{1}{2} V\Delta$
dU	$\frac{1}{2} P d\Delta$	$\frac{1}{2} M d\theta$	$\frac{1}{2} T d\phi$	$\frac{1}{2} V d\Delta$
DIFERENCIAL DE DEFORMACION	$\frac{P}{AE} dx$	$\frac{M}{EI} dx$	$\frac{T}{GJ} dx$	$\frac{K_s V}{AG} dx$
dU	$\frac{1}{2} \frac{P^2}{AE} dx$	$\frac{1}{2} \frac{M^2}{EI} dx$	$\frac{1}{2} \frac{T^2}{GJ} dx$	$\frac{1}{2} \frac{K_s V^2}{AG} dx$
U	$\int_0^L \frac{P^2}{2AE} dx$	$\int_0^L \frac{M^2}{2EI} dx$	$\int_0^L \frac{T^2}{2GJ} dx$	$\int_0^L \frac{K_s V^2}{2AG} dx$

LA ENERGIA INTERNA TOTAL ES IGUAL A LA SUMA DE LAS ENERGIAS -- INTERNAS DEBIDA A CADA TIPO DE SOLICITACION.

$$U = \int_0^L \frac{P^2}{2AE} dx + \int_0^L \frac{M^2}{2EI} dx + \int_0^L \frac{K_T T^2}{2EI} dx + \int_0^L \frac{K_s V^2}{2AG} dx$$

TEOREMA DE BETTI

SEA UN CUERPO EN EQUILIBRIO SUJETO A LA ACCION DE DOS FUERZAS EXTERIORES P_A y P_B Y LAS REACCIONES EN LOS APOYOS.

EL CUERPO PRESENTARA LAS MISMAS DEFORMACIONES FINALES SI SE APLICA PRIMERO LA FUERZA "P_A" Y POSTERIORMENTE "P_B" O PRIMERO "P_B" Y POSTERIORMENTE "P_A" POR LO TANTO EL TRABAJO DE ESTAS FUERZAS ES EL MISMO SIN IMPORTAR SU ORDEN DE APLICACION.

SI SE APLICA PRIMERO "P_A" EL TRABAJO DESARROLLADO SERA:

$$\zeta_A = \frac{1}{2} P_a \Delta_A$$

SI AHORA SE APLICA "P_B"

$$\zeta_B = \frac{1}{2} P_B \Delta_B + \frac{1}{2} P_A \Delta_{AB}$$

Δ_{AB} = DEFORMACION EN EL PUNTO DE APLICACION DE LA FUERZA "P_A" CON LA DIRECCION DE "P_A" PRODUCIDO POR LA FUERZA "P_B" POR LO TANTO EL TRABAJO TOTAL SERA:

$$\zeta = \frac{1}{2} P_A \Delta_A + \frac{1}{2} P_B \Delta_B + \frac{1}{2} P_A \Delta_{AB} \qquad (1)$$

SI SE INVIERTE EL ORDEN DE APLICACION DE LAS FUERZAS O SEA SI SE APLICA PRIMERO "P_B"

$$\zeta_B = \frac{1}{2} P_B \Delta_B$$

SI AHORA SE APLICA "P_A"

$$\zeta_A = \frac{1}{2} P_A \Delta_A + \frac{1}{2} P_B \Delta_{BA}$$

Δ_{BA} = DEFORMACION EN EL PUNTO DE APLICACION DE LA FUERZA "P_B" CON LA DIRECCION DE "P_B" PRODUCIDO POR LA FUERZA "P_A"

EL TRABAJO TOTAL SERA:

$$\zeta = \frac{1}{2} P_B \Delta_B + \frac{1}{2} P_A \Delta_A + \frac{1}{2} P_B \Delta_{BA} \qquad (2)$$

IGUALANDO (1) y (2)

$P_A \Delta_{AB} = P_B \Delta_{Ba}$ TEOREMA DE BETTI

TEOREMAS DE CASTIGLIANO

SEA UN CUERPO EN EQUILIBRIO SUJETO A LA ACCION DE DOS FUERZAS "P" Y "Q" Y LAS REACCIONES EN LOS APOYOS.

$$U = \frac{1}{2} P\Delta_P + \frac{1}{2} Q\Delta_Q \qquad (1)$$

SI SE AGREGA A "P" UNA PEQUEÑA CARGA ADICIONAL d_P SE CAUSARAN DEFORAMCIONES ADICIONALES AL CUERPO $d\Delta_P$ $d\Delta_Q$
EL INCREMENTO RESULTANTE DE LA ENERGIA DE DEFORAMCION SERA --- IGUAL A LA DEFLEXION PRODUCIDA POR LA CARGA PROMEDIO ACTUANTE PARA CADA PUNTO RESPECTIVAMENTE.

$$dU = \frac{P + (P + dP)}{2} d\Delta_P + Q\, d\Delta_Q$$

DESPRECIANDO EL PRODUCTO DE DOS DIFERENCIALES

$$dU = \frac{1}{2} Pd\Delta P + \frac{1}{2} Pd\Delta_P + Qd\,\Delta_Q$$

$$dU = Pd\,\Delta_P + Qd\Delta_Q \qquad (2)$$

SI SE APLICAN SIMULTANEAMENTE $(P + \Delta_P)$ Y Q LA ENERGIA DE DEFORMACION SERA:

$$U' = \frac{1}{2} (P + dP) (\Delta_P + d\Delta_P) + \frac{Q}{2} (\Delta_Q + d\Delta_Q)$$

DESPRECIANDO EL PRODUCTO DE DOS DIFERENCIALES

$$U' = \frac{1}{2} P\Delta_P + \frac{1}{2} Pd\Delta_P + \frac{1}{2} dP\Delta_P + \frac{1}{2} Q\Delta_Q + \frac{1}{2} Qd\Delta_Q \qquad (3)$$

PERO $dU = U' - U$ SUSTITUYENDO 2 Y 3

$$d\,U = \frac{1}{2} Pd\Delta_P + \frac{1}{2} dP\Delta_P + \frac{1}{2} Qd\Delta_Q \qquad (4)$$

SUSTITUYENDO (2) EN (4)

$$dU = \frac{1}{2}\ dP\Delta_P + \frac{1}{2}\ dU$$

$$\frac{1}{2}\ dU = \frac{1}{2}\ dP\ \Delta_P$$

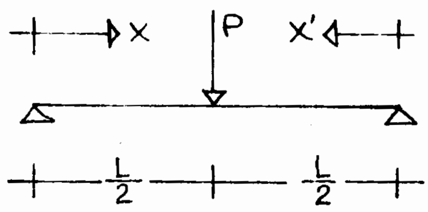
$$\frac{dU}{dP} = \Delta_P$$ PRIMER TEOREMA DE CASTIGLIANO

PARA FLEXIOS SE TIENE:

$$\Delta = \frac{\partial}{\partial P}\int_0^L \frac{M^2}{2EI}\,dx \quad \text{O SEA} \quad \Delta = \int_0^L \frac{1}{EI} M \frac{\partial M}{\partial P}\,dx$$

PARA CARGA AXIAL:

$$\Delta = \frac{\partial}{\partial P}\int_0^L \frac{S^2}{2AE}\,dx \quad \text{O SEA} \quad \Delta = \int_0^L \frac{1}{AE} S \frac{\partial S}{\partial P}\,dx$$

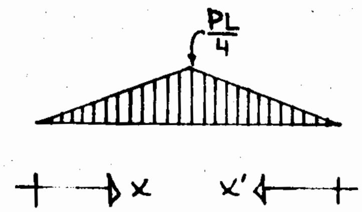


CALCULAR LA DEFORMACION DE LA VIGA EN EL CENTRO DEL CLARO.

ECUACIONES DE MOMENTO FLEXIONANTE

$0 \leq x \leq \frac{L}{2}$ $\qquad M_x = \frac{P}{2} x$

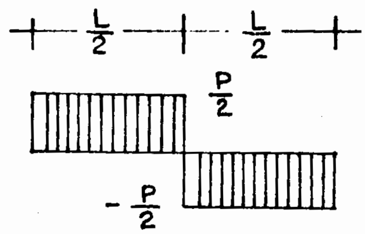
$0 \leq x' \leq \frac{L}{2}$ $\qquad M_{x'} = \frac{P}{2} x'$



ECUACIONES DE ESFUERZO CORTANTE

$0 \leq x \leq \frac{L}{2}$ $\qquad V_x = \frac{P}{2}$

$0 \leq x' \leq \frac{L}{2}$ $\qquad V_x = -\frac{P}{2}$



ENERGIA DE DEFORMACION:

$$U = \int_0^{\frac{L}{2}} \frac{(\frac{P}{2} x)^2 \, dx}{2EI} + \int_0^{\frac{L}{2}} \frac{(\frac{P}{2} x')^2 \, dx'}{2EI} + \int_0^{\frac{L}{2}} \frac{Ks \, (\frac{P}{2})^2 \, dx}{2AG}$$

$$+ \int_0^{\frac{L}{2}} \frac{Ks \, (-\frac{P}{2})^2 \, dx'}{2\,AG}$$

$$U = \int_0^{\frac{L}{2}} \frac{(\frac{P}{2} x)^2}{EI} \, dx + \int_0^{\frac{L}{2}} \frac{Ks \, (\frac{P}{2})^2}{AG} \, dx$$

$$\Delta = \frac{\partial U}{\partial P} = \int_0^{\frac{L}{2}} \frac{2 \, \frac{P}{2} \, x \, \frac{x}{2}}{EI} \, dx + \int_0^{\frac{L}{2}} \frac{Ks \; 2 \, \frac{P}{2} \, \frac{1}{2}}{2AG} \, dx$$

$$\Delta = \frac{\partial U}{\partial P} = \int_0^{\frac{L}{2}} \frac{Px^2}{2EI} \, dx + \int_0^{\frac{L}{2}} \frac{KsP}{2AG} \, dx$$

$$\Delta = \frac{\partial U}{\partial P} = \frac{P}{2EI} \left[\frac{x^3}{3} \right]_0^{\frac{L}{2}} + \frac{KsP}{2AG} \left[x \right]_0^{\frac{L}{2}}$$

$$\Delta = \frac{\partial U}{\partial P} = \frac{PL^3}{48EI} + \frac{KsPL}{4AG}$$

$$I = \frac{bh^3}{12} \qquad A = bh$$

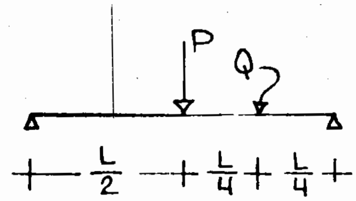
SUSTITUYENDO:

$$\Delta = \frac{12\ PL^3}{48Ebh^3} + \frac{3.9PL}{4Ebh}$$

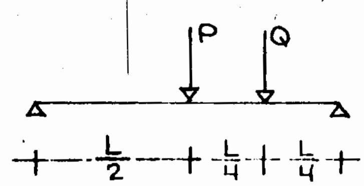
$$\Delta = \frac{PL}{4Ebh} \left[\left(\frac{L}{h} \right)^2 + 3.9 \right]$$

SE DEDUCE QUE EN VIGAS ORDINARIAS LA DEFORMACION POR CORTANTE ES RELATIVAMENTE BAJA COMPARANDOLA CON LA DEFORMACION POR FLEXIONANTE PERO EN VIGAS CORTAS LA DEFORMACION POR CORTANTE PUEDE SER DEL ORDEN DE LA DEFORMACION POR FLEXIONANTE.

ENCONTRAR LA DEFORMACION DEL PUNTO "Q" DEBIDA A LA FLEXION DE LA BARRA.



PARA CONOCER LA DEFORMACION DE LA BARRA EN EL PUNTO Q SE CONSIDERA UNA FUERZA "FICTICIA" APLICADA EN EL PUNTO "Q" DE LA SIGUIENTE MANERA:



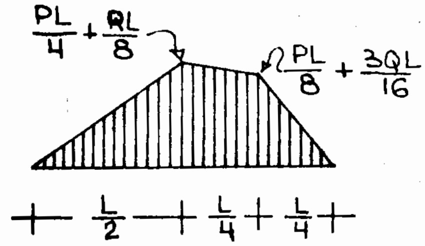
ECUACIONES DE MOMENTO FLEXIONANTE

$0 \leq x \leq \frac{L}{2}$ $\qquad M_x = \left(\frac{P}{2} + \frac{Q}{4} \right) x$

$\frac{L}{2} \leq x \leq \frac{3L}{4}$ $\qquad M_x = \left(\frac{P}{2} + \frac{Q}{4} \right) x - P \left(x - \frac{L}{2} \right)$

$0 \leq x' \leq \frac{L}{4}$ $\qquad M_{x'} = \left(\frac{P}{2} + \frac{3Q}{4} \right) x'$

DIAGRAMA DE MOMENTOS FLEXIONANTES



ENERGIA DE DEFORMACION

$$U = \int_0^{\frac{L}{2}} \frac{\left[\left(\frac{P}{2} + \frac{Q}{4}\right) x\right]^2}{2EI} dx + \int_{\frac{L}{2}}^{\frac{3L}{4}} \frac{\left[\left(\frac{P}{2} + \frac{Q}{4}\right) x - P\left(x - \frac{L}{2}\right)\right]^2}{2\ EI} dx$$

$$+ \int_0^{\frac{L}{4}} \frac{\left[\left(\frac{P}{2} + \frac{3Q}{4}\right) x'\right]^2}{2EI} dx'$$

$$\Delta = \frac{\partial U}{\partial Q} = \int_0^{\frac{L}{2}} \frac{2\left(\frac{P}{2} + \frac{Q}{4}\right) x \ \frac{x}{4}}{2EI} dx + \int_{\frac{L}{2}}^{\frac{3L}{4}} \frac{2\left[\left(\frac{P}{2} + \frac{Q}{4}\right)x - P\left(x - \frac{L}{2}\right)\right]\frac{x}{4}}{2EI} dx$$

$$+ \int_0^{\frac{L}{4}} \frac{2\left(\frac{P}{2} + \frac{3Q}{4}\right) x' \ \frac{3}{4} x'}{2EI} dx'$$

PERO "Q" ES FICTICIA O SEA Q = O POR LO TANTO

$$\Delta = \int_0^{\frac{L}{2}} \frac{Px^2}{8EI} dx + \int_{\frac{L}{2}}^{\frac{3L}{4}} \frac{\frac{Px^2}{8} - \frac{Px}{4}\left(x - \frac{L}{2}\right)}{EI} dx + \int_0^{\frac{L}{4}} \frac{\frac{3}{8} Px'^2}{EI} dx'$$

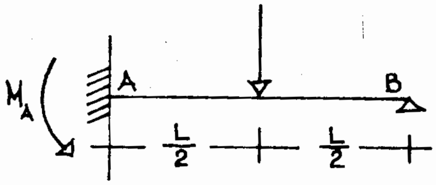
$$\Delta = \frac{1}{EI} \left\{ \frac{1}{3} \frac{P}{8} \left[x^3\right]_0^{\frac{L}{2}} + \frac{1}{3} \frac{P}{8} \left[x^3\right]_{\frac{L}{2}}^{\frac{3L}{4}} - \frac{1}{3} \frac{P}{4} \left[x^3\right]_{\frac{L}{2}}^{\frac{3L}{4}} + \frac{1}{2} \frac{PL}{8} \left[x^2\right]_{\frac{L}{2}}^{\frac{3L}{4}} \right.$$

$$\left. + \frac{1}{3} \frac{3P}{8} \left[x'^3\right]_0^{\frac{L}{4}} \right\}$$

$$\Delta = \frac{1}{EI} \left[\frac{P}{24} \frac{L^3}{8} - \frac{P}{24} \frac{27L^3}{64} + \frac{P}{24} \frac{L^3}{8} + \frac{PL}{16} \frac{9L^2}{16} - \frac{PL}{16} \frac{L^2}{4} + \frac{PL^3}{512} \right]$$

$$\Delta = \frac{PL^3}{EI} \left(\frac{1}{192} - \frac{27}{1536} + \frac{1}{192} + \frac{9}{256} - \frac{1}{64} + \frac{1}{512} \right)$$

$$\Delta = \frac{22}{1536} \frac{PL^3}{EI}$$



CALCULAR LAS REACCIONES EN LOS APOYOS

PARA RESOLVER PROBLEMAS HIPERESTATICOS DE 1ER. ORDEN COMO EL DE ESTE EJEMPLO SE CONSIDERA UNA DE LAS REACCIONES COMO FUERZA " REDUNDANTE."

LA DEFORMACION PRODUCIDA POR LA FUERZA REDUNDANTE SERA IGUAL A CERO.

ESCOGIENDO A R_B COMO REDUNDANTE : (RESULTA UN CANTILIVER)

$$\frac{\partial U}{\partial R_B} = 0$$

ECUACIONES DE MOMENTO FLEXIONANTE.

CONSIDERANDO LA SUPERPOSICION DE CAUSAS Y EFECTOS

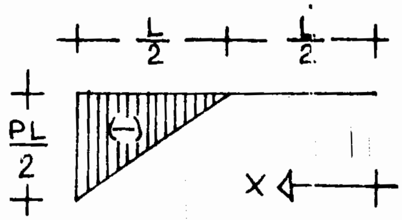


DIAGRAMA DE MOMENTOS FLEXIONANTES DEBIDO A LA FUERZA "P"

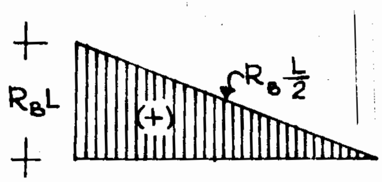


DIAGRAMA DE MOMENTOS FLEXIONANTES DEBIDO A LA FUERZA "R_B"

ECUACIONES DE MOMENTO FLEXIONANTE

$0 \leq x \leq \frac{L}{2}$ $\qquad M_x = R_B X$

$\frac{L}{2} \leq x \leq L$ $\qquad M_x = R_B X - P (X - \frac{L}{2})$

ENERGIA DE DEFORMACION

$$U = \int_0^{\frac{L}{2}} \frac{(R_B X)^2}{2EI} dx + \int_{\frac{L}{2}}^{L} \frac{\left[R_B X - P\left(X - \frac{L}{2}\right)\right]^2}{2EI} dx$$

$$\frac{\partial U}{\partial R_B} = \int_0^{\frac{L}{2}} \frac{(R_B X) X}{EI} dx + \int_{\frac{L}{2}}^{L} \frac{\left[R_B X - P\left(X - \frac{L}{2}\right)\right] X}{EI} dx$$

$$\frac{\partial U}{\partial R_B} = \frac{1}{3} \frac{R_B}{EI} \left[X^3\right]_0^{\frac{L}{2}} + \frac{1}{3} \frac{R_B}{EI} \left[X^3\right]_{\frac{L}{2}}^{L} - \frac{1}{3} \frac{P}{EI} \left[X^3\right]_{\frac{L}{2}}^{L} + \frac{1}{2} \frac{PL}{2} \left[X^2\right]_{\frac{L}{2}}^{L}$$

$$\frac{\partial U}{\partial R_B} = \frac{1}{3} \frac{R_B}{EI} \frac{L^3}{8} + \frac{1}{3} \frac{R_B}{EI} L^3 - \frac{1}{3} \frac{R_B}{EI} \frac{L^3}{8} - \frac{1}{3} \frac{P}{EI} L^3 + \frac{1}{3} \frac{P}{EI} \frac{L^3}{8}$$

$$+ \frac{1}{2} \frac{P}{2EI} L^3 - \frac{1}{2} \frac{P}{2EI} \frac{L^3}{4}$$

$$\frac{\partial U}{\partial R_B} = \frac{R_B}{3EI} L^3 - \frac{5P}{48EI} L^3$$

PERO $\frac{\partial U}{\partial R_B} = 0$ POR LO TANTO

$$\frac{R_B}{3EI} L^3 - \frac{5P}{48EI} L^3 = 0$$

$$\frac{R_B}{3} - \frac{5P}{48} = 0$$

$$R_B = 3 \frac{5P}{48} = \frac{5}{16} P$$

$$R_A = P - \frac{5}{16} P = \frac{11}{16} P$$

$$M_A = P \frac{L}{2} - \frac{5}{16} PL = \frac{3}{16} PL$$

$$R_A = \frac{11}{16} P \qquad R_B = \frac{5}{16} P \qquad M_A = \frac{3}{16} PL$$

CONSIDERANDO COMO REDUNDANTE AL MOMENTO (RESULTA UNA VIGA SIMPLEMENTE APOYADA) $\frac{\partial U}{\partial M} = 0$

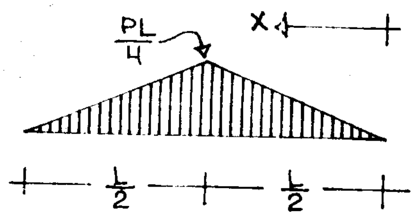


DIAGRAMA DE MOMENTOS FLEXIONANTES DEBIDO A LA FUERZA "P"

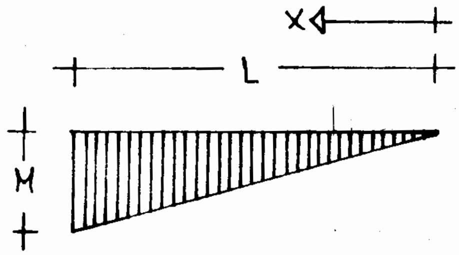


DIAGRAMA DE MOMENTOS FLEXIONANTES DEBIDO AL MOMENTO "M"

ECUACIONES DE MOMENTO FLEXIONANTE

$0 \leq x \leq \frac{L}{2}$ $\qquad M_x = -\frac{M}{L}x + \frac{P}{2}x$

$\frac{L}{2} \leq x \leq L$ $\qquad M_x = -\frac{M}{L}x + \frac{P}{2}x - P\left(x - \frac{L}{2}\right) =$

$$M_x = -\frac{M}{L}x - \frac{P}{2}x + \frac{PL}{2}$$

ENERGIA DE DEFORMACION

$$U = \int_0^{\frac{L}{2}} \frac{\left(-\frac{M}{L}x + \frac{P}{2}x\right)^2}{2EI}\,dx + \int_{\frac{L}{2}}^{L} \frac{\left(-\frac{M}{L}x - \frac{P}{2}x + \frac{PL}{2}\right)^2}{2EI}\,dx$$

$$\frac{\partial U}{\partial M} = \int_0^{\frac{L}{2}} \frac{\left(-\frac{M}{L}x + \frac{P}{2}x\right)\left(-\frac{x}{2}\right)}{EI}\,dx + \int_{\frac{L}{2}}^{L} \frac{\left(-\frac{M}{L}x - \frac{P}{2}x + \frac{PL}{2}\right)\left(-\frac{x}{2}\right)}{}\,dx$$

$$\frac{\partial U}{\partial M} = \frac{1}{3}\frac{M}{2LEI}\left[x^3\right]_0^{\frac{L}{2}} - \frac{1}{3}\frac{P}{4EI}\left[x^3\right]_0^{\frac{L}{2}} + \frac{1}{3}\frac{M}{2L}\left[x^3\right]_{\frac{L}{2}}^{L} + \frac{1}{3}\frac{P}{4}\left[x^3\right]_{\frac{L}{2}}^{L} - \frac{1}{2}\frac{PL}{4}\left[x^2\right]_{\frac{L}{2}}^{L}$$

$$\frac{\partial U}{\partial M} = \frac{1}{3}\frac{M}{2LEI}\frac{L^3}{8} - \frac{1}{3}\frac{P}{4EI}\frac{L^3}{8} + \frac{1}{3}\frac{M}{2LEI}L^3 - \frac{1}{3}\frac{M}{2LEI}\frac{L^3}{8} + \frac{1}{3}\frac{P}{4EI}L^3 - \frac{1}{3}$$

$$\frac{P}{4EI}\frac{L^3}{8} - \frac{1}{2}\frac{P}{4EI}L^3 + \frac{1}{2}\frac{P}{4EI}\frac{L^3}{4}$$

$$\frac{\partial U}{\partial M} = -\frac{P}{96EI}L^3 + \frac{M}{6LEI}L^3 + \frac{P}{12EI}L^3 - \frac{P}{96EI}L^3 - \frac{P}{8EI}L^3 + \frac{P}{32EI}L^3$$

$$\frac{\partial U}{\partial M} = -\frac{3P}{96EI}L^3 + \frac{M}{6LEI}L^3$$

PERO $\frac{\partial U}{\partial M} = 0$ POR LO TANTO

$$-\frac{3P}{96EI}L^3 + \frac{M}{6LEI}L^3 = 0$$

$$-\frac{3P}{96}+\frac{M}{6L}=0$$

$$M=\frac{6L(3P)}{96}=\frac{3}{16}PL$$

$$\sum M_B=0$$

$$-\frac{3}{16}PL+R_AL-P\frac{L}{2}=0$$

$$R_A=\frac{1}{L}\left(\frac{3}{16}PL+\frac{1}{2}PL\right)=\frac{11}{16}P$$

$$R_B=P-\frac{11}{16}P=\frac{5}{16}P$$

$$R_A=\frac{11}{16}P \qquad R_B=\frac{5}{16}P \qquad M=\frac{3}{16}PL$$

$$M_{\mathrm{\not{c}}}=-\frac{3}{16}PL+\frac{5}{16}P\left(\frac{L}{2}\right)=\frac{5}{32}PL$$

DIAGRAMA DE MOMENTOS FLEXIONANTES

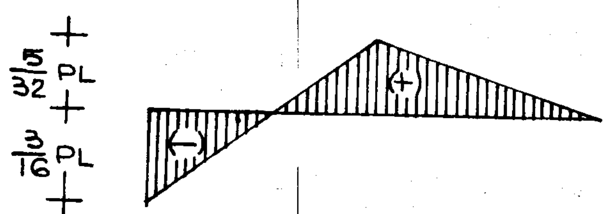
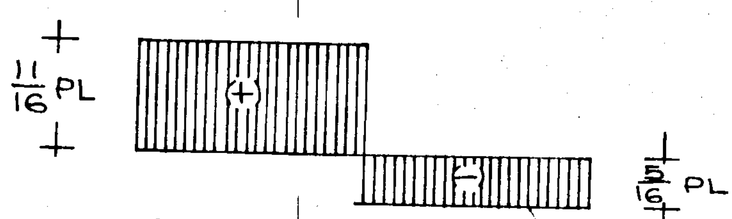
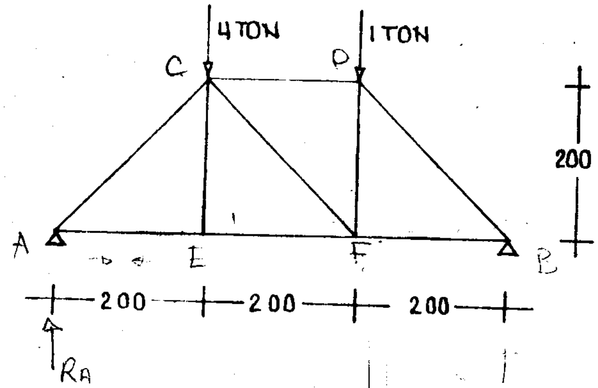


DIAGRAMA DE FUERZAS CORTANTES





CALCULAR LA DEFORMACION VERTICAL DEL NUDO "E"

AREA DE LAS SECCIONES - DE LAS BARRAS.

$E = 2100 \dfrac{TON}{CM^2}$

$A_{AC} = A_{DB} = A_{CD} = 10$ CM.

$A_{AE} = A_{EF} = A_{FB} = A_{CB} = 7$ CM.

$A_{CE} = A_{DF} = 9$ CM.

PARA TODA LA ARMADURA

$\Sigma M_B = 0$

$6\ R_A - 4 \times 4 - 1 \times 2 = 0$

$R_A = \dfrac{16 + 2}{6} = 3$

$\Delta = \dfrac{P_i L_i}{A_i E} \dfrac{\partial U}{\partial P}$

$\Sigma F_Y = 0$

$3 - 4 - 1 + R_B = 0$

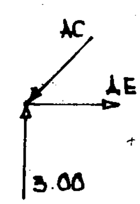
$R_B = 2$

NUDO A

$\Sigma F_Y = 0$

$AC \cos 45^o = 3$

$AC = \dfrac{3}{.7071} = 4.24$



$\Sigma F_X = 0$

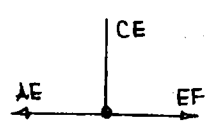
$AE = AC \cos 45^o$

$AE = 4.24 \times .7071 = 3$

NUDO E

$\Sigma F_Y = 0$

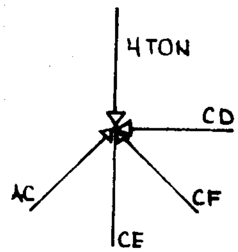
$CE = 0$



$\Sigma F_X = 0$

$AE = EF = 3$

NUDO C



$\Sigma F_Y = 0$

$4 = AC \cos 45^\circ + CF \cos 45^\circ$

$4 = 3 + .7071\ CF$

$CF = \frac{1}{.7071} = 1.41$

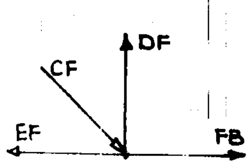
$\Sigma F_X = 0$

$AC \cos 45^\circ = CF \cos 45^\circ + CD$

$3 = 1 + CD$

$CD = 2$

NUDO F



$\Sigma F_Y = 0$

$DF = CF \cos 45^\circ$

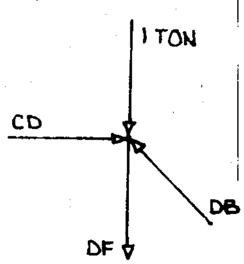
$DF = 1.41 \times .7071 = 1.0$

$\Sigma F_X = 0$

$F_B = EF - CF \cos 45^\circ$

$F_B = 3 - 1.41 \times .7071 = 3 - 1 = 2$

NUDO D



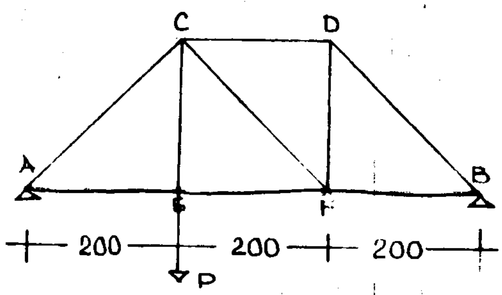
$\Sigma F_X = 0$

$DB \cos 45^\circ = CD$

$DB \times .7071 = 2$

$DB = \frac{2}{.7071} = 2.83$

CALCULO DE U



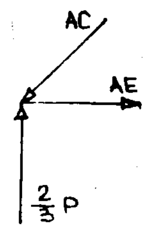
PARA TODA LA ARMADURA

$\Sigma M_B = 0$

$6\ R_A - 4p = 0$

$R_A = \frac{4}{6}\ P = \frac{2}{3}\ P$

NUDO A



$\Sigma F_Y = 0$

$AC\ \cos 45^\circ = \frac{2}{3}\ P$

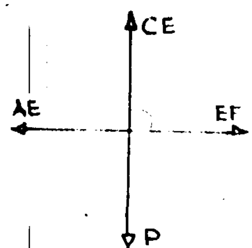
$AC = \frac{2}{3 \times .7071}\ P = 0.94\ P$

$\Sigma F_X = 0$

$AE = AC\ \cos 45^\circ$

$AE = 0.94P \times .7071 = 0.67\ P$

NUDO E



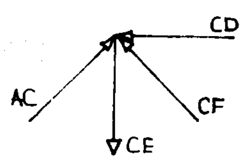
$\Sigma F_X = 0$

$EF = AE = 0.67\ P$

$\Sigma F_Y = 0$

$CE = P$

NUDO C



$\Sigma F_Y = 0$

$CF\ \cos 45^\circ = CE - AC\ \cos 45^\circ$

$CF = \frac{P - 0.94P \times .7071}{.7071}$

$CF = \frac{P - 0.67\ P}{.7071} = 0.47\ P$

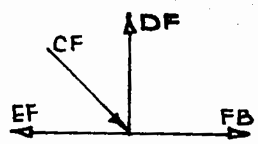
$\Sigma F_X = 0$

$CD = AC\ \cos 45^\circ - CF\ \cos 45^\circ$

$CD = 0.94\ P \times .7071 - 0.47\ P \times .7071$

$CD = 0.67\ P - 0.33\ P = 0.34\ P$

NUDO F



$\Sigma F_Y = 0$

$DF = CF \cos 45^\circ$

$DF = 0.47\ P \times .7071 = 0.33\ P$

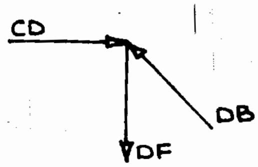
$\Sigma F_X = 0$

$F_B = EF - CF \cos 45^\circ$

$F_B = 0.37\ P - 0.47\ P \times .7071$

$F_B = 0.47\ P - 0.33\ P = 0.34\ P$

NUDO D



$\Sigma F_X = 0$

$DB \cos 45^\circ = CD$

$DB = \frac{0.34\ P}{.7071} = 0.47\ P$

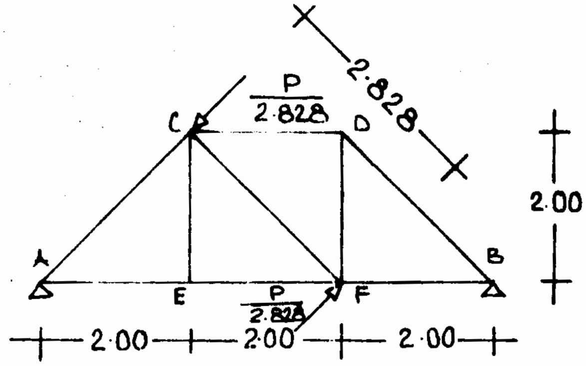
BARRA	L_i (CM)	A(CM^2)	P_i (TON)	$\frac{P_i L_i}{A_i}(\frac{TON}{CM})$	U(TON)	$\frac{U}{P}$	$\frac{P_i L_i}{A_i} \cdot \frac{U}{P}(\frac{TON}{CM})$
AC	283	10	-4.24	- 120	-0.94P	-0.94	+ 112.80
CD	200	10	-2.00	- 40	-0.34P	-0.34	+ 13.60
DB	283	10	-2.83	- 80	-0.47P	-0.47	+ 37.60
AE	200	7	+3.00	+ 86	+0.67P	+0.67	+ 57.62
EF	200	7	+3.00	+ 86	+0.67P	+0.67	+ 57.62
FB	200	7	+2.00	+ 57	+0.34P	+0.34	+ 19.38
CE	200	9	0.00	00	+1.00P	+1.00	0.00
DF	200	9	+1.00	+ 22	+0.33P	+0.33	+ 7.26
CF	283	7	-1.41	- 57	-0.47P	-0.47	+ 26.79
						Σ	+ 332.67

$\Delta = \frac{332.67}{2100} = 0.158$ CM.

CALCULAR EL GIRO EN EL MIEMBRO CF

SE PROCEDERA DE LA SIGUIENTE MANERA:

SE APLICAN FUERZAS EN LOS EXTREMOS DE LA BARRA CF COMO SIGUE:



LAS FUERZAS APLICADAS RESULTAN DE DIVIDIR "P" ENTRE LA LONGITUD DE LA BARRA CF Y LE PRODUCEN UN MOMENTO:

$$\frac{P}{2.828} \times 2.828 = P$$

PARA TODA LA ESTRUCTURA.

$$\Sigma M_B = 0$$

$$6R_A - \frac{P}{2.828} \cos 45^\circ \times 4 - \frac{P}{2.828} \cos 45^\circ \times 2 + \frac{P}{2.828} \cos 45^\circ \times 2 = 0$$

$$R_A = \frac{1}{6} \; \frac{4 \times .7071}{2.828} P = \frac{P}{6}$$

$$\Sigma F_Y = 0$$

$$\frac{P}{6} - \frac{P}{2.828} \cos 45^\circ + \frac{P}{2.828} \cos 45^\circ - R_B = 0$$

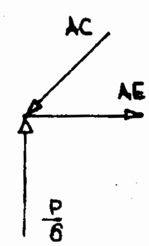
$$R_B = \frac{P}{6}$$

NUDO A

$$\Sigma F_Y = 0$$

$$AC \cos 45^\circ = \frac{P}{6}$$

$$AC = \frac{P}{6x.7071} = 0.236 \; P$$



$$\Sigma F_X = 0$$

$$AE = AC \cos 45^\circ$$

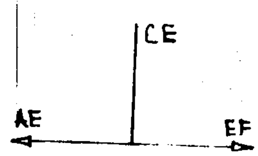
$$AE = \frac{P}{6x.7071} \times .7071 = \frac{P}{6}$$

$$AE = 0.167 \; P$$

NUDO E

$\Sigma F_Y = 0$

$CE = 0$



$\Sigma F_X = 0$

$EF = AE$

$EF = \frac{P}{6} = 0.167\ P$

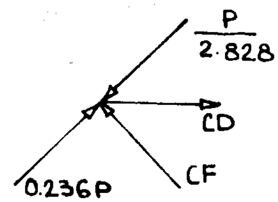
NUDO C

$\Sigma F_Y = 0$

$CD \cos 45^\circ = \frac{P}{2.828} - 0.236\ P$

$CD \cos 45^\circ = 0.354\ P - 0.236\ P$

$CD = \frac{0.118}{.7071}\ P = 0.167\ P$

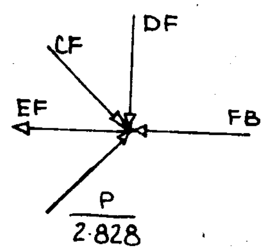


$\Sigma F_X = 0$

$CF = CD \cos 45^\circ$

$CF = \frac{0.118}{.7071}\ P \times .7071 = 0.118\ P$

NUDO F



$\Sigma F_Y = 0$

$DF = -\ CF \cos 45^\circ + \frac{P}{2.828} \cos 45^\circ$

$DF = -\ 0.118 \times .7071\ P + 0.350\ P = (-0.083 + 0.250)\ P = 0.167\ P$

$\Sigma F_X = 0$

$F_B = CF \cos 45^\circ + \frac{P}{2.828} \cos 45^\circ - EF$

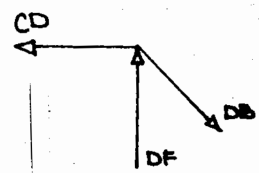
$F_B = +\ 0.083\ P + 0.250\ P - 0.67\ P = 0.166\ P$

NUDO D

$\Sigma F_X = 0$

$DB \cos 45^o = CD$

$DB = \frac{0.167}{.7071} P = 0.236 P$

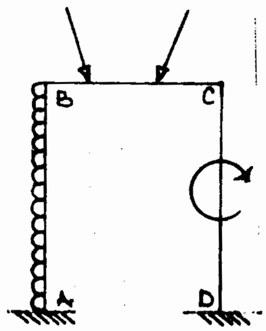


BARRA	L_i(CM)	A(CM^2)	P_i(TON)	$\frac{P_iL_i}{A_i}(\frac{TON}{CM})$	U(TON)	$\frac{U}{P}(\frac{1}{CM})$	$\frac{P_iL_i}{A_i} \frac{U}{P}(\frac{TON}{CM^2})$
AC	283	10	-4.24	- 120	-0.236P	-0.236	+ 28.32
CD	200	10	-2.00	- 40	+0.167P	-0.167	- 6.40
DB	283	10	-2.83	- 80	+0.236P	-0.236	- 18.88
AE	200	7	+3.00	+ 86	+0.167P	+0.167	+ 14.36
EF	200	7	+3.00	+ 86	+0.167P	+0.167	+ 14.36
FB	200	7	+2.00	+ 57	-0.166P	-0.166	- 9.52
CE	200	9	0.00	00	0.000	0.000	0.00
DF	200	9	+1.00	+ 22	-0.167P	-0.167	- 3.67
CF	283	7	-1.41	- 57	-0.118P	-0.118	+ 6.72
						$\sum$	= 25.29

$\theta = \frac{25.29}{2100} = 1.20 \times 10^{-2}$ RAD.

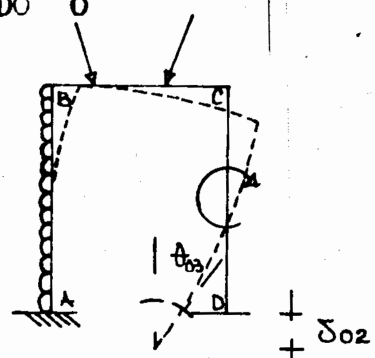
METODO DE LAS FUERZAS

SEA LA SIGUIENTE ESTRUCTURA



POR EL PRINCIPIO DE LA SUPERPOSICION DE CAUSAS Y EFECTOS SE PUEDE DESCOMPONER EN LOS SIGUIENTES ESTADOS:

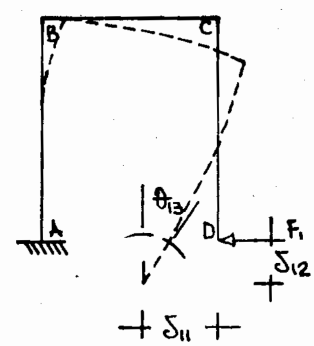
ESTADO 0



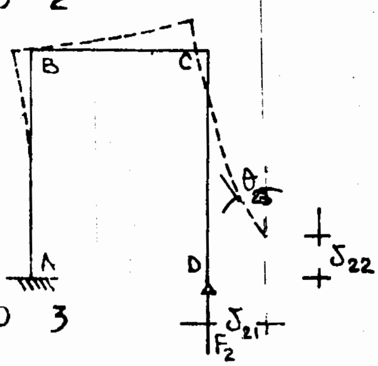
SE ESCOGE UNA ESTRUCTURA ISOSTATICA A LA QUE SE LE APLICA EL SISTEMA DE CARGAS DE LA ESTRUCTURA ORIGINAL.

ESTADO 1

A LA ESTRUCTURA ISOSTATICA ESCOGIDA SE APLICA UNA FUERZA F_1 UNITARIA.



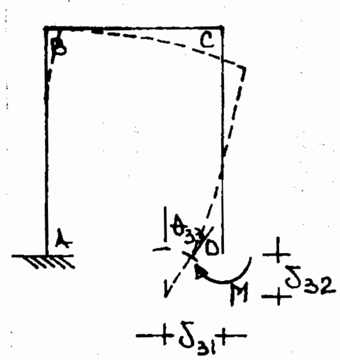
ESTADO 2



A LA ESTRUCTURA ISOSTATICA ESCOGIDA SE LE APLICA UNA FUERZA F_2 UNITARIA

ESTADO 3

A LA ESTRUCTURA ISOSTATICA ESCOGIDA SE LE APLICA UN MOMENTO M UNITARIO.



ECUACIONES DE COMPATIBILIDAD DE DEFORMACIONES:

$\delta_{HD} = 0$

$\delta_{VD} = 0$

$\theta_D = 0$

SUPERPONIENDO LOS ESTADOS Y DADO QUE LAS DEFORMACIONES SON PRO PORCIONALES A LAS FUERZAS QUE LAS PRODUCEN

$$- \delta_{01} + X_1 \delta_{11} + X_2 \delta_{21} + X_3 \delta_{31} = \delta_{HD} = 0$$

$$- \delta_{02} + X_1 \delta_{12} + X_2 \delta_{22} + X_3 \delta_{32} = \delta_{VD} = 0$$

$$- \theta_{03} + X_1 \theta_{13} + X_2 \theta_{23} + X_3 \theta_{33} = \theta_D = 0$$

O SEA

$$X_1 \delta_{11} + X_2 \delta_{21} + X_3 \delta_{31} = \delta_{01}$$

$$X_1 \delta_{12} + X_2 \delta_{22} + X_3 \delta_{32} = \delta_{02}$$

$$X_1 \theta_{13} + X_2 \theta_{23} + X_3 \theta_{33} = \theta_{03}$$

MATRICIALMENTE

$$\begin{bmatrix} \delta_{11} & \delta_{21} & \delta_{31} \\ \delta_{12} & \delta_{22} & \delta_{32} \\ \theta_{13} & \theta_{23} & \theta_{33} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} \delta_{01} \\ \delta_{02} \\ \theta_{03} \end{bmatrix}$$

SI A LA MATRIZ DE LAS REACCIONES LA LLAMAMOS X

$$X = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}$$

A LA MATRIZ DE FLEXIBILIDAD LA LLAMAMOS F

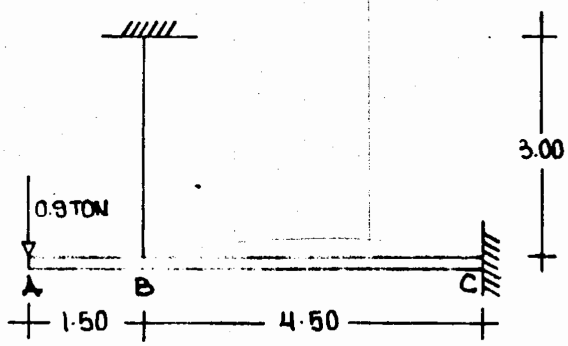
$$F = \begin{bmatrix} \delta_{11} & \delta_{21} & \delta_{31} \\ \delta_{12} & \delta_{22} & \delta_{32} \\ \theta_{13} & \theta_{23} & \theta_{33} \end{bmatrix}$$

A LA MATRIZ DE LAS DEFORMACIONES PRODUCIDAS POR EL SISTEMA DE FUERZAS LA LLAMAMOS

$FX = \Delta$

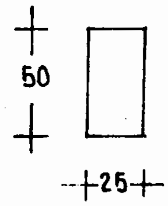
DESPEJANDO $X = F^{-1} \Delta$

CALCULAR LA TENSION EN EL CABLE



SECCION DE LA VIGA

$E_Y = 70 \frac{TON}{CM^2}$



SECCION DEL CABLE

$A_C = 1.3 \ CM^2$

$E_C = 2100 \frac{TON}{CM^2}$

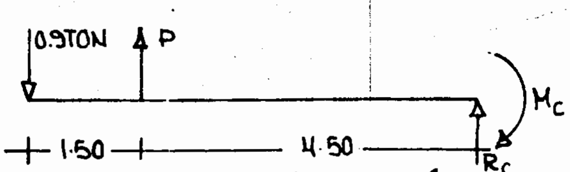
DEFORMACION DEL CABLE = Δ_C

DEFORMACION DE LA VIGA EN EL PUNTO "B" = Δ_V

ECUACION DE COMPATIBILIDAD DE DEFORMACIONES:

$\Delta_C + \Delta_V = 0$

CALCULO DE LA DEFORMACION VERTICAL DE LA VIGA EN EL PUNTO "B"



$0 \leq x \leq 1.5 \qquad M_x = 0.9\,x$

$1.5 \leq x \leq 6.0 \qquad M_x = 0.9x - P(x-1.5)$

$$U = \int_0^{1.5} \frac{(0.9\,x)^2}{2EI}\,dx + \int_{1.5}^{6} \frac{\left[0.9x - P(x-1.5)\right]^2}{2EI}\,dx$$

$$\Delta_V = \frac{\partial U}{\partial P} = \int_{1.5}^{6} \frac{\left[0.9x - P(x1.5)\right]\ (x-1.5)}{EI}\,dx$$

$$\Delta_V = \int_{1.5}^{6} \frac{0.9x^2 - 1.35x - Px^2 + 3\,Px - 2.25\,P}{EI}\,dx$$

$$\Delta_V = \left[\frac{0.9}{3}\left[x^3\right]_{1.5}^{6} - \frac{1.35}{2}\left[x^2\right]_{1.5}^{6} - \frac{P}{3}\left[x^3\right]_{1.5}^{6} + \frac{3P}{2}\left[x^2\right]_{1.5}^{6} - 2.25\,P\left[x\right]_{1.5}^{6}\right]$$

$$\Delta_V = \Big[0.3\,(216 - 3.38) - 0.63\,(36 - 2.25) - \frac{P}{3}\,(216 - 3.38)$$

$$+ \frac{3P}{2}\,(36 - 2.25) - 2.25\,P\,(6 - 1.5)\Big]\frac{1}{EI}$$

$$\Delta_V = \left[64 - 21 - 71\ P + 51\ P - 10\ P\right] \frac{1}{EI} \quad \left[43 - 30\ P\right]\frac{1}{EI}$$

$$I = \frac{0.25 \times 0.50^3}{12} = 0.0026$$

$$EI = 700\ 000 \times 0.0026 = 1820$$

$$\Delta_Y = 0.0236 - 0.0164\ P$$

DEFORMACION DEL CABLE

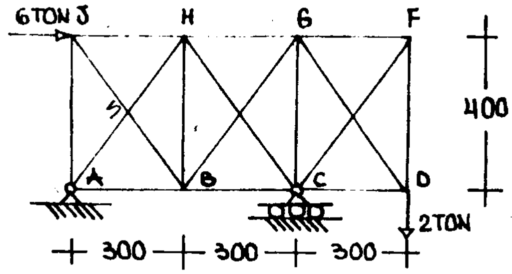
$$\Delta_C = \frac{PL_C}{A_C E_C} = \frac{3P}{1.3 \times 2100} = 0.0011\ P$$

SUSTITUYENDO EN LA ECUACION DE COMPATIBILIDAD DE DEFORMACIONES

$$0.0011\ P + 0.0236 - 0.0164\ P = 0$$

$$0.0236 - 0.0153\ P = 0$$

$$P = \frac{0.0236}{0.0153} = 1.54 \quad TON$$



ENCONTRAR LAS FUERZAS EN LAS BARRAS DE LA ARMADURA

E Y A CONSTANTES

EQUILIBRIO EXTERNO

$\Sigma M_A = 0$

$6 \times 4 + 2 \times 9 - R_C \times 6 = 0$

$R_C = \frac{24 + 18}{6} = \frac{42}{6} = 7$

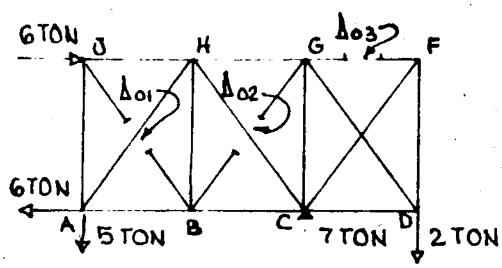
$\Sigma F_X = 0$

$R_{AX} = 6$ TON

$\Sigma F_Y = 0$

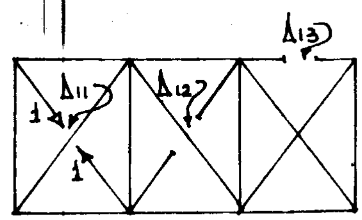
$R_{AY} = 7 - 2 = 5$

ESCOGIENDO COMO REDUNDANTES A LAS FUERZAS BJ, BG, Y FG SE TENDRA LA SIGUIENTE ESTRUCTURA ISOSTATICA

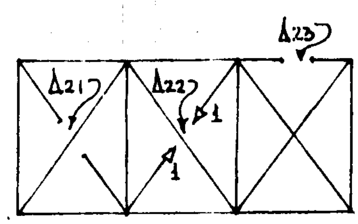


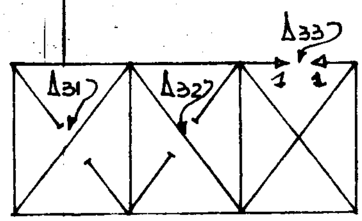
ESTADO 0

ESTADO 1



ESTADO 2





ESTADO 3

ECUACIONES DE COMPATIBILIDAD DE DEFORMACIONES.

$$\Delta_{01} + X_1 \Delta_{11} + X_2 \Delta_{21} + X_3 \Delta_{31} = 0$$

$$\Delta_{02} + X_1 \Delta_{12} + X_2 \Delta_{22} + X_3 \Delta_{32} = 0$$

$$\Delta_{03} + X_1 \Delta_{13} + X_2 \Delta_{23} + X_3 \Delta_{33} = 0$$

$$\Delta_{01} = \frac{S_{oi} \; U_{1i}}{A_i E_i} L_i \qquad \Delta_{11} = \frac{U_{1i} U_{1i}}{A_i E_i} L_i$$

$$\Delta_{02} = \frac{S_{oi} \; U_{2i}}{A_i E_i} L_i \qquad \Delta_{12} = \frac{U_{1i} U_{2i}}{A_i E_i} L_i$$

$$\Delta_{03} = \frac{S_{oi} \; U_{3i}}{A_i E_i} L_i \qquad \Delta_{13} = \frac{U_{1i} U_{3i}}{A_i E_i} L_i$$

$$\Delta_{21} = \frac{U_{2i} \; U_{1i}}{A_i E_i} L_i \qquad \Delta_{31} = \frac{U_{3i} U_{1i}}{A_i E_i} L_i$$

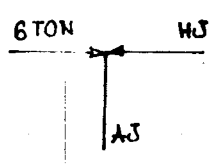
$$\Delta_{22} = \frac{U_{2i} \; U_{2i}}{A_i E_i} L_i \qquad \Delta_{32} = \frac{U_{3i} U_{2i}}{A_i E_i} L_i$$

$$\Delta_{23} = \frac{U_{2i} U_{3i}}{A_i E_i} L_i \qquad \Delta_{33} = \frac{U_{3i} U_{3i}}{A_i E_i} L_i$$

DETERMINACION DE LAS FUERZAS EN LAS BARRAS

ESTADC O

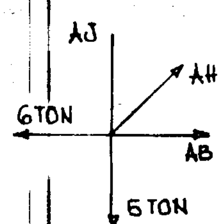
NUDO "J"



$F_Y = 0 \qquad F_X = 0$

$A_J = 0 \qquad H_J = 6.0$

NUDO "A"



$\Sigma F_Y = 0$

$AH \frac{4}{5} = 5$

$AH = \frac{5}{4} 5 = 6.25$

$\Sigma F_X = 0$

$AB = 6-6.25 \frac{3}{5}$

$AB = 2.25$

NUDO "B"

$\Sigma F_Y = 0$

$BH = 0$

$\Sigma F_X = 0$

$BC = 2.25$

NUDO "H"

$\Sigma F_Y = 0$

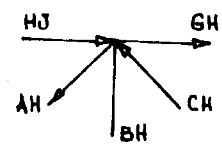
$CH \frac{4}{5} = 6.25 \frac{4}{5}$

$CH = 6.25$

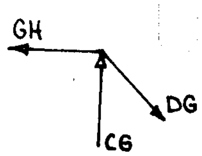
$\Sigma F_X = 0$

$GH = 6.25 \frac{3}{5} + 6.25 \frac{3}{5} - 6$

$GH = 7.5 - 6 = 1.5$



NUDO "G"



$\Sigma F_X = 0$

$D_G \frac{3}{5} = 1.5$

$D_G = 1.5 \times \frac{5}{3} = 2.5$

$\Sigma F_Y = 0$

$CG = 2.5 \frac{4}{5}$

$CG = 2.0$

NUDO "C"

$\Sigma F_Y = 0$

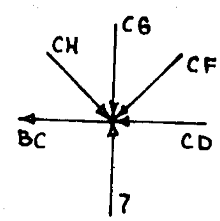
$CF \frac{4}{5} = 7-6.25\frac{4}{5}-2$

$CF = 0$

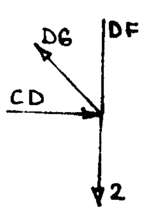
$\Sigma F_X = 0$

$CD = 6.25 \frac{3}{5} - 2.25$

$CD = 1.5$



NUDO "D"



$\Sigma F_Y = 0$

$DF = 2 - 2.5 \frac{4}{5}$

$DF = 0$

$\Sigma F_X = 0$

$2.5 \frac{3}{5} = 1.5$

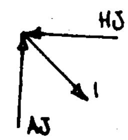
ESTADO 1

NUDO "J"

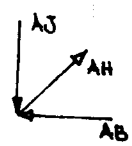
$\Sigma F_Y = 0$ $\qquad$ $\Sigma F_X = 0$

$AJ = 1.0 \times \frac{4}{5}$ $\qquad$ $HJ = 1.0 \times \frac{3}{5}$

$AJ = 0.8$ $\qquad$ $HJ = 0.6$



NUDO "A"



$\Sigma F_Y = 0$ $\qquad$ $\Sigma F_X = 0$

$AH \frac{4}{5} = 0.8$ $\qquad$ $AB = 1.0 \frac{3}{5}$

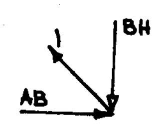
$AH = 0.8 \frac{5}{4} = 1.0$ $\qquad$ $AB = 0.6$

NUDO "B"

$\Sigma F_Y = 0$ $\qquad$ $\Sigma F_X = 0$

$BH = 1.0 \times \frac{4}{5}$ $\qquad$ $1.0 \frac{3}{5} = 0.6$

$BH = 0.8$



ESTADO 2

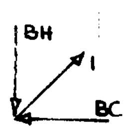
NUDO "J"

$\Sigma HJ = 0$ $\qquad$ $\Sigma BJ = 0$

NUDO "A"

$\Sigma AH = 0$ $\qquad$ $\Sigma AB = 0$

NUDO "B"



$\Sigma F_Y = 0$ $\qquad$ $\Sigma F_X = 0$

$BH = 1.0 \times \frac{4}{5}$ $\qquad$ $BC = 1.0 \times \frac{3}{5}$

$BH = 0.8$ $\qquad$ $BC = 0.6$

NUDO "H"

$\Sigma F_Y = 0$

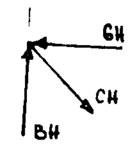
$CH \frac{4}{5} = 0.8$

$CH = 0.8 \frac{5}{4} = 1.0$

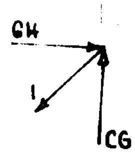
$\Sigma F_X = 0$

$GH = 1.0 \times \frac{3}{5}$

$GH = 0.6$



NUDO "G"



$\Sigma F_Y = 0$

$CG = 1.0 \times \frac{4}{5}$

$CG = 0.8$

ESTADO 3

NUDO "F"

$\Sigma F_Y = 0$

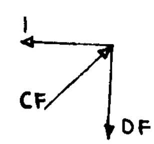
$DF = \frac{5}{3} \times \frac{4}{5}$

$DF = 1.33$

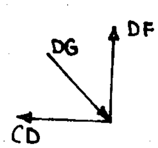
$\Sigma F_X = 0$

$CF \frac{3}{5} = 1.0$

$CF = 1.0 \times \frac{5}{3} = 1.67$



NUDO "D"



$\Sigma F_Y = 0$

$DG \frac{4}{5} = \frac{4}{3}$

$DG = \frac{5}{4} \frac{4}{3} = 1.67$

$\Sigma F_X = 0$

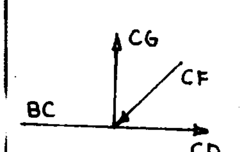
$CD = \frac{5}{5}$

$CD = 1.0$

NUDO "C"

$\Sigma F_Y = 0$

$CG = \frac{5}{3} \times \frac{4}{5} = 1.33$



BARRA	L	S_o	U_1	U_2	U_3	S_oU_1L	S_oU_2L	S_oU_3L	U_1U_1L	U_2U_2L	U_3U_3L
A B	3	+2.25	-0.6	0.0	0.0	-4.05			+1.08		
H J	3	-6.00	-0.6	0.0	0.0	+10.80			+1.08		
A J	4	0.00	-0.8	0.0	0.0				+2.56		
A H	5	+6.25	+1.0	0.0	0.0	+31.25			+5.00		
B J	5	0.00	+1.0	0.0	0.0				+5.00		
B H	4	0.00	-0.8	-0.8	0.0				+2.56	+2.56	
B C	3	+2.25	0.0	-0.6	0.0		-4.05			+1.08	
G H	3	+1.50	0.0	-0.6	0.0		-2.70			+1.08	
B G	5	0.00	0.0	+1.0	0.0					+5.00	
C H	5	-6.25	0.0	+1.0	0.0		-31.25			+5.00	
C G	4	-2.00	0.0	-0.8	+1.3		+6.40	-10.64		+2.56	+7.08
C D	3	-1.50	0.0	0.0	+1.0			-4.50			+3.00
D F	4	0.00	0.0	0.0	+1.3						+7.08
F G	3	0.00	0.0	0.0	+1.0						+3.00
C F	5	0.00	0.0	0.0	-1.7						13.95
D G	5	+2.50	0.0	0.0	-1.7			-20.88			13.95
					$\sum$	+38.00	-31.60	-36.02	+17.28	+17.28	+48.06

$\Sigma U_1U_1L = +0.8x0.8x0.4 = +2.56$

$\Sigma U_1U_3L = 0$

$\Sigma U_2U_3L = -0.8x1.33x4.0 = -4.26$

SUSTITUYENDO EN LAS ECUACIONES DE COMPATIBILIDAD DE DEFORMACIONES

$$+38.00 + 17.28\,X_1 + 2.56\,X_2 + 0.00\,X_3 = 0$$

$$-31.60 + 2.56\,X_1 + 17.28\,X_2 - 4.26\,X_3 = 0$$

$$-36.02 + 0.00\,X_1 - 4.26\,X_2 + 48.06\,X_3 = 0$$

MATRICIALMENTE

$$\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} +17.28 & +2.56 & 0.00 \\ +2.56 & +17.28 & -4.26 \\ 0.00 & -4.26 & +48.06 \end{bmatrix}^{-1} \begin{bmatrix} -38.00 \\ +31.60 \\ +36.02 \end{bmatrix}$$

$$F^{-1} = \begin{bmatrix} +0.0592 & -0.0090 & -0.0008 \\ -0.0090 & +0.0605 & +0.0054 \\ -0.0008 & +0.0054 & +0.0213 \end{bmatrix}$$

$$\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} +0.0592 & -0.0090 & -0.0008 \\ -0.0090 & +0.0605 & +0.0054 \\ -0.0008 & +0.0054 & +0.0213 \end{bmatrix} \begin{bmatrix} -38.00 \\ +31.60 \\ +36.02 \end{bmatrix}$$

EFECTUANDO EL PRODUCTO DE MATRICES

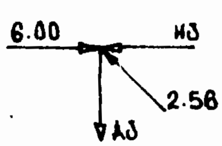
$$X_1 = -2.2496 - 0.2844 - 0.0288 = -2.5628$$

$$X_2 = +0.3420 + 1.9118 + 0.1945 = +2.4483$$

$$X_3 = +0.0304 + 0.1706 + 0.7672 = +0.9682$$

DETERMINACION DE LAS FUERZAS FINALES

NUDO "J"



$\Sigma F_Y = 0$

$$AJ = 2.56\,\frac{4}{5}$$

$$AJ = 2.04$$

$\Sigma F_X = 0$

$$HJ = 6 - 2.56\,\frac{3}{5}$$

$$HJ = 4.46$$

NUDO "A"

$\Sigma F_Y = 0$

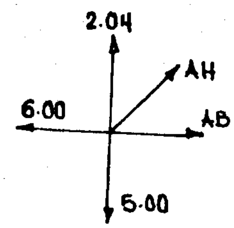
$AH \frac{4}{5} = 5 - 2.04$

$AH = 3.70$

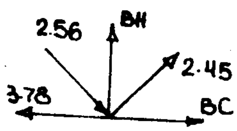
$\Sigma F_X = 0$

$AB = 6 - 3.70 \frac{3}{5}$

$AB = 3.78$



NUDO "B"



$\Sigma F_Y = 0$

$BH = 2.56 \frac{4}{5} - 2.45 \frac{4}{5}$

$BH = 0.09$

$\Sigma F_X = 0$

$BC = 3.78 - 2.56 \frac{3}{5} - 2.45 \frac{3}{5}$

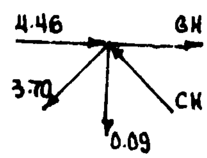
$BC = 0.77$

NUDO "H"

$\Sigma F_Y = 0$

$CH \frac{4}{5} = 3.70 \frac{4}{5} + 0.09$

$CH = 3.81$



$\Sigma F_X = 0$

$GH = 3.70 \frac{3}{5} + 3.81 \frac{3}{5} - 4.46$

$GH = 0.04$

NUDO "F"

$\Sigma F_X = 0$

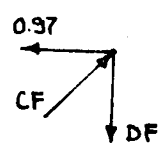
$CF \frac{3}{5} = 0.97$

$CF = 1.62$

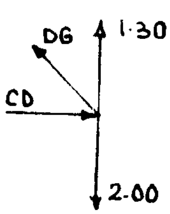
$\Sigma F_Y = 0$

$DF = 1.62 \frac{4}{5}$

$DF = 1.30$



NUDO "D"



$\Sigma F_Y = 0$

$DG \frac{4}{5} = 2.0 - 1.3$

$DG = 0.88$

$\Sigma F_X = 0$

$CD = 0.88 \frac{3}{5}$

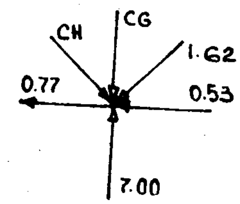
$CD = 0.53$

NUDO "C"

$\Sigma F_X = 0$

$CH \frac{5}{3} = 0.77 + 0.53 + 1.62 \frac{3}{5}$

$CH = 3.81$



$\Sigma F_Y = 0$

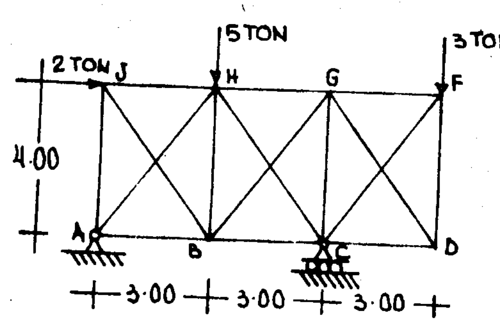
$CG = 7.00 - 3.81 \frac{4}{5} - 1.62 \frac{4}{5}$

$CG = 2.66$

FUERZAS FINALES EN LAS BARRAS

AJ = + 2.04 TON.	BJ = - 2.56 TON.	GH = + 0.04 TON.	CD = - 0.53 TON.
AH = + 3.70 TON.	BH = + 0.09 TON.	CH = - 3.81 TON.	DF = + 1.30 TON.
AB = + 3.78 TON.	BG = + 2.45 TON.	CG = - 2.66 TON.	DG = + 0.88 TON.
HJ = - 4.46 TON.	BC = + 0.77 TON.	CF = - 1.62 TON.	FG = + 0.97 TON.

SI SE TIENE OTRO SISTEMA DE CARGAS



$\Sigma M_C = 0$

$- 6 R_{AY} + 4 (2) - 5 (3) + 3 (3) = 0$

$R_{AY} = \frac{8 - 15 + 9}{6} = 0.33$

$\Sigma M_A = 0$

$2 (4) + 5 (3) + 3 (9) - 6 R_C = 0$

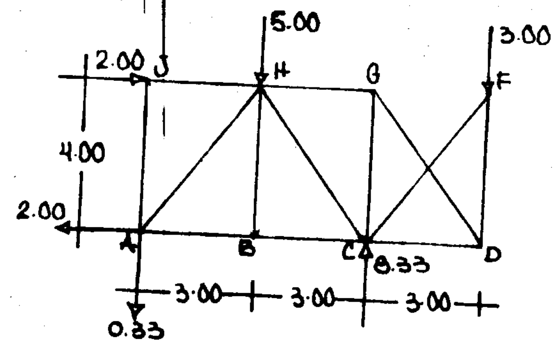
$R_C = \frac{8 + 15 + 27}{6} = 8.33$

$\Sigma F_X = 0$

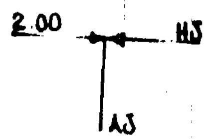
$R_{AX} = 2.00$

ESCOGIENDO LA MISMA ISOSTATICA

ESTADO "O"



NUDO "J"



$\Sigma F_Y = 0$

AJ = 0

$\Sigma F_X = 0$

HJ = 2.00

NUDO "A"

$\Sigma F_Y = 0$

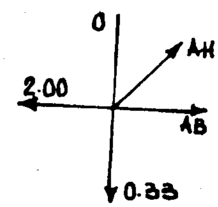
$AH \frac{4}{5} = 0.33$

AH = 0.42

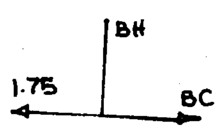
$\Sigma F_X = 0$

$AB = 2 - 0.42 \frac{3}{5}$

AB = 1.75



NUDO "B"



$\Sigma F_Y = 0$

BH = 0

$\Sigma F_X = 0$

BC = 1.75

NUDO "H"

$\Sigma F_Y = 0$

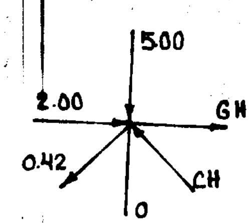
$CH \frac{5}{4} = 5 + 0.42 \frac{4}{5}$

CH = 6.67

$\Sigma F_X = 0$

$GH = 6.67 \frac{3}{5} + 0.42 \frac{3}{5} - 2.0$

GH = 2.25



NUDO "G"

$\Sigma F_X = 0$

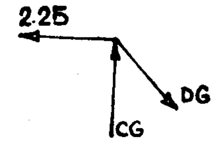
$DG \frac{3}{5} = 2.25$

$DG = 3.75$

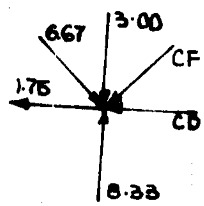
$\Sigma F_Y = 0$

$CG = 3.75 \frac{4}{5}$

$CG = 3.00$



NUDO "C"



$\Sigma F_Y = 0$

$CF \frac{4}{5} = 8.33 - 3.00 - 6.67 \frac{4}{5}$

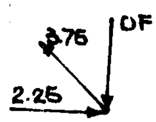
$CF = 0.00$

$\Sigma F_X = 0$

$CD = 6.67 \frac{3}{5} - 1.75$

$CD = 2.25$

NUDO "D"



$\Sigma F_Y = 0$

$DF = 3.75 \frac{4}{5}$

$DF = 3.00$

$S_oU_1L = - 3.15 + 3.60 + 2.10 = + 2.55$

$S_oU_2L = - 3.15 - 4.05 - 33.35 = - 30.95$

$S_oU_3L = - 15.96 - 6.75 - 15.96 - 31.31 = - 69.98$

SUSTITUYENDO VALORES EN LA ECUACION MATRICIAL

$$\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} +0.0592 & -0.0090 & -0.0008 \\ -0.0090 & +0.0605 & +0.0054 \\ -0.0008 & +0.0054 & +0.0213 \end{bmatrix} \begin{bmatrix} +2.55 \\ -30.95 \\ -69.98 \end{bmatrix}$$

EFECTUANDO LA MULTIPLICACION MATRICIAL

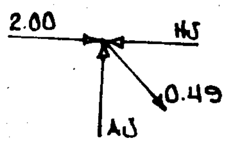
$X_1 = + 0.1510 + 0.2786 + 0.0560 = + 0.4856$

$X_2 = -\ 0.0230 - 1.8725 - 0.3779 = -\ 2.2734$

$X_3 = -\ 0.0020 - 0.1671 - 1.4906 = -\ 1.6597$

DETERMIANCION DE LAS FUERZAS FINALES

NUDO "J"



$\Sigma F_X = 0$

$HJ = 2 + 0.49\ \frac{3}{5}$

$HJ = 2.29$

$\Sigma F_Y = 0$

$AJ = 0.49\ \frac{4}{5}$

$AJ = 0.39$

NUDO "A"

$\Sigma F_Y = 0$

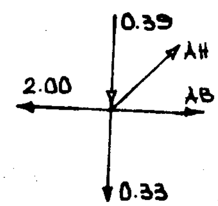
$AH\ \frac{4}{5} = 0.39 + 0.33$

$AH = 0.90$

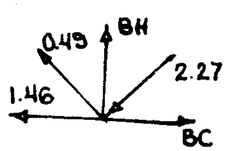
$\Sigma F_X = 0$

$AB = 2 - 0.90\ \frac{3}{5}$

$AB = 1.46$



NUDO "B"



$\Sigma F_Y = 0$

$BH = 2.27\ \frac{4}{5} - 0.49\ \frac{4}{5}$

$BH = 1.42$

$\Sigma F_X = 0$

$BC = 2.27\frac{3}{5} + 0.49\frac{3}{5} + 1.4$

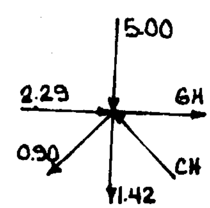
$BC = 3.12$

NUDO "H"

$\Sigma F_Y = 0$

$CH\ \frac{4}{5} = 50 + 1.42 + 0.90\ \frac{4}{5}$

$CH = 8.93$

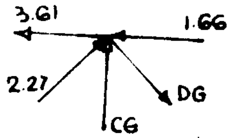


$\Sigma F_X = 0$

$GH = 8.93\ \frac{3}{5} + 0.90\ \frac{3}{5} - 2.29$

$GH = 3.61$

NUDO "G"



$\Sigma F_X = 0$

$DG \frac{3}{5} = 1.66 + 3.61 - 2.27 \frac{3}{5}$

$DG = 6.51$

$\Sigma F_Y = 0$

$CG = 6.51 \frac{4}{5} - 2.27 \frac{4}{5}$

$CG = 3.39$

NUDO "F"

$\Sigma F_Y = 0$

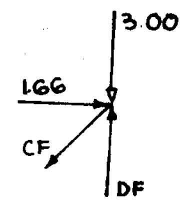
$DF = 3 + 3.78 \times \frac{4}{5}$

$DF = 5.22$

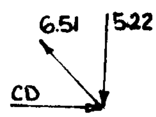
$\Sigma F_X = 0$

$CF \frac{3}{5} = 1.66$

$CF = 2.78$



NUDO "D"

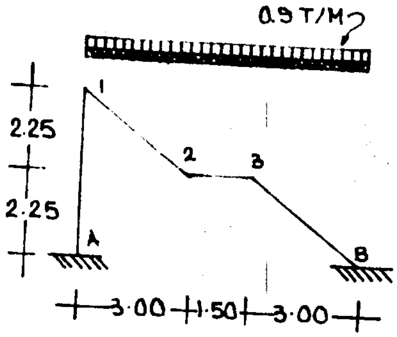


$\Sigma F_X = 0$

$CD = 6.51 \frac{3}{5}$

$CD = 3.91$

AJ = - 0.39	BJ = + 0.49	GH = + 3.61	CD = - 3.91
AH = + 0.90	BH = + 1.42	CH = - 0.93	DF = - 5.22
AB = + 1.46	BF = - 2.27	CG = - 3.39	DG = + 6.51
HJ = - 2.29	BC = + 3.12	CF = + 2.78	FG = - 1.66



OBTENER LOS DIAGRAMAS DE MOMENTOS FLEXIONANTES Y FUERZAS CORTANTES.

$EI = CTE$

TRAMO 12 $\quad Y = 4.5 - \frac{2.25}{3}x = 4.5 - 0.75x$

TRAMO 23 $\quad Y = 2.25$

TRAMO 3B $\quad Y = 5.63 - 0.75x$

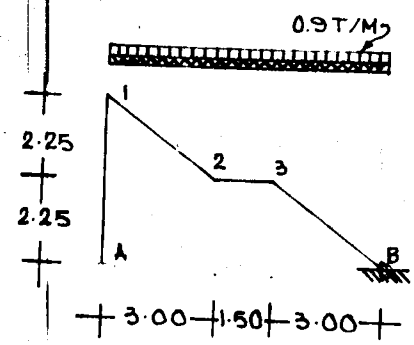
ESTADO "0"

TRAMO A1 $\quad M = 0$

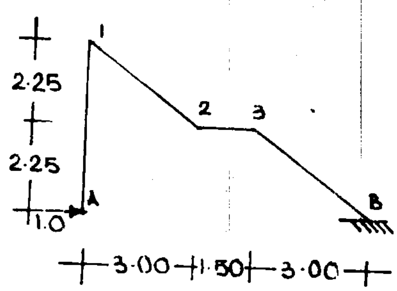
TRAMO 12 $\quad M = -\frac{0.9}{2}x^2$

TRAMO 23 $\quad M = -\frac{0.9}{2}x^2$

TRAMO 3B $\quad M = -\frac{0.9}{2}x^2$



ESTADO "1"



TRAMO A1 $\quad M = -1.0Y$

TRAMO 12 $\quad M = -1.0Y = -4.5+0.75x$

TRAMO 23 $\quad M = -1.0(2.25) = -2.25$

TRAMO 3B $\quad M = -1.0Y = -5.63+0.75x$

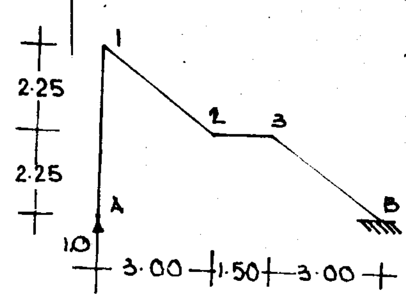
ESTADO "2"

TRAMO A1 $\quad M = 0$

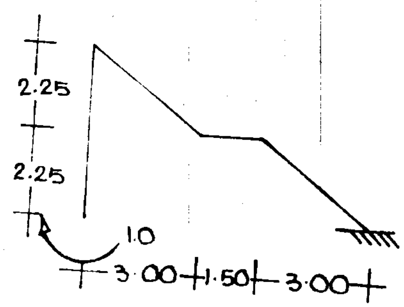
TRAMO 12 $\quad M = 1.0x$

TRAMO 23 $\quad M = 1.0x$

TRAMO 3B $\quad M = 1.0x$



ESTADO "3"



TRAMO A1	M = 1.0
TRAMO 12	M = 1.0
TRAMO 23	M = 1.0
TRAMO 3B	M = 1.0

$$\delta_{01} = \int \frac{M_o m_1}{EI} dx = \frac{1}{EI} \left[\int_0^3 (-0.45x^2)(-4.5+0.75x)\,dx + \int_3^{4.5} (-0.45x^2)(-2.25)\,dx + \int_{4.5}^{7.5} (-0.45x^2)(-5.63+0.75x)\,dx \right]$$

$$\delta_{01} = \frac{1}{EI} \left[\int_0^3 2.03x^2\,dx - \int_0^3 0.34x^3 dx + \int_3^{4.5} 1.01\,x^2 dx + \int_{4.5}^{7.5} 2.53x^2 dx - \int_{4.5}^{7.5} 0.34x^3 dx \right]$$

$$\delta_{01} = \frac{1}{EI} \left[\frac{2.03}{3} \left[x^3\right]_0^3 - \frac{0.34}{4} \left[x^4\right]_0^3 + \frac{1.01}{3} \left[x^3\right]_0^{4.5} + \frac{2.53}{3} \left[x^3\right]_{4.5}^{7.5} - \frac{0.34}{4} \left[x^4\right]_{4.5}^{7.5} \right]$$

$$\delta_{01} = \frac{1}{EI} (18.27 - 6.89 + 30.68 - 9.09 + 355.79 - 76.85 - 268.95 + 34.86)$$

$$\delta_{01} = \frac{77.82}{EI}$$

$$\delta_{02} = \int \frac{M_o m_2}{EI} dx = \frac{1}{EI} \left[\int_0^3 (-0.45x^2)\,x\,dx + \int_3^{4.5} (-0.45x^2)\,x\,dx + \int_{4.5}^{7.5} (-0.45x^2)\,x\,dx \right]$$

$$\delta_{02} = \frac{1}{EI} \int_0^{7.5} -0.45x^3 dx = -\frac{1}{EI} \frac{0.45}{4} \left[x^4\right]_0^{7.5}$$

$$\delta_{02} = -\frac{355.96}{EI}$$

$$\delta_{03} = \int \frac{M_o m_3}{EI} dx = \frac{1}{EI} \left[\int_0^3 -0.45x^2 dx + \int_3^{4.5} -0.45x^2 dx + \int_{4.5}^{7.5} -0.45x^2 dx \right]$$

$$\delta_{03} = \frac{1}{EI} \int_0^{7.5} -0.45x^2 dx = -\frac{1}{EI} \frac{0.45}{3} \left[x^3\right]_0^{7.5}$$

$$\delta_{03} = -\frac{63.28}{EI}$$

$$\delta_{11} = \int \frac{m_1^2}{EI} dx = \frac{1}{EI} \left[\int_0^{4.5} (-Y)^2 dy + \int_0^3 (-4.5+0.75x)^2 dx + \int_3^{4.5} (-2.25)^2 dx + \int_{4.5}^{7.5} (-5.63 + 0.75x)^2 dx\right]$$

$$\delta_{11} = \frac{1}{EI} \left[\int_0^{4.5} Y^2 dy + \int_0^3 20.25\, dx - \int_0^3 6.75\, x\, dx + \int_0^3 0.56x^2 dx + \int_3^{4.5} 5.06 dx + \int_{4.5}^{7.5} 31.70\, dx - \int_{4.5}^{7.5} 8.45x dx + \int_{4.5}^{7.5} 0.56x^2 dx\right]$$

$$\delta_{11} = \frac{1}{EI} \left(\frac{1}{3} \left[Y^3\right]_0^{4.5} + 20.25 \left[x\right]_0^3 - \frac{6.75}{2} \left[x^2\right]_0^3 + \frac{0.56}{3} \left[x^3\right]_0^3 + 5.06 \left[x\right]_3^{4.5} + 31.70 \left[x\right]_{4.5}^{7.5} - \frac{8.45}{2} \left[x^2\right]_{4.5}^{7.5} + \frac{0.56}{3} \left[x^3\right]_{4.5}^{7.5} \right)$$

$$\delta_{11} = \frac{1}{EI} (30.38 + 60.75 - 30.38 + 5.04 + 22.77 - 15.18 + 237.75 - 142.65 - 237.94 + 85.66 + 78.75 - 17.01)$$

$$\delta_{11} = + \frac{77.94}{EI}$$

$$\delta_{12} = \int \frac{m_1 m_2}{EI} dx = \frac{1}{EI} \left[\int_0^3 (-4.5+0.75x) x dx + \int_3^{4.5} -2.25x dx + \int_{4.5}^{7.5} (-5.63 + 0.75x)\, x\, dx\right]$$

$$\delta_{12} = \frac{1}{EI} \left[- \int_0^3 4.5x dx + \int_0^3 0.75x^2 dx - \int_3^{4.5} 2.25x dx - \int_{4.5}^{7.5} 5.63x dx + \int_{4.5}^{7.5} 0.75x^2 dx\right]$$

$$\delta_{12} = \frac{1}{EI} \left(- \frac{4.5}{2} \left[x^2\right]_0^3 + \frac{0.75}{3} \left[x^3\right]_0^3 - \frac{2.25}{2} \left[x^2\right]_3^{4.5} - \frac{5.63}{2} \left[x^2\right]_{4.5}^{7.5} + \frac{0.75}{3} \left[x^3\right]_{4.5}^{7.5} \right)$$

$$\delta_{12} = \frac{1}{EI} (-20.25+6.75-22.79+10.13-158.37+57.03+105.47-22.78)$$

$$\delta_{12} = -\frac{44.81}{EI}$$

$$\delta_{13} = \int \frac{m_1 m_3}{EI} dx = \frac{1}{EI}\left[\int_0^{4.5} -Y dy + \int_0^{3} (-4.5+0.75x) dx + \int_3^{4.5} -2.25\, dx + \int_{4.5}^{7.5} (-5.63 + 0.75x)\, dx\right]$$

$$\delta_{13} = \frac{1}{EI}\left(-\int_0^{4.5} Y dy - \int_0^{3} 4.5 dx + \int_0^{3} 0.75x dx - \int_3^{4.5} 2.25\, dx - \int_{4.5}^{7.5} 5.63 dx + \int_{4.5}^{7.5} 0.75x dx \right)$$

$$\delta_{13} = \frac{1}{EI}\left(-\frac{1}{2}\left[Y^2\right]_0^{4.5} - 4.5\left[x\right]_0^{3} + \frac{0.75}{2}\left[x^2\right]_0^{3} - 2.25\left[x\right]_3^{4.5} - 5.63\left[x\right]_{4.5}^{7.5} + \frac{0.75}{2}\left[x^2\right]_{4.5}^{7.5}\right)$$

$$\delta_{13} = \frac{1}{EI}\left(-10.13 - 13.50 + 3.38 - 10.13 + 6.75 - 42.23 + 25.34 + 21.10 - 7.60 \right)$$

$$\delta_{13} = -\frac{27.02}{EI}$$

$$\delta_{22} = \int \frac{m_2^2}{EI} dx = \frac{1}{EI}\left(\int_0^{3} x^2 dx + \int_3^{4.5} x^2 dx + \int_{4.5}^{7.5} x^2 dx \right)$$

$$\delta_{22} = \frac{1}{EI} \int_0^{7.5} x^2 dx = \frac{1}{EI}\,\frac{1}{3}\left[x^3\right]_0^{7.5} = \frac{140.63}{EI}$$

$$\delta_{22} = +\frac{140.63}{EI}$$

$$\delta_{23} = \int \frac{m_2 m_3}{EI} dx = \frac{1}{EI}\left(\int_0^{3} x dx + \int_3^{4.5} x dx + \int_{4.5}^{7.5} x dx \right)$$

$$\delta_{23} = \frac{1}{EI} \int_0^{7.5} x dx = \frac{1}{EI}\,\frac{1}{2}\left[x^2\right]_0^{7.5} = \frac{28.13}{EI}$$

$$\delta_{23} = +\frac{28.13}{EI}$$

$$\delta_{33} = \int \frac{m_3^2}{EI} dx = \frac{1}{EI}\left(\int_0^{4.5} dy + \int_0^{3} dx + \int_3^{4.5} dx + \int_{4.5}^{7.5} dx \right)$$

$$\delta_{33} = \frac{1}{EI}\left(\int_0^{4.5} dy + \int_0^{7.5} dx \right) = \frac{1}{EI}\left(\left[Y\right]_0^{4.5} + \left[x\right]_0^{7.5} \right) = \frac{12}{EI}$$

$$\delta_{33} = +\frac{12.00}{EI}$$

SUSTITUYENDO EN LAS ECUACIONES DE COMPATIBILIDAD DE DEFORMACIONES Y ELIMINANDO EI

$$+ 77.82 + 77.94\,X_A - 44.81\,Y_A - 27.02\,M_A = 0$$

$$- 355.96 - 44.81\,X_A + 140.63\,Y_A + 28.13\,M_A = 0$$

$$- 63.28 - 27.02\,X_A + 28.13\,Y_A + 12.00\,M_A = 0$$

MATRICIALMENTE

$$\begin{bmatrix} X_A \\ Y_A \\ M_A \end{bmatrix} = \begin{bmatrix} +77.94 & -44.81 & -27.02 \\ -44.81 & +140.63 & +28.13 \\ -27.02 & +28.13 & +12.00 \end{bmatrix}^{-1} \begin{bmatrix} -77.82 \\ +355.96 \\ +63.28 \end{bmatrix}$$

$$F^{-1} = \begin{bmatrix} +0.0798 & -0.0199 & +0.2263 \\ -0.0199 & +0.0183 & -0.0876 \\ +0.2263 & -0.0876 & +0.7989 \end{bmatrix}$$

SUSTITUYENDO

$$\begin{bmatrix} X_A \\ Y_A \\ M_A \end{bmatrix} = \begin{bmatrix} +0.0798 & -0.0199 & +0.2263 \\ -0.0199 & +0.0183 & -0.0876 \\ +0.2263 & -0.0876 & +0.7989 \end{bmatrix} \begin{bmatrix} -77.82 \\ +355.96 \\ +63.28 \end{bmatrix}$$

$$X_A = -6.21 - 7.08 + 14.32 = +1.03 \quad \text{TON}$$

$$Y_A = +1.55 + 6.51 - 5.54 = +2.52 \quad \text{TON}$$

$$M_A = -17.61 - 31.18 + 50.55 = +1.76 \quad \text{T x M}$$

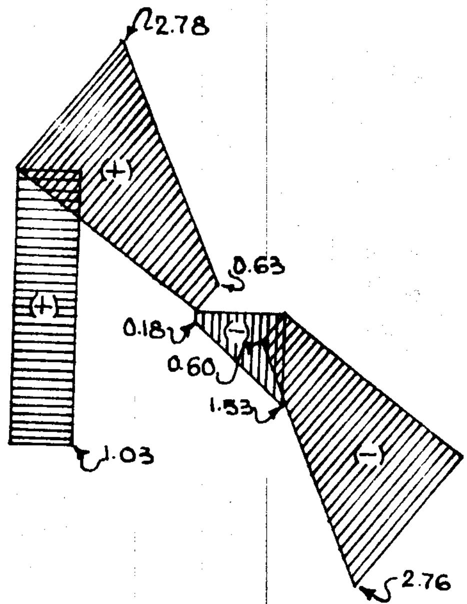
$$X_B = -1.03 \text{ TON}$$

$$Y_B = 6.75 - 2.52 = +4.23 \text{ TON}$$

$$M_B = +\frac{0.9}{2}\,7.5^2 - 1.76 - 2.52\,(7.5) = +25.32 - 1.76 - 18.90$$

$$M_B = +4.66 \text{ T x M}$$

DIAGRAMA DE FUERZAS CORTANTES.



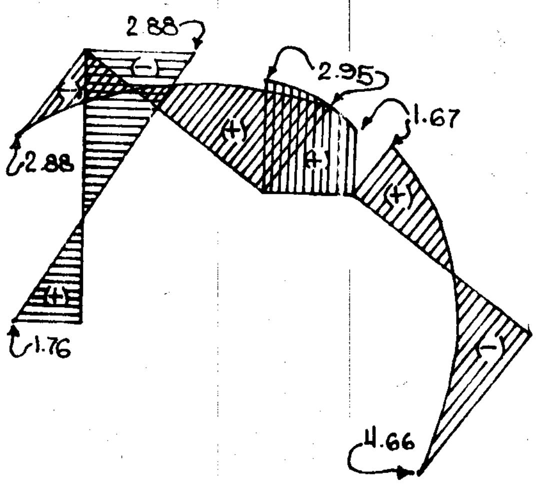


DIAGRAMA DE MOMENTOS FLEXIONANTES.

$M_1 = 1.76 - 1.03\ (4.5)$

$M_1 = 1.76 - 4.64 = -2.88$

$M_2 = 1.76 - 1.03(2.25) + 2.52(3.00) - 0.9\ \frac{3(3)}{2}$

$M_2\ 1.76 - 2.32 + 7.56 - 4.05$

$M_2 = +2.95$

$M_3 = 1.76 - 2.32 + 2.52(4.5) - 0.9\frac{4.5(4.5)}{2}$

$M_3 = 1.76 - 2.32 + 11.34 - 9.11$

$M_3 = +1.67$

$V_1 = 4.03$

$V'_1 = \frac{2.25}{3.75}\ 1.03 + \frac{3.00}{3.75}\ 2.52$

$V'_1 = 0.77 + 2.01 = 2.78$

$V_2 = 0.77 - \frac{3.00}{3.75}\ 0.18$

$V_2 = 0.77 - 0.14 = 0.63$

$V'_2 = 2.52 - 2.70 = -0.18$

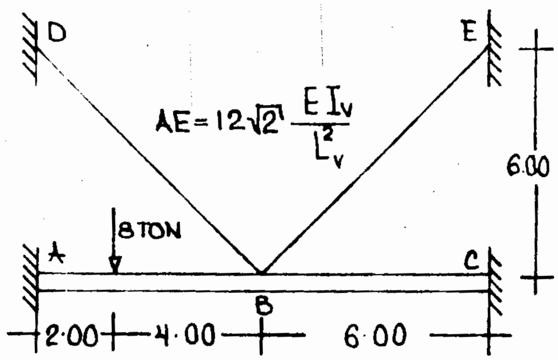
$V_3 = 2.52 - 4.05 = -1.53$

$V_B = -\frac{3.00}{3.75}\ 4.23 + \frac{2.25}{3.75}\ 1.03$

$V_B = -3.38 + 0.62 = -2.76$

$V'_3 = -2.76 + \frac{3.00}{3.75}\ 2.7$

$V'_3 = -2.76 + 2.16 = -0.60$



I_{VIGA} = CONSTANTE

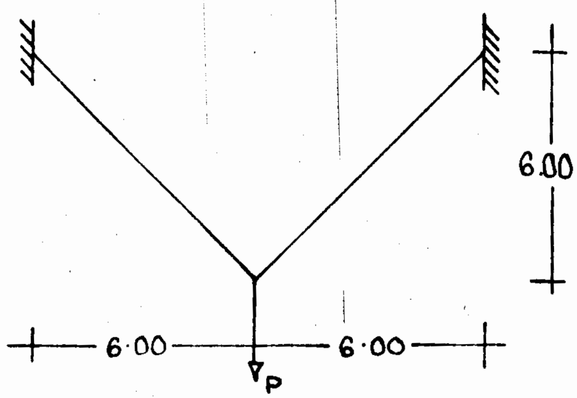
CALCULAS LAS TENSIONES EN LOS CABLES Y LOS DIAGRAMAS DE FUERZAS CORTANTES Y MOMENTOS FLEXIONANTES EN LA VIGA

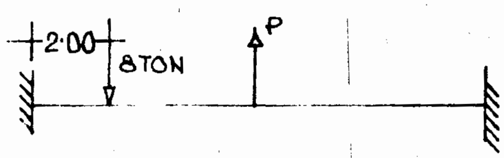
Δ_V = DEFORMACION VERTICAL DE LA VIGA EN PUNTO "B"

Δ_C = DEFORMACION VERTICAL DE LOS CABLES EN EL PUNTO "B"

ECUACION DE COMPATIBILIDAD DE DEFORMACIONES:

$$\Delta_V + \Delta_C = 0$$





$$\Delta_C = P \cos 45^\circ \; \frac{L_C}{AE} \frac{1}{\cos 45^\circ}$$

$$\Delta_C = \frac{PL_C}{AE}$$

SUSTITUYENDO

$$\Delta_C = \frac{PL_C}{12\sqrt{2}\,\frac{EI}{L_V^2}}$$

DE LA FIGURA

$$L_C = \sqrt{2\left(\frac{L_V}{2}\right)^2} = \frac{L_V}{\sqrt{2}}$$

$$\Delta_C = \frac{P\,\frac{L_V}{\sqrt{2}}\,L_V^2}{12\sqrt{2}\;EI} = \frac{P\,L_V^3}{24EI}$$

$$\Delta_V = \frac{P\,L_V^3}{192EI} - \frac{8(2)^2\left(\frac{L_V}{2}\right)^2}{6EI\;L^3}\left[3(10)\,L_V - 3(10)\,\frac{L_V}{2} - 2\,\frac{L_V}{2}\right]$$

$$\Delta_V = \frac{P\,L_V^3}{192EI} - \frac{32\;L_V^2}{24EI\;L^3}\left(30\,L_V - 15\,L_V - L_V\right)$$

$$\Delta_V = \frac{P\,L^3_V}{192EI} - \frac{18.67}{EI}$$

SUSTITUYENDO EN LA ECUACION DE COMPATIBILIDAD DE DEFORMACIONES

$$\frac{P\,L^3_V}{192EI} - \frac{18.67}{EI} + \frac{P\,L^3_V}{24EI} = 0$$

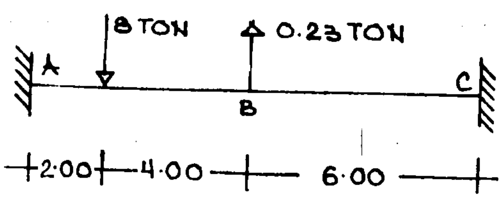
$$\frac{9P12^3}{192} = 18.67$$

$$P = \frac{18.67\,(192)}{9\,(1728)} = 0.23 \quad \text{TON}$$

TENSION EN LOS CABLES

$$T = \frac{P}{2\frac{1}{\sqrt{2}}} = \frac{0.23}{\sqrt{2}} = 0.163$$

REACCIONES EN LA VIGA



$$M_A = \frac{0.23(12)}{8} - \frac{8(2)\,10^2}{12^2}$$

$$M_A = 0.35 - 11.11 = -10.76 \text{ T x M}$$

$$Y_A = \frac{0.23}{2} + \frac{8(10)^2}{12^3}\left[3(2)+10\right]$$

$$Y_A = -0.12 + 7.41 = 7.29 \text{ TON}$$

$$X_A = 0$$

$$Y_C = -7.29 - 0.23 + 8.00 = 0.48 \quad \text{TON}$$

$$M_C = 10.76 + 8\,(10) - 7.29\,(12) - 0.23\,(6)$$

$$M_C = 10.76 + 80 - 87.48 - 1.38 = 1.90 \quad \text{T x M}$$

$$X_C = 0$$

DIAGRAMA DE FUERZAS CORTANTES

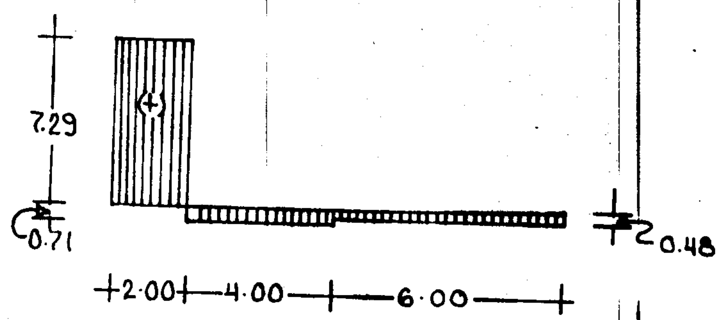
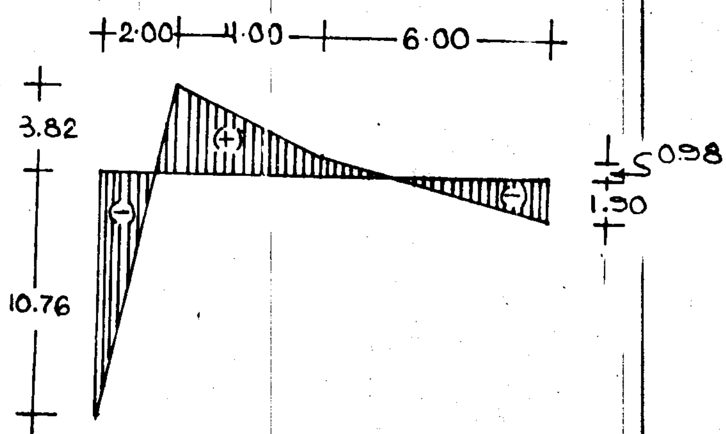
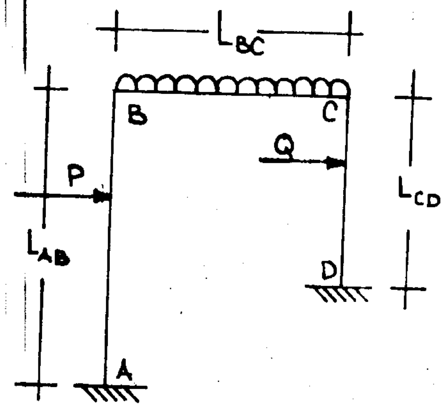


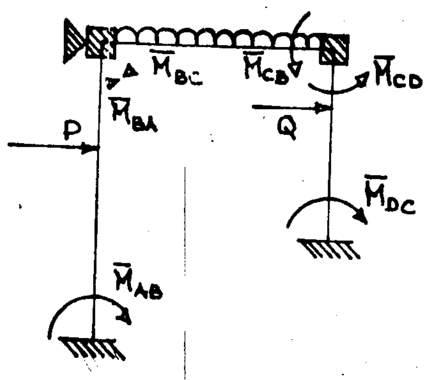
DIAGRAMA DE MOMENTOS FLEXIONANTES



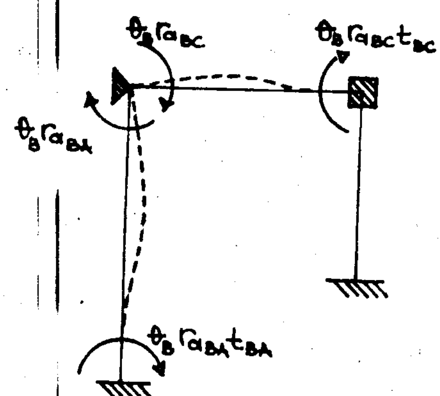
SEA LA SIGUIENTE ESTRUCTURA



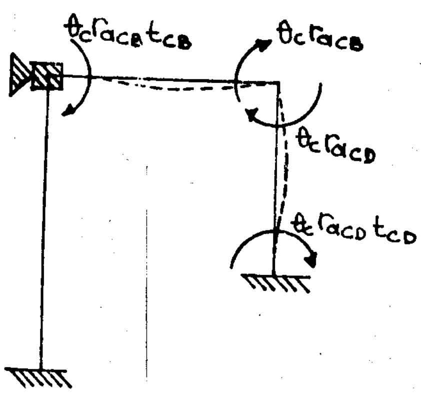
SI SE CONSIDERA VALIDO EL PRINCIPIO DE SUPERPOSICION DE CAUSAS Y EFECTOS LA ESTRUCTURA REAL SERA IGUAL A LA SUMA DE LOS SIGUIENTES ESTADOS:



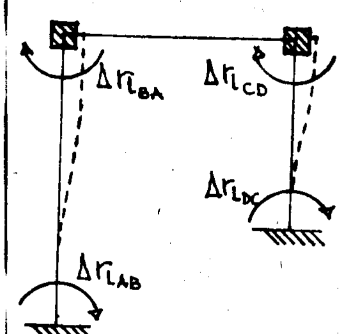
ESTADO "0"



ESTADO "1"



ESTADO "2"



ESTADO "3"

$$\Sigma M_B = 0$$

$$\overline{M}_{BC} + \overline{M}_{BA} + \theta_B\, r_a BC + \theta_B\, r_a BA + \theta_C\, r_a CB\, t_a CB + \Delta\, r_1 BA = 0$$

$$\theta_B\, (r_a BC + r_a BA) + \theta_C\, r_a CB\, t_a CB + \Delta\, r_1 BA + \overline{M}_{BC} + \overline{M}_{BA} = 0 \qquad (1)$$

$$\Sigma M_C = 0$$

$$\overline{M}_{CB} + \overline{M}_{CD} + \theta_B\, r_a BC\, t_a BC + \theta_C\, r_a CB + \theta_C\, r_a CD + \Delta\, r_1 CD = 0$$

$$\theta_B\, r_a BC\, t_a BC + \theta_C\, (r_a CB + r_a CD) + \Delta\, r_1 CD + \overline{M}_{CB} + \overline{M}_{CD} = 0 \qquad (2)$$

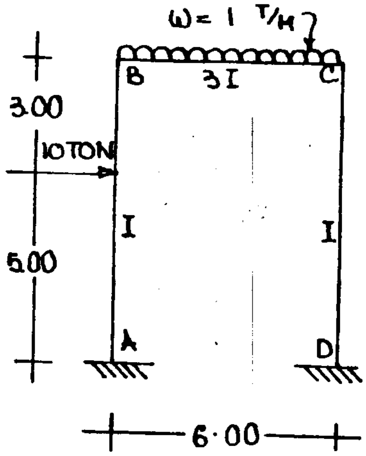
$$\Sigma V_{CABEZAL} = 0$$

$$RxB + RxC + \frac{\theta_B\, r_a BA + \theta_B\, r_a BA\, t_a BA}{L_{AB}} + \frac{\theta_C\, r_a CD + \theta_C\, r_a CD\, t_a CD}{L_{CD}}$$

$$+ \frac{\Delta\, r_1 BA + \Delta\, r_1 AB}{L_{AB}} + \frac{\Delta\, r_1 CD + \Delta\, r_1 DC}{L_{CD}} = 0$$

$$\frac{\theta_B\, r_a BA\, (1 + t_a BA)}{L_{AB}} + \frac{\theta_C\, r_a CD\, (1 + t_a CD)}{L_{BC}}$$

$$+ \Delta \left(\frac{r_1 BA + r_1 AB}{L_{AB}} + \frac{r_1 CD + r_1 DC}{L_{CD}} \right) + RxB + RxC = 0 \qquad (3)$$



OBTENER LOS DIAGRAMAS DE FUERZAS CORTANTES Y MOMENTOS FLEXIONANTES.

ESTADO "O"

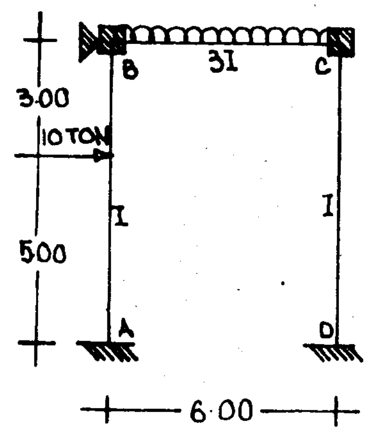
(MOMENTOS DE EMPOTRAMIENTO)

$\overline{M}_{AB} = + \frac{10 \times 5 \times 3^2}{8^2} = + 7.03$

$\overline{M}_{BA} = - \frac{10 \times 5^2 \times 3}{8^2} = - 11.72$

$\overline{M}_{BC} = + \frac{1 \times 6^2}{12} = + 3.00$

$\overline{M}_{CB} = - \frac{1 \times 6^2}{12} = - 3.00$



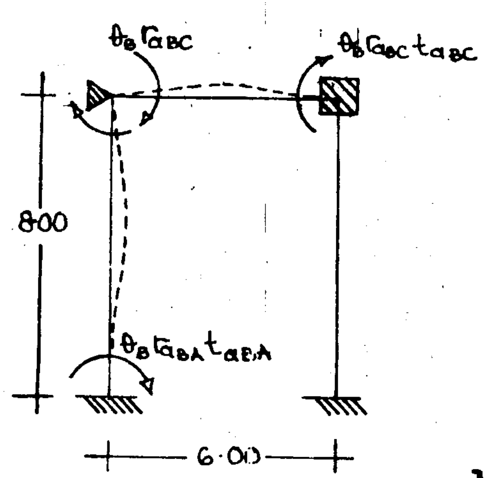
$M_{CD} = M_{DC} = 0$

$RxB = - \frac{10 \times 5}{8} = - 6.25$

$RxC = 0$

ESTADO "1"

(EL NUDO "B" GIRA UN ANGULO θ_B)



$r_a BA = \frac{4EI_{AB}}{L_{AB}} = \frac{4EI}{8} = 0.5\ EI$

$r_a BC = \frac{4EI_{BC}}{L_{BC}} = \frac{4E(3I)}{6} = 2.0\ EI$

$t_a BA = t_a BC = \frac{1}{2}$

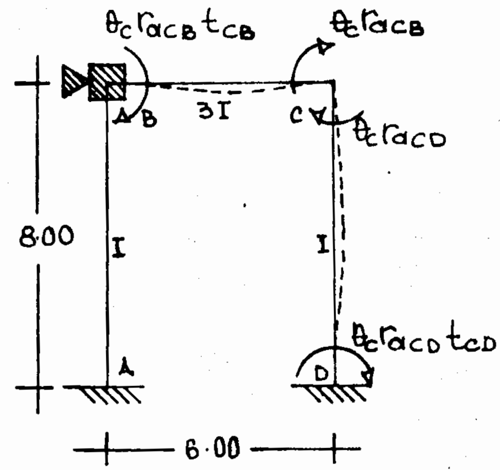
ESTADO "2"

(EL NUDO C GIRA UN ANGULO θ_C)

$$r_a CB = \frac{4EI_{CB}}{L_{CB}} = \frac{4EI(3I)}{6} = 2.0\ EI$$

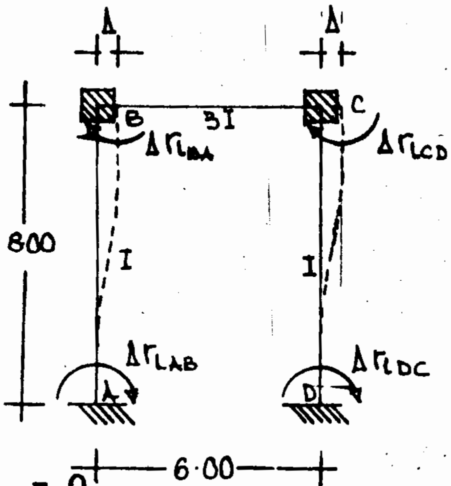
$$r_a CD = \frac{4EI_{CD}}{L_{CD}} = \frac{4EI}{8} = 0.5\ EI$$

$$t_a CB = t_a CD = 0.5$$



ESTADO "3"

(EL CABEZAL SUFRE UN DESPLAZAMIENTO)



$$r_1 AB = r_1 BA = \frac{6EI_{AB}}{L^2_{AB}} = \frac{6EI}{64} = 0.0938\ EI$$

$$r_1 CD = r_1 DC = \frac{6EI_{DC}}{L_{DC}} = \frac{6EI}{64} = 0.0938\ EI$$

$\Sigma M_B = 0$

$$\theta_B\ (0.5 + 2.0)\ EI + \theta_C\ (2.0)\ 0.5\ EI + \Delta\, 0.0938 + 3.00 - 11.72 = 0$$

$$2.5\ EI\ \theta_B + EI\ \theta_C + 0.0938\ EI\ \Delta - 8.72 = 0 \qquad (1)$$

$\Sigma M_C = 0$

$$\theta_B\ (2.0)\ 0.5\ EI + \theta_C\ (2.0 + 0.5)\ EI + \Delta\, 0.0938\ EI - 3.00 = 0$$

$$EI\ \theta_B + 2.5\ EI\ \theta_C + 0.0938\ EI\ \Delta - 3.00 = 0 \qquad (2)$$

$V_{CABEZAL} = 0$

$$\frac{\theta_B\ (0.5)\ (1+0.5)EI}{8} + \frac{\theta_C\ (0.5)(1+0.5)EI}{8} + \Delta\Big(\frac{0.0938+0.0938}{8}$$

$$+ \frac{0.0938 + 0.0938}{8}\Big)\ EI - 6.25 = 0$$

$$0.0938EI\,\theta_B + 0.0938\,EI\,\theta_C + 0.469\,EI\,\Delta - 6.25 = 0 \qquad (3)$$

MATRICIALMENTE

$$\begin{bmatrix} +2.5 & +1.0 & +0.0938 \\ +1.0 & +2.5 & +0.0938 \\ +0.0938 & +0.0938 & +0.0469 \end{bmatrix} \begin{bmatrix} \theta_B \\ \theta_C \\ \Delta \end{bmatrix} = \frac{1}{EI} \begin{bmatrix} 8.72 \\ 3.00 \\ 6.25 \end{bmatrix}$$

$$\begin{bmatrix} \theta_B \\ \theta_C \\ \Delta \end{bmatrix} = \frac{1}{EI} \begin{bmatrix} +0.4933 & -0.1734 & -0.6396 \\ -0.1734 & +0.4933 & -0.6396 \\ -0.6396 & -0.6396 & +23.8663 \end{bmatrix} \begin{bmatrix} 8.72 \\ 3.00 \\ 6.25 \end{bmatrix}$$

EFECTUANDO LA MULTIPLICACION

$$\theta_B = \frac{1}{EI}\,(4.3016 - 0.5202 - 3.9975) = -\frac{0.2161}{EI}$$

$$\theta_C = \frac{1}{EI}\,(-1.5120 + 1.4799 - 3.9975) = -\frac{4.0296}{EI}$$

$$\Delta = \frac{1}{EI}\,(-5.5773 - 1.9188 + 149.1644) = -\frac{141.6683}{EI}$$

CALCULO DE REACCIONES Y DIAGRAMAS

$$M_{AB} = +7.03 + 0.5\left(-\frac{0.2161}{EI}\right) 0.5\,EI + \frac{141.6683}{EI}\,0.0938\,EI$$

$$M_{AB} = +7.03 - 0.05 + 13.28 = +20.26 \quad T \times M$$

$$M_{BA} = -11.72 + \left(-\frac{0.2161}{EI}\right) 0.5\,EI + \frac{141.6683}{EI}\,0.0938\,EI$$

$$M_{BA} = -11.72 - 0.11 + 13.28 = +1.45 \quad T \times M$$

$$M_{BC} = +3.00 + \left(-\frac{0.2161}{EI}\right) 2.0\,EI + 0.5\left(-\frac{4.0296}{EI}\right) 2.0\,EI$$

$$M_{BC} = +3.00 - 0.43 - 4.02 = -1.45 \quad T \times M$$

$$M_{CB} = -3.00 + 0.5 \left(-\frac{0.2161}{EI} \right) 2.0\ EI + \left(-\frac{4.0296}{EI} \right) 2.0\ EI$$

$$M_{CB} = -3.00 - 0.21 - 8.06 = -11.27 \quad T \times M$$

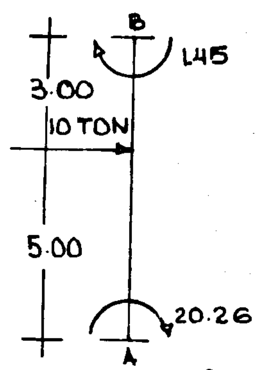
$$M_{CD} = \left(-\frac{4.0296}{EI} \right) 0.5\ EI + \frac{141.6683}{EI}\ 0.0938\ EI$$

$$M_{CD} = -2.01 + 13.28 = +11.27\ T \times M$$

$$M_{DC} = 0.5 \left(-\frac{4.0296}{EI} \right) 0.5\ EI + \frac{141.6683}{EI}\ 0.0938\ EI$$

$$M_{DC} = -1.01 + 13.28 = +12.27\ T \times M$$

CALCULO DE REACCIONES Y DIAGRAMAS



$$1.45 = -20.26 + 8\ X_A - 3\ (10)$$

$$X_A = \frac{1.45 + 20.26 + 30.00}{8} = \frac{51.71}{8}$$

$$X_A = 6.46 \quad TON$$

$$-11.27 = 1.45 - 1\ \frac{6^2}{2} + 6\ Y_B$$

$$Y_B = \frac{-11.27 - 1.45 + 18.00}{6} = \frac{5.28}{6}$$

$$Y_B = 0.88 \quad TON$$

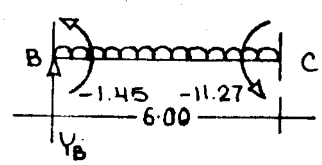
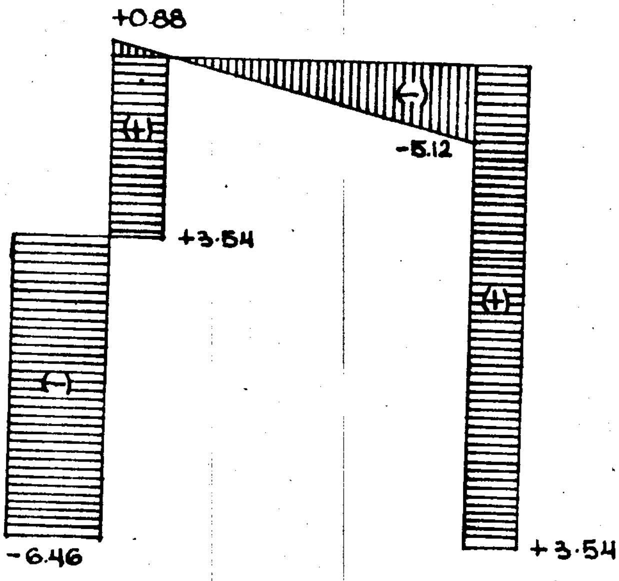


DIAGRAMA DE FUERZAS CORTANTES



REACCIONES

X_A = - 6.46 TON

Y_A = + 0.88 TON

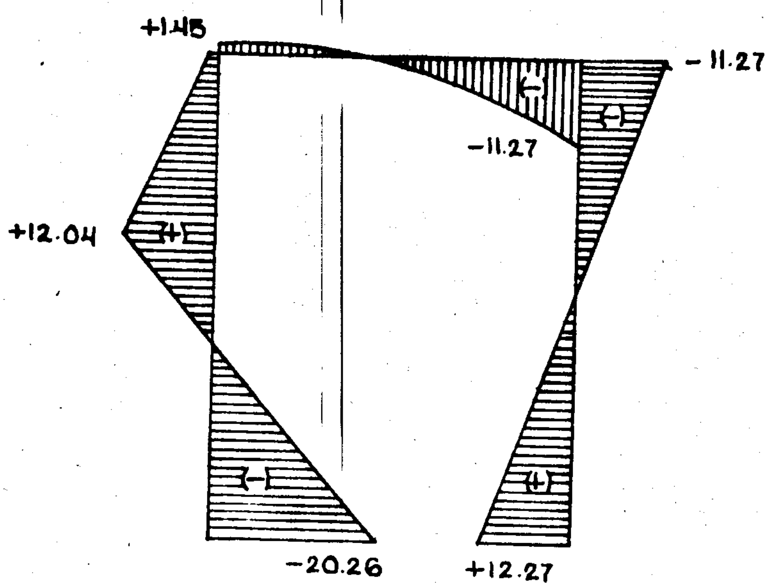
M_A = -20.26 T x M

X_D = - 3.54 TON

Y_D = + 5.12 T x M

M_D = -12.27 T x M

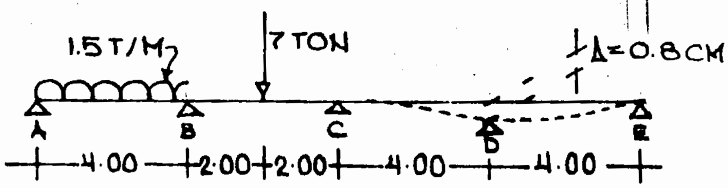
DIAGRAMA DE MOMENTOS FLEXIONANTES



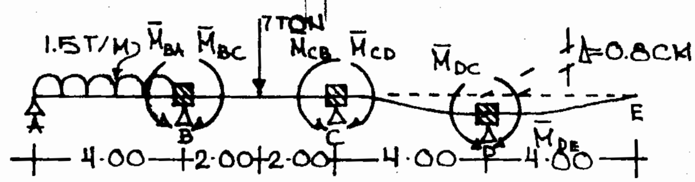
OBTENER LOS DIAGRAMAS DE FUERZAS COR

TANTES Y MOMENTOS FLEXIONANTES.

$EI = 2100\ T \times M^2$



ESTADO "O"



$\bar{M}_{BA} = -\frac{1.5\ (4)^2}{12} = -\ 1.33\ T \times M$

$\bar{M}_{BC} = +\frac{7(4)}{8} = \quad +\ 3.50\ T \times M$

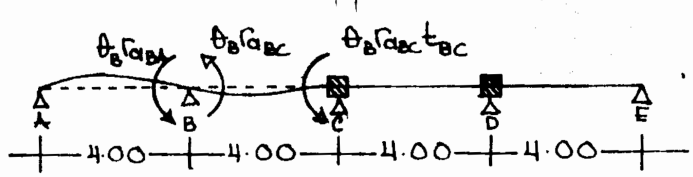
$\bar{M}_{CB} = -\frac{7(4)}{8} = \quad -\ 3.50\ T \times M$

$\bar{M}_{CD} = +\ 0.008\ \frac{6EI}{16} = +\ 0.003\ EI = +\ 6.30\ T \times M$

$\bar{M}_{DC} = +\ M_{CD} = +\ 0.003\ EI = +\ 6.30\ T \times M$

$\bar{M}_{DE} = -\ 0.008\ \frac{3EI}{16} = -\ 0.0015\ EI = -\ 3.15\ T \times M$

ESTADO "1"

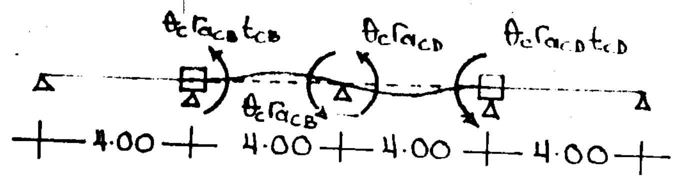


$r_a BA = \frac{3EI}{4} = 0.75\ EI$

$r_a BC = \frac{4EI}{4} = 1.00\ EI$

$t_a BC = 0.5$

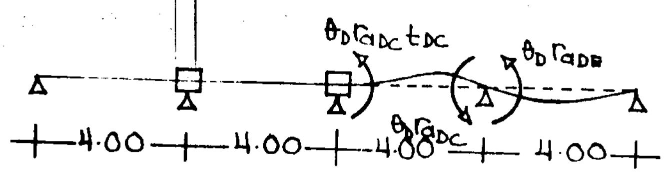
ESTADO "2"



$r_a CB = r_a CD = \frac{4EI}{4} = 1.00\ EI$

$t_a BC = t_a CD = 0.5$

ESTADO "3"



$r_a DC = \frac{4EI}{4} = 1.00\ EI$

$r_a DE = \frac{3EI}{4} = 0.75\ EI$

$t_a DC = 0.5$

$\Sigma M_B = 0$

$\theta_B\ (-0.75 - 1.00)\ EI + \theta_C\ (-1.00)\ 0.5EI - 1.33 + 3.50 = 0$

$- 1.75\ \theta_B\ EI - 0.50\ \theta_C\ EI + 2.17 = 0$ (1)

$\Sigma M_C = 0$

$\theta_B\ (-1.00)\ 0.5EI + \theta_C\ (-1.00 - 1.00)\ EI + \theta_D\ (-1.00)\ 0.5 - 3.50 + 6.3$

$- 0.5\ \theta_B\ EI - 2.0\ \theta_C\ EI - 0.5\ \theta_D + 2.80 = 0$ (2)

$\Sigma M_D = 0$

$$\theta_C (-1.00)\ 0.5\ EI + \theta_D (-1.00 - 0.75)EI + 6.30 - 3.15 = 0$$

$$-0.5\ \theta_C\ EI - 1.75\ \theta_D\ EI + 3.15 = 0 \qquad (3)$$

MATRICIALMENTE Y MULTRIPLICANDO POR - 1

$$\begin{bmatrix} +1.75 & +0.50 & 0 \\ +0.50 & +2.00 & +0.50 \\ 0 & +0.50 & +1.75 \end{bmatrix} \begin{bmatrix} \theta_B \\ \theta_C \\ \theta_D \end{bmatrix} = \frac{1}{EI} \begin{bmatrix} +2.17 \\ +2.80 \\ +3.15 \end{bmatrix}$$

$$\begin{bmatrix} \theta_B \\ \theta_C \\ \theta_D \end{bmatrix} = \frac{1}{EI} \begin{bmatrix} +0.6190 & -0.1666 & +0.0476 \\ -0.1666 & +0.5833 & -0.1666 \\ +0.0476 & -0.1666 & +0.6190 \end{bmatrix} \begin{bmatrix} +2.17 \\ +2.80 \\ +3.15 \end{bmatrix}$$

$$\theta_B = \frac{1}{EI} (+1.3432 - 0.4665 + 0.1499) = + \frac{1.0266}{EI}$$

$$\theta_C = \frac{1}{EI} (-0.3615 + 1.6332 - 0.5248) = + \frac{0.7469}{EI}$$

$$\theta_D = \frac{1}{EI} (+0.1033 - 0.4665 + 1.9499) = + \frac{1.5867}{EI}$$

CALCULO DE MOMENTOS FINALES.

$$M_{BA} = -1.33 + \frac{1.0266}{EI} (-0.75\ EI) = -1.33 - 0.77 = -2.10\ T \times M$$

$$M_{BC} = +3.50 + \frac{1.0266}{EI} (-1.00EI) + \frac{0.7469}{EI} (-1.00EI)0.5 = 2.10\ T \times M$$

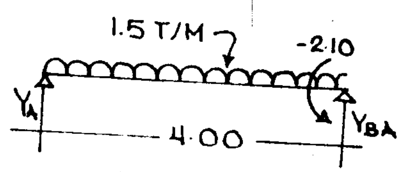
$$M_{CB} = -3.50 + \frac{1.0266}{EI} (-1.00EI)\ 0.5 + \frac{0.7469}{EI} (-1.00EI) = -4.76\ T \times M$$

$$M_{CD} = +6.30 + \frac{0.7469}{EI}(-1.00EI) + \frac{1.5867}{EI}(-1.00EI)\,0.5 = +4.76 \text{ T x M}$$

$$M_{DC} = +6.30 + \frac{0.7469}{EI}(-1.00EI)\,0.5 + \frac{1.5867}{EI}(-1.00EI) = +4.34 \text{ T x M}$$

$$M_{DE} = -3.15 + \frac{1.5867}{EI}(-0.75EI) = -3.15 - 1.19 = -4.34 \text{ T x M}$$

CALCULO DE REACCIONES Y DIAGRAMAS.



$$-2.10 = -1.5\,\frac{4^2}{2} + 4\,Y_A$$

$$Y_A = \frac{-2.10+12.00}{4} = \frac{9.90}{4} = +2.48 \text{ TON}$$

$$+2.10 + 1.5\,\frac{4^2}{2} - 4\,Y_{BA} = 0$$

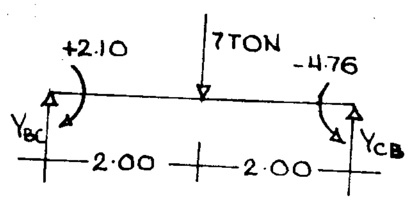
$$Y_{BA} = \frac{+2.10+12.00}{4} = \frac{14.10}{4} = +3.52 \text{ TON}$$

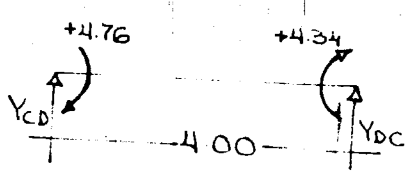
$$-2.10 + 4\,Y_{BC} - 7(2) = -4.76$$

$$Y_{BC} = \frac{-4.76+2.10+14.00}{4} = +2.83 \text{ TxM}$$

$$+4.76 - 4\,Y_{CB} + 7(2) = 2.10$$

$$Y_{CB} = \frac{4.76+14.00-2.10}{4} = +4.17 \text{ T x M}$$





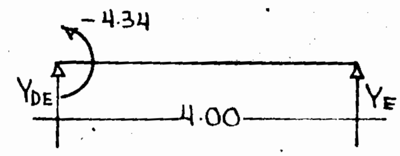
$$+4.34 = -4.76 + 4\,Y_{CD}$$

$$Y_{CD} = \frac{4.34+4.76}{4} = +2.27 \text{ TON}$$

$$+4.76 = -4.34 - 4\,Y_{DC}$$

$$Y_{DC} = \frac{-4.76-4.34}{4} = -2.27 \text{ TON}$$

$0 = 4.34 + 4\ Y_{DE}$



$Y_{DE} = -\dfrac{4.34}{4} = -1.09$ TON

$-4.34 = -4\ Y_E$

$Y_E = +1.09$ TON

$Y_A = +2.48$ TON

$Y_B = Y_{BA} + Y_{BC} = +3.52 + 2.83 = +6.35$ TON

$Y_C = Y_{CB} + Y_{CD} = +4.17 + 2.27 = +6.44$ TON

$Y_D = Y_{DC} + Y_{DE} = -2.27 - 1.09 = -3.36$ TON

$Y_E = +1.09$ TON

DIAGRAMA DE FUERZAS CORTANTES

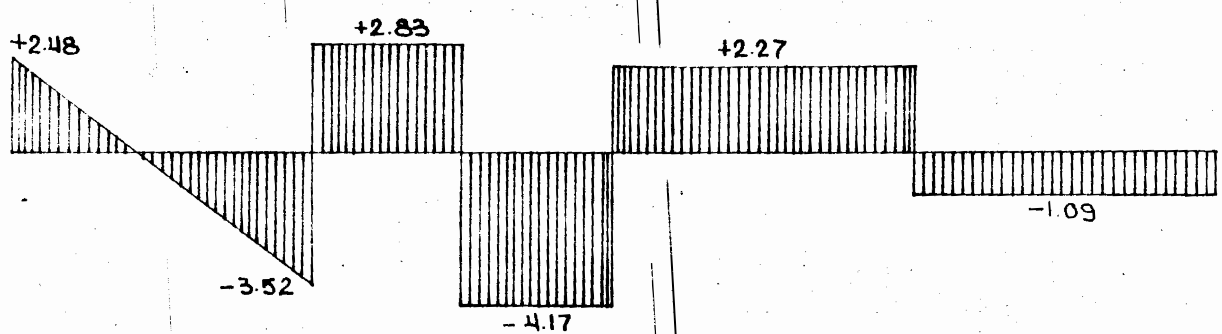
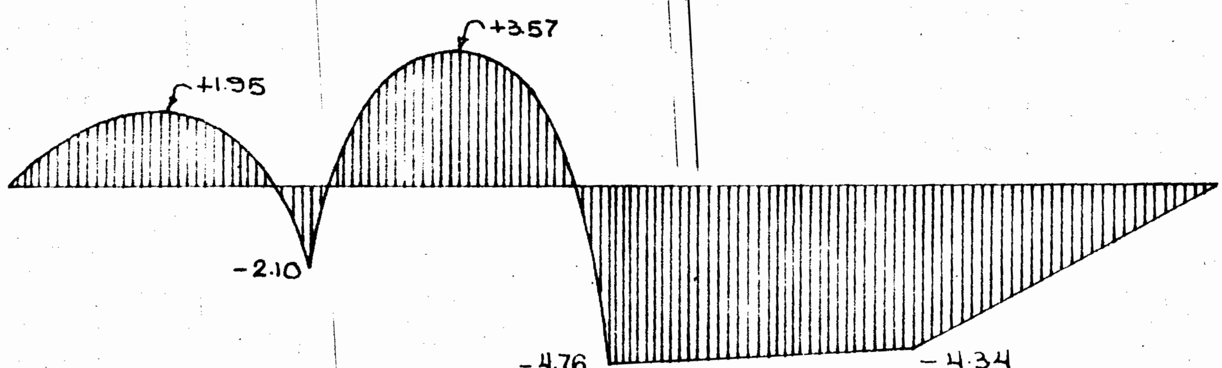
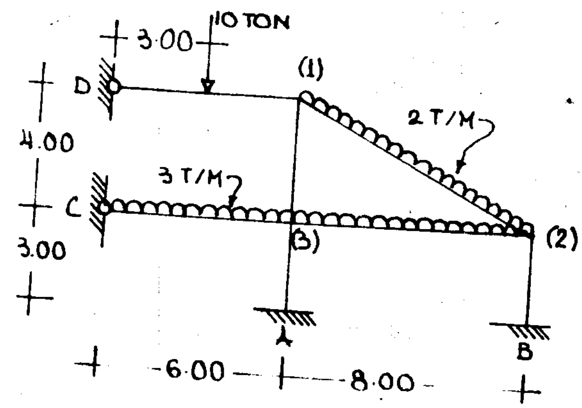


DIAGRAMA DE MOMENTOS FLEXIONANTES

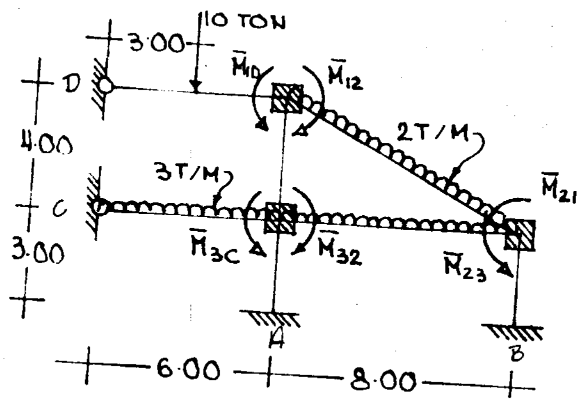




EI = CTE

OBTENER LOS DIAGRAMAS DE FUERZAS CORTANTES Y MOMENTOS FLEXIONANTES.

ESTADO "0"



$$\bar{M}_{1D} = -\frac{3\times10\times6}{16} = -11.25\ T\times M$$

$$\bar{M}_{12} = +\frac{2(8.9)^2}{12} = +13.20\ T\times M$$

$$\bar{M}_{21} = -\frac{2(8.9)^2}{12} = -13.20\ T\times M$$

$$\bar{M}_{3C} = -\frac{3(6)^2}{8} = -13.50\ T\times M$$

$$\bar{M}_{32} = +\frac{3(8)^2}{12} = +16.00\ T\times M$$

$$\bar{M}_{23} = -\frac{3(8)^2}{12} = -16.00\ T\times M$$

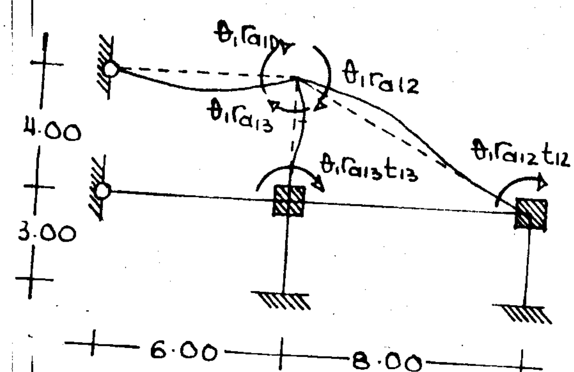
ESTADO "1"

$$r_a 1D = \frac{3EI}{6} = 0.50\ EI$$

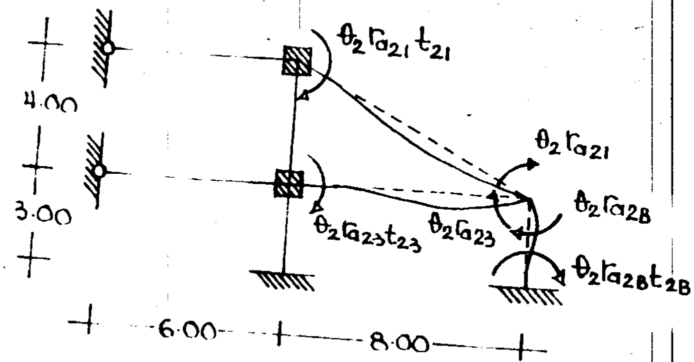
$$r_a 12 = \frac{4EI}{8.9} = 0.45\ EI$$

$$r_a 13 = \frac{4EI}{4} = 1.00\ EI$$

$$t_a 12 = t_a 13 = 0.5$$



ESTADO "2"



$r_a21 = \frac{4EI}{8.9} = 0.45\ EI$

$r_a2B = \frac{4EI}{3} = 1.33\ EI$

$r_a23 = \frac{4EI}{8} = 0.50\ EI$

$t_a21 = t_a2B = t_a23 = 0.5$

ESTADO "3"

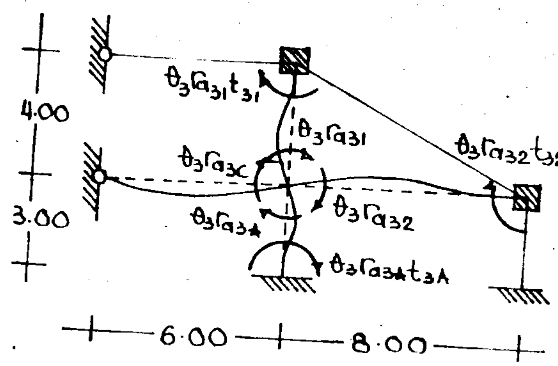
$r_a31 = \frac{4EI}{4} = 1.00\ EI$

$r_a32 = \frac{4EI}{8} = 0.50\ EI$

$r_a3A = \frac{4EI}{3} = 1.33\ EI$

$r_a3C = \frac{3EI}{6} = 0.50\ EI$

$t_a31 = t_a32 = t_a3A = 0.5$



$\Sigma M_1 = 0$

$\theta_1\ (0.50 + 0.45 + 1.00)EI + \theta_2\ (\ 0.45\ )\ 0.5\ EI + \theta_3\ (\ 1.00\ )\ 0.5\ EI$
$- 11.25 + 13.20 = 0$

$1.95\ \theta_1\ EI + 0.225\ \theta_2\ EI + 0.50\ \theta_3\ EI + 1.95 = 0 \qquad (1)$

$\Sigma M_2 = 0$

$\theta_1\ (0.45)0.5EI + \theta_2(0.45+1.33+0.50)EI + \theta_3(0.50)0.5EI - 13.20 - 16.00 = 0$

$0.225\ \theta_1 + 2.28\ \theta_2\ EI + 0.25\ \theta_3\ EI - 29.20 = 0 \qquad (2)$

$\Sigma M_3 = 0$

$\theta_1(1.00)0.5EI + \theta_2(0.5)0.5EI + \theta_3(1.00+0.50+1.33+0.50)EI - 13.50 + 16.00 = 0$

$$0.50\,\theta_1\,EI + 0.25\,\theta_2\,EI + 3.33\,\theta_3\,EI + 2.50 = 0 \qquad (3)$$

MATRICIALMENTE

$$\begin{bmatrix} 1.95 & 0.225 & 0.50 \\ 0.225 & 2.28 & 0.25 \\ 0.50 & 0.25 & 3.33 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} = \frac{1}{EI} \begin{bmatrix} -\ 1.95 \\ +\ 29.20 \\ -\ 2.50 \end{bmatrix}$$

$$\begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} = \frac{1}{EI} \begin{bmatrix} +0.5378 & -0.0446 & -0.0774 \\ -0.0446 & +0.4459 & -0.0268 \\ -0.0774 & -0.0268 & +0.3139 \end{bmatrix} \begin{bmatrix} -1.95 \\ +29.20 \\ -2.50 \end{bmatrix}$$

$$\theta_1 = \frac{1}{EI}(-1.0487 - 1.3023 + 0.1935) = -\frac{2.1575}{EI}$$

$$\theta_2 = \frac{1}{EI}(+0.0870 + 13.0203 + 0.0670) = +\frac{13.1743}{EI}$$

$$\theta_3 = \frac{1}{EI}(+0.1509 - 0.7826 - 0.7848) = -\frac{1.4165}{EI}$$

CALCULO DE MOMENTOS FINALES

$$M_{1D} = -11.25 - \frac{2.1575}{EI}\,0.50\,EI = -11.25 - 1.08 = -12.33\ T \times M$$

$$M_{12} = +13.20 - \frac{2.1575}{EI}\,0.45\,EI + \frac{13.1743}{EI}(0.45\,EI)\,0.5$$

$$M_{12} = +13.20 - 0.97 + 2.97 = +15.20\ T \times M$$

$$M_{13} = -\frac{2.1575}{EI}\,1.00\,EI - \frac{1.4165}{EI}(1.00\,EI)\,0.5 = -2.16$$

$$-0.71 = -2.87\ T \times M$$

$$M_{21} = -13.20 - \frac{2.1575}{EI}(0.45\,EI)\,0.5 + \frac{13.1743}{EI}\,0.45\,EI$$

$$M_{21} = -13.20 - 0.48 + 5.92 = -7.76\ T \times M$$

$$M_{2B} = + \frac{13.1743}{EI} 1.33\ EI = + 17.52\ T \times M$$

$$M_{23} = - 16.00 + \frac{13.1743}{EI} 0.50\ EI - \frac{1.4165}{EI} (0.50\ EI) 0.5$$

$$M_{23} = - 16.00 + 6.59 - 0.35 = - 9.76\ T \times M$$

$$M_{32} = + 16.00 + \frac{13.1743}{EI} (0.50\ EI) 0.5 - \frac{1.4165}{EI} 0.50\ EI$$

$$M_{32} = + 16.00 + 3.29 - 0.71 = + 18.57\ T \times M$$

$$M_{3A} = - \frac{1.4165}{EI} 1.33\ EI = 1.88\ T \times M$$

$$M_{3C} = - 13.50 - \frac{1.4165}{EI} 0.50\ EI = - 13.50 - 0.71 = - 14.21\ T \times M$$

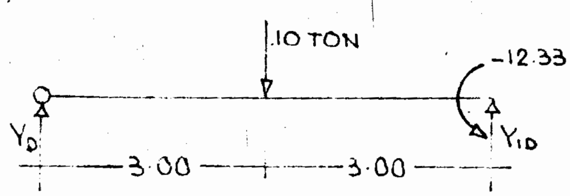
$$M_{31} = - \frac{2.1575}{EI} (1.00\ EI) 0.5 - \frac{1.4165}{EI} 1.00\ EI$$

$$M_{31} = - 1.07 - 1.41 = - 2.48\ T \times M$$

$$M_A = - \frac{1.4165}{EI} (1.33\ EI) 0.5 = - 0.94\ T \times M$$

$$M_B = + \frac{13.1743}{EI} (1.33\ EI) 0.5 = + 8.76\ T \times M$$

CALCULO DE REACCIONES Y DIAGRAMAS.



$$- 12.33 = 6\ Y_D - 10 (3.00)$$

$$Y_D = \frac{30.00 - 12.33}{6} = + 2.95\ TON$$

$$0 = + 12.33 + 10(3.00) - 6\ Y_{1D}$$

$$Y_{1D} = \frac{30.00 + 12.33}{6} = + 7.06\ TON$$

$$M_{ISOS} = \frac{10 \times 6}{4} = 15.00\ T \times M$$

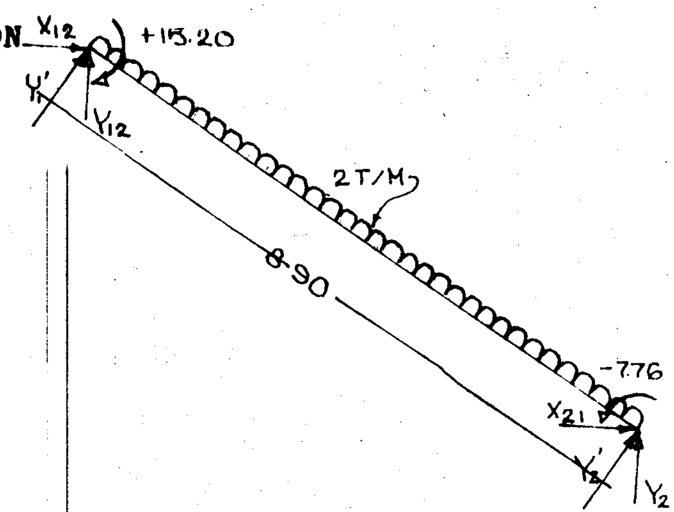
$$-7.76 = -15.20 + 8.90 Y_1' - \frac{2(8.9)^2}{2}$$

$$Y_1' = \frac{-7.76+15.20+79.21}{8.90} = +9.74 \text{ TON}$$

$$+15.20 = +7.76 - 8.90 Y_2' + \frac{2(8.9)^2}{2}$$

$$Y_2' = \frac{7.76+79.21-15.20}{8.90} = +8.06 \text{ TON}$$

$$M_{ISOS} = \frac{2(8.90)^2}{8} = 19.80$$

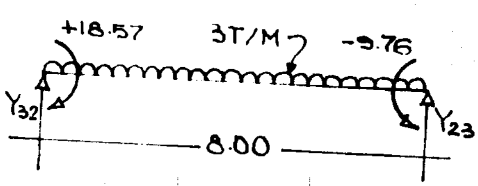


$$Y_{12} = \frac{8.0}{8.9} 9.72 = 8.74 \text{ TON}$$

$$Y_{21} = \frac{8.0}{8.9} 8.06 = 7.24 \text{ TON}$$

$$X_{12} = \frac{4.0}{8.9} 9.72 = 4.37 \text{ TON}$$

$$X_{21} = \frac{4.0}{8.9} 8.06 = 3.62 \text{ TON}$$



$$-9.76 = -18.57 - \frac{3(8)^2}{2} + 8 Y_{32}$$

$$Y_{32} = \frac{-9.76+18.57+96.00}{8} = +13.10 \text{ TON}$$

$$18.57 = +9.76 + \frac{3(8)^2}{2} - 8 Y_{23}$$

$$Y_{23} = \frac{9.76+96.00+18.57}{8} = +10.90 \text{ TON}$$

$$M_{ISOS} = \frac{3(8)^2}{8} = 24.00 \text{ T x M}$$

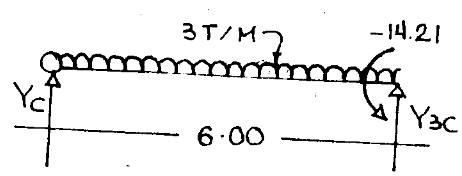
$$-14.21 = -\frac{3(6)^2}{2} + 6 Y_C$$

$$Y_C = \frac{-14.21 + 54.00}{6} = +6.63 \text{ TON}$$

$$0 = +14.21 + \frac{3(6)^2}{2} - 6 Y_{3C}$$

$$Y_{3C} = \frac{14.21+54.00}{6} = +11.37 \text{ TON}$$

$$M_{ISOS} = \frac{3(6)^2}{8} = 13.50 \text{ T x M}$$

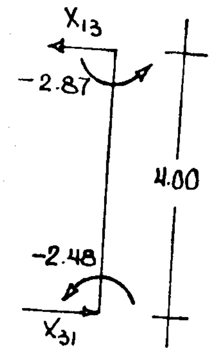


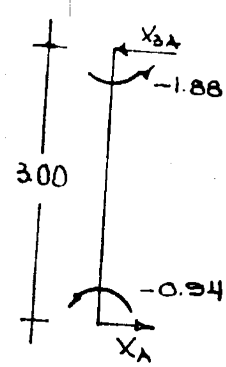
$$- 2.87 = + 2.48 - 4 X_{31}$$

$$X_{31} = \frac{2.48 + 2.87}{4} = + 1.34$$

$$- 2.48 = + 2.87 - 4 X_{13}$$

$$X_{13} = \frac{2.87 + 2.48}{4} = 1.34$$





$$- 1.88 = 0.94 - 3 X_A$$

$$X_A = \frac{1.88 + 0.94}{3} = + 0.94 \text{ TON}$$

$$- 0.94 = 1.88 - 3 X_{3A}$$

$$X_{3A} = \frac{1.88 + 0.94}{3} = 0.94 \text{ TON}$$

$$17.52 = - 8.76 + 3 X_B$$

$$X_B = \frac{17.52 + 8.76}{3} = 8.76 \text{ TON}$$

$$8.76 = - 17.52 + 3 X_{2B}$$

$$X_{2B} = \frac{8.76 + 17.52}{3} = 8.76 \text{ TON}$$

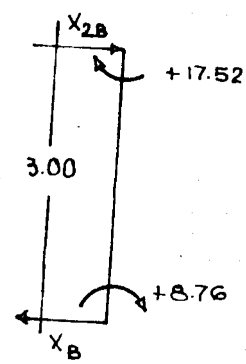
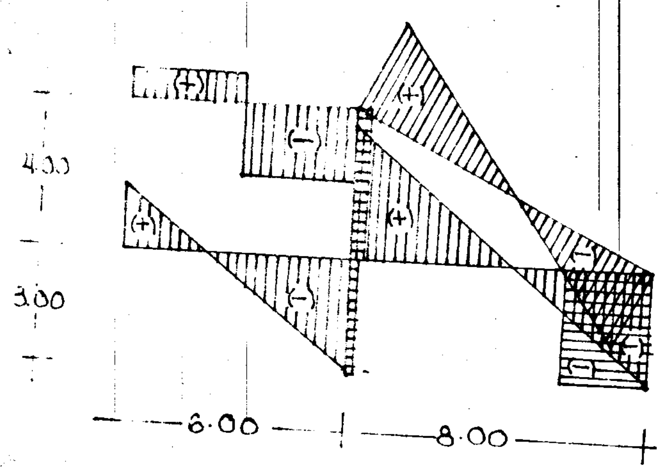


DIAGRAMA DE FUERZAS CORTANTES:



REACCIONES

$X_A = +0.94$ TON

$Y_A = 11.37 + 13.10 + 7.05 + 8.74 = +40.26$

$M_A = +0.94$ T x M

$X_B = -8.76$ TON

$Y_B = +10.90$ TON

$M_B = -8.76$ T x M

$X_C = +8.76 - 0.94 + 1.34 + 3.62 = +12.78$ TON

$Y_C = +6.63$ TON

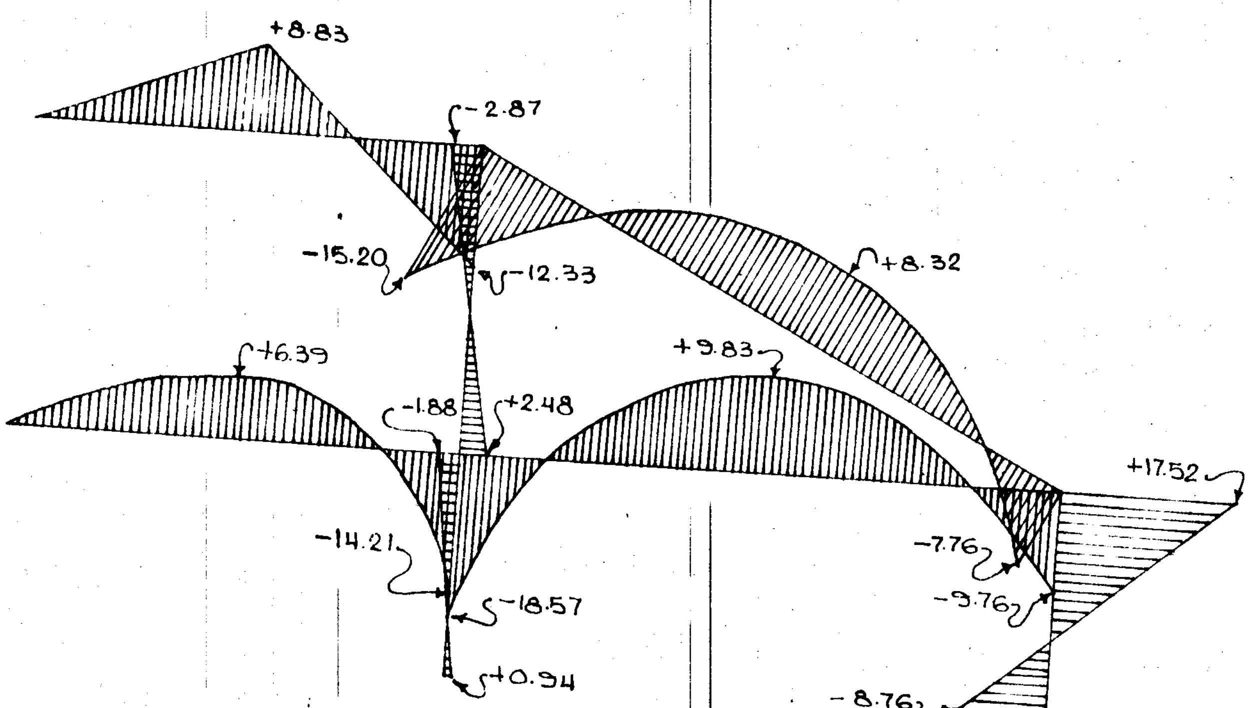
$M_C = 0$

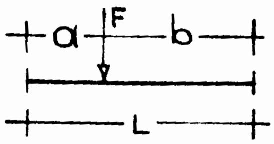
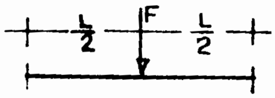
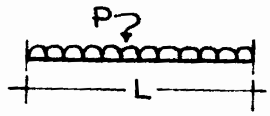
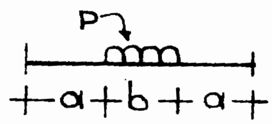
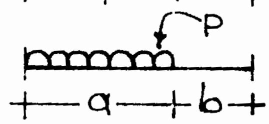
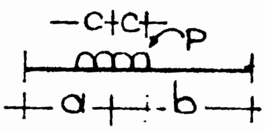
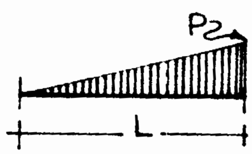
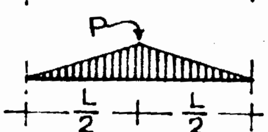
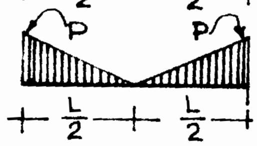
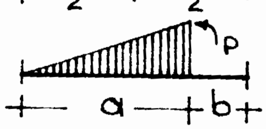
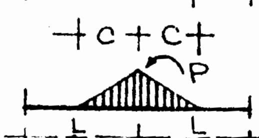
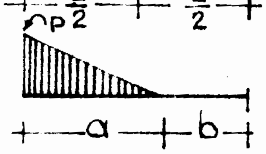
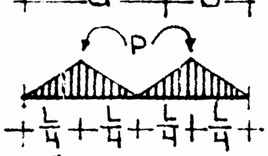
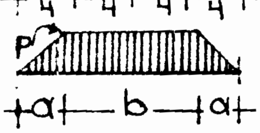
$X_D = -1.34 + 4.37 = +3.03$ TON

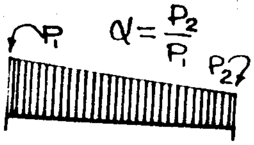
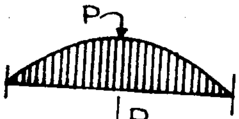
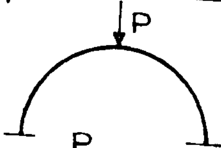
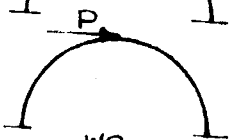
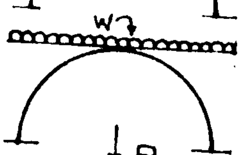
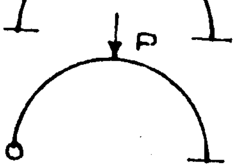
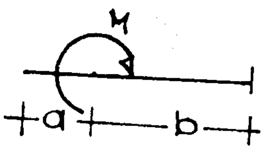
$Y_D = +2.95$ TON

$M_D = 0$

DIAGRAMA DE MOMENTOS FLEXIONANTES

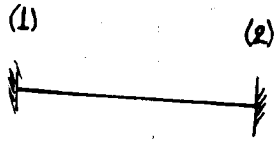


	CARGA	IZQUIERDO	DERECHO
1		$\frac{Fab^2}{L^2}$	$-\frac{Fba^2}{L^2}$
2		$\frac{1}{8}FL$	$-\frac{1}{8}FL$
3		$\frac{1}{12}PL^2$	$-\frac{1}{12}PL^2$
4		$\frac{1}{24}pbL(3-\frac{b^2}{L^2})$	$-\frac{1}{24}pbL(3-\frac{b^2}{L^2})$
5		$\frac{1}{12}pa^2\ 4\frac{b}{L}+2\frac{b^2}{L^2}+\frac{a^2}{L^2}$	$-\frac{1}{12}pa^2\ 3-2\frac{b}{L}-\frac{b^2}{L^2}-2\frac{a^2}{L^2}$
6		$\frac{2}{3}pc\left[2b(1-\frac{b^2}{L^2}-\frac{c^2}{L^2})-a(1-\frac{a^2}{L^2}-\frac{c^2}{L^2})\right]$	$-\frac{2}{3}pc\left[2a(1-\frac{a^2}{L^2}-\frac{c^2}{L^2})-b(1-\frac{b^2}{L^2}-\frac{c^2}{L^2})\right]$
7		$\frac{1}{30}pL^2$	$-\frac{1}{20}pL^2$
8		$\frac{5}{96}pL^2$	$-\frac{5}{96}pL^2$
9		$\frac{1}{32}pL^2$	$-\frac{1}{32}pL^2$
10		$\frac{pa^2}{30}\left[10-15\frac{a}{L}+6\frac{b^2}{L^2}\right]$	$-\frac{pa^2}{180}\left[45\frac{a}{L}-36\frac{a^2}{L^2}\right]$
11		$\frac{1}{24}pcL(3-2\frac{c^2}{L^2})$	$-\frac{1}{24}pcL(3-2\frac{c^2}{L^2})$
12		$\frac{pa^2}{60}\left[2+6\frac{b}{L}+2\frac{b^2}{L^2}+\frac{a^2}{L^2}\right]$	$\frac{pa^2}{60}\left[4-3\frac{b}{L}-\frac{b^2}{L^2}-2\frac{a^2}{L^2}\right]$
13		$\frac{17}{384}pL^2$	$-\frac{17}{384}pL^2$
14		$\frac{1}{96}pL(L+b)(5-\frac{b^2}{L^2})$	$-\frac{1}{96}pL(L+b)(5-\frac{b^2}{L^2})$

15		$\frac{p_1 L^2}{60}(3+2\alpha)$	$\frac{p_1 L^2}{60}(2+3\alpha)$
16		$\frac{1}{15}pL^2$	$-\frac{1}{15}pL^2$
17		$\frac{PR}{8}$	$\frac{PR}{8}$
18		0.131PR	0.817PR
19		$\frac{wR}{3}$	$\frac{wR}{3}$
20		0	0.067PR
21		$-\frac{Mb}{L^2}(2a-b)$	$-\frac{Ma}{L^2}(2b-a)$

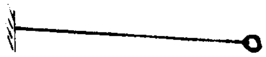
	MOMENTO ISOSTATICO	REACCIONES	
	VALOR MAXIMO	IZQUIERDA	DERECHA
1	$\frac{Fab}{L}$	$F\frac{b}{L}$	$F\frac{a}{L}$
2	$\frac{FL}{4}$	$\frac{1}{2}F$	$\frac{1}{2}F$
3	$\frac{pL^2}{8}$	$p\frac{L}{2}$	$p\frac{L}{2}$
4	$\frac{pb}{4}(L-\frac{b}{2})$	$p\frac{b}{2}$	$p\frac{b}{2}$
5	$\frac{pa^2}{8}\ \frac{(L+b)^2}{L^2}$	$pa\frac{L+b}{2L}$	$pa\frac{a}{2L}$
6	$2\frac{pabc}{L}(1-\frac{c}{L})$	$2pc\frac{b}{L}$	$2pc\frac{a}{L}$
7	$\frac{pL^2}{9\ 3}$	$p\frac{L}{6}$	$p\frac{L}{3}$
8	$\frac{pL^2}{12}$	$p\frac{L}{4}$	$p\frac{L}{4}$
9	$\frac{pL^2}{24}$	$p\frac{L}{4}$	$p\frac{L}{4}$
10	$\frac{pa^2}{3}(\frac{\frac{1}{2}a+b}{L})^{3/2}$	$pa\frac{\frac{1}{2}a+b}{2L}$	$pa\frac{a}{3L}$
11	$\frac{pc}{12}(3L-2c)$	$p\frac{c}{2}$	$p\frac{c}{2}$
12	$pa^2(\frac{1}{6}-\frac{a}{6L}+\frac{a}{9L}\ \frac{a}{3L})$	$pa\frac{\frac{2}{3}a+b}{2L}$	$pa\frac{a}{6L}$
13	$\frac{pL^2}{16}$	$p\frac{L}{4}$	$p\frac{L}{4}$
14	$\frac{p}{4}(\frac{L^2}{2}-\frac{2a^2}{3})$	$p\frac{a+b}{2}$	$p\frac{a+b}{2}$

15	$\frac{P_1 L^2}{6(1-\ )^2}$ - - ${}^2 + \frac{2}{3}(1+\ +\ {}^2)$ A	$P_2\frac{L}{2} + (P_1 - P_2)\frac{L}{3}$	$P_2\frac{L}{2} + (P_1 - P_2)\frac{L}{6}$
16	$\frac{5}{48}pL^2$	$P\frac{L}{3}$	$P\frac{L}{3}$
17	$\frac{PR}{2}$	$\frac{P}{2}$	$\frac{P}{2}$
18	$\frac{PR}{2}$	$\frac{P}{2}$	$\frac{P}{2}$
19	$\frac{wR^2}{2}$	wR	wR
20	$\frac{PR}{2}$	$\frac{P}{2}$	$\frac{P}{2}$
21	M	$-\frac{M}{L}$	$\frac{M}{L}$



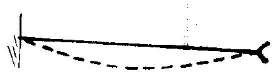
$r_a = \frac{4EI}{l}$ — $t_a(1-2) = \frac{1}{2}$

$r_1 = \frac{6EI}{l^2}$ — $t_1(1-2) = 1$

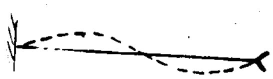


$r_a = \frac{3EI}{l}$ — $t_a(1-2) = 0$

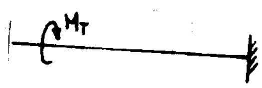
$r_1 = \frac{3EI}{l^2}$ — $t_1(1-2) = 0$



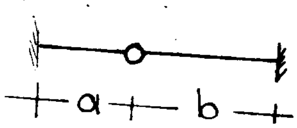
$r_a = \frac{2EI}{l}$ — $t_a(1-2) = 1$



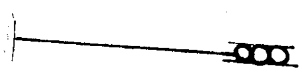
$r_a = \frac{6EI}{l}$ — $t_a(1-2) = 1$



$r_T = \frac{GJ}{l}$ — $t_T(1-2) = -1$



$r_a = \frac{3EIa^2}{a^3+b^3}$ — $t_a(1-2) = \frac{b}{a}$

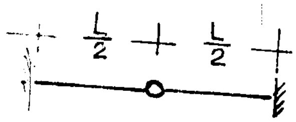


$r_a = \frac{4EI}{l}$ — $t_a(1-2) = \frac{1}{2}$

$r_1 = \frac{6EI}{l^2}$ — $t_1(1-2) = 1$

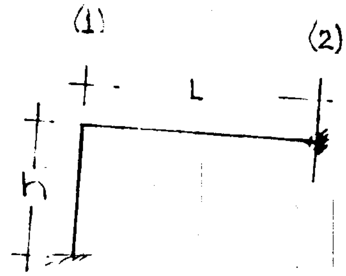
$r_a = \frac{EI}{l}$ — $t_a(1-2) = -1$

$r_1 = 0$ — $t_1(1-2) = 0$



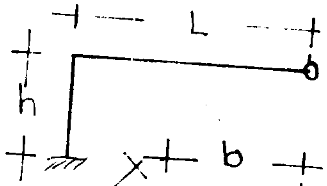
$r_a = \frac{3EI}{l}$ — $t_a = 1$

$r_1 = \frac{6EI}{l^2}$ — $t_1(1-2) = 1$

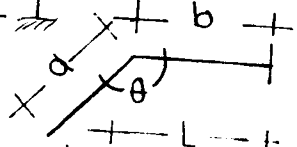


$r_a = \frac{(3l+4h)EI}{h(h+l)}$ $t_a(1-2) = \frac{h}{3l+4h}$

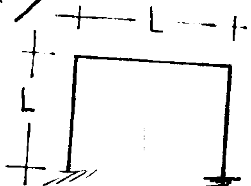
$r_1 = \frac{l}{l+h}$ $t_1(1-2) = \frac{h}{h+l}$



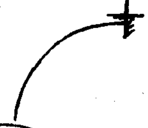
$r_a = \frac{12(h+l)EI}{h(4l+3h)}$ $t_a(1-2) = 0$



$r_a = \frac{3EI}{a}$ $t_a = -\frac{b}{a}\cos\theta$



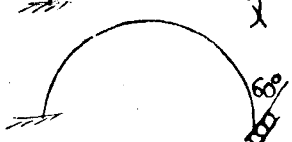
$r_a = \frac{2.04EI}{l}$ $t_a = -0.58$



$r_a = \frac{0.64EI}{r} + \frac{51.5EI}{r^2}$ $t_a(1-2) = -0.46$

$r_a = \frac{1.28EI}{r}$ $t_a = -1$

$r_a = \frac{3.36EI}{r}$ $t_a = -1$



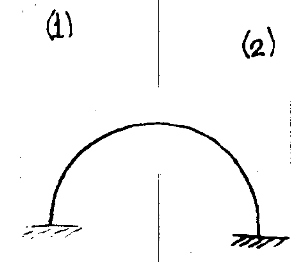
$r_a = \frac{1.14EI}{r}$ $t_a = 0.188$

$r_a(1-2) = 0.95EI/_r$ $t_a(1-2) = \frac{1}{3}$

$r_1(1-2) = 0.64EI/_{r^2}$ $t_1(1-2) = 1$

$r_a = 1.66EI/_r$

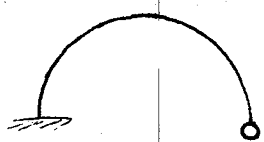
$r_1 = 2.12EI/_{r^2}$



$r_a = 2.3EI/_r$ $\quad$ $t_a(1\text{-}2) = -0.46$

$r_1 vert = 0.64EI/_{r}2$ $\quad$ $t_1(1\text{-}2) = 1$

$r_1 horiz = 2.13EI/_{r}2$ $\quad$ $t_1(1\text{-}2) = 1$



$r_a = 1.83EI/_r$ $\quad$ $t_a = 0$

$r_1 vert = 1.74EI/_{r}2$ $\quad$ $t_1 = 0$

$r_1 horiz = 1.17EI/_{r}2$ $\quad$ $t_1 = 0$